Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of Technology, Madras

Module - 3.5

Lecture - 21

Basic Matrix Concepts

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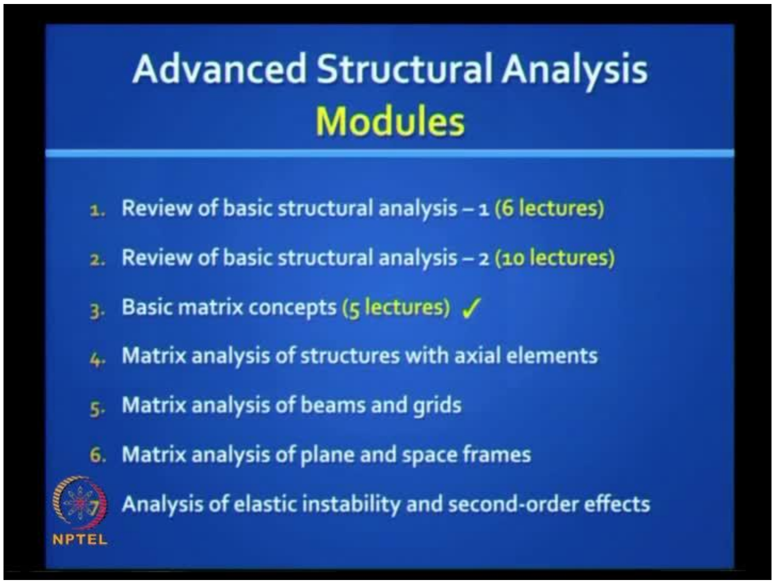

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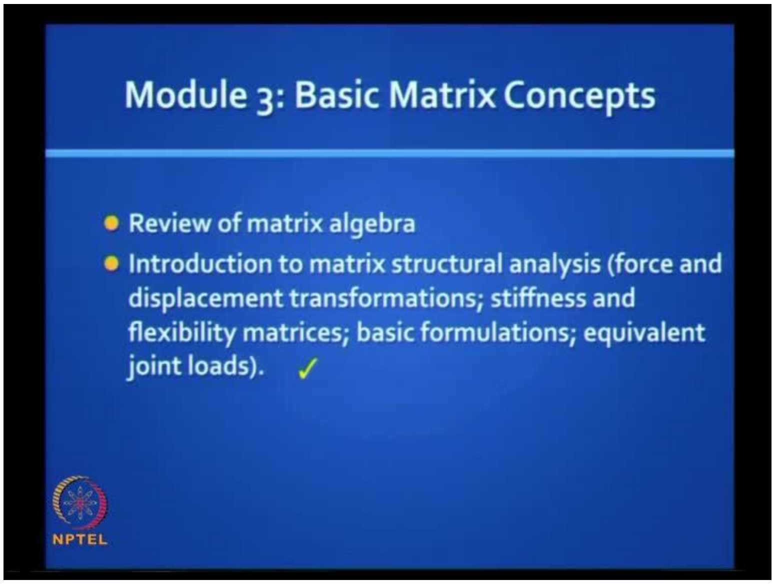

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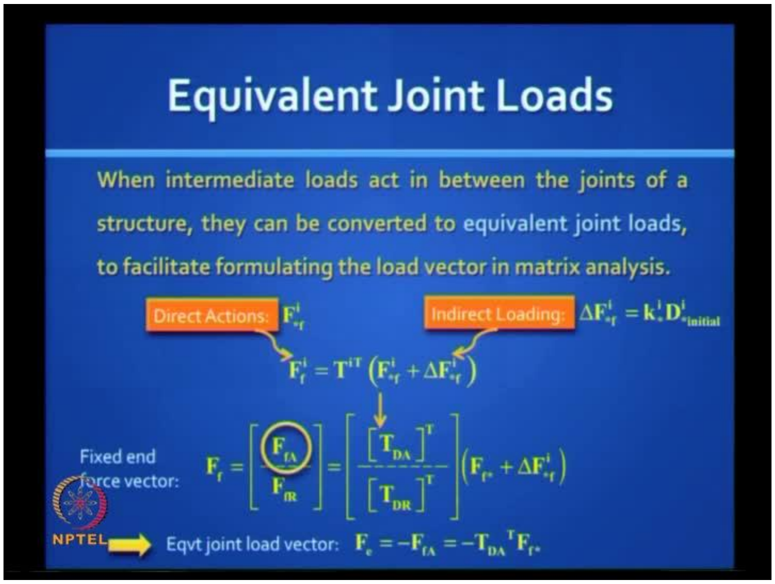

Good morning. This is lecture 21. I hope to cover basics matrix concepts with this lecture so this will be the fifth lecture on this topic. This is covered in the second and third chapters of the book on advanced structural analysis. There was one topic which we did not cover in the last session that is equivalent joints loads. You are already familiar with the need for this, you need this whenever you have intermediate loads. We have used it when we did slope deflection method and moment distribution method.

So when intermediate loads act in between the joints of a structure, they can be converted to equivalent joint loads to facilitate formulating the load vector in matrix analysis that is because we want to limit the degrees of freedom.

Ah so you have two types of problems, one where you have direct actions which are distributed and you can find the fixed end forces you have done it for beams but, it is possible you might get them in axial elements e.g. a column with self-weight that is a distributed load. We will see how to handle that.

So this notation f i star f what does it represent in the stiffness method? In the i th element the fixed end forces that small subscript f stands for fix end forces along the local axis, ok? so it is a local axis system, right? You can get fixed end forces from another problem indirect loading.

Take a truss when you let us say you have a lack of fit problem in the truss or let us say you have a temperature change problem that is indirect loading.

Ah how will you handle that? How will you convert into fixed end forces?

First find tension in that bar…

How will you find that tension?

Elongation (( )).

Where is elongation? Let us say it is statically determinate then why are you taking that as a fixed end force? Elongation will not give you the fixed end force it will give you elongation. Say l alpha delta t is a free elongation you get it in a bar from that where is the tension there is no tension in the bar yeah.

Then there is an elongation (( ))

Please understand the physics. I have a statically determinate truss let us say I am doing by the stiffness method. I heat all the bars let them all get heated. Where do I get tension?

No fixed force no tension.

No there is you are right but, you are not able to give the physics behind the problem what you said is right you might get tension or you might get compression that is a different issue but, you will get an axial force, when? In which structure will you get an axial force?

Sir if it is determinate we can move the structure at a.

We are doing by the stiffness method that is what you learnt till now, right? In the stiffness method how do you handle environmental change in bars in a truss for example? It is a little tricky we will study it in detail in the next module but, I will give you a clue. All structures are kinematically indeterminate even if they are statically determinate.

So we are always working as our base the primary structure. So in the primary structure no movements are allowed at the joint (remove 'so') you have to arrest all the joints. Now you try heating those bars you will get compression in those members or you cool those bars you get tension in those members. So in the primary structure you get them and they get passed on to the joints that is how you handle it in stiffness method it is very interesting.

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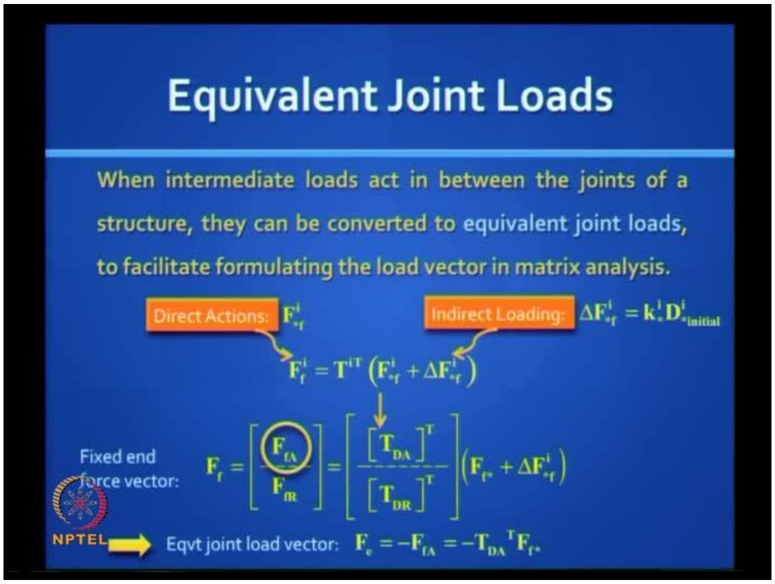

So do you agree I can write down a set of initial displacements either contraction or elongation in a truss and if I just multiply it with the elements stiffness of that I get the fixed end forces we will see this in detail later. So you got the local fixed end force vectors for all the elements either caused by direct actions that is intermediate loads or by indirect loading.

What do you do next in this conventional stiffness method which we studied? You have to convert these to the global axis system, how do you do that?

Transformation matrix

What is the transformation matrix you will use?

(( ))

Will you use t i will you pre multiply by t i? not t i then what? (remove ‘t i’)

T f

Don’t mix up problems. We are doing conventional stiffness method t f comes in flexibility method so it is easy to get confused. You remember the t i enables you a transformation from the global axis system to the local axis system both for forces and displacements.

Now we are trying to switch from the local to the global so, what is the transformation you have need to do?

T i transpose

There you are. t i transpose that is all you have to do. So add up all your elements fixed end forces, apply the transformation in the reverse direction so that you would go back to the fixed end forces and then you can generate the fixed end force vector for the structure by assembling it in some appropriate way and you will realize you will get some fixed end forces coming to the active degrees of freedom they are going to be your loads and some will pile on to the restrained degrees of freedom they will add on to your reactions

So you pick up your fix equivalent joint loads from f f a. a is at the active degrees of freedom and then you have to apply it in the opposite direction you remember in the classical stiffness method and that is your equivalent joint loads. I’m just brushing up what we have done earlier but, we are doing it in a matrix format.

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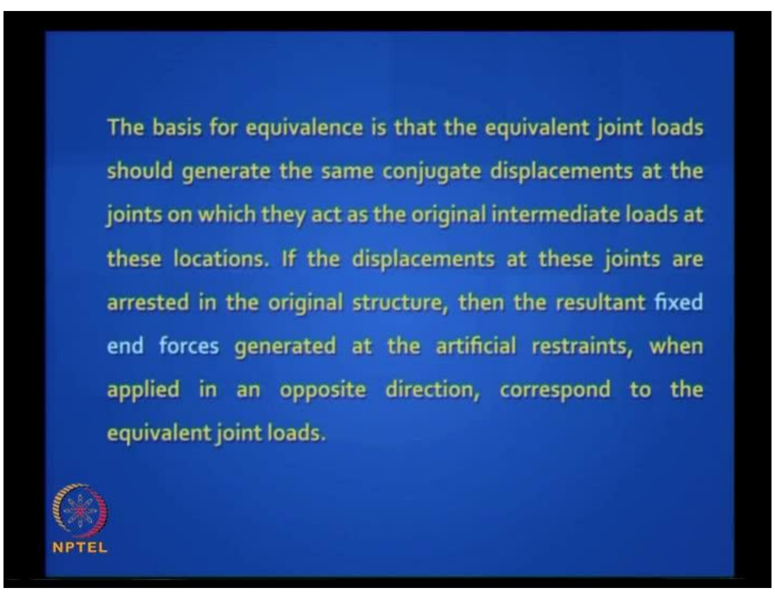

So the basis for equivalence is that the equivalent joint loads should generate the same conjugate displacements at the joints on which they act as the original intermediate loads at these locations. Remember we... I have shown you this earlier you have a original structure and you have first a restrained structure where you arrest all the degrees of freedom and then you have a structure in which you apply nodal loads.

