NAVAL SEAKEEPING & MANEUVERING - VOL 2 OF 2

Main Category:	Naval Engineering
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Course #:	NAV-122
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Exam Preview:

1. The phase angle, δ, gives the phase relationship between the motion and the wave, the maximum positive response occurs δ/ω seconds after the maximum wave depression.
   a. True
   b. False
2. Using Figure 4.15: Relative ship and wave heading angle, μ and the surrounding reference material, which of the following μ values corresponds to following seas?
   a. 270
   b. 180
   c. 90
   d. 0
3. According to the reference material, there is a great deal of effort put into minimizing roll motion in ships or roll mitigation. Which of the following stabilization techniques matches the description: reduces the resonant peak, sometimes by adding appendages to the hull?
   a. Tuning stabilization
   b. Damping stabilization
   c. Equilibrium stabilization
   d. Harmonic stabilization
4. According to the reference material, rudder dimensions are limited by the geometry of the ship's stern. However, it is not surprising that the larger the dimensions of the rudder, the more maneuverable the ship.
   a. True
   b. False

5. You will probably have noticed that a typical ship's rudder is limited to a range of angles from about ±__ degrees. This is because at greater angles than these the rudder is likely to stall.
    a. 55
    b. 25
    c. 35
    d. 45
6. According to the reference material, levels of slow speed maneuverability are specified in terms of turning circle and other quantifiable parameters at speeds below 3 knots.
    a. True
    b. False
7. Active fins are active roll stabilizers that are mounted on rotatable stocks at the turn of the bilge near the middle of the ship. Their effectiveness increases with the square of the speed. Reductions of at least __% in the average roll amplitudes are possible in moderate waves with a well-designed system.
    a. 25
    b. 40
    c. 50
    d. 60
8. Generally, the turning path of a ship is characterized by four numerical measures: advance, transfer, tactical diameter, and steady turning diameter. All but the last are related to heading positions of the ship rather than tangents to the turning path.
    a. True
    b. False
9. The response characteristics of a ship will depend upon the rudder angle ordered for a particular maneuver. It is common procedure for the levels of response to be specified with the ship using standard rudder. This is _ degrees of wheel for the USN.
    a. 10
    b. 20
    c. 30
    d. 40
10. According to the Constraints on Rudder Design section of the reference material, the usual section shape is NACA 0015 (see Figure 7.20) to 0021 (relatively thick). These foils have a relatively constant center of pressure and thick sections are better structurally, and have a max thickness at __% chord length?
    a. 70
    b. 20
    c. 40
    d. 30

Chapter 4

The System: Ship Dynamics

Learning Objectives:

1. **For Dynamics Review:**
   (a) Determine the equation of motion for a spring-mass-damper system with sinusoidal excitation and solve for motion amplitude, velocity, acceleration, and phase.
   (b) Explain the significance of under-damping, over-damping, or critically damping in a spring-mass-damper system.
   (c) Identify which of the seakeeping DOFs are under-damped or over-damped.
   (d) Explain what resonance is and why it is relevant to seakeeping
   (e) In your own words, describe the concepts of added mass and hydrodynamic damping

2. **For Model Testing in Regular Waves:**
   (a) Calculate the encounter frequency given the wavelength, heading, and ship speed.
   (b) Explain how encounter frequency depends on heading, ship speed, and wavelength.
   (c) Describe what a transfer function is.
   (d) Explain how the transfer function can help determine resulting ship motion given wave characteristics and why the limitation of linearity is important.
   (e) Describe the shape of a transfer function and what this means with respect to the magnitude of the ship response as the encounter frequency goes from very small though the natural frequency of the ship to very large.

3. **For Strip Theory:**
   (a) State the assumptions/limitations associated with strip theory and explain what they mean in your own words.
   (b) Identify Maxsurf Motions seakeeping output and explain what the results mean.

4. **For Roll Mitigation:**
    (a) Solve a roll seakeeping problem given a transfer function, vessel speed and wave heading, and wave frequency to determine best action (how to change speed or heading) to reduce roll motions.
    (b) Describe a typical roll transfer function.
    (c) Explain the effect of damping on the roll response.
    (d) Calculate the roll natural frequency for a given ship.
    (e) Identify and explain different devices for reducing roll motion.
    (f) Calculate the damping factor given the roll decay coefficient.
5. **Laboratory Objectives:**
    (a) Describe proper ballasting techniques for seakeeping experiments.
    (b) Set proper pitch and yaw mass moments of inertia for models.
    (c) Calculate the pitch gyradius using the knife edge method.
    (d) Calculate the yaw gyradius using the bifilar suspension method.
    (e) Measure the heave and pitch amplitude responses to given wave excitation.
    (f) Measure the excitation (encounter) frequency and compare with the predicted values.
    (g) Develop a pitch and heave transfer function plot from experimental measurements.
    (h) Describe the expected heave and pitch motion responses to "short" and "long" wavelengths.
    (i) Explain resonance and the effects of damping on resonant response.
    (j) Describe the effects of a passive tank roll stabilization device on the roll motion for a model in beam seas.
    (k) Describe the effect of roll resonance on the roll amplitude magnification.
    (l) Explain the relationship between the roll motion and the wave frequencies for a model experiencing regular waves coming from the starboard beam.
    (m) Develop a realistic test plan for a seakeeping experiment.

A ship can be considered a mass that has damping and stiffness and is experiencing an oscillating excitation force. Chapter 3 dealt with the specifics of the exciting force. This chapter will deal with how a mass with damping and stiffness responds to a sinusoidal excitation. Let's review the six degrees of freedom associated with ship motion:

1. Surge	4. Roll
2. Sway	5. Pitch
3. Heave	6. Yaw

Three of these motions experience a "restoring force" due to buoyancy: *heave, roll*, and *pitch*. But, what is a restoring force and why is it a problem for a ship in waves?

4.1 A Review of Dynamics

Let's start with Newton's Second Law:

$$\sum \vec{F} = m\vec{a}$$

The forces on the left-hand side include any forces acting on the body - most obviously any external force on the system, $\vec{F}_{\text{external}}$, but also forces due to springs, $\vec{F}_{\text{spring}}$, or damping, $\vec{F}_{\text{damping}}$. Consider a single point mass on a spring, as shown in Figure 4.1. There are two forces in the system: the force of gravity due to the mass and the spring force due to the spring compression or extension. Figure 4.1 also shows the Free Body Diagram (FBD) of the system.

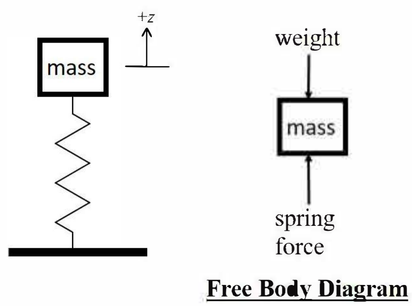

Figure 4.1: Point mass on a spring and the FBD (free body diagram)

The force due to gravity equals mg, where m is the mass and g is the acceleration due to gravity, and the force due to the spring equals $-kx$, where k is the spring stiffness and x is the distance the spring is stretched. Using Newton's Second Law and these forces, the Equation of Motion (EOM) for the system can be written:

$$-kx - mg = ma$$
$$0 = ma + kx + mg$$

Acceleration is the second time-derivative of position. If position is written as x, then acceleration can be written as $\ddot{x}$ and the EOM becomes:

$$0 = m\ddot{x} + kx + mg$$

For a ship, the stiffness is due to the buoyant force acting on the ship. Consider heave motion, for example. If you push the ship a foot down in the water, there is an extra buoyant force acting up on the ship in excess of the ship's displacement. If you then release the downward force on the ship, it will move up. Likewise, lifting a ship out of the water will result in *less* buoyant force than the ship's displacement, so when released the ship will move *down*. Thus, buoyancy is our restoring force, i.e. the spring in the system. Now, if we keep the gravity term, our x value equals the draft of a barge just for everything to be in equilibrium (in equilibrium there is no acceleration, so $\ddot{x} = 0$ and the position, x, equals the weight divided by the buoyant force). If we redefine the $x = 0$ to occur when the ship is in equilibrium (rather than when the ship is not in the water yet), the mg term goes away as we are only interested in *changes* to the equilibrium state. So, we can simplify to $0 = m\ddot{x} + kx$. Also, and just to mess with students who took classical dynamics, instead of using k to represent the stiffness coefficient, naval architects use c. So, the EOM now looks like,

$$0 = m\ddot{x} + cx$$

Ships don't just experience a buoyant force from the surrounding water, there is also hydrodynamic damping. Water is more viscous than air, so energy dissipates more quickly when moving in water. This damping force is proportional to the velocity, $-b\dot{x}$, where b is the damping coefficient and $\dot{x}$ is the velocity. This force is an additional force to the spring force.

$$0 = m\ddot{x} + b\dot{x} + cx$$

This equation can be considered *trivial* in the sense that if the ship is at its equilibrium draft and is not currently moving or accelerating then both sides of the equation are zero and $0 = 0$! However, if something gives the ship a bump, thus causing a positive change from equilibrium or an initial velocity, then we have a dynamic response that changes over time. For the case of spring-mass-damper with no external excitation force, the resulting motion is a decaying position that moves back towards the equilibrium position. If the system is **over-damped** the response will look like an exponential decay back to equilibrium. If the system is **under-damped** the position will be a decaying oscillation where the amplitudes of oscillation get smaller and smaller until equilibrium is finally reached. And if the system is **critically damped** the position returns to equilibrium in the shortest amount of time with one or no oscillations.

Ships, however, almost never operate in conditions where there is no heaving, rolling, or pitching because there is almost always an excitation force around - waves! We have to deal with the EOM that includes such external excitation forces. For now we will consider only regular waves - sinusoidal excitation with a single frequency and amplitude - moving on to more realistic waves in Chapter 5. Our wave excitation force can be written as:

$$F(t) = F_0 \sin \omega_e t$$

where F_0 is the forcing amplitude (related to the wave height) and ω_e is the frequency the wave moves past the ship. Figure 4.2 shows the updated free body diagram for our system including the mass, spring, damping, and excitation force.
Using the FBD and Newton's Second Law, our EOM for the full system becomes

$$F_0 \sin \omega_e t = m\ddot{x} + b\dot{x} + cx \tag{4.1}$$

The solution to this equation will be a system that has a transient oscillation until the damping has eliminated the ship's natural buoyant/damping response to the initial displacement and then an equilibrium solution that will have the same frequency as the excitation force. The solution will not necessarily have the same amplitude or phase as the excitation force, however. The solution can be written as

$$x(t) = X_0 \sin(\omega_e t - \phi)$$

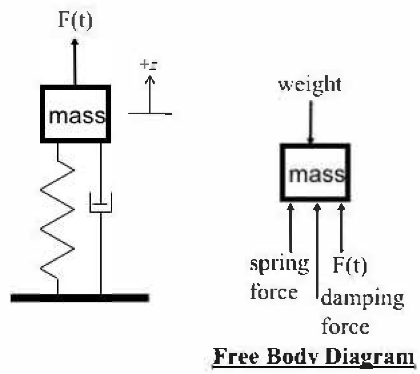

Figure 4.2: Point mass on a spring with damping and an excitation force and the FBD

where X_0 is the amplitude of the motion, ω_e is the frequency of the motion (and equal to the excitation frequency), and ϕ is the phase difference between the excitation sinusoidal motion and the resulting sinusoidal motion. The Appendix to this chapter goes through the derivation to solve for X_0 and ϕ. The solutions are:

$$X_0 = \frac{F_0}{\sqrt{(-\omega_e^2 m + c)^2 + (\omega_e b)^2}}$$

$$\tan\phi = \frac{\omega_e b}{-\omega_e^2 m + c}$$

The terms of the EOM can be described as:

$m\ddot{x}$:	can be considered an **inertial term**
$b\dot{x}$:	can be considered a **damping term**
cx:	can be considered a **stiffness term**
$F_0 \sin\omega_e t$:	can be considered the **excitation force term**

If the solution is $x(t) = X_0 \sin(\omega_e t + \phi)$, then the stiffness term has a sine phase ($cx = cX_0 \sin(\omega_e t + \phi)$), the damping term has a cosine phase ($b\dot{x} = b\omega_e X_0 \sin(\omega_e t + \phi)$), and the inertial term has a sine phase ($m\ddot{x} = -m\omega_e^2 X_0 \sin(\omega_e t + \phi)$). Figure 4.3 shows how each of these terms can be treated as a separate effect on the final motion of the mass.
These terms can be related to some common concepts used in seakeeping (and vibration) to describe systems that experience *simple harmonic motion* (i.e. the system motion is sinusoidal due to the presence of a restoring force).

Natural Frequency - the frequency at which the system oscillates on its own when disturbed from equilibrium

$$\omega_n \equiv \sqrt{\frac{c}{m}}$$

Damping Factor - the amount of damping in the system (> 1 for over-damped, < 1 for under-damped, and = 1 for critically damped)

$$\eta \equiv \frac{b}{2\sqrt{mc}}$$

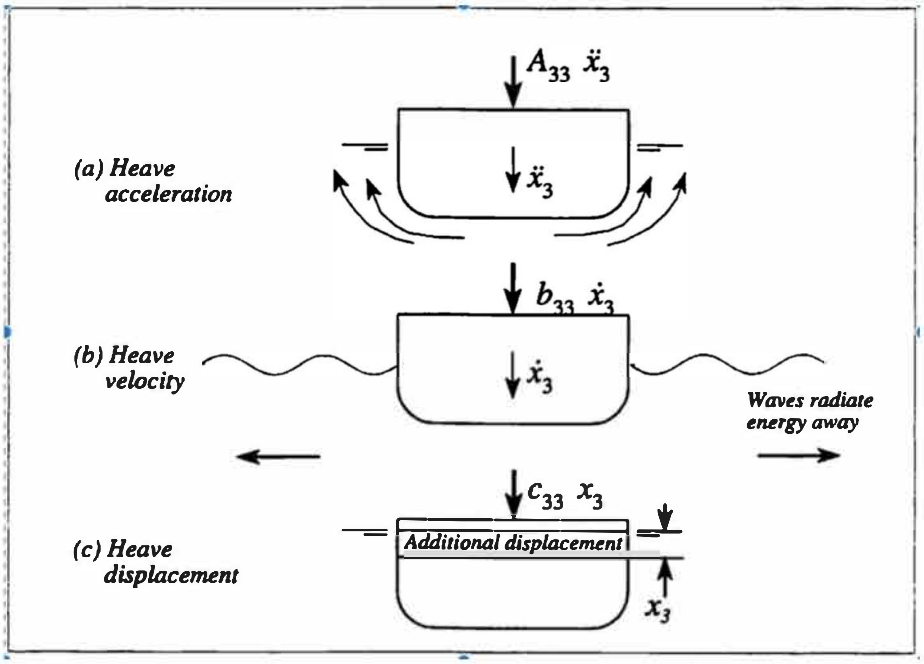

Figure 4.3: Effects of terms in the Equation of Motion (EOM) (Figure 3.4 in reference 2)

Tuning Factor - the ratio of the excitation frequency to the natural frequency

$$\Lambda = \frac{\omega_e}{\omega_n}$$

These terms can be used in the EOM giving an EOM that equals:

$$\ddot{x} + 2\eta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m}\sin\omega_e t.$$

The solution for the response amplitude (X_0) and response phase (ϕ) can also be written using these concepts:

$$X_0 = \frac{F_0/c}{\sqrt{(1-\Lambda^2)^2 + (2\eta\Lambda)^2}} \tag{4.2}$$

$$\tan\phi = \frac{2\eta\Lambda}{1-\Lambda^2}. \tag{4.3}$$

To understand what all is going on with the response amplitude solution, consider a spring-mass-damper system than is experiencing a constant force, F_0 (instead of a sinusoidally varying force). In this case, the equation of motion looks like

$$F_0 = m\ddot{x} + b\dot{x} + cx$$

However, if the force is a constant, the system will come to a state of equilibrium where the mass in not moving, i.e. $\ddot{x} = \dot{x} = 0$. Thus, the EOM simplifies to

$$F_0 = cX_0$$

and the solution is

$$X_0 = F_0/c.$$

Looking at the solution for X_0, this is the term in the numerator of equation 4.2. So, when a sinusoidal force is applied to a spring-mass-damper system, the resulting position amplitude is the "static" response (response if the force amplitude, F_0, were constantly applied) modified by the terms in the denominator. The modification of this static response is called the **Dynamic Magnification Factor** or MF:

$$\text{Dynamic Magnification Factor (MF)} = \frac{1}{\sqrt{(1-\Lambda^2)^2 + (2\eta\Lambda)^2}}. \tag{4.4}$$

This means that for a given forcing amplitude, F_0, the response amplitude changes depending on the damping factor (η) and the tuning factor (Λ). The damping factor relates to how much damping there is in the system. Looking at the MF, the larger η, the smaller the Magnification Factor. As one would expect, increasing damping reduces the magnitude of the response. The tuning factor relates to how close the excitation frequency (ω_e) is to the natural frequency (ω_n). Unlike for damping, it is not clear what happens as the tuning factor increases. Consider the case where the excitation frequency equals the natural frequency ($\omega_e = \omega_n$). In this case $\Lambda = 1$ and in the absence of damping ($\eta = 0$) the response amplitude would go to infinity. The presence of damping reduces the response amplitude, but the maximum response amplitude will still occur at the natural frequency ($\Lambda = 1$). The MF increases as Λ approaches 1 and then decreases as Λ becomes greater than 1. Figure 4.4 shows some example MF curves for different amounts of damping. As the damping increases the peak response amplitude decreases until the system becomes over-damped and there is no peak. This peak is called **resonance** and systems that are over-damped do not show any response amplitudes greater than the static response amplitude. For over-damped systems there is no magnification, no resonance.

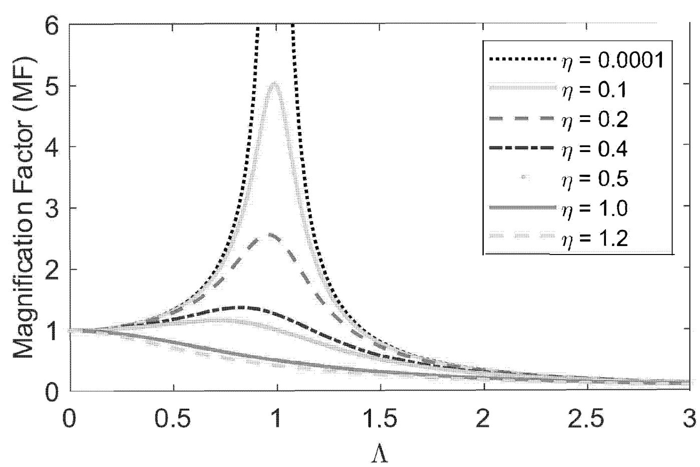

Figure 4.4: Example Dynamic Magnification Factor plots for different amounts of η (damping)

4.2 Added Mass and Hydrodynamic Damping

Moving through water is different than moving through air. When you push your flat palm through the air it does not feel like you are pushing on anything. The same motion feels very different when done under water. This is partly because water is more viscous than air (increased damping), but it is also because the water is denser than air and needs to be moved along with your palm. This motion requires the water surrounding your palm to be accelerated. The effect of this is to make your palm feel as if it had extra inertia (mass). This effect is known as **added mass**. This extra required force shows up in the equation of motion (EOM) as an addition to the mass of the object. The added mass represents the amount of fluid accelerated by the object. However, something to keep in mind is that the particles of fluid adjacent to the body will accelerate to varying degrees and the added mass value is a weighted integration of the entire fluid mass effected by the accelerating object. So, instead of $m\ddot{x} + b\dot{x} + cx = F_0 \sin \omega t$, the equation of motion becomes,

$$(a + m)\ddot{x} + b\dot{x} + cx = F_0 \sin \omega t.$$

The a stands for added mass (and now explains why naval architects have damping as b and stiffness as c, in contrast to the standard mechanical engineering expressions). Added mass depends primarily on the shape of the object, the type of motion (linear or rotation), and the direction of the motion. In this way, added mass differs from just *mass* since mass is a quantity independent of motion.

Hydrodynamic damping is related to the viscosity of the fluid (and hence the frictional drag), but when a free surface is involved the damping is dominated by the generation of waves. The larger the waves generated, the larger the hydrodynamic damping. This damping is proportional to the velocity in this direction as well. Both the added mass (a) and hydrodynamic damping coefficients (b) are a function of the frequency of oscillation.

Experimental Investigation in Sway

The concepts of damping and "added mass" are forces on an object moving in a fluid that are not explained by buoyancy or the mass of the object. To explore the concepts of added mass and hydrodynamic damping in a way that could be observed, we conducted an experiment on a U-shaped foam barge in the 120-ft towing tank in the USNA Hydromechanics Laboratory. Using the data collected, we were able to calculate the added mass and hydrodynamic damping coefficients and determined the dependence of the added mass and hydrodynamic damping on frequency. In this section we will go over the details, data, and analysis from this experiment to help explain the concepts of added mass and hydrodynamic damping.

The problem addressed in this experiment was a two-dimensional simplification of the problem of a ship moving in simple harmonic motion in a *calm* sea. The cross-section of the body had a semi-circular bottom with vertical sides, shown in the figure below. The instrumented model section had a length of 3 feet, a beam of 8 inches, and a draft of 5 inches. The displacement was about 15 pounds.

The model was attached to a scotch yoke that oscillated the model with pure sinusoidal sway motion. The only measurements taken were the sway **displacement** and the sway **force** over time.

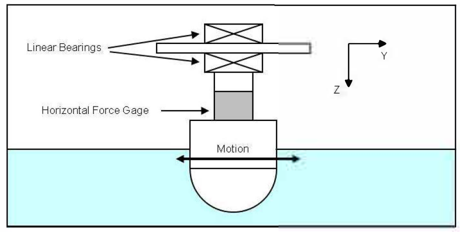

Figure 4.5: Cross-section of the "two-dimensional" ship model.

Experimental Procedure

To collect the data necessary to measure added mass and hydrodynamic damping, the following steps were taken:

1. We oscillated the model in sway in air and recorded the position and the force as a function of time
2. We oscillated the model in sway in water and recorded the position and the force as a function of time

Analysis Method

The damping force is the force component that is proportional to the velocity and the added mass force is the force component that is proportional to the acceleration. The equation of motion for the model is

$$(m + a_2)\ddot{x}_2 + b_2\dot{x}_2 + c_2x_2 = F_0 \sin \omega_e t$$

where m is the mass of the model, a_2 is the added mass coefficient in sway, $\ddot{x}_2$ is the acceleration of the model in sway, b_2 is the damping coefficient in sway, $\dot{x}_2$ is the sway velocity, c_2 is the stiffness coefficient in sway, x_2 is the sway displacement, F_0 is the force amplitude in sway, and ω_e is the sway excitation frequency in radians/second. For sway motion there is no buoyancy (stiffness), so $c_2 = 0$. The solution to the equation of motion (i.e. the motion of the model in time) is of the form

$$x_2(t) = X_2 \sin(\omega_e t - \phi).$$

To solve for the motion and force amplitudes, the data collected was analyzed using a FFT to identify the actual oscillation frequency and amplitude at that frequency. For example, consider the sample in-air sway displacement plot in Figure 4.6. The motion is fairly sinusoidal, although each oscillation varies slightly from the ones before it. However, performing an FFT on the data (see the Fourier Transform section in the **Irregular Waves Lab**), gives a pretty clear spike at a single amplitude (see Figure 4.7). We can then say that the sway amplitude of the data is 1.3 inches at an excitation frequency of 1.1 Hz. If we use that amplitude and frequency and create a sinusoidal function, we can compare that result with the actual data, see the plot in Figure 4.8. In this case, the perfect sine wave matches the actual data fairly closely.

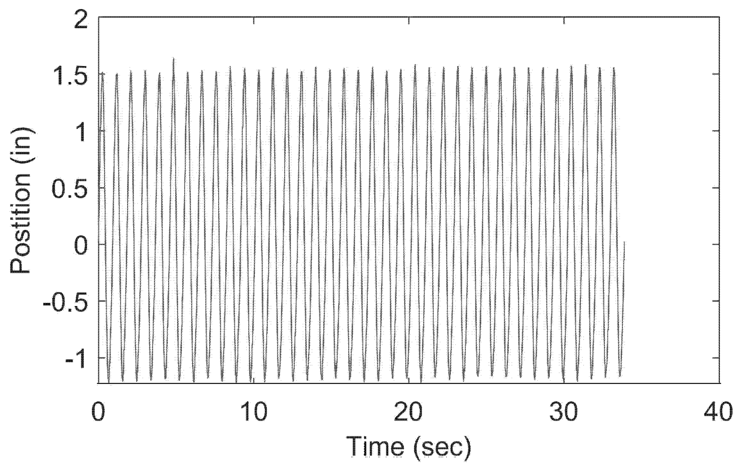

Figure 4.6: Sway Displacement in Air as a function of time

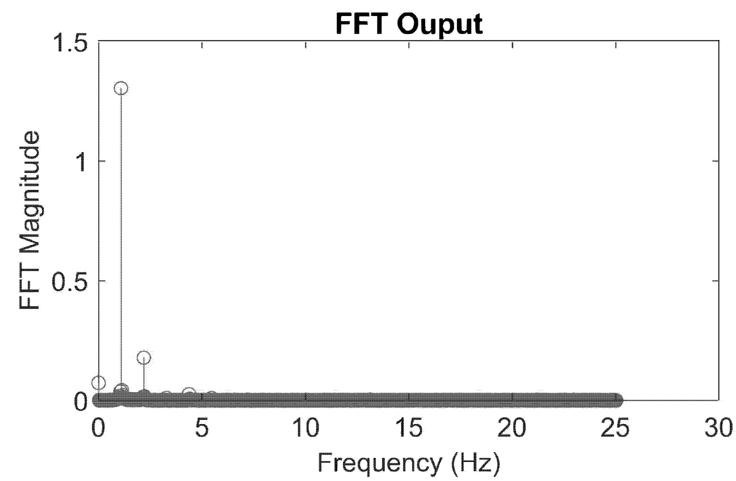

Figure 4.7: FFT of Sway Displacement in Air data

We can do the same for the force data - perform a FFT on the data and use the amplitude and frequency data to recreate the "pure" signal. The plots below (Figure 4.9) show the raw data signal and the FFT result. Compared to the sway displacement, the force FFT signal shows many more spikes. This means the raw data is not as good of a pure sinusoid and has higher frequency components. However, for this experiment, those higher frequency responses can be considered "noise" and we are only interested in the primary spike that occurs at the excitation frequency. Recreating the force as a function of time using the FFT result, we can compare that pure sine wave to the actual data, see Figure 4.10.

