Math in Living C O L O R !!

Complex Fractions Part II

Intermediate Algebra: One Step at a Time, Section 2.06, Pages 196-200:
College Algebra: One Step at a Time, Section 1.05, Pages 57-62:

 # 29, 30, 31, 33, 34, 36, 39, 40, 41, 43, 47, 49, 51, 53, extras.

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Intermediate Algebra: One Step at a Time
See Section 2.06 with explanations, examples, and exercises!

College Algebra: One Step at a Time
See Section 1.05 with explanations, examples, and exercises!

Remember, in general, to "unstack" the problem, use these formulas:

$$\frac{\frac{a}{b}}{\frac{c}{d}} \text{ means } \frac{a}{b} \div \frac{c}{d}$$

and

$$\frac{a}{b} \div \frac{c}{d} \text{ means } \frac{a}{b} \bullet \frac{d}{c}$$

P. 197 # 29. $$\frac{\frac{x+2}{x}+\frac{x}{x+2}}{\frac{x+2}{x}-\frac{x}{x+2}}$$

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\frac{x+2}{x}+\frac{x}{x+2}}{\frac{x+2}{x}-\frac{x}{x+2}} = \left(\frac{x+2}{x}+\frac{x}{x+2}\right)\div\left(\frac{x+2}{x}-\frac{x}{x+2}\right)$$

The LCD for the first (red) part is x(x+2), for the second (blue) part is also x(x+2), so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{x+2}{x}+\frac{x}{x+2}\right)\div\left(\frac{x+2}{x}-\frac{x}{x+2}\right)$$

$$= \left(\frac{x+2}{x}\bullet\frac{(x+2)}{(x+2)}+\frac{x}{x+2}\bullet\frac{x}{x}\right)\div\left(\frac{x+2}{x}\bullet\frac{(x+2)}{(x+2)}-\frac{x}{x+2}\bullet\frac{x}{x}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad}{x(x+2)}\right)\div\left(\frac{\qquad}{x(x+2)}\right)$$

Multiply out the numerators:

$$= \left(\frac{x^2+4x+4 \quad + \quad x^2}{x(x+2)}\right)\div\left(\frac{x^2+4x+4 \quad - \quad x^2}{x(x+2)}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{2x^2+4x+4}{x(x+2)}\right)\div\left(\frac{4x+4}{x(x+2)}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \frac{2(x^2+2x+2)}{x(x+2)}\bullet\frac{x(x+2)}{4(x+1)}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x, (x+2), and 2:

$$= \frac{x^2+2x+2}{2(x+1)}$$

P. 197 # 30. $$\frac{\frac{x+2}{x}+\frac{x}{x-2}}{\frac{x+2}{x}-\frac{x}{x-2}}$$

Solution: Method I. **The "Unstacking Method."**

$$\frac{\frac{x+2}{x}+\frac{x}{x-2}}{\frac{x+2}{x}-\frac{x}{x-2}} = \left(\frac{x+2}{x}+\frac{x}{x-2}\right) \div \left(\frac{x+2}{x}-\frac{x}{x-2}\right)$$

The LCD for the first (red) part is x(x−2), for the second (blue) part is also x(x−2), so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{x+2}{x}+\frac{x}{x-2}\right) \div \left(\frac{x+2}{x}-\frac{x}{x-2}\right)$$

$$= \left(\frac{x+2}{x}\bullet\frac{(x-2)}{(x-2)}+\frac{x}{x-2}\bullet\frac{x}{x}\right) \div \left(\frac{x+2}{x}\bullet\frac{(x-2)}{(x-2)}-\frac{x}{x-2}\bullet\frac{x}{x}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad\qquad}{x(x-2)}\right) \div \left(\frac{\qquad\qquad}{x(x-2)}\right)$$

Multiply out the numerators:

$$= \left(\frac{x^2-4 \quad + \quad x^2}{x(x-2)}\right) \div \left(\frac{x^2-4 \quad - \quad x^2}{x(x-2)}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{2x^2-4}{x(x-2)}\right) \div \left(\frac{-4}{x(x-2)}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \frac{2(x^2-2)}{x(x-2)}\bullet\frac{x(x-2)}{-4}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x, (x−2), and 2:

$$= \frac{x^2-2}{-2} \quad \text{or} \quad \frac{-1\bullet}{-1\bullet}\frac{(x^2-2)}{-2} \quad \text{or} \quad \frac{(2-x^2)}{2}$$

