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RESISTANCE & POWERING OF SHIPS

Main Category:	Naval Engineering
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Course #:	NAV-116
Course Content:	46 pgs
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OFFICIAL COURSE/EXAM

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NAV-116 EXAM PREVIEW

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Exam Preview:

1. Ordinarily in design, the Delivered Horsepower is estimated first, and then efficiencies are assumed for each portion of the drivetrain to estimate the required Brake Horsepower to be installed.
    a. True
    b. False
2. Using The Ship Drive Train section of the reference material, which of the following "types" of horsepower matches the description: is the power required to move the ship's hull at a given speed in the absence of propeller action?
    a. Brake Horsepower
    b. Shaft Horsepower
    c. Delivered Horsepower
    d. Effective Horsepower
3. According to the reference material, "Hull Efficiency" includes the interaction between the hull and the propeller, which varies with ship type.
    a. True
    b. False
4. The largest losses in the system are the thermodynamic and mechanical losses in the engines, which cause the loss of roughly __% of the fuel energy before it becomes rotational power at the output of the engine
    a. 35
    b. 50
    c. 60
    d. 70

5. Propeller pitch (P) is the ideal linear distance perpendicular to the direction of motion that would be traveled in one revolution of the propeller shaft.
   a. True
   b. False
6. A ship's fuel consumption curve is similar in shape to its horsepower and total resistance curves. The U.S. Navy generally requires that ships arrive with no less than __ percent fuel onboard as a reserve.
   a. 25
   b. 30
   c. 40
   d. 50
7. Using Figure 7.4 Power Curve of effective horsepower for a Navy YP, which of the following horsepower values most closely corresponds to a ship travelling at 12 knots?
   a. 200
   b. 300
   c. 400
   d. 500
8. According to the reference material, increasing the length of a ship, and reducing beam for a given speed tend to reduce the viscous resistance coefficient; however this increases wetted surface area.
   a. True
   b. False
9. Using Figure 7.15 Wave Patterns vs. Speed, which of the following wavelengths corresponds to the condition: worst speed to operate at?
   a. Wavelength = 2/3 Ship Length
   b. Wavelength = Ship Length
   c. Wavelength = 1.5 Ship Length
   d. Wavelength >> Ship Length
10. According to the reference material, resistance due to air is typically 4-8% of the total ship resistance but may be as much as __% in high sided ships such as aircraft carriers.
    a. 10
    b. 15
    c. 20
    d. 25

COURSE OBJECTIVES
CHAPTER 7

7. RESISTANCE AND POWERING OF SHIPS

1. Define effective horsepower (EHP) conceptually and mathematically

2. State the relationship between velocity and total resistance, and velocity and effective horsepower

3. Write an equation for total hull resistance as a sum of viscous resistance, wave making resistance and correlation resistance. Explain each of these resistive terms.

4. Draw and explain the flow of water around a moving ship showing the laminar flow region, turbulent flow region, and separated flow region

5. Draw the transverse and longitudinal wave patterns when a displacement ship moves through the water

6. Define Reynolds number with a mathematical formula and explain each parameter in the Reynolds equation with units

7. Be qualitatively familiar with the following sources of ship resistance:
    a. Steering Resistance
    b. Air and Wind Resistance
    c. Added Resistance due to Waves
    d. Increased Resistance in Shallow Water

8. Read and interpret a ship resistance curve including humps and hollows

9. State the importance of naval architecture modeling for the resistance on the ship's hull

10. Define geometric and dynamic similarity

11. Write the relationships for geometric scale factor in terms of length ratios, speed ratios, wetted surface area ratios and volume ratios

12. Describe the law of comparison (Froude's law of corresponding speeds) conceptually and mathematically, and state its importance in model testing

13. Qualitatively describe the effects of length and bulbous bows on ship resistance

14. Be familiar with the momentum theory of propeller action and how it can be used to describe how a propeller creates thrust

15. Define Coefficient of Thrust and Thrust Loading

16. Know the relationship between thrust loading and propeller efficiency

17. Define the following terms associated with the screw propeller:
    a. Diameter
    b. Pitch
    c. Fixed Pitch
    d. Controllable Pitch
    e. Reversible Pitch
    f. Right Handed Screw
    g. Left Handed Screw
    h. Pressure Face
    i. Suction Face
    j. Leading Edge
    k. Trailing Edge
    l. Blade Tip
    m. Root
    n. Variable Pitch

18. Be familiar with cavitation including the following:
    a. The relationship between thrust loading and cavitation
    b. The typical blade locations where cavitation occurs
    c. Spot Cavitation
    d. Sheet Cavitation
    e. Blade Tip Cavitation
    f. Operator action to avoid cavitation
    g. The effect of depth on cavitation
    h. The difference between cavitation and ventilation

7.1 Introduction to Ship Resistance and Powering

One of the most important considerations for a naval architect is the powering requirement for a ship. Once the hull form has been decided upon, it is necessary to determine the amount of engine power that will enable the ship to meet its operational requirements. Knowing the power required to propel a ship enables the naval architect to select a propulsion plant, determine the amount of fuel storage required, and refine the ship's center of gravity estimate.

Throughout history, naval architects have endeavored to increase the speed of ships. Increased speed enable a warship to close with its opponent, or conversely, to escape from an attack. Increased speed enables merchant vessels to reach port sooner and maximize profit for its owner.

Until the early 1800's, wind was the force used to propel ships through the water and ships could only go as fast as the wind would propel them. Additionally, because ships were constructed of wood, the structural limitations of wooden hull configurations drove hull designs to primarily meet the structural needs while hydrodynamics was only a secondary concern. With the advent of steam propulsion in the early 1800's, naval architects realized that ship speeds were no longer constrained by the wind and research began into the power required to propel a hull through the water using this new propulsion medium.

Testing of full-scale ships and models determined that the power required to propel a ship through the water was directly related to the amount of resistance a hull experiences when moving through the water.

The development of iron hull construction produced radical changes in hull strength and hull design. Gone were the blunt bows and full hull forms of early sailing vessels. Capitalizing on the added strength of iron hulls, naval architects could design ships with finer bows and as a result, ship speeds increased.

About the time of the Civil War, the modern screw propeller was developed, replacing the paddle wheel as the prime mode of ship propulsion. The screw propeller, with many modifications to its original design, remains the principle method of ship propulsion to this day.

This chapter will investigate the differing forms of hull resistance, ship power transmission, and the screw propeller. Additionally, we will investigate ship modeling and how full-scale ship resistance and performance can be predicted using models in a towing tank.

7.2 The Ship Drive Train

The purpose of the propulsion system on a ship is to convert fuel energy into useful thrust to propel the ship. Figure 7.1 shows a simplified picture of a ship's drive train.

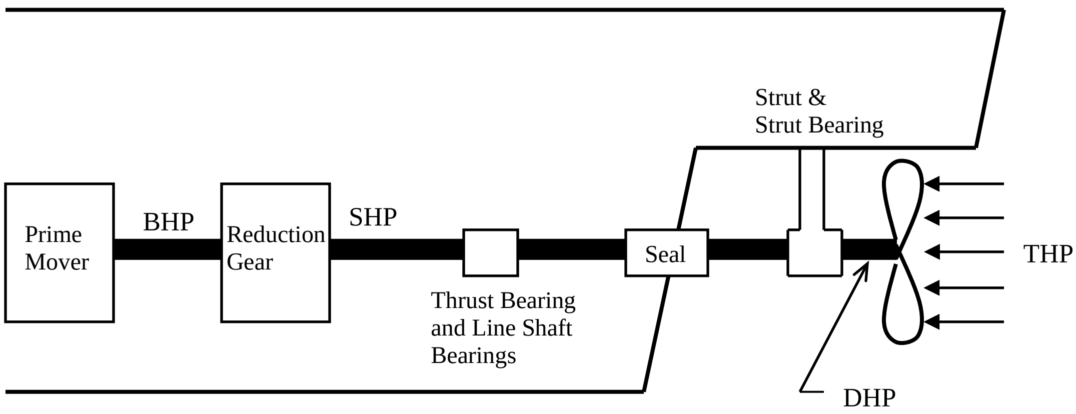


Figure 7.1 Simplified ship drive train

BHP – "Brake Horsepower" is the power output of the engine. It is called "brake" because engines are tested by applying a mechanical load to the shaft using a brake. The power of a rotating engine is the product of the torque (ft-lb) and the rotational speed (with suitable unit corrections).

SHP – "Shaft Horsepower" is equal to the Brake Horsepower minus any mechanical losses in the reduction gear. The reduction gear reduces the RPM (revolutions per minute) of the engine to an efficient propeller speed, such as reducing from a few thousand RPM for gas turbines to a few hundred RPM for a warship. Reduction gears are very large, heavy, and expensive.

DHP – "Delivered Horsepower" is the power delivered to the propeller, which includes the losses due to the gearbox, the bearings and the stern tube seal. The delivered horsepower is usually 95%-98% of the Brake Horsepower, depending on the propulsion system.

The propeller converts the rotational power into useful thrust. **THP – "Thrust Horsepower"** is the power from the propeller thrust, equal to the product of the speed of advance and the thrust generated by the propeller (with suitable unit conversions). This power includes the losses of the gearbox, shafting, and propeller.

EHP – "Effective Horsepower" is the power required to move the ship's hull at a given speed in the absence of propeller action. It is equal to the product of the resistance of a ship and the speed of the ship. This power is equal to the Brake Horsepower minus losses due to the gearbox, shafting and propeller, as well as interaction between the propeller and the hull.

Ordinarily in design, the Effective Horsepower is estimated first, and then efficiencies are assumed for each portion of the drivetrain to estimate the required Brake Horsepower to be installed.

Figure 7.2 shows a diagram of the energy losses in a typical shipboard propulsion system. The largest losses in the system are the thermodynamic and mechanical losses in the engines, which cause the loss of roughly 60% of the fuel energy before it becomes rotational power at the output of the engine (Brake Horsepower). This huge loss is why engineers study thermodynamics and also why mechanical engineers continually strive for more fuel efficient engines.

Following this are the losses in the gearbox, shafting and propellers, resulting in only one-quarter of the original fuel energy being converted to useful thrust energy to move the ship forward. The main areas that the Naval Engineer can control is the hull form to minimize the Effective Horsepower required to propel the ship, as well as the design of the propeller to minimize propeller losses.

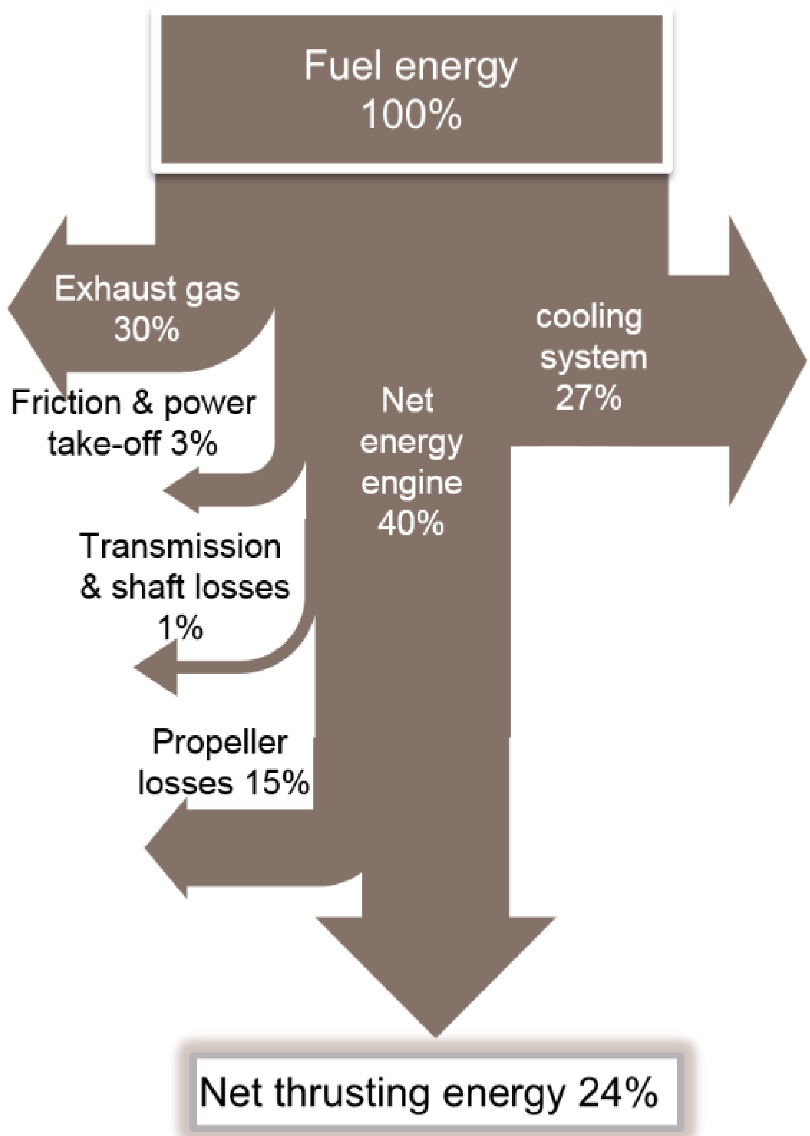


Figure 7.2 Typical Energy Losses in Shipboard Propulsion System
(Courtesy of John Gallagher, MTU engines)

7.3 Propulsive Efficiency

Figure 7.3 shows a block diagram of a ship's drive train, starting with the Brake Horsepower from the prime mover, and ending with the Effective Horsepower to drive the ship.

