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SHIP STABILITY

Main Category:	Naval Engineering
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Course #:	NAV-113
Course Content:	46 pgs
PDH/CE Hours:	4

OFFICIAL COURSE/EXAM

(SEE INSTRUCTIONS ON NEXT PAGE)

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NAV-113 EXAM PREVIEW

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Exam Preview:

1. According to the reference material, quasi-static means that the external moment heeling the ship over is doing so in infinitely small steps so that equilibrium is always present.
    a. True
    b. False
2. Using Figure 4.2 Curve of Intact Statical Stability, which of the following Righting Arm lengths corresponds to an angle of heel of 75°?
    a. 0 ft
    b. 2 ft
    c. 3 ft
    d. 4 ft
3. The actual location of the center of gravity of the ship will always be above the keel. This means that the _____ correction can always be subtracted from the value read off the cross curves.
    a. Cosecant
    b. Tangent
    c. Cosine
    d. Sine
4. According to the reference material, the smaller the range of stability, the less likely the ship will capsize.
    a. True
    b. False

5. The margin line is a line defining the highest permissible location on the side of the vessel of any damaged waterplane in the final condition of sinkage, trim and heel. It is in no case permitted to be less than __ inches below the top of the bulkhead deck at the side.
    a. 1.5
    b. 3
    c. 6
    d. 8
6. According to the reference material, ships 100 - 300 ft long are required to withstand flooding in any two adjacent compartments.
    a. True
    b. False
7. Using the table taken from Basic Ship Theory - 4th Edition, which of the following permeability corresponds to the machinery compartment of a ship?
    a. 60%
    b. 70%
    c. 85%
    d. 95%
8. According to Section 4.7.3 US Navy Damage Stability Design Criteria, the heel caused by damage shall not exceed ___ degrees. This angle is too great for continuous operation of equipment.
    a. 12.5
    b. 15
    c. 20
    d. 25
9. According to the reference material, tanks should be kept at least 70% full so that pocketing occurs. Pocketing is when the liquid hits the top of the tank thus reducing the free surface effects.
    a. True
    b. False
10. Using Figure 4.14 Positive, Neutral, and Negative Metacentric Height, what is the correct slope for the curve of intact statical stability when the Metacentric height is negative?
    a. Positive
    b. Negative
    c. 0
    d. N/A

COURSE OBJECTIVES
CHAPTER 4

4. STABILITY

1. Explain the concepts of righting arm and righting moment and show these concepts on a sectional vector diagram of the ship's hull that is being heeled over by an external couple.

2. Calculate the righting moment of a ship given the magnitude of the righting arm.

3. Read, interpret, and sketch a Curve of Intact Statical Stability (or Righting Arm Curve) and draw the sectional vector diagram of forces that corresponds to any point along the curve.

4. Discuss what tenderness and stiffness mean with respect to naval engineering.

5. Evaluate the stability of a ship in terms of:
    a. Range of Stability
    b. Dynamic Stability
    c. Maximum Righting Arm
    d. Maximum Righting Moment
    e. Angle at which Maximum Righting Moment Occurs

6. Create a Curve of Intact Statical Stability for a ship at a given displacement and assumed vertical center of gravity, using the Cross Curves of Stability.

7. Correct a GZ curve for a shift of the ship's vertical center of gravity and interpret the curve. Draw the appropriate sectional vector diagram and use this diagram to show the derivation of the sine correction.

8. Correct a GZ curve for a shift of the ship's transverse center of gravity and interpret the curve. Draw the appropriate sectional vector diagram and use this diagram to show the derivation of the cosine correction.

9. Determine the initial slope of the GZ curve using Metacentric Height.

10. Analyze and discuss damage to ships, including:
    a. Use added weight method to calculate ship trim, angle of list and draft
    b. Qualitatively discuss lost buoyancy method
    c. Navy Damage Stability Criteria for ships

11. Analyze and discuss free surface effects, including:
    a. Consequences of free surface on overall ship stability
    b. Ways to limit the effects of free surface
    c. Calculate the effective metacentric height
    d. The meaning of a negative metacentric height and show this condition on a sectional vector diagram of the ship’s hull.
    e. Correct the GZ curve

4.1 Introduction

In the last chapter we studied hydrostatics of a displacement ship. In that chapter there were only two internally produced forces and no external forces were considered. The resultant buoyant force and the resultant weight of the ship were in vertical alignment so that no moments were produced. The criteria for static equilibrium were met so that the displacement ship would forever sit motionless until external forces acted on the ship or a weight change occurred.

In this chapter we are concerned with the ability of the ship to remain upright when external forces are trying to roll it over. We are mostly concerned with the transverse movement or heeling because it is nearly impossible to tip a ship end to end (longitudinally). Here the resultant weight of the ship is very often not in vertical alignment with the resultant buoyant force so that internal moments are produced.

- First, we will study the general principle of a righting moment for a ship. We will see how the magnitude of the righting moment is a function of the heeling angle.

- Second, we will show how the righting moment is affected by changes in the vertical and transverse location of the center of gravity of the ship.

- Third, we will discuss how stability is affected by hull damage and learn ways to model a damaged ship.

- Fourth, we will study the effects of free surface (fluids in less than full tanks or compartments) on the righting moment.

- Finally, we will show the effects of a negative metacentric height on the stability of ship.

4.2 The Internal Righting Moment Produced by a Heeling Ship

Understanding overall stability comes down to understanding how the relative positions of the resultant weight of the ship and the resultant buoyant force change when a ship is heeled over by an external moment or couple.

4.2.1 The External Couple

The external couple can be caused by the action of wind pushing on one side of the ship, trying to translate the ship in that direction, and the water pushing back on the hull in the opposite direction. The resultant forces from these two distributed forces would be acting parallel to the water's surface. The resultant wind force would be above the water and the resultant water force would be below the water. Thus the two resultant forces would not be aligned. They would form an external couple or moment causing the ship to rotate. A good analogy can be made by picturing a steering wheel -- the wind is pushing at the top of the steering wheel and the water is pushing in the opposite direction at the bottom. The steering wheel will rotate when acted upon by these unbalanced forces. Refer to Figure 4.1.

4.2.2 The Internal Couple

A ship will also tend to rotate when acted upon by wind and water. However, as the ship heels over due to an external moment it also develops an internal moment. The internal moment acts in response to the external moment and in the opposite rotational direction. If the internal and external moments balance the ship will stay heeled at that angle of inclination, otherwise it will keep heeling until the ship capsizes.

To understand how the ship develops an internal moment, consider how the relative positions of the resultant weight of the ship and the resultant buoyant force change as the ship is heeled over.

The resultant weight of the ship acts vertically downward at the center of gravity. Only changes in the distribution of weight affect the location of the center of gravity. If no weight changes occur then no shifts in the center of gravity will occur.

The resultant buoyant force acts vertically upward at the center of buoyancy. The center of buoyancy is located at the centroid of the underwater volume of the ship. When the ship is heeled over by an external moment the underwater shape changes and thus the centroid moves. Where the center of buoyancy moves with respect to the center of gravity defines the stability characteristics of the ship as the ship is heeled over.

Figure 4.1 shows the sectional view of a ship that is being heeled over due to an external moment. It shows the relative positions of the center of gravity and center of buoyancy for a ship that has been designed properly. Notice the perpendicular distance between the lines of action of the resultant weight and resultant buoyant force. This distance is the "righting arm" (GZ).

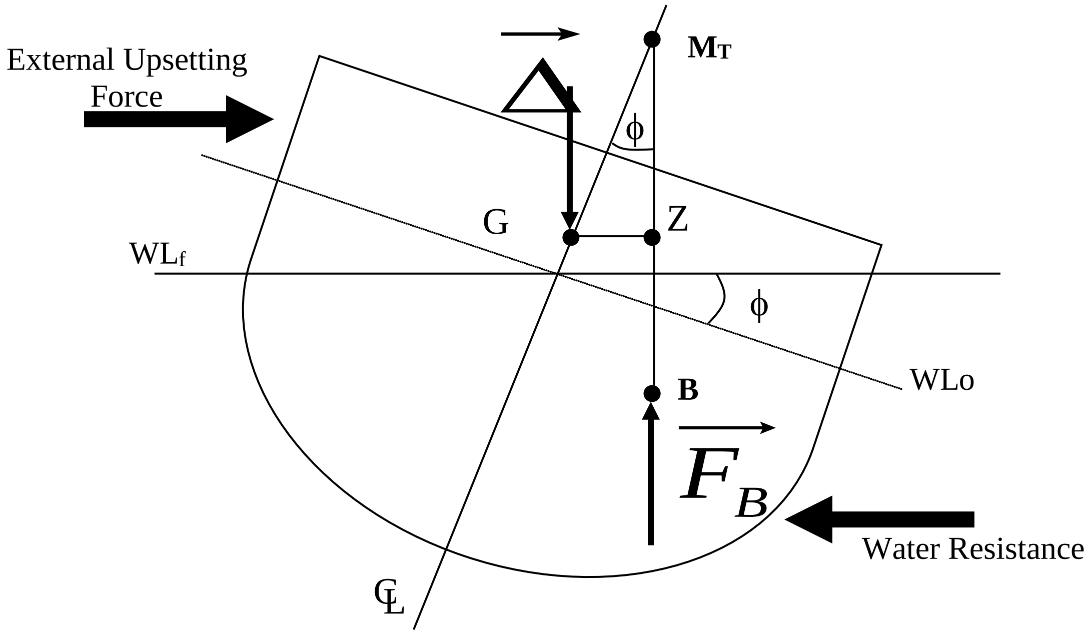


Figure 4.1 Heeled Ship due to an External Moment

(!) You should be able to draw Figure 4.1 without the use of your notes.

To find the internal righting moment multiply the righting arm by the magnitude of the resultant weight of the ship (or the magnitude of the resultant buoyant force since the magnitude of these forces are equal). The equation below shows this relationship.

$$RM = \overline{GZ}\,\Delta = \overline{GZ}\,F_B$$

where:	RM	is the internal righting moment of the ship (LT-ft)
	Δ_S	is the displacement of the ship (LT)
	F_B	is the magnitude of the resultant buoyant force (LT)
	GZ	is the righting arm (ft) which is the perpendicular distance between the line of action of the resultant buoyant force and the resultant weight of the ship. This distance is a function of the heeling angle.

4.3 The Curve of Intact Statical Stability

Figure 4.1 is only a snapshot of the total stability picture. We are really interested in how Figure 4.1 changes as the ship is heeled over from zero degrees to large enough angles of heel to make the ship capsize. To help us conceptualize this process, a graph of heeling angle (degrees) versus righting arm (GZ) is constructed. This graph is called the “curve of intact statical stability” or the “Righting Arm Curve”.

The curve of intact statical stability assumes the ship is being heeled over quasi-statically in calm water. Quasi-static means that the external moment heeling the ship over is doing so in infinitely small steps so that equilibrium is always present. Of course this is impossible, but it is an acceptable idealization in the modeling of ship stability. Be sure to realize that the predictions made by the curve of intact statical stability can not be directly applied to a rolling ship in a dynamic seaway. The dynamics of such a system, including the application of additional external forces and the presence of rotational momentum, are not considered in the intact statical stability curve. However, the intact statical stability curve is useful for comparative purposes. The stability characteristics of different hull shapes can be compared as well as differences in operating conditions for the same hull type.

Figure 4.2 shows a typical intact statical stability curve. When the ship is in equilibrium with no outside forces acting on it, the resultant weight of the ship will be vertically aligned with the resultant buoyant force. As an external moment heels the ship to port or starboard, the resultant weight and the resultant buoyant force will become out of vertical alignment creating the righting arm. The righting arm will obtain a maximum value and then decrease until the resultant weight of the ship and the resultant buoyant force are again in vertical alignment. Heeling any further will cause the ship to capsize. See Figure 4.3.

(!) You should be able to draw Figure 4.2 without the use of your notes and to draw the sectional vector diagram of forces (as shown by Figure 4.3) that corresponds to any point along the curve on Figure 4.2.

Typically only the starboard side of the intact statical stability curve is shown. The entire curve is shown in Figure 4.2 to give the entire picture of the statical stability curve. Notice how the port side is drawn in quadrant 3 since angles to port are assigned a negative and righting arms to port are assigned a negative. This is only a convention used to distinguish between port and starboard heeling.

Each intact statical stability curve is for a given displacement and given vertical center of gravity. The process of obtaining the actual intact statical stability curve is done by reading values off the “cross curves of stability” for a given displacement of the ship, and then making a sine correction to account for the proper vertical location of the center of gravity of the operating ship. You will learn about the sine correction later in this chapter.

