ENGINEERING-PDH.com
ONLINE CONTINUING EDUCATION

CLASSICAL PHYSICS - VOL 1 OF 2

Main Category:	General Engineering
Sub Category:	Physics
Course #:	ENG-110
Course Content:	62 pgs
PDH/CE Hours:	5

OFFICIAL COURSE/EXAM

(SEE INSTRUCTIONS ON NEXT PAGE)

WWW.ENGINEERING-PDH.COM
TOLL FREE (US & CA): 1-833-ENGR-PDH (1-833-364-7734)
SUPPORT@ENGINEERING-PDH.COM

ENG-110 EXAM PREVIEW

- TAKE EXAM! -

Instructions:

- At your convenience and own pace, review the course material below. When ready, click "Take Exam!" above to complete the live graded exam. (Note it may take a few seconds for the link to pull up the exam.) You will be able to re-take the exam as many times as needed to pass.
- Upon a satisfactory completion of the course exam, which is a score of 70% or better, you will be provided with your course completion certificate. Be sure to download and print your certificates to keep for your records.

Exam Preview:

1. According to the reference material, weight (a derived unit, not a fundamental unit) is a measurement that describes the force of gravity on the "mass" of an object.
    a. True
    b. False
2. Using TABLE 5 Approximate Masses of Familiar Objects, which of the following objects has an approximate mass of 2×10^5 kilograms?
    a. Quart of water
    b. Earth
    c. Car
    d. House
3. According to the reference material, the conversion factor for lbm to kg is 1 lbm = 0.5435 kg.
    a. True
    b. False
4. Most physical quantities have units that are combinations of the three fundamental dimensions of length, mass, and time. Which of the following derived units matches the description: the product of three lengths (e.g., length x width x depth for a rectangular solid); thus, it has the units of length cubed, such as cubic inches ($in.^3$) or cubic meters (m^3)?
    a. Velocity
    b. Area
    c. Volume
    d. Density

5. According to the reference material, the component addition method refers to the addition of vector coordinates on a rectangular (x,y) coordinate system.
   a. True
   b. False
6. Using Figure 5 Display Vectors Graphically – Direction, and the surrounding reference material, which cardinal direction is described by a degree measurement of 270°?
   a. North
   b. South
   c. West
   d. East
7. According to the reference material, vector components are added to determine the magnitude and direction of the resultant. Calculations using trigonometric functions are the most accurate method for making this determination.
   a. True
   b. False
8. According to the reference material, trigonometry may also be used to determine vector components. Which of the following trigonometric functions is used to find the X component of a resultant vector?
   a. Cotangent
   b. Tangent
   c. Cosine
   d. Sine
9. According to the reference material, typical examples of vector quantities are time, speed, temperature, and volume.
   a. True
   b. False
10. The SI system is made up of two related systems, the meter-kilogram-second (MKS) system and the centimeter-gram-second (CGS) system. Which of the following standard units of measure belongs to the CGS system?
    a. Meter
    b. Milligram
    c. Hour
    d. Centimeter

VOL 1 OF 2

Module 1 - Unit Systems

TABLE OF CONTENTS

LIST OF TABLES

REFERENCES

- Beiser, Arthur, Applied Physics Schaums Outline Series, McGraw-Hill Book Company, 1976.
- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 Given appropriate conversion tables, **CONVERT** between English and SI system units of measurement.

ENABLING OBJECTIVES

1.1 **DEFINE** the three fundamental dimensions: length, mass, and time.

1.2 **LIST** standard units of the fundamental dimensions for each of the following systems:
   a. International System of Units (SI)
   b. English System

1.3 **DIFFERENTIATE** between fundamental and derived measurements.

1.4 Given appropriate conversion tables, **CONVERT** between English and SI units of length.

1.5 Given appropriate conversion tables, **CONVERT** between English and SI units of mass.

1.6 **CONVERT** time measurements between the following:
   a. Years
   b. Weeks
   c. Days
   d. Hours
   e. Minutes
   f. Seconds

FUNDAMENTAL DIMENSIONS

Length, mass, and time are the three fundamental dimensions on which the measurement of all physical quantities is dependent.

EO 1.1 DEFINE the three fundamental dimensions: length, mass, and time.

EO 1.2 LIST standard units of the fundamental dimensions for each of the following systems:
- **a. International System of Units (SI)**
- **b. English System**

EO 1.3 DIFFERENTIATE between fundamental and derived measurements.

Fundamental Dimensions

Physics is a science based upon exact measurement of physical quantities that are dependent upon three fundamental dimensions. The three fundamental or primary dimensions are *mass*, *length,* and *time*. These three fundamental units must be understood in order to lay the foundation for the many concepts and principles presented in this material.

Mass

Mass is the amount of material present in an object. This measurement describes "how much" material makes up an object. Often, mass and weight are confused as being the same because the units used to describe them are similar. Weight (a derived unit, not a fundamental unit) is a measurement that describes the force of gravity on the "mass" of an object.

Length

Length is the distance between two points. The concept of length is needed to locate the position of a point in space and thereby describe the size of a physical object or system. When measuring a length of pipe, the ends of the pipe are the two points and the distance between the two points is the length. A typical unit used to describe length is the "foot."

Time

Time is the duration between two instants. The measurement of time is described in units of seconds, minutes, or hours.

Units

A number alone is not sufficient to describe a physical quantity. For example, to say that "a pipe must be 4 long to fit" has no meaning unless a unit of measurement for length is also specified. By adding units to the number, it becomes clear, "a pipe must be 4 feet long to fit."

The unit defines the magnitude of a measurement. If we have a measurement of length, the unit used to describe the length could be a foot or yard, each of which describes a different magnitude of length. The importance of specifying the units of a measurement for a number used to describe a physical quantity is doubly emphasized when it is noted that the same physical quantity may be measured using a variety of different units. For example, length may be measured in meters, inches, miles, furlongs, fathoms, kilometers, or a variety of other units.

Units of measurement have been established for use with each of the fundamental dimensions mentioned previously. The following section describes the unit systems in use today and provides examples of units that are used in each system.

Unit Systems

There are two unit systems in use at the present time, English units and International System of Units (SI).

In the United States, the English system is currently used. This system consists of various units for each of the fundamental dimensions or measurements. These units are shown in Table 1.

TABLE 1
English Units of Measurement

Length	Mass	Time
Inch	Ounce	* Second
* Foot	* Pound	Minute
Yard	Ton	Hour
Mile		Day
		Month
		Year

* Standard unit of measure

The English system is presently used in the field of engineering and throughout the United States. The foot-pound-second (FPS) system is the usual unit system used in the U.S. when dealing with physics.

Over the years there have been movements to standardize units so that all countries, including the United States, will adopt the SI system. The SI system is made up of two related systems, the meter-kilogram-second (MKS) system and the centimeter-gram-second (CGS) system.

The MKS and CGS systems are much simpler to use than the English system because they use a decimal-based system in which prefixes are used to denote powers of ten. For example, one kilometer is 1000 meters, and one centimeter is one one-hundredth of a meter. The English system has odd units of conversion. For example, a mile is 5280 feet, and an inch is one twelfth of a foot.

The MKS system is used primarily for calculations in the field of physics while both the MKS and CGS systems are used in the field of chemistry. The units for each of these systems are shown in Tables 2 and 3 below.

TABLE 2
MKS Units of Measurement

Length	**Mass**	**Time**
Millimeter	Milligram	* Second
* Meter	Gram	Minute
Kilometer	* Kilogram	Hour
		Day
		Month
		Year

* Standard unit of measure

TABLE 3
CGS Units of Measurement

Length	**Mass**	**Time**
* Centimeter	Milligram	* Second
Meter	* Gram	Minute
Kilometer	Kilogram	Hour
		Day
		Month
		Year

* Standard unit of measure

The following tables show approximate lengths, masses, and times for some familiar objects or events.

TABLE 4
Approximate Lengths of Familiar Objects

Object	Length (meters)
Diameter of Earth Orbit Around Sun	2×10^{11}
Football Field	1×10^{2}
Diameter of Dime	2×10^{-2}
Thickness of Window Pane	1×10^{-3}
Thickness of Paper	1×10^{-4}

TABLE 5
Approximate Masses of Familiar Objects

Object	Mass (kilograms)
Earth	6×10^{24}
House	2×10^{5}
Car	2×10^{3}
Quart of Water	1
Dime	3×10^{-3}
Postage Stamp	5×10^{-8}

TABLE 6
Approximate Times of Familiar Events

Event	Time (seconds)
Age of Earth	2×10^{17}
Human Life Span	2×10^{9}
Earth Rotation Around Sun	3×10^{7}
Earth Rotation Around Axis	8.64×10^{4}
Time Between Heart Beats	1

Derived Measurements

Most physical quantities have units that are combinations of the three *fundamental* dimensions of length, mass, and time. When these dimensions or measurements are combined, they produce what are referred to as *derived* units. This means that they have been "derived" from one or more fundamental measurements. These combinations of fundamental measurements can be the combination of the same or different units. The following are examples of various derived units.

Area

Area is the product of two lengths (e.g., width x length for a rectangle); thus, it has the units of length squared, such as square inches ($in.^2$) or square meters (m^2).

$1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$

$4 \text{ in.} \times 2 \text{ in.} = 8 \text{ in.}^2$

Volume

Volume is the product of three lengths (e.g., length x width x depth for a rectangular solid); thus, it has the units of length cubed, such as cubic inches ($in.^3$) or cubic meters (m^3). The MKS and CGS unit systems have a specific unit for volume called the liter (l). One liter is equal to 1000 cubic centimeters (1 l = 1000 cm^3).

$2 \text{ in.} \times 3 \text{ in.} \times 5 \text{ in.} = 30 \text{ in.}^3$

Density

Density is a measure of the mass of an object per unit volume; thus, it has units of mass divided by length cubed such as kilograms per cubic meter (kg/m^3) or pounds per cubic foot (lbs/ft^3).

$$15 \text{ lbs}/5 \text{ ft}^3 = 3 \text{ lbs/ft}^3$$

Velocity

Velocity is the change in length per unit time; thus, it has units such as kilometers per hour (km/h) or feet per second (ft/s).

Acceleration

Acceleration is a measure of the change in velocity or velocity per unit time; thus, it has units such as centimeters per second per second (cm/s^2) or feet per second per second (ft/s^2).

Summary

The main points of this chapter are summarized below.

Fundamental Dimensions Summary

The fundamental measurements consist of:

- Length - distance between two points
- Mass - amount of material in an object
- Time - duration between two instants

The English system of units is based on the following standard units:

- Foot
- Pound
- Second

The SI system of measurement consists of the following standard units:

MKS	CGS
• Meter	• Centimeter
• Kilogram	• Gram
• Second	• Second

Derived units are made up of a combination of units to describe various physical quantities. For example:

- Area - square inches (in.^{2})
- Volume - cubic inches (in.^{3}) or liters
- Density - mass per volume (lb/in.^{3})

UNIT CONVERSIONS

In order to apply measurements from the SI system to the English system, it is necessary to develop relationships of known equivalents (conversion factors). These equivalents can then be used to convert from the given units of measure to the desired units of measure.