So, for the data we collected in this experiment, Figures 4.6 to 4.12, we found the values for X_0, F_0, and ω_e for the in-air and in-water experiments. Using these values, the known mass, the zero stiffness coefficient, and the equations for magnification (see the appendix in this chapter), we can solve for the added mass, a_2, and damping, b_2, in the air and again in the water. For the data shown, the in-air total mass is 1.74 slugs, the in-air damping is 0.06 lb-s/ft, the in-water total mass is 5.75 slugs, and the in-water hydrodynamic damping

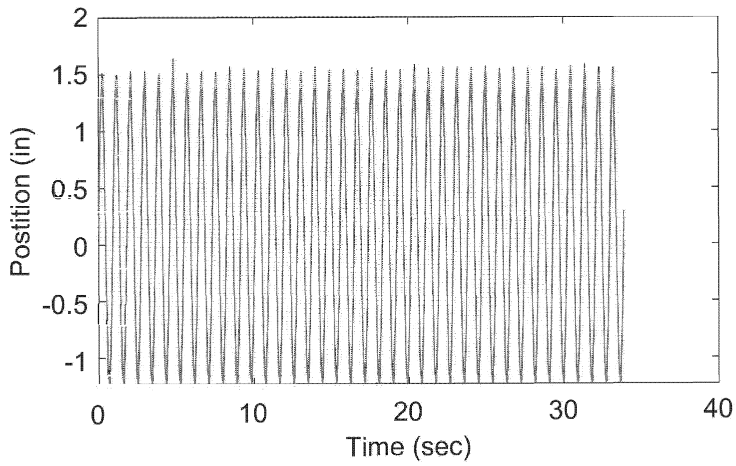

Figure 4.8: Data and Pure Sway Displacement in Air

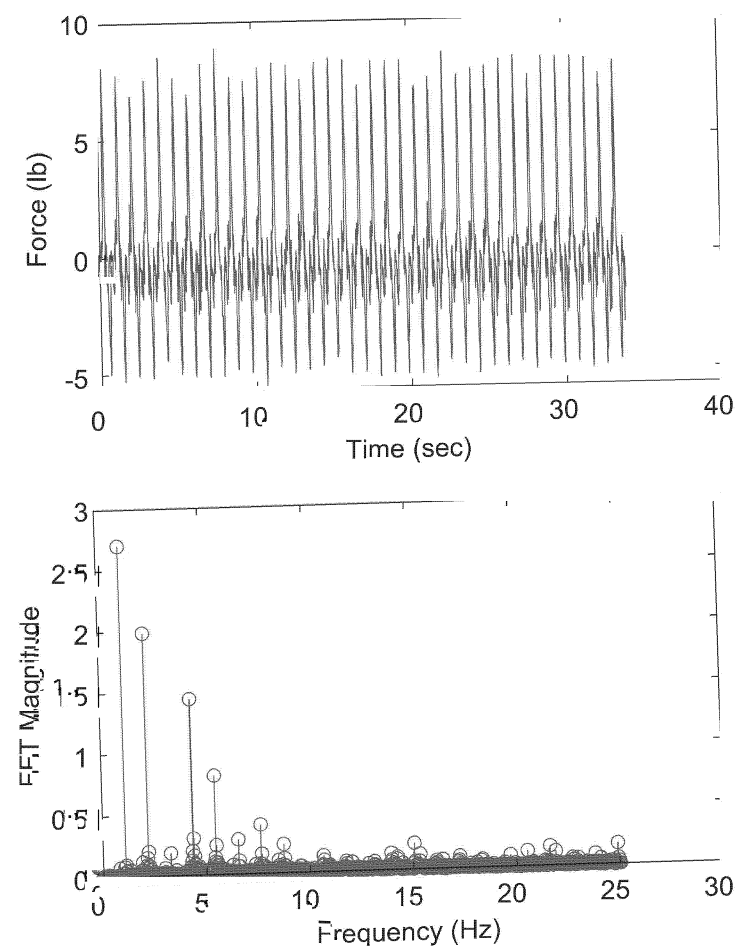

Figure 4.9: Sway Force as a function of time and FFT of Sway Force in Air data

is 0.08 lb-s^2/ft. Thus in-air added mass is almost negligible (air provides almost no added

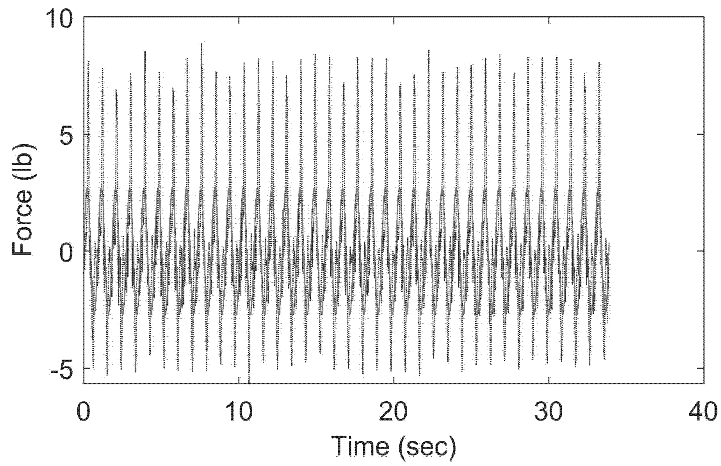

Figure 4.10: Data and Pure Sway Force in Air

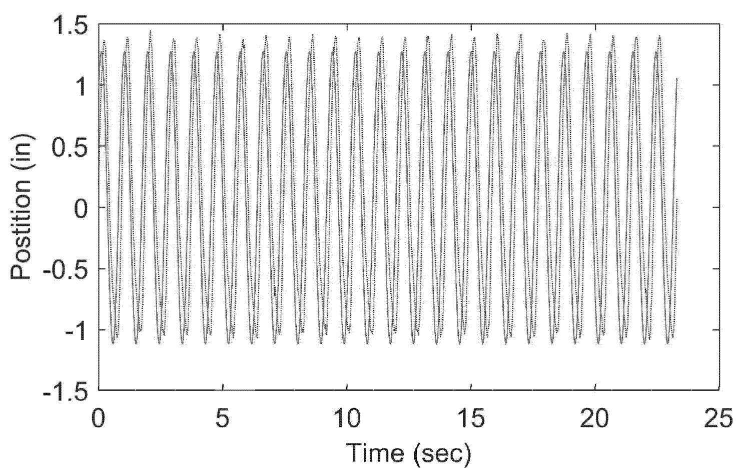

Figure 4.11: Sway Displacement Data for in Water Experiment

mass effect) meaning the mass in the equation of motion is really just equal to the mass of the model, while the added mass is very noticeable for the in-water experiment. There is a similar result for the damping coefficient, the hydrodynamic damping is significantly more than the aerodynamic damping.

By repeating this procedure at different excitation frequencies, we can see how added mass and damping vary with frequency of oscillation. Figure 4.13 shows the dependence of these coefficients on the frequency of oscillation. The added mass is close to zero in air and does not depend on the frequency. The same trends apply for the damping in air. In water, the added mass is frequency dependent and tends to decrease with increasing oscillation frequency. This means that at higher frequencies there is less of an effect from the added mass. The hydrodynamic damping in water also depends on frequency, but in this case the damping *increases* with increasing frequency. Therefore, the damping gets *more* significant at higher oscillation frequencies.

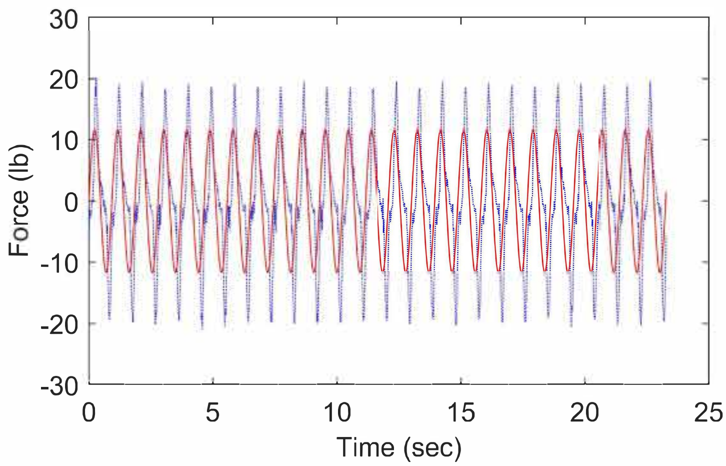

Figure 4.12: Sway Force Data for in Water Experiment

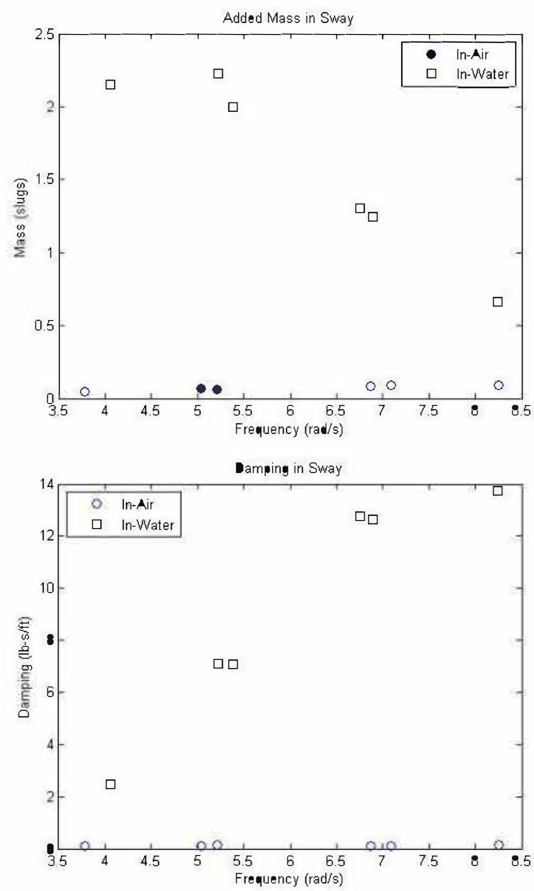

Figure 4.13: Added Mass and Damping of Sway Oscillation Model as a function of frequency

4.3 Ship Natural Frequencies

Each degree of freedom that has a restoring force has an associated natural frequency. So, for a ship, there is a natural frequency in heave, roll, and pitch. These natural frequencies depend on the mass and stiffness properties of the system.

To a first approximation, we will consider the heave and pitch motions to be uncoupled (i.e. *independent*). The natural frequency in heave is

$$\omega_{3n} = \sqrt{\frac{c_3}{m + a_3}} \tag{4.5}$$

and the natural frequency in pitch is

$$\omega_{5n} = \sqrt{\frac{c_5}{I_5 + a_5}} \tag{4.6}$$

We can actually reasonably estimate the added mass or inertia in heave and pitch, based on conventional ships, as equal to the actual mass or inertia. In other words,

$$a_3 \approx m$$

and

$$a_5 \approx I_5.$$

The natural frequency in roll can be similarly determined,

$$\omega_{4n} = \sqrt{\frac{c_4}{I_4 + a_4}}.$$

For roll, the estimated added inertia is about a quarter of the ship's roll moment of inertia, or

$$a_4 = 0.25 I_4.$$

However, the roll natural frequency of a ship is also strongly linked to the ship's metacentric height (as can be determined from the stiffness coefficient). So, the roll natural frequency can also be written as

$$\omega_{4n} = \sqrt{\frac{mg\bar{GM}}{1.25 I_4}}. \tag{4.7}$$

The roll damping increases with forward speed. The increase in damping results in a smaller maximum resonant peak, but also a slight reduction in the frequency at which the peak response will occur.

4.4 Ship Inertia - Pitch, Yaw, and Roll

The mass of a ship is determined by its total weight or displacement. The rotational inertia is determined by the distance of each weight from the combined center of gravity. The further the heaviest weights are from the CG, the larger the moment of inertia (the rotational inertia). If all the mass were located equidistant from the center of gravity the moment of inertia would be easy to calculate and would be equal to the total mass times the distance from the CG squared. Although the mass in a ship is never located equidistant from the center of gravity, we can find the representative distance the mass would need to be were the ship a sphere. This representative distance is the **radius of gyration**, k. If we have the radius of gyration, we can find the ship's moment of inertia,

$$I = mk^2.$$

There are three rotational degrees of freedom - roll, pitch, and yaw - and each have a subscript number associated with the direction. It turns out that for typical ship shapes, the radii of gyration have a relationship to the ship's geometry. So, in general,

$$\begin{aligned} k_4 &= 0.30\mathrm{B_{WL}} && \text{roll} \\ k_5 &= 0.25\mathrm{L_{PP}} && \text{pitch} \\ k_6 &= 0.25\mathrm{L_{PP}} && \text{yaw} \end{aligned}$$

4.5 Ship Transfer Function

To predict the ship motion in a set of regular waves, we need to have a way to predict the ship response as a function of the excitation amplitude and frequency. This is essentially the same as the dynamic magnification factor described in Section 4.1. Other names for this relationship include Frequency Response Function (FRF) and, for naval architects, Transfer Function. In all cases, the result can be represented as a plot with the ratio of ship response to excitation amplitude (X/ζ_0, where ζ_0 is the wave amplitude) on the vertical axis and the ratio excitation frequency to natural frequency ($\Lambda = \omega_e/\omega_n$) on the horizontal axis. Figure 4.14 shows a typical transfer function in roll.

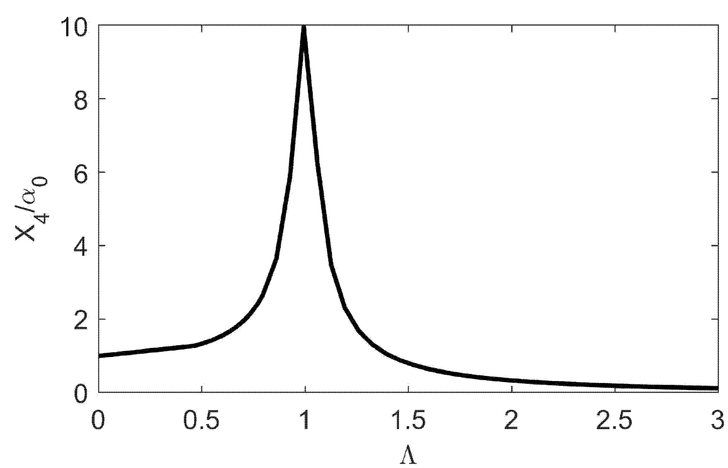

Figure 4.14: Typical Ship Transfer Function for Roll

The response depends on the ship mass, added mass, hydrodynamic damping, buoyancy, and excitation frequency in the direction of motion. If the ship were to not be moving forward (zero speed), the excitation frequency would match the wave frequency. However, when the ship has forward speed the excitation frequency depends on the ship speed, the wave frequency, and the relative direction of the ship and waves. This resulting excitation frequency is called the **encounter frequency** since it is the frequency at which the ship encounters the waves.

Encounter Frequency

Assuming the waves and ship are on a straight course, the frequency with which the ship will encounter a wave crest depends on the distance between the waves crests (λ - wavelength), the speed of the waves (c - which depends on the wavelength), the speed of the ship (U), and the relative angle between the ship heading and the wave heading (μ), see Figure 4.15. The encounter period is thus the distance traveled (λ) divided by the speed the ship encounters the waves ($c - U\cos\mu$). The encounter frequency is

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi}{\lambda}(c - U\cos\mu).$$

Manipulating the equation a bit and substituting in the relationships between wavelength, wave speed, and wave frequency, the encounter frequency can be written as

$$\omega_e = \omega - \frac{\omega^2 U}{g}\cos\mu. \tag{4.8}$$

The encounter period (the time between crests to pass) is given by

$$T_e = \frac{\lambda}{c - U\cos\mu}.$$

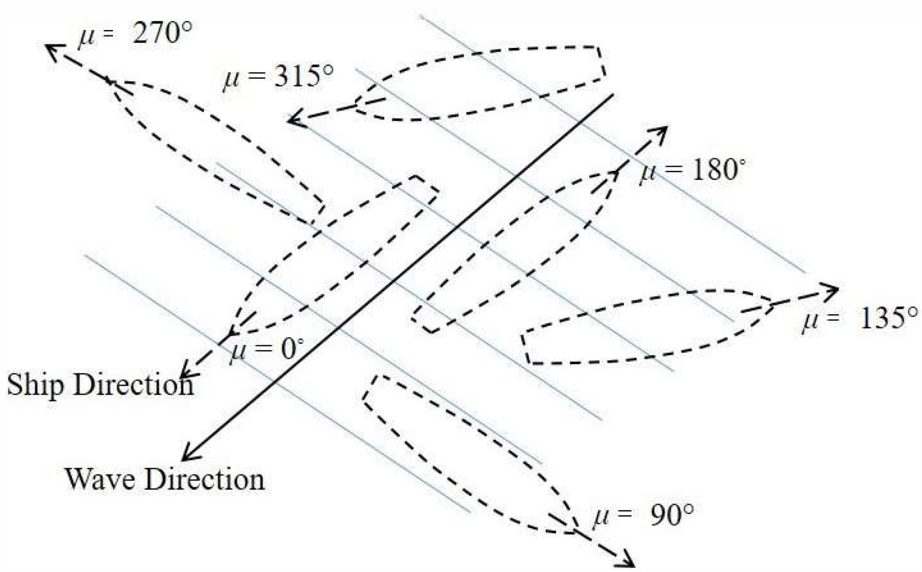

Figure 4.15: Relative ship and wave heading angle, μ

The heading angle determines the "type" of seas the ship experiences. For example, $\mu = 0°$ means the ship and waves are heading in the same direction and the ship is experiencing **following seas**. When the ship is traveling directly at the oncoming waves ($\mu = 180°$), the ship is experiencing **head seas**. Quartering waves on the ship's starboard side are when μ is between 0° and 90° and quartering waves on the ship's port side are when μ is between 270° and 360°. Starboard beam seas are when $\mu = 90°$ and port beam seas are when $\mu = 270°$. When μ is between 90° and 180° the ship is experiencing bow waves on the starboard side and bow waves on the port side are when μ is between 180° and 270°.

To calculate the transfer function computationally (as opposed to experimentally), the hydrodynamic coefficients need to be known (added mass, mass, damping, and buoyancy).

Ship Transfer Function Example

Consider the following roll characteristics for a particular ship:

- mass = 4,898,300 kg
- $\mathrm{L_{PP}} = 86.5$ m
- $k_4 = 0.25\mathrm{L_{PP}}$
- $a_4 = 0.25\mathrm{I}_4$
- $c_4 = 715{,}000{,}000$ N-m/rad
- $b_4 = 143{,}000{,}000$ N-m/(rad/s)

We would like to find the transfer function (or frequency response function) for this ship in roll. First we need to solve for all the terms in the EOM for roll: $(a_4 + I_4)$, ω_n, and η.

- To solve for the combined moment of inertia and added inertia in roll we need to find the ship's roll inertia, I_4.

$$I_4 = m\mathrm{k}_4^2 = (4898300) \cdot (0.25 \cdot 86.5) = 2290644073 \text{ kg} \cdot \text{m}^2$$

  We can then find the combined moment of inertia and added inertia using the relationships provided above:

$$a_4 + I_4 = 1.25 \cdot I_4 = 1.25(2290644073) = 2863305092 \text{ kg} \cdot \text{m}^2.$$

- To use the equation for the magnification factor (which gives us the ship transfer function), we also need to find the ship's natural frequency, ω_n

$$\omega_n = \sqrt{\frac{c_4}{a_4 + I_4}} = \sqrt{\frac{715000000}{2863305092}} = 0.50 \text{ rad/sec}$$

- and the damping factor, η

$$\eta = \frac{b_4}{2(a_4 + I_4)\omega_n} = \frac{143000000}{2 \cdot 2863305092 \cdot 0.50} = 0.05$$

- Now we can plug these values into the ship transfer function equation (i.e. the magnification equation) using $\Lambda = \omega_e/\omega_n$:

$$\text{ship transfer function} = \frac{1}{\sqrt{(1-\Lambda^2)^2 + (2\eta\Lambda)^2}} = \frac{1}{\sqrt{(1-\Lambda^2)^2 + (2 \cdot 0.05\Lambda)^2}}$$

- Let's consider some points of interest. What is the transfer function when the encounter frequency equals the natural frequency (i.e. $\omega_e = \omega_n$)? Plugging in $\Lambda = 1$, the transfer function is 10. What does this mean? It means that the roll response will be 10 times as large as the excitation magnitude. What about when the excitation frequency is close to zero? In this case, $\Lambda \approx 0$ and the ship transfer function equals 1. This means for very low excitation frequencies the roll response is the same as the excitation magnitude. Lastly, let's consider a very high excitation frequency, say $\Lambda = 10$. In this case the ship transfer function equals 0.01. This means the ship response is $1/100^{\text{th}}$ the magnitude of the excitation.
- To understand the ship's roll response over a range of frequencies, the best thing is to create a plot. Figure 4.16 shows the roll transfer function for this problem.

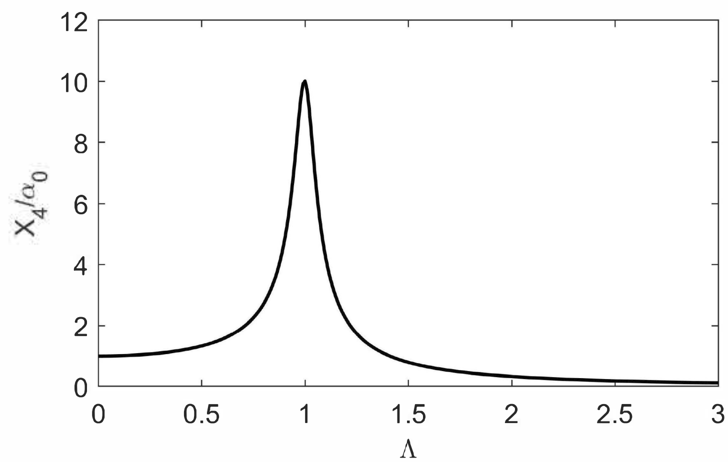

Figure 4.16: Example ship transfer function in roll

There are two ways to determine the transfer function for a ship - experimentally or theoretically. There are distinct limitations to both methods. We will be exploring both methods in this class.

Consider a wave elevation time history at point O that is of the form $\zeta = \zeta_0 \sin(\omega_e t)$ with the resulting ship motion described by $x_i = X_{i0} \sin(\omega_e t + \delta_i)$. The motion amplitude ($X_{i0}$) and phase ($\delta_i$) are functions of the ship speed (U), the ship heading relative to the waves (μ), and the encounter frequency (ω_e). The amplitudes are assumed to be proportional to the wave amplitude ($X_{i0} = \text{constant} \times \zeta_0$). Therefore, we typically express the motion amplitudes in non-dimensional form:

$\frac{X_{30}}{\zeta_0}$ heave transfer function

$\frac{X_{40}}{k\zeta_0}$ roll transfer function

$\frac{X_{50}}{k\zeta_0}$ pitch transfer function

where $k\zeta_0$ is the wave slope amplitude. Graphs of the resulting non-dimensional amplitudes are plotted as a function of the encounter frequency. In essence, a transfer function gives the proportion of wave amplitude or wave slope amplitude "transferred" by the ship "system" into ship motions.

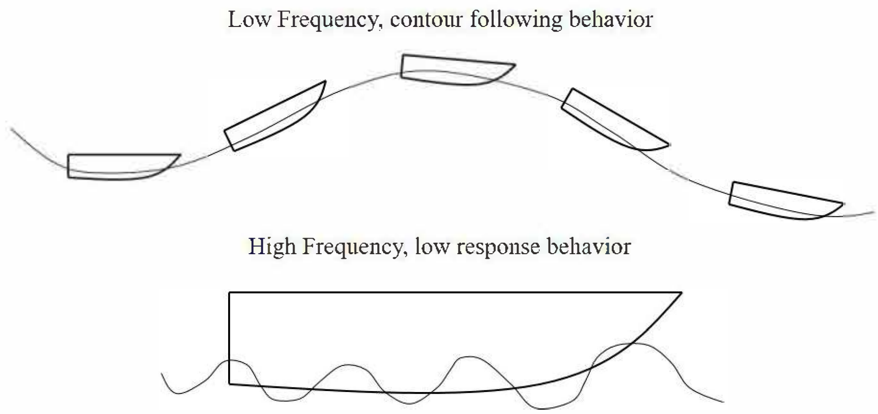

Figure 4.17: Relationship between wave length and ship response

The phase angle, δ_i gives the phase relationship between the motion and the wave, the maximum positive response occurs $+\delta_i/\omega_e$ seconds **before** the maximum wave depression. For $-\delta_i$ the motion lags the wave depression.

Let's start by examining the magnitude of a ship response operating in head seas ($\mu = 180°$). Consider very long waves (low frequency). When ω_e is very low, the dynamic effects associated with added mass and damping are virtually negligible. Therefore, the excitations and motion responses experienced by the ship are almost entirely attributed to the buoyancy changes as the wave passes by - the maximum pitch occurs at the wave nodes (inflections) and the maximum heave response occurs at the crests and troughs. The result is motion amplitudes on the same order as the wave amplitudes. If you think of the wave length as much longer than the ship length, the ship will always be aligned with the wave surface (top of Figure 4.17). In this scenario, the transfer function is equal to one (for example, in heave $\frac{X_{30}}{\zeta_0} = 1$ at low encounter frequencies).

Now consider very short waves (high frequency). When ω_e is very high, the ship responses are reduced because the short waves do not excite the ship very much. In this scenario, there are many wavelengths along the length of the ship, and the ship (bottom of Figure 4.17), in a way, can't decide if it wants to be up or down so it does neither. As the ship speed increases, the wavelengths which do not excite the ship are encountered over a wider range of frequencies. When operating in head seas, increasing the ship speed has the effect of increasing the encounter frequency for a given wave.

If the range of frequencies encountered includes the natural frequency of heave and/or pitch, the response may exhibit a resonant peak (see the Dynamics Review section). However, heave and pitch are both heavily damped so any peaks at resonance are never very pronounced.

What about the *phases* of the ship's response when operating in head seas? In the long waves (low ω_e) the heave motion is synchronized with the wave motions (maximum response at the crest and trough) while the pitch has a phase of 90°, so the maximum (positive) pitch occurs at the wave node. For short waves, there is little response so the phase is not very relevant. However, in theory the waves should be *out of phase* (that is, $\delta = 180°$) with the ship motions (for example, the maximum positive heave motion occurs at the wave trough and the maximum negative heave motion occurs at the wave crest).

