ELECTRIC CIRCUIT PROBLEMS WITH SOLUTIONS

Electric Circuit Problems with Solutions

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PREFACE TO THE FIRST EDITION

Electrical-engineering and electronic-engineering students have frequently to resolve and simplify quite complex circuits in order to understand them or to obtain numerical results and a sound knowledge of basic circuit theory is therefore essential. The author is very much in favour of tutorials and the solving of problems as a method of education. Experience shows that many engineering students encounter difficulties when they first apply their theoretical knowledge to practical problems.

Over a period of about twenty years the author has collected a large number of problems on electric circuits while giving lectures to students attending the first two post-intermediate years of University engineering courses. The purpose of this book is to present these problems (a total of 365) together with many solutions (some problems, with answers, given at the end of each Chapter, are left as student exercises) in the hope that they will prove of value to other teachers and students. Solutions are separated from the problems so that they will not be seen by accident. The answer is given at the end of each problem, however, for convenience.

Parts of the book are based on the author's previous work *Electrical Engineering Problems with Solutions* which was published in 1954. Very specialized material in the earlier book, such as electrochemistry, machine windings and illumination, has been omitted together with elementary problems on units and circuit parameters while other topics have now been introduced or covered much more fully. It has been necessary to limit the number of examples so some subjects, such as topology and signal-flow graphs, which readers may expect to find included have had to be left out and others have had less space devoted to them than one would have liked. Valve and transistor circuits have not been dealt with as these are covered in considerable detail in the author's existing book *Problems in Electronics with Solutions*. For the same reason coupled circuits, more advanced problems on transients in circuits and Fourier-series representation of non-sinusoidal waveforms have been omitted.

The author cannot possibly claim that all the problems in the collection are original and it is impossible to acknowledge the sources of those which are not. He is indebted to the late Professor E. W. Marchant, D.Sc., F.C.G.I., Hon. M.I.E.E., who granted permission for a set of problems, originally produced for students at the University of Liverpool, to form the basis of *Electrical Engineering Problems with Solutions*. Some of these problems have been used again. The author also wishes to thank his colleagues O. I. Butler, D.Sc., F.I.E.E., A.M.I.Mech.E., R. Brown, B.Eng., Ph.D., P. J. Spreadbury, M.A., M.Sc. and J. Dobson, B.Eng., Ph.D. for providing questions and solutions on certain topics which they have produced for tutorial classes and the University of Sheffield for permission to use some of the questions set in examination papers.

Although great care has been taken to try to eliminate mistakes some will inevitably have crept in and the author will be glad to have any brought to his notice.

F. A. BENSON

Electronic and Electrical Engineering Department,
The University of Sheffield,
1966.

PREFACE TO THE SECOND EDITION

Many changes have been made in this edition to bring the nomenclature up-to-date and to use S.I. units throughout. For example mho has been replaced by siemens, cycle per second by hertz and vector by phasor. Also British units have been deleted. In addition the circuit diagrams have been re-drawn to conform to the recommendations in B.S.S. 3939. Some further minor modifications and corrections have been made.

F. A. BENSON

Electronic and Electrical Engineering Department,
The University of Sheffield,
1975

CONTENTS

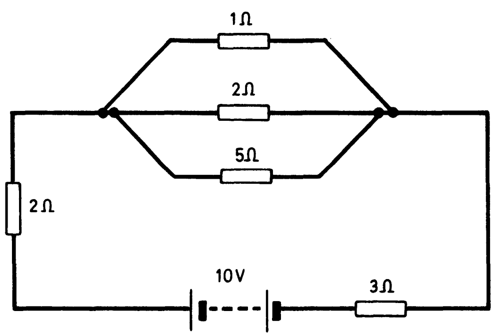
CHAPTER ONE

D.C. CIRCUITS

1. *mn* equal cells are arranged in *m* parallel branches, each branch containing *n* cells in series. If the resistance of a cell is *r* ohms and its e.m.f. *E* volts, find the current flowing in an external circuit of resistance *R* ohms. Prove also that the current is a maximum when $R = nr/m$ and show that the maximum current is $En/2R$ amperes.

[*Ans.* $E/(r/m + R/n)$ amperes]

2. For the arrangement shown determine the voltage across the parallel branch and the current in the main circuit.

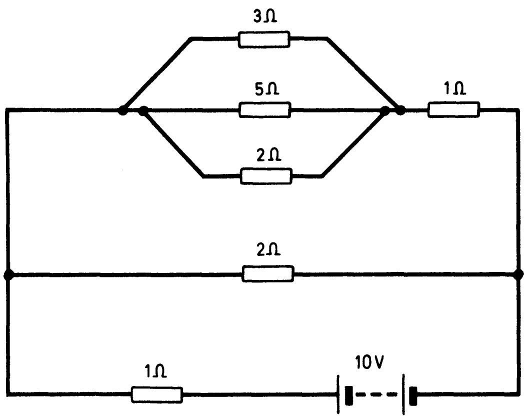


[*Ans.* 1·05 V; 1·79 A]

3. Two insulated cables are connected in parallel. One is made up of seven wires each 2 mm diameter and is 100 m long; the other of five wires each 1 mm diameter and 200 m long. A current of 100 A is fed into the parallel combination. Find the current flowing in each cable. Assume the resistivity of the conductors is $1{\cdot}7 \times 10^{-8}$ Ωm.

[*Ans.* 91·8 A; 8·2 A]

4. Calculate the current supplied by the battery in the following circuit:



[*Ans.* 5·02 A]

5. A moving-coil ammeter has a resistance of 0·7 Ω and gives its full-scale deflection of 150 divisions with a potential difference of 0·015 V. Calculate the resistance of the shunt necessary in order that the instrument may give full-scale deflection for (*a*) 15 A, (*b*) 300 A.

[*Ans.* (*a*) 7/6990 Ω, (*b*) 7/139 990 Ω]

6. A moving-coil ammeter has a resistance of 50 Ω and gives a full-scale deflection of 150 divisions with a current of 2·5 mA. Estimate the resistance of a shunt with which the instrument will give full-scale deflection for a current of 75 A.

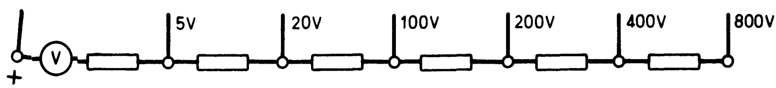
[*Ans.* 1·25/749·975 Ω]

7. A milliammeter gives full-scale deflection for 15 mA and has a resistance of 14·6 Ω. What resistance must be put in series with it if it is to be used as a voltmeter giving 250 V for full-scale deflection?

[*Ans.* 16 652 Ω]

8. A moving-coil voltmeter has a resistance of 20 Ω and gives full-scale deflection of 150 divisions with a potential difference of 300 mV.

The terminals are arranged as shown in the diagram. Find the resistance that must be put in series with the instrument and between each terminal and the next so that the instrument may give full-scale deflection for 5, 20, 100, 200, 400 and 800 V.



[*Ans.* 313·3, 1 000·3, 5 333·3, 6 666·6, 13 333·3 and 26 666·6 Ω]

9. The current passing to a Kelvin double-bridge network is 5A. The network has the following components:

Standard resistance = 0·1 Ω
Resistance to be measured = 0·16 Ω.
Ratio resistances = 10 and 17·1 Ω.
Galvanometer resistance = 10 Ω.
Resistance in remaining branch = 0·02 Ω.

Determine the current through the galvanometer.

[*Ans.* 0·001 A]

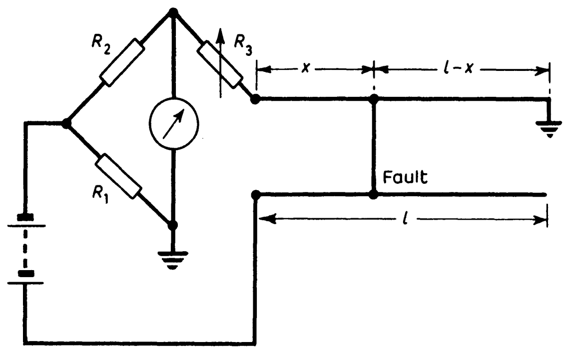
10. A Varley-Loop Test is used to locate an earth fault on a telephone line which is 60 km long. The resistance per km of line is found to be 43 Ω. The fixed resistors in the circuit each have a value of 300 Ω. At balance the variable resistor is set at 1760 Ω. Determine the position of the fault.

[*Ans.* 39·5 km from the end connected to the variable resistor]

11. A Murray-Loop Test is used to locate an earth fault on a telephone line which is 30 km long. The fixed resistor has a value of 500 Ω and at balance the variable resistor is set at 100 Ω. Find the position of the earth fault.

[*Ans.* 10 km from the end connected to the variable resistor]

12. The circuit shown is used to find a contact fault on a pair of telephone lines which are 30 km long. The resistors R_1 and R_2 are each made 300 Ω and the resistance per km of line is 40 Ω. Find the distance x at which the fault occurs if at balance $R_3 = 440$ Ω.



[*Ans.* 9·5 km]

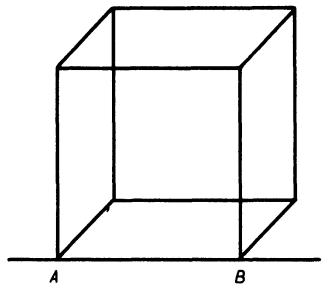
13. A hexagonal frame *ABCDEF* is made up of six conductors each having a resistance of 1 Ω. Diagonal wires are connected between *A* and *C*, *A* and *D*, and *A* and *E*, the wires having the same cross-section as the wires of the frame. A potential difference of 10 V is applied between *C* and *E*. Find the current flowing in each branch of the network and the effective resistance between *C* and *E*.

[*Ans.* $i_{CB} = i_{FE} = 2{\cdot}5$ A; $i_{CA} = i_{AE} = 2{\cdot}885$ A;
$i_{AD} = 0$; $i_{CD} = i_{DE} = 5$ A; 0·98 Ω]

14. A network is made up of five conductors. The resistance between *A* and *B* is 20 Ω, between *B* and *C* 25 Ω, between *A* and *D* 30 Ω, between *C* and *D* 35 Ω and between *B* and *D* 100 Ω. A potential difference of 10 V is applied between *A* and *C*. Find the current in each of the branches of the network and the total resistance between *A* and *C*.

[*Ans.* $AB = 0{\cdot}223$ A; $BC = 0{\cdot}222$ A;
$CD = -0{\cdot}153$ A; $BD = 0{\cdot}001$ A;
$AD = 0{\cdot}152$ A; 26·67 Ω]

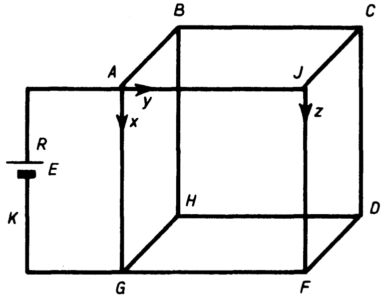
15. Estimate the resistance between points A and B on a framework made up of twelve wires, each of resistance 1 Ω, as shown in the diagram.*



[*Ans.* 7/12 Ω]

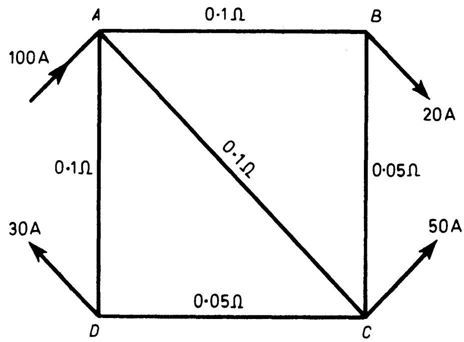
16. A cube is formed from twelve wires each having a resistance r as shown. Given currents x, y and z and assuming the resistance of the battery is R show that

$$\frac{y}{5} = \frac{x}{14} = \frac{z}{4} = \frac{E}{24R + 14r}$$



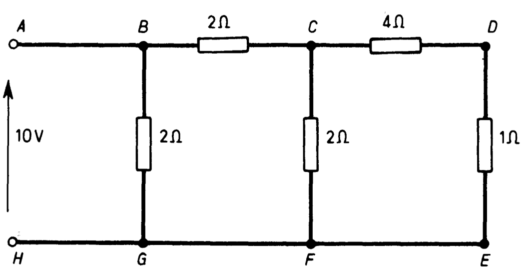
*A solution to a similar problem, where the resistance between two diagonally opposite points of the cube was required, has been given by A. J. Small (*Engineering News*, No. 190, March 18th, 1965, p. 2). This solution requires a minimum of mathematics. An alternative solution has been presented by H. Trencham (*Engineering News*, No. 192, April 1st, 1965, p. 2).

17. Apply (*a*) the Superposition-of-Currents method, (*b*) Kirchhoff's Laws, (*c*) Thévenin's Theorem, to find the current in branch *AC* of the network *ABCD* shown.



[*Ans.* 35·7 A]

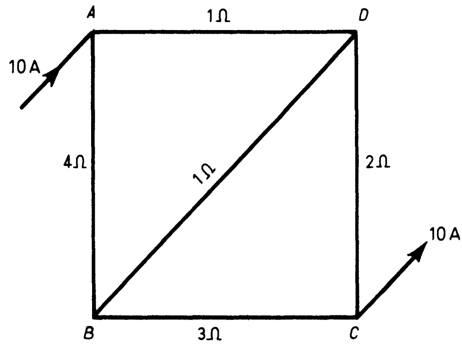
18. Use Thévenin's Theorem to calculate the current in branch *CF* of the network below and check the result by means of Kirchhoff's Laws. Find, also, the resistance of the circuit measured between terminals *A* and *H*.



[*Ans.* 2·08 A; 1·26 Ω]

19. A constant current of 10 A flows into and out of the circuit *ABCD* as shown in the figure.

Use Thévenin's Theorem to find the current in *BD*.

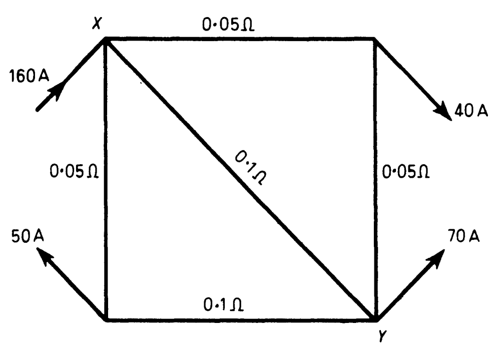


[*Ans.* 1·43 A]

20. *ABCD* is a uniform circular wire of resistance 4 Ω and *AOC BOD* are two wires forming diameters at right angles and joined at *O*. *AOC* and *BOD* each have a resistance of 2 Ω.

Prove that if a battery be placed in *AD*, the resistance of the network offered to it is 15/7 Ω.

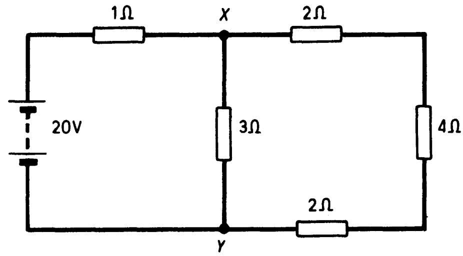
21. Use (*a*) Kirchhoff's Laws and (*b*) Thévenin's Theorem to find the current in *XY* in the circuit below.



[*Ans.* 40 A]

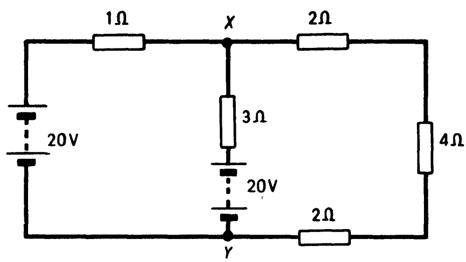
22. For the circuit shown calculate the current in XY by:

(*a*) Thévenin's Theorem.
(*b*) Kirchhoff's Laws.
(*c*) Maxwell's Cyclic-Current Rule.



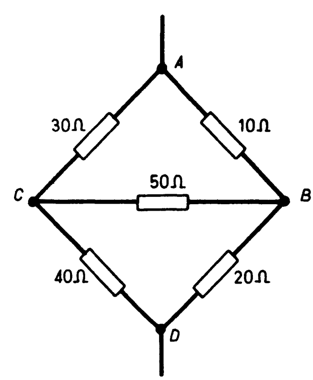
[*Ans.* 4·57 A]

23. Use Thévenin's Theorem to calculate the current flowing in branch XY of the circuit shown, and check the result by Kirchhoff's Laws and Maxwell's Cyclic-Current Rule.



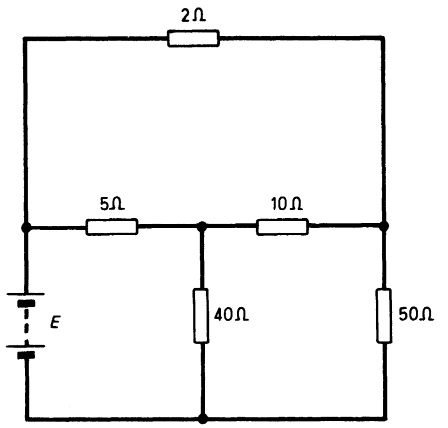
[*Ans.* 0·57 A]

24. Find the equivalent resistance of the network illustrated between points A and D by first using a delta-star transformation on mesh ABC.

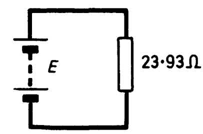


[*Ans.* 20·94 Ω]

25. Reduce the circuit shown at (*a*) to that of (*b*).

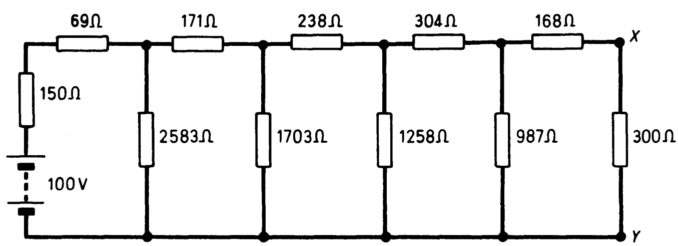


(*a*)



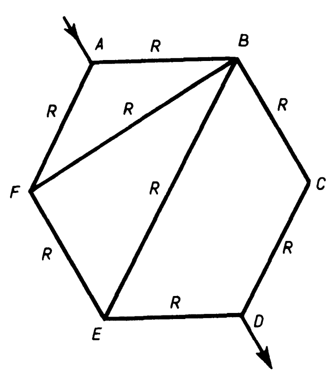
(*b*)

26. By the iterated use of Thévenin's Theorem find the current in the branch XY of the circuit illustrated.



[*Ans.* 35·7 mA]

27. Use the delta-star transformation to show that the equivalent resistance of the network illustrated between points A and D is $40R/29$. Check the result by Maxwell's Cyclic-Current Rule.



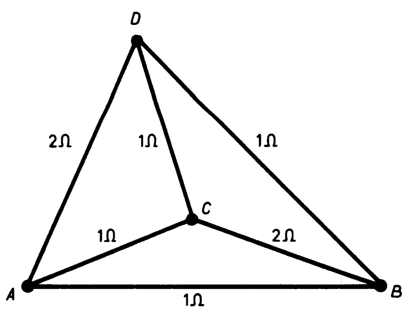
28. The sides of a hexagon *ABCDEF* are formed of wires each of resistance R; a wire of the same resistance R joins *BF*. Find the equivalent resistance of the circuit between the points A and D. Show that the result is independent of the resistance of the wire joining B and F.

[*Ans.* $3R/2$]

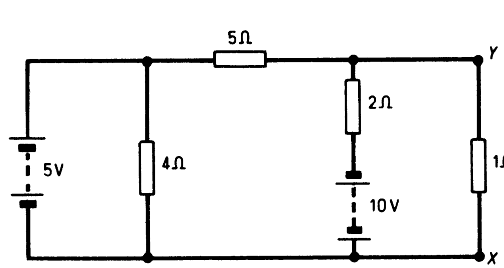
29. A square *ABCD* is formed from a uniform piece of wire and the centre is joined to the middle points of the sides by straight wires of the same material and cross-section. A current enters at A and leaves at the middle point of *BC*.

Use Maxwell's Cyclic-Current Rule to show that the equivalent resistance of the network between the points where the current enters and leaves is 29/48 times the resistance of a side of the square.

30. A triangular pyramid *ABCD* is built up of six wires whose resistances are shown on the sketch. Use Kirchhoff's Laws to prove that the effective resistance of the network between points A and B is $7/12\,\Omega$.



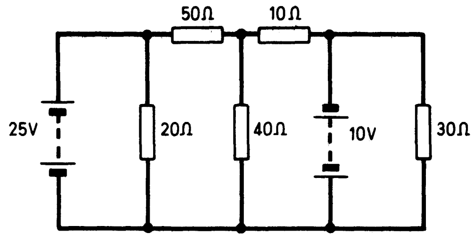
31. Find the voltage between points X and Y in the network shown using (*a*) Millman's Theorem*, (*b*) the Generalized Form of Norton's Theorem.*



[*Ans.* 2·35 V]

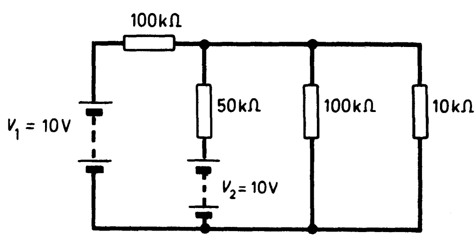
*See the solution to Problem 141.

32. Calculate the voltage across the 40-Ω resistor in the network illustrated using (*a*) Millman's Theorem, (*b*) the Generalized Form of Norton's Theorem. Check the results with Maxwell's Cyclic-Current Rule.



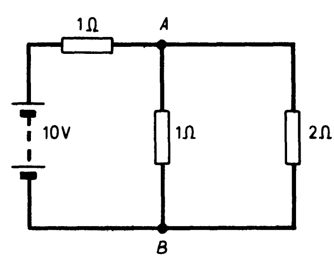
[*Ans.* 3·45 V]

33. Calculate the voltage across the 10 000-Ω resistor in the network shown if (*a*) the two batteries are in phase, (*b*) the two batteries are out of phase. Use both Millman's Theorem and then the Generalized Form of Norton's Theorem for the solution.



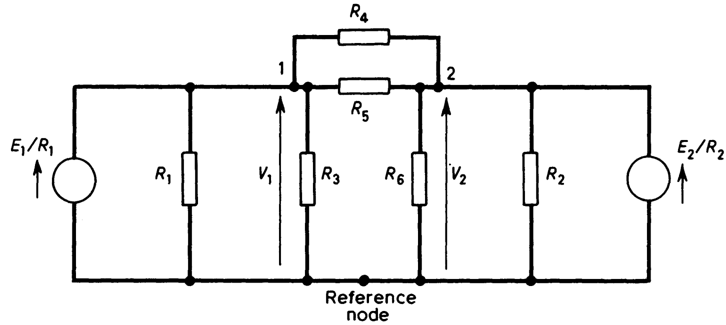
[*Ans.* 2·14 V; 0·71 V]

34. Use nodal analysis to find the voltage across *AB* in the circuit illustrated.



[*Ans.* 4 V]

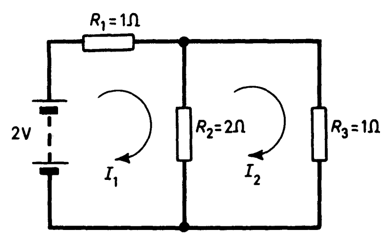
35. Write down the nodal equations for the circuit illustrated.



$$\left[Ans.\ \frac{E_1}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - V_2\left(\frac{1}{R_4} + \frac{1}{R_5}\right);\right.$$

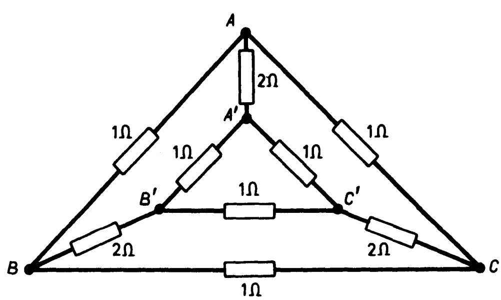
$$\left.\frac{E_2}{R_2} = -V_1\left(\frac{1}{R_4} + \frac{1}{R_5}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right)\right]$$

36. Use the Compensation Theorem to determine the change in current I_1 in the network shown when resistor R_3 increases by 20%.



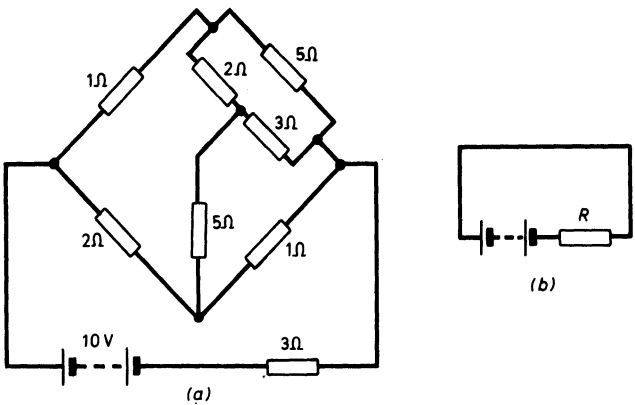
[*Ans.* (2/35) A]

37. A network of nine conductors connects six points A, B, C, A', B' and C' as shown. Determine the resistance between points A and C by using the delta-star transformation only.



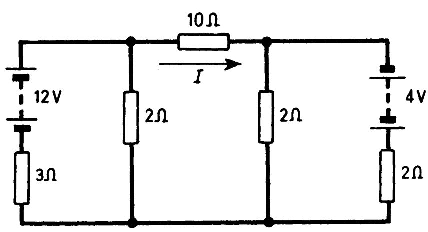
[*Ans.* (7/12) Ω]

38. With the aid of the delta-star transformation reduce the network given in (*a*) to the equivalent circuit shown at (*b*) and hence find *R*.



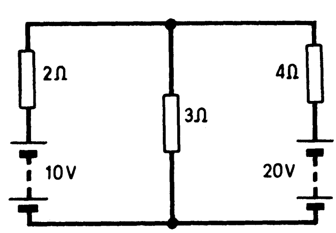
[*Ans.* $R = 4{\cdot}612\ \Omega$]

39. Use Thévenin's Theorem to find the current *I* in the circuit shown.



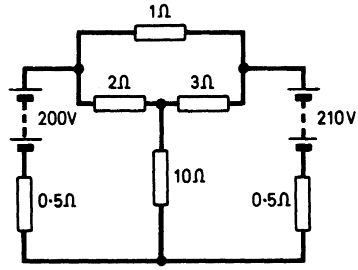
[*Ans.* (34/61) A]

40. Apply the Principle of Superposition to find the currents in the three branches of the network shown.



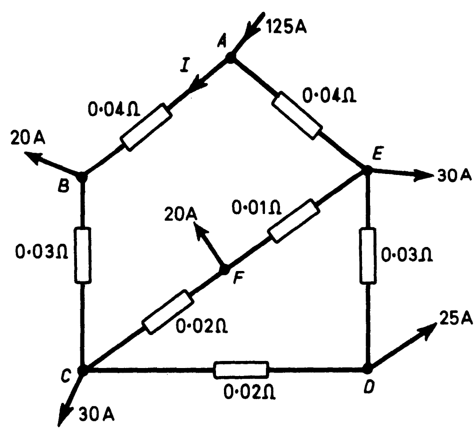
[*Ans.* (1) 0·38 A; (2) 3·08 A; (3) 2·7 A]

41. Use Maxwell's Cyclic Current Rule to find the current through the 10-Ω resistor in the circuit shown.



[*Ans.* 17·86 A]

42. Use Thévenin's Theorem to calculate the current *I* in the circuit shown.



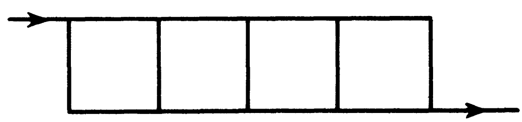
[*Ans.* 53·9 A]

43. Five points* are connected by ten wires, each pair being joined by a wire of the same resistance *R*. Make use of symmetry and employ Kirchhoff's Laws to show that the resistance of the network to current entering at one point and leaving at any other point is $2R/5$.

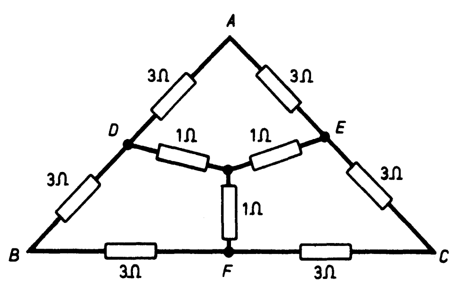
*Readers may like to extend this problem to *n* points to show that if every pair of *n* Points is connected by a conductor of resistance *R* the equivalent resistance of the network between any pair of electrodes is $2R/n$.

44. *A*, *B*, *C* and *D* are four points in succession at equal distances along a wire. Points *A* and *C* and points *B* and *D* are also joined by two other similar wires of the same length as the distances between these pairs of points measured along the original wire. Current enters the network, so formed, at *A* and leaves at *D*. Show that 1/5 of it passes along *BC*.

45. A network is made up of 13m of uniform wire, placed to form four equal squares side by side as shown in the diagram. Current enters the network at one extreme corner and leaves by the diagonally opposite corner. Use Maxwell's Cyclic-Current Rule to show that the total resistance of the network is equal to that of $2\frac{7}{19}$ m of the wire.

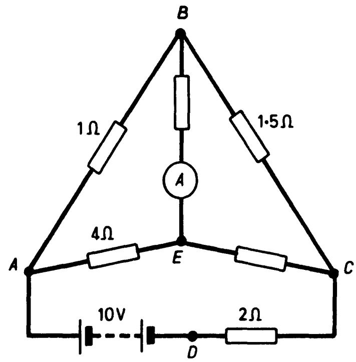


46. A network of conductors is arranged as shown. Determine the resistance between *A* and *C* by using delta-star and star-delta transformations, or a symmetry argument.

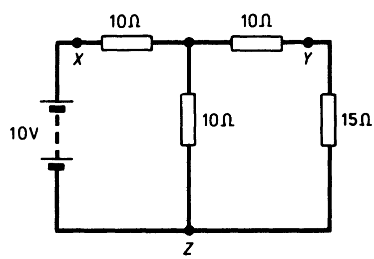


[*Ans.* $3\frac{1}{3}$ Ω]

47. In the circuit illustrated the values of the resistors in branches *EB* and *EC* are unknown. The ammeter in branch, *EB*, however, reads zero. Use Maxwell's Cyclic-Current Rule to show that the resistance of branch *EC* is 6 Ω and that the current supplied by the battery is 2·5 A.



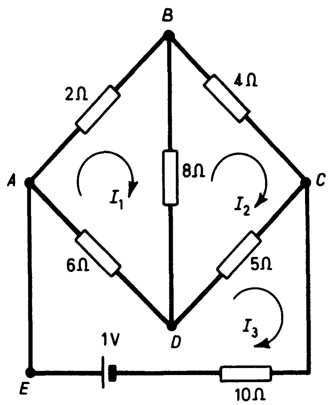
48. In the circuit shown, transform the star *XYZ* to a delta and then apply Thévenin's Theorem to find the current through the 15-Ω resistor. Assume that the supply has zero internal resistance.



[*Ans.* (1/6) A]

49. For the circuit illustrated it is found, by applying Maxwell's Cyclic-Current Rule, that $I_1 = 0{\cdot}0494$ A, $I_2 = 0{\cdot}0445$ A and $I_3 = 0{\cdot}0723$ A.

Use the Principle of Superposition and the Reciprocity Theorem together to evaluate the current in the 1-V battery circuit when an e.m.f. of 2 V is added in branch *BD* so as to oppose the flow of the original current in that branch.



[*Ans.* 0·0625 A from *C* to *E*]

50. A two-wire d.c. distributor 2000 m long is fed at one end at 410 V and at the other end at 390 V. Each conductor has a resistance of 0·125 Ω per 1000 m and the distributor supplies 9 equidistant loads each of 80 A. Determine which loads have the lowest supply voltage and the value of that voltage.

[*Ans.* 5th and 6th from 410 V end; 350 V]

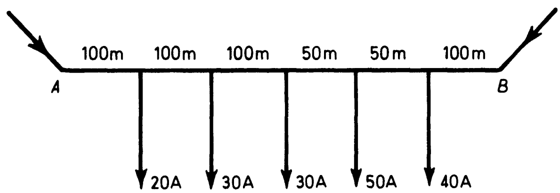
51. A two-wire distributor which is 400 m long and which is fed at one end is loaded as shown by the following table.

Distance from feeding point (m)	100	200	300	350	400
Load in amperes	20	30	30	50	40

If the resistance/m of the distributor is 10^{-4} Ω find the total voltage drop.

[*Ans.* 10·1 V]

52. The distributor shown in the sketch is fed at both ends. The feeding points *A* and *B* are the same potential. Determine the current distribution in the system.



[*Ans.* The currents in the various sections beginning at *A* are 69, 49, 19, 11, 61 and 101 A respectively]

53.* A capacitor of capacitance 8 μF is to be charged by a voltage of 400 V through a resistance of 100 000 Ω. How long will it take for the voltage across the capacitor to rise from its initial zero value to 300 V? What fraction of the final energy is stored in the capacitor at 300 V?

[*Ans.* 1·11 s; 0·56]

54. A coil having a self-inductance of 5 H and a resistance of 4 Ω is connected with a battery having an e.m.f. of 8 V. Draw a curve showing the rate of increase of current through the coil after switching on, and find how long it takes before the current reaches 1·9 A. What is the time constant of this circuit?

[*Ans.* 3·74 s; 1·25 s]

55. The four field coils of a shunt-wound generator, each having a self-inductance of 1·2 H, are connected in series, and carry a current of 1·6 A if connected to a 230-V supply. When the circuit is opened they are connected to a discharge resistance of 100 Ω. Draw the first part of the curve showing how the current dies away under these conditions.

56. A coil of resistance 10 Ω and inductance 0·4 H is connected to a 100-V, d.c. supply. Calculate:

(*a*) the rate of change of current at the instant of closing the mains switch.

*Other problems on transients may be found in the book: F. A. Benson, *Problems in Electronics with Solutions*, Spon, 4th Edition, 1965, Chapter 2.

(*b*) the final steady value of current.
(*c*) the time constant of the circuit.
(*d*) the time taken for the current to rise to half its final value.
(*e*) the energy finally stored in the magnetic field in joules.

[*Ans.* (*a*) 250 A s^{-1}, (*b*) 10 A, (*c*) 0·04 s, (*d*) 0·0277 s, (*e*) 20 J]

57. Two coils, having self inductances of 0·02 and 0·01 H, are mounted on a common iron core to have a coupling coefficient of 0·6. Determine the initial rates of rise of battery current when they are connected to a 12-V battery (*a*) in series, (*b*) in parallel.

[*Ans.* (*a*) 255 or 925 A s^{-1}, (*b*) 1221 or 4420 As^{-1}]

ADDITIONAL PROBLEMS

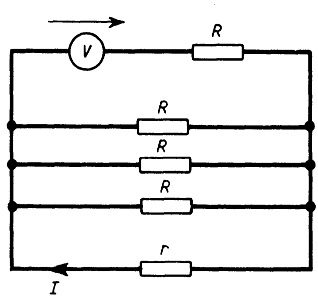
58. A battery of e.m.f. E and internal resistance r is connected to a heating element of resistance R. Find the value of R required for maximum heating effect, and the efficiency of the system. Where does the waste power go?

[*Ans.* $R = r$; 50%]

59. A battery of twelve cells (each of e.m.f. 1·4 V and internal resistance 6·3 Ω) is required to send the largest possible current through a 10-Ω resistor. Determine the grouping of the cells to give this current. What value has the current?

[*Ans.* Three parallel rows with four cells in each; 0·305 A]

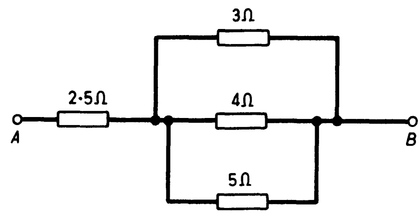
60. Find I as a function of R, r and V for the circuit shown and check the result using the Reciprocity Theorem.



[*Ans.* $I = V/(4r + R)$]

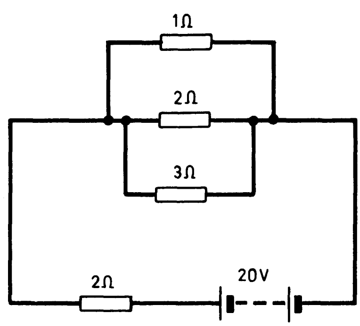
61. Find the equivalent resistance of the combination of resistors shown.

A 12-V battery with an internal resistance of 0·25 Ω is connected between terminals *A* and *B*. Calculate the current in each of the resistors.



[*Ans.* 1·277 Ω; 2·98 A; 1·27 A; 0·95 A; 0·76 A]

62. Calculate the voltage across the parallel branches of the circuit arrangement shown and the current in the main circuit.



[*Ans.* 4·29 V; 7·86 A]

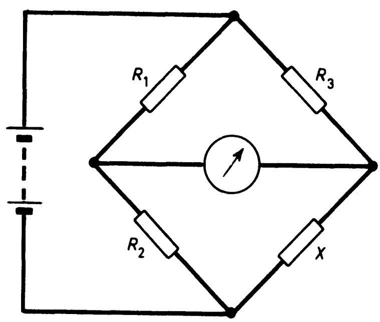
63. A milliammeter having a resistance of 15 Ω gives full-scale deflection for a current of 20 mA. Determine the value of the resistor which must be put in series with the instrument if it is to be used as a voltmeter giving 100 V for full-scale deflection.

[*Ans.* 4985 Ω]

64. A moving-coil ammeter has a resistance of 100 Ω and gives a full-scale deflection with a current of 5 mA. Evaluate the resistance of a shunt with which the instrument will give full-scale deflection for a current of 5 A.

[*Ans.* (1/9·99) Ω]

65. The bridge network shown balances at 10°C when $R_1 = 600\ \Omega$, $R_2 = 200\ \Omega$ and $R_3 = 1800\ \Omega$. The ambient temperature rises to 20°C, which brings about a change in R_1 of $-0{\cdot}2\%$ per °C and in R_2 of $+0{\cdot}15\%$ per °C. The bridge is re-balanced at the new temperature by decreasing R_3 to $1764\ \Omega$.
Find (*a*) the value of X at 10°C.
(*b*) the percentage change in X per °C rise in temperature.



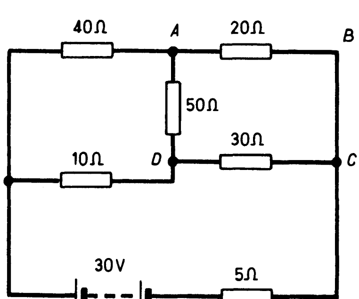
[*Ans.* (*a*) 600 Ω; (*b*) 0·167% per °C]

66. A pentagon *ABCDE* is made up of five lengths of uniform wire each of resistance 1 Ω. Calculate the resistance between points *A* and *C*.

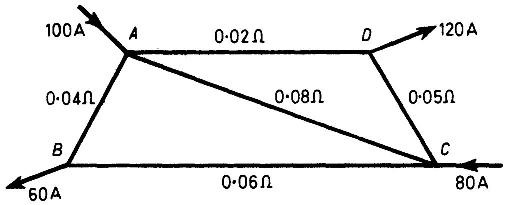
If now two lengths of wire, each of resistance 1 Ω, join *A* and *D* and *A* and *C* respectively, use the delta-star transformation on meshes *ABC* and *AED* to calculate the resistance between points *A* and *C*.

[*Ans.* 1·2 Ω; (10/21) Ω]

67. By first using a delta-star transformation on the mesh *ABCD* of the circuit shown, prove that the current supplied by the battery is (90/83) A.

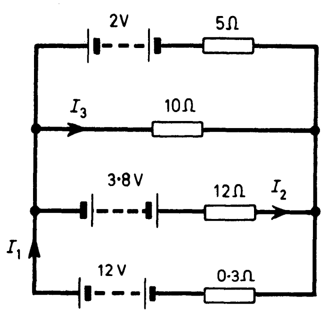


68. Use Thévenin's Theorem to calculate the current in conductor *AC* of the network *ABCD* shown.



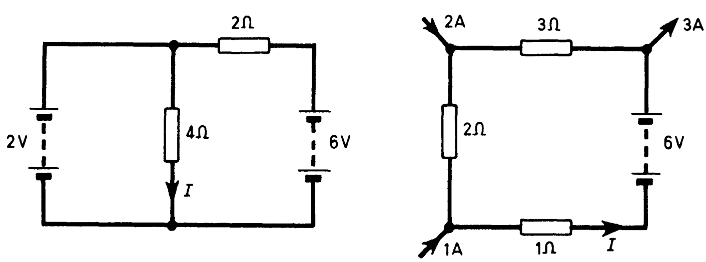
[*Ans.* 7·38 A flowing from *C* to *A*]

69. A circuit is made up as shown in the diagram. Find I_1, I_2 and I_3



[*Ans.* $I_1 = 4{\cdot}07$ A; $I_2 = 1{\cdot}215$ A ; $I_3 = 1{\cdot}085$ A]

70. Use the Principle of Superposition to find the current I in each of the circuits illustrated. The batteries each have zero internal resistance.



(a) (b)

[*Ans.* (*a*) 0·5 A; (*b*) $2\frac{5}{6}$ A]

71. A tetrahedron frame $ABCD$ is formed by six wires, the resistances of opposite edges being equal. Prove that the resistance of the frame for a current entering at A and leaving at D is $(r_1r_3 + 2r_1r_2 + r_2r_3)r_3/2(r_1 + r_3)(r_2 + r_3)$ where r_1 is the resistance of AB or CD, r_2 that of AC or BD and r_3 that of AD or BC.

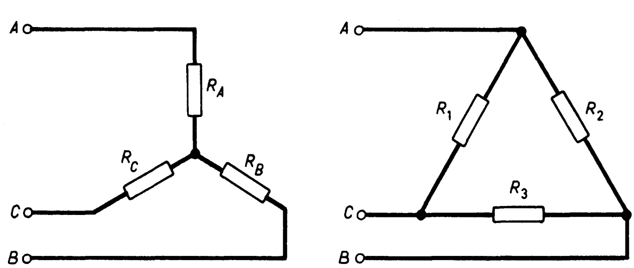
72. A cube is formed of twelve uniform wires of the same resistance r, the opposite corners are connected by wires of resistance r'. Prove that the resistance to a current which enters at one corner of the cube and leaves at the opposite corner is $rr'(2r + 5r')/2(r^2 + 4rr' + 3r'^2)$.

73. Six similar wires are connected so as to form a regular tetrahedron $ABCD$. A current enters at the middle point of AB and leaves from the middle point of CD. Show that the resistance of the arrangement is $3r/4$, where r is the resistance of one of the wires.

74. An octahedron is formed of twelve similar bars. A current enters the system at one end of a bar and leaves at the other end of the same bar. Show that the resistance of the octahedron is 5/12 of that of a single bar.

75. If the two networks illustrated are electrically equivalent show that:*

$$R_A = R_1R_2/(R_1 + R_2 + R_3)$$
$$R_B = R_2R_3/(R_1 + R_2 + R_3)$$
$$R_C = R_1R_3/(R_1 + R_2 + R_3)$$



*Proofs can be found in the following book: F. A. Benson and D. Harrison, *Electric Circuit Theory*, 2nd Edition, Arnold, 1963, pp. 27–30.

See also T. R. Walsh, 'Some Useful Procedures in Circuit and Line Theory', *Bull. Elect. Eng. Educ.*, No. 10, p. 24, June, 1953. (A proof is given which forms an interesting application of Maxwell's Cyclic-Current Rule.)

Prove also that:

$$R_1 = R_A R_C \left[\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right]$$

$$R_2 = R_A R_B \left[\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right]$$

$$R_3 = R_B R_C \left[\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right]$$

76. Points A, B, C, D are joined by five wires AB, BC, CD, DA and BD. The resistances of these wires are respectively 5, 5, 5, 3 and 8 Ω. Find the equivalent resistance of the network for a current entering at A and leaving at C.

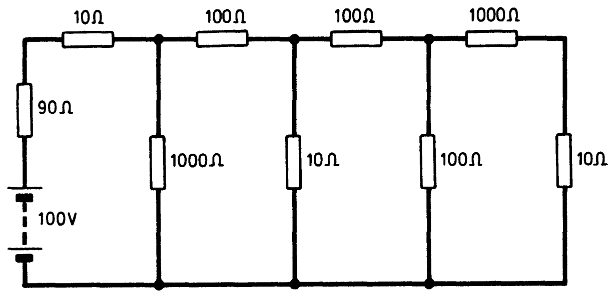
[*Ans.* $(4\frac{47}{112})$ Ω]

77. Two long straight parallel wires are joined by cross wires of the same material at equal distances, forming an infinite ladder of equal squares, the resistance of a side of a square being r. A current enters and leaves the network at the ends P and Q of one of the cross wires. If the currents in successive segments of one of the long wires, measured from P, are $i_1, i_2, i_3, \ldots$, show that

$$i_n - 4i_{n+1} + i_{n+2} = 0 \quad (n \geq 1).$$

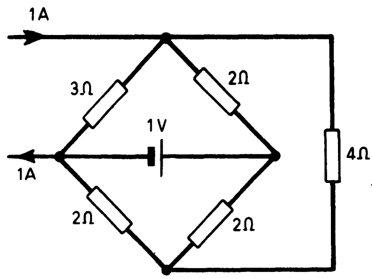
Show also that $i_n = i_1(2 - \sqrt{3})^{n-1}$ and the equivalent resistance of the network is $r/\sqrt{3}$.

78. By the iterated use of Thévenin's Theorem reduce the circuit shown to a single voltage source acting in series with a single resistor. Hence calculate the current in the 10-Ω resistor at the end of the network.



[*Ans.* 2·16 V and 1062 Ω; 2·03 mA]

79. Use Thévenin's Theorem and the Principle of Superposition to find the current in the 4-Ω resistor in the network illustrated.



[*Ans.* 0·21 A]

80. A two-wire distributor 1000 m long has the following loads:

Distance from feed (m)		100	300	500	700	1000
Load (A)	20	30	40	20	40	30

The feed is at 240 V and the end consumer is to receive at 220 V. Show that the cross-sectional area of the copper distributor must be 166 mm^2. The resistivity of copper is 1·7 × 10 Ωm.

81. The mutual inductance between two circuits *A* and *B* is 0·1 H and the resistance of each circuit is 100 Ω. The self inductance of circuit *B* is 0·8 H. The current in *A* is made to increase uniformly with time from 0 to 10 A in 0·1 s and to remain constant after this period. Plot the variation of the current in *B* with time measured from the instant that the current in *A* begins to flow. What is the value of the current in *B* after 0·01 s?

[*Ans.* 0·071 A]

82. A coil of resistance 40 Ω and self inductance 58·3 H is connected to a 60-V d.c. supply. Draw a curve showing the rise of current in the coil with time measured from the instant when the supply switch is closed. Show that the current reaches 0·95 A after a time 1·458 s.

83. A coil carrying a current of 10 A has a self inductance of 1·6 H. Calculate the energy stored in the coil and determine the capacitance which must be shunted across it so that the voltage rise on the coil does not exceed 200 V when the current is suddenly switched off.

[*Ans.* 80 J; 4030 μF]

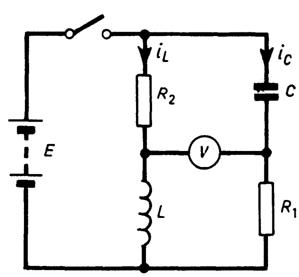
84. A coil of self inductance 0·6 H and resistance 2 Ω has a steady current of 10 A flowing through it. The battery supplying the circuit has an e.m.f. of 40 V and the current is controlled by a non-inductive resistor in series with it. If the resistance of the non-inductive resistor is suddenly reduced by 1 Ω find the current flowing in the coil 0·2s after the change has been made.

[*Ans.* 12·1 A]

85. A 4-μF capacitor is discharged suddenly through a coil of inductance 1 H and resistance 100 Ω. If the initial voltage on the capacitor is 10 V derive an expression for the resulting current and find the additional resistance required to give critical damping.

[*Ans.* $0{\cdot}02e^{-50t}\sin 497t$ amps; 900 Ω]

86. In the circuit illustrated currents i_L and i_C (functions of time) flow after closing the switch. Show that if $R_1R_2 = 2L/C$ the maximum value of the voltage V is $E/4$.



87. An electromagnet of inductance 1 H and a 100-Ω non-inductive resistor are connected in parallel across a 100-V d.c. supply and take a total steady current of 11 A. The d.c. supply is then suddenly switched off. After deriving an expression for the decay in the current flowing through the electromagnet find the maximum value of the voltage across the electromagnet.

The 100-V supply is reconnected after the magnet current has fallen to zero. Determine the time taken for the supply current to reach 10 A.

[*Ans.* 1000 V; 0·23 s]

88. The 1-μF storage capacitor of a 75-kV supply is automatically discharged, as the supply is switched off, by connecting a 10-MΩ resistor across it. Find the time taken for the voltage to fall to 100 V, the maximum current and the energy dissipated in the resistor.

[*Ans.* 66s; 7·5 mA; 2820 J]

89. The voltage of a supply connected across the terminals of a 100-μF capacitor is changing at a uniform rate of 100V s^{-1}. What is the supply current?

[*Ans.* 10 mA]

90. An 8-μF capacitor and a 500-Ω resistor are suddenly connected in series across a 200-V d.c. supply. Find:

(*a*) the initial current.
(*b*) the final current.
(*c*) the capacitor voltage as a function of time.
(*d*) the final energy stored in the capacitor.
(*e*) the energy dissipated by the resistor.

[*Ans.* (*a*) 0·4 A; (*b*) 0; (*c*) $200(1 - e^{-250t})$; (*d*) 0·16J; (*e*) 0·16 J]

91. A 10-H inductor and a 1-kΩ resistor are connected in parallel across a 200-A current source. The current supply is then suddenly switched off. Find (after switching off):

(*a*) the initial values of current and voltage in the resistor.
(*b*) the circuit time constant.
(*c*) the energy lost in the resistor.

[*Ans.* (*a*) 200 A; 200 kV; (*b*) 10ms; (*c*) 2×10^5 J]

92. A simple glow-discharge-tube time base* for a cathode-ray oscillograph employs a 300-kΩ resistor and a 0·016-μF capacitor. The striking and extinction voltages of the glow-discharge tube are 170 V and 140 V respectively. Calculate the frequency of the time base if the supply voltage is 200 V.

[*Ans.* 300 Hz]

*A few other problems on simple time bases may be found in the book: F. A. Benson, *Problems in Electronics with Solutions*, Spon, 4th Edition, 1965, Chapter 15.

CHAPTER TWO

SINGLE-PHASE A.C. CIRCUITS

93. An e.m.f. $e_1 = 50 \sin \omega t$ and an e.m.f. $e_2 = 30 \sin (\omega t - \pi/6)$ act together in the same circuit. Find the resultant e.m.f. by calculation and graphically.

[*Ans.* $77{\cdot}5 \sin (\omega t - 11^\circ\ 10')$]

94. The total e.m.f. acting in a circuit is $e_1 = 50 \sin \omega t$. The voltage drop in one part of the circuit is $e_2 = 30 \sin (\omega t - \pi/6)$. Find the voltage drop in the rest of the circuit by calculation and graphically.

[*Ans.* $28{\cdot}3 \sin (\omega t + 31^\circ\ 59')$]

95. The following four e.m.f.'s act together in the same circuit:

$e_1 = 10 \sin \omega t.$

$e_2 = 8 \sin (\omega t + \pi/3).$

$e_3 = 4 \sin (\omega t - \pi/6).$

$e_4 = 6 \sin (\omega t + 3\pi/4).$

Find the resultant e.m.f. by calculation and graphically.

[*Ans.* $16{\cdot}09 \sin (\omega t + 34^\circ\ 45')$]

96. Using the same four e.m.f.'s as in Question 95, calculate the e.m.f. represented by $e_1 - e_2 + e_3 - e_4$.

[*Ans.* $19 \sin (\omega t - 43^\circ\ 51')$]

97. Current flows through a series circuit consisting of two impedances *AB* and *BC*. The current lags by 30° behind the voltage of 150 V between *A* and *B* and leads by 50° on the voltage of 180 V between *B* and *C*. Find the voltage between *A* and *C*.

[*Ans.* 254 V lagging current by 14° 26′]

98. If the voltage represented by $V = 140 \sin \omega t$ is connected to a non-inductive resistance of 8 Ω, calculate the maximum value of the current and plot a curve showing the variation of this current with time. The frequency of the applied voltage is 50 Hz.

[*Ans.* 17·5 A]

99. A choking coil takes 10 A when connected to an a.c. supply of 230 V, 50 Hz. If the resistance of the coil is 2 Ω, find its inductance and the phase difference between the applied voltage and current.

[*Ans.* 0·073 H; 85°]

100. A coil of negligible resistance has an inductance of 0·01 H. A voltage represented by $140 \sin \omega t$ is applied to it, the frequency being 50 Hz. Calculate the current flowing through the coil and plot the curves of voltage and current with time.

[*Ans.* $44{\cdot}5 \sin (\omega t - \pi/2)$]

101. A voltage represented by $280 \sin \omega t$ is applied to a coil having a resistance of 20 Ω and an inductance of 0·02 H. The frequency is 50 Hz. Calculate the maximum value of the current and plot the curves of voltage and current with time. Find also the angle of phase difference between the voltage and the current.

[*Ans.* 13·4 A; 17° 26′]

102. An alternating voltage of 200 V r.m.s. value at a frequency of 50 Hz is applied to a circuit consisting of a coil whose resistance is 2 Ω and inductance 0·01 H. Find the current flowing and its angle of lag behind the voltage.

[*Ans.* 53·8 A; 57° 30′]

103. A coil having an inductance of 0·05 H and a resistance of 10 Ω is connected to a voltage of 200 V maximum value at a frequency of 50 Hz. Calculate the current and the angle of phase difference between the voltage and the current.

[*Ans.* 10·74 A; 57° 30′]

104. A voltage represented by $150 \sin \omega t$ is applied to a capacitor of capacitance 24μF. The frequency is 25 Hz. Calculate the maximum value of the current flowing through the capacitor and plot curves of current and voltage with time.

[*Ans.* 0·566 A]

105. A circuit consisting of a coil having an inductance of 0·25 H and a resistance of 3 Ω is arranged in series with a capacitor of capacitance 20 μF. Calculate at what frequency resonance will take place and the current flowing if an alternating voltage of 40 V at the resonant frequency is applied to the circuit. Find also the voltage across the capacitor.

[*Ans.* 71·2 Hz; 13·33 A; 1491 V]

106. A circuit consisting of a choking coil having an inductance of 0·05 H and a resistance of 10 Ω is connected to a 200–V r.m.s. supply at a frequency of 50 Hz and having a sine-wave shape. Draw curves showing the voltage applied, the current, and the power used in the circuit during a half cycle. Estimate the average power expended in the circuit. Calculate also the angle of lag of the current behind the voltage and the power-factor.

[*Ans.* 1160 W; 57° 30′; 0·5373]

107. A coil of inductance 0·64 H and resistance 40 Ω is connected in series with a capacitor of capacitance 12 μF.

Estimate:

(*a*) The frequency at which resonance will occur.

(*b*) The voltage across the coil and capacitor, respectively, and also the supply voltage when a current of 1·5 A at the resonant frequency is flowing.

(*c*) The three voltages in (*b*) with a current of 1·5 A flowing at a frequency of 50 Hz.

[*Ans.* (*a*) 57·5 Hz; (*b*) 352 V; 346·4 V; 60 V;
(*c*) 307 V; 398 V; 111 V]

108. The following particulars are for a certain coil:

Inductance 0·0271 H.
Resistance 10 Ω.

A voltage of 110 V r.m.s. at frequency 50 Hz is connected to the coil. Show that the power absorbed is given by both RI^2 and $VI \cos \phi$ and is 702 W.

109. A choking coil is required to enable a number of incandescent (non-inductive) lamps to take 3·2 A at 110 V from an a.c. supply of

210 V. If the resistance of the coil is 4 Ω, find its reactance and compare its efficiency with that of a series resistor for the same purpose.

[*Ans.* 53·2 Ω; 89·6%; 52·4%]

110. A coil of inductance 0·03 H and resistance 1 Ω is placed in series with a capacitor of capacitance 200 μF. Calculate at what frequency resonance will take place. If a voltage of 1 V is applied to the circuit, plot current as a function of frequency. If the coil is replaced by another one having the same inductance but a resistance of 0·5 Ω, plot the new current-frequency curve.

[*Ans.* 65 Hz]

111. A coil of resistance 1 Ω and impedance 8·06 Ω is placed in series with a second coil of resistance 1·24 Ω. When a voltage of 200 V is applied to the circuit the current flowing is 6·3 A. Find the inductance of the second coil. The frequency of the supply is 50 Hz.

[*Ans.* 0·075 H]

112. A coil with resistance is placed in series with a non-inductive resistance of 3 Ω. When a voltage of 104 V is applied to the circuit, the voltage across the coil is 66 V and that across the resistor 50 V. Calculate the power absorbed by the coil and its power-factor.

[*Ans.* 660 W; 0·6]

113. When a d.c. voltage of 30 V is applied to a given coil the power consumed is 150 W. When an a.c. voltage of 230 V r.m.s. is applied to the same coil the power consumed is 3174 W. Show that the reactance of the coil is 8 Ω.

114. Two coils each take a current of 4 A when connected to a 100-V, 50-Hz supply and the powers dissipated in them are 240 W and 320 W respectively. Find the current taken and the power-factor when the coils are connected in series with the 100-V, 50-Hz supply.

[*Ans.* 2·02 A; 0·707]

115. A circuit consisting of an inductive coil and a capacitor in series is connected across a 200-V variable frequency supply. An ammeter in the circuit reads 4 A when the frequency is 50 Hz and again when the frequency is 100 Hz. The current is 5 A at the resonant frequency.

Determine:

(*a*) the resistance of the coil,
(*b*) the inductance of the coil,
(*c*) the capacitance of the capacitor.

[*Ans.* 40 Ω; 0·0955 H; 53 μF]

116. A coil of resistance 10 Ω and inductance 1 H is connected in series with a 16-μF capacitor across a 100-V, variable-frequency supply.

Calculate the current drawn from the supply, the power-factor, the power, the voltage across the inductance and the voltage across the capacitance when the frequency is (*a*) 35 Hz, (*b*) resonant.

[*Ans.* (*a*) 1·53 A; 0·153; 23·4 W; 337 V; 435 V;
(*b*) 10 A; 1; 1000 W; 2502 V; 2500 V]

117. An a.c. circuit *ABCD* consists of a resistor *AB*, an inductor *BC* and a resistor *CD*, connected in series across a 200-V, 50-Hz supply. The current flowing is 10 A.

The voltages are as follows:

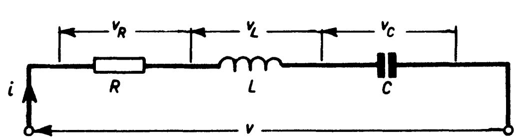
Across *AB* 80 V.
Across *BC* 100 V.
Across *AC* 145 V.

Draw a voltage phasor diagram to scale and determine

(*a*) the voltage across *BD*,
(*b*) the inductance and resistance of the inductor,
(*c*) the power-factor of the circuit.

[*Ans.* (*a*) 135·6 V; (*b*) 0·03 H; 2·89 Ω; (*c*) 0·88]

118. Draw the dual of the series circuit illustrated.



119. Calculate the total current taken by the following two coils in parallel and the phase difference between it and the applied voltage.

Coil 1 carries 12 A lagging 30° behind the applied voltage.

Coil 2 carries 22 A lagging 45° behind the applied voltage.

[*Ans.* 33·74 A; 39° 45′]

120. A capacitor is placed in parallel with a coil having an inductance of 0·03 H and a resistance of 5 Ω across a supply of 200 V at a frequency of 50 Hz. The current passing through the capacitor is 32 A leading the applied voltage by 90°. Calculate the total current flowing in the external circuit and the phase difference between it and the applied voltage.

[*Ans.* 17·75 A; 60° 21′ leading the voltage]

121. A capacitor passing 32·3 A leading by 90° on the applied voltage is placed in parallel with two coils, one of which carries 10 A lagging 30° behind the applied voltage and the other carries 20 A lagging 60° behind the applied voltage. Calculate the total current flowing in the external circuit and its angle of phase displacement. Is the current leading or lagging with respect to the applied voltage?

[*Ans.* 21·18 A; 28° 12′; leading]

122. A sinusoidal 50-Hz voltage, of 200 V r.m.s., supplies the following three circuits which are in parallel:

(*a*) a coil of inductance 0·03 H and resistance 3 Ω.

(*b*) a capacitor of 400 μF in series with a resistance of 100 Ω.

(*c*) a coil of inductance 0·02 H and resistance 7 Ω in series with a capacitor of 300 μF.

Determine the total current supplied and draw a complete phasor diagram.

[*Ans.* 29·5 A lagging the applied voltage by an angle of 12° 22′]

123. Two coils are connected in parallel across a 200-V, 50-Hz supply. At the supply frequency their impedances are 6 and 10 Ω respectively and their resistances are 2 and 3 Ω respectively.

Calculate:

(*a*) The current in each coil.

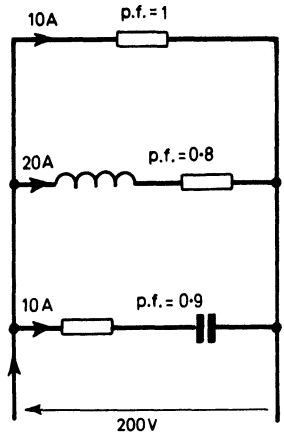
(*b*) The total current.

(*c*) The total power-factor.

Draw a complete phasor diagram for the system.