The joint displacements here are zero because you arrested everything. The joint displacement in the original structure are non zero so obviously if you do super position the equivalent joint load should act in such a way that you end up getting the same unknown displacements in the original structure at the joints not in between and that is the basic principle of equivalent joint loads.

If the displacements at these joints are arrested in the original structure then the resultant fixed end forces generated at the artificial restraints when applied in an opposite direction correspond to the equivalent joint loads. You have to add up these equivalent joint loads with whatever additional joint loads that you may get, (remove ‘ok’)

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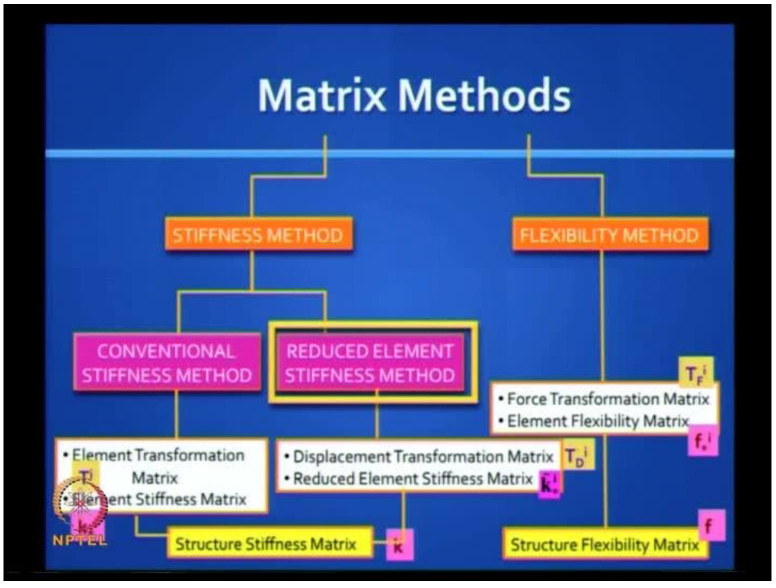

Ok. Now we move on. We had a quick look at the conventional stiffness method there is no way you fully understand it unless you apply to problems but, the idea here is to just lay road map now we are going to look at the reduced element stiffness method we will see the flexibility method after that they are similar so it is easy to see them.

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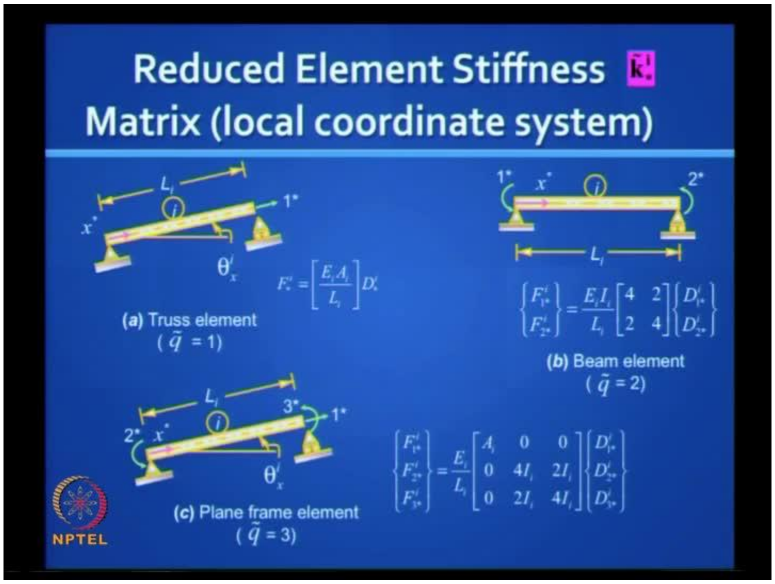

If you recall, we know why we can take advantage of these reduced element stiffness method because the rank of the element the conventional stiffness matrix at the element level is less than full so you can reduce it to the minimum to full rank and we've already discussed three types of elements we will see other types later but, this is elementary so we looked at the truss element you have only one degree of freedom reduced.

We looked at the beam element two degrees of freedom. We have chosen just for convenience a simply supported arrangement you can very well choose a cantilever but, let us not get confused we will take one at a time and the plane frame element is a combination of the truss element and the beam element so you have got three degrees of freedom and three displacements correspond to one translation and two end rotations in terms of forces one actual force and two end movements

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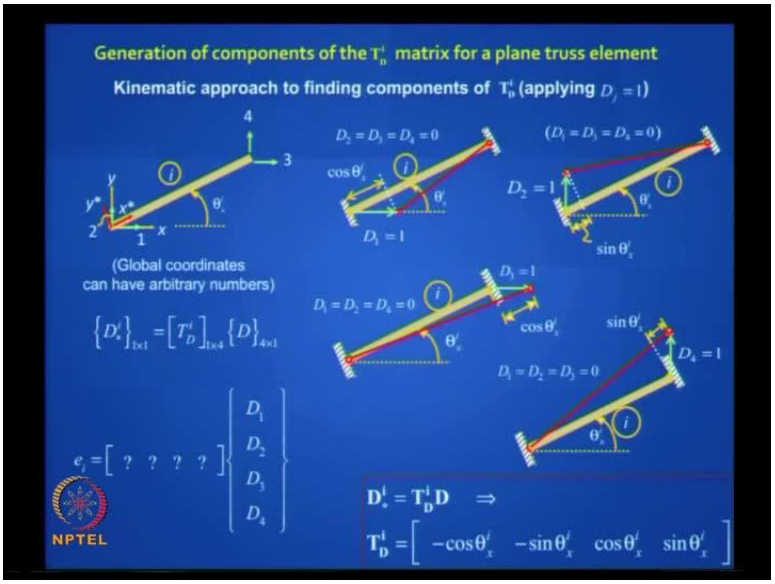

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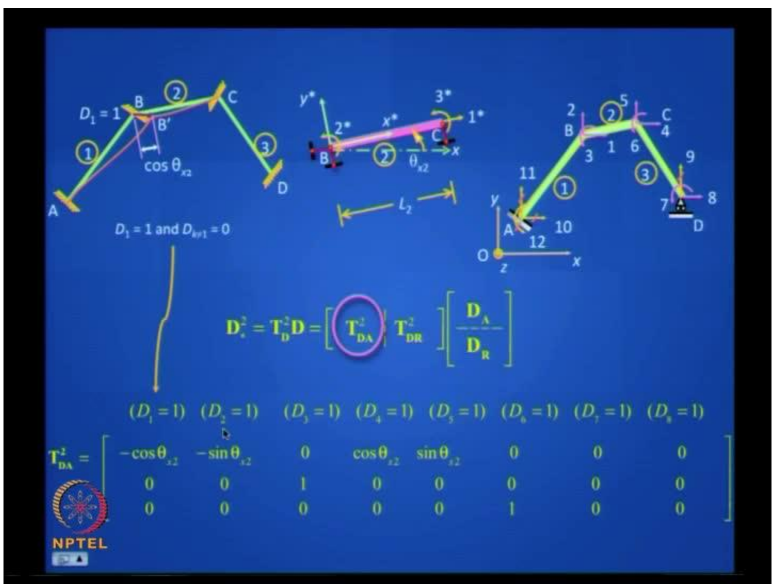

Now you may ask what about the shear forces and all that? All that is statically determinate. Once you got the end movements you got the shear forces, right? So we also looked at this recall we talked about the displacement transformation matrix and we derived this from first principles using what we call a kinematic approach. I’ll show you, you can also get the same matrix the t d i matrix from a static approach using this principle called the contra gradient principle which we are now going to look at.

But before that let us just do an application of the t d i matrix. Let us take this frame which we have been playing along all this while you have got this three element frame the primary structures are arrested we want to if you push d one equal to one what and if you have to generate the t two d a matrix which are put it with a circle here.

How will you get this matrix? What will be the size of this matrix? I'm just taking the second element which I have shown here. Second element has three degrees of freedom one two and three. When I apply a unit displacement to the structure here arresting all the other displacements this will effect elements one and two. Just see the effect in element two. how will you convert it to …what…What effect will it cause?

It will cause a compression in that member, agreed? And remember you have only one degree of freedom in that element in the reduce format and that displacement is what? We said it is an elongation, right? so we want to get that quantity what will it be? It will be…. it is a component of one you know would not it be cos theta? Will it be plus cos theta or minus cos theta? minus cos theta, right? And will you get… so we have got this value d i one star is going to be minus cos theta. What about d i two star for element two? Will you get any moments? Will you get any rotations? No because you are arrested that here. You won't get anything. There will you get any rotation here? No. ok(remove 'ok')

Ok. You'll understand when I show you the matrix but, at least tell me the size of the matrix. What is the size of t two d a matrix? You remember there are how many degrees of freedom here in this problem? There were eight active degrees of freedom and four restrained degrees of freedom, right?

So this is (remove 'this is') the matrix will look like this. Ok I am just pulling out the t two d a matrix here with the circle. For this element if I have to generate the matrix the first column this is a three by eight matrix first column is what I write down when I apply d one equal to one to this structure. Do you agree? When I apply d one equal to one to the structure I get just a contraction in that element and so corresponding to this first degree of freedom I get minus cos theta of that element two. I do not get a rotation at the left end. I do not get a rotation at the right end.

So do you understand the physical meaning of first row, second row, third row. The first row refers to what I get here when I keep applying a unit displacement to the structure. The second row refers to the rotation here. The third row refers to the rotation here. Likewise I can move ahead and go to apply d two equal to one which means I now lift up this joint and keep everything else restrained when I lift up this joint what do you think i will get? I will get minus sin theta.

Do you understand or you do not? You understand. Do not worry we will come back to this later.

Why the minus sign?

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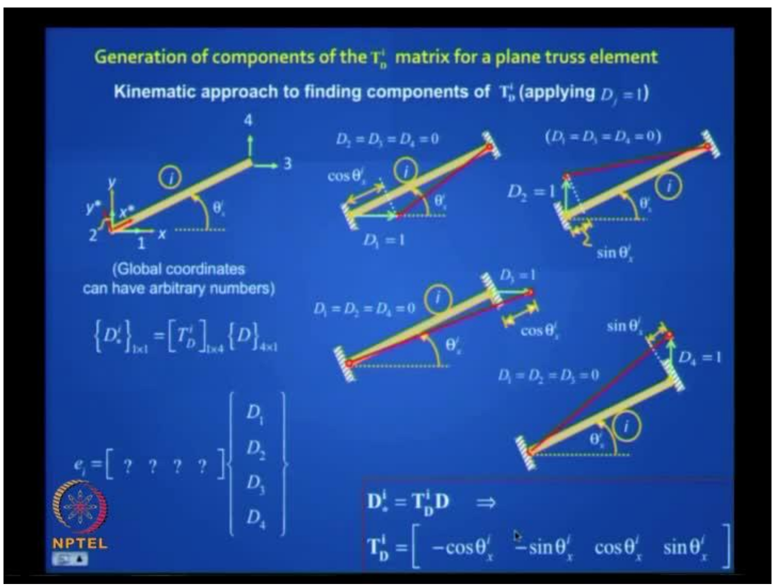

Because you will get a contraction, the length of the material… well that question you should have asked here. Why did you get a minus sin here? See when I push this horizontally here I got a contraction so this was a minus sin. When I lifted it up then also the length reduced, I got a minus sin. But when I pull the right end here right I get a plus sign because you can see the red line is elongated and when I lift it up at the right end I get elongation, right?

