# Noble gas magic numbers: from quarks to quasars

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The proton or atomic numbers, $Z$, of the stable noble gases (2, 10, 18, 36, 54, and 86 corresponding to He, Ne, Ar, Kr, Xe, and Rn respectively) are often referred to as the atomic magic numbers. This article highlights a collection of three interesting mathematical observations associated with these magic numbers.

## Mathematical observation 1: form of curve-of-best-fit through $Z$ values

If one fits a second-order polynomial of the form $Z = ax^2 + bx + c$ through the $Z$ values of interest, while setting corresponding $x$ values to be 0, 1, 2, 3, 4 and 5 respectively (Figure 1), a least-squares regression analysis ($R^2 = 0.997$, $p < 0.001$) results in the following equation-of-best-fit for which the values of $a = 3 - 1/7$, $b = 2$ and $c = 3 + 1/7$ are exact and have not been rounded:

$$Z = \left(3 - \tfrac{1}{7}\right)x^2 + 2x + \left(3 + \tfrac{1}{7}\right).$$

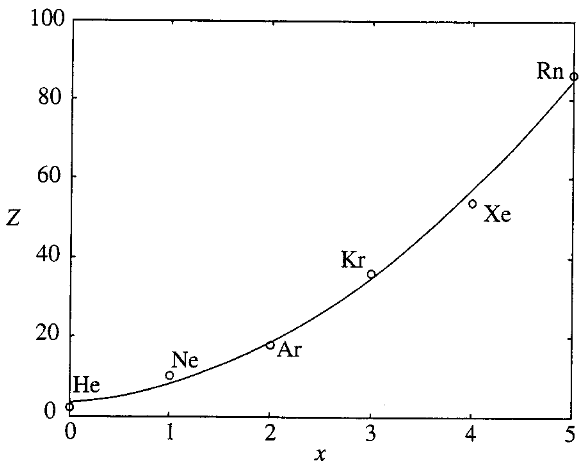


**Figure 1.** Atomic number, $Z$, versus $x$ for the noble gases, where $x$ is a monotonically increasing integer, commencing at $x = 0$, assigned to each successive noble gas in the periodic table.

The mathematical features of interest from Figure 1 include the values and exactness of $a$, $b$ and $c$, recalling that baryons (e.g. protons, neutrons) consist of three quarks[1], mesons (e.g. $\pi$ mesons) consist of two quarks, and $\pi$ mesons are approx-

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[1] Quarks are elementary particles that constitute the hadron family of particles which is divided into two groups, baryons (consisting of three quarks) and mesons (consisting of two quarks). Protons and neutrons are the most well-known baryons, while pions or $\pi$ mesons (which can be electrically neutral, $\pi^0$, positively charged, $\pi^+$, or negatively charged, $\pi^-$) are the most well-known mesons.

imately $\frac{1}{7}$th the mass of a proton or neutron (e.g. neutron/$\pi^0$ meson mass rounds to 7.0) [1]. Note that quoted masses to follow are expressed in energy equivalent units in accordance with $E = mc^2$, as is standard in particle physics.

## Mathematical observation 2: deviations of $Z$ values from curve-of-best-fit

The deviation of each $(x, Z)$ point from the curve-of-best-fit in Figure 1 is given by column 2 of Table 1, and the deviations are all integer multiples of $\frac{1}{7}$.

**Table 1.** Deviation of noble gas $Z$ values from the curve-of-best-fit, $Z = (3 - \frac{1}{7})x^2 + 2x + (3 + \frac{1}{7})$, in Figure 1. Also shown is this deviation multiplied by a scaling factor $C = 9.45/Z$ and the masses of the six known quarks with an additional mass of the largest bare quark calculated by Okubo [5]. The bracketed term indicates the electrical charge sign for each quark.

| Noble gas | Deviation | Deviation $\times$ $C$ | Quark type | Quark mass (GeV) |
|---|---|---|---|---|
| He | $-\frac{8}{7}$ | $-5.40$ | Bottom | $\approx 5.0$ (–) |
| Ne | $+\frac{14}{7}$ | $+1.89$ | Charmed | $\approx 1.7$ (+) |
| Ar | $-\frac{4}{7}$ | $-0.300$ | Down | $\approx 0.3$ (–) |
| Kr | $+\frac{8}{7}$ | $+0.300$ | Up | $\approx 0.3$ (+) |
| Xe | $-\frac{20}{7}$ | $-0.500$ | Strange | $\approx 0.5$ (–) |
| Rn | $+\frac{10}{7}$ | $+0.157$ | Top | $\approx 180$ (+) |
| | | | Bare | 0.156 |

Column 3 of Table 1 presents scaled deviations obtained by calculating the deviation per proton and multiplying by $7 \times 135/100$ (135 MeV is the mass of a $\pi^0$ meson), giving an overall scaling factor, $C$, of

$$C = \frac{9.45}{Z} \approx \frac{\pi c}{10^8 Z}.$$

Comparing columns 3 and 5 of Table 1 reveals a similarity between the scaled deviations for the six noble gases and the masses [1]–[4], for the six known quarks (down, up, strange, charmed, bottom and top). The marked exception to the observed similarity is for the top (relatively massive) quark. However, it is noted that the scaled deviation in this case matches to within 0.6% the theoretical largest bare quark mass (156 MeV) calculated by Okubo [5].