[*Ans.* (*a*) 33·3 A lagging applied voltage by 70° 32′
20 A lagging applied voltage by 72° 32′
(*b*) 53·3 A lagging applied voltage by 71° 16′
(*c*) 0·32]

124. For the parallel circuit shown obtain graphically the total current supplied.



[*Ans.* 35·6 A]

125. Three circuits *A*, *B* and *C* are connected in parallel across a 200-V a.c. supply. Circuit *A* consists of a bank of lamps taking a current of 10 A at unity power-factor, *B* consists of an inductive resistor taking a current of 20 A at a power-factor of 0·8 and *C* consists of a resistor and capacitor in series taking a current of 10 A at a power-factor of 0·9.

Find the power supplied to the whole circuit and the power-factor.

[*Ans.* 7000 W; 0·977]

126. Two coils are connected in parallel across a 200-V, 50-Hz supply. At the supply frequency their impedances are 5 Ω and 9 Ω respectively and their resistances are both 3 Ω.

Calculate:
(*a*) The current in each coil.
(*b*) The total current.
(*c*) The power-factor of the whole arrangement.
Sketch a complete phasor diagram for the system.

[*Ans.* (*a*) 40 A; 22·2 A, (*b*) 61·54 A, (*c*) 0·51]

127. A coil of resistance 10 Ω and inductance 0·02 H is placed in parallel with a 100-μF capacitor across a 200-V, r.m.s. 50-Hz, sinusoidal supply. Find the total current supplied and the power-factor of the arrangement.

If a second capacitor is then placed in parallel with the first one, what value of capacitance must it have so that the total current supplied is in phase with the input voltage?

[*Ans.* 14·57 A; 0·98 lagging; 43 μF]

128. A resistance coil of 100 Ω has an inductance of 20 μH. What value of the effective self-capacitance would make it accurately non-inductive at a frequency of 50 Hz?

Would this coil be accurate if used in a circuit carrying a current at a frequency of 1 MHz?

[*Ans.* 0·002 μF; No.]

129. Estimate the current that will flow through a coil having an inductance of 0·02 H and a resistance of 5 Ω when connected to a 200-V supply at a frequency of 50 Hz. Find the capacitance of a capacitor which, when connected in series with a 5-Ω resistor, will take the same current as the coil. Find, also, the current taken by the two circuits when connected in parallel.

[*Ans.* 24·9 A; 508 μF; 31 A]

130. An inductance coil takes a current of 10 A when it is connected across the 200-V, 50-Hz supply. When the coil is connected in parallel with a non-inductive resistor of 25 Ω across the same supply, the total current is 15 A. Find, graphically, the power-factors of the coil and the parallel combination of coil and resistor.

Calculate the capacitance which is necessary to give unity power-factor in the above parallel circuit, the capacitance being placed also across the supply voltage.

[*Ans.* 0·39; 0·79; 147 μF]

131. A coil having a resistance of 1 Ω and a reactance of 1 Ω is shunted by a resistance of 1 Ω. The combination is connected in series with an inductive reactance of 1 Ω. Draw to scale a complete phasor diagram for the arrangement and determine the applied voltage necessary to give a current of 1 A in the coil and show that it leads this current by 90°.

[*Ans.* 3 V]

132. A piece of equipment consumes 2000 W when supplied with 110 V and takes a lagging current of 25 A. Determine the equivalent series resistance and reactance of the equipment.

If a capacitor is connected in parallel with the equipment to make the power-factor unity, find its capacitance. The supply frequency is 100 Hz.

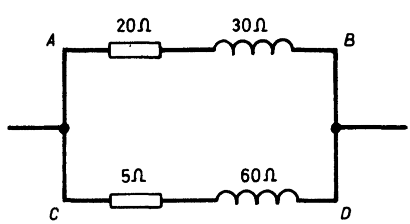
[*Ans.* 3·2 Ω; 3·02 Ω; 248 μF]

133. A coil having an impedance of 25 Ω and a resistance of 15 Ω is connected in parallel with a non-inductive resistance of 10 Ω. Calculate the total admittance, conductance and susceptance.

[*Ans.* 0·128 S; 0·124 S; 0·032 S]

134. For the circuit shown calculate the following particulars:

(*a*) the impedance of branch *AB*.
(*b*) the admittance of branch *AB*.
(*c*) the impedance of branch *CD*.
(*d*) the admittance of branch *CD*.
(*e*) the total conductance.
(*f*) the total susceptance.
(*g*) the resultant admittance.



[*Ans.* 36 Ω; 0·0278 S; 60·2 Ω; 0·0167 S; 0·0167 S;
0·0397 S; 0·043 S]

135. If a coil with reactance 10 Ω and resistance 50 Ω is placed in series with the circuit of Question 134, find the equivalent resistance, reactance and impedance of the whole arrangement.

[*Ans.* 59 Ω; 31·5 Ω; 66·9 Ω]

136. An admittance of 0·2S is connected in parallel with a pure reactance the susceptance of which is 0·15S. The combined admittance is 0·314S. Prove that the resistance in the circuit is 4 Ω.

137. Prove that the combined impedance of a resistance R and a reactance X in parallel is $RX/\sqrt{(R^2 + X^2)}$.

138. Two coils are connected in parallel across a voltage of 300 V. The frequency is 50 Hz. At the supply frequency the impedances of the coils are 8 Ω and 11 Ω respectively and their resistances are 7 Ω and 4 Ω respectively.

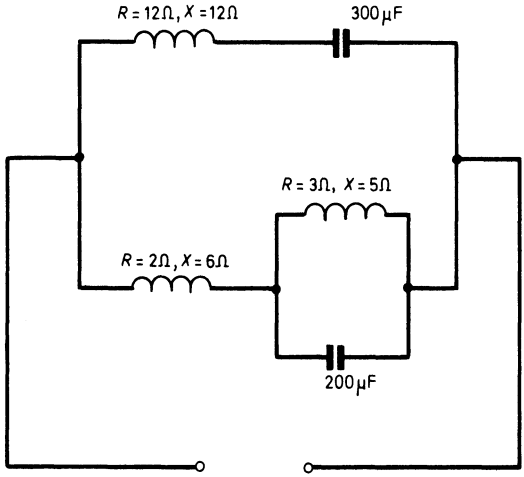
Calculate:
(*a*) The current in each coil.
(*b*) The total current.
(*v*) The total power-factor.

[*Ans.* 37·5 A; 27·3 A; 61·2 A; 0·697]

139. If a capacitor of capacitance 120 μF is connected in parallel with the coils of Question 138 across the same voltage find the total current.

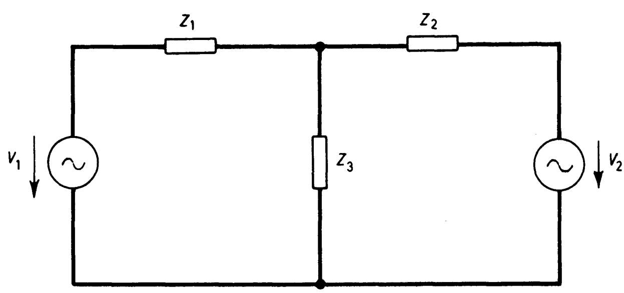
[*Ans.* 53·4 A]

140. A voltage of 200 V at a frequency of 50 Hz is applied to the circuit shown. Find the total current supplied.



[*Ans.* 28 A]

141. Use (*a*) Millman's Theorem and (*b*) the Generalized Form of Norton's Theorem to find the current through impedance Z_3 in the network shown.



[*Ans.* $Y_3(V_1Y_1 + V_2Y_2)/(Y_1 + Y_2 + Y_3)$]

142. A half-wave rectifier has a voltage given by 100 sin ωt applied to it. Estimate the average voltage on the d.c. side.

[*Ans.* 31·83 V]

143. The instantaneous values of an alternating current between the angles of 0 and 90° are given in the following table. Draw the graph and find the current which would be shown by an ammeter reading r.m.s. values placed in the circuit. The curve is symmetrical about the axis through 90°.

Angle ° . .	0	6	12	18	24	30	36	42
Current (A) .	0	9·5	16	20·5	22·5	22·5	21·5	19

Angle ° . .	48	54	60	66	72	78	84	90
Current (A) .	16	14	12·5	12·1	12·5	13·5	14·75	15

[*Ans.* 16·3 A]

144. (*a*) Prove that for a sinusoidal waveform the form factor is 1·11.

(*b*) Determine the r.m.s. value of a semi-circular waveform which has a maximum value of E.

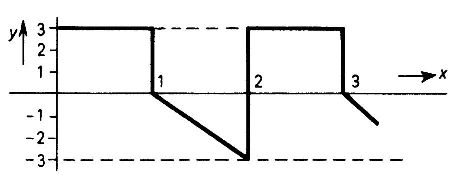
[*Ans.* 0·816 E]

145. Find the relative heating effects of two current waves which have the same maximum value, if one is sinusoidal and the other rectangular in waveform.

[*Ans.* 2:1]

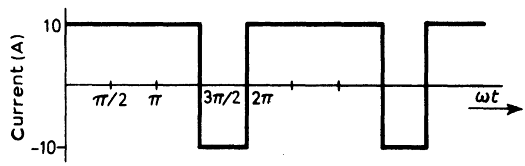
146. Show that the r.m.s. value of a triangular waveform of maximum value E is $E/\sqrt{3}$ and is independent of the lengths and slopes of the sides.

147. Find the r.m.s. and average values of the wave illustrated.



[*Ans.* 2·45; 0·75]

148. Determine the r.m.s. value of the current wave illustrated.



[*Ans.* 10 A]

149. The three-ammeter method is used to measure the power in an inductive load. The current in the main circuit is 6 A while the currents in the standard resistor and the load are 3 A and 4 A respectively. The supply voltage is 200 V.

Draw a phasor diagram for the arrangement and calculate the power absorbed by the load and its power-factor.

[*Ans.* 366·4 W; 0·458]

150. The three-voltmeter method is used to measure the power in an inductive load. The voltage across the load is 210 V, that across the standard resistor 180 V and the supply voltage is 290 V.

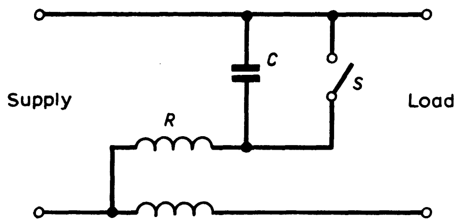
Draw a phasor diagram for the arrangement and calculate the power-factor of the load. Show that in order to calculate the power in the load either the current in the circuit or the value of the standard resistor must be known.

[*Ans.* 0·1]

151. (*a*) One method of measuring power-factor is to use a wattmeter connected as shown. Let the voltage, current and power-factor of the load be V, I and $\cos\phi$ respectively. Let the voltage coil of the wattmeter have a resistance R and a negligible reactance. If the readings of the wattmeter with the switch S closed and open are W_1 and W_2 respectively, show that:

$$\frac{W_2}{W_1} = \frac{VI}{\sqrt{[R^2+(1/\omega^2C^2)]}}\cos\left[\phi - \tan^{-1} 1/\omega CR\right] \Big/ \frac{VI}{R}\cos\phi$$

where $\omega = 2\pi \times$ frequency.



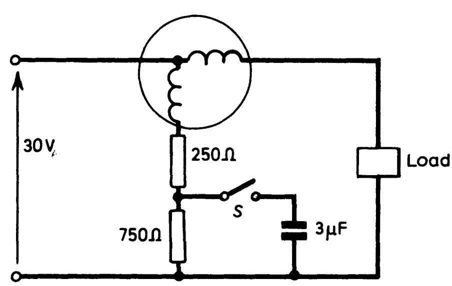
(*b*) If the circuit is arranged such that $R = 1/\omega C$, show that the power-factor $\cos\phi$ is given by $1/\sqrt{(4a^2 - 4a + 2)}$ where $a = W_2/W_1$.

152. A wattmeter was tested against an accurate standard one. The deflection with a given continuous load current was 50 divisions. With an alternating current of frequency 100 Hz the deflection was 52 divisions for the same power at a power-factor 0·707 (45° lagging current). Determine the inductance of the wattmeter shunt circuit if its resistance is 2000 Ω.

[*Ans.* 0·133 H]

153. In the circuit illustrated the wattmeter is connected for measuring power. The voltage applied to the circuit is 30 V at a frequency of 50 Hz and a 3-μF capacitor is connected in parallel with part of the wattmeter shunt circuit as indicated. The readings of the wattmeter are (*a*) W_1 when the switch *S* is open and (*b*) W_2 when *S* is closed.

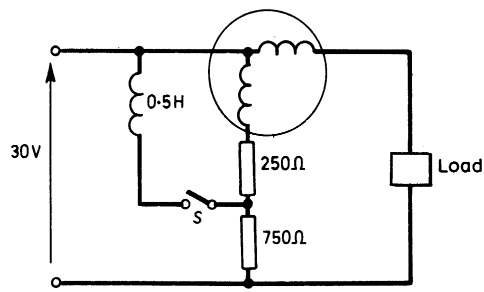
Derive an expression for the angle of lag of the phasor representing the current passing to the load referred to the voltage phasor, in terms of W_1 and W_2.



$$\left[Ans.\ \tan^{-1}\left(2{\cdot}13 - 1{\cdot}95\frac{W_2}{W_1}\right)\right]$$

154. A wattmeter is connected for measuring power in an alternating-current circuit. The applied voltage is 30 V at a frequency of 50 Hz. A coil of inductance 0·5 H and negligible resistance is connected in parallel with the voltage coil as illustrated in the diagram. The wattmeter readings are (*a*) W_1 when switch *S* is open and (*b*) W_2 when *S* is closed.

Derive a formula for calculating, from these wattmeter readings, the phase difference between the load-current and voltage phasors when the load current lags the applied voltage.



$$\left[\textit{Ans.}\ \tan^{-1}\left(0{\cdot}89 - 2{\cdot}1\,\frac{W_1}{W_2}\right)\right]$$

155. Determine the values of the admittance and the current source which result in a constant-current circuit being equivalent to a constant-voltage circuit of e.m.f. *E* and internal impedance *Z*.

[*Ans.* $1/Z$; E/Z]

156. A coil having a constant resistance of 4 Ω and an inductive reactance which can be varied between the limits 0 and 10 Ω is connected to a sinusoidal supply of 100 V maximum value. Draw the locus diagrams for the current and the impedance when the inductive reactance is varied.

157. A circuit consisting of a 50-Ω resistor in series with a variable reactor is shunted by a 100-Ω resistor. Draw the locus of the extremity of the total-current phasor to scale and determine the reactance and current corresponding to the minimum overall power-factor, the supply voltage being 100 V.

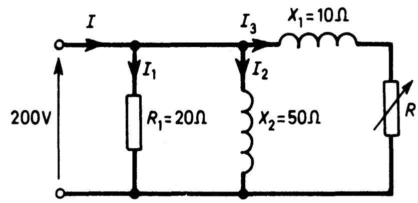
[*Ans.* 86·5 Ω; 1·732 A]

158. Two impedances of Z_1 and Z_2 are connected in parallel across a 200-V, 50-Hz supply. The impedance Z_1 consists of a 50-Ω resistor in series with a 0·2-H inductor, while Z_2 consists of a variable resistor R ohms in series with a 0·1-H inductor.

Show that the extremity of the total current phasor describes a circular locus as the resistance R is varied, and draw the diagram to scale. Determine the value of R to give the maximum power, and the corresponding total current and power-factor. What is the maximum power?

[*Ans.* 31·4 Ω; 7 A; 0·68; 952 W]

159. For the circuit illustrated draw the locus of the total-current phasor to scale and then determine the maximum power and the corresponding values of R and the power-factor.



[*Ans.* 4000 W; 10 Ω; 0·82]

ADDITIONAL PROBLEMS

160. Find the resultant e.m.f., both by calculation and graphically, if the following e.m.f.'s are added together:

$e_1 = 20 \sin \omega t,$ $e_2 = 10 \sin (\omega t + \pi/6),$
$e_3 = 15 \cos \omega t,$ $e_4 = 10 \sin (\omega t - \pi/3),$
$e_5 = 25 \cos (\omega t + 2\pi/3)$

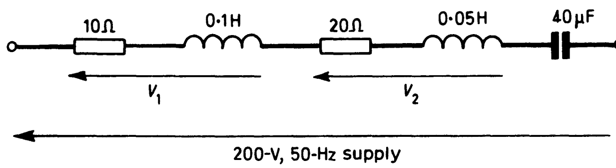
[*Ans.* 12·1 sin (ωt − 0·096)]

161. Find the sum of the following voltages: $v_1 = 50 \sin \omega t$, $v_2 = 40 \sin (\omega t + 60°)$, $v_3 = 60 \sin (\omega t - 30°)$.

[*Ans.* 122 sin (ωt + 2·2°)]

162. Draw a complete phasor diagram for the series circuit shown, indicating the various resistance and reactance voltage drops, the voltages V_1 and V_2, the supply voltage and the current. The diagram need not be drawn to scale but a brief explanation of it should be given.

Find the magnitudes of the voltages V_1 and V_2 and the current. Calculate also the power-factor of the circuit.



[*Ans.* 149 V; 115 V; 4·53 A; 0·679]

163. An inductive coil, in series with a non-inductive resistor, takes a current of 5 A when connected to a 100-V, a.c. supply. The voltages across the coil and resistor are 80 V and 30 V respectively. Find the power-factor and power for (*a*) the whole circuit (*b*) the coil alone.

[*Ans.* (*a*) 0·75; 375 W,
(*b*) 0·56; 225 W]

164. A series circuit consists of a 50-Ω resistor, a 2·5-mH inductor and a 250-$\mu\mu$F capacitor. A sinusoidal voltage of constant amplitude is connected to the series circuit and its frequency varied over a range including the resonant frequency. Calculate the frequency at which resonance will occur and the difference between the two frequencies at which the current is one half of the maximum value.

[*Ans.* 201 kHz; 5·5 kHz]

165. A coil of resistance 4 Ω and inductance 0·3 H is connected in series with a 30-μF capacitor across a 50-V a.c. supply. Find the supply frequency for resonance, the supply current and the voltage across the capacitor.

[*Ans.* 53 Hz; 12·5 A, 1251 V]

166. A coil takes 6 A and dissipates 200 W when connected to a 100-V, 50-Hz, supply, while another coil takes 8 A and dissipates 600 W when connected to a similar supply.

Calculate (*a*) the current taken (*b*) the power dissipated and (*c*) the circuit power-factor when the coils are joined in series and connected to a 200-V, 50-Hz supply.

[*Ans.* (*a*) 7·07 A; (*b*) 746 W; (*c*) 0·53]

167. A circuit *ABD* consists of: *AB*—a 100-Ω resistor, *BCD*—a 50-Ω resistor *BC* in series with a 1-μF capacitor *CD*, *BED*—a 100-Ω resistor *BE* in series with a 25-mH inductor *ED*. A 100-V 1000-Hz a.c. supply is connected across *AD*.

Determine the current taken and its power-factor.

[*Ans.* 0·328 A; 0·987]

168. Two circuits *P* and *Q* are connected in parallel across a 250-V, 50-Hz supply. *P* consists of a 50-Ω resistor in series with a 0·15-H inductor. *Q* comprises a 35-Ω resistor in series with a 50-μF capacitor. Calculate the current in each circuit, the total current and the total power-factor.

Sketch a complete phasor diagram for the arrangement.

[*Ans.* 3·64 A; 3·44 A; 4·34 A; 0·993]

169. Two circuits are connected in parallel across a 230-V, 50-Hz mains supply. The first one consists of a 50-Ω resistor in series with a 0·2-H inductor; the second comprises a 40-Ω resistor in series with a 50-μF capacitor.

Evaluate the total current supplied to the arrangement and sketch the phasor diagram.

[*Ans.* 3·43 A leading the supply voltage by an angle of 5° 51′]

170. A parallel combination of circuits is made up as follows:

Branch 1: Resistance 5 Ω, inductance 40 mH,

Branch 2: Resistance 6 Ω, capacitance 300 μF,

Branch 3: Resistance of 10 Ω only.

Determine the current in each branch if the combination is connected across a 200-V, 50-Hz supply. Sketch the phasor diagram. Find graphically, or by calculation, the total current and its phase angle.

[*Ans.* 14·8 A; 16·4 A; 20 A; 33·54 A; 54′ leading the supply voltage]

171. A 0·32-H inductor having a resistance of 100 Ω is connected across a 100-V, 50-Hz supply. Draw a phasor diagram for the arrangement and from it determine the magnitude and phase of the current.

A capacitor is to be connected to the inductor to reduce the phase angle of the current from the source to 30°. Use the phasor diagram to find the value of capacitance required and show how the capacitor would be connected to the circuit.

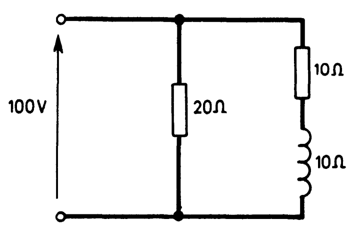
[*Ans.* 0·707 A; lags voltage by 45°; 75·3 μF]

172. A 100-V, 100-W lamp is correctly operated from a 240-V, 50-Hz supply by connecting it in series with an inductor L. A capacitor C is connected across the supply to make the power-factor unity. Find the values of L and C.

If a second similar lamp is connected in parallel with the first find the new supply frequency necessary to operate the lamps correctly and the phase of the supply current.

[*Ans.* 0·694 H; 12·2 μF; 25 Hz; 58·6° lagging]

173. An alternating voltage of 100 V r.m.s. is applied to the following circuit. Find the current in each limb, the total current (magnitude and phase) from the generator, and the power dissipated in the circuit.



[*Ans.* 5 A in phase with the voltage;
7·07 A lagging by 45°;
11·2 A lagging by 26·6°; 1000 W]

174. Show that the instantaneous current i in the circuit illustrated is:

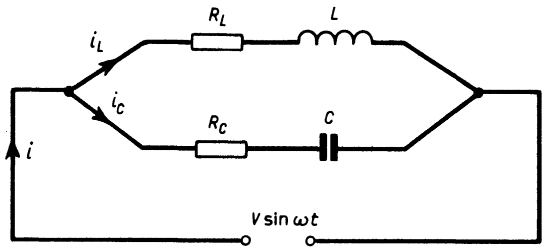
$$V\sqrt{\left[\left(\frac{R_L}{Z_L^2}+\frac{R_C}{Z_C^2}\right)^2+\left\{\frac{\omega L}{Z_L^2}-\frac{(1/\omega C)}{Z_C^2}\right\}^2\right]}\sin(\omega t-\beta)$$

where $Z_L = \sqrt{[R_L^2 + \omega^2 L^2]}$

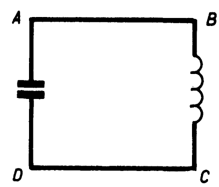
$Z_C = \sqrt{[R_C^2 + (1/\omega C)^2]}$

and $\tan\beta = \left(\frac{\omega L}{Z_L^2}-\frac{1/\omega C}{Z_C^2}\right)\Big/\left(\frac{R_L}{Z_L^2}+\frac{R_C}{Z_C^2}\right)$

Give a sketch of a phasor diagram for the system showing the applied voltage, the two branch currents and the resultant current.



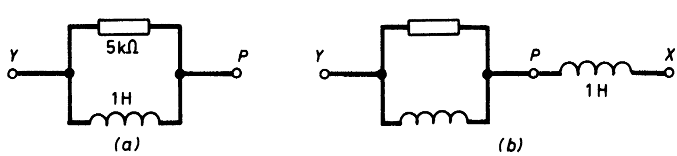
175. The circuit shown is driven as a parallel resonant circuit by an a.c. voltage source V connected between A and C. Show that the voltages and currents remain the same if V is replaced by an a.c. current source $I = V\sqrt{(C/L)}$ inserted into link AB.



176. A capacitor is connected in parallel with a coil of resistance 20Ω and inductance 0·07 H across a 50-Hz supply. Find the capacitance necessary to produce resonance.

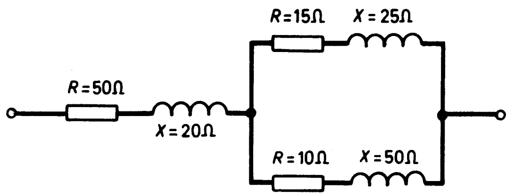
[*Ans.* 79·2 μF]

177. A 100-V, 500-Hz supply is connected across circuit (*a*). Draw a phasor diagram for the circuit. A second coil is now joined in series as in (*b*). By adding suitable phasors to the diagram find the voltage required across the whole circuit to maintain 100 V across *PY*.



[*Ans.* 210 V]

178. Find the equivalent resistance, reactance and impedance of the whole arrangement illustrated.



[*Ans.* 57·6 Ω; 37·2 Ω; 68·6 Ω]

179. Use (*a*) mesh analysis (*b*) nodal analysis to prove the validity of the Superposition Theorem for a restricted set of conditions which will suggest that the general proof is simply an enlargement of the logic followed.*

180. Verify the Reciprocity Theorem by proving that only one value of transfer impedance is associated with two pairs of terminals of a network.*

181. Write down the nodal equations for a linear network with n independent nodes if I_1, $I_2 \ldots I_n$ are the generator currents and V_1, $V_2 \ldots V_n$ are the various node voltages. The node admittances are Y_{11}, $Y_{22} \ldots Y_{nn}$ and the mutual admittances are Y_{12}, Y_{21}, etc.

Give the solution for any node voltage V_k in determinant form and define the open-circuit transfer and input impedances.

[*Ans.*
$$
\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 + \ldots + Y_{1n}V_n \\
I_2 &= Y_{21}V_1 + Y_{22}V_2 + \ldots + Y_{2n}V_n \\
I_3 &= Y_{31}V_1 + Y_{32}V_2 + \ldots + Y_{3n}V_n \\
\ldots & \quad \ldots \quad \ldots \quad \ldots \\
I_n &= Y_{n1}V_1 + Y_{n2}V_2 + \ldots + Y_{nn}V_n;
\end{aligned}
$$

$$
V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} \ldots & Y_{1k-1} & I_1 & Y_{1k+1} & \ldots Y_{1n} \\ Y_{21} & Y_{22} \ldots & Y_{2k-1} & I_2 & Y_{2k+1} & \ldots Y_{2n} \\ \ldots & & \ldots & & & \ldots \\ Y_{n1} & Y_{n2} \ldots & Y_{nk-1} & I_n & Y_{nk+1} & \ldots Y_{nn} \end{vmatrix}}{\Delta}
$$

*The solutions to these problems can be found in the book by W. H. Middendorf *Analysis of Electric Circuits*, Wiley, 1956.

$$\text{where } \Delta = \begin{vmatrix} Y_{11} & Y_{12} & Y_{13} \ldots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} \ldots & Y_{2n} \\ & \ldots & & \ldots \\ Y_{n1} & Y_{n2} & Y_{n3} \ldots & Y_{nn} \end{vmatrix}$$]

182. Calculate the r.m.s. value of the resultant current in a conductor which carries simultaneously a direct current of 1 A and a sinusoidal alternating current having a maximum value of 1 A.

[*Ans.* 1·22 A]

183. The current in a certain circuit has a waveform containing a 20% third harmonic, the fundamental having an amplitude of 100 A. Determine the r.m.s. value of the current. Sketch the shape of the waveform of the current which you would expect to observe on the screen of an oscillograph assuming that the third harmonic leads 30° on the fundamental.

[*Ans.* 70·1 A]

184. Find the r.m.s. value of the voltage which increases linearly from 0 to 5 V in 1s, drops to zero in negligible time and then repeats the variation.

[*Ans.* 2·887 V]

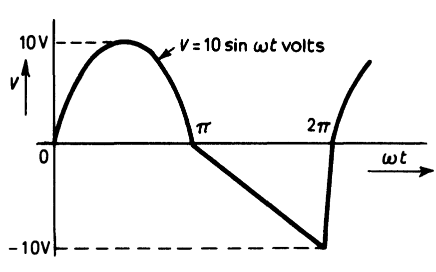
185. The voltage applied to a resistance of 1000 Ω varies as follows:

(*a*) increases uniformly from zero to 500 V in 5 μs.
(*b*) remains constant for 1 μs.
(*c*) falls to zero uniformly in 50 μs.

The cycle is repeated every 1000 μs.

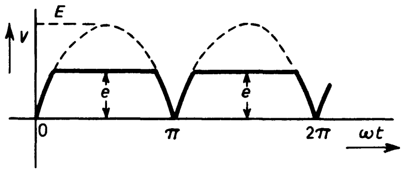
Find the r.m.s. current. [*Ans.* 0·08 A]

186. Determine the r.m.s. value of the voltage wave illustrated.



[*Ans.* 6·46 V]

187. A clipped full-wave rectifier waveform is illustrated.

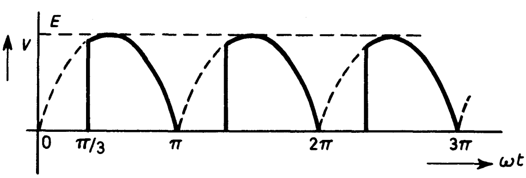


If $e = E\sqrt{2}$, determine the average and r.m.s. values of the waveform.

If $e = E/2$, what is the r.m.s. value?

[*Ans.* 0·54 E; 0·58 E; 0·44 E]

188. The output-voltage waveform from a controlled rectifier is as illustrated, where the delay angle is $\pi/3$. Determine the average and r.m.s. values of the waveform.



[*Ans.* 0·48 E; 0·63 E]

189. The power absorbed by an inductive load is measured by the three-ammeter method. The current in the main circuit is 5·6 A while the currents in the standard resistor and the load are 2·5 A and 4 A respectively. The supply voltage is 300 V. Calculate the power absorbed by the load and its power-factor.

[*Ans.* 550 W; 0·458]

190. The power absorbed by an inductive load is measured by the three-voltmeter method, the voltage across the load is 200 V, that across the standard resistor 180 V and the supply voltage is 300 V. Calculate the power-factor of the load.

[*Ans.* 0·244]

191.* Derive the conditions of balance for an a.c. bridge the arms of which are as follows:

AB an iron-cored choke, resistance r, inductance L.
BC a variable capacitor C_1 in series with a variable resistor R_1.
CD a mica capacitor C_2.
DA a standard resistor S.

For a given choke, balance was obtained with the following component values:

$C_1 = 0{\cdot}31\ \mu F$, $C_2 = 0{\cdot}057\ \mu F$,
$R_1 = 13\,100\ \Omega$, $S = 3470\ \Omega$.

Find the inductance and resistance of the choke.

[*Ans.* $r = SC_2/C_1$; $L = SC_2R_1$; 2·59 H; 638 Ω]

192. A bridge network *ABCD* is set up to measure the inductance and resistance of an r.f. choke. The bridge is supplied from a 10-V, 1-kHz, low-impedance source and the high-impedance detector gives a minimum discernible indication for an input of 2 mV.

At balance the other arms are made up as follows:

BC resistance of 100 Ω.

CD capacitance of 0·961 μF shunted by a resistance of 1195 Ω.
DA resistance of 100 Ω.

Determine the inductance and resistance of the choke and the maximum possible errors in these values.

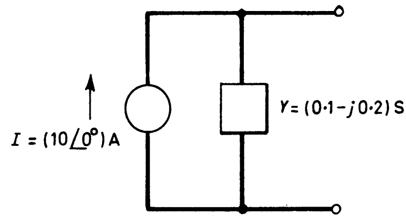
[*Ans.* 9·61 mH; 8·38 Ω; 0·086 %; 0·61 %]

193. A generator has an e.m.f. that may be represented by $E = 120\ \angle 0°$ V and an internal equivalent series impedance of $Z = (1 + j2)\ \Omega$. Determine the equivalent current generator, the terminal voltages if a load $Z_l = (6 - j8)\ \Omega$ is connected to each generator, the efficiency of each generator at that load and the regulation of each generator.

[*Ans.* The equivalent generator is made up of a source producing a current $\{53{\cdot}6\ \angle -63{\cdot}4°\}$ A in parallel with an admittance $\{0{\cdot}447\angle -63{\cdot}4°\}$ S; $\{130\angle -12{\cdot}6°\}$V; 85·7 % for constant-voltage source; 18·3 % for constant-current source; −7·7 %]

*A number of problems on a.c. bridges, with their solutions, can be found in the book: F. A. Benson, *Problems in Electronics with Solutions*, Spon, 4th edition, 1965, Chapter 24.

194. Change the constant-current generator illustrated to a constant-voltage one.



[*Ans.* An e.m.f. $E = \{44·7 \angle 63·5°\}$ V in series with an impedance $Z = \{4·47 \angle 63·5°\}$ Ω]

195. An a.c. circuit, supplied at 100 V, 50 Hz, consists of a variable resistor in series with a fixed 100-μF capacitor.

Show that the extremity of the current phasor moves on a circle. Determine the maximum power dissipated in the circuit and the corresponding power-factor and value of the resistor.

[*Ans.* 157 W; 0·707; 31·8 Ω]

196. A variable non-inductive resistor R of maximum value 10 Ω is placed in series with a coil which has a resistance of 3 Ω and a reactance of 4 Ω. The arrangement is supplied from a 240-V, a.c. supply. Show that the locus of the extremity of the current phasor is a semicircle. From the locus diagram calculate the current supplied when R is 5 Ω.

[*Ans.* 26·7 A]

197. A 20-Ω reactor is connected in parallel with a series circuit consisting of a reactor of reactance 10 Ω and a variable resistor R. Prove that the extremity of the total-current phasor moves on a circle if the supply voltage is constant at 100 V r.m.s. What is the maximum power-factor? Determine also the value of R when the power-factor has its maximum value.

[*Ans.* 0·5; 17·3 Ω]

198. Two circuits are connected in parallel. One circuit consists of a 100-Ω resistor and the other of a 25-Ω resistor in series with a fixed capacitor. An alternating-current supply of 100 V amplitude and variable frequency is connected across the combination.

Draw to scale the locus of the total current phasor and determine the capacitor reactance when the phase-angle between the supply voltage and current is a maximum.

[*Ans*. 56 Ω]

199. A circuit having two branches in parallel is connected to a 230-V, 50-Hz supply. One branch consists of an inductive resistance having $R = 40\ \Omega$ and $L = 0{\cdot}2$ H. The other branch has an inductance of 0·073 H in series with a variable resistance.

Draw to scale the locus diagram of the line current for all values of the variable resistance between zero and infinity.

Determine from the diagram:

(*a*) The value of variable resistance for which the power-factor of the circuit is a maximum,

(*b*) the power dissipated in each branch under this condition.

[*Ans*. (*a*) 65·5 Ω; (*b*) 384 W and 1080 W]

200. Draw the current locus of a circuit containing a variable resistance in series with a reactance of 1 Ω, across which a constant voltage of 200 V is applied. Find the current corresponding to a maximum power input in watts and state the value of resistance required to give the maximum power input.

[*Ans*. 141·4 A; 1 Ω]

CHAPTER THREE

COMPLEX QUANTITIES AND THEIR USE IN A.C. CIRCUITS

201. †Rotate the complex number $A = 3 + j4$ through $53{\cdot}2°$ clockwise.

[*Ans.* $5 + j0$]

202. Given

$$A = 3 + j4 = 5\angle 53{\cdot}2°$$

and

$$B = -12 - j5 = 13\angle 202{\cdot}6°$$

find

$$A + B,\ A - B,\ AB,\ A/B,\ A^2,\ \sqrt{B} \text{ and } \log_e A$$

[*Ans.* $-9 - j1$; $15 + j9$; $65\angle 255{\cdot}8°$; $0{\cdot}385\angle -149{\cdot}4°$; $25\angle 106{\cdot}4°$; $3{\cdot}6\angle 101{\cdot}3°$ and $3{\cdot}6\angle 281{\cdot}3°$; $1{\cdot}6 + j0{\cdot}93$]

203. Show that $(12 + j16)/(10 - j20)$ simplifies to $(-0{\cdot}4 + j0{\cdot}8)$.

204. Evaluate $(60 + j40) + (30 + j30)$ and determine the modulus and argument of the resultant.

[*Ans.* $90 + j70$; 114; 37° 53′]

205. Evaluate the following:

(*a*) $(20 + j20)(60 - j30)$

(*b*) $(100 + j70)/(60 + j10)$ and determine the modulus and argument of the result.

[*Ans.* (*a*) $1800 + j600$;
(*b*) $1{\cdot}81 - j0{\cdot}865$; 2·0; 25° 30′]

†Readers may wish to solve many of the problems given in Chapter 2 by employing complex quantities.

206. Find ZZ^* if:

(*a*) $Z = 3 - j4$

(*b*) $Z = 10\angle -40°$

(*c*) $Z = 2{\cdot}5e^{-j\pi/3}$

[*Ans.* 25; 100; 6·25]

207. If $Z = 3 - j2$, express Z, $1/Z$ and Z^2 in the polar form $r\angle\theta°$

[*Ans.* $3{\cdot}6\angle -33°\,41'$; $0{\cdot}277\angle 33°\,41'$; $13\angle -67°\,22'$]

208. Impedances $Z_1 = 4 - j3$ and $Z_2 = 5 + j2$ are connected in parallel. Express their combined impedance in the form $(a + jb)$.

[*Ans.* $2{\cdot}94 - j0{\cdot}45$]

209. An impedance of $(12 + j16)\,\Omega$ is connected in parallel with another of $(10 - j20)\,\Omega$ across a supply of $(120 + j160)$ V. Calculate the current in each branch, the current from the supply and the overall power-factor of the circuit.

[*Ans.* $10\angle 0°$ A; $8{\cdot}94\angle 116°\,34'$ A; $10\angle 53°\,8'$ A; 1]

210. A capacitor of 150-μF capacitance is connected in series with two parallel branches *A* and *B*. Branch *A* consists of an 80-Ω resistor, *B* consists of a coil having an inductance of 0·2 H and negligible resistance. The whole circuit is connected to a 230-V, 50-Hz source.

Find (*a*) the line current and its phase-difference from the supply voltage, (*b*) the voltage across the capacitor.

[*Ans.* (*a*) 6·4 A, 30° 24′ lagging,
(*b*) $136\angle -120°\,24'$ V with respect to supply voltage]

211. A circuit is made up of a coil and a capacitor in series. The impedance of the coil is $(5 + j44)\Omega$ and that of the capacitor is $-j159\ \Omega$. These impedances are calculated for a frequency of 50 Hz.

Calculate the current that will pass in this circuit and its phase relation to the supply voltage when

(*a*) the supply voltage is 200 V at 50 Hz.

(*b*) the supply voltage is 200 V at 150 Hz.

[*Ans.* (*a*) 1·74 A; 87° 30′ leading
(*b*) 2·53 A; 86° 23′ lagging]

212. Two coils A and B are arranged in series. The voltage across the circuit is 200 V at a frequency of 50 Hz. Coil A has an impedance of 15·5 Ω at this frequency and its resistance is 10 Ω. The resistance of coil B is 12·5 Ω. If the current passing through the circuit is 5·5 A find the inductance of coil B.

[*Ans*: 0·0532 H]

213. A circuit having an impedance given by $(10 + j15)\Omega$ is arranged in parallel with another circuit having an impedance given by $(8 - j25)\Omega$. Estimate the current that will flow in each of these branches and also the total current if a voltage of 200 V is applied. Find also the inductance of the coil in the first circuit and the capacitance of the capacitor in the second one if the frequency is 50 Hz.

[*Ans.* 11·3 A; 7·68 A; 8·7 A; 0·0478 H; 127·5 μF]

214. Determine the current that will flow through a circuit with an impedance of $(15 + j12)\Omega$ when the applied voltage is 100 V. Give the phase relationship of this current to the applied voltage.

A capacitor is put in parallel with this circuit. Find the reactance of this capacitor that will make the power-factor of the two circuits taken together equal to unity. If the frequency of the applied voltage is 50 Hz evaluate the capacitance of the capacitor in microfarads.

[*Ans.* 5·2 A; 38° 40′ lagging; 30·75 Ω; 103·5 μF]

215. Three circuits are arranged in parallel across a 200-V, a.c. supply at a frequency of 50 Hz. The impedances of the circuits are $(12 + j15)\Omega$, $(10 + j20)\Omega$, and $(5 - j25)\Omega$ respectively. Calculate the current in each branch and the total current passing to the arrangement with their phase relations to the applied voltage. Find also the inductances of the first two circuits and the capacitance of the third one if the above impedances are calculated for a frequency of 50 Hz.

[*Ans.* 10·4 A lagging by 51° 20′; 8·95 A lagging by 63° 26′;
7·85 A leading by 78° 41′; 14·7 A lagging by 35°;
0·0478 H; 0·0637 H; 127 μF]

216. The current in amperes in a circuit is given by $(4{\cdot}5 + j12)$ when the applied voltage is $(100 + j150)$. Find (a) the complex expression for the impedance, (b) the power, (c) the phase angle between the current and the applied voltage. State whether the current is leading or lagging.

[*Ans.* (*a*) $(13{\cdot}7 - j3{\cdot}2)\Omega$; (*b*) 2250 W; (*c*) 13° 10′ leading]

217. Three impedances of $(70{\cdot}7 + j70{\cdot}7)\Omega$, $(120 + j160)\Omega$ and $(120 + j90)\Omega$ respectively, are connected in parallel across a 250-V supply. Calculate the admittance of the combination and the total current taken. Determine, also, the value of the pure reactance which, when connected across the supply, will bring the overall power-factor to unity and find the new value of the total current. Take the voltage applied as the reference phasor.

[*Ans.* $\{0{\cdot}0215\angle{-44{\cdot}3°}\}$S; $\{5{\cdot}37\angle{-44{\cdot}3°}\}$ A; $-j66{\cdot}6\Omega$; 3·85 A]

218. A circuit consists of a coil represented by $(2 + j50)\Omega$ in series with an imperfect capacitor. If the total circuit impedance is $(2{\cdot}5 + j18{\cdot}2)\Omega$ and the frequency is 50 Hz, find the value of the capacitance and its equivalent series resistance.

[*Ans.* 100 μF; 0·5 Ω]

219. Find the parallel combination of resistance and capacitance which takes the same current at the same power-factor from a 50-Hz supply as an impedance of $(17{\cdot}3 - j10)\Omega$.

Would these circuits be equivalent at a different value of frequency?

[*Ans.* 23·1 Ω; 79·6 μF; No.]

220. Two voltage sources, $V_1 = 100(1 - j)$ and $V_2 = 100(1 + j)$ volts, of internal impedance $Z_1 = 30(1 + j)$ and $Z_2 = 50(1 + j0)$ ohms, respectively, are connected in parallel across an impedance of $Z_3 = 20(1 + j5)$ ohms. Determine the magnitude of the current in Z_3 and its phase relationship to V_2.

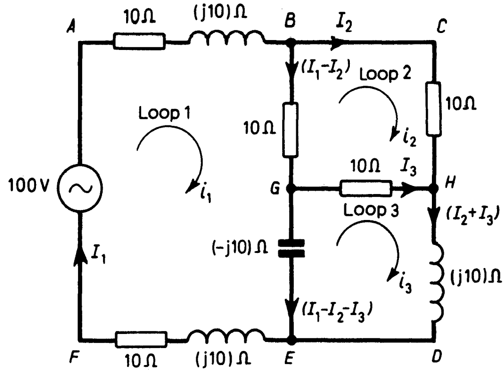
[*Ans.* 0·503 A; 123° 6′]

221. A parallel combination of resistance and inductance, which has an admittance of $(1{\cdot}25 - j1{\cdot}59)10^{-2}$S, is connected in series with a capacitor of impedance $-(j21{\cdot}2)\ \Omega$ across a 230-V a.c. supply. Calculate the magnitudes of the current taken from the supply and the voltage across the capacitor.

[*Ans.* 6·42 A; 75·9 V]

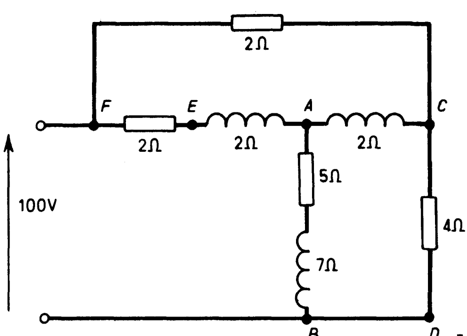
222. Use (*a*) Kirchhoff's Laws, (*b*) Maxwell's Cyclic-Current

Rule to find the currents I_1, I_2 and I_3 in the network illustrated. The generator 100 V (r.m.s.) should be taken as $(100 + j0)$ V acting in the direction F to A.



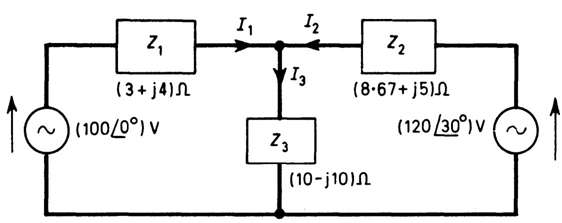
[*Ans.* $I_1 = (2 - j1)$A; $I_2 = (0{\cdot}5 - j1{\cdot}5)$A; $I_3 = -(1 + j2)$A]

223. Use Thévenin's Theorem to find the current in branch *CD* of the circuit illustrated.



[*Ans.* 17·15 A]

224. Use the Principle of Superposition to find the current I_2 in the network shown.



[*Ans.* $(3{\cdot}92 + j5)$A]

ADDITIONAL PROBLEMS

225. Evaluate:

(*a*) $(6 + j9) + (7 - j11)$

(*b*) $(100 - j100)/(8{\cdot}66 + j5)$

(*c*) $(4 - j2) - (3 + j3)$

(*d*) $(2 + j3)(-1 - j3)$

[*Ans.* (*a*) $13 - j2$; (*b*) $3{\cdot}66 - j13{\cdot}7$; (*c*) $1 - j5$; (*d*) $7 - j9$]

226. Express the following in polar form:

(*a*) $5 - j12$

(*b*) $-7 + j2$

(*c*) $-10 - j10$

(*d*) $6 + j2$

[*Ans.* (*a*) $13\angle -67{\cdot}3°$; (*b*) $7{\cdot}3\angle 164°$;
(*c*) $14{\cdot}1\angle -135°$; (*d*) $6{\cdot}3\angle 18{\cdot}4°$]

227. Evaluate the following:

(*a*) $6\angle 15° - 4\angle 40° + 7\angle -60°$

(*b*) $[50\angle 60°](7 - j9)$

(*c*) $(10 + j33)(4 + j5)(6 - j4)/(7 + j3)$

(*d*) $10\angle 53{\cdot}1° + (4 + j2)$

(*e*) $\sqrt{(5 + j8)}$

(*f*) $\log_e[20\angle 45°]$

[*Ans.* (*a*) $6{\cdot}23 - j7{\cdot}08 = 9{\cdot}33\angle -48°\,37'$;
(*b*) $571\angle 8° = 565 + j79{\cdot}4$;
(*c*) $209\angle 67°\,45'$;
(*d*) $10 + j10$;
(*e*) $3{\cdot}1\angle 29°$;
(*f*) $3 + j(\pi/4)$]

228. Show that:

(*a*) $3\angle 22° + 4\angle 112° = 5\angle 75{\cdot}1°$

(*b*) $(1 + j1)^4 = -4 + j0$

(*c*) $(3 - j2)/(1 + j3) = -0{\cdot}3 - j1{\cdot}1$

229. Express the following in terms of their modulus and argument:

$$\sqrt{(3)} + j1;\ 3 + j3;\ 2 - j2\sqrt{3}$$

[*Ans.* $2\angle 30°$; $3\sqrt{2}\angle 45°$; $4\angle -60°$]

230. An alternating current is represented by the phasor $(5 + j6)$A. Find the magnitude of this current and also the angle of phase difference between it and a voltage represented by the phasor $(200 - j15)$V.

[*Ans.* 7·8 A; 52° 31′]

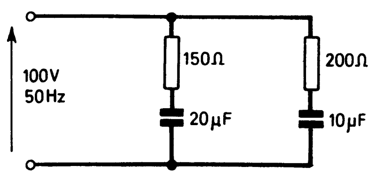
231. Determine the complex input impedance at 10 kHz of:

(*a*) *Circuit ABCD*. Input terminals *A* and *D* with connections, *A* to *B* = 50 Ω, *B* to *C* = 2 mH, *B* to *D* = 50 Ω, *C* to *D* = 50 Ω.

(*b*) *Circuit WXYZ*. Input terminals *W* and *Z* with connections. *W* to *X* = 50 Ω, *X* to *Y* = 30 Ω, *Y* to *Z* = 1 mH, *X* to *Z* = 0·2 μF.

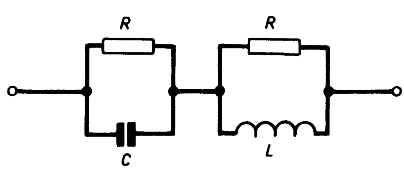
[*Ans.* (*a*) $(90{\cdot}4 + j12)\Omega$; (*b*) $(211 + j9{\cdot}7)\Omega$]

232. Determine the admittance, impedance, total current and power-factor of the following circuit:



[*Ans.* $(4{\cdot}65 + j5{\cdot}58)\ 10^{-3}$ S; $(89{\cdot}7 - j107)\Omega$; 0·72 A; 0·635]

233. Prove that if $R = \sqrt{(L/C)}$ the impedance of the circuit illustrated is independent of frequency, and determine the value of this impedance.*



[*Ans.* *R*]

*Two similar problems with their solutions may be found in the following book: F. A. Benson, *Problems in Electronics with Solutions*, Spon, 4th Edition, 1965, Problems 23 and 24.

234. A series circuit consisting of a 20-Ω resistor and an inductor of 10 Ω reactance is connected in parallel with another circuit consisting of a 10-Ω resistor and a capacitor of 20 Ω reactance in series. Reduce the combination to a simple series circuit.

[*Ans.* 15-Ω resistor in series with 5-Ω capacitive reactance]

235. A 60-Ω resistor, a coil of inductance $(1/\pi)$ henrys and a capacitor C each take a supply current of amplitude I when separately connected across a 240-V supply of frequency f. Find I, f and C and the total supply current taken by the three components connected across the supply (*a*) in series, (*b*) in parallel.

[*Ans.* 4 A; 30 Hz; 88·4 μF; 4 A; 4 A]

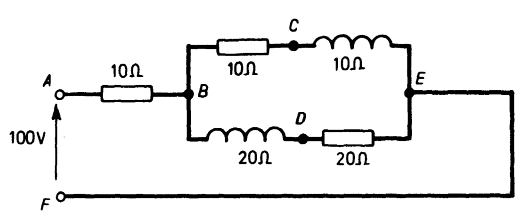
236. Two voltage sources $V_1 = 100(1 - j1)$V and $V_2 = 100(1 + j1)$V, of internal impedance $Z_1 = 30(1 + j1)\Omega$ and $Z_2 = 50(1 + j0)\Omega$, respectively, are connected in parallel across an impedance $Z_3 = 20(1 + j5)\Omega$. Find the magnitude of the current in Z_3 and its phase relationship to V_2.

[*Ans.* 0·503 A; 123° 6′]

237. Three impedances $(5 - j6)\Omega$, $(3 + j4)\Omega$ and $(0 - j3)\Omega$ are connected in parallel. Evaluate the total admittance of the arrangement.

[*Ans.* $(0{\cdot}202 + j0{\cdot}271)$ S]

238. For the circuit illustrated evaluate the currents in *AB*, *BCE* and *BDE*. Calculate also the voltage between points *C* and *D*.



[*Ans.* 5·57 A; 3·71 A; 1·86 A; 52·5 V]

239. A variable capacitor connected in series with a circuit comprising a 20-Ω non-inductive resistor in parallel with an inductor of resistance 15 Ω and inductance 0·02 H is supplied from 200-V, 100-Hz mains. Determine the value of capacitance to give a total power-factor of unity and the current then taken from the mains.

[*Ans.* 441 μF; 20·1 A]

240. Calculate the admittance, impedance and power-factor of the following circuit when the supply frequency is 500 Hz.

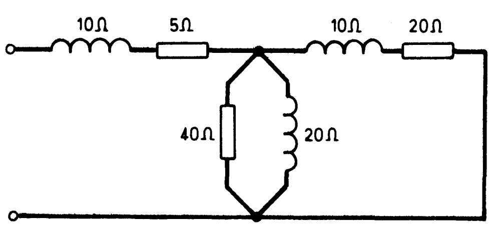
Branches *A* and *B* are connected in parallel across the supply, where *A* is a 100-Ω resistor in series with a 2-μF capacitor and *B* is a 100-Ω resistor in series with a 20-mH inductor.

[*Ans.* 0·01 S; 100 Ω; 1]

241. An a.c. circuit *ABC* consists of a 1-μF capacitor in parallel with a 1000-Ω resistor (*AB*) in series with a 1-H inductor in parallel with a 1000-Ω resistor (*BC*). Determine the impedance and power-factor of the circuit for a frequency of 50 Hz. Calculate also the phase-angle between the voltages across *AB* and *BC*.

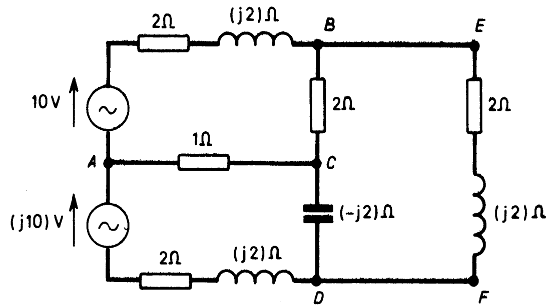
[*Ans.* 1000 Ω; 1; 88° 25′]

242. The circuit shown has values of resistance and reactance as illustrated. Determine the total admittance, conductance and susceptance and the total effective resistance and reactance.



[*Ans.* 0·0467 S; 0·0264 S; 0·0385 S; 12·1 Ω; 17·7 Ω]

243. Evaluate the current in branch *EF* of the circuit shown by using (*a*) Kirchhoff's Laws, (*b*) Maxwell's Cyclic-Current Rule, (*c*) Thévenin's Theorem, (*d*) The Principle of Superposition.



[*Ans.* (2·078 − *j*0·673)A]

CHAPTER FOUR

POLYPHASE CIRCUITS

244. A three-phase, star-connected system has 400 V between wires. Estimate the voltage between each wire and the neutral point.

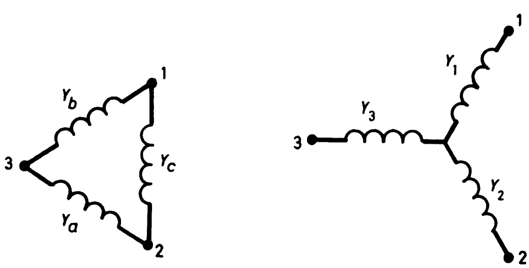
[*Ans.* 231 V]

245. A three-phase rectifier is connected to a three-phase transformer giving 460 V maximum between each line and the neutral point, and a current flows between the neutral point and the common cathode of the rectifier. Assuming no overlap, estimate the average voltage on the d.c. side if the drop in voltage in the rectifier is 15 V.

[*Ans.* 365 V]

246. Show that the star and mesh circuits illustrated are equivalent if the following conditions hold:

$$Y_a = Y_2 Y_3/(Y_1 + Y_2 + Y_3)$$
$$Y_b = Y_1 Y_3/(Y_1 + Y_2 + Y_3)$$
and
$$Y_c = Y_1 Y_2/(Y_1 + Y_2 + Y_3)$$



247. From a three-phase, star-connected synchronous generator the line current is 10 A. If the machine was delta-connected, find the current in each of its phases if the line current remains the same as before.

[*Ans.* 5·77 A]

248. Three similar coils, each of resistance 10 Ω and reactance 10 Ω, are connected (*a*) in star, (*b*) in delta across a 400-V, three-phase supply. Find, in each case, the line current and the sum of the readings of two wattmeters connected to measure the power.

[*Ans.* (*a*) 16·33 A; 8000 W;
(*b*) 48·97 A; 24 000 W]

249. Three 20-Ω non-inductive resistors are connected in star across a three-phase supply the line voltage of which is 480 V. Three other equal non-inductive resistors are connected in delta so as to take the same line current. What are the resistance values of these other resistors and what is the current flowing through each of them?

[*Ans.* 60 Ω; 8 A]

250. In a two-phase, three-wire system the two phases are unequally loaded. The current in the leading phase is 47 A in phase with its own voltage. The current in the lagging phase is 39 A and lags 22° 36′ behind its own voltage. Estimate the magnitude of the current in the common return and find the phase angle between it and the 47 A current.

[*Ans.* 48·17 A; 48° 22′]

251. A two-phase synchronous generator has a terminal voltage of 220 V to the neutral. The leading phase is loaded with a resistor of 11 Ω, the other with a coil of resistance 5 Ω and reactance 4 Ω.

If the connections are according to the three-wire system, calculate the magnitude of the current in the common return and the phase-angle difference between this current and the voltage of the lagging phase.

[*Ans.* 26·87 A; 3° 6′]

252. If two sets of coils with laminated iron cores are arranged perpendicularly to one another and are connected respectively to the two phases of a two-phase circuit, show that a rotating field is produced.

253. Show that three-phase currents can be applied to the production of a rotating field by using three coils exciting pole pieces placed at 120° to each other. Prove that the field rotates at constant speed and has a constant magnitude.

254. A 75 kW, three-phase, star-connected motor is connected to a supply whose line voltage is 3000 V. The efficiency is 0·92 and the power-factor 0·9. Calculate the line current.

[*Ans.* 17·4 A]

255. The one-wattmeter method is used to measure power in a three-phase balanced system. Find expressions for the total power and the power-factor in terms of the wattmeter readings W_1 and W_2.

[*Ans.* $W_1 + W_2$; $(W_1 + W_2)/2\sqrt{(W_1^2 - W_1W_2 + W_2^2)}$]

256. A three-phase load has a power-factor of 0·3. The two-wattmeter method is used to measure the power in the load which is known to be 20 kW. Calculate the reading on each wattmeter.

[*Ans.* 28·35 kW; −8·35 kW]

257. The power supplied to a three-phase delta-connected induction motor is measured by two wattmeters. The line voltage is 400 V. When the motor is running on a light load the wattmeter readings are 389 W and 271 W. Calculate the power taken by the motor and the power-factor. Find also the line current.

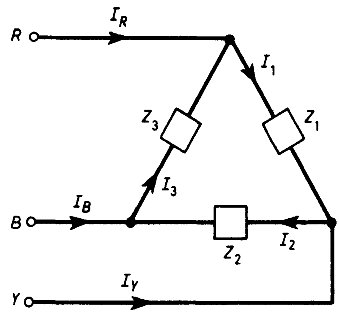
In a no-load test on the motor the wattmeter readings are 788·5 W and −288·5 W. Determine again the power taken by the motor, the power-factor and the line current.

[*Ans.* 660 W; 0·955; 0·996 A; 500 W; 0·26; 2·78 A]

258. A coil, a capacitor and a resistor are connected in star to a three-phase, 400-V, three-wire supply. The coil has a resistance of 30 Ω and a reactance of 40 Ω, the resistor has a resistance of 50 Ω and the capacitor has a reactance of 50 Ω. Determine the current in each component of the circuit given that the phase rotation is *R.Y.B.* and the coil is connected to *Y* and the capacitor to *B*.

[*Ans.* $7{\cdot}35\angle 11{\cdot}5°$ A; $2{\cdot}53\angle -133{\cdot}5°$ A; $5{\cdot}5\angle 175{\cdot}5°$ A]

259. Three impedances, $Z_1 = (10 + j10)\Omega$, $Z_2 = (8{\cdot}66 + j5)\Omega$ and $Z_3 = (12 + j16)\Omega$ are delta-connected as shown to a 380-V, three-phase system. Find the line currents. Take V_{RY} as the reference phasor.



[*Ans.* $I_R = 38{\cdot}2\angle -72{\cdot}5°$ A;
$I_Y = 51{\cdot}9\angle 180°$ A;
$I_B = 54{\cdot}3\angle 42{\cdot}1°$ A]

260. An unbalanced delta-connected load is connected to the terminals *A*, *B*, *C* of a 400-V, balanced, three-phase supply, whose order of phase-sequence is: V_{AB}, V_{BC}, V_{CA}.

The load consists of:

AB: A non-inductive 10-Ω resistor.

BC: A capacitor of reactance 20 Ω.

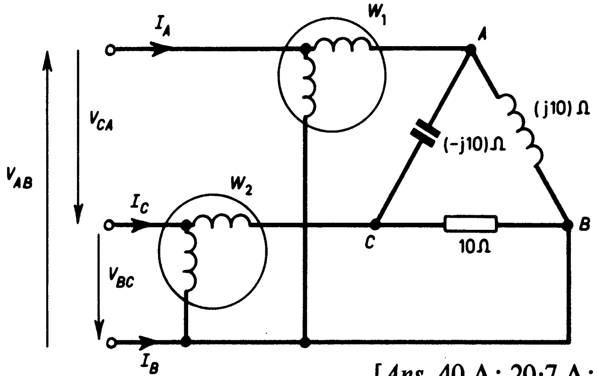
CA: An inductor of reactance 10 Ω and resistance 5 Ω.

By drawing a phasor diagram to scale, or by the use of complex quantities, determine the magnitudes of the line currents, and the reading given by a wattmeter connected with its current coil in line *A* and its voltage coil between lines *B* and *C*.

[*Ans.* 36·1 A; 24·8 A; 39·9 A; 6130 W]

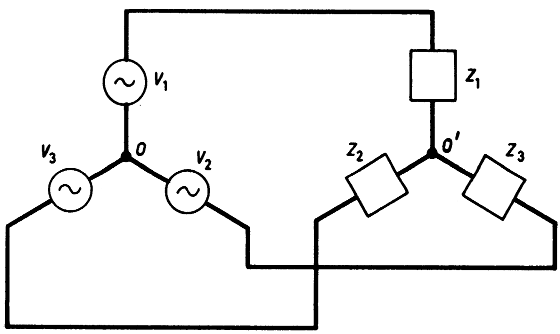
261. A balanced set of three-phase voltages, of r.m.s. line value 400 V and with positive phase-sequence, is applied to the circuit illustrated. Evaluate the magnitudes of the currents I_A, I_B, I_C, I_{CA}, I_{AB} and I_{BC}. Take V_{AB} as $(400 + j0)$V.

Find, also, the readings on the two wattmeters W_1 and W_2.