Next, let's consider following waves. As in head seas, for long waves ($\omega_e \to 0$) the transfer function is 1 (for heave and pitch). The heave phase is close to zero over most of the encounter frequencies (i.e. the motion is nearly synchronized with the wave). The pitch phase is $-270°$ or $+90°$ over most of the range of encounter frequencies. In following waves the maximum bow up motion now *leads* the trough of the wave. In oblique waves the motion is no longer confined to the vertical plane, we can now have roll, sway, and yaw motions. For *long* oblique waves the ship appears to be crawling over a succession of long, shallow hills. From the ship's perspective, the wave length appears longer. The "effective wave length" depends on the heading angle and is, thus,

$$\frac{\lambda}{\cos\mu}.$$

The "effective wave slope" is $k\zeta_0 \cos\mu$. For headings forward of the beam ($90° < \mu < 180°$), the responses are broadly similar to the head seas responses.

4.6 Model Testing in Regular Waves

For an experimentally determined transfer function - whether in roll, pitch, or heave - the results are only relevant to the specific ship model tested at the tested speed. Therefore, each new geometry must have a new model built and tested at relevant speeds to find the transfer functions. We can create a set of regular waves to send the ship model through and measure the results. For a straight, long tow tank (like the ones at USNA) we can only test in head seas with the model in motion. For a stationary model, we can also test beam seas. The model tests in regular waves are concerned with the experimental determination of the motion transfer functions. We will determine the motion amplitudes experienced for a variety of different wavelengths or frequencies.

Model testing in regular head seas is an important part of determination of full-scale ship responses. The usual procedure is to test the model at a variety of speeds covering the operating speed range of the vessel. At each speed the model is tested in regular waves with a range of frequencies (wavelengths) such that the expected wave frequencies in a typical seaway are covered. For each test the heave and pitch response are recorded along with information about the waves generated for each run.

The usual form of output for motions in regular waves is a plot of the *average response amplitude normalized by the average wave amplitude* against the *encounter frequency.* When all of the test runs have been plotted on the same graph, a curve is faired through the data. This faired curve represents the Transfer Function of the response for the specified motion of the vessel at that specific speed. Squaring the transfer function gives what is known as the *Response Amplitude Operator (RAO).* RAO's are used in the process of determining the ship's response in an irregular sea (as we will see later).

Typically we keep the *wave slope constant* while varying the wavelength. For these tests, the wave steepness must be kept small to ensure the responses are in the linear range (more on this requirement in the section on Strip Theory). To acquire ship phase information (with respect to the waves), it is necessary to measure the incident waves using a wave probe. The measurement must not be at a location where the model is influencing the waves and any wave generated by a probe must not influence the model.

Phase Shift for Wave Time History

In cases where we can't measure the waves at a location parallel to the model's CG, we need to introduce a time shift to the wave time history data. Consider the theoretical condition shown in Figure 4.18[1]. If the probe is located at position x_{1P} ahead of the CG and x_{2P} to starboard, the probe will record the waves x_P after (or before) they have passed the CG.

$$x_P = x_{1P} \cos \mu - x_{2P} \sin \mu.$$

If the waves are overtaking the model, the speed the waves encounter the ship is

$$c - U \cos \mu$$

a wave trough recorded at the probe would be at the CG at time

$$t_P = \frac{x_P}{c - U \cos \mu}.$$

So, the phase lead measured with reference to the waves recorded at the wave probe should

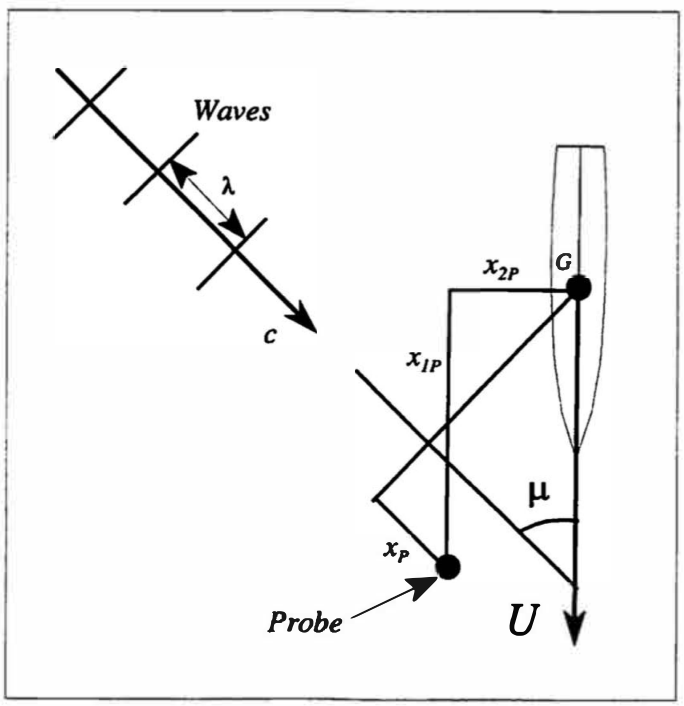

Figure 4.18: Time Shift for Wave Probe Data (Figure 10.15 from reference 2)

be *reduced* by the amount

$$\delta_P = \omega_e t_P = \frac{\omega_e (x_{1P} \cos \mu - x_{2P} \sin \mu)}{c - U \cos \mu}$$

[1]taken from reference 2

For a standard straight tow tank, the only waves that can be tested are in head seas where $\mu = 180°$. In this case $\cos 180° = -1$ and $\sin 180° = 0$ so

$$\delta_P = \frac{\omega_e(-x_{1P})}{c+U}$$

and we will *increase* the phase by $(\omega_e x_{1P})/(c+U)$ for the waves in the experiment if the probe is located forward of the model.

4.6.1 Experimental Model Ship Testing

Let's review the basics of resistance testing and then add the complexities introduced by the dynamic motions involved in testing in waves. Consider a full-scale boat with the following characteristics: You wish to build and test in waves a 5.35 scale model. We need to determine

Length Overall, ft	42.77
Max Beam, ft	13.1
Displacement, lbs	35,000
LCG (fwd of transom), ft	15.09
L_{PP}, ft	38.4
Pitch gyradius, ft	9.6
Speed, kts	40

Table 4.1: Model Testing Ship Characteristics

the **model length** (in feet), **model beam** (in feet), **displacement** (in pounds), **pitch gyradius** (k_5, in inches), and the **model speed** (in ft/s).

Remember from Resistance and Propulsion that there are three types of experimental "similarities" - *Geometric Similarity*, *Kinematic Similarity*, and *Dynamic Similarity*. To achieve **geometric similarity** we need to make sure all length measurements (for the model and the waves) have the same scale ratio between the full-scale and model. Since in this example the scale ratio is 5.35, the relationship between the model and full-scale geometric lengths is

$$R = \frac{L_S}{L_M} = 5.35$$
$$L_M = \frac{L_S}{R} = 8.0.$$

Therefore, the model must have the following geometric characteristics: The wave heights and wavelengths scale in a similar way,

$$H_M = \frac{H_S}{R}$$
$$\lambda_M = \frac{\lambda_S}{R}.$$

Area measurements scale by R^2 and mass (or displacement) scales by R^3. The area is two lengths multiplied together, so if each length is scaled by R then the area is scaled by

	Ship	Model
Length Overall, ft	42.77	8.0
Max Beam, ft	13.1	2.45
LCG (fwd of transom), ft	15.09	2.82
L_{PP}, ft	38.4	7.18
Pitch gyradius, ft	9.6	1.79

Table 4.2: Model Testing Model Characteristics

$R \cdot R = R^2$. The mass scales by R^3 because it is proportional to volume, which is *three* lengths multiplied together ($R \cdot R \cdot R = R^3$). The other concern between model and ship is that, generally, a full-scale ship is floating in salt water while a model is floating in fresh water. Therefore, we need to account for this difference in scaled displaced volume as well,

$$m_M = \frac{\rho_M m_S}{\rho_S R^3}$$

So, the moments of inertia scale by R^5! Remember, the moment of inertia can be expressed as mass times the gyradius squared ($m \cdot k^2$). So, if the mass is scaled by R^3, when we multiply by k^2 we get another R^2 in the relationship. So,

$$I_M = \frac{\rho_M I_S}{\rho_S R^5}.$$

To achieve **kinematic similarity** we need to match the Reynolds numbers of the full-scale and model ships. To achieve **Dynamic Similarity** we need to match the Froude numbers of the full-scale and model ships. Due to the physical properties of our Earth, we can't simultaneously satisfy both the Reynolds and Froude scaling requirements. For tank testing we neglect the Reynolds number scaling (friction matching) and stimulate the flow as necessary for turbulent flow. We do match the Froude scaling. The Froude number is

$$\mathrm{Fr} = \frac{U}{\sqrt{gL}}$$

So, to satisfy dynamic similarity, the model speed must be equal to

$$U_M = U_S \sqrt{\frac{L_M}{L_S}}$$
$$U_M = \frac{U_S}{\sqrt{R}} = \frac{40 \cdot 1.688}{\sqrt{5.35}}$$

So, for our model the speed must be 29.2 ft/s. The wave frequency (or wave period) is related to the wave velocity. We can relate the frequencies of the full-scale and model-scale using the wavelengths and the relationships between frequency and wavelength shown in

Chapter 3. So we find the model wave frequency from

$$
\begin{aligned}
\omega &= \sqrt{\frac{2\pi g}{\lambda}} \\
\omega_S &= \sqrt{\frac{2\pi g}{\lambda_S}} \\
\omega_M &= \sqrt{\frac{2\pi g}{\lambda_M}} \\
\omega_M &= \sqrt{\frac{2\pi g}{(\lambda_S/R)}} \\
\omega_M &= \sqrt{\frac{2\pi g}{\lambda_S}}\sqrt{R} \\
\omega_M &= \omega_S\sqrt{R}
\end{aligned}
$$

Using a similar process, the model wave period is

$$T_M = \frac{T_S}{\sqrt{R}}$$

Froude scaling gives us *velocity* scaling and geometric scaling gives us *position* scaling. What about *acceleration* scaling? Let's consider heave. Position scaling gives us

$$x_{3M} = \frac{x_{3S}}{R}$$

Velocity scaling gives us

$$\dot{x}_{3M} = \frac{\dot{x}_{3S}}{\sqrt{R}}$$

Why is there a square-root for scaling velocity when there isn't for position? This is because while position scales by R, *time* scales by $\sqrt{R}$ due to the requirements of Froude scaling. Since velocity has units of *length/time*, the scaling is $R/\sqrt{R}$ or $\sqrt{R}$. So, let's consider acceleration. The units of acceleration are *length/time*^{2}. Plugging in the length and time scaling relationships we get $R/(\sqrt{R})^2 = R/R = 1$. So, there is no scaling factor for acceleration! What you measure is what you get,

$$\ddot{x}_{3M} = \ddot{x}_{3S}$$

To prepare a model to be tested in waves, we need to have geometric scaling for the hull and waves, the model needs to ballasted to the scaled displacement and center of gravity location (correct calm water trim), and we need to have the correct scaled moments of inertia in pitch. This last point is different than your previous experience with resistance testing. To have the correct moment of inertia the *distribution* of the weight in the model must match the full-scale ship, not just the *average* of the weights. To get the moments of inertia correct in our model we need to **dynamically ballast the model**.

Dynamic Ballasting

When traditional model tests are conducted in still water for resistance and speed related sinkage and trim attitude changes, the geometrically similar model must be ballasted to the scaled waterline of the subject full-scale prototype. This means that both displacement, Δ, and longitudinal position of the center of gravity with respect to amidships, LCG, must be scaled geometrically; i.e.

$$\Delta_s = \Delta_m \times \frac{\rho_s}{\rho_m} \times \frac{R^3}{2240}$$

and

$$LCG_s = LCG_m \times R$$

where	Δ_s	= ship displacement in long tons
	Δ_m	= model displacement in pounds
	ρ_s	= density of water in which the ship floats
	ρ_m	= density of water in which the model floats
	R	= linear scale ratio

As long as the displacement and LCG scale as indicated, no other requirement must be met. The situation can be considered to be a steady state.

However, when models are to be tested in waves such that they oscillate in one or more of the six degrees of freedom possible for a rigid body, the added specification of weight distribution about the center of gravity becomes necessary. Specifically, when ship models are tested in long crested head (or following) sea conditions - as is typical in long, narrow towing tanks - the displacement, the LCG, and a quantitative measure of the distribution of weight (longitudinally) about the center must be modeled. The measure of longitudinal weight distribution about the center of gravity is the *longitudinal gyradius,* k_5 *(subscript* 5 *referring to pitch motion)*, and is defined and scaled as follows:

$$I_{55} = \int (x_{B1}^2 + x_{B3}^2)\, dm \qquad \text{mass moment of inertia about the } x_{B2} \text{ axis}$$

However, it is usually more practical to find the moment of inertia using:

$$I_{55} = mk_5^2$$
$$k_{5s} = k_{5m} \times R$$

Traditionally, a value of k_5 equal to about 25% of the length between perpendiculars, L_{PP}, is assumed for ships. Hence, models are ballasted accordingly. Ship models are ballasted to the correct displacement, LCG, and k_5, by the judicious placement of ballast weights within the test model.

When the weights are moved symmetrically away from the center of gravity of a ship, the gyradius of that ship will increase. No change in displacement or change in the position of the center of gravity will result from such a move. Increasing a ship's longitudinal gyradius should affect the natural pitching period and the magnitude of pitch response for a given wave excitation. Because of the strong coupling between pitch and heave motions for ships, it is likely that the heave motion will also be affected.

We will be learning two methods for measuring the pitch gyradius for ship models.

Knife Edge Method The model is hung as shown in Figure 4.19 so that it is supported by an installed transverse knife edge which is located at some known longitudinal position (e.g., Station 1) on the model. The pitch gyradius can then be computed from

$$k_5 = \sqrt{\frac{12g}{4\pi^2}T^2 a - a^2}$$

where T = swing period in seconds
a = distance from the knife edge to the model center of gravity (inches)

Experience has shown that timing 50 cycles of model oscillations and then dividing the time by 50 provides adequate precision for the calculation of the swing period. It should be noted that the Knife Edge Method is really only practical for small models whose ballasted weights are securely fastened.

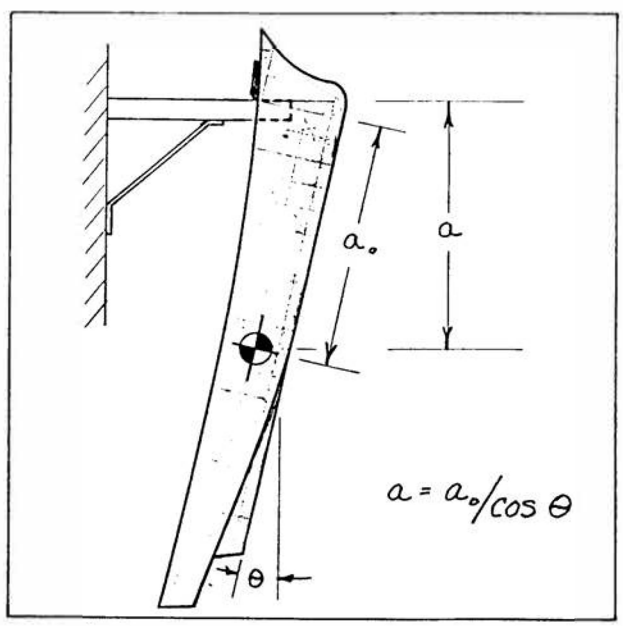

Figure 4.19: Knife Edge Method

Bifilar Suspension Method This method involves suspending the model horizontally from two eyes equidistant from the model center of gravity and at the same height above the model baseline as shown (Figure 4.20). The model is forced to oscillate horizontally about a vertical axis through the center of gravity. Thus the oscillation is in fact in YAW, not PITCH, and the resulting gyradius is k_6 vice k_5. For "normal ship forms" we tacitly *assume* that $k_6 = k_5$. This assumption loses validity as L/B decreases. Nonetheless, k_6 can be computed from

$$k_6 = \frac{T x_R}{2\pi}\sqrt{\frac{g}{h}} \qquad \text{ft, Eqn. 10.29, p.197 in Lloyd}$$

where
$2x_R =$ distance between support wires (ft)
$T =$ oscillation period (sec)
$h =$ length of the support wires (ft)

This method is much more tractable for large models.

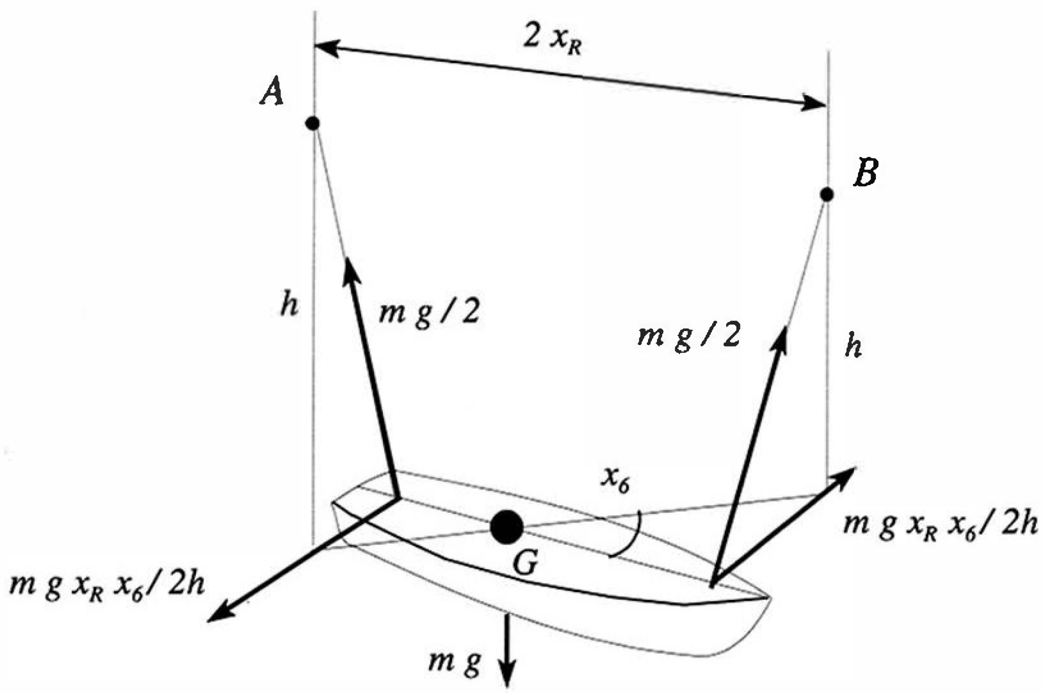

Figure 4.20: Bifilar Suspension Method (Lloyd, 1999)

Lamboley Method Developed by Gilbert Lamboley, this techniques involves swinging the model in pitch from pivots a known distance apart. The resultant period is a function of the distance from the pivots to the model center of gravity and the pitch gyradius. By employing two pivot heights and measuring periods for each, a simultaneous equation can be created to solve for both the distance to the CG and the pitch gyradius of the model. If the weight of the pivot gear is significant relative to the weight of the model, the pitch moment of inertia of the gear should be measured independently and accounted for in the final calculations.

$$T_1 = 2\pi\sqrt{\frac{d^2 + k_5^2}{gd}}$$

$$T_2 = 2\pi\sqrt{\frac{(d-x)^2 + k_5^2}{g(d-x)}}$$

where T is the swing period in seconds, d is the vertical distance from the pivot to the model CG, x is the vertical distance between the pivots, and k_5 is the pitch gyradius. Figure 4.21 shows how the rig is configured. If we solve for the intermediate quantity $c = g/(4\pi^2 x)$, then

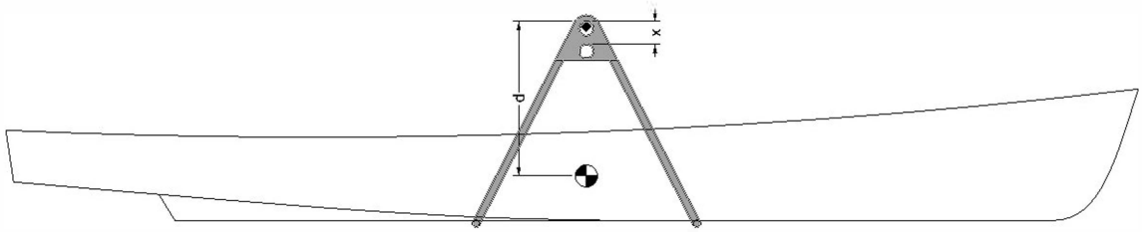

Figure 4.21: Set-up for Lamboley Method

$$d = \frac{x(cT_2^2 + 1)}{c(T_2^2 - T_1^2) + 2}$$

and

$$k_5 = \sqrt{(dxcT_1^2) - d^2}$$

In all of these methods, large amplitudes of motion are not necessary - there is very little damping due to air. **The oscillation periods measured in either method bear no direct relationship to the model's natural period in pitch in water**.

4.7 Strip Theory

The linearized equations of motion for the six degrees of freedom are as follows:

$$(m + a_{11})\ddot{x}_1 + b_{11}\dot{x}_1 = F_{w10}\sin(\omega_e t + \gamma_1) \quad \textbf{Surge}$$

$$(m + a_{22})\ddot{x}_2 + b_{22}\dot{x}_2 + a_{24}\ddot{x}_4 + a_{26}\ddot{x}_6 + b_{26}\dot{x}_6 + c_{26} = F_{w20}\sin(\omega_e t + \gamma_2) \quad \textbf{Sway}$$

$$(m + a_{33})\ddot{x}_3 + b_{33}\dot{x}_3 + c_{33}x_3 + a_{35}\ddot{x}_5 + b_{35}\dot{x}_5 + c_{35}x_5 = F_{w30}\sin(\omega_e t + \gamma_3) \quad \textbf{Heave}$$

$$a_{42}\ddot{x}_2 + b_{42}\dot{x}_2 + (I_4 + a_{44})\ddot{x}_4 + b_{44}\dot{x}_4 + c_{44}x_4 + a_{46}\ddot{x}_6 + b_{46}\dot{x}_6 + c_{46}x_6 = F_{w40}\sin(\omega_e t + \gamma_4) \quad \textbf{Roll}$$

$$a_{53}\ddot{x}_3 + b_{53}\dot{x}_3 + c_{53}x_3 + (I_5 + a_{55})\ddot{x}_5 + b_{55}\dot{x}_5 + c_{55}x_5 = F_{w50}\sin(\omega_e t + \gamma_5) \quad \textbf{Pitch}$$

$$a_{62}\ddot{x}_2 + b_{62}\dot{x}_2 + a_{64}\ddot{x}_4 + b_{64}\dot{x}_4 + (I_6 + a_{66})\ddot{x}_6 + b_{66}\dot{x}_6 + c_{66}x_6 = F_{w60}\sin(\omega_e t + \gamma_6) \quad \textbf{Yaw}$$

In these equations, the coefficients have two subscripts - one refers to the direction of motion and the other refers to the direction of force. For example, c_{35} in the heave equation refers to the buoyancy force in the heave direction (3) due to a change in position in the pitch direction (5). In this example, if the ship pitches bow down, the ship will experience a pitching restoring force (trying to return the ship to bow up). The ship will *also* experience a general upwards force on the ship due to the bow pushing down into the water. Therefore, the ship will experience a heave force due to a pitch motion. For coefficients with both subscripts being the same number (for example, c_{55}) that is the force and motion in the same direction (so c_{55} refers to the restoring force in pitch due to pitch displacement).

The six degrees of freedom have the following sign conventions:

6DOF

x_1	surge	x_4	roll
x_2	sway	x_5	pitch
x_3	heave	x_6	yaw

Motions for the ship measured at the ship's center of gravity.

Solving the linearized equations of motion requires evaluation of the coefficients and the excitation amplitudes and phases. Considerable effort has therefore been devoted to developing theoretical methods of determining the coefficients and excitations to allow ship motions to be calculated without recourse to experiment. Strip theory is a method for determining the coefficients and excitations theoretically to allow ship motions to be calculated.

There are limitations concerning what assumptions <u>must</u> be made to use strip theory. The basic principle behind strip theory is that the hydrodynamic properties of a vessel (that is added mass, damping, and stiffness) may be predicted by dividing the vessel into a series for two-dimensional transverse strips, for which these properties may be computed. The global hydrodynamic values for the complete hull are then computed by integrating the two-dimensional values of the strips over the length of the ship. Linear strip theory assumes the vessel's motions are linear and harmonic, in which case the response of the vessel in both pitch and heave, for a given wave frequency and speed, will be proportional to the wave amplitude and slope, respectively.

The basic assumptions (as stated in reference 2) required for linear strip theory are:

1. The fluid is inviscid, that is, viscous damping is ignored (although, the damping factor which the user enters in Maxsurf Motions for roll should include viscous roll damping, which is the primary source of damping for roll).

2. The ship is slender (i.e. the length is much greater than the beam or the draft, and the beam is much less than the wave length).

3. The hull is rigid so that no flexure of the structure occurs.

4. The speed is moderate so there is no appreciable planing lift.

5. The motions are small (or at least linear with wave amplitude).

6. The ship hull sections are wall-sided.

7. The water depth is much greater than the wave length so that deep water wave approximations may be applied.

8. The presence of the hull has no effect on the waves (Froude-Krilov hypothesis).

This theory is called **Strip Theory** because it represents the 3D underwater hull form by a series of 2D slices or strips. Each strip has associated local hydrodynamic properties (added mass, damping, and stiffness) which contribute to the coefficients for the complete hull in the equations of motion. Similarly the wave excitations experienced by the hull are composed of contributions from all of the strips.

