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REACTOR THEORY - VOL 2 OF 3 - NUCLEAR PARAMETERS

Main Category:	Nuclear Engineering
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Exam Preview:

1. Any reactor of a finite size will have neutrons leak out of it. Generally, the larger the reactor, the higher the fraction of neutron leakage.
    a. True
    b. False
2. According to the reference material, there are ___ factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This formula accurately represents the infinite multiplication factor
    a. 2
    b. 3
    c. 4
    d. 5
3. According to the reference material, reactivity is a dimensionless number. It does not have dimensions of time, length, mass, or any combination of these dimensions.
    a. True
    b. False
4. The reproduction factor (η) is defined as the ratio of the number of fast neutrons produces by thermal fission to the number of thermal neutrons absorbed in the fuel. Using TABLE 1 Average Number of Neutrons Liberated in Fission, what is the reproduction factor of Uranium-235 fast neutrons?
    a. 2.07
    b. 2.35
    c. 2.90
    d. 3.04

5. Using Figure 2 Effects of Over and Under Moderation on k_{eff}, which of the following statements is true based on the graph, and surrounding reference material?
    a. In an under moderated state, the resonance escape probability increases
    b. In an under moderated state, the thermal utilization factor decreases
    c. In an over moderated state, the thermal utilization factor increases
    d. In an over moderated state, the resonance escape probability increases
6. According to the reference material, non-burnable poisons are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as the result of neutron absorption.
    a. True
    b. False
7. The higher the power level, or flux, the higher the equilibrium xenon-135 concentration, but equilibrium xenon-135 is not directly proportional to power level. According to the reference material, equilibrium xenon-135 at 25% power is more than ____ the value for equilibrium xenon-135 at 100% power for many reactors.
    a. 1/3
    b. 1/2
    c. 2/3
    d. 3/4
8. According to the reference material, a control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them.
    a. True
    b. False
9. Using Figure 7 Behavior of Samarium-149 in a Typical Light Water Reactor and the surrounding reference material, at what time does Samarium-149 reach its peak concentration?
    a. 15 days
    b. 25 days
    c. 35 days
    d. 45 days
10. According to the reference material, there are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Which of the following rod type matches the description: used for fine adjustments and to maintain desired power or temperature?
    a. Shim rods
    b. Safety rods
    c. Tuning rods
    d. Regulating rods

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

LIST OF FIGURES

LIST OF TABLES

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TERMINAL OBJECTIVE

1.0 Using appropriate references, **DESCRIBE** the neutron life cycle discussed in this module.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

   a. Infinite multiplication factor, k_∞
   b. Effective multiplication factor, k_{eff}
   c. Subcritical
   d. Critical
   e. Supercritical

1.2 **DEFINE** each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

1.3 Given the macroscopic cross sections for various materials, **CALCULATE** the thermal utilization factor.

1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, **CALCULATE** the reproduction factor.

1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, **CALCULATE** the number of neutrons that will be present at any point in the life cycle.

1.6 **LIST** physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

1.7 **EXPLAIN** the effect that temperature changes will have on the following factors:

   a. Thermal utilization factor
   b. Resonance escape probability
   c. Fast non-leakage probability
   d. Thermal non-leakage probability

1.8 Given the number of neutrons in a reactor core and the effective multiplication factor, **CALCULATE** the number of neutrons present after any number of generations.

ENABLING OBJECTIVES (Cont.)

1.9 **DEFINE** the term reactivity.

1.10 **CONVERT** between reactivity and the associated value of k_{eff}.

1.11 **CONVERT** measures of reactivity between the following units:

a. $\Delta k/k$
b. $\%\Delta k/k$
c. $10^{-4}\ \Delta k/k$
d. Percent millirho (pcm)

1.12 **EXPLAIN** the relationship between reactivity coefficients and reactivity defects.

TERMINAL OBJECTIVE

2.0 From memory, **EXPLAIN** how reactivity varies with the thermodynamic properties of the moderator and the fuel.

ENABLING OBJECTIVES

2.1 **EXPLAIN** the conditions of over moderation and under moderation.

2.2 **EXPLAIN** why many reactors are designed to be operated in an under moderated condition.

2.3 **STATE** the effect that a change in moderator temperature will have on the moderator to fuel ratio.

2.4 **DEFINE** the temperature coefficient of reactivity.

2.5 **EXPLAIN** why a negative temperature coefficient of reactivity is desirable.

2.6 **EXPLAIN** why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.

2.7 **EXPLAIN** the concept of Doppler broadening of resonance absorption peaks.

2.8 **LIST** two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.

2.9 **DEFINE** the pressure coefficient of reactivity.

2.10 **EXPLAIN** why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.

2.11 **DEFINE** the void coefficient of reactivity.

2.12 **IDENTIFY** the moderator conditions under which the void coefficient of reactivity becomes significant.

TERMINAL OBJECTIVE

3.0 Without references, **DESCRIBE** the use of neutron poisons.

ENABLING OBJECTIVES

3.1 **DEFINE** the following terms:

- a. Burnable poison
- b. Non-burnable poison
- c. Chemical shim

3.2 **EXPLAIN** the use of burnable neutron poisons in a reactor core.

3.3 **LIST** the advantages and disadvantages of chemical shim over fixed burnable poisons.

3.4 **STATE** two reasons why fixed non-burnable neutron poisons are used in reactor cores.

3.5 **STATE** an example of a material used as a fixed non-burnable neutron poison.

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the effects of fission product poisons on a reactor.

ENABLING OBJECTIVES

4.1 **LIST** two methods of production and two methods of removal for xenon-135 during reactor operation.

4.2 **STATE** the equation for equilibrium xenon-135 concentration.

4.3 **DESCRIBE** how equilibrium xenon-135 concentration varies with reactor power level.

4.4 **DESCRIBE** the causes and effects of a xenon oscillation.

4.5 **DESCRIBE** how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.

4.6 **EXPLAIN** the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.

4.7 **STATE** the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."

4.8 **EXPLAIN** what is meant by the following terms:

a. Xenon precluded startup
b. Xenon dead time

4.9 **DESCRIBE** how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.

4.10 **DESCRIBE** how samarium-149 is produced and removed from the reactor core during reactor operation.

4.11 **STATE** the equation for equilibrium samarium-149 concentration.

4.12 **DESCRIBE** how equilibrium samarium-149 concentration varies with reactor power level.

ENABLING OBJECTIVES (Cont.)

4.13 **DESCRIBE** how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

4.14 **DESCRIBE** how samarium-149 concentration changes following a reactor startup.

4.15 **STATE** the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

TERMINAL OBJECTIVE

5.0 Without references, **DESCRIBE** how control rods affect the reactor core.

ENABLING OBJECTIVES

5.1 **DESCRIBE** the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

5.2 **EXPLAIN** why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

5.3 **EXPLAIN** why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

5.4 **DEFINE** the following terms:

a. Integral control rod worth
b. Differential control rod worth

5.5 **DESCRIBE** the shape of a typical differential control rod worth curve and explain the reason for the shape.

5.6 **DESCRIBE** the shape of a typical integral control rod worth curve and explain the reason for the shape.

5.7 Given an integral or differential control rod worth curve, **CALCULATE** the reactivity change due to a control rod movement between two positions.

5.8 Given differential control rod worth data, **PLOT** differential and integral control rod worth curves.

NEUTRON LIFE CYCLE

Some number of the fast neutrons produced by fission in one generation will eventually cause fission in the next generation. The series of steps that fission neutrons go through as they slow to thermal energies and are absorbed in the reactor is referred to as the neutron life cycle. The neutron life cycle is markedly different between fast reactors and thermal reactors. This chapter presents the neutron life cycle for thermal reactors.

EO 1.1 DEFINE the following terms:

a. Infinite multiplication factor, k_∞
b. Effective multiplication factor, k_{eff}
c. Subcritical
d. Critical
e. Supercritical

EO 1.2 DEFINE each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

EO 1.3 Given the macroscopic cross sections for various materials, CALCULATE the thermal utilization factor.

EO 1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, CALCULATE the reproduction factor.

EO 1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, CALCULATE the number of neutrons that will be present at any point in the life cycle.

EO 1.6 LIST physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

EO 1.7 EXPLAIN the effect that temperature changes will have on the following factors:

a. Thermal utilization factor
b. Resonance escape probability
c. Fast non-leakage probability
d. Thermal non-leakage probability

Infinite Multiplication Factor, k_∞

Not all of the neutrons produced by fission will have the opportunity to cause new fissions because some neutrons will be absorbed by non-fissionable material. Some will be absorbed parasitically in fissionable material and will not cause fission, and others will leak out of the reactor. For the maintenance of a self-sustaining chain reaction, however, it is not necessary that every neutron produced in fission initiate another fission. The minimum condition is for each nucleus undergoing fission to produce, on the average, at least one neutron that causes fission of another nucleus. This condition is conveniently expressed in terms of a multiplication factor.

The number of neutrons absorbed or leaking out of the reactor will determine the value of this multiplication factor, and will also determine whether a new generation of neutrons is larger, smaller, or the same size as the preceding generation. Any reactor of a finite size will have neutrons leak out of it. Generally, the larger the reactor, the lower the fraction of neutron leakage. For simplicity, we will first consider a reactor that is infinitely large, and therefore has no neutron leakage. A measure of the increase or decrease in neutron flux in an infinite reactor is the infinite multiplication factor, k_∞. The *infinite multiplication factor* is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation. This can be expressed mathematically as shown below.

$$k_\infty = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation}}$$

Four Factor Formula

A group of fast neutrons produced by fission can enter into several reactions. Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation. There are four factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This *four factor formula* accurately represents the infinite multiplication factor as shown in the equation below.

$$k_\infty = \epsilon \, p \, f \, \eta$$

where:

ϵ = fast fission factor
p = resonance escape probability
f = thermal utilization factor
η = reproduction factor

Each of these four factors, which are explained in the following subsections, represents a process that adds to or subtracts from the initial neutron group produced in a generation by fission.

Fast Fission Factor, (ε)

The first process that the neutrons of one generation may undergo is fast fission. Fast fission is fission caused by neutrons that are in the fast energy range. Fast fission results in the net increase in the fast neutron population of the reactor core. The cross section for fast fission in uranium-235 or uranium-238 is small; therefore, only a small number of fast neutrons cause fission. The fast neutron population in one generation is therefore increased by a factor called the fast fission factor. The *fast fission factor* (ε) is defined as the ratio of the net number of fast neutrons produced by all fissions to the number of fast neutrons produced by thermal fissions. The mathematical expression of this ratio is shown below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close enough to a fuel nucleus while it is a fast neutron. The value of ε will be affected by the arrangement and concentrations of the fuel and the moderator. The value of ε is essentially 1.00 for a homogenous reactor where the fuel atoms are surrounded by moderator atoms. However, in a heterogeneous reactor, all the fuel atoms are packed closely together in elements such as pins, rods, or pellets. Neutrons emitted from the fission of one fuel atom have a very good chance of passing near another fuel atom before slowing down significantly. The arrangement of the core elements results in a value of about 1.03 for ε in most heterogeneous reactors. The value of ε is not significantly affected by variables such as temperature, pressure, enrichment, or neutron poison concentrations. Poisons are non-fuel materials that easily absorb neutrons and will be discussed in more detail later.

Resonance Escape Probability, (p)

After increasing in number as a result of some fast fissions, the neutrons continue to diffuse through the reactor. As the neutrons move they collide with nuclei of fuel and non-fuel material and moderator in the reactor losing part of their energy in each collision and slowing down. While they are slowing down through the resonance region of uranium-238, which extends from about 6 eV to 200 eV, there is a chance that some neutrons will be captured. The probability that a neutron will not be absorbed by a resonance peak is called the resonance escape probability. The *resonance escape probability* (p) is defined as the ratio of the number of neutrons that reach thermal energies to the number of fast neutrons that start to slow down. This ratio is shown below.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

The value of the resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235 (if any is used). To undergo resonance absorption, a neutron must pass close enough to a uranium-238 nucleus to be absorbed while slowing down. In a homogeneous reactor the neutron does its slowing down in the region of the fuel nuclei, and this condition is easily met. This means that a neutron has a high probability of being absorbed by uranium-238 while slowing down; therefore, its escape probability is lower. In a heterogeneous reactor, however, the neutron slows down in the moderator where there are no atoms of uranium-238 present. Therefore, it has a low probability of undergoing resonance absorption, and its escape probability is higher.

The value of the resonance escape probability is not significantly affected by pressure or poison concentration. In water moderated, low uranium-235 enrichment reactors, raising the temperature of the fuel will raise the resonance absorption in uranium-238 due to the doppler effect (an apparent broadening of the normally narrow resonance peaks due to thermal motion of nuclei). The increase in resonance absorption lowers the resonance escape probability, and the fuel temperature coefficient for resonance escape is negative (explained in detail later). The temperature coefficient of resonance escape probability for the moderator temperature is also negative. As water temperature increases, water density decreases. The decrease in water density allows more resonance energy neutrons to enter the fuel and be absorbed. The value of the resonance escape probability is always slightly less than one (normally 0.95 to 0.99).

The product of the fast fission factor and the resonance escape probability (ε p) is the ratio of the number of fast neutrons that survive slowing down (thermalization) compared to the number of fast neutrons originally starting the generation.

Thermal Utilization Factor, (f)

Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel. The thermal utilization factor describes how effectively thermal neutrons are absorbed by the fuel, or how well they are utilized within the reactor. The *thermal utilization factor* (f) is defined as the ratio of the number of thermal neutrons absorbed in the fuel to the number of thermal neutrons absorbed in any reactor material. This ratio is shown below.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

The thermal utilization factor will always be less than one because some of the thermal neutrons absorbed within the reactor will be absorbed by atoms of non-fuel materials.

An equation can be developed for the thermal utilization factor in terms of reaction rates as follows.

$$f = \frac{\text{rate of absorption of thermal neutrons by the fuel}}{\text{rate of absorption of thermal neutrons by all reactor materials}}$$

$$f = \frac{\Sigma_a^U \, \phi^U \, V^U}{\Sigma_a^U \, \phi^U \, V^U + \Sigma_a^m \, \phi^m \, V^m + \Sigma_a^p \, \phi^p \, V^p}$$

The superscripts U, m, and p refer to uranium, moderator, and poison, respectively. In a heterogeneous reactor, the flux will be different in the fuel region than in the moderator region due to the high absorption rate by the fuel. Also, the volumes of fuel, moderator, and poisons will be different. Although not shown in the above equation, other non-fuel materials, such as core construction materials, may absorb neutrons in a heterogeneous reactor. These other materials are often lumped together with the superscript designation OS, for "other stuff." To be completely accurate, the above equation for the thermal utilization factor should include all neutron-absorbing reactor materials when dealing with heterogeneous reactors. However, for the purposes of this text, the above equation is satisfactory.

In a homogeneous reactor the neutron flux seen by the fuel, moderator, and poisons will be the same. Also, since they are spread throughout the reactor, they all occupy the same volume. This allows the previous equation to be rewritten as shown below.

