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HYDROSTATICS

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OFFICIAL COURSE/EXAM

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NAV-112 EXAM PREVIEW

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Exam Preview:

1. The amount of list is usually measured in degrees of incline from the level condition. When the ship lists to port the angles are assigned positive values and when the ship lists to starboard the angles are assigned negative values.
   a. True
   b. False
2. According to the reference material, the metacenter is a stationary point for small angles of inclination. We define "small" to be less than __ degrees.
   a. 20
   b. 5
   c. 15
   d. 10
3. Equilibrium of forces alone would not guarantee static equilibrium. The sum of the moments must also be zero!
   a. True
   b. False
4. Adding, removing, or shifting weight on a ship changes the location of G on a ship. According to the reference material, a ship's center of gravity ________ to a shift in weight onboard.
   a. Move toward
   b. Move away
   c. Move parallel
   d. Move perpendicular

5. If a ship is "trimmed by the bow," then the forward draft is bigger than the after draft. In other words, the stern sits deeper in the water than the bow.
    a. True
    b. False
6. According to the reference material, once the ship has been prepared, the inclining weights and apparatus are brought on board. Typically, the inclining weights are approximately __% of the displacement of the light-ship.
    a. 2
    b. 4
    c. 8
    d. 10
7. Due to the nature and complexity of repair and maintenance that must be performed on the underwater hull, openings, and sea-connected systems of ships, it is often necessary to perform this work in a drydock.
    a. True
    b. False
8. Adding, removing, or shifting weight on a ship changes the location of G on a ship. According to the reference material, a ship's center of gravity _______ from the location of removed weight.
    a. Move toward
    b. Move away
    c. Move parallel
    d. Move perpendicular
9. Rare does a weight change occur on board a ship that results in only a vertical movement of G or only a transverse movement of G. Usually, a weight change will result in both.
    a. True
    b. False
10. The metacenter is a convenient reference point for hydrostatic calculations at small angles. It was pointed out that the transverse metacenter is on the order of 10 to 30 feet above the keel whereas the longitudinal metacenter is on the order of ________ feet above the keel.
    a. 100 to 500
    b. 500 to 1000
    c. 100 to 1000
    d. 250 to 500

COURSE OBJECTIVES
CHAPTER 3

3. HYDROSTATICS

1. Explain a distributed force and a resultant force and relate them to a submerged ship's hull.

2. Calculate the absolute pressure below the surface of the water.

3. Apply Archimedes' Principle to a ship.

4. Know the necessary and sufficient conditions for static equilibrium and apply these conditions to various situations in Naval Engineering.

5. Draw a picture of a ships section at midships that has been inclined due to a transverse weight shift. Show all the relevant forces acting on this section and properly label the diagram.

6. Calculate a ship's vertical center of gravity from an Inclining Experiment. State the purpose and explain the process of an inclining experiment, including the derivation of relevant equations, figures, and diagrams. Complete calculations associated with an inclining experiment.

7. Due to the addition, removal or shift of weight on a ship:
   a. Qualitatively describe the direction of shift in a ship's center of gravity
   b. Calculate a ship's vertical center of gravity
   c. Calculate a ship's transverse center of gravity
   d. Calculate the angle of list (less than 10°)
   e. Calculate forward and aft drafts, including showing geometric relationships in the problems

8. Define trim.

9. Define, understand, and use Metacentric Height and Metacentric Radius.

10. Understand the dangers and basic procedures followed in drydocking.

3.1 Archimedes' Principle Revisited and Static Equilibrium

Most people find it truly amazing that steel ships weighing hundreds of thousands of tons can float in water. We know that they float because we have seen it with our own eyes, but what we have seen somehow seems contrary to other everyday experiences. Take a steel bar, throw it into the water and it will sink immediately. Why will a pound or so of metal sink, whereas several tons of the same metal will float?

From your study of Chapter 2 you realize that each object in the water is buoyed up with a force equal to the weight of the water displaced by the object. To get an object to float, the object must be able to displace a volume of water equal in weight to the weight of the object itself. With this knowledge you can build a concrete canoe!

At this point, you know the name of the Greek mathematician who discovered this principle of flotation – Archimedes'.

(!) Be sure that you can verbally and mathematically define Archimedes' Principle.

Let us combine the concepts of Archimedes' Principle with static equilibrium as applied to a free-floating ship in calm water.

3.1.1 Forces Acting on a Floating Body

The forces of concern on a freely floating ship are the distributed gravitational forces and the distributed buoyant forces. The forces are said to be distributed because they act over the entire ship. Some engineering analysis requires the use of the distributed force system to do the modeling (this will be used in Chapter 6). Other analysis allows the engineer to replace the distributed force system with an equivalent single resultant vector. The resultant vector is the sum of the distributed force system and is considered to act at such a location as to create the same effect on the body as the distributed system.

(!) In this chapter all distributed forces are replaced with resultant vectors to do the hydrostatic analysis.

3.1.1.1 Force due to Gravity

The force of gravity acts on each little part of the ship. Instead of dealing with millions of weights acting at millions of places throughout a ship, we resolve all of these weights into one resultant force, called the resultant weight or displacement (Δ_S) of the ship. This gravitational force, or resultant weight, is resolved to act at the center of gravity (G), which is simply the weighted average location of all of the weights that make up a ship. See Figure 3.1.

3.1.1.2 Force due to Buoyancy

The second system of distributed forces on a freely floating ship comes from the pressure exerted on the submerged part of the hull by the water. These hydrostatic forces act perpendicular to the surface of the hull and can be resolved into horizontal and vertical components with respect to the surface of the water.

The sum of the horizontal hydrostatic forces will be zero. This should make sense to you. If the horizontal forces didn't balance it would imply that a ship would move through the water all by itself without power or external forces. This kind of spontaneous movement does not occur.

The sum of the vertical hydrostatic forces is not zero. The net vertical force is called the resultant buoyant force (F_B). This force, like weight, is resolved to act at a unique point. The buoyant force acts at the center of buoyancy (B), which is the geometric centroid of the underwater volume. See Figure 3.1.

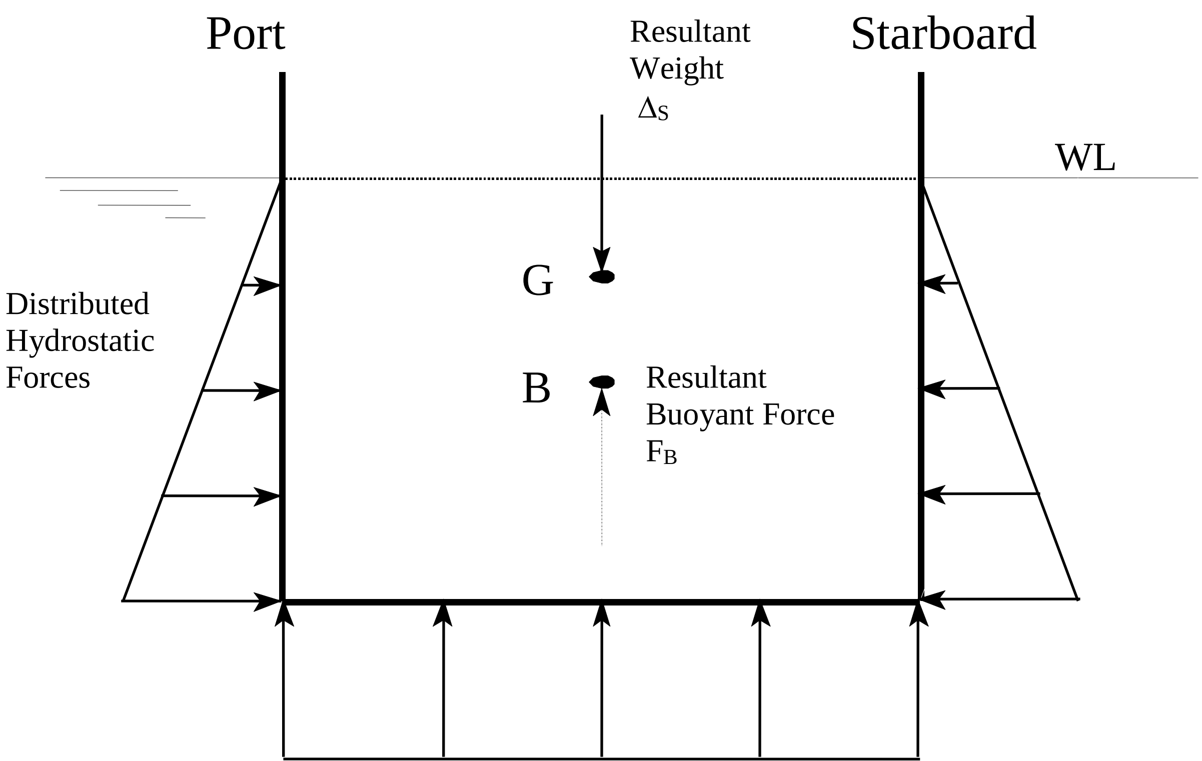


Figure 3.1 Ship at Static Equilibrium Showing Resultant Weight and Distributed & Resultant Buoyant Forces.

Notes on Figure 3.1:

- The distributed forces shown on the outside of the hull are being replaced by the resultant buoyant force. Normally you would not show both because it is redundant.

- The absolute pressure at depth "z" below the water surface is due to the atmospheric pressure plus the pressure from the column of water above the point of interest. This is shown in Equation 3-3.

$$P_{absolute} = P_{atm} + \rho\, g\, z \frac{1\, ft^2}{144\, in^2}$$

where:	$P_{absolute}$	absolute pressure at depth "z" (psi)
	P_{atm}	atmospheric pressure at the surface of the water (psi)
	ρ	density of the water ($lb \cdot s^2/ft^4$)
	g	magnitude of the acceleration of gravity (32.17 ft/s^2)
	ρgz	gage pressure (psi)

- The resultant weight and the resultant buoyant force always act perpendicular to the surface of the water. Resultant buoyant force acts upward while the resultant weight force acts downward.

- The vector arrows representing the resultant weight and resultant buoyant force must have their heads (or tails) attached to the center of gravity and center of buoyancy, be equal in length, and be labeled with symbols.

- We always use a capital "G" for the ship's center of gravity and a lower case "g" for the center of gravity of some object or fluid on the ship. You must use this convention in your diagrams.

- The magnitude of the resultant weight is the displacement (Δ_S). The resultant weight is a vector and the displacement is a scalar. Both have units of LT.

- The center of buoyancy is at the centroid of the submerged volume of the hull.

3.1.2 Static Equilibrium

Static Equilibrium is defined as a condition where:

"........the sum of the forces and the sum of the moments on a body are zero so that the body has no tendency to translate or rotate."

Each of the conditions is met in Figure 3.1. Let us explore each of them in the following paragraphs.