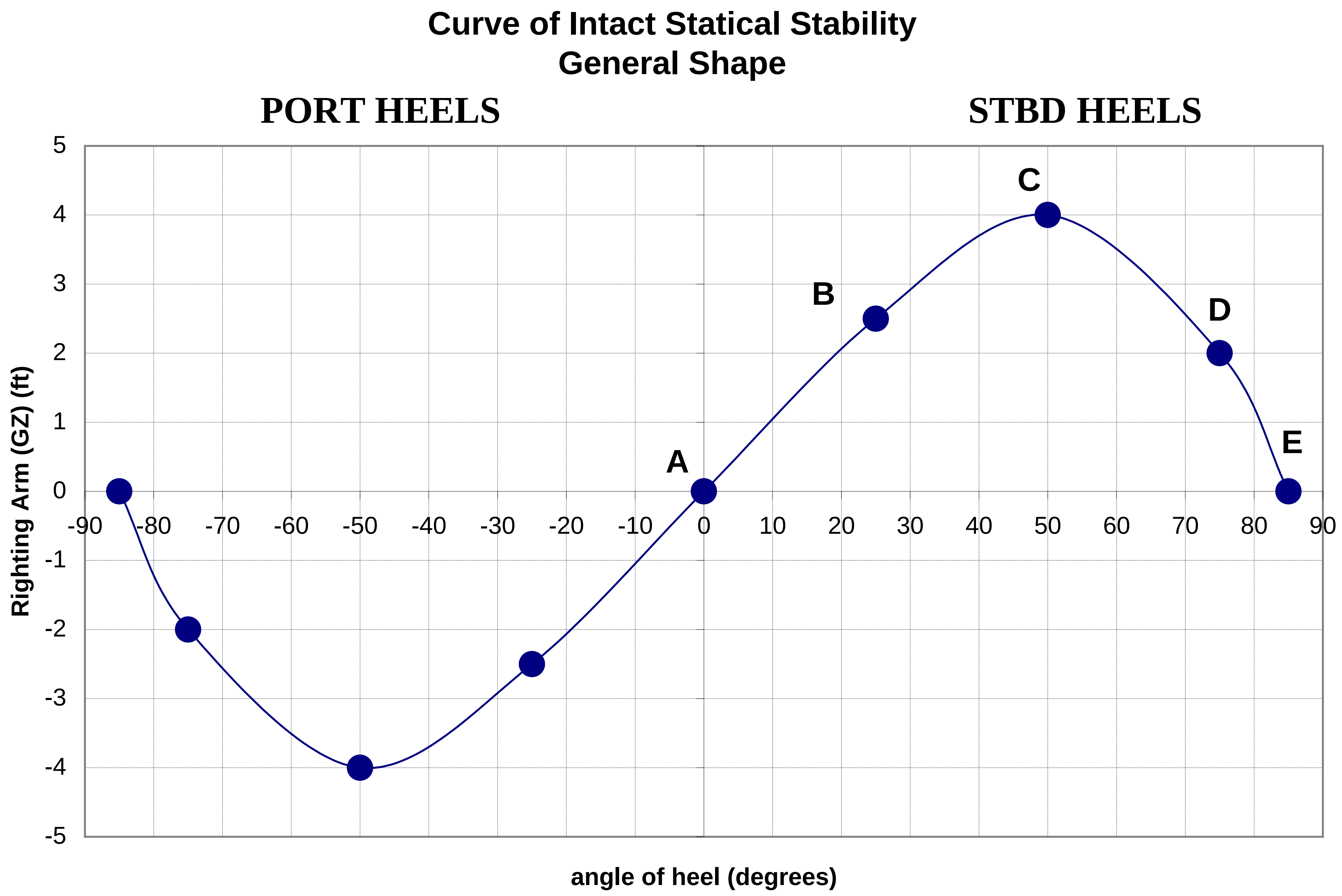
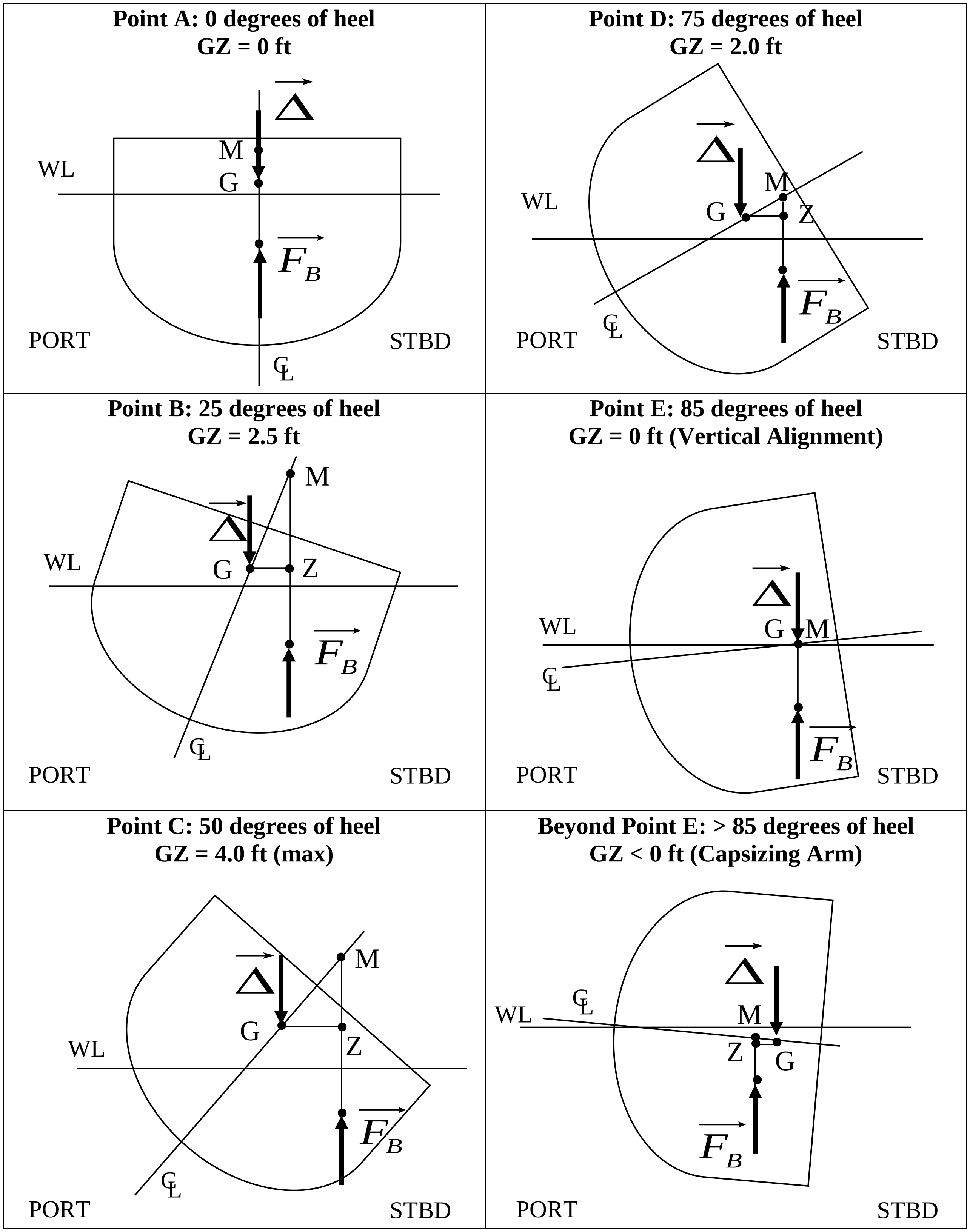


Figure 4.2 Curve of Intact Statical Stability



Point A: 0 degrees of heel
GZ = 0 ft

Point D: 75 degrees of heel
GZ = 2.0 ft

Point B: 25 degrees of heel
GZ = 2.5 ft

Point E: 85 degrees of heel
GZ = 0 ft (Vertical Alignment)

Point C: 50 degrees of heel
GZ = 4.0 ft (max)

Beyond Point E: > 85 degrees of heel
GZ < 0 ft (Capsizing Arm)

Figure 4.3 Vector Drawings Associated with Figure 4.2

4.3.1 Cross Curves of Stability

The cross curves of stability are a series of curves on a single set of axes. The X-axis is the displacement of the ship in LT. The Y-axis is the righting arm of the ship in feet. Each curve is for one angle of heel. Typically angles of heel are taken each 5 or 10 degrees. Figure 4.4 is a set of cross curves for the FFG-7. There are cross curves for some of the more common ships used in the Navy in the ship data section.

The entire series of curves assumes an arbitrary location for the vertical center of gravity of the ship. Sometimes the assumed location of the center of gravity is at the keel. This may seem strange to you at first but it makes sense when you consider the following. The actual location of the center of gravity of the ship will always be above the keel. This means that the sine correction can always be subtracted from the value read off the cross curves. Otherwise, the sine correction would sometimes be subtracted and sometimes be added. The actual location of the assumed value of the center of gravity of the ship will always be marked on the cross curves.

The cross curves are made by a series of integrations based on hull geometry. You had a hint of this in Chapter 2. It is beyond the scope of this course to explain in detail how the cross curves are derived from the basic geometry of the hull.

In summary, the intact statical stability curve, for a single displacement, comes from reading values off the cross curves of stability and using a sine correction for the actual location of the vertical center of gravity.

(!) Be able to sketch a set of cross curves with fictitious numbers without the use of your notes.

Student Exercise: On a separate piece of paper draw an intact static stability curve for a FFG-7 displacing 4000 LT. Assume the FFG-7 has a KG=0 so that a sine correction is unnecessary.

This is unrealistic, but for now you are learning how to read values off the cross curves to construct the intact statical stability curve. Later in this chapter you will learn how to do the sine correction to account for the actual location of the vertical center of gravity of the ship.

Insert this page in your notes.

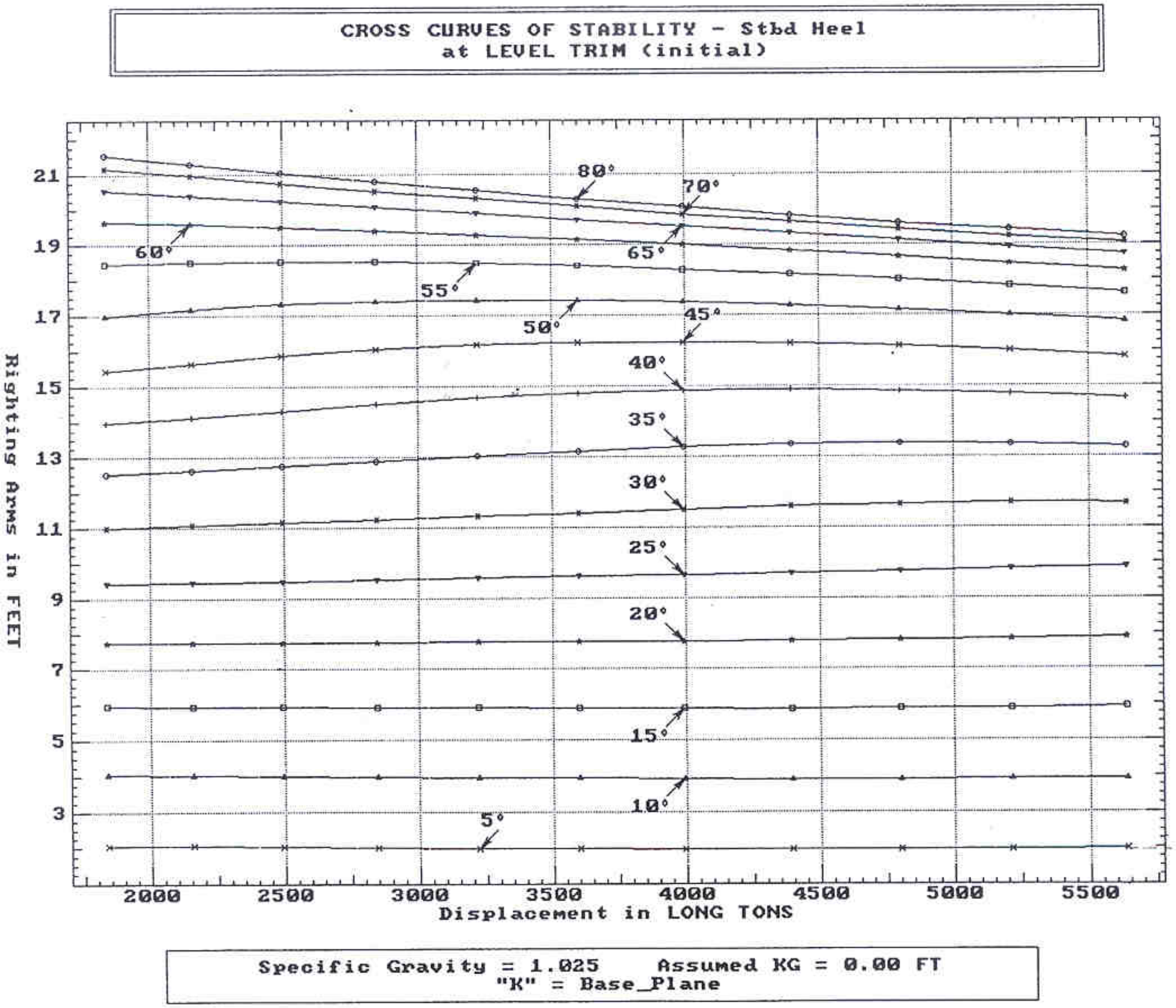


Figure 4.4 Cross Curves of Stability for FFG-7

4.4 Obtainable Stability Characteristics from the Curve of Intact Statical Stability

Several overall stability characteristics can be obtained from the curve of intact statical stability. A simplied GZ curve diagram is shown below in Figure 4.5:

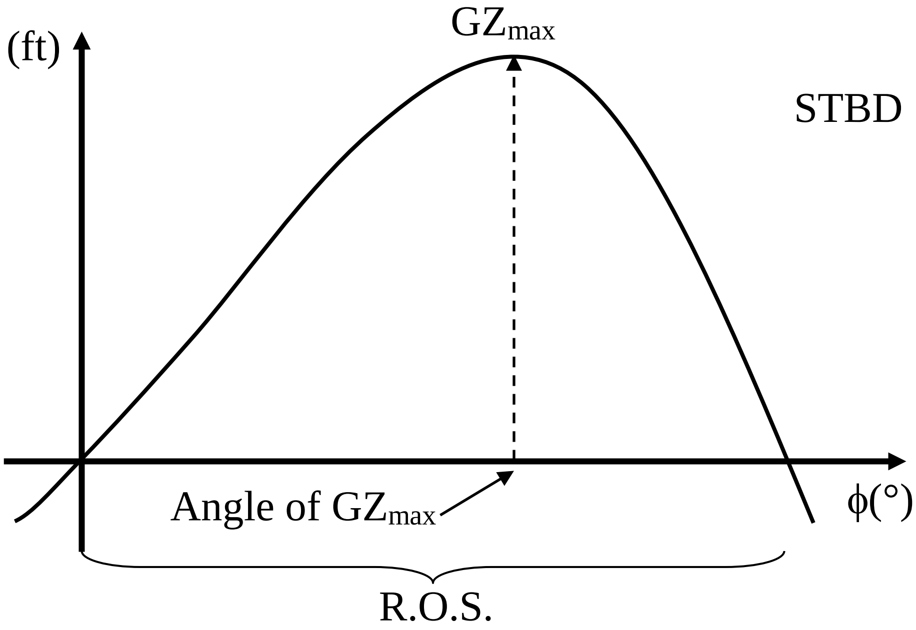


Figure 4.5 The Simplified Intact Statical Stability Curve depicting select stability characteristics

4.4.1 Range of Stability (R.O.S.)

This is the range of angles for which there exists a righting moment. The range starts at the angle corresponding to the ship's equilibrium position with no external moments applied to it and goes to the angle at which the ship will capsize. For a ship with no initial angle of list the starting angle would be zero degrees. If the ship has a permanent angle of list, then the range is given from that angle of list to the capsizing angle of the heeled side.

