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(Afternoon Session - 11.02.2018)

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General Aptitude

Q.1 to Q.5 Carry one mark each

Question 1

For non-negative integers, *a*, *b*, *c*, what would be the value of $a+b+c$ if $\log a+\log b+\log c=0$?

(A) 3 (B) 1 (C) 0 (D) −1

Ans. **(A)**

Sol. **Given :** $\log a+\log b+\log c=0$

To find : Value of $a+b+c$.

$$\log a+\log b+\log c=0$$

$$\log(a\times b\times c)=\log 1 \quad \text{[Applying log formula]}$$

$$a\times b\times c=1$$

As *a*, *b* and *c* are non-negative integers.

$\therefore$ $a=1$, $b=1$ and $c=1$.

Value of $a+b+c=1+1+1=3$

Question 2

"Although it does contain some pioneering ideas, one would hardly characterize the work as _______."

The word that best fills the blank in the above sentence is

(A) Innovative (B) Simple (C) Dull (D) Boring

Ans. **(A)**

Sol. Here, pioneering ideas means involving new methods or ideas for any project, work or any other thing, which requires some innovations or innovating views. That's why, the word that best fills the blank in the above sentence is innovative.

Question 3

"His face _______ with joy when the solution of the puzzle was _______ to him."

The words that best fill the blanks in the above sentence are

(A) Shone, shown (B) Shone, shone (C) Shown, shone (D) Shown, shown

Ans. **(A)**

Sol. **Shone :** Shone is the past tense and past participle of shine.

It means to glow, spark, or glitter.

Shown : Shown is the past participle of show.

It means to display, exhibit, or unveil.

Question 4

$\dfrac{\underbrace{a+a+a+\ldots+a}}{n\text{ times}}=a^2b$ and $\dfrac{\underbrace{b+b+b+\ldots+a}}{m\text{ times}}=ab^2$, where *a*, *b*, *n* and *m* are natural numbers. What is the value of $\left(\dfrac{\underbrace{m+m+m+\ldots+m}}{n\text{ times}}\right)\left(\dfrac{\underbrace{n+n+n+\ldots+n}}{m\text{ times}}\right)$?

(A) $2a^2b^2$ (B) a^4b^4 (C) $ab(a+b)$ (D) a^2+b^2

Ans. **(B)**

Sol. **Given :** $\underbrace{\frac{a+a+a+...+a}{n\text{ times}}}=a^2b$, $\frac{b+b+b+...+a}{m\text{ times}}=ab^2$

To find : Value of $\left(\frac{m+m+m+...+m}{n\text{ times}}\right)\left(\frac{n+n+n+...+n}{m\text{ times}}\right)$.

$$\frac{a+a+a+...+a}{n\text{ times}}=a^2b$$

In this, number of times a approach = na.

$\therefore$ $na=a^2b \Rightarrow n=ab$

$$\frac{b+b+b+...+a}{m\text{ times}}=ab^2$$

In this, number of times b approach = mb.

$\therefore$ $mb=ab^2 \Rightarrow m=ab$

So, $\left(\frac{m+m+m+...+m}{n\text{ times}}\right)=mn$

And $\left(\frac{n+n+n+...+n}{m\text{ times}}\right)=nm$

Then, $\left(\frac{m+m+m+...+m}{n\text{ times}}\right)\left(\frac{n+n+n+...+n}{m\text{ times}}\right)=mn\times nm$ ...(i)

Putting the value of n and m in equation (i), we get

$$mn\times nm=(ab\times ab)\times(ab\times ab)$$

$$mn\times nm=(a^2b^2)\times(a^2b^2)$$

$$mn\times nm=a^4b^4$$

Question 5

A three-member committee has to be formed a group of 9 people. How many such distinct committees can be formed?

(A) 27 (B) 72 (C) 81 (D) 84

Ans. **(D)**

Sol. **Given :** Number of member in a committee = 3

Number of people in group = 9

To find : Total number of committees to be formed.

By using binomial distribution,

$$^nC_r=\frac{n!}{(n-r)!\times r!}$$

where, n = Number of people, r = Number of member in a committee.

$$^{9}C_3 = \frac{9!}{(9-3)!\times 3!} = \frac{9!}{6!\times 3!}$$

$$^{9}C_3 = \frac{9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{(6\times 5\times 4\times 3\times 2\times 1)\times(3\times 2\times 1)}$$

$$^{9}C_3 = \frac{9\times 8\times 7}{6} = 84$$

Q.6 to Q.10 Carry two marks each

Question 6

A faulty wall clock is known to gain 15 minutes every 24 hours. It is synchronized to the correct time at 9 AM on 11th July. What will be the correct time to the nearest minute when the clock shows 2 PM on 15th July of the same year?

(A) 12:45 PM (B) 12:58 PM (C) 1:00 PM (D) 2:00 PM

Ans. (B)

Sol. **Given :** Clock gains 15 minutes in every 24 hours.

To find : Correct time to the nearest minute.

Clock shows correct time at 9 AM on 11th July up to 2 PM on 15th July.

Total hours on date 12, 13 and 14 $= 24\times 3 = 72$ hours

Total hours on date 11 = 15 hours

Total hours on date 15 = 14 hours

Total hours from 9 AM on 11th July up to 2 PM on 15th July = 72 + 15 + 14 = 101 hours

Clock gains 15 minutes in every 24 hours.

Gain (per hour) $= \frac{15}{24}$ minutes

Gain (Total hours 9 AM on 11th July up to 2 PM on 15th July)

$$= 101\times\frac{15}{24} = 63.125 \approx 63 \text{ minutes}$$

Correct time to the nearest minute (63 minutes back from 2 PM) $\approx 12:58$ PM

Question 7

Given that $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$ for $x \neq y \neq z$, what is the value of the product PQR?

(A) 0 (B) 1 (C) xyz (D) 10^{xyz}

Ans. (B)

Sol. **Given :** $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$

To find : Value of the product PQR.

$$\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$$

From the above equation,

$$\log P = 10(y - z)$$
$$\log Q = 10(z - x)$$
$$\log R = 10(x - y)$$
$$\log P + \log Q + \log R = (y - z) + (z - x) + (x - y)$$
$$\log P + \log Q + \log R = 0$$
$$\log(PQR) = \log 1$$

Value of the product $PQR = 1$

Question 8

In manufacturing industries, loss is usually taken to be proportional to the square of the deviation from a target. If the loss is Rs. 4900 for a deviation of 7 units, what would be the loss in Rupees for a deviation of 4 units from the target?

(A) 400 (B) 1200 (C) 1600 (D) 2800

Ans. (C)

Sol. **Given :** $(\text{Loss})_1 : \text{Rs.} 4900$

Number of units, $D_1 = 7$, $D_2 = 4$

To find : Loss for a deviation of 4 units from the target.

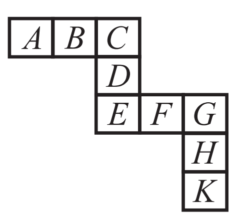
It is given that, loss is usually taken to be proportional to the square of the deviation from a target.

$$(\text{Loss})_1 = K \times D_1^2$$
$$4900 = K \times 7^2$$
$$K = 100$$
$$(\text{Loss})_2 = K \times D_2^2$$

Loss for a deviation of 4 units from the target, $(\text{Loss})_2 = 100 \times 4^2 = 1600$

Question 9

Each of the letters in the figure below represents a unique integer from 1 to 9. The letters are positioned in the figure such that each of $(A+B+C)$, $(C+D+E)$, $(E+F+G)$ and $(G+H+K)$ is equal to 13. Which integers does E represent?

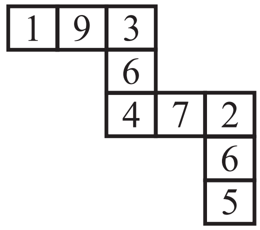


(A) 1 (B) 4 (C) 6 (D) 7

Ans. (B)

Sol. **Given :** Value of $(A+B+C)$, $(C+D+E)$, $(E+F+G)$ and $(G+H+K)$ = 13

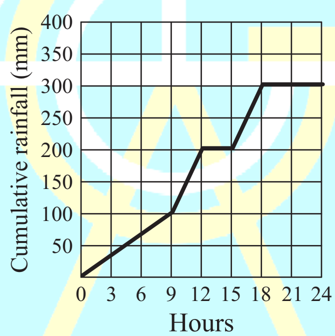
To find : E represent which integer.



E represent the integer 4.

Question 10

The annual average rainfall in a tropical city is 1000 mm. On a particular rainy day (24-hour period), the cumulative rainfall experienced by the city is shown in the graph. Over the 24-hour period, 50% of the rainfall falling on a rooftop, which had an obstruction-free area of $50\,m^2$ was harvested into a tank. What is the total volume of water collected in the tank in liters?



(A) 25,000 (B) 18,750 (C) 7500 (D) 3125

Ans. (C)

Sol. **Given :** Annual average rainfall = 1000 mm

Rainfall falling on a rooftop = 50%

Obstruction-free area = $50\,m^2$

To find : Total volume of water collected in the tank.

Total volume of water collected in the tank = Obstruction-free area × 50% of cumulative rainfall

Cumulative rainfall (from the graph) = 300 mm

50% of the cumulative rainfall $=\dfrac{50}{100}\times 300 = 150\,mm = 150\times 10^{-3}\,m$

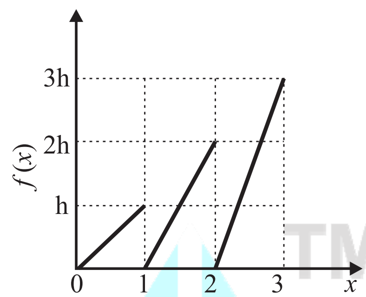
Total volume of water collected in the tank $=50\times 150\times 10^{-3} = 7.5\,m^3 = 7500\,liters$

Technical Section

Q.1 to Q.25 Carry one mark each

Question 1

The graph of a function $f(x)$ is shown in the figure



For $f(x)$ to be a valid probability density function, the value of h is

(A) 1/3 (B) 2/3 (C) 1 (D) 3

Ans. (A)

Sol. **Probability density function :** It is defined as a function of continuous random variables whose integral across an interval gives the probability that the value of variable lies within the same interval.

According to the drawn graph (in question), h lies between 0 and 1, $2h$ lies between 1 and 2 and $3h$ between 2 and 3.

$$\underbrace{\int_0^1 f(x)\,dx}_{1^{st}\text{ interval}}+\underbrace{\int_1^2 f(x)\,dx}_{2^{nd}\text{ interval}}+\underbrace{\int_2^3 f(x)\,dx}_{3^{rd}\text{ interval}}=1$$

$$\frac{h}{2}+\frac{2h}{2}+\frac{3h}{2}=1$$

$$0.5h+1h+1.5h=1$$

$$3h=1$$

$$h=\frac{1}{3}$$

Question 2

A culvert is designed for a flood frequency of 100 years and a useful life of 20 years. The risk involved in the design of the culvert (in percentage up to two decimal places) is _______.

Ans. (18.20)

Sol. **Given :** Flood frequency, T = 100 years, Useful life, n = 20 years

To find : Risk involved in the design of culvert.

$$\text{Risk } = 1-q^n$$

where, q = Complementary probability of a flood, $q = 1 - P$

where, P = Probability of a flood, $P = \frac{1}{T}$

$$P = \frac{1}{100}$$

$$\therefore \quad q = 1 - \frac{1}{100} = 0.99$$

Then, Risk $= 1 - q^n = 1 - 0.99^{20} = 0.182093$

Therefore, risk involved in the design of culvert equal to 18.20%.

Note :

Risk : It is defined as that probability in which the flood occurs at least once in n-successive years.