So what I have written here, the displacement transformation matrix gives me the element elongations caused by a unit movement, unit translation at the left end and the right end keeping all the other degrees of freedom, clear? So I get minus whenever I push the left end and I get plus when I push the right. Is that clear now? Ok? I do not seem to…. Yes or no?

yes.

Give me a resounding yes if you understood. But do not worry with time you will understand.

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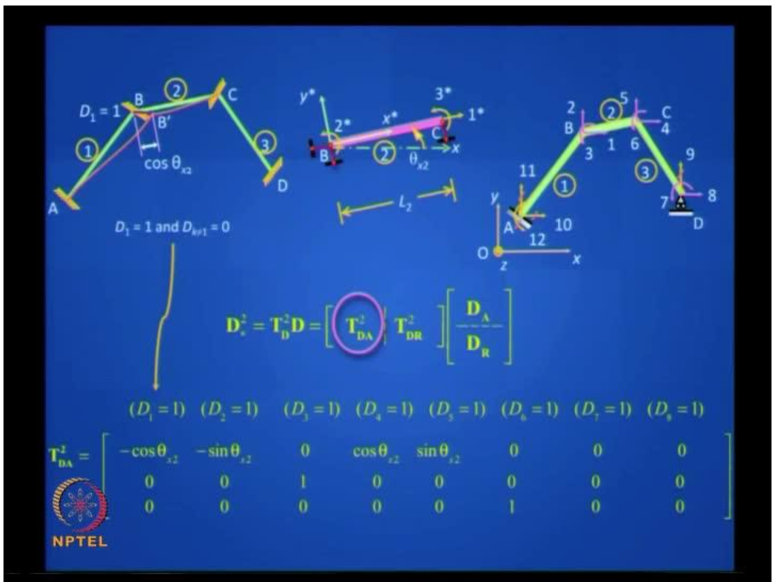

So broadly you get the picture hang of this matrix. now this (remove ‘now this’) When you do reduced element stiffness method and you would do flexibility method you cannot program it as blindly as you could the conventional stiffness method because you are trying to take a shortcut you have to apply your brain a little bit especially in generating the fixed end force vector which we will see later, so you can do this.

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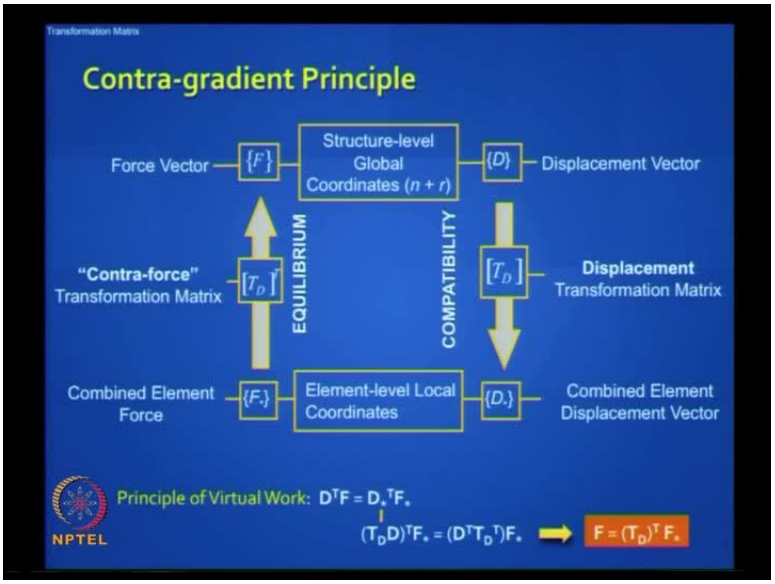

Ah now let us look at the contra gradient principle. I have shown this earlier but, here at the lower level I put the element level forces, the combined element force vector f star which actually refers in a case of a truss to all the axial forces in the truss. The combined element displacement vector d star which are the bar elongations in the truss. The structure level displacement vector which are the joint displacement in the truss and the structure level force vector f, ok?

F I can include both the active degrees of freedom and the restrained degrees of freedom or I can limit to the active degrees of freedom. Now the transformation we did was, if I know the joint displacements then I get the individual member deformations in this case the bar elongation by applying the t d matrix as a transformation I just demonstrated that. And this actually gives a compatibility relationship between the structure joint displacements and the member bar elongations, right?

Likewise there is a set of equilibrium relationships which relate f star with f. and.. Do you think this relationship is linked with the t d matrix? Can we prove that relationship? How do you prove that relationship?

This is a very important concept. How do you relate the element level forces to the structure level forces in the way we related the element level displacements to the structure level displacements? What principle will you use? Ah?

Principle of super position… like…

But whenever you are doing transformations of matrices you are using principle of super position whether you like it or not. that is how the…. It is a linear combination of vectors. So that is implicit. What is the answer you gave?

(( ))

But you have to talk matrix language. You are just saying what I wrote here. I am asking you. How do I build a relationship between this and this? There you are it has come out.

Using this. So you use the principle of virtual work your real work is that principle which ties together the force field with the displacement field. So do you realize, the principle virtual work says the total external virtual work product is equal to the total internal virtual work product?

Now, here all the forces are real, all the displacements are real but, it is the product is still virtual because I am not applying any factor half. I am just taking the forces. So do you agree to this equation that I have written here? That if I take the structure level forces and find the total work done, it is simply given by either d transpose f or f transpose d similarly, if I take the internal virtual work, I can write either d star transpose t f star or f star transpose d.

Now we will use the other one in the flexibility method. Now, how to proceed after this because you will end up with a beautiful equation. What can we now substitute in this equation?

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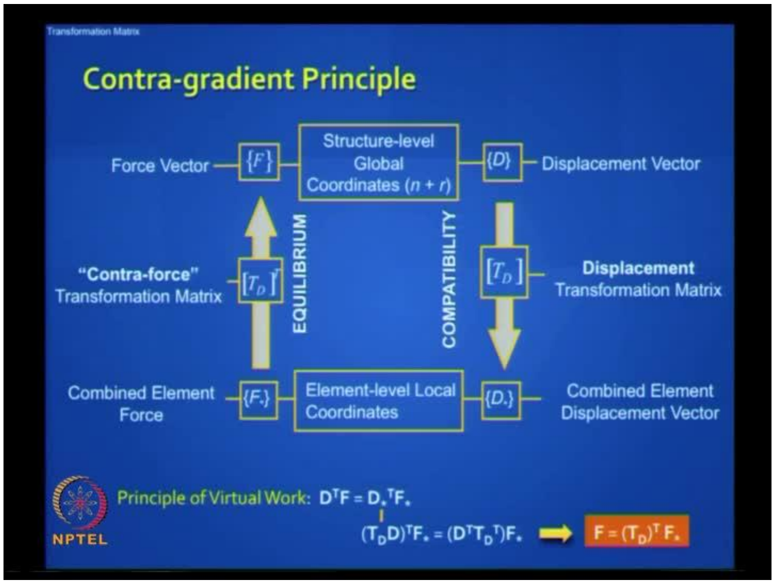

Instead of d with ah

Instead of….?

D star.

No, that is not what you need. You have to bring in t d into the picture, man.

(( )).

Equal to t d.

Ah

T d into d star.

You replace d star with?

T d into d.

T d into d that' s that is how you bring into picture and you have to take the transpose of it.

Now a b the whole transpose is b transpose a transpose, right? So do not you get this nice relationship? Then what? What is this establish?

(( ))

Then you keep your eyes open and compare the left hand side with the right hand side and then you arrive at a simple equation.

F is equal to.

F is equal to t d (( ))

So what does this mean? This means that is your relationship. That is a powerful relationship. That means once you (remove ‘would’) establish the t d matrix, you are actually tying… you (remove one ‘you’) are also linking simultaneously a relationship between the force vector at the element level to the joint force vector. In terms of a truss what we are saying, if you know the bar forces in the truss then the joint loads that could have caused those forces are related in this way but, it is not a very useful relationship in the sense of normal problem is, you have unknown bar forces when you have known loads. Ok?

The real problem is that but, this is useful in building up the force transformation matrix. Now because you are doing it in the reverse direction you know this was the normal direction for t d, you are (remove one ‘you are’) moving in that contra direction, the slope is moving in the reverse direction hence the word contra gradient. Gradient is the slope.

You know normally things flow downhill but, here it is going to flow uphill when you are dealing with the force field. Ok? and yeah.

(( )) x star into t d into f star

Sorry.

Similarly you can write f star equal to t d into…

You cannot do all that.

But t d is not (( ))

You cannot do all that. See in the stiffness method t d is unique. From first principle we derived it. What you are saying is something we will do in the flexibility method. There we will bring the t f matrix.

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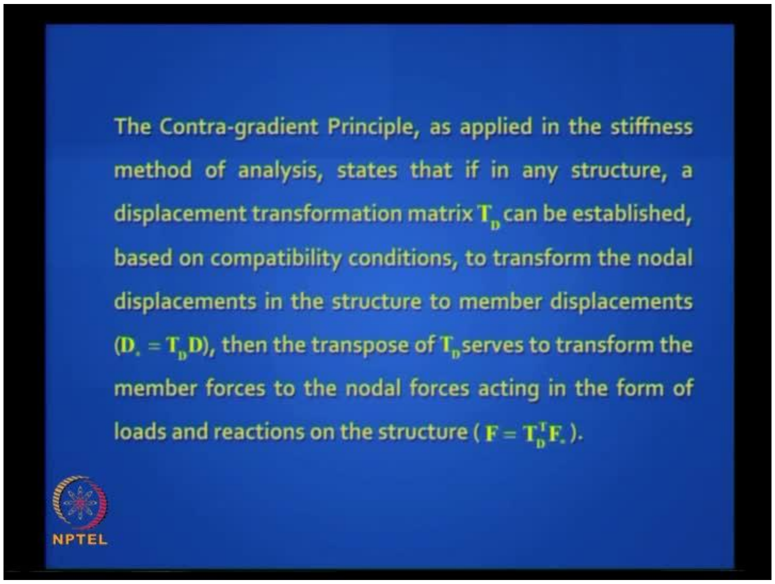

Right now, do not jump ahead because you will make mistakes. Do it slowly. Is this proof crystal clear? It is a simple proof beautiful proof. Let us apply this proof but, before that a statement. The contra gradient principle as applied in the stiffness method of analysis states that if in any structure displacement transformation matrix t d can be established based on compatibility conditions to transform the nodal displacements in the structure to member displacements d star equal to t d d then the transpose of t d serves to transform the member forces to the nodal forces acting in the form of loads and reactions on the structure. f is equal to td transpose f star. Ok.

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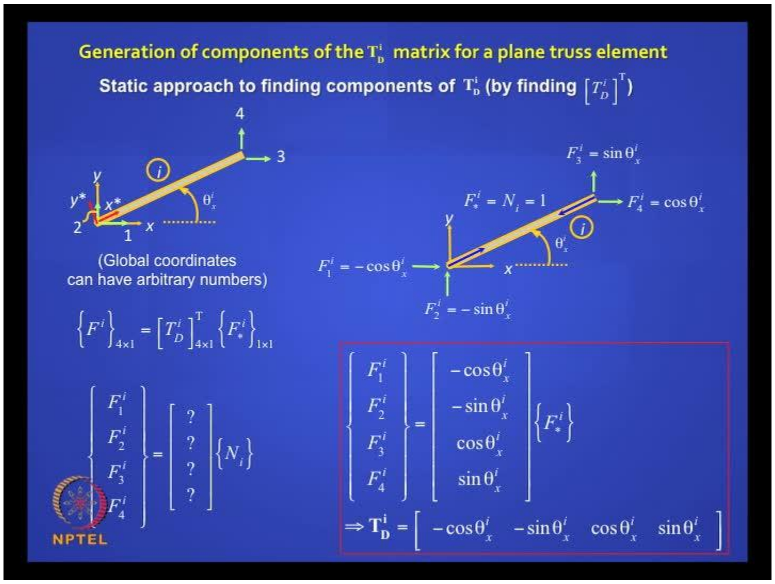

Remember this problem? We wanted to find the t d matrix. Well, if you use statics you can find the t d transpose matrix then you take the transpose of the transpose you will get the t d matrix. right? So now if I look at this relationship, f star in this you have four (remove 'a four') components here, right? f i one, f i two, f i three, f i four this force internal force here is actually n i you know that is what I called f that is what that is what(remove one 'that is what') the element force there is, right?

