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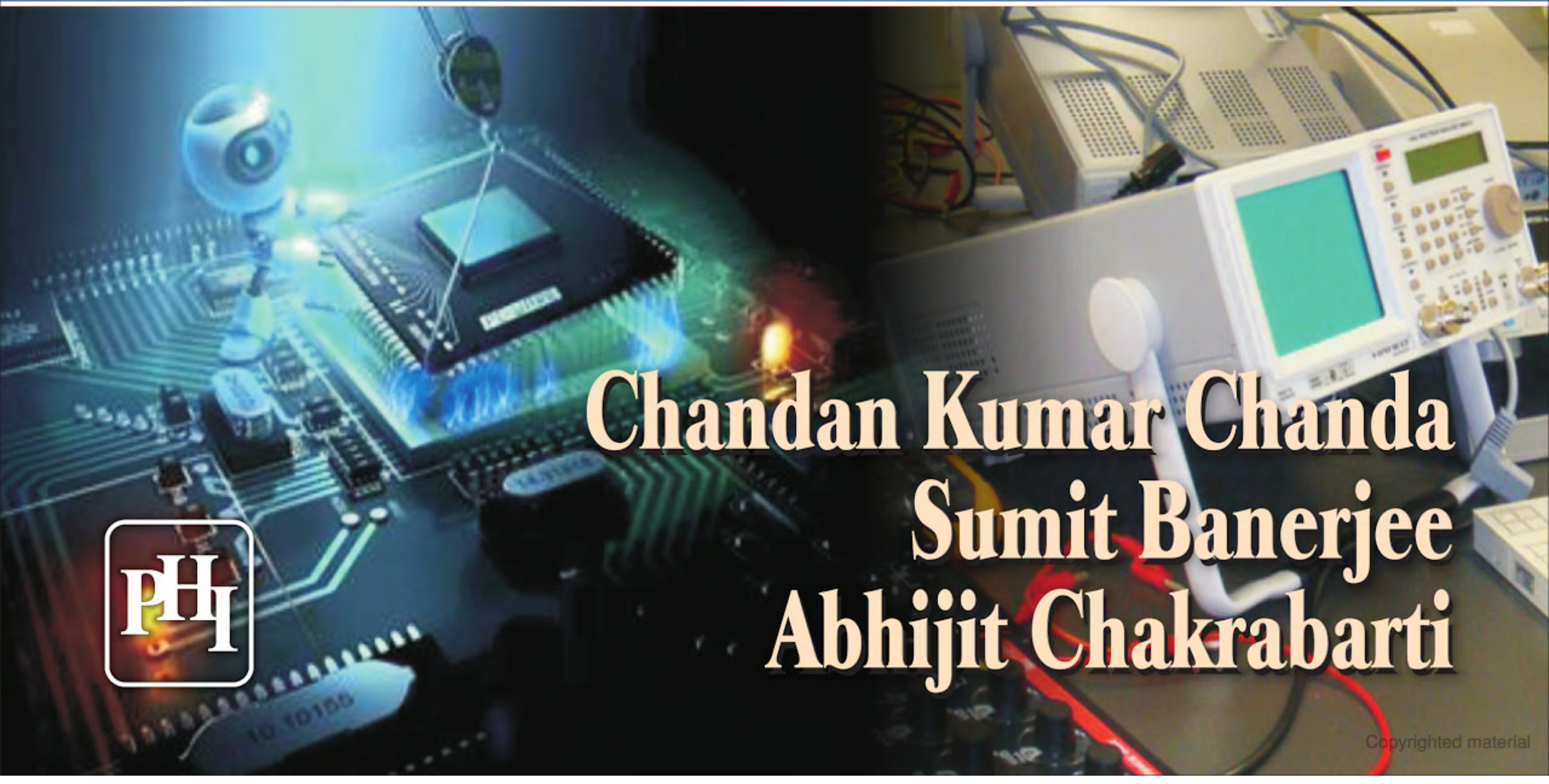

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PHI

GATE

FOR ELECTRICAL ENGINEERING

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GATE FOR ELECTRICAL ENGINEERING (with CD-ROM)
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Preface

In the present competitive scenario, where engineering colleges are mushrooming at every nook and corner of the country, the only yardstick to measure and test the actual ability of students is the GATE. That is why the number of aspirants for the GATE examination is increasing exponentially every year. There is a general misconception among students that GATE examination is only meant for pursuing ME/M.Tech from the prestigious IITs/NITs and finally ends up with a teaching career. But in this context, we want to mention here that the recent developments about the GATE score is very vital and instrumental to get different PSUs job with a high salary package.

Although there is no dearth of books in the market for some of these areas, since this is an academically high level examination, we have tried our level best to design the book in such a manner that an average student can score high if he/she solves an adequate number of problems based on the fundamental concepts. In fact, this book comprises the flow of topics arranged in such a way so that the students can grab the advanced concepts to tackle the tricky problems in this kind of competitive examination.

We are grateful to the authorities of Indian Institute of Engineering Science and Technology, Shibpur, Howrah and Dr. B.C. Roy Engineering College, Durgapur for providing the facilities necessary for writing this book. We are greatly thankful to our colleagues at the IIEST, Shibpur and BCREC, Durgapur for their encouragement and various useful suggestions.

We cannot express our thanks and gratefulness in words to our student Deblina Maity, Assistant Professor, Netaji Subhash Engineering College, Kolkata, who has helped us in many ways for this book.

Last, but not the least, we appreciate the patience, help, inspiration, constant motivation and encouragement by Gopa Chanda, Shampa Banerjee, Sayonsom Chanda, Soumyadeep Banerjee and other family members throughout the preparation of the book.

We would also like to thank the editorial team of PHI Learning, Delhi for their support.

We certainly hope that this book will prove to be a great aid to all the GATE/PSUs aspirants.

Constructive criticism and comments or suggestions for the improvement of the book are always welcome and will be greatly acknowledged.

Best of Luck!

Chandan Kumar Chanda
Sumit Banerjee
Abhijit Chakrabarti

How to prepare for GATE exam?

Preparation Strategy

To get success in GATE/PSUs examinations in a short span of time, you need a proper planning. The following steps are to be followed to prepare GATE examination:

- Preparation time for GATE is subjective and depends on individual's aptitude, fundamentals, attitude, concentration level, etc. Typically a rigorous preparation of 7–8 months is considered well enough for getting IITs, IISC or NITs.
- Channelise your energy to a particular goal.
- A good understanding of the basic concepts and their application is required.
- Prepare those chapters or topics in depth from which questions appeared frequently.
- As a thumb rule, while solving any GATE problem, if the solution takes more than 10 steps, you must re-look at the approach (generally GATE problems are little tricky but not lengthy).
- While solving the problem, students must have balance between speed and accuracy.
- Any problem can be tackled in a number of ways. Therefore, be innovative and intuitive also.
- Practice all varieties of questions from as many sources as possible, i.e., practice a lot.
- Take up regular mock tests to keep a check on your performance.
- It is advisable to directly jump upon the previous GATE question on the topic you just finished.
- Always remember "You Can Get If You Really Want". So, positive mind is the key.
- Remember the secret of success in GATE is not a brilliant mind, but rather a hard working mentality and constant honing of your skills which comes from practice.

What is special about the book?

- This book provides a complete analysis of the questions chapter-wise based on previous years' GATE examination. The book is according to the syllabus of the examination of DRDO, BARC, BHEL, DVC, NTPC, ONGC, SAIL, ISRO, GAIL, NHPC, PGCIL, IOCL, HAL and many more Public Sector Undertaking examination and also for the preparation for IES examination.
- Within depth and detailed explanations of the various concepts and techniques that have been made use of in order to solve each question. Solutions are presented in lucid and understandable language for an average student.
- Apart from subject topics, verbal ability, logical reasoning, numerical ability and engineering mathematics are covered in detail in this book. Explanations are provided in a simple and easy to understand method covering all the required topics for the complete preparation of the examinations.
- This book is accompanied by a free CD of last 13 years' solved GATE question papers with hints and solutions.
- Last but not the least, authors' 30 years of experience and devotion to write this book will surely help its readers to achieve success in various competitive examinations.

How to read this book?

- Before commencing a new chapter, first glance through the syllabus and weightage of the questions appeared in the previous exams.
- To make the theory sound and completely go through the fundamentals and formulae along with the key points.
- Points to remember after a brief theory is thus a ready reckoner in this book.
- To enhance your knowledge you can take the help of other resources and references mentioned in each chapter of this book.
- At first glance, if you cannot crack the problem in your own, then only you can refer to the solution which are solved in this book in a lucid and understandable manner.

1

Electric Circuits

1.1 SYLLABUS

Network graph, KCL, KVL, node and mesh analysis, transient response of dc and ac networks; sinusoidal steady state analysis, resonance, basic filter concepts; ideal current and voltage sources, Thevenin's, Norton's and superposition and maximum power transfer theorems, 2-port network, three phase circuits.

1.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	5	2	1	3	2	3	3	5	3	3	4	4
2 Marks Question	7	7	11	9	6	4	6	7	4	3	3	4
Total Marks	**19**	**16**	**23**	**21**	**14**	**11**	**15**	**19**	**11**	**9**	**10**	**12**

1.3 TOPICS TO BE FOCUSED

NETWORKS AND CIRCUITS

- The interconnection of two or more simple circuit elements forms an electrical network. If the network contains at least one closed path, it is also an electric circuit. So, every circuit is a network, but all networks are not circuits.
- A network that contains at least one active element, such as an independent voltage or current source, is an active network. A network that does not contain any active elements is a passive network.
- The independent and dependent voltage and current sources are circuit element. More additional circuit elements are resistor, inductor capacitor, transformer and the ideal operational amplifier (op-amp). These are all ideal elements.

NETWORK ELEMENTS

A network element is a component of a circuit having different characteristics as describe below:

Linear and Non-linear Elements

- A linear element shows linear characteristics of voltage vs. current. Simple resistors, inductors, and capacitors are linear elements and their resistances, inductances and capacitances do not change with a change in applied voltage or the circuit current. A linear network is one in which the principal of superposition (linearity test) holds.
- In an electric circuit, a nonlinear element or nonlinear device is an electrical element which does not have a linear relationship between current and voltage. A diode is a simple example.

Active and Passive Elements

- If a circuit element has the capability of enhancing the energy level of a signal passing through it, it is called an *active element*. Vacuum tubes and semiconductor devices (such as transistors, op-amps etc.) are active elements.
- On the other hand, resistors, inductors, capacitors, thermistors, etc. are passive elements as they do not have any intrinsic means of signal boosting.

Unilateral and Bilateral Elements

- If the magnitude of the current passing through an element is affected due to change in the polarity of the applied voltage, the element is called *unilateral element*. On the other hand, if the current magnitude remains the same even if the applied emf's polarity is changed, it is called a *bilateral element*.

- Unilateral elements offer varying impedances with variations in flow of current while bilateral elements offer same impedances irrespective of flow of current.

  Examples of unilateral element are diodes, transistor, etc. whereas examples of bilateral network elements are resistor, inductor, capacitor, etc.

CLASSIFICATION OF NETWORK

The network being a conducting path through which electric current flows or intends to flow, contains resistance, inductance, and capacitance. These elements are also called *network parameters* and may be in form of lumped or distributed. There are different types of classifications of networks.

Linear and Non-linear Circuits

- It is the circuit whose parameters remain constant with change in applied voltage or current. Examples are a resistance, inductance or capacitance.
- It is a circuit whose parameters change with voltage or current. A semiconductor resistor is an example of this circuit.
- A linear circuit obeys Ohm's law, while in non-linear circuit, Ohm's law is not satisfied.

Unilateral and Bilateral Circuits

- When the direction of current is changed, the characteristics or properties of the circuit may change. In this case, the circuit is a unilateral circuit (diode, transistor etc.).
- When with change in direction of current, the characteristics or properties of the circuit may not change; it is then called a bilateral circuit. Mostly, elements made of high conducting material are bilateral circuits.

Active and Passive Networks

- It is a network which contains one or more than one sources of emf. An active network consists of an active element like a battery or transistor.
- When the network does not contain any source of emf, it is called *passive network*. A passive network consists of resistance, inductance or capacitance as passive elements. A passive network does not contain any source of energy.

Lump and Distributed Networks

Physically separate network elements like R, L or C are known as lumped elements. A transmission line or a cable on the other hand, is an example of distributed parameter network as throughout the line they are not physically separate. If the network is fabricated with its elements in lump form, it is called a lumped network and if in distributed form it is called distributed network.

Recurrent and Non-recurrent Networks

When a large circuit consists of similar networks connected one after another, the network is called as *recurrent network* or *cascaded network*. It may also call as *ladder network*. On the other hand, a single network is called *non-recurrent network*.

CIRCUIT ELEMENTS

Resistance

Electrical resistance is the property of a material by virtue of which it opposes the flow of electrons through the material. Thus, resistance restricts the flow of electric current through the material. The unit of resistance (R) being 'ohm' (Ω).

$$R = \frac{V}{I}$$

When an electric current flows through any conductor, heat is generated due to collision of free electron with atoms. If I amperes (A) be the strength of current for a potential difference of V volts (V) across the conductor, the power absorbed by the resistor is given by

$$P = VI = (IR)R = I^2R \text{ watts (W)}$$

and the energy lost in the resistance in form of heat is then expressed as

$$W = \int_0^t P.dt = Pt = I^2Rt = \frac{V^2}{R}t$$

Inductance

Inductance is the property of a material by virtue of which it opposes any change of magnitude or direction of electric current passing through the conductor. The unit of inductance (L) being 'henry' (H).

$$V = L\frac{dI}{dt}$$

Where I is the current through the inductor in ampere.

Power absorbed by the inductor is given by

$$P = V.I = LI\frac{dI}{dt} \text{ watts}$$

Energy absorbed by the inductor will thus be given by:

$$W = \int_0^t Pdt = \int_0^t LI\frac{dI}{dt}dt = \frac{1}{2}LI^2$$

Note: The inductor can store finite amount of energy, even the voltage across it may be nil. A pure inductor does not dissipate energy but only store it.

Capacitance

It is the capacity of an element to store electric charge within it. A capacitor stores electric energy in the form of electric field being established by the two polarities of charges on the two electrodes of a capacitor. The unit of capacitance (C) being farad (F).

If q being the amount of charge that can be stored in a capacitor of capacitance C against a potential difference of V volts, then

$$C = \frac{q}{V}$$

$$\Rightarrow \quad I = C\frac{dV}{dt} \quad \left[\text{As } I = \frac{dq}{dt}\right]$$

$$\Rightarrow \quad dV = \frac{1}{C} I dt$$

$$\Rightarrow \quad \int_{V_0}^{V_t} dV = \frac{1}{C}\int_0^t I dt$$

[V_0 = initial voltage of the capacitor, V_t = final voltage of the capacitor]

$$\Rightarrow \quad V_t - V_0 = \frac{1}{C}\int_0^t I dt$$

$$\Rightarrow \quad V_t = \frac{1}{C}\int_0^t I dt + V_0$$

The power absorbed by the capacitor is given by $P = VI = V\,C\dfrac{dV}{dt}$

And energy stored by the capacitor is

$$W = \int_0^t P dt = \int_0^t VC\frac{dV}{dt}\,dt = \frac{1}{2}CV^2$$

Voltage-current relationship of circuit elements shown in the following table.

Circuit elements	Voltage (volts)	Current (amperes)	Power (watts)
$R(\Omega)$	$V = RI$	$I = \dfrac{V}{R}$	$P = I^2R$
L(H)	$V = L\dfrac{dI}{dt}$	$I = \dfrac{1}{L}\int V\,dt + I_0$ [I_0 being the initial current]	$P = LI\dfrac{dI}{dt}$
C(F)	$V = \dfrac{1}{C}\int I dt + V_0$ [V_0 being the initial voltage]	$I = C\dfrac{dV}{dt}$	$P = CV\dfrac{dV}{dt}$

VECTOR AND PHASOR

Vector is a generalized multidimensional quantity having both magnitude and direction. Phasor is a two dimensional vector used in electrical technology which relates to voltage and current.

SERIES AND PARALLEL NETWORKS

Resistors in Series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$$

Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n$$

Inductances in Series

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n$$

Inductances in Parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

CONTROLLED SOURCES

- A dependent source is a voltage source or a current source whose value depends on a voltage or current somewhere else in the network.
- Dependent or controlled sources are of the following types:
  (i) Voltage Controlled Voltage Source (VCVS)
  (ii) Current Controlled Voltage Source (CCVS)
  (iii) Voltage Controlled Current Source (VCCS)
  (iv) Current Controlled Current Source (CCCS)

DUALS AND DUALITY

In an electrical circuit itself there are pairs of terms which can be interchanged to get new circuits. Such pair of dual terms are as follows:

Current → Voltage
Open → Short
L → C
R → G
Series → Parallel
Voltage source → Current source
KCL → KVL

The following steps are to be followed to draw the dual of any network:

- In each loop of a network place a node; and place an extra node, called the reference node, outside the network.
- Draw the lines connecting adjacent nodes passing through each element, and also to the reference node,

by placing the dual of each element in the line passing through original elements.

KIRCHHOFF'S LAWS

- Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\text{Mathematically, } \sum_{n=1}^{N} I_n = 0$$

- Kirchhoff's second law is based on the principal of conservation of energy.
- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

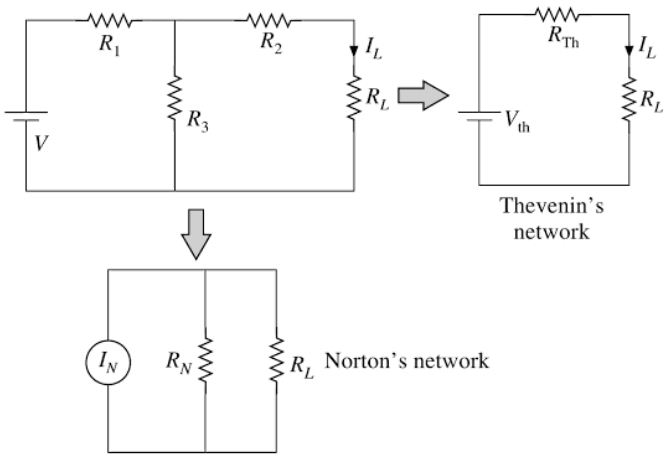
$$\text{Mathematically, } \sum_{m=1}^{M} V_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and V_m is the m^{th} voltage.

NETWORK THEOREMS

Thevenin's Theorem and Norton's Theorem

A linear active resistive bilateral network containing several voltage or current source can be replaced either by single voltage source [Thevenin's voltage (V_{Th})] with a series resistance [Thevenin's resistance (R_{Th})] or by a constant single current source [Norton's current (I_N)] with parallel resistance [Norton's resistance (R_N)].



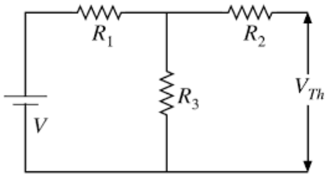
So the current (I_L) flowing through load resistance R_L connected across any two terminals A and B of the network shown in the figure is given by

$$\text{According to Thevenin's theorem, } I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

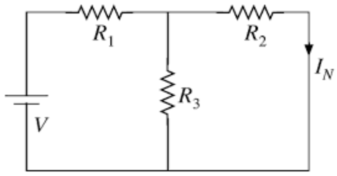
$$\text{According to Norton's theorem, } I_L = I_N \frac{R_N}{R_N + R_L}$$

V_{Th} is the open circuit voltage that is voltage across load terminals when R_L is removed.

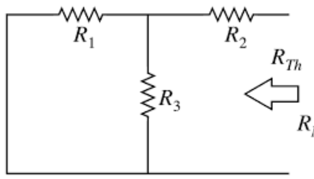
I_N is the short circuit current when load is removed and that path is to be shorted.



Finding Thevenin voltage



Finding Norton's current



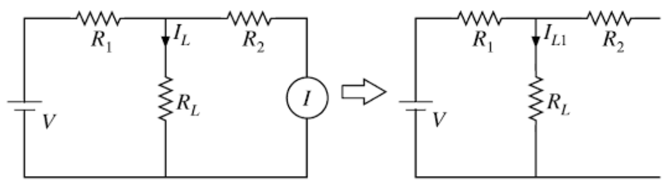
Finding Thevenin resistance or Norton's resistance

R_{Th} and R_N is the Thevenin's resistance and Norton's resistance which is the equivalent resistance of the network as viewed back into the open circuited network from load terminals deactivating all other sources.

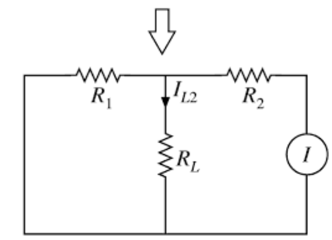
- Voltage source has zero internal resistance. When it is deactivated then the path should be shorted.
- Current source has infinite resistance. When it is deactivated then the path should be opened.

Superposition Theorem

For a linear system, the voltage or current in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the voltages or currents caused by each independent source acting alone, where all the other independent sources are replaced by their internal impedances.



Current source is deactivated. Only voltage source considered.



Voltage source is deactivated. Only current source considered.

When two sources are considered, load current I_L. According to superposition theorem, first one source is considered and other sources are deactivated. Then again calculate load current I_{L1} (considering voltage source). Then another source is considered and other source are deactivated and calculate load current I_{L2} (considering current source).This calculation is continued still all sources are considered individually. So load current $I_L = I_{L1} + I_{L2}$.

Limitations of superposition theorem

(i) This theorem cannot be used to measure power.
(ii) This theorem is not applicable to unbalanced bridge circuits.
(iii) Applicable only to linear circuits.
(iv) Applicable only for the circuits having more than one source.

Maximum Power Transfer Theorem

Maximum external power from a source is abstracted with a finite internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals (Thevenin or Norton resistance).

$$\text{Maximum power} = \frac{V_{Th}^2}{4R_L}$$

The average power delivered to a load Z_S from a sinusoidal source with voltage V_S and internal impedance $Z_i = R + jX$ is maximum when Z_S is equal to the complex conjugate of Z_i so that $Z_S = R - jX$.

Application of maximum power transfer theorem

(i) In communication system, maximum power transfer is always sought. For example in public address system, the circuit is adjusted for maximum power transfer by making load resistance (speaker) equal to the source resistance (amplifier). When source and load have the same resistance, they are said to be matched.
(ii) In car engines, the power delivered to the starter motor of the car will depend upon the effective resistance of the motor and the internal resistance of the battery. If the two resistances are equal, maximum power will be transferred to the motor to turn to the engine.

Note: Overall efficiency of a network supplying maximum power is 50%.

NETWORK GRAPH

Definitions

- ***Node:*** Node is a point in the circuit where two or more circuit elements are joined.
- ***Branch:*** Branch is a path that connects two nodes.
- ***Loop:*** It is a complete path i.e., starting from a point/node return back to the original point.
- ***Mesh:*** It is special type of loop i.e., it does not contain any other loop within it.
- ***Orient graph:*** Graph whose branch are oriented called directed/oriented graph.
- ***Rank of graph:*** It is $(n-1)$ where n is number of nodes.
- ***Sub-graph:*** It is the sub-set of nodes/branch of a graph. Sub-graph is said to be proper if it has no nodes and branch strictly less than that of original graph.
- ***Incident of a branch:*** Branch whose end falls on a node are said to be incident at that node.
- ***Path:*** It is particular sub-graph consisting a sequence of branch.
- ***Connected graph:*** When there exists at least one path between any two nodes of the graph.
- ***Tree:*** It is defined as a set of branches which is connected, contains all the nodes of the graph, but contains no circuits. The branches of a tree are known as *twigs*.
  Total number of twigs = Number of nodes – 1
- ***Co-tree:*** Those branches of a graph which are not included in a tree form a co-tree. Number of links = Number of branches – Number of nodes + 1

Note:
(i) Number of independent KVL equations = l (number of links)
(ii) Reducing the twig voltages to zero force, all the link voltages also to become zero. Thus, twig voltages are independent in a network.

Number of independent KCL equations = $n - 1$

Planar Graph

A planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. In other words, it can be drawn in such a way that no edges cross each other.

Network Variables

- The branch currents or branch voltages are the network variables.
- The minimum number of variables or unknown involved in analyzing a network on current basis is number of links d.
- The minimum number of variables or unknown involved in the analysis of network on the voltage basis is number of nodes.
- The currents and voltages in all branches of a network are completely known if we know link currents or branch voltages.

Link Current or Tie Set Schedule

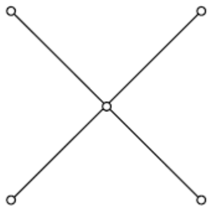
It is the set of branches contain in a loop such that each loop contains one link and remainder are tree branches.

Now, matrix which is formed using all the fundamental loops and branches of a directed graph known as *tie set* or fundamental loop matrix.

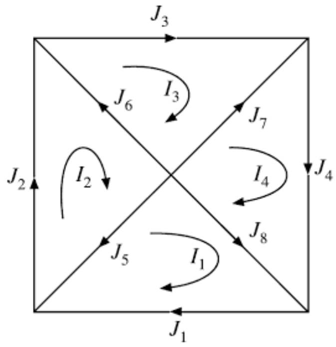
> ***Note:*** Orientation of the loop is determined by the orientation of the link for that loop.

So, Number of links = Number of fundamental loops

A corresponding tree for graph can be as show in the following figure.



Link current in above tree can be shown as



- Branches are numbered in a graph.
- Correspond tree is also shown.
- Addition of link 1 results in the formation of closed loop and flow of current I_1.
- These set of branches forming the closed loop on which the link current circulates is called tie set.
- Similarly addition of links 2, 3 and 4 result in flow of link currents I_2, I_3 and I_4 respectively.

The tie set matrix for the above shown graph is

Link number	Branch number							
	1	2	3	4	5	6	7	8
1	+1	0	0	0	–1	0	0	+1
2	0	+1	0	0	+1	–1	0	0
3	0	0	+1	0	0	+1	–1	0
4	0	0	0	+1	0	0	+1	–1

'–' sign corresponds to opposite direction of flow of current in that branch

I_5 and I_1 are flowing oppositely

J_7 and I_2 are flowing oppositely

Tree Branch Voltages or Cut Set Schedule

Tree branch voltages are treated as a set of independent variables in terms of which voltages of all branches can be expressed.

Incidence Matrix

Matrix between number of nodes in rows and number of branches in columns is called *incidence matrix.*

Incidence matrix is complete because sum of every column is zero.

Number of possible tree is determinant $[A_r, A_r^T]$

$A_r \rightarrow$ reduced incidence matrix

Reduced incidence matrix can be found by eliminating last row.

2-PORT NETWORK

Impedance Parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Open circuit impedance parameters:

$$Z_{11} = \left[\frac{V_1}{I_1}\right]_{I_2=0}, Z_{12} = \left[\frac{V_1}{I_2}\right]_{I_1=0}, Z_{21} = \left[\frac{V_2}{I_1}\right]_{I_2=0}, Z_{22} = \left[\frac{V_2}{I_2}\right]_{I_1=0}$$

Z_{11} = open circuit input impedance (driving point impedance)

Z_{12} = open circuit reverse transfer impedance

Z_{21} = open circuit forward transfer impedance

Z_{22} = open circuit output impedance

> ***Note:*** If the network under study is reciprocal or bilateral, then in accordance with the reciprocity principle,
> $$\left[\frac{V_2}{I_1}\right]_{I_2=0} = \left[\frac{V_1}{I_2}\right]_{I_1=0} \Rightarrow Z_{21} = Z_{12}$$

Admittance Parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Short circuit admittance parameters:

$$Y_{11} = \left[\frac{I_1}{V_1}\right]_{V_2=0}, Y_{12} = \left[\frac{I_1}{V_2}\right]_{V_1=0}, Y_{21} = \left[\frac{I_2}{V_1}\right]_{V_2=0}, Y_{22} = \left[\frac{I_2}{V_2}\right]_{V_1=0}$$

Y_{11} = short circuit input admittance (driving point admittance)

Y_{12} = short circuit reverse transfer admittance

Y_{21} = short circuit forward transfer admittance

Y_{22} = short circuit output admittance

> ***Note:*** If the network under study is reciprocal or bilateral, then
> $$\left[\frac{I_2}{V_1}\right]_{V_2=0} = \left[\frac{I_1}{V_2}\right]_{V_1=0} \Rightarrow Y_{21} = Y_{12}$$

Hybrid Parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Hybrid parameters:

$$h_{11} = \left[\frac{V_1}{I_1}\right]_{V_2=0}, h_{12} = \left[\frac{V_1}{V_2}\right]_{I_1=0}, h_{21} = \left[\frac{I_2}{I_1}\right]_{V_2=0}, h_{22} = \left[\frac{I_2}{V_2}\right]_{I_1=0}$$

h_{11} = short circuit input impedance
h_{12} = open circuit reverse voltage gain
h_{21} = short circuit forward current gain
h_{22} = open circuit output admittance

Since the h parameters represent dimensionally an impedance, an admittance, a voltage gain, and a current gain, these are called hybrid parameters.

Transmission Parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Transmission parameters

$$A = \left[\frac{V_1}{V_2}\right]_{I_2=0}, -B = \left[\frac{V_1}{I_2}\right]_{V_2=0}, C = \left[\frac{I_1}{V_2}\right]_{I_2=0}, -D = \left[\frac{I_1}{I_2}\right]_{V_2=0}$$

Now, $\frac{1}{A} = \left[\frac{V_2}{V_1}\right]_{I_2=0} = [G_{21}]_{I_2=0}$ is called the open circuit voltage gain, a dimensionless parameter.

$\frac{1}{C} = \left[\frac{V_2}{I_1}\right]_{I_2=0} = Z_{21}$, which is the open circuit transfer impedance.

$-\frac{1}{B} = \left[\frac{I_2}{V_1}\right]_{V_2=0} = Y_{21}$, which is the short circuit transfer admittance.

$-\frac{1}{D} = \left[\frac{I_2}{I_1}\right]_{V_2=0} = [\alpha_{21}]_{V_2=0}$, which is the short circuit current gain, a dimensionless parameter.

> ***Note:*** In a 2-port bilateral network, $AD - BC = 1$.

Expression of *Z* Parameters in Terms of *Y* Parameters and Vice-versa

$$Z_{11} = \frac{Y_{22}}{\Delta_Y}, Z_{12} = -\frac{Y_{12}}{\Delta_Y}, Z_{21} = -\frac{Y_{21}}{\Delta_Y}, Z_{22} = \frac{Y_{11}}{\Delta_Y}$$

$$Y_{11} = \frac{Z_{22}}{\Delta_Z}, Y_{12} = -\frac{Z_{12}}{\Delta_Z}, Y_{21} = -\frac{Z_{21}}{\Delta_Z}, Y_{22} = \frac{Z_{11}}{\Delta_Z}$$

Expression of *ABCD* Parameters in Terms of *Z* Parameters and *Y* Parameters

$$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{\Delta_Z}{Z_{21}}, \quad C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$$

$$A = -\frac{Y_{22}}{Y_{21}}, B = -\frac{1}{Y_{21}}, C = -\frac{\Delta_Y}{Y_{21}}, D = -\frac{Y_{11}}{Y_{21}}$$

DRIVING POINT IMMITTANCE (IMPEDANCE AND ADMITTANCE) FUNCTION

Driving point impedance $Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$.

Transfer function is the ratio of any output quantity (V_2 and I_2) and any input quantity (V_1 and I_1).

$\frac{V_2(s)}{V_1(s)} = G_{12}(s)$ = Voltage transfer function

$\frac{V_2(s)}{I_1(s)} = Z_{12}(s)$ = Impedance transfer function

$\frac{I_2(s)}{V_1(s)} = Y_{12}(s)$ = Admittance transfer function

$\frac{I_2(s)}{I_1(s)} = \alpha_{12}(s)$ = Current transfer function

CALCULATION PROCEDURE FOR A SERIES AC CIRCUITS

(i) Express R in Ω, L in H, C in F and f in Hz.
(ii) Determine $X_L = 2\pi f L$ Ω
(iii) Determine $X_C = \frac{1}{2\pi fC}$ Ω
(iv) Determine the impedance $Z = [R + j(X_L - X_C)]$ Ω
(v) In polar form $Z = |Z|\angle\theta$.
Here, $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$
and $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$.
(vi) Take supply voltage V as reference phasor, $V = V\angle 0°$.
(vii) Determine the circuit current
$$I = \frac{V}{Z} = \frac{V\angle 0°}{Z\angle\theta} = \frac{V}{Z}\angle -\theta \text{ A}.$$
(viii) Determine the power factor cosθ. Specify whether the power factor is lagging or leading. If θ is negative in Step (vii), the power factor is lagging. If θ is positive in Step (vii), the power factor is leading.
(ix) Determine the complex conjugate of I, i.e., I^*.
(x) Determine $S = VI^* = P + jQ$. The real part of VI^* gives active power P in watts and the imaginary part

gives the reactive volt-amperes in VAr. The magnitude of VI^* gives volt-amperes (VA).

$\therefore$ $P = \text{Re}(VI^*)$ W, $Q = \text{Im}(VI^*)$ VAr and $S = |VI^*|$ VA

So, $P = VI\cos\theta$, $Q = VI\sin\theta$ and $S = VI$

The lagging current gives a positive of Q, and the leading current gives a negative value of Q. By convention, lagging VAr is taken positive and leading VAr is taken negative.

(xi) Determine voltage drop across resistance, inductance and capacitance.

$$V_R = IR,\ V_L = I(jX_L) \text{ and } V_C = I(-jX_C)$$

SOLUTION OF THREE-PHASE CIRCUITS

It is always preferable to solve balanced three phase circuits on a per phase basis. Steps are

(i) Draw the circuit diagram
(ii) Draw one phase separately
(iii) Determine $X_{Lp} = 2\pi fL$
(iv) Determine $X_{Cp} = \dfrac{1}{2\pi fC}$
(v) Determine $X_p = X_{Lp} - X_{Cp}$
(vi) Determine $Z_p = (R_p^2 + X_p^2)^{1/2}$
(vii) Determine $\cos\theta = \dfrac{R_p}{Z_p}$

The power factor is lagging when $X_{Lp} > X_{Cp}$ and it is leading when $X_{Cp} > X_{Lp}$.

(viii) Determine V_p. For star connection, $V_p = \dfrac{V_L}{\sqrt{3}}$ and for delta connection $V_p = V_L$.
(ix) Determine $I_p = \dfrac{V_p}{Z_p}$.
(x) Determine the line current I_L. For star connection, $I_L = I_p$ and for delta connection $I_L = \sqrt{3}\,I_p$.
(xi) Determine $P = 3V_pI_p\cos\theta$.
(xii) Determine $Q = 3V_pI_p\sin\theta$.
(xiii) Determine $S = 3V_pI_p$.

FILTER

- A filter is an electrical network that can transmit signals within a specified frequency range. The range of frequency passed is called *pass band* and the rest frequency band where the signals are suppressed is called the *attenuation band* or *stop band*. The frequency that separates the pass band and the attenuation band is the *cut-off frequency*.
- An ideal filter would transmit signals under the pass band frequencies without attenuation and completely suppress the signal with attenuation band of frequencies with sharp cut-off profile. In real life, ideal filter is difficult to implement.

Properties of Filters

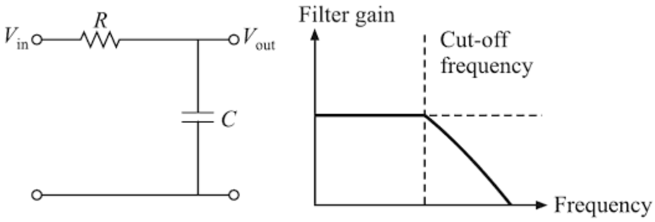
Filters are characterized by the following properties:

(i) *Pass band characteristics:* The filter should have minimum attenuation in its pass band range and high attenuation in the stop band range. Degree of attenuation is expressed with a constant which have the unit decibel or neper.
(ii) *Cut-off frequency characteristics:* It must have frequency distinguishing property capable of identifying the lower and higher cut-off frequencies.
(iii) *Characteristic impedances:* The characteristic impedance of the filter should matches with the circuit to which it connected throughout the pass band. It prevents reflection loss.

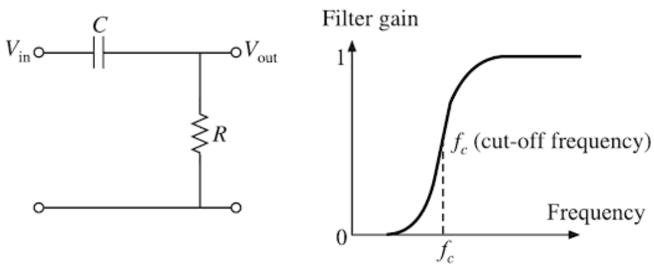
Types of Filters

(i) Low-pass
(ii) High-pass
(iii) Band-pass
(iv) Band-reject (notch)
(v) All-pass

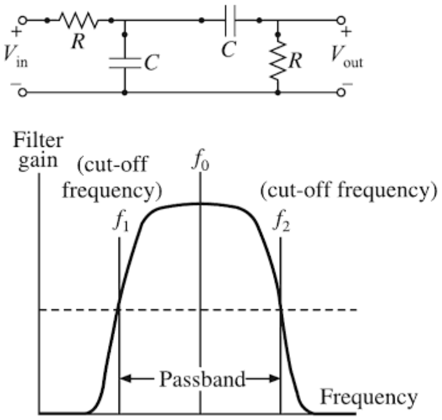
Low-pass filter: A low-pass filter passes signals with a frequency lower than a certain cut-off frequency and attenuates signals with frequencies higher than the cut-off frequency. The amount of attenuation for each frequency depends on the filter design.



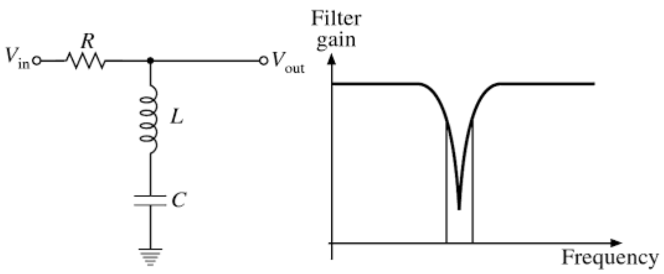
High-pass filter: A high-pass filter (HPF) passes high-frequency signals but attenuates signals with frequencies lower than the cut-off frequency.



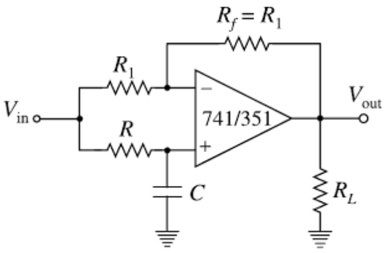
Band-pass: A band-pass filter allows a specific frequency range to pass, while blocking lower and higher frequencies. It allows frequencies between two cut-off frequencies while attenuating frequencies outside the cut-off frequencies.

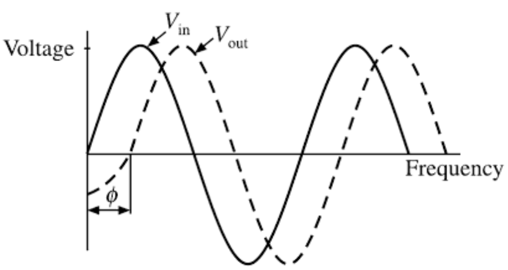


Band-reject: It also termed as band stop or notch filter. This kind of filter passes all frequencies above and below a particular range set by the component values. Stop band filters can be constructed using a low-pass and a high pass filter.



All-pass filter: All pass filter is that which passes all frequency components of the input signal without attenuation but provides predictable phase shifts for different frequencies of the input signals. The all-pass filters are also called *delay equalizers* or *phase correctors*. An all-pass filter with the output lagging behind the input is illustrated in figure. This type of filter is also called *phase-shift filter*.





RESONANCE

In RLC series circuit, resonance is defined as the condition in a circuit containing at least one inductor and one capacitor, when the supply voltage and the supply current are in phase.

Resonance in series circuit is called *series resonance* or *voltage resonance*.

Main Properties of Series Resonance Circuit

(i) $X_{L0} = X_{C0}$ and $V_{L0} = V_{C0}$.

(ii) $V_{L0}(=X_{L0}I_0)$ leads the current I_0 by 90° and $V_{C0}(=X_{C0}I_0)$ lags behind the current I_0 by 90°

(iii) The supply voltage and the supply current are in phase so that power factor angle is zero and power factor is unity.

(iv) The circuit impedance Z_0 is minimum and $Z_0 = R$.

(v) The supply current I_0 is maximum and is given by $I_0 = \frac{V}{R}$.

(vi) The power P_0 absorbed by the circuit is a maximum and $P_0 = \frac{V^2}{R}$.

(vii) Resonant frequency $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ Hz in series circuit.

(viii) Quality factor Q =

$$2\pi \times \frac{\text{maximum stored energy per cycle}}{\text{energy dissipated per cycle}}.$$

Now quality factor at resonance $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$.

(ix) Resonant frequency $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz in parallel circuit.

1.4 IMPORTANT POINTS TO REMEMBER

1. An electric circuit is an interconnection of electrical elements.
2. A short circuit is a circuit element with resistance approaching to zero. An open circuit is a circuit element with resistance approaching to infinity.
3. Kirchhoff's laws are valid for both linear and non-linear circuits.
4. Kirchhoff's laws are applicable to circuits with any excitation.
5. The law of conservation of charge state that charge can neither be created nor be destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.
6. The law of conservation of energy must be obeyed in any electric circuit. So, the algebraic sum of power in a circuit, at any instant of time, must be zero. Hence, the total power supplied to the circuit must balance the total power absorbed.

7. Charge is an electrical property of the atomic particles of which matter consists. It is measured in coulombs (C).
8. Electric current is the time rate of change of charge, measured in amperes (A). $\left[I = \frac{dq}{dt} \text{ and } q = \int_{t_0}^{t} Idt\right]$
9. Potential Difference is the energy required to move a unit charge through an element, measured in volts (V).

   Where, $V_{ab} = \frac{dW}{dq}$ and 1 volt = 1 joule/coulomb = 1 newton meter/coulomb
10. Power is the time rate of expending or absorbing energy, measured in watts (W).

$$\left[P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = VI\right]$$

11. The energy absorbed or supplied by an element from time t_0 to time t is $W = \int_{t_0}^{t} P\,dt = \int_{t_0}^{t} VI\,dt$.
12. The electric power utility companies measure energy in watt-hour (Wh), where 1 Wh = 3600 J.
13. A direct current (dc) is a current that remains constant with time.
14. An alternating current (ac) is a current that varies sinusoidally with time.
15. An ideal independent source is an active elements that provides a specified voltage or current that is completely independent of other circuit elements.
16. An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.
17. For a dc voltage, an inductor is virtually a short circuit.
18. The internal impedance of an ideal voltage source is zero.
19. The internal impedance of an ideal current source is infinite.
20. The internal impedance of a dependent voltage source is any unknown value.
21. An ideal voltage source will charge an ideal capacitor instantaneously.
22. If one of the resistors in a parallel circuit is removed, the total resistance increases.
23. All resistors do not obey Ohm's law. A resistor that obeys Ohm's law is known as linear resistor. It has a constant resistance and thus its current voltage characteristic is a straight line passing through the origin. A non-linear resistor does not obey Ohm's law. Examples of devices with non-linear resistance are the light bulb and diode. Although all practical resistors may exhibit non-linear behaviour under certain condition.
24. KCL follows law of conservation of charge.
25. Kirchhoff's law fails in case of distributed parameter networks.
26. Transient behaviour occurs in any circuit when
    - the circuit is connected or disconnected from the supply
    - there are sudden changes of applied voltage
    - the voltage source is shorted
27. The transient response occurs both in inductive circuits and capacitive circuits.
28. Inductor does not allow sudden changes in currents.
29. The time constant of a series RL circuit is $\frac{L}{R}$.
30. A capacitor does not allow sudden changes in voltages.
31. While drawing vector diagram for a series circuit, the reference vector is current.
32. While drawing vector diagram for a parallel circuit, the reference vector is voltage.
33. A tree has no closed paths.
34. The number of branches in a tree is less than the number of branches in a graph.
35. The tie set schedule gives the relation between branch currents and link currents.
36. The cut set schedule gives the relation between branch voltages and tree branch voltages.
37. Inductance affects the direct current flow at the time of turning on and off.
38. Zero degree phase difference exists between voltage and current in an ac circuit when voltage and current both reach zero and maximum at the same time.
39. Form factor of a sine wave is 1.11.
40. The ac measuring instruments measure rms value.
41. In a purely inductive circuit, actual power is zero.
42. For a given power factor of the load, if the power factor of the load decreases, it will draw more current from the supply.
43. In any ac circuit always apparent power is more than the actual power.
44. Mesh analysis is based on Kirchhoff's voltage law.
45. The nodal method of circuit analysis is based on Kirchhoff's current law and Ohm's law.
46. The reciprocity theorem is applicable to linear / bilateral networks.
47. A practical voltage source consists of an ideal voltage source in series with an internal resistance.
48. A practical current source consists of an ideal current source in parallel with an internal resistance.
49. If a network contains b branches and n nodes, then the number of mesh current equations would be $[b-(n-1)]$.
50. The compensation theorem is applicable to linear and non-linear networks
51. The superposition theorem is valid only for linear circuits.

52. The superposition theorem is valid for voltage responses.
53. The superposition theorem is valid for current responses.
54. The superposition theorem is not valid for power responses.
55. The superposition theorem is applicable to networks containing dependent voltage sources as well as dependent current sources.
56. When the superposition theorem is applied to any circuit, the dependent voltage source in that circuit is always active.
57. The superposition theorem is valid for both ac and dc circuits.
58. When applying the superposition theorem to any circuit, the voltage source is shorted and the current source is opened.
59. Norton's equivalent circuit consists of current source in parallel with impedance.
60. The maximum power transfer theorem can be applied to both dc and ac circuits.
61. Maximum power transfer occurs at a 50% efficiency.
62. In a complex impedance circuit, the maximum power transfer occurs when the load impedance is equal to complex conjugate of source impedance.
63. In series resonance, the circuit impedance is purely resistive.
64. When the circuit contains capacitance and inductance, then the power factor of the circuit will be zero.
65. In a series circuit on resonance, the following will happen:
    (a) $X_L = X_C$
    (b) $V_L = V_C$
    (c) $Z = R$
    (d) $V = V_R$
66. At resonance, the phase angle is equal to 0°.
67. In series resonance, current is maximum.
68. In series resonance, above resonant frequency, impedance is inductive.
69. In series resonance, below resonant frequency, impedance is capacitive.
70. In parallel resonance, impedance is maximum. (i.e., current minimum)
71. Wattless current is said to flow in an ac circuit when the phase angle between virtual current and virtual voltage is 90°.
72. In an ac circuit, the average value of the current over a full cycle is zero.
73. The average value of alternating current over half a cycle is $\frac{2}{\pi} I_0$.
74. The form factor in ac means the ratio of rms value to average value.
75. A low-pass filter is one which passes all frequencies up to cut-off frequency and attenuates all other frequencies.
76. A high-pass filter is one which attenuates all frequencies below a designated cut-off frequency, and passes all frequencies above cut-off.
77. A band-pass filter is one which passes frequencies between two designated cut-off frequencies and attenuates all other frequencies.
78. An ideal filter should have zero attenuation in the pass band.
79. For a reciprocal network, the 2-port *ABCD* parameters are related as $AD - BC = 1$.
80. For a symmetrical 2-port network $Z_{11} = Z_{22}$.
81. A 2-port network is reciprocal if and only if
$$AD - BC = 1.$$
82. When a number of 2-port networks are connected in cascade, the individual chain matrices are multiplied.
83. In terms of *ABCD* parameters, a 2-port network is symmetrical if and only if $A = D$.
84. The relation $AD - BC = 1$, where A, B, C, D are the elements of a transmission matrix of a network, is valid for passive and reciprocal network.
85. Two 2-port networks with transmission parameters A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 respectively are cascaded. The transmission parameter matrix of the cascaded network will be $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$.
86. The series resonant circuit is also called an acceptor circuit.
87. The resonant frequency is the geometric mean of the lower and upper half power frequencies.
88. The parallel resonant circuit is also called a rejector circuit.

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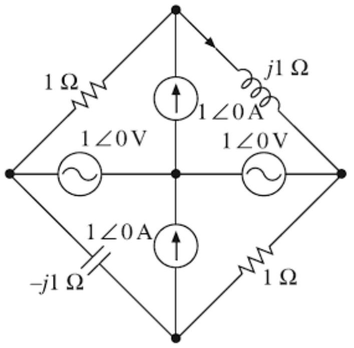
1.6 PREVIOUS YEARS' QUESTIONS

1. In the circuit shown below, the current through the inductor is



(a) $\dfrac{2}{1+j}$A (b) $\dfrac{-1}{1+j}$A

(c) $\dfrac{1}{1+j}$A (d) 0 A [Gate 2012]

2. A two-phase load draws the following phase current:

$I_1(t) = I_m \sin(\omega t - \phi_1)$, $I_2(t) = I_m \cos(\omega t - \phi_2)$

These currents are balanced if ϕ_1 is equal to

(a) $-\phi_2$ (b) ϕ_2

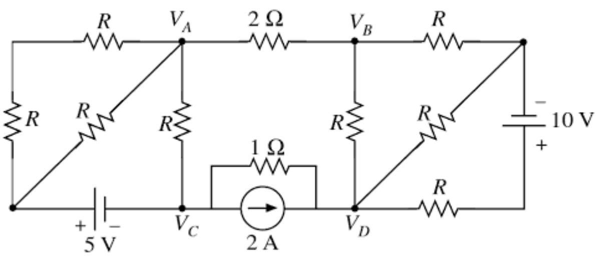
(c) $\dfrac{\pi}{2} - \phi_2$ (d) $\dfrac{\pi}{2} + \phi_2$ [Gate 2012]

3. The average power delivered to an impedance $(4 - j3)\Omega$ by a current $5\cos(100\pi t + 100)$ A is

(a) 44.2 W (b) 50 W

(c) 62.5 W (d) 125 W [Gate 2012]

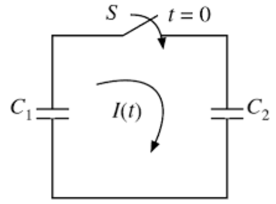
4. If $V_A - V_B = 6$ V, then $V_C - V_D$ is



(a) – 5 V (b) 2 V

(c) 3 V (d) 6 V [Gate 2012]

5. In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $I(t)$ for all t is



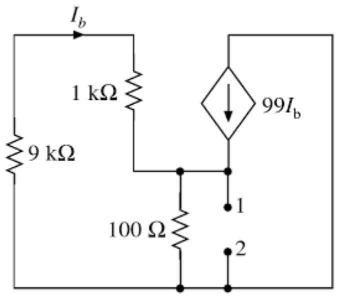
(a) Zero

(b) A step function

(c) An exponentially decaying function

(d) An impulse function [Gate 2012]

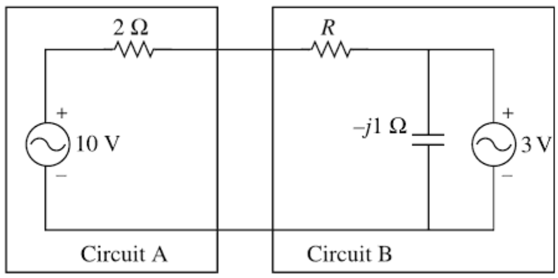
6. The impedance looking into nodes 1 and 2 in the given circuit is



(a) 50 Ω (b) 100 Ω

(c) 5 kΩ (d) 10.1 kΩ [Gate 2012]

7. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(a) 0.8 Ω (b) 1.4 Ω

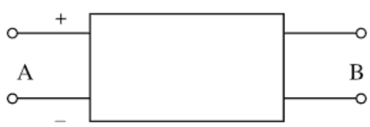
(c) 2 Ω (d) 2.8 Ω [Gate 2012]

Common Data Questions: 8 and 9

With 10 V dc connected at port A in the linear non-reciprocal 2-port network shown below, the following were observed:

(i) 1 Ω connected at port B draws a current of 3 A

(ii) 2.5 Ω connected at port B draws a current of 2 A



8. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V dc is connected to port A, the open circuit voltage at port B is

(a) 6 V (b) 7 V

(c) 8 V (d) 9 V [Gate 2012]

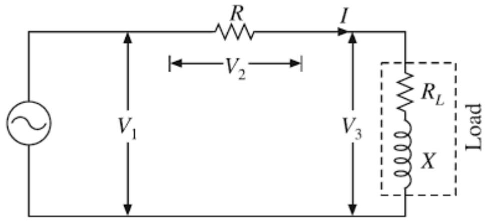
9. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port B is

(a) 3/7 A (b) 5/7 A

(c) 1 A (d) 9/7 A [Gate 2012]

Statement for Linked Answer Questions: 10 and 11

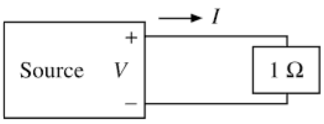
In the circuit shown, the three voltmeter readings are V_1 = 220 V, V_2 = 122 V, and V_3 = 136 V.



10. The power factor of the load is
(a) 0.45 (b) 0.50
(c) 0.55 (d) 0.60 [Gate 2012]

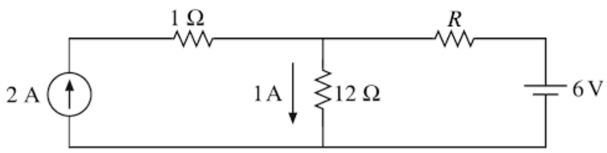
11. If R_L = 5 Ω, the approximate power consumption in the load is
(a) 700 W (b) 750 W
(c) 800 W (d) 850 W [Gate 2012]

12. As shown in figure, a 1 Ω resistance is connected across a source that has a load line $V + I = 100$. The current through the resistance is



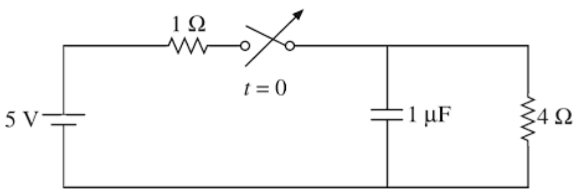
(a) 25 A (b) 50 A
(c) 100 A (d) 200 A [Gate 2010]

13. If the 12 Ω resistor draws a current of 1 A as shown in figure, the value of resistance R is



(a) 4 Ω (b) 6 Ω
(c) 8 Ω (d) 18 Ω [Gate 2010]

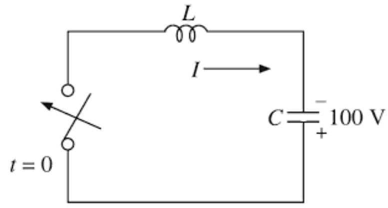
14. The switch in the circuit has been closed for a long time. It is opened at t = 0. At $t = 0^+$, the current through the 1 μF capacitor is



(a) 0 A (b) 1 A
(c) 1.25 A (d) 5 A [Gate 2010]

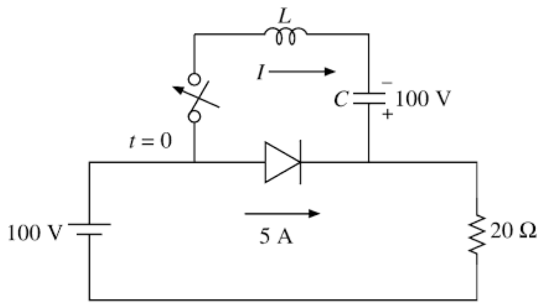
Statement for Linked Answer Questions: 15 and 16

The LC circuit shown in figure has an inductance L = 1 mH and a capacitance C = 10 μF.



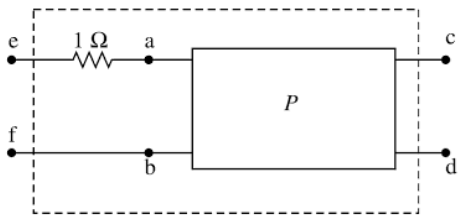
15. The initial current through the inductor is zero, while the initial capacitor voltage is 100 V. The switch is closed at t = 0. The current I through the circuit is:
(a) $5\cos(5\times10^3 t)$ A (b) $5\sin(10^4 t)$ A
(c) $10\cos(5\times10^3 t)$A (d) $10\sin(10^4 t)$ A
[Gate 2010]

16. The LC circuit of above question is used to commutate a thyristor, which is initially carrying a current of 5 A as shown in figure below. The values and initial conditions of L and C are the same as in above question. The switch is closed at t = 0. If the forward drop is negligible, the time taken for the device to turn off is



(a) 52 μs (b) 156 μs
(c) 312 μs (d) 26 μs [Gate 2010]

17. The 2-port network P shown in figure has ports 1 and 2, denoted by terminals (a, b) and (c, d) respectively. It has an impedance matrix Z with parameters denoted by Z_{ij}. A 1 Ω resistor is connected in series with the network at port 1 as shown in the figure. The impedance matrix of the modified 2-port network (shown as a dashed box) is

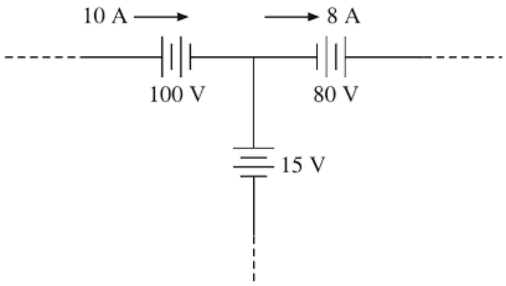


(a) $\begin{pmatrix} Z_{11}+1 & Z_{12}+1 \\ Z_{21} & Z_{22}+1 \end{pmatrix}$ (b) $\begin{pmatrix} Z_{11}+1 & Z_{12} \\ Z_{21} & Z_{22}+1 \end{pmatrix}$

(c) $\begin{pmatrix} Z_{11}+1 & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$ (d) $\begin{pmatrix} Z_{11}+1 & Z_{12} \\ Z_{21}+1 & Z_{22} \end{pmatrix}$

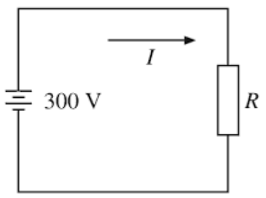
[Gate 2010]

18. The three circuit elements shown in figure are part of an electric circuit. The total power absorbed by the three circuit elements in watts is __________



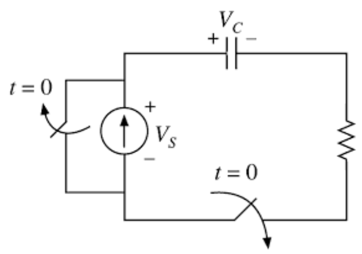
[Gate 2014 set 1]

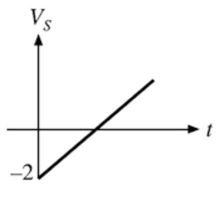
19. In figure, the value of resistor R is $(25 + I/2)$ ohms, where I is the current in amperes. The current I is ____________.

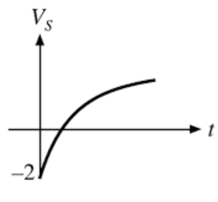


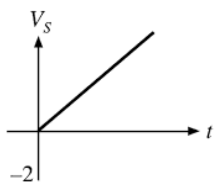
[Gate 2014 set 1]

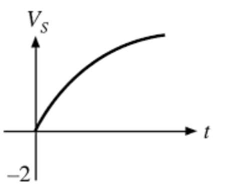
20. A combination of 1 μF capacitor with an initial voltage $V_C(0) = -2$ V in series with an 100 Ω resistor is connected to a 20 mA ideal dc current source by operating both switches at $t = 0$ s as shown in figure. Which of the following graphs shown in the options approximates the voltage V_S across the current source over the next few seconds?



(a) 

(b) 

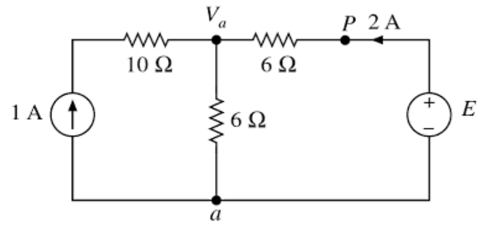
(c) 

(d) 

[Gate 2014 set 1]

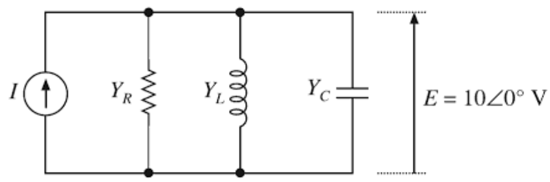
21. An incandescent lamp is marked 40 W, 240 V. If resistance at room temperature (26 °C) is 120 Ω and temperature coefficient of resistance is 4.5×10^{-3}/°C, then its 'ON' state filament temperature in °C is approximately ___________ . [Gate 2014 set 1]

22. In figure, the value of the source voltage is



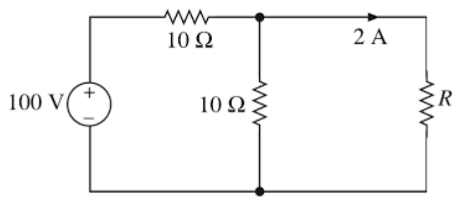
(a) 12 V (b) 24 V
(c) 30 V (d) 44 V [Gate 2004]

23. In figure, the admittance values of the elements in Siemens are $Y_R = 0.5 + j0$, $Y_L = 0 - j1.5$, $Y_C = 0 + j0.3$, respectively. The value of I as a phasor when the voltage E across the elements is $10 \angle 0°$ V is



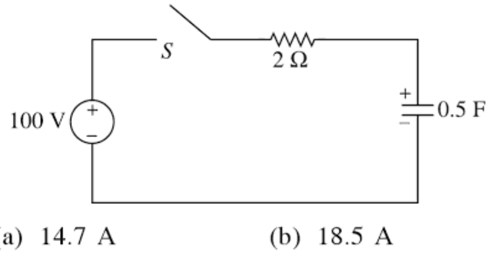
(a) $1.5 + j0.5$ (b) $5 - j18$
(c) $0.5 + j1.8$ (d) $5 - j12$ [Gate 2004]

24. In figure, the value of resistance R in Ω is



(a) 10 (b) 20
(c) 30 (d) 40 [Gate 2004]

25. In figure, the capacitor initially has a charge of 10 C. The current in the circuit one second after the switch S is closed will be



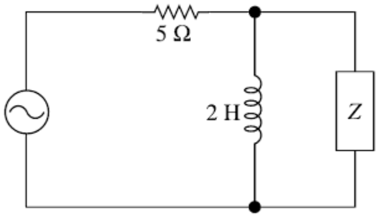
(a) 14.7 A (b) 18.5 A
(c) 40.0 A (d) 50.0 A [Gate 2004]

26. The Z matrix of a 2-port network as given by $\begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$. The element Y_{22} of the corresponding Y

matrix of the same network is given by

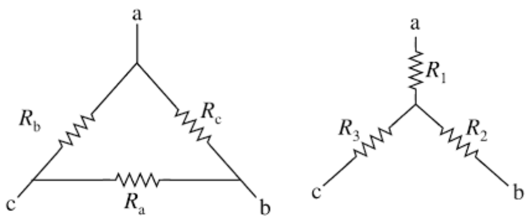
(a) 1.2 (b) 0.4
(c) – 0.4 (d) 1.8 [Gate 2004]

27. The value of Z in figure, which is most appropriate to cause parallel resonance at 500 Hz, is



(a) 125.00 mH (b) 304.20 μF
(c) 2.0 μF (d) 0.05 μF [Gate 2004]

28. In figure, R_a, R_b and R_c are 20 Ω, 10 Ω and 10 Ω respectively. The resistance R_1, R_2 and R_3 in Ω of an equivalent star–connection are



(a) 2.5, 5, 5 (b) 5, 2.5, 5
(c) 5, 5, 2.5 (d) 2.5, 5, 2.5 [Gate 2004]

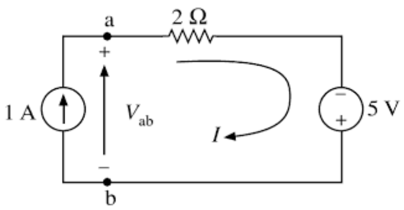
29. The rms value of the current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 A is

(a) 10 A (b) 14.14 A
(c) 15 A (d) 17.32 A [Gate 2004]

30. The Thevenin's equivalent of a circuit operating at $\omega = 5$ rad/s, has $V_{OC} = 3.71\angle -15.9$ °V and $Z_0 = 2.38 - j\,0.667$ Ω. At this frequency, the minimal realization of the Thevenin's impedance will have a

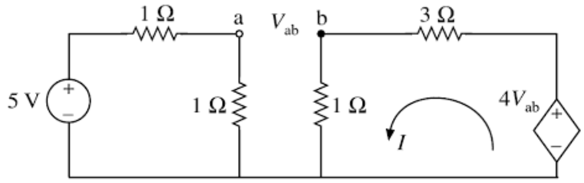
(a) Resistor and a capacitor and an inductor
(b) Resistor and a capacitor
(c) Resistor and an inductor
(d) Capacitor and an inductor [Gate 2008]

31. Assuming ideal elements in the circuit shown in figure, the voltage V_{ab} will be



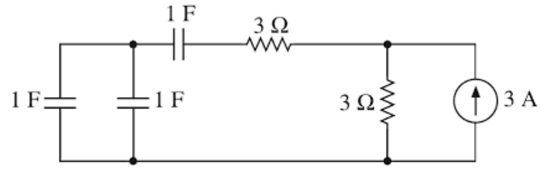
(a) – 3 V (b) 0 V
(c) 3 V (d) 5 V [Gate 2008]

32. In the circuit shown in figure, the value of the current I will be given by



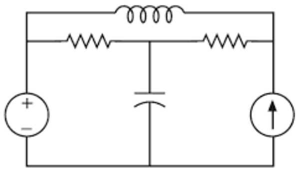
(a) 0.31 A (b) 1.25 A
(c) 1.75 A (d) 2.5 A [Gate 2008]

33. The time constant for the given circuit will be



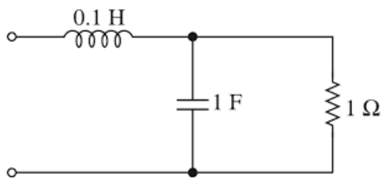
(a) $\frac{1}{9}$ s (b) $\frac{1}{4}$ s
(c) 4 s (d) 9 s [Gate 2008]

34. The number of chords in the graph of the given circuit will be



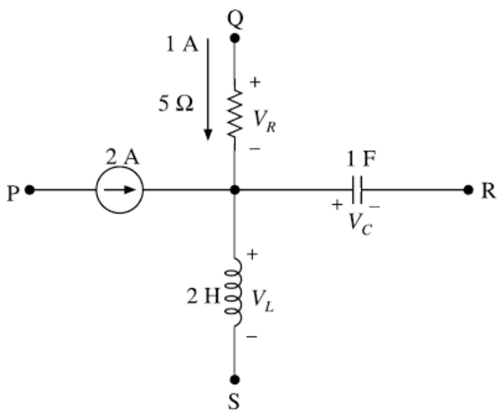
(a) 3 (b) 4
(c) 5 (d) 6 [Gate 2008]

35. The resonant frequency for the given circuit will be



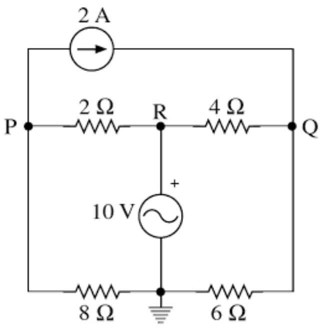
(a) 1 rad/s (b) 2 rad/s
(c) 3 rad/s (d) 4 rad/s [Gate 2008]

36. A segment of a circuit is shown in figure. $V_R = 5V$, $V_C = 4\sin 2t$. The voltage V_L is given by



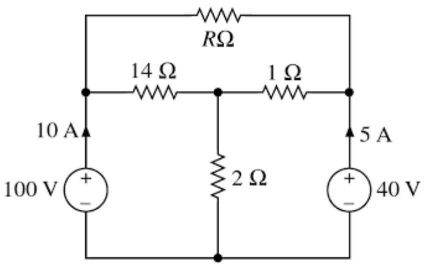
(a) $3 - 8\cos 2t$ (b) $32\sin 2t$
(c) $16\sin 2t$ (d) $16\cos 2t$ [Gate 2003]

37. In figure, the potential difference between points P and Q is



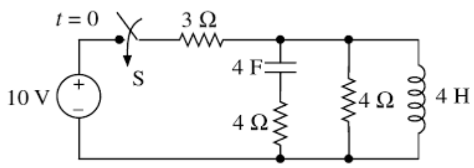
(a) 12 V (b) 10 V
(c) − 6 V (d) 8 V [Gate 2003]

38. In figure, the value of R is



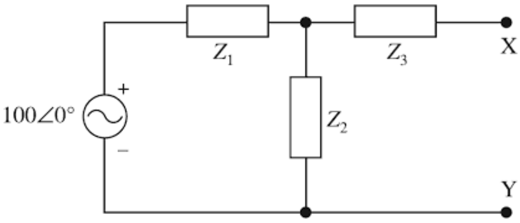
(a) 10 Ω (b) 18 Ω
(c) 24 Ω (d) 12 Ω [Gate 2003]

39. In the circuit shown in figure, the switch S is closed at time (t = 0). The voltage across the inductance at $t = 0^+$ is



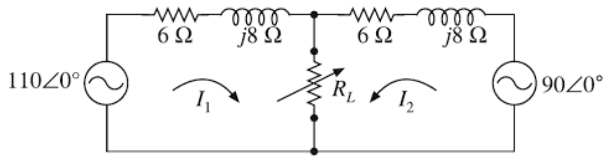
(a) 2 V (b) 4 V
(c) − 6 V (d) 8 V [Gate 2003]

40. In figure, $Z_1 = 10\angle{-60°}$, $Z_2 = 10\angle 60°$, $Z_3 = 50\angle 53.13°$. The Thevenin impedance seen from X − Y is



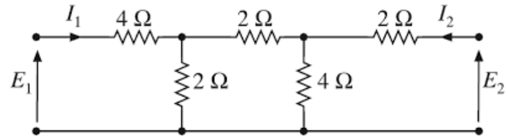
(a) $56.66\angle 45°$ (b) $60\angle 30°$
(c) $70\angle 30°$ (d) $34.4\angle 65°$ [Gate 2003]

41. Two ac sources feed a common variable resistive load as shown in figure. Under the maximum power transfer condition, the power absorbed by the load resistance R_L is



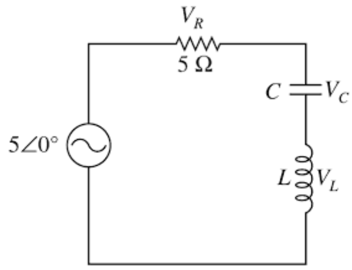
(a) 2200 W (b) 1250 W
(c) 1000 W (d) 625 W [Gate 2003]

42. The h parameters for a 2-port network are defined by $\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$. For the 2-port network shown in figure, the value of h_{12} is given by



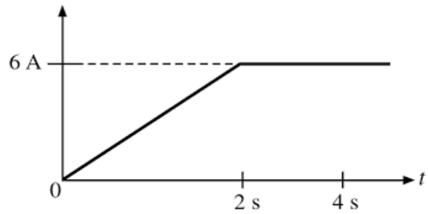
(a) 0.125 (b) 0.167
(c) 0.625 (d) 0.25 [Gate 2003]

43. In the circuit shown in figure, the magnitudes of V_L and V_C are twice that of V_R. Given that f = 50 Hz, the inductance of the coil is



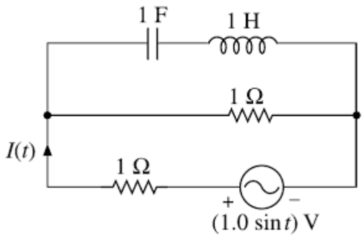
(a) 2.14 mH (b) 5.30 H
(c) 31.8 mH (d) 1.32 H [Gate 2003]

44. Figure shows the waveform of the current passing through an inductor of resistance 1 Ω and inductance 2 H. The energy absorbed by the inductor in the first four seconds is



(a) 144 J
(b) 98 J
(c) 132 J
(d) 168 J [Gate 2003]

45. The rms value of current $I(t)$ in the circuit shown in figure is



(a) $\frac{1}{2}$ A (b) $\frac{1}{\sqrt{2}}$ A

(c) 1 A (d) $\sqrt{2}$ A [Gate 2011]

46. The voltage applied to a circuit is $100\sqrt{2}\cos(100\pi t)$ V and the circuit draws a current of $10\sqrt{2}\sin(100\pi t + \pi/4)$ A. Taking the voltage as the reference phasor, the phasor representation of the current in amperes is

(a) $10\sqrt{2}\angle{-\pi/4}$ (b) $10\angle{-\pi/4}$

(c) $10\angle{+\pi/4}$ (d) $10\sqrt{2}\angle{+\pi/4}$

[Gate 2011]

Common Data Questions: 47 and 48

The input voltage given to a converter is

$$V_{\text{in}} = 100\sqrt{2}\sin(100\pi t)\text{ V}$$

The current drawn by the converter is

$$I = 10\sqrt{2}\sin(100\pi t - \pi/3) + 5\sqrt{2}\sin(300\pi t + \pi/4) + 2\sqrt{2}\sin(500\pi t - \pi/6)\text{ A}$$

47. The active power drawn by the converter is

(a) 181 W (b) 500 W

(c) 707 W (d) 887 W [Gate 2011]

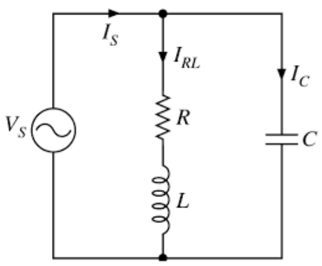
48. The input power factor of the converter is

(a) 0.31 (b) 0.44

(c) 0.5 (d) 0.71 [Gate 2011]

Common Data Questions: 49 and 50

An RLC circuit with relevant data is given in figure.



$$V_S = 1\angle 0\text{ V}$$

$$I_S = \sqrt{2}\angle\pi/4\text{ A}$$

$$I_{RL} = \sqrt{2}\angle -\pi/4\text{ A}$$

49. The power dissipated in the resistor R is

(a) 0.5 W (b) 1 W

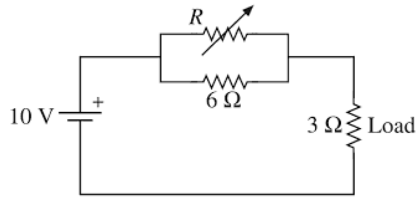
(c) $\sqrt{2}$ W (d) 2 W [Gate 2011]

50. The current I_C in the figure above is

(a) $-j2$ A (b) $-j\frac{1}{\sqrt{2}}$ A

(c) $+j\frac{1}{\sqrt{2}}$ A (d) $+j2$ A [Gate 2011]

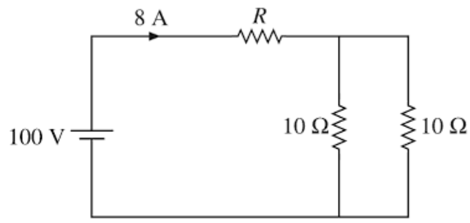
51. In the circuit given below, the value of R required for the transfer of maximum power to the load having a resistance of 3 Ω is



(a) 0 Ω (b) 3 Ω

(c) 6 Ω (d) ∞ Ω [Gate 2011]

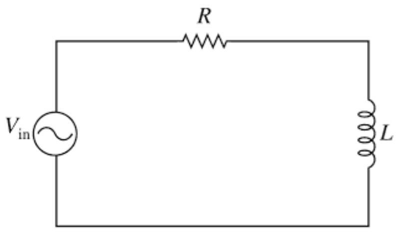
52. In figure, the value of R is



(a) 2.5 Ω (b) 5.0 Ω

(c) 7.5 Ω (d) 10.0 Ω [Gate 2005]

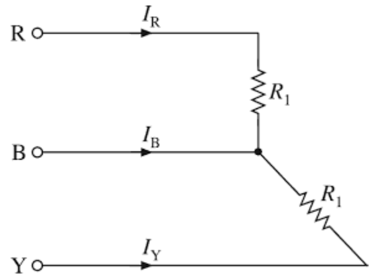
53. The RL circuit of figure is fed from a constant magnitude, variable frequency sinusoidal voltage source V_{in}. At 100 Hz, the R and L elements each have a voltage drop V_{rms}. If the frequency of the source is changed to 50 Hz, the new voltage drop across R is



(a) $\sqrt{\frac{5}{8}}V_{\text{rms}}$ (b) $\sqrt{\frac{2}{3}}V_{\text{rms}}$

(c) $\sqrt{\frac{8}{5}}V_{\text{rms}}$ (d) $\sqrt{\frac{3}{2}}V_{\text{rms}}$ [Gate 2005]

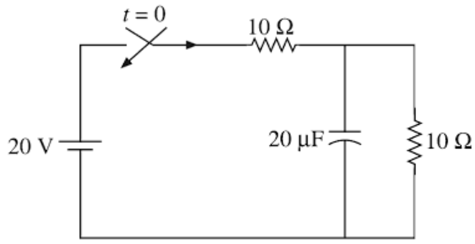
54. For the three phase circuit shown in figure, the ratio of the currents $I_R : I_Y : I_B$ is given by



(a) $1:1:\sqrt{3}$ (b) $1:1:2$

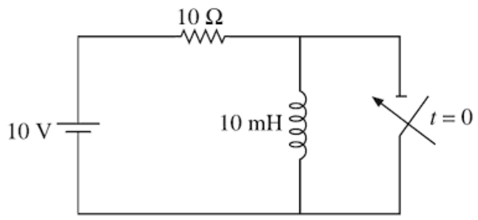
(c) $1:1:0$ (d) $1:1:\sqrt{\frac{3}{2}}$ [Gate 2005]

55. In figure given, for the initial capacitor voltage is zero. The switch is closed at $t = 0$. The final steady state voltage across the capacitor is



(a) 20 V (b) 10 V
(c) 5 V (d) 0 V [Gate 2005]

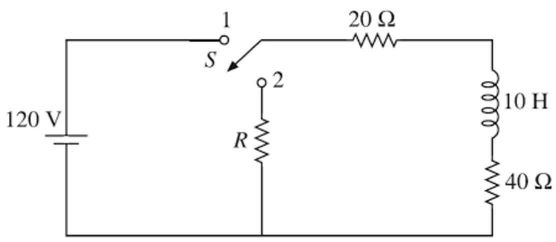
56. The circuit shown in figure is in steady state, when the switch is closed at $t = 0$. Assuming that the inductance is ideal, the current through the inductor at $t = 0^+$ equals



(a) 0 A (b) 0.5 A
(c) 1 A (d) 2 A [Gate 2005]

Statement for Linked Answer Questions: 57 and 58

A coil of inductance 10 H resistance 40 Ω is connected as shown in figure. After the switch S has been in contact with point 1 for a very long time, it is moved to point 2 at $t = 0$.



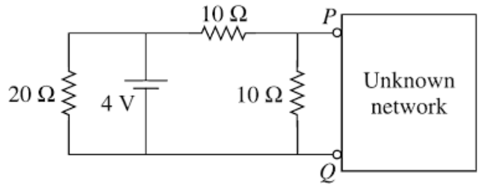
57. If at $t = 0^+$, the voltage across the coil is 120 V, the value of resistance R is

(a) 0 Ω (b) 20 Ω
(c) 40 Ω (d) 60 Ω [Gate 2005]

58. For the value of R obtained in Q.57, the time taken for 95% of the stored energy to be dissipated is close to

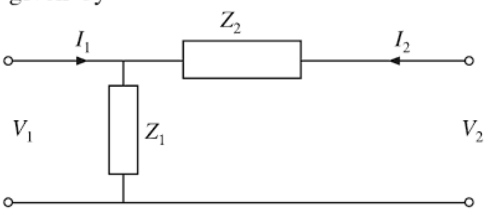
(a) 0.10 s (b) 0.15 s
(c) 0.50 s (d) 1.0 s [Gate 2005]

59. In the given figure, the Thevenin's equivalent pair (voltage, impedance), as seen at the terminals P and Q, is given by



(a) (2 V, 5 Ω) (b) (2 V, 7.5 Ω)
(c) (4 V, 5 Ω) (d) (4 V, 7.5 Ω)
[Gate 2005]

60. For the 2-port network shown in figure, the Z matrix is given by

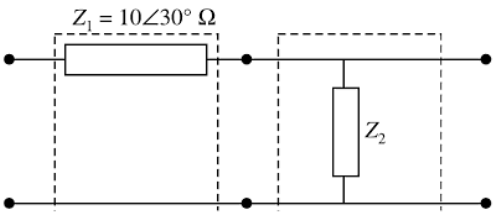


(a) $\begin{bmatrix} Z_1 & Z_1+Z_2 \\ Z_1+Z_2 & Z_2 \end{bmatrix}$ (b) $\begin{bmatrix} Z_1 & Z_1 \\ Z_1+Z_2 & Z_2 \end{bmatrix}$

(c) $\begin{bmatrix} Z_1 & Z_2 \\ Z_2 & Z_1+Z_2 \end{bmatrix}$ (d) $\begin{bmatrix} Z_1 & Z_1 \\ Z_1 & Z_1+Z_2 \end{bmatrix}$

[Gate 2005]

61. Two networks are connected in cascade as shown in figure. With the usual notations the equivalent A, B, C and D constants are obtained. Given that, $C = 0.025\angle 45°$, the value of Z_2 is



(a) $10\angle 30°$ Ω (b) $40\angle -45°$ Ω
(c) 1 Ω (d) 0 Ω [Gate 2005]

62. The rms value of the voltage $u(t) = 3 + 4\cos(3t)$ is

(a) $\sqrt{17}$ V (b) 5 V
(c) 7 V (d) $(3+2\sqrt{2})$ V
[Gate 2005]

63. Current I_1, I_2 and I_3 meet at a junction (node) in a circuit. All currents are marked as entering the node. If $I_1 = -6\sin(\omega t)$ mA and $I_1 = 8\cos(\omega t)$ mA, then I_3 will be

(a) $10\cos(\omega t + 36.87°)$ mA
(b) $14\cos(\omega t + 36.87°)$ mA
(c) $-14\sin(\omega t + 36.87°)$ mA
(d) $-10\cos(\omega t + 36.87°)$ mA [Gate 1999]

64. Given two coupled inductors L_1 and L_2, their mutual inductance M satisfies

(a) $M = \sqrt{L_1^2 + L_2^2}$ (b) $M > \dfrac{L_1 + L_2}{2}$
(c) $M > \sqrt{L_1 L_2}$ (d) $M \le \sqrt{L_1 L_2}$ [Gate 2001]

65. A voltage waveform $V(t) = 12t^2$ is applied across a 1H inductor for $t \ge 0$, with initial current through it being zero. The current through the inductor for $t \ge 0$ is given by

(a) $12t$ (b) $24t$
(c) $12t^3$ (d) $4t^3$ [Gate 2000]

66. The graph of an electrical network has N nodes and B branches. The number of links L, with respect to the choice of a tree, is given by

(a) $B - N + 1$ (b) $B + N$
(c) $N - B + 1$ (d) $N - 2B - 1$ [Gate 2002]

67. Superposition principle is not applicable to a network containing time–varying resistors. (True/False) [Gate 1994]

68. If a 2-port network is reciprocal, then we have, with the usual notation, the following relationship

(a) $h_{12} = h_{21}$ (b) $h_{12} = -h_{21}$
(c) $h_{11} = h_{22}$ (d) $h_{11}h_{22} - h_{12}h_{21} = 1$ [Gate 1994]

69. A 2-port device is defined by the following pair of equations:

$$I_1 = 2V_1 + V_2 \quad \text{and} \quad I_2 = V_1 + V_2$$

Its impedance parameters $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ are given by

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & +1 \\ -1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ [Gate 2000]

70. A series RLC circuit has the following parameter values:

$$R = 10\ \Omega,\ L = 0.01\ \text{H},\ C = 100\ \text{mF}$$

The Q-factor of the circuit at resonance is ________ [Gate 1995]

71. A circuit with a resistor, inductor and capacitor in series is resonant at f_0 Hz. If all the component values are now doubled, the new resonant frequency is

(a) $2f_0$ (b) Still f_0
(c) $\dfrac{f_0}{4}$ (d) $\dfrac{f_0}{2}$ [Gate 1998]

72. In a series RLC circuit at resonance, the magnitude of the voltage developed across the capacitor

(a) Is always zero
(b) Can never be greater than the input voltage
(c) Can be greater than the input voltage, however it is 90° out of phase with the input voltage
(d) Can be greater than the input voltage, and is in phase with the input voltage [Gate 2001]

73. A series RLC circuit has $R = 50\ \Omega$; $L = 100\ \mu$H and $C = 1\ \mu$F. The lower half power frequency of the circuit is

(a) 30.55 kHz (b) 3.055 kHz
(c) 51.92 kHz (d) 1.92 kHz [Gate 2002]

74. A coil (which can be modelled as a series RL circuit) has been designed for high Q-performance at a rated voltage and a specified frequency. If the frequency of operation is doubled and the coil operated at the same rated voltage then the Q-factor and the active power P consumed by the coil will be affected as follows.

(a) P is doubled, Q is halved
(b) P is halved, Q is doubled
(c) P remains constant, Q is doubled
(d) P is decreased four times, Q is doubled [Gate 1996]

75. A major advantage of active filters is that they can be realized without using

(a) Op–amps (b) Inductors
(c) Resistors (d) Capacitors [Gate 1997]

76. A practical current source is usually represented by

(a) A resistance in series with an ideal current source
(b) A resistance in parallel with an ideal current source
(c) A resistance in parallel with an ideal voltage source
(d) None of the above [Gate 1997]

77. An ideal voltage source will charge an ideal capacitor

(a) In infinite time (b) Exponentially
(c) Instantaneously (d) None of these [Gate 1997]

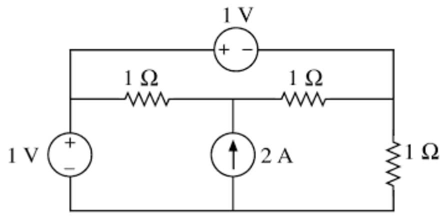
78. Energy stored in capacitor over a cycle, when excited by an ac source is

(a) The same as that due to a dc source of equivalent magnitude
(b) Half of that due to a dc source of equivalent magnitude
(c) Zero
(d) None of the above [Gate 1997]

79. A passive 2-port network is in a steady state. Compared to its input, the steady state output can never offer
(a) Higher voltage (b) Lower impedance
(c) Greater power (d) Better regulation
[Gate 2001]

80. Two incandescent light bulbs of 40 W and 60 W ratings are connected in series across the mains. Then
(a) The bulbs together consume 100 W
(b) The bulbs together consume 50 W
(c) The 60 W bulb glows brighter
(d) The 40 W bulb glows brighter [Gate 2001]

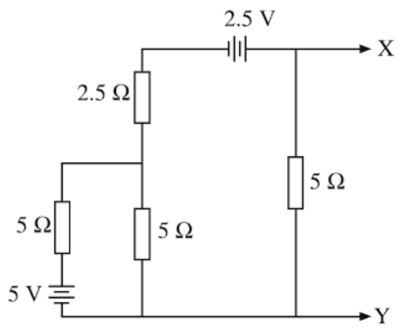
81. The power delivered by the current source, in figure, is __________.



[Gate 2014 set 3]

82. A non ideal voltage source V_s has an internal impedance of Z_s. If a purely resistive load is to be chosen that maximizes the power transferred to the load, its value must be
(a) 0
(b) Real part of Z_s
(c) Magnitude of Z_s
(d) Complex part of Z_s [Gate 2014 set 3]

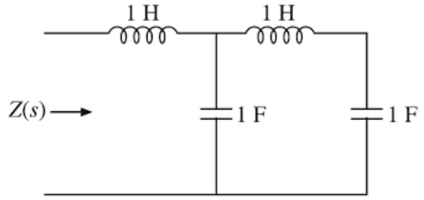
83. The Norton's equivalent source in amperes as seen into the terminals X and Y is ____________.



[Gate 2014 set 3]

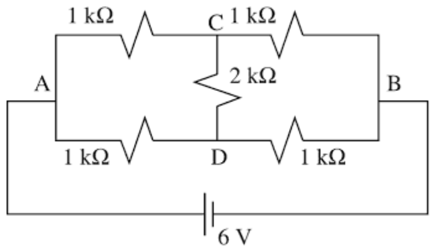
84. A series RLC circuit is observed at two frequencies. At ω_1 = 1 k rad/s, we note that source voltage $V_1 = 100\angle 0°$ V results in current $I_1 = 0.03\angle 31°$ A. At ω_2 = 2 k rad/s, the source voltage $V_2 = 100\angle 0°$ V results in a current $I_2 = 2\angle 0°$ A. The closest values for R, L, C out of the following options are
(a) R = 50 Ω; L = 25 mH; C = 10 μF
(b) R = 50 Ω; L = 10 mH; C = 25 μF
(c) R = 50 Ω; L = 50 mH; C = 5 μF
(d) R = 50 Ω; L = 5 mH; C = 50 μF
[Gate 2014 set 3]

85. The driving point impedance $Z(s)$ for the circuit shown below is



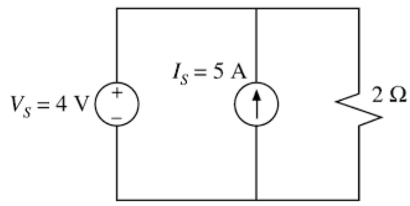
(a) $\dfrac{s^4+3s^2+1}{s^3+2s}$ (b) $\dfrac{s^4+2s^2+4}{s^2+2}$
(c) $\dfrac{s^2+1}{s^4+s^2+1}$ (d) $\dfrac{s^3+1}{s^4+s^2+1}$
[Gate 2014 set 3]

86. The current through the 2 kΩ resistance in the circuit shown is



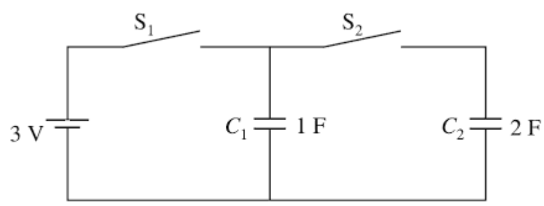
(a) 0 mA (b) 1 mA
(c) 2 mA (d) 6 mA [Gate 2009]

87. For the circuit shown, find out the current flowing through the 2 Ω resistance. Also identify the changes to be made to double the current through the 2 Ω resistance.



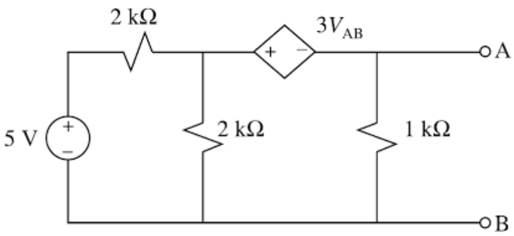
(a) (5 A, put V_S = 20 V)
(b) (2 A, put V_S = 8 V)
(c) (5 A, put I_S = 10 A)
(d) (7 A, put I_S = 12 A) [Gate 2009]

88. In figure shown, all elements used are ideal. For time $t < 0$, S_1 remained closed and S_2 open. At $t = 0$, S_1 is opened and S_2 is closed. If the voltage V_{C2} across the capacitor C_2 at $t = 0^-$ is zero, the voltage across the capacitor combination at $t = 0^+$ will be



(a) 1 V (b) 2 V
(c) 1.5 V (d) 3 V [Gate 2009]

Statement for Linked Answer Questions: 89 and 90

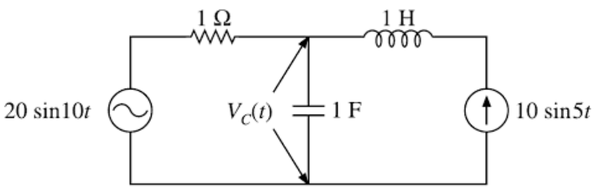


89. For the circuit given above, the Thevenin's resistance across the terminals A and B is
(a) 0.5 kΩ (b) 0.2 kΩ
(c) 1 kΩ (d) 0.11 kΩ [Gate 2009]

90. For the circuit given above, the Thevenin's voltage across the terminals A and B is
(a) 1.25 V (b) 0.25 V
(c) 1 V (d) 0.5 V [Gate 2009]

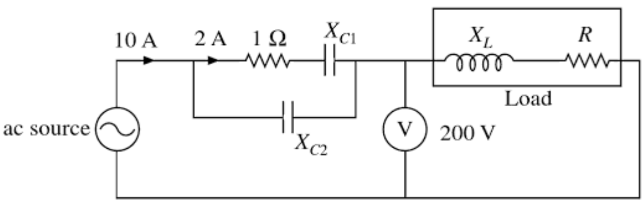
91. How many 200 W/220 V incandescent lamps connected in series would consume the same total power as a single 100 W/220 V incandescent lamp?
(a) Not possible (b) 4
(c) 3 (d) 2 [Gate 2009]

92. The voltage across the capacitor, as shown in figure, is expressed as $V_C(t) = A_1 \sin(\omega_1 t - \theta_1) + A_2 \sin(\omega_2 t - \theta_2)$



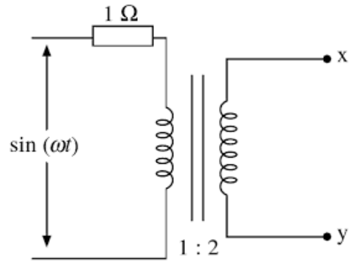
The value of A_1 and A_2 respectively, are
(a) 2.0 and 1.98 (b) 2.0 and 4.20
(c) 2.5 and 3.50 (d) 5.0 and 6.40
[Gate 2014 set 2]

93. The total power dissipated in the circuit, shown in figure, is 1 kW.



The voltmeter, across the load, reads 200 V. The value of X_L is __________. [Gate 2014 set 2]

94. Assuming an ideal transformer, the Thevenin's equivalent voltage and impedance as seen from the terminals x and y for the circuit in figure are



(a) $2\sin(\omega t)$, 4 Ω (b) $1\sin(\omega t)$, 1 Ω
(c) $1\sin(\omega t)$, 2 Ω (d) $2\sin(\omega t)$, 0.5 Ω
[Gate 2014 set 2]

95. Two identical coupled inductors are connected in series. The measured inductances for the two possible series connections are 380 μH and 240 μH. Their mutual inductance in μH is __________. [Gate 2014 set 2]

Solutions

1. Ans: (c)
Hint: Consider upper part of the circuit.
Now, applying current division rule

$$I_L = \frac{1}{1+j} \times 1\angle 0 = \frac{1}{1+j}\ \text{A}$$

2. Ans: (d)
Hint: $I_1 = I_m \sin(\omega t - \phi_1)$

$I_2 = I_m \cos(\omega t - \phi_2)$

$$= I_m \sin\left(\frac{\pi}{2} + \omega t - \phi_2\right)$$

The two currents are balanced.
So,

$$I_1 + I_2 = 0$$

or, $$I_m \sin(\omega t - \phi_1) + I_m \sin\left(\frac{\pi}{2} + \omega t - \phi_2\right) = 0$$

$$2\sin\frac{1}{2}\left(2\omega t - (\phi_1 + \phi_2) + \frac{\pi}{2}\right)\cos\frac{1}{2}\left(\frac{\pi}{2} - \phi_2 + \phi_1\right) = 0$$

or, $$\frac{1}{2}\left(\frac{\pi}{2} - \phi_2 + \phi_1\right) = \frac{\pi}{2}$$

$\therefore$ $$\phi_1 = \frac{\pi}{2} + \phi_2$$

3. Ans: (b)
Hint: $I = 5\cos(100\pi t + 100°)$ A

$\therefore$ $$I = \frac{5}{\sqrt{2}}\angle 100°\ \text{A}$$

$$Z = (4 - j3)\Omega = 5\angle -36.87°\ \Omega$$

$$V = IZ = \frac{25}{\sqrt{2}}\angle 63.13°\ \text{V}$$

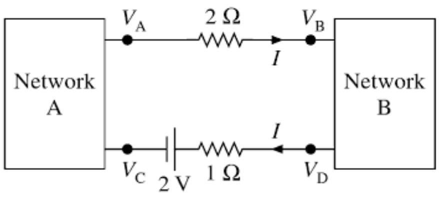
The average power is

$$P = \frac{1}{2} VI \cos\phi$$

$$= \frac{1}{2} \times 25 \times 5 \times \cos(36.87°) = 50\,\text{W}$$

4. Ans: (a)

Hint:



$$V_A - V_B = 2I$$

or $\quad 2I = 6$

or $\quad I = 3$ A

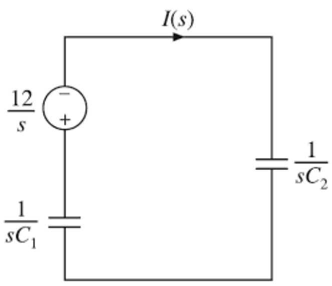
$$V_C + 2 + 1 \times 1 = V_D$$

$$V_C - V_D = -2 - 3 = -5\ \text{V}$$

5. Ans: (d)

Hint:

Circuit is s–domain



Applying KVL

$$\frac{12}{s} + I(s)\left(\frac{1}{sC_1} + \frac{1}{sC_2}\right) = 0$$

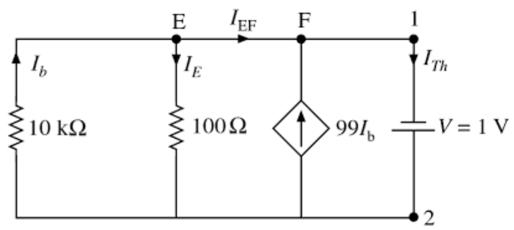
or, $\quad I(s) = -\dfrac{12C_1C_2}{C_1 + C_2} = K \quad \text{(constant)}$

or, $\quad I(t) = K\delta(t)$

$\therefore$ Current $I(t)$ is an impulse function.

6. Ans: (a)

Hint:



Connect a 1 V source across nodes 1 and 2 and find the current through voltage source.

Then, $\quad Z_{Th} = \dfrac{1}{I_{Th}}$

By applying KCL at nodes E and F, we get

$$I_b = I_E + I_{EF}$$

$$I_{EF} + 99I_b = I_{Th}$$

$\Rightarrow \quad I_b - I_E + 99I_b = I_{Th}$

$\Rightarrow \quad 100I_b - I_E = I_{Th} \quad$ (i)

Applying KVL in outer loop

$$10 \times 10^3 I_b = 1$$

$\Rightarrow \quad I_b = 10^{-4}$ A

And $\quad 10 \times 10^3 I_b = -100 I_E$

$\Rightarrow \quad I_E = -100\ I_b$

$\therefore$ From Eq. (i)

$$100 I_b + 100 I_b = I_{Th}$$

$\Rightarrow \quad I_{Th} = 200I_b$

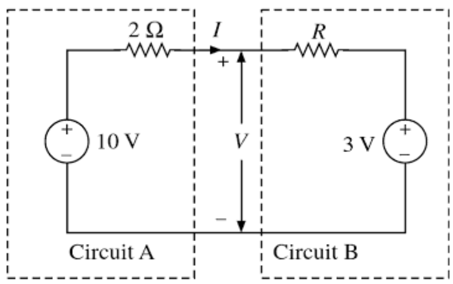
$$= 200 \times 10^{-4} = 0.02$$

$\therefore \quad Z_{Th} = \dfrac{1}{I_{Th}} = \dfrac{1}{0.02} = 50\ \Omega$

7. Ans: (a)

Hint:

Thevenin's equivalent circuit



Now, $\quad I = \dfrac{7}{R+2}$

and $\quad V = 10 - 2I$

$$= 10 - \frac{14}{R+2}$$

$$= \frac{10R+6}{R+2}$$

Power transferred from circuit A to circuit B

$$P = VI$$

$$= \frac{10R+6}{R+2} \times \frac{7}{R+2}$$

For P to be maximum $\quad \dfrac{dP}{dR} = 0$

$$(R+2)^2(10) - (10R+6) \times 2(R+2) = 0$$

$\Rightarrow \quad R = 0.8\ \Omega$

8. Ans: (b)

Hint:

(i) $V_1 = 10$ V

$$V_2 = 3 \text{ V}$$
$$I_2 = -3 \text{ A}$$
$$V_1 = AV_2 - BI_2$$
$$10 = 3A + 3B \quad \text{(i)}$$

(ii) $V_2 = 5$ V

$$I_2 = -2 \text{ A}$$
$$10 = 5A + 2B \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$A = \frac{10}{9} \quad \text{(iii)}$$

$$B = \frac{20}{9} \quad \text{(iv)}$$

Given $V_1 = 8$ V

$$(V_2)_{OC} = ?$$
$$I_2 = 0$$
$$V_1 = AV_2 - BI_2$$
$$\therefore \quad 8 = A(V_2)_{OC} - 0$$
$$\Rightarrow \quad (V_2)_{OC} = \frac{8}{A} = 7.2 \text{ V}$$

9. Ans: (c)

Hint:

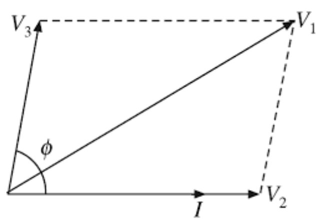
Given $V_1 = 10$ V

$$V_2 = (-7I_2)$$
$$V_1 = AV_2 - BI_2$$
$$10 = -7I_2A - BI_2$$
$$10 = -\frac{70}{9}I_2 - \frac{20}{9}I_2$$

$I_2 = -1$ A (current is drawn)

10. Ans: (a)

Hint:



$$V_1^2 = V_2^2 + V_2^2 + 2V_2V_3\cos\phi$$

or $(220)^2 = (122)^2 + (136)^2 + 2 \times 122 \times 136 \times \cos\phi$

or $\cos\phi = 0.45$

11. Ans: (b)

Hint:

Given $R_L = 5\ \Omega$

$$\therefore \quad \cos\phi = \frac{R_L}{|Z|}$$

$$\text{or} \quad |Z| = \frac{5}{0.45} = 11.11$$

Power consumed by load

$$P_L = I^2 R_L$$
$$= \left(\frac{V_3}{|Z|}\right)^2 R_L$$
$$= \left(\frac{136}{11.11}\right)^2 \times 5 = 749.1 \approx 750$$

12. Ans: (b)

Hint: A resistor has linear characteristics

i.e. $V = RI$

or $V = I$ [As $R = 1\ \Omega$]

Load line

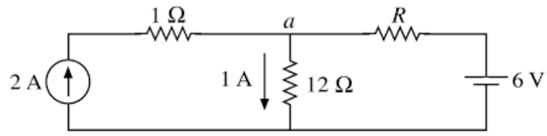
$$V + I = 100$$
$$I + I = 100$$

Current through resistance

$$I = \frac{100}{2} = 50 \text{ A}$$

13. Ans: (b)

Hint:



Voltage at node a

$$V = 1 \times 12 = 12 \text{ V}$$

Now, current through resistance R is

$$I = 1 \text{ A}$$

Again,

$$I = \frac{V-6}{R} = \frac{12-6}{R} = \frac{6}{R}$$

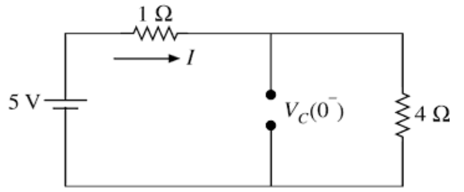
$$\text{or} \quad 1 = \frac{6}{R}$$

$$\text{or} \quad R = 6\ \Omega$$

14. Ans: (b)

Hint: The switch has been closed for a long time. So, the circuit is in steady state.

At steady state, capacitor is open circuited.



Circuit at $t = 0^-$

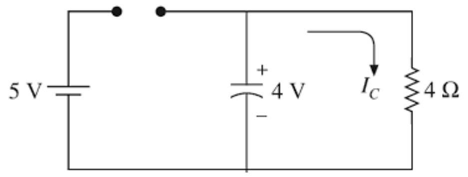
Using KVL

$$I = \frac{5}{5} = 1 \text{ A}$$

$\therefore \quad V_C(0^-) = 4 \times 1 = 4$ V

So, $\quad V_C(0^+) = V_C(0^-) = 4$ V

Circuit at $t = 0^+$



Current through capacitor at $t = 0^+$

$$I_C(0^+) = \frac{4}{4} = 1 \text{ A}$$

15. Ans: (d)

Hint: Given

$$I_L(0^-) = 0 \text{ A}$$

$$V_C(0^-) = 100 \text{ V}$$

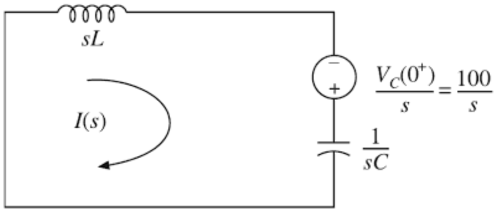
Now, current through inductor and voltage across capacitor cannot change instantaneously.

So, after closing the switch,

$$I_L(0^+) = I_L(0^-) = 0$$

And $\quad V_C(0^+) = V_C(0^-) = 100$ V

The circuit in s-domain



$$I(s) = \frac{100/s}{\left(sL + \frac{1}{sC}\right)}$$

$$= 100\sqrt{\frac{C}{L}}\left(\frac{\frac{1}{\sqrt{LC}}}{s^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}\right)$$

Taking inverse Laplace transform

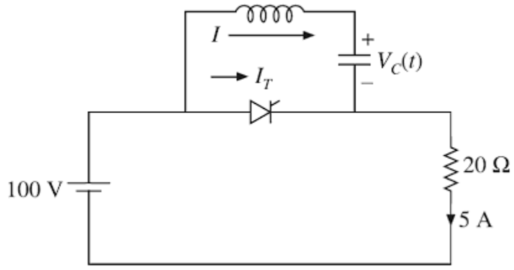
$$I(t) = 100\sqrt{\frac{C}{L}} \sin\frac{1}{\sqrt{LC}} t$$

$$= 100 \times \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} \times \sin\left(\frac{1}{\sqrt{1 \times 10^{-3} \times 10 \times 10^{-6}}} t\right)$$

$$I(t) = 10 \sin(10^4 t) \text{ A}$$

16. Ans: (a)

Hint:



Load current = I_L = 5 A

Net current through thyristor

$$I_T = I_L - I$$

or $\quad I_T = 5 - 10 \sin 10^4 t$

Let at $t = T$, circuit get turned off and current I_T becomes zero

$$I_T = 5 - 10 \sin 10^4 T = 0$$

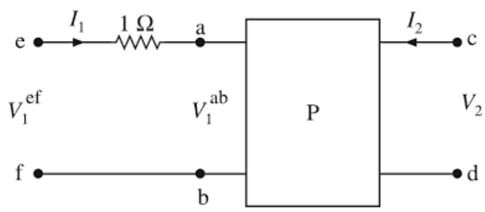
or $\quad 10 \sin 10^4 T = 5$

or $\quad \sin 10^4 T = 0.5$

or $\quad T = 52$ μs

17. Ans: (c)

Hint:



$$V_1^{ab} = Z_{11}I_1 + Z_{12}I_2 \qquad \text{(i)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \qquad \text{(ii)}$$

Now, 1 Ω resistor is connected in series with the network at port 1

Here, V_2 remains same.

$$V_1^{ef} = V_1^{ab} + I_1 \times 1$$

$$= Z_{11}I_1 + Z_{12}I_2 + I_1$$

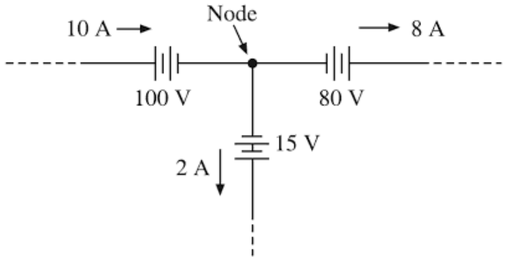
$$= (Z_{11} + 1)I_1 + Z_{12}I_2$$

Modified Z parameter

$$= \begin{bmatrix} Z_{11}+1 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

18. Ans: (330)

Hint:



Applying KCL at node, current through 15 V voltage source = 2 A

$\therefore$ Power absorbed by 100 V voltage source

$$= 10 \times 100 = 1000 \text{ W}$$

Power absorbed by 80 V voltage source

$$= -(80 \times 8) = -640 \text{ W}$$

and power absorbed by 15 V voltage source

$$= -(15 \times 2) = -30 \text{ W}$$

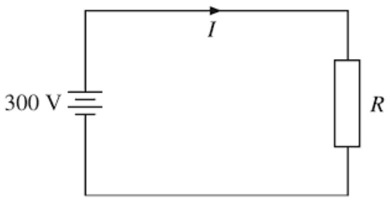
$\therefore$ Total power absorbed by the three circuit elements

$$= -(1000 - 640 - 30) \text{ W}$$
$$= 330 \text{ W}$$

19. Ans: (10)

Hint: Given, $R = \left(25 + \frac{I}{2}\right)\Omega$

or $I = (2R - 50)$



Applying KVL, $IR = 300$

or, $(2R - 50) \times R = 300$

or, $R = 30$ or $R = -5$

Since resistance can't be negative.

So, $R = 30\ \Omega$

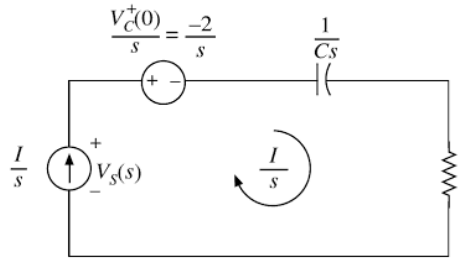
Hence, $I = (2R - 50) = 10$ A

$\therefore$ Current, $I = 10$ A

20. Ans: (c)

Hint: Given, $C = 1\ \mu$F, $V_C(0) = -2$ V, $R = 100\ \Omega$, $I = 20$ mA

Circuit for the given condition at time $t > 0$ is shown below:



Applying KVL, we have:

$$V_S(s) = \left(\frac{-2}{s}\right) + \frac{I}{s}\left(R + \frac{I}{Cs}\right)$$

$$\Rightarrow \quad V_S(s) = \frac{1}{s}\left[(IR - 2) + \frac{I}{Cs}\right]$$

$$\Rightarrow \quad V_S(s) = \frac{1}{s}\left[(20 \times 10^{-3} \times 200 - 2) + \left(\frac{20 \times 10^{-3}}{10^{-6}}\right) \times \frac{1}{s}\right]$$

$$\Rightarrow \quad V(s) = \frac{20 \times 10^3}{s^2}$$

$$\therefore \quad V_S(s) = \frac{20 \times 10^3}{s^2}$$

$$\therefore \quad V_S(t) = 20000t\ u(t)$$

This is the equation of a straight line passing through origin.

21. Ans: (2470 to 2471)

Hint: Let the resistance of incandescent lamp,

$$R = \frac{240^2}{40} = 1440\ \Omega$$

Given, $R_0 = 120\ \Omega$, $\alpha = 4.5 \times 10^{-3}/°$C

Let R be the resistance of the filament in ON state at temperature T.

Then, $R = R_0[1 + \alpha\Delta T]$

$$\Rightarrow [1 + \alpha\Delta T] = \frac{R}{R_0} = \frac{1440}{120} = 12$$

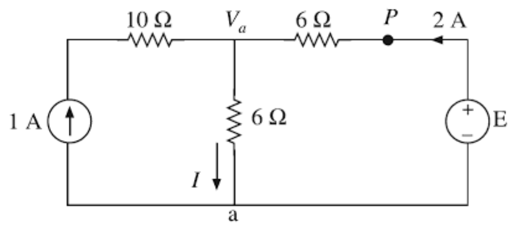
$$\Rightarrow \Delta T = 2444.44\ °\text{C}$$

$$\Rightarrow T = 2444.44° + 26° = 2470.44\ °\text{C}$$

Therefore, ON state temperature of filament = 2470.44 °C

22. Ans: (c)

Hint:



Applying KCL, we get

$$\frac{V_a - E}{6} + \frac{V_a}{6} - 1 = 0$$

or $\qquad 2V_a - E = 6 \qquad$ (i)

Again,

$$\frac{E - V_a}{6} = 2$$

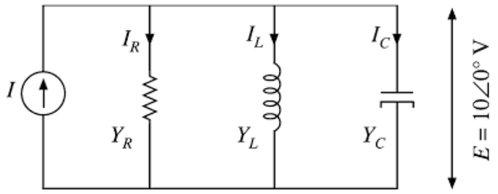
or $\qquad E - V_a = 12 \qquad$ (ii)

Solving Eqs. (i) and (ii), we get

$$V_a = 18 \text{ V and } E = 30 \text{ V}$$

23. Ans: (d)

Hint:



$$I_R = Y_R E = (0.5 + j0) \times 10\angle 0° = 5 \text{ A}$$

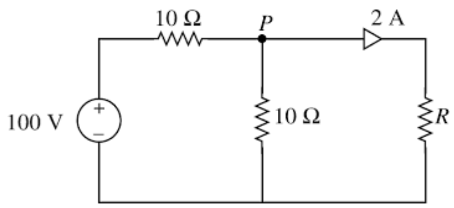
$$I_Y = Y_L E = (0 - j1.5) \times 10\angle 0° = -j15 \text{ A}$$

$$I_C = Y_C E = (0 + j0.3) \times 10\angle 0° = j3 \text{ A}$$

$\therefore \qquad I = 5 + (-j15) + j3 = (5 - j12)$ A

24. Ans: (b)

Hint:



Applying KCL at node P, we get

$$\frac{V_P - 100}{10} + \frac{V_P}{10} + 2 = 0$$

$\Rightarrow \qquad V_P = 40$ V

$\therefore \qquad R = \frac{V_P}{2} = \frac{40}{2} = 20\ \Omega$

25. Ans: (a)

Hint: Applying KVL, we get

$$-100 + R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$100C = RC\frac{dq}{dt} + q$$

$$\int_{q_0}^{q} \frac{dq}{100C - q} = \frac{1}{RC}\int_0^t dt$$

$$100C - q = (100C - q_0)e^{-t/RC}$$

$$I = \frac{dq}{dt} = \frac{(100C - q_0)}{RC}e^{-t/RC}$$

$$= \frac{(100 \times 0.5 - 10)}{2 \times 0.5}e^{-1/(2\times 0.5)} = 14.7 \text{ A}$$

26. Ans: (d)

Hint: $[Z] = \begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$

$$[Y] = [Z]^{-1} = \frac{\begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.9 \end{bmatrix}}{[0.9 \times 0.6 - 0.02 \times 0.02]}$$

$$= \begin{bmatrix} 1.2 & -0.4 \\ -0.4 & 1.8 \end{bmatrix}$$

$\therefore \quad Y_{22} = 1.8$

27. Ans: (d)

Hint: Z should be capacitive because at resonance, the power factor of the circuit should be unity.

Now, admittance of the parallel circuit

$$Y = \frac{1}{j\omega L} + j\omega C = 0$$

$$\therefore \quad C = \frac{1}{L \times \omega^2} = \frac{1}{2 \times (2\pi \times 500)^2} = 0.05\ \mu\text{F}$$

28. Ans: (a)

Hint:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 10}{20 + 10 + 10} = 2.5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{10 \times 20}{20 + 10 + 10} = 5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{20 \times 10}{20 + 10 + 10} = 5\ \Omega$$

29. Ans: (d)

Hint: The rms value of dc current $I_{dc} = 10$ A

The rms value of sinusoidal current $= I_{ac} = \left(\frac{20}{\sqrt{2}}\right)$ A

The rms value of resultant current, $I_T = \sqrt{I_{dc}^2 + I_{ac}^2}$

$$= \sqrt{10^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 17.32 \text{ A}$$

30. Ans: (b)

Hint: Thevenin's impedance

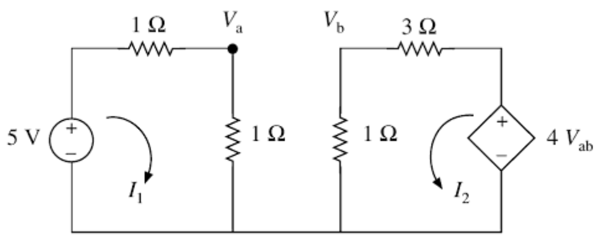
$$Z_0 = 2.38 - j0.667\ \Omega$$

$\text{Re}[Z_0] = 2.38\ \Omega$. So, Z_0 has resistor
$\text{Im}[Z_0] = -j0.667\ \Omega$. As $\text{Im}[Z_0]$ is negative and for minimal realization
Z_0 will have a resistor and a capacitor.

31. Ans: (a)
Hint: $i = I$ A
Applying KVL

$$V_{ab} = -5 + 2 \times 1 = -3 \text{ V}$$

32. Ans: (b)
Hint:



In left loop, applying KVL

$$-5 + I_1 + I_1 = 0$$

or $\quad I_1 = 2.5$ A

In right loop, applying KVL

$$-4V_{ab} + 3I_2 + I_2 = 0$$

or $\quad I_2 = V_{ab}$

∴ $\quad V_a = 2.5$ V

∴ $\quad V_b = 1 \times I_2 = V_{ab}$

Now,

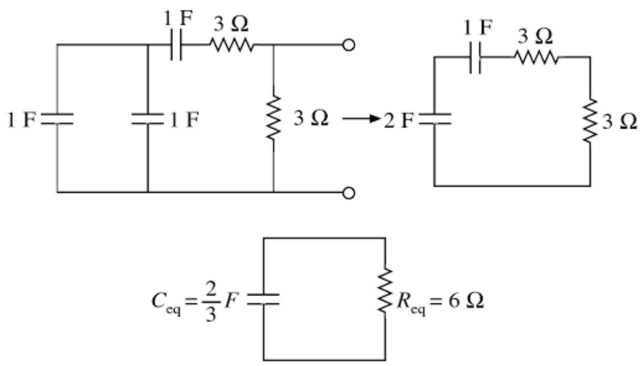
$$V_{ab} = V_a - V_b$$

or $\quad V_b = V_a - V_b$

or $\quad V_b = \dfrac{V_a}{2} = \dfrac{2.5}{2} = 1.25 \text{ V}$

or $\quad I = I_2 = V_{ab} = V_b = 1.25 \text{ A}$

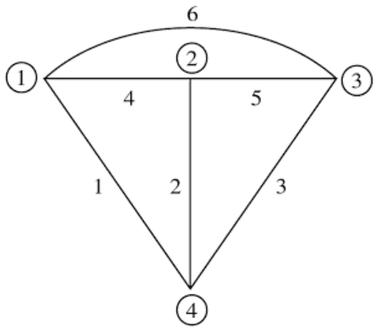
33. Ans: (c)
Hint: For finding time constant, we consider current source as an open circuit
∴ Circuit becomes



∴ Time constant $= R_{eq}C_{eq}$

$$= 6 \times \frac{2}{3} = 4 \text{ s}$$

34. Ans: (a)
Hint:

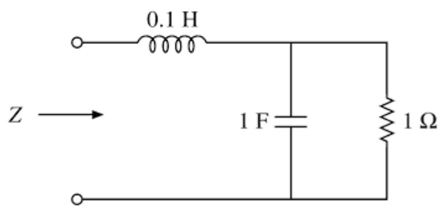


Number of branches $B = 6$
Number of nodes $N = 4$
So, number of chords $[B - (N - 1)] = 3$

35. Ans: (c)
Hint: Input impedance,

$$Z = j\omega L + \frac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\Rightarrow \quad Z = j\omega L + \frac{R}{1 + j\omega RC}$$



$$\therefore \quad Z = j0.1\omega + \frac{1}{1 + j\omega}$$

or $\quad Z = j0.1\omega + \dfrac{1}{1 + j\omega} \times \dfrac{1 - j\omega}{1 + j\omega}$

$$= \frac{1}{1 + \omega^2} + j\left(0.1\omega - \frac{\omega}{1 + \omega^2}\right)$$

At resonance, imaginary part should be zero.

$$0.1\omega - \frac{\omega}{1 + \omega^2} = 0$$

$$0.1 = \frac{1}{1 + \omega^2}$$

$$1 + \omega^2 = 10$$

∴ $\quad \omega^2 = 9$

∴ $\quad \omega = 3$ rad/s

36. Ans: (b)

Hint: Applying KCL at the junction, we get

$$2+1+I_C+I_L=0$$

Here, $I_C = C\dfrac{dV_C}{dt} = 1\times\dfrac{d}{dt}(4\sin 2t) = 8\cos 2t$

$\therefore \qquad I_L = -3-8\cos 2t$

$\therefore \quad V_L = L\dfrac{dI_L}{dt} = 2\times\dfrac{d}{dt}(-3-8\cos 2t) = 32\sin 2t$

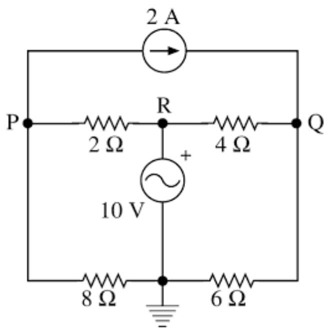
37. Ans: (c)

Hint: Given, V_R = 10 V

Applying KCL at nodes P and Q, we get

$$\frac{V_P-10}{2}+2+\frac{V_P}{8}=0 \qquad \text{(i)}$$

$$\frac{V_Q-10}{4}-2+\frac{V_Q}{6}=0 \qquad \text{(ii)}$$



$\therefore$ From Eqs. (i) and (ii)

$$V_P = 4.8$$
$$V_Q = 10.8$$

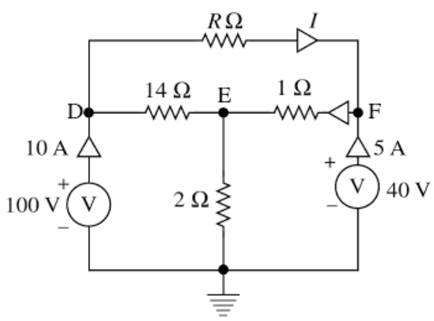
$\therefore \qquad V_P - V_Q = -6$ V

38. Ans: (d)

Hint: Applying KCL at node E, we get

$$\frac{V_E-40}{1}+\frac{V_E-100}{14}+\frac{V_E}{2}=0$$

or $\qquad V_E = 30$ V



Now, potential difference between nodes D and F = 60 V

Applying KCL at node F, we get

$$\frac{40-30}{1}-5-I=0$$

$\Rightarrow \qquad I = 5$ A

$\therefore \qquad R = \dfrac{60}{5} = 12\ \Omega$

39. Ans: (b)

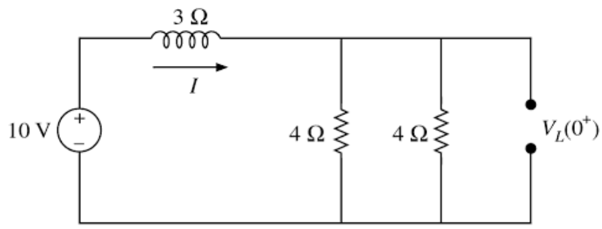
Hint: Here, the circuit was not energized before closing the switch.