3.1.2.1 Forces

In general, there are two ways to mathematically state that the sum of the forces is zero. The following expression shows the vector equation stating this.

$$\sum \vec{F} = 0$$

This vector expression may be broken into an equivalent set of scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

In Figure 3.1 there are only two vertical forces shown. Immediately we can see that these forces must be equal and opposite or else the ship would sink or fly! We can prove this formally by applying condition of static equilibrium of forces to the vector diagram shown in Figure 3.1.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 = F_B - \Delta_s$$

$$F_B = \Delta_s$$

where: ΣF_z is the sum of the forces in the vertical direction with positive “z” as the up direction

F_B is the magnitude of the resultant buoyant force (lb)

Δ_S is the magnitude of the resultant weight of the ship, called the displacement (lb)

Example 3.1 Calculate the submerged volume of a DDG51 floating at a draft of 21.0 ft and level trim in sea water. (ρ= 1.99 lb-s^2/ft^4) (g = 32.17 ft/s^2) (1LT = 2240 lb).

From DDG51 curves of form.

@ 21 ft draft - curve 1 = 144

$\Rightarrow \quad \Delta_S = 144 \times 60 \text{ LT}$

$\Delta_S = 8640 \text{ LT}$

From Principle of Static Equilibrium

$F_B = \Delta_S$

$\Rightarrow \quad F_B = 8640 \text{ LT}$

From Archimedes' Principle

$$F_B = \rho g \nabla$$

$$\nabla = \frac{F_B}{\rho g}$$

$$\nabla = \frac{8640\ LT * 2240\ lb/LT}{1.99\ lb\frac{s^2}{ft^4} * 32.17\ \frac{ft}{s^2}}$$

$$\nabla = 302{,}300\ ft^3$$

3.1.2.2 Moments

Equilibrium of forces alone would not guarantee static equilibrium. The sum of the moments must also be zero! For the forces shown in Figure 3.1, the sum of moments about any arbitrary reference point would be zero. This is because the two resultant vertical forces shown have equal magnitudes, opposite direction, and lines of action that are coincident.

The following expression shows how to mathematically state the sum of the moments is zero about any reference point “p”. Notice it is a vector equation. The direction of the vector is normal to the plane containing the lever arm and the force.

$$\sum \vec{M}_p = 0$$

(!) The concept of a moment was discussed in Chapter 1 Section 1.9.4. Please go back and re-read that section if you are not comfortable with the concept of a moment.

3.1.3 Summary

In summary, Figure 3.1 shows a ship in static equilibrium because the two necessary and sufficient conditions for static equilibrium have been met; the vector sum of the forces are zero and the vector sum of the moments are zero. This means that the ship will have no tendency to move either in translation or rotation. It will just sit in the same position until something changes with the ship or an outside force acts on it. Further, it means that Archimedes’ Principle can be used to find the displacement of a freely floating ship since it is equal to the magnitude of the buoyant force.

Student Exercise: Draw the same ship in static equilibrium assuming that a large weight has been shifted from port to starboard so that the center of gravity of the ship has moved off the centerline. Label this figure “Figure 3.2” and add a caption to describe what you are trying to show.

3.2 New States of Static Equilibrium Due to Weight Additions, Weight Removals and Weight Shifts on a Floating Ship

Section 3.1 applies static equilibrium to a freely floating ship. Now we want to be able to determine the new static equilibrium condition after changing the weight distribution on a ship.

An altered weight distribution will cause the Center of Gravity (G) to move. To fully identify the location of G before and after its movement, we must be able to reference it in 3D space in the three Cartesian directions. As with the other centroids, the location of G is referenced vertically from the keel (KG) or the Vertical Center of Gravity (VCG), transversely from the centerline with the Transverse Center of Gravity (TCG) and longitudinally from either of the perpendiculars or midships with the Longitudinal Center of Gravity (LCG). Recall that the correct sign convention is negative to port of the centerline and aft of midships.

The weight distribution on a ship can change whenever...

- A weight is shifted in any one of three separate directions
- A weight is added or removed from anywhere on a ship
- By some combination of the above.

At first, determining the effect of any of these changes upon the location of G may seem overwhelming. However, it is manageable if we break it down into a study of three separate directions and then further break it down into addition, removal, and shift of weight in each of these directions. This process will be stepped through over the following pages.

Think of how practical this study of hydrostatics could be. On a ship the distribution of weight is constantly changing, and it would be desirable to know the final static equilibrium position of your ship after these changes. If these final conditions are undesirable the captain can take actions to avoid or minimize the effects.

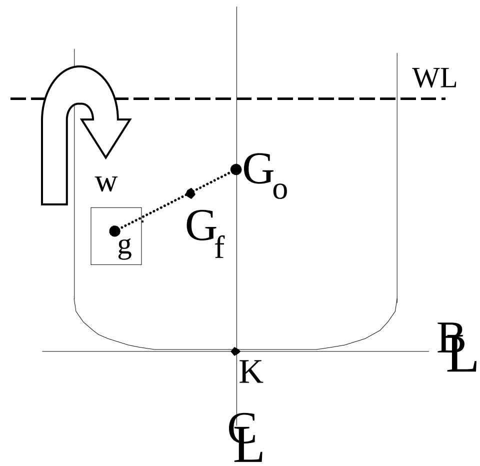
Student Exercise: With the help of your instructor make a list of ways weight is distributed differently over time from planned and unplanned evolutions:

3.2.1 Qualitative Analysis of Weight Additions, Removals and Shifts

Adding, removing, or shifting weight on a ship changes the location of G on a ship. A qualitative understanding of what is occurring can help in as a check upon the quantitative work that follows.

3.2.1.1 Weight Addition

A ship's center of gravity **moves towards** the location of added weight. Consequently, the Center of Gravity of the ship (G) will move in a straight line from its current position toward the center of gravity of the weight (g) being added. An example of this is shown in Figure 3.3.



F
F

Figure 3.3 Ship center of gravity (G) moves towards the location of added weight (g)

3.2.1.2 Weight Removal

A ship's center of gravity **moves away** from the location of removed weight. Consequently, the Center of Gravity of the ship (G) will move in a straight line from its current position away from the center of gravity of the weight (g) being removed. See Figure 3.4.

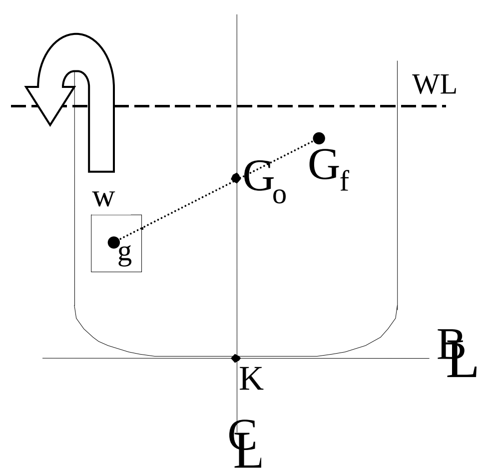


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Figure 3.4 The Effect of a weight Removal Upon the Center of Gravity of a Ship.

3.2.1.3 Weight Shift

A ship's center of gravity **moves parallel** to a shift in weight onboard. Ship's G will not move as far as the weight being shifted (g) because the weight is only a small fraction of the total weight of the ship. An example of this is shown in Figure 3.5.

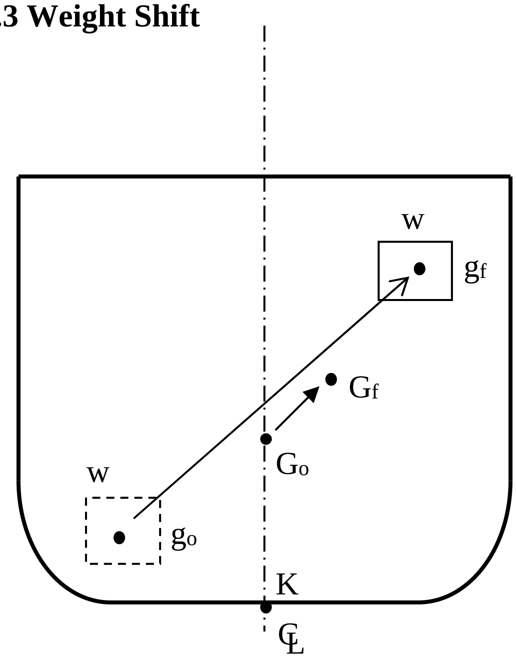


Figure 3.5 The Effects of a Weight Shift on the Center of Gravity of a Ship

A weight shift can be modelled as the removal of a weight from its previous position and the addition of a weight to its new position. Figure 3.6 demonstrates this principle using the rules governing weight additions and removals discussed previously.

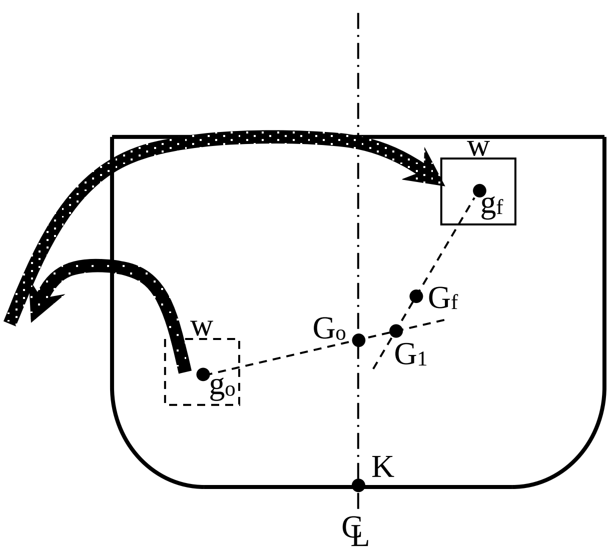


Figure 3.6 A Weight Shift Being Modeled as a Weight Removal Followed By a Weight Addition

3.2.2 Quantitative Changes in the Ship's Center of Gravity Due to Vertical Weight Additions, Removals, or Shifts

The same equation may be used to separately measure changes in the ship's center of gravity in the vertical and transverse directions. Both of these directions are calculated separately and then combined for a final location.

The same equation may be used for weight addition, removal, and shifts. Positive weight is weight added and negative weight is weight removed. Weight shifts are modelled as being removed from the previous location and added to the new location.

3.2.2.1 Weighted Average Technique

The KG_{new} of the ship can be calculated by doing a weighted average of the distances from the keel to ship's G_{old} and object's g with a weighting factor based on a weight ratio.

$$\overline{KG}_{new} \cdot \Delta_{s\ new} = \overline{KG}_{old}\ \Delta_{s\ old} + \sum_{i=1}^{N} (\pm w_i)(\overline{Kg}_i)$$

where: KG_{new} is the final vertical position of the center of gravity of the ship as referenced from the keel (ft)

KG_{old} is the initial vertical position of the center of gravity of the ship as referenced from the keel (ft)

$\Delta_{S\ new}$ is the final displacement of the ship (LT) (= $\Delta_{S\ old} + \Sigma +/- w_i$)

$\Delta_{S\ old}$ is the initial displacement of the ship (LT)

Kg_i is the vertical position of the center of gravity of the weight being added/removed/shifted as referenced from the keel (ft)

w_i is the object's weight (+ added/- removed) (LT)

(!) To shift weight, first remove it from its old position and then add it to its new position. Since the weight removed and added are the same, ship displacement is unchanged.