Actually this is written the wrong way, yeah the f i is four by one, right? Then what you get here what you get here (remove one 'what you get here') these are the four vector joint that I need. That means if I take this structure and imagine I have a bar and have a unit axial force in that bar. What are that load components that I get at the two ends of the bar which will which will (remove one 'which will') satisfy equilibrium, that is a static approach. Can you tell me what these four components… what will be the… what should this force be equal to?

(( ))

Plus or minus?

Minus.

Because there is axial tension.

Minus. Plus. Plus. Minus.

Then how will you satisfy sigma f x equal to zero?

Minus.

What about here? This one? It will be plus or minus?

Plus sir.

And what will this be?

Plus.

Plus, what?

Sine.

Sine theta. What will this be?

Minus.

There. I hope it does not take too long.

So you you (remove one ‘you’) see you have got this. That is your t d transpose matrix, you take the transpose of that you get t d you see, very clever way of getting the t d matrix in the reverse direction. This is statics, previously we did kinematics. You have a choice. Normally, students are weak in kinematics. They love doing sigma f x equal zero sigma f y equal to zero. You can get it this way and take the transpose you get the t d matrix but, if you are good in geometry you can do that. I want you to be good in both and you can do either way you are. Is this clear?

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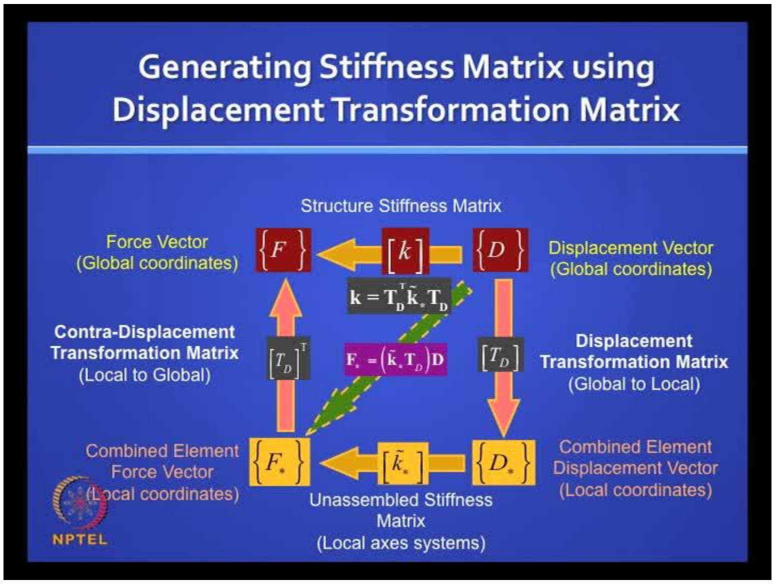

We will see examples of this in trusses later. Now you got you got (remove one ‘you got’) the stiffness matrix at the element level. You got the t d matrix which relates the element level displacements to the structure level displacements. Our job is to find the structure stiffness matrix.

Remember we were not very comfortable with the conventional stiffness method where, you took put the linking coordinates and you have to put it in the slot and so on. Here, you will have no problems because you got this relationship. This is t d. Now, you got the same playground. Everything is familiar to you. You got a new beautiful relationship from the contra gradient principle which is what? Which is this.

Does this make sense to you? The structure level on top, the element level below. t d flowing downward for displacements, t d transpose flowing upwards for forces, then take the diagonal. The diagonal just says that, if you tell me what the displacements are in a truss joint displacement immediately I will tell you what the bar forces are by doing this transpose. This matrix.

Remember I put a k tilde to to (remove one ‘to’) remind myself that I am dealing with the reduced stiffness matrix, so agree? So this is the unknown. Bar forces are nothing but, this times this times that. Is it clear? Then if I want to take this shortcut directly from d to f what do you think k will be? k will be t d transpose k tilde star t d.

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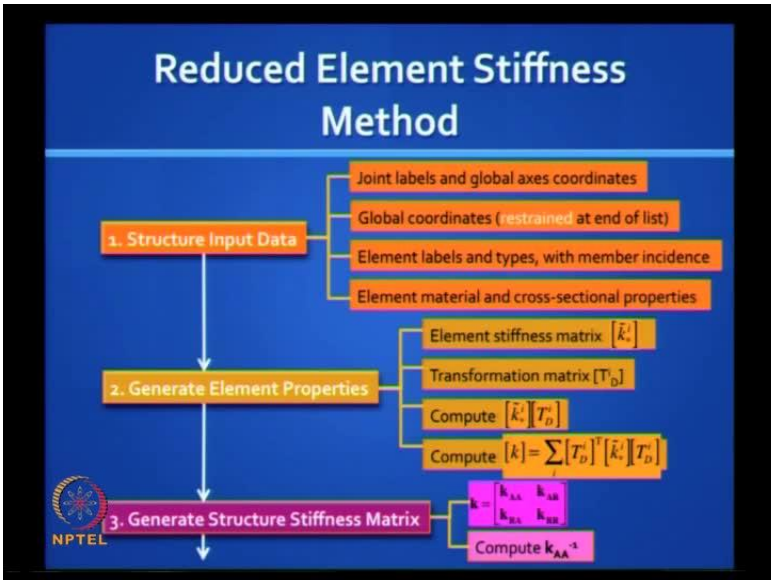

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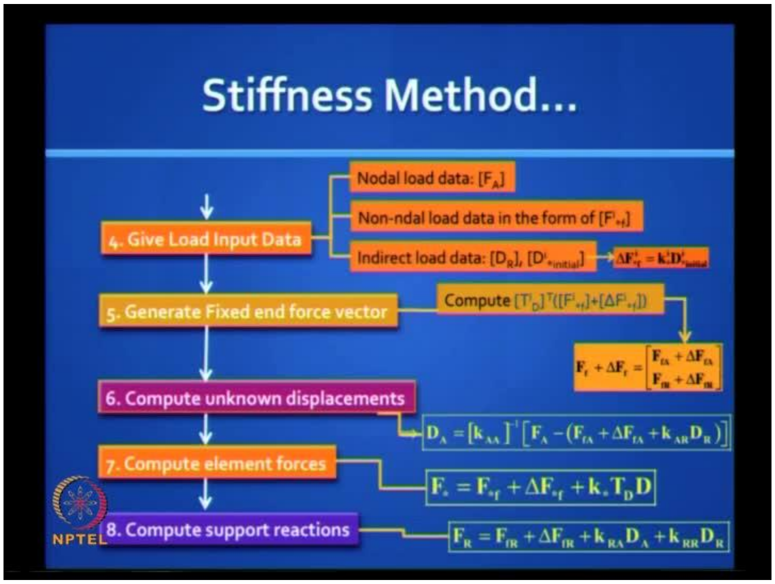

So you will find repeatedly this kind of pattern emerging you have to pre multiply the stiffness matrix whether it is a regular stiffness matrix or the conventional stiffness matrix by the transpose of some transformation matrix in this case t d and also post multiply with t d. So that is that is (remove on 'that is') what you do. When you do this you will get the full stiffness matrix without any effect. So that is to sum up the reduced element stiffness method. We will go through the details later. It similar to the conventional stiffness method. The steps are similar but, you will really understand this only when we do some problems.

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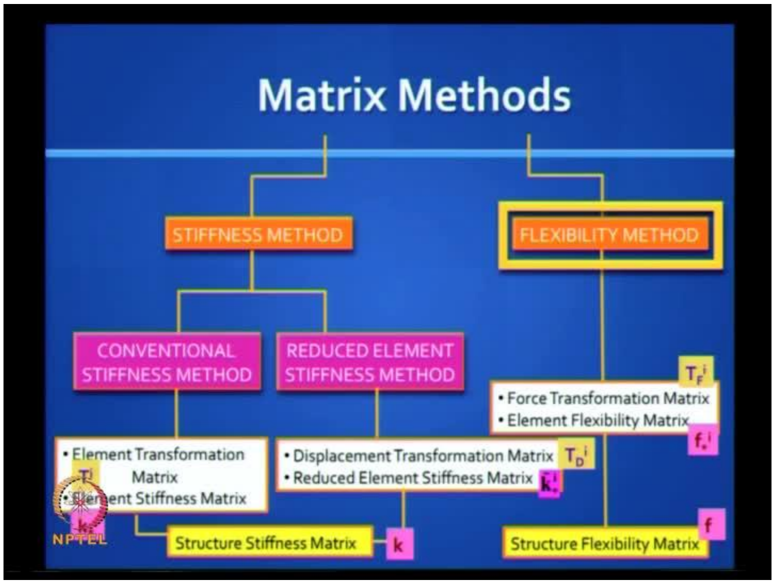

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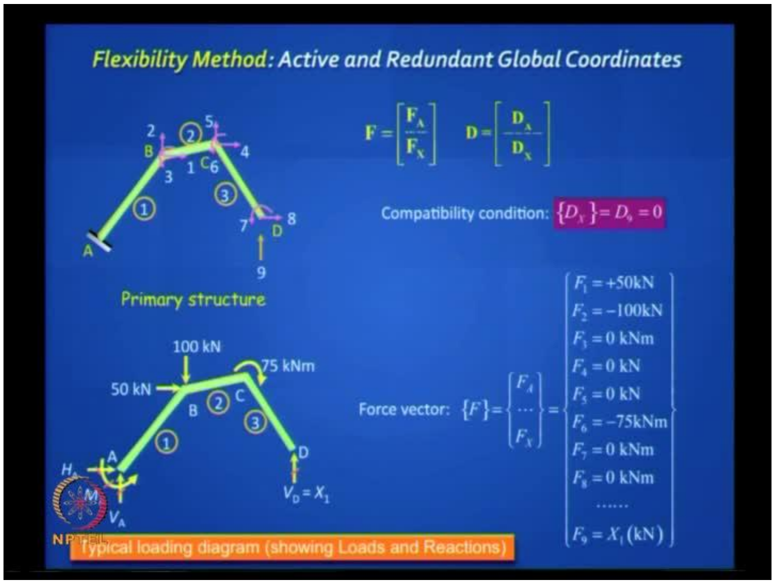

So we are now set to move on to the last topic which is the flexibility method. The first problem you will notice… the main [dis/difference] (remove this bracket) difference between flexibility and stiffness method is that this (remove 'this') you have… even at the global coordinates level you have to be careful. Why? because here you have a choice of redundance. Now this particular structure, is it statically indeterminate or determinate?

Indeterminate, sir.

It is determinate?

Indeterminate.

Indeterminate. What is the degree of indeterminacy?

One.

One. Which would you like to choose as a redundant?

The roller sir (( ))

The support is not… redundant is a force. Ok? you will… so corresponding to that redundant support… you want the primarily structure to be cantilever. Say that. That means, you want to get rid of that roller support, right?