P. 197: #31. $$\frac{\frac{x+2}{x}+\frac{x}{x+2}}{\frac{x+2}{x}-\frac{x}{x-2}}$$

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\frac{x+2}{x}+\frac{x}{x+2}}{\frac{x+2}{x}-\frac{x}{x-2}} = \left(\frac{x+2}{x}+\frac{x}{x+2}\right)\div\left(\frac{x+2}{x}-\frac{x}{x-2}\right)$$

The LCD for the first (red) part is x(x+2), for the second (blue) part is x(x−2), so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{x+2}{x}+\frac{x}{x+2}\right)\div\left(\frac{x+2}{x}-\frac{x}{x-2}\right)$$

$$= \left(\frac{x+2}{x}\bullet\frac{(x+2)}{(x+2)}+\frac{x}{x+2}\bullet\frac{x}{x}\right)\div\left(\frac{x+2}{x}\bullet\frac{(x-2)}{(x-2)}-\frac{x}{x-2}\bullet\frac{x}{x}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad\qquad}{x(x+2)}\right)\div\left(\frac{\qquad\qquad}{x(x-2)}\right)$$

Multiply out the numerators:

$$= \left(\frac{x^2+4x+4 \quad + \quad x^2}{x(x+2)}\right)\div\left(\frac{x^2-4 \quad - \quad x^2}{x(x-2)}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{2x^2+4x+4}{x(x+2)}\right)\div\left(\frac{-4}{x(x-2)}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \frac{2(x^2+2x+2)}{x(x+2)}\bullet\frac{x(x-2)}{-4}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x and 2:

$$= \frac{(x-2)(x^2+2x+2)}{-2(x+2)}$$ **also correct:** $$\frac{(2-x)(x^2+2x+2)}{2(x+2)}$$

P. 197: #33. $\dfrac{\dfrac{4}{x-4}-\dfrac{2}{x}}{\dfrac{-6}{x+1}+\dfrac{8}{x}}$

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\frac{4}{x-4}-\frac{2}{x}}{\frac{-6}{x+1}+\frac{8}{x}} = \left(\frac{4}{x-4}-\frac{2}{x}\right)\div\left(\frac{-6}{x+1}+\frac{8}{x}\right)$$

The LCD for the first (red) part is x(x−4), for the second (blue) part is x(x+1), so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{4}{x-4}-\frac{2}{x}\right)\div\left(\frac{-6}{x+1}+\frac{8}{x}\right)$$

$$= \left(\frac{4}{x-4}\bullet\frac{x}{x}-\frac{2}{x}\bullet\frac{(x-4)}{(x-4)}\right)\div\left(\frac{-6}{x+1}\bullet\frac{x}{x}+\frac{8}{x}\bullet\frac{(x+1)}{(x+1)}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad}{x(x-4)}\right)\div\left(\frac{\qquad}{x(x+1)}\right)$$

Multiply out the numerators:

$$= \left(\frac{4x-2x+8}{x(x-4)}\right)\div\left(\frac{-6x+8x+8}{x(x+1)}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{2x+8}{x(x-4)}\right)\div\left(\frac{2x+8}{x(x+1)}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \frac{2(x+4)}{x(x-4)}\bullet\frac{x(x+1)}{2(x+4)}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x , 2, and the x+4. The final answer is

$$= \frac{x+1}{x-4}$$

P.197: #34. $\dfrac{\dfrac{4}{x+y}-\dfrac{2}{x}}{\dfrac{4}{x-y}-\dfrac{2}{x}}$

Solution: **Method I.** **The "Unstacking Method."**

$$\frac{\frac{4}{x+y}-\frac{2}{x}}{\frac{4}{x-y}-\frac{2}{x}} = \left(\frac{4}{x+y}-\frac{2}{x}\right)\div\left(\frac{4}{x-y}-\frac{2}{x}\right)$$