Therefore, current through inductor as well as voltage across capacitor are zero.

At $t = 0^+$ (i.e., after closing the switch)

Inductor acts as open circuit and capacitor acts as short circuit.

So the equivalent circuit at $t = 0^+$ is shown below.

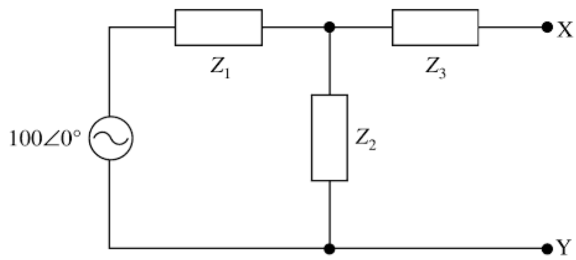


Now, $\qquad I = \dfrac{10}{3+\dfrac{4\times 4}{4+4}} = 2$ A

$\therefore \qquad V_L(0^+) = I\times\dfrac{4\times 4}{4+4} = 4$ V

40. Ans: (a)

Hint: By Thevenin's theorem



$$Z_{Th} = \frac{Z_1\times Z_2}{(Z_1+Z_2)}+Z_3$$

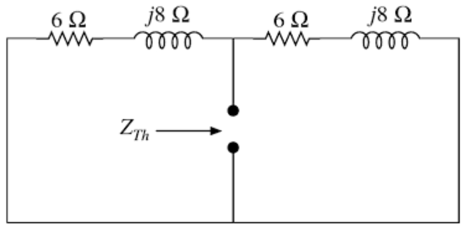
$$= \frac{10\angle-60\times 10\angle 60}{(10\angle-60+10\angle 60)}+(50\angle 53.13)$$

$$= 56.66\angle 45°$$

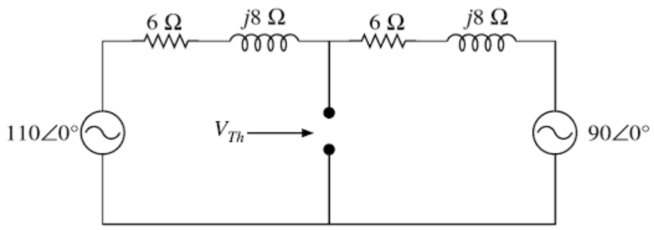
41. Ans: (d)

Hint: For obtaining power absorbed by R_L under maximum power transfer condition, we draw

Thevenin's equivalent circuit across R_L. (by short circuiting the voltage sources)

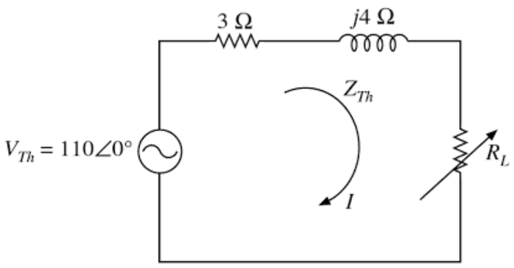


$$Z_{Th} = (6 + j8)\,\|\,(6 + j8) = 3 + j4\Omega$$



$$\frac{V_{Th} - 110\angle 0^\circ}{6 + j8} + \frac{V_{Th} - 90\angle 0^\circ}{6 + j8} = 0$$

$$V_{Th} = 100\angle 0^\circ$$



For maximum power transfer,

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = \sqrt{3^2 + 4^2} = 5\,\Omega$$

$$I = \frac{V_{Th}}{(3 + j4) + R_L} = \frac{100}{8 + j4} = 11.18\angle -26.56^\circ \text{ A}$$

Maximum power absorbed by R_L,

$$I^2 R_L = 11.18^2 \times 5 = 625 \text{ W}$$

42. Ans: (d)

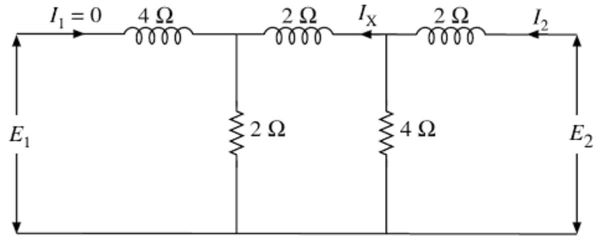
Hint: Here,

$$E_1 = h_{11} I_1 + h_{12} E_2$$

$$\therefore \qquad h_{12} = \left.\frac{E_1}{E_2}\right|_{I_1 = 0}$$

Taking $I_1 = 0$

Now, $$I_2 = \frac{E_2}{2 + \dfrac{4 \times (2 + 2)}{4 + (2 + 2)}} = \frac{E_2}{4}$$



$$I_X = I_2 \times \frac{4}{(2 + 2) + 4} = \frac{I_2}{2} = \frac{E_2}{8}$$

Here, $$E_1 = 2I_X = 2\left(\frac{E_2}{8}\right) = \frac{E_2}{4}$$

or $$\frac{E_1}{E_2} = 0.25$$

43. Ans: (c)

Hint: $$V = V_R + j(V_L - V_C)$$

Given $|V_L| = |V_C|$ and $|V_L| = 2|V_R|$

So, the circuit is at resonance condition and hence $V = V_R$

Now, quality factor $= \dfrac{V_L}{V} = \dfrac{V_L}{V_R} = \dfrac{2V_R}{V_R} = 2$

Again, we know

$$Q = \frac{\omega_0 L}{R}$$

or $$2 = \frac{2\pi f \times L}{5}$$

or $$L = 31.8 \text{ mH}$$

44. Ans: (c)

Hint: In $0 < t < 2$, current varies linearly with time and so $I(t) = 3\,t$ and for $2 < t < 4$, current is constant, $I(t) = 6$ A

The energy absorbed by the inductor (resistance neglected) in the first 2 s,

$$E_L = \int_0^t LI \frac{dI}{dt}\,dt = E_{L1} + E_{L2}$$

$$E_{L1} = \int_0^2 LI\left(\frac{dI}{dt}\right)dt$$

$$= \int_0^2 2 \times 3t \times 3dt = 36 \text{ J}$$

The energy absorbed by the inductor in 2 to 4 s

$$E_{L2} = \int_2^4 LI\left(\frac{dI}{dt}\right)dt$$

$$= \int_2^4 2 \times 6 \times 0\; dt = 0 \text{ J}$$

As pure inductor does not dissipate energy but only stores it. But, some energy is dissipated in the resistor. So, total energy absorbed by the inductor is the sum of energy stored in the inductor and the energy dissipated in the resistor.

The energy dissipated by the resistance in 4 s is

$$E_R = \int_0^t I^2 R dt = \int_0^2 (3t)^2 \times 1dt + \int_2^4 6^2 \times 1dt = 96 \text{ J}$$

The total energy absorbed by the inductor in 4 s

$$= 96\text{ J} + 36\text{ J} = 132\text{ J}$$

45. Ans: (b)

Hint: Source voltage

$$V_S = 1 \sin t$$

So, maximum value

$$V_m = 1 \text{ V} \quad \text{and} \quad \omega = 1 \text{ rad/s}$$

Impedance of the branch containing inductor and capacitor

$$Z = j(X_L - X_C)$$

$$= j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= j\left(1 \times 1 - \frac{1}{1 \times 1}\right) = 0$$

So, this branch acts as short circuit and the total current flows through it

$$\therefore \quad I(t) = \frac{1 \sin t}{1} = 1 \sin t$$

The rms value of current

$$= \frac{1}{\sqrt{2}} \text{ A}$$

46. Ans: (b)

Hint: $I(t) = 10\sqrt{2} \sin\left(100\pi t + \frac{\pi}{4}\right)$

$$= 10\sqrt{2} \sin\left(\frac{\pi}{2} - \frac{\pi}{4} + 100\pi t\right)$$

$$= 10\sqrt{2} \sin\left[\frac{\pi}{2} - \left(\frac{\pi}{4} - 100\pi t\right)\right]$$

$$= 10\sqrt{2} \cos\left(\frac{\pi}{4} - 100\pi t\right)$$

$$= 10\sqrt{2} \cos\left(100\pi t - \frac{\pi}{4}\right)$$

So, current phasor

$$= 10\angle -\frac{\pi}{4}$$

47. Ans: (b)

Hint: Given

$$V_{in} = 100\sqrt{2} \sin(100\pi t) \text{ V}$$

$$I_{in} = 10\sqrt{2} \sin\left(100\pi t - \frac{\pi}{3}\right) + 5\sqrt{2} \sin\left(300\pi t + \frac{\pi}{4}\right) + 2\sqrt{2} \sin\left(500\pi t - \frac{\pi}{4}\right) \text{ A}$$

For input voltage fundamental component is

$$(V_{in})_1 = 100\sqrt{2} \sin(100\pi t)$$

$$\therefore \quad (V_{in})_{1,rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

Fundamental component of current

$$(i_{in})_1 = 10\sqrt{2} \sin\left(100\pi t - \frac{\pi}{3}\right)$$

$$\therefore \quad (i_{in})_{1,rms} = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ A}$$

Phase difference between these two components

$$\theta_1 = \frac{\pi}{3}$$

So, $\cos\theta_1 = 0.5$

Active power due to fundamental components

$$P_1 = (V_{in})_{1,rms} \times (i_{in})_{1,rms} \cos\theta_1$$

$$= 100 \times 10 \times 0.5 = 500 \text{ W}$$

Now, 3^{rd} and 5^{th} harmonics are not present in input voltage.

Hence, there is no active power due to 3^{rd} and 5^{th} harmonics components.

Hence, active power drawn by the converter $P = 500$ W

48. Ans: (b)

Hint: The rms value of input voltage

$$V_{rms} = 100 \text{ V}$$

The rms value of current

$$I_{rms} = \sqrt{\left(\frac{10\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2} = 11.358 \text{ A}$$

Let input power factor be $\cos\theta$

Active power drawn by the converter $= V_{rms} I_{rms} \cos\theta$

or $\quad 100 \times 11.358 \times \cos\theta = 500$ W

or $\quad \cos\theta = 0.44$

49. Ans: (b)

Hint: Now, in the circuit, inductor and capacitor do not consume any power.

Therefore, power dissipated in resistor R = Power supplied by the source

$$\Rightarrow \quad P = V_S I_S \cos\phi$$

$$= 1 \times \sqrt{2} \times \cos\frac{\pi}{4} = 1 \text{ W}$$

50. Ans: (d)

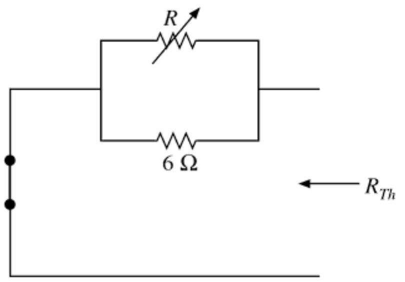
Hint: Applying KCL, we get

$$I_C = I_S - I_{RL}$$

or $$\sqrt{2}\angle\frac{\pi}{4} - \sqrt{2}\angle -\frac{\pi}{4} = j2 \text{ A}$$

51. Ans: (c)

Hint: To calculate R_{Th}, voltage source is short circuited.



Now, $$R_{Th} = \frac{6R}{6+R}$$

According to maximum power transfer theorem

$$R_L = R_{Th}$$

or $$3 = \frac{6R}{6+R}$$

$\therefore$ $$R = 6\ \Omega$$

52. Ans: (c)

Hint: $$R + \frac{10 \times 10}{10+10} = \frac{100}{8}$$

$\therefore$ $$R = 12.5 - 5 = 7.5\ \Omega$$

53. Ans: (c)

Hint: At f = 100 Hz

$$|V_R| = |V_L|$$

$$IR = IX_L = I\omega L$$

$$I = \frac{V_{in}}{\sqrt{R^2 + X_L^2}}$$

$$= \frac{V_{in}}{\sqrt{R^2 + R^2}} = \frac{V_{in}}{\sqrt{2}R}$$

$$V_R = V_{rms} = IR$$

$$= \left(\frac{V_{in}}{\sqrt{2}R}\right) \times R = \frac{V_{in}}{\sqrt{2}}$$

$$\Rightarrow \quad V_{in} = \sqrt{2}V_{rms} \qquad \text{(i)}$$

At f = 50 Hz

$$X_L \propto f$$

So, $$X'_L = X_L \times \frac{50}{100} = \frac{X_L}{2} = \frac{R}{2}$$

$$I' = \frac{V_{in}}{\sqrt{R^2 + (X'_L)^2}}$$

$$= \frac{V_{in}}{\sqrt{R^2 + \left(\frac{R}{2}\right)^2}} = \frac{2V_{in}}{\sqrt{5}R}$$

$$V'_R = I'R = \left(\frac{2V_{in}}{\sqrt{5}R}\right)R = \frac{2}{\sqrt{5}}V_{in}$$

$$= \frac{2}{\sqrt{5}} \times (\sqrt{2}V_{rms}) = \sqrt{\frac{8}{5}}V_{rms}$$

54. Ans: (a)

Hint: We are considering RYB phase sequence. Taking V_{RY} as the reference

$$V_{RY} = V\angle 0^\circ$$

$$V_{YB} = V\angle -120^\circ$$

$$V_{BR} = V\angle -240^\circ$$

$$I_R = \frac{V_{RB}}{R_1} = -\frac{V_{BR}}{R_1}$$

$$= -\frac{V\angle -240^\circ}{R_1} = \frac{V}{R_1}\angle -60^\circ$$

$$I_Y = \frac{V_{YB}}{R_1} = \frac{V\angle -120^\circ}{R_1}$$

Applying KCL,

$$I_R + I_Y + I_B = 0$$

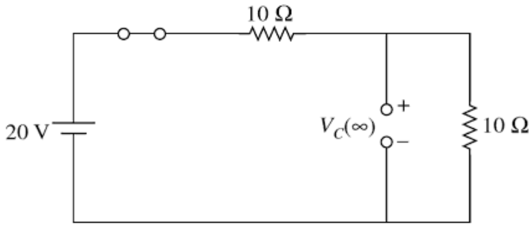
$$\frac{V}{R_1}\angle -60^\circ + \frac{V}{R_1}\angle -120^\circ + I_B = 0$$

or $$I_B = \sqrt{3}\frac{V}{R_1}\angle 90^\circ$$

So, $$I_R : I_Y : I_B = \frac{V}{R_1} : \frac{V}{R_1} : \sqrt{3}\frac{V}{R_1} = 1 : 1 : \sqrt{3}$$

55. Ans: (b)

Hint: At $t = 0^+$ the capacitor acts as short circuit. At steady state, the capacitor acts as open circuit.

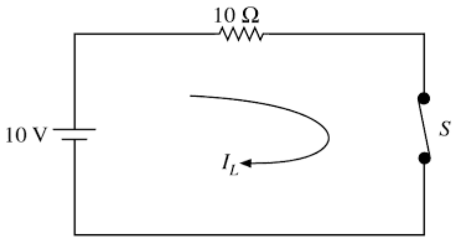


∴ Voltage across capacitor

$$= \frac{20}{10+10} \times 10 = 10 \text{ V}$$

56. Ans: (c)

Hint: At $t = 0^-$, the circuit is in steady state. So, inductor acts as short circuit.



$$I_L(0^-) = \frac{10}{10} = 1 \text{ A}$$

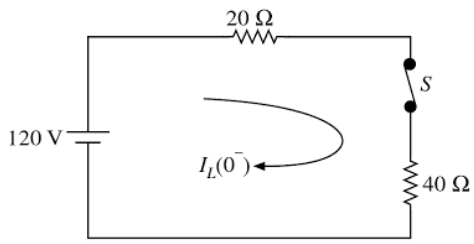
At $t = 0^+$

$$I_L(0^+) = I_L(0^-) = 1 \text{ A}$$

57. Ans: (c)

Hint: At $t = 0^-$, the circuit is in steady state. So, inductor acts as short circuit.

The circuit at $t = 0^-$



$$I_L(0^-) = \frac{120}{20+40} = 2 \text{ A}$$

After moving the switch, at $t = 0^+$

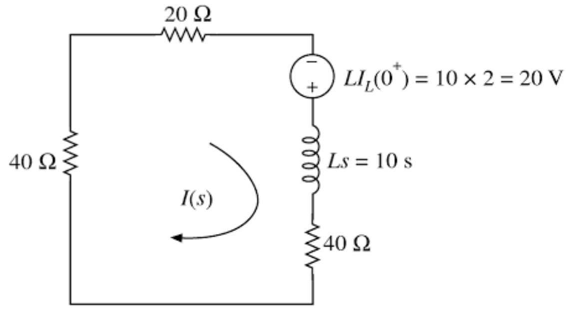
∴ $\quad I_L(0^+) = I_L(0^-) = 2 \text{ A}$

Now, $\quad 2 \times (20 + R) = 120$

⇒ $\quad R = 40 \ \Omega$

58. Ans: (b)

Hint: The circuit (in s–domain)



$$I(s) = \frac{20}{(20+40+40)+10s}$$

$$= \frac{20}{10s+100} = \frac{2}{s+10}$$

$$I(t) = L^{-1}[I(s)] = L^{-1}\left[\frac{2}{s+10}\right] = 2e^{-10t}$$

or, $\quad I(t) = I_L(0^+)e^{-\frac{R_{\text{eff}}}{L}t}$

$$= 2e^{-\frac{(20+40+40)}{10}t} = 2e^{-10t}$$

Initial stored energy in inductor

$$W_{\text{init}} = \frac{1}{2}LI_L^2(0^+) = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ J}$$

Remaining energy in inductor

$$W_{\text{rest}} = 0.05 \ W_{\text{init}} = 0.05 \times 20 = 1 \text{ J}$$

Now, $\quad \frac{1}{2}LI_1^2 = 1$

$$\frac{1}{2} \times 10 \times I_1^2 = 1$$

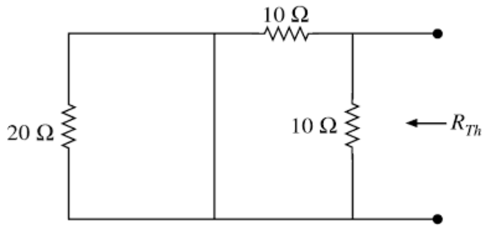
⇒ $\quad I_1 = \frac{1}{\sqrt{5}} = 0.4472$

Now, $\quad 0.4472 = 2e^{-10t}$

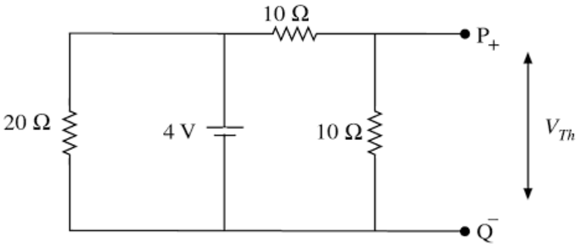
⇒ $\quad t \approx 0.15$ s

59. Ans: (a)

Hint:



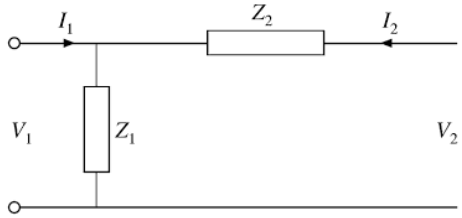
$$R_{Th} = \frac{10 \times 10}{10+10} = 5 \ \Omega$$



$$V_{Th} = \frac{4}{10+10} \times 10 = 2 \text{ V}$$

60. Ans: (d)

Hint:



$$V_1 = (I_1 + I_2)Z_1 = Z_1 I_1 + Z_1 I_2 \quad \text{(i)}$$

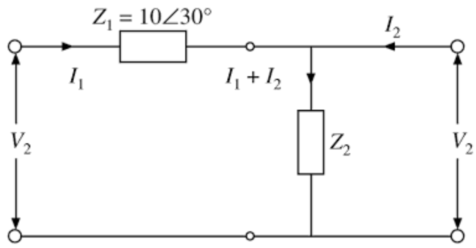
$$V_2 = Z_2 I_2 + (I_1 + I_2)Z_1 = Z_1 I_1 + (Z_1 + Z_2)I_2 \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$Z \text{ matrix} = \begin{bmatrix} Z_1 & Z_1 \\ Z_1 & Z_1 + Z_2 \end{bmatrix}$$

61. Ans: (b)

Hint:



$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$V_2 = Z_2(I_1 + I_2) \quad \text{(i)}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

Putting $I_2 = 0$ in Eq. (i)

$$V_2 = Z_2 I_1$$

or $$Z_2 = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{C} = \frac{1}{0.025\angle 45°}$$

$$Z_2 = 40\angle -45° \ \Omega$$

62. Ans: (a)

Hint: The rms value of dc voltage = V_{dc} = 3 V

The rms value of ac voltage = $V_{ac} = (4/\sqrt{2})$ V

∴ Total rms value of voltage

$$= \sqrt{3^2 + (4/\sqrt{2})^2} = \sqrt{9+8} = \sqrt{17} \text{ V}$$

63. Ans: (d)

Hint: $I_1 + I_2 + I_3 = 0$

$$I_3 = -I_1 - I_2$$

$$= -[-6\sin(\omega t)] - [8\cos(\omega t)]$$

$$= 10\left[\frac{6}{10}\sin(\omega t) - \frac{8}{10}\cos(\omega t)\right]$$

$$= -10[\cos(36.87°)\cos(\omega t) - \sin(36.87°)\sin(\omega t)]$$

$$= -10\cos(\omega t + 36.87°) \text{ mA}$$

64. Ans: (d)

Hint: $M = K\sqrt{L_1 L_2}$

Where K = coefficient of coupling

∵ $0 < K < 1$

∴ $M \le \sqrt{L_1 L_2}$

65. Ans: (d)

Hint: Current through inductor,

$$I(t) = \frac{1}{L}\int_{-\infty}^{t} V(t)dt$$

$$= \frac{1}{1}\int_{0}^{t} 12t^2 dt \text{ for } t \ge 0$$

$$= 4t^3 \text{ A}$$

66. Ans: (a)

Hint: Number of links = $B - (N - 1)$
= $B - N + 1$

67. Ans: (False)

Hint: Superposition principle is applicable on both time-variant and time-invariant resistors.

68. Ans: (b)

Hint: For reciprocity,

$$h_{12} = -h_{21}$$

For symmetry $\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$

69. Ans: (d)

Hint:

$$[Y] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[Z] = [Y]^{-1} = \frac{1}{2-1}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

70. Ans: (0.032)

Hint: For series RLC circuit,

$$Q\text{–factor at resonance } = \frac{\omega_0 L}{R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.01)\times(100\times 10^{-3})}} = 10\sqrt{10} \text{ rad/s}$$

$$Q = \omega_0 \frac{L}{R} = \frac{10\sqrt{10}\times 0.01}{10} = 0.032$$

71. Ans: (d)

Hint:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{(For series RLC resonance)}$$

$$f_{\text{new}} = \frac{1}{2\pi\sqrt{2L\times 2C}} \quad \text{(When all the components values are doubled)}$$

Hence, $f_{\text{new}} = \dfrac{f_0}{2}$

72. Ans: (c)

Hint: In a series RLC circuit, at resonance

$$V_L = jQV_{\text{source}}$$

And $$V_C = -jQV_{\text{source}}$$

Also for $Q > 1, |V_C| > |V_{\text{source}}|$

Hence, option (c) is correct.

73. Ans: (b)

Hint:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\times 10^{-6}\times 10^{-6}}} = 10^5 \text{ rad/s}$$

$$\Delta\omega = \frac{R}{L} = \frac{50}{100\times 10^{-6}} = 50\times 10^4 \text{ rad/s}$$

$$\omega_{\text{lower}} = \left[\sqrt{\omega_0^2 + \left(\frac{\Delta\omega}{2}\right)^2} - \frac{\Delta\omega}{2}\right]$$

$$= \left[\sqrt{(10^5)^2 + \left(\frac{5\times 10^5}{2}\right)^2} - \frac{5\times 10^5}{2}\right]$$

$$= 10^5[\sqrt{1+6.25} - 2.5]$$

$$= 0.193\times 10^5 \text{ rad/s}$$

Hence,

$$f_{\text{lower}} = \frac{\omega_{\text{lower}}}{2\pi} = \frac{0.193\times 10^5}{2\pi}$$

$$= 3065 \text{ Hz} \approx 3.065 \text{ kHz}$$

74. Ans: (d)

Hint: $Q = \dfrac{\omega L}{R}$

When frequency of operation is doubled, $\omega = 2\pi f$, also gets doubled

Consequently, Q also get doubled

$$P = I^2 R = \left[\frac{V}{\sqrt{R^2 + (\omega L)^2}}\right]^2 \cdot R$$

$$= \frac{V^2 \cdot R}{R^2\left[1 + \left(\frac{\omega L}{R}\right)^2\right]} = \frac{V^2}{R[1+Q^2]}$$

$\because$ It is given that Q is high.

$\therefore \quad Q^2 \gg 1$

or $\quad P \cong \dfrac{V^2}{RQ^2}$

$\because$ Q is doubled

$\therefore$ P decreased by 4 times.

75. Ans: (b)

Hint: Inductive coils are bulky in nature.

76. Ans: (b)

Hint: A practical current source is usually represented by a resistance in parallel with an ideal current source and a practical source is usually represented by a resistance in series with an ideal voltage source.

77. Ans: (c)

Hint:

$\because$ Ideal voltage has zero internal resistance.

$\therefore$ Time constant $\tau = RC = 0$

Hence, capacitor will charge instantaneously.

78. Ans: (c)

Hint: When excited by an ac source, capacitor stores the energy in one half cycle and delivers that energy in another half cycle. Hence, total energy stored in a capacitor over a complete cycle, when excited by an ac source is zero.

79. Ans: (c)

Hint: For a passive two-port network, output power cannot be greater than input power.

80. Ans: (d)

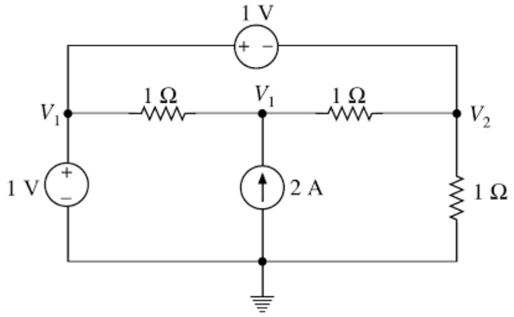
Hint:

$\because \quad P \propto \dfrac{1}{R}$

Therefore, resistance of 40 W bulb > resistance of 60 W bulb.

For series connection, current through both the bulbs will be same $P = I^2R$ (for series connection).
Power consumed by 40 W bulb > power consumed by 60 W bulb.
Hence, the 40 W bulb glows brighter.

81. Ans: (3)
Hint:



Applying nodal analysis at node P, we have

$$\frac{V_I - V_1}{1} + \frac{V_I - V_2}{1} - 2 = 0$$

or $$2V_I - (V_1 + V_2) = 2$$

or $$V_I = \left[\frac{2 + (V_1 + V_2)}{2}\right] \quad \text{(i)}$$

Also, $V_1 - V_2 = 1$ V and $V_1 = 1$ V

$\therefore$ $$V_2 = V_1 - 1 = 1 - 1 = 0 \text{ V}$$

Putting values of V_1 and V_2 in Eq. (i), we get

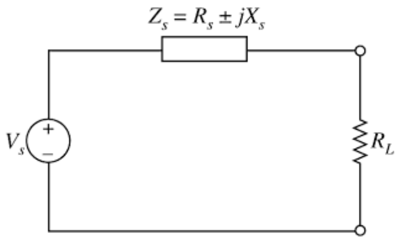
$$V_I = \left[\frac{2 + (1 + 0)}{2}\right] = \frac{3}{2} \text{ V}$$

$\therefore$ Power delivered by the current source

$$P = V_I \cdot I = \frac{3}{2} \times 2 = 3 \text{ W} \qquad [\because I = 2\text{A (given)}]$$

So, $V_{Th} = 2\sin(\omega t)$ and $Z_{Th} = R_{Th} = 4\ \Omega$

82. Ans: (c)
Hint: The solution of problem is shown in figure:



For the transfer of maximum power from source to load, $R_L = \sqrt{R_s^2 + X_s^2} = |Z_s|$

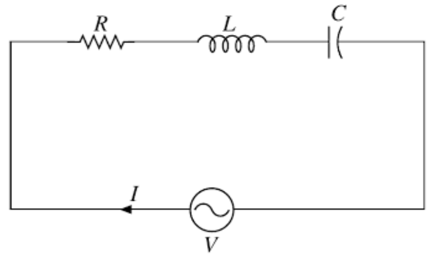
83. Ans: (1)
Hint: Shorted X and Y terminals. Calculate the current flowing through short circuit path.

84. Ans: (b)
Hint: Given, At $\omega_1 = 1$ k rad/s,

$$V_1 = 100\angle 0° \text{ V}, I_1 = 0.03\angle 31° \text{ A}$$

At $\omega_2 = 2$ k rad/s,

$$V_2 = 100\angle 0° \text{ V}, I_2 = 2\angle 0° \text{ A}$$



At $\omega_2 = 2$ k rad/s, voltage and current are in phase. So, series resonance occurs.

$$X_{L\omega_2} = X_{C\omega_2}$$

$\therefore$ $$Z = R = \frac{V_2}{I_2} = \frac{100\angle 0°}{2\angle 0°} = 50\ \Omega$$

Now, at $\omega_1 = 1$ k rad/s,

$\therefore$ $$Z = \frac{V_1}{I_1} = \frac{100\angle 0°}{0.03\angle 31°}$$

$$= 3333.33\angle -31°\ \Omega \quad \text{(i)}$$

At $\omega_1 = 1$ k rad/s, also

$$Z = |Z| \cdot \angle \tan^{-1}\left[\frac{X_L - X_C}{R}\right] \quad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\tan^{-1}\left[\frac{X_L - X_C}{R}\right] = -31°$$

or $$\left[\frac{X_L - X_C}{R}\right] = \tan(-31°)$$

or $$X_L - X_C = -30$$

$\therefore$ $$X_{L_{\omega_1}} - X_{C_{\omega_1}} = -30 \quad \text{(iii)}$$

Also, at $\omega_2 = 2$ k rad/s,

$\therefore$ $$X_{L_{\omega_1}} = X_{C_{\omega_1}}$$

or $$\omega_2 L = \frac{1}{\omega_2 C}$$

or $$L = \frac{1}{\omega_2^2 C} \quad \text{(iv)}$$

From Eq. (iii),

$$\omega_1 L - \frac{1}{\omega_1 C} = -30$$

or $$\omega_1\left(\frac{1}{\omega_2^2 C}\right)-\frac{1}{\omega_1 C}=-30 \quad \text{[From Eq. (iv)]}$$

or $$\left(\frac{10^{-3}}{4C}-\frac{10^{-3}}{C}\right)=-30$$

or $$C=\frac{3\times10^{-3}}{4\times30}$$

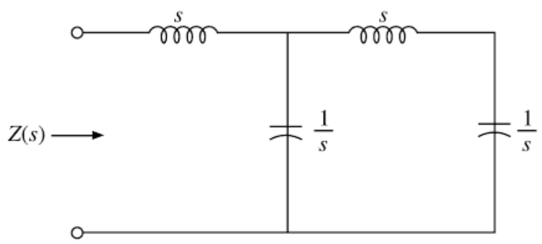
or $$C = 25\ \mu\text{F}$$

Substituting the value of C in Eq. (iv), we get

$$L=\frac{1}{\omega_2^2 C}=\frac{1}{(2\times10^3)^2\times25\times10^{-6}}=10\text{ mH}$$

85. Ans: (a)

Hint:



Driving point impedance, $Z(s)$ is

$$Z(s)=s+\left(\frac{\left(s+\frac{1}{s}\right)\times\frac{1}{s}}{s+\frac{1}{s}+\frac{1}{s}}\right)$$

or $$Z(s)=s+\left(\frac{s^2+1}{s^2}\right)\times\frac{s}{(s^2+2)}$$

or $$Z(s)=\frac{s^4+3s^2+1}{s^3+2s}$$

86. Ans: (a)

Hint: Bridge is balanced. So, node C and node D are at same potential.

Hence, no current flows through 2 kΩ resistor.

87. Ans: (b)

Hint: Voltage across 2 Ω resistance = V_S = 4 V

So, current through 2 Ω resistance

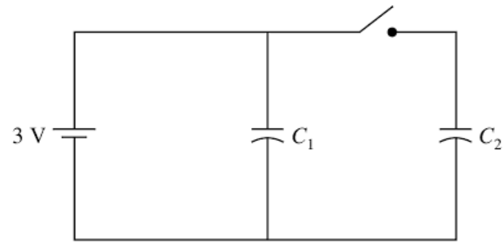
$$=\frac{V_S}{R}=2\text{ A}$$

When current source is connected across voltage source, it has no effect.

When current through 2 Ω resistance is doubled, then V_S = 8 V.

88. Ans: (d)

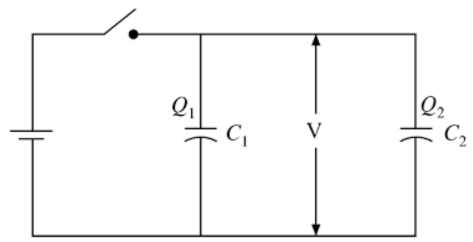
Hint: At $t = 0^-$, S_1 is closed, S_2 is open



C_1 gets charged upto 3 V

Voltage across C_2 is zero at $t = 0^-$, so no charge is stored in C_2.

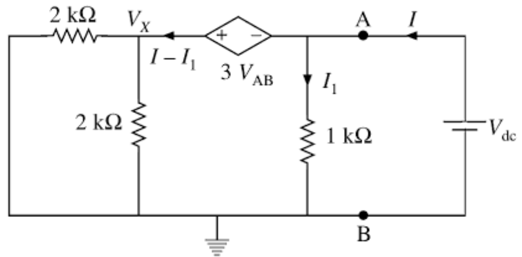
At $t > 0$, S_1 is open and S_2 is closed



Voltage across capacitor C_1 is 3 V which is connected in parallel across capacitor C_2. The capacitor C_2 cannot be instantly charged, so the voltage across it at time $t = 0^+$ will be 3V only.

89. Ans: (b)

Hint: To calculate Thevenin's resistance 5 V source is short circuited and V_{dc} source is connected at terminals A and B



$$R_{Th}=\frac{V_{dc}}{I}$$

Now,

$V_{AB} = V_{dc}$ and assuming I and I_1 in mA.

Current through 1 kΩ resistance

$$I_1=\frac{V_{dc}}{1}=V_{dc}$$

$$V_{AB}=V_{dc}$$

$$V_X=\frac{2\times2}{2+2}\times(I-I_1)$$

or $$V_X = I - V_{dc} \qquad \text{(i)}$$

Again, $$V_X = 3V_{AB} + I_1$$

or $$V_X = 3V_{dc} + V_{dc}$$

or $$V_X = 4V_{dc} \qquad \text{(ii)}$$

Comparing Eqs. (i) and (ii)

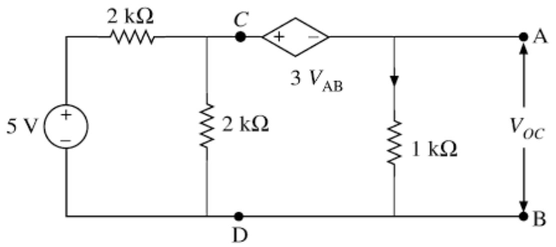
$$I - V_{dc} = 4V_{dc}$$

or $$5V_{dc} = I$$

$$\therefore\ R_{Th} = \frac{V_{dc}}{I} = \frac{1}{5} = 0.2\ \Omega \ \text{[I was assumed to be in mA]}$$

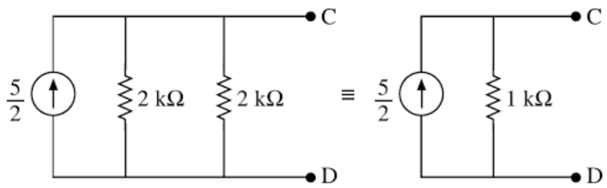
90. Ans: (d)

Hint:

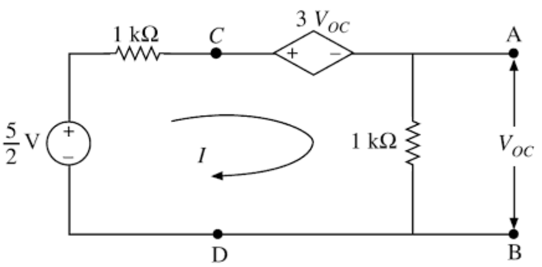


Terminals A and B are kept open.

By source transformation, voltage source is transformed into current source.



Applying source transformation, current source is transformed into voltage source.



Applying KVL, {I is assumed to be in mA}

$$-\frac{5}{2} + I \times 1 + 3V_{OC} + I \times 1 = 0$$

or $$6V_{OC} + 4I = 5 \qquad \text{(i)}$$

and $$I = \frac{V_{OC}}{1} = V_{OC} \qquad \text{(ii)}$$

Put value of I in Eq. (i), we get

$$6V_{OC} + 4V_{OC} = 5$$

or $$V_{OC} = 0.5\ \text{V}$$

91. Ans: (d)

Hint: Let resistance of a single incandescent lamp $= R$

So $P = \dfrac{V^2}{R}$

[Given P = 200 W and V = 220 V]

or $$R = 242\ \Omega$$

Let n number of lamps are connected is series across voltage V = 220 V

So, total resistance of lamps

$$R_{eq} = nR = 242n$$

Total power consumed

$$P = \frac{V^2}{R_{eq}}$$

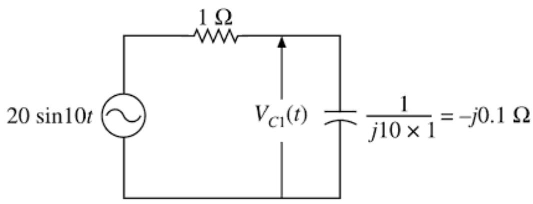
or $$100 = \frac{220^2}{242n}$$

or $$n = 2$$

92. Ans: (a)

Hint: Apply superposition theorem

Considering the voltage source $20\sin 10t$ acting alone and $10\sin 5t$ remains open circuited.



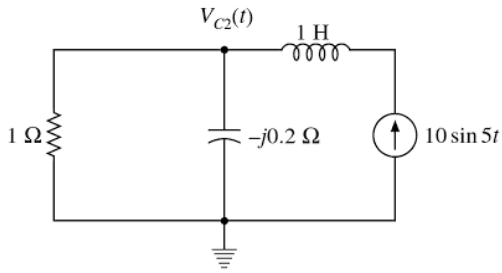
Let $V_{C1}(t)$ be the voltage across capacitor

$$\therefore \quad V_{C1}(t) = \left(\frac{-j0.1}{1 - j0.1}\right) \times 20\sin 10\,t$$

$$\Rightarrow \quad V_{C1}(t) = (1.99\angle -84.28°) \times 20\sin 10t$$

$$\therefore \quad V_{C1}(t) = 2\sin(10t - 84.28°)$$

Considering the current source $10\sin 5t$ acting alone and $20\sin 10t$ voltage source remains short circuited.



Let $V_{C2}(t)$ be the voltage across capacitor

Applying KCL at the node, we have

$$\frac{V_{C2}}{1} + \frac{V_{C2}}{(-j0.2)} - 10\sin 5t = 0$$

or $$V_{C2}(t) = \frac{10\sin 5t}{(1+j5)}$$

or $$V_{C2}(t) = 1.97\sin(5t - 78.69°)$$

Voltage across capacitor is

$$V_C(t) = V_{C1}(t) + V_{C2}(t)$$
$$= 2\sin(10t - 84.28°) + 1.97\sin(5t - 78.69°)$$

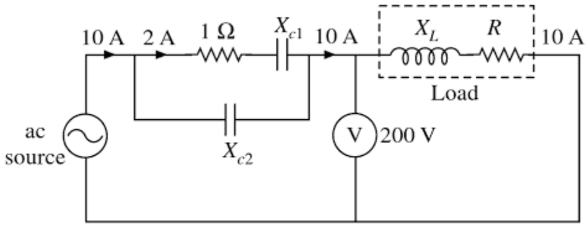
$$\therefore\; V_C(t) = 2\sin(10t - 84.28°) + 1.97\sin(5t - 78.69°) \quad \text{(i)}$$

Given,

$$V_C(t) = A_1 \sin(\omega_1 t - \theta_1) + A_2 \sin(\omega_2 t - \theta_2) \quad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we have
A_1 = 2 and A_2 = 1.97 ≈ 1.98 (closest answer)

93. Ans: (17.3 to 17.4)
Hint:



Given, total power dissipated in the circuit = 1000 W

$$\therefore\; 2^2 \times 1 + 10^2 \times R = 1000$$

or $$R = 9.98\ \Omega$$

Voltage drop across R, $V_R = IR = 10 \times 9.98 = 99.8$ V

Voltage across load, $V = 200V = \sqrt{V_R^2 + V_{X_L}^2}$

∴ Voltage drop across inductor,

$$V_{X_L} = \sqrt{V^2 - V_R^2} = \sqrt{(200)^2 - (99.8)^2} = 173.32$$

Now, $V_{X_L} = IX_L$

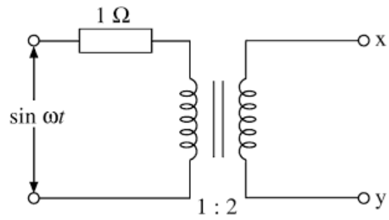
or $$X_L = \frac{V_{X_L}}{I} = 17.332\ \Omega$$

$$\therefore\; X_L = 17.332\ \Omega$$

94. Ans: (a)

Hint: Thevenin's equivalent voltage = voltage referred to secondary

Now, $\dfrac{\sin \omega t}{V_{Th}} = \dfrac{1}{2}$



So, Thevenin's voltage, $V_{Th} = 2\sin(\omega t)$
Thevenin's impedance

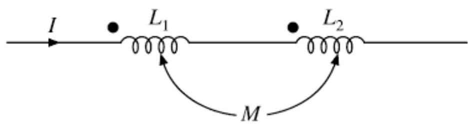
$$Z_{Th} = 2^2 \times 1 = 4\ \Omega\ldots$$

So, $V_{Th} = 2\sin(\omega t)$ *and* $Z_{Th} = R_{Th} = 4\ \Omega$

95. Ans: (35)

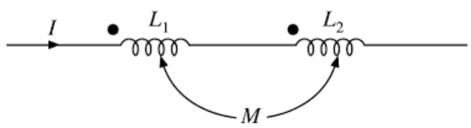
Hint: The two possible series connection are shown below:
Let the mutual inductance be M

(i) Additive connection



$$L_{eq} = L_1 + L_2 + 2M = 380\ \mu\text{H}$$

(ii) Subtractive connection



$$L_{eq} = L_1 + L_2 - 2M = 240\ \mu\text{H}$$

Thus, $$L_1 + L_2 + 2M = 380\ \mu\text{H} \quad \text{(i)}$$

And $$L_1 + L_2 - 2M = 240\ \mu\text{H} \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$M = 35\ \mu\text{H}$$

∴ Mutual inductance, M = 35 μH

Control System

2.1 SYLLABUS

Principles of feedback; transfer function; block diagrams; steady state errors; Routh and Nyquist techniques; Bode plots; root loci; lag, lead and lead lag compensation; state space model; state transition matrix, controllability and observability.

2.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

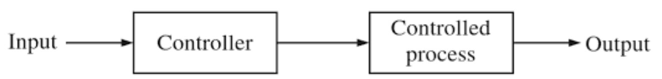
Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	3	1	1	1	4	2	3	1	2	2	2	2
2 Marks Question	6	3	7	8	5	3	3	4	4	2	3	3
Total Marks	**15**	**7**	**15**	**17**	**14**	**8**	**9**	**9**	**10**	**6**	**8**	**8**

2.3 TOPICS TO BE FOCUSED

INTRODUCTION

Control system is an interconnection of the physical component to provided a desired function, involving same kind of controlling action in it.

OPEN LOOP CONTROL SYSTEMS



The open loop control system is also known as without feedback control system or non-feedback control system. Control characteristic of open loop control system are independent of the output of the system.

Examples

(i) Automatic washing machine
(ii) Immersion rod

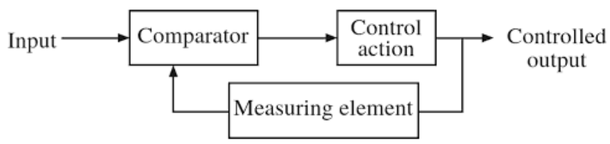
Advantages

(i) Open loop control systems are simple to design
(ii) Not troubled with the problem of stability

Disadvantages

(i) They are less accurate and not reliable
(ii) Optimization is not possible

CLOSED LOOP SYSTEMS



Closed loop system is a system in which controlling action is dependent on its output. Its means present output depends upon input and previous output. Such a system is called closed loop system.

Examples

(i) A missile launching system
(ii) Rader tracking system

Advantages

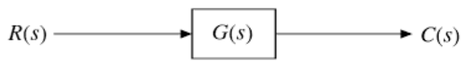
(i) They are more accurate, reliable and faster
(ii) Speed of the response can be greatly increased

Disadvantages

(i) They are more complex and hence cost of maintenance is high
(ii) They are less stable than open loop, their oscillations in the output may occur

TRANSFER FUNCTION

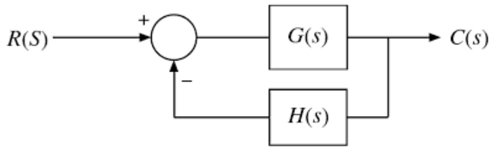
Open Loop Transfer Function



Transfer function, $G(s) = \frac{C(s)}{R(s)}$

$C(s)$ is the Laplace transform of output variables
$R(s)$ is the Laplace transform of input variables

Closed Loop Transfer Function (Negative Feedback)



Transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$

SIGNAL FLOW GRAPH (SFG)

A SFG is a graphical representation of the relationship between the variables of a set of linear algebraic equations. A few SFG terms are:

- *Node:* It represents a system variable, which is equal to the sum of all incoming signals at the node.
- *Branch:* A signal travels along a branch from one to another in the direction indicated by the branch arrow and in the process gets multiplied by the gain of the branch.
- *Path:* It is the transversal of connected branches in the direction of the branch arrows such that no node is traversed more than once.
- *Forward path:* It is the path from the input node to the output node.
- *Loop gain:* It is the product of branch gains encountered in traversing the loop.

MASON'S GAIN FORMULA

$$P = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

P = Overall transfer function
P_K = Path gain of K^{th} forward path
Δ = 1 – (sum of loop gains of all individual loops) + (sum of gain product of all possible combination of two non-touching loops) – (sum of gain products of all possible combination of three non-touching loops) + - - -
Δ_K = Value of Δ obtained by removing all the loops touching the K^{th} forward path.

TIME RESPONSE

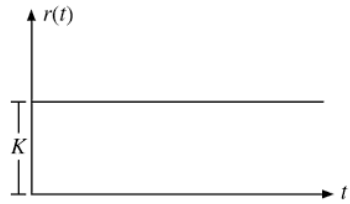
The time response of a control system consists of two parts namely,

(i) The transient response
(ii) The steady state response

The specified input test signals applied for analyzing a control system: are (i) step, (ii) ramp, (iii) parabolic, and (iv) impulse.

Specified Input Test Signals

Step function: Sudden application of input signal.

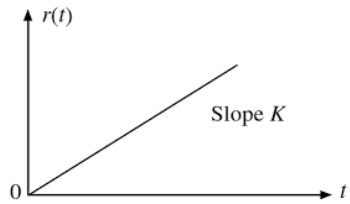


Step function

A unit step function $r(t)$ is defined as:

$$r(t) = 0, \quad t < 0$$
$$= K \quad t \geq 0$$

Ramp function: Gradual application of input signal.

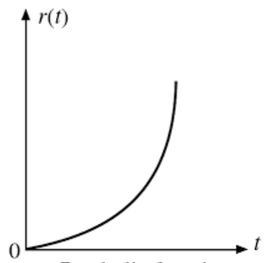


Ramp function

A unit ramp function $r(t)$ is defined as:

$$r(t) = 0, \quad t < 0$$
$$= Kt, \quad t \geq 0$$

Parabolic function: More gradual application of input in comparison with ramp function.



Parabolic function

A unit parabolic function $r(t)$ is defined as:

$$r(t) = 0, \quad t < 0$$
$$= K\frac{t^2}{2}, \quad t \geq 0$$

Impulse function: Input is suddenly applied as a shock for a very short duration of time.

Impulse function

A unit impulse function $r(t)$ is defined as:

$$\delta(t) = 0 \quad t \neq 0$$
$$\delta(t) = \infty \quad t = 0$$

Steady State Error

The steady state error for this system from either the open- or closed-loop transfer function using the final value theorem is calculated.

$$e_{ss}(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

Step input [$R(s) = 1/s$]:

$$e_{ss}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+K_p} \Rightarrow K_p = \lim_{s\to 0} G(s)$$

Ramp input $\left[R(s) = \frac{1}{s^2}\right]$:

$$e_{ss}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_{\text{vee}}} \Rightarrow K_{\text{vee}} = \lim_{s\to 0} sG(s)$$

Parabolic input $\left[R(s) = \frac{1}{s^3}\right]$:

$$e_{ss}(\infty) = \frac{1}{\lim_{s\to 0} s^2 G(s)} = \frac{1}{K_a} \Rightarrow K_a = \lim_{s\to 0} s^2 G(s)$$

Time Response of a Second Order System

Transfer function of a second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The denominator of the transfer function $s^2 + 2\xi\omega_n s + \omega_n^2$ should be equal to zero. This is called *characteristic equation*.

(i) Underdamped system ($0 < \xi < 1$)

$$c(t) = 1 - e^{-\xi\omega_n t} \frac{1}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right) \quad \text{for } t \geq 0$$

The error signal $e(t)$ will be

$$e(t) = e^{-\xi\omega_n t} \frac{1}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

The steady state error e_{ss} will be

$$e_{ss} = \lim_{t\to\infty} e(t)$$
$$= \lim_{t\to\infty} e^{-\xi\omega_n t} \frac{1}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

(ii) Critically damped system ($\xi = 1$)

$$c(t) = 1 - (1+\omega_n t)e^{-\omega_n t}$$

(iii) Undamped system ($\xi = 0$)

$$c(t) = 1 - \cos(\omega_n t)$$

(iv) Overdamped system ($\xi > 1$)

$$c(t) = 1 - \frac{1}{2\sqrt{\xi^2-1}(\xi+\sqrt{\xi^2-1})} e^{-(\xi-\sqrt{\xi^2-1})\omega_n t}$$

Time Response of Second Order System with Unit Ramp Input Function

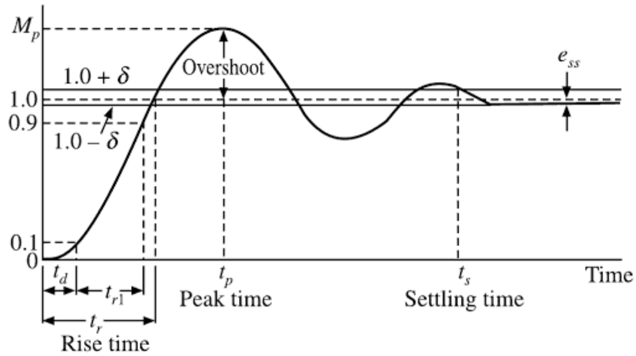
$$c(t) = t - \frac{2\xi}{\omega_n} + e^{-\xi\omega_n t}\left[\frac{2\xi}{\omega_n}\cos\alpha\omega_n t - \frac{1-2\xi^2}{\alpha\omega_n}\sin\alpha\omega_n t\right]$$

The error signal $e(t)$ will be

$$e(t) = \frac{2\xi}{\omega_n} - e^{-\xi\omega_n t}\left[\frac{2\xi}{\omega_n}\cos\alpha\omega_n t - \frac{1-2\xi^2}{\alpha\omega_n}\sin\alpha\omega_n t\right]$$

Steady state error, $e_{ss} = \lim_{t\to\infty} e(t) = \frac{2\xi}{\omega_n}$

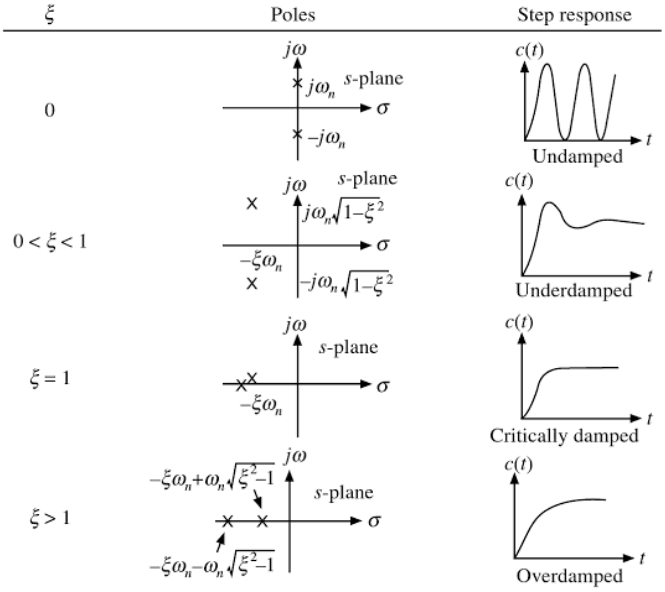
Time Response Specification of Second order System



Time Domain Specification

Delay Time (t_d)	Rise Time (t_r)	Peak Time (t_p)	Peak Overshoot (M_p)	Settling Time (t_s)	Steady State Error (e_{ss})
$t_d = \frac{1+0.7\xi}{\omega_n}$	$t_r = \left[\Pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] / \omega_d$	$t_p = \frac{n\pi}{\omega_d}$	$M_p = e^{\frac{-\Pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$	$(t_s) = \frac{3}{\xi\omega_n}$ $= 3T$	$e_{ss} = \lim_{t\to\infty} [r(t) - c(t)]$

Transient Response for Different Pole Location of the Transfer Function



ERROR CONSTANT (OR COEFFICIENT)

The static position error constant (K_p) is defined as

$$K_p = \lim_{s\to 0} G(s)H(s)$$

The static velocity error constant (K_v) is defined as

$$K_{\text{vee}} = \lim_{s\to 0} sG(s)H(s)$$

The static, acceleration error constant (K_a) is defined as

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

Steady State Errors for Different Types of Inputs

Input	Type '0'		Type '1'		Type '2'	
	Static error coefficient	Steady state error	Static error coefficient	Steady state error (e_{ss})	Static error coefficient	Steady state error (e_{ss})
Step	$K_p = K$	$1/(1+K)$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp	$K_v = 0$	∞	$K_v = K$	$1/K$	$K_v = \infty$	0
Parabolic	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = K$	$1/K$

SENSITIVITY

Effect of forward path transfer function parameter variation in a closed loop control system:

- Sensitivity of the open loop system

$$S_G^N = \frac{\partial G(s)/G(s)}{\partial G(s)/G(s)} = 1$$

- Sensitivity of the closed loop system

$$S_G^N = \frac{1}{[1+G(s)H(s)]}$$

- Sensitivity due to the variation of feedback path gain $H(s)$

$$S_H^N = \frac{-G(s)H(s)}{[1+G(s)H(s)]}$$

CONTROL ACTION

Proportional Controller

The equation of a proportional controller in time domain:

$$E_a(t) = K_p E(t)$$

where,

$E(t)$ = Input error signal
$E_a(t)$ = Actuating error signal

Proportional Plus Derivative Controller

The equation of a proportional plus derivative controller in time domain:

$$E_a(t) = K_p E(t) + K_d \frac{d}{dt} E(t)$$

Effects

(i) Damping factor increases
(ii) Natural frequency remains unchanged
(iii) Settling time reduces

Proportional Plus Integral Controller

The equation of a proportional plus integral controller in time domain:

$$E_a(t) = K_p E(t) + K_i \int_0^t E(\tau)d\tau$$

Where K_p and K_i are proportional and integral gain constant respectively.

Effects

(i) It increases type of system
(ii) Error of velocity input reduced considerably
(iii) It may improve steady state performance

Proportional Plus Integral Plus Derivative Controller

The equation of a proportional plus integral plus derivative controller in time domain:

$$E_a(t) = K_p E(t) + K_i \int_0^t E(\tau)d\tau + K_d \frac{d}{dt}E(t)$$

Effects

(i) It increases system order
(ii) Damping factor increases
(iii) Peak overshoot increases

All Pass and Minimum Phase System

When system having all the poles and zeros lie on the left-half s-plane then the system is defined as minimum phase system.

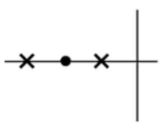
$G(s) = \dfrac{15(s+5)}{(s+3)(s+9)}$ is a minimum phase transfer function.

When one or more zeros lie on the right-half of the s-plane and all the poles lie on the left half s-plane, the system is called non-minimum phase system.

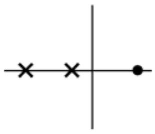
$G(s) = \dfrac{18(s-7)}{(s+4)(s+2)}$ is a non-minimum phase transfer function.

A transfer function having anti symmetric pole-zero pattern about the imaginary axis i.e. for every pole in the left half there is a zero in the right half at the mirror position is called *all pass transfer function*.

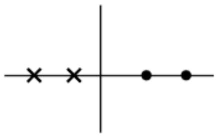
$G(s) = \dfrac{14(s-6)(s-8)}{(s+6)(s+8)}$ is the all pass transfer function.



Minimum phase system



Non-minimum phase system

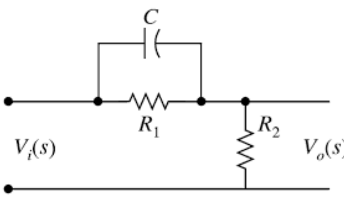


All pass system

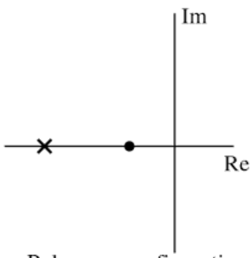
COMPENSATOR

The additional device added in control system to obtain the performance as per desired specification is known as *compensator*.

Phase Lead Compensator



Phase lead network



Pole-zero configuration

Transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+sT)}{(1+s\alpha T)} \qquad [\text{Where, } \alpha < 1]$$

$$\alpha = \frac{R_2}{R_1 + R_2} \text{ and } T = R_1 C$$

Here two corner frequency

$$\omega_1 = \frac{1}{T},\ \omega_2 = \frac{1}{\alpha T}$$

Maximum phase lead occurs at mid corner frequency

$$\omega_m = \frac{1}{2}\left[\log_{10}(\frac{1}{T}) + \log_{10}(\frac{1}{\alpha T})\right]$$

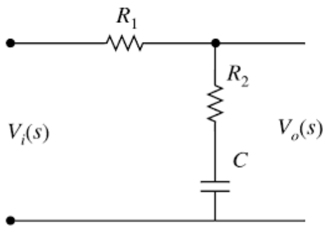
Phase angle

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

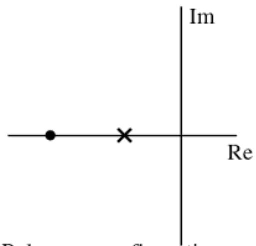
Properties

(i) It shifts gain crossover frequency to a higher value
(ii) Speed response is improved
(iii) Time constant is decreased

Phase–lag Compensation



Phase lead network



Pole-zero configuration

Transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{(1+sT)}{(1+s\beta T)}$$

$$\beta = \frac{R_1 + R_2}{R_2} \text{ and } T = R_2 C$$

Here two corner frequency

$$\omega_1 = \frac{1}{T},\ \omega_2 = \frac{1}{\beta T}$$

Maximum phase lead occurs at mid corner frequency

$$\omega_m = \frac{1}{\sqrt{3}T}$$

Phase angle

$$\tan\phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

Properties

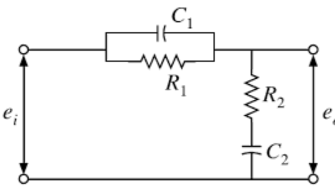
(i) Improve steady state error

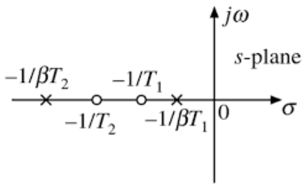
(ii) Time constant increased

(iii) Bandwidth is reduced

Phase Lead Lag Compensation

The speed of response and steady state error can be simultaneously improved by a lead-lag network. Circuit and pole-zero plot are given below.





STABILITY ANALYSIS

Stability analysis in a system implies the small changes in the system input, in initial condition or in system parameters do not result in large changes in system output.

Stability Condition

- When the system is excited by a bounded input, the output is bounded.
- In the absence of the input, the output tends to zero irrespective of initial condition. This is known as *asymptotic stability.*

SYSTEM RESPONSE

Roots in s-plane	Corresponding Impulse Response
Poles at negative real axis	Exponentially decreasing curve (stable)
Poles at positive real axis	Exponentially increasing curve (unstable)
Conjugate complex poles at LHS of s-plane	Exponentially decreasing curve (stable)
Conjugate complex poles at RHS of s-plane	Exponentially increasing curve (unstable)
Single conjugate complex poles at imaginary axis	Marginally stable (sustained oscillation)
Double conjugate complex poles at imaginary axis	Unstable
Single pole at origin	Marginally stable
Double poles at origin	Unstable

ROUTH HURWITZ STABILITY CRITERION

Write the polynomial as:

$$Q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

To determine whether this system is stable or not, check the following conditions:

1. Two necessary but not sufficient conditions that all the roots have negative real parts are:
   (i) All the polynomial coefficients must have the same sign.
   (ii) All the polynomial coefficients must be non-zero (root locus).
2. If condition (1) is satisfied, then compute the Routh-Hurwitz array as:

$$\begin{array}{c|l}
s^n & a_n \quad a_{n-2} \quad a_{n-4} \ \ldots \\
s^{n-1} & a_{n-1} \quad a_{n-3} \quad a_{n-5} \ldots \\
s^{n-2} & b_1 \quad b_2 \quad b_3 \ \ldots \\
s^{n-3} & c_1 \quad c_2 \quad c_3 \ \ldots \\
s^{n-4} & \ldots\ldots\ldots\ldots\ldots\ldots \\
.. & \ldots\ldots\ldots\ldots\ldots\ldots \\
s^0 & \ldots\ldots\ldots\ldots\ldots
\end{array}$$

$$b_1 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}, \quad b_2 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix},$$

$$b_3 = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}, \quad c_1 = \frac{-1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix},$$

$$c_2 = \frac{-1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}$$

3. The necessary condition that all roots have negative real parts is that all the elements of the first column of the array have the same sign. The number of changes of sign equals the number of roots with positive real parts.
4. *Special Case 1:* The first element of a row is zero, but some other elements in that row are non-zero. In this case, simply replace the zero element by e, complete the table development, and then interpret the results assuming that e is a small number of the same sign as the element above it. The results must be interpreted in the limit as e not equals to 0.
5. *Special Case 2*: All the elements of a particular row are zero. In this case, some of the roots of the polynomial are located symmetrically about the origin of the s-plane, e.g., a pair of purely imaginary roots. The zero row will always occur in a row associated with an odd power of s. The row just above the zero row holds the coefficients of the auxiliary polynomial. The roots of the auxiliary polynomial are the symmetrically placed roots. Be careful to remember that the coefficients in the array skip powers of s from one coefficient to the next.

ROOT LOCUS

The following point are remember for sketching the root locus using poles and zeros of $G(s)H(s)$, as K is varies from zero to infinity

The root locus is symmetrical about the horizontal real axis of s-plane.

Characteristic equation

$$1 + G(s)H(s) = 0$$
$$|G(s)H(s)| = 1$$
$$\angle G(s)H(s) = (2K+1)180°$$
$$K = 0, 1, 2, 3...$$

Number of asymptotes = $(P - Z)$

Angle of Asymptotes

The $(P - Z)$ branches of the root locus which go to infinity travel in straight line. The asymptotes are inclined to the real axis at an angle θ is given by

$$\theta = \frac{(2K+1)180°}{(P-Z)}$$

Centroid

The asymptotes cross the real axis at a common point are known as centroid.

$$\sigma = \frac{\sum(\text{real part of open loop poles}) - \sum(\text{real part of open loop zeros})}{(P-Z)}$$

Break Away Point

The break away point and break-in-point of root locus are determined from the relation:

$$\frac{dK}{ds} = 0 \quad \text{or} \quad \frac{dG(s)H(s)}{ds} = 0$$

Angle of Departure

The angle of departure of the root locus is given by

$$\Phi_d = 180° + \Phi$$

Angle of Arrival

The angle of arrival of the root locus is given by

$$\Phi_a = 180° - \Phi$$

Where $\Phi = \left(\sum \Phi_z - \sum \Phi_p\right)$

Φ_z = sum of all the angles subtended by remaining zeros
Φ_p = sum of all the angles subtended by poles

BODE PLOT

Bode plot consists two plot

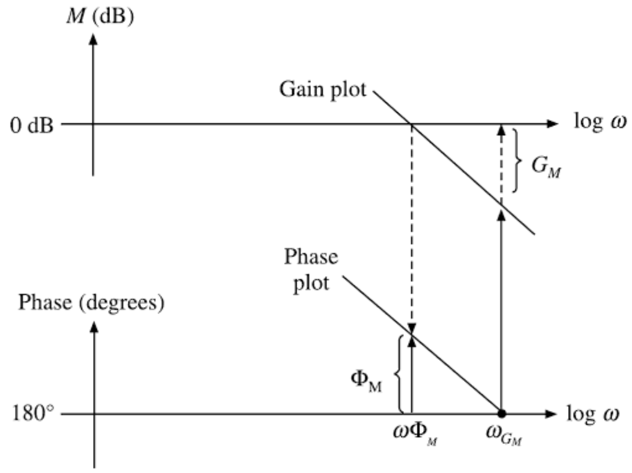
(i) Magnitude plot
(ii) Phase plot

- The magnitude can be expressed in its logarithmic values. The transfer function is required to be converted into 'time constant' form for corner frequency calculation. The frequency at which the change of slope from 0 dB to –20 dB per decade occurs is the corner frequency.

For magnitude plot, each terms of the transfer function has to consider in a proper sequence.

(i) Constant gain factor is considered first.
Its 'db' value = $20\log_{10}|K|$
(ii) Each integrator factor (pole at origin) provides –20 db decade.
(iii) Differentiator factor (zero at origin) provides 20 db decade.
(iv) Each numerator factors (pole) contributes –20 db decade which are considered as the increasing order of its corner frequencies.
(v) Each denominator factors (zero) contributes 20 db decade which are considered as the increasing order of its corner frequencies.

- The angle of $G(j\omega)$ is to be expressed in degrees which is to be plotted against log ω.



Gain Crossover Frequency

The frequency at which the magnitude plot crosses the 0 dB line is termed as gain crossover frequency.

Phase Crossover Frequency

The frequency at which the phase plot crosses –180° line is termed as phase crossover frequency.

Phase Margin

The marginal value of the phase plot to –180° at the frequency where the magnitude plot crosses the 0 dB line (gain crossover frequency ω_{gc}).

$$\Phi_M = 180° + \angle G(j\omega)H(j\omega)$$

Gain Margin

The marginal value of gain plot from 0 dB line at the frequency where the phase plot crosses –180° (phase crossover frequency ω_{pc}).

$$G_M = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} = \frac{1}{A}$$

$$G_M(\text{db}) = 20 \log \frac{1}{A}$$

For the stability analysis

When,

- Gain crossover frequency < Phase crossover frequency, Gain margin and phase margin both are positive, then system will be stable.
- Gain crossover frequency > Phase crossover frequency, Gain margin and phase margin both are negative, then system will be unstable.
- Gain crossover frequency = Phase crossover frequency, Gain margin and phase margin both are zero, then system will be marginally stable.

POLAR PLOT

It is the locus of tips of the phasor of magnitude plotted corresponding to the phase angle for different values of frequency from zero to infinity.

For polar plot magnitude and phase of the sinusoidal transfer function $[G(j\omega)H(j\omega)]$ is required to find.

Steps to Draw Polar Plot

Step 1: Determine the transfer function $G(s)$

Step 2: Put $s = j\omega$ in the $G(s)$

Step 3: At $\omega = 0$ and $\omega = \infty$, find $G(j\omega)H(j\omega)$

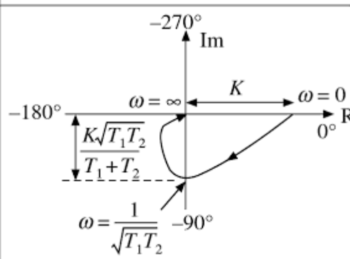
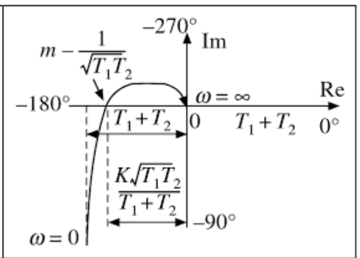
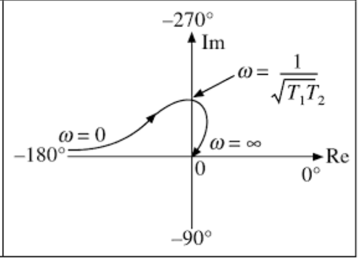
Step 4: At $\omega = 0$ and $\omega = \infty$, find angle of $G(j\omega)H(j\omega)$

Step 5: Rationalize the function $G(j\omega)$ and separate the real and imaginary parts

Step 6: Put Re $[G(j\omega)] = 0$, determine the frequency at which plot intersects the imaginary axis and calculate intersection value by putting the above calculated frequency in $G(j\omega)$

Step 7: Put Im $[G(j\omega)] = 0$, determine the frequency at which plot intersects the real axis and calculate intersection value by putting the above calculated frequency in $G(j\omega)$

Step 8: Sketch the polar plot with the help of above information

Transfer Function	**Polar Plot**
$G(s)H(s) = \dfrac{K}{(1+sT_1)(1+sT_2)}$ Type 0 system	
$G(s)H(s) = \dfrac{K}{s(1+sT_1)(1+sT_2)}$ Type 1 system	
$G(s)H(s) = \dfrac{K}{s^2(1+sT_1)(1+sT_2)}$ Type 2 system	

NYQUIST PLOT

The Nyquist method is used to examine the relative stability of the system.

For Nyquist plot comprising of two concepts: mapping and principle of argument.

Mapping: Any independent variable can produce a curve in independent s-plane then that curve can be plotted on a plane which is function of s, i.e, $G(s)H(s)$ plane.

Principle of argument: Number of encirclement of poles and zeroes in the s-plane refers no of encirclement of the origin in the $G(s)H(s)$ plane clockwise or anti-clockwise respectively. Therefore, number of encirclement (N) = number of poles – number of zeroes

Steps to Draw Nyquist Path

Step 1: Drawing of contour in s-plane which will cover the right half side. If a system has any pole at the origin then the semicircle has to draw at the origin. So that the contour will cover the right half side of the s-plane avoiding the origin.

Step 2: As per the contour each part has to be drawn in $G(s)H(s)$ plane with the help of polar plot and $Re^{j\theta}$ $(R \to \infty$ and $R \to 0)$.

Step 3: From the closed path $G(s)H(s)$ plane number of encirclement (N) of critical point $(-1 + j0)$ is required to count.

Step 4: For system to be stable $Z = N - P$

N = number of counter clockwise direction about the origin

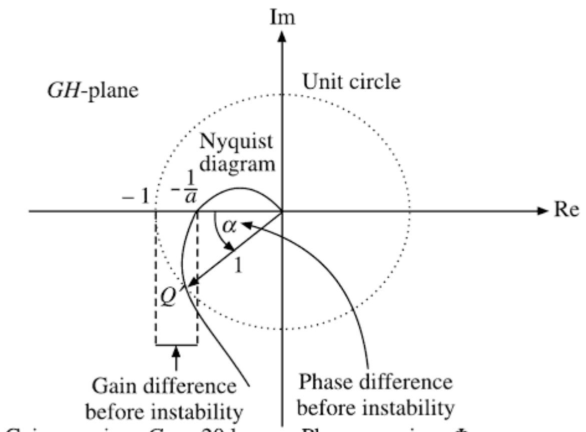
P = number of poles of characteristic equation inside contour, or

= number of poles of open-loop system(poles at RHS of s-plane)

Z = number of zeros of characteristic equation inside contour, or

= number of poles of closed-loop system

GAIN MARGIN AND PHASE MARGIN DETERMINATION



Gain margin = G_M = 20 log a Phase margin = $\Phi_M = \alpha$

FREQUENCY RESPONSE OF A CONTROL SYSTEM

Second order system with the transfer function

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The magnitude and angle of the frequency response is obtained as

$$|T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + 4\xi^2 u^2}}$$

And $\angle T(j\omega) = \phi = -\tan^{-1}\frac{2\xi u}{1-u^2}$

Where $u = \dfrac{\omega}{\omega_n}$

ω = normalising frequency,

ω_n = natural frequency

Resonance frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

Resonance peak $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

Bandwidth $\omega_b = \omega_n\sqrt{(1-2\xi^2) + \sqrt{2-4\xi^2+4\xi^4}}$

CONSTANT MAGNITUDE OF LOCUS (*M*-CIRCLES AND *N*-CIRCLES)

Radius of M-circle = $\dfrac{M}{1-M^2}$,

Centre of M-circle = $\left(\dfrac{M^2}{1-M^2}, 0\right)$,

Radius of N-circle = $\sqrt{\dfrac{1}{4} + \dfrac{1}{4N^2}}$,

Centre of N-circle = $\left(-\dfrac{1}{2}, \dfrac{1}{2N}\right)$

STATE SPACE ANALYSIS

- ***State:*** It is the minimal state of variables such that the knowledge of this variables at $t = t_0$ together with the knowledge of inputs for $t > t_0$ completely determined the behaviour of the system.
- ***State variable:*** The variables involved in determining state of dynamic system $x(t)$:

  Input equation, $\overline{X} = Ax + Bu$

  Output equation, $Y = Cx + Du$

  where

  A = System matrix

  B = Control matrix

  C = Output matrix

  D = Transmission matrix
- ***Transfer function***

$$\text{Transfer function, TF} = C[sI - A]^{-1}B + D$$

- ***Properties of state transition matrix***

  For time invariant system

$$\overline{X} = Ax$$

$$\phi(t) = e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \cdots$$

(i) $\phi(0) = e^{A(0)} = I$

(ii) $\phi(t) = e^{At} = (e^{-At})^{-1} = [\phi(-t)]^{-1}$

(iii) $\phi^{-1}(t) = \phi(-t)$

(iv) $\phi(t_1 + t_2) = e^{A(t_1+t_2)} = e^{At_1}e^{At_2} = \phi(t_1)\phi(t_2)$
$= \phi(t_2)\phi(t_1)$

(v) $[\phi(t)]^n = \phi(nt)$

(vi) $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$
$= \phi(t_1 - t_0)\phi(t_2 - t_1)$

CONTROLLABILITY

A system is completely controllable if it is possible to transfer the system from its initial state to any other desire state in specified time.

Controllability matrix $Q_0 = [B : AB : A^2B : \cdots : A^{n-1}B]$
where n is the order of the state matrix.
Condition for state controllability, $|Q_0| \neq 0$ or rank of Q_0 = order of Q_0

OBSERVABILITY

A system is completely observable if every state of the system can be identified by its output over a finite time.
Observability matrix Q_b =

$$[C^T : A^TC^T : (A^2)^TC^T : \cdots : (A^T)^{n-1}C^T]$$

where n is the order of the state matrix.
Condition for state observability, $|Q_b| \neq 0$ or rank of Q_b = order of Q_b

2.4 IMPORTANT POINTS TO REMEMBER

1. The transfer function is defined as the ratio of Laplace transform of output to Laplace transform of input considering initial condition as zero.
2. A signal flow graph is graphical representation of the relationships between the variables of a set of linear algebraic equations.
3. Basic properties of signal flow graph:
   - It is applicable to linear system only.
   - Nodes are arranged from left to right in a sequence.
   - The algebraic equations must be in terms of cause and effect relationship.
   - Signals travel along branches only in the marked direction and is multiplied by the gain of the branch.
4. Negative feedback is desirable in any control system.
5. Insertion of negative feedback in a control system affects the transient response to decay very fast.
6. For any system, the highest power of s in the characteristic equation is called order of the system.
7. Order of the system shows the number of energy storing elements present in the system.
8. A system can have more than one signal flow graph.
9. While identifying feedback loops, no node should be traced twice.
10. Generally, no system is linear.
11. The stability of a system decreases as the type of system increases.
12. The roots of the characteristic equation are same as closed loop poles.
13. The steady state output of a unity feedback control system is very near to reference input.
14. For step input, maximum overshoot of a second order system depends on damping ratio only.
15. If $M_p = 100\%$, the damping ratio is 0.
16. A high damping ratio will give low overshoot.
17. The ratio of damped frequency to natural frequency of the system having damping factor ξ is $\sqrt{1-\xi^2}$.
18. To reduce steady state error of the control system, increase its gain constant.
19. Type 0 system will have small steady state error.
20. Settling time of a system is inversely proportional to natural frequency.
21. The derivative controller improves the transient response.
22. To decrease time constant of the control system, decrease its inertia.
23. The integral controller improves the steady state response.
24. Derivative control increases damping ratio and decreases peak overshoot.
25. Derivative feedback control also increases damping ratio and decreases peak overshoot.
26. Derivative control decreases rise time.
27. Derivative feedback control increases rise time.
28. In derivative control, steady state error remains unchanged.
29. In derivative feedback control, steady state error is increased.
30. The proportional plus derivative controller reduces maximum overshoot.
31. The proportional plus integral controller increases the order and type of the system by one.
32. The proportional plus integral plus derivative controller combines the advantages of proportional, derivative and integral control action.
33. The Routh–Hurwitz criterion gives absolute stability.
34. When the poles lie on $j\omega$ axis, then the system is marginally stable.
35. The location of the closed loop conjugate pair of pole on $j\omega$ axis indicates that the system is marginally stable.
36. Transfer functions having no poles and zeros in the RHS of s-plane are called minimum phase transfer functions.
37. Transfer functions having at least one pole or zero in the RHS of s-plane is called non-minimum phase transfer functions.
38. Transfer functions having symmetric pole and zero about the imaginary axis in s-plane are called all pass transfer function.
39. A system is stable when both gain margin (G_M) and phase margin (Φ_M) are positive.
40. Loop gain K is zero at poles and infinity at zeros.
41. Gain margin and phase margin can be determined from the root locus plot also.
42. If the root locus branches cross the imaginary axis, the system becomes unstable.
43. If the root locus branches do not cross the imaginary axis, the system is inherently stable.
44. At gain factor $K = 0$, the roots of the characteristic equation are placed at open loop poles.
45. The starting points of root loci are open loop poles.
46. The root loci ends at open loop zeros.
47. The root locus plot is symmetrical about the real axis because complex roots occurs in conjugate pairs.
48. The root locus separates at a point between two open loop poles, the point is called break away point.
49. The intersection of the root locus branch with the imaginary axis is determined by using characteristic equation $1+G(s)H(s)=0$.
50. Centroid is the point on real axis where all asymptotes intersect.
51. The intersection point of root locus branches with imaginary axis of s-plane is determined using Routh–Hurwitz array.

52. Nyquist criterion enables to determine either absolute stability or relative stability.
53. In Nyquist plot, phase crossover frequency occurs when phase is –180°.
54. Nyquist plot analysis is suitable for both non-minimum phase and minimum phase transfer function.
55. Nyquist plot is drawn on polar graph paper for open loop transfer function.
56. The frequency at which the Nyquist diagram cuts (–1,0) circle is known as gain crossover frequency.
57. The frequency at which the Nyquist diagram crosses the negative real axis is known as phase crossover frequency.
58. Phase margin Φ_M is directly proportional to damping ratio ξ.
59. When phase margin Φ_M is 0, damping ratio ξ is 0.
60. Zero gain margin indicates that polar plot passes through (–1, 0) point.
61. Decibel is a ratio of power.
62. The Bode plot is obtained using open loop transfer function.
63. The initial slope of the Bode plot gives an indication of type of the system.
64. If initial slope in Bode plot is −40 dB per decade, then the open loop transfer function will have double pole at the origin.
65. The initial slope of the Bode plot for a transfer function having no poles at the origin is 0 dB per decade.
66. Bode plot analysis is suitable for minimum phase transfer function.
67. When gain crossover frequency is equal to phase crossover frequency, then the gain margin and phase margin are zero.
68. Unstable systems have negative gain margin.
69. The gain margin and phase margin can be determined directly from the Bode plot.
70. The frequency at which the magnitude of the Bode plot crosses 0 dB axis is termed as gain crossover frequency.
71. The gain crossover frequency is the frequency at which the gain is 1.
72. The frequency at which the phase curve of the Bode plot crosses –180° line is termed as phase crossover frequency.
73. The phase crossover frequency is the frequency at which the phase shift is 180°.
74. For a stable system, the gain crossover frequency occurs earlier than phase crossover frequency.
75. For unstable system, the phase crossover frequency occurs earlier than gain crossover frequency.
76. For marginally stable system, the gain crossover frequency and phase crossover frequency are coincident.
77. If the gain margin is positive and the phase margin is negative, the system is unstable.
78. A lag compensation network:
    - Increases the gain of the original network without affecting stability
    - Does not affect stability on increased gain
    - Reduces the steady state error
    - Reduces the speed of response
    - Permits the increase of gain if phase margin is acceptable
79. The lag compensation has a pole nearer to the origin.
80. The lag compensator decreases the bandwidth.
81. The lead compensation has a zero nearer to the origin.
82. Lead compensators improve transient performance.
83. A lead compensating network:
    - Improves response time
    - Increases resonant frequency
    - Stabilizes the system with low phase margin
    - Enables moderate increase in gain without affecting stability
84. The analysis of multiple input multiple output system is conveniently analyzed by state space approach.
85. The eigenvalues of a linear system are the location of poles of the system.

2.5 REFERENCES

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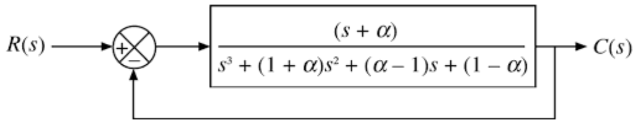
2.6 PREVIOUS YEARS' QUESTIONS

1. In the formation of Routh–Hurwitz array for a polynomial, all the elements of a row have zero values. This premature termination of the array indicates the presence of

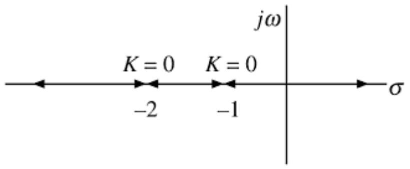
(a) Only one root at the origin
(b) Imaginary roots
(c) Only positive real roots
(d) Only negative real roots [Gate 2014 set 1]

2. For the given system, it is desired that the system be stable. The minimum value of α for this condition is____________



[Gate 2014 set 1]

3. The root locus of a unity feedback system is shown in figure.



The closed loop transfer function of the system is

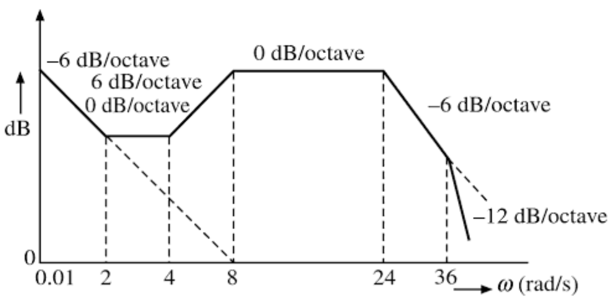
(a) $\dfrac{C(s)}{R(s)}=\dfrac{K}{(s+1)(s+2)}$

(b) $\dfrac{C(s)}{R(s)}=\dfrac{-K}{(s+1)(s+2)+K}$

(c) $\dfrac{C(s)}{R(s)}=\dfrac{K}{(s+1)(s+2)-K}$

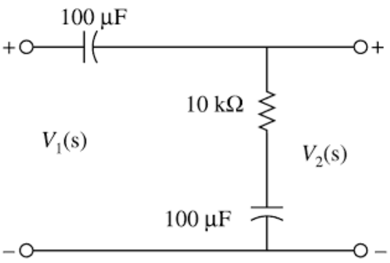
(d) $\dfrac{C(s)}{R(s)}=\dfrac{K}{(s+1)(s+2)+K}$ [Gate 2014 set 1]

4. The Bode magnitude plot of the transfer function $G(s)=\dfrac{K(1+0.5s)(1+as)}{s\left(1+\dfrac{s}{8}\right)(1+bs)\left(1+\dfrac{s}{36}\right)}$ is shown in figure.



Note that – 6 dB/octave = – 20 dB/decade. The value of $\dfrac{a}{bK}$ is __________. [Gate 2014 set 1]

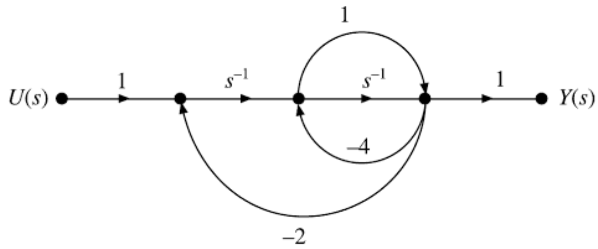
5. The transfer function $\dfrac{V_2(s)}{V_1(s)}$ of the circuit shown in figure is



(a) $\dfrac{0.5s+1}{s+1}$ (b) $\dfrac{3s+6}{s+2}$

(c) $\dfrac{s+2}{s+1}$ (d) $\dfrac{s+1}{s+2}$ [Gate 2013]

6. The signal flow graph for a system is given in figure. The transfer function, $\dfrac{Y(s)}{U(s)}$ for the system is

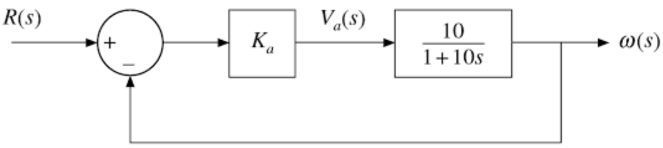


(a) $\dfrac{s+1}{5s^2+6s+2}$ (b) $\dfrac{s+1}{s^2+6s+2}$

(c) $\dfrac{s+1}{s^2+4s+2}$ (d) $\dfrac{1}{5s^2+6s+2}$

[Gate 2013]

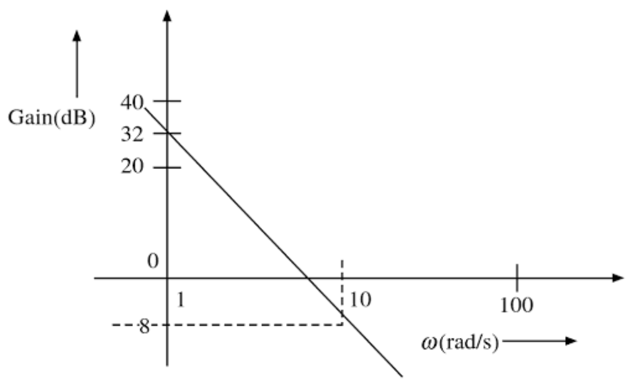
7. The open loop transfer function of a dc motor is given as $\dfrac{\omega(s)}{V_a(s)}=\dfrac{10}{1+10s}$. When connected in feedback as shown in figure, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open loop system is



(a) 1 (b) 5

(c) 10 (d) 100 [Gate 2013]

8. The Bode plot of a transfer function $G(s)$ is shown in figure.



The gain $(20\log|G(s)|)$ is 32 dB and – 8 dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all ω. Then G(s) is

(a) $\frac{39.8}{s}$ (b) $\frac{39.8}{s^2}$

(c) $\frac{32}{s}$ (d) $\frac{32}{s^2}$ [Gate 2013]

Common Data Questions: 9 and 10

The state variable formulation of a system is given as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u,\quad x_1(0)=0,\quad x_2(0)=0,$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

9. The system is

(a) Controllable but not observable
(b) Not controllable but observable
(c) Both controllable and observable
(d) Both not controllable and not observable

[Gate 2013]

10. The response $y(t)$ to the unit step input is

(a) $\frac{1}{2}-\frac{1}{2}e^{-2t}$ (b) $1-\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-t}$

(c) $e^{-2t}-e^{-t}$ (d) $1-e^{-t}$ [Gate 2013]

11. A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12(1-e^{-2t})$$

The response of the system as $t\to\infty$ is

(a) $x=6$ (b) $x=2$

(c) $x=2.4$ (d) $x=-2$ [Gate 2003]

12. A lead compensator used for a closed loop controller has the following transfer function

$$\frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$$

For such a lead compensator

(a) $a < b$ (b) $b < a$

(c) $a > Kb$ (d) $a < Kb$ [Gate 2003]

13. A second order system starts with an initial condition of $\begin{bmatrix}2\\3\end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-t}\end{bmatrix}$. The state of the system at the end of 1 s is given by

(a) $\begin{bmatrix}0.271\\1.100\end{bmatrix}$ (b) $\begin{bmatrix}0.135\\0.368\end{bmatrix}$

(c) $\begin{bmatrix}0.271\\0.736\end{bmatrix}$ (d) $\begin{bmatrix}0.135\\1.100\end{bmatrix}$ [Gate 2003]

14. A control system with certain excitation is governed by the following mathematical equation

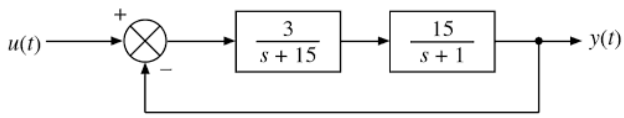
$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+5e^{-4t}+2e^{-5t}$$

The natural time constants of the response of the system are

(a) 2s and 5s (b) 3s and 6s
(c) 4s and 5s (d) 1/3s and 1/6s

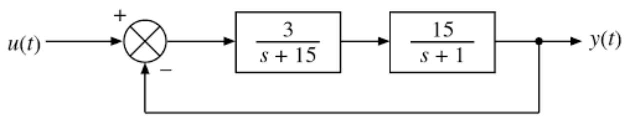
[Gate 2003]

15. The block diagram shown in figure gives a unity feedback closed loop control system. The steady state error in the response of the above system to unit step input is



(a) 25% (b) 0.75%
(c) 6% (d) 33% [Gate 2003]

16. The roots of the closed loop characteristic equation of the system shown in figure are



(a) – 1 and – 15 (b) 6 and 10
(c) – 4 and – 15 (d) – 6 and – 10

[Gate 2003]

17. The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$$

The above equation may be organized in the state space form as

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix}=P\begin{bmatrix}\frac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a$$

where the P matrix is given by

(a) $\begin{bmatrix} -\frac{B}{J} & -\frac{K^2}{LJ} \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{K^2}{LJ} & -\frac{B}{J} \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -\frac{K^2}{LJ} & -\frac{B}{J} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -\frac{B}{J} & -\frac{K^2}{LJ} \end{bmatrix}$

[Gate 2003]

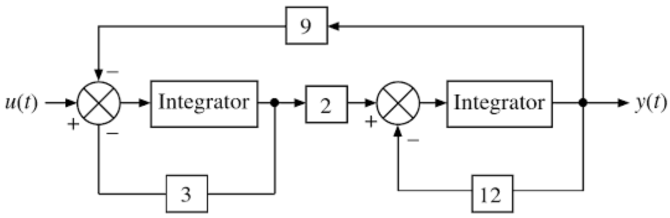
18. The loop gain of a closed loop system is given by the following expression

$$\frac{K}{s(s+2)(s+4)}$$

The value of K for which the system just becomes unstable is

(a) $K = 6$ (b) $K = 8$
(c) $K = 48$ (d) $K = 96$ [Gate 2003]

19. The block diagram of a control system is shown in figure. The transfer function $G(s) = Y(s)/U(s)$ of the system is

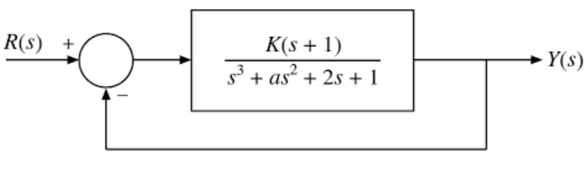


(a) $\dfrac{1}{18\left(1+\frac{s}{12}\right)\left(1+\frac{s}{3}\right)}$ (b) $\dfrac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$

(c) $\dfrac{1}{27\left(1+\frac{s}{12}\right)\left(1+\frac{s}{9}\right)}$ (d) $\dfrac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{3}\right)}$

[Gate 2003]

20. The feedback system shown in figure oscillates at 2 rad/s when



(a) $K = 2$ and $a = 0.75$
(b) $K = 3$ and $a = 0.75$
(c) $K = 4$ and $a = 0.5$
(d) $K = 2$ and $a = 0.5$ [Gate 2012]

21. A system with transfer function

$G(s) = \dfrac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$ is excited by $\sin(\omega t)$.

The steady state output of the system is zero at

(a) $\omega = 1$ rad/s (b) $\omega = 2$ rad/s
(c) $\omega = 3$ rad/s (d) $\omega = 4$ rad/s

[Gate 2012]

Statement for Linked Answer Questions: 22 and 23

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

22. $G_c(s)$ is a lead compensator if

(a) $a = 1, b = 2$ (b) $a = 3, b = 2$
(c) $a = 3, b = -1$ (d) $a = 3, b = 1$

[Gate 2012]

23. The phase of the above lead compensator is maximum at

(a) $\sqrt{2}$ rad/s (b) $\sqrt{3}$ rad/s
(c) $\sqrt{6}$ rad/s (d) $1/\sqrt{3}$ rad/s

[Gate 2012]

24. The state variable description of an LTI system is given by

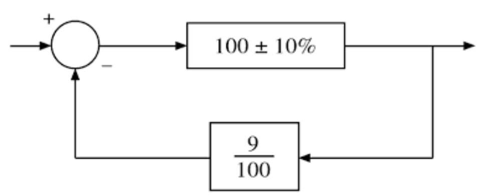
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Where y is the output and u is the input. The system is controllable for

(a) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$
(b) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$
(c) $a_1 = 0, a_2 \neq 0, a_3 = 0$
(d) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$ [Gate 2012]

25. As shown in figure, a negative feedback system has an amplifier of gain 100 with ±10% tolerance in the forward path, and an attenuator of value 9/100 in the feedback path. The overall system gain is approximately:



(a) 10 ± 1% (b) 10 ± 2%
(c) 10 ± 5% (d) 10 ± 10%

[Gate 2010]

26. For the system $\dfrac{2}{(s+1)}$, the approximate time taken for a step response to reach 98% of its final value is

(a) 1s (b) 2s

(c) 4s (d) 8s [Gate 2010]

27. The characteristic equation of a closed loop system is $s(s+1)(s+3)+K(s+2)=0$, $K>0$. Which of the following statements is true?

(a) Its roots are always real

(b) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$

(c) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$

(d) It may have complex roots in the right half plane

[Gate 2010]

28. The system $\dot{x} = Ax + Bu$ with $A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

(a) Stable and controllable

(b) Stable but uncontrollable

(c) Unstable but controllable

(d) Unstable and uncontrollable [Gate 2010]

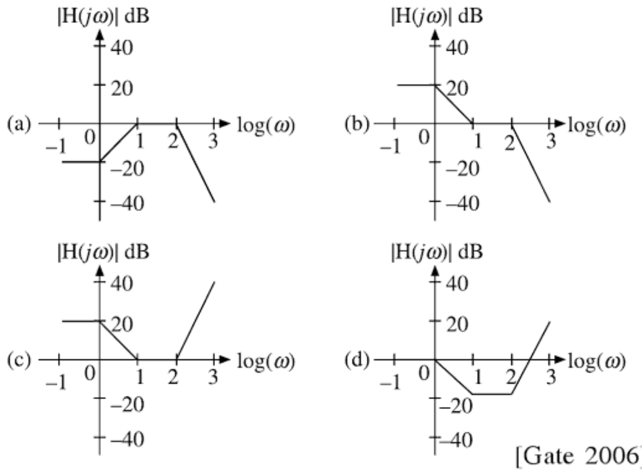
29. For a system with the transfer function $H(s) = \dfrac{3(s-2)}{s^3+4s^2-2s+1}$, the matrix A in the state space form $\dot{x} = Ax + Bu$ is equal to

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

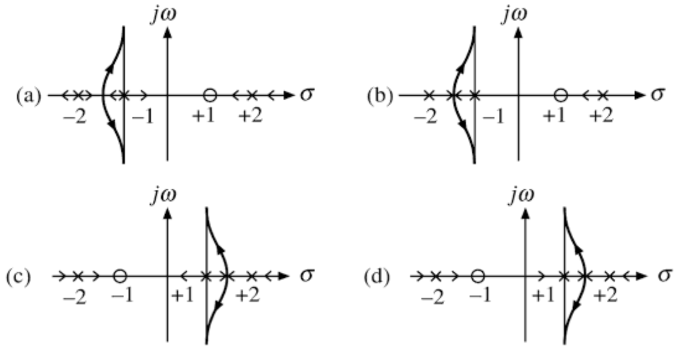
[Gate 2006]

30. The Bode magnitude plot of $H(j\omega) = \dfrac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$ is



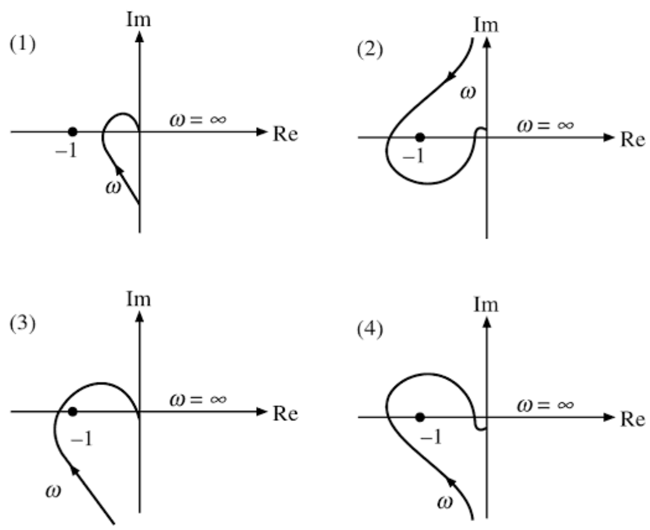
[Gate 2006]

31. A closed loop system has the characteristic function $(s^2-4)(s+1)+K(s-1)=0$. Its root locus plot against K is



[Gate 2006]

32. Consider the following Nyquist plots of loop transfer functions over $\omega = 0$ to $\omega = \infty$. Which of these plots represent a stable closed loop system?



(a) (1) only (b) All, except (1)

(c) All, except (3) (d) (1) and (2) only

[Gate 2006]

33. The closed loop transfer function of a system is $T(s) = \dfrac{4}{(s^2+0.4s+4)}$. The steady state error due to unit step input is __________. [Gate 1995]

34. A system with the open loop transfer function:

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

is connected in a negative feedback configuration with a feedback gain of unity. For the closed loop system to be marginally stable, the value of K is__________

[Gate 2014 set 2]

35. For the transfer function

$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

The values of the constant gain term and the highest corner frequency of the Bode plot respectively are

(a) 3.2, 5.0 (b) 16.0, 4.0
(c) 3.2, 4.0 (d) 16.0, 5.0

[Gate 2014 set 2]

36. The second order dynamic system

$$\frac{dx}{dt} = Px + Qu$$
$$y = Rx$$

has the matrices P, Q and R as follows:

$$P\begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}, Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, R = [0 \quad 1]$$

The system has the following controllability and observability properties:

(a) Controllable and observable
(b) Not controllable but observable
(c) Controllable but not observable
(d) Neither controllable nor observable

[Gate 2014 set 2]

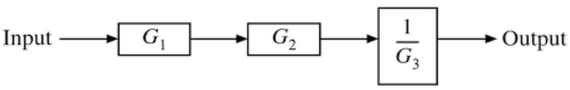
37. The state transition matrix for the system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u$ is

(a) $\begin{bmatrix} e^t & 0 \\ e^t & e^t \end{bmatrix}$ (b) $\begin{bmatrix} e^t & 0 \\ t^2e^t & e^t \end{bmatrix}$

(c) $\begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$ (d) $\begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$

[Gate 2014 set 2]

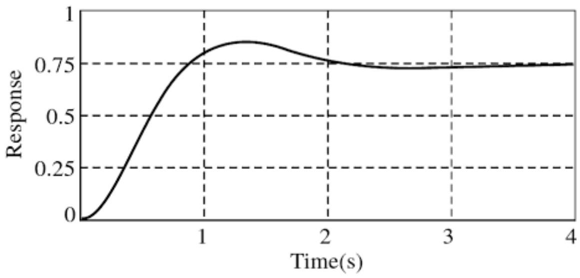
38. The measurement system shown in figure uses three subsystems in cascade whose gains are specified as G_1, G_2 and $1/G_3$. The relative small errors associated with each respective subsystem G_1, G_2 and G_3 are ε_1, ε_2 and ε_3. The error associated with the output is:



(a) $\varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$ (b) $\frac{\varepsilon_1\varepsilon_2}{\varepsilon_3}$

(c) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$ (d) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$ [Gate 2009]

39. The unit step response of a unity feedback system with open loop transfer function $G(s) = \frac{K}{(s+1)(s+2)}$ is shown in figure.



The value of K is

(a) 0.5
(b) 2
(c) 4
(d) 6 [Gate 2009]

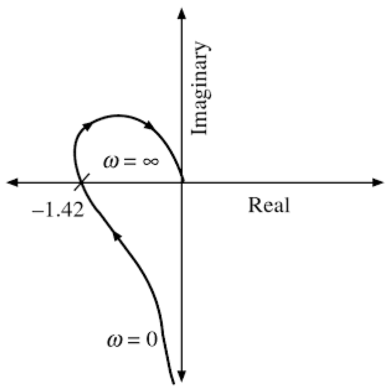
40. The first two rows of Routh's tabulation of a third order equation are as follows.

s^3	2	2
s^2	4	4

This means there are

(a) Two roots at $s = \pm j$ and one root in right half s-plane
(b) Two roots at $s = \pm j2$ and one root in left half s-plane
(c) Two roots at $s = \pm j2$ and one root in right half s-plane
(d) Two roots at $s = \pm j$ and one root in left half s-plane [Gate 2009]

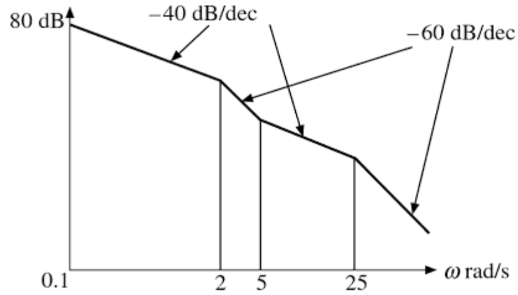
41. The polar plot of an open loop stable system is shown in figure. The closed loop system is



(a) Always stable
(b) Marginally stable
(c) Unstable with one pole on the RHS s-plane
(d) Unstable with two poles on the RHS s-plane

[Gate 2009]

42. The asymptotic approximation of the log magnitude vs frequency plot of a system containing only real poles and zeros is shown in figure. Its transfer function is



(a) $\dfrac{10(s+5)}{s(s+2)(s+25)}$ (b) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

(c) $\dfrac{100(s+5)}{s(s+2)(s+25)}$ (d) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

[Gate 2009]

43. The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{(e^{-0.1s})}{s}$. The gain margin of this system is

(a) 11.95 dB (b) 17.67 dB

(c) 21.33 dB (d) 23.9 dB [Gate 2009]

Common Data for Questions: 44 and 45

A system is described by the following state and output equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

$$y(t) = x_1(t)$$

Where $u(t)$ is the input and $y(t)$ is the output

44. The system transfer function is

(a) $\dfrac{s+2}{s^2+5s-6}$ (b) $\dfrac{s+3}{s^2+5s+6}$

(c) $\dfrac{2s+5}{s^2+5s+6}$ (d) $\dfrac{2s-5}{s^2+5s-6}$ [Gate 2009]

45. The state transition matrix of the above system is

(a) $\begin{bmatrix} e^{-3t} & 0 \\ e^{-2t}+e^{-3t} & e^{-2t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-3t} & e^{-2t}+e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{3t} & e^{2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

[Gate 2009]

46. The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $c(t)$ is

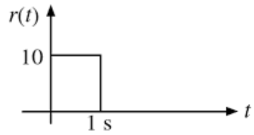
(a) $u(t) + e^{-t} + e^{-2t}$

(b) $(e^{-t} + e^{-2t})\, u(t)$

(c) $(1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$

(d) $e^{-t}\delta(t) + e^{-2t}\, u(t)$ [Gate 2011]

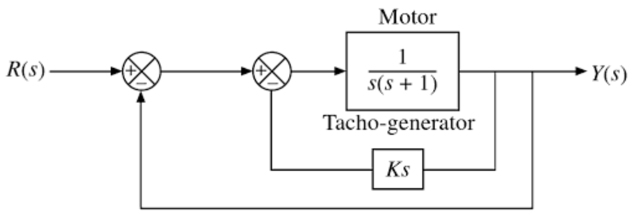
47. The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input $r(t)$ having a magnitude of 10 and a duration of one second, as shown in figure is



(a) 0 (b) 0.1

(c) 1 (d) 10 [Gate 2011]

48. A two loop position control system is shown in figure



The gain K of the tacho-generator influences mainly the

(a) Peak overshoot

(b) Natural frequency of oscillation

(c) Phase shift of the closed loop transfer function at very low frequencies ($\omega \to 0$)

(d) Phase shift of the closed loop transfer function at very high frequencies ($\omega \to \infty$)

[Gate 2011]

49. An open loop system represented by the transfer function $G(s) = \dfrac{(s-1)}{(s+2)(s+3)}$ is

(a) Stable and of the minimum phase type

(b) Stable and of the non-minimum phase type

(c) Unstable and of the minimum phase type

(d) Unstable and of the non-minimum phase type

[Gate 2011]

50. The open loop transfer function $G(s)$ of a unity feedback control system is given as

$$G(s) = \frac{K\left(s+\frac{2}{3}\right)}{s^2(s+2)}$$

From the root locus, it can be inferred that when K tends to positive infinity,

(a) Three roots with nearly equal real parts exist on the left half of the s-plane

(b) One real root is found on the right half of the s-plane

(c) The root loci cross the $j\omega$ axis for a finite value of K; $K \neq 0$

(d) Three real roots are found on the right half of the s-plane [Gate 2011]

51. The frequency response of a linear system $G(j\omega)$ is provided in the tubular form as

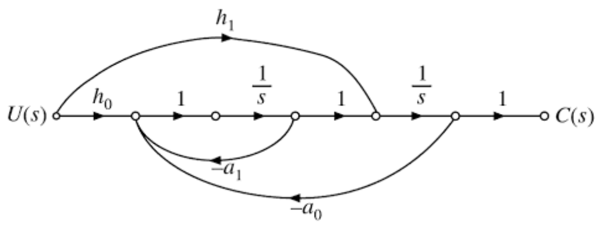
$\lvert G(j\omega)\rvert$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	$-130°$	$-140°$	$-150°$	$-160°$	$-180°$	$-200°$

The gain margin and phase margin of the system are

(a) 6 dB and 30° (b) 6 dB and – 30°

(c) – 6 dB and 30° (d) – 6 dB and – 30° [Gate 2011]

52. The signal flow graph of a system is shown in figure. $U(s)$ is the input and $C(s)$ is the output.



Assuming, $h_1 = b_1$ and $h_0 = b_0 - b_1a_1$, the input output transfer function, $G(s) = \dfrac{C(s)}{U(s)}$ of the system is given by

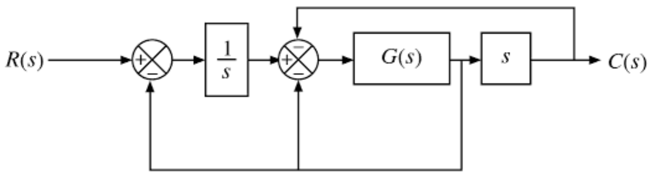
(a) $G(s) = \dfrac{b_0 s + b_1}{s^2 + a_0 s + a_1}$

(b) $G(s) = \dfrac{a_1 s + a_0}{a^2 + b_1 s + b_0}$

(c) $G(s) = \dfrac{b_1 s + b_0}{a^2 + a_1 s + a_0}$

(d) $G(s) = \dfrac{a_0 s + a_1}{a^2 + b_0 s + b_1}$ [Gate 2014 set 3]

53. The block diagram of a system is shown in figure.



If the desired transfer function of the system is $\dfrac{C(s)}{R(s)} = \dfrac{s}{s^2 + s + 1}$ then $G(s)$ is

(a) 1 (b) s

(c) $1/s$ (d) $\dfrac{-s}{s^3 + s^2 - s - 2}$

[Gate 2014 set 3]

54. A single-input single-output feedback system has forward transfer function $G(s)$ and feedback transfer function $H(s)$. It is given that $|G(s).H(s)| < 1$. Which of the following is true about the stability of the system?

(a) The system is always stable.

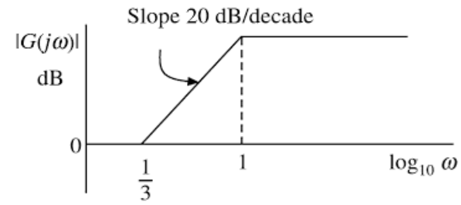
(b) The system is stable if all zeros of $G(s).H(s)$ are in left half of the s-plane.

(c) The system is stable if all poles of $G(s).H(s)$ are in left half of the s-plane.

(d) It is not possible to say whether or not the system is stable from the information given.

[Gate 2014 set 3]

55. The magnitude Bode plot of a network is shown in figure.



The maximum phase angle Φ_M and the corresponding gain G_M respectively, are

(a) – 30° and 1.73 dB (b) – 30° and 4.77 dB

(c) + 30° and 4.77 dB (d) + 30° and 1.73 dB

[Gate 2014 set 3]

56. Consider the system described by following state space equations

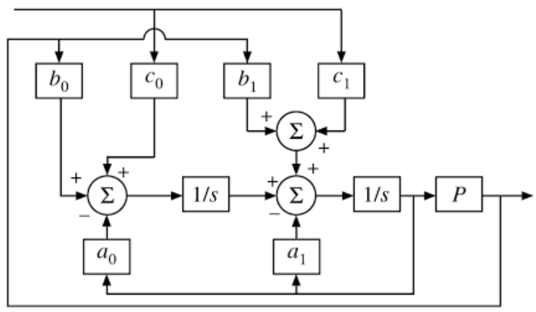
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u\ ;\ y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

If u is unit step input, then the steady state error of the system is

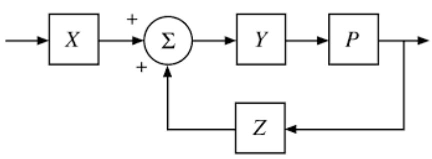
(a) 0 (b) 1/2

(c) 2/3 (d) 1 [Gate 2014 set 3]

57. The system shown in figure below.



can be reduced to the form



with

(a) $X = c_0 s + c_1$, $Y = \dfrac{1}{(s^2 + a_0 s + a_1)}$, $Z = b_0 s + b_1$

(b) $X = 1$, $Y = \dfrac{(c_0 s + c_1)}{(s^2 + a_0 s + a_1)}$, $Z = b_0 s + b_1$

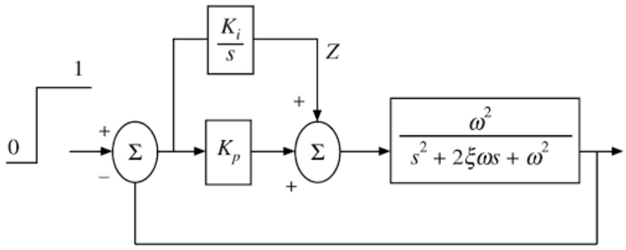
(c) $X = c_1 s + c_0$, $Y = \dfrac{(b_1 s + b_0)}{(s^2 + a_1 s + a_0)}$, $Z = 1$

(d) $X = c_1 s + c_0$, $Y = \dfrac{1}{(s^2 + a_1 s + a_0)}$, $Z = b_1 s + b_0$

[Gate 2007]

58. Consider the feedback control system shown in figure which is subjected to a unit step input. The system is stable and has following parameters

$K_p = 4$, $K_i = 10$, $\omega = 500$ and $\xi = 0.7$.



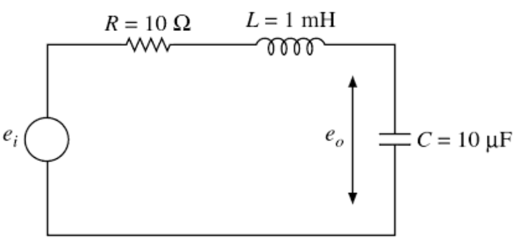
The steady state value of Z is

(a) 1 (b) 0.25

(c) 0.1 (d) 0 [Gate 2007]

Statement for Common Data Questions: 59 and 60

59. RLC circuit shown in figure.



For a step input e_i, the overshoot in the output e_o will be

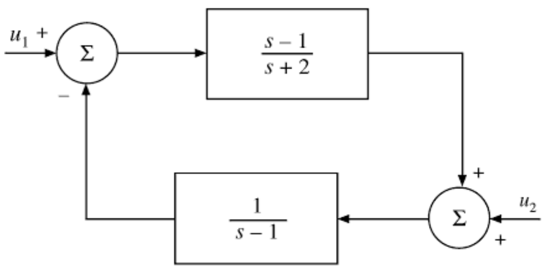
(a) 0, since the system is not underdamped

(b) 5%

(c) 16%

(d) 48% [Gate 2007]

60. The system shown in figure is



(a) Stable

(b) Unstable

(c) Conditionally stable

(d) Stable for input u_1, but unstable for input u_2

[Gate 2007]

61. If the loop gain K of a negative feedback system having a loop transfer function $\dfrac{K(s+3)}{(s+8)^2}$ is to be adjusted to induce a sustained oscillation then

(a) The frequency of this oscillation must be $\dfrac{4}{\sqrt{3}}$ rad/s

(b) The frequency of this oscillation must be 4 rad/s

(c) The frequency of this oscillation must be 4 or $\dfrac{4}{\sqrt{3}}$ rad/s

(d) Such K does not exist [Gate 2007]

62. If x = Re $[G(j\omega)]$ and y = Im $[G(j\omega)]$ then for $\omega \to 0^+$, the Nyquist plot for $G(s) = \dfrac{1}{s(s+1)(s+2)}$

(a) $x = 0$ (b) $x = -\dfrac{3}{4}$

(c) $x = y - \dfrac{1}{6}$ (d) $x = \dfrac{y}{\sqrt{3}}$ [Gate 2007]

63. The Nyquist plot of loop transfer function $G(s)H(s)$ of a closed loop control system passes through the point $(-1, j0)$ in the $G(s)H(s)$ plane. The phase margin of the system is

(a) 0° (b) 45°

(c) 90° (d) 180° [Gate 2004]

64. Consider the function $F(s) = \dfrac{5}{s(s^2 + 3s + 2)}$, where $F(s)$ is the Laplace transform of the function $f(t)$. The initial value of $f(t)$ is equal to

(a) 5 (b) $\dfrac{5}{2}$

(c) $\dfrac{5}{3}$ (d) 0 [Gate 2004]

65. For a tachometer, if $\theta(t)$ is the rotor displacement in radians, $e(t)$ is the output voltage and K_t is the tachometer constant in V/rad/s, then the transfer function, $\dfrac{E(s)}{\theta(s)}$ will be

(a) $K_t s^2$ (b) $\dfrac{K_t}{s}$

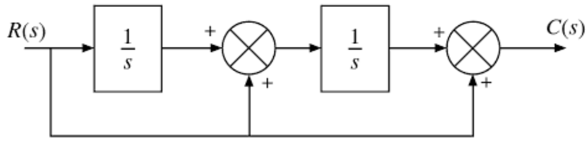
(c) $K_t s$ (d) K_t [Gate 2004]

66. For the equation, $s^3 - 4s^2 + s + 6 = 0$, the number of roots in the left half of s-plane will be

(a) 0 (b) 1

(c) 2 (d) 3 [Gate 2004]

67. For the block diagram shown in figure, the transfer function $\frac{C(s)}{R(s)}$ is equal to



(a) $\frac{s^2+1}{s^2}$ (b) $\frac{s^2+s+1}{s^2}$

(c) $\frac{1}{s^2+s+1}$ (d) $\frac{s^2+s+1}{s}$ [Gate 2004]

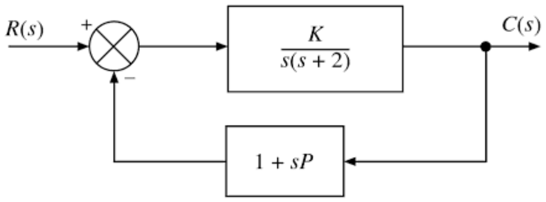
68. The state variable description of a linear autonomous system is $\dot{X} = AX$, where $\dot{X}$ is the two dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The roots of the characteristic equation are

(a) -2 and $+2$ (b) $-j2$ and $+j2$

(c) -2 and -2 (d) $+2$ and $+2$

[Gate 2004]

69. The block diagram of a closed loop control system is given in figure. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_n of 5 rad/s, are respectively equal to



(a) 20 and 0.3 (b) 20 and 0.2

(c) 25 and 0.3 (d) 25 and 0.2

[Gate 2004]

70. The unit impulse response of a second order underdamped system starting from rest is given by

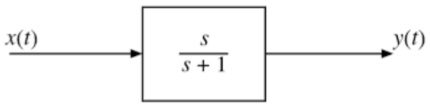
$$c(t) = 12.5e^{-6t}\sin 8t, t \geq 0$$

The steady state value of the unit step response of the system is equal to

(a) 0 (b) 0.25

(c) 0.5 (d) 1.0 [Gate 2004]

71. In the system shown in figure, the input $x(t) = \sin t$. In the steady state, the response $y(t)$ will be



(a) $\frac{1}{\sqrt{2}}\sin(t-45°)$ (b) $\frac{1}{\sqrt{2}}\sin(t+45°)$

(c) $\sin(t-45°)$ (d) $\sin(t+45°)$ [Gate 2004]

72. The open loop transfer function of a unity feedback control system is given as $G(s) = \frac{as+1}{s^2}$. The value of a to give a phase margin of 45° is equal to

(a) 0.141 (b) 0.441

(c) 0.841 (d) 1.141 [Gate 2004]

73. A function $y(t)$ satisfies the following differential equation:

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

where $\delta(t)$ is the delta function. Assuming zero initial condition and denoting the unit step function by $u(t)$, $y(t)$ can be of the form

(a) e^t (b) e^{-t}

(c) $e^t u(t)$ (d) $e^{-t}u(t)$ [Gate 2008]

74. The transfer function of a linear time invariant system is given as $G(s) = \frac{1}{s^2+3s+2}$. The steady state value of the output of the system for a unit impulse input applied at time instant $t = 1$ will be

(a) 0 (b) 0.5

(c) 1 (d) 2 [Gate 2008]

75. The transfer function of a system is given as

$$\frac{100}{s^2+20s+100}$$

The system is

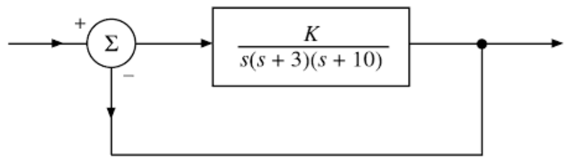
(a) An overdamped system

(b) An underdamped system

(c) A critically damped system

(d) An unstable system [Gate 2008]

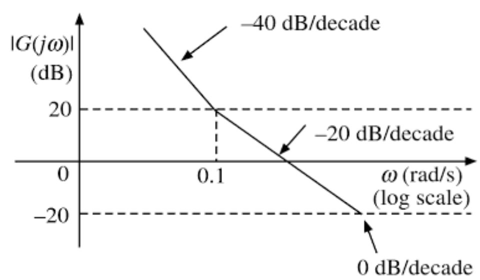
76. Figure shows a feedback system where $K > 0$. The range of K for which the system is stable will be given by



(a) $0 < K < 30$ (b) $0 < K < 39$

(c) $0 < K < 390$ (d) $K > 390$ [Gate 2008]

77. The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in figure. This transfer function has



(a) Three poles and one zero
(b) Two poles and one zero
(c) Two poles and two zeros
(d) One pole and two zeros [Gate 2008]

78. The transfer functions of two compensators are given as

$$C_1 = \frac{10(s+1)}{(s+10)},\ C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?
(a) C_1 is a lead compensator and C_2 is a lag compensator
(b) C_1 is a lag compensator and C_2 is a lead compensator
(c) Both C_1 and C_2 are lead compensators
(d) Both C_1 and C_2 are lag compensators [Gate 2008]

Statement for Linked Answer Questions: 79 and 80

The state space equation of a system is described by

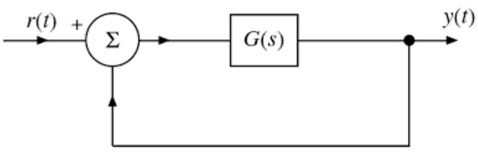
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where $\dot{x}$ is state vector, u is input, y is output and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

79. The transfer function $G(s)$ of this system will be
(a) $\frac{s}{(s+2)}$ (b) $\frac{s+1}{s(s-2)}$
(c) $\frac{s}{(s-2)}$ (d) $\frac{1}{s(s+2)}$ [Gate 2008]

80. A unity feedback is provided to the above system $G(s)$ to make it a closed loop system as shown in figure. For a unit step input $r(t)$, the steady state error in the output will be



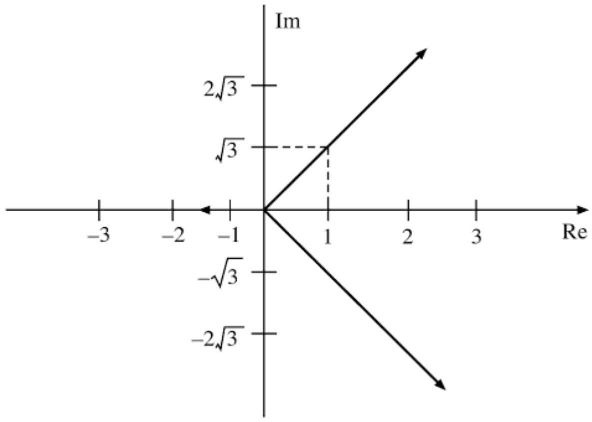
(a) 0 (b) 1
(c) 2 (d) ∞ [Gate 2008]

81. A system with zero initial conditions has the closed loop transfer function

$$T(s) = \frac{(s^2+4)}{(s+1)(s+4)}$$

The system output is zero at the frequency
(a) 0.5 rad/s
(b) 1 rad/s
(c) 2 rad/s
(d) 4 rad/s [Gate 2005]

82. Figure shows the root locus plot (location of plots not given) of a third order system whose open loop transfer function is

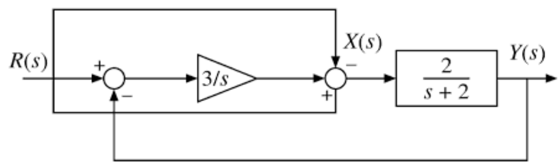


(a) $\frac{K}{s^3}$ (b) $\frac{K}{s^2(s+1)}$
(c) $\frac{K}{s(s^2+1)}$ (d) $\frac{K}{s(s^2-1)}$ [Gate 2005]

83. The gain margin of a unity feedback control system with the open loop transfer function $G(s) = \frac{(s+1)}{s^2}$ is
(a) 0 (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) ∞ [Gate 2005]

84. A unity feedback system, having an open loop gain $G(s)H(s) = \frac{K(1-s)}{(1+s)}$, becomes stable when
(a) $|K| > 1$ (b) $K > 1$
(c) $|K| < 1$ (d) $K < -1$ [Gate 2005]

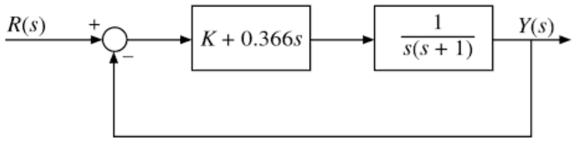
85. When subjected to a unit step input, the closed loop control system shown in figure will have a steady state error of



(a) − 1.0 (b) − 0.5
(c) 0 (d) 0.5 [Gate 2005]

86. In the $GH(s)$ plane, the Nyquist plot of the loop transfer function $G(s)H(s) = \frac{\pi\, e^{-0.25s}}{s}$ passes through the negative real axis at the point
(a) $(-0.25, j0)$ (b) $(-0.5, -j0)$
(c) $(-1, j0)$ (d) $(-2, j0)$ [Gate 2005]

87. If the compensated system shown in figure has a phase margin of 60° at the crossover frequency of 1 rad/s, then value of the gain K is



(a) 0.366 (b) 0.732
(c) 1.366 (d) 2.738 [Gate 2005]

Statement for Linked Answer Questions: 88 and 89

A state variable system

$\dot{X}(t)=\begin{bmatrix}0 & 1\\ 0 & -3\end{bmatrix}X(t)+\begin{bmatrix}1\\0\end{bmatrix}u(t)$, with the initial condition $X(0)=\begin{bmatrix}-1 & 3\end{bmatrix}^T$ and the unit step input $u(t)$ has:

88. The state transition matrix

(a) $\begin{bmatrix}1 & \frac{1}{3}(1-e^{-3t})\\ 0 & e^{-3t}\end{bmatrix}$ (b) $\begin{bmatrix}1 & \frac{1}{3}(e^{-t}-e^{-3t})\\ 0 & e^{-t}\end{bmatrix}$

(c) $\begin{bmatrix}1 & \frac{1}{3}(e^{-t}-e^{-3t})\\ 0 & e^{-3t}\end{bmatrix}$ (d) $\begin{bmatrix}1 & (1-e^{-t})\\ 0 & e^{-t}\end{bmatrix}$

[Gate 2005]

89. The state transition equation

(a) $X(t)=\begin{bmatrix}t-e^{-t}\\ e^{-t}\end{bmatrix}$

(b) $X(t)=\begin{bmatrix}t-e^{-t}\\ 3e^{-3t}\end{bmatrix}$

(c) $X(t)=\begin{bmatrix}t-e^{-3t}\\ 3e^{-3t}\end{bmatrix}$

(d) $X(t)=\begin{bmatrix}t-e^{-3t}\\ e^{-t}\end{bmatrix}$ [Gate 2005]

90. Signal flow graph is used to obtain
(a) Stability of a system
(b) Transfer function of a system
(c) Controllability of a system
(d) Observability of a system [Gate 1993]

91. The closed loop transfer function of a control system is given by

$$\frac{C(s)}{R(s)}=\frac{2(s-1)}{(s+2)(s+1)}$$

For a unit step input, the output is
(a) $-3e^{-2t}+4e^{t}-1$ (b) $-3e^{-2t}-4e^{t}+1$
(c) Zero (d) Infinity [Gate 1995]

92. A linear time invariant system initially at rest, when subjected to a unit step input, gives a response $y(t) = te^{-t}$, $t > 0$. The transfer function of the system is

(a) $\frac{1}{(s+1)^2}$ (b) $\frac{1}{s(s+1)^2}$

(c) $\frac{s}{(s+1)^2}$ (d) $\frac{1}{s(s+1)}$ [Gate 2000]

93. The transfer function of the system described by $\frac{d^2y}{dt^2}+\frac{dy}{dt}=\frac{du}{dt}+2u$ with u as input and y as output is

(a) $\frac{(s+2)}{(s^2+s)}$ (b) $\frac{(s+1)}{(s^2+s)}$

(c) $\frac{2}{(s^2+s)}$ (d) $\frac{2s}{(s^2+s)}$ [Gate 2002]

94. The unit impulse response of a unit feedback control system is given by $c(t)=-te^{-t}+2e^{-t}, (t\geq 0)$. The open loop transfer function is equal to

(a) $\frac{(s+1)}{(s+2)^2}$ (b) $\frac{(2s+1)}{s^2}$

(c) $\frac{s+1}{(s+1)^2}$ (d) $\frac{s+1}{s^2}$ [Gate 1996]

95. Feedback control systems are
(a) Insensitive to both forward and feedback path parameter changes
(b) Less sensitive to feedback path parameter change than to forward path parameter change
(c) Less sensitive to forward path parameter change than to feedback path parameter change
(d) Equally sensitive to forward and feedback path parameter changes [Gate 2000]

96. The steady state error due to a step input for type 1 system is .... [Gate 1995]

97. Consider the unit step response of a unity feedback control system whose open loop transfer function is

$$G(s)=\frac{1}{s(s+1)}$$

The maximum overshoot is equal to
(a) 0.143 (b) 0.153
(c) 0.163 (d) 0.173 [Gate 1996]

98. For a feedback control system of type 2, the steady state error for a ramp input is
(a) Infinite (b) Constant
(c) Zero (d) Indeterminate
[Gate 1996]

99. A unity feedback system has open loop transfer function $G(s)$. The steady state error is zero for
(a) Step input and type 1 $G(s)$
(b) Ramp input and type 1 $G(s)$
(c) Step input and type 0 $G(s)$
(d) Ramp input and type 0 $G(s)$ [Gate 2000]

100. A unity feedback system has open loop transfer function $G(s)=\dfrac{25}{s(s+6)}$. The peak overshoot in the step-input response of the system is approximately equal to

(a) 5% (b) 10%
(c) 15% (d) 20% [Gate 2000]

101. The number of positive real roots of the equation $s^3-2s+2=0$ is .... [Gate 1994]

102. Closed loop stability implies that $[1+G(s)H(s)]$ has all the .... in the left half of the s-plane. [Gate 1995]

103. The system represented by the transfer function

$$G(s)=\frac{s^2+10s+24}{s^4+6s^3-39s^2+19s+84}$$

has...............................pole(s) in the right half of s-plane. [Gate 1997]

104. None of the poles of a linear control system lie in the right half of s-plane. For a bounded input, the output of this system

(a) Is always bounded
(b) Could be unbounded
(c) Always tends to zero
(d) None of the above [Gate 1998]

105. The number of the roots on the equation $2s^4+s^3+3s^2+5s+7=0$ that lie in right half of s-plane is

(a) Zero (b) One
(c) Two (d) Three [Gate 1998]

106. A unity feedback system has the open loop transfer function

$$G(s)=\frac{1}{(s-1)(s+2)(s+3)}$$

How many time the Nyquist plot of $G(s)$ encircles the origin?