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p} \tag{3-1}$$

Equation (3-1) gives an approximation for a heterogeneous reactor if the fuel and moderator are composed of small elements distributed uniformly throughout the reactor.

Since absorption cross sections vary with temperature, it would appear that the thermal utilization factor would vary with a temperature change. But, substitution of the temperature correction formulas (see Module 2) in the above equation will reveal that all terms change by the same amount, and the ratio remains the same. In heterogeneous water-moderated reactors, there is another important factor. When the temperature rises, the water moderator expands, and a significant amount of it will be forced out of the reactor core. This means that N^m, the number of moderator atoms per cm^3, will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N^m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom. Because of this effect, the temperature coefficient for the thermal utilization factor is positive. The amount of enrichment of uranium-235 and the poison concentration will affect the thermal utilization factor in a similar manner as can be seen from the equation above.

Example:

> Calculate the thermal utilization factor for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.3020 cm^{-1}, the macroscopic absorption cross section of the moderator is 0.0104 cm^{-1}, and the macroscopic absorption cross section of the poison is 0.0118 cm^{-1}.

Solution:

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

$$= \frac{0.3020\ cm^{-1}}{0.3020\ cm^{-1} + 0.0104 cm^{-1} + 0.0118 cm^{-1}}$$

$$= 0.932$$

Reproduction Factor, (η)

Most of the neutrons absorbed in the fuel cause fission, but some do not. The *reproduction factor* (η) is defined as the ratio of the number of fast neurtons produces by thermal fission to the number of themal neutrons absorbed in the fuel. The reproduction factor is shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

The reproduction factor can also be stated as a ratio of rates as shown below.

$$\eta = \frac{\text{rate of production of fast neutrons by thermal fission}}{\text{rate of absorption of thermal neutrons by the fuel}}$$

The rate of production of fast neutrons by thermal fission can be determined by the product of the fission reaction rate ($\Sigma_f^u \phi^u$) and the average number of neutrons produced per fission (ν). The average number of neutrons released in thermal fission of uranium-235 is 2.42. The rate of absorption of thermal neutrons by the fuel is $\Sigma_a^u \phi^u$. Substituting these terms into the equation above results in the following equation.

$$\eta = \frac{\Sigma_f^U\ \phi^U\ \nu}{\Sigma_a^U\ \phi^U}$$

Table 1 lists values of ν and η for fission of several different materials by thermal neutrons and fast neutrons.

TABLE 1
Average Number of Neutrons Liberated in Fission

Fissile Nucleus	Thermal Neutrons		Fast Neutrons	
	ν	η	ν	η
Uranium-233	2.49	2.29	2.58	2.40
Uranium-235	2.42	2.07	2.51	2.35
Plutonium-239	2.93	2.15	3.04	2.90

In the case where the fuel contains several fissionable materials, it is necessary to account for each material. In the case of a reactor core containing both uranium-235 and uranium-238, the reproduction factor would be calculated as shown below.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}} \tag{3-2}$$

Example:

Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The atom density of uranium-235 is 4.83 x 10^{21} atoms/cm^3. The atom density of uranium-238 is 4.35 x 10^{22} atoms/cm^3. ν is 2.42.

Solution:

Use Equation (3-2) to calculate the reproduction factor.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}}$$

$$= \frac{\left(4.83 \text{ x } 10^{21} \frac{\text{atoms}}{\text{cm}^3}\right)\left(582 \text{ x } 10^{-24} \text{ cm}^2\right)\left(2.42\right)}{\left(4.83 \text{ x } 10^{21} \frac{\text{atoms}}{\text{cm}^3}\right)\left(694 \text{ x } 10^{-24} \text{ cm}^2\right) + \left(4.35 \text{ x } 10^{22} \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.71 \text{ x } 10^{-24} \text{ cm}^2\right)}$$

$$= 1.96$$

As temperature varies, each absorption and fission microscopic cross section varies according to the 1/v relationship (see Module 2). Since both the numerator and the denominator change equally, the net change in η is zero. Therefore, η changes only as uranium-235 enrichment changes. η increases with enrichment because there is less uranium-238 in the reactor making it more likely that a neutron absorbed in the fuel will be absorbed by uranium-235 and cause fission.

To determine the reproduction factor for a single nuclide rather than for a mixture, the calculation may be further simplified to the one shown below.

$$\eta = \frac{\sigma_f \, \nu}{\sigma_a}$$

Effective Multiplication Factor

The infinite multiplication factor can fully represent only a reactor that is infinitely large, because it assumes that no neutrons leak out of the reactor. To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out. The multiplication factor that takes leakage into account is the *effective multiplication factor* (k_{eff}), which is defined as the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

The effective multiplication factor may be expressed mathematically as shown below.

$$k_{eff} = \frac{\text{neutron production from fission in one generation}}{\begin{array}{c}\text{neutron absorption in the}\\ \text{preceding generation}\end{array} + \begin{array}{c}\text{neutron leakage in the}\\ \text{preceding generation}\end{array}}$$

So, the value of k_{eff} for a self-sustaining chain reaction of fissions, where the neutron population is neither increasing nor decreasing, is one. The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing is referred to as the *critical* condition and can be expressed by the simple equation $k_{eff} = 1$.

If the neutron production is greater than the absorption and leakage, the reactor is called supercritical. In a *supercritical* reactor, k_{eff} is greater than one, and the neutron flux increases each generation. If, on the other hand, the neutron production is less than the absorption and leakage, the reactor is called subcritical. In a *subcritical* reactor, k_{eff} is less than one, and the flux decreases each generation.

When the multiplication factor of a reactor is not equal to exactly one, the neutron flux will change and cause a change in the power level. Therefore, it is essential to know more about how this factor depends upon the contents and construction of the reactor. The balance between production of neutrons and their absorption in the core and leakage out of the core determines the value of the multiplication factor. If the leakage is small enough to be neglected, the multiplication factor depends upon only the balance between production and absorption, and is called the infinite multiplication factor (k_∞) since an infinitely large core can have no leakage. When the leakage is included, the factor is called the effective multiplication factor (k_{eff}).

The effective multiplication factor (k_{eff}) for a finite reactor may be expressed mathematically in terms of the infinite multiplication factor and two additional factors which account for neutron leakage as shown below.

$$k_{eff} = k_\infty \mathcal{L}_f \mathcal{L}_t$$

Fast Non-Leakage Probability ($\mathcal{L}_f$)

In a realistic reactor of finite size, some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process. The *fast non-leakage probability* ($\mathcal{L}_f$) is defined as the ratio of the number of fast neutrons that do not leak from the reactor core to the number of fast neutrons produced by all fissions. This ratio is stated as follows.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

Thermal Non-Leakage Probability ($\mathcal{L}_t$)

Neutrons can also leak out of a finite reactor core after they reach thermal energies. The *thermal non-leakage probability* ($\mathcal{L}_t$) is defined as the ratio of the number of thermal neutrons that do not leak from the reactor core to the number of neutrons that reach thermal energies. The thermal non-leakage probability is represented by the following.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

The fast non-leakage probability ($\mathcal{L}_f$) and the thermal non-leakage probability ($\mathcal{L}_t$) may be combined into one term that gives the fraction of all neutrons that do not leak out of the reactor core. This term is called the *total non-leakage probability* and is given the symbol $\mathcal{L}_T$, where $\mathcal{L}_T = \mathcal{L}_f \mathcal{L}_t$. $\mathcal{L}_f$ and $\mathcal{L}_t$ are both effected by a change in coolant temperature in a heterogeneous water-cooled, water-moderated reactor. As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability. Consequently, the temperature coefficient (defined later) for the non-leakage probabilities is negative, because as temperature rises, $\mathcal{L}_f$ and $\mathcal{L}_t$ decrease.

Six Factor Formula

With the inclusion of these last two factors it is possible to determine the fraction of neutrons that remain after every possible process in a nuclear reactor. The effective multiplication factor (k_{eff}) can then be determined by the product of six terms.

$$k_{eff} = \varepsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta \qquad (3\text{-}3)$$

Equation (3-3) is called the *six factor formula*. Using this six factor formula, it is possible to trace the entire neutron life cycle from production by fission to the initiation of subsequent fissions. Figure 1 illustrates a neutron life cycle with nominal values provided for each of the six factors. Refer to Figure 1 for the remainder of the discussion on the neutron life cycle and sample calculations. The generation begins with 1000 neutrons. This initial number is represented by N_o. The first process is fast fission and the population has been increased by the neutrons from this fast fission process. From the definition of the fast fission factor it is possible to calculate its value based on the number of neutrons before and after fast fission occur.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$= \frac{1040}{1000}$$

$$= 1.04$$

The total number of fast neutrons produced by thermal and fast fission is represented by the quantity $N_o \varepsilon$.

Next, it can be seen that 140 neutrons leak from the core before reaching the thermal energy range. The fast non-leakage probability is calculated from its definition, as shown below.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$= \frac{1040 - 140}{1040}$$

$$= 0.865$$

The number of neutrons that remain in the core during the slowing down process is represented by the quantity $N_o \, \varepsilon \, \mathcal{L}_f$.

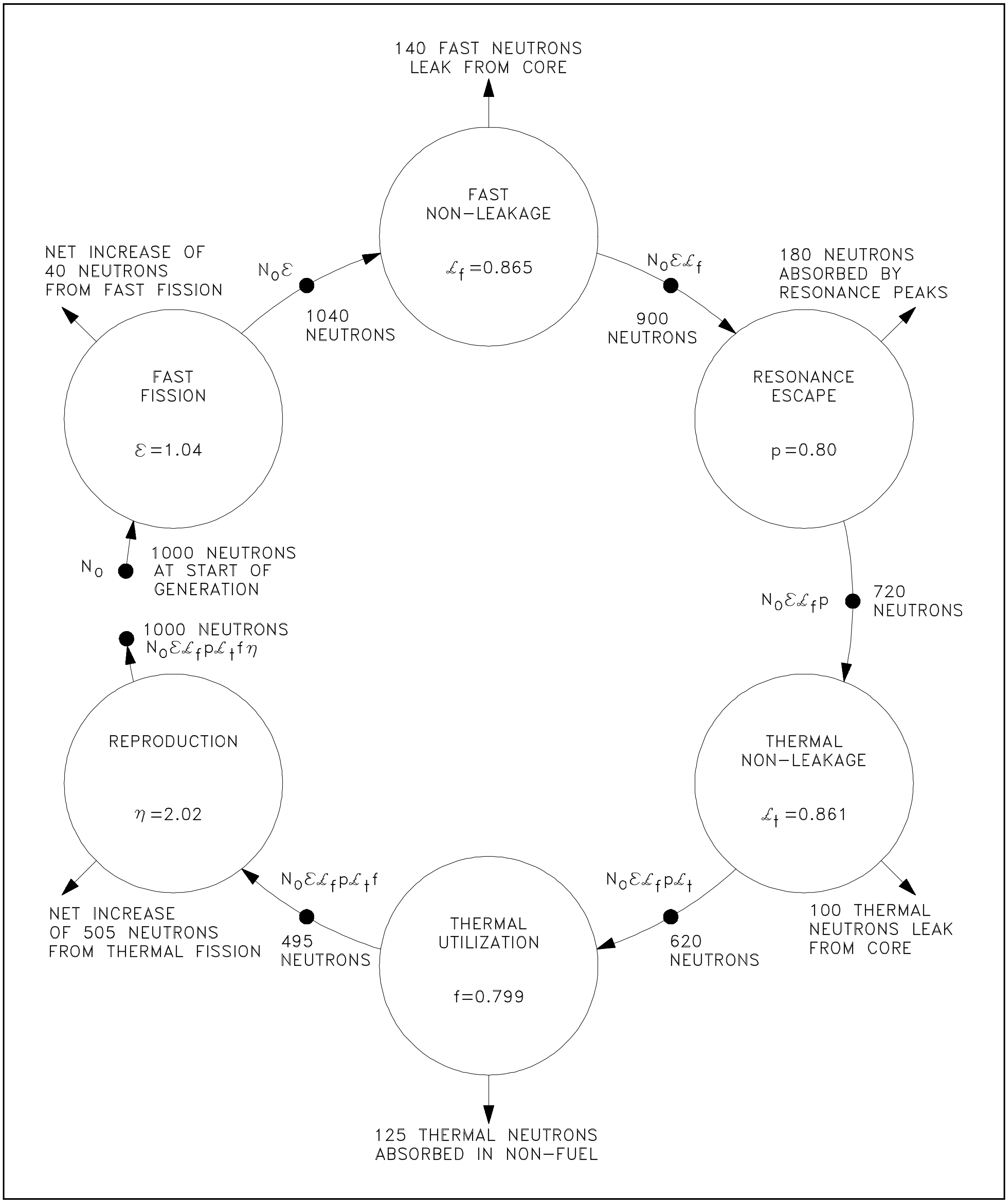

Figure 1 Neutron Life Cycle with $k_{eff} = 1$

The next step in the analysis is to consider the number of neutrons that are absorbed in the intermediate energy level. The probability of escaping this resonance absorption (p) is stated as follows.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$= \frac{720}{900}$$

$$= 0.80$$

The number of neutrons entering the thermal energy range is now represented by the quantity $N_o\,\boldsymbol{\varepsilon}\,\mathcal{L}_f\,p$.

After reaching thermal energies, 100 neutrons leak from the core. The value for $\mathcal{L}_t$ can be calculated by substitution of the known values in the definition as shown below.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$= \frac{620}{720}$$

$$= 0.861$$

The number of thermal neutrons available for absorption anywhere in the core is represented by the quantity $N_o\,\boldsymbol{\varepsilon}\,\mathcal{L}_f\,p\,\mathcal{L}_t$.

Figure 1 indicates that 125 neutrons were absorbed in non-fuel materials. Since a total of 620 thermal neutrons were absorbed, the number absorbed by the fuel equals 620 - 125 = 495. Therefore, the thermal utilization factor can be calculated as follows.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in any reactor material}}$$

$$= \frac{495}{620}$$

$$= 0.799$$

The final factor numerically describes the production of fission neutrons resulting from thermal neutrons being absorbed in the fuel. This factor is called the reproduction factor (η). The value for the reproduction factor can be determined as shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

$$= \frac{1000}{495}$$

$$= 2.02$$

The number of fission neutrons that exist at the end of the life cycle which are available to start a new generation and cycle is represented by the quantity $N_o \, \epsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta$.

In the example illustrated in Figure 1, k_{eff} is equal to one. Therefore, 1000 neutrons are available to start the next generation.

Example:

10,000 neutrons exist at the beginning of a generation. The values for each factor of the six factor formula are listed below. Calculate the number of neutrons that exist at the points in the neutron life cycle listed below.

1) Number of neutrons that exist after fast fission.
2) Number of neutrons that start to slow down in the reactor.
3) Number of neutrons that reach thermal energies.
4) Number of thermal neutrons that are absorbed in the reactor.
5) Number of thermal neutrons absorbed in the fuel.
6) Number of neutrons produced from thermal fission.