The LCD for the first (red) part is x(x+y), for the second (blue) part is x(x−y), so multiply numerator and denominator of each fraction by the appropriate missing factors:

$$= \left(\frac{4}{x+y}-\frac{2}{x}\right)\div\left(\frac{4}{x-y}-\frac{2}{x}\right)$$

$$= \left(\frac{4}{x+y}\bullet\frac{x}{x}-\frac{2}{x}\bullet\frac{(x+y)}{(x+y)}\right)\div\left(\frac{4}{x-y}\bullet\frac{x}{x}-\frac{2}{x}\bullet\frac{(x-y)}{(x-y)}\right)$$

Put each of these LCDs in place:

$$= \left(\frac{\qquad\qquad}{x(x+y)}\right)\div\left(\frac{\qquad\qquad}{x(x-y)}\right)$$

Multiply out the numerators:

$$= \left(\frac{4x\ -\ 2x-2y}{x(x+y)}\right)\div\left(\frac{4x\ -\ 2x+2y}{x(x-y)}\right)$$

Combine like terms for each of the numerators:

$$= \left(\frac{2x-2y}{x(x+y)}\right)\div\left(\frac{2x+2y}{x(x-y)}\right)$$

Factor the numerators (if possible!), invert the second fraction, and multiply:

$$= \frac{2(x-y)}{x(x+y)}\bullet\frac{x(x-y)}{2(x+y)}$$

Divide out factors of any numerator with corresponding factors from the denominators. In particular, divide out the factors of x and 2:

$$= \frac{(x-y)^2}{(x+y)^2}$$

P. 198 # 36. $\dfrac{2x^{-1} - (2y)^{-1}}{2xy^{-1}}$

Solution: First, eliminate the negative exponents.

$$\frac{2x^{-1} - (2y)^{-1}}{2xy^{-1}} = \frac{2 \bullet \frac{1}{x} - \frac{1}{2y}}{2x \bullet \frac{1}{y}}$$

The LCD for the whole problem is 2*xy*, so multiply numerator and denominator by $\dfrac{2xy}{1}$

$$= \frac{\frac{2xy}{1}\left(\frac{2}{x} - \frac{1}{2y}\right)}{\frac{2xy}{1}\left(\frac{2x}{y}\right)}$$

$$= \frac{\frac{2xy}{1} \bullet \frac{2}{x} - \frac{2xy}{1} \bullet \frac{1}{2y}}{\frac{2xy}{1} \bullet \frac{2x}{y}}$$

Although this looks terrible, it is really quite simple, and you can do it in your head. Normally, you don't even write down the previous step. It all simplifies to this:

$$= \frac{4y - x}{4x^2}$$

P. 198 # 39. $\dfrac{(4x)^{-2}+4^{-2}}{4x^{-2}-4^{-2}}$

Solution: First, eliminate the negative exponents.

$$\frac{(4x)^{-2}+4^{-2}}{4x^{-2}-4^{-2}} = \frac{\dfrac{1}{(4x)^2} + \dfrac{1}{4^2}}{4\bullet\dfrac{1}{x^2} - \dfrac{1}{4^2}}$$

$$= \frac{\dfrac{1}{16x^2} + \dfrac{1}{16}}{\dfrac{4}{x^2} - \dfrac{1}{16}}$$

The LCD for the whole problem is $16x^2$. Multiply numerator and denominator by $\dfrac{16x^2}{1}$.

$$= \frac{\dfrac{16x^2}{1}\left(\dfrac{1}{16x^2} + \dfrac{1}{16}\right)}{\dfrac{16x^2}{1}\left(\dfrac{4}{x^2} - \dfrac{1}{16}\right)}$$

Next, use the distributive property. You may want not need to show your work in the next step, but rather just multiply and simplify the fractions in your head.

$$= \frac{\dfrac{16x^2}{1}\bullet\dfrac{1}{16x^2} + \dfrac{16x^2}{1}\bullet\dfrac{1}{16}}{\dfrac{16x^2}{1}\bullet\dfrac{4}{x^2} - \dfrac{16x^2}{1}\bullet\dfrac{1}{16}}$$

It all simplifies down to this:

$$= \frac{1 + x^2}{64 - x^2}$$

P. 198 # 40. $\dfrac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}}$

Solution: First, eliminate the negative exponents.

$$\frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}} = \frac{\frac{1}{x^1} - \frac{1}{y^1}}{\frac{1}{x^2} - \frac{1}{y^2}}$$