(a) Never (b) Once
(c) Twice (d) Thrice [Gate 1992]

107. The closed loop transfer function of a control system is given by

$$\frac{C(s)}{R(s)}=\frac{1}{1+s}$$

For the input $r(t)=\sin t$, the steady state value of $c(t)$ is equal to

(a) $\dfrac{1}{\sqrt{2}}\cos t$ (b) 1

(c) $\dfrac{1}{\sqrt{2}}\sin t$ (d) $\dfrac{1}{\sqrt{2}}\sin\left(t-\dfrac{\pi}{4}\right)$

[Gate 1996]

108. Introduction of integral action in the forward path of a unity feedback system results in a

(a) Marginally stability
(b) System with no steady state error
(c) System with increased stability margin
(d) System with better speed of response

[Gate 1997]

109. A differentiator has transfer function whose

(a) Phase increases linearly with frequency
(b) Amplitude remains constant
(c) Amplitude increases linearly with frequency
(d) Amplitude decreases linearly with frequency

[Gate 1997]

110. The phase lead compensation used to

(a) Increase rise time and decrease overshoot
(b) Decrease both rise time and overshoot
(c) Increase both rise time and overshoot
(d) Decrease rise time and increase overshoot

[Gate 1998]

111. For $G(s)=\dfrac{(0.5s+1)}{(0.05s+1)}$, maximum phase lead of the compensator is

(a) 52 deg at 4 rad/s (b) 52 deg at 10 rad/s
(c) 55 deg at 12 rad/s (d) None of these

[Gate 1998]

112. The transfer function of a state variable representation

$$\dot{X}=AX+BU, Y=CX+DU$$

is given by

(a) $D+C(sI-A)^{-1}+B$ (b) $B(sI-A)^{-1}C+D$
(c) $D(sI-A)^{-1}B+C$ (d) $C(sI-A)^{-1}D+B$

[Gate 1993]

113. The eigenvalues of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are

(a) $(a+1)$, 0 (b) a, 0
(c) $(a-1)$, 0 (d) 0, 0 [Gate 1994]

114. Given the matrix

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

Its eigenvalues are .... [Gate 1995]

115. The state transition matrix for the system $\dot{X}=AX$ with initial state $X(0)$ is

(a) $(sI-A)^{-1}$
(b) $e^{AT}X(0)$
(c) Laplace inverse of $[(sI-A)^{-1}]$
(d) Laplace inverse of $[(sI-A)^{-1}X(0)]$ [Gate 2002]

116. For the system $\dot{X} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, which of the following statement is true?

(a) The system is controllable but unstable
(b) The system is uncontrollable and unstable
(c) The system is controllable and stable
(d) The system is uncontrollable and stable

[Gate 2002]

Solutions

1. Ans: (b)

Hint: When all the elements of a row in Routh–Hurwitz array ends abruptly i.e., all elements of that row have a zero values then, the system will be either unstable or marginally stable. Marginally stable means it will have imaginary roots (two equal and opposite complex roots on imaginary axis of s-plane).

2. Ans: 0.61 to 0.23

Hint: Given,

$$G(s) = \frac{(s+\alpha)}{s^3 + (1+\alpha)s^2 + (\alpha-1)s + (1-\alpha)}$$

and $H(s) = 1$

Characteristic equation of given control system is given by

$$1 + G(s)H(s) = 0$$

or, $1 + \left[\dfrac{(s+\alpha)}{s^3 + (1+\alpha)s^2 + (\alpha-1)s + (1-\alpha)}\right] = 0$

or, $s^3 + (1+\alpha)s^2 + (\alpha-1)s + (1-\alpha) + s + \alpha = 0$

or, $s^3 + (1+\alpha)s^2 + \alpha s + 1 = 0$

Routh array is

s^3	1	α
s^2	$1+\alpha$	1
s^1	$\left(\dfrac{\alpha^2+\alpha-1}{1+\alpha}\right)$	0
s^0	1	0

For the given system to be stable there should not be any sign change in the first column of Routh array.

Therefore, $1 + \alpha > 0$ or, $\alpha > -1$ (i)

Also, $\dfrac{\alpha^2+\alpha-1}{1+\alpha} > 0$ or, $\alpha^2 + \alpha - 1 > 0$

or, $(\alpha + 1.618)(\alpha - 0.618) > 0$

or, $\alpha < -1.618$

or, $\alpha > 0.618$

As $\alpha > -1$ [using Eq. (i)]

Therefore, $\alpha > 0.618$ (ii)

Combining conditions from Eqs. (i) and (ii),

$$-1 < \alpha < 0.618$$

Thus, for the system to be stable, minimum value of $\alpha = 0.618$.

3. Ans: (c)

Hint: This is converse root locus having no zero.

As, its $G(s)H(s) = \dfrac{K}{(s+1)(s+2)}$

Its converse root locus only valid when $K < 0$

So,
$$\text{CLTF} = \frac{G(s)H(s)}{1 - G(s)H(s)} = \frac{\dfrac{K}{(s+1)(s+2)}}{1 - \dfrac{K}{(s+1)(s+2)}} = \frac{K}{(s+1)(s+2) - K}$$

4. Ans: (0.7 to 0.8)

Hint:

Given, $G(s) = \dfrac{K(1+0.5s)(1+as)}{s\left(1+\dfrac{s}{8}\right)(1+bs)\left(1+\dfrac{s}{36}\right)}$

$(1 + as)$ is addition of zero to the transfer function whose contribution in slope = +20 dB/decade or −6 dB/octave.

$(1 + bs)$ is addition of pole to the transfer function whose contribution in slope = –20 dB/decade or –6 dB/octave.

Observing the change in the slope at different corner frequencies, we conclude that

$$a = \frac{1}{4} \text{ rad/s} \quad \text{and} \quad b = \frac{1}{24} \text{ rad/s.}$$

From $\omega = 0.01$ rad/s to $\omega = 8$ rad/s, slope = – 20 dB/decade

Let the vertical length in dB be y

$\therefore \quad -20 = \left(\dfrac{0-y}{\log 8 - \log 0.01}\right)$

or, $\quad -20 = \dfrac{y}{\log 8 - 2}$

or, $\quad y = 58 \text{ dB}$

Applying $y = mx + C$ at $\omega = 0.01$ rad/s

We have, $58 = -20 \log 0.01 + C$

or, $C = 58 - 40 = 18$

Now, $C = 20 \log K$

or, $\log K = \dfrac{18}{20} = 0.9$

$\therefore \quad K = \log^{-1}(0.9) = 10^{0.9} = 7.94$

$$\therefore \quad \frac{a}{bK} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{24}\right)\times 7.94} = \frac{24}{4\times 7.94} = 0.755$$

$$\therefore \quad \frac{a}{bK} = 0.755$$

5. Ans: (d)

Hint: $$\frac{V_2(s)}{V_1(s)} = \frac{R+\frac{1}{Cs}}{\frac{1}{Cs}+R+\frac{1}{Cs}}$$

$$= \frac{1+10\times 10^3\times 100\times 10^{-6}s}{2+10\times 10^3\times 100\times 10^{-6}s}$$

$$= \frac{1+s}{2+s}$$

6. Ans: (a)

Hint: Applying Mason's gain formulae

$$P_1 = s^{-2}$$

$$P_2 = s^{-1}$$

$$\Delta_1 = 1$$

$$\Delta_2 = 2$$

$$\therefore \quad \Delta = 1-[-2s^{-1}-2s^{-2}-4-4s^{-1}]$$

or $$\Delta = \frac{5s^2+6s+2}{s^2}$$

$$\frac{Y(s)}{U(s)} = \frac{\sum P_K\Delta_K}{\Delta}$$

$$= \frac{\frac{1}{s^2}\times 1+\frac{1}{s}\times 1}{\frac{5s^2+6s+2}{s^2}}$$

$$= \frac{s+1}{5s^2+6s+2}$$

7. Ans: (c)

Hint: $$\frac{\omega(s)}{R(s)} = \frac{10}{1+10s} = \frac{10}{1+\tau s}$$

$$\tau = 10$$

Closed loop system

$$\frac{\omega(s)}{R(s)} = \frac{\frac{10K_a}{1+10s}}{1+\frac{10K_a}{1+10s}} = \frac{10K_a}{1+10K_a+10s}$$

$$= \frac{\frac{10K_a}{1+10K_a}}{1+\frac{10}{1+10K_a}s}$$

According to question,

$$\frac{10}{1+10K_a} = \frac{1}{100}\times 10$$

$$1+10K_a = 100$$

$$K_a = 9.9 \simeq 10$$

8. Ans: (b)

Hint: Let $G(s) = \frac{K}{s^n}$

Put $s = j\omega$

$$G(j\omega) = \frac{K}{(j\omega)^n}$$

$\because$ Gain = 32 dB at ω = 1 rad/s

$$\therefore \quad 20\log\left|\frac{K}{1^n}\right| = 32$$

or $$20\log K = 32$$

or $$K = 39.8$$

Now,

$\because$ Gain = –8 dB at ω = 10 rad/s

$$\therefore \quad 20\log\left|\frac{39.8}{1^n}\right| = -82$$

or $$10^n = 100$$

or $$n = 2$$

9. Ans: (a)

Hint:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

For controllability, $|B : AB| \neq 0$

or, $$\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1-(-2) = 1 \neq 0$$

The system is controllable.

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A^TC^T = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

For observability, $|C^T : A^TC^T| \neq 0$

or, $$\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix} = 0$$

The system is not observable.

10. Ans: (a)

Hint: $[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$

$$= \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$[sI - A]^{-1}[B] = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+2} \\ \frac{1}{s+1} \end{bmatrix}$$

$$C[sI - A]^{-1}[B] = [1 \quad 0] \begin{bmatrix} \frac{1}{s+2} \\ \frac{1}{s+1} \end{bmatrix} = \frac{1}{s+2}$$

$$G(s) = \frac{1}{s+2}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

$$Y(s) = \frac{1}{s(s+2)} = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

$$y(t) = \frac{1}{2}(1 - e^{-2t}) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

11. Ans: (c)

Hint: Applying Laplace transform on both sides

$$(s^2 + 6s + 5)X(s) = 12\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

or $$X(s) = \frac{24}{s(s+2)(s+1)(s+5)}$$

Using final value theorem

$$\operatorname*{Lt}_{t\to\infty} X(t) = \operatorname*{Lt}_{s\to 0} sX(s) = 2.4$$

12. Ans: (a)

Hint: For lead compensator,

$$T(s) = \frac{K\left(1 + \frac{s}{a}\right)}{\left(1 + \frac{s}{b}\right)}$$

Now, expression of lead compensator

$T(s) = K_c \alpha \left(\frac{Ts+1}{\alpha Ts + 1}\right)$, where $0 < \alpha < 1$

Comparing, $$T = \frac{1}{a}$$

$$\alpha T = \frac{1}{b}$$

$$\alpha = \frac{a}{b}$$

$$0 < \alpha < 1 \Rightarrow 0 < \frac{a}{b} < 1 \Rightarrow a < b$$

13. Ans: (a)

Hint: Given, $X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

The state transition matrix $= \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix} = \phi(t)$

The state of system at time t is

$$X(t) = \phi(t)X(0) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ 3e^{-t} \end{bmatrix}$$

At t = 1s, $X(1) = \begin{bmatrix} 2e^{-2} \\ 3e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$

14. Ans: (b)

Hint: Natural time constants of the response of the system depend only on poles.

The given expression

$$M(s) = \frac{1}{s^2 + \frac{1}{2}s + \frac{1}{18}} = \frac{18}{18s^2 + 9s + 1} = \frac{18}{(6s+1)(3s+1)}$$

The general expression $= \dfrac{A}{(T_1 s + 1)(T_2 s + 1)}$

Hence, time constants T_1 and T_2 will be 3s and 6s.

15. Ans: (a)

Hint: Now, $G(s) = \left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)$ and $H(s) = 1$

Steady state error $e_{ss} = \dfrac{1}{1 + K_p}$

Where $K_p = \operatorname*{Lim}_{s\to 0} G(s)H(s) = 3$

So, $e_{ss} = \dfrac{1}{1+3} = 0.25$ or 25 %

16. Ans: (d)

Hint: The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{45}{(s+15)(s+1)} = 0$$

or $$s^2 + 16s + 60 = 0$$

$$s^2 + 10s + 6s + 60 = 0$$

$$(s+6)(s+10) = 0$$

Hence, the roots are (– 6, –10)

17. Ans: (a)

Hint: The given differential equation

$$\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$$

or $$\frac{d^2\omega}{dt^2}=-\frac{B}{J}\frac{d\omega}{dt}-\frac{K^2}{LJ}\omega+\frac{K}{LJ}V_a$$

And $$\frac{d\omega}{dt}=1\cdot\frac{d\omega}{dt}+0\cdot\omega$$

In the matrix form,

$$\begin{bmatrix}\dfrac{d^2\omega}{dt^2}\\ \dfrac{d\omega}{dt}\end{bmatrix}=\begin{bmatrix}-\dfrac{B}{J} & -\dfrac{K^2}{LJ}\\ 1 & 0\end{bmatrix}\begin{bmatrix}\dfrac{d\omega}{dt}\\ \omega\end{bmatrix}+\begin{bmatrix}\dfrac{K}{LJ}\\ 0\end{bmatrix}V_a$$

Comparing the given state space,

$$\begin{bmatrix}\dfrac{d^2\omega}{dt^2}\\ \dfrac{d\omega}{dt}\end{bmatrix}=P\begin{bmatrix}\dfrac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a$$

Hence, $$[P]=\begin{bmatrix}-\dfrac{B}{J} & -\dfrac{K^2}{LJ}\\ 1 & 0\end{bmatrix}$$

18. Ans: (c)

Hint: The characteristic equation is

$$1+G(s)H(s)=0$$

$$1+\frac{K}{s(s+2)(s+4)}=0$$

$$s(s^2+6s+8)+K=0$$

$$s^3+6s^2+8s+K=0$$

The Routh array is

s^3	1	8
s^2	6	K
s	$\dfrac{48-K}{6}$	0
1	K	

Now, for the system to be stable

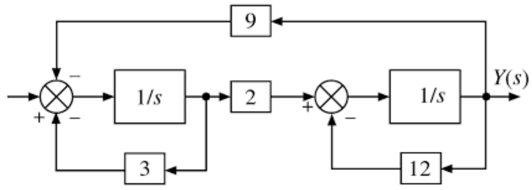
$$\frac{48-K}{6}>0 \quad\text{and}\quad K>0$$

So, $0 < K < 48$

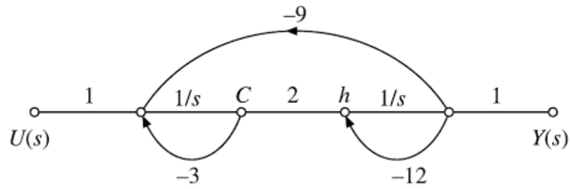
Hence, for $K = 48$, the system just becomes unstable.

19. Ans: (b)

Hint: By changing the integrator,



The corresponding signal flow graph



Using Mason's gain formula, the overall transfer function

$$=\frac{\dfrac{2}{s^2}}{1+\dfrac{3}{s}+\dfrac{12}{s}+\dfrac{18}{s^2}+\dfrac{36}{s^2}}$$

$$=\frac{2}{s^2+15s+54}$$

$$=\frac{2}{(s+9)(s+6)}$$

$$=\frac{2}{9\left(1+\dfrac{s}{9}\right)\times 6\left(1+\dfrac{s}{6}\right)}=\frac{1}{27\left(1+\dfrac{s}{9}\right)\left(1+\dfrac{s}{6}\right)}$$

20. Ans: (a)

Hint: The characteristic equation

$$1 + G(s)H(s) = 0$$

or $$s^3+as^2+(K+2)s+(K+1)=0$$

Routh array

s^3	1	$(K+2)$
s^2	a	$(K+1)$
s^1	$\dfrac{a(K+2)-(K+1)}{a}$	0
s^0	$(K+1)$	0

For oscillates at

$$\omega=2\text{ rad/s}$$

$$a(K+2)-(K+1)=0$$

or $$a=\frac{K+1}{K+2}$$

and $$as^2+(K+1)=0$$

Put $s = j2$

$$-4a+(K+1)=0$$

$$K=4a-1$$

or $a = 0.75$ and $K = 2$

21. Ans: (c)

Hint: Put $s = j\omega$

$$\therefore \qquad G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+1)(j\omega+3)(j\omega+4)}$$

Steady state output of the system is zero when

$$|G(j\omega)| = 0 = \frac{(-\omega^2+9)\sqrt{\omega^2+4}}{\left(\sqrt{\omega^2+1}\right)\left(\sqrt{\omega^2+9}\right)\left(\sqrt{\omega^2+16}\right)}$$

So, for zero steady state output

$$\omega^2 = 9$$

or $\qquad \omega = 3$ rad/s

22. Ans: (a)

Hint:

$$G_c(j\omega) = \frac{j\omega + a}{j\omega + b}$$

$$\angle G_c(j\omega) = \tan^{-1}\frac{\omega}{a} - \tan^{-1}\frac{\omega}{b}$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right)$$

For lead compensator

$$\angle G_c(j\omega) > 0$$

$$\frac{\omega}{a} > \frac{\omega}{b}$$

or $\qquad b > a$

23. Ans: (a)

Hint: For phase of the lead compensator to be maximum,

$$\frac{d\angle G_c(j\omega)}{dt} = 0$$

or $\qquad \left(1+\frac{\omega^2}{ab}\right)\left(\frac{1}{a}-\frac{1}{b}\right)-\left(\frac{\omega}{a}-\frac{\omega}{b}\right)\left(\frac{2\omega}{ab}\right)=0$

or $\qquad \left(1+\frac{\omega^2}{2}\right)\left(1-\frac{1}{2}\right)-\left(\frac{\omega}{1}-\frac{\omega}{2}\right)\left(\frac{2\omega}{2}\right)=0$

or $\qquad \frac{\omega^2}{2} = 1$

or $\qquad \omega = \sqrt{2}$ rad/s

24. Ans: (d)

Hint: For system to be controllable, Q must be non-singular.

$Q = [B \quad AB \quad A^2B]$

or $\qquad Q = \begin{bmatrix} 0 & 0 & a_1a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\therefore \qquad |Q| = -a_1a_2^2$$

$$|Q| \neq 0$$

$\therefore \qquad a_1 \neq 0$ and $a_2 \neq 0$ and $a_3 = 0$

25. Ans: (a)

Hint:

$$G = 100 \pm 10\%$$

$$\frac{dG}{G} = 10\% \text{ or } 0.1$$

$$H = \frac{9}{100}$$

Overall system gain

$$M = \frac{G}{1+GH} = \frac{100}{1+100\times\frac{9}{100}} = 10$$

$$\frac{dM}{dG} = \frac{(1+GH)-GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}.$$

or $\qquad dM = \frac{dG}{(1+GH)^2}$

$$\frac{dM}{M} = \frac{dG}{(1+GH)^2}\times\frac{(1+GH)}{G}$$

$$= 10\times\frac{1}{\left(1+100\times\frac{9}{100}\right)}\% = 1\%$$

So, overall system gain = 10 ± 1%

26. Ans: (c)

Hint:

$$\frac{C(s)}{R(s)} = \frac{2}{s+1}$$

$$R(s) = \frac{1}{s} \quad \text{(Step input)}$$

$$C(s) = \frac{2}{s(s+1)} = 2\left[\frac{1}{s}-\frac{1}{s+1}\right]$$

$$c(t) = 2[1-e^{-t}]$$

Final value of $c(t) = 2$

Let at $t = T$, the response reaches 98% of its final values.

$$2\times 0.98 = 2[1-e^{-T}]$$

or $\quad T \approx 4$ s.

27. Ans: (c)

Hint: The characteristic equation is

$s(s+1)(s+3)+K(s+2)=0$

or $$1+\frac{K(s+2)}{s(s+1)(s+3)}=0$$

Open loop transfer function $G(s)H(s)=\dfrac{K(s+2)}{s(s+1)(s+2)}$

Number of zeros = m = 1 [zero at s = –2]

Number of poles = n = 3 [poles at s = 0,–1,–3]

Number of branches terminating at infinity

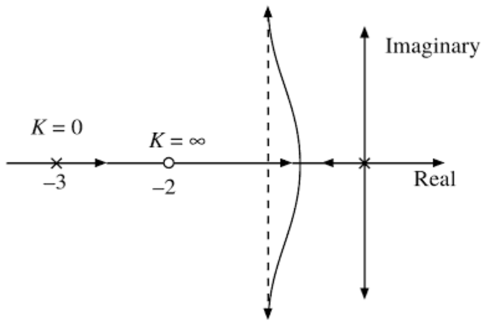
$$=n-m=3-1=2$$

Angle of asymptotes

$$=\frac{(2K+1)\times 180^\circ}{n-m}$$

$$= 90^\circ \text{ and } 270^\circ$$

Centroid $=\dfrac{0-1-3-(2)}{2}=-1$



Breakaway points lie in the range $-1 < \text{Re}[s] < 0$ and two of its roots tend to infinity along the asymptotes Re[s] = –1.

28. Ans: (c)

Hint:

Now,

$$[sI-A]=\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}-\begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} s+1 & -2 \\ 0 & s-2 \end{bmatrix}$$

$$[sI-A]^{-1}=\frac{\begin{bmatrix} s-2 & 2 \\ 0 & s+1 \end{bmatrix}}{(s+1)(s-2)} \quad \text{(i)}$$

Transfer function = $C[sI - A]^{-1}\, B$

Now, denominator of Eq. (i) gives poles of the system

So, poles are at

s = –1 and 2

As one pole lies in RHS of s-plane, hence, the system is unstable.

For controllability, $Q=[B \quad AB]=\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$

$\therefore$ $|Q| \neq 0$

Hence, the system is controllable.

29. Ans: (b)

Hint: $H(s)=\dfrac{Y(s)}{U(s)}=\dfrac{3(s-2)}{s^3+4s^2-2s+1}$

$$\frac{Y(s)}{X_1(s)}\times\frac{X_1(s)}{U(s)}=3(s-2)\times\frac{1}{s^3+4s^2-2s+1}$$

Let $$\frac{X_1(s)}{U(s)}=\frac{1}{s^3+4s^2-2s+1}$$

or $s^3X_1(s)+4s^2X_1(s)-2sX_1(s)+X_1(s)=U(s)$

Let $$\dot{x}_1=x_2$$
$$\dot{x}_2=x_3$$
$$\dot{x}_3=-x_1+2x_2-4x_3+u$$

$\therefore$ $$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$$

30. Ans: (a)

Hint: $H(j\omega)=\dfrac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)}$

$$=\frac{0.1(1+j\omega)}{\left(1+\dfrac{j\omega}{10}\right)\left(1+\dfrac{j\omega}{100}\right)^2}$$

Corner frequencies are $\omega_1 = 1$ rad/s

$\omega_2 = 10$ rad/s

$\omega_3 = 100$ rad/s

For $\omega < \omega_1$, as there is no pole at origin, so gain of the system is constant.

Gain $= 20 \log 0.1 = -20$ dB

So, Bode magnitude plot starts at –20 dB.

31. Ans: (b)

Hint: The characteristic equation is

$$1+\frac{K(s-1)}{(s+1)(s^2-4)}=0$$

$\therefore$ $$G(s)H(s)=\frac{K(s-1)}{(s+1)(s^2-4)}$$

Number of zero, m = 1 (s = 1)

Number of poles n = 3 (s = –1, –2, 2)

Angle of asymptotes $=\dfrac{(2K+1)\times 180^\circ}{n-m}$

$= 90^\circ, 270^\circ$

Centroid $=\dfrac{(-1-2+2)-(1)}{3-1}=-1$

32. Ans: (a)

Hint: For stable closed loop system, the Nyquist plot must not encircle the (–1, 0) point.

The plot of (1) does not enclose (–1, 0) point whereas other plots enclosed.

33. Ans: 0

Hint: Closed loop transfer function,

$$T(s) = \frac{4}{(s^2 + 0.4s + 4)}$$

or $$\frac{G(s)}{1+G(s)} = \left(\frac{4}{s^2 + 0.4s + 4}\right) \quad [H(s) = 1]$$

or $$G(s) = \frac{4}{s(s+0.4)}$$

It is the open loop transfer function for unity feedback system.

Given, input = $u(t) = r(t)$

$\therefore$ $$R(s) = \frac{1}{s}$$

Steady state error,

$$e_{ss} = \lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)H(s)}\right]$$

$$= \lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)}\right] \qquad (\text{For } H(s) = 1)$$

$$= \lim_{s\to 0}\left[\frac{s\times\frac{1}{s}}{1+\frac{4}{s(s+0.4)}}\right]$$

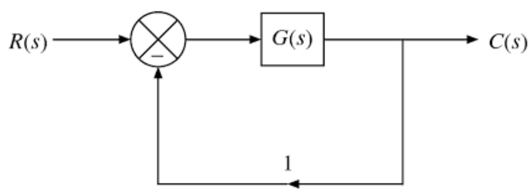
$$= \lim_{s\to 0}\left[\frac{s(s+0.4)}{s^2+0.4s+4}\right] = 0$$

$\therefore$ Steady state error for step input, $e_{ss} = 0$.

34. Ans: 5

Hint:

Given, $$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$



The characteristic equation of the given unity negative feedback control system is given by

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K}{s(s+2)(s^2+2s+2)} = 0$$

or $$s^4 + 4s^3 + 6s^2 + 4s + K = 0$$

Routh array

$$\begin{array}{cccc} s^4 & 1 & 6 & K \\ s^3 & 4 & 4 & 0 \\ s^2 & 5 & K & 0 \\ s^1 & \left(\frac{20-4K}{5}\right) & 0 & 0 \\ s^0 & K & 0 & 0 \end{array}$$

For stability of the system, $K > 0$ and

$$\frac{20-4K}{5} > 0 \text{ or } K < 5$$

$\therefore$ For stability, $0 < K < 5$

For the given system to be marginally stable,

$$K = 5$$

35. Ans: (a)

Hint:

Given, $$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

Reducing $G(s)$ in time constant form, we have:

$$G(s) = \frac{5\times 4\left(1+\frac{s}{4}\right)}{s\left[0.25\left(1+\frac{s}{0.25}\right)\right]\left[25\left(\frac{1}{25}s^2+\frac{4}{25}s+1\right)\right]}$$

$$= \frac{20(1+0.25s)}{s\left[0.25(1+4s)\right]\left[25(1+0.16s+0.04s^2)\right]}$$

or $$G(s) = \frac{3.2(1+0.25s)}{s(1+4s)(1+0.16s+0.04s^2)}$$

$\therefore$ The value of constant gain term = 3.2

Also, $$G(j\omega) = \frac{3.2\left(1+\frac{j\omega}{4}\right)}{(j\omega)\left(1+\frac{j\omega}{0.25}\right)\left(1+j0.16\omega-\frac{\omega^2}{25}\right)}$$

Corner frequencies are:

ω_1 = 4 rad/s

ω_2 = 0.25 rad/s

ω_3 = 5 rad/s

$\therefore$ Highest corner frequency = ω_3 = 5 rad/s.

36. Ans: (c)

Hint:

Given, $\dot{x}(t) = Px + Qu$

$$y = Rx$$

$$[sI-A]^{-1} = \frac{\begin{bmatrix} s+2 & -1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

Transfer function

$$= C[sI-A]^{-1}B + D$$

$$= [1 \quad 0]\frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0$$

$$= \frac{2s+5}{s^2+5s+6}$$

45. Ans: (b)

Hint:

$$[sI-A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)(s+3)}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)} - \frac{1}{(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

State transition matrix

$$\phi(t) = L^{-1}[(sI-A)^{-1}]$$

$$= \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

46. Ans: (c)

Hint: Transfer function

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

Now,

$$H(s) = \frac{C(s)}{R(s)} = \left(\frac{1}{s+1} + \frac{1}{s+2}\right)$$

$$R(s) = \frac{1}{s} \qquad \text{(Step input)}$$

$$C(s) = R(s)H(s) = \frac{1}{s}\left(\frac{1}{s+1} + \frac{1}{s+2}\right)$$

$$= \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2}$$

Hence, response

$$c(t) = (1.5 - e^{-t} - 0.5e^{-2t})u(t)$$

47. Ans: (a)

Hint: Steady state error

$$e_{ss} = \underset{s\to 0}{\text{Lt}}\, sE(s)$$

For unity feedback system with unit step input

$$e_{ss} = \underset{s\to 0}{\text{Lt}} \frac{s.\frac{1}{s}}{1+G(s)} = \underset{s\to 0}{\text{Lt}} \frac{1}{1+G(s)} = 0.1$$

For pulse input,

$$r(t) = 10[u(t) - u(t-1)]$$

$$R(s) = 10\left[\frac{1}{s} - \frac{e^{-s}}{s}\right] = \frac{10}{s}[1 - e^{-s}]$$

$$e_{ss} = \underset{s\to 0}{\text{Lt}} \frac{s \times \frac{10}{s}(1-e^{-s})}{1+G(s)}$$

or $$e_{ss} = \underset{s\to 0}{\text{Lt}} \frac{10(1-e^{-s})}{1+G(s)} = 0$$

48. Ans: (a)

Hint:

$$G(s) = \frac{\frac{1}{s(s+1)}}{1 + \frac{1}{s(s+1)}.Ks} = \frac{1}{s(s+1+K)} \quad \text{and } H(s) = 1$$

So, the characteristic equation

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{1}{s(s+1+K)} = 0$$

or $$s^2 + (K+1)s + 1 = 0$$

Comparing with standard equation

$$s^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$$

Natural frequency $\omega_n = 1 = \text{constant}$

$$2\xi\omega_n = K+1$$

$$\xi = \frac{K+1}{2}$$

So, damping ratio depends on K

Peak overshoot = $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

As, M_p depends on damping ratio ξ

So, M_p depends on K.

49. Ans: (b)

Hint: One zero at s = 1

Two poles at s = –2 and –3

So, it is non-minimum phase type system.

Since both poles lie in LHS of s-plane,

So, system is stable.

50. Ans: (a)

Hint: Number of zero m = 1

Number of poles n = 3

Number of branches terminating at infinity

$$= n - m = 3 - 1 = 2$$

Angle of asymptotes $= \dfrac{(2k+1)\times 180°}{n-m}$

$$= 90° \text{ and } 270°$$

Centroid $= \dfrac{\sum \text{poles} - \sum \text{zeros}}{n-m}$

$$= \frac{0+0-2-\left(-\frac{2}{3}\right)}{2} = -\frac{2}{3}$$

Now, the characteristic equation $1 + G(s)H(s) = 0$

$$\Rightarrow \qquad s^3 + 2s^2 + Ks + \frac{2K}{3} = 0$$

Routh Array

s^3	1	K
s^2	2	$\frac{2K}{3}$
s^1	$\dfrac{2K - \frac{2K}{3}}{2} = \dfrac{2K}{3}$	0
s^0	$\frac{2K}{3}$	0

For, $K > 0$, no sign change in the 1st column of Routh array as well as no row of array is zero.

So, all the three roots lie on LHS of s-plane and root loci does not cross the $j\omega$ axis.

So, all the three roots with nearly equal real parts exists on the left half of the s-plane.

51. Ans: (a)

Hint: Let gain crossover frequency = (ω_{gc}) and phase crossover frequency =(ω_{pc}),

So, at ω_{gc}, magnitude of $G(j\omega)$ is 1 and at ω_{pc}, phase of $G(j\omega)$ is –180°

$$\therefore \quad |G(j\omega_{gc})| = 1 \quad \text{and} \quad \angle G(j\omega_{pc}) = -180°$$

Phase margin $= 180° + \angle G(j\omega_{gc}) = 180° - 150° = 30°$

Gain margin $= 20 \log \dfrac{1}{M}$

$$= 20 \log \frac{1}{0.5} = 6 \text{ dB}$$

52. Ans: (c)

Hint: Applying Mason's gain formula,

Transfer function, $G(s) = \dfrac{C(s)}{U(s)} = \dfrac{\sum P_K \Delta_K}{\Delta}$

Forward paths are:

$$P_1 = \frac{h_0}{s^2}, P_2 = \frac{h_1}{s^2}$$

Individual loops are:

$$L_1 = \frac{-a_1}{s}, L_2 = \frac{-a_0}{s^2}$$

Non-touching loops are zero

$$\therefore \quad \Delta_1 = 1,$$

$$\Delta_2 = 1 - \left(\frac{-a_1}{s}\right) = \left(\frac{s+a_1}{s}\right)$$

$$\Delta = 1 - \left(\frac{-a_1}{s} - \frac{-a_0}{s^2}\right) = \frac{s^2 + a_1 s + a_0}{s^2}$$

$$\therefore \quad G(s) = \frac{C(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{\left(\frac{h_0}{s^2}\right)\times 1 + \left(\frac{h_1}{s}\right)\times\left(\frac{s+a_1}{a}\right)}{\left(\frac{s^2 + a_1 s + a_0}{s^2}\right)}$$

$$= \left[\frac{h_0 + h_1(s+a_1)}{s^2 + a_1 s + a_0}\right]$$

Given, $h_1 = b_1$ and $h_0 = b_0 - b_1 a_1$

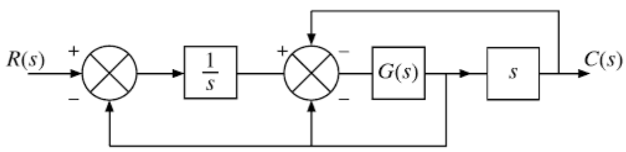
Thus, $\quad G(s) = \dfrac{(b_0 - b_1 a_1) + b_1(s+a_1)}{(s^2 + a_1 s + a_0)}$

$$= \left(\frac{b_1 s + b_0}{s_2 + a_1 s + a_0}\right)$$

$$G(s) = \left(\frac{b_1 s + b_0}{s_2 + a_1 s + a_0}\right)$$

53. Ans: (b)

Hint:



If $G(s) = s$

$$\frac{C(s)}{R(s)} = \frac{s}{s^2 + s + 2}$$

54. Ans: (a)

Hint: Given, $|\text{Open loop transfer function}| < 1$

i.e., $\quad |G(s)H(s)| < 1$

or, $\quad |G(j\omega)H(j\omega)| < 1$

or, $$|X| < 1$$

$\therefore$ Gain margin, $G_M = \dfrac{1}{|X|}$ i.e., $G_M > 1$

$\therefore$ Gain margin (in dB) > 0

Since gain margin of the given system is positive, therefore, the system is always stable.

55. Ans: (c)

Hint: From the given Bode plot, it is clear that corner frequencies

$$\omega_{c_1} = \frac{1}{3} \text{ and } \omega_{c_2} = 1$$

$\therefore$ Transfer function of given system is given by

$$T(s) = \frac{C(s)}{R(s)} = \left[\frac{1+3s}{1+s}\right] = \left[\frac{1+Ts}{1+\alpha Ts}\right]$$

Here, $T = 3$ and $\alpha T = 1$ or, $\alpha = \dfrac{1}{3}$

As $\alpha < 1$, therefore, the transfer function $T(s)$ represents a lead compensator having

$$\alpha = \frac{1}{3}$$

$\therefore$ Maximum phase shift,

$$\phi_m = \tan^{-1}\left[\frac{1-\alpha}{2\sqrt{\alpha}}\right] = \tan^{-1}\left[\frac{1-\dfrac{1}{3}}{\dfrac{2}{\sqrt{3}}}\right]$$

$$= \tan^{-1}\left[\frac{1}{\sqrt{3}}\right] = 30°$$

$\therefore$ $\phi_m = 30°$

Also,

$$T(j\omega) = \frac{1+j3\omega}{1+j\omega}$$

$\therefore$ $$|T(j\omega)| = \frac{\sqrt{1+9\omega^2}}{\sqrt{1+\omega^2}}$$

$\therefore$ Gain G_M in dB $= 20\log|T(j\omega)|$

$$= 20\log_{10}\left[\frac{\sqrt{1+9\omega_m^2}}{\sqrt{1+\omega_m^2}}\right]$$

Now, $$\omega_m = \frac{1}{T\sqrt{\alpha}} = \frac{1}{\sqrt{3}}$$

$\therefore$ $$G_M = 20\log_{10}\left[\sqrt{\frac{1+9\times\left(\dfrac{1}{\sqrt{3}}\right)^2}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2}}\right]$$

$$= 20\log_{10}\sqrt{3}$$

or, $$G_M = 4.77 \text{ dB}$$

56. Ans: (d)

Hint:

Transfer function of the given system is given by

$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B$$

Now, $$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix}$$

$\therefore$ $$(sI - A)^{-1} = \frac{1}{(s^2+s+1)}\begin{bmatrix} s+1 & 1 \\ -1 & s \end{bmatrix}$$

$\therefore$ Transfer function,

$$\frac{Y(s)}{U(s)} = \left(\frac{1}{s^2+s+1}\right)\left\{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s+1 & 1 \\ -1 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$$

$$= \left(\frac{1}{s^2+s+1}\right)$$

$\therefore$ $$\frac{Y(s)}{U(s)} = \left(\frac{1}{s^2+s+1}\right)$$

Given, input = unit step

$\therefore$ $$U(s) = \left(\frac{1}{s}\right)$$

$\therefore$ $$Y(s) = \frac{1}{s} \times \frac{1}{s^2+s+1} = \frac{1}{s(s^2+s+1)}$$

$\therefore$ Steady state error,

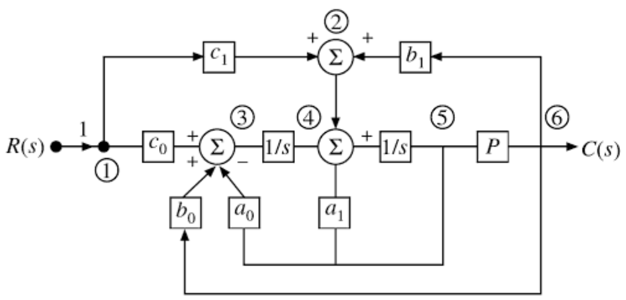
$$e_{ss} = \operatorname*{Lt}_{s\to 0}\{sY(s)\}$$

$$= \operatorname*{Lt}_{s\to 0}\left[\frac{1}{s^2+s+1}\right] = 1$$

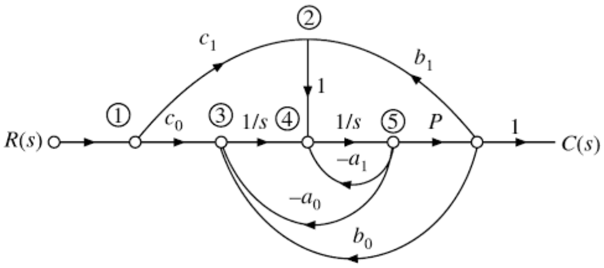
$\therefore$ $e_{ss} = 1$

57. Ans: (d)

Hint: The block diagram can be redrawn as



Signal flow graph of the block diagram shown as



Two forward paths

$$P_1 = 1 \times c_0 \times \frac{1}{s} \times \frac{1}{s} \times P = \frac{c_0 P}{s^2}$$

$$P_2 = 1 \times c_1 \times 1 \times \frac{1}{s} \times P = \frac{c_1 P}{s}$$

Four individual loops

$$L_1 = (-a_1) \times \frac{1}{s} = -\frac{a_1}{s}$$

$$L_2 = (-a_0) \times \frac{1}{s} \times \frac{1}{s} = -\frac{a_0}{s^2}$$

$$L_3 = b_0 \times P \times \frac{1}{s} \times \frac{1}{s} = \frac{b_0 P}{s^2}$$

$$L_4 = b_1 \times P \times \frac{1}{s} = \frac{b_1 P}{s}$$

All the loops touch forward paths

$$\Delta_1 = \Delta_2 = 1$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$$

$$= 1 + \frac{a_0}{s^2} + \frac{a_1}{s} - \frac{b_0 P}{s^2} - \frac{b_1 P}{s}$$

From Mason's gain formula

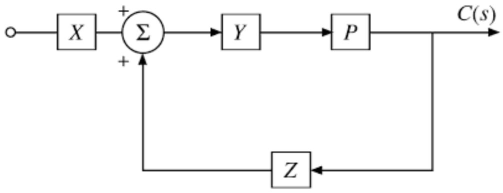
$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{s}$$

$$= \frac{\frac{c_0 P}{s^2} + \frac{c_1 P}{s}}{1 + \frac{a_0}{s^2} + \frac{a_1}{s} - \frac{b_0 P}{s^2} - \frac{b_1 P}{s}}$$

$$\frac{C(s)}{R(s)} = \frac{c_0 P + c_1 P_s}{s^2 + (a_0 - b_1)s + (a_0 - b_0 P)}$$

$$\frac{C(s)}{R(s)} = \frac{P(c_0 + c_1 s)/(s^2 + a_1 s + a_0)}{1 - (b_0 + b_1 s)P/(s^2 + a_1 s + a_0)} \quad \text{(i)}$$

$$\frac{C(s)}{R(s)} = \frac{XYP}{1 - YZP} \quad \text{(ii)}$$



Comparing Eqs (i) and (ii),

$$XY = \frac{c_0 + c_1 s}{s^2 + a_1 s + a_0}$$

$$YZ = \frac{b_0 + b_1 s}{s^2 + a_1 s + a_0}$$

58. Ans: (a)

Hint: Step input

$$R(s) = \frac{1}{s}$$

$$G(s) = \left(K_p + \frac{K_i}{s}\right)\left(\frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}\right)$$

And $H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{G(s)}{1 + G(s)}$$

$$E(s) = R(s) - C(s)$$

$$= \frac{R(s)}{1 + G(s)}$$

$$Z(s) = \frac{K_i}{s} E(s) = \frac{\frac{K_i}{s} R(s)}{1 + G(s)}$$

Steady state value of Z

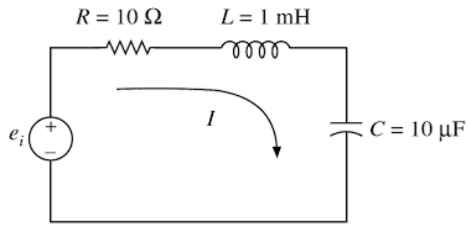
$$Z_{ss} = \operatorname*{Lt}_{s \to 0} sZ(s)$$

$$= \operatorname*{Lt}_{s \to 0} \frac{s \cdot \frac{K_i}{s} \cdot \frac{1}{s}}{1 + \left(K_P + \frac{K_i}{s}\right)\left(\frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}\right)}$$

$$= \frac{K_i}{K_i \cdot \frac{\omega^2}{\omega^2}} = 1$$

59. Ans: (c)

Hint:



$$e_i = R_I + L\frac{dI}{dt} + \frac{1}{C}\int I dt$$

Taking Laplace transform

$$E_i(s) = \left(R + Ls + \frac{1}{Cs}\right)I(s)$$

$$I(s) = \frac{E_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$e_o = \frac{1}{c}\int I dt$$

or $$E_o(s) = \frac{1}{Cs}I(s)$$

or $$E_o(s) = \frac{E_i(s)}{RCs + LCs^2 + 1}$$

or $$\frac{E_0(s)}{E_i(s)} = \frac{1}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

Characteristic equation

or $$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Comparing with

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{L}\times\frac{1}{2\omega_n} = \frac{R}{L}\times\frac{\sqrt{LC}}{2} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2}\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}} = 0.5$$

$$\text{Overshoot } = M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

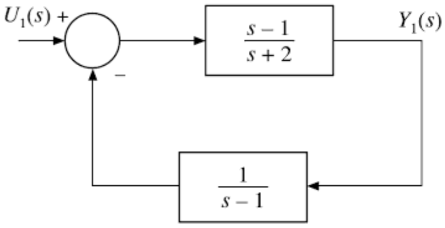
$$= e^{-\pi\times0.5/\sqrt{1-0.5^2}}$$

$$= 0.163$$

$$\approx 16\%$$

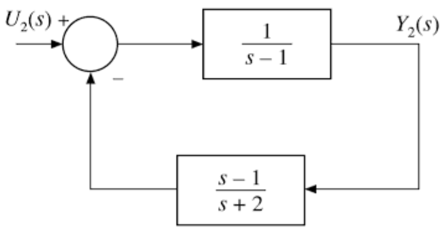
60. Ans: (d)

Hint:



$$\text{TF}_1 = \frac{Y_1(s)}{U_1(s)} = \frac{\frac{(s-1)}{(s+2)}}{1 + \frac{(s-1)}{(s+2)}\times\frac{1}{(s-1)}} = \frac{(s-1)}{(s+3)}$$

Pole is at $s = -3$, hence stable.



$$\text{TF}_2 = \frac{Y_2(s)}{U_2(s)} = \frac{\frac{1}{(s-1)}}{1 + \frac{1}{(s-1)}\times\frac{(s-1)}{(s+2)}} = \frac{(s+2)}{(s-1)(s+3)}$$

Poles are at $s = 1, -3$. So, unstable.

61. Ans: (b)

Hint: Loop transfer function

$$G(s)H(s) = K\frac{(s+3)}{(s+8)^2}$$

$\therefore$ Characteristic equation $= 1 + G(s)H(s) = 0$

$\Rightarrow$ $$s^2 + (16+K)s + 64 + 3K = 0$$

Routh array

s^2	1	$64+3K$
s^1	$16+K$	
s^0	$64+3K$	

For sustained oscillation

$$16 + K = 0$$

$\Rightarrow$ $$K = -16$$

Auxiliary equation is

$$s^2 + 64 + 3K = 0$$

$\Rightarrow$ $$s^2 + 64 + 3(-16) = 0$$

$\Rightarrow$ $$s = \pm j4$$

62. Ans: (b)

Hint: Put $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$= \frac{-j(1-j\omega)(2-j\omega)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3\omega - j(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3}{(1+\omega^2)(4+\omega^2)} + \frac{j(\omega^2-2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= x + jy$$

Using final value theorem, steady state output

$$\lim_{s\to 0} sC(s) = s \times \frac{1}{s} \times \frac{100}{s^2+12s+100} = 1$$

71. Ans: (b)

Hint: Given, $x(t) = \sin t$

So, here ω = 1 rad/s

$$H(s) = \frac{s}{s+1}$$

$$H(j\omega) = \frac{j\omega}{j\omega+1}$$

So, $$H(j1) = \frac{j}{j+1}$$

$$= \frac{1}{\sqrt{2}}\angle 90° - 45°$$

$$= \frac{1}{\sqrt{2}}\angle 45°$$

$$y(t) = \frac{1}{\sqrt{2}}\sin(t+45°)$$

72. Ans: (c)

Hint: Given that

Phase margin = 45°

Let ω_{gc} be the gain crossover frequency

$$180° + \angle G(j\omega)H(j\omega)\Big|_{\omega_{gc}} = 45°$$

$$\angle G(j\omega)H(j\omega)\Big|_{\omega_{gc}} = -135°$$

Given that

$$G(s)H(s) = \frac{as+1}{s^2}$$

or $$G(j\omega)H(j\omega) = \frac{aj\omega+1}{(j\omega)^2}$$

Now $\angle G(j\omega)H(j\omega) = -180° + \tan^{-1}(a\omega)$

$\therefore$ $$-180° + \tan^{-1}(a\omega_{gc}) = -135°$$

or $$\tan^{-1}(a\omega_{gc}) = 45°$$

or $$\omega_{gc} = \frac{1}{a}$$

Now, $$\left|G(s)H(s)\right|_{\omega_{gc}} = \frac{\sqrt{1+\omega^2 a^2}}{\omega^2} = \frac{\sqrt{1+\frac{1}{a^2}\cdot a^2}}{\frac{1}{a^2}} = 1$$

or $$\frac{1}{a^2} = \sqrt{2}$$

or $$a^2 = \frac{1}{2^{1/2}}$$

or $$a = 0.841$$

73. Ans: (d)

Hint: Taking Laplace transform on both sides

$$Y(s)(s+1) = 1$$

(Assuming zero initial condition)

$\therefore$ $$Y(s) = \frac{1}{s+1}$$

Taking inverse Laplace transform, we get

$$y(t) = e^{-t}u(t)$$

74. Ans: (a)

Hint: $r(t)$ = unit impulse input applied at time instant t = 1

$$= \delta(t-1)$$

$$R(s) = e^{-s}$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2+3s+2}$$

$$C(s) = \frac{e^{-s}}{s^2+3s+2}$$

Using final value theorem , steady state value of output,

$$c_{ss} = \operatorname*{Lt}_{s\to 0} sC(s)$$

$$= \operatorname*{Lt}_{s\to 0} \frac{se^{-s}}{s^2+3s+2} = 0$$

75. Ans: (c)

Hint: $$T(s) = \frac{100}{s^2+20s+100}$$

Comparing with standard form,

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\therefore$ $$2\xi\omega_n = 20$$

$$\omega_n^2 = 100$$

$\therefore$ $$\omega_n = 10$$

$$\xi = 1$$

$\therefore$ The system is critically damped.

76. Ans: (c)

Hint: $G(s) = \dfrac{K}{s(s+3)(s+10)} = 0$ and $H(s) = 1$

Characteristic equation is $1 + G(s)H(s) = 0$

or $$1 + \frac{K}{s(s+3)(s+10)} = 0$$

or $$s^3 + 13s^2 + 30s + K = 0$$

Routh array

$$\begin{array}{lll} s^3 & 1 & 30 \\ s^2 & 13 & K \\ s^1 & \dfrac{13\times 30-K}{13} & \\ s^0 & K & \end{array}$$

For a stable system, signs of first column do not change
So,

$$K > 0$$

and $$\frac{13\times 30-K}{13} > 0$$

Therefore for stable system $0 < K < 390$

77. Ans: (c)

Hint: Initial slope is – 40 dB/decade, so there are two poles at origin.

Slope changes from – 40 dB/decade to – 20 dB/decade. So, there is a zero.

Slope changes from – 20 dB/decade to 0 dB/decade. So, there is one more zero.

Therefore, transfer function has two poles and two zeros.

78. Ans: (a)

Hint: *Part* I

$$C_1 = \frac{10(s+1)}{(s+10)}$$

Zero at $s = -1$
Pole at $s = -10$

As zero is nearer to origin and zero dominates pole. Hence, C_1 is lead compensator.

Part II

$$C_2 = \frac{s+10}{10(s+1)}$$

Zero at $s = -10$
Pole at $s = -1$

As pole is nearer to origin and pole dominates zero. Hence, C_2 is lag compensator.

79. Ans: (d)

Hint:

$$[sI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{s(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}$$

Transfer function $= C[sI-A]^{-1}B + D$

$$= [1 \quad 0]\frac{\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s(s+2)} = \frac{1}{s(s+2)}$$

80. Ans: (a)

Hint: $G(s) = \dfrac{1}{s(s+2)}$ and $H(s) = 1$

$r(t) = u(t)$

or $$R(s) = \frac{1}{s}$$

Error, $$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

or $$E(s) = \frac{s+2}{s(s+2)+1}$$

Using final value theorem, steady state error

$$e_{ss} = \operatorname*{Lt}_{s\to 0} sE(s)$$

$$= \operatorname*{Lt}_{s\to 0} \frac{s(s+2)}{s(s+2)+1}$$

$$= 0$$

81. Ans: (c)

Hint: $T(s) = \dfrac{s^2+4}{(s+1)(s+4)}$

Putting $s = j\omega$,

$$T(j\omega) = \frac{(j\omega)^2+4}{(j\omega+1)(j\omega+4)} = \frac{-\omega^2+4}{(j\omega+1)(j\omega+4)}$$

System output to be zero when

$$-\omega^2+4 = 0$$

or $$\omega = 2 \text{ rad/s}$$

82. Ans: (a)

Hint: For given root locus, it indicates that the break away point is at 0.

Consider for first option,

$$G(s)H(s) = \frac{K}{s^3}$$

Characteristic equation is $1+G(s)H(s) = 0$

$$\Rightarrow \quad s^3 + K = 0$$

For break away point,

$$\frac{dK}{ds} = 0$$

$$3s^2 = 0$$

$$s = 0$$

83. Ans: (a)

Hint: $G(s)H(s) = \dfrac{s+1}{s^2}$

Putting $s = j\omega$,

$$G(j\omega)H(j\omega) = \frac{(1+j\omega)}{-\omega^2}$$

At phase crossover frequency ω_{pc},

$$\angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{pc}} = -180°$$

$$-180° + \tan^{-1}\omega_{pc} = -180°$$

or $\omega_{pc} = 0$

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}} = \frac{1}{0} = \infty$$

$$G_M = \frac{1}{\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}}} = \frac{1}{\infty} = 0$$

84. Ans: (c)

Hint: The characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(1-s)}{(1+s)} = 0$$

$$s + 1 + K - Ks = 0$$

$$s(1-K) + (1+K) = 0$$

Routh array is

$$\begin{array}{ll} s^1 & 1-K \\ s^0 & 1+K \end{array}$$

For system to be stable,

$$1 - K > 0$$
$$K < 1$$

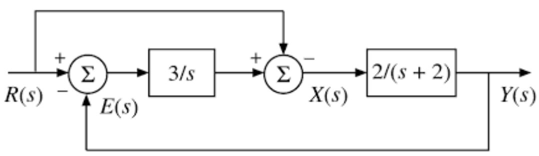
and
$$1 + K > 0$$
$$K > -1$$

So, $-1 < K < 1$

or $|K| < 1$

85. Ans: (c)

Hint: Replacing the integrator by $3/s$, the block diagram redrawn as



We can write

$$X(s) = \{R(s) - Y(s)\}\frac{3}{s} - R(s)$$

and $$Y(s) = \frac{2}{s+2}X(s)$$

Putting the value of $X(s)$,

$$Y(s) = \frac{2}{(s+2)}\left[\{R(s) - Y(s)\}\frac{3}{s} - R(s)\right]$$

$$\Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{2(3-s)}{s^2 + 2s + 6}$$

Error $\quad E(s) = R(s) - Y(s)$

$$\Rightarrow \quad E(s) = R(s)\left[\frac{s^2 + 4s}{s^2 + 2s + 6}\right]$$

Steady state error

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \times \frac{1}{s} \times \left[\frac{s^2 + 4s}{s^2 + 2s + 6}\right] = 0$$

86. Ans: (b)

Hint:

$$G(s)H(s) = \frac{\pi e^{-0.25s}}{s}$$

$$G(j\omega)H(j\omega) = \frac{\pi e^{-0.25(j\omega)}}{j\omega}$$

At negative real axis,

$$\angle G(j\omega)H(j\omega) = -180°$$

$$-90° - \frac{0.25 \times 180\omega}{\pi} = -180°$$

$$-90° - \frac{0.25\omega \times 180}{\pi} = -180°$$

$$-\frac{0.25\omega}{\pi} = \frac{-90}{180} = -\frac{1}{2}$$

$$G(j\omega)H(j\omega) = \frac{\pi\left[\cos(0.25\omega) - j\sin(0.25\omega)\right]}{j\omega}$$

$$= -\frac{\pi}{\omega}\sin(0.25\omega) - j\frac{\pi}{\omega}\cos(0.25\omega)$$

real part of

$$G(j\omega)H(j\omega)_{\omega=2\pi} = -\frac{\pi}{2}\sin(0.25 \times 2\pi) - j0 = -\frac{1}{2} - j0$$

87. Ans: (c)

Hint:

$$G(s)H(s) = \frac{K + j0.366s}{s(s+1)}$$

$$G(j\omega)H(j\omega) = \frac{K + j0.366\omega}{j\omega(j\omega+1)}$$

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{0.366\omega}{K}\right) - 90° - \tan^{-1}\omega$$

At $\quad \omega_{gc}, \Phi_M = 60°$

$$\therefore \; 180° + \tan^{-1}\left(\frac{0.366\omega_{gc}}{K}\right) - 90° - \tan^{-1}(\omega_{gc}) = 60°$$

Given $\omega_{gc} = 1$ rad/s

$$\therefore \; 180° + \tan^{-1}\left(\frac{0.366}{K}\right) - 90° - \tan^{-1}(1) = 60°$$

$$\tan^{-1}\left(\frac{0.366}{K}\right) = 90° + 60° + 45° - 180° = 15°$$

$$\Rightarrow \quad \frac{0.366}{K} = \tan(15)$$

$$K = \frac{0.366}{0.267} = 1.366$$

88. Ans: (a)

Hint:

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+3 & 1 \\ 0 & s \end{bmatrix}}{s(s+3) - 0 \times (-1)}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right) \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

State transition matrix

$$\phi(t) = \begin{bmatrix} 1 & \frac{1}{3}(1 - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$$

89. Ans: (c)

Hint: ZIR (zero input response) = $\phi(t).X(0)$

$$= \begin{bmatrix} 1 & \frac{1}{3}(1 - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 1 - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

$$= \begin{bmatrix} -e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

ZSR (zero state response)

$$= L^{-1}[(sI - A)^{-1} BU(s)]$$

$$= L^{-1}\left\{ \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{s} \right\}$$

$$= L^{-1}\begin{bmatrix} \frac{1}{s^2} \\ 0 \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$\therefore$ State transition equation

$$= ZIR + ZSR = \begin{bmatrix} -e^{-3t} \\ 3e^{-3t} \end{bmatrix} + \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

90. Ans: (b)

91. Ans: (a)

Hint: $$\frac{C(s)}{1/s} = \frac{2(s-1)}{(s+2)(s+1)}$$

$$C(s) = \frac{-1}{s} + \frac{-3}{s+2} + \frac{4}{s+1}$$

$$C(t) = -1 - 3e^{-2t} + 4e^{-t}$$

92. Ans: (c)

Hint: $X(s) = \text{Laplace of } u(t) = \dfrac{1}{s}$

$$Y(s) = \text{Laplace of } te^{-t} = \frac{1}{(s+1)^2}$$

$$TF = \frac{Y(s)}{X(s)} = \frac{s}{(s+1)^2}$$

93 Ans: (a)

Hint: $$\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{du}{dt} + 2u$$

or $\quad s^2Y(s) + sY(s) = sU(s) + 2U(s)$

$$\therefore \quad \frac{Y(s)}{U(s)} = \frac{s+2}{s^2+s}$$

94. Ans: (b)

Hint: Closed loop TF = Laplace of impulse response

= Laplace of $(te^{-t} + 2e^{-t})$

$$\text{CLTF} = \frac{2s+1}{(s+1)^2}$$

$\text{CLTF} = \dfrac{\text{OLTF}}{1 + \text{OLTF}}$ for unity feedback

$\text{CLTF} = \dfrac{G(s)}{1 + G(s)}$ when $H(s) = 1$

$G(s) = \dfrac{\text{CLTF}}{1 - \text{CLTF}}$ when $H(s) = 1$

On putting the value of CLTF, we get

$$G(s) = \frac{2s+1}{s^2}$$

95. Ans: (c)

Hint: $$S_G^{\text{TF}} = \frac{1}{1 + GH}$$

And $\quad S_H^{\text{TF}} = \dfrac{-GH}{1+GH} \approx -1$

Less sensitive to change in G than change in H.

96. Ans: Zero

Hint: $\lim\limits_{s\to 0} sE(s) = \lim\limits_{s\to 0} \dfrac{s.\frac{1}{s}}{1+\frac{G(s)H(s)}{s}} = 0$

97. Ans: (c)

Hint: $G(s) = \dfrac{1}{s(s+1)}$

Characteristic equation = 1 + $G(s)$

$= s^2 + s + 1$

On comparing with $s^2 + 2\xi\omega_n s + \omega_n^2$

We have $\xi = 0.5$

$$M_p = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}$$

$= 0.1630$

98. Ans: (c)

Hint: $\lim\limits_{s\to 0} sE(s) = \lim\limits_{s\to 0} \dfrac{s.\frac{1}{s^2}}{1+\frac{G(s)H(s)}{s^2}} = 0$

99. Ans: (a)

Hint: Steady state error is zero for step input with type 1 $G(s)$.

100. Ans: (b)

Hint: $G(s) = \dfrac{25}{s(s+6)}$

Characteristic equation = 1+ $G(s)$

$$1 + \frac{25}{s(s+6) = 0}$$

$\Rightarrow \quad s^2 + 6s + 25 = 0$

On comparing with $s^2 + 2\xi\omega_n s + \omega_n^2$

We get ω_n = 5 and ξ = 0.6

$$M_p = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}$$

$= 0.0948 \approx 9.48\% \approx 10\%$

101. Ans: Zero

Hint: $s^3 - 2s + 2 = 0$

$\therefore \quad s = -1.77, 0.88 \pm j0.59$

Hence, no positive real root axis.

102. Ans: Zeros

Hint: Because zeros of the characteristic equation represent closed loop transfer function poles which must lie in the left half of the s-plane.

103. Ans: two

Hint: Characteristic equation

$$s^4 + 6s^3 - 39s^2 + 19s + 84 = 0$$

Routh-Hurwitz criteria:

s^4	1	–39	84
s^3	6	19	
s^2	–42	84	
s^1	31		
s^0	84		

Two sign changes in first column.

Two poles in the right half of s-plane.

104. Ans: (a)

Hint: If the poles lie on imaginary axis then output is bounded but if none of the poles lie on imaginary axis, output tends to zero.

105. Ans: (c)

Hint: $2s^4 + s^3 + 3s^2 + 5s + 7 = 0$

Routh–Hurwitz criteria:

s^4	2	3	7
s^3	1	5	0
s^2	–7	7	
s^1	6		
s^0	7		

Two sign changes in first column.

Two poles in the right half of s-plane.

106. Ans: (b)

Hint: Number of encirclements about origin

= number of right sided poles

– number of zeroes of OLTF

= 1 – 0

= 1

107. Ans: (d)

Hint: $r(t) = \sin t$

ω = 1

$$\text{Gain} = \frac{1}{(1+s)} = \frac{1}{1+j\omega} = \frac{1}{1+j}$$

$$= \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$\text{Output} = \sin t \times \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$$

108. Ans: (b)

Hint: Integral controller is widely used to eliminate steady state error.

109. Ans: (c)

Hint: $X(s) \longrightarrow$ $\boxed{\text{Differentiator}}$ $\longrightarrow Y(s) = sX(s)$

$$H(s) = \frac{Y(s)}{X(s)} = s$$

Frequency response

$$H(j\omega) = j\omega$$

Hence, phase is constant at 90° for all ω but amplitude increases linearly with frequency.

110. Ans: (b)

Hint: Lead compensator increases bandwidth, hence decreases rise time. As it improves damping, so overshoot also decreases.

111. Ans: (c)

Hint: $G(s) = \dfrac{0.5s+1}{0.05s+1}$

On comparing with

$$G(s) = \frac{1+aTs}{1+Ts}$$

We get

$$T = 0.05 \quad \text{and} \quad a = 10$$

$$\phi_n = \sin^{-1}\left(\frac{a-1}{a+1}\right) = 55^\circ$$

And $\omega_n = \dfrac{1}{T\sqrt{a}}$

= 6.32 rad/s

112. Ans: (b)

113. Ans: (a)

Hint: Characteristic equation $|sI - A| = 0$

$$\begin{vmatrix} s-a & -1 \\ -a & s-1 \end{vmatrix} = 0$$

$$(s-a)(s-1) - a = 0$$

Eigenvalues are, $s = 0, (a+1)$

114. Ans: –1, –2, –3

Hint: $|sI - A| = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{vmatrix} = 0$

$$s^3 + 6s^2 + 11s + 6 = 0$$

Eigenvalues are $s = -1, -2, -3$

115. Ans: (c)

Hint: e^{AT} = Laplace inverse of $[sI - A]^{-1}$

116. Ans: (b)

Hint: $|Q_c| = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0$

or Uncontrollable

Characteristic equation

$$|sI - A| = 0$$

$$\begin{vmatrix} s-2 & -3 \\ 0 & s-5 \end{vmatrix} = 0$$

$$s^2 - 7s + 13 = 0$$

Eigenvalues, $s = 3.5 \pm j0.866$

i.e, roots lie right side of s-plane.

or Unstable

3

Power Systems

3.1 SYLLABUS

Basic power generation concepts; transmission line models and performance; cable performance, insulation; corona and radio interference; distribution systems; per unit quantities; bus impedance and admittance matrices; load flow; voltage control; power factor correction; economic operation; symmetrical components; fault analysis; principles of over current; differential and distance protection; solid state relays and digital protection; circuit breakers; system stability concepts, swing curves and equal area criterion; HVDC transmission and FACTs concepts.

3.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	4	4	4	3	2	3	5	3	1	3	2	3
2 Marks Question	7	10	9	8	5	5	4	2	3	5	4	3
Total Marks	**18**	**24**	**22**	**19**	**12**	**13**	**13**	**7**	**7**	**13**	**10**	**9**

3.3 TOPICS TO BE FOCUSED

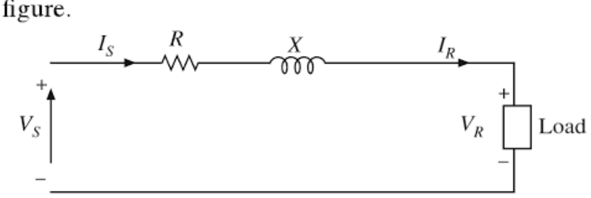
LINE MODEL AND PERFORMANCE OF TRANSMISSION LINES

The transmission lines are categorized in three types:

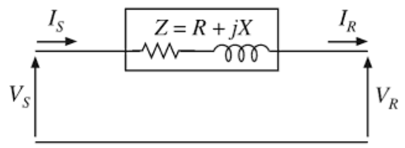
1. Short transmission line—the line length is up to 80 km.
2. Medium transmission line—the line length is between 80 km to 160 km.
3. Long transmission line—the line length is more than 160 km.

Short Transmission Line

The equivalent circuit for a short transmission line is shown in figure.



The transmission lines which have length less than 80 km are generally referred as ***short transmission lines***. For short length, the shunt capacitance of this type of line is neglected and other parameters like electrical resistance and inductor of these short lines are lumped. The equivalent circuit of short transmission line is

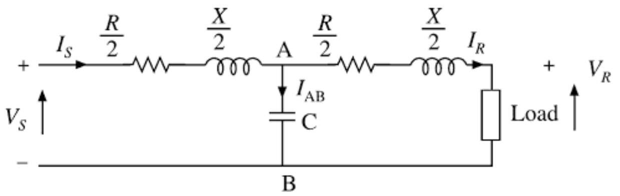


V_S = Sending end voltage
V_R = Receving end voltage
I_S = Sending end current
I_R = Receiving end current
Z = Short transmission line impedance

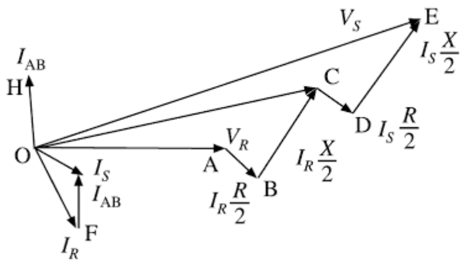
Medium Transmission Line

For the lines more than 80 km long and below 160 km in length are treated as medium length lines where the parameters are assumed to be lumped. In medium length lines, the line charging current becomes appreciable and the shunt capacitance must be considered. The shunt capacitance is either assumed to be concentrated at the middle point of the line or half of the total capacitance is concentrated at each end point of the line. The two configurations are known as nominal T model and nominal Π model respectively.

Nominal T model of a medium line

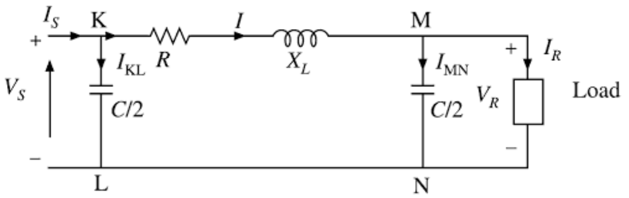


Nominal T model of a medium line

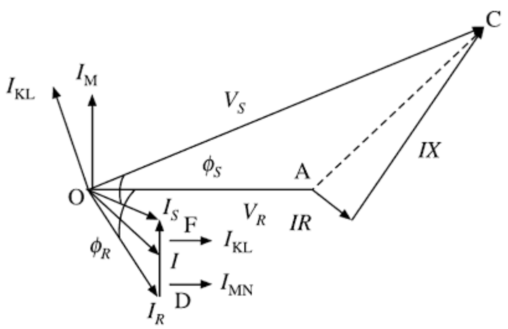


Phasor diagram of a nominal T network

Nominal Π model of a medium line



Normal Π model of a medium line



Phasor diagram of a nominal Π network

Long Transmission Line

A power transmission line with its effective length of around 250 km or above is referred to as a long transmission line. Calculations related to circuit parameters (ABCD parameters) of such a power transmission is not that simple, as was the case for a short transmission line or medium transmission line. The reason being that, the effective circuit length in this case is much higher than what it was for the former models (long and medium line) and, thus ruling out the approximations considered there like.

VOLTAGE REGULATION

Voltage regulation of a line is defined as the change in voltage at the receiving end, expressed in per cent of full load voltage, from no load to full load, keeping the voltage at the sending end constant.

$$\text{Per cent voltage regulation} = \frac{\left|V_R^{\text{NL}}\right| - \left|V_R^{\text{FL}}\right|}{\left|V_R^{\text{FL}}\right|} \times 100$$

where

$\left|V_R^{\text{NL}}\right|$ = magnitude of receiving end voltage at no load

$\left|V_R^{\text{FL}}\right|$ = magnitude of receiving end voltage at full load.

TRANSMISSION EFFICIENCY

The efficiency of lines is defined as

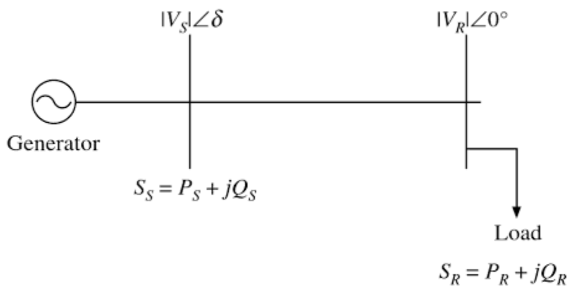
% Transmission efficiency,

$$\eta = \frac{\text{Power delivered at the receiving end}}{\text{Power sent at the sending end}} \times 100$$

$$= \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end + Losses}} \times 100$$

POWER FLOW THROUGH A TRANSMISSION LINE

A two bus system is shown in figure.



Let V_R be taken as the reference phasor, and V_S leads V_R by an angle δ.

i.e., $V_R = |V_R|\angle 0$ and $V_S = |V_S|\angle\delta$

Here, the angle δ is known as the torque angle.

The complex power leaving the receiving end and entering the sending end of the transmission line can be expressed on per phase basis as

$$S_R = P_R + jQ_R = V_R I_R^* \quad (1)$$

$$S_S = P_S + jQ_S = V_S I_S^* \quad (2)$$

We know, $$V_S = AV_R + BI_R \quad (3)$$

$$I_S = CV_R + DI_R \quad (4)$$

From Eq. (3), $$I_R = \frac{1}{B}V_S - \frac{A}{B}V_R \quad (5)$$

and from Eq. (4), we have

$$I_S = CV_R + D\left[\frac{V_S}{B} - \frac{A}{B}V_R\right]$$

$$= CV_R + \frac{D}{B}V_S - \frac{AD}{B}V_R$$

$$= \frac{D}{B}V_S - \left(\frac{AD - BC}{B}\right)V_R$$

$$= \frac{D}{B}V_S - \frac{1}{B}V_S \qquad \because [AD - BC = 1]$$

In equations, the receiving and sending end currents, can however be expressed in terms of receiving and sending end voltages.

Let A, B and D be the transmission line constants be written as

$$A = |A|\angle\alpha,\ B = |B|\angle\beta \text{ and } D = |D|\angle\alpha \text{ [as } A = D]$$

So, from Eq. (5), we have

$$I_R = \frac{|V_S|\angle\delta}{|B|\angle\beta} - \frac{|A|\angle\alpha}{|B|\angle\beta}.|V_R|\angle 0$$

or $$I_R = \frac{V_S}{B}\angle(\delta - \beta) - \frac{AV_R}{B}\angle(\alpha - \beta) \qquad (6)$$

and from previous equation, we have

$$I_S = \frac{|D|\angle\alpha}{|B|\angle\beta}|V_S|\angle\delta - \frac{1}{|B|\angle\beta}.|V_R|\angle 0$$

or $$I_S = \frac{D}{B}V_S\angle(\alpha + \delta - \beta) - \frac{V_R}{B}\angle(-\beta)$$

Putting the value of I_R from Eq. (7) in Eq. (1) we have

$$S_R = V_R I_R^*$$

$$= V_R\angle 0^\circ\left[\frac{V_S}{B}\angle(\beta - \delta) - \frac{AV_R}{B}\angle(\beta - \alpha)\right]$$

$$= \frac{V_S V_R}{B}\angle(\beta - \delta) - \frac{AV_R^2}{B}\angle(\beta - \alpha)$$

$$= \frac{V_S V_R}{B}[\cos(\beta - \delta) + j\sin(\beta - \delta)]$$

$$- \frac{AV_R^2}{B}[\cos(\beta - \alpha) + j\sin(\beta - \alpha)]$$

or $$S_R = P_R + jQ_R = \left[\frac{V_S V_R}{B}\cos(\beta - \delta) - \frac{AV_R^2}{B}\cos(\beta - \alpha)\right]$$

$$+ j\left[\frac{V_S V_R}{B}\sin(\beta - \delta) - \frac{AV_R^2}{B}\sin(\beta - \alpha)\right]$$

Now, comparing real and imaginary parts on both sides of the equation, we can write the real and reactive powers at the receiving end as

$$P_R = \frac{V_S V_R}{B}\cos(\beta - \delta) - \frac{AV_R^2}{B}\cos(\beta - \alpha)$$

and $$Q_R = \frac{V_S V_R}{B}\sin(\beta - \delta) - \frac{AV_R^2}{B}\sin(\beta - \alpha)$$

Similarly, putting the value of I_S from Eq. (7) in Eq. (2), we have

$$S_S = V_S I_S^*$$

$$= V_S\angle\delta\left[\frac{D}{B}V_S\angle(\beta - \alpha - \delta) - \frac{V_R}{B}\angle\beta\right]$$

$$= \frac{D}{B}V_S^2\angle(\beta - \alpha) - \frac{V_S V_R}{B}\angle(\beta + \delta)$$

$$= \frac{D}{B}V_S^2[\cos(\beta - \alpha) + j\sin(\beta - \alpha)]$$

$$- \frac{V_S V_R}{B}[\cos(\beta + \delta) + j\sin(\beta + \delta)]$$

or $$S_S = P_S + jQ_S = \left[\frac{D}{B}V_S^2\cos(\beta - \alpha) - \frac{V_S V_R}{B}\cos(\beta + \delta)\right]$$

$$+ j\left[\frac{D}{B}V_S^2\sin(\beta - \alpha) - \frac{V_S V_R}{B}\sin(\beta + \delta)\right]$$

Again, comparing real and imaginary parts on both sides of the equation, we can write the real and reactive powers at the sending end as

$$P_S = \frac{D}{B}V_S^2\cos(\beta - \alpha) - \frac{V_S V_R}{B}\cos(\beta + \delta)$$

and $$Q_S = \frac{D}{B}V_S^2\sin(\beta - \alpha) - \frac{V_S V_R}{B}\sin(\beta + \delta)$$

The received power P_R will be maximum at $\delta = \beta$ such that

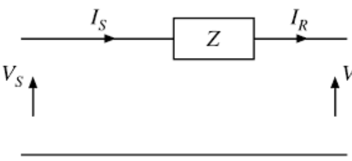
$$P_{R(\max)} = \frac{V_S V_R}{B} - \frac{A}{B}V_R^2\cos(\beta - \alpha)$$

and the corresponding Q_R at maximum P_R is

$$Q_R = -\frac{A}{B}V_R^2\sin(\beta - \alpha)$$

Hence, from equation, we can conclude that the load must draw this much leading VAr in order to receive the maximum real power.

Consider a short line with a series impedance Z.



Here $$V_S = 1V_R + ZI_R$$

$$I_S = 0V_R + 1I_R$$

$$\therefore \qquad A = D = 1\angle 0° \qquad (8)$$

$$B = Z = |Z|\angle\theta \qquad (9)$$

So, from Eqs. (8) and (9), $B = \theta, \alpha = 0$, $A = 1$, $B = Z$, $D = 1$

Substituting the values of A, D, B, β, α, we get the simplified results for the short line as

$$P_R = \frac{V_S V_R}{Z}\cos(\theta - \delta) - \frac{V_R^2}{Z}\cos\theta \qquad (10)$$

$$Q_R = \frac{V_S V_R}{Z}\sin(\theta - \delta) - \frac{V_R^2}{Z}\sin\theta$$

$$P_S = \frac{V_S^2}{Z}\cos\theta - \frac{V_S V_R}{Z}\cos(\theta + \delta)$$

and $$Q_S = \frac{V_S^2}{Z}\sin\theta - \frac{V_S V_R}{Z}\sin(\theta + \delta)$$

Now, from Eq. (10), the maximum receiving end power is received, when $\delta = \theta$ so that

$$P_{R(\max)} = \frac{V_S V_R}{Z} - \frac{V_R^2}{Z}\cos\theta$$

$$= \frac{V_S V_R}{Z} - \frac{V_R^2}{Z}.\frac{R}{Z}$$

$$= \frac{V_S V_R}{Z} - \frac{V_R^2}{Z^2}.R$$

We can neglect resistance R for transmission line with high X/R ratio. Thus, $Z = X$ and $\theta = \tan^{-1}\left(\frac{X}{R}\right) \cong 90°$, so the receiving end equations can then be approximated as

$$P_R = \frac{V_S V_R}{X}\cos(90° - \delta) - \frac{V_R^2}{X}\cos 90°$$

$$= \frac{V_S V_R}{X}\sin\delta$$

CABLES

Cables are used for underground transmission and distribution of electrical energy. These consist of (i) conductor for transmitting electrical power, (ii) insulation to insulate the conductor from direct contact with earth and other object, and (iii) external protection against mechanical damage, chemical attack, fire, etc.

Depending upon service, the cables are classified as (i) single core, (ii) double core, (iii) three core, and (iv) four core cables. For 3-phase service, either three single core or four core cables can be used.

Requirements for Insulators used in the Cables

(i) High insulation resistance
(ii) High dielectric strength
(iii) Good mechanical protection

Types of Insulations

(i) Vulcanized rubber
(ii) Butyl rubber
(iii) Silicon rubber
(iv) Neoprene rubber
(v) PVC
(vi) Impregnated paper

Insulation Resistance (*R*) of a Single Core Cable

If r_1 and r_2 are the internal and external radius of a single core cable, ρ is the resistivity of insulating material and l is length of the cable then $R = (\rho/2 \times \pi \times l) \times \ln(r_2/r_1)$

Capacitance (*C*) of a Single Core Cable

If d and D are the internal and external diameter of a single core cable, ε_0 (free space) and ε_r (relative) are the permittivity of the insulating medium then $C = (2 \times \pi \times \varepsilon_0 \times \varepsilon_r) / \ln(D/d)$ F/m.

CORONA

It is defined as the ionization of air surroundings the power conductor due to the electrons normally presents in free space because of radio activity and cosmic rays. It is the self sustained electric discharge in which the field intensified ionization is localized only over a portion of distance between the electrodes.

Factors Affecting Corona

(i) Electrical factors
(ii) Atmospheric condition
(iii) Factors connected with conductors
(iv) Number of conductors

Method of Reducing Corona

(i) Conductor having larger diameter
(ii) Increase the conductors spacing

Advantage of Corona

(i) Due to corona formation, the air surrounding the conductor becomes conducting and hence the virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.
(ii) Corona also reduces the transient's effect produced by surges.

Disadvantage of Corona

(i) Corona means energy loss so it affects the transmission efficiency of the line.
(ii) Ozone (O_3) is produced by corona so it causes the environment pollution.
(iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop in

the line. This may cause inductive interference with neighbouring communication line.

Important Terms Related with Corona

(i) Critical disruptive voltage $(V_C) = gr \ln \frac{d}{R}$; where g is breakdown strength of air at 76 cm pressure and temperature of 25 °C = 30 kV/cm (max), r = conductor radius, d = spacing between two conductors.

(ii) Visual critical voltage $(V_V) = mg\delta r\left(1 + \frac{3}{\sqrt{\delta r}}\right)\ln\left(\frac{d}{r}\right)$ kV/phase, where m = irregularity factor, δ = air density factor.

(iii) Power loss $(P) = 242.2\frac{(f+25)}{\delta}\sqrt{\frac{r}{d}}\left(V - V_C\right)2 \times 10^{-5}$ kW/kV/phase, where f = supply frequency, V = phase neutral voltage (rms).

Radio Interference (RI)

It is the adverse effect introduced by corona on wireless broadcasting. The corona discharge unit radiation which introduces noise signals in the communication lines, radio and television receivers. This leads to corona loss at voltages less than the critical voltage. Radio interference is considered as a field measured in rV/m at any distance from the transmission line and significant only at voltage >200 kV. There is a gradual increase in RI level till measurable corona loss takes place. Above this voltage RI increases rapidly. The amplitude of RI varies inversely as the frequency at which interference takes place. RI is considered during line designing.

DISTRIBUTION SYSTEM

Distribution system is that part of power system by which electric power is distributed among various customers for their local use.

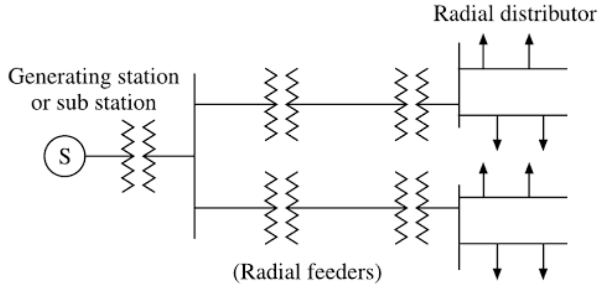
Classification of Distribution System

(i) According to current nature (ac and dc)
(ii) According to construction nature (OH and underground system)
(iii) According to connection scheme (radial, ring main and interconnected system)

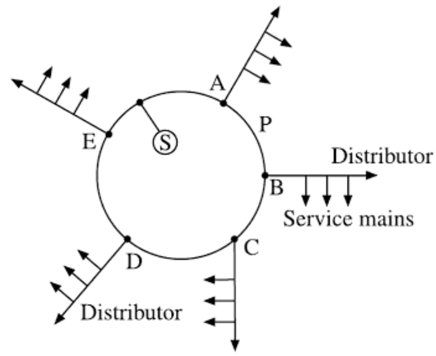
Radial electrical power distribution system

It has one major drawback that in case of any feeder failure, the associated consumers would not get any power as there was no alternative path to feed the transformer. In case of transformer failure also, the power supply is interrupted. In other words the consumer in the radial electrical distribution system would be in darkness until the feeder or transformer was rectified.

Radial feeder



Ring main feeder



Comparison of AC and DC Distribution

(i) In dc system, voltage drop is due to distributor resistance and in ac system voltage drop is due to combined effect of resistance, inductance and capacitance.
(ii) In dc system, all additions and subtraction of voltages and currents are done arithmetically but in ac system these are done by vectorically.

PER UNIT REPRESENTATION

In the process of computation of power system problems it is usual to express voltage, current, impedance and volt-amperes of an electrical circuit in per unit (or percentage) of base or reference values of these quantities rather than in volts, amperes, ohms and vars. The per unit value of any quantity is defined as

$$\text{Per unit value} = \frac{\text{The actual value in any units}}{\text{The base or reference value in the same units}}$$

Per unit values are dimensionless. Per cent values differ from per unit values by a factor of 100.

$$\text{Per cent value} = \text{Per unit value} \times 100$$

Let I = Actual current in amperes
I_B = Base current in amperes
V = Actual voltage in volts
V_B = Base voltage in volts
Z = Actual impedance in ohms
Z_B = Base impedance in ohms
S = Actual volt amperes
S_B = Base volt amperes

The per unit value of any quantity is defined as:
Per unit value = (Actual value)/(Base value)

So, per unit current, $I_{\text{p.u.}} = \dfrac{I}{I_B}$

Per unit voltage, $V_{\text{p.u.}} = \dfrac{V}{V_B}$

Per unit impedance, $Z_{\text{p.u.}} = \dfrac{Z}{Z_B}$

Now, $Z = R + jX$

$$Z_{\text{p.u.}} = \frac{Z}{Z_B} = \frac{R+jX}{Z_B} = \frac{R}{Z_B} + j\frac{X}{Z_B} = R_{\text{p.u.}} + jX_{\text{p.u.}}$$

where $R_{\text{p.u.}} = \dfrac{R}{Z_B}$

and $X_{\text{p.u.}} = \dfrac{X}{Z_B}$

Per unit volt amperes $S_{\text{p.u.}} = \dfrac{S}{S_B}$

Now, $S = P + jQ = VI^*$

$$\therefore \quad S_{\text{p.u.}} = \frac{S}{S_B} = \frac{P+jQ}{S_B} = \frac{P}{S_B} + j\frac{Q}{S_B} = P_{\text{p.u.}} + jQ_{\text{p.u.}}$$

where $P_{\text{p.u.}} = \dfrac{P}{S_B}$

and $Q_{\text{p.u.}} = \dfrac{Q}{S_B}$

Also, for single phase circuit,
Base impedance, Z_B = (Base voltage V_B)/(Base current I_B)
Base volt amperes, S_B = Base voltage × Base current

$$S_B = V_B I_B$$

The values of base quantities are taken according to convenience from equations, if any two of the four quantities are satisfied, the remaining two are determined easily. In power system calculations, usually base voltage and base volt amperes are selected. The base current I_B and base impedance Z_B are expressed in terms of base voltage V_B and base volt amperes S_B.

Now, select base voltage and base volt amperes

∴ Per unit voltage = (actual voltage)/(base voltage)

So, base current, I_B = (Base volt amperes, S_B)/(Base voltage, V_B) (in amperes)

and base impedance, Z_B = (Base voltage)/(Base current)
= (Base voltage) × {(Base voltage, V_B)/(Base volt amperes, S_B)}
= (Base voltage)^2/(Base volt amperes)
$= \dfrac{V_B^{\,2}}{S_B}$ (in ohms)

∴ Per unit impedance $Z_{\text{p.u.}}$ = (Actual impedance)/(Base impedance)
= Actual impedance × (Base volt ampere)/(Base voltage)^2
$= Z \times \dfrac{S_B}{(V_B)^2}$

Change of Base

It is sometimes necessary to convert per unit quantities from one base to another.

Let $Z_{\text{p.u.}}^{\text{old}}$ be the per unit impedance on the base volt amperes S_B^{old} and the base voltage V_B^{old}, which is expressed as

$$Z_{\text{p.u.}}^{\text{old}} = \frac{Z}{Z_B^{\text{old}}} = Z \times \frac{S_B^{\text{old}}}{(V_B^{\text{old}})^2}$$

If $Z_{\text{p.u.}}^{\text{new}}$ be the new per unit impedance on the new base volt amperes S_B^{new} and the new base voltage V_B^{new}, then

$$Z_{\text{p.u.}}^{\text{new}} = \frac{Z}{Z_B^{\text{new}}} = Z \times \frac{S_B^{\text{new}}}{(V_B^{\text{new}})^2}$$

From equation, the relationship between the old and the new per unit values is

$$Z_{\text{p.u.}}^{\text{new}} = Z_{\text{p.u.}}^{\text{old}} \cdot \frac{S_B^{\text{new}}}{S_B^{\text{old}}} \cdot \left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}}\right)^2$$

Three Phase Systems

Three phase systems are solved on single phase basis.

Base voltage represents line to neutral voltage and base current represents the line current. From this the equation for base impedance are as follows:

Now, Base kV is line to neutral kV.

$$\text{Base kVA} = \frac{3-\text{phase kVA}}{3}$$

$$\therefore \quad \text{Base current} = \frac{\text{Base kVA}}{\text{Base kV}}$$

and $$\text{Base impedance} = \frac{(\text{Base kV})^2}{(\text{Base kVA})} \times 1000$$

Suppose, we take a three-phase base kVA and base kV as line to line kV. On this basis,

$$\text{Base current} = \frac{\left(\dfrac{\text{Base kVA}}{3}\right)}{\left(\dfrac{\text{Base kV}}{\sqrt{3}}\right)} = \frac{\text{Base kVA}}{\sqrt{3} \times \text{Base kV}}$$

and

$$\text{Base impedance} = \frac{\left(\dfrac{\text{Base kV}}{\sqrt{3}}\right)^2}{\left(\dfrac{\text{Base kVA}}{3}\right)} \times 1000 = \frac{(\text{Base kV})^2}{\text{Base kVA}} \times 1000$$

Per Unit Representation of Transformer

Consider a single phase transformer in which
Base current in the primary = I_P
Base current in the secondary = I_S
Base voltage in the primary = V_P
Base voltage in the secondary = V_S

Base impedance in the primary $Z_{BP} = \dfrac{V_P}{I_P}$

Base impedance in the secondary $Z_{BS} = \dfrac{V_S}{I_S}$

Total series impedance of the two windings referred to the primary = Z_{PE}
Total series impedance of the two windings referred to the secondary = Z_{SE}
Primary turns = N_P
Secondary turns = N_S

Per unit impedance of the transformer referred to the primary

$$Z_{PE\,\text{p.u.}} = \frac{Z_{PE}}{Z_{BP}} = \frac{Z_{PE}}{\left(\dfrac{V_P}{I_P}\right)} = \frac{Z_{PE}\,I_P}{V_P}$$

The total series impedance of the two windings referred to the secondary

$$Z_{SE} = Z_{PE}\left(\frac{N_S}{N_P}\right)^2$$

Per unit impedance of the transformer referred to the secondary

$$Z_{SE\,\text{p.u.}} = \frac{Z_{SE}}{Z_{BS}} = \frac{Z_{SE}}{\left(\dfrac{V_S}{I_S}\right)} = \frac{Z_{SE}\,I_S}{V_S}$$

Now, $I_P N_P = I_S N_S$ or, $I_S = I_P \dfrac{N_P}{N_S}$

and $\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P}$ or, $V_S = V_P \dfrac{N_S}{N_P}$

$$\therefore \quad Z_{SE\,\text{p.u.}} = \frac{Z_{SE} I_S}{V_S} = Z_{PE}\left(\frac{N_S}{N_P}\right)^2 I_P\left(\frac{N_P}{N_S}\right)\frac{1}{\left(\dfrac{N_S}{N_P}\right)V_P}$$

$$= Z_{PE}\frac{I_P}{V_P} = Z_{PE\,\text{p.u.}}$$

The results of equation may alternatively be deduced as follows:

$$Z_{PE\,\text{p.u.}} = \frac{Z_{PE}}{Z_{BP}} = \frac{Z_{SE}\left(\dfrac{N_P}{N_S}\right)^2}{\left(\dfrac{V_P}{I_P}\right)} = \frac{Z_{SE}\left(\dfrac{N_P}{N_S}\right)^2}{V_S\left(\dfrac{N_P}{N_S}\right)} I_S\left(\dfrac{N_S}{N_P}\right)$$

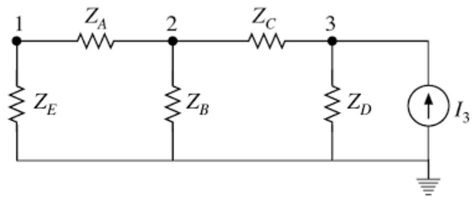
$$= \frac{Z_{SE}}{\left(\dfrac{V_S}{I_S}\right)} = \frac{Z_{SE}}{Z_{BS}} = Z_{SE\,\text{p.u.}}$$

So, the per unit equivalent impedance of a 2-winding transformer is the same whether the calculation is made from the primary side or the secondary side.

BUS ADMITTANCE MATRIX

Most power system networks are analyzed by first forming the admittance matrix. The admittance matrix is based upon Kirchhoff's current law (KCL), and it is easily formed and very sparse.

Consider the three-bus network shown in figure that has five branch impedances and one current source.



Applying KCL at the three independent nodes yields the following equations for the bus voltages (with respect to ground):

At bus 1, $\dfrac{V_1}{Z_E} + \dfrac{V_1 - V_2}{Z_A} = 0$

At bus 2, $\dfrac{V_2}{Z_B} + \dfrac{V_2 - V_1}{Z_A} + \dfrac{V_2 - V_3}{Z_C} = 0$

At bus 3, $\dfrac{V_3}{Z_D} + \dfrac{V_3 - V_2}{Z_C} = I_3$

Collecting terms and writing the equations in matrix form

$$\begin{bmatrix} \dfrac{1}{Z_E}+\dfrac{1}{Z_A} & -\dfrac{1}{Z_A} & 0 \\ -\dfrac{1}{Z_A} & \dfrac{1}{Z_A}+\dfrac{1}{Z_B}+\dfrac{1}{Z_C} & -\dfrac{1}{Z_C} \\ 0 & -\dfrac{1}{Z_C} & \dfrac{1}{Z_C}+\dfrac{1}{Z_D} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_3 \end{bmatrix}$$

or in matrix form, $Y\mathbf{V} = \mathbf{I}$

where Y is the admittance matrix, **V** is a vector of bus voltages (with respect to ground), and **I** is a vector of current injections. Voltage sources, if present, can be converted to current sources using the usual network rules. If a bus has a zero-impedance voltage source attached to it, then the bus voltage is already known, and the dimension of the problem is reduced by one. A simple observation of the structure of the above admittance matrix leads to the following rule for building Y:

(i) The diagonal terms of Y contain the sum of all branch admittances connected directly to the corresponding bus.

(ii) The off-diagonal elements of Y contain the negative sum of all branch admittances connected directly between the corresponding buses.

Impedance Matrix

The impedance matrix is the inverse of the admittance matrix.

$$Z = Y^{-1}$$

So that $\mathbf{V} = Z\mathbf{I}$

The reference bus both Y and Z is ground. Although the impedance matrix can be found via inversion, complete inversion is not common for matrices with more than a few hundred rows and columns because of the matrix storage requirements. In those instances, Z elements are usually found via Gaussian elimination, Kron reduction, or, less commonly, by a direct building algorithm. If only a few of the Z elements are needed, then Gaussian elimination or Kron reduction are the best.

Load Flow

Load flow solution of a network under steady state condition subject to certain inequality constraints under which the system operates. The constraints are—bus voltage, reactive power of alternators, tap setting of a tap changer etc. A load flow (LF) solution of power system (PS) requires mainly the following steps: (i) formulation of the network equations, and (ii) suitable mathematical method for solution of equation.

Bus Analysis

In a PS each bus or node is associated with four quantities i.e., real power (P) and reactive power (Q), bus voltage (V) and its phase angle (δ). In a LF solution, out of four two are specified and other two quantities are obtained.

Classification of Bus

(i) Load bus (P and Q are specified)
(ii) Generator bus or voltage control bus (V and P are specified)
(iii) Slack or swing or reference bus (V and δ are given)

SYMMETRICAL COMPONENTS

Three unbalanced vectors (means they may not be equal in magnitude and/or do not have same phase displacement) are $\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$ or $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$. These three vectors ($\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$ or $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$) can be resolved into three balanced vectors. The vectors of the balanced system are called symmetrical components of the original system. These are

(i) Positive sequence components: $\mathbf{V}_{a1}$, $\mathbf{V}_{b1}$, $\mathbf{V}_{c1}$ (or $\mathbf{I}_{a1}$, $\mathbf{I}_{b1}$, $\mathbf{I}_{c1}$) are three balanced vectors, displaced from each other by 120° in phase and having the same phase sequence as that of the original unbalanced system.
(ii) Negative sequence components: $\mathbf{V}_{a2}$, $\mathbf{V}_{b2}$, $\mathbf{V}_{c2}$ (or $\mathbf{I}_{a2}$, $\mathbf{I}_{b2}$, $\mathbf{I}_{c2}$) are three balanced vectors, displaced from each other by 120° in phase and having phase sequence opposite to that of the original unbalanced system.
(iii) Zero phase sequence components: $\mathbf{V}_{a0}$, $\mathbf{V}_{b0}$, $\mathbf{V}_{c0}$ (or $\mathbf{I}_{a0}$, $\mathbf{I}_{b0}$, $\mathbf{I}_{c0}$) are three equal vectors having zero phase displacement.

Thus,

$$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2}$$
$$\mathbf{V}_b = \mathbf{V}_{b0} + \mathbf{V}_{b1} + \mathbf{V}_{b2}$$
$$\mathbf{V}_c = \mathbf{V}_{c0} + \mathbf{V}_{c1} + \mathbf{V}_{c2}$$

α is an operator that causes a counter clockwise rotation of 120°. It has unit magnitude and angle 120°.

$$\alpha = 1\angle 120° = -0.5 + j0.866$$
$$\alpha^2 = 1\angle 240° = -0.5 - j0.866$$
$$\alpha^3 = 1\angle 360° = 1 + j0$$
$$1 + \alpha + \alpha^2 = 0$$

So,

$$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2}$$
$$\mathbf{V}_b = \mathbf{V}_{a0} + \alpha^2\mathbf{V}_{a1} + \alpha\mathbf{V}_{a2}$$
$$\mathbf{V}_c = \mathbf{V}_{a0} + \alpha\mathbf{V}_{a1} + \alpha^2\mathbf{V}_{a2}$$

In matrix form, $$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix}$$

or $$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}$$

Hence,

$$\mathbf{V}_{a0} = \frac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathrm{V}_c)$$
$$\mathbf{V}_{a1} = \frac{1}{3}(\mathbf{V}_a + \alpha\mathbf{V}_b + \alpha^2\mathbf{V}_c)$$
$$\mathbf{V}_{a2} = \frac{1}{3}(\mathbf{V}_a + \alpha^2\mathbf{V}_b + \alpha\mathbf{V}_c)$$

In star connected system with neutral return path for current

$$\mathbf{I}_n = (\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 3\mathbf{I}_{a0}$$

Without neutral path or neutral grounding, $\mathbf{I}_n = 0$.
So, zero sequence currents are zero.

In delta connected system, the line currents do not have return neutral path. So, zero sequence currents are zero.

UNSYMMETRICAL FAULTS

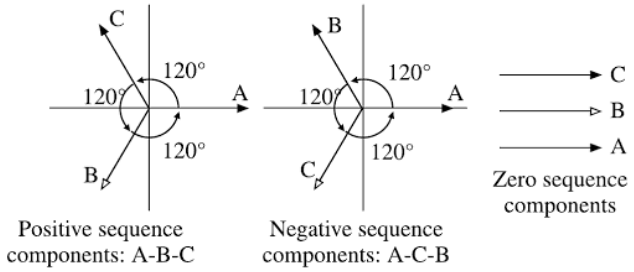
Unsymeetrical faults are of following types:

(i) Single line to ground (L-G) fault
(ii) Line to line (L-L) fault
(iii) Double line to ground (L-L-G) fault

For such faults, simple single phase representation is not valid. These are analysed by symmetrical components of the unbalanced currents.

POSITIVE, NEGATIVE AND ZERO SEQUENCE COMPONENTS

There are three sets of independent components in a three-phase system: positive, negative and zero for both current and voltage. Positive sequence voltages are supplied by generators within the system and are always present. A second set of balanced phasors are also equal in magnitude and displaced 120° apart, but display a counter-clockwise rotation sequence of A-C-B which represents a negative sequence. The final set of balanced phasors is equal in magnitude and in phase with each other, however, since there is no rotation sequence, this is known as a zero sequence.

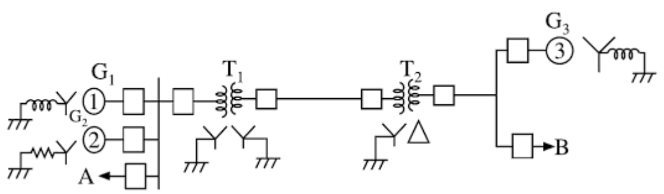


SINGLE LINE DIAGRAM

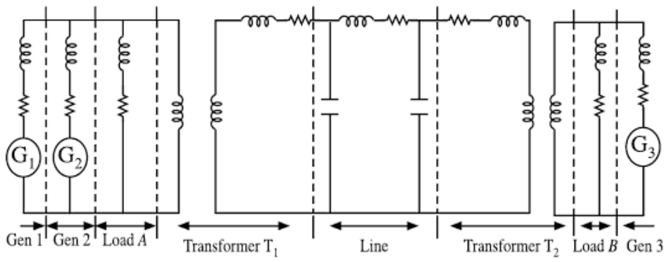
A single line diagram shows the essential connections and arrangements of components of a power system. Power system networks are represented by single line diagrams using suitable symbols for generators, motors, transformers and a combination of industrial, commercial and residential loads. The information supplied by a single line diagram varies according to the requirement. For example, circuit breakers need not be shown in a load flow study but these components are must for a protection study. For stability studies, circuit breakers and relay positions are shown in the single line diagram. In short circuit studies, three separate diagrams to represent positive, negative and zero sequence networks are shown.

A balanced three phase system is studied on a per phase basis. A single phase circuit consists of one of the three phase conductors and a neutral return conductor. The loop impedance of a single phase circuit may be considered to be concentrated in one conductor only by taking the impedance of the return conductor assumed to be zero. Thus, a balanced three phase system is replaced by a single line diagram. A single line diagram of a power system is shown in the following figure.

Here, G_1, G_2 and G_3 are three synchronous generators. T_1 and T_2 are two transformers. *A* and *B* are loads. Circuit breakers are numbered 1 to 9.



Under balanced operating conditions, the impedance diagram on single phase basis can be easily drawn from the single line diagram. In the impedance diagram, the different components of power system are replaced by their equivalent circuits. The generators are represented as voltage source with series resistance and inductive reactance. The transformer is represented by its equivalent circuit. The transmission line is represented by Π model. Loads are assumed to be passive (not involving rotating machines) and are represented by series resistance and inductive reactance. Neutral grounding impedances do not appear in the diagram as balanced conditions are assumed. In many power system studies, each of the resistance of synchronous generator, transformer winding and transmission lines, line charging and magnetizing circuits of transformers are neglected. The impedance diagram is then represented by reactance diagram. Impedance diagram of the power system is shown in the following figure.



PREPARATION OF PER UNIT IMPEDANCE DIAGRAM

We can directly draw the impedance diagram from single line diagram of a power system. The procedure is given as:

Step 1 Select an appropriate common kVA base for the entire power system.

Step 2 Consider the entire power system to be divided into a number of sections by the transformers. Select an appropriate kV base for one part of the system. Usually, a voltage base is selected for a transmission line in the power system. Compute kV bases of remaining parts of the power system in the ratio of transformation.

Step 3 Compute per unit values of voltages and impedances in each sections and joining them as per the topology of single line diagram.

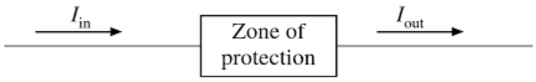
Advantages of Per Unit System

Per unit system has the following advantages:

(i) Calculations are simplified.
(ii) Per unit values give more meaningful information.
(iii) Per unit values are independent of the type of the power system, that is, whether the power system is single phase or three phase.
(iv) The per unit impedance referred to either side of a single phase transformer is the same.
(v) Using per unit values, analysis of power systems are simplified considerably.
(vi) The impedances of machines are specified by the manufacturers in terms of per unit values.

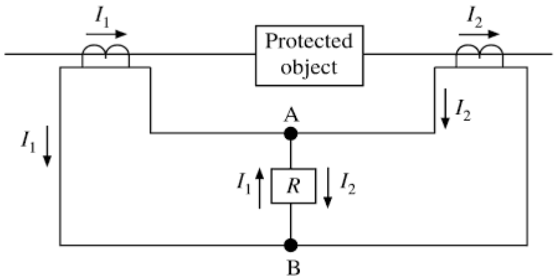
DIFFERENTIAL PROTECTION

Differential relaying systems are based on the premise that under normal conditions, current in equals to current out.



$$I_{in} = I_{out} \Rightarrow I_{diff} = I_{in} - I_{out} = 0$$

In reality provision has to be made for non-zero differential quantities under normal healthy conditions. These could result due to line charging current, CT mismatching, the transformer tap changer, etc. Provision is thus made for ways to prevent relay operation which could result due to differential current being present under normal system conditions. This is classically done by deriving a restraint quantity from the terminal currents (biased differential protection).



Normal condition, $I_1 = I_2$. By virtue of CT connections I_1 and I_2 subtract to zero through relay i.e., $I_{diff} = I_1 - I_2$. The secondary currents thus appear to circulate in the CT secondary's only circulating current differential protection. And lastly this type of relay always works for internal fault only.

DISTANCE PROTECTION

The impedance of a transmission line is proportional to its length, for distance measurement it is appropriate to use a relay capable of measuring the impedance of a line up to a predetermined point (the reach point). Such a relay is described as a distance relay and is designed to operate only for faults occurring between the relay location and the selected reach point, thus giving discrimination for faults that may occur in different line sections. The basic principle of distance protection involves the division of the voltage at the relaying point by the measured current. The apparent impedance so calculated is compared with the reach point impedance. If the measured impedance is less than the reach point impedance, it is assumed that a fault exists on the line between the relay and the reach point. The reach point of a relay is the point along the line impedance locus that is intersected by the boundary characteristic of the relay. Since this is dependent on the ratio of voltage and current and the phase angle between them, it may be plotted on an *R*/*X* diagram. The loci of power system impedances as seen by the relay during faults, power swings and load variations may be plotted on the same diagram and in this manner the performance of the relay in the presence of system faults and disturbances may be studied.

Impedance Relationship of Distance Relay

With electromechanical relay designs, the magnitude of input quantities particularly influenced both reach accuracy and operating time. It was customary to present information on relay performance by voltage/reach curves. System impedance ratios (SIRs) as Z_S/Z_L, where Z_S = system source impedance behind the relay location and Z_L = line impedance equivalent to relay reach setting.

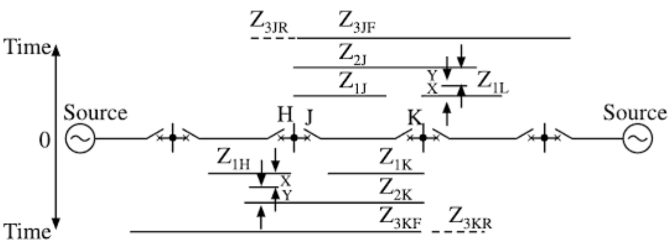
ZONE PROTECTION

Careful selection of the reach settings and tripping times for the various zones of measurement enables correct coordination between distance relays on a power system. Basic distance protection will comprise instantaneous directional Zone 1 protection and one or more time delayed zones. Typical reach and time settings for a 3-zone distance protection in digital and numerical distance relays may have up to five zones, some set to measure in the reverse direction. Typical settings for three forward-looking zones of basic distance protection are given in the following sub-sections.

(i) *Relay setting of Zone 1:* Electromechanical relays usually have a reach setting of up to 80% of the protected line impedance for instantaneous Zone 1 protection. For digital/numerical distance relays, settings of up to 85% may be safe. The resulting 15–20% safety margin ensures that there is no risk of the Zone 1 protection over-reaching the protected line due to errors in the current and voltage transformers, inaccuracies in line impedance data provided for setting purposes and errors of relay setting and measurement. Otherwise, there would be a loss of discrimination with fast operating protection on the following line section. Zone 2 of the distance protection must cover the remaining 15–20% of the line.

(ii) *Relay setting of Zone 2:* To ensure full cover of the line with allowance for the sources of error already listed in the previous section, the reach setting of the Zone 2 protection should be at least 120% of the protected line impedance. In many applications it is common practice to set the Zone 2 reach to be equal to the protected line section +50% of the shortest adjacent line. Where possible, this ensures that the resulting maximum effective Zone 2 reach does not extend beyond the minimum effective Zone 1 reach of the adjacent line protection. This avoids the need to grade the Zone 2 time settings between upstream and downstream relays. In electromechanical and static relays, Zone 2 protection is provided either by separate elements or by extending the reach of the Zone 1 elements after a time delay that is initiated by a fault detector. In most digital and numerical relays, the Zone 2 elements are implemented in software. Zone 2 tripping must be time-delayed to ensure grading with the primary relaying applied to adjacent circuits that fall within the Zone 2 reach. Thus complete coverage of a line section is obtained, with fast clearance of faults in the first 80–85% of the line and somewhat slower clearance of faults in the remaining section of the line.

(iii) *Relay setting of Zone 3:* Remote back-up protection for all faults on adjacent lines can be provided by a third zone of protection that is time delayed to discriminate with Zone 2 protection plus circuit breaker trip time for the adjacent line. Zone 3 reach should be set to at least 1.2 times the impedance presented to the relay for a fault at the remote end of the second line section. On interconnected power systems, the effect of fault current infeed at the remote bus bars will cause the impedance presented to the relay to be much greater than the actual impedance to the fault and this needs to be taken into account when setting Zone 3. In some systems, variations in the remote bus bar infeed can prevent the application of remote back-up Zone 3 protection but on radial distribution systems with single end infeed, no difficulties should arise. Typical distance characteristics of Zone 3 protection is shown in the following figure.



Zone 1 = 80–85% of protected line impedance
Zone 2 (minimum) = 120% of protected line
Zone 2 (maximum) < protected line + 50% of shortest second line
Zone 3F = 1.2 (protected line + longest second line)
Zone 3R = 20% of protected line
X = Circuit breaker tripping time
Y = Discriminating time

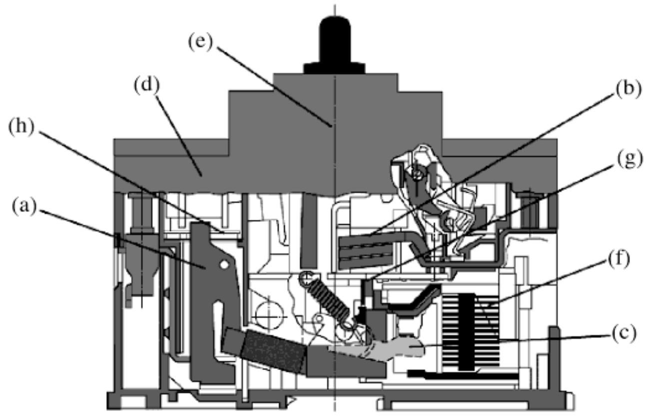
SOLID STATE RELAY (SSR)

A solid-state relay is an ON-OFF control device in which the load current is conducted by one or more semiconductors e.g., power transistor, SCR etc. Like all relays, the SSR requires relatively low control circuit energy to switch the output state from OFF to ON, or vice versa. Since this control energy is very much lower than the output power controllable by the relay at full load, "power gain" in an SSR is substantial–frequently much higher than in an electromagnetic relay of comparable output rating. To put it another way, the sensitivity of an SSR is often significantly higher than that of an electromagnetic relay (EMR) of comparable output rating.

CIRCUIT BREAKER (CB)

The circuit breaker is a mechanical switching device capable of protecting the circuit wiring, capable of making, carrying and breaking currents under normal circuit conditions and also making, carrying for a specified time and breaking currents under specified abnormal circuit conditions such as those of short-circuit. Especially, the circuit breakers have the capability of interrupting short-circuit currents.

Design of Circuit Breaker



(a) Thermal over current release
(b) Magnetic over current release
(c) Main contact system
(d) Auxiliary contacts
(e) Operating mechanism
(f) Arc chamber (splitter plates)
(g) Striker (hammer)
(h) Sliding piece for differential protection

Types of Circuit Breaker

(i) Oil circuit breaker
(ii) Air blast circuit breaker
(iii) SF_6 circuit breaker
(iv) Vacuum circuit breaker

Breaker used in 132 kV Grid Station

(i) Oil circuit breaker
(ii) Vacuum circuit breaker
(iii) SF_6 circuit breaker

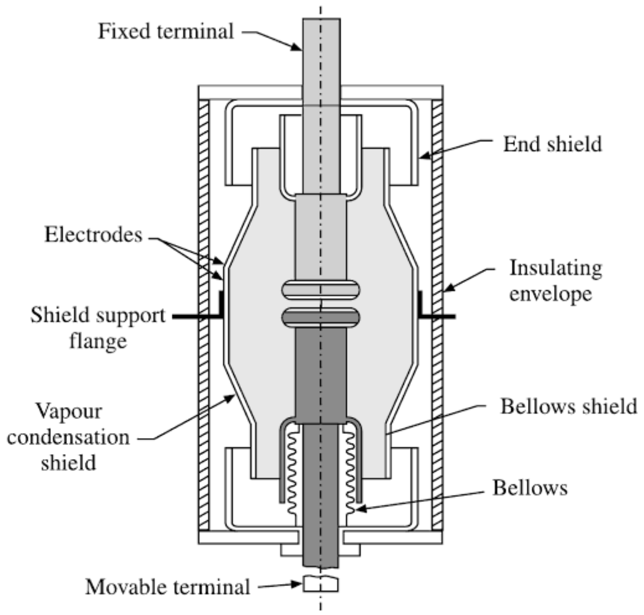
Oil circuit breaker: It consists of two parts, i.e., supporting chamber and circuit-breaking chamber (consist of fixed and moving contact).

Disadvantages of oil circuit breaker

(i) It is inflammable and there is a risk of fire.
(ii) It may form an explosive mixture with air.

(iii) It requires maintenance.
(iv) Absorbs moisture, so dielectric strength reduces.
(v) Oil leakage problem.
(vi) Oil has to be replaced after some operations because of the carbonization of oil.

Vacuum circuit breaker: In vacuum circuit breaker, vacuum is used as an arc quenching medium. It has the greatest insulating strength and 10^{-7} to 10^{-5} pressure is to be maintained. It is used in 11 kV panel in control room of grid station. Vacuum circuit breaker is shown in the following figure.



Advantages of vacuum circuit breaker
(i) Compact, reliable and have longer life.
(ii) No fire hazards.
(iii) No generation of gas during and after operation.
(iv) Can interrupt any fault current.
(v) No noise is produced while operating.
(vi) Require less power for control operation.

SF_6 circuit breaker: In SF_6 circuit breaker, sulphur hexafluoride (SF_6) gas is used as an arc quenching medium. SF_6 is an electronegative gas. It has strong tendency to absorb electrons. When contact are opened in a high pressure flow of SF_6 gas, arc produced. Free electron in the arc are captured by the gas which build up enough insulation strength to extinguish arc. It is much effective for high power and high voltages services.

Advantages of SF_6 circuit breaker
(i) Simple construction, less cost.
(ii) SF_6 gas is non-flammable, non-toxic and chemical inert gas.
(iii) Same gas is recirculated in the circuit.
(iv) Maintenance free circuit breaker.
(v) Ability to interrupt low and high fault current.
(vi) Excellent arc extinction.

Advantages of SF_6 over oil circuit breakers
(i) Short arcing time.
(ii) Can interrupt much larger currents.
(iii) Gives noiseless operation due to its closed gas circuit.
(iv) No moisture problem.
(v) No risk of fire.
(vi) No carbon deposits. So, no tracking and insulation problems.
(vii) Low maintenance cost.

CB Rating

(i) Breaking capacity: It is the current (rms) that a CB is capable of breaking at recovery voltage and under specified conditions (power factor, rate of rise of restriking voltage etc).

Breaking capacity = sqrt(3) × V × I × 10^{-6} MVA,

where
V = service voltage
I = rated breaking current.

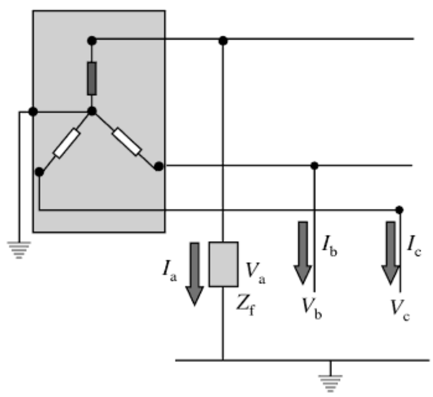
(ii) Making capacity: The peak value of current (including ac and dc component) during the 1st cycle of current wave after the closure of CB is called making capacity.

