

2.01 *Factoring, Factoring, Factoring!*

PART II: Factoring by Grouping, Review of Factoring

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

Factoring by Grouping

This lesson introduces the technique of factoring by grouping. It builds on the ideas that were presented in the section on factoring the common factor. There are many ways to group terms, some of which will be successful in the factoring process, others will not. The method of grouping then is one of trial and error, with a few insights that may be helpful. There is no substitute for practice.

EXAMPLE 1: Factor $x^3 + 2x^2 + 8x + 16$

SOLUTION: There are no common factors to all four terms. It is not a trinomial, and nothing discussed so far works to factor this. So, try grouping the first two terms together, and the last two terms together, and factor out the common factor within each grouping as follows:

DEFINITION

To **FACTOR** means to **EXPRESS AS A PRODUCT!**

$$(x^3 + 2x^2) + (8x + 16) = x^2(x + 2) + 8(x + 2)$$

Notice that there is a common factor of $(x+2)$ that can be factored out: $= (x + 2)(x^2 + 8)$

EXAMPLE 2: Factor $XY - 4Y + 3X - 12$

SOLUTION: Again, there are no common factors, and this is not a trinomial. Group the first two and the last two terms together, and factor out the common factor from each grouping:

$$(XY - 4Y) + (3X - 12) = Y(x - 4) + 3(x - 4)$$

Now, take out the common factor, which is $(x - 4)$:
 $= (x - 4)(Y + 3)$

EXAMPLE 3: Factor $XY - 4Y - 3X + 12$

SOLUTION: Group the first two and the last two terms together, and factor out the common factor from each grouping:

$$(XY - 4Y) + (-3X + 12) = Y(X - 4) + 3(-X + 4)$$

This time there is no common factor. Try again, this time factoring a -3 from the last grouping. This works!

$$\begin{aligned} XY - 4Y - 3X + 12 &= Y(X - 4) - 3(X - 4) \\ &= (X - 4)(Y - 3) \end{aligned}$$

EXAMPLE 4: Factor $XY - 4Y + 3X + 12$

SOLUTION: Group the first two and the last two terms:

$$XY - 4Y + 3X + 12 = Y(X - 4) + 3(X + 4)$$

At this point, it is important to realize that no common factor resulted. Do not try to factor out something that is not common to both groupings. In fact, there is no way to group this problem to get a common factor. This one cannot be factored by grouping. In fact, it can't be factored by any method. Remember, not all problems can be factored.

Remember that the entire process of "grouping" is one of trial and error, and, as you will see later, there are different types of grouping.

EXERCISES: Factor each of the following by grouping:

1. $XY + 7X + 4Y + 28$

$$= X(\quad) + 4(\quad)$$

$$= (\quad)(\quad)$$

2. $2XY + 5X + 10Y + 25$

$$=$$

$$=$$

3. $x^3 + 3x^2 + 9x + 27$

$$=$$

$$=$$

4. $x^3 - 3x^2 + 9x - 27$

$$=$$

$$=$$

$$\begin{aligned}
 5. \quad & XY - 5X - 2Y + 10 \\
 &= X(\quad) - 2(\quad) \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & X^2Y + XY^2 - 5X - 5Y \\
 &= XY(\quad) - 5(\quad) \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & X^3 - X^2 - 9X + 9 \\
 &= X^2(\quad) - 9(\quad) \\
 &= (\quad)(\quad) \\
 &= (\quad)(\quad)(\quad)
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & X^3 - 5X^2 - 4X + 20 \\
 &= X^2(\quad) - 4(\quad) \\
 &= \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & X^3 + 7X^2 - X - 7 \\
 &= X^2(\quad) - 1(\quad) \\
 &= \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & X^3 - 5X^2 + 25X - 125 \\
 &= \\
 &= \\
 &\text{Does } X^2 + 25 \text{ factor?}
 \end{aligned}$$

$$11. \quad X^3 + 5X^2 - 25X - 125$$

$$12. \quad X^3 - 5X^2 - 25X + 125$$

$$13. \quad X^3 - 4X^2 + 9X - 36$$

$$14. \quad X^3 + 4X^2 - 9X - 36$$

$$15. \quad x^3 - 4x^2 - 9x + 36$$

$$16. \quad x^3 - 9x^2 - 4x + 36$$

In the previous exercises, the first two terms and the last two terms were grouped in order to obtain a common factor. As you might imagine, other groupings are frequently convenient. As a prelude to such "other groupings," the following exercises, which may be thought of as advanced trinomials or difference of squares, will help set the stage for what is to come. Also, notice that you are moving to a higher level of abstraction in these exercises.