$\epsilon = 1.031$ $\mathcal{L}_f = 0.889$ $f = 0.751$
$p = 0.803$ $\mathcal{L}_t = 0.905$ $\eta = 2.012$

Solution:

1) $N = N_o \, \epsilon = 10{,}310$
2) $N = N_o \, \epsilon \, \mathcal{L}_f = 9{,}166$
3) $N = N_o \, \epsilon \, \mathcal{L}_f \, p = 7{,}360$
4) $N = N_o \, \epsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t = 6{,}661$
5) $N = N_o \, \epsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f = 5{,}002$
6) $N = N_o \, \epsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta = 10{,}065$

Neutron Life Cycle of a Fast Reactor

The neutron life cycle in a fast reactor is markedly different than that for a thermal reactor. In a fast reactor, care is taken during the reactor design to minimize thermalization of neutrons. Virtually all fissions taking place in a fast reactor are caused by fast neutrons. Due to this, many factors that are taken into account by the thermal reactor neutron life cycle are irrelevant to the fast reactor neutron life cycle. The resonance escape probability is not significant because very few neutrons exist at energies where resonance absorption is significant. The thermal non-leakage probability does not exist because the reactor is designed to avoid the thermalization of neutrons. A separate term to deal with fast fission is not necessary because all fission is fast fission and is handled by the reproduction factor.

The thermal utilization factor is modified to describe the utilization of fast neutrons instead of thermal neutrons. The reproduction factor is similarly modified to account for fast fission instead of thermal fission.

Summary

The important information in this chapter is summarized on the following pages.

Neutron Life Cycle Summary

- The infinite multiplication factor, k_∞, is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation.
- The effective multiplication factor, k_{eff}, is the ratio of the number of neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.
- Critical is the condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing.
- Subcritical is the condition in which the neutron population is decreasing each generation.
- Supercritical is the condition in which the neutron population is increasing each generation.
- The six factor formula is stated as $k_{eff} = \varepsilon \mathcal{L}_f\, p\, \mathcal{L}_t\, f\, \eta$. Each of the six factors is defined below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

Neutron Life Cycle Summary (Cont.)

- The thermal utilization factor can be calculated from the macroscopic cross section for absorption of reactor materials using Equation (3-1).

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

- The reproduction factor can be calculated based on the characteristics of the reactor fuel using Equation (3-2).

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}}$$

- The number of neutrons present at any point in the neutron life cycle can be calculated as the product of the number of neutrons present at the start of the generation and all the factors preceding that point in the life cycle.

- The thermal utilization factor is effected by the enrichment of uranium-235, the amount of neutron poisons, and the moderator-to-fuel ratio.

- The reproduction factor is effected by the enrichment of uranium-235.

- The resonance escape probability is effected by the enrichment of uranium-235, the temperature of the fuel, and the temperature of the moderator.

- An increase in moderator temperature will have the following effects.

  Increase the thermal utilization factor
  Decrease resonance escape probability
  Decrease fast non-leakage probability
  Decrease thermal non-leakage probability

REACTIVITY

Reactivity is a measure of the departure of a reactor from criticality. The reactivity is related to the value of k_{eff}. Reactivity is a useful concept to predict how the neutron population of a reactor will change over time.

EO 1.8 **Given the number of neutrons in a reactor core and the effective multiplication factor, CALCULATE the number of neutrons present after any number of generations.**

EO 1.9 **DEFINE the term reactivity.**

EO 1.10 **CONVERT between reactivity and the associated value of k_{eff}.**

EO 1.11 **CONVERT measures of reactivity between the following units:**

a.	**$\Delta k/k$**	**c.**	**10^{-4} $\Delta k/k$**
b.	**$\%\Delta k/k$**	**d.**	**Percent millirho (pcm)**

EO 1.12 **EXPLAIN the relationship between reactivity coefficients and reactivity defects.**

Application of the Effective Multiplication Factor

When k_{eff} remains constant from generation to generation, it is possible to determine the number of neutrons beginning any particular generation by knowing only the value of k_{eff} and the number of neutrons starting the first generation. If N_o neutrons start the first generation, then $N_o(k_{eff})$ neutrons start the second generation. Equation (3-4) can be used to calculate the number of neutrons after the completion of "n" generations.

$$N_n = N_o \left(k_{eff}\right)^n \tag{3-4}$$

Example:

The number of neutrons in the core at time zero is 1000 and k_{eff} = 1.002. Calculate the number of neutrons after 50 generations.

Solution:

Use Equation (3-4) to calculate the number of neutrons.

$$N_n = N_o \left(k_{eff}\right)^n$$

$$N_{50} = 1000 \text{ neutrons } \left(1.002\right)^{50}$$

$$= 1105 \text{ neutrons}$$

Reactivity

If there are N_o neutrons in the preceding generation, then there are $N_o(k_{eff})$ neutrons in the present generation. The numerical change in neutron population is $(N_o k_{eff} - N_o)$. The gain or loss in neutron population $(N_o k_{eff} - N_o)$, expressed as a fraction of the present generation $(N_o k_{eff})$, is shown below.

$$\frac{N_o\ k_{eff} - N_o}{N_o\ k_{eff}}$$

This relationship represents the fractional change in neutron population per generation and is referred to as *reactivity* (ρ). Cancelling out the term N_o from the numerator and denominator, the reactivity is determined as shown in the equation below.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{3-5}$$

From Equation (3-5) it may be seen that ρ may be positive, zero, or negative, depending upon the value of k_{eff}. The larger the absolute value of reactivity in the reactor core, the further the reactor is from criticality. It may be convenient to think of reactivity as a measure of a reactor's departure from criticality.

Example:

Calculate the reactivity in the reactor core when k_{eff} is equal to 1.002 and 0.998.

Solution:

The reactivity for each case is determined by substituting the value of k_{eff} into Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad \rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.002 - 1}{1.002} \qquad = \frac{0.998 - 1}{0.998}$$

$$= 0.001996 \qquad = -0.0020$$

Units of Reactivity

Reactivity is a dimensionless number. It does not have dimensions of time, length, mass, or any combination of these dimensions. It is simply a ratio of two quantities that are dimensionless. As shown in the calculation in the previous example, the value of reactivity is often a small decimal value. In order to make this value easier to express, artificial units are defined.

By definition, the value for reactivity that results directly from the calculation of Equation (3-5) is in units of $\Delta k/k$. Alternative units for reactivity are $\%\Delta k/k$ and pcm (percent millirho). The conversions between these units of reactivity are shown below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \text{ pcm} = 0.00001 \ \frac{\Delta k}{k}$$

Another unit of reactivity that is used at some reactors is equivalent to 10^{-4} $\Delta k/k$. This unit of reactivity does not have a unique name. Special units for reactivity that do have unique names are dollars and cents. These units and their applications will be described in a later chapter.

Example:

Convert the values of reactivity listed below to the indicated units.

a. 0.000421 $\Delta k/k$ = _____ pcm
b. 0.0085 $\Delta k/k$ = _____ % $\Delta k/k$
c. 16×10^{-4} $\Delta k/k$ = _____ $\Delta k/k$

Solution:

a. 42.1 pcm
b. 0.85% $\Delta k/k$
c. 0.0016 $\Delta k/k$

If the reactivity is known, the effective multiplication factor can be determined by solving Equation (3-5) for k_{eff} in terms of the reactivity. This results in the following relationship.

$$k_{eff} = \frac{1}{1 - \rho} \tag{3-6}$$

Reactivity must be in units of $\Delta k/k$ for use in Equation (3-6).

Example:

Given a reactivity of -20.0×10^{-4} $\Delta k/k$, calculate k_{eff}.

Solution:

$$\begin{aligned} k_{eff} &= \frac{1}{1 - \rho} \\ &= \frac{1}{1 - (-20.0 \times 10^{-4})} \\ &= 0.998 \end{aligned}$$

Reactivity Coefficients and Reactivity Defects

The amount of reactivity (ρ) in a reactor core determines what the neutron population, and consequently the reactor power, are doing at any given time. The reactivity can be effected by many factors (for example, fuel depletion, temperature, pressure, or poisons). The next several chapters discuss the factors affecting reactivity and how they are used to control or predict reactor behavior.

To quantify the effect that a variation in parameter (that is, increase in temperature, control rod insertion, increase in neutron poison) will have on the reactivity of the core, *reactivity coefficients* are used. Reactivity coefficients are the amount that the reactivity will change for a given change in the parameter. For instance, an increase in moderator temperature will cause a decrease in the reactivity of the core. The amount of reactivity change per degree change in the moderator temperature is the moderator temperature coefficient. Typical units for the moderator temperature coefficient are pcm/°F. Reactivity coefficients are generally symbolized by α_x, where x represents some variable reactor parameter that affects reactivity. The definition of a reactivity coefficient in equation format is shown below.

$$\alpha_x = \frac{\Delta \rho}{\Delta x}$$

If the parameter x increases and positive reactivity is added, then α_x is positive. If the parameter x increases and negative reactivity is added, then α_x is negative.

Reactivity defects ($\Delta\rho$) are the total reactivity change caused by a variation in a parameter. Reactivity defects can be determined by multiplying the change in the parameter by the average value of the reactivity coefficient for that parameter. The equation below shows the general method for relating reactivity coefficients to reactivity defects.

$$\Delta\rho = \alpha_x \, \Delta x$$

Example:

> The moderator temperature coefficient for a reactor is -8.2 pcm/°F. Calculate the reactivity defect that results from a temperature decrease of 5°F.

Solution:

$$\begin{aligned}\Delta \rho &= \alpha_T \, \Delta T \\ &= \left(-8.2 \ \frac{\text{pcm}}{{}^\circ\text{F}}\right) \left(-5 \ {}^\circ\text{F}\right) \\ &= 41 \ \text{pcm}\end{aligned}$$

> The reactivity addition due to the temperature decrease was positive because of the negative temperature coefficient.

Summary

The important information in this chapter is summarized below.

Reactivity Summary

- The number of neutrons present in the core after a given number of generations is calculated using Equation (3-4).

$$N_n = N_o \left(k_{eff}\right)^n$$

- Reactivity is the fractional change in neutron population per generation.
- Reactivity and k_{eff} are represented in Equation (3-5) and Equation (3-6), respectively.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad k_{eff} = \frac{1}{1 - \rho}$$

- The relationship between units of reactivity are listed below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \ \text{pcm} = 0.00001 \ \frac{\Delta k}{k}$$

- A reactivity coefficient is the amount of change in reactivity per unit change in the parameter. A reactivity defect is the total reactivity change caused by a change in the parameter. The reactivity defect is the product of the reactivity coefficient and the magnitude of the parameter change.

REACTIVITY COEFFICIENTS

Changes in the physical properties of the materials in the reactor will result in changes in the reactivity. Reactivity coefficients are useful in quantifying the reactivity change that will occur due to the change in a physical property such as the temperature of the moderator or fuel.

EO 2.1 EXPLAIN the conditions of over moderation and under moderation.

EO 2.2 EXPLAIN why many reactors are designed to be operated in an under moderated condition.

EO 2.3 STATE the effect that a change in moderator temperature will have on the moderator to fuel ratio.

EO 2.4 DEFINE the temperature coefficient of reactivity.

EO 2.5 EXPLAIN why a negative temperature coefficient of reactivity is desirable.

EO 2.6 EXPLAIN why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.

EO 2.7 EXPLAIN the concept of Doppler broadening of resonance absorption peaks.

EO 2.8 LIST two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.

EO 2.9 DEFINE the pressure coefficient of reactivity.

EO 2.10 EXPLAIN why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.

EO 2.11 DEFINE the void coefficient of reactivity.

EO 2.12 IDENTIFY the moderator conditions under which the void coefficient of reactivity becomes significant.

Moderator Effects

As discussed in the previous module, a moderator possesses specific desirable characteristics.

(a) large neutron scattering cross section
(b) low neutron absorption cross section
(c) large neutron energy loss per collision

With the exception of the Liquid Metal Fast Breeder Reactor (LMFBR), the remaining major reactor types that are currently employed use moderating materials to reduce fission neutron energies to the thermal range. Light moderators (composed of light nuclei) are found to be more effective than heavy moderators because the light moderator removes more energy per collision than a heavy moderator. Therefore, the neutrons reach thermal energy more rapidly and they are less likely to be lost through resonance absorption.

As discussed in a previous module, the ability of a given material to slow down neutrons is referred to as the macroscopic slowing down power (MSDP) and is defined as the product of the logarithmic energy decrement per collision (ξ) times the macroscopic scattering cross section for neutrons as follows.

$$\mathrm{MSDP} = \xi \, \Sigma_s$$

Macroscopic slowing down power indicates how rapidly slowing down occurs in the material in question, but it does not completely define the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but is a poor moderator. It is a poor moderator because of its high probability of absorbing neutrons, and may be accounted for by dividing the macroscopic slowing down power by the macroscopic absorption cross section. This relationship is called the moderating ratio (MR).

$$\mathrm{MR} = \frac{\xi \, \Sigma_s}{\Sigma_a}$$

The moderating ratio is merely the ratio of slowing down power to the macroscopic absorption cross section. The higher the moderating ratio, the more effectively the material performs as a moderator.

Another ratio, the *moderator-to-fuel ratio* (N^m/N^u), is very important in the discussion of moderators. As the reactor designer increases the amount of moderator in the core (that is, N^m/N^u increases), neutron leakage decreases. Neutron absorption in the moderator (Σ_a^m) increases and causes a decrease in the thermal utilization factor. Having insufficient moderator in the core (that is, N^m/N^u decreases) causes an increase in slowing down time and results in a greater loss of neutrons by resonance absorption. This also causes an increase in neutron leakage. The effects of varying the moderator-to-fuel ratio on the thermal utilization factor and the resonance probability are shown in Figure 2.