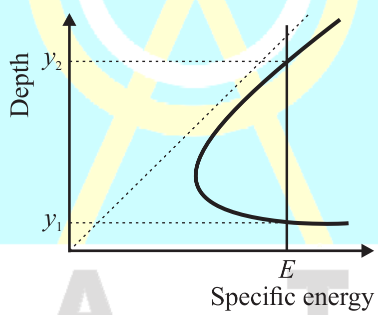
Question 3

For a given discharge in an open channel, there are two depths which have the same specific energy. These two depths are known as

(A) Alternate depths (B) Critical depths (C) Normal depths (D) Sequent depths

Ans. (A)

Sol. For the same value of specific energy, discharge is constant and there are two possible values of depth i.e., super critical depth (y_1) and subcritical depth (y_2). These two depths are called alternate depths.



For better understanding, we can also say that depths with same specific energy are called alternate depths.

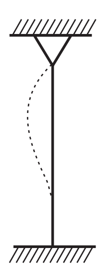
Question 4

A structural member subjected to compression has both translation and rotation restrained at one end, while only translation is restrained at the other end. As per IS 456 : 2000, the effective length factor recommended for design is

(A) 0.50 (B) 0.65 (C) 0.70 (D) 0.80

Ans. (D)

Sol.



From IS code 456 : 2000 (Table 28), in the case of structural member subjected to compression has both translation and rotation restrained at one end and only translation is restrained at the other end, the effective length is equal to 0.80 *L*.

Question 5

The initial concavity in the load-penetration curve of a CBR test is NOT due to

(A) Uneven top surface
(B) High impact at start of loading
(C) Inclined penetration plunger
(D) Soft top layer of soaked soil

Ans. **(B)**

Sol. In a CBR test, when the load-penetration curve is plotted, curve usually gets convex upwards although the initial portion of the curve may be concave due to surface irregularities.

The initial concavity is due to the following reasons :

- Top surface of soil sample.
- Top layer of soaked soil.
- Irregular contact between the plunger and soil.
- Uneven compaction.

Therefore, high impact at start of loading will not cause any change in the shape of curve.

Question 6

A reinforced-concrete slab with effective depth of 80 mm is simply supported at two opposite ends on 230 thick masonry walls. The centre-to-centre distance between the walls is 3.3 m. As per IS 456 : 2000, the effective span of the slab (in m, up to two decimal plates) is _______.

Ans. **(3.15)**

Sol. **Given :** Effective depth, $d_{eff} = 80\,\text{mm} = 0.08\,\text{m}$

Thickness of masonry wall = 230 mm = 0.23 m

c/c distance between the walls = 3.30 m

To find : Effective span of slab (L_{eff})

Clear span of slab, $L_c =$ c/c distance – Thickness of masonry wall

$$L_c = = 3.30 - 0.23 = 3.07\text{ m}$$

From IS code 456 : 2000 (by effective span criteria for simply supported slab), the effective span (L_{eff}) of a member shall be taken as

$$L_{eff} = L_c + d_{eff}$$

Or, c/c distance between the walls

Whichever is less.

$$L_{eff} = L_c + d_{eff}$$

$$= 3.07 + 0.08 = 3.15\text{ m}$$

And c/c distance between the walls = 3.30 m.

$\therefore$ Minimum effective span, $L_{eff} = 3.15\,\text{m}$

Question 7

Peak Hour (PHF) is used to represent the proportion of peak sub-hourly traffic flow within the peak hour. If 15-minute sub-hours are considered, the theoretically possible range of PHF will be

(A) 0 to 1.0 (B) 0.25 to 0.75 (C) 0.25 to 1.0 (D) 0.5 to 1.0

Ans. (C)

Sol. The peak hour factor is defined as the ratio of total hourly traffic volume to the maximum 15-minute traffic volume within an hour. It represents the flow within an hour.

If 15-minute period is used, the PHF may be computed as

$$\text{PHF} = \frac{V}{4 \times V_{15}}$$

where, PHF = Peak hour factor, V = Hourly volume (vph), and V_{15} = Volume during 15 min of the peak hour (vehicle/15 min).

The theoretical possible range of PHF will be $0.25 \le \text{PHF}_{15} \le 1$.

Question 8

Probability (up to one decimal place) of consecutively picking 3 red balls without replacement from a box containing 5 red balls and 1 white ball is _______.

Ans. (0.5)

Sol. Given : Number of red balls in the box = 5

Number of white balls in the box = 1

To find : Probability of consecutively picking 3 red balls without replacement.

Probability of first ball to be red $= \frac{5}{6}$

Probability of second ball to be red (out of remaining 5 balls) $= \frac{4}{5}$

Probability of third ball to be red (out of remaining 4 balls) $= \frac{3}{4}$

Probability of consecutively picking 3 red balls without replacement $= \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{2} = 0.5$

Question 9

As per IS 456 : 2000, the minimum percentage of tension reinforcement (up to two decimal places) required in reinforced-concrete beams of rectangular cross-section (considering effective depth in the calculation of area) using Fe 500 grade steel is _______.

Ans. (0.17)

Sol. Given : Grade of steel = Fe 500 $\Rightarrow$ $f_y = 500\text{ MPa}$

To find : Minimum percentage of steel

From IS code 456 : 2000 (by tension reinforcement criteria), the minimum percentage of tension reinforcement shall not be less than given below :

Minimum percentage $= \frac{85}{f_y}\% = \frac{85}{500}\% = 0.17\%$

Question 10

The solution of the equation $x\frac{dy}{dx}+y=0$ passing through the point (1, 1) is

(A) x (B) x^2 (C) x^{-1} (D) x^{-2}

Ans. (C)

Sol. Given : $x\frac{dy}{dx}+y=0$

To find : Solution of the above equation passing through point (1, 1).

$$x\frac{dy}{dx}=-y$$

$$\frac{dy}{y}=-\frac{dx}{x}$$

Integrating on both sides, we get

$$\int\frac{dy}{y}=-\int\frac{dx}{x}$$

$$\ln(y)=-\ln(x)+C \qquad \left[\because \int\frac{1}{x}dx=\ln(x)\right]$$

$$\ln(y)=\ln\left(\frac{1}{x}\right)+C \qquad \left[\because \int\frac{1}{x}dx=\ln(x)^{-1}=\ln\left(\frac{1}{x}\right)\right] \qquad \ldots(i)$$

Putting $x=1$ and $y=1$ in equation (i)

$$\ln(1)=\ln\left(\frac{1}{1}\right)+C$$

$$C=0$$

Putting the value of $C=0$ in equation (i), we get

$$\ln(y)=\ln\left(\frac{1}{x}\right)+0$$

$$y=\frac{1}{x} \qquad \text{[Cancelling (ln) on both sides]}$$

$$y=x^{-1}$$

Question 11

As per IS 10500 : 2012 for drinking water in the absence of alternate source of water, the permissible limits for chloride and sulphate in mg/L respectively are

(A) 250 and 200 (B) 1000 and 400 (C) 200 and 250 (D) 500 and 1000

Ans. (B)

Sol. From table 2 of IS code 10500 : 2012 (General parameters for substances),

For drinking water in the absence of alternate source of water, the permissible limits for chloride and sulphate in mg/L respectively are

Substances	Permissible limits (mg/L)
Chloride	1000
Sulphate	400

Question 12

Which one of the following statements is NOT correct?

(A) When the water content of soil lies between liquid limit and plastic limit the soil is said to be in plastic state.

(B) Boussinesq's theory is used for the analysis of stratified soil.

(C) The inclination of stable in cohesive soil can be greater than its angle of internal friction.

(D) For saturated dense fine sand, after applying overburden correction if the Standard Penetration Test value exceeds 15, dilatancy correction is to be applied.

Ans. (B)

Sol. For the analysis, Boussinesq assumed that the soil is elastic, isotropic and homogenous for the development of a point load formula. However, the soil is neither isotropic nor homogenous. The most common type of soils are met in the nature are water deposited sedimentary soils.

Therefore, Boussinesq's theory is only applicable for isotropic soil and not for stratified soil.

Question 13

The intensity of irrigation for the Kharif season is 50% for an irrigation project with culturable command area of 50,000 hectares. The duty for the Kharif season is 1000 hectare/cumec. Assuming transmission loss of 10%, the required discharge (in cumec, up to two decimal places) at the head of the canal is _______.

Ans. (27.78)

Sol. **Given :** Irrigation intensity for Kharif season = 50%

Culturable command area (CCA) = 50,000 hectares

Duty for Kharif season, D = 1000 hectares/cumec

Transmission loss = 10%

To find : Discharge at the head of the canal.

Area under Kharif season, A = 50% of CCA

$$= \frac{50}{100} \times 50,000 = 25,000 \text{ hectares}$$

Discharge (at head of the field),

$$Q = \frac{\text{Area under Kharif season}}{\text{Duty for Kharif season}}$$

$$Q = \frac{A}{D} = \frac{25,000}{1000} = 25 \text{ cumec}$$

Transmission loss = 10%

$\therefore$ Transmission efficiency, $\eta_{transmission} = 90\%$

Discharge (at head of the canal) $= \frac{\text{Discharge at the head of the field}}{\text{Transmission efficiency}}$

$$= \frac{Q}{\eta_{transmission}} = \frac{25}{0.90} = 27.778 \approx 27.78 \text{ cumec}$$

Question 14

The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton-Raphson method is _______.

Ans. (1)

Sol. Given : $f(x) = 2x^2 - 3x + 3$

$x_0 = 2$

From Newton-Raphson formula,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \ldots\text{(i)}$$

Taking the equation given in question

$f(x) = 2x^2 - 3x + 3$

Differentiating on both sides, we get

$$f'(x) = 4x - 3 \quad \ldots\text{(ii)}$$

Putting the value of $x_0 = 2$ in equation (ii), we get

$f(x_0) = 2(2)^2 - 3(2) + 3$

$f(x_0) = 8 - 6 + 3 = 5$

And $f'(x_0) = 4(2) - 3$

$f'(x_0) = 8 - 3 = 5$

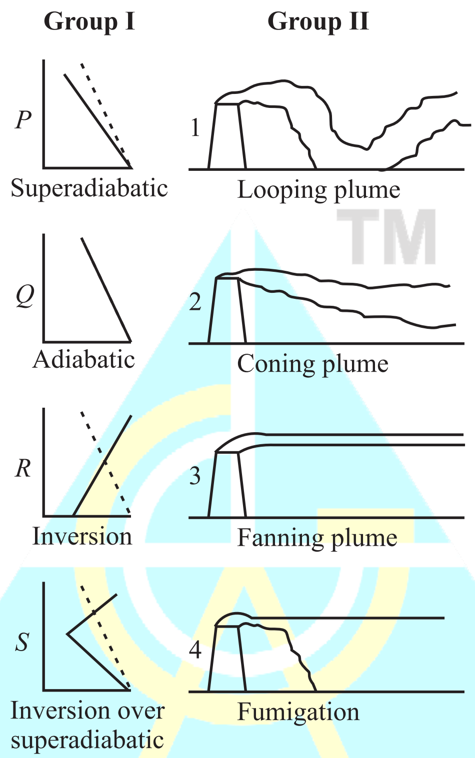
Putting the values of $f(x_0)$ and $f'(x_0)$ in equation (i)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{5}{5} = 1$$

Question 15

In the figures, Group I represents the atmospheric temperature profiles (*P*, *Q*, *R* and *S*) and Group II represent dispersion of pollutants from a smoke stack (1, 2, 3 and 4). In the figures of Group I, the dashed line represents the dry adiabatic lapse rate, whereas the horizontal axis represents temperature and the vertical axis represents the altitude.



(A) P-1, Q-2, R-3, S-4 (B) P-1, Q-2, R-4, S-3 (C) P-1, Q-4, R-3, S-2 (D) P-3, Q-1, R-2, S-4

Ans. (A)

Sol.