(iii) Short time rating: It is period for which the CB is able to carry fault current while remaining closed.

FAULT ANALYSIS

There are mainly three types of fault viz.
(i) Single line to ground fault (L-G)
(ii) Line to line fault (L-L)
(iii) Double line to ground fault (L-L-G).

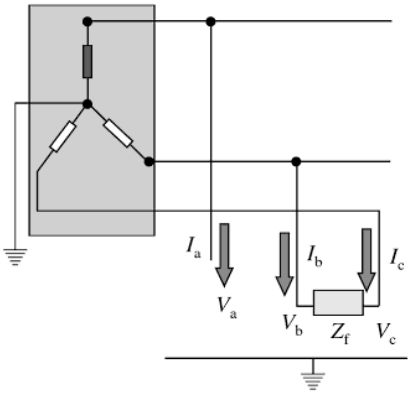
L-G Fault: Consider Fault at Phase a



$$I_b = 0 \quad \text{and} \quad I_c = 0$$
$$V_a = I_a Z_f$$
$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_a^0 = I_a^+ = I_a^- = \frac{I_a}{3}$$

L-L Fault: Consider Fault at Phase Band c



$$I_a = 0 \quad \text{and} \quad I_b = -I_c$$

$$V_b = V_c + I_b Z_f$$

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_a^0 = 0$$

$$I_a^+ = \frac{1}{3}(\alpha - \alpha^2)I_b$$

$$I_a^- = \frac{1}{3}(\alpha^2 - \alpha)I_b$$

$$I_a^- = -I_a^+$$

Note:

$$\alpha = 1\angle 120°$$

$$\alpha = -0.5 + j0.866$$

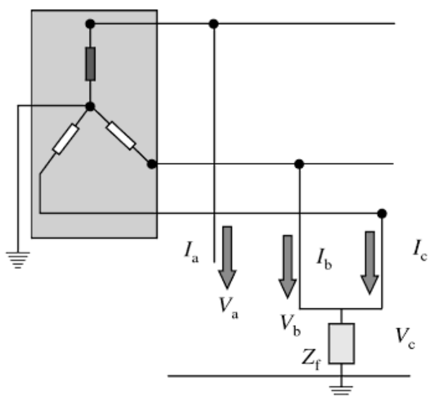
$$\alpha^2 = 1\angle 240°$$

$$\alpha^2 = -0.5 - j0.866$$

$$\alpha^2 - \alpha = -j\sqrt{3}$$

$$\alpha - \alpha^2 = j\sqrt{3}$$

L-L-G Fault: Consider Fault at Phases b and c and Ground



$$V_b = V_c$$

$$V_b = V_c = (I_b + I_c)Z_f$$

$$I_a = 0$$

$$I_a = I_a^0 + I_a^+ + I_a^- = 0$$

$$V_b = V_b^0 + V_b^+ + V_b^-$$

$$V_b = V_a^0 + \alpha^2 V_a^+ + \alpha V_a^-$$

$$V_c = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^-$$

$$V_b = V_c$$

$$V_a^0 + \alpha^2 V_a^+ + \alpha V_a^- = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^-$$

$$(\alpha^2 - \alpha)V_a^+ = (\alpha^2 - \alpha)V_a^-$$

$$V_a^+ = V_a^-$$

POWER SYSTEM STABILITY

It is the ability of a system to reach a normal or stable condition after being disturbed is called stability. There are mainly two types of stability viz. (i) steady state, and (ii) transient stability.

(i) *Steady state stability:* It is the ability of the power system to regain synchronism after small and slow disturbances such as gradual power changes etc.

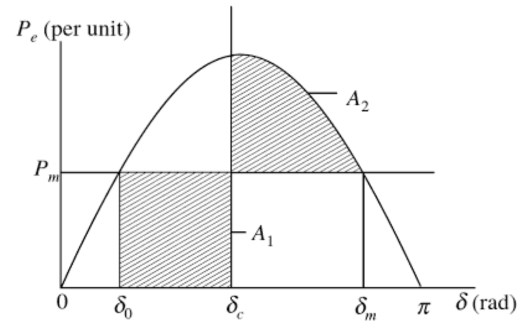
(ii) *Transient stability:* It is the ability of a system to maintain synchronous operation and to reach a stable or near to stable condition after a large disturbance.

SWING CURVE

The behaviour of a synchronous machine during transients is described by swing curve. It determines the adequacy of relay protection on power system with regard to the clearing of faults before one or more machines become unstable and fall out of synchronism. It is related with transient stability.

EQUAL AREA CRITERION

It is related with transient stability. It determines the maximum angle of swing of generator rotor, an estimate whether synchronism will be maintained and the maximum amount of disturbance that can be allowed without losing synchronism. It is applicable only 2 machines system or 1 machine connected with infinite bus. It is not applicable to multi machines system.



A_1 = accelerating area and A_2 = decelerating area if (i) $A_1 > A_2$ system unstable, (ii) $A_1 < A_2$ system stable, and (iii) $A_1 = A_2$ system marginally stable (critically stable).

ECONOMIC OPERATION

The economic operation of alternators aims to guarantee at all times the optimum combination of alternators connected to the system to supply the load demand.

The unit commitment is the selection of units that will supply anticipated load of the system over a required period of time at a minimum cost as well as provide a specified margin of spinning reserve.

The function of on line ELD is to distribute the load among the alternators actually parallel with the system in such a manner as to minimize the total cost of supplying the minute to minute requirements of the system.

$dC_1/dP_1 = dC_2/dP_2 = \ldots = dC_n/dP_n = \lambda$ (incremental fuel cost)

POWER FACTOR (Cos ϕ) CORRECTION

$I = \dfrac{P}{V\cos\phi}$, where P = power, and V supply voltage, hence current depends on power factor when V = constant. Due to many disadvantages, both capital and running costs are increased, operation at low power factor is uneconomical. So, higher power factor is economical.

Disadvantage of Lower Power Factor

(i) Large kVA rating of equipment
(ii) Greater conductor size
(iii) Large cu loss
(iv) Poor voltage regulation

Causes of Lower Power Factor

(i) Uses of induction motor (large inductance)
(ii) Uses of arc and electric discharge lamps etc.

The power factor may be improved by using capacitor bank, synchronous condenser, FACTS devices, etc. These devices are connected parallel with the network.

VOLTAGE CONTROL

Voltage control is associated with reactive power drop. If the reactive power drop in any network is large then the voltage profile of the network is very poor. For a long transmission line due to large distance and presence of series impedance at a certain distance reactive power injection is required. It is done by FACTS devices. On the other hand, in case of load center due to low voltage at the load end capacitor bank or FACTS devices are connected to the load end to improve the voltage.

3.4 IMPORTANT POINTS TO REMEMBER

1. The constants A, B, C and D are related for a transmission line as $AD - BC = 1$.
2. In short transmission lines, the effect of capacitance is neglected.
3. The presence of earth in case of overhead lines increases the capacitance.
4. The breakdown stress of atmospheric air is approximately 30 kV/cm.
5. The effect of corona is increased energy loss.
6. Corona acts as a safety valve.
7. Corona loss is less when the shape of the conductor is circular.
8. The current drawn by the line due to corona loss is non-sinusoidal.
9. Presence of ozone as a result of corona is harmful because it corrodes the material.
10. Corona effect can be detected by
    (a) Hissing sound
    (b) Faint luminous glow of bluish colour
    (c) Presence of ozone detected by odour
11. Corona usually occurs when the electrostatic stress in the air around the conductors exceeds 30 kV(maximum value)/cm.
12. Corona loss is directly proportional to f.
13. The effect of corona can be reduced by the use of bundle conductors.
14. As compared to plains, the corona loss in hilly areas is more.
15. Load factor is defined as average load/peak load.
16. Diversity factor is always greater than 1.
17. In a transmission line, the conductors take the form of catenary.
18. Regulating transformers are used in power system to control load flow.
19. The self GMD method is used to evaluate inductance.
20. The Buchholz relay protects a transformer from all types of internal fault.
21. The fact that a conductor carries more current on the surface as compared to core, is known as skin effect.
22. Skin effect is proportional to (diameter of conductor)^2.
23. Skin effect depends on
    (a) Size of the conductor
    (b) Frequency of the current
    (c) Resistivity of the conductor material
24. Skin effect results in increased effective resistance but reduced effective internal reactance of the conductor.
25. The skin effect of a conductor will reduce as the resistivity of conductor material increases.
26. Skin effect is proportional to frequency.
27. A fuse is normally a current limiting device.
28. A fuse wire possesses inverse time characteristics.
29. Most of the fuses operate due to heating effect of current.
30. A circuit breaker will normally operate, whenever fault in the line occurs.

31. Buchholz relay is operated by gas pressure.
32. Guy wire should have high tensile strength.
33. Guy wires generally make an angle of 45 to 60 degrees with the surface of the earth.
34. In ACSR conductors, aluminium conductors carry current and steel conductors provide strength.
35. In ACSR conductors, due to steel, the resistance of conductor increases.
36. Surge impedance of overhead line is of the order of hundreds.
37. The rate of rise of restriking voltage (RRRV) depends upon the inductance and capacitance of the system.
38. A fault is more severe from the view point of RRRV if it is a short line fault.
39. Mho relay is normally used for the protection of long transmission lines.
40. A directional relay senses power.
41. Trip circuit is normally a part of circuit breaker.
42. Switching and protection are served by switchgear.
43. Internal faults of transformer are detected and protected by Buchholz relay.
44. Swing equation gives the variation of torque angle S w.r.t. time.
45. Ferranti effect on transmission line is a phenomenon which represents rise in receiving end voltage at no load.
46. Load flow study is carried out for system planning.
47. If the loading of the line corresponds to the surge impedance loading, the voltage at the receiving end is equal to the sending end.
48. Merz–Price protection is used for the protection of transformers and generators.
49. Merz–Price current protection is also known as differential protection.
50. An isolator operates under no load condition.
51. Isolators are mainly used for providing disconnection for maintenance.
52. Maximum short-circuit occurs in case of three phase faults to ground.
53. Transposition of transmission lines is adopted to make the reactance uniform throughout the line.
54. The positive, negative and zero sequence impedances of a solidly grounded system under steady state condition always follow the relations $Z_1 > Z_2 > Z_0$.
55. Translay relay is used for feeders.
56. Convergence characteristic of Gauss–Seidel method is linear.

3.5 REFERENCES

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Husain, Ashfaq, *Electrical Power Systems*, 5^{th} ed., CBS Publishers, Delhi, 2009.

Kothari, D.P., and Nagrath, J., *Power System Engineering*, McGraw-Hill, India, 2011.

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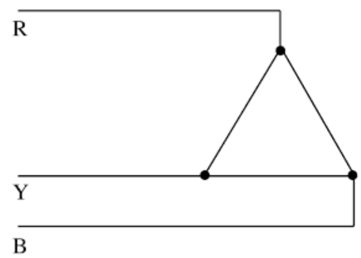
Wadhwa, C.L., *Electrical Power Systems*, New Age International, Delhi, 2010.

3.6 PREVIOUS YEARS' QUESTIONS

1. The phase sequence of the 3-phase system shown in figure is



(a) RYB (b) RBY
(c) BRY (d) YBR [Gate 2004]

2. A hydraulic turbine having rated speed of 250 rpm is connected to a synchronous generator. In order to produce power at 50 Hz, the number of poles required in the generator are

(a) 6 (b) 12
(c) 16 (d) 24 [Gate 2004]

3. The p.u. parameters for a 500 MVA machine on its own base are:
Inertia, M = 20 p.u.; reactance, X = 2 p.u.
The p.u. values of inertia and reactance on 100 MVA common base, respectively, are

(a) 4, 0.4 (b) 100, 10
(c) 4, 10 (d) 100, 0.4 [Gate 2005]

4. The parameters of transposed overhead transmission line are given as:
Self reactance X_s = 0.4 Ω/km and Mutual reactance X_m = 0.1 Ω/km
The positive sequence reactance X_1 and zero sequence reactance X_0, respectively, in Ω/km are

(a) 0.3, 0.2 (b) 0.5, 0.2
(c) 0.5, 0.6 (d) 0.3, 0.6 [Gate 2005]

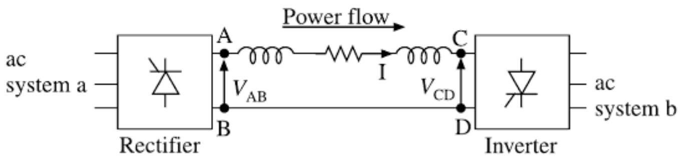
5. The concept of an electrically short, medium and long line is primarily based on the

(a) Nominal voltage of the line
(b) Physical length of the line
(c) Wavelength of the line
(d) Power transmitted over the line [Gate 2006]

6. For a fixed value of complex power flow in a transmission line having a sending end voltage V, the real power loss will be proportional to

(a) V (b) V^2
(c) $1/V^2$ (d) $1/V$ [Gate 2009]

7. Power is transferred from system a to system b by an HVDC link as shown in figure. If the voltages V_{AB} and V_{CD} are as indicated in the figure, and $I > 0$, then

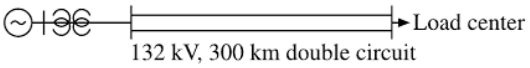


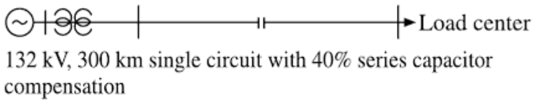
(a) $V_{AB} < 0$, $V_{CD} < 0$, $V_{AB} > V_{CD}$
(b) $V_{AB} > 0$, $V_{CD} > 0$, $V_{AB} < V_{CD}$
(c) $V_{AB} > 0$, $V_{CD} > 0$, $V_{AB} > V_{CD}$
(d) $V_{AB} > 0$, $V_{CD} < 0$ [Gate 2010]

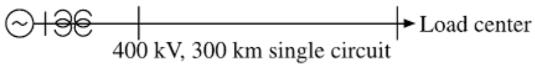
8. A three-phase, 33 kV oil circuit breaker is rated 1200 A, 2000 MVA, 3s. The symmetrical breaking current is
(a) 1200 A (b) 3600 A
(c) 35 kA (d) 104.8 kA [Gate 2010]

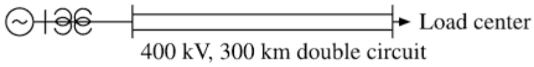
9. Consider two buses connected by an impedance of $(0 + j5)\ \Omega$. The bus 1 voltage is $100\angle 30°$ V, and bus 2 voltage is $100\angle 0°$ V. The real and reactive power supplied by bus 1, respectively, are
(a) 1000 W, 268 VAr
(b) – 1000 W, – 134 VAr
(c) 276.9 W, – 56.7 VAr
(d) – 276.9 W, 56.7 Var [Gate 2010]

10. A nuclear power station of 500 MW capacity is located at 300 km away from a load center. Select the most suitable power evacuation transmission configuration among the following options.

(a) 

(b) 

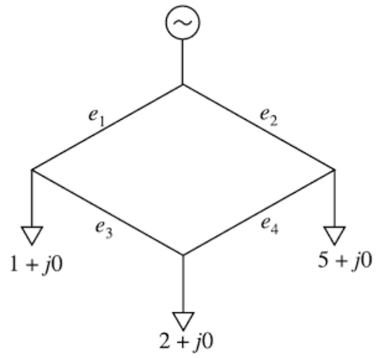
(c) 

(d) 

[Gate 2011]

11. An extra high voltage transmission line of length 300 km can be approximated by a lossless line having propagation constant $\beta = 0.00127$ rad/km. Then the percentage ratio of line length to wavelength will be given by
(a) 24.24% (b) 12.12%
(c) 19.05% (d) 6.06% [Gate 2008]

12. Single line diagram of a 4-bus single source distribution system is shown below. Branches e_1, e_2, e_3 and e_4 have equal impedances. The load current values indicated in figure are in per unit.



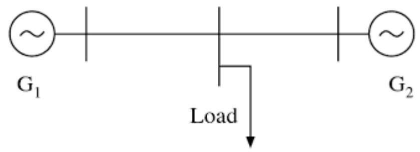
Distribution company's policy requires radial system operation with minimum loss. This can be achieved by opening of the branch
(a) e (b) e_2
(c) e_3 (d) e_4 [Gate 2008]

13. A lossless power system has to serve a load of 250 MW. There are two generators (G_1 and G_2) in the system with cost curves C_1 and C_2 respectively defined as follows

$$C_1(P_{G1}) = P_{G1} + 0.055 \times P_{G1}^2$$
$$C_2(P_{G2}) = 3P_{G2} + 0.03 \times P_{G2}^2$$

Where P_{G1} and P_{G2} are the MW injections from generators G_1 and G_2 respectively. Thus, the minimum cost dispatch will be
(a) P_{G1} = 250 MW; P_{G2} = 0 MW
(b) P_{G1} = 150 MW; P_{G2} = 100 MW
(c) P_{G1} = 100 MW; P_{G2} = 150 MW
(d) P_{G1} = 0 MW; P_{G2} = 250 MW [Gate 2008]

14. A load centre is at an equidistant from the two thermal generating stations G_1 and G_2 as shown in figure.

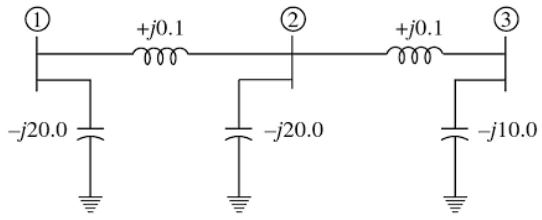


The fuel cost characteristics of the generating stations are given by

$$F_1 = a + bP_1 + cP_1^2 \text{ Rs/hr}$$
$$F_2 = a + bP_2 + 2cP_2^2 \text{ Rs/hr}$$

where P_1 and P_2 are the generation in MW of G_1 and G_2, respectively. For most economic generation to meet 300 MW of load, P_1 and P_2, respectively, are
(a) 150, 150
(b) 100, 200
(c) 200, 100
(d) 175, 125 [Gate 2005]

15. The network shown in figure has impedances in p.u. as indicated. The diagonal element Y_{22} of the bus admittance matrix Y_{bus} of the network is:



(a) $-j\ 19.8$ (b) $+j\ 20.0$
(c) $+j\ 0.2$ (d) $-j\ 19.95$ [Gate 2005]

16. An HVDC link consists of rectifier, inverter transmission line and other equipments. Which one of the following is true for this link?
(a) The transmission line produces/supplies reactive power
(b) The rectifier consumes reactive power and the inverter supplies reactive power from/to the respective connected ac systems
(c) Rectifier supplies reactive power and the inverter consumes reactive power to/from the respective connected ac systems
(d) Both the converters (rectifier and inverter) consume reactive power from the respective connected ac systems [Gate 2006]

17. The rated voltage of a 3-phase power system is given as
(a) rms phase voltage
(b) peak phase voltage
(c) rms line to line voltage
(d) peak line to line voltage [Gate 2004]

18. Incremental fuel costs (in some appropriate unit) for a power plant consists of three generating units are

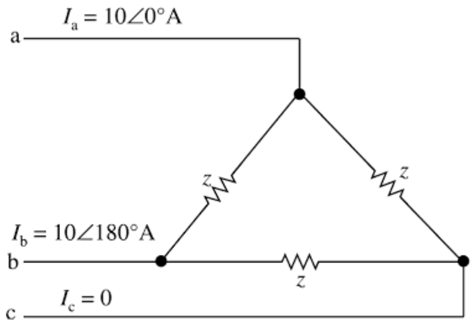
$IC_1 = 20 + 0.3\ P_1$, $IC_2 = 30 + 0.4\ P_2$, $IC_3 = 30$

where P_1 is the power in MW generated by unit I, for $I = 1$, 2 and 3.
Assume that all the three units are operating all the time. Minimum and maximum loads on each unit are 50 MW and 300 MW respectively. If the plant is operating on economic load dispatch to supply the total power demand of 700 MW, the power generated by each unit is
(a) P_1 = 242.86 MW; P_2 = 157.14 MW and P_3 = 300 MW
(b) P_1 = 157.14 MW; P_2 = 242.86 MW and P_3 = 300 MW
(c) P_1 = 300.00 MW; P_2 = 300.00 MW and P_3 = 100 MW
(d) P_1 = 233.3 MW; P_2 = 233.3 MW and P_3 = 233.40 MW [Gate 2003]

19. A power system consists of 300 buses out of which 20 buses are generator buses, 25 buses are ones with reactive power support and 15 buses are the ones with fixed shunt capacitors. All the other buses are load buses. It is proposed to perform a load flow analysis in the system using Newton–Raphson method. The size of the Newton–Raphson Jacobian matrix is
(a) 553 × 553 (b) 540 × 540
(c) 555 × 555 (d) 554 × 554 [Gate 2003]

20. Bundled conductors are mainly used in high voltage overhead transmission lines to
(a) Reduce transmission line losses
(b) Increase mechanical strength of the line
(c) Reduce corona
(d) Reduce sag [Gate 2003]

21. A 3-phase transmission line supplies Δ-connected load Z. The conductor 'c' of the line develops an open circuit fault as shown in figure. The currents in the lines are as shown on the diagram. The positive sequence current component in line 'a' will be



(a) $5.77\angle -30°$ (b) $5.78\angle 90°$
(c) $6.33\angle 90°$ (d) $10.00\angle -30°$
[Gate 2004]

22. An 800 kV transmission line has a maximum power transfer capacity of P. If it is operated at 400 kV with the series reactance unchanged, then new maximum power transfer capacity is approximately
(a) P (b) $2P$
(c) $P/2$ (d) $P/4$ [Gate 2004]

23. A lighting stroke discharges impulse current of 10 kA (peak) on a 400 kV transmission line having surge impedance of 250 Ω. The magnitude of transient over-voltage travelling waves in either direction assuming equal distribution from the point of lightning strike will be
(a) 1250 kV (b) 1650 kV
(c) 2500 kV (d) 2900 kV [Gate 2004]

24. The generalized circuit constants of a 3-phase, 220 kV rated voltage, medium length transmission line are

$$A = D = 0.936 + j0.016 = 0.936\angle 0.98°$$
$$B = 33.5 + j138 = 142.0\angle 76.4°\ \Omega$$
$$C = (-5.18 + j914) \times 10^{-6}\ \text{mho}$$

If the load at the receiving end is 50 MW at 220 kV with a power factor of 0.9 lagging, then magnitude of line to line sending end voltage should be
(a) 133.23 kV (b) 220.00 kV
(c) 230.78 kV (d) 246.30 kV [Gate 2004]

25. For harnessing low variable water heads, the suitable hydraulic turbine with high percentage of reaction and runner adjustable vanes is
(a) Kaplan (b) Francis
(c) Pelton (d) Impeller [Gate 2004]

26. In thermal power plants, the pressure in the working fluid cycle is developed by
(a) Condenser (b) Super heater
(c) Feed water pump (d) Turbine [Gate 2004]

27. The transmission line distance protection relay having the property of being inherently directional is
(a) Impedance relay (b) MHO relay
(c) OHM relay (d) Reactance relay
[Gate 2004]

28. A 800 kV transmission line is having per phase line inductance of 1.1 mH/km and per phase line capacitance of 11.68 nF/km. Ignoring the length of the line, its ideal power transfer capability in MW is
(a) 1204 MW (b) 1504 MW
(c) 2085 MW (d) 2606 MW [Gate 2004]

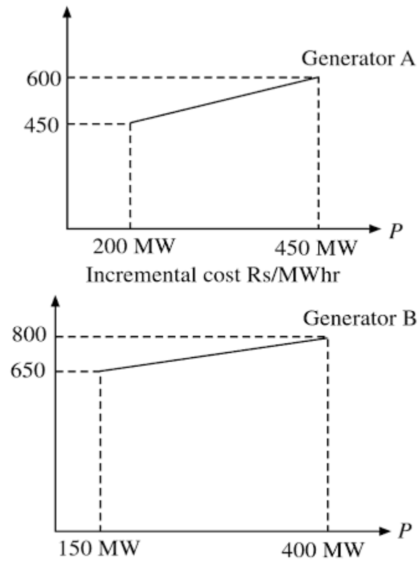
29. The insulation strength of an EHV transmission line is mainly governed by
(a) Load power factor (b) Switching over-voltages
(c) Harmonics (d) Corona
[Gate 2005]

30. High voltage dc (HVDC) transmission is mainly used for
(a) Bulk power transmission over very long distances
(b) Inter-connecting two systems with the same nominal frequency
(c) Eliminating reactive power requirement in the operation
(d) Minimizing harmonics at the converter stations
[Gate 2005]

31. At an industrial sub-station with a 4 MW load, a capacitor of 2 MVAr is installed to maintain the load power factor at 0.97 lagging. If the capacitor goes out of service, the load power factor becomes
(a) 0.85 lag (b) 1.00 lag
(c) 0.80 lag (d) 0.90 lag [Gate 2005]

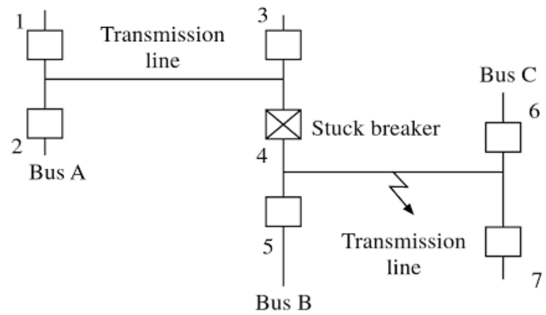
32. The Gauss–Seidel load flow method has following disadvantages. Tick the incorrect statement.
(a) Unreliable convergence
(b) Slow convergence
(c) Choice of slack bus affects convergence
(d) A good initial guess for voltages is essential for convergence [Gate 2006]

33. The incremental cost curves in Rs/MWhr for two generators supplying a common load of 700 MW are shown in figures. The maximum and minimum generation limits are also indicated. The optimum generation schedule is:



(a) Generator A: 400 MW, Generator B: 300 MW
(b) Generator A: 350 MW, Generator B: 350 MW
(c) Generator A: 450 MW, Generator B: 250 MW
(d) Generator A: 425 MW; Generator B: 275 MW
[Gate 2007]

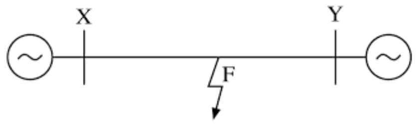
34. Consider the protection system shown in figure. The circuit breakers, numbered from 1 to 7 are of identical type. A single line to ground fault with zero fault impedance occurs at the midpoint of the line (at point F), but circuit breaker 4 fails to operate (stuck breaker). If the relays are coordinated correctly, a valid sequence of circuit breaker operations is



(a) 1, 2, 6, 7, 3, 5 (b) 1, 2, 5, 6, 7, 3
(c) 5, 6, 7, 3, 1, 2 (d) 5, 1, 2, 3, 6, 7
[Gate 2007]

35. A two machine power system is shown in figure. Transmission line XY has positive sequence impedance of Z_1 Ω and zero sequence impedance of Z_0 Ω.
An 'a' phase to ground fault with zero fault impedance occurs at the centre of the transmission line.

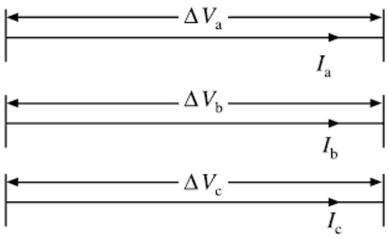
Bus voltage at X and line current from X to F for the phase 'a', are given by V_a volts and I_a amperes, respectively. Then, the impedance measured by the ground distance relay located at the terminal X of line XY will be given by



(a) $Z_1/2\ \Omega$ (b) $Z_0/2\ \Omega$
(c) $(Z_0 + Z_1)/2\ \Omega$ (d) $V_a/I_a\ \Omega$

[Gate 2008]

36. A 3-phase transmission line shown in figure.



Voltage drop across the transmission line is given by the following equation

$$\begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Shunt capacitance of the line can be neglected. If the line has positive sequence impedance of 15 Ω and zero sequence impedance of 48 Ω, then the values of Z_s and Z_m will be

(a) $Z_s = 31.5\ \Omega$; $Z_m = 16.5\ \Omega$
(b) $Z_s = 26\ \Omega$; $Z_m = 11\ \Omega$
(c) $Z_s = 16.5\ \Omega$; $Z_m = 31.5\ \Omega$
(d) $Z_s = 11\ \Omega$; $Z_m = 26\ \Omega$ [Gate 2008]

37. An isolated 50 Hz synchronous generator is rated at 15 MW which is also the maximum continuous power limit of its prime mover. It is equipped with a speed governor with 5% droop. Initially, the generator is feeding three loads of 4 MW each at 50 Hz. One of these loads is programmed to trip permanently if the frequency falls below 48 Hz. If an additional load of 3.5 MW is connected then the frequency will settle down to

(a) 49.417 Hz (b) 49.917 Hz
(c) 50.083 Hz (d) 50.583 Hz

[Gate 2007]

38. Consider the two power systems shown in figure A, which are initially not inter-connected, and are operating in steady state at the same frequency. Separate load flow solutions are computed individually for the two systems, corresponding to this scenario. The bus voltage phasors so obtained are indicated on figure A. These two isolated systems are now inter-connected by a short transmission line as shown in figure B, and it is found that $P_1 = P_2 = Q_1 = Q_2 = 0$.

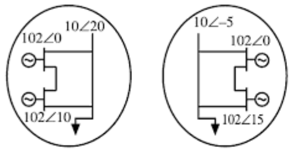


Figure A

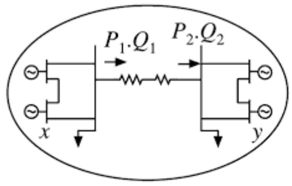


Figure B

The bus voltage phase angular difference between generator bus x and generator bus y after the inter-connection is

(a) 10° (b) 25°
(c) – 30° (d) 30° [Gate 2007]

39. Voltage phasors at the two terminals of a transmission line of length 70 km have a magnitude of 1.0 per unit but are 180° out of phase. Assuming that the maximum load current in the line is 1/5 th of minimum 3-phase fault current, which one of the following transmission line protection schemes will not pick up for this condition?

(a) Distance protection using mho relays with Zone 1 set to 80% of the line impedance
(b) Directional overcurrent protection set to pick up at 1.25 times the maximum load current
(c) Pilot relaying system with directional comparison scheme
(d) Pilot relaying system with segregated phase comparison scheme [Gate 2007]

40. A lossless transmission line having surge impedance loading (SIL) of 2280 MW is provided with a uniformly distributed series capacitive compensation of 30%. Then, SIL of the compensated transmission line will be

(a) 1835 MW
(b) 2280 MW
(c) 2725 MW
(d) 3257 MW [Gate 2008]

41. The interrupting time of a circuit breaker is the period between the instant of

(a) Initiation of short circuit and the arc extinction on an opening operation
(b) Energizing of the trip circuit and the arc extinction on an opening operation

(c) Initiation of short circuit and the parting of primary arc contacts
(d) Energizing of the trip circuit and the parting of primary arc contacts [Gate 2003]

42. A round rotor generator with internal voltage E_1 = 2.0 p.u. and X = 1.1 p.u. are connected to a round rotor synchronous motor with internal voltage E_2 = 1.3 p.u. and X = 1.2 p.u. The reactance of the line connecting the generator to the motor is 0.5 p.u. When the generator supplies 0.5 p.u. power, the rotor angle difference between the machines will be
(a) 57.42° (b) 1°
(c) 32.58° (d) 122.58° [Gate 2003]

43. Choose the appropriate auxiliary components of HVDC transmission system from the following
1. The dc line inductor
2. The ac line inductor
3. Reactive power sources
4. Distance relays on dc line
5. Series capacitance on ac line

(a) 1 and 2 (b) 1 and 3
(c) 2 and 4 (d) 4 and 5 [Gate 2003]

44. A list of relays and the power system components protected by the relays are given in List-I and List-II respectively. Choose the correct match from the four choices given below the lists:

List-I	List-II
A. Distance relay	1. Transformers
B. Under frequency relay	2. Turbines
C. Differential relay	3. Bus bars
D. Buchholz relay	4. Shunt capacitors
	5. Alternators
	6. Transmission lines

	A	B	C	D
(a)	6	5	3	1
(b)	4	3	2	1
(c)	5	2	1	6
(d)	6	4	5	3

[Gate 2003]

45. The ABCD parameters of a 3-phase overhead transmission line are $A = D = 0.9\angle 0$, $B = 200\angle 90°\Omega$ and $C = 0.95 \times 10^{-3}\angle 90°$S. At no-load condition and shunt inductive, reactor is connected at the receiving end of the line to limit the receiving-end voltage to equal to the sending-end voltage. The ohmic value of the reactor is
(a) ∞ Ω (b) 2000 Ω
(c) 105.26 Ω (d) 1052.6 Ω [Gate 2003]

46. A 20-MVA, 6.6 kV, 3-phase alternator is connected to a 3-phase transmission line. The per unit positive-sequence, negative-sequence and zero-sequence impedances of the alternator are j 0.1, j 0.1 and j 0.04 respectively. The neutral of the alternator is connected to ground through an inductive reactor of j 0.05 p.u. The per unit positive-, negative- and zero-sequence impedances of the transmission line are j 0.1, j 0.1 and j 0.3 respectively. All per unit values are based on the machine ratings. A solid ground fault occurs at one phase of the far end of the transmission line. The voltage of the alternator neutral with respect to ground during the fault is
(a) 513.8 V (b) 889.9 V
(c) 1112.0 V (d) 642.2 V [Gate 2003]

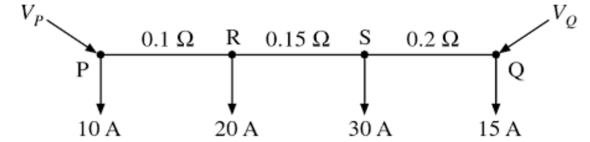
47. Total instantaneous power supplied by a 3-phase ac supply to a balanced R-L load is
(a) Zero
(b) Constant
(c) Pulsating with zero average
(d) Pulsating with non-zero average [Gate 2004]

48. A balanced delta connected load of (8 + j 6) Ω per phase is connected to a 400 V, 50 Hz, 3-phase supply lines. If the input power factor is to be improved to 0.9 by connecting a bank of star connected capacitors, the required kVAr of the bank is
(a) 42.7 (b) 10.2
(c) 28.8 (d) 38.4 [Gate 2003]

49. A 3-phase generator rated at 110 MVA, 11 kV is connected through circuit breakers to a transformer. The generator is having direct axis sub-transient reactance X''_d = 19%, transient reactance X'_d = 26% and synchronous reactance = 130%. The generator is operating at no load and rated voltage when a three phase short circuit fault occurs between the breakers and the transformer. The magnitude of initial symmetrical rms current in the breakers will be
(a) 4.44 kA (b) 22.20 kA
(c) 30.39 kA (d) 38.45 kA [Gate 2004]

50. A surge of 20 kV magnitude travels along a lossless cable towards its junction with two identical lossless overhead transmission lines. The inductance and the capacitance of the cable are 0.4 mH and 0.5 μF per km. The inductance and capacitance of the overhead transmission lines are 1.5 mH and 0.015 μF per km. The magnitude of the voltage at the junction due to surge is
(a) 36.72 kV
(b) 18.36 kV
(c) 6.07 kV
(d) 33.93 kV [Gate 2003]

51. A dc distribution system is shown in figure with load currents as marked. The two ends of the feeder are fed by voltage sources such that $V_P - V_Q = 3$ V. The value of the voltage V_P for a minimum voltage of 220 V at any point along the feeder is

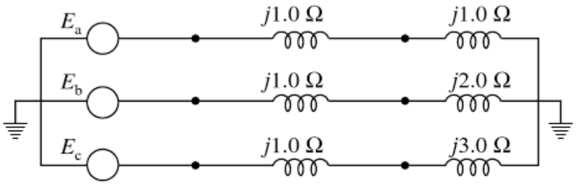


(a) 225.89 V (b) 222.89 V
(c) 220.0 V (d) 228.58 V [Gate 2003]

52. A 3-phase 11 kV generator feeds power to a constant power unity power factor load of 100 MW through a 3-phase transmission line. The line-to-line voltage at the terminals of the machine is maintained constant at 11 kV. The per unit positive sequence impedance of the line based on 100 MVA and 11 kV is j 0.2. The line to line voltage at the load terminals is measured to be less than 11 kV. The total reactive power to be injected at the terminals of the load to increase the line-to-line voltage at the load terminals to 11 kV is

(a) 100 MVAr (b) 10.1 MVAr
(c) – 100 MVAr (d) – 10.1 MVAr
[Gate 2003]

53. A three-phase alternator generating unbalanced voltages is connected to an unbalanced load through a 3-phase transmission line as shown in figure. The neutral of the alternator and the star point of the load are solidly grounded. The phase voltages of the alternator are $E_a = 10\angle 0°$, $E_b = 10\angle{-90°}$V, $E_c = 10\angle 120°$V. The positive-sequence component of the load current is



(a) $1.310\ \angle{-107°}$A (b) $0.332\ \angle{-120°}$A
(c) $0.996\ \angle{-120°}$A (d) $3.510\ \angle{-81°}$A
[Gate 2003]

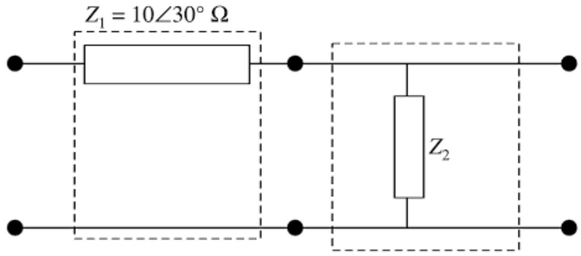
54. A 110 kV, single core coaxial, XLPE insulated power cable delivering power at 50 Hz, has a capacitance of 125 nF/km. If the dielectric loss tangent of XLPE is 2×10^{-4}, then dielectric power loss in this cable in W/km is

(a) 5.0 (b) 31.7
(c) 37.8 (d) 189.0 [Gate 2004]

55. A 500 MVA, 50 Hz, 3-phase turbo-generator produces power at 22 kV. Generator is Y-connected and its neutral is solidly grounded. Its sequence reactance are $X_1 = X_2 = 0.15$ and $X_0 = 0.05$ p.u. It is operating at rated voltage and disconnected from the rest of the system (no load). The magnitude of the sub-transient line current for single line to ground fault at the generator terminal in p.u. will be

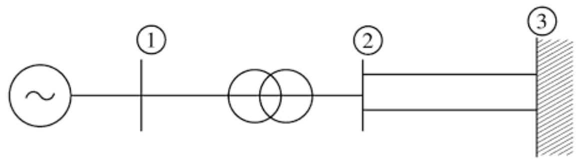
(a) 2.851 (b) 3.333
(c) 6.667 (d) 8.571 [Gate 2004]

56. Two networks are connected in cascade as shown in figure. With the usual notations the equivalent A, B, C and D constants are obtained. Given that $C = 0.025\angle 45°$, the value of Z_2 is



(a) $10\angle 30°\ \Omega$ (b) $40\angle -45°\ \Omega$
(c) 1 Ω (d) 0 Ω
[Gate 2005]

57. A generator with constant 1.0 p.u. terminal voltage supplies power through a step up transformer of 0.12 p.u. reactance and a double circuit line to an infinite bus bar as shown in figure. The infinite bus voltage is maintained at 1.0 p.u. Neglecting the resistances and susceptances of the system, the steady state stability power limit of the system is 6.25 p.u. If one of the double circuit is tripped, the resulting steady state stability power limit in p.u. will be



(a) 12.5 p.u. (b) 3.125 p.u.
(c) 10.0 p.u. (d) 5.0 p.u. [Gate 2005]

58. Keeping in view the cost and overall effectiveness, the following circuit breaker is best suited for capacitor bank switching

(a) Vacuum (b) Air blast
(c) SF_6 (d) Oil [Gate 2006]

59. In a biased differential relay, the bias is defined as a ratio of

(a) Number of turns of restraining and operating coil
(b) Operating coil current and restraining coil current
(c) Fault current and operating coil current
(d) Fault current and restraining coil current
[Gate 2006]

60. Out of the following plant categories

(i) nuclear (ii) run-of-river
(iii) pump storage (iv) diesel

The base load power plants are

(a) (i) and (ii)
(b) (ii) and (iii)
(c) (i), (ii) and (iv)
(d) (i), (iii) and (iv) [Gate 2009]

61. Match the items in List-I (To) with the items in List-II (Use) and select the correct answer using the codes given below the lists.

List-I	List-II
To	**Use**
a. Improve power factor	1. Shunt reactor
b. Reduce the current ripples	2. Shunt capacitor
c. Increase the power flow in line	3. Series capacitor
d. Reduce the Ferranti effect	4. Series reactor

(a) a → 2, b → 3, c → 4, d → 1
(b) a → 2, b → 4, c → 3, d → 1
(c) a → 4, b → 3, c → 1, d → 2
(d) a → 4, b → 1, c → 3, d → 2 [Gate 2009]

62. Match the items in List-I with the items in List-II and select the correct answer using the codes given below the lists.

List-I	List-II
Type of transmission line	Type of distance relay preferred
a. Short line	1. Ohm relay
b. Medium line	2. Reactance relay
c. Long line	3. Mho relay

(a) a → 2, b → 1, c → 3
(b) a → 3, b → 2, c → 1
(c) a → 1, b → 2, c → 3
(d) a → 1, b → 3, c → 2 [Gate 2009]

63. A 500 MW, 21 kV, 50 Hz, 3-phase, 2-pole synchronous generator having a rated power factor = 0.9, has a moment of inertia of $27.5 \times 10^3\,\text{kg m}^2$. The inertia constant ($H$) will be
(a) 2.44 s (b) 2.71 s
(c) 4.88 s (d) 5.42 s [Gate 2009]

64. For enhancing the power transmission in a long EHV transmission line, the most preferred method is to connect a
(a) Series inductive compensator in the line
(b) Shunt inductive compensator at the receiving end
(c) Series capacitive compensator in the line
(d) Shunt capacitive compensator at the sending end [Gate 2011]

65. The sequence components of the fault current are as follows:

$I_{\text{positive}} = j1.5$ p.u., $I_{\text{negative}} = -j0.5$ p.u., $I_{\text{zero}} = -j1$ p.u.

The type of fault in the system is
(a) L-G (b) L-L
(c) L-L-G (d) L-L-L-G [Gate 2012]

66. The inductance of a power transmission line increases with
(a) Decrease in line length
(b) Increase in diameter of conductor
(c) Increase in spacing between the phase conductors
(d) Increase in load current carried by the conductors [Gate 1992]

67. In a 400 kV network, 360 kV is recorded at a 400 kV bus. The reactive power absorbed by a shunt rated for 50 MVAr, 400 kV connected at the bus is
(a) 61.73 MVAr
(b) 55.56 MVAr
(c) 45 MVAr
(d) 40.5 MVAr [Gate 1994]

68. The insulation resistance of a cable of length 10 km is 1 MΩ. For a length of 100 km of the same cable, the insulation resistance will be
(a) 1 MΩ
(b) 10 MΩ
(c) 0.1 MΩ
(d) 0.01 MΩ [Gate 1992]

69. The incremental cost characteristic of two generators delivering 200 MW are as follows:

$$\frac{dF_1}{dP_1} = 20 + 0.1P_1$$

$$\frac{dF_2}{dP_2} = 16 + 0.2P_2$$

For economic operation, the generation P_1 and P_2 should be
(a) $P_1 = P_2 = 100$ MW
(b) $P_1 = 80$ MW, $P_2 = 120$ MW
(c) $P_1 = 200$ MW, $P_2 = 0$ MW
(d) $P_1 = 120$ MW, $P_2 = 80$ MW [Gate 2000]

70. For an unbalanced fault, with paths for zero sequence currents, at the point of fault
(a) The negative and zero sequence voltages are minimum
(b) The negative and zero sequence voltages are maximum
(c) The negative sequence voltage is minimum and zero sequence voltage is maximum
(d) The negative sequence voltage is maximum and zero sequence voltage is minimum [Gate 1996]

71. The transient stability of the power system can be effectively improved by
(a) Improving generator excitation
(b) Phase shifting transformer
(c) Single pole switching of circuit breakers
(d) Increasing the turbine valve opening [Gate 1996]

72. The insulation level of a 400 kV, EHV overhead transmission line is decided on the basis of
(a) Lighting over voltage
(b) Switching over voltage
(c) Corona inception voltage
(d) Radio and TV interference [Gate 1995]

73. Which material is used in controlling chain reaction in a nuclear reactor?
(a) Thorium (b) Heavy water
(c) Boron (d) Beryllium [Gate 1996]

Solutions

1. Ans: (b)
Hint: Here, RYB, BRY and YBR represent the same phase sequence.

2. Ans: (d)
Hint: $$p = \frac{120 \times 50}{250} = 24$$

3. Ans: (d)
Hint:
$$(M_{\text{p.u.}})_n = (M_{\text{p.u.}})_o \times \frac{(\text{MVA})_o}{(\text{MVA})_n} = 20 \times \frac{500}{100} = 100 \text{ p.u.}$$
$$[n \to \text{new}, \ o \to \text{old}]$$
$$(X_{\text{p.u.}})_n = (X_{\text{p.u.}})_o \times \frac{(\text{MVA})_n}{(\text{MVA})_o} \times \frac{(\text{kV})_o^2}{(\text{kV})_n^2} = 2 \times \frac{100}{500} = 0.4$$
$$[(\text{kV})_o = (\text{kV})_n]$$

4. Ans: (d)
Hint: For transmission line,
$$X_1 = X_s - X_m = 0.4 - 0.1 = 0.3 \ \Omega/\text{km}$$
$$X_0 = X_s + 2X_m = 0.4 + 2 \times 0.1 = 0.6 \ \Omega/\text{km}$$

5. Ans: (b)
Hint: *Short line:* upto 80 km
Medium line: 80 to 240 km
Long line: more than 240 km

6. Ans: (c)
Hint: Complex power $S = VI^*$
$$\Rightarrow \qquad I = \frac{S^*}{V^*}$$
Let R = resistance of transmission line
So, real power loss = $I^2 R = \left(\frac{S^*}{V^*}\right)^2 \times R$
Hence, real power loss $\propto \frac{1}{V^2}$
[As S and R are constant]

7. Ans: (c)
Hint: Current flows from high voltage to low voltage
So, $V_{AB} > 0$, $V_{CD} > 0$ and $V_{AB} > V_{CD}$

8. Ans: (c)
Hint: $\sqrt{3} \times$ Rated line voltage (kV) × Symmetrical breaking current (kA) = Rated MVA of circuit breaker
So, symmetrical breaking current = $\frac{200}{\sqrt{3} \times 33} = 35 \text{ kA}$

9. Ans: (a)
Hint: $V_1 = 100\angle 30° \text{ V}$ and $V_2 = 100\angle 0° \text{ V}$
$$Z = 0 + j5 \ \Omega$$
As $\delta_1 > \delta_2$
$$\therefore \quad I = \frac{V_1 - V_2}{Z} = \frac{100\angle 30° - 100\angle 0°}{j5} 10.35\angle 15° \text{ A}$$
So, $\qquad S = P + jQ = V_1 I^*$
$$P + jQ = 100\angle 30° \times (10.35\angle -15°) = 1000 + j268$$
P = 1000 W and Q = 268 VAr

10. Ans: (d)
Hint: To transmit bulk power to a very long distance, high voltage is used and to increase reliability, double circuit is preferred.

11. Ans: (d)
Hint: Wavelength $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.00127} = 4947.39 \text{ km}$
where β = propagation constant
$$\%\,\text{Ratio} = \frac{\text{line length}}{\text{wavelength}} \times 100 = \frac{300 \text{ km}}{4947.39 \text{ km}} \times 100 = 6.06\%$$

12. Ans: (d)
Hint: Let impedance of each branch is R
Case (I) e_1 is opened
Total losses = $8^2 R + 3^2 R + 1^2 R = 74R$
Case (II) e_2 is opened
Total losses = $8^2 R + 7^2 R + 5^2 R = 138R$
Case (III) e_3 is opened
Total losses = $1^2 R + 7^2 R + 2^2 R = 54R$
Case (IV) e_4 is opened
Total losses = $5^2 R + 3^2 R + 2^2 R = 38R$
So, minimum loss occurs when e_4 is opened.

13. Ans: (c)
Hint:
$$\frac{dC_1}{dP_{G1}} = 1 + 0.11P_{G1}$$
$$\frac{dC_2}{dP_{G2}} = 3 + 0.06P_{G2}$$
For minimum cost, $\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}}$
$$1 + 0.11P_{G1} = 3 + 0.06P_{G2}$$
$$P_{G1} + P_{G2} = 250 \text{ MW}$$

Solving equations, we get

$$P_{G1} = 100 \text{ MW} \quad \text{and} \quad P_{G2} = 150 \text{ MW}$$

14. Ans: (c)

Hint:
$$IC_1 = \frac{dF_1}{dP_1} = b + 2cP_1$$
$$IC_2 = \frac{dF_2}{dP_2} = b + 4cP_2$$

For economic generation

$$IC_1 = IC_2$$
$$b + 2cP_1 = b + 4cP_2$$
$$P_1 = 2P_2 \quad \text{(i)}$$

Again, $P_1 + P_2 = 300$ (ii)

Solving Eqs. (i) and (ii)

$$P_1 = 200 \text{ MW and}$$
$$P_2 = 100 \text{ MW}$$

15. Ans: (d)

Hint:
$$Y_{22} = Y_{20} + Y_{12} + Y_{23}$$
$$= \frac{1}{-j20} + \frac{1}{j0.1} + \frac{1}{j0.1}$$
$$= j0.05 - j10 - j10$$
$$= -j19.95 \text{ p.u.}$$

16. Ans: (b)

Hint: No reactive power can be transmitted over a dc link.

17. Ans: (c)

18. Ans: (a)

Hint: P_1 = 50 MW (minimum value)

$$IC_1 = 20 + 0.3 \times 50 = 35$$

Take P_2 = 50 MW (minimum value)

$$IC_2 = 30 + 0.4 \times 50 = 50$$

For minimum value of P_1 and P_2, $IC_1 > IC_3$ and $IC_2 > IC_3$

Therefore, P_3 = 300 MW (maximum load is assigned to unit 3)

So, $P_1 + P_2 = 700 - 300 = 400$ MW (i)

For optimal operation

$$IC_1 = IC_2$$

or, $20 + 0.3\, P_1 = 30 + 0.4\, P_2$

or, $3\, P_1 - 4\, P_2 = 100$ (ii)

Solving Eqs. (i) and (ii), we get

$$P_1 = 242.86 \text{ MW}$$

and $P_2 = 157.14$ MW

19. Ans: (b)

Hint: Total number of buses = N = 300

Number of generator buses = N_g = 20

Number of buses with reactive power support = N_r = 25

Number of buses with fixed shunt capacitors = N_c = 15

Number of voltage controlled buses = $N_r + N_c$ = 40

Number of PQ buses, $N_L = 2N - (N_g + N_r + N_c)$

$= 2 \times 300 - (20 + 40) = 540$

So, size = 540 × 540

20. Ans: (c)

21. Ans: (a)

Hint: Positive sequence current component in line 'a'

$$I_{a1} = \frac{1}{3}[I_a + \alpha I_b + \alpha^2 I_c] \text{ A}$$
$$= \frac{1}{3}[10\angle 0° + (1\angle 120°) \times (10\angle 180°) + (1\angle 240°) \times 0] \text{A}$$
$$I_{a1} = 5.77\angle -30° \text{ A}$$

22. Ans: (d)

Hint: Power transfer capacity ∝ (voltage)^2

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^2$$

or $$P_2 = P \times \left(\frac{400}{800}\right)^2 = \frac{P}{4}$$

23. Ans: (a)

Hint: Impulse current =10 kA (peak)

It will equally divided on both directions.

So, Magnitude of transient over voltage =

$$\left(\frac{10}{2}\right) \times 250 = 1250 \text{ kV}$$

24. Ans: (c)

Hint:

$$V_R = \frac{220}{\sqrt{3}}\angle 0° = 127\angle 0° \text{ kV}$$
$$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 220 \times 10^3 \times 0.9}\angle -\cos^{-1}(0.9)$$
$$= 145.8\,\angle -25.84° \text{ A}$$

Sending end voltage (phase voltage), $V_S = AV_R + BI_R$

or $$V_S = (0.936\angle 0.98°) \times (127\angle 0°) + (142\angle 76.4°) \times (0.1458\angle -25.84°)$$
$$= 133.23\angle 7.77°$$

Sending end voltage (line to line) = $|V_S|_{l-l}$

$= \sqrt{3} \times 133.23$ kV = 230.78 kV

25. Ans: (a)

26. Ans: (c)

27. Ans: (b)

28. Ans: (c)

Hint: Surge impedance of the line =

$$Z_S = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.1\times10^{-3}}{11.68\times10^{-9}}} = 306.88\ \Omega$$

Ideal power transfer capacity =

$$\frac{V_{l-l}^2}{Z_S} = \frac{(800\times10^3)^2}{306.88} = 2085\ \text{MW}$$

29. Ans: (b)

30. Ans: (a)

31. Ans: (c)

Hint: Active load = P_L = 4 MW

Let reactive load = Q_L.

Reactive power supplied by the capacitor = Q_C = 2 MVAr

Load power factor, $\cos\theta_1 = 0.97$

$$\therefore \quad \theta_1 = 14.07°$$

$$\text{Now,} \quad \tan\theta_1 = \frac{Q_L - Q_C}{P_L}$$

$$\text{or} \quad \frac{Q_L - 2}{4} = \tan 14.07°$$

$$\text{or} \quad Q_L = 3$$

When the capacitor goes out of service,

$$\tan\theta_2 = \frac{Q_L}{P_L} = \frac{3}{4} = 0.75$$

$$\theta_2 = 36.87°$$

Therefore, new power factor,

$$\cos\theta_2 = 0.8 \text{ lag}$$

32. Ans: (a)

33. Ans: (c)

Hint: For generator A,

Maximum incremental cost in Rs/MWhr = 600 at 450 MW

For generator B,

Minimum incremental cost in Rs/MWhr = 650 at 150 MW

As maximum value of incremental cost of generator A is less than minimum value of generator B,

$\therefore$ Optimum generation schedule is

Generator A: 450 MW

and Generator B: (700 – 450) = 250 MW.

34. Ans: (c)

Hint: Assume, the relays are coordinated correctly. Due to fault in a particular section, relay in that section must operate first.

The sequence is [5, 6, 7, 3, 1, 2].

35. Ans: (d)

Hint: Ground distance relay is located at the terminal X of line XY.

Therefore, impedance measured by the relay

$$= \frac{\text{Bus voltage}}{\text{current}} = \frac{V_a}{I_a}$$

36. Ans: (b)

Hint: Z_1 = Positive sequence impedance

$$= Z_s - Z_m = 15$$

Z_2 = Negative sequence impedance $= Z_s - Z_m = 15$

Z_0 = Zero-sequence impedance $= Z_s + 2Z_m = 48$

Solving, we get

$$Z_s = 26\ \Omega \text{ and } Z_m = 11\ \Omega$$

37. Ans: (a)

Hint: Change in frequency w.r.t. power

$$\left(\frac{5}{15}\right)\times 3.5 = 1.1667\ \%$$

$$\therefore \quad \Delta f = 0.583$$

So, frequency $f = 50 - 0.583 = 49.417$ Hz

38. Ans: (a)

Hint: As $P_1 = P_2 = 0$ and $Q_1 = Q_2 = 0$

There is no energy transfer, so both bus bar voltages and their phase angles should be same.

Assuming both bus bar phase angles are zero.

So, phase-angle of $x = \angle x = 30°$

And phase-angle of $y = \angle y = 20°$

Phase angular difference $= \angle x - \angle y = 30° - 20° = 10°$

39. Ans: (a)

40. Ans: (c)

Hint: Surge impedance loading, SIL = 2280 MW

Surge impedance = Z_C

$$\therefore \quad \text{SIL} = \frac{V^2}{Z_C}$$

As voltage is constant

$$\text{Or} \quad \text{SIL} \propto \frac{1}{Z_C}$$

Let C_1 be the series capacitance per unit length for series compensation.

Therefore, total series reactance

$$= j\omega L - \frac{1}{j\omega C_1} = j\omega L\left(1 - \frac{1}{j\omega L . j\omega C_1}\right)$$

$$= j\omega L\left(1 - \frac{X_{C_1}}{X_L}\right) = j\omega L(1-\gamma_s)$$

Where γ_s = degree of series compensation

Therefore, $Z_{C_1} = \sqrt{\dfrac{j\omega L}{j\omega C}}$ and $Z_{C_2} = \sqrt{\dfrac{j\omega L(1-\gamma_s)}{j\omega C}}$

$$\therefore \quad \text{SIL}_2 = \text{SIL}_1 \times \frac{Z_{C_1}}{Z_{C_2}} = \frac{\text{SIL}_1}{\sqrt{1-\gamma_s}} = \frac{2280}{\sqrt{1-0.3}}$$

$$= 2725.12 \text{ MW}$$

41. Ans: (b)

42. Ans: (c)

Hint: $x_{eq} = x_1 + x_{\text{line}} + x_2 = 1.1 + 0.5 + 1.2 = 2.8$ p.u.

$E_1 = 2.0$ p.u. and $E_2 = 1.3$ p.u.

Rotor angle difference, $\delta = (\delta_1 - \delta_2)$

Power transferred from generator to motor

$$P = \frac{E_1 E_2}{x_{eq}} \sin\delta$$

or $$0.5 = \frac{2 \times 1.3}{2.8} \sin\delta$$

$$\therefore \quad \delta = 32.58°$$

43. Ans: (b)

44. Ans: (a)

45. Ans: (b)

Hint: Given,

$$|A| = 0.9 \text{ and } \alpha = 0°$$

$$|B| = 200 \text{ and } \beta = 90°$$

$$S_R = \frac{|V_S||V_R|}{|\beta|} \angle(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \angle(\beta - \alpha)$$

$$\therefore \quad P_R = \frac{|V_S||V_R|}{|\beta|} \cos(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \cos(\beta - \alpha) \quad \text{(i)}$$

and $$Q_R = \frac{|V_S||V_R|}{|\beta|} \sin(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \sin(\beta - \alpha) \quad \text{(ii)}$$

At no-load condition, $P_R = 0$

Since, $P_R = 0$, Eq. (i) becomes

$$0 = \frac{|V_R|^2}{200} \cos(90 - \delta) - \frac{0.9}{200} |V_R|^2 \cos(90 - 0)$$

or $$0 = \frac{|V_R|^2}{200} \sin\delta$$

or $$\delta = 0°$$

From Eq. (ii),

$$Q_R = \frac{|V_R|^2}{200} \sin(90 - 0) - \frac{0.9}{200} |V_R|^2 \sin(90 - 0)$$

$$Q_R = \frac{0.1}{200} |V_R|^2 \quad \text{(iii)}$$

To make, receiving end voltage to be equal to sending end voltage, reactive power at the receiving end must be equal to reactive power absorbed by the reactor. Reactive power absorbed by reactor of reactance x

$$Q = \frac{|V_R|^2}{x} \quad \text{(iv)}$$

Equating Eqs. (iii) and (iv), we get

$$\frac{0.1}{200} |V_R|^2 = \frac{|V_R|^2}{x}$$

or $$x = 2000\ \Omega$$

46. Ans: (d)

Hint: For alternator,

$$Z_1^G = j\ 0.1 \text{ p.u.}$$
$$Z_2^G = j\ 0.1 \text{ p.u.}$$
$$Z_0^G = j\ 0.04 \text{ p.u.}$$

For line,

$$Z_1^L = j\ 0.1 \text{ p.u.}$$
$$Z_2^L = j\ 0.1 \text{ p.u.}$$
$$Z_0^L = j\ 0.3 \text{ p.u.}$$
$$Z_n = j\ 0.05 \text{ p.u.}$$

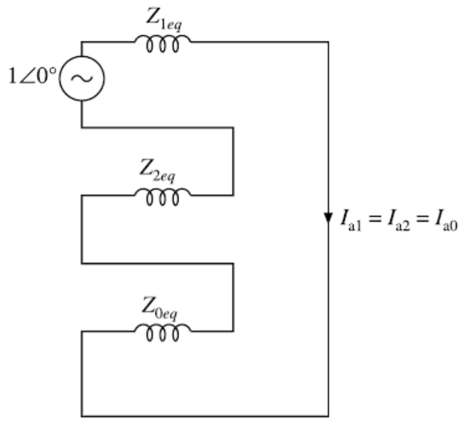
Equivalent sequence impedances;

$$Z_{1eq} = Z_1^G + Z_1^L = j0.1 + j0.1 = j0.2 \text{ p.u.}$$
$$Z_{2eq} = Z_2^G + Z_2^L = j0.1 + j0.1 = j0.2 \text{ p.u.}$$
$$Z_{0eq} = Z_0^G + Z_0^L + 3Z_n = j0.04 + j0.3 + 3 \times j0.05$$
$$= j0.49 \text{ p.u.}$$

For single line to ground fault

$$I_{a1} = I_{a2} = I_{a0} = \frac{1\angle 0°}{j0.2 + j0.2 + j0.49} = -j1.1236 \text{ p.u.}$$

Fault current, $I_{\text{fault}} = 3I_{a1} = -j\ 33708$ p.u.



Voltage of neutral w.r.t. ground

$$V_n = I_{\text{fault}} \times Z_n = -j\ 3.3708 \times j\ 0.05 = 0.16854 \text{ p.u.}$$

Per phase base voltage

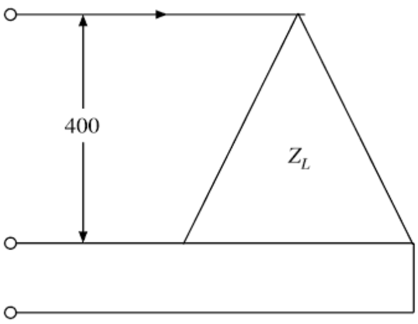
$$V_p = \frac{6.6}{\sqrt{3}} \text{ kV} = \frac{6.6}{\sqrt{3}} \times 10^3 \text{ V}$$

$$V_n = \frac{6.6}{\sqrt{3}} \times 0.16854 \times 10^3 = 642.2 \text{ V}$$

47. Ans: (b)

48. Ans: (b)

Hint: Load is connected to 3-ϕ supply



Load impedance = Z_L = 8 + j6 Ω

$V_p = V_L = 400\angle 0°$ V [As load is delta connected]

$$\text{Phase current} = I_p = \frac{V_p}{Z_L} = \frac{400\angle 0°}{8 + j6} = 40\angle -36.87° \text{ A}$$

Load complex power,

$$S_L = 3V_p I_p = 3 \times 400\angle 0° \times (40\angle -36.87°)$$

$$S_L = 38.4 + j28.8 \text{ kVA}$$

$$P_L + jQ_L = 38.4 + j28.8$$

or $\quad P_L = 38.4$ kW and $Q_L = 28.8$ kVAr

When capacitors banks are connected, power factor changes to 0.9.

Assuming required kVAr of the bank, Q_1

So, cos θ = 0.9

or, θ = 25.84°

$$\tan\theta = \frac{Q}{P_L} = \frac{Q_L - Q_1}{P_L}$$

or $$\tan 25.84° = \frac{28.8 - Q_1}{38.4}$$

$$Q_1 = 10.2 \text{ kVAr}$$

49. Ans: (c)

Hint: The generator operates at rated voltage and no load condition

So, $E_g = 1.0\angle 0°$ p.u.

Initial symmetrical current

$$= I = \frac{E_g}{X_d''} = \frac{1\angle 0°}{j0.19} = 5.26\angle -90° \text{ p.u.}$$

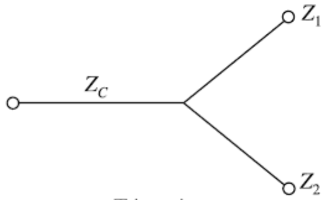
$$\text{Base current} = \frac{\text{Base MVA}}{\sqrt{3} \times \text{Base kV}} = \frac{100}{\sqrt{3} \times 11}$$

Magnitude of initial symmetrical current

$$= 5.26 \times \text{Base current} = 5.26 \times \frac{100}{\sqrt{3} \times 11} = 30.37 \text{ kA}$$

50. Ans: (d)

Hint:



T-junction

where $Z_1 = Z_2$

Parameter of cable

$$\text{Surge impedance of the cable} = Z_C = \sqrt{\frac{L_C}{C_C}} = \sqrt{\frac{0.4 \times 10^{-3}}{0.5 \times 10^{-6}}}$$

$$= 28.28\ \Omega.$$

Parameters of overhead transmission lines

Surge impedance of the lines

$$= Z_1 = Z_2 = \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{1.5 \times 10^{-3}}{0.015 \times 10^{-6}}} = 316.22\ \Omega.$$

For V = 20 kV

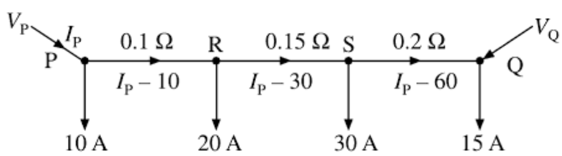
Junction voltage,

$$V_{\text{junction}} = \frac{\frac{1}{Z_C}}{\frac{1}{Z_C} + \frac{1}{Z_1} + \frac{1}{Z_2}} \times 2V$$

$$= \frac{\frac{1}{28.28}}{\frac{1}{28.28} + \frac{2}{316.22}} \times 2 \times 20 = 33.93 \text{ kV}$$

51. Ans: (a)

Hint: Let current injected at point $P = I_P$



$$V_P - V_Q = 0.1\ (I_P - 10) + 0.15\ (I_P - 30) + 0.2\ (I_P - 60)$$

or, $\quad 3 = 0.45\ I_P - 17.5$

or, $\quad I_P = 45.55$ A

In section SQ, direction of current will be opposite. So, point S shows minimum voltage, V_S = 220 V.

$V_P = V_S + 0.1\ (I_P - 10) + 0.15\ (I_P - 30)$
$= 220 + 0.1\ (45.55 - 10) + 0.15\ (45.55 - 30)$
$= 225.89$ V

52. Ans: (b)

Hint: $|V_S| = |V_R| = 11\text{ kV}$

Per unit impedance of the line = j 0.2 p.u.
Impedance of the line = p.u. impedance × base impedance

or, $$x = j0.2 \times \frac{11^2}{100} = j0.242\ \Omega$$

Active power at receiving end = P_R = 100 MW

$$P_R = \frac{|V_S||V_R|}{x}\sin\delta$$

or $$100 = \frac{11 \times 11}{0.242}\sin\delta$$

Reactive power at receiving end

$$Q_R = \frac{|V_S||V_R|}{x}\cos\delta - \frac{|V_R|^2}{x}$$

$$= \frac{11 \times 11}{0.242}\cos 11.54 - \frac{11^2}{0.242} = -10.1\ \text{MVAr}$$

For upf, load reactive power must be zero
Therefore, reactive power to be injected at terminals $= -Q_R = -(-10.1) = 10.1$ MVAr

53. Ans: (d)

Hint: Since both sides are grounded, we can take it as 3 independent circuits.

Hence, $$I_a = \frac{E_a}{Z_a} = \frac{10\angle 0^\circ}{2j} = -5j\ \text{A}$$

$$I_b = \frac{E_b}{Z_b} = \frac{10\angle -90^\circ}{3j} = 3.33\angle -180^\circ\ \text{A}$$

$$I_c = \frac{E_c}{Z_c} = \frac{10\angle 120^\circ}{4j} = 2.5\angle 30^\circ\ \text{A}$$

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}[5\angle -90^\circ + 3.33\angle(-180^\circ + 120^\circ)$$
$$+ 2.5\angle(240^\circ + 30^\circ)]$$
$$= 3.510\angle -81^\circ\ \text{A}$$

54. Ans: (b)

Hint: Phase voltage, $V = \dfrac{110}{\sqrt{3}}$ kV

C = 125 nF/km

$$\tan\delta = 2 \times 10^{-4}$$

Dielectric power loss in cable = $P = V^2\omega C \tan\delta$

$$= \left(\frac{110 \times 10^3}{\sqrt{3}}\right)^2 \times 2\pi \times 50 \times 125 \times 10^{-9} \times 2 \times 10^{-4}$$

$$= 31.7\ \text{W/Km}$$

55. Ans: (d)

Hint: Since, the generator is operating at no-load and rated voltage

$$E_a = 1\angle 0\ \text{p.u.}$$
$$Z_1 = jX_1 = j0.15\ \text{p.u.}$$
$$Z_2 = jX_2 = j0.15$$

And, $$Z_0 = jX_0 = j0.05\ \text{p.u.}$$

For single-line to ground fault

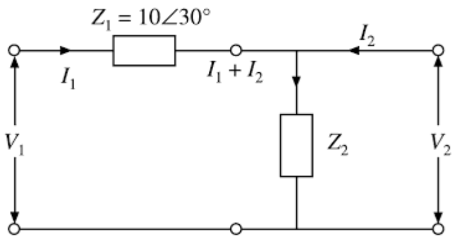
$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

$$= \frac{1\angle 0^\circ}{j0.15 + j0.15 + j0.05} = 2.857\angle 90^\circ\ \text{p.u.}$$

Fault current = $3I_{a1} = 3 \times 2.857 = 8.571$ p.u.

56. Ans: (b)

Hint:



$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

From the circuit,

$$V_2 = Z_2(I_1 + I_2) \qquad \text{(i)}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

Putting $I_2 = 0$ in Eq. (i), we get

$$V_2 = Z_2 I_1$$

$$Z_2 = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{C} = \frac{1}{0.025\angle 45^\circ} = 40\angle -45^\circ\ \Omega$$

57. Ans: (d)

Hint: Induced emf of the generator = $E = 1$ p.u.

Terminal voltage = $V = 1$ p.u.
Reactance of transformer = $X_t = 0.12$ p.u.

When double-circuit line is connected, the reactance of line $= \dfrac{X \times X}{X + X} = \dfrac{X}{2}$

Steady state stability power limit,

$$P_1 = \frac{EV}{X_t + \frac{X}{2}} = \frac{1}{0.12 + \frac{X}{2}}$$

or $$6.25 = \frac{1}{0.12 + \frac{X}{2}}$$

or $$X = 0.08 \text{ p.u.}$$

When one of the double circuit tripled, steady state stability power limit

$$P_2 = \frac{EV}{X_t + X} = \frac{1}{0.12 + 0.08} = 5 \text{ p.u.}$$

58. Ans: (a)

59. Ans: (a)

60. Ans: (c)

61. Ans: (b)

62. Ans: (a)

63. Ans: (a)

Hint: J = moment of inertia = 27.5×10^3 kg m^2.

Synchronous speed, $N_s = \dfrac{120 \times 50}{2} = 3000$ rpm

Kinetic energy of the rotor $= \dfrac{1}{2} J\omega_s^2$

$$= \frac{1}{2} \times 27.5 \times 10^3 \times (314.16)^2 = 1357 \text{ MJ}$$

G = Machine rating $= \dfrac{500}{0.9} = 555.55$ MVA

Inertial constant $= H = \dfrac{\text{K.E.}}{G} = \dfrac{1357}{555.55} = 2.44$ MJ/MVA

64. Ans: (c)

65. Ans: (c)

66. Ans: (c)

Hint: $$L \propto \ln\left(\frac{d}{r'}\right)$$

or $$L \propto d \quad \text{and} \quad L \propto \frac{1}{r'}$$

67. Ans: (d)

Hint: $$Q_{\text{absorbed}} = \frac{V^2}{X_{\text{rated}}} = \frac{(360)^2}{(400)^2/50} = 40.5 \text{ MVAr}$$

68. Ans: (c)

Hint: Insulation resistance,

$$R_{\text{ins}} = \frac{\rho}{2\pi l} \ln\left(\frac{R}{r}\right) \Omega$$

or $$\frac{R_2}{R_1} = \left(\frac{l_1}{l_2}\right)$$

$$R_2 = 1 \text{ M}\Omega \left(\frac{10}{100}\right) = 0.1 \text{ M}\Omega$$

69. Ans: (d)

Hint: $P_1 + P_2 = 200$ MW (i)

For economic operations

$$20 + 0.1P_1 = 16 + 0.2P_2$$
$$4 = 0.2P_2 - 0.1P_1$$
$$2P_2 - P_1 = 40 \text{ MW} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$P_2 = 80 \text{ MW and } P_1 = 120 \text{ MW}$$

70. Ans: (b)

Hint: Unbalanced fault → Negative sequence dominate

Ground fault → Zero sequence dominates

71. Ans: (c)

Hint: Using high speed circuit breakers, this can be achieved.

72. Ans: (b)

Hint: Up to 345 kV, insulation level is decided based on lighting over voltages and above 345 kV; insulation level is decided based on switching over voltage.

73. Ans: (c)

Hint: Boron and cadmium rods are used as control rods.

4

Electrical Machines

4.1 SYLLABUS

Single phase transformer—equivalent circuit, phasor diagram, tests, regulation and efficiency; three-phase transformers—connections, parallel operation; auto-transformer; energy conversion principles; dc machines—types, windings, generator characteristics, armature reaction and commutation, starting and speed control of motors; three-phase induction motors—principles, types, performance characteristics, starting and speed control; single phase induction motors; synchronous machines—performance, regulation and parallel operation of generators, motor starting, characteristics and applications; servo and stepper motors.

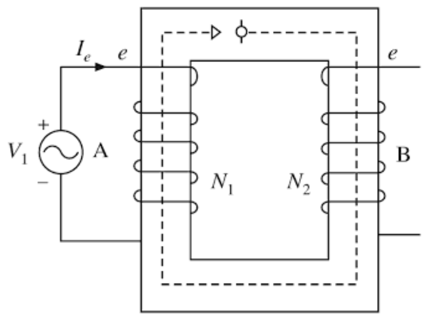
4.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	4	4	4	4	3	2	3	1	3	3	4	4
2 Marks Question	10	12	7	10	8	4	3	3	2	4	4	5
Total Marks	**24**	**28**	**18**	**24**	**19**	**10**	**9**	**7**	**7**	**11**	**12**	**14**

4.3 TOPICS TO BE FOCUSED

SINGLE PHASE TRANSFORMER

- Transformer is a device that transfers electrical energy from one electrical circuit to electrically isolated another electrical circuit through the medium of magnetic field.

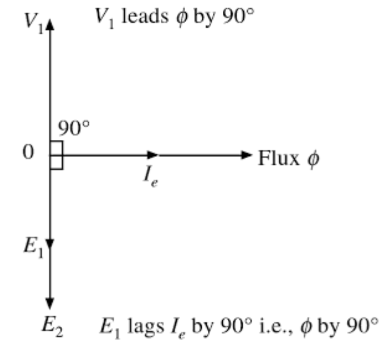


If ac voltage source is placed across winding A then electrical energy received by winding A is first converted to magnetic energy which focused through magnetic core and reaches winding B where it is reconnected electrical energy as shown in figure. When energy is transferred from winding A to B, winding A is called primary winding and winding B is called secondary winding and vice versa. Winding are of two types:

(a) Core type
(b) Shell type

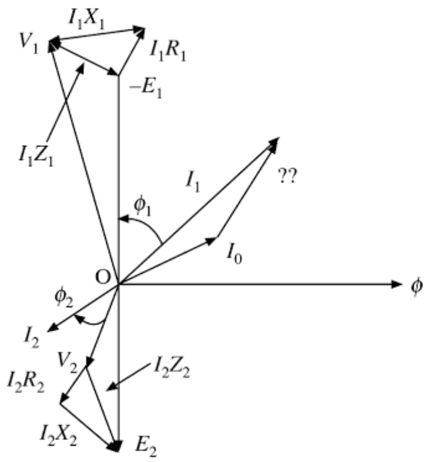
- In core, the winding surrounds a considerable part of the steel core and in shell type the steel core surrounds a major part of the windings.
- In figure, the primary winding A is connected to an alternating voltage source therefore, an alternating current I_e starts flowing through N_1 turns. The alternating m.m.f. $F_1 = N_1 I_e$ sets up alternating flux ϕ which is confined to the magnetic core. This flux induces voltage E_1 in A and E_2 in B. If load is connected then current starts flowing. Thus, energy is transferred from one winding to isolated another winding due to magnetic coupling.
- Ideal transformer has following assumptions:
  (i) no copper losses (no winding resistance)

(ii) no iron loss in core
(iii) no leakage flux
(iv) core has constant permeability i.e., linear magnetization curve



Now, if an alternating voltage V_1 is applied to the primary winding of an ideal transformer, counter emf E_1 will be induced in the primary winding. As windings are purely inductive, this induced emf E_1 lags behind ϕ by 90° (followed by $e_1 = -N_1 \dfrac{d\phi}{dt}$) and will be exactly equal to the apply voltage but in 180° phase opposition because of Lenz's law. Current drawn from the source produces required magnetic flux. This current is called magnetizing current of the transformer I_e.This flux ϕ gets linked with the secondary winding and emf E_2 gets induced by mutual induction. This mutually induced emf E_2 is in phase with E_1. If closed circuit is provided at secondary winding, E_2 causes current I_2 to flow in the circuit. For an ideal transformer, $E_1I_1 = E_2I_2$.

Inductive load phasor diagram is shown in the following figure.



Transformer Tests

The complete analysis of a given transformer can be done, if the equivalent parameters of that transformer are known. The equivalent parameters of the transformers include:

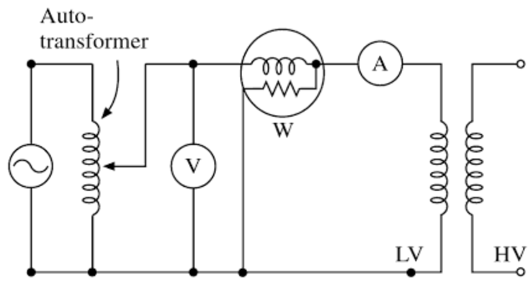
(i) Equivalent resistance r_{e1} as referred to primary or r_{e2} as referred to secondary
(ii) Equivalent leakage reactance x_{e1} as referred to primary or x_{e2} as referred to secondary
(iii) The core loss resistance R_C
(iv) The core magnetizing reactance X_m

Transformer tests are of two types:

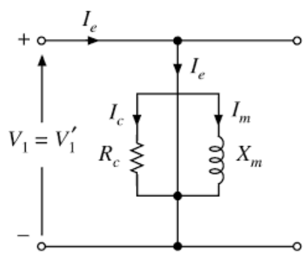
(a) Short circuit
(b) Open circuit

The short circuit and open circuit tests on a transformer helps to obtain (i) parameters of the equivalent circuit, (ii) voltage regulation, and (iii) efficiency of the transformer.

Open circuit or no load test: This test gives the no load loss or core loss, no load current and in turn helps in finding R_C and X_m of the equivalent circuit. These obtained parameters are referred to the side from where reading is taken.



(a)



(b)

The test is usually performed on the LV side of transformer with HV side left open circuited. At rated frequency, the voltage on LV side is gradually raised to rated voltage and the reading of wattmeter (P_{OC}), voltmeter (V_{OC}) and ammeter (I_{OC}) is taken.

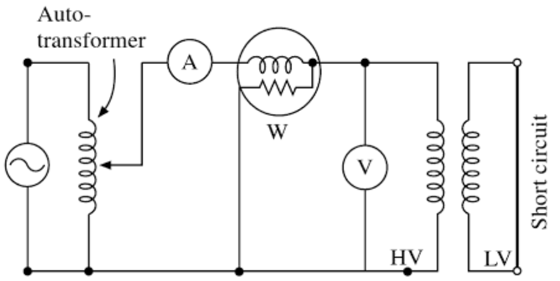
Now, $$P_{OC} = V_{OC}I_{OC}\cos\phi_{OC}$$

$$\cos\phi_{OC} = \frac{P_{OC}}{V_{OC}I_{OC}} \quad \text{and} \quad \sin\phi_{OC} = \sqrt{1-\cos^2\phi_{OC}}$$

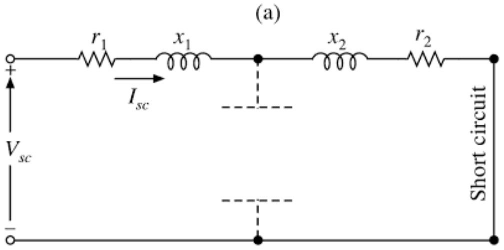
$\because$ The current I_{OC} has two components, $I_{OC}\sin\phi_{OC}$ flowing through X_m and $I_{OC}\cos\phi_{OC}$ flowing through R_C.

$\therefore$ Core loss resistance $R_C = \dfrac{V_{OC}}{I_{OC}\cos\phi_{OC}}$ and magnetizing reactance $X_m = \dfrac{V_{OC}}{I_{OC}\sin\phi_{OC}}$

Short circuit or impedance test: This test gives the full load copper loss of transformer and helps in finding the equivalent resistance r_e and leakage reactance x_e of transformer referred to the side where test is performed.



(a)



(b)

The test is usually performed on HV side with LV side solidly short circuited by thick conductor. 2–12% of the rated voltage is sufficient to circulate rated current in both HV and LV sides. As the applied voltage is only a small percentage of rated voltage, the mutual flux in the core is also very small and hence core losses are very small compared to rated value. Thus, the wattmeter reading shows mainly the full load copper loss of the whole transformer. If P_{SC}, V_{SC} and I_{SC} are the readings of the wattmeter, voltmeter and ammeter respectively, then

$$Z_{SC} = \frac{V_{SC}}{I_{SC}}; \quad R_{SC} = \frac{P_{SC}}{I_{SC}^2}; \quad X_{SC} = \sqrt{Z_{SC}^2 + R_{SC}^2}$$

where R_{SC}, X_{SC} and Z_{SC} are the equivalent resistance, equivalent reactance and equivalent impedance of the whole transformer referred to the side where test is performed.

Note: Voltage regulation of a transformer can be determined from data of short circuit test only whereas data of both short circuit and open circuit test is required to obtain all the parameters of equivalent circuit and efficiency of the transformer.

Voltage Regulation

If V_2 = secondary terminal voltage at full load, and
E_2 = secondary terminal voltage at no load

Then, % voltage regulation (down) $= \dfrac{E_2 - V_2}{E_2} \times 100\%$

and, % voltage regulation (up) $= \dfrac{E_2 - V_2}{V_2} \times 100\%$

The change in secondary terminal voltage with load current is due to the primary and secondary leakage impedances. The magnitude of voltage regulation depends on the load power factor, load current, total resistance and total reactance of the transformer.

Lesser the value of voltage regulation, better the transformer, because for a good transformer its secondary voltage should remain constant under all condition of load.

$$\% \text{ voltage regulation } = \frac{I_2 r_{e2} \cos\theta_2 - I_2 x_{e2} \sin\theta_2}{E_2} \times 100\%$$

where I_2 is secondary current at any load, r_{e2} and x_{e2} are the equivalent resistance and leakage reactance of transformer referred to secondary.

For zero voltage regulation, the load power factor should be $\cos\theta_2 = \dfrac{x_{e2}}{z_{e2}}$ leading.

For maximum voltage regulation, the load power factor should be $\cos\theta_2 = \dfrac{r_{e2}}{z_{e2}}$ lagging.

The magnitude of maximum voltage regulation is equal to the per unit value of equivalent leakage impedance of the transformer.

Percentage or Per Unit Resistance, Reactance and Impedance

(i) Per unit resistance at full load

Base impedance,

$$Z_{b1} = \frac{\text{Base voltage}}{\text{Base current}} = \frac{\text{rated voltage at side 1}}{\text{rated current at side 1}} = \frac{V_1}{I_1}$$

$$= \frac{V_1^2}{V_1 I_1} = \frac{(\text{rated voltage})^2}{\text{rated VA}}$$

$$R_{\text{p.u.}} = \frac{r_{e1}}{z_{b1}} = \frac{r_{e1} I_1}{z_{b1} I_1} = \frac{r_{e1} I_1}{V_1} = \frac{r_{e1} I_1^2}{V_1 I_1} = \frac{\text{Ohmic loss at rated current}}{\text{Rated VA}}$$

$$R_{\text{p.u.}} = \frac{r_{e2}}{z_{b2}} = \frac{r_{e2} I_2}{z_{b2} I_2} = \frac{r_{e2} I_2}{V_2} = \frac{r_{e2} I_2^2}{V_2 I_2} = \frac{\text{Ohmic loss at rated current}}{\text{Rated VA}}$$

Therefore, per unit R = per unit copper loss or, % R = % Cu Loss

(ii) Per unit leakage reactance at full load

$$X_{\text{p.u.}} = \frac{x_{e1}}{z_{b1}} = \frac{x_{e1} I_1}{z_{b1} I_1} = \frac{x_{e1} I_1}{V_1} = \frac{x_{e1} I_1^2}{V_1 I_1}$$

$$X_{\text{p.u.}} = \frac{x_{e2}}{z_{b2}} = \frac{x_{e2} I_2}{z_{b2} I_2} = \frac{x_{e2} I_2}{V_2} = \frac{x_{e2} I_2^2}{V_2 I_2}$$

(iii) Per unit leakage impedance at full load

$$Z_{\text{p.u.}} = \frac{z_{e1}}{z_{b1}} = \frac{z_{e1} I_1}{z_{b1} I_1} = \frac{z_{e1} I_1}{V_1} = \frac{z_{e1} I_1^2}{V_1 I_1}$$

$$Z_{\text{p.u.}} = \frac{z_{e2}}{z_{b2}} = \frac{z_{e2}I_2}{z_{b2}I_2} = \frac{z_{e2}I_2}{V_2} = \frac{z_{e2}I_2^2}{V_2I_2}$$

Relation among per unit resistance, reactance and impedance:

$$Z_{\text{p.u.}} = R_{\text{p.u.}} + jX_{\text{p.u.}} \text{ and } \%Z = \%R + j\%X$$

Efficiency of Transformer

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{\text{core}} + I_2^2 r_{e2}}$$

where V_2I_2 = output VA, $\cos\theta_2$ = load pf, P_{core} = core loss and $I_2^2 r_{e2}$ = ohmic loss.

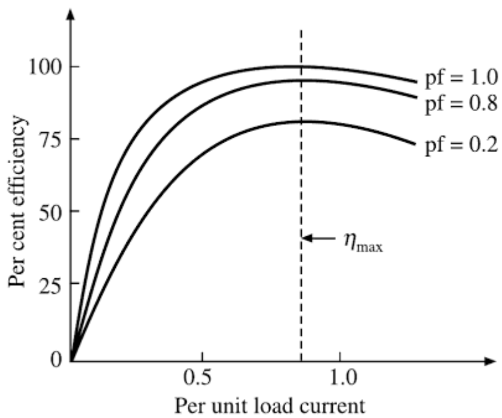
Condition for maximum efficiency: Variable loss = Fixed loss or Core loss = Copper loss.
Load at which maximum efficiency occurs:

$$I_2 = I_{fl}\sqrt{\frac{P_{\text{core}}}{I_{fl}^2 r_{e2}}} = \sqrt{\frac{\text{rated core loss}}{\text{rated copper loss}}}$$

Also, kVA at which maximum efficiency occurs:

$$\text{kVA}_{\max} = \text{kVA}_{\text{rated}}\sqrt{\left(\frac{P_{\text{core}}}{I_{fl}^2 r_{e2}}\right)_{\text{rated}}}$$

Note: For constant load current, the maximum efficiency occurs at unity power factor. Transformer efficiency is maximum at same load current, irrespective of the load pf.



Energy efficiency of a transformer is defined as the ratio of total energy output for a certain period to total energy input for the same period. If the period is for 1 day of 24 hours, the energy efficiency is called *all day efficiency*.

$$\text{All day } \eta = \frac{\text{Output in kWh}}{\text{Input in kWh}} \text{ (for 24 hours)}$$

THREE-PHASE TRANSFORMER

A bank of three transformer or a three-phase transformer, may have its primary and secondary winding connected in star (Y), delta (D) or zigzag (Z). Depending upon the type of connections and phase displacement between the HV and LV line emfs, four phasor groups are characterized. They are:

Group number 1: Zero phase displacement
The phase angle displacement between the HV and LV line emfs for this group is zero. Connections under this group are (i) Star-star zero (Yy0), (ii) Delta-delta zero (Dd0), and (iii) Delta-zigzag zero (Dz0).

Group number 2: 180° phase displacement
The phase angle displacement between the HV and LV line emfs for this group is 180°. Connections under this group are (i) Star-star six (Yy6), (ii) Delta-delta six (Dd6), and (iii) Delta-zigzag six (Dz6).

Group number 3: –30° phase displacement
A phase displacement of –30° means that the LV line phasor lags the HV line phasor by 30°. Connections under this group are (i) Star-delta one (Yd1), (ii) Delta-star one (Dy1), and (iii) Star zigzag one (Yz1).

Group number 4: +30° phase displacement
A phase displacement of +30° means that the LV line phasor leads the HV line phasor by 30°. Connections under this group are (i) Star-delta eleven (Yd11), (ii) Delta-star eleven (Dy11), and (iii) Star zigzag eleven (Yz11).

- Open delta connection
  Open delta kVA rating is 0.577 times the closed delta kVA rating.
- Utilization factor is defined as the ratio

$$\frac{\text{Actual available kVA rating}}{\text{Sum of the kVA rating of the transformer installed}}$$

For open delta, the utilization factor is 0.866
For closed delta, the utilization factor is 1

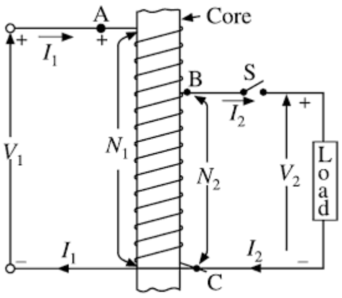
Harmonics in Transformer

We have come to conclusion that

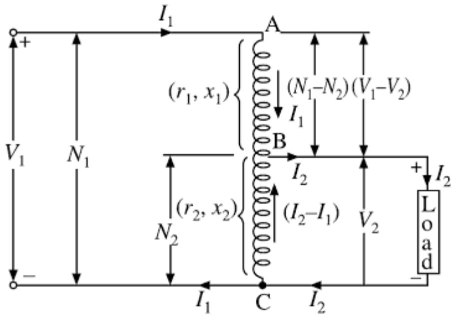
(i) For a sinusoidal applied voltage, flux is sinusoidal and magnetization current due to saturation is peaky in nature containing pronounced 3rd harmonic component.
(ii) For a sinusoidal magnetization current, the flux wave is flat-tapped and the induced emfs both in primary and secondary windings are peaky in nature containing pronounced 3rd harmonic current.

AUTO-TRANSFORMER

Auto-transformer has one winding only, a part of which is common to both primary and secondary circuit. In this transformer, primary and secondary are not electrically isolated.



(a)



(b)

Neglecting losses and no load current, $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = k$

The output VA of auto–transformer are of two types:
Conducted VA = (k) Input VA
Transformed VA = (1 – k) Input VA

Saving of Copper

Weight of Cu in auto-transformer, $W_a = (1 - k)$ weight of Cu in 2nd winding transformer, $W_{2\text{wdg}}$
Saving in auto-transformer = $W_{2\text{wdg}} - W_a = k \times W_{2\text{wdg}}$

Therefore, auto–transformer becomes more economical when k is near unity.

Note: The efficiency and voltage regulation of auto-transformer is better than 2nd winding transformer of same output.

Parallel Operation of Transformer

For supplying load demand beyond the rated kVA capacity of a transformer, extra transformers are needed to be connected in parallel with the existing transformer. To ensure that no circulating current is flowing and the transformers are sharing common load proportional to its kVA rating, few conditions must be satisfied. They are:

(i) The transformers must have same voltage ratios.
(ii) They must operate at same frequency.
(iii) Primary and secondary windings must be connected in same polarity.
(iv) Per unit impedance of all the transformers should be same.
(v) The ratio of equivalent leakage reactance to equivalent resistance i.e., x_e/r_e should be same for all the transformers.

Two additional conditions applies for three-phase transformer are:

(i) Relative phase displacement between secondary line voltages of the transformer must be zero.
(ii) Secondary line voltage should have same phase sequence.

Note:
(i) If x_e/r_e of parallel connected transformers are not equal, their full load kVA output is less than the sum of their individual transformer kVA ratings.
(ii) If $Z_{\text{p.u.}}$ of two transformers are not same, then transformer having greater Z_e shares less kVA and that having lower Z_e shares greater kVA. For transformer to share load in proportion to their kVA rating, their Z_e must be inversely proportional to their respective kVA rating.

DC MACHINES

Any machine which is fed by dc supply or gives dc voltage at the output is a type of dc machine.

Direct current (dc) machines are two types: (i) dc generator, and (ii) dc motor.

(i) ***A dc generator:*** It converts mechanical energy into electrical energy. It follows Fleming's right hand rule.
(ii) ***A dc motor:*** It converts electrical energy to mechanical energy. It follows Fleming's left hand rule.

EMF Equation of DC Machine

The generated voltage E is determined by the number of armature conductors in series in any one path between the brushes. Thus, the total voltage generated is given as

$$E = \frac{P\phi ZN}{60\,A}$$

Types of windings: There are two types of winding:

(i) Lap winding
(ii) Wave winding

(i) ***Lap winding:*** In lap winding, the total number of parallel paths (A) is equal to the total number of poles (P) in the machine i.e., $A = P$.

(ii) ***Wave winding:*** In the wave winding there are only two parallel paths (A) present across the brushes i.e., $A = 2$.

Armature Reaction in DC Machine

Armature reaction is the effect of magnetic flux set up by armature current upon the distribution of flux under the main poles. The armature flux ϕ_a distorts the main field flux by cross magnetizing flux i.e., $\phi_a \cos\theta$ (perpendicular to the main field flux ϕ) and weakens the main field flux ϕ by its demagnetizing flux $\phi_a \sin\theta$ (opposing the main field flux ϕ).

Methods of improving armature reaction: The armature flux produced by armature conductor can be compensated by placing compensating winding on the pole faces connected in series with armature winding.

Commutation

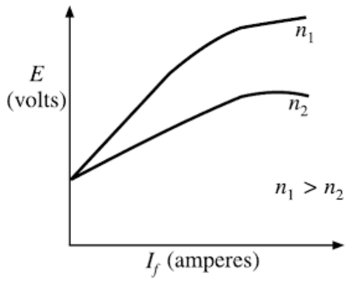
The process of producing a unidirectional or direct current from the alternating current generated *I*, the armature coils is called commutation.

Methods to improve commutation: Commutation can be improved by:

(i) using high resistance carbon brushes.
(ii) the use of interpoles.

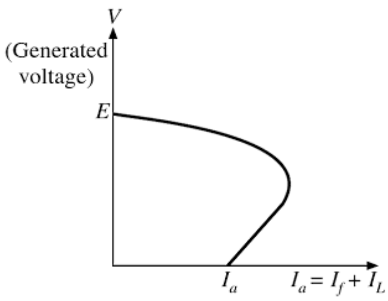
Characteristics of DC Generators

(i) Open circuit characteristics: The open circuit characteristic of a dc generator is the relation or plot between the open terminal voltage across armature and the field excitation current.

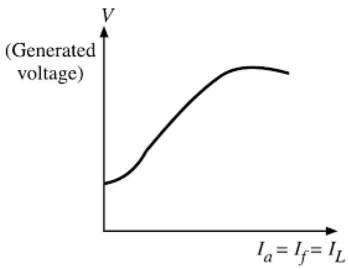


(ii) Load characteristics: The terminal voltage (V) verses the load current (I_L) characteristic is called the load or external characteristics of a dc generator.

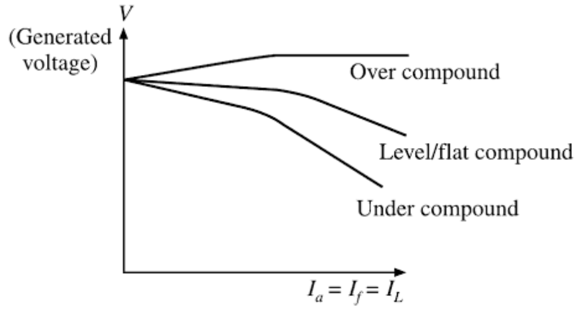
DC shunt generator



DC series generator



DC compound generator



DC MOTOR

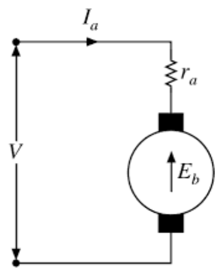
Operating principle: The working principle of a dc motor is based on the fact that when a current carrying conductor is placed in a magnetic field, a mechanical force is being experienced by that conductor, the direction of which is given by Fleming's left hand rule.

Back emf: When the dc motor starts to rotate, its armature conductors cut the magnetic flux produced by the field windings. This induces an emf on the armature conductors which is known as back emf, E_b as it opposes the applied voltage.

$$E_b = \frac{PNZ\phi}{60a} = \frac{PZ\phi}{a} \cdot \frac{\omega_m}{2\pi} = K_a \phi \omega_m \tag{1}$$

EMF equation

$$V = E_b + I_a r_a \tag{2}$$



Where V, I_a and r_a are the applied dc voltage, armature current and armature winding resistance respectively.

Power equation: Multiplying I_a on both side of Eq. (2), we get

$$VI_a = E_b I_a + I_a^2 r_a \tag{3}$$

i.e., input power = power developed at output + ohmic loss

For maximum power to be developed, $E_b = I_a r_a = \dfrac{V}{2}$

Torque equation

$$T_a = \frac{\text{Output power}}{\text{Angular speed}} = \frac{E_b I_a}{\omega_m} = \frac{\frac{PZ\phi\omega_m}{2\pi a} \cdot I_a}{\omega_m} = \frac{PZ\phi}{2\pi a} \cdot I_a = K_a \phi I_a \tag{4}$$

For shunt motor, ϕ = constant, so $T_a \propto I_a$

For series motor, $\phi \propto I_a$, so $T_a \propto I_a^2$

Operating characteristics of dc motors: The three important characteristics of dc motors of general interest are:

(i) Speed–armature current characteristic
(ii) Torque–armature current characteristic
(iii) Speed–torque characteristic

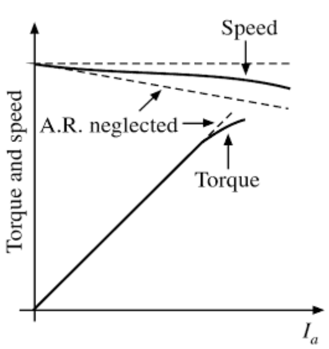
DC Shunt Motor

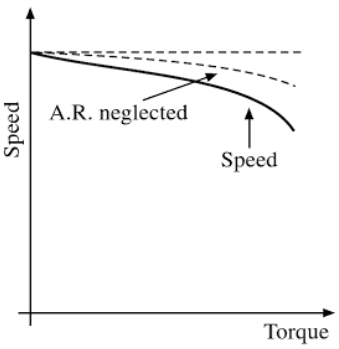
When the field winding is connected in parallel with the armature winding, this is called DC shunt motor.

Stator/Shunt windings: Input power is supplied to the stationary element of the motor, i.e., the shunt winding. The shunt field winding is made of several turns on the coil of fine gauge wire. As the turns are made up of thin wire, the shunt winding is quite small in size. Unlike heavier gauges of wire in the winding in series motors, the shunt winding in this motor cannot carry very high current.

Rotor/Armature: The armature, generally called the "rotor," handles the shaft load. It has a heavier gauge wire so it can support higher current. High current passes through the armature during the motor start up or when the motor is running at lower speed. As the motor's speed increases, the armature generates counter-electromagnetic force opposing the current in the armature.

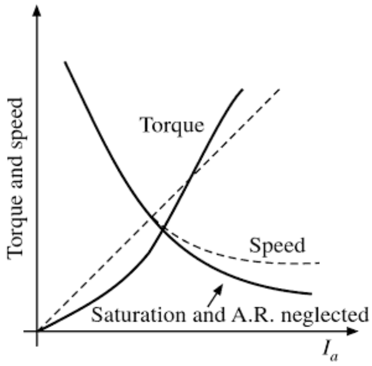
Commutator: The commutator and brush arrangement provide current from the static field windings to the rotor. Torque in the machine is generated by the interaction of the magnetic field of the windings and armature.

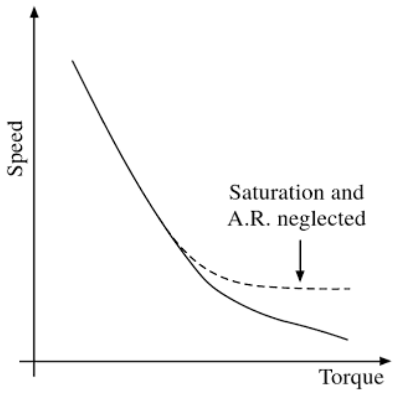




DC Series Motor

When the field winding is connected in series with the armature winding, it is called dc series motor. In case of a series wound self-excited dc motor or simply series wound dc motor, the entire armature current flows through the field winding as its connected in series to the armature winding.





DC Compound Motor

The compound excitation characteristic in a dc motor can be obtained by combining the operational characteristic of both the shunt and series excited dc motor. The compound wound self-excited dc motor or simply compound wound dc motor essentially contains the field winding connected both in series and in parallel to the armature winding.

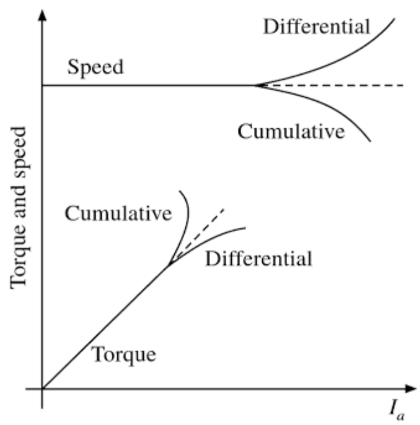
Cumulative compound DC motor: When the shunt field flux assists the main field flux, produced by the main field connected in series to the armature winding then its called cumulative compound dc motor.

$$\phi_{\text{total}} = \phi_{\text{series}} + \phi_{\text{shunt}}$$

Differential compound DC motor: In case of a differentially compounded self-excited dc motor i.e., differential compound dc motor, the arrangement of shunt and series winding is such that the field flux produced by the shunt field winding diminishes the effect of flux by the main series field winding.

$$\phi_{\text{total}} = \phi_{\text{series}} - \phi_{\text{shunt}}$$

The net flux produced in this case is lesser than the original flux and hence does not find much of a practical application.



Starting of DC Motor

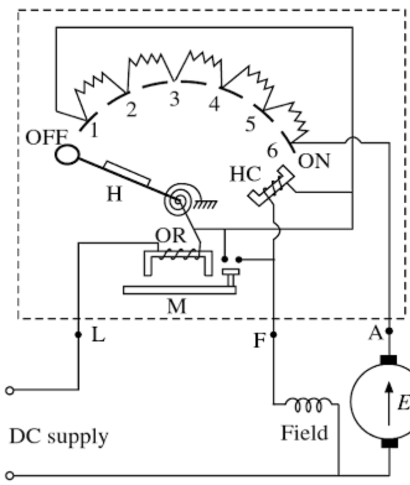
At the time of starting, the motor speed ω_m is zero. Therefore, the back emf $E_b = K_a\phi\omega_m$ is also zero. Now, from the Eq. (2), we get

$$V = E_b + I_a r_a$$

If the rated voltage is applied at the time of starting, then $V = I_a r_a$ and starting current will be $I_{a\,st} = \dfrac{V}{r_a}$ which is much larger than the rated current. This high starting current taken by motor causes sparking at the commutator, overheating of insulation, mechanical shock and supply voltage dip. Thus, a need for a proper starting technique of dc motor originated.

3-POINT STARTER

3-point starter reduces the starting current as well as provides the protection against no-volt fault and overloading. Three terminals i.e., L, A, F are available from starter which are connected to the any one of the two terminals of the supply mains, armature winding and field winding respectively. The other terminals of the supply, armature and field are connected together.



When the motor is at rest, the handle H is moved to stud 1 and after the motor gains sufficient speed, it is moved to stud 2 and so on. The field current flowing through holding coil HC produces force of attraction that holds the handle at stud 6 during normal condition. If no-volt or low-volt occurs, the HC gets de-energized and the handle comes back to OFF position due to spring pull. Also when the motor is overloaded and armature current exceeds the rated value, the overload release coil OR attracts the movable soft iron strip M with short circuit the HC coil and the handle comes back to OFF position.

SPEED CONTROL OF DC MOTORS

From voltage equation of dc motor,

$$V = E_b + I_a r_a$$

$$E_b = V - I_a r_a$$

$$K_a\phi\omega_m = V - I_a r_a$$

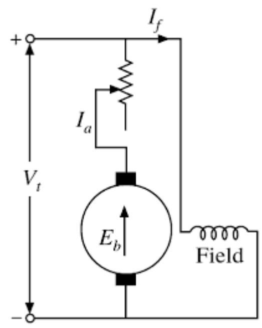
$$\omega_m = \frac{V - I_a r_a}{K_a\phi} \tag{5}$$

From Eq. (5), it becomes obvious that speed of dc motors can be controlled by

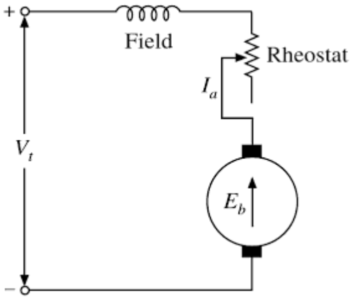
(i) Varying field flux per pole, ϕ
(ii) Varying resistance r_a of the armature circuit
(iii) Varying applied voltage V

Speed Control by Varying Armature Circuit Resistance

For shunt motors: As seen from Eq. (5), if the resistance of armature circuit i.e., r_a increases, the speed of the motor decreases. The figure shows a rheostat being inserted in armature circuit. By varying the resistance of armature circuit, various speeds are obtained. The method is used generally for achieving speed below the rated speed.

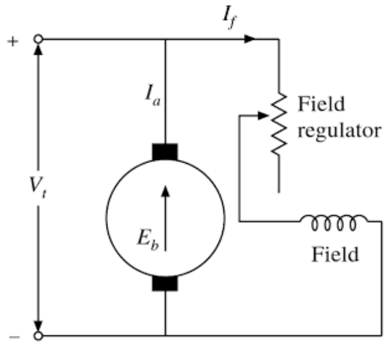


For series motors: From Eq. (5), it can be seen that if resistance of armature circuit r_a is varied, then the speed of the motor can also be regulated. By inserting series resistance with armature, the voltage applied across the armature terminals is decreased and proportionally the speed is also decreased.



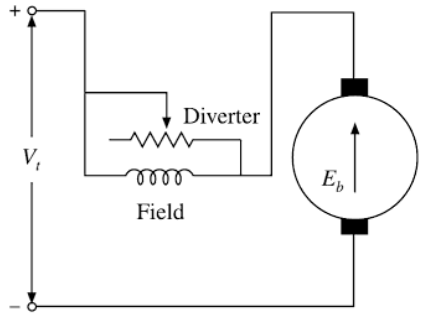
Speed Control by Varying Field Flux

For shunt motors: From Eq. (5), it can be observed that the speed is inversely proportional to main field flux. Therefore, by decreasing the field, speed increases and vice versa. As shown in figure, the flux can be changed by changing the field current I_f with the help of a field regulator. The method is used generally for achieving speed above the rated speed.

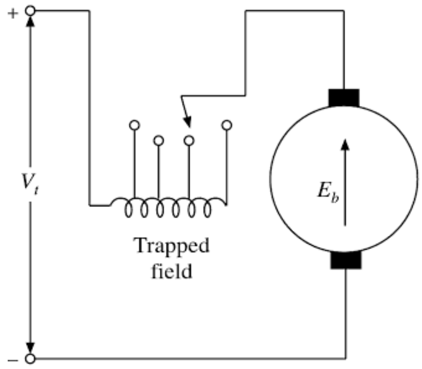


For series motors

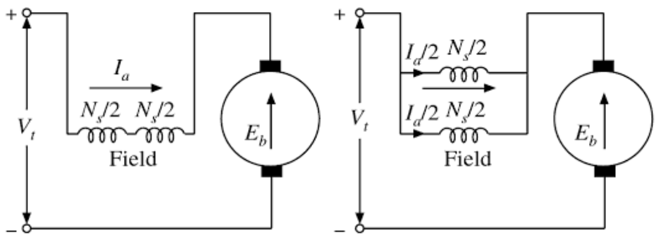
(i) Diverter field control: In this method, the series field winding is shunted with a variable resistor known as field diverter. By varying the diverter, any desired magnitude of current can be diverted from field winding, thus achieving variable field flux and in turn variable speed. In general, speed is increased by decreasing flux.



(ii) Tapped field control: When the field winding is tapped, the number of series field turns is changed and therefore, the series field m.m.f. and the speed are changed. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some series turns. This method is used in electric traction.



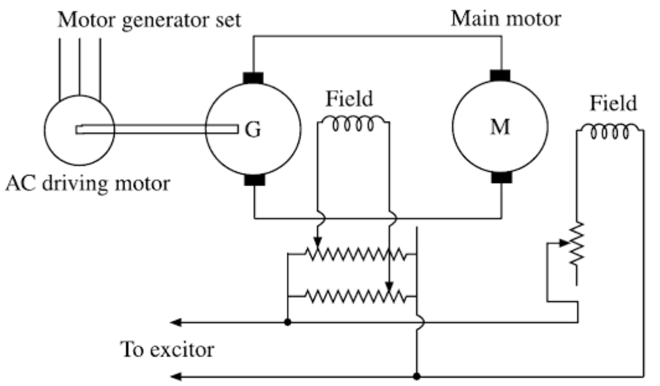
(iii) Series-parallel field control: In this method, the series field winding is divided into two equal halves. When these two halves are in series, then total field m.m.f. is $I_a\left(\frac{N_s}{2}+\frac{N_s}{2}\right)=I_aN_s$ and when in parallel, then total field m.m.f. is $\left(\frac{I_a}{2}\cdot\frac{N_s}{2}\right)\times 2=\frac{I_aN_s}{2}$.



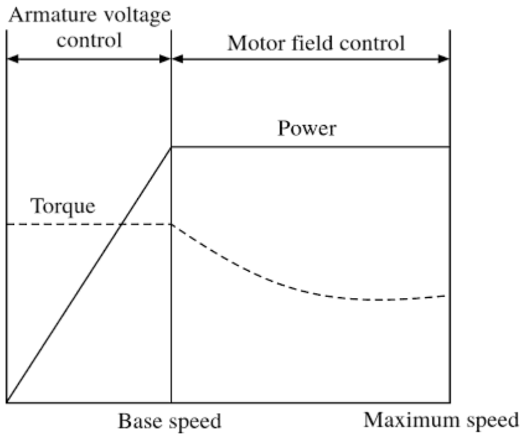
Therefore for parallel connection of field windings, the speed is double that for the series.

Speed Control by Varying Armature Terminal Voltage

Ward–Leonard system: The system consists of a motor generator set as shown in figure, which runs at constant speed. The field winding of dc generator G and dc motor M are fed by a dc exciter through a rheostat. The armature of the motor M is fed by the dc generator G.



A change in the generator field current by varying the rheostat varies the generated voltage and therefore, the voltage applied to the motor armature changes. This in turn changes the motor speed. The speed of motor can also be regulated by varying the resistance connected to the field winding of the motor.

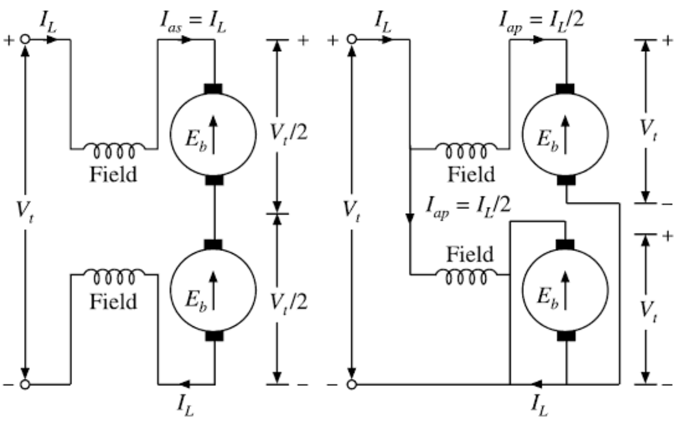


With the help of this method, speed control in both the direction can be obtained. In order to achieve wider speed control range, speeds below base speed are obtained by voltage control and above base speed by field flux control.

The only disadvantage of this method is the high initial cost.

Speed control with controlled rectifiers: This method is similar to Ward–Leonard system. In Ward–Leonard system, a motor–generator set is used to convert ac to dc whereas in this method, a controlled rectifier is used for supplying dc. The scheme is much more economical and requires less floor space, has higher efficiency and provide quicker control of output voltage. Single phase controlled rectifier are used for the speed control of dc motors below the base speed.

Series–parallel armature control: This method of armature voltage control requires two identical dc motors coupled together mechanically to a common load.



For low speeds, the motors are joined in series and for high speeds; the motors are connected in parallel. The method is generally used for series motors in electric traction.

THREE-PHASE INDUCTION MOTOR

Principle

When a three-phase winding is supplied with three-phase balanced supply, a rotating magnetic field is developed in the air gap. A closed rotor circuit, if placed in the air gap, will have flux cutting action and hence induces emf and current. The rotor field produced due to induced rotor current interacts with the air gap field and hence rotor starts to rotate in the direction of the magnetic field.

Types of Induction Motor

The induction motor has two main parts: (a) stator, and (b) rotor.

Stator is made of number of laminations stacked together and having slots for receiving the windings.

Rotor of induction motor is of two types based on its construction. They are: (i) squirrel cage rotor, and (ii) wound rotor or slip ring rotor.

The squirrel cage type is simpler and more economical on construction than wound type and also requires less maintenance. Wound rotor is used where the driven load requires speed control or high starting torque. In both, the types the rotor slots are skewed for obtaining a quieter and smoother operation of the motor.

Rotating Magnetic Field

For three-phase induction motor, the resultant flux in the air gap is constant to 1.5 times the maximum value of the flux produced due to any phase. The air gap flux rotates at the synchronous speed of $N_s = \dfrac{120f}{P}$.

Slip: The difference between the synchronous speed and rotor actual speed is called slip speed. The ratio of slip speed to synchronous speed is called slip.

Slip, $s = \dfrac{N_s - N_r}{N_s}$ where N_r is rotor speed and $N_s - N_r$ is slip speed.

Performance Equations

For any slip s, the per phase generated emf in rotor conductors $= sE_2$

If f_1, f_2 = supply and rotor frequency respectively, l_2 = rotor leakage inductance, r_2 = rotor resistance, then

- Rotor leakage reactance at standstill $= 2\pi f_1 l_2 = x_2$
- Rotor leakage reactance at any slip $s = 2\pi f_2 l_2 = 2\pi(sf_1)l_2 = sx_2$
- Rotor leakage impedance at standstill $= \sqrt{r_2^2 + x_2^2}$
- Rotor leakage impedance at any slip $s = \sqrt{r_2^2 + (sx_2)^2}$

- Per phase rotor current at standstill $= \dfrac{E_2}{\sqrt{r_2^2 + x_2^2}}$ and at any slip s,

$$I_2 = \frac{sE_2}{\sqrt{r_2^2 + (sx_2)^2}} = \frac{E_2}{\sqrt{(r_2/s)^2 + (x_2)^2}}$$

- The rotor current I_2 lags the rotor emf E_2 by rotor power factor angle $\theta_2 = \tan^{-1}\dfrac{sx_2}{r_2}$
- Per phase power input to rotor (air gap power),

$$P_g = E_2 I_2 \cos\theta_2$$

$$= E_2 \times I_2 \times \frac{r_2/s}{\sqrt{(r_2/s)^2 + (x_2)^2}}$$

$$= \frac{E_2}{\sqrt{(r_2/s)^2 + (x_2)^2}} \times I_2 \times \frac{r_2}{s} = I_2^2 \frac{r_2}{s}$$

$$P_g = I_2^2 \frac{r_2}{s} = I_2^2 r_2 + I_2^2 r_2 \left(\frac{1-s}{s}\right)$$

= Rotor ohmic loss + internal mechanical power developed in rotor (P_m)

$\therefore$ Rotor ohmic loss, $I_2^2 r_2 = sP_g$

and $\quad P_m = I_2^2 r_2 \left(\dfrac{1-s}{s}\right) = (1-s)P_g$

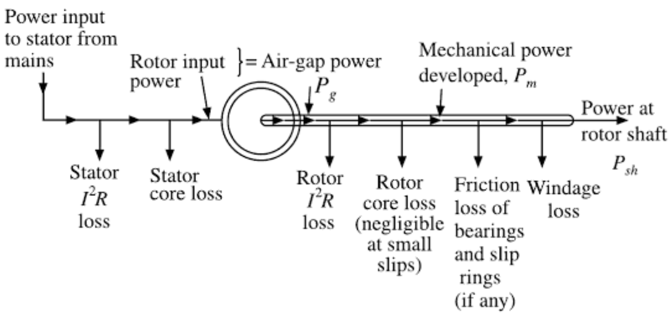
- Internal torque developed per phase

$$T_e = \frac{P_m}{\omega_r} = \frac{(1-s)P_g}{(1-s)\omega_s} = \frac{P_g}{\omega_s}$$

$$= \frac{P_g}{\omega_s} = \frac{1}{\omega_s} \cdot \frac{I_2^2 r_2}{s} = \frac{1}{2\pi n_s} \cdot \frac{I_2^2 r_2}{s}$$

- Power at shaft
  $P_{sh} = P_m$ – Mechanical losses = P_g – Rotor Cu loss – Mechanical losses
- Output or shaft torque

$$T_{sh} = \frac{P_{sh}}{\omega_r} = \frac{P_{sh}}{(1-s)\omega_s}$$



Losses and Efficiency

Fixed losses (P_f) = Power input at no load – Stator Cu loss at no load

Variable losses (P_v) = Stator Cu loss + Rotor Cu loss + Brush contact loss in WRIM + Stray loss

Efficiency $\qquad \eta = \dfrac{P_{sh}}{P_{sh} + P_f + P_v} \times 100\%$

Per Phase Torque Developed on Rotor Under Running Condition

$$T_e = \frac{1}{2\pi n_s} \cdot \left(\frac{E_2}{\sqrt{(r_2/s)^2 + (x_2)^2}}\right)^2 \cdot \frac{r_2}{s} = \frac{1}{2\pi n_s} \cdot \frac{E_2{}^2}{(r_2/s)^2 + (x_2)^2} \cdot \frac{r_2}{s}$$

Condition for maximum torque under running condition, $r_2 = sx_2$

And $\qquad T_{e\,\max} = \dfrac{1}{2\pi n_s} \cdot \dfrac{E_2{}^2}{2x_2}$

Per Phase Starting Torque (s = 1) Developed on Rotor

$$T_{e\,st} = \frac{1}{2\pi n_s} \cdot \frac{r_2 E_2{}^2}{(r_2)^2 + (x_2)^2}$$

Condition for maximum starting torque, $r_2 = x_2$

and $\qquad T_{e\,st\,\max} = \dfrac{1}{2\pi n_s} \cdot \dfrac{E_2{}^2}{2(r_2)}$

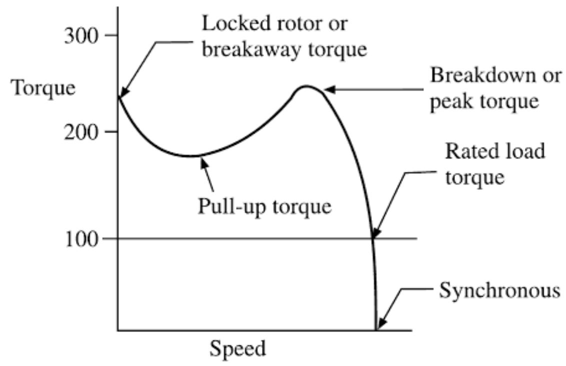
Full Load Torque and Maximum Torque

$T_e = T_{e\,\max}\left(\dfrac{2}{\dfrac{s_m}{s} + \dfrac{s}{s_m}}\right)$ where slip $s_m = \dfrac{r_2}{x_2}$ at which maximum torque occurs.

Starting Torque and Maximum Torque

$T_{e\,st} = T_{e\,\max}\left(\dfrac{2}{\dfrac{s_m}{1} + \dfrac{1}{s_m}}\right)$ where slip $s_m = \dfrac{r_2}{x_2}$ at which maximum torque occurs.

Complete Torque Slip Characteristic



Locked rotor or starting torque: The locked rotor torque or starting torque is the torque the electrical motor develop when its starts at rest or zero speed.

A high starting torque is more important for applications or machines which are hard to start such as positive displacement pumps, cranes etc. A lower starting torque can be accepted in applications such as centrifugal fans or pumps where the start loads is low or close to zero.

Pull-up torque: The pull-up torque is the minimum torque developed by the electrical motor when it runs from zero to full-load speed (before it reaches the break-down torque point).

When the motor starts and begins to accelerate the torque in general, decreases until it reach a low point at a certain speed, the pull-up torque. Before the torque increases until it reach the highest torque at a higher speed, the break-down torque point.

The pull-up torque may be critical for applications that need power to go through some temporary barriers achieving the working conditions.

Break-down torque: The break-down torque is the highest torque available before the torque decreases when the machine continues to accelerate to the working conditions.

Full-load (rated) torque or braking torque: The full-load torque is the torque required to produce the rated power of the electrical motor at full-load speed.

Plugging

An induction motor can be stopped quickly by interchanging the any two terminals of the stator supply leads. It reverses the direction of rotation of air gap magnetic flux and hence the direction of torque. During plugging brake is applied to the rotor and if left connected to supply, the rotor starts to rotate in reverse direction.

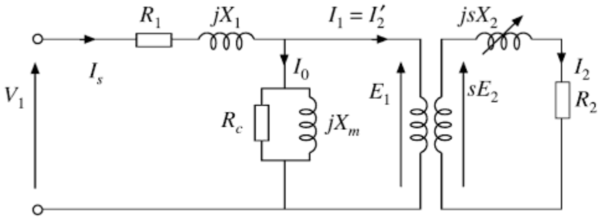
Equivalent Circuit of Induction Motor

One way to analyse and understand the operation of an induction motor is by the use of an equivalent circuit. Before jumping into the equivalent circuit, a few concepts are useful.

Slip: Induction motors are asynchronous. The stator magnetic field rotates at the motors synchronous speed (n_s). The rotor can never rotate at synchronous speed, otherwise there would be no induced current. Typically the rotor full speed will between 2 and 6% that of the synchronous speed. The difference between the motors synchronous and actual asynchronous speed is known as the *slip*.

The difference between the motors synchronous speed (n_s) and actual rotor speed (n_r) is known as the slip (s). Slip can be expressed as either a fraction or percentage:

$$s = \frac{n_s - n_r}{n_s}$$



In the equivalent circuit R_1 represents the resistance of the stator winding and X_1 represents the stator leakage reactance (flux that does not link with the air gap and rotor). Magnetising reactance required to cross the air gap is represented by X_m and core losses (hysteresis and eddy current) by R_c.

