Electrical conductance: Measurement Project

1. Theory[1] (Litz, 2005, pp. 354-359)

1.1.Resistance (revision + addition):

- Object (Resistor)
- Property: Materials of different geometry *(l, A)* and material quality hinder the movement of the charged particles contained in them to varying degrees.
- Physical quantity (Resistance): According to the measurement instruction, the resistance R of a conductor is understood as the ratio of the voltage *V* dropping across it and the current *I* flowing through it. It shows the voltage required to create and maintain a specific current intensity.

Ohm's law: $R = \frac{V}{I}$

Ohm found that the intensity of the current formed in the conductor under the influence of the electric field is directly proportional to the voltage. The I-V characteristics of most materials are straight.

We confirmed Ohm's law with measurement, our experimental setup consisted of a voltage source, resistance, ammeter and wires. Ohm based his measurement on contemporary knowledge and conducted it with the possibilities of contemporary measurement technology. [2]

Wire resistance: By measurement, it can be determined that the resistance of a homogeneous conductive wire of length l, cross-section A, is:

$$R = \rho \cdot \frac{l}{A}$$

ρ: *specific resistance*, depends on material quality. It shows the resistance of a wire of unit length with a unit cross section.

$$[\rho] = \Omega m$$

Let us qualitatively interpret the formula of resistance.

The longer the wire is, the ...., thus the resistance is higher. The larger the cross section is, the more ...., thus the resistance is lower.

If the specific resistance or cross-section changes along the wire, then the resistance of the wire is:

$$R = \int \rho \frac{dl}{A}$$

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[1] Source of theory: Litz, J. (2005). *Fizika II.* Nemzeti Tankönyvkiadó Rt.

[2] András Juhász et al. (2021): Teaching Physics II, 386-387. http://fiztan.phd.elte.hu/files/kiadvanyok/Fizika_tanitasa_II.pdf

Let's interpret this equation. The wire is divided into small dl sections on which the variable quantity is considered constant. We give the elementary resistance of the small section (dR), then summarize for the entire length of the wire, if dl → 0, we integrate.

Task 1: Resistance of the dielectric layer of a spherical capacitor. (Litz, 2005, p. 357.)

Between the armaments of the sphere capacitor with radius R_1 and R_2 ($R_2 > R_1$), there is a dielectric layer with ρ specific resistance. What is the resistance of the layer?

Resistors in electric circuits: Draw the circuit diagrams (with R_1 and R_2), explain the formulas you can find in the table and derive the resultant resistance – use Ohm's law to do this!

a) Series	b) Parallel
$I = I_1 = I_2$ $V = V_1 + V_2$ $R = R_1 + R_2$	$V = V_1 = V_2$ $I = I_1 + I_2$ $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

1.2.Conductance

Consider two conductors with resistance R_1 and R_2 resistance. What can we say about the current intensities formed at a given voltage, if $R_1 > R_2$?

....
....

True or false?

a) R_1 is a weaker conductor than R_2
b) R_2 allows less charge to flow than R_1
c) R_1 is a better conductor than R_2

When the resistor allows for a lower current at a given voltage, we say that it is a weaker conductor, so its conductance is lower. "Conductance is the opposite of resistance: the measure of how easy it is for electric current to flow through something. Conductance is symbolized with the letter "G" and is measured in unit of Siemens. Mathematically, conductance equals the reciprocal of resistance."[3]

[3] Definition of conductance: https://ecstudiosystems.com/discover/textbooks/lessons/DC/chapter5-4-conductance/

$$G = \frac{1}{R}$$

$$[G] = \frac{1}{[R]} = \frac{1}{\Omega} = 1\ S\ (Siemens)$$

To describe materials, the reciprocal of specific resistance (resistivity), *specific conductance (conductivity)*, is often used.

$$\sigma = \frac{1}{\rho} = \frac{l}{R \cdot A}$$

$$[\sigma] = 1\frac{S}{m} = \frac{1}{\Omega m}$$

Homework (Hungarian matura exam tasks – intermediate level)[4]

1. In an experiment, the conductance of metals was investigated at low temperatures. First, a piece of wire made of material 'A' was connected in a circuit and in the case of 1 A current intensity, the voltage was measured at different temperature values. Then the same experiment was repeated with the "B" material, also made of a piece of wire. The attached table contains the measured values.