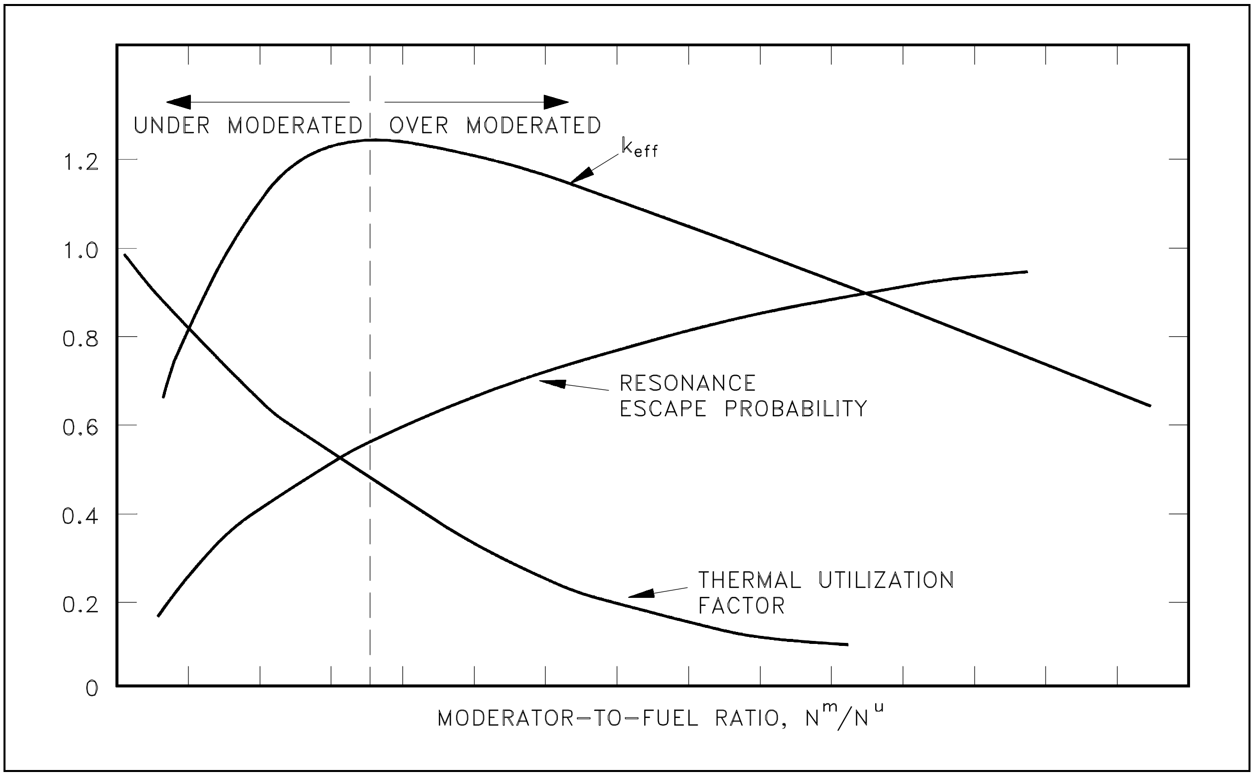

Figure 2 Effects of Over and Under Moderation on k_{eff}

Because the moderator-to-fuel ratio affects the thermal utilization factor and the resonance escape probability, it also affects k_{eff}. The remaining factors in the six factor formula are also affected by the moderator-to-fuel ratio, but to a lesser extent than f and p. As illustrated in Figure 2, which is applicable to a large core fueled with low-enriched fuel, there is an optimum point above which increasing the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the decreasing thermal utilization factor. Below this point, a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the increased resonance absorption in the fuel. If the ratio is above this point, the core is said to be over moderated, and if the ratio is below this point, the core is said to be under moderated.

In practice, water-moderated reactors are designed with a moderator-to-fuel ratio so that the reactor is operated in an under moderated condition. The reason that some reactors are designed to be under moderated is if the reactor were over moderated, an increase in temperature would decrease the N^m/N^u due to the expansion of the water as its density became lower. This decrease in N^m/N^u would be a positive reactivity addition, increasing k_{eff} and further raising power and temperature in a dangerous cycle. If the reactor is under moderated, the same increase in temperature results in the addition of negative reactivity, and the reactor becomes more self-regulating.

Moderator Temperature Coefficient

The change in reactivity per degree change in temperature is called the *temperature coefficient of reactivity*. Because different materials in the reactor have different reactivity changes with temperature and the various materials are at different temperatures during reactor operation, several different temperature coefficients are used. Usually, the two dominant temperature coefficients are the moderator temperature coefficient and the fuel temperature coefficient.

The change in reactivity per degree change in moderator temperature is called the *moderator temperature coefficient* of reactivity. The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of the moderator-to-fuel ratio. If a reactor is under moderated, it will have a negative moderator temperature coefficient. If a reactor is over moderated, it will have a positive moderator temperature coefficient. A negative moderator temperature coefficient is desirable because of its self-regulating effect. For example, an increase in reactivity causes the reactor to produce more power. This raises the temperature of the core and adds negative reactivity, which slows down, or turns, the power rise.

Fuel Temperature Coefficient

Another temperature coefficient of reactivity, the fuel temperature coefficient, has a greater effect than the moderator temperature coefficient for some reactors. The *fuel temperature coefficient* is the change in reactivity per degree change in fuel temperature. This coefficient is also called the "prompt" temperature coefficient because an increase in reactor power causes an immediate change in fuel temperature. A negative fuel temperature coefficient is generally considered to be even more important than a negative moderator temperature coefficient because fuel temperature immediately increases following an increase in reactor power. The time for heat to be transferred to the moderator is measured in seconds. In the event of a large positive reactivity insertion, the moderator temperature cannot turn the power rise for several seconds, whereas the fuel temperature coefficient starts adding negative reactivity immediately.

Another name applied to the fuel temperature coefficient of reactivity is the fuel doppler reactivity coefficient. This name is applied because in typical low enrichment, light water-moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the doppler effect, also called doppler broadening. The phenomenon of the doppler effect is caused by an apparent broadening of the resonances due to thermal motion of nuclei as illustrated in Figure 3. Stationary nuclei absorb only neutrons of energy E_o. If the nucleus is moving away from the neutron, the velocity (and energy) of the neutron must be greater than E_o to undergo resonance absorption. Likewise, if the nucleus is moving toward the neutron, the neutron needs less energy than E_o to be absorbed. Raising the temperature causes the nuclei to vibrate more rapidly within their lattice structures, effectively broadening the energy range of neutrons that may be resonantly absorbed in the fuel. Two nuclides present in large amounts in the fuel of some reactors with large resonant peaks that dominate the doppler effect are uranium-238 and plutonium-240.

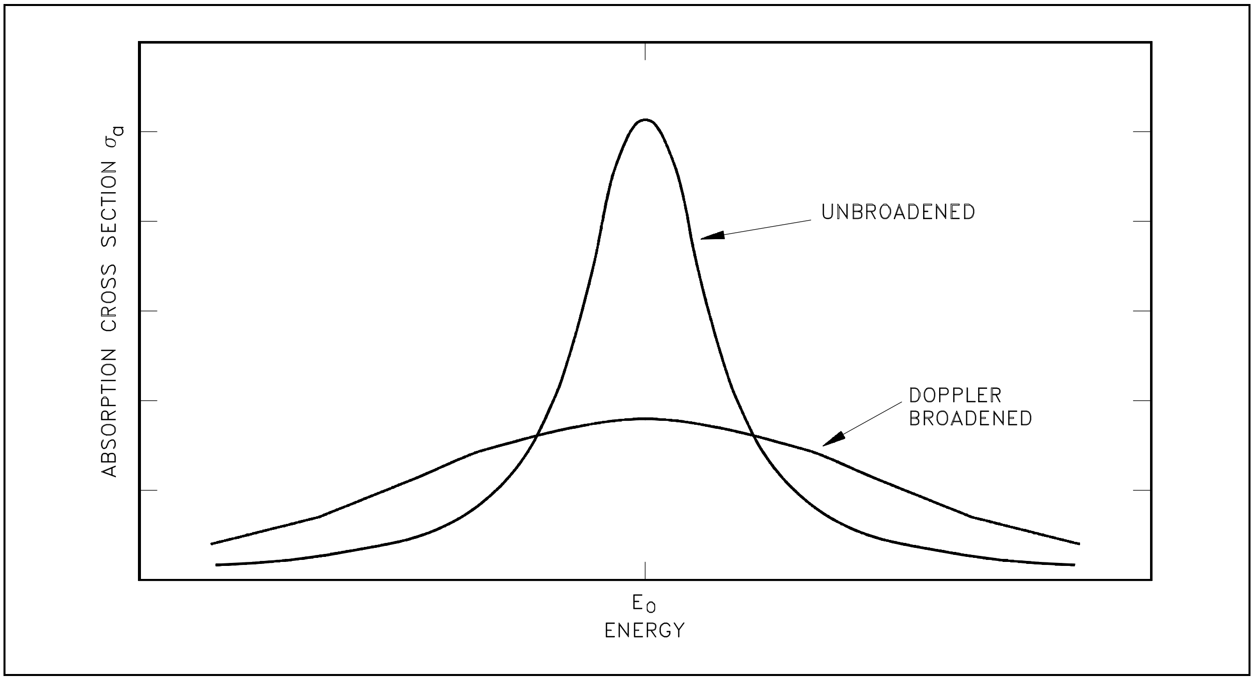

Figure 3 Effect of Fuel Temperature on Resonance Absorption Peaks

Pressure Coefficient

The reactivity in a reactor core can be affected by the system pressure. The *pressure coefficient* of reactivity is defined as the change in reactivity per unit change in pressure. The pressure coefficient of reactivity for the reactor is the result of the effect of pressure on the density of the moderator. For this reason, it is sometimes referred to as the moderator density reactivity coefficient. As pressure increases, density correspondingly increases, which increases the moderator-to-fuel ratio in the core. In the typical under moderated core the increase in the moderator-to-fuel ratio will result in a positive reactivity addition. In reactors that use water as a moderator, the absolute value of the pressure reactivity coefficient is seldom a major factor because it is very small compared to the moderator temperature coefficient of reactivity.

Void Coefficient

In systems with boiling conditions, such as boiling water reactors (BWR), the pressure coefficient becomes an important factor due to the larger density changes that occur when the vapor phase of water undergoes a pressure change. Of prime importance during operation of a BWR, and a factor in some other water-moderated reactors, is the void coefficient. The void coefficient is caused by the formation of steam voids in the moderator. The *void coefficient* of reactivity is defined as the change in reactivity per percent change in void volume. As the reactor power is raised to the point where the steam voids start to form, voids displace moderator from the coolant channels within the core. This displacement reduces the moderator-to-fuel ratio, and in an under moderated core, results in a negative reactivity addition, thereby limiting reactor power rise. The void coefficient is significant in water-moderated reactors that operate at or near saturated conditions.

Summary

The important information in this chapter is summarized below.

Reactivity Coefficients Summary

- The temperature coefficient of reactivity is the change in reactivity per degree change in temperature.
- A reactor is under moderated when a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the increased resonance absorption. A reactor is over moderated when an increase in the moderator-to-fuel ratio decreases k_{eff} due to the decrease in the thermal utilization factor.
- Reactors are usually designed to operate in an under moderated condition so that the moderator temperature coefficient of reactivity is negative.
- Increasing the moderator temperature will decrease the moderator-to-fuel ratio. Decreasing the moderator temperature will increase the moderator-to-fuel ratio.
- A negative temperature coefficient of reactivity is desirable because it makes the reactor more self-regulating. An increase in power, resulting in an increase in temperature, results in negative reactivity addition due to the temperature coefficient. The negative reactivity addition due to the temperature increase will slow or stop the power increase.
- The fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise because the fuel temperature immediately increases following a power increase, while the moderator temperature does not increase for several seconds.
- The Doppler broadening of resonance peaks occurs because the nuclei may be moving either toward or away from the neutron at the time of interaction. Therefore, the neutron may actually have either slightly more or slightly less than the resonant energy, but still appear to be at resonant energy relative to the nucleus.
- Uranium-238 and plutonium-240 are two nuclides present in some reactor fuels that have large resonance absorption peaks.

Reactivity Coefficients Summary (Cont.)

- The pressure coefficient of reactivity is the change in reactivity per unit change in pressure.
- The pressure coefficient of reactivity is usually negligible in reactors moderated by subcooled liquids because the density of the liquid does not change significantly within the operating pressure range.
- The void coefficient of reactivity is the change in reactivity per unit change in void volume.
- The void coefficient of reactivity becomes significant in a reactor in which the moderator is at or near saturated conditions.

NEUTRON POISONS

In some reactors, neutron-absorbing materials called poisons are intentionally designed into the reactor for specific purposes. Some of these poisons deplete as they absorb neutrons during reactor operation, and others remain relatively constant.

EO 3.1 DEFINE the following terms:

- **a. Burnable poison**
- **b. Non-burnable poison**
- **c. Chemical shim**

EO 3.2 EXPLAIN the use of burnable neutron poisons in a reactor core.

EO 3.3 LIST the advantages and disadvantages of chemical shim over fixed burnable poisons.

EO 3.4 STATE two reasons why fixed non-burnable neutron poisons are used in reactor cores.

EO 3.5 STATE an example of a material used as a fixed non-burnable neutron poison.

Fixed Burnable Poisons

During operation of a reactor the amount of fuel contained in the core constantly decreases. If the reactor is to operate for a long period of time, fuel in excess of that needed for exact criticality must be added when the reactor is built. The positive reactivity due to the excess fuel must be balanced with negative reactivity from neutron-absorbing material. Moveable control rods containing neutron-absorbing material are one method used to offset the excess fuel. Control rods will be discussed in detail in a later chapter. Using control rods alone to balance the excess reactivity may be undesirable or impractical for several reasons. One reason for a particular core design may be that there is physically insufficient room for the control rods and their large mechanisms.

To control large amounts of excess fuel without adding additional control rods, burnable poisons are loaded into the core. *Burnable poisons* are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as the result of neutron absorption. Due to the burnup of the poison material, the negative reactivity of the burnable poison decreases over core life. Ideally, these poisons should decrease their negative reactivity at the same rate the fuel's excess positive reactivity is depleted. Fixed burnable poisons are generally used in the form of compounds of boron or gadolinium that are shaped into separate lattice pins or plates, or introduced as additives to the fuel. Since they can usually be distributed more uniformly than control rods, these poisons are less disruptive to the core power distribution.

Soluble Poisons

Soluble poisons, also called *chemical shim*, produce a spatially uniform neutron absorption when dissolved in the water coolant. The most common soluble poison in commercial pressurized water reactors (PWR) is boric acid, which is often referred to as "soluble boron," or simply "solbor." The boric acid in the coolant decreases the thermal utilization factor, causing a decrease in reactivity. By varying the concentration of boric acid in the coolant (a process referred to as boration and dilution), the reactivity of the core can be easily varied. If the boron concentration is increased, the coolant/moderator absorbs more neutrons, adding negative reactivity. If the boron concentration is reduced (dilution), positive reactivity is added. The changing of boron concentration in a PWR is a slow process and is used primarily to compensate for fuel burnout or poison buildup. The variation in boron concentration allows control rod use to be minimized, which results in a flatter flux profile over the core than can be produced by rod insertion. The flatter flux profile is due to the fact that there are no regions of depressed flux like those that would be produced in the vicinity of inserted control rods.

DOE reactors typically do not use soluble neutron poisons during normal operation. Some DOE reactors do, however, include emergency shutdown systems that inject solutions containing neutron poisons into the system that circulates reactor coolant. Various solutions, including sodium polyborate and gadolinium nitrate, are used.

Fixed burnable poisons possess some advantages over chemical shim. Fixed burnable poisons may be discretely loaded in specific locations in order to shape or control flux profiles in the core. Also, fixed burnable poisons do not make the moderator temperature reactivity coefficient less negative as chemical shim does. With chemical shim, as temperature rises and the moderator expands, some moderator is pushed out of the active core area. Boron is also moved out, and this has a positive effect on reactivity. This property of chemical shim limits the allowable boron concentration because any greater concentration makes the moderator temperature coefficient of reactivity positive.