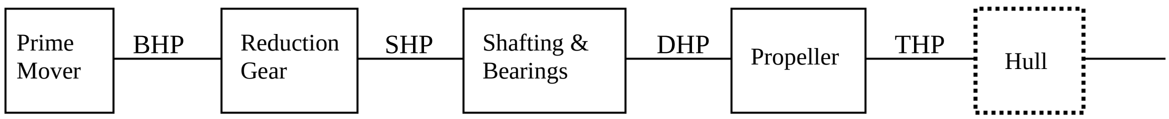


Figure 7.3 Block diagram of a ship's drive train

There are losses at each stage of the drivetrain, listed below:

Gear Efficiency $\eta_{gear} = \dfrac{SHP}{BHP} \approx 0.95 - 0.99$

Shaft Efficiency $\eta_{shaft} = \dfrac{DHP}{SHP} \approx 0.97 - 0.99$

Propeller Efficiency $\eta_{propeller} = \dfrac{THP}{DHP} \approx 0.65 - 0.75$

Hull Efficiency $\eta_{hull} = \dfrac{EHP}{THP}$

The gear, shaft and propeller efficiencies are all mechanical or fluid losses. "Hull Efficiency" includes the interaction between the hull and the propeller, which varies with ship type. Instead of having to deduce the effect of all the separate efficiencies of each component in the drive train, the separate efficiencies are often combined into a single efficiency called the *propulsive efficiency* (η_P) or propulsive coefficient (PC).

$$\eta_P = PC = \frac{EHP}{SHP} = \eta_{gear}\eta_{shaft}\eta_{propeller}\eta_{hull}$$

The propulsive efficiency is the ratio of effective horsepower to shaft horsepower, therefore allowing the designer to make a direct determination of the shaft horsepower required to be installed in the ship. Common values of propulsive efficiency typically range from 55% to 75%.

Example 7.1 Model testing has determined that a ship has an EHP of 30,000 HP at a speed of 19 knots. Assuming a propulsive efficiency of 70%, what SHP is required to be installed to achieve 19 knots?

$$\eta_P = \frac{EHP}{SHP}$$

$$SHP = \frac{EHP}{\eta_P} = \frac{30{,}000HP}{0.70}$$

$$SHP = 42{,}860HP$$

A total of 42,860 horsepower (43,000 HP) should be installed to achieve a speed of 19 knots.

Once a value of shaft horsepower has been determined, various combinations of prime movers can be considered based on power produced, weight, fuel consumption, etc for installation in the ship.

A “prime mover” is another term for an engine or motor (i.e. a source of mechanical power). Common prime mover types include steam turbines, gas turbines engines, diesel engines, and electric motors.

7.4 Effective Horsepower (EHP)

Shaft horsepower and brake horsepower are quantities that are purchased from the engine manufacturer. Likewise, the amount of thrust a propeller can produce is a product of analysis and calculation. However, the naval architect must still determine the amount of power (BHP or SHP) actually required to propel the ship through the water. The amount of power is determined through the concept of *Effective Horsepower* (EHP). Effective horsepower is defined as:

"The horsepower required to move the ship's hull at a given speed in the absence of propeller action."

Effective horsepower is often determined through model data obtained from towing tank experimentation. In these experiments, a hull model is towed through the water at a given speed while measuring the amount of force resisting the hull's movement through the water. Model resistance data can then be scaled up to full-scale ship resistance. Knowing a ship's total hull resistance and its speed through the water, the ship's effective horsepower can be determined using the following equation:

$$EHP = \frac{R_T V}{550 \frac{ft-lb}{\sec - HP}}$$

where: EHP is the effective horsepower (HP)
R_T is the total hull resistance (lb)
V_S is the ship's speed (ft/sec)

Model testing is carried out over the expected speed range of the ship with resistance data collected at each testing speed. Effective horsepower is then calculated and plotted as shown in Figure 7.4. It will be observed from the figure that the doubling of speed of the Navy YP from 7 to 14 knots increases the power by a factor of 10! Speed and power are not linearly related.

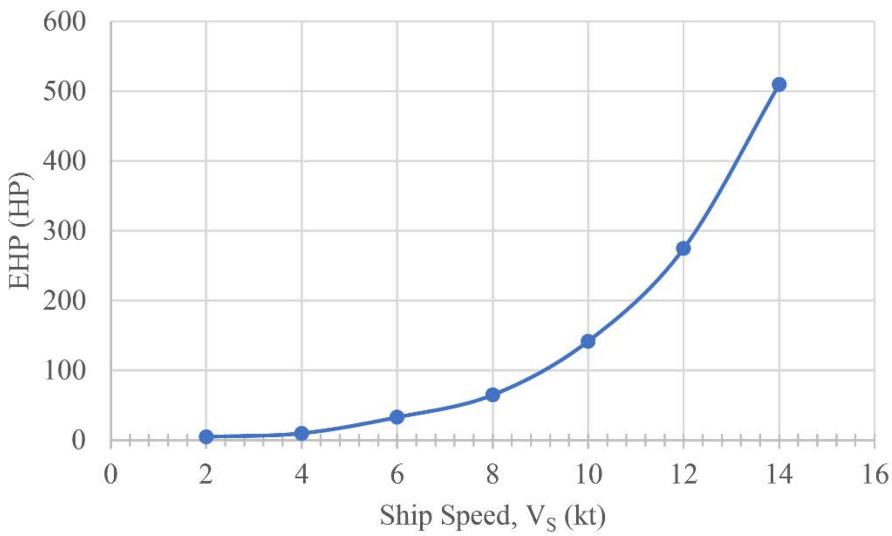


Figure 7.4 Power Curve of effective horsepower for a Navy YP

7.5 Total Hull Resistance (R_T)

As a ship moves through calm water, the ship experiences a force acting opposite to its direction of motion. This force is the water's resistance to the motion of the ship, which is referred to as "total hull resistance" (R_T). It is this resistance force that is used to calculate a ship's effective horsepower. A ship's calm water resistance is a function of many factors, including ship speed, hull form (draft, beam, length, wetted surface area), and water temperature.

Total hull resistance increases as speed increases as shown in Figure 7.5. Note that the resistance curve is not linear, but increases more steeply at higher speeds. In later sections of this chapter we will investigate why resistance increases so rapidly at high speeds. Also shown in Figure 7.5 is a bump, or "hump", in the total resistance curve. This hump is not a mistake, but a phenomenon common to nearly all ship resistance curves that will be discussed later. As shown in previous sections, the power required to propel a ship through the water is the product of total hull resistance and ship speed, and so engine power increases even more rapidly than resistance. Often, ship power is roughly proportional to the cube of the speed, so doubling (2x) the speed of a destroyer from 15 knots to 30 knots will require $2^3 = 8$ times as much power!

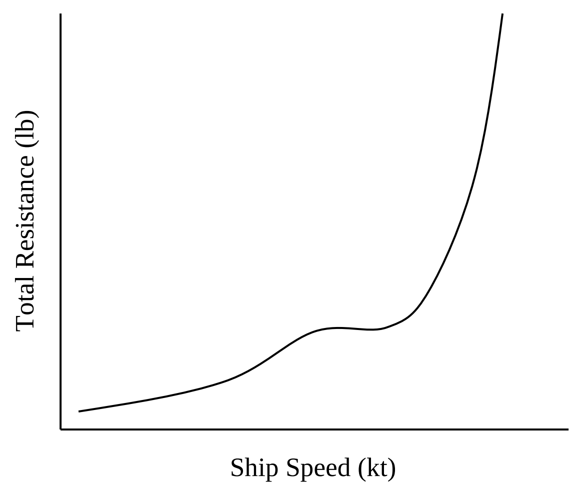


Figure 7.5 Typical curve of total hull resistance

For the ship operator planning a voyage, getting from Point A to Point B in a shortest amount of time (high speed) requires a lot more power than traveling the same distance at a slower speed. This increase in power is felt directly in the amount of fuel burned during the transit. A ship's fuel consumption curve is similar in shape to its horsepower and total resistance curves. Voyage planning requires careful attention to transit speed and fuel consumption rates to ensure that the ship arrives at its destination with an adequate supply of fuel onboard. The U.S. Navy generally requires that ships arrive with no less than 50 percent fuel onboard as a reserve.

7.6 Components of Total Hull Resistance

As a ship moves through calm water, there are many factors that combine to form the total resistance force acting on the hull. The principle factors affecting ship resistance are the friction and viscous effects of water acting on the hull, the energy required to create and maintain the ship's characteristic bow and stern waves, and the resistance that air provides to ship motion. In mathematical terms, total resistance can be written as:

$$R_T = R_V + R_W + R_{AA}$$

Where:
R_T = total hull resistance
R_V = viscous (friction) resistance
R_W = wave making resistance
R_{AA} = air resistance caused by ship moving through calm air

Other factors affecting total hull resistance will also be presented. Figure 7.6 shows how the magnitude of each component of resistance varies with ship speed. At low speeds viscous resistance dominates, and at high speeds the total resistance curve turns upward dramatically as wave making resistance begins to dominate.

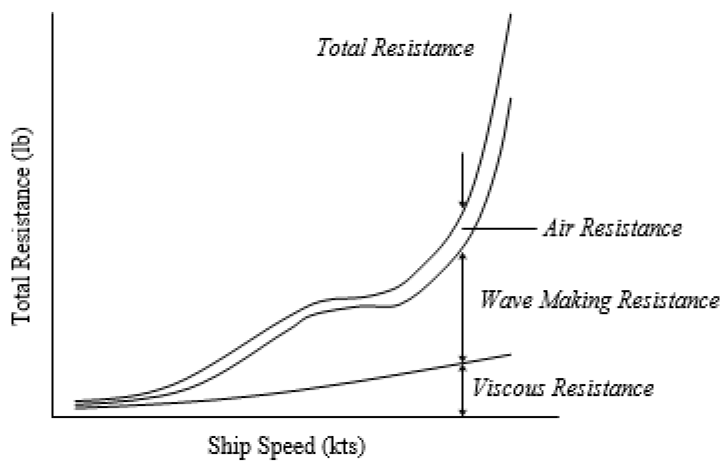


Figure 7.6 Components of Hull Resistance

7.6.1 Dimensionless Coefficients

Naval architects, as well as all engineers and scientists, use dimensionless coefficients to describe the performance of a system or to compare different systems to each other. Automotive engineers use a "drag coefficient" to describe the performance of a car. Aviators use the "Mach number" to compare the speed of an aircraft to the speed of sound. Naval architects use many dimensionless coefficients to describe the design and performance of a ship's hull.

Dimensionless coefficients allow the naval architect to compare model test data to full-scale ship data, or to compare the performance of several ship types. The field of ship resistance and propulsion makes extensive use of standard dimensionless coefficients. The derivation of these standard coefficients is accomplished through dimensional analysis. Dimensional analysis is beyond the scope of this text, however, you can learn about dimensional analysis from any text on fluid mechanics or from Volume 2 of "Principles of Naval Architecture" published by the Society of Naval Architects and Marine Engineers.

Just as total hull resistance is the sum of viscous, wave making, and air resistance, we can write an equation for total resistance in terms of dimensionless coefficients.

$$C_T = C_V + C_W$$

Where: C_T = coefficient of total hull resistance
C_V = coefficient of viscous resistance
C_W = coefficient of wave making resistance

Note that air resistance is not represented in dimensionless form. This dimensionless form of resistance is a product of model testing, and most models do not have superstructures.

Since total hull resistance is a function of hull form, ship speed, and water properties, the coefficient of total hull resistance is also a function of hull form, ship speed, and water properties. The coefficient of total hull resistance is found from the following equation:

$$C_T = \frac{R_T}{\frac{1}{2}\rho \cdot S \cdot V^2}$$

Where: R_T = total hull resistance (lb)
ρ = water density (lb-s^2/ft^4)
S = wetted surface area of the underwater hull (ft^2)
V = velocity (ft/s)

Naval architects also use a dimensionless form of velocity called the "*Froude number*" (F_n), named in honor of William Froude (1810-1878), one of the pioneers in ship model testing.

$$F_n = \frac{V}{\sqrt{gL}}$$

where: V = velocity (ft/s)
g = acceleration of gravity (ft/s^2)
L = length of ship or model (ft) (L_{PP}, LBP, or LWL)

Another common, although not dimensionless, way of expressing velocity is through the *speed-to-length* ratio. This ratio is similar to the Froude number except that the gravity term is omitted. This is a dimensional value and ***must use speed in knots***, and length in feet.

$$\text{speed-to-length ratio} = \frac{V_S}{\sqrt{L_S}}$$

7.6.2 Viscous Resistance (R_V)

Figure 7.7 shows a body submerged in an ideal (inviscid) fluid. As the fluid flows around the body, there is a pressure distribution normal to the body. In the forward section of the hull there is a component of pressure resisting motion, and in the aft section of the body there is a component of pressure assisting motion. In an ideal fluid these pressure forces are equal and the body experiences no resistance.

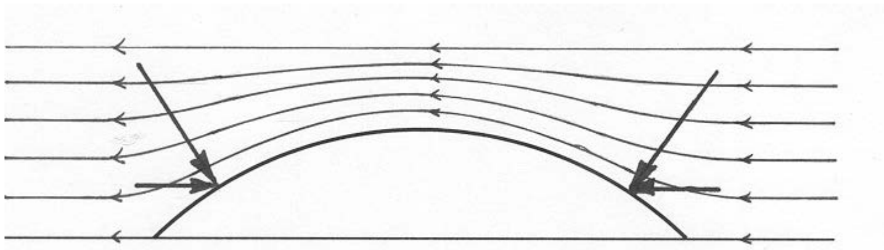


Figure 7.7 Ideal flow around a submerged body

Unfortunately, water is not an ideal fluid, and therefore the body will experience resistance. Figure 7.8 shows a hull submerged in a real fluid with viscosity. Fluid particles cling to the body, resulting in the formation of a "boundary layer," where the flow rapidly changes speed, from zero speed at the side of the body, to the free-stream speed. Two forms of resistance happen as a result of viscosity **Friction Resistance** and **Viscous Pressure Resistance**. Friction arises from the shear stresses in the fluid and acts tangential to the body. Viscous pressure resistance acts normal to the body.