EO 1.4 CONVERT between English and SI units of length.

EO 1.5 CONVERT between English and SI units of mass.

EO 1.6 CONVERT time measurements between the following:

- **a. Years**
- **b. Weeks**
- **c. Days**
- **d. Hours**
- **e. Minutes**
- **f. Seconds**

Personnel at DOE nuclear facilties are often exposed to both the English and SI systems of units in their work. In some cases, the measurements that are taken or read from an instrument will be different from those required by a procedure. This situation will require the conversion of measurements to those required by the procedure.

Conversion Factors

Conversion factors are based on relationships of equivalents from different measurement systems. These conversion factors are then applied to the given measurement in order to convert it to the units that are required. The equivalent relationships between different units of measurement are defined in conversion tables. Some examples from conversion tables are given below.

- 1 yard = .9144 meters
- 1 kilogram = 2.205 pounds mass (lbm)
- 1 hour = 3600 seconds

A typical conversion table is shown in Table 7.

TABLE 7
Conversion Table

Length	1 yd	=	0.9144 m
	12 in.	=	1 ft
	5280 ft	=	1 mile
	1 m	=	3.281 ft
	1 in.	=	0.0254 m
Time	60 sec	=	1 min
	3600 sec	=	1 hr
Mass	1 lbm	=	0.4535 kg
	2.205 lbm	=	1 kg
	1 kg	=	1000 g
Area	1 ft^2	=	144 $in.^2$
	10.764 ft^2	=	1 m^2
	1 yd^2	=	9 ft^2
	1 $mile^2$ =	3.098 X 10^6 yd^2	
Volume	7.48 gal	=	1 ft^3
	1 gal	=	3.785 l (liter)
	1 l	=	1000 cm^3

Unit Conversion

To convert from one measurement unit to another measurement unit (for example, to convert 5 feet to inches), first select the appropriate equivalent relationship from the conversion table (for this example, 1 foot = 12 inches). Conversion is basically a multiplication by 1. We can divide both sides of the equation 1 ft = 12 inches by 1 foot to obtain the following.

Then $\frac{1 \text{ ft}}{1 \text{ ft}} = \frac{12 \text{ inches}}{1 \text{ foot}}$ or $1 = \frac{12 \text{ inches}}{1 \text{ foot}}$

The relationship $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor which we can use in our example to convert 5 feet to inches.

Using the example, 5 feet is to be converted to inches. Start with the obvious equation

5 ft = 5 ft, and multiply the right hand side by $1 = \frac{12 \text{ inches}}{\text{foot}}$.

5 feet = 5 feet x 1

$$5 \text{ feet} = 5 \text{ feet} \times \frac{12 \text{ inches}}{\text{ft}} = 5 \times 12 \text{ inches} = 60 \text{ inches.}$$

Thus, 5 feet is equivalent to 60 inches.

Steps for Unit Conversion

Using the following example, we will step through the process for converting from a given set of units to a desired set of units.

Convert 795 m to ft.

Step 1. Select the equivalent relationship from the conversion table (Table 7).

1 meter = 3.281 ft

Step 2. Divide to obtain the factor 1 as a ratio $\left(\frac{\text{desired units}}{\text{present units}}\right)$.

$$1 = \frac{3.281 \text{ ft}}{1 \text{ m}}$$

Step 3. Multiply the quantity by the ratio.

$$(795 \text{ m}) \text{ x } \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = \left(\frac{795 \text{ m}}{1}\right) \text{ x } \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right)$$

= 795 x 3.281 ft

= 2608.395 ft

If an equivalent relationship between the given units and the desired units cannot be found in the conversion tables, multiple conversion factors must be used. The conversion is performed in several steps until the measurement is in the desired units. The given measurement must be multiplied by each conversion factor (ratio). After the common units have been canceled out, the answer will be in the desired units.

Example: Convert 2.91 sq miles to sq meters.

Step 1. Select the equivalent relationship from the conversion table. Because there is no direct conversion shown for square miles to square meters, multiple conversions will be necessary. For this example the following conversions will be used.

sq miles to sq yds to sq ft to sq m

1 sq mile = 3.098×10^6 sq yd

1 sq yd = 9 sq ft

10.764 sq ft = 1 sq m

Step 2. Express the relationship as a ratio (desired unit/present unit).

$$1 = \frac{3.098 \text{ x } 10^6 \text{ sq yd}}{1 \text{ sq mile}}$$

Step 3. Multiply the quantity by the ratio.

$$(2.91 \text{ sq miles}) \text{ x } \left(\frac{3.098 \text{ x } 10^6 \text{ sq yd}}{1 \text{ sq mile}}\right) = 9.015 \text{ x } 10^6 \text{ sq yd}$$

Step 4. Repeat the steps until the value is in the desired units.

$$1 = \frac{9 \text{ sq ft}}{1 \text{ sq yd}}$$

$$(9.015 \text{ x } 10^6 \text{ sq yd}) \text{ x } \left(\frac{9 \text{ sq ft}}{1 \text{ sq yd}}\right) = 8.114 \text{ x } 10^7 \text{ sq ft}$$

$$1 = \frac{1 \text{ sq m}}{10.764 \text{ sq ft}}$$

$$(8.114 \times 10^7 \text{ sq ft}) \times \left(\frac{1 \text{ sq m}}{10.764 \text{ sq ft}}\right) = \frac{(8.114 \times 10^7)(1 \text{ sq m})}{10.764}$$

$$= \frac{8.114 \times 10^7 \text{ sq m}}{10.764}$$

$$= 7.538 \times 10^6 \text{ sq m}$$

It is possible to perform all of the conversions in a single equation as long as all of the appropriate conversion factors are included.

$$(2.91 \text{ sq miles}) \times \left(\frac{3.098 \times 10^6 \text{ sq yd}}{1 \text{ sq mile}}\right) \times \left(\frac{9 \text{ sq ft}}{1 \text{ sq yd}}\right) \times \left(\frac{1 \text{ sq m}}{10.764 \text{ sq ft}}\right)$$

$$= \frac{(2.91) \times (3.098 \times 10^6)(9)(1 \text{ sq m})}{10.764}$$

$$= \frac{8.114 \times 10^7 \text{ sq m}}{10.764}$$

$$= 7.538 \times 10^6 \text{ sq m}$$

Example:

A Swedish firm is producing a valve that is to be used by an American supplier. The Swedish firm uses the MKS system for all machining. To conform with the MKS system, how will the following measurements be listed?

Valve stem	57.20 in.
Valve inlet and outlet	
I.D.	22.00 in.
O.D.	27.50 in.

Solution:

Valve stem
57.20 in. x 0.0254 m/in. = 1.453 m

Valve inlet and outlet
I.D. 22.00 x 0.0254 = 0.559 m
O.D. 27.50 x 0.0254 = 0.699 m

Examples of common conversion factors are shown in Table 8.

TABLE 8
Conversion Factors

CONVERSION FACTORS FOR COMMON UNITS OF MASS

		g	kg	t	lbm
1 gram	=	1	0.001	10^{-6}	2.2046×10^{-3}
1 kilogram	=	1000	1	0.001	2.2046
1 metric ton (t)	=	10^{6}	1000	1	2204.6
1 pound-mass (lbm)	=	453.59	0.45359	4.5359×10^{-4}	1
1 slug	=	14,594	14.594	0.014594	32.174

CONVERSION FACTORS FOR COMMON UNITS OF LENGTH

		cm	m	km	in.	ft	mi
1 centimeter	=	1	0.01	10^{-5}	0.39370	0.032808	$6.2137 \text{ X } 10^{-6}$
1 meter	=	100	1	0.001	39.370	3.2808	6.2137×10^{-4}
1 kilometer	=	10^{5}	1000	1	39,370	3280.8	0.62137
1 inch	=	2.5400	0.025400	2.5400×10^{-5}	1	0.083333	1.5783×10^{-5}
1 foot	=	30.480	0.30480	3.0480×10^{-4}	12.000	1	1.8939×10^{-4}
1 mile	=	1.6093×10^{5}	1609.3	1.6093	63,360	5280.0	1

TABLE 8 (Cont.)
Conversion Factors

CONVERSION FACTORS FOR COMMON UNITS OF TIME

		sec	min	hr
1 second	=	1	0.017	2.7×10^{-4}
1 minute	=	60	1	0.017
1 hour	=	3600	60	1
1 day	=	86,400	1440	24
1 year	=	3.15×10^{7}	5.26×10^{5}	8760

		day	year
1 second	=	1.16×10^{-5}	3.1×10^{-8}
1 minute	=	6.9×10^{-4}	1.9×10^{-6}
1 hour	=	4.16×10^{-2}	1.14×10^{-4}
1 day	=	1	2.74×10^{-3}
1 year	=	365	1

****VIDEO LEARNING****

- Click here for a 6.5 min video on Treating Units Algebraically & Dimensional Analysis -

Summary

Unit conversion is summarized below.

Unit Conversion Summary

- Conversion Tables list equivalent relationships.
- Conversion Factors are obtained by dividing to get a multiplying factor (1).

Unit Conversion Steps

- Step 1 - Select the equivalent relationship from the conversion table.
- Step 2 - Express the relationship as a conversion factor.
- Step 3 - Multiply the given quantity by the conversion factor.

Module 2 - Vectors

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

LIST OF FIGURES

LIST OF FIGURES (Cont.)

LIST OF TABLES

NONE

REFERENCES

- Beiser, Arthur, Applied Physics Schaums Outline Series, McGraw-Hill Book Company, 1976.
- Murray, Raymond and Cobb, Grover, Physics Concepts and Consequences, Prentice-Hall, Inc., 1970.
- Alexander, Joseph, et al., Physics for Engineering Technology, John Wiley and Sons, 1978.
- Sears, Francis and Zemansky, Mark, University Physics, 3rd edition, Addison-Wesley Publishing Co.
- Science and Fundamental Engineering (SAFE), Classical Physics, "Measure" Windsor, CT: Combustion Engineering, Inc., 1988.
- Academic Program for Nuclear Power Plant Personnel, Volume II, General Physics Corporation, Library of Congress Card #A 397747, April 1982.

TERMINAL OBJECTIVE

1.0 Using vectors, **DETERMINE** the net force acting on an object.

ENABLING OBJECTIVES

1.1 **DEFINE** the following as they relate to vectors:
   a. Scalar quantity
   b. Vector quantity
   c. Vector component
   d. Resultant

1.2 **DETERMINE** components of a vector from a resultant vector.

1.3 **ADD** vectors using the following methods:
   a. Graphical
   b. Component addition
   c. Analytical

SCALAR AND VECTOR QUANTITIES

Scalars are quantities that have magnitude only; they are independent of direction. Vectors have both magnitude and direction. The length of a vector represents magnitude. The arrow shows direction.

