DOES THE FIBONACCI SEQUENCE EXIST WITHIN OUR BRAIN WAVES?

RICARDO JOSEPH SIMEONI AND PETER MICHAEL MILLS

School of Physiotherapy and Exercise Science, Griffith University, Gold Coast 9726, Australia

The many occurrences in nature of numbers within the Fibonacci sequence, and the golden ratio (≈ 1.62) of consecutive numbers within this sequence, are remarkable. We have found evidence to show that the Fibonacci sequence may exist within the complexities of human brain waves.

Introduction

The Fibonacci sequence in nature

Leonardo de Pisa (nicknamed Fibonacci) was born in 1170 and died in 1250. He made many contributions to mathematics and society but is best known for discovering the sequence of numbers known as the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

The Fibonacci sequence is generated by adding two consecutive numbers to give the following number in the sequence (starting with 1 and 1 as the first two numbers).

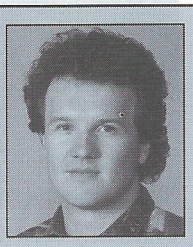
The occurrence in nature of numbers within the Fibonacci sequence, and the ratio of consecutive Fibonacci numbers, known as the golden ratio (≈ 1.62), is remarkable. Examples are found in:

- the bracts of a pine cone which spiral in two directions in 8 and 13 rows,
- the scales of a pineapple which spiral in three directions in 8, 13 and 21 rows,
- the number of petals in varieties of daisies (13 (Blue), 21 (English), 34 (Oxeye) and 55 (African)), and
- the ratio of chamber dimensions associated with the cyclic growth of a Nautilus shell.

Those above are just a few examples of the Fibonacci sequence in nature.[1] Many examples of the Fibonacci sequence in nature are associated with spiral-like growth.

Frequencies within EEG biosignals

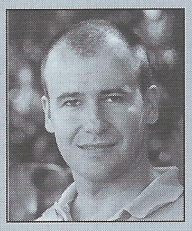
All biological voltage versus time waveforms (biosignals), whether they originate from the brain (electroencephalogram (EEG)), heart (electrocardiogram (ECG)) or muscle (electromyogram (EMG)), can be broken down into the sum of many pure sine waves each with its own frequency and amplitude. This process is known as Fourier analysis and each sine wave is known as an harmonic.



Ricardo Joseph Simeoni received his PhD in theoretical Atomic Physics in 1997 from the James Cook University of North Queensland. His thesis is entitled A Theoretical Investigation of Laser Induced Collisional Energy Transfer in Rare Gases and was obtained under the expert supervision of Associate Professor Ian Whittingham.

Ricardo has 4 years experience as a hospital Medical Physicist and holds a M.App.Sc. (Med. Phys.) from the Queensland University of Technology. He also holds a G.Dip.T. from the Australian Catholic University and has taught senior high school physics at St. John's College, Nambour.

Ricardo currently lectures Biophysics, Bioinstrumentation and Mathematics for Clinical Sciences at Griffith University.



Peter Mills received a Bachelor's degree in Exercise Science from Griffith University in 1998 and graduated with 1st Class Honours in Exercise Science in 1999. He is currently in the final year of a Doctor of Philosophy degree with the School of Physiotherapy and Exercise Science at Griffith University. His PhD research project is concerned with the effect of ageing on biomechanical variability and coordination during the swing phase of gait. Aside from his PhD project, his research interests are in the areas of postural control, and digital signal processing and analysis.

The Fourier (frequency) spectrum is a graph of amplitude versus frequency for all harmonics and allows dominant (in terms of amplitude) frequencies within a biosignal to be identified. An example of a filtered EEG biosignal and its corresponding frequency spectrum are shown in Figures 1 (a) and (b) respectively. Note that power (amplitude squared) is plotted on the y-axis in Figure 1 (b) and the resulting power spectrum is a common representation for the frequency spectrum. The raw EEG data used to generate Figures 1 (a) and (b) are from Nayak.[2]

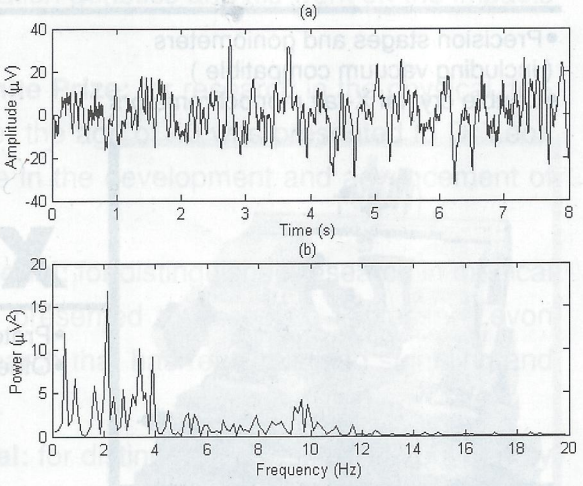


Figure 1. (a) An example of a filtered EEG biosignal and (b) the corresponding frequency spectrum based on the raw EEG data of Nayak.2