T (°C)	−273	−269	−268	−264	−260	−256	−252	−248
U_A (V)	0,28	0,29	0,30	0,34	0,38	0,42	0,46	0,50
U_B (V)	0	0	0,12	0,20	0,28	0,36	0,44	0,52

a) Represent the data graphically.
b) Which wire has a higher resistance at a temperature of −260 °C? Determine the value of this resistance.
c) At approximately what temperature will the resistance of the two wires be equal?
d) The resistance of which wire shows special behavior at low temperatures? Explain this special behaviour.

2. To build a spacecraft, you need a resistor R = 0.05 Ω, l = 5 m length of wire. Of course, you need a wire with the smallest possible mass made of available materials. You can find the density (ρ) and resistivity (let us now indicate it with ρ^*) of four metals that can be used for building the spacecraft in the table below. Which material is suitable for the given conditions (as light as possible to make a wire)?

Material	Density ρ (g/cm^3)	Resistivity ρ^*(Ωm)
aluminium	2.7	$2.67\cdot10^{-8}$
copper	8.9	$1.69\cdot10^{-8}$
silver	10.5	$1.63\cdot10^{-8}$
titanium	4.5	$5.40\cdot10^{-7}$

[4] Source of 1st matura exam task: https://dload-oktatas.educatio.hu/erettsegi/feladatok_2018tavasz_kozep/k_fizma_18maj_fl.pdf and source of the 2nd task: https://dload-oktatas.educatio.hu/erettsegi/feladatok_2017osz_kozep/k_fiz_17okt_fl.pdf

2. Warmer experiments

Examine which liquids conduct electricity (distilled water, tap water, salt water). Using the following tools, design and build an experimental setup with which you can qualitatively test the conductance of various liquids!

- Glass
- Distilled water (1st experiment)/tap water (2nd experiment)/salt water (3rd experiment)
- 2 metal electrodes
- Wires
- Crocodile clips
- Flashlight bulb
- Pocket battery

Experimental setup and circuit diagram:

Experiences

a) In which case does the bulb light up?

....
....

b) Explain what you have experienced.

....
....
....

c) In the case of salt water, what particles flows to the end of the wire connected to the positive and negative poles of the pocket battery?[5]

....
....

d) How does the increasing concentration of the solution affect the electrical resistance of the solution and the brightness of the bulb? Justify your answer in 3-5 sentences.

....
....
....
....
....
....

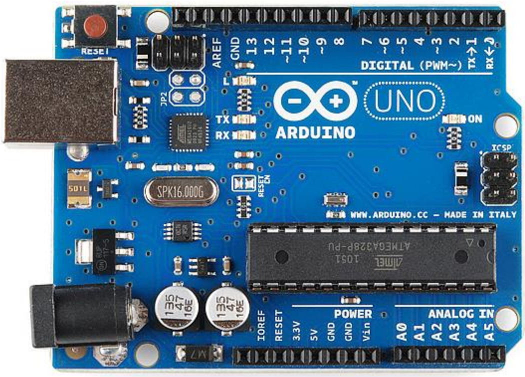
[5] Source of questions c) and d): https://dload-oktatas.educatio.hu/erettsegi/feladatok_2017tavasz_kozep/k_fizma_17maj_fl.pdf (Matura exam 3/B).

3. Measurement with Arduino

3.1. Briefly about Arduino[6]

Arduino is a platform for conducting electronic projects. It consists of a programmable computer (microcontroller) and a software (IDE = Integrated Development Environment). In the software, you write the program code (in a simplified language similar to C++), which you upload to the device. With codes you can operate the sensors connected to the Arduino board, and you can follow the status of the sensors over time.

The power supply of the Arduino board comes from the computer, to which we connect it with a USB cable. Uploading the code is only possible with a USB connection.



Arduino board – microcontroller

Source: https://learn.sparkfun.com/tutorials/what-is-an-arduino/all

On the Arduino, there are connectors (pins) marked with different numbers, to which sensors can be connected. In this way, we can refer to the given terminal based on the number in the program code with which we want to operate the particular sensor.

