A PRIMER ON NAVAL SEAKEEPING

Main Category:	Naval Engineering
Sub Category:	-
Course #:	NAV-117
Course Content:	45 pgs
PDH/CE Hours:	4

COURSE/EXAM PREVIEW
(HIT BACK TO RETURN TO ENGINEERING-PDH.COM)

NAV-117 EXAM PREVIEW

Instructions:

- Review the course & exam preview below.
- Click "Add to Cart" from the course page on the website. You can "Continue Shopping" to add additional courses, or checkout. Don't forget to apply your coupon code if you have one before checkout.
- After checkout you will be provided with links to download the official courses/exams.
- At your convenience and own pace, you can review the course material. When ready, select "Take Exam" to complete the live graded exam. Don't worry, you can take an exam as many times as needed to pass.
- Upon a satisfactory completion of the course exam, which is a score of 70% or better, you will be provided with your course completion certificate. Be sure to download and print your certificates to keep for your records.

Exam Preview:

1. 'Beamy' ships such as tugs and fishing vessels will suffer short period heave oscillations and high heave accelerations.
    a. True
    b. False
2. Wave systems are often at different phases in their development and at differing distances from the observation point. This modeling of the sea is made possible by the Superposition Theorem that implies that the complicated sea wave system is made up of many _____ wave components superimposed upon each other.
    a. Square
    b. Triangular
    c. Sinusoidal
    d. Sawtooth
3. Changing the viscosity of the fluid in the damper the level of damping can be altered. A low level of damping will allow several oscillations before the system comes to rest. In this instance the system is critically damped.
    a. True
    b. False
4. Using Table 8.1 NATO Sea State Numeral Table for the Open Ocean North Atlantic, which of the following values corresponds to the most probable modal wave period for a Sea State of 7?
    a. 9.7 seconds
    b. 15.0 seconds
    c. 16.4 seconds
    d. 20.0 seconds

5. The figure for significant wave height is often used to describe the height of the wave system. It corresponds to the average of the ___ highest waves. This value is typically estimated by observers of wave systems for the average wave height.
   a. 3/4
   b. 1/2
   c. 1/3
   d. ¼

6. The effect of both passive and active stabilizers is to lessen pitch motion. Their presence lessens the extreme pitch amplitudes that can be suffered by a ship by increasing pitch damping.
   a. True
   b. False

7. Using Figure 8.11 Values of μ for Different Ship Orientations, and the surrounding reference material, which of the following μ value corresponds to the ship being perpendicular to the direction of the waves?
   a. 0°
   b. 135°
   c. 180°
   d. 90°

8. Traditionally, the seakeeping dynamics of the hull form have been considered at a lower level of importance than hull resistance, strength, and space efficiency. The seakeeping characteristics of ships have been left to chance.
   a. True
   b. False

9. According to the reference material, most U.S. Navy ships are outfitted with a hard and soft rudder stop to prevent the rudder angle from exceeding ___°.
   a. 15
   b. 25
   c. 35
   d. 45

10. Using Figure 8.12 The 6 Degrees of Freedom and the reference material, which axis does "Pitch" act on a ship?
   a. X axis
   b. Y axis
   c. Z axis
   d. N/A

COURSE OBJECTIVES
CHAPTER 8

8.   SEAKEEPING

1. Be qualitatively familiar with the creation of waves including:
   a. The effects of wind strength, wind duration, water depth and fetch
   b. The wave creation sequence
   c. The superposition theorem and considering a sea as a wave spectrum

2. Be qualitatively familiar with simple harmonic motions including:
   a. Free oscillations
   b. Effect of damping
   c. Forced oscillations and the effects of forcing function frequency on motion amplitude
   d. Resonance

3. Calculate the ship-wave encounter frequency with a known ship speed and heading and known wave frequency and direction

4. Identify the 6 rigid body ship motions of Surge, Sway, Heave, Roll, Pitch and Yaw.

5. Qualitatively describe why heave, roll and pitch are simple harmonic motions

6. Qualitatively describe and calculate when rigid body motion resonance will occur

7. Qualitatively describe the structural response of a ship including:
   a. Longitudinal bending
   b. Torsional twisting
   c. Transverse stressing

8. Qualitatively describe non-oscillatory dynamic responses including:
   a. Shipping water
   b. Forefoot emergence
   c. Slamming
   d. Racing
   e. Added power
   f. Broaching
   g. Loss of stability

9. Qualitatively describe the way ship response can be improved including:
   a. Different hull shape
   b. Passive and active anti-roll devices
   c. Ship operation

8.1 Introduction

So far in this course we have considered the ship to be in calm water. All the hydrostatic data such as TPI and MT1" are calculated for the ship at level trim in calm water. The curve of intact statical stability is produced assuming level trim and calm conditions. Even powering calculations such as the Froude expansion assumes calm water. Unfortunately, this is true for only a small percentage of a ship's operating life, the majority of the time it will encounter some sort of wave system.

In the most simple of models, the ship can be considered as a system that is excited by external moments and forces. The ship then responds to these external influences. Figure 8.1 shows the block diagram of this system.
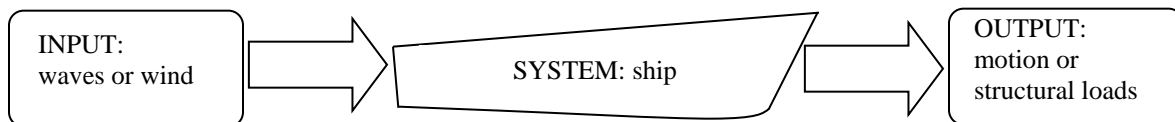

Figure 8.1 Block Diagram of Ship Response Model

For a ship, the external influences will be wind, waves, current and other natural phenomena. The responses of the ship will be the motions we associate with a ship underway such as roll, pitch, and slamming and its structural loads.

This chapter will examine the way the sea influences ship response, which responses are the most damaging to its operation and what the ship operator can do to reduce them. It will become evident that response will depend upon:

    1.   The size, direction and frequency of the external moments and forces.

    2.   The seakeeping and structural characteristics of the ship.

Only by considering the interaction of the two, will an understanding of the ship response be achieved.

8.2 Waves

As seen, the excitation forces and moments in the ship system shown at Figure 8.1 will be generated by wind and waves. Wind will play an important part in the response of any vessels that have significant height above the waterline; the motions of off shore structures are influenced by the direction and characteristics of the wind. It is also fairly obvious that the seakeeping and dynamic responses of yachts are wind dependent. However, to reduce the complexities of the study of ship response, we shall limit our examination of excitation forces and moments to those produced by wave systems.

Wave systems themselves can be very complicated. However, an understanding of them is vital if we are to predict ship responses with any level of accuracy.

8.2.1 Wave Creation

Waves will be created by anything that supplies energy to the water surface. Consequently, sources of wave systems are numerous. From our own experience we know that throwing a stone into a pool will generate a circular wave pattern. We also saw in the last chapter that a ship will generate several wave systems when traveling through water. The faster the ship speed, the larger the waves generated. It was also observed that the larger wave systems caused by higher ship speeds required an ever increasing amount of energy. This resulted in the rapid increase in EHP at high ship speeds. This phenomenon is caused by the relationship between wave height and wave energy.

$$Wave\ Energy = f(Wave\ Height)^2$$

Hence a doubling in wave height is indicative of a quadrupling of wave energy. This explains the rapid increase in C_w, and in turn, EHP at high ship speeds. Conversely, this relationship also tells us that the energy content of a wave increases rapidly with wave height.

8.2.1.1 Wave Energy Sources

The wave systems generated by a ship are insignificant compared with those found at sea. These systems must be generated by much larger energy sources.

- **Wind**   Wind is probably the most common wave system energy source. Waves created by wind will be examined in detail in the next section.

- **Geological Events**   Seismic activity on the sea bed can input significant quantities of

energy into the sea system and generate waves. Tsunamis (tidal waves) are the result of seismic activity in the deep ocean.

- **Currents**   The interaction of ocean currents can create very large wave systems. These systems are usually created by the shape of the coastline and are often highly localized. The interaction of ocean currents explains the large wave systems that can occur at the Cape of Good Hope and Cape Horn.

8.2.1.2 Wind Generated Wave Systems

We have seen that the most common energy source of wave systems is the wind. Hence we will limit our discussion to wind generated wave systems. The size of these systems is dependent upon a number of factors.

- **Wind Strength**   Obviously the energy content of the wind is a function of its strength or speed. The faster the wind speed, the larger its energy content and so the more energy is transferred to the sea. Quite simply, large waves are created by strong winds.

- **Wind Duration**   The length of time a wind has input energy into a sea will effect the energy content and hence the height of the wave system. As we will see, the longer the wind blows the greater the time the sea has to become fully developed at that wind speed.

- **Water Depth**   Although not covered in this course, the equations for deep water and shallow water waves are very different. Consequently, water depth can have a significant effect on wave height. This is easily verified by observing the ever increasing height of a wave as it travels from deep water to the shallow water of a beach.

- **Fetch**   Fetch is the area or expanse of water that is being influenced by the wind. The larger the fetch, the more efficient the energy transfer between the wind and sea. Hence large expanses of water will be rougher than small areas when subjected to the same wind.

The combination of these factors will then relate to the magnitude of the generated wave system.

8.2.1.3 Wave Creation Sequence

When examining the wave system creation sequence, it is important to be aware of the energy transfer that is constantly occurring in a wave. The energy of a wave is always being dissipated by the viscous friction forces associated with the viscosity of the sea. This energy dissipation increases with wave height. For the wave to be maintained, the energy being dissipated must be replaced by the energy source of the wave - the wind. Hence, without the continued presence of the wind, the wave system will die. Figure 8.2 demonstrates this principle.
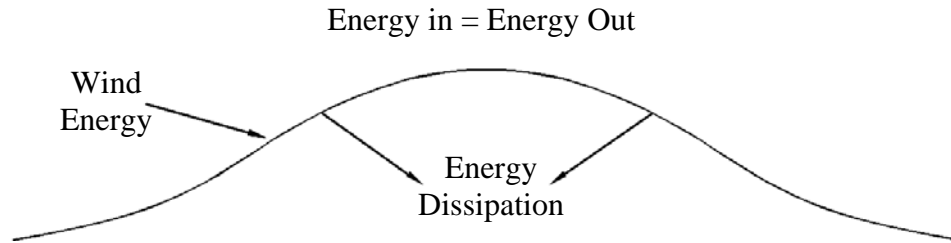
Energy in = Energy Out

Wind
Energy

Energy
Dissipation

Figure 8.2 The Wave Energy Cycle

The sequence of events in creating a wave system is as follows.

- **Initial**   At first, the action of the wind over the water surface creates small ripples or high frequency, low wave length waves.

- **Growing**   As the wind continues to blow, the wave frequency reduces and wave length increases with the energy content of the wave system.

  Wind Energy > Wave Energy Dissipation

- **Fully Developed**   In this condition, the sea has stopped growing and wave height and energy content is maximized.

  Wind Energy = Wave Energy Dissipation

- **Reducing**   When the wind begins to reduce, the wave system can no longer be maintained. High frequency waves disappear first with ever lower frequency waves disappearing along with the system's energy.

  Wind Energy < Wave Energy Dissipation

- **Swell**   Eventually, the wave system consists of the low frequency, long wave length waves associated with an ocean swell.