So in terms of statics you have to replace it with a force, right? And that is your single redundant here. So as far as your external loads are concerned there is an one to one similarity between flexibility method and stiffness method. You have eight degrees of freedom active degrees of freedom I have put the same numbering one, two, three, four, five, six, seven, eight. I am not showing the reactions yet. Actually flexibility method is more suited for manual analysis, so you can always get the reactions once you got the free body.

So you can always get the reaction once you got the free body (remove 'so….body') So for the present let us not worry about reactions but, I will show you can find reactions if really want to also. We will skip that. So what do you do, remove it. This is your primary structure and to complete the picture introduce introduce (remove on 'introduce') your redundant as the next coordinate. So you put the f x the redundant forces. Here there is only one supposing at three so you'll put nine ten eleven as your degrees of freedom in addition to the active degrees of freedom we will call them redundant degrees of freedom, ok?

So for example, if you had this problem which i showed you earlier, then v d is your redundant x one and this is how you would write the force vector, right? The first part the f a part we have seen earlier. You are just putting f nine now. f nine is your additional unknown redundant, ok?

You need to pay attention right (remove 'you…right') So f vector is f a f x. d vector is d a d x .so this is the major departure from the stiffness method because the stiffness method we have d a d r, f a f r. in flexibility method for statically indeterminate structure we have f a f x and d a d x as a force vector displacement vector. does it make sense to you? Right? Ok.

In these vectors what is known and what is unknown? Well, fa is known these loads are usually given to you f x is unknown this is the redundant you need to find. d a is not known and d x is known because d x gives you youe (remove 'youe') compatibility conditions right? So what is your compatibility condition? Well, in this particular problem d x is having only one element d nine and it is zero but, suppose you add a support movement settlement then it will not be zero, you'll put the correct value. We are familiar with this we did method of consistent deformation so it falls into place so far so good? Ok.

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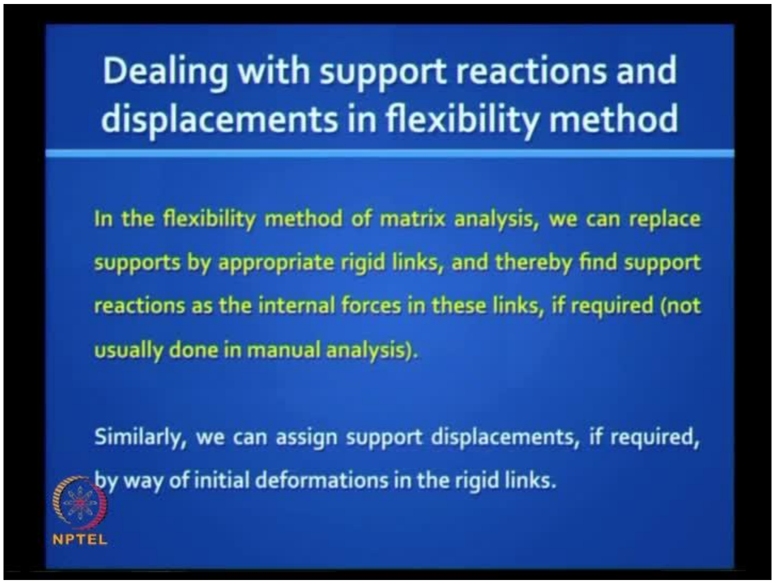

Next point if you want to find support reactions you can do it. We will see it later. In the flexibility method of matrix analysis we can replace supports by appropriate rigid links so the force in a rigid link becomes your… you convert the external reaction to an internal force in an imaginary rigid link. It is a little trick you can do but, it adds to your size of your matrix. You might not find it convenient to do in in practice. Ok.

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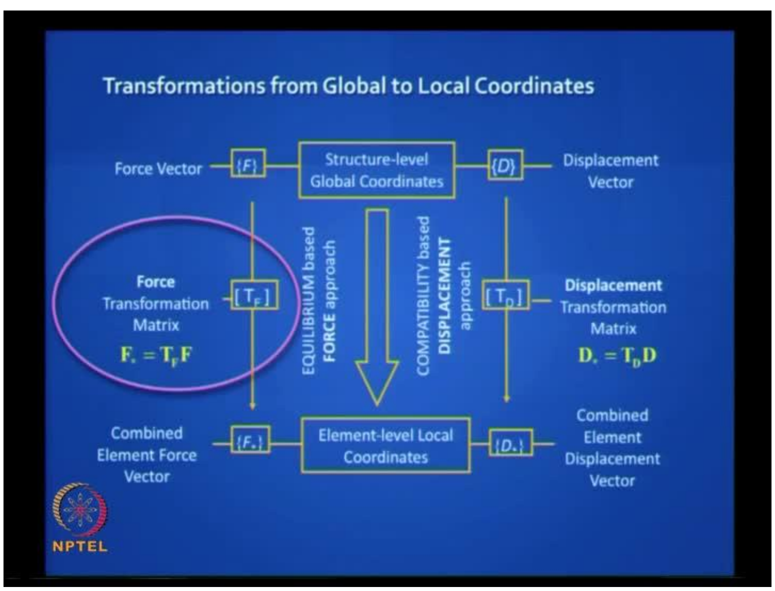

Similarly we can assign support displacement if required by way of initial deformations in the rigid links. Ok we will see that later. So now we are going to look at what one of you asked know? Can we do this transformations? So this is our t f transformation. Remember we did t d transformation we looked at t d transformation in the stiffness method both the conventional stiffness method and the reduce stiffness method. Now you do not worry about t d you look only at t f and we have based (remove 'we…based') obviously t f must be based on based on (remove one 'based on') equilibrium alone.

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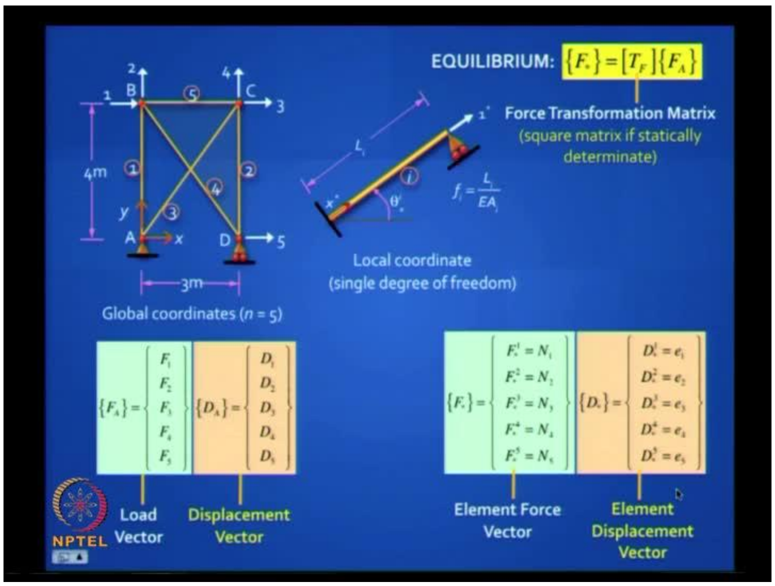

So you can uniquely define t f only for a statically determinate structure because you have a unique statically admissible solution. For a statically indeterminate structure you cannot uniquely define t f please note that. Demonstrate with an example, very familiar example. This is plane truss. You identify the active degrees of freedom. There only five of them, right? One, two, three, four, five. Ah (remove 'ah')

You have one element. Remember you done the reduced element stiffness method. The flexibility method follows the same as same (remove 'as same') system as the reduce element stiffness method. So what is what is (remove on 'what is') your unknown displacement in that element? It is a bar elongation. What is the unknown force in the element? It is a bar force. Tension assume positive. Elongation assume positive. Ok (remove 'ok')

Ok. So that is your degree of freedom. We want to write down the t f matrix. We want to write down the t f matrix, which is f star is equal to t f into f a. How do you write the t f matrix?

(( ))

What do you need to do to write the to generate the t f matrix?

Apply external…

Apply?

Unit load.

Unit load. How many of them? One at a time. How many of them? You answer the question that I have raised.

(( ))

What’s the size of t f matrix?

Five by five.

Five by five. by five (remove ‘by five’) so far so good. How do I get the first column in the t f matrix?

(( ))

Apply unit load. Where?

(( ))

At one. So it is called apply f one equal to one. Then how do I get the second column?

F two equal to one.

F two equal to one. So I apply… so I have to apply five times and one at a time. is it clear?

Let’s do one of them. So this is my first transformation matrix. Five by five, it turns out to be a square matrix. It is a unique matrix, it is interesting. So f a has five components, d a has five components they are referred to the joint movements, right? You know what f one is, you know what d one is. f two, d two etc. This is external. What about internal? f star has also five because there are five bars, right?

Now I have written here a f one star, f two star, f three star but, you can write if you want f one one star because really your degree of freedom is one star. So whether I write just star here or one star here in the case of a truss it does not matter you can choose either. I’ll show you alternatively how you can do it.

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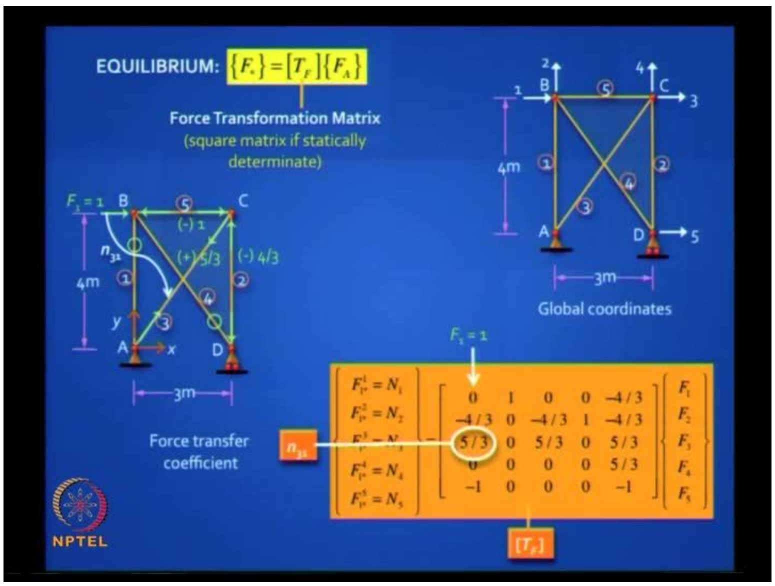

So this is nothing but, the bar force and the internal d one star is nothing but, the bar elongation, Do you understand what this means? Everything make sense? Now we need to generate the t f matrix so let us do that. So let us take that matrix apply f one equal to one. Can we quickly analyze this? Are there any zero bar forces here? By inspection tell me which bar force… ah.

One, four.

One and four. It is quite fast. well look at the support d roller support from there you can make out one of those two bar forces zero which is zero? Four is zero, right. Then you go up to the joint b, which is zero? One is zero. Then what is the force in bar five? One. Is it tension or compression?

Compression.

It's compression so minus one. Then you go to joint c, what is the force in bar three?

(())

The component… the horizontal component of the axial force in bar three must be equal to one. So how much would… plus or minus?

Plus. Plus.

Plus. Very good. Then the vertical component of this must be the force in bar two. How much is that?

Four by (( )) minus four by three.

Solved. Like this you do five times you can get that matrix and some of them are really easy. that is what it looks like so do not get scared t f matrix is very comfortable because these we were raised on statically determinate structures that is of orders so it is very easy to generate the… especially the second one f two equal to one has only one element. ok.