In Figure 4.2 the Range of Stability is	*0 - 85 degrees for stbd heels*
	0 - 85 degrees for port heels

The greater the range of stability, the less likely the ship will capsize. If the ship is heeled to any angle in the range of stability, the ship will exhibit an internal righting moment that will right the ship if the external moment ceases.

4.4.2 Maximum Righting Arm (GZmax)

This is the largest internal moment arm created by the vertical mis-alignment of the buoyant force and the resultant weight vectors. It is simply measured as the peak of the curve of intact statical stability.

In Figure 4.2 the Maximum Righting Arm is 4.1 ft

4.4.3 Maximum Righting Moment

This is the largest static moment the ship can produce. It is simply calculated from the product of the ship's displacement (Δ_S) by the maximum righting arm (GZmax). The units are LT-ft.

The larger the value of the maximum righting moment the less likely the ship will capsize. The maximum righting moment can't be shown directly on the curve of intact statical stability. Only the maximum righting arm can be shown. However, there is only a scaling difference between the righting arm and righting moment.

4.4.4 Angle of GZmax

This is the angle of heel at which the maximum righting moment occurs. Beyond this angle the righting moment decreases to zero.

In Figure 4.2 the Angle of GZmax is 50 degrees

It is desirable to have this angle occur at large degrees of heel so that a rolling ship will experience a righting moment that increases in magnitude over a greater range of heeling angles.

4.4.5 Dynamic Stability

This is the work done by quasi statically (very slowly) rolling the ship through its range of stability to the capsizing angle. Mathematically, this work is,

$$\Delta_S \int GZ \, d\phi$$

This is the product of the ship's displacement with the area under the curve of intact statical stability. The units are LT-ft. The dynamic stability can' t be shown directly on the curve of intact statical stability but the area under the curve can be shown.

The work represented by dynamical stability is not necessary representative of the work required to capsize a ship in a real seaway. This is because the statical stability curve does not account for rotational momentum, or additional forces that may be present on a real ship in a seaway. It is useful for a comparative basis with other ships or ships of the same type under different operating conditions.

4.4.6 A Measure of the Tenderness or Stiffness

The initial slope of the intact statical stability curve indicates the rate at which a righting arm is developed as the ship is heeled over.

If the initial slope is large, the righting arm develops rapidly as the ship is heeled over and the ship is said to be “stiff.” A stiff ship will have a short period of roll and react very strongly to external heeling moments. The ship will try to upright itself very quickly and forcefully. If the ship is too stiff, violent accelerations can damage ship structures and be harmful to personnel.

If the initial slope is small, the righting arm develops slowly as the ship is heeled over and the ship is said to be “tender”. A tender ship will have a long period of roll and react sluggishly to external heeling moments. Too tender of a ship can compromise stability and leave too little margin for capsizing.

4.5 The Effects of a Vertical Shift in the Center of Gravity of the Ship on the Righting Arm (GZ): Sine Correction

We have already seen that the Curve of Intact Statical Stability can be created from the Cross Curves of Stability. However, the Cross Curves assume a value for KG (regularly KG = 0 ft). To obtain the true Righting Arm Curve, the values from the cross curves must be corrected for the true vertical location of G. This is achieved using the sine correction.

There are 2 instances when the sine correction is necessary.

- Correcting the Curve of Intact Statical Stability for the true vertical location of G.
- Correcting the Curve of Intact Statical Stability for changes in KG.

The theory behind the sine correction can be seen by an analysis of Figure 4.5. It is obvious from the Figure that a rise in KG decreases the righting arm. If G_v is the final vertical location of the center of gravity, and G_0 is its initial location, then the value of G_vZ_v at each angle of heel may be found using the following relationship:

$$G_V Z_V = G_O Z_O - G_O G_V \sin\phi$$

where: G_vZ_v is the righting arm created by the final center of gravity (ft).
G_0Z_0 is the righting arm created by the initial center of gravity (ft).
G_0G_v is the vertical distance between G_0 and G_v (ft).
$G_0G_v \sin\phi$ is the sine correction term (ft).

This equation should be evident from Figure 4.5 by examining the right angled triangle G_0PG_v and by observing that the distance G_vZ_v is the same as the distance PZ_0.

$$G_V Z_V = PZ_O = G_O Z_O - G_O P \;\; and \;\; G_O P = G_O G_V \sin\phi$$

$$\Rightarrow G_V Z_V = G_O Z_O - G_O G_V \sin\phi$$

or:

$$\overline{GZ}_{final} = \overline{GZ}_{initial} - \delta\overline{KG}\sin\phi$$

(!) Students must be able to draw Figure 4.5 and be able to derive the sine correction from this Figure.

A similar analysis to Figure 4.6 should reveal that the sine correction term must be added if KG is reduced.

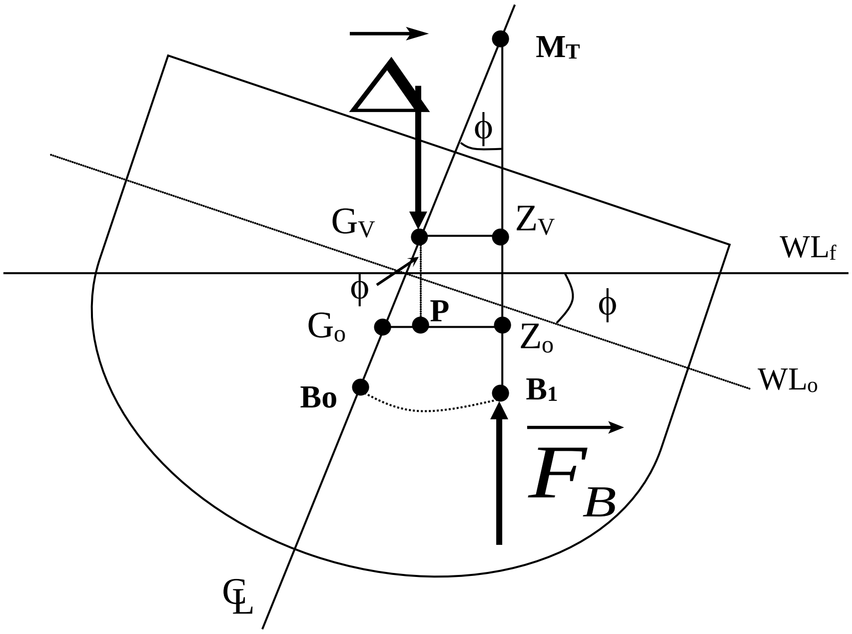


Figure 4.6 The Sine Correction Derivation

In this Figure, the following segments are defined:

WL_0	is the original waterline
WL_f	is the new waterline
G_0Z_0	is the righting arm prior to a shift in the center of gravity
G_vZ_v	is the righting arm after a shift in the center of gravity
B_1	is the center of buoyancy after the ship lists
B_0	is the center of buoyancy before the ship lists

Example 4.1 Draw the intact statical stability curve for the DDG51 assuming a displacement of 8600 LT and a vertical center of gravity of 23.84 ft above the keel. Graph both G_0Z_0 and G_vZ_v values as a function of heeling angle on the intact statical stability curve. The cross curves for the DDG51 are located in the ship data section.

Solution: The general form of the sine correction at each angle is

$$G_vZ_v = G_0Z_0 - 23.84 \sin \phi.$$

For instance, at 20 degrees:

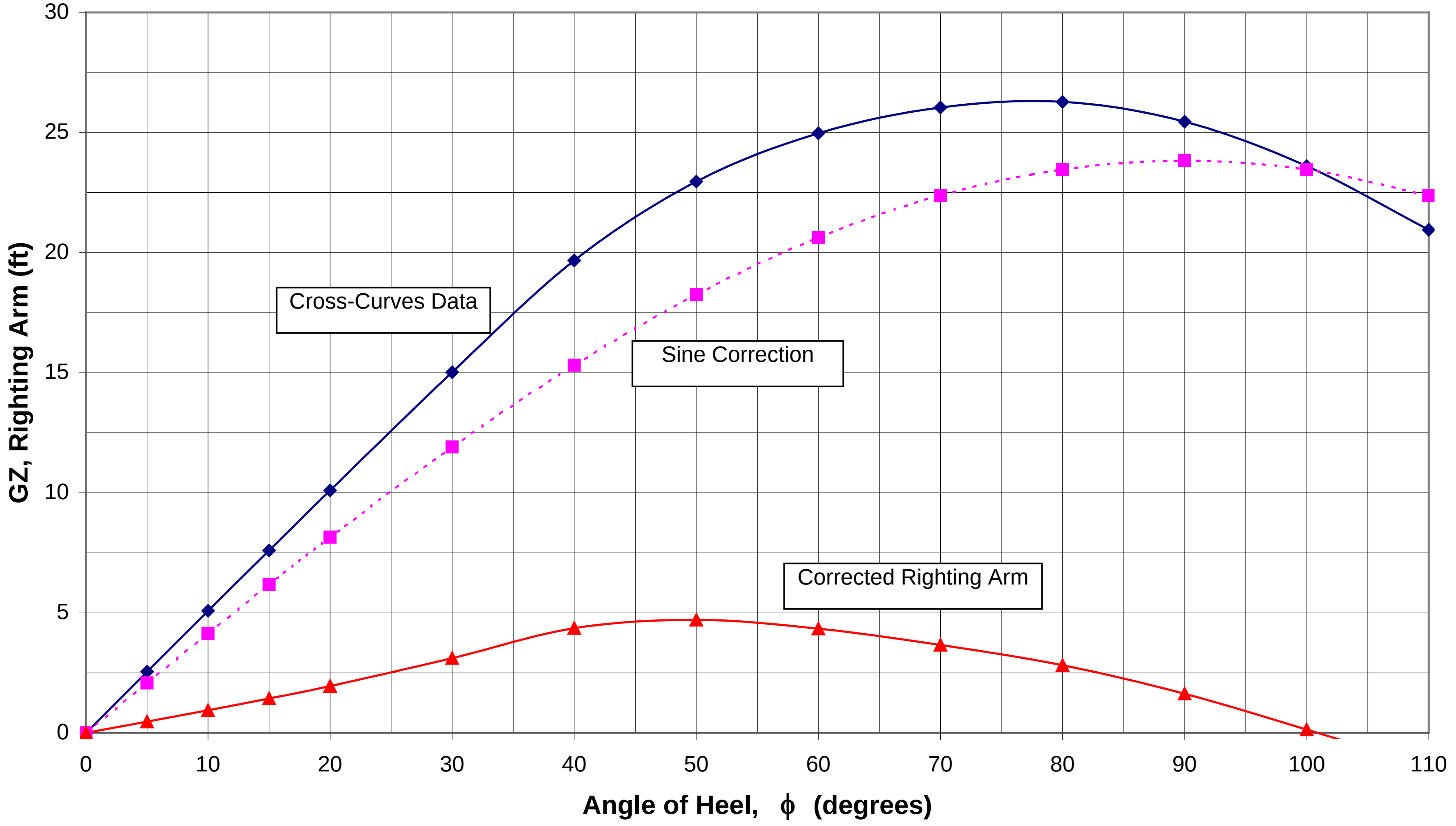
$$G_vZ_v = 10.10 - 23.84 \sin (20) = 1.95 \text{ ft.}$$

However, it often more convenient to use a table.

Angle of Heel, ϕ (degrees)	0	5	10	15	20	30	40
Righting Arm from Cross Curves, G_0Z_0 (ft)	0	2.55	5.08	7.60	10.10	15.02	19.67
Sine Correction Term (ft)	0	2.08	4.14	6.17	8.15	11.91	15.31
Corrected Righting Arm, G_vZ_v (ft)	0	0.47	0.94	1.43	1.95	3.11	4.36

Angle of Heel, ϕ (degrees)	50	60	70	80	90	100	110
Righting Arm from Cross Curves, G_0Z_0 (ft)	22.96	24.97	26.04	26.28	25.45	23.60 (given)	20.95 (given)
Sine Correction Term (ft)	18.25	20.63	22.38	23.46	23.82	23.46	22.38
Corrected Righting Arm, G_vZ_v (ft)	4.71	4.34	3.66	2.82	1.63	0.14	-1.43

When plotted, these new G_vZ_v values will give a Curve of Intact Statical Stability for DDG51 which is correct for a displacement of 8600 LT and KG = 23.84 ft. If displacement changes, then new G_0Z_0 values must be obtained from the Cross Curves and corrected for KG. If KG changes, then a sine correction can be made between 23.84 ft and the new value of KG.



Example 4.1 - Statical Stability Curve

DDG-51 @ 8600 LT, KG = 23.84 ft.

