Math in Living C O L O R !!

See Section 2.07 with explanations, examples, and exercises, coming soon!

2.07 Functions, Domain, and Range

College Algebra: One Step at a Time

Pages 265 - 273: #53, 70, 75

Pages 274 - 280: #2, 18, 25, 26, 35, 38, 44, 46

Pages 283 - 288: #3, 4, 18, 19, 25, E.C.

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Functions, Domain, Range Summary

Relation--any graph (in which two variables are "related").

Function--a relation in which each value of x has a unique value of y.

Domain--Set of all (permissible) x values

Range--Set of all (resulting) y values

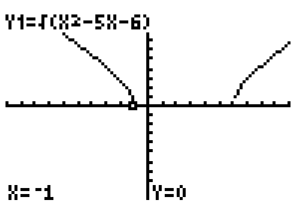
To find the Domain, solve for y in terms of x.

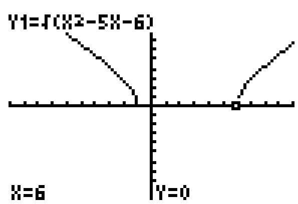
To find the Range, solve for x in terms of y.

Domain

Page 269. # 53. $y = \sqrt{(x^2 - 5x - 6)}$

Solution: Since this equation is in the form $y = \underline{\qquad}$, the graphing calculator will probably be a good way to find domain and range. Just enter this in the calculator. In the standard window, and the graph should look like this. In the two sketches below, it is clear that there are points at (-1,0) and (6,0) that will be critical to finding the domain and range. Notice that the graph actually touches the x axis at these two points, and from these points the graph extends upward from y=0. The graph extends to the left from x=-1, and to the right from x=6.





From this graph, it should be clear that the Domain is $(-\infty,-1]\cup[6,\infty)$. Likewise, the range is all values that are on or above the x-axis, or $y \geq 0$, or in interval notation $[0,\infty)$.

Page 270: #70. $5x-5xy=4$

Solution: In order to find the domain, you must solve for y in terms of x, in order to see if there are any restrictions on x. To do this, notice that there is only one y term already on the left side. You should start by adding $-5x$ to both sides to get all the non-y terms on the right side .

$$-5xy=4-5x$$

In order to solve for y, you must divide both sides by $-5x$,

$$\frac{-5xy}{-5x}=\frac{4-5x}{-5x}$$

$$y=\frac{4-5x}{-5x}$$

To find the domain, you are looking for restrictions on the denominator. Denominators can NOT be zero. In this case, the denominator is $-5x$ must never be zero, so $x\neq 0$.

Therefore, the domain is all values of x, such that $x\neq 0$.

Page 271: #75. $x^2y+4y=6x$

Solution: In order to find the domain, you must solve for y in terms of x, in order to see if there are any restrictions on x. To do this, notice that all the y terms are already on the left side, and you can start by factoring out the y.

$$y(x^2+4)=6x$$

In order to solve for y, you must divide both sides by x^2+4,

$$y=\frac{6x}{x^2+4}$$

To find the domain, you are looking for restrictions on the denominator. Denominators can NOT be zero. But in this case, the denominator is x^2+4, which can never be zero anyway, so there are no restrictions on x.

Therefore, the domain is all real x, or $(-\infty,\infty)$.

Range

Page 274. # 2. $7x+4y=16$

Solution: If you recognize that this is a straight line, then you may already know that the domain and range are both all real values. However, the general practice in finding the range is to solve for x in terms of y in order to determine what restrictions there might be for y To find the range, solve for x.

$$7x+4y=16 \qquad \text{Subtract } 4y \text{ from each side of the equation.}$$

$$7x=-4y+16 \qquad \text{Divide both sides by } 7$$

$$\frac{7x}{7}=\frac{-4y+16}{7} \quad \text{or} \quad \frac{7x}{7}=\frac{-4y}{7}+\frac{16}{7}$$

$$x=\frac{-4y+16}{7} \quad \text{or} \quad x=\frac{-4y}{7}+\frac{16}{7}$$

Either way you look at it, there are no denominators and no radicals, and therefore there are no restrictions on y. Range is therefore all real y, or in interval notation $(-\infty,\infty)$.