In the following exercises, notice the patterns that develop.

$$17a) \quad x^2 - 9$$

$$18a) \quad x^2 - 64$$

$$b) \quad \pi^2 - 9$$

$$b) \quad \pi^2 - 64$$

$$c) \quad (\text{Junk})^2 - 9$$

$$c) \quad (\text{Junk})^2 - 64$$

$$= [(\quad) - 3] [(\quad) + 3]$$

$$= (\quad) (\quad)$$

$$d) \quad (2X+3Y)^2 - 9$$

$$d) \quad (2X+3Y)^2 - 64$$

$$= [(\quad) - 3] [(\quad) + 3]$$

$$= (\quad) (\quad)$$

19a) $x^2 - 5x + 6$

20a) $x^2 - 5x - 6$

b) $(\text{Junk})^2 - 5(\text{Junk}) + 6$
= $[(\quad) - 2][(\quad) - 3]$
= $(\quad)(\quad)$

b) $(\text{Junk})^2 - 5(\text{Junk}) - 6$

c) $(2x+3y)^2 - 5(2x+3y) + 6$
= $[(\quad) - 2][(\quad) - 3]$
= $(\quad)(\quad)$

c) $(2x+3y)^2 - 5(2x+3y) - 6$

21. $(2x-3y)^2 - 25$

22. $(2x-3y)^2 - 169$

23. $(2x-3y)^2 - 4(2x-3y) - 60$

24. $(2x-3y)^2 - 14(2x-3y) + 40$

25a) $x^2 - 10x + 25$

26a) $x^2 + 16x + 64$

b) $(\text{Junk})^2 - 10(\text{Junk}) + 25$

b) $(\text{Junk})^2 + 16(\text{Junk}) + 64$

c) $(2x+3y)^2 - 10(2x+3y) + 25$

c) $(2x+3y)^2 + 16(2x+3y) + 64$

$$33. (x^2 - 7x)^2 + 16(x^2 - 7x) + 60 \quad 34. (x^2 - 5x)^2 - 2(x^2 - 5x) - 24$$

$$35. (x^2 - 5x)^2 - 36$$

$$36. (x^2 + 5x)^2 - 36$$

There are many different variations of **factoring by grouping**. If there are four terms, sometimes it is "appropriate" to group the first two terms and the last two terms together. Sometimes three of the terms "look good together," and it is appropriate to group three terms together, leaving the other term alone. Sometimes there are more than four terms, and different groupings are "appropriate." What makes a grouping "appropriate" is that the result factors. In other words, it works!

$$37. (x^2 + 2xy + y^2) - 49$$

$$38. x^2 + 4xy + 4y^2 - 25$$

$$39. x^2 - 6xy + 9y^2 - 36$$

$$40. x^2 - 8xy + 16y^2 - 81$$

$$\begin{aligned}
41. \quad & 25 - x^2 + 2xy - y^2 \\
& = 25 - (x^2 - 2xy + y^2) \\
& = 25 - (x - y)^2 \\
& = [5 - (\quad)] [5 + (\quad)] \\
& = (\quad) (\quad)
\end{aligned}$$

$$\begin{aligned}
42. \quad & 64 - x^2 + 4xy - 4y^2 \\
& = 64 - (\quad) \\
& = 64 - (\quad)^2 \\
& = [4 - (\quad)] [4 + (\quad)] \\
& = (\quad) (\quad)
\end{aligned}$$

$$43. \quad 16 - x^2 - 10xy - 25y^2$$

$$44. \quad 9 - x^2 - 4xy - 4y^2$$

$$45. \quad X^2 + 2XY + Y^2 + 7X + 7Y + 10$$

$$46. \quad X^2 - 2XY + Y^2 + 5X - 5Y + 6$$

$$x^2 + 2xy + y^2 + 7x + 7y + 10$$

$$(x + y)^2 + 7(x + y) + 10$$

$$[(x + y) + 5] [(x + y) + 2]$$

Find two numbers whose product is 10 and whose sum is 7.

$$[(x + y) + 5] [(x + y) + 2]$$

This cleans up to give you this for the final answer:

$$(x + y + 5) (x + y + 2)$$

$$47. \quad X^2 - 4XY + 4Y^2 + 7X - 14Y + 6$$

$$48. \quad X^2 - 4XY + 4Y^2 + 3X - 6Y + 2$$

49. $X^2 - 6XY + 9Y^2 - 5X + 15Y + 6$

50. $X^2 - 6XY + 9Y^2 - 8X + 24Y + 15$

51. $X^2 + 2XY + Y^2 + 9X + 9Y - 10$

52. $X^2 + 2XY + Y^2 + 3X + 3Y - 10$

REVIEW OF FACTORING

GUIDELINES TO FACTORING

1. **Common Factor** (Factor Common Factor First!)
2. **Trinomial** (F O I L rearranged to spell F L O I)
3. **Difference of Squares:** $X^2 - Y^2 = (X - Y)(X + Y)$
Diff of Cubes: $X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)$
Sum of Cubes: $X^3 + Y^3 = (X + Y)(X^2 - XY + Y^2)$
4. **Factoring by Grouping**