(!) If several separate weights are added, removed, or shifted, one equation may be used. Repeat the (Kg·w) term for each weight change

For a visual depiction of these changes to center of gravity, see Figures 3.7, 3.8, and 3.9.

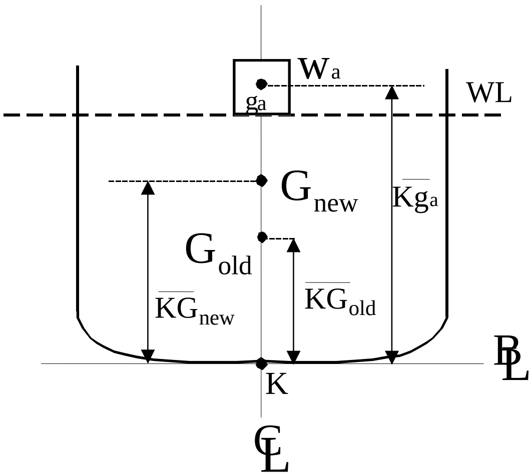


Figure 3.7 Vertical Weight Addition

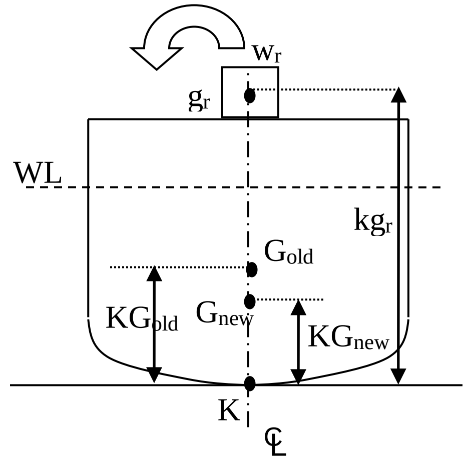


Figure 3.8 Vertical Weight Removal

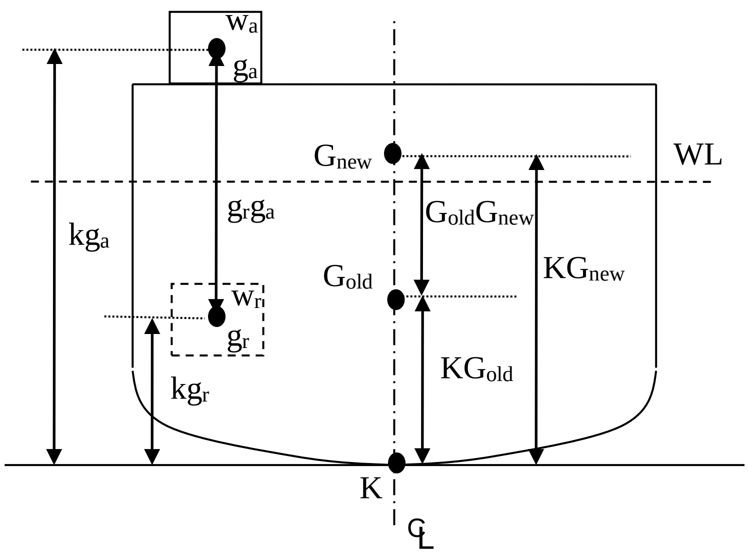


Figure 3.9 Vertical Weight Shift

After you calculate a new center of gravity, qualitatively check your answer. For example:

> Suppose your old KG is 18 feet and a fuel tank Kg is 14 feet. After "steaming" for some time, the fuel tank is half empty. Suppose you calculate a final KG of 15 feet. Immediately you should know you made a mistake because removing weight below the existing center of gravity of the ship should cause the center of gravity of the ship to rise. Your answer should have been something greater than 18 feet!
>
> You can also check the magnitude of the change. Suppose you calculated a new KG of the ship to be 100 feet. This change is much too large to be reasonable.

The moral of this story is *always check* your final answer. This implies you have a qualitative understanding of the physical processes involved in the calculation of the number! In exam, test and quizzes, you will be graded more when you show a qualitative understanding than simply submitting an answer which is obviously incorrect.

Example 3.2 An FFG-7 class frigate has an initial displacement of 4092 LT and an initial vertical location of the center of gravity of the ship of 18.9 feet above the keel. If 200 LT are added 10 feet above the keel, and 75 LT are removed 20 feet above the keel, what is the new vertical location of the center of gravity of the ship?

Solution:

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\ \Delta_{s\ old} - \overline{Kg}_r w_r + \overline{Kg}_a w_a}{\Delta_{s\ old} - w_r + w_a}$$

$$\overline{KG}_{new} = \frac{(18.9\ ft)(4092\ LT) - (20\ ft)(75\,LT) + (10\ ft)(200\,LT)}{4092\ LT - 75\ LT + 200\ LT}$$

$$\overline{KG}_{new} = \frac{77839\ \ ft - LT}{4217\ LT} = 18.5\ ft$$

Remember: Always check your final answer for reasonability and consistency of units.

- Final KG should be a smaller number since both the addition and removal lower the center of gravity of the ship. Adding 200 LT below the initial center of gravity of the ship causes the center of gravity of the ship to move lower (towards weight added). Removing 75 LT above the initial center of gravity of the ship should cause the center of gravity of the ship to move lower (away from weight removed).

- The direction and magnitude of the change are both reasonable.

- The units of the final answer are consistent with the parameter being found.

3.2.3 Quantitative Changes in the Ship's Center of Gravity Due to Transverse Weight Additions, Removals, or Shifts

Recall the transverse direction is the "side to side" direction (or the port to starboard direction). The centerline of the ship separates the port from the starboard. Recall that ***distances to the port are defined to be negative, and distances to the starboard are positive***. In general, we use the symbol "y" as the general variable to represent a transverse distance from the centerline of the ship. Other names you might hear in referencing this direction are "half breadth" and "athwartships."

Qualitatively, we know that should a weight be added or removed off center (not on the centerline) or a weight is shifted transversely across the ship, the ship will assume some angle of inclination. This angle is called an "angle of list". List is the condition where the ship is in static equilibrium and down by the port or starboard side. In other words, the ship is not level in the water from side to side. The list angle is created because the weight change has resulted in the Center of Gravity (G) of the ship to move from the centerline. There are no external forces acting on the ship to keep it down by the port or starboard. The angle is maintained because the resultant weight and buoyant force are vertically aligned as shown in Figure 3.2 and Figure 3.10.

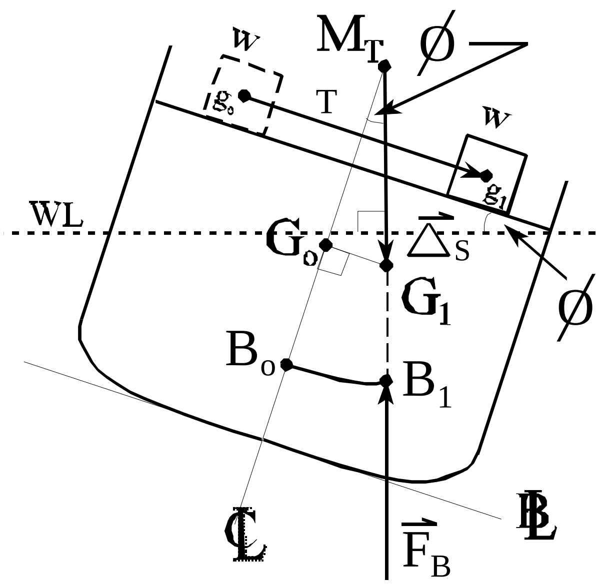


Figure 3.10 The Locations of G and B for a Listing Ship

The off center G causes a moment to be created within the ship that causes it to rotate. As the ship rotates, the underwater volume changes shape which causes the Center of Buoyancy (B) of the ship to move. At small angles of list, B moves in an arc, centered at the transverse metacenter (M). It continues to move until the shape of the underwater volume causes B to move directly vertically underneath G, causing the ship to be back in static equilibrium.

(!) The concept of the metacenter and B movement will be discussed in greater detail later in this chapter.

3.2.3.1 Measurement in the Transverse Direction

The amount of list is usually measured in degrees of incline from the level condition. When the ship lists to port the angles are assigned negative values and when the ship lists to starboard the angles are assigned positive values. In general, we use the symbol ϕ (phi) as the general variable to represent an angle of inclination to the port or starboard side.

The center of gravity (G) is referenced in the transverse direction from the centerline of the ship. The distance from the centerline of the ship to the center of gravity of the ship is called the transverse center of gravity (TCG) and is measured in units of feet.

3.2.3.2 Weighted Average Technique

The final TCG after a transverse weight change can be quantitatively determined by using a weighted average equation or by equating moments about the centerline before and after the change in a similar manner shown for vertical changes of weight. The equation takes on the same form as previously discussed with two differences.

- The first difference is that the KG terms have been replaced with TCG since we are working in the transverse direction.
- The second difference is that distances to port must have a negative sign. In the vertical case all distances were positive since the reference point was the keel. In the transverse case the reference point is the centerline so that the TCG can be either negative or positive.
- Typically, your TCG_0 is 0 ft at problem start (if the ship starts at even keel, even list, even trim), but this is not always the case.

The generalized equation for changes in the transverse center of gravity due to shifts, additions, and removals is:

$$TCG_{new} * \Delta_{s\ new} = \pm TCG_{old}\ \Delta_{s\ old} + \sum_{i=1}^{N} (\pm w_i)(tcg_i)$$

where: TCG_{new} is the new transverse position of the center of gravity <u>of the ship</u> as reference from the centerline (ft)

TCG_{old} is the old transverse position of the center of gravity <u>of the ship</u> as reference from the centerline (ft)

$\Delta_{s\ new}$ is the new displacement of the ship (LT) (= $\Delta_{S\ old} + \Sigma +/- w_i$)

$\Delta_{s\ old}$ is the old displacement of the ship (LT)

tcg_i is the transverse position of the center of gravity <u>of the weight being added or removed</u> as referenced from the centerline (ft)

w_i is the object's weight (+ added/- removed) (LT)

This equation works the same as the vertical shift equation (KG), except distance are measured from the centerline with ***negative to port and positive to starboard.***

Example 3.3 An FFG 7 ship has a displacement of 4092 LT, and an initial transverse center of gravity 2 feet starboard of the centerline. A 75 LT weight is moved from a position 10 feet port of the centerline to a position 20 feet port of centerline and a 50 LT weight is added 15 feet port of the centerline. What is the final location of the ship's transverse center of gravity?