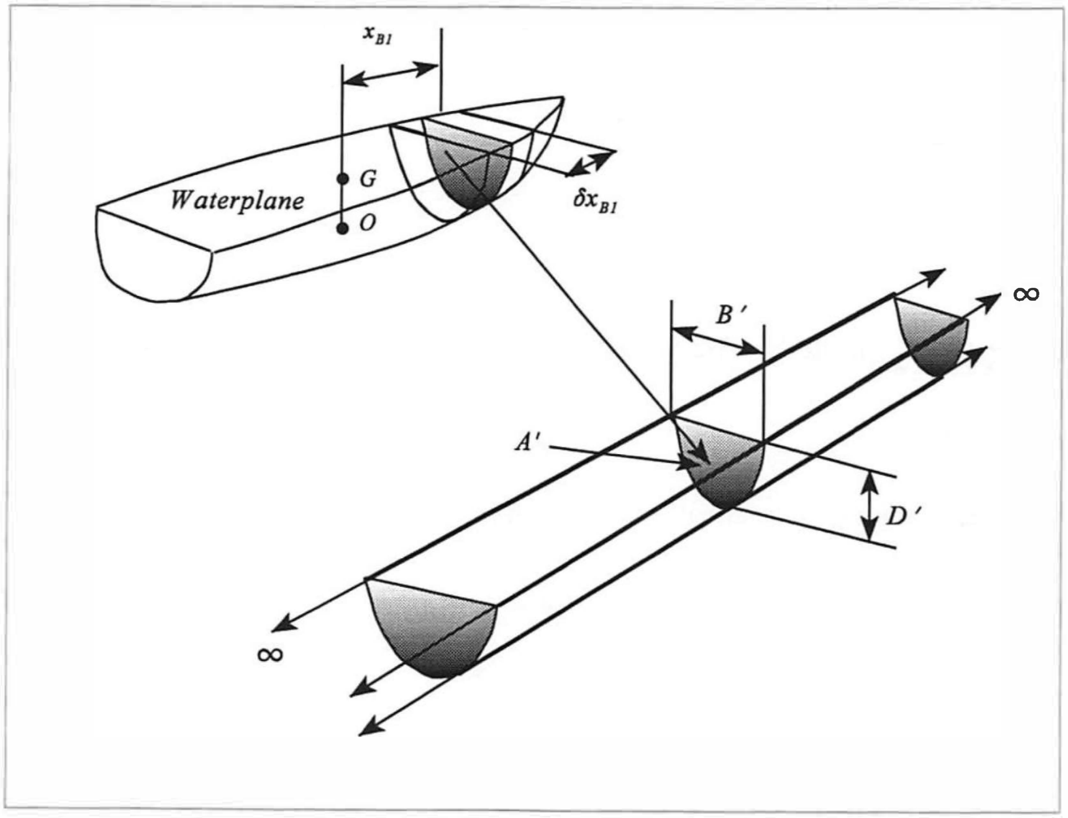

Figure 4.22: Concept of the ship "strips" used in Strip Theory (Figure 4.1 in reference 2)

Consider the hydrodynamic properties of each strip:

- added mass
- damping
- stiffness

These hydrodynamic properties for each strip contribute to the total hydrodynamic properties of the ship. How can we distinguish between a property for a *strip* and a property of the *ship*? By convention, properties of strips are written with a hashmark:

Ship	**Strip**
a_{33}	a'_{33}
b_{33}	b'_{33}
c_{33}	c'_{33}

Consider a barge where each strip has an added mass of a'_{33}.

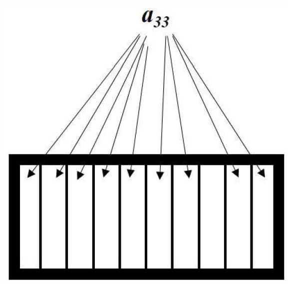

Figure 4.23: Strip Theory Barge Example

How do we find a_{33} of the total barge? How would we find the total **mass** if we had the mass of each strip? We could add each strip to find the total! Same principle for the added mass, except we will take the integral:

$$a_{33} = \int_0^{L_{PP}} a'_{33} \mathrm{d}x_{B1}.$$

Can a *strip* have an added mass in pitch? For strip theory to be successful, each strip must be extremely thin so that it can be considered a 2D strip. A 2D strip cannot have any pitch motions, so there can be no strip added mass in pitch! Which leaves us with the question of how to find the **total ship added mass in pitch**! Consider how we find the moment of inertia about G for a mass a'_{33} located x_{B1} away from the center of rotation: $I = mr^2$. We can use the same relationship for added mass in pitch of the total ship:

$$a_{55} = \int_0^{L_{PP}} (a'_{33} x_{B1}^2) \mathrm{d}x_{B1}.$$

Example: Consider a model barge with each section having a sectional added mass coefficient of 3.2 slugs/ft with 10 sections each of width 2 inches. Find the total ship added mass in heave (a_{33}) and pitch (a_{55}).

$$a_{33} = \int_0^{L_{PP}} a'_{33} \mathrm{d}x_{B1} = \int_0^{L_{PP}} 3.2 \mathrm{d}x_{B1} = 3.2 \int_0^{L_{PP}} \mathrm{d}x_{B1}$$

$$L = 10 * 2 = 20 \text{ inches} = 1.67 \text{ ft}$$

$$a_{33} = 3.2(1.67) = 5.34 \text{ slugs}$$

For added mass in pitch, I need to integrate each strip added mass over the distances from G. Given that each strip has the same added mass ($a'_{33} = 3.2$ slugs/ft), we can use Simpson's Rule to integrate this numerically,

x_{B1} (ft)	$a'_{33}x_{B1}^2$
−0.75	1.8
−0.58	1.09
−0.42	0.56
−0.25	0.2
−0.08	0.02
0.08	0.02
0.25	0.2
0.42	0.56
0.58	1.09
0.75	1.8

Using the symmetry in the problem, the Simpson's Rule integration looks like

$$a_{55} = 2\big[\frac{0.17}{3}(1.8 + 4(1.09) + 2(0.56) + 4(0.42) + 0.02)\big] = 0.918 \text{ slug} \cdot \text{ft}^2$$

4.7.1 Lewis Coefficients

Once the forces for inertia, damping, restoring, and exciting are known, the various expressions for the motion can be easily determined. In other words, before we can determine the heaving motion, we must evaluate the various strip coefficients! The strip inertial coefficient (added mass) can be expressed as a function of the added mass for a simple shape. In the case of Lewis coefficients, the shape is a circular segment of unit length and diameter B' (the beam of the "strip"). The added mass in heave for a circular segment of unit length and diameter B' is

$$\frac{1}{2}\rho\pi r^2 = \frac{\rho\pi(B')^2}{8}$$

where B' is twice the radius of the circular section. For shapes other than semi-circular, the added mass in heave is

$$a'_{33} = C\frac{\rho\pi(B')^2}{8}$$

where C is the ratio of the added mass of the section of unit length, section beam B', and section draft D' over half of the added mass for a circular segment of unit length and diameter B'. The C value is the coefficient for Lewis-form sections (derived through a method known as conformal mapping). There are plots that allow you to determine the appropriate Lewis-form coefficient given the section (or strip) beam at the waterline, draft, and cross-sectional area. **Lewis-forms have the same beam, draft and area as the actual ship section, but not the same shape.** There are similar charts for the damping Lewis-form coefficients.

4.8 Roll Mitigation

Because of the limited hydrodynamic damping available in roll (rolling causes only small waves, as opposed to the waves generated from ship heave or pitch), motions are generally large in roll and can be devastating (cause capsize, for example) if the encountered excitation wave frequency is too close to the ship's natural frequency in roll. Therefore, there is a great deal of effort put into minimizing roll motion in ships, or *roll mitigation.*

If the roll motions are too large crew comfort, economics, safety, and readiness can all be negatively affected. There are three main ways to reduce motions in waves:

Tuning stabilization Tuning stabilization changes the natural period (frequency) so that resonance is less likely to occur in the expected sea conditions. It is accomplished by hull form design and/or weight placement.

Damping stabilization Damping stabilization reduces the resonant peak. This is accomplished by increasing the damping of the system, sometimes by adding appendages to the hull.

Equilibrium stabilization Equilibrium stabilization creates an equal and opposite force approach. This is generally accomplished by applying a counter-acting force to maintain the ship in equilibrium, It relies on proper phasing of the forces and moments to reduce the motions.

In addition to the light amount of damping, roll is suitable for mitigation because it is a narrow-banded response. It means a stabilizer can be "tuned" to a single frequency. And being lightly damped means there can be large motions, but also that small increases in damping or counter-acting forces can make big differences. The total roll damping of a ship depends on four general categories: wave making (largest), viscous (eddies), skin friction, and appendage forces:

$$b_{44} = b_{\text{wave}} + b_{\text{visous}} + b_{\text{skin friction}} + b_{\text{appendage forces}}.$$

Methods of motion reduction are known as "stabilization". The term implies an increase in the stiffness coefficient (i.e., c_{44}), but it is more likely the "stabilization" method involves an increase in the motion damping (b_{44}). If a motion damper can double the decay coefficient,

$$\eta = \frac{b_{44}}{2\sqrt{c_{44}(I_4 + a_{44})}},$$

the roll amplitude at the natural frequency (ω_n) is halved, if the inherent damping is very small.

There are many different ways roll motion can be reduced, but in this chapter we are going to discuss three types of roll mitigation devices:

- Bilge Keels
- Active Fins

- Passive Tanks

For more roll mitigation devices, and further information on the three mentioned in this chapter, I refer you to the NSWC Carderock report "A Survey of Ship Motion Reduction Devices" by T.C. Smith and W.L. Thomas III.

Bilge Keels

Bilge keels are passive roll devices that increase the damping at all speeds and sea states. They consist of long narrow keels, mounted at the turn of the bilge. They work by generating drag forces which oppose the rolling motion of the ship. The advantages of bilge keels is that they are simple, inexpensive, and require no more maintenance than the hull. The disadvantage of bilge keels is that they increase the resistance of the ship (although the effects can be minimized by optimizing the design of the bilge keels for the design speed).

Active Fins

Active fins are active roll stabilizers that are mounted on rotatable stocks at the turn of the bilge near the middle of the ship. They work by using the angle of incidence between the fins and the flow of the water past the ship. The fins are continually adjusted by a control system that is sensitive to the rolling motion of the ship. The fins develop lift forces (due to the forward motion) that exert roll moments to oppose the moment applied by the waves. The advantages of active fins is that they are the most powerful and effective motion control device for high-speed applications. Their effectiveness increases with the square of the speed. Reductions of at least 50% in the average roll amplitudes are possible in moderate waves with a well-designed system. The disadvantages include that at low speeds the fins do not generate much lift (although they act somewhat like passive dampers). Also, their ability to reduce roll motion decreases in very severe sea states. They are a relatively sophisticated and expensive system and require considerable maintenance. Finally, at less than 10 knots they do not produce much lift, and at extreme speeds they can experience cavitation and flow separation.

Passive Tanks

Passive tanks are stabilizers that involve a sloshing liquid to produce damping and restoring forces. They work by shifting the weight of the liquid so that it exerts a roll moment on the ship and (by suitable design) this can be arranged to damp the roll motion. The natural frequency of the tank should be equal or near the ship's natural frequency. The tank is tuned by adjusting the amount of liquid in the tank or by the baffle design. A passive tank is a good choice if space and weight are not concerns. There are no moving parts and it requires very little maintenance. However, optimal tank placement is high in the ship and this makes access along the ship difficult. In addition, the free surface always reduces the metacentric height so roll stability is reduced. And all passive tanks *amplify* the roll motions at low encounter frequencies.

Motion Sickness Indices (MSI)

Ship roll motions can have very negative effects on passengers and crew. The people onboard can experience motion sickness and the roll motion can make it more difficult to move in a controlled and coherent manner so the performance of everyday tasks is impaired. The inner ear detects changes in magnitude and direction of apparent gravitational acceleration. Motion sickness is exacerbated if the person is

- confined below decks (can't see the horizon)
- facing diagonally across the ship
- anxious
- fatigued
- hungry
- smelling strong smells
- eating or smelling greasy foods
- reading
- drinking carbonated or alcoholic drinks

The symptoms of seasickness generally disappear after a few days at sea (the person becomes acclimated). Motions can impair the ability to work effectively even when there are no problems with seasickness. In these cases, the addage "one hand for the ship and one for yourself" is true.

The principle cause of motion sickness is believed to be the vertical acceleration experienced by the person (which varies with location on the ship). Other motions can cause motion sickness if sufficiently high, but are not common on conventional ships (the other motions aren't large enough). It is very difficult to predict the occurrence of seasickness. For one thing, individuals differ in their susceptibility to motions. Even a single individual's responses may vary from day to day, depending on the other factors mentioned above. Having a job to do versus thinking about how awful you feel can effect how much you suffer. Since a deterministic approach is not realistic (there exists no *if this, then that* relationship that is always true), a statistical approach is required.

The Motion Sickness Incidence is based on a 1974 experiment. The test measured the motion sickness response of over 300 American male college student (paid) volunteers who were not acclimated to motions. The students were tested in pairs in a ship motion simulator that had no windows and experienced sinusoidal vertical motion with amplitudes of about 3.5 meters ($\approx$ 23 ft overall height). The experimenters monitored the state of the participants nausea by having the students press buttons on a control panel. The experiments lasted up to 2 hours or until the subjects vomited. The results of this data allowed the Motion Sickness Incidence (defined as the percentage of subjects who vomited within two hours) equation:

$$\mathrm{MSI} = 100\big[0.5 + \mathrm{erf}\big(\frac{\log_{10}(\frac{|\ddot{s}_3|}{g}) - \mu_{\mathrm{MSI}}}{0.4}\big)\big] \tag{4.9}$$

where erf is the error function and equal to

$$\mathrm{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \mathrm{e}^{\frac{-z^2}{2}} \mathrm{d}z$$

and the values are typically stored in tables for reference. The other variables in the equation are

$$\mu_{\mathrm{MSI}} = -0.819 + 2.32(\log_{10} \omega_e)^2$$

and $|\ddot{s}_3|$, which is the absolute value of the vertical (heave) acceleration averaged over a half cycle. The MSI increases as the magnitude of the vertical acceleration increases and it is most severe at a frequency of about 1.07 rad/sec. This frequency is very close to the average frequency for the vertical motions for many ships and explains why seasickness is such a common problem.

The applications of these results to the real life environment of a ship in rough weather requires us to make assumptions about the equivalence of the random motions of the ship and the sinusoidal motions of the simulator used in the experiment. We can assume the ship accelerations are distributed according to a normal (or Gaussian) probability distribution function (just like the waves the ship is encountering). Therefore, the average acceleration is

$$|\ddot{s}_3| = 0.798\sqrt{m_4}$$

where m_4 is the variance of the *vertical acceleration.* The average frequency of the motion *peaks* is

$$\bar{\omega}_e = \sqrt{\frac{m_4}{m_2}}$$

where m_2 is the variance of the *vertical displacement.* These approximations allow an estimate of the proportion of people who will suffer from seasickness in a given set of conditions at sea.

Motion Induced Interruptions (MII)

Another measure of importance for motions related to crew performance is the Motion Induced Interruptions value. This measures when a member of a crew would have to stop working at the current task and hold on to some convenient anchorage to prevent loss of balance. The MII is relevant to the effectiveness of the ship and the crew. Typically, MII is given as number of interruptions per minute.

4.9 Appendix: EOM Solution Derivation

The basic set-up for simple harmonic motion consists of a mass, a spring, a damper, and an external harmonic excitation. For the basic equation, the mass is variable m, the spring coefficient is c, the damping coefficient is b and the external excitation has magnitude F_0 and frequency ω_e. The equation of motion is a 2$^{\text{nd}}$-order, linear, ordinary differential equation:

$$m\ddot{x} + b\dot{x} + cx = F_0 \sin \omega_e t$$

The basic solution to this equation is given as

$$x(t) = X_0 \sin(\omega_e t - \phi).$$

To solve for the unknown amplitude, X_0, and phase angle, ϕ, we need to plug the solution back into the governing equation. First we need to take the time-derivatives of the solution:

$$\begin{aligned} \dot{x}(t) &= \omega_e X_0 \cos(\omega_e t - \phi) \\ \ddot{x}(t) &= -\omega_e^2 X_0 \sin(\omega_e t - \phi) \end{aligned}$$

Now, plugging back into the governing equation:

$$-m\omega_e^2 X_0 \sin(\omega_e t - \phi) + b\omega_e X_0 \cos(\omega_e t - \phi) + cX_0 \sin(\omega_e t - \phi) = F_0 \sin \omega_e t.$$

We have two unknowns (X_0 and ϕ) and only one equation. We can separate the equation into two equations by choosing two times to evaluate at. To start, choose a time such that the $\sin \omega_e t$ term is equal to zero (i.e. $\omega_e t = 0$):

$$\begin{aligned} -m\omega_e^2 X_0 \sin(-\phi) + b\omega_e X_0 \cos(-\phi) + cX_0 \sin(-\phi) &= 0 \\ m\omega_e^2 X_0 \sin(\phi) + b\omega_e X_0 \cos(\phi) - cX_0 \sin(\phi) &= 0. \end{aligned}$$

This equation simplifies to

$$\boxed{\tan \phi = \frac{\omega_e b}{-\omega_e^2 m + c}.}$$

For the second equation, choose a time such that the $\sin \omega_e t$ term is equal to one (i.e. $\omega_e t = \pi/2$):

$$\begin{aligned} -m\omega_e^2 X_0 \sin(\pi/2 - \phi) + b\omega_e X_0 \cos(\pi/2 - \phi) + cX_0 \sin(\pi/2 - \phi) &= F_0 \\ -m\omega_e^2 X_0 \cos(\phi) + b\omega_e X_0 \sin(\phi) + cX_0 \cos(\phi) &= F_0. \end{aligned}$$

This equation simplifies to

$$Z = \frac{F_0}{(-\omega_e^2 m + c)\cos \phi + \omega_e b \sin \phi}.$$

Using the trigonometric identities for sin and cos in terms of tan:

$$\begin{aligned} \sin \phi &= \frac{\tan \phi}{\sqrt{1 + \tan^2 \phi}} \\ \cos \phi &= \frac{1}{\sqrt{1 + \tan^2 \phi}} \end{aligned}$$

we can plug the solution for $\tan\phi$ (from above) into the second equation:

$$X_0 = F_0 \frac{1}{(-\omega_e^2 m + c)\frac{1}{\sqrt{1+\tan^2\phi}} + \omega_e b \frac{\tan\phi}{\sqrt{1+\tan^2\phi}}}$$

$$X_0 = \frac{F_0}{\frac{1}{\sqrt{1+\tan^2\phi}}} \frac{1}{(-\omega_e^2 m + c) + \omega_e b \tan\phi}$$

$$X_0 = \frac{F_0}{\frac{1}{\sqrt{1+\tan^2\phi}}} \frac{1}{(-\omega_e^2 m + c) + \omega_e b \frac{\omega_e b}{-\omega_e^2 m + c}}$$

$$X_0 = \frac{F_0}{\frac{1}{\sqrt{1+\tan^2\phi}}} \frac{1}{(-\omega_e^2 m + c) + \frac{(\omega_e b)^2}{-\omega_e^2 m + c}}$$

Using the first equation, we can substitute

$$\sqrt{1+\tan^2\phi} = \sqrt{1 + (\frac{\omega_e b}{-\omega_e^2 m + +c})^2}$$

This leads to

$$X_0 = \frac{F_0 \sqrt{1 + (\frac{\omega_e b}{-\omega_e^2 (m+a) + c})^2}}{(-\omega_e^2 m + c) + \frac{(\omega_e b)^2}{-\omega_e^2 m + c}}$$

Simplifying this equation,

$$X_0 = \frac{F_0 \sqrt{(-\omega_e^2 m + c)^2 + (\omega_e b)^2}}{(-\omega_e^2 m + c)((-\omega_e^2 m + c) + \frac{(\omega_e b)^2}{-\omega_e^2 m + c})}$$

$$X_0 = F_0 \frac{\sqrt{(-\omega_e^2 m + c)^2 + (\omega_e b)^2}}{(-\omega_e^2 m + c)^2 + (\omega_e b)^2}$$

$$\boxed{X_0 = \frac{F_0}{\sqrt{(-\omega_e^2 m + c)^2 + (\omega_e b)^2}}}$$

The other form of the equation of motion for a *spring-mass-damper* system is written in terms of the natural frequency of the system (ω_n), the damping factor (η), and the tuning factor (Λ). These variables are related to the coefficients (mass, damping, and stiffness) in the following ways:

$$\omega_n^2 = \frac{c}{m}$$

$$\eta = \frac{b}{2m\omega_n}$$

$$\Lambda = \frac{\omega_e}{\omega_n}$$

Plugging these substitutions back into the equations for X_0 and ϕ requires reworking the equations so that the m, b, and c coefficients can be replaced by the ω_n, η, and Λ variables.

$$-\omega_e^2 m + c = (-\frac{\omega_e^2}{\omega_n^2} + 1)c$$
$$\omega_e b = \omega_e(2m\omega_n\eta) = 2\eta\frac{\omega_e}{\omega_n}\omega_n^2 m = 2\eta\frac{\omega_e}{\omega_n}\frac{c}{m}m = (2\eta\frac{\omega_e}{\omega_n})c$$

Reworking the X_0 and ϕ equations results in

$$X_0 = \frac{F_0}{\sqrt{c^2(1-\Lambda^2)^2 + (2\eta\Lambda)^2 c^2}}$$

$$\boxed{X_0 = \frac{F_0/c}{\sqrt{(1-\Lambda^2)^2 + (2\eta\Lambda)^2}} \qquad \tan\phi = \frac{2\eta\Lambda}{1-\Lambda^2}}$$

Chapter 5

The Output: Ship Motions in Waves

Learning Objectives:

1. Calculate the ship motion response spectrum given a sea state and relevant ship transfer function.
2. Calculate the motion variance from the motion response spectrum
3. Describe the difference between a transfer function and a Response Amplitude Operator (RAO)
4. Calculate the significant motion amplitude given the motion response spectrum
5. Calculate the probability of a particular motion amplitude being exceeded given the motion response spectrum
6. Identify the worst operating conditions given a polar plot with motion RMS or significant amplitudes
7. Explain what a Safe Operating Envelope is and how it is calculated.
8. **Laboratory Objectives:**
   (a) Calculate the significant pitch and vertical acceleration motions from an experimental time history record
   (b) Compare analytical, experimental, and simulation predictions for ship motions in irregular seas
   (c) Develop a realistic test plan for a seakeeping experiment

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Now that we have considered the input (waves) and the system characteristics (ship transfer functions), we are ready to put them together and determine ship motions in a realistic seaway! If we were to know the exact waves the ship will be encountering and we have access to a sophisticated CFD (computational fluid dynamics) code designed for seakeeping, we could predict (with reasonable accuracy) the actual motions the ship will make. However, we rarely know what exact waves the ship will encounter. Remember from Chapter 3 that when we deal with realistic sea conditions we have to work with probabilistic predictions. So, the best we will be able to do in terms of ship motions is predict the probable resulting motions for a given ship in a given wave spectrum.

5.1 The Electronic Filter Analogy

In 1951, St. Denis and Pierson suggested that the ship could be treated in much the same way as the "black box" in an electrical filter.

$$\text{Input} \rightarrow \boxed{\boxed{\text{Filter}}} \rightarrow \text{Output}$$
$$\text{Waves} \rightarrow \boxed{\boxed{\text{Ship}}} \rightarrow \text{Ship Motion}$$

The ship is a "black box" that receives the waves as input and generates ship motions as output. This analogy works as long as the filter is "linear." In other words, the output amplitude must be proportional to the input amplitude.

Consider the heave transfer function for a ship in head seas (see Figure 5.1). This transfer function looks much the same as a "low-pass" filter. For low ω_e the waves are translated into corresponding motions (same amplitudes and phase). For high ω_e, there are no resulting ship motions (i.e. the signal does not pass through the filter).

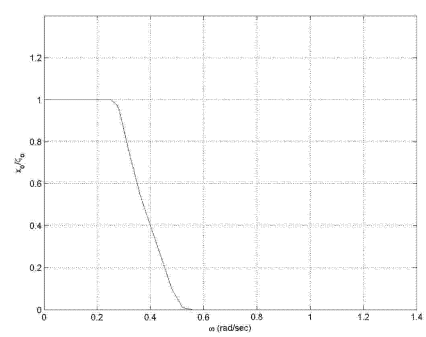

Figure 5.1: Typical Heave Transfer Function

5.2 First Challenge: Encounter Frequency Spectrum

The first challenge to apply the electronic filter analogy is that the ship experiences the **encounter frequency spectrum**, not the wave energy spectrum! Assuming the waves are long crested, the wave energy spectra formulae give the wave energy spectum for a fixed point in the ocean. We are going to need to transform this information to the reference frame of the moving ship. Remember the encounter frequency is found from

$$\omega_e = \omega - \frac{\omega^2 U \cos\mu}{g}$$

where encounter frequency (ω_e) is greater than the wave frequency (ω) in head seas and (generally) less than the wave frequency in following seas. Therefore, the wave energy spectrum will be *shifted* along the frequency axis to a *different* range of frequencies. For example, a typical open ocean wave spectrum might look like Figure 5.2. If we consider a ship operating in head seas for this wave energy spectrum, we would get an encounter frequency spectrum like the one shown in Figure 5.3. When we shift the wave frequency range from the wave energy spectrum to the equivalent encounter frequencies, we get the shaded areas in Figure 5.4. The relationship between the frequency spacing on the wave energy spectrum and the encounter frequency wave spectrum is given by

$$\delta\omega_e = \big(1 - \frac{2\omega U}{g}\cos\mu\big)\delta\omega.$$

The energy in a given range of wave frequencies must match the energy in the shifted set

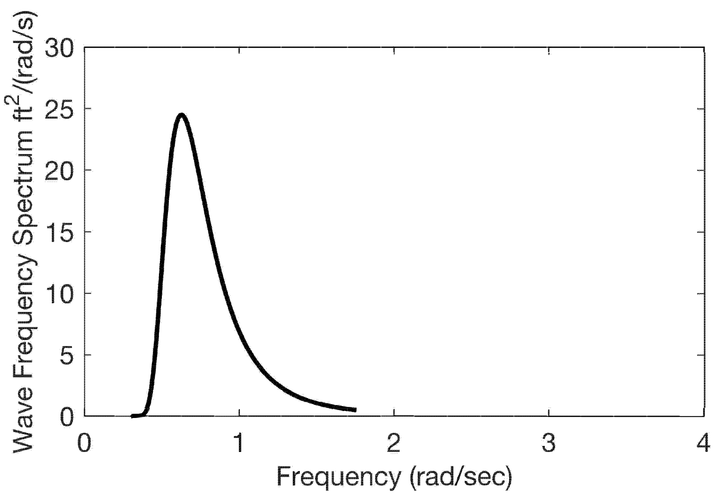

Figure 5.2: Typical Wave Energy Spectrum

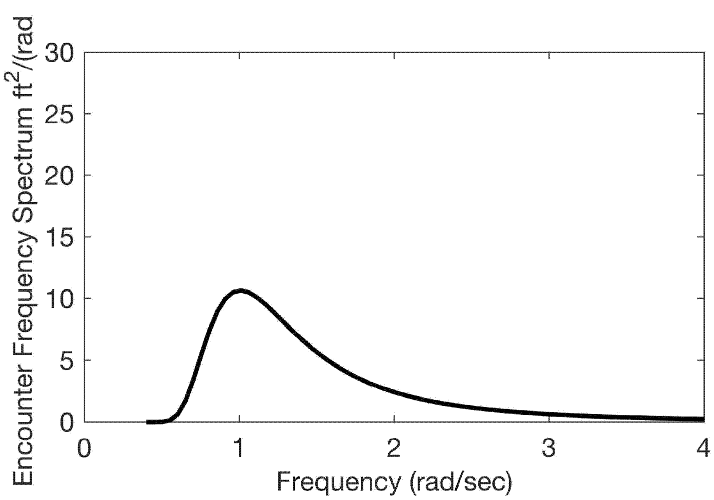

Figure 5.3: Encounter Frequency Spectrum

of encounter frequencies. In other words, the area in the shaded regions of Figure 5.4 must be equal. The areas must be the same since the total wave energy and the significant wave height must be the same whether the waves are measured by a stationary probe or one moving with the ship. So, the area of a section is,

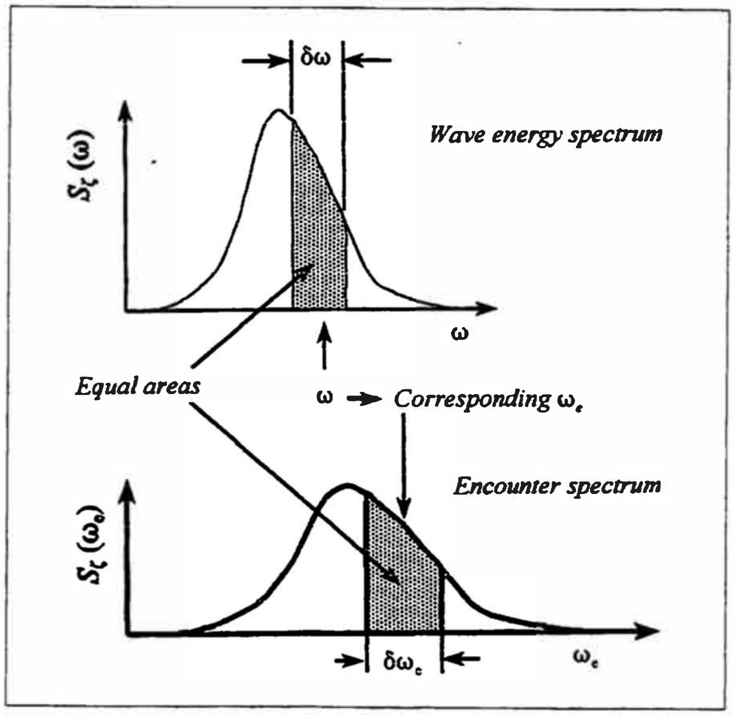

Figure 5.4: Areas under Wave Energy and Encounter Frequency Spectra (Figure 8.2 in reference 2)

$$S_\zeta(\omega_e)\delta\omega_e = S_\zeta(\omega)\delta\omega$$
$$S_\zeta(\omega_e) = S_\zeta(\omega)\frac{\delta\omega}{\delta\omega_e}$$

Plugging in the expression for $\delta\omega_e$ given above, the encounter wave energy spectrum can be found from

$$S_\zeta(\omega_e) = S_\zeta(\omega)\frac{g}{g - 2\omega U\cos\mu} \tag{5.1}$$

In head seas, the frequencies increase and the range widens which also results in reducing the height of the spectral ordinates (to keep the area under the curve the same).

