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# Mechanism and Machine Theory

**Ashok G. Ambekar**



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**MECHANISM AND MACHINE THEORY**

Ashok G. Ambekar

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# CONTENTS

*Preface*

xv

## **1. Introduction to Kinematics and Mechanisms 1–18**

1.1	Introduction	1
1.2	The Four-bar Mechanism	2
1.3	Motion of a Particle	2
1.4	Motion of a Rigid Body	3
1.5	Motion of Translation	3
1.6	Motion of Rotation	4
1.7	Planar Motion and Euler's Theorem	4
1.8	Degrees of Freedom	4
1.9	Vectors and Their Treatment	5
1.10	Methods of Expressing Vectors	8
1.11	Position Vectors	10
1.12	Displacement of a Particle	11
1.13	Rigid Body Displacement	11
1.14	Relative Displacement	12
	<i>Review Questions</i>	18

## **2. Planar Mechanisms and Geometry of Motion 19–60**

2.1	Introduction	19
2.2	Definitions and Basic Concepts	19
2.3	Classification of Links	22
2.4	Classification of Pairs	23
2.4.1	Classification of Pairs Based on Type of Relative Motion	23
2.4.2	Classification of Pairs Based on Type of Contact	25
2.4.3	Classification of Pairs Based on Degrees of Freedom	26
2.4.4	Classification of Pairs Based on Type of Closure	27

2.5	Mechanism and Machine	27
2.6	Inversions	28
2.7	Quadric Cycle Chain and Its Inversions	28
2.8	Inversion of Slider Crank Chain	33
2.9	The Double Slider Crank Chain and Its Inversion	35
2.10	Transmission of Torque and Force in Mechanisms	36
2.11	Constrained Motion and Degrees of Freedom of a Mechanism	38
2.12	Expression for D.O.F. of Chains and Mechanisms	38
2.13	Interpretation of Mobility Equation	40
2.14	Inconsistencies of Grubler's Equation	43
2.15	Degrees of Freedom Permitted by Joints other than Turning and Sliding	45
2.16	Equivalent Linkages	48
2.17	Number Synthesis	49
2.17.1	Effect of Even/Odd Number of Links on Degrees of Freedom	50
2.17.2	Minimum Number of Binary Links in a Mechanism	51
2.17.3	Maximum Possible Number of Turning Pairs on any of the $n$ Links in a Mechanism	52
2.18	Enumeration of Kinematic Chains	53
2.19	Spatial Mechanisms	57
2.20	Manipulators	58
	<i>Review Questions</i>	58

### **3. Velocity and Acceleration Analysis (Graphical Approaches) 61–126**

3.1	Introduction	61
3.2	Linear and Angular Velocity	61
3.3	Velocity of a Point on Rotating Rigid Body	63
3.4	Graphical Differentiation	64
3.5	Relative Velocity	67
3.6	Relative Velocity between Two Points on the Same Link	68
3.7	Velocity Image	68
3.8	Velocity Polygon	70
3.9	Velocity of Rubbing	75
3.10	Mechanical Advantage and Power Transmission	77
3.11	Instantaneous Centres of Rotation	78
3.12	Properties of Instantaneous Centre	79
3.13	Location of Velocity Pole (I.C.)	80
3.14	Instant Centres of 4-bar Mechanism	81
3.15	Aronhold-Kennedy's Theorem of Three Centres	82
3.16	Locating I.Cs. in Mechanisms	83
3.17	Acceleration in Mechanisms	87
3.18	Motion of a Particle along Curved Path	88

3.19	Acceleration of a Rigid Link	88
3.20	Acceleration Image of a Link	90
3.21	General Acceleration Equation for a Link	91
3.22	Necessary Conditions for $A^{cor}$ to Exist	94
3.23	Acceleration Polygon	94
3.24	Combined Four-Bar Chain and Slider-Crank Mechanism	101
3.25	Acceleration Polygon Involving Coriolis Component of Acceleration	103
3.26	Klein's Construction	109
3.27	Approximate Analytical Expression for Displacement, Velocity and Acceleration of Piston of Reciprocating Engine Mechanism	115
3.28	Kinematic Analysis of Complex Mechanisms	118
	<i>Review Questions</i>	121

#### **4. Velocity and Acceleration Analysis (Analytical Approach) 127–157**

4.1	Introduction	127
4.2	Vector Method	127
4.3	Types of Analysis Problems	128
4.4	The Loop Closure Equation	131
4.5	Algebraic Position Analysis	131
	4.5.1 Case of Slider-Crank Mechanism	132
	4.5.2 Case of Four-Bar Mechanism	133
4.6	Velocity and Acceleration Analysis Using Complex Algebra (Raven's Approach)	135
4.7	Application to Slider-Crank Mechanism	136
4.8	Application to Four-Bar Mechanism	137
4.9	Application to Quick-Return Mechanism	139
	<i>Review Questions</i>	153

#### **5. Mechanisms with Lower Pairs 158–202**

5.1	Introduction	158
5.2	Offset Slider-Crank Mechanism as a Quick Return Mechanism	158
5.3	The Pantograph	160
5.4	Straight Line Motion Mechanisms	162
5.5	Exact Straight Line Motion Mechanisms	163
	5.5.1 Condition for Generating Exact Straight Line Motion	163
	5.5.2 Peaucellier Mechanism	164
	5.5.3 The Hart Mechanism	165
	5.5.4 The Scott-Russel Mechanism	166
5.6	The Approximate Straight Line Motion	167
	5.6.1 The Watt Mechanism	167
	5.6.2 The Grasshopper Mechanism	168
	5.6.3 The Tchebicheff Straight Line Motion Mechanism	169
	5.6.4 The Roberts Straight Line Motion Mechanism	171

5.7	Engine Indicators	171
5.7.1	Simplex Indicator	171
5.7.2	Crosby Indicator	172
5.7.3	Thompson Indicator	173
5.7.4	Dobbie-McInnes Indicator	175
5.8	Motor Car Steering Gear	177
5.8.1	Condition of Correct Steering	177
5.8.2	Davis Steering Gear	178
5.8.3	Ackerman Steering Gear	181
5.9	Hooke's (Cardan) Joint or Universal Coupling	185
5.9.1	Transmission Characteristics	186
5.9.2	Double Hooke's Joint	192
5.10	Toggle Mechanism	197
5.11	Scotch Yoke Mechanism	199
	<i>Review Questions</i>	199

## 6. Elements of Kinematic Synthesis of Mechanisms (Graphical and Algebraic Methods) 203–240

6.1	Kinematic Synthesis	203
6.2	Approximate and Exact Synthesis	205
6.3	Chebyshev's Spacing of Accuracy Points	205
6.4	Graphical Methods of Dimensional Synthesis	208
<b>Motion Generation</b>		
6.5	Poles and Relative Poles	208
6.6	Motion Generation: Three Prescribed Positions	210
6.7	Relative Poles (Roto-Centres) of the 4-Bar Mechanism	212
6.8	Relative Poles of Slider-Crank Mechanism	214
6.9	Function Generation (Three Precision Points)	217
6.10	Algebraic Method of Function Generation	219
6.11	Comments on Design Parameters and Special Nature of Results	226
6.12	Coupler Curves	228
6.13	Synthesis for Path Generation	230
6.14	Graphical Synthesis for Path Generation (Three Specified Positions)	231
6.15	Roberts-Chebyshev Theorem (Cognate Linkages)	232
6.16	Coupler Curves from 5-Bar Mechanisms	235
	<i>Review Questions</i>	238

## 7. Cams 241–299

7.1	Introduction	241
7.2	Comparison between Cams and Lower Paired Mechanisms	241
7.3	Classification of Cams and Followers	242
7.3.1	Classification of Cams	242
7.3.2	Classification of Followers	244

7.4	Terminology for Radial Cam	247
7.5	Types of Follower Motion	248
7.5.1	Uniform Motion or Constant Velocity Follower Motion	249
7.5.2	Modified Uniform Motion	249
7.5.3	Simple Harmonic Motion	250
7.5.4	Uniformly Accelerated and Retarded (Parabolic) Follower Motion	254
7.5.5	Cycloidal Follower Motion	258
7.6	Pressure Angle ( $\psi$ )	260
7.7	Parameters Affecting Pressure Angle	261
7.8	Effect of Offset Follower Motion	263
7.9	Main Consideration Influencing Choice of Cam	266
7.9.1	Smaller Lateral Pressure on Guides	266
7.9.2	Smaller Force Required to Accelerated Follower	267
7.9.3	Smooth Jerkless Motion	268
7.9.4	Smaller Base-Circle	268
7.10	Radius of Curvature and Undercutting	269
7.11	Construction of Cam Profiles	269
7.12	Cam Layout: General Type of Problems	272
7.13	Translating Flat Face Follower: Analytical Design	280
7.14	Cam with Oscillating Roller Follower	284
7.15	Cams with Specified Contours	287
7.15.1	Circular Arc Cam with Tangent Follower	287
7.15.2	Tangent Cam with Roller Follower	292
	<i>Review Questions</i>	296

## 8. Gears

**300–365**

8.1	Introduction	300
8.2	Rolling Contact and Positive Drive	300
8.3	Classification of Gears	301
8.3.1	Gears Mounted on Parallel Axes	301
8.3.2	Gears Mounted on Intersecting Shaft Axes	303
8.3.3	Gears Mounted on Skew Shaft Axes	304
8.4	Nomenclature for Straight Spur Gears	305
8.5	Fundamental Law of Toothed Gearing	310
8.6	Conjugate Teeth	312
8.7	Tooth Profiles	313
8.7.1	Cycloidal Tooth Profile	314
8.7.2	Involute Tooth Profile	315
8.8	Length of Path of Contact	318
8.9	Length of Arc of Contact	320
8.10	Contact Ratio	321

8.11	Interference and Undercutting	324
8.12	Standard Proportions of Interchangeable Gears	327
8.13	Minimum Number of Teeth to Avoid Interference	328
8.14	Minimum Number of Teeth on Pinion to Avoid Interference with Rack	336
8.15	Comparison between Involute and Cycloidal Tooth Profiles	342
8.16	Methods of Reducing or Eliminating Interference	343
8.17	Helical Gears	344
8.18	Spiral Gears (Skew or Screw Gears)	347
8.19	The Efficiency of Spiral and Helical Gears	350
8.20	Worm and Worm Gear	355
8.21	Bevel Gears	360
8.22	Special Bevel Gears	362
	<i>Review Questions</i>	363

## 9. Gear Trains 366–403

9.1	Introduction	366
9.2	Classification	366
9.3	Epicyclic Gear Trains with Bevel Gears	372
9.4	Algebraic Method of Analysing Epicyclic Gear Trains	373
9.5	Tabulation Method for Analysing Epicyclic Gear Train	379
9.6	Torques and Tooth Loads in Epicyclic Gear Trains	384
9.7	Bevel Gear Differential	397
	<i>Review Questions</i>	398

## 10. Gyroscopic Effects 404–442

10.1	Introduction	404
10.2	Angular Motion and Conventional Vector Representation	405
10.3	Precessional Motion and Angular Acceleration	406
10.4	Gyroscopic Couple	410
10.5	Gyro-couple and Gyro-reaction Couple	411
10.6	Analogy with Motion of a Particle in Circular Path	411
10.7	Gyroscopic Effects on an Aeroplane	414
10.8	Stability Analysis of 4-Wheeler Vehicle	415
10.9	Stability Analysis of a Two-wheel Vehicle	423
10.10	Gyroscopic Effects on Naval Ships	427
10.11	Gyroscopic Ship Stabilization	432
10.12	Gyroscopic Analysis of a Disc Fixed Rigidly to a Rotating Shaft at Certain Angle	435
10.13	Gyroscopic Analysis of Grinding Mill	437
	<i>Review Questions</i>	440



**11. Friction Gears****443–512**

- 11.1 Introduction 443
- 11.2 Types of Friction 443
- 11.3 Dry Friction 444
- 11.4 Angle of Repose and Angle of Friction 446
- 11.5 Motion along Inclined Plane 446
- 11.6 Friction of Nut and Screw 450
- 11.7 Wedge 457
- 11.8 Rolling Friction 461
- 11.9 Pivot and Collar Friction 463
- 11.10 Axial Force and Friction Moment in Pivots and Collars 464
  - 11.10.1 Assumption of Uniform Pressure Intensity 466
  - 11.10.2 Assumption of Uniform Rate of Wear 467
- 11.11 Design Considerations in the Choice of Assumption 469
- 11.12 Thrust Bearing 470
- 11.13 Friction Clutches 474
- 11.14 Cone Clutch 475
- 11.15 Single Plate Clutch 479
- 11.16 Multi-Disc Clutch 481
- 11.17 Effect of Number of Pairs of Active Surfaces 482
- 11.18 Centrifugal Clutch 492
- 11.19 Friction Circle and Friction Axis 495
- 11.20 Lubricated Surfaces 500
- 11.21 Friction between Lubricated Surfaces 501
- 11.22 Film Lubrication in Rotating Shafts 504
- 11.23 Michel Thrust Bearing 505
- 11.24 Hydrostatic Lubrication 506
- 11.25 Rolling Contact Bearings 506
- 11.26 Advantages and Disadvantages of Rolling Contact Bearings 508
- Review Questions* 509

**12. Belt, Rope and Chain Drives****513–558**

- 12.1 Introduction 513
- 12.2 Velocity Ratio 513
- 12.3 Belt Length 517
- 12.4 Limiting Ratio of Belt-Tensions 519
- 12.5 Maximum Effective Tension and H.P. Transmitted 522
- 12.6 Centrifugal Tension and Stresses in Belts or Rope 525
- 12.7 Maximum Tension in Belt/Rope 526
- 12.8 Initial Tension and its Role in Power Transmission 526
- 12.9 Condition for Maximum Power Transmission 535
- 12.10 Power Transmitted by Belt: Further Comments 537
- 12.11 Idler and Jockey Pulleys 540

12.12 Timing Belt 541  
 12.13 Rope Drive 541  
 12.14 Materials of Belt and Rope 552  
 12.15 Chains 553  
 12.16 Inverted Tooth Chain (Silent Chain) 555  
*Review Questions* 556

**13. Brakes and Dynamometers 559–616**

13.1 Introduction 559  
 13.2 Classification of Brakes 559  
 13.3 The Simple Block or Shoe Brake 560  
 13.4 Short-Shoe Brakes (Condition of Self Energization) 561  
 13.5 Double Block Brakes 563  
 13.6 Long Shoe Brakes 566  
 13.7 Long Shoe Brakes (Shorter Method) 570  
 13.8 Internally Expanding Shoes 573  
 13.9 Band Brakes 583  
 13.10 Band and Block Brake 587  
 13.11 The Braking of a Vehicle 596  
 13.12 Types of Dynamometers 604

**Absorption Dynamometers**

13.13 Prony Brake Dynamometer 605  
 13.14 Rope Brake Dynamometer 606

**Transmission Dynamometers**

13.15 Epicyclic Train Dynamometers 608  
 13.16 Belt Transmission Dynamometer 609  
 13.17 Torsion Dynamometers 611  
*Review Questions* 614

**14. Dynamics of Machines, Turning Moment, Flywheel 617–704**

14.1 Role of Force Analysis in Design Calculations 617  
 14.2 Laws of Motion and D'Alembert's Principle 618  
 14.3 Static Force Analysis 620  
 14.4 Static Force Analysis for Mechanisms 621  
 14.5 Mass Moment of Inertia and Inertia Torques 622  
 14.6 Simple Harmonic Motion 625  
 14.7 Dynamically Equivalent Two Mass System 634  
 14.8 Centre of Percussion 636  
 14.9 Significance of Kinetic Equivalence 639  
 14.10 Equivalent Dynamic System: Graphical Determination 639  
 14.11 Correction Couple Required for Arbitrary Choice of Both the Mass Locations 640

14.12	The Effective Force and the Inertia Force	644
14.13	Reversed Effective (Inertia) Force and Force Analysis	646
14.14	Dynamic Force Analysis of a Four-link Mechanism	648
14.15	Analytical Expressions for Velocity and Acceleration of Slider in Slider-Crank Mechanism	654
14.16	Piston Effort, Crank Pin Effort and Crank Effort	657
14.17	Inertia Forces and Torques in Slider-Crank Mechanism	659
14.18	Dynamic Force Analysis of a Slider-Crank Mechanism	663
14.19	Turning Moment Diagram (Crank Effort Diagram)	675
14.20	Fluctuation of Crank Shaft Speed	678
14.21	The Flywheel	682
14.22	Flywheel for Punching Press	683
	<i>Review Questions</i>	701

## 15. Governors

705–766

15.1	Introduction	705
15.2	Functions of a Governor	705
15.3	Types of Governors	707
15.4	Terms Used in Governors	708
15.5	The Watt Governor	709
15.6	Effect of Mass of Arms in Watt Governor	711
15.7	The Porter Governor	713
15.8	Effect of Friction	717
15.9	Proell Governor	724
15.10	Spring Controlled Governors	729
15.11	Hartnell Governor	729
15.12	Governor with Spring Connected Balls (Wilson–Hartnell Governor)	732
15.13	Governor with Gravity and Spring Control	742
15.14	Hartung Governor	744
15.15	Pickering Governor	746
15.16	Inertia Governors	747
15.17	Characteristics of Centrifugal Governors	747
15.18	Quality of Governor: Definitions	748
	15.18.1 Controlling Force	748
	15.18.2 Stability and Isochronism	749
	15.18.3 Sensitiveness	752
	15.18.4 Hunting	752
15.19	Governor Effort and Power	752
15.20	Effect of Friction: Insensitiveness	754
	<i>Review Questions</i>	763

**16. Balancing****767–849**

16.1	Introduction	767
16.2	Balancing of Rotating Masses	767
16.3	Static and Dynamic Balancing Problem	768
16.4	Unbalanced Rotating Mass	769
16.5	Balancing of Several Masses Revolving in the Same Plane	772
16.6	Analytical Approach for Several Rotating Masses in Same Plane	773
16.7	Balancing of Several Masses Revolving in Different Planes	775
	16.7.1 First Method	775
	16.7.2 Second Method (Dalby's Method)	779
16.8	Balancing of Rotors	789
16.9	Static and Dynamic Balancing	790
16.10	Static Balancing Machines	790
16.11	Dynamic Balancing Machines	791
	16.11.1 Pivoted Cradle Balancing Machine	792
16.12	Field Balancing	794
	16.12.1 Balancing of a Twin Disc	795
	16.12.2 Balancing by Four Observations	796
16.13	Balancing of Reciprocating Masses	798
16.14	Inertia Effects of Reciprocating Masses in Engine Mechanism	799
16.15	Primary and Secondary Unbalanced Forces due to Reciprocating Masses	801
16.16	Inertia Effects of Crank and Connecting Rod	802
16.17	Partial Balancing of Primary Inertia Forces	804
16.18	Partial Balancing of Locomotives	807
16.19	Effect of Partial Balancing in Locomotives	807
16.20	Primary Balance of Multi-Cylinder In-Line Engine	822
16.21	Secondary Balance of Multi-Cylinder In-Line Engines	823
16.22	Balancing of 2-Stroke and 4-Stroke In-Line Engines	824
16.23	Firing Order	827
16.24	Direct and Reverse Cranks	836
16.25	Balancing V-Engines	841
	<i>Review Exercises</i>	845

**17. Vibration Analysis****850–935**

17.1	Introduction	850
17.2	Definitions	850
17.3	Simple Harmonic Motion and Rotating Vectors	852
17.4	Work Done in Harmonic Motion	853
17.5	Elements of Discrete (Lumped Parameter) Vibratory System	855
<b>Single Degree of Freedom Problems</b>		
17.6	Undamped Free Vibrations	856
	17.6.1 Method Based on Newton's Second Law of Motion	856

17.6.2	Energy Method	859
17.6.3	Rayleigh's Method	860
17.7	Equivalent Springs and Dashpots	864
17.8	Equivalent Length of Shaft	867
17.9	Damped Free Vibrations	868
17.10	Logarithmic Decrement	872
17.11	Forced Vibrations with Harmonic Excitation	877
17.12	Vibration Isolation and Transmissibility	883
17.13	Vibration Isolation without Dampers	883
17.14	Vibration Isolation Using Dampers	885
17.15	Motion Transmissibility	889
17.16	Whirling of Shafts	895
17.17	Critical Speed of Light Vertical Shaft with Single DISC (without Damping)	896
17.18	Critical Speed of Light Vertical Shaft Having Single Disc with Damping	900
17.19	Longitudinal and Transverse Vibrations	905
17.20	Natural Frequency of Free Transverse Vibrations due to a Point Load on a Simply-Supported Shaft	906
17.21	Transverse Vibration of a Uniformly Loaded Shaft	907
17.22	Transverse Vibrations of Shaft Carrying Several Loads	911
<b>Torsional Vibrations</b>		
17.23	Single Rotor System	917
17.24	Free Torsional Vibrations: Two Rotor System	918
17.25	Free Torsional Vibrations: Three Rotors	920
17.26	Torsional Vibration of Geared System	927
	<i>Review Questions</i>	931

<b>Appendix I</b>	<b>Units</b>	<b>937–938</b>
<b>Appendix II</b>	<b>Mathematics</b>	<b>939–941</b>
<b>Appendix III</b>	<b>S.I. and M.K.S. Units</b>	<b>942–974</b>
	<b>Bibliography</b>	<b>975–976</b>
	<b>Index</b>	<b>977–986</b>

## PREFACE

This book covers that field of engineering studies which conventionally goes under the name 'Theory of Machines'. Over the years, knowledge in this area has expanded manifold both in width as well as in depth. This is invariably reflected in the theme and scope of some of the prominent text/reference books published over the last 40 years or so. Departing from traditionally common titles, some of the relatively recent book titles are *Applied Kinematics*, *Mechanics of Machinery*, *Kinematics and Dynamics of Plane Mechanisms* and *Synthesis and Analysis of Mechanisms*. Details of these titles appear in the Bibliography provided at the end of this book. Needless to say that in some way, may be in different proportions, these trends should get reflected in our course curriculum.

On the analysis side, there is a visible shift of emphasis from graphical to analytical approach. Modern high-speed applications have greatly enhanced the importance of acceleration analysis for accessing the effect of inertia forces in the overall design of machine components. This is further accentuated by the availability of high speed computers. Besides accuracy and speed of computations, analytical methods help in establishing motion characteristics (e.g. velocity and acceleration) for large number of configurations of a mechanism (e.g. for different crank positions in an I.C. engine) with minimal extra time. This feature enables to establish maximum value of inertia force and corresponding crank angle of rotation, thereby rendering design calculations more rationale. In view of the importance of this area of learning, a separate chapter has been devoted to the analytical methods of velocity and acceleration analysis.

The above comments, however, should not be taken to mean that traditional graphical methods need to be replaced by analytical methods. It is generally believed that graphical methods provide a better insight into the subject matter. This is in fact true. A separate chapter on graphical analysis of velocity and acceleration analysis has, therefore, been included in this book.

A practising mechanical engineer is required to design a mechanism that satisfies the prescribed motion characteristics. The synthesis method calls for exposure to type synthesis, number synthesis or dimensional synthesis. An obvious outcome of number synthesis can be seen in the 'Enumeration of distinct chains of given number of links and degrees of freedom'. This aspect has been discussed in the chapter on planar mechanisms. Function and path

generation problems involve dimensional synthesis. Coupler curves provide a very useful tool in mechanism synthesis. A separate chapter has been dedicated to the studies on elements of mechanism synthesis in kinematics and it includes discussions on the coupler curves and cognate linkages.

In all conventional topics, attempt has been made to present the subject matter in a way commensurate with state-of-the-art presentation in the published literature. While teaching dynamic force analysis, a common observation is that the students do not have enough exposure to the application part of D'Alembert's principle. Therefore, in chapters involving dynamic force analysis, special emphasis has been laid to explain as to how the problem is converted into an equivalent problem of static equilibrium. Special efforts have been made to explain how a positive drive is really achieved through gearing action. Similarly, extra efforts have been made to convey conceptual meaning of 'interference' of involute gear through the principle of conjugate teeth.

Wherever deemed necessary, applications of 'principle of inversion' and 'instantaneous centre of rotation' have been highlighted. Throughout the text, special emphasis has been laid on developing conceptual understanding. This will be especially visible in the chapter on governors, cams and gyroscopes.

The book is designed to serve as a textbook for undergraduate engineering students studying in mechanical, production and aeronautical streams. The course content is intended to be covered as a two-semester course, requiring about 90 lecture hours. While preparing the manuscript of this book, I had in my mind needs of B.E./B.Tech. students of various universities and institutes of higher learning. This is also true in respect of students preparing for various technical boards, professional examining bodies (e.g. Institution of Engineers and Institution of Mechanical Engineers) and national level examinations like GATE and IES. A good number of solved problems in S.I. units appear in each chapter, through which theory is amplified and extended. I believe that providing good number of representative solved problems is an effective way of clarifying doubts, if any, and it also helps in presenting the subject matter in a more comprehensive way. Ample number of unsolved problems, with answers, have been provided at the end of each chapter as Review Questions.

The course material is largely based on my lectures delivered as a faculty at Shri G.S. Institute of Technology and Science, Indore for a period of over 35 years at undergraduate and postgraduate levels and also at Swami Vivekanand College of Engineering, Indore. I feel highly indebted to my teachers Dr. S.S. Rao and Dr. J.N. Chakraborty (during my studies in M.Tech. at IIT Kanpur) and to Dr. V.P. Agrawal and Dr. Raghavacharlu (during my Ph.D. studies at IIT Delhi) for enrichment of my knowledge. Without their classroom teaching and general guidance, a book writing project of this type would have been impossible. I acknowledge with thanks all other teachers and fellow students at both the IITs who have directly or indirectly contributed in refining my level of conceptual understanding in the subject.

I feel highly indebted to (Late) Dr. J.P. Shrivastava, the then Director and my teacher at S.G.S. Institute of Technology and Science, Indore, without whose inspiration this project would not have started. Words are inadequate to put on record the motivating role of my father, late G.W. Ambekar, in this endeavour. This project turned out to be his last desire. I also desire to place on record my heartfelt thanks to the publisher Prentice-Hall of India and their expert editorial team for the quality conscious editing work for this book. I am especially thankful to

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The preface will remain incomplete if I do not acknowledge the patience with which my wife Mrs. Sushama Ambekar, spared her valuable time and energy to share my day-to-day responsibilities during the period of book writing. Thanks are also due to my sons Chi. Aniruddha and Chi. Ashwin for their encouragement and an ever helping hand in this stupendous task.

Special efforts have been made to satisfy quest of knowledge of readers through this book. Also, great care has been taken to make the book 'error-free'. I will be grateful for constructive criticism and suggestions for further improvements in the book.

**Ashok G. Ambekar**



# INTRODUCTION TO KINEMATICS AND MECHANISMS

## 1.1 INTRODUCTION

A more traditional name of this subject is *Theory of Machines*. The subject comprises study of *Kinematics* and *Dynamics of Machines*. The former deals with relationship between the geometry of machine parts and their relative motion. Dynamics of machines, on the other hand, deals with the forces which produce these motions. The study of dynamics comprises *Statics* and *Kinetics*. In statics, one deals with the forces that act on the machine parts which are assumed to be massless. Kinetics, on the other hand, deals with the inertia forces arising out of the combined effect of mass and the motion of the parts.

An elementary definition of mechanism describes it as a mechanical device which has the purpose of transferring motion and/or force from the source to the output. Mechanisms pervades all walks of life. A few illustrative applications of mechanisms can be found in domestic appliances (e.g., sewing machines, washing machines, refrigerator, mixers, air-coolers, door-locks/latches, etc.), toys (e.g., peddling cars for kids, walking toys, etc.), agricultural implements (e.g., sprinkler mechanism, tractors, corn drills, ploughs, cultivators, mowers, forage harvesters, potato diggers, etc.), transport vehicles (on air, land and sea, incorporating steering, braking and transmission mechanism, I.C. engine mechanism, etc.), construction equipment (e.g., concrete mixers, cranes, excavators, rock-crushers, bulldozers, etc.), military equipment (e.g., armoured tank, recoil-less guns, self-loading guns, etc.), manufacturing machines and machines used in production processes (e.g., lathe, shaping, drilling, milling and grinding machines, lifting tackles, overhead cranes, conveyors, press, rivet and punching machines, robots and manipulators, etc). Mechanisms also find applications in bio-medical engineering in the form of artificial limbs (e.g., prosthetic knee mechanism, pace maker mechanism, etc.). In short, there is hardly any place where mechanisms are not present. Some of the representative mechanisms are illustrated in Figs. 1.17(a) through 1.17(w). In big cities readers can have a look at different types of construction equipment in operation at any construction cite of tall buildings or clusters of such buildings.

The studies under kinematics and mechanisms are fundamental to the studies in a much broader discipline called *Machine Design*. Developments in engineering applications over the last

few decades indicate a consistent trend towards increasing the machine speed. This trend can be readily seen in automobiles. As a direct sequel to this trend, consideration of inertia forces in design calculations have become much more essential. In a number of cases, inertia force consideration compels the designer to cut down the weight of the components. This necessitates a greater emphasis on acceleration part of motion analysis.

### 1.2 THE FOUR-BAR MECHANISM

Mechanisms play a key role in transferring motion from one part of a machine to some other part. Individual parts/components of a mechanism are called *bars* or, more precisely *links*, which are connected to one another through pin joints ensuring a relative motion of rotation. Based on similarity of relative motion between door panels and the frame in houses, these pin joints are also called hinges. A four-bar mechanism constitutes the simplest type of closed loop linkage. It consists of three mobile links and a fixed link called frame. One of the links is connected to power source/prime mover and is called the input link. An another link, called output link, delivers required type of motion to some other part of the machine. The input and output links are connected to the frame through pin joints called pivots. The input and output links are interconnected at their other ends through a link called coupler/floating link through pin joints. The coupler, input and output links are called the mobile links in this mechanism. A simple application of a four-bar mechanism is shown in the form of a treadle-operated grinding machine in Fig. 1.17(a). The foot pressure is indicated by  $F$  in the figure.

### 1.3 MOTION OF A PARTICLE

A particle, of the size of a point, is supposed to have no dimensions. In kinematics, the word 'particle' is reserved for those objects whose motion is not affected in any way by the dimensions of the object. The concept of 'motion of a particle' is of far-reaching consequence and therefore needs special attention.

When a particle moves in space, it occupies successive positions in the coordinate system with respect to time. When the successive positions of the particle are connected by a smooth curve, it produces a curved line. This is obvious because the particle does not have any dimension. This line, representing successive positions of the point, is called the path or locus of the point.

#### ***Spatial and Planar Motion***

When all the three coordinates are necessary to describe the path or locus, the particle is said to have *spatial motion*. When only two coordinates are necessary (the third coordinate always being zero or constant) to describe the path, the motion of particle is said to be a *planar motion*.

In the above definitions of planar and spatial motions of a particle, it is implied that frame of coordinate axis is so chosen in orientation that minimum possible number of coordinates is needed to describe the motion.

#### ***Rectilinear or Straight Line Motion***

In the same spirit, a particle is said to execute straight line or rectilinear motion when a single coordinate is required to describe the motion. For instance, let us assume that a particle moves

along a straight path  $AB$  as shown in Fig. 1.1. In such a case, re-orientation of axes of reference, with  $x$ -axis lying along  $AB$ , necessitates only one coordinate  $x'$  along  $AB$ . Therefore, it is a rectilinear motion.

In general, a plane curved path requires a minimum of two coordinates while a space (skew) curve requires three coordinates.

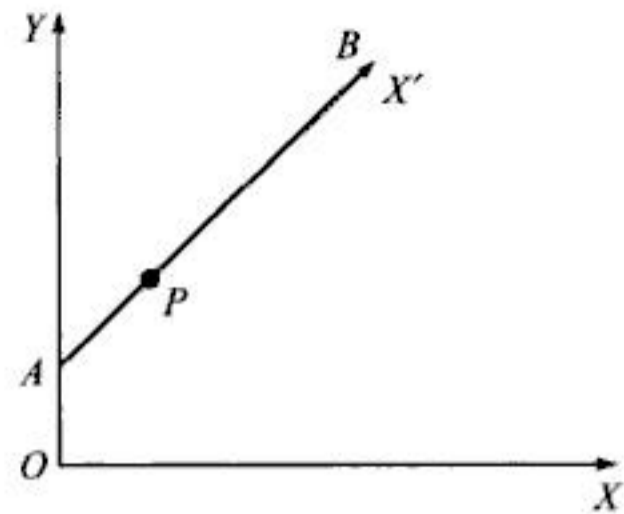


Fig. 1.1 Particle moving along straight path.

## 1.4 MOTION OF A RIGID BODY

When dimensions of a body become important in studying intricacies of motion, the motion can be idealised as a 'rigid-body motion'. The body in such a case is considered to consist of a large number of small particles.

To understand the concept of *motion of a particle*, consider the motion executed by a piston of an I.C. engine. The piston reciprocates along a straight line, but does not have any other motion (e.g., a motion of rotation about its own axis) associated with it. Thus, each and every particle of the piston, lying on the cylindrical surface or on the axis, has same motion characteristics. In such a motion, it is immaterial whether a piston is represented by a cylinder of specific dimensions or by a particle located at the piston pin on the axis. The resulting simplicity in analysis on account of idealisation of 'motion of a particle' is quite significant. As against this, motion of a connecting rod cannot be idealised as 'motion of a particle.' This is mainly because every particle in the body can have its own motion characteristics.

A very common assumption in studying motion of a rigid body is that any two particles in the body of the object always remain apart at the same distance, howsoever large the deforming forces can be. This is the concept of rigid body motion.

## 1.5 MOTION OF TRANSLATION

When each particle of a body has exactly the same motion as every other particle of which it is composed, the body is said to execute *motion of translation*. This implies that a straight line, drawn joining two particles, remains parallel to itself throughout the motion.

**Motion of translation can be of two types.**

- (A) **Rectilinear Motion of Translation.** When all the particles of a rigid body have exactly the same motion, and that motion is along a straight line, the motion is called rectilinear motion of translation. Piston of I.C. engine/steam engine and cross-head of a steam engine execute rectilinear motion of translation.
- (B) **Curvilinear Motion of Translation.** When motion of all the particles of a rigid body are exactly the same but take place along curved path, the motion of body is called curvilinear motion of translation. Figure 1.2 describes a double crank and slider mechanism in which the slider executes rectilinear motion of translation but the coupler  $AB$  executes curvilinear motion of translation. It can be verified that as cranks  $OA$  and  $CB$  have equal lengths, the coupler  $AB$  always moves parallel to itself, and any point  $K$  on this coupler actually moves along the arc of a circle of radius  $CB$  or  $OA$ , but with centre at  $O_k$ .

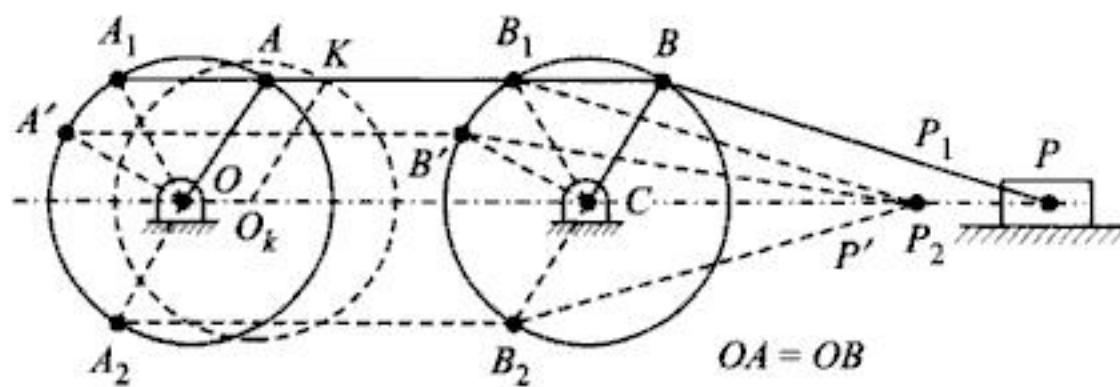


Fig. 1.2 Double crank-slider mechanism.

It follows therefore that if motion of slider  $P$  or coupler  $AB$  is required to be analysed, the entire shape and size can be ignored and the body be replaced by a point for the purpose of analysis.

## 1.6 MOTION OF ROTATION

A rigid body is said to have motion of rotation when any point in the body remains at the same distance called centre of rotation. Motion of links  $OA$  and  $CB$  in Fig. 1.2 are called motion of rotation. It can be verified that when a body rotates, a straight line joining any two points in the rotating body, does not remain parallel to itself, except perhaps for those lines lying along axis of rotation.

## 1.7 PLANAR MOTION AND EULER'S THEOREM

Motion of connecting rod  $PB$  in Fig. 1.2 is truly representative of a general complex motion of a rigid body moving always in a plane. The motion of connecting rod is a combination of motion of translation and motion of rotation. This is easy to understand because one end  $B$  of the connecting rod always moves along a circle, while the other end  $P$  reciprocates along the line of stroke. This is in accordance with *Euler's theorem* which states that—

*Any displacement of a rigid body is equivalent to the sum of a net translation of any point  $P$  and a net rotation of the body about that point.*

It follows that displacement difference between any two points on the same rigid body is attributed to motion of the rotation of the body, and not to the motion of translation.

## 1.8 DEGREES OF FREEDOM

Minimum number of independent coordinates or parameters needed to describe configuration of a body in motion is called its *degrees of freedom*. While specifying degrees of freedom, it is common to assume that lengths of links are known. A crank-slider mechanism constitutes a simple example for this purpose.

In case of slider-crank mechanism, shown in Fig. 1.3, if angle  $\theta$  made by the crank with line of stroke is known, the entire mechanism can be described in position. For instance, one may choose crank shaft centre  $O$  and line of stroke arbitrarily and, at an angle  $\theta$  to the line of stroke, draw the crank  $OA$  of known length. From  $A$  as centre, with compass set for a radius equal to

length of connecting rod  $AP$ , an arc can be drawn to cut line of stroke at  $P$ . This gives position of slider  $P$  as also the position of the connecting rod  $AP$ . Thus a single coordinate  $\theta$  is required to describe configuration and hence, degrees of freedom (abbreviated as d.o.f.) equals 1.

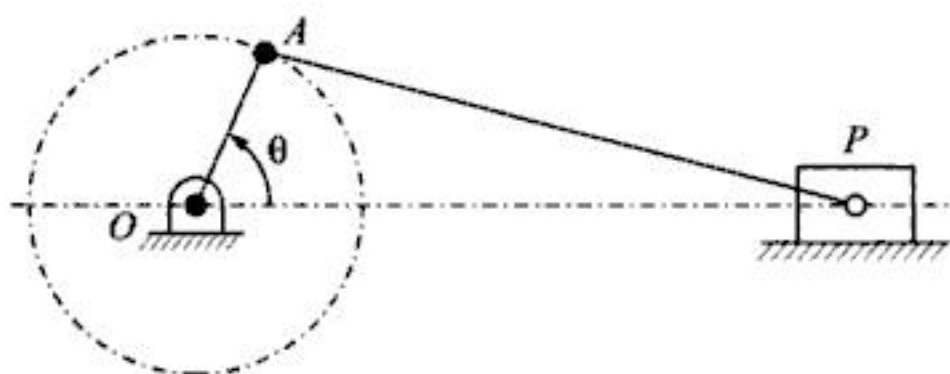


Fig. 1.3 A slider-crank mechanism.

Degrees of freedom can also be defined on the basis of number of independent motions which a body can have. In general, a body in space can have six independent motions; three of translation along  $x$ ,  $y$  and  $z$  axes and three of rotation about  $x$ ,  $y$  and  $z$  axes, as shown in Fig. 1.4. The body can have motion of translation  $u$ ,  $v$  and  $w$  parallel to  $x$ ,  $y$  and  $z$  axes respectively. The body can also rotate through  $\alpha$ ,  $\beta$  and  $\gamma$  about lines parallel to  $x$ ,  $y$  and  $z$  axes respectively. In forming chains and mechanism, some of these degrees of freedom are suppressed.

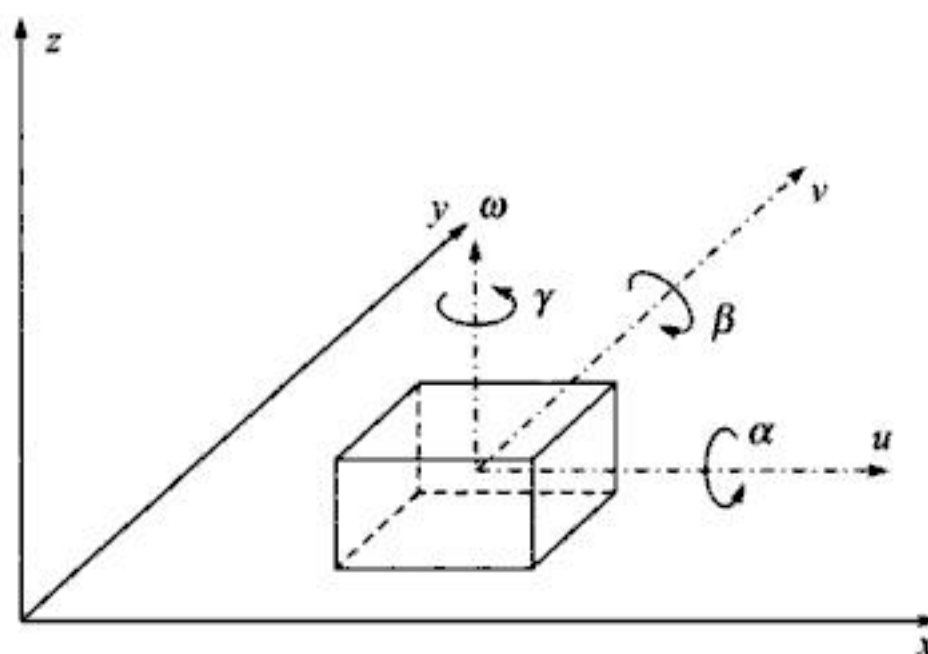


Fig. 1.4 Independent motions of a body in space.

## 1.9 VECTORS AND THEIR TREATMENT

The readers must be already conversant with the definitions of scalars and vectors. As distinct from scalars, a vector quantity has both a magnitude and direction. Thus, mass of a body is a scalar quantity but its weight is a vector quantity. A vector is also defined as a directed line segment which has specified length (magnitude) and a specified direction. The line segment for a vector must be drawn to some scale, with its length representing the magnitude of the vector quantity. An arrow, with its head pointing in the direction of vector quantity, is placed at the appropriate end of the above line segment.

(A) *Vector addition.* (Refer Fig. 1.5) While using letters of the alphabet to designate vectors, boldface capital characters will be reserved for vectors and italic characters for scalars. Thus, italic letters  $a, b, c$  will be used for scalar component of vectors  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  respectively.

The operation  $\mathbf{A} + \mathbf{B}$  is defined as a vector addition of  $\mathbf{A}$  and  $\mathbf{B}$ . To understand the physical significance of this operation, let us assume that a particle undergoes a displacement  $\mathbf{A}$  from  $P$  to  $Q$ . Let it be further subjected to a displacement  $\mathbf{B}$  from  $Q$  to  $R$ . Although these displacement do not take place simultaneously, the net (resultant) displacement could be obtained by giving displacement to the particle directly from  $P$  to  $R$ .

The displacements  $PQ$  and  $QR$  in Fig. 1.5 are represented by vectors  $\mathbf{A}$  and  $\mathbf{B}$  respectively, and the net displacement  $PR$  is represented by vector  $\mathbf{C}$ , so that vector equation, indicating vector addition, is

$$\mathbf{A} + \mathbf{B} = \mathbf{C} \quad (1.1)$$

Equation (1.1), when examined in the light of Fig. 1.5, leads to the conclusion that the resultant vector  $\mathbf{C}$  originates at the tail of the vector  $\mathbf{A}$  and terminates at the tip of the arrowhead of vector  $\mathbf{B}$ .

It follows from vector algebra that *commutative law* holds good in vectorial addition and subtraction. This can be further verified using parallelogram rule for vector addition. Fig. 1.6 demonstrates addition of vectors using parallelogram rule. Let the vectors  $\mathbf{A}$  and  $\mathbf{B}$  originate at a common point  $O$ , and let the parallelogram be completed on sides  $OP$  and  $OQ$ . The diagonal  $OR$  then represents the resultant  $\mathbf{C}$ .

Since  $PR$  is equal and parallel to  $OQ$  (i.e., vector  $\mathbf{B}$ ), it follows that vector  $\mathbf{B}$  can be truly represented by side  $PR$ . Thus the vector triangle  $OPR$ , confirms earlier vector equation (1.1),  $\mathbf{A} + \mathbf{B} = \mathbf{C}$

Again, side  $QR$  is equal and parallel to  $OP$  (which represents  $\mathbf{A}$ ), and as such, represents vector  $\mathbf{A}$ . Thus, from vector triangle  $OQR$  it follows that

$$\mathbf{B} + \mathbf{A} = \mathbf{C} \quad (1.2)$$

Comparing equations (1.1) and (1.2), it follows that *in vector addition, order of vectors is unimportant*. Again, it is concluded from vector triangle  $OPR$  of Fig. 1.6 that to find the vector sum  $\mathbf{A} + \mathbf{B}$ , tail of vector  $\mathbf{B}$  be placed at the tip of the arrow-head of vector  $\mathbf{A}$ . Similarly, for finding the vector sum  $\mathbf{B} + \mathbf{A}$  from vector triangle  $OQR$ , one needs to keep tail of the vector  $\mathbf{A}$  at the tip of the arrow-head of vector  $\mathbf{B}$ , and the resulting vector  $\mathbf{C}$  will have tip of its arrow-head touching tip of the arrowhead of  $\mathbf{A}$ .

(B) *Vector sum of more than two vectors.* When a vector sum of say three vectors  $\mathbf{A}, \mathbf{B}$  and  $\mathbf{C}$  is to be obtained, one may consider the sum  $(\mathbf{A} + \mathbf{B}) + \mathbf{C}$ . An expression of this type implies that the resultant of vector sum  $\mathbf{A} + \mathbf{B}$  and vector  $\mathbf{C}$  are to be added vectorially. Let the vectors  $\mathbf{A}, \mathbf{B}$  and  $\mathbf{C}$  be as shown in Fig. 1.7(a). Then, the resultant of vector sum  $\mathbf{A} + \mathbf{B}$  is represented by vector  $\mathbf{D}$  in Fig. 1.7(b), and Fig. 1.7(c) shows the addition of vectors  $(\mathbf{A} + \mathbf{B})$  and  $\mathbf{C}$ .

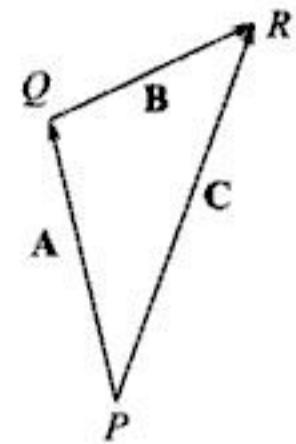


Fig. 1.5 Vector addition.

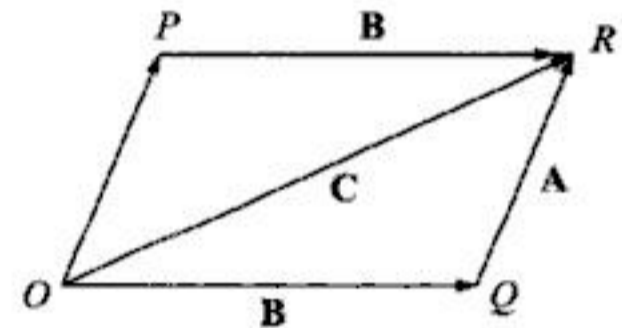


Fig. 1.6 Parallelogram of vectors.

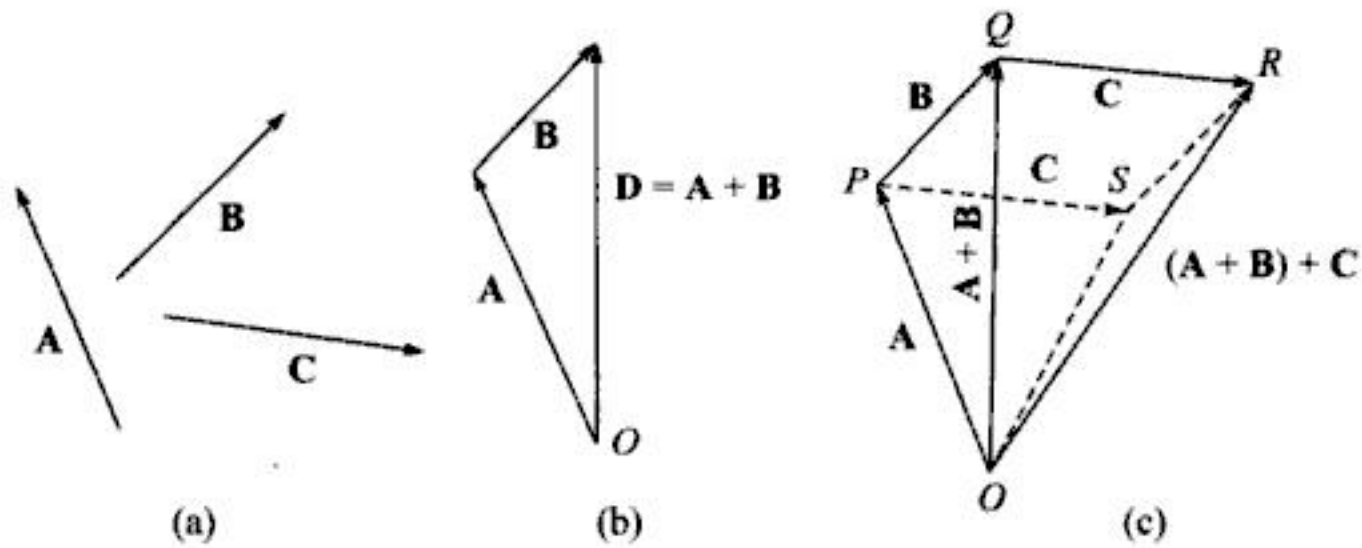


Fig. 1.7 Vector sum of three vectors.

It can be seen in Fig. 1.7(c) that the resultant vector  $(A + B) + C$  originates at the tail of the first vector  $A$  and terminates at the tip of the arrowhead of the last vector  $C$ .

By completing parallelogram  $PQRS$ , where  $PQ$  and  $QR$  represent vectors  $B$  and  $C$  respectively, it is possible to prove that commutative law is valid for this vector operation. For instance, in the parallelogram  $PQRS$ , the sides  $PS$  and  $SR$  can be taken to represent vectors  $C$  and  $B$ . Then, from vector polygon  $OPSR$ , line joining  $OS$  represents vector sum  $A + C$  and therefore vector  $OR = (A + C) + B$ .

It follows that

$$(A + B) + C = (A + C) + B \quad (1.3)$$

In other words, in vector addition, order of vectors is unimportant.

(C) *Vector subtraction.* It is more convenient to represent vector subtraction  $A - B$  as a problem of addition of a negative of a vector  $B$  to vector  $A$ . Thus, mathematically

$$A - B \equiv A + (-B) \quad (1.4)$$

Physically, negative of a vector  $B$  implies a vector pointing out in diametrically opposite side. Fig. 1.8 may be referred for complete understanding.

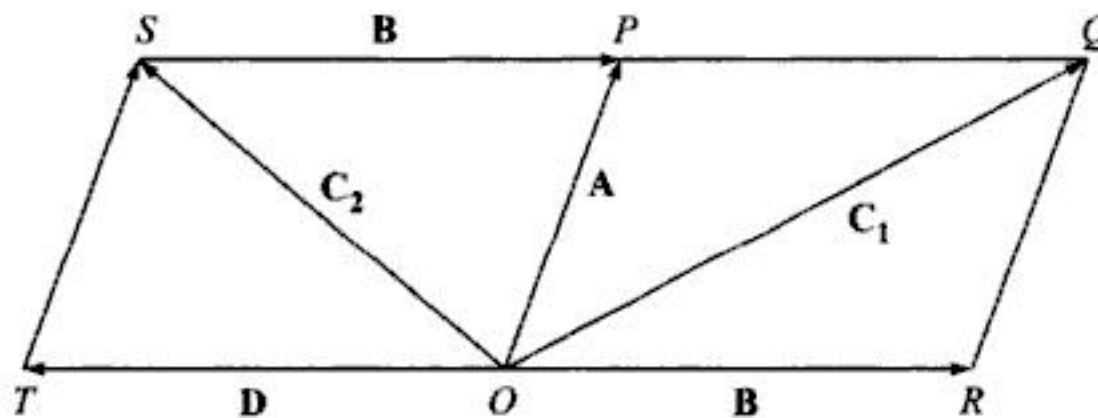


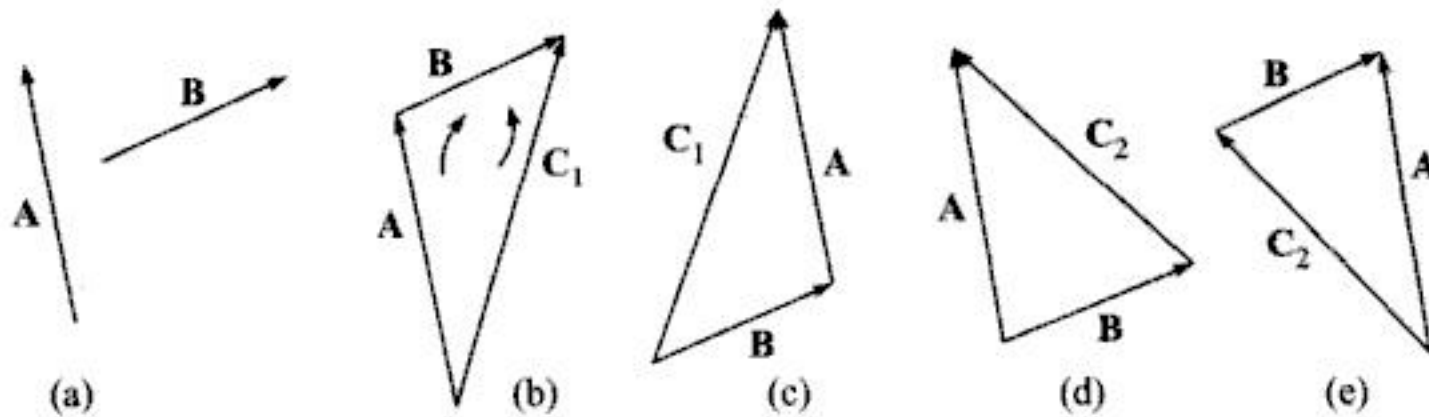
Fig. 1.8

The vector  $C_1$  in Fig. 1.8 is the resultant of vector summation  $A + B$  while the vector  $C_2$  represents resultant of vector addition  $A + D$ , where  $D \equiv -B$ . Thus, in effect, vector  $C_1$  and  $C_2$  represent resultant of vector addition and subtraction respectively of vectors  $A$  and  $B$ . Since side  $OT$  represents  $-B$ , the side  $SP$  in vector triangle  $OSP$  represents vector  $B$ .

To evolve law of vector subtraction, we consider the vector triangle  $OSP$ , keeping vectors  $A$  and  $C_2$  as they are, and interpret orientation of vector  $B$  vis-à-vis the above two vectors. We conclude that when vector  $B$  is arranged so that its tip of arrowhead touches tip of arrowhead of  $A$ , the closing side of the vector triangle, namely  $OS$ , represents resultant of vector subtraction. Next consider the vector triangle  $OST$  in which for given vectors  $A$  (as represented by sense  $TS$ ) and  $C_2$ , the side  $TO$  represents vector  $B$ . It follows that when vectors  $A$  and  $B$  are arranged such that their tails are at a common point, the closing side of vector triangle represents resultant of vector subtraction.

**Conclusions can be summed as under:**

- (1) For vector addition, vectors  $A$  and  $B$  should be so arranged that tail of one vector touches tip of the arrowhead of the other. The closing side of the vector triangle then represents resultant of vector addition. The arrowhead for the resultant vector  $C_1$  should be so placed that it opposes sense of the two vectors. Thus, in Fig 1.9 (b) vectors  $A$  and  $B$  are in a clockwise sense and the resultant  $C_1$  should therefore be in counterclockwise sense.
- (2) For vector subtraction, vectors  $A$  and  $B$  must be so arranged that either (a) their tips of arrowhead touch at a common point or (b) the tails of the two vectors touch at a common point. This is shown in Figs. 1.9(d) and (e).



**Fig. 1.9** Vector Addition (b) and (c) Vector Subtractions (d) and (e)

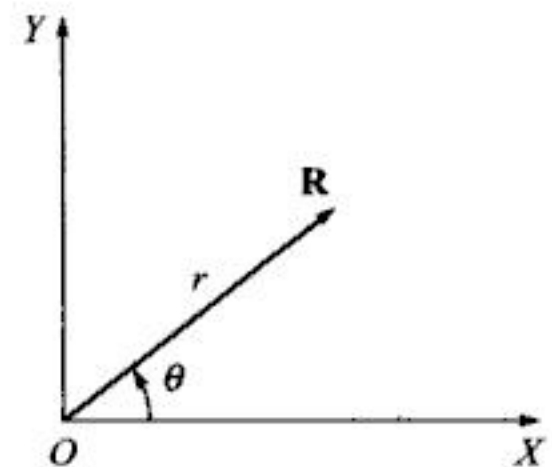
*Students are advised to study above vector operations and their graphical depiction carefully. This is basic for the proper understanding of velocity and acceleration polygons to be dealt with later.*

**1.10 METHODS OF EXPRESSING VECTORS**

(A) *Polar Notation.* Polar method of expressing a vector is quite convenient when the length of the vector and its angular direction from a given reference line are known. As shown in Fig. 1.10, a vector  $R$  of magnitude  $r$  and making an angle  $\theta$  with  $x$ -axis is shown. This vector is expressed mathematically as:

$$R = r \angle \theta \tag{1.5}$$

(B) *Rectangular Notation.* This method consists in defining a vector in terms of its components along  $x$  and  $y$  axes or, if necessary, along any other set of rectangular axes. This can be expressed mathematically as:



**Fig. 1.10**



$$\mathbf{R} = R^x + R^y \quad (1.6)$$

A vector  $\mathbf{R} = r \angle \theta$  in polar form can be expressed in terms of rectangular coordinates. Thus, the vector  $r \angle \theta$  has components along  $x$ -axes and  $y$ -axes as

$$r^x = r \cos \theta \text{ and } r^y = r \sin \theta$$

Thus,

$$r = \sqrt{(r^x)^2 + (r^y)^2} \quad (1.7)$$

and

$$\theta = \tan^{-1} \left( \frac{r^y}{r^x} \right) \quad (1.8)$$

where  $r^x$  and  $r^y$  are the magnitudes of  $R^x$  and  $R^y$ .

(C) *Complex Rectangular Notation.* The convention adopted in this case is that a vector always originates from the origin of co-ordinate system. It follows, therefore, that a vector will always act away from the origin. Any system, which specifies length and direction, defines a vector. Hence a complex rectangular notation, indicated on next page, will also define a vector. The vector  $\mathbf{A}$  is thus expressed as,

$$\mathbf{A} = a + jb \quad (1.9)$$

where  $a$  represents real part and  $b$  represents imaginary part.

It is usual to define  $x$ -axis as real axis and  $y$ -axis as imaginary axis. When  $j$  is prefixed to  $b$ , it indicates  $90^\circ$  c.c.w. rotation with respect to real axis along which  $a$  is measured. So, here  $j$  appears as an operator which indicates that  $b$  is to be measured along  $y$ -axis. Physical significance of equation (1.9) is illustrated in Fig. (1.11), where the magnitude of vector is,

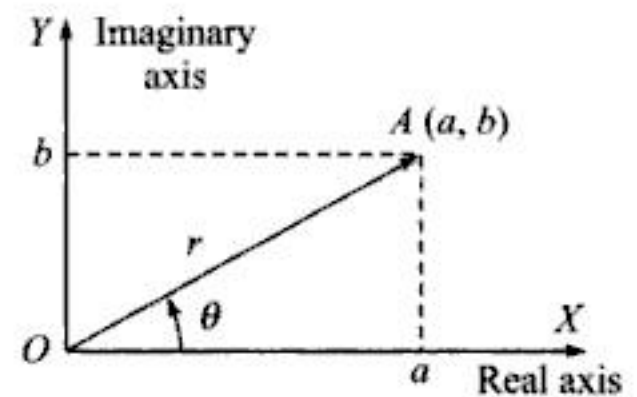


Fig. 1.11

$$r = \sqrt{a^2 + b^2}, \text{ and}$$

$$\theta = \tan^{-1} (b/a)$$

When the operator  $j$  has a positive sign, it indicates positive  $y$ -axis direction, but when  $j$  is negative, it indicates negative side of  $y$ -axis. As a rule, whenever the imaginary constant ( $j = \sqrt{-1}$ ) is prefixed to a number, it implies rotation through right angle in c.c.w. sense. The operator  $j$  may be used repeatedly, but each time its use implies a rotation through  $90^\circ$  in c.c.w. sense. For instance, consider versions  $R_2, R_3, R_4$ , as obtained by multiplying vector  $R_1$  successively by  $j$ . The result is depicted in Fig. 1.12.

(D) *Complex Polar Notation.* From De Moivre's theorem, we know that

$$e^{j\theta} = \cos \theta + j \sin \theta \quad (1.10)$$

where,

$$j = \sqrt{-1}$$

Multiplying equation (1.10) by  $r$  on the either sides, we find that

$$r e^{j\theta} = r (\cos \theta + j \sin \theta) \quad (1.11)$$

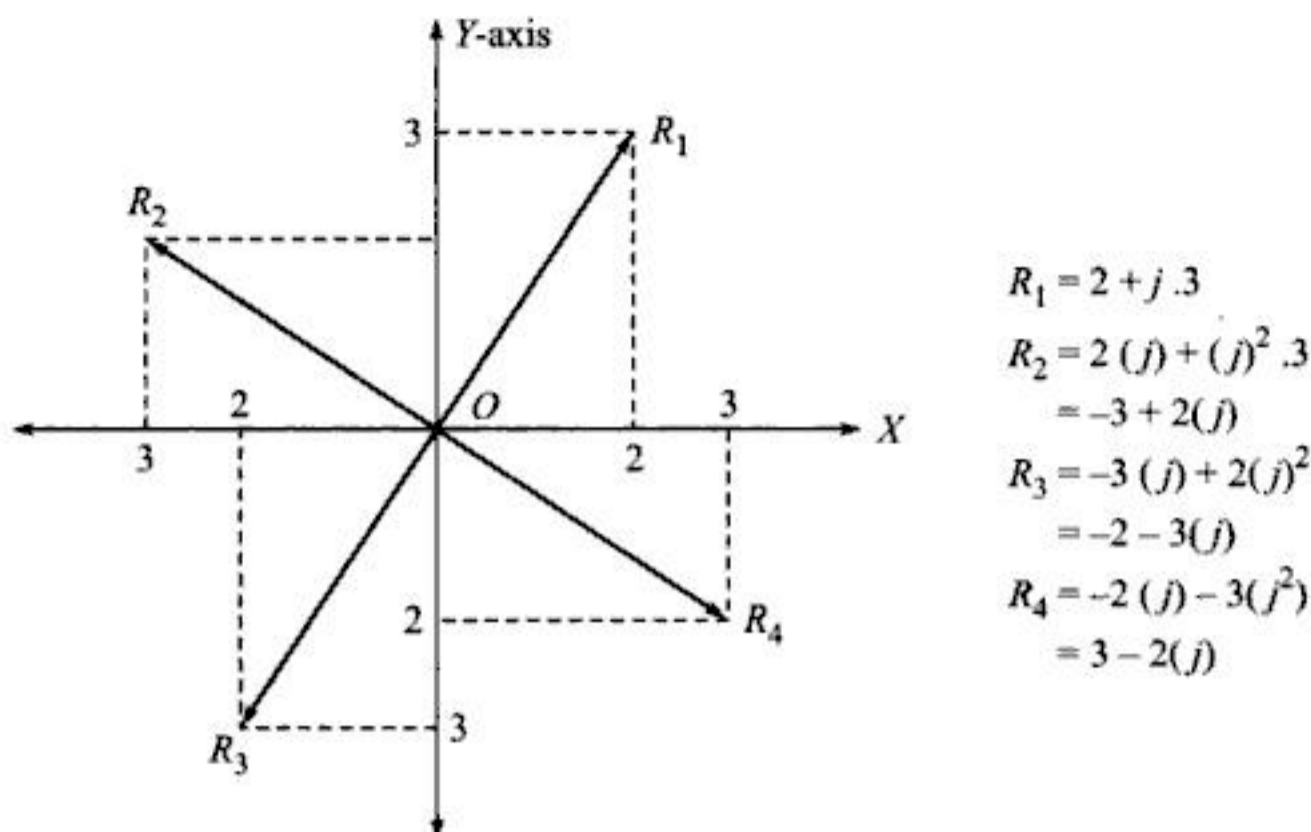


Fig. 1.12 Effect of successive multiplication by  $j$  on a vector.

In view of exposure to the concept of real and imaginary axes, it is rather simple to conclude from the right hand side of the equal to sign, in equation (1.11), that  $r \cos \theta$  and  $r \sin \theta$  are the components along real and imaginary axes respectively. Hence,

$$\mathbf{R} = r e^{j\theta} \tag{1.12}$$

represents exponential form of a vector, in which  $r$  is the magnitude and  $\angle \theta$  is the orientation (angle with positive  $x$ -axis) of the above vector.

*The exponential form and complex rectangular notation of vectors are very useful in analytical methods of velocity and acceleration analysis and, therefore, students should get themselves conversant with these notations.*

### 1.11 POSITION VECTORS

Location of a point is given by its co-ordinates. These co-ordinates can be either rectangular or polar. A position vector is defined as a vector which originates from origin of co-ordinate frame and defines the location or position of a point in reference frame. Position of a point is pivotal in defining displacement and other motion characteristics. And this speaks of importance of the concept of position vector. In Fig. 1.13 the vector  $\mathbf{R}_Q$  defines the position of point  $Q$  in the co-ordinate frame. If  $OQ = q$  and  $\theta$  is the angle made by  $OQ$  with  $x$ -axis and if  $x'$  and  $y'$  are the co-ordinates of point  $Q$ , the position vector can be expressed with all the four notations as,

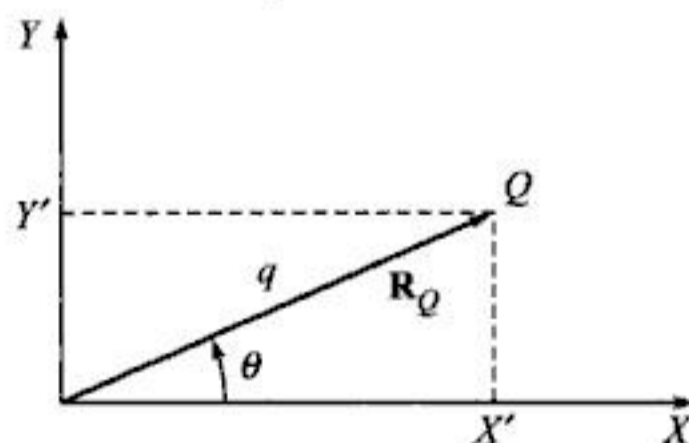


Fig. 1.13 Illustration for a position vector.

$$\begin{aligned}
 \mathbf{R}_Q &= q \angle \theta \\
 \mathbf{R}_Q &= R_Q^x + R_Q^y \\
 \mathbf{R}_Q &= x' + jy' \\
 \mathbf{R}_Q &= q \cdot e^{j\theta}
 \end{aligned}
 \tag{1.13}$$

## 1.12 DISPLACEMENT OF A PARTICLE

A particle may travel along a straight line or a curved path. Distance measured along the given path between two locations  $A$  and  $B$  is the distance moved through by the particle in the given time interval. Displacement, on the other hand, is treated as a vector, and is defined to be the net change of position of the particle over the given period of time interval. For instance in Fig. 1.14, although distance moved is as along  $R_1PQR_2$ , the displacement vector is  $\mathbf{R}_1\mathbf{R}_2 = \Delta\mathbf{R}$ .

Displacement as a vector can be conveniently expressed in terms of position vector of initial position  $\mathbf{R}_1$  and final position  $\mathbf{R}_2$ . Thus vector equation involving position vectors  $\mathbf{R}_1$  and  $\mathbf{R}_2$  and displacement vector  $\Delta\mathbf{R}$  is given by

$$\mathbf{R}_2 = \mathbf{R}_1 + \Delta\mathbf{R} \tag{1.14}$$

or 
$$\Delta\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1 \tag{1.15}$$

This defines displacement mathematically.

## 1.13 RIGID BODY DISPLACEMENT

A general planar rigid body motion may be conceived to consist of two component motions:

1. Motion of translation, and
2. Motion of rotation.

In motion of translation, every particle has identical motion and as such, displacement of a particle in body (say, its C.G.) also describes the translatory motion of the rigid body.

As a direct sequel to the definition of motion of rotation in article 1.4, a straight line  $AB$  joining two particles in the body, does not remain parallel to itself. A rigid body displacement involving motion of translation and rotation, is shown in Fig. 1.15. It can be seen that due to translation alone, there is no relative motion between different particles of the body  $AB$ . However, the angular-rotation component from  $A_1B_1$  to  $A_2B_1$  does produce relative displacement.

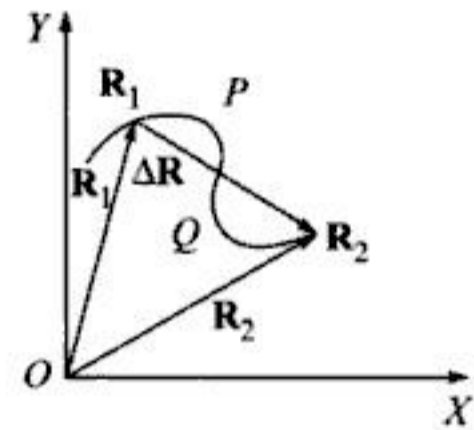


Fig. 1.14 Illustration for displacement vector.

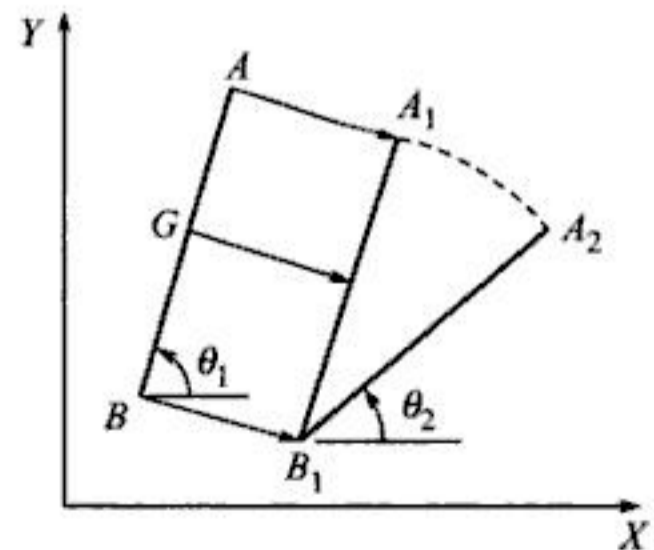


Fig. 1.15 Rigid body displacement.

ment between different points of the body. We, therefore, introduce the concept of angular displacement  $\Delta\theta$  and define it as the net angular change in the position of a line (such as  $AB$ ) in the rigid body. It must be noted, however, that we are not concerned here with the actual path points  $A$  and  $B$  have traversed.

As seen in figure, the net angular displacement is,

$$\Delta\theta = \theta_1 - \theta_2 \quad \text{in clockwise sense in radians.} \quad (1.16)$$

It is important to note that angular displacements do not follow commutative rule to addition and, as such, cannot be combined vectorially. It can be treated as scalar with c.w or c.c.w. sense.

### 1.14 RELATIVE DISPLACEMENT

Let  $A$  and  $B$  be two points defined with respect to origin by position vectors  $\mathbf{A}$  and  $\mathbf{B}$ , as shown in Fig. 1.16(a). Let us assume further that points  $A$  and  $B$  undergo small displacements of  $\Delta\mathbf{A}$  and  $\Delta\mathbf{B}$  respectively to positions  $A'$  and  $B'$  in small interval of time  $\Delta t$ , for position vectors  $\mathbf{A}$  and  $\mathbf{B}$ .

The vector  $\mathbf{AB} = \mathbf{B}_A$  denotes relative position  $B$  with respect to  $A$ . Complete parallelograms, first with sides  $AB$  and  $BB'$  and second one with sides  $AB$  and  $AA'$ . Join  $A'A''$  and  $B'B''$  as shown in Fig. 1.16(b).

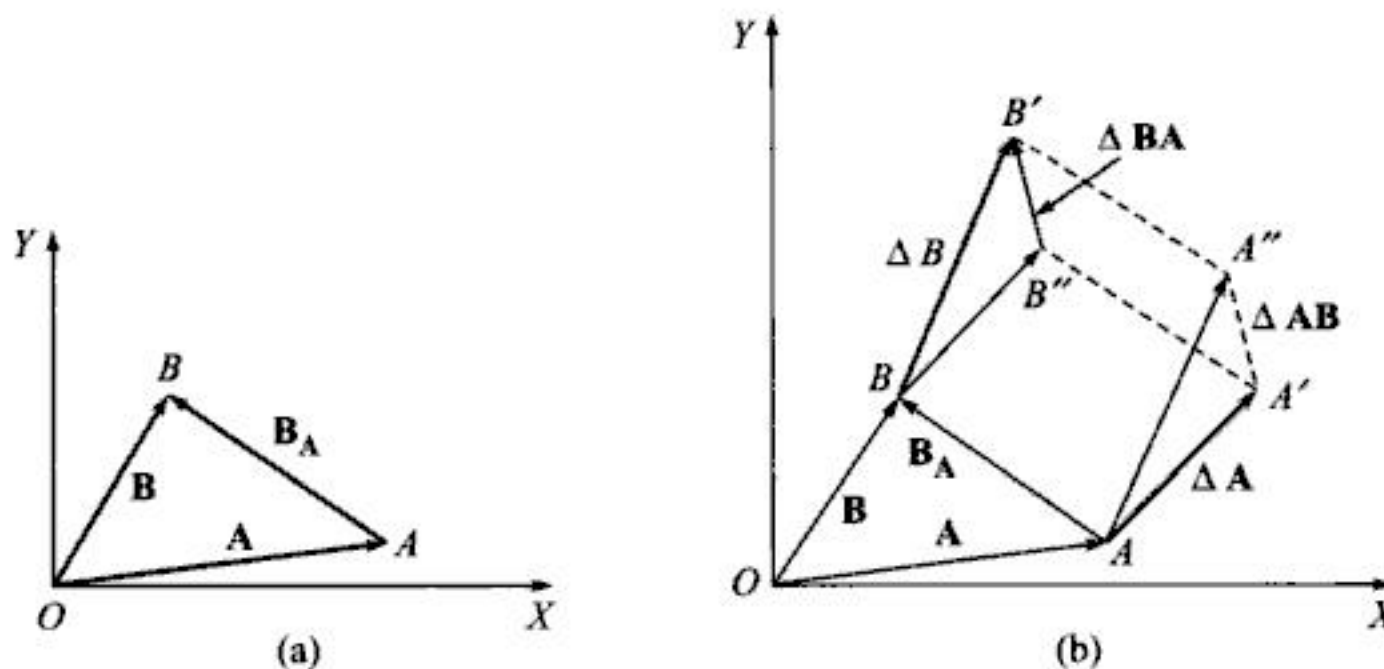


Fig. 1.16 Relative displacement.