So you understand what t f matrix is. Good. Now let us take any element in that. That element we used to call n three one remember small n for unit loads. n three one refers to the force in bar number three caused by f one equal to one. Remember, we have used this notation earlier. It is called the first transfer coefficient. Why it why it (remove one 'why it') is called transfer coefficient?

(( )) it transfers that much load to the … (( ))

Yeah it is like all these five bars form on organization a partnership and whatever money comes or goes out to that you know it is an immediate transaction online bank account some money come in some fraction of it goes. So that is exactly what is happening, you apply a load f one equal to one on the structure. Can you see what I have done? You apply f one equal to one immediately you got a transfer n three one to bar number three in this case five by three but, do not take that analogy too far because you are actually transferring more money than that came in. Right?

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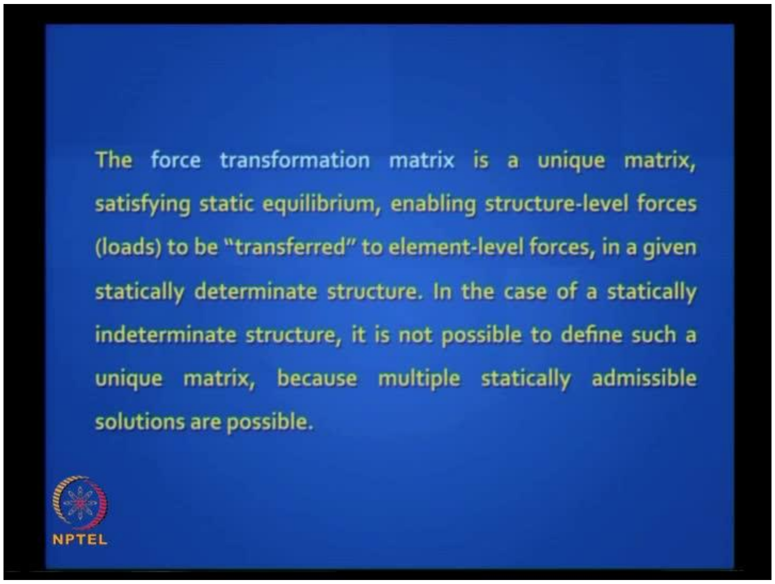

So this is the meaning of force transfer coefficient. Right. the force transformation matrix is a unique matrix satisfying static equilibrium enabling structure level forces or loads to be transferred in inverted commas to element level forces in a given statically determinate structure in the case of a statically indeterminate structure remember it is not possible to define such a unique matrix because multiple statical admissible solutions are possible.

So how do you handle an indeterminate structure? Primary structure.

(( ))

Primary structure. Primary structure is statically determinate. And apply?

(( )).

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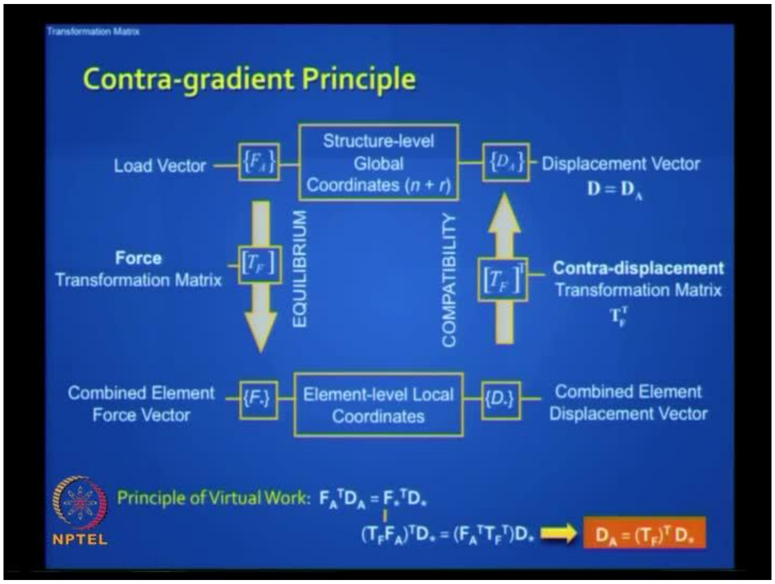

Apply super position. Right, that is what we did in consistent deformation. So that is what we are going to do. So here again you have a contra gradient principle and actually have (remove 'have') a principle which is the one that proves why you do not need to satisfy explicitly compatibility in statically determinate structure. you remember, all structures have to satisfy three requirements. What are the three requirements? Equilibrium, compatibility and force displacement relationship which include the stress-strain relationships, constitute relationships.

Statically indeterminate structures equilibrium alone cannot help you get the unknown forces, we have force to look into the other two. Statically determinate, you are getting away we just managing looking at equilibrium. The question arises… a good student should ask this question, does it mean somehow automatically compatibility satisfied? Yes, prove it.

So the contra gradient principle actually proves it. How do you prove it? We we (remove one 'we') did a similar proof earlier we did principle of virtual work. Can we do the same principle? Then we will get the same old result. No, there we proved t d transpose here we have to prove t f transpose.

So how do you use the same principle? You flip what?

(( )) f star n a

Ah you (remove 'you') there we did d transpose f now we do f transpose d. Now because we are keeping reactions out of this game anyway reactions do not do any work if the supports you are move. We have f a transpose d a, agreed? is equal to f star transpose d star, agreed? Total external virtual work product is equal to total internal virtual work product. What do you need to substitute now?

F star equal to t f.

There. Now you are good. This is simple logic. What do you end up proving by comparing?

(( ))

D a is stable.

(( ))

Fantastic. So without your knowledge when you are satisfying equilibrium in a statically determinate structure you are also simultaneously satisfying compatibility, provided your structure is statically determine. Ok.

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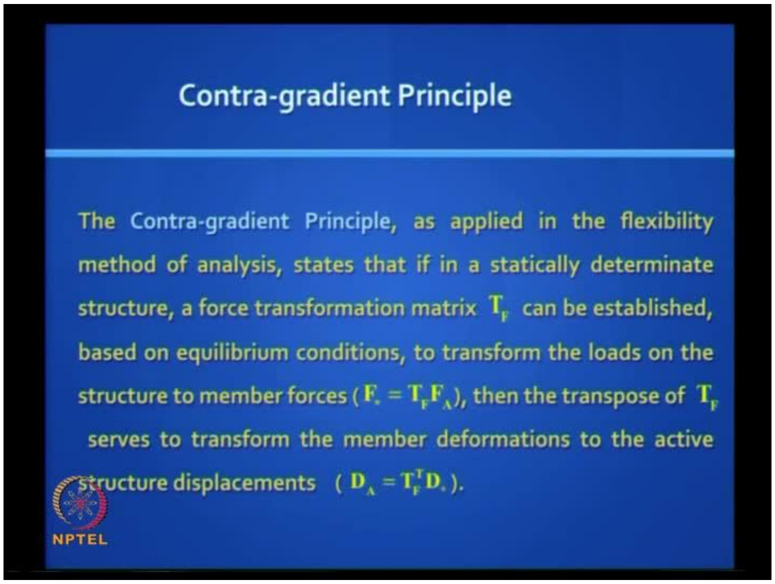

The contra gradient principle, as applied in the flexibility method of analysis, states that if in a statically determinate structure, a force transformation matrix t f can be established, based on equilibrium conditions, to transform the loads on the structure to the member forces f star is equal to t f into f a, then the transpose of t f star servers to transform the member deformations to the active structure displacements. d a is equal to t f transpose t star.

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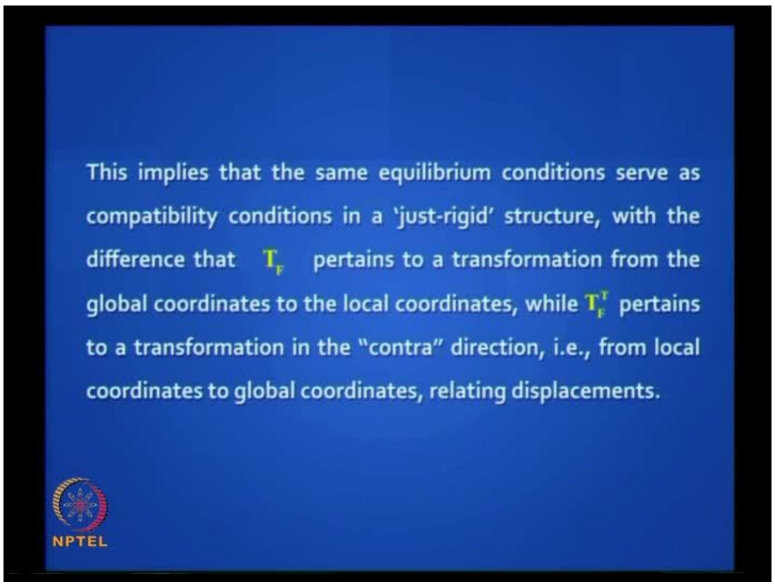

This implies that the same equilibrium conditions serve as compatibility conditions in a 'just rigid' structure, with a difference that t f pertains to a transformation from the global coordinates to the local coordinate, while t f transform transpose pertains to a transformation in the opposite direction in the contra direction in the contra gradient direction that is from local to global when you are relating displacement. That is it.

So you see you will never forget the contra gradient principle. It has two two (remove one 'two') parts to it, one part when you find t d transpose that is in stiffness method, the other part is when you find t f transpose. You will you will (remove one 'you will') be very comfortable with flexibility method. Unfortunately, our focus is not the flexibility method, because the computers do not to do that method for reasons we will see shortly.

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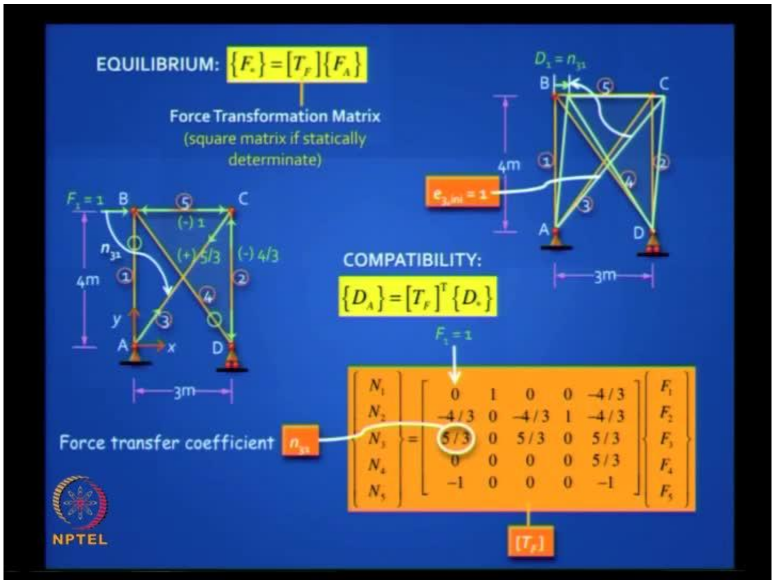

Let’s see the implication of this t f transpose. We have done this, right? Now what are the implications? Let us take a bar in that truss, the third bar. For the fun of it let us heat that bar. How does this t f transpose matrix help us find… find what?

(( ))

Ah

(( ))

Displacement at c? If I say e three initial that is the initial displacement. It could be a lack of fit, it could be heating that bar by say one mm. What do I end up finding? Let us take n three one. Five by three. We did a bank transfer recently, right? for the forces. What is five by three going to be? Displacement, it must be a displacement. Where? At b or c?