The LCD for the whole problem is x^2y^2. Multiply numerator and denominator by $\dfrac{x^2y^2}{x^2y^2}$.

$$= \frac{\frac{x^2y^2}{1}\left(\frac{1}{x} - \frac{1}{y}\right)}{\frac{x^2y^2}{1}\left(\frac{1}{x^2} - \frac{1}{y^2}\right)}$$

Next, use the distributive property. You may want not need to show your work in the next step, but rather just multiply and simplify the fractions in your head.

$$= \frac{\frac{x^2y^2}{1}\bullet\frac{1}{x} - \frac{x^2y^2}{1}\bullet\frac{1}{y}}{\frac{x^2y^2}{1}\bullet\frac{1}{x^2} - \frac{x^2y^2}{1}\bullet\frac{1}{y^2}}$$

It all simplifies down to this:

$$= \frac{xy^2 - x^2y}{y^2 - x^2}$$

which factors into this:

$$= \frac{xy(y - x)}{(y - x)(y + x)}$$

and reduces (divide out the (y − x) factor!) to this:

$$= \frac{xy}{y + x}$$

P. 199 # 41. $\dfrac{2x - (2x)^{-1}}{1 + (2x)^{-1}}$

Solution: First, eliminate the negative exponents.

$$\frac{2x - (2x)^{-1}}{1 + (2x)^{-1}} = \frac{2x - \dfrac{1}{2x}}{1 + \dfrac{1}{2x}}$$

The LCD for the whole problem is 2x. Multiply numerator and denominator by $\dfrac{2x}{2x}$.

$$= \frac{\dfrac{2x}{1}\left(2x - \dfrac{1}{2x}\right)}{\dfrac{2x}{1}\left(1 + \dfrac{1}{2x}\right)}$$

Next, use the distributive property. You may want not need to show your work in the next step, but rather just multiply and simplify the fractions in your head.

$$= \frac{\dfrac{2x}{1} \bullet 2x - \dfrac{2x}{1} \bullet \dfrac{1}{2x}}{\dfrac{2x}{1} \bullet 1 + \dfrac{2x}{1} \bullet \dfrac{1}{2x}}$$

It all simplifies down to this:

$$= \frac{4x^2 - 1}{2x + 1}$$

which factors into this:

$$= \frac{(2x-1)(2x + 1)}{(2x+1)}$$

and reduces (divide out the (2x + 1) factor!) to this:

$$= 2x - 1$$

P. 199 # 43. $\dfrac{2^{-1}x - 2x^{-1}}{4^{-1}x^2 - 2x^{-1}}$

Solution: First, eliminate the negative exponents.

$$\frac{2^{-1}x - 2x^{-1}}{4^{-1}x^2 - 2x^{-1}} = \frac{\frac{1}{2}\bullet x \ - \ 2\bullet\frac{1}{x}}{\frac{1}{4}\bullet x^2 + \ 2\bullet\frac{1}{x}}$$

The LCD for the whole problem is 4x. Multiply numerator and denominator by $\frac{4x}{4x}$.

$$= \frac{\frac{4x}{1}\left(\frac{x}{2} - \frac{2}{x}\right)}{\frac{4x}{1}\left(\frac{x^2}{4} - \frac{2}{x}\right)}$$

Next, use the distributive property. You may want not need to show your work in the next step, but rather just multiply and simplify the fractions in your head.

$$= \frac{\frac{4x}{1}\bullet\frac{x}{2} - \frac{4x}{1}\bullet\frac{2}{x}}{\frac{4x}{1}\bullet\frac{x^2}{4} - \frac{4x}{1}\bullet\frac{2}{x}}$$

It all simplifies down to this:

$$= \frac{2x^2 - 8}{x^3 - 8}$$

which factors into this:

$$= \frac{2(x^2 - 4)}{(x-2)(x^2+2x + 4)} \quad \text{or} \quad \frac{2(x-2)(x+2)}{(x-2)(x^2+2x + 4)}$$

and reduces (divide out the (x − 2) factor!) to this:

$$= \frac{2(x+2)}{x^2+2x + 4}$$

P. 199: #47. $\left(x^{-2} - y^{-2}\right)^{-1}$

Solution: Short cuts don't work here. Back to basics $x^{-2} = \frac{1}{x^2}$ and $y^{-2} = \frac{1}{y^2}$.