8.2.2 Wave Interaction

Unfortunately, the true shape and configuration of the sea is far more complicated than described above due to the interaction of several different wave systems. Observing an area of sea would lead us to believe that wave height and direction of travel is completely random. See an example in Figure 8.3.
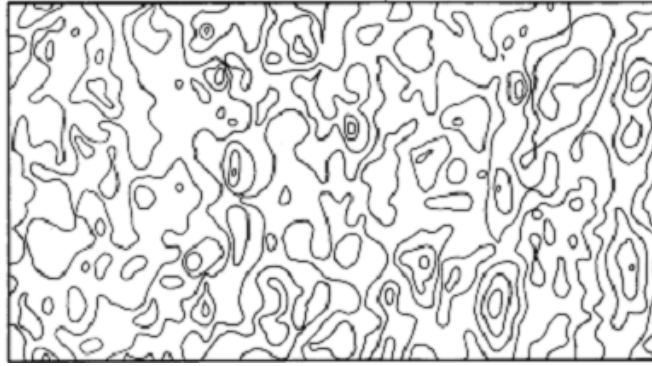
Figure 8.3 Topological Sketch of the North Atlantic
"Principles of Naval Architecture: Vol III" by SNAME

8.2.2.1 Superposition Theorem

The confused state of the sea at any point can be modeled as the destructive and constructive interference pattern created between several wave systems. These wave systems are often at different phases in their development and at differing distances from the observation point. This modeling of the sea is made possible by the **Superposition Theorem** that implies that the complicated sea wave system is made up of many sinusoidal wave components superimposed upon each other. Each component sine wave has its own wavelength, speed and amplitude and is created from one of the wave energy sources. This is shown diagrammatically in Figure 8.4.
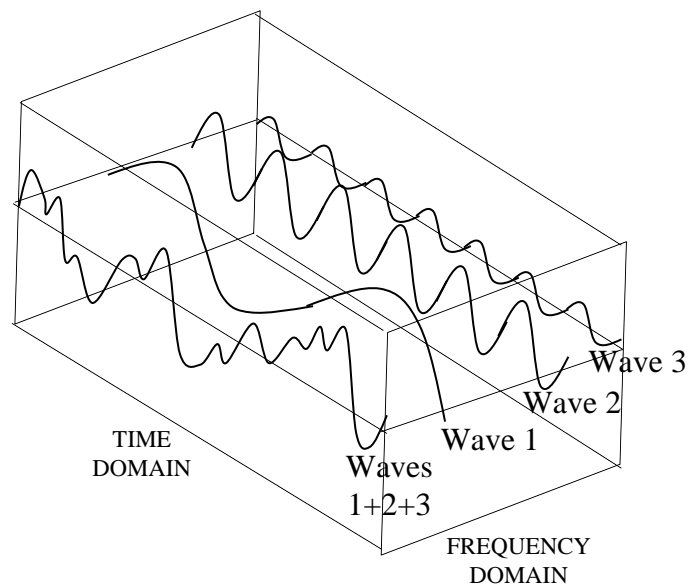

Figure 8.4 Wave Creation from Superposition Theorem

8.2.3 Wave Spectrum

Figure 8.4 also shows that it is also possible to analyze the sea in the frequency domain. This is called the **Sea Spectrum**. Figure 8.5 shows a typical sea spectrum obtained through analysis of wave measurements (using accelerometers)
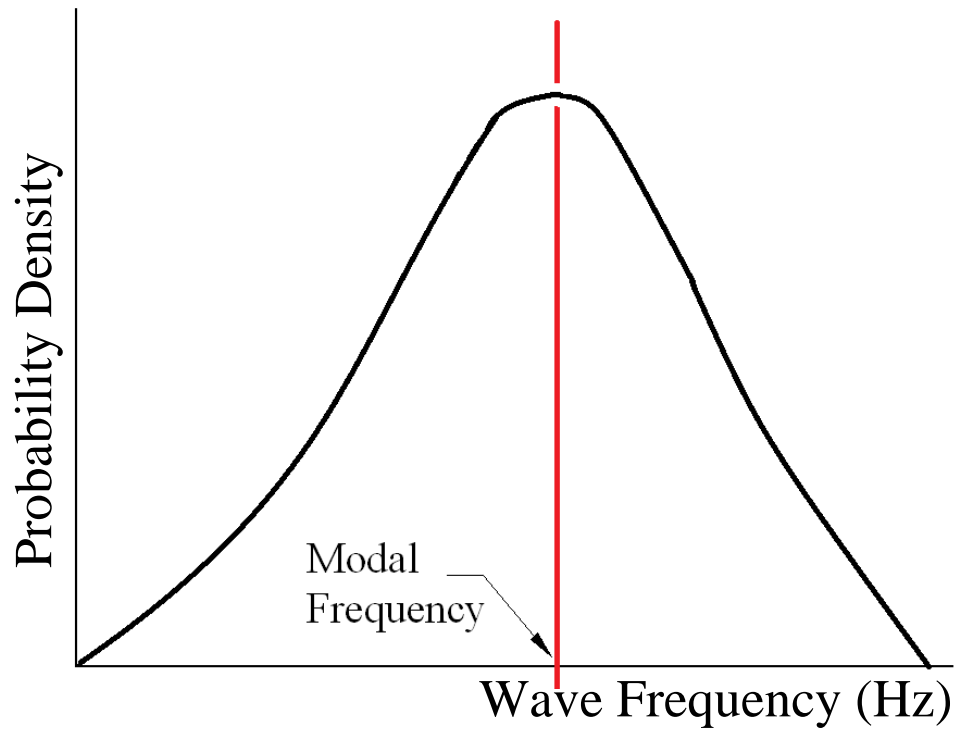

Figure 8.5 Typical Spectrum of a fully developed sea

8.2.4 Wave Data

Examination of sea spectra such as this allows the creation of tables of sea characteristics such as the NATO Unclassified table reproduced below.

Sea State Number	Significant Wave Height (ft)		Sustained Wind Speed (Kts)		Percentage Probability of Sea State	Modal Wave Period (s)	
	Range	Mean	Range	Mean		Range	Most Probable
0-1	0-0.3	0.2	0-6	3	0	-	-
2	0.3-1.5	1.0	7-10	8.5	7.2	3.3-12.8	7.5
3	1.5-4	2.9	11-16	13.5	22.4	5.0-14.8	7.5
4	4-8	6.2	17-21	19	28.7	6.1-15.2	8.8
5	8-13	10.7	22-27	24.5	15.5	8.3-15.5	9.7
6	13-20	16.4	28-47	37.5	18.7	9.8-16.2	12.4
7	20-30	24.6	48-55	51.5	6.1	11.8-18.5	15.0
8	30-45	37.7	56-63	59.5	1.2	14.2-18.6	16.4
>8	>45	>45	>63	>63	<0.05	15.7-23.7	20.0

Table 8.1 NATO Sea State Numeral Table for the Open Ocean North Atlantic.

Table 8.1 gives very useful information regarding likely wave system characteristics at different sea states. The modal wave periods are easily converted to modal wave frequencies via the following relationship.

$$\omega_w = \frac{2\pi}{T}$$

Wave frequency can then be used in further calculations (see 8.4.1).

The figure for significant wave height is often used to describe the height of the wave system. It corresponds to the average of the 1/3 highest waves. This value is typically estimated by observers of wave systems for the average wave height.

It is clear that although the sea appears very complicated, for each sea state there is a predominant modal frequency and wave height associated with that sea. Also, it is usual for these modal conditions to be generated by the wave energy source closest to the point of observation, this will almost always be the wind being experienced at the observation point. Consequently, as well as the modal period and wave height being known, its direction of movement will be in the same direction as the wind.

So the first part of the jig-saw is now in place. We know how to predict the magnitude, direction and frequency of excitation forces and moments in our simplified ship system discussed earlier. We now must study the way ships respond to these excitation forces and moments. However before we can proceed, we need to quickly review our knowledge of Simple Harmonic Motion.

8.3   Simple Harmonic Motion

Simple Harmonic Motion (SHM) is a natural motion that occurs in many engineering fields. Naval Engineering is no exception. A system will exhibit SHM when any displacement from its resting location causes it to experience a linear restoring force or moment.

- **Linear**   The size of force or moment must be proportional to the size of displacement.

- **Restoring**   The force or moment must oppose the direction of displacement.

A commonly used example of a system that exhibits SHM is the spring, mass, damper shown in Figure 8.6.
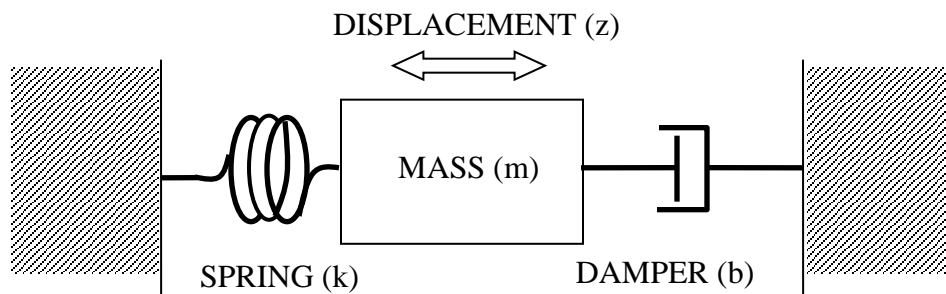
Figure 8.6 Spring-Mass-Damper System

If the mass is displaced in either direction, the spring will either be compressed or placed in tension. This will generate a force that will try to return the mass to its original location - a restoring force. Provided the spring remains within its linear operating region, the size of the force will be proportional to the amount of displacement - a linear force.

If the mass is let go, the linear restoring force will act to bring the mass back to its original location. However, because of inertia effects, the mass will overshoot its original position and be displaced to the other side. At this point the spring creates another linear restoring force in the opposite direction, again acting to restore the mass to its central position.

This motion is repeated until the effects of the damper dissipate the energy stored by the system oscillations. The important point to note is that no matter which side the mass moves, the mass always experiences a linear restoring force - it exhibits SHM.

The mathematics behind the motion involves the analysis of a differential equation involving displacement (z) with respect to time (t).

If the effects of the damper are ignored:

$$m\frac{d^2z}{dt^2} + kz = 0$$

the solution is a simple cosine.

$$Z = Z_0 \cos(\omega_n t)$$

where Z_0 is the initial displacement and ω_n is the natural frequency of the system, t is time.
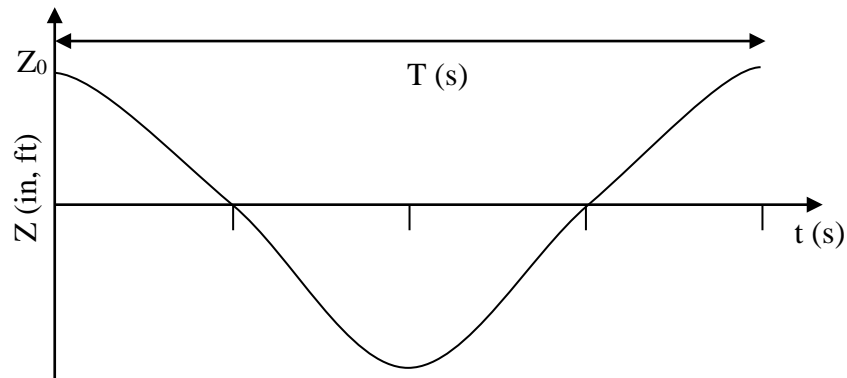
Figure 8.7 plots displacement (z) against time (t).

Figure 8.7 Displacement vs Time Plot Without Damping

From this plot it is possible to find the period (T) of the system motion and from this calculate the natural frequency. It is also possible to check the observed natural frequency against the known system parameters, mass (m) and spring constant (k).

$$\omega_n = \frac{2\pi}{T} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

8.3.1   Damping

In reality, the amplitude of oscillation of the spring, mass, damper system plotted in Figure 8.7 will reduce with time due to damping effects. The damper works by dissipating the energy of the system to zero.