Non-Burnable Poisons

A *non-burnable poison* is one that maintains a constant negative reactivity worth over the life of the core. While no neutron poison is strictly non-burnable, certain materials can be treated as non-burnable poisons under certain conditions. One example is hafnium. The removal (by absorption of neutrons) of one isotope of hafnium leads to the production of another neutron absorber, and continues through a chain of five absorbers. This absorption chain results in a long-lived burnable poison which approximates non-burnable characteristics. Absorbers with low neutron absorption cross sections can also be treated as non-burnable under most conditions.

It is possible to make the reactivity of a poison material that is usually a burnable poison more uniform over core life through the use of self-shielding. In self-shielding, the poison material is thick enough that only the outer layer of the poison is exposed to the neutron flux. The absorptions that take place in the outer layers reduce the number of neutrons that penetrate to the inner material. As the outer layers of poison absorb neutrons and are converted to non-poison materials, the inner layers begin absorbing more neutrons, and the negative reactivity of the poison is fairly uniform.

The normal use of fixed non-burnable poisons is in power shaping, or to prevent excessive flux and power peaking near moderator regions of the reactor.

Summary

The important information in this chapter is summarized below.

Neutron Poisons Summary

- A burnable neutron poison is a material that has a high neutron absorption cross section that is converted into a material of relatively low absorption cross section as the result of neutron absorption.

- A non-burnable neutron poison is a material that has relatively constant neutron absorption characteristics over core life. The absorption of a neutron by one isotope in the material produces another isotope that also has a high absorption cross section.

- Chemical shim is a soluble neutron poison that is circulated in the coolant during normal operation.

- Burnable neutron poisons are used in reactor cores to compensate for the excess positive reactivity of the fuel when the reactor is initially started up.

- Chemical shim has several advantages over fixed burnable poisons.
  Has a spatially uniform effect
  Possible to increase or decrease amount of poison in the core during reactor operation

- Fixed burnable poisons have several advantages over chemical shim.
  Can be used to shape flux profiles
  Do not have an adverse effect on moderator temperature coefficient

- Two reasons for using non-burnable neutron poisons in reactor cores are to shape power and to prevent excessive flux and power peaking near moderator regions.

- An example of a material that is used as a fixed non-burnable neutron poison is hafnium.

XENON

Xenon-135 has a tremendous impact on the operation of a nuclear reactor. It is important to understand the mechanisms that produce and remove xenon from the reactor to predict how the reactor will respond following changes in power level.

EO 4.1 **LIST two methods of production and two methods of removal for xenon-135 during reactor operation.**

EO 4.2 **STATE the equation for equilibrium xenon-135 concentration.**

EO 4.3 **DESCRIBE how equilibrium xenon-135 concentration varies with reactor power level.**

EO 4.4 **DESCRIBE the causes and effects of a xenon oscillation.**

EO 4.5 **DESCRIBE how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.**

EO 4.6 **EXPLAIN the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.**

EO 4.7 **STATE the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."**

EO 4.8 **EXPLAIN what is meant by the following terms:**

a. Xenon precluded startup
b. Xenon dead time

EO 4.9 **DESCRIBE how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.**

Fission Product Poisons

Fission fragments generated at the time of fission decay to produce a variety of fission products. Fission products are of concern in reactors primarily because they become parasitic absorbers of neutrons and result in long term sources of heat. Although several fission products have significant neutron absorption cross sections, xenon-135 and samarium-149 have the most substantial impact on reactor design and operation. Because these two fission product poisons remove neutrons from the reactor, they will have an impact on the thermal utilization factor and thus k_{eff} and reactivity.

Production and Removal of Xenon-135

Xenon-135 has a 2.6×10^6 barns neutron absorption cross section. It is produced directly by some fissions, but is more commonly a product of the tellurium-135 decay chain shown below. The fission yield (γ) for xenon-135 is about 0.3%, while γ for tellurium-135 is about 6%.

$$ {}^{135}_{52}\text{Te} \xrightarrow[19.0\ \text{sec}]{\beta^-} {}^{135}_{53}\text{I} \xrightarrow[6.57\ \text{hr}]{\beta^-} {}^{135}_{54}\text{Xe} \xrightarrow[9.10\ \text{hr}]{\beta^-} {}^{135}_{55}\text{Cs} \xrightarrow[2.3\text{x}10^{6}\ \text{yr}]{\beta^-} {}^{135}_{56}\text{Ba}\quad \text{(stable)} $$

The half-life for tellurium-135 is so short compared to the other half-lives that it can be assumed that iodine-135 is produced directly from fission. Iodine-135 is not a strong neutron absorber, but decays to form the neutron poison xenon-135. Ninety-five percent of all the xenon-135 produced comes from the decay of iodine-135. Therefore, the half-life of iodine-135 plays an important role in the amount of xenon-135 present.

The rate of change of iodine concentration is equal to the rate of production minus the rate of removal. This can be expressed in the equation below.

rate of change of iodine concentration = yield from fission - decay rate - burnup rate

or

$$ \frac{dN_I}{dt} = \gamma_I\, \Sigma_f^{fuel}\, \phi - \lambda_I\, N_I - \sigma_a^I\, N_I\, \phi $$

where:

N_I = ^{135}I concentration

γ_I = fission yield of ^{135}I

Σ_f^{fuel} = macroscopic fission cross section fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

σ_a^I = microscopic absorption cross section ^{135}I

Since the σ_a^I is very small, the burn up rate term may be ignored, and the expression for the rate of change of iodine concentration is modified as shown below.

$$ \frac{dN_I}{dt} = \gamma_I\, \Sigma_f^{fuel}\, \phi - \lambda_I\, N_I $$

When the rate of production of iodine equals the rate of removal of iodine, equilibrium exists. The iodine concentration remains constant and is designated N_I(eq). The following equation for the equilibrium concentration of iodine can be determined from the preceding equation by setting the two terms equal to each other and solving for N_I(eq).

$$N_I(eq) = \frac{\gamma_I \, \Sigma_f^{fuel} \, \phi}{\lambda_I}$$

Since the equilibrium iodine concentration is proportional to the fission reaction rate, it is also proportional to reactor power level.

The rate of change of the xenon concentration is equal to the rate of production minus the rate of removal. Recall that 5% of xenon comes directly from fission and 95% comes from the decay of iodine. The rate of change of xenon concentration is expressed by the following equations.

$$\begin{matrix}\text{rate of change of}\\ \text{xenon}-135\text{ concentration}\end{matrix} = \begin{pmatrix}\text{xenon}-135\text{ yield}\\ \text{from fission}\end{pmatrix} + \begin{pmatrix}\text{iodine}-135\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon}-135\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon}-135\\ \text{burnup}\end{pmatrix}$$

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} \, \Sigma_f^{fuel} \, \phi + \lambda_I \, N_I - \lambda_{Xe} \, N_{Xe} - \sigma_a^{Xe} \, N_{Xe} \, \phi$$

where:

N_{Xe} = ^{135}Xe concentration

γ_{Xe} = fission yield of ^{135}Xe

Σ_f^{fuel} = macroscopic fission cross section of the fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

N_I = ^{135}I concentration

λ_{Xe} = decay constant for ^{135}Xe

σ_a^{Xe} = microscopic absorption cross section^{135}Xe

The xenon burnup term above refers to neutron absorption by xenon-135 by the following reaction.

$$^{135}_{54}Xe + ^{1}_{0}n \rightarrow ^{136}_{54}Xe + \gamma$$

Xenon-136 is not a significant neutron absorber; therefore, the neutron absorption by xenon-135 constitutes removal of poison from the reactor. The burnup rate of xenon-135 is dependent upon the neutron flux and the xenon-135 concentration.

The equilibrium concentration of xenon-135 is designated N_{Xe}(eq), and is represented as shown below.

$$N_{Xe} \, (eq) = \frac{\gamma_{Xe} \, \Sigma_f^{fuel} \, \phi + \lambda_I \, N_I}{\lambda_{Xe} + \sigma_a^{Xe} \, \phi}$$

For xenon-135 to be in equilibrium, iodine-135 must also be in equilibrium. Substituting the expression for equilibrium iodine-135 concentration into the equation for equilibrium xenon results in the following.

$$N_{Xe}\ (eq) = \frac{\left(\gamma_{Xe} + \gamma_{I}\right) \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

From this equation it can be seen that the equilibrium value for xenon-135 increases as power increases, because the numerator is proportional to the fission reaction rate. Thermal flux is also in the denominator; therefore, as the thermal flux exceeds 10^{12} neutrons/cm^2-sec, the $\sigma_a^{Xe}\phi$ term begins to dominate, and at approximately 10^{15} neutrons/cm^2-sec, the xenon-135 concentration approaches a limiting value. The equilibrium iodine-135 and xenon-135 concentrations as a function of neutron flux are illustrated in Figure 4.

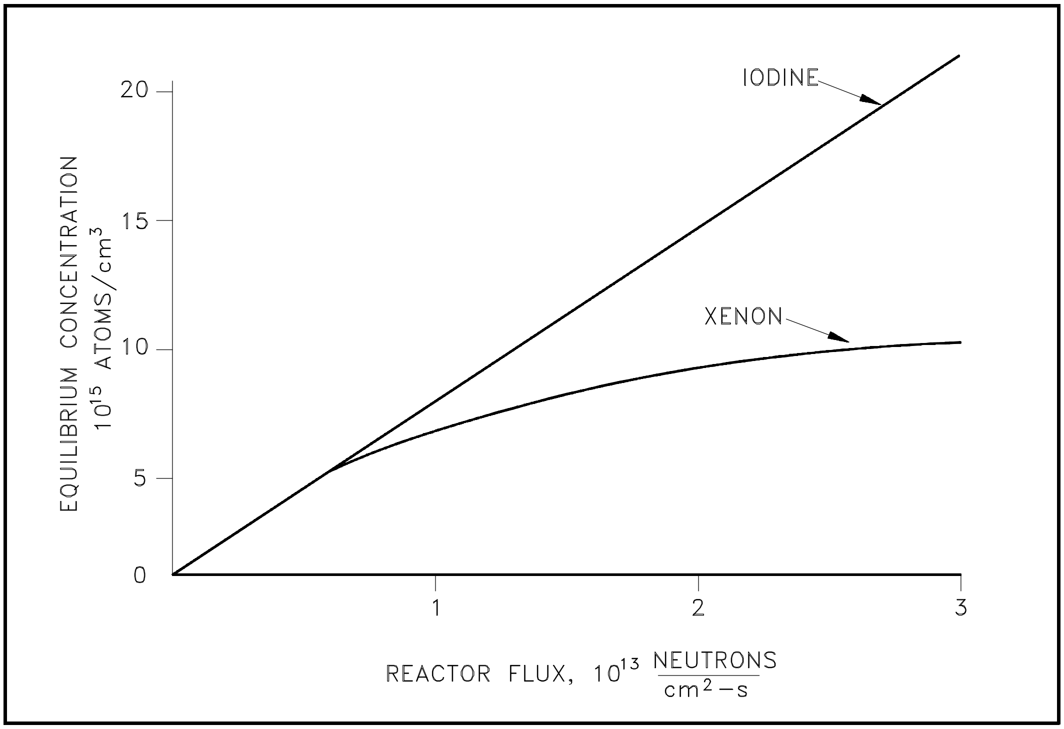

Figure 4 Equilibrium Iodine-135 and Xenon-135 Concentrations Versus Neutron Flux

The higher the power level, or flux, the higher the equilibrium xenon-135 concentration, but equilibrium xenon-135 is not directly proportional to power level. For example, equilibrium xenon-135 at 25% power is more than half the value for equilibrium xenon-135 at 100% power for many reactors. Because the xenon-135 concentration directly affects the reactivity level in the reactor core, the negative reactivity due to the xenon concentrations for different power levels or conditions are frequently plotted instead of the xenon concentration.

Xenon-135 Response to Reactor Shutdown

When a reactor is shutdown, the neutron flux is reduced essentially to zero. Therefore, after shutdown, xenon-135 is no longer produced by fission and is no longer removed by burnup. The only remaining production mechanism is the decay of the iodine-135 which was in the core at the time of shutdown. The only removal mechanism for xenon-135 is decay.

$$\frac{dN_{Xe}}{dt} = \lambda_I\ N_I - \lambda_{Xe}\ N_{Xe}$$

Because the decay rate of iodine-135 is faster than the decay rate of xenon-135, the xenon concentration builds to a peak. The peak is reached when the product of the terms $\lambda_I N_I$ is equal to $\lambda_{Xe} N_{Xe}$ (in about 10 to 11 hours). Subsequently, the production from iodine decay is less than the removal of xenon by decay, and the concentration of xenon-135 decreases. The greater the flux level prior to shutdown, the greater the concentration of iodine-135 at shutdown; therefore, the greater the peak in xenon-135 concentration after shutdown. This phenomenon can be seen in Figure 5, which illustrates the negative reactivity value of xenon-135 following shutdown from various neutron flux levels.