EO 1.1 DEFINE the following as they relate to vectors:
- **a. Scalar quantity**
- **b. Vector quantity**

Scalar Quantities

Most of the physical quantities encountered in physics are either scalar or vector quantities. A *scalar* quantity is defined as a quantity that has magnitude only. Typical examples of scalar quantities are time, speed, temperature, and volume. A scalar quantity or parameter has no directional component, only magnitude. For example, the units for time (minutes, days, hours, etc.) represent an amount of time only and tell nothing of direction. Additional examples of scalar quantities are density, mass, and energy.

Vector Quantities

A *vector* quantity is defined as a quantity that has both magnitude and direction. To work with vector quantities, one must know the method for representing these quantities.

Magnitude, or "size" of a vector, is also referred to as the vector's "displacement." It can be thought of as the scalar portion of the vector and is represented by the length of the vector. By definition, a vector has both magnitude and direction. Direction indicates how the vector is oriented relative to some reference axis, as shown in Figure 1.

Using north/south and east/west reference axes, vector "A" is oriented in the NE quadrant with a direction of 45º north of the EW axis. G iving direction to scalar "A" makes it a vector. The length of "A" is representative of its magnitude or displacement.

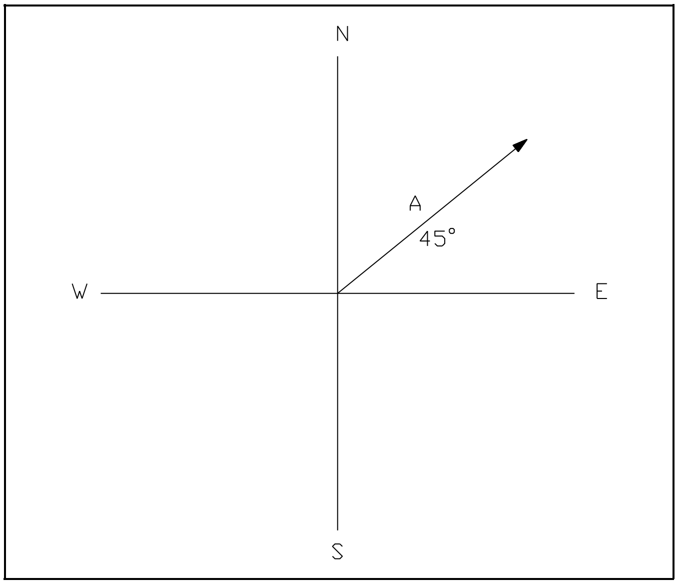

Figure 1 Vector Reference Axis

To help distinguish between a scalar and a vector, let's look at an example where the only information known is that a car is moving at 50 miles per hour. The information given (50 mph) only refers to the car's speed, which is a scalar quantity. It does not indicate the direction the car is moving. However, the same car traveling at 50 mph due east indicates the velocity of the car because it has magnitude (50 mph) and direction (due east); therefore, a vector is indicated. When a vector is diagrammed, a straight line is drawn to show the unit of length. An arrow is drawn on one end of the line. The length of the line represents the magnitude of the vector, and the arrow represents the direction of the vector.

Description of a Simple Vector

Vectors are simple straight lines used to illustrate the direction and magnitude of certain quantities. Vectors have a starting point at one end (tail) and an arrow at the opposite end (head), as shown in Figure 2.

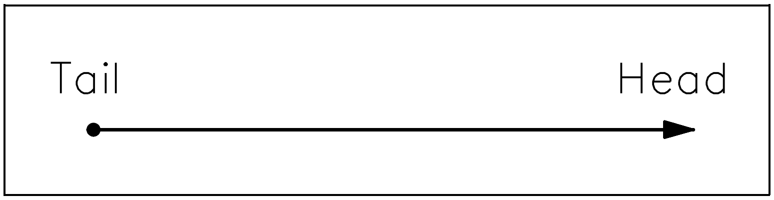

Figure 2 Vector

Examples of Vector Quantities

Displacement, velocity, acceleration, and force are examples of vector quantities. Momentum and magnetic field strength are also good examples of vector quantities, although somewhat more difficult to understand. In each of these examples, the main ingredients of magnitude and direction are present.

****VIDEO LEARNING****
- Click here for a 8.5 min video explaining the difference between Vectors & Scalars -

Summary

The important aspects of scalar and vector quantities are summarized below.

Scalar and Vector Quantities Summary

SCALAR QUANTITIES

- Magnitude only
- Independent of direction
- Examples of scalars include: time, speed, volume, and temperature

VECTOR QUANTITIES

- Both magnitude and direction
- Length represents magnitude
- Arrow shows direction
- Examples of vectors include: force, velocity, and acceleration

VECTOR IDENTIFICATION

Vectors are symbolized in specific ways in texts and on graphs, using letters or rectangular coordinates.

In Written Materials

In textbooks, vector quantities are often represented by simply using a boldfaced letter (**A, B, C, R**). Particular quantities are predefined (**F** - force, **V** - velocity, and **A** - acceleration). Vector quantities are sometimes represented by $\vec{A}, \vec{B}, \vec{C}, \vec{R}$. Regardless of the convention used, specific vector quantities must include magnitude and direction (for example, 50 mph due north, or 50 lbf at 90°).

Graphic Representation

Vector quantities are graphically represented using the rectangular coordinate system, a two-dimensional system that uses an x-axis and a y-axis. The x-axis is a horizonal straight line. The y-axis is a vertical straight line, perpendicular to the x-axis. An example of a rectangular system is shown in Figure 3.

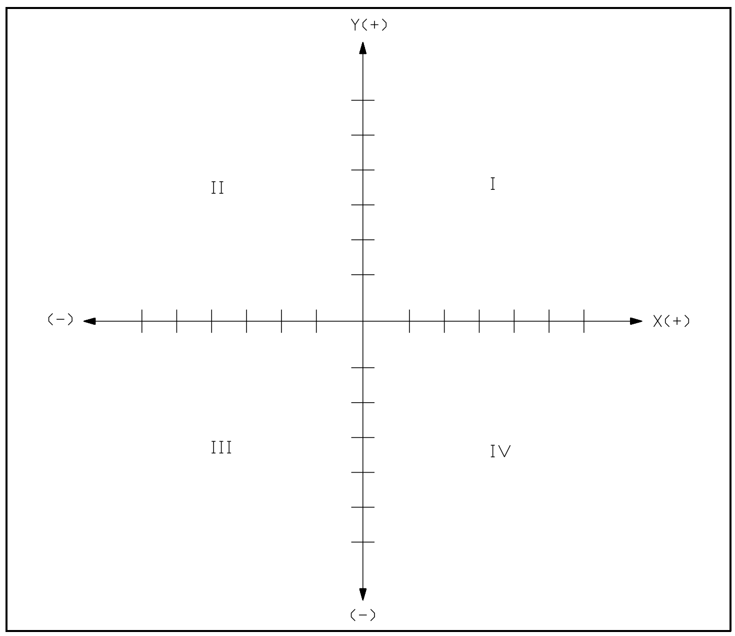

Figure 3 Rectangular Coordinate System

The intersection of the axes is called the point of origin. Each axis is marked off in equal divisions in all four directions from the point of origin. On the horizonal axis (x), values to the right of the origin are positive (+). Values to the left of the origin are negative (-). On the vertical axis (y), values above the point of origin are positive (+). Values below the origin are negative (-). It is very important to use the same units (divisions) on both axes.

The rectangular coordinate system creates four infinite quadrants. Quadrant I is located above and to the right of the origin. Quadrant II is located above and to the left of the origin. Quadrant III is situated to the left and below the origin, and quadrant IV is located below and to the right of the origin (see Figure 3).

Graphic Representation of Vectors

With the coordinate system defined, the following explanation will illustrate how to locate vectors in that system.

First, using a ruler and graph paper, a rectangular coordinate system is laid out as described in the previous section. The x- and y-axes are labeled. Equal divisions are marked off in all four directions. Those to the right and above the point of origin are labeled positive (+). Those to the left and below the point of origin are labeled negative (-).

Beginning at the point of origin (intersection of the axes), a line segment of the proper length is shown along the x-axis, in the positive direction. This line segment represents the vector magnitude, or displacement. An arrow is placed at the "head" of the vector to indicate direction. The "tail" of the vector is located at the point of origin (see Figure 4).

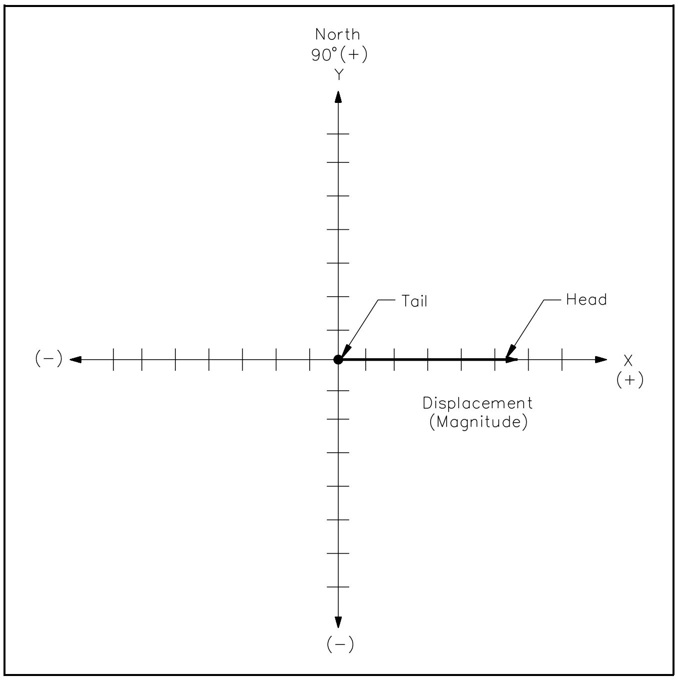

Figure 4 Displaying Vectors Graphically - Magnitude

When vectors are drawn that do not fall on the x- or y-axes, the tail is located at the point of origin. Depending on the vector description, there are two methods of locating the head of the vector. If coordinates (x,y) are given, these values can be plotted to locate the vector head. If the vector is described in degrees, the line segment can be rotated counterclockwise from the x-axis to the proper orientation, as shown in Figure 5.