Well-known frequencies within EEG biosignals are the δ, θ, α and β bands, corresponding to frequencies of 0.5 to 3.5, 4 to 7, 8 to 13 and >13 Hz respectively. The relative activities of these bands provide important information on brain function (e.g., the β band is relatively active when a person is fully alert and reduces in activity during relaxation).

An introduction to Bicoherence analysis

Bicoherence analysis detects for the existence of second-order phase coupling between the n harmonics, $X_n(f_n) = c_n \sin(2\pi f_n t + \phi_n)$, of an EEG biosignal, where c_n is amplitude, f_n is frequency, ϕ_n is phase and t is time.

The bicoherence between two frequencies, written $bic(f_p, f_q)$, equals 100 % if the harmonics, $X(f_p)$, $X(f_q)$ and $X(f_p+f_q)$, maintain a constant phase relationship in time (Figure 2 (a)). Such phase coupling is referred to as second-order. If the phases of the three harmonics in question vary independently, as represented by the different phase relationship for the time intervals either side of the vertical line in Figure 2 (b), then $bic(f_p, f_q)$ equals 0 %. bic(fp, fq) values between 0 and 100 % represent varying degrees of second-order phase coupling.

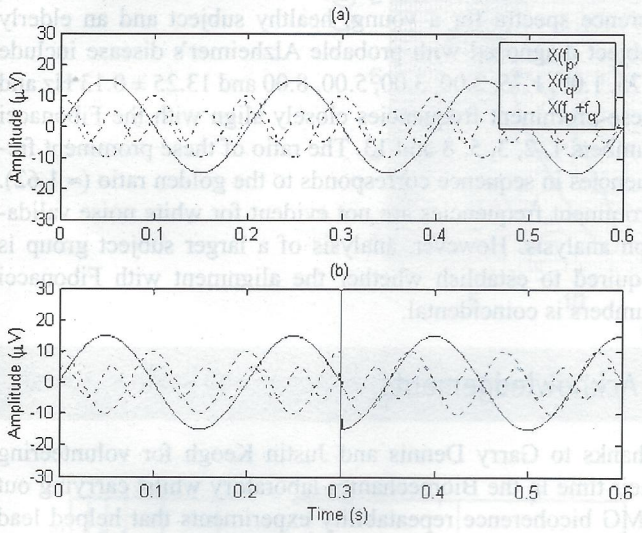


Figure 2. (a) Harmonics, X(fp), X(fq) and X(fp+fq), maintain a constant phase relationship in time and (b) the phases of the three harmonics vary independently in time as indicated by a different phase relationship for the time intervals either side of the vertical line.

To calculate $bic(f_p, f_q)$ over an extended time period, the EEG biosignal is divided into several epochs (e.g., 8 to 32 epochs are used by Muthuswamy et. al.[3], Lipton et. al.[4] and Ning and Bronzino[5]). Successive epochs generally overlap by 75 % for EEG analysis[6]. The phase information from all epochs is combined to calculate $bic(f_p, f_q)$ for all frequencies, and thus give the bicoherence spectrum, according to

$$BIC(f_p, f_q) = \frac{\left| \sum_{m=1}^{L} X_m(f_p)\, X_m(f_q)\, X_m^*(f_p + f_q) \right|}{\sum_{m=1}^{L} \sqrt{P_m(f_p)\, P(f_q)\, P_m(f_p + f_q)}} \times 100\%,$$

where L is the number of epochs, the index m is the m-th epoch, $X^*(f_p+f_q)$ is the complex conjugate of $X(f_p+f_q)$ and $P(f_n) = c_n^2$ is the power of $X(f_n)$. For a detailed explanation on the theory and calculation of $bic(f_p, f_q)$ see Sigl and Chamoun.[6]

Relatively high $bic(f_p, f_q)$ values within bands of frequencies indicate that brain function involves significant nonlinear processes (the brain is considered a nonlinear biological system). Information on - or a deeper understanding of - nonlinear biological systems like the brain may be achieved by observing changes in second-order phase couplings in response to various stimuli. For example, bicoherence analysis is used to calculate an index for the depth of brain anaesthesia.[6]

The aim of the present study is to see whether, on average, the combined bicoherence spectra from several different EEG channel (electrode) positions for an individual, display prominent frequencies (see Cameron and Skofronick[7] for a description of the international standard 10-20 referencing system of EEG channel position).