4.6 The Effects of a Transverse Shift in the Center of Gravity of the Ship on the Righting Arm (GZ): Cosine Correction

The stability analysis so far has considered the center of gravity on the centerline, or TCG = 0 ft. We saw in Chapter 3 that the center of gravity may be moved off the centerline by weight additions, removals, or shifts such as cargo loading, ordinance firing, and movement of personnel. When this occurs, there is an effect upon the stability of the ship.

The effect upon stability of a transverse shift in G can be calculated using the cosine correction.

There are 2 instances when the cosine correction is necessary.

- Correcting the Curve of Intact Statical Stability for the true transverse location of G.
- Correcting the Curve of Intact Statical Stability for changes in TCG.

An analysis of Figure 4.7 showing a shift in the transverse location of G from the centerline enables the cosine correction to be quantified. The new righting arm may be computed at each angle using the following equation.

$$G_tZ_t = G_VZ_V - G_VG_t \cos\phi$$

or

$$\overline{GZ}_{final} = \overline{GZ}_{initial} - \delta TCG \cos\phi$$

where:

G_tZ_t	is the corrected righting arm (ft)
G_vZ_v	is the uncorrected righting arm (ft)
G_vG_t	is the transverse distance from the centerline to the center of gravity (ft)
$G_vG_t \cos\phi$	is the cosine correction term (ft)

This equation should be evident from Figure 4.7 by examining the enlarged right angled triangle at the top of the Figure.

Students must be able to draw Figure 4.7 and be able to derive the cosine correction from this Figure.

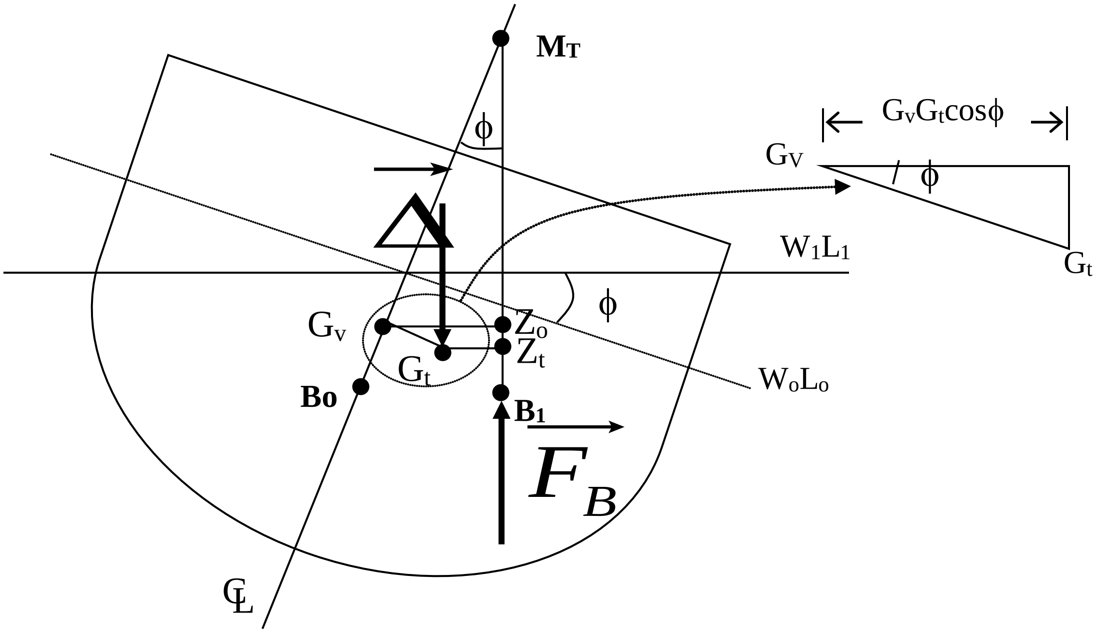


Figure 4.7 The Cosine Correction Derivation

The new righting arm (G_tZ_t) created due to the shift in the transverse center of gravity is smaller than the righting arm created if the transverse center of gravity had not been moved (G_0Z_0).

However, if heeling to port was considered the righting arm would increase. A similar diagram to Figure 4.7 can show that for the opposite side to the weight shift, the cosine correction is added to give the corrected righting arm.

Example 4.2 For a DDG51 with a displacement of 8600 LT, a vertical location of the center of gravity of 23.84 ft from the keel, and a transverse location of the center of gravity of 0.4 feet to the starboard of centerline, graph the intact statical stability curve.

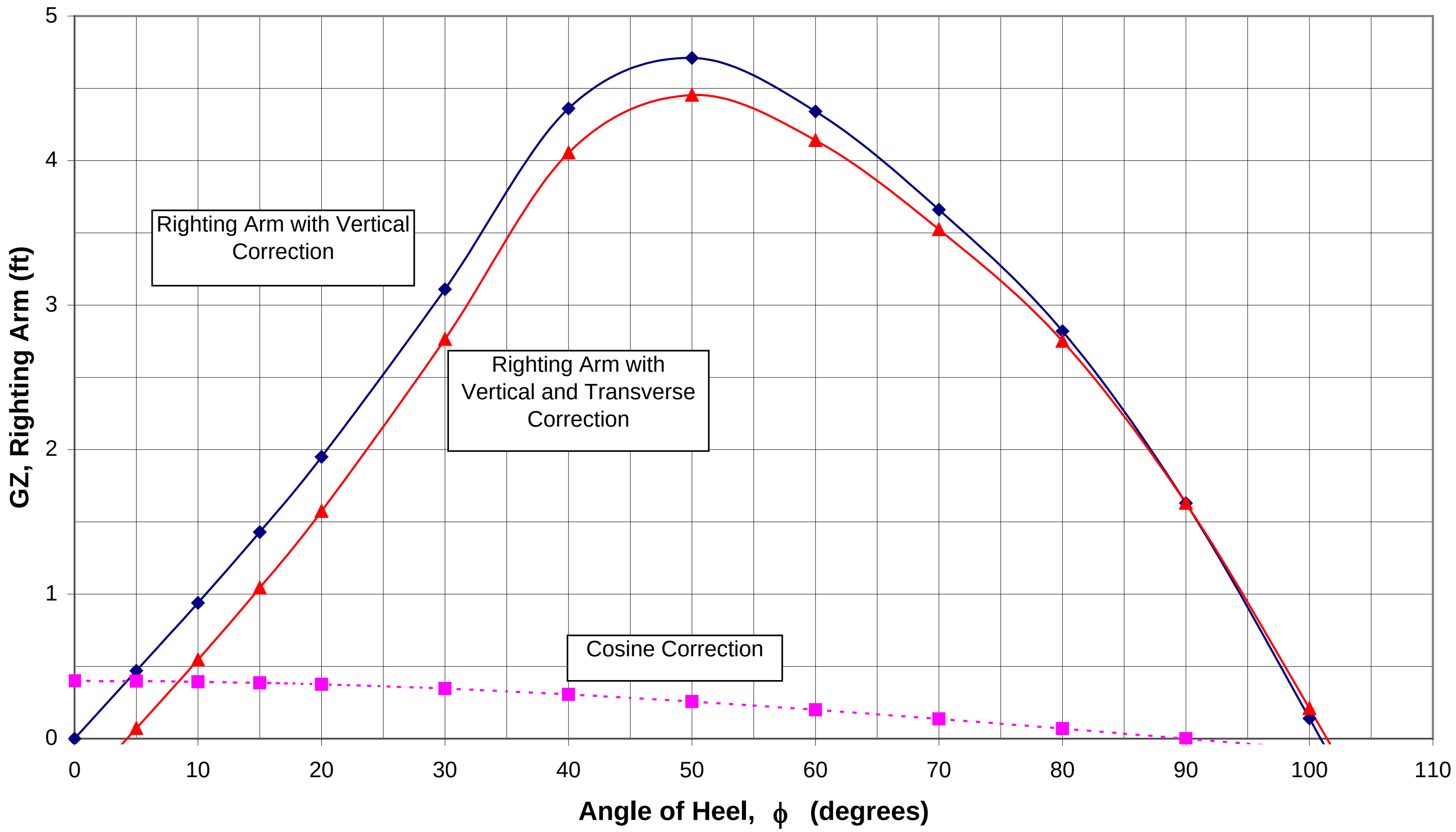
Solution: (First four rows are from Example 4.1)

Angle of Heel, ϕ (degrees)	0	5	10	15	20	30	40
Righting Arm from Cross Curves, G_0Z_0 (ft)	0	2.55	5.08	7.60	10.10	15.02	19.67
Sine Correction, (ft)	0	2.08	4.14	6.17	8.15	11.91	15.31
Vertically Corrected Righting Arm, G_vZ_v (ft)	0	0.47	0.94	1.43	1.95	3.11	4.36
Cosine Correction, (ft)	**0.40**	**0.40**	**0.39**	**0.39**	**0.38**	**0.35**	**0.31**
Transversely Corrected Righting Arm, G_tZ_t (ft)	**-0.40**	**0.07**	**0.55**	**1.04**	**1.57**	**2.76**	**4.05**

Angle of Heel, ϕ (degrees)	50	60	70	80	90	100	110
Righting Arm from Cross Curves, G_0Z_0 (ft)	22.96	24.97	26.04	26.28	25.45	23.60	20.95
Sine Correction, (ft)	18.25	20.63	22.38	23.46	23.82	23.46	22.38
Vertically Corrected Righting Arm, G_vZ_v (ft)	4.71	4.34	3.66	2.82	1.63	0.14	-1.43
Cosine Correction, (ft)	**0.26**	**0.20**	**0.14**	**0.07**	**0**	**-0.07**	**-0.14**
Transversely Corrected Righting Arm, G_tZ_t (ft)	**4.45**	**4.14**	**3.52**	**2.75**	**1.63**	**0.21**	**-1.29**

Example 4.2 - Statical Stability Curve

DDG-51 @ 8600 LT, KG = 23.84 ft., TCG = 0.4 ft.



The following should be considered, regarding transverse weight shifts:

- As in the case of a vertical increase in the center of gravity, a horizontal shift away from centerline results in worsened stability characteristics on the side to which G moves. This should be evident from the example.

- A horizontal shift results in improved stability characteristics on the side opposite to which G moves. This can explain the need to lean out when attempting to prevent a small sailing craft from capsizing.

- Capsizing. It is interesting to note that, according to the curves calculated, the ship will capsize at a greater angle with G off of the centerline. In reality, the ship will capsize before the angle at which GZ = 0 ft in any case. These curves do not account for the fact that at extreme angles non-watertight parts of the hull and superstructure will be immersed (in particular the gas turbine exhaust stacks), allowing water to enter the ship resulting in a capsize 10 - 20 degrees earlier than predicted by these curves. Also, at extreme angles equipment is likely to move within the ship, further decreasing stability.

Student Exercise: Figure 4.8 is a statical stability curve for the DDG-51 with a 0.4 ft starboard shift in the center of gravity with a displacement of 8600 LT and a KG of 23.84 ft. Fill in Figure 4.8 on the following page with the sectional diagrams for each of the points indicated on Figure 4.7. In your diagrams include G, B, Δ_S , F_B , etc.

(This is similar to Figure 4.2, but this time there is a transverse shift in the center of gravity.)

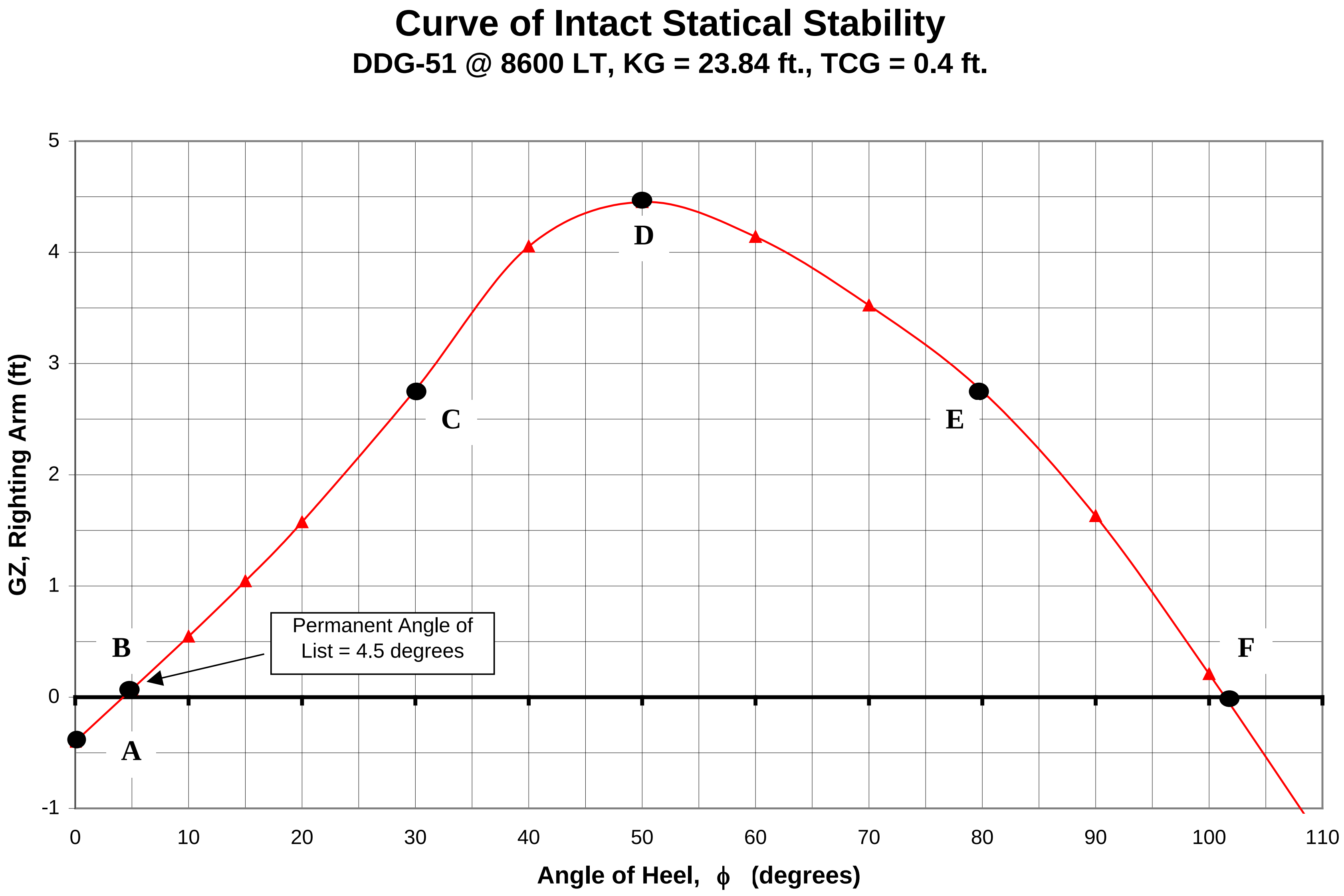


Figure 4.8 Curve of Intact Statical Stability for Student Exercise

Point A - 0 degrees of heel	**Point B -** 4.5 degrees of heel
Point C - 30 degrees of heel	**Point D -** 50 degrees of heel
Point E - 80 degrees of heel	**Point F -** 102 degrees of heel

Figure 4.9 Vector Diagrams Associated with Figure 4.7

4.7 Damage Stability

Naval ships are intended to go in harms way. When the shooting starts the object is to do harm to others, but sometimes damage to your ship is unavoidable. If the watertight portion of the hull is breached and water pours into the ship, the draft will increase, the trim will change, a permanent angle of list will result, and stability will be affected. In extreme circumstances the ship could be lost.