- Digital terminals: 0-13

- Analog terminals: A0-A5

For example, analog connectors can read the signal of an analog sensor (e.g. thermometer sensor, photoresistor, etc.), which is converted into a digital signal.

The digital terminals can be used both as digital inputs (signal reception - e.g. pressing a button) and as outputs (signal output - e.g. flashing of an LED).

In addition, we can still find the pins marked with GND and 5V/3V on the microcontroller.

GND (ground) provides the grounding, 5V/3V provides the voltage.

We can create the electrical circuit on the *breadboard*[7], the power supply of which can be solved by connecting a battery or an external power supply - in our case, connected to the Arduino (it is connected to the laptop with a USB cable). The + and – holes mark the terminals of the circuit. These two lanes are used for power connection. The holes in the middle of the breadboard are used to connect electrical components. These holes are connected in series vertically, there are 5 holes in a column, which means that a maximum of 5 devices can be connected in

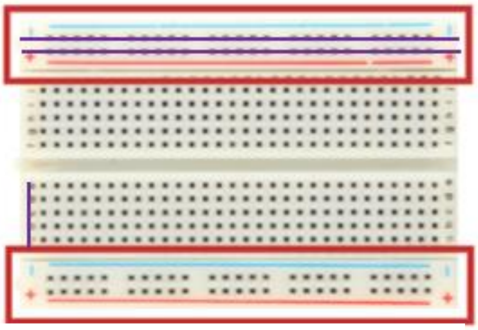


Image Source: https://www.seeedstudio.com/blog/2020/01/06/how-to-use-a-breadboard-wiring-circuit-and-arduino-interfacing/

[6] Source: https://learn.sparkfun.com/tutorials/what-is-an-arduino/all
[7] Source: https://www.seeedstudio.com/blog/2020/01/06/how-to-use-a-breadboard-wiring-circuit-and-arduino-interfacing/

one section. The connection of the holes is illustrated by the purple lines drawn in the figure.

3.2. Basic codes[8]

When programming the Arduino, we always want to get the sensor we are using to work, to read the data it detects, i.e. to retrieve the state of the sensor. So, of course, when using different sensors, when conducting different projects, we have to apply different codes, but at the same time the basics are there. All further problems can be built on the basis codes, on the Internet there is a lot of information and codes, there are also descriptions of ready-made projects.

Here we will give you just a few basic codes:

- **void setup:** This is where setting up the program begins.
- **Serial. begin (....):** Specifies how much data one can allocate per second (bits/sec). e.g. for Serial. begin (9600), one can receive 9600 data per second.
- **void loop { :** defines a loop, i.e. the code runs again at given intervals
- **int value = 10; :** We can define with the *int* code. The word "value" can henceforth be used instead of pin 10.
- **int value = analogRead (A0); :** Reads the analog value and defines it as "value"
- **float value :** A float is a floating-point number, which means it is a number that has a decimal place. Floats are used when more precision is needed.
- **pinMode (10, INPUT):** The sensor connected to pin 10 serves as input.
- **pinMode (10, OUTPUT):** The sensor connected to pin 10 is used to output a signal.
- **digitalWrite (10, LOW):** Specifies the specific state of the pin 10. e.g. turns off the LED connected to pin 10.
- **digitalWrite (10, HIGH):** Turns on the sensor connected to pin 10.
- **delay (1000) :** pause, 1000 ms =1 s wait.
- **Serial.print** ("Voltage (Volts)"): you can use the command to write different expressions to the machine, which are displayed on the serial monitor (can be opened by clicking on the small magnifying glass icon).
- **Serial. println (....):** Outputs the measured values one below the other and displays them on the serial monitor.

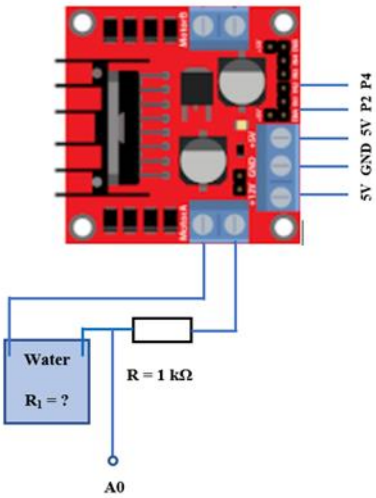
[8] Basic codes and more details can be found here: https://www.arduino.cc/reference/en/

3.3. The Arduino measurement — Conductance and conductivity of liquids

Compile a measurement setup using the listed tools based on the picture. P2 and P4 indicate Arduino pin 2 and pin 4, A0: analog pin.