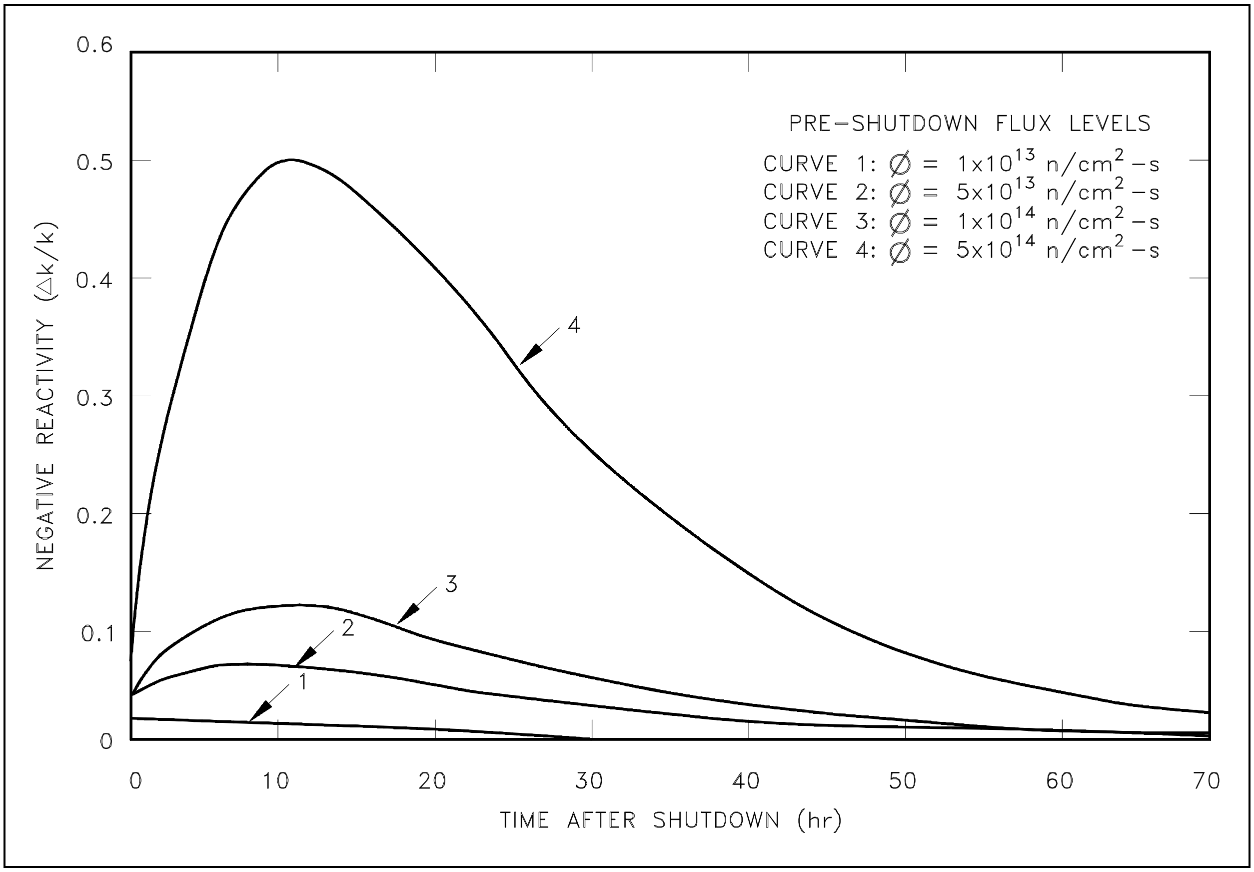

Figure 5 Xenon-135 Reactivity After Reactor Shutdown

Negative xenon reactivity, also called xenon poisoning, may provide sufficient negative reactivity to make the reactor inoperable because there is insufficient positive reactivity available from control rod removal or chemical shim dilution (if used) to counteract it. The inability of the reactor to be started due to the effects of xenon is sometimes referred to as a *xenon precluded startup*. The period of time where the reactor is unable to "override" the effects of xenon is called *xenon dead time*. Because the amount of excess core reactivity available to override the negative reactivity of the xenon is usually less than 10% $\Delta k/k$, thermal power reactors are normally limited to flux levels of about 5×10^{13} neutrons/cm^2-sec so that timely restart can be ensured after shutdown. For reactors with very low thermal flux levels (~5×10^{12} neutrons/cm^2-sec or less), most xenon is removed by decay as opposed to neutron absorption. For these cases, reactor shutdown does not cause any xenon-135 peaking effect.

Following the peak in xenon-135 concentration about 10 hours after shutdown, the xenon-135 concentration will decrease at a rate controlled by the decay of iodine-135 into xenon-135 and the decay rate of xenon-135. For some reactors, the xenon-135 concentration about 20 hours after shutdown from full power will be the same as the equilibrium xenon-135 concentration at full power. About 3 days after shutdown, the xenon-135 concentration will have decreased to a small percentage of its pre-shutdown level, and the reactor can be assumed to be xenon free without a significant error introduced into reactivity calculations.

Xenon-135 Oscillations

Large thermal reactors with little flux coupling between regions may experience spatial power oscillations because of the non-uniform presence of xenon-135. The mechanism is described in the following four steps.

(1) An initial lack of symmetry in the core power distribution (for example, individual control rod movement or misalignment) causes an imbalance in fission rates within the reactor core, and therefore, in the iodine-135 buildup and the xenon-135 absorption.

(2) In the high-flux region, xenon-135 burnout allows the flux to increase further, while in the low-flux region, the increase in xenon-135 causes a further reduction in flux. The iodine concentration increases where the flux is high and decreases where the flux is low.

(3) As soon as the iodine-135 levels build up sufficiently, decay to xenon reverses the initial situation. Flux decreases in this area, and the former low-flux region increases in power.

(4) Repetition of these patterns can lead to xenon oscillations moving about the core with periods on the order of about 15 hours.

With little change in overall power level, these oscillations can change the local power levels by a factor of three or more. In a reactor system with strongly negative temperature coefficients, the xenon-135 oscillations are damped quite readily. This is one reason for designing reactors to have negative moderator-temperature coefficients.

Xenon-135 Response to Reactor Power Changes

During periods of steady state operation, at a constant neutron flux level, the xenon-135 concentration builds up to its equilibrium value for that reactor power in about 40 to 50 hours. Figure 6 illustrates a typical xenon transient that occurs as a result of a change in reactor power level. At time zero, reactor power is raised from 50% power to 100% power. When the reactor power is increased, xenon concentration initially decreases because the burnup is increased at the new higher power level. Because 95% of the xenon production is from iodine-135 decay, which has a 6 to 7 hour half-life, the production of xenon remains constant for several hours. After a few hours (roughly 4 to 6 hours depending on power levels) the rate of production of xenon from iodine and fission equals the rate of removal of xenon by burnup and decay. At this point, the xenon concentration reaches a minimum. The xenon concentration then increases to the new equilibrium level for the new power level in roughly 40 to 50 hours. It should be noted that the magnitude and the rate of change of xenon concentration during the initial 4 to 6 hours following the power change is dependent upon the initial power level and on the amount of change in power level. The xenon concentration change is greater for a larger change in power level.

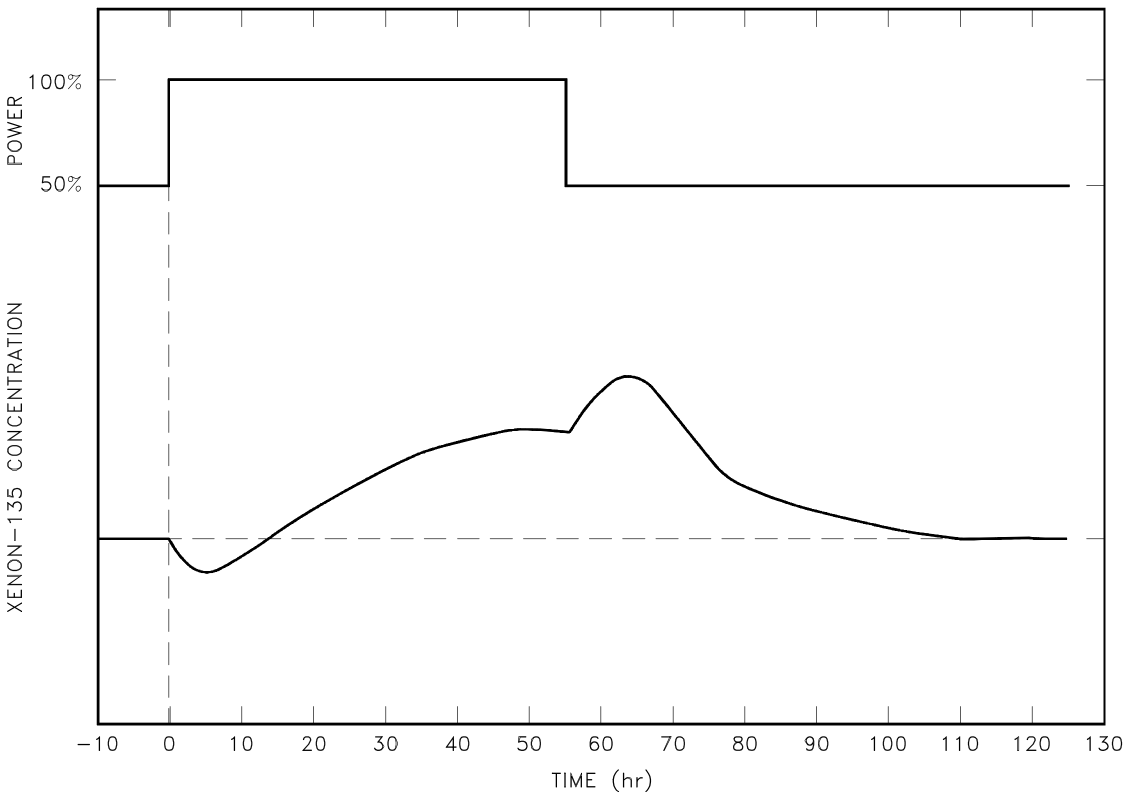

Figure 6 Xenon-135 Variations During Power Changes

When reactor power is decreased from 100% to 50% power (t = 55 hours), the process is reversed. There is an immediate decrease in xenon burnup, which results in an increase in xenon-135 concentration. The iodine-135 concentration is still at the higher equilibrium level for 100% power and is therefore still producing xenon-135 at the higher rate. The xenon-135 concentration continues to rise until the rate of production of xenon-135 becomes equal to the rate of removal (roughly 7 to 8 hours after the initial reduction in power level). The xenon-135 concentration then gradually decreases to the new equilibrium level in about 50 to 60 hours. The magnitude of the xenon peak is greatest if the initial power level is very high.

Maximum peak xenon occurs when a reactor that is operating at 100% equilibrium xenon concentration is suddenly shut down. The most rapid possible burnout of xenon occurs when a reactor is started up and operated at full power while this maximum peak xenon condition exists.

Summary

The important information in this chapter is summarized below.

Xenon Summary

- Xenon-135 is produced directly as a fission product and by the decay of iodine-135 during reactor operation. Xenon-135 is removed from the core by radioactive decay and by neutron absorption during reactor operation.

- The equilibrium concentration for xenon-135 is determined by the following equation.

$$N_{Xe}\ (eq) = \frac{\gamma_{Xe}\ \Sigma_f^{fuel}\ \phi + \lambda_I\ N_I}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi} \quad \text{or} \quad N_{Xe}\ (eq) = \frac{\left(\gamma_{Xe} + \gamma_I\right)\ \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

- The xenon-135 concentration increases with increasing power level in a non-linear manner. Equilibrium xenon-135 concentration reaches a maximum at a flux of about 10^{15} neutrons/cm^2-sec.

- After a power increase, xenon-135 concentration will initially decrease due to the increased removal by burnout. Xenon-135 will reach a minimum about 5 hours after the power increase and then increase to a new, higher equilibrium value as the production from iodine decay increases.

Xenon Summary (Cont.)

- A xenon-135 oscillation may be caused by a rapid perturbation in the core power distribution. The xenon-135 oscillation can change local power levels in the core by a factor of three or more.
- Following a reactor shutdown, xenon-135 concentration will increase due to the decay of the iodine inventory of the core. Xenon-135 will peak approximately 10 hours after the shutdown (from 100%) and then decrease as xenon-135 decay becomes greater than the iodine-135 decay.
- The greater the pre-shutdown power level, the greater the peak value of xenon.
- The core can be considered xenon-free about 3 days after shutdown.
- A xenon precluded startup occurs when there is insufficient reactivity in the control rods to overcome the negative reactivity of xenon-135.
- Xenon dead time is the period of time where the reactor is unable to override the effects of xenon.
- After a power decrease, xenon-135 concentration will initially increase due to production by iodine decay being greater than the burnout. Xenon-135 will reach a maximum about 8 hours after the power decrease and then decrease to a new, lower equilibrium value.

SAMARIUM AND OTHER FISSION PRODUCT POISONS

The fission product poison that has the most significant effect on reactor operations other than xenon-135 is samarium-149. Samarium-149 behaves significantly different from xenon-135 due to its different nuclear properties.

EO 4.10 DESCRIBE how samarium-149 is produced and removed from the reactor core during reactor operation.

EO 4.11 STATE the equation for equilibrium samarium-149 concentration.

EO 4.12 DESCRIBE how equilibrium samarium-149 concentration varies with reactor power level.

EO 4.13 DESCRIBE how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

EO 4.14 DESCRIBE how samarium-149 concentration changes following a reactor startup.

EO 4.15 STATE the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

Production and Removal of Samarium-149

Samarium-149 is the second most important fission-product poison because of its high thermal neutron absorption cross section of 4.1×10^4 barns. Samarium-149 is produced from the decay of the neodymium-149 fission fragment as shown in the decay chain below.

$${}^{149}_{60}\text{Nd} \xrightarrow[1.72\ \text{hr}]{\beta^-} {}^{149}_{61}\text{Pm} \xrightarrow[53.1\ \text{hr}]{\beta^-} {}^{149}_{62}\text{Sm} \quad \text{(stable)}$$

For the purpose of examining the behavior of samarium-149, the 1.73 hour half-life of neodymium-149 is sufficiently shorter than the 53.1 hour value for promethium-149 that the promethium-149 may be considered as if it were formed directly from fission. This assumption, and neglecting the small amount of promethium burnup, allows the situation to be described as follows.

Rate of change of ^{149}Pm = yield from fission - decay ^{149}Pm concentration

therefore:

$$\frac{dN_{Pm}}{dt} = \gamma_{Pm}\ \Sigma_f^{fuel}\ \phi - \lambda_{Pm}\ N_{Pm}$$

where:

N_{Pm} = ^{149}Pm concentration

γ_{Pm} = ^{149}Pm fission yield

λ_{Pm} = decay constant for ^{149}Pm

Solving for the equilibrium value of promethium-149 gives the following.

$$N_{Pm}(eq) = \frac{\gamma_{Pm}\ \Sigma_f^{fuel}\ \phi}{\lambda_{Pm}}$$

The rate of samarium-149 formation is described as follows.

Rate of change of ^{149}Sm = yield from fission + ^{149}Pm decay - ^{149}Sm burnup

therefore:

$$\frac{dN_{Sm}}{dt} = \gamma_{Sm}\ \Sigma_f^{fuel}\ \phi + \lambda_{Pm}\ N_{Pm} - N_{Sm}\ \sigma_a^{Sm}\ \phi$$

where:

N_{Sm} = ^{149}Sm concentration

γ_{Sm} = ^{149}Sm fission yield

σ_a^{Sm} = microscopic absorption cross section of ^{149}Sm

The fission yield of samarium-149, however, is nearly zero; therefore, the equation becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} \ N_{Pm} - N_{Sm} \ \sigma_a^{Sm} \ \phi$$

Solving this equation for the equilibrium concentration of samarium-149 and substituting $\gamma_{Pm}\Sigma_f^{fuel} \ \phi \ / \ \lambda_{Pm}$ for $N_{Pm}(eq)$ yields the following.

$$N_{Sm}(eq) = \frac{\gamma_{Pm} \ \Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

This expression for equilibrium samarium-149 concentration during reactor operation illustrates that equilibrium samarium-149 concentration is independent of neutron flux and power level. The samarium concentration will undergo a transient following a power level change, but it will return to its original value.

Samarium-149 Response to Reactor Shutdown

Since the neutron flux drops to essentially zero after reactor shutdown, the rate of samarium-149 production becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} \ N_{Pm}$$

Because samarium-149 is not radioactive and is not removed by decay, it presents problems somewhat different from those encountered with xenon-135, as illustrated in Figure 7. The equilibrium concentration and the poisoning effect build to an equilibrium value during reactor operation. This equilibrium is reached in approximately 20 days (500 hours), and since samarium-149 is stable, the concentration remains essentially constant during reactor operation. When the reactor is shutdown, the samarium-149 concentration builds up as a result of the decay of the accumulated promethium-149. The buildup of samarium-149 after shutdown depends upon the power level before shutdown. Samarium-149 does not peak as xenon-135 does, but increases slowly to a maximum value as shown in Figure 7. After shutdown, if the reactor is then operated at power, samarium-149 is burned up and its concentration returns to the equilibrium value. Samarium poisoning is minor when compared to xenon poisoning. Although samarium-149 has a constant poisoning effect during long-term sustained operation, its behavior during initial startup and during post-shutdown and restart periods requires special considerations in reactor design.