Page 275. # 18. $x^2=y$

Solution: If you recognize that this is a parabola that opens to the upward, then you may already know that the domain is all real values, and the range is $y\geq 0$. However, as always, the general practice in finding the range is to solve for x in terms of y in order to determine what restrictions there might be for y To find the range, solve for x.

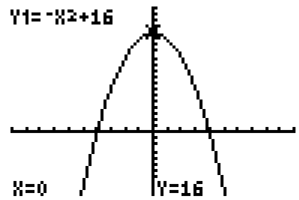
$$x^2=y \qquad \text{Take the square root of both sides.}$$

$$x=\pm\sqrt{y}$$

The restriction here is that, because of the square root, the radicand y must be greater than or equal to zero. That is, the range is $y\geq 0$, or in interval notation $[0,\infty)$.

Page 276. # 25. $y = -x^2 + 16$

Solution: You may recognize that this is a parabola that opens downward with vertex at (0, 16).



Window: x=[10,10] y=[-10,20]

If so, then you already know that the domain is all real values, and the range is all values BELOW the vertex, which would be $y \le 16$ or $(-\infty, 16]$.

However, as always, the general practice in finding the range is to solve for x in terms of y in order to determine what restrictions there might be for y To find the range, solve for x.

Start by adding x^2 to both sides.

$$\begin{array}{r} y = -x^2 + 16 \\ +x^2 \quad +x^2 \\ \hline x^2 + y = 16 \end{array}$$

Next add $-y$ to each side.

$$x^2 = 16 - y$$

Take the square root of both sides.

$$x = \pm\sqrt{16 - y}$$

The restriction here is that, because of the square root, the radicand $16 - y$ must be greater than or equal to zero. That is, the range is

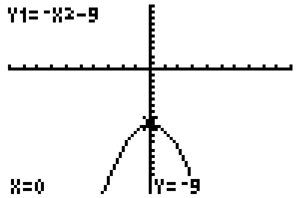
$$16 - y \ge 0$$
$$-y \ge -16$$

Divide by -1: $y \le 16$

Range: $(-\infty, 16]$

Page 276. # 26. $y = -x^2 - 9$

Solution: You may recognize that this is a parabola that opens downward with vertex at (0, −9).



Window: x=[10,10] y=[-20,10]

If so, then you already know that the domain is all real values, and the range is all values BELOW the vertex, which would be $y \le -9$ or $(-\infty, -9]$.

However, as always, the general practice in finding the range is to solve for x in terms of y in order to determine what restrictions there might be for y To find the range, solve for x.

Start by adding x^2 to both sides.

$$\begin{array}{r} y = -x^2 - 9 \\ +x^2 \quad +x^2 \\ \hline x^2 + y = -9 \end{array}$$

Next add $-y$ to each side.

$$x^2 = -9 - y$$

Take the square root of both sides.

$$x = \pm\sqrt{-9 - y}$$

The restriction here is that, because of the square root, the radicand $-9 - y$ must be greater than or equal to zero. That is, the range is

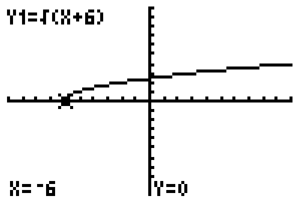
$$\begin{aligned} -9 - y &\ge 0 \\ -y &\ge 9 \end{aligned}$$

Divide by -1: $y \le -9$

Range: $(-\infty, -9]$

Page 278. # 35. $y = \sqrt{x+6}$

Solution: Since this equation is in the form $y =$ ______, the graphing calculator will probably be a good way to find **domain and range.** Just enter this in the calculator, and it should look like this:

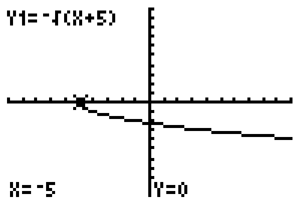


From this graph (or from squaring both sides of the equation!), you may recognize this as the upper half of a parabola that extends upward and to the right to infinity. The range is all values that are on or above the x-axis, or $y \geq 0$, or in interval notation $[0, \infty)$.