Solution:

$$TCG_{new} = TCG_{old}\frac{\Delta_{s\ old}}{\Delta_{s\ new}} - Tcg_{75\ ton\ r}\frac{w_{75\ ton}}{\Delta_{s\ new}} + Tcg_{75\ ton\ a}\frac{w_{75\ ton}}{\Delta_{s\ new}} + Tcg_{50\ ton}\frac{w_{50\ ton}}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{TCG_{old}\ \Delta_{s\ old} - Tcg_{75\ ton\ r}\ w_{75\ ton} + Tcg_{75\ ton\ a}\ w_{75\ ton} + Tcg_{50\ ton}\ w_{50\ ton}}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{TCG_{old}\ \Delta_{s\ old} + w_{75\ ton}\,(Tcg_{75\ ton\ a} - Tcg_{75\ ton\ r}) + Tcg_{50\ ton}\ w_{50\ ton}}{\Delta_{s\ old} - w_{75\ ton} + w_{75\ ton} + w_{50\ ton}}$$

$$TCG_{new} = \frac{+2\ ft\ 4092\ LT + 75\ LT\,(-20\ ft - -10\ ft) + -15\ ft\ \ 50\ LT}{4092\ LT - 75\ LT + 75\ LT + 50\ LT}$$

$$TCG_{new} = \frac{+2\ ft\ 4092\ LT + 75\ LT\,(-20\ ft + 10\ ft) + -15\ ft\ \ 50\ LT}{4092\ LT + 50\ LT}$$

$$TCG_{new} = \frac{8184\ LT - ft - 750\ LT - ft - 750\ LT - ft}{4142\ LT}$$

$$TCG_{new} = \frac{6684\ LT - ft}{4142\ LT} = 1.61\ ft\ to\ starboard\ of\ centerline$$

3.2.4 Combining Vertical and Transverse Weight Shifts

Rare does a weight change occur on board a ship that results in only a vertical movement of G or only a transverse movement of G. Usually, a weight change will result in both. Figure 3.11 shows an example with a weight addition.

Qualitatively, we know that G will move directly towards the location of the added weight. In this example, it results in an increase in KG and a TCG starboard of the centerline. Theoretically, it should be possible to calculate the new location of G in one step. However, significant simplification is achieved by breaking the problem down into the vertical and transverse directions.

The steps for carrying out an analysis of this situation would be:

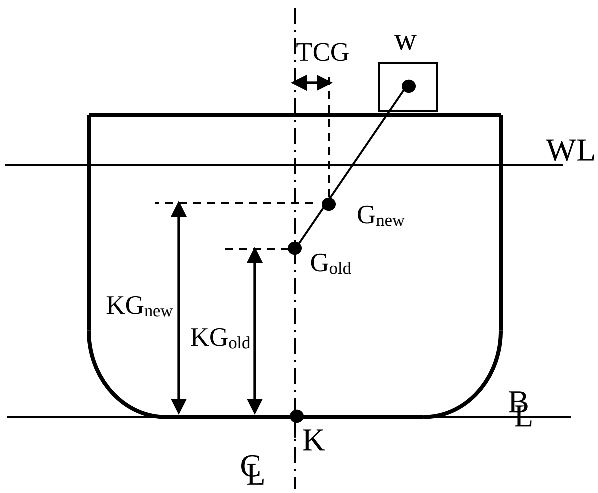


Figure 3.11 Combining Vertical and Transverse Weight Changes

- Qualitatively determine the approximate location of G_{new}.
- Perform a vertical analysis to calculate KG_{new}
- Perform a transverse analysis to calculate TCG_{new}
- Check your vertical and transverse answers with your qualitative work.

Using this type of method, you should be assured of success in weight shift, addition and removal problems. We will now move on and examine the listing ship created by an “off center” G in more detail. However, before we can do this, we must understand the meaning of the metacenter.

3.3 Transverse Metacentric Radius and the Transverse Metacentric Height

Figure 3.12 shows a typical sectional view of a ships hull when the ship is floating level in the water with no list or trim. The important points for hydrostatic calculations are the keel (K), the center of buoyancy (B), the center of gravity (G), and the transverse metacenter (M_T).

3.3.1 The Metacenter

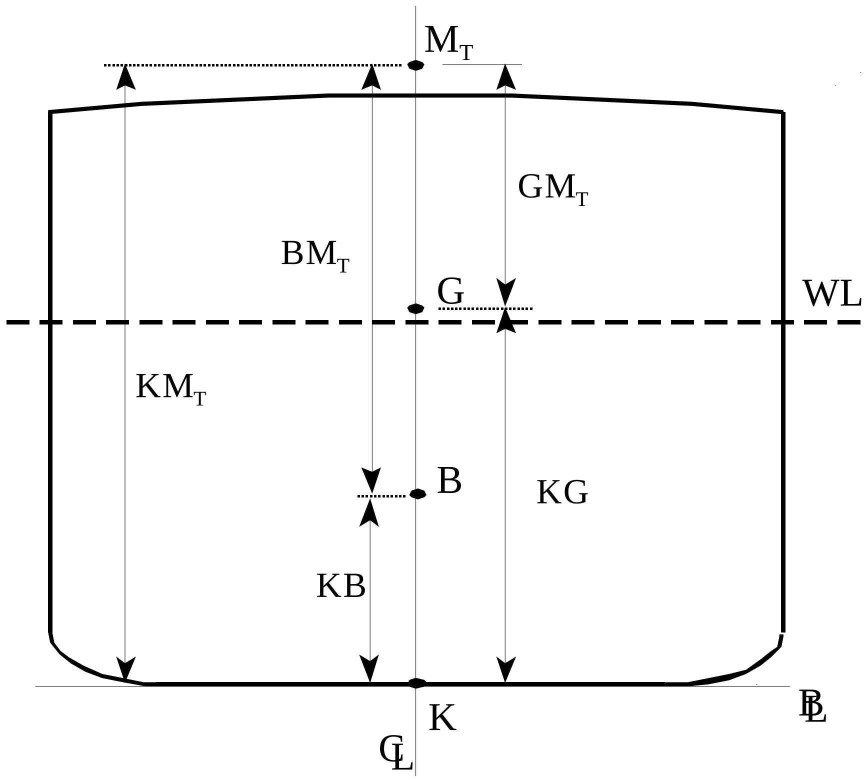


Figure 3.12 Important Locations and Line Segments used in Hydrostatic Calculations

The metacenter was briefly introduced in Section 2.10. It was stated there that the metacenter is a convenient reference point for hydrostatic calculations at small angles. Recall, there is one metacenter associated with rotating the ship in the transverse direction (M_T) and another one when rotating the ship in the longitudinal direction (M_L). It was pointed out that the transverse metacenter is on the order of 10 to 30 feet above the keel whereas the longitudinal metacenter is on the order of 100 to 1000 feet above the keel.

The metacenter is a stationary point for small angles of inclination. We define "small" to be less than 10 degrees. This is the reason the metacenter and the geometry derived here is only applicable to small angles of inclination. Beyond ~10 degrees the location of the metacenter moves off the centerline in a curved arc.

3.3.1.1 Metacentric Radius

To locate the metacenter for small angles requires the construction of two lines. The intersection of these lines defines the location of the transverse metacenter. The first line is the line of action of the buoyant force when the ship is upright with no list. The second line is the line of action of the buoyant force when the ship is inclined a small amount.

When a ship is inclined at small angles (<10 degrees), the center of buoyancy (B) moves in an arc. The center of this arc is the transverse metacenter (M_T). Picture in your mind a piece of string attached to the metacenter at the top and to the center of buoyancy at the other end. This is why the distance from the metacenter (M) to the center of buoyancy (B) is called the transverse metacentric radius (BM_T). The metacentric radius is a line segment measured in feet and it is a commonly used parameter in naval architecture calculations.

3.3.1.2 Metacentric Height

Another important line segment used in naval architecture calculations is the distance from the center of gravity (G) to the transverse metacenter (M_T). This line segment is called the transverse metacentric height (GM_T). As we shall see in the next chapter, the magnitude and sign of the metacentric height will reveal how strongly the ship will want to remain upright at small angles. The importance of this parameter will be made clear in the next chapter.

3.3.2 Calculations

Very often in the calculations you will be doing you will need the distance between two of the points shown on Figure 3.12. It is often the case that you know some of the distances but not others. To find any other distance you need, simply draw a quick sketch of Figure 3.12 and use your sketch to see the relationships between what you know and don't know.

For example, to find KG you could subtract KM - GM_T. KG is the line segment that gives the vertical distance to the center of gravity from the keel. The line segment KM_T is the "transverse metacentric height above the keel". You may recall that it can be found on the curves of form if you know the mean draft of the ship. We will see later in this chapter that the GM of a ship can be experimentally measured by doing an inclining experiment.

3.3.2.1 Advanced Calculations (OPTIONAL)

To obtain the values of KM in the curves of form, KB is added to BM. Recall that KB can be calculated by numerical integration of the table of offsets as was shown in Section 2.9.5. BM is related to the second moment of area of the waterplane and can be calculated by the following equation.

$$\overline{BM}_T = \frac{2}{3}\frac{\int y^2 \ ydx}{\nabla_s} = \frac{2}{3}\frac{\int y^3 dx}{\nabla_s} = \frac{I_T}{\nabla_s}$$

(the derivation of this equation is beyond the scope of this introductory course)

where: y is the half breadth distance (ft)
ydx is the area of the differential element on the operating waterplane (ft^2)
∇_s is the submerged volume of the ship's hull (ft^3)
I_T is the second moment of the operating waterplane area in the transverse direction with respect to the x-axis (ft^4)

(!) Physically the second moment of area in this case is a measure of the rotational resistance. The second moment of area is a "strong" function of the width of the ship since it is proportional to the half-breadth cubed. In general, this tells us that a wider ship will be harder to roll.

3.4 Calculating (small) Angles of List Due to Transverse Shift of Weight

For small angles of list (<10 degrees) we can easily relate the transverse shift in the center of gravity of the ship to the angle of inclination. The theory and derivation developed here are necessary components of the inclining experiment discussed in the next section.

3.4.1 Theory

As discussed previously, when the center of gravity of the ship shifts away from the centerline there is an instantaneous misalignment of the resultant weight of the ship with the resultant buoyant force. This causes a moment, rotating the ship to the side the shift occurred to. As the ship inclines the submerged volume changes form, resulting in a new location of the centroid of the underwater volume formed by the hull. The ship will continue to rotate until the centroid shifts far enough to once again be in vertical alignment with the line of action of the resultant weight of the ship.

To keep the following derivation simple we will assume that we always start with a ship that has no initial list so that the initial transverse center of gravity is zero feet. In other words, the initial center of gravity will lie on the centerline of the ship. We will label this point "G_0.". The final transverse center of gravity will be the distance from the centerline to a point we will label "G_t."

3.4.2 Diagram

The first thing we must do is to draw a typical cross section of a ship's hull inclined as a result of a transverse weight shift in the center of gravity. Figure 3.13 shows the inclined hull with the location of all the key points for our derivation. Additionally, the resultant weight of the ship, the resultant buoyant force, and the waterline are also shown.

You must be able to understand this diagram and be able to draw it without the use of your notes. If you understand the concepts it will be very easy to do so.