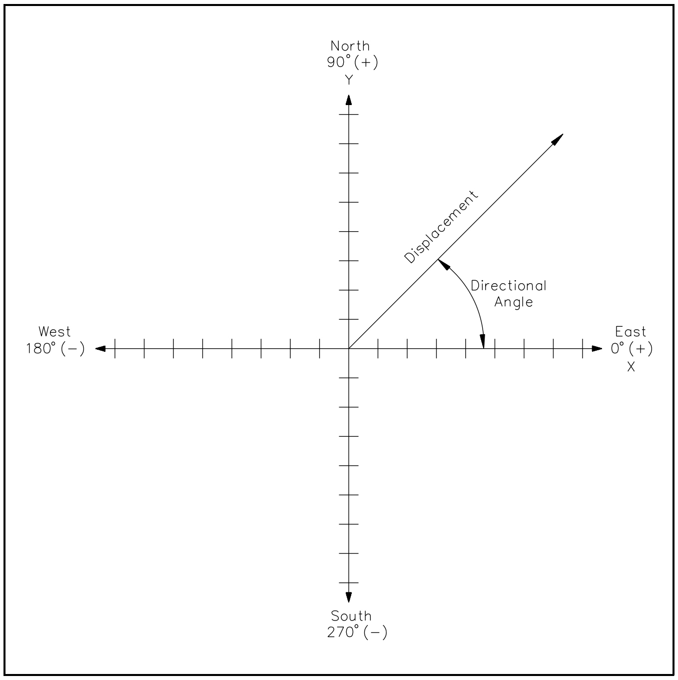

Figure 5 Display Vectors Graphically - Direction

Because the x- and y-axes define direction, conventional directional coordinates and degrees may also be used to identify the x- and y-axes (see Figures 6 and 7).

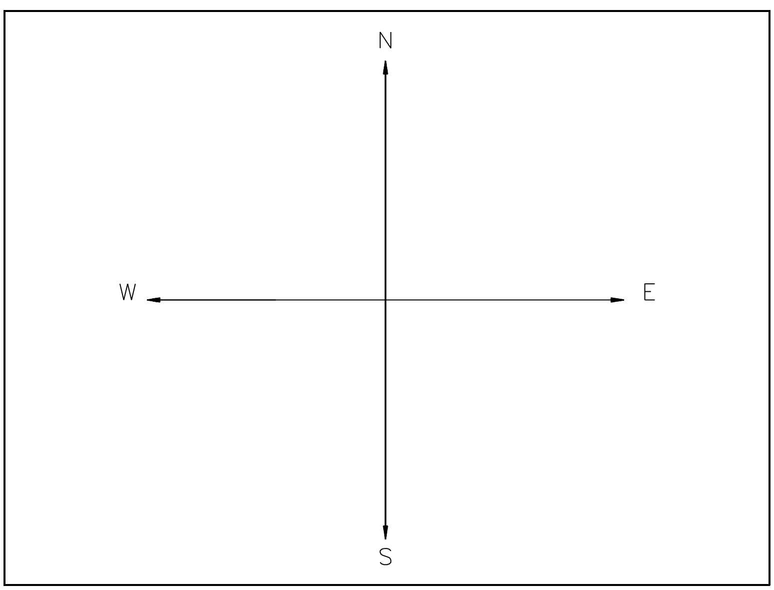

Figure 6 Directional Coordinates

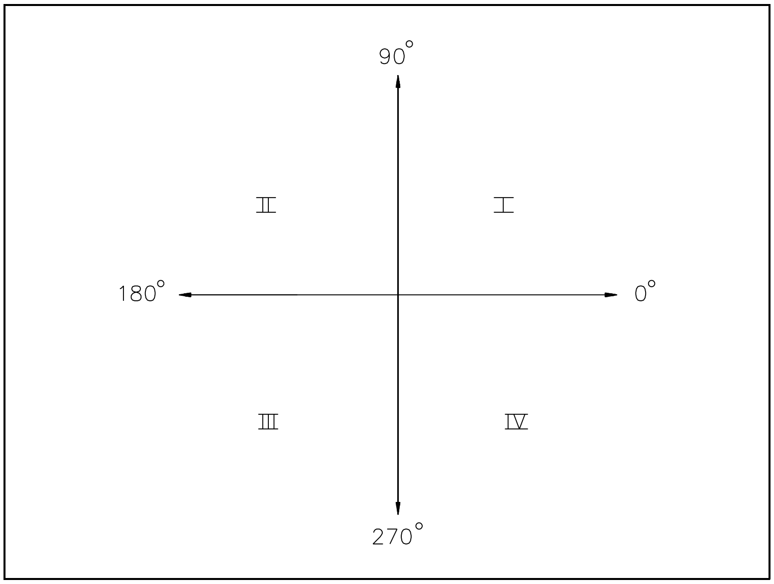

Figure 7 Degree Coordinates

Summary

The main points covered in this chapter are summarized below.

Vector Identification Summary

- In text:
  - Boldfaced letters (**A, F, R**)
  - Capital letters with arrows over ($\vec{A}$, $\vec{F}$, $\vec{R}$)
- Graphically:
  - (x,y) coordinates
  - Directional Coordinates
  - Degrees

VECTORS: RESULTANTS AND COMPONENTS

A resultant is a single vector which represents the combined effect of two or more other vectors (called components). The components can be determined either graphically or by using trigonometry.

EO 1.1 DEFINE the following as they relate to vectors:
- **c. Vector component**
- **d. Resultant**

EO 1.2 DETERMINE components of a vector from a resultant vector.

Resultant

When two or more vectors are added they yield the sum or *resultant* vector. A resultant vector is the result or sum of vector addition. Vector addition is somewhat different from addition of pure numbers unless the addition takes place along a straight line. In the latter case, it reduces to the number line of standards or scale addition. For example, if one walks five miles east and then three miles east, he is eight miles from his starting point. On a graph (Figure 8), the sum of the two vectors, i.e., the sum of the five miles plus the three mile displacement, is the total or resultant displacement of eight miles.

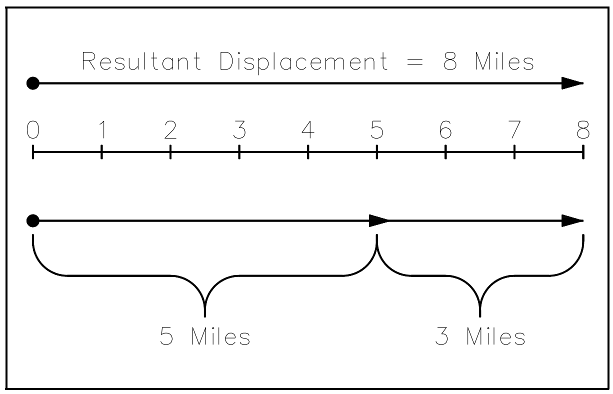

Figure 8 Vector Addition in Same Direction

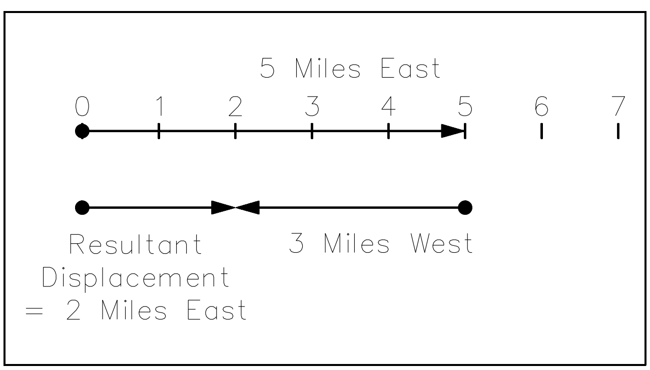

Figure 9 Vector Addition in Opposite Directions

Similarly, if one walks five miles east and then three miles west, the resultant displacement is two miles east (Figure 9).

The vector diagrams of Figure 8 and Figure 9 are basically scale diagrams of what is happening in the real world of addition of vector quantities.

Consider next the addition of vector quantities which are not in a straight line. For example, consider the resultant displacement when a person travels four miles east and then three miles north. Again a scale drawing (Figure 10) is in order. Use a scale of 1 inch = 1 mile.

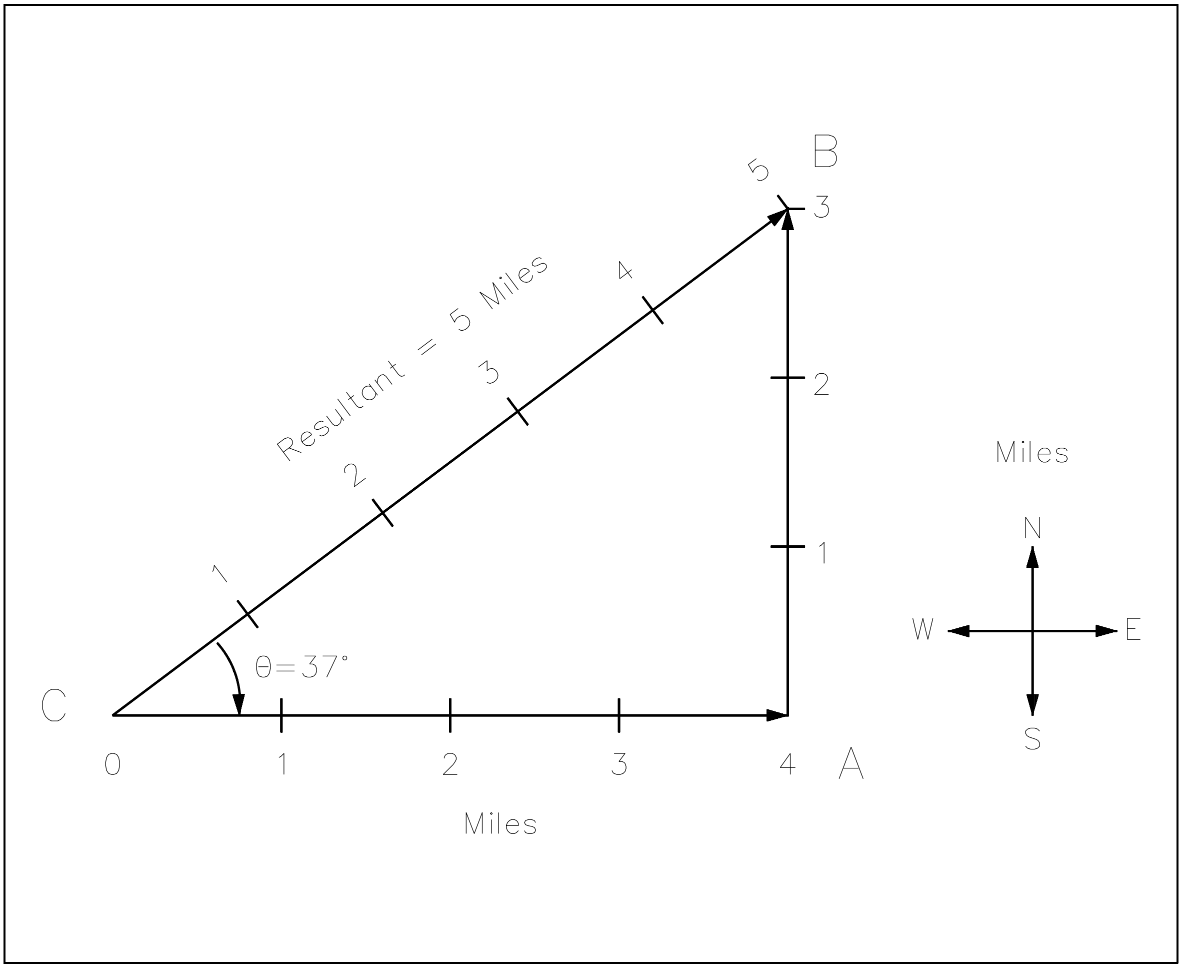

Figure 10 Vector Addition Not in Same Line

When drawing a scale drawing, one draws a straight line from the origin C to the final position B to represent the net or resultant displacement. Drawing the straight line CB and measuring its length, one should obtain about 5 inches. Then, since the scale of the drawing is 1 inch = 1 mile, this is used as a conversion factor giving $5 \text{ inches} \times \frac{1 \text{ mile}}{\text{inch}} = 5 \text{ miles}$ as the displacement.