An ideal transformer of N_1 and N_2 turns respectively represents the air gap. For the rotor side, the induced emf is affected by the slip (as the rotor gains speed, slip reduces and less emf is induced). The rotor resistance and reactance are represented by R_2 and X_2; with X_2 being dependant on the frequency of the inductor rotor emfs.

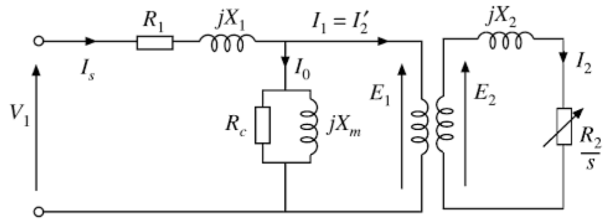
The rotor circuit, the current I_2 is given by:

$$I_2 = \frac{sE_2}{(R_2)^2 + (sX_2)^2}$$

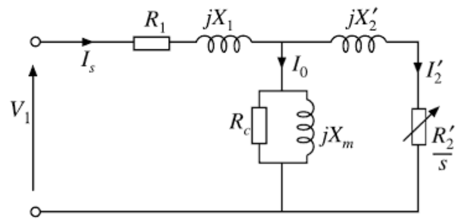
which can be rewritten as:

$$I_2 = \frac{E_2}{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}$$

The above equality allows the equivalent circuit to be drawn as:



Simplified equivalent circuit: The equivalent circuit shown above has removed the dependence on slip for determining the secondary voltage and frequency. Consequently the circuit can be simplified by eliminating the ideal transformer and referring the rotor's resistance and reactance to the primary (denoted by ′).



The referred values are calculated by multiplying them by k^2, where k is the effective stator/rotor turns ratio.

Starting of Induction Motor

Squirrel cage induction motor starting

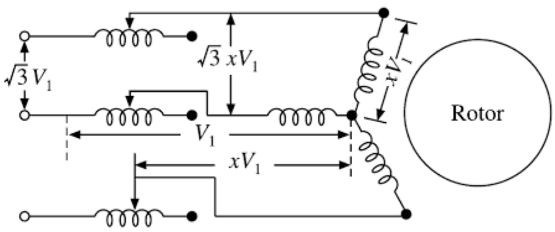
Direct on-line starting (DOL): This method is used for both SC and SR induction motor.

$$T_e = \frac{1}{\omega_s} \cdot I_2^2 \cdot \frac{r_2}{s}$$

$$\frac{T_{e\,st}}{T_{e\,fl}} = \frac{I_{2\,st}^2 \cdot \frac{r_2}{1}}{I_{2\,fl}^2 \cdot \frac{r_2}{s_{fl}}} = \left(\frac{I_{2\,st}}{I_{2\,fl}}\right)^2 s_{fl}$$

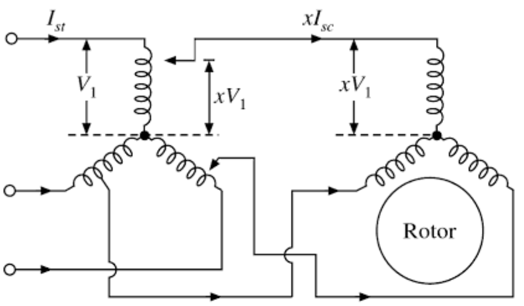
For direct switching, $I_{st} = I_{SC} = \frac{V_1}{Z_{SC}}$ where I_{st} = starting current drawn by stator.

(i) Stator resistor or reactor starting: In this method, a resistor is inserted between the supply mains and the stator terminals. At starting resistance is higher and as rotor speeds up, the resistance is cut-out of circuit.



$$I_{st} = \frac{xV_1}{Z_{SC}} = xI_{SC} \quad \text{and} \quad \frac{T_{e\,st}}{T_{e\,fl}} = x^2\left(\frac{I_{SC}}{I_{fl}}\right)^2 s_{fl}$$

(ii) Auto–transformer starting: A fraction of supply voltage is applied to the stator terminals during starting and as the motor speeds up, auto-transformer is disconnected and full line voltage is applied to the motor.

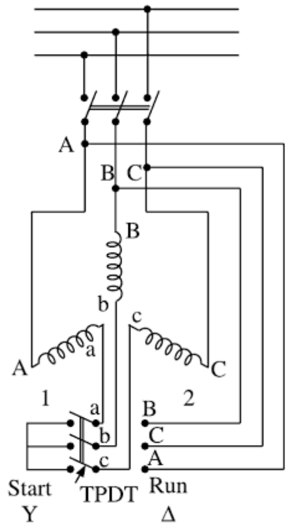


Per phase starting current in motor winding $= \frac{xV_1}{Z_{SC}} = xI_{SC}$

Per phase starting current from supply mains $I_{st} = x^2 I_{SC}$

Therefore, $\frac{T_{e\,st}}{T_{e\,fl}} = x^2\left(\frac{I_{SC}}{I_{fl}}\right)^2 s_{fl}$

(iii) Star–delta starting: In this method, the motor is started with star connection in stator winding and after motor speeds up, its connection is changed to delta.



$$I_{st\,Y} = \frac{V_L}{\sqrt{3}Z_{SC}} \quad \text{and} \quad I_{st\,D} = \frac{V_L}{Z_{SC}} = I_{SC\,D} \quad \text{Also,} \quad I_{st\,Y} = \frac{I_{st\,D}}{\sqrt{3}}$$

$$\frac{\text{Starting line current with star/delta starter}}{\text{Starting line current with direct switching in delta}} = \frac{I_{st\,Y}}{\sqrt{3}I_{st\,D}} = \frac{1}{3}$$

$$\frac{\text{Starting torque with star/delta starter}}{\text{Starting torque with direct switching in delta}} = \frac{T_{e\,st\,Y}}{T_{e\,fl\,D}} = \frac{1}{3}\left(\frac{I_{SC}}{I_{fl}}\right)^2 s_{fl}$$

Note: If starting to full load torque of auto-transformer starting and star/delta staring is compared, then in auto-transformer for $x = \sqrt{\frac{1}{3}} = 0.577$, the both method has same performance. Thus, star/delta method is equivalent to auto-transformer starting with tapping at 57.7%.

Speed Control of Induction Motor

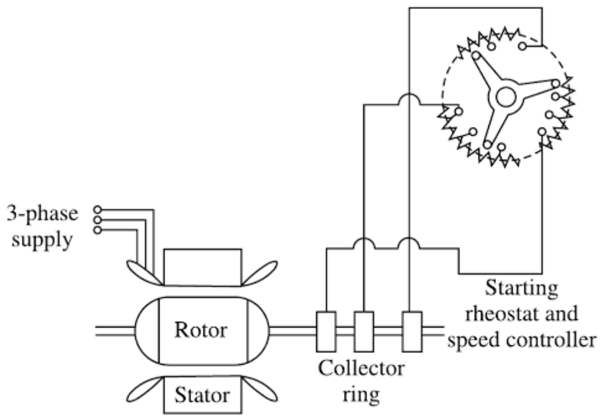
Speed of Induction motor can be controlled by following methods:

(i) Changing applied voltage: The method is rare in use because large change in voltage is required for relatively smaller change in speed. The large change in voltage causes a large variation in flux density which affects the overall performance of the motor.

(ii) Changing the applied frequency: The rotor of induction motor rotates very close to synchronous speed. If the synchronous speed of the motor i.e., $N_s = \dfrac{120f}{P}$ is changed by changing the applied frequency then the speed of the rotor will also change. This method is also rare because of difficulty in availability of various frequencies.

(iii) Changing the number of stator poles: The speed of the air gap flux i.e., synchronous speed also depends on the number of poles on stator. By changing the connection of stator windings only, we can change the number of stator poles and in turn the synchronous speed. This changes the speed of the rotor. This is a general method used in practice for elevator motors, traction motors and small driving machine tools.

(iv) Rotor rheostat control: This method is used in slip ring motors only. The motor speed is reduced by adding an external resistance in the rotor circuit through slip rings. Disadvantage of this method is the high Cu loss and lower efficiency.



SINGLE PHASE INDUCTION MOTOR

The single phase induction machine is the most frequently used motor for refrigerators, washing machines, clocks, drills, compressors, pumps, and so forth. It is not self-starting. It has pulsating air gap field and therefore, no starting torque. For starting torque purpose starter of single phase induction motor carries two winding

- Main winding
- Auxiliary winding

Auxiliary winding is used only for starting purpose. This is disconnected by centrifugal switch when the rotor attains 80% of synchronous speed.

Motor Operation

The single phase induction motor operation can be described by two methods:

- Double revolving field theory
- Cross-field theory

Double revolving field theory: A single phase ac current supplies the main winding that produces a pulsating magnetic field. Mathematically, the pulsating field could be divided into two fields, which are rotating in opposite directions with equal amplitude and moving at synchronous speed.

$$F_s = \frac{1}{2} F_{s\max} \sin \omega t \cos \alpha$$

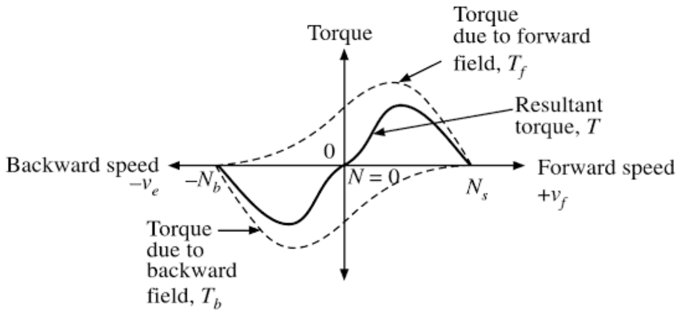
$$= \frac{1}{2} F_{s\max} \sin(\omega t + \alpha) + \frac{1}{2} F_{s\max} \sin(\omega t - \alpha)$$

where, α is the space displacement angle measured from the stator main winding axis.

$\frac{1}{2} F_{s\max} \sin(\omega t + \alpha)$ is the backward travelling wave.

$\frac{1}{2} F_{s\max} \sin(\omega t - \alpha)$ is the forward travelling wave.

The forward and backward flux waves are equal only at standstill. At all other rotor speed, the forward flux wave is much larger than the backward flux wave. Each flux wave produces their own torque. The superposition of two torque gives the torque speed characteristics.



Three-phase induction motor produces rotating magnetic field. But single phase induction motor produces pulsating magnetic field which divided into two oppositely rotating field which produces torque pulsating at double the line frequency. Thus, single phase induction motor is noisier.

$$\text{Forward slip} = s_f = \frac{N_s - N_r}{N_s} = s$$

$$\text{Backward slip} = s_b = \frac{N_s + N_r}{N_s} = 2 - s$$

The single phase motor starting torque is zero because of the pulsating single phase magnetic flux. The starting of the motor requires the generation of a rotating magnetic flux similar to the rotating flux in a three-phase motor. Two perpendicular coils that have currents 90° out of phase can generate the necessary rotating magnetic fields which start the motor. Therefore, single phase motors are built with two perpendicular windings.

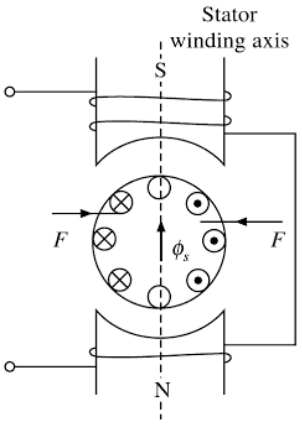
The phase shift is achieved by connecting:

- a resistance,
- an inductance, or a capacitance in series with the starting winding.

Most frequently used is a capacitor to generate the starting torque.

Cross Field Theory

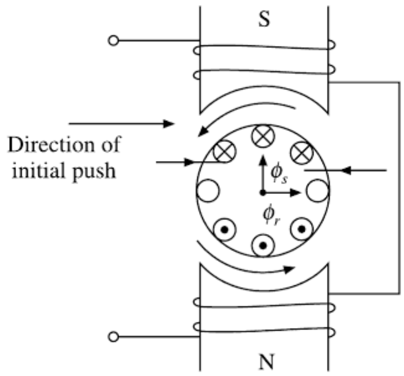
Consider a single phase induction motor with standstill rotor as shown in the following figure. The stator winding is excited by the single phase ac supply. This supply produces an alternating flux ϕ_s which acts along the axis of the stator winding. Due to this flux, emf, gets induced in the rotor conductors due to transformer action. As rotor is closed one, this emf circulates current through the rotor conductors. The direction of the rotor current is as shown in the figure. The direction of rotor current is so as to oppose the cause producing it, which is stator flux ϕ_s.



Now Fleming's left hand rule can be used to find the direction of the force experienced by the rotor conductors. It can be seen that when ϕ_s acts in upward direction and increasing positively, the conductors on left experience force from left to right while conductors on right experience force from right to left. Thus overall, the force experienced by the rotor is zero. Hence, no torque exists on the rotor and rotor cannot start rotating.

We have seen that there must exist two fluxes separated by some angle so as to produce rotating magnetic field. According to cross field theory, the stator flux can be resolved into two components which are mutually perpendicular. One acts along axis of the stator winding and other acts perpendicular to it.

Assume now that an initial push is given to the rotor anti-clockwise direction. Due to the rotation, rotor physically cuts the stator flux and dynamically emf gets induced in the rotor. This is called speed emf or rotational emf. The direction of such emf can be obtained by Fleming's right hand rule and this emf in phase with the stator flux ϕ_s. The direction of emf is shown in the following figure. This emf is denoted as E_{2N}. This emf circulates current through rotor which is I_{2N}. This current produces its own flux called rotor flux ϕ_r. This axis of ϕ_r is at 90° to the axis of stator flux hence this rotor flux is called *cross-field*.



Due to very high rotor reactance, the rotor current I_{2N} and ϕ_r lags the rotational emf by almost 90°. Thus, ϕ_r is in quadrature with ϕ_s in space and lags ϕ_s by 90° in time phase. Such two fluxes produce the rotating magnetic field. The direction of this rotating magnetic field will be same as the direction of the initial push given. Thus, rotor experiences a torque in the same direction as that of rotating magnetic field i.e., the direction of initial push. So, rotor accelerates in the anti-clockwise direction under the case considered and attains a sub-synchronous speed in the steady state.

ALTERNATOR

Basic Principle

The operation of an alternator is based on the principle of electromagnetic induction. It is a doubly excited ac machine with its field winding fed from dc source and armature winding feeding power to the ac bus.

Constructional Features

Constructional features are:

(i) *Stator frame and end–covers* provide only mechanical support to the stator core and not designed to carry the stator flux.

(ii) *Stator core* is build up of laminations of special magnetic iron or steel alloy.

(iii) *Rotor* are of two types

 (a) *Salient pole rotor* is used in low and medium speed alternators such as hydro generators and has small core length and large diameter.

 (b) *Cylindrical rotor* is used in high speed alternators such as turbo generators and has long core length and small diameter.

(iv) *Armature windings* are generally inserted in the stator core and on which emf is induced that we call as generated emf.

(v) *Field windings* are generally placed on the rotor.

(vi) *Damper windings:* The pole shoes present on the rotor of the salient pole alternators are slotted and copper bars are inserted in the slots. These copper bars are short circuited at both ends by heavy copper rings. This forms the dampers windings which helps to
(i) Prevent the hunting in alternators
(ii) Provide the starting torque in synchronous motors

Pitch/Chording Factor

It is defined as the ratio of resultant emf induced on a chorded coil to the resultant emf induced on the same coil when it is full-pitched.

$$k_p = \frac{\text{Resultant emf on chorded coil}}{\text{Resultant emf on full-pitched coil}} = \cos\frac{\varepsilon^\circ}{2}$$

where ε° is the chording angle for fundamental flux wave. For nth space field harmonic, the chording angle becomes $n\varepsilon^\circ$ and pitch factor for nth harmonic is $k_{pn} = \cos\frac{n\varepsilon^\circ}{2}$

$\therefore$ Pitch factor $k_p = \cos\frac{\varepsilon^\circ}{2}$, for fundamental

$k_{p3} = \cos\frac{3\varepsilon^\circ}{2}$, for 3rd harmonics

$k_{p5} = \cos\frac{5\varepsilon^\circ}{2}$, for 5th harmonics

Advantages

(i) Reduction in copper required for overhang.
(ii) Reduction of harmonics in generated emf i.e., the generated emf is made to approximate to a sine wave.
(iii) Due to elimination of high frequency harmonics, eddy current and hysteresis losses are also reduced and in turn efficiency increases.

Disadvantages: The total voltage induced around a chorded coil is somewhat less than full pitched coil.

Distribution/Breadth/Spread Factor

It is defined as the ratio of the phasor sum of coil emfs to the arithmetic sum of the coil emfs.

$$k_d = \frac{\text{Phasor sum of coils emfs}}{\text{Arithmetic sum of coil emfs}}$$

It is also given as:

$$k_d = \frac{\text{Resultant emf with coils distributed in slots}}{\text{Resultant emf with coils concentrated in one slot}}$$

$$k_d = \frac{\sin\left(m\frac{\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$$

where $\beta = \frac{180}{\text{Number of slots/pole}} = \frac{180}{n}$ and m = Number of slots/pole/phase

The distribution factor for nth harmonic is

$$k_{dn} = \frac{\sin\left(nm\frac{\beta}{2}\right)}{m\sin\left(n\frac{\beta}{2}\right)}$$

For fundamental $n = 1$, $k_{d1} = \frac{\sin\left(m\frac{\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$

For 3rd harmonic $n = 3$, $k_{d3} = \frac{\sin\left(3m\frac{\beta}{2}\right)}{m\sin\left(3\frac{\beta}{2}\right)}$

For 5th harmonic $n = 5$, $k_{d5} = \frac{\sin\left(5m\frac{\beta}{2}\right)}{m\sin\left(5\frac{\beta}{2}\right)}$

Equation of Induced EMF

RMS value of induced emf/phase $= 4.44 k_p k_d f \phi T$
where,

k_p = Pitch or coil span factor $= \cos\frac{\varepsilon^\circ}{2}$

k_d = Distribution factor $= \frac{\sin\left(m\frac{\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$

f = Frequency of induced emf in Hz
T = Number of coils or turns per phase
ϕ = Phase

Factors Affecting Alternator Size

(i) As the output power of alternator increases, its efficiency also increases. The size of alternator increases as the output capacity of it increases. The alternators of 1000 MW and above possess efficiencies of the order of 99%.
(ii) Power output per kilogram of alternator increases as the alternator output power capacity increases. Thus, larger alternators weigh relatively less than smaller ones and are, consequently cheaper.
(iii) With increase in size of alternator, the cooling problem also becomes more serious.
(iv) To give same frequency at the output, the low speed alternators requires larger number of poles compared to high speed alternators. Thus, low speed alternators

BC = Drop in voltage due to armature leakage reactance X_L i.e., IX_L.

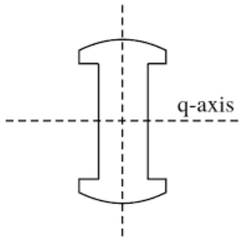
The triangle ABC is known as Potier triangle.

Operation of a Salient Pole Synchronous Machine

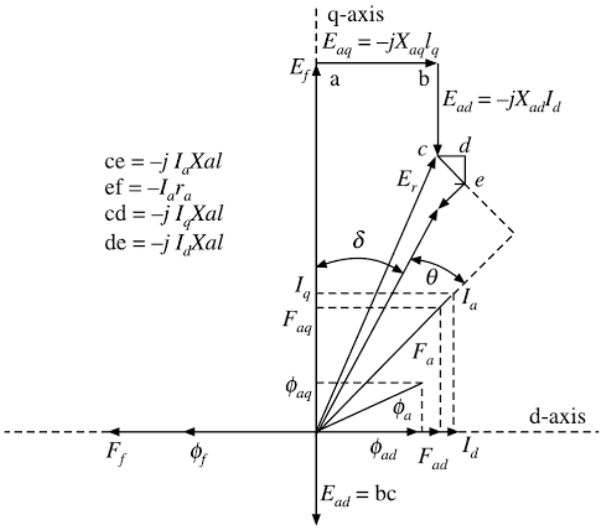
A salient pole synchronous machine has non–uniform air gap due to which its reactance varies with rotor position. A salient pole rotor machine possesses two axes of geometric symmetry.

(i) Field pole axis, called direct axis or d-axis

(ii) Axis passing through the centre of the interpolar space, called the quadrature axis or q-axis as shown in figure.



The armature reaction m.m.f. has two components, one along d-axis and other along q-axis. The field m.m.f. acts only along d-axis and has no component in the q-axis. Therefore, two m.m.fs, i.e., (i) field m.m.f., and (ii) armature m.m.f. act on the d-axis of a salient pole synchronous machine whereas only one m.m.f. i.e., armature m.m.f. acts on the q-axis. The magnetic reluctance is low along d-axis and high along q-axis.



According to **two-reaction theory** or **Blondel's theorem**

(i) Armature current I_a can be resolved into two components i.e., I_d along E_f and I_q perpendicular to E_f.

(ii) Armature reaction F_a has two components i.e., d-axis armature reaction F_{ad} associated with I_d and q-axis armature reaction F_{aq} linked with I_q.

As reluctance along the q-axis is higher due to the larger air gap, therefore, $X_d > X_q$.

Power Developed by a Synchronous Generator

The per phase power output of an alternator is $P_{out} = V_t I_a \cos\phi = P_d$

$$P_d = \frac{E_f V_t}{X_d}\sin\delta + \frac{1}{2}V_t^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

$$P_d = \frac{E_f V_t}{X_d}\sin\delta + \frac{V_t^2\left(X_d - X_q\right)}{2X_d X_q}\sin 2\delta$$

E_f = No load generated voltage
V_t = Terminal voltage
δ = Angle between E_f and V_t

The value of δ is positive for a generator and negative for a motor.

The power developed consists of two components, the first term gives electromagnetic power due to field excitation and second term gives the reluctance power due to saliency.

Parallel Operation of Alternators

The process of connecting an alternator in parallel with another alternator or with common bus-bars is known as *synchronizing*. The following three conditions must be satisfied for proper synchronization of alternators:

(i) The terminal voltage of the incoming alternator must be the same as the existing bus-bar voltage.

(ii) The speed of the incoming alternator must be such that its output frequency equals bus-bar frequency.

(iii) The phase sequence of the incoming alternator voltage must be identical with that of the bus-bar.

Synchronizing Power

$$I_{SY} = \frac{\delta E}{2X_s},\quad P_{SY} = \frac{\delta E^2}{2X_s}$$

$$E_1 = E_2 = E$$

$$P_{SY} = \frac{\delta E^2}{2X_s}\sin\phi$$

Total synchronising power for three phases

$$3P_{SY} = \frac{3\delta E^2}{2X_s}\left(\text{or } \frac{3\delta E^2 \sin\phi}{2X_s}\right)$$

Alternator Connected to Infinite Bus-bars

$$I_{SY} = \frac{\delta E}{X_s}$$

$$P_{SY} = \frac{\delta E^2}{X_s}\left(\text{or } P_{SY} = \frac{\delta E^2 \sin\phi}{X_s}\right)$$

Total synchronizing power for three phases = $3P_{SY}$.

Synchronizing Torque

Let T_{SY} be the synchronizing torque per phase in newton-metre (N-m),

(i) When there are two alternators in parallel,

$$T_{SY} \times \frac{2\pi N_s}{60} = P_{SY}$$

$$T_{SY} = \frac{P_{SY}}{\frac{2\pi N_s}{60}} = \frac{\frac{\delta E^2}{2X_s}}{\frac{2\pi N_s}{60}} \text{ N-m}$$

$$\text{Torque due to three phases } = \frac{3P_{SY}}{\frac{2\pi N_s}{60}} = \frac{\frac{3\delta E^2}{2X_s}}{\frac{2\pi N_s}{60}}$$

(ii) Alternator connected to infinite bus-bars,

$$T_{SY} \times \frac{2\pi N_s}{60} = P_{SY}$$

$$T_{SY} = \frac{P_{SY}}{\frac{2\pi N_s}{60}} = \frac{\frac{\delta E^2}{X_s}}{\frac{2\pi N_s}{60}} \text{ N-m}$$

$$\text{Torque due to three phases } = \frac{3P_{SY}}{\frac{2\pi N_s}{60}} = \frac{\frac{3\delta E^2}{X_s}}{\frac{2\pi N_s}{60}}$$

$$N_s = \text{Synchronous speed in rpm } = \frac{120f}{P}$$

Distribution of Load

When two or more alternators are running in parallel, the amount of load taken up by an alternator solely determined by its driving torque i.e., by the power input to its prime movers. The alternation in its excitation merely changes its kVA output, but not its kW output. In other words, with the change in excitation of the alternator, the power factor at which the load is delivered changes.

Some important points are:

(i) The load shared by an alternators directly depends upon its driving torque or in other words, upon the angular advance of its rotor.

(ii) The change in excitation changes the pf at which the load is delivered without affecting the load so long as steam supply remains unchanged.

(iii) If input of the prime mover of an alternator is kept constant, but its excitation is changed, then kVA component of its output is changed but not kW.

Maximum Power Output

(i) $I_{\max} = \dfrac{\sqrt{E^2 + V^2}}{X_s}$

(ii) $P_{\max} = \dfrac{E}{V} \cdot \dfrac{100}{\%X_s} \times$ F.L. power output at unity power factor

SYNCHRONOUS MOTOR

A synchronous motor is similar to that of an alternator. It is called synchronous motor because it runs at synchronous speed, given by $N_s = \dfrac{120f}{P}$. It does not produce any self-starting torque and needs certain auxiliary means for starting. Synchronous motor is capable of operating under a wide range of power factors both lagging and leading.

The mechanical power developed is given by

$$P_m = \frac{E_b V}{Z_s}\cos(\theta - \delta) - \frac{E_b^{\,2}}{Z_s}\cos\theta$$

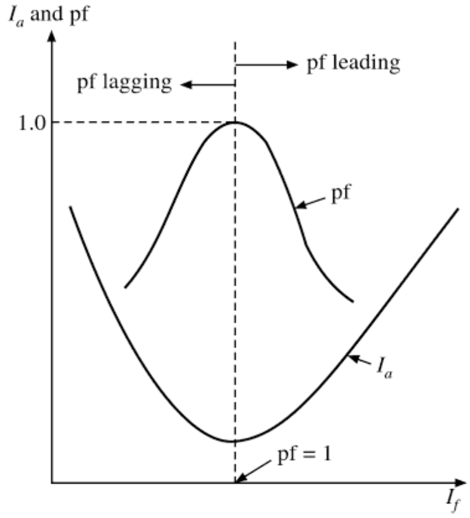
where δ = load angle, θ = internal angle.

Maximum power developed is $P_m = \dfrac{E_b V}{Z_s} - \dfrac{E_b^{\,2} R_a}{Z_s^{\,2}}$

The magnitude of armature current varies with excitation. An over-excited synchronous motor runs with leading pf whereas an under-excited one runs with lagging pf.

Maximum power developed is given by $P_m = \dfrac{V^2}{4R_a}$.

The V curves present the variation of armature current with excitation as shown in figure.



Methods of Starting Synchronous Motors

As the synchronous motor does not produce any self-starting torque, therefore, an external mean is necessary to start the motor. Some of the methods are described as follows:

(i) By means of dc motor: The synchronous motor is coupled with a dc motor, whose speed is adjusted by a speed regulator. The field winding of the synchronous motor is then excited with the dc supply. At the moment of synchronizing, the synchronous motor is switched on to ac mains and dc motor is disconnected from the dc supply.

(ii) By means of ac motor: Synchronous motor with an exciter is coupled with a small induction motor. Before switching the ac supply to the synchronous motor it must be synchronized to the bus-bar. The induction motor used for starting should have lesser number of poles than synchronous motor so that it may be capable of bringing the synchronous motor to the synchronous speed.

(iii) Self-starting: The synchronous motor can be made self-starting by providing a special winding on the rotor poles known as damper winding or squirrel cage winding. These windings are shorted on both ends by heavy copper rings. The ac supply given to the stator produces a rotating magnetic field in the air gap. These damper windings help the rotor to start rotating as squirrel cage induction motor. When the motor approaches close to synchronous speed, the rotor winding is connected to the exciter winding so that the rotor is magnetically locked by the rotating field of the stator and thus the motor begins to run as synchronous motor.

Damper windings also help to prevent hunting. During rotor oscillation, eddy currents are set up in the short circuited damper windings. The eddy currents in damper winding flows in such a direction so as to suppress the oscillations.

Applications

(i) It is used in power houses and substations in parallel with bus-bars to improve the power factor.

(ii) It is used as booster to control the voltage at the end of transmission line by varying its excitation.

(iii) It is used to drive continuously operating and constant speed equipment such as centrifugal pumps, blowers and motor generators.

4.4 IMPORTANT POINTS TO REMEMBER

1. Open circuit test of a transformer gives core loss or iron loss or sum of hysteresis and eddy current loss.
2. Short circuit test of a transformer gives copper loss at full load.
3. In a transformer, iron loss occurs in core and copper loss occurs in winding.
4. Transformer ratings are usually expressed in terms of kVA.
5. From the construction point of view, the transformers may be core type and shell type.
6. Transformer oil must be free from moisture.
7. The part of a transformer which is visible from outside is bush.
8. For a transformer, condition for maximum efficiency is copper loss is equal to iron loss.
9. A transformer has no friction and windage loss.
10. Consumers are provided connections from distribution transformer.
11. The third winding in a transformer, if provided, is known as tertiary.
12. An auto-transformer is used when the transformation ratio is small.
13. An auto-transformer has only one winding.
14. An auto-transformer finds its application extensively as variable voltage device.
15. Frequency does not change in an ordinary transformer.
16. In a transformer, the chemical contained inside the breather is silica gel.
17. The colour of dry silica gel is pale pink and when moist, its colour becomes blue.
18. The function of breather in a transformer is to arrest flow of moisture into the tank.
19. The silica gel is used to absorb moisture from air entering the transformer.
20. Function of transformer oil—cooling the primary coils and secondary coils, and providing additional insulation.
21. The power factor in a transformer depends on the power factor of the load.
22. In a distribution transformer, normally core losses are less than copper losses.
23. The phase difference between the primary and secondary voltage of a transformer is 180°.
24. In a transformer, the resistance between the primary and secondary must be infinite.
25. The material used in the construction of core is usually silicon steel.
26. In a transformer, open circuit test is conducted on low voltage side and short circuit test on high voltage side.
27. The path of the magnetic flux in a transformer should have low reluctance.
28. Transformer tanks are usually made of mild steel.
29. In a power transformer, low voltage winding is closer to core.
30. The noise produced by a transformer is termed as hum.
31. The hum in a transformer is due to magnetostriction.
32. Buchholz relay is a gas operated relay.
33. Buchholz relay is connected between the transformer tank and the conservator.
34. Buchholz relay is generally not provided on transformers below 500 kVA.
35. Buchholz relay is used on oil cooled transformers.

36. A Buchholz relay will operate in a transformer whenever there is large internal fault.
37. In case of minor fault, a Buchholz relay gives an alarm, and in case of major fault a Buchholz relay disconnects the transformer from the supply mains.
38. Operating time of Buchholz relay is of the order of 0.1 s.
39. Full load copper loss in a transformer is 1800 W. At half load, the loss will be 450 W.
40. A transformer working under the conditions of maximum efficiency has total losses of 2000 W. While operating at half load, the copper loss will be 250 W.
41. Mineral oil is suitable for transformer oil.
42. The function of conservator in a transformer is to take care of expansion and contraction of oil.
43. 1% The regulation of a good transformer
    5%
    25%
    100%
    Lower percentage gives the regulation of a good transformer.
44. In a step down transformer, secondary turns are less than primary turns.
45. In a power transformer, no load current is usually 2 to 5 % of full load current.
46. The flux involved in the emf equation of the transformer is maximum value.
47. Lubrication is not the function of oil in a transformer.
48. A transformer can have zero voltage regulation at leading power factor.
49. Eddy current loss is minimised by laminating the core of a transformer.
50. Scott connection is used for three-phase to two-phase conversion or vice versa.
51. In Scott connection, the transformer with 0.866 tap is known as teaser transformer.
52. Back to back test or Sumpner's test on a transformer provides information about regulation, efficiency and heating under load conditions.
53. Power transformers are usually designed to have maximum efficiency near full load.
54. The leakage flux in a transformer depends upon load current.
55. Distribution transformers are usually designed to have maximum efficiency near 50% of full load.
56. Eddy current loss ultimately appears in the form of heat.
57. The colour of fresh dielectric oil for a transformer is pale yellow.
58. Tappings are provided on transformers for varying the output voltage.
59. Silicon steel used for transformer core contains 4% of silicon.
60. A three-phase induction motor is most widely used type of ac motor because of its ruggedness and simplicity.
61. The rotor of a three-phase induction motor rotates in the same direction as the direction of revolving field.
62. The rotor of a three-phase induction motor always run less than synchronous speed.
63. Interchanging of any two phases of an induction motor reverses the direction of rotation.
64. The direction of rotation of an induction motor depends upon the phase sequence.
65. A three-phase induction motor always run with lagging power factor.
66. Three-phase slip ring induction motor has three slip rings.
67. A slip ring induction motor requires more maintenance as compared to a squirrel cage induction motor.
68. The starting torque of a three-phase slip ring induction motor can be increased by adding resistance to the rotor.
69. External resistance can be inserted in the rotor circuit of slip ring induction motor.
70. Uneven air gap in a three-phase induction motor is likely to result in heating of the motor.
71. A squirrel cage induction motor does not have slip rings.
72. In squirrel cage rotors, rotor conductors are not parallel to shaft but skewed.
73. Three-phase induction motors are self-starting.
74. A squirrel cage induction motor has low starting torque.
75. In a wound rotor induction motor, the motor windings are shorted through slip rings.
76. Rotor copper loss = rotor input × slip.
77. The stator frame in a three-phase induction motor is used as a return path for the flux.
78. The effect of introducing star-delta starter in induction motor current is the same as approximately 58% tapping of an auto-transformer.
79. Due to cogging, the three-phase induction motor refuses to start at no load.
80. The starting of a three-phase induction motor can be found from circle diagram.
81. Torque at any value of slip varies as the square of the applied voltage.
82. In the double cage motor, there is a low resistance winding in the bottom of the rotor slot and a high resistance winding at the top.
83. The core loss in an induction motor occurs primarily in stator iron.
84. The starting current of three-phase induction motor is 5 to 7 times the full load current.
85. The rating of generator is expressed in terms of kVA.
86. In an alternator, m.m.fs of the stator and rotor are stationary with respect to each other.

87. In short pitch windings fractional number of slots per pole can be used.
88. One of the advantages of short pitch winding is saving in copper.
89. The stator winding of a three-pole alternator is always star connected.
90. The rotor of an alternator has two slip rings for dc supply.
91. Salient pole synchronous generators normally operate at lower speeds.
92. The ac generators do not have commutators.
93. Synchronous reactance can be determined from open circuit and short circuit tests.
94. In alternators, armature is stationary.
95. The damper winding tends to maintain balanced three-phase voltage under unbalanced load conditions.
96. In large alternator, damper windings is used to improve stability.
97. Copper losses in a generator vary with load.
98. The power factor of an alternator depends on load.
99. Salient poles are generally used on low and medium speed prime movers only.
100. Fleming's left hand rule may be applied to an electric generator to find out direction of induced emf.
101. An alternator is said to be over excited when it is operating at lagging power factor.
102. Dampers in large generator increase stability.
103. A magnetisation curve represents the relation between exciting current and terminal voltage.
104. The Potier's triangle separates the armature leakage reactance and armature reaction m.m.f.
105. In an alternator, if the armature reaction produces demagnetisation of the main field, the power factor should be zero, lagging load.
106. A three-phase alternator has a phase sequence of RYB for its three output voltages. In case the field current is reversed, the phase sequence will become RYB.
107. For 50 Hz system, the maximum speed of an alternator can be 3000 rpm.
108. In an alternator, voltage drop occurs in armature resistance, leakage reactance and armature reaction.
109. If the input to the prime mover of an alternator is kept constant but the excitation is changed, then the reactive component of the output is changed.
110. For an alternator when the power factor of the load is unity the armature flux will be cross-magnetising.
111. Brushes of dc machine are usually made of carbon.
112. As the load of a dc shunt motor is increased, its speed reduces slightly.
113. A dc generator has
    - Wave winding
    - Lap winding
    - Duplex winding
114. Commutation in a dc generator causes ac changes to dc.
115. The output voltage of a simply dc generator is pulsating dc.
116. The critical resistance of a dc generator refers to the field resistance.
117. In the armature, dc generator generates ac voltage.
118. Field test can be used to measure stray losses of a dc motor.
119. Power developed by dc motor is maximum when the ratio of back emf and applied voltage is half.
120. Once residual magnetism of a shunt generator is lost, it may be restored by connecting its shunt field to an external battery.

4.5 REFERENCES

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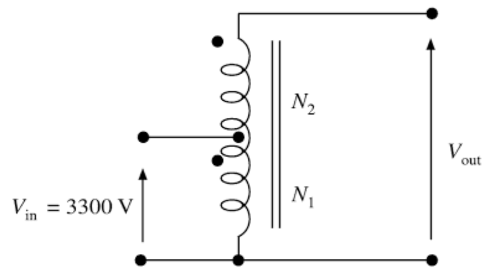
4.6 PREVIOUS YEARS' QUESTIONS

1. The armature resistance of a permanent magnet dc motor is 0.8 Ω. At no load, the motor draws 1.5 A from a supply voltage of 25 V and runs at 1500 rpm. The efficiency of the motor while it is operating on load at 1500 rpm drawing a current of 3.5 A from the same source will be

(a) 48.0% (b) 57.1%
(c) 59.2% (d) 88.8% [Gate 2004]

2. A 50 kVA, 3300/230 V single phase transformer is connected as an auto-transformer shown in figure. The nominal rating of the auto-transformer will be



(a) 50.0 kVA (b) 53.5 kVA
(c) 717.4 kVA (d) 767.4 kVA

[Gate 2004]

3. The resistance and reactance of a 100 kVA, 11000/400 V, Δ–Y distribution transformer are 0.02 and 0.07 p.u. respectively. The phase impedance of the transformer referred to the primary is

(a) $(0.02 + j\ 0.07)\ \Omega$
(b) $(0.55 + j\ 1.925)\ \Omega$
(c) $(15.125 + j\ 52.94)\ \Omega$
(d) $(72.6 + j\ 254.1)\ \Omega$ [Gate 2004]

4. A single phase, 230 V, 50 Hz, 4-pole, capacitor-start induction motor has the following standstill impedances
Main winding $Z_m = 6.0 + j\ 4.0\ \Omega$
Auxiliary winding $Z_a = 8.0 + j\ 6.0\ \Omega$
The value of the starting capacitor required to produce 90° phase difference between the currents in the main and auxiliary windings will be
(a) 176.84 μF (b) 187.24 μF
(c) 265.26 μF (d) 280.86 μF [Gate 2004]

5. A single phase induction motor with only the main winding excited would exhibit the following response at synchronous speed.
(a) Rotor current is zero
(b) Rotor current is non-zero and is at slip frequency
(c) Forward and backward rotating fields are equal
(d) Forward rotating fields is more than the backward rotating field [Gate 2003]

6. A 400 V, 15 kW, 4-pole, 50 Hz, Y-connected induction motor has full load slip of 4%. The output torque of the machine at full load is
(a) 1.66 N-m (b) 95.50 N-m
(c) 99.47 N-m (d) 624.73 N-m
[Gate 2004]

7. For a 1.8°, 2-phase bipolar stepper motor, the stepping rate is 100 steps/s. The rotational speed of the motor in rpm is
(a) 15 (b) 30
(c) 60 (d) 90 [Gate 2004]

8. An 8-pole, dc generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 Wb. The machine is running at 250 rpm. The induced armature voltage is
(a) 96 V (b) 192 V
(c) 384 V (d) 768 V [Gate 2004]

9. A 400 V, 50 kVA, 0.8 pf leading Δ-connected, and 50 Hz synchronous machine has a synchronous reactance of 2 Ω and negligible armature resistance. The friction and windage losses are 2 kW and the core loss is 0.8 kW. The shaft is supplying 9 kW load at a power factor of 0.8 leading. The line current drawn is
(a) 12.29 A (b) 16.24 A
(c) 21.29 A (d) 36.88 A [Gate 2004]

10. A 500 MW, three-phase Y-connected synchronous generator has a rated voltage of 21.5 kV at 0.85 pf. The line current when operating at full load rated conditions will be
(a) 13.43 kA (b) 15.79 kA
(c) 23.25 kA (d) 27.36 kA [Gate 2004]

11. A 500 kVA, three-phase transformer has iron losses of 300 W and full load copper losses of 600 W. The percentage load at which the transformer is expected to have maximum efficiency is
(a) 50.0% (b) 70.7%
(c) 141.4% (d) 200.0% [Gate 2004]

12. For a given stepper motor, which of the following torque has the highest numerical value?
(a) Detent torque (b) Pull-in torque
(c) Pull-out torque (d) Holding toque
[Gate 2004]

13. Which of the following motors definitely has a permanent magnet rotor?
(a) DC commutator motor
(b) Brushless dc motor
(c) Stepper motor
(d) Reluctance motor [Gate 2004]

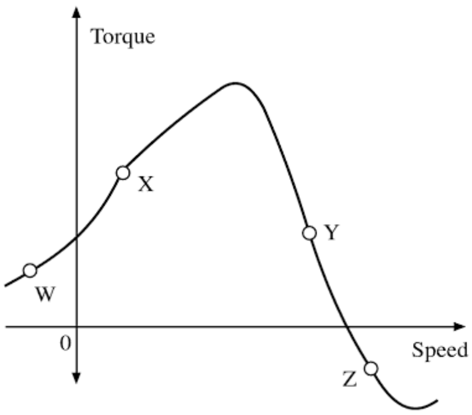
14. The type of single phase induction motor having the highest power factor at full load is
(a) Shaded pole type
(b) Split phase type
(c) Capacitor start type
(d) Capacitor run type [Gate 2004]

15. The direction of rotation of a three-phase induction motor is clockwise when it is supplied with three-phase sinusoidal voltage having phase sequence A-B-C. For counterclockwise rotation of the motor, the phase sequence of the power supply should be
(a) B-C-A
(b) C-A-B
(c) A-C-B
(d) B-C-A or C-A-B [Gate 2004]

16. For a linear electromagnetic circuit, which of the following statements is true?
(a) Field energy is equal to the co-energy
(b) Field energy is greater than the co-energy
(c) Field energy is lesser than the co-energy
(d) Co-energy is zero [Gate 2004]

17. The equivalent circuit of a transformer has leakage reactance x_1, x_2', and magnetizing reactance x_M. Their magnitudes satisfy
(a) $x_1 >> x_2' >> x_M$ (b) $x_1 << x_2' << x_M$
(c) $x_1 \approx x_2' >> x_M$ (d) $x_1 \approx x_2' << x_M$
[Gate 2005]

18. On the torque/speed curve of induction motor shown in figure, four points of operation are marked as W, X, Y and Z. Which one of them represents the operation at a slip greater than 1?
(a) W (b) X
(c) Y (d) Z [Gate 2005]



19. For an induction motor, operating at a slip s, the ratio of gross power output to air gap power is equal to
(a) $(1 - s)^2$ (b) $(1 - s)$
(c) $\sqrt{(1-s)}$ (d) $(1 - \sqrt{s})$ [Gate 2005]

20. In relation to dc machines, match the following and choose the correct combination.

Group 1	Group 2
Performance variables	Proportional to
(P) Armature emf (E)	(1) Flux (ϕ), speed (ω) and armature current (I_a)
(Q) Developed torque (T)	(2) ϕ and ω only
(R) Developed power (P)	(3) ϕ and I_a only
	(4) I_a and ω only
	(5) I_a only

(a) P – 3 Q – 3 R – 1
(b) P – 2 Q – 5 R – 4
(c) P – 3 Q – 5 R – 4
(d) P – 2 Q – 3 R – 1 [Gate 2005]

21. In relation to the synchronous machines, which one of the following statements is false?
(a) In salient pole machines, the direct-axis synchronous reactance is greater than the quadrature-axis synchronous reactance
(b) The damper bars help the synchronous motor to self-start
(c) Short circuit ratio is the ratio of the field current required to produce the rated voltage on open circuit to the rated armature current
(d) The V curve of a synchronous motor represents the variation in the armature current with the field excitation, at a given output power
[Gate 2005]

Statement for Linked Answer Questions: 22 and 23

A 1000 kVA, 6.6 kV, three-phase star connected cylindrical pole synchronous generator has a synchronous reactance of 20 Ω. Neglect the armature resistance and consider operation at full load and unity power factor.

22. The induced emf (line to line) is close to
(a) 5.5 kV (b) 7.2 kV
(c) 9.6 kV (d) 12.5 kV [Gate 2005]

23. The power (or torque) angle is close to
(a) 13.9° (b) 18.3°
(c) 24.6° (d) 33.0° [Gate 2005]

24. In transformers, which of the following statements is valid?
(a) In an open circuit test, copper losses are obtained while in short circuit test, core losses are obtained
(b) In an open circuit test, current is drawn at high power factor
(c) In a short circuit test, current is drawn at zero power factor
(d) In an open circuit test, current is drawn at low power factor [Gate 2006]

25. For a single phase capacitor start induction motor, which of the following statements is valid?
(a) The capacitor is used for power factor improvement
(b) The direction of rotation can be changed by reversing the main winding terminals
(c) The direction of rotation cannot be changed
(d) The direction of rotation can be changed by interchanging the supply terminals [Gate 2006]

26. In a dc machine, which of the following statements is true?
(a) Compensating winding is used for neutralizing armature reaction while interpole winding is used for producing residual flux
(b) Compensating winding is used for neutralizing armature reaction while interpole winding is used for improving commutation
(c) Compensating winding is used for improving commutation while interpole winding is used for neutralizing armature reaction
(d) Compensating winding is used for improving commutation while interpole winding is used for producing residual flux [Gate 2006]

27. A 220 V dc machine supplies 20 A at 200 V as a generator. The armature resistance is 0.2 Ω. If the machine is now operated as a motor at same terminal voltage and current but with the flux increased by 10%, then ratio of motor speed to generator speed is
(a) 0.87 (b) 0.95
(c) 0.96 (d) 1.06 [Gate 2006]

28. A synchronous generator is feeding a zero power factor (lagging) load at rated current. The armature reaction is
(a) Magnetizing (b) Demagnetizing
(c) Cross-magnetizing (d) Ineffective
[Gate 2006]

29. Two transformers are to be operated in parallel such that they share load in proportion to their kVA ratings. The rating of the first transformer is 500 kVA and its p.u. leakage impedance is 0.05 p.u. If the rating of second transformer is 250 kVA, then its p.u. leakage impedance is

(a) 0.20 (b) 0.10
(c) 0.05 (d) 0.025 [Gate 2006]

30. The speed of a 4-pole induction motor is controlled by varying the supply frequency while maintaining the ratio of supply voltage to supply frequency (*V/f*) constant. At rated frequency of 50 Hz and rated voltage of 400 V its speed is 1440 rpm. Find the speed at 30 Hz, if the load torque is constant.

(a) 882 rpm (b) 864 rpm
(c) 840 rpm (d) 828 rpm [Gate 2006]

31. A three-phase, 4-pole, 400 V, 50 Hz star connected induction motor has following circuit parameters:

$r_1 = 1.0\ \Omega$, $r_2 = 0.5\ \Omega$, $x_1 = x_2 = 1.2\ \Omega$, $x_m = 35\ \Omega$

The starting torque when the motor is started direct-on-line is (use approximate equivalent circuit model)

(a) 63.6 N-m (b) 74.3 N-m
(c) 190.8 N-m (d) 222.9 N-m
[Gate 2006]

32. A three-phase, 10 kW, 400 V, 4-pole, 50 Hz, star connected induction motor draws 20 A on full load. Its no load and blocked rotor test data are given as:

No load test:	400 V	6 A	1002 W
Blocked rotor test:	90 V	15 A	762 W

Neglecting copper loss in no load test and core loss in blocked rotor test, estimate motor's full load efficiency.

(a) 76% (b) 81%
(c) 82.4% (d) 85% [Gate 2006]

33. A three-phase, 400 V, 5 kW, star connected synchronous motor having an internal reactance of 10 Ω is operating at 50% load, unity pf. Now, the excitation is increased by 1%. What will be the new load in per cent, if the power factor is to be kept same? Neglect all losses and consider linear magnetic circuit.

(a) 67.9% (b) 56.9%
(c) 51% (d) 50% [Gate 2006]

Common Data for Questions: 34, 35 and 36

A 4-pole, 50 Hz, synchronous generator has 48 slots in which a double layer winding is housed. Each coil has 10 turns and is short pitched by an angle to 36° electrical. The fundamental flux per pole is 0.025 Wb.

34. The line-to-line induced emf (in volts), for a three-phase star connection is approximately

(a) 808 (b) 888
(c) 1400 (d) 1538
[Gate 2006]

35. The line-to-line induced emf (in volts) for a three-phase delta connection is approximately

(a) 1143 (b) 1332
(c) 1617 (d) 1791 [Gate 2006]

36. The fifth harmonic component of phase emf (in volts), for a three-phase star connection is

(a) 0 (b) 269
(c) 281 (d) 808 [Gate 2006]

Statement for Linked Answer Questions: 37 and 38

A 300 kVA transformer has 95% efficiency at full load 0.8 pf lagging and 96% efficiency at half load unity pf.

37. The iron loss (P_i) and copper loss (P_c) in kW, under full load operation are

(a) $P_c = 4.12$, $P_i = 8.51$
(b) $P_c = 6.59$, $P_i = 9.21$
(c) $P_c = 8.51$, $P_i = 4.12$
(d) $P_c = 12.72$, $P_i = 3.07$ [Gate 2006]

38. What is the maximum efficiency (in %) at unity pf load?

(a) 95.1 (b) 96.2
(c) 96.4 (d) 98.1 [Gate 2006]

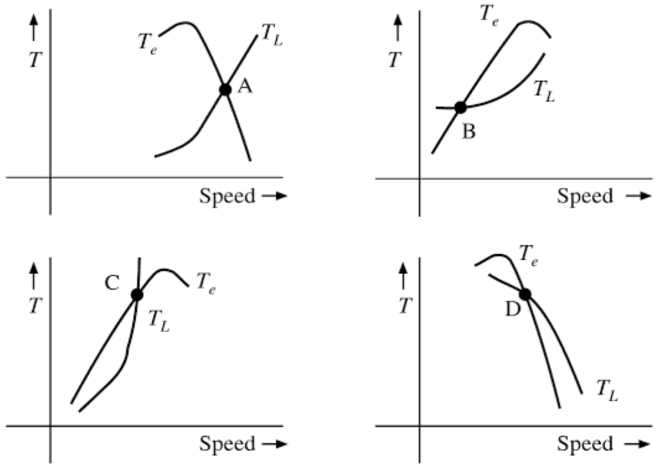
39. In a transformer, zero voltage regulation at full load is

(a) Not possible
(b) Possible at unity power factor load
(c) Possible at leading power factor load
(d) Possible at lagging power factor load
[Gate 2007]

40. The dc motor which can provide zero speed regulation at full load without any controller is

(a) Series
(b) Shunt
(c) Cumulative compound
(d) Differential compound [Gate 2007]

41. The electromagnetic torque T_e of a drive and its connected load torque T_L are as shown in figures. Out of the operating points A, B, C and D, the stable ones are



(a) A, C, D (b) B, C
(c) A, D (d) B, C, D [Gate 2007]

42. A three-phase synchronous motor connected to ac mains is running at full load and unity power factor. If its shaft load is reduced by half, with field current held constant, its new power factor will be
(a) Unity
(b) Leading
(c) Lagging
(d) Dependent on machine parameters [Gate 2007]

43. A three-phase, three-stack, variable reluctance step motor has 20 poles on each rotor and stator stack. The step angle of this step motor is
(a) 3° (b) 6°
(c) 9° (d) 18° [Gate 2007]

44. A three-phase squirrel cage induction motor has a starting torque of 150% and a maximum torque of 300% with respect to rated torque at rated voltage and rated frequency. Neglect the stator resistance and rotational losses. The value of slip for maximum torque is
(a) 13.48% (b) 16.42%
(c) 18.92% (d) 26.79% [Gate 2007]

Common Data for Questions: 45, 46, 47

A three-phase squirrel cage induction motor has a starting current of seven times the full load current and full load slip of 5%.

45. If an auto-transformer is used for reduced voltage starting to provide 1.5 per unit starting torque, the auto-transformer ratio (%) should be
(a) 57.77% (b) 72.56%
(c) 78.25% (d) 81.33% [Gate 2007]

46. If a star-delta starter is used to start this induction motor, the per unit starting torque will be
(a) 0.607 (b) 0.816
(c) 1.225 (d) 1.616 [Gate 2007]

47. If a starting torque of 0.5 per unit is required then the per unit starting current should be
(a) 4.65 (b) 3.75
(c) 3.16 (d) 2.13 [Gate 2007]

48. A 220 V, 15 kW, 1000 rpm shunt motor with armature resistance of 0.25 Ω, has a rated line current of 68 A and a rated field current of 2.2 A. The change in field flux required to obtain a speed of 1600 rpm while drawing a line current of 52.8 A and a field current of 1.8 A is
(a) 18.18% increase (b) 18.18% decrease
(c) 36.36% increase (d) 36.36% decrease
[Gate 2012]

49. A single phase 10 kVA, 50 Hz transformer with 1 kV primary winding draws 0.5 A and 55 W, at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power, respectively are
(a) 0.7 A, 77.8 W (b) 0.7 A, 155.6 W
(c) 1 A, 110 W (d) 1 A, 220 W
[Gate 2012]

50. The slip of an induction motor normally does not depend on
(a) Rotor speed (b) Synchronously speed
(c) Shaft torque (d) Core-loss component
[Gate 2012]

51. The locked rotor current in a three-phase, star connected 15 kW, four-pole, 230 V, 50 Hz induction motor at rated conditions is 50 A. Neglecting losses and magnetizing current, the approximate locked rotor line current drawn when the motor is connected to a 236 V, 57 Hz supply is
(a) 58.5 A (b) 45.0 A
(c) 42.7 A (d) 55.6 A [Gate 2012]

52. A four-point starter is used to start and control the speed of a
(a) dc shunt motor with armature resistance control
(b) dc shunt motor with field weakening control
(c) dc series motor
(d) dc compound motor [Gate 2011]

53. A 220 V dc shunt motor is operating at a speed of 1440 rpm. The armature resistance is 1.0 Ω and armature current is 10 A. If the excitation of the machine is reduced by 10%, the extra resistance to be put in the armature circuit to maintain the same speed and torque will be
(a) 1.79 Ω (b) 2.1 Ω
(c) 3.1 Ω (d) 18.9 Ω [Gate 2011]

54. A three-phase 400 V, 6 pole, 50 Hz squirrel cage induction motor is running at a slip of 5%. The speed of stator magnetic field with respect to rotor magnetic field and speed of rotor with respect to stator magnetic field are
(a) Zero, – 50 rpm
(b) Zero, 950 rpm
(c) 1000 rpm, – 50 rpm
(d) 1000 rpm, 950 rpm [Gate 2011]

55. A three-phase, salient pole synchronous motor is connected to an infinite bus. It is operated at no load at normal excitation. The field excitation of the motor is first reduced to zero and then increased in the reverse direction gradually. Then the armature current

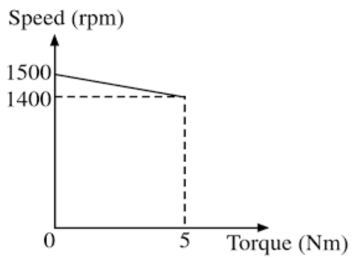
(a) Increases continuously
(b) First increases and then decreases steeply
(c) First decreases and then increases steeply
(d) Remains constant [Gate 2011]

56. The direct-axis and quadrature-axis reactances of a salient pole alternator are 1.2 p.u. and 1.0 p.u. respectively. The armature resistance is negligible. If this alternator is delivering rated kVA at upf and at rated voltage then its power angle is

(a) $30°$ (b) $45°$
(c) $60°$ (d) $90°$ [Gate 2011]

Common Data Questions: 57 and 58

A separately excited dc motor runs at 1500 rpm under no load with 200 V applied to the armature. The field voltage is maintained at its rated value. The speed of the motor, when it delivers a torque of 5 N-m, is 1400 rpm as shown in figure. The rotational losses and armature reaction are neglected.



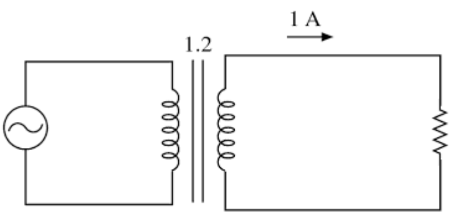
57. The armature resistance of the motor is

(a) 2 Ω (b) 3.4 Ω
(c) 4.4 Ω (d) 7.7 Ω [Gate 2010]

58. For the motor to deliver a torque of 2.5 N-m at 1400 rpm, the armature voltage to be applied is

(a) 125.5 V (b) 193.3 V
(c) 200 V (d) 241.7 V [Gate 2010]

59. A single phase transformer has a turn ratio of 1:2, and is connected to a purely resistive load as shown in figure. The magnetizing current drawn is 1 A, and the secondary current is 1 A. If core losses and leakage reactances are neglected, the primary current is



(a) 1.41 A
(b) 2 A
(c) 2.24 A
(d) 3 A [Gate 2010]

60. Distributed winding and short chording employed in ac machines will result in

(a) Increase in emf and reduction in harmonics
(b) Reduction in emf and increase in harmonics
(c) Increase in both emf and harmonics
(d) Reduction in both emf and harmonics [Gate 2008]

61. In a stepper motor, the detent torque means

(a) Minimum of the static torque with the phase winding excited
(b) Maximum of the static torque with the phase winding excited
(c) Minimum of the static torque with the phase winding unexcited
(d) Maximum of the static torque with the phase winding unexcited [Gate 2008]

62. It is desired to measure parameters of 230V/115V, 2 kVA, and a single phase transformer. The following wattmeters are available in a laboratory

W_1: 250 V, 10 A, low power factor
W_2: 250 V, 5 A, low power factor
W_3: 150 V, 10 A, high power factor
W_4: 150 V, 5 A, high power factor

The wattmeter used in open circuit test and short circuit test of the transformer will respectively be

(a) W_1 and W_2 (b) W_2 and W_4
(c) W_1 and W_4 (d) W_2 and W_3 [Gate 2008]

63. A 230 V, 50 Hz, 4-pole, single phase induction motor is rotating in the clockwise (forward) direction at a speed of 1425 rpm. If the rotor resistance at standstill is 7.8 Ω, then the effective rotor resistance in the backward branch of the equivalent circuit will be

(a) 2 Ω (b) 4 Ω
(c) 78 Ω (d) 156 Ω [Gate 2008]

64. A 400 V, 50 Hz, 30 hp, three-phase induction motor is drawing 50 A current at 0.8 power factor lagging. The stator and rotor copper losses are 1.5 kW and 900 W respectively. The friction and windage losses are 1050 W and the core losses are 1200 W. The air gap power of the motor will be

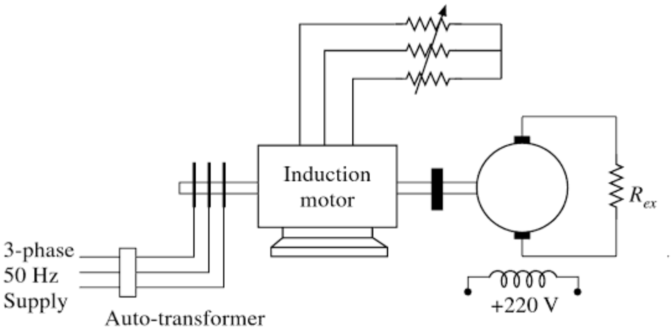
(a) 23.06 kW (b) 24.11 kW
(c) 25.01 kW (d) 26.21 kW [Gate 2008]

Common Data for Questions: 65 and 66

A three-phase, 440V, 50 Hz, 4-pole, slip ring induction motor is fed from the rotor side through an auto-transformer and the stator is connected to a variable resistance as shown in figure.

The motor is coupled to a 220 V, separately excited dc generator feeding power to fixed resistance of 10 Ω. Two-wattmeter method is used to measure the input power to induction motor. The variable resistance is adjusted such that the motor runs at 1410 rpm and the following readings were recorded

$$W_1 = 1800\text{ W},\ W_2 = -200\text{ W}.$$



65. The speed of rotation of stator magnetic field with respect to rotor structure will be
(a) 90 rpm in the direction of rotation
(b) 90 rpm in the opposite direction of rotation
(c) 1500 rpm in the direction of rotation
(d) 1500 rpm in the opposite direction of rotation [Gate 2008]

66. Neglecting all losses of both the machines, the dc generator power output and the current through resistance (R_{ex}) will respectively be
(a) 96 W, 3.10 A
(b) 120 W, 3.46 A
(c) 1504 W, 12.26 A
(d) 1880 W, 13.71 A [Gate 2008]

Statement for Linked Answer Questions: 67 and 68

A 240 V, dc shunt motor draws 15 A while supplying the rated load at a speed of 80 rad/s. The armature resistance is 0.5 Ω and the field winding resistance is 80 Ω.

67. The net voltage across the armature resistance at the time of plugging will be
(a) 6 V (b) 234 V
(c) 240 V (d) 474 V [Gate 2008]

68. The external resistance to be added in the armature circuit to limit the armature current to 125% of its rated value is
(a) 31.1 Ω (b) 31.9 Ω
(c) 15.1 Ω (d) 15.9 Ω [Gate 2008]

Statement for Linked Answer Questions: 69 and 70

A synchronous motor is connected to an infinite bus at 1.0 p.u. voltage and draws 0.6 p.u. current at unity power factor. Its synchronous reactance is 1.0 p.u. and resistance is negligible.

69. The excitation voltage (E) and load angle (δ) will respectively be
(a) 0.8 p.u. and 36.86° lag
(b) 0.8 p.u. and 36.86° lead
(c) 1.17 p.u. and 30.96° lead
(d) 1.17 p.u. and 30.96° lag [Gate 2008]

70. Keeping the excitation voltage same, the load on the motor is increased such that the motor current increases by 20%. The operating power factor will become
(a) 0.995 lagging (b) 0.995 leading
(c) 0.791 lagging (d) 0.848 leading
[Gate 2008]

71. A single phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load at the same power factor is
(a) 86.7% (b) 88.26%
(c) 88.9% (d) 87.8% [Gate 2003]

72. Match the following list I with list II and select the correct answer using the codes given below the lists.

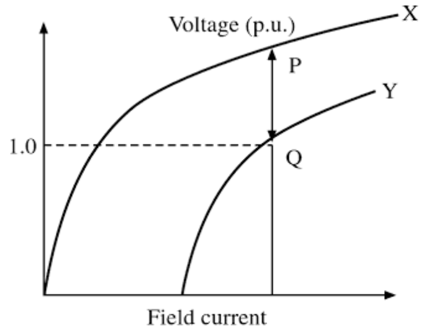
List I	List II
P. Food mixer	1. Permanent magnet dc motor
Q. Cassette tape recorder	2. Single phase induction motor
R. Domestic water pump	3. Universal motor
S. Escalator	4. Three-phase induction motor
	5. The dc series motor
	6. Stepper motor

	P	Q	R	S
(a)	3	6	4	5
(b)	1	3	2	4
(c)	3	1	2	4
(d)	3	2	1	4

[Gate 2003]

73. A stand-alone engine driven synchronous generator is feeding a partly inductive load. A capacitor is now connected across the load to completely nullify the inductive current. For this operating condition,
(a) The field current and fuel input have to be reduced
(b) The field current and fuel input have to be increased
(c) The field current has to be increased and fuel input left unaltered
(d) The field current has to be reduced and fuel input left unaltered [Gate 2003]

74. Curves X and Y in figure denote open circuit and full load zero power factor (ZPF) characteristics of a synchronous generator. Q is a point on the ZPF characteristics at 1.0 p.u. voltage. The vertical distance PQ in figure gives the voltage drop across
(a) Synchronous reactance
(b) Magnetizing reactance
(c) Potier reactance
(d) Leakage reactance [Gate 2003]



75. No load test on a three-phase induction motor was conducted at different supply voltages and a plot of input power versus voltage was drawn. This curve was extrapolated to intersect the y-axis. This intersection point yields

(a) Core loss
(b) Stator copper loss
(c) Stray load loss
(d) Friction and windage loss [Gate 2003]

76. Following are some of the properties of rotating electrical machines:

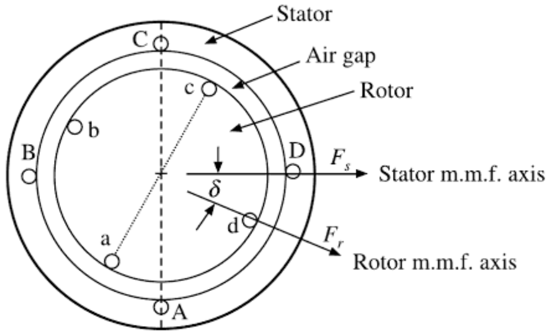
P: Stator winding current is dc, rotor winding current is ac
Q: Stator winding current is ac, rotor winding current is dc
R: Stator winding current is ac, rotor winding current is ac
S: Stator has salient poles and rotor has commutator
T: Rotor has salient poles and slip rings and stator is cylindrical
U: Both stator and rotor have poly phase windings

The dc machines, synchronous machines and induction machines exhibit some of the above properties as given in the following table. Indicate the correct combination from this table.

	DC machine	Synchronous machines	Induction machines
(a)	P.S	Q.T	R.U
(b)	Q.U	P.T	R.S
(c)	P.S	R.U	Q.T
(d)	R.S	Q.U	P.T

[Gate 2003]

77. When stator and rotor windings of a 2-pole rotating electrical machine are excited, each would produce a sinusoidal m.m.f. distribution in the air gap with peak values F_s and F_r respectively. The rotor m.m.f. lags stator m.m.f. by a space angle δ at any instant as shown in figure. Thus, half of stator and rotor surfaces will form one pole with the other half forming second pole. Further, the direction of torque acting on the rotor can be clockwise or counter clockwise.



The following table gives four sets of statement as regards poles and torque. Select the correct set corresponding to the m.m.f. axes as shown in figure.

	Stator surface ABC forms	Stator surface CDA forms	Rotor surface ABC forms	Rotor surface CDA forms	Torque surface
(a)	North pole	South pole	South pole	South pole	Clockwise
(b)	South pole	North pole	North pole	South pole	Counter clockwise
(c)	North pole	South pole	South pole	North pole	Counter clockwise
(d)	South pole	North pole	South pole	North pole	Clockwise

[Gate 2003]

78. A field excitation of 20 A in a certain alternator results in an armature current of 400 A in short circuit and a terminal voltage of 2000 V on open-circuit. The magnitude of the internal voltage drop within the machine at a load current of 200 A is

(a) 1 V (b) 10 V
(c) 100 V (d) 1000 V [Gate 2009]

79. Neglecting all losses, the developed torque (T) of a dc separately excited motor, operating under constant terminal voltage, is related to its output power (P) as

(a) $T \propto \sqrt{P}$ (b) $T \propto P$
(c) $T \propto P^3$ *(d)* T independent of P

[Gate 1992]

80. A differentially compounded dc motor with interpoles and with brushes on the neutral axis is to be driven as a generator in the direction with the same polarity of the terminal voltage. It will then

(a) Be a cumulatively compounded generator but the interpole coil connections are to be reversed
(b) Be a cumulatively compounded generator without reversing the interpole coil connections
(c) Be a differentially compounded generator without reversing the interpole coil connections
(d) Be a differentially compounded generator but the interpole coil connections are to be reversed

[Gate 1995]

Hence, kVA rating of auto-transformer

$$S = V_{in}\ I_{in} = 3300 \times 232.49 \times 10^{-3} = 767.4 \text{ kVA}$$

3. Ans: (d)

Hint: Rated kVA = 100 kVA

Primary phase voltage $V_p = 11000$ V

Primary phase current $I_p = \dfrac{100,000}{3 \times 11000} = 3.0303\text{A}$

Phase impedance (base value),

$$Z_b = \frac{V_p}{I_p} = \frac{11000}{3.0303} = 3630\ \Omega$$

p.u. phase impedance = 0.02 + j0.07 p.u.

Hence, phase impedance = phase impedance of base value × (0.02 + j0.07)

= 3630 × (0.02 + j0.07) Ω

= 72.6 + j254.1 Ω

4. Ans: (a)

Hint: Supply voltage V is taken as reference.

Current in main winding

$$I_m = \frac{V}{Z_m} = \frac{230\angle 0^\circ}{6 + j4} = 31.89\ \angle -33.7^\circ \text{ A}$$

Starting capacitor (C) will be connected in series with auxiliary winding. So, modified auxiliary winding impedance

$$Z_a = 8 + j6 + \frac{1}{j\omega C} = 8 - j\left(\frac{1}{\omega C} - 6\right)$$

Current in auxiliary winding

$$I_a = \frac{V}{Z_a} = \frac{230\angle 0}{8 - j\left(\dfrac{1}{\omega C} - 6\right)} = |I_a| \angle \tan^{-1}\left(\frac{\dfrac{1}{\omega C} - 6}{8}\right)$$

Capacitor is produced to make an angle of 90° between I_a and I_m.

So, $\angle I_a + \angle I_m = 90$

$$\angle I_a = 90 - 33.7 = 56.3$$

$$\therefore \qquad \tan^{-1}\left(\frac{\dfrac{1}{\omega C} - 6}{8}\right) = 56.3$$

or $\qquad C = 176.84\ \mu\text{F}$

5. Ans: (b)

6. Ans: (c)

Hint: Synchronous speed,

$$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Output torque developed,

$$T = \frac{P}{\omega_s (1 - s)} = \frac{15000}{\dfrac{2\pi}{60} \times 1500 \times (1 - 0.04)} = 99.47 \text{ N-m}$$

7. Ans: (b)

Hint: Stepping rate = 100 steps/s

For one revolution, step required = $\dfrac{360^\circ}{1.8^\circ} = 200$ steps

So, for one revolution time required = 2 s

Now, revolution/sec = 0.5 rps

Hence, revolution/min = 30 rpm

8. Ans: (c)

Hint: Total number of turns 32 × 6 = 192

Total number of conductors, Z = 192 × 2 = 384

Number of parallel paths, A = 2 (for wave wound armature)

Induced emf

$$E = \frac{PZ}{60A}\phi N = \frac{8 \times 384 \times 0.06 \times 250}{60 \times 2} = 384 \text{ V}$$

9. Ans: (c)

Hint: Power available at shaft = $P_{out} = 9$ kW

Losses = core losses + friction and windage losses + copper loss

For negligible armature resistance, copper loss is zero.

So, losses = 0.8 + 2 = 2.8 kW

Input power, $P_{in} = P_{out} + \text{losses} = 9 + 2.8 = 11.8$ kW

Line current drawn $= \dfrac{11.8 \times 1000}{\sqrt{3} \times 400 \times 0.8} = 21.29$ A

10. Ans: (b)

Hint: Rated line voltage = $I_L = \dfrac{P}{\sqrt{3V_L} \cos\phi}$

$$= \frac{500 \times 10^6}{\sqrt{3} \times 21.5 \times 1000 \times 0.85} = 15.79 \text{ A}$$

11. Ans: (b)

Hint: Copper losses at any load, $P_{Cu} = x^2 \times P_{fl.Cu}$

Where x = fraction of load.

For maximum efficiency, copper loss = iron loss

$$x^2 \times 600 = 300$$

$\Rightarrow \qquad x = 0.707$ or 70.7%

12. Ans: (c)

13. Ans: (c)

14. Ans: (d)

15. Ans: (c)

16. Ans: (a)

17. Ans: (d)

18. Ans: (a)

19. Ans: (b)

Hint: Let air gap power be P_g

P_{Cu} = Rotor copper loss = sP_g

Gross power output P_m = Air gap power – Rotor copper loss

$$\Rightarrow \quad P_m = P_g - sP_g = P_g(1-s)$$

or $$\frac{P_m}{P_g} = 1 - s$$

20. Ans: (d)

21. Ans: (c)

22. Ans: (b)

Hint: Generator is star connected

So, phase to neutral voltage, $V = \dfrac{6.6}{\sqrt{3}}\angle 0° = 3.81\angle 0°$ kV

Rated current, $I_a = \dfrac{1000 \times 1000}{\sqrt{3} \times 6.6 \times 1000} = 87.47$ A

Now, for unity power factor, $I_a = 87.47\angle 0°$ A

So, induced emf $E_f = V + I_aX_s$

$$= 3.81\angle 0° + 87.47\angle 0° \times j20 \times 10^{-3} \text{ kV}$$
$$= 4.19\angle 24.6° \text{ kV}$$

Line to line induced emf $= \sqrt{3} \times 4.19 = 7.2$ kV

23. Ans: (c)

Hint: From previous solution $\delta = 24.6°$

24. Ans: (d)

25. Ans: (b)

26. Ans: (b)

27. Ans: (a)

Hint: For generator,

$$E_g = V + I_aR_a = 200 + 20 \times 0.2 = 204 \text{ V}$$

For motor,

$$E_m = V - I_aR_a = 200 - 20 \times 0.2 = 196 \text{ V}$$

Now, $$\frac{E_g}{E_m} = \frac{N_g \times \phi_g}{N_m \times \phi_m}$$

or $$\frac{204}{196} = \frac{N_g \times 1}{N_m \times 1.1}$$

or $$\frac{N_m}{N_g} = 0.87$$

28. Ans: (b)

Hint: $E_f > V_t$, the generator is over-excited. Therefore, armature reaction is demagnetizing in nature.

29. Ans: (c)

Hint: The currents drawn by two transformers are proportional to their ratings if their per unit impedance on their own rating are equal.

$$Z_2(\text{p.u.}) = Z_1(\text{p.u.}) = 0.05 \text{ p.u.}$$

30. Ans: (c)

Hint: $$N_{s1} = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$s = \frac{N_{s1} - N_{r1}}{N_{s1}} = \frac{1500 - 1440}{1500} = 0.04$$

As $\dfrac{V}{f}$ = constant

$$\therefore \quad V_2 = \frac{V_1 \times f_2}{f_1} = \frac{400 \times 30}{50} = 240 \text{ V}$$

In induction motor, torque, $T \propto \dfrac{sV^2}{f}$

$$\frac{T_1}{T_2} = \frac{s_1}{s_2}\left(\frac{V_1}{V_2}\right)^2 \times \left(\frac{f_2}{f_1}\right)$$

As load torque is constant $T_1 = T_2$

$$s_2 = \frac{0.04 \times 400^2 \times 30}{240^2 \times 50} = 0.067$$

$$N_{s2} = \frac{120 f_2}{P} = \frac{120 \times 30}{4} = 900 \text{ rpm}$$

Rotor speed

$$N_{s2} = N_{s1}(1 - s_2) = 900(1 - 0.067) = 840 \text{ rpm}$$

31. Ans: (a)

Hint: Phase to neutral voltage, $V_p = \dfrac{400}{\sqrt{3}} = 230.94$ V

Synchronous speed $N_s = \dfrac{120 \times 50}{4} = 1500$ rpm

$$\omega_s = \frac{2 \times \pi \times 1500}{60} = 157.08 \text{ rad/s}$$

Starting torque, $T_{st} = \dfrac{3}{\omega_s} \times \dfrac{V_p^2 \; r_2}{(r_1 + r_2)^2 + (x_1 + x_2)^2}$

$$= \frac{3 \times (230.94)^2 \times 0.5}{157.08 \times \left\{(1 + 0.5)^2 + (1.2 + 1.2)^2\right\}} = 63.6 \text{ N-m}$$

32. Ans: (b)

Hint: From no load test data,

No load losses = 1.002 kW

From blocked rotor test data,

Copper loss at 15 A, $P_{Cu} = 762$ W

Now, full load current = 20 A

So, copper loss on full load $= 762 \times \left(\dfrac{20}{15}\right)^2 = 1.354$ kW

Total losses $= 1.002 + 1.354 = 2.356$ kW

Output power on full load, $P_o = 10$ kW

Input power = Output power + Losses = 10 + 2.356 = 12.356 kW

% efficiency $\eta = \dfrac{10}{12.356} \times 100 = 81\%$

Maximum torque occurs at $s_m = \dfrac{r_2'}{x_2'}$

and $$\frac{T_m}{T_s} = 0.5\left(\frac{s_m^2+1}{s_m}\right)$$

Now, $$\frac{3T_r}{1.5T_r} = 0.5\left(\frac{s_m^2+1}{s_m}\right)$$

$\Rightarrow$ $$s_m^2 - 4s_m + 1 = 0$$

$\Rightarrow$ $$s_m = 3.73 \quad \text{or} \quad 0.2679$$

$s_m = 3.73$ is rejected (braking mode). Therefore, for motoring mode $s_m = 0.2679$ or 26.79%

45. Ans: (c)

Hint: In auto-transformer,

$$\frac{T_s}{T_{fl}} = x^2 \times \left(\frac{I_{SC}}{I_{fl}}\right)^2 \times s_{fl}$$

or $$\frac{1.5\, T_{fl}}{T_{fl}} = x^2 \times \left(\frac{7I_{fl}}{I_{fl}}\right)^2 \times 0.05$$

Auto-transformer ratio $x = 0.7825$ or 78.25%

46. Ans: (b)

Hint: In star-delta starter,

$$\frac{T_s}{T_{fl}} = \frac{1}{3} \times \left(\frac{I_{SC}}{I_{fl}}\right)^2 \times s_{fl}$$

or $$\frac{T_s}{T_{fl}} = \frac{1}{3} \times \left(\frac{7I_{fl}}{I_{fl}}\right)^2 \times 0.05$$

or $$T_s = 0.816\, T_{fl}$$

47. Ans: (c)

Hint: $T_s = 0.5\, T_{fl}$ and $s_{fl} = 0.05$ (given)

Now, $$\frac{T_s}{T_{fl}} = \left(\frac{I_{SC}}{I_{fl}}\right)^2 \times s_{fl}$$

or $$\frac{0.5\, T_{fl}}{T_{fl}} = \left(\frac{I_{SC}}{I_{fl}}\right)^2 \times 0.05$$

or $$I_{SC} = 3.16\, I_{fl}$$

48. Ans: (d)

Hint: Given $r_a = 0.25\ \Omega$

$$I_{f1} = 2.2\text{ A}, I_{f2} = 1.8\text{ A}$$

$$I_{a1} = 68 - 2.2 = 65.8\text{ A}$$

$$I_{a2} = 52.8 - 1.8 = 51\text{ A}$$

$$E_{a1} = 220 - 65.8 \times 0.25 = 203.55\text{ V}$$

$$E_{a2} = 220 - 51 \times 0.25 = 207.25\text{ V}$$

Now, $$\frac{E_{a1}}{E_{a2}} = \frac{\phi_1}{\phi_2} \cdot \frac{\omega_{m1}}{\omega_{m2}}$$

$$\frac{203.55}{207.25} = \frac{\phi_1}{\phi_2} \times \frac{1000}{1600}$$

$$\frac{\phi_1}{\phi_2} = 1.571$$

So, field flux reduces

$\therefore$ % change in flux = $\dfrac{\phi_2 - \phi_1}{\phi_1} = -36.36\%$

Hence, decrease in flux.

49. Ans: (b)

Hint: $E = 4.44 N\phi_m f$

or $E \propto \phi_m$ [As N and f are constant]

$\therefore$ $$\frac{E_1}{E_2} = \frac{\phi_{m1}}{\phi_{m2}}$$

or $$\phi_{m2} = 2\phi_{m1}$$

and

$$\phi_m = \frac{\mu NIA}{d}$$

or $$\frac{\phi_{m1}}{\phi_{m2}} = \frac{I_1 A_1 d_2}{I_2 A_2 d_1} = \frac{\sqrt{2}\, I_1}{2I_2} = \frac{1}{2}$$

or $$I_2 = \sqrt{2}\, I_1$$

or $$I_2 = \sqrt{2} \times \frac{1}{2} = 0.7\text{ A}$$

Core loss $\propto$ Volume of core

$\therefore$ $$P_{c2} = \frac{V_2}{V_1} P_{c1}$$
$$= 2\sqrt{2} \times 55$$
$$= 155.6\text{ W}$$

50. Ans: (d)

51. Ans: (b)

Hint: At standstill, the rotor current is

$$I_2 = \frac{E_2}{\sqrt{R^2 + X^2}} = \frac{E_2}{j\omega L}$$

[neglecting losses, $R = 0$]

$\therefore$ $$\frac{I_2}{I_2'} = \frac{E_2}{E_2'} \times \frac{\omega'}{\omega}$$

or $$I_2' = \frac{236 \times 50}{230 \times 57} \times 50 = 45\text{ A}$$

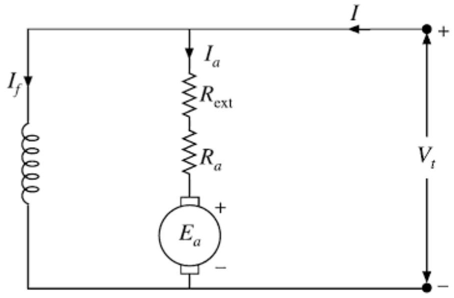
52. Ans: (b)

53. Ans: (a)

1st Case

$R_a = 1\Omega$ $\quad I_{a1} = 10$ A; $N_1 = 1440$ rpm; $\quad V_t = 220$ V;

$E_{a1} = Vt - I_{a1}R_a \{R_{ext} = 0\} = 210$ V



2nd Case

Back EMF $\phi_2 = 0.9\phi_1$ and $N_2 = N_1$

As speed is constant $E_{a2} = \frac{\phi_2}{\phi_1}E_{a1} = \frac{0.9\phi_1}{\phi_1}\times 210$

$\Rightarrow$ $E_{a2} = 189$ V

Torque $T \propto \phi I_a$

As torque is constant

$T_1 = T_2$

or $\phi_1 I_{a1} = \phi_2 I_{a2}$

or $\phi_1 \times 10 = 0.9\phi_1 I_{a2}$

or $I_{a2} = 11.11$ A

$E_{a2} = V_t - I_{a2}(R_a + R_{ext})$

or $189 = 220 - 11.11(1 + R_{ext})$

External resistance $= R_{ext} = 1.79\ \Omega$

54. Ans: (a)

Hint: Stator and rotor magnetic fields rotate at same speed. So, difference in speed is zero. Speed of stator magnetic field is synchronous speed, N_s

$$N_s = \frac{120\times 50}{6} = 1000 \text{ rpm}$$

Rotor speed $N_r = N_s(1-s) = 950$ rpm

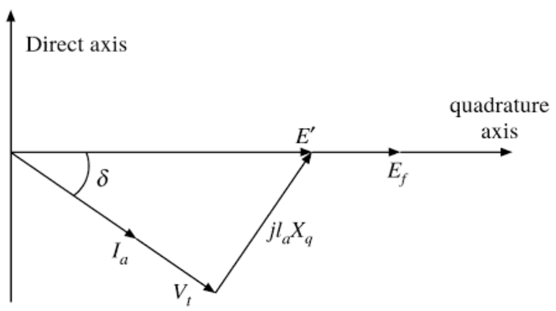
So, speed of rotor with respect to stator magnetic field

$$= N_r - N_s$$
$$= 950 - 1000$$
$$= -50 \text{ rpm}$$

55. Ans: (b)

56. Ans: (b)

Hint:



The angle between E_f and V_t is power angle δ.

Now, E' and E_f are in phase, so, angle between E' and V_t is also power angle δ.

Rated power is delivered at upf, so I_a and V_t are in phase.

$$I_a = 1\angle 0° \text{ p.u.}$$

Quadrature axis reactance $X_q = 1$ p.u.

$$E' = V_t + jI_aX_q$$
$$= 1 + j1\times 1 = \frac{1}{\sqrt{2}}\angle 45°$$
$$\delta = 45°$$

57. Ans: (b)

Hint: Assuming $E_a \propto N$,

At $N = 1400$ rpm

or $E_a = 200\times\frac{1400}{1500} = 186.67$ V

$$I_a = \frac{T\omega}{E_a} = \frac{5\times\frac{2\pi}{60}\times 1400}{186.67} = 3.925 \text{ A}$$

Now,

$$V = E_a + I_a r_a$$

or $r_a = \frac{200 - 186.67}{3.925} = 3.39\ \Omega \simeq 3.4\ \Omega$

58. Ans: (b)

Hint: For $N = 1400$ rpm, $E_a = 186.67$ V

$$V = E_a + I_a r_a = 186.67 + I_a(3.39)$$

To develop a torque of 5 N-m, it draws 3.925 A. For torque of 2.5 N-m,

$$I_a = 3.925\left(\frac{2.5}{5}\right) = 1.9625 \text{ A}$$
$$V = 186.67 + (1.9625)(3.39) = 193.3 \text{ V}$$

59. Ans: (c)

Hint: Secondary current $I_2 = 1\angle 0°$ A

Secondary current referred to primary side

$$I_2' = \left(\frac{N_2}{N_1}\right)I_2$$
$$= \left(\frac{2}{1}\right)\times 1\angle 0° = 2\angle 0° \text{ A}$$

Given, the core losses are neglected, magnetizing current I_m will be in phase with flux ϕ. Therefore, I_m lags the induced emf by 90°

$$I_m = 1\angle -90°$$

Primary current $I_1 = I_m + I_2'$

$$= 1\angle -90° + 2\angle 0°$$
$$= 2.24\angle -26.56° \text{ A}$$

60. Ans: (d)

61. Ans: (d)

62. Ans: (d)

Hint: In open circuit test, no load current is very small (it is usually 2–5% of the rated current). No load current I_0 lags E by slightly less than 90 degree. So, power factor is very low. So, wattmeter W_2 is suitable for the open circuit test.

In short circuit test, short circuit voltage (to circulate the full load current) is very low. As I_0 is highly lagging but it is very small as compared to full load current, therefore, the power factor is high. So, wattmeter W_3 is suitable for the short circuit test.

63. Ans: (a)

Hint: Rotor resistance at standstill $r_2 = 7.8\ \Omega$

Synchronous speed, $N_s = \dfrac{120 \times 50}{4} = 1500$ rpm

The slip of rotor with respect to forward field

$$s = \frac{1500 - 1425}{1500} = 0.05$$

So, the slip of rotor with respect to backward field

$$= 2 - s = 2 - 0.05 = 1.96$$

Effective rotor resistance

$$= \frac{r_2}{2(2-s)} = \frac{7.8}{2 \times 1.96} = 2\ \Omega$$

64. Ans: (c)

Hint: Input power,

$$P_1 = \sqrt{3} \times 400 \times 50 \times 0.8 = 27.71 \text{ kW}$$

Air gap power, P_g = input power – core loss – copper loss of stator

$$= 27.71 - 1.5 - 1.2 = 25.01 \text{ kW}$$

65. Ans: (d)

Hint: Three-phase supply is given to the rotor windings through an auto-transformer.

Now, the three-phase rotor current will generate a rotating field in the air gap, which rotates at synchronous speed with respect to rotor.

If the rotor is kept stationary, this rotating field will also rotate in the air gap at the synchronous speed.

So, voltage and current will be induced in the stator winding and a torque will be developed.

If the rotor is allowed to move, it will rotate, opposing the rotation of the rotating field decreasing the induced voltage in the stator winding.

Thus at a particular speed, the frequency of the stator circuit will correspond to the slip speed.

Synchronous speed, $N_s = \dfrac{120 \times 50}{4} = 1500$ rpm

Rotor speed, $N_r = 1410$ rpm

$$\text{Slip} = \frac{1500 - 1410}{1500} = 0.06$$

Slip frequency induced in stator $= 0.06 \times 50 = 3$ Hz

$$\text{Slip speed} = \frac{120 \times 30}{4} = 90 \text{ rpm}$$

As stator magnetic field rotates 90 rpm in the opposite direction of the rotation of rotor, therefore, the speed of the stator field with respect to rotor

$$= 1410 + 90 = 1500 \text{ rpm}$$

66. Ans: (c)

Hint: Slip $= \dfrac{90}{1500} = 0.06$

Total input power to induction motor = 1800 – 200 = 1600 W

Output power of induction motor

$$= 1600(1 - 0.06) = 1504 \text{ W}$$

Given, all losses are neglected.

Induction motor output = dc generator input = dc generator output = 1504 W

$$I^2 R_{ex} = 1504$$

or $$I^2 \times 10 = 1504$$

or $$I = 12.26 \text{ A}$$

67. Ans: (d)

Hint: Field current, $I_f = \dfrac{240}{80} = 3$A

Armature current, $I_a = 15 - 3 = 12$ A

Back emf, $E_b = V - I_a r_a = 240 - 12 \times 0.5 = 234$ V

The connection of armature winding is reversed at the time of plugging. A strong braking torque is achieved by maintaining the supply voltage to the armature with connection reversed.

So, the net voltage across the armature resistance at the time of plugging

$$E_b + V = 240 + 234 = 474 \text{ V}$$

68. Ans: (a)

Hint: Normally, armature current = 12 A

During braking, armature current

$$I' = 1.25 \times 12 = 15 \text{ A}$$

During braking, $r_b = r_a + r_{ext}$

Again, $r_b = \dfrac{E_b + V}{I'} = \dfrac{474}{15} = 31.6\ \Omega$

$\therefore\ r_{ext} = 31.6 - 0.5 = 31.1\ \Omega$

69. Ans: (d)

Hint: Motor draws 0.6 p.u. current at upf

$$I_a = 0.6\angle 0^\circ$$

5

Electrical and Electronic Measurements

5.1 SYLLABUS

Bridges and potentiometers; PMMC, moving iron, dynamometer and induction type instruments; measurement of voltage, current, power, energy and power factor; instrument transformers; digital voltmeters; phase, time and frequency measurement; Q-meters, oscilloscopes; potentiometric recorders, error analysis.

5.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	3	2	1	1	2	2	3	3	2	2	2	2
2 Marks Question	5	4	1	2	2	1	1	1	1	2	2	2
Total Marks	**13**	**10**	**3**	**5**	**6**	**4**	**5**	**5**	**4**	**6**	**6**	**6**

5.3 TOPICS TO BE FOCUSED

ELECTRICAL MEASUREMENT

Electrical measurement involves the use of various electrical instruments as a physical means of determining quantities or variables. Electrical parameters such as voltage, current, resistance, capacitance, inductance, power, power factor and energy are measured by means of electrical instruments.

POTENTIOMETERS

A potentiometer is an instrument for measuring the potential (voltage) in a circuit. Before the introduction of the moving coil and digital voltmeters, potentiometers were used in measuring voltage, hence the '-meter' part of their name. The slide wire is provided with a scale and by sliding the contact, the unknown emf is adjusted with the standard cell. When no current flows through the galvanometer, the proportional length of the slide wire is calculated. The scale of slide wire is calibrated directly in term of measured emf. To measure higher voltage ranges, a precision potential divider called a volt ratio box is used. It provides multiple ranges. Potentiometers are also used for calibration of voltmeter and ammeter, measurement of resistance and power.

AC Potentiometers

In ac potentiometers, for getting balance point the magnitude and phase of the unknown voltage and the standard cell are made equal.

These are classified into two types:

(i) Polar type
(ii) Coordinate type

- Example for polar type is the Drysdale polar potentiometer. Here a Drysdale phase shifting transformer is used.
- Example for coordinate potentiometer is the Gall Tinsley ac potentiometer.

BRIDGES

Bridges are the most widely used means for precise measurement of capacitance, resistance, self and mutual inductance. Bridges are broadly classified into types: for measuring capacitance and inductance called ac bridge and the bridges used for measuring resistance.

AC Bridges are of different types which are listed as:

(a) Bridges used for measuring self-inductance are as follows:
   (i) Hay's bridge
   (ii) Maxwell's bridge

(iii) Anderson's bridge
(iv) Owen's bridge

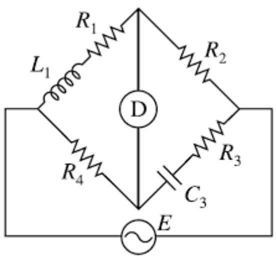
(b) Bridges used for measuring mutual inductance are as follows:
(i) Heaviside bridge
(ii) Heaviside–Campbell equal ratio bridge

(c) Bridges used for measuring capacitance are as follows:
(i) Schering bridge
(ii) De Sauty's bridge
(iii) Wein bridge

Wein bridge is also used to measure frequency.

Measurement of Self-inductance

Hay's bridge: Hay's bridge is used for measuring high Q-inductors.



At balance condition,

$$L_1 = \frac{R_4 R_2 C_3}{1+\omega^2 C_3^2 R_3^2} \quad \text{and} \quad R_1 = \frac{\omega^2 R_4 R_3 R_4 C_3^2}{1+\omega^2 C_3^2 R_3^2}$$

The Q-factor is determined by $Q = \dfrac{\omega L_1}{R_1} = \dfrac{1}{\omega C_3 R_4}$

Maxwell's inductance bridge: It is basically constructed by resistance, inductance and capacitance. There are some particular arrangements that these elements are connected in each arm as shown in figures.

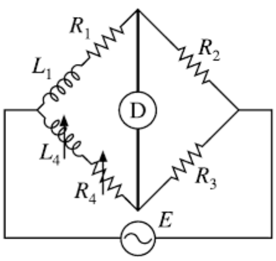


Figure (a)

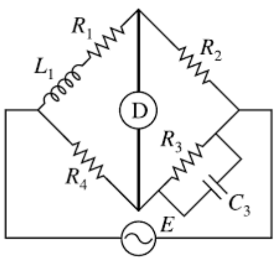


Figure (b)

The balance condition for bridge shown in Figure (a),

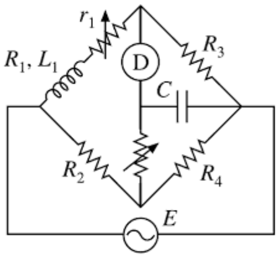
$$L_1 = \frac{R_2 L_4}{R_3} \quad \text{and} \quad R_1 = \frac{R_2 R_4}{R_3}$$

The balance condition for bridge shown in Figure (b),

$$L_1 = R_2 R_4 C_3 \quad \text{and} \quad R_1 = \frac{R_2 R_4}{R_3}$$

Maxwell bridge works well for medium Q values not for small one like $Q < 1$.

Anderson's bridge: It is a modified Maxwell's bridge used for the measurement of very low Q-inductors.

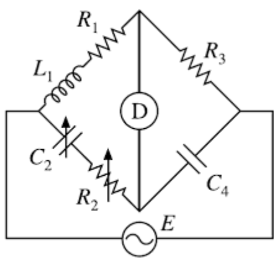


At balanced condition,

$$L_1 = C\frac{R_3}{R_4}\left[r\left(R_4 + R_2\right) + R_2 R_4\right]$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Owen's bridge: Owen's bridge is basically for measuring wide range of inductance. Here inductance is measured in terms of capacitance.

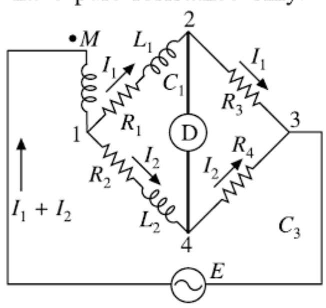


At balanced condition,

$$L_1 = R_2 R_3 C_4 \quad \text{and} \quad R_1 = R_3 \frac{C_4}{C_2}$$

Mutual Inductance

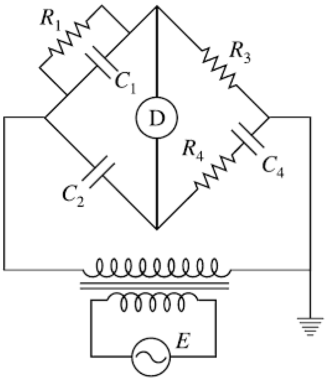
Heaviside bridge: It is a mutual induction bridge and is used for measuring self-inductance over a wide range in terms of mutual inductor meter readings. The connections for Heaviside's bridge employing a standard variable mutual inductance as shown in figure. The primary of the mutual inductor meter is inserted in the supply circuit and the secondary having self-inductance and resistance is put in arm 2 of the bridge. The unknown inductive impedance having self inductance of and resistance is placed in arm 1. The other two arms have pure resistance only.



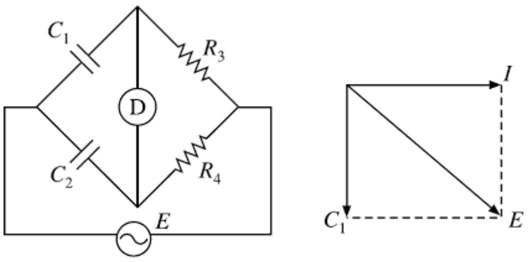
Heaviside–Campbell equal ratio bridge: It is updated version of Heaviside bridge and is used for measuring mutual inductance. Mutual inductance is measured in terms of self-inductance.

Measurement of Capacitance

Schering bridge: Schering bridge is used for dielectric loss and power factor of a lossy capacitor at high voltages.



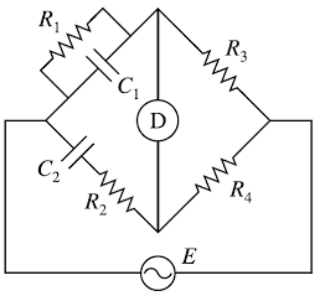
De Sauty's bridge: It is the simplest method for comparing two capacitances.



At balance condition,

$$C_1 = \frac{C_2 R_4}{R_3}$$

Wien's bridge: It is used for measurement of capacitance and frequency of the circuit.



At balanced condition,

$$C_1 = \frac{\dfrac{R_4 C_2}{R_3}}{1+\omega^2 C_2^2 R_2^2} \quad \text{and} \quad R_1 = \frac{R_3\left(1+\omega^2 C_2^2 R_2^2\right)}{R_4 R_2 \omega^2 C_2^2}$$

Frequency, $f = \dfrac{1}{2\pi C_2 R_2}$, when $R_3 = R_4$, $C_1 = \dfrac{C_2}{2}$ and $R_1 = 2R_2$

MEASUREMENT OF RESISTANCE

Resistances are classified into three categories namely low, medium and high.

(a) Low resistance means resistance below 1 Ω.
(b) Medium resistance is the range between 1 Ω and 100 kΩ.
(c) High resistance is the type resistance which has a value above 100 kΩ.

Measurement of Low Resistance

There are two major techniques to measure low resistance, they are:

(i) Ammeter–voltmeter method
(ii) Kelvin double bridge method

Ammeter–Voltmeter method: There are two types of connections in this method, they are shown as follows:

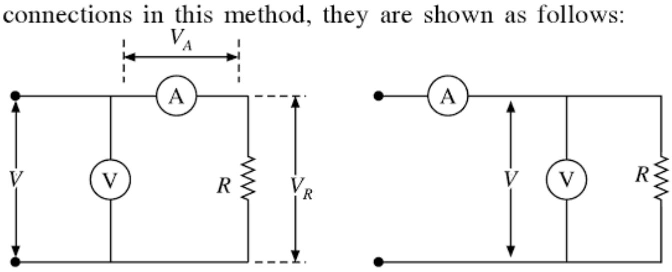


Figure (a) Figure (b)

Measured value of resistance, $R = \dfrac{V}{I}$

For Figure (a):

$$R = \frac{E}{I} - R_A$$

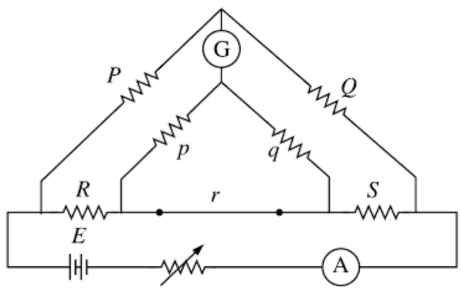
where R_A = resistance of ammeter

For Figure (b):

$$R = \frac{E}{I\left(1 - \dfrac{E}{IR_V}\right)}$$

where R_V = resistance of voltmeter

Kelvin double bridge: It is the best method to measure low resistance.



For zero galvanometer defection,

$$R = \frac{P}{Q}S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$$

If $\frac{P}{Q} = \frac{p}{q}$, then equation becomes, $R = \frac{P}{Q}S$

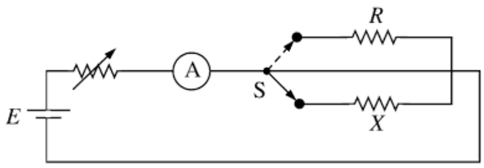
Measurement of Medium Valued Resistance

The methods to measure medium resistance are:

(i) Ammeter–voltmeter method
(ii) Substitution method
(iii) Wheatstone bridge method
(iv) Ohmmeter method

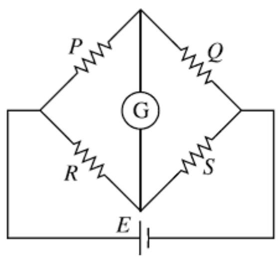
Ammeter–Voltmeter method: This is same as the measurement of low valued resistance.

Substitution method: The arrangement is made in the following way.



$X \rightarrow$ unknown resistance
$R \rightarrow$ standard known resistance
A $\rightarrow$ ammeter used to measure the current in the circuit
S $\rightarrow$ switch

Wheatstone bridge method: The Wheatstone bridge is used for making comparison measurements and operates upon a null indication principle. The arrangement is shown as in figure.



Unknown resistance, $S = \frac{QR}{P}$

Ohmmeter method: An ohmmeter is an instrument used for measuring medium value resistance. Its accuracy is 3% to 4%.

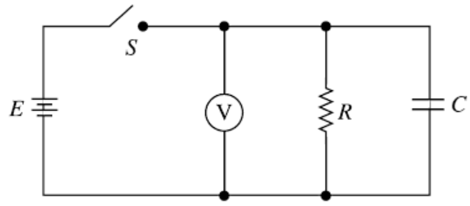
Methods for Measurement of High Resistance

There are four methods:

(i) Direct deflection method
(ii) Loss of charge method
(iii) Megohm bridge
(iv) Megger

Direct deflection method: It is basically used to measure the insulation resistance of cables with the help of a sensitive galvanometer of d'Arsonval type having a current sensitivity of 1000 mm/μA at a scale of 1 m.

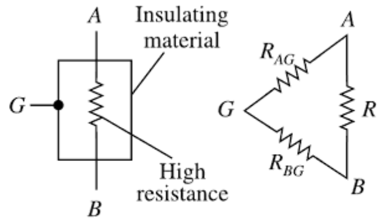
Loss of charge method: Here an insulation resistance is connected in parallel with a capacitor and an electrostatic voltmeter as shown in the following figure.



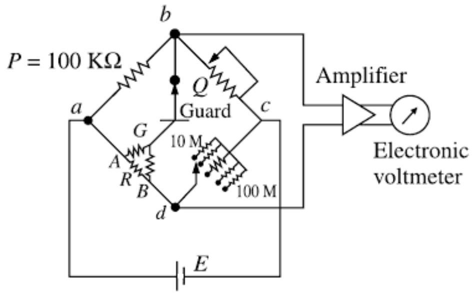
Here, insulation resistance is given by, $R = \frac{0.4343t}{C\log_{10}\frac{E}{V}}$

where, E is the initial voltage across capacitor at time, $t = 0$.
$V \rightarrow$ voltage across capacitor at t seconds (while discharging)
$C \rightarrow$ capacitance in farad

Megohm bridge: Megohm bridge is another important method for measurement of high resistances. It has three high resistance terminals located in one arm of the bridge. Figure shows the very high resistance with terminals A and B, and a guard terminal, which is put on the insulation. So it forms a three terminal resistance.



Let us consider the hypothetical case of a 100 MΩ resistance. Let us assume that this resistance is measured by an ordinary Wheatstone bridge. It is clear that Wheatstone will measure a resistance of 100 × 200/(100 + 200) = 67 MΩ instead of 100 MΩ thus the error is 33%.



However, if the same resistance is measured by a modified Wheatstone bridge as shown in figure with the guard connection G connected as indicated, the error in measurement will be reduced and this modified Wheatstone bridge is called Megohm bridge.

Where $M \rightarrow$ mutual inductance between fixed and moving coils deflection θ is proportional to square of current.

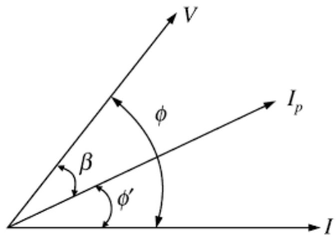
Error in electro-dynamometer wattmeter

$$\beta = \tan^{-1}\left(\frac{\omega L}{r_p + R}\right)$$

$$\phi' = \phi - \beta$$

where

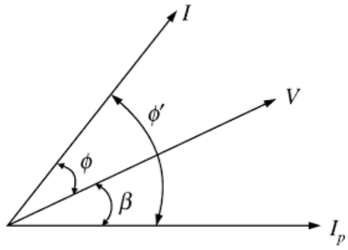
r_p = Resistance of pressure coil
L = Inductance of pressure coil
R = Resistance in series with pressure coil



$\therefore$ True power $= \dfrac{\cos\phi}{\cos\beta\cos(\phi-\beta)} \times$ actual wattmeter reading

So, for leading pf

$$\text{Correction factor} = \frac{\cos\phi}{\cos\beta\cos(\phi+\beta)}$$



Moving Iron Instruments

The working principle of moving iron instrument depends upon the deflection of a soft iron vane in a magnetic field created by a fixed coil which acts as an electromagnet. These types of instruments are classified into two types: attraction and repulsion type.

The deflecting torque is given by,

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$T_c = K\theta$$

$$\therefore \quad \theta = \frac{1}{2K} I^2 \frac{dL}{d\theta}$$

So, the instrument has a square law response.

Thermal Instruments

Thermal instruments depend on some property of a circuit element which is heated by flow of current. It has two types: thermocouple type and hot wire type.

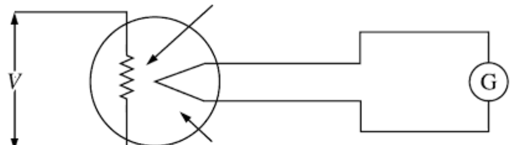
Thermocouple instruments: Thermocouple converts heat energy to electrical energy. In a thermocouple there are two metals with separate working principle placed together. The junction gets heated due to the current flowing in the circuit.

$$\text{Emf } E = a(T_1 - T_2) + b(T_1 - T_2)^2$$

a and b are constants depend upon metals used.

$T_1 - T_2 = \Delta\theta \rightarrow$ is the temperature difference.

These instruments are popular for ac measurements at high frequency.



Hot wire instruments: The current to be measured is passed through a fine platinum iridium wire. On passage of current, the wire gets heated and it expands. This expansion of wire causes the pointer to deflect. Hot wire instruments have many disadvantages and are commercially obsolete. They are replaced by thermo-electric instruments.

Electrostatic Instruments

The deflecting torque is produced by action of electric field on charged conductors. The deflecting torque is given by,

$$T_d = \frac{\varepsilon_0 r^2}{d} V^2$$

Their deflection is directly proportional to square of the rms voltage.

where

ε_0 is the permittivity of air
r is the radius
d is the distance between the plates

MEASUREMENT OF POWER

Power is measured by wattmeter. Dynamometer consists of two types of coil, two fixed coils and one moving coil. The fixed coils are connected in series with the load so they are called 'current coils'. On the other hand the moving coil carries current proportional to the voltage and it is called 'voltage coil'.

The deflecting torque is given by,

$$T_d = \frac{VI}{R}\cos\phi\frac{dM}{d\theta}$$

where,

$R \rightarrow$ resistance of pressure coil

$M \rightarrow$ mutual inductance between the current coils and pressure coil

$\phi \rightarrow$ phase angle which the current lags with voltage in current coil circuit

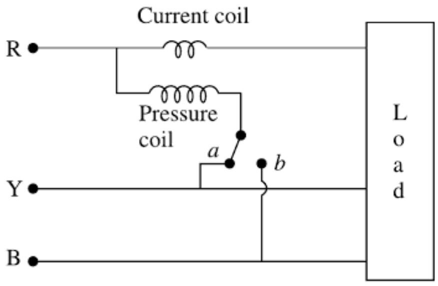
Controlling torque,

$$T_c = K\theta$$

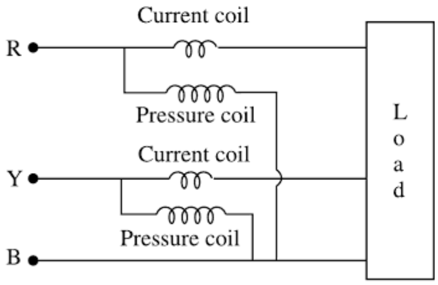
$K \rightarrow$ spring constant

$\theta \rightarrow$ final steady deflection

Measurement of power in three-phase circuit is done by three-wattmeter method.



The most common method of power measurement in three-phase circuit is the two-wattmeter method.



MEASUREMENT OF POWER FACTOR

Measurement of power factor is done in similar way as power is measured. These instruments also work in electrodynamics principle. But there are two moving coils instead of one along with two fixed coils. The two moving coils produce two torques.

$$T_{d1} \propto VI\cos\phi\sin\theta$$

$$T_{d2} \propto VI\sin\phi\cos\theta$$

$\theta \rightarrow$ angle of deflection

$\phi \rightarrow$ phase angle of the circuit

In equilibrium, $T_{d1} = T_{d2}$

$$\therefore \qquad \theta = \phi$$

Thus, deflection becomes equal to phase angle.

MEASUREMENT OF ENERGY

Energy is the total power delivered or consumed over a time span. Motor meters are used for measurement of energy in both dc and ac circuits. In a motor meter, the moving system revolves continuously. The speed of rotation is proportional to the power. The total number of revolutions made by the meter in a given interval of time is proportional to the energy supplied. In this context, the term meter constant comes.

Testing

The energy meters are tested generally at 5%, 50%, 100% and 120% of its rated current both for unity and 0.5 power factors.

Meter constant is defined as the number of revolutions made per kWh.

Creep

Sometimes the disc of the energy meter makes slow but continues rotation at no load, i.e., when potential coil is excited but no current flowing in the load. This is called creeping. This error may be caused due to the overcompensation for friction, excessive supply voltage, vibration, stray magnetic field, etc.

To prevent creeping, two diametrically opposite holes are drilled in the disc.

INSTRUMENT TRANSFORMERS

Transformers used in conjunction with measuring instruments for measurement are called *instrument transformers*. The transformer used for current measurement is called "current transformer" or "CT" and transformers used for voltage measurement are called "potential transformers" or simply "PT".

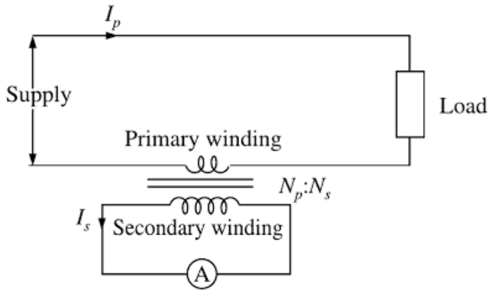
Burden

Burden is the load imposed on the secondary of the CT at rated current and is measured in VA (product of volts and amperes). The accuracy class applies only to loads at rated VA and below, down to one quarter VA .

The burden on the secondary of a CT includes the effect of pilot leads, connections etc, as well as the instrument burden itself.

Current Transformers

They are operated in short circuit condition. The current in the secondary of a CT is decided by the primary current and not by the impedance of secondary circuit. Figure shown below shows a current transformer.



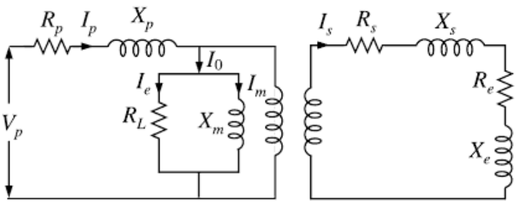
$$N_p I_p = N_s I_s$$
$$E_s = I_s Z_s$$

where,

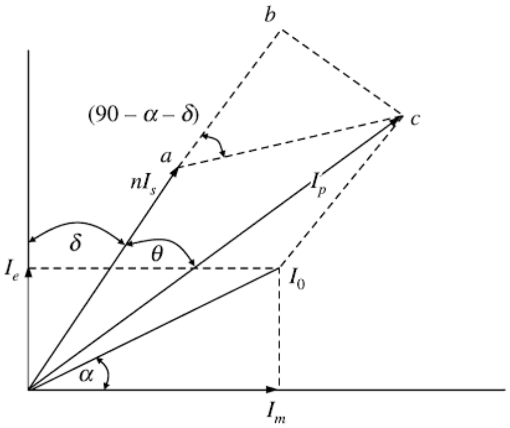
$N_p \rightarrow$ Number of turns of primary winding
$N_s \rightarrow$ Number of turns of secondary winding
$I_p \rightarrow$ Current in primary
$I_s \rightarrow$ Current in secondary
$Z \rightarrow$ Impedance
$E \rightarrow$ Induced emf

Transformation ratio

$$R = n + \frac{I_0}{I_s}\sin(\delta + \alpha)$$



$$R = n + \frac{I_m \sin\delta + I_e \cos\delta}{I_s}$$
$$I_m = I_0 \cos\alpha$$
$$I_e = I_0 \sin\alpha$$



Phase angle

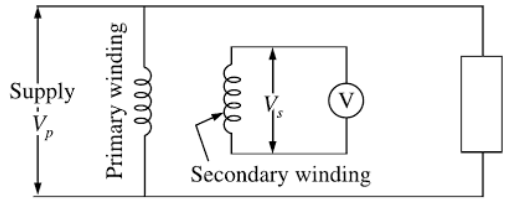
$$\theta = \frac{180}{\pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{nI_s}\right)\text{degree}$$

Errors in current transformers

$$\%\text{ ratio error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio} \times 100} \times 100$$
$$= \frac{K_n - R}{R} \times 100$$

Potential Transformer

Potential transformer or voltage transformer gets used in electrical power system for stepping down the system voltage to a safe value which can be fed to low ratings meters and relays. Commercially available relays and meters used for protection and metering, are designed for low voltage. The primary winding of the transformer is connected across the line carrying the voltage to be measured and the voltage circuit is connected across the secondary winding.

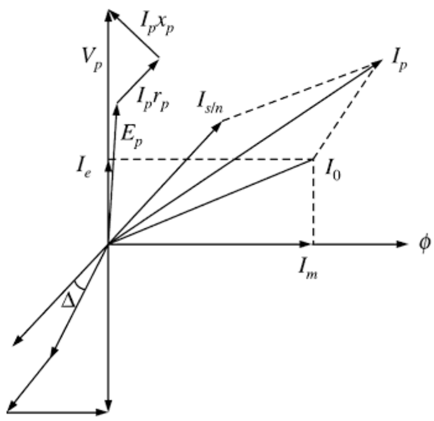


Transformation ratio: Actual transformation ratio = R

$$R = n + \frac{I_{s/n}\left[R_p \cos\Delta + X_p \sin\Delta + I_e r_p + I_m X_p\right]}{V_s}$$

where

$$R_p = n^2 r_s + r_p$$
$$X_p = n^2 x_s + x_p$$

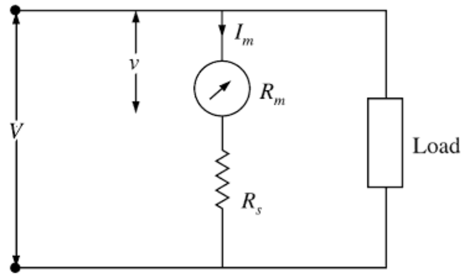


Phase angle error

$$\theta = \frac{I_s}{V_s}\left(X_s \cos\Delta - R_s \sin\Delta\right) + \frac{I_e X_p - I_m r_p}{\eta V_s}\ \text{rad.}$$

VOLTMETER MULTIPLIERS

A d'Arsonal basic meter movement is converted into a voltmeter by connecting a series resistance with it.
The series resistance is called multiplier.



It can be shown

$$m = \frac{V}{v} = 1 + \frac{R_s}{R_m}$$

$\therefore$ Resistance $R_s = (m - 1)R_m$

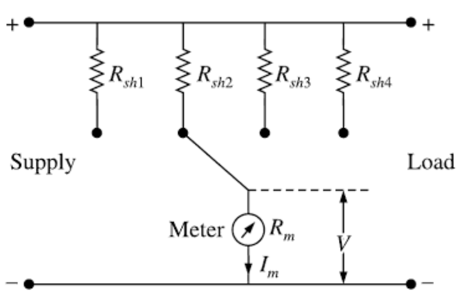
Hence, for the measurement of voltage m times the voltage range of instrument, the series multiply.

$$m = 1 + \frac{R_s}{R_m}$$

Multi-range DC Voltmeters

$$R_{s1} = (m_1 - 1)R_m$$
$$R_{s2} = (m_2 - 1)R_m$$
$$R_{s3} = (m_3 - 1)R_m$$
$$R_{s4} = (m_4 - 1)R_m$$

$$m_1 = \frac{V_1}{v}$$
$$m_2 = \frac{V_2}{v}$$
$$m_3 = \frac{V_3}{v}$$
$$m_4 = \frac{V_4}{v}$$



DIGITAL TO ANALOG

D/A performance characteristic are:

FSV (Full Scale Voltage)

$$e_0 = \frac{V_R}{2^n}$$

where

n = Number of bits

If there are n bits, output is the sum of individual bit of outputs

$$\text{FSV} = V_R\left(1 - \frac{1}{2^n}\right)$$

Resolution

$$\%\ \text{resolution} = \frac{\text{Step size}}{\text{FSV}} \times 100 = \frac{1}{\text{Total number of steps}} \times 100$$

$$\%\ \text{resolution} = \frac{1}{2^n - 1} \times 100$$

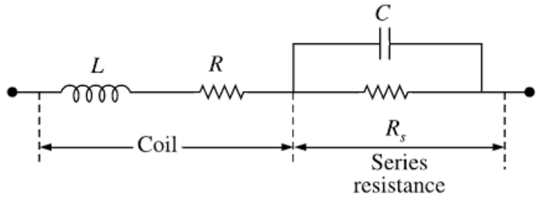
Accuracy

$$\pm\frac{1}{2}\text{LSB}$$

ERROR IN MI METERS

Frequency Error

Changes in frequency may cause error due to changes of reactance of working coil and also due to changes of magnitude of eddy currents set up in metal parts of instrument.

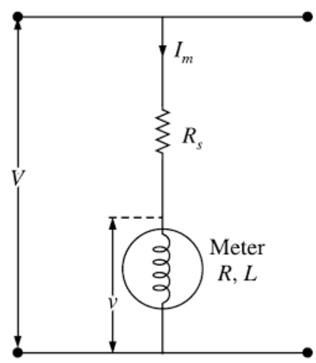


When the frequency error is eliminated then $C = 0.41\dfrac{L}{R_s^{\,2}}$

Frequency ranges upto 125 Hz.

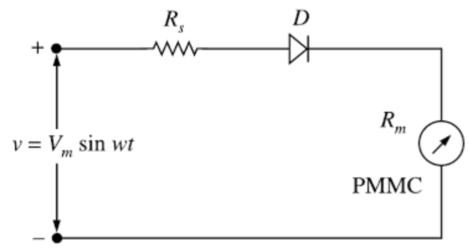
Multipliers for MI Meters

$$m = \frac{V}{v} = \frac{\sqrt{(R + R_s)^2 + \omega^2 I^2}}{\sqrt{R^2 + \omega^2 I^2}}$$



RECTIFIER TYPE OF INSTRUMENTS

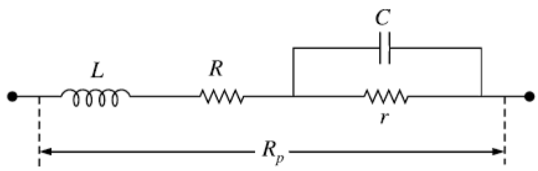
Extension of range of rectifier instruments as voltmeters:



$R_s = 0.45 S_{de} V - R_m R_d$ (for half wave rectification)

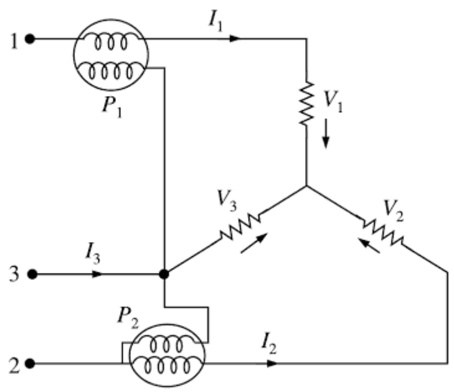
$R_s = 0.9 S_{de} V - 2R_d$ (for full wave rectification)

COMPENSATION FOR INDUCTANCE OF PRESSURE COIL



$$L = Cr^2$$

3-PHASE POWER MEASUREMENT BY WATTMETER



Reading of wattmeter $P_1 = \sqrt{3}VI\cos(30-\phi)$

Reading of wattmeter $P_2 = \sqrt{3}VI\cos(30+\phi)$

$$P = P_1 + P_2 = 3VI\cos\phi$$

$$P_1 - P_2 = 3VI\sin\phi$$

$$\phi = \tan^{-1}\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}$$

(i) With unity power factor (UPF), readings of the two wattmeters are equal, each wattmeter reads half of total power.
(ii) When power factor is 0.5, one of the wattmeter reads zero and other reads total power.
(iii) With zero power factor reading of the two wattmeters are equal but of opposite in sign.

CABLE FAULTS

Two types of cable fault that can occur are:

(i) Ground fault
(ii) Short circuit fault

Ground Fault

In an electric power system, a fault is any abnormal electric current. For example, a short circuit is a fault in which current bypasses the normal load. An open-circuit fault occurs if a circuit is interrupted by some failure. In three-phase systems, a fault may involve one or more phases and ground, or may occur only between phases. In a "ground fault" or "earth fault", charge flows into the earth.

The magnitude of fault currents differ widely depending on the type of earthing system used, the installation's supply type and earthing system, and its proximity to the supply. For example, for a domestic UK—230 V, 60 A, TN-S or USA—120 V/240 V supply, fault currents may be a few thousand amperes. Large low-voltage networks with multiple sources may have fault levels of 300,000 amperes. A high-resistance-grounded system may restrict line to ground fault current to only 5 amperes. Prior to selecting protective devices, prospective fault current must be measured reliably at the origin of the installation and at the furthest point of each circuit, and this information applied properly to the application of the circuits.

In a high resistance grounded distribution system, a feeder may develop a fault to ground but the system continues in operation. The faulted, but energized feeder can be found with a ring-type current transformer collecting all the phase wires of the circuit, only the circuit containing a fault to ground will show a net unbalanced current. To make the ground fault current easier to detect, the grounding resistor of the system may be switched between two values so that the fault current passes.

Short Circuit Fault

A short circuit (sometimes abbreviated to short or s/c) is an electrical circuit that allows a current to travel along an unintended path, often where essentially no (or a very low) electrical impedance is encountered.

A short circuit is an abnormal connection between two nodes of an electric circuit intended to be at different voltages. This results in an excessive electric current limited only by the Thévenin equivalent resistance of the rest of the network and potentially causes circuit damage, overheating, fire or explosion. Although usually the result of a fault, there are cases where short circuits are caused intentionally, for example, for the purpose of voltage-sensing crowbar circuit protectors.