5.3 Second Challenge: Motion Energy Spectrum

The second (and main) challenge is to get from the encounter frequency wave spectrum to the ship motion energy spectrum. The electronic filter analogy means the encounter frequency wave energy spectrum is the input, the ship transfer function is the filter, and the ship motion energy spectrum will be our output. But how do we combine the input and the filter to arrive at the output? The equation is actually fairly simple, you multiply the encounter frequency wave energy value (at each encounter frequency) times the square of the ship transfer function value (at the same encounter frequency). For example, the heave motion energy spectrum is found from

$$S_{x_3}(\omega_e) = S_\zeta(\omega_e)\big(\frac{X_{30}}{\zeta_0}|_{\omega_e}\big)^2 \tag{5.2}$$

The square of the transfer function is also referred to as the Response Amplitude Operator (RAO). However, the RAO is sometimes used to refer to the transfer function directly (for example, as in Maxsurf Motions), so it is important to double-check when something is called an RAO which the term is actually referring to. To summarize, the ship heave motion energy spectrum is found from

$$S_{x_3}(\omega_e) = S_\zeta(\omega_e)RAO_{x_3}.$$

To find the ship pitch or roll motion energy spectrum we need to find the encounter *wave slope* energy spectrum. The wave slope energy spectrum is

$$S_\alpha(\omega) = \frac{\omega^4}{g^2}S_\zeta(\omega)$$

and the encounter wave slope energy spectrum is equal to

$$S_\alpha(\omega_e) = S_\alpha(\omega)\frac{g}{g - 2\omega U\cos\mu}.$$

The resulting heave, pitch, and roll motions will be sinusoidal (just as the irregular waves that are in the wave energy spectrum), so we can determine the variance (measure of data spread) for these motions,

Heave	$m_0 = \int_0^\infty s_{x_3}(\omega_e)\mathrm{d}\omega_e$
Pitch	$m_0 = \int_0^\infty s_{x_5}(\omega_e)\mathrm{d}\omega_e$
Roll	$m_0 = \int_0^\infty s_{x_4}(\omega_e)\mathrm{d}\omega_e$

With the variance (or zeroth spectral moment), we can use the same techniques as for ocean waves (see section 3.2.2) to determine information about the velocity, acceleration, and motions using spectral moments! Remember, the spectral moments of an energy spectrum are equal to

$$m_n = \int_0^\infty \omega_e^n S_{x_3}(\omega_e)\mathrm{d}\omega_e$$

The root-mean-square (RMS) motion values are equal to the square-root of the variances. So, the RMS value for the motion amplitude is $\sigma_0 = \sqrt{m_0}$, the RMS value for velocity is

$\sigma_2 = \sqrt{m_2}$, and the RMS value for the acceleration is $\sigma_4 = \sqrt{m_4}$. The mean motion period, mean peak motion period, and mean zero-crossing period have the same equations as the irregular waves in Chapter 3,

$$\bar{T} = 2\pi \frac{m_0}{m_1}$$
$$\bar{T}_p = 2\pi \sqrt{\frac{m_2}{m_4}}$$
$$\bar{T}_z = 2\pi \sqrt{\frac{m_0}{m_2}}$$

Figure 5.5 shows an example of a heave motion energy spectrum determined from an encounter wave energy frequency for a ship in head seas. When the energy in the encountered

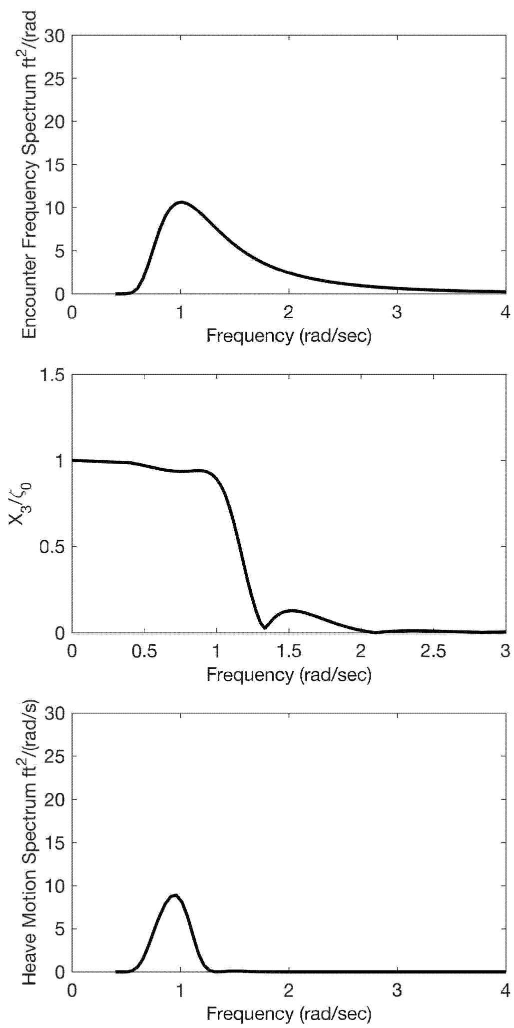

Figure 5.5: Calculation of heave energy spectrum

wave matches the largest response frequencies in the transfer function, you get large ship motions. When the energies do not overlap, only small ship motions result. Figure 5.6 shows the same ship transfer function (head seas at a single speed) when paired with different encounter frequency energy spectra (i.e. same ship operating in different sea conditions). As you can see, the ship response varies greatly based on what frequencies it is encountering in the seas. If you have large motions, one option is to change the encountered wave spectrum (change heading and/or speed) so that the high energy wave frequencies no longer align with the peak in your ship's transfer function.

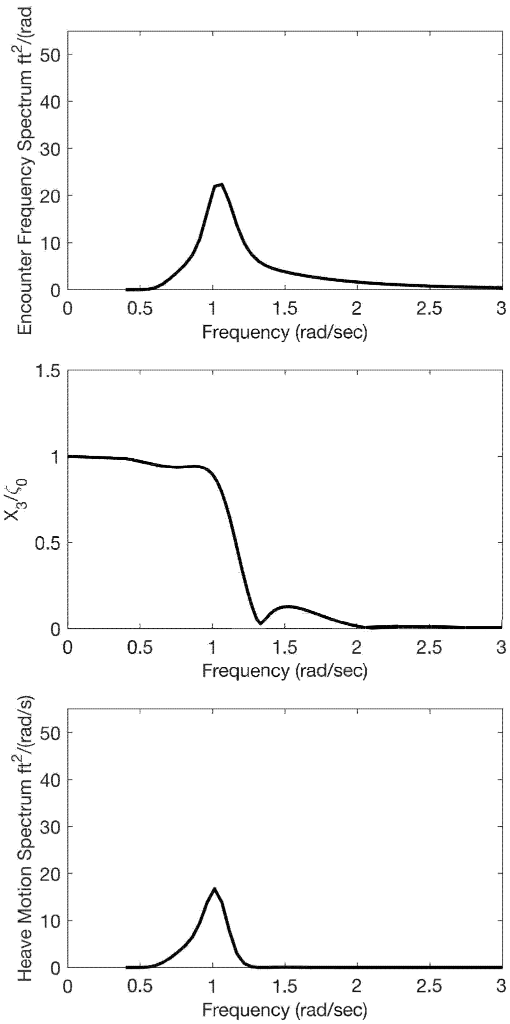

Figure 5.6: Heave motion energy spectra generated for the same ship transfer function but different encounter frequency energy spectra

5.4 Ship Motion Probability Distributions

Just as for the irregular waves, a key question in seakeeping is "what is the likelihood of a particular event occurring (such as a particular motion amplitude being exceeded)? We can use the ship motion energy spectrum to help us predict this type of probability.

Let's assume we are interested in the probability of the heave motion exceeding 2 m in a sea state 3. We can assume we have the relevant heave transfer function.

1. Begin by building an ITTC (Bretschneider) wave energy spectrum for a sea state 3 (determine the desired modal period and significant wave height from the NOAA tables). The NOAA tables show that the mean significant wave height for a SS3 is 2.85 ft and the most probable (modal) period is 7.5 sec. The equation we need is

$$S_\zeta(\omega) = \frac{1.25}{4}\left(\frac{\omega_0}{\omega}\right)^4 \frac{H_{1/3}^2}{\omega} e^{-1.25(\frac{\omega_0}{\omega})^4}$$

   where $\omega_0 = 2\pi/T_0$

2. Next convert the wave energy spectrum into the encounter frequency spectrum

$$S_\zeta(\omega_e) = S_\zeta(\omega)\frac{g}{g - 2\omega U \cos\mu}$$

3. Determine the RAO from the transfer function

$$RAO = \left(\frac{X_3}{\zeta_0}\right)^2$$

4. Find the motion energy spectrum

$$S_{x_3}(\omega_e) = S_\zeta(\omega_e) \cdot RAO_{x_3}$$

5. Find the variance (m_0) and RMS (σ_0)

$$m_0 = \int_0^\infty S_{x_3}(\omega_e)\mathrm{d}\omega_e$$
$$\sigma_0 = \sqrt{m_0}$$

6. Find the probability $X_3 > 2$ m

$$P(X_3 > 2\text{ m}) = e^{-\frac{1}{2}\left(\frac{x_3}{\sigma_0}\right)^2} = e^{-\frac{1}{2}\left(\frac{2}{\sigma_0}\right)^2}$$

Calculation of Probability of Exceedance Example Consider the pitching motion of a 520 ft ship. For the time history recorded (a total of 23.5 minutes), the variance of the pitching motion was 6.522 deg^2, the variance of the pitching velocity was 3.562 deg^2/sec^2, and the variance of the pitching acceleration was 2.276 deg^2/sec^4. Find the significant pitching magnitude and the probability of the pitching motion exceeding 6 degrees.
How did we get m_0? It was found either by taking the variance of the time history or by finding the area under the pitch motion energy spectrum. Once we have it we can find the RMS pitching motion from

$$X_{\text{RMS}} = \sqrt{m_0} = \sqrt{6.522\text{ deg}^2} = 2.55\text{ deg.}$$

The significant pitching amplitude is twice the RMS value, so

$$\bar{X}_{1/3} = 2(2.55\text{ deg}) = 5.10°$$

To find the probability of exceedance we plug this into the equation 3.12 from Chapter 3.

$$P(X_5 > 6°) = e^{-\frac{1}{2}(\frac{6°}{\sigma_0})^2} = e^{-\frac{1}{2}(\frac{6°}{2.55°})^2} = 0.0628$$

So, for this sea condition, there is a 6.28% probability the pitch motion will exceed 6°.

5.5 Polar Plots

The Response Motion Spectrum is one way to describe the ship motion in a realistic sea state, but it is not easy to get useful information directly from the plot. To find information about the magnitudes of probable motions the area under the plot needs to be taken and that only gives information on a single wave/ship heading combination. To determine the ship response in other headings (or the same heading but at different speeds), the process needs to be repeated. To understand the ship response for all headings and multiple speeds you will need to find the information from many response spectrum plots. One way to consolidate the information is using a polar plot.

There are two main types of polar plots and both have the heading angle as the angle around the plot. In the version we will be using in class, the radius of the polar plot represents the speed of the ship and the color of the lines represent the magnitude of the response. (In the other type of polar plot the radius represent the response magnitude and each curve is for a different ship speed). Figure 5.7 shows a typical example of a polar plot. The magnitudes for this plot are the RMS responses. To find the significant amplitudes you need to double the values found on this plot.

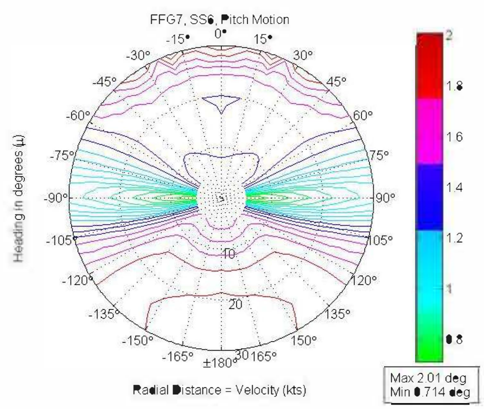

Figure 5.7: Sample Polar Plot

The information on the polar plot can be converted into a probability of exceedence based on the target magnitude plotted. For example, a plot of the significant motion amplitude gives a plot showing the motion amplitudes that have about a 14% chance of being exceeded. If desired, a polar plot showing the 10% or 1% chance of being exceeded could be displayed. The polar plot also has the advantage of showing under what operating conditions (speed and heading) for a particular sea the ship will have the largest motions. This will be true at any probability level. So, if you are experiencing large rolls in a quartering sea and your ship's polar plot for that condition shows beam seas are the worst likely response, it gives you better guidance on the best course of action for minimizing your ship's motion response.

5.5.1 Safe Operating Envelopes (SOE)

The calculations necessary for a polar plot are done using computational tools to determine response amplitudes for a large number of sea states, sea directions, and ship speeds. The results are often categorized by some limits (set by experience or some other guidance) and summarized in plots known as Safe Operating Envelopes (SOE). These polar plots show regions of marginal and unacceptable motions. Below are some examples from Comstock and Keane (1980) showing slamming predictions for CV-41 based on a change in sponson design[1].

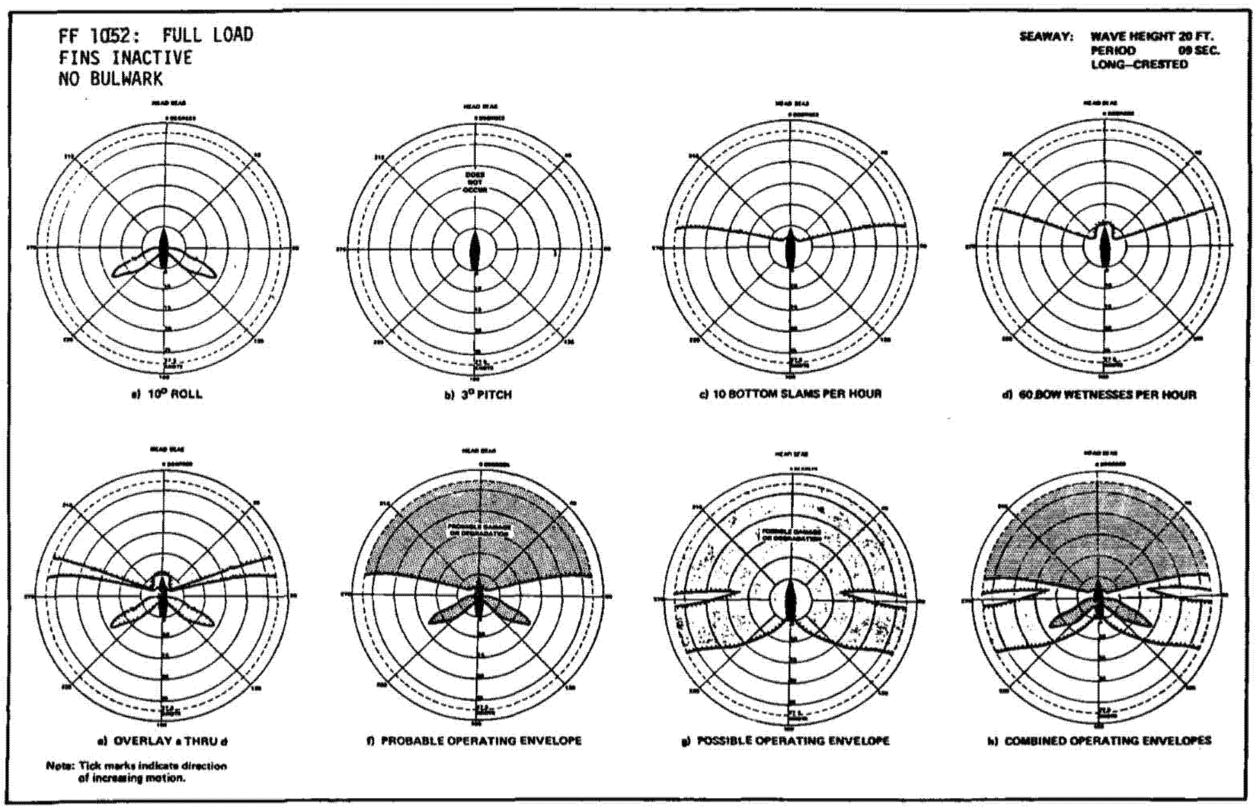

Figure 5.8: Development of FF 1042 Class Heavy Weather Seakeeping Operating Envelopes (SOEs)

5.6 Model Testing in Irregular Seas

Model testing in irregular head seas is a realistic scenario for seakeeping. The goal of seakeeping is to evaluate motions and accelerations in different sea conditions. Model testing is only possible for limited geometries and sea conditions, however, so it is important to be able to use other methods to predict motions and accelerations.

[1]Comstock, Edward N. and Keane, Robert G., Jr. (1980) "Seakeeping by Design," *Naval Engineers Journal*, vol. 92, no. 2, pp. 157-178.

Chapter 6

Introduction to Maneuvering

Learning Objectives:

1. Be qualitatively familiar with the 3 broad requirements for ship maneuverability.
   (a) Controls fixed straightline stability
   (b) Response
   (c) Slow speed maneuverability
2. Qualitatively describe what each requirement (listed above) is dependent upon.
3. Briefly describe the various common types of rudder.
4. Understand the various dimensions of the spade rudder, in particular
   (a) chord
   (b) span
   (c) rudder stock
   (d) root
   (e) tip
5. Qualitatively describe the meaning of
   (a) unbalanced rudder
   (b) balanced rudder
   (c) semi-balanced rudder
6. Qualitatively describe the sequence of events that causes a ship to turn.
7. Qualitatively describe a rudder stall and understand what it means.
8. Qualitatively describe the arrangements and devices that can be used to provide a ship with maneuverability at slow speeds.
   (a) rudder position
   (b) twin propellers
   (c) lateral thrusters
   (d) rotational thrusters

Welcome to the second part of this course - ship maneuverability! While seakeeping focused on heave, roll, and pitch, ship maneuverability focuses primarily on the three remaining degrees of freedom - surge, sway, and yaw. This text has broken the topic into a broad introductory overview (Chapter 6) and then a chapter delving more deeply into the topics we will focus on in this course (Chapter 7). As with the review on Stability (Chapter 2), this chapter is primarily taken from the Principles of Ship Performance (EN400) course notes.

6.1 Introduction

Ship maneuverability is a very complex and involved subject involving the study of equations of motion involving all 6 degrees of freedom. Analysis of these motion equations allows predictions of ship maneuverability to be made. However, many assumptions are made, so model testing is required to verify analytical results. Once built, a ship's maneuvering characteristics are quantified during its Sea Trials.

To limit the level of complexity covered in this chapter, the analytical study of the equations of motion will be limited to Chapter 7. However, the maneuverability requirements a ship designer strives to meet will be discussed along with the devices and their arrangements that can provide them.

After completing this chapter you will have an understanding of how a ship's rudder makes a ship turn and an appreciation of other devices that improve a ship's slow speed maneuverability.

6.2 Maneuverability Requirements

When given the task of designing a ship, the naval architect is given a number of design requirements to meet. These include the obvious dimensions (such as L_{PP}, beam, and draft), but also other requirements such as top speed, endurance, etc. Some of the more complicated requirements involve maneuverability. These can be split into 3 broad categories.

6.2.1 Directional Stability

In many operational circumstances, it is more important for a ship to be able to proceed in a straight line than turn. That is, with the rudder set at amidships, and in the absence of external forces, the ship will travel in a straight line. This is termed **controls fixed straight line stability**. Our experience indicates that this scenario is rarely the case, and anything but a sea directly on the bow will create a yawing moment that has to be compensaated for by movement of the ship's rudder. However, in principle, ships should be designed to achieve controls fixed straight line stability. An illustration of this stability is given in Figure 6.1[1]

Despite this requirement, many hull forms do not have this level of directional stability. In particular, ships which are relatively short and wide, such as tugs or harbor utility vessels,

[1]Figure 9.1 from EN400 course notes.

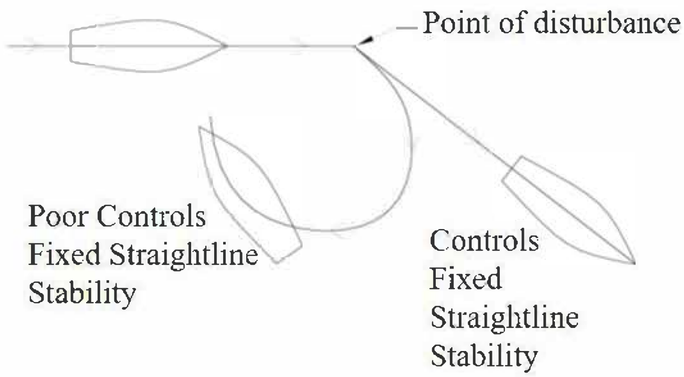

Figure 6.1: Hull Forms with Different Levels of Directional Stability (from EN400 course notes)

and, in certain circumstances, small combatants tend to have poor controls fixed straight line stability. This can be overcome by increasing the amount of deadwood of the hull (fin-like vertical surface) at the stern. This is directly analogous to the flights on an arrow or dart. Without flights, the arrow will tend to yaw in flight. With flights, the arrow maintains a straight line. Hence, increasing the amount of deadwood will increase the directional stability properties of a hull. For a ship, the problem is worsened because the ship is pushed rather than pulled along. Try pushing the end of your pencil and maintain its motion in a straight line.

Controls fixed straight line stability is quantified during sea trials by the spiral maneuver where rate of turn is compared with rudder direction.

6.2.2 Response

In opposition to the requirement for controls fixed straight line stability, it is also required for the ship to turn in a satisfactory manner when a rudder order is given. In particular:

- The ship must respond to its rudder and change heading in a specified minimum time.
- There should be minimum overshoot of heading after a rudder order is given.

In practice, both these response quantities are dependent upon the magnitude of the rudder's dimensions, the rudder angle, and the ship speed.

Rudder Dimensions

As we will see in the next section, rudder dimensions are limited by the geometry of the ship's stern. However, it is not surprising that the larger the dimensions of the rudder, the more maneuverable the ship. Increasing the rudder dimensions decreases the response time and overshoot experienced by the hull.

The level of response required by a ship is driven by its operational role. For example, the ratio of rudder area to the product of length and draft ranges from 0.017 for a cargo ship to 0.025 for destroyers.

$$\text{Rudder Area Ratio} = \frac{\text{Rudder Area}}{L_{PP}T}$$

Rudder Angle

Clearly, the response characteristics of a ship will depend upon the rudder angle ordered for a particular maneuver. It is common procedure for the levels of response to be specified with the ship using standard rudder. This is 20 degrees of wheel for the USN.

Ship Speed

Ship speed will also influence the level of maneuverability being experienced by the ship. In practice, for the majority of hull forms, greater ship speed will reduce response time, but increase overshoot. This is because greater ship speed increases the rudder force being generated by a given rudder angle.

For this reason, during sea trials, ship response and overshoot are quantified at several ship speeds. Ship response is usually assessed by the zig-zag maneuver.

6.2.3 Slow Speed Maneuverability

It is usually the case that it is most important for ships to be maneuverable when traveling at slow speeds. This is because evolutions such as canal transits and port entrances are performed at slow speeds for safety reasons. Unfortunately, this is when the ship's rudder is least effective.

Levels of slow speed maneuverability are specified in terms of turning circle and other quantifiable parameters at speeds below 5 knots. Devices that can improve slow speed maneuverability will be discussed later.