Bracketed within column 5 of Table 1 is the electric charge sign for each quark and these charge signs correspond with the signs of the deviations in columns 2 and 3. Figure 2 plots quark mass (multiplied by charge sign) versus scaled deviation for all except the top quark mass, which for the plot is replaced by the theoretical bare quark mass noted previously. The resulting bare quark mass point is not included within the line-of-best-fit least squares regression analysis ($R^2 = 1$, $p < 0.001$).

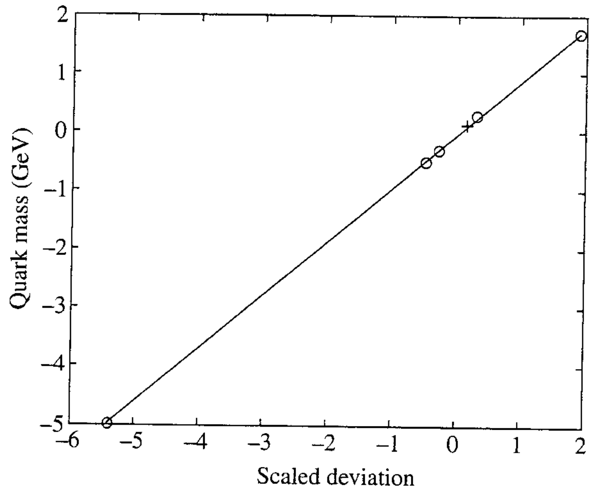


**Figure 2.** Quark mass versus scaled deviations (arbitrary units) of Table 1 (o). Top quark mass is replaced by a theoretical bare quark mass (+). Quark masses are multiplied by their corresponding electrical charge sign.

It should be noted that quark mass values vary within the literature as highlighted by the most comprehensive of reviews [6]. The quark masses presented in Table 1 are thus indicted as approximate, but are generally consistent between references cited and the rudimentary manner of quark mass determination, viz. up and down quark masses are approximately one third the mass of a proton, charmed quark mass is approximately half the mass of the charmonium (charmed–anticharmed quark pair) mesons. Following this rudimentary basis, the presented charmed quark mass in Table 1 matches some references [2] and agrees with the mid-range charmed pole mass based on a perturbative theory [6], but lies at the upper end of range values reported by others.

## Mathematical observation 3: quantisation of quasar redshifts

Quasars, traditionally thought to be the energetic cores of galaxies, emit immense amounts of energy from a relatively small region of space and are suggestive of a formation process involving the accretion of matter onto a super-massive black hole. Quasars also display relatively large Doppler redshifts, making them extremely distant based on a traditional expanding universe/Hubble law interpretation (viz. the greater the redshift the greater the distance from Earth) [3]. There is, however, evidence to suggest that the redshifts of some quasars are quantised [7], with grouped quasars seemingly demonstrating redshift parameters (essentially a relative change in wavelength) of $z = 0.06$, 0.30, 0.60, 0.96, 1.41 or 2.1 (1.96 is also often quoted in place of 2.1 for this series). This redshift periodicity or quantisation observation has provided a basis for arguments, headed by Arp [7], against an expanding universe theory and for many quasars being closer to Earth than the Hubble law indicates. The present article does not aim to advocate any particular school of thought re the above but instead simply draws a modest association between the quasar 'quantum numbers' and magic numbers under investigation.

If the above $z$ values are arbitrarily divided by 0.03 (or $\approx 10^{-10}c$), then the $(z, Z)$ pairs formed by parallel progression through the $z$ and $Z$ series fit the following equation:

$$z/0.03 = Z \text{ (He and Ne)}$$
$$= Z \pm 2^n \text{ (all other noble gases)},$$

where $n = 1, 2, 3$ and 4 respectively for Ar, Kr, Xe and Rn, as displayed by Table 2. Note however that the relationship is only approximate for $(z = 1.41, Z = 54)$, for which a value of $z = 1.38$ is required for exactness.