$\left(\frac{1}{x^2} - \frac{1}{y^2}\right)^{-1}$ You must have a single fraction, so find the LCD = x^2y^2.

$\left(\frac{y^2 - x^2}{x^2y^2}\right)^{-1}$ A single fraction raised to the -1 power means invert the fraction.

$\frac{x^2y^2}{y^2 - x^2}$ Final answer, unless you prefer to factor the denominator:

$\frac{x^2y^2}{(y-x)(y+x)}$

P. 200: #49. $\left(x^{-1} - y^{-1}\right)^{-2}$

Solution: Short cuts don't work! Back to basics $x^{-1} = \frac{1}{x}$ and $y^{-1} = \frac{1}{y}$.

$\left(\frac{1}{x} - \frac{1}{y}\right)^{-2}$ You must have a single fraction, so find the LCD = xy.

$\left(\frac{y-x}{xy}\right)^{-2}$ A single fraction raised to the -2 power means to invert the fraction and square it.

Now, invert the fraction: $\left(\frac{xy}{y-x}\right)^2$,

and square numerator and denominator: $\frac{x^2y^2}{(y-x)^2}$

NOTE: You could write it $\frac{x^2y^2}{y^2 - 2xy + x^2}$, but do NOT write $\frac{x^2y^2}{y^2 - x^2}$!!

P. 200: #51. $(3x^{-1}-3y^{-1})^{-1}$

Solution: Short cuts don't work. Back to basics $3x^{-1}=3\bullet\frac{1}{x}$ and $3y^{-1}=3\bullet\frac{1}{y}$.

$\left(\frac{3}{x}-\frac{3}{y}\right)^{-1}$ You must have a single fraction, so find the LCD = xy.

$\left(\frac{3y-3x}{xy}\right)^{-1}$ A single fraction raised to the -1 power means invert the fraction.

$\frac{xy}{3y-3x}$ Final answer, unless you prefer to factor out the 3: $\frac{xy}{3(y-x)}$

P. 200: #53. $\left[(3x)^{-1}-(3y)^{-1}\right]^{-1}$

Solution:

$\left[\frac{1}{3x}-\frac{1}{3y}\right]^{-1}$ LCD = 3xy

$\left[\frac{1}{3x}\bullet\frac{(\ )}{(\ )}-\frac{1}{3y}\bullet\frac{(\ )}{(\ )}\right]^{-1}$ The first fraction is "missing" *y*; second missing the x.

$\left[\frac{1}{3x}\bullet\frac{(y)}{(y)}-\frac{1}{3y}\bullet\frac{(x)}{(x)}\right]^{-1}$ The LCD is 3xy, and the numerator is y − x

$\left[\frac{y-x}{3xy}\right]^{-1}$ The last step is to invert the fraction!

$\frac{3xy}{y-x}$ Final Answer!

Extra Problem #1. $\dfrac{\left(t-\frac{1}{t}\right)}{\left(t+\frac{1}{t}\right)}$

Solution: **Method I.** Both methods work equally well here, but a lot of students just prefer the "unstacking" method, until there is time to see some of the advantages of Method II. This explanation will use Method I.

$$\frac{\left(t-\frac{1}{t}\right)}{\left(t+\frac{1}{t}\right)} = \left(t-\frac{1}{t}\right) \div \left(1+\frac{1}{t}\right)$$

You must find the LCD for the fractions on the left side (RED), which is t, and an LCD for the fractions on the right side (BLUE) , which is also t. You will have to play "what's missing" in these fractions.

$$\left(t-\frac{1}{t}\right) \div \left(1+\frac{1}{t}\right)$$

$$\left(\frac{t}{1}\bullet\frac{t}{t}-\frac{1}{t}\right) \div \left(\frac{t}{1}\bullet\frac{t}{t}+\frac{1}{t}\right)$$

$$\left(\frac{t^2-1}{t}\right) \div \left(\frac{t^2+1}{t}\right)$$

Invert the second fraction. Factoring the difference of squares in the first fraction will not reduce the fraction.

$$\left(\frac{t^2-1}{t}\right) \bullet \left(\frac{t}{t^2+1}\right)$$

The final answer is: $\dfrac{t^2-1}{t^2+1}$

Extra Problem #2. $$\frac{\left(\frac{1}{x+3}-\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