EXERCISES. Factor completely:

1. $X^2 - 5X - 24$
2. $3X^2 + 12XY$
3. $n(n-1) + 2(n-1)$
4. $X^3 - 5X^2 + X - 5$
5. $a^3 - 8$
6. $X^2 - X - 42$
7. $-X^3 + 7X^2$
8. $a(a-8) + 9(a-8)$
9. $X^4 - 13X^2 + 36$
10. $Y^3 + 125$
11. $n^2 + 9n - 10$
12. $27n^2 - 90mn$

$13. x^2(x-2) - 16(x-2) \quad 14. x^3 + 2x^2 + 4x + 8 \quad 15. 8x^3 - 27y^3$

$16. a^2 + 10a - 39 \quad 17. 15b^2 - 70b^3 \quad 18. 2b(b+3) - 5(b+3)$

$19. x^3 - 2x^2 + 4x - 8 \quad 20. 2a^3 + 250b^3 \quad 21. x^2 - 9x - 36$

$22. 8a^2 + 12a \quad 23. (4c+5)a + (4c+5)b \quad 24. 2p^4 - 54p$

$25. c^2 - 11c + 24 \quad 26. 75a^2 c - 3c^3 \quad 27. 4Y(2Y+7) - 9(2Y+7)$

$28. x^3 + 2x^2 - 4x - 8$

$29. 40x^5y^3 - 5x^2$

$30. y^2 - 8y - 20$

$31. 9a^3b - a^2b$

$32. 9a(a-3) + (a-3)$

$33. x^6 + 27y^9$

$34. n^2 + 10n + 21$

$35. 7x^2 + 21x^3$

$36. x^3 - 2x^2 - 4x + 8$

$37. 3x^{10} + 81xy^6$

$38. 13n(2n+1) - (2n+1)$

$39. b^2 + 13b + 36$

40. $-12x^2 - 6x$

41. $5a^3 - 6a^2 + a$

42. $x^2 - 18x + 45$

43. $12n^2 + 18n - 30$

44. $15x^2 + 20x - 35$

45. $x^3 + 5x^2 - 9x - 45$

46. $x^4 + 27xy^3$

47. $x^4 - 64x^2$

48. $x^3 - 5x^2 - 6x$

49. $a(a-b) + b(a-b) + c(a-b)$

50. $7x^2(x+1) + 3x(x+1) - 4(x+1)$

$$51. \quad x^5 + 9x^4 - x - 9$$

$$52. \quad (x^2+2xy+y^2) + (4x+4y) + 4$$

$$53. \quad (x^2-7x)^2 + 16(x^2-7x) + 60$$

$$54. \quad (x^2-3x)^2 - 14(x^2-3x) + 40$$

$$55. \quad x^5 + 5x^4 - 81x - 405$$

$$56. \quad x^5 - 9x^4 - x + 9$$

ANSWERS 2.01 Part II

- p.132-138:** 1. $(Y+7)(X+4)$; 2. $(2Y+5)(X+5)$; 3. $(X+3)(X^2+9)$; 4. $(X-3)(X^2+9)$;
5. $(Y-5)(X-2)$; 6. $(X+Y)(XY-5)$; 7. $(X-1)(X-3)(X+3)$;
8. $(X-5)(X-2)(X+2)$; 9. $(X+7)(X-1)(X+1)$; 10. $(X-5)(X^2+25)$, No;
11. $(X+5)^2(X-5)$; 12. $(X-5)^2(X+5)$; 13. $(X-4)(X^2+9)$;
14. $(X+4)(X-3)(X+3)$; 15. $(X-4)(X-3)(X+3)$; 16. $(X-9)(X-2)(X+2)$; 17a)
 $(X-3)(X+3)$; b) $(\pi-3)(\pi+3)$; c) $(\text{Junk}-3)(\text{Junk}+3)$;
17d) $(2X+3Y-3)(2X+3Y+3)$; 18a) $(X-8)(X+8)$; b) $(\pi-8)(\pi+8)$;
18c) $(\text{Junk}-8)(\text{Junk}+8)$; d) $(2X+3Y-8)(2X+3Y+8)$;
19a) $(X-2)(X-3)$; b) $(\text{Junk}-2)(\text{Junk}-3)$; c) $(2X+3Y-2)(2X+3Y-3)$;
20a) $(X-6)(X+1)$; b) $(\text{Junk}-6)(\text{Junk}+1)$; c) $(2X+3Y-6)(2X+3Y+1)$;