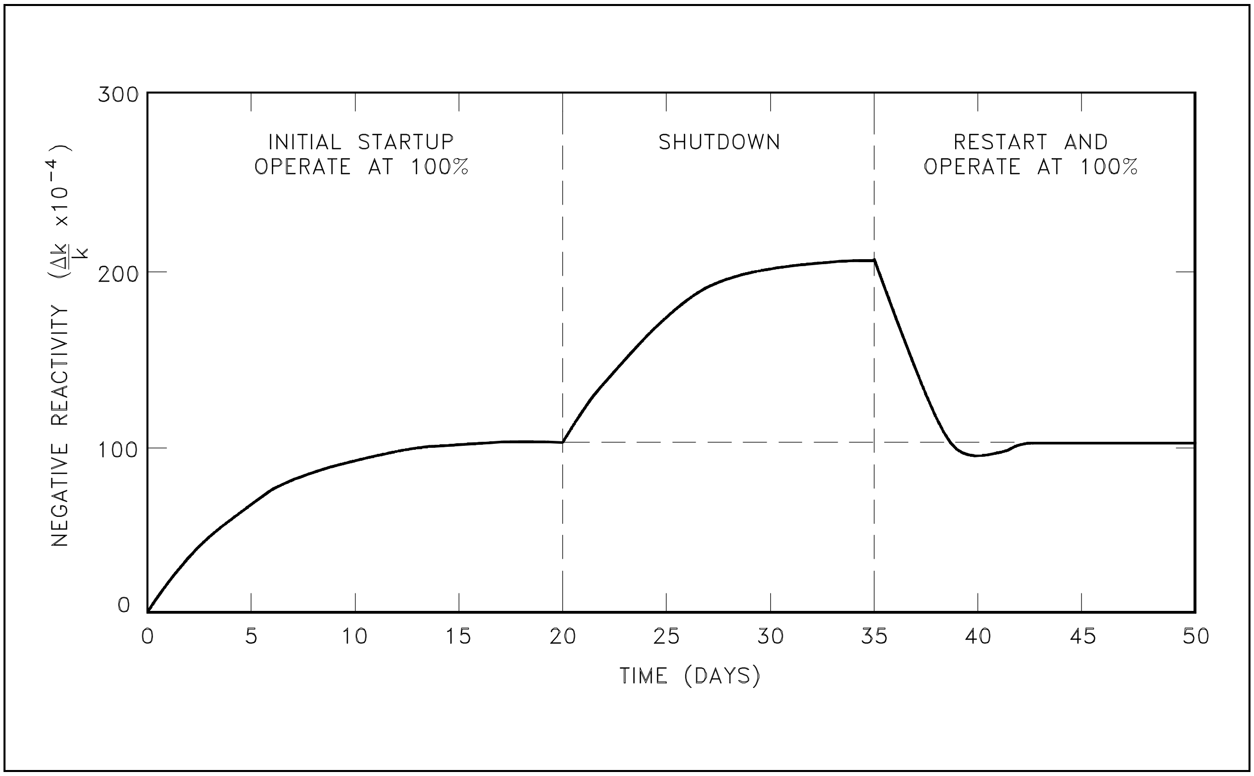

Figure 7 Behavior of Samarium-149 in a Typical Light Water Reactor

The xenon-135 and samarium-149 mechanisms are dependent on their very large thermal neutron cross sections and only affect thermal reactor systems. In fast reactors, neither these nor any other fission products have a major poisoning influence.

Other Neutron Poisons

There are numerous other fission products that, as a result of their concentration and thermal neutron absorption cross section, have a poisoning effect on reactor operation. Individually, they are of little consequence, but "lumped" together they have a significant impact. These are often characterized as "lumped fission product poisons" and accumulate at an average rate of 50 barns per fission event in the reactor.

In addition to fission product poisons, other materials in the reactor decay to materials that act as neutron poisons. An example of this is the decay of tritium to helium-3. Since tritium has a half-life of 12.3 years, normally this decay does not significantly affect reactor operations because the rate of decay of tritium is so slow. However, if tritium is produced in a reactor and then allowed to remain in the reactor during a prolonged shutdown of several months, a sufficient amount of tritium may decay to helium-3 to add a significant amount of negative reactivity. Any helium-3 produced in the reactor during a shutdown period will be removed during subsequent operation by a neutron-proton reaction.

Summary

The important information in this chapter is summarized below.

Samarium and Other Fission Product Poisons Summary

- Samarium-149 is produced directly from fission and from the decay of promethium-149 during reactor operation. Samarium-149 is removed from the core by neutron absorption.
- The equation for equilibrium samarium-149 concentration is stated below.

$$N_{Sm}(eq) = \frac{\gamma_{Pm} \Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

- The equilibrium samarium-149 concentration is independent of power level.
- Following a reactor shutdown, the samarium-149 concentration increases due to the decay of the promethium-149 inventory of the core and the loss of the burnup factor.
- If the reactor is restarted following a shutdown, the samarium-149 concentration decreases as samarium is burned up and returns to its equilibrium operating value.
- Helium-3 will become a significant neutron poison if significant amounts of tritium are left in a reactor during a shutdown period that lasts longer than several months.

CONTROL RODS

Most reactors contain control rods made of neutron absorbing materials that are used to adjust the reactivity of the core. Control rods can be designed and used for coarse control, fine control, or fast shutdowns.

EO 5.1 DESCRIBE the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

EO 5.2 EXPLAIN why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

EO 5.3 EXPLAIN why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

EO 5.4 DEFINE the following terms:

- **a. Integral control rod worth**
- **b. Differential control rod worth**

EO 5.5 DESCRIBE the shape of a typical differential control rod worth curve and explain the reason for the shape.

EO 5.6 DESCRIBE the shape of a typical integral control rod worth curve and explain the reason for the shape.

EO 5.7 Given an integral or differential control rod worth curve, CALCULATE the reactivity change due to a control rod movement between two positions.

EO 5.8 Given differential control rod worth data, PLOT differential and integral control rod worth curves.

Selection of Control Rod Materials

Rods of neutron-absorbing material are installed in most reactors to provide precise, adjustable control of reactivity. These rods are able to be moved into or out of the reactor core and typically contain elements such as silver, indium, cadmium, boron, or hafnium.

The material used for the control rods varies depending on reactor design. Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly). The ability of a control rod to absorb neutrons can be adjusted during manufacture. A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them. While it takes more grey rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause smaller depressions in the neutron flux and power in the vicinity of the rod. This leads to a flatter neutron flux profile and more even power distribution in the core.

If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited. Material with a very high absorption cross section may not be desired for use in a control rod, because it will burn out rapidly due to its high absorption cross section. The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material. This also results in a rod that does not burn out as rapidly.

Another factor in control rod material selection is that materials that resonantly absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections. Resonance neutron absorbers absorb neutrons in the epithermal energy range. The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons. Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber. This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile.

Types of Control Rods

There are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Three purposes of control rods are listed below.

Shim rods	-	used for coarse control and/or to remove reactivity in relatively large amounts.
Regulating rods	-	used for fine adjustments and to maintain desired power or temperature.
Safety rods	-	provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip."

Not all reactors have different control rods to serve the purposes mentioned above. Depending upon the type of reactor and the controls necessary, it is possible to use dual-purpose or even triple-purpose rods. For example, consider a set of control rods that can insert enough reactivity to be used as shim rods. If the same rods can be operated at slow speeds, they will function as regulating rods. Additionally, these same rods can be designed for rapid insertion, or scram. These rods serve a triple function yet meet other specifications such as precise control, range of control, and efficiency.

Control Rod Effectiveness

The effectiveness of a control rod depends largely upon the value of the ratio of the neutron flux at the location of the rod to the average neutron flux in the reactor. The control rod has maximum effect (inserts the most negative reactivity) if it is placed in the reactor where the flux is maximum. If a reactor has only one control rod, the rod should be placed in the center of the reactor core. The effect of such a rod on the flux is illustrated in Figure 8.

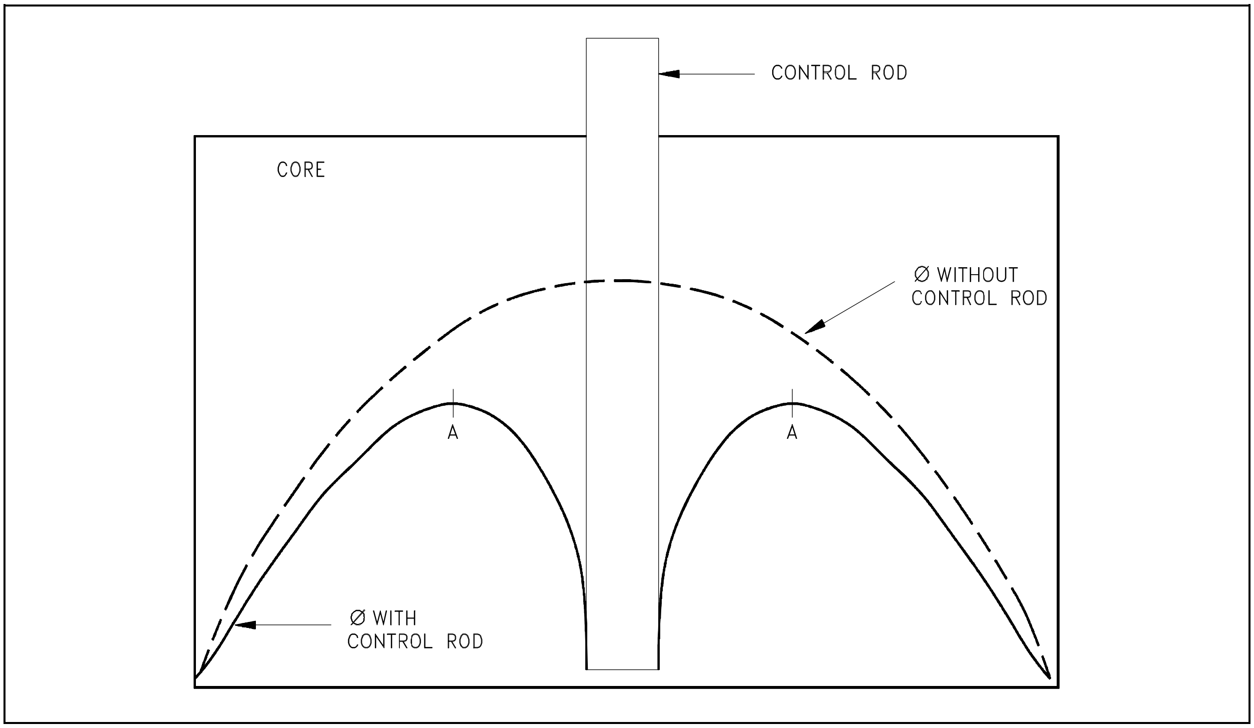

Figure 8 Effect of Control Rod on Radial Flux Distribution

If additional rods are added to this simple reactor, the most effective location is where the flux is maximum, that is, at point A. Numerous control rods are required for a reactor that has a large amount of excess reactivity (that amount of reactivity in excess of that needed to be critical). The exact amount of reactivity that each control rod inserts depends upon the reactor design. The change in reactivity caused by control rod motion is referred to as control rod worth.

Integral and Differential Control Rod Worth

The exact effect of control rods on reactivity can be determined experimentally. For example, a control rod can be withdrawn in small increments, such as 0.5 inch, and the change in reactivity can be determined following each increment of withdrawal. By plotting the resulting reactivity versus the rod position, a graph similar to Figure 9 is obtained. The graph depicts integral control rod worth over the full range of withdrawal. The *integral control rod worth* is the total reactivity worth of the rod at that particular degree of withdrawal and is usually defined to be the greatest when the rod is fully withdrawn.

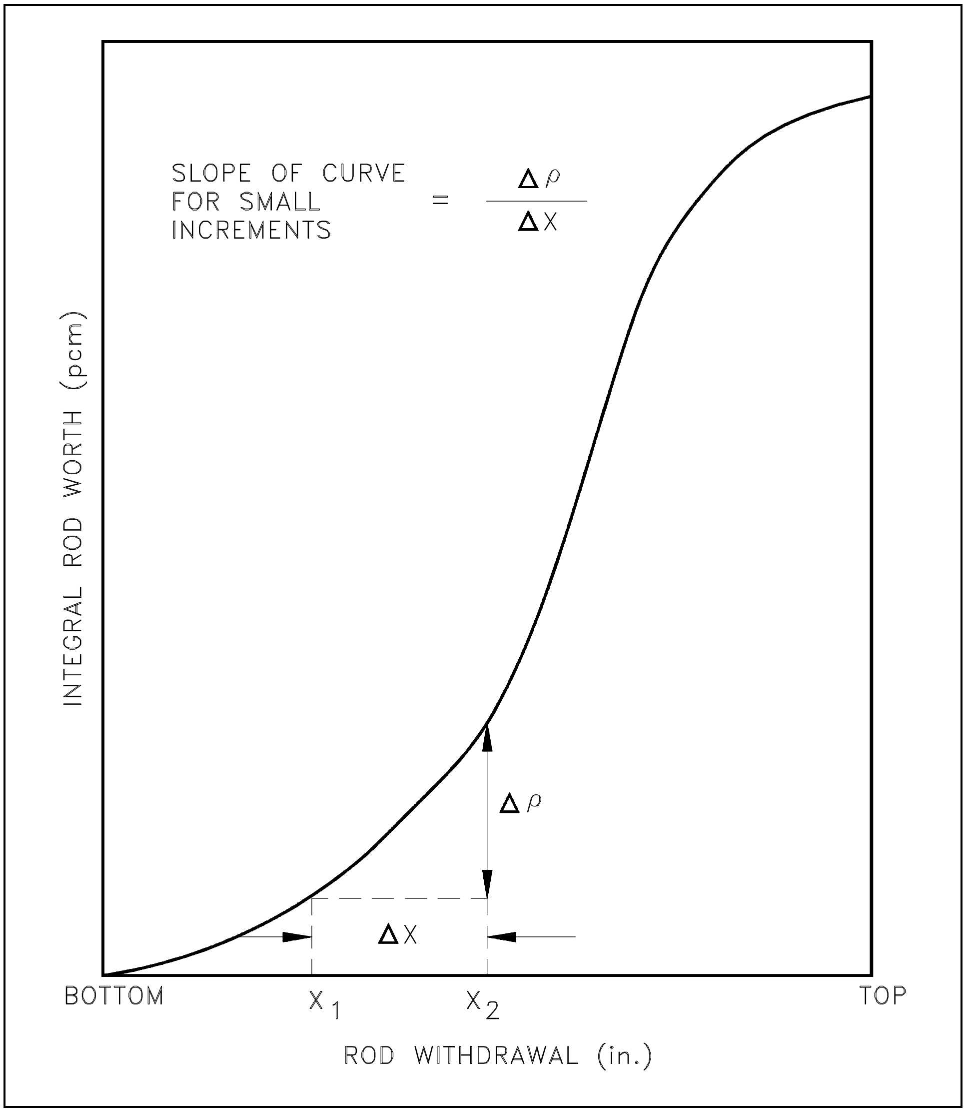

Figure 9 Integral Control Rod Worth

The slope of the curve ($\Delta\rho/\Delta x$), and therefore the amount of reactivity inserted per unit of withdrawal, is greatest when the control rod is midway out of the core. This occurs because the area of greatest neutron flux is near the center of the core; therefore, the amount of change in neutron absorption is greatest in this area. If the slope of the curve for integral rod worth in Figure 9 is taken, the result is a value for rate of change of control rod worth as a function of control rod position. A plot of the slope of the integral rod worth curve, also called the differential control rod worth, is shown in Figure 10. At the bottom of the core, where there are few neutrons, rod movement has little effect so the change in rod worth per inch varies little. As the rod approaches the center of the core its effect becomes greater, and the change in rod worth per inch is greater. At the center of the core the differential rod worth is greatest and varies little with rod motion. From the center of the core to the top, the rod worth per inch is basically the inverse of the rod worth per inch from the center to the bottom.