The equation connecting position vectors of  $A$  and  $B$  and their relative position vector is

$$\mathbf{B} = \mathbf{A} + \mathbf{B}_A \quad (1.17)$$

The equation says that position of point  $B$  is equal to the position of point  $A$  plus the position of point  $B$  with respect to  $A$ .

The relative displacement of  $B$  with respect to  $A$  is the displacement which  $B$  would have in a co-ordinate frame which is attached to  $A$  and moves with it. Thus, in Fig. 1.16(b), if the frame  $XOY$  is assumed to be attached to point  $A$  and moving with it, point  $B$  will move through  $\Delta\mathbf{A}$  parallel to  $AA'$  to position  $B''$ . Then  $B'B''$  gives displacement of  $B$  relative to  $A$ . In other words,

$$\Delta\mathbf{B}_A = B'B''$$

Since,  $BB''$  is equal and parallel to  $AA'$ , it represents  $\Delta A$  truly. Hence from vector triangle  $B'BB''$  we have

$$\Delta B = \Delta A + \Delta B_A \quad (1.18)$$

or

$$\Delta B_A = \Delta B - \Delta A \quad (1.19)$$

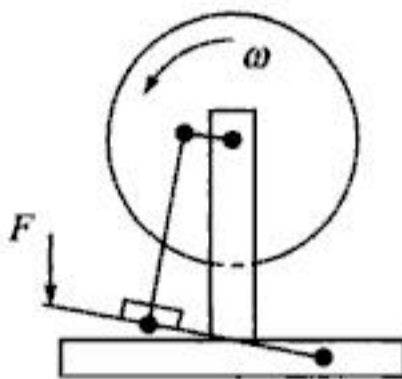
Similarly, relative displacement of  $A$  with respect to  $B$  is the displacement of  $A$  in a co-ordinate frame, which is attached to  $B$  and moves with it. Thus, when co-ordinate frame  $XOY$  is attached to  $B$  and moves it, the point  $A$  in the co-ordinate frame moves through a distance  $\Delta B$  to the position  $A''$ . As  $AA''$  is equal and parallel to  $BB'$  it follows that  $AA''$  truly represents vector  $\Delta B$ . Then  $A''A'$  represents displacement of  $A$  relative to  $B$ . Thus from vector triangle  $A A' A''$  we have.

$$\Delta A = \Delta B + \Delta A_B \quad (1.20)$$

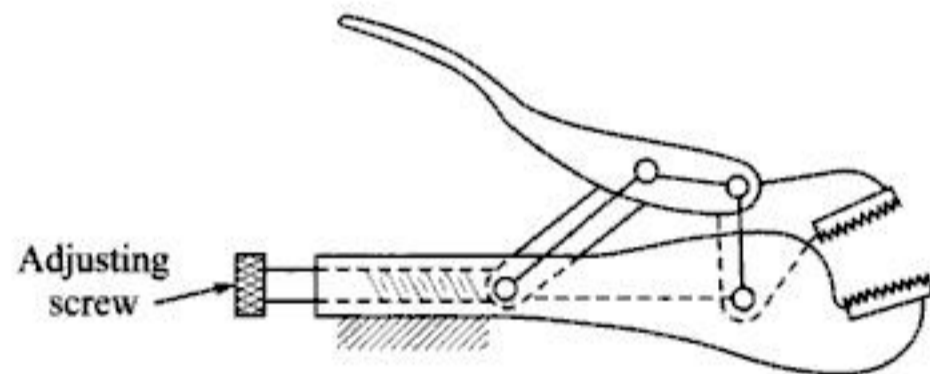
and

$$\Delta A_B = \Delta A - \Delta B \quad (1.21)$$

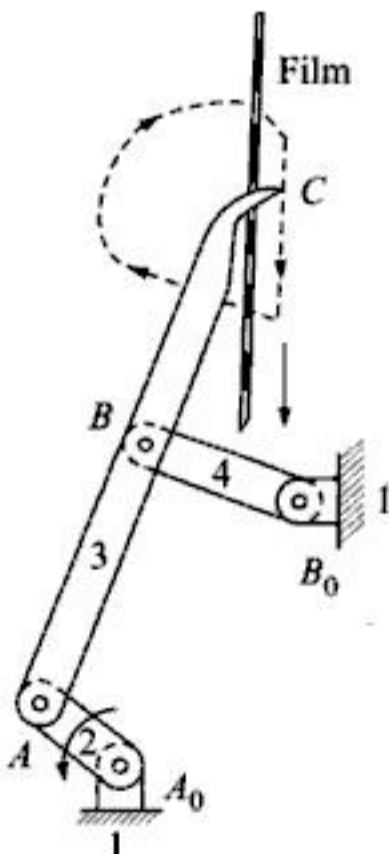
Students are advised to get themselves familiarised with the above concept before proceeding to the chapter on velocity and acceleration analysis.



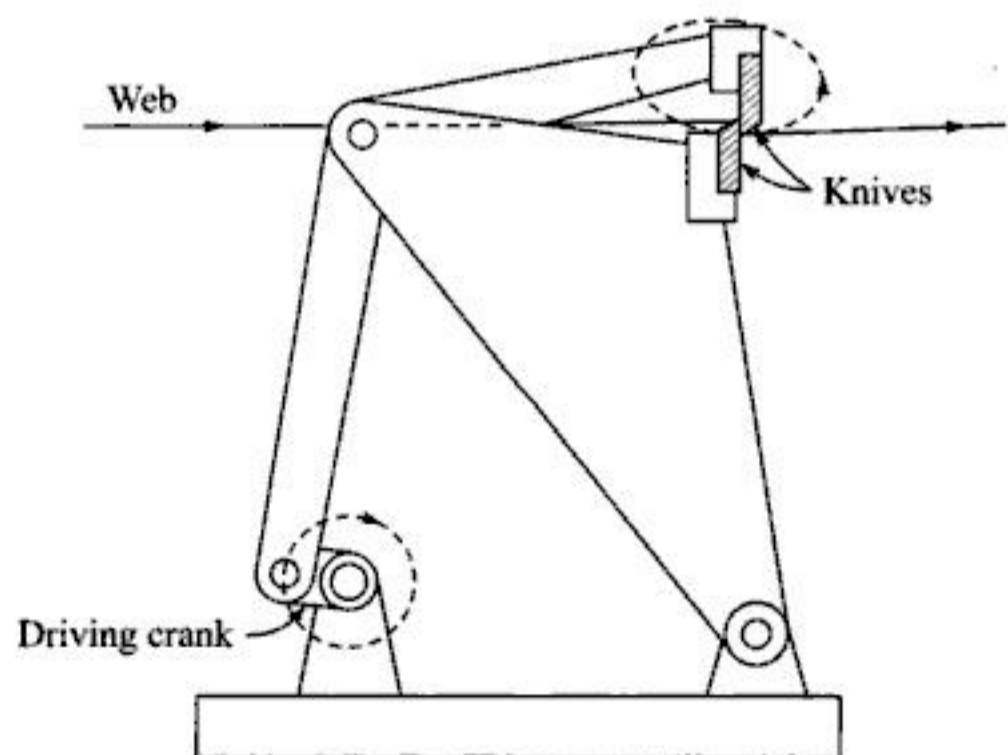
(a) Treadle operated grinding machine



(b) A pair of locking toggle pliers

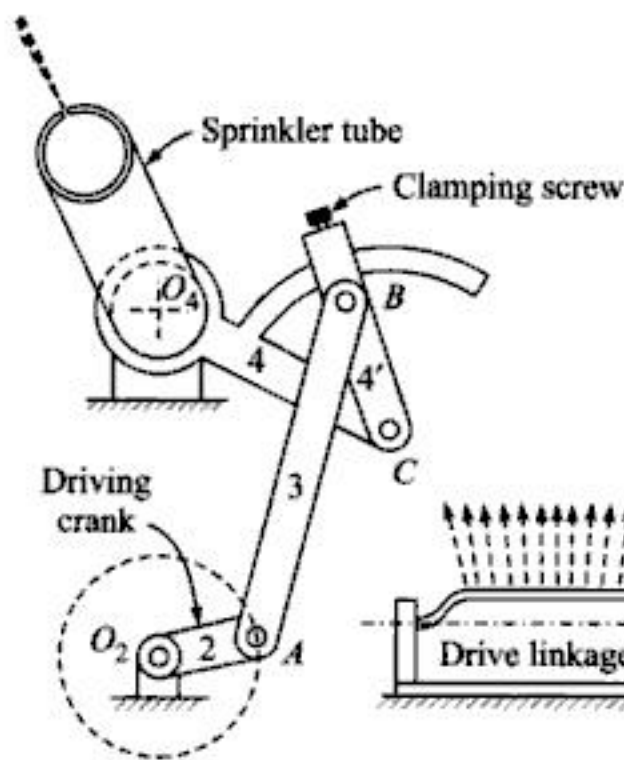


(c) A film advance mechanism of a movie camera/projector

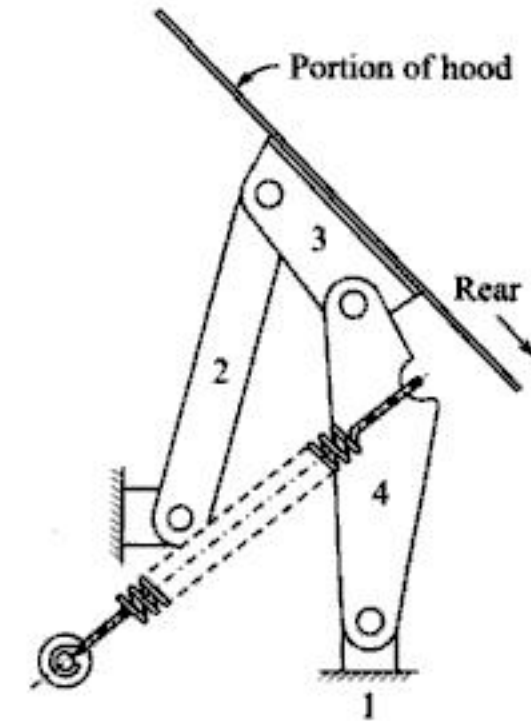
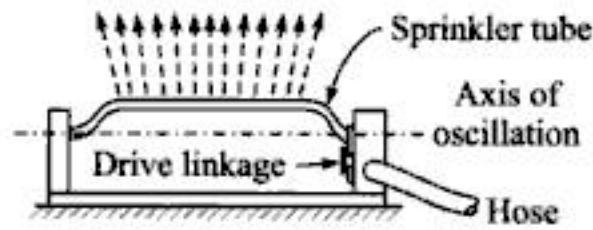


(d) A web-cutter mechanism

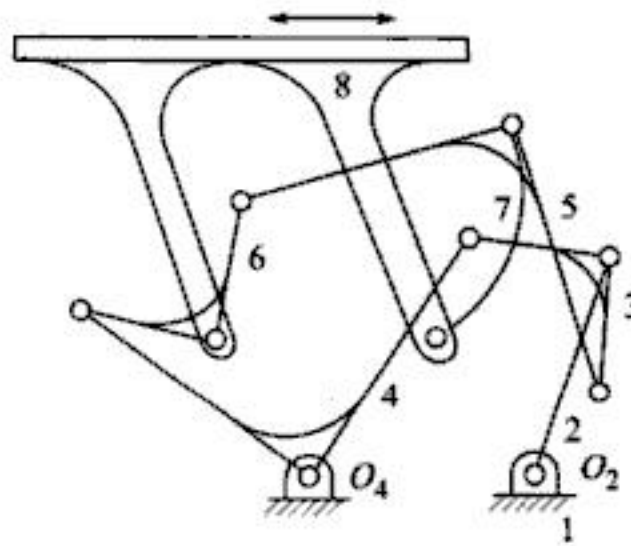
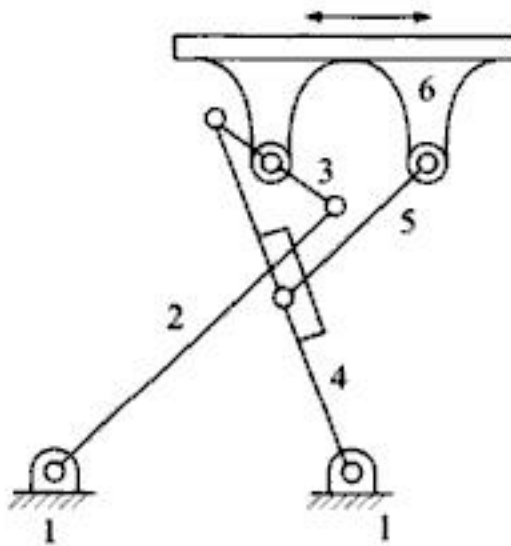
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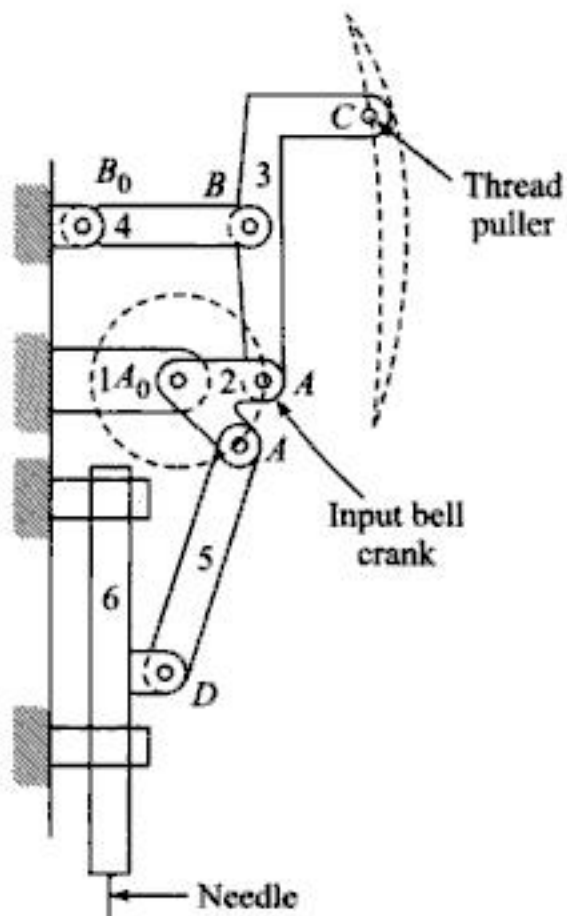
(e) A 4-bar mechanism used in lawn-sprinkler oscillator



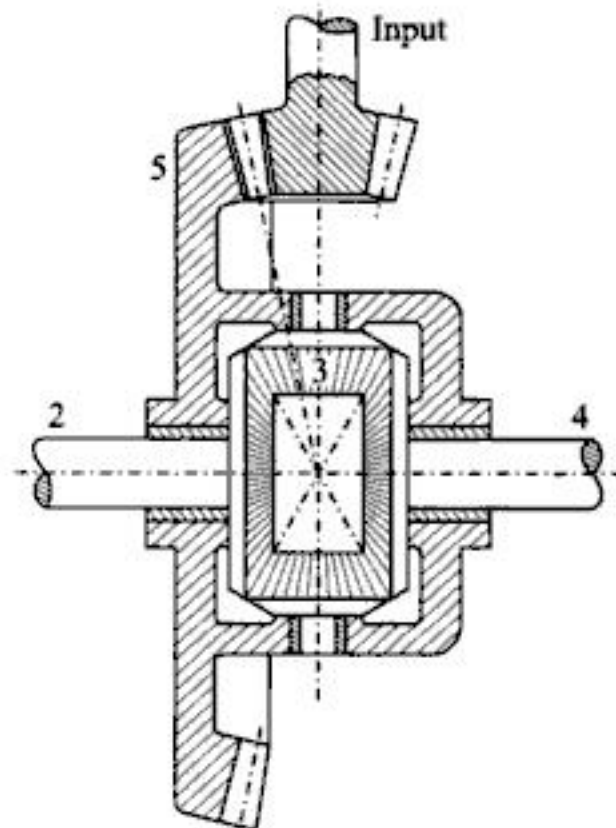
(f) A 4-bar linkage for auto-hood



(g) Straight line motion mechanisms on 6 links & 8 links

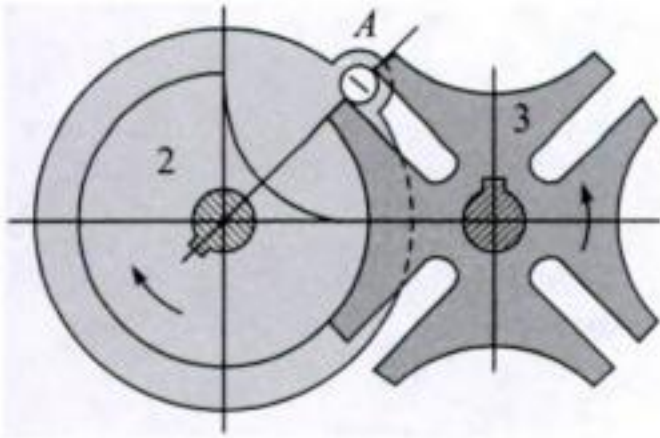


(h) A sewing machine mechanism

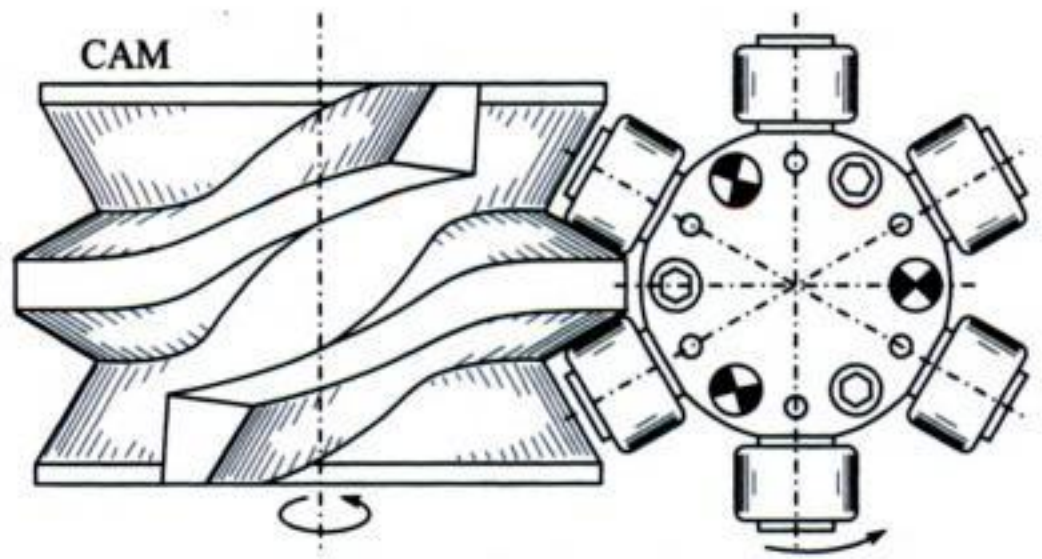


(i) A bevel gear differential mechanism for a 4-wheeler

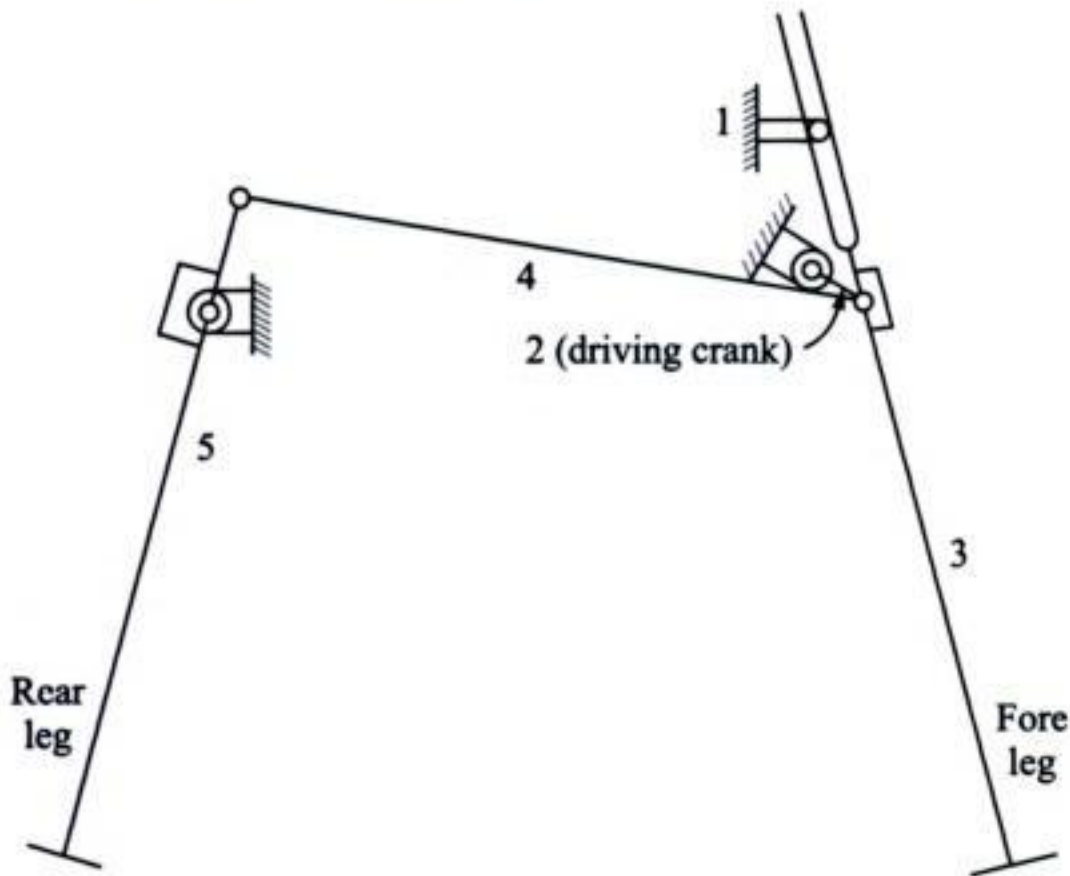
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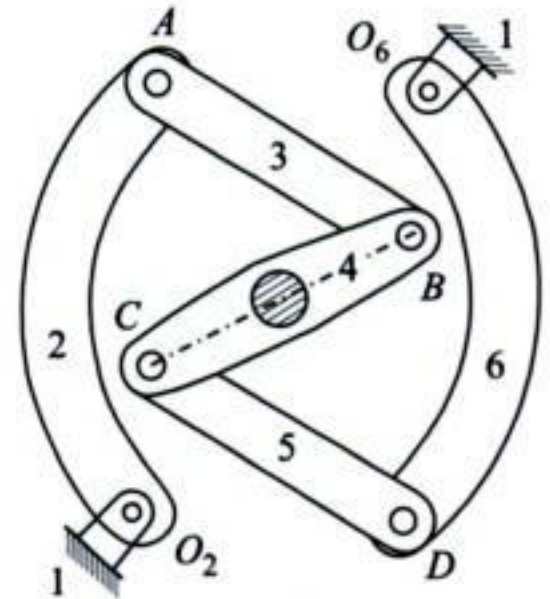
(j) Geneva wheel mechanism



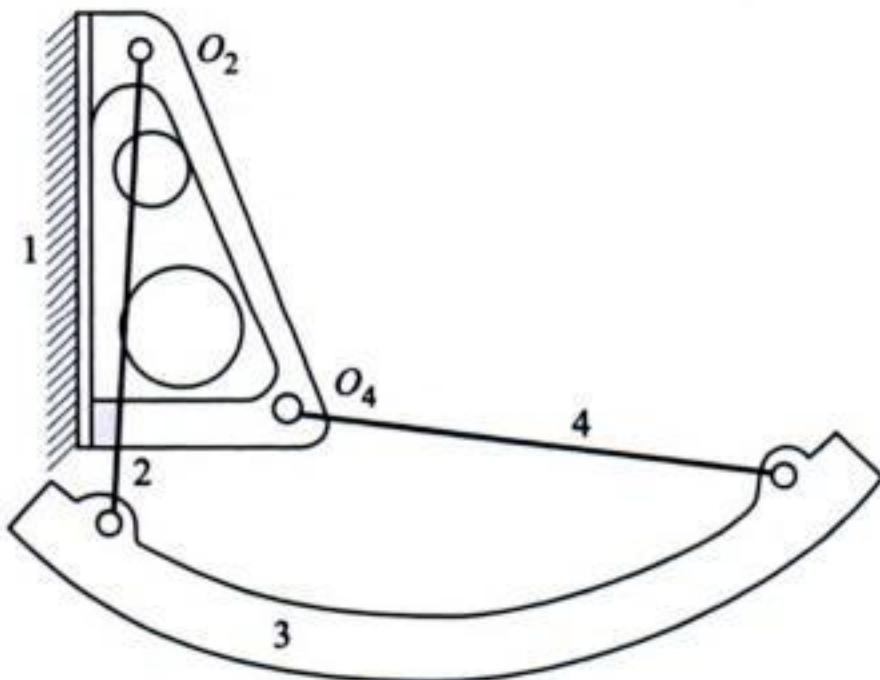
(k) Ferguson indexing drive



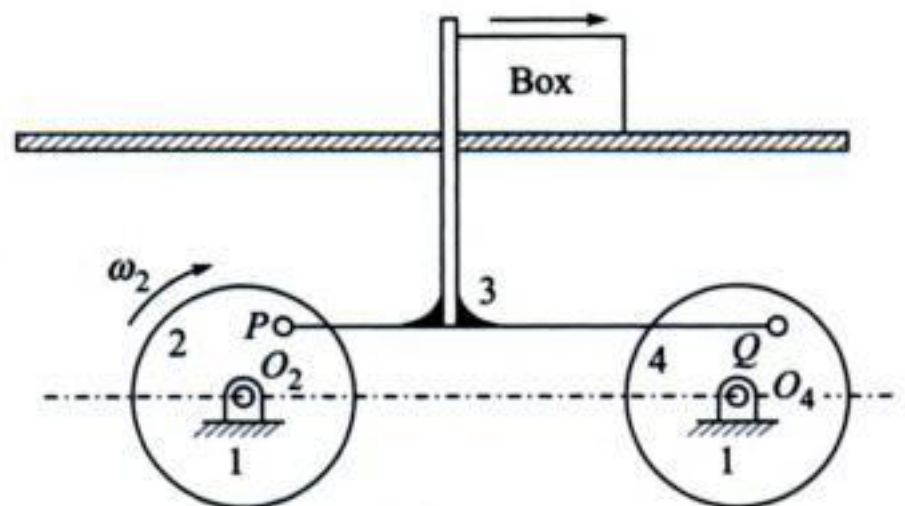
(l) Walking mechanism for a toy



(m) Drum brake mechanism

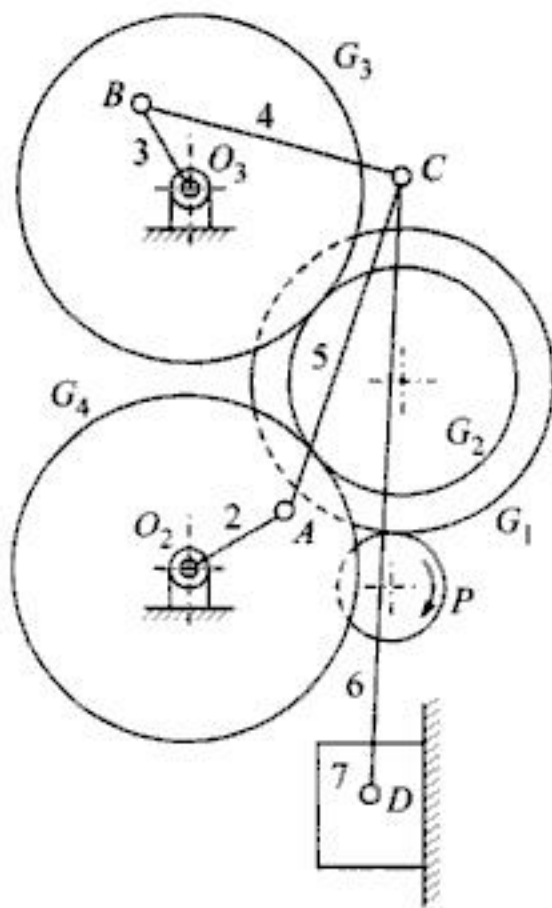


(n) An air-craft overhead bin mechanism

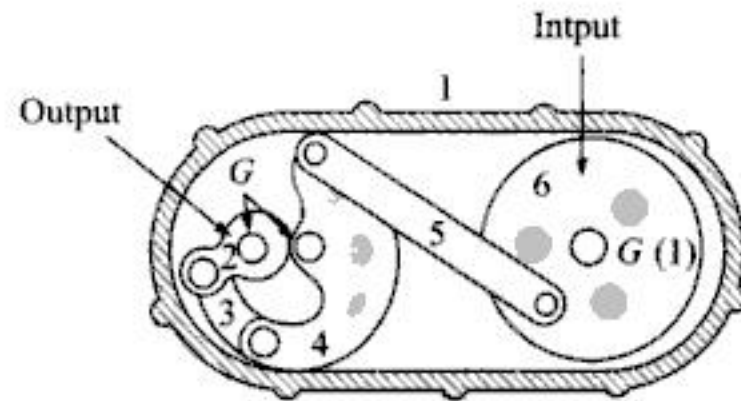


(o) Walking-beam mechanism

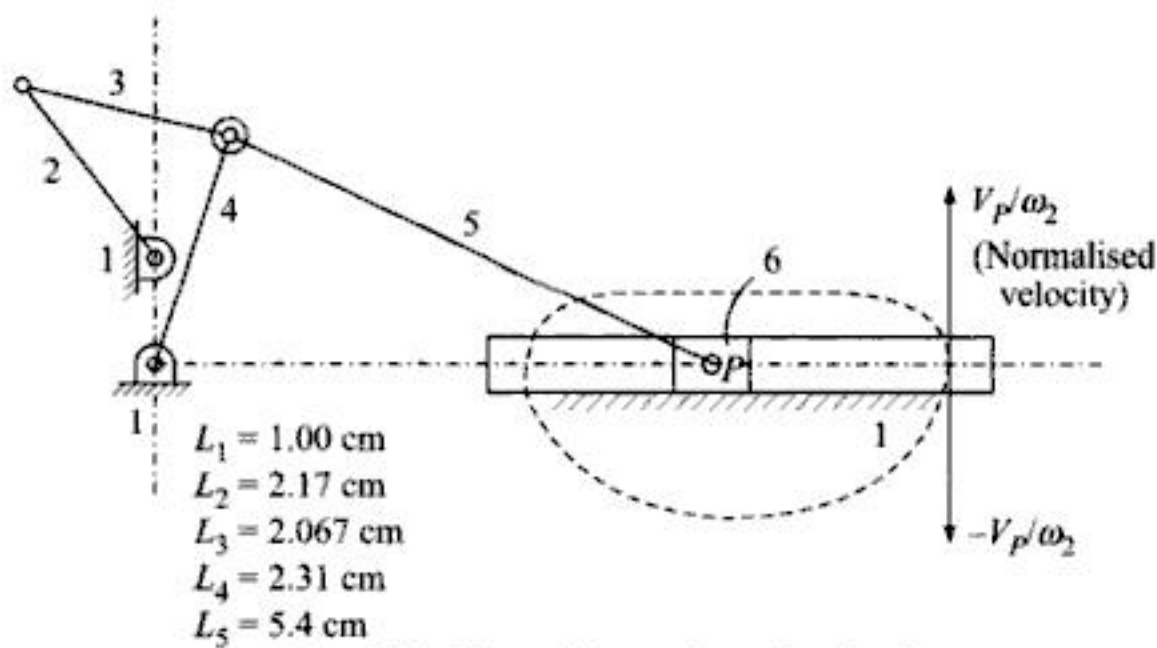
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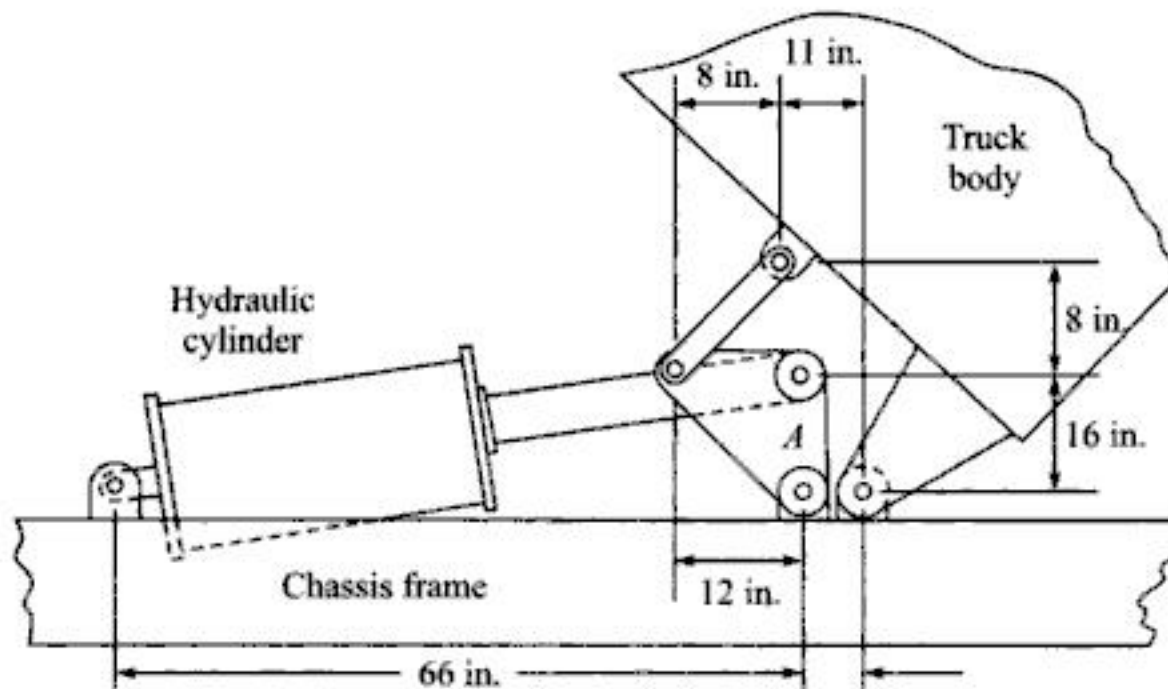
(p) Deep drawing press mechanism



(q) An agitator mechanism for a washing machine (G denotes the ground pivot)



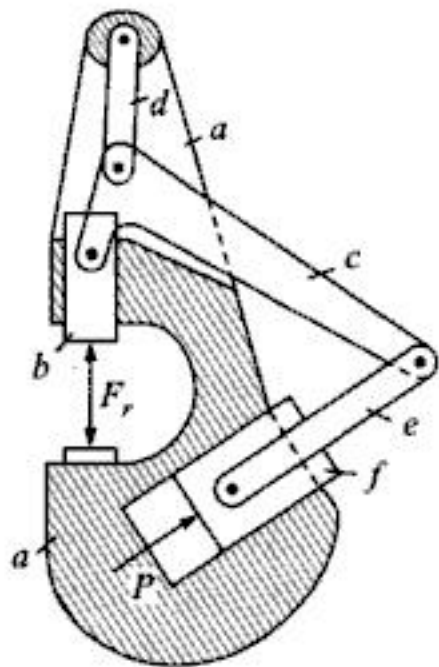
(r) Draglink driven slider crank mechanism for generating approximate constant velocity



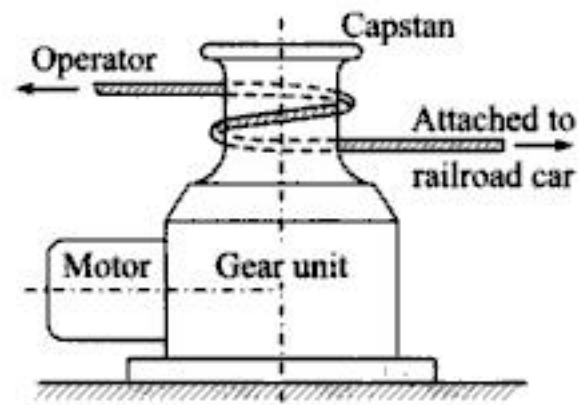
(s) A simplified mechanism for hydraulic dump truck

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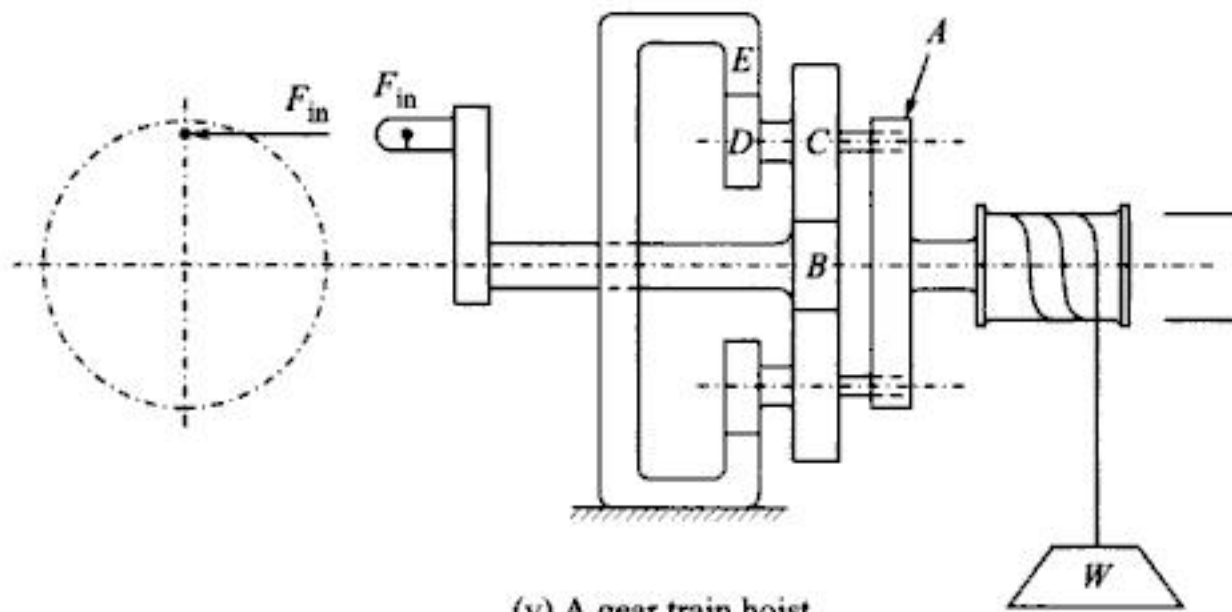




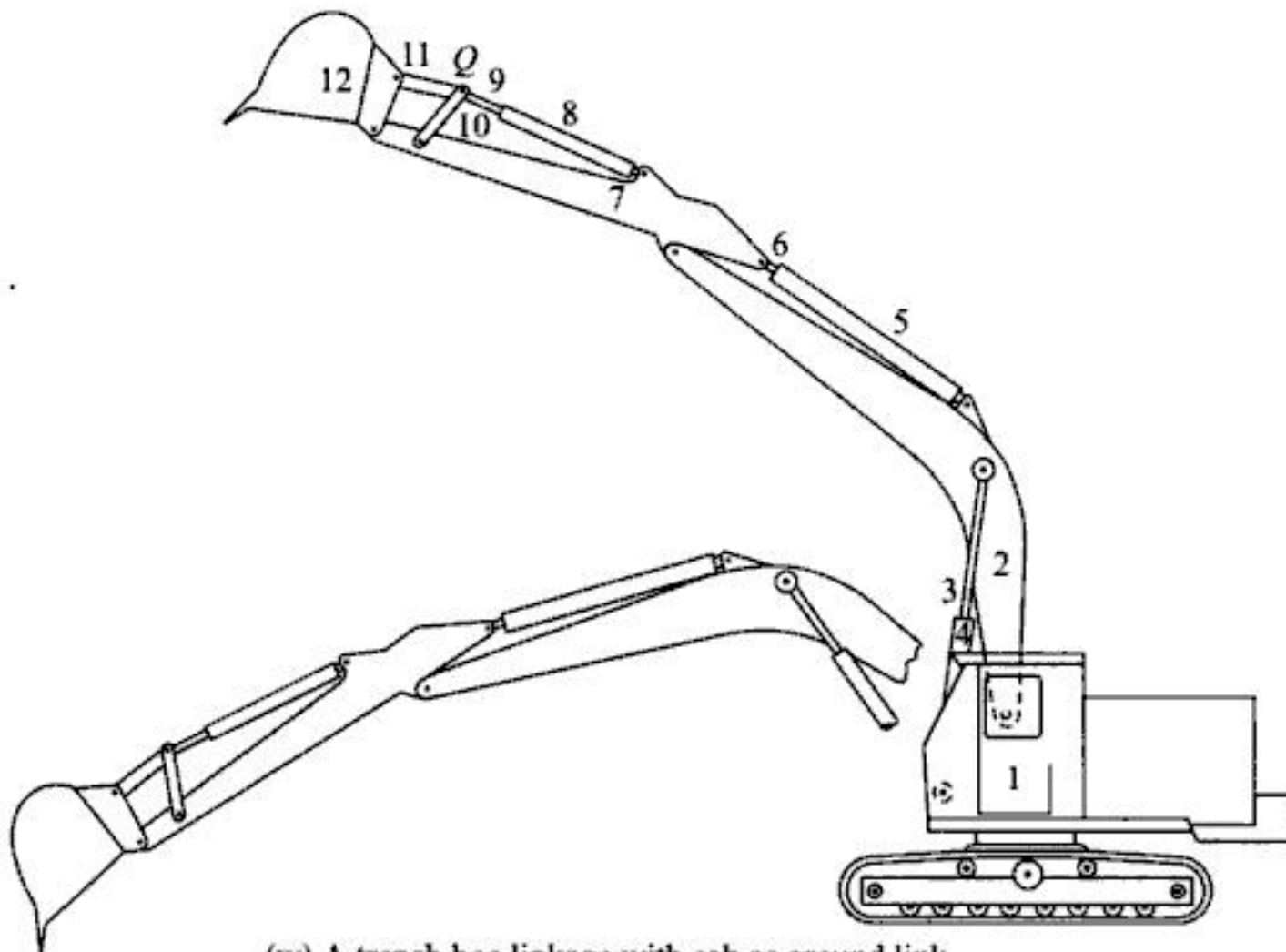
(t) Yoke riveter mechanism



(u) The car spotter which positions the railroad car for loading and unloading



(v) A gear train hoist



(w) A trench hoe linkage with cab as ground link

Fig. 1.17 A few applications of mechanisms.

**REVIEW QUESTIONS**

- 1.1 Distinguish clearly between the concepts of 'motion of a point' and 'motion of a rigid body.'
- 1.2 What do you understand by the term *Translation*. Giving suitable examples, distinguish between rectilinear and curvilinear motions of translation.
- 1.3 Find the sum of vectors  $\mathbf{A} = 4 \angle 45^\circ$  and  $\mathbf{B} = 3 \angle -30^\circ$  using graphical methods.  
(Ans.  $5.6 \angle 14^\circ$ )
- 1.4 Find the resultant of vectors  $\mathbf{A} = 10 \angle 60^\circ$ ,  $\mathbf{B} = 8 \angle 150^\circ$  and  $\mathbf{C} = 9 \angle 90^\circ$ , using graphical methods.  
(Ans.  $18.76 \angle -84.12^\circ$ )
- 1.5 Solve the equation  $P = A + B + C$  when,  $P$  has a slope of 2 in 5 in fourth quadrant,  $A$  has a slope of 4 in 1 in the third quadrant,  $B = 9 \angle 30^\circ$ , and  $C = 6 \angle -60^\circ$ .  
(Ans.  $A = (0.82) -j (3.29)$ ;  $P = - (9.97) -j (3.99)$ )

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# PLANAR MECHANISMS AND GEOMETRY OF MOTION

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## 2.1 INTRODUCTION

As the book is devoted to the study of mechanisms and machines, it is necessary to understand what these terms are. To a beginner, the term *machine* may be defined as a device/contrivance which receives energy in some available form and uses it to do certain particular kind of work. In a similar way, *mechanism* may be defined as a contrivance which transforms motion from one form to another.

## 2.2 DEFINITIONS AND BASIC CONCEPTS

**A Link.** A link may be defined as a single part (or an assembly of rigidly connected parts) of a machine, which is a resistant body having a motion relative to other parts of the machine.

Even if two or more connected parts are manufactured separately, they cannot be treated as different links unless there is a relative motion between them. For instance, piston, piston-rod and cross-head of a steam engine constitute a single link as there is no relative motion between them. From the point of view of a kinematician therefore, slider-crank mechanisms of a steam engine and I.C. engine are just the same.

A link need not necessarily be rigid but it must necessarily be resistant. It may be recalled that a *resistant body is one which transmits force with negligible deformation in the direction of force transmission*. Based on above considerations, a spring which has no effect on the kinematics of a device, and has significant deformation in the direction of applied force, is not treated as a link but only as a device to apply force. They are usually ignored during kinematic analysis, and their 'force-effects' are introduced during dynamic analysis.

There are machine members which possess one-way rigidity. For instance, because of their resistance to deformation under tensile load, belts, ropes and chains are treated as links only when they are in tension. Similarly, liquids on account of their incompressibility can be treated as links only when transmitting compressive force. Figure 2.1 shows a few representative mechanisms in which links have been numbered to bring home the above concept.

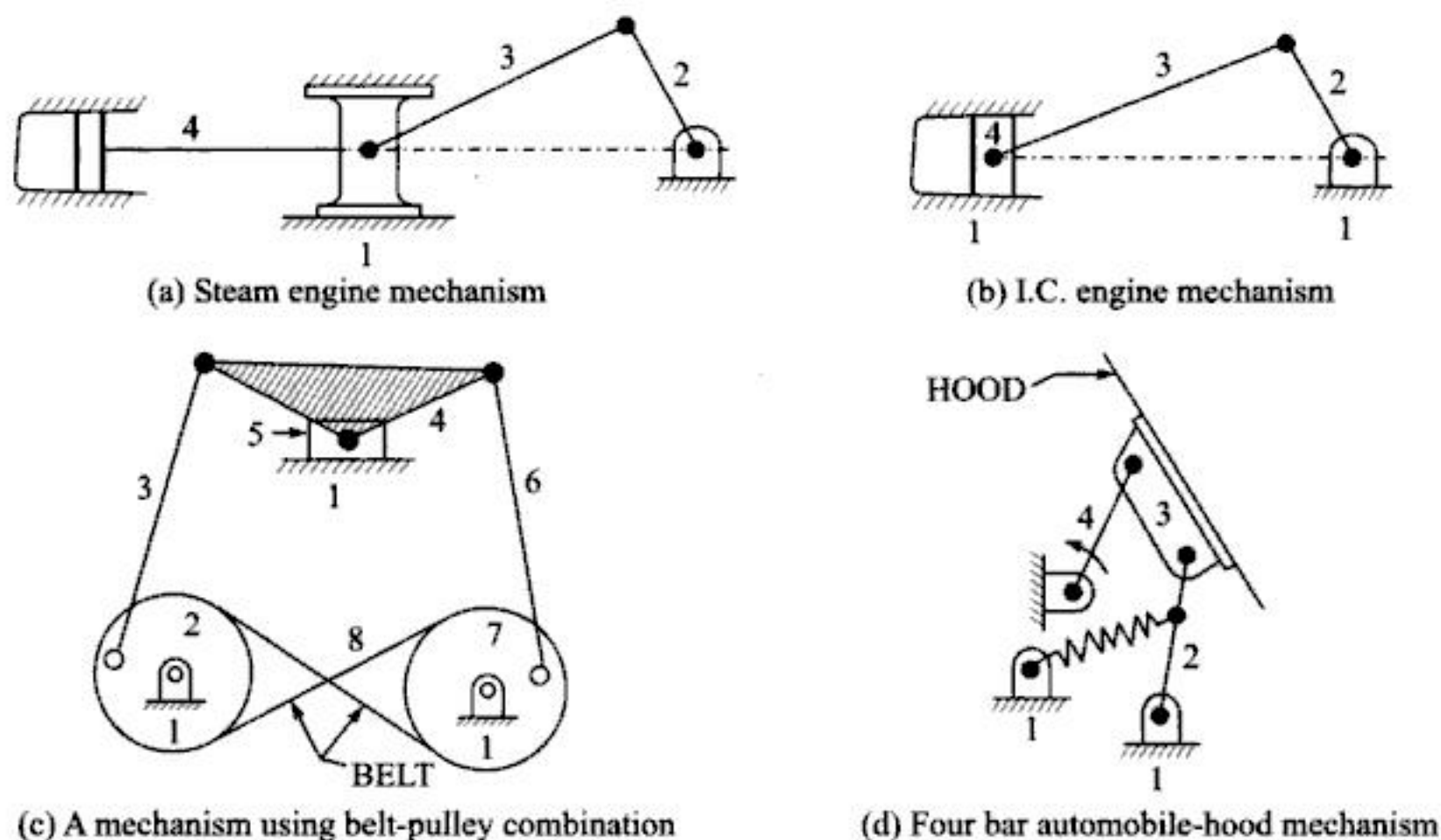


Fig. 2.1

**An Element.** The kind of relative motion between links of a mechanism is controlled by the form of the contacting surfaces of the adjacent (connected) links. These contacting surfaces may be thought of as 'working surfaces' of the connection between adjacent links. For instance, the connection between a lathe carriage and its bed is through 'working surfaces' (ways) which are so shaped that only motion of translation is possible. Similarly, the working surfaces of I.C. engine piston and connecting rod at piston pin are so shaped that relative motion of rotation alone is possible. Each of these working surface is also called an *element*.

An element may therefore be defined as a geometrical form provided on a link so as to ensure a working surface that permits desired relative motion between connected links.

**A Pair.** The two contacting elements of a connection constitute a kinematic pair. A pair may also be defined as a connection between two adjacent links that permits a definite relative motion between them. It may be noted that the above statement is generally true. In the case of multiple joint, however, more than two links can be connected at a kinematic pair (also known as joint). Cylindrical contacting surfaces between I.C. engine cylinder and piston constitute a pair. Similarly, cylindrical contacting surfaces of a rotating shaft and a journal bearing also constitute a pair.

When all the points in different links in a chain move in planes which are mutually parallel, the chain is said to have a planar motion. A motion other than planar motion is a spatial motion.

**Kinematic Diagram (Skeleton Diagram).** When the links are assumed to be rigid in kinematics, there can be no change in relative positions of any two arbitrarily chosen points on the same link. In particular, relative position(s) of pairing element(s) on the same link does not change. As a consequence of assumption of rigidity, many of the intricate details, shape and size

of the actual part (link) become unimportant in kinematic analysis. For this reason, it is customary to draw highly simplified schematic diagrams which contain only the important features in respect of the shape of each link (e.g., relative locations of pairing elements). This necessarily requires to completely suppress the information about real geometry of manufactured parts. Schematic diagrams of various links, showing relative locations of pairing elements, are shown in Fig. 2.5. Conventions followed in drawing kinematic diagram are also shown there.

In drawing a kinematic diagram, it is customary to draw the parts (links) in the most simplified form so that only those dimensions are considered which affect the relative motion. One such simplified kinematic diagram of slider-crank mechanism of an I.C. engine is shown in Fig. 2.2 in which connecting rod 3 and crank 2 are represented by lines joining their respective pairing elements. The piston has been represented by the slider 4 while cylinder (being a stationary member) has been represented by frame link 1.

It may be noted, however, that these schematics have a limitation in that they have little resemblance to the physical hardware. And, one should remember that kinematic diagrams are particularly useful in kinematic analysis and synthesis but they have very little significance in designing the machine components of such a mechanism.

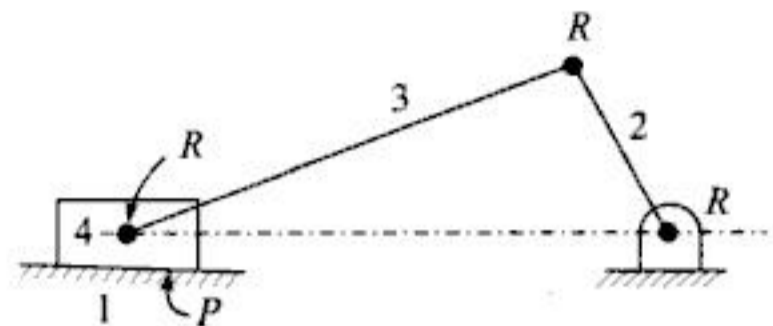


Fig. 2.2 Kinematic Diagram of an I.C. engine Mechanism.

**Kinematic Chain.** A kinematic chain can be defined as an assemblage of links which are interconnected through pairs, permitting relative motion between links. A chain is called a closed chain when links are so connected in sequence that first link is connected to the last, ensuring that all pairs are complete because of mated elements forming working surfaces at joints. As against this, when links are connected in a sequence, with first link not connected to the last (leaving incomplete pairs), the chain is called an open chain.

Examples of planar open loop chain are not many but they have many applications in the area of robotics and manipulators as space mechanisms. An example of a planar open-loop chain, which permits the use of a singular link (a link with only one element on it), is the common weighing scale shown in Fig. 2.3. Various links are numbered in the figure. Links 3, 1 and 4 are singular links.

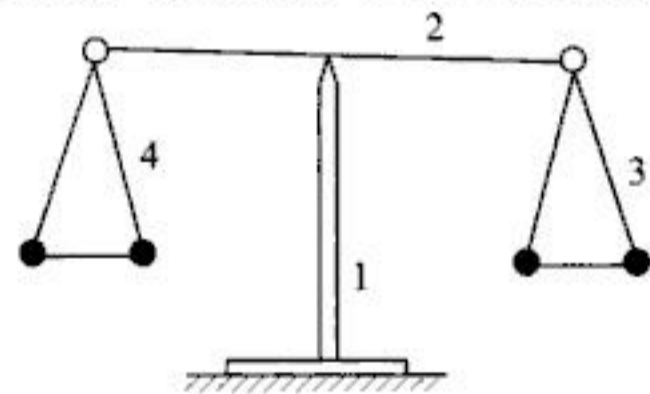


Fig. 2.3 Common weighing scale (An example of an open chain).

**Structure.** Geometric considerations sometimes prevent relative motion between links of a chain after closure. In such a situation the chain is called a structure. The structure can be statically determinate or indeterminate (*see* Fig. 2.4). It may be observed that a minimum of three links with three kinematic pairs are necessary to form a simple closed chain. However, if all the three pairs are such that no space-motion is possible, and one of the links is fixed, there cannot be any relative movement of links, such an arrangement is called 'structure'.

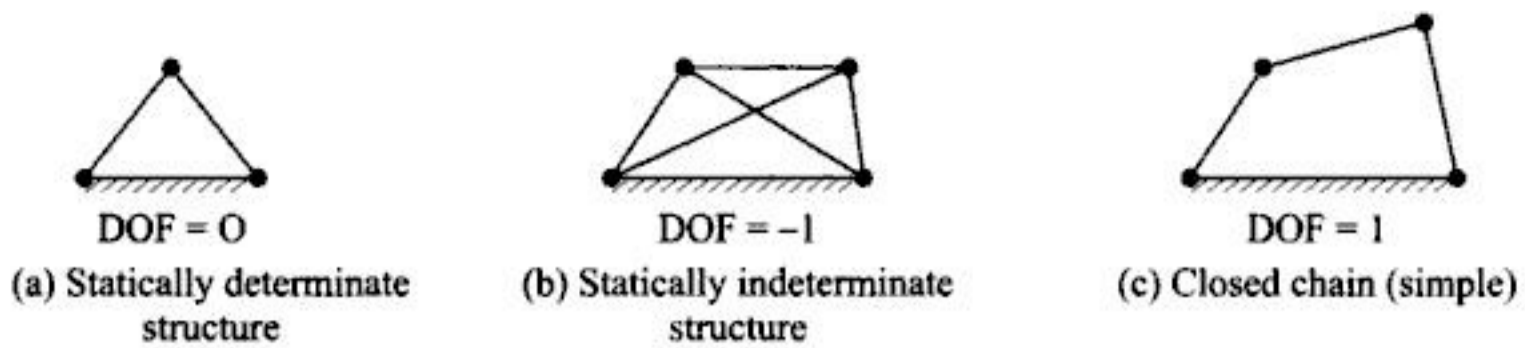


Fig. 2.4

**A Mechanism.** In the light of above discussions, a mechanism may now be defined as a movable closed kinematic chain with one of its links stationary (fixed).

### 2.3 CLASSIFICATION OF LINKS

A link can be called singular (unitary), binary, ternary, quaternary (etc.) link depending on the number of elements it has for pairing with other links. Thus a link carrying a single element is called a singular (unitary) link and a link with two elements is called a binary link. Similarly, a link having three elements is called a ternary link while a link having four elements is called a quaternary link. These links, along with their conventional representation, are shown in Fig. 2.5.

Type of Link	Typical Form	Schematic Representation
Single link (Typical shapes)		
Singular (Unitary) link		
Binary link		
Ternary		
Quaternary link		

Fig. 2.5 Conventional Representation of different types/shapes of Links in Kinematic Diagram.

## 2.4 CLASSIFICATION OF PAIRS

Kinematic pairs are classified on the basis of any of the following characteristics:

- (a) Type of relative motion between contacting elements.
- (b) Type of contact between contacting elements.
- (c) Number of degrees of freedom.
- (d) Type of closure (*i.e.* whether self-closed or force-closed).

### 2.4.1 Classification of Pairs Based on Type of Relative Motion

The relative motion of a point on one element relative to the other on mating element can be that of turning, sliding, screw (helical direction), planar, cylindrical or spherical. The controlling factor that determines the relative motions allowed by a given joint is the shapes of the mating surfaces or elements. Each type of joint has its own characteristic shapes for the elements, and each permits a particular type of motion, which is determined by the possible ways in which these elemental surfaces can move with respect to each other. The shapes of mating elemental surfaces restrict the totally arbitrary motion of two unconnected links to some prescribed type of relative motion.

- (i) *Turning Pair.* (Also called a hinge, a pin joint or a revolute pair). This is the most common type of kinematic pair and is designated by the letter  $R$ .  
A pin joint has cylindrical element surfaces and assuming that the links cannot slide axially, these surfaces permit relative motion of rotation only. A pin joint allows the two connected links to experience relative rotation about the pin centre. Thus, the pair permits only one degree of freedom. Kinematic pairs, marked  $R$  in Fig. 2.2, represent turning or revolute pairs. Thus, the pair at piston pin, the pair at crank pin and the pair formed by rotating crank-shaft in bearing are all examples of turning pairs.
- (ii) *Sliding or Prismatic Pair.* This is also a common type of pair and is designated as  $P$ . This type of pair permits relative motion of sliding only in one direction (along a line) and as such has only one degree of freedom. Pairs between piston and cylinder, cross-head and guides, die-block and slot of slotted lever are all examples of sliding pairs.
- (iii) *Screw Pair.* This pair permits a relative motion between coincident points, on mating elements, along a helix curve. Both axial sliding and rotational motions are involved. But as the sliding and rotational motions are related through helix angle  $\alpha$ , the pair has only one degree of freedom. The pair is commonly designated by the letter  $S$ . Examples of such pairs are to be found in translatory screws operating against rotating nuts to transmit large forces at comparatively low speed, *e.g.* in screw-jacks, screw-presses, valves and pressing screw of rolling mills. Other examples are rotating lead screws operating in nuts to transmit motion accurately as in lathes, machine tools, measuring instruments, etc.
- (iv) *Cylindrical Pair.* A cylindrical pair permits a relative motion which is a combination of rotation  $\theta$  and translation  $s$  parallel to the axis of rotation between the contacting elements. The pair has thus a degree of freedom of two and is designated by a letter  $C$ . A shaft free to rotate in a bearing and also free to slide axially inside the bearing provides example of a cylindrical pair.

- (v) *Globular or Spherical Pair.* Designated by the letter  $G$ , the pair permits relative motion such that coincident points on working surfaces of elements move along spherical surface. In other words, for a given position of spherical pair, the joint permits relative rotation about three mutually perpendicular axes. It has thus three degrees of freedom. A ball and socket joint (*e.g.*, the shoulder joint at arm-pit of a human being) is the best example of spherical pair.
  - (vi) *Flat pair (Planar Pair).* A flat or planar pair is seldom, if ever, found in mechanisms. The pair permits a planar relative motion between contacting elements. This relative motion can be described in terms of two translatory motions in  $x$  and  $y$  directions and a rotation  $\theta$  about third direction  $z$ ,  $-x, y, z$  being mutually perpendicular directions. The pair is designated as  $F$  and has a degree of freedom of 3.
- All the above six types of pairs, illustrated in Fig. 2.6, are representative of a particular class.

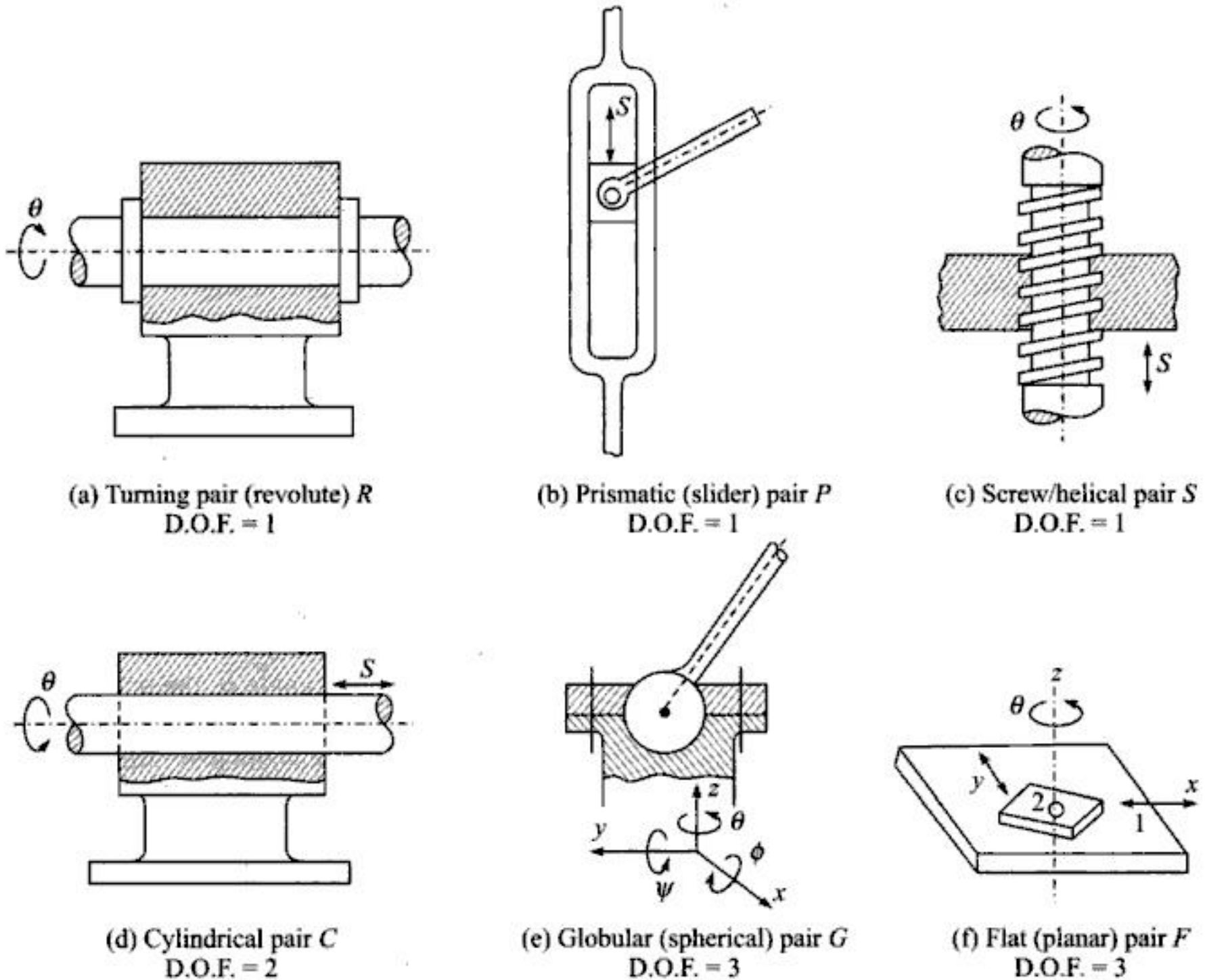


Fig. 2.6



- (vii) *Rolling Pair.* When surfaces of mating elements have a relative motion of rolling, the pair is called a rolling pair. Castor wheel of trolleys, ball and roller bearings, wheels of locomotive/wagon and rail are a few examples of this type.

### 2.4.2 Classification of Pairs Based on Type of Contact

This is the best known classification of kinematic pairs on the basis of nature of contact and was suggested by the great kinematician Reuleaux. He classified kinematic pairs in two categories:

- (i) *Lower Pair.* Kinematic pairs in which there is a surface (area) contact between the contacting elements are called lower pairs. All revolute pairs, sliding pairs, screw pairs, globular pairs, cylindrical pairs and flat pairs fall in this category.
- (ii) *Higher Pair.* Kinematic pairs in which there is a point or line contact between the contacting elements are called higher pairs. Meshing gear-teeth, cam follower pair, wheel rolling on a surface, ball and roller bearings and pawl and ratchet are a few examples of higher pairs.

Since lower pairs involve surface contact rather than line or point contact, it follows that lower pairs can be more heavily loaded for the same unit pressure. They are considerably more wear-resistant. For this reason, development in kinematics have involved more and more number of lower pairs. As against this, use of higher pairs implies lesser friction.

According to Hartenberg R.S. and Denavit J., the real concept of lower pairs lies in the particular kind of relative motion permitted by the connected links. For instance, let us assume that two mating elements  $P$  and  $Q$  form a kinematic pair. If the path traced by any point on the element  $P$ , relative to element  $Q$ , is identical to the path traced by a corresponding (coincident) point in the element  $Q$  relative to element  $P$ , then the two elements  $P$  and  $Q$  are said to form a lower pair. Elements not satisfying the above condition obviously form the higher pairs.

Since a turning pair involves relative motion of rotation about pin-axis, coincident points on the two contacting elements will have circular areas of same radius as their path. Similarly elements of sliding pair will have straight lines as the path for coincident points. In the case of screw pair, the coincident points on mating elements will have relative motion along helices. As against this a point on periphery of a disk rolling along a straight line generates cycloidal path, but the coincident point on straight line generates involute path when the straight line rolls over the disk. The two paths are thus different and the pair is a higher pair. *As a direct sequel to the above consideration, unlike a lower pair, a higher pair cannot be inverted. That is, the two elements of the pair cannot be interchanged with each other without affecting the overall motion of the mechanism.*

Lower pairs are further subdivided into linear motion and surface motion pairs. The distinction between these two sub-categories is based on the number of degrees of freedom of the pair. Linear motion lower pairs are those having one degree of freedom, *i.e.* each point on one element of the pair can move only along a single line or curve relative to the other element. This category includes turning pairs, prismatic pairs and screw pairs.

Surface-motion lower pairs have two or more degrees of freedom. This category includes cylindrical pair, spherical pair and the planar (Flat) pair.

### 2.4.3 Classification of Pairs Based on Degrees of Freedom

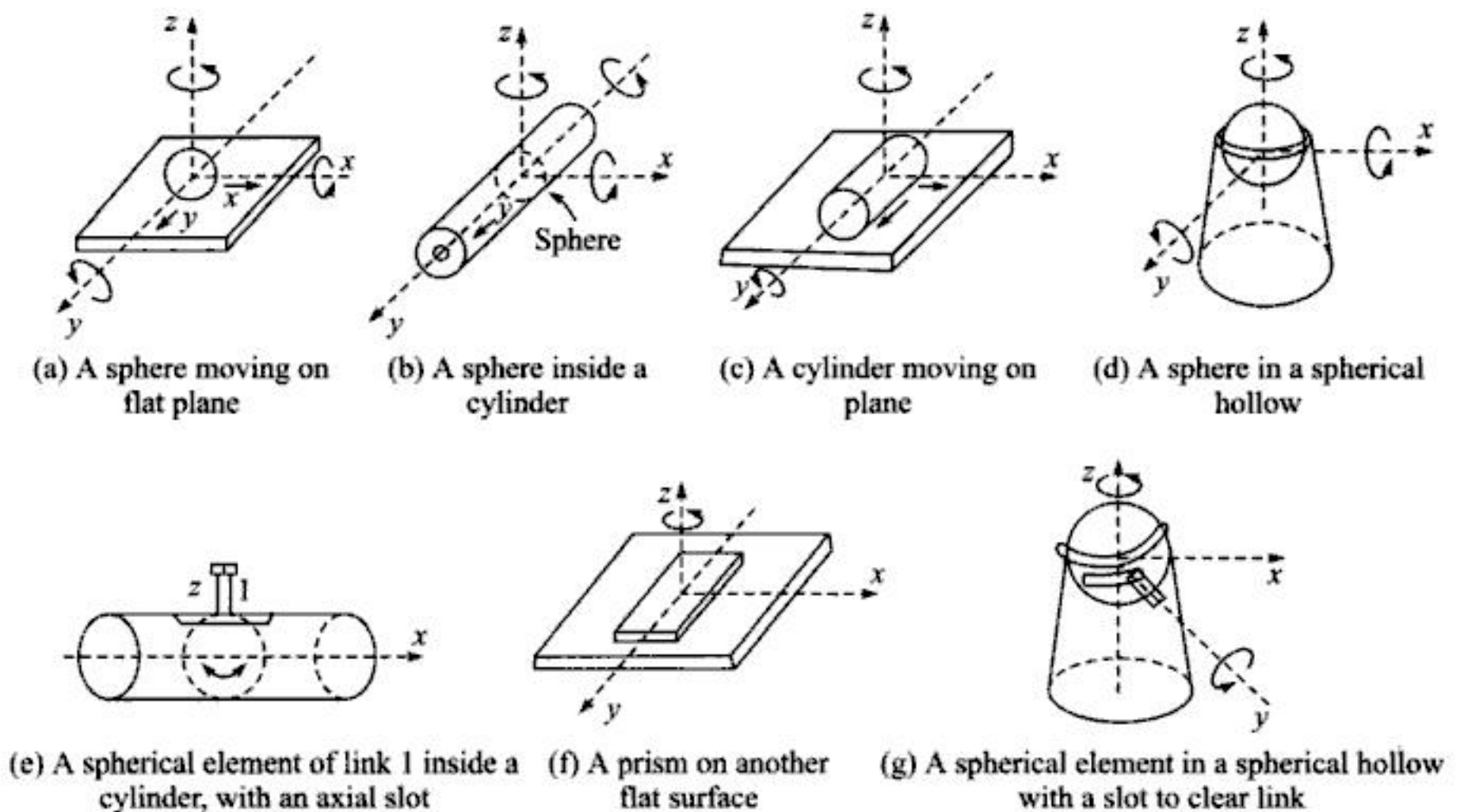
A free body in space has six degrees of freedom. In forming a kinematic pair, one or more degrees of freedom are lost. The remaining degrees of freedom of the pair can then be used to classify pairs. Thus,

$$\text{d.o.f. of a pair} = 6 - (\text{Number of restraints}).$$

A kinematic pair can therefore be classified on the basis of number of restraints imposed on the relative motion of connected links. This is done in Table 2.1 for different forms of pairing element shown in Fig. 2.7.

Table 2.1

S.No. in Fig. 2.7	Geometrical shapes of elements in contact	Number of Restraints on Translatory motion	Rotary motion	Total Number of Restraints	Class of pair
(a)	Sphere and plane	1	0	1	I
(b)	Sphere inside a cylinder	2	0	2	II
(c)	Cylinder on plane	1	1	2	II
(d)	Sphere in spherical socket	3	0	3	III
(e)	Sphere in slotted cylinder	2	1	3	III
(f)	Prism on a plane	1	2	3	III
(g)	Spherical ball in slotted socket	3	1	4	IV
(h)	Cylinder in cylindrical hollow	2	2	4	IV
(i)	Collared cylinder in hollow cylinder	3	2	5	V
(j)	Prism in prismatic hollow	2	3	5	V



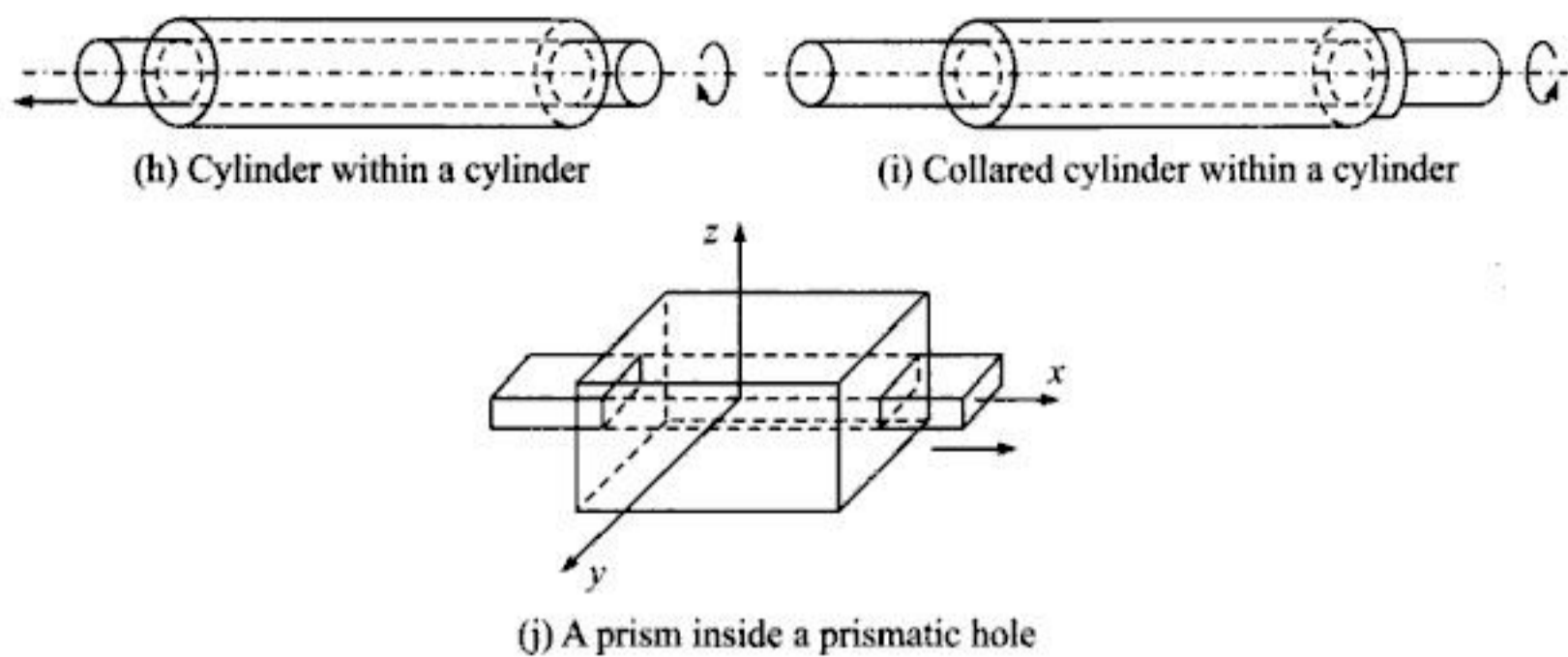


Fig. 2.7

#### 2.4.4 Classification of Pairs Based on Type of Closure

Another important way of classifying pairs is to group them as (a) closed kinematic pairs and (b) open kinematic pairs.