6.2.4 Maneuverability Trade-Off

Unfortunately, the need for good directional stability (in particular controls fixed straight line stability) and minimum ship response oppose each other. For example, for a fixed rudder area, increasing the length of a ship will make it more directionally stable but less responsive to its rudder. As discussed, a similar effect is created by increasing the amount of vertical flat surface at the stern (deadwood).

However, increasing rudder area will always improve the response characteristics of a hull form and usually improve its directional stability as well. Unfortunately, rudder dimensions are limited by stern geometry. Also, larger rudders will increase drag and so reduce ship speed for a given DHP from the propeller.

6.3 The Rudder

6.3.1 Rudder Types

Clearly, the rudder is the most important control surface on the hull. There are a multiplicity of different types. Figure 6.2 (repoduced from reference 5) shows some of them.

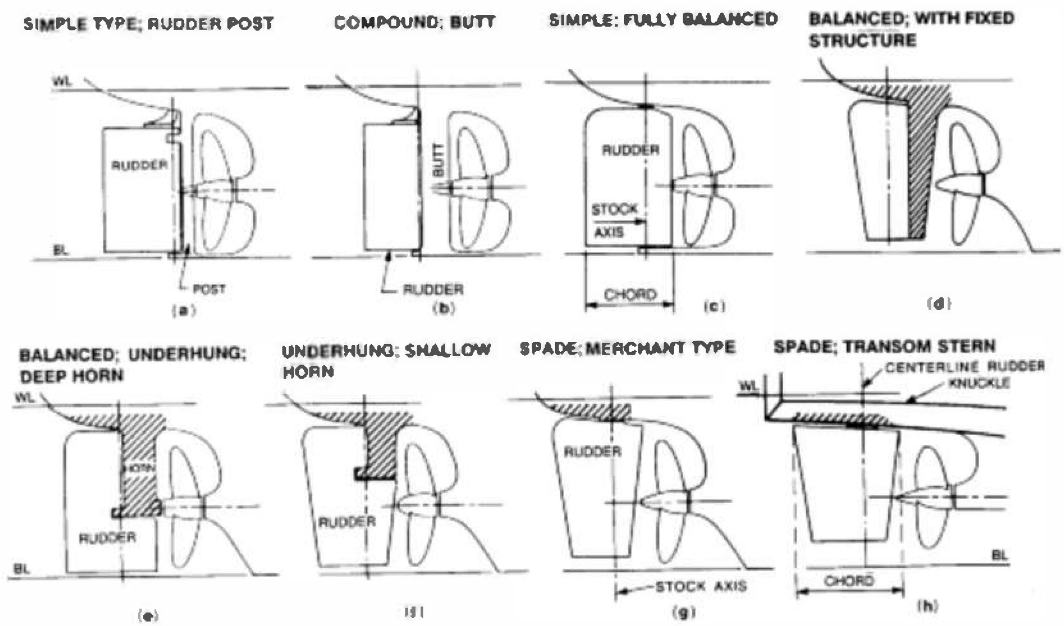

Figure 6.2: Different Rudder Types

The magnitude of all the rudder types dimensions are limited by the stern geometry.

- **Chord**: The chord is limited by the position of the propeller (propeller/rudder clearance is specified by ABS) and the edge of the stern. It is fairly obvious that a rudder protruding beyond the stern is inadvisable.
- **Span**: The span is limited by the hull and the need for the rudder to remain above the ship baseline. This is a "grounding" consideration.

The Spade Rudder

The most common type of rudder found on military vessels is the spade rudder. Figure 6.2 (reproduced from reference 5) shows the geometry of a typical semi-balanced spade rudder.

Rudder Balance Whether a rudder is balanced or not is dependent upon the relationship of the center of pressure of the rudder and the position of the rudder stock.

- When they are vertically aligned, the rudder is said to be "fully balanced". This arrangement greatly reduces the torque required by the tiller mechanism to turn the rudder.

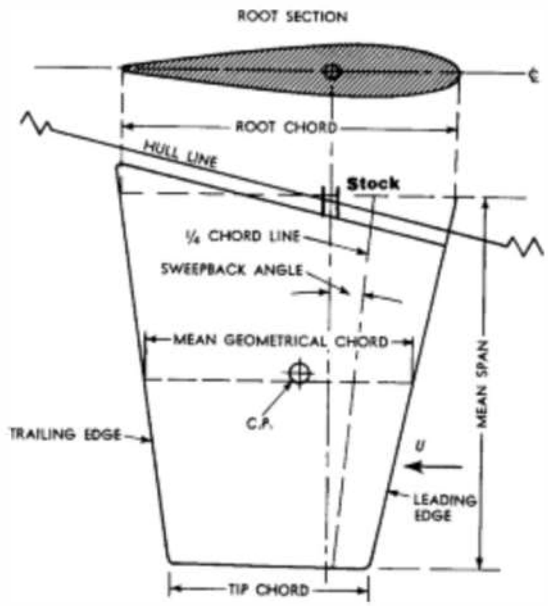

Figure 6.3: A Semi-balanced Spade Rudder

- When the rudder stock is at the leading edge, the rudder is "unbalanced" as in Figure 6.2(a). This is a common arrangement in merchant ships where rudder forces are not excessive.
- The spade rudder in Figure 6.3 is semi-balanced. This is a sensible arrangement as it limits the amount of torque required by the tiller mechanism yet should ensure the rudder returns to amidships after the occurrence of a tiller mechanism failure.

Rudder Performance

It is a common misconception that the rudder turns a ship. In fact, the rudder is analogous to the flaps on an aircraft wing. The rudder causes the ship to orientate itself at an angle of attack to its forward motion. It is the hydrodynamics of the flow past the ship that causes it to turn. Figure 6.4 shows the stages of a ship turn. The ship will continue to turn until the rudder angle is removed.

Rudder Stall You will probably have noticed that a typical ship's rudder is limited to a range of angles from about ±35 degrees. This is because at greater angles than these the rudder is likely to stall. Figure 6.5 (reproduced from reference 5) shows the development of stall as the rudder angle increases. At small angles, the rudder lift is created due to the differences in flow rate across the port and starboard sides of the rudder. However, as the rudder angle increases, the amount of flow separation increases until a full stall occurs at 45 degrees.

The amount of lift achieved by the rudder reduces significantly after a stall and is matched by a rapid increase in drag. Consequently, the rudder angle is limited to values less than the stall angle. Figure 6.6 shows how rudder lift alters with rudder angle.

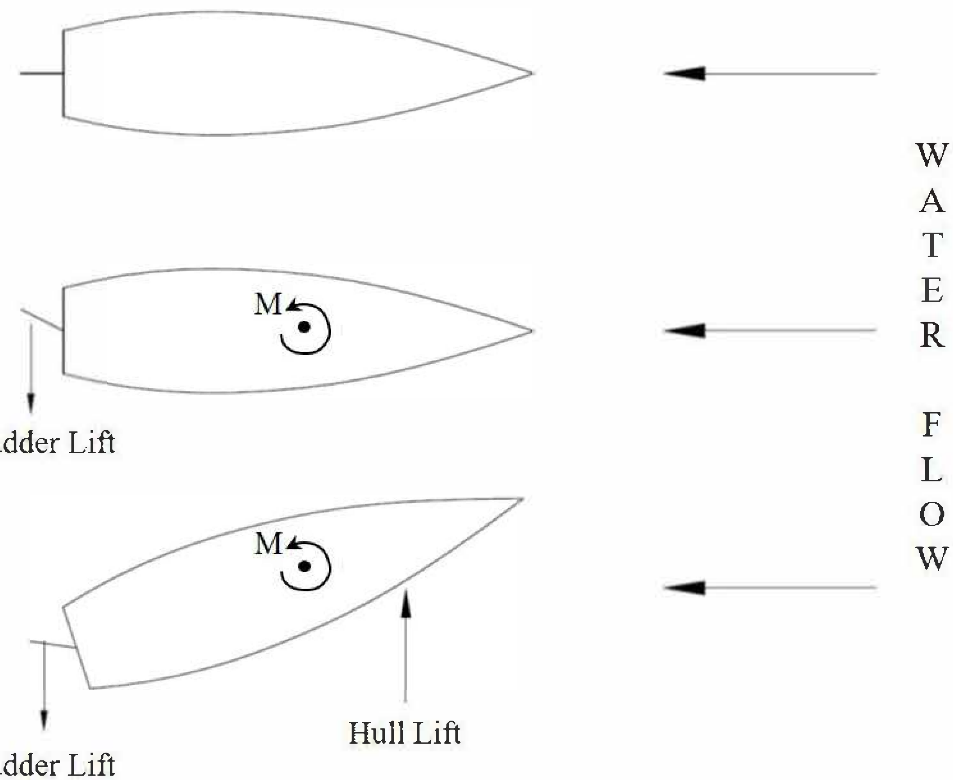

Figure 6.4: The Stages of a Ship's Turn

6.4 Slow Speed Maneuverability

As mentioned previously, it is at slow speeds when ships need to be the most maneuverable. Unfortunately, at slow speeds the rudder is limited in its effectiveness due to the lack of flow across its surfaces. However, there are several things that can be done to improve the situation.

6.4.1 Rudder Position

To improve the low flow rate experienced by the rudder at slow speeds, the rudder is often positioned directly behind the propeller. In this position, the thrust from the propeller acts directly upon the control surface. A skilled helmsman can then combine the throttle control and rudder angle to vector thrust laterally and so create a larger turning moment.

6.4.2 Twin Propellers

The presence of 2 propellers working in unison can significantly improve slow speed maneuverability. By putting one prop in reverse and the other forward, very large turning moments can be created with hardly any forward motion.

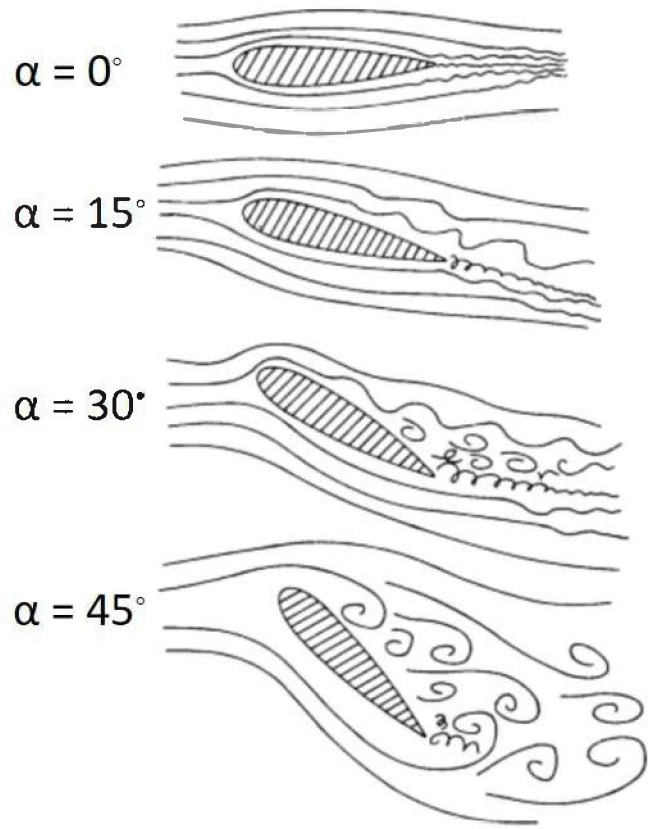

Figure 6.5: Rudder Flow Patterns at Increasing Rudder Angle

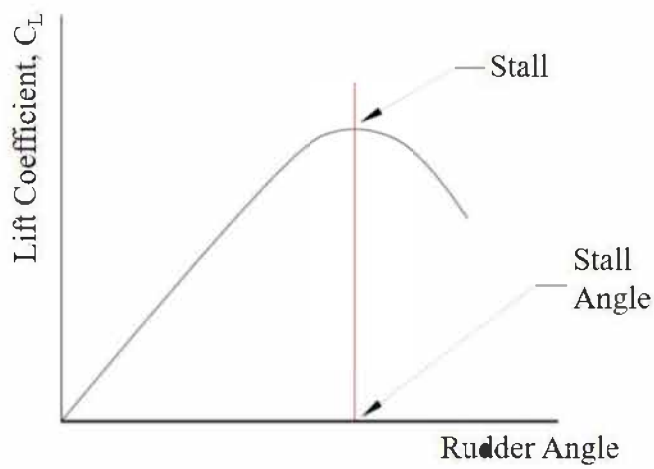

Figure 6.6: How Lift Alters with Rudder Angle

6.4.3 Lateral Thrusters

Lateral thrusters (or bow thrusters as they are usually positioned at the bow) consist of a tube running athwart ships inside of which is a propeller. They are usually electrically driven. With a simple control from the bridge, you can create a turning moment in either direction. Figure 6.7 shows a photograph of 2 lateral thrusters positioned in the bulbous bow of a ship. The photo is reproduced from reference 5.

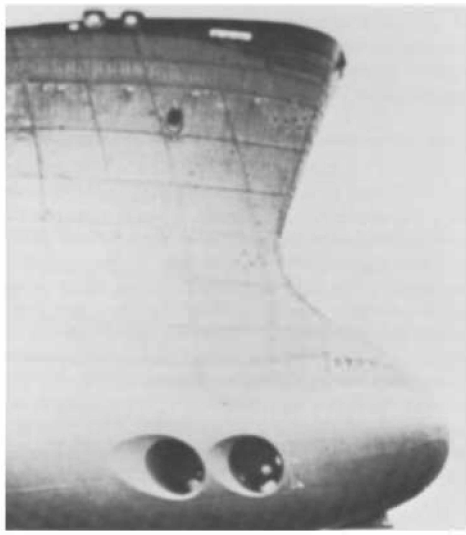

Figure 6.7: Lateral Thrusters in the Bow of a Ship

6.4.4 Rotational Thrusters

These provide the ultimate configuration for slow speed maneuverability. Rotational thrusters' appearance and operation resembles an outboard motor. They consist of pods that can be lowered from within the ship hull. Once deployed, the thruster can be rotated through 360 degrees allowing thrust to be directed at any angle. Figure 6.8 shows a typical "ro-thruster" design produced by Kværner Masa-Azipod of Finland.

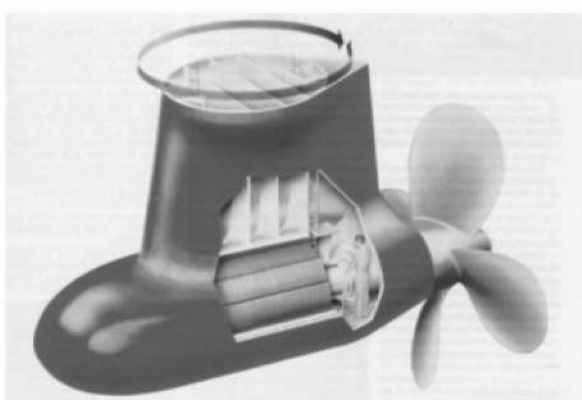

Figure 6.8: Typical Rotational Thruster Design (from EN400 course notes)

Some highly specialized ships use "ro-thrusters" as their only means of propulsion. Two or three "ro-thrusters" coupled with a complicated GPS centered control system can keep a ship in a geostationary position over the sea bed and at the same heading in quite considerable tide and wave conditions. These ships are often associated with diver, salvage, or seabed drilling operations.

Figure 6.9 shows a typical auxiliary propulsion unit used on ships and submarines. It can be used for both emergency propulsion and maneuverabilty.

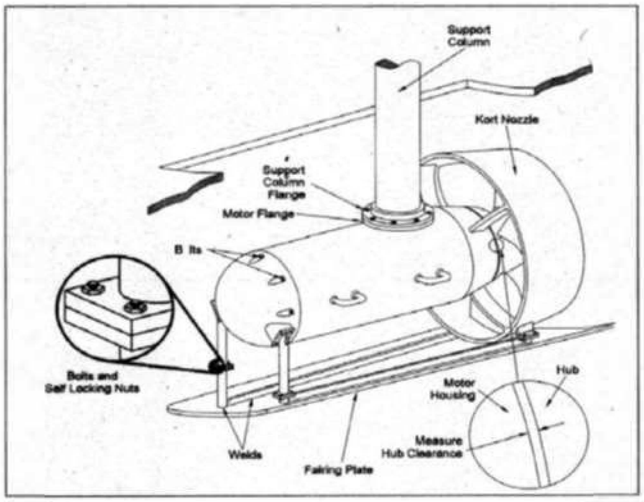

Figure 6.9: Auxiliary Propulsion Unit (from EN400 course notes)

In practice, the amount of slow speed maneuverability exhibited by a ship is largely dependent upon the amount of money the designer is willing to spend on lateral or rotational thrusters in the ship design. This economic question is highly involved and includes estimates of ship docking rates, costs of hiring tugs, etc.

Chapter 7

Maneuvering Theory

Learning Objectives:

1. Explain the concepts of directional stability
2. Explain the relationship between controls-fixed straight line stability and the linearized equations of motion for maneuvering.
3. Describe the forces on a ship in a turn.
4. Describe the effect of the ship characteristics and rudder on turning ability.
5. Calculate the maneuvering hydrodynamic derivatives for a ship from experimental measurements.
6. Use the maneuvering hydrodynamic derivatives to determine the controls fixed straight line stability of a ship.
7. Use the maneuvering hydrodynamic derivatives to determine the steady turning radius of a ship.
8. Estimate appropriate rudder dimensions based on standard guidelines.
9. **Laboratory Objectives:**
   (a) Explain the process for determining the maneuvering hydrodynamic derivatives dependent on sway and yaw velocities and accelerations.
   (b) Describe the procedure for static and dynamic PMM testing and how that data is used to determine the hydrodynamic derivatives Y_v, N_v, $Y_{\dot{v}}$, $N_{\dot{v}}$, Y_r, N_r, $Y_{\dot{r}}$, and $N_{\dot{r}}$.

In this chapter we will go much more in-depth on the theory behind maneuvering, including the equations of motion and the hydrodynamic derivatives. We will discuss how to determine these derivatives experimentally and discuss how a ship turns in more detail. Finally, we will cover some rudder design considerations.

7.1 Elements of Controllability

1. **Coursekeeping** (or Steering) - The maintenance of a steady mean course or heading. Interest centers on the ease with which the ship can be held to the course.

2. **Maneuvering** - The controlled change in the direction of motion (turning or course changing). Interest centers on the ease with which change can be accomplished and the radius and distance required to accomplish the change.

3. **Speed Changing** - The controlled change in speed including stopping and backing. Interest centers on the ease, rapidity and distance covered in accomplishing changes.

Performance varies with water depth, channel restrictions, and hydrodynamic interference from nearby vessels or obstacles. Coursekeeping and maneuvering characteristics are particularly sensitive to ship trim. For conventional ships, the two qualities of coursekeeping and maneuvering may tend to work against each other: an easy turning ship may be difficult to keep on course whereas a ship which maintains course well may be hard to turn. Fortunately a practical compromise is nearly always possible.

Since controllability is so important, it is an essential consideration in the design of any floating structure. Controllability is, however, but one of many considerations facing the naval architect and involves compromises with other important characteristics. Some solutions are obtained through comparison with the characteristics of earlier successful designs. In other cases, experimental techniques, theoretical analyses, and rational design practices must all come into play to assure adequacy.

Three tasks are generally involved in producing a ship with good controllability:

1. Establishing realistic specifications and criteria for coursekeeping, maneuvering, and speed changing.

2. Designing the hull control surfaces, appendages, steering gear, and control systems to meet these requirements and predicting the resultant performance.

3. Conducting full-scale trials to measure performance for comparison with required criteria and predictions.

7.2 Basic Equations of Motion

For the axis fixed with respect to the Earth, the equations of motion for maneuvering are

$$
\begin{aligned}
X_0 &= m_\Delta \ddot{x}_{0G} && \text{Surge} \\
Y_0 &= m_\Delta \ddot{y}_{0G} && \text{Sway} \\
N &= I_z \ddot{\psi} && \text{Yaw}
\end{aligned}
$$

However, for convenience we want to discuss the ship forces and motions from the ship-fixed reference frame. To do that, we need to express the variables in the previous equations from the ship-fixed coordinate system rather than in the Earth reference frame.

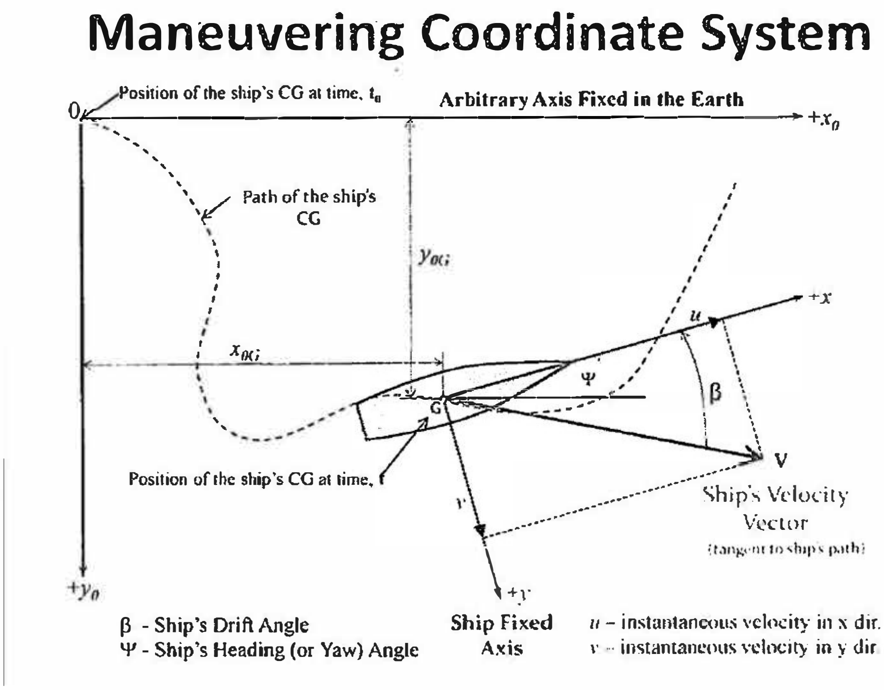

Figure 7.1: Coordinate System for Maneuvering (from reference 1)

Consider the velocities (draw a picture):

$$\dot{x}_{0G} = u\cos\psi + v\sin\psi$$
$$\dot{y}_{0G} = -u\sin\psi + v\cos\psi$$

To get accelerations we need to take the derivative of the velocities:

$$\ddot{x}_{0G} = \dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi}$$
$$\ddot{y}_{0G} = -\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi}$$

Plugging these into the equations of motion (*Note: the forces are still in the Earth reference frame*):

$$X_0 = m_\Delta(\dot{u}\cos\psi + \dot{v}\sin\psi + (-u\sin\psi + v\cos\psi)\dot{\psi})$$
$$Y_0 = m_\Delta(-\dot{u}\sin\psi + \dot{v}\cos\psi - (u\cos\psi + v\sin\psi)\dot{\psi})$$

Now consider the forces in the ship-fixed reference frame (same transformation as for the velocities):

$$X_0 = X\cos\psi + Y\sin\psi$$
$$Y_0 = -X\sin\psi + Y\cos\psi$$

Plugging into the previous equations and simplifying gives the equations of motion in the forces, velocities, and accelerations measured in the ship-fixed reference frame:

$$X = m_\Delta(\dot{u} + v\dot{\psi})$$
$$Y = m_\Delta(\dot{v} - u\dot{\psi})$$

The angular equation is unchanged by the shift in coordinate system. Since the other variables (u, v) are velocities, let's replace the angular velocity ($\dot{\psi}$) with r (now velocities have no dot and accelerations are all represented with one dot). Now, the equations of motion are:

$$\boxed{\begin{aligned} X &= m_\Delta(\dot{u} + vr) \\ Y &= m_\Delta(\dot{v} - ur) \\ N &= I_z\dot{r} \end{aligned}}$$

The forces and moments (left hand side) of the equations of motion consist of four types of forces that act on a ship during a maneuver:

1. Hydrodynamic forces acting on the hull and appendages due to ship velocity and acceleration, rudder deflection, and propeller rotation
   (a) Due to relative velocity and acceleration of the surrounding fluid
   (b) Due to rudder deflection
   (c) Due to propeller action
2. Inertial reaction forces caused by ship acceleration
   (a) Rigid body forces acting on a moving body - due to body accelerations
3. Environmental forces due to wind, waves and currents
4. External forces such as tugs, thrusters, mooring lines, etc.

We will only deal with the top two!

7.2.1 Linear Equations

The force components X, Y, and moment component N are assumed to be composed of several parts, some of which are functions of the velocities and accelerations of the ship. For now, we will assume that the forces are composed only of the forces and moments arising from motions of the ship which, in turn, have been excited by disturbances whose details we need not be concerned with here.

$$\begin{aligned} X &= F_x(u, \dot{u}, v, \dot{v}, r, \dot{r}) \\ Y &= F_y(u, \dot{u}, v, \dot{v}, r, \dot{r}) \\ N &= F_\psi(u, \dot{u}, v, \dot{v}, r, \dot{r}) \end{aligned}$$

The forces are comprised of velocity dependent forces arising from hull drag through the water (in surge, sway and yaw) and acceleration dependent forces arising from the mass of the ship and the added mass of the fluid being accelerated in surge, sway, and yaw.