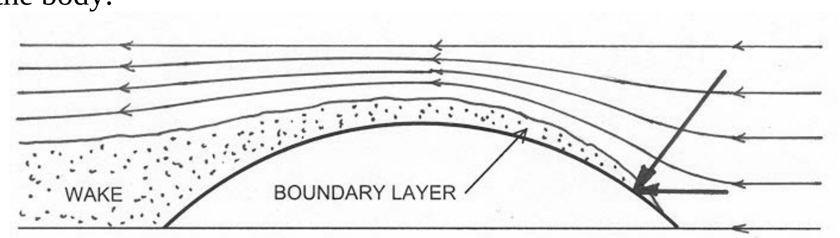


Figure 7.8 Flow around a body submerged in water

7.6.2.1 Friction Resistance

As a ship moves through the water, the friction of the water acting over the entire wetted surface of the hull causes a net force opposing the ship's motion. This frictional resistance is a function of the hull's wetted surface area, surface roughness, and water viscosity. Viscosity is a temperature dependent property of a fluid that describes its resistance to flow. Syrup is said to be a very viscous liquid; the fluid particles in syrup being very resistant to flow between adjacent particles and to other bodies. On the other hand, alcohol has a low viscosity with little interaction between particles.

Although water has low viscosity, water produces a significant friction force opposing ship motion. Experimental data have shown that water friction can account for up to 85% of a hull's total resistance at low speed ($F_n \leq 0.12$ or speed-to-length ratio less than 0.4 if ship speed is expressed in knots), and 40-50% of resistance for some ships at higher speeds.

7.6.2.2 Viscous Pressure Resistance

In the forward portion of the hull pressure forces act normal to the surface; however, in the aft portion of the hull the boundary layer reduces the forward acting component of pressure. This reduction in the forward acting component results in a net resistance force due to pressure acting on the hull. This increase in resistance due to pressure is called "viscous pressure drag" or "form drag", and is sometimes also referred to as the normal component of viscous resistance.

As you might expect, from looking at Figure 7.8, the shape of a ship's hull can influence the magnitude of viscous pressure drag. Ships that are short in length with wide beams (a low length to beam ratio) will have greater form drag than those with a larger length to beam ratio. Also, ships that are fuller near the bow (e.g. bulk oil tanker) will have greater form drag than ships with fine bows (e.g. destroyer).

7.6.2.3 Laminar and Turbulent Flow

The flow of fluid around a body can be divided into two general types of flow: laminar flow and turbulent flow. The extent of the viscous resistance on a body depends on the type of flow it is experiencing. A typical flow pattern around a ship's hull showing laminar and turbulent flow is shown in Figure 7.9

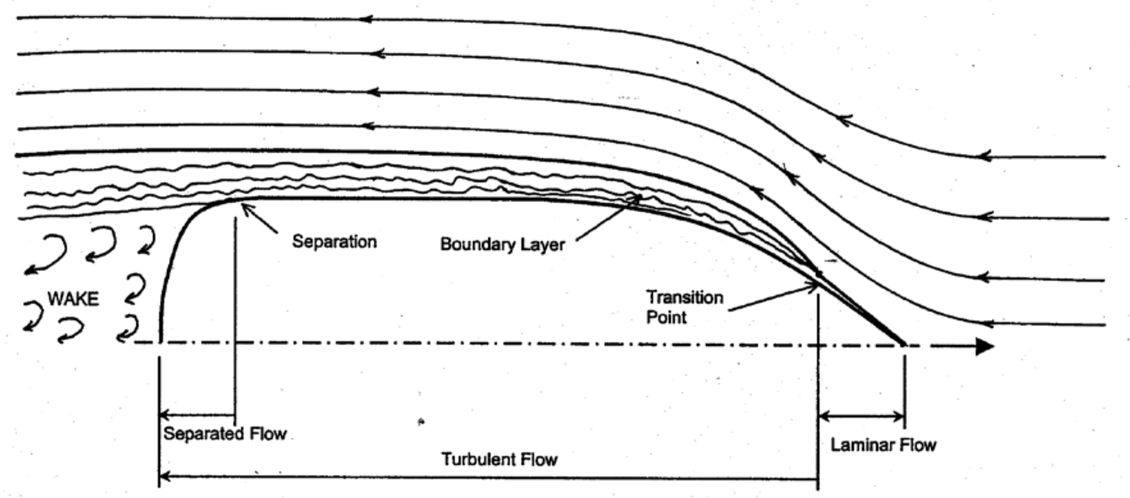


Figure 7.9 Typical water flow pattern around a ship's hull

Laminar flow is characterized by fluid flowing along smooth lines in an orderly fashion with a minimal amount of frictional resistance. For a typical ship, laminar flow exists for only a very small distance along the hull. As water flows along the hull, the laminar flow begins to break down and become chaotic and well mixed. This chaotic behavior is referred to as turbulent flow and the transition from laminar to turbulent flow occurs at the transition point shown in Figure 7.9.

Turbulent flow is characterized by the development of a layer of water along the hull moving with the ship along its direction of travel. This layer of water is referred to as the "boundary layer." Water molecules closest to the ship are carried along with the ship at the ship's velocity.

Moving away from the hull, the velocity of water particles in the boundary layer becomes less, until at the outer edge of the boundary layer velocity is nearly that of the surrounding ocean. Formation of the boundary layer begins at the transition point and the thickness of the boundary layer increases along the length of the hull as the flow becomes more and more turbulent. For a ship underway, the boundary layer can be seen as the frothy white band of water next to the hull. Careful observation of this band will reveal the turbulent nature of the boundary layer, and perhaps you can see some of the water actually moving with the ship. As ship speed increases, the thickness of the boundary layer will increase, and the transition point between laminar and turbulent flow moves closer to the bow, thereby causing an increase in frictional resistance as speed increases.

Mathematically, laminar and turbulent flow can be described using the dimensionless coefficient known as the Reynolds Number in honor of Sir Osborne Reynolds' (1883) contribution to the study of hydrodynamics. For a ship, the Reynolds Number is calculated using the equation below:

$$\mathcal{R}_n = \frac{LV}{\nu}$$

Where: $\mathcal{R}_n$ is the Reynolds number
L = length (ft) (L_{PP}, LBP, or LWL)
V = velocity (ft/sec)
ν = kinematic viscosity of water (ft^2/sec)

For external flow over flat plates (or ship hulls), typical Reynolds number magnitudes are as follows:

Laminar flow:	$\mathcal{R}_n < 5 \times 10^5$
Turbulent flow:	$\mathcal{R}_n > 1 \times 10^6$
Transition from Laminar to Turbulent Flow:	$5 \times 10^5 < \mathcal{R}_n < 1 \times 10^6$

Example 7.2 A ship 250 feet in length is traveling at 15 knots in salt water at 59°F (ν = 1.2791×10^{-5} ft^2/sec). Calculate the ship's Reynolds number at this speed.

$$\mathcal{R}_n = \frac{LV}{\nu} = \frac{(250ft)(15kt)\left(1.688\frac{ft}{s-kt}\right)}{1.2791\times10^{-5}\frac{ft^2}{s}}$$

$\mathcal{R}_n = 4.949 \times 10^8$ water flow around the ship is definitely turbulent

Example 7.3 A model 5 feet in length is being towed at a speed of 5 ft/sec in fresh water at 59°F ($\nu = 1.092 \times 10^{-5}$ ft^2/s). Calculate the model's Reynolds number.

$$\mathcal{R}_n = \frac{L \cdot V}{\nu} = \frac{(5ft)\left(5\frac{ft}{s}\right)}{1.092\times10^{-5}\frac{ft^2}{s}}$$

$\mathcal{R}_n = 2.29 \times 10^6$ the model is also operating in the turbulent regime

Note: Ships have turbulent flow over nearly their entire length except when operating at very low speeds. Even at low speeds, laminar flow is present for only one or two feet of the hull length.

7.6.2.4 Coefficient of Viscous Resistance (Cv)

The dimensionless form of viscous resistance is the coefficient of viscous resistance (C_V). This coefficient is a function of the same properties that influence viscous resistance itself: hull form, speed, and water properties. The equations for the coefficient of viscous resistance that follow are empirical products of many years of towing tank testing, and are internationally recognized by the International Towing Tank Conference (ITTC). The coefficient of viscous resistance takes into account the friction of the water on the ship as well as the influence of hull form on viscous pressure drag.

$$C_V = C_F + K \cdot C_F$$

where: C_V = coefficient of viscous resistance
C_F = tangential (skin friction) component of viscous resistance
$K \cdot C_F$ = normal (viscous pressure drag) component of viscous resistance

The skin friction coefficient (equation below) is based on the assumption that the hull is a flat plate moving through the water, and is a function of Reynolds number (ship speed, length, and water properties). The form factor (K) accounts for the effect of hull form on viscous resistance. Both the skin friction coefficient and the form factor equation are empirically derived from many tests on flat plates and ships.

$$C_F = \frac{0.075}{[(\log_{10} \mathcal{R}_n) - 2]^2}, \quad \mathcal{R}_n = \frac{LV}{\nu}$$

$$K \approx 19\left(\frac{\nabla}{LWL \cdot B \cdot T} \times \frac{B}{LWL}\right)^2$$

7.6.2.5 Reducing the Viscous Resistance

The viscous resistance of a ship is:

$$R_V = C_V \tfrac{1}{2} \rho V^2 S$$

where: C_V = coefficient of viscous resistance
ρ = water density ($lb\text{-}s^2/ft^4$)
V = velocity (ft/s)
S = wetted surface area of the underwater hull (ft^2)

Ships are often designed to carry a certain amount of payload (weight and volume) at a given speed. Therefore, the means of reducing Viscous Resistance for a design is to reduce the coefficient of viscous resistance or to reduce the surface area for a given volume. A sphere has the smallest wetted surface area per unit volume, but it would be expected to have a lot of separation and a high form factor, K, also it would create a lot of waves at the surface.

Increasing the length of a ship, and reducing beam for a given speed tend to reduce the viscous resistance <u>coefficient</u>; however this increases wetted surface area. Thus, the design of a ship is a trade-off between a sphere (minimal wetted area) and a toothpick (minimum viscous coefficient), with suitable concerns for stability and seakeeping added in.

7.6.3 Wave Making Resistance (R_W)

The second major component of hull resistance is the resistance due to wave making. The creation of waves requires energy. As ship speed increases, the height of the waves produced by the ship increases and therefore the energy required to produce these waves also increases. This lost energy is referred to as wave making resistance and often becomes a limiting factor in the speed of a ship.

An object moving through the water creates both divergent waves, which spread outward from the ship, and transverse waves, illustrated in Figure 7.10.

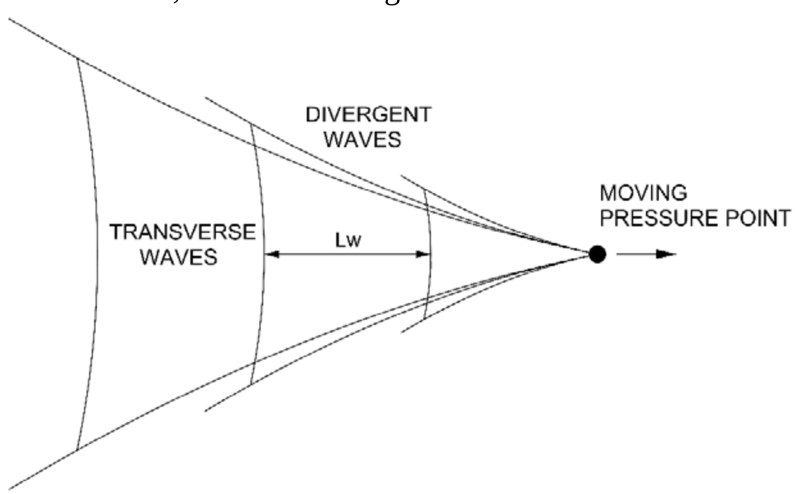


Figure 7.10 Wave pattern generated by a moving object in the water

Sir William Froude (1810-1878) did much of the early research in wave making resistance and his results and conclusions in this field are used to this day. Figure 7.11 is Froude's 1877 sketch of the wave patterns produced by a ship. Compare Froude's sketch to the photographs of actual ships in Figures 7.12 and 7.13 and note the similarities.