1. **Super adiabatic :** Super adiabatic atmosphere is highly unstable. During the high degree of turbulence, the dispersion of plume would be rapid (in a type of loop), yet higher concentrations near the ground may occur due to turbulence, before the dispersion is finally completed.
2. **Adiabatic :** It is also known as sub-adiabatic condition. Under these conditions, the atmosphere is slightly stable, and there is a limited vertical mixing, thereby increasing the probability of air pollution in the area. The neutral plume tends to get a shape of cone.
3. **Inversion :** In this, the emission will spread horizontally, as it cannot lift due to extremely stable atmosphere. In this case, there will be no vertical mixing, and the plume will simply extend horizontally over large distances. Such a plume pattern is known as fanning plume.
4. **Inversion over super adiabatic :** In such a case, the pollutants cannot escape above the top of the stack because of inversion, and they will be brought down near the ground due to turbulence in the region above the ground and below the inversion, caused by strong lapse rate. This is the case of fumigation.

Question 16

The clay mineral, whose structural units are held together by potassium bond is

(A) Halloysite (B) Illite (C) Kaolinite (D) Smectite

Ans. **(B)**

Sol. In the hydrous mica clay minerals such as illite, the 2 : 1 layers are held together by potassium ions with no hydration water in the interlayer space. With loss of this structural potassium, illite weathers to montmorillonite.

Question 17

A fillet weld is simultaneously subjected to factored normal and shear stresses of 120 Mpa and 50 MPa, respectively. As per IS 800 : 2007 the equivalent stress (in MPa, up to two decimal place) is _______.

Ans. **(147.99)**

Sol. **Given :** Factored normal stress, $f_a = 120\text{MPa}$

Factored shear stress, $q = 50\text{MPa}$

To find : Equivalent stress of fillet weld.

From IS code 800 : 2007 (by fillet weld criteria), equivalent stress formula is given by

$$f_e = \sqrt{f_a^2 + 3q^2} \leq \text{Design strength of fillet weld } (f_{wd})$$

$$f_e = \sqrt{f_a^2 + 3q^2} \leq \frac{f_u}{\gamma_{mw}\sqrt{3}}$$

where, $f_u =$ Ultimate stress ($f_u = 410\,\text{MPa}$)

$\gamma_{mw} =$ Partial safety factor for welds ($\gamma_{mw} = 1.25$)

$$f_e = \sqrt{f_a^2 + 3q^2} = \sqrt{120^2 + 3\times 50^2}$$

$$f_e = \sqrt{21900} = 147.99\,\text{MPa}$$

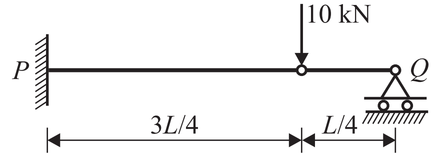
And $$f_e \leq \frac{f_u}{\gamma_{mw}\sqrt{3}}$$

$$f_e \leq \frac{410}{1.25\times\sqrt{3}} = 189.37\,\text{MPa}$$

$\therefore$ Equivalent stress of fillet weld, $f_e = 147.99\,\text{MPa}$

Question 18

A vertical load of 10 kN acts on a hinge located at a distance of $L/4$ from the roller support Q of a beam of length L (see figure)

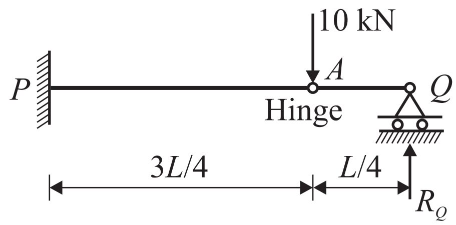


The vertical reaction at support Q is

(A) 0.0 kN (B) 2.5 kN (C) 7.5 kN (D) 10.0 kN

Ans. **(A)**

Sol. Considering all the reaction in the beam due to loads.



Taking from right hand side of the beam,

$$-R_Q \times \frac{L}{4} = 0$$ [Taking anticlockwise (–ve)]

$$R_Q = 0$$

Question 19

As per IRC : 37-2012, in order to control subgrade rutting in flexible pavements, the parameter to be considered is

(A) Horizontal tensile strain at the bottom of bituminous layer

(B) Vertical compressive strain on to subgrade

(C) Vertical compressive stress on top of granular layer

(D) Vertical deflection at the surface of the pavement

Ans. (B)

Sol. From IRC : 37-2012 (by rutting model criteria), the two equations are :

Allowable number of load repetitions to control permanent distortion

$$N = 4.1656 \times 10^{-8} \left(\frac{1}{\varepsilon_v}\right)^{4.5337}$$

$$N = 1.41 \times 10^{-8} \left(\frac{1}{\varepsilon_v}\right)^{4.5337}$$

where, N = Number of cumulative standard axle

ε_v = Vertical subgrade strain

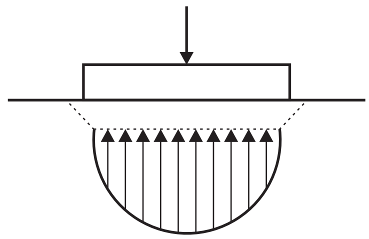
The model considers the vertical strain in subgrade as the only variable for rutting, which is a measure of bearing capacity of the subgrade.

Note :

Rutting : It is defined as long deep track made by the repeated moving of the wheels of vehicles. Rutting is the permanent deformation in pavement usually occurring longitudinally along the wheel path.

Question 20

The contact pressure and settlement distribution for a footing are shown in the figure.



The figure corresponds to a

(A) Rigid footing on granular soil
(B) Flexible footing on granular soil
(C) Flexible footing on saturated clay
(D) Rigid footing on cohesive soil

Ans. (A)

Sol. From the figure (in question), the contact pressure increases from zero at the edges to maximum at the centre and the settlement is uniform. Therefore, it is the case of rigid footing on granular (sandy) soil.

Note :

In flexible footing, the contact pressure is same throughout in both granular and cohesive soil. And settlement varies in both granular and cohesive soil.

Question 21

Dupuit's assumptions are valid for

(A) Artesian aquifer
(B) Confined aquifer
(C) Leaky aquifer
(D) Unconfined aquifer

Ans. (D)

Sol. Dupuit derived a radial flow solution for unconfined aquifers by neglecting the vertical flow component. He estimated the steady-state head difference between two distances from the pumping well for unconfined aquifers as

$$h^2(r_2) - h^2(r_1) = \frac{Q}{\pi K}\log\left(\frac{r_2}{r_1}\right)$$

where, h = Head, r_1 and r_2 = Radial distance from the pumping well

Q = Volumetric discharge, K = Hydraulic conductivity.

Note :

Dupuit's assumptions for unconfined aquifers :

- The bottom of the aquifer is a horizontal plane.
- Groundwater flow towards the pumping wells is horizontal with no vertical hydraulic gradient component.
- The horizontal component of the hydraulic gradient is constant with depth and equal to the water table slope.
- There is no seepage face at the borehole.

These assumptions are one of the main approaches to simplify the unconfined flow problems and making them analytically manageable.

Question 22

A flow net below a dam consists of 24 equipotential drops and 7 flow channels. The difference between the upstream and downstream water levels is 6 m. The length of the flow line adjacent to the toe of the dam at exit is 1 m. The specific gravity and void ratio of the soil below the dam are 2.70 and 0.70, respectively. The factor of safety against piping is

(A) 1.67 (B) 2.5 (C) 3.4 (D) 4

Ans. **(D)**

Sol. **Given :** Number of equipotential drops, $N_d = 24$

Number of flow channels, $N_f = 7$

Difference between upstream and downstream water levels, H = 6 m

Length of flow line, l = 1 m

Specific gravity of soil, $G_s = 2.7$

Void ratio, $e = 0.7$

To find : Factor of safety against piping.

From the formula,

$$\text{Factor of safety (FOS)} = \frac{\text{Critical hydraulic gradient}}{\text{Exit gradient}} = \frac{i_{critical}}{i_{exit}} \quad \ldots\text{(i)}$$

Critical hydraulic gradient, $i_{critical} = \dfrac{G_s - 1}{1+e}$

$$i_{critical} = \frac{2.7-1}{1+0.7} = \frac{1.7}{1.7} = 1$$

Exit gradient, $i_{exit} = \dfrac{\Delta h}{l}$

where, $\Delta h =$ Head loss per drop

$$\Delta h = \frac{h}{N_d} = \frac{6}{24} = \frac{1}{4}\text{ m}$$

$\therefore$ $$i_{exit} = \frac{1/4}{1} = \frac{1}{4}$$

Putting the value of $i_{critical}$ and i_{exit} in equation (i), we get

$$\text{FOS} = \frac{i_{critical}}{i_{exit}} = \frac{1}{1/4} = 4$$

Question 23

A probability distribution with right skew is shown in the figure.

The correct statement for the probability distribution is

(A) Mean is equal to mode

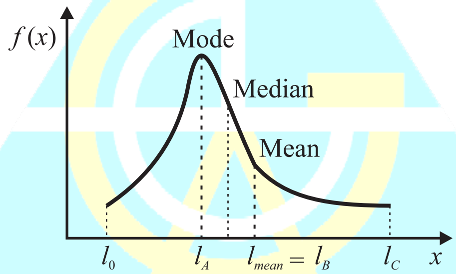
(B) Mean is greater than median but less than mode

(C) Mean is greater than median and mode

(D) Mode is greater than median

Ans. (C)

Sol. In a right skew distribution curve, mean is always right of the peak. Mode comes at the peak and median comes in between mean and mode.



Therefore, for a right skew curve (from the graph)

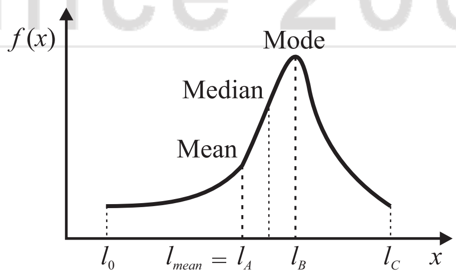
$$l_B > l_A$$

$\Rightarrow$ Mean > Mode

$\Rightarrow$ Mean is greater than mode and median.

Note :

Similarly, in a left skew distribution curve, mean is always left of the peak. Mode comes at the peak and median comes in between mean and mode.



For a left skew curve (from the graph)

$$l_A < l_B$$

$\Rightarrow$ Mean < Mode

$\Rightarrow$ Mean is lesser than mode and median.

Question 24

The setting of cement is determined by using

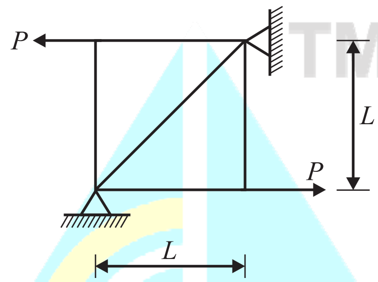
(A) Le Chatelier apparatus (B) Briquette testing apparatus
(C) Vicat apparatus (D) Casagrande's apparatus

Ans. (C)

Sol. **Vicat apparatus :** It is a device used for determining the setting time (initial and final) of cement and consistency that consists of a rod weighing 300 gm, having a needle at each end, and supported in frame with a graduated scale to measure the distance to which the needle penetrates the cement.

Question 25

All the members of the planar truss (see figure), have the same properties in terms of area of cross-section (A) and modulus of elasticity (E).