For stability analyses, we need to consider a vessel moving in equilibrium that experiences a disturbance. To consider the effect of a disturbance on the forces acting on the vessel, we can use the Taylor Series expansion technique. "If the function of a variable, x, and all its derivatives are continuous at a particular value of x, say x_1, then the value of the function at the value of x not far removed from x_1 can be expressed as follows":

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx} + \frac{(x - x_1)^2}{2!}\frac{d^2 f(x_1)}{dx^2} + \frac{(x - x_1)^3}{3!}\frac{d^3 f(x_1)}{dx^3} + ...$$

If the change in the variable $(x - x_1)$ is kept small, the higher order terms (HOT) can be neglected, leaving

$$f(x) = f(x_1) + (x - x_1)\frac{df(x_1)}{dx}$$

For multivariable functions,

$$f(x, y) = f(x_1, y_1) + (x - x_1)\frac{\partial f(x_1, y_1)}{\partial x} + (y - y_1)\frac{\partial f(x_1, y_1)}{\partial y}$$

So, if we write the linearized sway force we get

$$Y = F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) + (u - u_1)\frac{\partial Y}{\partial u} + (v - v_1)\frac{\partial Y}{\partial v} + ... + (\dot{r} - \dot{r}_1)\frac{\partial Y}{\partial \dot{r}}$$

For *Straight Line Stability*, many of the velocities and accelerations are zero. For example, for a vessel moving at constant forward speed, there are no acceleration terms, no sway or yaw velocities and no Y force before a disturbance. The forward velocity is equal to the ship speed, U.

$$\begin{aligned}
&u_1 = U \\
&v_1 = r_1 = 0 \\
&\dot{u}_1 = \dot{v}_1 = \dot{r}_1 = 0 \\
&F_y(u_1, \dot{u}_1, v_1, \dot{v}_1, r_1, \dot{r}_1) = 0
\end{aligned}$$

Because of symmetry, there can be no Y force due to forward velocity or acceleration, so

$$\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial \dot{u}} = 0$$

The Sway Force Equation now becomes,

$$Y = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r}$$

We can perform the same technique to get the linearized Surge and Yaw equations:

$$\begin{aligned}
X &= \frac{\partial X}{\partial u}(u - U) + \frac{\partial X}{\partial \dot{u}}\dot{u} \\
N &= \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r}
\end{aligned}$$

Now we have the forces for the basic equations of motion, we can combine (and move everything over to the right hand side) and get

$$
\begin{aligned}
0 &= m_\Delta \dot{u} + m_\Delta vr - \frac{\partial X}{\partial u}(u - U) - \frac{\partial X}{\partial \dot{u}}\dot{u} && \text{Surge} \\
0 &= m_\Delta \dot{v} - m_\Delta Ur - \frac{\partial Y}{\partial v}v - \frac{\partial Y}{\partial \dot{v}}\dot{v} - \frac{\partial Y}{\partial r}r - \frac{\partial Y}{\partial \dot{r}}\dot{r} && \text{Sway} \\
0 &= I_z \dot{r} - \frac{\partial N}{\partial v}v - \frac{\partial N}{\partial \dot{v}}\dot{v} - \frac{\partial N}{\partial r}r - \frac{\partial N}{\partial \dot{r}}\dot{r} && \text{Yaw}
\end{aligned}
$$

The partial derivatives are called the *Hydrodynamic Derivatives* and we need to find them to solve the equations of motion!

7.2.2 Notes on Notation

To define a standard notation for maneuvering (rather than writing out the partial derivatives every time), SNAME (1952) specifed the following rule:

- Replace the partial derivative with the letter for force (or moment) and the subscript with the motion. For example,

$$
\begin{aligned}
\frac{\partial Y}{\partial v} &\equiv Y_v \\
\frac{\partial N}{\partial \dot{r}} &\equiv N_{\dot{r}}
\end{aligned}
$$

Rewriting the equations of motion using this notation gives the official Linearized Maneuvering Equations of Motion:

$$
\boxed{
\begin{aligned}
&- X_u(u - U) + (m_\Delta - X_{\dot{u}})\dot{u} + m_\Delta vr = 0 \\
&- Y_v v + (m_\Delta - Y_{\dot{v}})\dot{v} - (Y_r + m_\Delta U)r - Y_{\dot{r}}\dot{r} = 0 \\
&- N_v v - N_{\dot{v}}\dot{v} - N_r r + (I_z - N_{\dot{r}})\dot{r} = 0
\end{aligned}
}
$$

For convenience in analysis, we will non-dimensionalize the equations. For maneuvering the main effects are on sway and yaw - we can neglect surge since changes in forward velocity will be small relative to the mean forward velocity, U.

$$
\begin{aligned}
&- Y'_v v' + (m'_\Delta - Y'_{\dot{v}})\dot{v}' - (Y'_r + m'_\Delta)r' - Y'_{\dot{r}}\dot{r}' = 0 \\
&- N'_v v' - N'_{\dot{v}}\dot{v}' - N'_r r' + (I'_z - N'_{\dot{r}})\dot{r}' = 0
\end{aligned}
$$

(*The U disappeared in the sway equation since the velocities are non-dimensionalized by U, so $U' = 1$.*)

7.2.3 Control Forces and Moments

It is important to note that all the terms in the previous equations must include the effect of the ship's rudder held at zero degrees (on the centerline). On the other hand, if we want to consider the path of a ship with controls working, we must include terms expressing the control forces and moments created by rudder deflection (and any other control devices) as functions of time. The linearized y-component of the force created by rudder deflection is $Y_\delta \delta_R$. The linearized component of the moment created by rudder deflection about the z-axis of the ship is $N_\delta \delta_R$.

δ_R	=	rudder-deflection angle, measured from xz-plane of the ship to plane of the rudder; positive deflection corresponds to a turn to port for rudder(s) located at the stern.
Y_δ, N_δ	=	linearized derivatives of Y and N with respect to rudder-deflection angle δ_R

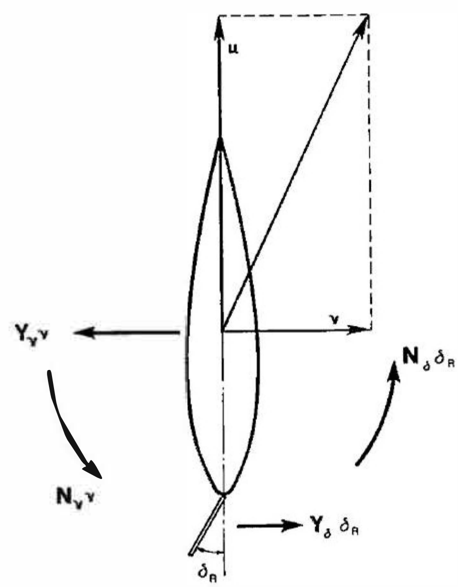

Figure 7.2: Rudder Induced Turning moments (from reference 1)

For small rudder deflections (due to small disturbances, for example) and for usual ship configurations,

$$Y'_{\dot{r}} \approx 0$$
$$N'_{\dot{v}} \approx 0$$

Applying these assumptions and including the rudder force and moment, the equations of motion become:

$$(I'_z - N_{\dot{r}})\dot{r}' - N'_v v' - N'_r r' = N'_\delta \delta_R \qquad \text{Yaw Moment}$$
$$(m'_\Delta - Y'_{\dot{v}})\dot{v}' - Y'_v v' - (Y'_r + m'_\Delta) r' = Y'_\delta \delta_R \qquad \text{Sway Force}$$

For conventional ship configurations, we can simplify the mass and inertial terms as follows:

$$(m'_{\Delta} - Y'_{\dot{v}}) \cong 2m'_{\Delta}$$
$$(I'_z - N'_{\dot{r}}) \cong 2I'_z$$

We can evaluate the hydrodynamic derivatives for the effect of the rudder on the hull, where δ_R is the rudder angle in radians (positive to **port**):

$$N'_{\delta} = \frac{\partial N}{\partial \delta_R}$$
$$Y'_{\delta} = \frac{\partial Y}{\partial \delta_R}$$

To make numerical predictions it is necessary to obtain values for some or all of the coefficients or derivatives involved. This is primarily done by means of captive model tests.

7.3 Captive Model Tests (PMM)

First let us consider what forces are acting on the vessel due to maneuvering motions and how these forces relate to the *Hydrodynamic Derivatives*.

Consider a ship experiencing transverse acceleration, $\dot{v}$ (see Figure 7.3). If the acceleration is to starboard (positive), there will be a reaction force $Y_{\dot{v}}$ to port due to the resistance of the water. For a transverse acceleration the force will always resist the direction of acceleration. This is shown in Figure 7.4 with the sway force versus sway acceleration showing a negative slope.

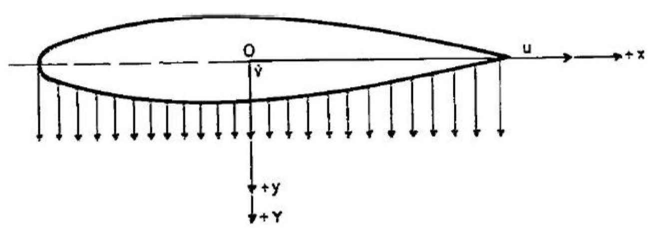

Figure 7.3: Ship Experiencing Transverse Acceleration

Consider a ship experiencing angular acceleration, $\dot{r}$ (see Figure 7.5). If the acceleration is positive (bow to starboard), there will be a reaction moment $N_{\dot{r}}$ in the negative direction due to the resistance of the water. For an angular acceleration the moment will always resist the direction of acceleration. Therefore, a plot of yaw moment versus yaw acceleration will always have a negative slope and will look like Figure 7.4.

Figure 7.6 shows the forces on a body with a sway velocity, v, added to a forward velocity, u. Both the bow and the stern experience a lift force oppositely directed to v. Therefore, Y_v is always negative (see Figure 7.7). However, the bow contribution is usually larger than that of the stern, so there is a negative moment contribution N_v. Yet the addition of a rudder at the stern will increase the magnitude of the stern force and so decrease the negative

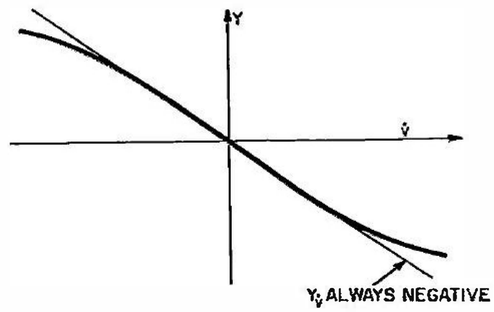

Figure 7.4: Sway Force versus Sway Acceleration

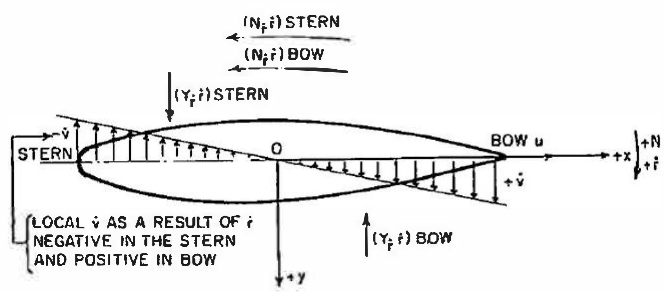

Figure 7.5: Ship Experiencing Angular Acceleration

magnitude of N_v. If the rudder force were sufficiently large, it might even cause N_v to be positive (not usually the case). Figure 7.7 show the possible relationships between N_v and v.

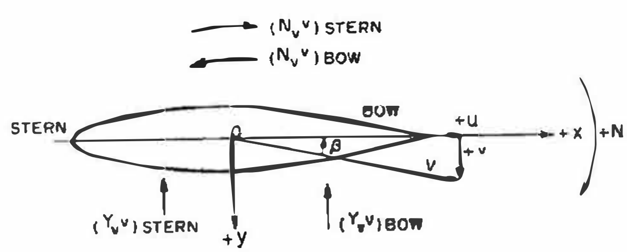

Figure 7.6: Ship Experiencing Forward Velocity and Transverse Velocity

Figure 7.8 shows the effect of an angular velocity, r, in addition to forward velocity, u, on Y and N. Due to the angular velocity, point B near the bow has a positive transverse velocity, v_B, producing a negative Y-force and a negative N-moment. Point S near the stern has a negative transverse velocity, v_S, producing a positive Y-force and a negative N-moment. So the moments can combine to produce a large negative moment, but the sway forces oppose each other resulting in a small positive or negative Y-force. Figure 7.9 shows the relationship between Y and N and r.

So, how can we use captive model tests and this information to find the hydrodynamic

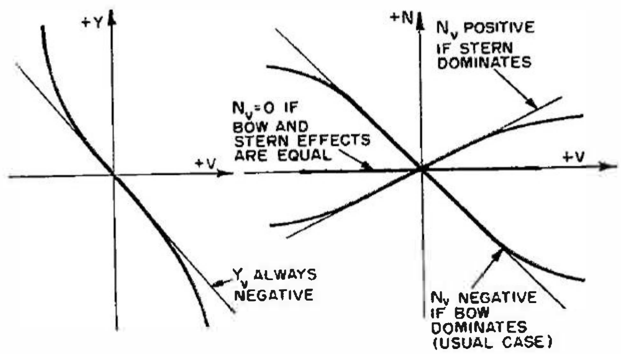

Figure 7.7: Sway Force and Yaw Moment versus Transverse Velocity

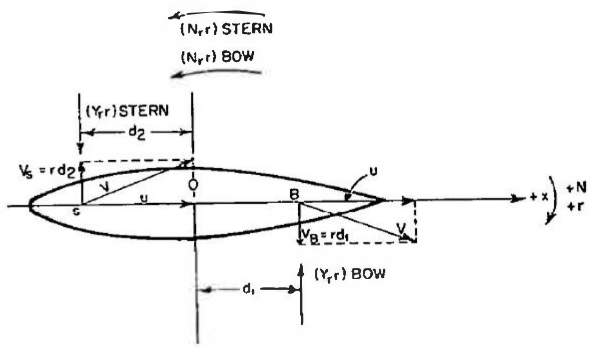

Figure 7.8: Ship Experiencing Forward Velocity and Angular Velocity

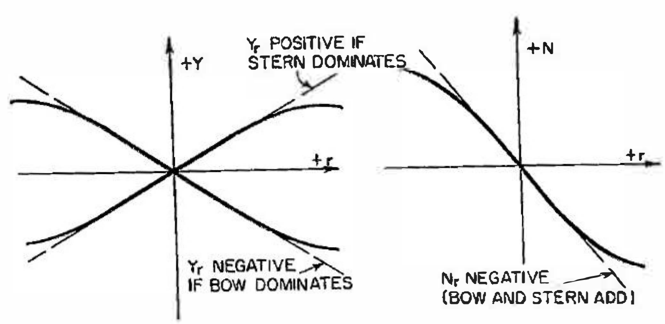

Figure 7.9: Sway Force and Yaw Moment versus Angular Velocity

derivatives?

7.3.1 Straight-Line Tests in a Towing Tank

The velocity-dependent derivatives Y_v and N_v of a ship at any draft and trim can be determined from measurements on a model of the ship, ballasted to a geometrically similar draft and trim, towed in a conventional towing tank at a constant velocity, U, corresponding to a given ship Froude number, at various angles of attack, β, to the model path. The figure below (Figure 7.10) shows the orientation of the model with respect to the tow tank. From the figure you can see that the transverse velocity component (from the vessel coordinate system) is produced along the y-axis such that

$$v = -U \sin\beta$$

where the negative sign is due to the sign convention (see Figure 7.1). The Y force and N

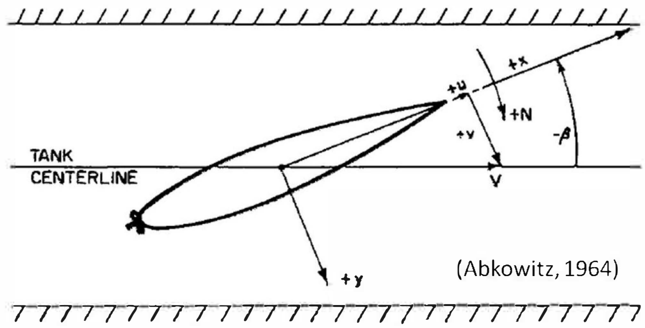

Figure 7.10: Straight Line Tow Testing

moment are measured on the model for each value of β tested. The force or moment versus sway velocity is then plotted and the hydrodynamic coefficient is the slope of the curve near $v = 0$. Figure 7.7 shows an example of sway force (Y) and yaw moment (N) versus sway velocity (v). The slope of the straight line through the curve at $v = 0$ is the hydrodynamic coefficient. So, for the plot Y versus v, you can find the coefficient Y_v and for the plot N versus v, you can find the coefficient N_v. Let's review:

1. Test a model fixed in yaw (specified drift angle, β) at a constant forward velocity, U.
2. The sway velocity felt by the model is equal to $-U \sin\beta$
3. The sway force and yaw moment are measured on the model
4. For a given U and β you have one point on the Y versus v plot and one point on the N versus v plot. To get additional points, run the test at various drift angles.

The propeller will usually exert an important influence on the hydrodynamic derivatives. Therefore, the model tests to determine these derivatives should be conducted with the propeller operating, preferably at the ship propulsion point. Also, since the undeflected rudder contributes significantly to the derivatives the model tests should also include the rudder in the amidships position.

The technique described above can also be used to determine the control derivatives Y_δ and N_δ. If the model is oriented with zero angle of attack ($\beta = 0$), but the model were towed down the tank at various values of rudder angle, δ_R, the force and moment measurements would determine the force Y and moment N as a function of rudder angle. Plots of these against rudder angle will indicate the values of the derivatives Y_δ and N_δ.

Straight-line tests can also be used to determine the cross-coupling effect of v on Y_δ and N_δ and of δ_R on Y_v and N_v.

7.3.2 Rotating-Arm Technique

To measure the rotating derivatives Y_r and N_r on a model a special type of towing tank and apparatus called a rotating-arm facility is occasionally employed.

- An angular velocity is imposed on the model by fixing it to the end of a radial arm and rotating the arm about a vertical axis fixed in the tank.
- The model revolves about the tank axis, rotates at rate r while its transverse velocity component v is zero at all times (yaw angle of attack or drift angle - $\beta = 0$).
- The model is rotated at a constant linear speed at various radii R and the sway force Y and yaw moment N are recorded.
- The angular velocity is given by $r = U/R$, so the only way to vary r at constant U is to vary R.
- The plots of Y and N versus r provide the hydrodynamic derivatives Y_r and N_r.

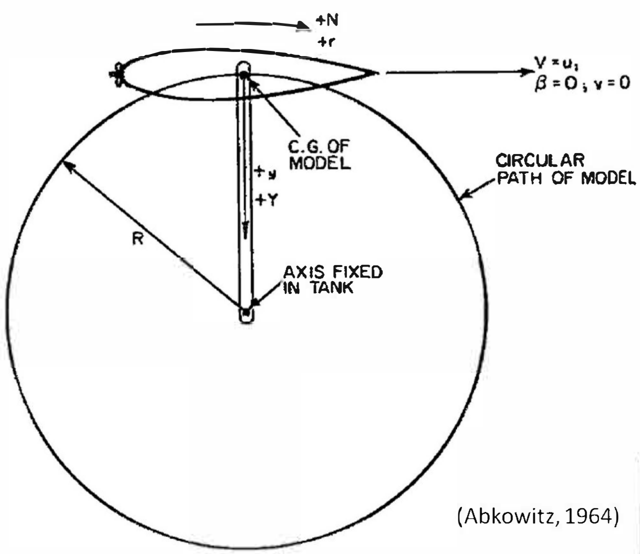

Figure 7.11: Model in Rotating-Arm Facility (from reference 1)

Some disadvantages of rotating-arm tests:

1. Require a specialized facility of substantial size. (There are only a few rotating-arm facilities in the world. One is at the David Taylor Research Center in Carderock, MD. Another was at the Davidson Laboratory at Stevens Institute of Technology.)
2. The model must be accelerated and data obtained within a single revolution. Otherwise the model will be running in its own wake and its velocity with respect to the fluid will not be accurately known.

3. In order to obtain values of the derivatives Y_r, N_r, Y_v, and N_v at $r = 0$, data at small values of r are necessary. This means that the ratio of the radius of turn, R to the model length L must be large.

7.3.3 Planar Motion Mechanism (PMM) Technique

To avoid the large expense of a rotating-arm facility, a device known as a Planar Motion Mechanism (PMM) was developed for use in the conventional long and narrow towing tank to measure the velocity-dependent and acceleration derivatives.

Basically the PMM consists of two oscillators, one of which produces a transverse oscillation at the bow and the other produces a transverse oscillation at the stern while the model moves down the towing tank at a constant forward velocity, U_0 (measured along the centerline of the tank). Figure 7.12 shows a sample model in a PMM. The forces required from each oscillator are recorded along with the transverse position of the model at each oscillator. The point B near the bow is oscillated transversely with a small amplitude, a_0,

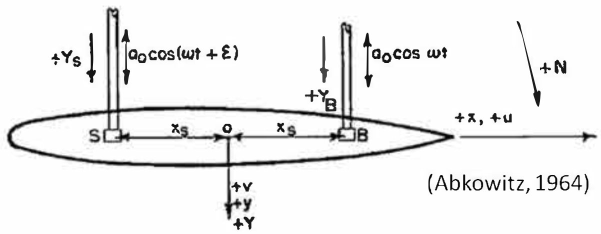

Figure 7.12: Model setup for planar motion tests

and at frequency ω. Point S near the stern is oscillated transversely with the same amplitude, a_0, and the same frequency, ω. The phase difference between the oscillations allows the model to experience yaw. If $\epsilon = 0$, the model experiences pure sway with zero yaw, as shown in Figure 7.13. For a pure sway test, the model is moving transversely in a sinusoidal

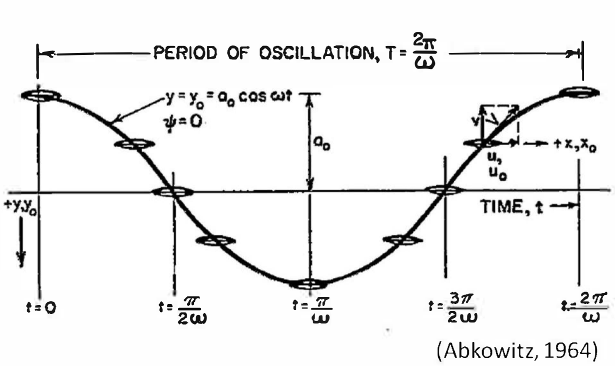

Figure 7.13: Path and orientation of model for pure sway motion

motion. The sway velocity and acceleration can be found by taking the time derivatives of the position.

$$\begin{aligned} y &= a_0 \sin \omega t \\ \frac{dy}{dt} &= v = \omega a_0 \cos \omega t \\ \frac{d^2 y}{dt^2} &= \dot{v} = -\omega^2 a_0 \sin \omega t \end{aligned}$$

Therefore, the magnitude of the velocity and acceleration is given by

$$\begin{aligned} v &= a_0 \omega \\ \dot{v} &= \omega^2 a_0 \end{aligned}$$

Each oscillator measures the Y-forces experienced by the model as a result of the swaying motion (Y_B and Y_S). To find the Y_v derivative, we need to consider the Y-force in-phase with the velocity (or 90° out-of-phase with the position). To get the magnitude of the Y-force in-phase with the velocity we need to do a FFT of the signal (YIPPEE! I hear you cry ☺). This time, however, we will find the sine and cosine components of the signal, rather than the total magnitude. Once we have the components in-phase with the velocity (the cosine components) we can find the derivative Y_v by plotting the Y_{vel} term versus the sway velocity.

$$Y_{vel} = Y_{B_{\cos}} + Y_{S_{\cos}}$$

For the yaw moment derivative, a similar procedure can be applied. In this case, the sway force at each oscillator must be multiplied by a distance to get the moment. The distance, x_s, is typically chosen as measured from ⊠ (and each point B and S must be equidistant from ⊠). This means the hydrodynamic derivative N_v can be determined from plotting the cosine component of the yaw moment versus the sway velocity.

$$N_{vel} = (Y_{B_{\cos}} - Y_{S_{\cos}}) x_s$$

The components of the sway force and yaw moment that are in-phase with the acceleration are the sine components. Therefore, the derivatives $Y_{\dot{v}}$ and $N_{\dot{v}}$ are found by plotting the Y_{acc} and N_{acc} versus the sway acceleration $\dot{v}$.

$$\begin{aligned} Y_{acc} &= Y_{B_{\sin}} + Y_{S_{\sin}} \\ N_{acc} &= (Y_{B_{\sin}} - Y_{S_{\sin}}) x_s \end{aligned}$$

To obtain the angular derivatives Y_r and N_r from planar motion tests, the measurements must be made when $\dot{r} = 0$, $v = 0$, and $\dot{v} = 0$. Similarly, for $Y_{\dot{r}}$ and $N_{\dot{r}}$, the measurements need to be taken when $r = 0$, $v = 0$, and $\dot{v} = 0$. To impose an angular velocity and an angular acceleration on a body with v and $\dot{v}$ equal to zero, the model must be towed down the tank with the centerline of the model always tangent to its path, see Figure 7.14. This means the sway velocity (relative to the model) is always zero. To obtain pure yaw motion using the two oscillators in the PMM, the phase angle, ϵ, must be equal to

$$\tan \epsilon / 2 = \frac{\omega x_s}{U}.$$

(Abkowitz, 1954 and Gertler, 1959)

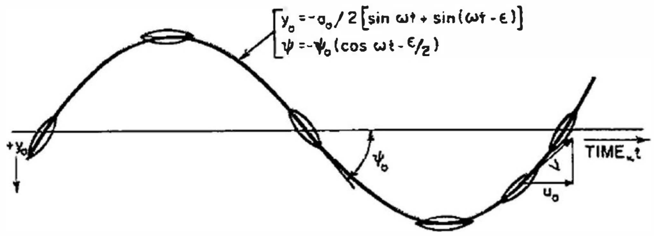

Figure 7.14: Path and orientation of model for pure yaw motion

The yaw oscillation is a sinusoidal motion and of the form

$$\begin{aligned} \psi &= -\psi_0 \sin(\omega t - \epsilon/2) \\ r = \dot{\psi} &= -\omega\psi_0 \cos(\omega t - \epsilon/2) \\ \dot{r} = \ddot{\psi} &= \omega^2\psi_0 \sin(\omega t - \epsilon/2) \end{aligned}$$

The yaw velocity, r is out-of-phase with the angle ψ and the angular accleration $\dot{r}$ is in-phase with the angle ψ. Therefore, the amplitudes of Y_B and Y_S measured 90° out-of-phase with ψ will determine the force and moment due to rotation r and the amplitudes of Y_B and Y_S in-phase with the ψ will determine the forces and moment due to angular acceleration $\dot{r}$.

$$\begin{aligned} Y_{angvel} &= Y_{B_{\cos}} + Y_{S_{\cos}} \\ N_{angvel} &= (Y_{B_{\cos}} - Y_{S_{\cos}})x_s \\ Y_{angacc} &= Y_{B_{\sin}} + Y_{S_{\sin}} \\ N_{angacc} &= (Y_{B_{\sin}} - Y_{S_{\sin}})x_s \end{aligned}$$

Plotting these forces versus velocity and acceleration can provide the yaw derivatives. The slope of Y_{angvel} versus r gives $(Y_r + m_\Delta U)$, the slope of N_{angvel} versus r gives N_r, the slope of Y_{angacc} versus $\dot{r}$ gives $Y_{\dot{r}}$, and the slope of N_{angacc} versus $\dot{r}$ gives $(N_{\dot{r}} - I_z)$.

7.4 Directional Stability

Now that we have experimental values for our hydrodynamic derivatives, we can solve the linear sway and yaw equations of motion. Solutions to the linear sway and yaw equations provide linear transfer functions permitting review of the stability of motion.