In closed pairs, one element completely surrounds the other so that it is held in place in all possible positions. Restraint is achieved only by the form of pair and, therefore, the pair is called closed or self-closed pair. The other possible definition of closed pair is: "closed pairs are those pairs in which elements are held together mechanically." All the lower pairs and a few higher pairs fall in the category of closed pairs.

As against this, open kinematic pairs maintain relative positions only when there is some external means to prevent separation of contacting elements. Open pairs are also sometimes called as 'unclosed pairs'. A cam and roller-follower mechanism, held in contact due to spring force, is an example of this type.

### 2.5 MECHANISM AND MACHINE

A mechanism can be defined as a combination of resistant bodies, so shaped and connected that as they move upon each other, a definite relative motion results. A beginner usually finds it difficult to conceive the term *mechanism* as distinct from a machine. Basically, a *mechanism* is a device to transform one motion into another. If the device transmits substantial forces as well, it can be called a machine. It follows, therefore, that all machines are mechanisms in spirit.

	<i>A Machine</i>	<i>A Mechanism</i>
(i)	It is an assemblage of links having relative motion and is capable of modifying available energy in a suitable way.	It is an assemblage of links used to transmit and modify motion.
(ii)	A machine may consist of one or more mechanisms for accomplishing the above task.	It represents skeleton outline of a machine. Only those link-dimensions are considered which are important from the point of view of motion transmission.
(iii)	All machines are mechanisms in spirit.	If the device transmits substantial forces as well, it can be called a machine.

**Linkage.** Many kinematicians of repute prefer to reserve the term *linkage* to describe mechanisms consisting of lower pairs only. But on a number of occasions this term has been used rather loosely synonymous to the term mechanism.

## 2.6 INVERSIONS

A mechanism is formed by fixing one of the links of a chain. Clearly, when different links of the same chain are chosen to become frame-link, different mechanisms will result. The process of choosing different links of a kinematic chain for becoming frame is known as kinematic inversion.

### Properties of Inversion

1. Number of inversions possible for a kinematic chain equals the number of links in the parent kinematic chain.
2. Relative motion (displacement, velocity and acceleration) between any two links does not change with inversion. This is simply because relative motion between different links is a property of parent kinematic chain.
3. Absolute motion of points on various links (measured with respect to the frame-link) may, however, change drastically from one inversion to the other, even in direct inversion.

As explained in Section 2.4 *unlike lower paired mechanisms, a higher paired mechanism cannot be inverted. This is simply because the two elements of a higher pair cannot be interchanged with each other without affecting overall motion of the mechanism.*

### Importance of Inversion

Important aspects of the concept of inversion can be summarised as under:

1. The concept of inversion enables us to categorise a group of mechanisms arising out of inversions of a parent kinematic chain as a family of mechanisms. Members of this family have a common characteristic in respect of relative motion.
2. In case of direct inversions, as relative velocity and relative acceleration between two links remain the same, it follows that complex problems of velocity/acceleration analysis may often be simplified by considering a kinematically simpler direct inversion of the original mechanism. Such procedure is the basis of Goodman's ingenious method of indirect acceleration analysis. The concept is also useful in converting motion analysis problem of an epi-cyclic gear-train to that of a simpler gear train by fixing arm and freeing the fixed member.
3. In many cases of inversions, by changing proportions of lengths of links, desirable features of the inversion may be accentuated and many useful mechanisms may be developed.

## 2.7 QUADRIC CYCLE CHAIN AND ITS INVERSIONS

A kinematic chain consisting of turning pairs only (*i.e.*, a linkage) must have a minimum of four links and four pairs. It is customary to call a fully rotating link a 'crank', an oscillating link a

'Rocker/lever' and the connecting link a 'coupler' or 'connecting rod'. The coupler is the link which is not connected to the frame.

An important consideration when designing a mechanism is to see whether an electric motor is a driving member. In such a case, naturally there must be a crank member in the mechanism to receive power from motor. For a four-bar linkage, Grashof's law provides a very simple test to check whether any of the links in the chain can be a crank.

**Grashof's Law.** *Grashof's law states that for a planar four-bar linkage, sum of the shortest and longest link-lengths must be less than or equal to the sum of the remaining two link-lengths, if there is to be a continuous relative rotation between two members. Thus, if  $s$  and  $l$  be the lengths of shortest and longest links respectively and  $p$  and  $q$  be the remaining two link-lengths, then one of the links, in particular the shortest link, will rotate continuously relative to the other three links, if and only if,*

$$s + l \leq p + q \quad (2.1)$$

If this inequality is not satisfied, the chain is called non-Grashof chain in which none of the links can have complete revolution relative to other links.

It is important to note that the Grashof's law does not specify the order in which the links are to be connected. Thus any of the links having length  $l$ ,  $p$  and  $q$  can be the link opposite to the link of length  $s$ . A chain satisfying Grashof's law generates three distinct inversions only. A *non-Grashof chain, on the other hand, generates only one distinct inversion, namely the "Rocker-Rocker mechanism."*

### Inversions of a Grashof's Chain

1. *Drag link or Double-crank Mechanism.* When the shortest link of a Grashof-chain is fixed, it gives rise to a drag-link or double-crank mechanism in which both the links connected to the frame rotate continuously. [Fig. 2.8(a)]
2. *Double Rocker/Lever Mechanism.* When the link opposite to the shortest link is fixed, a double rocker/lever mechanism results. None of the two links (driver and driven) connected to the frame can have complete revolution but the coupler link can have full revolution. [Fig. 2.8(b)]
3. *Crank Rocker/Lever Mechanism.* When any of the two remaining links (adjacent to the shortest link) is fixed, a crank-lever/rocker mechanism results and one of the two links (driver or driven) directly connected to the frame, is capable of having full revolution. [Fig. 2.8(c)]

In Fig. 2.8(b)  $OP_1 Q_1 C$  and  $OP_2 Q_2 C$  represent extreme positions of the double-rocker mechanism. Similarly in Fig. 2.8(c) it is seen that as crank  $OP$  continues to rotate c.c.w. from position  $OP_1$  onwards, the lever  $CQ_1$  reverses its direction of motion and at crank position  $OP_2$  takes back the position  $CQ_2$ .

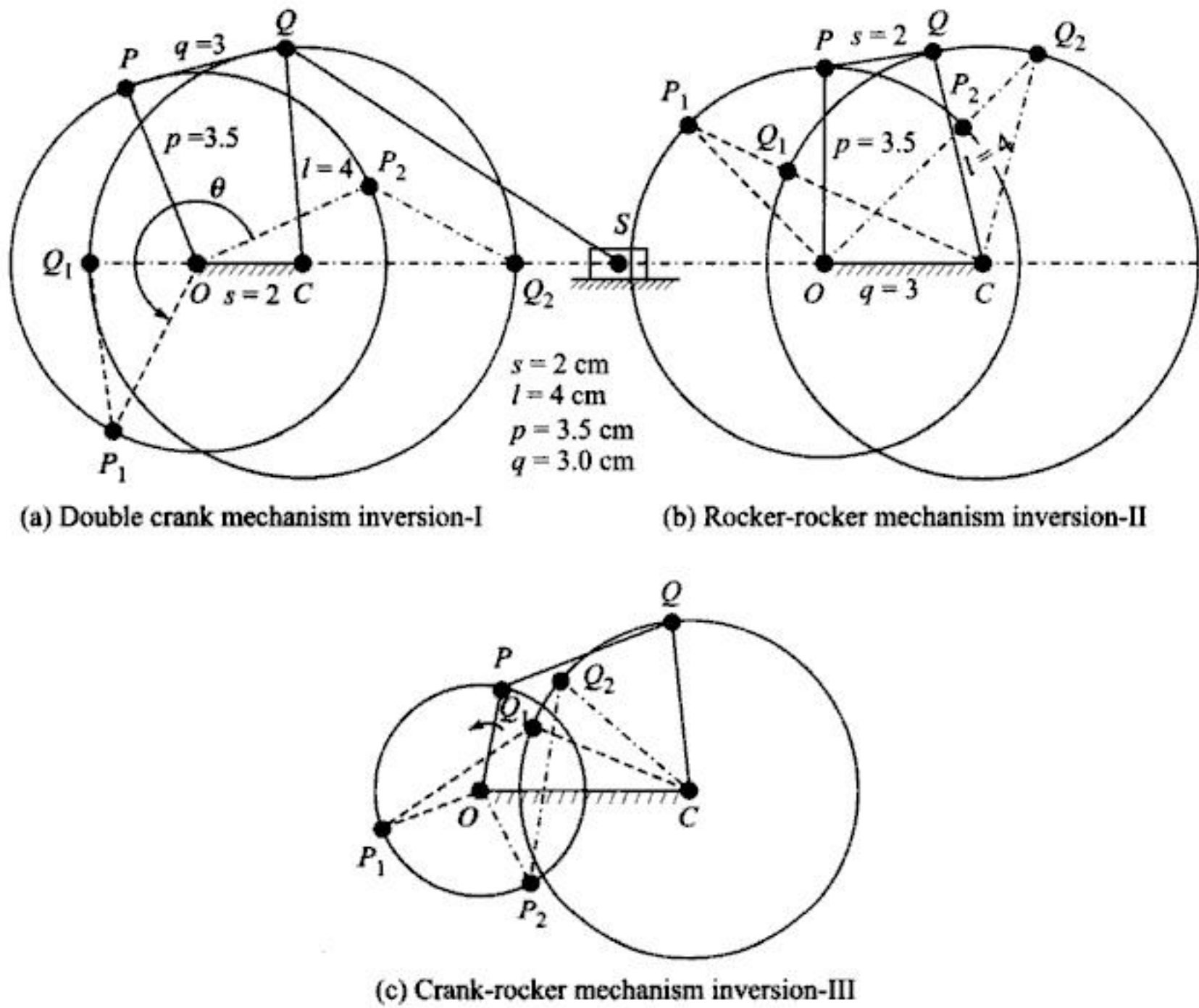


Fig. 2.8 Inversions of Quadric cycle chain.

**EXAMPLE 2.1** Figure 2.9 shows a planar mechanism with link-lengths given in some unit. If slider A is the driver, will link CG revolve or oscillate? Justify your answer. (A.M.I.E. Summer 1993)

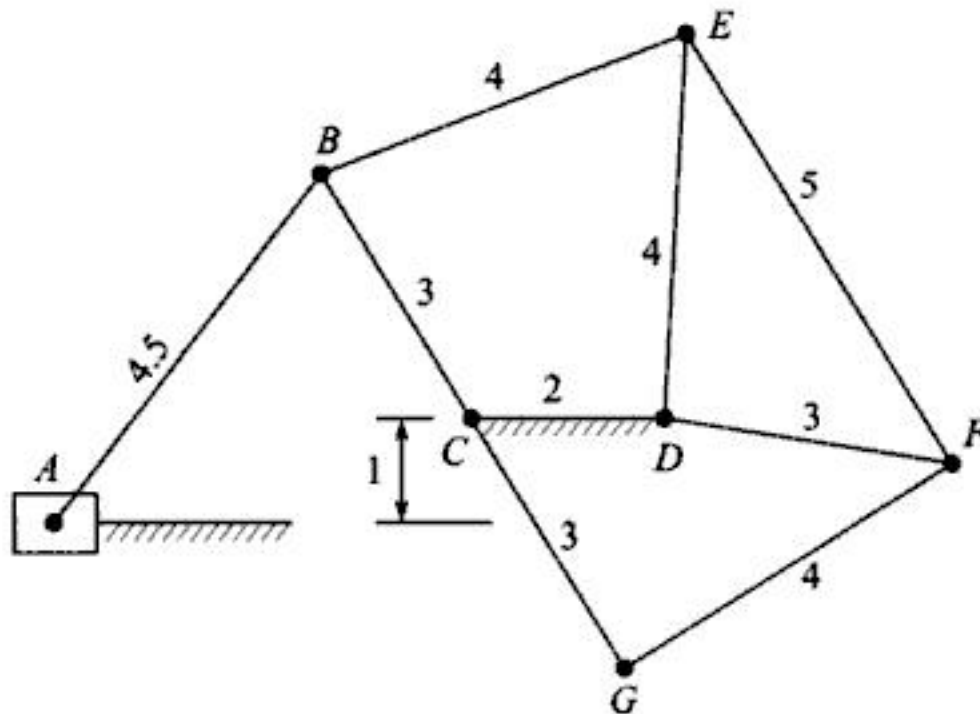


Fig. 2.9

**Solution:** The loop formed by three links  $DE$ ,  $EF$  and  $FD$  represents a structure. Thus the loop can be taken to represent a ternary link.

Now in the 4-link loop  $CDEB$ ,  $s = 2$ ;  $l = 4$ ; and  $p + q = 7$ . Thus the 4-link loop portion  $CDEB$  satisfies Grashof's criterion. And as the shortest link  $CD$  is fixed, link  $CB$  is capable of complete revolution. Also, 4-link loop  $CDFG$  satisfies Grashof's criterion ( $l + s = p + q$ ) and the shortest link  $CD$  is fixed. Thus whether considered a part of 4-link loop  $CDEB$  or that of  $CDFG$ , link  $BCG$  is capable of full revolution. **Ans.**

**EXAMPLE 2.2** In a 4-bar mechanism, the lengths of driver crank, coupler and follower link are 150 mm, 250 mm and 300 mm respectively. The fixed link-length is  $L_o$ . Find the range of values for  $L_o$ , so as to make it a –

- (i) Crank-rocker mechanism (ii) Crank-crank mechanism. (SGSITS: 1999)

**Solution:** (A) For a crank-rocker mechanism the conditions to be satisfied are:

- (a) Link adjacent to fixed link must be the smallest link and, (b)  $s + l \leq p + q$

We have to consider both the possibilities, namely, (i) when  $L_o$  is the longest link and (ii) when  $L_o$  is not the longest link.

When  $L_o$  is the longest link, from Grashof's criterion,

$$L_o + 150 \leq 250 + 300$$

or

$$L_o \leq 400 \text{ mm}$$

When  $L_o$  is not the longest link, from Grashof's criterion,

$$300 + 150 \leq L_o + 250$$

or

$$L_o \geq 200$$

Thus, for crank-rocker mechanism, range of values for  $L_o$  is

$$200 \leq L_o \leq 400 \text{ mm}$$

**Ans.**

(B) For crank-crank mechanism, the conditions to be satisfied are

(a) shortest link must be the frame link, and

- (b)  $s + l \leq p + q$ .

Thus,  $L_o + 300 \leq 150 + 250$

or

$$L_o \leq 100 \text{ mm}$$

**Ans.**

### Applications:

**First Inversion.** (Drag Link Quick Return Mechanism). For understanding the application of drag link mechanism, consider Fig. 2.8(a) which shows its extension in the form of a Drag Link Quick Return Mechanism. For this purpose, let the reciprocating ram  $S$  (representing cutting tool-post) be connected through a coupler  $QC$  to the quadric cycle chain through turning pairs at  $Q$  and  $S$ .

For extreme left hand position of ram  $S$ , the driven link will be located in position  $CQ_1$  and the driving link  $OP$  in position  $OP_1$ . As the crank  $OP$  rotates c.c.w., the ram moves to the right and as the driven link takes up position  $CQ_2$ , the ram completes its forward stroke taking up

extreme right hand position. For this position, the driver link is located in position  $OP_2$ . Thus for completing forward stroke of ram, the crank  $OP$  has to rotate through say  $136^\circ$  (which is less than  $180^\circ$ ). For completing return stroke, clearly the driver crank will have to rotate from  $OP_2$  to  $OP_1$  c.c.w. through an angle say  $224^\circ$  (which is larger than  $180^\circ$ ). It must be noted that crank rotation for completing forward and return stroke of ram together, cannot take more than  $360^\circ$ .

Thus it is seen that as crank  $OP$  (which is driven by a constant speed drive) requires smaller angle of rotation for moving slider from left to right, and the stroke will require smaller time, and hence may be used as a return (idle) stroke. The other stroke, requiring larger time, may be reserved for cutting stroke in a shaper machine tool. This arrangement results in a better utilization of available time for cutting operation.

Angle  $\theta_C$ , the larger of the two angles, corresponds to cutting/working stroke while the smaller one  $\theta_R$  corresponds to the return/idle stroke. Since the crank rotates uniformly, the time of cutting stroke bears a ratio to time of idle stroke, given by

$$\frac{\theta_C}{\theta_R} = \frac{\text{Time of cutting stroke}}{\text{Time of Return stroke}} \quad (2.2)$$

**Second Inversion.** There are a number of applications of double lever/rocker mechanisms. Some of them are Pantograph, Ackerman steering gear mechanism and Watt's approximate straight line motion mechanism. These are shown in Figs. 2.10(a), (b) and (c).

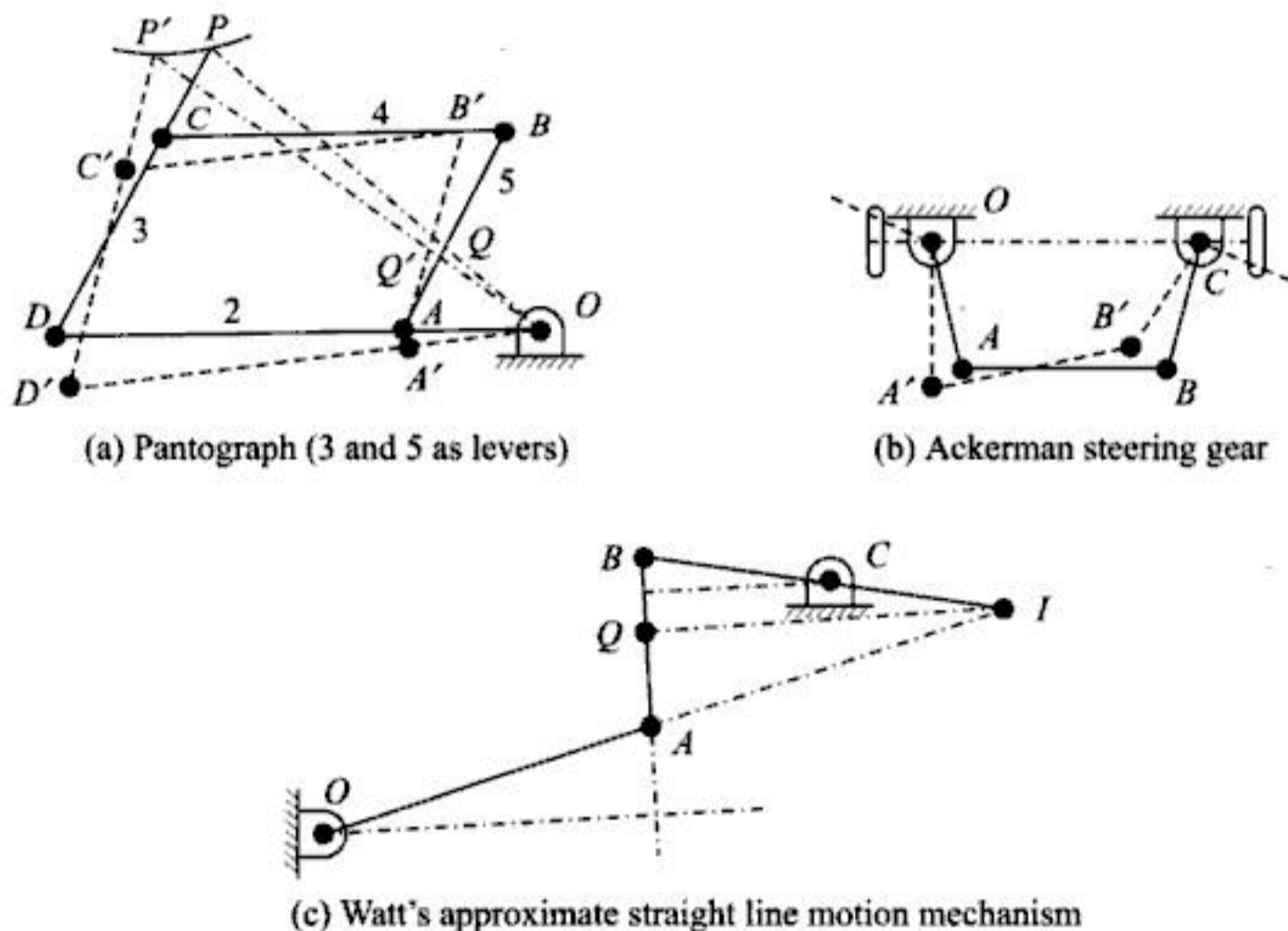


Fig. 2.10

**Third Inversion (Crank-Rocker Mechanism).** Beam engine mechanism is used for converting rotary motion of crank into reciprocating motion of piston. The line diagram explains the principle of a beam engine (Fig. 2.11). The link 1 is the crank while link 3 is the rocker.



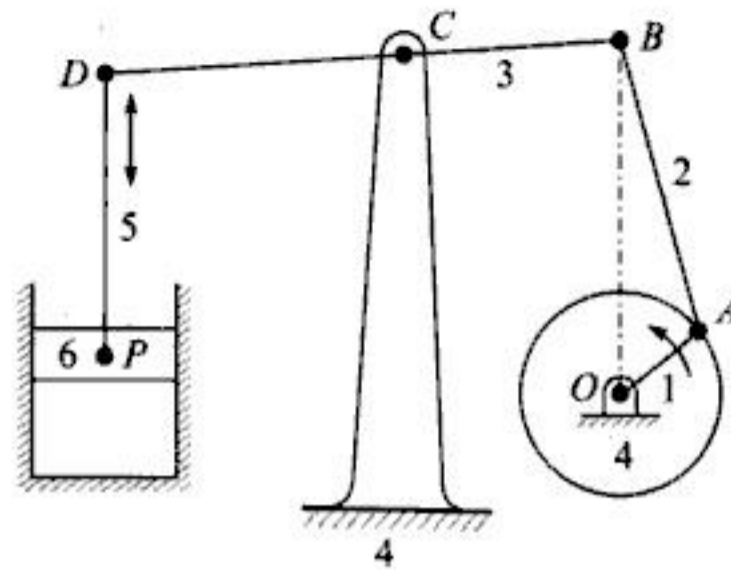


Fig. 2.11 Beam engine mechanism.

### Limiting condition of Grashof's chain-comments:

Experience reveals that mechanisms with proportions approaching the limits of Grashof's rule are not satisfactory in practice. During a motion-cycle, links assume relative positions approaching to "toggling" (dead centre positions), producing chatter and possibly excessive stresses. Further, such linkages require tighter machining tolerances on links, if undesirable effects on motion-characteristics are to be avoided, and as such are more expensive to manufacture.

## 2.8 INVERSION OF SLIDER CRANK CHAIN

A slider crank chain is as shown in Fig. 2.12(a).

1. **First Inversion.** It is obtained by fixing link 1 of the chain and the result is the crank-slider mechanism as shown in Fig. 2.12(b). This mechanism is very commonly used in I.C. engines, steam engines and reciprocating compressor mechanism.
2. **Second Inversion.** It is obtained by fixing link 3, the connecting rod. The mechanism obtained by 'verbatim inversion', as shown in Fig. 2.12(c), has some practical difficulties. For instance, the oscillating cylinder will have to be slotted for clearing the pin through which slider is pivoted to frame. The problem is resolved, if one remembers that any suitable alteration in shapes of members, ensuring same type of pairs between links 3 and 4 and also between links 1 and 4, is permissible. This gives rise to an inversion at Fig. 2.12(d). The resulting mechanism is oscillating cylinder engine mechanism. It is used in hoisting engine mechanism and also in toys. In hoisting purposes its chief advantage lies in its compactness of construction as it permits simple scheme of supplying steam to the cylinder.  
Second application of the above inversion lies in 'Slotted Lever Quick Return Mechanism', shown in Fig. 2.12(e). The extremum positions of lever 4 is decided by the tangents drawn from lever-pivot to the crank-circle on either side. Corresponding positions of crank 1 include angles  $\theta_C$  and  $\theta_R$ , which correspond to cutting stroke angle and return stroke angle.
3. **Third Inversion.** The third inversion is obtained by fixing crank 2. It is the slider-crank equivalent of Drag-link mechanism and forms the basis of Whit-Worth Quick Return Mechanism. Basic inversion is given by portion OAS. To derive advantage however, the

slotted link 1 is extended upto  $P$  and here it is connected to reciprocating tool-post through a connecting link  $PQ$  and two turning pairs. The cutting stroke angle  $\theta_C$  and return stroke angle  $\theta_R$  are shown in Fig. 2.12(f).

A yet another application of third inversion is in Gnome (Rotary cylinders) engine. This was extensively used as aero-engine during World War I.

**Fourth Inversion.** The fourth inversion is obtained by fixing slider, the link 4. Fixing of slider implies that the slider should be position-fixed and also fixed in respect of rotation. The verbatim inversion is shown in Fig. 2.12(g). This form has certain practical difficulties. As explained earlier, the cylinder will have to be slotted so as to clear piston pin of connecting rod as cylinder slides past piston. To overcome this difficulty, the shapes of piston and cylinder are exchanged as shown in Fig. 2.12(h). This gives a hand pump mechanism. Lever 2 is extended

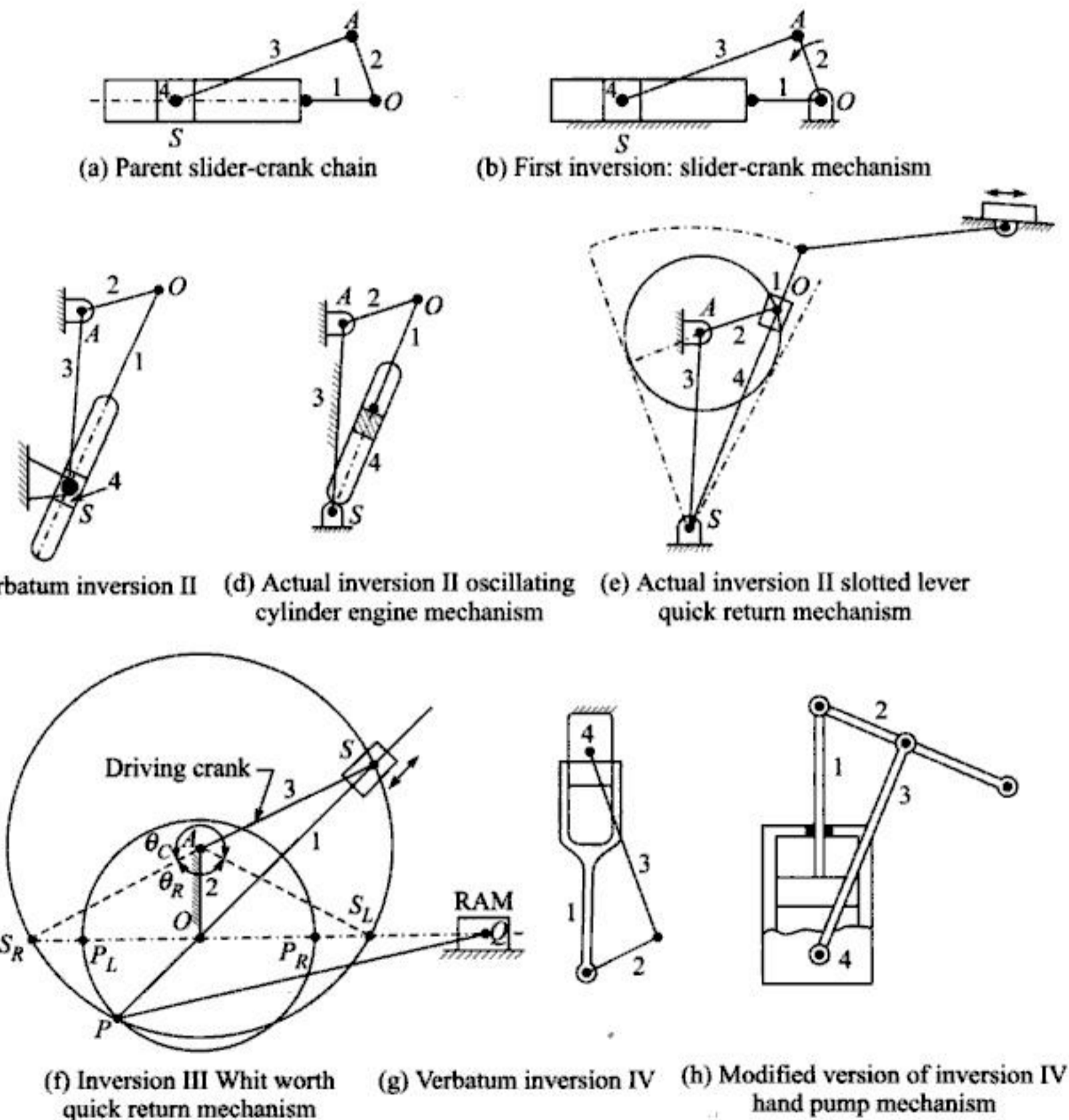


Fig. 2.12 Inversions of a slider crank chain.

## 2.9 THE DOUBLE SLIDER CRANK CHAIN AND ITS INVERSION

This consists of four binary links with two revolute and two sliding pairs placed in the order stated. There are three distinct inversions.

**First Inversion.** The first inversion occurs when the link, consisting of two perpendicular slots, becomes the frame-link. The result is an elliptical trammel. As the two sliders which are connected to the binary link 3 continue to slide in the slots, any point  $S$  on the extension of this link describes an ellipse. Clearly  $QS$  and  $PS$  are the semi-major and minor axes of the ellipse. It can be proved as under:

$$x_s = (QS) \sin \theta$$

or,  $\sin \theta = (x_s / QS)$

Also,  $y_s = (PS) \cos \theta$

or,  $\cos \theta = (y_s / PS)$

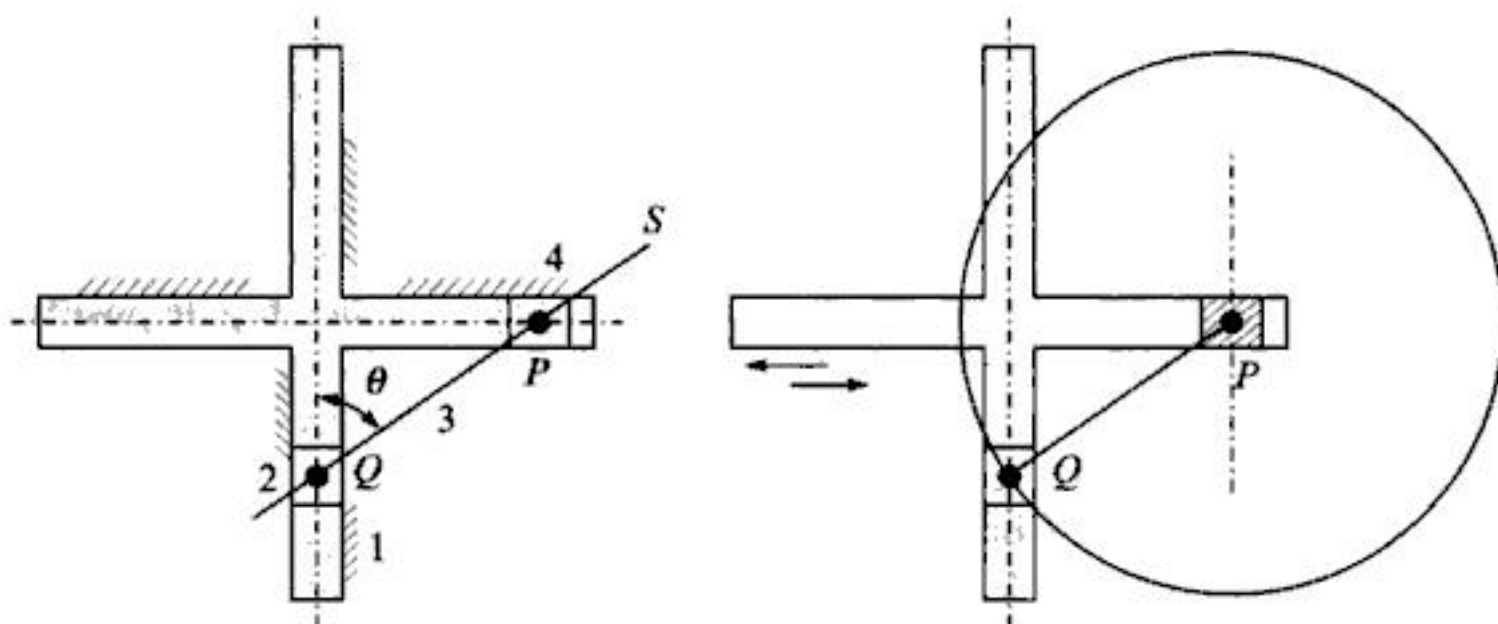
Squaring and adding on either side,

$$\sin^2 \theta + \cos^2 \theta = \left( \frac{x_s}{QS} \right)^2 + \left( \frac{y_s}{PS} \right)^2 \quad (2.3)$$

Thus  $\left( \frac{x_s}{QS} \right)^2 + \left( \frac{y_s}{PS} \right)^2 = 1$ , which is an equation to ellipse with semi-major axis =  $QS$  and semi-minor axis =  $PS$ .

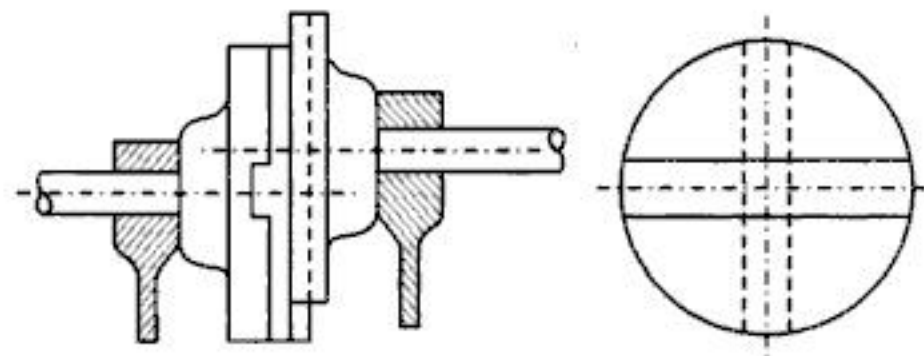
This inversion is shown in Fig. 2.13(a).

**Second Inversion.** The second inversion occurs when one of the two sliding blocks  $P$  and  $Q$  is fixed. This is shown in Fig. 2.13(b). The block  $P$  is shown fixed so that  $PQ$  can be treated as a crank, pivoted to frame at  $P$  and carrying a sliding block  $Q$  at the other end of this crank. This causes the frame to reciprocate. The fixed block  $P$  guides the frame. This results in scotch yoke mechanism. Due to guidance of sliding block  $P$ , the mechanism converts rotary motion of  $PQ$  into reciprocating motion of cross.



(a) First inversion: elliptical trammel

(b) Second inversion: scotch yoke mechanism



(c) Third inversion oldham's coupling

Fig. 2.13 Inversions of Double slider-crank chains

**Third Inversion.** The third inversion is obtained when connecting link 3 is fixed. This inversion leads to oldham's coupling, which is used to transmit uniform angular velocity when shaft axes are parallel but eccentric. This is shown in Fig. 2.13(c).

## 2.10 TRANSMISSION OF TORQUE AND FORCE IN MECHANISMS

Ability to transmit torque or force effectively is one of the major criteria which a designer must consider. Four-bar linkages are being widely used in practice and, as such, we need to develop a criteria to judge the quality of such a linkage for its intended application. Some mechanisms, such as a gear train, transmit a constant torque ratio from input to output shaft. Apparently, this is possible because there is a constant speed ratio between input and output shaft. In the case of a normal four-bar linkage however, this is not possible because torque ratio is a function of geometric parameters which generally change during the course of mechanism motion.

**Mechanical advantage** of a linkage may be defined as the ratio of the output torque, supplied by the driven link, to the input torque, required to be supplied to the driver link. It will be shown later that the mechanical advantage of the four-bar linkage varies directly as the sine of the angle  $\gamma$  between the coupler and the follower and varies inversely as the sine of the angle  $\beta$  between coupler and driving link. Needless to say that as the mechanism goes on changing positions, both these angles change continuously.

When angle  $\beta$  becomes quite small its sine also approaches zero value, and the mechanical advantage becomes infinite. Physically, this implies that at this position of linkage a small input torque will be sufficient to overcome large resisting torque at driven member. This condition is obtainable when the driving link  $OA$  is directly in-line with the coupler  $AB$ . In the crank-rocker mechanism of Fig. 2.14, this condition is obtained when crank is in position  $OA_1$  and again when it is in position  $OA_2$ . It can be verified that these two positions of crank correspond to the extreme positions of the rocker  $CB_1$  and  $CB_2$ . Clearly, in either of the above two positions of crank, the mechanical advantage is infinite and such a position of linkage is called **toggle position**. In such a position, large output torques can be generated by providing smaller input torque.

Similarly, **transmission angle**  $\gamma$  is defined as the acute angle between the coupler and the driven link. As this angle becomes too small,  $\sin \gamma$  approaches zero and mechanical advantage approaches zero value. Physical interpretation of a very small mechanical advantage implies that the torque/force available at the driven member (for a given input torque) may be so small as not enough to overcome friction and may cause the mechanism to lock or jam. As a thumb rule a four-bar mechanism should neither be designed for intended use, nor be used in regions where

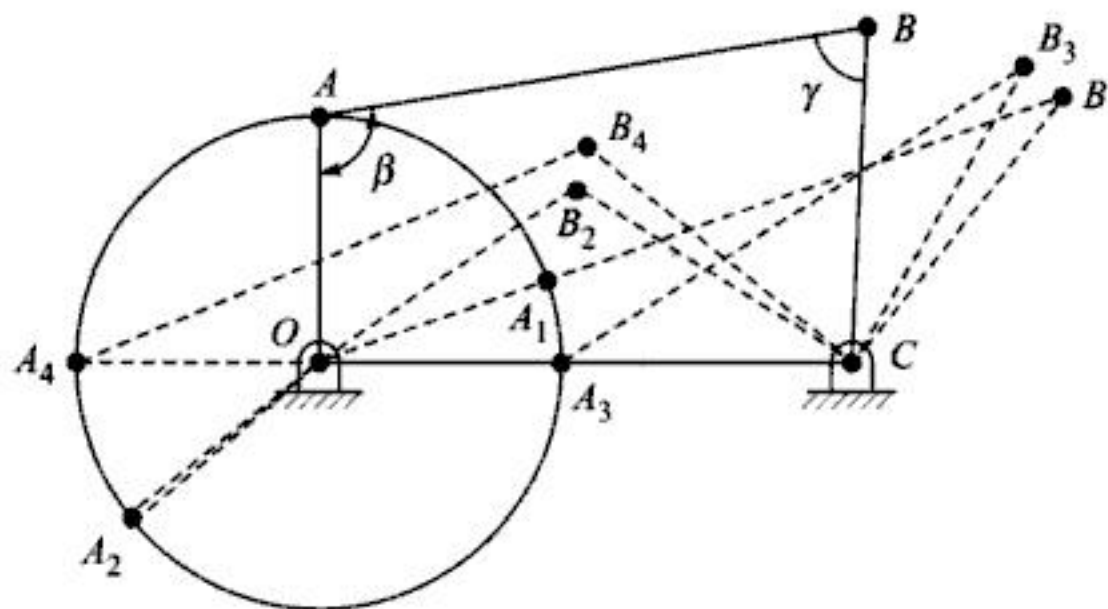


Fig. 2.14 Index of mechanical advantage: Transmission angle  $\gamma$ .

transmission angle  $\gamma$  is less than  $45\text{--}50^\circ$ . The smallest transmission angle occurs when the crank  $OA$  lies along line of centres  $OC$  in position  $OA_3$ , maximum value of transmission angle occurs when crank takes up position  $OA_4$  along line of centres.

Transmission angle criteria as an index of assessing mechanical advantage has become more popular on account of ease with which it can be inspected visually.

Besides transmission angle, in some of the mechanisms like meshing gear teeth or a cam-follower system, the **pressure angle** is used as an index of merit. In a four-bar mechanism, pressure angle is taken to be the angle which is complement of transmission angle.

**EXAMPLE 2.3** Mechanism shown in Fig. 2.15 is driven by turning link  $A_6 A$ . Find out geometrically the maximum and minimum transmission angles. (A.M.I.E. Summer 1993)

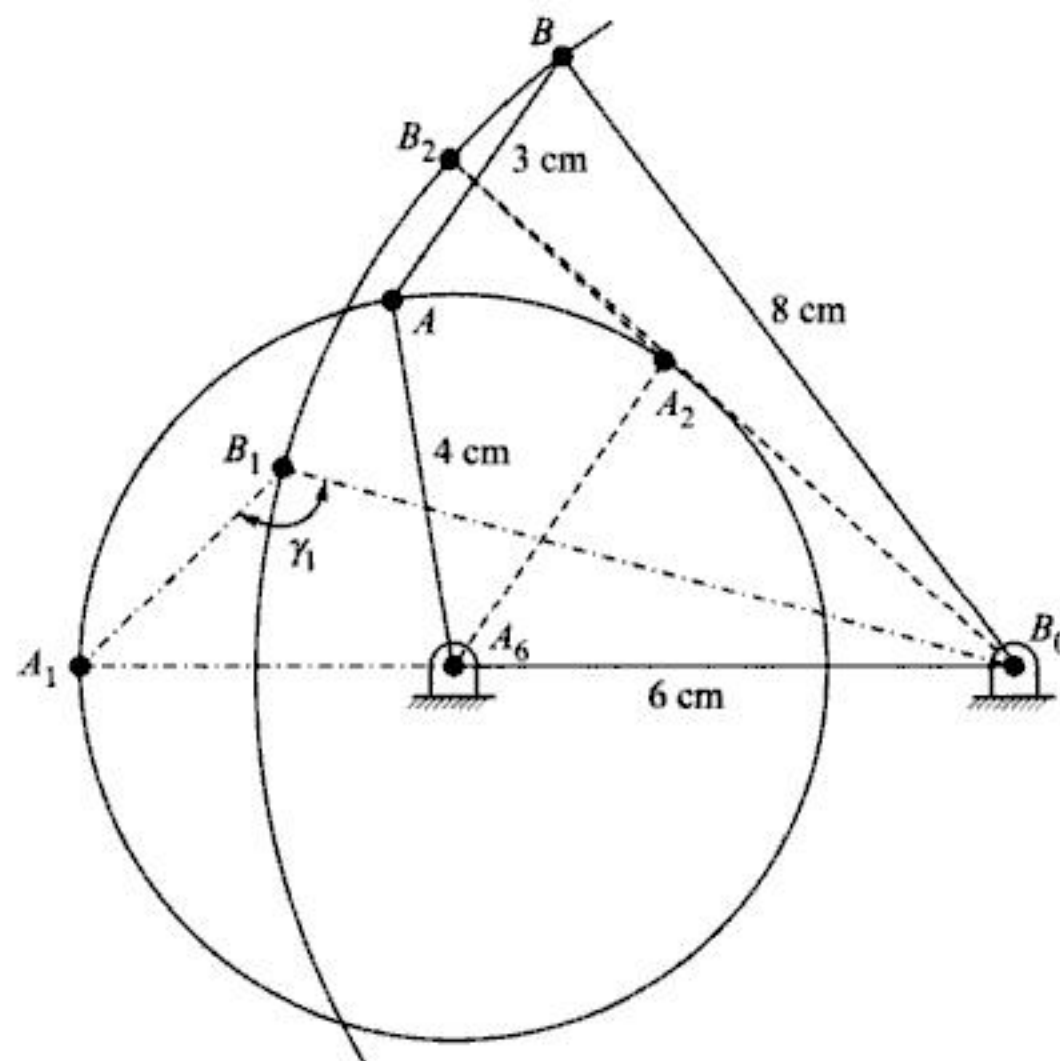


Fig. 2.15

**Solution:** Clearly maximum value of transmission angle  $\gamma = \gamma_1$  is reached when the input crank  $A_6A_1$  lies at the dead centre position. The configuration  $A_6A_1B_1B_0$  (a) corresponds to the maximum transmission angle as shown.

Since the mechanism cannot exist in configuration corresponding to other dead centre position of crank, the other extreme position is obtained by deciding location  $A_2$  of crank for which arc with  $A_2$  as centre and  $A_2B_2$  as radius just touches the arc with  $B_0B$  as radius and  $B_0$  as centre. The transmission angle corresponding to this position is  $2^\circ$ . Also  $\gamma_1 = 122^\circ$ .

Transmission angle being the acute angle between the coupler and driven link,

Maximum transmission angle =  $180^\circ - 122^\circ = 58^\circ$

**Ans.**

Minimum transmission angle =  $02^\circ$

**Ans.**

## 2.11 CONSTRAINED MOTION AND DEGREES OF FREEDOM OF A MECHANISM

*Constrained motion* is defined as that motion in which all points move in predetermined paths, irrespective of the directions and magnitudes of the applied forces. Mechanisms may be categorised in a number of ways to emphasise their similarities and differences. One such grouping can be to divide mechanisms into planar, spherical and spatial categories. As seen earlier, a planar mechanism is one in which all particles, on any link of a mechanism, describe plane curves in space and all these curves lie in parallel planes.

**Mobility/Degree of Freedom of Mechanisms.** In the design or analysis of a mechanism, one of the prime concerns is the number of degrees of freedom or mobility of the mechanism. The number of independent input parameters which must be controlled independently so that a mechanism fulfills its useful engineering purpose is called its *degree of freedom* or *mobility*. Degree of freedom equal to 1 (d.o.f. = 1) implies that when any point on the mechanism is moved in a prescribed way, all other points have uniquely determined (constrained) motions. When d.o.f. = 2, it follows that two independent motions must be introduced at two different points in a mechanism, or two different forces or moments must be present as output resistances (as is the case in automotive differential). Corresponding conditions apply for d.o.f. more than two.

## 2.12 EXPRESSION FOR D.O.F. OF CHAINS AND MECHANISMS

A rigid body free to move any where in space has six degrees of freedom; three of translation parallel to  $x$ ,  $y$  and  $z$  directions and three of rotations about these axes. As against this, when a rigid body is restrained to have planar motion, it has only three degree of freedom; two of translation along  $x$  and  $y$  axes and third of rotation about third axis. Thus, if there are unconnected  $n$  links, they possess a total of  $3n$  degrees of freedom. Figures 2.16 a, b, c, d, e illustrate the process of losing degrees of freedom, each time a turning pair is introduced, *i.e.* adding constraints, between two unconnected links.

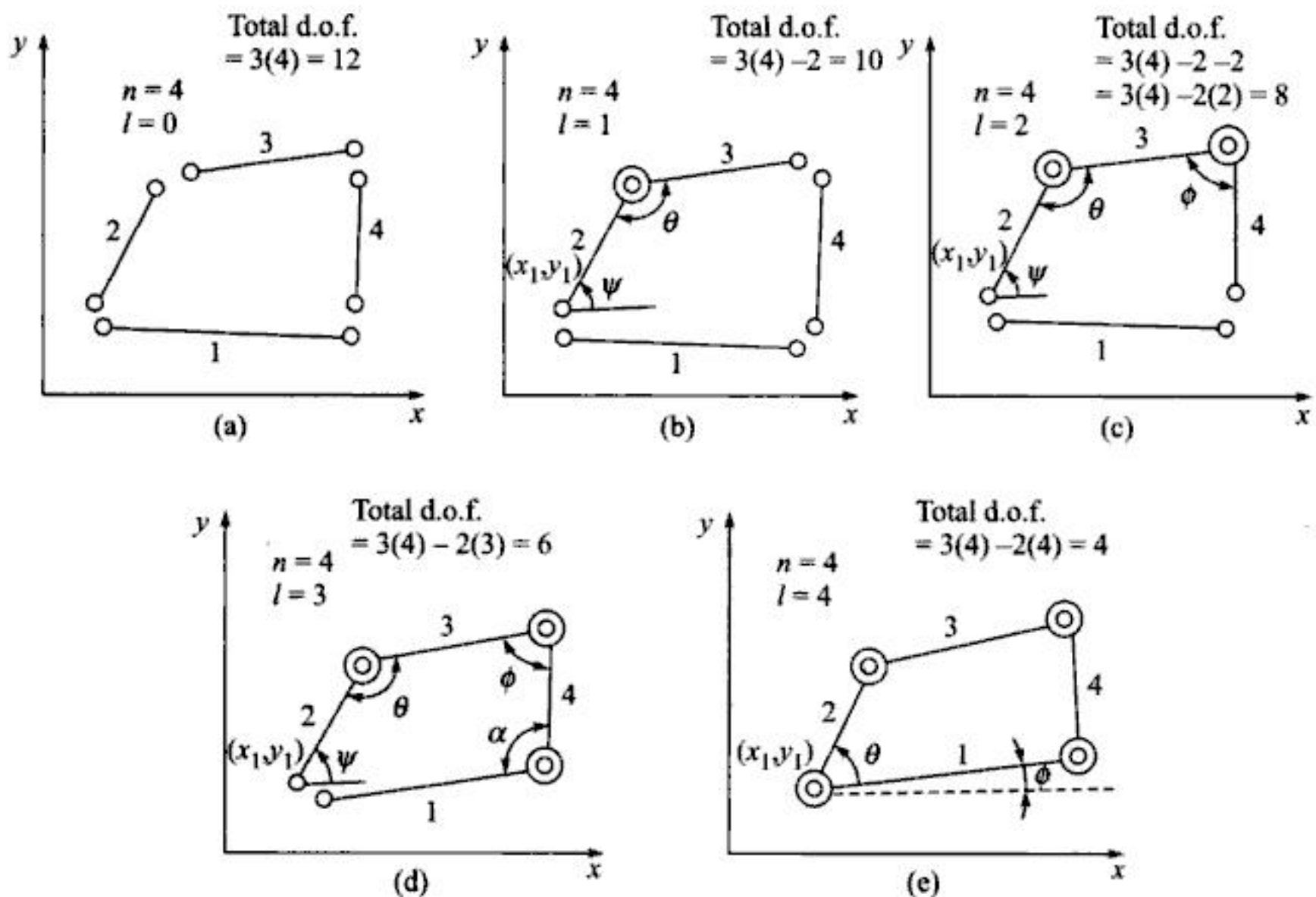
By introducing a turning pair between links 2 and 3 in Fig. 2.16(b), combined d.o.f. of links 2 and 3 are cut down from 6 to 4, while total d.o.f. of all the links reduces to 10. This is easy

to understand as two coordinates  $(x_1, y_1)$  and angle  $\psi$ , measured with respect to say,  $x$  axis is required to fix link 2 in position. Additionally, only angle  $\theta$  between links 2 and 3 will be required to fix link 3 in position relative to link 2.

By introducing turning pairs between 2 and 3 and also between 3 and 4, the total d.o.f. are further cut down from 12 to 8. This is easy to verify in Fig. 2.16(c). Besides coordinates  $(x_1, y_1)$  and angle  $\psi$  for fixing link 2 in position and angle  $\theta$  for fixing position of link 3 w.r. to 2 we now require angle  $\phi$  for fixing position of link 4 in relation to 2 and 3.

In Fig. 2.16(d), after inserting 3rd turning pair we require  $x_1, y_1, \psi, \theta, \phi$  and  $\alpha$  for locating connected links 2, 3, 4 in two-dimensional space and hence the combined d.o.f. is 6, which tallies with the figure worked out in Fig. 2.16(d).

With the insertion of the 4th turning pair between links 1 and 4 loop closure is complete. The completed chain requires four parameters for uniquely describing its members in  $x$ - $y$  plane; these are  $x_1, y_1, \phi$  and  $\theta$  only. This gives four d.o.f. to the chain which tallies with the figures worked out in Fig. 2.16(e).



**Fig. 2.16** Figures illustrating process of losing degrees of freedom each time a constraint is added by way of a turning pair between two unconnected links.

We thus conclude that introduction of each lower pair (more precisely, turning pairs) between a pair of links cut down degree of freedom by 2 from an assembly of  $n$  unconnected mobile links. The process of adding kinematic pairs between links thus amounts to constraining the motion so as to produce a resultant motion which is useful for some purpose. Based on above discussions, expression for degree of freedom of a planar kinematic chain, consisting of lower pairs (of d.o.f. = 1) only, is given by –

$$F = 3n - 2(l) \quad (2.4)$$

where

$n$  = number of mobile links

and

$l$  = total number of lower pairs.

In case of a mechanism which is obtained from a chain by fixing one link, number of mobile links reduces to  $(n - 1)$  and therefore, expression for degrees of freedom of a mechanism, consisting of lower pairs only, is given by—

$$\text{d.o.f. of a mechanism, } F = 3(n - 1) - 2(l) \quad (2.5)$$

Equation (2.5) is known as Grubler's equation, and is one of the most popular mobility equations.

**Effect of Higher Pair.** Just as a lower pair (linear motion lower pair) cuts down 2 d.o.f., a higher pair cuts only 1 d.o.f. (this is because invariably rolling is associated with slipping, permitting 2 d.o.f.). Hence equation (2.5) can be further modified to include the effect of higher pairs also. Thus, for a mechanism having lower and higher pairs,

$$\text{d.o.f., } F = 3(n - 1) - 2(l) - h \quad (2.6)$$

where,

$l$  = total number of lower pair

and

$h$  = total number of higher pairs.

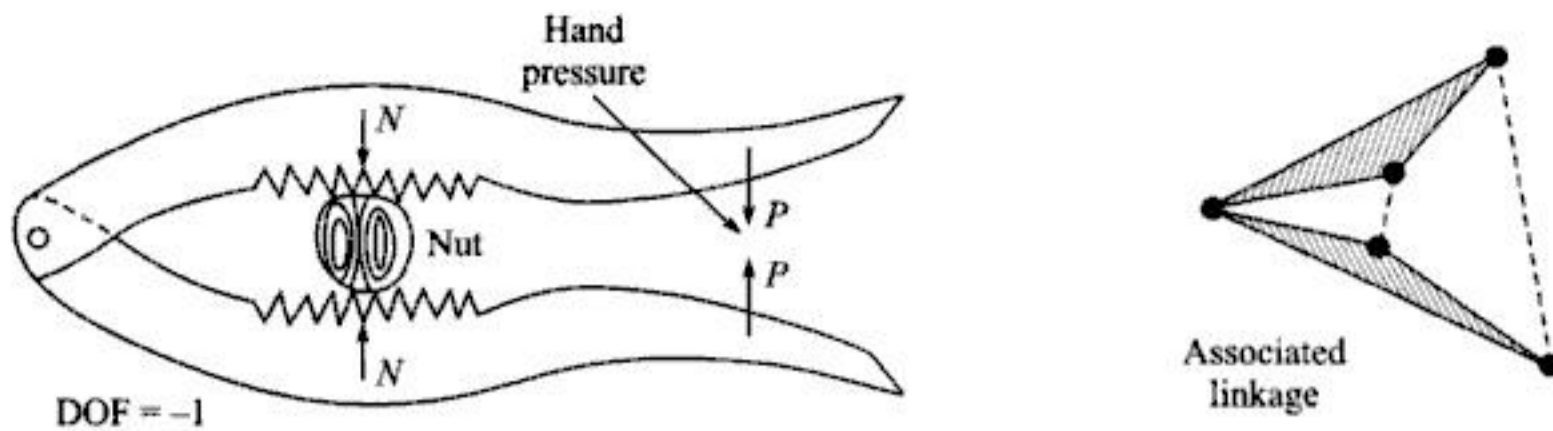
Equation (2.6) is the modified Grubler's equation. It is also known as Kutzbach criterion for the mobility of a planar mechanism. It would be more appropriate to define, in equations (2.5) and (2.6),  $l$  to be the number of pairs of 1 d.o.f and  $h$  to be number of pairs of 2 d.o.f.

**Movability.** German literature prefers to distinguish the terms 'mobility' and 'movability'. *Movability*, in German literature, refers to the condition of a device assuming that ground link was not fixed. In other words, the term 'movability' refers to degree of freedom of parent kinematic chain. Thus equation (2.4) can be taken to represent movability. The English literature, however, seldom recognises this difference and terms 'movability' and 'mobility' are freely interchanged.

## 2.13 INTERPRETATION OF MOBILITY EQUATION

1. When mobility or d.o.f. of mechanism turns out to be zero, no motion is possible and the mechanism forms a structure for supporting or resisting external loads (*see* Fig. 2.4 a).
2. When the mobility from Grubler's equation turns out to be  $-1$ , it results in an indeterminate structure. It follows that the linkage has an additional (redundant) constraint, beyond what is necessary to establish uniquely the position of the assemblage. Therefore, with the technique of replacing internal forces of a device with binary links (*see* Fig. 2.17) in the associated linkage,  $F = -1$  is characteristic of both pre-loaded structure and internal force exerting devices (*e.g.* clamps).
3. If the mobility  $F$  turns out to be  $+1$ , the mechanism can be driven by a single input motion. If  $F = 2$ , two separate input motions are necessary to produce constrained motion of the mechanism.





**Fig. 2.17** A Nut Cracker. In associated linkage, the dashed line indicate imaginary binary links which represent the effect of forces  $N$  and  $P$ .

**Effect of Spring Connection.** Purpose of a spring is to produce a mutual force between the two links which it connects but, in no way, does it kinematically constrain the relative motion between the two links. The action of spring, to elongate or compress, can also be simulated by a pair of binary links and three pin joints which form the instantaneous-velocity-equivalent lower-pair model to the spring, allowing same degrees of freedom of relative motion between the connected links.

**Effect of Multiple Joint.** A multiple joint is called a  $p$ -tuple joint when a given link is connected to  $p$  other links at that joint. Thus, a double joint connects a given link to two other links, and a triple joint connects a given link to three other links and so on. Many a time a multiple joint is preferred in practice on account of smaller construction cost and, quite often, smaller space requirement. While finding degree of freedom of a mechanism with multiple joints, a double joint should be considered equivalent to two simple joints and a triple joint be considered equivalent to three simple joints, etc.

**EXAMPLE 2.4** Find out degrees of freedom of mechanism shown in Figs. 2.18(a), (b), (c), (d) and (e).

**Solution:** (a) Here  $n = 9$ ;  $l = 11$

Therefore  $\text{d.o.f.} = 3(9 - 1) - 2(11) = 2$

(b) Here  $n = 8$

$$l = 9 + 2 \text{ (on account of a double joint)}$$

$$= 11$$

Therefore  $\text{d.o.f.} = 3(8 - 1) - 2(11)$   
 $= 21 - 22 = -1$

**Ans.**

*i.e.* the mechanism at Fig. 2.18(b) is a statically indeterminate structure.

(c) As in case (b), here too there are double joints at  $A$  &  $B$ . Hence

$$n = 10; l = 9 + 2(2) = 13$$

Therefore  $\text{d.o.f.} = 3(10 - 1) - 2(13) = 1$

**Ans.**

(d) The mechanism at Fig. 2.18(d) has three ternary links (links 2, 3 and 4) and 5 binary links (links 1, 5, 6, 7 and 8) and one slider. It has 9 simple turning pairs marked  $R$ , one sliding pair marked  $P$  and one double joint at  $J$ . Since the double joint  $J$  joins 3 links, it may be taken equivalent to two simple turning pairs. Thus,

$$n = 9; l = 11$$

Therefore  $\text{d.o.f.} = 3(9 - 1) - 2(11) = 2$

**Ans.**

(e) The mechanism at Fig. 2.18(e) has a roller pin at *E* and a spring at *H*. The spring is only a device to apply force, and is not a link. Thus, there are 7 links numbered 1 through 7, one sliding pair, one rolling (higher) pairs at *E* besides 6 turning pairs

Thus

$$n = 7; l = 7 \text{ and } h = 1$$

Therefore d.o.f.,

$$F = 3(7 - 1) - 2(7) - (1) \\ = 18 - 14 - 1 = 3$$

Ans.

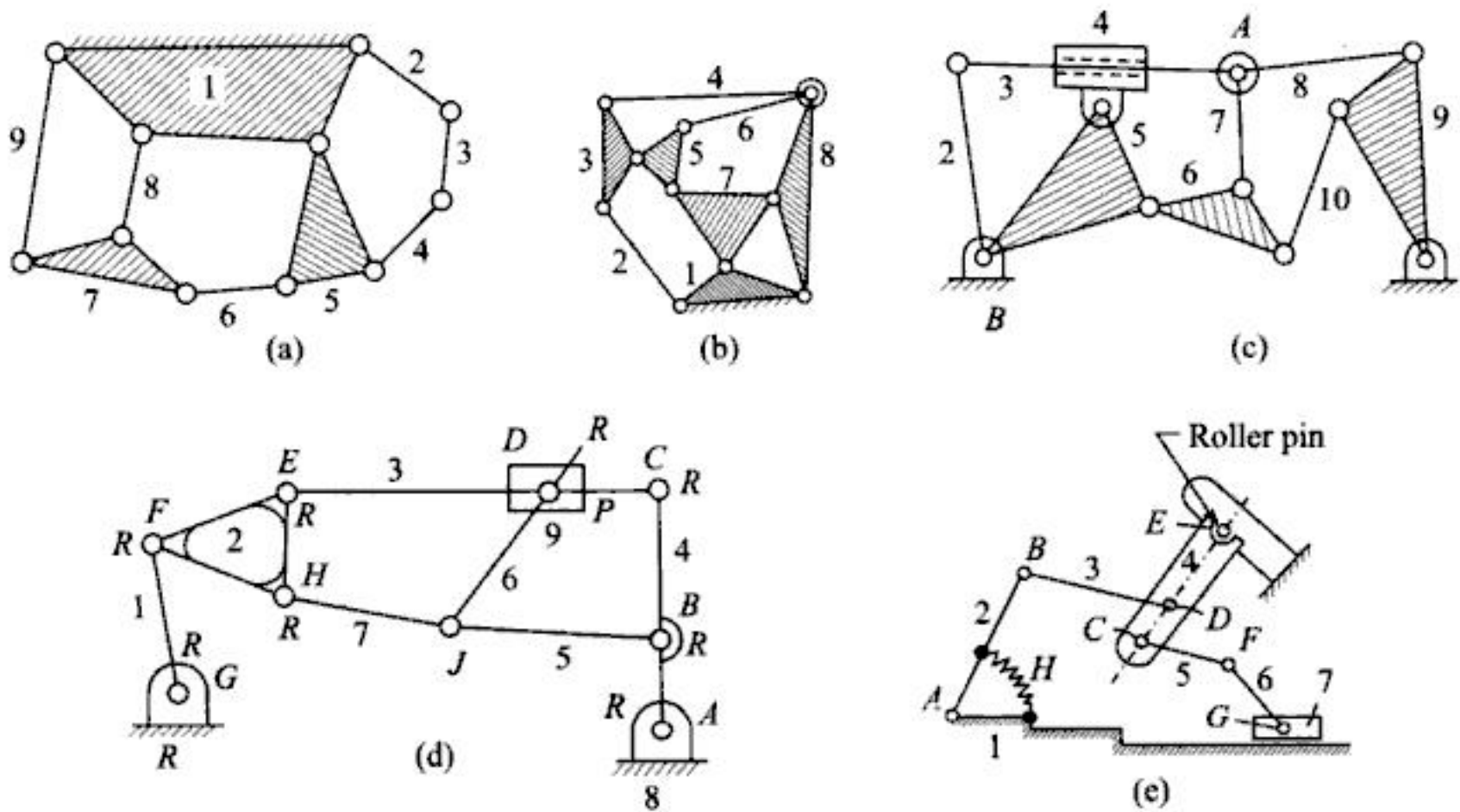


Fig. 2.18

**EXAMPLE 2.5** Find out degrees of freedom of the mechanism shown in Figs. 2.19(a), (b) and (c).

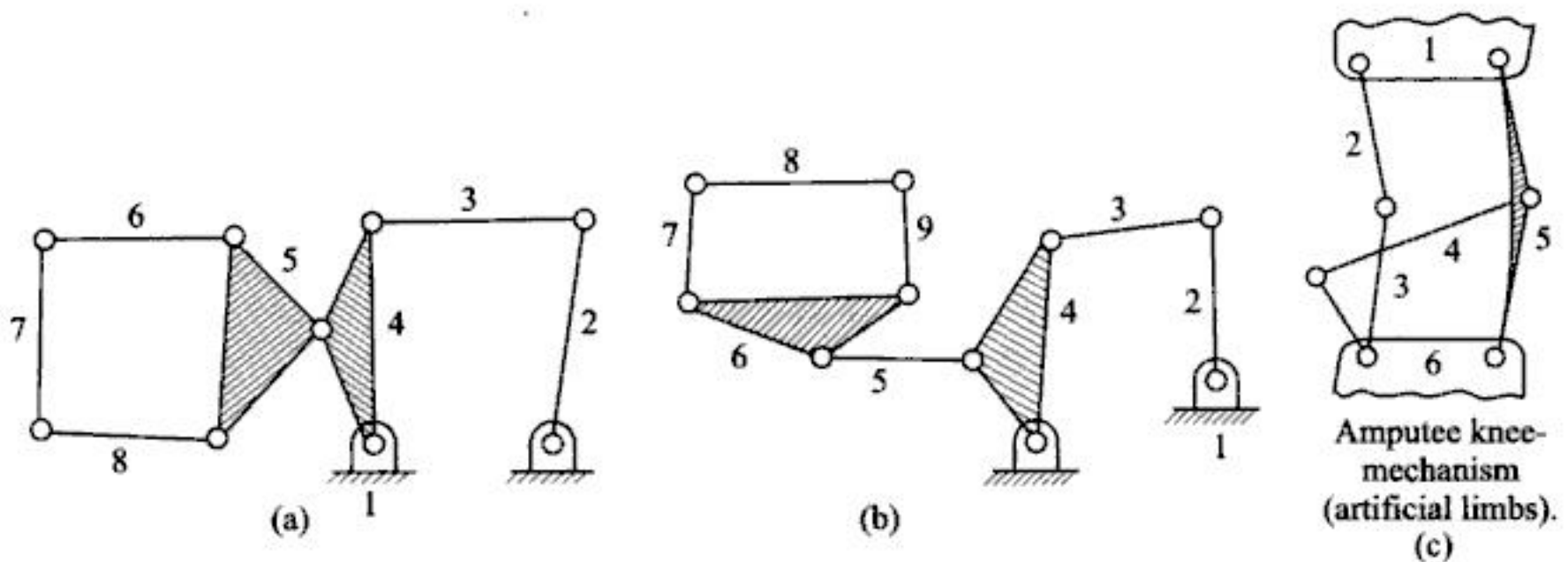


Fig. 2.19

**Solution:** (a)  $n = 8; l = 9$

Therefore d.o.f. =  $3(8 - 1) - 2(9) = 3$

Ans.

$$(b) \quad n = 9 ; l = 10$$

$$\text{d.o.f.} = 3(9 - 1) - 2(10) = 24 - 20 = 4$$

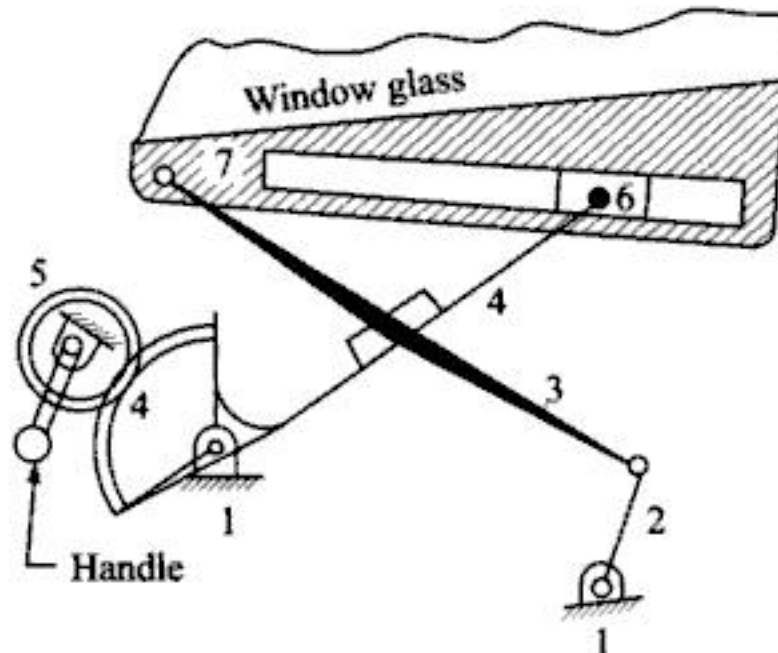
Ans.

$$(c) \quad n = 6 ; l = 7$$

$$\text{Therefore d.o.f.} = 3(6 - 1) - 2(7) = 1.$$

Ans.

**EXAMPLE 2.6** Show that the automobile window glass guiding mechanism in Fig. 2.20 has a single degree of freedom.



**Fig. 2.20** Automobile window guidance linkage.

**Solution:** As numbered, there are total 7 links. There are seven revolute pairs between link pairs (1, 2), (2, 3), (3, 4), (3, 7), (4, 6), (4, 1) and (1, 5). Besides, there is one sliding pair between links 6 and 7 and a geared pair between links 4 and 5.

Thus,

$$l = 8 \text{ and } h = 1$$

Hence,

$$\text{d.o.f.} = 3(7 - 1) - 2(8) - 1$$

$$= 18 - 16 - 1 = 1$$

Ans.

## 2.14 INCONSISTENCIES OF GRUBLER'S EQUATION

In a good number of cases, Grubler's equation appears to give incorrect results, particularly when—

- (1) the mechanism has a lower pair which could be replaced by a higher pair, without influencing output motion.
- (2) the mechanism has a kinematically redundant pair, and
- (3) there is a link with redundant degree of freedom.

Inconsistency at (1) may be illustrated with the help of Figs. 2.21(a) and (b). Figure 2.21(a) depicts a mechanism with three links and three sliding pairs. According to Grubler's theory, this combination of links has a degree of freedom of zero. But by inspection, it is clear that the links have a constrained motion, because as the link 2 is pushed to the left, link 3 is lifted due to wedge action. A little consideration shows that the sliding pair between links 2 and 3 can be replaced

by a slip rolling pair [Fig. 2.21(b)], ensuring constrained motion. In the latter case,  $n = 3$ ,  $l = 2$  and  $h = 1$  which, according to Grubler's equation, gives d.o.f. = 1.

Figure 2.21(c) demonstrates inconsistency at (2). The cam follower mechanism has 4 links, 3 turning pairs and a rolling pair, giving d.o.f. as 2. However, a close scrutiny reveals that as a function generator, oscillatory motion of follower is a unique function of cam rotation, i.e.  $\phi = f(\theta)$ . In other words, d.o.f. of the above mechanism is only 1. It may be noted, however, that the function of roller in this case is to minimise friction, it does not in any way influence the motion of follower. For instance, even if the turning pair between follower and roller is eliminated (rendering roller to be an integral part of follower), the motion of follower will not be affected. Thus the kinematic pair between links 2 and 3 is redundant. Therefore, with this pair eliminated,  $n = 3$ ,  $l = 2$  and  $h = 1$ , gives d.o.f. as one.

If a link can be moved without producing any movement in the remaining links of mechanism, the link is said to have *redundant degree of freedom*. Link 3 in mechanism of Fig. 2.21(d), for instance, can slide and rotate without causing any movement in links 2 and 4. Since the Grubler's equation gives d.o.f. as 1, the loss due to redundant d.o.f. of link 3 implies effective d.o.f. as zero, and Fig. 2.21(d) represents a locked system. However, if link 3 is bent, as shown in Fig. 2.21(e), the link 3 ceases to have redundant d.o.f. and constrained motion results for the mechanism. Figure 2.21(f) shows a mechanism in which one of the two parallel links  $AB$  and  $PQ$  is a redundant link, as none of them produces additional constraint. By removing any of the two links, motion remains the same. It is logical therefore to consider only one of the two links in calculating degrees of freedom. Another example where Grubler's equation gives zero mobility is the mechanism shown in Fig. 2.21(g), which has a constrained motion.

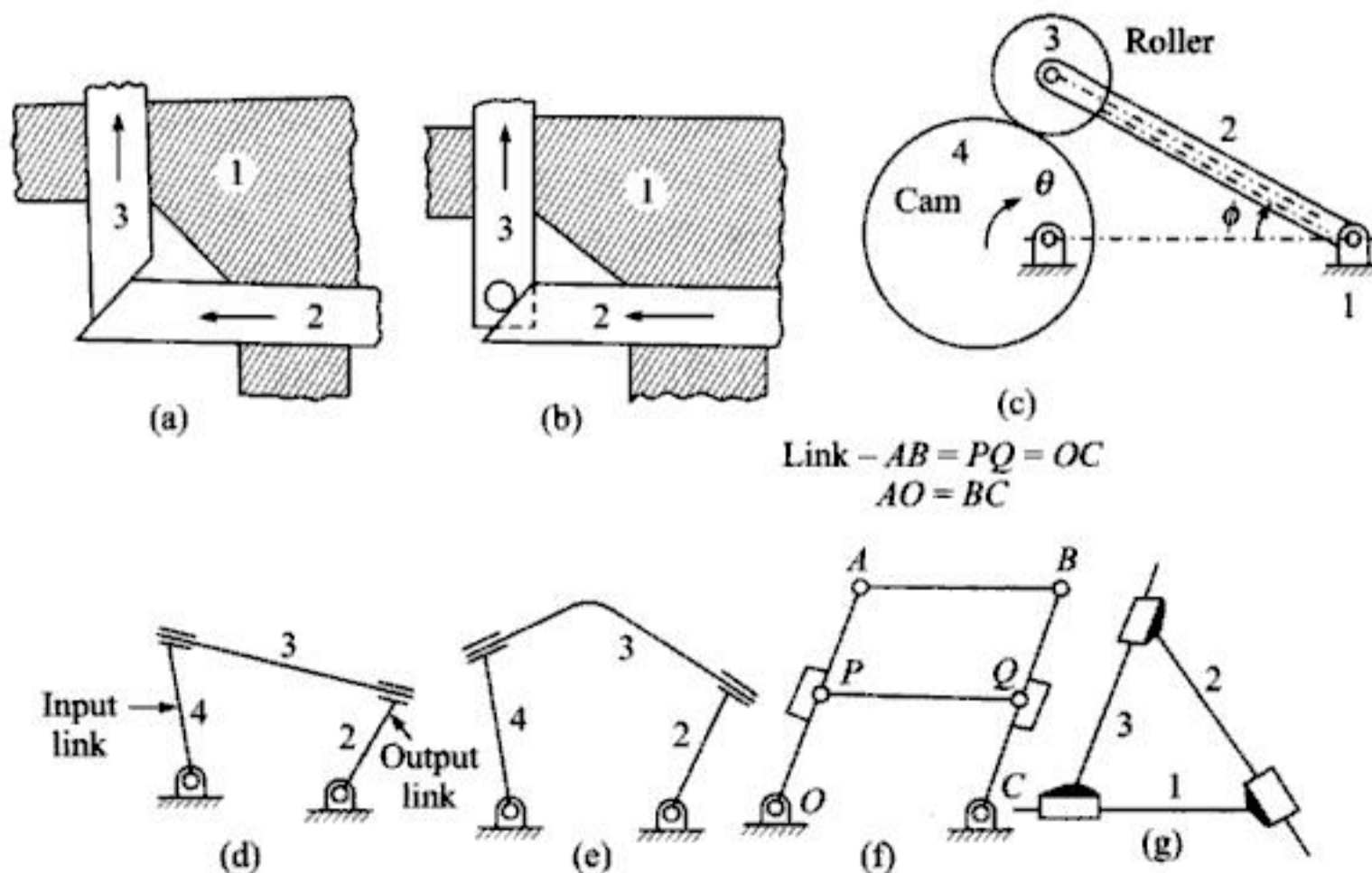


Fig. 2.21 Examples of inconsistencies of Grubler's equation.