Method

The raw EEG data used for the present study are for a young, healthy subject and an elderly subject diagnosed with probable Alzheimer's disease. The data is available on the world-wide-web.[2] A total of 38 biosignals (19 EEG channel positions with recordings for eyes open and closed) are available for each subject. The EEG data were sampled at 128 Hz for 8 s.

Each EEG biosignal was adjusted to a mean of zero and filtered with a fourth-order, zero phase Butterworth filter (24 dB/octave rolloff) with low- and high-pass filter cut-offs set to 40 and 0.5 Hz respectively. The biosignals were divided into five epochs, each overlapping the next by 75%. Ideally, a larger number of epochs would be used, with the restriction set by the length of the available data. A Blackman smoothing window[8] was applied to each epoch to reduce possible distortion effects due to the finite length of the epochs and this window subsequently gave an overall smoothing effect to bicoherence spectra. A 512 point Fast Fourier transform (FFT) was then performed on each epoch and $bic(f_p, f_q)$ calculated for all f_p and f_q above 0.5 Hz such that $(f_p + f_q) \leq 30.00$ Hz, with a frequency resolution of 0.25 Hz. The 30 Hz maximum was chosen in accordance with the spectral edge frequency (SEF) of the EEG frequency spectra (see results section), where SEF is defined as the frequency below which 95% of the power exists.

To identify prominent bicoherence frequencies, the highest three $bic(f_p, f_q)$ values in a given bicoherence spectrum were identified and their corresponding f_p, f_q and $f_p + f_q$ values recorded. This was repeated for all 38 bicoherence spectra from each subject. The recorded frequencies were then combined to give a frequency histogram to show the most frequently occurring frequencies (i.e., the frequencies most often involved with the strongest second-order phase coupling). A similar procedure was employed for the highest six $bic(f_p, f_q)$ values. Since each bicoherence spectrum is made up of approximately 3500 points, the highest three and six bicoherence values respectively represent 0.086 and 0.17% of the points within each bicoherence spectrum. The frequency histogram generation procedures were repeated for 1000 seg-

ments of white noise data, each equal in length to an EEG biosignal.

Results and discussion

Median frequency (MF) and SEF are common indices for EEG frequency spectra. The average (± SD) MF and SEF for the 38 biosignals are respectively 7.5 ± 3.2 and 24.1 ± 3.9 Hz (young) and 6.0 ± 1.7 and 25.4 ± 4.2 Hz (elderly). Average MF statistically differs between the subjects ($t = 2.6$, $df = 74$, $p < 0.02$).

A typical bicoherence spectrum, which displays $bic(f_p, f_q)$ calculated for all f_p and f_q, is shown in Figure 3 (see inside back cover). The typical triangular shape is a consequence of the possible mathematical permutations of $f_p + f_p$ that can equal the maximum frequency in the EEG frequency spectrum. Bicoherence spectra for the young and elderly subjects are similar in general appearance and in terms of their average bicoherence, $\overline{bic(f_p, f_q)}$, which is a simple measure of the overall bicoherence for a given spectrum. $\overline{bic(f_p, f_q)}$ is 55.5 ± 1.3 % (young) and 55.2 ± 1.2 % (elderly).

Highlighted in Figure 3 is one of many scattered ridge-like structures at approximately 45° to the x-axis, typical of normal ECG, EEG and EMG bicoherence spectra.[4, 6, 9] Such ridges appear chaotic in nature.[4, 9] However, a ridge at approximately 45° suggests that a number of frequencies are phase coupled to one particular frequency, at least for a given instant in time. Hence, it is possible that some frequencies become more prominent than others when averaging over a large number of bicoherence spectra.

Figures 4 (a) and (b) respectively show for the young and elderly subjects, frequency histograms (n = 342) for all frequencies based on the highest three $bic(f_p, f_q)$ values within each bicoherence spectrum. The vertical lines represent Fibonacci numbers. The histogram in Figure 4 (c) represents the same analysis for 1000 segments of white noise data with results normalised to n = 342. The trend of a gradual fall-off in the number of occurrences as frequency increases in these figures, is related to the fact that the lowest frequency occurs most often within a bicoherence spectrum and therefore has the highest probability of chance occurrence for our methodology.