In circuit analysis, a short circuit is a connection between two nodes that forces them to be at the same voltage. In an ideal short circuit, this means there is no resistance and no voltage drop across the short. In real circuits, the result is a connection with almost no resistance. In such a case, the current is limited by the rest of the circuit.

MURRAY LOOP TEST

$$X = \frac{Q}{P+Q}(R+X)$$

where P, Q, R and X be resistances.

If l_1 represents the length of fault from test end and l be length of each cable. Then X = unknown resistance.

$$l_1 = \left(\frac{Q}{P+Q}\right)2l$$

VARLEY LOOP TEST

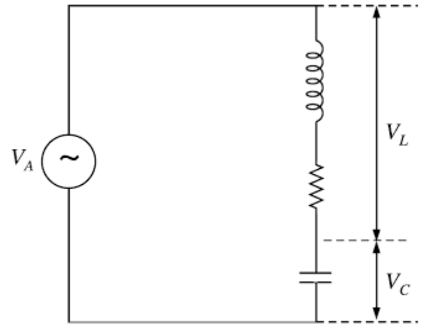
Here $$l_1 = \left(\frac{X}{R+X}\right)2l$$

R, X are the similar meaning.

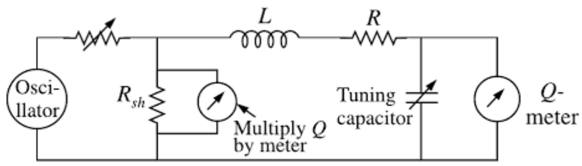
Q-METER

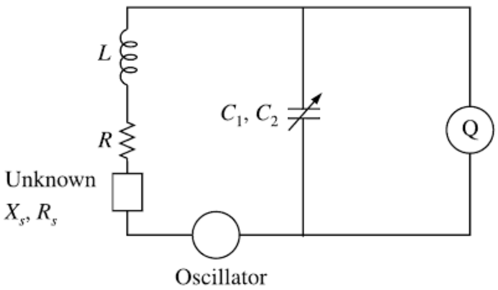
Q-meter are intended to measure Q of inductor or capacitor.

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR} = \frac{V_C}{V_A}$$



Basic Circuit of Q-meter



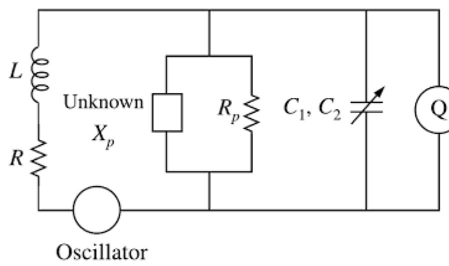


$$\omega L = \frac{1}{\omega C_1} \text{ and } Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

$$X_s = \frac{C_1 - C_2}{\omega C_1 C_2}$$

$$R_s = \frac{Q_1 C_1 - Q_2 C_2}{\omega C_1 C_2 Q_1 Q_2}$$

$$Q_x = \frac{X_s}{R_s} = \frac{(C_1 - C_2) Q_1 Q_2}{Q_1 C_1 - Q_2 C_2}$$



$$\frac{1}{R_p} = \frac{\omega C_1}{Q_2} - \frac{1}{R Q_1^2}$$

$$X_p = \frac{1}{\omega (C_2 - C_1)}$$

$$Q_x = \frac{R_p}{R_x} = \frac{(C_2 - C_1) Q_1 Q_2}{Q_1 C_1 - Q_2 C_2}$$

Q measured at two frequencies f_1 and f_2, ratio of frequencies is n.

$$C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}; \; C_d = \text{Distributed frequency capacitance}$$

5.4 IMPORTANT POINTS TO REMEMBER

1. Direct method is used to measure length.
2. Ampere is one of the base units.
3. Creeping is observed in watt hour meter.
4. A CRO can display both ac and dc signals.
5. The internal resistance of an ammeter should be very small.
6. Ideally the internal resistance of an ammeter should be zero.
7. A CRO uses electrostatic focusing.
8. A 10 MHz CRO has 10 MHz horizontal oscillator.
9. A CT is overloaded when its terminals are open circuited.
10. PMMC type instruments normally use eddy current damping.
11. The errors in CT are mainly due to core loss.
12. A PT is a device which is electromagnetically coupled.
13. Usually a CT has a higher overload capacity than a PT.
14. The standard secondary voltage for a PT is 110 V.
15. The VA rating of instrument transformers is normally near 10 VA.
16. A galvanometer has eddy current damping.
17. It can be stated that, CT operates with considerably lower flux density than a PT.
18. A megger is basically a moving coil type instrument.
19. In a megger, when not in operation, the needle shows the resistance of infinity.
20. The instrument used normally to check the insulation resistance is megger.
21. A vector meter measures
    (a) Power factor
    (b) Current
22. Wien bridge is useful for measuring high frequency.
23. An oscilloscope indicates peak to peak value of voltage.
24. Ionic wind voltmeter is used for measuring high voltage.
25. Schering bridge is used to determine dielectric loss.
26. No eddy current and hysteresis losses occur in electrostatic instruments.
27. Electrostatic instruments are normally used for high voltage measurements.
28. A two-wattmeter method can be used
    (a) For balanced power

(b) For unbalanced power
(c) For reactive power measurement

29. In three-phase power measurement, the power factor of load will be

$$\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

30. In a two-wattmeter method of measuring three-phase power, power factor is 0.5, then one of the wattmeters will read zero.
31. A piezometer is used to measure very low pressure.
32. Hay's bridge can be used for measurement of high Q inductive circuits.
33. The deflection depends on the average value in rectifier meter.
34. To measure flux, devices used are based on Hall effect.
35. In a ballistic galvanometer, the deflecting torque is proportional to the current through coil.
36. The resistance can be measured most accurately by bridge method.
37. The ratio error of a current transformer is due to exciting current.
38. The number of turns in the primary of a current transformer is usually 1 to 5.
39. Permanent magnets used in instruments are generally made of Alnico.
40. Wien bridge is used to determine frequency.
41. The absolute measurement of resistance is done by Lorenz method.
42. The dielectric loss can be measured by Electrostatic meter.
43. Schering bridge is used for measurement of capacitance.
44. The maximum deflection that is possible with a PMMC movement is 360°.
45. Lissajous patterns are used to measure frequency and phase shift.
46. The maximum reading on a $3\frac{1}{2}$ digit voltmeter is 1999.
47. The resolution of a digital ammeter with a 4-digit display is $\frac{1}{10000}$.
48. Low resistance is measured by Kelvin double bridge.
49. Manganin is an alloy used to make standard resistors.
50. The error of an instrument is normally given as a percentage of full scale value.
51. Different bridges used to measure different parameters:

(i) Kelvin double bridge $\rightarrow$	Low resistance
(ii) Maxwell's bridge $\rightarrow$	Self-inductance
(iii) Wien bridge $\rightarrow$	Frequency
(iv) Schering bridge $\rightarrow$	Capacitance

5.5 REFERENCES

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Ghosh, Arun K., *Introduction to Measurements and Instrumentation*, 4th ed., PHI Learning, Delhi, 2012.

Gupta, J.B., *A Course in Electronics and Electrical Measurements and Instrumentation*, S.K. Kataria & Sons, Delhi, 2013.

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5.6 PREVIOUS YEARS' QUESTIONS

1. A manganin swap resistance is connected in series with a moving coil ammeter consisting of a milli–ammeter and a suitable shunt in order to
(a) Minimise the effect of temperature variation
(b) Obtain large deflecting torque
(c) Reduce the size of the meter
(d) Minimise the effect of stray magnetic fields
[Gate 2003]

2. The effect of stray magnetic fields on the actuating torque of a portable instrument is maximum when the operating field of the instrument and the stray fields are
(a) Perpendicular (b) Parallel
(c) Inclined at 60° (d) Inclined at 30°
[Gate 2003]

3. The item in Group I represent the various types of measurements to be made with a reasonable accuracy using a suitable bridge. The items in Group II represent the various bridges available for this purpose. Select the correct choice of the item in Group II for the corresponding item in Group I from the following:

Group I	Group II
P. Resistance in the mΩ range	**1.** Wheatstone bridge
Q. Low values of capacitance	**2.** Kelvin bridge
R. Comparison of resistances which are nearly equal	**3.** Schering bridge
S. Inductance of a coil with a large time constant	**4.** Wien's bridge
	5. Hay's bridge
	6. Carey–Foster bridge

(a) P – 2 Q – 3 R – 6 S – 5
(b) P – 2 Q – 6 R – 4 S – 5
(c) P – 2 Q – 3 R – 5 S – 4
(d) P – 1 Q – 3 R – 2 S – 6
[Gate 2003]

4. A moving coil of a meter has 100 turns and a length and depth of 10 mm and 20 mm respectively. It is positioned in a uniform radial flux density of 200 mT. The coil carries a current of 50 mA. The torque on the coil is

(a) 200 μNm (b) 100 μNm
(c) 2 μNm (d) 1 μNm [Gate 2004]

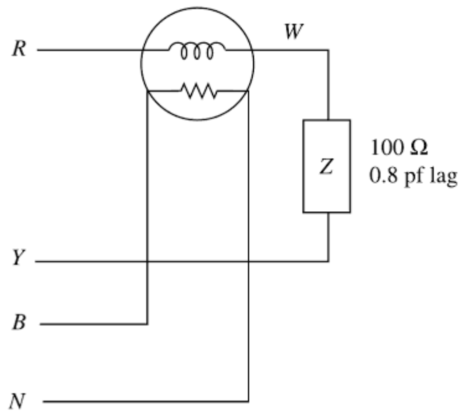
5. A dc A-h meter is rated for 15 A, 250 V. The meter constant is 14.4 A s/rev. The meter constant at rated voltage may be expressed as

(a) 3750 rev/kWh (b) 3600 rev/kWh
(c) 1000 rev/kWh (d) 960 rev/kWh
[Gate 2004]

6. A moving iron ammeter produces a full scale torque of 240 μNm with a deflection of 120° at a current of 10 A. The rate of change of self-inductance (μH/rad) of the instrument at full scale is

(a) 2.0 μH/rad (b) 4.8 μH/rad
(c) 12.0 μH/rad (d) 114.6 μH/rad
[Gate 2004]

7. A single-phase load is connected between R and Y terminals of a 415 V, symmetrical, 3-phase, 4-wire system with phase sequence RYB. A wattmeter is connected in the system as shown in figure. The power factor of the load is 0.8 lagging. The wattmeter will read



(a) – 795 W (b) – 597 W
(c) + 597 W (d) + 795 W [Gate 2004]

8. A 50 Hz, bar primary CT has a secondary with 50 turns. The secondary supplies 5 A current into a purely resistive burden of 1 Ω. The magnetizing ampere–turns is 200. The phase angle between the primary and secondary current is

(a) 4.6° (b) 85.4°
(c) 94.6° (d) 175.4° [Gate 2004]

9. The core flux in the CT of Problem 8 under the given operating conditions is

(a) 0
(b) 45.0 μWb
(c) 22.5 mWb
(d) 100.0 mWb [Gate 2004]

10. The Q–meter works on the principle of

(a) Mutual inductance (b) Self-inductance
(c) Series resonance (d) Parallel resonance
[Gate 2005]

11. A PMMC voltmeter is connected across series combination of a dc voltage source $V_1 = 2$ V and ac voltage source $V_2(t) = 3\sin(4t)$ V. The meter reads

(a) 2 V (b) 5 V
(c) $(2 + 3\sqrt{2})$ V (d) $(\sqrt{17}/2)$ V
[Gate 2005]

12. A digital-to-analog converter with a full scale output voltage of 3.5 V has a resolution close to 14 mV. Its bit size is

(a) 4 (b) 8
(c) 16 (d) 32 [Gate 2005]

13. Two wattmeters, which are connected to measure the total power on a three-phase system supplying a balanced load, read 10.5 kW and – 2.5 kW respectively. The total power and the power factor respectively are

(a) 13.0 kW, 0.334 (b) 13.0 kW, 0.684
(c) 8.0 kW, 0.52 (d) 8.0 kW, 0.334
[Gate 2005]

14. The pressure coil of a dynamometer type wattmeter is

(a) Highly inductive (b) Highly resistive
(c) Purely resistive (d) Purely inductive
[Gate 2009]

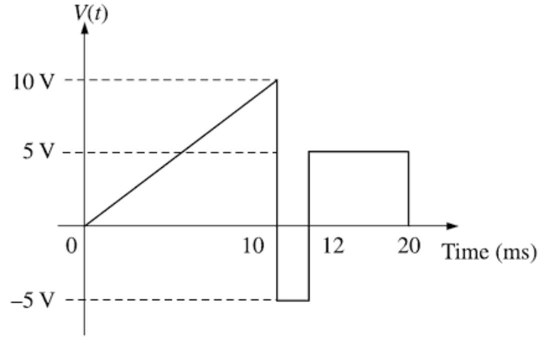
15. The two inputs of a CRO are fed with two stationary periodic signals. In the X–Y mode, the screen shows a figure which changes from ellipse to circle and back to ellipse with its major axis changing orientation slowly and repeatedly. The following inference can be made from this:

(a) The signals are not sinusoidal
(b) The amplitudes of the signals are very close nut not equal
(c) This signals are sinusoidal with their frequencies very close but not equal
(d) There is a constant but small phase difference between the signals [Gate 2009]

16. An average-reading digital multimeter reads 10 V when fed with a triangular wave, symmetric about the time-axis. For the same input an rms reading meter will read

(a) $\frac{20}{\sqrt{3}}$ V (b) $\frac{10}{\sqrt{3}}$ V
(c) $20\sqrt{3}$ V (d) $10\sqrt{3}$ V [Gate 2009]

17. A periodic voltage waveform observed on an oscilloscope across a load is shown in figure. A permanent magnet moving coil (PMMC) meter connected across the same load reads



(a) 4 V (b) 5 V
(c) 8 V (d) 10 V [Gate 2012]

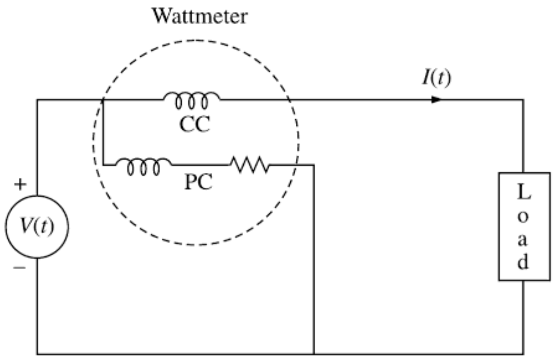
18. The bridge method commonly used for finding mutual inductance is
(a) Heaviside–Campbell bridge
(b) Schering bridge
(c) De Sauty bridge
(d) Wien bridge [Gate 2012]

19. For the circuit in figure, the voltage and current expressions are:

$V(t) = E_1 \sin(\omega t) + E_3 \sin(3\omega t)$ and

$I(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t - \phi_3) + I_5 \sin(5\omega t)$

The average power measured by the wattmeter is



(a) $\frac{1}{2} E_1 I_1 \cos\phi_1$

(b) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_1 I_3 \cos\phi_3 + E_1 I_5\right]$

(c) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_3 I_3 \cos\phi_3\right]$

(d) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_3 I_1 \cos\phi_1\right]$ [Gate 2012]

20. Consider the following statements:
(i) The compensating coil of a low power factor wattmeter compensates the effect of the impedance of the current coil.
(ii) The compensating coil of a low power factor wattmeter compensates the effect of the impedance of the voltage coil circuit.

(a) (i) is true but (ii) is false
(b) (i) is false but (ii) is true
(c) Both (i) and (ii) are true
(d) Both (i) and (ii) are false [Gate 2011]

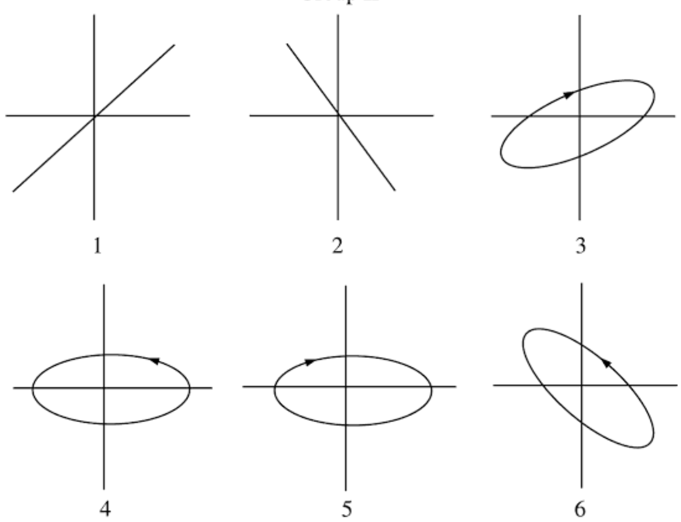
21. R_1 and R_4 are the opposite arms of a Wheatstone bridge as are R_3 and R_2. The source voltage is applied across R_1 and R_3. Under balanced conditions, which one of the following is true?
(a) $R_1 = R_3 R_4 / R_2$ (b) $R_1 = R_2 R_3 / R_4$
(c) $R_1 = R_2 R_4 / R_3$ (d) $R_1 = R_2 + R_3 + R_4$
[Gate 2006]

22. An ammeter has a current range of 0–5 A, and its internal resistance is 0.2 Ω. In order to change the range to 0–25 A, we need to add a resistance of
(a) 0.8 Ω in series with the meter
(b) 1.0 Ω in series with the meter
(c) 0.04 Ω in parallel with the meter
(d) 0.05 Ω in parallel with the meter [Gate 2010]

23. Group II represents the figures obtained on a CRO screen when the voltage signals $V_x = V_{xm} \sin \omega t$ and $V_y = V_{ym} \sin(\omega t + \phi)$ are given to its x and y plates respectively and ϕ is changed. Choose the correct value of ϕ from Group I to match with the corresponding figure of Group II.

Group I
P. $\phi = 0$
Q. $\phi = \pi/2$
R. $\pi < \phi < 3\pi/2$
S. $\phi = 3\pi/2$

Group II



(a) P – 1 Q – 3 R – 6 S – 5
(b) P – 2 Q – 6 R – 4 S – 5
(c) P – 2 Q – 3 R – 5 S – 4
(d) P – 1 Q – 5 R – 6 S – 4
[Gate 2003]

24. A dc potentiometer is designed to measure up to about 2 V with a slide wire of 800 mm. A standard cell of

6

Analog and Digital Electronics

6.1 SYLLABUS

Characteristics of diodes, BJT, FET; amplifiers-biasing, equivalent circuit and frequency response; oscillators and feedback amplifiers; operational amplifiers-characteristics and applications; simple active filters; VCOs and timers; combinational and sequential logic circuits; multiplexer, Schmitt trigger; multi-vibrators; sample and hold circuits; A/D and D/A converters; 8-bit microprocessor basics, architecture, programming and interfacing.

6.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**		
										Set 1	Set 2	Set 3
1 Mark Question	4	2	4	1	4	3	3	4	3	3	3	3
2 Marks Question	10	8	4	9	4	5	4	3	5	4	5	4
Total Marks	**24**	**18**	**12**	**19**	**12**	**13**	**11**	**10**	**13**	**11**	**13**	**11**

6.3 TOPICS TO BE FOCUSED

DIGITAL ELECTRONICS

Digital electronics involves circuits and systems in which there are only two possible states that are typically represented by voltage levels. Other circuit conditions, such as current levels, open or closed switches and ON or OFF lamps, can also represent the two states. In digital systems, the two states are used to represent numbers, symbols, alphabetic characters, and other types of information. In the two state number system, called binary, the two digits are 0 and 1. These binary digits are called *bits*.

Logic circuit is a digital circuit, a switching circuit, or any kind of two state circuits that duplicates mental processes. The gate is a digital circuit with one or more input voltages but only one output voltage. Logic gates are the building blocks of digital circuits. The OR, AND and NOT gates can be built using different types of switching elements.

Truth Table of 2-input OR Gate

A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table of 2-input AND Gate

A	*B*	*Y*
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table of NOT Gate

A	*Y*
0	1
1	0

Truth Table of 2-input NAND Gate

A	*B*	*Y*
0	0	1
0	1	1
1	0	1
1	1	0

Truth Table of 2-input NOR Gate

A	*B*	*Y*
0	0	1
0	1	0
1	0	0
1	1	0

Truth Table of 2-input XOR Gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table of 2-input XNOR Gate

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Boolean Addition

- 0 + 0 = 0
- 0 + 1 = 1
- 1 + 0 = 1
- 1 + 1 = 1

Boolean Multiplication

- 0 × 0 = 0
- 0 × 1 = 0
- 1 × 0 = 0
- 1 × 1 = 1

Rules of Boolean Algebra

- $A+0=A$
- $A+1=1$
- $A\times 0=0$
- $A\times 1=A$
- $A+A=A$
- $A+\overline{A}=1$
- $A\times A=A$
- $A\times\overline{A}=0$
- $\overline{\overline{A}}=A$

DeMorgan's Theorem

- $\overline{A.B}=\overline{A}+\overline{B}$
- $\overline{A+B}=\overline{A}\,\overline{B}$

Truth Table for Half Adder

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Truth Table for Half Subtractor

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Truth Table for S-R Flip-flop

S_n	R_n	Q_{n+1}	Q'_{n+1}
0	0	Q_n	Q'_n
0	1	0	1
1	0	1	0
1	1	?	?

Truth Table for JK flip-flop

J_n	K_n	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	Q'_n

Truth Table of D flip-flop

D_n	Q_{n+1}
0	0
1	1

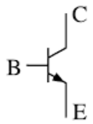
Truth Table of T flip-flop

T_n	Q_{n+1}
0	Q_n
1	Q'_n

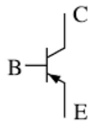
ANALOG ELECTRONICS

Bipolar Junction Transistor (BJT)

NPN transistor



PNP transistor



For NPN

$$I_E = I_C + I_B$$

Total power in the two junctions

$$P_T = V_{BE}I_B + V_{CE}I_C$$

where,

I_E = Emitter current
I_C = Collector current
I_B = Base current
V_{BE} = Base emitter voltage
VCE = Collector emitter voltage

- Forward current gain

$$\alpha = \frac{I_C}{I_E},\ \alpha \text{ varies from 0.95 to 0.99}$$

- Current gain

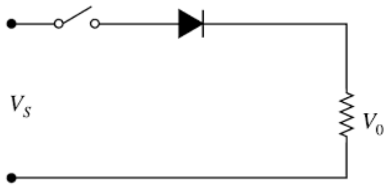
$$\beta = \frac{I_C}{I_B}$$

- Relation between α and β

$$\beta = \frac{\alpha}{1-\alpha}$$

Diode Circuit with DC Source

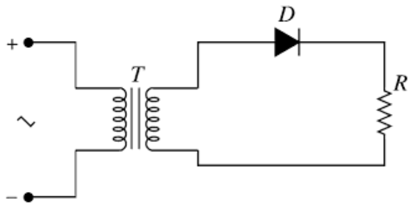
Resistive load



where,

V_S = Source voltage
V_0 = Load voltage
R = Load resistance

Single Phase Half Wave Rectifier



- Average output voltage and current

$$V_{\text{dc}} = \frac{V_m}{\Pi}$$

$$I_{\text{dc}} = \frac{V_m}{\Pi R}$$

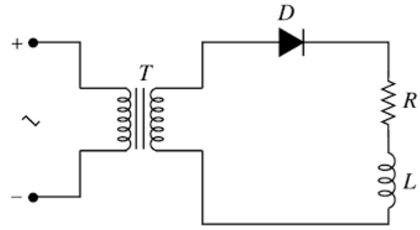
- The rms value of output voltage (V_{rms}) and current (I_{rms})

$$V_{\text{rms}} = \frac{V_m}{2}$$

$$I_{\text{rms}} = \frac{V_m}{2R}$$

- Efficiency (η) in percentage = 40.5%
- Form factor (FF) of output voltage = 1.57
- Ripple factor (RF) of output voltage = 1.21
- Transformer utilization factor (TUF) = 0.286
- Peak inverse voltage (PIV) = V_m
- Harmonic factor (HF) = $\left[\left(\frac{I_2}{I_{2\text{Fundamental}}}\right)^2 - 1\right]^{\frac{1}{2}}$
- Displacement factor (DF) = $\cos\phi$
- Crest factor (CF) = $\frac{I_2(\text{maximum})}{I_2}$

Single Phase Half-wave Rectifier with *RL* Load



- Average output voltage

$$V_{\text{dc}} = \frac{V_m}{2\Pi}[1-\cos(\Pi+u)]$$

u being the additional conduction angle

- Average load current

$$I_{\text{dc}} = \frac{V_m}{2\Pi R}[1-\cos(\Pi+u)]$$

- The rms value of load voltage

$$V_{\text{rms}}(\text{dc}) = \frac{V_m}{\sqrt{2\Pi}}\left[\beta - \frac{\sin(2\beta)}{2}\right]^{\frac{1}{2}}, \beta = (\Pi + \mu)$$

β being the additional conduction angle.

Single Phase Full Wave Diode Rectifier

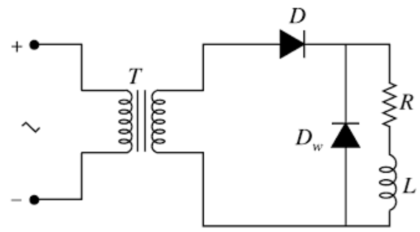
- Average output voltage

$$V_{\text{dc}} = \frac{2V_m}{\Pi}$$

- The rms value of output voltage

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

Addition of Free-wheeling Diode (D_w) in a Half Wave Rectifier



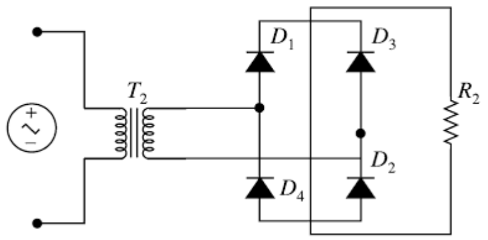
- The voltage across the main diode (D) during the free-wheeling action is given as

$$V_{\text{dc}(av)} = \frac{V}{\sqrt{2\Pi}}[\cos\beta - 1]$$

- The rms value of the diode voltage is given by

$$V_{\text{dc(rms)}} = \frac{V}{\sqrt{2\Pi}}\left[2\Pi - \beta + \frac{\sin 2\beta}{2}\right]^{\frac{1}{2}}$$

Single Phase Full Wave Rectifier using Bridge Rectifier



- Average output voltage and current

$$V_{dc} = \frac{2V_m}{\Pi} = 0.637V_m$$

$$I_{dc} = \frac{2V_m}{\Pi R} = \frac{0.637V_m}{R}$$

- The rms value of output voltage (V_{rms}) and output current (I_{rms})

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

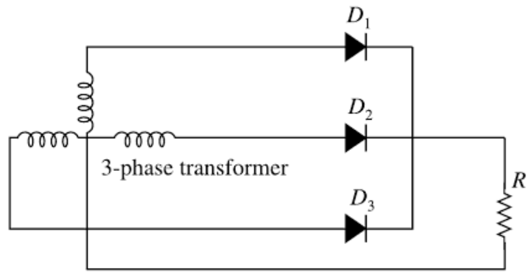
$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707V_m}{R}$$

- Efficiency (η) in percentage = 81.3%
- Form factor (FF) of output voltage = 1.11
- Ripple factor (RF) of output voltage = 0.482
- Transformer utilization factor (TUF) = 0.573
- Peak inverse voltage (PIV) = V_m
- For centre tapped connection PIV = 2 V_m

Comparison of Various Single Phase Diode Rectifiers

Parameter	Half Wave	Full Wave	
		Centre-tap	Bridge
The dc output voltage	$\frac{V_m}{\Pi}$	$\frac{2V_m}{\Pi}$	$\frac{2V_m}{\Pi}$
The rms output voltage	$\frac{V_m}{2}$	$\frac{V_m}{\sqrt{2}}$	$\frac{V_m}{\sqrt{2}}$
Ripple voltage	0.3856 V_m	0.3077 V_m	0.3077 V_m
Number of diode	1	2	2
Ripple factor voltage	1.211	0.482	0.482
Rectifier efficiency	40.53%	81.06%	81.06%
TUF	0.2865	0.672	0.8106
Peak inverse voltage (PIV)	V_m	$2V_m$	V_m
Crest factor (CF)	2	$\sqrt{2}$	$\sqrt{2}$
Ripple frequency	f	$2f$	$2f$

Three-phase Half Wave Diode Rectifier



- Average output voltage and current

$$V_{dc} = 1.17V_m$$

$$I_{dc} = \frac{0.827V_m}{R}$$

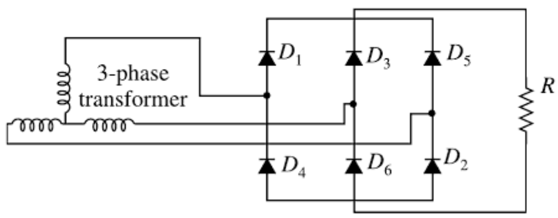
- The rms value of output voltage (V_{rms}) and output current (I_{rms})

$$V_{rms} = 0.84V_m$$

$$I_{rms} = \frac{0.84V_m}{R}$$

- Efficiency (η) in percentage = 96.5%
- Ripple factor (RF) of output voltage = 0.185
- Transformer utilization factor (TUF) = 0.573
- Peak Inverse voltage (PIV) = 2.09 V_{dc}

Three-phase Full Wave Bridge Rectifier



- Average output voltage and current

$$V_{dc} = 1.654V_m$$

$$I_{dc} = \frac{1.6547V_m}{R}$$

- The rms value of output voltage (V_{rms}) and output current (I_{rms})

$$V_{rms} = 1.6554V_m$$

$$I_{rms} = \frac{1.6554V_m}{R}$$

- Efficiency (η) in percentage = 99.8%
- Form factor (FF) of output voltage = 1.0008
- Ripple factor (RF) of output voltage = 0.04
- Transformer utilization factor (TUF) = 0.954
- Peak inverse voltage (PIV) = $\sqrt{3}$ V_m

Comparison of Various Three-Phase Diode Rectifiers

Parameter	3-pulse rectifier	6-pulse rectifier M-6 type	6-pulse rectifier B-6 type	12-pulse rectifier
The dc output voltage	$\frac{3V_m}{2\Pi}$	$\frac{3V_m}{2\Pi}$	$\frac{3\sqrt{3}V_m}{2\Pi}$	$191V_m$
The rms output voltage	$0.4854\ V_m$	$0.55185\ V_m$	$0.9558\ V_m$	$1.9101V_m$
Ripple voltage (V_r)	$0.0872\ V_m$	$0.02356\ V_m$	$0.0408\ V_m$	$0.019545\ V_m$
Ripple factor voltage	0.1826	0.043	0.0427	0.01023
Rectifier efficiency, η	96.765%	99.82%	99.82%	–
Transformer utilization factor (TUF)	0.6644	0.551	0.9541	–
Peak inverse voltage (PIV)	V_m	$1.155V_m$	V_m	–
Form factor (FF)	1.0165	1.0009	1.0009	1.00005

6.4 IMPORTANT POINTS TO REMEMBER

1. A transistor has two P-N junctions.
2. A transistor is operating in the active region. Under this condition, emitter-base junction is forward biased and collector-base junction is reverse biased.
3. In a transistor, base is very thin.
4. The arrowhead on the transistor symbol always points in the direction of hole flow in the emitter region.
5. The leakage currents in a transistor are due to minority carriers.
6. In a transistor, $I_E = I_B + I_C$ irrespective of transistor configuration.
7. If emitter current in a transistor is 1 mA, then collector current is nearly 1 mA.
8. The most commonly used transistor circuit arrangement is CE circuit.
9. The input resistance of a transistor is much less than its output resistance.
10. The function of a transistor is to do amplification.
11. The value of α is less than unity.
12. The base of a transistor is lightly doped.
13. For proper operation of a transistor, base-emitter junction should be forward biased and collector-base junction reverse biased.
14. Common collector (CC) arrangement is generally used for impedance matching.
15. Feedback means transferring a portion of energy from output of the same device to its input.
16. If the feedback voltage is in phase with the signal voltage, then it is called positive feedback.
17. The operation of power amplifier is non-linear.
18. Emitter follower is used in electronic instruments because its output impedance is low and input impedance is high.
19. Emitter follower is used for impedance matching.
20. The current gain of emitter follower is less than unity.
21. A Darlington pair has very high input impedance.
22. An oscillator has high efficiency.
23. An oscillator is a non-rotating device.
24. An oscillator produces undamped oscillations.
25. An oscillator employs positive feedback.
26. Heat sink is used to cool down power transistors.
27. Negative feedback reduces distortion in amplifiers.
28. Negative feedback reduces gain.
29. The gain of an amplifier with negative feedback is less than without feedback.
30. With negative feedback, output impedance of amplifier is decreased.
31. With negative feedback, bandwidth of the amplifier is increased.
32. The value of feedback fraction is less than unity.
33. Feedback circuit is essentially a potential divider.
34. A power transistor differs from other transistors in size.
35. Wien bridge uses both positive and negative feedback.
36. The negative feedback increases input impedance of the amplifier.
37. The 1's complement of binary number 11010 is 00101.
38. A byte consists of 8 bits.
39. $\overline{A} . \overline{B} . \overline{C} . \overline{D} = E$ represents NOR gate.
40. The data word of 4 bits is known as nibble.
41. The number of nibbles in a byte is 2.
42. The statement for commutative law is $A + B = B + A$.
43. NAND gate is a universal gate.
44. The radix of the binary number is 2.
45. The radix of the octal system is 8.

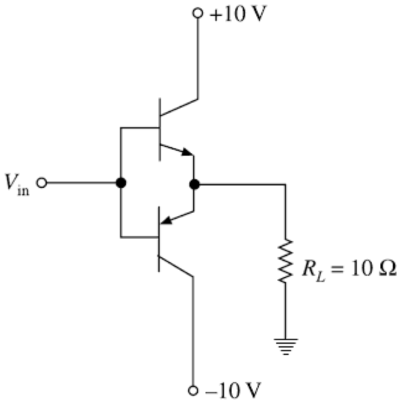
46. ASCII code is used as an alphanumeric code.
47. NOR and NAND gates can realize any logic function.
48. The base of the binary number is 2.
49. The base of the decimal number is 10.
50. The radix of the hexadecimal system is 16.
51. $A\overline{B} + \overline{A}\,B = F$ represents an XOR gate.
52. The number of binary bits required to represent a digit of octal number is 3.
53. The BCD equivalent of decimal number $(83)_{10}$ is 1000 0011.
54. The excess 3 code is also known as self-complementing code.
55. The decimal 7 in excess 3 code is given by 1010.
56. The number of binary bits required to represent a digit of hexadecimal number is 4.
57. $\overline{A} + \overline{B} + \overline{C} + \overline{D} = F$ represents a NAND gate.
58. A multiplexer is also known as data selector.
59. A sequential circuit consists of combinational circuits and memory.
60. Multiplexer is not a sequential circuit.
61. NOR function can be realized from NAND gates.
62. Both the octal and hexadecimal systems can be used as "shorthands" for displaying binary numbers.
63. The output of a combinational circuit depends upon present input only.
64. NOR gate is known as a universal gate because all logical functions can be realized using NOR gates.
65. The output of a sequential circuit depends upon both present and past inputs.
66. The range of numbers which can be stored in an 8-bit register (with left bit for sign) is –128 to 127.
67. A demultiplexer is used to route the data from single input to one of many outputs.
68. The minimum number of NAND gates required to realize an exclusive OR gate is 4.
69. The minimum number of NAND gates required to realize an OR gate is 3.
70. The minimum number of NAND gates required to realize an AND gate is 2.
71. The minimum number of NAND gates required to realize an XNOR gate is 5.
72. The minimum number of NAND gates required to realize a NOR gate is 4.
73. The minimum number of NAND gates required to realize a NOT gate is 1.
74. The minimum number of NOR gates required to realize a NAND gate is 4.
75. The minimum number of NOR gates required to realize an OR gate is 2.
76. The minimum number of NOR gates required to realize an AND gate is 3.
77. The minimum number of NOR gates required to realize a NOT gate is 1.
78. The minimum number of NOR gates required to realize an XOR gate is 5.
79. The minimum number of NOR gates required to realize an XNOR gate is 4.
80. A 6-variable Boolean function has a total maximum number of terms equal to 64.
81. The number of control lines required for a 1 to 8 MUX is 3.
82. $F = AB + \overline{A\,B}$ represents XNOR gate.
83. A multiplexer has only one output line.
84. A circuit that transforms decimal number to binary code is encoder.
85. A flip-flop can store 1 bit of data.
86. The number of flip-flops needed to divide the input frequency by 32 is 5.
87. The functional difference between SR and JK flip-flop is that JK flip-flop can accept both inputs one.
88. T flip-flop is commonly used as digital counter and frequency divider.
89. Master-slave JK flip-flop is free from race around problem.
90. The flip-flops which operate in synchronism with external clock pulses are known as synchronous flip-flops.
91. Asynchronous sequential circuits are difficult to design because they have stability problem.
92. A SR flip-flop does not allow the inputs such that $S = 1$, $R = 1$.
93. The fan-out of TTL logic gate is about 10.
94. The clear data and present input of the JK flip-flop are known as either direct inputs or synchronous inputs.
95. The m bit parallel adder consists of m full adders.
96. The circuit used for parallel to serial conversion of data is known as multiplexer.
97. The number of binaries required in a decade counter is 4.
98. The minimum number of binaries required in a mod 100 counter is 7.
99. Synchronous counters have the highest speed.
100. A flip-flop has two outputs which are always complementary.

101. When a flip-flop is set, its output will be $Q = 1, \overline{Q} = 0$.
102. A flip-flop may be built with NAND gates.
103. Shift register most commonly use D flip-flop.
104. Flip-flops can be used to make registers, bounce elimination switches and latches.
105. The minimum number of flip-flops required for a synchronous decade counter is 4.
106. If two Mod-3 counters are cascaded, the circuit will behave as a Mod-9 counter.
107. A decade ripple counter requires a minimum of 4.
108. Design of a sequential circuit requires the use of state tables/state diagram.
109. Sequential circuits contain at least one memory element.
110. Sequential circuits as compared to combination circuits are slower in speed.
111. An AND gate is equivalent to a serial switching circuit.
112. The NAND-NAND realization is equivalent to AND-OR realization.
113. The only function of a NOT gate is invert an input.
114. A half adder circuit consists of one XOR gate and one AND gate.
115. In a clocked SR flip-flop, S point is joined to R point through NOT gate, the circuit will become D flip-flop.
116. Master-slave configuration is used in flip-flop to eliminate race around condition.
117. In an asynchronous sequential circuit, all data transfers are unclocked.
118. To construct a state diagram of a sequential circuit, it is necessary to know the number of states and the number of input conditions.
119. In sequential circuits, memory elements are bistable.
120. For a flip-flop with preset and clear, while presetting, clear is disabled and while clearing, preset is disabled
121. In constructing a MOD-10 counter, we require 4 flip-flops and by pass 6 states.
122. A serial adder is an example of combinational logic element.
123. The most suitable gate for comparing two bits is XOR gate.
124. Minimization of logical expressions, while designing digital systems, helps in reducing power, cost and space required.
125. Asynchronous sequential circuit consists of combinational circuit and number of delay elements.
126. A PLA is LSI device.
127. The capacity or size of a PLA is specified by the number of inputs, number of product terms and number of outputs.
128. A PLA consists of AND matrix, OR matrix and invert/non-invert matrix.
129. A PLA can be used to realize combinational logic.
130. A field programmable logic array can be programmed by a user only once.
131. Fusible link is associated with PROM.
132. A PLA is mask programmable.
133. CMOS logic consists of P-channel and N-channel MOS devices.
134. If a logic circuit has a fan-out of 4, then the circuit has 4 outputs.
135. Recommended fan out for TTL gate is 10.
136. ECL is a non-saturated bipolar logic family.
137. RTL consists of resistors, diodes and bipolar junction transistors.
138. The main disadvantage of TTL with totem-pole output is that wire ANDing operation is not allowed.
139. Standard TTL is a current sink logic.
140. Tri state logic has three output states; 0, 1 and high impedance.
141. Tri state logic is used for bus oriented system.
142. Schottky TTL is a non-saturated bipolar logic.
143. MOS logic circuit consists of only MOS devices.
144. Gray code is also referred to as a reflected code.
145. The octal number system is composed of eight digits.
146. Two major integrated circuit types are TTL and CMOS.
147. A multiplexer with 4-bit data select input is a 16:1 multiplexer.
148. In a full adder, the sum is obtained by an exclusive OR function.
149. CMOS has excellent noise immunity.
150. The BiCMOS logic is mainly available in 8-bit configuration.
151. An octet eliminates three variables and their complements.
152. Adjacent cells in a Karnaugh map must not differ by more than one variable.
153. In Karnaugh map, the permitted size of loops is any power of two.
154. NAND gates can be replaced by bubbled OR gates.
155. In a hexadecimal system, letters are chosen to represent values greater than 9.

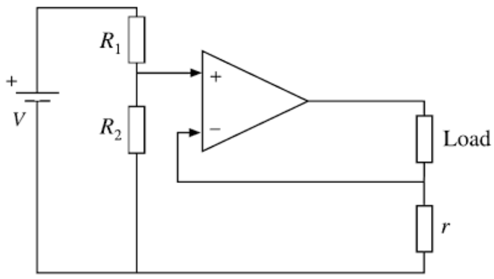
6.5 REFERENCES

Boylestad, Robert L., and Nashelsky, Louis, *Electronic Devices and Circuits Theory*, 10th ed., Pearson, India, 2009.

Donald, Leach P., *Digital Principles and Applications*, 7th ed., Tata McGraw-Hill, India, 2010.



32. The circuit shown in figure is



(a) A voltage source with voltage $\dfrac{rV}{R_1 \| R_2}$

(b) A voltage source with voltage $\dfrac{r \| R_2}{R_1} V$

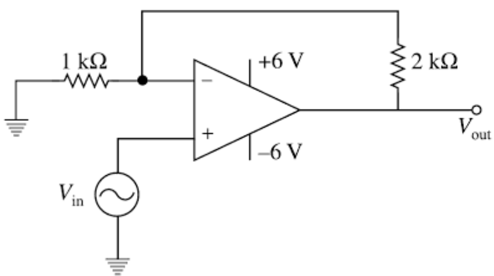
(c) A current source with current $\dfrac{r \| R_2}{R_1 + R_2} \cdot \dfrac{V}{r}$

(d) A current source with current $\dfrac{R_2}{R_1 + R_2} \cdot \dfrac{V}{r}$

[Gate 2007]

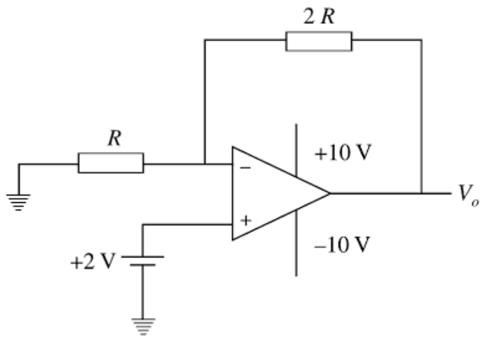
33. The complete set of only those logic gates designated as universal gates is
(a) NOT, OR and AND gates
(b) XNOR, NOR and NAND gates
(c) NOR and NAND gates
(d) XOR, NOR and NAND gates [Gate 2009]

34. The nature of feedback in the op-amp circuit shown is



(a) Current-current feedback
(b) Voltage-voltage feedback
(c) Current-voltage feedback
(d) Voltage-current feedback [Gate 2009]

35. Given that op-amp is ideal, the output voltage V_o is



(a) 4 V (b) 6 V
(c) 7.5 V (d) 12.12 V [Gate 2010]

36. A low-pass filter with a cut-off frequency of 30 Hz is cascaded with a high-pass filter with a cut-off frequency of 20 Hz. The resultant system of filters will function as
(a) An all-pass filter
(b) An all-stop filter
(c) A band-stop (band-reject) filter
(d) A band-pass filter [Gate 2011]

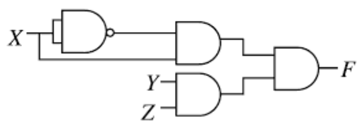
Statement for Linked Answer Questions: 37 and 38

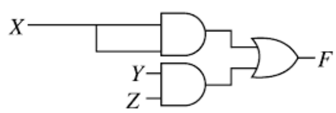
The following Karnaugh map represents a function F.

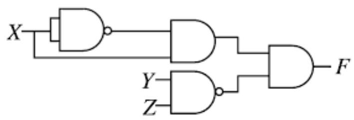
X \ YZ	00	01	11	10
0	1	1	1	0
1	0	0	1	0

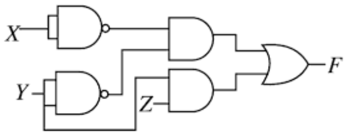
37. A minimized form of the function F is
(a) $F = \overline{X}Y + YZ$ (b) $F = \overline{X}\,\overline{Y} + YZ$
(c) $F = \overline{X}\,\overline{Y} + Y\overline{Z}$ (d) $F = \overline{X}\,\overline{Y} + \overline{Y}Z$
[Gate 2010]

38. Which of the following circuits is a realization of the above function F?

(a)


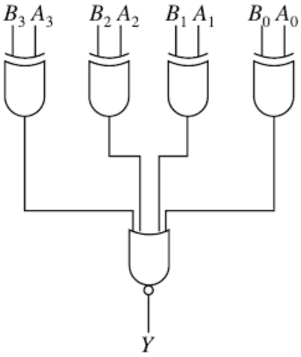
(b)


(c)


(d) 

[Gate 2010]

39. A digital circuit which compares two numbers, $A_3A_2A_1A_0$, $B_3B_2B_1B_0$ is shown in figure. To get output $Y = 0$, choose one pair of correct input numbers.



(a) 1010, 1010 (b) 0101, 0101
(c) 0010, 0010 (d) 1010, 1011

[Gate 2004]

40. Transformer and emitter follower can both be used for impedance matching at the output of an audio amplifier. The basic relationship between the input power P_{in} and output power P_{out} in both the cases is

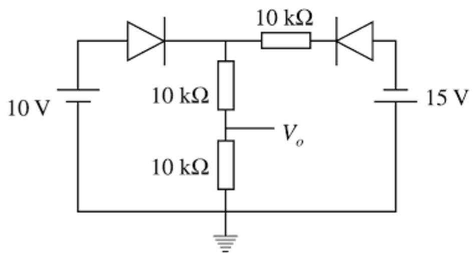
(a) $P_{in} = P_{out}$ for both transformer and emitter follower
(b) $P_{in} > P_{out}$ for both transformer and emitter follower
(c) $P_{in} < P_{out}$ for transformer and $P_{in} = P_{out}$ for emitter follower
(d) $P_{in} = P_{out}$ for transformer and $P_{in} < P_{out}$ for emitter follower [Gate 2009]

41. In an 8085 microprocessor, the contents of the accumulator, after the following instructions are executed will become

XRA A
MVI B, F0H
SUB B

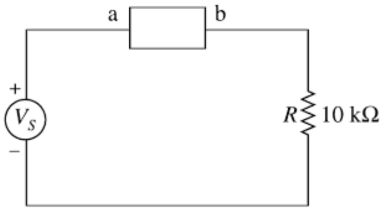
(a) 01 H (b) 0F H
(c) F0 H (d) 10 H [Gate 2009]

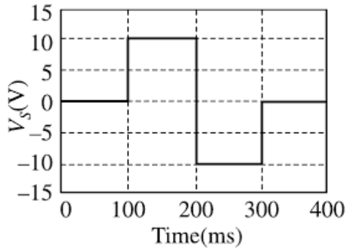
42. Assuming that the diodes in the given circuit are ideal, the voltage V_o is

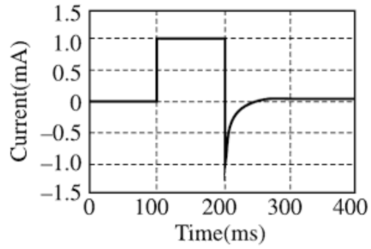


(a) 4 V (b) 5 V
(c) 7.5 V (d) 12.12 V [Gate 2010]

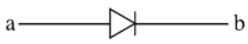
43. The following circuit has a source voltage V_S as shown in figure. The current through the circuit is also shown.

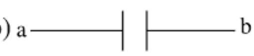


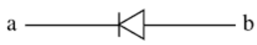


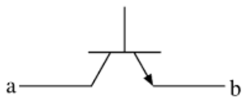


The element connected between points a and b could be

(a) 

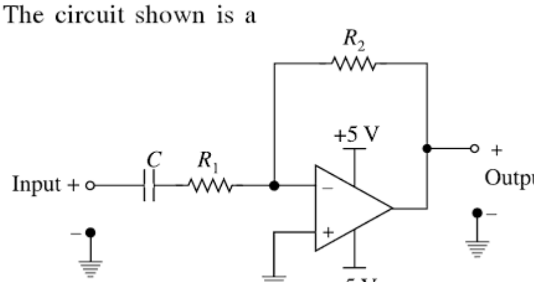
(b) 

(c) 

(d) 

[Gate 2009]

44. The circuit shown is a



(a) Low-pass filter with $f_{3dB} = \dfrac{1}{(R_1 + R_2)C}$ rad/s

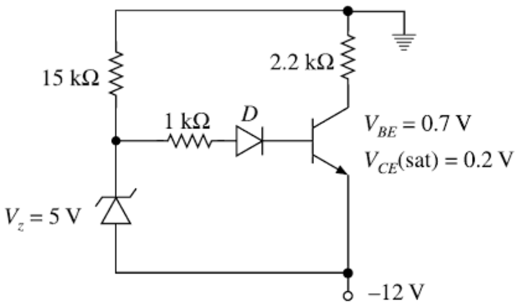
(b) High-pass filter with $f_{3dB} = \dfrac{1}{R_1C}$ rad/s

(c) Low-pass filter with $f_{3dB} = \dfrac{1}{R_1C}$ rad/s

(d) High-pass filter with $f_{3\text{dB}} = \dfrac{1}{(R_1 + R_2)C}$ rad/s

[Gate 2012]

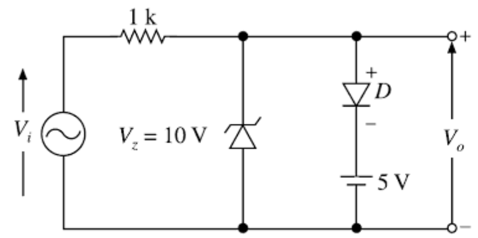
45. The transistor used in the circuit shown below has a β of 30 and I_{CBO} is negligible.



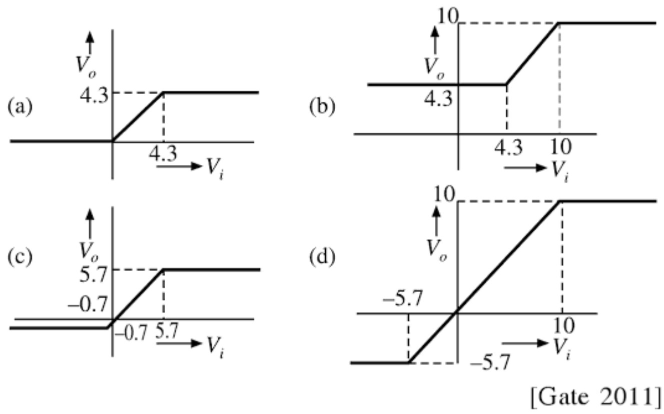
If the forward voltage drop of diode is 0.7 V, then the current through collector will be

(a) 168 mA (b) 108 mA
(c) 20.54 mA (d) 5.36 mA [Gate 2011]

46. A clipper circuit is shown as



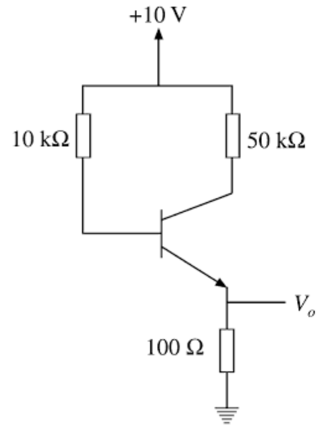
Assuming forward voltage drops of the diodes to be 0.7 V, the input-output transfer characteristics of the circuit is



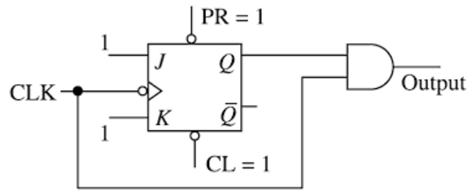
[Gate 2011]

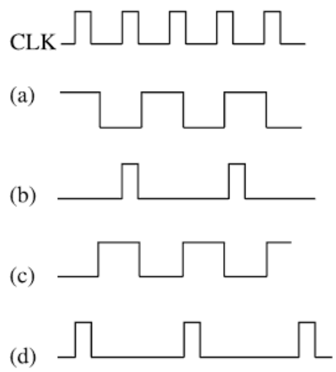
47. The transistor circuit shown in figure uses a silicon transistor with $V_{BE} = 0.7$ V, $I_C \approx I_E$ and a dc current gain of 100. The value of V_o is

(a) 4.65 V
(b) 5 V
(c) 6.3 V
(d) 7.23 V [Gate 2010]



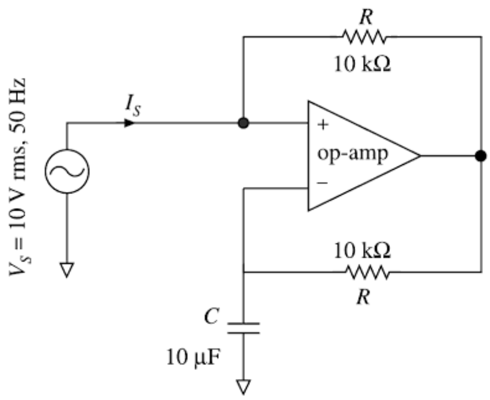
48. The digital circuit shown in figure generates a modified clock pulse at the output. Choose the correct output waveform from the options given below.





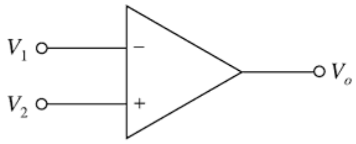
[Gate 2004]

49. The following circuit has R = 10 kΩ, C = 10 μF. The input voltage is a sinusoidal at 50 Hz with an rms value of 10 V. Under ideal conditions, the current I_S from the source is



(a) 1.25 V (b) 1.35 V
(c) 2.50 V (d) 5.00 V [Gate 2004]

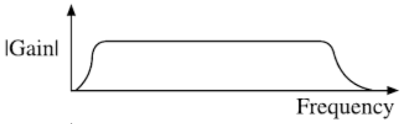
56. The voltage comparator shown in figure can be used in the analog-to-digital conversion as

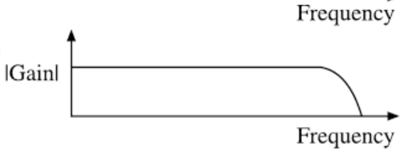


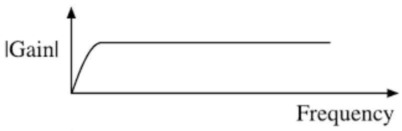
(a) 1-bit quantizer (b) 2-bit quantizer
(c) 4-bit quantizer (d) 8-bit quantizer
[Gate 2004]

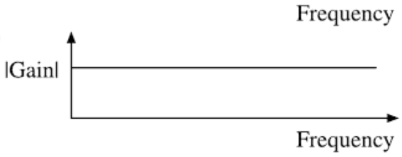
57. The 8085 assembly language instruction that stores the content of H and L registers into the memory locations 2050H and 2051H, respectively is
(a) SPHL 2050H
(b) SPHL 2051H
(c) SHLD 2050H
(d) STAX 2050H [Gate 2005]

58. The typical frequency response of a two-stage direct coupled voltage amplifier is shown in

(a)


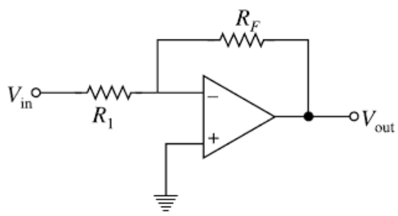
(b)


(c)


(d)


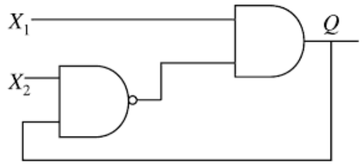
[Gate 2005]

59. Consider the inverting amplifier, using an ideal operational amplifier shown in figure. The designer wishes to realize the input resistance seen by the small-signal source to be as large as possible, while keeping the voltage gain between –10 and –25. The upper limit on R_F is 1 MΩ. The value of R_1 should be



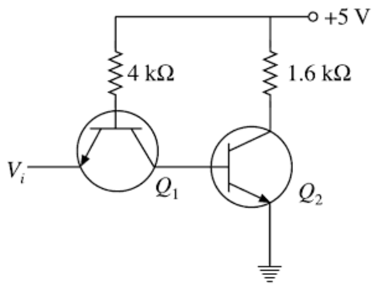
(a) Infinity (b) 1 MΩ
(c) 100 kΩ (d) 40 kΩ [Gate 2005]

60. In figure, as long as $X_1 = 1$ and $X_2 = 1$, the output Q remains



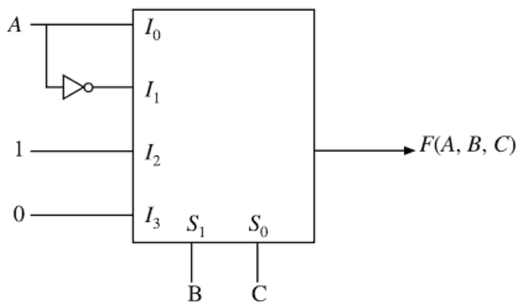
(a) At 1
(b) At 0
(c) At its initial value
(d) Unstable [Gate 2005]

61. A TTL NOT gate circuit is shown in figure. Assuming $V_{BE} = 0.7$ V of both the transistors, if $V_i = 3.0$ V, then the states of the two transistors will be



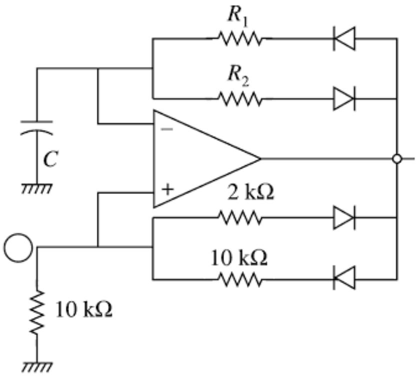
(a) Q_1 ON and Q_2 OFF
(b) Q_1 reverse ON and Q_2 OFF
(c) Q_1 reverse ON and Q_2 ON
(d) Q_1 OFF and Q_2 reverse ON [Gate 2006]

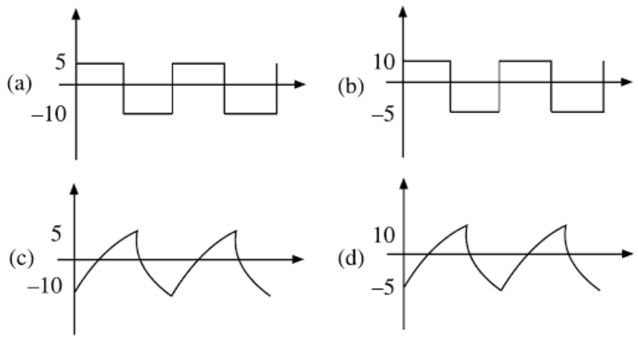
62. A (4 × 1) MUX is used to implement a 3-input Boolean function as shown in figure. The Boolean function $F(A, B, C)$ implemented is



(a) $F(A, B, C) = \Sigma(1, 2, 4, 6)$
(b) $F(A, B, C) = \Sigma(1, 2, 6)$
(c) $F(A, B, C) = \Sigma(2, 4, 5, 6)$
(d) $F(A, B, C) = \Sigma(1, 5, 6)$ [Gate 2006]

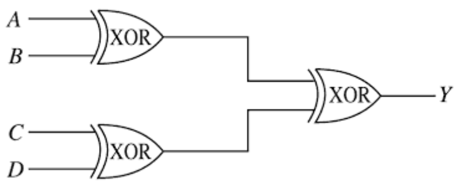
63. A relaxation oscillator is made using op-amp as shown in figure. The supply voltage of the op-amp is ±12 V. The voltage waveform at point P will be





[Gate 2006]

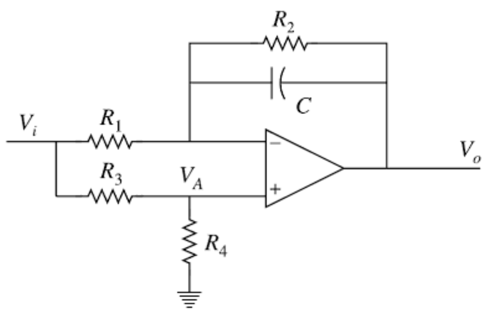
64. A, B, C and D are input and Y is the output bit in the XOR gate circuit of the figure below. Which of the following statements about the sum S of A, B, C, D and Y is correct?



(a) S is always with zero or odd
(b) S is always either zero or even
(c) $S = 1$ only if the sum of A, B, C and D is even
(d) $S = 1$ only if the sum of A, B, C and D is odd

[Gate 2007]

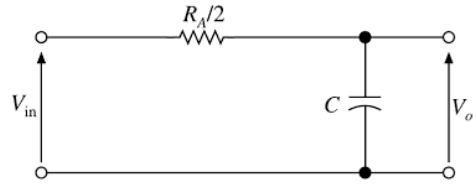
Statement for Linked Answer Questions: 65 and 66
A general filter circuit is shown in figure.

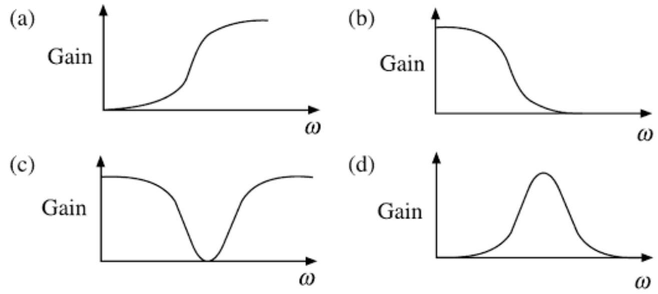


65. If $R_1 = R_2 = R_A$ and $R_3 = R_4 = R_B$, the circuit acts as a
(a) All-pass filter (b) Band-pass filter
(c) High-pass filter (d) Low-pass filter

[Gate 2008]

66. The output of the filter in Q. 65 is given to the circuit shown in figure. The gain vs frequency characteristic of the output (V_o) will be





[Gate 2008]

67. When a "CALL Addr" instruction is executed, the CPU carries out the following sequential operations internally.

Note:
(R) means content of register R
((R)) means content of memory location pointed to by R
PC means program counter
SP means stack pointer

(a) (SP) incremented
(PC) ← Addr
((SP)) ← (PC)
(b) (PC) ← Addr
((SP)) ← (PC)
(SP) incremented
(c) (PC) ← Addr
(SP) incremented
((SP)) ← (PC)
(d) ((SP)) ← (PC)
(SP) incremented
(PC) ← Addr

[Gate 2010]

68. An XY flip-flop, whose characteristic table is given below is to be implemented using a JK flip-flop

X	Y	Q_{n+1}
0	0	1
0	1	Q_n
1	0	$\overline{Q_n}$
1	1	0

This can be done by making

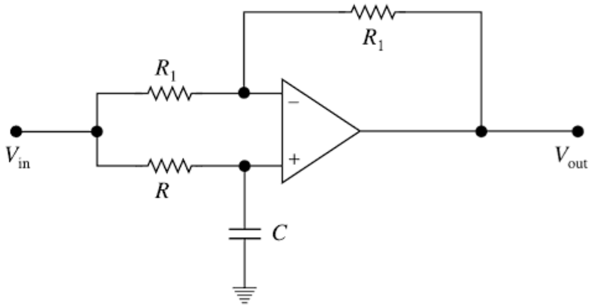
(a) $J = X$, $K = \overline{Y}$

(b) $J = \overline{X}$, $K = Y$

(c) $J = Y$, $K = \overline{X}$

(d) $J = \overline{Y}$, $K = X$ [Gate 2003]

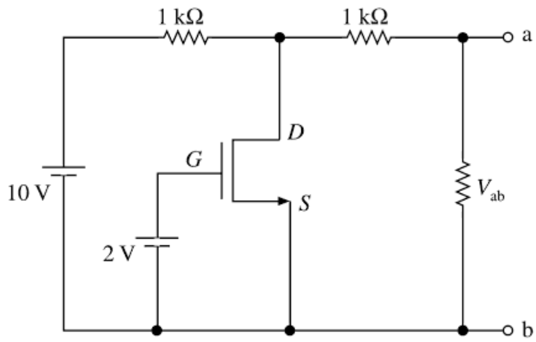
69. For the circuit of figure with an ideal operational amplifier, the maximum phase shift of the output V_{out} with reference to the input V_{in} is



(a) 0° (b) –90°

(c) +90° (d) ±180° [Gate 2003]

70. Assume that the N-channel MOSFET shown in figure is ideal, and that its threshold voltage is + 1.0 V. The voltage V_{ab} between nodes a and b is



(a) 5 V (b) 2 V

(c) 1 V (d) 0 V [Gate 2005]

Solutions

1. Ans: (a)

Hint: $Y = \left(\overline{A}BC + D\right)\left(\overline{A}D + \overline{B}\overline{C}\right)$

$$= \overline{A}BCD + 0 + \overline{A}D + \overline{B}\,\overline{C}D$$

$$= \overline{A}D\left(BC + 1\right) + \overline{B}\,\overline{C}D$$

$$= \overline{A}D + \overline{B}\,\overline{C}D$$

2. Ans: (c)

Hint: If D_1 is ON, Kirchhoff's law is not satisfied.

∴ D_1 is OFF and D_2 is ON

∴ The current in diode D_1 is 0 mA.

3. Ans: (a)

Hint: For both states (0, 1), the system is stable. Therefore, it is bistable multi-vibrator.

4. Ans: (b)

Hint: Hexadecimal number (AB × CD)

$$\overbrace{1010}^{A}\overbrace{1011}^{B}\cdot\overbrace{1100}^{C}\overbrace{1101}^{D}$$

$$\underbrace{010}_{2}\underbrace{101}_{5}\underbrace{011}_{3}\cdot\underbrace{110}_{6}\underbrace{011}_{3}\underbrace{010}_{2}$$

∴ Equivalent octal number: $(253 \times 632)_8$

5. Ans: (a)

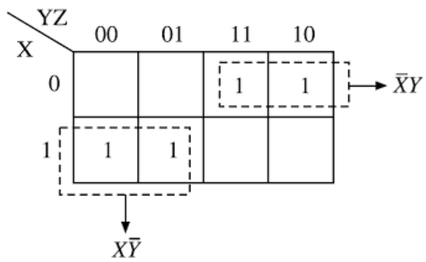
Hint:

X	$\overline{X}$	Y
1	0	1
0	1	1

$$Y = X\overline{\overline{X}} + \overline{X}\,\overline{X} = X + \overline{X} = 1$$

6. Ans: (a)

Hint: $f(X, Y, Z) = \Sigma(2, 3, 4, 5)$



∴ $$f\left(X,Y,Z\right) = X\overline{Y} + \overline{X}Y$$

7. Ans: (d)

Hint: Applying KVL, we get

$$10 = 10^3 I + V$$

or $$10 = 10^3\left(\frac{V - 0.7}{500}\right) + V$$

or $$V = 3.8$$

or $$I = \frac{10 - 3.8}{10^{-3}} = 6.2 \text{ mA}$$

8. Ans: (a)

Hint: This is SR flip-flop circuit. The race around condition does not occur here. It occurs in JK flip-flop.

9. Ans: (b)

Hint:

A	*B*	Number of cases
01	00	1
10	00, 01	2
11	00, 01, 10	3

∴ Total number of cases = 1 + 2 + 3 = 6

10. Ans: (c)

Hint: This is the characteristics of a p-channel depletion mode device.

11. Ans: (d)

Hint: Program counter contains the memory address of the instruction that is to be executed next.

12. Ans: (b)

Hint:

$$\overline{X}Y\overline{Z}+\overline{X}\,\overline{Y}Z+XY\overline{Z}+X\overline{Y}Z+XYZ$$
$$=\left(X+\overline{X}\right)Y\overline{Z}+\left(X+\overline{X}\right)\overline{Y}Z+XYZ$$
$$=Y\overline{Z}+\overline{Y}Z+XYZ$$
$$=\overline{Y}Z+Y\left(\overline{Z}+XZ\right)$$
$$=\overline{Y}Z+Y\left(\overline{Z}+Z\right)\left(\overline{Z}+X\right)$$
$$=\overline{Y}Z+Y\left(\overline{Z}+X\right)$$
$$=Y\overline{Z}+\overline{Y}Z+XY$$

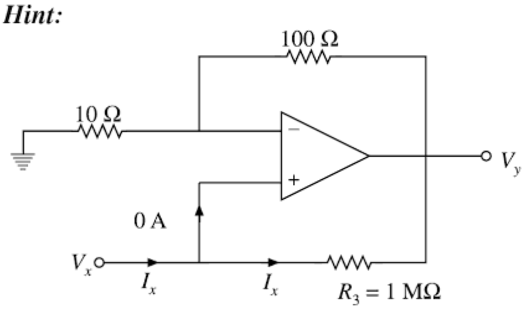
13. Ans: (c)

Hint: $$\Delta I_Z=\frac{\Delta V_Z}{R_Z}=\frac{3.5-3.3}{0.1\times10^3}=2\text{ mA}$$

14. Ans: (b)

15. Ans: (b)

Hint:



Here $$I_x=\frac{V_x-V_y}{R_3}$$

But $$V_y=\left(1+\frac{100\times1000}{10\times1000}\right)V_x=11V_x$$

$\therefore$ $$I_x=\frac{-10V_x}{R_3}$$

or $$\frac{V_x}{I_x}=-\frac{R_3}{10}=-\frac{1\times10^3}{10}\text{ k}\Omega=-100\text{ k}\Omega$$

16. Ans: (d)

Hint: Here V_S = +4 V

V_G = 0 V

V_T = –1 V

Therefore,

V_{SG} = 4 V

V_{SD} = $V_S - V_D$

= 4 – I_aR

Now, for linear region

$V_{SD} < (V_{SG} + V_T)$

or $4 - I_aR < (4 - 1)$

or $I_aR > 1$

or $1 \times 10^{-3} \times R > 1$

or $R > 1000\ \Omega$

17. Ans: (c)

Hint: Truth table of circuit

X	$Q(t)$	$Q(t+1)$
0	0	0
0	1	1
1	0	1
1	1	0

18. Ans: (a)

Hint: Number of instructions that are executed = 4

So, four instruction cycles.

19. Ans: (a)

Hint: Here, D_1 is forward biased and D_2 is reverse biased.

Now, current will flow through low resistance path.

$\therefore$ Value of current = 0 mA.

20. Ans: (b)

Hint: $$\beta=\frac{I_C}{I_B}$$

or $$I_B=\frac{1\times10^{-3}}{100}=10\ \mu\text{A}$$

21. Ans: (a)

Hint: Total size of the memory = 4 × 8 × 2^{12}

= 128 kilobits

= 16 kilobytes

22. Ans: (c)

Hint:

LXI	H, IFFE	[Load 1FFE in H-L memory]
MOV	B, M	[Move content of memory location (1FFE) to register B]
INR	L	[Increment the content of HL by 1; so, 1FFE becomes 1FFF]
MOV	A, M	[Move content of 1FFF memory location to accumulator]
ADD	B	[Add content of register B with accumulator, result stored in accumulator]
INR	L	[FF + 1 → 00; so, HL → 1F00]
MOV	M, A	[Move content of accumulator to memory location 1F00]
XRA	A	[Content of accumulator becomes zero]

23. Ans: (b)

Hint: Applying KVL, we get [base emitter loop]

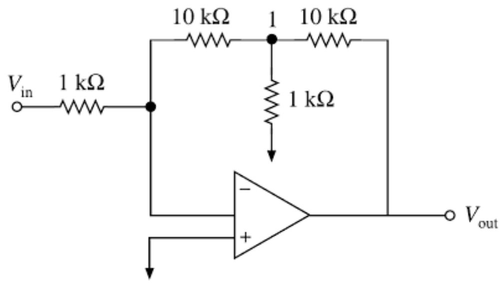
$$-4+33\times I_B+0.7+3.3\times100I_B=0$$

$\therefore$ $$I_B=\frac{4-0.7}{(33+330)}$$

$$\therefore \qquad I_C = \frac{3.3 \times 100}{(33+330)}$$
$$= \frac{3.3}{(0.33+3.3)}$$

24. Ans: (d)

Hint: Applying KCL at node 1, we get



$$\frac{V_1 - 0}{10} + \frac{V_1}{1} + \frac{V_1 - V_{out}}{10} = 0$$
$$12V_1 = V_{out} \qquad \text{(i)}$$

Again, applying KCL at inverting node, we get

$$\frac{V_{in} - 0}{1} = \frac{0 - V_1}{10}$$
$$V_1 = -V_{in} \times 10 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{V_{out}}{V_{in}} = -120$$

25. Ans: (a)

Hint: Truth table

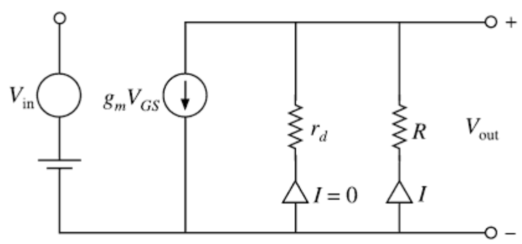
X_1	X_2	Q
1	0	1
0	0	1
0	1	0

26. Ans: (b)

Hint: $$g_m = \frac{\Delta I_{dc}}{\Delta V_{GS}}$$
$$= \frac{(4-3)\text{mA}}{(4-3)\text{V}} = 1 \text{ ms}$$

27. Ans: (d)

Hint:



Since $r_d >> R$, so, all current will pass through R

$$V_{out} = -g_m V_{GS} \cdot R$$
$$= -1 \times 10^{-3} \times 2 \times 10^{-3} \times 10 \times 10^{-3}$$
$$= -20 \text{ mV}$$
$$\therefore \text{ Voltage gain} = -\frac{20 \text{ mV}}{2 \text{ mV}} = -10$$

28. Ans: (a)

Hint: By inspection, D_1 and D_2 are in forward bias and D_3 is in reverse bias. But no current will flow through D_2, because current gets shortest path through diode D_1.

29. Ans: (d)

Hint: $Y_1 = \overline{X}_1$

Output: $Q = \overline{Y_1 + X_2}$
$$= \overline{(\overline{X}_1 + X_2)}$$
$$\Rightarrow \qquad Q = \overline{\overline{X_1}} \,.\, \overline{X_2} = X_1 \,.\, \overline{X_2}$$

30. Ans: (a)

Hint: The address of the memory location is stored in DE register pair. But the command INR M will increase the content of memory location M, and this command will execute only on HL pair. Hence, we have to exchange the address of memory location in HL pair from DE pair.

31. Ans: (d)

Hint: It is a class C amplifier, so efficiency must be high.

32. Ans: (d)

Hint: A current source

$$\text{Where current } I = \frac{\left(\dfrac{V \times R_2}{R_1 + R_2}\right)}{r} = \frac{R_2}{R_1 + R_2} \cdot \frac{V}{r}$$

33. Ans: (c)

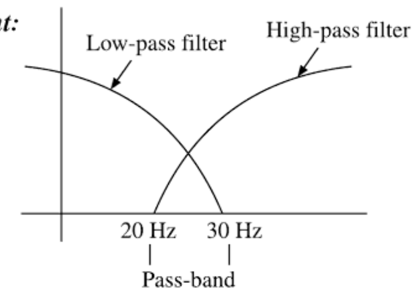
34. Ans: (b)

35. Ans: (b)

Hint: $$V_o = \left(1 + \frac{2R}{R}\right) V_i$$
$$= (1+2) \times 2 = 6 \text{ V}$$

36. Ans: (d)

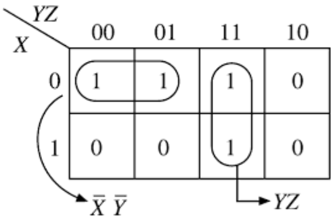
Hint:



It is a band-pass filter.

37. Ans: (b)

Hint:



$$F = \overline{X}\overline{Y} + YZ$$

38. Ans: (d)

Hint: For getting $\overline{X}$ and $\overline{Y}$, two NAND gates required.

Two AND gates required: one AND gate for inputs $\overline{X}$ and $\overline{Y}$ and another AND gates for inputs Y and Z.

One OR gate is used to sum them and generate the F.

39. Ans: (d)

40. Ans: (d)

Hint: For emitter follower

A_V is approximately equal to 1.

Now, $A_P = A_V A_I$

$\therefore$ A_P is high as A_I is high

So, $$P_{out} > P_{in}$$

For transformer $P_{in} = P_{out}$

41. Ans: (d)

Hint:

XRA	A	[Accumulator is cleared, accumulator contents 00H]
MVI	B F0H	[The contents of register B is F0H]
SUB	B	[Subtract content of register B from accumulator, results stored in accumulator]

42. Ans: (b)

Hint: $$V_o = 10 \times \frac{10}{10 + 10}$$

$= 5$ V [D_1 is ON and D_2 is OFF]

43. Ans: (a)

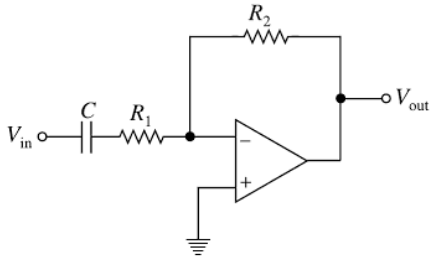
Hints: Diode acts as a switch.

Under forward biased condition, it is short circuited.

But under reverse biased condition, current does not become zero instantly. Initially the same current flows in opposite direction and after some time (turn-off time) it will become zero.

44. Ans: (b)

Hint:



$$\frac{V_{in} - 0}{R_1 + \frac{1}{sC}} = \frac{0 - V_{out}}{R_2}$$

$$\Rightarrow \qquad \frac{V_{out}}{V_{in}} = -\left(\frac{R_2}{R_1 + \frac{1}{sC}}\right)$$

It is a high-pass filter with

$$f_{3dB} = \frac{1}{R_1 C} \text{ rad/s}$$

45. Ans: (d)

Hint: $$I_{C(sat)} = \frac{0 - (-12) - 0.2}{2.2 \times 10^3} = 5.36 \text{ mA}$$

46. Ans: (c)

Hint: For positive voltage the waveform clips at + 5 V + 0.7 V. For negative voltage at – 0.7 V, the zener diode conducts and clips out.

47. Ans: (a)

Hint: Applying KVL

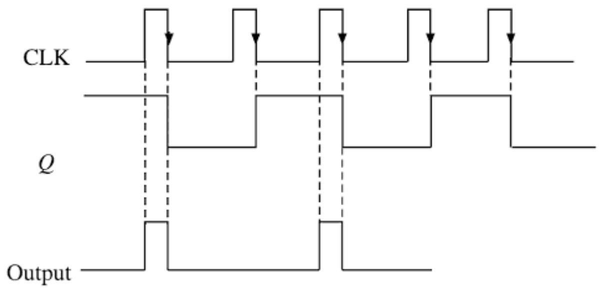
$$10 \times 10^3 \times I_B + 0.7 + 100 \times 100 \times I_B = 10$$

or $I_b = 0.465$ mA

$$V_o = 100 \times 0.465 \times 10^{-3} \times 100 = 4.65 \text{ V}$$

48. Ans: (b)

Hint:



60. Ans: (d)

Hint: The output Q always changes the state from '1' to '0' and from '0' to '1'.

61. Ans: (c)

Hint: When V_i = 3 V then Q_1 will be in reverse active mode and Q_2 will be ON.

62. Ans: (a)

Hint:

A	B	C	Output
0	0	0	0
1	0	0	1
0	0	1	1
1	0	1	0
0	1	0	1
1	1	0	1
0	1	1	0
1	1	1	0

$\therefore \quad F(A, B, C) = \Sigma(1, 2, 4, 6)$

63. Ans: (a)

Hint: Voltage at point P $= 12 \times \dfrac{10}{10+10} = 6$ V

[supply voltage = 12 V]

Voltage at point P $= -12 \times \dfrac{10}{10+2} = -10\ V$

[supply voltage = –12 V]

64. Ans: (b)

Hint: From figure, $Y = A \oplus B \oplus C \oplus D$

We know, sum of any number of bits is XOR of all bits.

So, $\quad S = A \oplus B \oplus C \oplus D \oplus Y$

$S = Y \oplus Y$

S = Either zero or even because LSB is zero (always)

65. Ans: (c)

Hint: $\quad V_A = \dfrac{R_4}{R_4 + R_3} \times V_i$

or $\quad V_A = \dfrac{R_B}{R_B + R_B} \times V_i = \dfrac{V_i}{2}$

Case I: V_i only on inverting terminal

$$V_{o1} = -\frac{R_f}{R_1} \times V_i$$

Here, $\quad R_f = \dfrac{R_2 \times \dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}} = \dfrac{R_2}{1 + sCR_2}$

Case II: V_i only on non-inverting terminal

$$V_{o2} = \left(1 + \frac{R_f}{R_1}\right)\frac{V_i}{2}$$

Now, $\quad V_o = V_{o1} + V_{o2}$

$$= -\frac{R_f}{R_1} \times V_i + \left(1 + \frac{R_f}{R_1}\right)\frac{V_i}{2}$$

$$= \left(1 - \frac{R_f}{R_1}\right)\frac{V_i}{2}$$

or $\quad V_o = \left(1 - \dfrac{R_2}{R_1(1 + sCR_2)}\right)\dfrac{V_i}{2}$

Here ($R_1 = R_2 = R_A$)

$\therefore \quad V_o = \left(\dfrac{sCR_A}{1 + sCR_A}\right)\dfrac{V_i}{2}$

$$= \frac{1}{1 + \dfrac{1}{sCR_A}} \times \left(\frac{V_i}{2}\right)$$

So, it is a high-pass filter.

66. Ans: (d)

Hint:

$$V_{\text{in}} = \left(\frac{sCR_A}{(1 + sCR_A)}\right)\frac{V_i}{2}$$

$\therefore \quad V_o = \dfrac{\dfrac{1}{sC}}{\dfrac{1}{sC} + \dfrac{R_A}{2}} V_{\text{in}} = \dfrac{1}{\left(1 + sR_A \dfrac{C}{2}\right)} V_{\text{in}}$

$\therefore \quad V_o = \dfrac{1}{1 + sR_A \dfrac{C}{2}} \cdot \dfrac{sR_AC}{(1 + sR_AC)} V_{\text{in}}$

So, it is a band-pass filter.

67. Ans: (d)

Hint: Content of PC is loaded into stack i.e., address of next instruction to be executed is loaded into stack.

SP is decremented then PC is loaded by address given in call instruction.

68. Ans: (d)

Hint: XY truth table

X	Y	Q_{n+1}
0	0	1
0	1	Q_n
1	0	$\overline{Q}_n$
1	1	0

JK truth table

J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\overline{Q}_n$

Excitation table

$Q(t)$	$Q(t+1)$	J	K	X	Y
0	0	0	×	×	1
0	1	1	×	×	0
1	0	×	1	1	×
1	1	×	0	0	×

To make XY flip-flop using JK flip-flop,

$$J = \overline{Y},\ K = X$$

69. Ans: (d)

Hint: Non-inverting terminal,

$$V_1 = \frac{V_{in} \times \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{V_{in}}{1 + j\omega RC}$$

For ideal op-amp, $V_1 = V_2$ [V_2 = inverting terminal]

$$\frac{V_{in} - V_2}{R_1} = \frac{V_2 - V_{out}}{R_1}$$

$$V_{out} = 2V_2 - V_{in} = 2V_1 - V_{in}$$

$$= \left(\frac{2}{1 + j\omega RC} - 1\right)V_{in}$$

$$= \left(\frac{1 - j\omega RC}{1 + j\omega RC}\right)V_{in}$$

$$\angle\left(\frac{V_{out}}{V_{in}}\right) = -2\tan^{-1}\omega RC$$

For $-90° \le \theta \le 90°$, phase shift maximum occur is ($\pm 180°$).

70. Ans: (d)

Hint: N-channel MOSFET is assumed.

Gate through source is so connected that MOSFET will be in enhance mode and so conductivity of the channel will be increased very much and effectively β terminal act as short circuited.

So, $V_{ab} = 0$ V

7

Power Electronics

7.1 SYLLABUS

Semiconductor power diodes, transistors, thyristors, triacs, GTOs, MOSFETs and IGBTs—static characteristics and principles of operation; triggering circuits; phase control rectifiers; bridge converters—fully controlled and half controlled; principles of choppers and inverters; basic concepts of adjustable speed dc and ac drives.

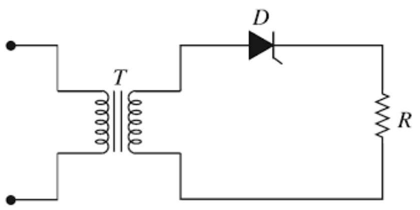
7.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	3	1	4	2	1	2	2	2	–	2	1	–
2 Marks Question	5	8	7	6	4	–	3	3	7	3	3	3
Total Marks	**13**	**17**	**18**	**14**	**9**	**2**	**8**	**8**	**14**	**8**	**7**	**6**

7.3 TOPICS TO BE FOCUSED

PHASE CONTROL RECTIFIER

Single Phase Half Wave Controlled Rectifier with Load *R*



- Average load voltage and current for half wave controlled rectifier

$$V_{dc} = \frac{V_m}{2\Pi}(1+\cos\alpha)$$

$$I_{dc} = \frac{V_m}{2\Pi R}(1+\cos\alpha)$$

where α is the firing angle.
When $\alpha = 0$, output voltage is maximum and current $I_{dc} = \frac{V_m}{\Pi R}$

When $\alpha = \frac{\Pi}{2}$, output voltage $V_{dc} = 0.1592V_m$ and current $I_{dc} = \frac{0.1592V_m}{R}$

- The rms value of output voltage (V_{rms}) and output current (I_{rms})

$$V_{rms} = V_m\left[\frac{\Pi-\alpha}{4\Pi}+\frac{\sin 2\alpha}{8\Pi}\right]^{1/2}$$

$$I_{rms} = \frac{V_m}{R}\left[\frac{\Pi-\alpha}{4\Pi}+\frac{\sin 2\alpha}{8\Pi}\right]^{1/2}$$

When $\alpha = 0$, output voltage $V_{rms} = \frac{V_m}{2}$ and current $I_{rms} = \frac{V_{rms}}{R}$

When $\alpha = \frac{\Pi}{2}$, output voltage $V_{rms} = 0.3536V_m$ and current $I_{rms} = \frac{0.3536V_m}{R}$

- Rectifier efficiency
When $\alpha = 0°$, efficiency $(\eta)\% = 64\%$

When $\alpha = 90°$, efficiency $(\eta)\% = 20.27\%$

- Form factor
  When $\alpha = 0°$, form factor (FF) = 1.57
  When $\alpha = 90°$, form factor (FF) = 2.22
- Ripple factor
  When $\alpha = 0°$, ripple factor (RF) = 1.465
  When $\alpha = 90°$, ripple factor (RF) = 1.983
- Transformer utilization factor
  When $\alpha = 0°$, transformer utilization factor (TUF) = 0.287
  When $\alpha = 90°$, transformer utilization factor (TUF) = 0.1014

Single Phase Half Wave Controlled Rectifier with *R-L* Load

- Average load voltage and current for half wave controlled rectifier

$$V_{dc} = \frac{V_m}{\Pi}.\sin\frac{\beta}{2}.\sin\left(\alpha + \frac{\beta}{2}\right)$$

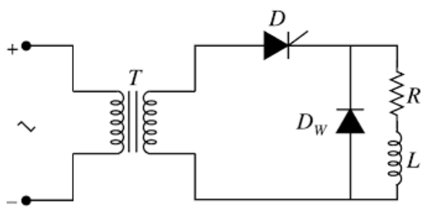
$$I_{dc} = \frac{V_m}{2\Pi R}(\cos\alpha - \cos\beta)$$

Conduction angle extending upto $\omega t = \gamma = (\alpha + \beta)$

- The rms value of output voltage (V_{rms})

$$V_{rms} = \frac{V_m}{2\sqrt{\Pi}}[\beta - \sin\beta\cos(2\alpha + \beta)]^{1/2}$$

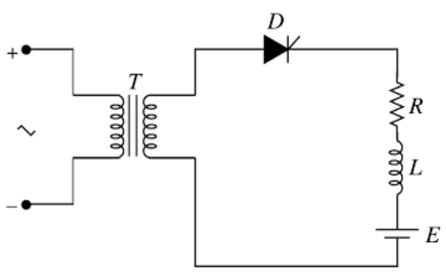
Role of Free-wheeling of Controlled Rectifier



The mean load voltage is given by

$$V_{dc} = \frac{1}{2\Pi}\int_{\alpha}^{\Pi}\frac{\sqrt{2}V}{2\Pi}(1 + \cos\alpha)$$

Single Phase Half Wave Controlled Rectifier with Active Load (*R-L-E*)



Expression of average voltage and current

$$V_{dc} = E\left(1 - \frac{\beta}{2\pi}\right) + \frac{V_m}{\pi}\sin\left(\alpha + \frac{\beta}{2}\right)\sin\frac{\beta}{2}$$

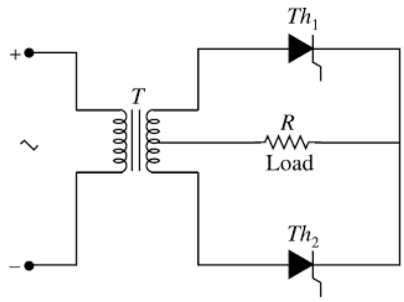
$$I_{dc} = \frac{1}{2\pi R}\left[2V_m\sin\left(\alpha + \frac{\beta}{2}\right)\sin\frac{\beta}{2} - E\beta\right]$$

Single Phase Full Wave Controlled Rectifier with Load *R*

Types of single phase full wave controlled rectifier with load *R* are:

(i) Midpoint converter
(ii) Bridge type converter

Midpoint converter



- Expression of average voltage and current

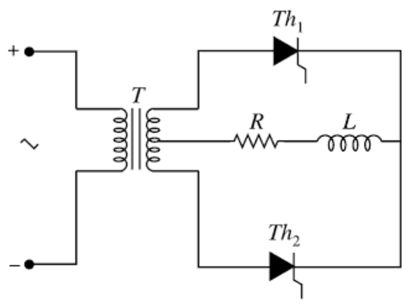
$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$I_{dc} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

- Expression of rms load voltage and current

$$V_{rms} = V_m\left[\frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2}$$

Midpoint converter with R-L load

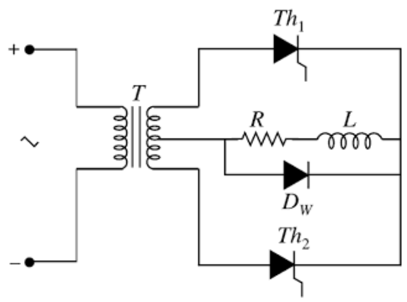


Expression of average voltage and current

$$V_{dc} = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

$$I_{dc} = \frac{\sqrt{2}V}{z}\sin(\omega t - \phi)$$

Role of free-wheeling diode in case of midpoint converter



- Expression of average voltage and current

$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)$$

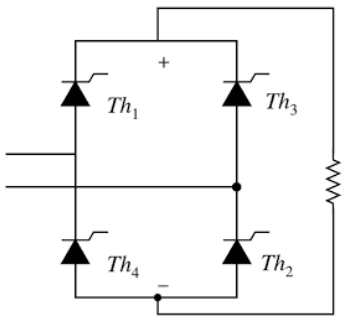
$$I_{dc} = \frac{V_m}{\pi R}(1+\cos\alpha)$$

- Free-wheeling diode current

$$I_{D_w} = \frac{V_m}{\pi^2 R}(\alpha + \alpha\cos\alpha)$$

Bridge type converter

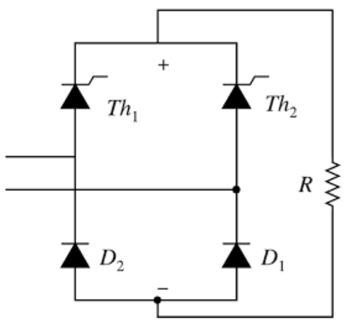
Full controlled bridge



Average output dc voltage

$$V_{dc} = \frac{2V_m}{\pi}(\cos\alpha)$$

Half controlled bridge



- Expression of average voltage and current

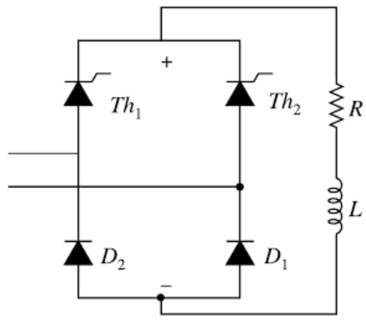
$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$I_{dc} = \frac{V_m}{\pi R}(1+\cos\alpha)$$

- Expression for rms load voltage

$$V_{rms} = V_m\left[\frac{\pi-\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2}$$

Half controlled bridge with R-L load



Expression of average voltage

$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)$$

Full wave bridge with R-L-E load

- ***Case (a):*** Load current discontinuous

$$V_{dc} = \frac{\sqrt{2}V}{\pi}\left[(\cos\alpha - \cos\beta) + (\pi - \beta + \alpha)\frac{E}{\sqrt{2}V}\right]$$

- ***Case (b):*** Continuous load current

$$V_{dc} = \frac{2\sqrt{2}V}{\pi}(\cos\alpha)$$

7.4 IMPORTANT POINTS TO REMEMBER

1. A thyristor is a four layer device.
2. The number of P-N junctions in a thyristor is 3.
3. The anode current in a thyristor is made up of electrons and holes.
4. Thyristor is equivalent to two transistors.
5. Thyristor can be used as dc switch.
6. In a thyristor, turn-on time is less than turn-off time.
7. TRIAC is equivalent to two thyristors.
8. Thyristor is a current controlled.
9. The minimum value of current required to maintain conduction of thyristor is known as holding current.
10. Thyristor is a unidirectional device.

11. The commutation of thyristor means turn-off.
12. The three terminals of TRIAC are MT1, MT2 and gate.
13. The ratio of latching current to holding current is 2.5.
14. Snubber circuit is used for $\frac{dV}{dt}$ protection of thyristor.
15. After turn on a thyristor, the gate pulse should be removed to reduce losses and junction temperature.
16. The turn-on time of thyristor is equal to sum of delay time, rise time and spread time.
17. A power semiconductor device may be damaged due to high $\frac{dI}{dt}$.
18. SCR is a unidirectional switching device.
19. TRIAC is bidirectional switching device.
20. TRIAC is just like two anti-parallel connected thyristors.
21. In the gate triggering of a thyristor, the pulse width of gate current must be larger than the turn-on time of the device.
22. The total turn-off time of a thyristor is sum of reverse recovery time and gate recovery time.
23. In the forward blocking state, a thyristor is associated with large voltage and low current.
24. In the forward conducting state, a thyristor is associated with low voltage and large current.
25. Controlled rectifier is used to convert ac to dc.
26. Inverters are used to convert dc to ac.
27. Cyclo-converters are used to convert high frequency ac to low frequency ac.
28. Cyclo-converter is known as one stage frequency changer.
29. A converter circuit which converts input power from one frequency to output power at different frequency is called cyclo-converter.
30. A cyclo-converter can convert power from supply frequency to variable frequency which is fraction of supply frequency.
31. A chopper is a converter which converts fixed dc voltage to variable dc voltage.
32. In a dc chopper using thyristor, the best performance can be obtained from voltage commutation.
33. In a type A chopper, output voltage and current are positive.
34. In a thyristor based dc chopper circuit, voltage commutation gives the best performance.
35. A class E chopper is a four quadrant chopper.
36. A class C chopper is a two quadrant chopper.
37. In a step up chopper, duty cycle is greater than zero and less than unity.
38. In a step up chopper, output voltage is always greater than input voltage.
39. In a step down chopper, output voltage is always less than input voltage.
40. The first quadrant chopper is a class A chopper.
41. The second quadrant chopper is a class B chopper.
42. In a dc to dc converter, the input voltage and output voltage waveforms are continuous and discontinuous respectively.
43. The output voltage of dc to dc converter can be controlled by pulse width modulation and frequency modulation.
44. In a class B chopper, output voltage is positive and current is negative.
45. In a class D chopper, the output current remains positive but output voltage is either positive or negative.
46. In a voltage source inverter, the output voltage waveform does not depend on load impedance but load current waveform depends on load impedance.
47. In a current source inverter, the output voltage waveform depends on load impedance but load current waveform does not depend on load impedance.
48. The output voltage waveform of a three-phase square wave inverter consists of only odd harmonics.
49. An inverter is a converter circuit which is used to convert dc power into ac power at desired output voltage and frequency.
50. The current source inverter is commonly used in very high power applications such as induction motor drives.

7.5 REFERENCES

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Bimbhra, P.S., *Power Electronics*, Khanna Publishers, Delhi, 2012.

Chakrabarti, Abhijit, *Fundamentals of Power Electronics and Drives*, Dhanpat Rai & Co., Delhi, 2013.

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7.6 PREVIOUS YEARS' QUESTIONS

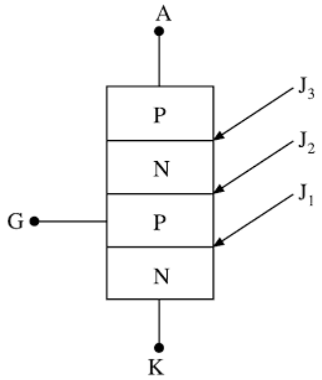
1. Figure shows a thyristor with the standard terminations of anode (A), cathode (K), gate (G) and the different junctions named J_1, J_2, and J_3. When the thyristor is turned on and conducting

(a) J_1 and J_2 are forward biased and J_3 is reverse biased

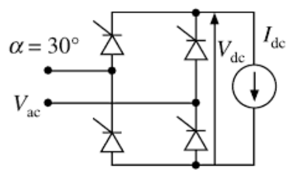
(b) J_1 and J_3 are forward biased and J_2 is reverse biased

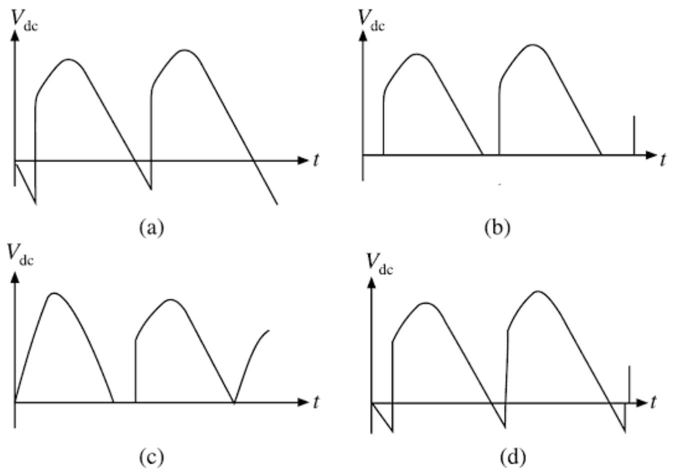
(c) J_1 is forward biased and J_2 and J_3 are reverse biased

(d) J_1, J_2 and J_3 are all forward biased [Gate 2003]



2. A single phase half controlled converter is shown in figure. The control angle $\alpha = 30°$. The output dc voltage wave shape will be as shown in





(a) Figure A (b) Figure B
(c) Figure C (d) Figure D [Gate 2003]

3. A fully controlled natural commuted three-phase bridge rectifier is operating with a firing angle $\alpha = 30°$. The peak to peak voltage ripple expressed as a ratio of the peak output dc voltage at the output of the converter bridge is

(a) 0.5 (b) $\frac{\sqrt{3}}{2}$

(c) $\left(1-\frac{\sqrt{3}}{2}\right)$ (d) $\sqrt{3}-1$ [Gate 2003]

4. A bipolar junction transistor (BJT) is used as a power control switch by biasing it in the cut-off region (OFF state) or in the saturation region (ON state). In the ON state, for the BJT

(a) Both the base–emitter and base–collector junctions are reverse biased
(b) The base–emitter junction is reversed biased and the base–collector junction is forward biased
(c) The base–emitter junction is forward biased and the base–collector junction is reversed biased
(d) Both the base–emitter junction and base–collector junctions are forward biased [Gate 2004]

5. The conduction loss versus device current characteristic of a power MOSFET is best approximated by
(a) A parabola
(b) A straight line
(c) A rectangular hyperbola
(d) An exponentially decaying function

[Gate 2005]

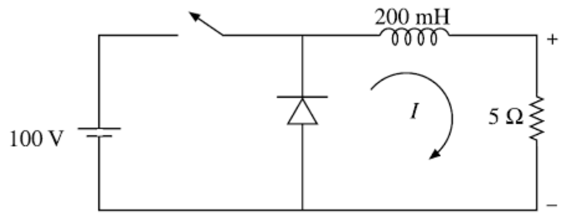
6. A three-phase diode bridge rectifier is fed from a 400 V rms, 50 Hz, and three-phase ac source. If the load is pure resistive, the peak instantaneous output voltage is equal to

(a) 400 V (b) $400\sqrt{2}$ V

(c) $400\sqrt{\frac{2}{3}}$ V (d) $\frac{400}{\sqrt{3}}$ V [Gate 2005]

7. The output voltage waveform of a three-phase square wave inverter contains
(a) Only even harmonics
(b) Both odd and even harmonics
(c) Only odd harmonics
(d) Only triplen harmonics [Gate 2005]

8. Figure shows a step down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak to peak ripple in the load current is close to



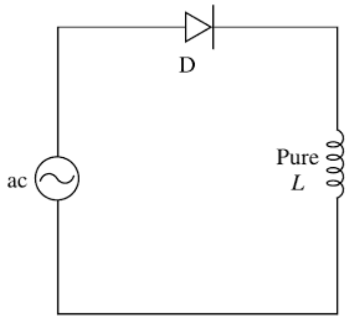
(a) 10 A (b) 0.5 A
(c) 0.125 A (d) 0.25 A [Gate 2005]

9. A three-phase, fully controlled thyristor bridge converter is used as line commutated inverter to feed 50 kW power at 420 V dc to a three-phase, 415 V (line), 50 Hz ac mains. Consider dc link current to be constant. The rms current of the thyristor is
(a) 119.05 A
(b) 79.37 A
(c) 68.73 A
(d) 39.68 A [Gate 2007]

10. "Six MOSFETs connected in a bridge configuration (having no other power device) must be operated as a voltage source inverter (VSI)". This statement is

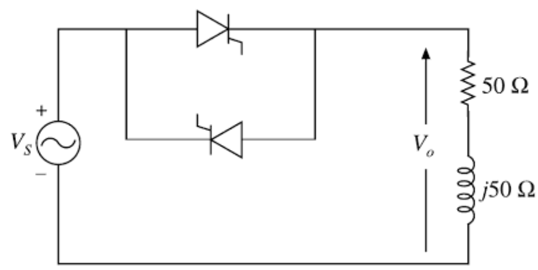
(a) True, because being majority carrier devices, MOSFETs are voltage driven
(b) True, because MOSFETs have inherently anti-parallel diodes
(c) False, because it can be operated both as current source inverter (CSI) or a VSI
(d) False, because MOSFETs can be operated as excellent constant current sources in the saturation region [Gate 2007]

11. In the circuit of adjacent figure, the diode connects the ac source to a pure inductor L.



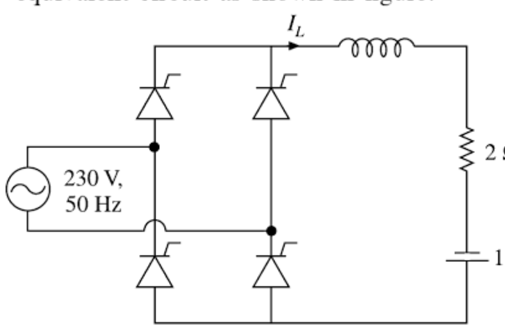
The diode conducts for
(a) 90° (b) 180°
(c) 270° (d) 360° [Gate 2007]

12. In the single phase voltage controller circuit shown in figure, for what range of triggering angle (α), the output voltage (V_o) is not controllable?



(a) $0° < \alpha < 45°$ (b) $45° < \alpha < 135°$
(c) $90° < \alpha < 180°$ (d) $135° < \alpha < 180°$
[Gate 2008]

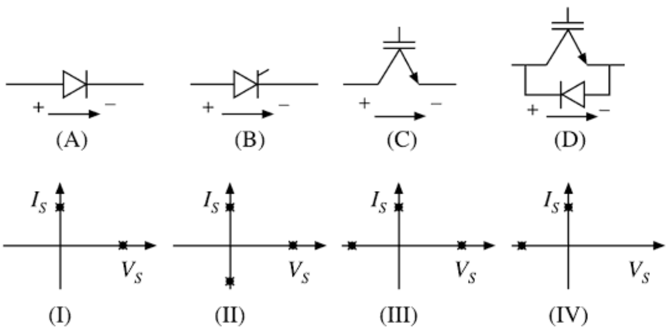
13. A single phase fully controlled converter bridge is used for electrical braking of a separately excited dc motor. The dc motor load is represented by an equivalent circuit as shown in figure.



Assume that the load inductance is sufficient to ensure continuous and ripple free load current. The firing angle of the bridge for a load current of I_L = 10 A will be
(a) 44° (b) 51°
(c) 129° (d) 136° [Gate 2008]

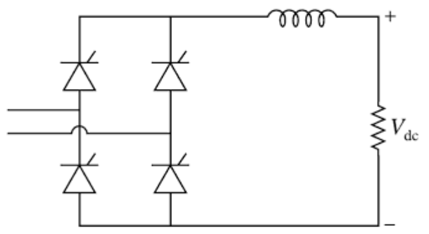
14. An SCR is considered to be a semi-controlled device because
(a) It can be turned OFF but not ON with a gate pulse
(b) It conducts only during one half cycle of an alternating current wave
(c) It can be turned ON but not OFF with a gate pulse
(d) It can be turned ON only during one half cycle of an alternating voltage wave [Gate 2009]

15. Match the switch arrangements on the top row to the steady state *V-I* characteristics on the lower row. The steady state operating points are shown by large black dots.



(a) A – I, B – II, C – III, D = IV
(b) A – II, B – IV, C – I, D = III
(c) A – IV, B – III, C – I, D = II
(d) A – IV, B – III, C – II, D = I [Gate 2009]

16. The fully controlled thyristor converter shown in figure is fed from a single phase source. When the firing angle is 0°, the dc output voltage of the converter is 300 V. What will be the output voltage for a firing angle of 60°, assuming continuous conduction?



(a) 150 V (b) 210 V
(c) 300 V (d) 100 π V [Gate 2010]

17. Circuit turn-off time of an SCR is defined as the time
(a) Taken by the SCR turn-off
(b) Required for the SCR current to become zero

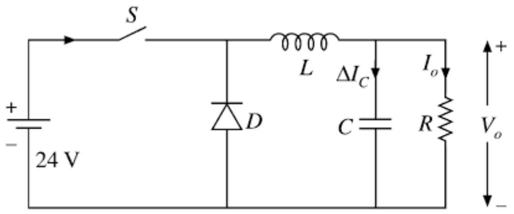
(c) For which the SCR is reversed biased by the commutation circuit
(d) For which the SCR is reverse biased to reduce its current below the holding current [Gate 2011]

18. A half controlled single phase bridge rectifier is supplying an R-L load. It is operated at a firing angle α and the load current is continuous. The fraction of cycle that the free-wheeling diode conducts is

(a) $\frac{1}{2}$ (b) $\left(1-\frac{\alpha}{\pi}\right)$
(c) $\frac{\alpha}{2\pi}$ (d) $\frac{\alpha}{\pi}$ [Gate 2012]

19. The typical ratio of latching current to hold current in a 20 A thyristor is
(a) 5.0 (b) 2.0
(c) 1.0 (d) 0.5 [Gate 2012]

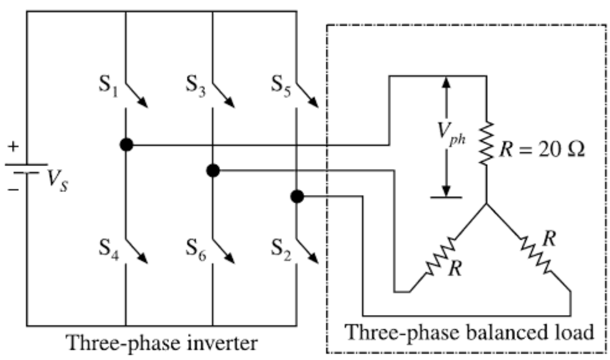
20. In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio or 50%. Given that ΔI_C is 1.6 A peak to peak and I_o is 5 A dc, the peak current in S is



(a) 6.6 A (b) 5.0 A
(c) 5.8 A (d) 4.2 A [Gate 2012]

Common Data Questions: 21 and 22

In the three-phase inverter circuit shown in figure, the load is balanced and the gating scheme is 180° conduction mode. All the switching devices are ideal



21. The rms value of load phase voltage is
(a) 106.1 V
(b) 141.4 V
(c) 212.2 V
(d) 282.8 V [Gate 2012]

22. If the dc bus voltage $V_S = 300$ V, the power consumed by three-phase load is
(a) 1.5 kW (b) 2.0 kW
(c) 2.5 kW (d) 3.0 kW [Gate 2012]

23. The MOSFET switch in its ON state may be considered equivalent to
(a) Resistor (b) Inductor
(c) Capacitor (d) Battery [Gate 1998]

24. In a commutation circuit employed to turn-off an SCR, satisfactory turn-off is obtained when
(a) Circuit turn-off time < device turn-off time
(b) Circuit turn-off time > device turn-off time
(c) Circuit time constant > device turn-off time
(d) Circuit time constant < device turn-off time [Gate 1998]

25. In a three-phase controlled bridge rectifier, with an increase of overlap angle, the output dc voltage
(a) Decreases
(b) Increases
(c) Does not change
(d) Depends upon load inductance [Gate 1996]

26. A dc to dc transistor chopper supplied from a fixed voltage dc source feeds a fixed resistive-inductive load and a free-wheeling diode. The chopper operates at 1 kHz and 50% duty cycle. Without changing the value of the average dc current through the load, if it is desired to reduce the ripple constant of load current, the control action needed will be
(a) Increased the chopper frequency keeping its duty cycle constant
(b) Increased the chopper frequency and duty cycle in equal ratio
(c) Decreased only the chopper frequency
(d) Decreased only the duty cycle [Gate 1995]

27. A step down chopper is operated in the continuous conduction mode in steady state with a constant duty ratio D. If V_o is the magnitude of the dc output voltage and if V_i is the magnitude of the dc input voltage, the ratio V_o/V_i is given by
(a) D (b) $1-D$
(c) $1/(1-D)$ (d) $D/(1-D)$ [Gate 2002]

28. When a line commutated converter operates in the inverter mode?
(a) It draws both real and reactive power from the ac supply
(b) It delivers both real and reactive power to the ac supply
(c) It delivers real power to the ac supply
(d) It draws reactive power from the ac supply [Gate 1993]

29. A three-phase semiconverter feeds the armature of separately excited dc motor, supplying a non-zero torque for steady state operation, the motor armature current is found to drop to zero at certain instances of time. At such instances, the voltage assumes a value that is
(a) Equal to the instantaneous value of the ac phase voltage
(b) Equal to the instantaneous value of the motor back emf
(c) Arbitrary
(d) Zero [Gate 2001]

30. If a diode is connected in anti-parallel with a thyristor, then
(a) Both turn-off power loss and turn-off time decreases
(b) Turn-off power loss decreases but turn-off time increases
(c) Turn-off power loss increases but turn-off time decreases
(d) None of the above [Gate 1997]

Solutions

1. Ans: (b)
Hint: Forward blocking mode and forward conduction mode: Anode is positive w.r.t. the cathode. So, junctions J_1 and J_3 are forward biased but junction J_2 is reverse biased.
Reverse blocking mode: Cathode is more positive w.r.t. anode. Junctions J_1 and J_3 are reverse biased whereas junction J_2 is forward biased.

2. Ans: (a)
Hint: Current source is connected as load. So, load current continuously flows either through pair upper left, lower right or upper right, lower left.
Upper left, lower right are fired at angle $\alpha = 30°$. So, these SCR's will get turned ON as firing angle is 30°. Upper right, lower left will be fired at $\pi + \alpha$.

So, upper left, lower right conducts for $\alpha < \omega t < \pi + \alpha$. Here, V_{dc} is positive.

Upper right, lower left are fired at $\pi + \alpha$. The supply voltage turns OFF upper left, lower right SCRs' by natural commutation and the load current is transferred from upper left, lower right to upper right, lower left.

V_{dc} is negative when upper right, lower left conduct.

3. Ans: (b)
Hint: For converter,

Average output voltage, $V_o = \dfrac{3V_m}{\pi}\cos\alpha$
At $\alpha = 30°$

$$\therefore \quad V_o = \frac{3V_m}{\pi}\cos 30°$$

Peak output dc voltage, $(V_o)_{\text{peak}} = \dfrac{3V_m}{\pi}$
Peak to peak voltage ripple

$$\frac{V_o}{(V_o)_{\text{peak}}} = \frac{\dfrac{3V_m}{\pi}\cos 30°}{\dfrac{3V_m}{\pi}} = \frac{\sqrt{3}}{2}$$

4. Ans: (d)
5. Ans: (a)
6. Ans: (b)
Hint: Maximum value of input voltage

$$V_m = 400\sqrt{2}\ \text{V}$$

Now, load is purely resistive, therefore, peak instantaneous output voltage

$$V_m = 400\sqrt{2}\ \text{V}$$

7. Ans: (c)
8. Ans: (c)

Hint: Here, $T = \dfrac{1}{1\times 10^3} = 10^{-3}$ s

$$T_a = \frac{L}{R} = \frac{200\times 10^{-3}}{5} = 40\ \text{ms}$$

$$\text{Ripple} = \frac{V_S}{R}\left[\frac{\left(1-e^{-\alpha T/T_a}\right)\left(1-e^{-(1-\alpha)T/T_a}\right)}{\left(1-e^{-T/T_a}\right)}\right]$$

$$(\Delta I)_{\max} = \frac{V_S}{4fL} = \frac{100}{4\times 10^3\times 200\times 10^{-3}} = 0.125\ \text{A}$$

9. Ans: (c)
Hint: Let dc link current = I_{dc}
The dc voltage applied, V_{dc} = 420 V
Power fed to the inverter

$$P = V_{dc}I_{dc}$$

or $\quad 420 I_{dc} = 50\times 10^3$

or $\quad I_{dc} = 119.05$ A

Current through each thyristor flows for period of $\dfrac{2\pi}{3}$.
So, rms current of thyristor

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/3} I_{dc}^2\, d(\omega t)}$$

$$= \frac{I_{dc}}{\sqrt{3}} = \frac{119.05}{\sqrt{3}} = 68.73\ \text{A}$$

10. Ans: (b)
11. Ans: (b)

8

Signals and Systems

8.1 SYLLABUS

Representation of continuous and discrete time signals; shifting and scaling operations; linear, time invariant and causal systems; Fourier series representation of continuous periodic signals, sampling theorem; Fourier, Laplace and Z-transforms.

8.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014		
										Set 1	Set 2	Set 3
1 Mark Question	–	3	2	2	1	2	2	2	6	2	2	3
2 Marks Question	4	4	5	7	3	4	2	4	1	3	2	3
Total Marks	**8**	**11**	**12**	**16**	**7**	**10**	**6**	**10**	**8**	**8**	**6**	**9**

8.3 TOPICS TO BE FOCUSED

INTRODUCTION TO SIGNALS

- A signal is the function of one or more independent variables that carries some information to represent a physical phenomenon.
- A continuous-time signal, also called an analog signal, is defined along a continuum of time.

CAUSALITY

A system is causal if the output at any time depends upon the values of the input at the present and past time.

STABILITY

A system is said to be stable if the responses due to small inputs do not diverge.

INVERTIBILITY AND INVERSE SYSTEM

A system is said to be invertible if distinct inputs lead to distinct outputs, and for the discrete time system if a system is invertible then an inverse system exists.

CONTINUOUS TIME LINEAR TIME INVARIANT (LTI) SYSTEM

A system of which both input and output are continuous time signals and possesses two properties namely linearity and time invariance is referred to as LTI system. This system follows the superposition theorem. These system characterized either impulse response $h(t)$ or by their transfer function $H(s)$. Transfer function of a LTI system is the ratio of Laplace transform of output $y(t)$ and Laplace transform of input $s(t)$ of a system when the system is initially relaxed and consider all initial conditions of a system are zero.

Convolution

Convolution is referred as superposition integral. It is used to compute the response of a continuous time LTI system with impulse response $h(t)$ and for any arbitrary input $s(t)$. Impulse response $h(t)$ is the output of system when unit impulse function $\delta(t)$ is applied as input.

Causality of LTI System

Impulse response of a causal LTI system should satisfy the following condition $h(t) = 0$ for all $t < 0$. This condition suggests that for a causal LTI system, convolution sum exists.

Stability of LTI System

A system is stable if every bounded input produces a bounded outputs. In order to check whether LTI systems are to be stable, the system should satisfy the following condition $|x[t]| < B$ for all t, where B = boundary of the outputs.

Some Important Properties of LTI System

(i) Commutative property: $y(t) = s(t) \times h(t)$

(ii) Distributive property: $y(t) = s(t) \times [h_1(t) + h_2(t)]$
$= s(t) \times h_1(t) + s(t) \times h_2(t)$

(iii) Associative property: $y(t) = s(t) \times [h_1(t) \times h_2(t)]$
$= s(t) \times h_1(t) \times h_2(t)$

Unit Step Response of a LTI System

Unit step response is the output of a LTI system for input which is equal to unit step function or sequence.

$g(t) = u(t) \times h(t)$, where $u(t)$ = unit step response, $g(t)$ = unit step output.

STATIC AND DYNAMIC SYSTEM

A system is called static if its output at any time depends only on the value of the input at the same time. It is also called *memory less system.*

If $h(t) = 0$ for $t \neq 0$ then the memory less system is defined as $y(t) = k \times s(t)$, where k = constant. If $k = 1$ then the system is called *identity system.* If impulse response $[h(t)]$ of LTI system is not identically zero for $t \neq 0$, then the system is called *dynamic system* or *system having memory.*

Z-TRANSFORM

For a discrete time linear time-invariant system with impulse response $h[n]$, the response $y[n]$ of the system to a complex exponential input of the form z^n is $y[n] = H[z].z^n$, where $H[z] = \Sigma(n = -\infty \text{ to } + \infty)\ h[n].z^{-n}$.

The Z-transform of a discrete time signal $x[n]$ is defined as $X[z] = \sum (n = -\infty \text{ to } + \infty)\ x[n].\ z^{-n}$, where z is a complex variable.

The Region of Convergence (ROC) of Z-transforms

Region of convergence for a discrete time signal $x[n]$ is defined as a continuous region in z-plane where the Z-transform converges. In order to determine ROC, it is convenient to represent the Z-transform,

$$X(z) = \frac{P(z)}{Q(z)}$$

The roots of the equation $P(z) = 0$ correspond to the 'zeros' of $X(z)$.

The roots of the equation $Q(z) = 0$ correspond to the 'poles' of $X(z)$.

Properties of ROC

(i) ROC of $X(z)$ consists of a ring in the z-plane centred about origin.

(ii) The ROC does not contain any poles.

(iii) If $x[n]$ is of finite duration then ROC is the entire z-plane except $z = 0$ and $z = \infty$.

(iv) If $x[n]$ is a right sided sequence and if the circle $|z| = r$ is in the ROC, then all finite values of z for which $|z| > r$ will also be in the ROC.

(v) If $x[n]$ is two sided, and if the circle $|z| = r$ is in ROC, then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r$.

(vi) If the Z-transform $X(z)$ is rational, then its ROC is bounded by poles or extends to infinity.

(vii) If the Z-transform $X(z)$ of $x[n]$ is rational and if $x[n]$ is right sided then the ROC is the region in the z-plane outside the outermost pole.

(viii) If the Z-transform $X(z)$ is rational, and if $x[n]$ is left sided then ROC is the region in the z-plane inside the investment to non-zero pole i.e., inside the circle of radius equal to the smallest magnitudes of the poles of $X(z)$ other than any at $z = 0$ and extending inward to and possibly including $z = 0$.

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_n(n - k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1 - z^{-1}}$
4.	$\frac{1}{s+a}$	$e^{-\omega t}$	$e^{-\omega kT}$	$\frac{1}{1 - e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1 - z^{-1})^2}$
				(*Contd.*)

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^2z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{at}$	$1-e^{akt}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-at}z^{-1}}{(1-e^{-at}z^{-1})^2}$
11.	$\frac{s^2}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akt)e^{-at}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$(1-at)e^{-at}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-aT}$	$akt-1+e^{-akT}$	$\frac{\lvert(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}\rvert z^{-1}}{(1-z^{-1})(1-d^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kt$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kt$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kt$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kt$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^k $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-1})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^3}$
24.	–	–	$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$

FOURIER TRANSFORM

The Fourier transform is important in mathematics, engineering, and the physical sciences. Its discrete counterpart, the Discrete Fourier Transform (DFT), which is normally computed using the so-called Fast Fourier Transform (FFT), has revolutionized modern society, as it is ubiquitous in digital electronics and signal processing.

The Fourier transform is a reversible, linear transform with many important properties. For any function $f(x)$ (which in astronomy is usually real-valued, but $f(x)$ may be complex), the Fourier transform can be denoted as F(s), where the product of x and s is dimensionless. Often x is a measure of time t (i.e., the *time-domain* signal) and so s corresponds to inverse time, or frequency ν (i.e., the *frequency-domain* signal).

The Fourier transform is defined by

$$\mathrm{F}(s) = \int_{-\infty}^{\infty} f(x)e^{-2\pi isx}\,dx$$

The Discrete Fourier Transform

- Function sampled at N discrete points
  – sampling at evenly spaced intervals
  – Fourier transform estimated at discrete values:

$$X_k = \sum_{j=0}^{N-1} x_j e^{-ij2\pi k/N}$$

- Almost the same symmetry properties as the continuous Fourier transform.