## 2.15 DEGREES OF FREEDOM PERMITTED BY JOINTS OTHER THAN TURNING AND SLIDING

As against one degree of freedom of relative motion permitted by turning and sliding pairs, higher pairs may permit a higher number of degrees of freedom of relative motion. *Each such higher pair is equivalent to as many lower pairs as the number of degrees of freedom of relative motion permitted by the given higher pair.* This is elaborated for different types of higher pairs, as discussed below:

- (a) *Rolling Contact Without Sliding.* This allows only one d.o.f. of relative motion as only relative motion of rotation exists. A pure rolling type of joint can therefore be taken equivalent to a lower pair with one d.o.f. The lower pair equivalent for instantaneous velocity is given by a simple hinge joint at the relative instant centre, which is the point of contact between rolling links. Note that instantaneous velocity implies that in case a higher pair is replaced by a lower pair equivalent, the instantaneous relative velocity between the connecting links remains the same, but the relative acceleration may, in general, change.
- (b) *Roll-Slide Contact.* Due to sliding motion associated with rolling, only one out of three planar motions is constrained. Thus a lower pair equivalence for instantaneous velocity is given by a slider and pin joint combination between the connected links. This implies that such a pair can be replaced by a link and two lower pairs. This allows for two degrees of freedom of relative motion. Such a joint is also taken care of, in Grubler's equation, by making contribution to the term  $h$ .
- (c) *Gear-Tooth Contact (Roll-Slide).* Gear tooth contact is a roll-slide pair and therefore makes a contribution to the term  $h$  in Grubler's equation. Thus, on account of two turning pairs at gear centres together with a higher pair at contacting teeth,

$$\text{d.o.f.} = 3(3 - 1) - 2(2) - 1 = 1$$

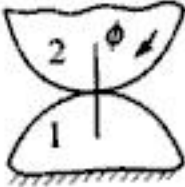
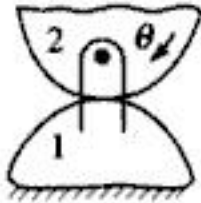
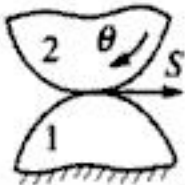
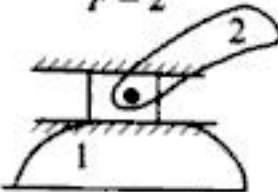
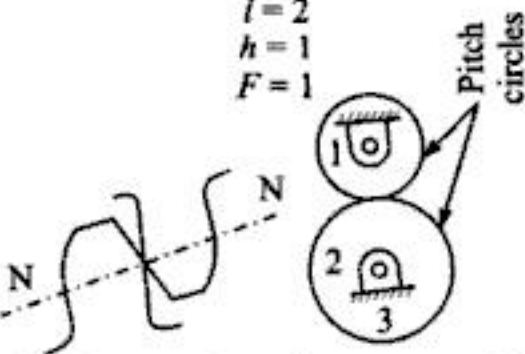
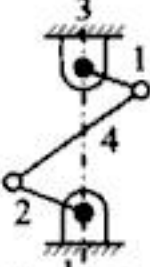
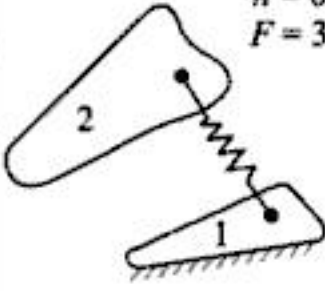
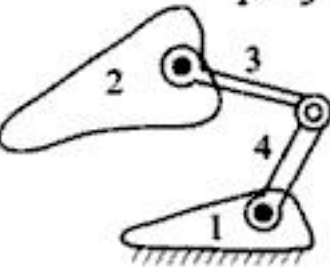
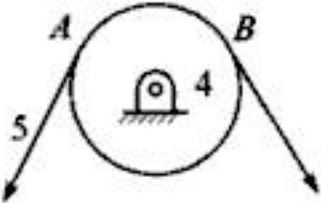
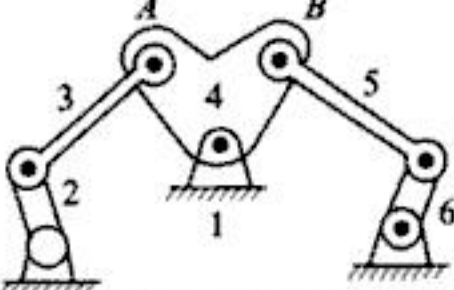
Lower pair equivalent for instantaneous velocity of such a pair is a 4-bar mechanism with fixed pivots at gear centres and moving pivots at the centres of curvature of contacting tooth profiles. *In case of involute teeth, these centres of curvature will coincide with points of tangency of common tangent drawn to base circles of the two gears. Such a 4-bar mechanism retains that d.o.f. equal to 1.*

- (d) *A Spring Connection.* Purpose of a spring is to exert force on the connected links, but it does not participate in relative motion between connected links actively. Since the spring permits elongation and contraction in length, a pair of binary links, with a turning pair connecting them, can be considered to constitute instantaneous velocity equivalent lower pair mechanism. A pair of binary links with a turning pair permits variation in distance between their other ends (unconnected), and allows same degree of freedom of relative motion between links connected by the spring (for  $n = 4$ ,  $l = 3$ ,  $F = +3$ ). It may be noted that in the presence of spring, ( $n = 2$ ,  $l = 0$ ,  $h = 0$ ) the d.o.f. would be 3.
- (e) *The Belt and Pulley or Chain and Sprockets Connection.* When the belt or chain is maintained tight, it provides planar connections. Instantaneous velocity, lower pair

equivalent can be found in a ternary link with three pin joints (sliding is not allowed). It can be verified from Table 2.2 that d.o.f. of equivalent six bar linkage is

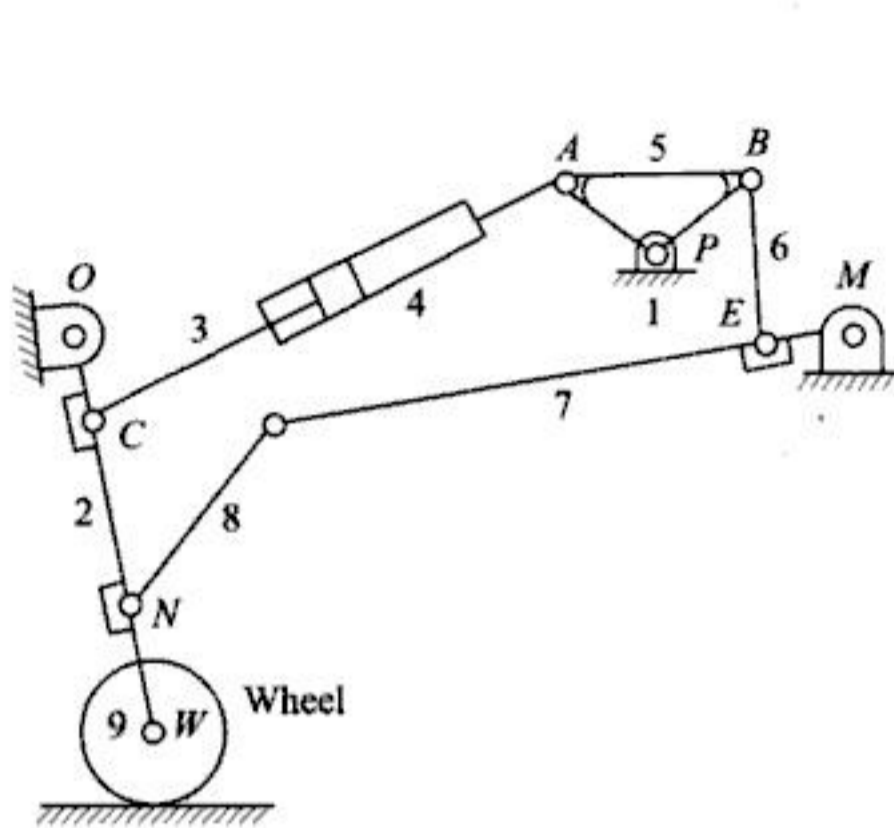
$$F = 3(6 - 1) - 2(7) = 1$$

**Table 2.2** Lower pair equivalents of higher pairs.

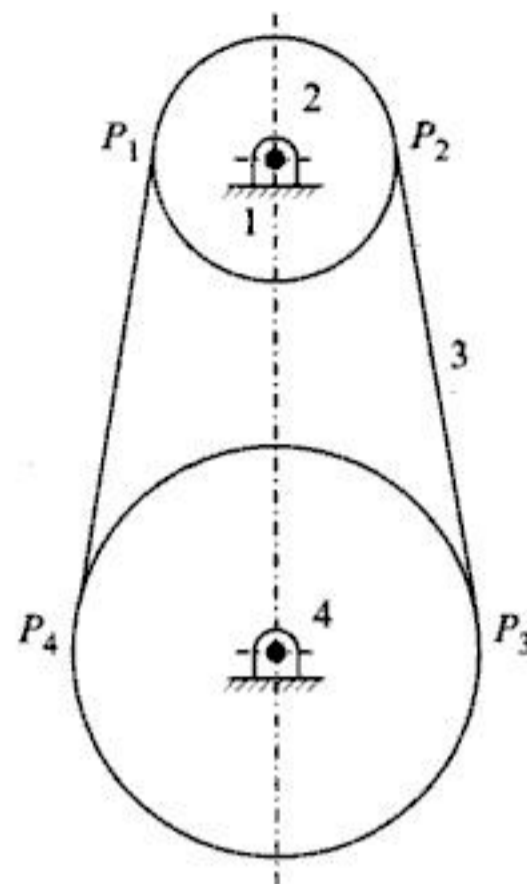
Name of Joint	Diagram original representation	Lower pair equivalent for instantaneous velocity
Rolling contact (without sliding)	$N=2$ $F=1$ 	$N=2$ $l=1$ $F=1$ 
Rolling-sliding contact	$N=2$ $l=0$ $h=1$ $F=2$ 	$N=3$ $l=2$ $h=0$ $F=2$ 
Gear contact (involving roll-slide contact)	$N=3$ $l=2$ $h=1$ $F=1$ 	$N=4$ $l=4$ $F=1$ 
Spring connection	$N=2$ $l=0$ $h=0$ $F=3$ 	$N=4$ $l=3$ $h=0$ $F=3$ 
Belt-pulley (no sliding) contact on chain-sprocket connection	$F=1$ 	$N=6$ $l=7$ $F=1$ 

**EXAMPLE 2.7** Find out degrees of freedom of mechanisms shown in Figs. 2.22(a), (b), and (c).

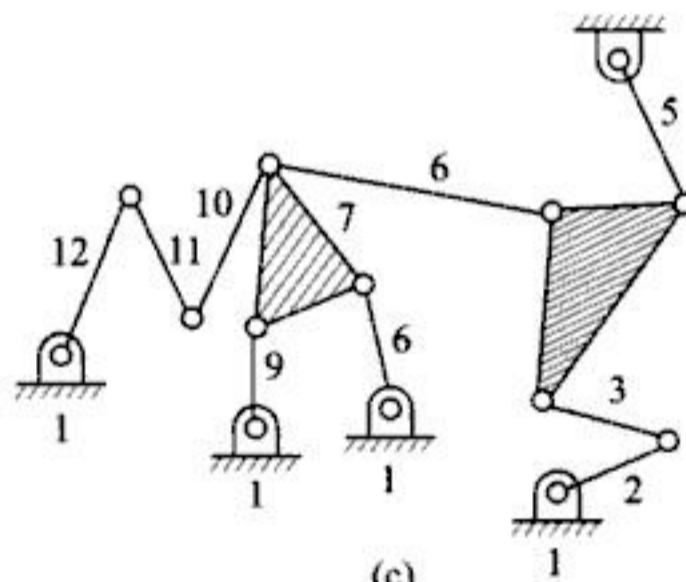
**Solution:** (a) In the case of undercarriage mechanism of aircraft in Fig. 2.22(a), we note that



(a) Undercarriage mechanism of an aircraft



(b) Belt-pulley mechanism (assume belt to be tight)



(c)

**Fig. 2.22**

Total number of links  $n = 9$

Total number of pairs of single d.o.f. = 11

Higher pair of 2 d.o.f. (between wheel and runway) = 01

$$\begin{aligned} \therefore \text{d.o.f. } F &= 3(9 - 1) - 2(11) - 1(1) \\ &= 24 - 22 - 1 = 01 \\ F &= 1 \end{aligned}$$

(b) In the case of belt-pulley drive, assuming the belt to be tight, the four links are marked as 1, 2, 3 and 4. The two distinct lower (turning) pairs are pivots of pulley 2 and 4. The points  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$ , at which belt enters/leaves pulley, constitute 4 higher pairs. Thus

$$n = 4 ; l = 2 ; h = 4$$

Therefore

$$F = 3(4 - 1) - 2(2) - 4 = 1.$$

(c) In the case of mechanism at Fig. 2.22, there is a double joint between links 6, 7 and 10. Therefore, this joint is equivalent to two simple joints. Besides above, there are 13 turning pairs.

Hence,

$$n = 12; l = 13 + 2 = 15$$

Therefore

$$F = 3(12 - 1) - 2(15) \\ = 33 - 30 = 3.$$

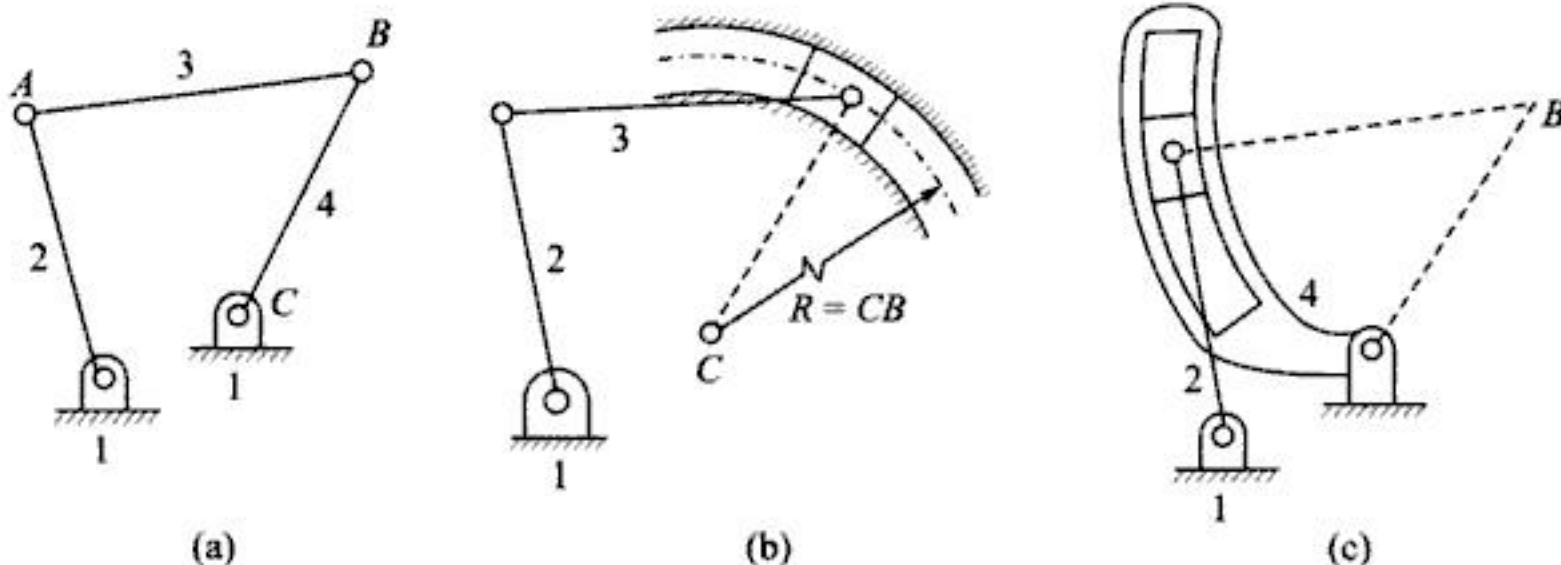
## 2.16 EQUIVALENT LINKAGES

Equivalent linkages are commonly employed to duplicate instantaneously the position, velocity, and perhaps acceleration of a direct-contact (higher pair) mechanism by a mechanism with lower pairs (say, a four-bar mechanism). The dimensions of equivalent linkages are obviously different at various positions of given higher paired mechanism. This is evident because for every position of a higher paired mechanism, different equivalent linkages are expected.

Much of the developments in kinematics in the subject of theory of machines are centred around four-bar mechanism. Some of the reasons are as under:

- (1) A four-bar mechanism is the simplest possible lower paired mechanism and is widely used.
- (2) Many mechanisms which do not have any resemblance with a four-bar mechanism have four bars for their basic skeletons, so a theory developed for the four-bar applies to them also.
- (3) Many mechanisms have equivalence in four-bar mechanism in respect of certain motion aspects, Thus, as far as these motions are concerned, four-bar theory is applicable.
- (4) Several complex mechanisms have four-bar loop as a basic element. Theory of four-bar mechanism is, therefore, useful in the design of these mechanisms.

Point (2) above, is illustrated in Figs. 2.23(a), (b) and (c). In Fig. 2.23(b), the link 4 in Fig (a) is replaced by a curved slot and slider, with slot radius equal to link length. In Fig. 2.23(c) the link 3 is replaced by a slider, sliding in a curved slotted link 4 ensuring relative motion of rotation of pinned end *A* relative to *B*.



**Fig. 2.23** Equivalent mechanisms (kinematically identical mechanisms having the 4-bar as the basic skeleton).



Point (3) is illustrated in Figs. 2.24(a), (b) and (c). Mechanisms in which relative motion between driver and driven links 2 and 4 is identical are illustrated in Fig. 2.24.

In Fig. 2.24(b) the centres of curvature of circular cam and roller constitute the end point of link  $AB$ ; link 3 becomes roller and link 2 become circular cam. For  $d.o.f. = 1$  however, the rolling pair in (b) should be without slip.

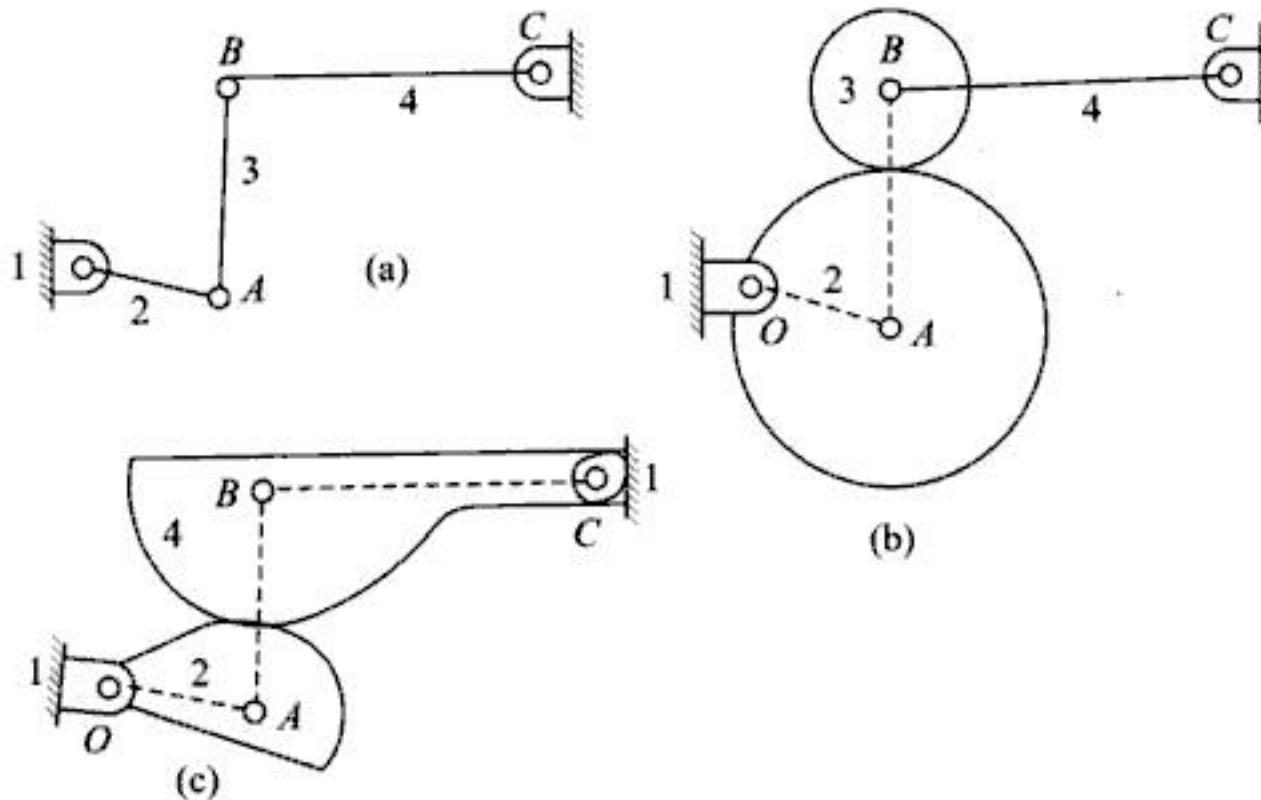


Fig. 2.24 Mechanisms having identical relative motions between links 2 and 4.

**Use of Spring to Replace Turning Pairs.** Extension and compression in a spring is comparable to variation in length between the turning pairs accomplished by a pair of binary links connected through an another turning pair. For instance pair of binary links 4 and 5 of a Stephenson's chain can be replaced by a spring to obtain an equivalent mechanism. This is shown in Figs. 2.25(a) and (b).

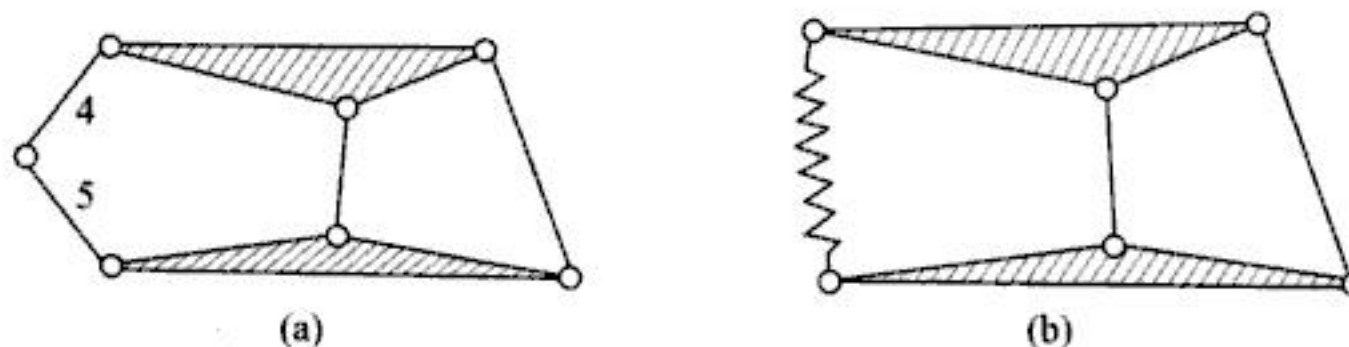


Fig. 2.25 Spring to replace a pair of binary links and ternary pairs.

**A Ternary with Turning Pairs to Replace Belt/Pulley or Chain/Sprocket Pair.** When the belt or chain is maintained tight, a ternary link with three turning pairs is the instantaneous-velocity equivalent lower pair connection to the belt and pulley (sliding/slipping is disallowed).

## 2.17 NUMBER SYNTHESIS

Whereas kinematic analysis aims at analysing the motion inherent in a given machine or mechanism, kinematic synthesis aims at determining mechanisms that are required to fulfill certain motion

specifications. Kinematic synthesis can, therefore, be thought of as a reverse problem to kinematic analysis of mechanisms. Synthesis is very fundamental of a design as it represents creation of a new hardware to meet particular needs of motion, namely displacement, velocity or acceleration—singly or in combination.

Probably, the most obvious external characteristics of a kinematic chain or mechanism are: the number of links and number of joints. Movability studies based on only these two parameters come under the name 'number synthesis.' The oldest and still the most useful (although with limitations) estimate of movability/mobility is known as the 'Grubler's criterion'. Effect of link lengths, directions and locations of axis, position of instantaneous centres of velocity, complexity of connections, etc. are neglected in this approach.

Grubler's criterion for planar mechanisms, consisting of lower pairs only (of d.o.f. = 1), can be remodelled for producing constrained motion of d.o.f. = 1. Thus equation (2.5) can be rewritten (For  $F = 1$ ) as,

$$1 = 3(n - 1) - 2(l)$$

or,

$$3n - 2l - 4 = 0 \quad (2.7)$$

This is Grubler's criterion for planar mechanisms of lower pairs. It is, however, possible to derive a criterion that predicts more than what  $F = 1$  condition dictates.

Mechanism number-synthesis is applied basically to linkages having turning pairs (pin joints) only. This does not, however, restrict its application to mechanisms with turning pairs alone. For it has been shown by kinematicians that, having once developed complete variety of pin jointed mechanisms, the method can most readily be converted to accommodate cams, gears, belt drives, hydraulic cylinder mechanisms and clamping devices.

Following deductions will be useful in deriving possible link combinations of a given number of links for a given degree of freedom. It is assumed that all joints are simple and there is no singular link.

### 2.17.1 Effect of Even/Odd Number of Links on Degrees of Freedom

From equation (2.5),

$$F = 3(n - 1) - 2l$$

Rewriting this equation, we have

$$\begin{aligned} l &= \text{number of turning pairs} \\ &= \left[ \frac{3(n - 1) - F}{2} \right] \end{aligned} \quad (2.8)$$

Since total number of turning pairs must be an integer number, it follows that either  $(n - 1)$  and  $F$  should be both even or both odd. Thus, for  $l$  to be an integer number:

- (1) If d.o.f.  $F$  is odd (say, 1, 3, 5 ...),  $(n - 1)$  should also be odd. In other words,  $n$  must be even.
- (2) If d.o.f.  $F$  is even (say 2, 4 ...),  $(n - 1)$  should also be even. In other words for  $F$  to be even,  $n$  must be odd.

*Summing up, for d.o.f.  $F$  to be even,  $n$  must be odd and for  $F$  to be odd,  $n$  must be even.*

### 2.17.2 Minimum Number of Binary Links in a Mechanism

Let

$$\begin{aligned} n_2 &= \text{number of binary links} \\ n_3 &= \text{number of ternary links} \\ n_4 &= \text{number of quaternary links} \\ &\vdots \\ n_k &= \text{number of } k\text{-nary links.} \end{aligned}$$

The above number of links must add up to the total number of links in the mechanism. Thus,

$$n = n_2 + n_3 + n_4 + \dots + n_k \quad (2.9)$$

or

$$n = \sum_{i=2}^k n_i$$

Since discussions are limited to simple jointed chains, each joint/pair consists of two elements. Thus, if  $e$  is total number of elements in the mechanism, then

$$e = 2(l) \quad (2.10)$$

By definition binary, ternary, quaternary, etc. links consist of 2, 3, 4 elements respectively. Hence, total number of elements are also given by

$$e = 2n_2 + 3n_3 + 4n_4 + \dots + k(n_k) \quad (2.11)$$

Comparing right hand side of equations (2.10) and (2.11), we have

$$2l = 2n_2 + 3n_3 + 4n_4 + \dots + k(n_k) \quad (2.12)$$

Substituting for  $n$  and  $2l$  from (2.9) and (2.12) in (2.5), we have

$$F = 3 [(n_2 + n_3 + n_4 + \dots + n_k) - 1] - [2n_2 + 3n_3 + 4n_4 + \dots + kn_k]$$

Simplifying further,

$$F = [n_2 - n_4 - 2n_5 - 3n_6 - \dots - (k - 3) n_k] - 3 \quad (2.13)$$

or rearranging,

$$n_2 = (F + 3) + [n_4 + 2n_5 + 3n_6 + \dots + (k - 3) n_k] \quad (2.14)$$

Thus, number of binary links required in a mechanism depend on d.o.f. and also on the number of links having elements  $> 3$ . Sacrificing exactness for fear of a complex relation, minimum number of binary links can be deduced from eq. (2.14) as:

$$\begin{aligned} n_2 &\geq 4, \text{ for d.o.f.} = 1 \\ n_2 &\geq 5, \text{ for d.o.f.} = 2 \\ n_2 &\geq 6, \text{ for d.o.f.} = 3, \text{ etc.} \end{aligned} \quad (2.15)$$

This proves that minimum number of binary links for d.o.f. = 1 is 4, while the minimum number of binary links required for d.o.f. = 2 is 5.

### 2.17.3 Maximum Possible Number of Turning Pairs on any of the $n$ Links in a Mechanism

The problem is approached in an indirect manner. We pose the problem to be that of finding minimum number of links  $n$  required for closure when one of the links has largest number of elements =  $k$ . An attempt is now made to close the chain in Fig. 2.26 having a link  $A$  of  $k$  elements.

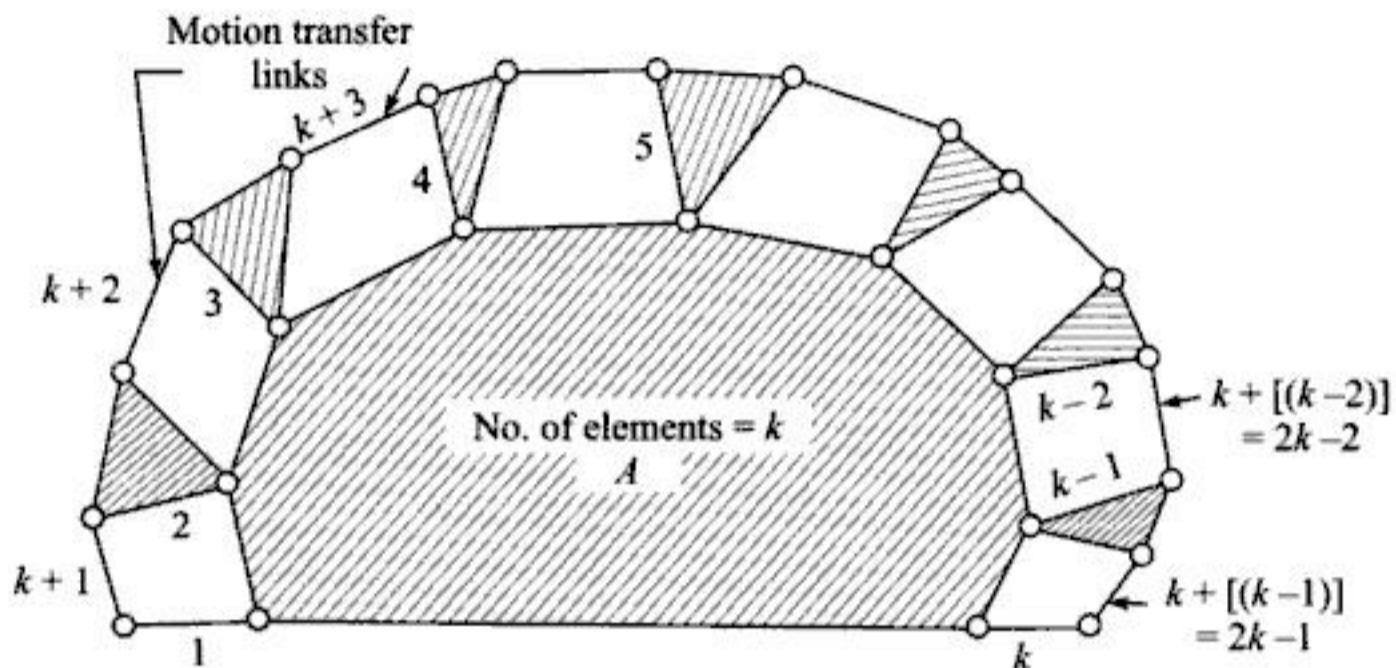


Fig. 2.26 Minimum number of link's required for closure.

For completing the chain with a minimum number of links involving no multiple joint, it is necessary to interconnect ternary links at all the elements of link  $A$  except the first and last element. Connecting ternaries at intermediate elements ensures a continuity of motion from link 1 to link  $k$ . Links directly connected to link  $A$  are labelled 1 through  $k$ , while the motion transfer links shown in Fig. 2.26 are numbered as  $(k + 1)$ ,  $(k + 2)$ ,  $(k + 3)$ , ...  $[k + (k - 2)]$ ,  $[k + (k - 1)]$ . The last motion transfer link is thus numbered as  $(2k - 1)$ . Clearly, minimum number of links required to complete the chain is  $(2k - 1)$ , besides the link of highest elements.

In other words, for a given number of links  $n = 2k$ , a link can have a maximum of  $k$  elements. Hence,

$$(k) = \frac{n}{2} \quad (2.16)$$

Thus when  $n$  is even, maximum possible number of elements which a link can have is  $n/2$ .

An important conclusion emerging out of eq. (2.13) is that the number of ternary links does not have any influence on degrees of freedom of a mechanism. According to B. Paul, for a mechanism with d.o.f. = 1,

$$C = \frac{1}{2} (n) - 1 \quad (2.17)$$

where  $C$  = number of independent circuit or loops  
and,  $n$  = number of links.

## 2.18 ENUMERATION OF KINEMATIC CHAINS

Let  $N$  be the number of links and  $F$  degree of freedom, for which all possible planar chains are needed to be established.

**Step 1.** For given  $N$  and  $F$  establish the number of joints (hinges) using

$$l = \frac{3N - (F + 3)}{2} \quad (2.18)$$

For d.o.f. = 1, this reduces to

$$l = \frac{3N - 4}{2}$$

**Step 2.** For given  $N$ , establish maximum number of elements permissible on any link, using

$$k = \frac{N}{2}, \text{ for } F = 1, 3, 5$$

and

$$k = \frac{(N + 1)}{2}, \text{ for } F = 2, 4, \dots$$

**Step 3.** Substituting expression for  $N$  and  $2l$ , namely,

$$N = n_2 + n_3 + n_4 + \dots + n_k$$

and

$$2l = 2n_2 + 3n_3 + 4n_4 + \dots + k(n_k)$$

in Grubler's eq. (2.5),

$$F = 3 [(n_2 + n_3 + n_4 + \dots + n_k) - 1] - [2n_2 + 3n_3 + 4n_4 + \dots + kn_k] \quad (2.19)$$

we get,

$$F = [n_2 - (n_4 + 2n_5 + \dots + (k - 3)n_k) - 3] \quad (2.20)$$

Thus for  $F = 1$ ,

$$n_2 - n_4 - 2n_5 + \dots + (k - 3)n_k = 4 \quad (2.21)$$

Above equations may be used to list all possible combinations of  $n_2, n_4, n_5, \dots$  which satisfy above conditions.

**EXAMPLE 2.8** Enumerate all chains possible with  $N = 6$  and  $F = 1$ .

**Solution:** Total number of hinges  $l = \frac{3(6) - 4}{2} = 7$

Also, for even number of links ( $N = 6$ ), maximum number of hinges on any link =  $6/2 = 3$ . Thus the chains will consist of binary and ternary links only. Hence, we have from eqs. (2.9) and (2.12),

$$n_2 + n_3 = N = 6 \quad \text{and} \quad 2n_2 + 3n_3 = 2l = 14 \quad (2.22)$$

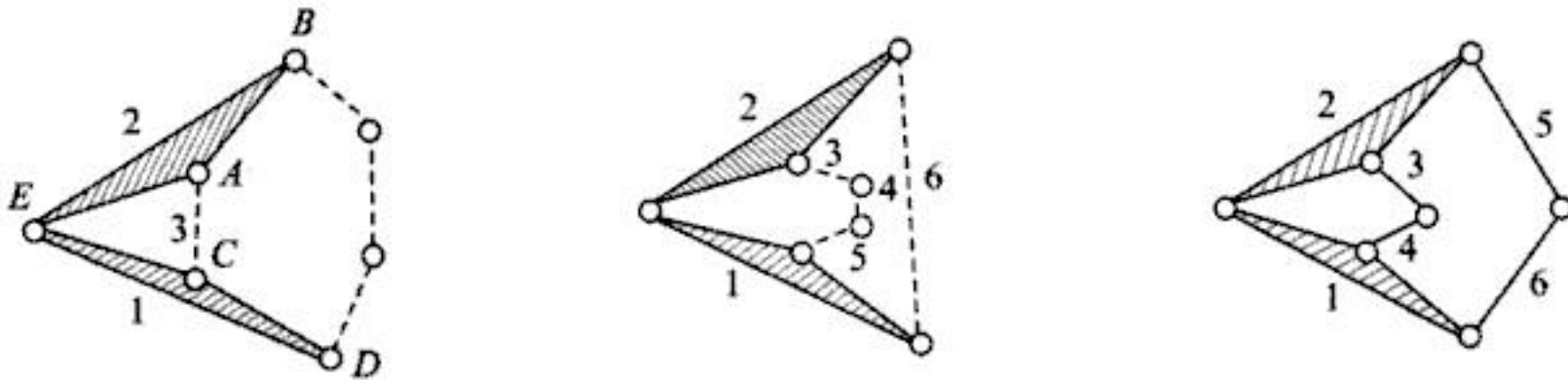
Substituting in Grubler's criterion,  $3N - 2l - 4 = 0$ ,

we have,  $3(n_2 + n_3) - (2n_2 + 3n_3) - 4 = 0$  or  $n_2 - 4 = 0$

Thus,  $n_2 = 4$  and from eq. (2.22),  $n_3 = 2$ .

We begin by considering the ways in which links of highest degree (i.e., links having largest number of elements) can be interconnected. The two ternaries can be either connected directly through a common pair or can be connected only through one or two binary links.

In Fig. 2.27 we consider the first possibility. The two ternaries of Figs. 2.27(a) and (b) can not be connected through a single link as it amounts to forming a structural loop (3-link loop). The only way to connect them through 4 binaries (avoiding formation of a 3-link loop) is therefore, as shown at Figs. 2.27(c), which gives Watt's chain.



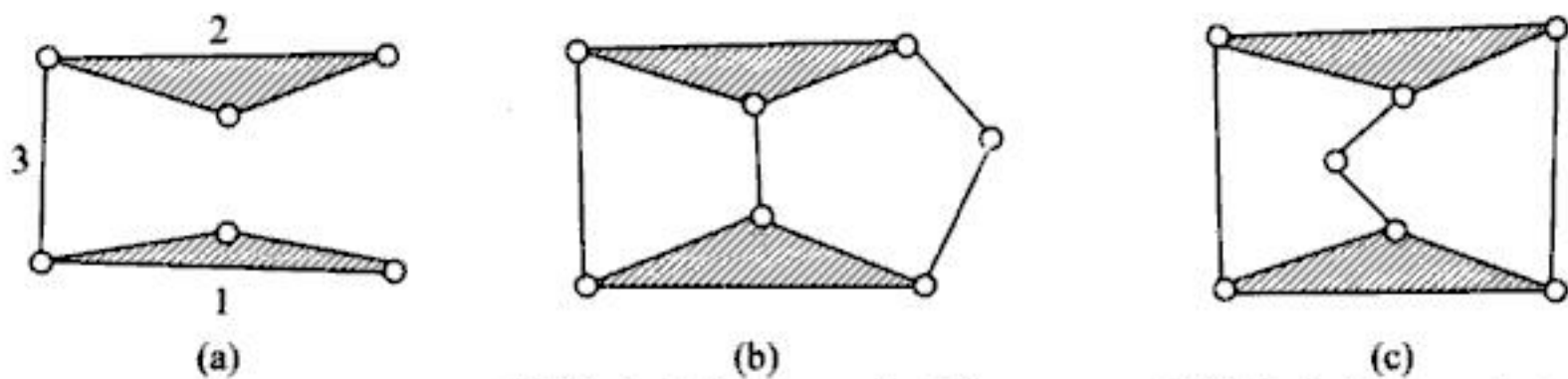
(a) Undesirable due to existence of 3-link loop 1-2-3

(b) Undesirable due to formation of 3-link loop 1-2-6

(c) Permissible combination as no 3-link loop exists

Fig. 2.27

Considering the second alternative, Fig. 2.28(a) shows the two ternary links 1 and 2 being connected through a single binary link 3. Then, between one of remaining two pairs of elements of links 1 and 2, we may introduce a single binary and between the other pair of elements, two remaining binaries. The resulting arrangements are as at Fig. 2.28(b) and (c). It is easy to verify that arrangements at Figs. 2.28(b) and (c) are structurally the same.



(a)

(b)

(c)

Valid chain Stephenson's chain

Valid chain (same as that at b)

Fig. 2.28

**EXAMPLE 2.9** Enumerate all possible chains of  $N = 7$  and d.o.f. = 2.

**Solution:** Total number of hinges  $l = \frac{3(N) - (F + 3)}{2} = \frac{3(7) - (2 + 3)}{2} = 8$

Maximum number of elements on any link :  $\leq (N + 1)/2 = 4$ .

Hence, only binary, ternary and quaternary links are possible. Thus, from eqs. (2.9) and (2.12),

$$\begin{aligned}n_2 + n_3 + n_4 &= N (= 7) \\ 2n_2 + 3n_3 + 4n_4 &= 2l (= 16)\end{aligned}$$

Substituting in Grubler's equation,

$$F = 3(N - 1) - 2l$$

we have

$$2 = 3[(n_2 + n_3 + n_4) - 1] - (2n_2 + 3n_3 + 4n_4)$$

or

$$n_2 - n_4 = 5$$

Thus the possible combinations are (Note that for any mechanism with  $F = 2$ ,  $n_2 \geq 5$ ):

$$\begin{aligned}n_4 = 1; & \quad n_2 = 6 \\ n_4 = 0; & \quad n_2 = 5, \quad (\text{Also check that } n_2 + n_3 + n_4 = 7).\end{aligned}$$

Obviously, remaining links in above combinations will be the ternaries. Thus the two combinations possible are:

$n_4$	$n_3$	$n_2$	Total $N$
1	0	6	7
0	2	5	7

Different chains that can be formed are as shown in Figs. (a), (b), (c) and (d) of 2.29. Chain at Fig. 2.29(a) involves a quaternary link with remaining 6 binary links forming two independent loops of d.o.f. = 1. A mechanism of 2 d.o.f. is possible only when any link other than quaternary is fixed. Chain at Fig. 2.29(b) involves two ternary links that are directly connected. A binary cannot be used singly to connect these ternaries at any of the remaining pairs of elements as that leads to a 3-link loop. Therefore, the only option is to connect these ternaries through two binaries and through three binaries at remaining pairs of elements. This is shown in Fig. 2.29(b).

When the two ternaries are connected through a single binary, the two possible ways of incorporating remaining 4 binaries are shown at Figs. 2.29(c) and (d).

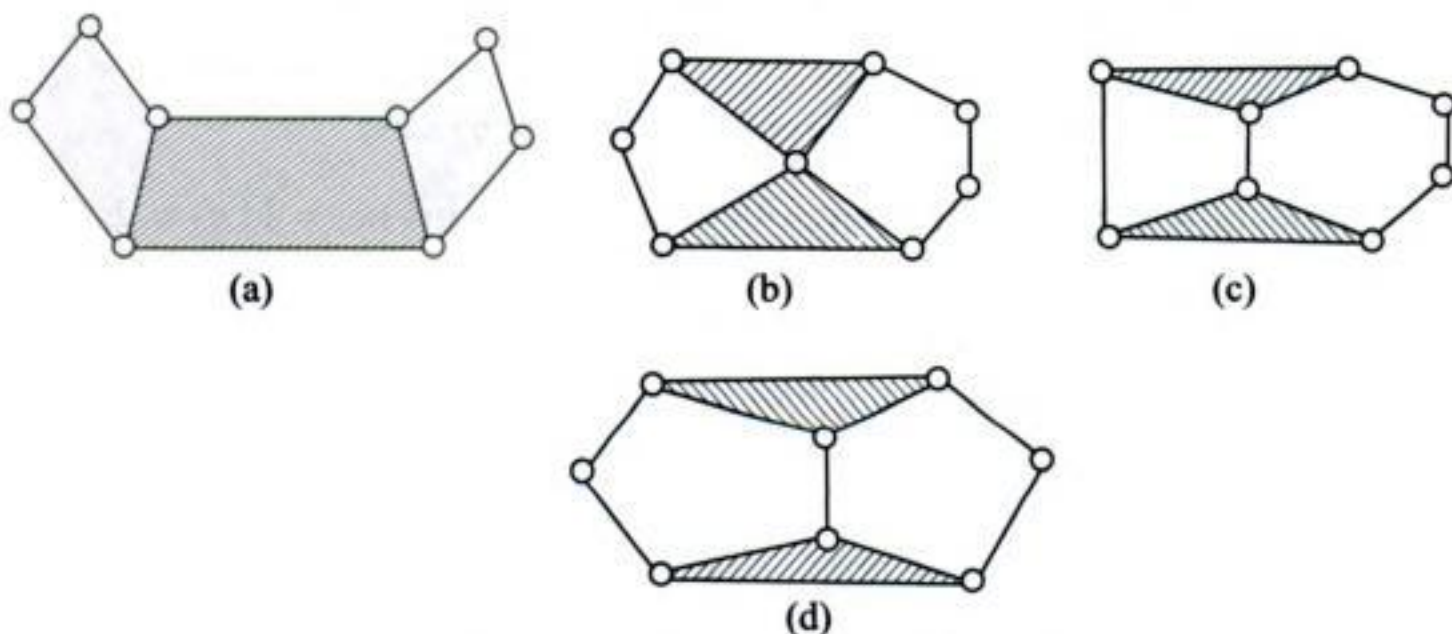


Fig. 2.29 Feasible chains on seven links.

**EXAMPLE 2.10** Enumerate combination of links possible in case of a 8-link chain with  $F = 1$ .

**Solution:** No. of hinges  $l = \frac{3(8) - 4}{2} = 10$

Maximum number of elements on one link =  $\frac{8}{2} = 4$ .

Hence the chains can have binary, ternary and quaternary links only. From equations (2.9) and (2.12).

$$n_2 + n_3 + n_4 = 8 \quad \text{and} \quad 2n_2 + 3n_3 + 4n_4 = 20$$

Substituting in Grubler's criterion,

$$3(n_2 + n_3 + n_4) - (2n_2 + 3n_3 + 4n_4) - 4 = 0 \quad \text{or} \quad (n_2 - n_4) = 4$$

We study various combinations indicated by above expression in respect of their viability:

$n_4$ (assumed)	$n_2 = (n_4 + 4)$	$n_3 = 8 - (n_2 + n_4)$	Remark
4	8	-	Not acceptable as $n_2 + n_4 > N$
3	7	-	Not acceptable as $n_2 + n_4 > N$
2	6	-	acceptable
1	5	2	acceptable
0	4	4	acceptable

Thus the three valid combination of links are:

- (i)  $n_4 = 2; n_3 = 0; n_2 = 6$
- (ii)  $n_4 = 1; n_3 = 2; n_2 = 5$
- (iii)  $n_4 = 0; n_3 = 4; n_2 = 4$

The first combination yields the following two chains ( $n_4 = 2; n_3 = 0; n_2 = 6$ ). (Fig. 2.30)

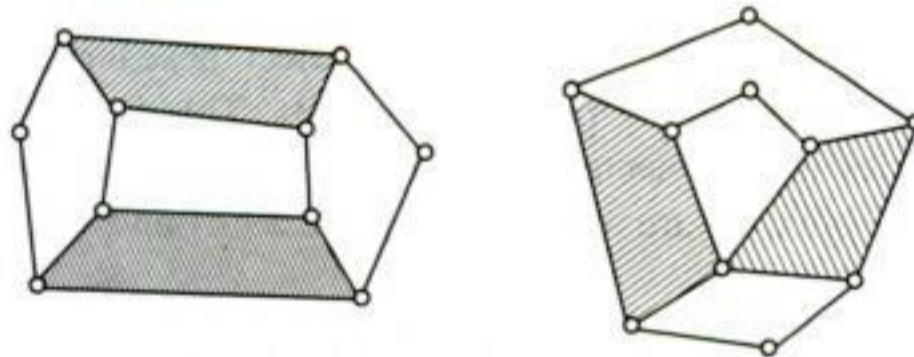


Fig. 2.30 Feasible chains for first combination.

The second combination of links yields the following five chains ( $n_4 = 1; n_3 = 2; n_2 = 5$ ) (see Fig. 2.31).

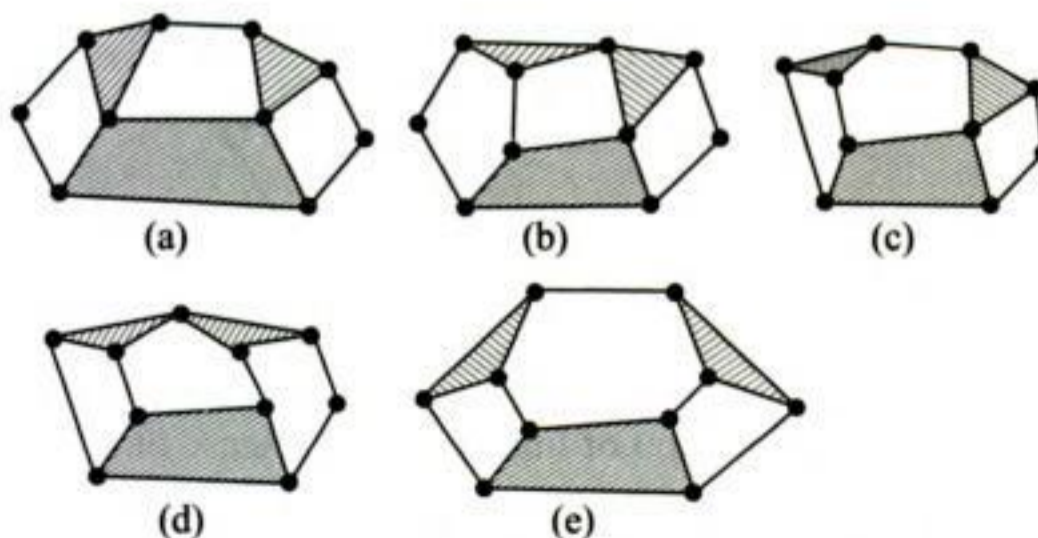


Fig. 2.31 Feasible chains for second combination.



The third combination ( $n_4 = 0 ; n_3 = 4 ; n_2 = 4$ ) of links yields following chains (see Fig. 2.32).

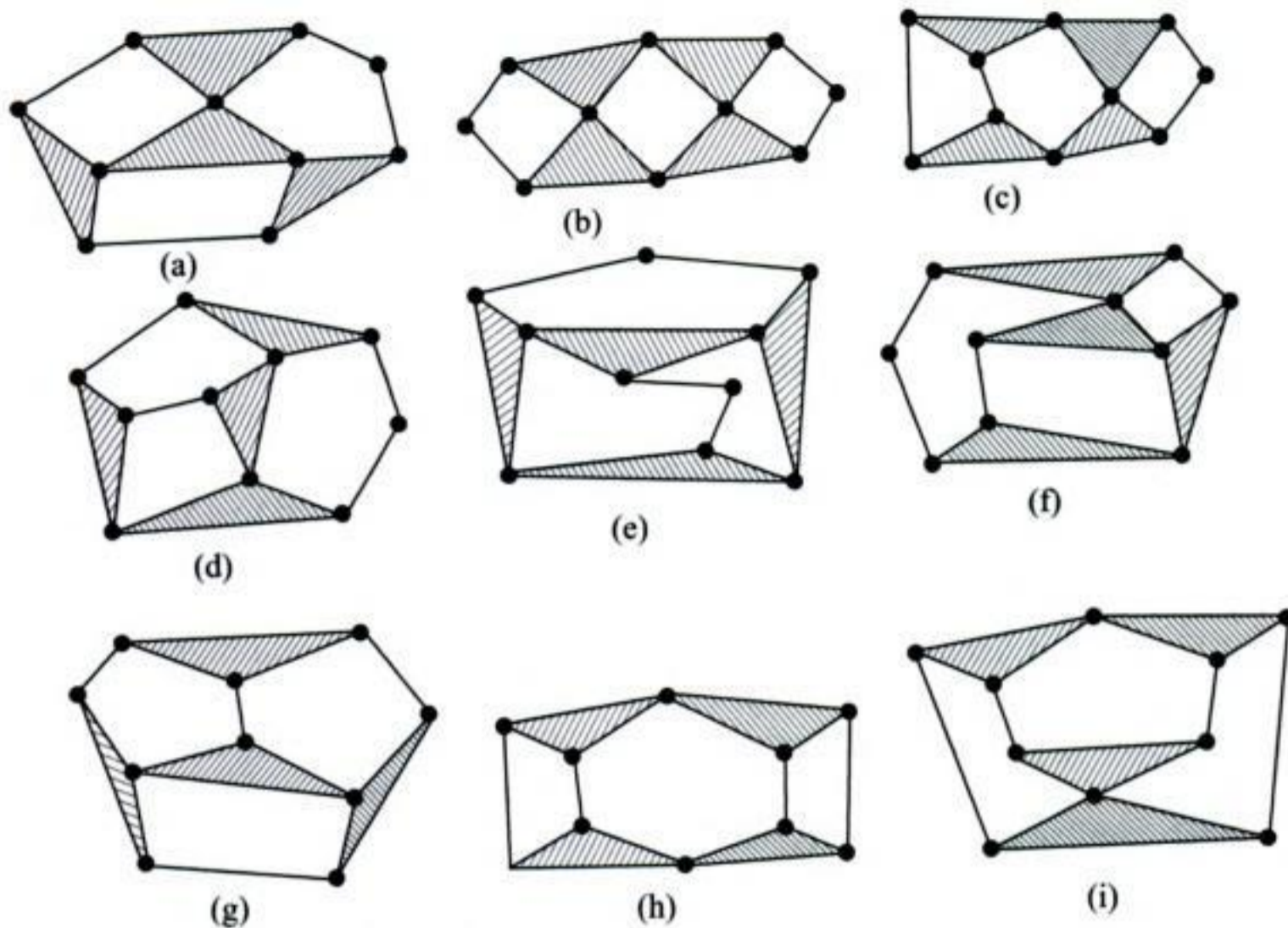


Fig. 2.32 Feasible chains for third combination.

## 2.19 SPATIAL MECHANISMS

As distinct from a planar mechanism, in a spherical mechanism, each link has some point which remains stationary as the links move and in which the stationary points of all the links lie at a common location (the centre of an imaginary sphere). The locus of each point on a link is a curve lying on an imaginary spherical surface. These spherical mechanisms are constructed out of revolute pairs, and the axes of all revolute pairs must intersect at a point.

Spatial mechanisms do not incorporate any restriction on the relative motions of the particles. A spatial mechanism may have particles describing paths of double curvature.

Grubler's criterion was originally developed for planar mechanisms. If similar criterion is to be developed for spatial mechanisms, we must remember that an unconnected link has six d.o.f. in place of 3. As such, by fixing one link of a chain the total d.o.f. of  $(n - 1)$  links separately will be  $6(n - 1)$ . Again a revolute and prismatic pair would provide 5 constraints (permitting 1 d.o.f.), rolling pairs will provide 4 constraints, and so on. Hence, an expression for d.o.f. of a closed spatial mechanism can be written as:

$$F = 6(N - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5 \quad (2.23)$$

where

$N$  = total number of links

$j_1$  = number of joints providing 5 constraints

$j_2$  = number of joints providing 4 constraints

$j_3$  = number of joints providing 3 constraints

$j_4$  = number of joints providing 2 constraints and

$j_5$  = number of joints providing only one constraint.

## 2.20 MANIPULATORS

Manipulators are attracting wide interest in many branches of manufacturing industry and assembly sections, especially where computer-control is advantageous. A more common dictionary meaning of the word *manipulation* is 'handling with skill'. In engineering practice, a manipulator can be described as a mechanical device which is suitably actuated and controlled so that it can function as a substitute for (with in built improvements over) human manipulation by fingers on a hand supported or a wrist at the end of an arm connected to a fixed base through a joint, comparable with human shoulder.

Last three to four decades have witnessed dramatic advances in tools, extending limited capability of fingers and hands, mainly due to the advent of sensors and highly precise servo-devices. These devices function in response to instructions given either by an operator on a master system, guiding his 'slave' or independently of an operator, solely through the medium of a digital computer. Master slave system was used in past to handle radioactive substances in an environment, similar to a chemistry laboratory, which the operator can observe through a window or on a television monitor. There are other kinds of tasks which an operator is physically capable of performing directly, but being repetitive and boring, can be handled through manipulators.

Subject of kinematics relates closely to manipulators because, their mechanical functioning must be based on a system of relatively moving bodies. Components of manipulators are assumed to be rigid and the joints between them are kinematic pair – lower pairs in particular. Kinematic behaviour, especially the freedom and constraints of joints, divides the capability and potential of a manipulator-arm.

### REVIEW QUESTIONS

- 2.1 Define the term *Link*. Can spring, belt, liquids be treated as Links? Justify your answer.
- 2.2 Explain how I.C. engine mechanism and steam engine mechanism are kinematically identical.
- 2.3 Define the terms *Element* and *Pair*.
- 2.4 Explain the term *kinematic* with a schematic diagram. What is its role in kinematic analysis and synthesis?
- 2.5 Define the terms *machine* and *mechanism*. How do they differ?
- 2.6 Explain various methods of classifying pairs, giving three examples of each category.
- 2.7 Distinguish between higher and lower pairs. Is there some more convincing definition than the conventional one, based on point/line and area contact basis?
- 2.8 'Slider crank mechanism is a special case of 4-bar mechanism'. Justify the statement.
- 2.9 Define the term *Inversion*. What are the properties of Inversion? Explain advantages arising out of the concept of Inversion.

- 2.10 Explain with suitable sketches 'Inversions of slider crank chain.'
- 2.11 Explain Grashof's criterion and describe inversions of 4-bar chain giving suitable sketches.
- 2.12 Define terms *degrees of freedom* and *constrained motion* of a mechanism.
- 2.13 Distinguish between a chain structure and a mechanism.
- 2.14 Proceeding systematically, derive expression for degrees of freedom of a chain and mechanism.
- 2.15 Distinguish between the terms *mobility* and *moveability*. State Grubler's equation for mobility of a mechanism. What is the interpretation when mobility turns out to be  $-1$ ,  $0$ ,  $+1$  and  $+2$ ?
- 2.16 In a slotted lever quick-return mechanism, distance between the fixed centres is  $7.5$  cm and radius of driving crank is  $4$  cm. Find the ratio of the time taken during cutting stroke and return stroke. (Ans. 2.105)
- 2.17 Find out degrees of freedom of mechanisms shown in Fig. 2.33.

[Ans. (a) = 3 ; (b) 3; (c) = 2]

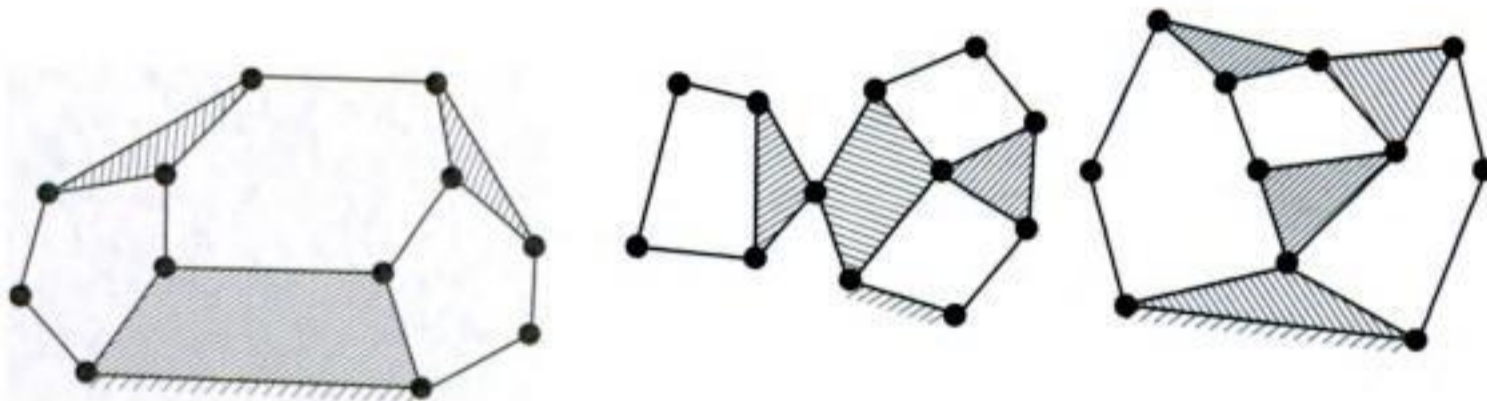


Fig. 2.33

- 2.18 Determine mobility of the mechanisms shown in Fig. 2.34.

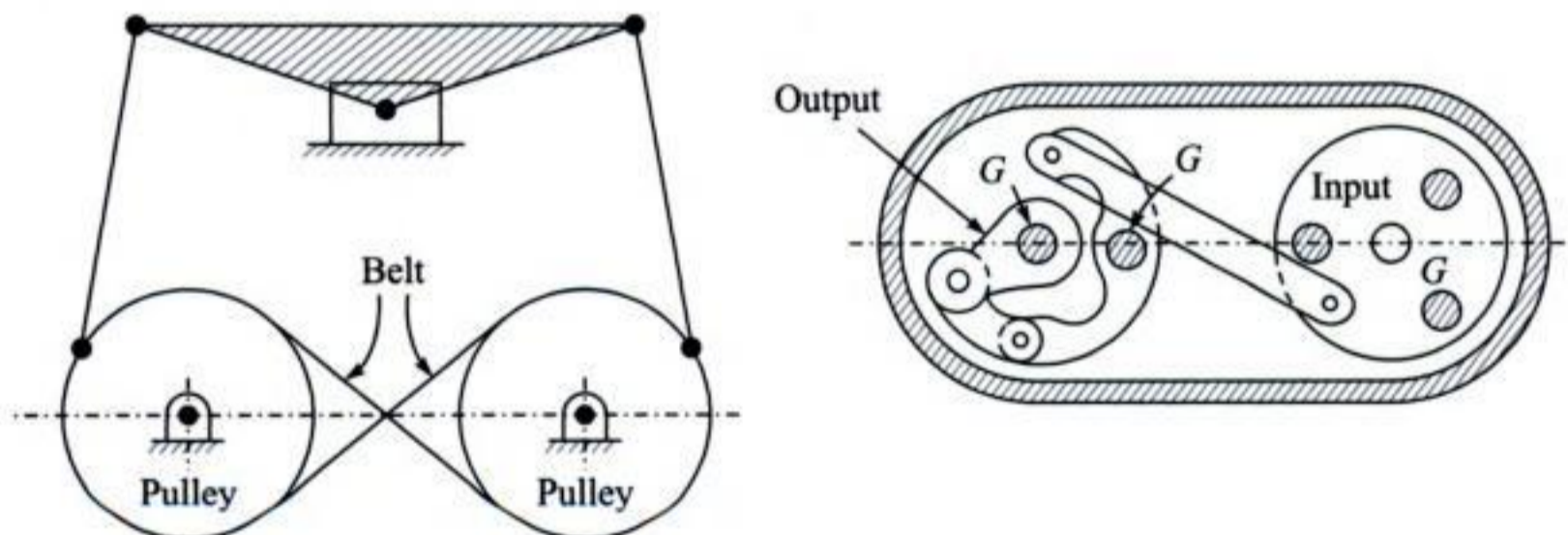


Fig. 2.34

- 2.19 What do you understand by mechanical advantage in a mechanism? How is it measured and what are the desirable limits on range of operation of mechanisms?
- 2.20 For  $N = 8$  and d.o.f. = 3, list all possible combination of links.
- 2.21 Enumerate all possible chains for—
- $N = 6$  and d.o.f. = 1
  - $N = 7$  and d.o.f. = 2

- 2.22 Show that number of ternary links has no effect on d.o.f. of a mechanism.  
 2.23 Show that maximum possible number of elements on any link of a chain on  $N$  links is  $N/2$ .  
 2.24 A grab-bucket, shown diagrammatically in Fig. 2.35, consists of eight links. Find its d.o.f.

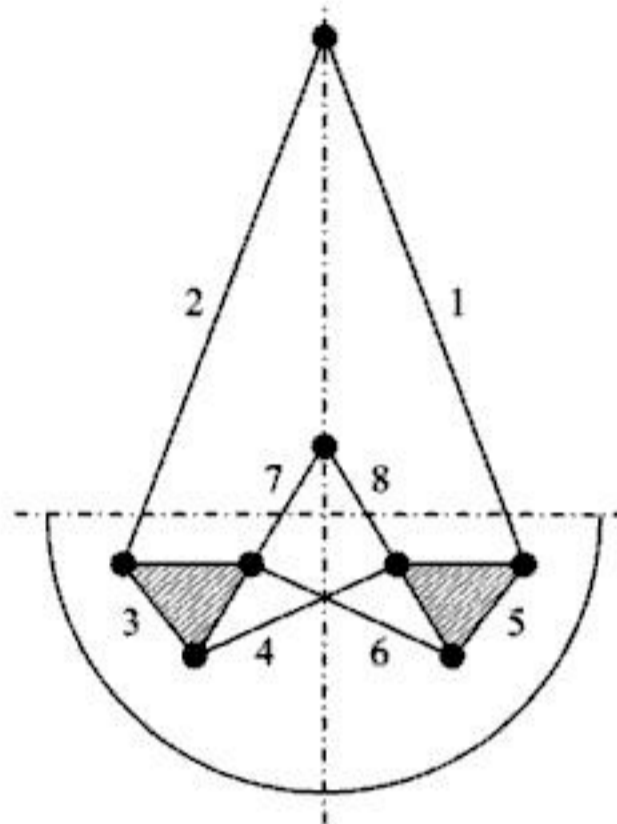


Fig. 2.35

# VELOCITY AND ACCELERATION ANALYSIS (GRAPHICAL APPROACHES)

## 3.1 INTRODUCTION

Design and analysis of machinery and mechanisms relies heavily on the skill and ability of a designer to visualise relative motion between machine components. With the ever increasing use of high speed machines, velocity and acceleration analysis have become indispensable in machine design process. Kinematic analysis usually aims at determining motion characteristics (like displacement, velocity and acceleration) of various links for a given input motion. But in yet another category of problems, a kinematician may also be required to establish the type of input motion needed for providing a desired output motion.

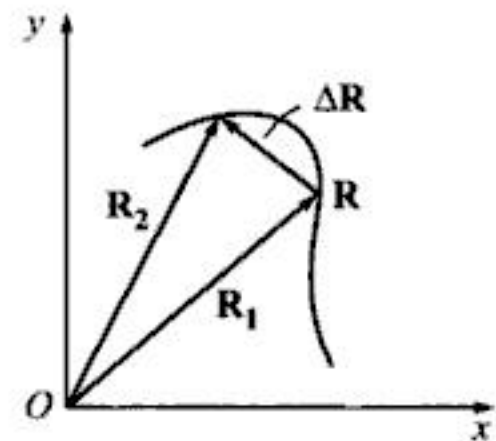
Present chapter is devoted to velocity and acceleration analysis of lower paired mechanisms only. The methods can, however, be applied to equivalent lower paired mechanisms in case kinematic analysis is needed for higher paired mechanisms. With the advent of high speed digital computers, coupled with ease in availability of PCs in technical institutions, analytical methods of velocity and acceleration analysis are becoming more and more popular. But this does not reduce the importance of graphical methods. Graphical methods are essential in developing a conceptual understanding about the subject matter. Besides, they provide the fastest method of checking results of machine computation.

## 3.2 LINEAR AND ANGULAR VELOCITY

**(A) Linear Velocity.** Consider a particle  $R$  to move along a curved path and let it shift its position from  $R_1$  to  $R_2$  along the curved path during a time interval  $\Delta t$ . The small displacement  $\Delta R$  occurring during this interval can be expressed in terms of position vectors as

$$\Delta R = R_2 - R_1$$

If displacement  $\Delta R$  occurs over a small interval of time  $\Delta t$ , (Fig. 3.1), the rate of change of displacement or the instantaneous velocity is given by



**Fig. 3.1** Displacement of a particle.

$$V = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta R}{\Delta t} \right) = \left( \frac{dR}{dt} \right) \quad (3.1)$$

The instantaneous value of velocity (which will be called simply as 'velocity' henceforth) is thus given by time derivative of position vector.

Many a time it is quite convenient to use Newtonian notations for time derivatives. Thus,

$$V = \frac{dR}{dt} = \dot{R} \quad (3.2)$$

when direction of motion is implied in the problem, it may be sufficient to deal with magnitudes only. In such cases, the velocity is given by

$$v = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta r}{\Delta t} \right) = \frac{dr}{dt} \quad (3.3)$$

$$\therefore v = \dot{r}$$

**(B) Angular Velocity.** Discussions for linear velocity can be extended to cover angular velocity aspect also. Let  $\Delta\theta$  be the angular displacement in time interval  $\Delta t$  of a rotating line, rotating position vector or of a rotating rigid body. The average angular velocity during this time  $\Delta t$  is,

$$(\omega)_{\text{average}} = \left( \frac{\Delta\theta}{\Delta t} \right)$$

The instantaneous value of angular velocity may be obtained by taking a limit. Thus,

$$\omega = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta\theta}{\Delta t} \right)$$

$$\text{or } \omega = \frac{d\theta}{dt} = \dot{\theta} \quad (3.4)$$

The angular velocity  $\omega$  can act either in clockwise or counterclockwise sense. When the c.w. and c.c.w. sense of rotation is taken care of by assigning positive and negative signs, it is not necessary to treat them as vectors. The usual unit of angular velocity is radians per second.

At times, it becomes appropriate to treat angular velocity also as a vector. In such a case, the angular velocity vector is represented by right hand screw rule. Let us assume that the line or body is rotating about origin in  $x - y$  plane in, say, clockwise sense with angular velocity  $\omega$ . Such angular velocity is then represented by a vector placed normal to the plane of rotation ( $x - y$ ), with arrowhead pointing in the direction in which tip of the right handed screw moves. The screw is assumed to be held normal to the plane of rotation and rotated in the sense of rotation  $\Delta\theta$ . The length of vector is chosen to represent magnitude to some scale. (Fig. 3.2)

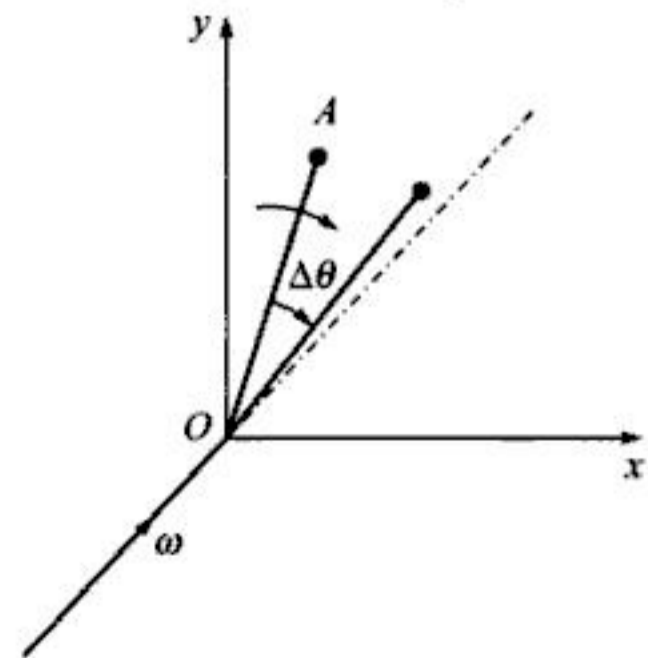


Fig. 3.2 Vector representation of angular velocity.