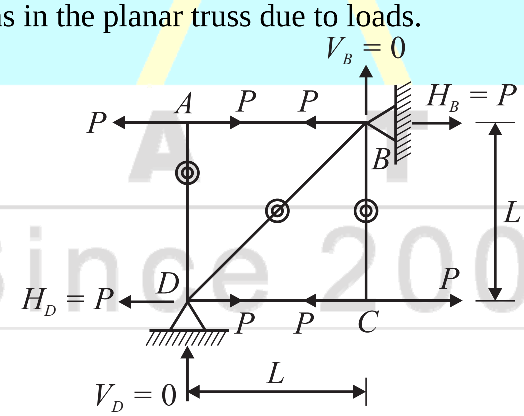


For the loads shown on the truss, the statement that correctly represents the nature of forces in the members of the truss is

(A) There are 3 members in tension and 2 members in compression
(B) There are 2 members in tension 1 member in compression and 1 zero-force members
(C) There are 2 members in tension 1 member in compression and 2 zero-force members
(D) There are 2 members in tension and 3 members

Ans. (D)

Sol. Considering all the reactions in the planar truss due to loads.



Calculation of tension members :

From the figure, only members AB and DC consists of tension force P.

Therefore, AB and DC are the two tension members.

Calculation of zero force members :

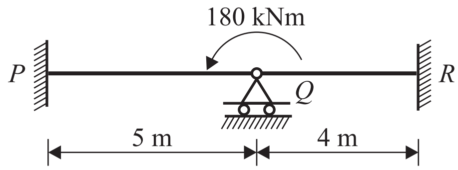
From the figure, member AD consist of only reaction V_D i.e., zero. Member BC consist of only reaction V_B, which is also zero. Member BD having no force, and the member neither expand nor contract, which means it is also zero.

Therefore, AD, BC and BD are the three zero force members.

Q.26 to Q.55 Carry two marks each

Question 26

A prismatic beam *P*-*Q*-*R* of flexural rigidity $EI = 1\times10^4$ kNm^2 is subjected to a moment of 180 kNm at *Q* as shown in the figure.



The rotation at *Q* (in rad, up to two decimal places) is _______.

Ans. **(0.01)**

Sol. **Given :** Flexural rigidity, $EI = 1\times10^4$ kNm^2, Moment = 180 kNm

To find : Rotation at *Q*.

Step 1 : Fixed end moments

$$M_{FQP} = 0$$

$$M_{FQR} = 0$$

Step 2 : Slope deflection equations

For span *QP* :

[Since, slope $\theta_P = 0$ as it is fixed end and there is no settlement therefore, deflection $\Delta = 0$].

$$M_{QP} = M_{FQP} + \frac{2EI}{L}\left(\theta_P + 2\theta_Q - \frac{3\Delta}{L}\right)$$

$$M_{QP} = 0 + \frac{2\times1\times10^4}{5}(0 + 2\theta_Q - 0)$$

$$M_{QP} = \frac{2\times10^4}{5}\times2\theta_Q = 8000\,\theta_Q$$

For span *QR* :

[Since, slope $\theta_R = 0$ as it is fixed end and there is no settlement therefore, deflection $\Delta = 0$].

$$M_{QR} = M_{FQR} + \frac{2EI}{L}\left(\theta_R + 2\theta_Q - \frac{3\Delta}{L}\right)$$

$$M_{QR} = 0 + \frac{2\times1\times10^4}{4}(0 + 2\theta_Q - 0)$$

$$M_{QR} = \frac{2\times10^4}{4}\times2\theta_Q = 10000\,\theta_Q$$

Step 3 : Taking moment about *Q*

$$\sum M_Q = 0$$

$$M_{QP} + M_{QP} - 180 = 0 \qquad \text{[Taking anticlockwise (–ve)]}$$

$$8000\,\theta_Q + 10000\,\theta_Q - 180 = 0$$

$$18000\,\theta_Q = 180$$

$$\theta_Q = \frac{180}{18000} = 0.01\,\text{rad}$$

Question 27

At a small water treatment plant which has 4 filters, the rates of filtration and backwashing are $200\,\text{m}^3/\text{dm}^2$ and $1000\,\text{m}^3/\text{dm}^2$, respectively. Backwashing is done for 15 min per day. The maturation, which occurs initially as the filter is put back into service after cleaning takes 30 min. It is proposed to recover the being wasted during backwashing and maturation. The percentage increase in the filtered water produced (up to two decimal places) would be _______.

Ans. (7.52)

Sol. Given : Number of filters = 4

Rate of filtration $= 200\,\text{m}^3/\text{dm}^2$

Rate of backwashing $= 1000\,\text{m}^3/\text{dm}^2$

Time of backwashing = 15 min per day $= \dfrac{15}{24 \times 60}$ days

Time wasted in maturation = 30 min per day $= \dfrac{15}{24 \times 60}$ days

To find : Percentage increase in the filtered water produced.

Let the area of filter to be unity i.e.,1m^2.

Time left after backwashing and maturation $= 24 - \dfrac{15}{60} - \dfrac{30}{60} = 23.25$ hours

$\therefore$ Total quantity of water to be produced $= 200 \times \dfrac{23.25}{24} = 193.75\,\text{m}^3$

Water used for backwashing $= 1000 \times \dfrac{15}{24 \times 60} = 10.416\,\text{m}^3$

Water used for maturation $= 200 \times \dfrac{30}{24 \times 60} = 4.167\,\text{m}^3$

Total quantity of water = 193.75 + 10.416 + 4.167 = 208.33 m^3

Percentage increase in the filtered water produced $= \dfrac{208.33 - 193.75}{193.75} \times 100 = 7.52\%$

Question 28

The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has

(A) Real eigenvalues and eigenvectors

(B) Real eigenvalue but complex eigenvectors

(C) Complex eigenvalue but real eigenvectors

(D) Complex eigenvalue and eigenvectors

Ans. **(D)**

Sol.

$$P = \begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}$$

$$[P - \lambda I] = 0$$

$$\begin{vmatrix} 2-\lambda & -4 \\ 4 & -2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(2-\lambda) - (-4)(4) = 0$$

$$-4 - 2\lambda + 2\lambda + \lambda^2 + 16 = 0$$

$$\lambda^2 + 12 = 0$$

On solving the equation, we get

$$\lambda = +2\sqrt{3}\,i \text{ and } \lambda = -2\sqrt{3}\,i$$

These are the two complex eigenvalues of the matrix.

First case : For $\lambda = 2\sqrt{3}\,i$

Consider the equation $(P - \lambda I)X = 0$.

$$\begin{bmatrix} 2-2\sqrt{3}\,i & -4 \\ 4 & -2-2\sqrt{3}\,i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2 - 2\sqrt{3}\,i x_1 - 4x_2 = 0$$

$$2 - 2\sqrt{3}\,i x_1 = 4x_2$$

$$\frac{x_1}{4} = \frac{x_2}{2-2\sqrt{3}\,i}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2-2\sqrt{3}\,i \end{bmatrix}$$

This is the first complex eigenvector of the matrix.

Second case : For $\lambda = -2\sqrt{3}\,i$

Consider the equation $(P-\lambda I)X = 0$.

$$\begin{bmatrix} 2+2\sqrt{3}i & -4 \\ 4 & -2+2\sqrt{3}i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2+2\sqrt{3}ix_1 - 4x_2 = 0$$

$$2+2\sqrt{3}ix_1 = 4x_2$$

$$\frac{x_1}{4} = \frac{x_2}{2+2\sqrt{3}i}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2+2\sqrt{3}i \end{bmatrix}$$

This is the second complex eigenvector of the matrix.

$\therefore$ The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has complex eigenvalues and complex eigenvectors.

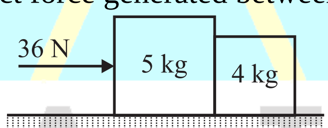
Note :

Eigenvalues : They are the special set of scalars associated with a matrix (linear transformation) equation, which are sometimes also known as characteristic roots or values.

Eigenvectors : They are the non-zero vectors of a matrix (linear transformation) that only changes by scalar factors, when that linear transformation is applied to it, which are sometimes also known as characteristic vectors.

Question 29

Two rigid bodies of mass 5 kg and 4 kg are at rest on a frictional surface until acted upon by a force of 36 N as shown in the figure. The contact force generated between the two bodies is



(A) 4.0 N (B) 7.2 N (C) 9.0 N (D) 16.0 N

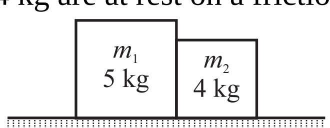
Ans. (D)

Sol. **Given :** Mass of first body, $m_1 = 5\,\text{kg}$

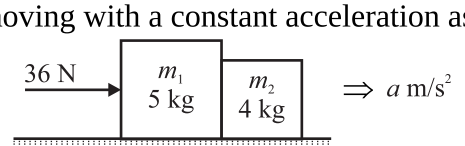
Mass of second body, $m_2 = 4\,\text{kg}$

Force acting on first body, F = 36 N

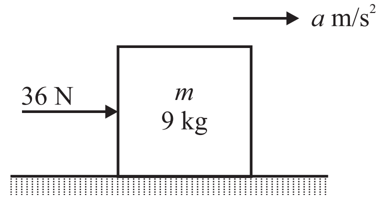
Two rigid bodies of mass 5 kg and 4 kg are at rest on a frictional surface as shown in figure.



After that, a force of 36 N is applied to first body (mass 5 kg). The action of force causes a disturbance to both the bodies and both are moving with a constant acceleration as shown in figure.



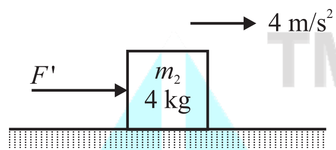
Applying Newton's second law, by considering both the bodies in a system,



Force, $F = m \times a$

$$a = \frac{F}{m} = \frac{36}{9} = 4\,\text{m/s}^2$$

This calculated acceleration causes a force (F') to move the second body as shown in figure.



Applying Newton's second law,

$$F' = m_2 \times a$$

$$F' = 4 \times 4 = 16\,\text{N}$$

Question 30

A car follows a slow moving truck (travelling at a speed of 10 m/s) on a two-lane two-way highway. The car reduces its speed to 10 m/s and follows the truck maintaining a distance of 16 m from the truck. On finding a clear gap in the opposing traffic stream, the car accelerates at an average rate of $4\,\text{m/s}^2$, overtakes the truck and returns to its original lane. When it returns to its original lane, the distance between the car and the truck is 16 m. The total distance covered by the car during this period (from the time it leaves its lane and subsequently returns to its lane after overtaking) is

(A) 64 m (B) 72 m (C) 128 m (D) 144 m

Ans. (B)

Sol. Given : Speed of car, $u = 10\,\text{m/s}$

Distance (spacing) between car and truck, $s = 16\,\text{m}$

Acceleration of car, $a = 4\,\text{m/s}^2$

To find : Total distance covered by the car.

From second equation of motion,

Total distance covered by the car $= uT + \frac{1}{2}aT^2$

where, T = Duration (time) of overtaking of car, $T = \sqrt{\frac{4s}{a}}$

$$T = \sqrt{\frac{4 \times 16}{4}}$$

$$T = \sqrt{16} = 4\,\text{sec}$$

$\therefore$ Total distance covered by the car $= uT + \frac{1}{2}aT^2 = 10 \times 4 + \frac{1}{2} \times 4 \times 4^2 = 40 + 32 = 72\,\text{m}$.

Question 31

A singly reinforced rectangular concrete beam of width 300 mm and effective depth 400 mm is to be designed using M25 grade concrete and Fe500 grade reinforcing steel. For the beam to be under-reinforced, the maximum number of 16 mm diameter reinforcing bars that can be provided is

(A) 3 (B) 4 (C) 5 (D) 6

Ans. (C)

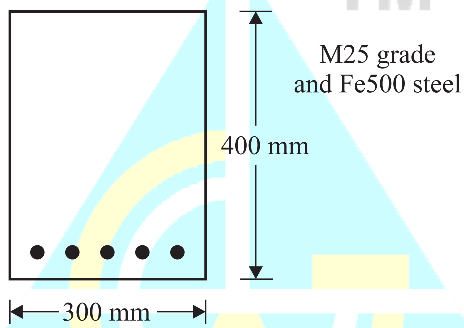
Sol. **Given :** Width of beam, $b = 300$ mm

Effective depth of beam, $d = 400$ mm

Grade of concrete = M25 and grade of reinforcing steel = Fe500

Number of diameter bars, $D = 16$ mm

To find : Maximum number of 16 mm diameter reinforcing bars.