Using a protractor or trigonometry, the acute angle ACB can be determined to be about 37°. Thus, the resultant (or vector sum) of traveling 4 miles east plus 3 miles north is a displacement of 5 miles at 37 degrees north of east.

It is left as an exercise for the student to show that vector addition is commutative, using the above example. Specifically, make a scale drawing showing that traveling 3 miles north and then 4 miles east yields the same resultant as above.

It is also reasonably obvious that more than two vectors can be added. One can travel three miles east and then three miles north and then three miles west and arrive at a point three miles north of the starting point. The sum of these three displacements is a resultant displacement of three miles north. (If this is not immediately apparent, sketch it.)

A student problem is to find the net or resultant displacement if a person travels 9 miles south and then 12 miles east and then 25 miles north. Make a scale drawing and determine the magnitude and direction of the resultant displacement. A scale of 2 miles per centimeter or 4 miles per inch will fit the drawing on standard paper.

Answer: About 20 miles at 53^o north of east.

Vector Components

Components of a vector are vectors, which when added, yield the vector. For example, as shown in the previous section (Figure 10), traveling 3 miles north and then 4 miles east yields a resultant displacement of 5 miles, 37^o north of east. This example demonstrates that component vectors of any two non-parallel directions can be obtained for any resultant vector in the same plane. For the purposes of this manual, we restrict our discussions to two dimensional space. The student should realize that vectors can and do exist in three dimensional space.

One could write an alternate problem: "If I am 5 miles from where I started northeast along a line 37^o N of east, how far north and how far east am I from my original position?" Drawing this on a scale drawing, the vector components in the east and north directions can be measured to be about 4 miles east and 3 miles north. These two vectors are the components of the resultant vector of 5 miles, 37^o north of east.

Component vectors can be determined by plotting them on a rectangular coordinate system. For example, a resultant vector of 5 units at 53^o can be broken down into its respective x and y magnitudes. The x value of 3 and the y value of 4 can be determined using trigonometry or graphically. Their magnitudes and position can be expressed by one of several conventions including: (3,4), (x=3, y=4), (3 at 0^o, 4 at 90^o), and (5 at 53^o). In the first expression, the first term is the x-component (F_x), and the second term is the y-component (F_y) of the associated resultant vector.

As in the previous example, if only the resultant is given, instead of component coordinates, one can determine the vector components as illustrated in Figure 11. First, plot the resultant on rectangular coordinates and then project the vector coordinates to the axis. The length along the x-axis is F_x, and the length along the y-axis is F_y. This method is demonstrated in the following example.

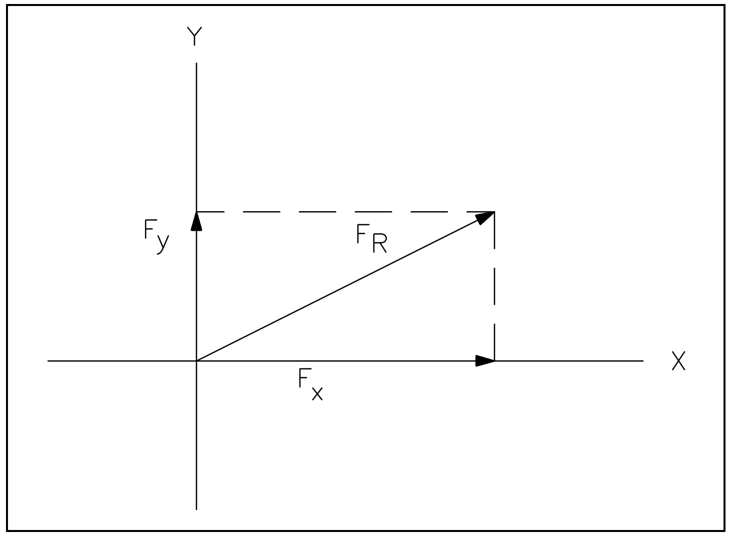

Figure 11 Vector Components

For the resultant vector shown in Figure 12, determine the component vectors given F_R = 50 lbf at 53°. First, project a perpendicular line from the head of F_R to the x-axis and a similar line to the y-axis. Where the projected lines meet, the axes determine the magnitude size of the component vectors. In this example, the component vectors are 30 lbf at 0° (F_x) and 40 lbf at 90° (F_y). If F_R had not already been drawn, the first step would have been to draw the vector.

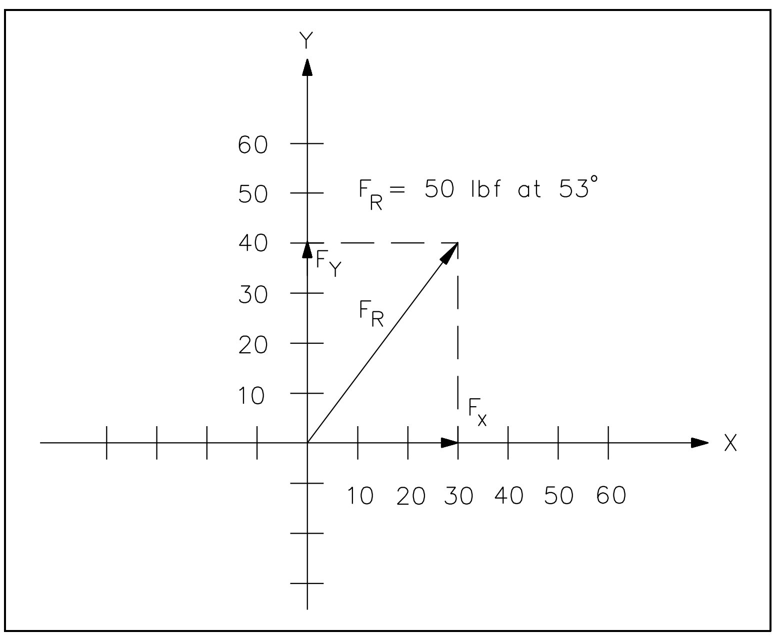

Figure 12 Component Vectors

As an exercise, the student should graphically find the easterly and northerly components of a 13 mile displacement at 22.6º north of east. The correct answer is 5 miles east and 12 miles north.

Trigonometry may also be used to determine vector components. Before explaining this method, it may be helpful to review the fundamental trigonometric functions. Recall that trigonometry is a branch of mathematics that deals with the relationships between angles and the length of the sides of triangles. The relationship between an acute angle of a right triangle, shown in Figure 13, and its sides is given by three ratios.

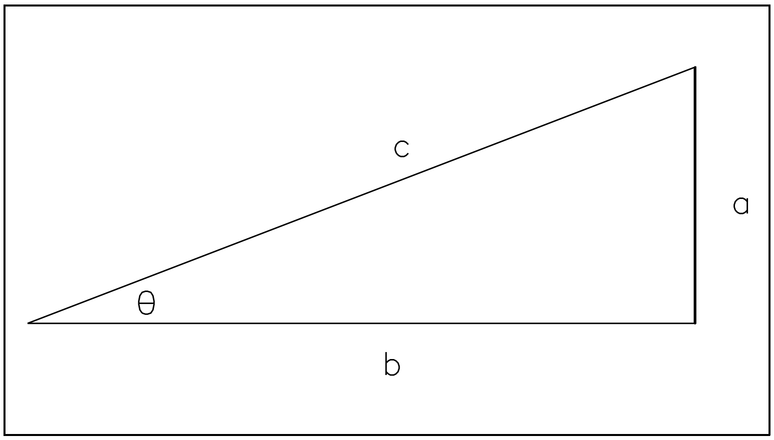

Figure 13 Right Triangle

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} \tag{2-1}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} \tag{2-2}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} \tag{2-3}$$

Before attempting to calculate vector components, first make a rough sketch that shows the approximate location of the resultant vector in an x-y coordinate system. It is helpful to form a visual picture before selecting the correct trigonometric function to be used. Consider the example of Figure 12, that was used previously. This time the component vectors will be calculated.

Example 1: Determine the component vectors, F_x and F_y, for F_R = 50 lbf at 53° in Figure 14. Use trigonometric functions.

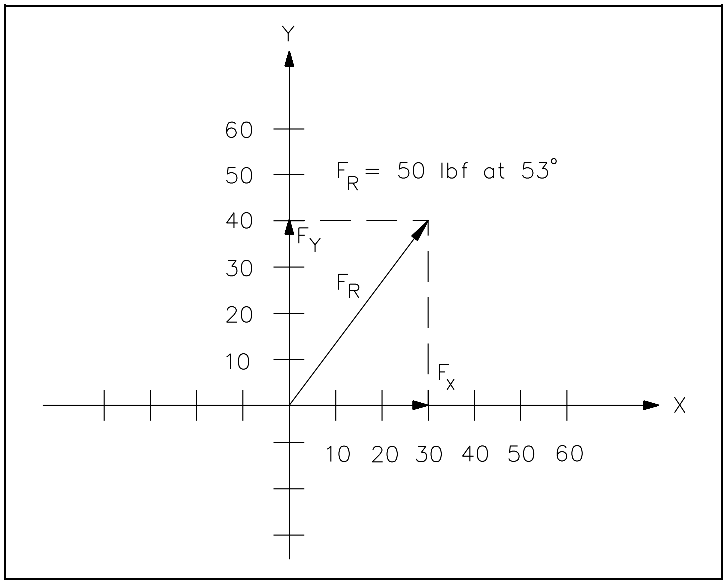

Figure 14 F_R = 50 lbf at 53°

F_x is calculated as follows:

$\cos\theta$ = adjacent/hypotenuse
$\cos\theta = F_x/F_R$ or $F_x = F_R \cos\theta$
F_x = (50)(cos53°)
F_x = (50)(0.6018)
F_x = 30 lbf on x-axis

F_y is calculated as follows:

$\sin\theta$ = opposite/hypotenuse
$\sin\theta = F_y/F_R$ or $F_y = F_R \sin\theta$
$F_y = (F_R)(\sin\theta)$
F_y = (50)(sin 53°)
F_y = (50)(0.7986)
F_y = 40 lbf on y-axis

Therefore, the components for F_R are F_x = 30 lbf at 0° and F_y = 40 lbf at 90°. Note that this result is identical to the result obtained using the graphic method.