Properties of Fourier Transform

(i) Linearity: For any constants a, b, the following equality holds:

$$\mathrm{F}\{af(t)+bg(t)\} = a\mathrm{F}\{f(t)\}+b\mathrm{F}\{g(t)\} = a\mathrm{F}(\omega)+bG(\omega)$$

(ii) Scaling: For any constant c, the following equality holds:

$$\mathrm{F}\{f(ct)\} = \frac{1}{|c|}\mathrm{F}\left(\frac{\omega}{c}\right)$$

(iii) Time shifting

$$\mathrm{F}\{f(t-t_0)\} = e^{-i\omega t_0}\mathrm{F}(\omega)$$

(iv) Frequency shifting

$$\mathrm{F}\{e^{i\omega t_0} f(t)\} = \mathrm{F}(\omega-\omega_0)$$

(v) Symmetry

$$\mathrm{F}\{\mathrm{F}(t)\} = 2\pi f(-\omega)$$

Formulas of DFT

$$H(f_n) = \int_{-\infty}^{\infty} h(t)\exp[2\pi i f_n t]dt \approx \sum_{k=0}^{N-1} h_k \exp[2\pi i f_n t_n]\Delta$$

$$= \Delta\sum_{k=0}^{N-1} h_k \exp[2\pi ikn/N]$$

$$H_n \equiv \sum_{k=0}^{N-1} h_k \exp[2\pi ikn/N]$$

$$h_k = \frac{1}{N}\sum_{n=0}^{N-1} H_n \exp[-2\pi ikn/N]$$

8.4 IMPORTANT POINTS TO REMEMBER

1. A signal is the function of one or more independent variables that carries some information to represent a physical phenomenon.
2. A set of things working together as parts of a mechanism or an interconnecting network in a complex whole is called system.
3. A continuous-time signal is a varying quantity (a signal) whose domain, which is often time, is a continuum (e.g., a connected interval of the reals).
4. A discrete signal or discrete-time signal is a time series consisting of a sequence of quantities.
5. A system of which both input and output are continuous time signals and possesses two properties namely linearity and time invariance is referred to as LTI system.
6. LTI system follows the superposition theorem. These system characterised either impulse response $h(t)$ or by their transfer function $H(s)$.
7. Transfer function of a LTI system is the ratio of Laplace transform of output $y(t)$ and Laplace transform of input $s(t)$ of a system when the system is initially relaxed and consider all initial conditions of a system are zero.
8. A filter whose output also depends on future inputs is non-causal
9. A filter whose output depends only on future inputs is anti-causal.
10. Systems (including filters) that are realisable (i.e., that operate in real time) must be causal because such systems cannot act on a future input.
11. A system is said to be invertible if distinct inputs lead to distinct outputs.
12. A system is said to be stable if the responses due to small inputs do not diverge.
13. A nonlinear system, in contrast to a linear system, is a system which does not satisfy the superposition principle, i.e., the output of a nonlinear system is not directly proportional to the input.
14. Impulse response of a causal LTI system should satisfy the following condition $h(t) = 0$ for all $t < 0$. This condition suggests that for a causal LTI system, convolution sum exists.
15. A system is stable if every bounded input produces a bounded output. It order to check whether LTI

systems are to be stable, the system should satisfy the following condition

$|x[t]| < B$ for all t, where B = boundary of the outputs.

16. A system is called static if its output at any time depends only on the value of the input at the same time. It is also called memory less system.
17. If $h(t) = 0$ for $t \neq 0$ then the memory less system is defined as $y(t) = k \times s(t)$, where k = constant. If $k = 1$ then the system is called identity system.
18. If impulse response $[h(t)]$ of LTI system is not identically zero for $t \neq 0$, then the system is called dynamic system or system having memory.
19. Commutative property: $y(t) = s(t) \times h(t)$
20. Distributive property: $y(t) = s(t) \times [h_1(t) + h_2(t)] = s(t) \times h_1(t) + s(t) \times h_2(t)$
21. Associative property: $y(t) = s(t) \times [h_1(t) \times h_2(t)] = s(t) \times h_1(t) \times h_2(t)$
22. Laplace transform of 1 is $1/s$
23. Laplace transform of t is $1/s^2$
24. Laplace transform of e^{at} is $\dfrac{1}{s-a}$
25. Laplace transform of te^{at} is $\dfrac{1}{(s-a)^2}$
26. Laplace transform method is very useful to solve differential equation because it gives particular solution directly. Specially it is useful for solving linear differential equations with constant coefficient.
27. In signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency domain representation.
28. ROC of $X(z)$ consists of a ring in the z-plane centred about origin.
29. If $x[n]$ is a right sided sequence and if the circle $|z| = r$ is in the ROC, then all finite values of z for which $|z| > r$ will also be in the ROC.
30. If $x[n]$ is two sided, and if the circle $|z| = r$ is in ROC, then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r$.
31. If the Z-transform $X(z)$ is rational, then its ROC is bounded by poles or extends to infinity.
32. If a periodic function $f(x)$ with period 2π is piecewise continuous in the interval $-\pi \le x \le \pi$ and has a left hand derivative and right hand derivative at each point of that interval, then the Fourier series

$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of $f(x)$ with coefficient a_0, a_n, b_n.

8.5 REFERENCES

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Oppenheim, Alan V., Willsky, Alan S. and Nawab, S. Hamid, *Signals and Systems*, 2nd ed., PHI Learning, Delhi, 1996.

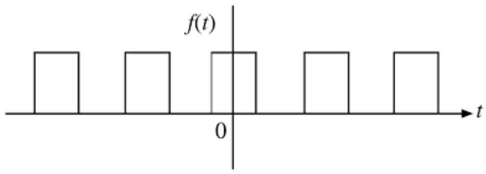
Sharma, Sanjay, *Signals and Systems*, S.K. Kataria & Sons, Delhi, 2012.

8.6 PREVIOUS YEARS' QUESTIONS

1. Consider the differential equation $\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} + y(t) = \delta(t)$ with $y(t)\big|_{t=0^-} = -2$ and $\dfrac{dy}{dt}\Big|_{t=0^-} = 0$. The numerical value of $\dfrac{dy}{dt}\Big|_{t=0^+}$ is

(a) -2 (b) -1
(c) 0 (d) 1 [Gate 2012]

2. The Fourier expansion $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$ of the periodic signal shown below will contain the following non-zero terms



(a) a_0 and b_n, $n = 1, 3, 5, \ldots \infty$
(b) a_0 and a_n, $n = 1, 2, 3, \ldots \infty$
(c) a_0, a_n and b_n, $n = 1, 2, 3, \ldots \infty$
(d) a_0 and a_n, $n = 1, 3, 5, \ldots \infty$ [Gate 2011]

3. Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t) * y(t)$ is

(a) $e^{-t} - e^{-2t}$ (b) e^{-3t}
(c) e^{+t} (d) $e^{-t} + e^{-2t}$ [Gate 2011]

4. The unilateral Laplace transform of $f(t)$ is $\dfrac{1}{s^2+s+1}$. The unilateral Laplace transform of $tf(t)$ is

(a) $-\dfrac{s}{(s^2+s+1)^2}$ (b) $-\dfrac{2s+1}{(s^2+s+1)^2}$

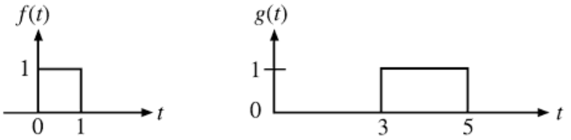
(c) $\dfrac{s}{(s^2+s+1)^2}$ (d) $\dfrac{2s+1}{(s^2+s+1)^2}$

[Gate 2012]

5. Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n\, u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals
(a) 0 (b) 1/2
(c) 1 (d) 3/2 [Gate 2012]

Common Data for Questions: 6 and 7

Given $f(t)$ and $g(t)$ as shown below:

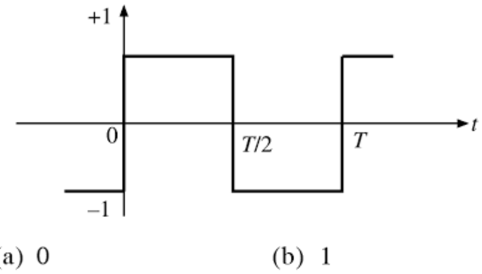


6. $g(t)$ can be expressed as
(a) $g(t) = f(2t - 3)$ (b) $g(t) = f\left(\frac{t}{2} - 3\right)$
(c) $g(t) = f\left(2t - \frac{3}{2}\right)$ (d) $g(t) = f\left(\frac{t}{2} - \frac{3}{2}\right)$
[Gate 2010]

7. The Laplace transform of $g(t)$ is
(a) $\frac{1}{s}(e^{3s} - e^{5s})$ (b) $\frac{1}{s}(e^{-5s} - e^{-3s})$
(c) $\frac{e^{-3s}}{s}(1 - e^{-2s})$ (d) $\frac{1}{s}(e^{5s} - e^{3s})$
[Gate 2010]

8. The Z-transform of a signal $x[n]$ is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$. It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$. Let the output be $y(n)$. Which of the following is true?
(a) $y(n)$ is non-causal with finite support
(b) $y(n)$ is causal with infinite support
(c) $y(n) = 0;\ |n| > 3$
(d) $\text{Re}[Y(z)]_{z = e^{j\theta}} = -\text{Re}[Y(z)]_{z = e^{-j\theta}}$
$\text{Im}[Y(z)]_{z = e^{j\theta}} = \text{Im}[Y(z)]_{z = e^{-j\theta}};\ -\pi \le \theta < \pi$
[Gate 2009]

9. The second harmonic component of the periodic waveform given in figure has an amplitude of



(a) 0 (b) 1
(c) $2/\pi$ (d) $\sqrt{5}$ [Gate 2010]

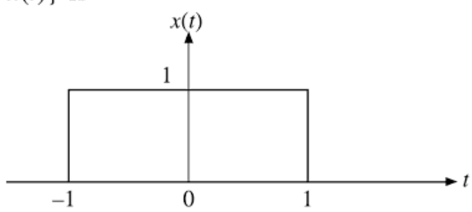
10. The impulse response of a causal linear time invariant system is given as $h(t)$. Now, consider the following two statements:
Statement (I): Principle of superposition holds
Statement (II): $h(t) = 0$ for $t < 0$.
Which one of the following statements is correct?
(a) Statement (I) is correct and Statement (II) is wrong
(b) Statement (II) is correct and Statement (I) is wrong
(c) Both Statement (I) and Statement (II) are wrong
(d) Both Statement (I) and Statement (II) are correct [Gate 2008]

11. $X(z) = 1 - 3z^{-1}$, $Y(z) = 1 + 2z^{-2}$ are Z-transforms of two signals $x[n]$, $y[n]$ respectively. A linear time invariant system has the impulse response $h[n]$ defined by these two signals as $h[n] = x[n - 1] * y[n]$ where * denotes discrete time convolution. Then the output of the system for the input $\delta[n - 1]$
(a) Has Z-transform, $z^{-1} X(z)Y(z)$
(b) Equals, $\delta[n - 2] - 3\delta[n - 3] + 2\delta[n - 4] - 6\delta[n - 5]$
(c) Has Z-transform, $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$
(d) Does not satisfy any of the above three
[Gate 2007]

12. A linear time invariant system with an impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. When the input $x(t - \tau)$ is applied to a system with impulse response $h(t - \tau)$, the output will be
(a) $y(\tau)$ (b) $y[2(t - \tau)]$
(c) $y(t - \tau)$ (d) $y(t - 2\tau)$ [Gate 2009]

13. Given $X(z) = \frac{z}{(z - a)^2}$ with $|z| > a$, the residue of $X(z)Z^{n-1}$ at $z = a$ for $n \ge 0$ will be
(a) a^{n-1} (b) a^n
(c) na^n (d) na^{n-1} [Gate 2008]

14. $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ {where $X(\omega)$ is the Fourier transform of $x(t)$} is



(a) 2 (b) 2π
(c) 4 (d) 4π [Gate 2010]

15. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its Z-transform in the z-plane will be
(a) $\frac{1}{3} < |z| < 3$ (b) $\frac{1}{3} < |z| < \frac{1}{2}$
(c) $\frac{1}{2} < |z| < 3$ (d) $\frac{1}{3} < |z|$ [Gate 2012]

16. Given a sequence $x[n]$, to generate the sequence $y[n] = x[3 - 4n]$, which one of the following procedures would be correct?
   (a) First delay $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4th sample of $z_1[n]$ to generate $z_2[n]$, and then finally time reverse $z_2[n]$ to obtain $y[n]$
   (b) First advance $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4th sample of $z_1[n]$ to generate $z_2[n]$, and then finally time reverse $z_2[n]$ to obtain $y[n]$
   (c) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$ by 3 samples to obtain $y[n]$
   (d) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$ by 3 samples to obtain $y[n]$ [Gate 2008]

17. A discrete real all pass system has a pole at $z = 2\angle 30°$, it therefore,
   (a) Also has a pole at $0.5\angle 30°$
   (b) Has a constant phase response over the z-plane: $\arg|H(z)| = \text{constant}$
   (c) Is stable only if it anti-causal
   (d) Has a constant phase response over the unit circle: $\arg|H(e^{i\Omega})| = \text{constant}$ [Gate 2006]

18. A continuous time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input. $y(t)$ is bounded
   (a) Only when $x(t)$ is bounded
   (b) Only when $x(t)$ is non-negative
   (c) Only for $t \le 0$ if $x(t)$ is bounded for $t \ge 0$
   (d) Even when $x(t)$ is not bounded [Gate 2006]

Statement for Linked Answer Questions: 19 and 20

19. A signal is processed by a causal filter with transfer function $G(s)$. For a distortion free output signal waveform, $G(s)$ must
   (a) Provide zero phase shift for all frequency
   (b) Provide constant phase shift for all frequency
   (c) Provide linear phase shift that is proportional to frequency
   (d) Provide a phase shift that is inversely proportional to frequency [Gate 2007]

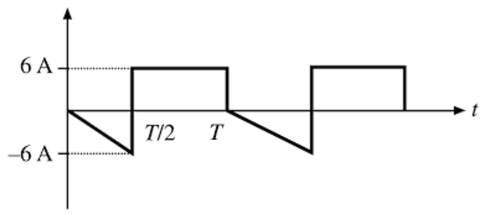
20. $G(z) = \alpha z^{-1} + \beta z^{-3}$ is allow pass digital filter with a phase characteristics same as that of the above question if
   (a) $\alpha = \beta$
   (b) $\alpha = -\beta$
   (c) $\alpha = \sqrt[3]{\beta}$
   (d) $\alpha = -\sqrt[3]{\beta}$ [Gate 2007]

21. A zero mean random signal is uniformly distributed between limits $-a$ and $+a$ and its mean square value is equal to its variance. Then rms value of the signal is
   (a) $\dfrac{a}{\sqrt{3}}$ (b) $\dfrac{a}{\sqrt{2}}$
   (c) $a\sqrt{2}$ (d) $a\sqrt{3}$ [Gate 2011]

22. The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is
   (a) Time invariant and stable
   (b) Stable and not time invariant
   (c) Time invariant and not stable
   (d) Not time invariant and not stable [Gate 2012]

23. Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t - \tau)$ be $F_2(s) \cdot F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set as $s = \sigma + j\omega$. If $G(s) = \dfrac{F_2(s)F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is
   (a) An ideal impulse $\delta(t)$
   (b) An ideal delayed impulse $\delta(t - \tau)$
   (c) An ideal step function $u(t)$
   (d) An ideal delayed step function $u(t - \tau)$ [Gate 2011]

24. The rms value of the periodic waveform given in figure is



   (a) $2\sqrt{6}$ A (b) $6\sqrt{2}$ A
   (c) $\sqrt{4/3}$ A (d) 1.5 A [Gate 2004]

25. The rms value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 A is
   (a) 14.1 A (b) 17.3 A
   (c) 22.4 A (d) 30.0 A [Gate 2004]

26. Consider the function, $F(s) = \dfrac{5}{s(s^2 + 3s + 2)}$ where $F(s)$ is the Laplace transform of the function $f(t)$. The initial value of $f(t)$ is equal to
   (a) 5 (b) $\dfrac{5}{2}$
   (c) $\dfrac{5}{3}$ (d) 0 [Gate 2004]

or $$y(t) = -2e^{-t} - te^{-t}$$

$$\frac{dy(t)}{dt} = 2e^{-t} - (-te^{-t} + e^{-t})$$

$$= e^{-t} + te^{-t}$$

$$\therefore \quad \left.\frac{dy(t)}{dt}\right|_{t=0^+} = 1 + 0 = 1$$

2. Ans: (d)

Hint: The given signal is even and half wave symmetry.

So, $b_n = 0$

Again, $a_n = 0$ for $n = 2, 4, 6 \ldots \infty$

Hence, a_0, a_n for $n = 1, 3, 5, \ldots \infty$ is present

3. Ans: (a)

Hint: $$X(s) = \frac{1}{s+1}$$

$$Y(s) = \frac{1}{s+2}$$

Now,

$$Z(s) = X(s)\, Y(s)$$

$$= \frac{1}{s+1} \times \frac{1}{s+2}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$z(t) = e^{-t} - e^{-2t}$$

4. Ans: (d)

Hint: As, $\mathrm{L}\left[f(t)\right] = \mathrm{F}(s)$

$$\therefore \quad \mathrm{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathrm{F}(s)$$

Given,

$$\mathrm{L}\left[f(t)\right] = \frac{1}{s^2 + s + 1} = \mathrm{F}(s)$$

So, $$\mathrm{L}\left[t\, f(t)\right] = (-1)^n \frac{d^1}{ds^1}\mathrm{F}(s)$$

$$= -\frac{d}{ds}\mathrm{F}(s)$$

$$= -\left[\frac{-1}{(s^2+s+1)^2}\right] \times (2s+1)$$

$$= \left[\frac{2s+1}{(s^2+s+1)^2}\right]$$

5. Ans: (a)

Hint: Given $g[n]$ is a causal sequence

$\therefore$ $g[n]$ will be zero for $n < 0$

$$y[n] = \sum_{k=-\infty}^{\infty} g[k]h[n-k]$$

and $$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & ; \; n \geq 0 \\ 0 & ; \; n < 0 \end{cases}$$

So,

$$h[0] = 1,\; h[1] = \frac{1}{2},\; h[2] = \frac{1}{4}, \ldots$$

Now, $y[0] = g[0]h[0] + g[1]h[-1] + \cdots$

or $$1 = g[0] \times 1 + 0 + 0 + \cdots$$

or $$g[0] = 1$$

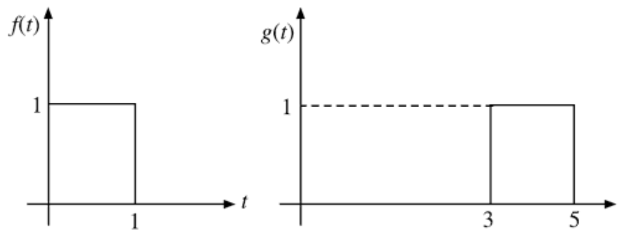
Again, $y[1] = g[0]h[1] + g[1]h[0] + g[2]h[-1] + \cdots$

or $$\frac{1}{2} = 1 \cdot \frac{1}{2} + g[1] \cdot 1 + 0 \ldots$$

or $$g[1] = 0$$

6. Ans: (d)

Hint:



Now, $g(t)$ has width of 2 units and $f(t)$ has 1 unit.

Therefore, $f_1(t) = f\left(\frac{1}{2}t\right)$

Now, shift $f_1(t)$ by three units to get $g(t)$.

$$\therefore \quad g(t) = f_1(t-3)$$

or $$g(t) = f\left(\frac{t-3}{2}\right)$$

$$= f\left(\frac{t}{2} - \frac{3}{2}\right)$$

7. Ans: (c)

Hint: Now, $g(t) = u(t) \quad 3 < t < 5$

$= 0 \quad t < 3,\; t > 5$

So, $$g(t) = u(t-3) - u(t-5)$$

$$\mathrm{L}\{g(t)\} = \mathrm{L}\{u(t-3)\} - \mathrm{L}\{u(t-5)\}$$

$$= \frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s}$$

$$= \frac{e^{-3s}}{s}\left(1 - e^{-2s}\right)$$

8. Ans: (a)

Hint: $y(z) = x(z)H(z)$

$$y(z) = (4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3)(3z^{-1} - 2)$$
$$= -4z^3 + 18z^2 - 18z + 12z^{-4} + 9z^{-2} - 4 - 8z^{-3}$$

So,
$$y[n] \neq 0 \text{ for } n < 0$$

Therefore, $y(n)$ is non-causal with finite support.

9. Ans: (a)

Hint: The given periodic waveform is odd as well as having half wave symmetry.

So, it has only sine terms with odd harmonics.

So, for second harmonic component has an amplitude = 0.

10. Ans: (d)

Hint: System is causal

So, its impulse response will be zero for $t < 0$.

Hence, Statement II is correct.

Again, system is linear

So, it satisfies the principle of superposition.

$\therefore$ Both the statements are correct.

11. Ans: (b)

Hint: $h[n] = x(n-1) * y[n]$

or $H[z] = z^{-1}X(z)\cdot Y(z)$

or $H[z] = z^{-1}(1 - 3z^{-1})(1 + 2z^{-2})$

or $H[z] = z^{-1}(1 + 2z^{-2} - 3z^{-1} - 6z^{-3})$

or $H[z] = (z^{-1} - 3z^{-2} + 2z^{-3} - 6z^{-4})$

When input

$$R(n) = \delta[n-1]$$

then $R[z] = z^{-1}$

Therefore, output $Y(z) = H(z)\ R(z)$

or $Y(z) = (z^{-2} - 3z^{-3} + 2z^{-4} - 6z^{-5})$

$\therefore\ Y(n) = \delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$

12. Ans: (d)

Hint: $Y(s) = H(s)\cdot X(s)$

When input $= x(t-\tau)$, L.T. $= X(s)\cdot e^{-s\tau}$

Impulse response $= h(t-\tau)$, L.T. $= H(s)\cdot e^{-s\tau}$

Now, output $Y(s) = X(s)\cdot e^{-s\tau}\cdot H(s)\cdot e^{-s\tau}$

$$= X(s)\cdot H(s)\cdot e^{-2s\tau}$$

So, inverse L.T. gives output, $y(t-2\tau)$

13. Ans: (d)

Hint: $X(z) = \dfrac{z}{(z-a)^2}$

$\therefore\quad z^{n-1}X(z) = \dfrac{z^n}{(z-a)^2}$

Since, $z = a$ is a pole of second order

Therefore, residue at $z = a$

$$= \frac{1}{1!}\left[\frac{d}{dz}\left\{(z-a)^2\frac{z^n}{(z-a)^2}\right\}\right]_{z=a}$$

$$= na^{n-1}$$

14. Ans: (d)

Hint: $\displaystyle\int_{-\infty}^{\infty}\left|X(\omega)^2\right|d\omega = \int_{-\infty}^{\infty}X(\omega)\cdot X^*(\omega)d\omega$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt\right]\cdot X^*(\omega)d\omega$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}X^*(\omega)e^{-j\omega t}d\omega\right]x(t)dt$$

$$= 2\pi\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\frac{1}{2\pi}\left\{X(\omega)e^{-j\omega t}\right\}^* d\omega\right]x(t)dt$$

$$= 2\pi\int_{-\infty}^{\infty}x^*(t)x(t)dt$$

$$= 2\pi\int_{-\infty}^{\infty}|x(t)|^2 dt$$

$$= 2\pi\int_{-1}^{1}1dt$$

$$= 4\pi$$

15. Ans: (c)

Hint: Let $x_1[n] = \left(\dfrac{1}{3}\right)^{|n|}$

and $x_2[n] = \left(\dfrac{1}{2}\right)^n u[n]$

So, $x_1[n] = \left(\dfrac{1}{3}\right)^n u[n] + \left(\dfrac{1}{3}\right)^{-n} u[-n-1]$

$$\left(\frac{1}{3}\right)^n u[n] \xleftrightarrow{z} \frac{1}{1-\frac{1}{3}z^{-1}};\ \text{ROC}: |z| > \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^{-n} u[-n-1] \xleftrightarrow{z} \frac{-1}{1-\left(\frac{1}{3}\right)^{-1}z^{-1}};\ \text{ROC}: |z| < 3$$

$$\left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{z} \frac{1}{1-\frac{1}{2}z^{-1}};\ \text{ROC}: |z| > \frac{1}{2}$$

$\therefore$ ROC is $\dfrac{1}{2} < |z| < 3$

16. Ans: (b)

Hint: $y[n] = x[3-4n] = x[-4n+3]$

Now, to obtain $y[n]$ we first advance $x[n]$ by 3 units

So, $z_1[n] = x[n+3]$

Now, we will pick every fourth sample of $z_1[n]$

So, $z_2[n] = z_1[4n] = x[4n+3]$

Now, time reverse $z_2[n]$ will give

$$y[n] = z_2[-n] = x[-4n+3]$$

17. Ans: (c)

Hint: For causal system, if all the poles are inside the unit circle then system is stable. Converse is true for anti-causal system.

18. Ans: (d)

Hint: e^{-x} is always convergent even when x is not bounded.

$\therefore$ e^{-x} is bounded even though x is not bounded.

19. Ans: (c)

Hint: For distortion free output, phase shift must be proportional to frequency; this is because delay to all frequency components will be equal.

20. Ans: (a)

Hint: For distortion free output, phase shift must be linear function of ω as well as all the frequency component must be amplified by same amount.

$z^{-1} = e^{-j\omega}$ corresponds to frequency ω.

$z^{-3} = e^{-3j\omega}$ corresponds to frequency 3ω.

So, for same amplification of frequency component at ω and 3ω, $\alpha = \beta$.

21. Ans: (a)

Hint: Mean square value $= \dfrac{1}{2a}\displaystyle\int_{-a}^{a} x^2 dx = \dfrac{a^2}{3}$

The rms value $= \dfrac{a}{\sqrt{3}}$

22. Ans: (d)

Hint: $$y = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)\,d\tau$$

$y'(t)$ for input $x(t-t_0)$ is

$$y'(t) = \int_{-\infty}^{t} x(\tau - t_0)\cos(3\tau)d\tau$$

$$y'(t) = \int_{-\infty}^{t-t_0} x(\tau)\cos 3(\tau + t_0)d\tau$$

$y'(t) \neq y(t-t_0)$ so system is not time invariant for input $x(\tau) = \cos(3\tau)$ bounded input.

$$y(t) = \int_{-\infty}^{t} \cos^2(3\tau)\,d\tau \to \infty \quad \text{as} \quad t \to \infty$$

So for bounded input, output is not bounded, therefore, system is not stable.

23. Ans: (b)

Hint: $\mathrm{F}_2(t) = \mathrm{L}\{f(t-\tau)\} = e^{-s\tau}\mathrm{F}_1(s)$

$$G(s) = \frac{e^{-s\tau}\mathrm{F}_1(s)\mathrm{F}_1^*(s)}{|\mathrm{F}_1(s)|^2} = e^{-s\tau}$$

$$G(t) = \delta(t-\tau)$$

24. Ans: (a)

Hint: $$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T i^2(t)dt\right]^{1/2}$$

$$\therefore \quad I_{\text{rms}}^2 = \left[\frac{1}{T}\int_0^{T/2}\left(-\frac{12t}{T}\right)^2 dt + \frac{1}{T}\int_{T/2}^{T} 6^2\,dt\right]$$

$$= \left[\frac{1}{T}\int_0^{T/2}\frac{144t^2}{T^2}dt + \frac{1}{T}\int_{T/2}^{T} 36dt\right]$$

$$= \frac{144}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2} + \frac{36}{T}[t]_{T/2}^{T}$$

$$= \frac{144}{8\times 3} + \frac{36}{2} = 24$$

$$\therefore \quad I_{\text{rms}} = 2\sqrt{6}\text{ A}$$

25. Ans: (b)

Hint: The rms value of dc current = 10 A

The rms value of alternating current

$$= 20/\sqrt{2} = 10\sqrt{2}\text{ A}$$

$\therefore$ rms value of resultant current

$$= \sqrt{10^2 + \left(10\sqrt{2}\right)^2} = 17.32\text{ A}$$

26. Ans: (d)

Hint: $$\mathrm{F}(s) = \frac{5}{s(s^2+3s+2)}$$

By initial value theorem,

$$\underset{t\to 0}{\text{Lt}}\, f(t) = \underset{s\to\infty}{\text{Lt}}\, s\mathrm{F}(s)$$

$$= \underset{s\to\infty}{\text{Lt}}\, \frac{5\times s}{s(s^2+3s+2)} = 0$$

27. Ans: (a)

Hint: From the wave symmetry

$$V_{\text{rms}} = \left[\frac{1}{T}\left\{\int_0^{T/2}\left(\frac{2t}{T}\right)^2 dt + \int_{T/2}^{T} 0dt\right\}\right]^{1/2}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^{T/2}\frac{4t^2}{T^2}dt = \frac{4}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2} = \frac{1}{6}$$

$\therefore \quad V_{rms} = \sqrt{\frac{1}{6}}$ V

28. Ans: (d)

Hint: $f(x) = \sin^2 x$

For finding the Fourier series expansion

$$f(x) = \sin^2 x = \frac{1-\cos 2x}{2}$$

$$= 0.5 - 0.5\cos 2x$$

29. Ans: (a)

Hint: By final value theorem,

$$\underset{t\to\infty}{\text{Lt}}\, f(t) = \underset{s\to 0}{\text{Lt}}\, s\text{F}(s) = \frac{6}{2} = 3$$

30. Ans: (b)

Hint: $\text{F}(z) = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$

So, the inverse Z-transform of $f(z)$

$$= \delta(k) - (-1)^k u(k)$$

31. Ans: (d)

Hint: $V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \frac{1}{C}\int_{-\infty}^{t} 5\delta(t)\,dt$

$$= \frac{5}{C}u(t)$$

32. Ans: (a)

Hint: $y(t) = u(t) * h(t)$ (always)

where $h(t)$ is impulse response

$\because$ $s(t)$ is step response (given)

or $\quad \frac{d}{dt}s(t) = h(t)$

$\therefore \quad y(t) = u(t) * \frac{d}{dt}s(t)$

$= \frac{d}{dt}[u(t) * s(t)]$ (using property)

$= \frac{d}{dt}\int_{-\infty}^{\infty} u(\tau)s(t-\tau)d\tau$ (by definition of convolution)

$= \frac{d}{dt}\int_{0}^{t} u(\tau)s(t-\tau)d\tau$ ($\because u(t)$ and $s(t)$ are causal functions)

33. Ans: (c)

Hint: $H(s) = \text{L}\left[e^{-2t}u(t)\right] = \frac{1}{s+2}$

$Y(s) = \text{L}\left[te^{-2t}u(t)\right] = \frac{1}{(s+2)^2}$

or $\quad X(s) = \frac{Y(s)}{H(s)} = \frac{1}{s+2}$

Hence, input $x(t) = \text{L}^{-1}\left[X(s)\right] = e^{-2t}u(t)$

34. Ans: (b)

Hint: Impulse response $= -4e^{-t} + 6e^{-2t}$

Step response $= \int_{-\infty}^{t}(\text{impulse response})\,dt$

$= 4e^{-t} - 3e^{-2t}$ + Constant

Hence, option (b) is correct.

35. Ans: (d)

36. Ans: (d)

Hint: $\because \quad \text{F}(s) = \frac{\omega}{s+\omega^2}$

or $f(t) = \sin(\omega t)u(t)$ which is a periodic signal so final value theorem is not applicable on it.

Final value of $f(t)$ may be any value between – 1 and + 1.

37. Ans: (c)

Hint: $\text{L}(t^2 - 2t)u(t-1) = \text{L}[(t-1)^2 u(t-1) - u(t-1)]$

$$= \frac{2e^{-s}}{s^3} - \frac{e^{-s}}{s}$$

38. Ans: (c)

Hint: Final value $= \underset{s\to 0}{\text{Lim}}\, sY(s)$

9

Engineering Mathematics

9.1 SYLLABUS

Linear algebra: Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.

Calculus: Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, fourier series, vector identities, directional derivatives, line, surface and volume integrals, Stokes, Gauss and Green's theorems.

Differential equations: First order equations (linear and non-linear), higher order linear differential equations with constant coefficients, method of aviation of parameters, Cauchy's and Euler's equations, initial and boundary value problems, partial differential equations and variable separable method.

Complex variables: Analytic functions, Cauchy's integral theorem and integral formula, Taylor and Laurent series, residue theorem and solution integrals.

Probability and statistics: Sampling theorems, conditional probability, mean, median, mode and standard deviation, random variables, discrete and continuous distributions, Poisson, normal and binomial distribution, correlation and regression analysis.

Numerical methods: Solutions of non-linear algebraic equations, single and multi-step methods of differential equations.

Transform theory: Fourier transforms, Laplace transforms, and Z-transforms.

9.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**		
										Set 1	Set 2	Set 3
1 Mark Question	4	–	1	3	1	2	3	4	4	5	5	3
2 Marks Question	5	5	8	5	5	4	4	3	4	4	3	3
Total Marks	**14**	**10**	**17**	**13**	**11**	**10**	**11**	**10**	**12**	**13**	**11**	**9**

9.3 TOPICS TO BE FOCUSED

LINEAR ALGEBRA

Matrix

Arthur Cayley (1821–1895) developed matrix. Heisenberg used matrices to explain his quantum theory.

Definition with example

$$x + y + z = 12$$
$$2x - z = -5$$
$$5x + 2y + 3z = 27$$

If we arrange the co-efficient of x, y, z in the order in which they occur in the given equations and enclose those brackets, we get

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ 5 & 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 12 \\ -5 \\ 27 \end{bmatrix}$$

$\therefore$ A matrix is not just a collection of elements but is a collection in which each array has a very definite position i.e., it is an ordered array of elements/entries.

Symbol: [ ],( ),‖ ‖

If A is a matrix of order $m \times n$, then we shall write $A = [a_{ij}]_{m\times n}$

Types of Matrices

1. Rectangular matrix: If $m \neq n$ i.e., if the number of rows and number of columns are not equal.

2. Square matrix: If $m = n$

3. Row matrix: If $m = 1$, $n = n$, i.e., a single row matrix is called a row matrix or row vector. $[1 \ \ -2 \ \ 5]$

4. Column matrix: If $m = m$, $n = 1$, i.e., a single column matrix is called a column matrix or column vector.

5. Null matrix: If all the elements are zero i.e., $0_{m \times n} = 0$.

6. Diagonal matrix: A square matrix in which all its elements are zero except those in the leading diagonal.

Thus, $a_{ij} = 0$ if $i \neq j$

Example: $A = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

7. Unit matrix or identity matrix: If all the elements of the leading diagonal is 1, then it is called unit matrix or identity matrix.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

8. Scalar matrix: A square matrix in which all the diagonal elements are equal and all other elements equal to zero is called a scalar matrix.

In a scalar matrix

$$a_{ij} = k \text{ for } i = j$$

And $$a_{ij} = 0 \text{ for } i \neq j$$

Thus, $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ is a scalar matrix.

9. Equal matrices

$A = B$ will be equal if

(i) They are both of the same order that is comparable.

(ii) The elements in the corresponding places of the two matrices are the same.

10. Singular and non-singular matrix

Singular matrix

(i) Matrix should be square

(ii) $|A| = 0$ i.e., $\det A = 0$

Non-singular matrix

(i) Matrix should be square

(ii) $|A| \neq 0$ i.e., $\det A \neq 0$

11. Symmetric and skew–symmetric matrix

Symmetric matrix

(i) Must be square matrix

(ii) If $a_{ij} = a_{ji}$ is true

(iii) $A = A^T$

(iv) $\frac{1}{2}\left(A + A^T\right)$ is a symmetric matrix

Skew–symmetric matrix

(i) Must be square matrix

(ii) If $a_{ij} = -a_{ji}$ is true

(iii) $A^T = -A$

(iv) $\frac{1}{2}\left(A - A^T\right)$ is a skew–symmetric matrix

12. Transpose of a matrix: Let A be a $m \times n$ matrix, then the matrix of order $n \times m$ obtained by changing rows into columns and columns into rows and it is called transpose of A. It is designated by $A^T = A' = A^t$.

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \therefore \quad A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Properties:

(i) $(A^T)^T = A$

(ii) $(A + B)^T = A^T + B^T$

(iii) $(A - B)^T = A^T - B^T$

(iv) $(AB)^T = B^T . A^T$

(v) $(kA)^T = kA^T$ if k is any number.

(vi) $A = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right)$

13. Operations of matrices

A. *Scalar multiplication*

Let $A = [a_{ij}]_{m\times n}$ be a matrix and k is a scalar, then the matrix obtained by multiplying each element of matrix A by k is called the scalar multiplication. It is denoted by kA or Ak.

If $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \end{bmatrix}$ then $3A = \begin{bmatrix} 3 & 12 & 15 \\ 9 & 6 & 18 \end{bmatrix}$

B. *Addition*

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$

$\therefore \quad A + B = [a_{ij} + b_{ij}]_{m\times n}$ where $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$

Caution: If both the matrix are not of same order, then they cannot be added up i.e., they are not defined from addition point of view.

C. *Subtraction*

$$A - B = A + (-B)$$
$$= A + (-1)B$$

And then similar as addition.

Properties

(i) $A + B = B + A$
(ii) $(A + B) + C = A + (B + C)$
(iii) $k(A + B) = kA + kB$ (k is a scalar)
(iv) $A + 0 = 0 + A = A$
(v) $A + (-A) = (-A) + A = 0$
(vi) If $A + C = B + C$ then $A = B$ must be true

D. *Multiplication*

Let $A=[a_{ij}]_{m\times n}$ and $B=[a_{jk}]_{n\times p}$, then $C=[c_{ik}]_{m\times p}$

Example:

$$X=\begin{bmatrix}4 & 2\\ -1 & 3\end{bmatrix},\ Y=\begin{bmatrix}3 & -1\\ 5 & 2\end{bmatrix}$$

$$\therefore\quad XY=\begin{bmatrix}4\times3+2\times5 & 4\times(-1)+2\times2\\ -1\times3+3\times5 & -1\times-1+3\times2\end{bmatrix}=\begin{bmatrix}22 & 0\\ 12 & 7\end{bmatrix}$$

Properties

(i) $AB\neq BA$ (i.e., does not hold the commutative law generally)
(ii) $(AB)C = A(BC)$
(iii) $A(B + C) = AB + AC$
(iv) If $CA = CB$, then $A = B$ may not be true
(v) $A\cdot0=0\cdot A=0$
(vi) $A\cdot I=I\cdot A=A$
(vii) If $A\neq0, B\neq0$ then also $AB = 0$ can be true

14. Adjoint matrix/adjugate matrix

(i) A must be a square matrix.
(ii) Determine the co-factors. If A is a square matrix, then co-factor of a_{ij} in $|A|$ denoted by c_{ij} is $(-1)^{i+j}$ times the determinant obtained by leaving the ith row and jth column passing through the element a_{ij} in $|A|$.
(iii) Transpose the co-factor matrix is called the *Adj* or *adj A*.
(iv) Calculate det $A = |A|$. If $|A| = 0$, matrix inverse does not exists.
(v) If $|A|\neq0$, then $A^{-1}=\dfrac{AdjA}{|A|}$

[***Remember:*** Minor and co-factors are defined for square matrix only]

Properties

(i) $(A^{-1})^{-1} = A$
(ii) $(A^{-1})^T = (A^T)^{-1}$
(iii) $(AB)^{-1}=B^{-1}\cdot A^{-1}$
(iv) $A^{-1}\cdot A=A\cdot A^{-1}=I$
(v) Only square matrix have inverse

15. Orthogonal matrix

Condition

(i) If $A\cdot A^T=I=A^T\cdot A$, then this is orthogonal matrix.
(ii) If $|A| = 1$, then this is proper orthogonal.
(iii) If $|A| = -1$, then this is improper orthogonal.

Remember

(i) Product of two orthogonal matrix are also orthogonal.
(ii) If A is orthogonal, then $\overline{A}$ and A^{-1} is an orthogonal matrix.

16. Idempotent matrix: If $A^2 = A$, for a square matrix A.

17. Involuntary matrix: If $A^2 = A$.

18. Nilpotent matrix: If $A^P = 0$ when P is a positive number

19. Rank of a matrix: The rank of a matrix is the order of any highest order non-singular matrix.

20. Conjugate of a matrix

If $A=\begin{bmatrix}1 & 2+3i\\ 2 & 2+7i\end{bmatrix}$ then conjugate of $A=\overline{A}=\begin{bmatrix}1 & 2-3i\\ 2 & 2-7i\end{bmatrix}$

21. Hermitian and Skew–Hermitian matrix: If $A=\overline{A}$, then matrix is called Hermitian matrix.

If $A=-\overline{A}$, then matrix is called Skew–Hermitian matrix.

22. Triangular matrix: If every element above or below the leading diagonal is zero, then the matrix is called a triangular matrix.

Example:

$$A=\begin{bmatrix}4 & 0 & 0\\ 8 & 7 & 0\\ 3 & 2 & 1\end{bmatrix}$$

Again it may be of two types: (i) Upper triangular matrix, and (ii) Lower triangular matrix.

23. Comparable matrices: Two matrices are said to be comparable, when each of them has as many rows and columns as the other.

Example:

$$A=\begin{bmatrix}1 & 3 & 0\\ 5 & 6 & 3\\ 2 & 1 & 4\end{bmatrix}\quad\text{and}\quad B=\begin{bmatrix}4 & 6 & 1\\ 3 & 9 & 2\\ 1 & 5 & 6\end{bmatrix}$$

24. Submatrix: A matrix which is obtained by deleting any number of rows or columns from a given matrix is called a submatrix of the given matrix.

Example

$$\begin{bmatrix}1 & 4\\ 3 & 6\\ 7 & 8\end{bmatrix}\text{ is a submatrix of }\begin{bmatrix}1 & 4 & 0\\ 3 & 6 & 3\\ 7 & 8 & 2\end{bmatrix}$$

Now, if we write $b = a + h$ then since $a < k < b$, $k = a + \theta h$ where $0 < \theta < 1$, then it can be stated as $f(a + h) = f(a) + hf'(a + \theta h)$, at least one number $\theta (0 < \theta < 1)$.

Cauchy's Mean Value Theorem

Suppose $f(x)$ and $g(x)$ be two real valued functions such that

(i) If $f(x)$ and $g(x)$ both are continuous in $[a, b]$
(ii) If $f'(x)$ and $g'(x)$ both exist in (a, b)
(iii) $g'(x) \neq 0$ for all values of $x \in (a,b)$

Then there exists at least one point $k \in (a,b)$ such that

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(k)}{g'(k)}$$

Cauchy's mean-value theorem is a generalisation of Lagrange's mean value theorem.

Taylor's Theorem (Generalised mean value theorem)

Suppose $f(x)$ be a real valued function and if

(i) $f(x), f'(x), f''(x) \ldots f^{n-1}(x)$ are all continuous in $[a, a + h]$ and
(ii) $f^n(x)$ exists for every value of x in $(a, a + h)$ where $h = b - a$

And if there exists at least one number $\theta(0 < \theta < 1)$ then

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{(2)!} f''(a) + \cdots + \frac{h^n}{(n)!} f^n(a+\theta h)$$

Here, $R_n(h) = \frac{h^n}{(n)!} f^n(a+\theta h)$ is called the Lagrange's form of remainder in Taylor's theorem.

Note: Taking $n = 1$, then we have

$$f(a+h) = f(a) + hf'(a+\theta h)$$

which is known as Lagrange's mean value theorem. Putting $a = 0$, $h = x$, Taylor's theorem becomes

$$f(x) = f(0) + xf'(0) + \frac{x^2}{(2)!} f''(0) + \cdots$$
$$+ \frac{x^n}{(n)!} f^n(\theta x), \quad \text{where } 0 < \theta < 1$$

which is called Maclaurin's theorem with Lagrange's form of remainder.

If $f(x)$ can be expanded as an infinite series, then

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \cdots \infty$$

because R_n tends to zero when $n \to \infty$.

Taylor's Infinite Series

If $f(x)$ be a function such that $f(x), f'(x), f''(x)$.... exist in $[a, b]$ and the remainder $R_n = \frac{h^n}{(n)!} f^n(a+\theta h) \to 0$ as $n \to \infty$, then

$$f(a+h) = f'(a) + hf''(a) + \frac{h^2}{(2)!} f''(a) + \cdots \infty$$

Try to remember: If $a + h = x$, then Taylor's infinite series can be written as

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{(2)!} f''(a) + \cdots \infty$$

If we put $a = 0$, we get Maclaurin's series.

MAXIMA AND MINIMA

Let a function $f(x)$ be defined on its domain D, and assume that $f(x)$ has a relative extreme at an interior point c of D. If $f'(c)$ exists, then $f'(c) = 0$.

Local maximum: A function $f(x)$ is said to attain a local maximum at $x = a$ if there exists a neighbourhood $(a - \delta, a + \delta)$ of a such that,

$f(x) < f(a)$ for all $x \in (a-\delta, a+\delta)$, $x \neq a$

or, $f(x) - f(a) < 0$ for all $x \in (a-\delta, a+\delta)$, $x \neq a$

In such a case $f(a)$ is called to attain a local maximum value of $f(x)$ at $x = a$.

Local minimum: $f(x) > f(a)$ for all $x \in (a-\delta, a+\delta)$, $x \neq a$
or, $f(x) - f(a) > 0$ for all $x \in (a-\delta, a+\delta)$, $x \neq a$.

In such a case $f(a)$ is called to attain a local minimum value of $f(x)$ at $x = a$.

Some important points to be remembered

(i) The points at which a function attains either the local maximum value or local minimum values are known as the extreme points or turning points and both local maximum and local minimum values are called the extreme values of $f(x)$. Thus, a function attains an extreme value at $x = a$ if $f(a)$ is either a local maximum value or a local minimum value. Consequently at an extreme point a, $f(x) - f(a)$ keeps the same sign for all values of x in a deleted neighbourhood (nbd) of a.
(ii) A necessary condition for $f(a)$ to be an extreme value of a function $f(x)$ is that $f'(a) = 0$ in case it exists.
(iii) This condition is only a necessary condition for the point $x = a$ to be an extreme point. It is not sufficient i.e., $f'(a) = 0$ does not necessarily imply that $x = a$ is an extreme point. There are functions for which the derivatives vanish at a point but do not have an extreme value. For example, the function $f(x) = x^3$, $f'(0) = 0$ but at $x = 0$ the function does not attain an extreme value.
(iv) Geometrically the above condition means that the tangent to the curve $y = f(a)$ at a point where the

ordinate is maximum or minimum is parallel to the x-axis.

(v) A function may have a local extreme at a point but may not be differentiable at that point.

(vi) Vanishing of the derivative of a function at a point is not sufficient condition for the existence of extreme at that point.

First order derivative test

(a) $x = a$ is a point of local maximum of $f(x)$ if

(i) $f'(a) = 0$

(ii) $f'(x)$ changes sign from positive to negative as x passes through a i.e., $f'(x) > 0$ at every points in the left nbd $(a - \delta, a)$ of a and $f'(x) < 0$ at every point in the right nbd $(a, a + \delta)$ of a.

(b) $x = a$ is a point of local minimum of $f(x)$ if

(i) $f'(a) = 0$

(ii) $f'(x)$ changes sign from negative to positive as x passes through a i.e., $f'(x) < 0$ at every points in the left nbd $(a - \delta, a)$ of a and $f'(x) > 0$ at every point in the right nbd $(a, a + \delta)$ of a.

(c) If $f'(a) = 0$, but $f'(x)$ does not change sign, that is, $f'(a)$ has the same sign in the complete nbd of a, then a is neither a point of local maximum nor a point of local minimum.

Second order derivative test: Let a function $f(x)$ be differentiable in an interval I and interior point of I such that $f'(c) = 0$, then

(i) $f(x)$ has a local maximum at $x = c$ if $f''(c) < 0$.

(ii) $f(x)$ has a local minimum at $x = c$ if $f''(c) > 0$.

(iii) Nothing can be predicted about the occurrence of local maximum or minimum at $x = c$ if $f''(c) = 0$.

Higher order derivative test: Let f be a differentiable function on an interval I and let c be an interior point of I such that

(i) $f'(c) = f''(c) = f'''(c) = \cdots = f^{n-1}(c) = 0$, and

(ii) $f^n(c)$ exists and is non-zero. Then,

If n is even and $f^n(c) < 0 \Rightarrow x = c$ is a point of local maximum

If n is even and $f^n(c) > 0 \Rightarrow x = c$ is a point of local minimum

If n is odd $\Rightarrow x = c$ is neither a point of local maximum nor a point of local minimum

Properties of maxima and minima

(i) If $f(x)$ is continuous function in its domain, then at least one maxima and one minima must lie between two equal values of x.

(ii) Maxima and minima occur alternatively, that is, between two maxima there is one minima and vice-versa.

(iii) If $f(x) \to \infty$ as $x \to a$ or b and $f'(x) = 0$ only for one value of x (say c) between a and b, then $f(c)$ is necessarily the minimum and the least value.

(iv) If $f(x) \to -\infty$ as $x \to a$ or b and $f(c)$ is necessarily the maximum and the greatest value.

Maximum and minimum values in a closed interval: Let $y = f(x)$ be a real valued function defined on $[a, b]$. By a local maximum (or local minimum) value of a function at a point $c \in [a,b]$ we mean the greatest (or the least) value in the immediate neighbourhood of $x = c$. It does not mean the greatest or absolute maximum (or the least or absolute minimum) of $f(x)$ in the interval $[a, b]$. A function may have a number of local maxima or local minima in a given interval and even a local minimum may be greater than a relative maximum.

PARTIAL DIFFERENTIATION

Three variables u, x, y are so related that for pair of variables of x and y are within the defined domain.

- Let $u = f(x, y)$ be a function of two independent variables x, y in a defined domain R. If y is treated as a constant, then $u = f(x, y)$ becomes a function of x alone. Its derivative (if it exists) is called the partial derivative of u with respect to x i.e., if $\underset{\Delta x \to 0}{\text{Lt}} \dfrac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$ exists then the limiting value is called the partial derivative of $u = f(x, y)$ with respect to x. It is denoted by $\partial u/\partial x$. The symbols $f_x(x, y)$ or briefly $f_x, u_x, \dfrac{\partial f}{\partial x}$ etc. are also used.
- If the partial derivative of $u = f(x, y)$ exists at an interior point (a, b) of R, then it is denoted by $f_x(a, b)$. If x is treated as a constant, then $u = f(x, y)$ becomes a function of y alone, its derivative (if it exists) is called the partial derivative of u with respect to y i.e., if $\underset{\delta y \to 0}{\text{Lt}} \dfrac{f(x+y+\delta y) - f(x,y)}{\delta y}$, exists then the limiting value is called the partial derivative of $u = f(x, y)$ with respect to y. It is denoted by $\partial u/\partial y$. The symbols $\dfrac{\partial f}{\partial y}, f_y(x,y), u_y$ etc. are also used.

Illustrations

Let $u = x^2 + 2xy + 3y^2$; then $\dfrac{\partial u}{\partial x} = 2x + 2y; \dfrac{\partial u}{\partial y} = 6y + 2x$

Remember: The curl δ is generally used to denote the symbol of partial derivatives.

If $u = f(x_1, x_2, ..., x_n)$ i.e., a function of n variables $(x_1, x_2, ..., x_n)$, then the derivative of f with respect to x_1 is the ordinary derivative of f when all the variables except x_1 are regarded as constants. The partial differential coefficient of f with respect to x_1 is denoted by $\partial f/\partial x$.

Since, the partial derivative f_x and f_y are also functions of x and y, they can be differentiated partially with respect to each of the independent variables. For the function $u = f(x, y)$, we have the following second order partial derivatives:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial x^2}=f_{xx}=\lim_{\Delta x\to 0}\frac{f_x(x+\Delta x,y)-f_x(x,y)}{\Delta x}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial y\partial x}=f_{xy}=\lim_{\Delta y\to 0}\frac{f_x(x,y+\Delta y)-f_x(x,y)}{\Delta y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial x\partial y}=f_{yx}=\lim_{\Delta x\to 0}\frac{f_y(x+\Delta x,y)-f_y(x,y)}{\Delta x}$$

And, $$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial y^2}=f_{yy}=\lim_{\Delta y\to 0}\frac{f_x(x,y+\Delta y)-f_y(x,y)}{\Delta y}$$

Note: $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}$

$f(x, y)$ may have the above mentioned four second order partial derivatives. These, in turn, yield Eigen third order partial derivative.

INTEGRATION

Primitive or Anti-derivative

A function $\phi(x)$ is called a primitive or anti-derivative of a function $f(x)$ if $\phi'(x) = f(x)$.

Indefinite Integral

(i) Let $f(x)$ be a function. Then the collection of all its primitives is called indefinite integral of $f(x)$ and is denoted by $\int f(x)\,dx$.

(ii) $\dfrac{d}{dx}\big(\phi(x)+C\big)=f(x)\Leftrightarrow\int f(x)dx=\phi(x)+C,$ where $\phi(x)$ is primitive of $f(x)$ and C is an arbitrary constant known as the constant of integration.

(iii) Integration of a function $f(x)$ means finding a function $\phi(x)$ such that $\dfrac{d}{dx}\big(\phi(x)\big)=f(x)$

Fundamental Integration Formulas

(i) $\dfrac{d}{dx}\left(\dfrac{x^{n+1}}{n+1}\right)=x^n,\ n\neq -1 \quad \Rightarrow \int x^n dx=\dfrac{x^{n+1}}{n+1}+C, n\neq -1$

(ii) $\dfrac{d}{dx}(\log x)=\dfrac{1}{x} \quad \Rightarrow \int \dfrac{1}{x}dx=\log|x|+C,\ x\neq 0$

(iii) $\dfrac{d}{dx}(e^x)=e^x \quad \Rightarrow \int e^x dx=e^x+C$

(iv) $\dfrac{d}{dx}\left(\dfrac{a^x}{\log_e a}\right)=a^x,\ a>1, a\neq 1$

$\Rightarrow \int a^x dx=\dfrac{a^x}{\log a}+C$

(v) $\dfrac{d}{dx}(-\cos x)=\sin x \quad \Rightarrow \int \sin x dx=-\cos x+C$

(vi) $\dfrac{d}{dx}(\sin x)=\cos x \quad \Rightarrow \int \cos x dx=\sin x+C$

(vii) $\dfrac{d}{dx}(\tan x)=\sec^2 x \quad \Rightarrow \int \sec^2 x dx=\tan x+C$

(viii) $\dfrac{d}{dx}(-\cot x)=\text{cosec}^2 x \quad \Rightarrow \int \text{cosec}^2 x dx=-\cot x+C$

(ix) $\dfrac{d}{dx}(\sec x)=\sec x\tan x \quad \Rightarrow \int \sec x\tan x dx=\sec x+C$

(x) $\dfrac{d}{dx}(-\text{cosec}\, x)=\text{cosec}\, x\cot x$

$\Rightarrow \int \text{cosec}\, x\cot x dx=-\text{cosec}\, x+C$

(xi) $\dfrac{d}{dx}(\log\sin x)=\cot x \quad \Rightarrow \int \cot x dx=\log|\sin x|+C$

(xii) $\dfrac{d}{dx}(-\log\cos x)=\tan x \quad \Rightarrow \int \tan x dx=-\log|\cos x|+C$

(xiii) $\dfrac{d}{dx}\big(\log(\sec x+\tan x)\big)=\sec x$

$\Rightarrow \int \sec x dx=\log|\sec x+\tan x|+C$

(xiv) $\dfrac{d}{dx}\big(\log(\text{cosec}\, x-\cot x)\big)=\text{cosec}\, x$

$\Rightarrow \int \text{cosec}\, x dx=\log|\text{cosec}\, x-\cot x|+C$

(xv) $\dfrac{d}{dx}\left(\sin^{-1}\dfrac{x}{a}\right)=\dfrac{1}{\sqrt{a^2-x^2}}$

$\Rightarrow \int \dfrac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\dfrac{x}{a}\right)+C$

(xvi) $\dfrac{d}{dx}\left(\cos^{-1}\dfrac{x}{a}\right)=\dfrac{-1}{\sqrt{a^2-x^2}}$

$\Rightarrow \int \dfrac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\dfrac{x}{a}\right)+C$

(xvii) $\dfrac{d}{dx}\left(\dfrac{1}{a}\tan^{-1}\dfrac{x}{a}\right)=\dfrac{1}{a^2+x^2}$

$\Rightarrow \int \dfrac{1}{a^2+x^2}dx=\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+C$

(xviii) $\dfrac{d}{dx}\left(\dfrac{1}{a}\cot^{-1}\dfrac{x}{a}\right)=-\dfrac{1}{a^2+x^2}$

$\Rightarrow \int \dfrac{-1}{a^2+x^2}dx=\dfrac{1}{a}\cot^{-1}\left(\dfrac{x}{a}\right)+C$

(xix) $\dfrac{d}{dx}\left(\dfrac{1}{a}\sec^{-1}\dfrac{x}{a}\right)=\dfrac{1}{x\sqrt{x^2-a^2}}$

$\Rightarrow \int \dfrac{1}{x\sqrt{x^2-a^2}}dx=\dfrac{1}{a}\sec^{-1}\left(\dfrac{x}{a}\right)+C$

$$= \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdot\dfrac{2}{3} & \text{Only if } n \text{ is odd} \\ \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\cdot\dfrac{1}{2}\cdot\dfrac{\pi}{2} & \text{Only if } n \text{ is even} \end{cases}$$

(ii) $$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma(m+1)/2\Gamma(n+1)}{2\Gamma(m+n+2)/2}/2$$

Improper Integral

If $f(x)$ is continuous on $[a, \infty]$, then $\int_0^{\infty} f(x)dx$ is called an improper integral and is defined as:

$$\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$

If there exists a finite limit on the right hand side of the equation, then we can say that the improper integral is convergent otherwise it is divergent.

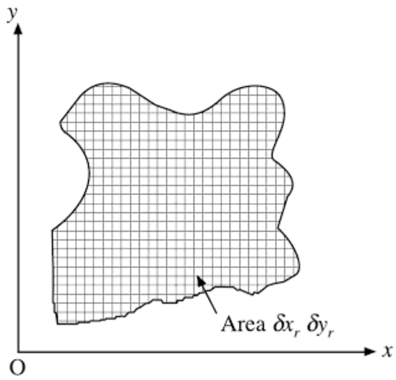
MULTIPLE INTEGRALS AND THEIR APPLICATIONS

Double Integrals

Consider a function $f(x, y)$ be a single valued real function of x and y bounded and defined in the region R of the xy-plane. Divide the region R into mn rectangles by drawing m lines parallel to x-axis and n lines parallel to y-axis. Consider the sum

$$f(x_1, y_1)\delta x_1\cdot\delta y_1 + f(x_2, y_2)\delta x_2\cdot\delta y_2 + \cdots$$
$$+ f(x_n, y_n)\delta x_n\cdot\delta y_n \text{ i.e. } \sum_{r=1}^{n} f(x_r, y_r)\delta x_r \cdot \delta y_r$$

where δx_r, δy_r are the dimensions of rth rectangle and (x_r, y_r) is any point on or within the rth rectangle.



The double integral of $f(x, y)$ over the region R is written as:

$$\iint_R f(x,y)\,dxdy$$

How to Evaluate Double Integrals

Let R be a region bounded below by the curve $\phi_1(x)$ and above by $\phi_2(x)$ and the line $x = x_1$, $x = x_2$ $(x_1 < x_2)$.

∴ The double integral of $f(x, y)$ over R is given by

$$I_1 = \iint_R f(x,y)\,dxdy = \int_{x_1}^{x_2}\left[\int_{\phi_1(x)}^{\phi_2(x)} f(x,y)dy\right]dx$$

where the integration is carried from the inner to the outer rectangle

If the region of integration R is bounded on the left by $x = \phi_1(y)$ and on the right by $x = \phi_2(y)$ and the lines $y = y_1$, $y = y_2$ $(y_1 < y_2)$, then the double integral of $f(x, y)$ over R is given by

$$I_2 = \iint_R f(x,y)\,dxdy = \int_{y_1}^{y_2}\left[\int_{\phi_1(y)}^{\phi_2(y)} f(x,y)dx\right]dy$$

Change of Variables

If $x = f_1(u, v)$, $y = f_2(u, v)$, then the double integral

$$\iint f(x,y)\,dxdy = \iint_R f\{f_1(u,v), f_2(u,v)\}\frac{\partial(x,y)}{\partial(u,v)}du\,dv$$

Where, $$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} \neq 0$$

This is known as Jacobian transformation from (x, y) to (u,v) coordinates.

Application: Calculation of Surface Area by Double Integration

Cartesian coordinate

(a) The area bounded by the curve $y = f(x)$, x-axis and the lines $x = x_1$, $x = x_2$ $(x_1 < x_2)$ is given by

$$A = \int_{x_1}^{x_2}\int_0^{f(x)} dydx$$

(b) The area bounded by the curve $x = g(y)$, y-axis and the lines $y = y_1$, $y = y_2$ $(y_1 < y_2)$ is given by

$$A = \int_{y_1}^{y_2}\int_0^{g(y)} dx\,dy$$

Polar coordinate

$A = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r,\theta)drd\theta$ is the area bound by the curve $f(r, \theta)$ and the lines $\theta = \theta_1$ and $\theta = \theta_2(\theta_1 < \theta_2)$. Here we first integrate w.r.t. r between limits $r = r_1$ and $r = r_2$ keeping θ fixed and the resulting expression is integrated w.r.t. to θ from θ_1 to θ_2. In this case, r, r_2 are functions of θ and θ_1, θ_2 remains constant.

Calculation of Volume by Double Integration

Consider a surface $z = f(x, y)$ be a continuous function of x, y over a region A in the xy-plane, then $\iint_A f(x,y)dxdy$ express the volume over the region A. In polar coordinate $\iint zrd\theta dr$:

(i) If the region A is bounded by the curves $y = f_1(x)$, $y = f_2(x)$ and the lines $x = x_1$, $x = x_2$, then the volume is given by

$$V = \int_{x_1}^{x_2}\int_{f_1(x)}^{f_2(x)} f(x, y)\,dydx$$

(ii) If the region A is bounded by the curves $x = f_1(y)$, $x = f_2(y)$ and the lines $y = y_1$, $y = y_2$, then the volume is given by

$$V = \int_{y_1}^{y_2}\int_{f_1(y)}^{f_2(y)} f(x, y)\,dxdy$$

Triple Integration

Consider a function $f(x, y, z)$ be a continuous and defined function of x, y and z over a three-dimensional finite region V. Divided this V into elementary volumes by drawing planes parallel to the coordinate planes. Suppose n be the number of elementary volumes lying completely within the region V. Then the triple integral of $f(x, y, z)$ over the region V is defined as

$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} f(x_i, y_i, z_i)\delta x_i\cdot\delta y_i\cdot\delta z_i$, where $\delta x_i\cdot\delta y_i\cdot\delta z_i$ are the dimensions of the ith volume and (x_i, y_i, z_i) is any point on or within the ith subdivision.

For the purpose of evaluation it can also be expressed as the repeated integration.

The triple integral of $f(x, y, z)$ over the region V is written as $\iiint f(x,y,z)\,dv$.

How to evaluate triple integrals: If the region V is bounded by the surfaces $x = x_1$, $x = x_2$, $y = y_1$, $y = y_2$, $z = z_1$, $z = z_2$, then

$$V = \iiint_V f(x,y,z)\,dxdydz = \int_{x_1}^{x_2}\int_{y_1(x)}^{y_2(x)}\int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)dzdydx$$

Application: Calculation of Volume of Solids by Triple Integration

The volume V of a solid is given by

$\sum V = \iiint_V dxdydz = \text{Lt}\sum\limits_{\delta x\to 0}\sum\limits_{\delta y\to 0}\sum\limits_{\delta z\to 0}\delta x\delta y\delta z$ with appropriate limit of integration.

DIFFERENTIAL EQUATIONS

Basic Definitions

Differential equation: A relation connecting an independent variable x, a dependent variable y and one or more of their differential coefficient or differentials is called differential equation.

Ordinary differential equation (ODE): A differential equation which involves only one independent variable with all the differential coefficients is called ordinary differential equation.

e.g., the equations $8\frac{dy}{dx} + 5xy = 0$ and $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + 3 = 0$ are ordinary differential equation.

Partial differential equations (PDE): A differential equation which involves two or more independent variables and partial differential coefficients is called partial differential equation.

e.g., the equation $x^3\frac{\partial z}{\partial x} + 4xy\frac{\partial z}{\partial y} = b$ is a partial differential equation.

The order of a differential equation: The order of a differential equation is the order of the highest derivative appearing in the equation.

The degree of a differential equation is the degree of the highest order derivative occurring in the equation, when the equation is free from radicals and fractional powers as far as the derivatives are concerned.

For e.g.,

$$\frac{dy}{dx} + 9xy = b \qquad \ldots(1)$$

$$\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} = 5 \qquad \ldots(2)$$

$$\frac{d^2y}{dx^2} = 5\left[4 + \left(\frac{dy}{dx}\right)^5\right]^{1/3} \qquad \ldots(3)$$

Equation (1) is of first order and first degree. Equation (2) is second order and first degree. Equation (3) is of second order and third degree.

Solution: The general (or complete) solution of a differential equation is that solution which contains arbitrary constants as many as the order of the differential equation.

A particular solution of a differential equation is that solution, which is obtained from general solution by giving particular values to the arbitrary constants involved there in.

The general (or complete) solution of a differential equation of order n contains exactly n independent arbitrary constants.

Geometrical significance of differential equation: The differential equation represents a single parameter family of curves and each number of their family is called an integral curve.

The equation of each curve is thus a particular solution of the differential equation, the whole system of curves being its general solution in the totality makes the locus of the differential equation.

Types of Differential Equations: Differential Equations of First Order and First Degree

Type (a): Separation of variable form: The equation of this type can be written in the form

$$f(x)dx + g(y)dy = 0$$

By integrating both sides, we have the solution

$$\int f(x)dx + \int g(y)dy = C$$

Type (b): Linear differential equations: A differential equation is called linear if the dependent variable and its derivative occurring in it are of the first degree and are not multiplied together. The standard form of linear differential equation of the first order is

$$\frac{dy}{dx} + Py = Q$$

where P and Q are the functions of x only.
The solution of the linear differential equation (multiplying both sides by $e^{\int Pdx}$) is

$$y \cdot e^{\int Pdx} = \int Q \cdot e^{\int Pdx} \cdot dx + C$$

where $e^{\int Pdx}$ is called the integrating factor (I.F.) of the linear equation.

Type (c): Bernoulli's equation (Leibnitz's linear equation): Bernoulli equation is of the form

$$\frac{dy}{dx} + Py = Qy^n \qquad \ldots(4)$$

where P and Q are the functions of x only.

Dividing throughout by y^n, Eq. (4) can be written as:

$$y^{-n}\frac{dy}{dx} + Py^{-n+1} = Q \qquad \ldots(5)$$

Putting $z = y^{1-n}$, we get

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$$

Therefore from Eq. (5), we have

$$\frac{1}{1-n} \cdot \frac{dz}{dx} + Pz = Q$$

$$\frac{dz}{dx} + P(1-n)z = Q(1-n)$$

This is Leibnitz's linear equation which can be solved easily.

Type (d): Homogeneous equations: If a function $f(x, y)$ can be expressed in the form $x^n\phi\left(\frac{y}{x}\right)$ or in the form $y^n\psi\left(\frac{x}{y}\right)$, then $f(x, y)$ is said to be a homogeneous function of degree n in x and y.

Consider a non-homogeneous equation of the form

$$(a_1x + b_1y + c_1)dx = (a_2x + b_2y + c_2)dy$$

i.e.,
$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2} \qquad \ldots(6)$$

in which $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

This equation can be made homogeneous by the substitution.

$$x = x' + h \quad \text{and} \quad y = y' + k$$

where h and k are constants and are so chosen that

$$a_1h + b_1k + c_1 = 0 \text{ and } a_2h + b_2k + c_2 = 0$$

This relation determine the constants h and k.

Equation (6) is then reduced to the homogeneous equation

$$\frac{dy'}{dx'} = \frac{a_1x' + b_1y'}{a_2x' + b_2y'}$$

which can be solved after the substitution $y' = vx'$.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, then after substituting

$$a_1x + b_1y = z$$

i.e.,
$$a_1 + b_1\frac{dy}{dx} = \frac{dz}{dx}$$

will transform the equation to a form, which can be easily solved.

Type (e): Exact differential equation: A differential equation which can be obtained by direct differentiation of some function of x and y is called exact differential equation. Let us consider the following equation

$Mdx + Ndy = 0$ is exact

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

where M and N are the functions of x and y. This is necessary and sufficient condition for the differential equation to be exact. Solution of an exact differential equation is

$$\int Mdx + \int Ndy = C$$

Integrating factor (I.F.): Sometimes an equation which is not exact can be made exact i.e., differential coefficient of a single function by multiplying the equation by a function of x and y called the integrating factor. It is important to remember that $e^{\int P dx}$ is called the integrating factor (I.F.) of the linear equation.

For e.g. (try to remember these exact equations and can apply directly to solve the sum)

$$xdy + ydx = d(xy)$$

$$\frac{xdy - ydx}{y^2} = -d\left(\frac{x}{y}\right)$$

$$\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$$

$$\frac{xdy - ydx}{xy} = d\log\left|\frac{y}{x}\right|$$

$$\frac{ydx - xdy}{x^2 + y^2} = d\left(\tan^{-1}\frac{x}{y}\right)$$

$$\frac{2xydy - y^2 x}{x^2} = d\left(\frac{y^2}{x}\right)$$

$$\frac{xdy + ydx}{\sqrt{1 - x^2 y^2}} = d[\sin^{-1}(xy)]$$

Type (f): Change of variable: Let the equation

$$\frac{dy}{dx} = f(ax + by + c) \qquad \ldots(7)$$

Putting $ax + by + c = z$, we have

$$a + b\frac{dy}{dx} = \frac{dz}{dx}$$

Equation (7) is then reduced to

$$\frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(z)$$

$$\frac{dz}{bf(z) + a} = dx$$

Then after integrating we find the required solution easily.

Type (g): Differential equation of first order and higher degree: An equation of the first order and of nth degree is

$$\left(\frac{dy}{dx}\right)^n + P_1\left(\frac{dy}{dx}\right)^{n-1} + P_2\left(\frac{dy}{dx}\right)^{n-2} + \cdots + P_n = 0$$

or, $$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \cdots + P_n = 0 \qquad \ldots(8)$$

where P_1, P_2, ..., P_n are functions of x and y and the parameter $p = \frac{dy}{dx}$.

Equation (8) can be solved for either p or x or y.

Type (h): Equation solvable for p: Splitting up the left hand side of Eq. (8) into n linear factors, it can be written as:

$$[p - f_1(x,y)][p - f_2(x,y)][p - f_3(x,y)]\ldots[p - f_n(x,y)] = 0$$

Equating each of the factors to zero, we have n equations

$$p = \frac{dy}{dx} = \psi_i(x,y),\ 1 \le i \le n$$

Let the solution be $F_i(x,y,c_i) = 0,\ 1 \le i \le n$, then the general solution of Eq. (8) is

$$F_1(x,y,c)\cdot F_2(x,y,c)\ldots F_n(x,y,c) = 0$$

Type (i) Equation solvable for y: Let the given equation be $y = f(x, p)$.

Differentiate it w.r.t. x, we have the equation of the form

$$p = \frac{dy}{dx} = \phi\left(x, p, \frac{dp}{dx}\right)$$

This differential equation is in two variables x and the parameter p. Let the solution of this equation be

$$F(x, p, c) = 0$$

After elimination of p from this equation, we get the desired solution. This method is very much useful for equations which do not contain x.

Type (j) Equation solvable for x: Let the given equation be $x = \phi(y, p)$.
Differentiate it w.r.t. y, we get

$$\frac{1}{p} = \frac{dx}{dy} = \Psi\left(y, p, \frac{dp}{dy}\right)$$

This differential equation is in two variables y and the parameter p. Let the solution of this equation be

$$F(y, p, c) = 0$$

After elimination of p from this equation, we get the desired solution. This method is very much useful for equations which do not contain y.

Type (k) Clairaut's equation: An equation of the form $y = px + f(p)$ is known as Clairaut's equation.
Differentiating with respect to x, we get

$$p = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx} = 0$$

or, $$[x + f'(p)]\frac{dp}{dx} = 0$$

$\therefore$ $$\frac{dp}{dx} = 0 \quad \text{or,} \quad x + f'(p) = 0$$

$$\frac{dp}{dx} = 0 \Rightarrow p = c$$

Thus, eliminating the parameter p from $p = c$ and the given equation, we get $y = cx + f(c)$ as a general solution of the Clairaut's equation.

Eliminating p from $x + f'(p) = 0$ and $y = px + f(p)$, we get an equation in x and y, which is known as singular solution. Many equations of the first order but not of first degree can easily be reduced to this equation form by making proper substitutions.

Type (l) Trajectory: A curve which cuts every member of a given family of curves according to a specified law is called a trajectory of the given family of curves.

Type (m) Orthogonal trajectories: Two families of curves are said to be orthogonal trajectories, when every member of one family cuts every member of the other at right angles.

The orthogonal trajectories of a family of a curves $F(x, y, c) = 0$ represented by the Cartesian differential equation $f\left(x, y, \dfrac{dy}{dx}\right) = 0$ by eliminating c.

The orthogonal trajectories of a family of a curves represented by the polar differential equation $f\left(r, \theta, \dfrac{dr}{d\theta}\right) = 0$ are given by $f\left(r, \theta, -r^2\dfrac{dr}{d\theta}\right) = 0.$

Type (n) Linear differential equations with constant coefficients: A differential equation in which the dependent variable and its derivatives appear only in the first degree and are not multiplied together.
Thus, the general form is

$$\frac{d^n y}{dx^n} + k_1\frac{d^{n-1}y}{dx^{n-1}} + k_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + k_n y = X$$

where $k_1, k_2, ..., k_n$ are constants and X is any function of x. The operator d/dx is denoted by D.

$\therefore$ $$(D^n + k_1D^{n-1} + \cdots + k_n)y = X$$

or, $$f(D)y = X$$

where an operator $f(D) = D^n + k_1D^{n-1} + \cdots + k_n$

Type (o) Solution of the differential equation: If the given equation is

$$\frac{d^n y}{dx^n} + k_1\frac{d^{n-1}y}{dx^{n-1}} + k_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + k_n y = 0 \qquad \ldots(9)$$

or, $$(D^n + k_1D^{n-1} + k_2D^{n-2} + \cdots + k_n)y = 0 \qquad \ldots(10)$$

Let $y = e^{mx}$, where C is an arbitrary constant.

$\Rightarrow$ $$D^r y = m^r e^{mx}, \quad 1 \le r \le n$$

$\therefore$ Then from Eq. (10), we have

$$(m^n + P_1m^{n-1} + P_2m^{n-2} + \cdots + P_n)e^{mx} = 0$$

$y = e^{mx}$ is a solution of Eq. (9), if

$$(m^n + P_1m^{n-1} + P_2m^{n-2} + \cdots + P_n) = 0$$

This equation is called the auxiliary equation or characteristic equation.

Case I: When auxiliary equation has distinct and real roots Let $m_1, m_2, m_3 ..., m_n$ are distinct roots of the auxiliary equation, then the general solution is

$$y = C_1e^{m_1x} + C_2e^{m_2x} + \cdots + C_ne^{m_nx}$$

where $C_1, C_2, ..., C_n$ are arbitrary constants.

Case II: When auxiliary equation has real and some equal roots If the auxiliary equation has two roots equal, say $m_1 = m_2$ and others are distinct say $m_3, m_4, ..., m_n$. In this case P.I. plus the complementary function (C.F.) of Eq. (9) is

$$y = (C_1 + C_2x)e^{m_1x} + C_3e^{m_3x} + \cdots + C_ne^{m_nx}$$

where $C_1, C_2, ..., C_n$ are arbitrary constants. If the auxiliary equation has equal roots m, then the P.I. plus the complementary function is

$$y = (C_1 + C_2x + \cdots + C_rx^{r-1})e^{mx} + C_{r+1}e^{m_{r+1}x} + \cdots + C_ne^{m}n^x$$

Case III: When the auxiliary equation has some imaginary roots If there are one pair of imaginary roots say $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$ and others distinct roots are $m_3, m_4, ..., m_n$. Then the P.I. plus complementary function of equation is

$$y = C_1e^{(\alpha+i\beta)x} + C_2e^{(\alpha-i\beta)x} + C_3e^{m_3x} + \cdots + C_ne^{m_nx} = 0$$

Applying Euler's theorem, we have

$$y = C_1e^{\alpha x}(\cos\beta x + i\sin\beta x) + C_2e^{\alpha x}(\cos\beta x - i\sin\beta x) + C_3e^{m_3x} + \cdots + C_ne^{m_nx}$$

$$y = (C_1 + C_2)e^{\alpha x}\cos\beta x + (C_1 - C_2)ie^{\alpha x}\sin\beta x + C_3e^{m_3x} + \cdots + C_ne^{m_nx}$$

Suppose $C_1 + C_2 = A_1$ and $(C_1 - C_2)i = B_1$
where A_1 and B_1 are arbitrary constants, then the general complete solution is

$$y = (A\cos\beta x + B\sin\beta x)e^{\alpha x} + C_3e^{m_3x} + \cdots + C_ne^{m_nx}$$

Note: Applications of linear differential equations are simple harmonic motion, oscillations of spring, oscillatory electrical circuits, etc.

COMPLEX NUMBER AND FUNCTIONS

Complex Numbers

A complex number of the form $a + ib$ or $x + iy$, where a and b or x and y are real numbers and $i = \sqrt{-1}$ (iota) is called a complex number. A complex number z is an ordered pair (a, b) of real numbers a and b.

$z \equiv (a, b) = a + ib$ is known as variable complex number and $2 + 3i$ (say) is constant complex number.

Re $z = a$ (real part), Im $z = b$ (imaginary part), $i = \sqrt{-1}$ (imaginary unit or iota)

(a) $c + iy = a + ib \Rightarrow c = a$ and $d = b$

(b) For $z = a + ib$ if $a = 0$ then $z = ib$ (purely imaginary) If $b = 0$ then $z = a$ (purely real)

Addition of two complex numbers: Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$, then $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$

Addition of two complex numbers is always complex except they are conjugate with each other.

Multiplication of two complex numbers

$$z_1 z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$$

Multiplication of two complex numbers is always complex except they are conjugate with each other.

Subtraction of two complex numbers

$$z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

It is always complex.

Division of two complex numbers

$$z = \frac{z_1}{z_2} = a + ib, \text{ where } x = \frac{a_1 a_2 + b_1 b_2}{x_2^2 + y_2^2}, y = \frac{a_2 b_1 + a_1 b_2}{a_2^2 + b_2^2}, z_2 \neq 0$$

Complex Conjugate and its Properties

The complex conjugate of the number

$$z = a + ib \text{ is } \bar{z} = z - ib$$

(a) $\text{Re}\, z = a = \frac{1}{2}(z + \bar{z})$ and $\text{Im}\, z = b = \frac{1}{2i}(z - \bar{z})$

(b) When z is real, $z = a$ then $z = \bar{z}$

(c) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}, \overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

(d) $\overline{z_1 z_2} = \overline{z_1}\, \overline{z_2}, \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

(e) $\left(\overline{z_n}\right) = (\bar{z})^n$

(f) $z_1 \overline{z_2} + \overline{z_1} z_2 = 2\,\text{Re}\left(\overline{z_1} z_2\right) = 2\,\text{Re}\left(z_1 \overline{z_2}\right)$

(g) If $z = f(z_1)$, then $\bar{z} = f(\overline{z_1})$

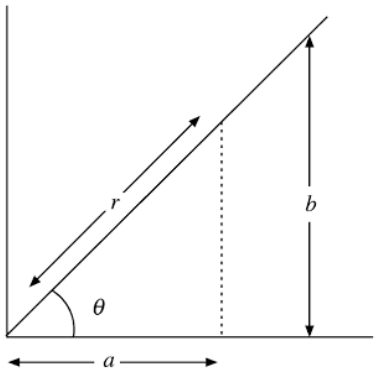
Modulus and Argument of a Complex Number

Let us consider (a, b) be the coordinate in Cartesian coordinate system and (r, θ) be the coordinates in polar coordinates, then

$$a = r \cos\theta, \; b = r \sin\theta$$

$$\therefore \quad z = a + ib = r(\cos\theta + i \sin\theta)$$

Here $r = |z| = +\sqrt{a^2 + b^2} = \sqrt{z\bar{z}}$ is called the modulus of z and $\theta = \arg z = \tan^{-1}\frac{b}{a}$ is called the argument or amplitude of z.



Triangle Inequality

$$|z_1 + z_2 + \cdots + z_n| \leq |z_1| + |z_2| + \cdots + |z_n|$$

In mathematics, the triangle inequality states that for any triangle, the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side.

Abraham De Moivre's Theorem

(i) $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$ where n is positive, negative or fractional number.

(ii) $Z = (\cos\theta_1 + i \sin\theta_1)(\cos\theta_2 + i \sin\theta_2) \ldots (\cos\theta_n + i \sin\theta_n) = \cos(\theta_1 + \theta_2 + \cdots + \theta_n) + i \sin(\theta_1 + \theta_2 + \cdots + \theta_n)$

(iii) qth root of z: $z^{1/q} = \sqrt[q]{r}\left(\cos\frac{\theta + 2k\pi}{n} + i \sin\frac{\theta + 2k\pi}{n}\right)$

$k = 0, 1, \ldots, (n - 1)$

Few Elementary Complex Functions: It may be Single valued or Multiple-valued functions

Single valued functions

(i) $z^n = (x + iy)^n \quad n \in N \quad (z \neq 0 \text{ if } n < 0)$

(ii) $e^z = e^x e^{iy} = e^x (\cos y + i \sin y)$, period $= 2\pi i$ since $\sin(z + 2n\pi) = \sin z$

(iii) Hyperbolic functions

- $\cosh z = \frac{1}{2}(e^z + e^{-z})$ here z be a real complex.
- $\sinh z = \frac{1}{2}(e^z - e^{-z})$
- $\tanh z = \frac{\sinh z}{\cosh z}, z \neq \left(k + \frac{1}{2}\right)\pi i$
- $\coth z = \frac{\cosh z}{\sinh z}, z \neq k\pi i$

(iv) Trignometric functions

- $\cos z = \frac{1}{2}(e^{iz} + e^{-iz})$
- $\sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$

- $\tan z = \dfrac{\sin z}{\cos z}, z \neq \left(k + \dfrac{1}{2}\right)\pi i$
- $\cot z = \dfrac{\cos z}{\sin z}, z \neq k\pi i$

Multiple valued functions

(i) Logarithmic function of a complex variable
$\log z = \log|z| + i \arg z = lr + i(\theta + 2n\pi)$
(sometimes known as capital log).

(ii) $\cosh^2 x - \sinh^2 x = 1$; $\tanh^2 x + \sinh^2 x = 1$; $\cosh^2 x - \text{cosech}^2 x = 1$

Analytic Functions

If w be a complex function then $w = f(a + ib) = u(a, b) + iv(a, b) = \int(z)$, where u, v are real functions of a and b.

Limit, Continuity and Derivatives of Complex Functions

Limit

A function $w = f(z)$ has a limit l as z approaches a point z_0, if $\lim\limits_{z \to z_0} f(z) = l$ exists. Here z approaches z_0 from any direction in the z-plane.

Continuity

A function $w = f(z)$ is said to be continuous at $z = z_0$. At z_0, if $f(z_0)$ is defined as $\lim\limits_{z \to z_0} f(z) = f(z_0)$, $f(z)$ is said to be continuous in any region R of the z-plane, if it is continuous at every point of that region R.

Derivative of $f(z)$: Consider a function $f(z)$ be a single valued function of the variable $z = x + iy$ at z_0, if

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} = \underset{\delta z \to 0}{\text{Lt}} \frac{f(z + \delta z) - f(z)}{\delta z}$$

Then $f(z)$ has a derivative $f'(z_0)$ at z_0.

$$f'(z) = u_x + iv_x = v_y - iu_y$$

Cauchy–Riemann Equation

If $f(z) = u(x, y) + iv(x, y)$, then **Cauchy–Riemann condition**

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Remember: A point at which an analytic function ceases to possess a derivative is called a singular point of the function. An analytic functions sometimes known as regular function or holomorphic function. Singularities of an analytic function may be (i) isolated singularity, (ii) removable singularity, (iii) poles, and (iv) essential singularity.

Let $f(z) = u + iv$ be an analytic function in some region of the z-plane, then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

This is known as harmonic functions and its theory is called potential theory.

Few important properties

(i) If $f(z) = u(x, y) + iv(x, y)$ is analytic in domain D, its satisfies Cauchy–Riemann equation and its partial derivative exist i.e., $u_x = v_y$ and $u_y = -v_x$.

(ii) If $f(z) = u(x, y) + iv(x, y)$ is analytic in domain D, then u and v satisfies Laplace equation i.e.,

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Similarly $$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Conformal Transformation

Suppose two curves C_1, C_2 on the z-plane intersect at the point P and the corresponding curves C_1' and C_2' on the w-plane intersect at P'. If the angle of intersection of the curves at P is the angle of intersection of the curves at P'in magnitude and sence then the transformation is said to be conformal. A point at which $f''(z) = 0$ is called a critical point of the transformation.

Complex Integration

The complex z line integral of a continuous function $f(z)$ of the complex variable taken over the path C: $z(t) = x(t) + iy(t)$ is $\int_c f(z)dz$ and for the closed path C, it is $\oint_C f(z)dz$.

Writing $f(z) = u(x, y) + iv(x, y)$ and noting that $dz = dx + idy$, we have

$$\oint_C f(z)dz = \int_C (udx - vdy) + i\int_C (vdx + udy)$$

Actually it is reduced to two line integrals.

Cauchy's Theorem

Cauchy's integral theorem: If $f(z)$ is an analytic function in a simple connected domain D, $f'(z)$ is continuous at each point, then for every simple closed path C in D, $\oint_C f(z)dz = 0$.

Cauchy's integral formula: Let $f(z)$ be an analytic within the simple connected closed domain D, and let C be a simple closed positively oriented contour that lies in D. Then for any point $a \in D$ and a lies interior to C,

$$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z - a)}dz$$

Which is desired Cauchy's integral formula.

In general $f^n(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz$

Remember: Converse of Cauchy's theorem is Morera's theorem.

Cauchy's inequality: Let $f(z)$ be an analytic function in the simple connected domain D, that contains the circle C: $|z - a| = r$.

If $|f(z)| \le M$ holds for all z on C, then

$\left|f^{(n)}(a)\right| \le \frac{(n)!M}{r^n}$ for n = 1, 2, ..., taking n = 1 and replacing a by z, above equation yields $f'(z) \le \frac{M}{r}$. As $r \to \infty$, it gives $f'(z) = 0$ is Liouville's theorem.

Taylor Series (Series of Complex Terms)

Taylor series: A Taylor series of an analytic function $f(z)$ inside a circle with centre at a, then for z inside C,

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$$

where $a_n = \frac{1}{n!} f^{(n)}(a)$

or $$a_n = \frac{1}{2\pi i} \oint_C \frac{f(\xi)}{(\xi - a)^{n+1}} d\xi$$

With positive orientation around a simple closed contour C that contains a in its interior and $f(z)$ is analytic on and everywhere inside C.

Maclaurin series: A Taylor series with centre $a = 0$ is called Maclaurin series.

Pierre Alphonse Laurent (simply Laurent series)

If $f(z)$ is analytic in the ring-shaped region R bounded by two concentric circles C_1 and C_2 with centre a and in the annulus between them, then for all z in R, $f(z)$ can be represented by the Laurent series,

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=0}^{\infty} \frac{b_n}{(z-a)^n}$$

$$= \sum_{n=-\infty}^{\infty} c_n (a-z)^n$$

Consisting of non-negative powers and negative power (principal part). The coefficients of this series are the integrals

$$c_n = a_n = \frac{1}{2\pi i} \oint_T \frac{f(t)}{(t-a)^{n+1}} dt$$

where T being any curve in region R, encircling the inner circles C_1.

THEORY OF PROBABILITY

- ***Random experiment:*** An experiment whose outcome cannot be predicted with certainty is called a random experiment.
- ***Sample spaces:*** The set of all possible results of a random experiment.
- ***Events:*** Subsets of sample space that contain outcome only.
- ***Elementary event:*** Subsets of sample space that contain one outcome only.
- ***Classical definition:*** If the sample space of an experiment consists of exhaustive, mutually exclusive, finitely many outcomes that are equally likely, then the probability $P(E)$ of the happening of the event E is

$$P(E) = \frac{\text{number of point in } E}{\text{number of point in } S} = \frac{n(E)}{n(S)}$$

- ***Probability:*** Given a sample space S, with each event E of S ($E \subset S$), there is associated a number $P(E)$, called probability of E, such that following axioms of probability are satisfied
  - (i) For every $E \subset S$
  $$0 \le P(E) \le 1$$
  - (ii) For the entire sample space
  $$P(S) = 1$$
  - (iii) For mutually exclusive events A and B
  $A \cap B = \phi$ [addition rule for mutually exclusive events]
  $$P(A \cup B) = P(A) + P(B)$$

Few Basic Theorems on Probability

(i) ***Complementary rule:*** For an event E and its complement E^C in sample space S, $P(E) + P(E^C) = 1$

(ii) ***Addition rule or theorem of total probability*** (for mutually exclusive events only): For mutually exclusive events A_1, ..., A_m in a sample space S,

$$P(A_1 \cup A_2 \cup \ldots \cup A_m) = P(A_1) + P(A_2) + \cdots + P(A_m)$$

(iii) ***Addition rule for not mutually exclusive events:*** For two events A and B in a sample space,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(iv) For three events,
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(v) $(A \cap \overline{B}) \cup (\overline{A} \cap B)$ denotes the occurrence of exactly one of A and B.

(vi) $(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$ denotes the occurrence of exactly two of A, B and C. $E_1 = \phi, E_1E_2 \ne \phi, \ldots E_1E_2E_3 \ldots E_{n-1} = \phi$ then

$$P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right) \ldots P\left(\frac{E_n}{E_1 E_2} \cdots E_{n-1}\right)$$

Conditional Probability

The probability of an event E_1 under the condition that an event E_2 occurs (probability of E_1 given E_2).

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_2 \cap E_1)}{P(E_2)}$$

Bayes' Theorem (Theorem of Inverse Probability): If $B_1, ..., B_n$ be n mutually and exhaustive events and E is any event such that $P(B_i) \neq 0$ for $i = 1, ..., n$ then

$$P\left(\frac{B_r}{A}\right) = \frac{P(B_r) \cdot P(A/B_r)}{\sum_{r=1}^{n} P(B_r) \cdot P(A/B_r)}, r = 1, \ldots, n$$

Probability Distribution

Let X be a random variable and $x_1, x_2 \ldots x_n$ be the real numbers. If in some way P (say), some other real numbers $k_1, k_2, \ldots k_n$ are assigned respectively to the numbers $x_1, x_2 \ldots x_n$, then the following system of numbers:

$$X: x_1 x_2 \ldots x_n$$

$$P(X): k_1 k_2 \ldots k_n$$

Bernoulli (Jacob Bernoulli) distribution: If an experiment of a repetitive nature has two possible outcomes namely success or failure, acceptance or rejection, yes or no with probabilities p and $q = 1 - p$ then the number of successes 0 or 1 has a Bernoulli distribution.
The Bernoulli distribution is given by

$$f(r, p) = p^r(1 - p)^{1-r}$$
$$= p^r q^{1-r}, \ r = 0, 1$$

Mean: $\mu = p$
Variance: $\sigma^2 = pq$

Binomial distribution: A random variable X has a Binomial distribution and is a Binomial random variable if its distribution is given by

$$b(r; n, p) = \binom{n}{r} p^r q^{n-r}, \ r = 0, 1, \ldots, n \quad \text{where } p + q = 1$$

The number of successes in n trials of repetitive nature is a random variable, having Binomial distribution with parameters p and n. The term $b(r; n, p)$ is a successive term in Binomial expansion $[p + q]^n$.
Mean: $\mu = np$
Variance: $\sigma^2 = npq$

Applications of binomial distribution

(i) Estimation of reliability of systems, (ii) Rader detection etc.

Poisson distribution: This is a limiting case of Binomial distribution and the Poisson distribution is given by

$P(r; m) = \dfrac{e^{-m} m^r}{r!}$, $r = 0, 1, 2, 3, \ldots$, here m is known as the parameter of the distribution and $m > 0$.

Mean: $\mu = m$
Variance: $\sigma^2 = m$

Applications of Poisson distribution

(i) Defectives cars in a workshop, (ii) Telephone calls etc.

Normal distribution (Gaussian distribution): It is one of the most frequently used distributions to model the random phenomenon. The limiting form of Binomial distribution for large values of n trials, the probability density function for normal distribution is

$$f(x) = P[X = x] = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], (\sigma > 0),$$
$$-\infty < x < \infty$$

where

(i) $-\infty < \mu < \infty$ is the mean and σ the standard deviation respectively.

(ii) $\dfrac{1}{\sigma\sqrt{2\pi}}$ is a constant factor that makes the area under the curve equal to 1.

(iii) The normal curve is bell-shaped and symmetrical about $x = \mu$.

(iv) The exponential function trends to zero very fast, the faster the smaller the standard deviation σ is.
The probability distribution function,

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2\right] du$$

Normal distribution can also be obtained as a limiting case of Poisson distribution with parameters $m \to \infty$.

Applications of normal distribution

(i) Computation of hit probability of a shot, (ii) Calculation of errors in experimental experiments.

NUMERICAL METHODS

Introduction

In engineering and scientific studies, we frequently face problems of the equations of the type $f(x) = 0$ to be solved. If $f(x) = 0$ is linear, quadratic or bi-quadratic, then we have a simple solution, but when $f(x)$ is a polynomial of higher order or a transcendental functions, it is not a simple task to get the solutions, then the method used to get the solution is known as numerical methods. Sometimes graphical method can solve this type of problem but with a low degree of accuracy. Numerical methods are often, of a respective nature i.e., it involves with iteration process till the result is obtained with desired degree of accuracy.

Solution of Algebraic and Transcendental Equations

(i) Bisection method/Bolzano method/Interval halving methods: Suppose f is continuous function defined on the interval $[a, b]$, with $f(a)$ and $f(b)$ of opposite sign. By the intermediate value theorem, there exists a number x_1 in (a, b) with $f(x_1) = 0$.

Important: This procedure will work for the case when $f(a)$ and $f(b)$ have opposite signs and there is more than one root in the interval (a, b).

The bisection method calls for a repeated halving of sub-intervals of $[a, b]$ and locating the half containing x_1.

To find a solution to $f(x) = 0$ given the continuous function f in the interval $[a, b]$, where $f(a)$ and $f(b)$ have opposite signs.

Set $a_1 = a$ and $b_1 = b$

And let $$x_1 = \frac{1}{2}(a+b)$$

If $f(x_1) = 0$, then $x = x_1$

If not, then $f(x_1)f(a) < 0$ or $f(x_1)f(b) > 0$

If $f(x_1)f(a_1) < 0$

Then $x_1 \in (a_1, x_1)$ and

If $f(x_1)$ is positive, so that the root lies between a and x_1.

Then the second approximation to the root $x_2 = \frac{1}{2}(a+x_1)$.

If $f(x_2)$ is negative, the root lies between x_1 and x_2. Then the third approximation to the root is $x_3 = \frac{1}{2}(x_1+x_2)$ and so on.

(ii) Method of false position or Regula–Falsi method: Here unlike bisection method, choose the abscissa of the point of intersection of AB where the point $A[x_0, f(x_0)]$ and $B[x_1, f(x_1)]$ on the curve $y = f(x)$ on the x-axis as the first approximation to the root of $f(x) = 0$ lying between x_0 and x_1 and $f(x_0)f(x_1) < 0$.

Equation of the chord joining the points AB is

$$\frac{y - f(x_0)}{f(x_1) - f(x_0)} = \frac{x - x_0}{x_1 - x_0}$$

Then the first approximation to the root is

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

If now $f(x_0)$ and $f(x_1)$ are of opposite signs, then the root lies between x_0 and x_2, therefore, by replacing x_1 to x_2, we get the next approximation and it is being repeated till we get the desired accuracy.

Second Order Quadratic Convergence of Newton's Method

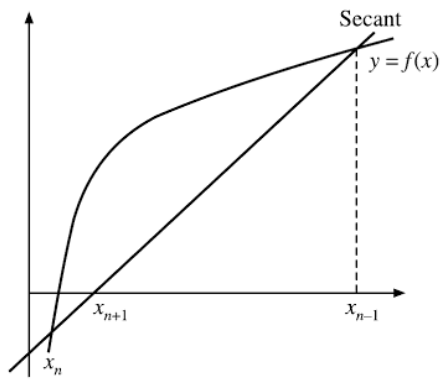
If $f(x)$ is three times differentiable and f', f'' are not zero at a solution of $f(x) = 0$. Then for x_0 sufficiently close to s, Newton's method is of second order.

Theorem: Let f in continuous in $[a, b]$ in xy-plane. If $p \in [a, b]$ is such that $f(p) = 0$ and $f'(p) \neq 0$, then there exist a $\delta > 0$ such that Newton's method generates a sequence $\{p_n\}_{n=1}^{\infty}$ converging to p for any initial approximation $p_0 \in [p - \delta, p + \delta]$.

Secant method (Regula–Falsi): Replacing the derivative $f'(x_n)$ by the difference quotient, in Newton's method, we have

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$x_{n+1} = x_n - f'(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$



Newton's Iteration Method or Newton's Raphson Method

Let us consider x^0 be an approximate root of the given equation $f(x) = 0$. If $x^1 = x^0 + a$ be the exact root, then $f(x') = 0$.

$\therefore$ Expanding $f(x^0 + a)$ by Taylor's series, we get

$$f(x^0) + af'(x^0) + \frac{a^2}{2!} f''(x^0) + \cdots = 0$$

Here a is a very small quantity and hence we can ignore a^2 and higher powers of a, we have

$$f(x^0) + af'(x^0) = 0 \quad \text{or} \quad a = -\frac{f(x^0)}{f'(x^0)}$$

A closer approximation to the root is given by

$$x^1 = x^0 - \frac{f(x^0)}{f'(x^0)}$$

Similarly, starting with x', a new better approximation x^2 is given by

$$x^2 = x^1 - \frac{f(x^1)}{f'(x^1)}$$

In general we can write

$$x^{n+1} = x^n - \frac{f(x^n)}{f'(x^n)}$$

[*Remember:* Here the power of x indicates the upper suffix only]

Newton's method is very much useful in cases of large values of $f'(x)$ and this method has second order of quadratic convergence.

Solution of Linear System of Equations (Gauss Elimination Method)

Given system of equations,

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \quad \ldots(11)$$

Step 1 Eliminate x from second and third equation assuming $a_1 \neq 0$.

Here multiply first equation by a_2/a_1 and then subtract it from second equation.

Here multiply first equation by a_3/a_1 and then subtract it from third equation.

The resulting system is,

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ b_2'y + c_2'z &= d_2' \\ b_3'y + c_3'z &= d_3' \end{aligned}\right\} \quad \ldots(12)$$

Here the first equation is pivotal equation and a_1 is the first pivot.

Step 2 Eliminate y from third equation in Eq. (12), assuming $b_2' \neq 0$

Multiply second equation in Eq. (12) by b_3'/b_2' and then subtract it from third equation.

The obtained equations are

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ b_2'y + c_2'z &= d_2' \\ c_3''z &= d_3'' \end{aligned}\right\} \quad \ldots(13)$$

Here the second equation is pivotal equation and b_2 is the new pivot.

Step 3 The value of x, y, z can be obtained from the reduced system of Eq. (13) by back substitution.

Numerical Integration

It is the numerical evaluation of integrals $I = \int_a^b f(x)dx$, where a and b are given and f is a function given analytically by a formula or empirically by table of values. i.e.,

$$I = \int_a^b f(x)dx \approx \int_a^b \phi(x)dx$$

where $\phi(x)$ is a polynomial which is equivalent to $f(x)$. This process when applied to a function of a single variable is known as quadrature.

Rectangular rule: Subdivide the interval of integration $a \le x \le b$ into n subintervals of equal length $h = \frac{b-a}{n}$ and in each subinterval approximate f by $f(x_i^*)$ = value of f at the midpoint x_i^* of the jth subinterval.

$$I = \int_a^b f(x)dx \approx h[f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)]$$

where $h = \frac{b-a}{n}$

Trapezoidal rule: Subdivide the interval $a \le x \le b$ into n subintervals of equal length $h = \frac{b-a}{n}$, where $x_0 = a$ and $x_0 + nh = b$

$$\begin{aligned} I &= \int_a^b f(x)dx \\ &= \int_{x_0}^{x_0+nh} f(x)dx \\ &= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})] \end{aligned}$$

Simpson's one-third rule: Subdivide the interval $a \le x \le b$ into even number of equal intervals $n = 2k$ of length $h = \frac{b-a}{2k}$.

$$\begin{aligned} I &= \int_a^b f(x)x \\ &= \int_{x_0}^{x_0+nh} f(x)dx \\ &= \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})] \end{aligned}$$

It is known as simply Simpson's rule.

Simpson's three-eight rule: Subdivide the interval $a \le x \le b$ into multiple of 3 of equal intervals $n = 3k$ of length $h = \dfrac{b-a}{3k}$.

$$I = \int_a^b f(x)dx$$
$$= \int_{x_0}^{x_0+nh} f(x)dx$$
$$= \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

Weddle's rule: Subdivide the interval $a \le x \le b$ into multiple of 6 of equal intervals $n = 6k$ of length $h = \dfrac{b-a}{6k}$.

$$I = \int_a^b f(x)dx$$
$$= \int_{x_0}^{x_0+nh} f(x)dx$$
$$= \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + 5y_7 + y_8 + \cdots)$$

Numerical Solution of Ordinary Differential Equation

Picard's process: Given the initial value problem

$$\frac{dy}{dx} = f(x,y)$$

Given $y(x_0) = y_0$

Integrating both sides, $\displaystyle\int_{y_0}^{y} dy = \int_{x_0}^{x} f(x,y)dy$

or $$y = y_0 + \int_{x_0}^{x} f(x,y)\,dx$$

This is known as integral equation.

For first approximation

$$y_1 = y_0 + \int_{x_0}^{x} f(x,y_0)\,dx$$

For second approximation

$$y_2 = y_0 + \int_{x_0}^{x} f(x,y_1)\,dx$$

For nth approximation

$$y_n = y_0 + \int_{x_0}^{x} f(x,y_{n-1})\,dx$$

Taylor's series method: Given the initial value problem

$$\frac{dy}{dx} = f(x,y) \qquad \ldots(14)$$

Given $y(x_0) = y_0$

Then $\dfrac{d^2y}{dx^2} = f_x + f_y y' = f_x + f_y f = \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y}\cdot\dfrac{df}{dx}$

$$y''' = \frac{d^3y}{dx^3} = f_{xx} + 2f_{xy}f + f_{yy}f^2 + f_x f_y + f_y^2 f \qquad \ldots(15)$$

Differentiating this successively, we can get $\dfrac{d^4y}{dx^4}$ etc.

Putting $x = x_0$ and $y = 0$, the values of $(y')_0, (y'')_0, (y''')_0$ can be obtained. Therefore, the Taylor's series about a point $x = x_0$ is

$$y = y_0 + (x - x_0)(y')_0 + \frac{(x-x_0)^2}{2!}(y'')_0 + \frac{(x-x_0)^3}{3!}(y''')_0 + \cdots \qquad \ldots(16)$$

From Eq. (16) we can find the value y_1 of y and $x = x_1$ and $y', y'', y''', \ldots$ can be found at $x = x_1$ with Eqs. (14) and (15) and so on.

Euler's method: Given initial value problem

$$\frac{dy}{dx} = f(x,y) \text{ where } y(x_0) = y_0$$

By Taylor series, for $h \to 0$,

$$y(x+h) \approx y(x) + hy'(x)$$
$$= y(x) + hf(x,y)$$

which yields

$$y_{n+1} = y_n + hf(x_n, y_n)$$

where $h = \dfrac{x_n - x_0}{n}$ i.e., $x_n = x_0 + nh$

Modified ruler method (Predictor–Corrector method)
Given the initial value problem

$$y' = \frac{dy}{dx} = f(x,y) \text{ and given } y(x_0) = y_0$$

$y_{n+1}^* = y_n + hf(x_n, y_n)$ (predicted value)

$y_{n+1} = y_n + h[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^*)]$ (corrected value)

where $h = \dfrac{x_n - x_0}{n}$ i.e., $x_n = x_0 + nh$

Runge–Kutta method: Consider the initial value problem

$$\frac{dy}{dx} = f(x,y) \text{ and given } y(x_0) = y_0$$

Here, $k_1 = hf(x_n, y_n)$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf\left(x_n + \frac{1}{2}h, y_n + k_3\right)$$

Finally compute $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

and $\quad y_{n+1} = y_n + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

where $h = \frac{x_n - x_0}{n}$, or $x_n = x_0 + nh$

LAPLACE TRANSFORM AND FOURIER TRANSFORM

Laplace Transform

Laplace transform method is very useful to solve differential equation because it gives particular solution directly without the necessity of first finding the general solution. Especially it is useful for solving linear differential equations with constant coefficients.

Consider $f(t)$ be a function for all $t \geq 0$. Then the Laplace transform of $f(t)$ denoted by $L\{f(t)\}$ defined by

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

provided that the integral exists. s may be a real or complex variable parameter. It is also denoted by $\bar{f}(s)$. The symbol L is called Laplace transform operator.

Inverse Laplace transform: $f(t)$ is the inverse transform of $\bar{f}(s)$.

$$f(t) = L^{-1}\{\bar{f}(s)\}$$

Existence condition for Laplace transform: Let $f(t)$ be a function that is piecewise continuous on every finite interval in the range $t \geq 0$ and satisfies $|e^{at}f(t)|$ is finite for $s > a$, for all $t \geq 0$.

Algebraic laws of Laplace transform

$$L\{mf(t) \pm ng(t)\} = mL\{f(t)\} \pm nL\{g(t)\}$$

Transforms of derivatives

If $f'(t)$ be continuous and $L\{f(t)\} = f(s)$, then

$$L\{f'(t)\} = s\bar{f}(s) - f(0)$$

$$L\{f''(t)\} = s^2\bar{f}(s) - sf(0) - f'(0)$$

In general

$$L\{f^{(n)}(t)\} = s^n\bar{f}(s) - s^{n-1}f(0) - s^{n-1}f'(0) - \cdots - f^{n-1}(0)$$

Transform of integrals

If $L\{f(t)\} = \bar{f}(s)$ then

$$L\left\{\int_0^t f(u)du\right\} = \frac{1}{s}\bar{f}(s)$$

Differentiation of inverse transform

$$L\{tf(t)\} = -\frac{d}{ds}\{f(s)\}$$

Integration of inverse Laplace transform

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$$

Convolution theorem

$$(f * g)(t) = (g * f)(t)$$

$$= \int_0^t f(u)g(t-u)du$$

$$= \int_0^t f(t-u)g(u)du$$

$$L\{f(t) * g(t)\} = L\{f(t)\}L\{g(t)\}$$

Here, $f(t) * g(t)$ is called the convolution or fatling of $f(t)$ and $g(t)$.

Some important Laplace transforms (Try to remember)

S.No.	$\bar{f}(s) = L\{f(t)\}$	$f(t)$
1.	$1/s$	1
2.	$1/s^2$	t
3.	$1/s^n$, (n = 1, 2,...)	$t^{n-1}/(n-1)!$
4.	$\frac{1}{s-a}$	e^{at}
5.	$\frac{1}{(s-a)^2}$	te^{at}
6.	$\frac{1}{(s-a)^n}$ (n = 1, 2, ...)	$\frac{1}{(n-1)!}t^{n-1}e^{at}$
7.	$\frac{1}{s^2+\omega^2}$	$\frac{1}{\omega}\sin\omega t$
8.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
9.	$\frac{1}{s^2-a^2}$	$\frac{1}{a}\sinh at$
10.	$\frac{a}{s^2+a^2}$	$\sin at$
11.	$\frac{s}{s^2+a^2}$	$\cos at$
12.	$\frac{a}{s^2-a^2}$	$\sinh at$
13.	$\frac{s}{s^2-a^2}$	$\cosh at$
14.	$\frac{b}{(s-a)^2+b^2}$	$e^{at}\sin bt$

15.	$\dfrac{s-a}{(s-a)^2+b^2}$	$e^{at}\cos bt$
16.	$\dfrac{b}{(s-a)^2-b^2}$	$e^{at}\sinh bt$
17.	$\dfrac{s-a}{(s-a)^2-b^2}$	$e^{at}\cosh bt$
18.	$\dfrac{s}{(s^4-a^4)}$	$\dfrac{1}{2a^2}(\cosh at-\cos at)$

Application: For the solution of simultaneous linear differential equations, Laplace transform is very helpful.

Fourier Transform

The integral transform of a function $f(x)$ is denoted by I$\{f(x)\}$ and defined by $\overline{f}(s)=\int_{x_1}^{x_2} f(x)k(s,x)dx$ where $k(s, x)$ is called kernel of the transform. If $k(s, x) = e^{isx}$, we get the Fourier transform of $f(x)$ i.e., $F(s)=\int_{-\infty}^{\infty} f(x)e^{isx}dx$

Convergence and sum of Fourier series

Theorem: If a periodic function $f(x)$ with period 2π is piecewise continuous in the interval $-\pi \le x \le \pi$ and has a left hand derivative and right had derivative at each point of the given interval, then the Fourier series

$f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$ of $f(x)$ with coefficient

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx$$

is convergent. Its sum is $f(x)$, except at a point x_0 at which $f(x)$ is discontinuous and the sum of the series is the average of the left and right hand limits of $f(x)$ at x_0. Like Laplace transform, Fourier transform hold the linear property.

Few elementary Fourier transforms: (Try to remember)

S.No.	$f(t)$	$F(\omega)$
1.	$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$
2.	$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega)e^{i\omega t}d\omega$	$F(\omega)$
3.	$af(t)+bg(t)$	$a\mathrm{F}(\omega)+b\mathrm{G}(\omega)$
4.	$f(t-T)$ (T real)	$e^{-i\omega T}\mathrm{F}(\omega)$
5.	$\left(\dfrac{d}{dt}\right)^n f(t)$	$(i\omega)^n\mathrm{F}(\omega)$
6.	$(-it)^n f(t)$	$\left(\dfrac{d}{d\omega}\right)^n \mathrm{F}(\omega)$
7.	$\int_{-\infty}^{\infty} f(\tau)d\tau$	$\dfrac{\mathrm{F}(\omega)}{i\omega}+\pi\mathrm{F}(0)\delta(\omega)$
8.	$f(t)*g(t)=\int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$	$\mathrm{F}(\omega)\mathrm{G}(\omega)$
9.	$f(t)g(t)$	$\dfrac{1}{2\pi}\mathrm{F}(\omega)*\mathrm{G}(\omega)$

Relation between Laplace and Fourier Transform

If $f(t)=\begin{cases} e^{-xt}g(t)\,, & t>0 \\ 0 & , t<0 \end{cases}$

Then F$\{f(t)\}$ = L$\{\phi(t)\}$

We have $\quad \mathrm{F}\{f(t)\}=\int_{-\infty}^{\infty} e^{ist}f(t)dt$

$$=\int_{-\infty}^{0} e^{ist}.0.dt+\int_{0}^{\infty} e^{ist}.e^{-xt}\phi(t)dt$$

$$=\int_{0}^{\infty} e^{(is-x)t}\phi(t)dt=\int_{0}^{\infty} e^{-kt}\phi(t)dt=\mathrm{L}\{\phi(t)\}$$

where $k = x - is$

Hence, the Fourier transform of $f(t)$ is the Laplace transform of $\phi(t)$.

The theory of integral transform *s* helps to get the solutions of numerous boundary value problems of engineering such as conduction of heat, electrical transmission line problems etc.

9.4 IMPORTANT POINTS TO REMEMBER

1. Iota (i) is neither 0, nor greater than 0, nor less than 0.
2. The complex number do not possess the property of order i.e., $2 + 3i > 1 + 2i$ makes no sense.
3. A matrix cannot be reduced to a number but determinant can be reduced to a number.
4. Every invertible matrix possesses a unique inverse.
5. The rank of a zero matrix is zero and the rank of an identity matrix of order n is n.
6. If A and B are unitary matrices then AB is also unitary.
7. A function satisfying the Laplace equation is said to be a harmonic function.
8. The rule for differentiation under the integral sign of an infinite integral is the same for a definite integral.
9. The change of the order of the double integration does not affect the value of the area in any way.

10. The complex form of a Fourier series is especially useful in problems on electrical circuits having impressed periodic voltage.
11. Differential equation helps to formulate the equation of the physical system (modelling), then solutions and finally offers the physical interpretation of the solution.
12. De Moivre's theorem is true for all theta (real or complex).
13. The inverse hyperbolic functions like other inverse functions are multi-valued but we will consider only their principal value.
14. Logarithm of a real quantity is also multi-valued. Its principal value is real while all other values are imaginary. In complex number theory, the logarithm of a negative quantity can be calculated.
15. If a complex function is once known to be analytic, it can be differentiated just in the ordinary way.
16. Laplace transforms reduce the problem of solving differential equations to mere algebraic equations.
17. Unit step function is known as Heaviside's unit function.
18. Depending on the kernel of the transform in integral transform, it can give Mellin transform, Bessel's function and Hankel transform.
19. The classical definition of probability fails when the number of outcomes is infinite (not exhaustive) and outcomes are not equally likely.
20. Jacobi iteration method cannot be used to solve a system of non-linear equations.
21. The convergence in the Gauss–Seidel method is faster than Jacobi's method.
22. The Runge–Kutta method is better than Taylor's series method because it does not require prior calculations of higher derivatives, as the Taylor's method does.
23. By applying elementary transformations to a matrix, its rank does not change.
24. The eigenvalues of a triangular matrix are the elements of its leading diagonal.
25. Hyperbolic functions are not periodic.
26. An analytic function with constant modulus is constant.
27. The mean and variance of a Poisson distribution are equal.
28. If A is a skew-symmetric matrix, then the elements along with the principal diagonal are always zero.
29. A hypothesis is true, but is rejected. This is an error of type I.
30. Out of Regula–Falsi method and Newton–Raphson method, the rate of convergence is faster for Newton-Raphson method.
31. For a symmetrical distribution—mean, median and mode coincide.
32. For a frequency distribution, if there exists only one model, then the distribution is called unimodel or otherwise it is called multimodel.
33. Mean deviation is least when taken from median.
34. Every square matrix can be uniquely expressed as sum of Hermitian and Skew-Hermitian matrices.
35. Every orthogonal matrix is a non-singular matrix.
36. If A, B are any two square matrices such that $AB = A$ and $BA = B$, then A and B are idempotent matrices.
37. A homogeneous system of linear equations in n unknowns whose determinant of coefficient does not equal to zero, has only the trivial solutions.
38. Power series method is the method for solving linear differential equations with variable coefficients.
39. A polynomial function is always a continuous function.
40. The order of the partial differential equation is the order of the highest derivative appear in it.

9.5 REFERENCES

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9.6 PREVIOUS YEARS' QUESTIONS

1. Let $X(s) = \dfrac{3s+5}{s^2+10s+21}$ be the Laplace transform of a signal $x(t)$. Then, $x(0^+)$ is

(a) 0 (b) 3

(c) 5 (d) 21

[Gate (EE) 2014 set 1]

2. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then, after one step of Newton's method, estimate x_1 of the solution which will be given by

(a) 0.71828 (b) 0.36784

(c) 0.20587 (d) 0.00000

[Gate (EE) 2008]

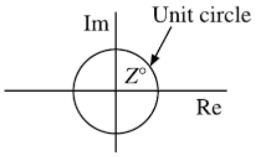
3. Let $x^2 - 117 = 0$. The iterative steps for the solution using Newton–Raphson's method is given by

(a) $x_{k+1} = \dfrac{1}{2}\left(x_k + \dfrac{117}{x_k}\right)$

(b) $x_{k+1} = \left(x_k - \dfrac{117}{x_k}\right)$

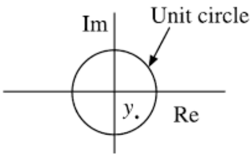
(c) $x_{k+1} = \left(x_k - \dfrac{x_k}{117}\right)$

19. A point Z has been plotted in the complex plane, as shown in figure.

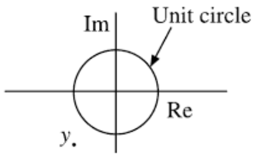


The plot of the complex number $y = \dfrac{1}{Z}$ is

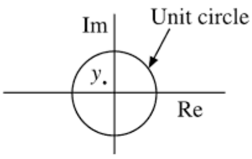
(a)



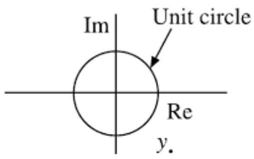
(b)



(c)



(d)



[Gate (EE) 2011]

20. If $x = \sqrt{-1}$, then the value of x^x is
(a) $e^{-\pi/2}$ (b) $e^{\pi/2}$
(c) x (d) 1 [Gate (EE) 2012]

21. Given $f(z) = \dfrac{1}{z+1} - \dfrac{2}{z+3}$. If C is a counter clockwise path in the z-plane such that $|z + 1| = 1$, the value of $\dfrac{1}{2\pi j}\oint_C f(z)dz$ is
(a) -2 (b) -1
(c) 1 (d) 2 [Gate (EE) 2012]

22. Square roots of $-i$, where $i = \sqrt{-1}$ are
(a) $i, -i$
(b) $\cos\left(-\dfrac{\pi}{4}\right) + i\sin\left(-\dfrac{\pi}{4}\right), \cos\left(\dfrac{3\pi}{4}\right) + i\sin\left(\dfrac{3\pi}{4}\right)$
(c) $\cos\left(\dfrac{\pi}{4}\right) + i\sin\left(\dfrac{3\pi}{4}\right), \cos\left(\dfrac{3\pi}{4}\right) + i\sin\left(\dfrac{\pi}{4}\right)$
(d) $\cos\left(\dfrac{3\pi}{4}\right) + i\sin\left(-\dfrac{3\pi}{4}\right), \cos\left(-\dfrac{3\pi}{4}\right) + i\sin\left(\dfrac{3\pi}{4}\right)$
[Gate (EE) 2013]

23. $\oint \dfrac{z^2-4}{z^2+4}dz$ evaluated anti-clockwise around the circle $|z - i| = 2$, where $i = \sqrt{2}$, is
(a) -4π (b) 0
(c) $2 + \pi$ (d) $2 + 2i$
[Gate (EE) 2013]

24. The differential equation $\dfrac{dx}{dt} = \dfrac{1-x}{\tau}$ is discretised using Euler's numerical integration method with a time step $\Delta T > 0$. What is the maximum permissible value of ΔT to ensure stability of the solution of the corresponding discrete time equation?
(a) 1 (b) $\tau/2$
(c) τ (d) 2τ [Gate (EE) 2007]

25. A differential equation $\dfrac{dx}{dt} = e^{-2t}u(t)$ has to be solved using trapezoidal rule of integration with a step size $h = 0.01$ s. Function $u(t)$ indicates a unit step function. If $x(0^-) = 0$, then value of x at $t = 0.01$ s will be given by
(a) 0.00099 (b) 0.00495
(c) 0.0099 (d) 0.0198
[Gate (EE) 2008]

26. For the differential equation $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 8x = 0$ with initial conditions $x(0) = 1$ and $\left.\dfrac{dx}{dt}\right|_{t=0} = 0$, the solution is
(a) $x(t) = 2e^{-6t} - e^{-2t}$ (b) $x(t) = 2e^{-2t} - e^{-4t}$
(c) $x(t) = e^{-6t} - 2e^{-4t}$ (d) $x(t) = e^{-2t} - 2e^{-4t}$
[Gate (EE) 2010]

27. With K as a constant, the possible for the first order differential equation $\dfrac{dy}{dx} = e^{-3x}$ is
(a) $-\dfrac{1}{3}e^{-3x} + K$ (b) $-\dfrac{1}{3}e^{3x} + K$
(c) $-3e^{-3x} + K$ (d) $-3e^{-x} + K$
[Gate (EE) 2011]

28. With initial condition $x(1) = 0.5$, the solution of the differential equation $t\dfrac{dx}{dt} + x = t$ is
(a) $x = t - \dfrac{1}{2}$ (b) $x = t^2 - \dfrac{1}{2}$
(c) $x = \dfrac{t^2}{2}$ (d) $x = \dfrac{t}{2}$
[Gate (EE) 2012]

29. The solution for the differential equation $\dfrac{d^2x}{dt^2} = -9x$ with initial conditions $x(0) = 1$ and $\left.\dfrac{dx}{dt}\right|_{t=0} = 1$, is
(a) $t^2 + t + 1$
(b) $\sin 3t + \dfrac{1}{3}\cos 3t + \dfrac{2}{3}$

(c) $\frac{1}{3}\sin 3t + \cos 3t$

(d) $\cos 3t + t$ [Gate (EE) 2014 set 1]

30. Consider the differential equation $x^2 \frac{d^2y}{dt^2} + x\frac{dy}{dx} - y = 0.$

Which of the following is a solution to this differential equation for $x > 0$?