Changing the viscosity of the fluid in the damper the level of damping can be altered. A low level of damping will allow several oscillations before the system comes to rest. In this instance

the system is **under damped**. An important level of damping to the control engineer is **critical damping** where the system is allowed to overshoot once and then return to rest. Critically damped system return to rest in the shortest period of time. Large amounts of damping will cause the system to be **over damped**. No oscillations occur; the motion is a slow return to the resting position. Figure 8.8 shows a displacement v time plot for the 3 levels of damping.
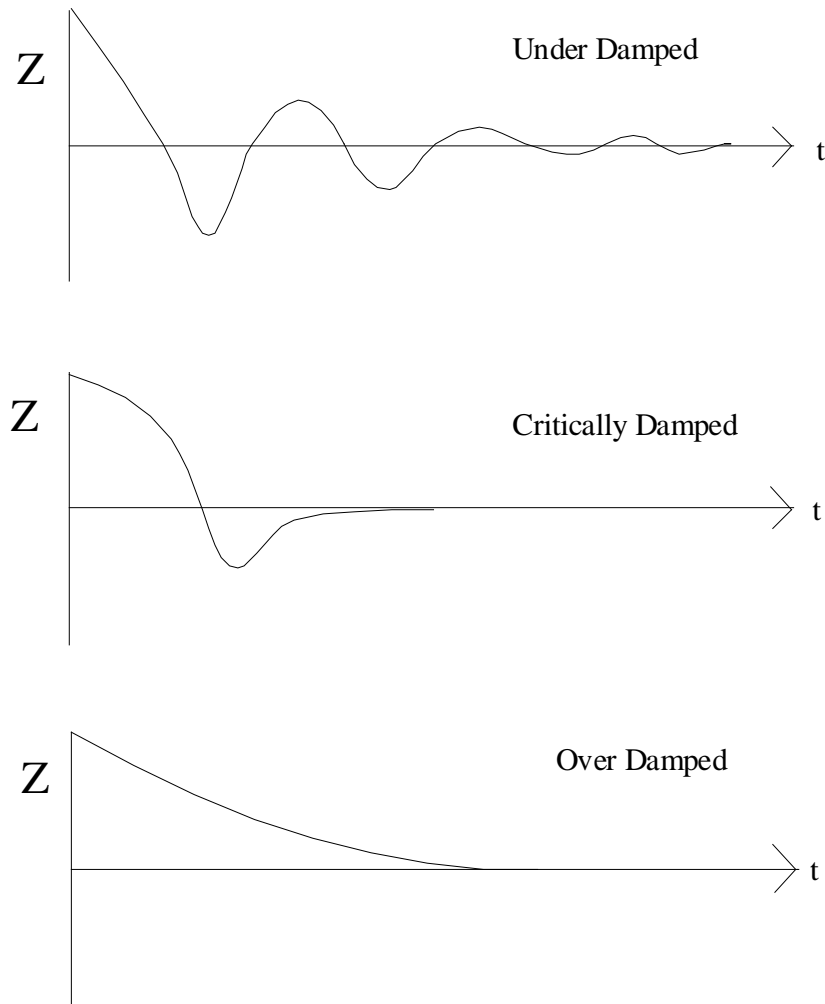
Figure 8.8 Displacement vs Time Plots for the three types of damping

8.3.2   Forcing Function and Resonance

The plots in Figures 8.7 and 8.8 consider SHM without any external exciting force or moment (apart from the force that initially displaces the system). To enable the spring, mass damper system to remain oscillating, it is necessary to inject energy into the system. This energy is required to overcome the energy being dissipated by the damper. In this system it would be applied as an external force, often called an **external forcing function**. Unfortunately, the presence of an external forcing function adds further complications to the analysis of system.

To create maximum displacement, the forcing function has to inject its energy to coincide with the movement of the mass, otherwise it is likely to inhibit oscillation rather than encourage it. So to maintain system oscillation, a cyclical force is required that is at the same frequency as the SHM system. When this occurs, the system is at **resonance** and maximum amplitude oscillations will occur. If the forcing function is applied at any other frequency, the amplitude of oscillation is diminished.

A mathematical analysis of the equations of motion supports this. The differential equation for the mass-spring system with forcing function (assuming the effects of the damper are ignored) becomes:

$$m\frac{d^2z}{dt^2} + kz = F\cos\omega t$$

where F is the size of the forcing function and ω is the frequency at which it is applied.

The solution becomes:

$$Z = \frac{F}{k}\frac{1}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\cos\omega t$$

when $\omega \ll \omega_n$ $Z = F/k$

when $\omega \gg \omega_n$ $Z = 0$

when $\omega = \omega_n$ $Z = \infty$

Figure 8.9 shows a plot of system motion amplitude against the frequency of the forcing function. It is obvious where the natural frequency of the system falls.
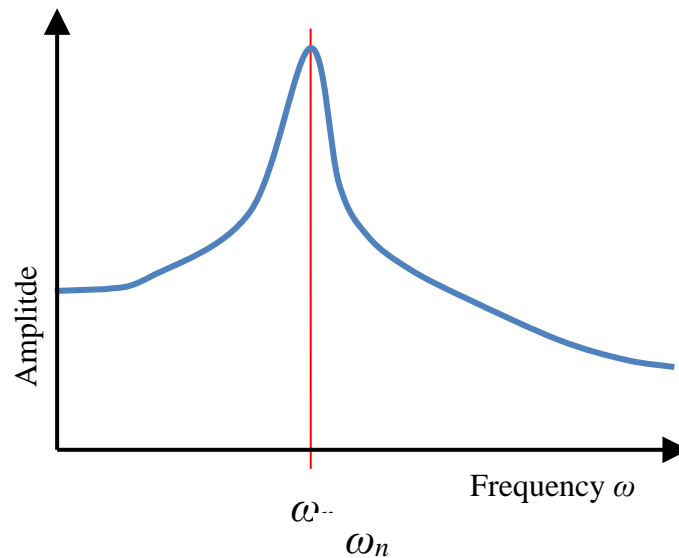

Figure 8.9 Motion Amplitude vs. Forcing Function Frequency

Figure 8.10 compares this type of plot for a system that is sharply tuned and one that is not. In general, SHM systems that are lightly damped will be more sharply tuned than those possessing higher levels of damping. Lightly damped systems are far more sensitive to the frequency of the forcing function.
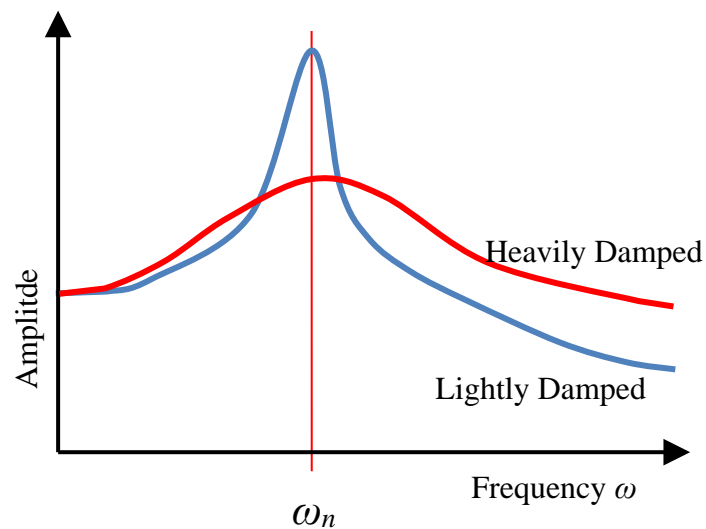

Figure 8.10 Comparison of Frequency Response between Lightly and more Heavily Damped Systems

8.4   Ship Response

The predictions of ship response when encountering a wave system is very complicated due to the confused state of typical wave systems. However, in most seas it is possible to determine the predominant direction of wave travel and a wave period. See Table 8.1. With this known, it is possible to work out ship response.

8.4.1   Encounter Frequency

When we examined the SHM of a mass, spring, damper system we saw that the motion created by the excitation force was dependent upon the magnitude of the excitation force and its frequency. The response of a ship to its excitation force is no different. However, the frequency of the excitation force is not only dependent upon wave frequency and direction, but also the speed and heading of the ship. The important parameter is the **encounter frequency ω_e** that allows for the relative velocity of the ship and sea waves.

$$\omega_e = \omega_w - \frac{\omega_W^2 \cdot V_S \cdot \cos(\mu)}{g}$$

where V_S is the ship speed in ft/s
μ is the heading of the ship relative to the direction of the sea (° Rel)

Figure 8.11 shows the value of μ for different sea - ship orientations. Hence for a given wave frequency (ω_w), the ship handler can alter ω_e by changing course or speed. $\mu = 0°$ when the ship direction and the wave direction are the same. Note: due to the symmetry of the cosine function, it is sufficient to find μ as the smaller angle formed from placing the ship's heading vector ***tail-to-tail*** with the vector representing the wave direction. $[\cos(\mu) = \cos(360° - \mu)]$. If you find $\mu > 180°$, use the reflex angle (the balance of the circle).
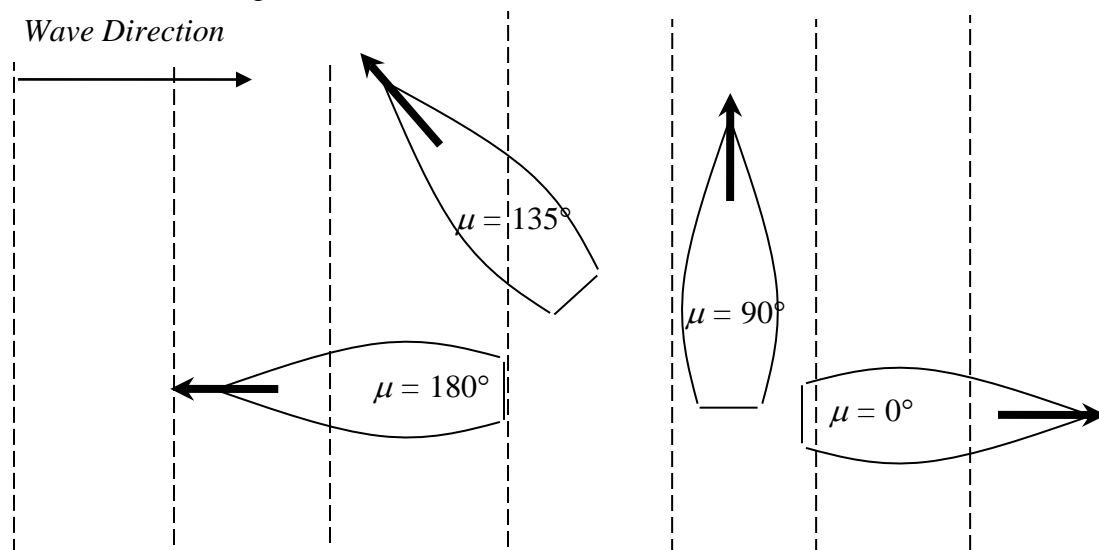

Figure 8.11 Values of μ for Different Ship Orientations

Example 8.1 A ship traveling at 20 knots on a course of 120 degrees is encountering waves coming from the north with a wave period of 12 seconds. What is the encounter frequency? (1 kt = 1.689 ft/s)

Solution:

$$\omega_w = \frac{2\pi}{T} = \frac{2\pi}{12s} = 0.52\,rad/s$$

The difference between the sea heading (180 degrees) and the ship heading (120 degrees) is 60 degrees.

$$\Rightarrow \mu = 60° \quad also \quad V_S = 20\ kts \cdot \frac{1.689\ ft/s}{1\,kt} = 33.78\ ft/s$$

$$\omega_e = \omega_w - \frac{\omega_w^2 V \cos\mu}{g}$$

$$\omega_e = 0.52\,rad/s - \frac{(0.52\ rad/s)^2\ (33.78\ ft/s)\cos(60)}{32.17\ ft/s^2}$$

$$\omega_e = 0.52\,rad/s - 0.14\,rad/s = 0.38\,rad/s$$

With the encounter frequency known it is possible to make a prediction about ship responses. They can be grouped into 3 major sets.

- Rigid Body Motions.

- Structural Responses.

- Non-oscillatory Dynamic Responses.

8.4.2 Rigid Body Motions

You may recall that the ship has 6 degrees of freedom about the xyz axis system, 3 rotary and 3 translatory. Figure 8.12 illustrates them, all can be considered as **rigid body motions**.
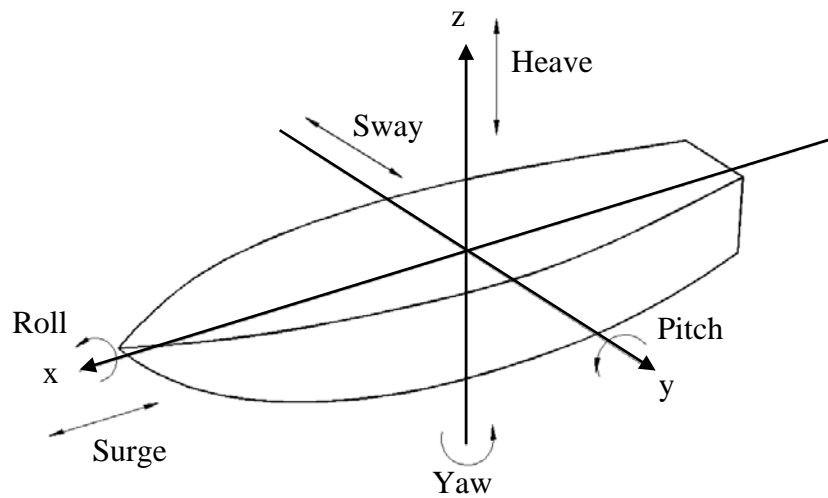
Figure 8.12 The 6 Degrees of Freedom

Of the 6 rigid body motions, 3 exhibit SHM because they experience a linear restoring force. They are the motions of Heave, Pitch and Roll. Each will be examined in turn.