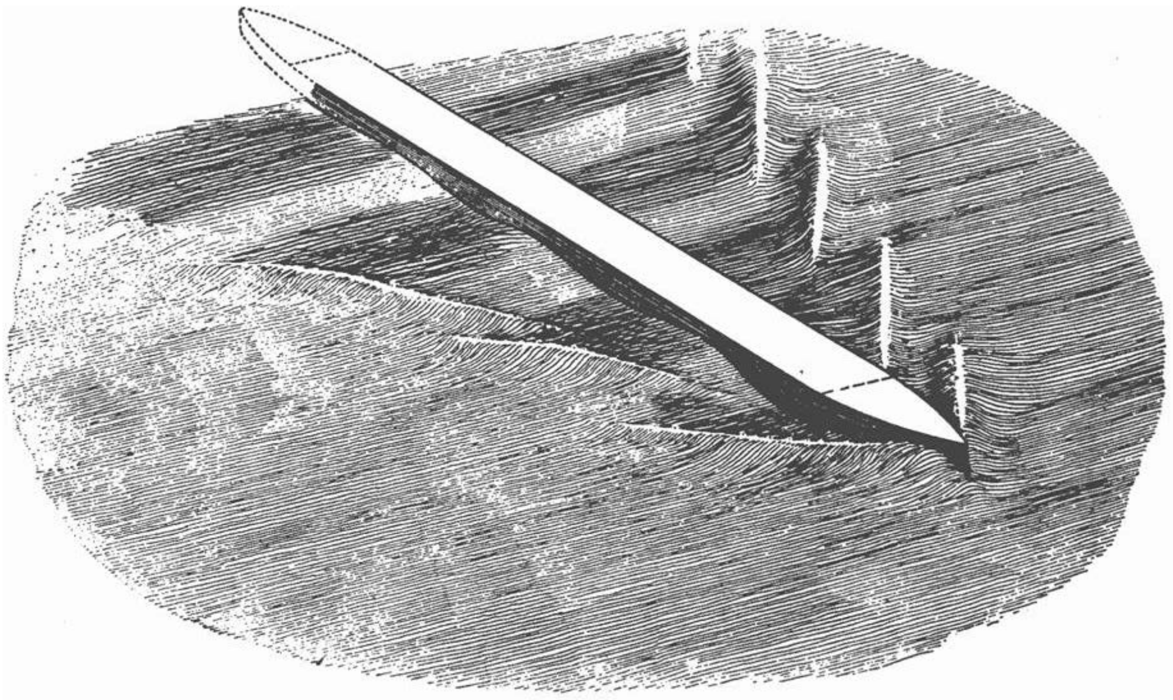


Figure 7.11 Froude's sketch of a characteristic wave train for ships. (reproduced from "Principles of Naval Architecture, Volume 2" published by the Society of Naval Architects and Marine Engineers)

Figure 7.12 USNS SPICA (left) conducting vertical replenishment with another ship. Note the divergent wave patterns emanating from SPICA. (U.S. Navy photo)

Figure 7.13 Transverse wave pattern along the hull of a replenishment ship (U.S. Navy photo)

Unlike the simple wave pattern developed by a moving pressure point (Figure 7.10), a real ship creates many wave systems, most prominently the bow and stern wave systems, shown in Figure 7.14. These wave systems can interact with each other, either partially canceling the waves made by a ship (and reducing the wavemaking resistance) or by adding and increasing the wavemaking resistance. The effects waves have on eachother waves as they collide and overlap is called constructive (adding) or destructive (reducing) interference.

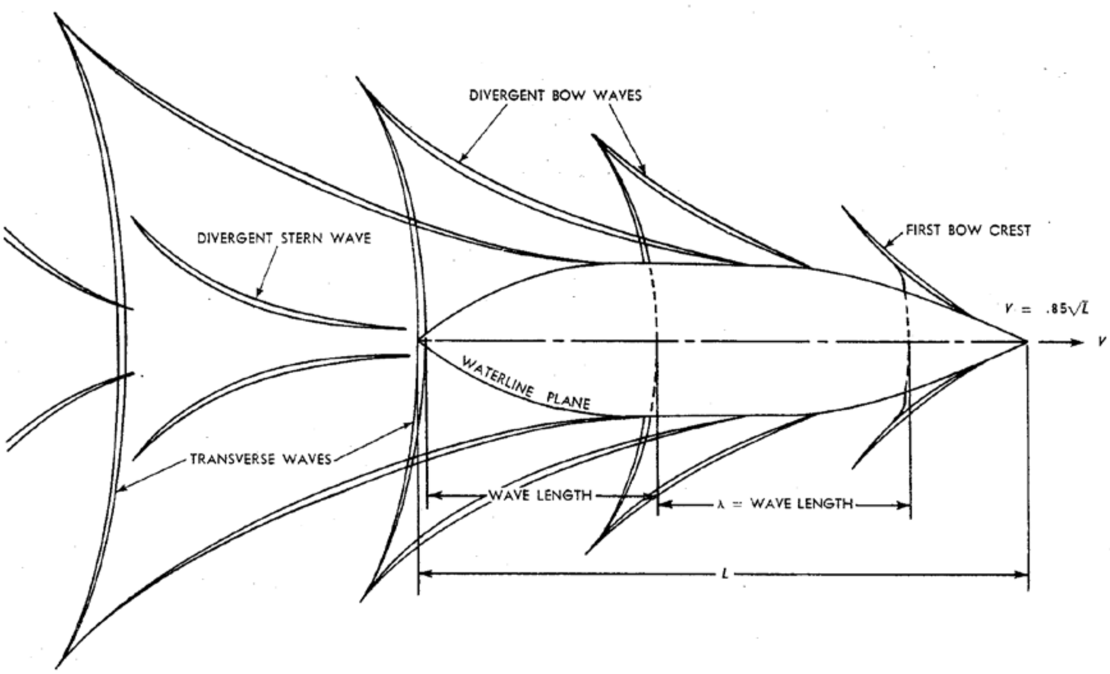


Figure 7.14 The bow and stern wave systems generated by a ship
(reproduced from "Introduction to Naval Architecture" by Gillmer and Johnson)

7.6.3.1 Wave Length versus Ship Speed

The transverse wave travels at approximately the same speed as the ship, because the ship is producing this wave. At slow speeds the transverse waves have short wave length and several crests can be seen along the ship's length. From wave theory, the length of a free wave on the surface is related to velocity as follows:

$$L_w = \frac{2\pi V^2}{g}$$

where: L_w = wave length (ft)
V = ship velocity (ft/s)
g = acceleration due to gravity (ft/s^2)

Figure 7.15 shows the appearance of the wave patterns along the side of the ship at various speeds. At low speeds, there are more wave crests on the side of the hull. At high speeds, the wavelength increases. The figure shows that at certain speeds, there is a crest at the stern, and at others, there is a trough. These crests and troughs can either partially cancel the stern wave system, or partially add to it, resulting in some speeds with higher resistance due to interference. This fact causes the plot of total resistance coefficient versus speed (Figure 7.16) to have "humps and hollows." It is best to operate the ship in a hollow for fuel economy. The figure shows a large increase in resistance at a speed-to-length ratio of 1.34. This is the speed at which the wavelength is equal to the length of the ship. It is known as "hull speed," which is the last efficient speed for a displacement ship.

The speed where wavelength equals ship length can be solved for in the above equations and is:

$$V_S = 1.34\sqrt{L_S} \quad kt/\sqrt{ft}$$

where: V_S = ship speed (knots)
L_S = ship length (ft) (LWL or L_{PP})

This is called the "hull speed rule of thumb" and units are typically not expressed, but the units are in kt·ft$^{-1/2}$.

Just above the "hull speed," the stern of the ship drops into a large wave trough, and the ship runs at a high trim angle. Anyone who has been on a planing boat will have noticed a particular speed, where the bow comes up very high. This is the worst speed to operate at in terms of efficiency and preventing hull damage. As the boat goes faster, the wavelength increases still further, and the hull begins to plane, reducing wavemaking resistance. The plot in Figure 7.16 shows the great increase in resistance that occurs above hull speed, and also its drop at even higher speeds. Ships rarely have enough power to reach these speeds.

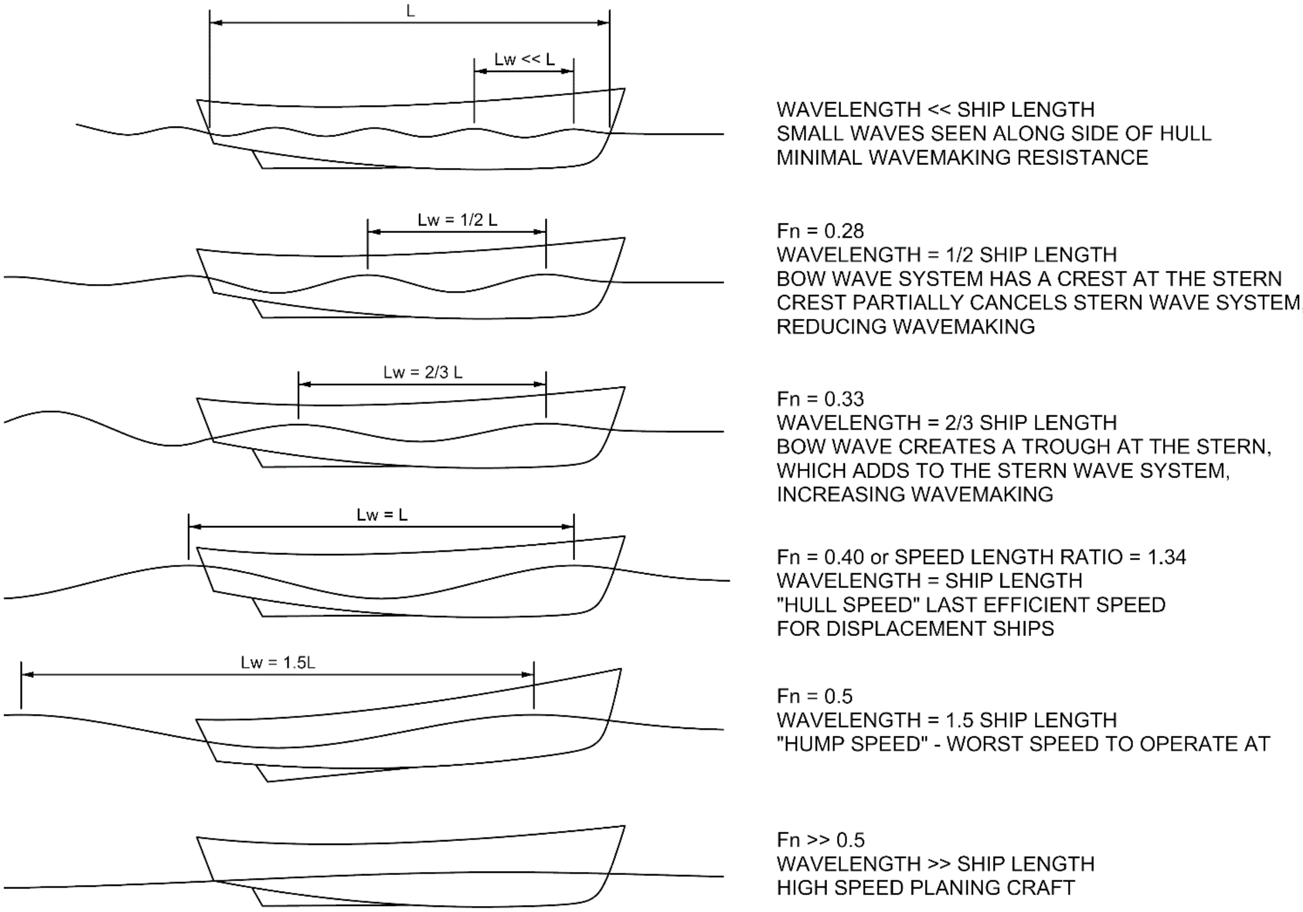


Figure 7.15 Wave Patterns vs. Speed

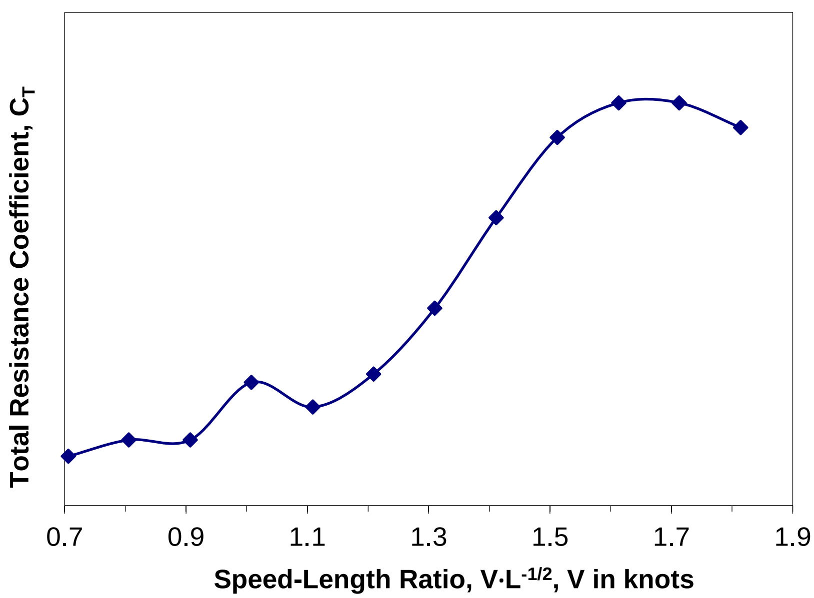


Figure 7.16 Typical relationship between C_T and speed

7.6.3.2 Understanding and Reducing Wave Resistance

Wave theory states that the energy in a wave is proportional to the square of the wave height. Since the energy in a wave depends on the square of the wave height, any increase in wave height requires a subsequent increase in energy required to create the wave and an increase in wave making resistance. Thus, if wave height doubles, a four-fold increase in energy required to create the wave occurs. Therefore, as ship speed increases and wave height increases, wave making resistance becomes dominant.

$$E_{wave} = f(H^2)$$

So how does wave making resistance affect a ship and its operation? To illustrate, consider the following example:

> The FFG-7 class ship has a waterline length of 408 ft and is powered by two gas turbine engines that produce approximately 41,000 SHP for a published maximum speed of 29 knots. At a speed of approximately 27 knots the length of the transverse wave is approximately the same length as the ship. With one gas turbine in operation (20,000 SHP), the ship is capable of speeds approaching 25 knots. It takes an additional 20,000 SHP (double the shaft horsepower) to increase speed by 4 knots! That increase in required horsepower is directly related to the effects of wave making resistance.

The question arises as how to reduce the effects of wave making resistance. In the design phase of a ship there are two things that can be done to reduce the effects of wave making, and therefore improve the performance of the ship:

- **Increasing length** of the ship increases the speed at which the length of the wave system generated by the ship is equal to ship length and therefore reduces the impact of wave making resistance.