Formula for calculating area of reinforcement steel is given by

$$0.87 f_y A_{st,\lim} = 0.36 f_{ck}.b.x_{u\lim}$$

$$A_{st,\lim} = \frac{0.36 f_{ck}.x_{u\lim}.b}{0.87 f_y}$$

$$A_{st,\lim} = \frac{0.36 \times 25 \times 0.46 \times 400 \times 300}{0.87 \times 500}$$

$$A_{st,\lim} = 1142.64\,\text{mm}^2$$

Maximum number of 16 mm diameter reinforcing bars,

$$n = \frac{A_{st,\lim}}{\frac{\pi}{4} \times d^2}$$

$$n = \frac{1142.64}{\frac{\pi}{4} \times (16)^2} = \frac{1142.64}{201.062} = 5.68 \approx 5\,\text{bars}$$

Question 32

A prismatic propped cantilever beam of span L and plastic moment capacity M is subjected to a concentrated load at its mid-span. If the collapse load of the beam is $\alpha \frac{M_p}{L}$, the value of α is _______.

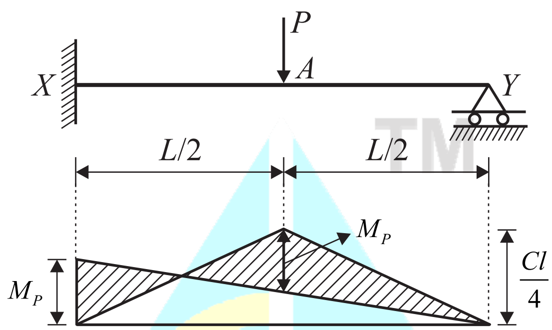
Ans. (6)

Sol. **Given :** Collapse load,

$$C = \frac{\alpha M_p}{L} \qquad \ldots(i)$$

To find : Value of α.

Consider a propped cantilever beam of span L and plastic moment capacity M is subjected to a concentrated load P at its mid-span.



At the collapse load C, B.M.D. is shown in the above figure. Moments at X and A are equal, each having a value M_P, the plastic moment of resistance of section.

$$\frac{Cl}{4} = M_P + \frac{M_P}{2}$$

$$\frac{Cl}{4} = 1.5M_P$$

$\therefore$ Collapse load,

$$C = \frac{6M_P}{L} \qquad \ldots(ii)$$

Equating equation (i) and (ii), we get

$$\frac{\alpha M_p}{L} = \frac{6M_P}{L}$$

$\therefore$ Value of $\alpha = 6$.

Question 33

An 8 m long simply supported elastic beam of rectangular cross-section (100 mm $\times$ 200 mm) is subjected to a uniformly distributed load of 10 kN/m over its entire span. The maximum principal stress (in MPa up to two decimal places) at a point located at the extreme compression edge of a cross-section and at 2 m from the support is _______.

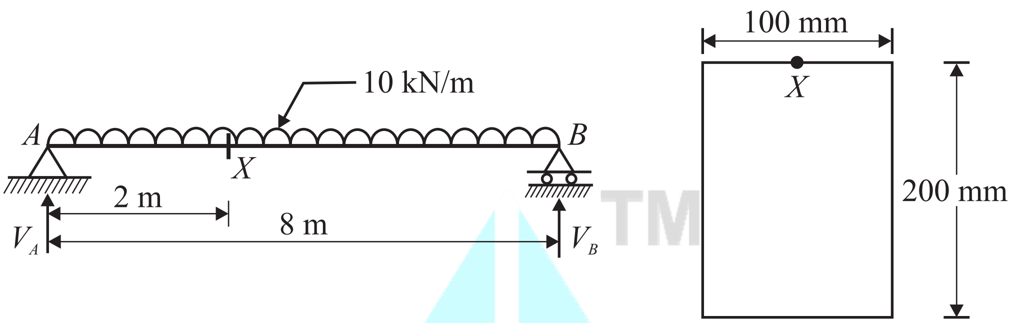
Ans. **(90)**

Sol. **Given :** Length of beam, l = 6 m

Dimension of beam = 100 mm × 200 mm

Uniformly distributed load, w = 10 kN/m

To find : Maximum principal stress at a point located at the extreme compression edge of a cross-section and at 2 m from the support.



Taking moment about point X

$$\sum M_X = 0$$

$$M_X = -V_A \times 2 + (w \times 2 \times 1) \quad \ldots(i)$$

Calculating the reactions :

$$V_A + V_B = 10 \times 8 = 80$$

Due to symmetry, $V_A = V_B$

$\therefore$ $$2V_A = 80$$

$$V_A = 40\,\text{kN} \text{ and } V_B = 40\,\text{kN}$$

Putting the value of $V_A = 40\,\text{kN}$ in equation (i), we get

$$M_X = -40 \times 2 + (10 \times 2 \times 1)$$

$$M_X = -80 + 20 = -60\,\text{kN}$$

Stress in bending, $\sigma = \dfrac{M}{I} \times y$

$$\sigma = \frac{60 \times 10^6}{100 \times \dfrac{200^3}{12}} \times \frac{200}{2}$$

$$\sigma = \frac{60 \times 10^6}{6.67 \times 10^7} \times 100 \approx 90\,\text{MPa}$$

Direct shear stress, $\tau = 0$

Therefore, maximum principal stress at a point located at the extreme compression edge equal to 90 MPa.

Question 34

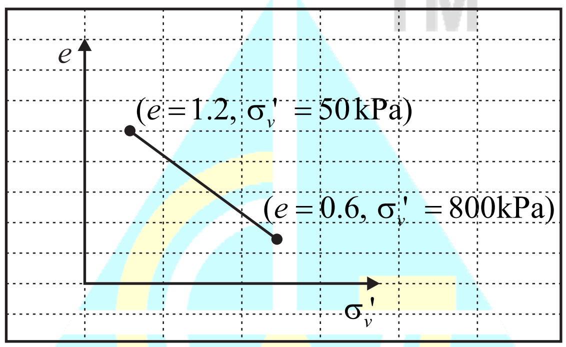
The compression curve (void ratio, e vs. effective stress, σ_v') for a certain clayey soil is a straight line in a semi-logarithmic plot and it passes through the points $(e = 1.2,\ \sigma_v' = 50\,\text{kPa})$ and $(e = 0.6,\ \sigma_v' = 800\text{kPa})$. The compression index (up to two decimal places) of the soil is _______.

Ans. (0.49)

Sol. **Given :** Void ratio, $e_1 = 1.2$ and $e_2 = 0.6$

Effective stress, $(\sigma_v')_1 = 50\,\text{kPa}$ and $(\sigma_v')_2 = 800\,\text{kPa}$

To find : Compression index of soil.



Compression index of soil, $C_c = \dfrac{\Delta e}{\Delta \log(\sigma_v')} = \dfrac{e_1 - e_2}{\log(\sigma_v')_2 - \log(\sigma_v')_1}$

$$C_c = \frac{e_1 - e_2}{\log\left[\dfrac{(\sigma_v')_2}{(\sigma_v')_1}\right]} = \frac{1.2 - 0.6}{\log\left[\dfrac{800}{50}\right]}$$

$$C_c = \frac{0.6}{1.2041} = 0.4983$$

$\therefore \quad C_c = 0.49$

Note :

Compression index : It is defined as the ratio of void ratio to the effective stress in a semi-logarithmic scale. It is an important parameter for evaluation of settlement due to primary settlement of clays.

Question 35

The Laplace transform $F(s)$ of the exponential function, $f(t) = e^{at}$ when $t \geq 0$, where a is a constant and $(s - a) > 0$ is

(A) $\dfrac{1}{s+a}$ (B) $\dfrac{1}{s-a}$ (C) $\dfrac{1}{a-s}$ (D) ∞

Ans. (B)

Sol. **Given :** $f(t) = e^{at}$

To find : Laplace transform $F(s)$ of the exponential function.

$$L\{f(t)\} = \int_0^{\infty} e^{-st}.f(t)\,dt$$

$$L\{f(t)\} = \int_0^{\infty} e^{-st}.e^{at}\,dt \qquad [t \geq 0]$$

$$L\{f(t)\} = \int_0^{\infty} e^{-(s-a)t}\,dt \qquad [(s-a) > 0]$$

$$L\{f(t)\} = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^{\infty}$$

$$L\{f(t)\} = \frac{1}{s-a}(-0+1)$$

$\therefore$ Laplace transform, $F(s) = \frac{1}{s-a}$

Question 36

A flocculation tank contains $1800\,\text{m}^3$ of water, which is mixed using paddles at an average velocity gradient *G* of 100/s. The water temperature and the corresponding dynamic viscosity are 30^0C and $0.798 \times 10^{-3}\,\text{Ns/m}^2$, respectively. The theoretical power required to achieve the stated value of *G* (in kW, up to two decimal places) is _______.

Ans. **(14.364)**

Sol. **Given :** Volume of water in flocculation tank, $V = 1800\,\text{m}^3$

Average velocity gradient, *G* = 100/sec

Temperature of water, $T = 30^0\text{C}$

Dynamic viscosity, $\mu = 0.798 \times 10^{-3}\ \text{Ns/m}^2$

To find : Theoretical power required to achieve the stated value of *G*.

Theoretical power, $P = \mu V G^2$

$$P = 0.798 \times 10^{-3} \times 1800 \times 100^2$$

$$P = 14364\,\text{W} = 14.364\,\text{kW}$$

Question 37

The total rainfall in a catchment of area $1000\,\text{km}^2$, during a 6-hour storm, is 19 cm. The surface runoff due to this storm computed from triangular direct runoff hydrograph is $1 \times 10^8\,\text{m}^3$. The ϕ-index for this storm (in cm/h, up to one decimal place) is _______.

Ans. **(1.5)**

Sol. **Given :** Area of catchment, $A = 1000\,\text{km}^2 = 1000 \times 10^{10}\ \text{cm}^2$

Total duration of storm, $t = 6\,\text{hours}$

Total rainfall (precipitation), *P* = 19 cm

Surface runoff volume $= 1 \times 10^8\,\text{m}^3 = 1 \times 10^{14}\,\text{cm}^3$

To find : ϕ-index for the storm.

Formula for ϕ-index is given by

$$\phi\text{-index} = \frac{P-R}{t}$$

where, R = Total surface runoff, $R = \frac{\text{Surface runoff volume}}{\text{Catchment area}\,(A)}$

$$R = \frac{1\times10^{14}}{1000\times10^{10}} = 10\,\text{cm}$$

$\therefore$ $$\phi\text{-index} = \frac{P-R}{t}$$

$$\phi\text{-index} = \frac{19-10}{6} = 1.5\,\text{cm/h}$$

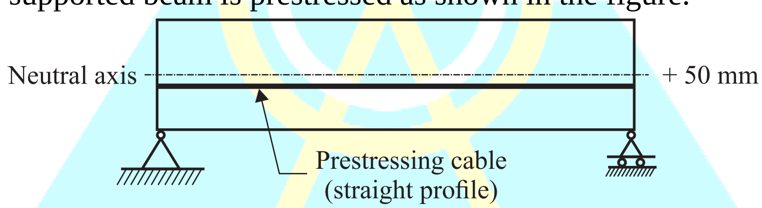
Note :

ϕ-index : It is defined as the rate of infiltration above which the rainfall volume equals runoff volume.

W-index : It is the average infiltration rate during the time when the rainfall intensity exceeds the infiltration rate.