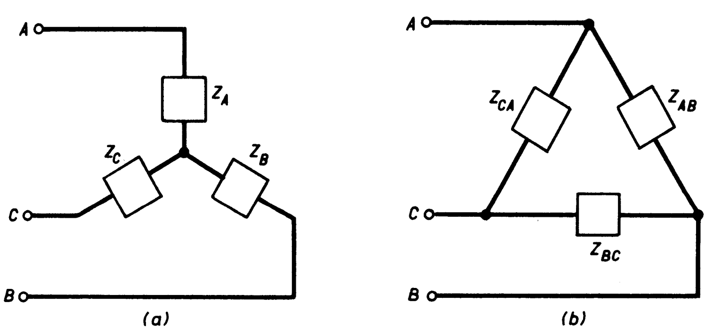
[*Ans.* 40 A; 20·7 A; 20·7 A;
40 A; 40 A; 40 A;
13 856 W; 2130 W]

262. Use Millman's Theorem to find the voltage between O and O' for the three-phase system illustrated.



[*Ans.* $(V_1Y_1 + V_2Y_2 + V_3Y_3)/(Y_1 + Y_2 + Y_3)$,
where $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$ and $Y_3 = 1/Z_3$]

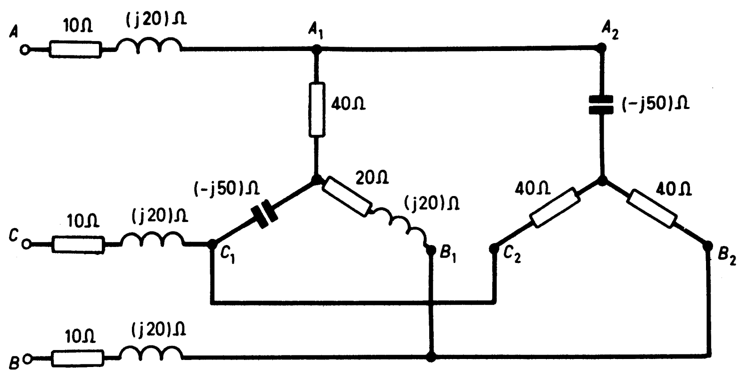
263. The star-connected set of impedances at (*a*) in the illustration may be transformed into the equivalent delta set at (*b*).



Show that:

$$Z_A = (Z_{AB}Z_{CA})/(Z_{AB} + Z_{BC} + Z_{CA})$$
$$Z_B = (Z_{BC}Z_{AB})/(Z_{AB} + Z_{BC} + Z_{CA})$$
$$Z_C = (Z_{CA}Z_{BC})/(Z_{AB} + Z_{BC} + Z_{CA})$$
$$Z_{AB} = Z_AZ_B(1/Z_A + 1/Z_B + 1/Z_C)$$
$$Z_{BC} = Z_BZ_C(1/Z_A + 1/Z_B + 1/Z_C)$$
$$Z_{CA} = Z_CZ_A(1/Z_A + 1/Z_B + 1/Z_C)$$

264. Calculate the line currents I_A, I_B and I_C for the circuit shown if $V_{AB} = 400$ V, $V_{BC} = a^2400$ V and $V_{CA} = a400$ V.



[*Ans.* $(3{\cdot}93 - j6{\cdot}30)$A;
$(-2{\cdot}54 - j1{\cdot}80)$A;
$(-1{\cdot}39 + j8{\cdot}10)$A]

265. A set of three-phase voltages has the following values: $V_A = (100 + j0)$ V, $V_B = (-100 - j100)$ V and $V_C = (+j50)$ V. Determine the symmetrical components of voltage V_A.

[*Ans.* $(-j16{\cdot}67)$ V; $(93{\cdot}3 - j20{\cdot}5$ V; $(6{\cdot}7 + j37{\cdot}19)$ V]

ADDITIONAL PROBLEMS

266. Three groups of heating elements used for de-icing a road surface are connected to a three-phase transformer. Find the three values of power dissipation provided by delta-star switching of load and transformer.

[*Ans.* W; $W/3$; $W/9$]

267. A three-phase star-connected generator, whose line voltage is 1730 V, supplies current to three non-inductive mesh-connected resistors with resistances of 3, 4 and 5 Ω respectively. Calculate the current in each of the three phases of the generator.

[*Ans.* 878 A; 676 A; 802 A]

268. Three coils, each having a resistance of 10 Ω and an inductance of 0·02 H are connected (*a*) in star, (*b*) in delta, to a three-phase, 50-Hz supply, the line voltage being 500 V. Calculate, for each case, the line current and the total power absorbed.

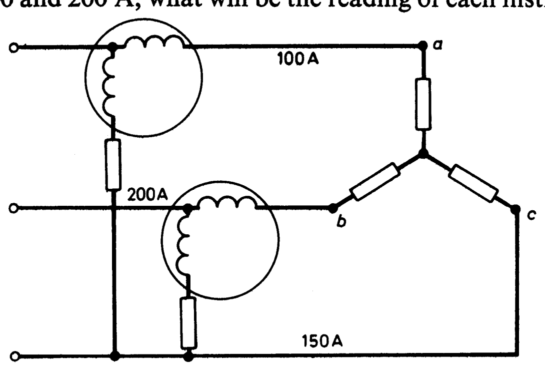
[*Ans.* (*a*) 24·44 A; 17·93 kW;
(*b*) 73·32 A; 53·89 kW]

269. A star-connected load is arranged on a three-phase supply with 1730 V between lines. The phase resistances of the load are 10, 20 and 30 Ω respectively.

Find both graphically and analytically the potential difference between each line and the neutral point of the load and the potential difference between the neutral point of the load and the neutral point of the star-connected generator.

[*Ans.* 680 V; 1140 V; 1250 V; 320 V]

270. Find the power absorbed in the star-connected circuit *abc* shown, which is formed of non-inductive resistors, when the line currents are 100, 200 and 150 A respectively and the supply voltage between lines is 380 V. If this power be measured by the two-wattmeter method, the current coils being inserted in the lines carrying 100 and 200 A, what will be the reading of each instrument?



[*Ans.* 94 kW; 24 kW; 70 kW]

271. Two wattmeters measure the total power in a three-phase circuit and are correctly connected. One reads 4800 W while the other reads backwards. On reversing the latter it reads 400 W. What is the total power absorbed by the circuit and the power-factor?

[*Ans.* 4400 W; 0·439]

272. A two-phase, 200-V (per phase), induction motor is to be rewound with a star-connected three-phase winding suitable for a line voltage of 400 V, producing the same air-gap flux. The two-phase winding has 350 turns per phase, the magnetizing current being 6 A. Determine the approximate number of turns per phase required for the three-phase winding and the magnetizing current.

[*Ans.* 405; 3·45 A]

273. A star-connected balanced load is supplied from a three-phase balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of the load consists of a resistor and a capacitor in series. The readings of two wattmeters connected to measure the total power supplied are 782 W and 1980 W (both positive). Calculate (*a*) the power-factor of the circuit, (*b*) the line current and (*c*) the capacitance of each capacitor.

[*Ans.* (*a*) 0·8; (*b*) 4·8 A; (*c*) 106 μF]

274. A 440-V, three-phase motor has an output of 60 kW, an efficiency of 90% and a power-factor of 0·87. Find the line current and the readings on each of the two wattmeters connected to measure the power input.

[*Ans.* 100 A; 44·01 kW; 22·3 kW]

275. A 440-V, 50-Hz, star-connected induction motor takes a line current of 50 A at a power-factor of 0·8 lagging. A three-phase capacitor bank, connected in star, is to be used to improve the power-factor to 0·95 lagging. Determine the capacitance of each capacitor.

Show that if the capacitors had been connected in delta the capacitance of each would have needed to be one third of the previous value.

[*Ans.* 211·3 μF]

276. The voltages between the terminals A, B, C of a three-phase supply, in order of phase-sequence are: $AB = 400$ V; $BC = 380$ V, $CA = 360$ V. An unbalanced delta-connected load is supplied from ABC and consists of:

AB a 20-Ω resistor

BC a 15-Ω inductor in series with a 15-Ω resistor

CA a 30-Ω capacitor.

Draw a phasor diagram of the line voltages and of the phase and line currents and determine the magnitudes of the line currents and the readings of wattmeters, one with its current-coil in line A and voltage-coil across AB, the other with its current-coil in line C and voltage-coil across CB.

[*Ans.* $|I_A| = 31$ A; $|I_B| = 39$ A; $|I_C| = 8{\cdot}5$ A; 12 100 W; 800 W]

277. A star-connected transformer of phase voltage 100 V (r.m.s.) with line terminals A, B, C and neutral point N, supplies balanced voltages to a star-connected load consisting of:

A to X a 50-Ω resistor

B to X a capacitor of 120-Ω reactance

C to X a 50-Ω resistor in series with an inductor of 100-Ω reactance.

The transformer voltage phase-sequence is AN, BN, CN. Determine the line currents and the voltage between N and X.

[*Ans.* $|I_A| = 0{\cdot}72$ A; $|I_B| = 1{\cdot}74$ A; $|I_C| = 1{\cdot}785$ A; 134·5 V]

278. The line voltages of an unbalanced three-phase system are $V_{AB} = (200 + j0)$V, $V_{BC} = -200(1 + j1)$V, $V_{CA} = j200$ V.

Determine graphically, and by calculation, the symmetrical components.

[*Ans.* $V_1 = (215{\cdot}4 - j57{\cdot}8)$V; $V_2 = (-15{\cdot}4 + j57{\cdot}8)$V; 0]

CHAPTER FIVE

NON-SINUSOIDAL WAVES

279.* Calculate, from first principles, the r.m.s. reading which would be indicated by an ammeter in a circuit whose current waveform is given by 10 sin ωt + 3 sin $3\omega t$ + 2 sin 5 ωt.

[*Ans.* 7·52]

280. A voltage is represented by (2000 sin ωt + 500 sin $3\omega t$ +300 sin $5\omega t$)V. Determine the maximum current corresponding to each harmonic of the voltage that will flow through a 20-μF capacitor. Find also the r.m.s. value of the current. Assume that $\omega = 100\pi$.

[*Ans.* 9·43 A; 9·43 A; 12·95 A]

281. A voltage represented by (50 sin ωt + 20 sin $3\omega t$ + 15 sin $5\omega t$)V is applied to a series *LCR* circuit in which $L = 0{\cdot}506$ H, $R = 5\ \Omega$ and $C = 20\ \mu$F.

Calculate the r.m.s. fundamental current and the currents corresponding to each harmonic. The frequency of the fundamental component of the applied voltage is 50 Hz. Determine also the three component voltages across the capacitor.

[*Ans.* 7·07 A; 0·0333 A; 0·0139 A; 1125 V; 1·76 V; 0·442 V]

282. A voltage represented by (150 sin ωt + 15 sin $3\omega t$ + 7·5 sin $5\omega t$)V is applied to a series circuit having a resistance of 15 Ω and an inductance of 0·05 H. The frequency of the fundamental is 50 Hz.

Determine the maximum amplitudes of the three current components and the ratios of the amplitudes of the third and fifth harmonics to the fundamental.

[*Ans.* 6·91 A; 0·304 A; 0·0938 A; 0·0441; 0·0136]

*There are no problems in this Chapter on Fourier-series representation of waveforms as this topic is dealt with in the following book: F. A. Benson, *Problems in Electronics with Solutions*, 4th Edition, Spon, 1965, Chapter 3.

283. A voltage represented by (150 sin ωt + 15 sin $3\omega t$ + 7·5 sin $5\omega t$) V is applied to a series circuit having a resistance of 5 Ω and a capacitance of 30 μF. The fundamental frequency is 50 Hz.

Determine the maximum amplitudes of the three component currents and the ratios of the amplitudes of the third and fifth harmonics to the fundamental.

[*Ans.* 1·41 A; 0·422 A; 0·344 A; 0·299; 0·244]

284. A circuit has a resistance of 10 Ω and an inductance of 0·035 H and the current passing is given by {5 sin ωt + 0·3 sin ($3\omega t$ + 70·1°) + 0·1 sin ($5\omega t$ + 159°)}A.

Calculate the amplitudes of the three component voltages required to maintain this current and show that they are in phase with one another. The fundamental frequency is 50 Hz.

[*Ans.* 74·4 V; 10·35 V; 5·59 V]

285. The voltage applied to a circuit is represented by (2000 sin ωt + 600 sin $3\omega t$ + 400 sin $5\omega t$)V. Estimate the r.m.s. current that will flow in this series circuit if the resistance is 10 Ω, the capacitance is 30 μF and the inductance is of such a value that there is resonance with the third harmonic of the voltage wave. The frequency of the fundamental is 50 Hz.

Calculate also the r.m.s. voltage between the terminals of the inductor under these conditions.

[*Ans.* 45·6 A; 2220 V]

286. A voltage wave-shape is given by the following Table:

Phase-angle θ (degrees)	0	30	60	90	120	150	180
Voltage v (V)	0	100	200	300	200	100	0

Find the magnitudes of the fundamental and of the third, fifth and seventh harmonics.

[*Ans.* 244 V; 27 V; 9·8 V; 5 V]

287. A voltmeter (reading r.m.s. values) is connected to a generator giving a voltage represented by:

$$8000 \sin \omega t + 540 \sin 3\omega t + 350 \sin 5\omega t.$$

What will the voltmeter read?

If the voltage is applied to (*a*) a capacitor of capacitance 125 μF (*b*) a coil having negligible resistance and an inductance of 0·05 H, find the r.m.s. value of the current flowing in each case. Assume $\omega = 100\pi$.

[*Ans.* 5680 V; (*a*) 232 A; (*b*) 361 A]

288. A resistance of 10 Ω is connected in series with a coil of inductance 6·36 mH. The supply voltage is given by

$$V = (300 \sin 314t + 50 \sin 942t + 40 \sin 1570t) \text{ volts}$$

Find:

(*a*) An expression for the instantaneous value of the current.

(*b*) The power dissipated.

[*Ans.* $29{\cdot}4 \sin(\omega t - 11{\cdot}3°) + 4{\cdot}28 \sin(3\omega t - 31°) + 2{\cdot}83 \sin(5\omega t - 45°)$; 4454 W]

289. A capacitor of 3·18 μF capacitance is connected in parallel with a resistance of 1000 Ω, the combination being connected in series with a 1000-Ω resistor to a voltage which is given by:

$$v = \{350 \sin \omega t + 150 \sin(3\omega t + 30°)\}\text{V}$$

Determine (*a*) the power dissipated in the circuit if $\omega = 314$ rad s^{-1}.

(*b*) the voltage across the series resistor.

(*c*) the percentage harmonic content of the resultant current.

[*Ans.* (*a*) 46·3 W;
(*b*) $\{222 \sin(\omega t + 18{\cdot}5°) + 131 \sin(3\omega t + 5{\cdot}3°)\}$V;
(*c*) 59%]

290. Voltage and current waveforms are represented by the expressions:

$v = (100 + 50 \sin 314t + 20 \sin 942t)$ volts.

$i = \{10 + 3{\cdot}54 \sin(314t - 45°) + 0{\cdot}635 \sin(942t - 71{\cdot}6°)\}$A

Determine the r.m.s. values of voltage and current.

[*Ans.* 107 V; 10·34 A]

291. A voltage of $v = \{10 + 20 \sin 250t + 15 \sin(500t + 30°)\}$ volts is applied to a branch composed of a resistance of 10 Ω. Find the average current, the r.m.s. value of the current and the power delivered to the resistor.

[*Ans.* 1 A; 2·03 A; 41·2 W]

292. A voltage is represented by the expression

$$v = (200 \cos 314t - 40 \sin 628t) \text{ volts}$$

and is connected to a circuit consisting of a 20-Ω resistor in series with a 100-μF capacitor. Derive the expression for the steady-state current and determine its r.m.s. value.

[*Ans.* 5·33 cos(314t + 57° 48′) − 1·565 sin (628t + 38° 33′); 3·94 A]

ADDITIONAL PROBLEMS

293. A voltage represented by $v = \{100 \sin 314t + 10 \sin 1570t\}$V is applied to the terminals of a 1-μF capacitor. Obtain an expression for the current which flows.

[*Ans.* {0·031 cos 314t + 0·016 cos 1570t}A]

294. A voltage which can be represented by $v = \{400 \sin \omega t + 30 \sin 3\omega t\}$V is applied to a rectifier having a resistance of 50 Ω in one direction and 200 Ω in the other direction. Find the average and r.m.s. values of the current.

[*Ans.* 1·96 A; 4·1 A]

295. A circuit consisting of a 200-μF capacitor in series with a 7-Ω resistor is supplied with a voltage whose instantaneous value is given by {200 sin (314t) + 20 sin (942t − 90°)}V.

Derive an expression for the current in the circuit and evaluate the total r.m.s. current, the total power and the power-factor.

[*Ans.* {11·5 sin (314t + 66° 14′) + 2·28 sin (942t − 52° 57′)}A; 8·3 A; 483 W; 0·41]

296. A circuit, consisting of a 10-Ω resistor in series with a 0·015-H inductor, carries a current represented by i = (10 sin 314t + 5 cos

942t)A. Determine the expression for the voltage across the circuit and calculate its r.m.s. value and the power-factor.

What is the total power absorbed?

[*Ans.* $\{110{\cdot}5 \sin(314t + 25° 13') + 86{\cdot}5 \cos(942t + 54° 39')\}$V; 99 V; 0·8; 625 W]

297. A circuit consisting of a 100-μF capacitor in series with a 20-Ω resistor is supplied with a voltage whose instantaneous value is $\{100 \sin 314t - 50 \cos 942t\}$V. Derive an expression for the steady-state instantaneous current and determine its r.m.s. value and the total power.

[*Ans.* $\{2{\cdot}66 \sin(314t + 57° 50') - 2{\cdot}21 \cos(942t + 27° 55')\}$A; 2·445 A; 120 W]

298. A voltage represented by the expression $v = \{100 \sin 314t - 25 \sin(628t - 60°)\}$V is connected to a circuit consisting of a 20-Ω resistor in series with a 0·1-H inductor and a 40-μF capacitor. Obtain an expression for the steady-state current flowing in the circuit and find its r.m.s. value.

[*Ans.* $\{1{\cdot}915 \sin(314t + 67{\cdot}5°) + 0{\cdot}82 \cos(628t - 19° 6')\}$A; 1·47 A]

299. A current represented by the expression $i = \{5 \sin 105t + 2 \sin(315t + 30°)\}$A is flowing in a circuit consisting of a 60-Ω resistor in series with a 0·9-H inductor and a 100-μF capacitor. Derive an expression for the voltage across the circuit as a function of time and calculate the total power dissipated.

[*Ans.* $\{300 \sin(105t - 40') + 516 \sin(315t + 106° 36')\}$V; 870 W]

300. A voltage $v = \{100 \cos 314t + 50 \sin(1570t - 30°)\}$V is applied to a series circuit consisting of a 10-Ω resistor, a 0·02-H inductor and a 50-μF capacitor. Determine the instantaneous current and the r.m.s. values of voltage and current.

[*Ans.* $\{1{\cdot}71 \cos(314t + 80° 10') + 2{\cdot}36 \sin(1570t + 91° 48']\}$A; 79·2 V; 2·06 A]

CHAPTER SIX

TRANSFORMERS AND ELECTRIC MACHINES

301.* A single-phase transformer has a primary voltage of 2000 V, a secondary voltage of 440 V and a full-load output of 20 kVA. The secondary winding has 130 turns. Calculate the number of primary turns and the primary and secondary full-load currents, neglecting losses.

[*Ans.* 591; 10 A; 45·5 A]

302. The primary of a transformer has 500 turns and is supplied at a voltage of 2000 V r.m.s. at a frequency of 50 Hz. Estimate the maximum value of the flux through the core.

[*Ans*, 0·018 Wb]

303. The primary of a transformer consists of 1000 turns of wire and produces a maximum flux of 0·03 Wb alternating at 50 Hz in the iron core. The secondary winding has 35 turns. Estimate the r.m.s. values of primary and secondary e.m.f.'s on the assumption that the flux change is sinusoidal.

[*Ans.* 6660 V; 233 V]

304. A three-phase transformer has its primary windings delta-connected and its secondary star-connected. The primary and secondary line voltages are 6600 and 380 V r.m.s. respectively. The maximum flux is 0·02 Wb and the frequency is 50 Hz. Determine the number of turns on each primary and secondary winding. Neglect losses.

[*Ans.* 1486; 49]

*Problems on coupled-circuits, which are not included here, can be found in the book: F. A. Benson, *Problems in Electronics with Solutions*, 4th Edition, 1965, Spon, Chapter 1.

305. The voltage on the secondary of a single-phase transformer is 200 V when supplying a load of 8 kW at a power-factor of 0·8. The secondary resistance is 0·04 Ω and the secondary leakage reactance is 0·8 Ω. Calculate the induced e.m.f. in the secondary winding.

[*Ans.* 227·5 V]

306. If the transformer of Question 305 has 500 primary turns and 50 secondary turns, find the induced e.m.f. in the primary winding. If the primary resistance is 4 Ω and the primary leakage reactance is 70 Ω, estimate the primary terminal voltage. The magnetizing current can be ignored.

[*Ans.* 2275 V; 2520 V]

307. A 50-kVA transformer, which steps down from 6600 V to 220 V, has a primary resistance of 10 Ω and a secondary resistance of 0·01 Ω.

Calculate: (*a*) the total resistance referred to the secondary;

(*b*) the total resistance referred to the primary;

(*c*) the full-load copper loss.

[*Ans.* 0·021 Ω; 19 Ω; 1091 W]

308. When the secondary of a transformer is short-circuited and a voltage of 30 V is applied to the primary, the primary current is 20 A and the power supplied is 200 W. Find the total impedance, reactance and resistance referred to the primary.

[*Ans.* 1·5 Ω; 1·41 Ω; 0·5 Ω]

309. A farm, whose electrical load can be represented by a resistance of 1 Ω in series with an inductive reactance of 1 Ω, is supplied from an 11 000-V single-phase line through a transformer of turns ratio 50:1. The resistance and leakage reactance of the transformer are 125 Ω and 250 Ω respectively when referred to its primary and its magnetizing current may be neglected. Determine the magnitude of the current taken from the secondary terminals of the transformer, the potential difference between those terminals, the magnitude and power-factor of the primary current.

[*Ans.* 145 A; 205 V; 2·9 A; 0·69]

310. A 100-kVA single-phase transformer steps down from 2000/400 V. It has a primary resistance of 0·17 Ω and a secondary resistance of 0·0068 Ω; the reactances are 0·25 Ω and 0·01 Ω respectively.

Calculate the resistance, reactance and impedance referred to the secondary. Hence find the percentage regulation on full secondary load of 250 A at a p.f. of 0·8 lagging.

[*Ans.* 0·0136 Ω; 0·02 Ω; 0·024 Ω; 1·43]

311. A star-star transformer of total X and R referred to the secondary, per phase, 0·75 Ω and 0·1 Ω respectively is connected in parallel with a delta-delta transformer of the same no-load line-voltage ratio with total X and R referred to the secondary, per phase, of 0·9 Ω and 0·3 Ω respectively. If the total load is 100 kVA, determine the load on each transformer.

[*Ans.* 29·3 kVA; 70·7 kVA]

312. A 50-kVA, three-phase, 3300/440-V transformer gave the following test results:

(*a*) *Primary open-circuited* and *secondary* supplied at 440 V; power input = 0·6 kW, line current = 5·4 A.

(*b*) *Secondary short-circuited* and *primary* supplied at 160 V, line current = 9 A, power input = 0·8 kW.

Determine the efficiency and input current when the transformer is delivering:

(1) Full load at 0·8 p.f. lagging.

(2) ¼ Full load at 0·9 p.f. lagging.

[*Ans.* (1) 96·7%; 9·27 A;
(2) 94·6%; 2·65 A]

313. A 300-kVA, three-phase transformer of no-load voltage ratio 11 000/500 has leakage reactance 4% and resistance 1% and is supplied at 11 000 V. Two balanced three-phase loads are supplied from the secondary:

(*a*) 200 kVA, 0·8 p.f. lagging at the end of a cable of R and X = 0·04 Ω and 0·05 Ω per phase respectively.

(*b*) 50 kVA, 0·6 p.f. lagging at the end of a cable of R and X $= 0{\cdot}1\ \Omega$ and $0{\cdot}2\ \Omega$ per phase respectively.

Calculate the *line* voltages at the loads and at the transformer secondary.

[*Ans.* 461·4 V (200-kVA load), 464·2 V (50-kVA load) 486·2 V (transformer secondary)]

314. A 50-kVA, 2000/500 V three-phase transformer when tested with the secondary short-circuited gave the following results:

Primary line voltage,	180 V
Primary line current,	25 A
Input power,	1600 W

Calculate the regulation at 0·8 p.f. lagging and at 0·8 p.f. leading.

[*Ans.* 3·89%; −2·19%]

315. A 10-kVA, 200/400 V, 50 Hz, single-phase transformer gave the following test results:

O.C. test: 200 V, 1·3 A, 120 W, on l.v. side.
S.C. test: 22 V, 30 A, 200 W, on h.v. side.

Calculate:

(*a*) the magnetizing current and the component corresponding to core loss at normal frequency and voltage.
(*b*) the percentage regulation when supplying full load at 0·8 p.f. leading.
(*c*) the load which gives maximum efficiency and the value of this efficiency at unity p.f.

[*Ans.* (*a*) 1·15 A, 0·6 A;
(*b*) −1·5%
(*c*) 9·25 kW, 97·4%]

316. Two three-phase transformers, both of voltage ratio 3300/400 are connected in parallel on the primary and secondary sides and supply a balanced load of 250 kVA at 0·7 power-factor lagging. The first transformer is rated at 100 kVA and has 2% resistance and 6% leakage reactance, while the second is rated at 200 kVA and has 1% resistance and 4% leakage reactance. Determine the load taken by each transformer.

[*Ans.* 59 kVA; 182 kVA]

317. A three-phase distribution cable is fed from the secondary of a 100-kVA transformer and supplies balanced loads of 60 A at 0·8 power-factor lagging at 250 m and 50 A at 0·9 power-factor lagging at 750 m from the transformer. The no-load secondary voltage of the latter is 440, and its total resistance and leakage reactance are 1% and 4% respectively. The resistance and reactance of the cable are 0·25 Ω and 0·5 Ω per phase per 1000 m respectively. Neglecting the voltage drop up to the transformer primary, determine the line voltages at the load points.

[*Ans.* 406·5 V; 387·3 V]

318. Calculate the percentage voltage drop for a transformer with a percentage resistance of 2% and a percentage reactance of 4%, of rating 550 kVA, when it is delivering 450 kVA at 0·6 p.f. lagging.

[*Ans.* 3·6%]

319. The power input to the rotor of a 400-V, 50-Hz, six-pole, three-phase induction motor is 80 kW. The rotor e.m.f. is observed to make 100 complete alternations per minute Calculate (*a*) the slip, (*b*) the rotor speed, (*c*) the mechanical power developed, (*d*) the rotor copper loss per phase, (*e*) the rotor resistance per phase if the rotor current is 65 A.

[*Ans.* (*a*) 3·3%; (*b*) 966·7 r.p.m.;
(*c*) 77·3 kW; (*d*) 0·9 kW;
(*e*) 0·21 Ω]

320. A 22·4 kW*, 400-V, three-phase, 50-Hz, six-pole induction motor has a delta-connected stator (primary) winding and the following impedances, all referred to the primary, per phase:

$R_1 = 0{\cdot}5\ \Omega$, $R_2' = 0{\cdot}39\ \Omega$, total leakage reactance $X = X_1 + X_2' = 4{\cdot}92\ \Omega$, no-load (magnetizing) current (line) = 10 A at 0·2 power-factor.

Determine the full-load line current, power-factor, speed and torque, also the maximum torque and the starting torque and current. Neglect friction and windage.

[*Ans.* 40·3 A; 0·888; 979 r.p.m.; 217·4 Nm; 422·1 Nm;
71·4 Nm; 148·6 A]

*This question was originally set using a figure of 30 h.p. i.e. 22·4 kW.

321. A three-phase, star-connected induction motor, with a 1:1 winding ratio, working on a 200-V circuit, takes a current of 3 A per line at a power-factor of 0·32 when running light. When the rotor is locked it takes 16 A at a power-factor of 0·36, the line voltage being reduced to 140 V for this purpose.

Draw the circle diagram and find:

(*a*) the maximum output

(*b*) the maximum power-factor.

[*Ans.* (*a*) 2·55 kW; (*b*) 0·82]

322. The following particulars refer to a three-phase, delta-connected induction motor whose full-load current per phase is 30 A and whose winding ratio is unity.

Phase conditions:

Rotor resistance = 0·453 Ω

Stator resistance = 0·68 Ω

No-load test: 200 V, 5 A, ϕ = 77°

Standstill test: 200 V; 80 A, ϕ = 65°

Draw the circle diagram and determine the output per phase, the total output, the efficiency, the power-factor and the slip.

[*Ans.* 4250 W; 12·75 kW; 80%; 0·885; 8·1%]

323. A 75 000-kVA turbo-generator, with 45% synchronous reactance and negligible resistance, is supplying 50 000 kVA at 0·9 power-factor lagging to a large supply network. If the machine is then made to supply its full-load current without the excitation being changed determine the actual power it supplies and its power-factor.

[*Ans.* 74 000 kW; 0·986]

324. A 75-kW, three-phase, 1000-V, star-connected synchronous motor requires a field current of 25 A when operating at half full-load, with an input power-factor of 0·8 leading and efficiency 80%. If the load is increased to full-load, determine the excitation required to give the same leading reactive kVA as before, if the efficiency is then 85%. The synchronous reactance per phase is 4 Ω and the armature resistance is negligible. The e.m.f. may be taken to be directly proportional to the excitation.

[*Ans.* 25·9 A]

325. The maximum power required by a colliery is 2500 kVA at 0·7 power-factor lagging. A 150-kW synchronous motor is to be installed to drive a fan and also to correct the power-factor to an overall value of 0·95 lagging. If the motor efficiency is 86%, determine its kVA and the power-factor at which it will operate.

[*Ans.* 1170 kVA; 0·148 leading]

ADDITIONAL PROBLEMS

326. A transformer for a low-voltage electric blanket has primary to secondary turns ratio of 12/1 and has total winding resistance of 12 Ω and leakage reactance of 60 Ω, both referred to the primary. The blanket heating element takes 7 A at 20 V and is completely resistive: it is connected to the secondary circuit of the transformer. What must the primary voltage be to get 140 W dissipation in the blanket? Neglect the magnetizing losses of the transformer.

[*Ans.* 249·5 V]

327. The terminals of a transformer are marked as follows:

Primary	200 V, 50 Hz	
Secondaries	250–0–250 V at	100 mA
	5 V at	1 A
	6·3 V at	4 A

The primary has 1000 turns. Find the approximate number of turns on the secondaries and the primary current neglecting losses.

[*Ans.* 2500; 25; 31·5; 401 mA]

328. A single-phase transformer has a primary input voltage of 2000 V, a secondary output voltage of 440 V, and full-load output of 20 kVA. Given that the secondary has 130 turns, determine the number of primary turns, and the primary and secondary currents (neglecting losses, leakage impedance and magnetizing current).

Given that the secondary load has a power-factor of 0·8, and that the magnetizing current is 1·5 A lagging the primary voltage by

84°, determine the total primary current, the total power taken from the supply and the efficiency of the transformer.

[*Ans.* 591; 10 A; 45·5 A; 11·1 A; 16·3 kW; 98%]

329. A transformer, with twice as many turns on its primary as on its secondary winding, is fed from a 440-V supply, its magnetizing current being 8 A lagging 90°. The resistance of the transformer is negligible and its leakage reactance referred to the primary is 2 Ω. The load connected to its secondary terminals has a resistance of 2 Ω and an inductive reactance of 1 Ω.
Determine:

(*a*) the magnitude and power-factor of the secondary current
(*b*) the magnitude and power-factor of the primary current
(*c*) the secondary output voltage.

[*Ans.* (*a*) 80 A; 0·9; (*b*) 45·3 A; 0·707; (*c*) 179 V]

330. A transformer having a ratio of primary to secondary turns of 1/2 has a total resistance and leakage reactance referred to the primary side of 0·01 Ω and 0·02 Ω respectively. It feeds, from a 200-V supply, a load having a resistance of 0·9 Ω, and an inductive reactance of 0·75 Ω. Determine the load current and load voltage.

[*Ans.* 318 A; 373 V]

331. A transformer for a frequency of 50 Hz has 800 primary and 160 secondary turns. The full-load output is 150 kVA at 2200 V. Calculate:

(*a*) the transformation ratio
(*b*) the primary voltage
(*c*) the full-load secondary current
(*d*) the full-load primary current
(*e*) the maximum flux in the core.

Neglect the magnetizing current.

[*Ans.* (*a*) 0·2; (*b*) 11 kV;
(*c*) 68·3 A; (*d*) 13·8 A;
(*e*) 0·062 Wb]

332. An ideal transformer having a secondary/primary turns ratio of 20 to 1 operates from a 250-V supply. If the magnetizing current is 30 A determine the output voltage, current and power, and the input current if the load impedance consists of a resistance of 400 Ω in series with an inductive reactance of 300 Ω.

[*Ans.* 5 kV; ($10\angle -36{\cdot}2°$) A; 40 kW; {$219{\cdot}3\angle -43{\cdot}2°$}A]

333. A transformer having a secondary/primary turns ratio 20/1 has a magnetizing current of 12 A lagging 80° and total resistance and leakage reactance referred to the primary of 0·025 Ω and 0·06 Ω respectively. The primary potential is 400 V and the actual load current is 12 A lagging by 35°. Determine the secondary voltage and the primary current, giving an approximate phasor diagram.

[*Ans.* 7·78 kV; 250 A]

334. A transformer has a primary/secondary turns ratio of 2/1 and operates from a 230-V supply. On no-load the power taken from the supply is 100 W. When the secondary winding is short-circuited and a p.d. of 22 V applied to the primary, the primary current is 15 A, and the power supplied is 150 W. Calculate the efficiency and the secondary voltage, when the transformer delivers 40 A to a resistive load, and the primary voltage is 230 V.

[*Ans.* 92·3 %; 108 V]

335. A three-phase transformer is used to supply a balanced resistive load consisting of sixty 240-V, 5-kW floodlights. The lamps are connected in delta and the transformer is normally star-connected.

Find the current in each transformer winding for normal operation and for the transformer re-connected in delta.

[*Ans.* 722 A; 241 A]

336. A single-phase transformer has 200-V output voltage when supplying a 10-kVA load at 0·8 p.f. lagging. Secondary resistance and leakage reactance are 0·04 Ω and 0·8 Ω respectively. What is the secondary induced e.m.f. and its phase angle to the secondary current?

[*Ans.* 228 V; 45°]

337. A transformer, with primary/secondary turns ratio of 1/2, has a total resistance and leakage reactance referred to the primary of 0·65 Ω and 1·8 Ω respectively and is fed from a 230-V supply. If the magnetizing current is 1 A lagging the primary voltage by 85° and the total primary current is 8 A lagging the primary voltage by 30°, calculate the secondary output voltage, current and power.

[*Ans.* 441·2 V; 3·74 A; 1·54 kW]

338. A current transformer has a bar primary and 200 secondary turns. The secondary has a resistance of 0·2 Ω and leakage reactance of 0·16 Ω. The burden is an ammeter of resistance 0·44 Ω and reactance 0·32 Ω. At a secondary e.m.f. of 2 V the useful and loss components of the magnetizing current are equivalent to 2 and 1·5 ampere turns respectively. Assuming a linear magnetization characteristic find the true transformation ratio and the phase-angle error.

[*Ans.* 201; 0]

339. A single-phase transformer of 50-kW output at 200 V and power-factor 0·8 lagging has an efficiency of 97% and is supplied at a voltage of 2000 V at a frequency of 50 Hz. The primary winding has 200 turns and a resistance of 0·225 Ω. The secondary winding has 20 turns and a resistance of 0·00234 Ω. During a short-circuit test the full current flows in the primary and secondary for a voltage of 45 V across the primary. Calculate the percentage regulation when the secondary current is 322 A at a power-factor of 0·8 lagging.

[*Ans.* 1·87%]

340. A transformer designed for 100 kW at unity power-factor and to transform from 6600 V to 200 V needs 210 V applied to the primary to send 500 A through the short-circuited secondary. Under these conditions the power supplied to the primary is 1 kW and the primary current is 15·5 A. Calculate the voltage on the secondary with 6600 V applied to the primary and a load on the secondary of 70 kW at a power-factor of 0·85 lagging.

[*Ans.* 196]

341. A 12-kVA, 400/230-V, 50-Hz single-phase transformer gave the following test results:

No-load test: 400 V, 1·15 A, 120 W.
S/C test: 25 V, 25 A, 80 W.

Find (i) the leakage reactance and resistance referred to the primary, the magnetizing current and its power-factor.
(ii) the power loss, efficiency and primary regulation drop for full 230-V output to a resistive load.

[*Ans.* 0·99 Ω; 0·128 Ω; 1·15 A; 0·26; 235 W; 98·2%; 4·8 V]

342. A 50-kVA transformer has an iron loss of 1 kW and at full load a copper loss of 2 kW. Plot efficiency against kW output for the range no-load to 25% overload (*a*) at unity p.f. (*b*) at 0·8 p.f. lagging.

Calculate the loading at which the transformer has its maximum efficiency and hence determine this maximum efficiency at (*a*) unity p.f. (*b*) 0·8 p.f. lagging.

[*Ans.* 35·35 kVA; 94·6%; 93·5%]

343. A 1000-kVA, 2000/400-V, single-phase transformer has a maximum efficiency of 97·72% which occurs at 60% of full load when the load p.f. is unity. The winding resistances are:

primary 0·04 Ω
secondary 0·0015 Ω

The leakage reactances are:

primary 0·25 Ω
secondary 0·08 Ω

The magnetizing current at 2000 V is 55 A. If open-circuit and short-circuit tests with suitable instruments are made on this transformer, determine the readings on the instruments.

[*Ans.* (*a*) *Open-circuit test.*
Voltmeter reads 2000 V
Ammeter reads 55·1 A
Wattmeter reads 7 kW

(*b*) *Short-circuit test.*
Voltmeter reads 228·5V
Ammeter reads 500 A
Wattmeter reads 19·38 kW]

344. A 200-kVA single-phase transformer has an efficiency of 98 % at full load. If the maximum efficiency occurs at three-quarters of full load, calculate (*a*) the iron loss, (*b*) the copper loss at full load, (*c*) the efficiency at half load. Ignore magnetizing current and assume a power-factor of 0·8 at all loads.

[*Ans.* (*a*) 1170 W; (*b*) 2100 W; (*c*) 97·9 %]

345. A 400-kVA, single-phase transformer has an efficiency of 97 % at full load, 0·8 power-factor. If the maximum efficiency occurs at three-quarters rated current calculate (*a*) the iron loss, (*b*) the copper loss at full load, (*c*) the efficiency at half-rated current and 0·7 power-factor. Ignore the magnetizing current as a component giving copper loss.

[*Ans.* 3480 W; 6420 W; 93·9 %]

346. Two transformers *A* and *B* both of no-load voltage ratio 1000/500 are connected in parallel and supplied at 1000 V. *A* is rated at 100 kVA, its total reactance and resistance being 5 % and 1 % respectively. *B* is rated at 250 kVA with 2 % reactance and 2 % resistance. Determine the secondary voltage when the transformers supply a total load of 300 kVA at 0·8 p.f. lagging. Determine also the load (kVA) on each transformer.

[*Ans.* 486 V; 56 kVA; 250 kVA]

347. Two 100-kVA single-phase transformers are connected in parallel on both the primary and secondary sides. One has a resistance drop of 1 % and a reactance drop of 5 % while the other has a resistance drop of 2 % and a reactance drop of 3 %. How will they share a load of 160 kW at 0·8 power-factor lagging?

[*Ans.* 54 kW; 106 kW]

348. A three-phase, 400-V, 4-pole, 50-Hz induction motor develops a total torque of 108 Nm, the frequency of the rotor current being 2 Hz. Calculate:

(*a*) the slip

(*b*) the rotor copper loss

(*c*) the stator line current if the power-factor is 0·88 and the total stator losses are 1 kW.

[*Ans.* (*a*) 4 %; (*b*) 680 W; (*c*) 29·6 A]

349. The following test results were obtained on a 3·75 kW, 220-V, three-phase, squirrel-cage motor:

(*a*) running light with rated applied voltage, line current = 5 A, input power = 490 W,

(*b*) with locked rotor, line voltage = 60 V, line current = 15 A, input power = 570 W.

Draw the circle diagram to scale and determine therefrom the efficiency and power-factor for full load and the maximum power developed.

[*Ans.* 83·5%; 0·857; 7 kW]

350. Draw the circle diagram for a six-pole, 15 kW, 400-V, 50-Hz, three-phase induction motor from the following data (line values):

No-load test: 400 V, 9 A, $\cos\phi = 0{\cdot}2$

Short-circuit test: 200 V, 50 A, $\cos\phi = 0{\cdot}4$

From the diagram find (*a*) the line current and power-factor at full load, (*b*) the maximum power developed.

[*Ans.* (*a*) 30·5 A; 0·85; (*b*) 22·2 kW]

351. A 7·5 kW, 400-V, three-phase, four-pole, 50-Hz, slip-ring induction motor takes a no-load current of 6 A at 0·2 power-factor. The motor has delta-connected stator and star-connected rotor windings. With the rotor open-circuited and full voltage applied to the stator, the voltage between slip-rings is 150 V. The stator and rotor leakage reactances are 8 Ω and 0·3 Ω per phase respectively and the resistances are 2 Ω and 0·2 Ω per phase. Determine the full-load speed, torque, current and power-factor, neglecting friction losses.

[*Ans.* 1375 r.p.m.; 51·5 Nm; 16·4 A; 0·825]

352. A 440-V, 7·5 kW, 50-Hz induction motor is working at full load. Two wattmeters correctly connected to the motor to measure its input power show 6450 W and 2550 W. What is the efficiency of the motor at full load and what is its power-factor? What value capacitors connected in star will improve the power-factor to 0·9 lagging? The capacitors can be assumed to be loss-free.

[*Ans.* 83%; 0·8; 40·1 μF]

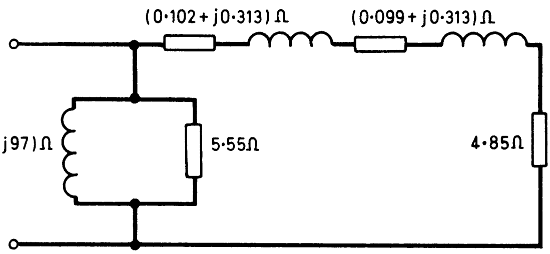
353. A 400-V, six-pole, three-phase, 50-Hz induction motor has a wound rotor of resistance 0·2 Ω/phase and is found on test to have the following speed/current and speed/torque relationships.

Shaft speed (r.p.m.)	950	900	860
Line current (A)	24	45	56
Shaft torque (Nm)	117	198	229

The motor takes rated current of 30 A at 0·8 power-factor; what is the shaft output power and the efficiency of the machine then? What added rotor resistance is needed to make the motor start with only 60% over rated current? Show that the machine would then start with about 40% over rated torque.

[*Ans.* 14·4 kW; 86·7%; 1·62 Ω]

354. An induction motor has the equivalent circuit shown in the diagram. Determine the simplest series equivalent circuit.



[*Ans.* (2·68 + *j*0·246)Ω]

355. The power input to a three-phase induction motor is 60 kW. The stator losses total 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.

[*Ans.* 56 kW; 0·59 kW]

356. A three-phase induction motor with star-connected rotor has an induced e.m.f. of 60 V between slip rings at standstill on open-circuit with normal voltage applied to the stator. The resistance and standstill reactance of each rotor phase are 0·6 Ω and 4 Ω respectively. Calculate the current per phase in the rotor (*a*) when at standstill and connected to a star-connected rheostat of resistance 5 Ω and reactance 2 Ω per phase; (*b*) when running short-circuited with 4% slip.

[*Ans.* (*a*) 4·2 A; (*b*) 2·23 A]

357. A three-phase induction motor with wound rotor has 816 conductors on the stator and 504 conductors on the rotor. The stator is delta-connected and the rotor star-connected. The line voltage is 200 V. What voltage would be measured on the open-circuited rotor at standstill?

If the stator was star-connected and the rotor delta-connected, what would the voltage then be?

[*Ans.* 214 V; 71·3 V]

358. A 2·25 kW, 440-V, four-pole, three-phase induction motor gave the following test results at the stator terminals:

(i) Standstill test (locked rotor) 142 V, 5 A, 205 W.

(ii) No-load test, 440 V, 2·21 A, 122 W.

From this data construct the circle diagram for the machine and hence determine the full-load stator current, power-factor, speed and efficiency.

The ratio of stator to rotor copper loss is 1·2 to 1.

[*Ans.* 4·3 A; 0·74 lagging; 1460 r.p.m.; 92%]

359. A three-phase, 220-V star-connected induction motor gave the following test results:

No load:	Line current = 10 A at 0·2 p.f.
Rotor clamped and short circuited:	Line current = 80 A at 0·5 p.f.

The stator resistance per phase is 0·4 Ω.

Determine:

(*a*) the power-factor at full load of 7·5 kW.

(*b*) the ratio of starting torque to full-load torque.

[*Ans.* (*a*) 0·7; (*b*) 0·58]

360. A three-phase, star-connected motor takes a current of 22 A per phase at a lagging power-factor of 0·8 at full load. The voltage between each line and the neutral is 230 V (frequency 50 Hz). Estimate the capacitance of the capacitors required to be connected between each line and the neutral, to improve the power-factor to unity. Find also the necessary capacitance if the capacitors are connected between the lines.

[*Ans.* 183 μF; 61 μF]

361. A three-phase star-connected synchronous generator is connected to busbars of negligible impedance, having a line voltage of 200. The generator has an equivalent reactance of 2 Ω per phase and is excited to give an e.m.f. of 250 V between lines.

Calculate:

(*a*) the maximum load which the machine can supply,

(*b*) the power-factor at which the machine would generate 20 kW,

(*c*) the e.m.f. necessary for the generation of 20 kW at unity power-factor.

[*Ans.* (*a*) 25 kW; (*b*) 0·97; (*c*) 283 V]

362. An isolated synchronous generator with 40% synchronous reactance and negligible resistance is supplying $\frac{3}{4}$ full-load current at 0·7 power-factor lagging at normal terminal voltage. If the current rises to the full-load value at 0·6 power-factor lagging, determine the percentage change in the terminal voltage if the excitation is unchanged.

[*Ans.* A fall of 12·4%]

363. A 3300-V, three-phase, star-connected synchronous motor, whose synchronous reactance is 12 Ω per phase and whose resistance is negligible, takes 100 kW input power at 0·6 power-factor leading. If the load on the motor is increased, without any change in excitation, until the input power-factor is unity, determine the input power.

[*Ans.* 515 kW]

364. A three-phase, star-connected synchronous motor has an effective resistance of 1 Ω per phase and an effective reactance of 5 Ω per phase. If the applied voltage is 2000 V between lines, estimate the back e.m.f. per phase necessary when the machine is taking 60 kW in order that the power-factor may be unity.

[*Ans.* 1141 V]

365. A three-phase, star-connected synchronous motor is excited to give a back e.m.f. of 240 V per phase between each line and neutral. The resistance per phase is 0·1 Ω and the equivalent reactance per phase is 2 Ω. If the motor is connected to a three-phase supply with 400 V between the lines, determine the power taken if the current per phase is 10 A.

[*Ans.* 6·11 kW]

1. The e.m.f. of each branch $= n \times E$ volts.

The resistance of each branch $= n \times r$ ohms.

$\therefore$ e.m.f. when m branches are connected in parallel is still nE volts but the total resistance of the parallel combination is nr/m.

$$\therefore \quad \text{Total current } I = \frac{nE}{(nr/m) + R} = \underline{\frac{E}{(r/m) + (R/n)}} \text{ amperes.}$$

$$I = \frac{E}{(r/m) + R/n} = \frac{E}{\{\sqrt{(r/m)} - \sqrt{(R/n)}\}^2 + 2\sqrt{(Rr/mn)}}$$

$\therefore$ I is a maximum when $r/m = R/n$ i.e. $\underline{R = nr/m}$

$$\therefore \quad I_{\max} = \frac{E}{(r/m) + (nr/nm)} = Em/2r = Emn/2mR = \underline{En/2R.}$$

2. The parallel combination may be replaced by a single resistor R where:

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5}$$

$$\therefore \quad \frac{1}{R} = \frac{10 + 5 + 2}{10} = \frac{17}{10}$$

$$\therefore \quad R = 10/17\,\Omega$$

$$\therefore \quad \text{Current in the circuit} = \frac{10}{5 + \frac{10}{17}}\text{ A} = \underline{1{\cdot}79\text{ A.}}$$

Voltage across parallel branch $= 1{\cdot}79 \times R = \underline{1{\cdot}05\text{ V.}}$

3. *For cable* 1.

$$\text{The resistance of each wire} = \frac{ls}{A} = \frac{100 \times 1{\cdot}7 \times 10^{-8}}{\pi(0{\cdot}001)^2}\,\Omega$$

$$= \frac{100 \times 1{\cdot}7 \times 10^{-8}}{7\,\pi\,(0{\cdot}001)^2}\,\Omega$$

$$= 0{\cdot}077\,\Omega$$

For cable 2.

$$\text{The resistance of each wire} = \frac{ls}{A} = \frac{200 \times 1{\cdot}7 \times 10^{-8}}{\pi\,(0{\cdot}0005)^2}\ \Omega$$

The resistance for 5 wires in parallel (R_2)

$$= \frac{200 \times 1{\cdot}7 \times 10^{-8}}{5\pi\,(0{\cdot}0005)^2}\ \Omega$$

$$= 0{\cdot}866\ \Omega$$

Let the current through cable 1 be I_1 amperes
Then the current through cable 2 is $(100 - I_1)$ amperes

Now $R_1 I_1 = R_2(100 - I_1)$

$$\therefore \qquad I_1 = \frac{100R_2}{R_1 + R_2} = \frac{86{\cdot}6}{0{\cdot}943} = 91{\cdot}8$$

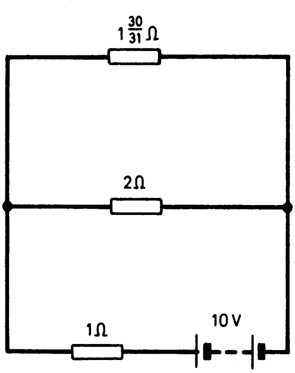
$$\therefore \qquad I_2 = 100 - I_1 = 8{\cdot}2\text{A}.$$

4. The parallel branch of three resistors can be replaced by a single resistor R where:

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{5} + \frac{1}{3} = \frac{15 + 6 + 10}{30} = \frac{31}{30}$$

$$\therefore \qquad R = \frac{30}{31}\ \Omega.$$

The circuit becomes, therefore:



The 2-Ω and $1\frac{30}{31}$-Ω resistors can be replaced by a single one of value

$$\frac{2 \times 1\frac{30}{31}}{2 + 1\frac{30}{31}} = \frac{2 \times 61}{123} = \frac{122}{123}\ \Omega$$

$\therefore$ Current supplied by battery

$$= \frac{10}{1 + \dfrac{122}{123}}\ \text{A}$$

$$= \underline{5{\cdot}02\ \text{A}}.$$

5. Let R_a be the resistance of the ammeter and R_s that of the shunt. Let V be the voltage to give full-scale deflection of ammeter.

$\therefore$ current taken by ammeter $= V/R_a$

and current taken by shunt $= V/R_s$

Total current $I = V(1/R_a + 1/R_s)$

Hence $1/R_s = I/V - 1/R_a$.

In the first example given:

$$1/R_s = 15/0{\cdot}015 - 1/0{\cdot}7$$

$$\underline{R_s = 7/6990\,\Omega}$$

In the second example given:

$$1/R_s = 300/0{\cdot}015 - 1/0{\cdot}7$$

$$\underline{R_s = 7/139\,990\,\Omega.}$$

6. Voltage across instrument for full-scale deflection

$$= 50 \times 0{\cdot}0025 = 0{\cdot}125\text{ V}$$

Current taken by shunt $= 75 - 0{\cdot}0025 = 74{\cdot}9975$ A

$\therefore$ Resistance of shunt $= 0{\cdot}125/74{\cdot}9975\,\Omega$

$$= \underline{1{\cdot}25/749{\cdot}975\,\Omega.}$$

7. Current in voltmeter circuit $I = 250/(R + 14{\cdot}6)$ where R is the series resistance.

$\therefore$ $R + 14{\cdot}6 = 250/I = 250/0{\cdot}015$

$\therefore$ $\underline{R = 16\,652\,\Omega.}$

8. The current through the voltmeter to produce full-scale deflection $= 0{\cdot}300/20 = 0{\cdot}015$ A.

Hence, if R is the resistance which requires to be put in series with the instrument to give full-scale deflection for V volts:

$$0{\cdot}015 = V/(R + 20)$$

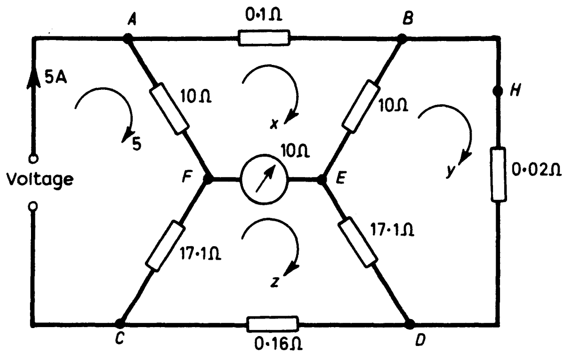
i.e. $$R = \frac{V}{0{\cdot}015} - 20$$

Thus if:

(*a*) $V = 5$, $R = 313{\cdot}3\ \Omega$
(*b*) $V = 20$, $R = 1313{\cdot}3\ \Omega$
(*c*) $V = 100$, $R = 6646{\cdot}6\ \Omega$
(*d*) $V = 200$, $R = 13\,313{\cdot}3\ \Omega$
(*e*) $V = 400$, $R = 26\,646{\cdot}6\ \Omega$
(*f*) $V = 800$, $R = 53\,313{\cdot}3\ \Omega$

If the terminals are arranged as in the given diagram, then the resistances between consecutive terminals are: 313·3, 1000·3, 5333·3, 6666·6, 13 333·3 and 26 666·6 Ω, respectively.

9.



Let the currents circulate as shown:

For loop ABEFA:

$$0{\cdot}1x + 10(x - y) + 10(x - z) + 10(x - 5) = 0 \qquad (1)$$

For loop FEDCF:

$$10(z - x) + 17{\cdot}1(z - y) + 0{\cdot}16z + 17{\cdot}1(z - 5) = 0 \qquad (2)$$

For loop BHDEB:

$$0{\cdot}02y + 17{\cdot}1(y - z) + 10(y - x) = 0 \qquad (3)$$

From (3),

$$y = \frac{10x + 17{\cdot}1z}{27{\cdot}12} \qquad (4)$$

Substituting (4) in (1) gives:

$$71{\cdot}63x - 44{\cdot}22z = 135{\cdot}6 \qquad (5)$$

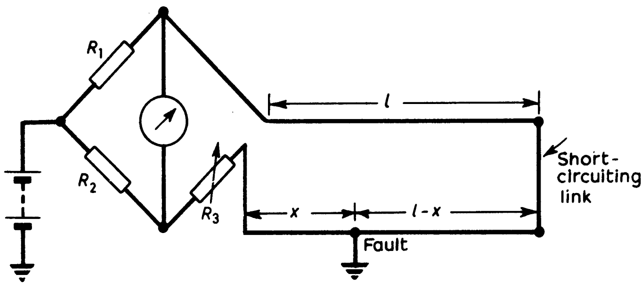
Substituting (4) in (2) gives:

$$910{\cdot}6z - 442{\cdot}2x = 2318{\cdot}8 \qquad (6)$$

From (5) and (6) $x = 4{\cdot}949$ A, $z = 4{\cdot}95$ A.

Therefore, the current through the galvanometer $= z - x = \underline{0{\cdot}001\ \text{A}}$

10.



The circuit for the Varley-Loop Test is as shown. At balance:

$$\frac{R_1}{R_2} = \frac{R[l + (l - x)]}{R_3 + Rx}$$

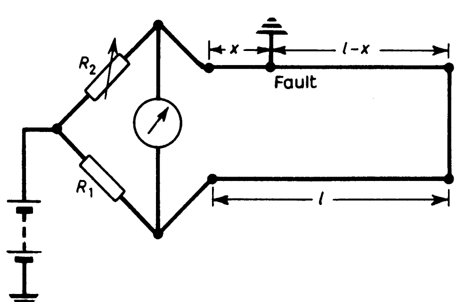
where R is the resistance per km of line and l and x are measured in kilometres.

In this case $R_1 = R_2 = 300\ \Omega$; $R = 43\ \Omega$, $l = 60$ km, $R_3 = 1760\ \Omega$

$$1760 + 43x = 43(120 - x)$$

$$\therefore \qquad \underline{x = 39{\cdot}5 \text{ km}}$$

11.



The circuit for the Murray-Loop Test is as shown.
At balance:

$$\frac{R_2}{R_1} = \frac{x}{2l - x}$$

If $R_1 = 500\ \Omega$, $R_2 = 100\ \Omega$ and $l = 30$ km

$$x = \frac{2R_2 l}{R_1 + R_2} = \underline{10 \text{ km}}$$

12. Let the resistance per km of line be R and let x and l be measured in km.

At balance:

$$\frac{R_2}{R_1} = \frac{R_3 + xR}{R(l - x)}$$

$\therefore$ If $R_2 = R_1$ as in this case: $R(l - x) = R_3 + Rx$

$$\therefore \qquad x = Rl - R_3/2R$$

In this case $R = 40\ \Omega$, $l = 30$ km, $R_3 = 440\ \Omega$

$$\therefore \qquad x = \frac{1200 - 440}{80} = \frac{76}{8} = \underline{9{\cdot}5 \text{ km.}}$$

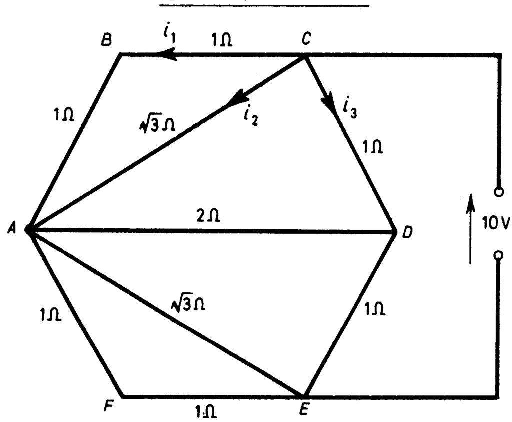
13. From considerations of symmetry:

$$i_{CB} = i_{FE} = i_1 \text{ say.}$$
$$i_{CA} = i_{AE} = i_2 \text{ say.}$$
$$i_{CD} = i_{DE} = i_3 \text{ say.}$$

whilst $i_{AD} = 0$.

Thus, round circuit $CBAFE$ and the battery

$$\underline{i_1 = 10/4 = 2{\cdot}5 \text{ A}}$$



and similarly:

$$\underline{i_2 = 10/2\sqrt{3} = 2{\cdot}885 \text{ A}}$$

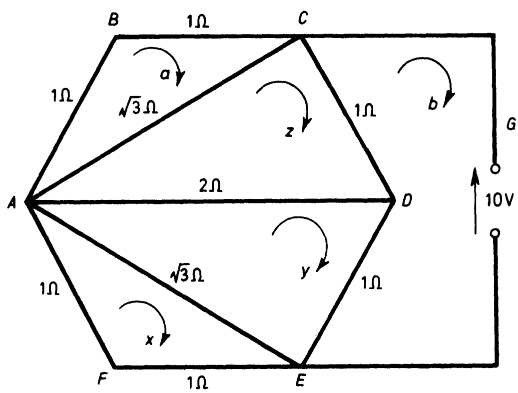
and

$$\underline{i_3 = 10/2 = 5 \text{ A}.}$$

The total current from the battery $= i_1 + i_2 + i_3 = 10{\cdot}38$ A.

∴ the effective resistance between points C and $E = 10/10{\cdot}38 = \underline{0{\cdot}98\Omega.}$

Alternative solution using Maxwell's Cyclic-Current Rule.



Let the currents circulate as shown.

From A to B to C to A:

$$\sqrt{3}z = a(2 + \sqrt{3}) \tag{1}$$

From A to C to D to A and using (1):

$$b + 2y = a\left[-\sqrt{3} + \frac{(3 + \sqrt{3})(2 + \sqrt{3})}{\sqrt{3}}\right] \tag{2}$$

From A to D to E to A and using (1):

$$y(\sqrt{3} + 3) - \frac{2a}{\sqrt{3}}(2 + \sqrt{3}) - b = \sqrt{3}x \tag{3}$$

From A to E to F:

$$y\sqrt{3} = x(2 + \sqrt{3}) \tag{4}$$

From C to G to E to D to C:

$$10 + 2b = y + z \tag{5}$$

From these equations:

$a = -2{\cdot}5$ A, $x = -2{\cdot}5$ A

$b = -10{\cdot}38$ A, $z = -5{\cdot}38$ A, $y = -5{\cdot}38$ A

giving the currents in the wires as before.

14. Let the currents circulate as shown.

From A to B to C to D to A:

$$20x + 25y + 35(y - z) + 30(x - z) = 0$$

i.e. $10x + 12y - 13z = 0$ (1)

From A to B to D to A:

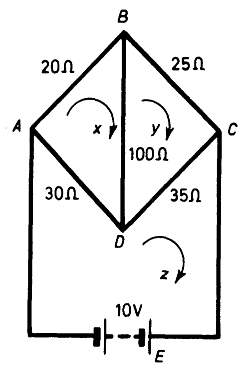
$$20x + 100(x - y) + 30(x - z) = 0$$

i.e. $15x - 10y - 3z = 0$ (2)

From A to B to C to E to A:

$$20x + 25y = 10$$

i.e. $4x + 5y = 2$ (3)



From (1), (2) and (3)

$$x = 0{\cdot}223 \text{ A}$$

$$y = 0{\cdot}222 \text{ A}$$

$$z = 0{\cdot}375 \text{ A}$$

Thus, the currents in the branches are:

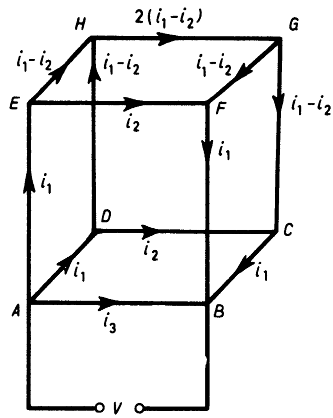
$AB = 0{\cdot}223$ A, $BC = 0{\cdot}222$ A, $BD = 0{\cdot}001$ A,

$CD = -0{\cdot}153$ A and $AD = 0{\cdot}152$ A.