Differential control rod worth is the reactivity change per unit movement of a rod and is normally expressed as ρ/inch, $\Delta k/k$ per inch, or pcm/inch. The integral rod worth at a given withdrawal is merely the summation of all the differential rod worths up to that point of withdrawal. It is also the area under the differential rod worth curve at any given withdrawal position.

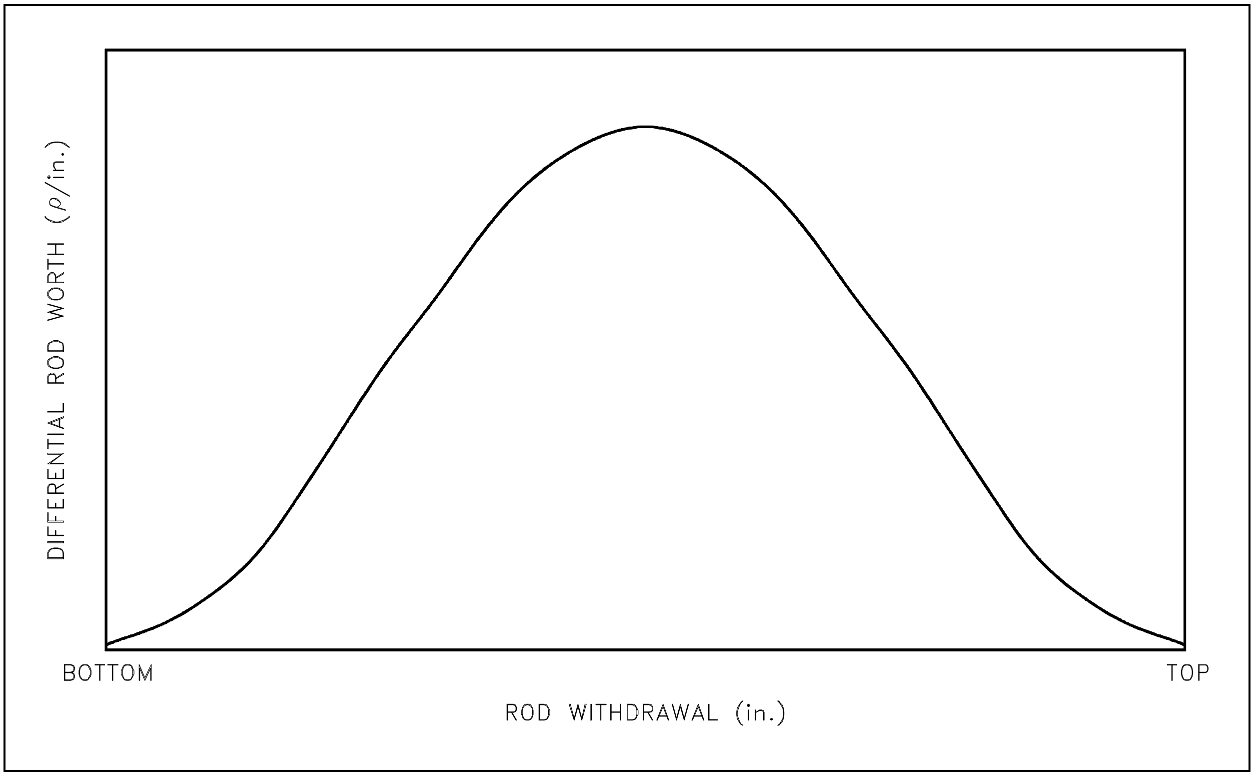

Figure 10 Differential Control Rod Worth

The following exercises are intended to reinforce an understanding of the concepts of integral and differential rod worth.

Example 1:

Using the integral rod worth curve provided in Figure 11, find the reactivity inserted by moving the rod from 12 inches withdrawn out to 18 inches withdrawn.

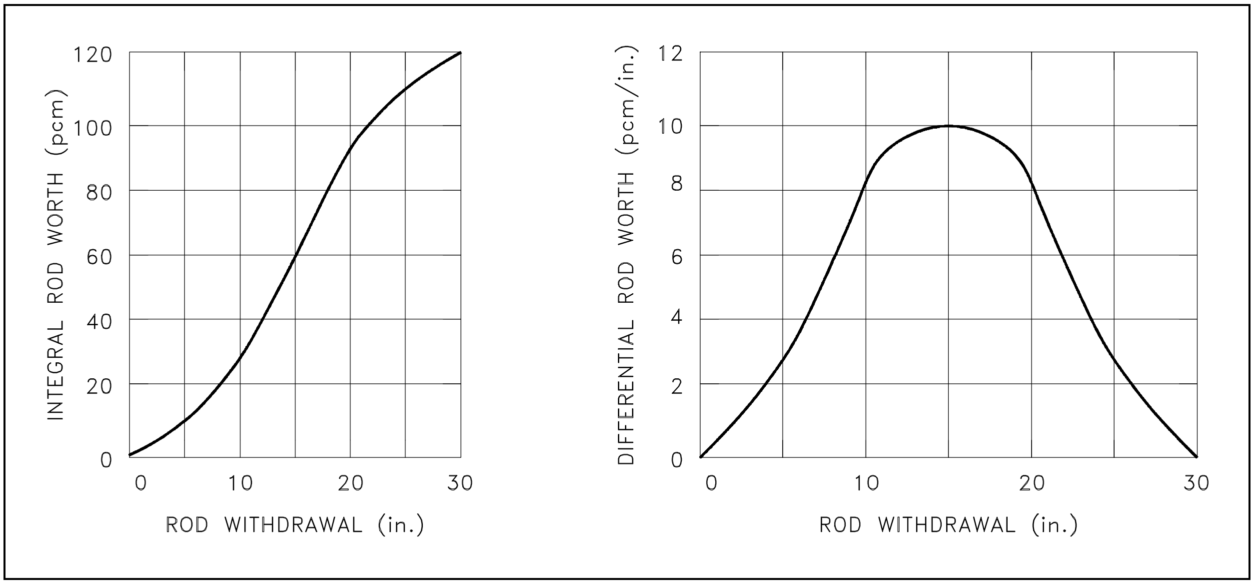

Figure 11 Rod Worth Curves for Example Problems

Solution:

The integral rod worth at 12 inches is 40 pcm and the integral rod worth at 18 inches is 80 pcm.

$$\begin{aligned}\Delta\rho &= \rho_{\text{final}} - \rho_{\text{initial}} \\ &= \rho_{18} - \rho_{12} \\ &= 80\ \text{pcm} - 40\ \text{pcm} \\ &= 40\ \text{pcm}\end{aligned}$$

Example 2:

Using the differential rod worth curve provided in Figure 11, calculate the reactivity inserted by moving the rod from 10 inches withdrawn to 6 inches withdrawn.

Solution:

The solution is basically given by the area under the curve for the interval. The answers obtained in the following approximation may vary slightly depending upon the degree of approximation.

Method 1. Treating the range from 10 inches to 6 inches as a trapezoid, that is, taking the end values of pcm/inch and multiplying their average by the 4 inches moved yields the following.

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)\left(4\ \text{inches}\right) = -22\ \text{pcm}$$

This is negative because the rod was inserted.

Method 2. Using the central value of rod position at 8 inches yields an average rod worth of 5.5 pcm/inch. Multiplying by the 4 inches of rod travel yields the answer.

(5.5 pcm/in.)(4 in.) = -22 pcm

Method 3. Breaking the rod travel total into two parts (10 inches to 8 inches and 8 inches to 6 inches) yields:

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 5.5\ \frac{\text{pcm}}{\text{inch}}}{2}\right)\left(-2\ \text{inches}\right) = -13.5\ \text{pcm}$$

$$\left(\frac{5.5\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)\left(-2\ \text{inches}\right) = -8.5\ \text{pcm}$$

(- 13.5 pcm) + (- 8.5 pcm) = -22 pcm

In this example the various approximations used did not cause any difference because the problem deals with a section of the curve with an approximately constant slope. To obtain the value over the interval between 8 inches and 20 inches, however, would require the use of several subintervals (as in the last approximation) to obtain an accurate answer.

Example 3:

For the differential rod worth data given below, construct differential and integral rod worth curves.

Interval (inches)	Reactivity Inserted (pcm)
0 to 2	10
2 to 4	20
4 to 6	40
6 to 8	60
8 to 10	60
10 to 12	40
12 to 14	20
14 to 16	10

Solution:

Differential rod worth:

For each interval, the number of pcm/inch must be determined. For example, in the first interval (0 inches to 2 inches), 10 pcm is added. Therefore, the differential rod worth equals an average 5 pcm/inch. This value of differential rod worth is plotted at the center of each interval. The center of the interval 0 inches to 2 inches is 1 inch. The values of pcm/inch for each interval are then listed as shown below and plotted on Figure 12.

Interval Center	pcm/inch
1	5
3	10
5	20
7	30
9	30
11	20
13	10
15	5

Integral rod worth:

To plot the integral rod worth, merely develop a cumulative total of the reactivity added after each interval and plot the summed reactivity insertion vs. rod position as shown in Figure 12.

Interval Endpoint	Summed Reactivity
2	10
4	30
6	70
8	130
10	190
12	230
14	250
16	260

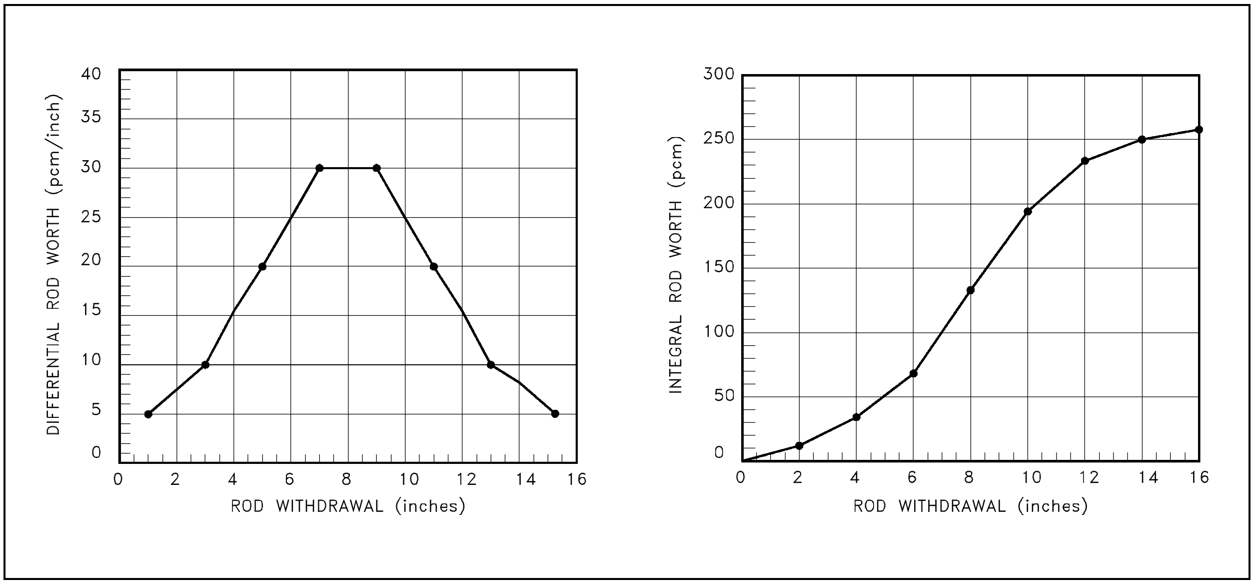

Figure 12 Rod Worth Curves From Example 3

If an integral rod worth curve is supplied, a differential rod worth curve can be generated from the integral rod worth data. Merely select a convenient interval of rod withdrawal, such as 1 inch or 2 inches. Then, determine from the curve the amount of reactivity added for each constant interval of rod withdrawal. A plot of this reactivity addition versus rod withdrawal represents differential rod worth.

Rod Control Mechanisms

The control rod insertion rates on a scram are designed to be sufficient to protect the reactor against damage in all transients that are expected to occur during the life of the reactor.

During normal rod motion, the control rods must be able to move rapidly enough to compensate for the most rapid rate at which positive reactivity is expected to build within the reactor in order to provide positive control. The transient that is normally considered when setting this minimum rod speed is the burnout of maximum peak xenon while at full power. Xenon burnout is usually the most rapid, non-accident transient expected. The maximum rod speed is normally limited in order to reduce the severity of an accident involving the continuous withdrawal of control rods.

Summary

The important information in this chapter is summarized on the following page.

Control Rods Summary

- A black neutron-absorbing material absorbs essentially all incident neutrons. A grey neutron-absorbing material absorbs only part of the incident neutrons.

- A grey neutron-absorbing material may be preferable to a black neutron-absorbing material in the construction of control rods because the grey absorber causes smaller depressions in neutron flux and power in the vicinity of the rod.

- Resonance absorbers are sometimes preferred to thermal absorbers as control rod materials because they have a larger area of influence and result in a flatter flux profile.

- Integral control rod worth is the total reactivity worth of the control rod at a particular degree of withdrawal from the core.

- Differential control rod worth is the reactivity change per unit movement of a control rod.

- The typical differential control rod worth curve has a bell shape. It has very low values at the top and bottom of the core and a maximum value at the center of the core. The curve has this shape because rod worth is related to neutron flux, and flux is highest in the center of the core.

- The typical integral control rod worth curve has an "S" shape. It has a relatively flat slope at the top and bottom of the core and a maximum slope at the center of the core.

- Integral or differential control rod worth curves can be used to determine the reactivity change due to a control rod movement between two positions.

- Integral or differential control rod worth curves can be plotted based on measured control rod worth data.