Question 38

A 6 m long simply supported beam is prestressed as shown in the figure.



The beam carries a uniformly distributed load of 6 kN/m over its entire span. If the effective flexural rigidity $EI = 2\times10^4\,\text{kNm}^2$ and the effective prestressing force is 200 kN, the net increase in length of the prestressing cable (in mm, up to two decimal places) is ________.

Ans. (0.12)

Sol. Given : Length of beam, L = 6 m = 6000 mm

Uniformly distributed load, w = 6 kN/m

Effective flexural rigidity, $EI = 2\times10^4\,\text{kNm}^2 = 2\times10^4\times1000\,\text{N}\times1000^2\,\text{mm}^2 = 2\times10^{13}\,\text{Nmm}^2$

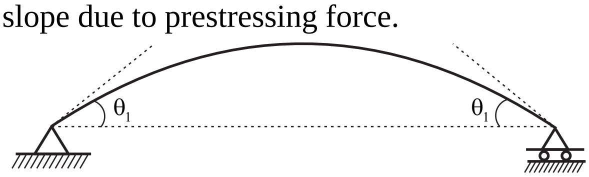
Effective prestressing force, P = 200 kN $= 200\times10^3$ N

Eccentricity, e = 50 mm

To find : Net increase in length of the prestressing cable.

Here, two cases arise.

Case I : We have to find slope due to prestressing force.



B.M.D. due to prestressing force

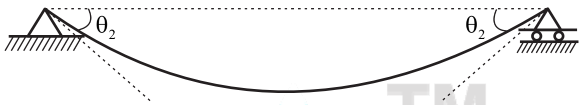
Slope due to prestressing force, $\theta_1 = \frac{PeL}{2EI}$

$$\theta_1 = \frac{200\times10^3\times50\times6000}{2\times2\times10^{13}}$$

$$\theta_1 = \frac{5\times3}{10^4} = 1.5\times10^{-3}$$

Value of θ_1 is negative due to upward movement.

Case II : Similarly, we have to find slope due to uniformly distributed load.



B.M.D. due to uniformly distributed load

Slope due to uniformly distributed load, $\theta_2 = \frac{1}{24}\frac{wL^3}{EI}$

$$\theta_2 = \frac{1}{4}\times\frac{6^3}{2\times10^4} = 2.7\times10^{-3}$$

Value of θ_2 is positive due to downward movement.

Now, net slope of the beam, $\theta = \theta_1 + \theta_2$

$$\theta = -1.5\times10^{-3} + 2.7\times10^{-3} = 1.2\times10^{-3}$$

Net increase in length of the prestressing cable = 2 × Eccentricity × Net slope of beam

$$= 2\times e\times\theta$$

$$= 2\times50\times1.2\times10^{-3} = 0.12\,\text{mm}$$

Question 39

A rough pipe of 0.5 m diameter, 300 m length and roughness height of 0.25 mm, carries water (kinematic viscosity $=0.9\times10^{-6}\,\text{m}^2/\text{s}$) with velocity of 3 m/s. Friction factor (f) for laminar flow is given by $f = 64/R_e$, and for turbulent flow it is given by $\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{r}{k}\right) + 1.74$ where, R_e = Reynolds number, r = radius of pipe, k = roughness height and $g = 9.81\,\text{m/s}^2$. The head loss (in m, up to three decimal places) in the pipe due to friction is _______.

Ans. (4.596)

Sol. **Given :** Diameter of rough pipe, D = 0.5 m, length of pipe, L = 300 m

Roughness height of pipe, k = 0.25 mm $= 0.25\times10^{-3}$ m

Kinematic viscosity of water, $\upsilon = 0.9\times10^{-6}\,\text{m}^2/\text{s}$

Velocity of water, V = 3 m/s, Acceleration due to gravity, $g = 9.81\,\text{m/s}^2$

To find : Head loss in the pipe due to friction.

Formula for Reynolds number is given by

$$R_e = \frac{VD}{\upsilon}$$

$$R_e = \frac{3\times 0.5}{0.9\times 10^{-6}} = 1.67\times 10^6$$

We know that Reynolds number, if greater than 2000 then, the flow is said to be turbulent.

Here, $R_e > 2000$. Therefore, the flow is turbulent.

So, friction factor for turbulent flow is given by the following equation :

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{r}{k}\right) + 1.74 \qquad \text{[Given in question]}$$

where, r = Radius of rough pipe, $r = \frac{0.5}{2} = 0.25\,\text{m}$

Then,
$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.25}{0.25\times 10^{-3}}\right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 6 + 1.74$$

$$\sqrt{f} = \frac{1}{7.74}$$

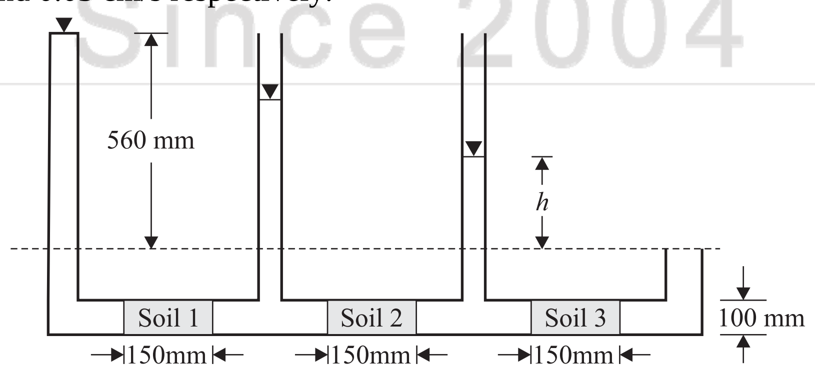
$$f = \left(\frac{1}{7.74}\right)^2 = 0.0167$$

Head loss in the pipe due to friction, $h_f = \frac{fLV^2}{2gD}$

$$h_f = \frac{0.0167\times 300\times 3^2}{2\times 9.81\times 0.5} = 4.596\,\text{m}$$

Question 40

Three soil specimens (Soil 1, Soil 2, and Soil 3), each 150 mm long and 100 mm diameter are placed in series in a constant head flow set-up as shown in the figure. Suitable screens are provided at the boundaries of the specimens to keep them intact. The values of coefficient of permeability of soil 1, soil 2, and soil 3 are 0.01, 0.003 and 0.03 cm/s respectively.



The value of h in the setup is

(A) 0 mm (B) 40 mm (C) 255 mm (D) 560 mm

Ans. (B)

Sol. **Given :** Length of all three soil specimens = 150 mm

Diameter of all three soil specimens = 100 mm

Coefficient of permeability of soil 1, K_1 = 0.01 cm/s

Coefficient of permeability of soil 2, K_2 = 0.003 cm/s

Coefficient of permeability of soil 3, K_3 = 0.03 cm/s

Total head loss, $h_L = 560\,\text{mm}$

To find : Value of h in the setup.

Since, it is given that all the three soil specimens are placed in series in a constant head flow, the discharge in each specimen is equal.

$$Q_1 = Q_2 = Q_3$$

$$K_1 i_1 A = K_2 i_2 A = K_3 i_3 A$$

$$K_1 \frac{h_1}{L} A = K_2 \frac{h_2}{L} A = K_3 \frac{h_3}{L} A$$

where, h_1, h_2 and h_3 are the head loss of soil 1, 2 and 3.

Length and area is same of all the three soil specimens.

$$\therefore \quad K_1 h_1 = K_2 h_2 = K_3 h_3 = t$$

$$h_1 = \frac{t}{0.01},\ h_2 = \frac{t}{0.003} \text{ and } h_3 = \frac{t}{0.03}$$

$$h_1 + h_2 + h_3 = h_L$$

$$\frac{t}{0.01} + \frac{t}{0.003} + \frac{t}{0.03} = 560$$

$$t = 1.2$$

Value of $h_1 = \frac{t}{0.01} = \frac{1.2}{0.01} = 120\,\text{mm}$

Value of $h_2 = \frac{t}{0.003} = \frac{1.2}{0.003} = 400\,\text{mm}$

Value of $h_3 = \frac{t}{0.03} = \frac{1.2}{0.03} = 40\,\text{mm}$

Value of h in the setup = 560 – (Head loss of soil 1 + Head loss of soil 2)

= 560 – (120 + 400)

= 560 – 520 = 40 mm

Question 41

A level instrument at a height of 1.320 m has been placed at a station having a Reduced Level (RL) of 112.565 m. The instrument reads – 2.835 m on a levelling staff held at the bottom of a bridge deck. The RL (in m) of the bottom of the bridge deck is

(A) 116.720 (B) 116.080 (C) 114.080 (D) 111.050

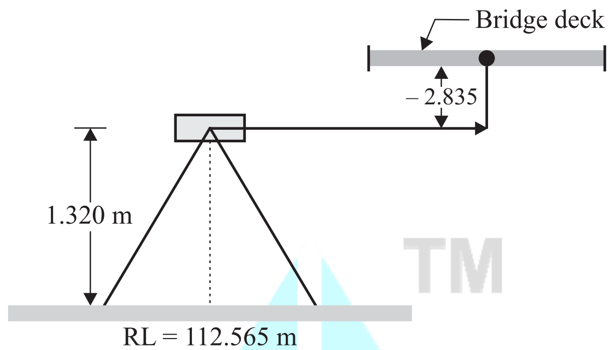
Ans. (A)

Sol. **Given :** Height of level instrument = 1.320 m

Reduced level (RL) = 112.565 m

Reading on a levelling staff = – 2.835 m

To find : RL of the bottom of the bridge deck.

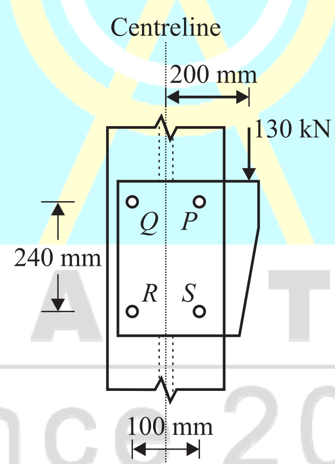


Total height of instrument = 112.565 + 1.320 = 113.885 m

RL of the bottom of the bridge deck = 113.885 + 2.835 = 116.720 m

Question 42

Four bolts *P*, *Q*, *R* and *S* equal diameter are used for a bracket subjected to a load of 130 kN as shown in the figure.



The force in bolt *P* is

(A) 32.50 kN (B) 69.32 kN (C) 82.50 kN (D) 119.32kN

Ans. **(B)**

Sol. **Given :** Load, P = 130 kN

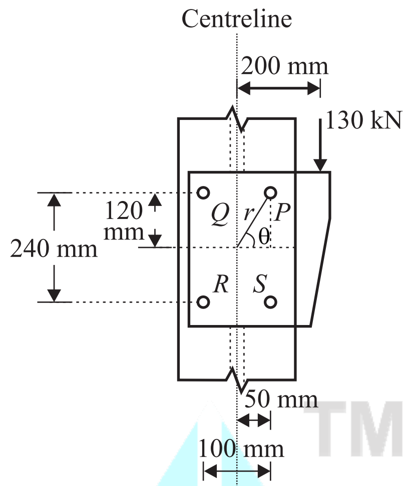
Eccentricity, e = 200 mm

Number of bolts, n = 4

Vertical distance between bolts *Q-R* and *P-S* = 240 mm

Horizontal distance between bolts *Q-R* and *P-S* = 100 mm

To find : Force in the bolt *P*.



Force in the bolt due to direct load, $F_1 = \dfrac{P}{n}$

$$F_1 = \frac{130}{4} = 32.5\,\text{kN}$$

Force in the bolt due to moment *M*,

$$F_2 = \frac{P \times e \times r_P}{\sum r_i^2}$$

where, $r_P =$ Distance of bolt *P* from the C.G., $r_P = \sqrt{50^2 + 120^2}$

$$r_P = \sqrt{16900} = 130\,\text{mm}$$

Since, bolts *P*, *Q*, *R* and *S* are at equal distances from the C.G.