### 3.3 VELOCITY OF A POINT ON ROTATING RIGID BODY

In Section 1.4 motion of rotation of a body has been defined as a motion of a rigid body such that a line within the body does not remain parallel to itself. Fig. 3.3 shows a rigid body rotating in c.c.w. direction about an axis through  $O$  perpendicular to the plane of the paper. Let  $A$  be a point with coordinates  $x = r_a \cos \theta$  and  $y = r_a \sin \theta$  and the position vector of this point be

$$R_a = r_a e^{j\theta}$$

Differentiating  $x$  and  $y$  coordinates, the rectangular velocity components are

$$v^x = \frac{d}{dt} (r_a \cos \theta) \quad \text{and} \quad v^y = \frac{d}{dt} (r_a \sin \theta)$$

or

$$v^x = -r_a \left( \frac{d\theta}{dt} \right) \sin \theta \quad \text{and} \quad v^y = r_a \left( \frac{d\theta}{dt} \right) \cos \theta$$

Noting that  $\frac{d\theta}{dt} = \omega$ , the angular speed of rotation of body 1 w.r. to 2, we have

$$v^x = -r_a \omega \sin \theta \tag{3.5}$$

and

$$v^y = r_a \omega \cos \theta \tag{3.6}$$

The resultant of the above two vectors give resultant velocity as

$$v = \sqrt{(v^x)^2 + (v^y)^2}$$

or

$$v = \sqrt{(-r_a \omega \sin \theta)^2 + (r_a \omega \cos \theta)^2}$$

$$= (r_a \omega) \sqrt{\sin^2 \theta + \cos^2 \theta}$$

or

$$v = (r_a) \omega \tag{3.7}$$

The resultant velocity  $v$  makes an angle  $\phi$  with  $x$ -axis given by

$$\tan \phi = \left( \frac{v^y}{v^x} \right) = \frac{(r_a \omega) \cos \theta}{-(r_a \omega) \sin \theta}$$

or

$$\tan \phi = -\cot \theta \tag{3.8}$$

Thus

$$\tan \phi = \tan \left( \frac{\pi}{2} + \theta \right) \tag{3.9}$$

Therefore

$$\phi = \left( \frac{\pi}{2} + \theta \right)$$

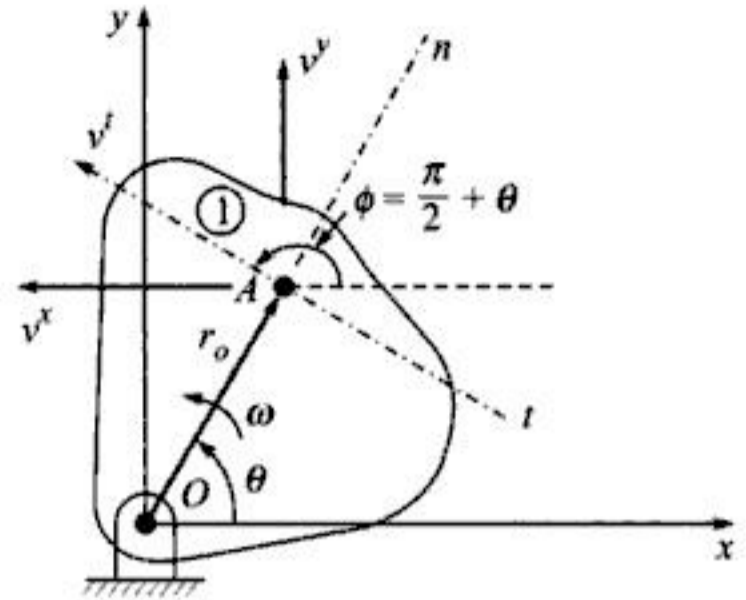


Fig. 3.3 Rotating rigid body.

Since this is the angle measured with respect to positive direction of  $x$ -axis, the resultant velocity  $V$  is at the right angles to the line  $OA$  and its sense will be commensurate with that of angular speed  $\omega$ .

We conclude therefore that in a rigid body, rotating about an axis through  $O$ , resultant velocity of a point  $A$  is equal to the product of radius  $r_a$  at point  $A$  and angular velocity  $\omega$ . Further, the direction of resultant velocity is perpendicular to the radial line in a sense commensurate with the sense of  $\omega$ . This resultant velocity  $V$  can therefore be treated as tangential velocity.

Above results can also be derived on the basis of complex polar notations. Thus, the position vector of point  $A$  is given by

$$R = (r_a) e^{j\theta}$$

Remembering that  $r_a$  is constant and  $j = \sqrt{-1}$ , differentiating w.r. to time,

$$\begin{aligned} V &= \frac{d}{dt}(R) = \frac{d}{dt}(r_a e^{j\theta}) \\ &= (0) e^{j\theta} + r_a \left( j \frac{d\theta}{dt} \right) e^{j\theta} \end{aligned}$$

or 
$$V = j(r_a \omega) e^{j\theta} \quad (3.10)$$

Magnitude of resultant velocity is given by  $(r_a \omega)$  while  $j(e^{j\theta})$  represents direction of this vector. Clearly, the direction indicated is the one obtained by rotating position vector through right angle in c.c.w. sense.

### 3.4 GRAPHICAL DIFFERENTIATION

When displacement as a function of time is available in the form of analytical expression, analytical method of differentiation can be conveniently used to obtain velocity. Sometimes, however, displacement v/s time data are available in a graphical or numerical form. Numerical methods of differentiation are best suited if data are available in numerical form.

There are two types of situations in which graphical differentiation can be used:

- (a) When displacement-time relationship  $x = f(t)$  is known and,
- (b) When displacement-crank angle of rotation, *i.e.*,  $x = f(\theta)$ , relationship is known.

In the latter category of the problem, graphical differentiation is of use only if crank speed  $\omega$  is assumed to be constant. For instance, consider the relation

$$x = f(\theta)$$

Differentiating w.r. to time,

$$\frac{dx}{dt} = \frac{df}{d\theta} \frac{d\theta}{dt}$$

or 
$$v = f'(\theta) \times \omega \quad (3.11)$$

As crank speed  $\omega$  is constant, it becomes a scaling factor and equation (3.11) can also be written as,

$$v \propto f'(\theta) \quad (3.12)$$



Differentiating (3.11) again w.r. to time,

$$\frac{dv}{dt} = \frac{d}{d\theta} [f'(\theta)] \times \frac{d\theta}{dt} \times \omega + f'(\theta) \times \frac{d\omega}{dt}$$

or  $a = \omega^2 f''(\theta) + (0)f'(\theta)$

or  $a = \omega^2 f''(\theta)$  (3.13)

This can also be rewritten as:

$$a \propto f''(\theta) \quad (3.14)$$

where  $f'(\theta) = \frac{df}{d\theta}$  and  $f''(\theta) = \frac{d^2f}{d\theta^2}$

In such problems, slope values of  $\theta$  v/s  $f(\theta)$  diagram can only give values which are proportional to velocity values on a common scale, but do not give actual values of velocities. This is apparent from eq. (3.11) in which  $\omega$  is not known. In a similar way, slopes of  $\theta$  v/s  $f'(\theta)$  diagram do not give actual acceleration values.

First category of problems are, however, comparatively straight forward. This is because,

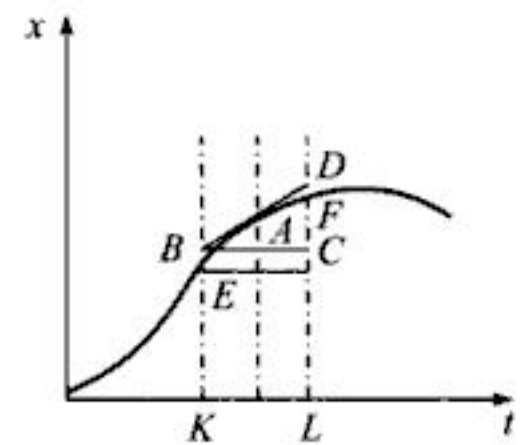
$$x = f(t)$$

and  $\frac{dx}{dt} = \text{velocity} = \frac{df}{dt}$  (3.15)

= slope of displacement curve

Thus, slope of the time v/s displacement curve itself gives actual velocity at the given time.

As seen in Fig. 3.4, if two ordinates  $KE$  and  $LF$  enclose a strip on the displacement–time curve, the slope of the chord  $EF$  of the curve will be approximately equal to the slope of the tangent to the curve on a point mid-way between  $K$  and  $L$ . And therefore, the slope of the chord  $EF$  also represents velocity approximately. Smaller is the error involved in this approximation, shorter the interval  $KL$  is. On the other hand, smaller the interval  $KL$  is, greater is the percentage error in determining difference in length of the ordinates  $KE$  and  $LF$ . A better method of drawing velocity/time diagram and acceleration-time diagram is therefore as under:



**Fig. 3.4** Displacement v/s time diagram.

For the displacement-time curve, as shown in Fig. 3.5(b), choose a pole distance  $Pa$  in Fig. 3.5(a) as per convenience along time axis, and draw a line  $ab$  parallel to displacement axis. Now, if from the pole  $P$  lines are drawn parallel to tangents at various points on the displacement time curve, we get triangles on same base  $Pa$ . For any point  $Q$  on the curve,  $Pq$  is the line drawn parallel to tangent to the displacement curve. In that case  $aq/Pa$  gives slope of the displacement curve at  $Q$  and therefore, the speed at point  $Q$ . Similarly, for point  $R$  on the displacement curve  $Pr$  is the line drawn parallel to the slope of curve at  $R$  and so  $ar/Pa$  gives the slope of curve and the velocity at point  $R$  on displacement curve.

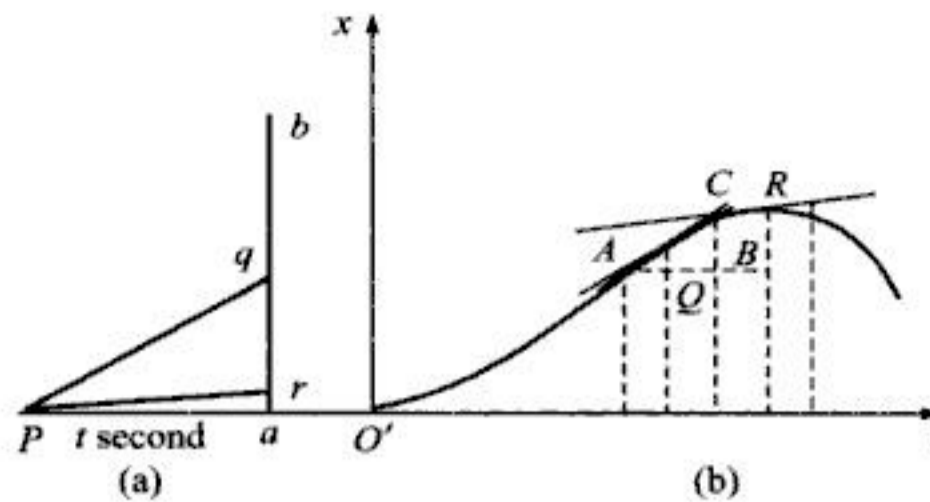


Fig. 3.5 Graphical Differentiation.

In Fig. 3.5(b) the strip width  $AB$  and the line of tangency at  $Q$  encloses a triangle  $ABC$ . In Fig. 3.5(a), line  $Pq$  is drawn parallel to tangent  $AC$  in Fig. 3.5(b). Hence, triangles  $Pqa$  and  $ACB$  are similar. Therefore,

$$v_Q = \frac{\delta x}{\delta t} = \frac{BC}{AB} = \left( \frac{aq}{Pa} \right) \quad (3.16)$$

And so,

$$aq = (Pa)v_Q$$

The method of graphical differentiation thus consists in dividing. Dividing the speed diagram into a number of vertical strips, not necessarily of same width, lines radiating from  $P$  may be drawn parallel to tangent at mid-point of strip on the curve. Since the base  $(Pa)$  remains same in Fig. 3.5(a), the intercepts made on  $x$ -axis by all such radiating lines are clearly proportional to the mean speeds for respective strips. Therefore, using information in Fig. 3.5(a) a speed-time curve can be quickly drawn by projecting these intercepts on  $ab$  in Fig. 3.5(a) on to the mid-ordinates of the corresponding strip.

Let us now establish the scale to which the ordinates of the speed-time curve represents the speed. Let the displacement scale be  $C_x$  m per cm and the time scale be  $C_t$  seconds per cm. Further, let the speed scale to which speeds are plotted in speed-time curve be  $C_v$  m/s per cm. Let the polar distance be taken as equal to  $t$  seconds.

Then, the velocity at a mid-point of a strip like  $Q$ ,

$$V_Q = \left( \frac{\delta x}{\delta t} \right) = \left( \frac{C_x BC}{C_t AB} \right) \quad (3.17)$$

But 
$$\frac{BC}{AB} = \left( \frac{aq}{Pa} \right) \quad (3.18)$$

As  $C_t$  is the scale along time axis,

$$(C_t)(Pa) = t$$

Therefore 
$$(Pa) = (t/C_t) \text{ cm}$$

Substituting for  $(BC/AB)$  and  $(Pa)$  in equation (3.17),

$$V_Q = \left( \frac{C_x}{C_t} \right) \left( \frac{aq}{t/C_t} \right)$$

Therefore 
$$V_Q = \left( \frac{C_x}{t} \right) aq \quad (3.19)$$

But, 
$$V_Q = (C_v) aq$$

Substituting for  $V_Q$  in eq. (3.19),

$$(C_v) aq = \left( \frac{C_x}{t} \right) aq \quad \text{or} \quad C_v = \left( \frac{C_x}{t} \right) \quad (3.20)$$

Advantage of the above method lies in its flexibility. The method permits variation in the strip width (*i.e.* time interval) depending on the curvature of displacement-time curve. Thus, a shorter strip width may be employed where radius of curvature of displacement curve is small. This increases accuracy. Similar discussions are possible for acceleration-time curve also.

### 3.5 RELATIVE VELOCITY

One of the dilemmas faced while preparing write-up of this chapter was in respect of the need to draw distinction between motion of two different points on the same body (*e.g.*, between two pin axes of a connecting rod) and motion between two different points on different bodies (*e.g.*, motion between two coincident points on sliding elements of slider and slotted lever of quick return mechanism). It is probably more appropriate to call the former as 'motion-difference' and the latter as 'apparent motion', rather than including both of them in a single common term 'relative motion'. However, it is rather too early to expose students to this subtle difference at U.G. level. Hence, we propose to continue with the use of conventional terms *Relative motion*, *Relative velocity*, etc., in chapters on velocity and acceleration analysis.

Figure 3.6 shows two independent particles  $A$  and  $B$  which are capable of moving with velocities  $V_A$  and  $V_B$  in the directions shown. Then, velocity of  $B$  relative to  $A$  is the velocity with which point  $B$  appears to move in a coordinate frame which is attached to  $A$  and moves with it. Relative velocity of  $B$  w.r. to  $A$  can also be defined as the velocity with which  $B$  appears to move to an observer situated at  $A$  and moving with it. Apart from the method illustrated in section (1.10), there is another way to illustrate the concept of relative velocity. In this method, particle  $A$  is brought to standstill by applying a velocity equal and opposite to  $V_A$  to both points  $A$  and  $B$ . In doing so, the relative motion between  $A$  and  $B$  is not changed but absolute motion of  $A$  is brought to zero. With  $A$  brought to standstill condition, resultant of velocities  $V_B$  and  $(-V_A)$  applied at  $B$  gives relative velocity of  $B$  w.r. to  $A$ . This is shown by vector  $V_{BA}$  in Fig. 3.6. It follows that vector sum of  $V_B$  and  $(-V_A)$  gives  $V_{BA}$ , the relative velocity of  $B$  w.r. to  $A$ , *i.e.*

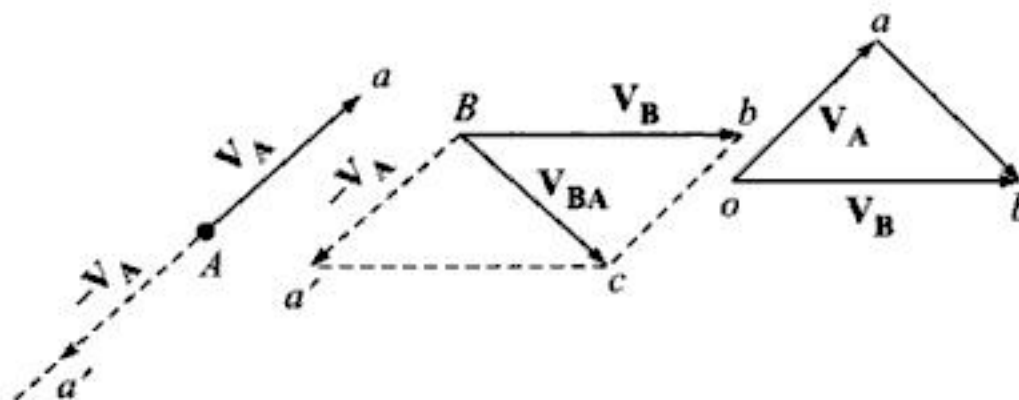


Fig. 3.6 Relative velocity of particle  $B$  w.r. to particle  $A$ .

$$\mathbf{V}_{BA} = \mathbf{V}_B + (-\mathbf{V}_A)$$

or 
$$\mathbf{V}_{BA} = \mathbf{V}_B - \mathbf{V}_A \quad (3.21)$$

Figure 3.6 also shows a triangle  $oab$  in which  $ob$  and  $ab$  represent velocities  $\mathbf{V}_B$  and  $\mathbf{V}_{BA}$  and the side  $oa$  represents  $\mathbf{V}_A$  in positive direction. It can be verified that this vector triangle represents equation (3.21).

### 3.6 RELATIVE VELOCITY BETWEEN TWO POINTS ON THE SAME LINK

When  $A$  and  $B$  are two points on a given link, the velocities  $\mathbf{V}_A$  and  $\mathbf{V}_B$  cannot be chosen arbitrarily for both points  $A$  and  $B$ . This is simply because points  $A$  and  $B$  remain apart at a fixed distance throughout the motion transmission. Thus relative motion between points  $A$  and  $B$  is not possible along the line  $AB$ . In other words, relative motion between points  $A$  and  $B$  can occur only in a direction perpendicular to  $AB$ , which is possible only when the body  $AB$  has a motion of rotation w.r. to point  $A$ .

This can also be explained in yet another way. In Section 1.9 it was shown that any displacement of a rigid body  $AB$  consists partly of translation and partly of rotation. By definition, motion of translation must be the same for points  $A$  and  $B$ . Hence, the relative motion between points  $A$  and  $B$  can be attributed only to the motion of rotation of link  $AB$ .

In view of the above explanation, problem of finding velocity of point  $B$  for a given velocity of  $A$  resolves itself in drawing a vector triangle  $oab$  in which side  $oa$  represents velocity  $\mathbf{V}_A$  in magnitude and direction. A line from  $a$  parallel to the direction of velocity  $\mathbf{V}_{BA}$  completes the triangle. The side  $ob$  represents  $\mathbf{V}_B$  while side  $ab$  represents velocity  $\mathbf{V}_{BA}$ .

Figure (3.7) shows a link  $AB$ . Velocity of point  $A$  is completely known but velocity of point  $B$  is known only in direction, which coincides with the tangent to the path followed by point  $B$ . So eq. (3.21),

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad (3.22)$$

In Fig. 3.7, vector  $oa$  is drawn to scale to represent velocity  $\mathbf{V}_A$ . The direction of absolute velocity vector  $\mathbf{V}_B$  is drawn from  $o$  while line parallel to the direction of  $\mathbf{V}_{BA}$  is drawn from the tip of vector  $\mathbf{V}_A$ , to complete the triangle  $oab$ . Since velocity  $\mathbf{V}_{BA}$  adds to the velocity vector  $\mathbf{V}_A$  in equation (3.22), the direction of vector  $\mathbf{V}_{BA}$  is from  $a$  to  $b$ , as shown in the figure. Also, as vector  $\mathbf{V}_B$  subtracts from vector  $\mathbf{V}_A$ , the direction of  $\mathbf{V}_B$  is from  $o$  to  $b$ .

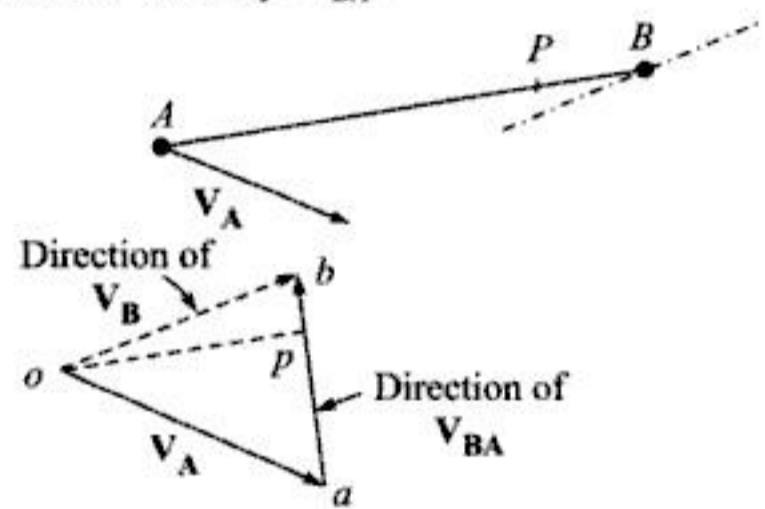


Fig. 3.7

### 3.7 VELOCITY IMAGE

Side  $ab$  of the velocity triangle  $oab$ , which represents  $\mathbf{V}_{BA}$ , i.e., velocity of  $B$  w.r. to  $A$ , is called *velocity image* of link  $AB$ . A point  $P$  on link  $AB$  is said to have a velocity image in point  $p$  on

$ab$ , if  $p$  divides  $ab$  in proportion same as  $P$  does in the link  $AB$ . Thus a necessary condition for point  $p$  on  $ab$  to become velocity image of point  $P$  is,

$$ap : ab = AP : AB$$

The above property of velocity image is justified on the basis of the discussion, below.  $P$ , being a point on link  $AB$ , has a motion of rotation similar to  $B$  about point  $A$ . Clearly,  $V_{PA}$  can act only in a direction perpendicular to  $AB$  and therefore, like  $b$ , point  $p$  must lie on line  $ab$ . Similarly, for a given angular speed of rotation  $\omega$  of link  $AB$  about  $A$ ,

$$(V_{BA}/V_{PA}) = (ab/ap) = (AB/AP)$$

It follows therefore that,

$$ap : ab = AP : AB \quad (3.23)$$

An interesting case of velocity image occurs when a point  $X$  on link  $AB$  does not lie on the line joining  $AB$ . This is illustrated in Fig. 3.8. Point  $X$  on link  $AB$  subtends, angles  $\alpha$  and  $\beta$  at  $A$  and  $B$  respectively. To complete velocity polygon  $oab$ , draw  $ob$  and  $oa$  parallel to  $V_B$  and  $V_A$  respectively, with their lengths representing magnitudes of the respective vectors. The relative velocity vector  $V_{BA}$  represented by vector  $ab$  will be perpendicular to the link centre line  $AB$ . Remembering that velocity of  $X$  w.r. to  $A$  can exist in a direction perpendicular to  $AX$ ,  $ax$  has been drawn perpendicular to  $AX$ . Similarly, velocity of  $X$  w.r. to  $B$  can exist only in a direction perpendicular to  $BX$ . Hence line  $bx$  is drawn perpendicular to  $BX$ . The intersection point defines velocity image  $x$  of point  $X$ .

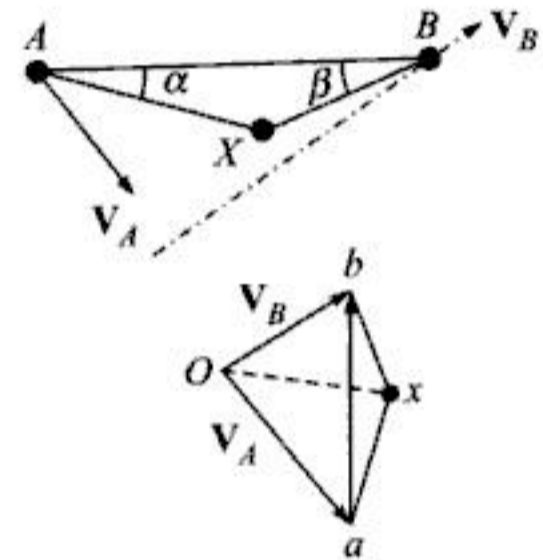


Fig. 3.8 Velocity Image of Point  $X$ .

Since, lines  $ab$ ,  $ax$  and  $bx$  have been drawn in directions perpendicular to sides  $AB$ ,  $AX$  and  $BX$  respectively, triangles  $abx$  and  $ABX$  are similar so that  $\angle bax = \alpha$  and  $\angle abx = \beta$ . Again, if link  $AB$  is assumed to rotate with angular speed  $\omega$  about  $A$ , then

$$\left( \frac{V_{BA}}{AB} \right) = \left( \frac{V_{XA}}{AX} \right)$$

Therefore  $\frac{AX}{AB} = \left( \frac{V_{XA}}{V_{BA}} \right)$ ;  $V_{BA}$  and  $V_{XA}$  being the magnitudes of corresponding velocities.

Also, from similar triangles  $abx$  and  $ABX$ ,

$$\frac{AX}{AB} = \frac{ax}{ab}$$

or

$$ax = ab \left( \frac{AX}{AB} \right)$$

or

$$V_{XA} = \left( \frac{AX}{AB} \right) V_{BA} \quad (3.24)$$

Join  $o$  to  $x$ . Now as  $oa = V_A$  and  $ax = V_{XA}$ ,

$$V_X = V_A + V_{XA} \quad (3.25)$$

It follows that  $ox$  represents absolute velocity  $V_X$  of point  $X$ .

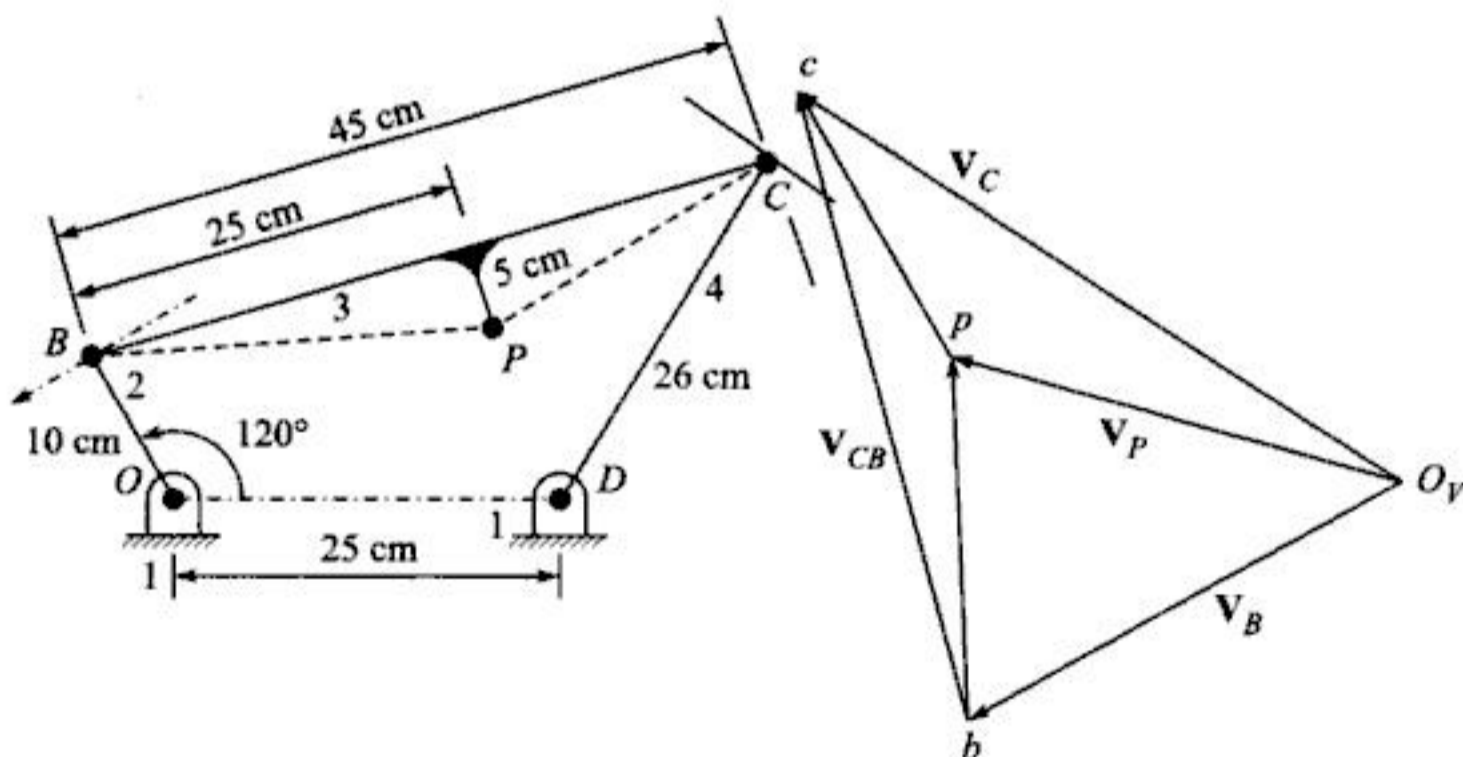
### 3.8 VELOCITY POLYGON

A *vector polygon* for velocities is a graphical depiction of vector equation, relating velocities of two or more points. When constructed to a scale, a velocity polygon contains information about magnitudes and directions of velocities of respective points.

Constructing velocity polygon in thick black lines makes them easy to read. However, when velocity polygon is intended to give graphical solution to vector equation, use of thick lines may result in inaccuracies. So, it is advisable to draw the polygon using thin sharp lines, made with hard drawing pencils. The solution in the form of vector polygon is started invariably by choosing a suitable velocity scale and a convenient location for a point  $O_v$ , called *velocity pole*. This point represents zero velocity. In general, absolute velocity vectors like  $V_A$ ,  $V_B$ ,  $V_C$  are constructed with their origin at this velocity pole  $O_v$ . Terminals of absolute velocity vectors  $V_A$ ,  $V_B$ ,  $V_C$  etc. will be labelled respectively as  $a$ ,  $b$ ,  $c$  and so on. The lines not originating from velocity pole and having terminals such as  $a$ ,  $b$  and  $c$  then represent relative velocity vectors. It may be useful to remember that vector  $V_{AB}$  is read as velocity of  $A$  w.r. to  $B$  and is represented by a vector showing direction from  $b$  to  $a$ . Conversely, vector  $V_{BA}$  is read as velocity of  $B$  w.r. to  $A$  and is represented by a vector showing direction from  $a$  to  $b$  in the velocity polygon.

It is always advisable to first write down vector equation connecting unknown and known velocities. This practice eliminates chances of error due to any confusion.

**EXAMPLE 3.1** A four-bar mechanism, with dimensions as shown in Fig. 3.9, is driven by a crank 2 at a constant angular speed of  $\omega_2 = 600$  r.p.m. in c.c.w. sense. Find the instantaneous velocities of coupler point  $P$  and angular velocities of links 3 and 4 in the position shown.



(a) Mechanism configuration, Scale: 1 cm = 7.5 cm

(b) Velocity polygon, Scale: 1 cm = 148 cm/s

**Fig. 3.9**

**Solution:**  $\omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.83 \text{ rad/s}$

The vector equation connecting known velocity  $V_B$  to unknown velocity  $V_C$  on same link  $BC$  is

$$V_C = V_B + V_{CB} \quad (3.26)$$

- (a) Velocity of crank pin  $B$  is known both in magnitude and direction.

$$V_B = (10) (62.83) = 628.3 \text{ cm/s}$$

- (b) Velocity of point  $C$  is not known in magnitude but it acts in a direction tangential to path of  $C$ , *i.e.* perpendicular to  $DC$ .
- (c) Velocity of point  $C$  w.r. to  $B$  is not known in magnitude, but its direction must be along a perpendicular to link  $BC$ .

**Construction.** For velocity polygon in Fig. 3.9(b), select a suitable velocity pole  $o_v$  and draw first velocity vector which is completely known. Thus, draw  $o_v b$  in a direction perpendicular to  $OB$ , to represent velocity  $V_B = 628.3 \text{ cm/s}$  to suitable scale. From  $o_v$  next draw the direction of absolute velocity vector  $V_C$  which is known only in direction. Thus draw  $o_v c$  from  $o_v$  which is in a direction perpendicular to crank  $DC$ . Finally from the tip  $b$  of vector  $o_v b$ , draw a line in a direction perpendicular to  $BC$  to cut above line at  $c$ . Note that  $bc$  represents  $V_{CB}$  which is now known in magnitude & direction both. This completes velocity polygon  $o_v bc$ . The directions of the velocity vectors  $V_C$  and  $V_{CB}$  are obtained by referring back to velocity equation (3.26).

According to the velocity equation, relative velocity vector  $V_{CB}$  is to be additive to  $V_B$ . Hence the arrowhead on line  $bc$  should be from  $b$  to  $c$ . Also, from the equation,  $V_C$  is subtractive from  $V_B$ . Hence arrowhead on line  $o_v c$  should be from  $o_v$  to  $c$ .

By measurement  $o_v c = V_C = 915 \text{ cm/s}$

And so,  $\omega_4 = \frac{V_C}{DC} = \frac{915}{26} = 35.19 \text{ rad/s}$  **Ans.**

Also from measurement,  $bc = V_{CB} = 835 \text{ cm/s}$

Therefore  $\omega_3 = \frac{V_{CB}}{BC} = \frac{835}{45} = 18.55 \text{ rad/s}$  **Ans.**

For obtaining linear velocity of coupler point  $P$ , we use the principle of velocity image. Thus, from  $b$  draw a line  $bp$  in a direction perpendicular to the line joining  $BP$ . Similarly, from point  $c$  draw a line  $cp$  in a direction perpendicular to the line joining  $CP$ . The intersection of the two lines define velocity image  $p$  of coupler point  $P$ . It can be verified that triangles  $BPC$  and  $bpc$  are similar.

Line  $o_v p$  on velocity polygon represents absolute velocity  $V_P$  of coupler point  $P$ . The sense of  $V_P$  is decided by referring to the velocity equation:

$$V_P = V_B + V_{PB}$$

As  $V_P$  and  $V_B$  are to be subtractive in nature, direction of  $V_P$  is, as shown, from  $o_v$  to  $P$ . Velocity of coupler point  $P$  measures:

$$o_v p = V_P = 600 \text{ cm/s} \quad \text{Ans.}$$

**EXAMPLE 3.2** In the slider-crank mechanism as shown in Fig. 3.10, the crank 2 makes 80 r.p.m. in c.w. sense. Determine the linear velocity of slider and angular velocity of connecting rod 3. Also find out the linear velocity of point Q on connecting rod. Lengths of crank and connecting rod are 8 cm and 32 cm respectively.

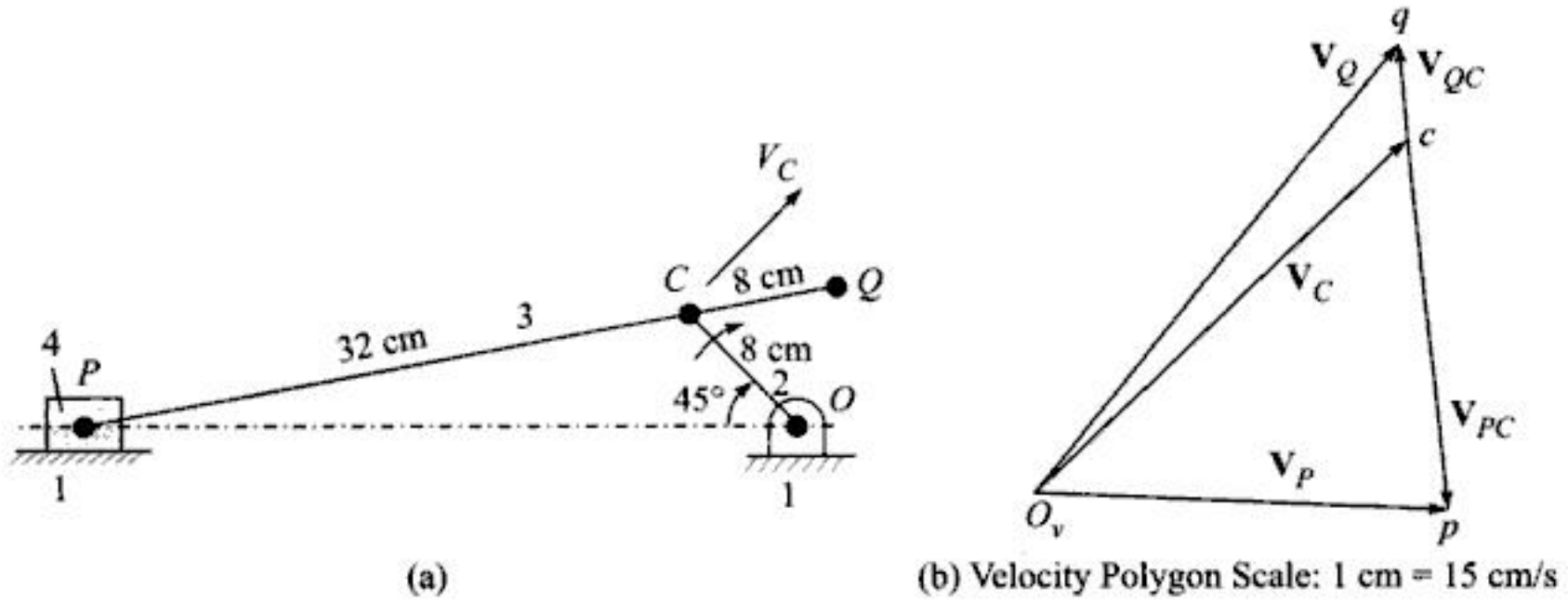


Fig. 3.10

**Solution:** Crank speed =  $\frac{2\pi \times 80}{60} = 8.377 \text{ rad/s}$

Tangential (linear) velocity of crank pin C =  $8 \times 8.377 = 67.0 \text{ cm/s}$

The velocity equation connecting known velocity of point C to unknown velocity of piston pin P on the connecting rod is

$$\mathbf{V}_P = \mathbf{V}_C + \mathbf{V}_{PC}$$

The velocity  $\mathbf{V}_C$  has a magnitude of 67.0 cm/sec and direction of  $\mathbf{V}_C$  is at right angles to OC as shown. Velocity of P w.r. to C (i.e.  $\mathbf{V}_{PC}$ ) is not known in magnitude but its direction is known to be perpendicular to PC. Similarly, velocity of piston pin P is not known in magnitude, but its direction must be along the line of stroke.

**Construction.** For velocity polygon in Fig. 3.10(b), select a suitable point  $o_v$  as velocity pole. From  $o_v$  draw  $o_v c$  to represent  $\mathbf{V}_C = 67.0 \text{ cm/s}$  to scale in a direction perpendicular to crank OC. After representing the velocity vector, which is completely known, from  $o_v$  draw line  $o_v p$  parallel to line of stroke. Note that  $\mathbf{V}_P$  is known only in direction. Similarly from c draw a line  $cp$  in a direction perpendicular to CP. Note that  $\mathbf{V}_{PC}$  was known only in direction. Intersection of the two lines locate point p.

Directions of velocities  $\mathbf{V}_P$  and  $\mathbf{V}_{PC}$  will be obtained by referring back to the velocity equation. Since  $\mathbf{V}_{PC}$  is additive to  $\mathbf{V}_C$ , the arrowhead should be as shown in figure. Again,  $\mathbf{V}_P$  being on the other side of equal to sign, should be subtractive from  $\mathbf{V}_C$ . Hence the arrowhead on  $\mathbf{V}_P$  and  $\mathbf{V}_{PC}$  should be as shown.

By measurement  $\mathbf{V}_P = 54.0 \text{ cm/s}$  **Ans.**  
 $\mathbf{V}_{PC} = 47 \text{ cm/s}$



The angular velocity of link  $CP$  is obtained thus: velocity  $V_{PC}$  represents velocity of  $P$  w.r. to  $C$ . Hence connecting rod can be assumed to rotate about  $C$  as centre. So,

$$\omega_{PC} = \left( \frac{V_{PC}}{PC} \right) = \frac{48}{32} = 1.5 \text{ rad/s} \quad \text{Ans.}$$

Velocity of point  $Q$  is obtained by using the principle of velocity image. Extend  $pc$  to  $q$  such that  $pq: pc = PQ: PC$ .

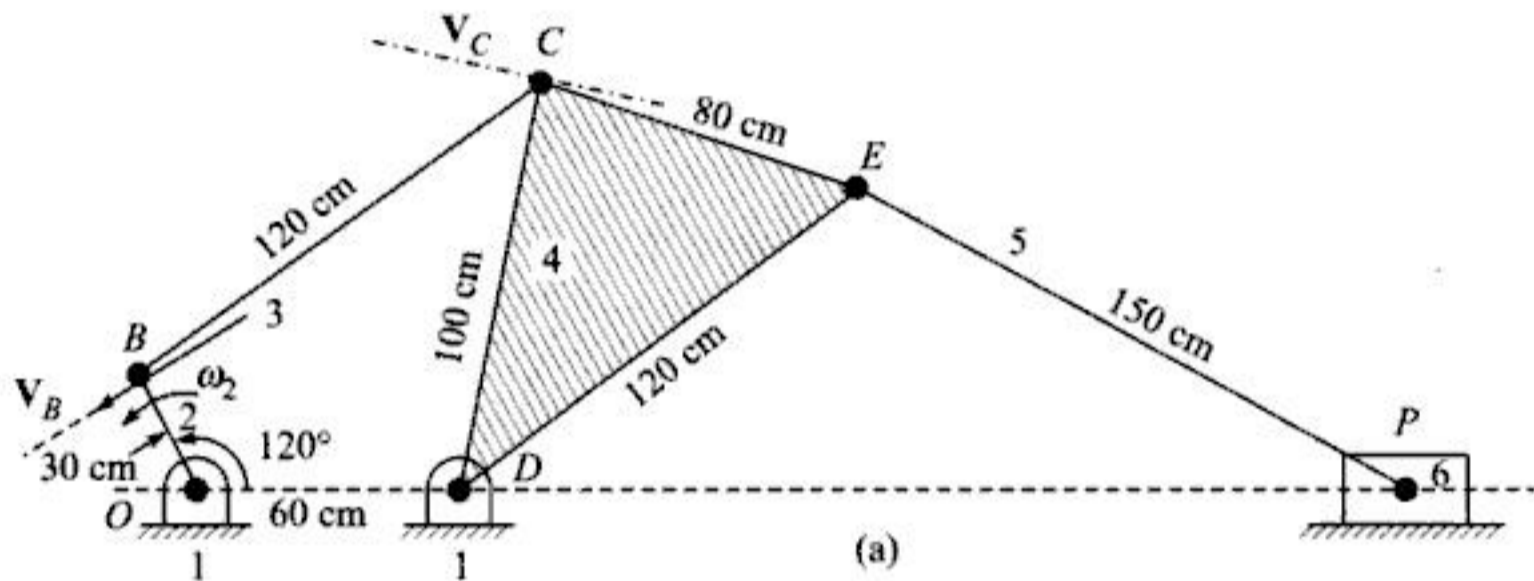
Velocity of  $Q$  is then given by line joining  $O_vq$ .

By measurement  $V_Q = 74 \text{ cm/s}$  Ans.

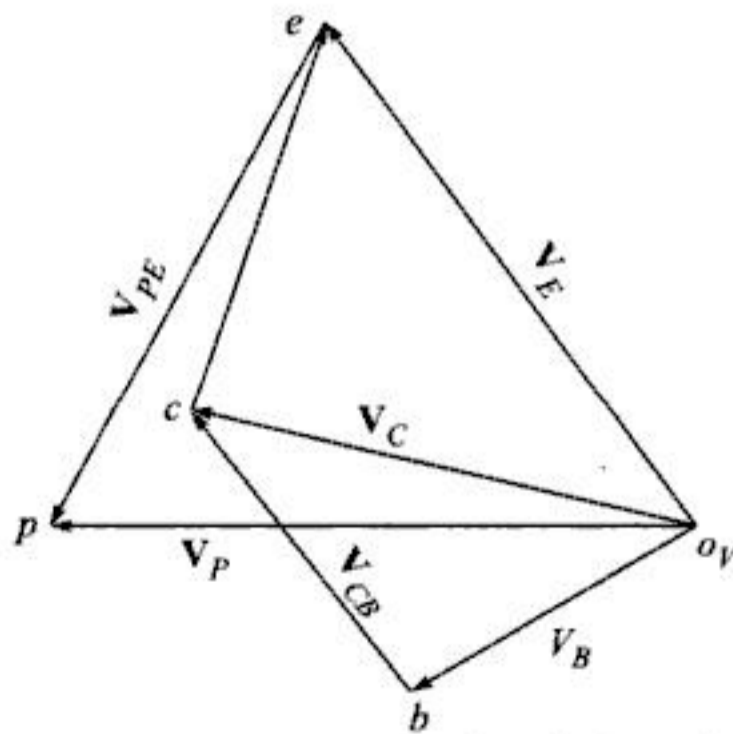
Direction of  $V_Q$ , which is as shown in figure, is decided based on equation  $V_Q = V_C + V_{QC}$ .

Readers are advised to verify that the direction of vector  $V_Q$ , shown in velocity polygon, confirms with the above equation.

**EXAMPLE 3.3** For the mechanism shown in Fig. 3.11, determine velocities of points  $C$ ,  $E$  and  $P$  and the angular velocities of links 3, 4 and 5. Crank  $OB$  rotates at 120 r.p.m. c.c.w.



Scale 1 cm = 29.5 cm



Scale 1 cm = 122 cm/s

(b) Velocity Polygon.

**Fig. 3.11**

**Solution:** 
$$\omega_2 = \frac{2\pi(120)}{60} = 12.570 \text{ rad/s}$$

$$V_B = 376.99 \text{ cm/s} = 377.0 \text{ cm/s, say}$$

There are two four-link loops 1-2-3-4 and 1-4-5-6. Therefore, we may solve velocity polygon problem in two stages. In the first stage, we solve velocity polygon using vector equation,

$$V_C = V_B + V_{CB}$$

This connects unknown velocity  $V_C$  on link 3 to known velocity  $V_B$ .

In the second stage, after establishing velocity of  $C$ , and hence of  $E$  (using velocity image concept), we solve vector equation,

$$V_P = V_E + V_{PE}$$

For solving first vector equation graphically, draw line  $o_v b$  from velocity pole to represent  $V_B$  in magnitude and direction. From  $b$  draw a line in a direction perpendicular to  $BC$  to represent direction of  $V_{BC}$  and from  $o_v$  draw line  $o_v c$  in a direction perpendicular to  $CD$  to cut it at point  $c$ . Then,  $o_v bc$  represents required velocity polygon. Since  $V_{CB}$  is additive to  $V_B$ , the direction of vector  $V_{CB}$  is as shown in Fig. 3.11(b). Also as vector  $V_C$  is subtractive from  $V_B$ , its direction is as shown.

Line  $o_v c$  is the velocity image of link  $DC$ . Make angles  $\angle co_v e = \angle CDE$  and  $\angle ceo_v = \angle DCE$ . Then  $o_v ce$  is the velocity image of link 4 and,  $o_v e = V_E$ . With this, we proceed to solve second vector equation namely,

$$V_P = V_B + V_{PE}$$

For this, draw  $o_v p$  parallel to the line of stroke to represent direction of  $V_P$ . Also, from  $e$  draw  $ep$  in a direction perpendicular to  $EP$ . The two lines intersect at  $p$ . Thus

$$o_v p = V_P \quad \text{and} \quad ep = V_{PE}$$

By measurement:

$$V_C = o_v c = 585 \text{ cm/s} \quad \text{Ans.}$$

$$V_E = o_v e = 702 \text{ cm/s} \quad \text{Ans.}$$

$$V_P = o_v p = 730 \text{ cm/s} \quad \text{Ans.}$$

Also, 
$$\omega_3 = \frac{V_{CB}}{BC} = \frac{380}{120} = 3.167 \text{ rad/s (c.c.w.)} \quad \text{Ans.}$$

$$\omega_4 = \frac{V_C}{CD} = \frac{585}{100} = 5.85 \text{ rad/s (c.c.w.)} \quad \text{Ans.}$$

$$\omega_5 = \frac{V_{PE}}{PE} = \frac{640}{150} = 4.267 \text{ rad/s (c.w.)} \quad \text{Ans.}$$

### 3.9 VELOCITY OF RUBBING

Elements on two links forming a turning pair are shown in Fig. 3.12. As can be seen, a pin forms an integral part of one element and this pin fits into the mating hole in the second link, permitting a relative motion of rotation.

When the two links are connected at the elements, surface of hole of one link rubs against the surface of pin of the other link. The rubbing velocity between the two depends on the angular velocity of one link relative to the other. This is illustrated below for pin joints  $O$ ,  $B$ ,  $C$  and  $D$  of the four-bar mechanism in Fig. 3.9.

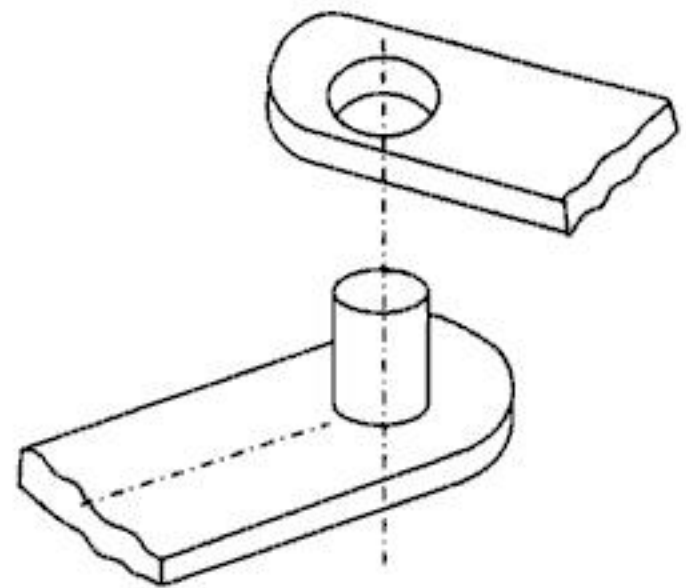


Fig. 3.12

**Pin at  $O$ .** One of the connecting links  $OD$  is fixed. Hence, angular velocity at  $O$  between connected link is only due to link  $OB$  which rotates in c.c.w. sense with angular velocity  $\omega_2$ . Therefore, rubbing velocity at  $O$  is  $(r_0\omega_2)$ , where  $r_0$  = pin radius at  $O$ .

**Pin at  $B$ .** Link 2 rotates in c.c.w. sense. Therefore, relative to link  $BC$ , link  $BO$  rotates in c.c.w., i.e.  $\omega_{OB}$  is c.c.w.

Also, from velocity polygon of Fig. 3.9(b),  $V_{cb}$  acts upwards, i.e. angular velocity of  $C$  w.r. to  $B$ ,  $\omega_{CB}$  is c.c.w.

Since  $\omega_{OB}$  and  $\omega_{CB}$  both act c.c.w. about  $B$ , their relative angular speed is  $(\omega_{OB} \sim \omega_{CB})$  and so, for a pin radius of  $r_B$ , the velocity of rubbing is

$$= r_B (\omega_{OB} \sim \omega_{CB}) \quad (3.27)$$

**Pin at  $C$ .** From velocity polygon of Fig. 3.9(b), it is clear from the direction of  $V_C$  that angular velocity of link 4,  $\omega_{CD}$  is c.c.w. Hence angular velocity of  $D$  w.r. to  $C$ , i.e.  $\omega_{DC}$  is also c.c.w.

Again from velocity polygon  $V_{BC}$  must be opposite to  $V_{CB}$ . Hence  $V_{BC}$  acts downwards and therefore,  $\omega_{BC}$  is c.c.w. Since  $\omega_{DC}$  and  $\omega_{BC}$  are both c.c.w. at  $C$ , the rubbing velocity,

$$v = (\omega_{DC} \sim \omega_{BC}) \times r_C$$

where,

$$r_C = \text{pin radius at } C.$$

**Pin at  $D$ .** Link  $OD$  is stationary and velocity  $V_{CD}$  from velocity polygon of Fig. 3.9(b) has a direction given by  $V_C$ . Hence  $\omega_{CD}$  acts c.c.w., and the velocity of rubbing at  $D$ ,

$$= (r_D) \omega_{CD}$$

In all the above cases, either only one of the two links was rotating or both the links were rotating in the same sense. In case both the links rotate in opposite sense, rubbing velocity  $= r \times (\omega_1 + \omega_2)$ .

**EXAMPLE 3.4** In the mechanism shown in Fig. 3.13, the crank  $AB$  rotates about  $A$  at a uniform speed of 120 r.p.m. c.w. The lever  $DC$  oscillates about a fixed point  $D$ , which is connected to  $AB$  by the coupler  $BC$ . The sliding block  $P$  moves in the horizontal guides being driven by the link  $EP$ . Determine:

- Velocity of block  $P$ ;
- Angular velocity of lever  $DC$ , and
- Rubbing speed of the pin  $C$  which is 5 cm in diameter.

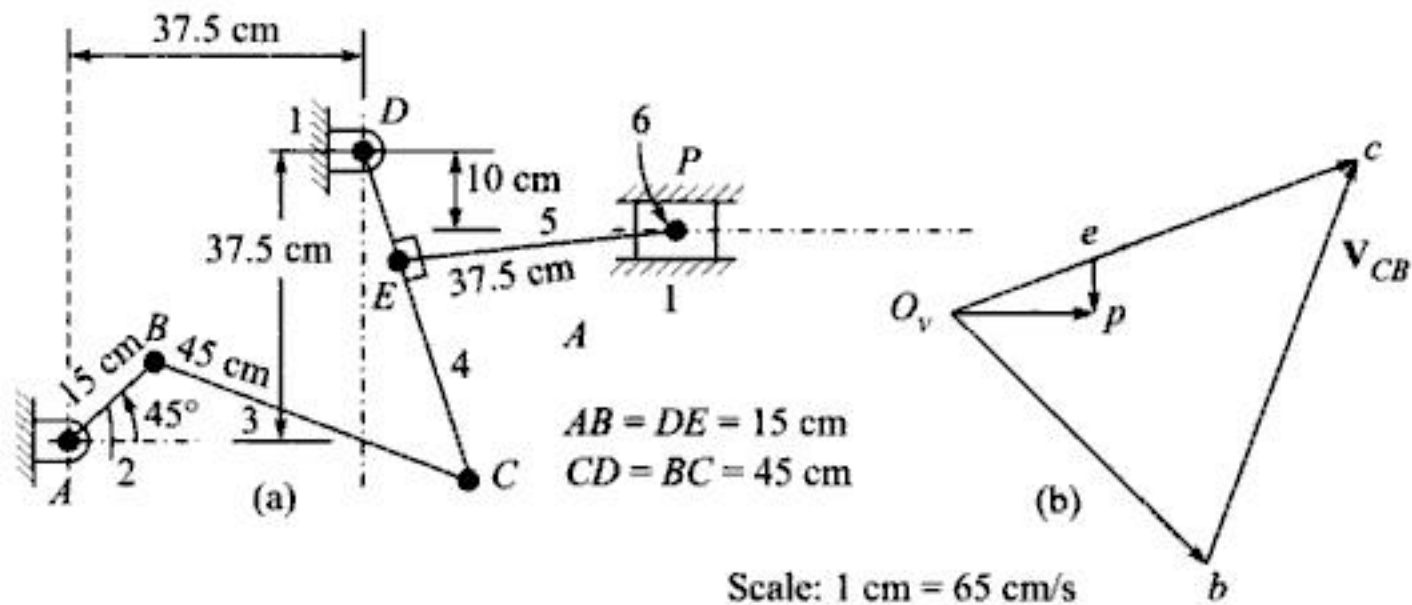


Fig. 3.13

**Solution:** Angular speed of crank  $\omega_2 = \frac{2\pi \times 120}{60} = 12.567 \text{ rad/s}$

Therefore Tangential speed of B,  $V_B = 15 \times 12.567 = 188.5 \text{ cm/s}$

The velocity polygon will be solved in two stages.

In the first stage, velocity polygon for links 1, 2, 3 and 4 will be constructed using equation,

$$V_C = V_B + V_{CB} \quad (a)$$

In the second stage with  $V_C$  known, velocity polygon will be solved for links 1, 4, 5 and 6 using velocity equation,

$$V_P = V_E + V_{PE} \quad (b)$$

**Construction.** From arbitrarily selected velocity pole  $o_v$  draw line  $o_v b$  perpendicular to direction AB and measure velocity  $V_B = 188.5 \text{ cm/s}$  along it, to a suitable scale. Since  $V_{CB}$  is additive to  $V_B$ , from  $b$  draw a line in a direction perpendicular to link BC. Similarly from  $o_v$  draw a line in direction perpendicular to DC (to represent  $V_C$ ) to cut above line at  $c$ . Then, in accordance with equation (a) line 'bc' represents  $V_{CB}$  and  $o_v c$  represents  $V_C$ . Locate point  $e$  on  $o_v c$  such that  $o_v e : o_v c = DE : DC$ . Then  $o_v e$  represents  $V_E$ . Now, in accordance with equation (b), draw  $ep$  in a direction perpendicular to EP and from  $o_v$  draw  $o_v p$  parallel to  $V_P$ , to meet above line at  $p$ . Then  $o_v p$  represents piston velocity  $V_P$ .

By measurements  $V_P = 74 \text{ cm/s}$  **Ans.**

and  $V_{CB} = 220 \text{ cm/s}$

Also  $V_C = 224 \text{ cm/s}$

$\therefore \omega_4 = \frac{224}{45} = 4.98 \text{ rad/s}$  **Ans.**

Also  $\omega_3 = \frac{V_{CB}}{BC} = \frac{220}{45} = 4.89$

**To find robbing speed at pin C, we proceed as under:**

Since  $\omega_4 = \frac{V_C}{45}$  is c.c.w., angular speed of CD about  $c$  is also c.c.w.

Also,  $\omega_{CB} = \frac{V_{CB}}{BC}$  and is c.c.w. and so angular speed of B about C is also c.c.w.

Thus about point  $C$ , both  $\omega_3$  and  $\omega_4$  are c.c.w. Hence rubbing velocity at  $C$ ,

$$V_{rub} = r_C (\omega_3 - \omega_4)$$

$$V_{rub} = \frac{5}{2}(4.89 - 4.98) = 0.225 \text{ cm/s}$$

**Ans.**

**EXAMPLE 3.5** Find out the rubbing velocity at pin point  $E$  in the mechanism shown in Fig. 3.11(a). Assume pin diameter at  $E$  to be 2 cm.

**Solution:** From the velocity polygon of Fig. 3.11(b),

$$\omega_{ED} = \frac{702}{120} = 5.85 \text{ rad/s (c.c.w.)}$$

Therefore

$$\omega_{DE} = 5.85 \text{ rad/s (c.c.w.)}$$

Also

$$\omega_{PE} = \frac{V_{PE}}{150} = \frac{640}{150} = 4.267 \text{ rad/s (c.w.)}$$

$$\begin{aligned} \text{Nett (relative) angular velocity between links 4 and 5 at } E &= \omega_{DE} + \omega_{PE} \\ &= 5.85 + 4.267 = 10.117 \text{ rad/s} \end{aligned}$$

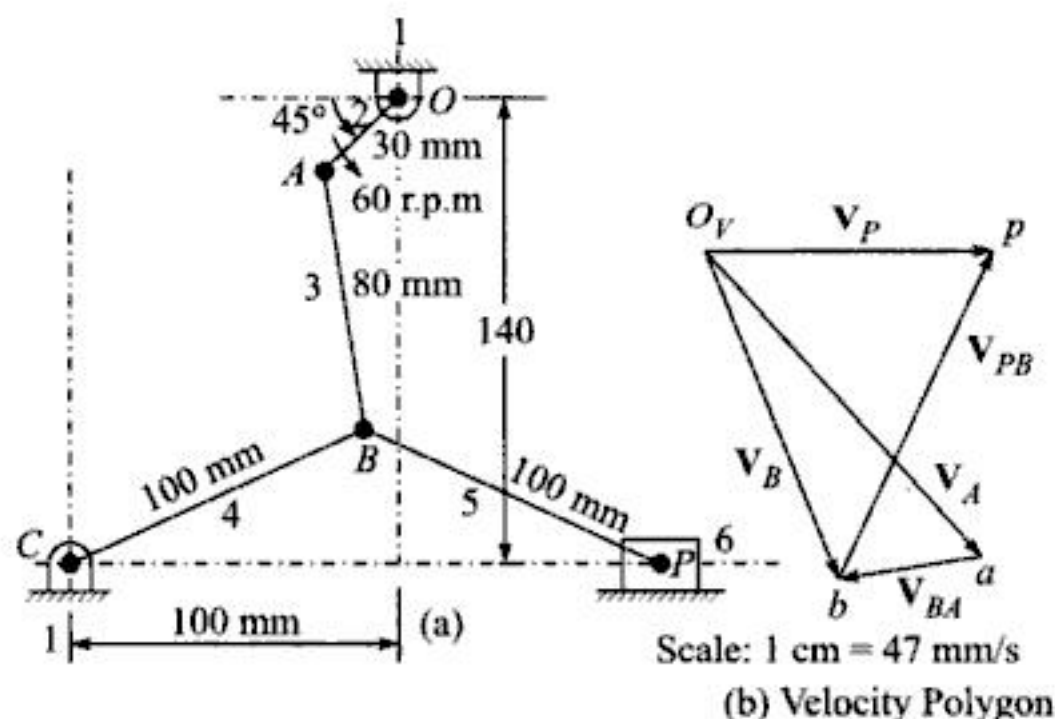
$$\text{Therefore rubbing velocity at } E = \left(\frac{2}{2}\right) \times 10.117 = 10.117 \text{ cm/s}$$

**Ans.**

### 3.10 MECHANICAL ADVANTAGE AND POWER TRANSMISSION

*Mechanical advantage* is defined as the ratio of the magnitude of output force to input force. Sometimes, mechanisms are needed for applying large forces for doing desired work. Examples of this type are toggle mechanisms used in stone crushing mechanisms, presses, pump mechanism, etc. In all such cases velocity ratio may be sacrificed for attaining higher values of mechanical advantage.

**EXAMPLE 3.6** A toggle mechanism shown in Fig. 3.14, has dimensions in mm as shown in Fig. 3.14. Find the velocities at pins  $B$  and  $P$  and the angular velocities of links  $AB$ ,  $BC$  and  $BP$ . Assume the crank to rotate at 60 r.p.m. in counter clockwise direction. Also find the mechanical advantage.



**Fig. 3.14**

**Solution:** Velocity of pin A,  $V_A = 30 \left( \frac{2\pi \times 60}{60} \right) = 188.5$  mm/s. We first solve velocity polygon involving links 1, 2, 3, 4. Velocity equation for this portion of linkage is

$$V_B = V_A + V_{BA}$$

From velocity pole  $o_v$ , therefore draw  $o_v a$  perpendicular to  $OA$  and measure along it a velocity of 188.5 mm/s to some scale (say, 1 cm = 47 mm/s) From  $a$  draw a line  $ab$  in a direction perpendicular to  $AB$  and from  $o_v$  draw another line parallel to direction of  $V_B$  (which is perpendicular to  $CB$ ), to cut earlier line at  $b$ . Then  $ab = V_{BA} = 69$  mm/s.

Therefore  $\omega_{BA} = \frac{69}{80} = 0.863$  rad/s **Ans.**

Also, by measurement  $V_B = o_v b = 162$  mm/s **Ans.**

Therefore  $\omega_4 = \frac{162}{100} = 1.62$  rad/s (c.w.) **Ans.**

With  $V_b$  known, we can proceed to solve velocity polygon for the remaining part of linkage. Thus, velocity equation is

$$V_P = V_B + V_{PB}$$

Hence, from point  $b$  on velocity polygon, draw  $bp$  perpendicular to  $BP$  to represent direction of  $V_{PB}$  and from  $o_v$ , draw  $o_v p$  to intersect it in a direction parallel to the line of stroke. Directions of vectors are to be assigned so as to satisfy the above velocity equation.

By measurement,

$$V_P = o_v p = 132.0 \text{ mm/s} \quad \text{Ans.}$$

and  $V_{PB} = 165$  mm/s **Ans.**

Therefore  $\omega_{PB} = \frac{165}{100} = 1.65$  rad/s (c.c.w.) **Ans.**

Now assuming 100% efficiency, work input must equal work output. Assuming  $F_A$  to be the tangential force at crank pin A and  $F_P$  the linear force at P, we have, for 100% efficiency,

$$F_A V_A = F_P V_P$$

or  $\frac{F_P}{F_A} = \frac{V_A}{V_P} = \text{Mechanical advantage.}$

Hence, mechanical advantage =  $\frac{188.5}{132.0} = 1.428.$  **Ans.**

### 3.11 INSTANTANEOUS CENTRES OF ROTATION

A *general planar motion* is a motion which is neither a pure translatory motion nor a pure rotary motion about a fixed axis. It will be shown in this and subsequent sections that when velocities alone are concerned, a general plane motion may be replaced by an instantaneous motion of rotation about a virtual axis. It may be noted that in regards to velocity such instantaneous rotation is equivalent to superposition of motion of translation and rotation.

It is quite interesting to conceive of relative motion between two rigid bodies to be that of rotation about an instantaneous (but imaginary) axis which is common to both the bodies. Since we intend to mainly deal with planar motions, and these axes are normal to plane of motion, they will be referred to as *instantaneous centres*, *centro* or *velocity poles*.

As seen in Fig. 3.15, finite displacement of a link from an initial position  $A_1B_1$  to the position  $A_2B_2$ , accomplished by moving  $A_1$  and  $B_1$  along prescribed paths, could also be brought about by rotation of the entire body through an angle  $\Delta\theta$  about an axis through  $O'$  which is the point of intersection of normal bisectors of  $A_1A_2$  and  $B_1B_2$ . It must be noted, however, that in view of comparatively large displacements involved, the discrepancy between actual paths of motion of  $A_1$  and  $B_1$  and corresponding circular arcs can be appreciable. For closest approximation between actual paths and corresponding arcs of circle, we may have to go for the limiting case of an infinitesimal displacement, indicated by a condition  $\Delta\theta \rightarrow 0$ . The virtual axis of rotation, under such limiting condition, is called *velocity pole* or *instant centre* of rotation,

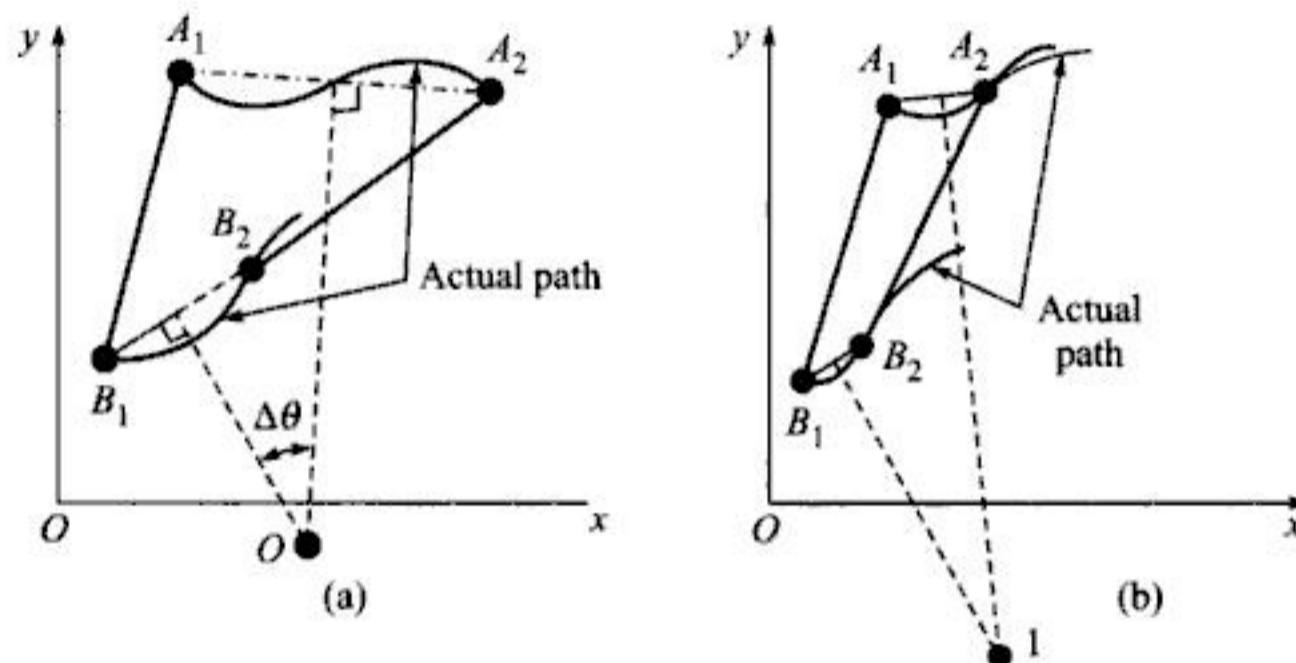


Fig. 3.15 Finite and Infinitesimal displacements.

**Definition.** For two bodies having relative motion with respect to one another, instantaneous centre (abbreviated as I.C.) of rotation is an imaginary point, common to two bodies such that any of the two bodies can be assumed to have motion of rotation with respect to the other about the imaginary point.

### 3.12 PROPERTIES OF INSTANTANEOUS CENTRE

Let  $x$ - $y$  coordinate frame in Fig. 3.16 represent plane (body) 1 and another plane (body) 2 have motion with respect to body 1. Let the relative motion of body 2 w.r. to 1 be represented by velocities  $V_A$  and  $V_B$  of two points  $A$  and  $B$  on the body 2. Let the perpendicular to the two velocity vectors  $V_A$  and  $V_B$  at  $A$  and  $B$  meet point  $I$ , the instantaneous centre of rotation of body 2 w.r. to 1.

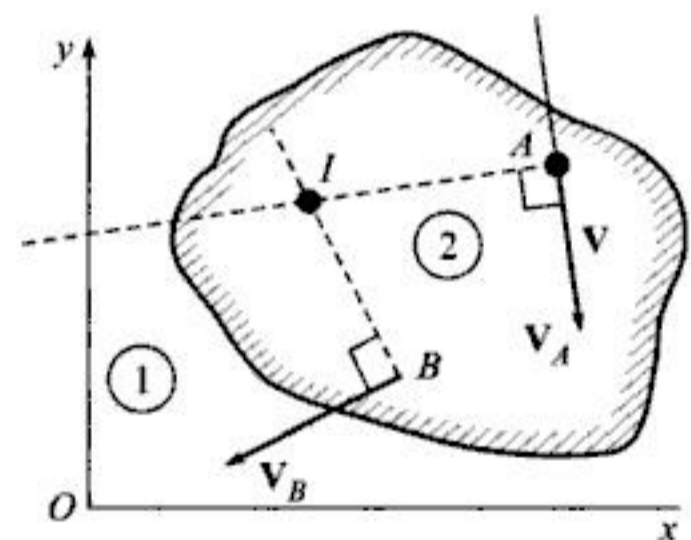


Fig. 3.16 Bodies in relative motion.

In order to bring out subtle meaning of I.C., let us consider  $I$  to define position of two coincident points, one on each body, having no relative motion instantaneously. Thus  $I$  may be considered to be a point on body 1 about which body 2 is instantaneously rotating; or it may be considered as a point on body 2 about which body 1 is instantaneously rotating. Again as the two coincident points at  $P$  (belonging to bodies 1 and 2) lie on the axis of relative rotation, they have zero relative velocity at  $P$ . In other words, when bodies 1 and 2 move with respect to a third body, keeping zero relative velocity at coincident points at  $I$ , they have the same velocity of  $I$  w.r. to body 3, whether it is considered to be a point on body 1 or 2.

Thus, the properties of I.C. can be summed up as under:

1. An I.C. of rotation is a point common to two links (bodies extended, if necessary) having a relative motion;
2. I.C. is a point about which one body can be assumed to rotate with respect to the other;
3. It is an imaginary point at which the two bodies have same absolute velocity. It follows that the two bodies have zero relative velocity at the coincident points  $I$ ;
4. I.C. in general, is not a stationary point. This is because as mechanism moves from one position to another, the velocities of points like  $A$  and  $B$  keep on changing.

### 3.13 LOCATION OF VELOCITY POLE (I.C.)

Readers may recall from Section 3.11 that general motion of any body can be thought of as superimposition of motion of translation and motion of rotation. Thus, let point  $A$  on body 1 has a linear velocity  $\mathbf{V}_A$  and the body itself has an angular velocity  $\omega$  as shown in Fig. 3.17(a). Then, velocities  $\mathbf{V}_A$  and  $\omega$  completely define velocities of all the other points. To appreciate this point, let a perpendicular be erected at  $A$  to  $\mathbf{V}_A$  and a distance  $r_A = (\mathbf{V}_A/\omega)$  measured along it to locate point  $I$ . Then  $I$  is the I.C. Magnitude of linear velocity of any other point  $D$ , at the given instant, will then be given as

$$\mathbf{V}_D = (ID) \times \omega$$

Again, when linear velocities  $\mathbf{V}_A$  and  $\mathbf{V}_C$  of any two points  $A$  and  $C$  on body 1 are known, the I.C. is located by point of intersection of perpendiculars drawn at  $A$  and  $C$  to  $\mathbf{V}_A$  and  $\mathbf{V}_C$  respectively [see Fig. 3.17(b)].

When velocities of two points  $A$  and  $B$  are parallel [as at Fig. 3.17(c)], the I.C. is located at a point of intersection of a line perpendicular to both  $\mathbf{V}_A$  and  $\mathbf{V}_B$  and an another line joining terminal points  $a$  and  $b$  of the velocity vectors  $\mathbf{V}_A$  and  $\mathbf{V}_B$ .

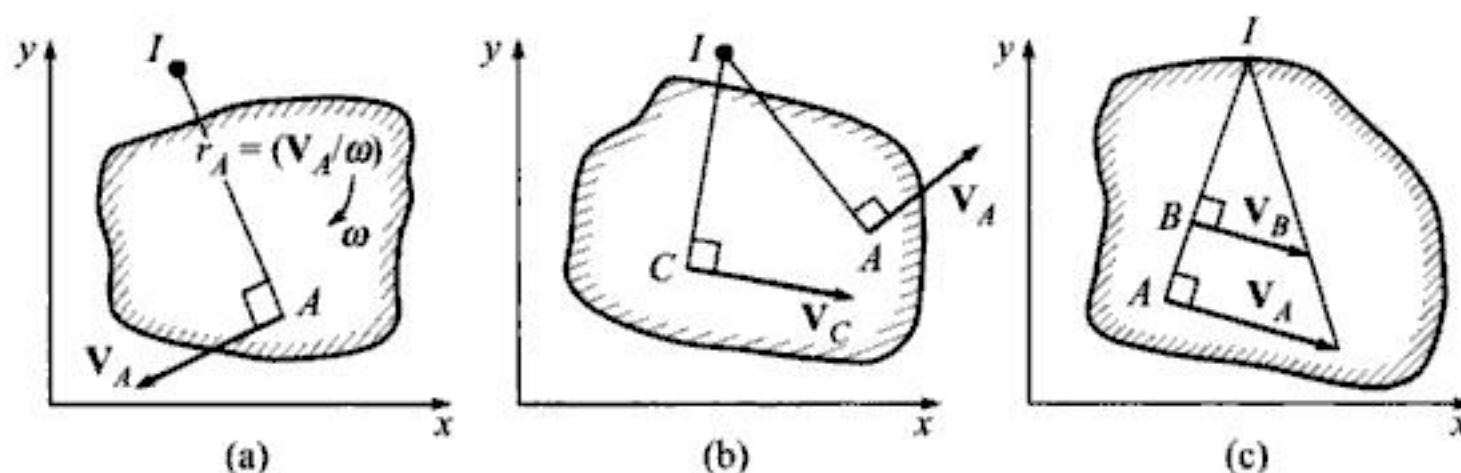


Fig. 3.17 Location of Instantaneous centres.



Figure 3.18 illustrates a few typical combinations of link motions and corresponding I.C.'s. A slider, sliding along a curved surface, is equivalent to motion of rotation about centre of curvature. Hence centre of curvature in Fig. 3.18(a) becomes an I.C. In Fig. 3.18(b) a slider moves along a straight path. Again, a straight line is an arc of a circle of infinite radius of curvature. Hence I.C. in such a case lies at infinity along a line perpendicular to the direction of motion. In Fig. 3.18(c) the slider slides along cylindrical surface. This motion is equivalent to the motion of rotation of slider about centre of circle. Therefore, I.C. lies at centre. In Fig. 3.18(d), the cylindrical disk rolls without slipping along straight line. For small angle of oscillation, any point  $P$  on disk moves in arc of circle with point of contact  $I$  as the centre. Hence the point  $I$  becomes an I.C.