The domain (it's easy, and no extra charge!) just from looking at the graph is $[-6, \infty)$.

Page 278. # 38. $y = -\sqrt{x+5}$

Solution: The best bet here is to graph the function, since it is in the form $y =$ ______. Just enter this in the calculator, and it should look like this:

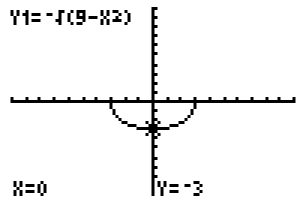


From this graph (or from squaring both sides of the equation!), you may recognize this as the lower half of a parabola that extends downward and to the right to infinity. The range is all values below the x-axis, or $y \leq 0$, or in interval notation $(-\infty, 0]$.

The domain (it's easy, and no extra charge!) just from looking at the graph is $[-5, \infty)$.

Page 280. # 44. $y = -\sqrt{9 - x^2}$

Solution: The best bet here is to graph the function, since it is in the form $y =$ ______. Just enter this in the calculator, and it should look like this:

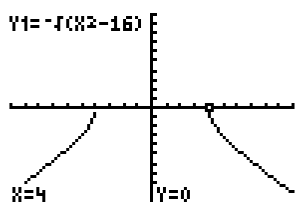


From this graph (or from squaring both sides of the equation!), you may recognize this as the lower half of a circle that extends 3 units down, 3 units to the right, and 3 units to the left. The range is all values from -3 up to 0, or in interval notation $[-3,0]$.

The domain (again, it's easy, and no extra charge!) just from looking at the graph is $[-3,3]$.

Page 280. # 46. $y = -\sqrt{x^2 - 16}$

Solution: The best bet here is to graph the function, since it is in the form $y =$ ______. Just enter this in the calculator, and it should look like this:



From this graph you probably won't recognize this graph, but it doesn't matter, does it? It should be clear that the values of y extend from negative infinity up to zero. The range is therefore in interval notation $(-\infty,0]$.

The domain (again, it's easy, and no extra charge!) just from looking at the graph is $(-\infty,-4]\cup[4,\infty)$.

Page 283. #3. $y = xy + 4$

Solution: To find the domain, you must solve for y in terms of x. This means you must first get all the y terms on one side, and the non-y terms on the other side. To do this, subtract xy from each side.

$$y - xy = 4$$

Factor out the y:

$$y(1 - x) = 4$$

Divide both sides by $(1 - x)$

$$y = \frac{4}{1 - x}$$

Because the denominator must never equal zero, this means that $x \neq 1$.
Domain is all $x \neq 1$.

Also, this IS a function, since it can be expressed in the form y = _____.

To find the range, you must solve for x in terms of y by subtracting 4 from each side.

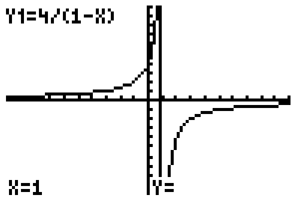
$$y = xy + 4$$
$$y - 4 = xy$$

Next, divide both sides by y in order to solve for x.

$$\frac{y - 4}{y} = x$$

From this equation, you can see that the denominator cannot equal zero. That is, the range is all $y \neq 0$.

If you choose to look at the graph of $y = \frac{4}{1 - x}$ with a graphing calculator, it looks like this:



Notice from this graph, that domain is all values of x except the asymptote at x=1 (the value of f(x) is undefined at x=1), Likewise, the range is all values of y except $y \neq 0$.

D: all $x \neq 1$. R: all $y \neq 0$.

P 283: #4. $4x = xy + 8$

Solution: To find the domain, you must solve for y in terms of x. In this case there is only one y term, so subtract 8 from each side to isolate the y term.

$$4x - 8 = xy$$

Now to solve for y, you must divide both sides by x.

$$\frac{4x-8}{x} = y$$

From this equation, you can see that the denominator cannot equal zero. That is, the domain is all $x \neq 0$.

Also, this IS a function, since it can be expressed in the form y = _____.