$\therefore$ $\quad r_P = r_Q = r_R = r_S = 130\,\text{mm}$

Then, $\quad r_i = 130^2$

And $\quad \sum r_i^2 = 4 \times 130^2 = 67600$

$$F_2 = \frac{130 \times 200 \times 130}{67600} = 50\,\text{kN}$$

$$\cos\theta = \frac{50}{130} = 0.3846$$

Resultant force, $F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$F_R = \sqrt{32.5^2 + 50^2 + 2 \times 32.5 \times 50\cos(0.3846)}$$

$$F_R = \sqrt{4806.2} = 69.32\,\text{kN}$$

Therefore, force in the bolt *P* = 69.32 kN.

Question 43

An aerial photograph of a terrain having an average elevation of 1400 m is taken at a scale of 1 : 7500. The focal length of the camera is 15 cm. The altitude of the flight above mean sea level (in m, up to one decimal place) is _______.

Ans. **(2525)**

Sol. **Given :** Average elevation, h = 1400 m

Scale of photograph = 1 : 7500

Focal length of camera, f = 15 cm = 0.15 m

To find : Altitude of the flight above mean sea level.

Formula of scale of photograph is given by

$$\text{Scale} = \frac{\text{Focal length}}{\text{Altitude} - \text{Average elevation}}$$

$$\text{Scale} = \frac{f}{H-h}$$

Altitude, $$H = \frac{f}{\text{Scale}} + h$$

$$H = \frac{0.15}{1/7500} + 1400$$

$$H = 1125 + 1400 = 2525\,\text{m}$$

Question 44

The value (up to two decimal places) of line integral $\int_c \vec{F}(\vec{r})\cdot d\vec{r}$, for $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$ along C which is a straight line joining (0, 0) to (1, 1) is _______.

Ans. **(0.67)**

Sol. **Given :** $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$

Coordinates of straight line = (0, 0) to (1, 1)

To find : Value of line integral $\int_c \vec{F}(\vec{r})\cdot d\vec{r}$.

$$\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$$

Value of $d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

$$\vec{F}(\vec{r})\cdot d\vec{r} = \left(x^2\hat{i} + y^2\hat{j}\right)\times\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$\vec{F}(\vec{r})\cdot d\vec{r} = x^2dx + y^2dy$$

Integrating both sides and integrating right side equation from (0, 1),

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \int_0^1 x^2 dx + y^2 dy$$

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \int_0^1 x^2 dx + \int_0^1 y^2 dy$$

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{y^3}{3}\right]_0^1$$

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \frac{1}{3}+\frac{1}{3}$$

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \frac{2}{3} = 0.67$$

Question 45

A 7.5 m wide two-lane road on a plain terrain is to be laid along a horizontal curve of radius 510 m. For a design speed of 100 kmph, super-elevation is provided as per IRC : 73-1980. Consider acceleration due to gravity as $9.81\,\text{m/s}^2$. The level difference between the inner and outer edges of the road (in m, up to three decimal places) is _______.

Ans. (0.525)

Sol. **Given :** Width of two-lane road, B = 7.5 m

Radius of horizontal curve, R = 510 m

Design speed, V = 100 kmph

Acceleration due to gravity, $g = 9.81\,\text{m/s}^2$

To find : Level difference between the inner and outer edges of the road.

Super-elevation required on horizontal curves should be calculated from the following formula. This assumes that centrifugal force corresponding to (3/4th) the design speed is balanced by super-elevation and rest counteracted by side friction :

$$e = \frac{V^2}{225R}$$

For plain and rolling terrain, value of super-elevation is limited to 7%.

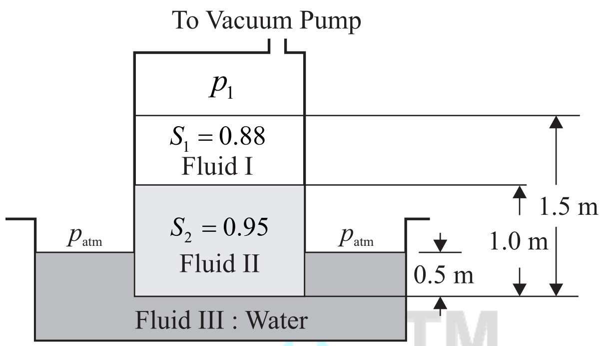
$$e = \frac{100^2}{225\times 510} = 0.0871 > 0.07$$

Level difference between the inner and outer edges of the road, $h = e\times B$

$$h = 0.07\times 7.5 = 0.525\,\text{m}$$

Question 46

A three-fluid system (immiscible) is connected to a vacuum pump. The specific gravity values of the fluids (S_1, S_2) are given in the figure.



Unit weight of water, $\gamma_w = 9.81\,\text{kN/m}^3$

Atmospheric pressure, $p_{atm} = 95.43\,\text{kPa}$

The gauge pressure value (in kN/m^2, up to two decimal places) of p_1 is _______.

Ans. (– 8.73)

Sol. Given : Specific gravity of fluid I, $S_1 = 0.88$

Specific gravity of fluid II, $S_2 = 0.95$

Unit weight of water, $\gamma_w = 9.81\,\text{kN/m}^3$

Atmospheric pressure, $p_{atm} = 95.43\,\text{kPa}$

To find : Gauge pressure value of p_1.

Pressure (at bottom of fluid II) = $0.5\gamma_w + p_{atm}$

$$= 0.5 \times 9.81 + 95.43$$

$$= 100.335\,\text{kPa}$$

$$p_1 + S_1 \times 0.5\gamma_w + S_2\,\gamma_w = \text{Pressure (at bottom of fluid II)}$$

$$p_1 + (0.88 \times 0.5 \times 9.81) + (0.95 \times 9.81) = 100.335$$

$$p_1 + 4.316 + 9.319 = 100.335$$

$$p_1 = 100.335 - 13.635 = 86.70\,\text{kPa}$$

Gauge pressure value $= p_1 - p_{atm}$

$$= 86.70 - 95.43 = -8.73\,\text{kN/m}^2$$

Question 47

A 3 m high vertical earth retaining wall retains a dry granular backfill with angle of internal friction of 30^0 and unit weight of $20\,\text{kN/m}^3$. If the wall is prevented from yielding (no movement), the total horizontal thrust (in kN per unit length) on the wall is

(A) 0 (B) 30 (C) 45 (D) 270

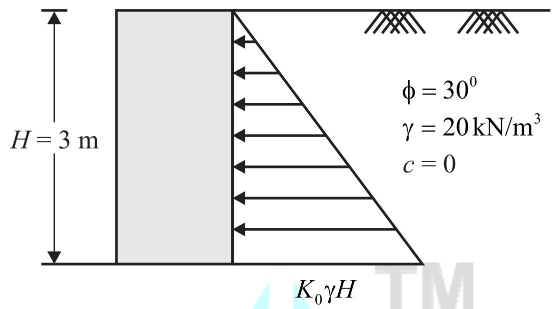
Ans. (C)

Sol. **Given :** Height of wall, $H = 3$ m

Angle of internal friction, $\phi = 30^0$

Unit weight, $\gamma = 20\,\text{kN/m}^3$

To find : Total horizontal thrust on the wall.



Since granular backfill is there therefore, soil is of sand type.

Coefficient of horizontal thrust, $K_0 = 1 - \sin\phi$

$$K_0 = 1 - \sin 30^0$$

$$K_0 = 1 - 0.5 = 0.5$$

Total horizontal thrust on the wall, $P_0 = \frac{1}{2}K_0\gamma H^2$

$$P_0 = \frac{1}{2}\times 0.5\times 20\times 3^2 = 45\,\text{kN/m}$$

Question 48

The space mean speed (kmph) and density (vehicles/km) of a traffic stream are linearly related. The free flow speed and jam density are 80 kmph and 100 vehicles/km respectively. The traffic flow (in vehicles/h, up to one decimal place) corresponding to a speed of 40 kmph is _______.

Ans. (2000)

Sol. **Given :** Free flow speed, $V_f = 80\,\text{kmph}$

Jam density, $k_j = 100\,\text{vehicles/km}$

Mean speed, $V = 40$ kmph

To find : Traffic flow corresponding to a speed of 40 kmph.

As per Green shield's macroscopic stream model,

$$V = V_f - \left[\frac{V_f}{k_j}\right]\times k$$

$$k = (V_f - V)\times\left[\frac{k_j}{V_f}\right]$$

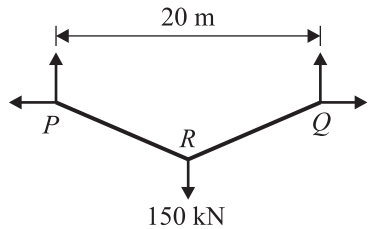
$$k = (80-40)\times\left[\frac{100}{80}\right] = 50\,\text{vehicles/km}$$

Traffic flow corresponding to a speed of 40 kmph, $q = V\times k$

$$q = 40\times 50 = 2000\,\text{vehicles/h}$$

Question 49

A cable *PQ* of length 25 m is supported at two ends at the same level as shown in the figure. The horizontal distance between the supports is 20 m. A point load of 150 kN is applied at point *R* which divides it into two equal parts.



Neglecting the self-weight of the cable, the tension (in kN, in integer value) in the cable due to the applied load will be _______.

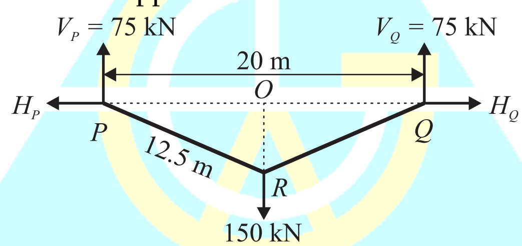
Ans. **(125)**

Sol. **Given :** Total length of cable *PQ* = 25 m

Horizontal distance between the supports *P* and *Q* = 20 m

Point load = 150 kN

To find : Tension in the cable due to applied load.



For determining the distance *OR*, by using the Pythagoras theorem in ΔPOR

$$PR^2 = PO^2 + OR^2$$

$$12.5^2 = 10^2 + OR^2$$

Distance, $OR = \sqrt{12.5^2 - 10^2} = 7.5\,\text{m}$

Taking moment about *R* and taking the left hand side

$$\sum M_R = 0$$

$$-H_P \times 7.5 + 75 \times 10 = 0$$ [Taking anticlockwise (–ve) and clockwise (+ve)]

$$H_P = \frac{750}{7.5} = 100\,\text{kN}$$

Tension in the cable $= \sqrt{H_P^2 + V_P^2} = \sqrt{100^2 + 75^2} = 125\,\text{kN}$

Question 50

A coal containing 2% sulfur is burned completely to ash in a brick kiln at a rate of 30 kg/mm. The sulfur content in the ash was found to be 6% of the initial amount of sulfur present in the coal fed to the brick kiln. The molecular weights of S, H and O are 32, 1 and 16 g/mole, respectively. The annual rate of sulfur dioxide (SO_2) emission from the kiln (in tons/year, up to two decimal places) is _______.

Ans. **(592.88)**

Sol. **Given :** Percentage of sulfur in unburned coal (initial amount) = 2%

Rate of burning = 30 kg/min

Percentage of sulfur in cash = 6% of the initial amount of sulfur

Molecular weight of sulfur (S) = 3.2 gm/mole

Hydrogen (H) = 2 gm/mole

Oxygen (O) = 16 gm/mole

To find : Annual rate of sulfur oxide (SO_2) emission.