There are various kinds of motion stability associated with ships and they are classified by the attributes of their initial state of equilibrium that are retained in the final path of their centers of gravity. For example, consider Figure 7.15.

In each of the cases, the ship is assumed to be traveling at a constant speed along a straight path.

1. For case I – Straight Line Stability: the final path after the disturbance is finished retains the straight-line attribute of the initial state of equilibrium, but not its direction.

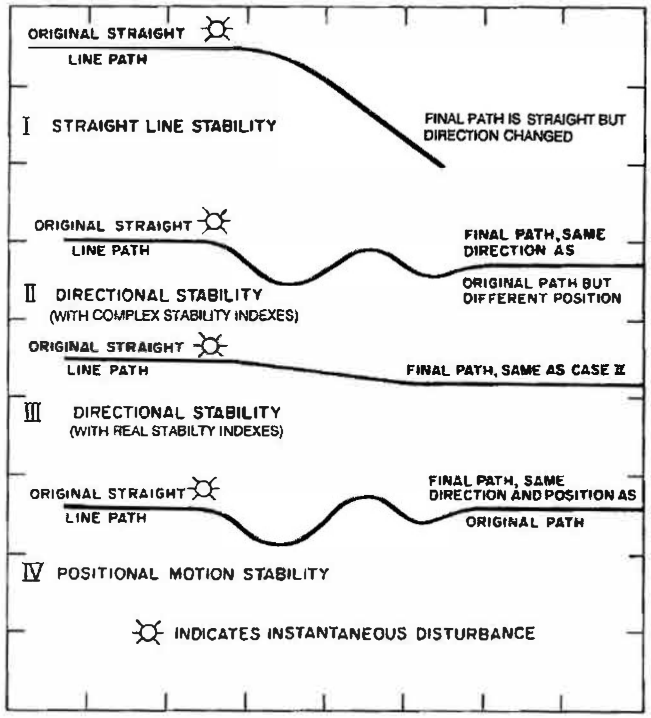

Figure 7.15: Various kinds of motion stability (PNA III, Arentzen 1960)

2. For case II – Directional Stability: the final path after the disturbance is finished retains not only the straight-line attribute of the initial path, but also its direction.

3. For case III - Directional Stability: the result is the same as for Case II, but without the oscillations.

4. For case IV – Positional Motion Stability: the ship returns to the original path - not only does the final path have the same direction as the original path, but also its same transverse position relative to the surface of the earth.

When operating with **controls-fixed** in the horizontal plane in the open ocean with stern propulsion, a surface ship does not have directional stability (i.e. if disturbed from its original course it will not return to that course by itself). However, the ship can have **Straight-Line Stability** (i.e. if disturbed from its original straight-line course, the ship will settle on a final path that is also a straight line).

When operating with **controls working** you can achieve directional stability. You want the ship to have directional stability with controls working, but also to have straight-line stability with controls fixed. This results in a compromise between rudder size and deadwood size.

We will start by using the ***linear equations of motion*** to evaluate the *straight-line stability* characteristics of a ship.

- We want to understand the effect of ship design features on maneuverability.

- With the rudder fixed on the centerline, we want the ship to have *straight-line stability*, but just barely.
- This will reduce the size of the rudder and steering gear needed for good maneuverability.

The simultaneous solution of the sway and yaw equations for the sway and yaw velocities yields a second-order differential equation. Working with non-dimensional variables, the solutions for v' and r' correspond to the standard solutions of second-order differential equations:

$$v' = V_1 e^{\sigma_1 t} + V_2 e^{\sigma_2 t}$$
$$r' = R_1 e^{\sigma_1 t} + R_2 e^{\sigma_2 t}$$

The variables V_1, V_2, R_1, and R_2 are constants of integration and σ_1 and σ_2 are the **stability indexes**. If both values of σ are negative, v' and r' will approach zero with increasing time which means that the path of the ship will eventually resume a new straight-line direction. If either σ_1 or σ_2 are positive, v' and r' will increase with increasing time and a straight-line path will never be resumed. We can relate these stability indexes, σ, to the hydrodynamic derivatives by substituting the solutions back into the equations of motion. If this is done, a quadratic equation in σ is obtained:

$$A\sigma^2 + B\sigma + C = 0$$

A, B, and C are as follows:

$$A = (Y'_{\dot{v}} - m'_{\Delta})(N'_{\dot{r}} - I'_z) - Y'_{\dot{r}} N'_{\dot{v}}$$
$$B = Y'_v(N'_{\dot{r}} - I'_z) + N'r(Y'_{\dot{v}} - m'_{\Delta}) - N'_{\dot{v}}(Y'_r + m'_{\Delta}) - Y'_{\dot{r}} N'_v$$
$$C = Y'_v N'_r - N'_v(Y'_r + m'_{\Delta})$$

The two roots, both of which must be negative for ***controls-fixed stability*** are:

$$\sigma_{1,2} = \frac{-B/A \pm [(B/A)^2 - 4C/A]^{1/2}}{2}$$

For both stability roots to be negative (all changes with respect to time are decreasing), two conditions must be met:

$$\frac{B}{A} > 0$$
$$\frac{C}{A} > 0$$

- For conventional ships A is *large* and *positive.*
- It can be shown that B is usually *large* and *positive* and on the same order of magnitude as A.
- This means that the **determining factor will be** C.

For both stability roots to be negative, $C > 0$! Therefore,

$$C = Y_v' N_r' - N_v'(Y_r' + m_\Delta') > 0$$

Rewriting we can say,

$$\frac{N_r'}{Y_r' + m_\Delta'} - \frac{N_v'}{Y_v'} > 0.$$

We can calculate the directional straight-line stability after having performed the PMM tests on a model, but what can we say generally about controls-fixed straight-line stability from what we know about the hydrodynamic derivatives?

The terms N_r' and Y_v' are always negative, and generally large relative to Y_r' and N_v'. If the bow is dominate (the usual condition), Y_r' and N_v' are negative. So, in a conventional craft, the ration $\frac{N_v'}{Y_v'}$ will be small and since $\frac{N_r'}{Y_r'+m_\Delta'}$ is likely to be larger, the ship *will have* directional stability. For a conventional hull (where the bow dominates), directional stability can be increased by increasing the magnitude of Y_v' and N_r'. Adding a larger rudder in the stern of the ship increases the directional stability of the ship by decreasing the magnitudes of Y_r' and N_v'.

7.5 Analysis of Turning Ability

The response of the ship to deflection of the rudder, and the resulting forces and moments produced by the rudder, can be divided into 2 portions:

1. An initial transient one in which significant surge, sway and yaw accelerations occur

2. A steady turning portion in which rate of turn and forward speed are constant and the path of the ship is circular

Figure 7.16 shows the turning path of a ship. Generally, the turning path of a ship is characterized by four numerical measures: **advance**, **transfer**, **tactical diameter**, and **steady turning diameter**. All but the last are related to heading positions of the ship rather than tangents to the turning path. The *advance* is the distance from the origin at "execute" to the x-axis of the ship when that axis has turned 90°. The *transfer* is the distance from the original approach course to the origin of the ship when the x-axis has turned 90°. The *tactical diameter* is the distance from the approach course to the x-axis of the ship when that axis has turned 180°. These parameters of a ship's turning circle are useful for characterizing maneuvers in the open sea.

7.5.1 The Three Phases of a Turn

Phase I:
The first phase starts the instant the rudder begins to deflect and may be completed by the time the rudder reaches full deflection. The rudder force ($Y_\delta \delta_R$) and the rudder moment ($N_\delta \delta_R$) produce accelerations and are opposed solely by the inertial reaction of the ship (hydrodynamic responses have not yet materialized). For this phase the ship has not changed

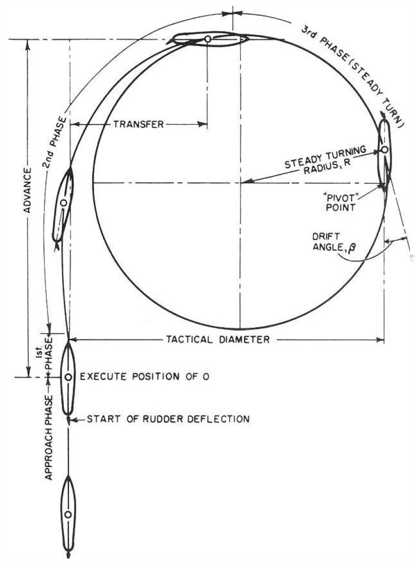

Figure 7.16: Turning Path of a Ship

direction, so $\beta = v/U = r = 0$. The linearized, dimensional equations for the first phase of turning are

$$(m_\Delta - Y_{\dot{v}})\dot{v} - Y_{\dot{r}}\dot{r} = Y_\delta \delta_R$$
$$(I_z - N_{\dot{r}})\dot{r} - N_{\dot{v}}\dot{v} = N_\delta \delta_R$$

These accelerations ($\dot{v}$ and $\dot{r}$) exist only for a moment, for they quickly give rise to a drift angle, β, and a rotation, r, of the ship.

Phase II:
The second phase starts with the introduction of the drift angle, β, and a rotation, r, of the ship. Here the accelerations of the ship *coexist* with the velocities and all the terms of the equations of motion along with the excitation terms $Y_\delta \delta_R$ and $N_\delta \delta_R$ are fully operative. The crucial event at the beginning of the second phase of the turn is the **creation of a $Y_v v$-force positively directed towards the center of the turn**. This force is introduced due to the drift angle, β. The magnitude of this force soon becomes larger than the $Y_\delta \delta_R$-force which is directed to the outside of the circle. The acceleration $\dot{v}$ ceases to grow to the outside of the circle and eventually becomes zero as the inwardly directed $Y_v v$-force comes into balance with the outwardly directed force of the ship. In the second phase of the turn, the path of the center of gravity of the ship at first responds to the $Y_\delta \delta_R$-force and tends towards the outside of the circle before the $Y_v v$-force grows large enough to enforce the inward turn.

Phase III:
Finally, after some oscillation (some of which is due to the settling down of the main propulsion machinery and is characteristic of the particular type of machinery and its control system) the second phase of turning ends with the establishment of the final equilibrium of forces. When this equilibrium is reached, the ship settles down to a turn of constant radius. This is the third, or steady, phase of the turn. In this phase v and r have non-zero values,

but $\dot{v}$ and $\dot{r}$ are zero. For this phase of the turn, the linearized equations of motion are:

$$-Y_v v - (Y_r + m_\Delta U)r = Y_\delta \delta_R$$
$$-N_v v - N_r r = N_\delta \delta_R$$

These two simultaneous equations can be solved for r and v provided that the stability derivatives (Y_v, Y_r, N_v, and N_r) and the control derivatives (Y_δ and N_δ) are known. Note that

$$r' = \frac{rL}{U} \qquad r = \frac{U}{R} \qquad r' = \frac{L}{R}$$

Solving the non-dimensional version of the linearized equations of motion shown above, we can solve for the turning radius, R, and the sway velocity, v':

$$\frac{R}{L} = -\frac{1}{\delta_R}\left[\frac{Y_v' N_r' - N_v'(Y_r' + m_\Delta')}{Y_v' N_\delta' - N_v' Y_\delta'}\right]$$

$$v' = -\beta = \delta_R\left[\frac{N_\delta'(Y_r' + m_\Delta') - Y_\delta' N_r'}{Y_v' N_r' - N_v'(Y_r' + m_\Delta')}\right]$$

A positive R denotes a starboard turn. The equation for the turn radius shows

- The steady turning radius is proportional to ship length and inversely proportional to rudder angle.
- Side velocity is equal to the drift angle and that is directly proportional to the rudder angle.
- Denominator in the equation for R introduces the effect of the rudder on the hull (N_δ' and Y_δ)
  - Sign of denominator is always positive
  - If the numerator is negative (straight-line stability) and the rudder is at the stern, a negative δ_R will give a positive R.

To **decrease the turning radius** we can:

1. Decrease Y_v' - could increase L/T ratio, but this is de-stabilizing
2. Generally increase N_v' (if N_v' is negative) - this is a result of different bow and stern shapes. Changes could be made by cutting away skeg and deadwood aft or increasing forefoot forward.
3. Increase N_δ' (obvious choice) - the trick is to do it without increasing $1/\delta$ too much. Want to move the rudder as far aft as possible and make the rudder as efficient as possible.
4. Increase Y_δ' (only if N_v' is negative) - can do this with a larger and/or more efficient rudder.

7.6 Rudder Design Considerations

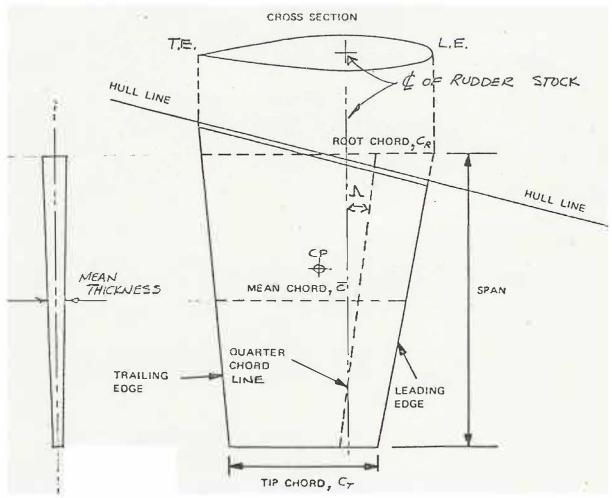

Figure 7.17: Rudder Definitions

7.6.1 Rudder Definitions

Figure 7.18 shows some important dimensions on a standard spade rudder.

- **Mean Span** - average of leading and trailing edge spans
- **Mean Chord** - average of the root and tip chords
- **Profile Area** - product of mean span and mean chord
- **Aspect Ratio** - ratio of mean span to the mean chord
- **Taper Ratio** - ratio of the tip chord to the root chord
- **Sweepback Angle** - angle between 1/4 chord line and vertical
- **Mean Thickness** - average of the max thickness of the foil at the root and tip

7.6.2 Lift, Drag and Angle of Attack

The lift (L) from an airfoil is defined as the component of force perpendicular to the free-stream velocity vector. The drag (D) from an airfoil is defined as the component of force parallel to the free-stream velocity.

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c}$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 c}$$

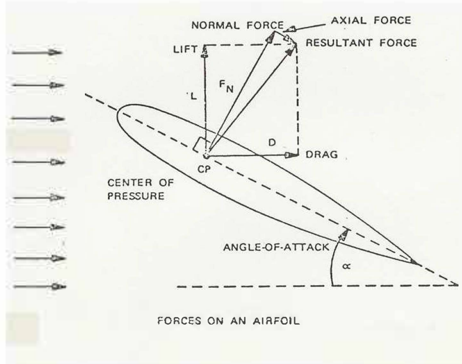

Figure 7.18: Forces on an Airfoil

The lift increases with increasing angle of attack. However, the lift cannot increase indefinitely with angle of attack. Eventually the adverse pressure gradient causes separation over the entire upper surface of the foil, resulting in a loss of lift. The maximum obtainable lift coefficient is called $C_{L,max}$.

- rudder stall often precedes a broach

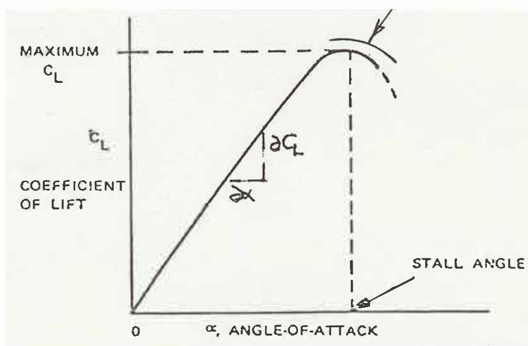

Figure 7.19: Lift Curve

7.6.3 Constraints on Rudder Design

In profile, the rudder needs to fit within dimensions dictated by the hull shape.

- The span is limited by the vessel draft.
  - It shouldn't extend below the baseline
  - It shouldn't penetrate the water surface
- The chord is limited by propeller clearance and stern shape.
  - The usual distance between the propeller and the rudder is 0.2·propeller diameter

The rudder should be designed for minimum drag at all speeds.

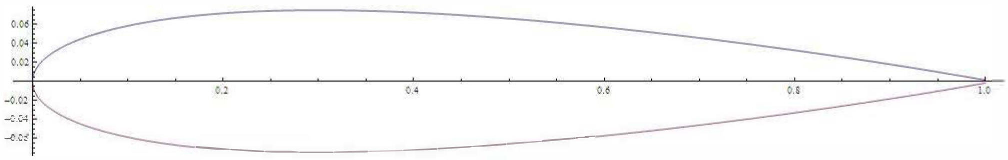

Figure 7.20: Typical Rudder Section

- The usual section shape is NACA 0015 (see Figure 7.20) to 0021 (relatively thick). These foils have a relatively constant center of pressure and thick sections are better structurally.
  - thickness-to-chord ratio is 0.15 to 0.21
  - symmetric shape
  - relatively low drag
  - max thickness at 30% chord length
- High aspect ratio
  - α = span/chord
  - very good lift-to-drag ratio

The rudder, rudder stock, rudder support and steering engine are considered together.

- Minimize size and weight of steering equipment
  - keep rudder weight as small as possible
  - keep torque on rudder stock as small as possible
    - balanced rudder - allows for smaller stock
    - semi-balanced rudder - support vs. moment
- Keep equipment as simple as possible
  - reduced repairs
  - simplifies layout

Undesirable effects of the rudder on the ship should be kept to a tolerable level (i.e. rudder induced vibration). From a hydrodynamic perspective, the basic considerations in rudder design can be summarized as follows:

- Full form ships need larger rudders
- Large rudders provide superior performance
- Put the rudder in the propeller wake to improve efficiency
- High aspect ratios give better efficiency

Table 31—General vessel hull form coefficients

Vessel Type	Typical Form Coefficients and Ratios C_B	L/B	B/T	Speed V, knots	Froude No. $V/\sqrt{gL}$	Number of Propellers/ Rudders	Rudder Area Ratios[a]	Dynamic Course Stability[b]
Harbor tug	0.50	3.3	2.1	10	0.25	1/1	0.025	S
Tuna seiner	0.50	5.5	2.4	16	0.31	1/1	0.025	S
Car ferry	0.55	5.1	4.5	20	0.34	2/2	0.020	S[e]
Container high speed	0.55	8.3	3.0	28.5	0.53	2/2	0.015	S
Container high speed	0.55	8.3	3.0	28.5	0.53	2/1	0.025	S[c]
Cargo liners	0.58	6.9	2.4	21	0.29	1/1	0.015	S
RO/RO	0.59	6.9	3.0	22	0.26	1/1	0.015	S
Barge carrier	0.64	7.5	2.9	19	0.20	1/1	0.015	S
Container Med. Speed	0.70	7.1	2.8	22	0.25	1/1	0.015	S
Offshore supply	0.71	4.7	2.75	13	0.28	2/2	0.016	S[d,e]
General cargo low speed	0.73	6.7	2.4	15	0.20	1/1	0.015	S
Lumber low speed	0.77	6.7	2.6	15	0.20	1/1	0.025	S
LNG (125 000 m^3)	0.78	6.8	3.7	20	0.20	1/1	0.015	U
OBO (Panamax)	0.82	7.5	2.4	16	0.17	1/1	0.018	U
OBO (150 000 dwt)	0.85	6.4	2.4	15	0.15	1/1	0.017	U
OBO (300 000 dwt)	0.84	6.0	2.5	15	0.14	1/1	0.015	U
Tanker (Panamax)	0.83	7.1	2.4	15	0.16	1/1	0.015	U
Tanker 100 000 to 350 000 dwt	0.84	6.2	2.4	16	0.15	1/1	0.015	U
Tanker 350 000 dwt	0.86	5.7	2.8	16	0.13	1/1	0.015	U
U.S. river towboat	0.65	3.5	4.5	10	0.25	2/2	...	U[d,e]

[a] Not for design guidance.
[b] U = unstable course stability; S = stable course stability.
[c] Although the vessel is directionally stable, maneuvering is difficult at low speeds when the propeller wash is not effective over the rudder.
[d] Maneuverability is good owing to installation of Kort nozzles, flanking rudders, and other capabilities.
[e] Twin screw because of restricted draft.

Figure 7.21: From PNA III (1989), p.346

  - limited by hull shape (span by draft; chord by stern shape)

- Rate of swing
  - increased rate of swing is good for small ships
  - large ships benefit more from rudder area than from swing rate

A good first estimate of rudder area can be achieved using the 1975 Det Norske Veritas (DNV) Rules.

$$A_R = \frac{T \times L_{BP}}{100}[1 + 25(\frac{B}{L_{BP}})^2] \tag{7.1}$$

The formula only applies for single rudders operating in a propeller wake. For all other arrangements DNV requires a 30% increase in area. (You want to put rudders behind propellers to increase the flow over the rudder at low speeds - makes the rudder more effective).

The equation gives (essentially) a rudder area coefficient. It is useful to compare values from the equation with values used in industry (see Figures 7.21 and 7.22). In choosing a design, the rudder performance is more affected by span length than chord length. An increase in the aspect ratio increases the lift/drag ratio.

Seakeeping Notation

Dynamics Review		Added Mass & Damping	
m	mass	DFT	Discrete Fourier Transform
b	damping coefficient	FFT	Fast Fourier Transform
c	stiffness coefficient	a	added mass
x	position	T	period (sec)
$\dot{x}$	velocity	f	frequency (Hz)
$\ddot{x}$	acceleration	D	draft
F_0	force amplitude	T_n	natural period (sec)
X	motion amplitude	**Regular Waves**	
ω	wave frequency (rad/s)	ζ_0	wave amplitude
ω_e	excitation (or encounter) frequency (rad/s)	c	wave celerity
ϕ	phase angle	u_g	group velocity
Λ	Tuning Factor, ω_e/ω_n	k	wave number
η	Damping Factor, $b/[2(m+a)\omega_n]$	α_0	wave slope
$X/(F_0/c)$	Magnification Factor	d	water depth
ω_n	natural frequency (rad/sec)	λ	wavelength
Dynamic Ballasting		**DDG-51 in Head Seas**	
I_5	pitch mass moment of inertia	ω_{n3}	heave natural frequency (rad/sec)
k_5	pitch gyradius	ω_{n5}	pitch natural frequency (rad/sec)
I_6	yaw mass moment of inertia	X_3/ζ_0	Heave Transfer Function
k_6	yaw gyradius	X_5/α_0	Pitch Transfer Function
R	model scale ratio	U	ship speed
Irregular Waves			
ϵ	bandwidth parameter	$S_\zeta(\omega)$	wave energy spectra

Subscripts

1	Surge (linear)
2	Sway (linear)
3	Heave (linear)
4	Roll (rotational)
5	Pitch (rotational)
6	Yaw (rotational)
M	model
S	ship
e	excitation or encounter
n	natural (as in natural frequency)

Maneuvering Notation

Reference Frame Maneuvering Notation

$\boldsymbol{x_{0G}}$	CG x-position from Earth Reference Frame
$\boldsymbol{y_{0G}}$	CG y-position from Earth Reference Frame
$\boldsymbol{\beta}$	drift angle
$\boldsymbol{\psi}$	yaw angle
$\boldsymbol{\ddot{x}_{0G}}$	CG x-direction acceleration from Earth Reference Frame
$\boldsymbol{\ddot{y}_{0G}}$	CG y-direction acceleration from Earth Reference Frame
$\boldsymbol{\dot{\psi}}$	angular yaw velocity
$\boldsymbol{r}$	angular yaw velocity
$\boldsymbol{\ddot{\psi}}$	angular yaw acceleration
$\boldsymbol{\dot{r}}$	angular yaw acceleration
$\boldsymbol{x}$	direction through ship bow
$\boldsymbol{y}$	direction to ship starboard
$\boldsymbol{u}$	velocity in x-direction
$\boldsymbol{v}$	velocity in y-direction
$\boldsymbol{\dot{u}}$	acceleration in x-direction
$\boldsymbol{\dot{v}}$	acceleration in y-direction
$\boldsymbol{X_0}$	Total force in Earth x-direction
$\boldsymbol{Y_0}$	Total force in Earth y-direction
$\boldsymbol{N}$	Moment in yaw
$\boldsymbol{I_z}$	yaw mass moment of inertia

SNAME (1952) Derivative and Non-dimensional Notation

$\boldsymbol{X_u}$	$\frac{\partial X}{\partial u}$		$\boldsymbol{X_{\dot{u}}}$	$\frac{\partial X}{\partial \dot{u}}$
$\boldsymbol{Y_v}$	$\frac{\partial Y}{\partial v}$		$\boldsymbol{Y_{\dot{v}}}$	$\frac{\partial Y}{\partial \dot{v}}$
$\boldsymbol{N_v}$	$\frac{\partial N}{\partial v}$		$\boldsymbol{N_{\dot{v}}}$	$\frac{\partial N}{\partial \dot{v}}$
$\boldsymbol{Y_r}$	$\frac{\partial Y}{\partial r}$		$\boldsymbol{Y_{\dot{r}}}$	$\frac{\partial Y}{\partial \dot{r}}$
$\boldsymbol{N_r}$	$\frac{\partial N}{\partial r}$		$\boldsymbol{N_{\dot{r}}}$	$\frac{\partial N}{\partial \dot{r}}$
$\boldsymbol{m'_\Delta}$	$\frac{m_\Delta}{1/2\rho L^3}$		$\boldsymbol{I'_z}$	$\frac{I_z}{1/2\rho L^5}$
$\boldsymbol{v'}$	$\frac{v}{U}$		$\boldsymbol{r'}$	$\frac{rL}{U}$
$\boldsymbol{\dot{v}'}$	$\frac{\dot{v}L}{U^2}$		$\boldsymbol{\dot{r}'}$	$\frac{\dot{r}L^2}{U^2}$
$\boldsymbol{Y'_v}$	$\frac{Y_v}{1/2\rho L^2 U}$		$\boldsymbol{Y'_r}$	$\frac{Y_r}{1/2\rho L^3 U}$
$\boldsymbol{N'_v}$	$\frac{N_v}{1/2\rho L^3 U}$		$\boldsymbol{N'_r}$	$\frac{N_r}{1/2\rho L^4 U}$
$\boldsymbol{Y'_{\dot{v}}}$	$\frac{Y_{\dot{v}}}{1/2\rho L^3}$		$\boldsymbol{Y'_{\dot{r}}}$	$\frac{Y_{\dot{r}}}{1/2\rho L^4}$
$\boldsymbol{N'_{\dot{v}}}$	$\frac{N_{\dot{v}}}{1/2\rho L^4}$		$\boldsymbol{N'_{\dot{r}}}$	$\frac{N_{\dot{r}}}{1/2\rho L^5}$