To find the range, you must solve for x in terms of y. To do this, you must first get all the x terms on one side by subtracting xy from each side.

$$4x - xy = 8$$

Next, factor out the x: $x(4-y) = 8$

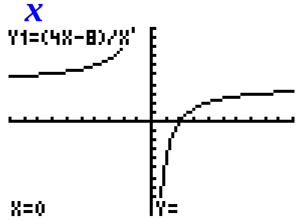
Next, divide both sides by $(4-y)$ in order to solve for x.

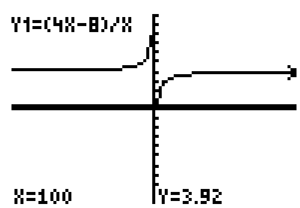
$$x = \frac{8}{4-y}$$

Because the denominator must never equal zero, this means that $y \neq 4$.

Range is all $y \neq 4$.

If you choose to look at the graph of $y = \dfrac{4}{1-x}$ with a graphing calculator, it looks like this: $y = \dfrac{4x-8}{x}$.



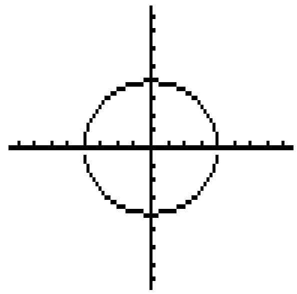


The first graph above is the standard window, illustrating that if x=0, the value of y is undefined. In the second graph, the x window is extended from [-100,100] to illustrate that the value of y is approaching but never quite reaches y=4. Notice from these graphs, that domain is all values of x except the asymptote at x=0 (the value of f(x) is undefined at x=0), Likewise, the range is all values of y except $y \neq 4$. D: all $x \neq 0$. R: all $y \neq 4$.

Page 285. # 18. $x^2 + y^2 = 16$

Solution: **Since this equation is in the form of a circle, you already know what the graph looks like. You don't even need your graphing calculator for this one.**

$$x^2 + y^2 = 16$$



From this graph of a circle with center at the origin and of radius 4, you can see that the values of x extend from -4 to 4 inclusive, and the values of y also extend from -4 to 4 inclusive. It is NOT a function

Domain: $[-4, 4]$.
Range: $[-4, 4]$
Function: **NO!**

Page 286. # 19. $x^2 - y^2 = 16$

Solution: While this is NOT in the form $y = \_\_\_\_\_$, you can always solve for y in terms of x, and then use the graphing calculator as a way to find domain and range. Begin by adding $-x^2$ to each side:

$$-y^2 = -x^2 + 16$$

Next, divide both sides by -1, which gives you

$$y^2 = x^2 - 16$$

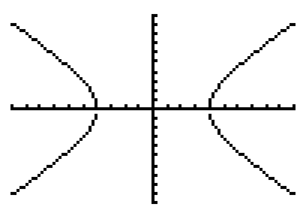
Take the square root of each side:

$$y = \pm\sqrt{(x^2 - 16)}$$

Of course to graph this you must use two graphs:

$$y1 = \sqrt{(x^2 - 16)} \quad \text{and} \quad y2 = -\sqrt{(x^2 - 16)}$$

Just enter these in the calculator, and the graph should look like this:



Although you entered the equation as an upper half and lower half graph, the actual graph looks like a left half and a right half graph. The domain consists of all values to left of and including -4 and to the right of and including 4 $(-\infty, -4] \cup [4, \infty)$. The range extends all the way down to negative infinity and all the way up to positive infinity. This would be all real values or $(-\infty, \infty)$. This is NOT a function.

Domain: $(-\infty, -4] \cup [4, \infty)$.

Range: $(-\infty, \infty)$

Function: NO!

P. 287 #25. $xy - 10x = 5$

Solution: To find the **domain**, you must solve for y in terms of x. In this case there is only one y term, so add $+10x$ to each side to isolate the y term.

$$xy = 5 + 10x$$

Now to solve for y, you must divide both sides by x.

$$\frac{xy}{x} = \frac{5+10x}{x}$$

$$y = \frac{5+10x}{x}$$

From this equation, you can see that the denominator cannot equal zero. That is, the **domain is all** $x \neq 0$.