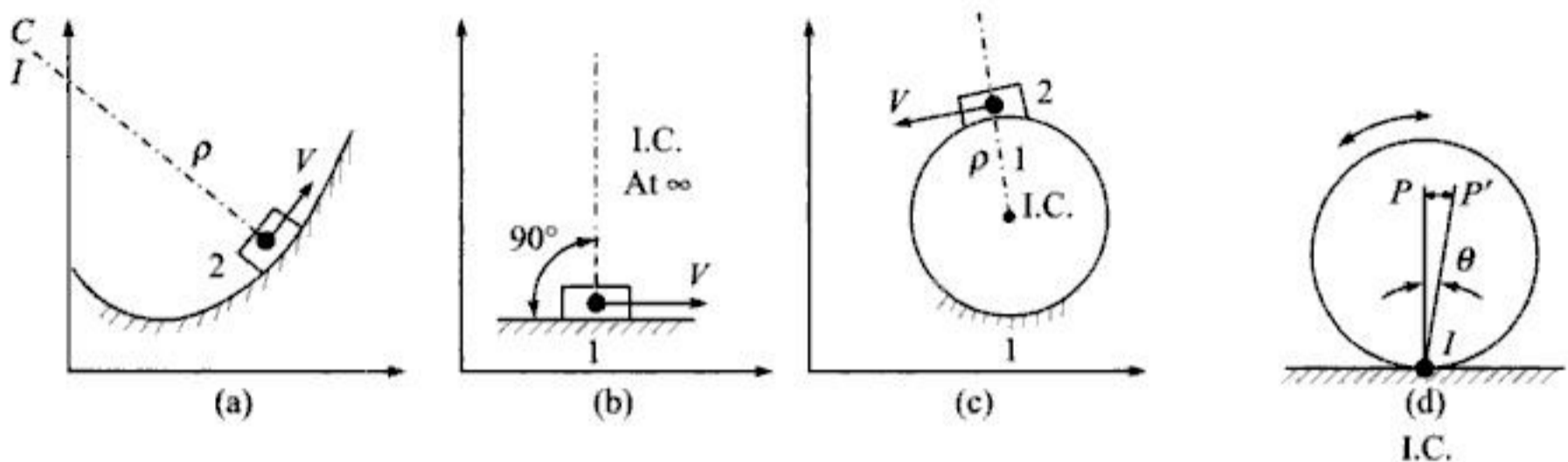


Fig. 3.18

### 3.14 INSTANT CENTRES OF 4-BAR MECHANISM

In the case of mechanisms and chains, the pin joints (hinges) become obvious choice for instant centres. This is because hinges are common to two bodies about which one body can be assumed to have motion of rotation with respect to the other. This is further explained as under.

For studying relative motion of any link  $A$  w.r. to  $B$  a convenient approach is to assume link  $B$  as fixed and link  $A$  as free to move. Thus in Fig. 3.19 relative motion of link 2 w.r. to 1 is obtained by assuming link 1 to be fixed. In that case, clearly, link 2 will revolve with pin  $A$  as centre. Similarly, assuming 2 to be stationary and link 1 to be free, link 1 can rotate w.r. to 2 about  $A$  as centre. Thus pin joint  $A$  is common to links 1 and 2. Further link 1 can revolve w.r. to 2 with  $A$  as centre. Similarly, link 2 can revolve w.r. to link 1 with  $A$  as centre. Hence pin joint  $A$  becomes I.C. for links 1 and 2. Arguing in a similar way, it can be shown that  $B$  is I.C. for links 2 and 3 and  $C$  is an I.C. for links 3 and 4. Finally pin  $D$  is an I.C. for links 1 and 4.

**Notation.** The most convenient way to represent an instantaneous centre is to denote it by a pair of digits that indicates corresponding links. Thus, in Fig. 3.19 the I.C.'s will be denoted as under:

Pin joint	Notation for I.C.
$A$	12 or 21
$B$	23 or 32
$C$	34 or 43
$D$	41 or 14

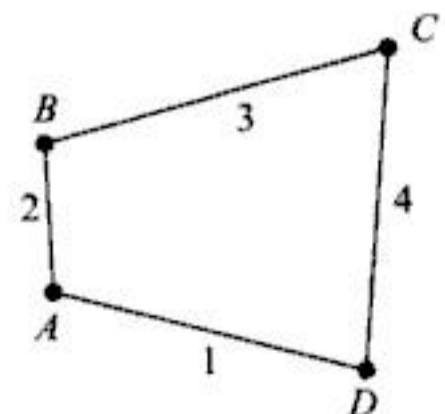


Fig. 3.19

**Total Number of I.C.'s.** Since we were able to identify four I.C.'s of a 4-bar chain, it is quite obvious to ask the question: How many I.C.'s can be reasonably expected in the above case? To answer this question, we realise that an instantaneous centre exists for a pair of links. Hence, there can be as many I.C.'s as are the possible number of pairs of links. Hence, total number of I.C.'s in chain of  $n$  links is—

$$\text{No. of I.C.'s} = {}^nC_2 = \frac{n(n-1)}{2} \quad (3.28)$$

Thus with  $n = 4, 5, 6, 7, 8$ , total number of I.C.'s are:

No. of links	Total No. of I.C.'s
4	6
5	10
6	15
7	21
8	28

In the case of 4-bar chain of Fig. 3.19, only 4 out of a total of 6 I.C.'s were found by visual inspection. The next question therefore is: How to establish remaining I.C.'s? The theorem described in the next section would be useful in answering this question.

### 3.15 ARONHOLD-KENNEDY'S THEOREM OF THREE CENTRES

This theorem, which is sometimes called Kennedy's Rule, states that *three bodies, having relative motion with respect to one another, have three instantaneous centres, all of which lie on the same straight line.* When extended to chains and mechanisms, the above statement implies that with every combination of 3 links, there are 3 I.C.'s and, if two of them are known, the third one will lie on line joining them. In effect, the above theorem suggests the selection of two combinations of 3 links in such a way that unknown I.C. is common to both the combinations and the remaining 4 I.C.'s are known. For instance, in Fig. 3.19, if one selects combination of links (1,2,3) and (1,4,3) then I.C.'s 12, 23, 14, 34 are known but unknown I.C. 13 is common to both the groups. Hence, intersection of lines joining 12 – 23 and 14 – 34 will yield unknown I.C. 13.

**Proof.** Consider three links 1, 2 and 3 to have relative motion with respect to one another. Let link 1 be fixed and links 2 and 3 be pivoted at points  $O$  and  $C$  respectively. This is shown in Fig. 3.20. Pivots  $O$  and  $C$  obviously become I.C.'s 12 and 13. It remains to show that the third I.C. 23 lies on line joining I.C.'s 12 and 13, extended if necessary. This is done in an indirect way by considering situation that would exist when all the three I.C.'s do not lie on the same straight line.

Let us assume that the third I.C. lies at the actual point of contact  $P$ . Now if  $P$  has to be an I.C., the velocity of  $P$  as a point on link 2 and as a point on link 3 must be equal. Thus, velocity of  $P$  as a point on link 2,

$$V_{p_2} = (OP) \omega_2$$

which is perpendicular to  $OP$ .

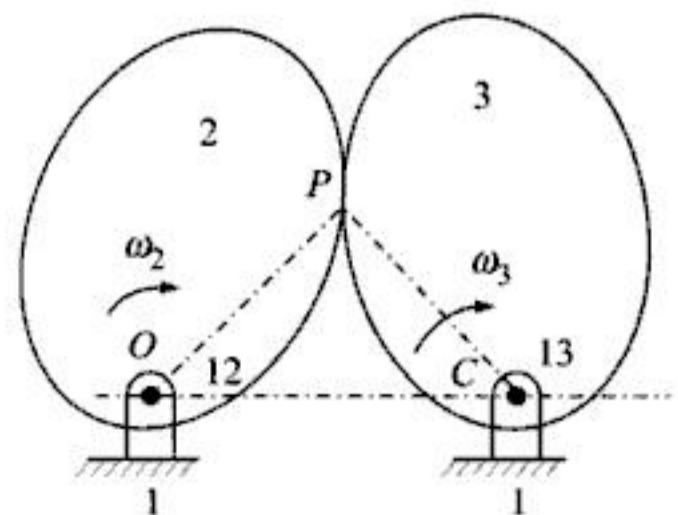


Fig. 3.20

Similarly, velocity of  $P$  as a point on body 3 is,

$$\mathbf{V}_{p_3} = (CP) \times \omega_3$$

which is perpendicular to  $CP$ .

As vectors  $\mathbf{V}_{p_2}$  and  $\mathbf{V}_{p_3}$  are not parallel, they cannot be equal. And this is true for any position of  $P$  which does not lie on line  $OC$ . In the given case, where  $\omega_2$  and  $\omega_3$  are acting in same sense, the I.C. will lie on  $OC$  extended. But when  $\omega_2$  and  $\omega_3$  are in opposite sense, any point  $P$  on line  $OC$  will produce same direction for  $\mathbf{V}_{p_2}$  and  $\mathbf{V}_{p_3}$ . It proves, therefore, that the third I.C. 23 must lie on the line joining the other two I.C.'s.

### 3.16 LOCATING I.Cs. IN MECHANISMS

Figures 3.21 (a) and (b) show a 4-bar mechanism and a slider crank mechanism with links numbered as 1, 2, 3 and 4. As we know, pin/hinge axes give I.C.'s by inspection. Pin joints between links have been marked as 12, 23, 34 and 14. In the case of slider crank mechanism of Fig. 3.21(b), the instantaneous centre 14 is shown at infinity in a direction perpendicular to line of stroke.

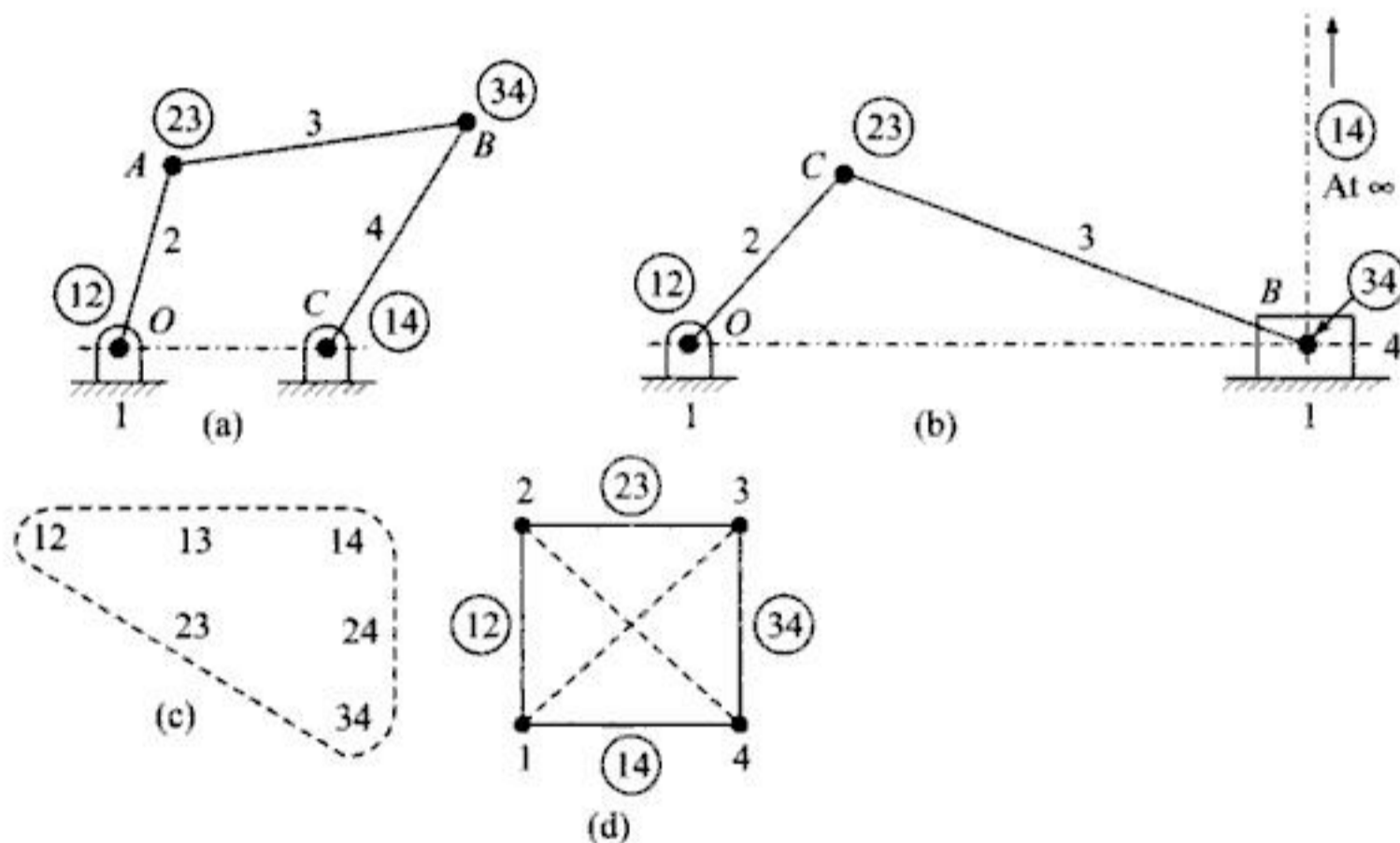


Fig. 3.21

For identifying the remaining I.C.'s one may opt for some sort of book keeping system. Such a system is particularly needed when number of links exceeds four. Thus the I.C.'s may be tabulated [Fig. 3.21 (c)] in the form of upper triangular square matrix, by considering possible combination of each link with every other link of higher index. For instance, after listing combinations of link 1 with links 2 to 4 (e.g. 12, 13, 14), combinations for links 2 will be 23, 24. This means that combinations (like 11, 22, 33, ...) of a link with itself are to be avoided. Further, as I.C. 12 was listed, the I.C. 21 (which is same as 12) is to be omitted from second row. This applies to row 3 and 4 also. The I.C.'s may be checked off as they are located. However, the polygon representation of I.Cs. shown in Fig. 3.21 (d) is much more useful. In

this representation each vertex (dot) represents a link and line joining any pair of vertices represents corresponding I.C. The process begins by arranging dots in an ordered sequence (mainly for the sake of neatness) and the I.C.'s located by inspection are indicated by full lines connecting two dots, denoting corresponding links. The I.C.'s which have to be located in the process can be denoted by dotted lines.

In the light of Kennedy's theorem, polygon method of identifying unknown I.C.'s becomes more aggressive than tabular array method. Any dotted line that can become a common side of two triangles (whose other sides are full lines) can be used to locate unknown I.C. using theorem of three centres. Following examples illustrate the method.

**EXAMPLE 3.7** Figure 3.22 shows a machine linkage in part where  $O$  and  $C$  are pivots and  $G$  is the centre of gravity of  $AB$ . The crank  $OA$  rotates uniformly at  $30 \text{ rad/s}$  clockwise.  $OA = 10 \text{ cm}$ ;  $AB = 28 \text{ cm}$ ;  $BC = 24 \text{ cm}$ ;  $CE = 12 \text{ cm}$ ;  $BG = 16 \text{ cm}$ . For the position shown, find velocity of  $G$  and  $E$  and indicate their direction. Also find the angular velocities of the link  $AB$  and the bell-crank lever  $BCE$ . Find the kinetic energy of bell-crank lever if its moment of inertia about an axis through  $C$  perpendicular to its plane of motion is  $2.1 \text{ kg}\cdot\text{cm}^2$ .

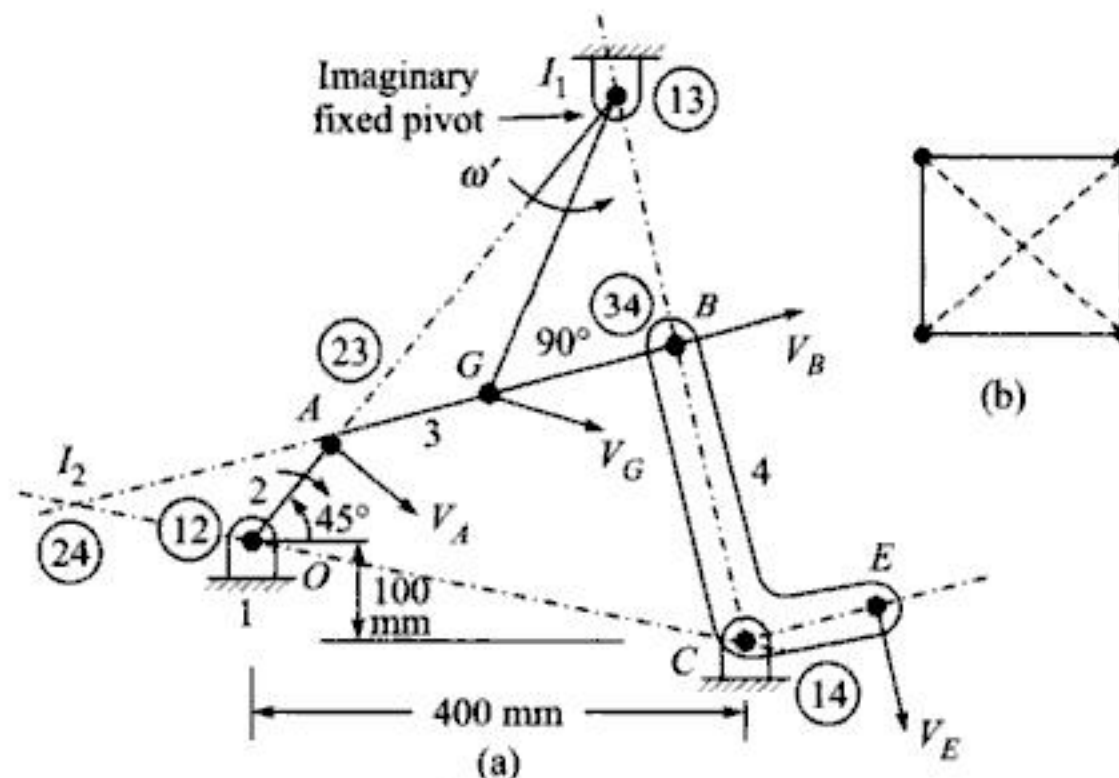


Fig. 3.22

**Solution:** Links in Fig. 3.22(a) have four pin joints at  $O$ ,  $A$ ,  $B$  and  $C$  and they have four I.C.'s 12, 23, 34 and 14 respectively, which can be located easily by visual inspection. These are shown by four full lines of polygon in Fig. 3.22(b). The unconnected diagonals 13 and 24 denote unknown I.C.'s. To locate the I.C.: 13, shown by dotted line, one needs to consider two triangles 123 and 143 in Fig. 3.22(b), to which line 13 can be a common side. Thus, point of intersection of lines joining 12 with 23 and 14 with 34 gives I.C. 13, which will be called as  $I_1$ .

Similarly, in locating I.C.: 24, shown by dotted line in Fig. 3.22(b), one needs to consider triangles 234 and 212 to which line 24 is a common side. Point of intersection of lines joining 12 with 14 and 23 with 34 gives I.C. 24, which will be referred to as  $I_2$ . Then,

$$V_A = 10 \times \omega_2 = 10 \times 30 = 300 \text{ cm/s}$$

As link 1 is frame, instant centre 13 can be considered to be a fixed pivot with link  $AB$  swinging about it. Hence, the angular velocity  $\omega'$ , with which link oscillates about I.C. 13, is also the angular speed of rotation of link 3. Thus,

$$\omega' = \frac{V_A}{I_1A} = \frac{300}{I_1A}$$

By measurement, from Fig. 3.22(a),  $I_1A = 4.38 \times 8 = 35.04$  cm

$$I_1B = 2.3 \times 8 = 18.4 \text{ cm}; \quad I_1G = 3.2 \times 8 = 25.6 \text{ cm}$$

And, as  $A$ ,  $B$  and  $G$  lie on the same link 3,

$$\frac{V_A}{I_1A} = \frac{V_B}{I_1B} = \frac{V_G}{I_1G} = \omega'$$

$$\therefore \omega' = \frac{300}{35.04} = 8.56 \text{ rad/s (c.c.w.)}$$

$$\therefore V_B = I_1B (\omega') = 18.4 \times 8.56 = 157.5 \text{ cm/s}$$

$$V_G = I_1G (\omega') = 25.6 (8.56) = 219.14 \text{ cm/s}$$

Also 
$$\omega_4 = \frac{V_B}{CB} = \frac{157.5}{24} = 6.56 \text{ rad/s (c.w.)}$$

$$\therefore V_E = (CE) \times \omega_4 = 12 \times 6.56 = 78.72 \text{ cm/s}$$

K.E. of angular motion of bell-crank lever

$$= \frac{1}{2} I \omega_4^2 = \frac{1}{2} (2.1) (6.56)^2 = 45.19 \text{ N}\cdot\text{cm} \quad \text{Ans.}$$

**EXAMPLE 3.8** Crank of a slider-crank mechanism is 48 cm long and rotates at 190 r.p.m. in c.c.w. direction. The connecting rod is 144 cm long. When the crank has turned through 50 degrees from the inner dead centre position, determine, using instantaneous centre method, the velocity of point  $E$  located at distance of 45 cm on the connecting rod extended. Also find point  $F$  on the connecting rod having the least absolute velocity and angular velocity of the connecting rod.

**Solution:** It may be noted that 14 is located at infinity along a line perpendicular to line of stroke from  $P$ . However, the distance  $OP$  is too small in comparison with infinity and as such the distance  $OP$  can be neglected in comparison to infinity for all practical purposes. Thus a line drawn from  $O$  perpendicular to line of stroke can be assumed to locate I.C. 14 at infinity. Hence point of intersection of line joining 23 with 34 and the line joining 12 with 14 along  $OK$  locates I.C. 24 at  $I_2$ . Similarly line joining I.C.'s 12 and 23 intersects the line joining I.C.'s 34 and 14 at 13 ( $= I_1$ ).

Any of the instant centres 13 or 24 can be used to find velocity of the slider, but only I.C. 13 can be used to find the velocity of any point on link  $AP$ . First we will see as to how I.C. 24 can help in finding  $V_P$ .



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# CAMS

## 7.1 INTRODUCTION

A cam is a higher paired machine member which is used to impart specified motion, by direct contact, to another machine element, called follower. A cam may also be defined as a convenient device transforming one type of motion into another. Usually the cam either oscillates or rotates while the follower has the motion of either translation or oscillation. The cam has a curved or grooved surface which mates with the follower to impart a specified motion to it.

Cam-follower mechanisms are simple and less expensive. They have fewer moving parts and require a very small working space. Many modern-day automatic machines depend upon cams to provide proper timing of the machine components. Many machines require complicated motion which can be obtained only by means of cam-follower mechanisms. Requirement of complex motion, however, does not complicate the design process for cams. Because of desired features, cam mechanisms are used in printing machines, paper cutting machines, packaging and processing machinery, spinning and weaving textile machinery, and internally expanding shoe-brakes. Cams are also used in automatic screw cutting machines, feed mechanism of automatic lathes, in presses and for operating inlet and exhaust valves of internal combustion engines. Cam mechanisms can also be designed with adjustability, as in duplicating machines. Cam mechanisms often find application where linkages would be suitable. These include instruments, typewriters, computers, and measuring apparatus which often require oscillating motion whose accuracy is not critical.

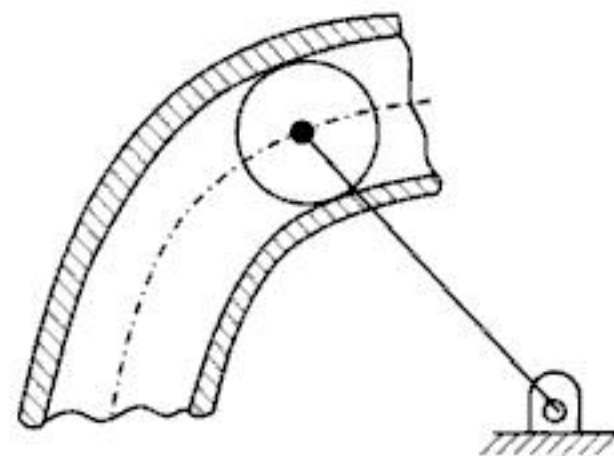
## 7.2 COMPARISON BETWEEN CAMS AND LOWER PAIRED MECHANISMS

Like lower paired mechanisms, cam and follower combination may be designed for motion generation (rigid body guidance), path generation or function generation. In majority of applications, however, cam and follower combination is used for function generation. A comparison between the two types of mechanisms may greatly facilitate the type synthesis.

1. Being a higher paired mechanism, wear between cam and follower is greater than that taking place between links connected through lower pairs in mechanisms. As a result of continuous wear, cam-contour is modified until required function is no longer generated.

This may lead to increased value of 'peak-accelerations' and may create excessive inertia forces, which will adversely affect the performance of the machine.

2. In case of force closure, a follower is held in continuous contact with cam through a spring force. Thus, even when there is no load to be driven, there is a fluctuating spring load because of which it may become impossible to connect it to a simple and desirable input. As against this, close-paired/track cam, where the pairing elements are held together mechanically (see Fig. 7.1), results in increase in the cost of manufacture. Reversals of pressure occur in this type of cam as the roller shifts from one side of the track to the other. This causes accelerated wear and premature failure in certain areas of track. No such problem is anticipated in lower paired mechanism.
3. In majority of applications, cam surface remains unprotected from dust and chips. This is in sharp contrast to linkage mechanism where it is much easier to enclose the joint. Linkage mechanisms are also well recognised for smooth and quiet action. They are preferable in applications requiring tight control on noise production.
4. In machines requiring periodic motions, an important aspect is to modify motions produced by various mechanisms. While it is difficult to adjust motion of a cam mechanism, the motion of a linkage mechanism can be modified easily by adjusting link lengths.
5. One of the major plus points of cam mechanism is that it can be designed to generate specified function/motion exactly at every position of the mechanism. Allowance must, of course, be provided in the above statement for deviations on account of manufacturing tolerances and dimensional changes arising out of wear. As against this, a linkage mechanism can generate motion/function which matches exactly with the specified one only at a limited number of precision points.
6. Another plus point of cam mechanisms is that it can produce exact dwell (*i.e.*, stand still position of follower) of finite duration, which is not possible with linkage mechanism.
7. A yet another plus point in favour of a cam mechanism is that unlike cam, the design of linkage is not that easy. Further, in the case of linkage, the design must be reviewed from the point of view of transmission angle. A review of design, from the point of view of space requirement, may also necessitate redesign of cam mechanism.



**Fig. 7.1** Closed cam pair with track cam and roller.

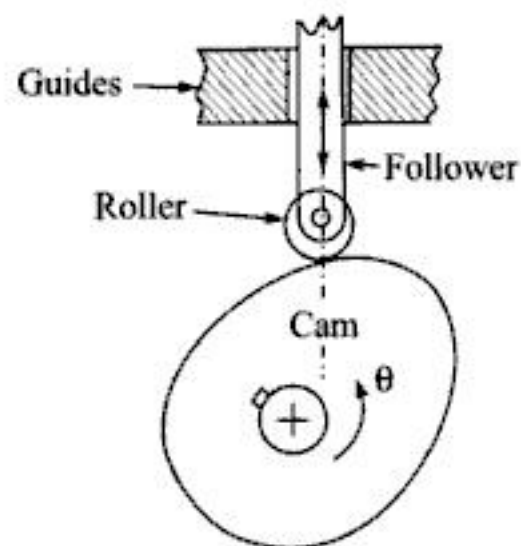
## 7.3 CLASSIFICATION OF CAMS AND FOLLOWERS

### 7.3.1 Classification of Cams

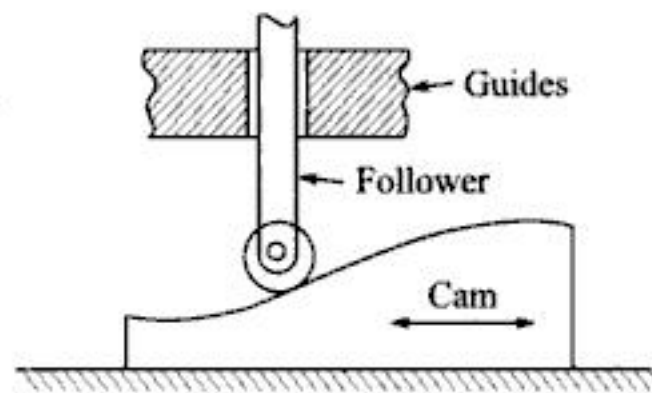
Cams can be classified according to their shapes. Different types of cams are:

1. A plate cam, which is also called a disc cam or a radial cam (see Fig. 7.2a)
2. A wedge cam (see Fig. 7.2b)

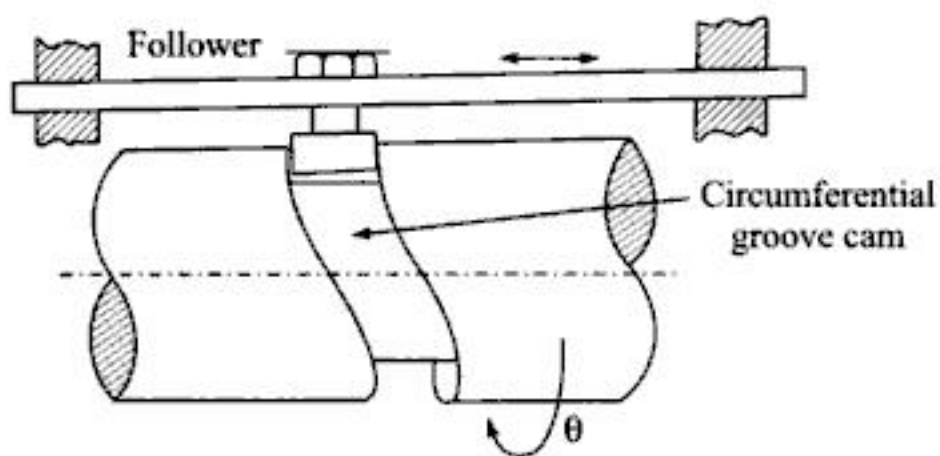
3. A cylindrical cam or barrel cam (see Fig. 7.2c)
4. An end or face cam (see Fig. 7.2d)
5. A conical cam (see Fig. 7.2e)
6. A globoidal cam (see Fig. 7.2f).



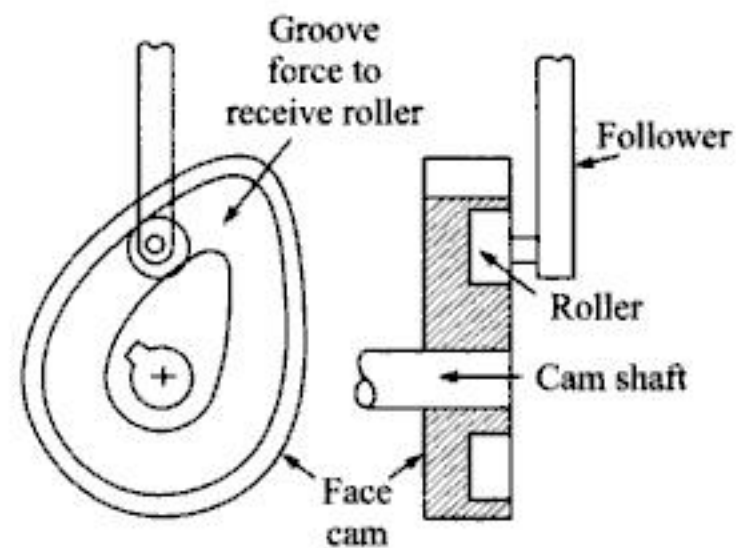
(a) Plate disc or radial cam with translating follower



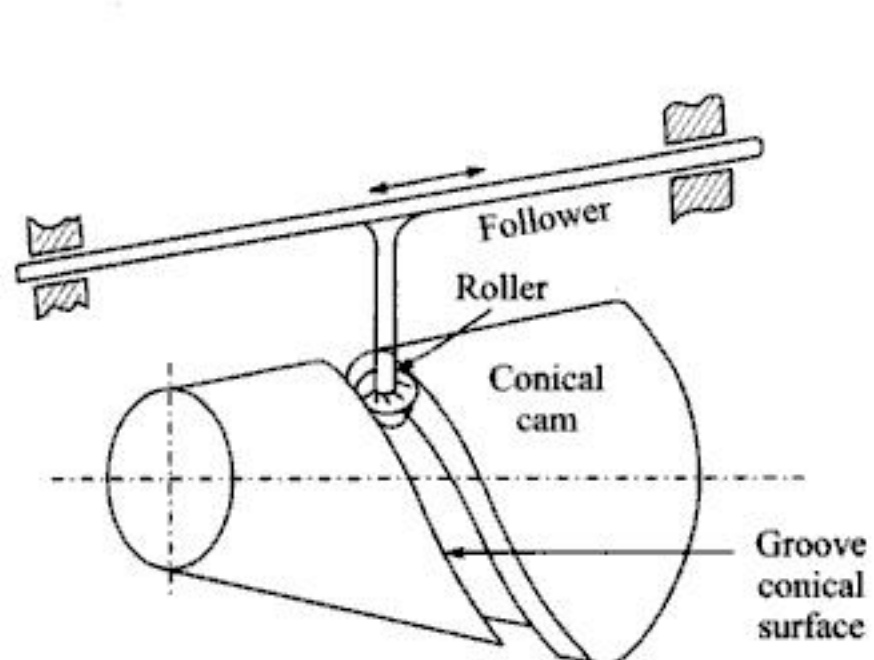
(b) Wedge cam with translating follower



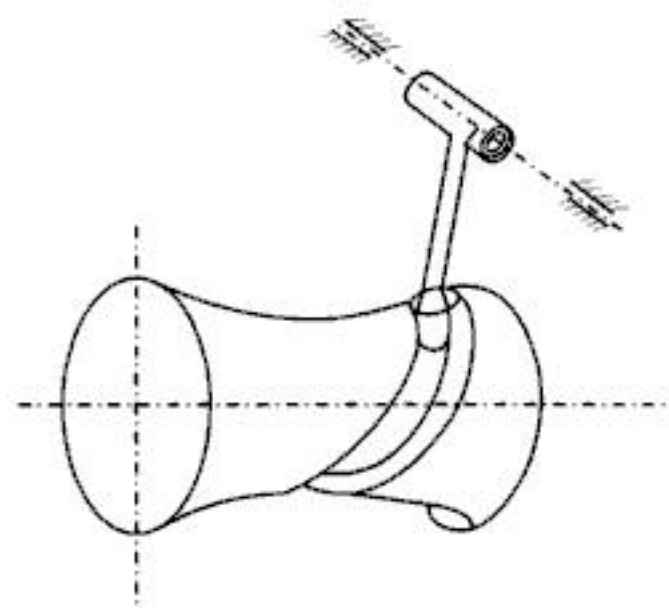
(c) Cylindrical/Barrel cam with translating follower



(d) An end or face cam



(e) Conical cam



(f) Globoidal concave cam with oscillating follower

Fig. 7.2 Illustration for different types of cams.

The **plate/disc cam** is the one most commonly used, while the least common category is the **wedge cam** as it requires reciprocating rather than continuous rotary motion. In view of this, most of the discussions in this chapter are directed towards the study of plate or disc cam, although the principles involved are equally useful in other types of cams.

Figure 7.2(a) shows a radial (plate) or disc cam whose working surface is so shaped that the follower reciprocates, *i.e.* translates in a plane perpendicular to the axis of cam. Figure 7.2(b) shows a wedge cam which lifts roller follower, through wedging action, as the cam translates. The direction of translation of cam and follower are mutually perpendicular but in the same plane. Figure 7.2(c) shows a barrel or cylindrical cam in which follower reciprocates in a direction parallel to the axis of cam. Figure 7.2(d) shows a face or an end cam whose roller moves along the corresponding groove cut in the face of the cam. The follower here reciprocates in a plane perpendicular to the axis of cam. A conical cam is shown in Figure 7.2(e) where the roller moves along a groove cut on the conical surface. The follower reciprocates in a direction parallel to an end-generator. Figure 7.2(f) shows a concave globoidal cam where the follower oscillates in a plane passing through the longitudinal axis of the cam. This type of cam is used for indexing purpose.

### 7.3.2 Classification of Followers

Followers can be classified in three ways:

- (a) On the basis of line/angle of action of follower,
- (b) On the basis of motion characteristics of follower, and
- (c) On the basis of geometrical shape provided on the surface of the element in contact with cam.

#### ***Based on Line/Angle of Action of Follower***

Motion of follower can be either of translation or oscillation. Again, the motion of translation can take place along a line passing through the cam axis or can be placed with some offset.

- (i) *Radial Translatory Followers.* When the line of action of follower passes through cam axis of rotation, the follower is called *radial followers*. These followers are shown in Figs. 7.2(a) and 7.3(a, b, c).
- (ii) *Offset Translatory Followers.* When the line of action of follower does not pass through the axis of cam rotation but is offset, say by an amount of  $e$ , the follower is called *offset follower*. These followers are shown in Figs. 7.3(d) and (e). *An offset is usually provided on a side so as to decrease pressure angle at the point of maximum velocity during outstroke. Readers will learn later that a smaller pressure angle decreases side thrust in guides of follower and increases useful component along line of stroke of follower.*
- (iii) *Oscillatory Follower.* Followers in this category are subjected to motion of oscillation about some fixed axis as the cam rotates. This type of follower are shown in Fig. 7.2(f) and 7.3(g, h).



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Graphically, as shown in Fig. 7.8(a), simple harmonic motion is obtained by constructing a semicircle on the displacement axis of diameter equal to  $h$ , the stroke-length. The semicircle is divided into as many equal parts as are used to divide outstroke angle  $\theta_0$  and return stroke angle  $\theta_R$  along  $\theta$ -axis. Intersection of horizontal projectors from points 1, 2, 3, 4, 5, and 6 with vertical projectors from points 1, 2, 3, 4, 5, and 6 of cam angles  $\theta_0$  and  $\theta_R$  then give the points on simple harmonic displacement diagram. S.H.M. curve is then obtained by joining all these points in correct sequence by a smooth curve.

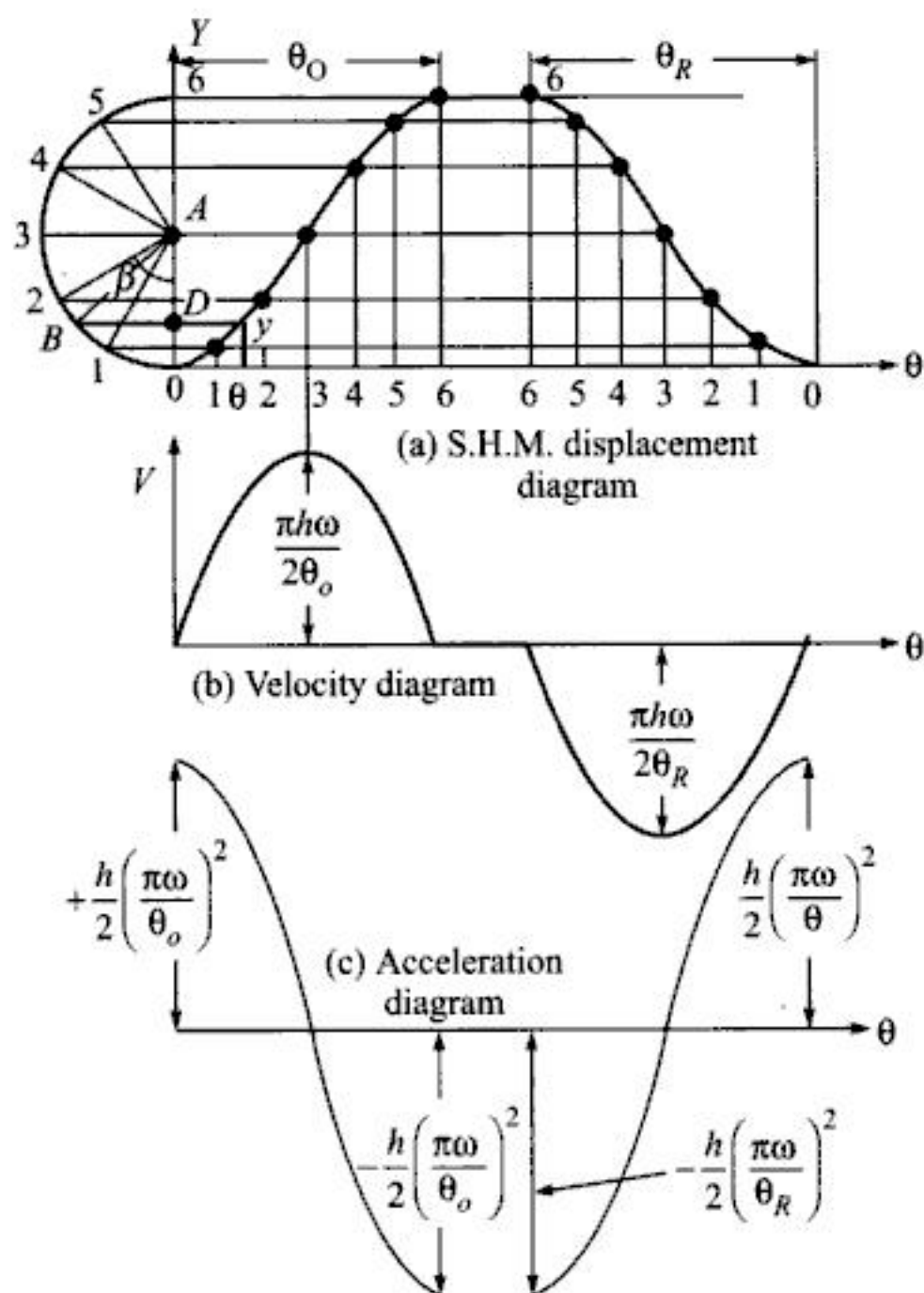


Fig. 7.8

To derive the displacement, velocity and acceleration equation for simple harmonic displacement diagram, let  $AB$  be a radius vector at an angle  $\beta$  from vertical diameter  $AO$  in Fig. 7.8(a). Further, let  $BD$  be the perpendicular dropped from  $B$  on vertical diameter. Then, the follower displacement during outstroke.

$$y = OD = AO - AD$$

$$\text{or } y = \frac{h}{2} - AB \cos \beta = \frac{h}{2} (1 - \cos \beta) \quad (7.4)$$

Now, as the radius  $AB$  rotates through  $\pi$  radians, the outstroke is complete and, during this period, cam turns through outstroke angle  $\theta_0$ . Hence, the angle  $\beta$  can be expressed in terms corresponding cam angle  $\theta$  as

$$\beta/\pi = \theta/\theta_0$$

Therefore

$$\beta = (\pi/\theta_0)\theta \quad (7.5)$$

Substituting for  $\beta$  in equation (7.4),

$$y = \frac{h}{2}[1 - \cos(\pi/\theta_0)\theta] \quad (7.6)$$

Equation (7.6) gives the displacement equation for follower, where  $h$ ,  $\pi$  and  $\theta_0$  are constant and the only variable on R.H.S. is  $\theta$ . Differentiating equation (7.6) w.r. to time.

$$\frac{dy}{dt} = V_0 = \frac{h}{2} \left[ 0 + \frac{\pi}{\theta_0} \left( \frac{d\theta}{dt} \right) \sin \left( \frac{\pi}{\theta_0} \right) \theta \right]$$

Therefore

$$V_0 = \left( \frac{\pi h \omega}{2\theta_0} \right) \sin(\pi/\theta_0)\theta \quad (7.7)$$

where  $d\theta/dt \equiv \omega =$  angular speed of cam which is constant.

Similarly, replacing  $\theta_0$  by  $\theta_R$  in equation (7.7), the expression for velocity of follower during return stroke is

$$V_R = \left( \frac{\pi h \omega}{2\theta_R} \right) \sin(\pi/\theta_R)\theta \quad (7.8)$$

Clearly, maximum velocity of follower during outward stroke and return stroke occurs when  $\theta = \frac{\theta_0}{2}$  and  $\theta = \frac{\theta_R}{2}$  respectively. Thus,

$$(V_0)_{\max} = \left( \frac{\pi h \omega}{2\theta_0} \right) \quad (7.9)$$

and

$$(V_R)_{\max} = \left( \frac{\pi h \omega}{2\theta_R} \right) \quad (7.10)$$

Again, differentiating equation (7.7) w.r. to time,

$$\frac{dV_0}{dt} = A_0 = \left( \frac{\pi h \omega}{2\theta_0} \right) \left( \frac{\pi}{\theta_0} \right) \frac{d\theta}{dt} \cos \left( \frac{\pi}{\theta_0} \right) \theta$$

or

$$A_0 = \frac{h}{2} \left( \frac{\pi \omega}{\theta_0} \right)^2 \cos(\pi/\theta_0)\theta \quad (7.11)$$

The acceleration equation for return stroke is obtained by replacing  $\theta_0$  by  $\theta_R$  in equation (7.11). Thus,

$$A_R = \frac{h}{2} \left( \frac{\pi \omega}{\theta_R} \right)^2 \cos(\pi/\theta_R)\theta \quad (7.12)$$

maximum value of accelerations during outstroke and return strokes are obtained when  $\theta = 0$  or  $\theta_0$  in equation (7.11) and  $\theta = 0$  or  $\theta_R$  in equation (7.12). Thus, maximum acceleration during outstroke and return strokes are

$$(A_O)_{\max} = \pm \frac{h}{2} \left( \frac{\pi \omega}{\theta_0} \right)^2 \quad (7.13)$$

and

$$(A_R)_{\max} = \pm \frac{h}{2} \left( \frac{\pi \omega}{\theta_R} \right)^2 \quad (7.14)$$

The displacement, velocity and acceleration diagrams for the S.H.M. are shown in Fig. 7.8(a), (b) and (c). Remembering that,

$$\frac{dy}{dt} = \frac{dy}{d\theta} \frac{d\theta}{dt},$$

it follows that the slope of the displacement diagram ( $dy/d\theta$ ) is given by  $\left( \frac{dy}{dt} \right) / \omega$ , and since most of the plate cams are driven at constant speeds  $\omega$ , it follows that the velocity diagram of Fig. 7.8 also represents slope of the displacement diagram to a scale ( $1/\omega$ ). Clearly, maximum slope of displacement diagram occurs at  $\theta = (\theta_0/2)$  and also at the middle point of return stroke. These are the points of inflection for the curve, where displacement diagram is the steepest. Readers should note that steepness aspect of cam contour is an important aspect in kinematic design of cam.

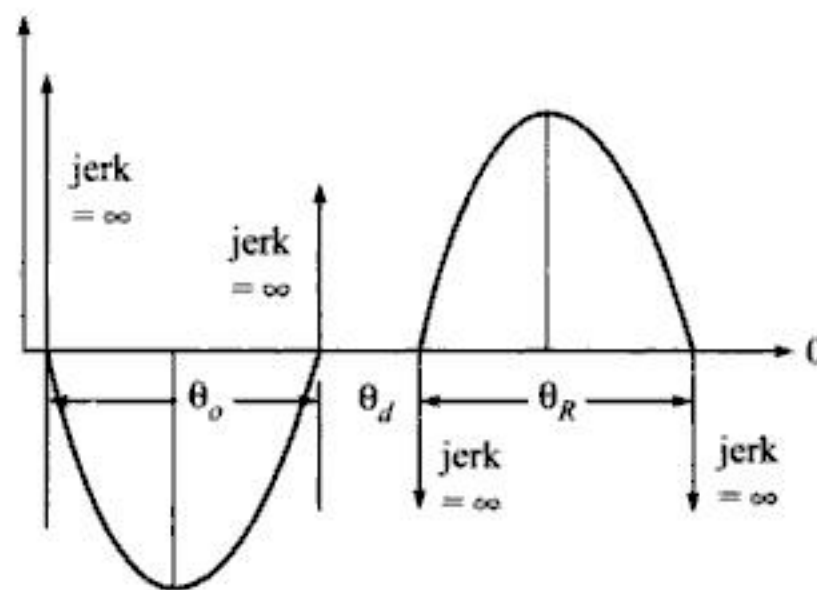


Fig. 7.9 Jerk diagram for simple harmonic motion.

Differentiating equation (7.11) further,

$$\text{jerk} = \frac{dA_O}{dt} = -\frac{h}{2} \left( \frac{\pi \omega}{\theta_0} \right)^3 \sin \left( \frac{\pi}{\theta_0} \right) \theta \quad (7.15)$$

The jerk at the beginning and end of each of the outstroke and return stroke is thus infinite, because a finite value of acceleration is to be generated in no time (Ref. Fig. 7.9). Hence, the jerk  $\left( = \frac{dA}{dt} \right)$  is infinite at the beginning and end of the outstroke/return stroke. It follows from the relation

$$F = mA$$



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Substituting in equation (7.18),  $a = (2h/\theta_0^2)$

$$\text{Hence, from equation (7.18), } y = 2h\left(\frac{\theta}{\theta_0}\right)^2 \quad (7.19)$$

For velocity and acceleration, differentiating successively w.r. to time,

$$V = \frac{dy}{dt} = \frac{4h\omega}{\theta_0} \left(\frac{\theta}{\theta_0}\right) \quad (7.20)$$

where

$$\omega = \frac{d\theta}{dt} = \text{cam speed}$$

and

$$A = \frac{d^2y}{dt^2} = \frac{4h\omega^2}{\theta_0^2} \quad (7.21)$$

It is to be noted that the acceleration is independent of cam angle of rotation  $\theta$ , and hence acceleration  $A$  is uniform throughout. Also, differentiating equation (7.19) w.r. to  $\theta$ , the slope of the displacement diagram is

$$\left(\frac{dy}{d\theta}\right) = \left(\frac{4h\theta}{\theta_0^2}\right) \quad (7.22)$$

$$\text{which is maximum at } \theta = \left(\frac{\theta_0}{2}\right) \text{ and is equal to } (dy/d\theta) = \left(\frac{2h}{\theta_0}\right) \quad (7.23)$$

Also from equation (7.20), maximum velocity occurs when  $\theta = \theta_0/2$  given by

$$V_{\max} = \left(\frac{2h\omega}{\theta_0}\right) \quad (7.24)$$

Equation of follower displacement for the latter half of outstroke, which is beyond the inflection point, is obtained from general equation:

$$y = a'\theta^2 + b'\theta + c' \quad (7.25)$$

where  $a'$ ,  $b'$  and  $c'$  are arbitrary constants. Differentiating w.r. to time,

$$V = (2a'\theta + b')\omega \quad (7.26)$$

where

$$\omega = \frac{d\theta}{dt} = \text{cam speed which is assumed constant}$$

Applying end conditions that at  $\theta = \theta_0$ ,  $V = 0$  and at  $\theta = \frac{\theta_0}{2}$ ,  $V = \left(\frac{2h\omega}{\theta_0}\right)$ , i.e. same as in first half of outstroke.

Thus, for  $V = 0$ , we have

$$0 = (2a'\theta_0 + b')\omega \quad (7.27)$$

and, at  $\theta = \frac{\theta_0}{2}$ ,

$$\left(\frac{2h\omega}{\theta_0}\right) = (a'\theta_0 + b')\omega$$

Cancelling  $\omega$  on either side of this equation,  $a'\theta_0 + b' = (2h/\theta_0)$

$$(7.28)$$





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Conversely to offset the effect of different values of  $(V_y)_{\max}$  and to obtain same value of maximum pressure angle (for a given follower lift), parabolic and cycloidal follower motion require largest size ( $r_b$ ) of cam while the cam with uniform follower motion requires smallest cam size ( $r_b$ ). The simple harmonic follower motion requires moderate cam size. Readers should note further that even for a uniform follower motion where velocity is constant, the pressure angle  $\psi$  changes with follower displacement.

## 7.8 EFFECT OF OFFSET FOLLOWER MOTION

Sometimes the line of action of follower is offset so as to clear another part of machine. However, the main reason in opting for an offset follower motion is to reduce the side thrust on the follower. Direction of offset however is very important. This is because *an offset follower arrangement reduces maximum pressure angle in one of the (outward or return) strokes but increases the same in the other stroke. As a rule, when the line of action of follower is offset to the right of cam-shaft centre, the cam must rotate in counterclockwise direction. Similarly, when the offset is provided to the left cam-shaft centre, the cam must rotate in clockwise direction.* This arrangement result in smaller value of maximum pressure angle for outstroke during which larger forces act on the follower.

Figure 7.14 shows again a radial cam with an offset knife-edge follower. Let the amount of offset be  $e$ . Further, let  $(V_y)_{\max}$  be the maximum velocity of the follower and  $\psi_{\max}$  be the maximum pressure angle. As in the previous case, the instantaneous centre 23 lies at the point  $P$  at the intersection of normal  $n-n$  to the cam profile at  $M$  and a line from cam centre  $O$  drawn perpendicular to the line of stroke. Thus,  $P$  being a point common to links 2 and 3, we have

$$V_P = V_y = (OP)\omega$$

which gives

$$\left(\frac{V_y}{\omega}\right) = \frac{dy}{d\theta} = OP \quad (7.45)$$

Again from Fig. 7.14

$$OP = e + QP \quad (7.46)$$

Further, as

$$\angle PMQ = \psi, \text{ the pressure angle,}$$

$$QP = MQ \tan \psi$$

or

$$QP = (LM + QL) \tan \psi = (y + \sqrt{r_b^2 - e^2}) \tan \psi \quad (7.47)$$

Substituting for  $OP$  and  $QP$  from equations (7.45) and (7.47) in equation (7.46),

$$(dy/d\theta) = e + (y + \sqrt{r_b^2 - e^2}) \tan \psi$$

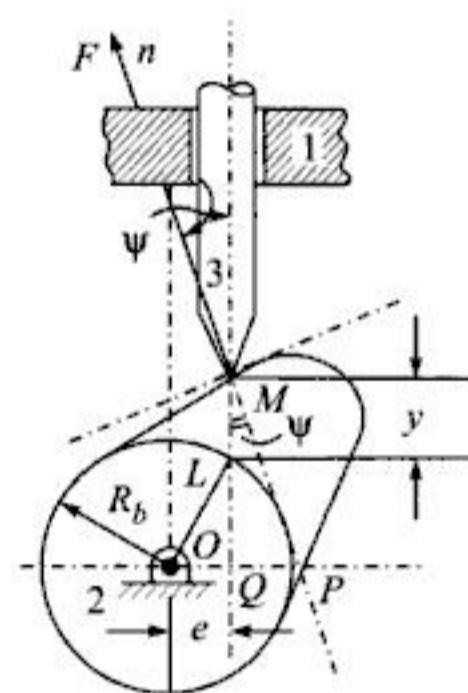


Fig. 7.14 Radial cam with off-set knife edge follower.

Rearranging

$$\tan \psi = \frac{(dy/d\theta) - e}{(\sqrt{r_b^2 - e^2} + y)} \quad (7.48)$$

Multiplying numerator and denominator of equation (7.48) by  $\omega$  and remembering that  $\omega(dy/d\theta) = dy/dt$ , we have

$$\tan \psi = \frac{(dy/dt) - e\omega}{(\sqrt{r_b^2 - e^2} + y)\omega} \quad (7.49)$$

When the follower is at pitch point,  $\psi = \psi_{\max}$  and  $dy/dt = (V_y)_{\max}$  and therefore,

$$\tan(\psi_{\max}) = \frac{(V_y)_{\max} - e\omega}{(\sqrt{r_b^2 - e^2} + y)\omega} \quad (7.50)$$

It follows from equation (7.48) that having chosen a displacement equation for a follower, parameters  $r_b$  (the base circle radius) and offset amount  $e$  may be selected suitably to ensure a satisfactory value of maximum pressure angle. It also follows from equation (7.48) that an increase in  $e$  either increases or decreases the magnitude of numerator depending on sign of slope  $(dy/d\theta)$ . During outstroke, the slope of  $(dy/d\theta)$  is positive and the numerator decreases, reducing the pressure angle  $\psi$  for a given value of  $e$ . As against this, during return stroke  $(dy/d\theta)$  is negative and adds to  $e$  increasing pressure angle  $\psi$ .

It follows from equation (7.50) that maximum pressure angle ( $\psi_{\max}$ ) during outstroke will be smallest when,

$$(V_y)_{\max} - e\omega \approx 0 \quad (7.51)$$

Calling the corresponding value of offset as the critical value  $e_c$ , it follows from equation (7.51) that,

$$e_c \approx (V_y)_{\max}/\omega \quad (7.52)$$

Equations (7.50) and (7.52) also suggest that it is possible to reduce pressure angle  $\psi$  to zero value, resulting in substantial gain in efficiency of transmission by offsetting follower by an amount  $e = e_c$ . Practical considerations, however, restrict achievable offset  $e$  to a maximum value of 50 per cent of the critical value  $e_c$ . Mathematically,

$$e \leq (0.5)e_c \quad (7.53)$$

It may be noted that equations similar to (7.48) and (7.50) can be obtained for roller follower by replacing the term  $r_b$  by prime circle radius  $r_0$ . Thus, for roller follower,

$$\tan \psi = \frac{(dy/d\theta) - e}{(\sqrt{r_0^2 - e^2} + y)} \quad (7.54)$$

and

$$\tan(\psi_{\max}) = \frac{(V_y)_{\max} - e\omega}{(\sqrt{r_0^2 - e^2} + y)} \quad (7.55)$$



### Reduction in Pressure Angle by Reduction in Steepness $dy/d\theta$

Equation (7.44) suggests that the pressure angle  $\psi$  can be decreased for outstroke by decreasing the slope  $dy/d\theta$  of the displacement diagram. In other words, steepness of displacement diagram can be decreased to reduce pressure angle  $\psi$ . It follows from Fig. 7.15 that any reduction in slope  $dy/d\theta$ , for same follower lift  $h$  requires a larger cam angle. Any step to reduce slope ( $dy/d\theta$ ) without increasing base circle radius  $r_b$ , amount to extending cam angle of rise in the next cam angle. This naturally results in reducing cam angle next in the cycle. In many practical situations this is considered undesirable.

To illustrate this point, consider Fig. 7.15 in which uniform follower motion is assumed for simplicity. It is clear from the figure that any step to reduce pressure angle by reducing steepness of displacement diagram, shifts point  $Q$  on displacement diagram to a position  $Q'$  increasing cam angle of rise from  $\theta_0$  to  $\theta'_0$ . For a given angle of return and the subsequent dwell angle  $\theta_w$ , the above step amounts to reducing dwell angle  $\theta_d$  to  $\theta'_d$ . In petrol/gas engine, this necessarily implies smaller time for inducting charge, which is undesirable. Instead of decreasing steepness of displacement diagram, in such cases, use of a flat faced follower is a better option.

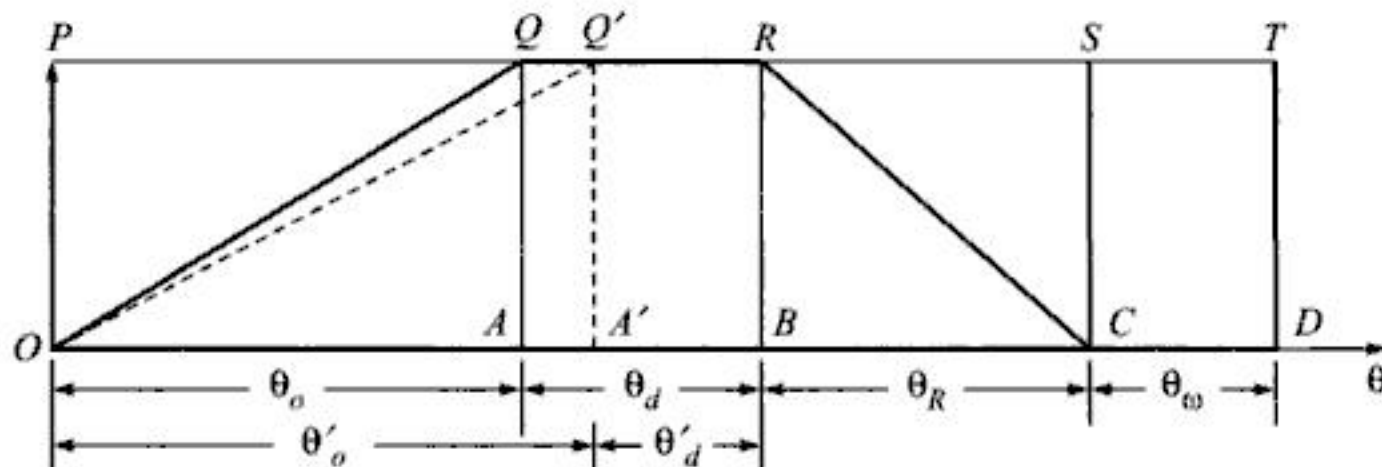


Fig. 7.15 Effect of reducing steepness on pressure angle.

Because common normal to the cam follower interface has to be normal to the flat face of follower, the angle between common normal  $n-n$  and follower axis is zero when the flat face is normal to follower axis. Thus, pressure angle for such flat faced translating follower is always zero. It is for this reason that a flat faced follower is used for relatively steep cam curves, and also where space is limited.

### Nomogram Relating $\psi_{\max}$ to Base Circle Radius

Since a knife-edge follower is a special case of roller follower, with radius of roller reduced to zero, it follows that by replacing  $r_b$  by the prime circle radius  $r_o$ , equation (7.44) leads to the following equation for roller follower.

$$\tan \psi = \frac{dy/d\theta}{(r_o + y)} \quad (7.56)$$

It follows from equation (7.44) that maximum value of  $\psi$  occurs when  $(dy/d\theta)$  is maximum and  $y$  is minimum. To find out maximum values of  $\psi$ , equation (7.44) can be differentiated with respect to  $\theta$  and equated to zero. Such a procedure then yields values of  $\theta$  at which maximum and minimum values of  $\psi$  are obtained. This is, however, not an easy proposition.

A more convenient approach is to construct nomogram by searching on digital computer  $\psi_{\max}$  using equation (7.44) for each of the standard follower motions. One such nomogram is shown in Fig. 7.16. With such a nomogram it is possible to obtain maximum pressure angle if the follower lift  $h$  and base circle radius  $r_b$  are known. A line drawn joining given value of  $(h/r_b)$  to 'total-cam angle  $\theta$ ', then give  $\psi_{\max}$  in the other segment.

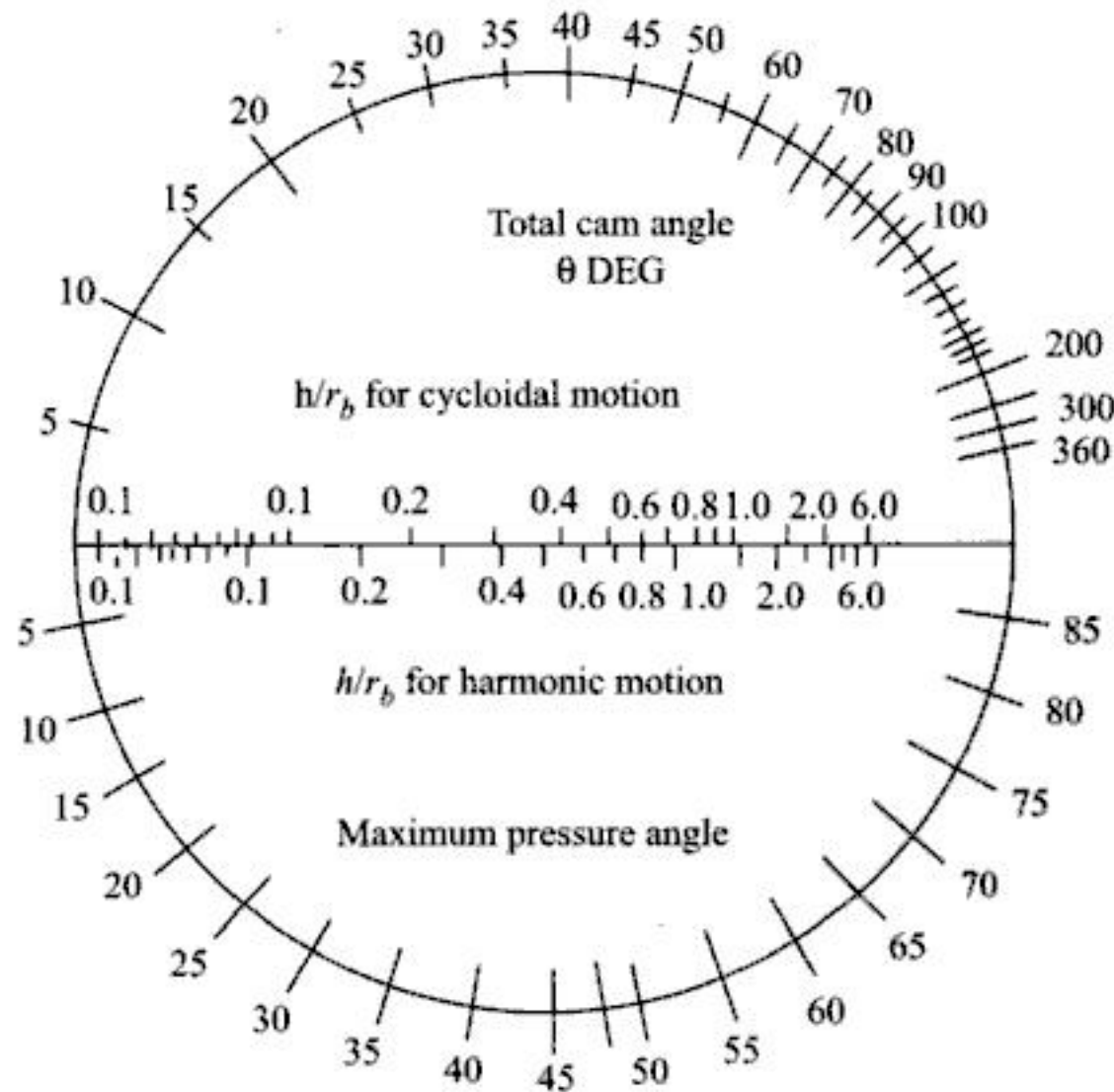


Fig. 7.16 Nomogram relating  $\psi_{\max}$  to cam angle.

## 7.9 MAIN CONSIDERATION INFLUENCING CHOICE OF CAM

For lifting follower through a stroke length  $h$ , for a given cam angle of rotation, innumerable cam profiles are possible. An answer to this question, namely which one of the cam profile to choose, must be based on the following consideration. An attempt will be made to show that majority of these considerations are related in some way to the derivatives of follower displacement diagram.

These considerations are:

- Smaller lateral pressure on guides.
- Force required to accelerate the follower should be smaller.
- Smooth (jerkless) follower motion and long-life.
- Small base circle, consistent with space requirement.

### 7.9.1 Smaller Lateral Pressure on Guides

For smaller lateral pressure on guides, one must ensure a smaller value for 'maximum-pressure angle' during outstroke. A number of steps to reduce 'maximum pressure angle' during outstroke



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	<i>Follower motion</i>	<i>Maximum acceleration during outstroke</i>
a.	Uniform follower motion	zero
b.	Simple harmonic motion	$4.935 \left( \frac{h\omega^2}{\theta_0^2} \right)$
c.	Parabolic follower motion	$4.0 \left( \frac{h\omega^2}{\theta_0^2} \right)$
d.	Cycloidal follower motion	$6.283 \left( \frac{h\omega^2}{\theta_0^2} \right)$

Thus, leaving aside uniform follower motion, parabolic motion requires smallest value of *maximum acceleration*, while cycloidal motion requires largest value of *maximum acceleration*. Stated in other words, every other thing remaining the same, maximum inertia force required to be overcome in parabolic motion is least, while in cycloidal motion it is maximum. Uniform follower motion is excluded from above discussions for obvious reasons.

### 7.9.3 Smooth Jerkless Motion

Except for cycloidal follower motion, all other follower motions produce infinite jerk. Simple harmonic motion involves infinite jerk at the beginning and end of outstroke, while parabolic follower motion involves infinite jerk at the beginning, end as well as at the point of inflection during outstroke. Thus, in this respect, parabolic follower motion is inferior to S.H.M. *Besides jerky motion, presence of infinite jerk introduces abruptly changing contact stresses at the bearings and on the cam surface and leads to noise, surface wear and eventual failure. This is especially true for cams running at high speeds.* Needless to say that in cycloidal motion, jerk is not infinite anywhere in the outstroke. However, even in cycloidal motion jerk is present in the third power of cam speed  $\omega$ . Thus as the speed increases, this component increases at a very fast rate. In general, however, a cycloidal follower motion is preferable to any of the standard follower motions.

### 7.9.4 Smaller Base-circle

Besides occupying small space, cams with smaller base-circle produce less of unbalance at higher speeds and the follower has a smaller path to follow. Smaller cams, however, tend to have larger pressure angle for the same lift  $h$ . As the cam profile is steeper, it tends to bend the follower sideways. It has been shown that for a common value of maximum pressure angle of  $30^\circ$  (at pitch point), circumference of pitch circles required for standard follower motion are:

- (a) Uniform follower motion =  $1.73 h$
- (b) Modified uniform motion =  $2.27 h$
- (c) Simple harmonic motion =  $2.72 h$
- (d) Parabolic and cycloidal motion =  $3.46 h$

Clearly, uniform follower motion requires smallest size of cam while parabolic and cycloidal motion of follower requires largest size of cam.

## 7.10 RADIUS OF CURVATURE AND UNDERCUTTING

Even if a cam is proportioned for giving satisfactory pressure angle, the follower still may not produce specified follower motion if the pitch curve is too sharp. Knife-edge follower has the advantage in that there is no restriction on the radius of curvature of the cam profile. *With a flat-footed mushroom follower, the cam profile must not have a concave shape. When using a roller follower, the roller must be free to touch all the points on cam profile. This requires that concave portion, occurring anywhere on the cam profile, must have a radius of curvature greater than that of the roller.*

If  $r_o$  be the prime circle radius,  $r_r$  be the roller radius and  $s_\rho$  be the radius of curvature of pitch curve in convex portion, then,

$$s_\rho = \rho + r_r \quad (7.57)$$

where

$$\rho = \text{radius of curvature of cam profile}$$

Now if roller radius  $r_r$  be increased so that  $r_r \rightarrow s_\rho$ , then under this situation  $\rho \rightarrow 0$ . In other words, the cam profile becomes pointed. Figure 7.17(a) envisages such a situation where roller radius is equal to the radius of curvature of pitch curve. A still more interesting situation arises if  $r_r$  exceeds the value of  $s_\rho$ . In such a case, as shown in Fig. 7.17(b), the cam-curve loops over itself which is impossible. This situation is known as *undercutting*. Presence of undercutting leads

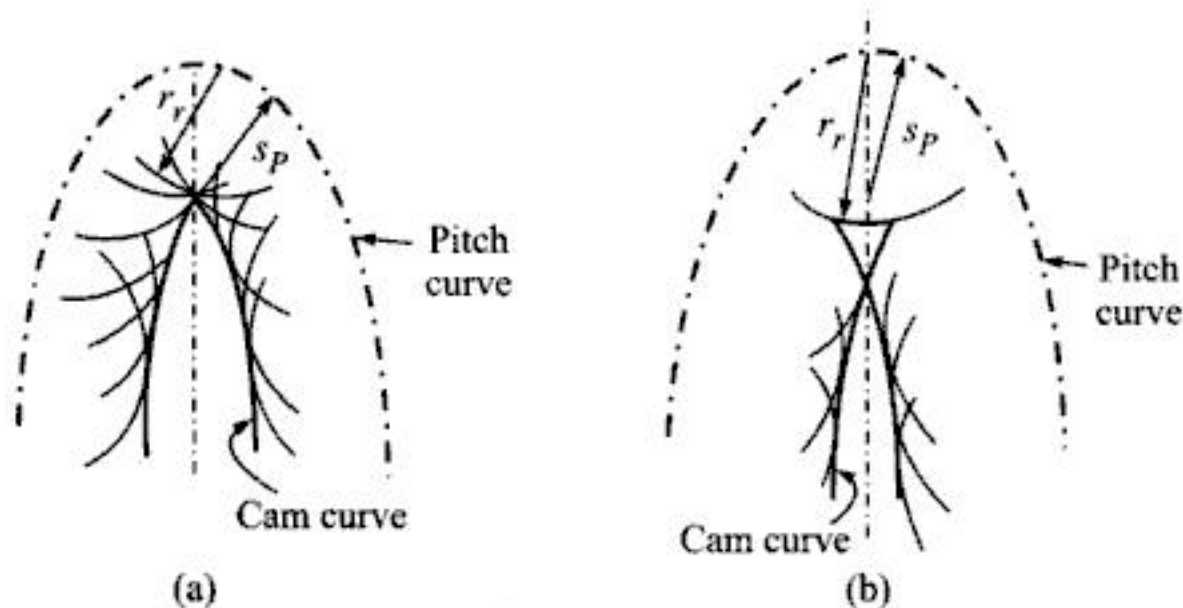


Fig. 7.17 Illustrations depicting conditions (a)  $\rho = 0$  and (b)  $r_r > s_\rho$ .

to deviation of roller centre from pitch curve. Thus, for satisfactory operation (avoiding both the conditions cited),

$$(S_\rho)_{\min} > r_r \quad (7.58)$$

Thus, the minimum radius of curvature of pitch curve must be greater than the roller radius.