**Table 2.** Noble gas $Z$ values compared to 'quantised' quasar redshift values, $z$, and $z/0.03$ expressed in terms of $Z$. The bracketed term represents the $z$ value that is required to complete the identified $z/0.03{=}Z{\pm}2^n$ pattern.

| $Z$ | $z$ | $z/0.03$ |
|---|---|---|
| 2 | 0.06 | 2 |
| 10 | 0.30 | 10 |
| 18 | 0.60 | $18 + 2^1$ |
| 36 | 0.96 | $36 - 2^2$ |
| 54 | 1.41(1.38) | $(54 - 2^3)$ |
| 86 | 2.1 | $86 - 2^4$ |

## Summary

The $Z$ values of the noble gases have been examined in a new light and, in so doing, previously unidentified 'magical' mathematical properties of these numbers have been found. A discussion of the physical significance of the mathematical observations is beyond the scope of the present article. However, Australian Mathematical Society members are welcome to contact the author for a free copy of his book [8] (while available or see reference for web link) which gives further detail on, and a physical interpretation of, the presented observations.

## References

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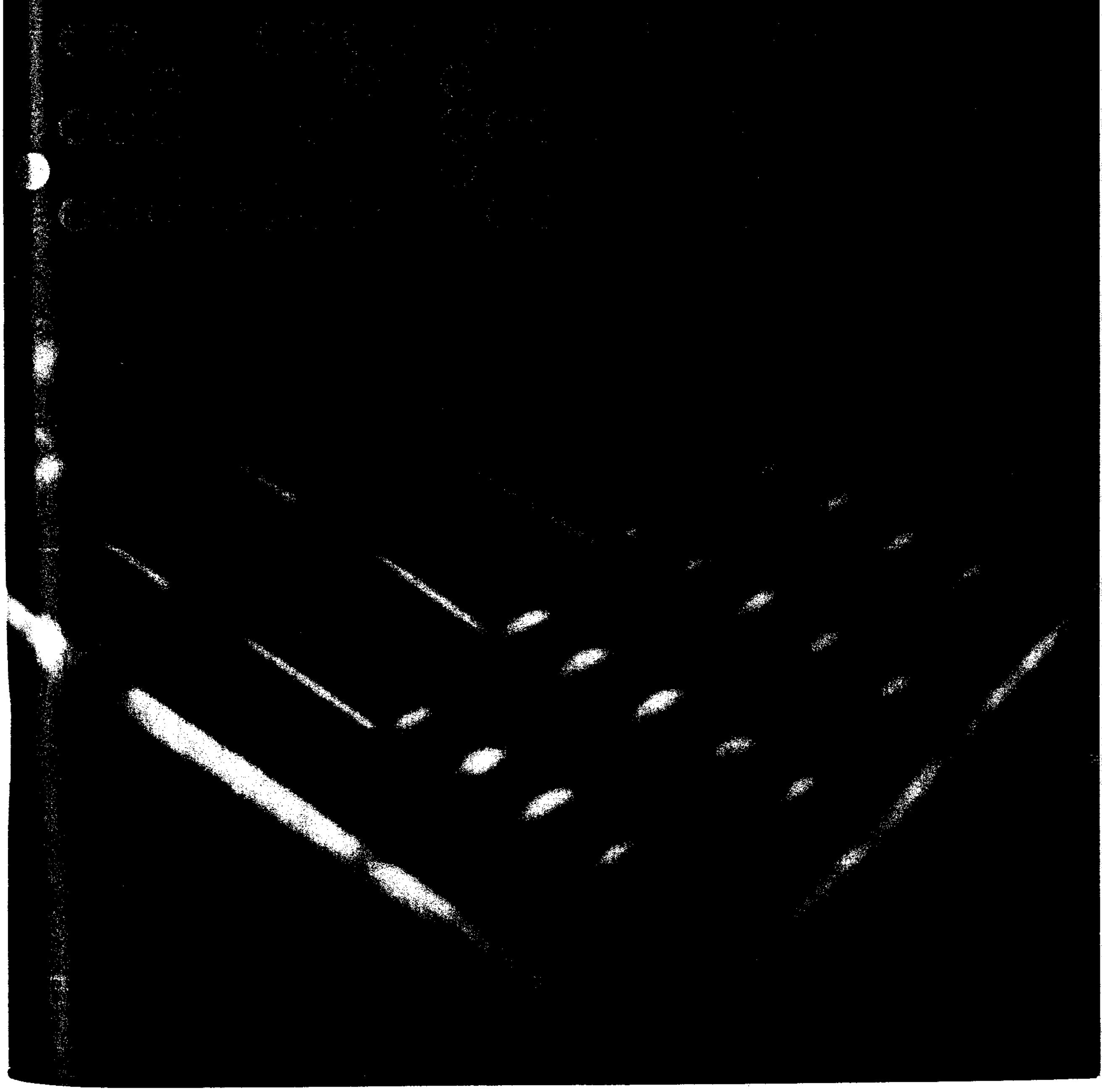

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