This section discusses the fundamental behavior of a damaged ship and introduces two techniques that allow its analysis.

- The Lost Buoyancy Method.

- The Added Weight Method.

The lost buoyancy method will be discussed only briefly. However, the added weight method will be covered in a little more depth. You will be required to perform simplified damaged ship calculations using the added weight method.

US Navy Damage Stability standards will also be covered so that you will have some idea how your ship will respond, and how much it is designed to take.

4.7.1 Lost Buoyancy Method

One method to examine the behavior of a damaged ship is by the lost buoyancy method. In the lost buoyancy method we analyze changes in buoyancy rather than the center of gravity or displacement. Simply stated, the center of gravity remains the same (the ship weight, metal etc is constant) and any changes due to damage effect the distribution of the buoyancy volume. The total buoyant volume must remain constant since the weight of the ship is not changing. The draft will increase and the ship will list and trim until the lost buoyant volume is regained.

The lost buoyancy method allows a damaged ship to be modeled mathematically so that the final drafts, list, and trim can be determined from assessed damage. The engineer can analyze every conceivable damage scenario and produce a damage stability handbook that may be used by the crew in the event of flooding. Using the lost buoyancy method allows “a prior” knowledge of the resulting stability condition of the ship so that appropriate procedures can be written and followed in the event of a breach in the ship’s hull.

4.7.2 The Added Weight Method

Another method of examining the damaged ship is with the “Added Weight” method. As the name suggests, in this technique, the ship is assumed undamaged, but part of it is filled with the water the ship is floating in. This is equivalent to a weight addition and can be modeled using the techniques for shifts in the center of gravity of the ship (G) covered in Chapter 3.

Provided the volume of the damaged compartment, its average location from the centerline, Keel & midships and the water density is known, the shift in G can be predicted along with the consequences of this shift upon the draft, trim and list of the ship.

4.7.2.1 Permeability

An added complication to the analysis of a damaged ship is the space available in a damaged compartment for the water to fill.

When a compartment is flooded, it is rare for the total volume of this compartment to be completely filled with water. This is because the compartment will already contain certain equipment or stores depending upon its use. The ratio of the volume that can be occupied by water to the total gross volume is called the “permeability”.

$$Permeability = \frac{volume\ available\ for\ flooding}{total\ gross\ volume} = \mu$$

For calculating the affected volumne,

$$\nabla actual = \nabla total \cdot (\mu)(\%\ \text{Flooded})$$

The table below from “Basic Ship Theory - 4th Edition” by Rawson & Tupper lists some typical ship compartment permeabilities.

Space	**Permeability (%)**
Watertight Compartment (Warship)	97
Watertight Compartment (Merchant Ship)	95
Accommodation Spaces	95
Machinery Compartments	85
Dry Cargo Spaces	70
Bunkers, Stores or Cargo Holds	60

We should now be in a position to perform simple added weight damage calculations.

Example 4.3 An FFG-7 displacing 3992 LT and of length 408 ft has KG = 18.5 ft, TCG = 0 ft. It is floating in sea-water at level trim with a draft of 16.0 ft. At this draft, TPI = 33.0 LT/in, MT1" = 793.4 LT-ft/in and LCF = 24.03 ft aft of midships.

A collision causes the complete flooding of the auxiliary machinery space. This space has a volume of 6400 ft^3, permeability of 85% and a centroid on the centerline, 6.6 ft above the keel and 30ft fwd of midships.

Calculate:

a. The KG in the damaged condition.

b. TCG in the damaged condition.

c. The T_{fwd} in the damaged condition.

Solution:

a. $$\text{Volume available for flooding} = \text{permeability x volume} = 0.85 \text{ x } 6400 \text{ ft}^3 = 5400\,ft^3$$

$$\text{Weight of water in compartment} = \rho g(\text{flooded volume})$$

$$= 1.99\frac{lb-s^2}{ft^4}32.17\frac{ft}{s^2}5400\,ft^3\,\frac{1\,LT}{2240\,lb} = 155\,LT$$

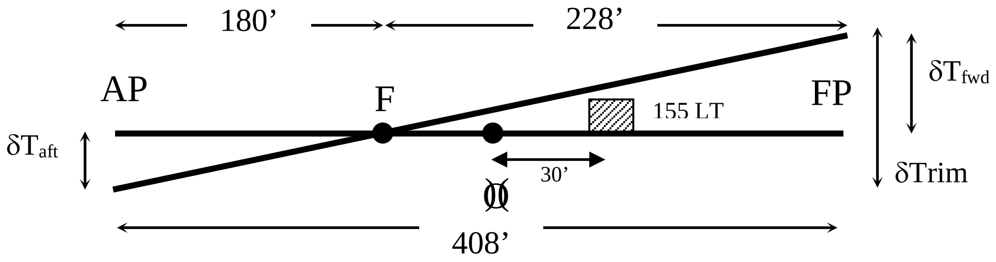
$$KG_{damaged} = \frac{KG_{old}\Delta_{old} + w_{flooding}kg}{\Delta_{damaged}}$$

$$KG_{damaged} = \frac{3992LT \cdot 18.5\,ft + 155LT \cdot 6.6\,ft}{3992 + 155\,LT}$$

$$KG_{damaged} = 18.06\,ft$$

b. $TCG_{damaged} = TCG_{old} = 0\,ft$ (Damaged compartment centroid is on centerline)

c.



$$\delta Trim = \frac{wl}{MT1''} = \frac{155LT \cdot (24+30)\,ft}{793.4\,{}^{ftLT}\!/_{in}} \cdot \frac{1\,ft}{12in} = 0.88\,ft$$

$$\delta T_{PS} = \frac{w}{TPI} = \frac{155LT}{33.0\,{}^{LT}\!/_{in}} \cdot \frac{1\,ft}{12in} = 0.39\,ft$$

From the trim diagram

$$\frac{\delta T_{fwd}}{d_{fwd}} = \frac{\delta Trim}{L_{pp}}$$

$$\delta T_{fwd} = d_{fwd}\,\frac{\delta Trim}{L_{pp}} = 228\,ft\,\frac{0.88\,ft}{408\,ft} = 0.49\,ft$$

So:

$$T_{fwd\ damaged} = T_{fwd,old} + \delta T_{ps} + \delta T_{fwd}$$

$$T_{fwd\ damaged} = 16.0\,ft + 0.39\,ft + 0.49\,ft$$

$$T_{fwd\ damaged} = 16.88\,ft$$

4.7.3 US Navy Damage Stability Design Criteria

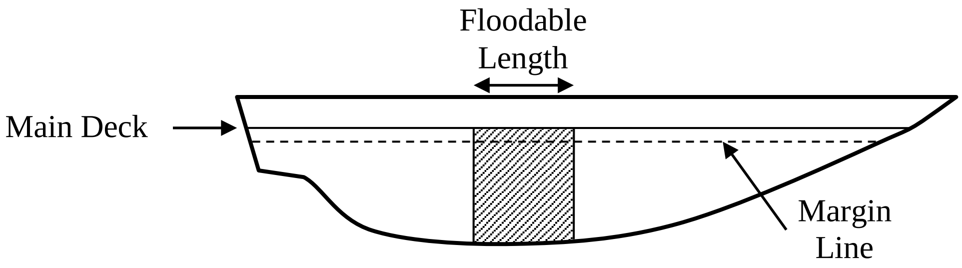


Figure 4.10 Illustration of Damage Stability Terms

Margin Line The margin line is a line defining the highest permissible location on the side of the vessel of any damaged waterplane in the final condition of sinkage, trim and heel. It is in no case permitted to be less than 3 inches (0.075 m) below the top of the bulkhead deck at the side. (PNA pp178)

List The heel caused by damage shall not exceed 20 degrees. This angle is too great for continuous operation of equipment. Naval machinery is designed to operate indefinitely at a permanent list of 15 degrees, although most equipment will probably remain functional up to about 25 degrees for at least a few hours. Personnel can continue damage control efforts effectively at a permanent list of 20 degrees. At a permanent list of 20 degrees, the ship will possess adequate stability against wind and waves to be towed at the very least.

Floodable Length

1. Ships less than 100 ft long are required to withstand flooding in one compartment.
2. Ships 100 - 300 ft long are required to withstand flooding in any two adjacent compartments.
3. Warships, troop transports and hospital ships over 300 ft long are required to withstand a hull opening of 15 % of the length between perpendiculars.
4. Any other ship over 300 ft long are required to withstand a hull opening of 12.5% of the length between perpendiculars.

4.7.4 Foundering and Plunging

A damaged ship could be lost in one of several ways.

- If the ship is left with inadequate maximum righting moment or dynamical stability, it could simply be overwhelmed by the seaway and the weather.

- If the angle of list or trim is too great, placing non-watertight parts of the ship underwater, then additional flooding will occur. In this case the ship could lose transverse stability, roll over and capsize.

- Longitudinal stability could also be lost in a similar manner causing the ship to plunge (go down bow or stern first). One of the most notable examples of plunging is the *Titanic*.

- A ship may be lost even if stability is not compromised. It may simply sink. This is called *foundering*.

The preceding discussion concerned ships which were in a static condition, meaning that the damage had occurred and the ship was in equilibrium. From the time damage occurs until equilibrium is reached the ship is in a very vulnerable state. The water rushing into the ship and the sudden changes in effective volume cause a number of dynamic effects in the face of reduced stability.

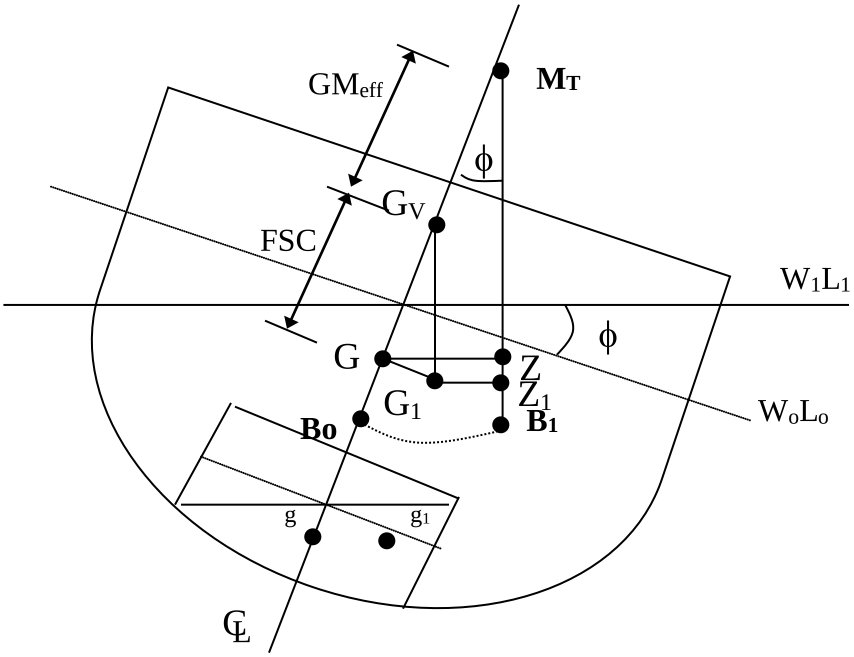
In some cases it is useful to flood a tank on the side of the ship opposite the damage in order to reduce the angle of list and lower KG. This is called *counter flooding*. However, counter flooding can be very dangerous.

Counter flooding results in an increase in displacement, causing ship's draft to increase. The increase in draft results in a loss of freeboard and a reduction in the angle of heel at which the deck edge will go underwater. The increase in displacement may also make the ship deeper than its limiting draft, which may cause further stability and structural problems. Additionally, if counter flooding is not done correctly, the possibility exists of adding an additional free surface to the ship, a very serious stability problem.

4.8 Free Surface Correction at Small Angles of Heel

A free surface is fluid that is allowed to move freely, such as water in a partially filled tank. As the ship lists, the fluid in the tank moves. The fluid movement acts like a weight shift, causing the center of gravity of the fluid to move which causes the ship's center of gravity to shift in both the vertical and horizontal directions. The effect of the vertical shift is negligible at small angles ($\phi < 5°$ to $7°$) and is discounted, but the horizontal (transverse) shift of the center of gravity causes a decrease in the righting arm (GZ).