Example 2: What are the component vectors, given F_R = 80 lbf at 220º? See Figure 15.

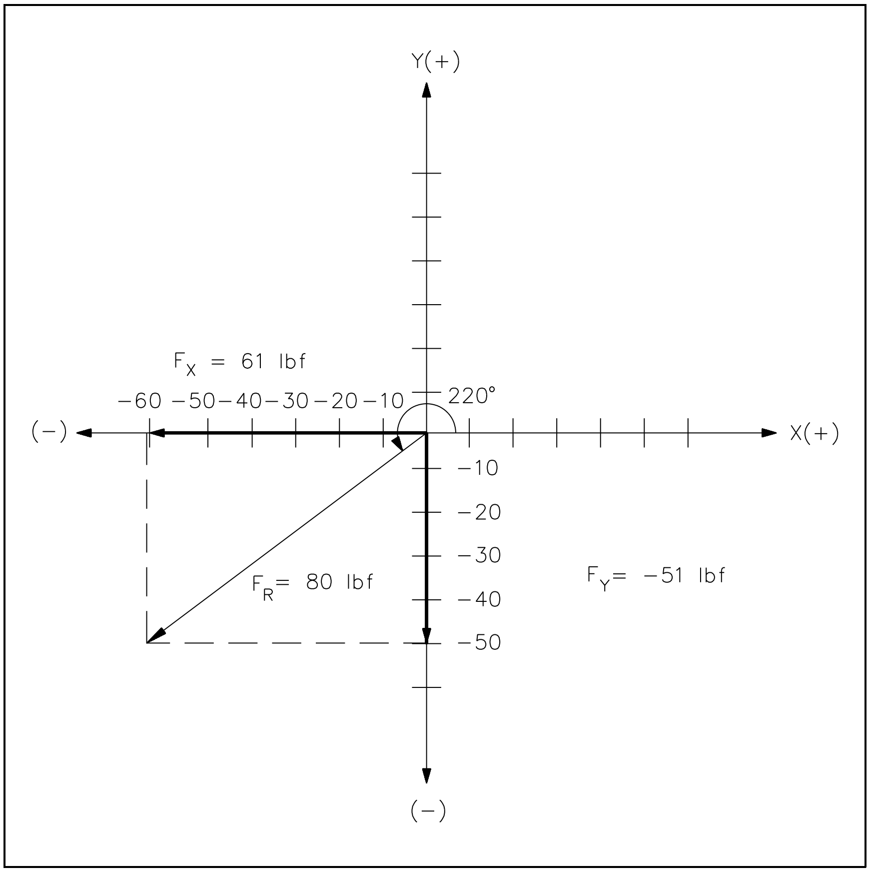

Figure 15 F_R = 80 lbf at 220º

F_x is calculated as follows:

$\cos \theta$ = adjacent/hypotenuse

$\cos \theta = F_x/F_R$ or $F_x = F_R \cos \theta$

$F_x = (F_R)(\cos \theta)$

$F_x = (80)(\cos 220^o)$

$F_x = (80)(-0.766)$

F_x = -61 lbf at 0º or 61 lbf at 180º

F_y is calculated as follows:

$\sin \theta$ = opposite/hypotenuse
$\sin \theta = F_y/F_R$ or $F_y = F_R \sin \theta$
$F_y = F_R \sin \theta$
$F_y = (80)(\sin 220^o)$
$F_y = (80)(-0.6428)$
F_y = -51 lbf at 90º or 51 lbf at 270º

Therefore, the components for F_R are F_x = 61 at 180º and F_y = 51 lbf at 270º.

Summary

Vector terminology is summarized below.

Vector Terminology Summary

- A resultant is a single vector that can replace two or more vectors.
- Components can be obtained for any two non-parallel directions if the vectors are in the same plane. Restricting the treatment to perpendicular directions and two dimensional space, the components of a vector are the two vectors in the x and y (or east-west and north-south) directions which produce the same effect as the original vector (or add to produce the original vector).
- Components are determined from data, graphically or analytically.

GRAPHIC METHOD OF VECTOR ADDITION

Vectors are added to determine the magnitude and direction of the resultant.

EO 1.3 ADD vectors using the following methods:
a. Graphical

Vector Addition

Component vectors are added to determine the resultant vector. For example, when two or more forces are acting on a single object, vector addition is used to determine the direction and magnitude of the net (resultant) force on the object. Consider an airplane that travels due east for 100 miles at 500 mph, then NE for 50 miles at 400 mph, and finally north for 500 miles at 500 mph. Vector addition can be used to determine the net distance the airplane is from its point of origin or to predict when it will arrive at its destination.

Methods Used to Add Vectors

Several methods have been developed to add vectors. In this chapter, the graphic method will be explained. The next chapter will explain the component addition method. Either one of these methods will provide fairly accurate results. If a high degree of accuracy is required, an analytical method using geometric and trigonometric functions is required.

Using the Graphic Method

Before attempting to use this method, the following equipment is needed: standard linear (nonlog) graph paper, ruler, protractor, and pencil. The graphic method utilizes a five-step process.

Step 1. Plot the first vector on the rectangular (x-y) axes.

a. Ensure that the same scale is used on both axes.

b. Place the tail (beginning) of the first vector at the origin of the axes as shown in Figure 16.

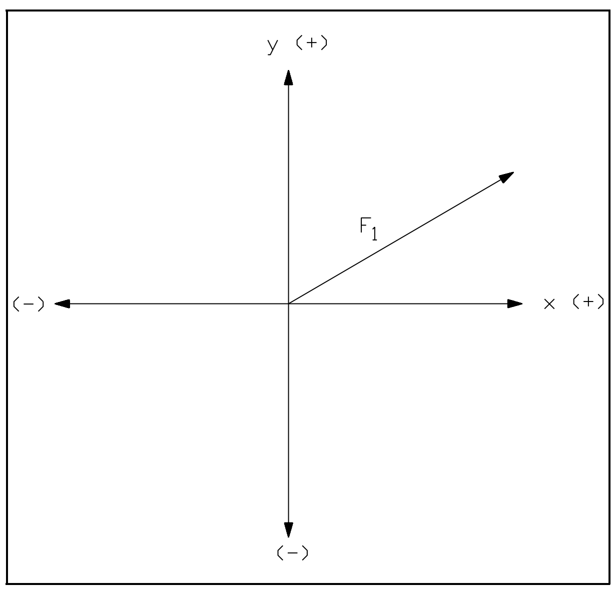

Figure 16 Rectangular Coordinate System

Step 2. Draw the second vector connected to the end of the first vector.

a. Start the tail of the second vector at the head of the first vector.

b. Ensure that the second vector is also drawn to scale.

c. Ensure proper angular orientation of the second vector with respect to the axes of the graph (see Figure 17).

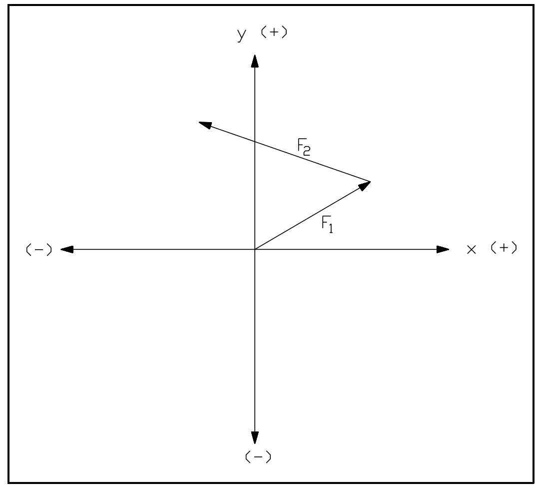

Figure 17 Vector F_2

Step 3. Add other vectors sequentially.

a. Add one vector at a time.

b. Always start the tail of the new vector at the head of the previous vector.

c. Draw all vectors to scale and with proper angular orientation.

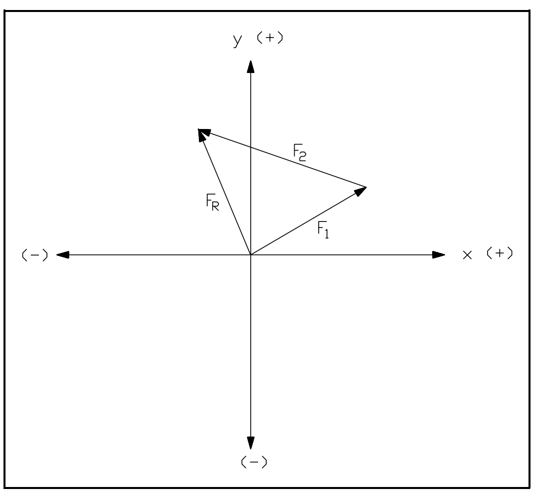

Figure 18 Resultant

Step 4. When all given vectors have been drawn, draw and label a resultant vector, F_R, from the point of origin of the axes to the head of the final vector.

a. The tail of the resultant is the tail of the first vector drawn as shown in Figure 18.

b. The head of the resultant is at the head of the last vector drawn.

Step 5. Determine the magnitude and direction of the resultant.

a. Measure the displacement and angle directly from the graph using a ruler and a protractor.

b. Determine the components of the resultant by projection onto the x- and y-axes.

Example 1: What are the magnitude and direction of the resultant for the following: F_1 = 3 units at 300°, F_2 = 4 units at 60°, and F_3 = 8 units at 180°? The three vectors and their resultant are shown in Figure 19.

Answer: F_R = 4 units at 150°

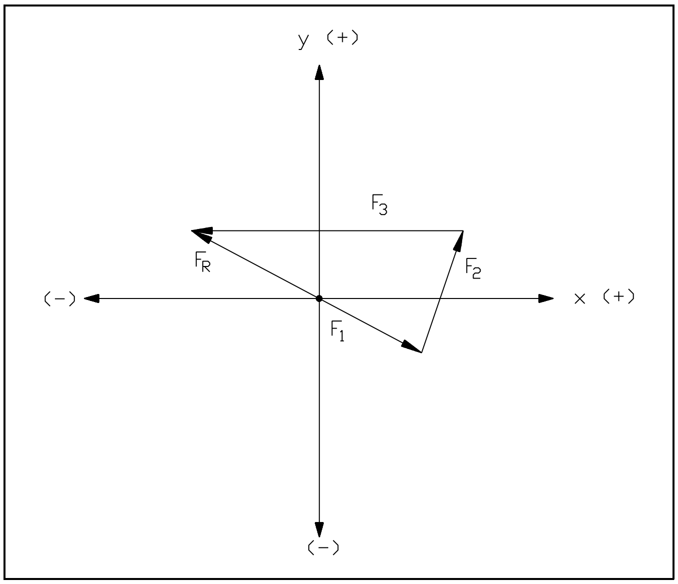

Figure 19 Graphic Addition - Example 1

Example 2: Given X_L = 50 Ohms at 90°, R = 50 Ohms at 0°, and X_c = 50 Ohms at 270°, what is the Resultant Z? (See Figure 20) Note: X_L is inductive reactance, X_c is capacitive reactance and Z is impedance.