If the effective resistance of the network between A and C is R, then $R = 10/0{\cdot}375\ \Omega$

i.e. $R = 26{\cdot}67\ \Omega$.

15. From considerations of symmetry all the currents can be expressed in terms of three quantities i_1, i_2 and i_3, as shown.



For branch $ABCD$: $\quad i_3 = 2i_1 + i_2$ (1)

For branch $EFGH$: $\quad i_2 = 4(i_1 - i_2)$ (2)

$\therefore \quad i_2 = (4/5)\, i_1$ and $i_3 = (14/5)\, i_1$

For the battery circuit:

$$i_3 = V \tag{3}$$

and if the effective resistance of the cube between A and B is R, then

$$R(2i_1 + i_3) = V \tag{4}$$

$$\therefore \quad R(V + 2i_1) = V = R\left(V + \frac{10i_3}{14}\right) = RV\left(1 + \frac{10}{14}\right)$$

$$\therefore \quad \underline{R = 7/12\,\Omega.}$$

16. From considerations of symmetry the currents in the various wires are as shown in terms of x, y and z given.

Applying Kirchhoff's Second Law:

For loop PAG:

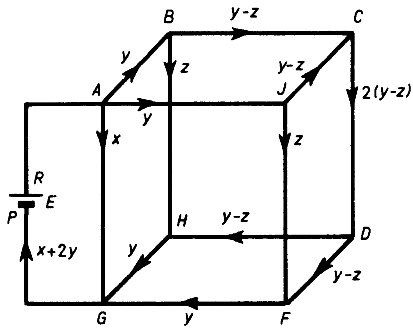
$$R(x + 2y) + rx = \mathrm{E} \tag{1}$$

For loop BCDH:

$$r(y - z) + r\,.\,2(y - z) + (y - z)r = rz.$$

$$\therefore \quad z = 4(y - z)$$

$$\text{i.e.} \quad y = 5z/4 \tag{2}$$



For loop ABHG:

$$ry + rz + ry - rx = 0$$

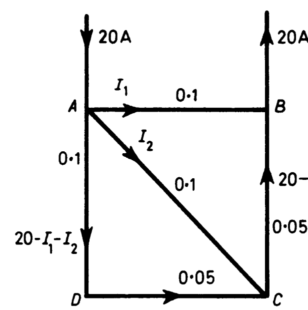
$$\therefore \qquad x = 2y + z \qquad (3)$$

From (1), (2) and (3)

$$\underline{\frac{y}{5} = \frac{x}{14} = \frac{z}{4} = \frac{E}{24R + 14r}.}$$

17. (*a*) *Consider first the* 20 A *load acting alone.*

Let the currents circulate as shown. It is required to find the current I_2 from A to C.



For loop ADC:

$$(20 - I_1 - I_2)0{\cdot}15 = 0{\cdot}1I_2 \qquad (1)$$

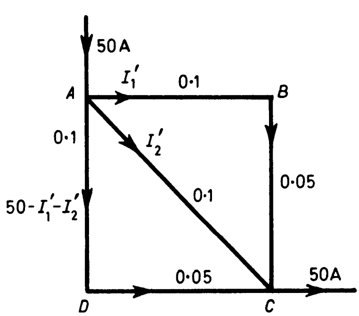
For loop ABC:

$$0{\cdot}1I_1 - (20 - I_1)0{\cdot}05 = 0{\cdot}1I_2 \qquad (2)$$

From (1) and (2)

$$I_2 = 40/7 \qquad (3)$$

Consider now the 50 A *load acting alone.*



Let the currents circulate as shown. It is required to find I_2'.

For loop ABC:

$$0{\cdot}15I_1' = 0{\cdot}1I_2' \qquad (4)$$

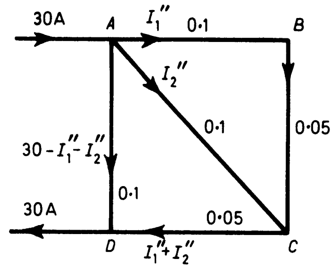
For loop ADC:

$$0{\cdot}15(50 - I_1' - I_2') = 0{\cdot}1I_2' \qquad (5)$$

From (4) and (5) $\qquad I_2' = 150/7 \qquad (6)$

Consider now the 30 A *load acting alone.*

Let the currents circulate as shown. It is required to find I_2''.



For loop ABC:

$$0{\cdot}15I_1'' = 0{\cdot}1I_2'' \qquad (7)$$

For loop ADC:

$$0{\cdot}1(30 - I_1'' - I_2'') = 0{\cdot}1I_2'' + 0{\cdot}05(I_1'' + I_2'') \qquad (8)$$

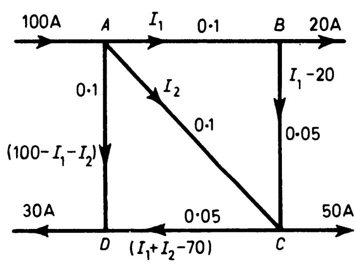
From (7) and (8), $\qquad I_2'' = 60/7 \qquad (9)$

From (3), (6) and (9), total current in AC due to all the loads acting simultaneously is $I_2 + I_2' + I_2''$.

$$= \frac{40}{7} + \frac{150}{7} + \frac{60}{7}$$

$$= \underline{35{\cdot}7 \text{ A.}}$$

(*b*) Let the currents in AB and AC be I_1 and I_2 respectively. Then, using Kirchhoff's first law, the currents in the remaining conductors can be written down in terms of I_1 and I_2 as shown.



For loop ABC:

$$0{\cdot}1I_1 + 0{\cdot}05(I_1 - 20) - 0{\cdot}1I_2 = 0 \qquad (1)$$

i.e. $\qquad 3I_1 = 20 + 2I_2 \qquad (2)$

For loop ADC:

$$0{\cdot}1(100 - I_1 - I_2) - 0{\cdot}05(I_1 + I_2 - 70) - 0{\cdot}1I_2 = 0 \qquad (3)$$

i.e. $\qquad 270 - 3I_1 - 5I_2 = 0 \qquad (4)$

From (2) and (4)

$$270 = 20 + 2I_2 + 5I_2$$

i.e. $\qquad I_2 = 250/7$

$$= \underline{35{\cdot}7 \text{ A.}}$$

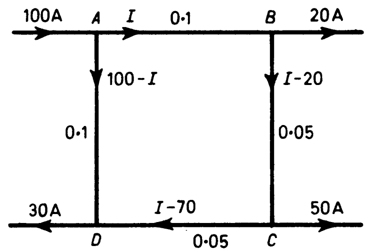
(*c*) To use Thévenin's Theorem, remove AC as shown. Then the voltage E_{AC} between A and C, and the resistance R_{AC} between A and C are required with AC removed.

The current in AC is $\dfrac{E_{AC}}{R_{AC} + 0{\cdot}1}$

since $0{\cdot}1\ \Omega$ is the resistance of the link removed.

$$R_{AC} = \frac{0{\cdot}15}{2}$$

Let the current in $AB = I$. Then the currents in the remaining conductors are as shown.



For loop ABCDA:

$$0{\cdot}1I + 0{\cdot}05(I - 20) + (I - 70)0{\cdot}05 = (100 - I)0{\cdot}1.$$

$$\therefore \qquad I = \frac{145}{3}.$$

$$\therefore \qquad E_{AC} = 0{\cdot}1I + 0{\cdot}05(I - 20)$$

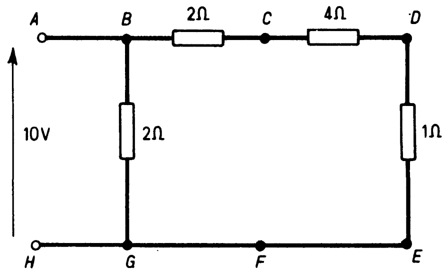
$$= \left(0{\cdot}1 \times \frac{145}{3}\right) + 0{\cdot}05\left(\frac{145}{3} - 20\right)$$

$$= 6{\cdot}25\text{ V}.$$

$$\therefore \text{ Current in } AC = \frac{6{\cdot}25}{\left(\frac{0{\cdot}15}{2}\right) + 0{\cdot}1} = \frac{250}{7} = \underline{35{\cdot}7\text{ A.}}$$

18. To use Thévenin's Theorem CF is removed. Then the resistance between C and F, R_{CF} is calculated with the battery short-circuited and the voltage E_{CF} between C and F is found.

The current in CF is $\dfrac{E_{CF}}{R_{CF} + 2}$ since the resistance of the link removed is $2\ \Omega$.



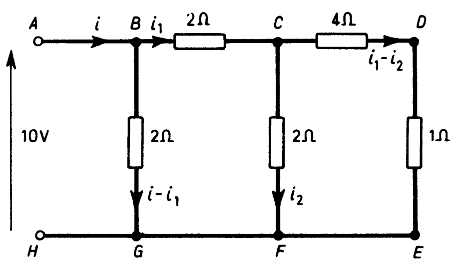
With the battery short-circuited $R_{CF} = \frac{2 \times 5}{2 + 5} = \frac{10}{7}\,\Omega$

The current in the circuit $BCDEG = \frac{10}{7}$ A

$\therefore \quad E_{CF} = 10 - 2\left(\frac{10}{7}\right) = \frac{50}{7}$ V.

$\therefore \quad$ Current in $CF = \frac{50/7}{(10/7) + 2} = \frac{50}{24} = \underline{2{\cdot}08\text{ A.}}$

Check by Kirchhoff's Laws.



Let the currents be as shown.

For loop ABGH:

$$2(i - i_1) = 10 \qquad (1)$$

For loop ACFH:

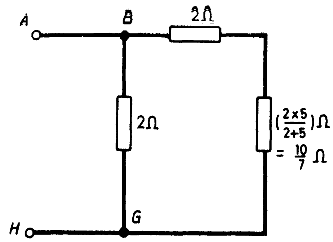
$$2i_1 + 2i_2 = 10 \qquad (2)$$

For loop CDEF:

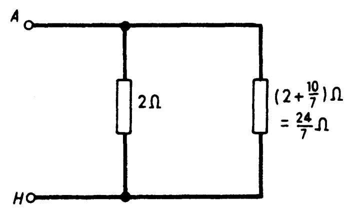
$$5(i_1 - i_2) - 2i_2 = 0 \qquad (3)$$

From (2) and (3) $\qquad i_2 = \frac{25}{12} = \underline{2{\cdot}08\text{ A.}}$

The circuit can be reduced to the following:



This can further be simplified to:



The equivalent resistance between A and H is, therefore,

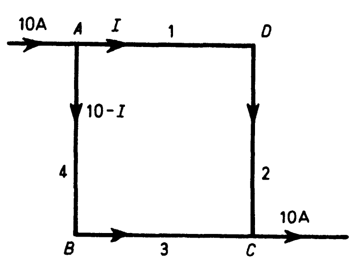
$$\frac{2 \times (24/7)}{2 + (24/7)} = \frac{24}{19} = \underline{1{\cdot}26\ \Omega.}$$

19. To use Thévenin's Theorem BD is removed. Then the resistance between B and D, R_{BD}, is found, and the voltage between B and D, E_{BD}.

The current in BD is $\dfrac{E_{BD}}{R_{BD} + 1}$ since the resistance of the link removed is 1 Ω.

$$R_{BD} = \frac{5 \times 5}{5 + 5} = 2{\cdot}5.$$

Let the current in AD be I.



Then for $ABCD$:

$$I + 2I = 7(10 - I)$$

i.e. $I = 7$ A

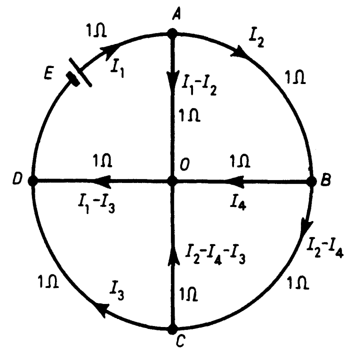
$$\therefore\quad E_{BD} = 3(10 - 7) - 2(7) = -5\ \text{V}$$

$$\therefore\quad I_{BD} = \frac{-5}{2{\cdot}5 + 1} = \underline{-1{\cdot}43\ \text{A}}$$

i.e. the current flows from D to B.

20. *Solution* 1. *Using Kirchhoff's Laws.*

Let the currents be as shown.



For loop AOB:

$$I_2 + I_4 = I_1 - I_2 \qquad (1)$$

For loop BOC:

$$I_4 - I_2 + I_4 + I_3 - I_2 + I_4 = 0 \qquad (2)$$

For loop DOC:

$$I_1 - I_3 - I_3 + I_2 - I_4 - I_3 = 0 \qquad (3)$$

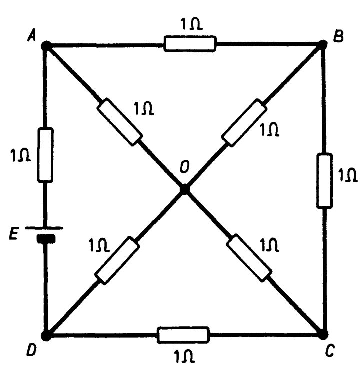
For loop AOD:

$$I_1 + I_1 - I_2 + I_1 - I_3 = E \qquad (4)$$

From these equations $E/I_1 = \underline{15/7\ \Omega}$ which is the resistance of the network offered to the battery.

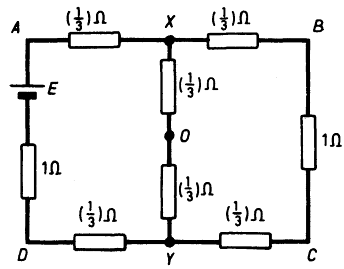
Solution 2. *Using network simplification.*

The original diagram can be redrawn as follows:

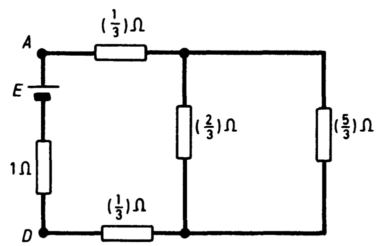


The problem is to find the total resistance across AD.

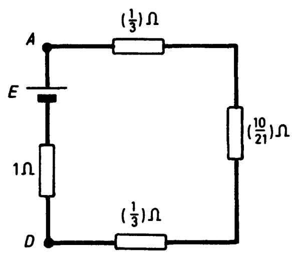
Replace mesh DOC by a star and likewise mesh AOB. Call the star points X and Y. Then the following circuit results:



This simplifies to:



and then:



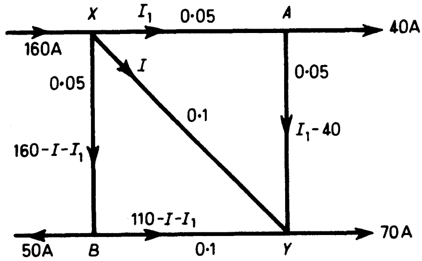
∴ the total resistance between A and D

$$= 1 + \frac{1}{3} + \frac{1}{3} + \frac{10}{21}$$

$$= \underline{15/7\ \Omega.}$$

21. (*a*) *Kirchhoff's Laws.*

Let the currents be as shown.



For loop XAY:

$$0{\cdot}05I_1 + 0{\cdot}05(I_1 - 40) = 0{\cdot}1I \qquad (1)$$

For loop XYB:

$$0{\cdot}1I - 0{\cdot}1(110 - I - I_1) - 0{\cdot}05(160 - I - I_1) = 0 \qquad (2)$$

From (1) and (2) $\underline{I = 40 \text{ A.}}$

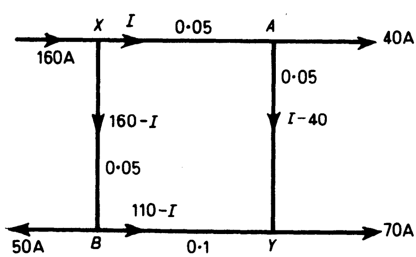
(*b*) *Thévenin's Theorem*

To use the theorem, XY is removed. Then the resistance R_{XY} between X and Y is found and the voltage E_{XY} between X and Y is calculated.

The current in XY is $\dfrac{E_{XY}}{R_{XY} + 0{\cdot}1}$ since the resistance of the link removed is $0{\cdot}1\ \Omega$.

$$R_{XY} = \frac{0{\cdot}1 \times 0{\cdot}15}{0{\cdot}1 + 0{\cdot}15} = \frac{0{\cdot}015}{0{\cdot}25} = 3/50\ \Omega.$$

Let the current in XA be I. Then the currents in the other conductors are as shown.



For loop XAYBX:

$$0{\cdot}05I + 0{\cdot}05(I - 40) - 0{\cdot}1(110 - I) - 0{\cdot}05(160 - I) = 0.$$

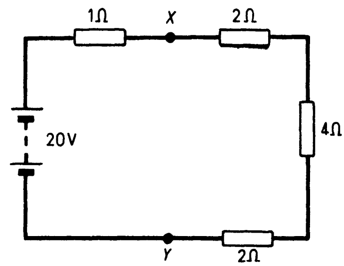
$$\therefore \qquad I = 84 \text{ A.}$$

$$\therefore \qquad E_{XY} = (0{\cdot}05 \times 84) + 0{\cdot}05\,(44) = 6{\cdot}4 \text{ V.}$$

$$\therefore \qquad \text{Current in } XY = \frac{6{\cdot}4}{(3/50) + 0{\cdot}1}$$

$$= \underline{40 \text{ A.}}$$

22. (*a*) *Thévenin's Theorem.*



XY is removed; then the resistance R_{XY} between X and Y is found with the battery short-circuited. The voltage E_{XY} between X and Y is also found with XY removed. The current in XY is E_{XY} / $(R_{XY} + 3)$ since the resistance of the link removed is 3 Ω.

$$\therefore \qquad R_{XY} = \frac{1 \times 8}{1 + 8} = \frac{8}{9}\,\Omega.$$

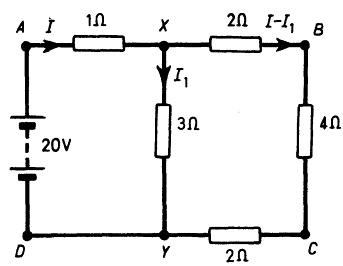
Current in circuit = 20/9 A.

$$\therefore \qquad E_{XY} = 8 \times (20/9) = 160/9$$

$$\therefore \qquad \text{Current in } XY = \frac{160/9}{(8/9) + 3} = \underline{4{\cdot}57\text{ A.}}$$

(*b*) *Kirchhoff's Laws.*

Let the currents be as shown.



For loop ABCD:

$$I + 8(I - I_1) = 20 \qquad (1)$$

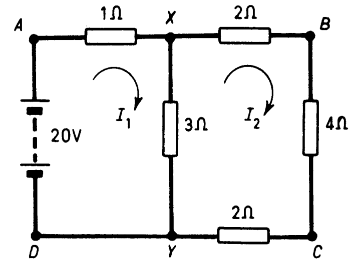
For loop AXYD:

$$I + 3I_1 = 20 \qquad (2)$$

From(1) and (2) $\qquad \underline{I_1 = 4{\cdot}57\text{ A.}}$

(*c*) *Maxwell's Cyclic-Current Rule.*

Let the currents circulate as shown.



For loop AXYD:

$$I_1 + 3(I_1 - I_2) = 20 \qquad (1)$$

For loop XBCY:

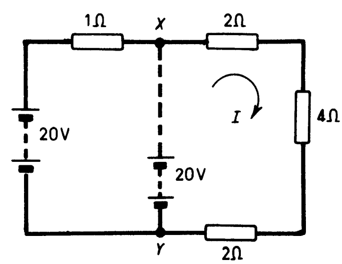
$$8I_2 + 3(I_2 - I_1) = 0 \qquad (2)$$

From (1) and (2) $I_2 = 12/7$ A and $I_1 = 44/7$ A.

The current in $XY = I_1 - I_2 = 32/7$ A
$= \underline{4{\cdot}57 \text{ A.}}$

23. To use Thévenin's Theorem XY is removed. Then the resistance R_{XY} between X and Y is found with the batteries short-circuited. The voltage between X and Y, E_{XY}, is also calculated.

The current in XY is $E_{XY}/(3 + R_{XY})$ since the resistance of the link removed is 3 Ω.



$$R_{XY} = \frac{8 \times 1}{8 + 1} = \frac{8}{9}$$

Current $I = 20/9$ A

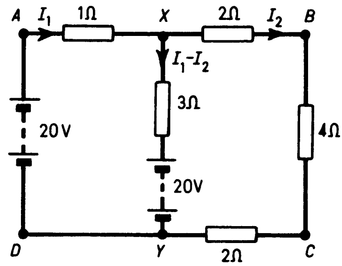
$\therefore \quad E_{XY} = 8I - 20 = 160/9 - 20 = -20/9$ V

$\therefore \quad$ Current in $XY = \dfrac{-20/9}{(8/9) + 3} = -4/7 \text{ A} = \underline{-0{\cdot}57 \text{ A.}}$

<u>i.e. current flows from Y to X.</u>

Check by Kirchhoff's Laws.

Let the currents be as shown.



For loop AXYD:

$$I_1 + 3(I_1 - I_2) + 20 - 20 = 0$$

i.e. $$4I_1 = 3I_2 \quad (1)$$

For loop ABCD:

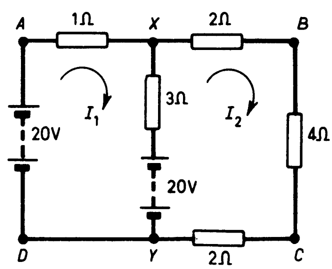
$$I_1 + 8I_2 - 20 = 0 \quad (2)$$

From (1) and (2) $I_1 = 12/7$ A and $I_2 = 16/7$ A

$\underline{I_1 - I_2 = 4/7 \text{ A as before.}}$

Check by Maxwell's Cyclic-Current Rule.

Let the currents circulate as shown.



For loop AXYD:

$$I_1 + 3(I_1 - I_2) + 20 - 20 = 0 \quad (1)$$

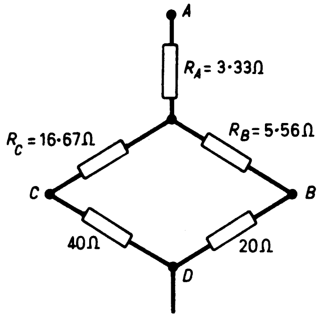
For loop ABCD:

$$I_1 + 8I_2 - 20 = 0 \quad (2)$$

From (1) and (2) $I_1 = 12/7$ A and $I_2 = 16/7$ A

$\therefore$ $\underline{I_1 - I_2 = -4/7 \text{ A as before.}}$

24. Using a delta-star transformation on mesh *ABC* of the given circuit the simplified circuit(*a*) results.



(*a*)

$$R_A = \frac{30 \times 10}{30 + 10 + 50} = 3{\cdot}33\ \Omega$$

$$R_B = \frac{50 \times 10}{30 + 10 + 50} = 5{\cdot}56\ \Omega$$

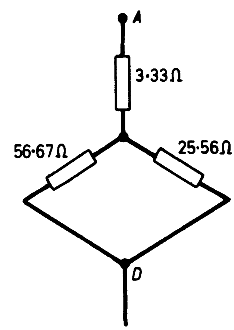
$$R_C = \frac{50 \times 30}{50 + 30 + 10} = 16{\cdot}67\ \Omega$$

Circuit (*a*) therefore reduces to that of (*b*).

The equivalent resistance between *A* and *D* is:

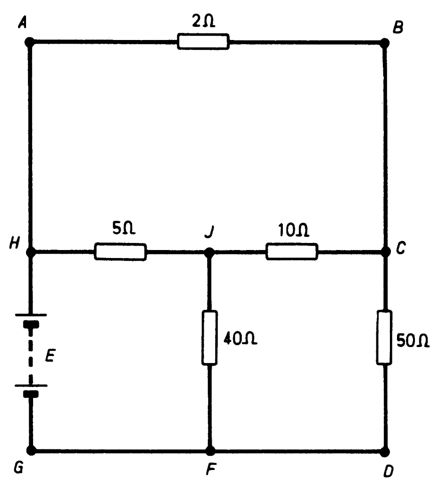
$$3{\cdot}33 + \left(\frac{56{\cdot}67 \times 25{\cdot}56}{56{\cdot}67 + 25{\cdot}56}\right)\Omega$$

$$= \underline{20{\cdot}94\ \Omega.}$$



(*b*)

25. A star-mesh transformation is first carried out on mesh *JCDF* giving the circuit shown at (*b*).



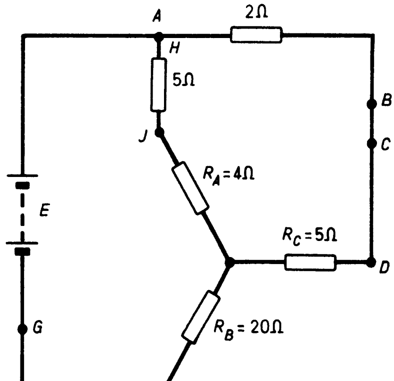
(*a*)

$$R_A = \frac{40 \times 10}{40 + 10 + 50} = 4\,\Omega$$

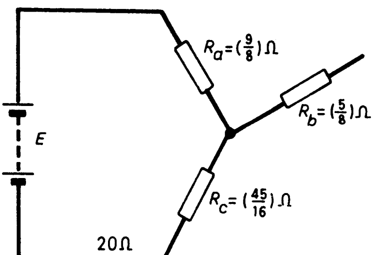
$$R_B = \frac{50 \times 40}{100} = 20\,\Omega$$

$$R_C = \frac{50 \times 10}{100} = 5\,\Omega$$

Carrying out a further transformation on mesh *ADJ*, the circuit at (*c*) results.



(b)



(c)

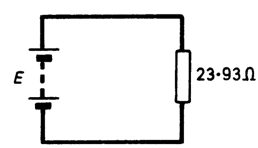
$$R_a = \frac{2 \times 9}{2 + 9 + 5} = \frac{9}{8}\,\Omega$$

$$R_b = \frac{2 \times 5}{16} = \frac{5}{8}\,\Omega$$

$$R_c = \frac{9 \times 5}{16} = \frac{45}{16}\,\Omega$$

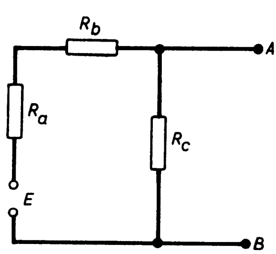
$$R_a + R_c + 20 = (9/8) + (45/16) + 20 = 23{\cdot}93\,\Omega$$

∴ the circuit is equivalent to (*d*).

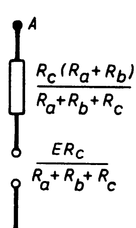


(d)

26. Thévenin's Theorem shows that the two following arrangements are equivalent:

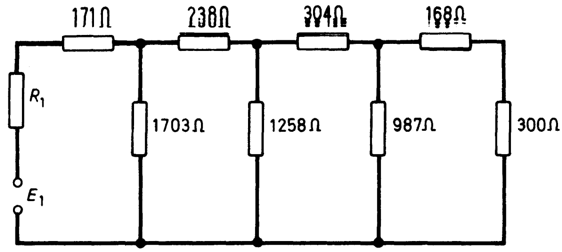


(a)



(b)

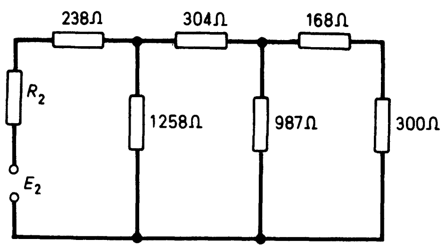
Using this simplication, the original circuit can be reduced to that of (*c*).



(*c*)

$$R_1 = 2583 \times 219/2802 = 202\ \Omega$$
$$E_1 = 100 \times 2583/2802 = 92{\cdot}1\ \text{V}.$$

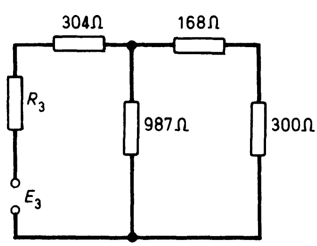
Similarly (*c*) can be reduced to (*d*).



(*d*)

$$R_2 = 1703 \times 373/2076 = 306\ \Omega$$
$$E_2 = 92{\cdot}1 \times 1703/2076 = 75{\cdot}5\ \text{V}.$$

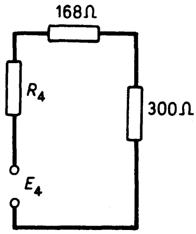
Similarly (*d*) can be reduced to (*e*).



(*e*)

$$R_3 = 1258 \times 544/1802 = 380\ \Omega$$
$$E_3 = 75{\cdot}5 \times 1258/1802 = 52{\cdot}7\ \text{V}.$$

Similarly (*e*) can be reduced to (*f*).



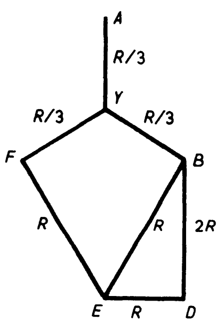
(*f*)

$$R_4 = 987 \times 684/1671 = 403\ \Omega$$
$$E_4 = 52{\cdot}7 \times 987/1671 = 31{\cdot}1\ \text{V}.$$

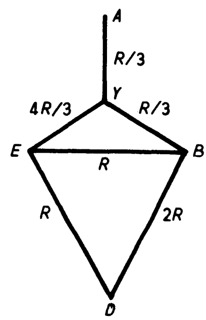
The current through the 300-Ω resistor is, therefore,

$$\frac{31{\cdot}1}{(403 + 168 + 300)}\ \text{A}$$
$$= \underline{35{\cdot}7\ \text{mA}.}$$

27. First use a delta-star transformation on *ABF*. This gives circuit (*a*). This simplifies to the circuit (*b*).

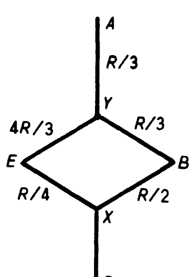


(*a*)

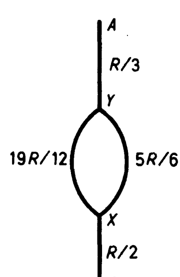


(*b*)

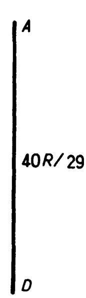
Now use a delta-star transformation on *EBD*, and circuit (*c*) results. (*c*) simplifies to (*d*) and (*d*) to (*e*).



(*c*)



(*d*)



(*e*)

i.e. the equivalent resistance of the original network between points A and D is $\underline{40R/29}$.

Check by Maxwell's Cyclic-Current Rule.

Let the battery voltage be e and the currents circulate as shown.

For loop ABF:

$$Ry + R(y - z) + R(y - x) = 0.$$

i.e. $$x + z = 3y \qquad (1)$$

For loop GAFED:

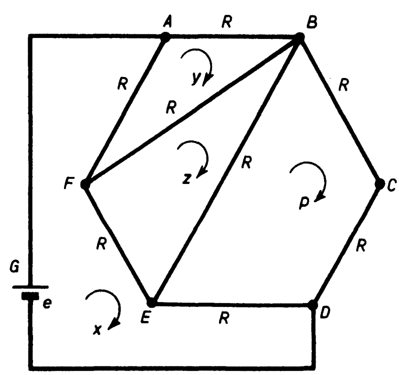
$$e = R(x - y) + R(x - z) + R(x - p)$$

i.e. $$e = 3Rx - Ry - Rz - Rp \qquad (2)$$

For loop BCDE:

$$2Rp + R(p - x) + R(p - z) = 0.$$

i.e. $$4p - x - z = 0. \qquad (3)$$



For loop BEF:

$$R(z - p) + R(z - x) + R(z - y) = 0.$$

i.e. $$3z - p - x - y = 0. \qquad (4)$$

From (1) and (3) $$3y = 4p \qquad (5)$$

Substituting (5) in (4) $$12z - 7y = 4x \qquad (6)$$

From (1) and (6) $$x = 29y/16 \qquad (7)$$

From (1) and (7) $$z = 19y/16 \qquad (8)$$

From (2), therefore,

$$e = R(87y/16 - y - 3y/4 - 19y/16)$$

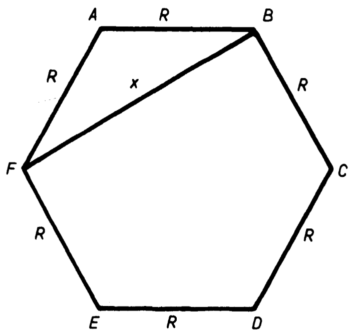
i.e. $$e = 40Ry/16 \qquad (9)$$

From (7) and (9)

$$\frac{e}{x} = \frac{40R}{29}$$

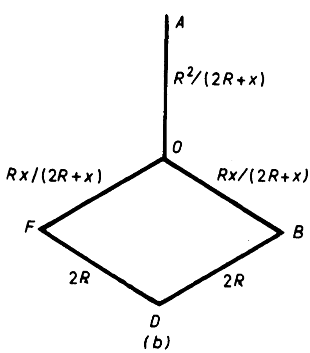
$\therefore$ equivalent resistance between A and D is $40R/29$ as before.

28. Let resistance of BF be x for the general case. Replacing mesh ABF by an equivalent star-circuit (b) results with O as the star point.

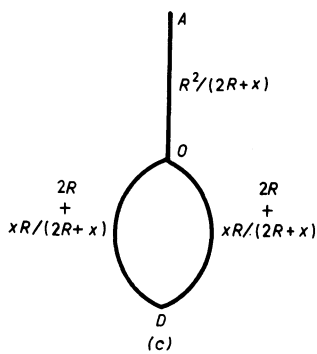


(a)

This leads to circuit (c).



(b)



(c)

$\therefore$ equivalent resistance of network between A and D is

$$\frac{R^2}{2R+x} + R + \frac{xR}{2(2R+x)}$$

$$= \frac{2R^2 + 2R(2R+x) + Rx}{2(2R+x)} = \frac{6R^2 + 3Rx}{2(2R+x)}$$

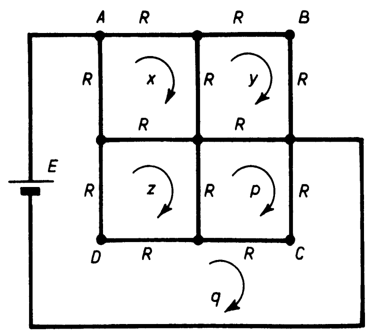
$= 3R/2$ and is independent of x.

29. Let the resistance of a side of the square be $2R$. Let the currents in the meshes be x, y, z, p and q, as shown. Using Maxwell's Cyclic-Current Rule, the following equations are obtained.

For the x mesh:

$$Rx + R(x - y) + R(x - z) + R(x - q) = 0$$

$$\therefore \quad 4x - y - z - q = 0 \tag{1}$$



For the y mesh:

$$Ry + Ry + R(y - p) + R(y - x) = 0$$

$$\therefore \quad 4y - p - x = 0 \tag{2}$$

For the p mesh:

$$2R(p - q) + R(p - z) + R(p - y) = 0$$

$$\therefore \quad 4p - 2q - z - y = 0 \tag{3}$$

For the z mesh:

$$R(z - x) + R(z - p) + 2R(z - q) = 0$$

$$\therefore \quad 4z - x - p - 2q = 0 \tag{4}$$

For the q mesh:

$$Rx + 2Ry = E \tag{5}$$

From (1) $\quad z = 4x - y - q \quad$ (6)

From (2) $\quad p = 4y - x \quad$ (7)

Substituting (6) and (7) in (3) and (4),

$$16y - 8x = q \tag{8}$$

and

$$16x - 8y = 6q \tag{9}$$

From (8) and (9) $\quad y = q/3$ and $x = 13q/24 \quad$ (10)

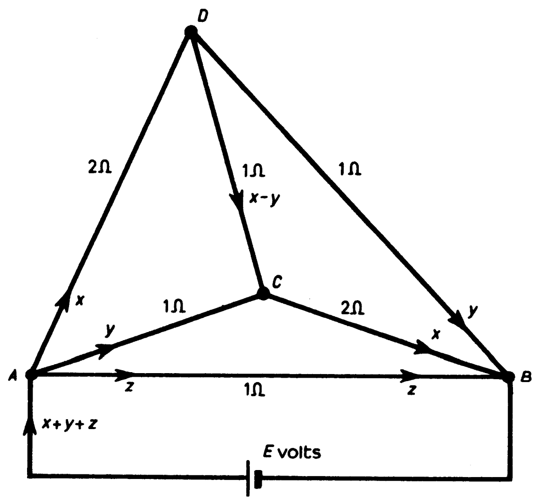
From (5) and (10) $R(13q/24 + 2q/3) = E.$

i.e. $E/q = 29R/24$ which is the equivalent resistance required.

But the resistance of a side of the square $R' = 2R$.

$$\therefore \qquad \underline{E/q = 29R'/48.}$$

30. Let the battery supply voltage be E. Let the currents in AD, AC and AB, be x, y and z respectively. Due to the symmetry of the networks, the currents in the remaining conductors can be written down in terms of x, y and z only as shown.



For mesh, ADC:

$$2x + (x - y) - y = 0$$

$$\therefore \qquad x = 2y/3 \qquad (1)$$

For mesh ADB and the battery:

$$2x + y = E \qquad (2)$$

For conductor AB and the battery:

$$E = z \qquad (3)$$

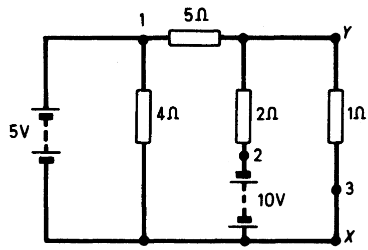
From (1) and (2) $x = 2E/7$ and $y = 3E/7$.

Resistance of network between A and B = $E/(x + y + z)$

$$= E\bigg/\left(\frac{2E}{7} + \frac{3E}{7} + E\right) = \underline{\frac{7}{12}\,\Omega.}$$

31. Mark points 1, 2 and 3 in the network as shown. Then Millman's Theorem states:

$$V_{XY} = \frac{V_{X1}Y_1 + V_{X2}Y_2 + V_{X3}Y_3}{Y_1 + Y_2 + Y_3}$$



where $V_{X1} = -5\text{ V}$

$V_{X2} = +10\text{ V}$

$V_{X3} = 0$

and $Y_1 = (1/5)\text{ S}, Y_2 = (1/2)\text{ S and } Y_3 = (1/1)\text{ S}.$

$$\therefore \quad V_{XY} = \frac{-\frac{5}{5} + \frac{10}{2}}{\frac{1}{5} + \frac{1}{2} + \frac{1}{1}} = \underline{+2{\cdot}35\text{ V.}}$$

To apply Norton's Theorem, the current through a short-circuit connected between X and Y is found.

Current in the short-circuit due to the 5-V battery $= \frac{5}{5} = 1\text{ A}$

Current in the short-circuit due to the 10-V battery $= -\frac{10}{2}$

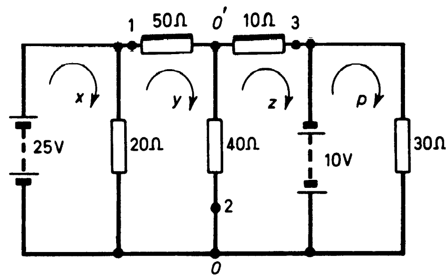
$= -5\text{ A}$

$$\therefore \quad I = 1 - 5 = -4\text{ A}$$

Also $\quad \frac{1}{Z} = \frac{1}{5} + \frac{1}{2} + \frac{1}{1} = \frac{17}{10}\text{ S}$

$$\therefore \quad V_{YX} = IZ = -4/(17/10) = \underline{-2{\cdot}35\text{ V} = -V_{XY}}.$$

32. To use Millman's Theorem, mark off points O, O′, 1, 2 and 3.



Then $V_{01} = -25\text{ V},\ V_{02} = 0,\ V_{03} = +10\text{ V}$

$$Y_1 = (1/50)\text{ S},\ Y_2 = (1/40)\text{ S},\ Y_3 = (1/10)\text{ S}.$$

$$\therefore \quad V_{00'} = (V_{01}Y_1 + V_{02}Y_2 + V_{03}Y_3)/(Y_1 + Y_2 + Y_3)$$

$$= \left(-25\,.\,\frac{1}{50} + 10\,.\,\frac{1}{10}\right)\bigg/\left(\frac{1}{50} + \frac{1}{40} + \frac{1}{10}\right)$$

$$= 100/29 = \underline{3{\cdot}45\text{ V}}.$$

To apply Norton's Theorem, short-circuit O and O′. The current in the short-circuit due to the 25-V battery = 25/50.

$$= 1/2\text{ A from O}'\text{ to O}.$$

The current in the short-circuit due to the 10-V battery

$$= -\frac{10}{10} = -1\text{ A from O}'\text{ to O}.$$

∴ Current in short-circuit = 1/2 A from O to O′.

Also, $\dfrac{1}{Z} = \dfrac{1}{50} + \dfrac{1}{40} + \dfrac{1}{10}$ i.e. Z = 200/29 Ω

$$\therefore \quad V_{00'} = \frac{1}{2}\,.\,\frac{200}{29} = \frac{100}{29} = \underline{3{\cdot}45\text{ V}}$$

To apply Maxwell's Cyclic-Current Rule, let the currents, x, y, z and p circulate as shown.

For the x mesh:

$$20(x - y) = 25 \tag{1}$$

For the y mesh:

$$20(y - x) + 50y + 40(y - z) = 0 \tag{2}$$

For the z mesh:

$$40(z - y) + 10z = 10 \qquad (3)$$

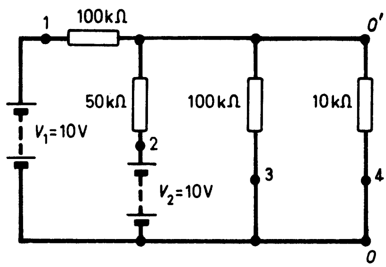
Solving these equations:

$$y = 33/58 \text{ A and } z = 38/58 \text{ A}$$

$\therefore$ Current from O to O′ $= z - y = 5/58$ A.

$\therefore$ $V_{00'} = 40 \,.\, 5/58 = 100/29 = \underline{3{\cdot}45 \text{ V.}}$

33. To use Millman's Theorem, mark points O, O′, 1, 2, 3 and 4.



$$V_{01} = \pm V_1,\ V_{02} = \pm V_2,\ V_{03} = 0,\ V_{04} = 0$$

$$Y_1 = 10^{-3}/100,\ Y_2 = 10^{-3}/50,\ Y_3 = 10^{-3}/100,\ Y_4 = 10^{-3}/10$$

$$\therefore \quad V_{00'} = \frac{\pm V_1\, 10^{-3}/100 \pm V_2\, 10^{-3}/50}{10^{-3}\left[\frac{1}{100} + \frac{1}{50} + \frac{1}{100} + \frac{1}{10}\right]}$$

$$\therefore \quad V_{00'} = \frac{\pm V_1/100 \pm V_2/50}{(1 + 2 + 1 + 10)/100} = \frac{\pm V_1 \pm 2V_2}{14}$$

If $V_1 = +10$V and $V_2 = +10$V $\quad V_{00'} = (15/7) = \underline{2{\cdot}14\text{V.}}$

If $V_1 = +10$V and $V_2 = -10$V $\quad V_{00'} = -(5/7) = \underline{-0{\cdot}71\text{V.}}$

To apply Norton's Theorem, short-circuit O and O′.
The current in the short-circuit due to $V_1 = \pm V_1\, 10^{-3}/100$.
The current in the short-circuit due to $V_2 = \pm V_2\, 10^{-3}/50$.

$\therefore$ Total short-circuit current $= \pm\ V_1.\ 10^{-3}/100 \pm V_2.\ 10^{-5}/50.$

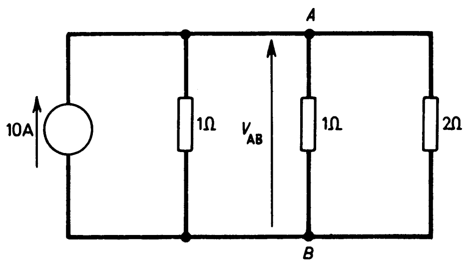
$$\text{Also } \frac{1}{Z} = \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{50} + \frac{1}{100}\right) 10^{-3}$$

$\therefore \qquad \frac{1}{Z} = 14 \,.\, 10^{-3}/100.$

$$\therefore \qquad V_{OO'} = \frac{100}{14 \,.\, 10^3}\left[10^{-3}\right]\left[\pm\frac{V_1}{100} \pm \frac{V_2}{50}\right]$$

$$= (\pm V_1 \pm 2V_2)/14 \text{ as before.}$$

34. The current-source equivalent circuit is shown.



The nodal equation obtained by applying Kirchhoff's first law at A is:

$$10 = \frac{V_{AB}}{1} + \frac{V_{AB}}{1} + \frac{V_{AB}}{2}$$

i.e. $$V_{AB} = 4\text{V}$$

35. The equations are obtained by applying Kirchhoff's first law at nodes 1 and 2:

At 1

$$\frac{E_1}{R_1} = \frac{V_1}{R_1} + \frac{V_1}{R_3} + (V_1 - V_2)\left(\frac{1}{R_4} + \frac{1}{R_5}\right)$$

At 2

$$\frac{E_2}{R_2} = \frac{V_2}{R_2} + \frac{V_2}{R_6} - (V_1 - V_2)\left(\frac{1}{R_4} + \frac{1}{R_5}\right)$$

i.e. $$\frac{E_1}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - V_2\left(\frac{1}{R_4} + \frac{1}{R_5}\right)$$

and $$\frac{E_2}{R_2} = -V_1\left(\frac{1}{R_4} + \frac{1}{R_5}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}\right)$$

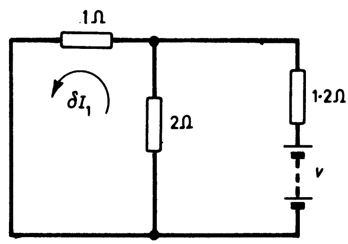
36. For the original circuit the two loop equations are:

$$2 = 3I_1 - 2I_2$$

$$3I_2 = 2I_1$$

from which $I_2 = 4/5$ A.

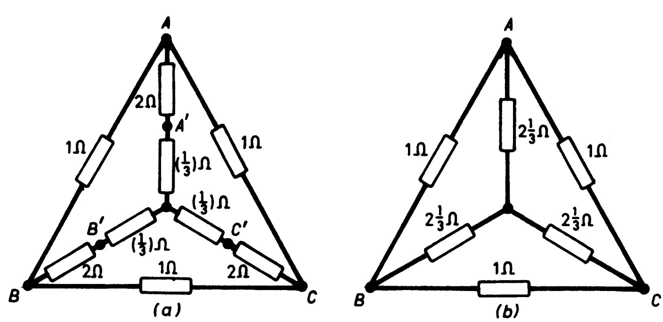
The new circuit to be solved by the Compensation Theorem is as



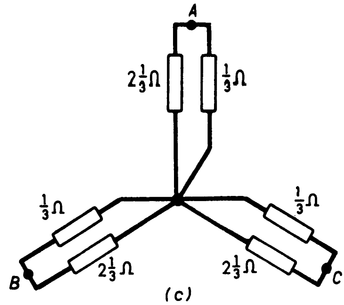
shown where R_3 has been increased by 20% to 1·2 Ω and voltage $v = I_2 \delta R_3 = 4/25$ A.

$\therefore$ $$\underline{\delta I_1 = 2/35 \text{ A}}$$

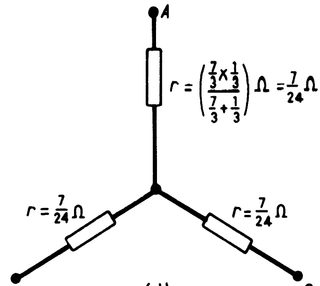
37. The original circuit reduces to (*a*) below when a delta-star transformation is applied to mesh A' B' C'. This is redrawn at (*b*).



If a delta-star transformation is now applied to the three 1-Ω resistors circuit (*c*) is obtained and this reduces to (*d*).



(c)



(d)

The resistance between A and $C = 2r = \underline{7/12\ \Omega.}$

38. The delta network ABC may be replaced by R_1, R_2, and R_3 where:

$$R_1 = \left(\frac{2 \times 5}{2 + 3 + 5}\right)\Omega = 1\ \Omega$$

$$R_2 = \left(\frac{3 \times 5}{2 + 3 + 5}\right)\Omega = 1{\cdot}5\ \Omega$$

$$R_3 = \left(\frac{2 \times 3}{2 + 3 + 5}\right)\Omega = 0{\cdot}6\ \Omega$$

Therefore, circuit (b) results.

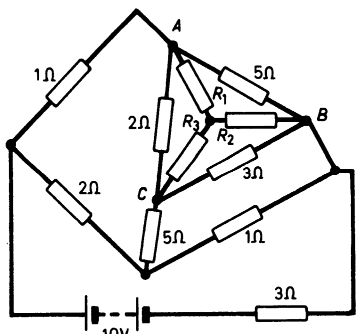
Delta network XYZ may be replaced by r_1, r_2 and r_3 where:

$$r_1 = \frac{5{\cdot}6 \times 1{\cdot}5}{5{\cdot}6 + 1{\cdot}5 + 1} = 1{\cdot}037\ \Omega$$

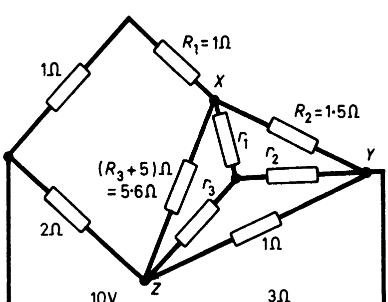
$$r_2 = \frac{1{\cdot}5 \times 1}{5{\cdot}6 + 1{\cdot}5 + 1} = 0{\cdot}185\ \Omega$$

$$r_3 = \frac{5{\cdot}6 \times 1}{5{\cdot}6 + 1{\cdot}5 + 1} = 0{\cdot}691\ \Omega$$

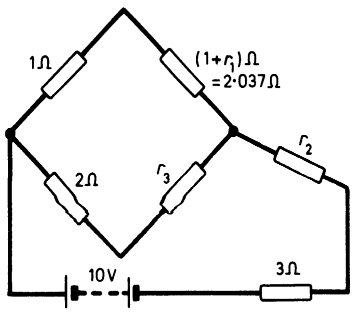
Therefore, circuit (c) results which in turn gives (d).



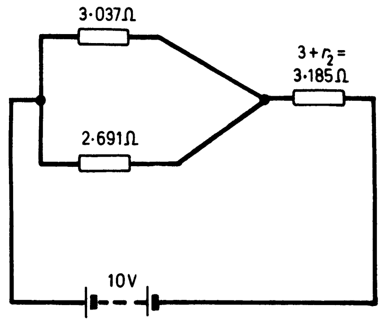
(a)



(b)



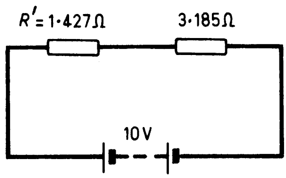
(c)



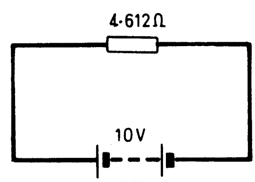
(d)

Circuit (*d*) reduces to (*e*) and then (*e*) to (*f*) since

$$R' = \left(\frac{3{\cdot}037 \times 2{\cdot}691}{3{\cdot}037 + 2{\cdot}691}\right)\Omega = 1{\cdot}427\ \Omega$$



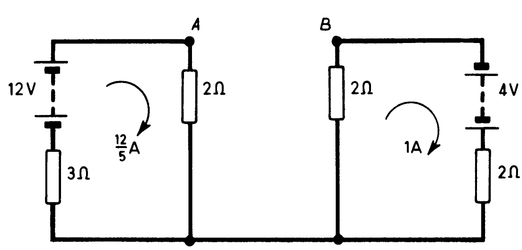
(e)



(f)

39. To use Thévenin's Theorem the 10-Ω resistor is removed to give the arrangement below. Then the resistance R_{AB} between A and B is found with the batteries short-circuited. The voltage between A and B, V_{AB}, is also calculated.

The current $I = V_{AB}/(10 + R_{AB})$ since the resistance of the link removed is 10 Ω.

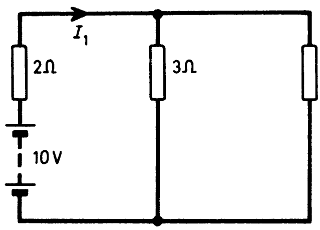


$$R_{AB} = \left(\frac{3 \times 2}{3 + 2} + \frac{2 \times 2}{2 + 2}\right)\Omega = 11/5\Omega$$

$$V_{AB} = \left(2 \times \frac{12}{5} + 2 \times 1\right)\text{V} = 34/5\text{V}$$

$$\therefore \qquad I = \frac{34/5}{10 + 11/5} = \underline{34/61 \text{ A.}}$$

40. Consider first that the 20-V source is short-circuited giving circuit (*a*) below:



(*a*)

$$\text{Current } I_1 = \frac{10}{2 + \left(\frac{3 \times 4}{3 + 4}\right)} \text{ A}$$

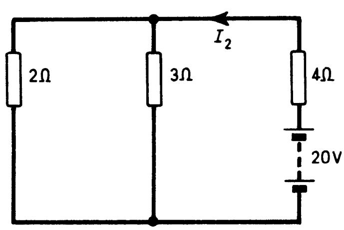
$$= 2{\cdot}69 \text{ A}$$

The current through the 3-Ω resistor $= 4I_1/(3 + 4) = 1{\cdot}54$ A.

The current through the 4-Ω resistor $= 2{\cdot}69 - 1{\cdot}54$ A

$= 1{\cdot}15$ A.

If now the 10-V source is short-circuited, circuit (*b*) results.



(*b*)

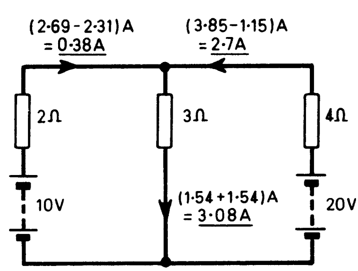
$$\text{Current } I_2 = \frac{20}{4 + \left(\frac{2 \times 3}{2 + 3}\right)} \text{ A}$$

$$= 3{\cdot}85 \text{ A}$$

The current through the 3-Ω resistor $= 2I_2/(2 + 3) = 1{\cdot}54$ A.

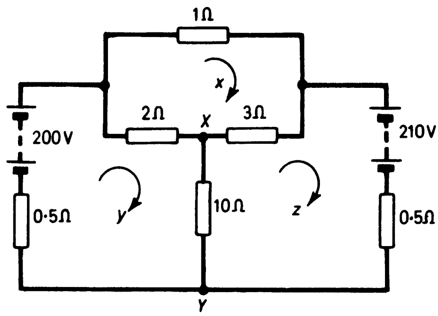
The current through the 2-Ω resistor is $(3{\cdot}85 - 1{\cdot}54)$ A $= 2{\cdot}31$ A.

When both voltage sources are present the currents are as shown in circuit (*c*), namely $\underline{0{\cdot}38 \text{ A}}$, $\underline{3{\cdot}08 \text{ A and } 2{\cdot}7 \text{ A.}}$



(*c*)

41. Let the loop currents be x, y and z as illustrated.



For the x loop:

$$6x - 2y - 3z = 0 \quad (1)$$

For the y loop:

$$12{\cdot}5y - 2x - 10z = 200 \quad (2)$$

For the z loop:

$$13{\cdot}5z - 3x - 10y + 210 = 0 \quad (3)$$

From (1),

$$x = (2y + 3z)/6 \quad (4)$$

Substituting (4) in (2):

$$35{\cdot}5y - 33z = 600 \quad (5)$$

Substituting (4) in (3):

$$12z - 11y + 210 = 0 \quad (6)$$

From (6):

$$z = (11y - 210)/12 \quad (7)$$

From (7) and (5)

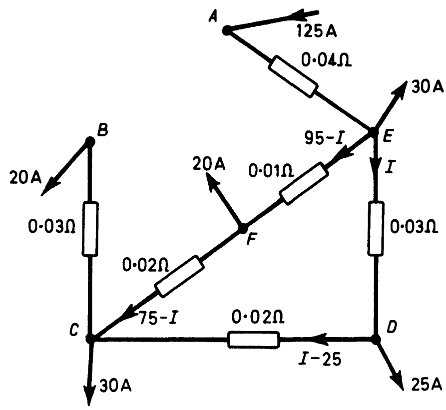
$$y = 30/7$$

$$\therefore \quad z = -95/7$$

Current through 10-Ω resistor $= y - z = 125/7$ A

$=$ <u>17·86 A flowing from X to Y.</u>

42. To use Thévenin's Theorem AB is removed. Then the resistance R_{AB} between A and B is found and the voltage V_{AB} between A and B is also calculated.



The current in AB originally is then $I_{AB} = V_{AB}/(R_{AB} + 0{\cdot}04)$, since the resistance of the link removed is 0·04 Ω.

In loop EDC, if I is the current in ED:

$$0{\cdot}01(95 - I) + 0{\cdot}02(75 - I) = 0{\cdot}03I + 0{\cdot}02(I - 25)$$

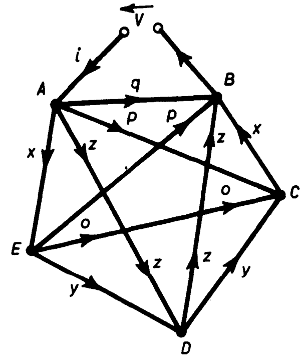
$$\therefore \quad I = 36{\cdot}9 \text{ A}.$$

$$V_{AB} = 0{\cdot}04 \times 125 + 0{\cdot}03I + 0{\cdot}02(I - 25) + 0{\cdot}03 \times 20$$
$$= 6{\cdot}94 \text{ V}$$

$$R_{AB} = 0{\cdot}04 + 0{\cdot}03 + \left(\frac{0{\cdot}03 \times 0{\cdot}05}{0{\cdot}03 + 0{\cdot}05}\right) = 0{\cdot}08875\ \Omega$$

$$\therefore \quad I_{AB} = 6{\cdot}94/(0{\cdot}08875 + 0{\cdot}04) = \underline{53{\cdot}9 \text{ A.}}$$

43. Let the currents be as shown with the supply connected between A and B.



For mesh BCD:

$$R(x + y) = Rz \qquad (1)$$

For mesh ABC:

$$R(x + p) = Rq \qquad (2)$$

For mesh ABD:

$$Rq = 2Rz \qquad (3)$$

For mesh *ADC:*

$$R(z + y) = Rp \qquad (4)$$

For mesh *AECB:*

$$Rq = 2Rx \qquad (5)$$

Also,

$$V = Rq \qquad (6)$$

From (3) and (5) $x = z$ (which could have been assumed by symmetry).

From (2) and (5) $x = p$ (which could have been assumed by symmetry).

Resistance of network (R') is V/i

$$\therefore \qquad R' = \frac{V}{x + z + p + q} = \frac{V}{x + x + x + 2x} = \frac{V}{5x} \qquad (7)$$

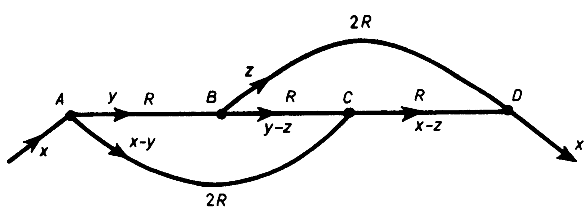
From equations (5) and (6):

$$V/q = R = V/2x$$

so using equation (7):

$$R' = V/5x = \underline{2R/5}$$

44.



Let the resistances of AB, BC, CD be R. Then the resistances of AC and BD are each $2R$.

Let the currents be as shown.

For loop ABCA:

$$Ry + R(y - z) = 2R(x - y)$$

i.e.

$$4y - z = 2x \qquad (1)$$

For loop BDCB:

$$2Rz = R(y - z) + R(x - z)$$

i.e.

$$4z = y + x \qquad (2)$$

From (1) $z = 4y - 2x$ (3)

From (2) and (3) $4(4y - 2x) = y + x$

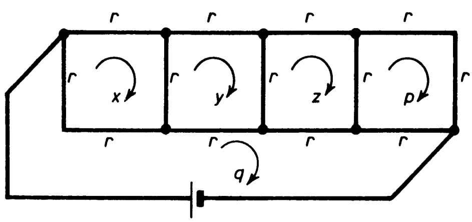
$\therefore$ $y = 3x/5$ (4)

From (3) and (4) $z = 2x/5$ (5)

Current along $BC = y - z = x/5$ from (4) and (5).

i.e. current along BC = 1/5 of current entering network.

45.



Let the resistance of one side of a square be r ohms and let the currents circulate as shown.