It is shown graphically in Figure 4.11 that a vertical rise in the center of gravity also causes a shortened righting arm. The distance the center of gravity would have to rise to cause a reduction in the righting arm equivalent to that caused by the actual transverse shift is called the *Free Surface Correction* (FSC). The position of this new center of gravity is called the "virtual" center of gravity (G_v). The distance from the virtual center of gravity to the metacenter is called the *Effective Metacentric Height* (GM_{eff}).



Note: $\phi < 5 – 7$ degrees (list angle in drawing is exaggerated to show geometry)

Figure 4.11 The Free Surface Correction

4.8.1 Static Effects

The static effects of free surface are adverse resulting in a virtual rise in the center of gravity, a smaller range of stability, a smaller maximum righting arm, a small angle at which the maximum righting arm occurs, and an exaggerated list and trim if the ship is listing or trimming.

4.8.2 Dynamic Effects

It should be noted that the preceding analysis is referring to only the static effects of free surface. Free surfaces also produce dynamic effects of the water rushing back and forth. This is also detrimental but is not described by the free surface correction. ***It is a common misconception to mix the dynamic effects of free surface with the static analysis and the FSC.***

To understand the dynamic effects fill a Tupperware plastic container half full of water, close it with a lid, and put it in the palm of your hand. Move the container so that it lists and trims. Notice how the geometry affects the magnitude of the roll and momentum of the fluid mass when the container is rolled in a listing condition versus a trimming condition. Another example of the dynamic effect is a fire engine carrying water down the road. If baffles are not put in the tank the truck will literally jump from side to side because of the water moving back and forth. Baffles are a good way to minimize the dynamic effects of free surface.

4.8.3 Calculating the FSC and GM_{eff}

The free surface correction (FSC) created by a tank within a ship is given by the following equation:

$$FSC = \frac{\rho_t i_t}{\rho_s \nabla_s}$$

where: ρ_t is the density of the fluid in the tank (lb-s^2/ft^4)
ρ_s is the density of the water the ship is floating (lb-s^2/ft^4)
∇_s is the underwater volume of the ship (ft^3)
i_t is the transverse second moment of area of the tank's free surface area (ft^4)

The formula for i_t is given on the next page.

Note: “Tank” is used synonomously with “compartment” or “space” when discussing/calculating FSC for damage stability.

4.8.3.1 The Second Moment of Area (i_t)

The formula for the second moment of area of a rectangle is given by the following equation. The distances refer to Figure 4.12.

$$i_t = \frac{(length)(width)^3}{12}$$

$$i_t = \frac{l \cdot b^3}{12}$$

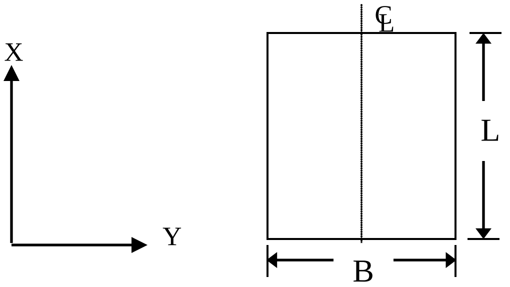


Figure 4.12 Tank Geometry for FSC

The free surface correction is applied to the original metacentric height to find the effective metacentric height:

$$\overline{GM}_{eff} = \overline{GM} - FSC = \overline{KM} - \overline{KG} - FSC$$

4.8.4 Minimizing the Effects of Free Surface

- **Compartmentalization:** A quick observation of the equation for i_t and FSC above should reveal that splitting a tank transversely with dividers running longitudinally will reduce the distance B and consequently have a major effect upon the magnitude of the FSC.

- **Pocketing:** Tanks should be kept at least 90% full so that pocketing occurs. Pocketing is when the liquid hits the top of the tank thus reducing the free surface effects. Pocketing therefore is a desirable physical event.

- **Compensated Fuel Oil Tanks:** Some ships use a water compensated fuel oil system to minimize the free surface effect (a small FSC remains due to the density difference between oil and seawater). This system replaces used fuel with salt water so no free surface occurs. The salt water is immiscible with the oil so no mixing occurs. Typically at least two tanks are used so that the boundary between salt water and the oil always stays one tank away from the engine. The intermediate tank is often referred to as a *clean fuel oil tank*. These types of tanks are used on CG-47, DDG-51, and nuclear sumarines (shielding).

- **Empty Tanks:** Obviously, the FSC is reduced completely if the tanks are empty!

Flooding aboard a ship can create compartments with free surface. This can affect the stability of the ship. Flooding can be caused by fire fighting as well as breaches in the hull. Putting fires out by a fire hose can add weight high in the ship and create free surface. Both of these will cause a rise in the center of gravity, smaller righting arms and less overall stability.

Example 4.4 An FFG-7 class ship displacing 4092 LT has KG=18.9 ft and KM=22.49 ft. There is a tank filled with fuel oil with a density of 1.5924 lb-s^2/ft^4 creating a free surface 30 ft wide and 60 ft long. The ship is floating in salt water with a density of 1.9905 lb-s^2/ft^4. What is the effective metacentric height?

Solution:

$$\Delta_s = \rho g \nabla_s$$

$$\nabla_s = \frac{\Delta_s}{\rho g} = \frac{4092 LT \cdot 2240 lb/LT}{1.9905 lb-s^2/ft^4 \cdot 32.17\, ft/s^2} = 143{,}143\, ft^3$$

$$i_t = \frac{l \cdot b^3}{12} = \frac{60\, ft \cdot (30\, ft)^3}{12} = 135{,}000\, ft^4$$

$$FSC = \frac{\rho_t i_t}{\rho_s \nabla_s}$$

$$FSC = \frac{1.5924 lb-s^2/ft^4 \cdot 135{,}000\, ft^4}{1.9905 lb-s^2/ft^4 \cdot 143{,}143\, ft^3} = 0.75\, ft$$

$$\overline{GM_{eff}} = \overline{KM} - \overline{KG} - \overline{FSC}$$

$$\overline{GM_{eff}} = 22.49\, ft - 18.9\, ft - 0.75\, ft$$

$$\overline{GM_{eff}} = 2.84\, ft$$

4.8.5 Effect of a Free Surface on GZ and Angle of List

As discussed earlier in this section, and shown in Figure 4.9, a free surface causes a reduction in the ship's righting arm, range of stability, and dynamic stability. With a free surface, the ship now behaves as if the center of gravity were located at the virtual center of gravity. To calculate the effective righting arm of a ship with a free surface, the original righting arm must be corrected for the virtual rise in G caused by the free surface. Fortunately, you already have the tool with which to make this correction: the sine correction. Using Figure 4.9 as a guide, the effective righting arm of a ship may be given as:

$$\overline{G_1Z_1} = \overline{GZ} - \overline{GG_v} \sin\phi \qquad \text{or} \qquad \overline{G_1Z_1} = \overline{GZ} - FSC \sin\varphi$$

The worst case for a free surface is when the ship's transverse center of gravity is located off of the centerline. Section 4.6 demonstrated that a transverse shift in G resulted in a reduction in the righting arm and overall stability. A free surface coupled with G being off the centerline is an especially bad case. Not only has the overall stability been reduced by the transverse location of G, but the effective rise in G due to the free surface further reduces righting arms, range of stability, and dynamic stability. To correct the righting arm curve for a free surface and a transverse change in G, one must first correct *GZ* for the virtual rise in G caused by the free surface using the sine correction, then correct *GZ* for the transverse location of G using the cosine correction. This correction is given by the following equation:

$$\overline{G_1Z_1} = \overline{GZ} - FSC \sin\phi - TCG \cos\phi$$

A free surface will also exaggerate a list angle. Recall from Chapter 3 that the angle of list for a transverse change in the center of gravity can be found by:

$$\phi = \tan^{-1}\left(\frac{TCG}{GM_T}\right)$$

When a free surface is present, the angle of list is now found using:

$$\phi = \tan^{-1}\left(\frac{TCG}{GM_{eff}}\right)$$

Example 4.5 The FFG-7 in Example 4.4 has a righting arm of 1.33 feet at a heeling angle of 20° and KG = 18.9 ft. What is the effective righting arm of the ship with the free surface present?
From Example 4.4: KM = 22.49 ft, GM_{eff} = 2.84 ft, FSC = 0.75 ft

Solution:

$$\begin{aligned}\overline{GZ}_{eff} &= \overline{GZ} - FSC \sin\phi \\ \overline{GZ}_{eff} &= 1.33\,ft - (0.75\,ft)(\sin 20) \\ \overline{GZ}_{eff} &= 1.07\,ft\end{aligned}$$

If the ship's transverse center of gravity is located 0.5 ft starboard of the centerline, calculate the ship's righting arm and angle of list.

$$\begin{aligned}\overline{GZ}_{eff} &= \overline{GZ} - FSC \sin\phi - TCG \cos\phi \\ \overline{GZ}_{eff} &= 1.33\,ft - (0.75\,ft)(\sin 20) - (0.5\,ft)(\cos 20) \\ \overline{GZ}_{eff} &= 0.60\,ft\end{aligned}$$

Notice the effect of the transverse location of G on the ship's righting arm!

$$\phi = \tan^{-1}\left(\frac{TCG}{GM_{eff}}\right)$$

$$\phi = \tan^{-1}\left(\frac{0.5\,ft}{2.84\,ft}\right)$$

$$\phi = 9.98^\circ$$

4.8.6 – Damage Control and its Effect on Stability and Buoyancy

Naval and commercial ships are designed to resist varying degrees of accidental and battle damage. Design features to mitigate or prevent damage include structural strength members (Chapter 6), watertight compartments, and the stability and buoyancy criteria discussed in this chapter. Maintaining these features at their optimum capabilities requires a constant state of vigilance which you will be partly responsible for whether you are the Damage Control Assistant (DCA) in charge of most of the maintenance on these systems and training the crew or an embarked Marine ensuring that the watertight door you just passed through is shut and dogged.

(!) Conventional wisdom says that 90 percent of the damage control needed to save the ship takes place before the ship is damaged (training, drills, inspection, and maintenance) and only 10 percent can be accomplished after the damage has occurred.

However, once damage has occurred the damage control efforts on the ship are a vitally important all-hands evolution which may often mean the difference between losing or saving the ship:

USS Cole (DDG-67), Gulf of Aden, Yemen, 2000

USS Cole suffered a large hole in its side while refueling in the harbor as a result of a terrorist attack. The explosion ripped through one of the ship's engine rooms and resulted in massive amounts of flooding, a severe list, and loss of electrical power (i.e. no electric bilge pumps). Three days of valiant damage control efforts by the crew kept the ship afloat in the harbor. Damage control methods ranged from judicious use of *counter-flooding* to “bucket-brigades” bailing water out of flooded spaces.

RMS Titanic, 1912

The “practically unsinkable” ship had a two/three compartment standard with many watertight compartments to minimize the effects of flooding but rapid crack propagation in the brittle hull plating led to *progressive flooding* in six adjacent watertight compartments. This flooding alone would eventually sink the ship; however, experts estimate that the ship could have stayed on the surface several hours longer than it did had the crew plugged the cracks in the hull which were only several inches wide with mattresses or some other material. These vital hours could have been long enough to allow the deployment of lifeboats in an orderly fashion and for help to arrive.

SS Normandie, 1942

This ship caught fire in New York City harbor while being converted from a luxury passenger liner to a troop transport to support the war effort. The resulting firefighting efforts from off-hull led to massive weight additions high on the upper decks and large free-surfaces inside the ship. After the fire was extinguished, the ship capsized in calm water pier side as a result of the negative stability introduced by the free-surface and vertical weight shift. This would have been avoided had the ship been *de-watered* following the fire.

4.9 Metacentric Height and the Curve of Intact Statical Stability

So far in this chapter we have considered the overall stability of a ship through all angles by creating and analyzing the curve of intact statical stability. However, in chapter 3 we often used the quantity called the metacentric height (GM), the distance from the center of gravity (G) to the metacenter (M). We also stated that the metacentric height was a measure of a ships initial stability, its ability to remain upright at small angles. Clearly, there must be some link between GM and the curve of intact statical stability.

Recall, that when G is below M, the metacentric height is considered to be positive and when G is above M it is considered to be negative.

4.9.1 The Link Between GM and the Righting Arm Curve

The link can be determined from an analysis of Figure 4.13 showing a ship heeling at small angles.