(( )) at c

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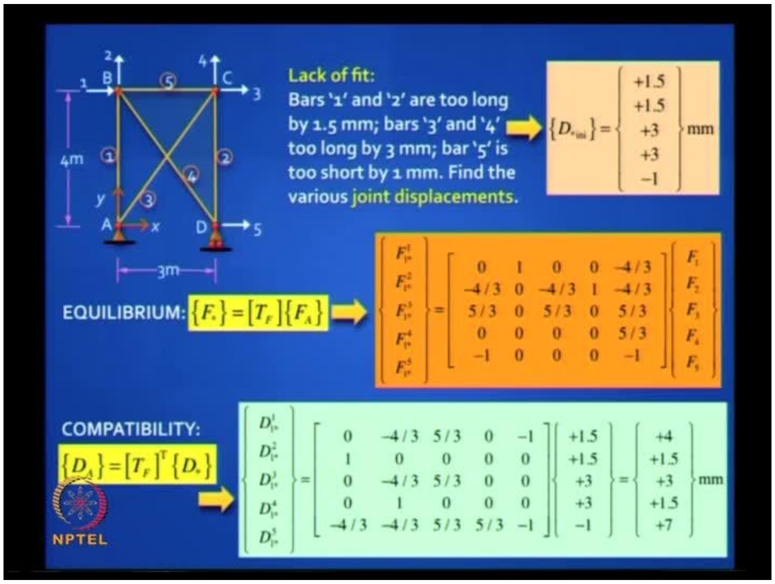

Well you (remove 'you') what you need to do is to see. Where all the bars will move? I have heated you know, the compatibility will satisfy. Remember this five by three is the force you got in bar three when you applied f one equal to one. So when you change the length of the bar three you will get the displacement at one. Where is one where is f one? D. This much. So it is fantastic. Where (remove 'where') You derived from statics of matrix you can get that matrix to get joint displacements. and let us give a (remove 'and...a') So d a is t t f transpose d star let us give another demonstration of it. Same truss. Now I have a lack of fit. I arbitrarily have different, you know? It it (remove one 'it') came to the site the bars came with different lengths with (remove 'with') that is how bad manufacturing is.

So bars one and two are found be two long by one point five mm, bars three and four two long by three mm, bar five too short by one mm, find the various joint displacements. Well, most people will have to necessarily either do this by geometry or actually do the experiment and then measure how much it moved.

But if you are intelligent, if you are brilliant, you can invoke this beautiful contra gradient principle and get it in a jiffy. How do you get it in a jiffy? Well, first you have to convert these into the language of matrices. How will you write that? You have an initial displacement element level matrix which (remove 'matrix which') vector which you can write like that. i n i short for initial. Do you agree? I put plus when I have a larger length put minus when I have a shorter length. Does it make sense? Right?

Then I invoke this powerful principle. these are my (remove 'these…my') This is my t f matrix and I take the transpose of that matrix I have got all the answers in one shot. If I am using matlab. Correct (remove 'correct') You you (remove one 'you') have to agree matrix method is very powerful, if you know how to use it, when to use it and when not to use it. When not to abuse it, when not to misuse it and in practice the misuse and abuse is wide spread. Ok.

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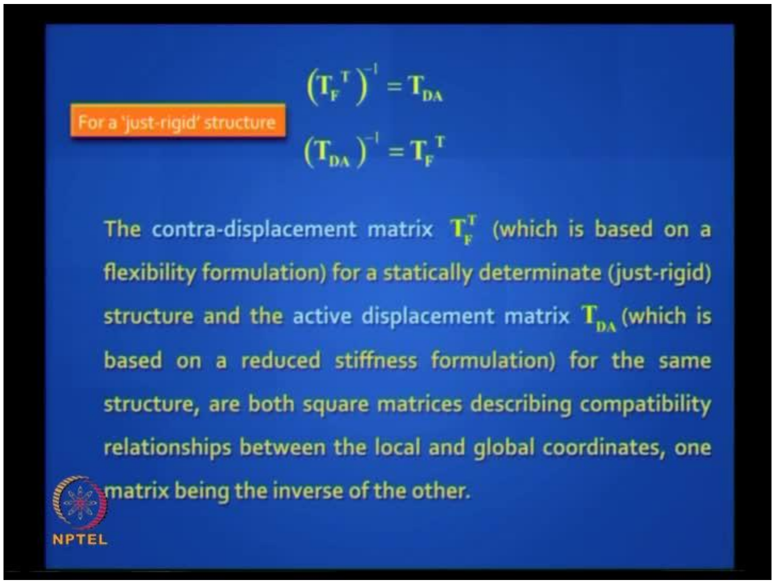

So if you compare now the two transformations, you get another interesting result for a statically determinate structure. t f transpose inverses t d a and t d a inverse is t f transpose. So you can get a potential question in the exam where I give you one of them and ask you to figure out the other you have to bring the same.

The contra displacement matrix that is a name I have given but, they remove ('they') it is not a very common name. You can say t f transpose matrix which is based on a flexibility formulation for a statically determinate just rigid structure and the active displacement matrix t d a which is based on a reduce elements stiffness formulation for the same structure are both square matrices describing compatibility relationship between the local and global coordinates one being the inverse of the other.

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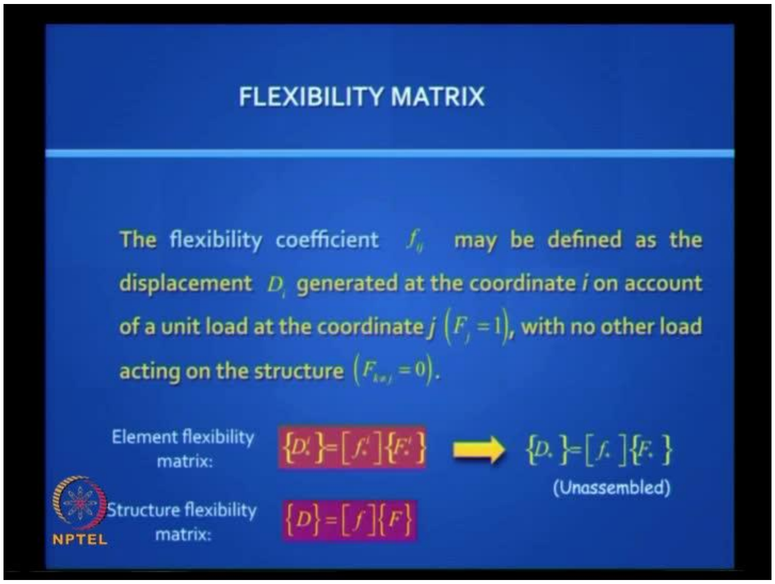

Flexibility matrix definition you know. Just like the stiffness matrix, each element is called the flexibility coefficient. f i j is defined as a displacement d i generated at the coordinate i on account of a unit load at the coordinate j that is f j is equal to one with no other loads acting on the structure. In stiffness method, we said the same thing as it with, all other displacement arrested then only the definition is complete. You can write it this way. You can put all the elements together this way. Ok?

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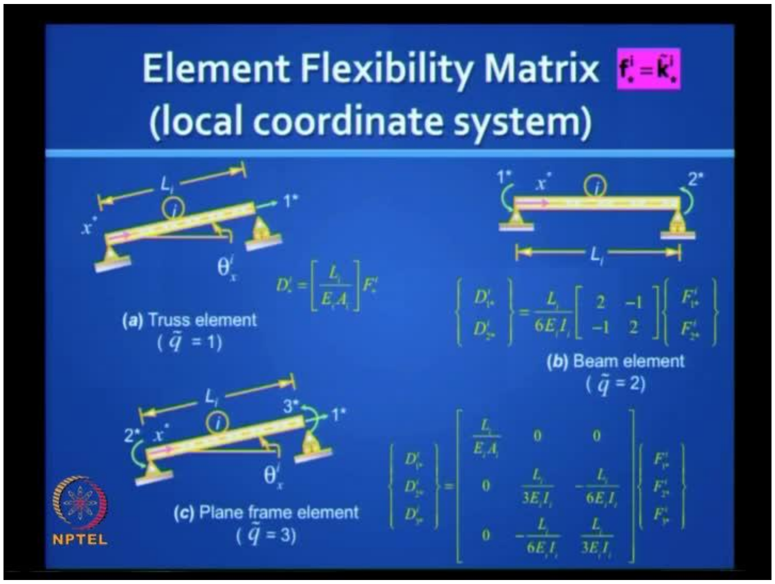

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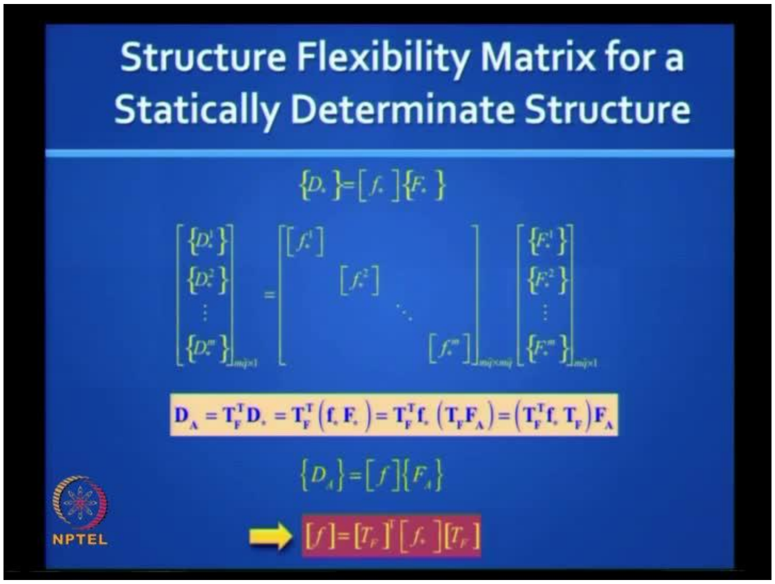

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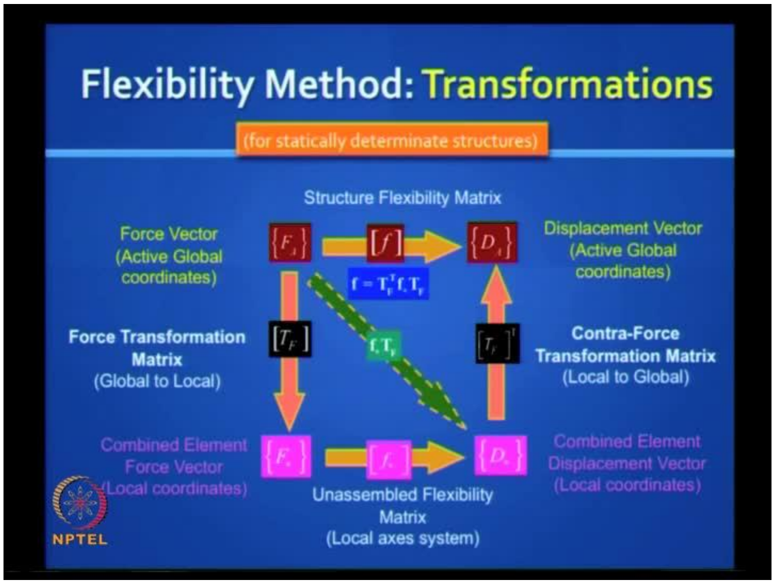

It’s called the unassembled, With (remove ‘with’) the diagonal form and you need to get the structure’s flexibility method. Now it is simple. We have played this game so many times, it is not difficult. You know the element flexibility stiffness is is (remove one ‘is’) just the inverse of what we did for the reduce element stiffness matrix and so this we know if you expand it will look (remove ‘it…look’) it will have a diagonal form like this. Remember, we did it recently. And then you can either write equations, if you have fond of writing equations or you can play this game.