## 7.11 CONSTRUCTION OF CAM PROFILES

Having learnt standard follower motions and their peculiarities, it remains now to generate (*i.e.* lay out) the cam curves based on specified follower motions. *A true cam profile must ensure that every point on it is accessible to the follower. Method of constructing cam profile for a specified follower motion, is based on the principle of inversion.*



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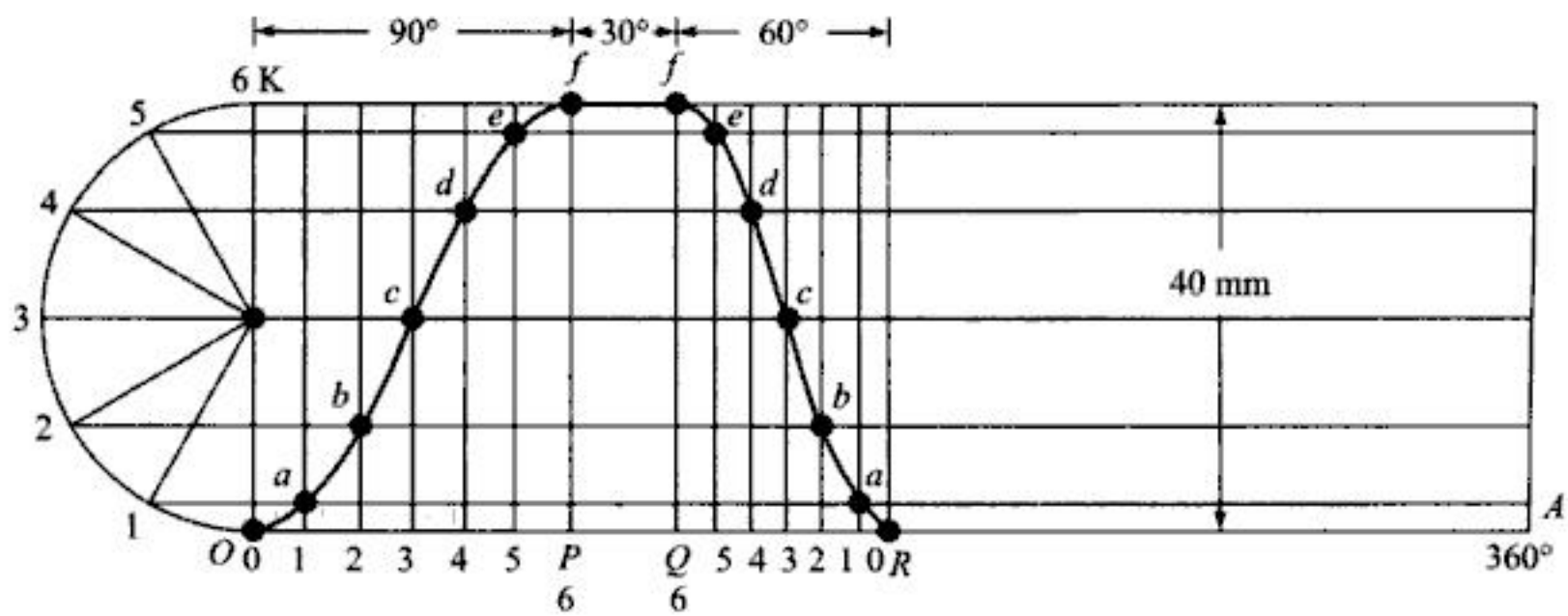


Fig. 7.18 Displacement diagram.

diagram, the cam profile of Fig. 7.19 may be drawn as detailed below. Though the follower is not stated to be a radial one, in the absence of any information about offset, the line of action of follower is assumed to pass through cam-shaft axis. During the dwell period  $PQ$  the follower does not move. The displacement diagram during the dwell period is therefore represented by straight line  $ff$ .

- (a) Draw the base circle of 40 mm diameter with  $C$  as the centre. With  $CO$  as the assumed initial position of follower, mark angle  $\angle OCP = 90^\circ$  for cam angle of rise. Also mark angle  $\angle PCQ = 30^\circ$  as angle of dwell and angle  $\angle QCR = 60^\circ$  as the angle of return.

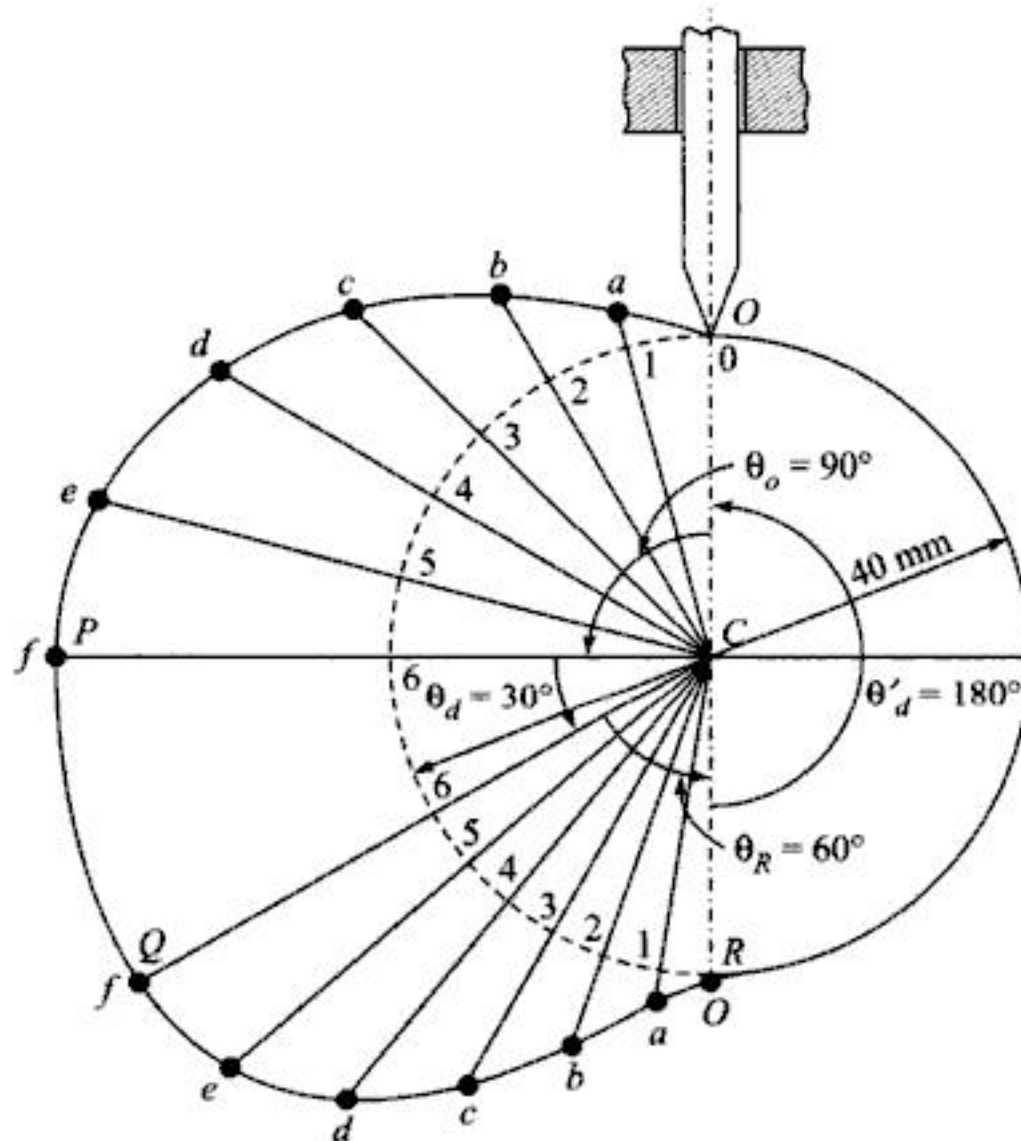


Fig. 7.19 Radial cam with knife edge follower.

- (b) Divide the angles  $\angle OCP$  and  $\angle QCR$  each in six equal parts, and draw the lines  $C-1$ ,  $C-2$ ,  $C-3$ ,  $C-4$ ,  $C-5$  and  $C-6$  in each of the rise and return portion.
- (c) The ordinates of displacement diagram  $1-a$ ,  $2-b$ , etc., represent the displacement of the follower from the zero displacement position. Hence, transfer the distance  $1-a$ , from displacement diagram on radial lines  $C-1$ , beyond base circle. Similarly, transfer the distance  $2-b$  from displacement diagram on  $C-2$  in Fig. 7.19 beyond base circle. Proceeding in this way  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  are the points obtained for the rise and return portion.
- (d) Since the knife-edge itself is the trace point, points  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  in Fig. 7.19 are the points on cam profile. Join these points in correct order by a smooth curve to obtain cam curve for rise and return period.
- (e) For the dwell period  $PCQ$ , the follower does not move along line of stroke and therefore, cam profile for dwell segment  $PCQ$  is given by an arc of circle with  $Cf$  as radius. Similarly, for the angle of dwell  $\angle RCO = 180^\circ$ , the cam profile is given by arc of a circle with  $CR$  as radius, (*i.e.* the base circle itself).

Students are advised to ensure that same scale is used for drawing displacement diagram and the cam profile.

For simple harmonic motion, maximum velocity during ascent (Rise) and descent (Return) is given for S.H.M. by

$$V_o = \frac{\pi \omega h}{2\theta_o} \quad \text{and} \quad V_R = \frac{\pi \omega h}{2\theta_R}$$

Now as 
$$\omega = \frac{2\pi \times 240}{60} = 8\pi \text{ rad/s}$$

and 
$$\theta_o = \frac{\pi \times 90}{180} = \frac{\pi}{2} \text{ radian}$$

with 
$$\theta_R = \frac{\pi \times 60}{180} = \frac{\pi}{3} \text{ radian}$$

Therefore 
$$V_o = \frac{\pi(8\pi)4.0}{2(\pi/2)} = 32\pi = 100.53 \text{ cm/s} \quad \text{Ans.}$$

and, 
$$V_R = \frac{\pi(8\pi)4.0}{2(\pi/3)} = 48\pi = 150.796 \text{ cm/s} \quad \text{Ans.}$$

## 7.12 CAM LAYOUT: GENERAL TYPE OF PROBLEMS

Problems involving knife-edge follower and that too, with its line of action passing through cam-axis of rotation, represent simplest type of problems on cam profile layout. General type of cam-layout problems require few more parameters, depending on the type of problems. These parameters are: prime circle radius, offset distance, roller-radius, and so on. Even the displacement diagram can be more involved.

Again in the case of knife-edge follower, contact between cam and follower must necessarily occur at the knife-edge, which falls on the line of action of the follower. It was therefore possible





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**Solution:** The radius  $r$  of circle, which rolls along displacement axis to generate cycloid, is given by

$$2\pi r = \text{stroke length} = 44 \text{ mm}$$

or, 
$$r = 44 \times \frac{1}{2} \times \left(\frac{7}{22}\right) = 7 \text{ mm}$$

Also, cam speed 
$$\omega = \frac{2\pi \times 500}{60} = 52.36 \text{ rad/s}$$

**Important Note:** In some of the problems involving eccentricity of line of action of follower, it is not given whether follower centre-line is off-set to the right or left of cam centre. In such cases, the direction of off-set should be such that rotating cam (in c.w. or c.c.w. sense) tends to push the follower-axis so as to make it radial.

### Construction of Displacement Diagram

1. Select a suitable scale, say  $1 \text{ cm} = 30^\circ$ , along  $\theta$ -axis and mark  $OA = 180^\circ$ ;  $AP = 150^\circ$  and  $PR = 30^\circ$  to scale, to represent the angles of rise, return and dwell respectively.
2. Draw  $OD = 44 \text{ mm}$  to represent stroke length to full scale. Complete the rectangle  $OPQD$ . Draw  $AB$  perpendicular to  $OA$ . Divide  $OA$  and  $AP$  each in six equal parts and number these divisions as 1 through 6 and  $1'$  through  $6'$  as shown in Fig. 7.21(a). From each of these divisions, draw lines perpendicular to side  $OP$ .
3. With  $B$  as the centre describe a circle of diameter 14 mm and divide this circle in six equal parts, in such a way that at least one division point is located on the horizontal diameter. Number the divisions as 1 through 6 as shown in Fig. 7.21. Let  $p$ ,  $q$  and  $s$  be the feet of perpendiculars dropped on vertical diameter from pair of points (4, 5); (3, 6) and (1, 2) respectively.
4. Join point  $q$  to the points  $O$  and  $P$  on displacement diagram. These lines cut vertical lines from division points 3, in outward and return strokes, at points  $c$  and  $c'$  respectively. From point  $s$  draw a line parallel to  $oq$  to cut vertical lines, from divisions 1 and 2 on  $OA$ , at points  $a$  and  $b$ . Similarly, from point  $p$  draw a line parallel to  $oq$  to cut vertical lines from divisions 4 and 5 at points  $d$  and  $e$  respectively. Join points  $o$ ,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $q$  by a smooth curve to get displacement diagram for outstroke.
5. From point  $q$  draw a line  $qp$  to cut vertical from point  $3'$ , on line  $AP$  at point  $c'$ . Again from points  $p$  and  $s$  draw lines parallel to line  $pq$  to cut vertical lines from divisions  $5'$ ,  $4'$  and  $2'$ ,  $1'$  at points  $e'$ ,  $d'$ ,  $b'$  and  $a'$  respectively, as shown in Fig. 7.10(a). Join points  $P$ ,  $a'$ ,  $b'$ ,  $c'$ ,  $d'$ ,  $e'$  and  $q$  by a smooth curve to complete the displacement diagram.

To draw the cam profile, draw from a suitable point  $C$  as centre, following circles of radii as indicated below:

- (i) Offset circle of radius 1 cm
- (ii) Base circle of radius 1.5 cm
- (iii) Prime circle of radius 2.0 cm, and then proceed as under:
  1. Draw a vertical line tangent to offset circle to the right. Consider this as the reference position of zero displacement of the follower. Let  $L$  be the point of tangency and let line of action of follower for this position cut the prime circle at  $O$ .



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2. The direction of rotation of cam is not given, but the same can be determined considering the direction in which offset is given. This is a stage at which mistake is most likely to creep in. *Offset is always so provided that the rotating cam tends to push it so as to make it a radial follower.* Thus for the offset provided on the right hand side of cam centre, the cam must rotate in counterclockwise sense. Thus,  $\omega$  is c.c.w.
3. Considering cam mechanism, therefore, the follower must be moved in c.w. sense around the cam. Thus measure angle  $\angle LCM = 180^\circ$  in c.w. sense from  $CL$  to denote outstroke and  $\angle MCN = 150^\circ$  in c.w. sense from  $CM$  to denote return stroke angle.
4. Divide the arc  $LM$  (representing outstroke) of the offset circle in six equal parts and draw tangents to offset circle from these points to cut prime circle at points 1, 2, 3, 4, 5 and 6 as shown in Fig. 7.21(b). Similarly, divide the arc  $MN$  in six equal parts and from each division draw lines tangent to offset circle to cut the prime circle at points  $0', 1', 2', 3', 4'$  and  $5'$  as shown in Fig. 7.21(b).
5. Transfer the follower displacements, as measured from displacement diagram, and plot them beyond prime circle along corresponding lines of action of follower to obtain points  $a, b, c, d, e, q$  and  $a', b', c', d', e'$ . These points give the location of respective trace point.
6. A smooth curve joining points  $o, a, b, c, d, e, q, e', d', c', b', a'$  and back to  $o$ , gives the pitch curve.
7. To obtain the cam curve, as in example 7.3, describe roller surfaces with centre at points  $a$  through  $q$  and  $a'$  through  $e'$ . A smooth curve touching all these follower surfaces gives cam profile. This is shown in Fig. 7.21(b).

#### Important Points

- (a) While describing roller surfaces, it is preferable to describe complete circles. If this is considered difficult or time-consuming, the arcs representing roller surfaces should preferably extend on either sides of the follower centre line by at least  $60^\circ$ .
- (b) It is equally important that the curve representing cam profile must touch roller surfaces for all the positions of roller centres on pitch curve.

Maximum velocity and acceleration during outstroke and return stroke are:

$$\text{Outstroke: } V_{\max} = \frac{2h\omega}{\theta_0} = \frac{2(4.4)(52.36)}{\pi} = 146.67 \text{ cm/s}$$

$$A_{\max} = \frac{2\pi h\omega^2}{\theta_0^2} = \frac{2\pi \times (4.4)(52.36)^2}{\pi^2} = 7679.48 \text{ cm/s}^2$$

$$\text{Return stroke: } V_{\max} = \frac{2h\omega}{\theta_R} = \frac{2(4.4)(52.36)}{5\pi/6} = 176.00 \text{ cm/s}$$

$$A_{\max} = \frac{2\pi h\omega^2}{\theta_R^2} = \frac{2\pi \times 4.4(52.36)^2}{(5\pi/6)^2} = 11058.46 \text{ cm/s}^2$$

**EXAMPLE 7.5** (a) A radial cam, operating a flat mushroom follower, rotates at 200 r.p.m. The follower rises through 20 mm with S.H.M. during  $120^\circ$  of cam rotation. It dwells for 30 degrees of cam rotation and returns to the initial position by S.H.M. in next  $150^\circ$  of cam rotation. Assuming a minimum radius for cam to be 25 mm, draw the cam profile. (DAVV Indore, Nov. 1987)

(b) Determine  $V_{\max}$  &  $A_{\max}$  during outstroke. Also determine minimum face width of follower required.

**Solution:** *Construction* As explained in example 7.2 construct displacement diagram for S.H.M. follower motion during outstroke and return stroke with  $h = 20$  mm;  $\theta_o = 120^\circ$ ;  $\theta_d = 30^\circ$ ;  $\theta_R = 150^\circ$ ; and  $\theta'_d = 60^\circ$ . This is illustrated in Fig. 7.22(a).

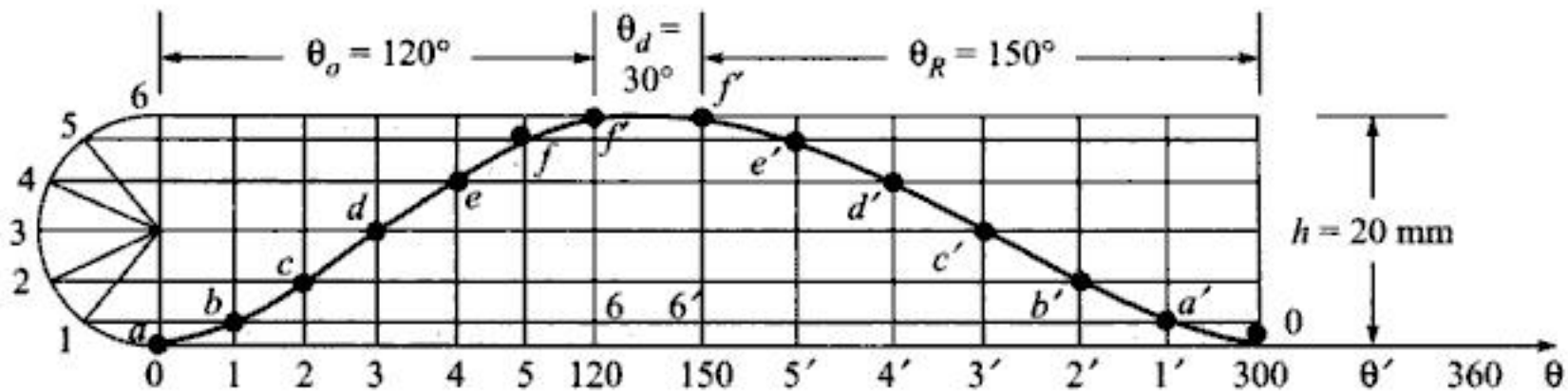


Fig. 7.22(a) Displacement diagram for S.H.M. follower motion.

### Cam Layout

1. Draw the base circle of 25 mm radius with  $C$  as the centre. Assume the rotation of cam in c.w. sense.
2. Take reference position, corresponding to zero displacement, of follower at  $O$ , the extremity of vertical diameter. Divide the circle in angles  $\angle OCP = 120^\circ$  for outstroke  $\angle PCQ = 30^\circ$  for dwell,  $\angle QCR = 150^\circ$  for return stroke and  $\angle RCO = 60^\circ$  for dwell again.
3. Divide angles  $\angle OCP$  and  $\angle QCR$  in six equal parts. Let the corresponding division marks for outstroke by 1, 2, 3, 4, 5 and 6 and for return strokes by 1', 2', 3', 4', 5' and 6' as shown in Fig. 7.22(b).
4. Draw radial lines from divisions 1 from 6 and also through divisions 1' through 6' to denote lines of action of follower. Note that cam is assumed to rotate in c.w. direction and therefore, follower is being moved around cam in c.c.w. direction.
5. Along the radial lines  $C-1$ ,  $C-2$ ,  $C-3$ ,  $C-4$ ,  $C-5$  and  $C-6$  transfer amount of follower displacement away from base circle, to obtain points  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$ . Similarly plot points  $a'$ ,  $b'$ ,  $c'$ ,  $d'$ ,  $e'$  and  $f'$  for return stroke.
6. From points  $a$ ,  $b$ ,  $c$ ,.....  $f$  and  $a'$ ,  $b'$ ,  $c'$ ,.....  $f'$  draw straight lines perpendicular to follower centre lines to represent flat faces of follower.
7. Draw a smooth curve touching each and every follower face so described. The resulting cam curve is shown in Fig. 7.22(b). Note that if any follower face does not touch cam, corresponding follower position is not obtainable and the resulting cam profile is rendered incorrect.

Readers must note that if the follower's flat-face is not sufficiently wide, it cannot rest on cam in the corresponding follower position. Thus, consider follower position along  $C-3$ , in which the point of contact between cam and flat-face is farthest in c.c.w. sense at a distance of 14 mm. If flat face of follower on this side does not extend by a distance greater than or equal to 14 mm the prescribed follower position is not obtainable. Similarly, in position  $C-3'$  during return stroke of follower, the follower cannot rest on cam unless on the other side flat face does not extend

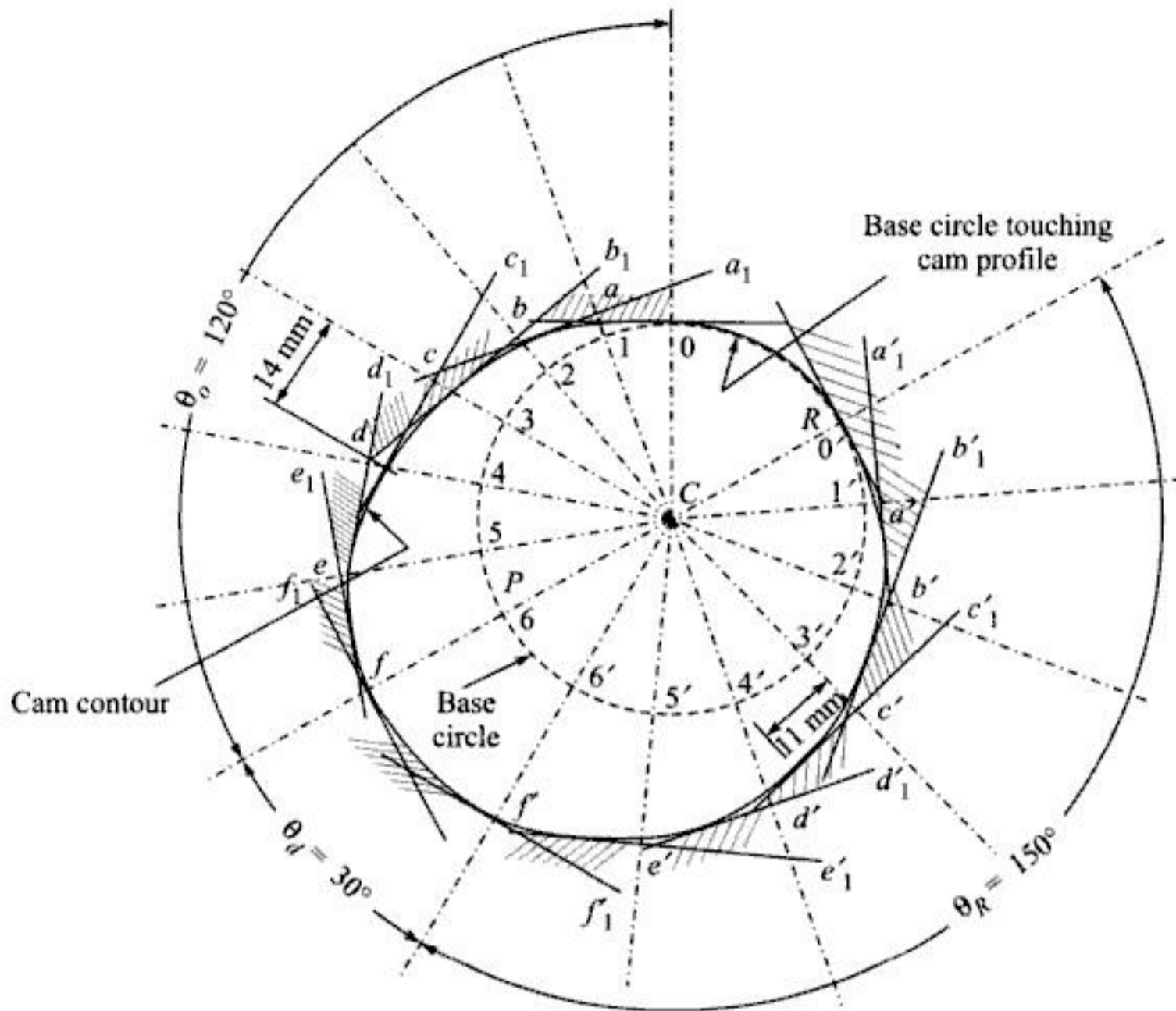


Fig. 7.22(b) Cam profile layout for flat faced follower.

by an amount greater than or equal to 11 mm. In every other position this requirement is smaller than 11 mm. Thus the follower must have a width (14 mm + margin) on left hand side and (11 mm + margin) on the right hand side, as shown in Fig. 7.23. Maximum velocity and acceleration during outstroke for S.H.M. are given by

$$V_{\max} = \frac{\pi h \omega}{2\theta_0} = \frac{\pi(2.0)}{2(2\pi/3)} \times \left( \frac{2\pi \times 200}{60} \right) = 31.4159 \text{ cm/s}$$

and, 
$$A_{\max} = \frac{h}{2} \left( \frac{\pi \omega}{\theta_0} \right)^2$$

or, 
$$A_{\max} = \frac{2.0}{2} \left( \frac{\pi}{2\pi/3} \right)^2 \omega^2 = 1.0 \left( \frac{3}{2} \right)^2 \omega^2 = 987 \text{ cm/s}^2$$

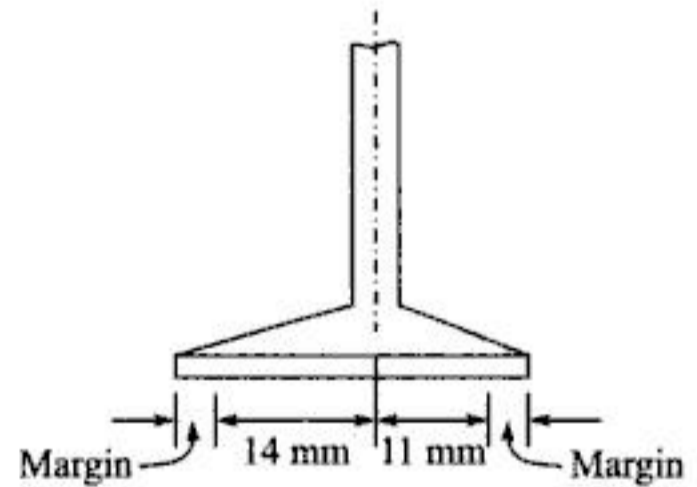


Fig. 7.23 Facewidth of follower.

### 7.13 TRANSLATING FLAT FACE FOLLOWER: ANALYTICAL DESIGN

Whenever a large follower lift is attempted in too little cam rotation with too small a cam, in all probability the cam profile crosses over itself. This phenomenon was described earlier in section





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Further, from the definition of gear ratio,

$$\text{Gear ratio} \quad G = \frac{T}{t} = \frac{R_3}{R_2} \quad (\text{from eq. 8.5}) \quad (8.6)$$

Again, for same pitch line velocity at pitch-point,

$$V_P = R_2 \omega_2 = R_3 \omega_3 \quad (8.7)$$

Hence, from equations (8.6) and (8.7),

$$\text{Gear ratio} \quad G = (T/t) = (R_3/R_2) = (\omega_2/\omega_3) \quad (8.8)$$

**EXAMPLE 8.1** A spur gear drive transmits a gear ratio of 4.0 over a centre distance of 50 cms. If module is 10 mm, determine the pitch circle diameters of pinion and gear. If pinion transmits a torque of 6000 N·cm, find (i) pressure between the teeth and (ii) load transmitted to bearing if pressure angle is  $20^\circ$ .

**Solution:** If  $D$  and  $d$  be the pitch circle diameters and  $T$  and  $t$  be the number of teeth on gear and pinion respectively, then for a centre distance of  $C = 500$  mm,

$$\begin{aligned} 500 &= (d + D)/2 \\ &= \frac{1}{2}(mt + mT) = \frac{1}{2}mt \left(1 + \frac{T}{t}\right) \\ &= \frac{1}{2}(10)t(1 + G) \end{aligned}$$

As gear ratio  $G = 4.0$ , above relation gives,

$$t = \frac{500 \times 2}{10 \times (1 + 4)} = 20$$

And therefore

$$T = G \times t = 80$$

Therefore

$$d = 20 \times m = 200 \text{ mm}$$

and

$$D = 80 \times m = 800 \text{ mm.} \quad \text{Ans.}$$

Further, assuming 100% transmission efficiency,

$$(\text{Torque} \times \omega)_p = (\text{Torque} \times \omega)_g$$

$$\text{Hence, Torque on gear} = \left(\frac{\omega_2}{\omega_3}\right) \times 6000 = 24000 \text{ N·cm}$$

If  $F_t$  and  $F_r$  be the tangential and radial components of tooth load on gear and  $F_n$  be the load transmitted by pinion,

$$F_t = \left(\frac{\text{Torque on gear}}{D/2}\right) = \frac{24000}{(80/2)} = 600 \text{ N}$$



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Join  $O_2$  and  $O_3$  and draw line  $LN$  normal to the contacting surfaces at  $A$ , and let the two lines intersect at  $P$ . Draw  $O_2M$  and  $O_3N$  as perpendiculars to common normal  $LN$  from  $O_2$  and  $O_3$  respectively. Join  $A$  to  $O_2$  and  $O_3$ . Let  $\angle AO_2M = \alpha$  and  $\angle AO_3N = \beta$ .

At the given instant a point  $A$ , considered to be point on body 2, is moving with the velocity  $V_{A2} = (O_2A)\omega_2$  in a direction perpendicular to line  $O_2A$ . Similarly the point  $A$ , now considered to be a point on body 3, is moving with velocity  $V_{A3} = (O_3A)\omega_3$  in a direction perpendicular to the line  $O_3A$  as shown in Fig. 8.14. Note that  $V_{A2}$  is perpendicular to  $O_2A$  and  $MN$  is perpendicular to line  $O_2M$ . Hence  $\angle CAN = \angle AO_2M = \alpha$ . Similarly, it can be shown that  $\angle BAN = \angle AO_3N = \beta$ .

The component of velocities  $V_{A2}\cos\alpha$  and  $V_{A3}\cos\beta$  of velocity vectors  $V_{A2}$  and  $V_{A3}$  respectively, along the common normal  $LN$  must be equal. If they were not equal, either body 2 would dig into body 3 or, it would move away from it (which means broken contact). Thus,

$$V_{A2}\cos\alpha = V_{A3}\cos\beta \quad (8.9)$$

But,  $V_{A2} = (O_2A)\omega_2$  and  $V_{A3} = (O_3A)\omega_3$ . Substituting in equation (8.9),

$$(O_2A)\omega_2\cos\alpha = (O_3A)\omega_3\cos\beta \quad (8.10)$$

From right angled triangles  $O_2MA$  and  $O_3NA$ ,

$$(O_2A)\cos\alpha = O_2M; \text{ and } (O_3A)\cos\beta = O_3N$$

Substituting in equation (8.10) and rearranging,

$$(\omega_2/\omega_3) = (O_3N)/(O_2M) \quad (8.11)$$

The triangles  $O_2MP$  and  $O_3NP$  are similar as besides right angles at  $M$  and  $N$ ,  $\angle O_2PM = \angle O_3PN$ . Hence,

$$(O_3N/O_2M) = (O_3P)/(O_2P) = (NP)/(MP) \quad (8.12)$$

Substituting for  $(O_3N/O_2M)$  in equation (8.11), we have

$$(\omega_2/\omega_3) = (O_3P)/(O_2P) \quad (8.13)$$

Thus the ratio of angular velocities  $(\omega_2/\omega_3)$  varies inversely as the ratio of distances of point  $P$  from centres  $O_2$  and  $O_3$ . Clearly, the ratio  $(\omega_2/\omega_3)$  will remain constant as long as the position of point  $P$  is fixed along the line  $O_2O_3$ . This leads us to following law of gearing.

*In order that a pair of curved surface (tooth profiles) may transmit a constant angular velocity ratio, the shape of contacting tooth profiles must be such that the common normal passes through a fixed point  $P$  on the line of centres. The point  $P$  divides the line of centres in an inverse proportion as the ratio of angular velocities. The fixed point  $P$  is called the pitch point and the line  $MN$  (common normal to contacting surfaces) is called the line of action.*

**The Velocity of Sliding.** Component of velocities  $V_{A2}$  and  $V_{A3}$  along the common tangent  $t-t$  are given by  $V_{A2}\sin\alpha$  and  $V_{A3}\sin\beta$ . The velocity of sliding of surface of body 3 relative to the surface of body 2 at the point of contact is given by

$$\begin{aligned} \text{Velocity of sliding} &= V_{A3}\sin\beta \sim V_{A2}\sin\alpha \\ &= \omega_3(O_3A)\sin\beta \sim \omega_2(O_2A)\sin\alpha \\ &= \omega_3(AN) \sim \omega_2(AM) \\ &= \omega_3(AP + PN) \sim \omega_2(PM - AP) \\ &= (\omega_2 + \omega_3)AP + \omega_3PN \sim \omega_2PM \end{aligned} \quad (8.14)$$

From equation, (8.12) and (8.13) it follows that,

$$\frac{\omega_2}{\omega_3} = \frac{O_3P}{O_2P} = \frac{NP}{MP} \text{ or } (\omega_2)MP = \omega_3(NP)$$

Substitution as above in equation (8.14) last two terms cancels out, yielding,

$$\text{Velocity of sliding} = V_s = (\omega_2 + \omega_3)AP. \quad (8.15)$$

This is obvious because point  $P$  is an instantaneous centre of rotation.

It follows from equation (8.15) that velocity of sliding is zero at the pitch point where  $AP \rightarrow 0$ , and is maximum at the farthest point along the line of action.

### The Pitch Point as an I.C.

Referring to Fig. 8.14, let frame link be 1. Clearly, the gear centres  $O_2$  and  $O_3$  are the instant centres 12 and 13 respectively. From the theorem of 3 centres, it follows that the third instant centre of rotation of link 2 with respect to link 3 will always lie on the line joining  $O_2$  and  $O_3$ . Further, since this instant centre  $I$  is a point common to links 2 and 3 at which both bodies have same absolute velocity, it follows that,

$$(O_2I)\omega_2 = (O_3I)\omega_3$$

A comparison of this equation with (8.13) shows that this instantaneous centre  $I$  (23) and the pitch point  $P$  are the same. The distances  $O_2I$  and  $O_3I$  are infact the pitch circle radii. There being no relative motion between gears 2 and 3 at this point, the velocity of sliding is always zero at the pitch point. Also as the pitch point  $P$  is the I.C. of rotation of 2 w.r. to 3, the velocity of sliding at any other point of contact  $A$  (on the path of contact) is –

$$AP(\omega_2 + \omega_3)$$

Note that due to opposite sense of rotation of the two gears the relative rotational speed is  $(\omega_2 + \omega_3)$ .

## 8.6 CONJUGATE TEETH

When mating tooth profiles are so shaped that they produce a constant angular velocity ratio during mesh, the tooth surfaces are said to be conjugate with respect to one another. *It is in fact possible to specify any tooth profile and then to find a profile for mating tooth so that the two tooth profiles are conjugate. One of these tooth profiles is that of involute, which is very widely used in gearing. A yet another solution consists of cycloidal profile which was first to be used and is still being used in clocks and watches. Even though a large number of conjugate curves are possible, real problem lies in producing these tooth profiles in large quantities on gear-blanks of steel and other materials using existing machinery. This apart, economics of reproducing these gear tooth profiles restricts the choice mostly in favour of involute tooth profiles.*

We now describe below graphical construction to emphasize the fact that for any assumed gear tooth curve, it is always possible to obtain conjugate tooth curve. For the sake of brevity, construction has been carried out for establishing three points on curve. The procedure is



however, general, and can be repeated to establish many more points on conjugate curve so that it can be described accurately.

Figure 8.15 shows two gear wheels  $G$  and  $G'$  in mesh with their axes at  $O_A$  and  $O_B$  respectively. For simplicity, the tooth profile of gear wheel  $G$  is assumed to be a circular arc  $A_1PA_2$  of radius  $OP$ . Then, as gear tooth moves around  $O_A$  with gear rotation, the location of centre of curvature of tooth profile must maintain same relative position of gear tooth curve  $A_1PA_2$ . In other words as the gear  $G$  rotates, the centre of curvature of tooth profile will move around in a circle with  $O_AO$  as radius. It is now required to find out a tooth profile that is conjugate in respect of tooth profile  $A_1PA_2$ . Rotate the gear wheel  $G$  about wheel axis  $O_A$  so that the tooth profile moves from position  $A_1PA_2$  to  $B_1BB_2$ . In doing so the centre of curve of tooth profile  $A_1PA_2$

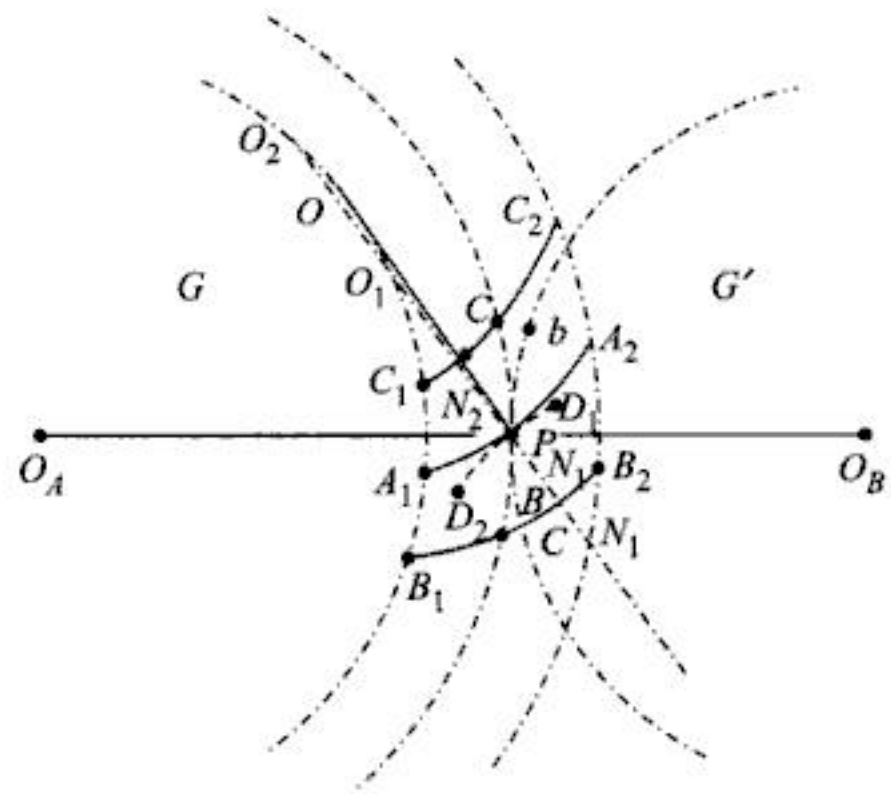


Fig. 8.15 Illustration for generating conjugate profile.

shifts from position  $O$  to  $O_1$  keeping same distance from  $O_A$ . In other words point  $O$  and  $O_1$  will be located on same circle of radius  $O_AO$ . Position of normal to new position is obtained by joining  $O_1$  to  $P$  which, when extended, cuts profile  $B_1BB_2$  at point  $N_1$ . Since  $O_1P$  is the common normal, it follows that for the given position of profile  $N_1$  is the point of contact. To transfer position of this point of contact to the reference position  $A_1PA_2$ , draw arc from  $N_1$  with radius  $O_BN_1$ . This step is self-explanatory the moment it is realised that  $N_1$ , being a point on gear tooth of wheel  $B$ , must move in an arc of circle with radius  $O_BN_1$ . As point  $B$  is moved back to position  $P$  a point coinciding with  $P$  on the pitch circle of gear  $G'$  moves to position  $b$ , such that  $Pb = PB$ . From  $b$  strike arc of radius  $PN_1$  to cut arc of circle  $N_1D_1$  at  $D_1$ . This gives a point on conjugate tooth profile besides the point  $P$ .

To locate a third point on conjugate tooth profile, consider  $C_1CC_2$  to be third position of tooth profile. Let  $O_2$  be the centre of curvature for this position of tooth profile. Line joining  $O_2$  and  $P$  cuts the current tooth profile position at point  $N_2$ .  $O_2P$ , being a common normal to the two tooth profiles,  $N_2$  gives yet another point of contact between the two tooth profiles. To transfer this position to the reference position, mark point  $C$  on pitch circle of  $G'$  such that arc  $PC =$  arc  $PC$ . Draw an arc of circle from  $N_2$  with  $O_BN_2$  as radius and strike arc on it of radius  $CD_2 = PN_2$ . Thus,  $D_2$  is the third point on the conjugate profile, besides points  $D_1$  and  $P$ . A more accurate tooth profile can be established by taking a few more points  $D_3, D_4, D_5$ , etc. and passing a smooth curve through them.

## 8.7 TOOTH PROFILES

It should be evident from discussions of section 8.6 that theoretically any suitable curve can be selected for gear tooth profile and a conjugate curve can be established for mating gear teeth.

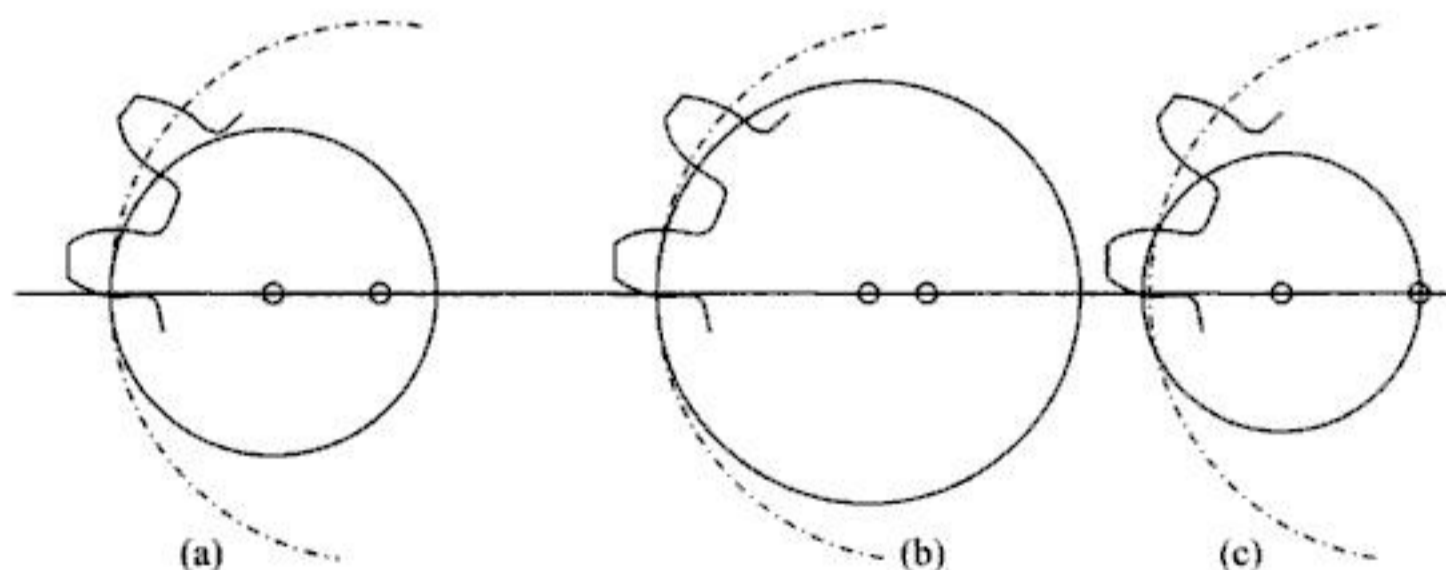
But owing to interchangeability aspect, cost and ease of production on large scale, together with many other advantages, the choice practically narrows down to involute curve.

### 8.7.1 Cycloidal Tooth Profile

Cycloidal tooth profile was extensively used for gear manufacture in distant past because it is easy to form by casting. Besides, compared to involute gearing, cycloidal gearing permits smaller values for minimum number of teeth, better contact and wear characteristics, and lower value of contact stresses. In spite of these advantages, its use was restricted to gear pumps, blowers, clock and watch gearing, for rack jacks, etc.

Despite its limited application, cycloidal profile needs our attention as involute is simply a limiting case of cycloid and, as such, may provide solution to the unsolved and unknown problems of the future. It may be noted that when a generating circle (rolling on the outside of a directing circle) has an infinite radius of curvature, it becomes a straight line, and a fixed point on this rolling line then generates an involute curve.

Interchangeability requires that the same describing (generating) circle must be used in generating all the faces and flanks. The size of the describing circle depends on the properties of the hypocycloid. The epicyclic curve forms the face while the hypocycloid forms the flanks of the teeth. Further, when the describing circle is half the size of directing circle, the flanks will be radial and the resulting tooth will be relatively weak at the root (Fig. 8.16a). If the describing circle is made smaller the hypocycloid curve is different from the radial lines giving spreading tooth (Fig. 8.16c) at the root, which is stronger. However, when describing circle is larger the hypocycloid curves the other way, passing inside the radial lines (Fig. 8.16b), producing a weaker tooth that is difficult to shape on milling machine. It follows that the diameter of the describing circle should not be larger than the radius of the directing circle of the smallest gear of the interchangeable set.



**Fig. 8.16** Effect of size of generating circle on flank portion.

*The pressure angle in cycloidal gears varies constantly; it diminishes during the approach, becomes zero at the pitch point, and then increases during the recess. The variation in pressure angle leads to additional noise and wear, and also to changes in bearing reactions at the shaft supports. Further, in order to run properly, cycloidal gears must be operated at exactly the correct centre distance, otherwise the contacting portions of the profiles will not produce conjugate action. It must be noted that in view of deflections produced by load, being transmitted, it is*

virtually impossible to maintain correct centre distance between gears. For gears used in heavy work, experience suggests a maximum permissible pressure angle of  $30^\circ$ , giving a mean value of  $15^\circ$ .

### 8.7.2 Involute Tooth Profile

It may be recalled that an involute curve is defined in two ways.

- (i) It is the locus of a point on a taut string as the string is unwrapped from the circumference of a circle.
- (ii) It is the locus of a fixed point on a straight line which rolls without slipping on a circle.

The circle from which the taut string is unwrapped or over which the line rolls without slipping, is called the *base circle*. It follows from the definition of involute curve that the involute curve cannot exist inside the base circle. It also follows that involute curves drawn from circles of different diameter (as base circle) are all different.

Figure 8.17 shows an involute curve generated by rolling a line  $BC$  without slipping on the base circle. Points  $C_1, C_2, C_3 \dots C_5$  are the points on the curve corresponding to the points of tangency  $P_1, P_2, P_3, \dots P_5$  on the base circle. It follows that each of the points  $P_j$  ( $j = 1, 2, 3, \dots 5$ ) is an instantaneous centre of curvature for small arc around corresponding point  $C_j$  on the involute curve. Since  $P_1C_1, P_2C_2, P_3C_3$ , etc. were drawn tangent to base circle, following important property of involute curve becomes evident:

*Tangent to the base circle is also the normal to the involute curve at a corresponding point.*

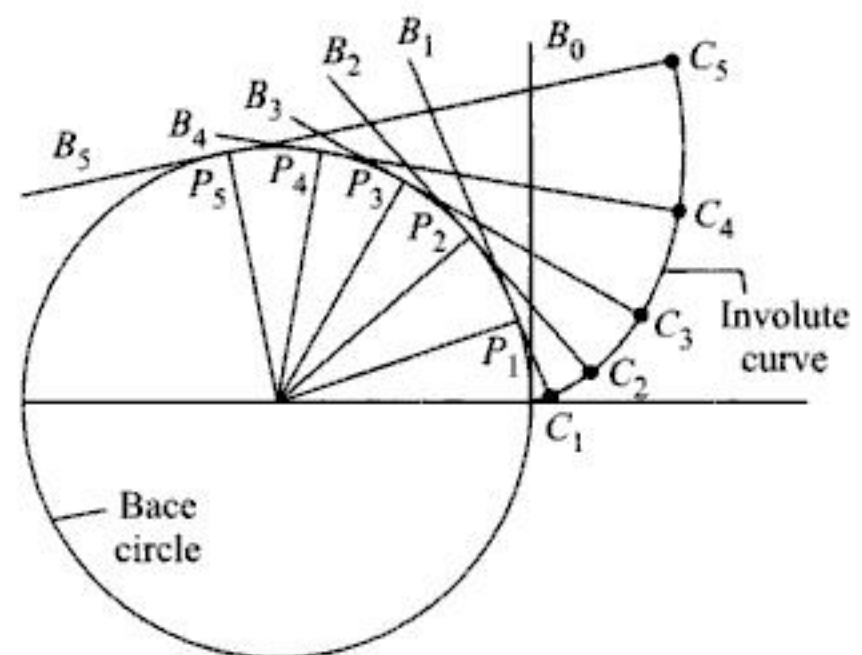


Fig. 8.17 Involute curve as obtained by rolling a straight line  $BC$  without slipping.

#### To Show That Involute Gears Satisfy Law of Gearing

To show that involute tooth profile satisfies the law of gearing, consider a string whose end points are attached to two rotating cylinders  $A$  and  $B$  corresponding to base circles. Cylinder  $A$  is free to rotate about axis through  $O_A$  and  $B$  is free to rotate about axis  $O_B$ .  $N$  represents a fixed point or knot on the string through which a pencil point projects out. Keeping cylinder  $A$  fixed and string taut, if  $N$  is moved around cylinder  $A$  (cylinder  $B$  being free to rotate) in c.c.w. sense, it describes the involute curve  $a'a$  on the upper disc. Similarly, keeping wheel  $B$  fixed (and, releasing cylinder  $A$ ) and string taut, if the point  $N$  is moved around cylinder  $B$ , it describes the involute curve  $bb'e$ .

Since motion of point  $N$  is always perpendicular to corresponding radius of rotation (like  $LN$  or  $MN$ ), the string  $LM$  is normal to the two involute curves for all positions of contact.

Finally, consider curved surface  $a'a$  to be an integral part of and rotating with cylinder  $A$ . Similarly, let the curved surface  $be$  to be an integral part of and moving with cylinder  $B$ . Now keeping the two curved surfaces in contact, back up both the cylinders so that a point  $d$  on curved surface  $a'a$  comes in contact with point  $e$  on the curved surface  $bb'e$ . Note that for this to occur

arcual length  $a'd$  must be equal to arcual length  $b'e$ . In order to locate the point  $N'$  at which the two points  $d$  and  $e$  will come in coincidence in Fig. 8.18, draw arcs of circle of radius  $O_A d$  and  $O_B e$  with  $O_A$  and  $O_B$  as centres respectively. They meet at  $N'$ . It is worth noting that point  $N'$  lies on the line (string)  $LM$  which is common tangent to both the cylinders. This can be checked further by selecting another pair of points  $f$  and  $g$  on curved surfaces  $a'a$  and  $be$  with the condition that arcual length  $f a' =$  arcual length  $g b'$ .

Above discussions lead us to the conclusion that whenever two involute profiles of mating gear teeth come in contact, the point of contact always lie on the common tangent to the base circles, which is also the common normal to the contacting tooth profiles. So long as gear centres remain the same, the common tangent to base circles remain unchanged in position and always cuts the line of centre at the same point  $P$ , which is the pitch point. This shows that involute tooth profiles in contact satisfy law of gearing.

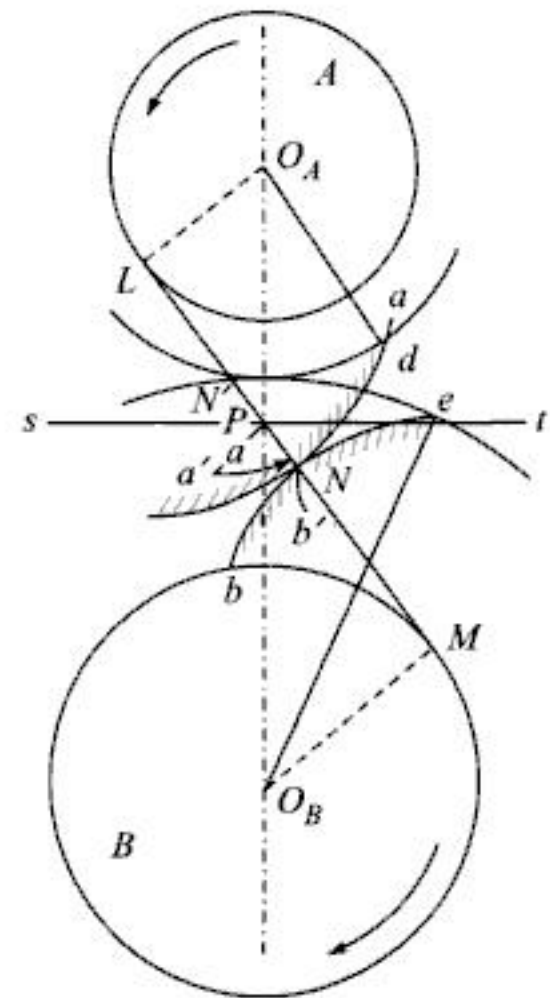


Fig. 8.18 Illustration to show that involute tooth satisfies law of gearing.

**Relation between Base Circle Radii, Pitch Circle Radii and Centre Distance**

Let  $A$  and  $B$  be two involute gears in mesh and let them be represented through their respective base circles and pitch circles as shown in Fig. 8.19. Let  $O_A$  and  $O_B$  be the axis of rotation of gears  $A$  and  $B$ . Further, let  $r_p$  and  $r_b$  be the pitch circle radius and base circle radius of gear  $A$  and  $R_p$  and  $R_b$  be the pitch circle radius and base circle radius of gear  $B$ . If  $\Psi$  is the pressure angle, then  $O_A O_B$  being perpendicular to the common tangent  $t-t$  and  $O_B M$  and  $O_A L$  being perpendicular to  $LM$ ,

$$\angle LO_A P = \angle MO_B P = \Psi$$

From the right angled triangle  $O_A LP$

$$O_A L = (O_A P) \cos \Psi$$

or  $r_b = r_p \cos \Psi$  (8.16)

Also, from right angled triangle  $O_B MP$ ,

$$O_B M = (O_B P) \cos \Psi$$

or  $R_b = R_p \cos \Psi$  (8.17)

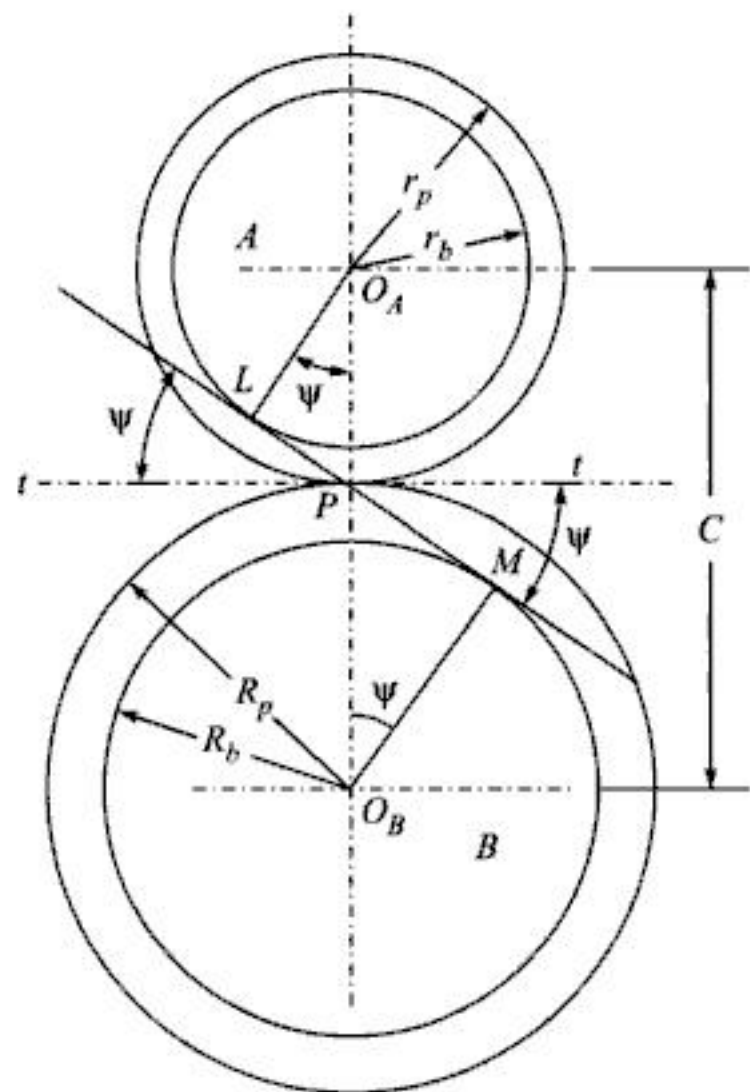


Fig. 8.19 Illustration showing relation of centre distance with base and pitch circle radii.

Adding on the corresponding sides of equations (8.16) and (8.17),

$$(r_b + R_b) = (r_p + R_p) \cos \Psi \quad (8.18)$$

Noting that

$$r_p + R_p = O_A P + O_B P = \text{centre distance} = C,$$

it follows from equation (8.18) that

$$(r_b + R_b) = C \cos \Psi \quad (8.19)$$

or

$$C = \frac{(r_b + R_b)}{\cos \Psi} \quad (8.20)$$

and

$$\cos \psi = \frac{(r_b + R_b)}{C} \quad (8.21)$$

Equation (8.21) indicates that for a given pair of gears in mesh (for which  $r_b$  and  $R_b$  are constant), pressure angle  $\psi$  increases if centre distance  $C$  is increased within limits. It must be noted here that as the tooth profile (which is involute here) does not change, the size of base circle remains unaltered.

### Effect of Increasing Centre-Distance on Velocity Ratio

Consider a pair of meshing gears having involute teeth with each tooth in contact with mating tooth on either side. Clearly, any attempt to bring the two centres closer will lead to jamming or deforming the teeth. Therefore, consider the case in which the centre distance is slightly increased. This leads to clearance and backlash between the mating teeth.

Base circles are fundamental characteristic of the gears and unless they are changed, tooth profiles remain unchanged. Thus, when the centre distance is increased, we have new pitch circles of larger radius for each gear, keeping the ratio of pitch circle radii the same. This is necessary because the pitch circles of mating gears are imaginary circles and, are always tangent to each other. Following discussions are aimed at elaborating this point.

As shown in Fig. 8.20, let a pinion with centre at  $O_1$  and a gear with centre at  $O$  be in mesh with pitch point at  $P$ . For this position of gears, the common tangent  $LM$  to base circles passes through  $P$ . Let

$$O_1 L = r_b \quad O_1 P = r_p \quad O M = R_b \quad \text{and} \quad O P = R_p$$

For this position, the common tangent  $LM$  represents line action and  $\angle LO_1 P = \angle M O P = \psi$ , the pressure angle. Using similarity of triangles  $PO_1 L$  and  $P O M$ , the velocity ratio is given by

$$\frac{\omega_1}{\omega_2} = \frac{O P}{O_1 P} = \frac{O M}{O_1 L} \quad (8.22)$$

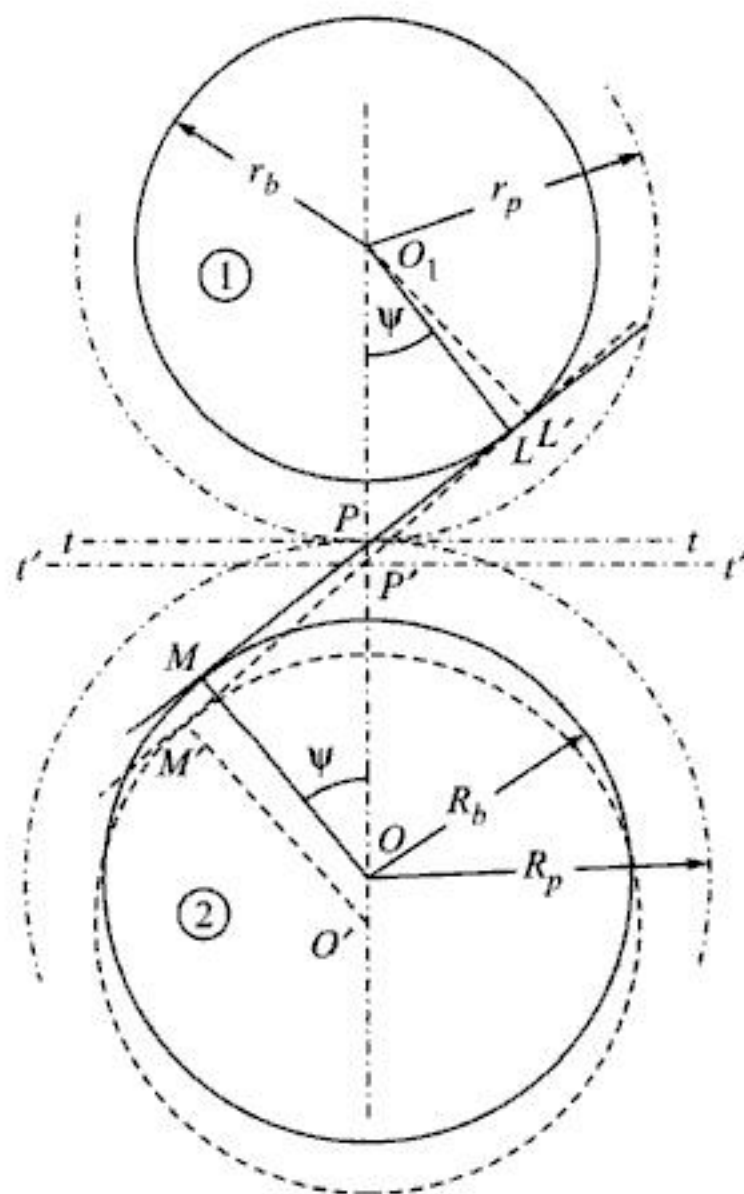


Fig. 8.20

Let the centre of gear 2 be now shifted slightly from  $O$  and  $O'$ . The base circle size remains the same, but with changed position of centre of gear 2, the base circle changes its position to the one shown by dotted lines. As a consequence, common tangent to the two base circles takes up the position  $L'M'$  and the pitch point changes to  $P'$ . Clearly, the new pressure angle  $\angle M'O'P'$  is larger than  $\psi$ . This is also clear from the new position of common tangent.

Even in the new position of centre of gear 2, from similar triangles  $O'M'P'$  and  $O_1L'P'$  we have

$$\frac{O'P'}{O_1P'} = \frac{O'M'}{O_1L'} \quad (8.23)$$

But  $O'M' = OM = R_b$  and  $O_1L' = O_1L = r_b$ , and

$$\text{therefore, from equation (8.23), } \frac{O'P'}{O_1P'} = \frac{OM}{O_1L} \quad (8.24)$$

Comparing equations (8.22) and (8.24),

$$\frac{\omega_1}{\omega_2} = \frac{OM}{O_1L} = \frac{O'P'}{O_1P'} \quad (8.25)$$

This shows that even if the centre distance is changed within limits, the velocity ratio remains unchanged. Pressure angle, however increases as the centre distance is increased.

## 8.8 LENGTH OF PATH OF CONTACT

For a smoother transmission of power, it is necessary that the length of arc of contact between a pair of teeth must at least be equal to the circular pitch of the teeth. When length of arc of contact equals circular pitch, a second pair of gear teeth begins to engage before the engagement between the preceding pair is over. However, as will be seen later, the length of path of contact should be more than the circular pitch. It is necessary therefore to establish a relation which enables us in computing length of path of contact. This is important because it will pave the way for calculating contact ratio, which is one of the measures of quality of gear drive.

Consider a pinion  $A$  to drive a gear  $B$  as shown in Fig. 8.21. As the pinion rotates in clockwise direction, the flank portion near the base circle of pinion comes in contact with addendum circle of mating gear tooth at point  $N$ . As the gear pair rotates further, the point of contact shifts, along the common tangent  $LM$  of the base circles. The contact continues upto point  $Q$ , where the addendum of pinion is in contact with the flank portion of the tooth of gear  $B$ .

The length  $NP$  is known as length of path of approach while length of path of recess is the length  $PQ$ .

Let  $R_a$  and  $R$  be the radius of addendum circle and radius of pitch circle of gear respectively. Similarly, let  $r_a$  and  $r$  be the radius of addendum circle and radius of pitch circle respectively of pinion  $A$ . Join  $O_AQ$  and  $O_BN$ . From the right angled triangle  $O_AQL$ ,

$$\begin{aligned} QL &= \sqrt{(O_AQ)^2 - OL^2} \\ &= \sqrt{r_a^2 - (O_AP \cos \Psi)^2} \end{aligned}$$



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which simplifies to,

$$C = \left( \frac{m'}{2} \right) (T + t \cot \lambda) \quad (8.83)$$

*Efficiency of Worm Gearing:* Since worm gearing is similar to spiral gearing, equation (8.73) can be adopted, as it is to represent efficiency of worm gearing. Thus,

$$\eta = \frac{\cos(\alpha' + \phi) \cos \alpha}{\cos(\alpha + \phi) \cos \alpha'}$$

Substituting  $\alpha' = \lambda$  and  $\alpha = (90 - \lambda)$  from equations (8.81) and (8.82),

$$\begin{aligned} \eta &= \frac{\cos(\lambda + \phi) \cos(90 - \lambda)}{\cos[90 - (\lambda + \phi)] \cos \lambda} \\ &= \frac{\cos(\lambda + \phi) \sin \lambda}{\sin(\lambda + \phi) \cos \lambda} \quad \text{or, } \eta = \frac{\tan \lambda}{\tan(\lambda + \phi)} \end{aligned} \quad (8.84)$$

Also, from equation (8.75) for spiral gears, maximum efficiency is given by

$$(\eta)_{\max} = \frac{\cos(\theta + \alpha) + 1}{\cos(\theta - \alpha) + 1}$$

As  $\theta = 90^\circ$  (usually), this gives maximum efficiency for worm gearing as

$$(\eta)_{\max} = \left( \frac{1 - \sin \alpha}{1 + \sin \alpha} \right) \quad (8.85)$$

In order to study the effect of coefficient of friction  $\mu = \tan \phi$  and angle of helix  $\alpha$  on efficiency, Table 8.3 is obtained from equation (8.84), which can be simplified to:

$$(\eta) = \frac{\tan(90 - \alpha)}{\tan[90 - (\alpha - \phi)]}$$

**Table 8.3**

$\mu = \tan \phi$	$\alpha = 10^\circ$	$20^\circ$	$30^\circ$	$40^\circ$	$50^\circ$	$60^\circ$	$70^\circ$	$80^\circ$	$90^\circ$
$0.24 = \tan 15^\circ$	—	0.24	0.464	0.556	0.587	0.577	0.52	0.378	0
$0.1584 = \tan 10^\circ$	0	0.484	0.63	0.688	0.704	0.688	0.63	0.484	0
$0.0787 = \tan 5^\circ$	0.496	0.736	0.807	0.834	0.839	0.825	0.78	0.658	0
$0.0393 = \tan 2.5^\circ$	0.746	0.866	0.902	0.914	0.916	0.906	0.879	0.795	0

It can be seen that in the useful range of  $20 \leq \alpha \leq 80^\circ$ , percentage variation in the efficiency for different friction angles are:

$$\begin{aligned} \phi = 15^\circ; \% \text{ variation in } \eta &= 59.1\% & \phi = 10^\circ; \% \text{ variation in } \eta &= 31.25\% \\ \phi = 5^\circ; \% \text{ variation in } \eta &= 21.57\% & \phi = 2.5^\circ; \% \text{ variation in } \eta &= 13.2\% \end{aligned}$$

It is easy to see that per cent variation in efficiency over the working range decreases with  $\mu$  and is least for  $\phi = 2.5^\circ$ . Further, coefficient of friction seems to be more important than helix

angle in increasing the efficiency of the drive. Though the table shows that maximum efficiency is obtainable for  $40^\circ \leq \phi \leq 50^\circ$  highest efficiency in this range can be obtained only with a low coefficient of friction.

**EXAMPLE 8.14** A two thread worm drives a worm wheel of 30 teeth. The worm has a pressure angle of  $14\frac{1}{2}^\circ$ , a pitch of 19 mm and a p.c.d of 50 mm. If the worm rotates at 1000 r.p.m., for  $\mu = 0.05$  find,

- (i) the helix angle of the worm,
- (ii) the speed of worm wheel,
- (iii) the centre distance,
- (vi) the efficiency of the set, and
- (v) the lead angle for maximum efficiency.

**Solution:** The lead angle of worm,

$$\lambda = \tan^{-1}\left(\frac{1}{\pi d}\right) = \tan^{-1}\left(\frac{2 \times 19}{\pi \times 50}\right) = 13.599^\circ \approx 13.6^\circ$$

Hence, the helix angle =  $(90 - \lambda) = 76.4^\circ$

**Ans.**

The speed of worm gear =  $N \times T/t = 1000 \times \frac{2}{30} = 66.666$  r.p.m.  $\approx 66.67$  r.p.m.

**Ans.**

$$\phi = \tan^{-1}(\mu) = \tan^{-1}(0.05) = 2.862^\circ$$

The centre distance,

$$C = \frac{m'}{2}(T + t \cot \lambda) \text{ where, } m' = \frac{P_c}{\pi} = \frac{19}{\pi} = 6.048 \text{ mm}$$

Therefore

$$C = \frac{19}{2\pi}(30 + 2 \cot 13.6) = 115.72 \text{ mm}$$

**Ans.**

$$\eta = \frac{\tan \lambda}{\tan(\lambda + \phi)} = \frac{\tan 13.6}{\tan(13.6 + 2.862)} = 81.87\%$$

**Ans.**

For maximum efficiency,

$$\alpha = \frac{\theta + \phi}{2} = \frac{90 + 2.862}{2} = 46.43^\circ$$

Hence,

$$\alpha' = 90 - 46.43 = 43.57^\circ$$

Lead angle for this condition,

$$\lambda = \alpha' = 43.57^\circ$$

**Ans.**

Maximum efficiency for this case is given by,

$$(\eta)_{\max} = \frac{\tan 43.57}{\tan(43.57 + 2.862)} = 90.49\%$$

## 8.21 BEVEL GEARS

Bevel gears are used to transmit power between shafts whose axes intersect and whose pitch surfaces are rolling cones. The pitch surfaces in this case are defined as the conical surfaces with common apex, which transmit same velocity ratio as the given pair of bevel gears over same centre distance by a pure rolling action. Interlocking projection called teeth are provided on pitch surfaces so as to have a positive drive. As in the case of spur gears, tooth profile of bevel gears may either have a cycloidal or an involute profile.

If the generating cone rolls on the outside of the pitch cone, an element (generator) on generating cone generates tooth profile of face. When the generating cone rolls on the inside of the pitch cone, the element sweeps out the flank portion of the tooth. Figure 8.41 shows involute tooth developed from a base cone. For this, one imagines a piece of paper wrapped around a cone. Let  $OP$  be a slit cut along a generator. Now let the paper be unwrapped along  $OP$  from the cone, keeping the paper taut. The cut edge  $OC$  then describes the involute tooth profile. Any point, like point  $C$ , on unwrapping slit remains at a constant distance  $OC$  from the apex  $O$  and therefore, the curve described by it lies on the surface of a sphere of radius  $OC$ . A generator curve of this type (say  $CP$ ) is therefore called a spherical involute. Readers will appreciate that it is not difficult to conceive that curve  $CP$  cannot lie in the end-face  $ABP$  or extension thereof.

As the curve generated by any point like  $C$ , on the generator  $OC$ , does not lie on plane surface, it is difficult to understand tooth action using true shape of tooth profile. In practice, Tredgold's approximation is employed, which consists in substituting a conical surface for the actual spherical surface, such that the conical surface is tangential to the actual spherical surface.

Figure 8.42 shows a pair of bevel gears in mesh. The back cone of these gears have elements that are perpendicular to the corresponding elements of pitch cone. The surface of the back cone gives an approximation to the spherical surface. For studying the tooth action of bevel gears, these back cones are laid out flat (not shown in Figure). Each cone forms a portion of a spur gear, whose pitch circle radius equals back cone radius.

As shown in Fig. 8.42,  $d$  and  $D$  are the pitch circle diameter at the bigger ends of smaller and bigger gears respectively. The module  $m$  and circular pitch  $p_c$  are defined in the same manner as spur gear at the larger end,

Thus, 
$$m = \frac{D}{T} = \frac{d}{t}$$

and 
$$p_c = (\pi D/T)$$

where  $T$  and  $t$  are the number of teeth on larger and smaller gear respectively. Also,

$$\frac{\omega}{\omega'} = \frac{D}{d} = \frac{T}{t}$$

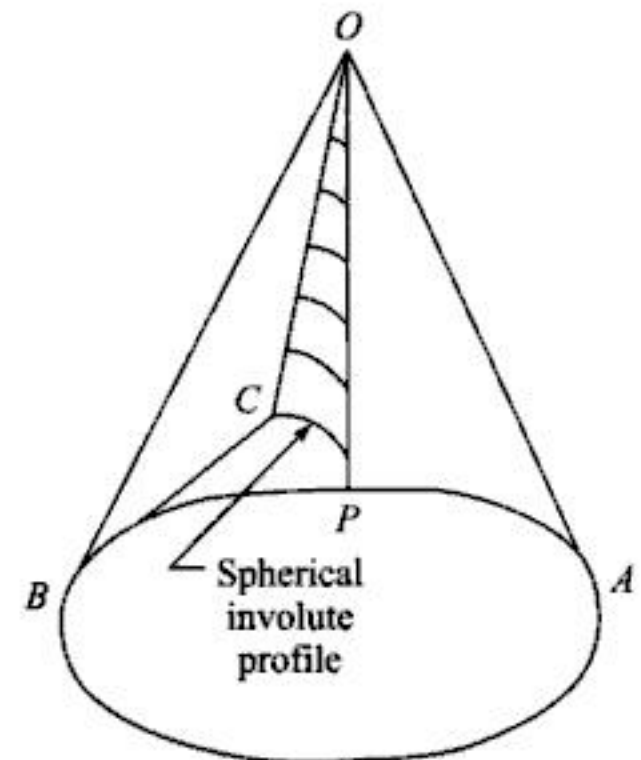


Fig. 8.41 Development of involute tooth from base cone.