Figure 5 is analogous to Figure 4 but for frequency histograms (n = 684) based on the highest six $bic(f_p, f_q)$ values within each bicoherence spectrum.

Figures 4 (a) and 5 (a) for the young subject display distribution peaks that are not present for white noise analysis and several of the distribution peaks, (e.g., at 1.00, 1.75, 3.00, 5.00, 8.00 and 13.25 ± 0.13 Hz) closely align with Fibonacci numbers. Since the unit of Hz is arbitrary, the ratio of these prominent frequencies in sequence and its correspondence to the golden ratio is perhaps of more interest than any absolute alignment with the Fibonacci numbers. Analysis of a larger subject group is required to establish whether the finding is coincidental. However, a similar finding is reported in an EMG bicoherence analysis study[9] that involves 18 adult males and EMG frequencies from 10 to 300 Hz.

Distribution peaks are observed at 0.75, 2.00 and 5.00 ± 0.13 Hz in Figure 5 (b) for the elderly subject but close alignment of distribution peaks with Fibonacci numbers is strongest for the young subject, raising the possibility that such alignment is desirable. Also, it is interesting to note that within individual bicoherence spectra, frequencies that match Fibonacci numbers were, in a number of instances, found to be strongly phase coupled to one another. If the Fibonacci sequence does exist within the complexities of various biosignals, then perhaps the helical nature of DNA is somehow related, since there are numerous examples in nature of the Fibonacci sequence occurring where there is spiral-like growth.

Higher-order phase couplings are also likely to occur within the EEG bicoherence spectra.[3] However, the consideration of higher-order couplings is beyond the scope of the present study.

Conclusion

Prominent frequencies found within two sets of 38 EEG bicoherence spectra for a young, healthy subject and an elderly subject diagnosed with probable Alzheimer's disease include 0.75, 1.00, 1.75, 2.00, 3.00, 5.00, 8.00 and 13.25 ± 0.13 Hz and these prominent frequencies closely align with the Fibonacci numbers 1, 2, 3, 5, 8 and 13. The ratio of these prominent frequencies in sequence corresponds to the golden ratio (≈ 1.62). Prominent frequencies are not evident for white noise validation analysis. However, analysis of a larger subject group is required to establish whether the alignment with Fibonacci numbers is coincidental.

Acknowledgements

Thanks to Garry Dennis and Justin Keogh for volunteering their time in the Biomechanics laboratory whilst carrying out EMG bicoherence repeatability experiments that helped lead to this study.

References

1. Tannenbaum, P. and Arnold, R., *Excursions in Modern Mathematics*, 3rd ed., Prentice-Hall, Upper Saddle River, New Jersey, 301-317, 1998.
2. Nayak, K., Electroencephalogram (EEG) Data, *SCRI, Florida State University*, http://*www.scri.fsu.edu/~nayak/chaos/data.html*. Accessible on May 1, 2002 (date when first made accessible on the world-wide-web is unknown).
3. Muthuswamy, J., Sherman, D.L., and Thakor, N.V., Higher-Order Spectral Analysis of Burst Patterns in EEG, *IEEE Transactions on Biomedical Engineering*. 46: 92-99, 1999.
4. Lipton, J.M., Dabke, K.P., Alison, J.F., Cheng, H., Yates, L. and Brown, T.I.H., Use of the Bispectrum to Analyse Properties of the Human Electrocardiograph, *Australasian*

Physical and Engineering Sciences in Medicine. 21: 1-10, 1998.

5. Ning, T. and Bronzino, J.D., Autoregressive and Bispectral Analysis Techniques: EEG Applications, *IEEE Engineering in Medicine and Biology Magazine*. March: 47-50, 1990.
6. Sigl, J.C. and Chamoun N.G., An Introduction to Bispectral Analysis for the Electroencephalogram, *Journal of Clinical Monitoring*. 10: 392-404, 1994.
7. Cameron, J. and Skofronick, J., *Medical Physics*, 1st ed., John Wiley and Sons, 205, 1978.
8. Harris, F.J., On the use of windows for harmonic analysis with the discrete Fourier transform, *Proceedings of the IEEE*. 66: 51-83, 1978.
9. Simeoni, R.J. and Mills, P., Bicoherence Analysis of Quadriceps EMG During Isometric Knee Extension, *Australasian Physical and Engineering Sciences in Medicine*. (Accepted for publication subject to changes, September 28, 2002.)