Do not attempt to blindly memorize the diagrams in this text. They must be constructed using the fundamental concepts in a logical progression of thought. Further, you should practice drawing each figure because it takes a little artistic skill to do them correctly.

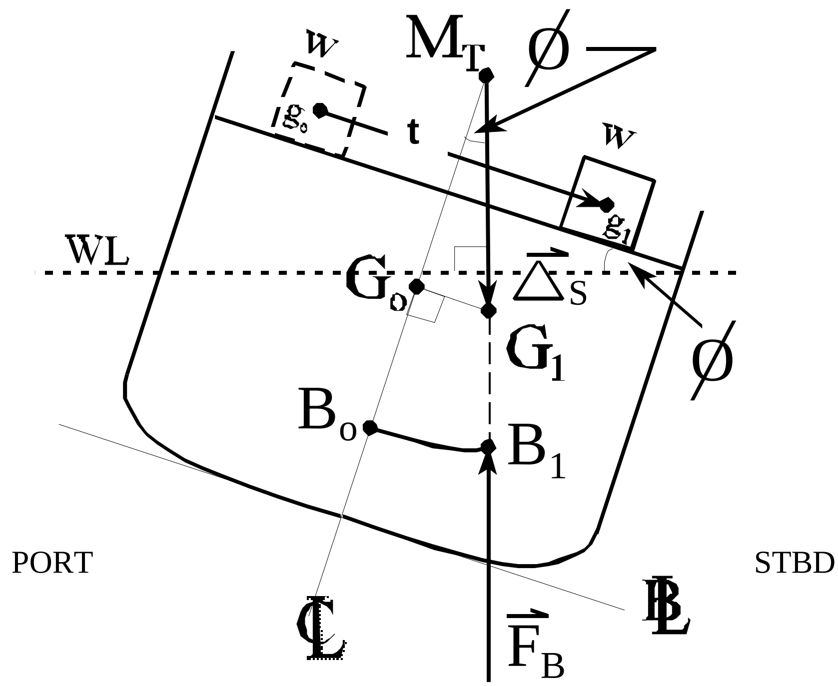


Figure 3.13 Inclined ship to the starboard side due to a shift in the center of gravity

You should notice the following key items on your diagram when you draw it. Very often these are the items that students get wrong on exams.

- By convention, the right side of the paper is starboard. Rotate the ship, which also rotates the baseline and centerline with the ship.
- Waterline remains horizontal (parallel to top of paper). Resultants of force of buoyancy (up) and force of gravity (Displacement), (down), remain vertical (perpendicular to waterline). In static equilibrium, these two forces are co-linear, located with their heads at their respective points B and G.
- The shift in the center of gravity of the ship is perpendicular to the centerline because the weight shift was perpendicular to the centerline. If your diagram doesn't look like it is then put a small square indicating perpendicularity to your instructor.
- All items should be labeled with the proper symbols including:
    - the waterline (WL), centerline (C_L), and keel (K) or Baseline (BL), PORT, STBD
    - the ship's initial center of gravity (G_0), and initial center of buoyancy (often but not always on the centerline)
    - the resultant weight of the ship (Δ_S) at the ship's final center of gravity (G_1), and the resultant buoyant force (F_B) at the final center of buoyancy (B)
    - the transverse metacenter (M_T), and angle of inclination (ϕ)

3.4.3 Relationship

Once you have sketched Figure 3.13 the derivation of the relationship between the "shift in the center of gravity of the ship" and the "angle of inclination" is evident. Notice the right triangle formed by the points ($M_T G_0 G_1$). The line segment G_0G_1 (of length δTCG) opposite from the angle of inclination. The metacentric height (G_0M_T) is adjacent to the angle of inclination. The opposite side over the adjacent side of a right triangle defines the tangent of the angle. Solving for (G_0G_1) yields:

$$\tan\phi = \frac{opp}{adj} = \frac{\overline{G_0G_1}}{\overline{G_0M_T}}$$

$$\overline{G_0G_1} = \overline{G_0M_T}\ \tan\varphi$$

Since G_0 is on the centerline, distance G_0G_1 is the same as TCG_{new}. And g_0g_1 is the same as tcg, which will be simplified to just distance 't'. Substitution of the above expression into the equation for a single transverse weight shift (from Section 3.2.3.2) yields:

$$TCG_{new} \cdot \Delta_{s\ new} = \pm TCG_{old}\ \Delta_{s\ old} + \sum_{i=1}^{N} (\pm w_i)(tcg_i)$$

$$\overline{G_0G_1} \cdot \Delta_s = (0\ ft \cdot \Delta_s) + w\,\overline{g_0g_1}$$

$$\overline{G_0G_1} = \frac{w\,\overline{g_0g_1}}{\Delta_s}$$

Setting the two derived G_0G_1 equations equal to each other yields:

$$\overline{G_0M_t}\ \tan\phi = \frac{w\ t}{\Delta_S}$$

where: "t" is the transverse distance the weight is shifted (g_0g_1) (ft)

This is the relationship we sought. It relates the transverse shift in the center of gravity of a ship to the angle of inclination for angles less than 10 degrees. This is the basic relationship used in the inclining experiment in the very next section.

3.5 The Inclining Experiment

The goal of the Inclining Experiment is to use small angle hydrostatics to compute the vertical center of gravity of a ship as referenced from the keel (KG). The basic process of an inclining experiment is straight-forward.

1. A known weight (w_i) is moved a known transverse distance (t_i). This transverse weight shift causes a transverse shift in the center of gravity of the ship, which in turn causes the ship to list to the side of the weight shift.
2. The amount of weight used (w_i), the distance it is shifted (t_i), and the resulting angle of list (ϕ_i) are measured and recorded. The process is repeated moving different weights different distances, port and starboard, causing port and starboard angles of list. This yields sets of (w_i , t_i , $\tan\phi_i$) data where the subscript " i " is just a counting variable.
3. The data from several weight shifts is plotted and the slope is used to reduce error from just one data point. Slope of the curve allows solving GM_T.
4. With KM_T from Curves of Form, one may solve KG_{incl} for the experimentally inclined ship. With a simple weight removal calculation, one may solve the KG_{light} for the ship without experimental gear onboard.

Before this process can begin, the ship has to be prepared for the experiment. The experiment is conducted alongside, in calm water with the ship free to list. It is usually performed with the ship in its light-ship condition. The light-ship displacement (Δ_{light}) is defined by Gilmer and Johnson as:

> *"the weight of the ship complete in every respect, including hull, machinery, outfit, equipment, water in the boilers at steaming level, and liquids in machinery and piping, but with all tanks and bunkers empty and no crew, passengers, cargo, stores, or ammunition on board."*
>
> *Introduction to Naval Architecture, p131.*

It is necessary to determine the displacement of the light-ship (Δ_{light}). This is achieved by observing the fwd and aft draft marks and consulting the ship's curves of form. In this step it is also important to find the density of the water the ship is floating in so that a correction can be made to the displacement read from the curves of form for the true water density.

Once the ship has been prepared, the inclining weights and apparatus are brought on board. Typically, the inclining weights are approximately 2% of the displacement of the light-ship (Δ_{light}). With the inclining weights and apparatus on board, the ship is said to be in an inclined condition. All quantities are then given the inclined suffix. For example Δ_{incl} , KG_{incl} .

With the inclining weights and equipment on board, the experiment can then proceed as described above. This often requires a great deal of co-ordination and the use of riggers etc. For larger ships, it is common to use a crane to move the inclining weights from and to different transverse locations. 2% of the displacement of a ship is a considerable weight to move.

3.5.1 Finding $G_0M_{T\ inclined}$

The equation to find list angle from a single transverse weight shift is expressed in terms of the metacentric height (G_0M_T) as:

$$\overline{G_oM}_{T\,incl} = \frac{w_i\,t_i}{\tan\,\phi_i}\frac{1}{\Delta_{S\ incl}}$$

Any one set of (w_i , t_i , ϕ_i) could be used in this equation to find a value for the inclined transverse metacentric height. Each set should yield the same value of metacentric height for small angles. However, there are experimental errors and deviations from the ideal that will yield a slightly different value for each set of (w_i , t_i , ϕ_i) used.

To achieve an average value for the transverse metacentric height (G_0M_T) the slope from a graph of "tangent of the inclining angle" (tan ϕ_i) versus the "inclining moment" (w_i t_i) is calculated. See Figure 3.14. The first group of parameters in the equation above is the slope of this graph. By dividing the slope by the displacement of the ship, the average value of G_0M_T is obtained as shown below:

$$\overline{G_oM_T} = \frac{w_i\,t_i}{\tan\,\phi_i}\frac{1}{\Delta_{S\ incl}}$$

$$Average\ \overline{G_0M_T} = \frac{(\text{slope of the "}\tan\phi_i\text{ vs }w_it_i\text{" curve})}{\Delta_{S\ incl}}$$

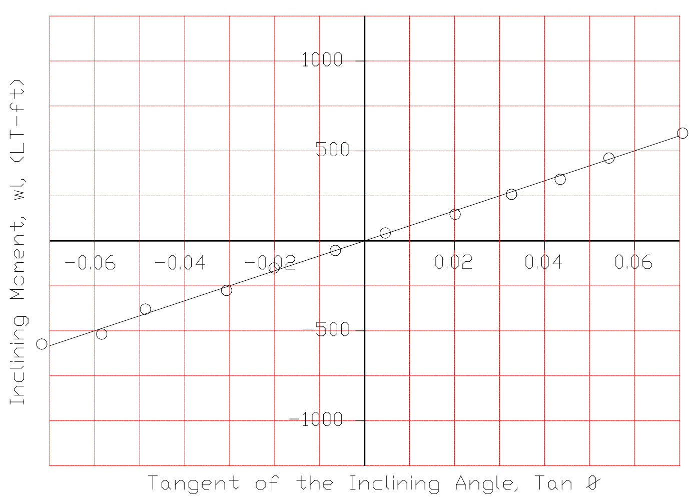


Figure 3.14 A Typical Plot of Data from an Inclining Experiment

The slope is calculated by picking any two points on the line of best fit and doing a change in "y" over a change in "x" calculation. Be sure to pick points on the line of best fit! A common student mistake is to use the original data points to calculate the slope. It is possible that none of these data points will be on the line you have drawn, the line represents the average of the data! An advantage of analyzing the data in this manner is that one stray data point can be “thrown out” or “ignored” as a bad point.

$$slope\ of\ \ a\ line = \frac{Rise}{Run} = \frac{dy}{dx} \equiv \frac{\delta y}{\delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

There is also a mathematical technique to do the linear regression called “least squares”. The mathematical technique is less subjective since no matter who does the calculation it will yield the same results. The linear regression by the least squares method can be easily done with a spreadsheet program on a computer. The computer will give the entire equation of the straight line to many decimal places. This technique minimizes the sum of the “squares of the error” between each data point and the line, thus the name least squares method.