For the x mesh: $4x - 2q - y = 0$ (1)

For the y mesh: $4y - x - q - z = 0$ (2)

For the z mesh: $4z - y - q - p = 0$ (3)

For the p mesh: $4p - z - q = 0$ (4)

For the q mesh: $E = rx + ry + rz + 2pr$ (5)

From (1) $y = 4x - 2q$ and from (4) $z = 4p - q$

$\therefore$ from (2) $15x - 9q = 4p - q$ i.e. $15x - 8q = 4p$ (6)

and from (3) $16p - 4q - 4x + 2q - q - p = 0$

i.e. $15p - 3q = 4x$ (7)

and from (5) $E/r = x + 4x - 2q + 4p - q + 2p = 5x - 3q + 6p$ (8)

From (6) $p = (15x - 8q)/4$ (9)

so from (7) $\dfrac{15(15x - 8q)}{4} - 3q = 4x,$ i.e. $x = 12q/19$ (10)

$\therefore$ from (9) $p = 7q/19$ (11)

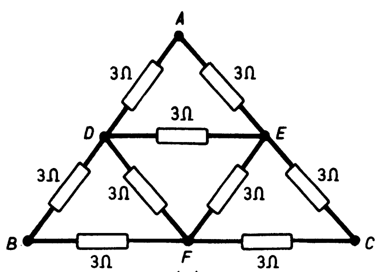
Thus (8) gives:

$$E/r = 5x - 3q + 6p = \frac{60q}{19} - 3q + \frac{42q}{19} = \frac{45q}{19} \qquad (12)$$

$$\therefore \qquad E/q = 45r/19 = \left(2\frac{7}{19}\right)r.$$

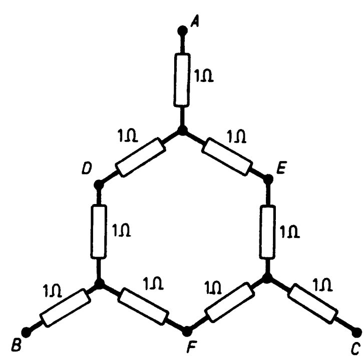
Now E/q = resistance of network and r is the resistance of 1 m of wire so the network resistance is equal to that of $2\frac{7}{19}$ m of the wire.

46. If star-connected 1-Ω resistors are replaced by an equivalent delta connection circuit (a) below is obtained.

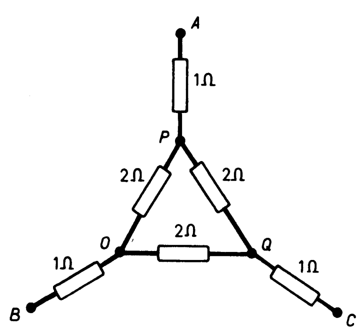


(a)

If the meshes AED, ECF and DFB are replaced by equivalent star connections circuit (b) results and this reduces to circuit (c).

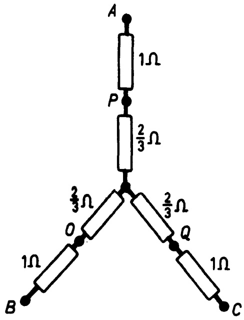


(b)



(c)

Mesh connection OPQ may be replaced by an equivalent star to give circuit (d).



(d)

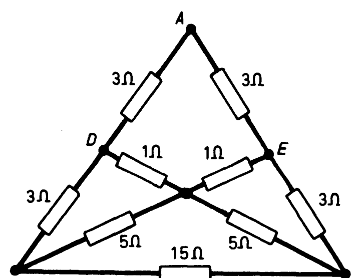
Resistance between A and C

$$= (2 \times 1\tfrac{2}{3})\Omega$$

$$= \underline{3\tfrac{1}{3}\ \Omega}$$

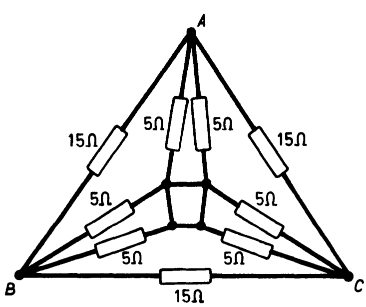
Alternative Solution

Replace the star between B, C and the star point of the given circuit by a delta connection. This gives circuit (*a*) below.

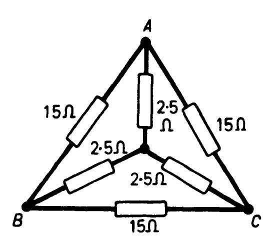


(a)

Repeating this procedure on the stars between A, B and the star point and A, C and the star point gives circuit (*b*) which simplifies to circuit (*c*).

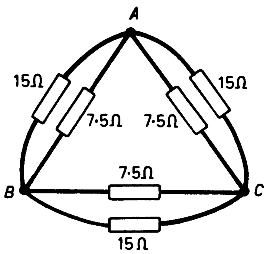


(b)

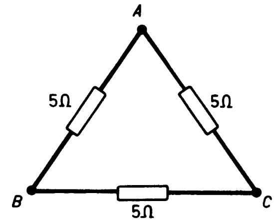


(c)

Circuit (*d*) is equivalent to that at (*c*) which can be redrawn as at (*e*).



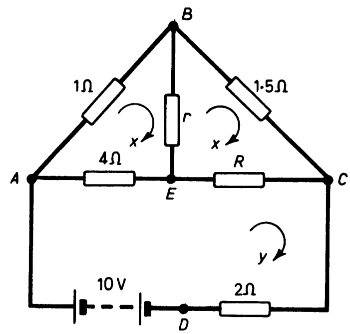
(*d*)



(*e*)

Resistance between A and $C = (5 \times 10)/(5 + 10)\Omega = \underline{3\frac{1}{3}\,\Omega.}$

47. Let the currents circulate as shown. As there is no current in *BE*, meshes *ABE* and *BCE* have the same current x.



For mesh *ABE:*

$$5x - 4y = 0 \qquad (1)$$

For mesh *BCE:*

$$(1{\cdot}5 + R)x - Ry = 0 \qquad (2)$$

For mesh *AECD:*

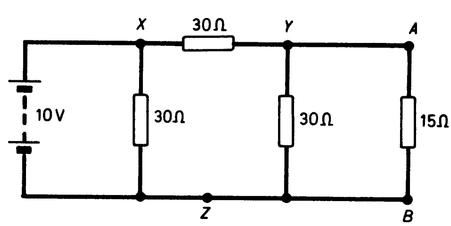
$$(6 + R)y - 4x - Rx = 10 \qquad (3)$$

From (1), $x = 4y/5$ (4)

From (4) and (2), $\underline{R = 6\,\Omega}$

From (4) and (3), $y = \underline{5/2\text{ A}}$

48. After transforming the star XYZ to a delta the circuit illustrated results.



To use Thévenin's Theorem the 15-Ω resistor is removed. Then the resistance R_{AB} between A and B is found with the battery short-circuited. The voltage between A and B, V_{AB}, is also calculated. The current in AB is $V_{AB}/(15 + R_{AB})$, since the resistance of the link removed is 15 Ω.

Now $R_{AB} = 15\ \Omega$ and $V_{AB} = 5$ V.

$\therefore$ the current in $AB = 5/(15 + 15) = \underline{1/6 \text{ A}}.$

49. Current through BD is $I_1 - I_2 = 0{\cdot}0049$ A flowing from B to D.

Using the Principle of Superposition this solution becomes one part of the required solution because the current in CEA due to the 1-V source is 0·0723 A flowing from E to A.

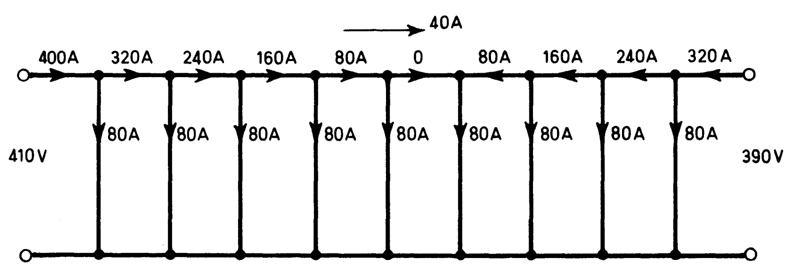
Using the Reciprocity Theorem it is seen that if the original e.m.f. of 1 V had been transferred to branch BD it would have given rise to a current in EA of the same value as that originally in BD, i.e. 0·0049 A.

Thus, an e.m.f. of 2 V in branch BD gives by proportion a current of $-(2 \times 0{\cdot}0049)$ A in EA, the minus sign being included because the e.m.f. of 2 V opposes the original current flow in BD.

$$\therefore \quad \text{resultant current in } EA = 0{\cdot}0723 + (-2 \times 0{\cdot}0049) \text{ A}$$

$$= \underline{0{\cdot}0625 \text{ A from } E \text{ to } A.}$$

50.



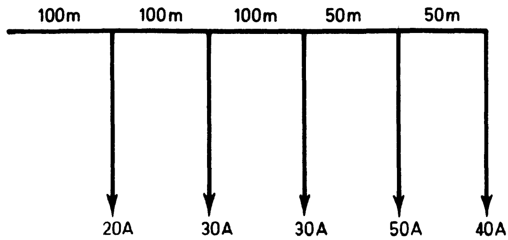
$$\text{Total cable resistance} = 4 \times 0{\cdot}125\ \Omega = 0{\cdot}5\ \Omega$$

$$\text{Circulating current} = \left(\frac{410 - 390}{0{\cdot}5}\right)\text{A} = 40\text{A}$$

Hence the current distribution by symmetry is as illustrated and the fifth and sixth loads from the 410-V end have the lowest voltage.

This voltage is $390 - 0{\cdot}05\ (320 + 240 + 160 + 80) = \underline{350 \text{ V.}}$

51.

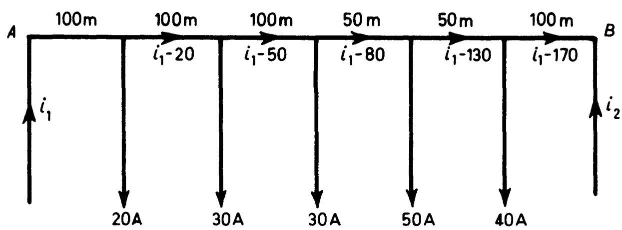


For each wire the voltage drop

$$= 10^{-4}(20 \times 100 + 30 \times 200 + 30 \times 300 + 50 \times 350 + 40 \times 400)\ \text{V}$$
$$= 5{\cdot}05\ \text{V}$$

$\therefore$ Total voltage drop $= 2 \times 5{\cdot}05$ V $=$ 10·1 V.

52.



Let the resistance/metre be r ohms. Let the current fed in at A be i_1 amperes and that at B i_2 amperes.

The total voltage drop between A and B is 0.

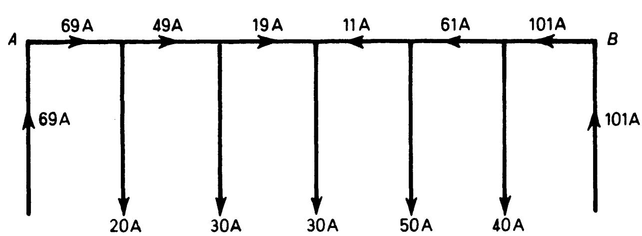
$$\therefore\ r\{100i_1 + 100(i_1 - 20) + 100(i_1 - 50) + 50(i_1 - 80) + 50(i_1 - 130) + 100(i_1 - 170)\} = 0.$$

$$\therefore\ 500i_1 = 34\,500$$

$$\therefore\ i_1 = 69\ \text{A}$$

It follows that $i_2 = 170 - i_1 = 101$ A.

The current distribution is, therefore, as shown in the sketch below:



53. The voltage v at any time t sec is given by $v = V(1 - e^{-t/CR})$ where V is the final voltage = 400 volts.

C is the capacitance in farads = $8/10^6$.

and R is the resistance in ohms = 100 000

$$\therefore \quad 300 = 400(1 - e^{-t/CR}) = 400(1 - e^{-\frac{t}{0\cdot 8}})$$

$$\therefore \quad \tfrac{3}{4} = 1 - e^{-\frac{t}{0\cdot 8}}$$

$$\therefore \quad e^{-\frac{t}{0\cdot 8}} = \tfrac{1}{4} \quad \text{or} \quad e^{\frac{t}{0\cdot 8}} = 4$$

$$\therefore \quad t = 0\cdot 8 \log_e 4 = \frac{0\cdot 8 \log_{10} 4}{\log_{10} e} \text{ s.} = \underline{1\cdot 11 \text{ s.}}$$

The reader should refer to the graphical methods of solution for this type of problem (see Problem No. 54).

$$\text{Final energy stored} = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \cdot \frac{8}{10^6} (400)^2 \text{ J}$$

$$= \underline{0\cdot 64 \text{ J.}}$$

When the voltage is 300 V the energy stored is $\tfrac{1}{2} \cdot \frac{8}{10^6}(300)^2$ J

$$= \underline{0\cdot 36 \text{ J.}}$$

∴ fraction of final energy stored at 300 V is 36/64 = $\underline{0\cdot 56.}$

54. The time constant of the circuit is L/R second = 5/4 = $\underline{1\cdot 25 \text{ s.}}$ The flow of current after switching on is given by the expression:

$$i = \frac{E}{R}\left\{1 - e^{-\frac{Rt}{L}}\right\} \qquad (1)$$

Substituting the given values in this expression the time t taken for the current to reach 1·9 A is obtained by putting $i = 1\cdot 9$.

$$\therefore \quad 1\cdot 9 = \frac{8}{4}\left\{1 - e^{-\frac{4t}{5}}\right\}$$

$$\therefore \quad e^{-\frac{4t}{5}} = 0\cdot 05$$

$$\therefore \quad t = \frac{5}{4} \log_e 20 = \frac{5 \log_{10} 20}{4 \log_{10} e} = \underline{3\cdot 74 \text{ s.}}$$

Two graphical methods for obtaining the current-time curves are given below.

Method 1

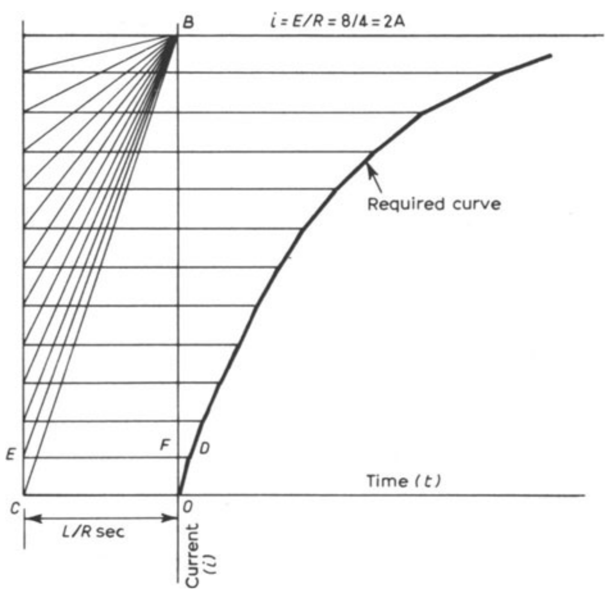
From equation (1)

$$\frac{di}{dt} = \frac{(E/R) - i}{L/R} \qquad (2)$$

Let OB equal the steady current which will flow through the circuit after the e.m.f. has been applied for a long time. In this case $OB = 2\text{A} = 8/4$ A. OC represents the time constant L/R in seconds and determines the scale of time on the figure. Join BC.

$$\text{The slope of } BC = BO/OC = (E/R)/(L/R) \qquad (3)$$

It will be seen from (2) that, since $i = 0$ at the start, this slope is equal to di/dt when $i = 0$. When current begins to flow through the circuit, therefore, it will increase at a rate corresponding to the slope of BC and if a short line OD is drawn parallel to BC this will represent the beginning of the required curve.



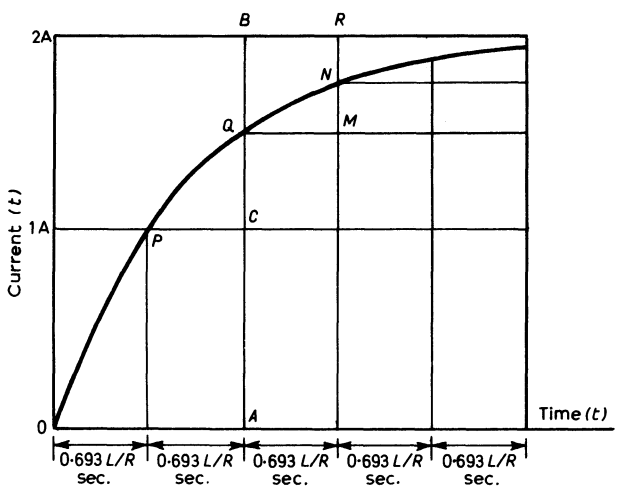
Draw a horizontal line DE cutting the vertical ordinate at F and the vertical through C at E. Join BE.

$$\text{The slope of } BE = \frac{E/R - i}{L/R} \tag{4}$$

i.e. slope of BE is equal to the value of di/dt when the current has the value OF. If now a second short line is drawn parallel to BE from D it will represent the continuation of the current curve. By repeating this process a close approximation may be made to the curve required as illustrated in the figure.

Method 2.

It is easily shown from equation (1) that the current i reaches half its steady value in a time $0{\cdot}693L/R$ after switching on. This fact enables a quick construction of the required curve to be produced as below.



Point P is the first point on the curve. Then point Q is obtained by drawing vertical AB and making $CQ = QB$. Similarly $MN = NR$ and so on.

55. Two graphical methods are given below.

Method 1.

The equation connecting current and time may be written:

$$L\frac{di}{dt} + Ri = 0 \qquad (1)$$

where R is the total resistance in the circuit.

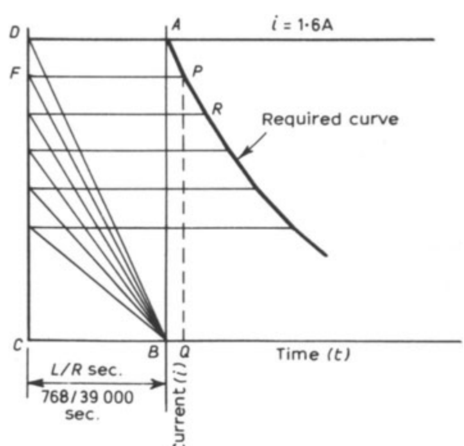
(1) may be written:

$$\frac{di}{dt} = \frac{-i}{L/R} \qquad (2)$$

L/R is the time constant of the circuit and is measured in seconds. In this case $L/R = \dfrac{1{\cdot}2 \times 4}{100 + (230/1{\cdot}6)}$

i.e. $$L/R = \frac{1{\cdot}2 \times 4 \times 1{\cdot}6}{390} = \frac{768}{39\,000}$$

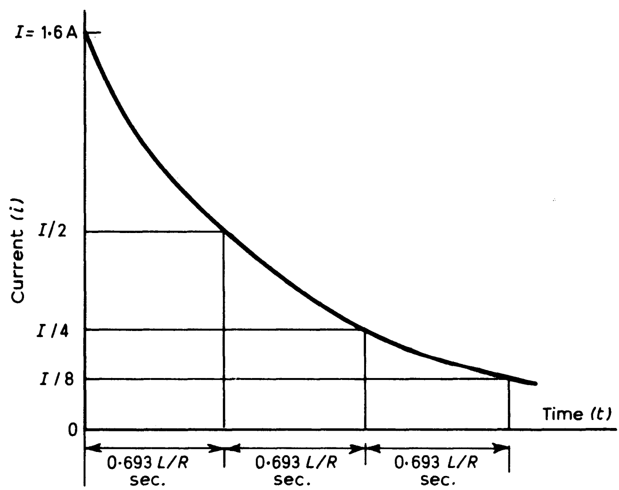
In drawing the curve let $AB = DC$ represent the initial value of the current (1·6 A) when the circuit is connected to the discharge resistor. Let $BC = L/R$. Then the slope of the line BD represents the value of di/dt when the current begins to die away. Draw a short line AP parallel with BD; the current will fall to the value PQ after a short time BQ. Draw a line from P parallel with BC, cutting CD in F and join BF. The slope of FB equals $[-i/(L/R)]$, that is, it equals di/dt, when



the current has fallen to *QP*. Draw a short line *PR* parallel to *FB*; this will represent the slope of the current curve during the next short interval. By continuing this process the complete curve of decay of current may be drawn.

Method 2.

The construction below is similar to that given for the rise of current in an inductive circuit as one solution to problem 54. This solution should first be consulted before studying the construction below.



56. The flow of current after switching on is given by the expression:

$$i = (V/R)\{1 - e^{-Rt/L}\}$$

(*a*) Initial rate of change of current $= V/L = 100/0{\cdot}4$

$$= \underline{250 \text{ A s}^{-1}}$$

(*b*) Final current $I = V/R = 100/10 = \underline{10 \text{ A}}$

(*c*) Time constant $= L/R = 0{\cdot}4/10 = \underline{0{\cdot}04 \text{ s}}$

(*d*) $i = I(1 - e^{-Rt/L})$

$\therefore \quad 5 = 10(1 - e^{-25t})$

$\therefore \quad \underline{t = 0{\cdot}0277 \text{ s}}$

(*e*) Energy stored $= \frac{1}{2}LI^2$ joules

$$= \frac{1}{2} \times 0{\cdot}4 \times 10^2$$

$$= \underline{20\ \text{J}}$$

57. $L_1 = 0{\cdot}02$ H, $L_2 = 0{\cdot}01$ H

$\therefore$ $M = 0{\cdot}6\sqrt{(0{\cdot}02 \times 0{\cdot}01)} = 0{\cdot}00848$ H

(*a*) *Coils in series*

$$(L_1 + L_2 \pm 2M)\frac{di}{dt} = 12$$

i.e.

$$\frac{di}{dt} = \frac{12}{0{\cdot}047} \quad \text{or} \quad \frac{12}{0{\cdot}013}$$

$$= \underline{255\ \text{A s}^{-1}} \quad \text{or} \quad \underline{925\ \text{A s}^{-1}}$$

(*b*) *Coils in parallel*

$$L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt} = 12$$

$$L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 12$$

$\therefore$

$$\frac{di_1}{dt} = 140\ \text{A s}^{-1} \quad \text{or} \quad 1740\ \text{A s}^{-1}$$

$$\frac{di_2}{dt} = 1081\ \text{A s}^{-1} \quad \text{or} \quad 2680\ \text{A s}^{-1}$$

Rate of change of battery current $= \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$

$$= \underline{1221\ \text{A s}^{-1} \quad \text{or} \quad 4420\ \text{A s}^{-1}}$$

93. The graphical solution is self-explanatory from the figure. Resolving the e.m.f.'s in the X and Y directions:

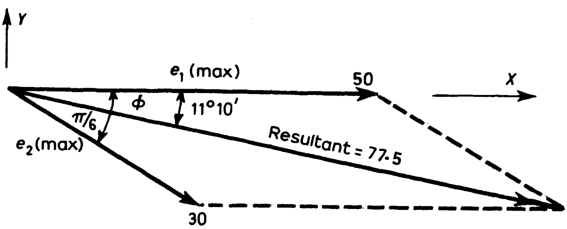
$$X = 50 + 30 \cos 30° = 75{\cdot}98,$$

$$Y = 0 - 30 \sin 30° = -15.$$

$$\text{The magnitude of the resultant} = \sqrt{(X^2 + Y^2)} = \sqrt{[(75{\cdot}98)^2 + (15)^2]} = \underline{77{\cdot}5.}$$

$$\tan \phi = \frac{Y}{X} = \frac{-15}{75{\cdot}98}$$

i.e. $\underline{\phi = -11^\circ\ 10'.}$



i.e. $\underline{\text{resultant e.m.f. is } 77{\cdot}5 \sin (\omega t - 11^\circ\ 10').}$

94. The graphical solution is self-explanatory from the figure. Resolving the e.m.f.'s in the X and Y directions:

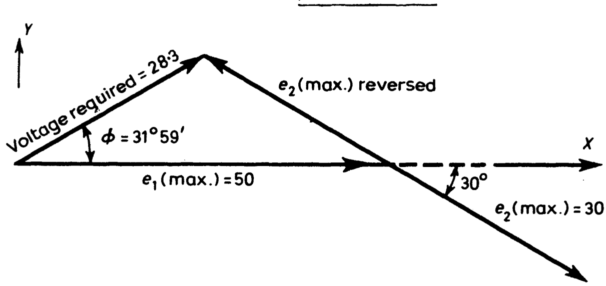
$$X = 50 - 30 \cos 30^\circ = 24{\cdot}02,$$

$$Y = 30 \sin 30^\circ = 15.$$

$$\text{The magnitude of the resultant} = \sqrt{(X^2 + Y^2)} = \sqrt{[(24{\cdot}02)^2 + (15)^2]} = \underline{28{\cdot}3.}$$

$$\tan \phi = Y/X = 15/24{\cdot}02$$

$\therefore$ $\underline{\phi = 31^\circ\ 59.}$



$\underline{\text{Voltage required is } 28{\cdot}3 \sin (\omega t + 31^\circ\ 59').}$

95. The graphical solution is self-explanatory from the figure. Resolving the e.m.f.'s in the X and Y directions:

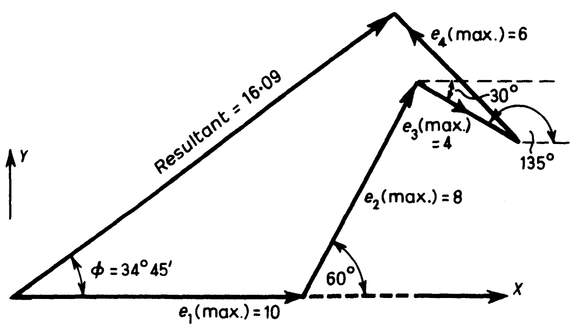
$$X = 10 + 8\cos 60° + 4\cos 30° - 6\cos 45°$$
$$= 13{\cdot}22,$$
$$Y = 0 + 8\sin 60° - 4\sin 30° + 6\sin 45°$$
$$= 9{\cdot}17$$

$\therefore$ resultant e.m.f. $= \sqrt{(X^2 + Y^2)} = \sqrt{[(13{\cdot}22)^2 + (9{\cdot}17)^2]}$

$= \underline{16{\cdot}09}$

$\tan\phi = Y/X = 9{\cdot}17/13{\cdot}22$

$\therefore$ $\underline{\phi = 34°\,45'.}$



$\therefore$ $\underline{\text{resultant e.m.f.} = 16{\cdot}09 \sin(\omega t + 34°\,45').}$

96. The graphical solution is self-explanatory from the figure. Resolving the e.m.f.'s in the X and Y directions:

$$X = 10 + 4\cos 30° - 8\cos 60° + 6\cos 45°$$
$$= 10 + 3{\cdot}464 - 4 + 4{\cdot}243 = 13{\cdot}71,$$
$$Y = -8\sin 60° - 4\sin 30° - 6\sin 45°$$
$$= -6{\cdot}928 - 2 - 4{\cdot}243 = -13{\cdot}17$$

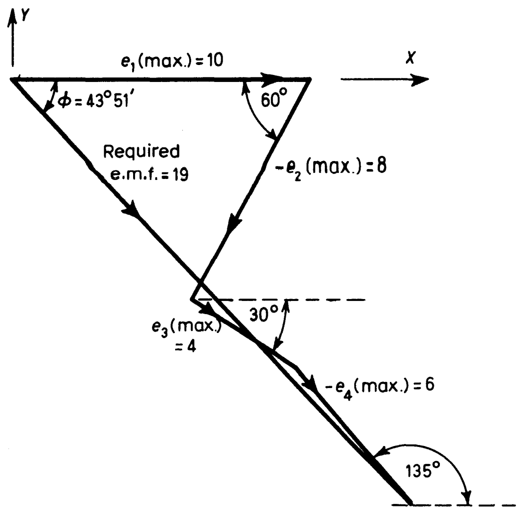
$\therefore$ e.m.f. required $= \sqrt{(X^2 + Y^2)}$

$= \sqrt{[(13{\cdot}71)^2 + (13{\cdot}17)^2]}$

$= \underline{19.}$

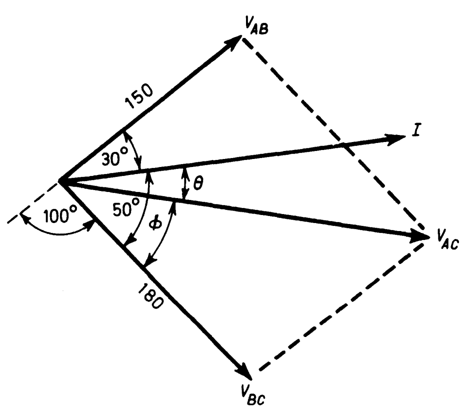
$$\tan \phi = Y/X = -13{\cdot}17/13{\cdot}71$$

$$\therefore \quad \underline{\phi = -43^\circ\ 51'.}$$



$\therefore$ $\underline{\text{required e.m.f. is } 19 \sin(\omega t - 43^\circ\ 51').}$

97. A sketch of the phasor diagram is shown. The voltage V_{AC} between A and C is the phasor sum of V_{AB} and V_{BC}. Thus V_{AC} and angle θ can be found either mathematically or by constructing a phasor diagram to scale.



$$V_{AC}^2 = V_{AB}^2 + V_{BC}^2 - 2V_{AB}V_{BC}\cos 100°$$
$$= 150^2 + 180^2 + 2 \,.\, 150 \,.\, 180 \cos 80°.$$

$\therefore \quad \underline{V_{AC} = 254 \text{ V.}}$

Also $\quad \dfrac{150}{\sin\phi} = \dfrac{254}{\sin 100°} = \dfrac{254}{\sin 80°}$

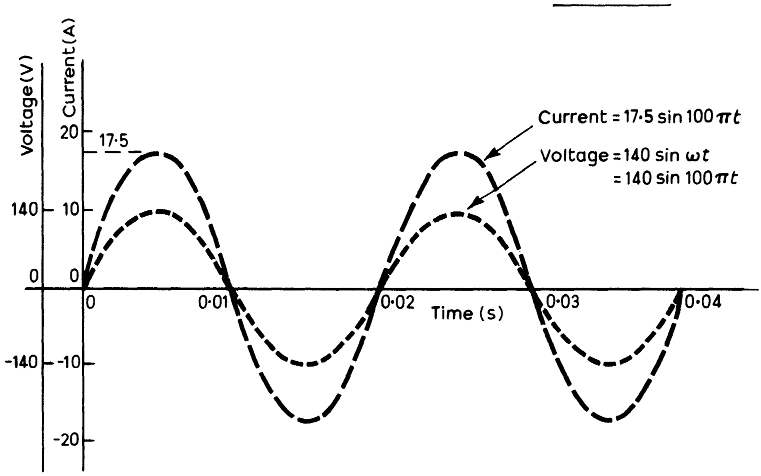
$\therefore \quad \phi = 35° 34'$

$\therefore \quad \theta = 50 - \phi = 14° 26'$

$\therefore$ resultant voltage between A and C lags current by an angle 14° 26′.

98. $V = 140 \sin \omega t$

$$\text{Maximum current} = \frac{\text{Maximum voltage}}{R} = \frac{140}{8}$$
$$= \underline{17{\cdot}5 \text{ A.}}$$



At any instant the current $i = 17{\cdot}5 \sin \omega t = \underline{17{\cdot}5 \sin 100\pi t.}$

99. $\quad I = V/\sqrt{(R^2 + L^2\omega^2)}$

$\therefore \quad 10 = 230/\sqrt{[4 + L^2(100\pi)^2]}$

$\therefore \quad \underline{L = 0{\cdot}073 \text{ H.}}$

Let ϕ be the angle of phase difference.

Then $\tan\phi = L\omega/R = (0{\cdot}073 \times 100\pi)/2 = 11{\cdot}455.$

$\therefore \quad \phi = 85°.$

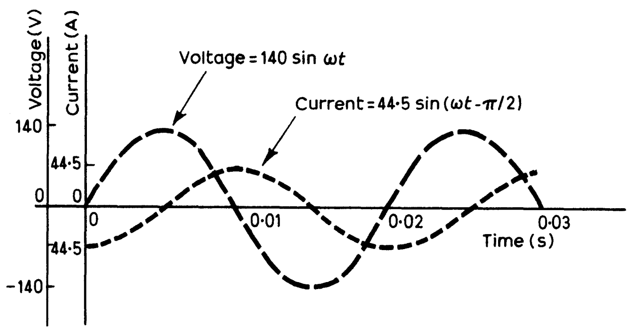
i.e. current lags the voltage by 85°.

100. $I_{max} = \dfrac{V}{\sqrt{(R^2 + L^2\omega^2)}}$ and $R = 0$ and $\omega = 100\pi$

$$\therefore \qquad I_{max} = \frac{140}{0{\cdot}01 \times 100\pi} = 44{\cdot}5 \text{ A}.$$

But current lags voltage by 90°.

$\therefore$ current flowing $= I_{max} \sin(\omega t - \pi/2) = \underline{44{\cdot}5 \sin(\omega t - 90°)}.$



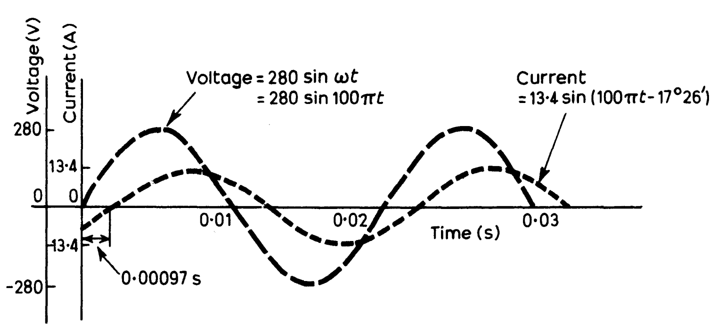
101. $$V = 280 \sin \omega t$$

Maximum value of current $= \dfrac{280}{\sqrt{[20^2 + (100\pi \times 0{\cdot}02)^2]}} = \underline{13{\cdot}4 \text{ A}.}$

Angle of lag of current behind voltage $= \phi$, where

$$\tan\phi = \frac{\omega L}{R} = \frac{100\pi \times 0{\cdot}02}{20} \qquad \text{i.e. } \underline{\phi = 17° 26'.}$$

$$\therefore \qquad \underline{\text{Current} = 13{\cdot}4 \sin(\omega t - 17° 26').}$$



$17° 26' = 0{\cdot}304$ radian

This corresponds to a time of $0{\cdot}304/100\pi = 0{\cdot}00097$ s.

102. $$I = \frac{V}{\sqrt{(R^2 + L^2\omega^2)}} = \frac{200}{\sqrt{[2^2 + (0{\cdot}01 \times 100\pi)^2]}}$$
$$= \underline{53{\cdot}8\ \text{A}.}$$

The angle of lag of the current behind the voltage is ϕ, where $\tan\phi = \omega L/R$.

$$\therefore \qquad \tan\phi = \frac{0{\cdot}01 \times 100\pi}{2} = 1{\cdot}57$$

$$\therefore \qquad \underline{\phi = 57^\circ\ 30'.}$$

103. $$I = \frac{V}{\sqrt{[R^2 + (L\omega)^2]}} = \frac{200}{\sqrt{[100 + (0{\cdot}05 \times 100\pi)^2]}}$$

$$\therefore \qquad \underline{I = 10{\cdot}74\ \text{A}.}$$

If ϕ is the angle of lag of the current behind the voltage:

$$\tan\phi = \frac{\omega L}{R} = \frac{0{\cdot}05 \times 100\pi}{10} = \frac{\pi}{2} = 1{\cdot}57$$

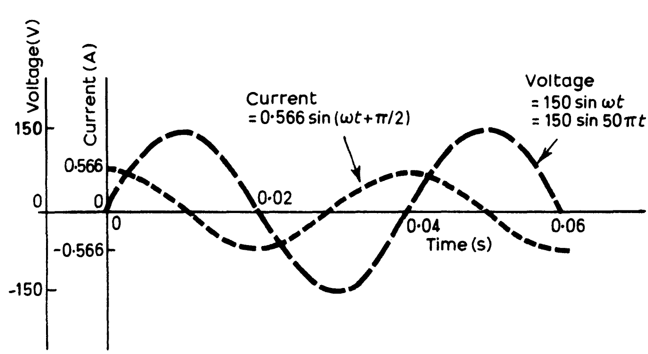
$$\therefore \qquad \underline{\phi = 57^\circ\ 30'.}$$

104. $$V = 150 \sin \omega t.$$

Maximum value of current $= V_{max}C\omega = 150 \times 24 \times 10^{-6} \times 50\pi$

$$= \underline{0{\cdot}566\ \text{A}.}$$

Current leads voltage by 90°.



105. For resonance $L\omega = \dfrac{1}{C\omega}$, i.e. $LC\omega^2 = 1$

$$\therefore \qquad \frac{20}{10^6} \cdot \frac{25}{10^2}\,\omega^2 = 1$$

$$\therefore \qquad \omega = 447{\cdot}2$$

$$\text{Now } f = \omega/2\pi = 447{\cdot}2/2\pi = \underline{71{\cdot}2 \text{ Hz}}$$

At the resonant frequency $I = V/R = 40/3$

$$\therefore \qquad \underline{I = 13{\cdot}33 \text{ A.}}$$

The voltage across the capacitor $= V_c = I/C\omega$

$$\therefore \qquad V_c = \frac{13{\cdot}33 \times 10^6}{20 \times 2\pi \times 71{\cdot}2} = \underline{1491 \text{ V.}}$$

106. Let angle of lag be ϕ.

Then $\tan\phi = \dfrac{L\omega}{R} = \dfrac{0{\cdot}05 \times 100\pi}{10} = 1{\cdot}571$

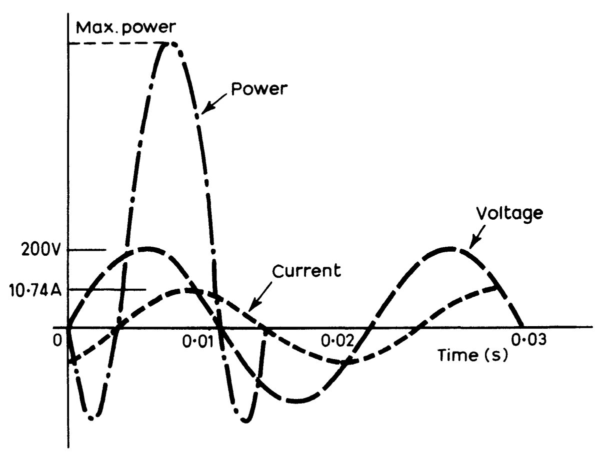
$$\therefore \qquad \phi = \underline{57^\circ\, 30'.}$$

Power-factor is $\cos\phi = \cos 57^\circ\, 30' = \underline{0{\cdot}5373.}$

Average power supplied $= VI\cos\phi$.

$$I = \frac{V}{\sqrt{(R^2 + L^2\omega^2)}} = \frac{200}{\sqrt{(100 + 5\pi^2)}} = 10{\cdot}74 \text{ A.}$$

$$\therefore \qquad \text{Power} = 10{\cdot}74 \times 200 \times 0{\cdot}5373 = \underline{1160 \text{ W.}}$$



The current lags the voltage by 0·0032 s,

since 57° 30′ = 1·0036 radian

∴ current lags by $(1{\cdot}0036/100\pi)$ s = 0·0032 s

Instantaneous power = instantaneous voltage × instantaneous current

Power curve, therefore, has shape shown.

107. (*a*) For resonance $L\omega = \dfrac{1}{C\omega}$

$$\therefore \quad \omega^2 = 1/LC = (2\pi f)^2$$

$$\therefore \quad f^2 = \frac{1}{(2\pi)^2}\,\frac{10^6}{0{\cdot}64 \times 12}$$

$$\therefore \quad f = \underline{57{\cdot}5 \text{ Hz}}$$

(*b*) At resonance the supply voltage = IR = 1·5 × 40 = <u>60 V.</u>

Voltage across coil $= [\sqrt{(R^2 + L\omega^2)}]\, I$

$$= 1{\cdot}5 \times \sqrt{[1600 + (2\pi \times 57{\cdot}5 \times 0{\cdot}64)^2]}$$

$$= \underline{352 \text{ V.}}$$

Voltage across capacitor $= I/C\omega$

$$= 1{\cdot}5/(12 \times 10^{-6} \times 2\pi \times 57{\cdot}5)$$

$$= \underline{346{\cdot}4 \text{ V.}}$$

(*c*) When the frequency is 50 Hz, $\omega = 100\pi$.

Voltage across coil $= I\sqrt{(R^2 + L^2\omega^2)}$

$$= 1{\cdot}5\sqrt{[1600 + (0{\cdot}64 \times 2\pi \times 50)^2]}$$

$$= \underline{307 \text{ V.}}$$

Voltage across capacitor $= I/C\omega$

$$= 1{\cdot}5/(12 \times 10^{-6} \times 100\pi)$$

$$= \underline{398 \text{ V.}}$$

For the combined system:

$$I = \frac{V}{\sqrt{\left[R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2\right]}}$$

$$\therefore \quad 1{\cdot}5 = \frac{V}{\sqrt{\left[1600 + \left(\frac{10^4}{12\pi} - 64\pi\right)^2\right]}}$$

$$\therefore \quad V = \underline{111 \text{ volts} = \text{supply voltage.}}$$

108. $I = \dfrac{V}{\sqrt{[R^2 + (\omega L)^2]}} = \dfrac{110}{\sqrt{[100 + (100\pi \times 0{\cdot}0271)^2]}} = 8{\cdot}4\text{ A}$

$$\cos\phi = \frac{\text{Resistance}}{\text{Impedance}} = \frac{10}{\sqrt{172}} = 0{\cdot}763.$$

$\therefore$ Power $= VI\cos\phi = 110 \times 8{\cdot}4 \times 0{\cdot}763 = \underline{702\text{ W.}}$

Power $= RI^2 = 10 \times 8{\cdot}4^2 = \underline{702\text{ W.}}$

Note:

$$\begin{aligned} \text{Power} &= VI\cos\phi. \\ &= (IZ)I\,.\,(R/Z) \\ &= \underline{RI^2}. \end{aligned}$$

109. Resistance of lamps $= 100/3{\cdot}2 = 34{\cdot}4\ \Omega$.

$\therefore$ Total resistance in circuit $= 34{\cdot}4 + 4\ \Omega = 38{\cdot}4\ \Omega$.

Total impedance $= 210/3{\cdot}2 = 65{\cdot}6\ \Omega$.

$\therefore$ reactance of coil $= \sqrt{(65{\cdot}6^2 - 38{\cdot}4^2)} = \underline{53{\cdot}2\ \Omega.}$

Efficiency $= 34{\cdot}4/38{\cdot}4 = 0{\cdot}896 = \underline{89{\cdot}6\%}$

With a series resistor, the total resistance in the circuit $= 210/3{\cdot}2$
$= 65{\cdot}6\ \Omega$

$\therefore$ series resistance $= 65{\cdot}6 - 34{\cdot}4 = 31{\cdot}2\ \Omega$.

$\therefore$ efficiency $= 34{\cdot}4/65{\cdot}6 = 0{\cdot}524 = \underline{52{\cdot}4\%.}$

Alternative solution.

Power absorbed by lamps = 110 × 3·2 = 352 W.

Power absorbed by coil = $3{\cdot}2^2 \times 4 = 41$ W.

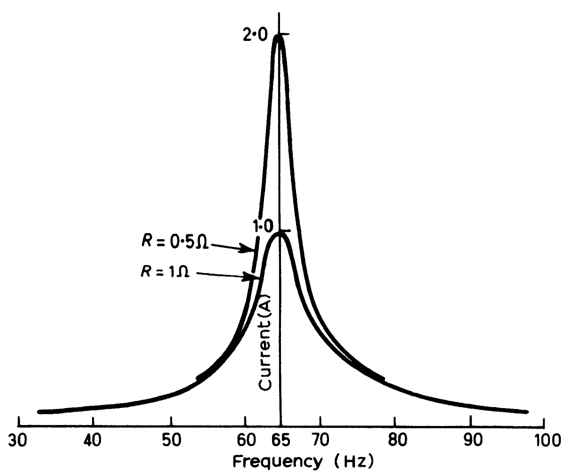
$$\therefore \quad \text{Efficiency of coil} = \frac{352 \times 100}{352 + 41} = \underline{89{\cdot}6\%}$$

Power absorbed by series resistor = (210 − 110) × 3·2 = 320 W.

$$\therefore \quad \text{Efficiency} = \frac{352 \times 100}{352 + 320} = \underline{52{\cdot}4\%}.$$

110. The resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{(LC)}}$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{(0{\cdot}03 \times 200 \times 10^{-6})}} = \frac{10^3}{2\pi\sqrt{6}} = \underline{65 \text{ Hz}}$$



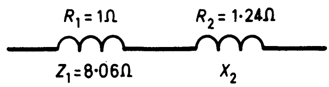
At any other frequency f the current is given by

$$I = \frac{1}{\sqrt{\left[R^2 + \left(L\omega - \dfrac{1}{\omega C}\right)^2\right]}}$$

$$= \frac{1}{\sqrt{\left[R^2 + \left\{(0.03)\,(2\pi f) - \dfrac{10^6}{2\pi f\,.\,200}\right\}^2\right]}}$$

Thus the required *I-f* curves can be drawn for the cases when $R = 1\ \Omega$ and $R = 0{\cdot}5\ \Omega$, as shown.

111.



Let reactance of first coil be X_1.

Then $X_1 = \sqrt{(Z_1^2 - R_1^2)} = \sqrt{(8{\cdot}06^2 - 1)}$

$= 8\ \Omega.$

The current $I = \dfrac{200}{\sqrt{[(R_1 + R_2)^2 + (X_1 + X_2)^2]}} = 6{\cdot}3$ A.

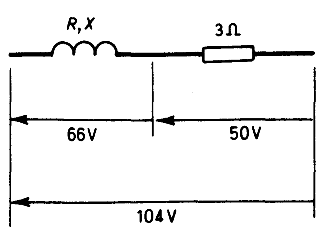
$\therefore\quad 200/6{\cdot}3 = \sqrt{[(1 + 1{\cdot}24^2) + (8 + X_2)^2]}$

$\therefore\quad X_2 = 23{\cdot}67.$

$\therefore\quad \omega L_2 = X_2 = 23{\cdot}67$ and $\omega = 2\pi\ .\ 50.$

$\therefore\quad L_2 = 23{\cdot}67/100\pi$ H.

i.e. $\underline{L_2 = 0{\cdot}075\ \text{H}.}$



112. Let resistance of coil be R and its reactance X.

For the non-inductive resistor

$I = 50/3$ A (1)

For the coil

$$66 = I\sqrt{(R^2 + X^2)} = (50/3)\sqrt{(R^2 + X^2)} \quad (2)$$

For the whole circuit

$$104 = I\sqrt{[(R + 3)^2 + X^2]}$$

$$= (50/3)\sqrt{[(R + 3)^2 + X^2]} \quad (3)$$

From (2) and (3) eliminating X, $R = 2{\cdot}375\ \Omega$

$\therefore$ Power absorbed by coil $= RI^2$

$$= 2{\cdot}375 \times \left(\frac{50}{3}\right)^2 \text{W}$$

$$= \underline{660 \text{ W.}}$$

Power absorbed by coil is also $= VI\cos\phi$

$$\therefore \quad 66 \,.\, (50/3) \,.\, \cos\phi = 660$$

$$\therefore \quad \cos\phi = 3/5 = \underline{0{\cdot}6.}$$

113. Let the resistance and reactance of the coil be R and X respectively.

With d.c. applied:

$$\text{Power} = V^2/R = 150$$

$$\therefore \quad 900/R = 150 \text{ or} R = 6\,\Omega.$$

With a.c. applied:

$$\text{Power} = VI\cos\phi = 3174.$$

$$\therefore \quad 230\,I\cos\phi = 3174.$$

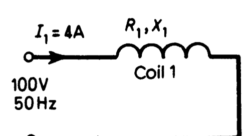
Also $I = 230/\sqrt{(36 + X^2)}$ and $\cos\phi = 6/\sqrt{(36 + X^2)}$

$$\therefore \quad 230 \,.\, \frac{230}{\sqrt{(36 + X^2)}} \,.\, \frac{6}{\sqrt{(36 + X^2)}} = 3174$$

$$\therefore \quad X^2 = 64$$

i.e. $\underline{X = 8\,\Omega.}$

114. Power dissipated in coil 1 = $R_1 I_1{}^2 = 240$ W



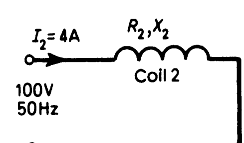
$\therefore$ $R_1 = 240/16 = 15\,\Omega.$

Impedance of coil 1 $= \sqrt{(R_1{}^2 + X_1{}^2)}$

$$= 100/4$$

$$\therefore \quad X_1 = 20\,\Omega.$$

Similarly, for coil 2:



$$R_2 = 320/16 = 20\,\Omega$$

and

$$\sqrt{(R_2{}^2 + X_2{}^2)} = 100/4$$

$$\therefore \quad X_2 = 15\,\Omega.$$

When the coils are connected in series:

$$\text{Total reactance } X = X_1 + X_2 = 35\,\Omega.$$

$$\text{Total resistance } R = R_1 + R_2 = 35\,\Omega.$$

$$\therefore \qquad \text{current } I = 100\sqrt{(35^2 + 35^2)} = \underline{2{\cdot}02\text{ A.}}$$

If the phase angle between the voltage and current is ϕ then $\tan\phi = X/R = 1$

$$\therefore \qquad \text{the power-factor, } \cos\phi = \underline{0{\cdot}707}$$

115. Current $I = 4$ A when frequency $f = 50$ Hz and 100 Hz $I = 5$ A at resonance.

$$\therefore \qquad \text{resistance } R = 200/5 = \underline{40\,\Omega}$$

At 50 Hz and 100 Hz impedance $Z = 200/4 = 50\ \Omega$.

At 50 Hz: $\omega L < 1/\omega C$

$$Z_{50} = \sqrt{\left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]} = 50$$

i.e. $$50 = \sqrt{\left[40^2 + \left(\frac{1}{100\pi C} - 100\pi L\right)^2\right]} \qquad (1)$$

At 100 Hz: $\omega L > 1/\omega C$

$$\therefore \qquad Z_{100} = 50 = \sqrt{\left[40^2 + \left(200\pi L - \frac{1}{200\pi C}\right)^2\right]} \qquad (2)$$

Squaring both sides of (1) and (2) gives:

$$50^2 = 40^2 + \left(\frac{1}{100\pi C} - 100\pi L\right)^2 \qquad (3)$$

$$50^2 = 40^2 + \left(200\pi L - \frac{1}{200\pi C}\right)^2 \qquad (4)$$

(3) gives $$\left(\frac{1}{100\pi C} - 100\pi L\right) = 30 \qquad (5)$$

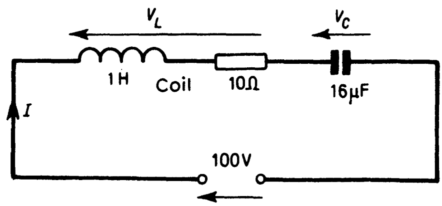
(4) gives: $$\left(200\pi L - \frac{1}{200\pi C}\right) = 30 \qquad (6)$$

From (5) and (6) $$L = \underline{0{\cdot}0955\text{ H}}$$

$$C = 1/6000\pi\text{ F}$$

$$= \underline{53\ \mu\text{F.}}$$

116.



(*a*) $f = 35$ Hz so $\omega = 2\pi \times 35 = 70\pi$

$$I = \frac{100}{\sqrt{\left[10^2 + \left(70\pi - \frac{10^6}{70\pi \times 16}\right)^2\right]}}$$

$$= \frac{100}{\sqrt{[100 + (219{\cdot}94 - 284{\cdot}3)^2]}}$$

$$\therefore \qquad I = \frac{100}{\sqrt{(100 + 64{\cdot}4^2)}} = \underline{1{\cdot}53 \text{ A.}}$$

If the phase angle between I and the supply voltage is ϕ,

$$\tan\phi = -(64{\cdot}4)/(10) = -6{\cdot}44 \text{ so } \phi = 81^\circ\, 12'$$

$$\text{Power-factor, } \cos\phi = \underline{0{\cdot}153.}$$

Power supplied $= 100 \times 1{\cdot}53 \times 0{\cdot}153 = \underline{23{\cdot}4 \text{ W.}}$

[Note: power supplied is also $10 \times 1{\cdot}53^2 = 23{\cdot}4$ W.]

$$V_L = I\sqrt{[R_L{}^2 + \omega^2 L^2]} = 1{\cdot}53\sqrt{[10^2 + (70\pi)^2]}$$
$$= \underline{337 \text{ V.}}$$

$$V_C = I/\omega C = (1{\cdot}53 \times 10^6)/(70\pi \times 16) = \underline{435 \text{ V.}}$$

(*b*) *At the resonant frequency* (f_r)

At resonance, $\omega L = 1/\omega C$

$$\therefore \qquad f_r = 1/2\pi\sqrt{(LC)}$$

$$\therefore \qquad f_r = \frac{1}{2\pi\sqrt{(1 \times 16 \times 10^{-6})}} = 39{\cdot}8 \text{ Hz}$$

At resonance $I = 100/10 = \underline{10 \text{ A.}}$

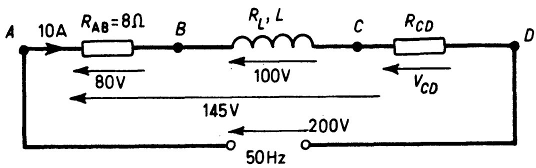
and power-factor $= \underline{1.}$

Power supplied $= 10 \times 10^2 = \underline{1000 \text{ W.}}$

$$V_L = 10\sqrt{[10^2 + (2\pi \times 39{\cdot}8 \times 1)^2]} = \underline{2502\ \text{V.}}$$

$$V_C = 10/\omega C = \underline{2500\ \text{V.}}$$

117.



$$R_{AB} = 80/10 = 8\ \Omega.$$

$$100 = 10\sqrt{[R_L{}^2 + \omega^2 L^2]} \quad (1)$$

$$145 = 10\sqrt{[(8 + R_L)^2 + \omega^2 L^2]} \quad (2)$$

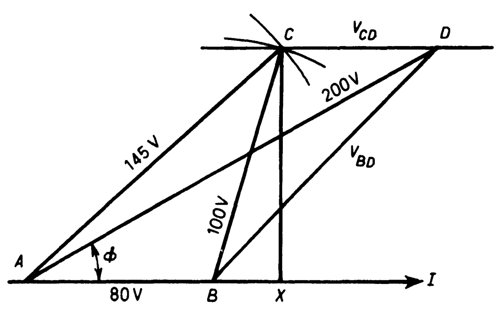
$\therefore$ $\underline{R_L = 2{\cdot}89\ \Omega.}$

and $\underline{L = 0{\cdot}03\ \text{H}}$ since $\omega = 100\pi$.

Also, $200 = 10\sqrt{[(8 + 2{\cdot}89 + R_{CD})^2 + \omega^2 L^2]}$

$\therefore$ $R_{CD} = 6{\cdot}7\ \Omega$ and $V_{CD} = 67$ V.

The phasor diagram is as shown



$$\text{Power-factor} = \cos\phi = (AB + BX + CD)/200$$

$$= (80 + 29 + 67)/200$$

$$= \underline{0{\cdot}88.}$$

Voltage across BD is represented by BD on the phasor diagram.

$$BD^2 = CX^2 + (BX + CD)^2$$

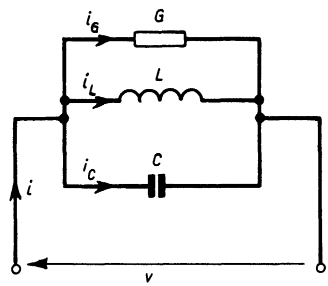
$\therefore$ $V_{BD} = \underline{135{\cdot}6\ \text{V.}}$

118. For the series circuit given:

$$v = v_R + v_L + v_C = Ri + L\frac{di}{dt} + \left(\int idt\right)/C$$

The dual is the parallel circuit shown for which:

$$i = i_G + i_L + i_C = Gv + C\frac{dv}{dt} + \left(\int vdt\right)/L$$



119. A sketch of the phasor diagram is shown. The resultant current is I, the phasor sum of the 12A and 22A currents. The current I lags the voltage V by $\phi°$

Resolving in the X and Y directions:

$X = 12 \cos 30° + 22 \cos 45°$

$= 10{\cdot}39 + 15{\cdot}56$

$= 25{\cdot}95$ A.

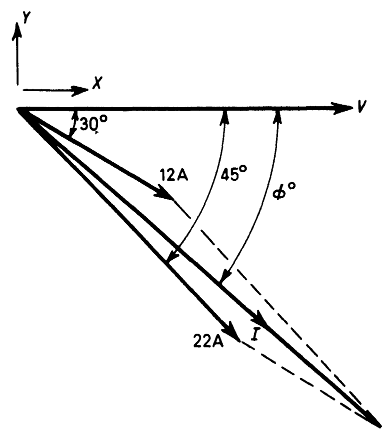
$Y = -(12 \sin 30° + 22 \sin 45°)$

$= -(6 + 15{\cdot}56)$

$= -21{\cdot}56$ A.

$\therefore I = \sqrt{(25{\cdot}95^2 + 21{\cdot}56^2)}$

$= \underline{33{\cdot}74 \text{ A.}}$



$\tan \phi = 21{\cdot}56/25{\cdot}95$ $\therefore \phi = \underline{39° 45'}.$

120. Current through coil

$$= \frac{200}{\sqrt{[25 + (0 \cdot 03 \times 100\pi)^2]}}$$

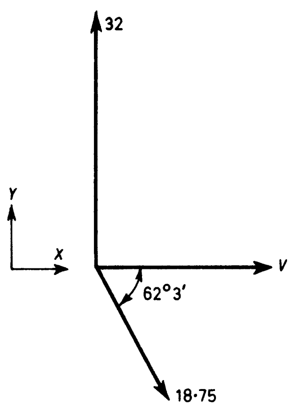
$$= \underline{18 \cdot 75 \text{ A}} = I_L \text{ say.}$$

Angle of lag of I_L behind applied voltage is ϕ where $\tan \phi = \omega L/R$

i.e. $\tan \phi = (0 \cdot 03 \times 100\pi)/5 = 3\pi/5$

$\therefore$ $\underline{\phi = 62° 3'}$.

A sketch of the phasor diagram is shown.



Total current flowing in the external circuit is the phasor sum of the two separate currents. Resolving these currents in the X and Y directions:

$$X = 18 \cdot 75 \cos 62° 3' = 8 \cdot 788 \text{ A.}$$

$$Y = 32 - 18 \cdot 75 \sin 62° 3' = 15 \cdot 44 \text{ A}$$

$\therefore$ Resultant current $= \sqrt{(X^2 + Y^2)}$

$$= \sqrt{(8 \cdot 788^2 + 15 \cdot 44^2)} = \sqrt{315 \cdot 5}$$

$$= \underline{17 \cdot 75 \text{ A.}}$$

Phase angle between this current and the applied voltage is ϕ where $\tan \phi = Y/X = 15 \cdot 44/8 \cdot 788$

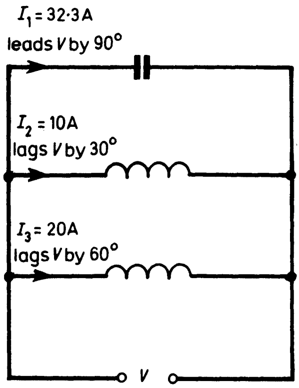
$\therefore$ $\underline{\phi = 60° 21'}$.

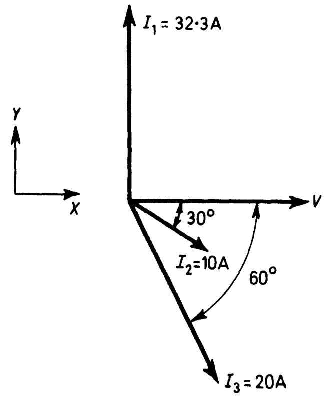
<u>The current leads the voltage by 60° 21′.</u>

121. A sketch of the phasor diagram is shown. The total current flowing is the phasor sum of I_1, I_2 and I_3. Resolving in the X and Y directions:

$$X = 10 \cos 30° + 20 \cos 60° = 18 \cdot 66 \text{ A.}$$

$$Y = 32 \cdot 3 - 10 \sin 30° - 20 \sin 60° = 10 \text{ A.}$$



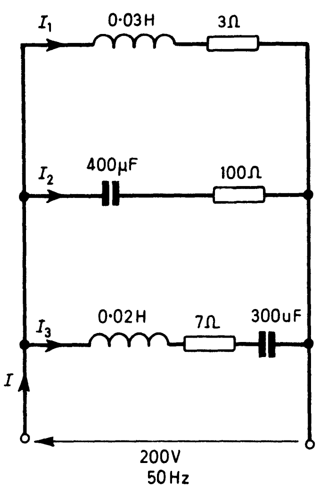


$\therefore$ Resultant current $= \sqrt{(X^2 + Y^2)} = \sqrt{(10^2 + 18{\cdot}66^2)} = \underline{21{\cdot}18\text{ A}}$

The resultant current leads V by an angle ϕ, where $\tan\phi \doteq Y/X = 10/18{\cdot}66$

i.e. $\underline{\phi = 28°\ 12'.}$

122. Let the currents be I, I_1, I_2, and I_3 as shown.



$$I_1 = \frac{200}{\sqrt{[3^2 + (0{\cdot}03 \times 100\pi)^2]}}$$

i.e. $I_1 = 20{\cdot}2$ A.

If ϕ_1 is the angle by which I_1 lags the applied voltage then

$\tan\phi_1 = 0{\cdot}03 \times 100\pi/3 = \pi = 3{\cdot}142$

$\therefore\ \phi_1 = 72°\ 18'.$

$$I_2 = \frac{200}{\sqrt{\left[100^2 + \left(\dfrac{10^6}{400\ .\ 100\pi}\right)^2\right]}}$$

i.e. $I_2 = 1{\cdot}99$ A.