(a) e^x (b) x^2

(c) $1/x$ (d) $\ln x$

[Gate (EE) 2014 set 2]

31. If $S = \int_1^{\infty} x^{-3} dx$, then S has the value

(a) $-\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1 [Gate (EE) 2005]

32. For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{2}$, magnitude of the gradient at the point (1, 3) is

(a) $\sqrt{\frac{13}{9}}$ (b) $\sqrt{\frac{9}{2}}$

(c) $\sqrt{5}$ (d) $\frac{9}{2}$ [Gate (EE) 2005]

33. The expression $V = \int_0^H \pi R^2 \left(1 - \frac{h}{H}\right)^2 dh$ for the volume of a cone is equal to

(a) $\int_0^R \pi R^2 \left(1 - \frac{h}{H}\right)^2 dr$ (b) $\int_0^R \pi R^2 \left(1 - \frac{h}{H}\right)^2 dh$

(c) $\int_0^H 2\pi rH \left(1 - \frac{r}{R}\right) dh$ (d) $\int_0^R 2\pi rH \left(1 - \frac{r}{R}\right)^2 dr$

[Gate (EE) 2006]

34. The integral $\frac{1}{2\pi}\int_0^{2\pi} \sin(t-\tau)\cos\tau\, d\tau$ equals

(a) $\sin t \cos t$ (b) 0

(c) (1/2) $\cos t$ (d) (1/2) $\sin t$

[Gate (EE) 2007]

35. Divergence of the three-dimensional radial vector field **r** is

(a) 3 (b) $1/r$

(c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (d) $3(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

[Gate (EE) 2010]

36. In the matrix equation $Px = q$, which of the following is a necessary condition for the existence of at least one solution for the unknown vector x

(a) Augmented matrix $[Pq]$ must have the same rank as matrix P

(b) Vector q must have only non-zero elements

(c) Matrix P must be singular

(d) Matrix P must be square [Gate (EE) 2005]

37. $x = [x_1 x_2 \ldots x_n]^T$ is an n-tuple non-zero vector. The $n \times n$ matrix $V = xx^T$

(a) Has rank zero (b) Has rank 1

(c) Is orthogonal (d) Has rank n

[Gate (EE) 2007]

38. Let x and y be two vectors in a three-dimensional space and $\langle x, y\rangle$ denote their dot product. Then the determinant is

$$\det\begin{bmatrix} \langle x,x\rangle & \langle x,y\rangle \\ \langle y,x\rangle & \langle y,y\rangle \end{bmatrix}$$

(a) Zero when x and y are linearly independent

(b) Positive when x and y are linearly independent

(c) Non-zero for all non-zero x and y

(d) Zero only when either x or y is zero

[Gate (EE) 2007]

39. The linear operation $L(x)$ is defined by the cross product $L(x) = bX_x$, where $b = [0\ 1\ 0]^T$ and $x = [x_1\ x_2\ x_3]^T$ are three-dimensional vectors. The 3×3 matrix M of this operation satisfies

$$L(x) = M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then the eigenvalues of M are

(a) 0, + 1, − 1 (b) 1, − 1, 1

(c) i, − i, 1 (d) i, − i, 0

[Gate (EE) 2007]

40. If the rank of a 5×6 matrix Q is 4, then which one of the following statements is correct?

(a) Q will have four linearly independent rows and four linearly independent columns

(b) Q will have four linearly independent rows and five linearly independent columns

(c) QQ^T will be invertible

(d) Q^TQ will be invertible [Gate (EE) 2008]

41. The eigenvalues of the matrix $\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix}$ are

(a) 2, −2, 1, −1 (b) 2, 3, −2, 4

(c) 2, 3, 1, 4 (d) None of these

[Gate(ECE) 2000]

42. Given the matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$, the eigenvector is

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

[Gate (ECE) 2005]

43. Let $A = \begin{bmatrix} 2 & -0.01 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$. Then $(a + b) =$

(a) 7/20 (b) 3/20
(c) 19/60 (d) 11/20

[Gate (ECE) 2005]

44. Given an orthogonal matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$, $[AA^T]^{-1}$ is

(a) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$

[Gate(ECE) 2005]

45. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) 3 [Gate(ECE) 2006]

46. The eigenvalues and the corresponding eigenvectors of a 2×2 matrix are given by

Eigenvalue Eigenvector

$\lambda_1 = 8$ $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_1 = 4$ $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

[Gate(ECE) 2006]

47. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, the eigenvalue corresponding to the eigenvector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

(a) 2 (b) 4
(c) 6 (d) 8 [Gate (ECE) 2006]

48. It is given that $X_1, X_2, ..., X_M, -X_1, -X_2, ..., -X_M$ is

(a) $2M$
(b) $M + 1$
(c) M
(d) Dependent on the choice of $X_1, X_2, ..., X_M$

[Gate (ECE) 2007]

49. All the four entries of the 2×2 matrix $p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ are non-zero, and one of its eigenvalues is zero. Which of the following statements is true?

(a) $p_{11}p_{22} - p_{12}p_{21} = 1$
(b) $p_{11}p_{22} - p_{12}p_{21} = -1$
(c) $p_{11}p_{22} - p_{12}p_{21} = 0$
(d) $p_{11}p_{22} + p_{12}p_{21} = 0$ [Gate (ECE) 2008]

50. The system of linear equations

$$4x + 2y = 7$$
$$2x + y = 6$$

has

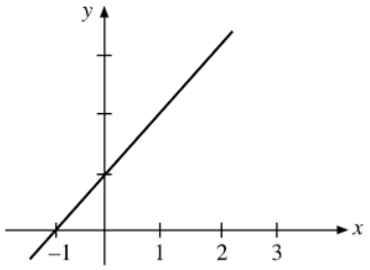
(a) A unique solution
(b) No solution
(c) An infinite number of solutions
(d) Exactly two distinct solutions

[Gate (ECE) 2008]

51. Consider the matrix $p = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$. The value of e^p is

(a) $\begin{bmatrix} 2e^{-2} - 3e^{-1} & e^{-1} - e^{-2} \\ 2e^{-2} - 2e^{-1} & 5e^{-2} - e^{-1} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-1} + e^{-2} & 2e^{-2} - e^{-1} \\ 2e^{-1} - 4e^{-2} & 3e^{-1} + 2e^{-2} \end{bmatrix}$



(a) 1.0 (b) 2.5
(c) 4.0 (d) 5.0 [Gate(ECE) 2007]

64. For $|x| << 1$, coth(x) can be approximated as
(a) x (b) x^2
(c) $1/x$ (d) $1/x^2$
[Gate (ECE) 2007]

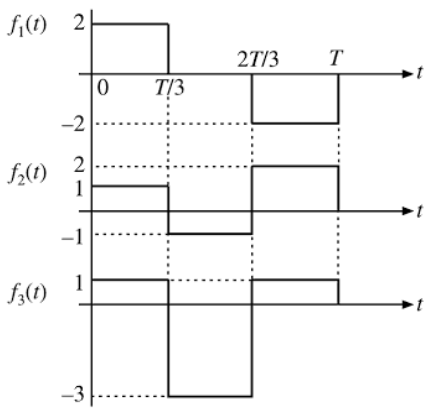
65. $\lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta}$ is
(a) 0.5 (b) 1
(c) 2 (d) Not defined
[Gate (ECE) 2007]

66. Which one of the following functions is strictly bounded?
(a) $1/x^2$ (b) e^x
(c) x^2 (d) e^{-x^2}
[Gate (ECE) 2007]

67. For the function e^{-x}, the linear approximation around $x = 2$ is
(a) $(3 - x)e^{-2}$
(b) $1 - x$
(c) $\left[3+2\sqrt{2}-(1+\sqrt{2})x\right]e^{-2}$
(d) e^{-2} [Gate(ECE) 2007]

68. Consider the function $f(x) = x^2 - x - 2$. The maximum value of $f(x)$ in the closed interval [–4, 4] is
(a) 18 (b) 10
(c) – 2.25 (d) Indeterminate
[Gate (ECE) 2007]

69. Three functions $f_1(t)$, $f_2(t)$ and $f_3(t)$, which are zero outside the interval [0, T], are shown in the figure. Which of the following statements is correct?



(a) $f_1(t)$ and $f_2(t)$ are orthogonal
(b) $f_1(t)$ and $f_3(t)$ are orthogonal
(c) $f_2(t)$ and $f_3(t)$ are orthogonal
(d) $f_1(t)$ and $f_2(t)$ are orthonormal
[Gate (ECE) 2007]

70. For real values of x, the minimum value of the function $f(x) = \exp(x) + \exp(-x)$ is
(a) 2 (b) 1
(c) 0.5 (d) 0 [Gate (ECE) 2008]

71. The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (1, 2) in the xy-plane is
(a) 33 (b) 35
(c) 40 (d) 56 [Gate (ECE) 2008]

72. Consider point P and Q in the xy-plane, with P = (1, 0) and Q = (0, 1). The line integral $2\int_P^Q (xdx + ydy)$ along the semicircle with the line segment PQ as its diameter is
(a) – 1
(b) 0
(c) 1
(d) Dependent on the direction (clockwise or anti-clockwise) of the semicircle
[Gate (ECE) 2008]

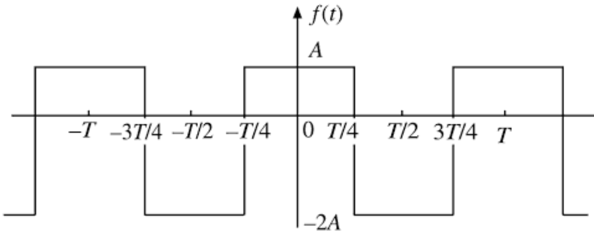
73. The Fourier series of a real periodic function has only
(P) cosine terms if it is even
(Q) sine terms if it is odd
(R) cosine terms if it is odd
(S) sine terms if it is odd
Which of the above statements are correct?
(a) P and S (b) P and R
(c) Q and S (d) Q and R
[Gate (ECE) 2009]

74. A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is true?
(a) f has frequency components at 0 and $1/2\pi$ Hz
(b) f has frequency components at 0 and $1/2\pi$ Hz
(c) f has frequency components at $1/2\pi$ and $1/\pi$ Hz
(d) f has frequency components at 0, $1/2\pi$ and $1/\pi$ Hz [Gate (ECE) 2008]

75. If a vector field **V** is related to another vector field **A** through $\mathbf{V} = \nabla \times \mathbf{A}$, which of the following is true?
Note: C and S_C refer to any closed contour and any surface whose boundary is C.
(a) $\oint_C \mathbf{V} \cdot d\mathbf{l} = \iint_{S_c} \mathbf{A} \cdot d\mathbf{S}$
(b) $\oint_C \mathbf{A} \cdot d\mathbf{l} = \iint_{S_c} \mathbf{V} \cdot d\mathbf{S}$
(c) $\oint_C \nabla \times \mathbf{V} \cdot d\mathbf{l} = \iint_{S_c} \nabla \times \mathbf{A} \cdot d\mathbf{S}$

(d) $\oint_C \nabla \times A \cdot dl = \iint_{S_C} \mathbf{V} \cdot dS$ [Gate (ECE) 2009]

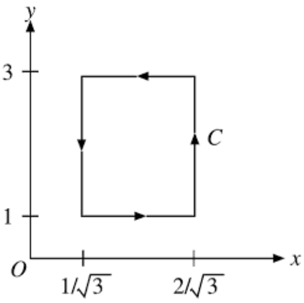
76. The trigonometric Fourier series for the waveform $f(t)$ shown below contains



(a) Only cosine terms and zero value for the dc component
(b) Only cosine terms and a positive value for the dc component
(c) Only cosine terms and a negative value for the dc component
(d) Only sine terms and a negative value for the dc component [Gate(ECE) 2010]

77. If $e^y = x^{\frac{1}{x}}$, then y has a
(a) Maximum at $x = e$
(b) Minimum at $x = e$
(c) Maximum at $x = e^{-1}$
(d) Minimum at $x = e^{-1}$ [Gate (ECE) 2010]

78. If $A = xy\mathbf{a}_x + x^2\mathbf{a}_y$, then $\oint_C \mathbf{A} \cdot dl$ over the path shown in the figure is



(a) 0 (b) $\dfrac{2}{\sqrt{3}}$
(c) 1 (d) $2\sqrt{3}$
[Gate (ECE) 2010]

79. Consider a closed surface S surrounding a volume V. If $\mathbf{r}$ is the position vector of a point inside S, with $\mathbf{n}$ the unit normal on S, the value of the integral $\oiint_S 5\mathbf{r} \cdot \mathbf{n} d\mathbf{S}$ is
(a) 3 V (b) 5 V
(c) 10 V (d) 15 V
[Gate (ECE) 2011]

80. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is
(a) 21 (b) 25
(c) 41 (d) 46 [Gate (ECE) 2012]

81. The direction of vector $\mathbf{A}$ is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla \cdot \mathbf{A} = 0$ is
(a) –2 (b) 2
(c) 1 (d) 0 [Gate (ECE) 2012]

82. The following differential equation has

$$3\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dt}\right)^3 + y^2 + 2 = x$$

(a) Order = 2, degree = 1
(b) Order = 3, degree = 2
(c) Order = 4, degree = 3
(d) Order = 2, degree = 3 [Gate (ECE) 2005]

83. A solution of the following differential equation is given by $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dt} + 6y = 0$
(a) $y = e^{2x} + e^{-3x}$ (b) $y = e^{2x} - e^{2x}$
(c) $y = e^{-2x} + e^{3x}$ (d) $y = e^{-2x} + e^{-3x}$
[Gate (ECE) 2005]

84. A solution for the differential equation $\dot{x}(t) + 2x(t) = \delta(t)$ with initial condition $x(0^-) = 0$ is
(a) $e^{-2t}u(t)$ (b) $e^{2t}u(t)$
(c) $e^{-t}u(t)$ (d) $e^{t}u(t)$
[Gate (ECE) 2006]

85. For the differential equation $\dfrac{d^2y}{dx^2} + k^2y = 0$, the boundary conditions are
(i) $y = 0$ for $x = 0$
(ii) $y = 0$ for $x = a$
The form of non–zero solution of y (where m varies over all integers) are

(a) $y = \sum_m A_m \sin\dfrac{m\pi x}{a}$

(b) $y = \sum_m A_m \cos\dfrac{m\pi x}{a}$

(c) $y = \sum_m A_m x^{\frac{m\pi}{a}}$

(d) $y = \sum_m A_m e^{-\frac{m\pi x}{a}}$ [Gate (ECE) 2006]

86. The order of the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dt}\right)^3 + y^4 = e^{-t} \text{ is}$$

(a) 1 (b) 2
(c) 3 (d) 4 [Gate (ECE) 2009]

87. The equation $\sin(z) = 10$ has
(a) No real or complex solution
(b) Exactly two distinct complex solutions
(c) A unique solution
(d) An infinite number of complex solutions
[Gate (ECE) 2008]

88. Which of the following functions would have only odd powers of x in its Taylor series expansion about the point $x = 0$?
(a) $\sin(x^3)$ (b) $\sin(x^2)$
(c) $\cos(x^3)$ (d) $\cos(x^2)$
[Gate (ECE) 2008]

89. The residue of the function $f(z) = \dfrac{1}{(z+2)^2(z-2)^2}$ at $z = 2$ is
(a) $-\dfrac{1}{32}$ (b) $-\dfrac{1}{16}$
(c) $\dfrac{1}{16}$ (d) $\dfrac{1}{32}$
[Gate (ECE) 2008]

90. In the Taylor series expansion of $\exp(x) + \sin(x)$ about the point $x = \pi$, the coefficient of $(x - \pi)^2$ is
(a) $\exp(\pi)$ (b) $0.5\exp(\pi)$
(c) $\exp(\pi) + 1$ (d) $\exp(\pi) - 1$
[Gate (ECE) 2008]

91. The value of the integral $\oint_c \dfrac{-3z+4}{(z^2+4z+5)}dz$ where c is the circle $|z| = 1$ is given by
(a) 0 (b) 1/10
(c) 4/5 (d) 1 [Gate (ECE) 2011]

92. Three companies X, Y and Z supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are tabulated as:

Company	% of computer supplied	Probability of being defective
X	60	0.01
Y	30	0.02
Z	10	0.03

Given that a computer is defective, the probability that it was supplied by Y is
(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4
[Gate (ECE) 2006]

93. If E denotes expectation, the variance of a random variable X is given by
(a) $E[X^2] - E^2[X]$ (b) $E[X^2] + E^2[X]$
(c) $E[X^2]$ (d) $E^2[X]$
[Gate (ECE) 2007]

94. An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is
(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06
[Gate (ECE) 2007]

95. $P_x(x) = M\exp(-2|x|) + N\exp(-3|x|)$ is the probability density function for the real random variable x, over the entire x-axis. M and N are both positive real numbers. The equation relating M and N is
(a) $M + \dfrac{2}{3}N = 1$ (b) $2M + \dfrac{1}{3}N = 1$
(c) $M + N = 1$ (d) $M + N = 3$
[Gate (ECE) 2008]

96. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?
(a) $\left(\dfrac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\dfrac{1}{2}\right)^2$
(c) $\left(\dfrac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\dfrac{1}{2}\right)^{10}$
[Gate (ECE) 2009]

97. Match the following and choose the correct combination:

Group 1	Group 2
(E) Newton–Raphson method (F) Runge–Kutta method (G) Simpson's rule (H) Gauss elimination	1. Solving non-linear equations 2. Solving linear simultaneous equations 3. Solving ordinary differential equation 4. Numerical integration 5. Interpolation 6. Calculation of eigenvalues

(a) E – 6, F – 1, G – 5, H – 3
(b) E – 1, F – 6, G – 4, H – 3
(c) E – 1, F – 3, G – 4, H – 2
(d) E – 5, F – 3, G – 4, H – 1 [Gate (ECE) 2005]

98. The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton–Raphson method. If $x = 2$ is taken as the initial approximation of the solution, then the next approximation using this method will be
(a) 2/3 (b) 4/3
(c) 1 (d) 3/2 [Gate (ECE) 2007]

99. The recursion relation to solve $x = e^{-x}$ using Newton–Raphson method is
(a) $x_{n+1} = e^{-x_n}$
(b) $x_{n+1} = x_n - e^{-x_n}$
(c) $x_{n+1} = (1+x_n)\dfrac{e^{-x_n}}{1+e^{-x_n}}$

(d) $x_{n+1} = \dfrac{x_n^{\,2} - e^{-x_n}(1+x_n) - 1}{x_n - e^{-x_n}}$ [Gate(ECE) 2008]

100. Consider a differential equation $\dfrac{dy(x)}{dx} - y(x) = x$ with the initial condition $y(0) = 0$. Using Euler's first order method with a step size of 0.1, the value of $y(0.3)$ is
(a) 0.01 (b) 0.031
(c) 0.0631 (d) 0.1
[Gate (ECE) 2010]

101. A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton–Raphson method. If the starting value is $x = 2$ for the iteration, the value of x that is to be used in the next step is
(a) 0.306 (b) 0.739
(c) 1.694 (d) 2.306
[Gate (ECE) 2011]

102. In what range should Re(s) remain so that the Laplace transform of the function $e^{(a+2)t+5}$ exists?
(a) Re(s) > a + 2 (b) Re(s) > a + 7
(c) Re(s) < 2 (d) Re(s) > a + 5
[Gate (ECE) 2005]

103. Consider the function $f(t)$ having Laplace transform
$F(s) = \dfrac{\omega_0}{s^2 + \omega_0^{\,2}}$ Re[s] > 0.
The final value of $f(t)$ would be
(a) 0 (b) 1
(c) $-1 \le f(\infty) \le 1$ (b) ∞
[Gate (ECE) 2006]

104. Given $f(t) = L^{-1}\dfrac{3s+1}{s^3 + 4s^2 + (k-3)s}$. If $\lim_{t\to\infty} f(t) = 1$, then the value of k is
(a) 1 (b) 2
(c) 3 (d) 4 [Gate (ECE) 2010]

105. If $F(s) = L[f(t)] = \dfrac{2(s+1)}{s^2 + 4s + 7}$ then the initial and final value of $f(t)$ are respectively
(a) 0, 2 (b) 2, 0
(c) 0, 2/7 (d) 2/7, 0
[Gate (ECE) 2011]

106. The unilateral Laplace transform of $f(t)$ is $\dfrac{1}{s^2 + s + 1}$. The unilateral Laplace transform of $tf(t)$ is

(a) $-\dfrac{s}{(s^2+s+1)^2}$ (b) $-\dfrac{2s+1}{(s^2+s+1)^2}$

(c) $\dfrac{s}{(s^2+s+1)^2}$ (d) $\dfrac{2s+1}{(s^2+s+1)^2}$

[Gate(ECE) 2012]

107. Evaluate the sum and product of the eigenvalues of the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.
(a) 4, –14 (b) 4, –13
(c) 3, –14 (d) 3, –13

108. Find the characteristic roots of the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$.
(a) 1, –1 (b) 0, 1
(c) 0, –1 (d) 1, 0

109. Calculate the eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
(a) –2, 4, 8 (b) –2, 3, 6
(c) 5, –4, –1 (d) 3, 9, –2

110. Find the characteristic equation of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
(a) $\lambda^3 - 15\lambda^2 + 45\lambda = 0$
(b) $\lambda^3 - 25\lambda^2 + 45\lambda = 0$
(c) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$
(d) $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

111. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{bmatrix}$.
(a) 3 (b) 4
(c) 2 (d) 1

112. Calculate $I = \int_0^1 x^2(1-x^2)^{3/2}\,dx$.
(a) $\dfrac{\pi}{16}$ (b) $\dfrac{\pi}{48}$
(c) $\dfrac{\pi}{32}$ (d) $\dfrac{\pi}{64}$

113. Solve $(D^2 + 1)y = x$
(a) $c_1 \cos x + c_2 \sin x + x$
(b) $c_1 \cos x + c_2 \sin x - x$
(c) $c_1 \cos x - c_2 \sin x + x$
(d) $c_1 \cos x - c_2 \sin x - x$

114. Calculate $I = \int_0^{1/2}\int_0^1 \dfrac{x}{\sqrt{1-x^2y^2}}\,dydx$
(a) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{2} + 1$ (b) $\dfrac{\pi}{12} - \dfrac{\sqrt{3}}{2} + 1$

(c) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{2} + 2$ (d) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{2} - 1$

115. Evaluate $I = \int_0^{\pi} \int_0^{a\cos\theta} r \sin\theta dr d\theta$

(a) $\frac{a^2}{27}$ (b) $\frac{a^2}{9}$

(c) $\frac{a^2}{3}$ (d) $\frac{a^2}{54}$

116. Calculate $I = \int_0^{\pi} \int_0^{\pi/2} \int_0^{1} r^2 \sin\theta dr d\theta d\phi$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{9}$

(c) $\frac{\pi}{18}$ (d) $\frac{\pi}{27}$

117. Find J if $x + y + z = u$; $y + z = uv$; $z = uvw$.

(a) u^2v (b) v^2u

(c) uv (d) $\frac{uv}{2}$

118. Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2i - j - 2k$.

(a) $\frac{13}{3}$ (b) $\frac{13}{9}$

(c) $-\frac{13}{3}$ (d) $\frac{13}{27}$

119. Find the directional derivative of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of $i + 2j + 2k$.

(a) $-\frac{11}{3}$ (b) $\frac{11}{3}$

(c) $-\frac{11}{9}$ (d) $-\frac{11}{18}$

120. Find $\nabla(\nabla . f)$ at the point $(-1, 2, 1)$ if $f = 3xyz^2 i + 4x^3yj - xy^2zk$.

(a) $4i + 3j + 2k$ (b) $4i + 3j - 2k$

(c) $4i + 14j + 12k$ (d) $8i + 7j + 12k$

121. Calculate $\nabla^2\phi$ if $\phi = x^2 - y^2$.

(a) 0 (b) –1

(c) 1 (d) 2

122. Calculate $\int_C f.dr$ where $f = (2xy + z^3)i + x^2 j + 3xz^2 k$ along the straight line joining $(1, -2, 1)$ and $(3, 1, 4)$.

(a) 205 (b) 204

(c) 200 (d) 202

123. Evaluate $\int_C (xy - x^2)dx + x^2 y dy$ along the closed curve c formed by $y = 0$, $x = 1$ and $y = x$.

(a) $-\frac{1}{14}$ (b) $-\frac{1}{12}$

(c) $-\frac{1}{16}$ (d) $-\frac{1}{18}$

124. Calculate $\int_C (yzdx + zxdy + xydz)$ where C is the curve and $x^2 + y^2 = 1$, and $z = y^2$.

(a) 1 (b) 0

(c) –1 (d) 2

125. Calculate $I = \int_0^1 x^{5-1}(1-x)^{4-1} dx$.

(a) $-\frac{1}{180}$ (b) $-\frac{1}{280}$

(c) $\frac{1}{280}$ (d) $-\frac{1}{189}$

126. If 1, 2, 3 are the roots of the equation $x^3 - 6x^2 + ax - 6 = 0$, find the value of a.

(a) 12 (b) 13

(c) 11 (d) 10

127. Find the P.I. in the solution of $(D^2 + 16)y = e^{-4x}$.

(a) $\frac{e^{-4x}}{32}$ (b) $\frac{e^{-4x}}{16}$

(c) $\frac{e^{-2x}}{32}$ (d) $\frac{e^{4x}}{32}$

Solutions

1. Ans: (b)

Hint: Given, $X(s) = \left[\frac{3s+5}{s^2 + 10s + 21}\right]$

Using initial value theorem, we have

$$x(0^+) = \lim_{s\to\infty}[sX(s)]$$

$$\therefore \quad x(0^+) = \lim_{s\to\infty}\left[\frac{s(3s+5)}{s^2+10s+21}\right]$$

$$= \lim_{s\to\infty}\left[\frac{3+\frac{5}{s}}{1+\frac{10}{s}+\frac{21}{s^2}}\right]$$

$$= \frac{3}{1} = 3$$

2. Ans: (a)

Hint: Here $f(x) = e^x - 1$

$$f'(x) = e^x$$

The Newton–Raphson iterative equation is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = e^{x_i} - 1$$

$$f'(x_i) = e^{x_i}$$

$$\therefore \quad x_{i+1} = x_i - \frac{e^{x_i} - 1}{e^{x_i}}$$

i.e.,
$$x_{i+1} = \frac{x_i e^{x_i} - (e^{x_i} - 1)}{e^{x_i}}$$

$$= \frac{e^{x_i}(x_i - 1) + 1}{e^{x_i}}$$

Now, put $i = 0$

$$x_1 = \frac{e^{x_0}(x_0 - 1) + 1}{e^{x_0}}$$

Put $x_0 = -1$ as given,

$$x_1 = \frac{[e^{-1}(-2) + 1]}{e^{-1}} = 0.71828$$

3. Ans: (a)

Hint:
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{x_k^2 - 117}{2x_k}$$

$$= \frac{1}{2}\left[x_k + \frac{117}{x_k}\right]$$

4. Ans: (b)

Hint: $u(x_1, x_2) = 10x_2 \sin x_1 - 0.8 = 0$

$$v(x_1, x_2) = 10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0$$

The Jacobian matrix is

$$\begin{bmatrix} \dfrac{\partial u}{\partial x_1} & \dfrac{\partial u}{\partial x_2} \\ \dfrac{\partial v}{\partial x_1} & \dfrac{\partial v}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 10x_2 \cos x_1 & 10 \sin x_1 \\ 10x_2 \sin x_1 & 20x_2 - 10\cos x_1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

5. Ans: (c)

Hint: $f'(x) = 3x^2 + 2$

$$f'(x_0) = 3(1.2)^2 + 2 = 6.32$$

$$f(x) = (1.2)^3 + 2 \times 1.2 - 1 = 3.128$$

$$f'(x_1) = x_0 - \frac{f(x_0)}{f'(x_1)}$$

$$= 1.2 - \frac{3.728}{6.32} = 0.705$$

6. Ans: (0.05 to 0.07)

Hint: Given, $f(x) = e^x - 1$

or, $f(x_k) = (e^{xk} - 1)$ and $x_0 = 1$

In Newton–Raphson method, we have

$$x_{k+1} = \left[x_k - \frac{f(x_k)}{f'(x_k)}\right]$$

$$\therefore \quad x_1 = \left[x_0 - \frac{f(x_0)}{f'(x_0)}\right] \quad \text{(i)}$$

Now, $f(x_0) = e^{x_0} - 1 = e^1 - 1 = (e - 1)$

and $f'(x) = e^x$

$\therefore \quad f'(x_0) = e^1 = e$

Putting the values, we get

$$x_1 = \left[1 - \frac{(e-1)}{e}\right]$$

$$= \left[\frac{e - e + 1}{e}\right] = e^{-1}$$

Also
$$x_2 = \left[x_1 - \frac{f(x_1)}{f'(x_1)}\right] \quad \text{(ii)}$$

Now, $x_1 = \dfrac{1}{e} = e^{-1}$ and $f(x_1) = (e^{e^{-1}} - 1)$

$$f'(x_1) = e^{e^{-1}}$$

Putting the values, we get

$$x_2 = \left[e^{-1} - \frac{(e^{e^{-1}} - 1)}{e^{e^{-1}}}\right]$$

$$= \left[e^{-1} - \frac{(e^{0.37} - 1)}{e^{0.37}}\right] = 0.06$$

Therefore, the absolute error observed at second iteration = 0.06.

7. Ans: (c)

Hint: x is uniformly distributes in [0, 1]

$\therefore$ Probability density function

$$f(x) = \frac{1}{b-a} = \frac{1}{1-0} = 1$$

$\therefore \quad f(x) = 1,\ 0 < x < 1$

$\quad = 0$, elsewhere

Now,
$$E(x^3) = \int_0^1 x^3 f(x)dx$$

$$= \int_0^1 x^3 \times 1 \times dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

8. Ans: (b)

Hint: Let N people in room. So number of event that at least two people in room born at same date = $^NC_2$

$$0.5 \geq \frac{^NC_2 \cdot {}^NC_4, \ldots, {}^NC_N}{N!}$$

So, $N = 7$

9. Ans: (c)

Hint: P(II is red | I is white)

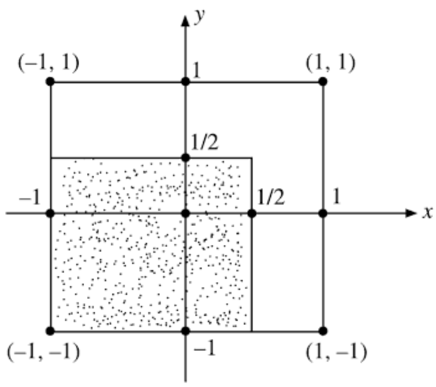
$$= \frac{P(\text{II is red and I is white})}{P(\text{I is white})}$$

$$= \frac{P(\text{I is white and II is red})}{P(\text{I is white})}$$

$$= \frac{4/7 \times 3/6}{4/7} = \frac{3}{6} = \frac{1}{2}$$

10. Ans: (b)

Hint: $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ is the entire rectangle. The region in which maximum of $\{x, y\}$ is less than 1/2 is shown below as shaded region inside this rectangle



$$P\left(\max\{x, y\} < \frac{1}{2}\right) = \frac{\text{Area of shaded region}}{\text{Area of entire rectangle}}$$

$$= \frac{3/2 \times 3/2}{2 \times 2} = \frac{9}{16}$$

11. Ans: (c)

Hint: P (number of tosses is odd)

$= P$ (number of tosses is 1, 3, 5, 7....)

P (number of toss is 1) = P (head in first toss)

$$= \frac{1}{2}$$

P (number of toss is 3) = P (tail in first toss, tail in second toss and head in third toss)

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

P (number of toss is 5) = P (T, T, T, T, H)

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32} \cdots \text{etc}$$

So, P (number of tosses is odd) $= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

Sum of infinite geometric series with

$$a = \frac{1}{2} \text{ and } r = \frac{1}{32}$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

12. Ans: (a)

Hint: $P = \int_1^\infty f(x)dx = \int_1^\infty e^{-x} dx$

$$= e^{-x}\Big|_\infty^1 = e^{-1} = 0.368$$

13. Ans: (b)

Hint: $(x + y)^n = \sum_{r=0}^{n} {}^nC_n x^{n-r} y^n$

When n = probability of occurrence of head

y = probability of occurrence of tail

Let number of head is P

Number of tail is q

$$P + q = n \quad \text{(total number of tails)} \quad \text{(i)}$$

Given $|P - q| = n - 3$

$|P - (n - P)| = n - 3$

$n = n - 3$ which is not possible

Here, required probability is zero.

14. Ans: (0.13 to 0.15)

Hint: Let probability of occurrence of one dot is P. So, writing total probability

$$P + 2P + 3P + 4P + 5P + 6P = 1$$

$$P = \frac{1}{21}$$

Hence, problem of occurrence of 3 dots is

$$3P = \frac{3}{21} = \frac{1}{7} = 0.142$$

15. Ans: (0.35 to 0.45)

Hint: Probability $(0.5 < n < 5) = \int_{0.5}^{5} f(x)dx$

$$= \int_{0.5}^{1} 0.2dx + \int_{1}^{4} 0.1dx + \int_{4}^{5} 0dx$$

$$= 0.2[1 - 0.5] + 0.1[4 - 1] + 0[5 - 4]$$

$$x = \int e^{-2t}u(t)dt = \int f(t)dt$$

At $t = 0.0015$, x = area of trapezoidal

$$= \frac{h}{2}[f(0) - f(0.01)]$$

$$= \frac{0.01}{2}[1 + 3^{-0.02}]$$

$= 0.0099$

26. Ans: (b)

Hint: Given: $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$

$$x(0) = 1 \text{ and } \left.\frac{dx}{dt}\right|_{t=0} = 0$$

$$D^2 + 6D + 8 = 0$$

$$(D + 4)(D + 2) = 0$$

$$D = -2 \text{ and } D = -4$$

$\therefore$ Solution is

$$x = C_1e^{-2t} + C_2e^{-4t}$$

Since, $x(0) = 1$

We have $\quad C_1 + C_2 = 1 \quad$ (i)

$$\frac{dx}{dt} = -2C_1e^{-2t} - 4C_2e^{-4t}$$

Since, $\left[\frac{dx}{dt}\right]_{t=0} = 0$, we have

$$-2C_1 - 4C_2 = 0 \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii) we have, $C_1 = 2$ and $C_2 = -1$

So, the solution is

$$x(t) = 2e^{-2t} - e^{-4t}$$

27. Ans: (a)

Hint: $\frac{dy}{dx} = e^{-3x}$

$$\int dy = \int e^{-3x}dx$$

$$y = \frac{e^{-3x}}{-3} + K$$

$$y = -\frac{1}{3}e^{-3x} + K$$

28. Ans: (d)

Hint: The given differential equation is $\frac{tdx}{dt} + x = t$ with initial condition $x(1) = \frac{1}{2}$ which is same as

$$\frac{dx}{dt} + \frac{x}{t} = 1$$

Which is a linear differential equation

$$\frac{dx}{dt} + Px = Q$$

Where $P = \frac{1}{t}$ and $Q = 1$

Integrating factor $= e^{\int Pdt = e^{\int \frac{1}{t}dt}}$

$$= e^{\log_e t} = t$$

Solution is

$$X \cdot (\text{IF}) = \int Q \cdot (\text{IF})dt + C$$

$$x \cdot t = \int 1 \cdot t \cdot dt + C$$

$$xt = \frac{t^2}{2} + C$$

$$x = \frac{t}{2} + \frac{C}{t}$$

Put $x(1) = \frac{1}{2}$

or $\quad \frac{1}{2} + \frac{C}{1} = \frac{1}{2}$

or $\quad C = 0$

So, $x = \frac{t}{2}$ is the solution.

29. Ans: (c)

Hint: $\frac{d^2x}{dt^2} = -9x; \qquad \frac{d}{dt} = D$

$$\frac{d^2x}{dt^2} + 9x = 0; \qquad (D^2 + 9x) = 0$$

Auxiliary equation is $m^2 + 9 = 0$

$$m = \pm 3i$$

$$x = C_1 \cos 3t + C_2 \sin 3t$$

$x(0) = 1$ i.e., $x \to 1$ when $t \to 0$

$\therefore \quad C_1 = 1$

$$\frac{dx}{dt} = -3C_1 \sin 3t + 3C_2 \cos 3t$$

$x'(0) = 1$ i.e., $x' \to 1$ when $t \to 0$

$\therefore \quad 1 = 3C_2$

or $\quad C_2 = \frac{1}{3}$

$\therefore \quad x = \cos 3t + \frac{1}{3}\sin 3t$

30. Ans: (c)

Hint: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

Let $\quad x = e^z$

$\therefore \quad z = \log x$

$$x\frac{d}{dx} = xD = \theta = \frac{d}{dz}$$

$$x^2D^2 = \theta(\theta-1)$$

$$(x^2D^2 + xD - 1)y = 0$$

$$[\theta(\theta-1)+\theta-1]y = 0$$

$$(\theta^2-\theta+\theta-1)y = 0$$

$$(\theta^2-1)y = 0$$

Auxiliary equation is $m^2 - 1 = 0$

$$m = \pm 1$$

C.F. is $C_1e^{-z} + C_2e^z$

Solution is $\quad y = C_1e^{-z} + C_2e^z$

$$y = C_1x^{-1} + C_2x^1$$

$$y = C_1\frac{1}{x} + C_2x$$

One independent solution is $1/x$.

Another independent solution is x.

31. Ans: (c)

Hint: $\delta = \int_1^\infty x^{-3}dx = \left[\frac{x^{-2}}{-2}\right]_1^\infty = -\left[\frac{1}{2x^2}\right]_1^\infty$

$$= -\left[\frac{1}{\infty} - \frac{1}{2}\right] = \frac{1}{2}$$

32. Ans: (c)

Hint: $\text{Grad} = i\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x\mathbf{i} + \frac{2}{3}y\mathbf{j}$

At $(1,3)$, $\text{Grad} = \sqrt{1^2 + \left[\left(\frac{2}{3}\right)\times 3\right]^2} = \sqrt{5}$

33. Ans: (d)

Hint: We consider options (a) and (d) only because it contains variable r.

By integrating (d), we get

$1/3\pi r^2H$, which is volume of cone.

34. Ans: (b)

Hint: By property of definite integral

$$\int_0^{2\pi} f(x) = \int_0^{2\pi} t(2\pi - x) = -f(x) = 0$$

35. Ans: (a)

Hint: $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$$\text{div}\,\mathbf{r} = \nabla\cdot\mathbf{r}$$

$$= \left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)\cdot(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$$

$$= \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right)$$

$$= 1 + 1 + 1 = 3$$

36. Ans: (a)

Hint: rank$[Pq]$ = rank$[P]$ is necessary for existence of at least one solution $Px = q$

37. Ans: (d)

Hint: The $n \times n$ matrix $V = xx^T$ has rank n.

38. Ans: (d)

Hint: $\det\begin{bmatrix} x\cdot x & x\cdot y \\ y\cdot x & y\cdot y\end{bmatrix} = \begin{bmatrix} x\cdot x & x\cdot y \\ y\cdot x & y\cdot y\end{bmatrix}$ is zero only when either x or y is zero.

39. Ans: (d)

Hint: The eigenvalues of M are $(i, -i, 1)$

40. Ans: (a)

Hint: If the rank of (5×6) matrix is 4, then surely it must have exactly four linearly independent rows as well as four linearly independent columns.

41. Ans: (b)

Hint: If matrix is A, then characteristic equation of matrix is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & 0 & 0 \\ 0 & 3-\lambda & 0 & 0 \\ 0 & 0 & -2-\lambda & 0 \\ 0 & 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

$$\lambda = 2, 3, -2, 4$$

42. Ans: (c)

Hint: Eigenvalue can be determined by

$$|\lambda I - A| = 0$$

$$\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}-4 & 2\\ 4 & 3\end{bmatrix} = \lambda I - A$$

$$\lambda I - A = \begin{bmatrix}\lambda+4 & -2\\ -4 & \lambda-3\end{bmatrix}$$

$$|\lambda I - A| = 0$$

$$(\lambda+4)(\lambda-3) - 8 = 0$$

$$\lambda^2 + \lambda - 20 = 0 \Rightarrow \lambda = 4, -5$$

Eigenvector for $\lambda_1 = 4$

$$(\lambda_1 I - A)X = 0$$

$$\begin{bmatrix} \lambda_1 + 4 & -2 \\ -4 & \lambda_1 - 3 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = 0$$

$$\begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = 0$$

We get only one independent equation

$$8x_{11} - 2x_{12} = 0$$
$$-4x_{11} + x_{12} = 0$$
$$4x_{11} = x_{12}$$

Suppose

$$x_{11} = K$$
$$x_{12} = 4K$$

Eigenvector will be

$$\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} K \\ 4K \end{bmatrix} = K\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Now, eigenvector for $\lambda_2 = -5$

$$(\lambda_2 I - A)X = 0$$

$$\begin{bmatrix} \lambda_2 + 4 & -2 \\ -4 & \lambda_2 - 3 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -2 \\ -4 & -8 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = 0$$

We get only one independent equation

$$-x_{21} - 2x_{22} = 0$$
$$x_{21} = -2x_{22}$$

Suppose

$$x_{22} = K$$
$$x_{21} = -2K$$

Eigenvector will be

$$\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} -2K \\ K \end{bmatrix} = -K\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

43. Ans: (a)

Hint: $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0.1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0/2 & 0.1/6 \\ 0 & 1/3 \end{bmatrix}$$

$$a = \frac{0.1}{6} = \frac{1}{60} \text{ and } b = \frac{1}{3}$$

$$a + b = \frac{1}{60} + \frac{1}{3} = \frac{7}{20}$$

44. Ans: (c)

Hint: From the definition of orthogonal matrix

$$[AA^T]^{-1} = I$$

where A is orthogonal matrix and I is identity matrix.

45. Ans: (c)

Hint: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{3\times 3}$

The rank of a matrix is the largest order of any non-vanishing minor of the matrix.

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0$, so the 3rd order minor of A vanishes.

Now, we check the 2nd order minor formed by its 1st and 2nd rows.

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \neq 0$$

$\therefore\ \rho(A) = 2$. Hence, the rank of the given matrix is 2.

46. Ans: (a)

Hint: Eigenvalues are $\lambda_1 = 8$, $\lambda_2 = 4$

Trace (A) = sum of diagonal elements of matrix A

Trace (A) = 12

Option (a) satisfies this relation.

47. Ans: (c)

Hint: $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$$|\lambda I - A| = 0$$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} \lambda - 4 & -2 \\ -2 & \lambda - 4 \end{bmatrix}$$

$$|\lambda I - A| = (\lambda - 4)^2 - 4 = \lambda^2 + 16 - 8\lambda - 4$$

$$|\lambda I - A| = \lambda^2 - 8\lambda + 12$$

$$|\lambda I - A| = 0$$

$$\lambda^2 - 8\lambda + 12 = 0 \Rightarrow \lambda_1 = 6, \lambda_2 = 2$$

Now, $(\lambda I - A)x = 0$

For $\lambda_1 = 6$, we have

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

We get only one independent equation

$$2x_1 - 2x_2 = 0$$
$$x_1 = x_2$$

Here we can choose $x_1 = x_2 = 101$

So, eigenvector is $\begin{bmatrix}101\\101\end{bmatrix}$.

48. Ans: (c)

Hint: If there are two orthogonal vectors, we require two dimensions to define them (say X and Y). If there are three orthogonal vectors, we require three dimensions to define them (say X, Y and Z). The $2M$ vectors

$$\begin{matrix} X_1, & X_2, & X_3 & \ldots & X_M, \\ -X_1, & -X_2, & -X_3 & \ldots & -X_M, \end{matrix}$$

are basically M orthogonal vectors. Hence, we require M dimensions to define them.

49. Ans: (c)

Hint: Product of eigenvalue = determinant of matrix, since one of the eigenvalue is zero thus the determinant of matrix = 0.

Hence, $$p_{11}p_{22} - p_{12}p_{21} = 0$$

50. Ans: (b)

Hint: Rank of matrix A; $\rho(A) = 1$

Rank of matrix AB; $\rho(AB) = 2$

Since $\rho(A) \neq \rho(AB)$

Therefore, it has no solution.

51. Ans: (d)

Hint: State transition matrix

$$e^p = \mathrm{L}^{-1}[(sI - A)^{-1}]$$

$$e^p = \mathrm{L}^{-1}\begin{bmatrix} s & -1 \\ +2 & (s+3) \end{bmatrix}^{-1}$$

$$= \mathrm{L}^{-1}\begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\ \dfrac{-2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-1} - e^{-2} & e^{-1} - e^{-2} \\ -2e^{-1} + 2e^{-2} & -e^{-1} + 2e^{-2} \end{bmatrix}$$

52. Ans: (d)

Hint: Matrix $\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

Sum of eigenvalues is the trace of matrix i.e.

Trace of matrix = $\lambda_1 + \lambda_2 + \lambda_3 = -1 - 1 + 3 = 1$

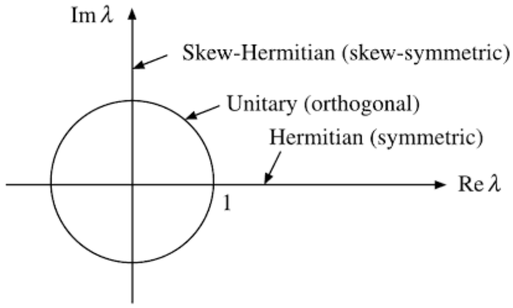
Trace of matrix = sum of diagonal elements of matrix from option (d).

$$\lambda_1 = 3,\ \lambda_2 = -1 + 3j,\ \lambda_3 = -1 - 3j$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 3 - 1 + 3j - 1 - 3j = 1$$

53. Ans: (c)

Hint: The location of the eigenvalues of Hermitian, Skew–Hermitian and unitary matrices in the complex λ-plane is shown in figure.



(i) The eigenvalues of a Hermitian matrix (symmetric matrix) are real.

(ii) The eigenvalues of a Skew–Hermitian matrix (skew–symmetric matrix) are pure imaginary or zero.

(iii) The eigenvalues of a unitary matrix (orthogonal–matrix) have absolute value 1.

54. Ans: (b)

Hint: Coefficient matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 6 \\ 1 & 4 & \lambda \end{bmatrix}$

Rank of coefficient matrix = $\rho(A)$

Augmented matrix $k = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 4 & 4 & \lambda & \mu \end{bmatrix}$

Rank of augmented matrix = $\rho(k)$

System of equation has no solution when $\rho(A) \neq \rho(k)$

In A, $R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 6 \\ 0 & 0 & \lambda - 6 \end{bmatrix}$$

$$\rho(A) = 2, \text{ if } \lambda = 6$$

In k, $R_3 \to R_3 - R_2$

$$k = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda - 6 & \mu - 20 \end{bmatrix}$$

If $\lambda = 6$

$$k = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & 0 & \mu - 20 \end{bmatrix}$$

$$\rho(k) = 3, \text{ if } \mu \neq 20$$

For $\lambda = 6$ and $\mu \neq 20, \rho(A) \neq \rho(k)$

55. Ans: (b)

Hint: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$$

By hit and trial, we can check the correct option.
In option (b), we have

$$19A + 30I = 19\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} + 30\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -95 & -57 \\ 38 & 0 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$$

56. Ans: (c)

Hint: Fourier series analysis is only applicable on periodic signals.
$3\sin(25t)$ is periodic.
$4\cos(20t + 3) + 2\sin(10t)$ is sum of two periodic signals so it is periodic.
$\exp(-|t|)\,\sin(25t)$ is exponentially decaying signal so it is a periodic.

57. Ans: (a)

Hint: We know that Gaussian PDF is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-m)^2}{2\sigma^2}} \quad \text{for } -\infty \le x \le \infty$$

Where σ^2 = variance, m = mean value
For $m = 0$, we have

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x^2}{2\sigma^2}}$$

If $\sigma^2 = 4$

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{\frac{-x^2}{8}} = \frac{1}{2\sqrt{2\pi}} e^{\frac{-x^2}{8}}$$

We also know that

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

(Area under the curve of PDF is 1)

$$\int_{-\infty}^{\infty} \frac{1}{2\sqrt{2\pi}} e^{\frac{-x^2}{8}} dx = 1$$

$$2\int_{0}^{\infty} \frac{1}{2\sqrt{2\pi}} e^{\frac{-x^2}{8}} dx = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{8}} dx = 1$$

58. Ans: (c)

Hint: In given signal, magnitude is constant near to origin so its derivative near to origin has to be zero. Only option (b), (c), and (d) are satisfying this. In given signals, right hand side has negative slope so its derivate has to be negative in magnitude and left hand side has positive slope so its derivative has to be positive in magnitude and only option (c) satisfies this.

59. Ans: (d)

Hint: $\nabla \times \nabla \times \mathbf{P} = \nabla(\nabla \cdot \mathbf{P}) - \nabla^2 \mathbf{P}$
We use this relation to find out the equation of electric field component of uniform plane wave.

60. Ans: (a)

Hint: Stokes theorem gives the relation between line and surface integrals i.e.,

$$\int_s (\nabla \times \mathbf{P}) \cdot ds = \oint \mathbf{P} \cdot dl$$

61. Ans: (c)

Hint: $\int_0^{\pi} \sin^3\theta d\theta = \int_0^{\pi} \sin\theta(1 - \cos^2\theta)d\theta$

$$= \int_0^{\pi} \sin\theta d\theta - \int_0^{\pi} \sin\theta\cos^2\theta d\theta$$

$$= [-\cos\theta]_0^{\pi} - \int_0^{\pi} \sin\theta\cos^2\theta d\theta$$

$$= -[-1-1] - \int_0^{\pi} \sin\theta\cos^2\theta d\theta$$

$$= 2 - \int_0^{\pi} \sin\theta\cos^2\theta d\theta$$

Now $\cos\theta = t$,

$-\sin\theta d\theta = dt$ and $\theta = \pi,\ t = -1,\ \theta = 0,\ t = 1$

$$= 2 + \int_1^{-1} t^2 dt = 2 - \int_{-1}^{1} t^2 dt$$

$$= 2 - 2\int_0^1 t^2 dt, \qquad \text{since } t^2 \text{ in even}$$

$$= 2 - 2\left[\frac{t^3}{3}\right]_0^1 = 2 - \frac{2}{3}[1] = \frac{4}{3}$$

62. Ans: (a)

Hint: $f(x)=\dfrac{e^x}{1+e^x}$

If we take x from $-\infty$ to $+\infty$ the value of $f(x)$ monotonically increases.

63. Ans: (b)

Hint: The equation of the line shown in the figure is $y = x + 1$.

$$I=\int_1^2 y dx=\int_1^2 (x+1)dx$$

$$I=\left[\frac{x^2}{2}\right]_1^2+[x]_1^2=\left[\frac{4}{2}-\frac{1}{2}\right]+[2-1]=1.5+1=2.5$$

64. Ans: (c)

Hint: $\coth(x)=\dfrac{\cosh x}{\sinh x}=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}=\dfrac{e^{2x}+1}{e^{2x}-1}$

$$=\frac{1+2x+\frac{(2x)^2}{2!}+\cdots+1}{1+2x+\frac{(2x)^2}{2!}+\cdots-1}=\frac{2+2x+\frac{(2x)^2}{2!}+\cdots}{2x+\frac{(2x)^2}{2!}+\cdots}$$

Given that $|x| << 1$, so neglecting higher power of x

$$\coth(x)=\frac{2+2x}{2x}=\frac{1}{x}+1$$

Now, $|x| << 1$ or $1<<\dfrac{1}{|x|}$

Hence, $\coth(x)\approx\dfrac{1}{x}$

65. Ans: (a)

Hint: $\lim\limits_{\theta\to 0}\dfrac{\sin(\theta/2)}{\theta}\quad\left(\dfrac{0}{0}\text{ form}\right)$

Applying L' Hospital rule, we have

$$\lim_{\theta\to 0}\frac{(1/2)\cos(\theta/2)}{1}=\frac{1}{2}=0.5$$

66. Ans: (d)

Hint: Any function $f(x)$ is bounded if $|f(x)|<\infty$, for any value of x. Only e^{-x^2} is satisfying the condition for bounded function.

67. Ans: (a)

Hint: $f(x) = e^{-x}$ linear approximation around $x = 2$

$$f(x)=e^{-(x-2)}e^{-2}$$

$$=\left[1-(x-2)+\frac{(x-2)^2}{2!}\right]e^{-2}$$

On linear approximation, we have

$f(x) = [1 - (x - 2)]e^{-2} = (3 - x)e^{-2}$

68. Ans: (d)

Hint: $f(x) = x^2 - x - 2$

$f'(x)=2x-1$

For $f(x)$ to be maximum or minimum $f'(x)$ must be zero in that point.

or $2x - 1 = 0$

or $x=\dfrac{1}{2}$

$f''(x)=2$, which is positive so at $x = 1/2$, $f(x)$ has minimum value we cannot find maximum value of $f(x)$.

69. Ans: (c)

Hint: The common area between the signals is zero for the orthogonal signals.

$$\int_{-\infty}^{\infty} f_1(t)\cdot f_2(t)dt=0$$

70. Ans: (a)

Hint: Given $f(x) = e^x + e^{-x}$

Maximum value at $x = 0$; $f(x) = 2$

71. Ans: (a)

Hint: $g(x, y) = 4x^3 + 10y^4$

Integration of $g(x, y)$ along $y = 2x$

$\therefore\quad g(x, 2x) = 4x^3 + 10(2x)^4 = 4x^3 + 160x^4$

$$\therefore\quad \int_0^1 g(x,2x)\,dx=\int_0^1(4x^3+160x^4)\,dx=[x^4+32x^5]_0^1=33$$

72. Ans: (b)

Hint: According to Green's theorem

$$\int_C(\phi dx+\phi dy)=\iint_S\left(\frac{\partial\phi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$$

where S = closed surface area (i.e., semicircle here)

$$\therefore\quad 2\int_P^Q(xdy+ydx)=2\iint_S\left(\frac{\partial(y)}{\partial x}-\frac{\partial(x)}{\partial y}\right)dxdy=0$$

73. Ans: (a)

Hint: If $x(t)$ is real and periodic then Fourier series of $x(t)$

$$x(t)=\frac{a_0}{2}+\sum_{k=1}^{\infty}a_k\cos\frac{2\pi}{T_0}kt+\sum_{k=1}^{\infty}b_k\sin\frac{2\pi}{T_0}kt$$

where $\dfrac{a_0}{2}=\dfrac{1}{T_0}\displaystyle\int_{T_0}x(t)\,dt$

$$a_k=\frac{1}{T_0}\int_{T_0}x(t)\cos\frac{2\pi}{T_0}ktdt$$

and $b_k=\dfrac{2}{T_0}\displaystyle\int_{T_0}x(t)\sin\frac{2\pi}{T_0}kt\,dt$

For path (3) to (4)

$$I_3 = \int_{2/\sqrt{3}}^{1/\sqrt{3}} (xy\mathbf{a}_x + x^2\mathbf{a}_y)\cdot dx\mathbf{a}_x$$

$$I_3 = \int_{2/\sqrt{3}}^{1/\sqrt{3}} xydx = \int_{2/\sqrt{3}}^{1/\sqrt{3}} 3xdx, \quad \text{since } y = 3,$$

$$I_3 = \frac{3}{2}[x^2]_{2/\sqrt{3}}^{1/\sqrt{3}} = \frac{3}{2}\left[\frac{1}{3} - \frac{4}{3}\right] = \frac{-3}{2}$$

For path (4) to (1)

$$I_4 = \int_3^1 \left(xy\mathbf{a}_x + x^2\mathbf{a}_y\right)\cdot dy\mathbf{a}_y$$

$$I_4 = \int_3^1 x^2 dy = \int_3^1 \frac{1}{3}dy, \text{ since } x = \frac{1}{\sqrt{3}}$$

$$I_4 = \frac{1}{3}[y]_3^1 = \frac{1}{3}[1-3] = \frac{-2}{3}$$

So, $I = \oint_C \mathbf{A}\cdot d\mathbf{l} = I_1 + I_2 + I_3 + I_4$

$$I = \oint_C \mathbf{A}\cdot d\mathbf{l} = \frac{1}{2} + \frac{8}{3} + \left(-\frac{3}{2}\right) + \left(-\frac{2}{3}\right) = -1 + 2 = 1$$

or $\quad I = \oint_C \mathbf{A}\cdot d\mathbf{l} = \int (\nabla\times\mathbf{A})\cdot d\mathbf{S}$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & x^2 & 0 \end{vmatrix} = 0 + 0 + \mathbf{a}_z(2x - x) = x\mathbf{a}_z$$

$$I = \int (\nabla\times\mathbf{A})\cdot d\mathbf{S} = \iint (x\mathbf{a}_z)\cdot(dxdy\mathbf{a}_z) = \iint xdxdy$$

$$= \int_{x=1/\sqrt{3}}^{x=2/\sqrt{3}} \int_{y=1}^{y=3} xdxdy = \int_{x=1/\sqrt{3}}^{x=2/\sqrt{3}} 2xdx = \frac{2}{2}[x^2]_{1/\sqrt{3}}^{2/\sqrt{3}}$$

$$I = \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1$$

79. Ans: (d)

Hint: $\mathbf{r}$ is a position vector

$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

By divergence theorem

$$\oiint_s \mathbf{r}.\mathbf{n}dS = \int_v (\overline{\nabla}\cdot\mathbf{r})dV \qquad \text{(i)}$$

$$\nabla\cdot\mathbf{r} = \left(\frac{d}{dx}\mathbf{a}_x + \frac{d}{dy}\mathbf{a}_y + \frac{d}{dz}\mathbf{a}_z\right)\cdot(x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z)$$

$$\nabla\cdot\mathbf{r} = 1 + 1 + 1 = 3$$

And volume is V, so right hand side of Eq. (i) is

$$\int_V (\nabla\cdot\mathbf{r})dV = 3V$$

We have to find out

$$5\oiint_S \mathbf{r}.\mathbf{n}dS = 5\times 3V = 15V$$

80. Ans: (c)

Hint: Given function is

$$f(x) = x^3 - 9x^2 + 24x + 5$$

In the interval [1, 6] or $1 \le x \le 6$

Differentiating $f(x)$ w.r.t. x, we get

$$\frac{df(x)}{dx} = 3x^2 - 18x + 24$$

$$\frac{df(x)}{dx} = 0$$

$$3x^2 - 18x + 24 = 0$$

or $\quad x_1 = 4,\ x_2 = 2$

Now differentiating $\frac{df(x)}{dx}$ w.r.t. x, we have

$$\frac{d^2 f(x)}{dx^2} = 6x - 18$$

At $x = x_1 = 4$, $\frac{d^2 f(x_1)}{dx^2} = 6\times 4 - 18 = +\text{ve}$

It means at $x = 4$, there is a minima.

At $x = x_2 = 2$, $\frac{d^2 f(x_2)}{dx^2} = 6\times 2 - 18 = -\text{ve}$

It means at $x = 2$, there is a maxima.

The value of function $f(x)$ at $x = 2$ is

$$f(2) = 2^3 - 9\times 2^2 + 24\times 2 + 5$$

$$f(2) = 8 - 36 + 48 + 5 = 41$$

Hence, the maximum value of $f(x)$ is 41.

81. Ans: (a)

Hint: $\mathbf{A} = |A|\mathbf{a}_r$

$\mathbf{A} = kr^n\mathbf{a}_r$

In spherical coordinate,

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\mathbf{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

Here A_θ and A_ϕ are zero.

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr^n)$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^{n+2}) = \frac{1}{r^2}(n+2)kr^{n+2-1}$$

$$\nabla\cdot\mathbf{A} = (n+2)kr^{n-1}$$

Company	*Probability of being defective*	*Defective computer*
X	0.01	0.6
Y	0.02	0.7
Z	0.03	0.8

When we take relative defectiveness into consideration it two times for X, two times for Y and one time for Z. Probability being supplied by Y is

$$= \frac{0.6}{0.6+0.6+0.3} = \frac{0.6}{1.5} = 0.4$$

93. Ans: (a)

Hint: Variance of random variable $X = \sigma_X^2$

Mean of random variable X is $\overline{X} = E[X]$

$$\sigma_X^2 = E[X-\overline{X}]^2$$

$$\sigma_X^2 = E[X^2 + \overline{X}^2 - 2X\overline{X}]$$

$$\sigma_X^2 = E[X^2] + E[\overline{X}^2] - E[2X\overline{X}]$$

$$\sigma_X^2 = E[X^2] + \overline{X}^2 - 2E[X]E[\overline{X}]$$

$$\sigma_X^2 = E[X^2] + \overline{X}^2 - 2\overline{X}^2$$

$$\sigma_X^2 = E[X^2] - \overline{X}^2$$

$$\sigma_X^2 = E[X^2] + E^2[X]$$

94. Ans: (c)

Hint: Probability of failing in paper 1 is $P(A) = 0.3$

Probability of failing in paper 2 is $P(B) = 0.2$

$$P(A/B) = 0.6$$

We have to find $P(A \cap B)$.

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

or $\quad P(A\cap B) = P\left(\frac{A}{B}\right)\cdot P(B) = 0.6\times 0.2 = 0.12$

95. Ans: (a)

Hint: For probability density function $f(x)$,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-\infty}^{\infty} \left(Me^{-2|x|} + Ne^{-3|x|}\right)dx = 1$$

$$\int_{-\infty}^{0} (Me^{2x} + Ne^{3x})dx + \int_{0}^{\infty} (Me^{-2x} + Ne^{-3x})dx = 1$$

$$2\left(\frac{M}{2} + \frac{N}{2}\right) = 1$$

$$M + \frac{2}{3}N = 1$$

96. Ans: (a)

Hint: Probability that only first two tosses yield heads

$$= H\cdot H\cdot (T)^8 = \frac{1}{2}\times\frac{1}{2}\times\left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{10}$$

97. Ans: (c)

98. Ans: (b)

Hint: Newton–Raphson rule

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For $\quad x_n = 2,\ f(x) = x^3 - x^2 + 4x - 4$

$$x_{n+1} = 2 - \frac{(2^3 - 2^2 + 4\cdot 2 - 4)}{(3\cdot 2^2 - 2\cdot 2 + 4)} = \frac{4}{3}$$

99. Ans: (c)

Hint: Given $x = e^{-x}$

Thus, $\quad f(x) = x - e^{-x} = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \left(\frac{x_n - e^{-x_n}}{1+e^{-x_n}}\right)$$

$$x_{n+1} = \frac{(1+x_n)e^{-x_n}}{1+e^{-x_n}}$$

100. Ans: (b)

Hint: By Euler's first order method new

$$y = \text{old } y + (0.1)\frac{dy}{dx}$$

Given step size is 0.1

x	y	$\frac{dy}{dx} = x + y$	new y = old y + $(0.1)\frac{dy}{dx}$
0	0	0	0
0.1	0	0.1	0.01
0.2	0.01	0.21	0.031
0.3	0.031	0.331	

$$y_1 = y_0 + hf(xy) = y_0 + h(x + y)$$

$$\frac{dy}{dx} = x + y(x) = f(xy)$$

$$y_1 = 0 + 0.1(0) = 0$$

$$y_2 = 0 + (0.1)(0.1) = 0.01$$

$$y_3 = 0.01 + 0.1(0.2 + 0.01) = 0.031$$

101. Ans: (c)

Hint: By Newton–Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_0 = 2, x_1 = ?$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x + \sqrt{x} - 3$$

$$f(x_0) = x_0 + \sqrt{x_0} - 3 = 2 + \sqrt{2} - 3 = 0.4142$$

$$f'(x_0) = 1 + \frac{1}{2}x^{\frac{1}{2}}$$

$$f'(x) = 1 + \frac{1}{2\sqrt{x_0}} = 1 + \frac{1}{2\sqrt{2}} = 1.3535$$

$$x_1 = 2 - \frac{0.4142}{1.3535} = 1.6939$$

102. Ans: (a)

Hint: $x(t) = e^{(a+2)t+5} = e^5 \cdot e^{(a+2)t}$

Considering $x(t)$ exists for $0 \le t \le \infty$

$$X(s) = \int_0^\infty x(t)e^{-st}dt$$

$$X(s) = \int_0^\infty e^5 \cdot e^{(a+2)t}e^{-st}dt = \int_0^\infty e^{-t(s-a-2)}dt$$

$$= e^5 \frac{1}{a+2-s}\left[e^{-t(s-a-2)}\right]_0^\infty$$

$$= e^5 \frac{1}{a+2-s}\left[e^{-\infty(s-a-2)} - 1\right]$$

Now to make $e^{-\infty(s-a-2)}$ zero real value of $(s - a - 2)$ should be positive

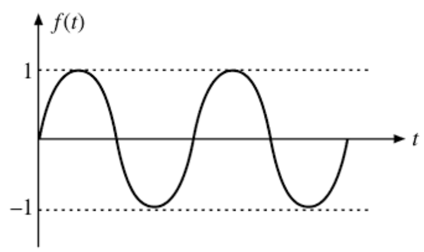
or $\text{Re}(s - a - 2) > 0$ or $\text{Re}(s) > a + 2$

$$X(s) = e^5 \frac{1}{a+2-s}[0-1] = \frac{e^5}{s-a-2}$$

103. Ans: (c)

Hint: $F(s) = \dfrac{\omega_0}{s_2 + \omega_0^2}$, $\text{Re}(s) > 0$

$f(t) = \sin \omega_0 \, tu(t)$



At $t = \infty$, value of $f(t)$ is in between – 1 and 1.

104. Ans: (d)

Hint: If Laplace transform of $f(t)$ is $F(s)$ then

$$F(s) = \frac{3s+1}{s^3 + 4s^2 + (k-3)s}$$

By final value theorem, we have

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} s\frac{3s+1}{[s^3 + 4s^2 + (k-3)s]}$$

$$= \lim_{s\to 0} \frac{3s+1}{s^2 + 4s + (k-3)}$$

$$\lim_{t\to\infty} f(t) = \frac{1}{k-3}$$

And given that $\lim_{t\to\infty} f(t) = 1$

So, $\dfrac{1}{k-3} = 1 \Rightarrow k = 4$

105. Ans: (b)

Hint: $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s\dfrac{2(s+1)}{s^2 + 4s + 7}$

$$f(0) = \lim_{s\to\infty} \frac{2s^2 + 2s}{s^2 + 4s + 7} = 2$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2s^2 + 2s}{s^2 + 4s + 7} = 0$$

106. Ans: (d)

Hint: Given that

$$F(s) = \frac{1}{s^2 + s + 1}$$

$$f(t) \xleftrightarrow{\text{L.T.}} F(s)$$

$$tf(t) \xleftrightarrow{\text{L.T.}} -\frac{dF(s)}{ds}$$

If $y(t) = tf(t)$

Then $Y(s) = -\dfrac{dF(s)}{ds} = -\dfrac{d}{ds}\left[\dfrac{1}{(s^2 + s + 1)}\right]$

$$Y(s) = -\left[\frac{(s^2 + s + 1)\cdot 0 - (2s+1)\cdot 1}{(s^2 + s + 1)^2}\right] = \frac{(2s+1)}{(s^2 + s + 1)^2}$$

107. Ans: (a)

Hint: Let $A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

Sum of the eigenvalues = trace of A = 3 + (–2) + 3 = 4

Product of the eigenvalues = det $|A|$ = –14

108. Ans: (a)

Hint: The characteristic equation of A is given by $|A - \lambda I| = 0$

So $$\begin{vmatrix} \cos\theta-\lambda & -\sin\theta \\ -\sin\theta & -\cos\theta-\lambda \end{vmatrix}=0$$

$\therefore$ $$\lambda^2-(\cos^2\theta+\sin^2\theta)=0$$

or $$\lambda^2-1=0$$

or $$\lambda=1,-1$$

109. Ans: (b)

Hint: The characteristic equation of A is given by $|A - \lambda I| = 0$

$\therefore$ $$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix}=0$$

$$(\lambda+2)(\lambda^2-9\lambda+18)=0$$

or $$(\lambda+2)(\lambda-6)(\lambda-3)=0$$

or $$\lambda=-2,3,6$$

110. Ans: (c)

Hint: The characteristic equation of A is given by $|A - \lambda I| = 0$

$\therefore$ $$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix}=0$$

or $(8-\lambda)(\lambda^2-10\lambda+5)+6(6\lambda-10)+2(2\lambda+10)=0$

or $$\lambda^3-18\lambda^2+45\lambda=0$$

111. Ans: (c)

Hint:

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & 0 \\ 0 & 3 & 4 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 4 & 2 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ 4 & 1 & 2 \\ 0 & 3 & 2 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ 4 & 0 & 2 \\ 0 & 4 & 2 \end{vmatrix}=0$$

Every 3×3 submatrix of A has determinant zero.

Also, $$\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix}=-3\neq 0$$

Hence, there exists at least one 2×2 sub-matrix whose determinant is not zero.

Rank of A = 2.

112. Ans: (c)

Hint: Put $x = \sin\theta$

Hence $dx = \cos\theta d\theta$

When $x = 0$, $\theta = 0$ and when $x=1,\ \theta=\dfrac{\pi}{2}$

$\therefore$ $$I=\int_0^{\pi/2}\sin^2\theta(\cos^2\theta)^{3/2}\cos\theta d\theta$$

$$=\int_0^{\pi/2}\sin^2\theta(\cos^4\theta)d\theta$$

$$=\frac{1.3.1}{6.4.2}\left(\frac{\pi}{2}\right)=\frac{\pi}{32}$$

113. Ans: (a)

Hint: The auxiliary equation is $m^2 + 1 = 0$

or $$m = \pm i$$

$$\text{C.F.} = c_1 \cos x + c_2 \sin x$$

$$\text{P.I.}=\left(\frac{1}{D^2+1}\right)x=(1+D^2)^{-1}x=(1-D^2)x=x$$

General solution $y = \text{C.F.} + \text{P.I.} = c_1 \cos x + c_2 \sin x + x$

114. Ans: (d)

Hint:

$$I=\int_0^{1/2}x\left[\frac{\sin^{-1}xy}{x}\right]_0^1 dx$$

$$=\int_0^{1/2}\sin^{-1}x dx$$

$$=\left[x\sin^{-1}x+\sqrt{1-x^2}\right]_0^{1/2}=\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$$

115. Ans: (c)

Hint: $$I=\int_0^{\pi}\sin\theta\left[\frac{r^2}{2}\right]_0^{a\cos\theta}d\theta$$

$$=\frac{1}{2}\int_0^{\pi}a^2\cos^2\theta\sin\theta d\theta$$

$$=\frac{-a^2}{2}\int_0^{\pi}\cos^2\theta d(\cos\theta)$$

$$=-\frac{a^2}{6}[\cos^3\theta]_0^{\pi}=\frac{a^2}{3}$$

116. Ans: (a)

Hint: $$I=\int_0^{\pi}\int_0^{\pi/2}\sin\theta\left[\frac{r^3}{3}\right]_0^1 d\theta d\phi$$

$$=\frac{1}{3}\int_0^{\pi}\int_0^{\pi/2}\sin\theta d\theta d\phi$$

$$=\frac{1}{3}\int_0^{\pi}[-\cos\theta]_0^{\pi/2}d\phi$$

$$=\frac{1}{3}\int_0^{\pi}d\phi=\frac{1}{3}[\phi]_0^{\pi}=\frac{\pi}{3}$$

117. Ans: (a)

Hint: From the given three transformations, we get

$$x = u - uv;\ y = uv - uvw;\ z = uvw$$

Now $$J=\frac{\partial(x,y,z)}{\partial(u,v,w)}=\begin{vmatrix} 1-v & -u & 0 \\ v(1-w) & u(1-w) & -uv \\ vw & uw & uv \end{vmatrix}$$

$$= u^2v$$

10

General Aptitude

10.1 SYLLABUS

Verbal ability: English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

Numerical ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

10.2 WEIGHTAGE IN PREVIOUS GATE EXAMINATION (MARKS)

Examination Year	2010	2011	2012	2013	2014		
					Set 1	Set 2	Set 3
1 Mark Question	5	5	5	5	5	5	5
2 Marks Question	5	5	5	5	5	5	5
Total Marks	**15**	**15**	**15**	**15**	**15**	**15**	**15**

10.3 VERBAL ABILITY

COMMON ERRORS

S. no.	Incorrect	Correct
1.	He turned a deaf ear to the advices of his father.	He turned a deaf ear to the advice of his father.
2.	He has no issues.	He has no issue.
3.	I have many works to do.	I have much work to do.
4.	Road closed for repair.	Road closed for repairs.
5.	How many son-in-laws have you?	How many sons-in-law have you?
6.	Second hand furnitures were put to auction.	Second hand furniture was put to auction.
7.	My house is out of repairs.	My house is out of repair.
8.	One must keep one's words.	One must keep one's word.
9.	There is no place in this compartment.	There is no room in this compartment.
10.	The girls are walking in the centre of the road.	The girls are walking in the middle of the road.
11.	All but I were present at the meeting.	All but me were present at the meeting.
12.	I qualified myself for the job.	I qualified for the job.
13.	You should avail of this opportunity.	You should avail yourself of this opportunity.
14.	They enjoyed in the fair.	They enjoyed themselves in the fair.
15.	Those who came late they were not allowed to sign the register.	Those who came late were not allowed to sign the register.
16.	I am junior than you by two years.	I am junior to you by two years.
17.	I have given you the most complete account of the incident.	I have given you a complete account of the incident.
18.	I gave him a few books I had.	I gave him the few books I had.
		(Contd.)

S. no.	Incorrect	Correct
19.	No less than fifty ships were destroyed by the storm.	No fewer than fifty ships were destroyed by the storm.
20.	Choose the least of the two evils.	Choose the lesser of the two evils.
21.	Suman is elder than Sujoy.	Suman is older than Sujoy.
22.	Verbal instructions will not serve the purpose.	Oral instructions will not serve the purpose.
23.	Agra is a worth seeing place.	Agra is a place worth seeing.
24.	The Times of India has the largest circulation of any paper.	The Times of India has the largest circulation of all the papers.
25.	I have ordered for dinner.	I have ordered dinner.
26.	He reached at his office late.	He reached his office late.
27.	Let us discuss about the problem.	Let us discuss the problem.
28.	Sign on this paper.	Sign this paper.
29.	Of course, you will get this job.	You will certainly get this job.
30.	I met him four weeks before.	I met him four weeks ago.
31.	He went directly to his house.	He went direct to his house.
32.	I only engaged this servant for a week.	I engaged this servant for a week only.
33.	Copy this letter word by word.	Copy this letter word for word.
34.	Send this letter on my address.	Send this letter to my address.
35.	It is a quarter to seven in my watch.	It is a quarter to seven by my watch.
36.	He got only passing marks.	He got only pass marks.
37.	When five years old, his father died.	When he was five years old, his father died.
38.	He is a coward man.	He is a coward.
39.	He informed his programme to me.	He informed me of his programme.
40.	He went to foreign for higher studies.	He went abroad for higher studies.
41.	He got fifty numbers.	He secured fifty marks.
42.	He got down his bicycle.	He got down from his bicycle.
43.	He gave me a sound advice.	He gave me a piece of sound advice.
44.	He does all his works regularly.	He does his work regularly.
45.	Delhi is a capital of India.	Delhi is the capital of India.
46.	None but fools says so.	None but fools say so.
47.	Please kindly help me.	Please help me.
48.	I prefer to write than to read.	I prefer writing to reading.
49.	He is enjoying bad health.	He is enjoying good health.
50.	I hope I will fail.	I fear I will fail.
51.	She had a strong headache.	She had a severe headache.
52.	I reached to Kolkata in time.	I reached Kolkata in time.
53.	I shall meet with you soon.	I shall meet you soon.
54.	He was absent at the examination.	He was absent from the examination.
55.	He is absent at the meeting.	He is absent from the meeting.
56.	I invite you at the meeting.	I invite you to the meeting.
57.	I have no ink to write with.	I have no ink to write in.
58.	I have no pen to write.	I have no pen to write with.
59.	I suggest you this.	I suggest this to you.
60.	I tried with heart and soul.	I tried heart and soul.
61.	I read Bengali near him.	I read Bengali with him.
62.	He was elected as President of the meeting.	He was elected President of the meeting.
63.	Devote your time in reading.	Devote your time to reading.
64.	You are appointed in that post.	You are appointed to that post.
65.	He got five rupees for his profit.	He got five rupees to his profit.
66.	Our examination begins from 3^{rd} June.	Our examination begins on 3^{rd} June.
67.	He is presently working in our Kolkata branch.	He is working in our Kolkata branch at present.
68.	He was even blamed by his colleagues.	He was blamed even by his colleagues.
69.	He will not listen what you say.	He will not listen to what you say.
70.	Have you disposed the old furniture you wanted to?	Have you disposed the old furniture you wanted to?
		(*Contd.*)

S. no.	Incorrect	Correct
71.	He is desirous to go abroad.	He is desirous of going abroad.
72.	Please remind me this.	Please remind me of this.
73.	He made an interesting lecture.	He delivered an interesting lecture.
74.	What are your future prospects in this firm?	What are your prospects in this firm?
75.	I am looking forward to work with you.	I am looking forward to working with you.
76.	Now the economical condition of India is quite sound.	Now the economic condition of India is quite sound.
77.	He worked for one and a half hours.	He worked for one hour and a half.
78.	Who hanged this picture on the wall?	Who hung this picture on the wall?
79.	This book comprises of ten chapters.	This book comprises ten chapters.
80.	He demanded for a reduction in price.	He demanded a reduction in price.
81.	I have come to the final conclusion.	I have come to the final decision.
82.	He went to his office but returned back immediately.	He went to his office but returned immediately.
83.	He did many mischiefs.	He made much mischief.
84.	A dog is very faithful to its master.	A dog is very faithful to its master.
85.	He was appointed on the post.	He was appointed to the post.
86.	His illness was owing to overwork.	His illness was due to overwork.
87.	Let us sit in the shadow of the tree.	Let us sit in the shade of the tree.
88.	The Chairman will arrive just now.	The Chairman will arrive shortly.
89.	Please send the reply on the address given below.	Please send the reply to the address given below.
90.	I doubt that he will come.	I doubt whether he will come.
91.	Not only he is honest but also sincere.	He is not only honest but also sincere.
92.	Because he is kind hearted, therefore he is very popular.	Because he is kind hearted, he is very popular.
93.	Although he ran very fast but he did not win the race.	Although he ran very fast yet he did not win the race.
94.	I did not see him since he wrote last.	I have not seen him since he wrote last.
95.	This is the reason I do not help you.	This is the reason why I do not help you.
96.	One should keep his word.	One should keep one's word.
97.	Let you and I volunteer ourselves.	Let you and me volunteer ourselves.
98.	The train is running in time.	The train is running on time.
99.	My father insisted me to read Sanskrit.	My father insisted on my reading Sanskrit.

ONE WORD SUBSTITUTIONS

1.	A man who is womanish in habits	Effeminate
2.	One who helps in trouble	Samaritan
3.	State of being married	Matrimony
4.	A child born after the death of his father	Posthumous child
5.	One who is well versed in everything	Versatile
6.	One who believes in fate	Fatalist
7.	Married only to one person at a time	Monogamy
8.	Person who eats human flesh	Cannibal
9.	Person abstaining from liquor	Teetotaler
10.	One who is unable to read and write	Illiterate
11.	Killing of self	Suicide
12.	One who eats too much	Glutton
13.	Something no longer in use	Obsolete
14.	A battle or game in which neither side wins	Drawn battle
15.	One who is having less blood	Anemic
16.	Something contrary to law	Illegal
17.	Beyond any doubt	Unquestionable
18.	Beyond all price	Invaluable
19.	Liable to be easily broken	Brittle
20.	Capable of being seen through	Transparent
21.	A disease commonly caught from others	Infections
22.	Something that can be eaten	Edible
23.	Surrounded on all sides	Enveloped
24.	Rule by a tyrant person	Dictatorship
25.	Not paying attention to	Inattentive
26.	Something that deserves all praise	Laudable
27.	A period of two weeks	Fortnight
28.	A long distance race	Marathon
29.	The ceremonial burning of dead body	Funeral cremation
30.	A person who readily believes others	Credulous
31.	Rule by single person	Autocracy
32.	Rule by officers	Bureaucracy
33.	To give and receive mutually	Reciprocate
34.	Something that cannot be overcome	Insurmountable

35. Something that will last for ever — Eternal
36. Science of crimes — Criminology
37. Incapable of being controlled — Uncontrollable
38. Incapable of being repaired — Irreparable
39. A place where birds are kept — Aviary
40. The last part of speech — Conclusion
41. One who walks in sleep — Somnambulist
42. Murder of father — Patricide
43. Murder of mother — Matricide
44. Bones separated from flesh and preserved in natural position — Skelton
45. Handwritten copy of book — Manuscript
46. A post for which no salary is paid — Honorary
47. A six sided figure — Hexagonal
48. One working in the same office — Colleague
49. A person who looks at the dark side of everything — Pessimist
50. A large scale departure of people from a place — Migration

ANTONYMS

Sl. No.	Words	Antonyms
1.	Absent	Present
2.	Answer	Question
3.	Accustomed	Unaccustomed
4.	Against	For
5.	Ancient	Modern
6.	Always	Never
7.	Alive	Dead
8.	Ago	Hence
9.	All	None
10.	Apart	Together
11.	Accept	Reject
12.	Agree	Differ
13.	Actual	Imaginary
14.	Attention	Inattention
15.	Appear	Disappear
16.	Antonym	Synonym
17.	Arrival	Departure
18.	Angry	Differ
19.	Accurately	Inaccurately
20.	Big	Small
21.	Beginning	End
22.	Blunt	Sharp
23.	Bright	Dull
24.	Broad	Narrow
25.	Birth	Death
26.	Beautiful	Ugly
27.	Back	Front
28.	Better	Worse
29.	Best	Worst
30.	Below	Above
31.	Blame	Praise
32.	Bitter	Sweet
33.	Bottom	Top
34.	Clean	Dirty
35.	Common	Uncommon
36.	Confirm	Deny
37.	Create	Destroy
38.	Cheap	Dear
39.	Casual	Regular
40.	Civilized	Uncivilized
41.	Confess	Deny
42.	Complete	Incomplete
43.	Complex	Simple
44.	Certainly	Uncertainly
45.	Cheerfully	Cheerlessly
46.	Cool	Hot
47.	Comfort	Discomfort
48.	Clever	Stupid
49.	Capable	Incapable
50.	Combine	Separate
51.	Cruel	Kind
52.	Compulsory	Voluntary
53.	Dark	Light
54.	Dawn	Dusk
55.	Dream	Reality
56.	Drown	Float
57.	Different	Similar
58.	Destruction	Construction
59.	Directly	Indirectly
60.	Difficult	Easy
61.	Discipline	Indiscipline
62.	Distant	Near
63.	Deny	Confess
64.	Descending	Ascending
65.	Dumb	Vocal
66.	Dull	Bright
67.	Dense	Thin
68.	Dangerous	Safe
69.	Decrease	Increase
70.	Deep	Shallow
71.	Definite	Indefinite
72.	Demand	Supply
73.	Discourage	Encourage
74.	Explored	Unexplored
75.	Entirely	Partially
76.	Extend	Contract
77.	Enormous	Tiny
78.	Earth	Heaven
79.	Employment	Unemployment
80.	Elder	Younger
81.	Exact	Inexact
82.	Essential	Inessential
83.	Ever	Never
84.	End	Beginning
85.	Even	Odd

86.	Export	Import
87.	Exterior	Interior
88.	Famous	Unknown
89.	Float	Sink
90.	Forget	Remember
91.	Found	Lost
92.	Friend	Enemy
93.	Full	Empty
94.	Fertile	Barren
95.	Finish	Begin
96.	Forgive	unish
97.	Farewell	Welcome
98.	Female	Male
99.	Foolish	Wise
100.	Fresh	Stale
101.	Frequent	Seldom
102.	Few	Many
103.	Great	Small
104.	Gentleness	Roughness
105.	General	Particular
106.	Gain	Lose
107.	Gentle	Wild
108.	Guilty	Innocent
109.	Guest	Host
110.	Grant	Refuse
111.	Giant	Tiny
112.	Glad	Sad
113.	Gloomy	Cheerful
114.	Generous	Stingy
115.	Humble	Arrogant
116.	Hard	Soft
117.	Hope	Despair
118.	Huge	Small
119.	Handsome	Ugly
120.	Harmful	Harmless
121.	Heaven	Hell
122.	Husband	Wife
123.	Hero	Coward
124.	Hill	Valley
125.	Horizontal	Vertical
126.	Harsh	Mild
127.	Include	Exclude
128.	Important	Trivial
129.	Inner	Outer
130.	Inferior	Superior
131.	Inmate	Outsider
132.	Invaluable	Worthless
133.	Identical	Different
134.	Joy	Woe
135.	Jolly	Sad
136.	Justice	Injustice
137.	Judicious	Injudicious
138.	Just	Unjust
139.	Join	Separate
140.	Junior	Senior
141.	Known	Unknown
142.	Kill	Revive
143.	Knowledge	Ignorance
144.	Love	Hate
145.	Liberty	Bondage
146.	Late	Early
147.	Like	Dislike
148.	Life	Death
149.	Lazy	Busy
150.	Locked	Unlocked
151.	Lonely	Crowded
152.	Leave	Stay
153.	Lonesome	Crowded
154.	Modern	Ancient
155.	Merit	Demerit
156.	Move	Stay
157.	Maximum	Minimum
158.	Masculine	Feminine
159.	Mute	Vocal
160.	Moral	Immoral
161.	Mindful	Unmindful
162.	Merry	Sad
163.	Mercy	Cruelty
164.	Married	Unmarried
165.	Net	Gross
166.	Necessary	Unnecessary
167.	Neat	Clumsy
168.	Noisy	Silent
169.	Novel	Traditional
170.	Necessity	Uselessness
171.	Never	Ever
172.	Noise	Silence
173.	Optimism	Pessimism
174.	Often	Seldom
175.	Older	Younger
176.	Obey	Disobey
177.	Order	Disorder
178.	Plain	Complex
179.	Proud	Humble
180.	Problem	Solution
181.	Present	Absent
182.	Particular	General
183.	Public	Private
184.	Primitive	Modern
185.	Prominent	Obscure
186.	Pretty	Ugly
187.	Punish	Reward
188.	Promotion	Demotion
189.	Plenty	Scarcity
190.	Painful	Pleasurable
191.	Practical	Theoretical

192.	Preserve	Destroy
193.	Pleasure	Pain
194.	Pleasant	Unpleasant
195.	Part	Whole
196.	Precious	Cheap
197.	Quiet	Agitated
198.	Quickly	Slowly
199.	Quantitative	Qualitative
200.	Rudely	Softly
201.	Rapidly	Slowly
202.	Rational	Irrational
203.	Roughly	Exactly
204.	Remove	Retain
205.	Rare	Common
206.	Recollect	Forget
207.	Regular	Irregular
208.	Responsible	Irresponsible
209.	Remember	Forget
210.	Reward	Punishment
211.	Supply	Demand
212.	Sufficient	Insufficient
213.	Stout	Weak
214.	Social	Unsocial
215.	Settle	Unsettle
216.	Submerge	Emerge
217.	Spend	Earn
218.	Smile	Weep
219.	Shy	Smart
220.	Sweet	Bitter
221.	Scarce	Ample
222.	Spiritual	Physical
223.	Servant	Master
224.	Strict	Lenient
225.	Sinking	Rising
226.	Strange	Familiar
227.	Suddenly	Slowly
228.	Starve	Eat
229.	Suffer	Enjoy
230.	Silently	Noisily
231.	Scorn	Love
232.	Swift	Slow
233.	Sink	Float
234.	Still	Noisy
235.	Tiny	Big
236.	Tide	Ebb
237.	Tenderly	Roughly
238.	Tight	Loose
239.	Tyranny	Democracy
240.	Thrust	Withdraw
241.	Tenant	Landlord
242.	Unfit	Fit
243.	Use	Misuse
244.	Under	Above
245.	Vast	Small
246.	Vanish	Appear
247.	Virtue	Vice
248.	Vagrant	Settled
249.	Wrap	Unwrap
250.	Wisdom	Ignorance
251.	Wide	Narrow
252.	Weal	Ill-being
253.	Weak	Strong
254.	Wax	Wane
255.	Wild	Tame
256.	Wise	Foolish
257.	Whole	Part
258.	Willing	Unwilling
259.	Weakness	Strength
260.	Young	Old

SYNONYMS

S. No.	Words	Synonyms
1.	Big	Large
2.	Accord	Agree
3.	Board	Wide
4.	Cunning	Clever
5.	Avoid	Ignore
6.	Fertile	Fruitful
7.	Glad	Happy
8.	Huge	Enormous
9.	Intelligent	Clever
10.	Polite	Courteous
11.	Poor	Destitute
12.	Interesting	Fascinating
13.	Damp	Moist
14.	Hurt	Impair
15.	Exciting	Thrilling
16.	Funny	Comical
17.	Terrific	Extraordinary
18.	Sometimes	Occasionally
19.	Boring	Tedious
20.	Easy	Effortless
21.	Cool	Chill
22.	Important	Substantial
23.	Fast	Swift
24.	Isolated	Detached
25.	Apart	Alone
26.	Broken	Abandoned
27.	Deserted	Bare
28.	Detached	Divided
29.	Brief	Concise
30.	Calamity	Disaster
31.	Aggression	Attack
32.	Budget	Allot
33.	Inaugurate	Opening ceremony
34.	Avert	Avoid
35.	Match	Agree

19. Which of the following options is the closest in meaning to the word **reveal**?
(a) Desire (b) Show
(c) Doubt (d) Define

20. Which of the following options is the closest in meaning to the word **trivial**?
(a) Unimportant (b) Logical
(c) Amusing (d) Changeable

21. Which of the following options is the closest in meaning to the word **dreadful**?
(a) Horrible (b) Pleasing
(c) Illusory (d) Lifeless

22. Which of the following options is the closest in meaning to the word **subtle**?
(a) Sympathetic (b) Amiable
(c) Clever (d) Unwilling

23. Which of the following options is the closest in meaning to the word **ratify**?
(a) Approve (b) Deny
(c) Negotiate (d) Demonstrate

24. Which of the following options is the closest in meaning to the word **vigilant**?
(a) Active (b) Awake
(c) Careful (d) Watchful

25. Which of the following options is the closest in meaning to the word **untoward**?
(a) Untidy (b) Retreating
(c) Unfurled (d) Unfortunate

Solutions

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (a) **6.** (d)
7. (a) **8.** (b) **9.** (a) **10.** (d) **11.** (a) **12.** (a)
13. (d) **14.** (d) **15.** (b) **16.** (d) **17.** (c) **18.** (a)
19. (b) **20.** (a) **21.** (a) **22.** (c) **23.** (a) **24.** (d)
25. (d)

Section B

Choose the most appropriate word from the options given to complete the following sentence:

1. He had no choice …………… to yield.
(a) But (b) Than
(c) Except (d) Still

2. He had no other claim …………… his services.
(a) Except (b) But
(c) Still (d) Than

3. I explained …………… .
(a) The matter to him (b) Him the matter
(c) The matter him (d) The matter of him

4. She …………… his mother.
(a) Resembles with (b) Resembles to
(c) Resembles (d) Resembles on

5. The examination will begin …………… Friday.
(a) On (b) From
(c) Till (d) Until

6. I congratulated him …………… .
(a) On his success (b) For his success
(c) At his success (d) Of his success

7. No sooner had he come …………… his enemy challenged him.
(a) When (b) Then
(c) Than (d) Unless

8. There is no such magazine …………… you mention.
(a) That (b) As
(c) Who (d) Which

9. He is nothing else …………… a rogue.
(a) Except (b) But
(c) Than (d) Despite

10. He did not stop any where he went …………… to the house.
(a) Directly (b) Direct
(c) Straightly (d) Pointedly

11. I have not seen him …………… .
(a) For the last six days. (b) Since the last six days
(c) About last six days (d) Of last six days

12. Such was his anger …………… cannot be calmed down.
(a) That (b) But
(c) Which (d) As

13. He is an/a …………… dancer.
(a) Adept (b) Adopt
(c) Adapt (d) Skilled

14. You are …………… for this post.
(a) Legible (b) Eligible
(c) Ineligible (d) Illegible

15. He is …………… and kind hearted.
(a) Graceful (b) Grateful
(c) Gaseous (d) Gracious

Solutions

1. (c) **2.** (a) **3.** (a) **4.** (c) **5.** (a) **6.** (a)
7. (c) **8.** (b) **9.** (b) **10.** (b) **11.** (a) **12.** (d)
13. (a) **14.** (b) **15.** (d)

Section C

In each of the following sentence a word or phrase is given in bold letters and it is followed by four words marked (a), (b), (c) and (d). Choose the word most nearly opposite in meaning to the bold lettered word.

1. Every self-respecting circus needs at least one **dwarf**.
(a) Pigmy (b) Undersized
(c) Giant (d) Decent

2. Of all the companions of our joyous **ascent**, there were only the two of us left.
(a) Descant (b) Descendant
(c) Descent (d) Decent

3. The people being so charming the **veracity** of their utterances is difficult to estimate.
(a) Difference (b) Uniformity
(c) Falsehood (d) Diversity

4. Our knowledge of the past is still largely a matter of informed **conjecture**.
(a) Certainty (b) Guess
(c) Position (d) Form

5. In all places, and at all times, there is a **profusion of talents**.
(a) Plenty (b) Aversion
(c) Scarcity (d) Generosity

6. It was **altercation** throughout and there was no discussion.
(a) Consonance (b) Alternative
(c) Inconsistency (d) Resonance

7. We were surprised by the **hostile** attitude of the villagers.
(a) Gentle (b) Friendly
(c) Soft (d) Forgiving

8. His style was **smooth**, but there was no mistaking his determination.
(a) Hard (b) Dense
(c) Harsh (d) Rough

9. Unlike the other candidates, his manner was entirely **languid**.
(a) Energetic (b) Lazy
(c) Liquid (d) Slow

10. People who are actually running the system often take a **myopic** view of the situation.
(a) Farsighted (b) Visionary
(c) Blind (d) Glassy

11. The distant hills looked so **beautiful**.
(a) Dark (b) Strange
(c) Small (d) Ugly

12. I love wearing a **loose** dress.
(a) Narrow (b) Tight
(c) Small (d) Fit

13. I was amazed at his **cowardly** behaviour.
(a) Determined (b) Enthusiastic
(c) Brave (d) Prompt

14. They had **smooth** voyage to Europe.
(a) Rough (b) Tiring
(c) Boring (d) Troublesome

15. I thanked him for his **prompt** reply.
(a) Careless (b) Slow
(c) Irregular (d) Abrupt

16. Evil practices in society should be **abolished**.
(a) Revived (b) Encouraged
(c) Established (d) Continued

17. He could **confirm** the receipt of the letter.
(a) Refuse (b) Deny
(c) Confess (d) Claim

18. The farmers have **adopted** new methods of production.
(a) Missed (b) Admitted
(c) Denied (d) Rejected

19. Being a **spendthrift** he wasted all his father's money.
(a) Conservative (b) Liberal
(c) Miser (d) Philanthropy

20. **Religious** institutions should not interfere in politics.
(a) Immoral (b) Social
(c) Secular (d) State

21. People of the world have been protesting against the **indiscriminate** killings but the Government of Peter Botha of South Africa prides itself on this.
(a) Systematic (b) Discreet
(c) Brutal (d) Vengeful

22. When man is born free, why should he suffer **servility**?
(a) Affluence (b) Diffidence
(c) Dependence (d) Disturbed

23. **Crestfallen**, he returned and shut himself up in his room.
(a) Vainglorious (b) Indignant
(c) Triumphant (d) Laborious

24. Soumyadeep is very **industrious**.
(a) Vivacious (b) Enthusiastic
(c) Indolent (d) Laborious

25. Our land is very **fertile**.
(a) Barren (b) Smooth
(c) Extensive (d) Dry

26. My brother is a **cautious** driver.
(a) Careless (b) Rush
(c) Slow (d) Dull

27. He was in a **dejected** mood.
(a) Jubilant (b) Rejected
(c) Irritable (d) Romantic

28. Philosophers say that the world is an **illusion**.
(a) A fact (b) A reality
(c) An actuality (d) A truth

29. There was a marked **deterioration** in his condition.
(a) Improvement (b) Revision
(c) Reformation (d) Amendment

30. This offer has come as a great **boon** to me.
(a) Curse (b) Blemish
(c) Trouble (d) Misfortune

31. I liked the poem for its **literal** meaning.
(a) Deep (b) Complex
(c) Fictitious (d) Figurative

32. His book has a short but useful **introduction**.
(a) End (b) Conclusion
(c) Termination (d) Deduction

33. Monika is a **smart** girl.
(a) Lazy (b) Active
(c) Indecent (d) Casual

34. India is a **heterogeneous** country.
(a) Strange (b) Homogeneous
(c) Complex (d) Vast

35. There is a good deal of **spurious** family affection.
(a) Genuine (b) Obvious
(c) Fictitious (d) Authentic

36. That police officer is known to be **humane** in his approach.
(a) Uncivilised (b) Indigenous
(c) Unsympathetic (d) Uncompromising

37. He has a weakness for **foreign** goods.
(a) Exported (b) Indigenous
(c) Fashionable (d) Exotic

38. There is only the most **tenuous** evidence for it.
(a) Enough (b) Reasonable
(c) Less (d) Abundant

39. The new government has **abolished** the Gold Control Act.
(a) Removed (b) Approved
(c) Passed (d) Restored

40. This is not ideology but **pragmatic** language teaching.
(a) Impossible (b) Imperfect
(c) Improper (d) Impractical

41. Machine civilization has made human life **artificial**.
(a) Authentic (b) True
(c) Natural (d) Genuine

42. There are reports that many poor people **abandon** female children.
(a) Help (b) Keep
(c) Reject (d) Like

43. Earthquakes are **frequent** in Japan.
(a) Rate (b) Few
(c) Unusual (d) Extinct

44. The Minister is **optimistic** about the new project just launched.
(a) Cynical (b) Pessimistic
(c) Dubious (d) Stoical

45. Will these steps **raise** the standard of living of our villagers?
(a) Down (b) Decrease
(c) Fall (d) Lower

46. **Foreign** travels have made him sophisticated.
(a) Inland (b) Spurious
(c) Homely (d) Frequent

47. He made a **comprehensive** study of rural problems.
(a) Slipshod (b) Superficial
(c) Exhaustive (d) Sketchy

48. It is **convenient** for me to work here.
(a) Desirable (b) Inadvisable
(c) Objectionable (d) Troublesome

49. Do they **love** their neighbour?
(a) Hate (b) Blame
(c) Envy (d) Indulge

50. From the cliff we saw high waves rising, falling and **advancing**.
(a) Going back (b) Returning
(c) Falling (d) Receding

51. Not all her children are **naughty**.
(a) Clever (b) Good-tampered
(c) Intelligent (d) Well behaved

52. **Marriages** are talked about openly, but not the ....
(a) Affairs (b) Courtships
(c) Engagements (d) Divorcees

53. They **deduced** the expenses from his salary, but........ an ad-hoc allowance to it.
(a) Adduced (b) Reduced
(c) Added (d) Deduced

54. His experience in business has been **narrow**, but he has .... experience in administration.
(a) Interesting (b) Unrestricted
(c) Wide (d) Long

55. Her coming is **definite**, but about him the situation is ....
(a) Doubtful (b) Regrettable
(c) Questionable (d) Unpredictable

56. The explosion and the plane's take off occurred **simultaneously**.
(a) Contemporary (b) Coincident
(c) Separately (d) Momentarily

57. Old people are usually more **conservative** than young people.
(a) Modern (b) Old fashioned
(c) Dynamic (d) Liberal

58. His family has **accumulated** wealth over the years.
(a) Amassed (b) Collected
(c) Drained (d) Squandered

59. The painting is full of **radiant** colours.
(a) Rare (b) Bright
(c) Dull (d) Delicate

60. The new officer is a **brash** young man.
(a) Handsome (b) Arrogant
(c) Kind (d) Polite

61. He has been **acquitted** of the charge of theft.
(a) Convicted (b) Exonerated
(c) Released (d) Punished

10. The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.
It can be inferred from the passage that horses were
(a) Given immunity to diseases
(b) Generally quite immune to diseases
(c) Given medicines to fight toxins
(d) Given diphtheria and tetanus serums
[Gate 2011]

11. Which one of the following options is the closest in meaning to the word **latitude**?
(a) Eligibility (b) Freedom
(c) Coercion (d) Meticulousness
[Gate 2012]

12. One of the parts (a, b, c, d) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?
I requested that he should be given the driving test today instead of tomorrow.
(a) Requested that (b) Should be given
(c) The driving test (d) Instead of tomorrow
[Gate 2012]

13. Choose the most appropriate alternative from the options given below to complete the following sentence:
If the tired soldier wanted to lie down, he .... the mattress out on the balcony.
(a) Should take (b) Shall take
(c) Should have taken (d) Will have taken
[Gate 2012]

14. Choose the most appropriate word from the options given below to complete the following sentence:
Given the seriousness of the situation that he had to face, his .... was impressive.
(a) Beggary (b) Nomenclature
(c) Jealousy (d) Nonchalance
[Gate 2012]

15. One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.
Which one of the following statements best sums up the meaning of the above passage?
(a) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances
(b) The legions were treated inhumanly as if the men were animals
(c) Disciple was the army's inheritance from their seniors
(d) The harsh disciple to which the legions were subjected to led to the odds and conditions being against them [Gate 2012]

16. Choose the grammatically CORRECT sentence:
(a) Two and two add four
(b) Two and two become four
(c) Two and two are four
(d) Two and two make four [Gate 2014]

17. **Statement**: You can always give me a ring whenever you need.
Which one of the following is the best inference from the above statement?
(a) Because I have a nice caller tune
(b) Because I have a better telephone facility
(c) Because a friend in need is a friend indeed
(d) Because you need not pay towards the telephone bills when you give me a ring [Gate 2013]

18. Complete the sentence:
Dare .... mistakes.
(a) Commit (b) To commit
(c) Committed (d) Committing
[Gate 2013]

19. They were requested not to quarrel with others.
Which one of the following options is the closest in meaning to the word **quarrel**?
(a) Make out (b) Call out
(c) Dig out (d) Fall out [Gate 2013]

20. **Statement**: There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists and so on.
Which one of the following is the best inference from the above statement?
(a) The emergence of nationalism in colonial India led to our Independence
(b) Nationalism in India emerged in the context of colonialism
(c) Nationalism in India is homogeneous
(d) Nationalism in India is heterogeneous
[Gate 2013]

21. Which of the following options is the closest in meaning to the phrase underlined in the sentence below?
It is fascinating to see life forms <u>cope with</u> varied environment conditions.
(a) Adopt to (b) Adapt to
(c) Adept in (d) Accept with
[Gate 2014 set 1]

22. Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite ....

(a) Superb (b) Medium
(c) Mediocre (d) Exhilarating

[Gate 2014 set 1]

23. In a press meet on the recent scam, the minister said, “the buck stops here”. What did the minister convey by the statement?

(a) He wants all the money
(b) He will return the money
(c) He will assume final responsibility
(d) He will resist all enquiries [Gate 2014 set 1]

24. The Palghat Gap (or Palakkad Gap), a region about 300 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures. What can be inferred from this passage?

(a) The Palghat gap is caused by high temperatures in southern Tamil Nadu and Kerala
(b) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low lying
(c) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala
(d) Higher summer temperatures result in higher rainfall near the Palghat Gap area

[Gate 2014 set 1]

25. Geneticists say that are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

On which of the following assumptions does the statement above rely?

(a) Strategies are now available for eliminating psychiatric illness
(b) Certain psychiatric illness have a genetic basis
(c) All human disease can be traced back to genes and how they are expressed
(d) In the future, genetics will become the only relevant field for identifying psychiatric illness

[Gate 2014 set 1]

26. Choose the most appropriate phrase from the options given below to complete the following sentence.

India is a post-colonial country because

(a) It was a former British colony
(b) Indian Information Technology processional have colonized the world
(c) India does not follow any colonial practices
(d) India has helped other countries gain freedom

[Gate 2014 set 2]

27. Who .... was coming to see us this evening?

(a) You said (b) Did you say
(c) Did you say that (d) Had you said

[Gate 2014 set 2]

28. Match the columns:

Column 1	Column 2
1. Eradicate	P. Misrepresent
2. Distort	Q. Soak completely
3. Saturate	R. Use
4. Utilize	S. Destroy utterly

(a) 1 – S, 2 – P, 3 – Q, 4 – R
(b) 1 – P, 2 – Q, 3 – R, 4 – S
(c) 1 – Q, 2 – R, 3 – S, 4 – P
(d) 1 – S, 2 – P, 3 – R, 4 – Q [Gate 2014 set 2]

29. The old city of Konigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the South and West and Lithuania to the East respectively. Which of the statements below can be inferred from this passage?

(a) Kaliningrad was historically Russian in its ethnic make up
(b) Kaliningrad is a part of Russian despite it not being contiguous with the rest of Russia
(c) Konigsberg was renamed Kaliningrad, as that was its original Russian name
(d) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

[Gate 2014 set 2]

30. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in North India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.

Which one of the following statements, if true, does not contradict this conclusion?

(a) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
(b) More cases of dengue are now reported because of an increase in the Municipal office’s administrative efficiency
(c) Many more cases of dengue are being diagnosed this year since the introduction of a new effective diagnostic test

(d) The number of people with malarial fever (also contracted from mosquito bites) has increased this year [Gate 2014 set 2]

31. While trying to collect (I) an envelope from under the table (II), Mr. X fell down (III) and was losing consciousness (IV)
Which one of the above underlined parts of the sentence is NOT appropriate?
(a) I (b) II
(c) III (d) IV [Gate 2014 set 3]

32. If she .... how to calibrate the instrument, she .... done the experiment.
(a) Knows, will have
(b) Knew, had
(c) Had known, could have
(d) Should have known, would have
[Gate 2014 set 3]

33. Choose the word that is opposite in meaning to the world "**coherent**"
(a) Sticky (b) Well-connected
(c) Rambling (d) Friendly
[Gate 2014 set 3]

34. A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is
(a) Students should come at 9.00 am and parents should come at 10.00 am
(b) Participating students should come at 9.00 am accompanied by a parent, and other parents and students should come by 10.00 am.
(c) Students who are not participating should come by 10.00 am and they should not bring their parent. Participating students should come at 9.00 am.
(d) Participating students should come before 9.00 am. Parents who accompany them should come at 9.00 am. All other should come at 10.00 am. [Gate 2014 set 3]

35. By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun. Which of the following inference(s) may be drawn from the above passage?
(i) Out understanding of the universe changes based on the positions of stars
(ii) Paradigm shifts usually occur at the beginning of centuries
(iii) Stars are important objects in the universe
(iv) Experimental evidence was important in confirming this paradigm shift
(a) (i), (ii) and (iv) (b) (iii) only
(c) (i) and (iv) (d) (iv) only
[Gate 2014 set 3]

Solutions

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (a)
7. (c)	**8.** (c)	**9.** (c)	**10.** (c)	**11.** (b)	**12.** (a)
13. (c)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (c)	**18.** (a)
19. (d)	**20.** (d)	**21.** (b)	**22.** (c)	**23.** (c)	**24.** (c)
25. (b)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (b)	**30.** (d)
31. (d)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (d)	

10.5 NUMERICAL ABILITY

Numerical analysis is used for numerical approximation for the problems of mathematical analysis. Direct methods which is used for computing the solution to a problem is a finite number of steps. These would give the precise answer if they were performed in infinite precision arithmetic. For example, Gaussian elimination, the QR factorization method for solving systems of linear equations, and the simplex method of linear programming.

Sometimes continuous problems must be replaced by a discrete problem whose solution is known to approximate that of the continuous problem. The study of errors forms an important part of numerical analysis.

One of the simplest problems is the evaluation of a function at a given point. It is necessary to estimate and control round-off errors. Interpolation solves the following problem of given the value of some unknown function at a number of points. Extrapolation is very similar to interpolation, except that it is used to find the value of the unknown function at a point which is outside the given points. Regression is also similar, but it takes into account that the data is imprecise.

In a **numerical reasoning** test, it is required to answer questions using facts and figures presented in statistical tables. In each question it is given a number of options to choose from. Only one of the options is correct in each case. The numerical reasoning test measures your ability to interpret, analyse and draw logical conclusions based on numerical data presented in graphs and tables. It is important to note that this numerical reasoning test is not designed to measure your mathematical ability but your ability to use numerical data as a tool to prepare reasoned decisions and solve problems.

Several important problems can be in terms of eigenvalue decompositions or singular value decompositions. For instance, the spectral image compression algorithm is based on the singular value decomposition. The corresponding tool in statistics is called principal component analysis.

The purpose of the **data analysis and interpretation** phase is to transform the data collected into credible evidence about the development of the intervention and its performance.

This process usually includes the following:

(i) Organizing the data for analysis (data preparation)
(ii) Analyzing the data
(iii) Interpreting the data (assessing the findings against the adopted evaluation criteria) statistical analysis can
- help measure the degree of change that has taken place
- allow an assessment to be made about the consistency of data

One of the most important issues in interpreting research findings is understanding how outcomes related to the intervention that is being evaluated. This involves making the difference between association and causation and the role that can be played by confounding factors in skewing the evidence.

(i) **Association:** An association exists when one happening event is more likely to occur because another event has taken place.
(ii) **Causation:** A causal relationship exists when one event which is cause is necessary for a second event which is effect to occur.
(iii) **Confounding:** To rule out that a relationship between two events has been distorted by other, external factors, it is necessary to control for confounding.

Short, medium and long terms are as follows:

(i) Short term outcomes include changes in skills, attitudes and knowledge.
(ii) Medium-term outcomes include changes in behaviour and decision-making.
(iii) Long-term outcomes include persistence of behaviours and broader lifestyle changes.

Short- and long-term outcomes can be measured by using different methodologies for collecting data.

Cross-sectional studies include the measurement at a single point in time after the intervention has been applied and allow short-term results to be measured.

Longitudinal study designs, on the other hand, follow progress over longer periods and allow measurements to be taken at two or more different points in time.

ALGEBRAIC FORMULAE

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- $(a+b)^2 - (a-b)^2 = 4ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)^3 + 3ab(a-b) = (a-b)(a^2 + ab + b^2)$

PROGRESSION

Arithmetic Progression (AP)

Let first term be a and common difference be d.

So, nth term $t_n = a + (n-1)d$

If last term be l, then nth term from the end of an AP $= l - (n-1)d$

Sum of n terms, $S_n = \frac{n}{2}[a+l] = \frac{n}{2}[2a + (n-1)d]$

Geometric Progression (GP)

Let first term be a and common ratio be r

So, nth term $t_n = ar^{n-1}$

If last term be l, then nth term from the end of a GP $= \frac{l}{r^{n-1}}$

Sum of n terms, $S_n = na$ when $r = 1$

$$S_n = \frac{a(1-r^n)}{(1-r)} \quad \text{when} \quad r < 1$$

$$S_n = \frac{a(r^n - 1)}{(r-1)} \quad \text{when} \quad r > 1$$

Note:

- **Sum of first n natural numbers**

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$$

- **Sum of the squares of first n natural numbers**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

- **Sum of the cubes of first n natural numbers**

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{1}{2}n(n+1)\right]^2$$

- **Sum of the cubes of first n odd natural numbers**

$$1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1)$$

- **Sum of an infinite GP with the first term a and the common ratio r, where $|r| < 1$ is**

$$S = \frac{a}{1-r}$$

Logarithms

If a is any positive real number (except 1), n is any rational number and $a^n = b$, then n is called logarithm of b to the base a, which is written as $\log_a b$.

$$a^n = b \text{ if and only if } \log_a b = n$$

Thus, $a^n = b$ is called the exponential form and $\log_a b = n$ is called the logarithmic form.
Since, $a^0 = 1$, $\log_a 1 = 0$ and $a^1 = a$, $\log_a a = 1$

Properties of logrithms

(i) Logarithms to the base 10 are called common logarithms.
(ii) If no base is given, the base is always taken as 10.
(iii) Product law: $\log_x ab = \log_x a + \log_x b$
(iv) Quotient law: $\log_x \frac{a}{b} = \log_x a - \log_x b$
(v) Power law: $\log_x a^b = b \log_x a$
(vi) $\log_b b^n = n$
(vii) $a^{\log_a y} = y$
(viii) Base changing formula: $\log_a b = \frac{\log_x b}{\log_x a}$
(ix) Reciprocal formula: $\log_a b = \frac{1}{\log_b a}$

10.6 IMPORTANT POINTS TO REMEMBER (NUMERICAL ABILITY)

1. An exterior angle of a triangle is equal to sum of its interior opposite angles.
2. If two sides of a triangle are unequal, then the longer side has greater angle opposite to it.
3. If two angles of a triangle are unequal, the greater angle has longer side opposite to it.
4. The sum of any two sides of a triangle is greater than the third side.
5. Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.
6. The line segment joining the mid points of any two sides of a triangle is parallel to the third side and is equal to half of it.
7. All congruent figures are similar but similar figures may not be congruent.
8. A diagonal of a parallelogram divides it into two triangles of equal areas.
9. Parallelograms on the same base (or equal bases) and between the same parallel lines are equal in area.
10. Area of a triangle is half that of a parallelogram on the same base and between same parallel lines.
11. A median of a triangle divides it into two triangles of equal areas.
12. Triangles on the same base (or equal bases) and between the same parallel lines are equal in area.
13. A simple closed plane figure made up entirely of line segments is called a polygon.
14. A polygon is called regular polygon if all its sides have equal lengths and all its angles have equal size.
15. The sum of interior of a convex polygon of n sides is $(2n - 4)$ right angles.
16. The sum of exterior angles of a convex polygon is four right angles.
17. Each interior angle of a regular polygon of n sides $= \frac{2n-4}{n}$ right angles.
18. Each exterior angle of a regular polygon of n sides $= \left(\frac{360}{n}\right)^{\circ}$.
19. If each exterior angle of a regular polygon is x°, then the number of sides in the polygon $= \left(\frac{360}{x}\right)$.
20. Sum of interior angles of a quadrilateral = 360°.
21. Two circles are called concentric if they have same centre.
22. Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
23. Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are lengths of sides and $s = \frac{1}{2}(a+b+c)$
24. Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is its side.
25. Area of an isosceles triangle $= \frac{b}{a}\sqrt{4a^2 - b^2}$, where b is the base and a is an equal side.
26. Area of a quadrilateral (when diagonals intersect at right angles) $= \frac{1}{2} \times \text{product of diagonals}$.
27. Area of a rectangle = length × breadth.
28. Area of a square = $(\text{side})^2$
29. Area of a parallelogram = base × height.
30. Area of a rhombus $= \frac{1}{2} \times \text{product of diagonals}$.
31. Area of a trapezium
$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}.$$
32. Length of diagonal of a square $= \sqrt{2}\,a$, where a is its side.

33. Length of diagonal of a rectangle $= \sqrt{l^2 + b^2}$, where l and b are its sides.
34. Length of diagonal of a cube $= \sqrt{3}\,a$, where a is its side.
35. Length of diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$, where l, b and h are its sides.
36. If r is the radius of a circle, then circumference of circle $= 2\pi r$ and area of circle $= 2\pi r^2$.
37. If R and r are the radii of the outer and inner circles, then area of circular ring $= \pi(R^2 - r^2)$.
38. If r is the radius of a circle, then perimeter of semicircle $= (\pi + 2)r$ and area of semicircle $= \frac{1}{2}\pi r^2$.
39. Surface area of a cube $= 6a^2$, where a is its side.
40. Surface area of a cuboid $= 2(lb + bh + lh)$, where l, b and h are its sides.
41. Surface area of four walls (lateral surface area) of a cuboid $= 2h(l + b)$.
42. Volume of a cube $= a^3$, where a is its side.
43. Volume of a cuboid = length × breadth × height = $l \times b \times h$
44. Let r and h be the radius and height of a solid cylinder, then
    (a) Curved (lateral) surface area $= 2\pi rh$
    (b) Total surface area $= 2\pi r(h + r)$
    (c) Volume $= \pi r^2 h$
45. Let R and r be the external and internal radii, and h be the height of a hollow cylinder, then
    (a) External curved surface area $= 2\pi Rh$
    (b) Internal curved surface area $= 2\pi rh$
    (c) Total surface area $= 2\pi(Rh + rh + R^2 - r^2)$
    (d) Volume of material $= \pi(R^2 - r^2)h$

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10.8 PREVIOUS YEARS' QUESTIONS (NUMERICAL ABILITY)

1. 25 persons are in a room, 15 of them play hockey, 17 of them play football and 10 of them play hockey and football. Then the number of persons playing neither hickey nor football is:
(a) 2 (b) 17
(c) 13 (d) 3 [Gate 2010]

2. If 137 + 276 = 435, how much is 731 + 672?
(a) 534 (b) 1403
(c) 1623 (d) 1531 [Gate 2010]

3. 5 skilled workers can build a wall in 20 days; 8 semiskilled workers can build a wall in 25 days; 10 unskilled workers cab build a wall in 30 days. If a team has 2 skilled, 6 semiskilled and 5 unskilled workers, how long will it take to build the wall?
(a) 20 days (b) 18 days
(c) 16 days (d) 15 days [Gate 2010]

4. Three friends R, S and T shared toffees from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?
(a) 38 (b) 31
(c) 48 (d) 41 [Gate 2011]

5. Given that $f(y) = |y|y$, and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is:
(a) 0 (b) -1
(c) 1 (d) 2 [Gate 2011]

6. The sum of n terms of the series 4 + 44 + 444 + ... is:
(a) $(4/81)[10^{n+1} - 9n - 1]$
(b) $(4/81)[10^{n-1} - 9n - 1]$
(c) $(4/81)[10^{n+1} - 9n - 10]$
(d) $(4/81)[10^{n} - 9n - 10]$ [Gate 2011]

7. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321}$
(a) 2.23 (b) 4.33
(c) 11.37 (d) 27.64 [Gate 2012]

8. The data given in the following table summarizes the monthly budget of an average household.

Category	Amount
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Others	1800

The approximate percentage of the monthly budget NOT spent on savings is:

(a) 10% (b) 14%
(c) 81% (d) 86% [Gate 2012]

9. Raju has 14 currency notes in his pocket consisting of only ₹20 notes and ₹10 notes. The total money value of the notes is ₹230. The number of ₹10 notes that Raju has is

(a) 5 (b) 6
(c) 9 (d) 10 [Gate 2012]

10. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 41°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was

(a) 40 (b) 43
(c) 46 (d) 49 [Gate 2013]

11. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is:

(a) 30 (b) 36
(c) 40 (d) 24 [Gate 2013]

12. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is:

(a) $(-\infty, 0)$ (b) $(0, 1)$
(c) $(1, \infty)$ (d) $(0, \infty)$ [Gate 2013]

13. If $\left(z+\frac{1}{z}\right)^2 = 98$, compute $\left(z^2+\frac{1}{z^2}\right)$.

[Gate 2014 set 1]

14. Which number does not belong in the series below?

2, 5, 10, 17, 26, 37, 50, 64

(a) 17 (b) 37
(c) 64 (d) 26 [Gate 2014 set 3]

15. Consider the equation $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(a) 1634 (b) 1737
(c) 3142 (d) 3162

[Gate 2014 set 3]

16. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include

(a) Half the sum of the base angles
(b) Half the difference of the base angles
(c) Sum of the base angles
(d) Difference of the base angles

17. A person sold two pipes at ₹12 each. His profit on one was 20% and his loss on the other was 20%. On the whole, he

(a) Neither lost nor gained
(b) Lost ₹1
(c) Gained ₹1
(d) Gained ₹2

18. A circular wire of diameter 42 cm is bent in the form of a rectangle whose sides are in the ratio 6:5. The area of the rectangle is

(a) 540 sq-cm (b) 1080 sq-cm
(c) 2160 sq-cm (d) 4320 sq-cm

19. $\left[\frac{(0.73)^3+(0.27)^3}{(0.73)^2+(0.27)^2-(0.73)(0.27)}\right]$ simplifies to

(a) 1 (b) 4.087
(c) 0.73 (d) 0.27

20. $\frac{2}{3}\times\frac{3}{\frac{5}{6}/\frac{2}{3}\text{ of }1\frac{1}{4}}$ is

(a) 2 (b) 1
(c) 1/2 (d) 2/3

21. The value of $\sqrt[3]{2^4\sqrt{2^{-5}\sqrt{2^6}}}$ is

(a) 2 (b) $2^{\frac{5}{3}}$
(c) 2^5 (d) 1

22. 1397 × 1397 = ?

(a) 1951609 (b) 1981709
(c) 18362619 (d) 2031719

23. If a = 4.36, b = 2.39 and c = 1.97 then the value of $a^3 - b^3 - c^3 - 3abc$

(a) 3.94 (b) 2.39
(c) 0 (d) 1

24. It is being given that (232 + 1) is completely divisible by a whole number. Which of the following numbers is completely divisible by this number?

(a) $2^{16}+1$ (b) $2^{16}-1$
(c) 7×2^{23} (d) $2^{96}+1$

25. A regular hexagon is inscribed in a circle with centre O. Then the angle subtended by each side of square at the centre O is

(a) 80° (b) 90°
(c) 60° (d) 45°

26. (112 × 54) = ?

(a) 67000 (b) 70000
(c) 76500 (d) 77200

27. The largest 4 digit number exactly divisible by 88 is

(a) 9944 (b) 9768
(c) 9988 (d) 8888

28. Equal chords of a circle subtend equal angles at the

(a) Circumference (b) Centre
(c) Radius (d) Diameter

29. Suman can do a work in 3 days. Gautam can do the same work in 2 days. Both of them finish the work together and get ₹150. What is the share of Suman?
(a) 30 (b) 60
(c) 70 (d) 75

30. The length of the diagonal of a square is a cm. Which of the following represents the area of the square?
(a) $2a$ (b) $a/\sqrt{2}$
(c) $a^2/2$ (d) $a^2/4$

31. How many of the following numbers are divisible by 132?
264, 396, 462, 792, 968, 2178, 5184, 6336
(a) 4 (b) 5
(c) 6 (d) 7

32. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number?
(a) 240 (b) 270
(c) 295 (d) 360

33. Simplify $1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{5}}}$
(a) 7/4 (b) 4/7
(c) 7/5 (d) 3/7

34. $1-[5-\{2+(-5+6-2)2\}]$ is equal to
(a) –4 (b) 2
(c) 0 (d) –2

35. If $\sqrt{x}/\sqrt{441}=0.02$ then x is equal to
(a) 1.64 (b) 2.64
(c) 1.764 (d) 0.1764

36. $2\sqrt[3]{32}-3\sqrt[3]{4}+\sqrt[3]{500}$ is equal to
(a) $4\sqrt[3]{6}$ (b) $3\sqrt{24}$
(c) $6\sqrt[3]{4}$ (d) 916

37. The sixth term of the sequence 11, 13, 17, 19, 23, __, 29 is
(a) 24 (b) 19
(c) 25 (d) 22

38. 72519 × 9999 = ?
(a) 725117481 (b) 674217481
(c) 685126481 (d) None of these

39. For what values of a is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square?
(a) $\pm\frac{1}{18}$ (b) $\pm\frac{1}{8}$
(c) $-\frac{1}{5}$ (d) $\frac{1}{4}$

40. If the HCF of two numbers (each greater than 13) be 13 and LCM 273 , then the sum of the numbers will be
(a) 288 (b) 290
(c) 130 (d) 286

41. A digit number is formed by repeating a 2 digit number such as 2525, 3232 etc. Any number of this form is always exactly
(a) 7
(b) 11
(c) 13
(d) Smallest 3 digit prime number

42. A number divided by 68 gives the quotient 269 and remainder zero. If the same number is divided by 67, the remainder is
(a) 0 (b) 1
(c) 2 (d) 3

43. (12345679 × 72) = ?
(a) 88888888 (b) 888888888
(c) 898989898 (d) 9999999998

44. If the number 97215 * 6 is completely divisible by 11, then the smallest whole number in place of * will be
(a) 3 (b) 2
(c) 1 (d) 5

45. The sum $(5^3+6^3+\cdots+10^3)$ is equal to
(a) 2295 (b) 2425
(c) 2495 (d) 2925

Q. 46 to Q. 50: The following table shows the number of new employees added to different categories of employees in a company and also the number of employees from these categories who left the company every year since the foundation of the company in 1995.

Year	Managers		Technicians		Operators		Accountants		Peons	
	New	Left	New	Left	New	Left	New	Left	New	Left
1995	706	–	1200	–	880	–	1160	–	820	–
1996	280	120	272	120	256	104	200	100	184	96
1997	179	92	240	128	240	120	224	104	152	88
1998	148	88	236	96	208	100	248	96	196	80
1999	160	72	256	100	192	112	272	88	224	120
2000	193	96	288	112	248	144	260	92	200	104

46. During the period between 1995 and 2000 the total number of operators who left the company is what per cent of the total number of operators who joined the company?
(a) 19% (b) 21%
(c) 27% (d) 29%

47. For which of the following categories, the percentage increases in the number of employees working in the company from 1995 to 2000 was the maximum?
(a) Manager (b) Technician
(c) Operator (d) Accountants

48. What is the difference between the total number of the technicians added to the company and the total

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