  As noted previously, the speed at which the wave length approaches ship length for an FFG-7 (Lpp = 408 ft, Δ = 4,000 LT, rated at 41,000 SHP) is approximately 27 knots, whereas speed at which wave length approaches ship length for a NIMITZ-class carrier (L = 1090 ft, Δ = 97,000 LT, approximately 280,000 SHP) is approximately 44 knots. At the FFG's top speed of 29 knots, the aircraft carrier is still in the relatively flat portion of the resistance/SHP curve. It would be very difficult to add enough propulsion machinery to the hull (space, weight, fuel, and center of gravity concerns) to increase the FFG-7's speed to an equivalent speed for the aircraft carrier. Therefore, longer ships use proportionally smaller engines to do the same speed as ships of less length. In other words, it requires fewer horsepower per ton to make the aircraft carrier (2.9 HP/LT) to achieve 30 knots than it does to make FFG-7 (10.3 HP/LT) achieve 29 knots. Figure 7.16 compares these factors. At a speed of 29 knots, the FFG-7 has a speed-length ratio of 1.4, giving the ship a large resistance coefficient. Compare the FFG to the aircraft carrier at 30 knots and a speed-length ratio of 0.90. The aircraft carrier has a much lower resistance coefficient and therefore requires significantly less horsepower per ton of displacement to achieve the same speed as the FFG.

- **Bulbous Bows.** Bulbous bows are one attempt to reduce the wave making resistance of surface ships by reducing the size of the bow wave system. The bulbous bow was developed by RADM David Taylor and was used as early as 1907 on the battleship *USS DELAWARE*. The "ram bows" of late 1800's battle ships and even those of early Greek and Roman warships could also be considered early versions of the bulbous bow even though their bow designs were intended for other purposes. The idea behind a bulbous bow is to create a second bow wave that interferes destructively with the bow divergent wave, resulting in little to no wave at the bow. A smaller resultant bow wave improves the ship's attitude in the water by producing less squat and trim by the stern. This more evenly trimmed ship results in less projected wetted surface area (i.e., less viscous resistance) and reduces the ship's tendency to try to "climb" over its own bow wave as speed increases (i.e., delays the inception of large wave making resistance). A well-designed bulbous merchant ship bulb has been shown to reduce total resistance by up to 15%. This reduction in resistance translates into lower operating costs and higher profits for those merchant vessels that employ this design enhancement. Many warships also have bulbous bows. These bows often house the sonar transducers and keep them as far away as possible from the ship's self-radiated noise. The bulbous bows of warships offer some reduction in wave making resistance and fuel savings. However, most warships cannot take full advantage of the bulbous bow since each bulb is generally "tuned" to the expected operating speed of the ship -- an easy task for a merchant which usually operates at a constant speed between ports, but not so simple for a warship whose operations necessitate frequent speed changes.

7.6.4 Air Resistance (R_{AA})

Air resistance is the resistance caused by the flow of air over the ship with no wind present. This component of resistance is affected by the shape of the ship above the waterline, the area of the ship exposed to the air, and the ship's speed through the water. Ships with low hulls and small "sail area" or projected area above the waterline will naturally have less air resistance than ships with high hulls and large amounts of sail area. Resistance due to air is typically 4-8% of the total ship resistance, but may be as much as 10% in high sided ships such as aircraft carriers. Attempts have been made reduce air resistance by streamlining hulls and superstructures, however; the power benefits and fuel savings associated with constructing a streamlined ship tend to be overshadowed by construction costs.

7.6.5 Other Types of Resistance Not Included in Total Hull Resistance

In addition to viscous resistance, wave making resistance, and air resistance, there are several other types of resistance that will influence the total resistance experienced by the ship.

7.6.5.1 Appendage Resistance

Appendage resistance is the drag caused by all the underwater appendages such as the propeller, propeller shaft, struts, rudder, bilge keels, pit sword, and sea chests. In Naval ships appendages can account for approximately 2-14% of the total resistance. Appendages will primarily affect the viscous component of resistance as the added surface area of appendages increases the surface area of viscous friction.

7.6.5.2 Steering Resistance

Steering resistance is added resistance caused by the motion of the rudder. Every time the rudder is moved to change course, the movement of the rudder creates additional drag. Although steering resistance is generally a small component of total hull resistance in warships and merchant ships, unnecessary rudder movement can have a significant impact. Remember that resistance is directly related to the horsepower required to propel the ship. Additional horsepower is directly related to fuel consumed (more horsepower equals more fuel burned). A warship traveling at 15 knots and attempting to maintain a point station in a formation may burn up to 10% more fuel per day than a ship traveling independently at 15 knots.

7.6.5.3 Wind and Current Resistance

The environment surrounding a ship can have a significant impact on ship resistance. Wind and current are two of the biggest environmental factors affecting a ship. Wind resistance on a ship is a function of the ship's sail area, wind velocity and direction relative to the ship's direction of travel. For a ship steaming into a 20-knot wind, ship's resistance may be increased by up to 25-30%.

Ocean currents can also have a significant impact on a ship's resistance and the power required to maintain a desired speed. Steaming into a current will increase the power required to maintain speed. For instance, the Kuroshio Current (Black Current) runs from South to North off the coast of Japan and can reach a speed of 4-5 knots. What is the impact of this current? For a ship heading south in the current and desiring to travel at 15 knots it is not uncommon to have the propulsion plant producing shaft horsepower for speeds of 18-19 knots. Therefore, the prudent mariner will plan his or her voyage to avoid steaming against ocean currents whenever possible, and to steam with currents wherever possible.

7.6.5.4 Added Resistance Due to Waves

Added resistance due to waves refers to ocean waves caused by wind and storms, and is not to be confused with wave making resistance. Ocean waves cause the ship to expend energy by increasing the wetted surface area of the hull (added viscous resistance), and to expend additional energy by rolling, pitching, and heaving. This component of resistance can be very significant in high sea states.

7.6.5.5 Increased Resistance in Shallow Water

Increased resistance in shallow water (the Shallow Water Effect) is caused by several factors.

- The flow of water around the bottom of the hull is restricted in shallow water, therefore the water flowing under the hull speeds up. The faster moving water increases the viscous resistance on the hull.
- The faster moving water decreases the pressure under the hull, causing the ship to “squat”, increasing wetted surface area and increasing frictional resistance.
- The waves produced in shallow water tend to be larger than do waves produced in deep water at the same speed. Therefore, the energy required to produce these waves increases, (i.e. wave making resistance increases in shallow water). In fact, the characteristic hump in the total resistance curve will occur at a lower speed in shallow water.

The net result of traveling in shallow water is that it takes more horsepower (and fuel) to meet your required speed. Another more troublesome effect of high speed operation in shallow water is the increased possibility of running aground. One notable occurrence was in 1992 when the liner *QUEEN ELIZABETH II*, ran aground at a speed of 25 knots on a reef near Cuttyhunk Island in Massachusetts. The ship’s nominal draft was 32 ft-4 inches and the charted depth of the reef was 39 feet. The 6.5-foot increase in draft was due to the shallow water effect known as *squat*.

Just as shallow water will adversely affect a ship’s resistance, operating in a narrow waterway such as a canal can produce the same effect. Therefore when operating in a canal, the ship’s resistance will increase due to the proximity of the canal walls and the decrease in pressure along the ships sides is likely to pull the ship towards the edge of the canal. The prudent mariner is advised to operate at moderate speeds when steaming in shallow and/or narrow waters.

7.6.6 Resistance and the Operator

There is a direct correlation between a ship’s curve of total hull resistance, the EHP curve, the SHP curve, and the fuel consumption curve for a ship. What can the ship operator do to reduce the effects of viscous and wave making resistance?

- **Hull Cleaning.** The easiest method to reduce the effect of viscous resistance is to keep the hull clean and free of barnacles and underwater grasses. Section 7.6.2 indicated that frictional resistance is a function of surface roughness. Fouling of the hull can increase fuel consumption up to 15 percent. Keeping the underwater hull clean will reduce surface roughness and help minimize the effects of viscous resistance and conserve fuel. The Navy requires its ships to undergo periodic hull inspections and cleanings in order to reduce surface roughness. Ships are also periodically dry-docked and their bottoms are stripped and repainted to return the ships hull to a smooth condition.

- **Operate at a prudent speed.** To reduce the effects of wave making resistance, the operator should transit at speeds away from the humps in the resistance curve. From Figure 7.16 one such speed to avoid is when the speed-length ratio is approximately 1.0. For the FFG-7 this equates to a speed of approximately 20 knots. For best performance, operating at a speed-length ratio less than 0.9 is preferable. The standard transit speed for the US Navy is 14 knots, which puts most ships well below any point where the effects of wave making become a problem. For the FFG-7, a transit speed of 14 knots results in a speed-length ratio of approximately 0.7, well below the wave making threshold. At a speed of 14 knots, and aircraft carrier has a speed-length ratio of approximately 0.45, a speed where viscous resistance is the dominant component of resistance!

Traveling at high speeds corresponding to the humps in the C_T curve requires that the ship produce enough shaft horsepower to overcome the rapidly increasing wave making resistance. This rapidly increasing horsepower requirement means that fuel consumption will increase just as rapidly. For example, an FFG-7 with a clean hull traveling at 14 knots (speed-length ratio of 0.7) with one engine running will burn approximately 10,000 gallons of fuel per day. The same ship traveling at 29 knots (speed-length ratio of 1.4) with both engines in operation burns approximately 3,000 gallons of fuel **per hour**! Consider that the FFG-7 has a total fuel capacity of about 190,000 gallons, you can do the math and see why ships do not travel at high speeds for sustained periods.

Unlike warships whose maximum speed is determined by mission requirement, merchant ships are designed to travel at a speed corresponding to a hollow in the resistance curve. In fact, the service speed of a merchant ship is usually below the first hump speed. Most merchant ships have a service speed of approximately 15 knots, and if a length of 600 feet is assumed (speed-length ratio is 0.6), the ship is well below hump speed. Therefore less horsepower is required to propel the ship. Less horsepower equals smaller propulsion machinery, less fuel storage requirements, more cargo storage space, and therefore more savings in operating cost and more room for payload/mission systems.

7.7 Determining the Total Hull Resistance and EHP Curves

Now that you have learned about the various components of ship resistance and how ship speed, resistance, and power are related; we now need to study how the EHP curve for a ship is obtained. One of the key phases in the design process for a ship is the determination of the amount of power required to propel a ship at either its maximum speed or service speed. This is necessary so the type and size of propulsion plant can be determined. Propulsion plant size is critical to the estimate of the location of the ship's center of gravity (stability concerns), and the amount of space to be set aside to accommodate the propulsion plant. Recall from section 7.4 that a ship's effective horsepower is related to total hull resistance by the following equation:

$$EHP = \frac{R_T V_S}{550 \frac{ft-lb}{sec-HP}}$$

Therefore, to determine effective horsepower for a given speed all the naval architect needs to do is to determine the total hull resistance at that speed. This presents a problem during design, as the ship only exists on paper and/or in the computer. There are two methods of predicting resistance and EHP curves for a ship during the design process: computer modeling and traditional tow tank testing with a model of the ship.

7.7.1 Computer Modeling

This method of determining a ship's resistance curve involves modeling the ship's hull in a computer and then solving three-dimensional fluid flow equations for the flow of water around the ship's hull. These equations are solved through a method called "computational fluid dynamics" using the finite element method of analysis. This method requires a large amount of computer memory and the ability to solve thousands of simultaneous equations. Computer modeling of the hull and the flow of water around the hull produces fairly accurate results (if you have a large enough computer) and can be used to compare many different hull designs.

Computer modeling does, however, have its drawbacks. Fluid flow around a ship's hull is very complex, especially near the stern where the hull's shape changes rapidly, and in many cases the flow in this region is difficult to analyze with the computer. This often necessitates the other method of determining a ship's resistance curve: tow tank testing of a model.

7.7.2 Theory Behind Ship Modeling and Tank Testing

Tow tank testing of a ship model is the traditional method of determining a ship's total hull resistance and its EHP curves. In this method, a model of the ship's hull is built and towed in a towing tank, measuring hull resistance at various speeds. The model results are then scaled up to predict full-scale hull resistance and EHP.

In order for model test results and full-scale ship predictions to be useable, two fundamental relationships between the model and ship must be met: geometric and dynamic similarity.

7.7.2.1 Geometric Similarity

Geometric similarity is obtained when all characteristic dimensions of the model are directly proportional to the ship's dimensions. The model is then a scaled version of the real ship – a very accurately scaled version of the ship. The ratio of the length of the ship to the length of the model is typically used to define the scaling factor (λ).

$$\textit{Scale Factor} = \lambda = \frac{L_S}{L_M}$$

where: L_S = length of the ship, ft
L_M = length of the model, ft

> Note: the subscript "*S*" will be used to denote values for the full-scale ship and the subscript "*M*" will be used to denote values for the model.

From this it follows logically that the ratio of areas is equal to the scale factor squared and the ratio of volumes is equal to the cube of the scale factor. The characteristic area of importance for modeling is the wetted surface area of the underwater hull (S), ft^2, and the characteristic volume of importance is the underwater volume of the ship (∇), ft^3. These relationships are shown below:

$$\lambda^2 = \frac{S_S}{S_M} \qquad\qquad \lambda^3 = \frac{\nabla_S}{\nabla_M}$$

7.7.2.2 Dynamic Similarity

In addition to geometric similarity between model and full-scale ship, there must also be dynamic similarity between the model and its environment and the full-scale ship and its environment. Dynamic similarity means that the velocities, accelerations, and forces associated with fluid flow around both the model and full-scale ship have scaled magnitudes and identical directions at corresponding locations along the hull. The model must behave in exactly the same manner as the full-scale ship.