If ϕ_2 is the angle by which I_2 leads the applied voltage, then

$$\tan \phi_2 = \frac{10^6}{(400 \,.\, 100\pi) \,.\, 100} = 0{\cdot}0796$$

i.e. $\phi_2 = 4° \, 32'$.

$$I_3 = \frac{200}{\sqrt{\left[7^2 + \left(0{\cdot}02 \times 100\pi - \frac{10^6}{300 \times 100\pi}\right)^2\right]}}$$

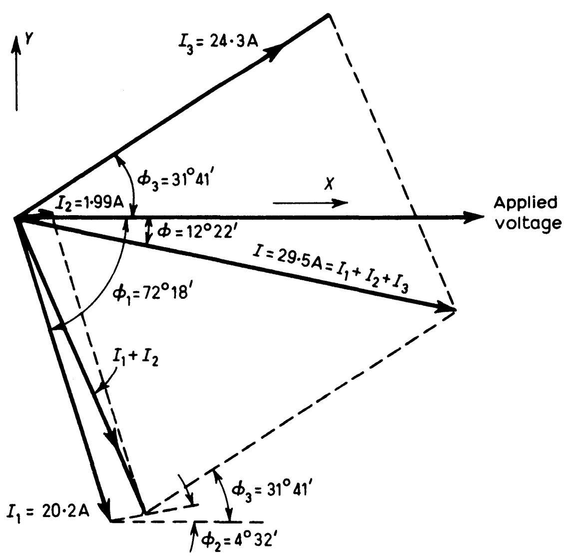
$$= \frac{200}{\sqrt{[7^2 + (6{\cdot}284 - 10{\cdot}6)^2]}}$$

i.e. $I_3 = 24{\cdot}3$ A.

I_3 leads the applied voltage by an angle ϕ_3, where

$$\tan \phi_3 = (10{\cdot}6 - 6{\cdot}284)/7 = 0{\cdot}617$$

i.e. $\phi_3 = 31° \, 41'$.



The complete phasor diagram can thus be drawn as shown, and the resultant current I and the resultant phase angle ϕ can be obtained graphically if desired. I and ϕ can also be calculated as follows:

Resolving in the X and Y directions (see phasor diagram)

$$X = I_1 \cos \phi_1 + I_2 \cos \phi_2 + I_3 \cos \phi_3$$
$$= 20{\cdot}2 \cos 72^\circ 18' + 1{\cdot}99 \cos 4^\circ 32' + 24{\cdot}3 \cos 31^\circ 41'$$
$$= 6{\cdot}142 + 1{\cdot}985 + 20{\cdot}67$$
$$= 28{\cdot}797 \text{ A.}$$

$$Y = I_2 \sin \phi_2 + I_3 \sin \phi_3 - I_1 \sin \phi_1$$
$$= 1{\cdot}99 \sin 4^\circ 32' + 24{\cdot}3 \sin 31^\circ 41' - 20{\cdot}2 \sin 72^\circ 18'$$
$$= 0{\cdot}157 + 12{\cdot}77 - 19{\cdot}24$$
$$= -6{\cdot}313 \text{ A.}$$

$\therefore$ Resultant current $I = \sqrt{[X^2 + Y^2]} = \sqrt{[(28{\cdot}797)^2 + (6{\cdot}313)^2]}$

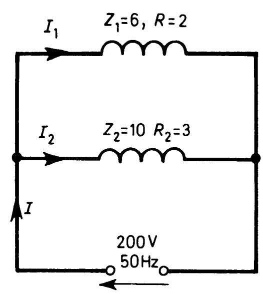
$$= \underline{29{\cdot}5 \text{ A.}}$$

The angle of lag of I behind the applied voltage is ϕ, where

$$\tan \phi = 6{\cdot}323/28{\cdot}797$$

i.e. $\underline{\phi = 12^\circ 22'.}$

123. Let the currents be I_1, I_2, and I as shown.



Then

$$I_1 = 200/Z_1 = 200/6 = \underline{33{\cdot}3} \text{ A.}$$

This current will lag the applied voltage by an angle ϕ_1, where

$$\cos \phi_1 = R_1/Z_1 = 2/6 = 1/3.$$

i.e. $\underline{\phi_1 = 70^\circ 32'.}$

Similarly

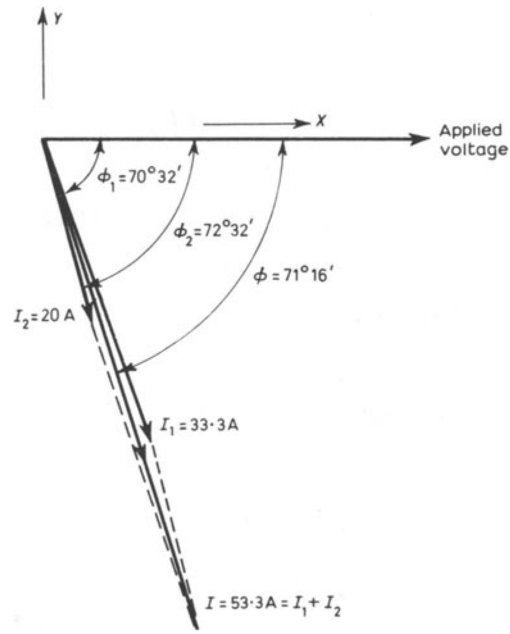
$$I_2 = 200/Z_2 = 200/10 = \underline{20 \text{ A.}}$$

I_2 will lag the applied voltage by an angle ϕ_2, where

$$\cos \phi_2 = R_2/Z_2 = 3/10 = 0{\cdot}3$$

i.e. $\phi_2 = \underline{72^\circ 32'.}$

The complete phasor diagram can thus be drawn as shown, and the resultant current I and the resultant phase angle ϕ can be obtained graphically if desired. I and ϕ can also be calculated as follows:



Resolving in the X and Y directions (see phasor diagram)

$$X = I_1 \cos 70°\,32' + I_2 \cos 72°\,32'$$

$$= \frac{200}{6} \cdot \frac{1}{3} + 20 \cdot \frac{3}{10}$$

$$= 17{\cdot}1 \text{ A.}$$

$$Y = -(I_1 \sin 70°\,32' + I_2 \sin 72°\,32')$$

$$= -(33{\cdot}3 \sin 70°\,32' + 20 \sin 72°\,32')$$

$$= -50{\cdot}46 \text{ A.}$$

The resultant current $I = \sqrt{(X^2 + Y^2)}$

$$= \sqrt{[(17{\cdot}1)^2 + (50{\cdot}46)^2]}$$

$$= \underline{53{\cdot}3 \text{ A.}}$$

The resultant current I lags the applied voltage by an angle ϕ, where

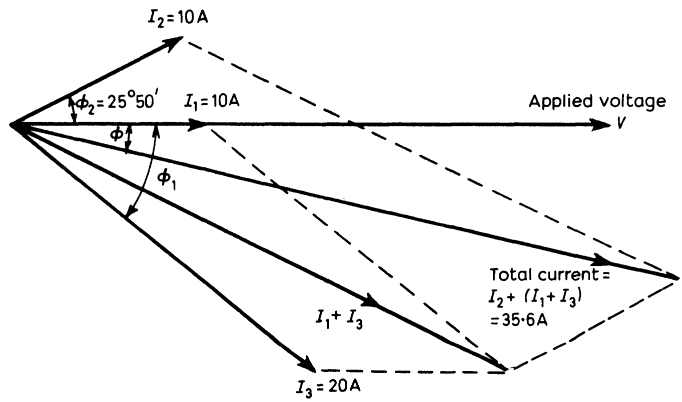
$$\tan\phi = Y/X = 50{\cdot}46/17{\cdot}1$$

$$\therefore \qquad \phi = \underline{71^\circ\, 16'.}$$

$$\therefore \qquad \text{Total power-factor} = \cos\phi = \cos 71^\circ\, 16' = \underline{0{\cdot}32.}$$

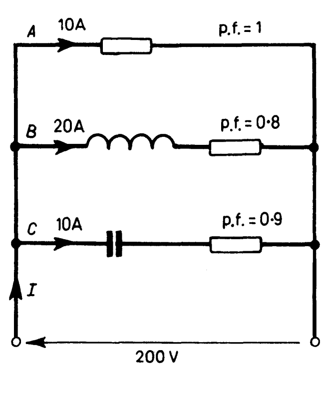
124. The 20-A current through the coil and resistor will lag the applied voltage by an angle ϕ_1, where $\cos\phi_1 = 0{\cdot}8$ i.e. $\phi_1 = 36^\circ\, 52'$. The 10-A current through the capacitor branch leads the applied voltage, by an angle ϕ_2, where $\cos\phi_2 = 0{\cdot}9$ i.e. $\phi_2 = 25^\circ\, 50'$.

Thus, the phasor diagram is as shown and the total current supplied is the phasor sum of the three individual currents, i.e. 35·6 A.

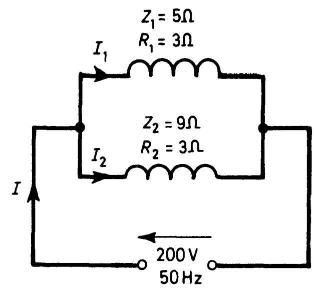


125. The 20-A current lags the applied voltage by an angle ϕ_B where $\cos\phi_B = 0{\cdot}8$, so $\sin\phi_B = 0{\cdot}6$ ($\phi_B = 36^\circ\, 52'$).

The 10-A current in circuit C leads the applied voltage by an angle ϕ_C where $\cos\phi_C = 0{\cdot}9$, so $\sin\phi_C = 0{\cdot}4357$ ($\phi_C = 25^\circ\, 50'$.)



The phasor diagram is the same therefore as the one given in Solution 124.



Resolving the currents in the X and Y directions as in previous solutions i.e. in the direction of the applied voltage and also at right angles to it.:

$$X = 10 + 10\cos\phi_C + 20\cos\phi_B$$

$$= (10 + 9 + 16)\text{ A} = 35\text{ A}$$

$$Y = 10\sin\phi_C - 20\sin\phi_B$$

$$= (4{\cdot}357 - 12)\text{ A} = -7{\cdot}643\text{ A}$$

Resultant current $I = \sqrt{(X^2 + Y^2)} = \sqrt{(35^2 + 7{\cdot}643^2)} = \underline{35{\cdot}8\text{ A}}.$

If the angle between I and the applied voltage is ϕ, $\tan\phi = Y/X = -7{\cdot}643/35$ so $\phi = -12^\circ\ 19'$.

The power-factor, $\cos\phi = \underline{0{\cdot}977}.$

Power supplied $= 200 \times 35{\cdot}8 \times 0{\cdot}977 = \underline{7000\text{ W}}.$

Note.

Power absorbed in circuit A $= 200 \times 10$ W $= 2000$ W

Power absorbed in circuit B $= 200 \times 20 \times 0{\cdot}8 = 3200$ W

Power absorbed in circuit C $= 200 \times 10 \times 0{\cdot}9 = 1800$ W

$\therefore$ total power absorbed $= \underline{7000\text{ W}}$

126.

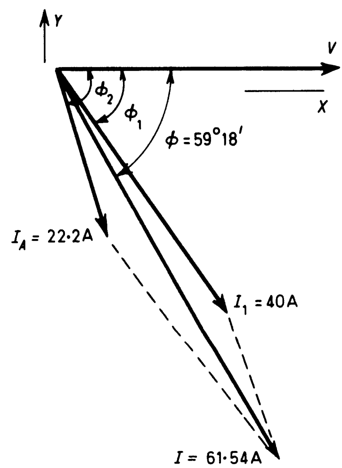
$I_1 = 200/5 = \underline{40\ \text{A}}$

$I_2 = 200/9 = \underline{22{\cdot}2\ \text{A}}$

$$\text{Reactance } X_1 = \sqrt{(Z_1{}^2 - R_1{}^2)}$$
$$= \sqrt{(5^2 - 3^2)}\ \Omega$$
$$= 4\ \Omega$$

$$\text{Reactance } X_2 = \sqrt{(Z_2{}^2 - R_2{}^2)}$$
$$= \sqrt{(9^2 - 3^2)}\ \Omega$$
$$= 8{\cdot}485\ \Omega.$$

The phasor diagram is shown below:



$\tan \phi_1 = X_1/R_1 = 4/3 = 1{\cdot}333,$ so $\phi_1 = 53°\ 7'$

$\tan \phi_2 = X_2/R_2 = 8{\cdot}485/3 = 2{\cdot}828,$ so $\phi_2 = 70°\ 30'$

Resolving in the X and Y directions:

$$X = 40 \cos 53°\ 7' + 22{\cdot}2 \cos 70°\ 30'$$
$$= 24{\cdot}01 + 7{\cdot}41 = 31{\cdot}42$$
$$Y = -40 \sin 53°\ 7' - 22{\cdot}2 \sin 70°\ 30'$$
$$= -32 - 20{\cdot}92 = -52{\cdot}92$$

The resultant current $I = \sqrt{(X^2 + Y^2)} = \sqrt{(31{\cdot}42^2 + 52{\cdot}92^2)}$

$$= \underline{61{\cdot}54\ \text{A.}}$$

The phase angle ϕ is given by $\tan\phi = Y/X = -52{\cdot}92/31{\cdot}42$

$$= -1{\cdot}685$$

The power-factor $= \cos\phi = \underline{0{\cdot}51.}$

Alternative Solution

Admittance of coil 1, $Y_1 = 1/5$ S

The conductance G_1 and susceptance B_1 are therefore given by:

$$G_1 = R_1/Z_1{}^2 = 3/25 \text{ S}$$

and $$B_1 = X_1/Z_1{}^2 = 4/25 \text{ S}$$

Similarly for coil 2, $Y_2 = 1/9$ S

$\therefore$ $$G_2 = 3/81 \text{ S}$$

and $$B_2 = 6\sqrt{2}/81 \text{ S}$$

Total conductance $G_T = G_1 + G_2 = 0{\cdot}157$ S

Total susceptance $B_T = B_1 + B_2 = 0{\cdot}265$ S

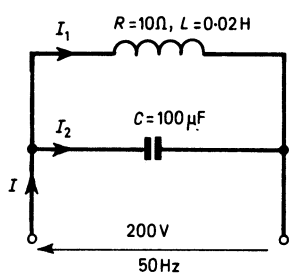
Total admittance $Y_T = \sqrt{(G_T{}^2 + B_T{}^2)} = 0{\cdot}308$ S

Total impedance $Z_T = 1/Y_T = 3{\cdot}26\ \Omega$

$\therefore$ $$I = 200/3{\cdot}26 = \underline{61{\cdot}54 \text{ A.}}$$

and $$\cos\phi = 0{\cdot}157/0{\cdot}308 = \underline{0{\cdot}51.}$$

127



$$I_1 = \frac{200}{\sqrt{(R^2 + \omega^2L^2)}}$$

$$= \frac{200}{\sqrt{[10^2 + (0{\cdot}02 \times 2\pi \times 50)^2]}}$$

$= 16{\cdot}93$ A.

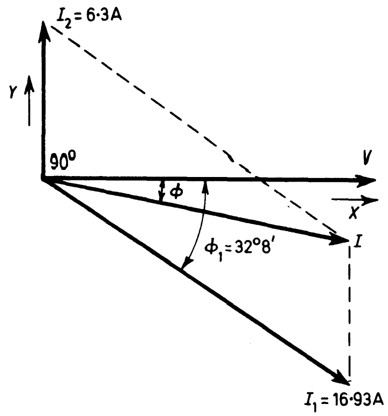
I_1 lags the applied voltage by an angle ϕ_1, where $\tan\phi_1 = 0{\cdot}02 \times 2\pi \times 50/10$

$\therefore$ $$\phi_1 = 32°\ 8'.$$

$$I_2 = 200\omega C = 200(2\pi \times 50)\ 100 \times 10^{-6} = 6{\cdot}3 \text{ A.}$$

I_2 leads the applied voltage by 90^0.

A sketch of the phasor diagram is given below:



Resolving in the X and Y directions:

$Y = 16{\cdot}93 \cos 32° \, 8' = 14{\cdot}33$

$Y = 6{\cdot}3 - 16{\cdot}93 \sin 32° \, 8'$
$= -2{\cdot}7$

Resultant current I

$= \sqrt{(X^2 + Y^2)}$

$= \sqrt{(14{\cdot}33^2 + 2{\cdot}7^2)}$

$= \underline{14{\cdot}57 \text{ A.}}$

The phase-angle $\phi = \tan^{-1}(Y/X) = \tan^{-1}(-2{\cdot}7/14{\cdot}33)$ so $\phi = -10° \, 40'$.

The power-factor $\cos \phi = \underline{0{\cdot}98 \text{ (lagging).}}$

Let the capacitance of the second capacitor be $C' \mu$F.

Then $200\omega C'/10^6 = I \sin \phi = 14{\cdot}57 \sin 10° \, 40'$

$$\therefore \qquad C' = \frac{14{\cdot}57 \sin 10° \, 40' \times 10^6}{200(2\pi \times 50)}$$

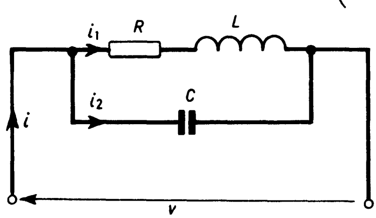
i.e. $\underline{C' = 43 \mu\text{F.}}$

128. The circuit to be analysed is shown in the illustration where C is the effective self-capacitance.

Now
$$i_1 = \frac{v}{R + j\omega L} = \frac{v(R - j\omega L)}{R^2 - \omega^2 L^2}$$

and
$$i_2 = v(j\omega C)$$

$$\therefore \; i = \frac{vR}{R^2 - \omega^2 L^2} + jv\left(\omega C - \frac{\omega L}{R^2 - \omega^2 L^2}\right)$$



In order that the coil circuit may be 'non-inductive' i must be in phase with v so that:

$$\omega C = \omega L/(R^2 - \omega^2 L^2)$$

or

$$C = L/(R^2 - \omega^2 L^2)$$

This value of C depends on ω, i.e. on frequency so the answer to the second part of the question is "No".

At a frequency of 50 Hz:

$$C = 20 \times 10^{-6}/\{100^2 - (2\pi \times 50 \times 20 \times 10^{-6})^2\}\text{F}$$

$$= \underline{0{\cdot}002\mu\text{F}.}$$

129. Current $I_L = \dfrac{V}{\sqrt{[R^2 + L^2\omega^2]}} = \dfrac{200}{\sqrt{[25 + (0{\cdot}02)^2\, 100^2\pi^2]}}$

$$= \underline{24{\cdot}9\text{ A}.}$$

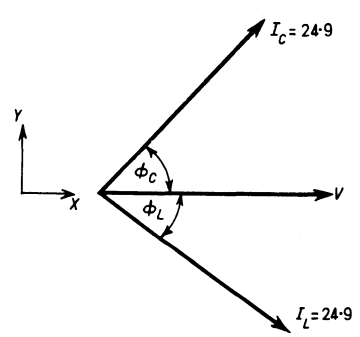
In the case of the capacitor circuit,

$$I_C = \frac{V}{\sqrt{[R^2 + (1/C\omega)^2]}} = 24{\cdot}9\text{ A}.$$

$$\therefore \quad 24{\cdot}9 = \frac{200}{\sqrt{[25 + (1/100^2\pi^2C^2)]}}$$

$$\therefore \quad C = \frac{1}{314{\cdot}2\sqrt{39{\cdot}48}}\text{ F} = \underline{508\ \mu\text{F}.}$$

The current I_C will lead the voltage V by an angle ϕ_c, and current I_L will lag V by an angle ϕ_L.



$$\tan \phi_L = \omega L/R = (0{\cdot}02 \times 100\pi)/5$$

$$\therefore \qquad \phi_L = 51^\circ\, 30'.$$

$$\tan \phi_C = 1/\omega CR = 10^6/(100\pi \times 508 \times 5)$$

$$\therefore \qquad \phi_C = 51^\circ\, 24'.$$

When the circuits are in parallel, the resultant current is the phasor sum of I_C and I_L.

Resolving in the X and Y directions:

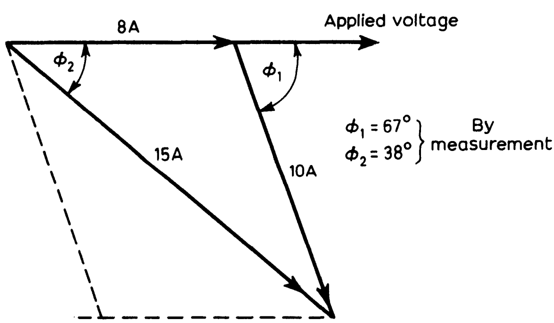
$$X = 24{\cdot}9\,\{\cos 51^\circ\, 24' + \cos 51^\circ\, 30'\}$$

$$= 24{\cdot}9\,\{1{\cdot}2464\} = 31{\cdot}02 \text{ A}.$$

$$Y = 24{\cdot}9\,\{\sin 51^\circ\, 24' - \sin 51^\circ\, 30'\} = 0 \text{ approx.}$$

$\therefore$ Resultant current is approximately 31 A and is in phase with V.

130. The resistor takes a current of (200/25) A = 8 A and this is in phase with the applied voltage. Thus for the parallel combination of coil and resistor the phasor diagram is as follows:



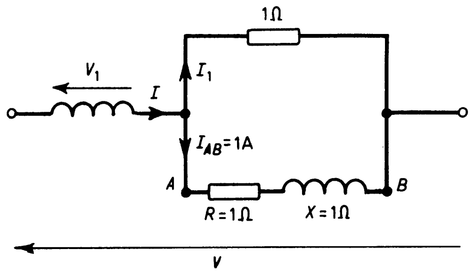
The power-factor of the coil is $\cos \phi_1 = 0{\cdot}39$.
The power-factor of the combination is $\cos \phi_2 = 0{\cdot}79$.
When a capacitor is added, it will take a current leading the applied voltage by 90°. The resultant of this current and the 15A one must be in phase with the applied voltage for unity power-factor.

i.e. capacitor current $I_c = 15 \sin \phi_2 = 15 \sin 38^\circ = 9{\cdot}234$ A.

If capacitance $= C$ then $200 = 9{\cdot}234/C\,.\,100\pi$

$$\therefore \qquad C = \frac{9{\cdot}234}{200 \times 100\pi} \times 10^6\ \mu\text{F} = \underline{147\ \mu\text{F}}.$$

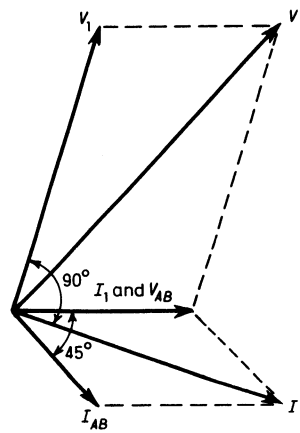
131. The voltage across the coil $V_{AB} = 1\sqrt{(1^2 + 1^2)}$ $= \sqrt{2}$ volts.



This leads the current I_{AB} by 45°.

Current I_1 is therefore $\sqrt{2}$ A in phase with V_{AB}.

Current I is the phasor sum of I_1 and I_{AB} and voltage $V_1 = 1 \times I$.



V is the phasor sum of V_1 and V_{AB}. The phasor diagram is therefore as shown. V is found to be 3 volts and leads I_{AB} by 90°.

132. Let the impedance, resistance and reactance be Z, R and X respectively and denote the current by I and the applied voltage by V. Then

$$RI^2 = VI\cos\phi = 2000 \text{ W}$$

$$\therefore \quad R(25)^2 = 2000, \quad \text{so} \quad R = \underline{3{\cdot}2\ \Omega}$$

Also,

$$Z = V/I = (110/25)\ \Omega = (22/5)\ \Omega$$

$$R = 3{\cdot}2\ \Omega = (16/5)\ \Omega$$

$$X = \sqrt{(Z^2 - R^2)} = \sqrt{(22^2 - 16^2)}/5 = \underline{3{\cdot}02\ \Omega}$$

If the phase angle between I and V is ϕ,

$$\cos\phi = R/Z = 8/11$$

$$\therefore \quad \sin\phi = \sqrt{(57)}/11$$

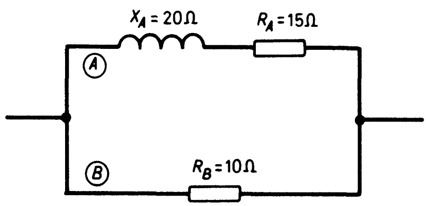
Parallel capacitor must have capacitance C such that

$$V\omega C = 25 \sin \phi = 25\sqrt{(57)}/11$$

where ω is the angular frequency $= 2\pi \times 100$.

$$\therefore \qquad C = \frac{25\sqrt{57}}{11 \times 110 \times 2\pi \times 100} \text{ F} = \underline{248 \cdot 2 \,\mu\text{F}.}$$

133.



For coil A,

$X_A = \sqrt{(25^2 - 15^2)} = 20\Omega$

$$G_A = 15/(15^2 + 20^2) = 0 \cdot 024 \text{ S}$$
$$G_B = 1/10 = 0 \cdot 1 \text{ S}$$
$$B_A = 20/(15^2 + 20^2) = 0 \cdot 032 \text{ S}$$
$$B_B = 0.$$

Total conductance $G_T = G_A + G_B = 0 \cdot 124$ S

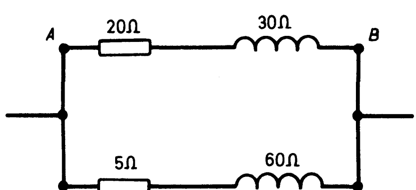
Total susceptance $B_T = B_A + B_B = 0 \cdot 032$ S

Total admittance $T_T = \sqrt{(G_T{}^2 + B_T{}^2)} = \sqrt{(0 \cdot 124^2 + 0 \cdot 032^2)}$

$$= \underline{0 \cdot 128 \text{ S}}$$

134. Impedance of branch $AB = \sqrt{(20^2 + 30^2)} = \underline{36\ \Omega.}$

$\therefore$ Admittance of branch $AB \quad = 1/36 = \underline{0 \cdot 0278 \text{ S}}$

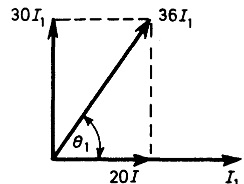


(a)

The phasor diagram for branch *AB* is shown in Fig. (*b*)

$$\tan \theta_1 = 30/20$$

$$\therefore \quad \theta_1 = 56° \, 19'.$$

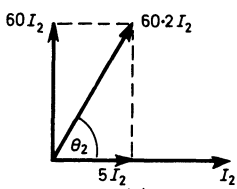


(b)

Impedance of branch *CD* = $\sqrt{(5^2 + 60^2)}$ = 60·2 Ω.

∴ Admittance of branch *CD* = 1/60·2 = 0·0167 S

The phasor diagram for branch CD is shown in Fig. (c)



(c)

$$\tan \theta_2 = 60/5$$

$$\theta_2 = 85° \, 14'.$$

Total conductance = 0·0278 cos 56° 19′ + 0·0167 cos 85° 14′

= 0·0167 S

Total susceptance = 0·0278 sin 56° 19′ + 0·0167 sin 85° 14′

= 0·0397 S

∴ Resultant admittance = $\sqrt{[(0{\cdot}0167)^2 + (0{\cdot}0397)^2]}$

= 0·043 S

135. Resultant admittance of circuit in question 134 is 0·043 S.

∴ Resultant impedance = 1/0·043 = 23·3 Ω.

The admittance consists of two components, a conductance = 0·0167 S and a susceptance = 0·0397 S.

Let $\alpha = \tan^{-1}$ (0·0397/0·0167) i.e. α = 67° 11′.

$\therefore$ equivalent resistance of new circuit $= 23{\cdot}3 \cos 67° 11' + 50$

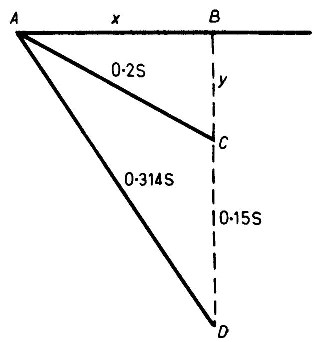
$= \underline{59\ \Omega.}$

Equivalent reactance of circuit $= 23{\cdot}3 \sin 67° 11' + 10$

$= \underline{31{\cdot}5\ \Omega.}$

Resultant impedance $= \sqrt{(59^2 + 31{\cdot}5^2)} = \underline{66{\cdot}9\ \Omega.}$

136. Let the admittance of 0·2 S have components x and y, as shown.



From triangle ABC:

$$x^2 + y^2 = 0{\cdot}2^2 \qquad (1)$$

From triangle ABD:

$$x^2 + (y + 0{\cdot}15)^2 = (0{\cdot}314)^2 \qquad (2)$$

From (1) and (2)

$$x = 0{\cdot}16 \qquad (3)$$

and

$$y = 0{\cdot}12 \qquad (4)$$

Let R and X be the resistance and reactance of the 0·2-S admittance.

Then

$$x = R/(R^2 + X^2) = 0{\cdot}16 \qquad (5)$$

and

$$y = X/(R^2 + X^2) = 0{\cdot}12 \qquad (6)$$

From (5) and (6) dividing,

$$R/X = 4/3 \quad \text{i.e. } X = 3R/4 \qquad (7)$$

Substituting (7) in (5),

$$R = 0{\cdot}16\left[R^2 + \frac{9R^2}{16}\right] = \underline{4\ \Omega.}$$

137. The admittance of the R branch $= 1/R$

The admittance of the X branch $= 1/X$

These admittances are 90° out of phase, so that the resultant

$$\text{admittance} = \sqrt{\left(\frac{1}{R^2} + \frac{1}{X^2}\right)} = \frac{\sqrt{(R^2 + X^2)}}{RX}$$

$\therefore$ the resultant impedance $= \underline{RX/\sqrt{(R^2 + X^2)}}$.

138. One method of solving this type of problem has been given already (see Problem 123). An alternative method is used below.

The admittance of the first coil $= Y_1 = 1/8 = 0{\cdot}125$ S

The power-factor of this coil $= \cos\phi_1 = 7/8 = 0{\cdot}875$.

The conductance of this coil $G_1 = Y_1 \cos\phi_1 = 0{\cdot}125 \times 0{\cdot}875$

$= 0{\cdot}109$ S

The susceptance of this coil $B_1 = \sqrt{(Y_1^2 - G_1^2)}$

$= \sqrt{[(0{\cdot}125)^2 - (0{\cdot}109)^2]}$

$= 0{\cdot}061$ S

Similarly, for the second coil

$$Y_2 = 1/11 = 0{\cdot}091 \text{ S}$$

$$\cos\phi_2 = 4/11 = 0{\cdot}364.$$

$$G_2 = Y_2 \cos\phi_2 = 0{\cdot}033 \text{ S}$$

$$B_2 = \sqrt{(Y_2^2 - G_2^2)} = 0{\cdot}085 \text{ S}$$

The total admittance is the phasor sum of Y_1 and Y_2

$$= Y_T = \sqrt{[(G_1 + G_2)^2 + (B_1 + B_2)^2]}$$

$$= \sqrt{[(0{\cdot}109 + 0{\cdot}033)^2 + (0{\cdot}061 + 0{\cdot}085)^2]}$$

i.e. $Y_T = 0{\cdot}204$ S

$\therefore$ the total current $= Y_T \times$ voltage

$= 0{\cdot}204 \times 300 = \underline{61{\cdot}2 \text{ A.}}$

Current in first coil $= 300 \times Y_1 = 300 \times 0{\cdot}125 = \underline{37{\cdot}5 \text{ A.}}$

Current in second coil $= 300 \times Y_2 = 300 \times 0{\cdot}091 = \underline{27{\cdot}3 \text{ A.}}$

If the angle between Y_T and the applied voltage is ϕ, then

$$\tan\phi = \frac{B_1 + B_2}{G_1 + G_2} = \frac{0{\cdot}061 + 0{\cdot}085}{0{\cdot}109 + 0{\cdot}033} = \frac{0{\cdot}146}{0{\cdot}142}.$$

$$\therefore \qquad \phi = 45^\circ\, 48'.$$

The total power-factor = $\cos\phi$ = $\underline{0{\cdot}697.}$

139. The admittance of the capacitor $= \omega C = 2\pi 50 \,.\, 120 \,.\, 10^{-6}$
$= 0{\cdot}038$ S

This is a pure susceptance, of course.

$\therefore$ total susceptance now $= B_1 + B_2 - 0{\cdot}038$

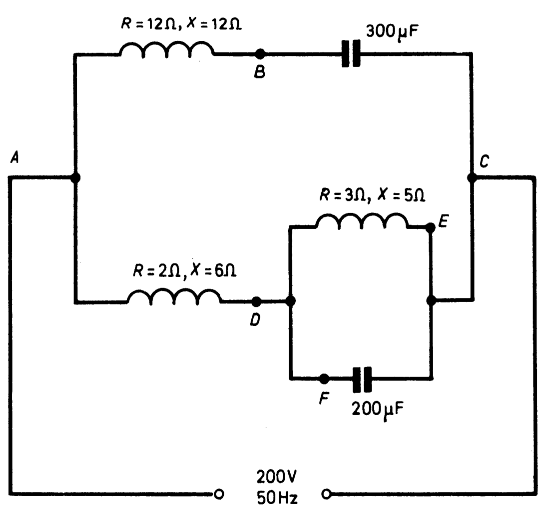
where B_1 and B_2 are defined in solution 138

$\therefore$ total susceptance $= 0{\cdot}061 + 0{\cdot}085 - 0{\cdot}038$ S
$= 0{\cdot}108$ S

$\therefore$ Total admittance $= \sqrt{[(G_1 + G_2)^2 + (0{\cdot}108)^2]}$
$= \sqrt{[(0{\cdot}109 + 0{\cdot}033)^2 + (0{\cdot}108)^2]}$
$= 0{\cdot}178$ S

$\therefore$ Total current $= 300 \times 0{\cdot}178$
$= \underline{53{\cdot}4 \text{ A.}}$

140.



(a)

$$\text{Conductance of } DEC = \frac{R}{R^2 + X^2} = \frac{3}{9 + 25} = 0{\cdot}088 \text{ S}$$

$$\text{Susceptance of } DEC = \frac{R}{R^2 + X^2} = \frac{5}{9 + 25} = 0{\cdot}147 \text{ S}$$

$$\text{Susceptance of } DFC = \frac{-(2\pi\ 50)200}{10^6} = -\,0{\cdot}063 \text{ S}$$

Total susceptance of $DC = 0{\cdot}147 - 0{\cdot}063 = 0{\cdot}084$ S

Total admittance of $DC = \sqrt{[(0{\cdot}088)^2 + (0{\cdot}084)^2]} = 0{\cdot}122$ S

Power-factor of circuit $DC = 0{\cdot}088/0{\cdot}122 = \cos\phi$.

∴ equivalent resistance of $DC = (1/0{\cdot}122)\cos\phi = 5{\cdot}9\ \Omega$.

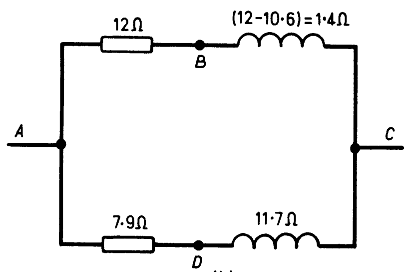
and equivalent reactance of $DC = (1/0{\cdot}122)\sin\phi = 5{\cdot}7\ \Omega$.

∴ total resistance of $ADC = 2 + 5{\cdot}9 = 7{\cdot}9\ \Omega$.

and total reactance of $ADC = 6 + 5{\cdot}7 = 11{\cdot}7\ \Omega$.

$$\text{The reactance of } BC = \frac{-10^6}{(2\pi 50)300} = -10{\cdot}6\ \Omega.$$

∴ the original circuit can be replaced by the simplified one shown at (*b*)



(*b*)

$$\text{Conductance of } ADC = \frac{7{\cdot}9}{(7{\cdot}9)^2 + (11{\cdot}7)^2} = 0{\cdot}040 \text{ S}$$

$$\text{Conductance of } ABC = \frac{12}{(12)^2 + (1{\cdot}4)^2} = 0{\cdot}082 \text{ S}$$

$\therefore$ total conductance = 0·122 S

$$\text{Susceptance of } ACD = \frac{11{\cdot}7}{(7{\cdot}9)^2 + (11{\cdot}7)^2} = 0{\cdot}059 \text{ S}$$

$$\text{Susceptance of } ABC = \frac{1{\cdot}4}{(12)^2 + (1{\cdot}4)^2} = 0{\cdot}010 \text{ S}$$

$\therefore$ total susceptance = 0·069 S

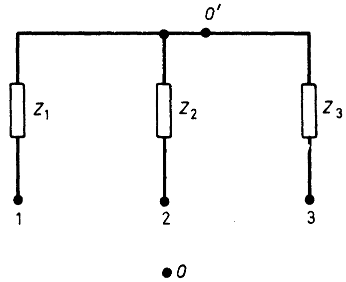
$$\therefore \quad \text{total admittance} = \sqrt{[(0{\cdot}122)^2 + (0{\cdot}069)^2]}$$
$$= 0{\cdot}140 \text{ S}$$

$$\therefore \quad \text{total current} = 0{\cdot}140 \times 200$$
$$= \underline{28 \text{ A.}}$$

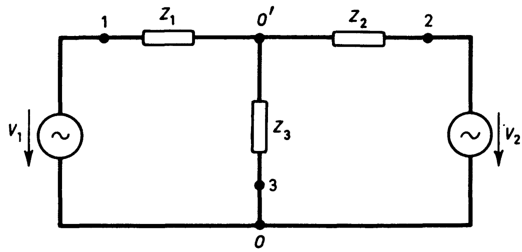
141. Millman's Theorem will first be stated. Consider a number of impedances Z_1, Z_2, Z_3, etc., which terminate at a common point O′. The other ends of the impedances are numbered 1, 2, 3, etc. O is any other point in the network. The Theorem states that the voltage drop from O to O′ ($V_{OO'}$) is given by:

$$V_{OO'} = \frac{V_{O1}Y_1 + V_{O2}Y_2 + V_{O3}Y_3 + \dots}{Y_1 + Y_2 + Y_3 + \dots}$$

where V_{On} (n = 1, 2, 3 etc.) is the voltage drop from 0 to the nth impedance and Y_n is the admittance ($= 1/Z_n$) corresponding to the nth impedance.



Applying this theorem to the network below which is given in the Question, let the point O, O′, 1, 2 and 3 be as shown.



Then $$V_{OO'} = \frac{V_1Y_1 + V_2Y_2}{Y_1 + Y_2 + Y_3}$$

where $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$, and $Y_3 = 1/Z_3$.

$\therefore$ Current through $Z_3 = Y_3V_{OO'}$

$$= \underline{Y_3(V_1Y_1 + V_2Y_2)/(Y_1 + Y_2 + Y_3).}$$

The Generalized Form of Norton's Theorem states that the voltage between two points in a linear network is equal to the product IZ where I is the current which flows in a short-circuit placed between these terminals and Z is the impedance between these points (generators replaced by their internal impedances).

Applying this to the given problem:

If a short-circuit is placed between O and O′, the current in this short-circuit due to V_1 is V_1/Z_1 and that due to V_2 is V_2/Z_2.

$$\therefore \qquad I = V_1/Z_1 + V_2/Z_2 = V_1Y_1 + V_2Y_2.$$

Assume each generator has zero impedance then

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = Y_1 + Y_2 + Y_3$$

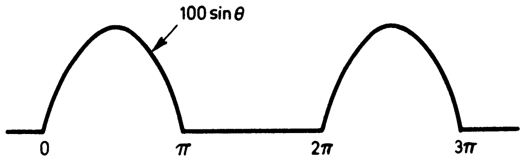
$$\therefore \qquad Z = 1/(Y_1 + Y_2 + Y_3)$$

$$\therefore \qquad V_{OO'} = IZ = (V_1Y_1 + V_2Y_2)/(Y_1 + Y_2 + Y_3)$$

The current through $Z_3 = Y_3V_{OO'}$

$$= \underline{Y_3(V_1Y_1 + V_2Y_2)/(Y_1 + Y_2 + Y_3).}$$

142. The waveform on the d.c. side is as shown.

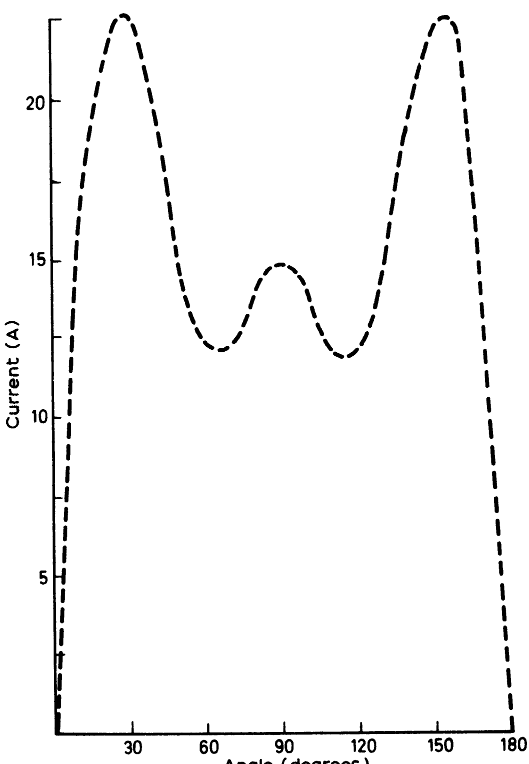


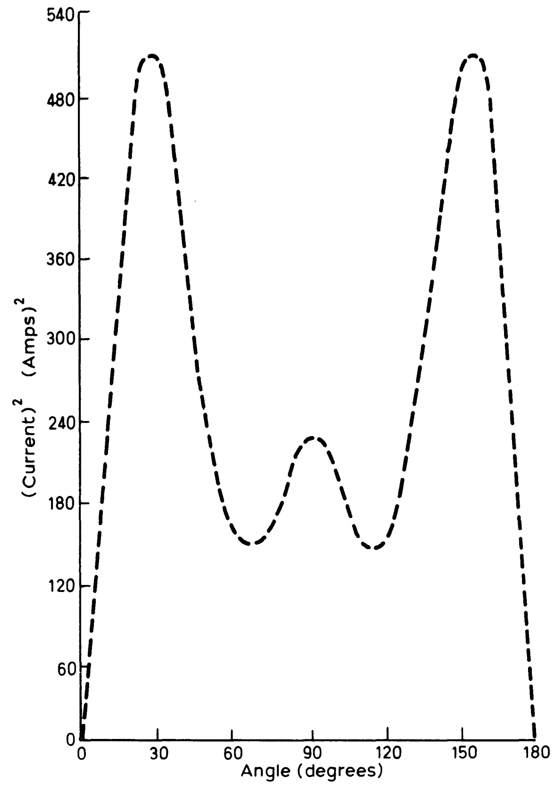
$$\text{Mean d.c. voltage} = \frac{1}{2\pi}\int_0^{\pi} 100 \sin\theta \, d\theta$$

$$= \frac{100}{2\pi}\Big[-\cos\theta\Big]_0^{\pi}$$

$$= \underline{31{\cdot}83 \text{ V.}}$$

143. The r.m.s. value is the square root of the average of the squares.





From this graph the average value of the squares of the ordinates

$$= 266$$

$\therefore$ r.m.s. value $= \sqrt{266}$ A $=$ $\underline{16 \cdot 3 \text{ A.}}$

144. (*a*) Form factor is the ratio of the r.m.s. value of the wave to its average value.

Let the waveform be represented by $i = I \sin \theta$.

$$(\text{r.m.s. value})^2 = \frac{1}{2\pi} \int_0^{2\pi} I^2 \sin^2 \theta \, d\theta$$

$$= \frac{1}{2\pi} \cdot I^2 \cdot \int_0^{2\pi} \left[\frac{1 - \cos 2\theta}{2}\right] d\theta$$

$$= I^2/2.$$

$\therefore$ r.m.s. value $= I/\sqrt{2}$.

The average value for a complete cycle = 0.
The average value for a half cycle is the quantity of interest.

$$\text{Average value} = \frac{1}{\pi}\int_0^{\pi} I \sin\theta \, d\theta$$

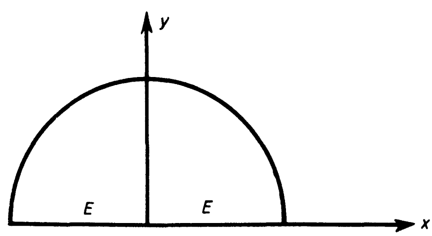
$$= \frac{I}{\pi}\Big[-\cos\theta\Big]_0^{\pi} = \frac{2I}{\pi}.$$

$$\therefore \qquad \text{Form factor} = \left(\frac{I}{\sqrt{2}}\right)\Big/\left(\frac{2I}{\pi}\right)$$

$$= \underline{1{\cdot}11.}$$

(*b*) The equation of the semicircle is:

$$x^2 + y^2 = E^2.$$



$$\text{The r.m.s. value} = \sqrt{\left(\frac{1}{2E}\int_{-E}^{+E} y^2\, dx\right)}$$

$$= \sqrt{\left(\frac{1}{2E}\int_{-E}^{+E} (E^2 - x^2)\, dx\right)}$$

$$= \sqrt{\left(\frac{1}{2E}\left[E^2x - \frac{x^3}{3}\right]_{-E}^{+E}\right)}$$

$$= E\sqrt{\left(\frac{2}{3}\right)} = \underline{0{\cdot}816\, E.}$$

145. The heating effect is proportional to the square of the r.m.s. current.

For the sine waveform, r.m.s. current = $I/\sqrt{2}$, where I is the maximum value.

For the rectangular waveform,

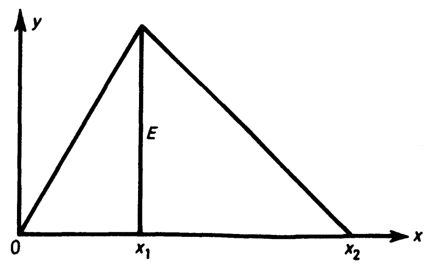
$$\text{r.m.s. value} = \sqrt{\left(\frac{1}{2\pi}\left[\int_0^{\pi} I^2\,d\theta + \int_{\pi}^{2\pi} (-I)^2\,d\theta\right]\right)}$$

$$= \sqrt{\left(\frac{1}{2\pi}\,.\,I^2\,.\,(\pi + \pi)\right)} = I.$$

$\therefore$ ratio of r.m.s. values $= I/(I/\sqrt{2}) = \sqrt{2} : 1$

$\therefore$ ratio of heating effects $= 2 : 1.$

146.



Let the triangular waveform be as shown.

From 0 to x_1 $\quad y = Ex/x_1,$

From x_1 to x_2 $\quad y = E(x_2 - x)/(x_2 - x_1).$

$$(\text{r.m.s. value})^2 = \frac{1}{x_2}\left[\int_0^{x_1}\left(\frac{Ex}{x_1}\right)^2 \mathrm{d}x + \int_{x_1}^{x_2}\frac{E^2(x_2 - x)^2}{(x_2 - x_1)^2}\,dx\right]$$

$$= \frac{E^2}{x_2}\left[\frac{1}{x_1^2}\,.\,\frac{x_1^3}{3} + \frac{1}{(x_2 - x_1)^2}\,(x_2^3/3 - x_2^2x_1 + x_2x_1^2 - x_1^3/3)\right]$$

$= E^2/3$

$\therefore$ r.m.s. value $= E/\sqrt{3}$ which is independent of x_1 and x_2.

147. $\quad y = 3$ for $0 \leq x \leq 1$

$y = -3x + 3$ for $1 \leq x \leq 2$

$$\text{r.m.s. value} = \sqrt{\left\{\tfrac{1}{2}\left[\int_0^1 3^2\,dx + \int_1^2 (-3x + 3)^2\,dx\right]\right\}}$$

$= 2{\cdot}45$

$$\text{Average value} = \tfrac{1}{2}\left[\int_0^1 3dx + \int_1^2 (-3x+3)dx\right]$$
$$= \underline{0{\cdot}75.}$$

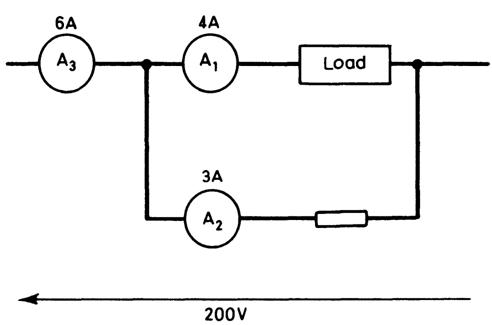
148. Denote current by y and ωt by x.

For $0 \leq x \leq 3\pi/2$, $\quad y_1 = 10$

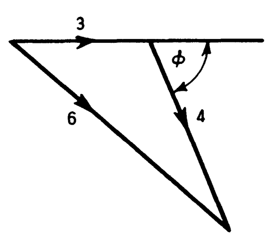
For $3\pi/2 \leq x \leq 2\pi$, $\quad y_2 = -10$

$$\therefore \text{ r.m.s. value of current} = \sqrt{\left\{\frac{1}{2\pi}\left[\int_0^{3\pi/2} y_1^2\,dx + \int_{3\pi/2}^{2\pi} y_2^2\,dx\right]\right\}}$$
$$= \sqrt{\left\{\frac{1}{2\pi}\left[10^2(3\pi/2) + 10^2(\pi/2)\right]\right\}}$$
$$= \underline{10\text{ A.}}$$

149.



A sketch of the phasor diagram is shown. From this diagram:



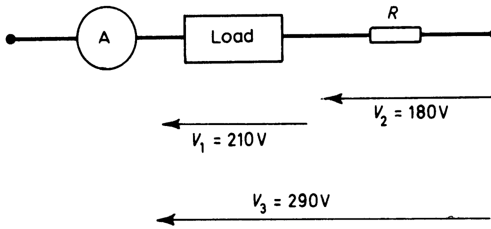
$$6^2 = 3^2 + 4^2 + 2\,.\,3\,.\,4 \cos\phi.$$

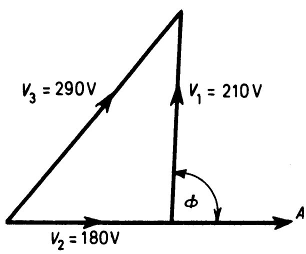
$$\therefore \cos\phi = (36 - 9 - 16)/24 = \underline{0{\cdot}458.}$$

The power absorbed by the load $= P = 4(200)\cos\phi$

$$\therefore \qquad \underline{P = 8 + 45{\cdot}8 = 366{\cdot}4\text{ W.}}$$

150.





A sketch of the phasor diagram is shown. From this diagram:

$$290^2 = 180^2 + 210^2 + 2{\cdot}180 \,.\, 210 \cos.\, \phi$$

$$\therefore \cos\phi = \frac{290^2 - 180^2 - 210^2}{2 \times 180 \times 210} = \underline{0{\cdot}1}$$

The power in the load $= P = V_1 A \cos\phi.$

$$= \underline{210 \times 0{\cdot}1\, A.}$$

$$= 210 \times 0{\cdot}1\, V_2/R$$

$$= \underline{210 \times 0{\cdot}1 \times 180/R.}$$

Thus in order to calculate the power either A or R must be known.

151. (*a*) With S closed, the reading of the wattmeter $W_1 = VI\cos\phi$. With S open, the voltage across the voltage coil of the wattmeter $= RV/\sqrt{(R^2 + 1/\omega^2C^2)}$ and the current through the current coils is I. The voltage across R is now out of phase with V by an angle $\tan^{-1}(1/\omega CR)$.

$$\therefore\quad W_2 = \frac{VR}{\sqrt{\left(R^2 + \dfrac{1}{\omega^2C^2}\right)}}\, I\cos\left(\phi - \tan^{-1}\frac{1}{\omega CR}\right).$$

$$\therefore \quad \frac{W_2}{W_1} = \frac{VIR}{\sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}} \cos\,[\phi - \tan^{-1} 1/\omega CR]/VI \cos \phi.$$

(*b*)

$$\frac{W_2}{W_1} = a = R\left(R + \frac{\tan\phi}{\omega C}\right)/(R^2 + 1/\omega^2 C^2).$$

If $R = 1/\omega C$,

$$a = R(R + R\tan\phi)/2R^2 = \frac{R^2 + R^2 \tan\phi}{2R^2}$$

$$\therefore \qquad \tan\phi = 2a - 1.$$

$$\therefore \qquad \underline{\cos\phi = 1/\sqrt{(4a^2 - 4a + 2)}.}$$

152. It may be shown* that

$$\frac{\text{True reading}}{\text{Actual reading}} = \frac{1 + \tan^2\psi}{1 + \tan\phi \tan\psi}$$

where ϕ = angle of lag of current in main circuit
and ψ = angle of lag of current in voltage coil behind the voltage across it.
Here $\phi = 45°$ (lagging) $\quad \therefore \tan\phi = 1$

$$\therefore \qquad \frac{50}{52} = \frac{1 + \tan^2\psi}{1 + \tan\psi}$$

$$\therefore \qquad \tan\psi = (25 \pm 22{\cdot}83)/52.$$

The wattmeter will be designed to give a small ψ so the value of $\tan\psi$ required is the smaller one.

i.e. $\qquad \tan\psi = 2{\cdot}17/52 = 0{\cdot}0417$

But $\tan\psi = \omega L/R = (2\pi \times 100)L/2000$

$$\therefore \qquad \underline{L = 0{\cdot}133 \text{ H.}}$$

*See, for example, H. Cotton, *Electrical Technology*, 5th Edition, Pitman, 1945, pp. 250–252.

153. Admittance of capacitor $= j\omega C$*
Admittance of 750-Ω resistor $= (1/750)$ S $= 1{\cdot}33 \times 10^{-3}$ S.
With S closed, admittance of parallel circuits

$$= 1{\cdot}33 \times 10^{-3} + j2\pi \times 100 \times 3 \times 10^{-6}$$

$$= (13{\cdot}3 + j9{\cdot}42)10^{-4} \text{ S}$$

$$\therefore \text{ impedance of parallel circuits} = \frac{10^4}{13{\cdot}3 + j9{\cdot}42}$$

$$= (500 - j352)\ \Omega.$$

Impedance of complete wattmeter shunt circuit with S closed $= Z_v = (250 + 500 - j352)\Omega = (750 - j352)\ \Omega$

$$\therefore \qquad |Z_v| = \sqrt{(750^2 + 352^2)} = 829\ \Omega.$$

Current in voltage coil $I_v = 30/Z_v = 30/829 = 0{\cdot}0362$ A and I_v leads the applied voltage V of 30 V by $\psi = \tan^{-1}(352/750) = 25°10'$.

Now wattmeter reading is $W = kI_vI_c \cos\phi$ where k is a constant for the meter, I_v is the current in the voltage coil, I_c is the current in the current coil and ϕ is the phase-angle between I_v and I_c.

With S closed Wattmeter reading $W_2 = k\,0{\cdot}0362\ I_c \cos(\phi_L + 25°\ 10')$ where ϕ_L is the load phase-angle.

With S open $I_v = 30/100 = 0{\cdot}03$ A and $\phi = \phi_L$

$$\therefore \qquad \text{wattmeter reading } W_1 = k\,0{\cdot}03\ I_c \cos\phi_L$$

$$\therefore \quad \frac{W_2}{W_1} = \frac{0{\cdot}0362}{0{\cdot}03}\,\frac{\cos(\phi_L + 25°\ 10')}{\cos\phi_L} = 1{\cdot}09 - 0{\cdot}514 \tan\phi_L$$

$$\therefore \qquad \underline{\phi_L = \tan^{-1}\left(2{\cdot}13 - 1{\cdot}95\frac{W_2}{W_1}\right).}$$

154. Impedance of coil $= j\omega L$* $= j100\pi \times 0{\cdot}5 = (j157)\ \Omega$.

Admittance of coil $= 1/j157 = -j0{\cdot}00637$ S

Admittance of 250-Ω resistor $= 1/250 = 0{\cdot}004$ S

With S closed, admittance of parallel circuits

$$= (0{\cdot}004 - j0{\cdot}00637) \text{ S}$$

*Complex quantities and their use in a.c. circuits are considered in the next chapter.

$\therefore$ impedance of parallel circuits $= \left(\dfrac{1}{0{\cdot}004 - j0{\cdot}00637}\right)\Omega$

$$= (71 + j113)\,\Omega.$$

Impedance of complete wattmeter shunt circuit

$$= (750 + 71 + j113)\,\Omega = (821 + j113)\,\Omega$$

Current i through 750-Ω resistor $= \left(\dfrac{30}{821 + j113}\right)$ A

$$= (0{\cdot}036 - j0{\cdot}00495)\text{ A}$$

$$\therefore \qquad |i| = \sqrt{(0{\cdot}036^2 + 0{\cdot}00495^2)} = 0{\cdot}0363\text{ A}$$

and i lags the applied voltage V by angle

$$\phi_1 = \tan^{-1}\left(\frac{0{\cdot}00495}{0{\cdot}036}\right)$$

$$\therefore \qquad \phi_1 = 7{\cdot}83°.$$

The voltage across the 750-Ω resistor, $V_1 = 750i = 27{\cdot}2$ volts. With respect to phasor V,

$$V_1 = 27{\cdot}2\cos 7{\cdot}83° - j27{\cdot}2\sin 7{\cdot}83° = 27 - j3{\cdot}71.$$

Voltage across parallel circuits,

$$V_2 = V - V_1 = 30 - (27 - j3{\cdot}71) = (3 + j3{\cdot}71)\text{ volts.}$$

$$\therefore \qquad |V_2| = \sqrt{(3^2 + 3{\cdot}71^2)} = 4{\cdot}78\text{ volts}$$

and V_2 leads V by angle $\phi_2 = \tan^{-1}(3{\cdot}71/3) = 48{\cdot}33°$.

Current through 250-Ω resistor $I_v = V_2/250 = 4{\cdot}78/250 = 0{\cdot}0191$ A and I_v is in phase with V_2, i.e. leads V by 48·33°.

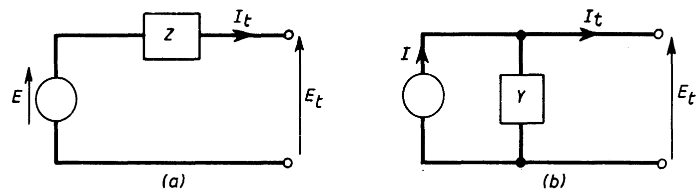
Wattmeter reading $W_2 = kI_vI_c\cos(\phi_L + 48{\cdot}33°)$ where k, I_c and ϕ_L have the same meaning as in the previous solution.

Also, $$W_1 = k\frac{30}{1000}I_c\cos\phi_L$$

$$\therefore \qquad \frac{W_2}{W_1} = \frac{0{\cdot}0191}{0{\cdot}03}\,\frac{\cos(\phi_L + 48{\cdot}33°)}{\cos\phi_L}$$

$$\therefore \qquad \phi_L = \tan^{-1}\left(0{\cdot}89 - 2{\cdot}1\frac{W_1}{W_2}\right)$$

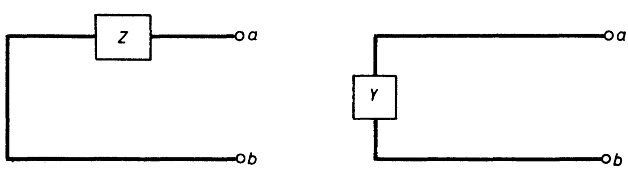
155. Two active networks are said to be equivalent when a given impedance connected to the output terminals receives the same current and has the same voltage drop across it.



From (*a*): $E_t = E - ZI_t \qquad \therefore \; I_t = \dfrac{E - E_t}{Z} = \dfrac{E}{Z} - \dfrac{E_t}{Z}$

From (*b*): $I_t = I - YE_t \qquad \therefore \; E_t = \dfrac{I - I_t}{Y} = \dfrac{I}{Y} - \dfrac{I_t}{Y}$

Now, for true equivalence, the circuits must remain the same, even if the voltage and current sources are reduced to zero, (accomplished by s.c. voltage source and o.c. current source) i.e.



The impedance between *a* and *b* must be the same in both instances to maintain an equivalent circuit.

Thus,

$$Z_{ab} = Z = 1/Y$$

$\therefore$ using the above equations too:

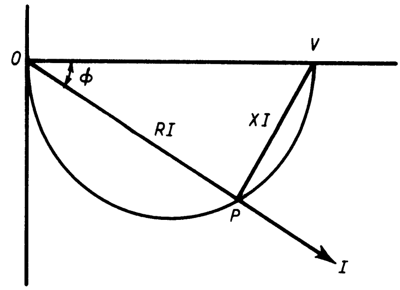
$$E - E_t = ZI_t \quad \text{and} \quad IZ - ZI_t = E_t$$

$$\therefore \qquad E = ZI$$

$$\therefore \qquad \underline{I = E/Z} \quad \text{and} \quad \underline{Y = 1/Z}$$

156. *Current locus.*

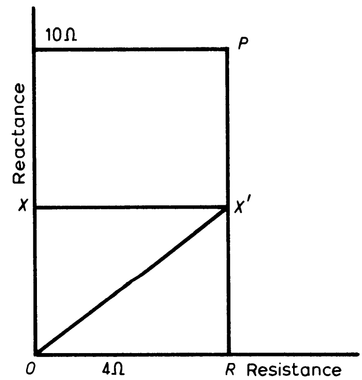
Let OV represent the applied voltage to a given scale. $PV(=XI)$ is the reactance drop to the same scale and $OP(=RI)$ is the resistance drop. Then as the reactance is varied, the angle OPV will always be a right angle. Hence, locus diagram of current is a semi-circle on OV as diameter, i.e., the diameter



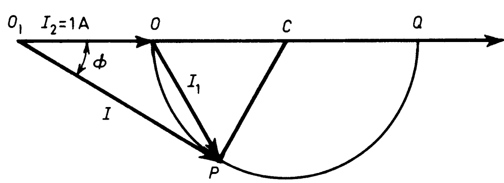
$$= 100/4 = 25 \text{ V}.$$

Impedance locus.

Let the horizontal distance OR represent the constant resistance of 4 Ω to a suitable scale. The reactance X varies from zero to 10 Ω and so the locus of the impedance phasor OX' is the straight line perpendicular to OR at R, i.e., PR.



157. Current I_1 in the reactive branch has a semicircular locus OPQ as illustrated; diameter OQ representing 2A. The current I_2 in the resistive branch is 1A. The resultant current I is therefore the phasor sum of I_1 and I_2.



Minimum power-factor, corresponding to maximum phase-angle ϕ occurs evidently when O_1P is tangential to the circle. Thus O_1P is then perpendicular to radius PC.

So, $\sin \phi = CP/O_1C = 0{\cdot}5$ and $\phi = 30°$.

Power-factor $\cos \phi = 0{\cdot}866$.