Obtaining the average value of the transverse metacentric height (G_0M_T) is not the objective of the inclining experiment. Keep in mind the objective is to find the vertical location of the center of gravity of the ship without inclining gear aboard (KG_{light}). Two more steps are required once the average value of G_0M_T is obtained.

3.5.2 Finding KG_{incl} and Correcting this for the Removal of Inclining Apparatus

The first step is find the vertical location of the center of gravity of the ship with the inclining gear on board by subtracting the average metacentric height from the value of KM_T. The value of KM_T is found on the curves of form as a function of mean draft.

$$\overline{KG}_{incl} = \overline{KM}_T - \overline{G_0M}_T$$

The second step is to calculate the vertical location of the center of gravity of the ship without the inclining weights aboard (KG_{light}). This is accomplished by doing a weight removal calculation as explained earlier in Chapter 3.

$$\overline{KG}_{light} = \frac{\overline{KG}_{incl}\,\Delta_{incl} - \sum \overline{Kg}_{incl\ weights} \cdot w_{incl\ weights}}{\Delta_{light}}$$

$$\overline{KG}_{light} = \frac{\overline{KG}_{incl}\,\Delta_{incl} - \sum \overline{Kg}_{incl\ weights} \cdot w_{incl\ weights}}{\Delta_{inclined} - \sum w_{incl\ weights}}$$

3.5.3 Inclining Experiment Practicalities

The inclining experiment is easily performed on a ship and it is likely that you will see it carried out or be a part of the evolution sometime in your career.

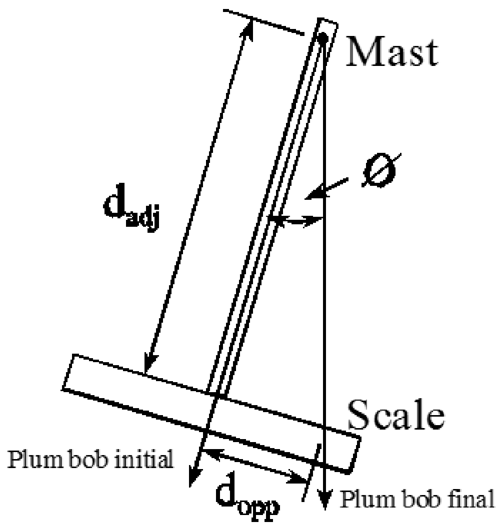


Figure 3.15 Measurement of tan ϕ during an inclining experiment

The tangent of the inclining angle for each placement can be measured by attaching a “plum bob” on a long wire suspended from a tall mast. The plum bob will always hang vertically downward and perpendicular to the waterplane. This plum bob can be used to measure the number of inches of deflection the bob makes when the ship is inclined from the level position. Figure 3.15 shows the right triangle formed by the mast, wire and horizontal scale. The tangent of the inclining angle can be calculated from this right triangle by dividing the deflection distance by the length of the wire as shown below:

$$\tan\phi_i = \frac{opposite\ side\ of\ right\ triangle}{adjacent\ side\ of\ right\ triangle} = \frac{d_{opp}}{d_{adj}}$$

These are the more common problems in doing an inclining experiment:

- Keeping track of all the weights onboard before and during the evolution.
- The presence of liquids in less than full tanks creates errors in the measurements. The shift in the fluid in a less than full tank creates a virtual rise in the center of gravity of the tank. This is called the “free surface effect” and it will be discussed in Chapter 4.
- The test must be done in calm conditions (test performed pierside typically).
- Potentially dangerous in that adding weights high on a ship reduces stability and/ or the deck may not be able to support the inclining weights. Additionally, moving large weights creates a safety concern to personnel involved. (These concerns are evaluated before the procedure ever takes place.)

Example 3.4 A ship undergoes an inclining experiment resulting in a graph of “the tangent of the list angle” versus “the inclining moment” (similar to Figure 3.14) with a slope of 28591 ft-LT. The displacement is 7986 LT and KM = 22.47 ft. What is the KG of the ship without the inclining gear aboard if the center of mass of the inclining gear is 30 feet above the keel with a weight of 50 LT?

Solution:

Finding $\overline{GM}_{inclined}$

$$\overline{GM}_{inclined} = \frac{\text{slope of "}\tan\phi \; vs \; wt\text{" curve}}{\Delta_{inclined}}$$

$$\overline{GM}_{inclined} = \frac{28591 \text{ LT} - \text{ft}}{7986 \text{ LT}}$$

$$\overline{GM}_{inclined} = 3.58 \text{ ft}$$

Finding $\overline{KG}_{inclined}$

$$\overline{KG}_{inclined} = \overline{KM}_{inclined} - \overline{GM}_{inclined}$$

$$\overline{KG}_{inclined} = 22.47 \text{ ft} - 3.58 \text{ ft} = 18.89 \text{ ft}$$

Finding $\overline{KG}_{light}$

$$\overline{KG}_{light} = \overline{KG}_{inclined}\Delta_{inclined} - \overline{Kg}_{inclining\,weight} \cdot w_{inclining\,weight}$$

$$\overline{KG}_{light} = \frac{18.89 \text{ ft } 7986 \text{ LT} - 30 \text{ ft } 50 \text{ LT}}{7986 \text{ LT} - 50 \text{ LT}}$$

$$\overline{KG}_{light} = \frac{150856 \text{ LT} - \text{ft} - 1500 \text{ LT} - \text{ft}}{7936 \text{ LT}} = 18.82\, ft$$

3.6 Longitudinal Changes in the Ship's Center of Gravity Due to Weight Shifts, Weight Additions, and Weight Removals

So far we have calculated vertical and transverse weight shifts, weight additions, and weight removals. In this section we will look at longitudinal weight shifts, weight additions, and weight removals. Longitudinal problems are done in a different manner because we are usually not concerned with the final position of G, but the new trim condition of the ship.

The consequence of longitudinal shifts, additions, and removals of weight is that the ship undergoes a change in the forward and after drafts. When the forward and after drafts have different magnitudes the ship is said to have trim. Recall from Chapter 2, that trim is defined by the difference between the forward and after drafts. Also recall, that T is the symbol for draft.

$$\text{Trim} = T_{aft} - T_{fwd}$$

If a ship is "trimmed by the bow," then the forward draft is bigger than the after draft. In other words, the bow sits deeper in the water than the stern. A ship "trimmed by the stern" has an after draft bigger than the forward draft. Recall that the ship rotates about the center of flotation (F) which is the centroid of the waterplane area. (It does not rotate about midships!) When the centroid of the waterplane area is aft of midships the forward draft will change by a larger amount than the after draft. This is usually the case since a typical ship is wider aft of midships than forward of midships.

The curves of form assume the ship is level with no trim, but they may be used for a ship in a trimmed condition, so long as the trim is not too large. If the ship is trimmed, the entering argument to the curves of form is the mean draft:

$$T_{mean} = \frac{T_{fwd} + T_{aft}}{2}$$

The goal of a longitudinal problem is to determine the final drafts forward and aft given the initial drafts and a description of the weight shifts, weight additions, and weight removals that occurred.

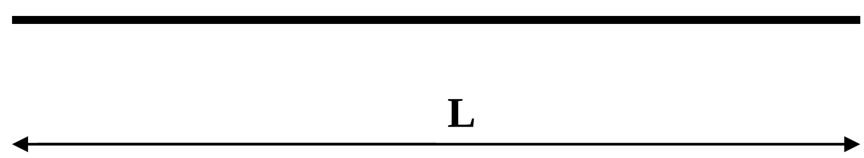
It is helpful in the modeling process to physically visualize the weight shift occurring. Picture a large wooden crate on the weather deck of a ship that is being pushed more forward or more aft. Try to predict if the ship will go down by the bow or go down by the stern from your mental picture.

- Notice it doesn't matter what position the crate starts from on the ship only that it moves forward or aft.
- Remember to visualize the weight shift. Pushing a weight forward makes the bow go down and the forward draft increase. Pushing a weight aft makes the stern go down and the after draft increase. Use this knowledge to determine when to add to or subtract from a draft. Additionally, test your final answer for reasonability and consistency. If you add weight forward, the bow's draft is likely to increase, and so on.

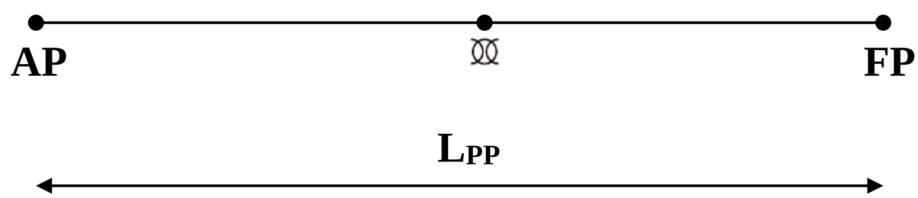
3.6.1 Trim Diagram

To quantify the changes in the forward and after drafts from a weight change requires an engineering analysis of the process. The analysis starts by developing a picture that shows all the geometric relationships that exist. This picture is developed logically in a step wise procedure.

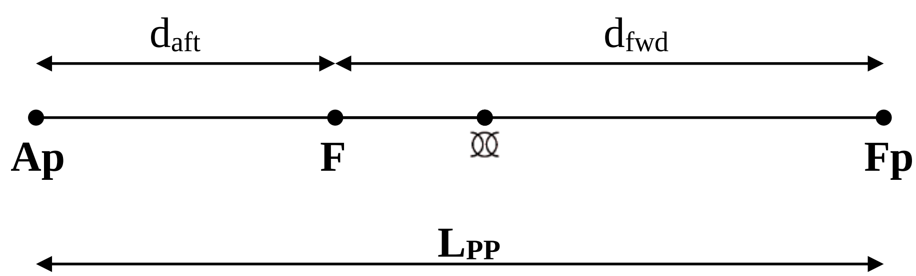
1. Draw a single horizontal line that represents the waterplane of the ship from the sheer plan view. The length of the line represents the length of the ship.



2. Decide which end is the bow and which is the stern, label them. Show the midpoint of the line and label it as midships.

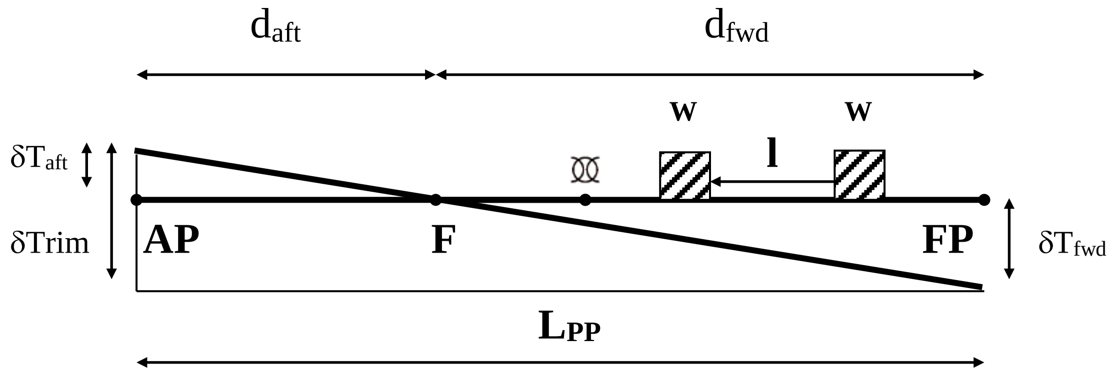


3. Show the center of flotation (F) and label it. Normally assume it is located aft of midships. Dimension and label the distances from the AP to the center of flotation (d_{aft}) and the FP to the center of flotation (d_{fwd}).