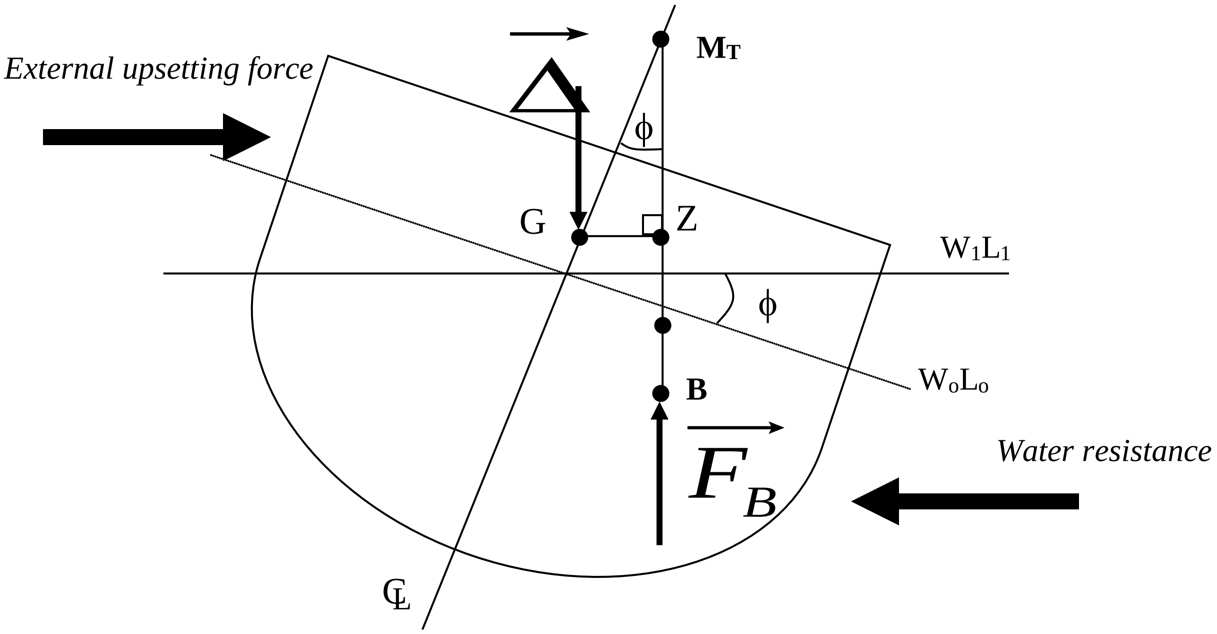


Figure 4.13 A Ship Heeling at Small Angles

At small angles, the right angled triangle (G, Z, M) reveals the following equation for the righting arm.

$$\overline{GZ} = \overline{GM} \sin \phi$$

In the limit as ϕ approaches 0 radians, where the metacenter is defined, the expression may be simplified to GZ = GM ϕ if the angle is given in radians. This is because

$$\sin\phi = \phi \quad \text{(in radians)}$$

Using this, at small angles the equation above becomes:

$$\overline{GZ} = \overline{GM}\phi$$

$$\overline{GM} = \frac{\overline{GZ}}{\phi}$$

The smallest angle that can be achieved is zero radians = zero degrees. Consequently, the magnitude of GM is equal to the magnitude of the initial slope of the Curve of Intact Statical Stability.

Hence the link between GM and the righting arm curve has been established. We will now examine three different ship conditions.

- A ship with positive GM
- A ship with zero GM
- A ship with negative GM

To find the magnitude of the initial slope on the curve of intact statical stability construct two lines and use the intersection of those two lines to determine the magnitude off the "y-axis". The first line is a line tangent to the slope at zero degrees of heel. The statical stability curve must run through zero for this technique to work. If it doesn't go through zero you can draw a parallel line to the tangent line to the slope that does go through zero and proceed with the rest of the steps. The second line is a vertical line at one radian or 57.3 degrees. Where these two lines cross, read over horizontally to the "y-axis" the value of the righting arm. This will be the magnitude of GM.

4.9.2 A Positive Metacentric Height (GM)

This is the ship condition that all the stability examples have been worked with so far. The center of gravity is below the metacenter so that as soon as the ship heels, a righting arm will begin to develop.

Figure 4.14 shows the configuration of the centroids for a ship with positive GM and a typical curve of intact statical stability created by this configuration. The ship has one position where it is static equilibrium which is at zero degrees of heel (provided TCG = 0 ft).

The stability condition is analogous to a marble rolling in a dish. A displacement of the marble to the left or right will result in the marble rolling back to its central stable position. It is in a state of positive stability.

4.9.2.1 Tenderness, Stiffness and the Magnitude of GM

Figure 4.14 also shows the way the magnitude of GM affects the shape of the righting arm curve.

- Large positive GM creates a curve with a steep slope passing through zero degrees of heel. This creates a "stiff" ship, a ship that develops a large righting arm very quickly - the ship is very stable.

- Small positive GM creates a curve with a shallow slope passing through zero degrees of heel. This creates a "tender" ship that develops a righting arm very slowly - the ship is not very stable.

The subject of stiffness verses tenderness will be covered in greater detail when the seakeeping properties of a ship are discussed in chapter 8.

4.9.3 Zero Metacentric Height (GM)

A ship with zero metacentric height is a very rare ship condition. It is where the center of gravity (G) coincides with the ship metacenter (M), there is zero distance between the two points.

Figure 4.14 shows this configuration. It is clear that at small angles of heel, the lines of action of the weight of the ship and the buoyant force remain in vertical alignment. Consequently there is no internal couple created to return the ship to zero degrees of heel. So if the external upsetting force is removed, the ship will remain at this angle!

This condition can be represented by the righting arm curve at Figure 4.12. At small angles of heel to port and starboard, there is zero righting arm developed. The shape of this curve also reaffirms the initial slope being equivalent to the magnitude of GM.

$$\overline{GM} = 0 \quad \Rightarrow \quad \text{Initial slope} = 0 \quad \Rightarrow \quad \overline{GZ}(\phi) \text{ is a horizontal line}$$

Consequently, there is a range of angles of heel where the ship is in static equilibrium.

The condition is analogous to a marble rolling on a flat surface. A displacement of the marble to the left or right will cause the marble to remain in this new position. It is in a state of neutral stability.

Once the ship heels beyond small angles of heel, the movement of M causes a misalignment between the buoyant force and the weight of the ship and a righting arm is developed. However, the curve is very tender.

4.9.4 A Negative Metacentric Height (GM)

A ship with a negative metacentric height has its center of gravity (G) above its metacenter (M). This condition can be created whenever weight shifts, removal or additions significantly elevate G.

Figure 4.12 shows the ship in this condition. As soon as the ship moves beyond zero degrees of heel, the misalignment of the buoyant force and ship's weight vectors tend to help the external upsetting force and continue to roll the ship. The ship is initially unstable.

The righting arm curve for the ship in this condition is also shown at Figure 4.12. Notice that the slope of the curve is negative at zero degrees of heel, supporting the negative value of GM. This condition is analogous to a marble rolling on an upturned bowl. A displacement of the marble to the left or right will cause the marble to continue to roll away from its initial position. It is in a state of negative stability.

4.9.4.1 Lolling

At larger angles of heel, the movement of M causes a righting arm to develop that opposes the roll motion. The curve of intact statical stability in Figure 4.12 supports this. This creates 2 angles of heel where the ship is in static equilibrium, one on the port side and one to starboard. When moving in this condition the ship will oscillate between these 2 conditions creating a very unfavorable motion for those on board. This is called Lolling. The 2 angles of heel at which the ship naturally sits are both called the "angle of loll".

Lolling is an unacceptable situation at sea. Often commercial tankers that are empty can have their center of gravity sufficiently elevated to have a negative metacentric height so that lolling occurs. To stop the lolling, the ship can take on ballast low to lower the center of gravity of the ship to obtain a righting moment at small angles.

Navy ships are designed so that lolling should not occur. If it does, it is telling you that something is wrong operationally and the cause should be determined. If a ship with a negative metacentric height is not lolling it will at least have an initial list.

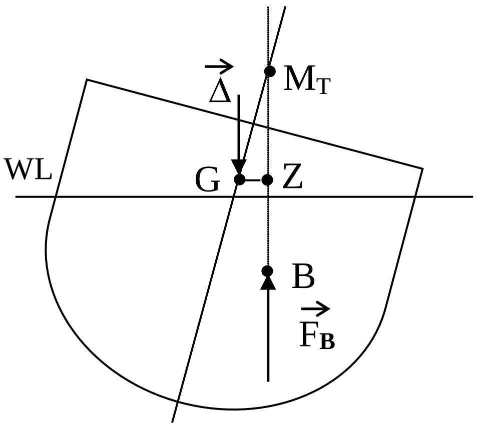
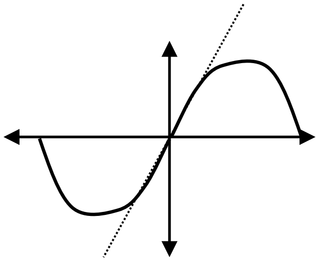
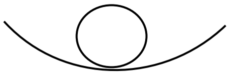
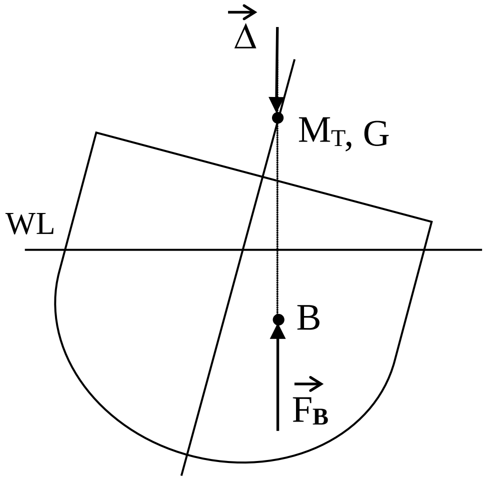
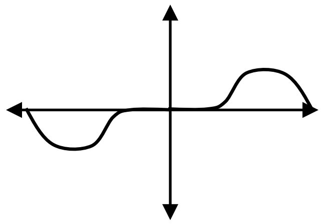
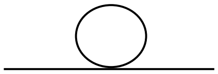
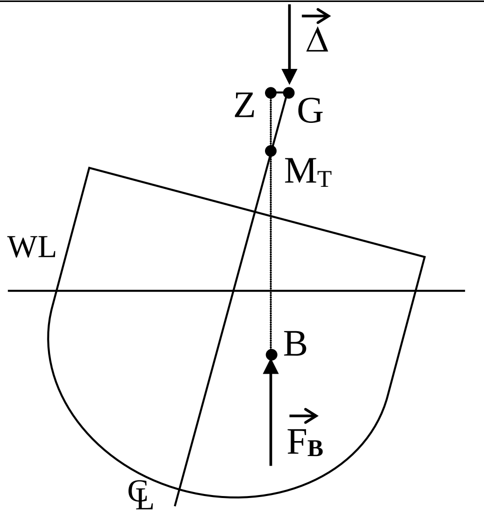
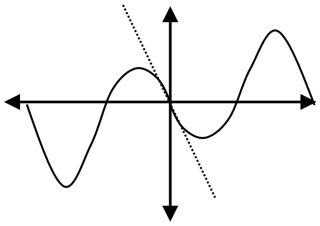
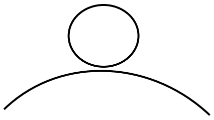
Metacentric Height (GM)	**Section Cut Vector Diagram**	**Curve of Intact Statical Stability GZ (ft) vs. ϕ (deg)**	**Marble Analogy**
Positive		Slope = GM (+) 	
Neutral		Slope = GM = 0 	
Negative		Slope = GM (-) 	

Figure 4.14 Positive, Neutral, and Negative Metacentric Height

4.9.5 Summary

It is critically important to remember that overall ship stability can never be assessed by the sign and magnitude of the metacentric height (GM) alone. The overall measures of statical stability were discussed in Section 4.4. They were:

- Range of stability
- Dynamic stability
- Maximum righting arm
- Maximum righting moment
- The Angle at which the maximum righting moment occurs.

It is incorrect to use GM as the sole yardstick for ship stability. Metacentric Height only indicates whether or not the ship will remain upright over small angles of heel. Additional indicators of ship stability include KG, and draft with respect to limiting draft.

To ensure adequate stability for a ship under all loading conditions every ship has limits on the maximum KG, minimum GM, maximum draft (displacement), and a minimum range of stability. The location of G and ship's displacement with respect to limiting draft can place a ship into one of four distinct stability categories. These categories will determine, for each ship, the amount of weight that can be added or removed from the ship, and the location at which the weight addition or removal may occur.

Status 1 - The ship has adequate weight and stability margins and a weight change at any height is generally acceptable.

Status 2 - The ship is close to limiting draft and stability (KG) limits. Any weight increase or rise in G is unacceptable.

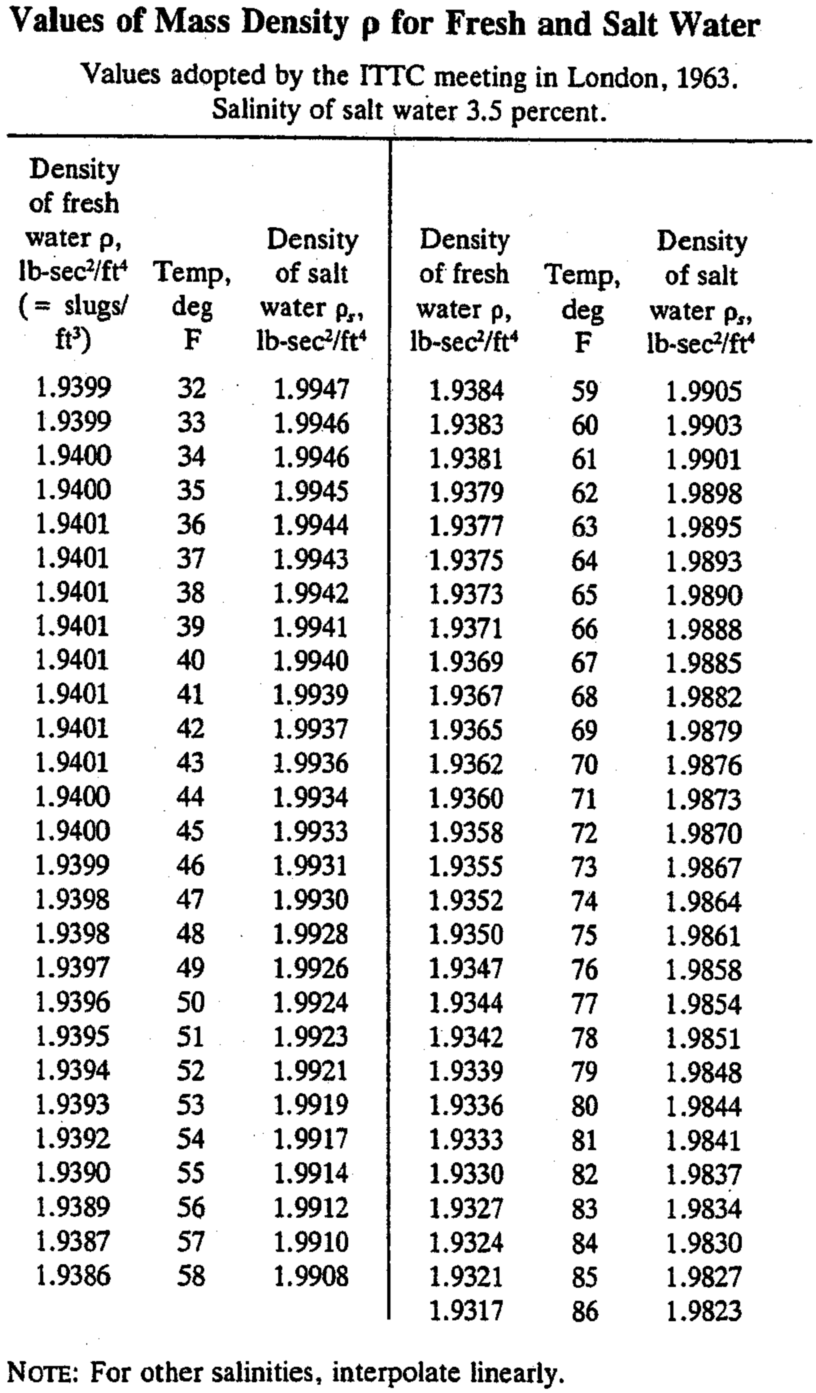
Status 3 - The ship is very close to its stability limit but has adequate weight margin. If a weight change is above the allowable KG value and would cause a rise in G, the addition of solid ballast (lead or concrete) low in the ship may be used to compensate for the weight addition high in the ship.