Now $I/O_1C = \cos\phi$, so $I = \underline{1{\cdot}732 \text{ A.}}$

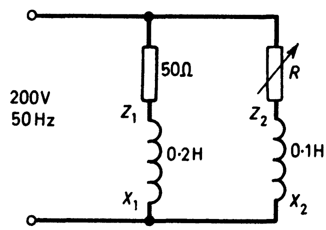
From triangle OCP, $I_1 = 1$ A.

If the reactance is X ohms

$$\sqrt{(50^2 + X^2)} = 100/I_1 = 100$$

so $$\underline{X = 86{\cdot}5 \text{ A.}}$$

158.



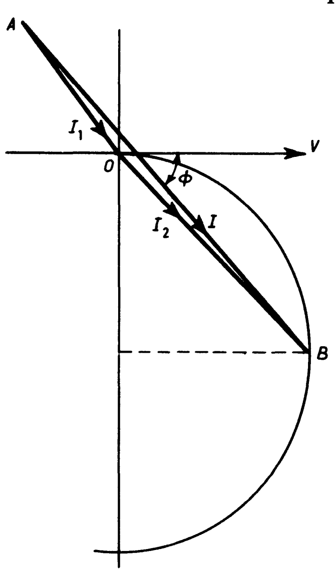
$$X_1 = 62{\cdot}8\ \Omega, \qquad X_2 = 31{\cdot}4\ \Omega$$

$$|Z_1| = \sqrt{[50^2 + (62{\cdot}8)^2]} = 80{\cdot}3\ \Omega$$

$$|I_1| = (200/80{\cdot}3)\text{ A} = 2{\cdot}49\text{ A}$$

I_1 lags the supply voltage by $\cos^{-1}(50/80{\cdot}3)$ i.e. by $51° \, 40'$.

Maximum value of $|I_2| = (200/31{\cdot}4)\text{ A} = 6{\cdot}37$ A.
This is the diameter of the I_2 circle shown. The phasor diagram is shown drawn to scale and the problem can then be solved graphically.



Total current for maximum power, $I = \underline{7 \text{ A.}}$

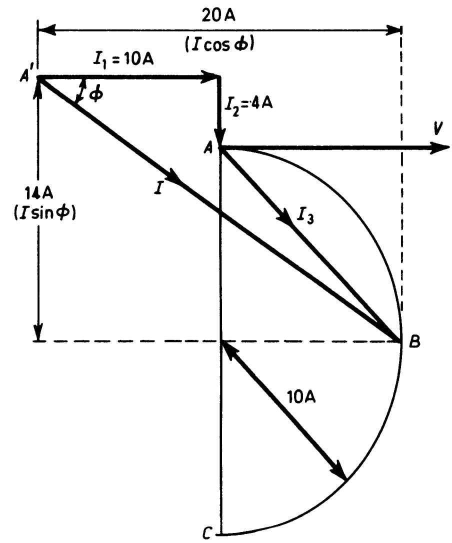
Power-factor $\cos\phi = \underline{0{\cdot}68.}$

Power $= 200 \times 0{\cdot}68 \times 7$

$= \underline{952 \text{ W.}}$

Value of R for maximum power = value of $X_2 = \underline{31{\cdot}4\,\Omega.}$

159. The locus diagram is as shown. The diameter of semicircle $ABC = 200/X_1 = 200/10 = 20$ A. Current $I_1 = 10$ A in phase with the applied voltage. Current $I_2 = 4$ A and lags the voltage by 90°. Point B is the one to give maximum power; here $R = X_1 = \underline{10\ \Omega}$.

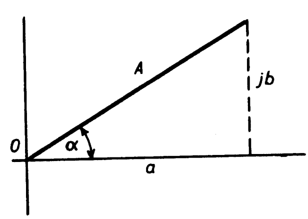


Maximum power $= V(I\cos\phi) = 200 \times 20 = \underline{4000\ \text{W}}$.

Total current $I = \{\sqrt{[(20^2 + 14^2)]}\}$ A $= 24{\cdot}4$ A.

Total power-factor $= (20/24{\cdot}4) = \underline{0{\cdot}82}$.

201. $A = a + jb = A(\cos\alpha + j\sin\alpha)$. Thus $(\cos\alpha + j\sin\alpha)$ can be thought of as an operator which turned a horizontal line of length A through an angle α in an anti-clockwise direction. For clockwise rotation α is replaced by $(-\alpha)$.



Since $\cos(-\alpha) = \cos\alpha$ and $\sin(-\alpha) = -\sin\alpha$:

$$A' = (3 + j4)(\cos 53{\cdot}2° - j\sin 53{\cdot}2°)$$
$$= (3 + j4)(0{\cdot}6 - j0{\cdot}8)$$
$$= \underline{5 + j0.}$$

202. $A + B = 3 + j4 - 12 - j5 = \underline{-9 - j1}$

$A - B = 3 + j4 + 12 + j5 = \underline{15 + j9}$

$AB = 5\angle 53{\cdot}2° \times 13\angle 202{\cdot}6° = \underline{65\angle 255{\cdot}8°}$

$A/B = 5\angle 53{\cdot}2°/13\angle 202{\cdot}6° = \underline{0{\cdot}385\angle -149{\cdot}4°}$

$A^2 = 5^2\angle(2 \times 53{\cdot}2°) = \underline{25\angle 106{\cdot}4°}$

$\sqrt{B} = \sqrt{(13)}\ \angle 202{\cdot}6°/2$ and $\underline{\sqrt{(13)}\ \angle 202{\cdot}6° + 360°/2}$

$= \underline{3{\cdot}6\angle 101{\cdot}3°}$ and $\underline{3{\cdot}6\angle 281{\cdot}3°}$

If $A = |A|\, e^{j(\omega t + \alpha)}$

$\log_e A = \log_e |A| + j(\omega t + \alpha).$

In this case: $\log_e A = \log_e 5 + j(53{\cdot}2 \times \pi/180)$

$= \underline{1{\cdot}6 + j0{\cdot}93.}$

203.
$$\frac{12 + j16}{10 - j20} = \frac{(12 + j16)(10 + j20)}{(10 - j20)(10 + j20)}$$
$$= \frac{120 + j240 + j160 - j^2 320}{10^2 - (j20)^2}$$
$$= \frac{120 - 320 + j240 + j160}{10^2 + 20^2}$$
$$= \frac{-200 + j400}{500} = \underline{-0{\cdot}4 + j0{\cdot}8.}$$

Alternative solution:

$$\frac{(12 + j16)}{(10 - j20)} = \frac{20\angle 53^\circ\, 18'}{22{\cdot}4\angle -63^\circ\, 26'} = 0{\cdot}893\angle 116^\circ\, 44'$$

$$= 0{\cdot}893\,(\cos 116^\circ\, 44' + j \sin 116^\circ\, 44')$$

$$= 0{\cdot}893\,(-0{\cdot}45 + j0{\cdot}89)$$

$$= \underline{-0{\cdot}4 + j0{\cdot}8.}$$

204. Resultant $= (60 + 30) + j(40 + 30)$

$$= 90 + j70.$$

Modulus $= \sqrt{(90^2 + 70^2)} = \underline{114.}$

Argument $= \tan^{-1}(70/90) = \underline{37^\circ\, 53'.}$

205. (*a*)

$$(20 + j20)(60 - j30) = 1200 + j1200 - j600 - j^2 600$$

$$= \underline{1800 + j600.}$$

Alternative solution.

$$(20 + j20)(60 - j30) = 600(1 + j)(2 - j)$$

$$= 600(1 + j)(2 - j)$$

$$= 600[\sqrt{2}\ \angle 45^\circ][\sqrt{5}\ \angle -26^\circ\, 34']$$

$$= 1900\angle 18^\circ\, 26'$$

$$= 1900(\cos 18^\circ\, 26' + j \sin 18^\circ\, 26')$$

$$= 1900(0{\cdot}9487 + j0{\cdot}3162)$$

$$= \underline{1800 + j600.}$$

(*b*)
$$\frac{(100 + j70)}{(60 + j10)} = \frac{(100 + j70)(60 - j10)}{60^2 + 10^2}$$

$$= \frac{(6000 + j4200 - j1000 + 700)}{3700}$$

$$= \underline{1{\cdot}81 - j0{\cdot}865.}$$

Modulus $= \sqrt{(3{\cdot}275 + 0{\cdot}75)} = \underline{2{\cdot}0.}$

Argument $= \tan^{-1}(0{\cdot}865/1{\cdot}81) = \underline{25^\circ\ 30'.}$

Alternative solution

$$(100 + j70) = \sqrt{(14\,900)}\ \angle 35^\circ$$

$$(60 + j10) = \sqrt{(3700)}\ \angle 9^\circ\ 30'$$

$$\therefore \quad (100 + j70)/(60 + j10) = \sqrt{\left(\frac{14\,900}{3700}\right)}\ \angle(35^\circ - 9^\circ\ 30')$$

$$= \underline{2{\cdot}0 \angle 25^\circ\ 30'.}$$

206. (*a*) The conjugate of the phasor

$$Z = (a + jb) \text{ is } Z^* = (a - jb), \text{ or if}$$

$$Z = z(\cos\theta + j\sin\theta) = ze^{j\theta}$$

$$Z^* = z(\cos\theta - j\sin\theta) = ze^{-j\theta}$$

Thus $ZZ^* = a^2 + b^2 = z^2$.

Here $a = 3$, $b = -4$ $\quad \therefore\ ZZ^* = \underline{25.}$

(*b*) It follows from the above statements that here $ZZ^* = 10^2 = \underline{100.}$

(*c*) Similarly, $ZZ^* = 2{\cdot}5^2 = \underline{6{\cdot}25.}$

207. $Z = 3 - j2$

$\therefore\ \ r\cos\theta = 3$ and $r\sin\theta = -2$

$\therefore\ \ r = \sqrt{[3^2 + (-2)^2]} = \sqrt{13}$

and $\tan\theta = -2/3$ so $\theta = -33^\circ\ 41'$

i.e. $$Z = \sqrt{13}\ \angle -33^\circ\ 41' = \underline{3{\cdot}6\ \angle -33^\circ\ 41'}$$

$$1/Z = (1/\sqrt{13})\ \angle 33^\circ\ 41' = \underline{0{\cdot}277\ \angle 33^\circ\ 41'}$$

$$Z^2 = [\sqrt{13}\ \angle -33^\circ\ 41']^2 = \underline{13\ \angle -67^\circ\ 22'.}$$

208. Let the combined impedance be Z.

Then

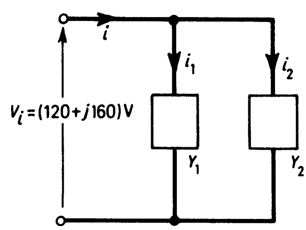
$$\frac{1}{Z} = \frac{1}{(4-j3)} + \frac{1}{(5+j2)}$$

$$= \frac{(5+j2)+(4-j3)}{(4-j3)(5+j2)}$$

$$\therefore \quad Z = \frac{(4-j3)(5+j2)}{(5+j2)+(4-j3)} = \frac{(20-j15+j8+6)(9+j1)}{(9-j1)(9+j1)}$$

$$= (241-j37)/82 = \underline{2{\cdot}94 - j0{\cdot}45.}$$

209.



$$Y_1 = \frac{1}{12+j16}$$

$$= \frac{1}{\sqrt{(12^2+16^2)}\ \angle\tan^{-1}(16/12)}$$

$$= 0{\cdot}05\ \angle -53°\ 8'$$

$$Y_2 = \frac{1}{10-j20}$$

$$= \frac{1}{\sqrt{(10^2+20^2)}\ \angle\tan^{-1}(-2)}$$

$$= 0{\cdot}0447\ \angle 63°\ 26'$$

$$V_i = (120+j160) = \sqrt{(120^2+160^2)}\ \angle\tan^{-1}(160/120)$$

$$= 200\ \angle 53°\ 8'$$

$$i_1 = Y_1V_i = 0{\cdot}05\ \angle -53°\ 8'\ .\ 200\ \angle 53°\ 8' = \underline{10\ \angle 0°\ \text{A}}$$

$$i_2 = Y_2V_i = 0{\cdot}0447\ \angle 63°\ 26'\ .\ 200\ \angle 53°\ 8'$$

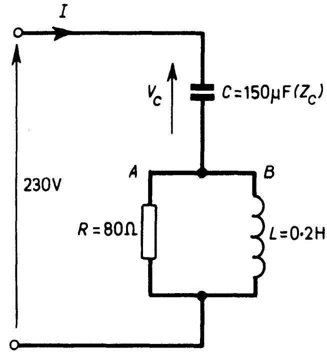
$$= \underline{8{\cdot}94\ \angle 116°\ 34'\ \text{A}}$$

$$i = i_1 + i_2 = 10 + 8{\cdot}94\,(\cos 116°\ 34' + j\sin 116°\ 34')\ \text{A}$$

$$= 5{\cdot}95 + j8{\cdot}04 = \underline{10\ \angle 53°\ 8'\ \text{A}}$$

$\therefore$ overall power-factor of circuit = 1.

210.



$$Z_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$= -j/(2\pi 50 \times 150 \times 10^{-6})$$

$$= -j21{\cdot}2$$

Admittance of parallel circuit

$$= \frac{1}{R} + \frac{1}{j\omega L}$$

$$= \frac{1}{80} + \frac{1}{j(314 \times 0{\cdot}2)}$$

$$= 10^{-2}(1{\cdot}25 - j1{\cdot}59)$$

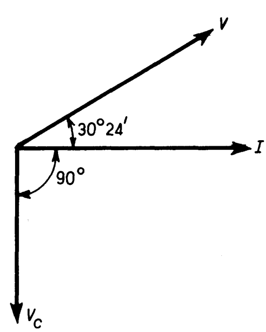
$\therefore$ impedance of parallel circuit $(Z) = \dfrac{1}{10^{-2}(1{\cdot}25 - j1{\cdot}59)}$

$$= 50 \angle 51^\circ\, 48' = 30{\cdot}9 + j39{\cdot}3.$$

Total impedance $= Z - j21{\cdot}2 = 30{\cdot}9 + j39{\cdot}3 - j21{\cdot}2$

$$= 30{\cdot}9 + j18{\cdot}1 = 35{\cdot}8 \angle 30^\circ\, 24'$$

The supply current has a magnitude of $230/35{\cdot}8 = \underline{6{\cdot}4\ \text{A}}$ and lags the voltage by 30° 24′.



$V_c = 6{\cdot}4 \angle -30^\circ\, 24' \times (-j21{\cdot}2)$

$= 6{\cdot}4 \angle -30^\circ\, 24' \times 21{\cdot}2 \angle -90^\circ$

$= 136 \angle -120^\circ\, 24'$ V with respect to supply voltage.

211. (*a*) *Frequency* = 50 Hz.

Total impedance of circuit = $(5 + j44) - j159 = (5 - j115)\ \Omega$.

$$\text{Current in circuit} = \frac{200}{\sqrt{(5^2 + 115^2)}} = \frac{40}{\sqrt{(1 + 23^2)}}$$

$$= \underline{1{\cdot}74 \text{ A.}}$$

Current leads applied voltage by angle $\phi = \tan^{-1}(23/1) = \underline{87° \, 30'}$.

(*b*) *Frequency* = 150 Hz.

At $f = 50$ Hz, $\omega L = 44$, so at 150 Hz, $\omega L = 3 \times 44 = 132\ \Omega$.

At $f = 50$ Hz, $1/\omega C = 159$, so at 150 Hz, $1/\omega C = 159/3 = 53\ \Omega$.

Total impedance of circuit $= (5 + j132) - j53 = (5 + j79)\ \Omega$.

$$\text{Current in circuit} = \frac{200}{\sqrt{(5^2 + 79^2)}} = \underline{2{\cdot}53 \text{ A.}}$$

Current lags applied voltage by angle $\phi = \tan^{-1}(79/5) = \underline{86° \, 23'}$.

212. Impedance of coil $A = 15{\cdot}5\ \Omega = 10 + jX_1$ where X_1 is the reactance of the coil.

$$\therefore \qquad X_1 = \sqrt{(15{\cdot}5^2 - 10^2)} = 11{\cdot}8\ \Omega.$$

Total impedance of coils A and B is 200/5·5

$$= \sqrt{[(10 + 12{\cdot}5)^2 + (11{\cdot}8 + X_2)^2]}$$

where X_2 is the reactance of coil B.

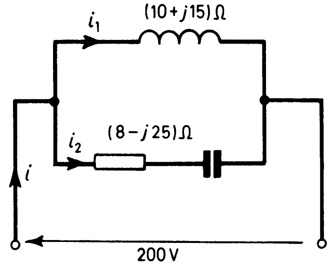
$$\therefore \qquad (11{\cdot}8 + X_2)^2 = (200/5{\cdot}5)^2 - (22{\cdot}5)^2$$

i.e. $X_2 = 16{\cdot}7\ \Omega$

and $L_2 = 16{\cdot}7/(2\pi \times 50) = \underline{0{\cdot}0532 \text{ H}}$

where L_2 is the inductance of coil B.

213.



$$i_1 = \frac{200}{10 + j15} = \frac{200(10 - j15)}{10^2 - (j15)^2} = (6{\cdot}15 - j9{\cdot}23) \text{ A}.$$

$$i_2 = \frac{200}{8 - j25} = \frac{200(8 + j25)}{8^2 - (j25)^2} = (2{\cdot}29 + j7{\cdot}26) \text{ A}.$$

$\therefore \quad i = i_1 + i_2 = (8{\cdot}44 - j1{\cdot}97)$ A.

Numerically:

$$i_1 = \sqrt{(6{\cdot}15^2 + 9{\cdot}23^2)} = \underline{11{\cdot}3 \text{ A}}$$

$$i_2 = \sqrt{[2{\cdot}29^2 + (7{\cdot}26)^2]} = \underline{7{\cdot}68 \text{ A}}$$

$$i = \sqrt{[8{\cdot}44^2 + (1{\cdot}97)^2]} = \underline{8{\cdot}7 \text{ A}.}$$

The inductance of the coil, $L_1 = X_1/\omega = 15/(2\pi \times 50)$

$= \underline{0{\cdot}0478 \text{ H}.}$

The capacitance of the capacitor, $C_2 = 1/\omega X_2$

$= 1/(2\pi \times 50 \times 25)$

$= 0{\cdot}1275 \times 10^{-3}$ F

$= \underline{127{\cdot}5\ \mu\text{F}.}$

214. Current i_1 through impedance $(15 + j12)\ \Omega = 100/(15 + j12)$

$$= \frac{100(15 - j12)}{15^2 - (j12)^2} = 4{\cdot}07 - j3{\cdot}25$$

$\therefore$ magnitude of $i_1 = \sqrt{[4{\cdot}07^2 + (3{\cdot}25)^2]} = \underline{5{\cdot}2 \text{ A}.}$

This current lags the applied voltage by $\phi_1 = \tan^{-1}(3{\cdot}25/4{\cdot}07)$

$= \underline{38^\circ\ 40'.}$

Let the capacitance of the capacitor be C farads.

Current i_2 through the capacitor $= 100/[-j(1/\omega C) = 100\, j\omega C$.

For unity power-factor of combined circuit:

$j100\omega C = j3{\cdot}25$

$\therefore \quad C = 3{\cdot}25/(100 \times 2\pi \times 50) = 0{\cdot}1035 \times 10^{-3}$ F

$= \underline{103{\cdot}5\ \mu\text{F}.}$

Reactance of capacitor $X = 1/\omega C$

$$= 1/(2\pi \times 50 \times 103{\cdot}5 \times 10^{-6})$$

$$= \underline{30{\cdot}75\,\Omega.}$$

215.

Current i_1 through impedance $(12 + j15)\,\Omega = \dfrac{200}{(12 + j15)}$

$$= \frac{200(12 - j15)}{12^2 - (j15)^2}$$

$$= (6{\cdot}69 - j8{\cdot}13)\text{ A.}$$

Current i_2 through impedance $(10 + j20)\,\Omega = \dfrac{200}{(10 + j20)}$

$$= \frac{200(10 - j20)}{10^2 + 20^2}$$

$$= (4 - j8)\text{ A.}$$

Current i_3 through impedance $(5 - j25)\,\Omega = \dfrac{200}{(5 - j25)}$

$$= \frac{200(5 + j25)}{5^2 + 25^2}$$

$$= (1{\cdot}54 + j7{\cdot}7)\text{ A.}$$

Total current $i = i_1 + i_2 + i_3 = (12{\cdot}23 - j8{\cdot}43)$ A.

Numerically:

$$i_1 = \sqrt{(6{\cdot}69^2 + 8{\cdot}13^2)} = \underline{10{\cdot}4\text{ A.}}$$

This lags the applied voltage by $\phi_1 = \tan^{-1}(15/12) = \underline{51^\circ\,20'}$

$$i_2 = \sqrt{(4^2 + 8^2)} = \underline{8{\cdot}95\text{ A.}}$$

This lags the applied voltage by $\phi_2 = \tan^{-1}(20/10) = \underline{63^\circ\,26'}$

$$i_3 = \sqrt{[1{\cdot}54^2 + (7{\cdot}7)^2]} = \underline{7{\cdot}85\text{ A.}}$$

This leads the applied voltage by $\phi_3 = \tan^{-1}(25/5) = \underline{78^\circ\,41'}$

$$i = \sqrt{(12{\cdot}23^2 + 8{\cdot}43^2)} = \underline{14{\cdot}7\text{ A.}}$$

This lags the applied voltage by $\phi = \tan^{-1}(8{\cdot}43/12{\cdot}23) = \underline{35°}$.

In the first branch the inductance $L_1 = 15/100\pi = \underline{0{\cdot}0478 \text{ H}}$.

In the second branch the inductance $L_2 = 20/100\pi = \underline{0{\cdot}0637 \text{ H}}$.

In the third branch the capacitance $C = 10^6/(25 \times 100\pi)\ \mu\text{F}$
$= \underline{127\ \mu\text{F}}$.

216.

(*a*) Impedance $Z = \dfrac{100 + j150}{4{\cdot}5 + j12}$

$$= \frac{(100 + j150)(4{\cdot}5 - j12)}{(4{\cdot}5 + j12)(4{\cdot}5 - j12)} = \underline{(13{\cdot}7 - j3{\cdot}2)\ \Omega}$$

(*b*) Power $= (100 \times 4{\cdot}5) + (150 \times 12) = \underline{2250 \text{ W}}$.

(*c*) Phase-angle of current with respect to reference phasor $= \tan^{-1}(12/4{\cdot}5) = 69°\ 30'$.
Phase-angle of voltage with respect to reference phasor $= \tan^{-1}(150/100) = 56°\ 20'$.
Phase difference between current and voltage $= \underline{13°\ 10'}$.

The current leads the voltage by $(69°\ 30' - 56°\ 20')$ i.e. by $13°\ 10'$.

217. Let $Z_1 = (70{\cdot}7 + j70{\cdot}7)\ \Omega = 100\ \angle 45°\ \Omega$.

Then $Y_1 = 1/Z_1 = 0{\cdot}01 \angle{-45°} = (0{\cdot}00707 - j0{\cdot}00707)$ S

Similarly $Z_2 = (120 + j160)\ \Omega = 200\ \angle 53{\cdot}1°\ \Omega$

so $Y_2 = 1/Z_2 = 0{\cdot}005 \angle{-53{\cdot}1°} = (0{\cdot}003 - j0{\cdot}004)$ S

and $Z_3 = (120 + j90)\ \Omega = 150\ \angle 36{\cdot}9°\ \Omega$

so $Y_3 = 1/Z_3 = 0{\cdot}00667 \angle{-36{\cdot}9°} = (0{\cdot}0053 - j0{\cdot}004)$ S
mho.

Admittance of combination, $Y = Y_1 + Y_2 + Y_3$
$= (0{\cdot}0154 - j0{\cdot}015)$ S
$= \underline{0{\cdot}0215 \angle{-44{\cdot}3°} \text{ S}}$

Total current $= [250\ \angle 0°][0{\cdot}0215\ \angle{-44{\cdot}3°}]$ A
$= \underline{5{\cdot}37\ \angle{-44{\cdot}3°} \text{ A}}$.

For unity power-factor pure susceptance required $= +j0{\cdot}015$ S

i.e. reactance required $= (1/j0{\cdot}015)\,\Omega = \underline{-j66{\cdot}6\,\Omega.}$

Y then becomes 0·0154 mho and the total current is now $[250\ \angle 0°]$ $[0{\cdot}0154\ \angle 0°]$ A $= \underline{3{\cdot}85\text{ A.}}$

218. Let the resistance and capacitance of the capacitor be R ohms and C μF respectively.

Then

$$(2{\cdot}5 + j18{\cdot}2) = (2 + j50) + (R - j/\omega C)$$

where

$$\omega = 2\pi \times \text{(the supply frequency)}$$

$$\therefore \quad R - j/\omega C = 0{\cdot}5 - j31{\cdot}8$$

$$\therefore \quad \underline{R = 0{\cdot}5\,\Omega.}$$

Also

$$1/\omega C = 31{\cdot}8\,\Omega$$

$$\therefore \quad C = \frac{10^6}{2\pi \times 50 \times 31{\cdot}8}\,\mu\text{F} = \underline{100\,\mu\text{F.}}$$

219. Admittance of impedance $(17{\cdot}3 - j10)\ \Omega = [1/17{\cdot}3 - j10)]$ S $= (0{\cdot}0433 + j0{\cdot}025)$ S.

If the parallel combination consists of R_p and C_p in parallel the admittance of it $= 1/R_p + j\omega C_p$

where $\quad \omega = 2\pi \times 50$

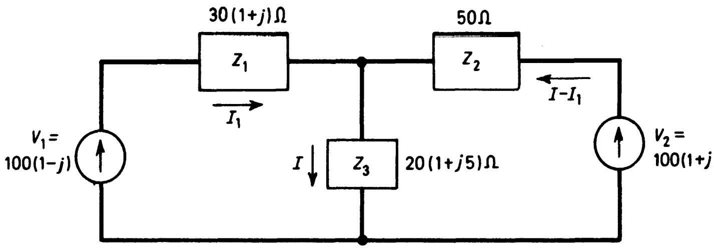
$$\therefore \quad R_p = (1/0{\cdot}0433)\,\Omega = \underline{23{\cdot}1\,\Omega}$$

and $\quad \omega C_p = 0{\cdot}025$ S

i.e. $\quad C_p = (0{\cdot}025/100\pi)$ F $= \underline{79{\cdot}6\,\mu\text{F.}}$

<u>Since ωC_p depends on frequency the circuits would not be equivalent at another value of frequency.</u>

220.



Let the currents be as shown above. Using Kirchhoff's second law for the two meshes:

$$100(1-j) = 30(1+j)I_1 + 20(1+j5)I \qquad (1)$$

and
$$100(1+j) = 50(I - I_1) + 20(1+j5)I \qquad (2)$$

Eliminating I_1:

$$I = (0{\cdot}104 - j0{\cdot}493)\ \text{A}$$

$$\therefore \qquad |I| = \sqrt{[0{\cdot}104^2 + (0{\cdot}493)^2]} = \underline{0{\cdot}503\ \text{A.}}$$

Angle of phase difference between I and V_2

$$= \tan^{-1}(0{\cdot}493/0{\cdot}104) + 45° = 78°\,6' + 45° = \underline{123°\,6'.}$$

221. Impedance of parallel combination $= 1/\{10^{-2}(1{\cdot}25 - j1{\cdot}59)\}$

$$= (30{\cdot}9 + j39{\cdot}3)\ \Omega$$

Total impedance $= (30{\cdot}9 + j39{\cdot}3 - j21{\cdot}2)\ \Omega$

$$= (30{\cdot}9 + j18{\cdot}1)\ \Omega$$

$$= (35{\cdot}8\ \angle 30°\,24')\ \Omega$$

Magnitude of supply current $= (230/35{\cdot}8)$ A $= \underline{6{\cdot}42\ \text{A}}$

Capacitor voltage $= (35{\cdot}8\ \angle 30°\,24')(-j21{\cdot}2)$ V

$$= (75{\cdot}9\ \angle -59°\,36')\ \text{V}$$

Magnitude of capacitor voltage $= \underline{75{\cdot}9\ \text{V}}$

222. (*a*) *Using Kirchhoff's Laws*

The currents in the various branches are shown on the illustration given with the problem. Given I_1, I_2 and I_3 the remaining currents are obtained by using Kirchhoff's first law.

Kirchhoff's second law gives:

For loop ABGEFA

$$(100 + j0) = I_1(10 + j10) + (I_1 - I_2)10 + (I_1 - I_2 - I_3)(-j10) + I_1(10 + j10) \quad (1)$$

i.e.
$$100 = I_1(30 + j10) + I_2(-10 + j10) + I_3(0 + j10) \quad (2)$$

For loop BCHGB

$$0 = I_2(10) - I_3(10) - (I_1 - I_2)(10) \quad (3)$$

i.e.
$$0 = -I_1 + 2I_2 - I_3 \quad (4)$$

For loop GHDEG

$$0 = I_3(10) + (I_2 + I_3)(j10) - (I_1 - I_2 - I_3)(-j10) \quad (5)$$

i.e.
$$0 = I_1(j10) + 10I_3 \quad (6)$$

From (2), (4) and (6) it is found that

$$\underline{I_1 = (2 - j)\ \text{A},\ I_2 = (0{\cdot}5 - j1{\cdot}5)\ \text{A}}$$

and
$$\underline{I_3 = -(1 + j2)\ \text{A}.}$$

(*b*) *Using Maxwell's Cyclic-Current Rule*

To use Maxwell's Cyclic-Current Rule the loop currents i_1, i_2 and i_3 are used and not the actual currents in the branches.

The loop equations are:

Loop 1

$$(100 + j0) = i_1(10 + j10 + 10 - j10 + j10 + 10) - i_2(10) - i_3(-j10) \quad (1)$$

Loop 2

$$i_2(10 + 10 + 10) - i_1(10) - i_3(10) = 0 \quad (2)$$

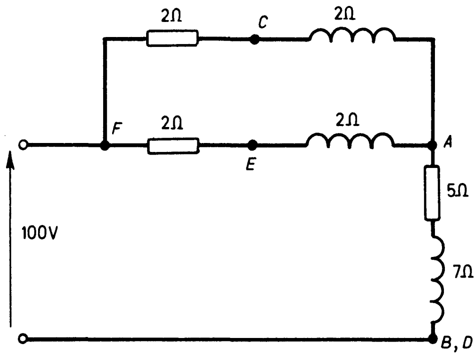
Loop 3

$$i_3(10 + j10 - j10) - i_2(10) - i_1(-j10) = 0 \quad (3)$$

Also $i_1 = I_1, i_2 = I_2$ and $i_3 = (I_2 + I_3)$ (4)

<u>From (1), (2), (3) and (4), I_1, I_2 and I_3 can be found.</u>

223. To use Thévenin's Theorem branch CD is removed to give the diagram below



$$\text{Total impedance} = \{(1 + j1) + (5 + j7)\}\ \Omega = (6 + j8)\ \Omega$$

$$\text{Current supplied} = 100/(6 + j8)\ \text{A} = (6 - j8)\ \text{A}$$

Voltage of C above $D = \{100 - (6 - j8)\}\ \text{V} = (94 + j8)\ \text{V}$.
The impedance between C and D is now required with the input terminals short-circuited. The combined impedance of branch FEA and AD in parallel is

$$(2 + j2)(5 + j7)/\{(2 + j2) + (5 + j7)\} = (1{\cdot}445 + j1{\cdot}72)\ \Omega.$$

Impedance between C and D is therefore

$$\frac{2\ (1{\cdot}445 + j1{\cdot}72 + j2)}{2 + 1{\cdot}445 + j1{\cdot}72 + j2} = 1{\cdot}463 + j0{\cdot}579.$$

Thus by Thévenin's Theorem:

Current through CD in original circuit (I_{CD})

$$= \frac{(94 + j8)}{4 + (1{\cdot}463 + j0{\cdot}579)}$$

since the resistance of the link CD is 4 Ω

$$\therefore \qquad |I_{CD}| = \underline{17{\cdot}15\ \text{A}.}$$

224. First short-circuit the 100-V generator. The impedance presented to the other generator is then $Z_2 + Z_1Z_3/(Z_1 + Z_3)$

i.e. $$(8{\cdot}67 + j5) + \frac{(3 + j4)(10 - j10)}{(3 + j4) + (10 - j10)}$$

$$= (12{\cdot}81 + j7{\cdot}69)\,\Omega = [14{\cdot}9 \angle 31^\circ)\,\Omega.$$

The current through Z_2 is then

$$I_2' = \{[120 \angle 30^\circ]/[14{\cdot}9 \angle 31^\circ]\}\text{A} = [8{\cdot}05 \angle -1^\circ]\,\text{A}.$$

Next short-circuit the 120-V generator. The impedance presented to the 100-V generator is then

$$Z_1 + Z_2Z_3/(Z_2 + Z_3) = (3 + j4) + \frac{(8{\cdot}67 + j5)(10 - j10)}{(8{\cdot}67 + j5) + (10 - j10)}$$

$$= (10{\cdot}33 + j4)\,\Omega = [11{\cdot}1 \angle 21{\cdot}2^\circ]\,\Omega.$$

The current through Z_1 is then

$$I_1'' = \{[100 \angle 0^\circ]/[11{\cdot}1 \angle 21{\cdot}2^\circ]\}\text{A} = [9{\cdot}0 \angle -21{\cdot}2^\circ]\,\text{A}.$$

Voltage across $Z_3 = V_3'' = \left(\frac{Z_2Z_3}{Z_2 + Z_3}\right)I_1'' = 7{\cdot}33[9{\cdot}0 \angle -21{\cdot}2^\circ]$ V

$$= [65{\cdot}97 \angle -21{\cdot}2^\circ]\,\text{V}.$$

Current through Z_2 is now

$$I_2'' = -V_3''/Z_2 = [65{\cdot}97 \angle -21{\cdot}2^\circ]/[8{\cdot}67 + j5)$$

$$= [65{\cdot}97 \angle -21{\cdot}2^\circ]/[10 \angle 30^\circ]$$

$$= -[6{\cdot}6 \angle -51{\cdot}2^\circ]\,\text{A}.$$

By the Principle of Superposition:

$$I_2 = I_2' + I_2'' = \{8{\cdot}05 \angle -1^\circ + [-6{\cdot}6 \angle -51{\cdot}2^\circ]\}\,\text{A}$$

$$= (8{\cdot}05 - j0{\cdot}14 - 4{\cdot}13 + j5{\cdot}14)\,\text{A}$$

$$= \underline{(3{\cdot}92 + j5{\cdot}0)\,\text{A}.}$$

244. The voltage between each wire and the neutral

$$= \text{line voltage}/\sqrt{3} = 400/\sqrt{3}$$

$$= \underline{231\ \text{V}.}$$

245.

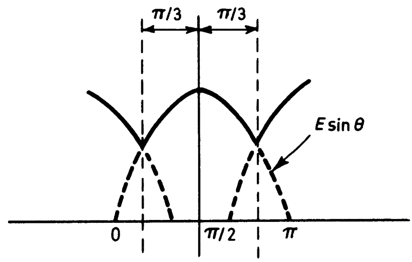
Mean d.c. voltage

$$= \left[\frac{1}{2\pi/3}\int_{\pi/2-\pi/3}^{\pi/2+\pi/3} E\sin\theta\, d\theta\right] - 15$$

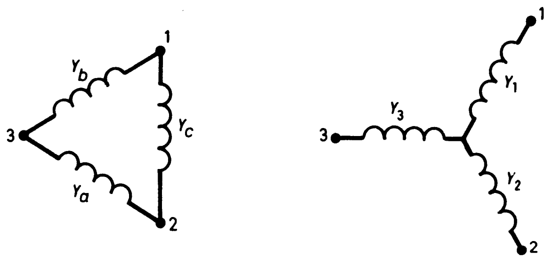
$$= \frac{3E}{2\pi}\left[-\cos\theta\right]_{\pi/6}^{5\pi/6} - 15$$

$$= (380 - 15)\text{ V}$$

$$= \underline{365\text{ V.}}$$



246. Write down the admittance between each line in turn and the other two connected together.



Between terminal 1 *and terminals* 2 *and* 3 *connected:*

$$Y_b + Y_c = Y_1(Y_2 + Y_3)/(Y_1 + Y_2 + Y_3) \quad (1)$$

Between terminal 2 *and terminals* 1 *and* 3 *connected:*

$$Y_c + Y_a = Y_2(Y_1 + Y_3)/(Y_1 + Y_2 + Y_3) \quad (2)$$

Between terminal 3 *and terminals* 1 *and* 2 *connected:*

$$Y_a + Y_b = Y_3(Y_1 + Y_2)/(Y_1 + Y_2 + Y_3) \quad (3)$$

Add (1), (2) and (3)

$$2(Y_a + Y_b + Y_c) = 2(Y_1Y_2 + Y_2Y_3 + Y_1Y_3)/(Y_1 + Y_2 + Y_3) \quad (4)$$

Subtract from (4) each of the first three equations in turn:

$$\underline{Y_a = Y_2Y_3/(Y_1 + Y_2 + Y_3),}$$

$$\underline{Y_b = Y_1Y_3/(Y_1 + Y_2 + Y_3),}$$

and $\underline{Y_c = Y_1Y_2/(Y_1 + Y_2 + Y_3).}$

247. For a star-connection, line current = phase current.

For a delta-connection, line current = $\sqrt{3}$ phase current.

$$\therefore \qquad \text{phase current} = 10/\sqrt{3} = \underline{5{\cdot}77\ \text{A.}}$$

248. (*a*) Phase voltage $V_p = 400/\sqrt{3} = I\sqrt{(10^2 + 10^2)}$ where I is the line or phase current.

$$\therefore \qquad I = \underline{16{\cdot}33\ \text{A.}}$$

Power absorbed in one phase = $V_pI\cos\phi$ where $\cos\phi$ is the power-factor.

$\therefore$ Power absorbed in one phase $= (400/\sqrt{3}) \times 16{\cdot}33 \times 1/\sqrt{2}$

$$= (8000/3)\ \text{W.}$$

Total power absorbed = sum of readings of the two wattmeters $= 3 \times (8000/3)\ \text{W} = \underline{8000\ \text{W.}}$

(*b*) Phase voltage $V_p = 400$ V

Phase current $I_p = 400/\sqrt{(10^2 + 10^2)} = 28{\cdot}28$ A

Line current $I_l = \sqrt{(3)}I_p = \sqrt{(3)} \times 28{\cdot}28$

$$= \underline{48{\cdot}97\ \text{A.}}$$

Power absorbed in one phase $= V_pI_p\cos\phi$

$$= (400 \times 28{\cdot}28 \times 1\sqrt{2})\ \text{W}$$

Total power absorbed $= (3 \times 400 \times 28{\cdot}28 \times 1/\sqrt{2})$ W

$$= \underline{24\,000\ \text{W.}}$$

249. For the star connection the line current = the phase current $= [480/\sqrt{(3)} \times 20]\ \text{A} = 8\sqrt{(3)}$ A.

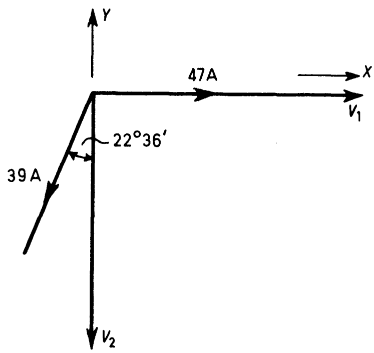
For the delta connection, if each resistor has a resistance R ohms, the phase current $= [8\sqrt{(3)}/\sqrt{(3)}]\ \text{A} = \underline{8\ \text{A.}}$

Also, $R = 480/8 = \underline{60\ \Omega.}$

250. The phasor diagram for the system is as shown. The current in the common return will be the resultant of the currents 47 A and 39 A. Resolving the currents in the X and Y directions:

$$X = 47 - 39 \sin 22°\,36' = 32 \text{ A}.$$

$$Y = -39 \cos 22°\,36' = 36 \text{ A}.$$



$\therefore$ the resultant current $= \sqrt{(X^2 + Y^2)} = \sqrt{(32^2 + 36^2)} = \underline{48{\cdot}17\,\text{A}}.$

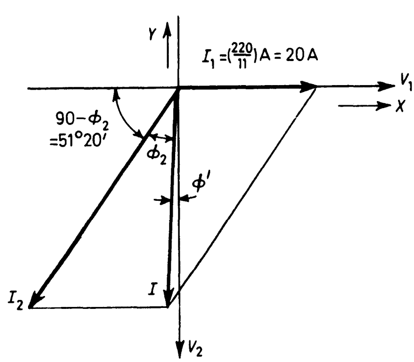
Let the phase-angle between this and the 47 A current be θ.

Then $\tan\theta = Y/X = 36/32 = 1{\cdot}125$

i.e. $\theta = 48°\,22'$.

i.e. the current in the common return lags the 47 A current by 48° 22′.

251. The phasor diagram is shown below:



$$I_1 = (220/11)\text{ A} = 20\text{ A}$$

$$I_2 = 220/\sqrt{(5^2 + 4^2)} = 34{\cdot}36\text{ A.}$$

$$\tan\phi_2 = 4/5 = 0{\cdot}8 \quad \text{so} \quad \phi_2 = 38^\circ\,40'.$$

Resolving in the X and Y directions:

$$X = 20 - 34{\cdot}36\cos 51^\circ\,20' = -1{\cdot}47$$

$$Y = -34{\cdot}36\cos 38^\circ\,40' = -26{\cdot}83.$$

The resultant current $I = \sqrt{(X^2 + Y^2)} = \sqrt{(1{\cdot}47^2 + 26{\cdot}83^2)}$

$$= \underline{26{\cdot}87\text{ A.}}$$

The phase-angle ϕ is given by $\tan\phi = Y/X = 26{\cdot}83/1{\cdot}47 = 18{\cdot}25$

$$\therefore \qquad \phi = 86^\circ\,54'$$

Angle required $\phi' = 90^\circ - 86^\circ\,54' = \underline{3^\circ\,6'}$.

252. Let the component fluxes due to the two phases be $\phi_1 = \phi\cos\omega t$ and $\phi_2 = \phi\sin\omega t$ since they are 90° out of phase.

The resultant flux $= \phi_R = \sqrt{(\phi_1{}^2 + \phi_2{}^2)}$

$$= \sqrt{(\phi^2\cos^2\omega t + \phi^2\sin^2\omega t)} = \phi.$$

Thus ϕ_R is constant in magnitude.

The angle between the resultant field and ϕ_1 is θ where

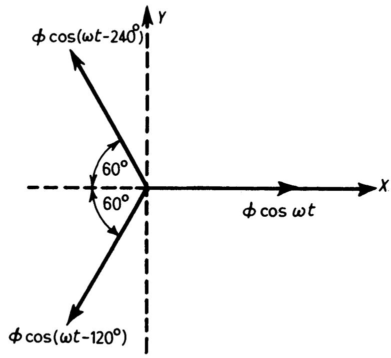
$$\tan\theta = \frac{\phi_2}{\phi_1} = \frac{\phi\sin\omega t}{\phi\cos\omega t} = \tan\omega t.$$

$$\therefore \qquad \theta = \omega t.$$

i.e. the angle between the resultant field and ϕ_1 is proportional to t. Thus, the resultant field rotates at uniform angular speed ω.

253. Let the component fluxes be $\phi\cos\omega t$, $\phi\cos(\omega t - 120^\circ)$ and $\phi\cos(\omega t - 240^\circ)$ at any instant. These fluxes act along axes which are mutually inclined to each other at 120° as shown in the sketch.

To find the resultant flux, resolve the component fluxes in the X and Y directions.



$$X = \phi \cos \omega t - \phi \cos (\omega t - 120°) \cos 60°$$
$$- \phi \cos (\omega t - 240°) \cos 60° = (3\phi \cos \omega t)/2.$$
$$Y = \phi \cos (\omega t - 240°) \sin 60° - \phi \cos (\omega t - 120°) \sin 60°$$
$$= -(3\phi \sin \omega t)/2.$$

$\therefore$ The resultant flux $\phi_R = \sqrt{(X^2 + Y^2)}$
$$= \sqrt{\{(\phi\, 3/2 \cos \omega t)^2 + [\phi\, (-3/2) \sin \omega t\,]^2\}}$$
$$= 3\phi/2.$$

Let the angle between the resultant flux and X at any instant be θ, then $\tan \theta = Y/X$

$$= \frac{-(3/2)\, \phi \sin \omega t}{(3/2)\, \phi \cos \omega t} = -\tan \omega t.$$

i.e. $$\theta = -\omega t.$$

$\therefore$ resultant flux is constant in magnitude and rotates at constant speed.

254. The input to the motor $= W = \dfrac{\text{Output}}{\text{Efficiency}}$

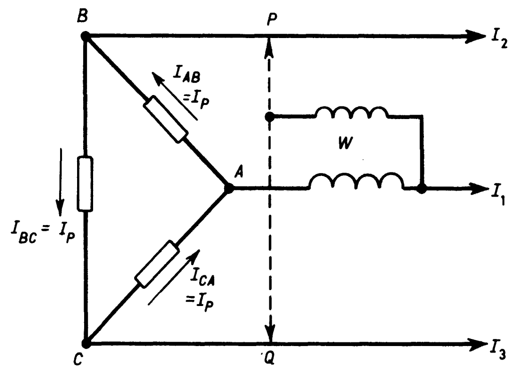
$$= \frac{75\,000}{0{\cdot}92} = 81\ 530\ \text{W}.$$

Now $W = \sqrt{(3)}\, V_l I_l \cos\phi$, where V_l is the line voltage and I_l is the line current

$\therefore$ $$81\,530 = \sqrt{(3)} \,.\, I_l \,.\, 3000 \,.\, 0{\cdot}9.$$

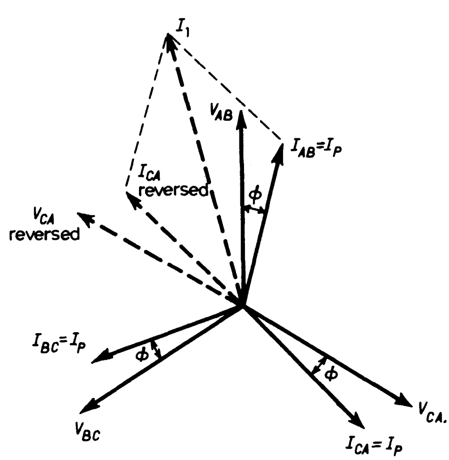
$\therefore$ $$= \underline{I_l = 17{\cdot}4\ \text{A}.}$$

255. The connections for the one-wattmeter method are shown, together with the phasor diagram.



The current I_1 through the wattmeter current coil is the phasor *difference of* I_{AB} and I_{CA} i.e. the phasor *sum* of I_{AB} and I_{CA} reversed.

When the wattmeter voltage coil is connected to P, the voltage across it is V_{AB} and the phase difference between it and the current is $(30 - \phi)$.



$$\therefore \text{ the reading } W_1 = V_{AB}I_1 \cos(30 - \phi)$$
$$= \sqrt{(3)}V_{AB}I_p \cos(30 - \phi)$$
$$= \sqrt{(3)}V_pI_p \cos(30 - \phi),$$

where V_p and I_p refer to the phase voltage and phase current respectively.

When the wattmeter voltage coil is connected to Q, the phase relation between the current and voltage for the wattmeter is between I_1 and V_{CA} reversed. The angle between I_1 and V_{CA} reversed is $(30 + \phi)$.

Thus if the wattmeter reading is now W_2:

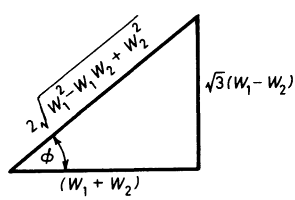
$$W_2 = V_{CA}I_1 \cos(30 + \phi).$$

$$\therefore \quad W_2 = \sqrt{(3)}I_pV_p \cos(30 + \phi).$$

$$\therefore \quad W_1 + W_2 = \sqrt{(3)}V_pI_p\{\cos(30 - \phi) + \cos(30 + \phi)\}$$
$$= 3V_pI_p \cos\phi.$$
$$= \underline{\text{the total power.}}$$

Also $W_1 = W_2 = \sqrt{(3)}V_pI_p \sin\phi$,

$$\therefore \quad \tan\phi = \frac{\sqrt{(3)}(W_1 - W_2)}{(W_1 + W_2)}.$$



It follows that the power-factor $\cos\phi = \underline{\dfrac{W_1 + W_2}{2\sqrt{(W_1^2 - W_1W_2 + W_2^2)}}}$.

256. The two formulae required are:

$$W_1 + W_2 = \text{total power}$$

and

$$\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\sqrt{3} = \tan\phi,$$

where W_1 and W_2 are the two wattmeter readings and ϕ is the phase-angle.

$$\therefore \qquad W_1 + W_2 = 20 \text{ kW} \qquad (1)$$

If $\cos\phi = 3/10$ $\tan\phi = \sqrt{(91)}/3$

$$\therefore \qquad \frac{\sqrt{91}}{3} = \frac{\sqrt{(3)}(W_1 - W_2)}{20}$$

$$\therefore \qquad W_1 - W_2 = 36{\cdot}7 \text{ kW.} \qquad (2)$$

From (1) and (2), $2W_1 = 56{\cdot}7$ kW

$$\therefore \qquad \underline{W_1 = 28{\cdot}35 \text{ kW.}}$$

$$\therefore \qquad W_2 = -(36{\cdot}7 - 28{\cdot}35) \text{ kW}$$

$$= \underline{-8{\cdot}35 \text{ kW.}}$$

257. *Induction motor on light load*

Wattmeter readings are: $W_1 = 389$, $W_2 = 271$.

If ϕ is the phase-angle of the load current

$$\tan\phi = \sqrt{(3)}(W_1 - W_2)/(W_1 + W_2)$$

$$\tan\phi = \sqrt{(3)} \times 118/660 = 0{\cdot}31$$

i.e. $\phi = 17° \, 12'$

and the power-factor $\cos\phi = \underline{0{\cdot}955.}$

Total power $= W_1 + W_2 = \underline{660 \text{ W.}}$

Also, if V and I are the line voltage and current

$$\sqrt{(3)}VI\cos\phi = W_1 + W_2 = 660$$

$$\therefore \qquad I = 660/[\sqrt{(3)} \times 400 \times 0{\cdot}955] = \underline{0{\cdot}996 \text{ A.}}$$

Induction motor on no-load

$$W_1 = 788{\cdot}5, \; W_2 = -288{\cdot}5$$

$$\therefore \qquad \tan\phi = \sqrt{(3)} \times 1077/500 = 3{\cdot}73$$

i.e. $\phi = 75°$

and the power-factor $\cos\phi = \underline{0{\cdot}26.}$

Total power $= W_1 + W_2 = \underline{500\text{ W.}}$

Line current $I = 500/[\sqrt{(3)} \times 400 \times 0{\cdot}26] = \underline{2{\cdot}78\text{ A.}}$

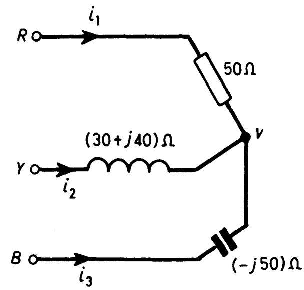
258. Voltages of lines with respect to earth are:

V, $V\angle -120°$ and $V\angle -240°$

where

$$V = 400/\sqrt{(3)} = 230 \text{ volts}$$

Let potential of star-point be v.



Then $i_1 = (V_1 - v)/50$

$$i_2 = (V_2 - v)/(30 + j40)$$

and $i_3 = (V_3 - v)/(-j50)$

Also $i_1 + i_2 + i_3 = 0$

so
$$v\left\{\frac{1}{50} + \frac{1}{(30 + j40)} + \frac{1}{(-j50)}\right\}$$
$$= \frac{V_1}{50} + \frac{V_2}{(30 + j40)} + \frac{V_3}{(-j50)}$$
$$= \frac{V}{50}\{1 + (0{\cdot}6 - j0{\cdot}8)\angle -120° + j\angle -240°\}$$

i.e.
$$v = \frac{V(-0{\cdot}86 - j0{\cdot}62)}{1{\cdot}6 + j0{\cdot}2}$$

$\therefore$
$$i_1 = \frac{V_1 - v}{50} = 0{\cdot}032V\angle 11{\cdot}5°$$
$$= 0{\cdot}032 \times 230\angle 11{\cdot}5° = \underline{7{\cdot}35\angle 11{\cdot}5°}$$
$$i_2 = 0{\cdot}011\text{ V }\angle -133{\cdot}5° = \underline{2{\cdot}53\angle -133{\cdot}5°}$$

and
$$i_3 = 0{\cdot}024\text{ V }\angle 175{\cdot}5° = \underline{5{\cdot}5\angle 175{\cdot}5°.}$$

259. $V_{YB} = 380\angle 240°$ volts, $V_{BR} = 380\angle 120°$ volts

$$I_1 = \frac{V_{RY}}{Z_1} = \frac{380\angle 0°}{10 + j10} = \frac{380\angle 0°}{14{\cdot}14\angle 45°} = 26{\cdot}8\angle -45°$$

$$= (19 - j19)\text{ A.}$$

$$I_2 = \frac{V_{YB}}{Z_2} = \frac{380\angle 240°}{8{\cdot}66 + j5} = \frac{380\angle 240°}{10\angle 30°} = 38\angle 210°$$

$$= -(32{\cdot}9 + j19)\text{ A.}$$

$$I_3 = \frac{V_{BR}}{Z_3} = \frac{380\angle 120°}{12 + j16} = \frac{380\angle 120°}{20\angle 53{\cdot}1°} = 19\angle 66{\cdot}9°$$

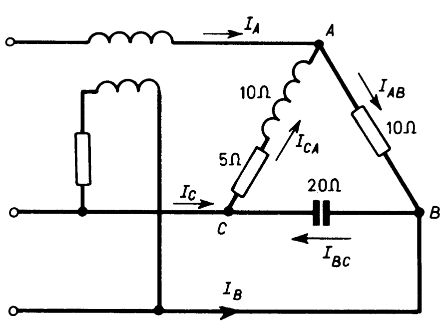
$$= (7{\cdot}45 + j17{\cdot}5)\text{ A.}$$

$\therefore \quad I_R = I_1 - I_3 = (11{\cdot}5 - j36{\cdot}5)\text{ A} = \underline{38{\cdot}2\angle -72{\cdot}5°\text{ A}}$

$I_Y = I_2 - I_1 = (-51{\cdot}9 - j0)\text{ A} = \underline{51{\cdot}9\angle 180°\text{ A}}$

$I_B = I_3 - I_2 = (40{\cdot}4 + j36{\cdot}5)\text{ A} = \underline{54{\cdot}3\angle 42{\cdot}1°\text{ A.}}$

260.



$V_{AB} = 400(1 + j0)$, $V_{BC} = 400\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$,

$V_{CA} = 400\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$.

$I_{AB} = 40$

$$I_{BC} = 400\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)/(-j20) = -j10 + 17{\cdot}3$$

$$I_{CA} = 400\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)/(5 + j10) = 19{\cdot}7 + j29{\cdot}8$$

$$I_A = I_{AB} - I_{CA} = 20{\cdot}3 - j29{\cdot}8$$

$\therefore \quad \underline{|I_A| = 36{\cdot}1 \text{ A}}$

$$I_B = I_{BC} - I_{AB} = -22{\cdot}7 - j10$$

$\therefore \quad \underline{|I_B| = 24{\cdot}8 \text{ A}}$

$$I_C = I_{CA} - I_{BC} = 2{\cdot}4 + j39{\cdot}8$$

$\therefore \quad \underline{|I_C| = 39{\cdot}9 \text{ A}}$

I_A lags V_{AB} by $\tan^{-1}(29{\cdot}8/20{\cdot}3)$ i.e. by 55° 53′.

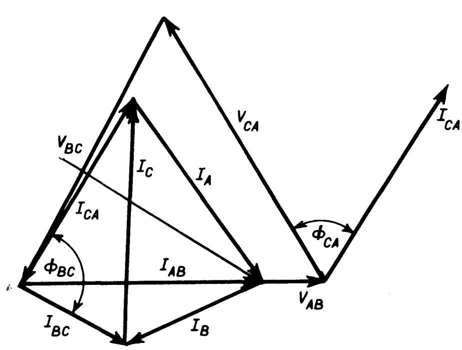
V_{BC} lags V_{AB} by 120°

$\therefore$ Angle between I_A and V_{BC} = 64° 7′.

Wattmeter reading = 400 × 36·1 × cos 64° 7′

= <u>6130 W.</u>

Alternative Solution by Drawing Phasor Diagram to Scale



$|I_{AB}| = 40$ A, $|I_{BC}| = 20$ A, $\phi_{BC} = 90°$

$|I_{CA}| = 400/\sqrt{125} = 35{\cdot}8$ A, $\phi_{CA} = 63{\cdot}5°$

$\therefore \quad \underline{|I_A| = 36{\cdot}1 \text{ A}, |I_B| = 24{\cdot}8 \text{ A}, |I_C| = 39{\cdot}9 \text{ A}.}$

Wattmeter reading $= 400 \times$ projection of I_A on V_{BC}

$$= \underline{6130\ \text{W.}}$$

261. Kirchhoff's first law gives the following equations:

$$I_A + I_B + I_C = 0$$

$$I_A = I_{AB} - I_{CA}$$

$$I_B = I_{BC} - I_{AB}$$

and $$I_C = I_{CA} - I_{BC}$$

Now $I_{AB} = V_{AB}/(j10) = -jV_{AB}/10 = -j400/10 = -j40$

so $$|I_{AB}| = \underline{40\ \text{A}}$$

$$I_{BC} = V_{BC}/10 = a^2 V_{AB}/10$$

where a is the '120° operator' $= \exp(j2\pi/3) = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

$\therefore$ $$I_{BC} = \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)40 = (-20 - j20\sqrt{3})$$

so $$|I_{BC}| = \underline{40\ \text{A}}$$

$$I_{CA} = V_{CA}/(-j10) = ja\,V_{AB}/10$$

$$= j\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)40 = (-j20 - 20\sqrt{3})$$

so $$|I_{CA}| = \underline{40\ \text{A}}$$

$\therefore$ $$I_A = -j20 + 20\sqrt{3}, \text{ so } |I_A| = \underline{40\ \text{A}}$$

$$I_B = -20 + j(40 - 20\sqrt{3}), \text{ so } |I_B| = \underline{20{\cdot}7\ \text{A}}$$

and $$I_C = -20[\sqrt{(3)} - 1] + j20[\sqrt{(3)} - 1],$$

so $$|I_C| = \underline{20{\cdot}7\text{A.}}$$

This type of problem can also easily be solved graphically.

$$W_1 \text{ reads } |I_A|\,|V_{AB}| \cos\phi_{AB} = \underline{13\,856\ \text{W}}$$

where ϕ_{AB} is the phase-angle between I_A and V_{AB}.

$$W_2 \text{ reads } |I_C|\,|V_{CB}| \cos\phi_{BC} = \underline{2130\ \text{W}}$$

where ϕ_{BC} is the phase-angle between I_C and V_{CB}.

Alternatively,

W_1 reads $\mathscr{R}[V_{AB} \,.\, I_A^*] = \mathscr{R}[400(j20 + 20\sqrt{3})] = \underline{13\,856 \text{ W.}}$

Similarly, W_2 reads $\mathscr{R}[V_{CB} \,.\, I_C^*]$

$$= \mathscr{R}\left[\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(-14{\cdot}64 - j14{\cdot}64)400\right]$$

$$= 2130 \text{ W.}$$

262. Let $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$ and $Y_3 = 1/Z_3$
The Theorem gives the voltage $V_{00'}$ between O and O′ directly, for

$$\underline{V_{00'} = (V_1Y_1 + V_2Y_2 + V_3Y_3)/(Y_1 + Y_2 + Y_3)}$$

263. The formulae for delta-star and star-delta transformations for resistor networks are quoted in Problem 75 and the same formulae, with resistances replaced by complex impedances, are used in a.c. circuits.

264. The method of solution to be used is first to transform each star-connected load to a delta, to combine these into a single delta and then solve in the usual way.

For star $A_1B_1C_1$

Using the results of the previous problem

$$Z_{A_1B_1} = 40(20 + j20)(10 - j)/200 = 4(11 + j9)\,\Omega$$

$$Z_{B_1C_1} = (20 + j20)(-j50)(10 - j)/200 = 5(9 - j11)\,\Omega$$

$$Z_{C_1A_1} = (-j50)(40)(10 - j)/200 = -10(1 + j10)\,\Omega$$

since

$$1/Z_{A_1} + 1/Z_{B_1} + 1/Z_{C_1} = 1/40 + (1 - j)/40 + j/50 = (10 - j)/200 \text{ S}$$

For star $A_2B_2C_2$

$$Z_{A_2B_2} = (-j50)(40)(5 + j2)/100 = 20(2 - j5)\,\Omega$$

$$Z_{B_2C_2} = (40)(40)(5 + j2)/100 = 16(5 + j2)\,\Omega$$

$$Z_{C_2A_2} = (40)(-j50)(5 + j2)/100 = 20(2 - j5)\,\Omega$$

since

$$1/Z_{A_2} + 1/Z_{B_2} + 1/Z_{C_2} = j/50 + 1/40 + 1/40 = (5 + j2)/100 \text{ S}$$

Combining the impedances of the two stars between lines:

$$Z_{A'B'} = Z_{A_1B_1}Z_{A_2B_2}/(Z_{A_1B_1} + Z_{A_2B_2}) = (64{\cdot}2 + j17{\cdot}4)\ \Omega$$

Similarly $Z_{B'C'} = (38{\cdot}4 - j\,24{\cdot}4)\ \Omega$

and $\quad Z_{C'A'} = (17{\cdot}3 + j19{\cdot}8)\ \Omega$

$$V_{AB} = (10 + j20)I_A + (64{\cdot}2 + j17{\cdot}4)I_{AB} - (10 + j20)I_B$$

where I_{AB} is the current through the branch of the resultant delta between A_1 and B_1

i.e. $$400 = (10 + j20)(I_{AB} - I_{CA}) + (64{\cdot}2 + j17{\cdot}4)I_{AB} - (10 + j20)(I_{BC} - I_{AB})$$

Similarly

$$a^2V_{AB} = V_{BC} = (10 + j20)I_B + (38{\cdot}4 - j24{\cdot}4)I_{BC} - (10 + j20)I_C$$

or

$$400\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = (10 + j20)(I_{BC} - I_{AB}) + (38{\cdot}4 - j24{\cdot}4)I_{BC} - (10 + j20)(I_{CA} - I_{BC})$$

and

$$aV_{AB} = V_{CA} = 400\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = (10 + j20)(I_{CA} - I_{BC})$$
$$= (17{\cdot}3 + j19{\cdot}8)I_{CA} - (10 + j20)(I_{AB} - I_{CA})$$

From these equations it is found that:

$$\underline{I_A = (3{\cdot}93 - j6{\cdot}30)\ \text{A}}$$

$$\underline{I_B = (-2{\cdot}54 - j1{\cdot}80)\ \text{A}}$$

and $$\underline{I_C = (-1{\cdot}39 + j8{\cdot}10)\ \text{A}}$$

265. It may be shown that*:

(*a*) the positive-sequence component $V_1 = (V_A + aV_B + a^2V_C)/3$

(*b*) the negative-sequence component $V_2 = (V_A + a^2V_B + aV_C)/3$

(*c*) the zero-sequence component $\quad V_0 = (V_A + V_B + V_C)/3$

*See, for example, F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, Second Edition, 1963, pp. 204–5.