Equipment

- Arduino (board and breadboard and software[9])
- USB cable, laptop
- Wires + duct tape
- Resistor (1 kΩ)
- Glass + tap water (app. 2 dl)
- Salt
- Digital scale
- H-bridge



Description of the task

Pour 2 dl tap water into the glass, build the circuit based on the picture, then test the conductance of the liquid using Arduino. Use the code below. Measure the relationship between the conductance, salt content and pH of different solutions.

Note: we do not measure the conductance, but the voltage *dropping across the known resistor! The conductance of the liquid can be calculated.*

You can find the code [here] as well. (CONDUCTANCE MEASUREMENT – ADVANCED LEVEL)

```
int input = A0; // analog input
int IAA = 2; // Define as Switch A
int IAB = 4; // Define as Switch B

void setup() {
  pinMode(IAA,OUTPUT);// digital output happens on a pin
  pinMode(IAB,OUTPUT);//
  Serial.begin (9600);
}

void plus(){  // switch on A and switch off B
  digitalWrite(IAA,HIGH);
  digitalWrite(IAB,LOW);
  delay(500);
}
void minus(){  // switch off A and switch on B
  digitalWrite(IAA,LOW);
  digitalWrite(IAB,HIGH);
  delay(500);
}
```

[9] https://www.arduino.cc/en/software

```
float measureVoltage(int numberOfReads){ //reads a given number of data
  float sumVoltage = 0.00; //includes the sum of the read data on the Arduino scale
  for(int i=0;i<numberOfReads;i++){
    sumVoltage = sumVoltage + analogRead(input);  //reads voltage data on the Arduino scale
  }
  float averageVoltage = sumVoltage/numberOfReads; // Average voltage value on the Arduino scale
  return averageVoltage*5.00/1024; // Average voltage measured in volts
}

void loop() {
    plus();
    Serial.print("Plus: ");
    Serial.println(measureVoltage(70)); //reads 70 data, print their average
    minus();
    Serial.print("Minus: ");

}
```

Questions and tasks

1. Draw the circuit diagram.

2. Interpretation of the switch: Water and resistance (R = 1 kΩ) are connected to each other in series. Based on this, what can we say about the intensity of current flowing through the resistors, and about the voltages?
(For uniform marking: $V_0 = 5V$ the voltage of the power source, V_1 is the voltage dropping across the solution, V is the voltage dropping across the resistor, the current intensity (I) is similarly marked.)

....
....
....

3. **Based on the given setup and quantities how can you determine the conductance of the investigated fluid (solution)? Give a theoretical model. Explain it in 2-3 sentences.**

....
....
....
....

3. **Use the program code and measure the values (current, voltage, conductance of water).**

V (V)	V_1 (V)	I (A)	I_1 (A)	G_1 (mS)

4. **Add salt to the water and in this case also determine the quantities you are looking for. For different salt contents (6-7 measurements), measure the conductance of the solution.**

	m (g)	V (V)	V_1 (V)	I (A)	I_1 (A)	G_1 (mS)
1.	0.00					
2.	0.05					
3.	0.10					
4.	0.15					
5.	0.20					
6.	0.25					
7.	0.30					

5. **Represent the data graphically – conductance of salt water in the function of mass of salt. For graphical representation and analysis, use Excel.**

6. **Qualitatively interpret the diagram. How does conductance of salt water depend on salt content? Why?**

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***Bonus 1): Fit a function to the data points.**

Function:

f(x) =

7. **Identify possible sources of error.**

………………………………………………………………………………………………
………………………………………………………………………………………………

****** Only for advanced-level students:** Do a literature review. What is the theoretical model that describes the relation between NaCl-content of water and its conductance? Compare the results of your measurements with the theory.

Bonus 2.) How could you determine the specific conductance (conductivity)? Give an order-of magnitude estimation. Come up with a theoretical model.

You can find more ideas and references here: https://www.arduino.cc/ https://create.arduino.cc/projecthub