4. Show the weight change that is occurring and the new waterplane that would exist after the weight change. To draw this correctly simply rotate your paper in a clockwise or counter clockwise direction and draw a horizontal line through the center of flotation. By rotating your paper you have the advantage of simulating the bow or the stern going down and the water surface remaining level with the bottom of your desk.

In this example we will consider a weight shifted more aft.



5. Put your paper level again. Any distance above the first waterline is positive and any distance below is negative. According to this convention the after draft increased by a positive number which is consistent with what actually happens when weight is shifted more aft. Draw vertical lines from the ends of the first waterline to the second waterline forming two similar triangles. Label those vertical distances with "δT_{aft}" and "δT_{fwd}."

6. Form the third similar triangle by drawing a third waterline parallel to the first and starting with the upper or lower most draft. The vertical leg of this third largest triangle should be labeled "δTrim" since the change in trim is equal to the change in draft aft minus the change in draft forward (See note 3 below). Label the angle of trim with the symbol "θ." Avoid using "ϕ" since that is used to express angles of rotation in the transverse direction.

Each time a change in trim problem is performed, this diagram must be completed in full. All the expressions that follow can only be written if you have a diagram.

Note 1: Notice what happens to the change in trim when the ship goes down by the stern. The change in draft aft is positive and the change in draft forward is negative. You're subtracting a positive number minus a negative number to get a larger positive number. This is consistent with the idea that trim down by the stern is positive by convention.

Note 2: It is really not necessary to follow all the sign conventions in a formal sense if you use your diagram and a little common sense. The procedure has been written very formally here to show you that the sign conventions and definitions are consistent throughout.

Note 3: The following is the derivation of the "δtrim" equation. Recall a change in a property is always the final value of the property minus the initial value of the property. You can always find a change in any parameter using this definition.

3.6.2 Trim Calculation

The starting equation to calculate the final draft forward or aft is based on an accounting concept. To find the final balance in a bank account you need to start with the initial balance, add the receipts and subtract the debits. Similarity, the final draft forward (or aft) is equal to the initial draft forward (or aft) minus any decreases in the draft forward (or aft), plus any increases in the draft forward (or aft).

$$T_{fwd\ new} = T_{fwd\ old} \pm \delta T_{fwd\ due\ to\ trimming\ moment} \pm \delta T_{fwd\ due\ to\ parallel\ rise\ or\ sinkage}$$

$$T_{aft\ new} = T_{aft\ old} \pm \delta T_{aft\ due\ to\ trimming\ moment} \pm \delta T_{aft\ due\ to\ parallel\ rise\ or\ sinkage}$$

We have discussed one way for the drafts to change, by a shift in a weight which creates a moment about the center of flotation ($\delta T_{\text{fwd due to wl}}$ or $\delta T_{\text{aft due to wl}}$). There are other ways to change the drafts forward or aft, specifically by adding and/or removing weight. First, we will go over a single weight shift and then discuss adding and/or removing weight.

To decide if the change in draft forward should be added or subtracted refer to your trim diagram and common sense. For example shifting weight forward increases the forward draft so the change in draft forward should be added making the final draft larger than the initial. Let's call this first equation the "accounting equation." It is shown by the preceding equations for the final forward draft and the final after draft.

- The first term these equations are the initial drafts. These are typically given as an initial condition of the problem. Most of your problems start even keel (zero list, zero trim) and $T_{\text{mean}} = T_{\text{fwd}} = T_{\text{aft}}$. This is not *always* the case.

- The second term in these equations must be calculated by using the similar triangles shown by the diagram previously developed.

- The third term in these equations will be found by dividing the weight added or removed by the TPI.

- By looking at the trim diagram we can develop the following equation from the similar triangles.

$$\frac{\delta T_{aft\ due\ to\ wl}}{d_{aft}} = \frac{\delta T_{fwd\ due\ to\ wl}}{d_{fwd}} = \frac{\delta Trim}{\mathrm{L_{PP}}}$$

The magnitudes of the distances shown above are evident in the trim diagram. If we can find the magnitude of the "δtrim" parameter, we can solve for both the change in draft aft and forward due to the trimming moment, "wl".

The change in trim is found by dividing the moment creating the change in trim (wl) by a parameter called MT1". The MT1" has unit of LT-ft per inch and is on the curves of form as a function of mean draft.

$$\delta Trim = \frac{w\,l}{MT1''}$$

Where:	w	=	weight added/removed or shifted to produce the trim moment (LT)
	MT1”	=	moment to change trim 1” (LT·ft/in)
	l	=	for a weight shift, the total longitudinal distance that weight “w” is shifted (ft) -or- for a weight addition/removal, the distance the weight lies from F (F acts as the fulcrum for the trim problem) (ft)

At this point you are ready to do any weight shift problem by drawing your picture and solving for the unknowns. Note for a weight shift problem the last term in “accounting” trim equations is zero.

Weight additions or removals are modeled as a two step process.

- For a weight addition, step one is to assume the weight is added at the center of flotation. Step two is to assume the weight is moved from the center of flotation to the resting position of the weight.

- For a weight removal, step one is to assume the weight is shifted from its resting position to the center of flotation. Step two is to assume the weight is removed from the center of flotation.

Weight additions require you to do all the work that you would do for a weight shift problem and to do one additional calculation. The additional calculation is to find the *change in draft aft or forward due to adding or removing weight at the center of flotation*. Since the center of flotation is at the pivot point of a floating ship, adding or removing weight at this location only causes the ship to sink or rise in a “parallel” fashion. In other words, there will be no change in trim, the after and forward drafts will change by the same amount. The resulting waterline, after the addition or removal of weight from the center of flotation, is parallel to the original waterline. This occurrence is called “parallel change” or in the case of weight addition “parallel sinkage.”

- The change in draft aft or forward due to adding or removing weight at the center of flotation (δT_{PS}) can be found as shown below and it is the last term in “accounting” trim equation.

$$\delta T_{PS} = \frac{w}{TPI}$$

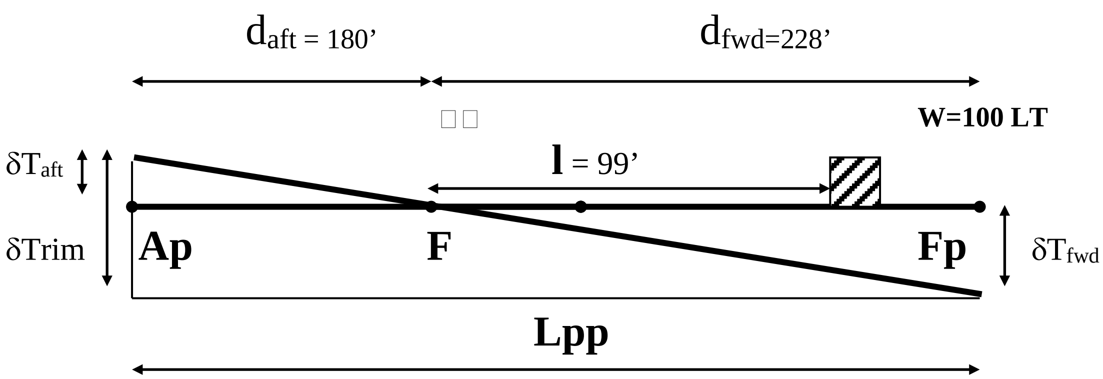
Where: δT_{PS} is the change in draft due to parallel sinkage, adding or removing weight (in)

w is the amount of weight added or removed at the center of flotation (LT)

TPI is the tons per inch immersion conversion factor (LT/in)

Exercise 3.5: An FFG7 is originally at a draft of 16.25 ft in level trim. 100 LT are removed from a location 75 ft forward of amidships. What are the final forward and after drafts? An FFG7 is 408 ft long and has the following characteristics:

T (ft)	Δ (LT)	TPI (LT/in)	MT1" (ft-LT/in)	LCF (ft) aft amidships
16.00	3992	33.0	793.4	24.03
16.25	4092	33.2	800.7	24.09



$$\delta T_{PS} = \frac{w}{TPI} = \frac{100\ LT}{33.2\ LT/in}$$

$$\delta T_{PS} = 3.01\ in = 0.25\ ft$$

$$\delta Trim = \frac{wl}{MT1''} = \frac{100\ LT \cdot 99\ ft}{800\ LT - ft/in}$$

$$\delta Trim = 12.38\, in = 1.03\ ft$$

$$\frac{\delta trim}{Lpp} = \frac{\delta T_{aft}}{d_{aft}} = \frac{\delta T_{fwd}}{d_{fwd}} \Rightarrow \quad \delta T_{aft} = \delta Trim\ \frac{d_{aft}}{Lpp} = 1.03\ ft\ \frac{180\ ft}{408\ ft} = 0.45\ ft$$

$$\delta T_{fwd} = \delta Trim\ \frac{d_{fwd}}{Lpp} = 1.03\ ft\ \frac{228\ ft}{408\ ft} = 0.58\ ft$$

$$T_{aft\ new} = T_{aft\ old} - \delta T_{PS} + \delta T_{aft} = 16.25\ ft - 0.25\ ft + 0.45\ ft = 16.45\ ft$$
$$T_{fwd\ new} = T_{fwd\ old} - \delta T_{PS} - \delta T_{fwd} = 16.25\ ft - 0.25\ ft - 0.58\ ft = 15.42\ ft$$

3.7 Correction to Displacement for Trim (Optional)

The curves of form are calculated assuming a ship with zero trim. So long as the trim is not significant, most of the quantities found will be sufficiently accurate.

Since the entering argument for the curves of form is mean draft, it will be useful to see what the effect of trim is on the displacement gained from the curves of form. The LCF is normally aft of amidships. If the ship trims by the stern, then the mean draft will be less than if the ship were in level trim. Therefore, you will enter the curves at a smaller draft and read a displacement smaller than the actual displacement.

$$\delta\Delta = \Delta_{T_{mean}} + (\delta\Delta_{1\,ft})(Trim)$$

The correction to displacement for trim is made in the following manner:

where: $\delta\Delta$ is the correction to displacement

Δ_{Tmean} is the displacement read from the curves of form at the mean draft

$\delta\Delta_{1ft}$ is the correction to displacement for a 1 ft trim read at T_{mean} on the curves of form

Trim is the difference between the fore and aft drafts.

Example 3.5 DDG51 has a mean draft of 20.75 ft and is trimming 1.5 ft by the stern. What is the displacement?