I i(remove one ‘I’) suggest you write this game. It is very easy, right? First at the element level f star is… now you see the direction is moving to the right because flexibility is always opposed to stiffness, they are two different paths. So if you know the forces, element level forces you multiply by the flexibility matrix, you get the element level displacement, right? In in (remove one ‘in’) a bar you multiply the bar force in a truss, multiply the bar force with its flexible (remove ‘flexible’) axial flexibility you get the bar elongation, right.

At the structure level,you need to find f, relating f a with d a. how do you get f from f star? small f. I have put small f because capital f goes to force. Well, let us do it the right royal . We will play the game by its rules so first this this (remove one ‘this’) transformation. It flows this way and on the left side t f transpose and contra gradient says this flows this way. You take the diagonal it will flow this way and you take the shortcut it goes that way. That is your proof t f transpose f star t f is your structure flexibility matrix.

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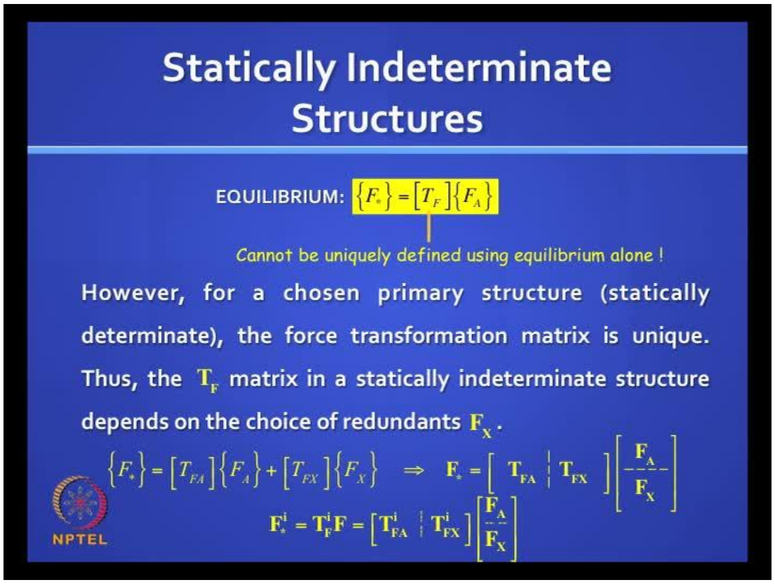

If you got these matrices with you, you do not multiply those matrices. put(remove 'put) Give kill (remove 'kill') that damn computer the job to do it. Press the button get your f matrix. So you got statically indeterminate structures. Here the problem is d f cannot be uniquely defined. So what you need to do? Primary structure. For a chosen primary structure d f is unique, what does it mean? Each student in this class can arrive at the different t f matrix so it is difficult for the examiner to check the results. So he has to wait for the final results to come out.

But whoever does it has to follow the same path. Now I say t f a and t f r. I you know (remove 'you know') have partitioned it. t f a refers to that part which is dealing with the loads on the primary structure and t f r. Sorry its t f x is that which deals with the unknown redundant and a (remove 'a') you can do it at the individual element level or you can take the combine level. Right?

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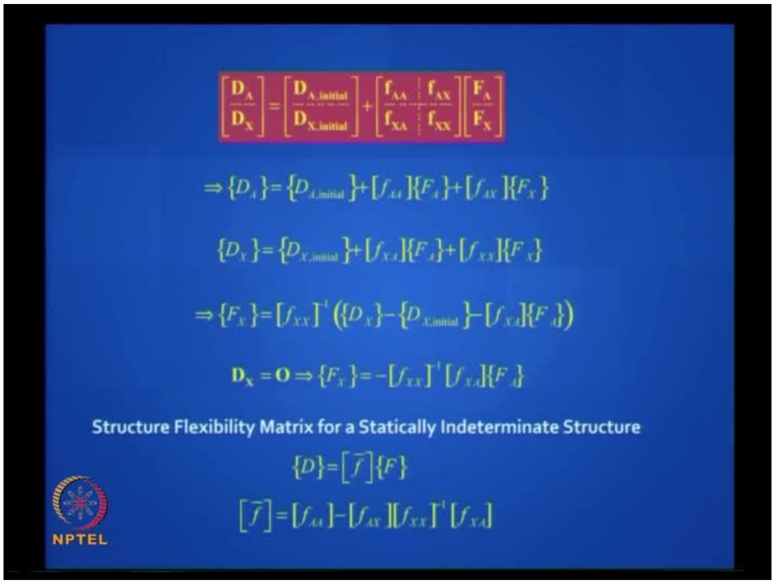

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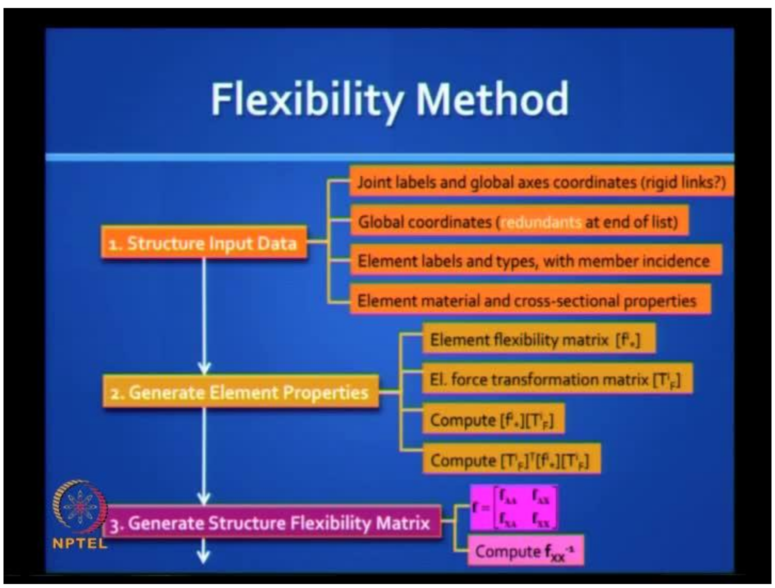

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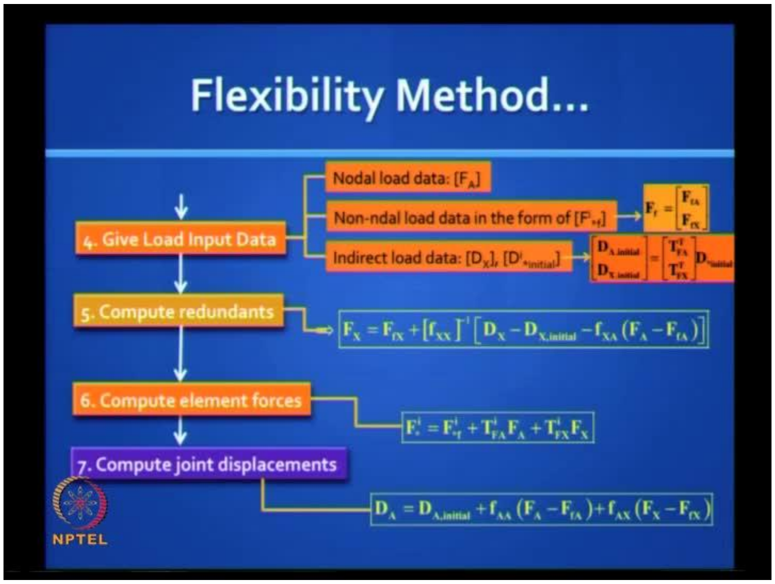

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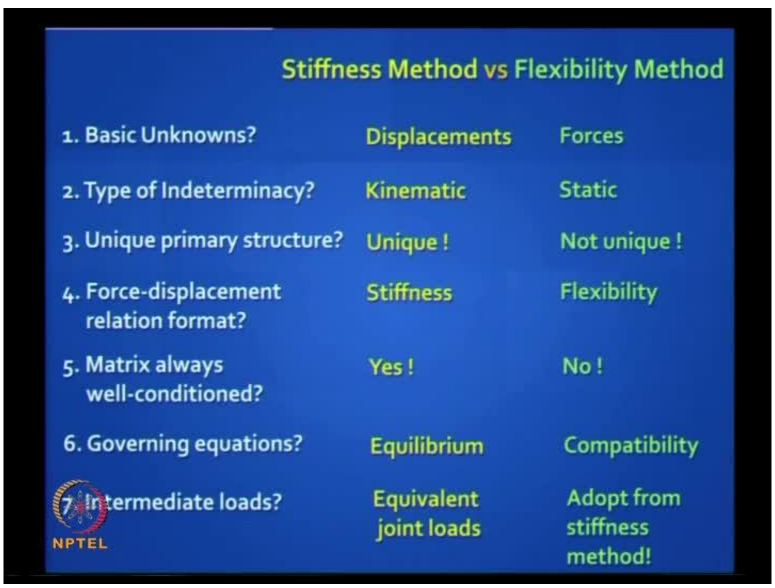

So you can write down these equations. They are compatibility equations. We will study them at (remove ‘at’) in great depth. I just want to rush through it because this is just a road map and you can write down the procedure just as we wrote the procedure earlier for the flexibility method. I want to conclude with this last slide summing up all that we’ve learnt till now.

Broadly, stiffness and flexibility method differ in some some (remove one ‘some’) terms. What are the basic unknowns in stiffness method?

(( ))

In Stiffness methods, they are forces?

Forces (( ))

In stiffness method, the unknown are?

Displacements.

In flexibility method, the unknowns are? This is child’s play. Ok.

Next one. What is the type of indeterminacy in stiffness method?

Kinematic

Kinematic verses.

Static. static. (remove one ‘static’)

Brilliant. what is a unique (remove what…unique’) Do you have a unique primary structure in the stiffness method.

No.

Do you have a unique primary structure, that means will all the students in a class...

Yes.

Yes, what about the flexibility method?

No.

Well depends. if all of you (remove ‘if…you’) If all fools think alike then it will be unique but, otherwise potentially it will be different, right.

What about the force displacement relation format? What is the format you have in stiffness method?

f is equal to kd

Yeah but, if you want to write it in word, what is a format?

f is equal to kd

It’s a stiffness format and flexibility format, come on. Is the matrix that you get the stiffness or flexibility always well-conditioned? Yes, in the stiffness method.

No

Well, sometimes yes though (remove ‘though’) but, you cannot say always, right?

Governing equations?

Ah compatibility.

Compatibility in which one?

(( )) flexibility method compatibility.

Equilibrium in stiffness. Compatibility in flexibility. Lastly how do you deal with the intermediate loads?

Equivalent loads

In stiffness method, equivalent joint loads. In flexibility method?

(( ))

No no (remove one ‘no’) on the computer. Your number of degrees of… this is the real… this is the achilles heel of flexibility method. This is what destroys the beauty of the method you have to borrow the same damn concept from the…? It is a tragedy because there is no other way you can convert to… that means you have a you have a (remove one ‘you have a’) statically determinate portal frame with the uniformly distributed load load (remove one ‘load’) on top you have to pretend that, that beam is fix-fix. Make it statically indeterminate and borrow from stiffness method how they calculate the equivalent and then put it here. That is a problem. So flexibility method is fantastic for trusses and not particularly great when you deal with intermediate loads.

Thank you.

Keywords: Basic Matrix Analysis, Displacement Transformation Matrix, Conventional Stiffness Method, Reduced Element Stiffness Method, Equivalent Joint Loads