Solution: **Method I.** Both methods work equally well here, but a lot of students just prefer the "unstacking" method, until there is time to see some of the advantages of Method II. This explanation will use Method I.

$$\frac{\left(\frac{1}{x+3}-\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \left(\frac{1}{x+3}-\frac{1}{x}\right) \div \left(\frac{1}{x}\right)$$

You must find the LCD for the fractions on the left side **(RED)**, which is $x(x+3)$, and an LCD for the fractions on the right side **(BLUE)** , which is x. You will have to play "what's missing in these fractions.

$$\left(\frac{1}{x+3}-\frac{1}{x}\right) \div \left(\frac{1}{x}\right)$$
$$\left(\frac{1}{x+3}\bullet\frac{x}{x}-\frac{1}{x}\bullet\frac{(x+3)}{(x+3)}\right) \div \left(\frac{1}{x}\right)$$
$$\left(\frac{x-(x+3)}{x(x+3)}\right) \div \left(\frac{1}{x}\right)$$

Simplify the first numerator, and invert the second fraction.

$$\left(\frac{x-x-3}{x(x+3)}\right) \bullet \left(\frac{x}{1}\right)$$
$$\left(\frac{-3}{x(x+3)}\right) \bullet \left(\frac{x}{1}\right)$$

Divide out an x from the first denominator with the x in the second numerator, and divide out the (1-x) factors. What is left is

$$\frac{-3}{\cancel{x}(x+3)} \bullet \frac{\cancel{x}}{1} \quad \text{or} \quad \frac{-3}{x+3} \quad \text{or} \quad -\frac{3}{x+3}$$

Extra Problem #3. $\dfrac{\left(1+\dfrac{1}{6-b}\right)}{\left(1-\dfrac{1}{6+b}\right)}$

Solution: **Method I.** **The "Unstacking Method".**

$$\frac{\left(1+\frac{1}{6-b}\right)}{\left(1-\frac{1}{6+b}\right)} = \left(\frac{1}{1}+\frac{1}{6-b}\right) \div \left(\frac{1}{1}-\frac{1}{6+b}\right)$$

You must find the LCD for the fractions on the left side (RED), which is $6-b$, and an LCD for the fractions on the right side (BLUE) , which is $6+b$. You will have to play "what's missing in these fractions.

$$\left(\frac{1}{1}+\frac{1}{6-b}\right) \div \left(\frac{1}{1}-\frac{1}{6+b}\right)$$

$$\left(\frac{1}{1}\bullet\frac{(6-b)}{(6-b)}+\frac{1}{6-b}\right) \div \left(\frac{1}{1}\bullet\frac{(6+b)}{(6+b)}-\frac{1}{6+b}\right)$$

$$\left(\frac{(6-b)+1}{6-b}\right) \div \left(\frac{(6+b)-1}{6+b}\right)$$

$$\frac{(7-b)}{6-b} \div \frac{(5+b)}{6+b}$$

Invert the second fraction.

$$\frac{7-b}{6-b} \bullet \frac{6+b}{5+b}$$

The final answer is $\dfrac{(7-b)(6+b)}{(5+b)(6-b)}$.

You may write the factors in any order!

Extra Problem #4. $\frac{x^{-1}+2}{x+2x^2}$

Solution: First, eliminate the negative exponents.

$$\frac{x^{-1}+2}{x+2x^2} = \frac{\frac{1}{x}+2}{x+2x^2}$$

Use the "unstacking method", rewriting as the red fraction divided by the blue fraction.

$$= \left(\frac{1}{x}+2\right) \div \left(x+2x^2\right)$$

$$= \left(\frac{1}{x}+2\frac{x}{x}\right) \div \left(\frac{x+2x^2}{1}\right)$$

$$= \left(\frac{1+2x}{x}\right) \div \left(\frac{x(1+2x)}{1}\right)$$

$$= \left(\frac{1+2x}{x}\right) \bullet \left(\frac{1}{x(1+2x)}\right)$$

$$= \left(\frac{\cancel{1+2x}}{x}\right) \bullet \left(\frac{1}{x\,\cancel{(1+2x)}}\right)$$

$$= \left(\frac{1}{x}\right) \bullet \left(\frac{1}{x}\right)$$

$$= \frac{1}{x^2}$$