8.4.2.1 Heave

The action of the sea can cause the ship to move bodily out of the water or sink below its waterline. This causes an imbalance between displacement and the buoyant force that creates a resultant force which attempts to restore the ship to its original waterline. Figure 8.13 illustrates the generation of the restoring force.
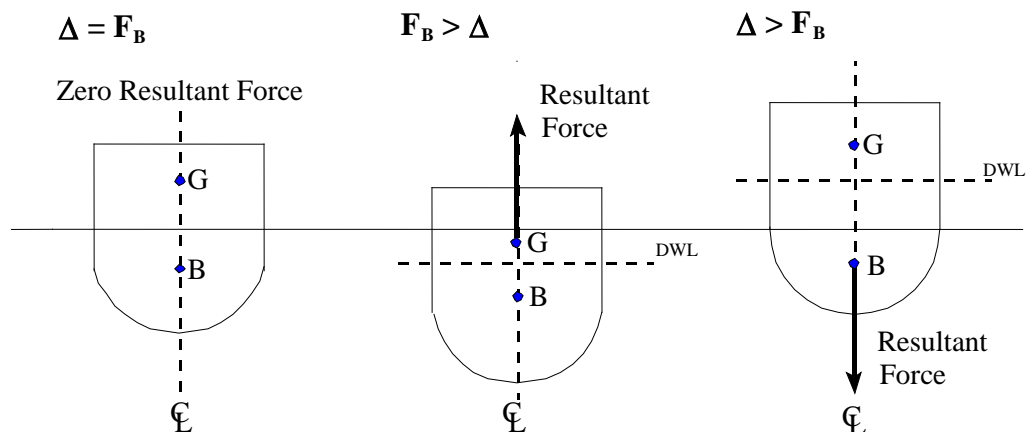

Figure 8.13 Generation of Heave Restoring Force

The restoring force is proportional to the distance displaced since the disparity between displacement and buoyant force is linear for different waterlines. The quantity that measures this force directly is the TPI of the ship.

Hence, heave motion has a linear restoring force. It is a SHM.

The up and down motion is completely analogous to the mass, spring, damper system we have studied.

$$Spring\ Constant\ (k) \equiv TPI \qquad Mass\ (m) \equiv \frac{\Delta}{g}$$

So taking this analogy further, it is possible to predict the natural heave frequency (ω_{heave}) of a ship.

$$\omega_{heave} \propto \sqrt{\frac{TPI}{\Delta}}$$

It should also be apparent that the TPI is heavily dependent upon the A_{WP} at the DWL, in fact the following relationship exists.

$$TPI = \frac{\rho g A_{WP}}{2240 \times 12} \qquad TPI \propto A_{WP} \text{ at DWL}$$

You may recall that TPI and A_{WP} are read off the same line on the curves of form.

Consequently, ships that have a large water plane area for their displacement will experience much greater heave restoring forces than ships with small water plane areas. So 'beamy' ships such as tugs and fishing vessels will suffer short period heave oscillations and high heave accelerations. Conversely, ships with small water plane areas will have much longer heave periods and experience lower heave accelerations. In general the lower the motion acceleration, the more comfortable the ride and the less chance of damage to equipment and personnel. This concept is taken to extremes in the case of off-shore floating platforms that have very small A_{WP} compared to their displacement.

The heave motion is quite heavily damped as the energy of the ship moving up and down creates waves that quickly dissipate the energy of the system away.

8.4.2.2 Roll

The rolling of ships has been studied in depth in earlier chapters so you should be aware that any transverse misalignment of B and G will create an internal righting moment. The misalignment can be created by a wave slope. Figure 8.14 refers.
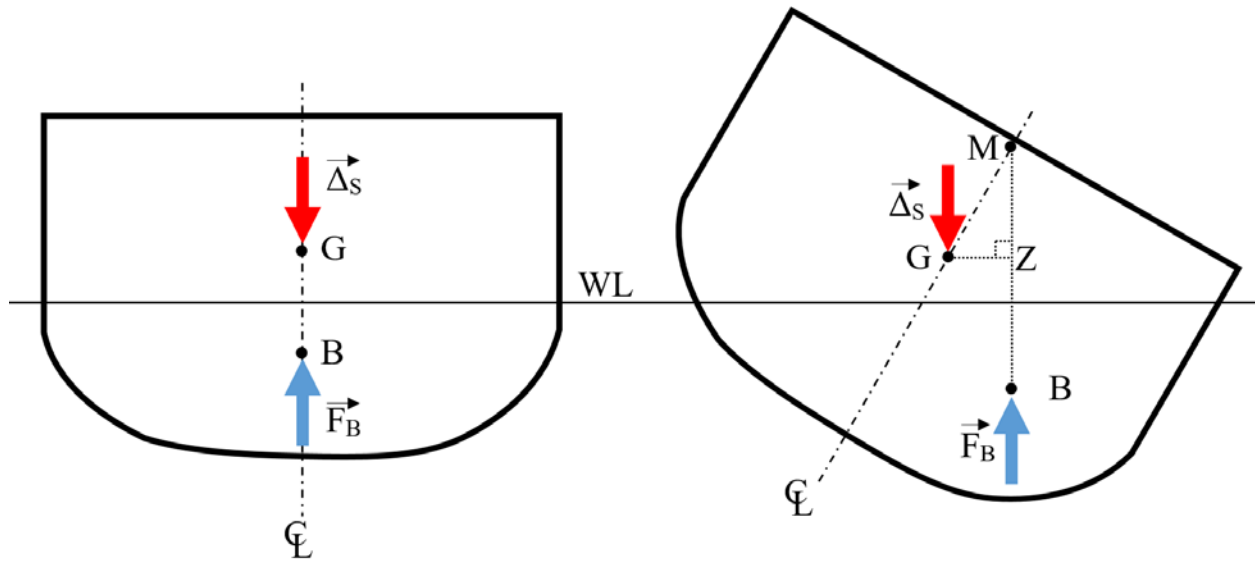

Figure 8.14 The Creation of the Internal Righting Moment

The magnitude of this righting moment depends upon the righting arm and ship displacement.

$$RM = \Delta \cdot \overline{GZ}$$

For small angles ($\phi < 10°$) this becomes:

$$RM = \Delta \cdot \overline{GM}_T \cdot \phi$$

So this time we see the creation of a linear restoring moment, and hence the presence of rotational SHM. Hence roll is also analogous to the linear motion of the mass, spring, damper system.

$$Spring\ Constant\ (k) \equiv \Delta \cdot \overline{GM}_T \qquad\qquad Mass\ (m) \equiv I_{xx}$$

Taking this analogy a little further we can find an expression for the natural roll frequency (ω_{roll}).

$$\omega_{roll} \propto \sqrt{\frac{\Delta \cdot \overline{GM}_T}{I_{xx}}}$$

By rearranging this expression and knowing the relationship between natural roll frequency (ω_{roll}) and period of roll T_{roll}.

$$T_{roll} = \frac{CB}{\sqrt{\overline{GM_T}}}$$

where:
- B   is the ship's Beam (ft)
- GM_T is the transverse metacentric height (ft)
- C   is a constant whose value can range from 0.35 - 0.55 (s/ft$^{\frac{1}{2}}$) when GM_T and beam are measured in ft. The value of C is dependent upon the roll damping of the ship. When unknown, a value of 0.44 gives good results.

The equation shows that ships with large transverse metacentric heights will experience small period oscillations, large restoring forces and large transverse angular accelerations. As with the other rigid body motions, the high accelerations are more likely to cause damage to equipment and personnel.

You may recall that the GM_T can be found by measuring the slope of the GZ curve at the origin. Hence stiff GZ curves are indicative of large GM_T, tender GZ curves indicate small GM_T. Figure 8.15 indicates the difference between the 2 types. So stiff ships will tend to have violent roll motions. This is typical of short 'beamy' ships that have low length to beam ratios.
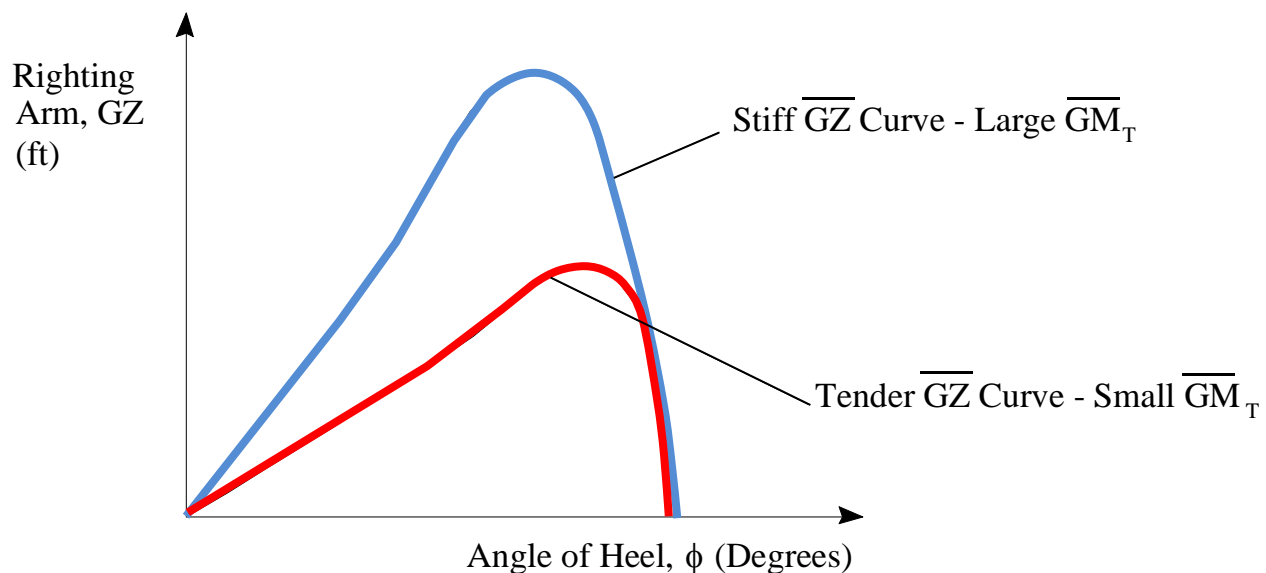

Figure 8.15 Comparison of Stiff and Tender Curves of Intact Statical Stability

From this you should realize that the ideal value of GM_T for a ship is a compromise between good sea keeping qualities (small GM_T) and good stability characteristics (large GM_T). This compromise is a common feature in many engineering fields. In this instance, the naval aArchitect aims for a GM_T of between 5 - 8% of a ship's beam. This offers a good compromise between seakeeping and stability.

Unlike the SHM of heave and pitch, roll motion experiences low damping effects because only low amplitude wave systems are generated by roll.

> ⚠ **"Sallying" Experiment -** The equation for roll period above has another use in the 'sallying' experiment. It is possible to induce roll motion in a ship by the cyclical transverse movement of personnel, known as 'sallying the ship'. Recording the roll period then allows an estimation of GM_T to be made.

8.4.2.3 Pitch

A ship heading into a sea (or in a stern sea) is liable to have situations where the slope of the waterline causes a movement in the center of buoyancy either forward or aft. This immediately creates an internal righting moment that attempts to restore the vertical alignment of B and G. Figure 8.16 illustrates this point.
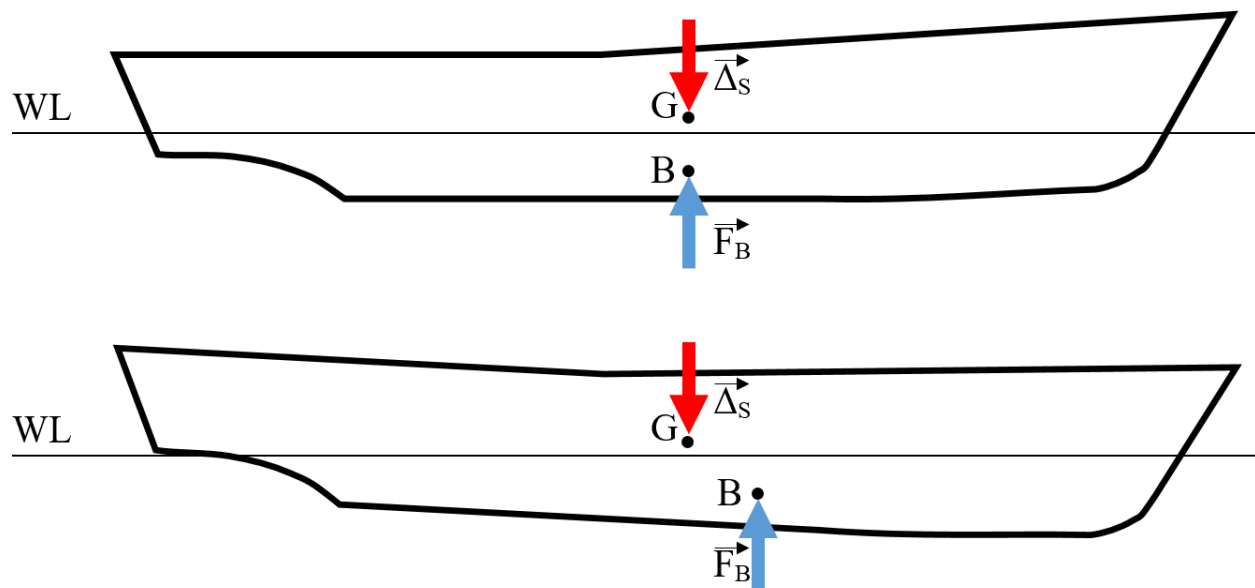

Figure 8.16 Generation of Pitch Restoring Moment

The internal righting moment is always acting to restore the ship and it is linear because it is dependent upon the MT1" value for the hull form. Hence, pitch motion is SHM.