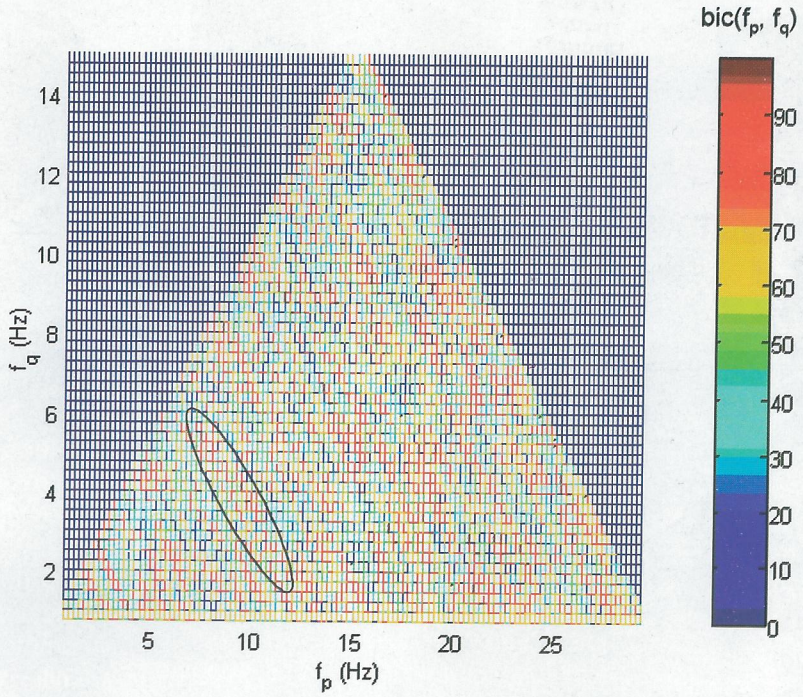


Figure 3. A typical EEG bicoherence spectrum, which displays bic(fp, fq) calculated for all fp and fq. A typical ridge-like feature is highlighted.

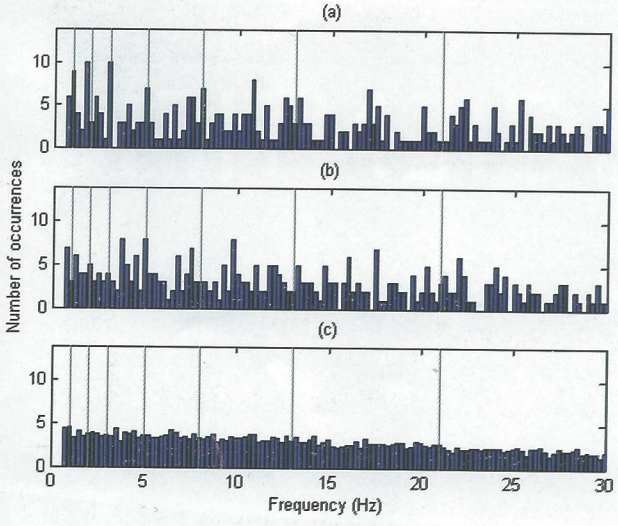


Figure 4. Frequency histograms (n = 342) showing the most frequently occurring frequencies based on the highest three bic(f_p, f_q) values within each EEG bicoherence spectrum for (a) young subject, (b) elderly subject and (c) white noise data. The vertical lines show the position of Fibonacci numbers.

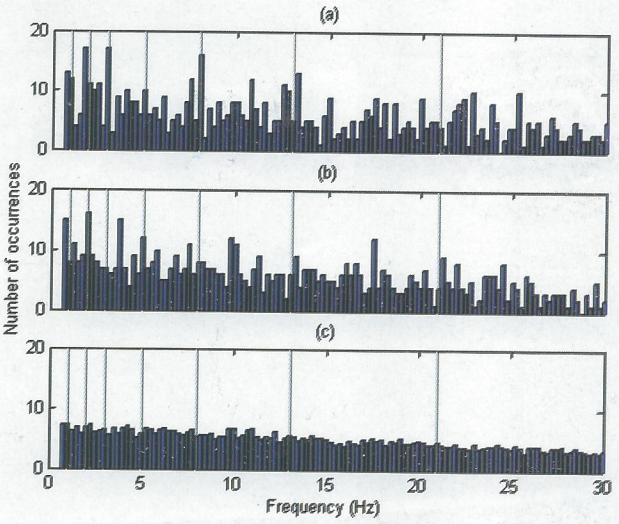


Figure 5. Frequency histograms (n = 684) showing the most frequently occurring frequencies based on the highest six bic(f_p, f_q) values within each EEG bicoherence spectrum for (a) young subject, (b) elderly subject and (c) white noise data. The vertical lines show the position of Fibonacci numbers.