Here $V_0 = -j50/3 = \underline{(-j16 \cdot 67)\text{V}}$

$$V_1 = \left\{100 + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(-100 - j100) + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(j50)\right\}/3$$

$$= \underline{(93 \cdot 3 - j20 \cdot 5)\text{ V}}$$

$$V_2 = \left\{100 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(-100 - j100) + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(j50)\right\}/3$$

$$= \underline{(6 \cdot 7 + j37 \cdot 19)\text{ V.}}$$

279. The instantaneous current

$$i = 10 \sin \omega t + 3 \sin 3\omega t + 2 \sin 5\omega t$$

Now $i^2 = (10 \sin \omega t + 3 \sin 3\omega t + 2 \sin 5\omega t)^2$

$$\therefore \quad i^2 = 10^2 \sin^2 \omega t + 3^2 \sin^2 3\omega t + 2^2 \sin^2 5\omega t + 2 . 10 . 3 \sin \omega t \sin 3\omega t + 2 . 10 . 2 \sin \omega t \sin 5\omega t + 2 . 3 . 2 \sin 3\omega t \sin 5\omega t.$$

The r.m.s. value of the current $= \sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)\right)}$

$\therefore$ r.m.s. value

$$= \sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi}\left[10^2 \sin^2 \omega t + 3^2 \sin^2 3\omega t + 2^2 \sin^2 5\omega t + \text{etc.}\right]d(\omega t)\right)}$$

All the terms containing the product of two sines when integrated over the range 0 to 2π disappear. This is easily seen by splitting such terms into the difference of two cosines.

$\therefore$ r.m.s. value =

$$\sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi}\frac{10^2(1 - \cos 2\omega t)}{2} + \frac{3^2}{2}(1 - \cos 6\omega t) + \frac{2^2}{2}(1 - \cos 10\omega t)d(\omega t)\right.}$$

$$= \sqrt{\left(\frac{1}{2\pi}\frac{(10^2 + 3^2 + 2^2)}{2} \cdot 2\pi\right)}$$

$$= \sqrt{[(10^2 + 3^2 + 2^2)/2]} = \sqrt{56 \cdot 5} = \underline{7 \cdot 52}.$$

280. $v = 2000 \sin \omega t + 500 \sin 3\, \omega t + 300 \sin 5\omega t.$

$$\text{Fundamental current (maximum value)} = \frac{2000}{1/(100\pi \times 20 \times 10^{-6})}$$

$$= 12 \cdot 6 \text{ A}$$

Third harmonic current (maximum value)

$$= \frac{500}{1/(300\pi \times 20 \times 10^{-6})}$$

$$= \underline{9 \cdot 43 \text{ A}}$$

Fifth harmonic current (maximum value)

$$= \frac{300}{1/(500\pi \times 20 \times 10^{-6})}$$

$$= \underline{9 \cdot 43 \text{ A}}$$

$$\text{R.m.s. value of current} = \frac{1}{\sqrt{2}}[\sqrt{(12 \cdot 6^2 + 9 \cdot 43^2 + 9 \cdot 43^2)}]$$

$$= \underline{12 \cdot 95 \text{ A.}}$$

281. $v = 50 \sin \omega t + 20 \sin 3\omega t + 15 \sin 5\omega t.$

The reactances of the capacitor and choke for the fundamental and harmonics are:

	Fundamental	Third Harmonic	Fifth Harmonic
Capacitor	−159 Ω	−53 Ω	−31·8 Ω
Choke	159 Ω	477 Ω	795 Ω

R.m.s. value of fundamental current

$$= \frac{50}{\sqrt{2}\sqrt{(25 + 0)}} = \underline{7 \cdot 07 \text{ A}}$$

R.m.s. value of third-harmonic current

$$= \frac{20}{\sqrt{2}\sqrt{[25 + (477 - 53)^2]}}$$

$$= \underline{0{\cdot}0333 \text{ A}}$$

R.m.s. value of fifth-harmonic current

$$= \frac{15}{\sqrt{2}\sqrt{[25 + (795 - 31{\cdot}8)^2]}}$$

$$= \underline{0{\cdot}0139 \text{ A.}}$$

The r.m.s. voltages across the capacitor are:

(*a*) Fundamental $= 7{\cdot}07 \times 159 = \underline{1125 \text{ V.}}$

(*b*) Third harmonic $= 0{\cdot}0333 \times 53 = \underline{1{\cdot}76 \text{ V.}}$

(*c*) Fifth harmonic $= 0{\cdot}0139 \times 31{\cdot}8 = \underline{0{\cdot}442 \text{ V.}}$

282. $v = 150 \sin \omega t + 15 \sin 3\omega t + 7{\cdot}5 \sin 5\omega t.$

Reactance of circuit to fundamental current

$$= 2\pi \times 50 \times 0{\cdot}05 = (5\pi)\,\Omega$$

Reactance of circuit to third-harmonic current

$$= 2\pi \times 150 \times 0{\cdot}05 = (15\pi)\,\Omega.$$

Reactance of circuit to fifth-harmonic current

$$= 2\pi \times 250 \times 0{\cdot}05 = (25\pi)\,\Omega.$$

Fundamental-current amplitude

$$i_1 = \frac{150}{\sqrt{[15^2 + (5\pi)^2]}} = \underline{6{\cdot}91 \text{ A.}}$$

Third-harmonic-current amplitude

$$i_3 = \frac{15}{\sqrt{[15^2 + (15\pi)^2]}} = \underline{0{\cdot}304 \text{ A.}}$$

Fifth-harmonic-current amplitude

$$i_5 = \frac{7 \cdot 5}{\sqrt{[15^2 + (25\pi)^2]}} = \underline{0 \cdot 0938 \text{ A.}}$$

$\therefore$ $$i_3/i_1 = 0 \cdot 304/6 \cdot 91 = \underline{0 \cdot 0441}$$

and $$i_5/i_1 = 0 \cdot 0938/6 \cdot 91 = \underline{0 \cdot 0136}$$

283. $v = 150 \sin \omega t + 15 \sin 3\omega t + 7 \cdot 5 \sin 5\omega t.$

Reactance of circuit to fundamental current

$$= 10^6/(100\pi \times 30) = 106\ \Omega.$$

Reactance of circuit to third-harmonic current

$$= 10^6/(300\pi \times 30) = 35 \cdot 3\ \Omega.$$

Reactance of circuit to fifth-harmonic current

$$= 10^6/(500\pi \times 30) = 21 \cdot 2\ \Omega.$$

Fundamental-current amplitude

$$i_1 = \frac{150}{\sqrt{(5^2 + 106^2)}} = \underline{1 \cdot 41 \text{ A.}}$$

Third-harmonic-current amplitude

$$i_3 = \frac{15}{\sqrt{(5^2 + 35 \cdot 3^2)}} = \underline{0 \cdot 422 \text{ A.}}$$

Fifth-harmonic-current amplitude

$$i_5 = \frac{7 \cdot 5}{\sqrt{(5^2 + 21 \cdot 2^2)}} = \underline{0 \cdot 344 \text{ A.}}$$

$\therefore$ $$i_3/i_1 = 0 \cdot 422/1 \cdot 41 = \underline{0 \cdot 299}$$

and $$i_5/i_1 = 0 \cdot 344/1 \cdot 41 = \underline{0 \cdot 244.}$$

284. $i = 5 \sin \omega t + 0 \cdot 3 \sin (3\omega t + 70 \cdot 1°) + 0 \cdot 1 \sin (5\omega t + 159°)$

Reactance of circuit to fundamental current is

$$X_1 = \omega L = 100\pi \times 0 \cdot 035 = 3 \cdot 5\pi = 11\ \Omega.$$

Reactance of circuit to third-harmonic current is

$$X_3 = 11 \times 3 = 33\,\Omega.$$

Reactance of circuit to fifth-harmonic current is

$$X_5 = 11 \times 5 = 55\,\Omega.$$

Amplitude of fundamental voltage

$$= V_1 = 5\sqrt{(10^2 + 11^2)} = \underline{74{\cdot}4\text{ V.}}$$

Amplitude of third-harmonic voltage

$$= V_3 = 0{\cdot}3\sqrt{(10^2 + 33^2)} = \underline{10{\cdot}35\text{ V.}}$$

Amplitude of fifth-harmonic voltage

$$= V_5 = 0{\cdot}1\sqrt{(10^2 + 55^2)} = \underline{5{\cdot}59\text{ V.}}$$

Fundamental current lags fundamental voltage by angle ϕ_1

$$= \tan^{-1}(X_1/10) = \tan^{-1}(11/10) = 47^\circ\,43'$$

Third-harmonic current lags third-harmonic voltage by angle ϕ_3

$$= \tan^{-1}(X_3/10) = \tan^{-1}(33/10) = 73^\circ\,15'$$

Fifth-harmonic current lags fifth-harmonic voltage by angle ϕ_5

$$= \tan^{-1}(X_5/10) = \tan^{-1}(55/10) = 79^\circ\,42'$$

$\therefore$ voltage wave is given by:

$$\begin{aligned} v &= V_1 \sin(\omega t + 47{\cdot}7^\circ) + V_3 \sin(3\omega t + 70{\cdot}1^\circ + 73{\cdot}25^\circ) \\ &\quad + V_5 \sin(5\omega t + 159^\circ + 79{\cdot}7^\circ) \\ &= V_1 \sin(\omega t + 47{\cdot}7^\circ) + V_3 \sin(3\omega t + 143{\cdot}35^\circ) \\ &\quad + V_5 \sin(5\omega t + 238{\cdot}7^\circ). \end{aligned}$$

Fundamental voltage component is zero when $\omega t = -47{\cdot}7^\circ$ and at this instant third-harmonic voltage is

$$V_3 \sin\{3 \times (-47{\cdot}7^\circ) + 143{\cdot}35^\circ\} = 0$$

i.e. <u>third-harmonic voltage is in phase with fundamental-voltage component.</u>

Similarly, when $\omega t = -47{\cdot}7^\circ$, fifth-harmonic voltage is

$$V_5 \sin\{5 \times (-47{\cdot}7^\circ) + 238{\cdot}7^\circ\} = 0$$

i.e. <u>fifth-harmonic voltage is in phase with fundamental-voltage component.</u>

285. $v = (2000 \sin \omega t + 600 \sin 3\omega t + 400 \sin 5\omega t)$.

Capacitive reactance to fundamental current is

$$X_1 = 1/\omega C = 1/(100\pi \times 30 \times 10^{-6}) = 106\,\Omega.$$

Capacitive reactance to third-harmonic current is

$$X_3 = (106/3)\,\Omega = 35{\cdot}3\,\Omega.$$

Capacitive reactance to fifth-harmonic current is

$$X_5 = (106/5)\,\Omega = 21{\cdot}2\,\Omega.$$

The resonance condition gives:

$$3\omega L = 35{\cdot}3$$

i.e.

$$L = 35{\cdot}3/(3 \times 100\pi) = 0{\cdot}0375\text{ H}.$$

Inductive reactance to fundamental current is

$$X_1' = \omega L = 100\pi \times 0{\cdot}0375 = 11{\cdot}77\,\Omega.$$

Inductive reactance to fifth-harmonic current is

$$X_5' = (11{\cdot}77 \times 5)\,\Omega = 58{\cdot}85\,\Omega.$$

$\therefore$ fundamental-current amplitude

$$I_1 = \frac{2000}{\sqrt{[10^2 + (11{\cdot}77 - 106)^2]}} = 21{\cdot}2\text{ A}.$$

third-harmonic current amplitude

$$I_3 = \frac{600}{10} = 60\text{ A}.$$

fifth-harmonic current amplitude

$$I_5 = \frac{400}{\sqrt{[10^2 + (58{\cdot}85 - 21{\cdot}2)^2]}} = 10{\cdot}36\text{ A}.$$

$$\therefore\ I_{\text{r.m.s.}} = \frac{1}{\sqrt{2}}\sqrt{(I_1{}^2 + I_3{}^2 + I_5{}^2)} = \frac{1}{\sqrt{2}}\sqrt{(4156)} = \underline{45{\cdot}6\text{ A.}}$$

R.m.s. voltage across L

$$= \frac{1}{\sqrt{2}}\left[(X_1'I_1)^2 + (X_3'I_3)^2 + (X_5'I_5)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}\left[(11{\cdot}77 \times 21{\cdot}2)^2 + (35{\cdot}3 \times 60)^2 + (58{\cdot}85 \times 10{\cdot}36)^2\right]^{\frac{1}{2}}$$

$$= \underline{2220\text{ V.}}$$

286. The wave-shape of the $v - \theta$ curve is triangular and is symmetrical about $\pi/2$.

$\therefore$ let $\quad a_n = 2 \times$ mean ordinate of $v \sin n\theta$

$$a_n = 2 \times \frac{2}{\pi}\int_0^{\pi/2} v \sin n\theta \, d\theta.$$

But $\quad \dfrac{v}{\theta} = \dfrac{300}{\pi/2}$

$$\therefore \quad a_n = \frac{2400}{\pi^2}\int_0^{\pi/2} \theta \sin n\theta \, d\theta = \frac{2400}{n^2\pi^2}$$

Thus, $a_1 = 2400/\pi^2 = \underline{244.}$

$a_3 = 244/9 = \underline{27.}$

$a_5 = 244/25 = \underline{9{\cdot}8.}$

and $a_7 = 244/49 = \underline{5.}$

The waveform of the voltage can therefore be represented by:

$$\underline{v = 244 \sin \theta + 27 \sin 3\theta + 9{\cdot}8 \sin 5\theta + 5 \sin 7\theta + \ldots}$$

287. $v = 8000 \sin \omega t + 540 \sin 3\omega t + 350 \sin 5\omega t$, where $\omega = 100\pi$.

R.m.s. value of $v = \dfrac{1}{\sqrt{2}}[\sqrt{(8000^2 + 540^2 + 350^2)}] = \underline{5680 \text{ V.}}$

(*a*) *Current through capacitor*

Let fundamental and harmonic currents be i_1, i_3 and i_5 respectively.

$$\therefore \quad i_1 = 8000 \times 100\pi \times 125 \times 10^{-6} = (100\pi) \text{ A}$$

$$i_3 = 540 \times 300\pi \times 125 \times 10^{-6} = (20{\cdot}25\pi) \text{ A}$$

$$i_5 = 350 \times 500\pi \times 125 \times 10^{-6} = (21{\cdot}9\pi) \text{ A}$$

$$\therefore \quad \text{r.m.s. value of current} = \frac{\pi}{\sqrt{2}}[\sqrt{(100^2 + 20{\cdot}25^2 + 21{\cdot}9^2)}]$$

$$= \underline{232 \text{ A.}}$$

(*b*) *Current through coil*

Let fundamental and harmonic currents be i_1, i_3 and i_5 respectively.

$$\therefore \quad i_1 = 8000/(100\pi \times 0{\cdot}05) = (1600/\pi)\ \text{A}$$

$$i_3 = 540/(300\pi \times 0{\cdot}05) = (36/\pi)\ \text{A}$$

$$i_5 = 350/(500\pi \times 0{\cdot}05) = (14/\pi)\ \text{A}$$

$$\therefore \text{ r.m.s. value of current} = \frac{1}{\pi\sqrt{2}}[\sqrt{(1600^2 + 36^2 + 14^2)}]$$

$$= \underline{361\ \text{A.}}$$

288. *When the fundamental angular frequency* $\omega = 314$ rad s^{-1}

Inductive reactance $X_1 = \omega L = 314 \times 6{\cdot}36 \times 10^{-3} = 2\ \Omega$

Impedance $Z_1 = \sqrt{(10^2 + 2^2)} = 10{\cdot}2\ \Omega.$

Phase-angle $= \tan^{-1}(2/10) = 11{\cdot}3°$ lagging.

At the third-harmonic frequency

Inductive reactance $X_3 = 3\omega L = 6\ \Omega.$

Impedance $Z_3 = \sqrt{(10^2 + 6^2)} = 11{\cdot}7\ \Omega.$

Phase-angle $= \tan^{-1}(6/10) = 31°$ lagging.

At the fifth-harmonic frequency

Inductive reactance $X_5 = 5\omega L = 10\ \Omega.$

Impedance $Z_5 = \sqrt{(10^2 + 10^2)} = 14{\cdot}1\ \Omega.$

Phase-angle $= \tan^{-1}(10/10) = 45°$ lagging.

$$\therefore \text{ current } i = \frac{300}{10{\cdot}2}\sin(\omega t - 11{\cdot}3°) + \frac{50}{11{\cdot}7}\sin(3\omega t - 31°) + \frac{40}{14{\cdot}1}\sin(5\omega t - 45°)$$

$$= \underline{29{\cdot}4\sin(\omega t - 11{\cdot}3°) + 4{\cdot}28\sin(3\omega t - 31°) + 2{\cdot}83\sin(5\omega t - 45°)}$$

$$\text{Power dissipated} = 10\left(\frac{29{\cdot}4^2}{2} + \frac{4{\cdot}28^2}{2} + \frac{2{\cdot}83^2}{2}\right)$$

$$= \underline{4454\ \text{W.}}$$

289. Capacitive reactance at fundamental frequency

$$= 1/(314 \times 3{\cdot}18 \times 10^{-6}) = 1000\,\Omega.$$

Impedance of circuit at fundamental frequency,

$$Z_1 = 1000 + \frac{1000(-j1000)}{1000 - j1000} = 1500 - j500 = 1580\angle -18{\cdot}5°\,\Omega.$$

Capacitive reactance at third-harmonic frequency = (1000/3) Ω.

Impedance of circuit at this frequency,

$$Z_3 = 1000 + \frac{1000(-j333)}{1000 - j333} = 1100 - j300 = 1140\angle -15{\cdot}3°\,\Omega$$

$$\therefore \text{ current } i = \frac{350}{1580}\sin(\omega t + 18{\cdot}5°) + \frac{150}{1140}\sin(3\omega t + 15{\cdot}3°)$$

$$= 0{\cdot}222 \sin(\omega t + 18{\cdot}5°) + 0{\cdot}131 \sin(3\omega t + 15{\cdot}3°)$$

(*a*) Total power $= \frac{350}{\sqrt{2}} \times \frac{0{\cdot}222}{\sqrt{2}} \cos 18{\cdot}5° + \frac{150}{\sqrt{2}} \times \frac{0{\cdot}131}{\sqrt{2}} \cos 15{\cdot}3°$

$= \underline{46{\cdot}3 \text{ W}}.$

(*b*) Voltage across the series resistor = $1000i$

$= \underline{\{222 \sin(\omega t + 18{\cdot}5°) + 131 \sin(3\omega t + 15{\cdot}3°)\} \text{ V}}$

(*c*) Percentage harmonic content $= \frac{131}{222} \times 100\% = \underline{59\%}.$

290. $v = (100 + 50 \sin 314t + 20 \sin 942t)$ volts

$i = \{10 + 3{\cdot}54 \sin(314t - 45°) + 0{\cdot}635 \sin(942t - 71{\cdot}6°)\}$ amps.

It is evident from the solution to Problem 279 that

$$V_{rms} = \sqrt{\left(100^2 + \frac{50^2 + 20^2}{2}\right)} = \underline{107 \text{ V}}$$

and

$$I_{rms} = \sqrt{\left(10^2 + \frac{3{\cdot}54^2 + 0{\cdot}635^2}{2}\right)} = \underline{10{\cdot}34 \text{ A}}.$$

291. The current at each frequency is found by dividing the corresponding voltage by the resistance.

$\therefore$ current $i = 1 + 2\sin 250t + 1{\cdot}5\sin(500t + 30°)$ amps.

Average current, $I_{dc} = \underline{1\text{ A}}$.

R.m.s. current, $I_{r.m.s.} = \sqrt{\left(1^2 + \frac{2^2 + 1{\cdot}5^2}{2}\right)}$ A (see solution to Problem 279)

$= \underline{2{\cdot}03\text{ A}}$.

Power delivered to resistor

$$= (10 \times 1) + \left(\frac{20 \times 2}{2}\right) + \left(\frac{15 \times 1{\cdot}5}{2}\right) = \underline{41{\cdot}2\text{ W}}$$

or

Power delivered to resistor $= 10 \times (I_{r.m.s.})^2$

$= 10 \times 2{\cdot}03^2 = \underline{41{\cdot}2\text{ W}}$.

292. $v = (200\cos 314t - 40\sin 628t)$ volts.

When the angular frequency $\omega = 314$, $1/\omega C = 10^6/(314 \times 100) = 31{\cdot}83\ \Omega$.

When the angular frequency $\omega = 628$, $1/\omega C = 15{\cdot}915\ \Omega$.

The impedance when $\omega = 314$ is $\sqrt{(20^2 + 31{\cdot}83^2)} = 37{\cdot}55\ \Omega$ and the corresponding maximum current $= (200/37{\cdot}55)$ A $= 5{\cdot}33$ A.

Also if the phase-angle between voltage and current is ϕ, $\cos\phi = 20/37{\cdot}55 = 0{\cdot}533$ so $\phi = 57°\,48'$

The impedance when $\omega = 628$ is $\sqrt{(20^2 + 15{\cdot}915^2)} = 25{\cdot}55\ \Omega$ and the maximum current is then $(40/25{\cdot}55)$ A $= 1{\cdot}565$ A.

Now $\cos\phi = 20/25{\cdot}55$ and $\phi = 38°\,33'$

$\therefore$ steady-state current is:

$$\underline{i = 5{\cdot}33\cos(314t + 57°\,48') - 1{\cdot}565\sin(628t + 38°\,33')}$$

Rm.s. current $= \frac{1}{\sqrt{2}} \times \sqrt{(5{\cdot}33^2 + 1{\cdot}565^2)} = \underline{3{\cdot}94\text{ A}}$.

301. Let N_1 = primary turns

Let N_2 = secondary turns = 130.

Let I_1 = primary current.

Let I_2 = secondary current.

Let E_1 = primary e.m.f.

Let E_2 = secondary e.m.f.

Then $I_1N_1 = I_2N_2$ and $E_1/E_2 = N_1/N_2$

$\therefore \quad 2000/440 = N_1/130 \qquad \therefore \quad N_1 = \underline{591 \text{ turns.}}$

$$\text{Power} = 20 \times 1000 \text{ W.}$$

$$\therefore \quad I_1 = 20 \times 1000/2000 = \underline{10 \text{ A.}}$$

$$I_2 = \frac{I_1N_1}{N_2} = 10 \cdot \frac{591}{N_2} = 10 \cdot \frac{591}{130} = \underline{45{\cdot}5 \text{ A.}}$$

302. Max^m. e.m.f. = $N\phi_{max}\,\omega$ webers, where N is the number of turns, ϕ_{max} is the maximum flux and $\omega = 2\pi \times$ frequency.

$$\therefore \quad \sqrt{(2)} \times 2000 = 500 \times \phi_{max} \times 2\pi \times 50.$$

$$\therefore \quad \underline{\phi = 0{\cdot}018 \text{ Wb.}}$$

303. $\text{E.m.f.} = N\dfrac{d\phi}{dt} = e$

Let $\phi = \phi_{max} \sin \omega t$

$$\therefore \quad \frac{d\phi}{dt} = \phi_{max}\,\omega \cos \omega t$$

$$\therefore \quad e_{max} = N\phi_{max}\omega \text{ volts}$$

where ϕ_{max} is in webers and $\omega = 2\pi f$

$$\therefore \quad e_{r.m.s.} = \frac{N}{\sqrt{2}}\phi_{max}\, 2\pi f \text{ volts}$$

$$= 4{\cdot}44\phi_{max} fN \text{ volts}$$

For the primary.

$$e_{r.m.s.} = 4{\cdot}44 \times 0{\cdot}03 \times 50 \times 1000\text{V}$$

$$= \underline{6660 \text{ V.}}$$

For the secondary.

$$e_{\text{r.m.s.}} = 4{\cdot}44 \times 0{\cdot}03 \times 50 \times 35\text{V}$$
$$= \underline{233\text{ V.}}$$

304. As in the previous question:

$$e_{\text{r.m.s.}} = 4{\cdot}44\phi_{\text{max}}fN \text{ volts}$$

For the primary winding the line voltage = the phase voltage = 6600 V.

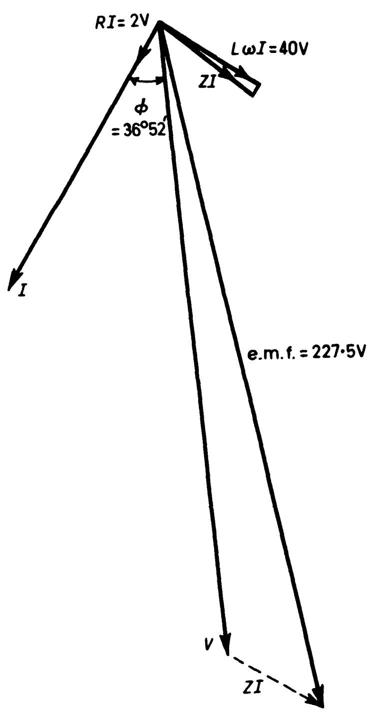
$$\therefore \quad 6600 = 4{\cdot}44 \times 0{\cdot}02 \times 50 \times N_p$$

$$\therefore \quad N_p = 6600/4{\cdot}44 = \underline{1486.}$$

For the secondary winding the phase voltage = $1/\sqrt{3}$ times the line voltage = $380/\sqrt{3}$.

$$\therefore \quad \frac{380}{\sqrt{3}} = 4{\cdot}44 \times 0{\cdot}02 \times 50 \times N_s$$

$$\therefore \quad N_s = \frac{380}{\sqrt{(3)}\,.\,4{\cdot}44} = \frac{380}{7{\cdot}69} = \underline{49.}$$



305. To find the secondary current, use $VI \cos \phi$ = watts,

$$\therefore \quad 200 \times I \times 0{\cdot}8 = 8000$$

$$\therefore \quad I = 50\text{ A.}$$

The secondary resistance drop is, therefore, $RI = 0{\cdot}04 \times 50 = 2$ V. The reactance drop = $0{\cdot}8 \times 50$ = 40 V.

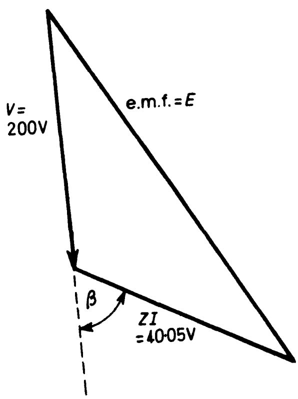
Assuming the current lags the voltage, the phasor diagram for the secondary quantities is as shown. The e.m.f. can be measured from this diagram and found to be $\underline{227{\cdot}5\text{ V.}}$

The e.m.f. can also be calculated as follows:

The impedance drop $ZI = \sqrt{(40^2 + 2^2)} \, . \, 50$ V

$$= 40{\cdot}05 \text{ V.}$$

ZI leads I by an angle θ, where $\tan\theta = \dfrac{L\omega}{R} = \dfrac{0{\cdot}8}{0{\cdot}04} = 20$



$\therefore \quad \theta = 87° \, 8'.$

$\therefore \quad \beta$ (in the diagram)

$= 50° \, 16' \, (87° \, 8' - 36° \, 52')$

$\therefore E^2 = 200^2 + 40{\cdot}05^2 + 2 \times 40{\cdot}05 \times 200 \cos\beta$

$= 40\,000 + 1604 + 10\,242$

$= 51\,846$

$\therefore \quad E = 227{\cdot}5$ V.

306. The induced e.m.f. in the primary = induced e.m.f. in the secondary $\times \dfrac{\text{primary turns}}{\text{secondary turns}}$

$$= 227{\cdot}5 \times 500/50 = \underline{2275 \text{ V.}}$$

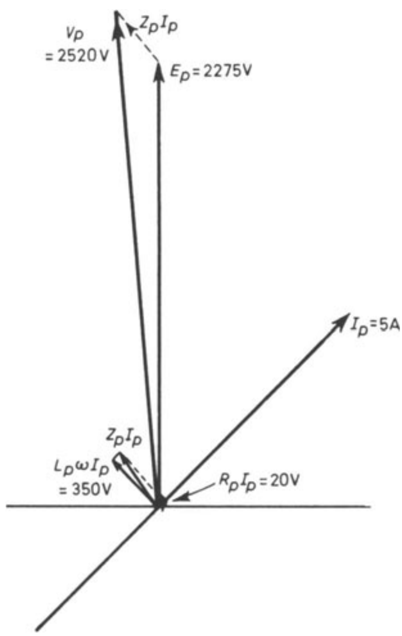
The primary current = secondary current × 50/500 = 5A = I_p.

The primary resistance drop = $R_p I_p = 4 \times 5 = 20$ V.

The primary leakage-reactance drop = 70 × 5 = 350 V. Knowing the angle between the secondary e.m.f. and the secondary current from the previous question, the phasor diagram for the primary quantities can be drawn as shown. The angle between the primary current and primary e.m.f. is the same as for the secondary quantities, since the magnetizing current can be neglected.

Adding the primary resistance drop and the primary leakage-reactance drop to the primary e.m.f., the primary voltage is found by measurement to be 2520 V.

The primary voltage can also be calculated, if desired, in a similar manner to the calculation in the previous question.



307. The total resistance referred to the primary R_{T1}

$$= R_1 + (N_1/N_2)^2R_2$$
$$= 10 + (6600/220)^2 0{\cdot}01 = \underline{19\ \Omega.}$$

Similarly, the total resistance referred to the secondary

$$= R_{T2} = R_2 + (N_2/N_1)^2R_1$$
$$= 0{\cdot}01 + (220/6600)^2 10 = \underline{0{\cdot}021\ \Omega.}$$

The full-load current $= 50\,000/6600 = 7{\cdot}58$ A.

$\therefore$ Copper loss $= R_{T1} \times 7{\cdot}58^2$

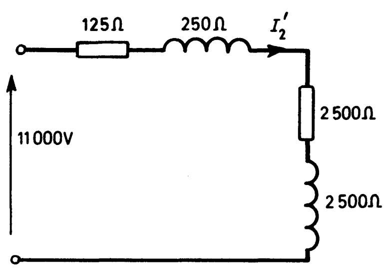
$$= 19 \times 7{\cdot}58^2 = \underline{1091\ \text{W.}}$$

308. The impedance $Z_1 = \dfrac{\text{Volts}}{\text{Current}} = \dfrac{30}{20} = \underline{1{\cdot}5\ \Omega.}$

The resistance $R_1 = \dfrac{\text{Watts}}{(\text{Current})^2} = \dfrac{200}{(20)^2} = \underline{0{\cdot}5\ \Omega}$

$$\therefore \quad \text{Reactance} = \sqrt{(Z_1^{\,2} - R_1^{\,2})}$$
$$= \sqrt{(1{\cdot}5^2 - 0{\cdot}5^2)}$$
$$= \underline{1{\cdot}41\ \Omega.}$$

309. Referring all quantities to the primary side the following circuit results:



$$I_2' = \frac{11\,000}{\sqrt{[(2500 + 125)^2 + (2500 + 250)^2]}} = \frac{11\,000}{3800}$$
$$= 2{\cdot}9\ \text{A}$$

$\therefore$ the secondary current $I_2 = (2{\cdot}9 \times 50)$ A

$= \underline{145\ \text{A}}$

P.d. between secondary terminals $V_2 = [\sqrt{(1^2 + 1^2)}]\ 145$

$= \underline{205\ \text{V}}$

Primary current $I_1 = I_2' = \underline{2{\cdot}9\ \text{A}}$

Power-factor $= \cos\phi_1 = (2500 + 125)/3800 = \underline{0{\cdot}69.}$

310. Turns ratio $\dfrac{N_2}{N_1} = \dfrac{400}{2000} = \dfrac{1}{5}$

Resistance referred to secondary, $R_2' = R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1$

$$= \left(0{\cdot}0068 + \frac{0{\cdot}17}{25}\right)\Omega = \underline{0{\cdot}0136\ \Omega.}$$

Reactance referred to secondary, $X_2' = X_2 + \left(\frac{N_2}{N_1}\right)^2 X_1$

$$= \left(0{\cdot}01 + \frac{0{\cdot}25}{25}\right)\Omega = \underline{0{\cdot}02\ \Omega.}$$

$\therefore$ impedance referred to secondary, Z_2'

$$= \sqrt{[(R_2')^2 + (X_2')^2]} = \underline{0{\cdot}0242\ \Omega}$$

$$\frac{R_2'I_2}{V_2} \times 100 = \frac{0{\cdot}0136 \times 250}{400} \times 100 = 0{\cdot}85$$

$$\frac{X_2'I_2}{V_2} \times 100 = \frac{0{\cdot}02 \times 250}{400} \times 100 = 1{\cdot}25$$

$$\therefore \qquad V_2' = V_2 - R_2'I_2 \cos\phi_2 - X_2'I_2 \sin\phi_2$$

where ϕ_2 is $\cos^{-1}(0{\cdot}8)$

i.e. V_2' (per cent) $= 100 - (0{\cdot}85 \times 0{\cdot}8) - (1{\cdot}25 \times 0{\cdot}6)$

$$= 98{\cdot}57$$

Thus, percentage regulation $= (100 - 98{\cdot}57)\%$

$$= \underline{1{\cdot}43\%.}$$

311. Transforming the delta-delta transformer to the equivalent star-star one its reactance and resistance become 0·3 and 0·1 Ω respectively.

The impedances are therefore:

$$Z_A = (0{\cdot}1 + j0{\cdot}75)\ \Omega$$

$$Z_B = (0{\cdot}1 + j0{\cdot}3)\ \Omega$$

$$\therefore \qquad Z_A + Z_B = (0{\cdot}2 + j1{\cdot}05)\ \Omega$$

$$|Z_A| = \sqrt{(0{\cdot}1^2 + 0{\cdot}75^2)} = 0{\cdot}756\ \Omega$$

$$|Z_B| = \sqrt{(0{\cdot}1^2 + 0{\cdot}3^2)} = 0{\cdot}316\ \Omega$$

$$|Z_A + Z_B| = \sqrt{(0{\cdot}4^2 + 1{\cdot}05^2)} = 1{\cdot}07\,\Omega$$

$$kVA_A = (100 \times 0{\cdot}316)/1{\cdot}07 = \underline{29{\cdot}3\text{ kVA}}$$

$$kVA_B = (100 \times 0{\cdot}756)/1{\cdot}07 = \underline{70{\cdot}7\text{ kVA.}}$$

312. *O.C. Test:*

Iron loss = 600 W.

Current/phase referred to primary = (5·4 × 440)/3300 = 0·72 A.

Power-factor, $\cos\phi = 600/[\sqrt{(3)} \times 3300 \times 0{\cdot}72] = 0{\cdot}146$

so $\sin\phi = 0{\cdot}988$

Power component of current = 0·72 × 0·146 = 0·105 A.

Quadrature component of current = 0·72 × 0·988 = 0·711 A.

S.C. Test:

Copper loss for primary current of 9A = 800 W.

Full-load current/phase = $50\,000/[\sqrt{(3)} \times 3300] = 8{\cdot}75$ A.

(primary neglecting

magnetizing current)

Full-load copper loss = $(8{\cdot}75/9)^2 \times 800$ W = 758 W.

Efficiency at Full-load, 0·8 *p.f. lagging*

Copper loss = 758 W
Iron loss = 600 W } Total = 1358 W.

Output = (50 000 × 0·8) W = 40 000 W.

Input = (40 000 + 1358) W = 41 358 W.

$$\text{Efficiency} = \left(1 - \frac{1358}{41\,358}\right) \times 100\% = \underline{96{\cdot}7\%.}$$

Efficiency at ¼ Full-load, 0·9 *p.f. lagging*

Copper loss = (758/16) W = 46·8 W
Iron loss = 600 W } Total = 646·8 W.

Output = 11 270 W.

Input = 11 917 W.

$$\text{Efficiency} = \left(1 - \frac{646{\cdot}8}{11\,917}\right) \times 100\% = \underline{94{\cdot}6\%.}$$

Full-load, 0·8 *p.f. lagging*

In-phase input current = (8·75 × 0·8) + 0·105 = 7·105 A.

Quadrature input current = (8·75 × 0·6) + 0·711 = 5·961 A.

Total current = $\sqrt{(7{\cdot}105^2 + 5{\cdot}961^2)}$ = 9·27 A.

¼ *Full-load,* 0·9 *p.f. lagging*

In-phase input current = (8·75 × 0·9)/4 + 0·105 = 2·070 A.

Quadrature input current = (8·75 × 0·435)/4 + 0·711 = 1·662 A

Total current = $\sqrt{(2{\cdot}070^2 + 1{\cdot}662^2)}$ = 2·65 A.

313. For the 200-kVA load, kW = 160, kVA_r = 120.

For the 50-kVA load, kW = 30, kVA_r = 40.

∴ total kW = 190 and total kVA_r = 160.

I cos ϕ component of current = (190 000 × $\sqrt{3}$)/(3 × 500) = 219·5 A.

I sin ϕ component of current = (160 000 × $\sqrt{3}$)/(3 × 500) = 185 A.

Full-load current of transformer = (100 000 × $\sqrt{3}$)/500 = 347 A.

Percentage resistance drop = (1 × 219·5)/347 = 0·63%.

Percentage reactance drop = (4 × 185)/347 = 2·13%.

Total voltage drop = 0·63 + 2·13 = 2·76%

$$= \left(\frac{2{\cdot}76}{100} \times 500\right) \text{V} = 13{\cdot}8 \text{ V (line)}$$

∴ line voltage at transformer secondary

= (500 − 13·8) V = 486·2 V.

200-*kVA line:*

I cos ϕ component of current = 160 000/[$\sqrt{(3)}$ × 500] = 185 A.

I sin ϕ component of current = 139 A.

$$\left.\begin{aligned} RI\cos\phi &= 0{\cdot}04 \times 185 = 7{\cdot}4 \\ XI\sin\phi &= 0{\cdot}05 \times 139 = 6{\cdot}95 \end{aligned}\right\} \text{Total } 14{\cdot}35 \text{ V/phase.}$$

Line voltage drop = $\sqrt{(3)}$ × 14·35 V = 24·8 V.

∴ voltage at 200-kVA load = (486·2 − 24·8) V

= 461·4 V.

50-kVA line:

I cos ϕ component of current = $30\,000/[\sqrt{(3)} \times 500] = 34{\cdot}7$ A.

I sin ϕ component of current = $46{\cdot}3$ A.

$$\left.\begin{aligned} RI\cos\phi &= 34{\cdot}7 \times 0{\cdot}1 = 3{\cdot}47 \\ XI\sin\phi &= 46{\cdot}3 \times 0{\cdot}2 = 9{\cdot}26 \end{aligned}\right\}\text{Total} = 12{\cdot}73 \text{ V/phase.}$$

Line voltage drop = $\sqrt{(3)} \times 12{\cdot}73$ V = 22 V.

$\therefore$ voltage at 50-kVA load = $(486{\cdot}2 - 22)$ V = $\underline{464{\cdot}2 \text{ V.}}$

314. The S.C. test gives:

Volts/phase = $180/\sqrt{3} = 104$ V

Current/phase = 25 A

Power/phase = $1600/3 = 533$ W

Impedance $Z = (104/25)\,\Omega = 4{\cdot}16\,\Omega$

Resistance $R = (533/25^2)\,\Omega = 0{\cdot}853\,\Omega$

$\therefore$ reactance $X = \sqrt{(4{\cdot}16^2 - 0{\cdot}853^2)} = 4{\cdot}07\,\Omega$

$$\text{Primary full-load current } (I) = \frac{50\,000}{\sqrt{(3)} \times 2000} = 14{\cdot}4 \text{ A}$$

$\therefore$ $RI\cos\phi = 0{\cdot}853 \times 14{\cdot}4 \times 0{\cdot}8 = 9{\cdot}82$ V

$XI\sin\phi = 4{\cdot}07 \times 14{\cdot}4 \times 0{\cdot}6 = 35{\cdot}1$ V

Power-factor lagging:

Volts drop/phase = $(9{\cdot}82 + 35{\cdot}1)$ V = $44{\cdot}92$ V

$$\text{Regulation} = \frac{44{\cdot}92 \times \sqrt{(3)} \times 100\%}{2000} = \underline{3{\cdot}89\%.}$$

Power-factor leading:

Volts drop/phase = $(9{\cdot}82 - 35{\cdot}1)$ V = $-25{\cdot}28$ V

$$\text{Regulation} = \frac{-25{\cdot}28 \times \sqrt{(3)} \times 100\%}{2000} = \underline{-2{\cdot}19\%.}$$

315.

(*a*) O.C. test gives power-factor p.f. $(\cos\phi_0) = \dfrac{120}{200 \times 1{\cdot}3}$

$= 0{\cdot}462$

$\therefore \qquad I_0 \sin \phi_0 = 1{\cdot}3 \times 0{\cdot}886 = \underline{1{\cdot}15\text{ A}}$

and $\qquad I_0 \cos \phi_0 = 1{\cdot}3 \times 0{\cdot}462 = \underline{0{\cdot}6\text{ A.}}$

(*b*) Impedance Z (referred to h.v. side) $= 22/30 = 0{\cdot}733\ \Omega$.
Resistance R (referred to h.v. side) $= 200/30^2 = 0{\cdot}222\ \Omega$.
$\therefore$ reactance $X = \sqrt{(0{\cdot}733^2 - 0{\cdot}222^2)} = 0{\cdot}698\ \Omega$.
Full-load current on h.v. side (I_2) $=$ (10 000/400) A $=$ 25 A.
Regulation $\simeq RI_2 \cos \phi - XI_2 \sin \phi$
where $\cos \phi = 0{\cdot}8$

$$\therefore \text{ regulation} = (0{\cdot}222 \times 25 \times 0{\cdot}8 - 0{\cdot}698 \times 25 \times 0{\cdot}6)\text{ V} = -6\text{ V}$$

$$\text{Percentage regulation} = \frac{-6}{400} \times 100\% = \underline{-1{\cdot}5\%}.$$

(*c*) O.C. test gives full-voltage iron loss $= P_i = 120$ W.

S.C. test gives full-voltage copper loss $= P_c = 200 \times (25/30)^2 = 140$ W.

Let m be the fraction of full-load kVA for which maximum efficiency occurs. At maximum efficiency the copper and iron losses are equal.

$\therefore$ under this condition $m^2 P_c = P_i$

i.e. $\qquad m = \sqrt{(120/140)} = 0{\cdot}925$

Thus, kVA for maximum efficiency $= 0{\cdot}925 \times 10 = 9{\cdot}25$ kVA.

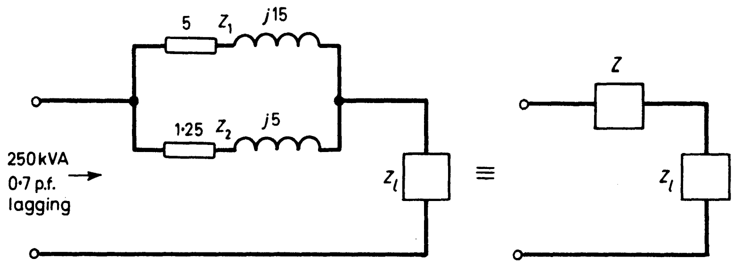
Efficiency is a maximum when load has unity power-factor too so this load $= \underline{9{\cdot}25\text{ kW.}}$

$$\text{Maximum efficiency} = \left(\frac{9250}{9250 + 120 + 120}\right) \times 100\% = \underline{97{\cdot}4\%}.$$

316. Re-rate the 100-kVA transformer to 250 kVA giving percentage resistance $= 5\%$ and percentage reactance $= 15\%$.

Re-rate the 200-kVA transformer to 250 kVA giving percentage resistance $= 1{\cdot}25\%$ and percentage reactance $= 5\%$.

The equivalent circuit then becomes:



Combining the parallel impedances:

$$Z = \frac{(5 + j15)(1{\cdot}25 + j5)}{(5 + j15) + (1{\cdot}25 + j5)} = 1{\cdot}013 + j3{\cdot}75$$

Let the load impedance (% on 250 kVA basis)

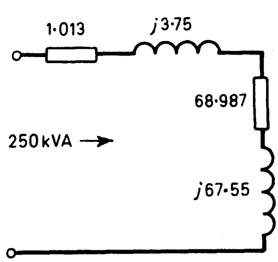
$$= Z_l = R_l + jX_l$$

Then $\sqrt{[(R_l + 1{\cdot}013)^2 + (X_l + 3{\cdot}75)^2]} = 100\%$

and $\dfrac{(R_l + 1{\cdot}013)}{\sqrt{[(R_l + 1{\cdot}013)^2 + (X_l + 3{\cdot}75)^2]}} = 0{\cdot}7$

∴ $R_l = 68{\cdot}987\%$

and $X_l = 67{\cdot}55\%$



$$\frac{\text{kVA}_{\text{load}}}{\text{kVA}_{\text{total}}} = \frac{|\,68{\cdot}987 + j67{\cdot}55\,|}{100}$$

$$= 96{\cdot}3/100$$

$$\therefore\ \text{kVA}_{\text{load}} = 241 \text{ kVA}$$

For the 100-*kVA transformer:* $Z_1 = 5 + j15$, so $|Z_1| = 15{\cdot}8$.

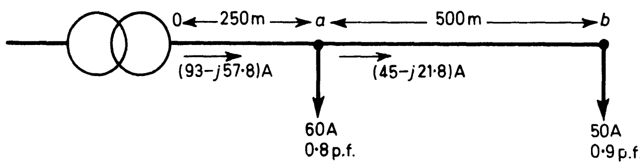
For the 200-*kVA transformer:* $Z_2 = 1{\cdot}25 + j5$, so $|Z_2| = 5{\cdot}154$.

$$Z_1 + Z_2 = 6{\cdot}25 + j20, \text{ so } |Z_1 + Z_2| = 20{\cdot}95$$

∴ kVA for first transformer = (5·154/20·95) × 241 = 59 kVA.

and kVA for second transformer = (15·8/20·95) × 241 = 182 kVA.

317.



Voltage drop from a to b per phase

$$= (0{\cdot}125 \times 45) + (0{\cdot}25 \times 21{\cdot}8) = 11{\cdot}075 \text{ V}.$$

Voltage drop from 0 to a per phase

$$= \left(\frac{0{\cdot}25}{4} \times 93\right) + \left(\frac{0{\cdot}5}{4} \times 57{\cdot}8\right) = 13{\cdot}04 \text{ V}.$$

Consider the transformer star-connected, then full-load current $=(100 \times 10^3)/(440\sqrt{3}) = 131$ A.

Rated secondary voltage = 254·5 V/phase.

Percentage resistance drop = $(1 \times 93)/131 = 0{\cdot}71\%$ or 1·81 V/phase.

Percentage reactance drop = $(4 \times 57{\cdot}8)/131$

$= 1{\cdot}763\%$ or 4·5 V/phase.

Total transformer voltage drop = 6·31 V/phase.

∴ phase voltage drop to a = 6·31 + 13·04 = 19·35 V.

Line voltage at a = $440 - [\sqrt{(3)} \times 19{\cdot}35]$ = <u>406·5 V.</u>

Phase voltage drop to b = 19·35 + 11·075 = 30·425 V.

Line voltage at b = $440 - [\sqrt{(3)} \times 30{\cdot}425]$

= <u>387·3 V.</u>

318.

The percentage voltage drop

$$= \frac{(\%R)I\cos\phi}{I_{f.l.}} + \frac{(\%X)I\sin\phi}{I_{f.l.}}$$

where $I_{f.l.}$ is the full-load current and I is the actual current.

$\therefore$ percentage voltage drop $= (\%R)\text{kW/kVA rating} + (\% X)(\text{kVA})_r/\text{kVA rating.}$

In this case kW $= 450 \times 0{\cdot}6 = 270$

$(\text{kVA})_r = 450 \times 0{\cdot}8 = 360$

$$\therefore \text{ percentage voltage drop} = \left\{\left(2 \times \frac{270}{550}\right) + \left(\frac{4 \times 360}{550}\right)\right\}\%$$

$$= \underline{3{\cdot}6\%.}$$

319.

(*a*) Slip frequency $= 100/60$ per second

$\therefore$ fractional slip $s = (100/60)/50 = 1/30$
$= \underline{3{\cdot}3\%.}$

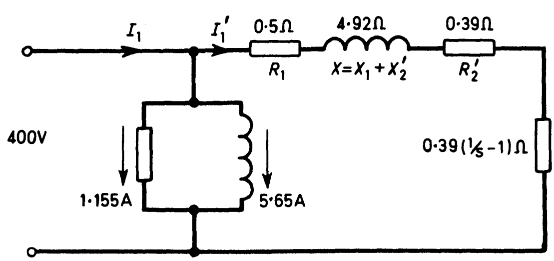
(*b*) Rotor speed $= 1000(1 - s)$
$= 1000 \times 29/30 = \underline{966{\cdot}7 \text{ r.p.m.}}$

(*c*) Mechanical power $= (1 - s) \times$ input to rotor
$= (29/30) \times 80 \text{ kW} = 77{\cdot}3 \text{ kW}$

(*d*) Copper loss/rotor phase $= (80 - 77{\cdot}3)/3 = \underline{0{\cdot}9 \text{ kW.}}$

(*e*) Rotor resistance/phase $= 900/65^2 = \underline{0{\cdot}21\ \Omega.}$

320. Magnetizing current/phase has components of 1·155 A and 5·65 A in phase with and in quadrature with the voltage respectively. The equivalent circuit for one phase is illustrated.



$$I_1' = \sqrt{[400^2/4{\cdot}92^2 + (0{\cdot}5 + 0{\cdot}39/s)^2]}$$

$$\text{Output per phase} = 0{\cdot}39(1/s - 1)(I_1')^2 = \frac{400^2 \times 0{\cdot}39(1/s - 1)}{4{\cdot}92^2 + (0{\cdot}5 + 0{\cdot}39/s)^2}$$

$$= (22{\cdot}4/3)\,\text{kW} = 7460\,\text{W}$$

$$\therefore \qquad s = 0{\cdot}0208 \quad \text{or} \quad 0{\cdot}2250.$$

The smaller value of s is the desired one, i.e. $s = 0{\cdot}0208$.

Full-load speed $= 1000(1 - 0{\cdot}0208) = \underline{979\text{ r.p.m.}}$

$$I_1' \text{ (full-load)} = 400/\sqrt{[4{\cdot}92^2 + (0{\cdot}5 + 0{\cdot}39/0{\cdot}0208)^2]}$$

$$= 20{\cdot}1\text{ A}.$$

This current lags by $\cos^{-1}\left\{\dfrac{(0{\cdot}5 + 0{\cdot}39/0{\cdot}0208)}{\sqrt{[4{\cdot}92^2 + (0{\cdot}5 + 0{\cdot}39/0{\cdot}0208)^2]}}\right\}$

so that the in-phase and quadrature components are 19·5 A and 4·98 A respectively.

$\therefore$ total-current components are:

$$I_1 \cos\phi_1 = (19{\cdot}5 + 1{\cdot}155)\text{ A} = 20{\cdot}655\text{ A}$$

$$I_1 \sin\phi_1 = (4{\cdot}98 + 5{\cdot}65)\text{ A} = 10{\cdot}63\text{ A}$$

$$\therefore \qquad I_1 = \sqrt{(20{\cdot}655^2 + 10{\cdot}63^2)} = 23{\cdot}25\text{ A}$$

and $\qquad \cos\phi_1 = \underline{0{\cdot}888.}$

Full-load line current $= [\sqrt{(3)} \times 23{\cdot}25]$ A

$= \underline{40{\cdot}3\text{ A.}}$

Full-load torque $= (I_1')^2 0{\cdot}39/s$

$= (20{\cdot}1)^2 \times 0{\cdot}39/0{\cdot}0208$

$= 7600$ synchronous watts/phase.

Total full-load torque $= 3 \times 7600$ synchronous watts/phase

$= \underline{217{\cdot}4\text{ Nm}}$

Maximum torque occurs when $0{\cdot}39/s = \sqrt{(4{\cdot}92^2 + 0{\cdot}5^2)}$

i.e. $\quad s = 0{\cdot}0792$ so speed $= 920{\cdot}8$ r.p.m.

$T_{max} = 400^2/2(4{\cdot}93 + 0{\cdot}5) = 14\,750$ synchronous watts/phase

$= \underline{422{\cdot}1\text{ Nm}}$

At starting $s = 1$

$$\therefore \qquad I_1' = 400/\sqrt{(4{\cdot}92^2 + 0{\cdot}89^2)} = 80 \text{ A}$$

$$\cos\phi_1' = 0{\cdot}178, \sin\phi_1' = 0{\cdot}985$$

$$I_1'\cos\phi_1' = 14{\cdot}25 \text{ A}, I_1'\sin\phi_1' = 78{\cdot}8 \text{ A}$$

$$\therefore \qquad I_1\cos\phi_1 = 15{\cdot}405 \text{ A/phase}$$

and $$I_1\sin\phi_1 = 84{\cdot}45 \text{ A/phase.}$$

$$\therefore \qquad I_1 = \sqrt{(15{\cdot}404^2 + 84{\cdot}45^2)} = 85{\cdot}8 \text{ A/phase}$$

$$\text{Line current} = 85{\cdot}8 \times \sqrt{3} = \underline{148{\cdot}6 \text{ A}}$$

$$\text{Starting torque} = (I_1')^2R_2' = 80^2 \times 0{\cdot}39$$

$$= 2495 \text{ synchronous watts/phase.}$$

$$\text{Total starting torque} = \underline{71{\cdot}4 \text{ Nm}}$$

321. Good accounts of the circle diagram of the induction motor can be found in many textbooks.*

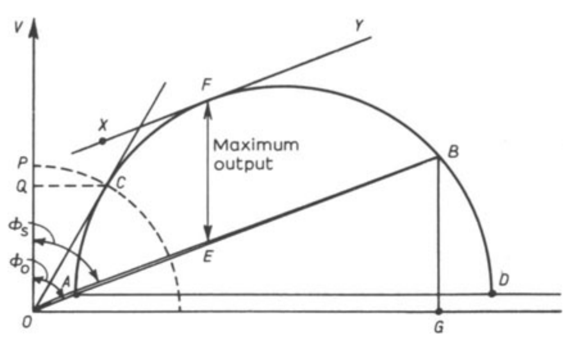
When the motor is running light the current = 3 A per line = 3 A per phase.

Line voltage/$\sqrt{3}$ = phase voltage

$$\cos\phi_0 = 0{\cdot}32 \qquad \therefore \phi_0 = 71{\cdot}4°.$$

At standstill the current = 16(200/140) = 22·85 A

and $$\cos\phi_s = 0{\cdot}36 \qquad \therefore \phi_s = 68{\cdot}9°.$$



*E.g. see F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, Second Edition, 1963, pp. 385–387.

Draw OA = 3A and ϕ_0 = 71·4°, also OB = 22·5 A and ϕ_s = 68·9°. Join AB and draw semicircle ABD (base AD). Draw OC the tangent to the circle at C. With O as centre and OC as radius draw the power-factor circle. OQ/OP = 0·82, ∴ maximum power-factor =0·82. AB is the output line. Maximum power output is given by EF. At F, line XY is parallel to AB.

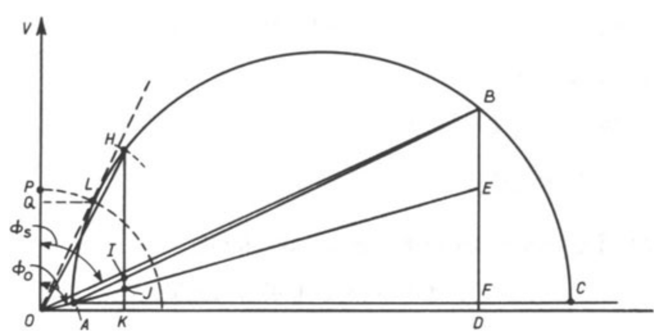
$$BG = \frac{200}{\sqrt{3}} \times 22{\cdot}85 \times 0{\cdot}36 = 950 \text{ W/phase}$$

$$\therefore \text{ maximum output} = \left(950 \times \frac{EF}{BG}\right) \text{W} = (950 \times 0{\cdot}894) \text{ W}$$

$$= 850 \text{ W}$$

Total maximum output = (850 × 3) W = 2550 W = 2·55 kW

322. Draw OA = 5 A and ϕ_0 = 77°, also OB = 80 A and ϕ_s = 65°. Join AB and draw semicircle ABC as in the previous solution. Draw BD perpendicular to AC crossing AC at F. Divide BF at E so that BE/EF = rotor resistance/stator resistance = 0·453/0·68. From O draw OH = 30 A and draw the perpendicular HK as illustrated. Draw in the power-factor circle and project point L on to OV.



At full-load:

Output per phase = HI = 4250 W.

Total output = (3 × 4250) W = 12 750 W = 12·75 kW ι.p.

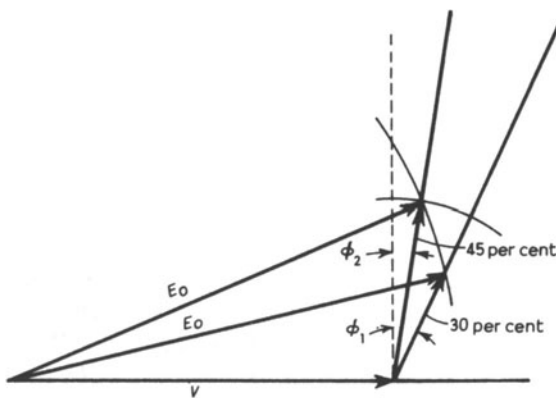
Efficiency = HI/HK = 0·8 = 80%.

Power-factor = 0·885 ($\cos^{-1} \angle VOH$).

Slip = IJ/HJ = 0·081 = 8·1%.

323. Percentage synchronous reactance drop when supplying 50 000 kVA = 5/7·5 × 45% = 30%.

Power-factor $\cos \phi_1 = 0{\cdot}9 \quad \therefore \phi_1 = 25° 50'$.



The phasor diagram is illustrated.

Angle $\phi_2 = 9{\cdot}6°$ so $\cos \phi_2 = 0{\cdot}986$

kVA = 75 000

∴ kW = 75 000 × 0·986

= 74 000.

324. The input power per phase originally

= (75 000/2)/(3 × 0·8) = 15 500 W.

The voltage per phase = 1000/√3 = 577 V.

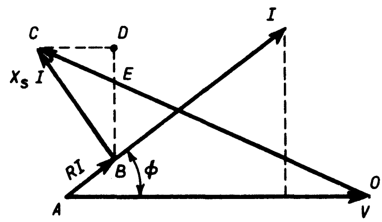
Power component of current per phase = 15 500/577

= 26·9 A.

Total current per phase (I) = 26·9/0·8 = 33·5 A.

∴ $X_s I = 4 \times 33{\cdot}5 = 134$ V (X_s is the synchronous reactance per phase).

and $\phi = \cos^{-1}(0{\cdot}8) = 36° 54'$.



The phasor diagram takes the form illustrated.* If this is constructed to scale with OA = 577 V, AB = 0, BC = 134 V, ϕ = 36° 54′, $OC = E$ is found to be 677 V. and this corresponds to 25 A excitation.

Under the new conditions OA is unchanged but C will move to C' and D to D'. The leading reactive kVA is to be unchanged so $X_s I \sin\phi = C'D'$ will be the same as CD. Thus, C' must lie on a line through C perpendicular to AO.

The new power input per phase

$$= (75\,000)/(0{\cdot}85 \times 3) = 29\,300 \text{ W}.$$

The power component of the input current per phase = 29 300/577 = 50·75 A.

So $X_s I \cos\phi = 4 \times 50{\cdot}75 = 203$ V.

If BD' is drawn to scale of length 203 V and $C'D'$ is drawn parallel to OA, C' is located. Measurement gives OC' = 690 V.

$$\therefore \text{ new excitation current} = \{(25 \times 690)/677\} \text{ A} = 25{\cdot}9 \text{ A}.$$

325. Load is 2500 kVA; power-factor 0·7

$$\therefore \qquad \text{kW} = 1750, \ \text{kVA}_r = 1783.$$

Output of motor = 150 kW

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*An explanation of the diagram can be found in many text books, e.g. see F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, Second Edition, 1963. p. 372.

Input to motor $= (150/0{\cdot}86)$ kW $= 173$ kW

Total kW required $= 1750 + 173 = 1923$

$\therefore$ total kVA $= 1923/0{\cdot}95 = 2025.$

Since p.f. $= \cos\phi = 0{\cdot}95$, $\sin\phi = 0{\cdot}308$

$\therefore$ $kVA_r = 2025 \times 0{\cdot}308 = 625.$

Motor leading $kVA_r = 1783 - 625 = 1158.$

Total motor kVA $= \sqrt{(1158^2 + 173^2)} = \underline{1170\text{ kVA.}}$

Motor p.f. $= 173/1170 = \underline{0{\cdot}148\text{ leading.}}$

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Page numbers are given first in this list, followed by problem numbers set in bold and in parentheses