Answer: Z = 50 Ohms at 0°

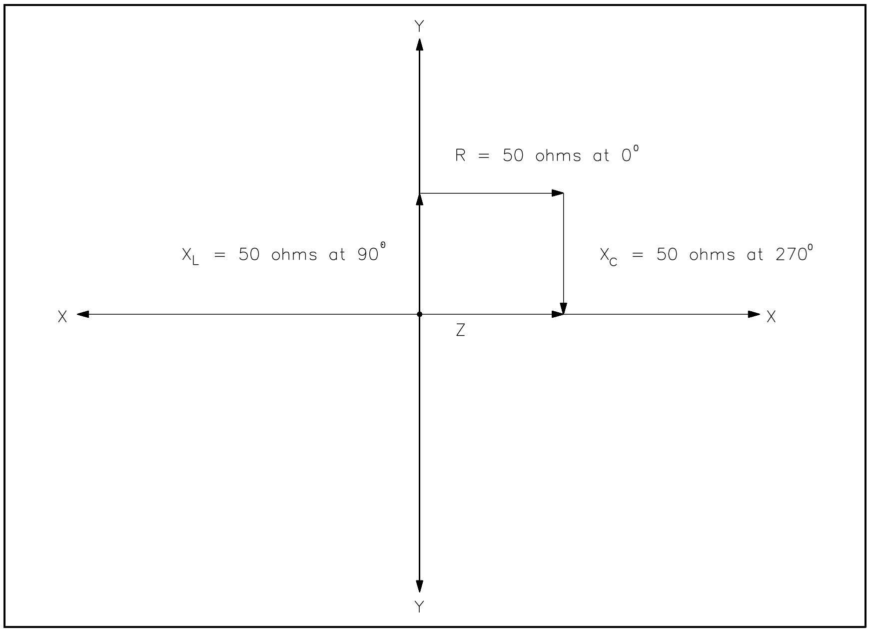

Figure 20 Graphic Addition - Example 2

Summary

The steps of the graphic method of vector addition are summarized below.

Graphic Method Summary

- Draw rectangular coordinates.
- Draw first vector.
- Draw second vector connected to the end (head) of first vector with proper angular orientation.
- Draw remaining vectors, starting at the head of the preceding vector.
- Draw resultant vector from the origin of axes to head of final vector.
- Measure length of resultant.
- Measure angle of resultant vector addition.

COMPONENT ADDITION METHOD

Vector components are added along each axis to determine the magnitude and direction of the resultant.

EO 1.3 ADD vectors using the following methods:
b. Component addition

An Explanation of Components

The component addition method refers to the addition of vector coordinates on a rectangular (x,y) coordinate system. Coordinates, as seen in previous examples, locate a specific point in the system. Relative to vectors, that specific point is the head of the vector. There are two ways to locate that point. The head can be located by counting the units along the x-axis and the units along the y-axis, as illustrated in Figure 21, where the point has coordinates (4,3); i.e., the x component has a magnitude of 4 and the y component has a magnitude of 3.

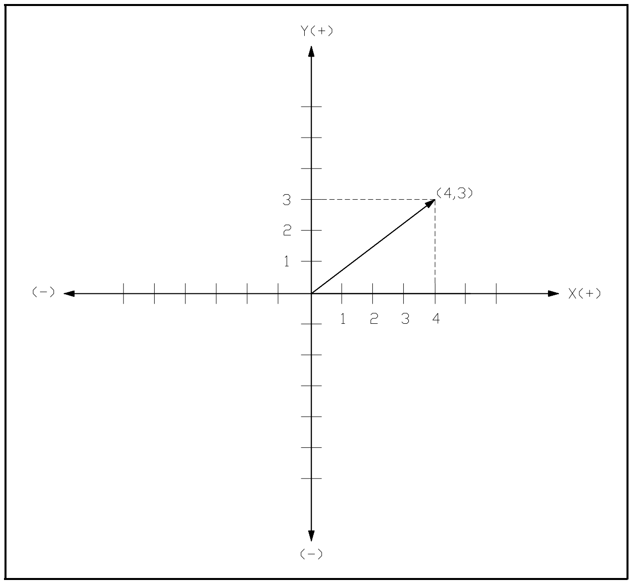

Figure 21 Vector Addition Component Method

The head can also be found by locating a vector of the proper length on the positive side of the x-axis, with its tail at the intersection of the x- and y- axes. Then the vector is rotated a given number of degrees in the counterclockwise direction. In this example, the head of the vector is located five units at 36.9^o. Five units is the length of the vector.

Using the Component Addition Method

To add vectors using the component addition method, use the following four step method.

Step 1. Determine x- and y-axes components of all original vectors.

Step 2. Mathematically combine all x-axis components.

Note: When combining, recognize that positive x components at 180^o are equivalent to negative x components at 0^o ($+x$ at $180^o = -x$ at 0^o).

3. Mathematically combine all y-axis components (+y at 270° = -y at 90°).

4. Resulting (x,y) components are the (x,y) components of the resulting vector.

The following examples illustrate vector addition using the component addition method.

Example 1:

Given the following vectors what are the coordinates of the resultant vector, that is, the sum of the vectors?

$F_1 = (4,10)$, $F_2 = (-6,4)$, $F_3 = (2,-4)$, and $F_4 = (10,-2)$

Step 1. Determine the x- and y-axes components of all four original vectors.

x-axes components = 4, -6, 2, 10
y-axes components = 10, 4, -4, -2

Step 2. Mathematically combine all x-axis components.

$F_x = 4 + (-6) + 2 + 10$
$F_x = 4 - 6 + 2 + 10$
$F_x = 10$

Step 3. Mathematically combine all y-axis components.

$F_y = 10 + 4 + (-4) + (-2)$
$F_y = 10 + 4 - 4 - 2$
$F_y = 8$

Step 4. Express the resultant vector.

The resultant components from the previous additions are the coordinates of the resultant, that is, $F_R = (10,8)$.

Example 2: Determine the resultant, F_R.

Given:
$F_1 = 30$ lbf at 0°, 10 lbf at 90°
$F_2 = 50$ lbf at 0°, 50 lbf at 90°
$F_3 = 45$ lbf at 180°, 30 lbf at 90°
$F_4 = 15$ lbf at 0°, 50 lbf at 270°

Follow the sequence used in the first example. Remember that x at 180° is -x at 0°, and y at 270° is -y at 90°.

F_x = 30 + 50 +(-45) + 15 = 50 lbf
F_y = 10 + 50 + 30 + (-50) = 40 lbf
F_R = 50 lbf at 0°, 40 lbf at 90°

Summary

The sequence of steps used in the component addition method of adding vectors is summarized below.

Component Addition Method Summary

- Determine the x- and y- axes of all original vectors.
- Mathematically combine all x-axis components.
- Mathematically combine all y-axis components.
- The results are the components of the resultant vector.

ANALYTICAL METHOD OF VECTOR ADDITION

Vector components are added to determine the magnitude and direction of the resultant. Calculations using trigonometric functions are the most accurate method for making this determination.

EO 1.3 ADD vectors using the following methods:
c. Analytical

The graphic and components addition methods of obtaining the resultant of several vectors described in the previous chapters can be hard to use and time consuming. In addition, accuracy is a function of the scale used in making the diagram and how carefully the vectors are drawn. The analytical method can be simpler and far more accurate than these previous methods.

Review of Mathematical Functions

In earlier mathematics lessons, the Pythagorean Theorem was used to relate the lengths of the sides of right triangles such as in Figure 22. The Pythagorean Theorem states that in any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. This expression may be written as given in Equation 2-4.

$$c^2 = a^2 + b^2 \quad \text{or} \quad c = \sqrt{a^2 + b^2} \tag{2-4}$$

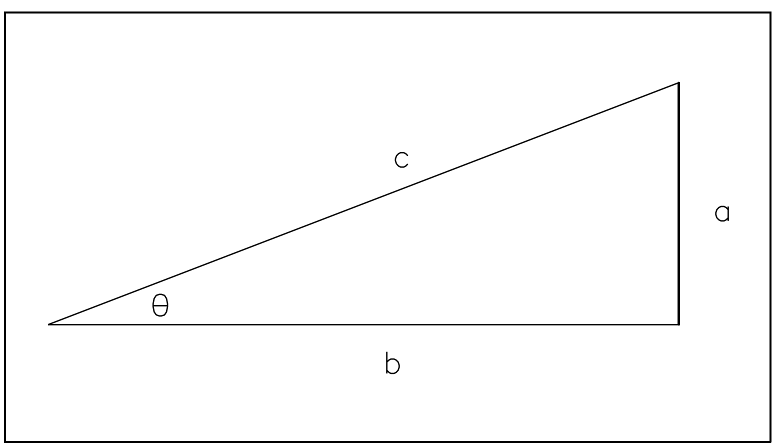

Figure 22 Right Triangle

Also, recall the three trigonometric functions reviewed in an earlier chapter and shown in Figure 23. The cosine will be used to solve for F_x. The sine will be used to solve for F_y. Tangent will normally be used to solve for θ, although sine and cosine may also be used.

$$\text{Sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\text{Cosine } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{Tangent } \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Figure 23 Trigonometric Functions

On a rectangular coordinate system, the sine values of θ are positive (+) in quadrants I and II and negative (-) in quadrants III and IV. The cosine values of θ are positive (+) in quadrants I and IV and negative (-) in quadrants II and III. Tangent values are positive (+) in quadrants I and III and negative (-) in quadrants II and IV.

When mathematically solving for tan θ, calculators will specify angles in quadrants I and IV only. Actual angles may be in quadrants II and III. Each problem should be analyzed graphically to report a realistic solution. Quadrant II and III angles may be obtained by adding or subtracting 180^o from the value calculated.

Using the Analytical Method

To illustrate this method, consider this example: a man walks 3 miles in one direction, then turns 90^o and continues to walk for an additional 4 miles. In what direction and how far is he from his starting point? The first step in solving this problem is to draw a simple sketch as shown in Figure 24.