Coal burned in one year = 30 kg/min × 24 hours × 60 minutes × 365 days

$$= 1.5768 \times 10^7 \text{ kg/year} = 1.5768 \times 10^4 \text{ tons/year}$$

Sulfur content (in unburnt coal) $= \frac{2}{100} \times 1.5768 \times 10^4 = 315.36$ tons/year

Sulfur content (in ash) $= \frac{6}{100} \times 315.36 = 18.92$ tons/year

$\therefore$ Sulfur converted to $SO_2 = 315.36 - 18.92 = 296.44$ tons/year

From the chemical formula,

$$S + O_2 \rightarrow SO_2$$

1 mole of S is present in 1 mole of SO_2.

Molecular weight of S = 32 gm/mole

Then, molecular weight of $SO_2 = 32 + (16 \times 2) = 64$ gm

$\therefore$ 32 gm of S is present in 64 gm of SO_2

Then, rate of SO_2 emission $= \frac{64 \text{ gm of } SO_2}{32 \text{ gm of } S} \times \text{Sulfur converted to } SO_2$

$$= \frac{64}{32} \times 296.44 = 592.88 \text{ tons/year}$$

Question 51

In a 5 m wide rectangular channel, the velocity distribution in the vertical direction y is given by $u = 1.25 y^{1/6}$. The distance y is measured from the channel bed. If the flow depth is 2 m, the discharge per unit width of the channel is

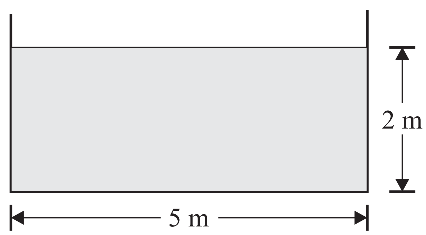
(A) $2.40 \text{ m}^3/\text{s/m}$ (B) $2.80 \text{ m}^3/\text{s/m}$ (C) $3.27 \text{ m}^3/\text{s/m}$ (D) $12.02 \text{ m}^3/\text{s/m}$

Ans. **(A)**

Sol. **Given :** Width of rectangular channel, b = 5 m

Flow depth, d = 2 m

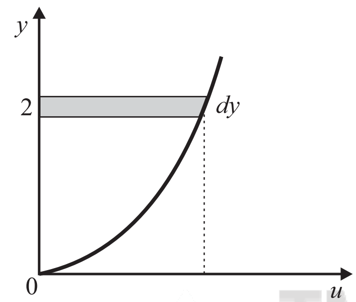
To find : Discharge per unit width of the channel.



The velocity distribution in the vertical direction y is given by

$$u = 1.25 y^{\frac{1}{6}}$$

For this velocity distribution equation, the graph is drawn below.



Consider a small element dy at a distance from the rectangular channel bed.

$$dq = u.dy \ \ = 1.25 y^{\frac{1}{6}}.dy$$

Integrating on both sides, we get

$$\int_0^q dq = 1.25 \int_0^2 y^{\frac{1}{6}}.dy$$

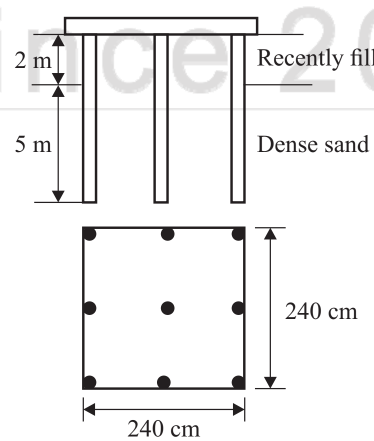
$$q = 1.25 \int_0^2 y^{\frac{1}{6}}.dy \ = 1.25 \left(\frac{y^{\frac{7}{6}}}{7/6} \right)_0^2$$

$$q = 1.25 \times \frac{6}{7} \times \left[y^{\frac{7}{6}} \right]_0^2 = 1.25 \times \frac{6}{7} \times \left[2^{\frac{7}{6}} - 0 \right]$$

Discharge per unit width, $q = 2.40 \text{ m}^3/\text{s/m}$

Question 52

A group of nine piles in a 3×3 square pattern is embedded in a soil strata comprising dense sand underlying recently filled clay layer, as shown in the figure. The perimeter of an individual pile is 126 cm. The size of pile group is $240 \text{ cm} \times 240 \text{ cm}$. The recently filled clay has undrained shear strength of 15 kPa unit weight of 16 kN/m^3.



The negative frictional load (in kN, up to two decimal places) acting on the pile group is _______.

Ans. **(472.32)**

Sol. **Given :** Number of piles, $n = 9$ $(3\times3$ square pattern)

Perimeter of an individual pile = 126 cm = 1.26 m

Size of pile group $= 240\,\text{cm}\times240\,\text{cm} = 2.4\,\text{m}\times2.4\,\text{m}$

Undrained shear strength of clay, S = 15 kPa

Unit weight of clay, $\gamma = 16\,\text{kN/m}^3$

To find : Negative frictional load acting on the pile group.

Negative frictional load acting on the pile group,

$$Q_{nf} = SLP + AL\gamma$$

where, P = Perimeter of the pile group, $P = 4B$

$$P = 4\times2.4 = 9.6\,\text{m}$$

For recently filled clay, L = 2 m

A = Area of the pile group, $A = 2.4\times2.4 = 5.76\,\text{m}^2$

$$Q_{nf} = SLP + AL\gamma$$

$$Q_{nf} = 15\times2\times9.6 + 5.76\times2\times16$$

$$Q_{nf} = 288 + 184.32 = 472.32\,\text{kN}$$

Question 53

The rank of the following matrix is :

$$\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$$

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol.

$$P = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 4R_1$

$$P = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{pmatrix}$$

Applying $R_3 \to R_3 - \frac{3}{2}R_2$

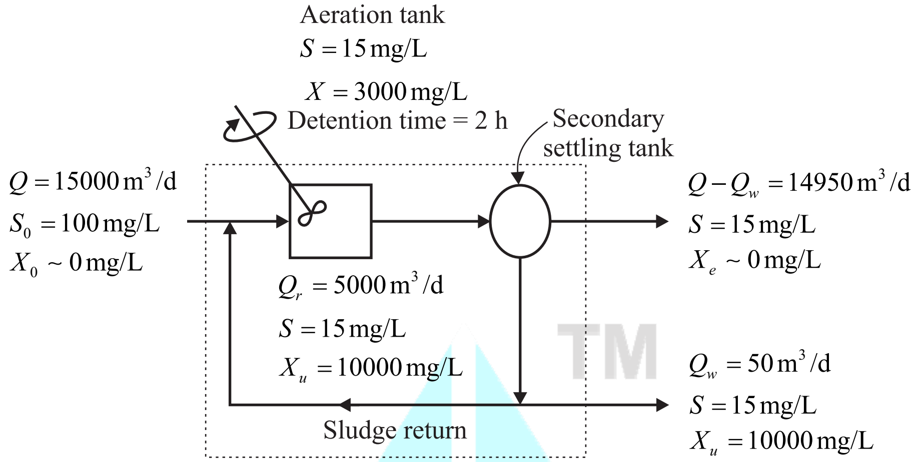
$$P = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

From this above matrix, we see that there are 2 non-zero rows.

Therefore, rank of matrix P = 2.

Question 54

A schematic flow diagram of a completely mixed biological reactor with provision for recycling of solids is shown in the figure.



So, $S =$ Readily biodegradable soluble BOD, mg/L

$Q, Q_r, Q_w =$ Flow rates, m^3/d

$X_0, X, X_e, X_n =$ Microorganism concentrations (mixed-liquor volatile suspended solids or MLVSS), mg/L

The mean cell residence time (in days, up to one decimal place) is ________.

Ans. (7.5)

Sol. Given : Microorganism concentration, X = 3000 mg/L

Detention time, t = 2 hours $= \frac{2}{24}$ days

Discharge or Flow rate, $Q = 15000\ m^3/d$

Flow rate of removing microorganism, $Q_w = 50\ m^3/d$

Microorganism solids concentration, $X_u = 10000$ mg/L

To find : Mean cell residence time (MCRT).

$$\text{MCRT} = \frac{\text{Microorganism in the system}}{\text{Microorganism lost or wasted}}$$

$$\text{MCRT} = \frac{XV}{Q_w X_u + (Q - Q_w) X_e}$$

There are no microorganism in the effluent, $X_e = 0$.

$$\therefore \quad \text{MCRT} = \frac{XV}{Q_w X_u} \quad \ldots\text{(i)}$$

Volume of reactor, V = Flow rate × Detention time

$$V = Q \times t$$

$$V = 15000 \times \frac{2}{24} = 1250\ m^3$$

Putting all the required values in equation (i), we get

$$\text{MCRT} = \frac{XV}{Q_w X_u} = \frac{3000 \times 1250}{50 \times 10000}$$

$$\text{MCRT} = \frac{375}{50} = 7.5 \text{ days}$$

Note :

MCRT : The mean cell residence time or MCRT is the amount of time (in days) in which solids or bacteria are maintained in the activated sludge system. The MCRT is also known as the solids retention time (SRT).

Question 55

The total horizontal and vertical stresses at point *X* in a saturated sandy medium are 170 kPa and 300 kPa respectively. The static pore-water pressure is 30 kPa. At failure, the excess pore-water pressure is measured to be 94.50 kPa, and the shear stresses on the vertical and horizontal planes passing through the point *X* are zero. Effective cohesion is 0 kPa and effective angle of internal friction is 36^0. The shear strength (in kPa, up to two decimal places) at point *X* is _______.

Ans. (52.52)

Sol. Given : Total horizontal stresses at point *X* = 170 kPa

Total vertical stresses at point *X* = 300 kPa

Static pore-water pressure = 30 kPa

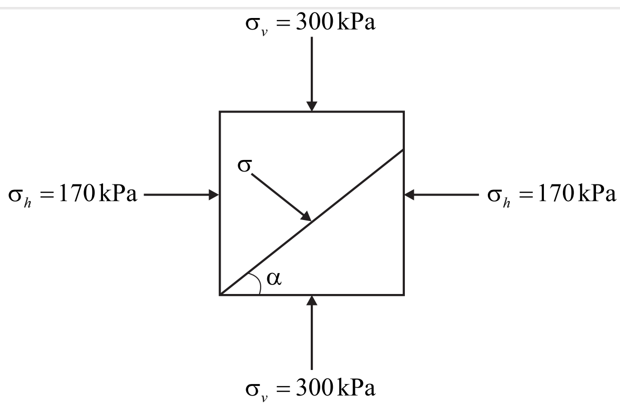
Excess pore-water pressure = 94.50 kPa

Shear stress on the vertical and horizontal planes = 0

Effective cohesion = 0

Effective angle of internal friction, $\phi = 36^0$

To find : Shear strength at point *X*.



An angle (α) is made, when shear failure occurs on a plane, with respect to horizontal.

$$\alpha = 45^0 + \frac{\phi}{2} = 45^0 + \frac{36^0}{2} = 63^0$$

Total pore water pressure = (Static + Excess) pore water pressure

= 30 + 94.5 = 124.5 kPa

$$\sigma_3' = \sigma_3 - \text{ Total pore water pressure}$$

$$\sigma_3' = 170 - 124.5 = 45.5\,\text{kPa}$$

$$\sigma_1' = \sigma_1 - \text{ Total pore water pressure}$$

$$\sigma_1' = 300 - 124.5 = 175.5\,\text{kPa}$$

Effective normal stress, $\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha$

$$\sigma' = \frac{175.5 + 45.5}{2} + \frac{175.5 - 45.5}{2}\cos(2\times 63^0)$$

$$\sigma' = 110.5 - 65\times 0.5877 = 72.29\,\text{kPa}$$

Shear strength at point $X = C' + \sigma' \tan\phi$

$$= 0 + 72.29\times \tan 36^0 = 52.52\,\text{N/mm}^2$$

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