21. $(2X-3Y-5)(2X-3Y+5)$; 22. $(2X-3Y-13)(2X-3Y+13)$;
23. $(2X-3Y-10)(2X-3Y+6)$; 24. $(2X-3Y-10)(2X-3Y-4)$;
25a) $(X-5)^2$; b) $(\text{Junk}-5)^2$; c) $(2X+3Y-5)^2$;
26a) $(X+8)^2$; b) $(\text{Junk}+8)^2$; c) $(2X+3Y+8)^2$;
27. $(2X-3Y-7)^2$; 28. $(2X-3Y+5)^2$; 29. $(2X-3Y+10)^2$; 30. $(2X-3Y-8)^2$;
31. $(X+2)(X-4)(X-1)^2$; 32. $(X-3)(X-2)(X-4)(X-1)$;
33. $(X-6)(X-1)(X-5)(X-2)$; 34. $(X-6)(X+1)(X-4)(X-1)$;
35. $(X-6)(X+1)(X-3)(X-2)$; 36. $(X+6)(X-1)(X+2)(X+3)$;
37. $(X+Y-7)(X+Y+7)$; 38. $(X+2Y-5)(X+2Y+5)$; 39. $(X-3Y-6)(X-3Y+6)$;
40. $(X-4Y-9)(X-4Y+9)$; 41. $(5-X+Y)(5+X-Y)$; 42. $(8-X+2Y)(8+X-2Y)$;
43. $(4-X-5Y)(4+X+5Y)$; 44. $(3-X-2Y)(3+X+2Y)$; 45. $(X+Y+2)(X+Y+5)$;
46. $(X-Y+3)(X-Y+2)$; 47. $(X-2Y+6)(X-2Y+1)$; 48. $(X-2Y+2)(X-2Y+1)$;
49. $(X-3Y-3)(X-3Y-2)$; 50. $(X-3Y-5)(X-3Y-3)$; 51. $(X+Y+10)(X+Y-1)$;
52. $(X+Y+5)(X+Y-2)$.
- p.139-142:** 1. $(X-8)(X+3)$; 2. $3X(X+4Y)$; 3. $(n-1)(n+2)$; 4. $(X-5)(X^2+1)$;
5. $(a-2)(a^2+2a+4)$; 6. $(X-7)(X+6)$; 7. $-X^2(X-7)$ or $X^2(-X+7)$;
8. $(a-8)(a+9)$; 9. $(X-3)(X+3)(X-2)(X+2)$; 10. $(Y+5)(Y^2-5Y+25)$;
11. $(n+10)(n-1)$; 12. $9n(3n-10m)$; 13. $(X-2)(X-4)(X+4)$;
14. $(X+2)(X^2+4)$; 15. $(2X-3Y)(4X^2+6XY+9Y^2)$; 16. $(a+13)(a-3)$;
17. $5b^2(3-14b)$; 18. $(b+3)(2b-5)$; 19. $(X-2)(X^2+4)$;
20. $2(a+5b)(a^2-5ab+25b^2)$; 21. $(X-12)(X+3)$; 22. $4a(2a+3)$;
23. $(4c+5)(a+b)$; 24. $2p(p-3)(p^2+3p+9)$; 25. $(c-8)(c-3)$;

ANSWERS 2.01 Part II (Continued)

p.139-142:

26. $3c(5a-c)(5a+c)$; 27. $(2Y+7)(4Y-9)$; 28. $(X+2)^2(X-2)$;
29. $5X^2(2XY-1)(4X^2Y^2+2XY+1)$; 30. $(Y-10)(Y+2)$;
31. $a^2b(9a-1)$; 32. $(a-3)(9a+1)$;
33. $(X^2+3Y^3)(X^4-3X^2Y^3+9Y^6)$; 34. $(n+7)(n+3)$;
35. $7X^2(1+3X)$; 36. $(X-2)^2(X+2)$;
37. $3X(X^3+3Y^2)(X^6-3X^3Y^2+9Y^4)$; 38. $(2n+1)(13n-1)$;
39. $(b+9)(b+4)$; 40. $-6X(2X+1)$; 41. $a(5a-1)(a-1)$;
42. $(X-15)(X-3)$; 43. $6(2n+5)(n-1)$; 44. $5(3X+7)(X-1)$;
45. $(X+5)(X-3)(X+3)$; 46. $X(X+3Y)(X^2-3XY+9Y^2)$;
47. $X^2(X-8)(X+8)$; 48. $X(X-6)(X+1)$; 49. $(a-b)(a+b+c)$;
50. $(X+1)^2(7X-4)$; 51. $(X+9)(X-1)(X+1)(X^2+1)$;
52. $(X+Y+2)^2$; 53. $(X-6)(X-1)(X-5)(X-2)$;
54. $(X-5)(X+2)(X-4)(X+1)$; 55. $(X+5)(X-3)(X+3)(X^2+9)$;
56. $(X-9)(X-1)(X+1)(X^2+1)$.