Unfortunately, it is physically impossible to achieve true dynamic similarity between the model and full-scale ship. Resistance is a function of velocity, water and air pressure, kinematic viscosity of water (ν), air and water density, and the acceleration due to gravity. It is impossible to scale gravity (think of a model having a scale ratio of 36 … now, try to establish a lab environment whose acceleration of gravity is 1/36th of 32.17 ft/sec^2). Similarly, it is impossible to scale water and its properties. Two fluids that come close to being scale versions of water are gasoline and liquid mercury; both of which pose serious health and safety issues.

So, if true dynamic similarity cannot be achieved, how can towing tanks exist, let alone produce meaningful results? The answer lies in achieving partial dynamic similarity between model and ship and Froude's "Law of Comparison", also referred to as the "Law of Corresponding Speeds".

7.7.2.3 The Law of Comparison and Tow Tank Testing

In previous sections of this chapter, we discussed ship resistance and ship performance in terms of dimensionless coefficients:

$$C_T = C_V + C_W$$

where: C_T = coefficient of total hull resistance
C_V = coefficient of viscous resistance
C_W = wave making coefficient

In an ideal world when comparing a geometrically similar ship and model, the coefficients of total resistance, viscous resistance, and wave making resistance would be equal between ship and model. However, due to the viscous effects of water, this is not possible. The question is how to effectively take model data and calculate a coefficient of total hull resistance for the full-scale ship. This question was answered by Froude through his research on ship performance.

After many towing tank tests, Froude noticed that the wave pattern produced by a geometrically similar model and ship looked the same when the model and ship were traveling at the same speed to square root of length ratio. This is the Law of Corresponding Speeds, and is written as:

$$\frac{V_S}{\sqrt{L_S}} = \frac{V_M}{\sqrt{L_M}}$$

where: V_S = ship velocity (ft/s)
V_M = model velocity (ft/s)
L_S = ship length (ft)
L_M = model length (ft)

Because the wave patterns of the model and ship were similar using this relationship, Froude determined that it would be correct to use the same value of wave making coefficient (C_W) for both the model and ship when operating under these conditions, and therefore partial dynamic similarity between model and ship could be obtained. This can be summarized in the following mathematical relationships:

$$C_{WS} = C_{WM}$$

$$\text{if,} \qquad V_M = \frac{V_S\sqrt{L_M}}{\sqrt{L_S}} = \frac{V_S}{\sqrt{L_S}}$$

Example 7.4 A new type of destroyer is undergoing model testing in the tow tank. The ship has a length of 435 feet and the model has a length of 18 feet. The ship is to have a maximum speed of 35 knots. At what speed should the model be towed to achieve partial dynamic similarity for a speed of 35 knots?

Model speed is found using the law of corresponding speeds:

$$V_M = \frac{V_S\sqrt{L_M}}{\sqrt{L_S}}$$

V_S = (35 *knots*)×(1.688 (*ft/sec)/kt*) = 59.08 ft/sec

$$V_M = \frac{\left(59.08\,\frac{ft}{\sec}\right)\left(\sqrt{18\,ft}\right)}{\sqrt{435\,ft}} = 12.02\,\frac{ft}{\sec}$$

Therefore, partial dynamic similarity ($C_{WM} = C_{WS}$) is achieved for a speed of 35 knots if the model is towed at a speed of 12.02 ft/sec.

The purpose of towing tank testing is to tow the model at speeds that correspond to full-scale ship speeds, measure the model's resistance and determine the model's coefficient of wave making resistance. Knowing that the coefficient of wave making resistance of the model and full-scale ship are equal, one can easily determine the coefficient of total hull resistance for the ship. Once the full-scale resistance coefficient is known, the total hull resistance and EHP for the ship are calculated.

To summarize, resistance testing of a model in a towing tank utilizes the following generalized procedure:

- Determine the full-scale ship speed range for the test: minimum ship speed to a desired maximum speed.
- Determine towing speeds for the model using the Law of Comparison.
- Tow the model at each speed, recording the total hull resistance of the model.
- Determine the coefficient of total hull resistance for the model at each speed.
- Determine the coefficient of viscous resistance for the model at each speed.
- Calculate the wave making coefficient of the model at each speed
- $C_{WS} = C_{WM}$
- Determine the coefficient of viscous resistance for the ship at ship speeds corresponding to model towing speeds
- Determine the coefficient of total hull resistance for the ship at each speed.
- Determine the total hull resistance of the ship for each speed.
- Determine and plot the effective horsepower of the ship at each speed

Once the full-scale EHP curve is known, a similar shaft horsepower curve can be determined based on the assumed propulsive coefficient. The bottom line of EHP testing in the towing tank is to determine the amount of shaft horsepower that must be installed in the full scale ship in order to drive at its maximum speed. Once the maximum shaft horsepower is determined, the physical size and weight of the ship's propulsion plant can be resolved as well as the fuel storage requirements based on the expected steaming range (miles) of the ship. These factors are important in estimating the location of the ship's center of gravity as well as the design of the ship's structure.

7.8 The Screw Propeller

The screw propeller is the device most commonly used to transmit the power produced by the prime mover into the water and drives the ship. Screw design theory is very complicated, however, this course includes a few definitions, some basic theory, and propeller characteristics.

7.8.1 Screw Propeller Definitions

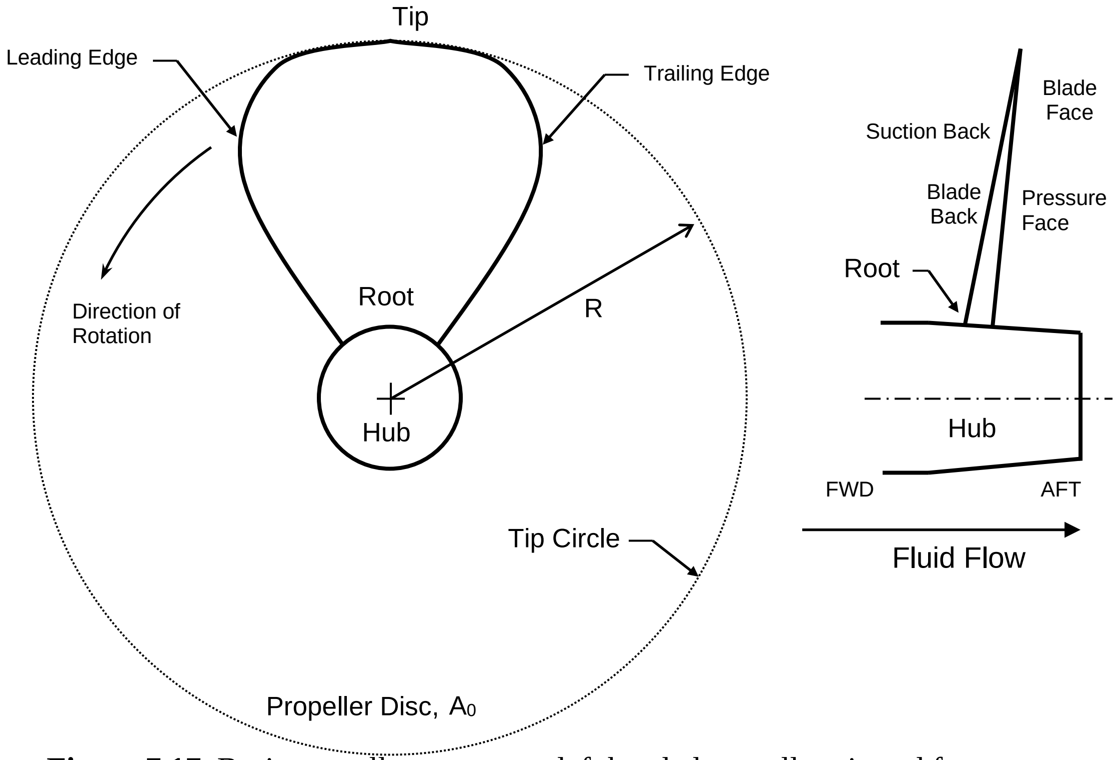


Figure 7.17 Basic propeller geometry; left handed propeller viewed from astern.

- Propeller Radius (R) — Distance from the propeller axis to the blade tip.
- Hub — Connection between the blades and the propeller shaft
- Blade Tip — Furthest point on the blade from the hub
- Blade Root — Point where the blade joins the hub
- Tip Circle — Circle described by the blade tips as the propeller rotates
- Propeller Disc (A_0) — Area described by the tip circle (propeller area, A_0)
- Leading Edge — First portion of the blade to encounter the water
- Trailing Edge — Last portion of the blade to encounter the water
- Pressure Face — High pressure side of the propeller blade. Astern side of the blade when moving the ship forward
- Suction Back — Low pressure side of the blade. Most of the pressure difference developed across the blade occurs on the low pressure side.
- Left Handed Screw — Rotates counterclockwise when viewed from astern. Single screw naval vessels use this type of propeller.
- Right Handed Screw — Rotates clockwise when viewed from astern. Twin screw naval vessels use one left handed and one right handed propeller.

7.8.2 Propeller Pitch (P)

Many times a propeller is referred to by its pitch. So, what is propeller pitch? Assuming the propeller shaft is rotating at a constant angular rate, and the ship is moving at a constant speed, as the propeller rotates and moves through the water, any point on the surface of a propeller blade will describe a helix in one 360° rotation of the shaft.

Propeller pitch (*P*) is the ideal **linear distance** parallel to the direction of motion that would be traveled in one revolution of the propeller shaft; similar to what happens when you turn a wood screw one revolution into a block of wood.

The pitch angle (ϕ) of a propeller is the angle that any portion of the blade makes from perpendicular to the water flow. Since any point on a propeller blade describes a helix, sections taken at varius radii of a propeller with a constant linear pitch (*P)* will have varying pitch angles (ϕ) depending on the radius from the hub. This gives propellers their characteristic blade twist. The pitch of a propeller (P) and pitch angle are related through the following equation:

$$\tan\phi = \frac{P}{2\pi r}$$

where: P = propeller pitch (ft)
ϕ = pitch angle (degrees)
r = radial distance of any point on the blade from the propeller shaft axis (ft)

This relationship is shown below in Figure 7.18.

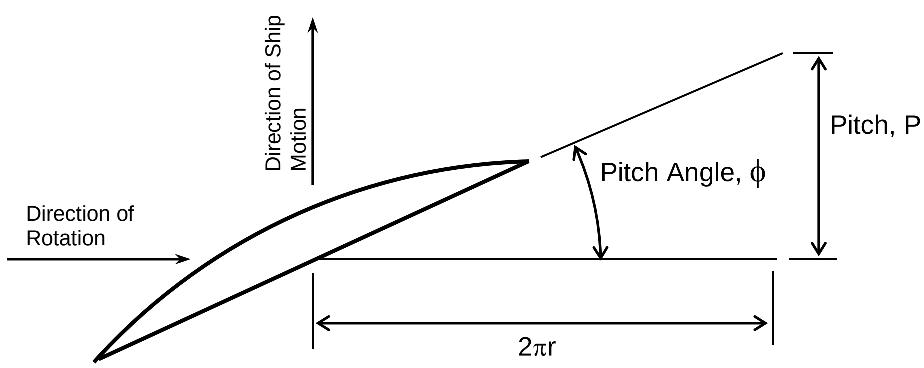


Figure 7.18 Relationship between propeller pitch and pitch angle

Each point on a constant linear pitch (*P)* propeller blade will travel the same linear distance in one rotation of the propeller. Therefore, when looking at a constant pitch propeller you will notice that the angle of the blades (ϕ) changes from root to tip.

Consider a constant pitch propeller that is 14 feet in diameter with a pitch of 15 feet. The hub has a diameter of 5 feet. Using the above equation, one can easily determine the pitch angle at any point along the blade. For instance, at the root the blade has a pitch angle of 43.6° and at the tip the blade has a pitch angle of 18°.

A typical propeller with constant linear pitch is illustrated below in Figure 7.19. Note how points 'A', 'B', and 'C' each travel the same distance in one revolution of the propeller. If a propeller had a flat blades with a constant pitch angle (ϕ) instead of helical blades with a constant linear pitch (*P*), the tips would travel much farther than the root in one revolution through a solid, and the propeller would essentially fight against itself.

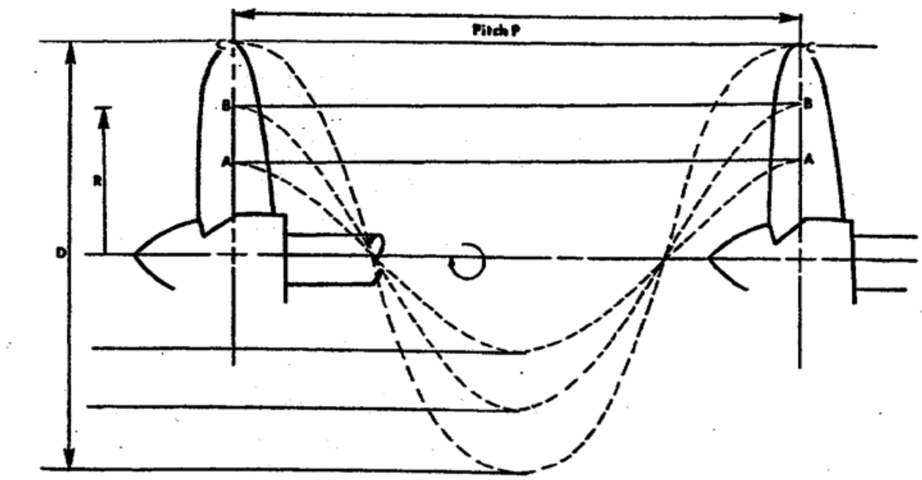


Figure 7.19 Typical propeller operating through one revolution.
(reproduced from "Introduction to Naval Architecture" by Gillmer and Johnson)

The linear pitch (*P*) of real ship propellers is more-or-less constant all the way from the blade root to the blade tip. Some propellers have small variations in linear pitch *(P)* at different radii to adapt to irregularities in the flow going into the propeller, or to minimize certain types of cavitation. These variations are usually less than 20%. Because of these small differences, the nominal pitch value for a propeller is taken at seventy percent of the blade radius (0.7*R*).