Draft (T)	Displacement Δ	Corr. to Disp. for 1 ft Trim
20.75 ft	8443 LT	31.1 LT/ft

Solution:

$\delta\Delta$ = (31.1 LT/ft)(1.5 ft) = 46.7 LT

Δ = 8443 LT + 46.7 LT = 8490 LT

3.8 Drydocking

Due to the nature and complexity of repair and maintenance that must be performed on the underwater hull, openings, and sea-connected systems of ships, it is often necessary to perform this work in a drydock. The object of drydocking is to properly support the ship while it is out of the water. There are three distinct phases to drydocking: preparation, docking, and undocking. An error during any phase may lead to catastrophe: ship tilting, hull structural damage, damage to appendages, and possibly, personnel injury.

- Preparation is critical to the success of all phases. The Dockmaster and Docking Officer must carefully evaluate the type of ship to be docked and where to place the supports on the ship. This task is accomplished by evaluating the ship's lines plans, structural drawings, and all of the underwater appendages on the ship. A Predocking Conference is held between the drydock and the ship to discuss plans, responsibilities, and procedures.

- Docking is a slow, closely orchestrated evolution. Once the dock is flooded above the blocks and the Docking Officer is ready, the ship is carefully pushed and/or pulled into the dock by tugs, workboats and dockside lines. Once the ship is in the correct position over the blocks (this is often verified by divers) pumping of the drydock can commence. Landing the ship on the blocks is a critical step in this evolution and as such, it is carefully approached. As the ship lands (usually stern first), part of the ship is supported by the blocks (P) and part of the ship is supported by the buoyant force. This causes a virtual rise in the center of gravity and a decreased metacentric height.

$$\overline{G_v M_T} = \overline{KM_T} - \frac{\overline{KG} \cdot \Delta}{\Delta - P}$$

Where:
G_vM_T = virtual metacentric height of ship at current waterline (ft)
P = upward force exerted by the keel blocks (LT)
KM_T = distance from keel to meta-center at the current waterline (ft)
KG = distance from keel to center of gravity (ft)
Δ = displacement of waterborne ship at current waterline (LT)

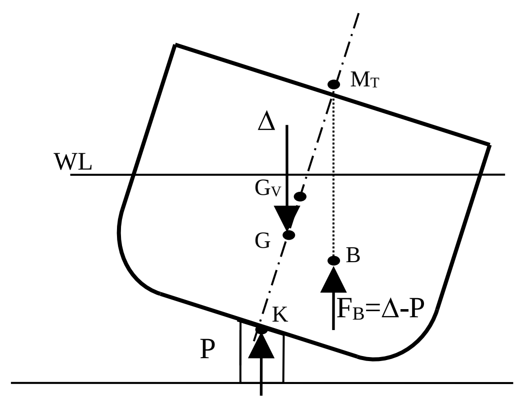


Figure 3.16 Stability in Drydock

If a list develops as the ship lands and continues to increase, pumping operations are stopped until the cause is found and corrected. There is a possibility during landing that the ship may develop a negative metacentric height and capsize (this will be explained more in Chapter 4). If all goes well, the ship lands on the blocks and work can start.

- Undocking can be just a precarious as the docking phase if not done carefully. Additionally, the hull and its openings must be tested for watertight integrity before the ship is floated and leaves the dock. Undocking follows the same basic procedure as docking, but in reverse.

APPENDIX A

TABLE of FRESH and SALT WATER DENSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Mass Density ρ for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

Density of fresh water ρ, $lb\text{-}sec^2/ft^4$ (= slugs/ft^3)	Temp, deg F	Density of salt water ρ_s, $lb\text{-}sec^2/ft^4$	Density of fresh water ρ, $lb\text{-}sec^2/ft^4$	Temp, deg F	Density of salt water ρ_s, $lb\text{-}sec^2/ft^4$
1.9399	32	1.9947	1.9384	59	1.9905
1.9399	33	1.9946	1.9383	60	1.9903
1.9400	34	1.9946	1.9381	61	1.9901
1.9400	35	1.9945	1.9379	62	1.9898
1.9401	36	1.9944	1.9377	63	1.9895
1.9401	37	1.9943	1.9375	64	1.9893
1.9401	38	1.9942	1.9373	65	1.9890
1.9401	39	1.9941	1.9371	66	1.9888
1.9401	40	1.9940	1.9369	67	1.9885
1.9401	41	1.9939	1.9367	68	1.9882
1.9401	42	1.9937	1.9365	69	1.9879
1.9401	43	1.9936	1.9362	70	1.9876
1.9400	44	1.9934	1.9360	71	1.9873
1.9400	45	1.9933	1.9358	72	1.9870
1.9399	46	1.9931	1.9355	73	1.9867
1.9398	47	1.9930	1.9352	74	1.9864
1.9398	48	1.9928	1.9350	75	1.9861
1.9397	49	1.9926	1.9347	76	1.9858
1.9396	50	1.9924	1.9344	77	1.9854
1.9395	51	1.9923	1.9342	78	1.9851
1.9394	52	1.9921	1.9339	79	1.9848
1.9393	53	1.9919	1.9336	80	1.9844
1.9392	54	1.9917	1.9333	81	1.9841
1.9390	55	1.9914	1.9330	82	1.9837
1.9389	56	1.9912	1.9327	83	1.9834
1.9387	57	1.9910	1.9324	84	1.9830
1.9386	58	1.9908	1.9321	85	1.9827
			1.9317	86	1.9823

NOTE: For other salinities, interpolate linearly.

APPENDIX B

TABLE of FRESH and SALT WATER KINEMATIC VISCOSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Kinematic Viscosity ν for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

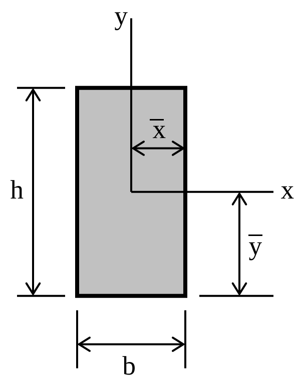
Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$	Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$
1.9231	32	1.9681	1.2260	59	1.2791
1.8871	33	1.9323	1.2083	60	1.2615
1.8520	34	1.8974	1.1910	61	1.2443
1.8180	35	1.8637	1.1741	62	1.2275
1.7849	36	1.8309	1.1576	63	1.2111
1.7527	37	1.7991	1.1415	64	1.1951
1.7215	38	1.7682	1.1257	65	1.1794
1.6911	39	1.7382	1.1103	66	1.1640
1.6616	40	1.7091	1.0952	67	1.1489
1.6329	41	1.6807	1.0804	68	1.1342
1.6049	42	1.6532	1.0660	69	1.1198
1.5777	43	1.6263	1.0519	70	1.1057
1.5512	44	1.6002	1.0381	71	1.0918
1.5254	45	1.5748	1.0245	72	1.0783
1.5003	46	1.5501	1.0113	73	1.0650
1.4759	47	1.5259	0.9984	74	1.0520
1.4520	48	1.5024	0.9857	75	1.0392
1.4288	49	1.4796	0.9733	76	1.0267
1.4062	50	1.4572	0.9611	77	1.0145
1.3841	51	1.4354	0.9492	78	1.0025
1.3626	52	1.4142	0.9375	79	1.9907
1.3416	53	1.3935	0.9261	80	0.9791
1.3212	54	1.3732	0.9149	81	0.9678
1.3012	55	1.3535	0.9039	82	0.9567
1.2817	56	1.3343	0.8931	83	0.9457
1.2627	57	1.3154	0.8826	84	0.9350
1.2441	58	1.2970	0.8722	85	0.9245
			0.8621	86	0.9142

NOTE: For other salinities, interpolate linearly.

APPENDIX C

PROPERTIES of COMMON GEOMETRIC SHAPES

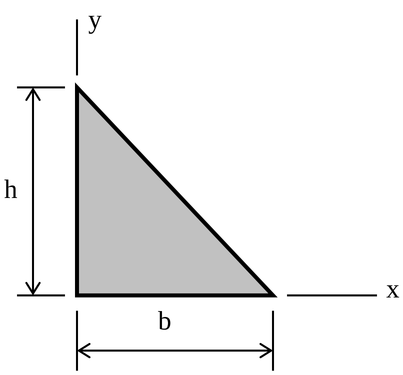
Rectangle (origin of axes at centroid)



$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

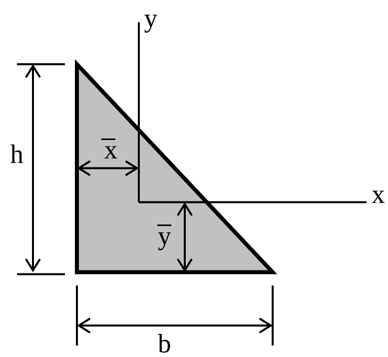
$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

Right Triangle (origin of axes at vertex)



$$A = \frac{bh}{2} \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

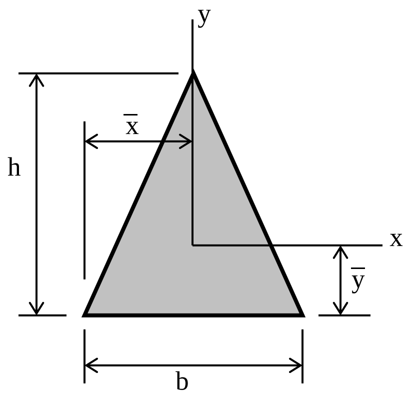
Right Triangle (origin of axes at centroid)



$$\bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36}$$

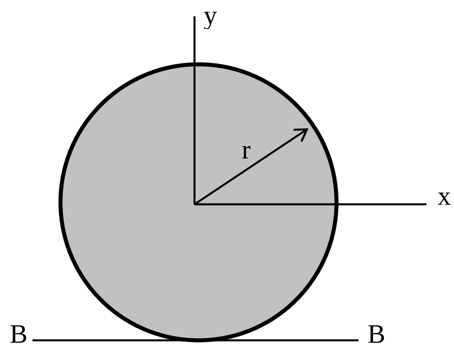
Isosceles Triangle (origin of axes at centroid)



$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48}$$

Circle (origin of axes at center)

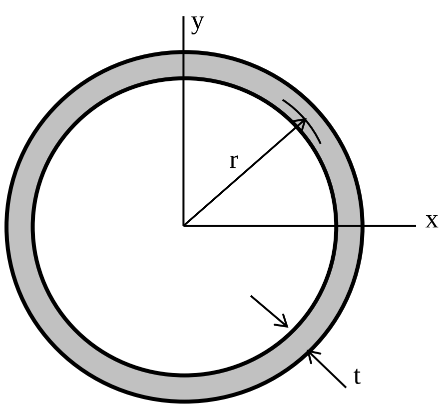


$$d = 2r \qquad A = \pi r^2 = \frac{\pi d^2}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

Circular Ring with thickness " *t* " (origin of axes at center)
Approximate formulas for the case when *t* is small



$$A = 2\pi rt = \pi dt$$

$$I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$