Once again the situation is analogous to the mass, spring, damper system although we are now comparing angular motion with linear.

$$\textit{Spring Const } (k) \equiv \textit{MT1}^{''} \qquad\qquad \textit{Mass } (m) \equiv \textit{I}_{yy}$$

Ships that have large MT1" when compared with I_{yy} will experience large pitch restoring moments and as a consequence, large angular pitch accelerations. This will occur with ships that are long and slender and have the majority of their weight close to midships.

As with heave, pitch motions are quickly damped as the oscillation causes the generation of large wave systems pulling energy from the SHM.

8.4.2.4 Resonance

You will recall that a SHM system will experience maximum amplitude oscillations when the frequency of the forcing function is equal to the natural frequency of the system. This condition is called resonance. So to reduce the amount of motion it is important to ensure that resonance does not occur.

This can be applied to a ship. We have seen that the rigid body motions of heave, pitch and roll are SHMs and that it is possible to estimate their natural frequencies, ω_{heave}, ω_{pitch} and ω_{roll} respectively. To limit ship motion it is important that these do not coincide with the frequency of the ships forcing function, the encounter frequency (ω_e).

Fortunately, the SHMs of pitch and heave are well damped and as such are not sharply tuned. However, the low damping experienced by roll motion cause it to be sharply tuned and very susceptible to ω_e. Figure 8.17 indicates the differences between the 3 motions.

Resonance can occur with all of them, however, extreme motions are ***more likely to occur with roll than pitch and heave.***

Because of relatively small restoring forces and its sharply tuned response characteristic, many devices are used to try and limit the roll motion. These 'anti-roll' devices will be examined later in this chapter.
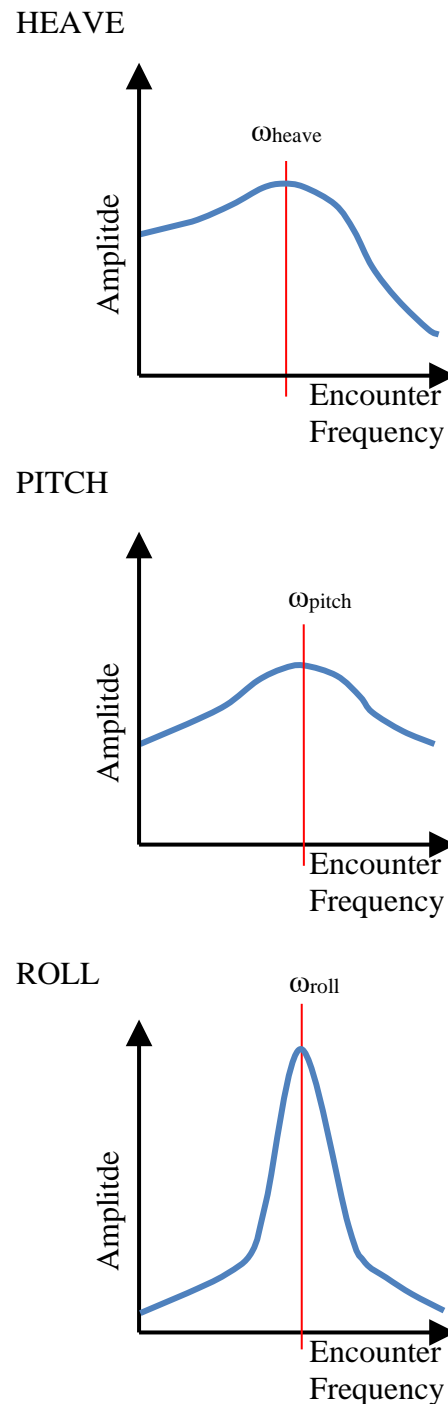
HEAVE

PITCH

ROLL

Figure 8.17 Comparison of the Frequency Response of the Motions of Heave, Pitch & Roll

8.4.3　Structural Responses

We have already seen that the sea can have a considerable effect upon the stresses that a ship structure has to withstand. These include:

1. Longitudinal bending caused by the ship being placed in a 'sagging' or 'hogging' condition by the position of wave crests.

2. Torsion. Waves systems can create a twisting effect upon the ship structure. This is particularly evident in ships with large hold openings such as container ships.

3. Transverse stresses caused by the hydrostatic pressure of the sea.

Due to the cyclical nature of the external forces and moments generated by the wave system, the ship structure is subjected to these stresses listed above in a cyclical manner, the frequency of which being equivalent to the encounter frequency (ω_e).

As with any structure, the ship structure will have its own natural frequency. In fact it will have many, one associated with each of the major loads - longitudinal bending, torsion and transverse stresses. It will also have numerous others associated with the elements of ship structure such as stiffeners plates, machinery mounts etc. Just as with the rigid body motions, the amplitude of structural oscillations will be maximized when their natural frequency coincide with the encounter frequency (ω_e). This must be avoided otherwise it is possible that yield stresses and endurance limits will be exceeded causing plastic deformation and a much higher risk of fatigue failure.

8.4.4　Non-Oscillatory Dynamic Responses

In addition to the oscillatory responses discussed above, the ship will experience a number of other dynamic responses. These are typically non-oscillatory and are caused by the relative motions of the ship and sea. The relative motions can be extreme. They are maximized when a movement of the ship out of the water due to heave, pitch or roll combines with a lowering of the sea surface (a trough) or vice versa. When this occurs the following severe non-oscillatory dynamic responses may result.

- **Shipping Water ("Green Deck")**　The relative motion of the ship and sea system can cause situations where the bow of the ship becomes submerged. This is called 'shipping water' or 'deck wetness'. As well as the obvious safety hazard to personnel, the extra weight of shipped water can place considerable loads on the ship structure.

- **Forefoot Emergence**　This is the opposite case to 'shipping water' where the bow of the ship is left unsupported. The lack of support creates severe structural loads.

- **Slamming**　Slamming is often the result of forefoot emergence. As the bow re-

enters the sea, the sudden impact of flat horizontal surfaces in the bow region creates a severe structural vibration felt through the length of the ship. See Figure 8.18.

- **Racing**   Racing is the sterns version of forefoot emergence. It occurs when the relative motion of ship and sea causes the propeller to leave the water. The sudden reduction in resistance causes the whole ship power train to race. This causes severe wear and tear on propulsion machinery and auxiliaries.

- **Added Power**   The effects of all these responses is to increase the effective resistance of the hull, consequently more power is required to drive the ship through the sea system.

As mentioned, the non-oscillatory dynamic responses above are all a consequence of the relative motion of the ship and sea. In particular, the larger the amplitude of the ship's heave and pitch motions the greater the possibility of shipping water, slamming etc. Hence, the frequency of these adverse ship responses can be reduced by making every attempt to reduce heave and pitch motions. This can be done by ensuring neither of these motions are at resonance. Changing the encounter frequency by alterations in course heading and speed may help.

- **Broaching**   Broaching is the sudden and uncontrollable turning of a ship to a beam on orientation to the sea. If the sea is big enough and has sufficient wave slope, there is then a high risk of capsize.

- **Loss of Stability**   In certain circumstances when in large sea systems, it is possible for a ship to surf. The prolonged change in shape of the water plane can have an adverse effect on stability.

Both these responses are the result of the ship traveling in large following seas at speeds close to the wave celerity. This should be avoided if at all possible.

Figure 8.18 SEAL Team 6 operators experience slamming in high-speed assault craft

8.5   Ship Response Reduction

We have now examined the various responses associated with a ship when excited by a wave system's forcing function. Unfortunately, the presence of this forcing function cannot be avoided, in fact it is very rare for there to be no forcing function. See Table 8.1. Consequently, despite the Naval Architects best efforts, it is impossible to prevent ship response. However, there are a number of things that can be done to minimize it.

8.5.1   Hull Shape

When examining the simple harmonic rigid body responses of heave, pitch and roll, the type of hull shape particularly susceptible to each motion was considered. Taking this knowledge a little further, it is fairly obvious to deduce that the creation of a hull shape optimized for minimum response can be created.

Traditionally, the seakeeping dynamics of the hull form have been considered at a lower level of importance than hull resistance, strength and space efficiency. The seakeeping characteristics of ships have been left to chance. This is unfortunate when one considers the impact poor seakeeping can have upon the operational effectiveness of the ship.

This state of affairs was true for the design of USN ships until the arrival of DDG-51. This hull form was the first to be created with seakeeping considered as a high priority. The differences between the hull of the DDG-51 and hulls designed previously are obvious on comparison.

- Forward and aft section are V shaped - limits MT1" reducing pitch accelerations.

- Volume is distributed higher from keel - limits A_{WP} and TPI reducing heave accelerations.

- Large degree of flare in hull design - limits the half-breadths at the design waterline, which in turn limits I_{xx}, reducing the stiffness of the GZ curve, thereby reducing roll accelerations.

The result is a ship that should have increased operability in heavy seas by a reduction in angular and vertical motion accelerations. There should also be a reduction in the frequency at which the non-oscillatory dynamic responses such as slamming occur due to the reduction in heave and pitch motions.

8.5.2   Anti-Roll Devices

When examining the roll response of a ship in 8.4.2.3, it was evident that the low level of damping experienced by roll caused it to have a highly tuned response characteristic. This made it highly susceptible to the encounter frequency of the ship. In an attempt to damp roll motion more effectively, a number of devices can be incorporated into the ship design.

8.5.2.1 Passive Anti-Roll Devices

Passive devices are as named, devices that require no external input to damp roll motion.

- **Bilge Keel**   Bilge keels are very common features on ships. They are typically located in pairs port and starboard and consist of flat plates projecting out from the hull at the point where the bilge turns up to the side wall of the ship. They can be very effective, reducing roll amplitudes by up to 35%.

- **Tank Stabilizers**   In chapter 4 we discovered that the presence of free fluid surfaces have a detrimental effect upon ship stability by reducing the effective transverse metacentric height. However, if the flow of fluid from one side of the tank to the other is 'throttled', the relative motion of the C of G of the fluid can damp roll motion. The level of throttling is critical to the tank effectiveness and usually has to be altered on a 'suck it and see' basis while the ship is underway. The conventional shape of passive tank stabilizers are long and fairly narrow orientated with their longest side transversely. However, many other shapes have been tried, including a 'U' tube running from the port side weather deck all the way to the keel and back up to the starboard weather deck.

- **Others**   Various other passive systems have been tried including delayed swinging pendulums, shifting weights and large gyroscopes. All suffer serious limitations with regard to the space they take up and safety.
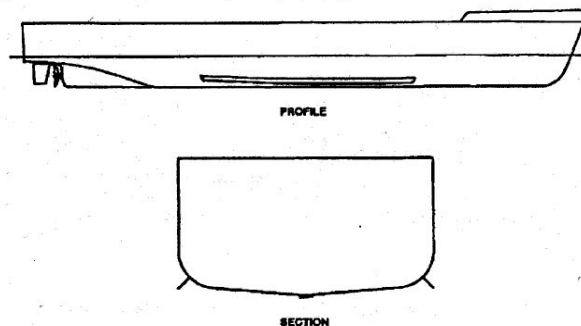

PROFILE

SECTION

Figure 8.19 Bilge Keels

8.5.2.2 Active Stabilizers

Active stabilizers rely on a control system that can detect ship motions and cause the stabilizer to respond with an action that limits the detected motions.