There are two main types of propellers seen in Naval applications – Fixed Pitch Propellers and Controllable Pitch Propellers:

Fixed Pitch Propeller: A fixed pitch propeller is a propeller whose blade is fixed with respect to the hub and cannot be changed while the propeller shaft is rotating. Most propellers in service today, from those attached to outboard engines or to the large screws of aircraft carriers, are fixed variable pitch propellers.

Controllable Pitch Propeller: This type of propeller design allows the position of the propeller blade with respect to the hub to be changed while the propeller shaft is rotating. This is accomplished by using an electro-hydraulic system to change the pitch angle of the blades.

Sometimes this can be referred to as Controllable Reversible Propeller, or CRP, (on LSD c*lass* ships). While the entire propeller is classified as a controllable pitch propeller, the blades can also be variable pitch, producing a controllable variable pitch propeller. A controllable pitch propeller can significantly improve the control and ship handling capabilities of a ship. It also obviates the need for a prime mover reversing mechanism because the pitch angle can be changed such that the blades provide reverse thrust without changing the direction of shaft rotation. This type of propeller is found on DDG 51, CG 47, LPD-19, DDG-1000 and LSD 49 classes of ship.

7.8.3 How a Propeller Blade Works

A propeller blade works in the same manner as an aircraft wing. Water flow over the propeller blade creates a pressure differential across the blade which creates a lifting or thrust force that propels the ship through the water. If we were to make a cut through a propeller blade, we would see that the blade has a shape similar to an aircraft wing. Figure 7.20 illustrates this concept. Water velocity over the suction back of the blade is greater than the velocity across the high-pressure face of the blade. Using Bernoulli's equation (from Chapter 1), this velocity differential across the blade results in a pressure differential across the blade. The resultant lifting force can be resolved into thrust and resistance vectors. It is the thrust vector that pushes the ship through the water.

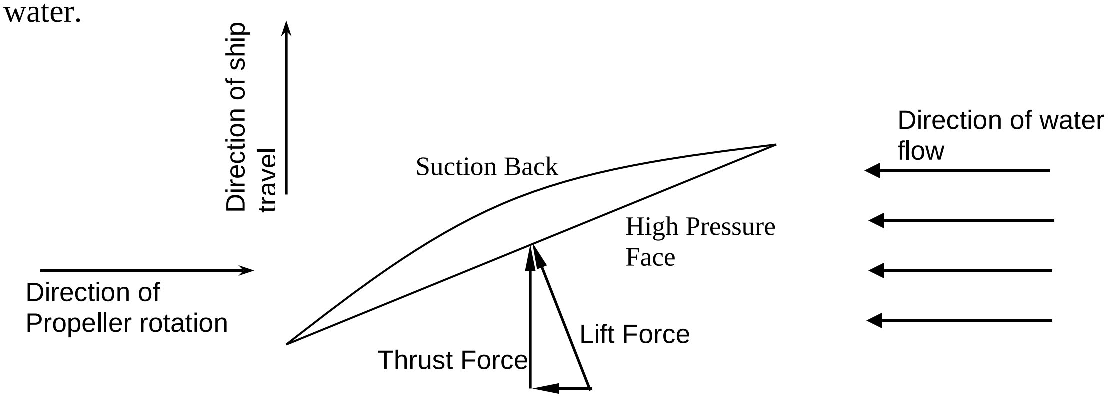


Figure 7.20 Forces acting on a propeller blade

7.8.4 Theory of Propeller Action (DERIVATION OPTIONAL)

There are several theories on fluid dynamics used to describe the operation of a screw propeller. These include momentum theory, impulse theory, blade element theory, and circulation theory. Each of these theories is used by naval architects to design a propeller and analyze its performance. The momentum theory is presented here because it gives some valuable insight into the operation of a propeller without the burden of advanced mathematics.

7.8.4.1 Speed of Advance (V_A) (DERIVATION OPTIONAL)

Before a study of momentum theory can proceed, it is necessary to understand the concept of the speed of advance (V_A) of a propeller. As a ship moves through the water at some velocity (V_S), it drags the surrounding water with it as explained earlier in the section on viscous resistance. At the stern of the ship, this causes the wake to follow along with the ship at a wake speed (V_W). Consequently the propeller is experiencing a flow velocity less than the ship's velocity. The flow velocity through the propeller is called the speed of advance (V_A). Figure 7.21 illustrates this concept.

$$V_A = V_S - V_W$$

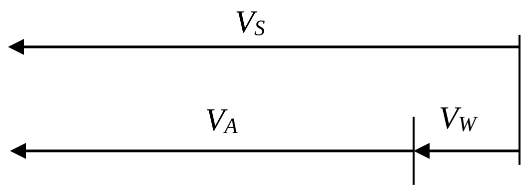


Figure 7.21 Speed of advance

7.8.4.2 Momentum Theory (DERIVATION OPTIONAL)

The momentum theory is used to describe the action of an "ideal" propeller. In this theory, the exact nature of the propeller (pitch, number of blades, shaft rpm, etc) is not important. The propeller itself is assumed to be a "disc" of area A_0 (disc area). The propeller causes an abrupt increase in pressure as the fluid passes through the disc coupled with an increase in fluid velocity. The method by which this occurs is ignored.

Momentum theory makes the following assumptions regarding the propeller and the flow through the propeller:

1. The propeller imparts a uniform acceleration to the water passing through it and the thrust generated by the propeller is uniformly distributed over the entire disc.
2. The flow is frictionless.
3. There is an unlimited supply of water available to the propeller.

Figures 7.22 through 7.24 look at the propeller and a control volume of water around the propeller. The control volume extends to some station 1 ahead of the propeller to some station 3 astern of the propeller. Station 2 represents the propeller disc area (A_0). Since the assumption is made that the propeller imparts a uniform acceleration to the water, this implies that the cross section area of the control volume must decrease from station 1 to station 3.

- Cross section area of the flow decreases through the propeller.

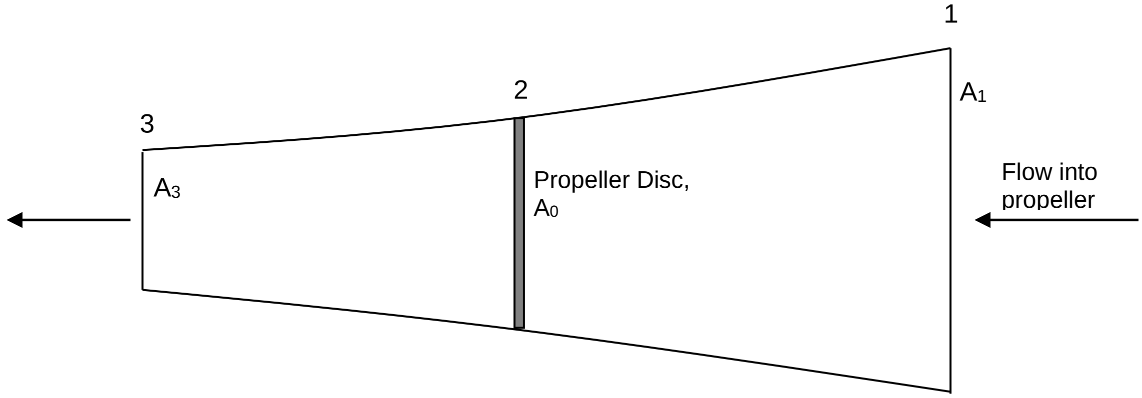


Figure 7.22 Flow cross-sectional area

- Flow velocity increases from the speed of advance (V_A) at point 1 to velocity V_3 as cross section area of the flow decreases from station 1 to station 3. The "*a*" and "*b*" terms are an axial-inflow factor and will be discussed later.

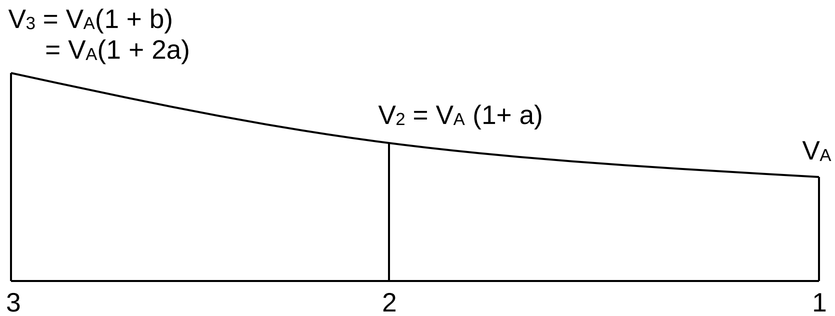


Figure 7.23 Flow velocity

- Pressure decreases as fluid velocity increases through the propeller. Note that the propeller causes an increase in pressure between the suction back (P_2) and pressure face (P_2'). Note that at some distance ahead and astern of the propeller, pressures are equal.

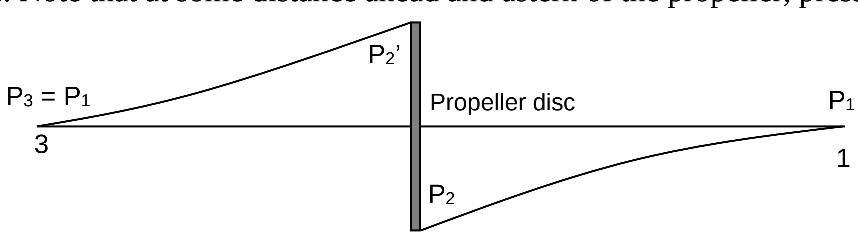


Figure 7.24 Flow pressure

7.8.4.3 Determination of Propeller Thrust (DERIVATION OPTIONAL)

Momentum theory states that the thrust produced by a propeller is equal to the change in momentum of the fluid per unit time as it passes through the control volume from station 1 to station 3.

The volume flow rate of water through the propeller disc is:

$$Q = V_A(1+a)A_0$$

Neglecting any effect of rotation that may be imparted to the flow by the propeller disc, the change in momentum per unit time between stations 1 and 3 is:

$$\rho Q(V_3 - V_A)$$

$$= \rho Q[V_A(1+b) - V_A]$$

Therefore, the thrust produced by the propeller is:

$$T = \rho Q[V_A(1+b) - V_A] = \rho Q V_A b$$

Therefore, propeller thrust is a function of the mass flow rate of water through the propeller and the change in fluid velocity through the propeller. Thrust can be increased by either increasing the flow rate through the propeller disc or by increasing the velocity differential between stations 1 and 3.

Substituting the above expression for flow rate through the disc produces the following expression for propeller thrust:

$$T = \rho A_0 V_A^2 (1+a) b$$

The terms "*a*" and "*b*" are axial-inflow factors and are used to describe the increase in fluid velocity through the propeller from station 1 to station 3. It can be shown that axial inflow factors "*a*" and "*b*" are related by the following expression:

$$b = 2a \quad \text{or} \quad a = b/2$$

The significance of this relation is that one half of the increase in fluid velocity through the propeller is obtained prior to the fluid reaching the propeller disc. In other words, the decrease in pressure at the suction back of the propeller causes fluid velocity to increase. Substituting $a = b/2$ into the equation for thrust produces the following expression for propeller thrust:

$$T = \rho A_0 V_A^2 b(1 + b/2)$$

Rearranging this expression produces the following:

$$T = \frac{1}{2}\rho A_0 V_A^2\left(2b + b^2\right)$$

We will use the above relationship for propeller thrust to describe thrust loading on the propeller and the efficiency of the propeller.

7.8.4.4 Thrust Loading Coefficient (C_T) (DERIVATION OPTIONAL)

The coefficient of thrust loading (C_T) is the dimensionless form of thrust and is defined by the following equation:

$$C_T = \frac{T}{\frac{1}{2}\rho A_0 V_A^2}$$

Comparing this equation to the above relation for thrust reveals that:

$$C_T = 2b + b^2$$

7.8.4.5 Ideal Propeller Efficiency (η_I) (DERIVATION OPTIONAL)

The ideal efficiency of a propeller is the ratio of useful work obtained from the propeller and work expended by the propeller on the water. The work done on the water by the propeller thrust is $TV_A(1 + a)$, and the work obtained from the propeller is TV_A. The ideal propeller efficiency is:

$$\eta_I = \frac{TV_A}{TV_A\left(1 + a\right)}$$

However, $a = b/2$, and substituting this expression into the equation for ideal efficiency produces:

$$\eta_I = \frac{TV_A}{TV_A\left(1 + a\right)} = \frac{1}{1 + a} = \frac{1}{1 + b/2} = \frac{2}{2 + b} = \frac{2}{1 + \left(1 + b\right)}$$

Rearranging the expression for C_T produces the following expression for C_T and b:

$$\sqrt{1 + C_T} = \left(1 + b\right)$$

Substituting into the above expression for ideal propeller efficiency produces:

$$\eta_I = \frac{2}{1 + \sqrt{1 + C_T}}$$

7.8.5 Propeller Characteristics

Momentum theory, derived above, gives us the following relationships for the ideal efficiency:

$$\eta_I = \frac{2}{1+\sqrt{1+C_t}} \quad \text{where} \qquad C_t = \frac{T}{\frac{1}{2}\rho A_0 V_A^2}$$

where,

η_I = ideal propeller efficiency
C_t = Thrust Coefficient
T = Propeller Thrust (lbf)
ρ = density of water (lbf-s^2/ft^4)
A_o =Disc area of propeller = $\pi D^2/4 = \pi R^2$, (ft^2)
D = propeller diameter = 2R, (ft)
V_A = Speed of advance of propeller through fluid (ft/s)

From these two relationships we can look at how different factors affect the performance of a propeller. For a given thrust (T) and speed of advance (V_A), the thrust loading coefficient will decrease as the propeller disc area (A_0) increases. A decrease in propeller thrust loading results in an increase in the ideal propeller efficiency. Thus, **larger propellers reduce thrust loading and improve efficiency**. Propeller diameter is often constrained by the ship's draft, or by other limitations, such as rotational speed, but in general, the larger propeller will be more efficient.