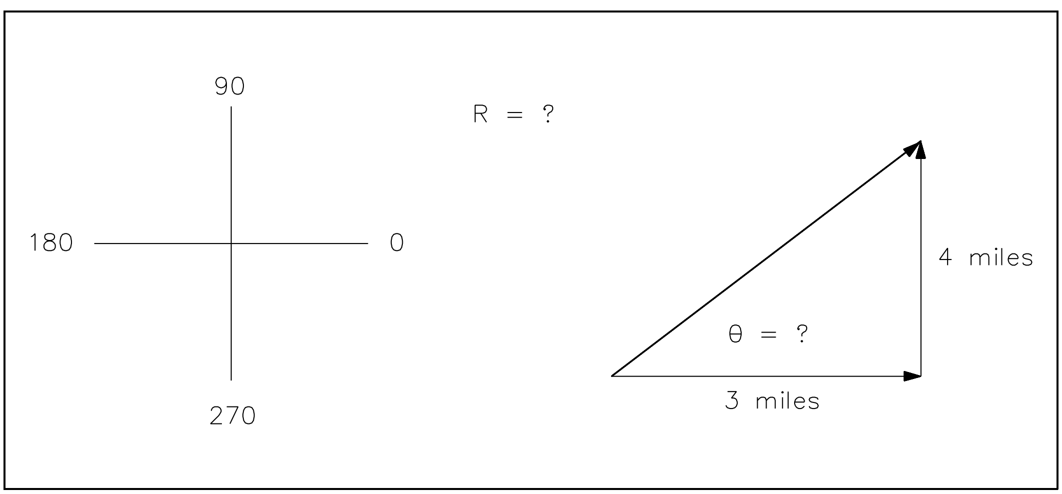

Figure 24 Hypotenuse and Angle

His net displacement is found using equation 2-4.

$$R = \sqrt{a^2 + b^2}$$

$$R = \sqrt{3^2 + 4^2}$$

$$R = \sqrt{25}$$

$$R = 5 \text{ miles}$$

His direction (angle of displacement) is found using the tangent function.

$\tan \theta$ = opposite/adjacent
$\tan \theta = a/b$
$\tan \theta = 4/3$
$\tan \theta = 1.33$
$\theta = \tan^{-1} 1.33$
$\theta = 53^o$

Therefore, his new location is 5 miles at 53^o from his starting point.

By carrying this approach a step further, a model has been developed for finding the resultant of several vectors. For the purpose of developing the model, consider three forces (F_1, F_2, and F_3) acting on an object as shown in Figure 25. The goal is to find the resultant force (F_R).

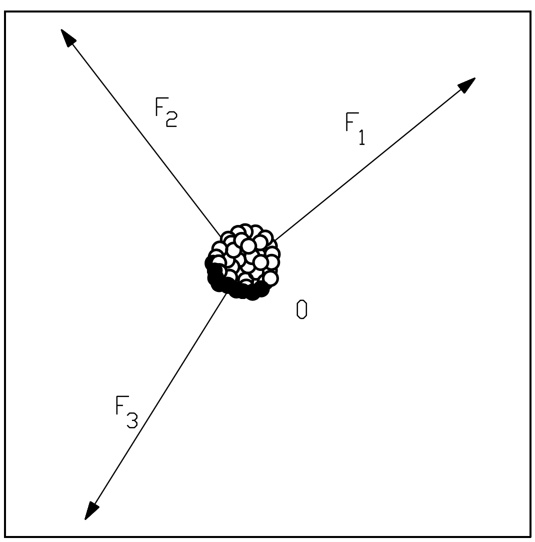

Figure 25 Example Model 1

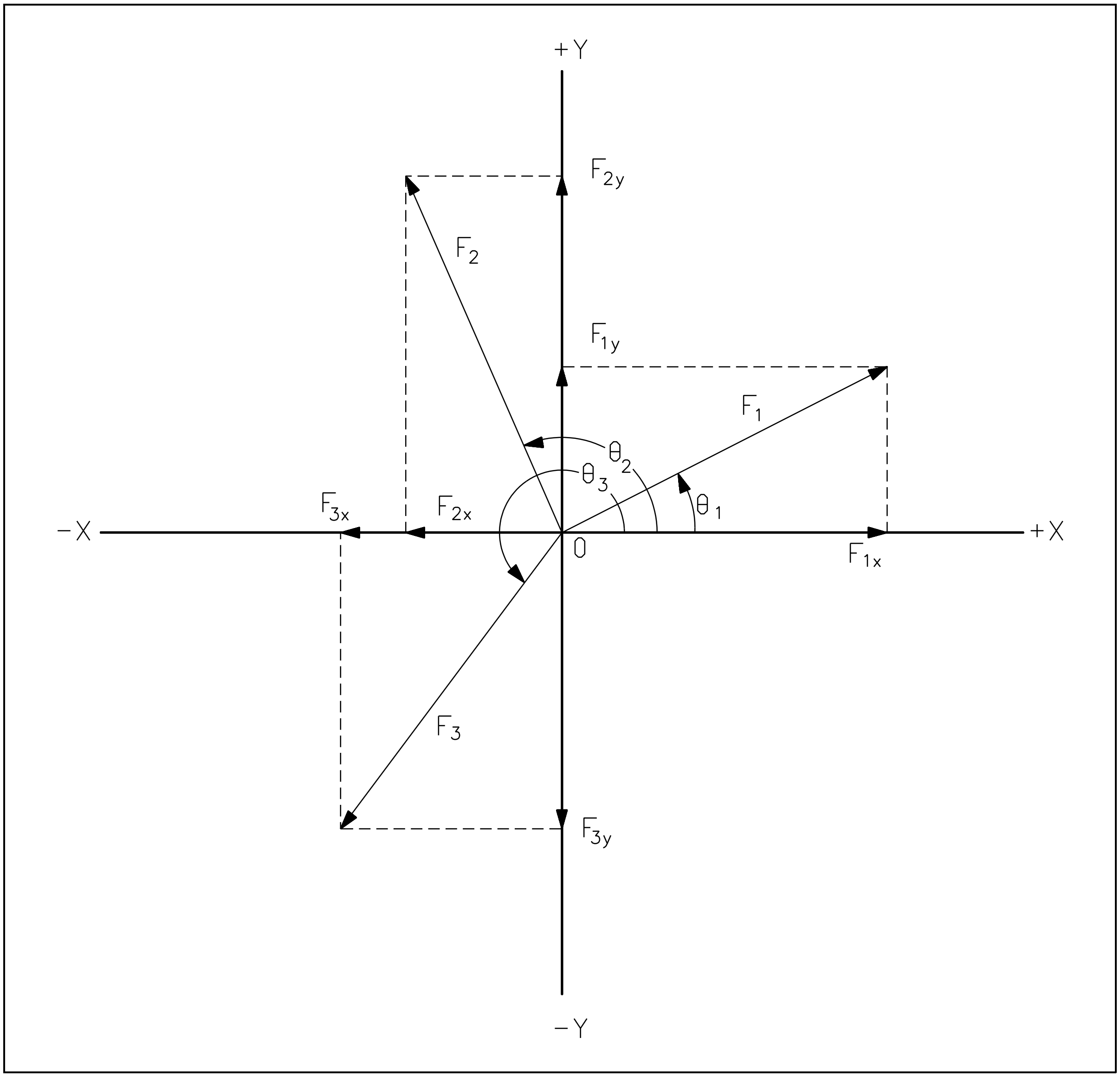

Figure 26 Example Model 2

Step 1: Draw x and y coordinates and the three forces from the point of origin or the center of the object, as shown in Figure 26. Component vectors and angles have been added to the drawing to aid in the discussion.

Step 2: Resolve each vector into its rectangular components.

Vector	Angle	x component	y component
F_1	θ_1	$F_{1x} = F_1\cos\theta_1$	$F_{1y} = F_1\sin\theta_1$
F_2	θ_2	$F_{2x} = F_2\cos\theta_2$	$F_{2y} = F_2\sin\theta_2$
F_3	θ_3	$F_{3x} = F_3\cos\theta_3$	$F_{3y} = F_3\sin\theta_3$

Step 3: Sum the x and y components.

$$F_{Rx} = \Sigma F_x = F_{1x} + F_{2x} + F_{3x}$$

$$F_{Ry} = \Sigma F_y = F_{1y} + F_{2y} + F_{3y}$$

Where "Σ" means summation

Step 4: Calculate the magnitude of F_R.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Step 5: Calculate the angle of displacement.

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}}$$

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}}$$

Here is an example using this model. Follow it through step by step.

Example: Given three forces acting on an object, determine the magnitude and direction of the resultant force F_R.

F_1 = 90 lbf at 39^o

F_2 = 50 lbf at 120^o

F_3 = 125 lbf at 250^o

Step 1: First draw x and y coordinate axes on a sheet of paper. Then, draw F_1, F_2, and F_3 from the point of origin. It is not necessary to be totally accurate in placing the vectors in the drawing. The approximate location in the right quadrant is all that is necessary. Label the drawing as in the model (Figure 26).

Step 2: Resolve each force into its rectangular coordinates.

Force	Magnitude	Angle	x component	y component
F_1	90 lbf	39^o	F_{1x} = 90 cos 39^o F_{1x} = (90) (.777) F_{1x} = 69.9 lbf	F_{1y} = 90 sin 39^o F_{1y} = (90) (.629) F_{1y} = 56.6 lbf
F_2	50 lbf	120^o	F_{2x} = 50 cos 120^o F_{2x} = (50) (-.5) F_{2x} = -25 lbf	F_{2y} = 50 sin 120^o F_{2y} = (50) (.866) F_{2y} = 43.3 lbf
F_3	125 lbf	250^o	F_{3x} = 125 cos 250^o F_{3x} = (125) (-.342) F_{3x} = -42.8 lbf	F_{3y} = 125 sin 250^o F_{3y} = (125) (-.94) F_{3y} = -117.5 lbf

Step 3: Sum the x and y components.

$F_{Rx} = F_{1x} + F_{2x} + F_{3x}$

$F_{Rx} = 69.9 \text{ lbf} + (-25 \text{ lbf}) + (-42.8 \text{ lbf})$

$F_{Rx} = 2.1 \text{ lbf}$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y}$

$F_{Ry} = 56.6 \text{ lbf} + 43.3 \text{ lbf} + (-117.5 \text{ lbf})$

$F_{Ry} = -17.6 \text{ lbf}$

Step 4: Calculate the magnitude of F_R.

$F_R = \sqrt{F_x^2 + F_y^2}$

$F_R = \sqrt{(2.1)^2 + (-17.6)^2}$

$F_R = \sqrt{314.2}$

$F_R = 17.7 \text{ lbf}$

Step 5: Calculate the angle of displacement.

$\tan \theta = F_{Ry}/F_{Rx}$

$\tan \theta = -17.6/2.1$

$\tan \theta = -8.381$

$\theta = \tan^{-1} (-8.381)$

$\theta = -83.2^o$

Therefore, $F_R = 17.7$ lbf at -83.2^o or 276.8^o.

Note: A negative angle means a clockwise rotation from the zero axis.

It is left to the student to try the previous example using the other methods of vector addition described in earlier chapters.

Summary

The steps followed when using the analytical method to find the resultant of several vectors are summarized below.

Analytical Method of Adding Vectors Summary

- Draw x and y coordinate axes.
- Draw component vectors from point of origin.
- Resolve each vector into rectangular components.
- Sum x and y components.
- Calculate magnitude of F_R.
- Calculate angle of displacement.

****VIDEO LEARNING****

- Click here for a 7 min video on Vector Addition -