Also, **this IS a function**, since it can be expressed in the form y = _____.

To find the **range**, you must solve for x in terms of y. To do this, you must first get all the x terms on one side of the equation. However, surprise!! All the x terms are already on the left side!!

$$xy - 10x = 5$$

Now, just factor out the x

$$x(y - 10) = 5$$

Next, divide both sides by $(y-10)$ in order to solve for x.

$$\frac{x(y-10)}{(y-10)} = \frac{5}{(y-10)}$$

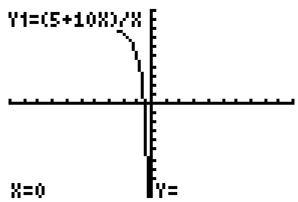
$$x = \frac{5}{y-10}$$

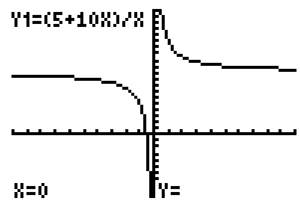
Because the denominator must never equal zero, this means that $y \neq 10$.

Range is all $y \neq 10$.

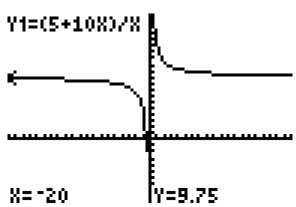
P. 287 #25 continued. $xy - 10x = 5$ or $y = \dfrac{5+10x}{x}$

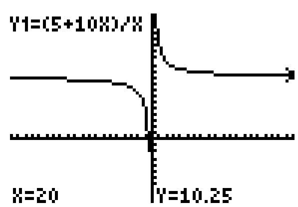
If you choose to solve this with a graphing calculator, the standard window will be absolutely NO help to you at all. to look at the graph of $y = \dfrac{5+10x}{x}$ with a graphing calculator, it looks like this: $y = \dfrac{5+10x}{x}$.





The first graph above is the standard window, which does not accurately reveal the behavior of this graph!! In the second graph, the y window is extended from [-10, 20]. From this graph, you can see that the domain is all values of x except the asymptote at x=0 (the value of f(x) is undefined at x=0). To find the range, you may suspect that there is a horizontal asymptote—that is, a forbidden value of y that might be determined by looking at points on the right and/or left edge of the graph. The next two graphs show the x window from [-20,20] and y window [-10,20]. See if this helps, by doing [TRACE] at the extreme edges of the window.





Can you see from looking at the right and left sides of this window that the value of y is approaching but never quite reaches y=10. Therefore, the range is all values of y except $y \neq 10$.

D: all $x \neq 0$. R: all $y \neq 10$.

Extra Credit Problem:

Find the domain of $k(x) = \dfrac{x-3}{\sqrt{3x+15}} - \dfrac{\sqrt{x-4}}{x+6}$

Solution: There are actually three restrictions here:

1. The first denominator is also a radical, so

$$3x+15>0$$
$$3x>-15$$
$$x>-5\ .$$

2. The second numerator is a radical, so

$$x-4\geq 0$$
$$x\geq 4$$

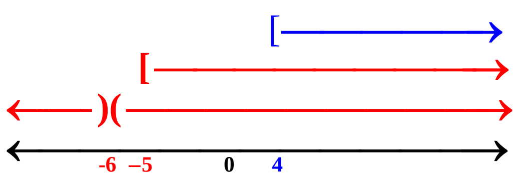
3. The second denominator cannot equal zero, so

$$x+6\neq 0$$
$$x\neq -6$$

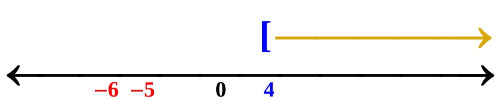
The domain of this function is the intersection of all three of these restrictions. However, in this case, since x must be greater than -5 and greater than or equal to 4, it will automatically not equal -6. The third restriction does not affect the solution.

$x>-5$ and $x\geq 4$ and $x\neq -6$

$[-5,\infty)\cap[4,\infty)$



"And" means "intersection", so choose only the regions that are common to all three graphs:



Interval notation: **[4, ∞)**