The equation for ideal propeller efficiency given above should not be confused with actual propeller efficiency discussed in section 7.2.4. The ideal propeller efficiency is based on frictionless, irrotational flow through the propeller. Because of friction, rotational factors, and other losses, the actual efficiency of a propeller is approximately 20% less than the ideal efficiency given above. However, the idea of increasing the size of the propeller to improve efficiency is still valid.

Momentum theory does not address such matters as the pitch of a propeller, the number of blades in the propeller, or even propeller shaft rotation (rpm).

- Pitch is determined based on the propeller rotation speed and the speed of the ship. Slow turning propellers with a higher pitch are generally more efficient; however they require larger reduction gears. Propeller pitch rarely exceeds 1.4 times the diameter.
- RPM is related to the pitch and the speed of the ship.
- Number of blades depends mainly on vibration characteristics. Submarines can have seven-bladed propellers to be extra quiet.

For further information on propellers and propeller theory, read Volume 2 of "Principles of Naval Architecture", published by the Society of Naval Architects and Marine Engineers.

7.8.6 Propeller Cavitation

Cavitation is the formation and subsequent collapse of vapor bubbles in regions on propeller blades where pressure has fallen below the vapor pressure of water. Cavitation occurs on propellers that are heavily loaded, or are experiencing a high thrust loading coefficient.

7.8.6.1 Types of Cavitation

There are three main types of propeller cavitation:

Tip Blade tip cavitation is the most common form of cavitation. Tip cavitation forms because the blade tips are moving the fastest and therefore experience the greatest dynamic pressure drop.

Sheet Sheet cavitation refers to a large and stable region of cavitation on a propeller, not necessarily covering the entire face of a blade. The suction face of the propeller is susceptible to sheet cavitation because of its low pressures. Additionally, if the angle of attack of the blade is set incorrectly (on a controllable pitch propeller, for instance) it is possible to cause sheet cavitation on the pressure surface.

Spot Spot cavitation occurs at sites on the blade where there is a scratch or some other surface imperfection.

7.8.6.2 Consequences of Cavitation

The consequences of propeller cavitation are not good and can include the following:

- Reduction in the thrust produced by the propeller.
- Erosion of the propeller blades. As cavitation bubbles form and collapse on the tip and face of a propeller blade, pressure wave formed causes a small amount of metal to be eroded away. Excessive cavitation can erode blade tips and cause other imperfections on the blade's surface.
- Vibration in the propeller shafting.
- Increase in ship's radiated noise signature.

In the case of a warship, cavitation is to be avoided because the noise of cavitation can compromise the location of the vessel. This is especially important when operating in the vicinity of enemy submarines. The Prairie-Masker system, used on several different classes of warship (DDG-51 and CG-47, for instance) is highly effective at reducing machinery and cavitation noise. The Prairie portion of the system routes compressed air from the turbines to the leading edges and propeller blade tips (the most likely location for cavitation to form), where it is released into the water through small holes. The air bubbles released from the propeller helps reduce cavitation and dampen the effect of collapsing vapor bubbles caused by cavitation.

7.8.6.3 Preventing Cavitation

Several actions can be taken to reduce the likelihood of cavitation occurring:

- **Fouling** The propeller must be kept unfouled by marine organisms and free of nicks and scratches. Fouling causes a reduction in propeller efficiency as well as the increased chance for cavitation. Even a small scratch can cause significant spot cavitation and result in an increase in radiated noise as well as erosion of the blades. The Navy conducts regular underwater inspections and cleaning of its propellers to prevent the effects of fouling.

- **Speed** Every ship has a cavitation inception speed, a speed where tip cavitation begins to form. Unless operationally necessary, ships should be operated at speeds below cavitation inception.

- **Thrust** For ships with manual throttles (steam turbine), the Throttleman must not increase shaft speed and thrust too quickly when accelerating the ship. An analysis of the equation for the thrust coefficient (C_T) reveals that high propeller thrust (T) and low speed through the propeller (V_A) increases the thrust loading coefficient which may result in cavitation.

$$C_t = \frac{T}{\frac{1}{2}\rho A_0 V_A^2}$$

  When accelerating the ship, the Throttleman should open the throttle slowly, allowing flow velocity to increase or decrease proportionally with propeller thrust. Ships may use an acceleration table to guide the Throttleman in opening throttles or hydrophones calibrated to detect cavitation from the propeller.

- **Pitch** Operators of ships with controllable pitch propellers must take care that propeller pitch is increased or decreased in a smooth manner. This is usually done as part of the ship's propulsion control system. Incorrect operation of the pitch control system may cause high thrust loading on the propeller blades and increase the likelihood of cavitation.

- **Depth** Since cavitation is a function of hydrostatic pressure, increasing hydrostatic pressure (i.e. depth) will reduce the likelihood of cavitation. Submarines are uniquely susceptible to depth effects and cavitation as the depth of the submarine affects hydrostatic pressure at the propeller blades. When operating at shallow depth, hydrostatic pressure is decreased and the propeller cavitates at lower shaft rpm and low thrust loading. As a submarines depth increases, hydrostatic pressure increases and cavitation inception is delayed. Therefore, a submarine can operate at higher speeds at deeper depths with little worry about cavitation noise.

7.8.6.4 Propeller Ventilation

Ventilation is a propeller effect often confused with cavitation. If a propeller operates too close to the surface of the water, the localized low pressure created by the propeller blades can draw air under the water and cause effects similar to those mentioned for cavitation.

Ventilation is most likely to occur when operating in a very light displacement condition (a condition common to merchant ships transiting in ballast), ships operating in rough seas where ship motion causes the propeller to go in and out of the water, and in ships with a large negative trim (trim down by the bow).

APPENDIX A

TABLE of FRESH and SALT WATER DENSITY

(reprinted from ‘Introduction to Naval Architecture’ by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Mass Density ρ for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

Density of fresh water ρ, lb-sec^2/ft^4 (= slugs/ft^3)	Temp, deg F	Density of salt water ρ_s, lb-sec^2/ft^4	Density of fresh water ρ, lb-sec^2/ft^4	Temp, deg F	Density of salt water ρ_s, lb-sec^2/ft^4
1.9399	32	1.9947	1.9384	59	1.9905
1.9399	33	1.9946	1.9383	60	1.9903
1.9400	34	1.9946	1.9381	61	1.9901
1.9400	35	1.9945	1.9379	62	1.9898
1.9401	36	1.9944	1.9377	63	1.9895
1.9401	37	1.9943	1.9375	64	1.9893
1.9401	38	1.9942	1.9373	65	1.9890
1.9401	39	1.9941	1.9371	66	1.9888
1.9401	40	1.9940	1.9369	67	1.9885
1.9401	41	1.9939	1.9367	68	1.9882
1.9401	42	1.9937	1.9365	69	1.9879
1.9401	43	1.9936	1.9362	70	1.9876
1.9400	44	1.9934	1.9360	71	1.9873
1.9400	45	1.9933	1.9358	72	1.9870
1.9399	46	1.9931	1.9355	73	1.9867
1.9398	47	1.9930	1.9352	74	1.9864
1.9398	48	1.9928	1.9350	75	1.9861
1.9397	49	1.9926	1.9347	76	1.9858
1.9396	50	1.9924	1.9344	77	1.9854
1.9395	51	1.9923	1.9342	78	1.9851
1.9394	52	1.9921	1.9339	79	1.9848
1.9393	53	1.9919	1.9336	80	1.9844
1.9392	54	1.9917	1.9333	81	1.9841
1.9390	55	1.9914	1.9330	82	1.9837
1.9389	56	1.9912	1.9327	83	1.9834
1.9387	57	1.9910	1.9324	84	1.9830
1.9386	58	1.9908	1.9321	85	1.9827
			1.9317	86	1.9823

NOTE: For other salinities, interpolate linearly.

APPENDIX B

TABLE of FRESH and SALT WATER KINEMATIC VISCOSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)

Values of Kinematic Viscosity ν for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

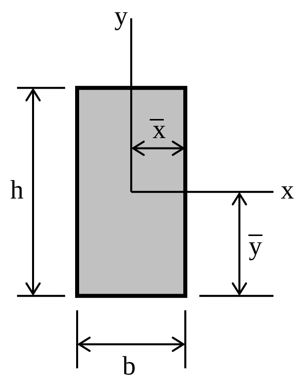
Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$	Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$
1.9231	32	1.9681	1.2260	59	1.2791
1.8871	33	1.9323	1.2083	60	1.2615
1.8520	34	1.8974	1.1910	61	1.2443
1.8180	35	1.8637	1.1741	62	1.2275
1.7849	36	1.8309	1.1576	63	1.2111
1.7527	37	1.7991	1.1415	64	1.1951
1.7215	38	1.7682	1.1257	65	1.1794
1.6911	39	1.7382	1.1103	66	1.1640
1.6616	40	1.7091	1.0952	67	1.1489
1.6329	41	1.6807	1.0804	68	1.1342
1.6049	42	1.6532	1.0660	69	1.1198
1.5777	43	1.6263	1.0519	70	1.1057
1.5512	44	1.6002	1.0381	71	1.0918
1.5254	45	1.5748	1.0245	72	1.0783
1.5003	46	1.5501	1.0113	73	1.0650
1.4759	47	1.5259	0.9984	74	1.0520
1.4520	48	1.5024	0.9857	75	1.0392
1.4288	49	1.4796	0.9733	76	1.0267
1.4062	50	1.4572	0.9611	77	1.0145
1.3841	51	1.4354	0.9492	78	1.0025
1.3626	52	1.4142	0.9375	79	1.9907
1.3416	53	1.3935	0.9261	80	0.9791
1.3212	54	1.3732	0.9149	81	0.9678
1.3012	55	1.3535	0.9039	82	0.9567
1.2817	56	1.3343	0.8931	83	0.9457
1.2627	57	1.3154	0.8826	84	0.9350
1.2441	58	1.2970	0.8722	85	0.9245
			0.8621	86	0.9142

NOTE: For other salinities, interpolate linearly.

APPENDIX C

PROPERTIES of COMMON GEOMETRIC SHAPES

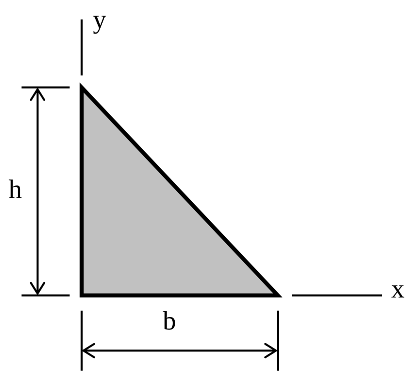
Rectangle (origin of axes at centroid)



$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

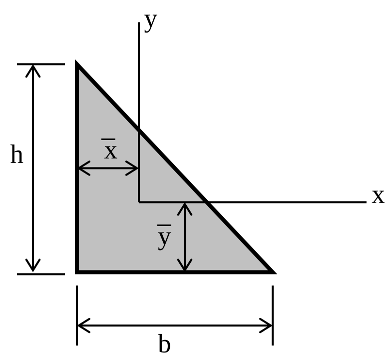
$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

Right Triangle (origin of axes at vertex)



$$A = \frac{bh}{2} \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

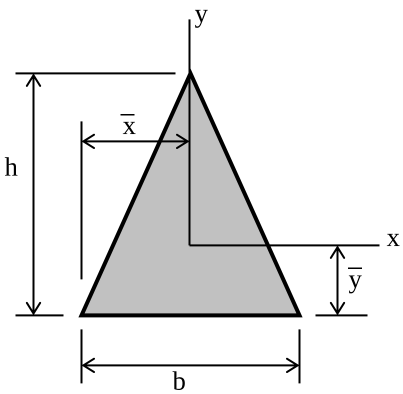
Right Triangle (origin of axes at centroid)



$$\bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36}$$

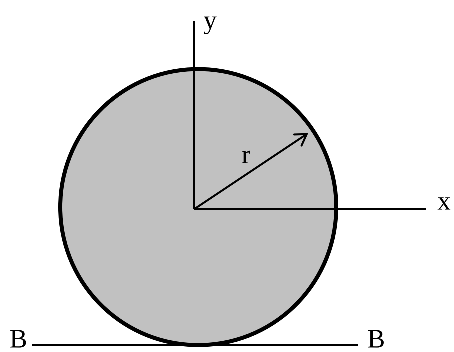
Isosceles Triangle (origin of axes at centroid)



$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48}$$

Circle (origin of axes at center)

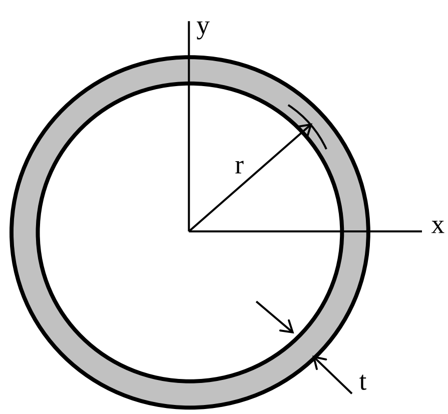


$$d = 2r \qquad A = \pi r^2 = \frac{\pi d^2}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

Circular Ring with thickness " *t* " (origin of axes at center)
Approximate formulas for the case when *t* is small



$$A = 2\pi rt = \pi dt$$

$$I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$