- **Fin Stabilizers**   Fin stabilizers are very common systems found on many ships. They are positioned in similar locations to bilge keels and work in

pairs port and starboard. They incorporate their own control circuitry that immediately detects when the ship begins to roll. A control signal is then sent to the fin hydraulics that create an angle of attack to the oncoming water. Lift is generated creating a moment opposite in direction to the internal moment produced by the transverse movement of B. These were employed on the FFG-7 class.

- **Others** There have been several other attempts including active tanks and the hydraulic movement of weights within the hull. All incur the same disadvantages suffered by weight shifting passive systems.
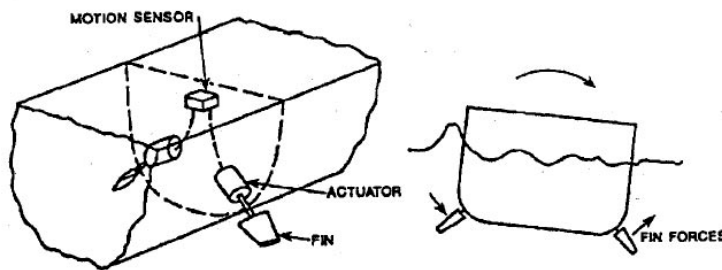
Figure 8.20 Active Fin Stabilizers

8.5.2.3 Stabilizer Effects

The effect of both passive and active stabilizers are to lessen roll motion. Their presence lessens the extreme roll amplitudes that can be suffered by a ship by increasing roll damping. The increase in damping reduces the susceptibility of the ship to the encounter frequency of the ship. Resonance can still occur, however roll amplitude at resonance is reduced.

As the name suggests, anti-roll devices have little impact upon the motions of heave and pitch. The heavily damped nature of these motions negate the need for any prescribed anti-heave or anti-pitch device.

8.5.3 Ship Operation

This chapter has demonstrated the responses associated with a ship excited by the sea's forcing function. It should be apparent by now that nearly all responses are significantly influenced by the encounter frequency the ship is experiencing. If the encounter frequency is close to any of the rigid body natural frequencies significant angular or vertical accelerations can be experienced. This in turn can greatly increase the possibility of non-oscillatory dynamic motions. Similarly, structural loads can be severe if resonance occurs between their natural frequency and the encounter frequency.

As a consequence, the ship handler can have a significant effect upon the response of a ship by altering the encounter frequency. We have seen that encounter frequency (ω_e) is given by:

$$\omega_e = \omega_w - \frac{\omega_w^2 \cdot V_S \cdot \cos(\mu)}{g}$$

So by altering course or speed or both, the ship handler can reduce motion accelerations being created by the sea and structural loading.

Once the ship has been designed and built, the naval architect has done their part. Ship response reduction is then up to the ship handler. It is in your hands!

COURSE OBJECTIVES
CHAPTER 9

9.   SHIP MANEUVERABILITY

1.   Be qualitatively familiar with the 3 broad requirements for ship maneuverability:
   a.   Controls fixed straightline stability
   b.   Response
   c.   Slow speed maneuverability

2.   Qualitatively describe what each requirement is dependant upon.

3.   Briefly describe the various common types of rudder.

4.   Understand the various dimensions of the spade rudder, in particular
   a.   Chord
   b.   Span
   c.   Rudder stock
   d.   Root
   e.   Tip

5.   Qualitatively describe the meaning of:
   a.   Unbalanced Rudder
   b.   Balanced Rudder
   c.   Semi-balanced rudder

6.   Qualitatively describe the sequence of events that causes a ship to turn

7.   Qualitatively describe a rudder stall and understand what it means

8.   Qualitatively describe the arrangement and devices that can be used to provide a ship with maneuverability at slow speeds:
   a.   Rudder position
   b.   Twin propellers
   c.   Lateral thrusters
   d.   Rotational thrusters

9.1　Introduction

Ship maneuverability is a very complex and involved subject involving the study of equations of motion involving all six degrees of freedom. Analysis of these motion equations allows for predictions of ship maneuverability to be made. However, many assumptions are made, so model testing is required to verify analytical results. Once built, a ship's maneuvering characteristics are quantified during its Sea Trials.

To limit the level of complexity covered in this chapter, the analytical study of the equations of motion will be ignored. However, maneuverability requirements a ship designer strives to meet will be discussed along with the devices and their arrangements that can provide them.

After completing this chapter you will have an understanding of how a ship's rudder makes a ship turn and an appreciation of other devices that improve a ship's slow speed maneuverability.

Keep in mind that some ships do not have rudders, like the LCS, which uses high powered water jets for both propulsion and maneuvering.

9.2   Maneuverability Requirements

When given the task of designing a ship, the Naval Architect is given a number of design requirements to meet. These include the obvious dimensions such as L_{PP}, Beam and Draft, but also other requirements such as top speed, endurance, operational mission, etc. Some of the more complicated requirements involve maneuverability. These can be split into three broad categories.

9.2.1   Directional Stability

In many operational circumstances, it is more important for a ship to be able to proceed in a straight line than turn. That is, with the rudder set at midships, and in the absence of external forces, the ship will travel in a straight line. This is termed **controls fixed straight line stability**. Our experience indicates that this scenario is rarely the case, and anything but a sea directly on the bow will create a yawing moment that has to be compensated for by movement of the ship's rudder. However, in principle, ships should be designed to achieve controls fixed straight line stability. An illustration of this stability is at Figure 9.1.
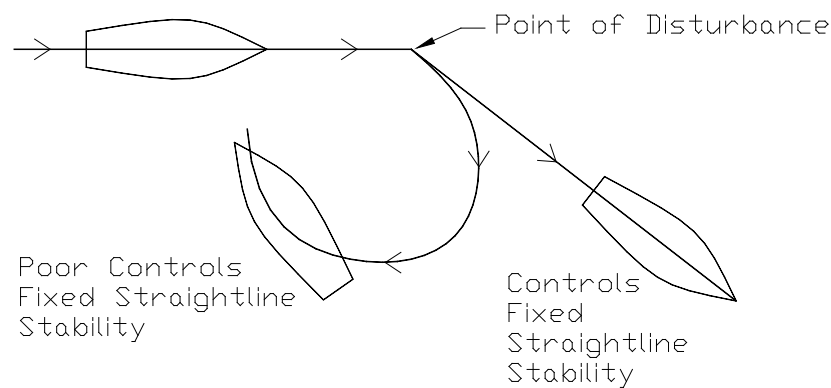
Figure 9.1 Hull Forms with Different Levels of Directional Stability

Despite this requirement, many hull forms do not have this level of directional stability. In particular, ships which are relatively short and wide such as tugs or harbor utility vessels and in certain circumstances, small combatants tend to have poor controls fixed straight line stability. This can be overcome by increasing the amount of deadwood of the hull (fin-like vertical surface) at the stern. This is directly analogous to the flights on an arrow or dart. Without flights, the arrow will tend to yaw in flight. With flights, the arrow maintains a straight line direction of flights. Hence increasing the amount of deadwood will increase the directional stability properties of a hull. For a ship, the problem is worsened because the ship is pushed rather than pulled along. Try pushing the end of your pencil and maintain its motion in a straight line!

Controls fixed straight line stability is quantified during sea trials by the spiral maneuver where rate of turn is compared with rudder direction.

9.2.2   Response

In opposition to the requirement for controls fixed straightline stability, it is also required for the ship to turn in a satisfactory manner when a rudder order is given. In particular:

- The ship must respond to its rudder and change heading in a specified minimum time.
- There should be minimum overshoot of heading after a rudder order is given.

In practice, both these response quantities are dependant upon the magnitude of the rudder's dimensions, the rudder angle and ship speed.

9.2.2.1 Rudder Dimensions

As we will see in the next section, rudder dimensions are limited by the geometry of the ship's stern. However, it is not surprising that the larger the dimensions of the rudder, the more maneuverable the ship. Increasing the rudder dimensions decreases the response time and overshoot experienced by the hull.

The level of response required by a ship is driven by its operational role. For example, the ratio of rudder area to the product of length and draft ranges from 0.017 for a cargo ship to 0.025 for destroyers.

$$\text{Rudder Area Ratio} = \frac{Rudder Area}{L_{PP}T}$$

9.2.2.2 Rudder Angle

Clearly, the response characteristics of a ship will depend upon the rudder angle ordered for a particular maneuver. It is common procedure for the levels of response to be specified with the ship using standard rudder. This is 20 degrees of wheel for the USN, but may vary for different ship types.

9.2.2.3 Ship Speed

Ship speed will also influence the level of maneuverability being experienced by the ship. In practice, for the majority of hull forms, greater ship speed will reduce response time but increase overshoot. This is because greater ship speed increases the rudder force being generated by a given rudder angle.

For this reason, during sea trials, ship response and overshoot is quantified at several ship speeds. Ship response is usually assessed by the zig-zag maneuver.

9.2.3   Slow Speed Maneuverability

It is most important for ships to be maneuverable when traveling at slow speeds. This is because evolutions such as canal transits and port entrances are performed at slow speeds for safety reasons. Unfortunately, this is when the ship's rudder is least effective.

Levels of slow speed maneuverability are specified in terms of turning circle and other quantifiable parameters at speeds below 5 knots. Devices that can improve slow speed maneuverability will be discussed later.

9.2.4   Maneuverability Trade-Off

Unfortunately, the need for good directional stability (in particular controls fixed straightline stability) and minimum ship response oppose each other. For example, for a fixed rudder area, increasing the length of a ship will make it more directionally stable but less responsive to its rudder. As discussed, a similar effect is created by increasing the amount of vertical flat surface at the stern (deadwood).

However, increasing rudder area will always improve the response characteristics of a hull form and usually improve its directional stability as well. Unfortunately, rudder dimensions are limited by stern geometry. Also larger rudders will increase drag and so reduce ship speed for a given DHP from the propeller.

9.3   The Rudder

9.3.1   Rudder Types

Clearly, the rudder is the most important control surface on the hull. There are a multiplicity of different types. Figure 9.2 reproduced from the SNAME publication "Principles of Naval Architecture" shows some of them.
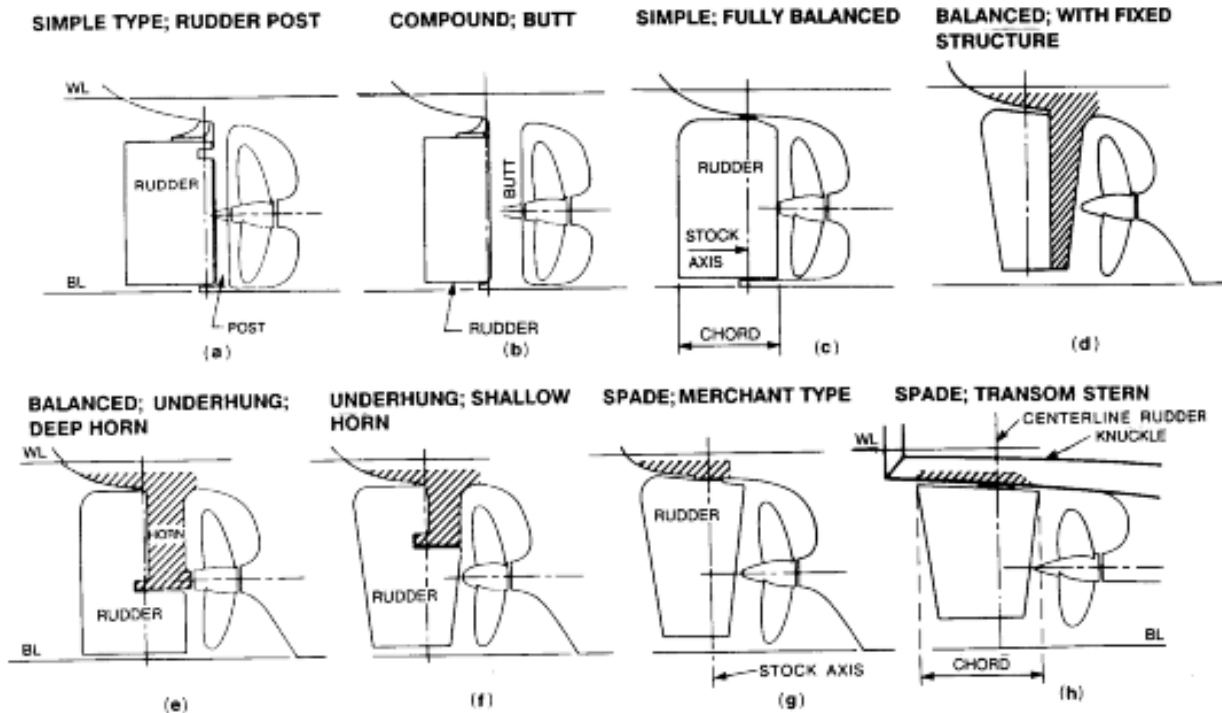
Figure 9.2 Different Rudder Types

The magnitude of all the rudder types dimensions are limited by the hull, specifically the stern's properties.