Status 4 - An adequate stability margin exists, but the ship is departing port very close to its limiting draft. This condition generally applies to tankers and amphibious landing craft.

APPENDIX A

TABLE of FRESH and SALT WATER DENSITY

(reprinted from 'Introduction to Naval Architecture' by Gillmer and Johnson, U.S. Naval Institute, 1982)



Values of Mass Density ρ for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

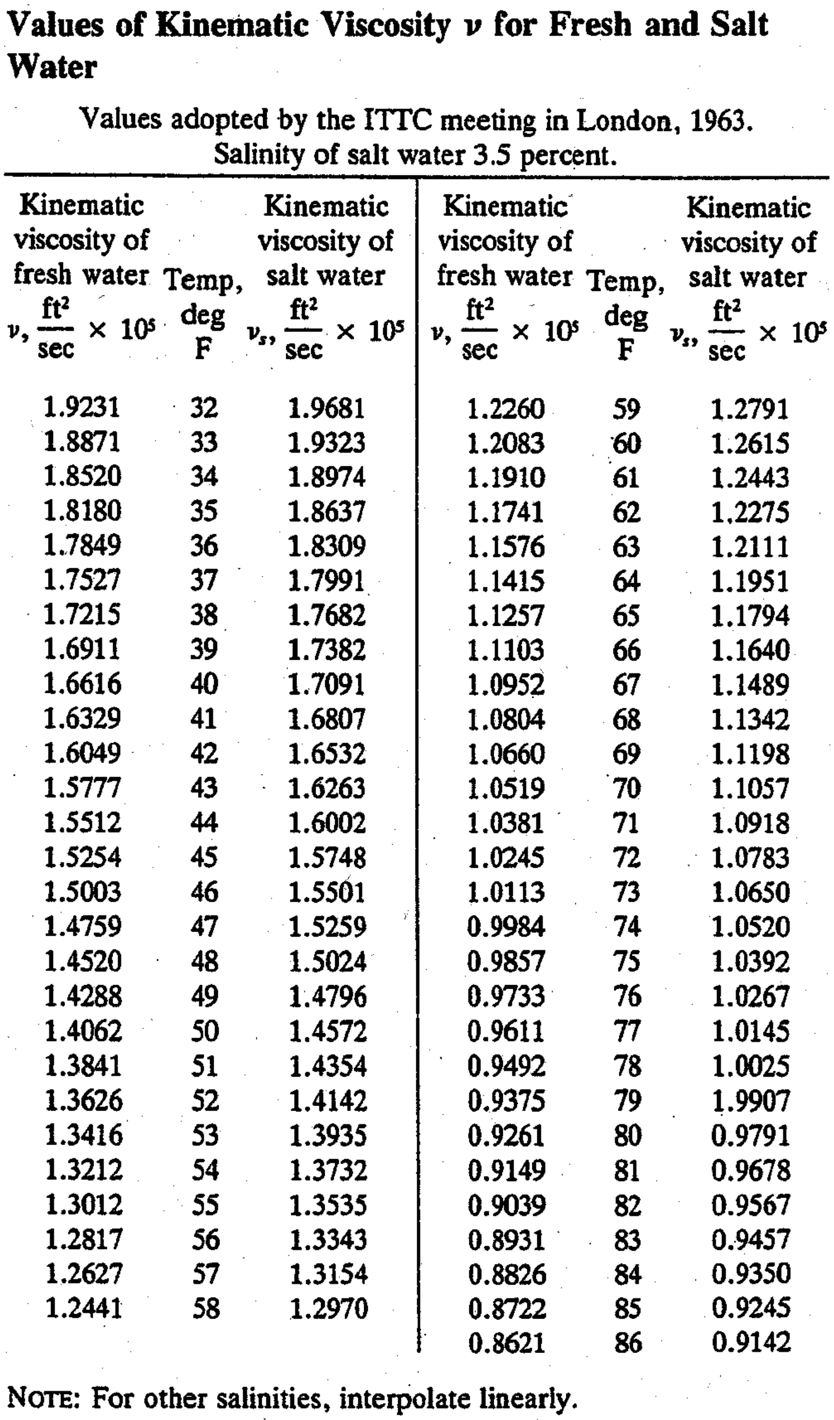
Density of fresh water ρ, lb-sec^2/ft^4 (= slugs/ft^3)	Temp, deg F	Density of salt water ρ_s, lb-sec^2/ft^4	Density of fresh water ρ, lb-sec^2/ft^4	Temp, deg F	Density of salt water ρ_s, lb-sec^2/ft^4
1.9399	32	1.9947	1.9384	59	1.9905
1.9399	33	1.9946	1.9383	60	1.9903
1.9400	34	1.9946	1.9381	61	1.9901
1.9400	35	1.9945	1.9379	62	1.9898
1.9401	36	1.9944	1.9377	63	1.9895
1.9401	37	1.9943	1.9375	64	1.9893
1.9401	38	1.9942	1.9373	65	1.9890
1.9401	39	1.9941	1.9371	66	1.9888
1.9401	40	1.9940	1.9369	67	1.9885
1.9401	41	1.9939	1.9367	68	1.9882
1.9401	42	1.9937	1.9365	69	1.9879
1.9401	43	1.9936	1.9362	70	1.9876
1.9400	44	1.9934	1.9360	71	1.9873
1.9400	45	1.9933	1.9358	72	1.9870
1.9399	46	1.9931	1.9355	73	1.9867
1.9398	47	1.9930	1.9352	74	1.9864
1.9398	48	1.9928	1.9350	75	1.9861
1.9397	49	1.9926	1.9347	76	1.9858
1.9396	50	1.9924	1.9344	77	1.9854
1.9395	51	1.9923	1.9342	78	1.9851
1.9394	52	1.9921	1.9339	79	1.9848
1.9393	53	1.9919	1.9336	80	1.9844
1.9392	54	1.9917	1.9333	81	1.9841
1.9390	55	1.9914	1.9330	82	1.9837
1.9389	56	1.9912	1.9327	83	1.9834
1.9387	57	1.9910	1.9324	84	1.9830
1.9386	58	1.9908	1.9321	85	1.9827
			1.9317	86	1.9823

NOTE: For other salinities, interpolate linearly.

APPENDIX B

TABLE of FRESH and SALT WATER KINEMATIC VISCOSITY

(reprinted from ‘Introduction to Naval Architecture’ by Gillmer and Johnson, U.S. Naval Institute, 1982)



Values of Kinematic Viscosity ν for Fresh and Salt Water

Values adopted by the ITTC meeting in London, 1963.
Salinity of salt water 3.5 percent.

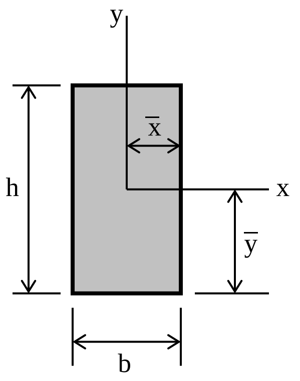
Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$	Kinematic viscosity of fresh water $\nu, \frac{ft^2}{sec} \times 10^5$	Temp, deg F	Kinematic viscosity of salt water $\nu_s, \frac{ft^2}{sec} \times 10^5$
1.9231	32	1.9681	1.2260	59	1.2791
1.8871	33	1.9323	1.2083	60	1.2615
1.8520	34	1.8974	1.1910	61	1.2443
1.8180	35	1.8637	1.1741	62	1.2275
1.7849	36	1.8309	1.1576	63	1.2111
1.7527	37	1.7991	1.1415	64	1.1951
1.7215	38	1.7682	1.1257	65	1.1794
1.6911	39	1.7382	1.1103	66	1.1640
1.6616	40	1.7091	1.0952	67	1.1489
1.6329	41	1.6807	1.0804	68	1.1342
1.6049	42	1.6532	1.0660	69	1.1198
1.5777	43	1.6263	1.0519	70	1.1057
1.5512	44	1.6002	1.0381	71	1.0918
1.5254	45	1.5748	1.0245	72	1.0783
1.5003	46	1.5501	1.0113	73	1.0650
1.4759	47	1.5259	0.9984	74	1.0520
1.4520	48	1.5024	0.9857	75	1.0392
1.4288	49	1.4796	0.9733	76	1.0267
1.4062	50	1.4572	0.9611	77	1.0145
1.3841	51	1.4354	0.9492	78	1.0025
1.3626	52	1.4142	0.9375	79	1.9907
1.3416	53	1.3935	0.9261	80	0.9791
1.3212	54	1.3732	0.9149	81	0.9678
1.3012	55	1.3535	0.9039	82	0.9567
1.2817	56	1.3343	0.8931	83	0.9457
1.2627	57	1.3154	0.8826	84	0.9350
1.2441	58	1.2970	0.8722	85	0.9245
			0.8621	86	0.9142

NOTE: For other salinities, interpolate linearly.

APPENDIX C

PROPERTIES of COMMON GEOMETRIC SHAPES

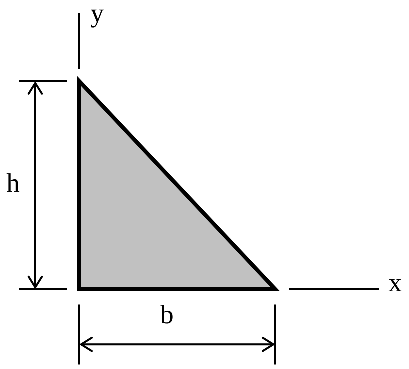
Rectangle (origin of axes at centroid)



$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

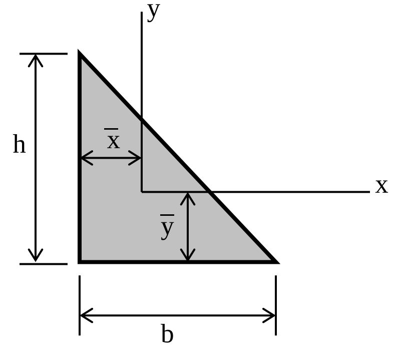
$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

Right Triangle (origin of axes at vertex)



$$A = \frac{bh}{2} \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12}$$

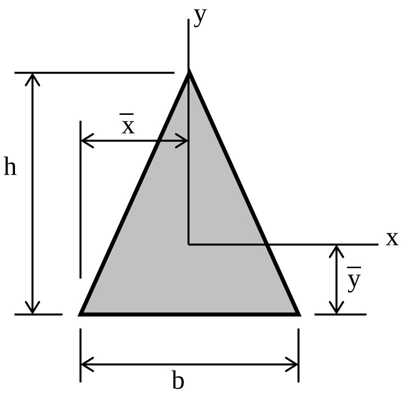
Right Triangle (origin of axes at centroid)



$$\bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36}$$

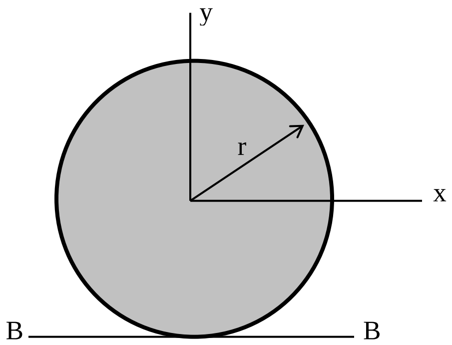
Isosceles Triangle (origin of axes at centroid)



$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48}$$

Circle (origin of axes at center)

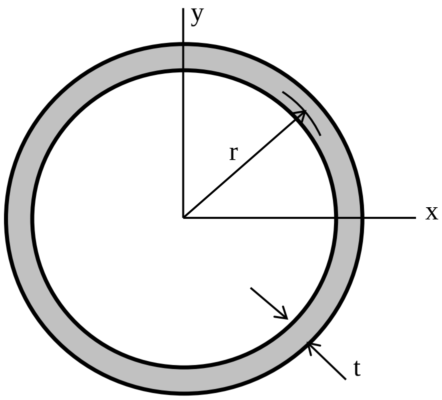


$$d = 2r \qquad A = \pi r^2 = \frac{\pi d^2}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

Circular Ring with thickness " *t* " (origin of axes at center)
Approximate formulas for the case when *t* is small



$$A = 2\pi rt = \pi dt$$

$$I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$