- **Chord**   The chord is limited by the position of the propeller (propeller/rudder clearance is specified by the American Bureau of Shipping for US ships) and the edge of the stern. Rudder protruding beyond the stern is inadvisable.

- **Span**   The span (or the height of the rudder from the root to tip) is limited by the draft and the need for the rudder to remain above the ship baseline (or keel). This is a "grounding" consideration. Typically, the span of the rudder is approximately the size of the propeller diameter.

9.3.2 The Spade Rudder

The most common type of rudder found on military vessels is the spade rudder. Figure 9.3 from "Introduction to Naval Architecture" by Gilmer and Johnson shows the geometry of a typical semi-balanced spade rudder.
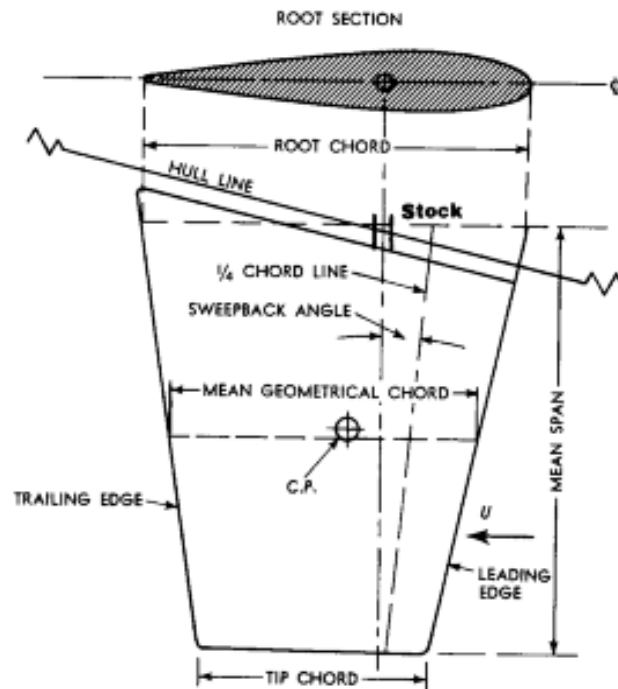
Figure 9.3 A Semi-balanced Spade Rudder

The CVN and LPD-17 class has a balanced underhung rudder due to their size.

9.3.2.1 Rudder Balance

Whether a rudder is balanced or not is dependant upon the relationship of the center of pressure of the rudder and the position of the rudder stock.

- When they are vertically aligned, the rudder is said to be "fully balanced". This arrangement greatly reduces the torque required by the tiller mechanism to turn the rudder.

- When the rudder stock is at the leading edge, the rudder is "unbalanced" as in Figure 9.2(a). This is a common arrangement in merchant ships where rudder forces are not excessive.

- The spade rudder in Figure 9.3 is semi-balanced. This is a sensible arrangement as it limits the amount of torque required by the tiller mechanism yet should ensure

the rudder returns to midships after the occurrence of a tiller mechanism failure.

9.3.3 Rudder Performance

It is a common misconception that the rudder turns a ship. In fact, the rudder is analogous to the flaps on an aircraft wing. The rudder causes the ship to orientate itself at an angle of attack to its forward motion. It is the hydrodynamics of the flow past the ship that causes it to turn. Figure 9.4 shows the stages of a ship turn.
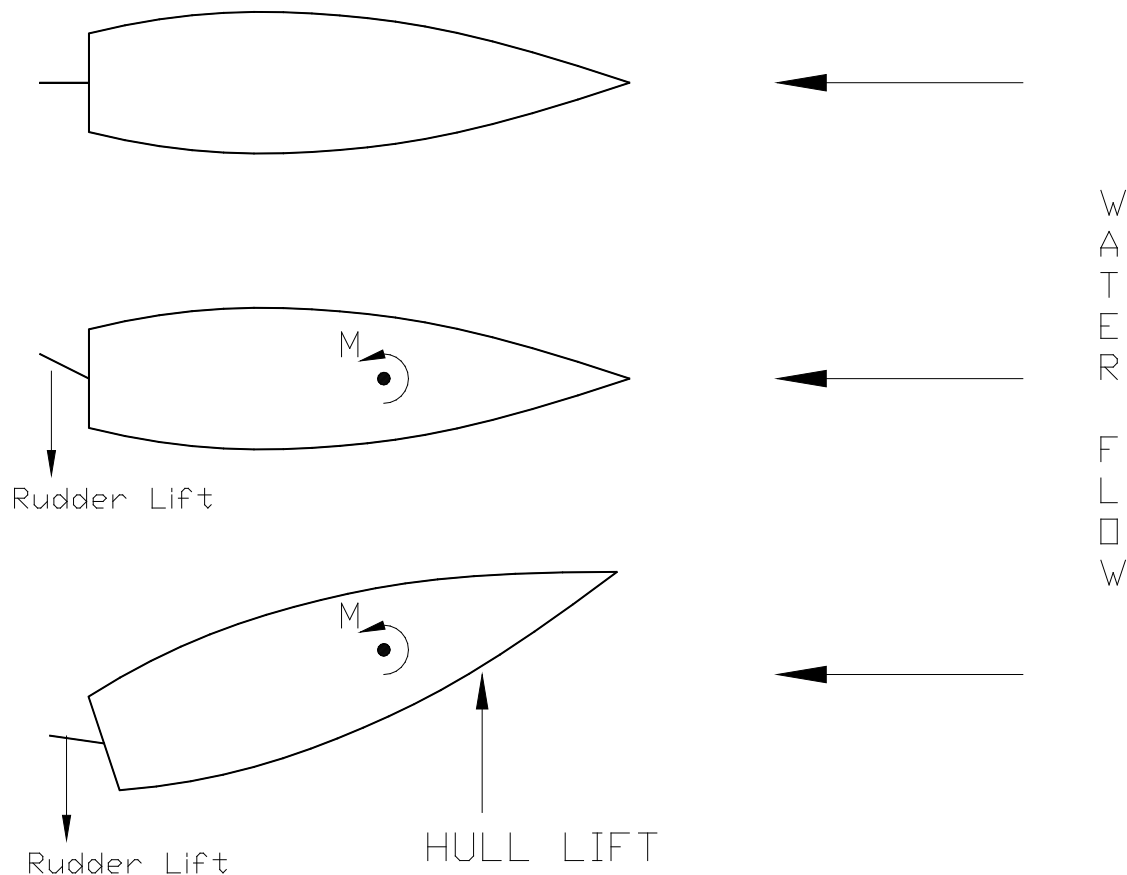
Figure 9.4 The Stages of a Ship's Turn

The ship will continue to turn until the rudder angle is removed.

9.3.3.1 Rudder Stall

You will notice that a typical ship's rudder is limited to a range of angles up to about ± 35°. This is because at greater angles than these the rudder is likely to stall. Most U.S. Navy ships are outfitted with a hard and soft rudder stop to prevent the rudder angle from exceeding 35°. Figure 9.5 from Gilmer & Johnson shows the development of stall as rudder angle increases. At small angles, rudder lift is created due to the difference in flow rate across the port and starboard sides

of the rudder. However, as rudder angle increases, the amount of flow separation increases until a full stall occurs at 45 degrees.
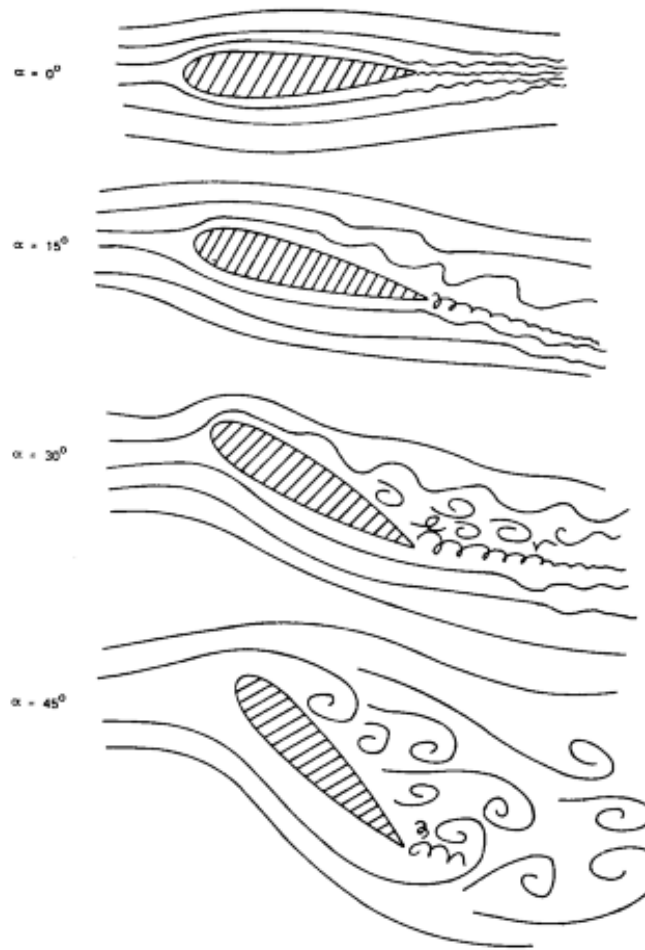
Figure 9.5 Rudder Flow Patterns at Increasing Rudder Angle

The amount of lift achieved by the rudder reduces significantly after a stall and is matched by a rapid increase in drag. Consequently, rudder angle is limited to values less than the stall angle. Figure 9.6 shows how rudder lift alters with rudder angle.
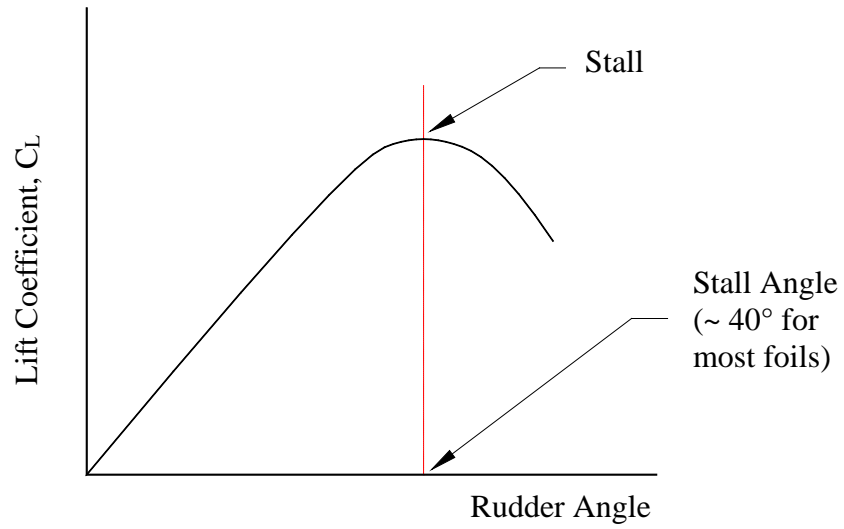
Figure 9.6 How Lift Alters with Rudder Angle

9.4   Slow Speed Maneuverability

As mentioned previously, it is at slow speeds when ships need to be the most maneuverable. Unfortunately, at slow speeds the rudder is limited in its effectiveness due to the lack of flow across its surfaces. However, there are several modifications that have been developed to improve the situation.

9.4.1   Rudder Position

To improve the low flow rate experienced by the rudder at slow speeds, the rudder is often positioned directly behind the propeller. In this position, the thrust from the propeller acts directly upon the control surface. A skilled helmsman can then combine the throttle control and rudder angle to vector thrust laterally and create a larger turning moment with minimal advance and transfer.

9.4.2   Twin Propellers

The presence of two propellers working 3in unison can significantly improve slow speed maneuverability. By turning one propeller in the reverse direction and the other in the forward direction, very large turning moments can be created with hardly any forward motion. This maneuver is called a "twist."

9.4.3   Lateral Thrusters

Lateral thrusters (or bow thrusters as they are usually positioned at the bow) consist of a tube running athwart ships inside of which is a propeller. They are usually electrically driven. With a simple control from the bridge, the helmsman can create a significant turning moment in either direction. Figure 9.7 shows a photograph of 2 lateral thrusters positioned in the bulbous bow of a ship. The photo is reproduced from "Introduction to Naval Architecture" by Gilmer and Johnson.
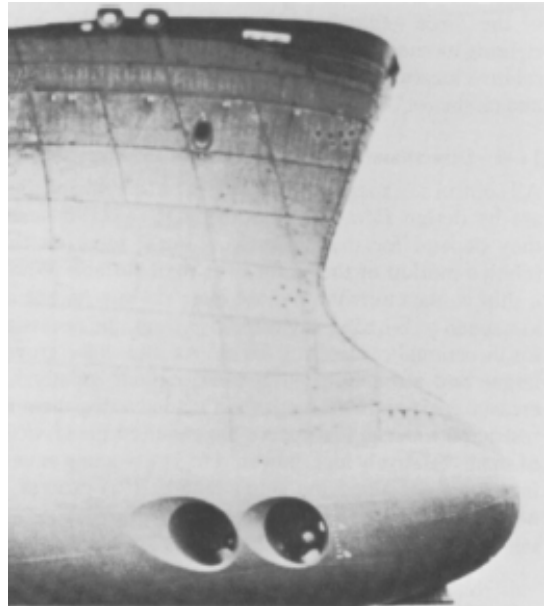

Figure 9.7 Lateral Thrusters in the Bow of a Ship

9.4.4   Rotational Thrusters

These provide the ultimate configuration for slow speed maneuverability. Rotational thrusters' appearance and operation resembles an outboard motor. They consist of pods that can be lowered from within the ship hull. Once deployed, the thruster can be rotated through 360 degrees

allowing thrust to be directed at any angle. Figure 9.8 shows a typical "ro-thruster" design produced by Kværner Masa-Azipod of Finland.
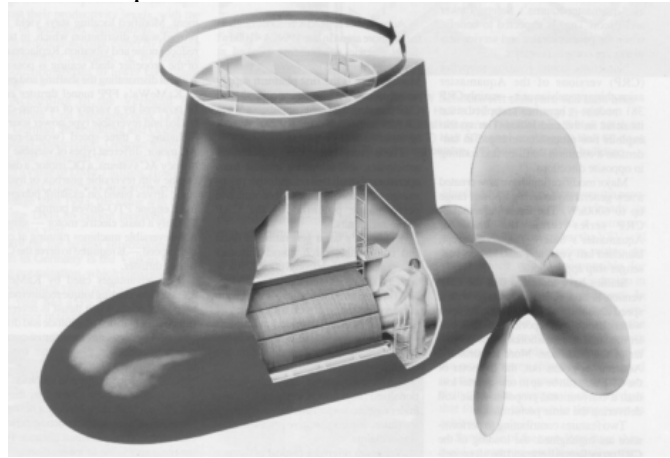

Figure 9.8 Typical Rotational Thruster Design

Some highly specialized ships use "ro-thrusters" as their only means of propulsion. 2 or 3 "ro-thrusters' coupled with a complicated G.P.S. centered control system can keep a ship in a geostationary position over the sea bed and at the same heading in quite considerable tide and wave conditions. These ships are often associated with diver, salvage or seabed drilling operations.

Figure 9.9 on the following page shows a typical auxiliary propulsion unit used on ships and submarines. It can be used for both emergency propulsion and maneuverability.

⚠ In practice, the amount of slow speed maneuverability exhibited by a ship is largely dependent upon the amount of money the designer is willing to spend on lateral or rotational thrusters in the ship design. This economic question is highly involved and includes estimates of ship docking rates, the costs of hiring tugs etc.
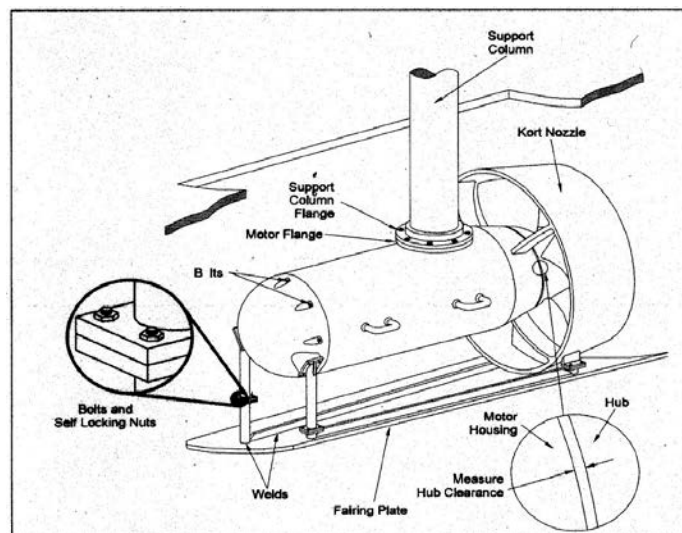
Figure 9.9 Auxiliary Propulsion Unit

APPENDIX A

TABLE of FRESH and SALT WATER DENSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Mass Density ρ for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

Density of fresh water ρ, lb-sec²/ft⁴ (= slugs/ ft³)	Temp, deg F	Density of salt water ρ_s, lb-sec²/ft⁴	Density of fresh water ρ, lb-sec²/ft⁴	Temp, deg F	Density of salt water ρ_s, lb-sec²/ft⁴
1.9399	32	1.9947	1.9384	59	1.9905
1.9399	33	1.9946	1.9383	60	1.9903
1.9400	34	1.9946	1.9381	61	1.9901
1.9400	35	1.9945	1.9379	62	1.9898
1.9401	36	1.9944	1.9377	63	1.9895
1.9401	37	1.9943	1.9375	64	1.9893
1.9401	38	1.9942	1.9373	65	1.9890
1.9401	39	1.9941	1.9371	66	1.9888
1.9401	40	1.9940	1.9369	67	1.9885
1.9401	41	1.9939	1.9367	68	1.9882
1.9401	42	1.9937	1.9365	69	1.9879
1.9401	43	1.9936	1.9362	70	1.9876
1.9400	44	1.9934	1.9360	71	1.9873
1.9400	45	1.9933	1.9358	72	1.9870
1.9399	46	1.9931	1.9355	73	1.9867
1.9398	47	1.9930	1.9352	74	1.9864
1.9398	48	1.9928	1.9350	75	1.9861
1.9397	49	1.9926	1.9347	76	1.9858
1.9396	50	1.9924	1.9344	77	1.9854
1.9395	51	1.9923	1.9342	78	1.9851
1.9394	52	1.9921	1.9339	79	1.9848
1.9393	53	1.9919	1.9336	80	1.9844
1.9392	54	1.9917	1.9333	81	1.9841
1.9390	55	1.9914	1.9330	82	1.9837
1.9389	56	1.9912	1.9327	83	1.9834
1.9387	57	1.9910	1.9324	84	1.9830
1.9386	58	1.9908	1.9321	85	1.9827
			1.9317	86	1.9823

NOTE: For other salinities, interpolate linearly.

APPENDIX B

TABLE of FRESH and SALT WATER KINEMATIC VISCOSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Kinematic Viscosity ν for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

Kinematic viscosity of fresh water $\nu, \dfrac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \dfrac{ft^2}{sec} \times 10^5$	Kinematic viscosity of fresh water $\nu, \dfrac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \dfrac{ft^2}{sec} \times 10^5$
1.9231	32	1.9681	1.2260	59	1.2791
1.8871	33	1.9323	1.2083	60	1.2615
1.8520	34	1.8974	1.1910	61	1.2443
1.8180	35	1.8637	1.1741	62	1.2275
1.7849	36	1.8309	1.1576	63	1.2111
1.7527	37	1.7991	1.1415	64	1.1951
1.7215	38	1.7682	1.1257	65	1.1794
1.6911	39	1.7382	1.1103	66	1.1640
1.6616	40	1.7091	1.0952	67	1.1489
1.6329	41	1.6807	1.0804	68	1.1342
1.6049	42	1.6532	1.0660	69	1.1198
1.5777	43	1.6263	1.0519	70	1.1057
1.5512	44	1.6002	1.0381	71	1.0918
1.5254	45	1.5748	1.0245	72	1.0783
1.5003	46	1.5501	1.0113	73	1.0650
1.4759	47	1.5259	0.9984	74	1.0520
1.4520	48	1.5024	0.9857	75	1.0392
1.4288	49	1.4796	0.9733	76	1.0267
1.4062	50	1.4572	0.9611	77	1.0145
1.3841	51	1.4354	0.9492	78	1.0025
1.3626	52	1.4142	0.9375	79	1.9907
1.3416	53	1.3935	0.9261	80	0.9791
1.3212	54	1.3732	0.9149	81	0.9678
1.3012	55	1.3535	0.9039	82	0.9567
1.2817	56	1.3343	0.8931	83	0.9457
1.2627	57	1.3154	0.8826	84	0.9350
1.2441	58	1.2970	0.8722	85	0.9245
			0.8621	86	0.9142

NOTE: For other salinities, interpolate linearly.

APPENDIX C

PROPERTIES of COMMON GEOMETRIC SHAPES

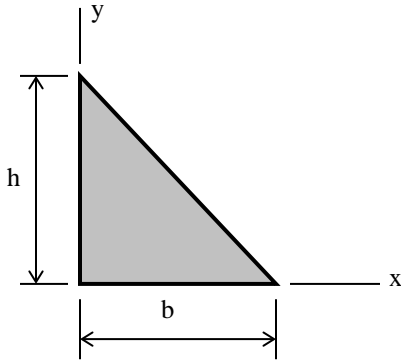**Rectangle** (origin of axes at centroid)
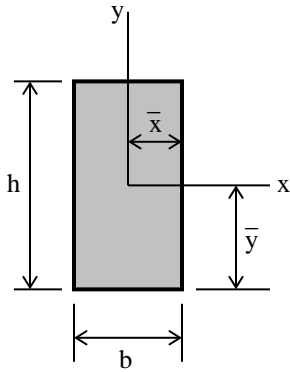
$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

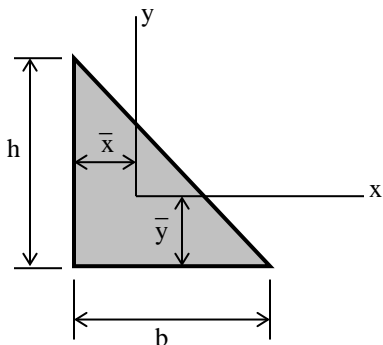**Right Triangle** (origin of axes at vertex)

$$A = \frac{bh}{2} \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$
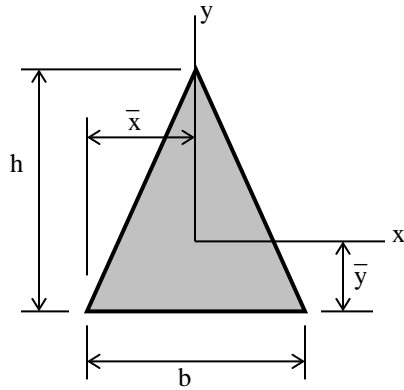
Right Triangle (origin of axes at centroid)

$$\bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

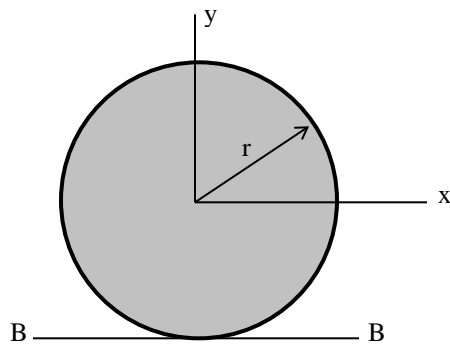$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36}$$

Isosceles Triangle (origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48}$$

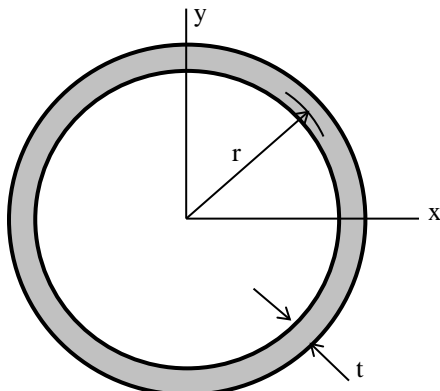## Circle (origin of axes at center)

$$d = 2r \qquad A = \pi r^2 = \frac{\pi d^2}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

Circular Ring with thickness " *t* " (origin of axes at center)
Approximate formulas for the case when *t* is small

$$A = 2\pi rt = \pi dt$$

$$I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$