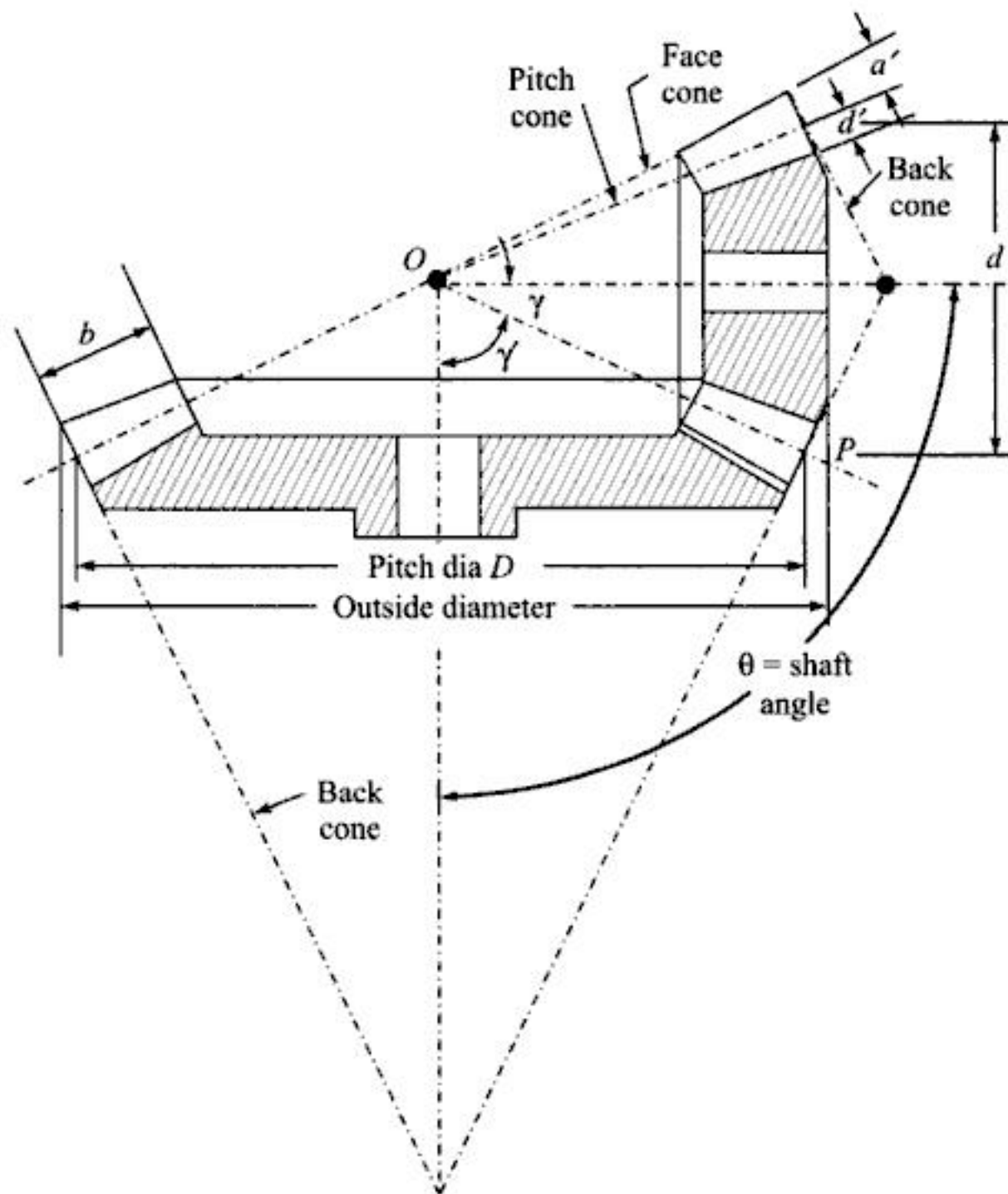


Fig. 8.42 A Pair of Bevel gears in mesh.

Pitch cone angles  $\lambda$  and  $\lambda'$  are in fact the semi-cone angles for the smaller and bigger gears.

The tooth elements are straight lines which converge at the apex of cone. Thus the tooth thickness decrease continuously towards inside; being maximum at the outside.

If  $\gamma$  and  $\gamma'$  be the pitch angles for pinion and gear respectively and  $d$  and  $D$  be their respective pitch circle diameters at the larger end, we have

$$\sin \gamma' = \left( \frac{D/2}{OP} \right) = \frac{D/2}{(d/2)/\sin \gamma} = \left( \frac{D}{d} \right) \sin \gamma \quad (8.86)$$

If  $\theta$  is the shaft angle, then from  $\theta = \gamma + \gamma'$ ,

$$\sin \gamma' = \frac{D}{d} \sin(\theta - \gamma')$$

Expanding R.H.S. and dividing both sides by  $\cos \lambda'$ ,

$$\tan \gamma' = \frac{D}{d \cos \gamma} (\sin \theta \cos \gamma' - \cos \theta \sin \gamma') = \left( \frac{D}{d} \right) (\sin \theta - \cos \theta \tan \gamma')$$

By transposition

$$\left(1 + \frac{D}{d} \cos \theta\right) \tan \gamma' = \left(\frac{D}{d}\right) \sin \theta$$

or 
$$\frac{D}{d} \left(\frac{d}{D} + \cos \theta\right) \tan \gamma' = \left(\frac{D}{d}\right) \sin \theta$$

Hence 
$$\tan \gamma' = \frac{\sin \theta}{(d/D) + \cos \theta} \quad (8.87)$$

But 
$$(d/D) = (\omega'/\omega)$$

where  $\omega$  and  $\omega'$  are the angular speeds of pinion and gear respectively.

Substituting for  $(d/D)$

$$\tan \gamma' = \frac{\sin \theta}{(\omega'/\omega) + \cos \theta} \quad (8.88)$$

In a similar way 
$$\tan \gamma' = \frac{\sin \theta}{(\omega/\omega') + \cos \theta} \quad (8.89)$$

## 8.22 SPECIAL BEVEL GEARS

Depending upon their intended use and design, bevel gears are given various names:

- (i) *Miter Gears*: Bevel gears having equal size and operating on shafts with included angle of  $90^\circ$  are called miter gears.
- (ii) *Angular Bevel Gears*: These are the bevel gears of unequal size which generally operate with an included shaft angle  $\theta$  which is other than  $90^\circ$ .
- (iii) *Crown Gears*: When the pitch angle of one of a pair of bevel gears is made equal to  $90^\circ$ , the pitch cone becomes a flat surface and the resulting gear is called a crown gear. The crown gear can be considered to be a counterpart of a rack in spur gearing.
- (iv) *Spiral Bevel Gears*: Spiral bevel gears consist of obliquely curved teeth. The spiral angle is such that the face advance of the tooth is greater than the circular pitch of the tooth. The spiral bevel gear thus, is much like a helical gear. The straight tooth bevel gears become noisy at high pitch-line velocities. Spiral bevel gears provide a distinctly better choice in such situations.
- (v) *Zerol Bevel Gear*: A bevel gear is a patented gear which has curved teeth such that the spiral angle of the tooth is tangent to a cone element giving a zero degree spiral angle.
- (vi) *Hypoid Gears*: As in the case of automobile-differential applications, it is frequently desirable to have a gear similar to bevel gear but with shafts offset. Such gears are called hypoid gears because their pitch surfaces are hyperboloids of revolution. This form of bevel gears was introduced for use with automotive rear-axle drive systems known as the ring gear and pinion.

**EXAMPLE 8.15** Two straight bevel gears are assembled with  $90^\circ$  shaft angle. The gears have a module of 10 mm with 42 and 79 teeth. Determine the pitch cone angles.

**Solution:** Given:  $\theta = 90^\circ$ ;  $m = 10$  mm;  $t = 42$ ;  $T = 79$

From 
$$\tan \gamma' = \frac{\sin \theta}{(\omega'/\omega) + \cos \theta}, \text{ as } \frac{\omega'}{\omega} = \frac{t}{T} = \frac{42}{79} = 0.5316$$

$$\gamma' = \tan^{-1} \left( \frac{\sin 90}{0.5316 + 0} \right) = 62.0^\circ \quad \text{Ans.}$$

Therefore 
$$\gamma = 90 - 62 = 28^\circ \quad \text{Ans.}$$

## REVIEW QUESTIONS

- 8.1** The following data relate to two meshing gears:  
Velocity ratio = 1 : 3; Module: 4mm; Pressure Angle:  $20^\circ$ ; Center distance: 200 mm  
Determine the number of teeth and the base circle radius of the gear wheel.  
(Ans.  $t = 25$ ;  $T = 75$ ;  $R_b = 141$  mm)
- 8.2** Two meshing spur gears with  $20^\circ$  pressure angle have a module of 4 mm. The centre distance is 220 mm and the number of teeth on the pinion is 40. What should be the new centre distance so that pressure angle is increased to  $22^\circ$ ?  
(Ans. 221.9 mm).
- 8.3** State and prove law of gear tooth action for constant velocity ratio and show how the involute teeth profile satisfies the condition. Derive an expression for the velocity of sliding between a pair of involute teeth. State the advantages of involute profile as a gear tooth profile. (AMIE, Winter 1978)
- 8.4** Derive the formula for the length of the path of contact for two meshing spur gears having involute profile. A pinion having 10 teeth of involute form  $20^\circ$  pressure angle and 6 mm module drives a gear having 40 teeth of addendum = module, find (i) addendum and pitch circle radii of the two gears, (ii) The length of path of approach, (iii) The path of contact and (iv) arc of contact. (AMIE, Winter 1983)  
(Ans.  $r = 6$  cm;  $R = 12$  cm;  $r_a = 6.6$  cm;  $R_a = 12.6$  cm; 1.535 cm; 2.921 cm; 3.11 cm).
- 8.5** Two  $20^\circ$  involute spur gear mesh externally and give a velocity ratio of 3, module is 3 mm and rotates at 120 r.p.m. determine  
(i) The minimum number of teeth on gear and pinion  
(ii) Number of pairs of teeth in contact. (Univ. Amravati)
- 8.6** A pair of gear, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m. Determine the velocity of sliding between the gear teeth face at the point of engagement, at the pitch point and at the point of disengagements if the smaller gear is the driver. The distances of engagement and disengagement from

the pitch point along a common tangent to the base circles are 1.3 cm and 1 cm respectively. (DAV Indore, 1983)

(Ans. 408.4 and 314.16 cm/s).

- 8.7 A pair of involute gears with  $16^\circ$  pressure angle and 6 mm module is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is just 1.75, find, in order that the interference is just avoided,

- (a) the addenda on pinion and gear wheel  
 (b) the length of path of contact, and  
 (d) the maximum velocity of sliding of teeth on either side of the pitch point.

(DAV Indore, 1989)

(Ans.  $a_p = 9.6$  mm;  $a_g = 4.2$  mm; 35.41 mm;  $V = 95.58$  cm/s)

- 8.8 A pair of gears, having 40 and 30 teeth respectively, are of  $25^\circ$  involute form. The addendum length is 5 mm and the module is 5.5 mm. If smaller wheel is the driver and rotates at 1500 r.p.m. find the velocity of sliding at the point of engagement and at the point of disengagement.

(Ans. 3 m/s; 2.4 m/s)

- 8.9 Two-spur wheels each have 30 teeth of involute shape. The circular pitch is 25 mm. and the pressure angle is  $20^\circ$ . If the arc of contact is to be  $2 \times$  pitch, determine the minimum addendum of the teeth.

- 8.10 Two equal involute gear wheels of 40 teeth each, 3 mm module and  $20^\circ$  pressures angle are in mesh. Determine the minimum addendum if 2 pairs of teeth are always in mesh. The gear wheels rotate at 750 r.p.m. and transmit 4.6 kW. Find the normal force at each pair assuming equal direction of load between the pairs. Neglect friction.

- 8.11 Determine the number of pairs of teeth in contact at a given instant, if two equal involute gears of 18 teeth of pressure angle  $19^\circ \frac{1}{n} 28'$ , have addendum of 0.8 module.

- 8.12 Two wheels with standard involute teeth of 5 mm module are to gear together with a velocity ratio of 4.5, the pressure angle being  $15^\circ$ . Find (i) the least permissible number of teeth in the pinion, if interference is to be avoided; (ii) the pressure between the teeth, when such a pinion is transmitting a torque of 6000 N·cm.

- 8.13 Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with,

- (a) a wheel to give a gear ratio of 3:1; and (b) an equal wheel

The pressure angle is  $20^\circ$  and a standard module of 1 module for the wheel may be assumed.

(Ans. 15; 13)

- 8.14 A gear wheel having 24 involute teeth of 5 mm module is generated by a straight sided rack cutter. The addendum of the cutter and the wheel is one module. Determine the minimum pressure angle if interference is to be avoided. If such 24 teeth wheel should mesh with a 40 teeth wheel correctly, determine the length of arc of contact and the minimum number of pairs of teeth in contact at any time.

(Ans.  $\Psi = 16.8^\circ$ ; 27.5 mm; 1.75).

- 8.15 A pair of spiral gears connect two non-intersecting shafts which are inclined at  $45^\circ$ . The speed ratio required is 2.5 to 1 and the least distance between axes of the shafts should

be 18 cm. The normal module is 5 mm and pinion should have 18 teeth. Determine the spiral angle for the gear and their pitch circle diameters. The spirals should have the same hand. If the coefficient of friction is 0.05 at the gear teeth determine the power lost in friction if the input into the pinion is 3.7 kW at 1000 r.p.m. Calculate also the sliding velocity at the teeth.

(Ans.  $\alpha' = 12.3$ ;  $d = 92.3$  mm;  $D = 267.7$  mm; 0.20 HP; 4.01 m/s)

- 8.16** Two non-intersecting shafts are inclined at  $60^\circ$  and the least distance between their axes is approximately 20 cm. They are connected by a pair of spiral gear, whose normal module is 6 mm to give a reduction in speed of 3 to 1. If the spiral angles of the pinion and gear are determined by the condition of maximum efficiency and the limiting angle of friction is  $3^\circ$ , determine the spiral angles of the gears, the pitch circle diameters of the gears, the corresponding numbers of teeth and the efficiency of transmission. Also calculate the axial thrust of both the shafts if the input torque on the pinion is 98 N·m.

(Ans.  $\alpha = 31.5^\circ$ ;  $\alpha' = 28.5^\circ$ ;  $t = 16$ ;  $T = 48$ ;  $d = 112.7$  mm;  $D = 294$  mm;  $c = 203.35$  mm;  $\eta = 0.942$ ;  $f_a = 946$  N; and 1036 N)

- 8.17** A quadruple thread worm is driving a 160 teeth worm wheel with the shafts at  $90^\circ$ . Circular pitch of the worm gear is 2.86 cm pitch diameter of the worm is 6.98 cm. Find out the worm lead angle, helix angle of the worm gear, and the distance between shaft centers.

(Ans.  $\lambda = 27.51^\circ = \text{helix} = \text{angle of gear}$ ;  $c = 76.258$  cm;  $N/N' = 40$ )

# GEAR TRAINS

## 9.1 INTRODUCTION

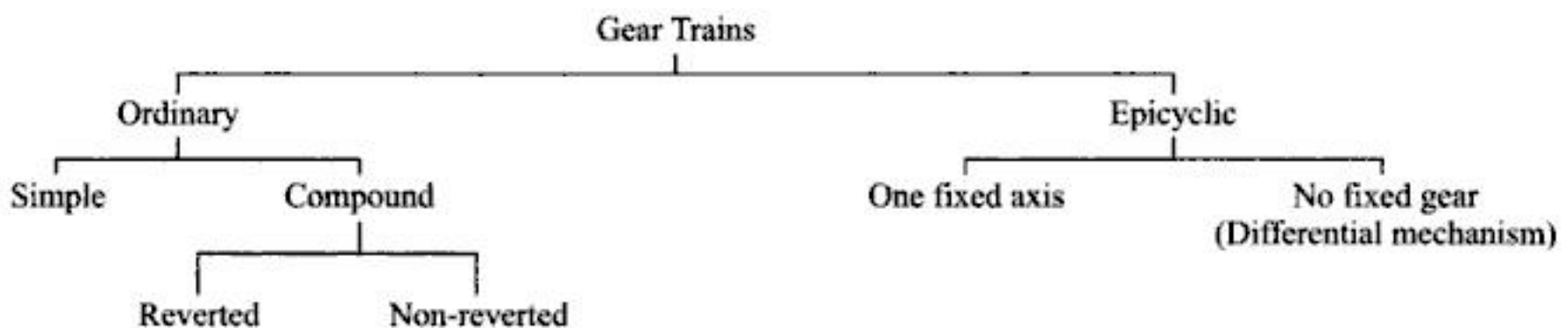
An assembly of gear wheels by means of which motion is transmitted from one shaft to another shaft is called gear train. A gear train may include any or all kinds of gears—spur, bevel, spiral, etc. Ordinary (or usual) gear trains are those in which the axes of none of the gears move relative to the frame. Ordinary gear trains comprise simple and compound gear trains. The other gear trains are the epicyclic gear trains in which relative motion between axes of gears is possible.

Gear trains are necessary when—

- (a) a large velocity reduction or mechanical advantage is desired,
- (b) the distance between two shafts is not too great and, at the same time, is not short enough to permit the use of a single large gear,
- (c) when certain specific velocity ratio is desired.

## 9.2 CLASSIFICATION

Gear trains can be classified as under:

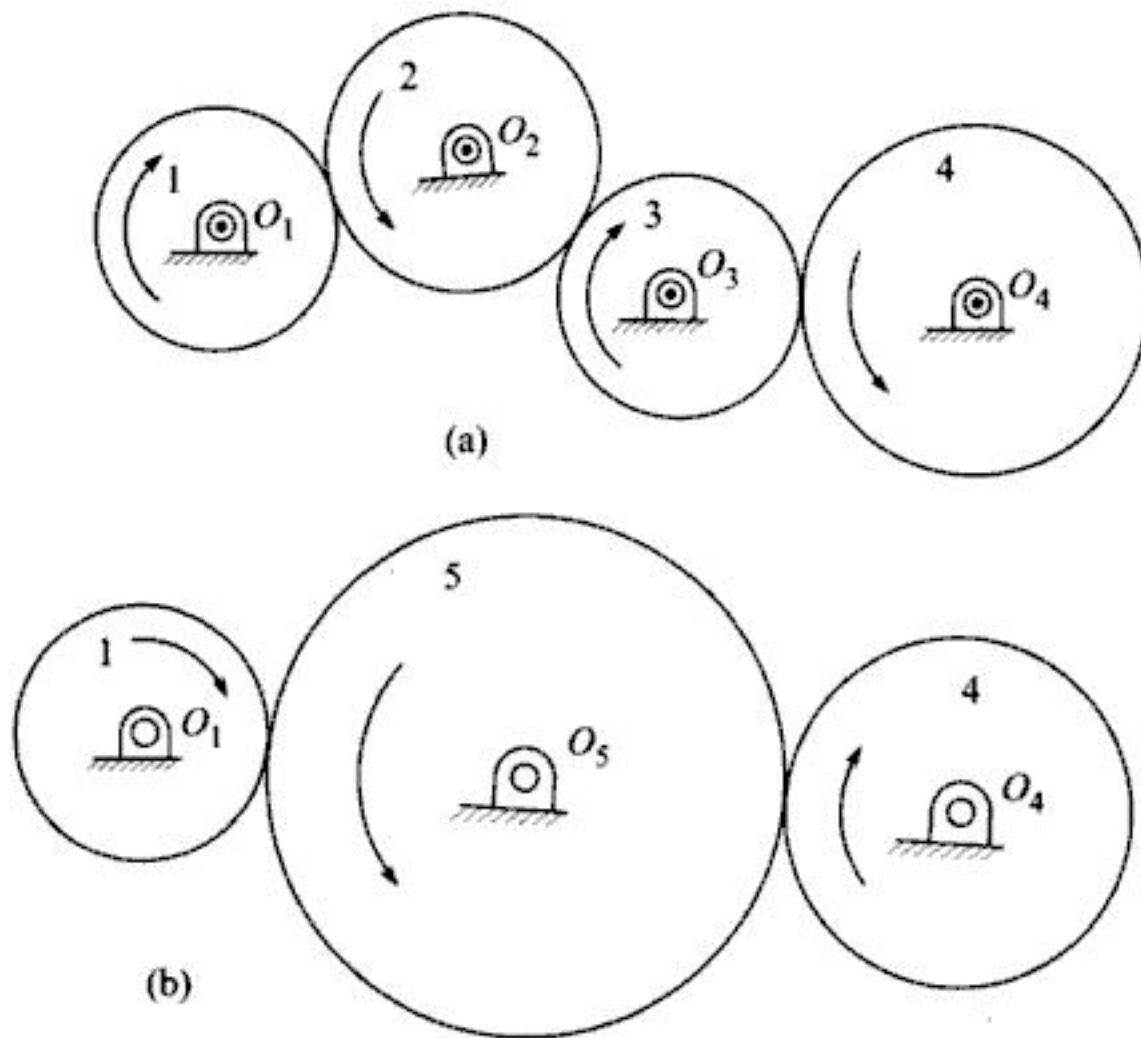


### **Simple Gear Trains**

*Effect of number of intermediate gears:* As the name implies, this is the simplest type of gear train for transmitting motion from one shaft to the other. Characteristic features of this type of train are: (a) All the gear axes remain position fixed with respect to frame and (b) each gear is mounted on a separate shaft. However, while in Fig. 9.1(a) two gears connect gears on shaft-

axes through  $O_1$  and  $O_4$ , in Fig. 9.1(b) a single gear connects gears on shaft-axes through  $O_1$  and  $O_4$ . To understand the role of number of gears separating the connected gears, we proceed as follows:

Let  $T_1, T_2, T_3, T_4$  and  $T_5$  be the number of teeth on gears with shaft axes at  $O_1, O_2, O_3, O_4$  and  $O_5$  respectively. Also let  $N_1, N_2, N_3, N_4$  and  $N_5$  be the r.p.m. of gears mounted on shafts through  $O_1, O_2, O_3, O_4$  and  $O_5$  respectively. Then for Fig. 9.1(a),



**Fig. 9.1** Simple Gear Train: Effect of number of intermediate gears.

$$N_2 = N_1 \left( \frac{T_1}{T_2} \right) \quad (9.1)$$

$$N_3 = N_2 \left( \frac{T_2}{T_3} \right) \quad (9.2)$$

and

$$N_4 = N_3 \left( \frac{T_3}{T_4} \right) \quad (9.3)$$

Substituting for  $N_3$  from equation (9.2) in equation (9.3),

$$N_4 = N_2 \left( \frac{T_2}{T_3} \right) \left( \frac{T_3}{T_4} \right) \quad (9.4)$$

Substituting for  $N_2$  from equation (9.1) in (9.4),

$$N_4 = N_1 \left( \frac{T_1}{T_2} \right) \left( \frac{T_2}{T_3} \right) \left( \frac{T_3}{T_4} \right)$$

Therefore 
$$N_4 = N_1 \left( \frac{T_1}{T_4} \right) \quad (9.5)$$

Also, for Figure 9.1(b),

$$N_5 = N_1 \left( \frac{T_1}{T_5} \right) \quad (9.6)$$

and 
$$N_4 = N_5 \left( \frac{T_5}{T_4} \right) \quad (9.7)$$

Substituting for  $N_5$  from equation (9.6) in (9.7),

$$N_4 = N_1 \left( \frac{T_1}{T_5} \right) \left( \frac{T_5}{T_4} \right)$$

or 
$$N_4 = N_1 \left( \frac{T_1}{T_4} \right) \quad (9.8)$$

If number of teeth on gears 1 and 4 in Fig 9.1(a) and Fig. 9.1(b) are equal, it follows that for the same input speed  $N_1$ , the output speed at gears 4 in the two trains is the same. Stated in other words, in simple gear trains, magnitude of final output speed is not affected by—

- (i) number of intermediate gears, and
- (ii) number of teeth on intermediate gears.

However, direction of rotation of output motion does depend on the number of intermediate gears. For instance, in the gear train of Fig. 9.1(a), there are even number (two) of intermediate gears, and the input and the output gears rotate in opposite sense. As against this, in the gear train of Fig. 9.1(b) there are odd (one) number of intermediate gears, and the input and output gears rotate in the same sense. Above results can be verified for other even and odd numbers of intermediate gears.

*Train value.* The ratio of output speed to input speed is called the train value and it varies inversely as the number of teeth on input and output gears.

We see from equations (9.5) and (9.8) that velocity ratio for a simple gear train depends only on the number of teeth on the first and last gears in the train. The intermediate gears do not have any influence on the velocity ratio and become *idler gears*. The idler gears serve the following two purposes:

- (i) Number of intermediate gears control direction of rotation of output gear.
- (ii) They serve the purpose of bridging the gap between the input and output shaft, when this centre distance is large. A single large gear to bridge this gap requires larger space in transverse direction and is usually uneconomical.

### Compound Gear Train

An alternative way of transmitting motion from one shaft to another using gear train is to compound one or more pairs of gears. A gear pair is compounded if they are mounted on the



same shaft and are made into an integral part in some way. A compound gear train is one which consists of one or more compound gears.

There are many applications in which power is supplied through high speed motor/prime mover. Such motors/prime movers are smaller in size and are usually cheaper than their low speed counterparts. Such applications involve large speed reductions. Simple gear trains in which the input and output gear alone decide the reduction ratio is of little use in such applications. A compound gear train in which each shaft, except the first and last, carries two wheels is more useful in such cases. A higher speed reduction is conveniently obtained with compound gear train. Figure 9.2 shows a compound gear train with two intermediate shafts. Gears 2 and 3 are mounted on shaft through  $O_2$  in such a way that there is no relative rotational motion between them. In other words their r.p.m.,  $N_2$  and  $N_3$ , is same. Similarly, gears 4 and 5 are mounted on shaft through  $O_4$  such that there is no relative rotational motion between gears 4 and 5. In other words their r.p.m.,  $N_4$  and  $N_5$ , are same. Let  $N_j$  and  $T_j$  ( $j = 1, 2, 3, \dots, 6$ ) denote r.p.m. and number of teeth respectively of  $j^{\text{th}}$  gear wheel.

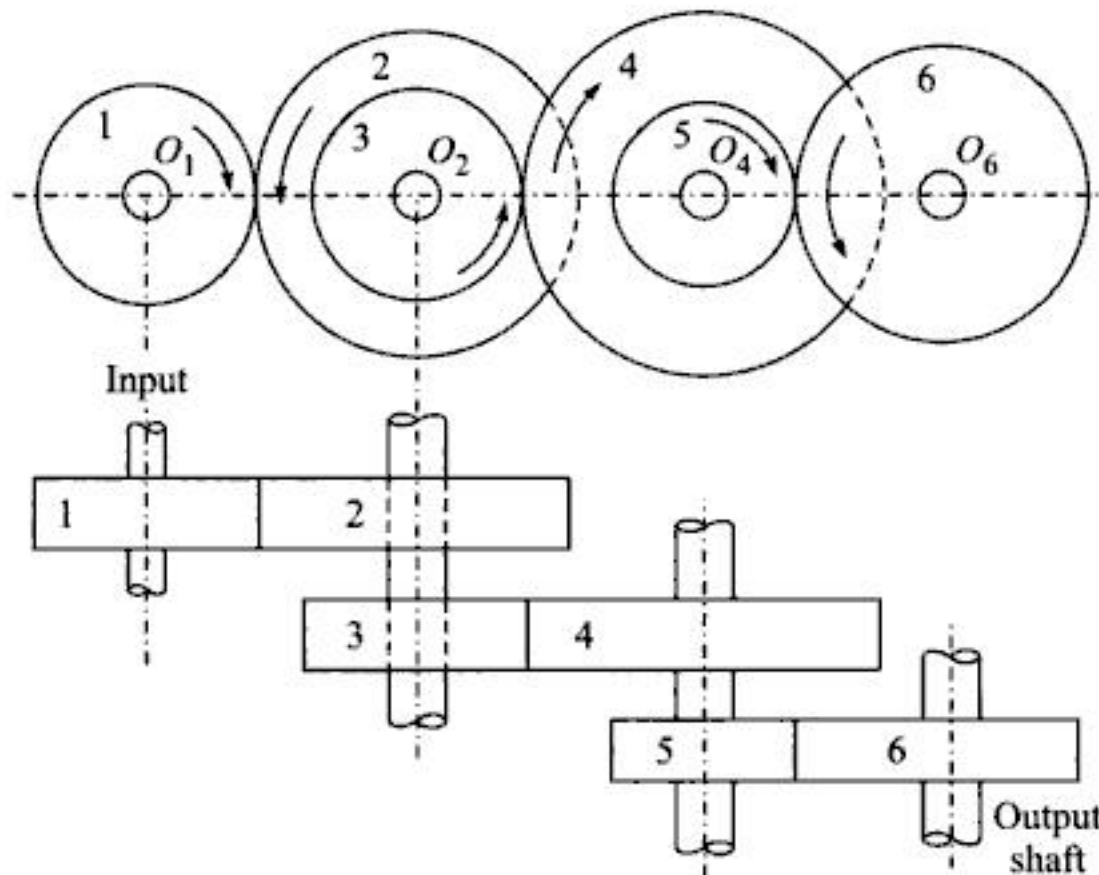


Fig. 9.2 A Compound Gear Train.

Then

$$\left(\frac{N_2}{N_1}\right) = \left(\frac{T_1}{T_2}\right) \quad (9.9)$$

and,

$$\left(\frac{N_4}{N_3}\right) = \left(\frac{T_3}{T_4}\right) \quad (9.10)$$

But gears 2 and 3 are keyed to the same shaft and therefore,  $N_2 = N_3$ . Thus, multiplying equations (9.9) and (9.10) on corresponding sides,

$$\left(\frac{N_4}{N_1}\right) = \left(\frac{N_4}{N_3}\right)\left(\frac{N_2}{N_1}\right) = \left(\frac{T_3}{T_4}\right)\left(\frac{T_1}{T_2}\right) \quad (9.11)$$

Also,

$$\left(\frac{N_6}{N_5}\right) = \left(\frac{T_5}{T_6}\right) \quad (9.12)$$

Remembering that gears 4 and 5 are keyed to the same shaft,  $N_4 = N_5$ . Hence multiplying equations (9.11) and (9.12) on corresponding sides, the gear train value is

$$\left(\frac{N_6}{N_1}\right) \equiv \left(\frac{N_6}{N_5}\right)\left(\frac{N_4}{N_1}\right) = \left(\frac{T_5}{T_6}\right)\left(\frac{T_3}{T_4}\right)\left(\frac{T_1}{T_2}\right) \quad (9.13)$$

Thus gear train value equals ratio of product of driving tooth numbers to product of driven tooth numbers. The velocity ratio (overall reduction ratio) is given by

$$\text{V.R} = \left(\frac{N_1}{N_6}\right) = \left(\frac{T_6}{T_5}\right)\left(\frac{T_4}{T_3}\right)\left(\frac{T_2}{T_1}\right) \quad (9.14)$$

### Reverted Gear Train

Compound gear trains are further subdivided into two classes: (i) Reverted gear train and (ii) Non-reverted gear train.

A **reverted gear train** is a compound gear train in which, the first and last gears are co-axial. Any compound gear train in which first and last gears are not co-axial, are called **non-reverted type gear train**. Reverted gear trains find applications in clocks and in simple lathes where back gear is used to impart slow speed to the chuck.

For the reverted gear train of Fig. 9.4(a), in which input gear 1 and output gear 4 are co-axial, we have

$$\left(\frac{N_2}{N_1}\right) = \left(\frac{T_1}{T_2}\right) \text{ and } \left(\frac{N_4}{N_3}\right) = \left(\frac{T_3}{T_4}\right)$$

Remembering that  $N_2 = N_3$ , a multiplication on corresponding sides yield,

$$\left(\frac{N_4}{N_1}\right) = \left(\frac{T_1}{T_2}\right)\left(\frac{T_3}{T_4}\right) \quad (9.15)$$

Further, if  $R_j$  ( $j = 1, 2, 3, 4$ ) be the pitch circle radii of  $j$ th gear, then, for common axis of gears 2 and 3,

$$R_1 + R_2 = R_3 + R_4 \quad (9.16)$$

Assuming common module for all the four wheels, and

remembering that  $R = m \frac{T}{2}$ , equation (9.16) yields,

$$\left(m \frac{T_1}{2}\right) + \left(m \frac{T_2}{2}\right) = \left(m \frac{T_3}{2}\right) + \left(m \frac{T_4}{2}\right)$$

Cancelling out common terms,

$$T_1 + T_2 = T_3 + T_4 \quad (9.17)$$

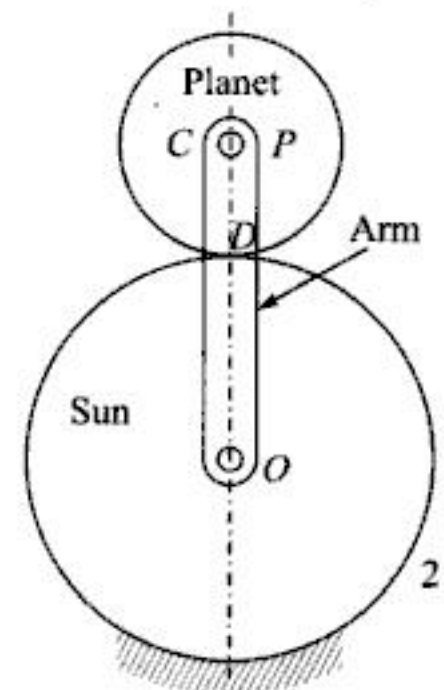
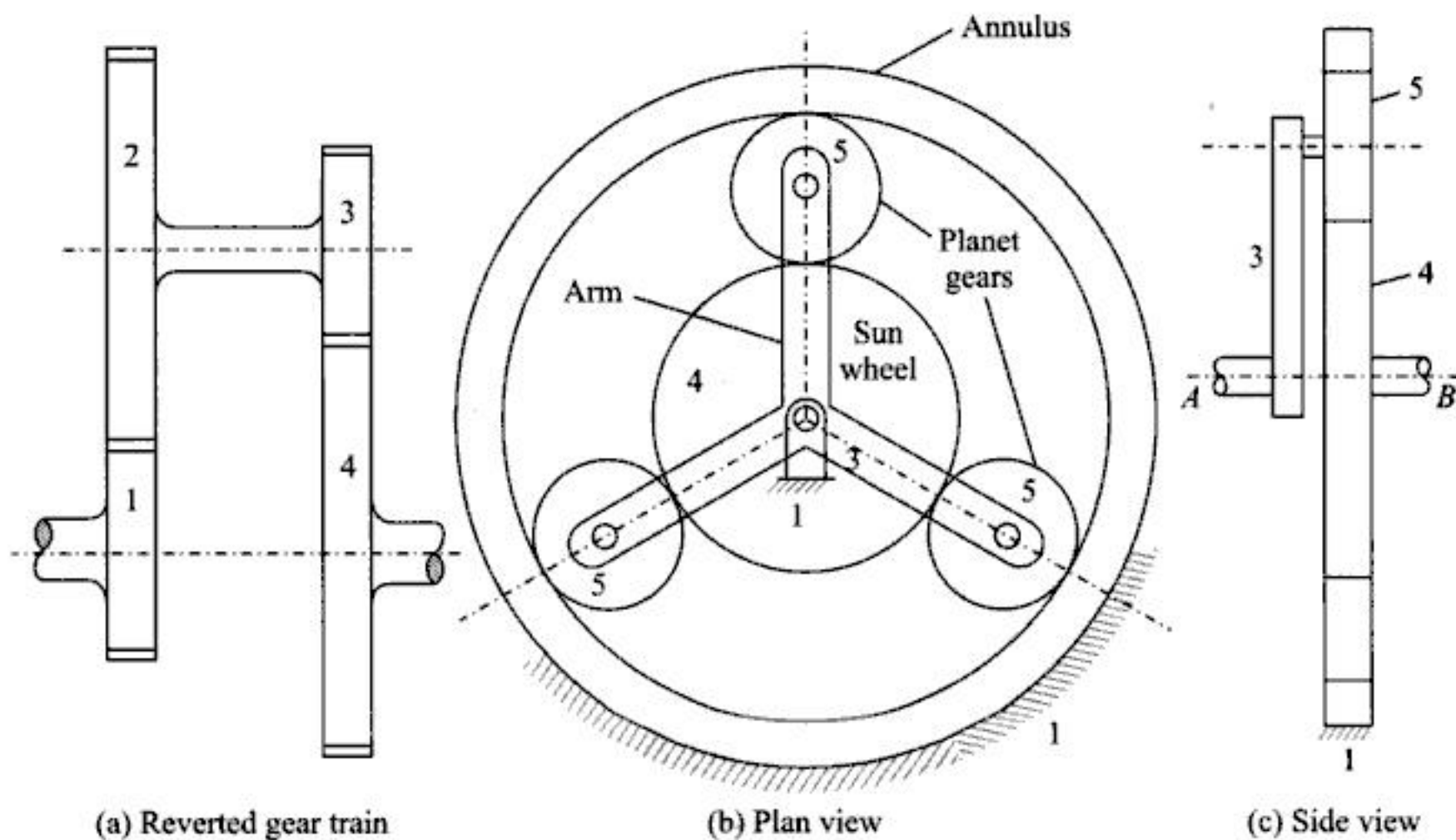


Fig. 9.3 Epicyclic Gear Train.

### Planetary or Epicyclic Gear Train

If axis of rotation of one or more gears is allowed to rotate about another axis the gear train is known as Planetary or Epicyclic gear train. A gear whose axis is permitted to move in an arc of a circle about the fixed axis is called a planet wheel. A gear whose axis is fixed in position, and about which axis of planet wheel revolves, is called a Sun wheel. The planet wheel is carried by an arm which is free to revolve about the fixed axis, and revolves with the input shaft. Sun wheel is also free to revolve about this fixed axis and is mounted on driven shaft. The planet gear  $P$  is free to revolve with respect to arm on a pin attached to it. A simple epicyclic gear train is shown in Fig. 9.3. The sun and planet wheels are shown to mesh at point  $D$ . Usually more than one planet wheel are required for dynamic balancing but, for the sake of simplicity and convenience, only one planet wheel is shown in sketches. A little consideration shows that if the arm in Fig. 9.3 is held stationary and the gear 2 is released, the inversion gives a simple reverted gear train [Fig. 9.4(a)]. In some of the applications, the fixed sun wheel is annular (*note: an internal gear is called as Annulus*) wheel and another (planet) wheel rolls on the inside of it. In such a case, the point  $D$  on planet wheel traces hypocycloidal path. It is customary, however, to call all such gear trains as epicyclic gear trains. Figure 9.4(b) shows an epicyclic gear train in which three planet gears, carried on an arm, are in mesh with annulus, which is fixed. The sun gear is free to revolve. Shaft  $A$  is the input shaft and shaft  $B$  is the output or driven shaft.



**Fig. 9.4** Epicyclic gear train with three planet gears.

In an epicyclic gear train, usually one wheel is fixed, but this is not necessary. There exists epicyclic gear trains in which no wheel is fixed. Epicyclic gear trains permit large speed reduction. An annulus may be used in an epicyclic gear train to make it compact.

A single epicyclic gear train is one in which a single planet gear rolls over another gear, producing epicyclic path. Three common type of single epicyclic gear trains are shown at

Figs. 9.5(a), (b) and (c). The one shown in Fig. 9.5(a) is the simplest one where  $P$  and  $Q$  are the input and output shafts. These are connected by a simple reverted train. The block of gear (called cluster gear) 2 is free to rotate on a shaft which is supported in casing. If the casing is fixed, the epicyclic gear train is reduced to a simple reverted train. When the casing is free to revolve about the axis of shafts  $P$  and  $Q$ , it becomes an epicyclic gear train. Casing  $C$  in this case is called *planet wheel carrier*.

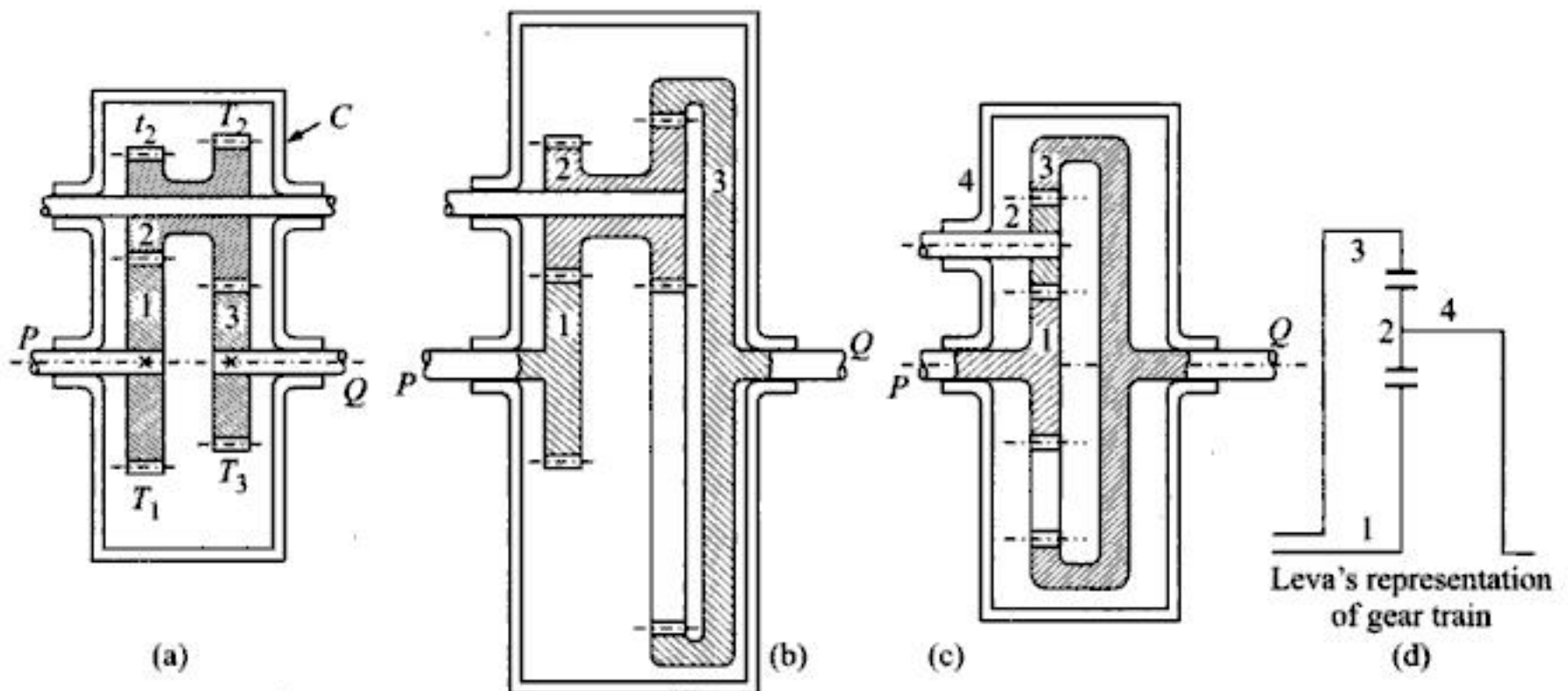


Fig. 9.5 Three types of Single Epicyclic Gear Trains.

The epicyclic gear train of Fig. 9.5(b) consists of an internal (annulus) gear mounted on output shaft  $Q$ . A cluster gear 2 meshes with input gear 1 on one hand and with the internal gear on the other. As against this in Fig. 9.5(c) a single planet wheel 2 meshes with input gear 1 as well as with the internal gear mounted on the output shaft. This epicyclic gear train is same as that shown in Fig. 9.4(b) except that the two redundant planet wheels (which are provided for dynamic balancing only) are not shown in the sketch in Fig. 9.5(c). These redundant planetary gears do not contribute anything additional to the kinematic performance and, for this reason, only one planetary gear is shown in illustration. The epicyclic gear train of Fig. 9.5(c) is represented at Fig. 9.5(d) using simplified designation of Levai, Z.L. A total of twelve simple epicyclic gear trains are listed in literature.

### 9.3 EPICYCLIC GEAR TRAINS WITH BEVEL GEARS

Essentially, bevel gear epicyclic trains are same as epicyclic gear trains using spur gears and are used quite frequently. Figure 9.6 shows Humpage's reduction gear and represents a double epicyclic gear train. As will be seen in the next section, the method of analysis of such trains is the same as for spur gear trains. However, when the axes of bevel gears is inclined to the main axis the terms 'clockwise' and 'counterclockwise' lose much of their significance and as such, usual convention of using plus and minus signs for indicating c.w. and c.c.w. sense is not followed in such cases. Instead, it is much more convenient to indicate sense by small

arrowheads at the pitch line. Bevel gears are used to provide a compact planetary system that permits high-speed reduction with few gears.

In solving problems on epicyclic gear trains with bevel gears, the most important point to note is that the sense of rotation of a pair of bevel gears, mounted on same/parallel axes and connected by an intermediate bevel gear, is always opposite. For instance, gears *A* and *F* in Fig. 9.6 are co-axial and are connected through gear *B*. Thus, if gear *A* rotates *c.c.w.*, gear *F* will rotate in *c.w.* sense. Since gears *A*, *B* and *F* have external gearing, gear *B* will be assumed to have sense of rotation same as that of *A*.

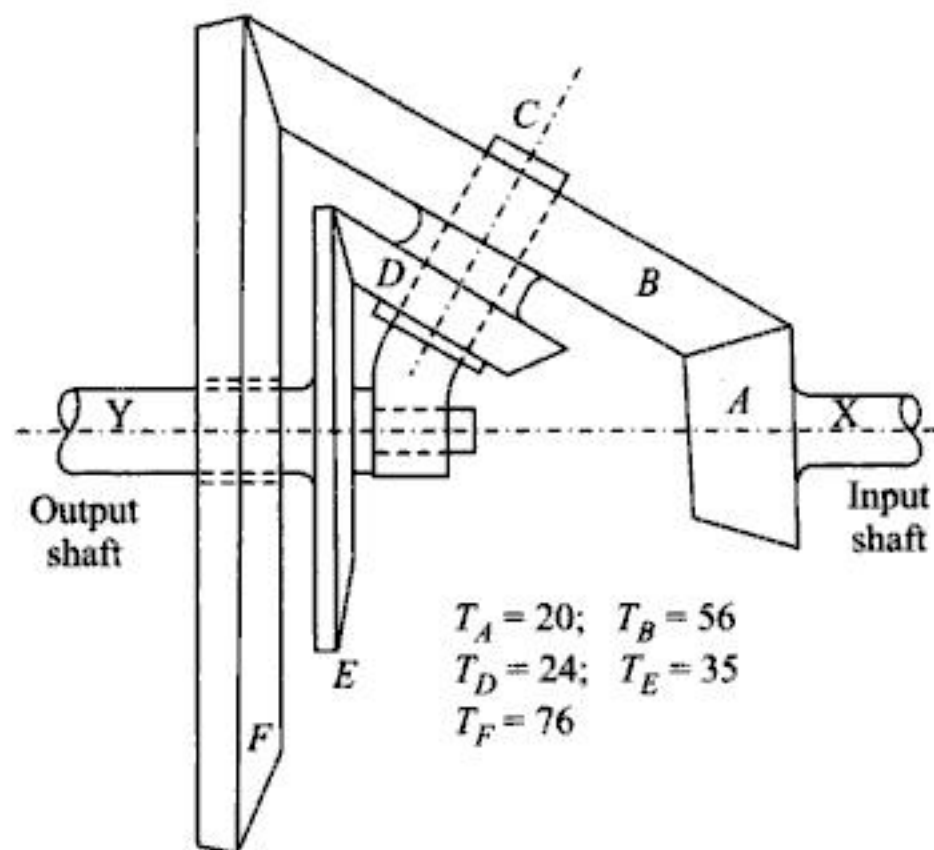


Fig. 9.6 Humpage gear train.

## 9.4 ALGEBRAIC METHOD OF ANALYSING EPICYCLIC GEAR TRAINS

Consider simple epicyclic gear train shown in Fig. 9.7, where the sun gear 2 is fixed to the ground and arm 3 carries the axis of planet gear 4 along an arc of circle of radius  $O_2O_4$  around  $O_2$ .

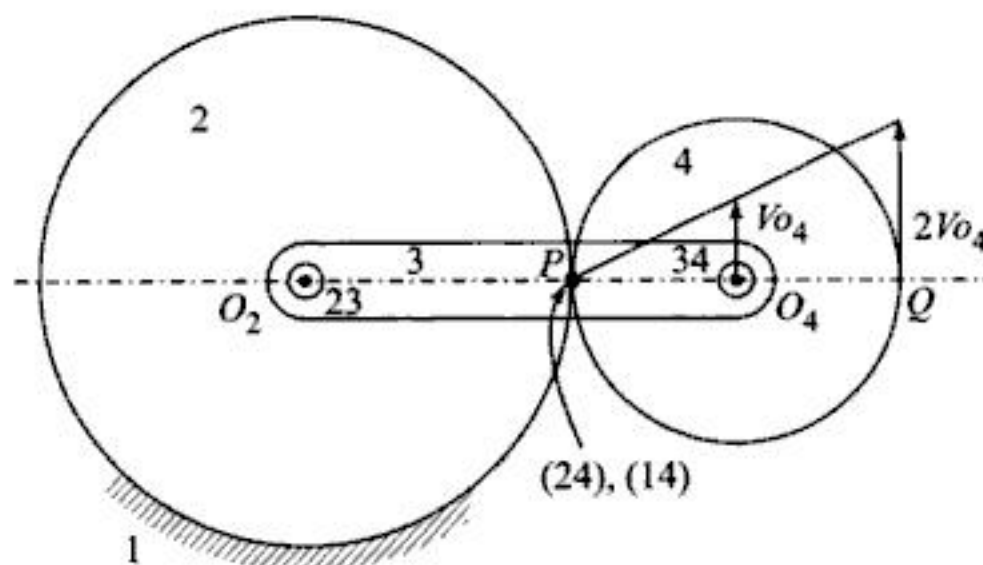


Fig. 9.7 A Planetary Gear Train.

A little consideration shows that this is the inversion of a simple gear train in which arm 3 is fixed and sun gear 2 is released to revolve about  $O_2$ . Note that  $O_2$  and  $O_4$  being lower pairs, link 3 can be fixed or released to obtain inversions. Remembering that relative velocity between links does not change with inversion, it follows that relative speeds of wheels 2 and 4 with respect to arm 3 remains same in both the inversions. However, the absolute speed of gears 2 and 4 with respect to the frame does change with inversion.

Hence, the absolute angular velocity  $\omega_{41}$  of planet wheel 4 w. r. to frame 1, may be obtained using expression

$$\omega_{41} = \omega_{31} + \omega_{43} \quad (9.18)$$

where  $\omega_{43}$  is the angular speed of gear 4 w. r. to arm 3.

Dividing by  $\omega_{31}$  and remembering that  $\omega_{32} = \omega_{31}$ ,

$$\left(\frac{\omega_{41}}{\omega_{31}}\right) = 1 + \left(\frac{\omega_{43}}{\omega_{32}}\right) \quad (9.19)$$

Again, remembering that  $\omega_{23} = -\omega_{32}$ , we have

$$\left(\frac{\omega_{41}}{\omega_{31}}\right) = 1 - \left(\frac{\omega_{43}}{\omega_{23}}\right) \quad (9.20)$$

The ratio  $\left(\frac{\omega_{43}}{\omega_{23}}\right)$  represents ratio of relative angular speeds of wheels 4 and 2 with respect to arm 3. Thus, when arm 3 is fixed in inversion,

$$\left(\frac{\omega_{43}}{\omega_{23}}\right) = \left(\frac{N_4}{N_2}\right) = -\left(\frac{T_2}{T_4}\right)$$

The negative sign appears on account of external gearing. Substituting in equation (9.20), we get

$$\omega_{41} = \omega_{31} \left(1 + \frac{T_2}{T_4}\right)$$

or 
$$\omega_{41} = \omega_{31} \left(1 + \frac{R_2}{R_4}\right) \quad (9.21)$$

Above expression can also be obtained as under. Remembering that  $P$  is an I.C. 24, we have  $V_q = 2V_{04}$ .

But, 
$$V_{04} = \omega_{31}(O_2 O_4) = \omega_{31}(R_2 + R_4)$$

Again as sun gear 2 is fixed to frame link 1, point  $P$  is also the I.C. 14. Hence

$$\omega_{41} = V_Q/(2R_4) = (2V_{04})/(2R_4)$$

Substituting for  $V_{04}$  
$$\omega_{41} = \omega_{31}(R_2 + R_4)/R_4 = \omega_{31}(1 + R_2/R_4)$$

or 
$$\omega_{41} = \omega_{31}(1 + T_2/T_4) \quad (9.22)$$

Planetary gear trains are quite useful in making the reduction unit more compact than a compound gear train. They are also useful in applications requiring two degrees of freedom. An epicyclic gear train of 2 degrees of freedom can be obtained from Fig. 9.4(b) by releasing the annulus, or from Fig. 9.7 by releasing sun wheel 2. Figure 9.8 shows the gear train obtained from Fig. 9.7 with Sun wheel 2 released now.

The epicyclic gear train of Fig. 9.8, has two degrees of freedom and, therefore, requires two input motions so as to determine the motion of epicyclic gear train completely. For instance, if the absolute angular speed  $\omega_{21}$  of sun wheel and  $\omega_{31}$  of the arm be given, it is possible to derive expression for the absolute speed  $\omega_{41}$  of the planet wheel.

Thus, as before, absolute angular speeds of sun wheel, planet wheel and arm are related as

$$\omega_{41} = \omega_{31} + \omega_{43} \quad (9.23)$$

and 
$$\omega_{21} = \omega_{31} + \omega_{23} \quad (9.24)$$

Rearranging above equations,

$$\omega_{43} = \omega_{41} - \omega_{31} \quad (9.25)$$

$$\omega_{23} = \omega_{21} - \omega_{31} \quad (9.26)$$

Dividing equation (9.25) by (9.26) on respective sides,

$$\left( \frac{\omega_{43}}{\omega_{23}} \right) = \left( \frac{\omega_{41} - \omega_{31}}{\omega_{21} - \omega_{31}} \right) \quad (9.27)$$

Noting that L.H.S. of equation (9.27) gives ratio of relative speeds of planet and sun wheel w.r. to arm and remembering that this ratio will remain the same in the inversion, obtained by fixing arm 3, it follows that

$$\left( \frac{\omega_{43}}{\omega_{23}} \right) = \left( -\frac{T_2}{T_4} \right) \quad (9.28)$$

Hence, from equation (9.27),

$$\left( \frac{\omega_{41} - \omega_{31}}{\omega_{21} - \omega_{31}} \right) = -\left( \frac{T_2}{T_4} \right) \quad (9.29)$$

Solving for the planetary gear speed  $\omega_{41}$ ,

$$\omega_{41} = -\left( \frac{T_2}{T_4} \right) \omega_{21} + \left( 1 + \frac{T_2}{T_4} \right) \omega_{31} \quad (9.30)$$

Equation (9.30) shows that resultant absolute angular velocity of planet gear depends on absolute angular velocities of sun wheel ( $\omega_{21}$ ) and of arm ( $\omega_{31}$ ) and, therefore, represents a 2 degree of freedom system. The contribution from the rotation of sun wheel is  $(-T_2/T_4)\omega_{21}$  and

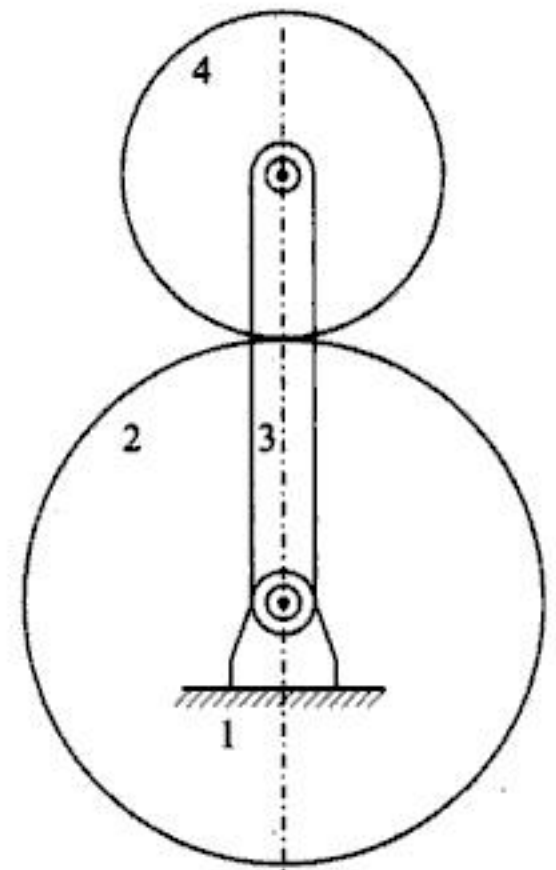


Fig. 9.8 Epicyclic Gear Train of 2 d.o.f.

from the arm is  $(1 + T_2/T_4)\omega_{31}$ . If the arm is fixed,  $\omega_{31}$  is equal to zero and the angular velocity of planet wheel is  $(-T_2/T_4)\omega_{21}$  which corresponds to a simple gear train. Instead, if the sun wheel is fixed (see Fig. 9.7) the system has only 1 degree of freedom and the resultant absolute angular velocity of planet-wheel is given by  $(1 + T_2/T_4)\omega_{31}$ . Note that this expression agrees with the one at (9.22). Readers may also note that use of epicyclic gear train, with input from rotating arm, gives higher velocity ratio compared to the simple gear train.

Considering the original gear train of 2 d.o.f., with arm 3 of Fig. 9.8 revolving, there are two input motions. Let us consider sun wheel 2 to be the first gear and the planet gear 4 to be the last gear. Since link 3 is the arm, the angular speeds of gears 2 and 4 w.r. to arm can be designated as

$$\begin{aligned}\omega_{43} &\equiv \omega_{LA} = \text{angular velocity of last gear w.r. to arm } A. \\ \omega_{23} &\equiv \omega_{FA} = \text{angular velocity of first gear w.r. to arm } A.\end{aligned}$$

The equation (9.27) can be re-written as,

$$\left(\frac{\omega_{LA}}{\omega_{FA}}\right) = \left(\frac{\omega_L - \omega_A}{\omega_F - \omega_A}\right) \quad (9.31)$$

where

$$\frac{\omega_{LA}}{\omega_{FA}} = -\left(\frac{T_2}{T_4}\right)$$

and  $\omega_L \equiv \omega_4 = \text{angular velocity of planet wheel (last gear) w.r. to ground link.}$

$\omega_F \equiv \omega_2 = \text{angular velocity of sun wheel (first gear) w.r. to ground link.}$

and  $\omega_A \equiv \text{angular velocity of arm link w.r. to ground link.}$

Before applying equation (9.31) to the problems on epicyclic gear trains, it is necessary to ensure that

- first and last gears are mounted on shafts supported on frame link, and
- the first and last gears must be in mesh either directly or through intermediate gears whose axis is carried by revolving arm.

The above method is also called 'Formula method'. *The conditions at (a) and (b) above imply that the planet gears must not be taken as the 'first' and the 'last' gears. Further, the first and the last gears must mesh with gears having planetary motion.*

**EXAMPLE 9.1** An internal wheel  $B$  with 80 teeth is keyed to a shaft  $F$ . A fixed internal wheel  $C$  with 82 teeth is concentric with  $B$ . A compound wheel  $DE$  gears with the two internal wheels;  $D$  has 28 teeth and gears with  $C$  while  $E$  gears with  $B$ . The compound wheel rotates freely on a pin which projects from disc which is keyed to a shaft  $A$  co-axial with  $F$ . If all the wheels have the same pitch and the shaft  $A$  makes 800 r.p.m.; what is the speed of  $F$ ? (DAV Indore, 1977, 1987, 1990)

**Solution:** Refer to Fig. 9.9. Since all the gears have same pitch, for common axis of compound wheel  $DE$ ,

$$T_B - T_E = T_C - T_D$$

or

$$T_E = (T_B + T_D - T_C) = (80 + 28 - 82) = 26$$



Let  $B$  be the first gear and  $C$  be the last gear which are co-axial. There are two intermediate gear reductions between first and last gears. Hence the left hand side of equation (9.31) is

$$\frac{\omega_{LA}}{\omega_{FA}} = \left(\frac{T_D}{T_C}\right)\left(\frac{T_B}{T_E}\right) = \left(\frac{28}{82}\right)\left(\frac{80}{26}\right)$$

The right-hand side of equation (9.31) can be written in terms of corresponding r.p.m. Thus,

$$\frac{(28)(80)}{(82)(26)} = \frac{N_L - N_A}{N_F - N_A}$$

As the gear  $C$  is fixed,  $N_L = 0$  and as per data  $N_A = 800$  r.p.m. Thus,

$$-(82)(26)N_A = (28)(80)(N_F - N_A)$$

or 
$$N_F = \left[1 - \frac{(82)(26)}{(28)(80)}\right] N_A = 0.0482 N_A = 38.57 \text{ r.p.m.} \quad \text{Ans.}$$

**EXAMPLE 9.2** In the gear train of Fig. 9.10 the inputs are the sun gear 5 and the ring gear 2. For the given angular velocities of  $N_{51} = 300$  r.p.m. and  $N_{21} = 600$  r.p.m. both c.c.w. as seen from the right, find the resulting motion or arm 6.

**Solution:** This is a 2 d.o.f. system involving two input motions of  $N_2$  and  $N_5$  as shown. Let us consider gear 5 to be the first gear and gear 2 to be the last gear.

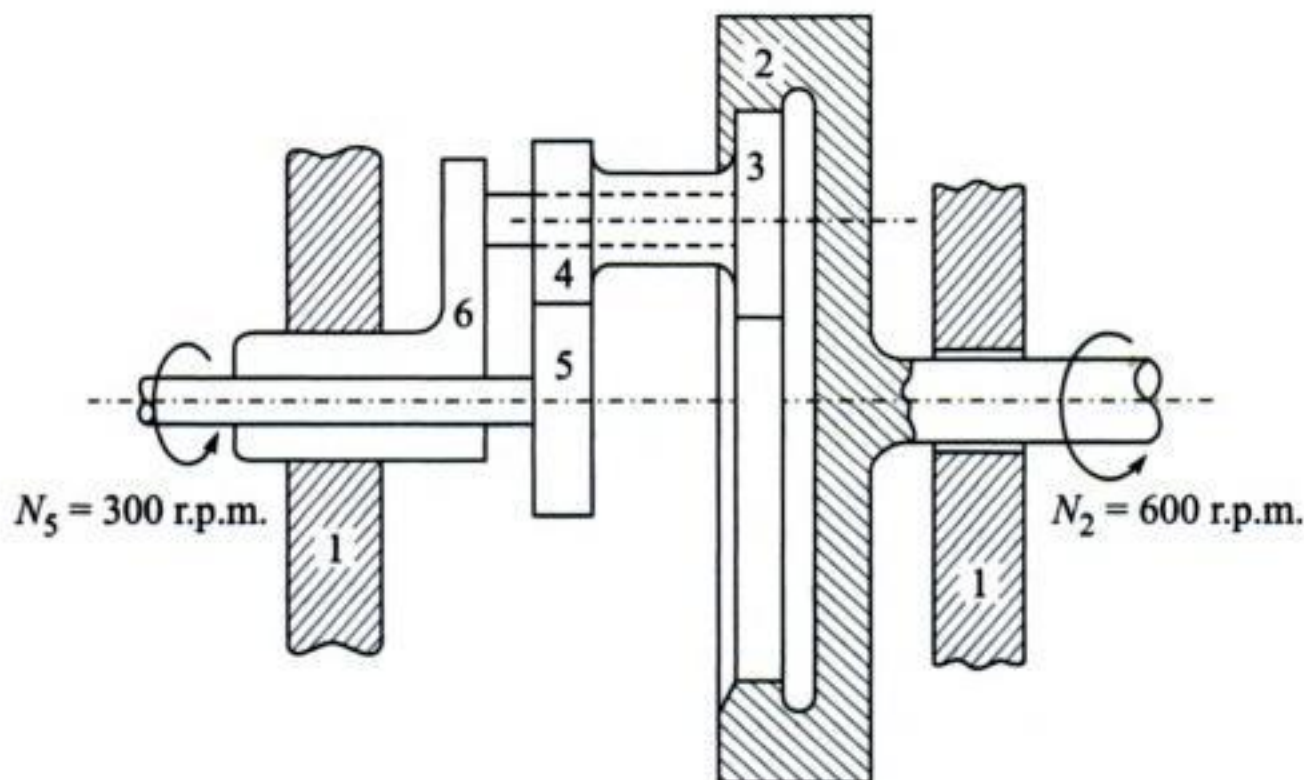


Fig. 9.10 Epicyclic Gear Train of 2 d.o.f.

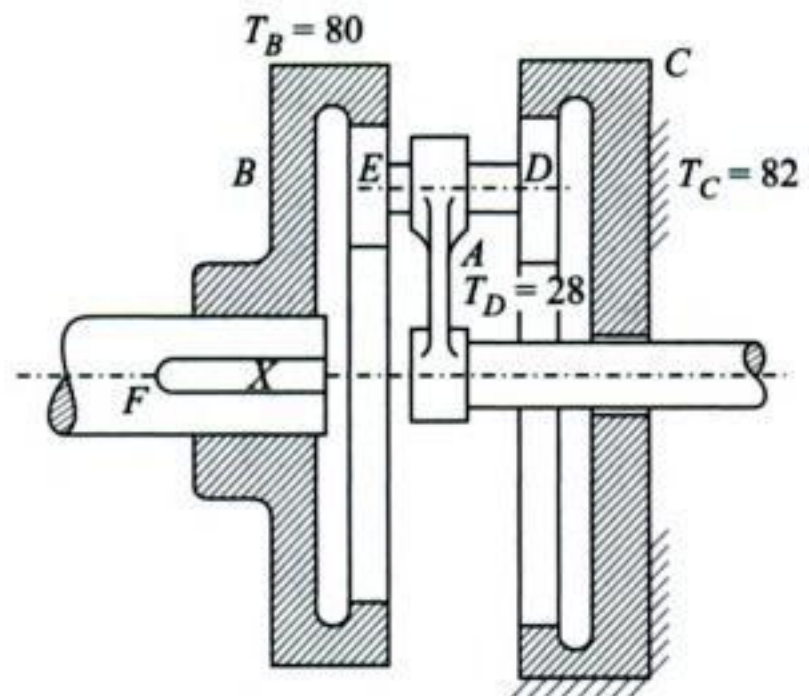


Fig. 9.9 Planetary Gear Train with internal wheels.

Then

$N_2 = 600$  r.p.m.;  $N_5 = 300$  and the L.H.S. of equation (9.31) is

$$\frac{\omega_{2A}}{\omega_{5A}} = \left( \frac{T_3}{T_2} \right) \left( \frac{-T_5}{T_4} \right) = \left( \frac{45}{120} \right) \left( -\frac{48}{27} \right) = -\frac{2}{3}$$

Hence, from equation (9.31),

$$-\frac{2}{3} = \frac{\omega_L - \omega_A}{\omega_F - \omega_A} \quad \text{or} \quad -\frac{2}{3} = \frac{600 - \omega_6}{300 - \omega_6}$$

or  $-600 + 2\omega_6 = 1800 - 3\omega_6$  or  $\omega_6 = \frac{2400}{5} = 480$  r.p.m.

Thus  $\omega_6 = 480$  r.p.m. (c.c.w.), as seen from right. **Ans.**

**EXAMPLE 9.3** If shaft A of the gear system shown in Fig. 9.11 turns at 100 r.p.m., obtain the speed of shaft B and its direction of rotation. Also find the degrees of freedom of the gear system. (AMIE, Summer 1993)

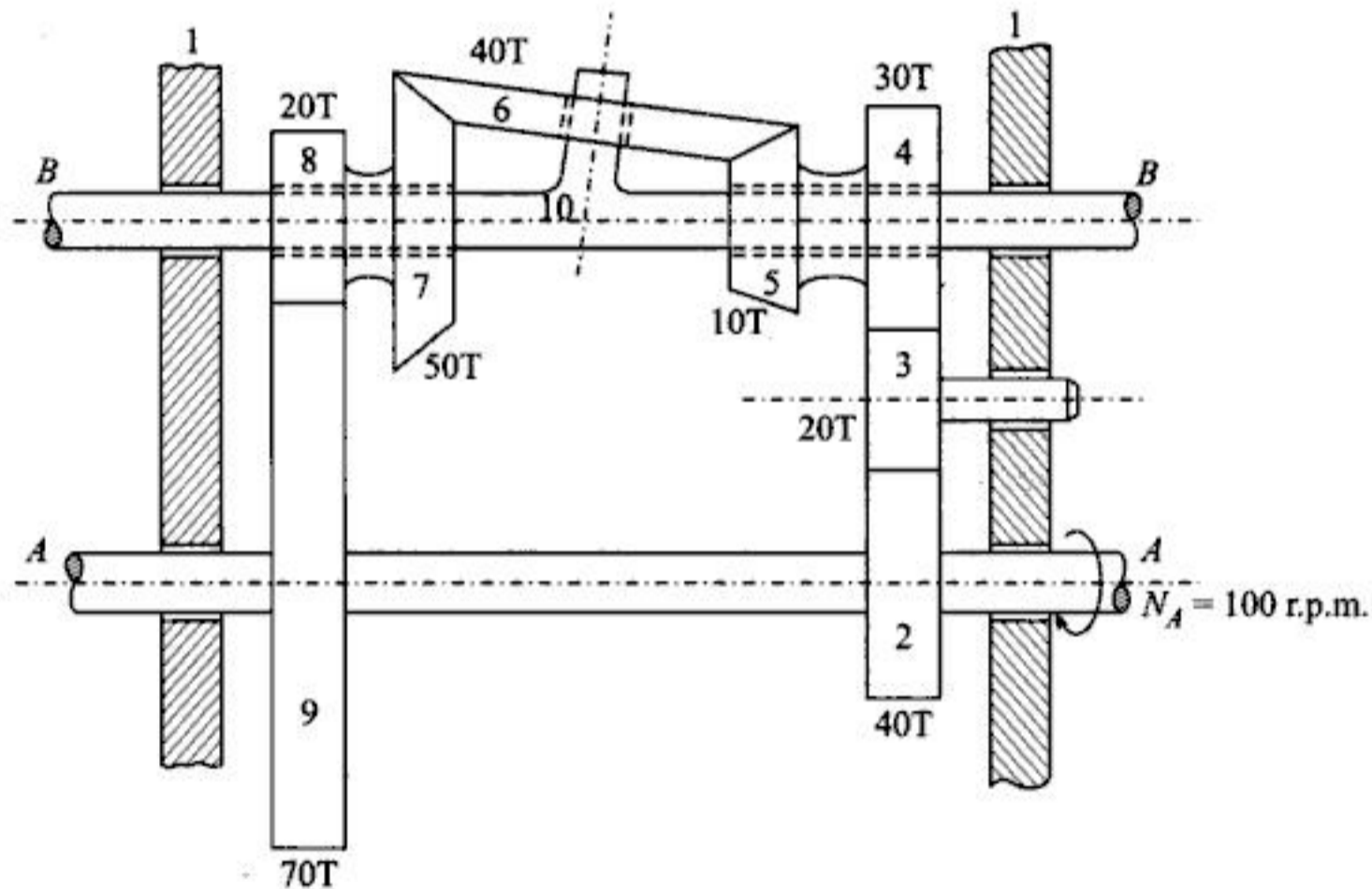


Fig. 9.11 A Gear system of 2 d.o.f.

**Solution:** Taking into account the entire system at Fig. 9.11, the total number of links is only seven. Note that shaft A with gears 2 and 9 mounted rigidly on it constitutes a single link. There are six turning (lower) pairs and five gear (higher) pairs. Thus, apparently

$$\text{d.o.f.} = 3(7 - 1) - 2(6) - 5 = 1.$$

However, a little consideration shows that the links consisting of input shaft A and gears, 2, 3 and 9 do not constitute part of epicyclic gear train; the purpose of gears 2, 3 and 9 along with shaft A is only to provide two input motions. Thus, considering links 1, 4/5, 6, 7/8 and 10

(total 5 links) only, there are 4 turning pairs (the two bearings of shaft 10 in frame 1 give only one pair) and 2 gear pairs between links 5, 6 and 7. Thus, d.o.f.,

$$F = 3(5 - 1) - 2(4) - 2 \\ = 12 - 8 - 2 = 2$$

Thus, the degree of freedom of epicyclic gear train is 2. **Ans.**

This is obvious because gears 2 and 9 on input shaft A are meant for providing two different speeds to gears 4 and 8 of epicyclic gear train.

This being a problem of 2 d.o.f., algebraic method is better suited. Considering gear wheel 5 to be the first gear, gear 7 to be the last gear and shaft 10, carrying planet wheel 6, to be the arm, we have from

$$\frac{\omega_L - \omega_{10}}{\omega_F - \omega_{10}} = \frac{\omega_{L10}}{\omega_{F10}}$$

or, 
$$\frac{\omega_L - \omega_{10}}{\omega_F - \omega_{10}} = \left( \frac{T_6}{T_7} \frac{T_5}{T_6} \right) = \frac{40 \times 10}{50 \times 40} = \frac{1}{5}$$

Again 
$$\omega_5 \equiv \omega_4 = \omega_2 \left( \frac{T_2}{T_5} \right)$$

or 
$$\omega_5 \equiv \omega_4 = 100 \left( \frac{40}{30} \right) = \frac{400}{3} \text{ r.p.m. in clockwise sense, as seen from left}$$

and 
$$\omega_7 \equiv \omega_8 = \omega_9 \left( \frac{T_9}{T_8} \right)$$

Therefore, 
$$\omega_7 \equiv \omega_8 = 100 \left( \frac{70}{20} \right) = 350 \text{ r.p.m. (c.c.w. as seen from left)}$$

Thus 
$$\omega_F \equiv \omega_5 = \frac{-400}{3} \text{ r.p.m.}$$

and, 
$$\omega_L \equiv \omega_7 = 350 \text{ r.p.m.}$$

Hence

$$\frac{350 - \omega_{10}}{-(400/3) - \omega_{10}} = \frac{1}{5} \text{ or } 4\omega_{10} = 5(350) + \frac{400}{3}$$

Therefore 
$$\omega_{10} = 470.8 \text{ r.p.m.}$$

**Ans.**

## 9.5 TABULATION METHOD FOR ANALYSING EPICYCLIC GEAR TRAIN

As indicated earlier, analysis of epicyclic gear train becomes involved because the arm/carriage carrying planet wheel (s) rotates. This suggests that principle of kinematic inversion can be of

great use in analysing the epicyclic gear trains. By fixing the arm and releasing the fixed link (gear), the epicyclic gear train gets converted into a simple reverted train which is simpler to analyse. It must be noted that in kinematic inversion, relative motion between any two links does not change but the absolute motion does change. Since only lower pairs can be inverted [vide section 2.4 B (ii)], and further because arm is the only link in the train having all lower paired connections, it follows that a meaningful inversion results by fixing arm only.

The motion of gear-wheels may be considered in two parts

- (i) Motion in which all the wheels are relatively at rest but are carried round bodily by the arm; and
- (ii) Motion in which the arm is fixed but the gear-wheels are in motion relative to each other. It is this part which corresponds to the inversion of epicyclic gear train.

To understand the first part of motion, which is entirely due to the mobile condition of arm/carriage, consider Fig. 9.12. The figure illustrates four positions of a planet wheels carried by arm 3. Imagine that the sun and planet wheels do not have the teeth and are represented by their pitch cylinders, with contact at pitch point  $P$ . Let the planet wheel be rigidly attached to the arm and let it be carried bodily through 1 revolution c.c.w. around the sun wheel. A radius  $CQ$  on planet wheel then sweeps through  $360^\circ$  as shown in the figure. Clearly, in a given problem if the arm revolves at  $y$  r.p.m. the planet wheel (together with other wheels assumed fixed to the arm) will also undergo  $y$  revolutions in one minute. This is contemplated in the third step of tabulation method.

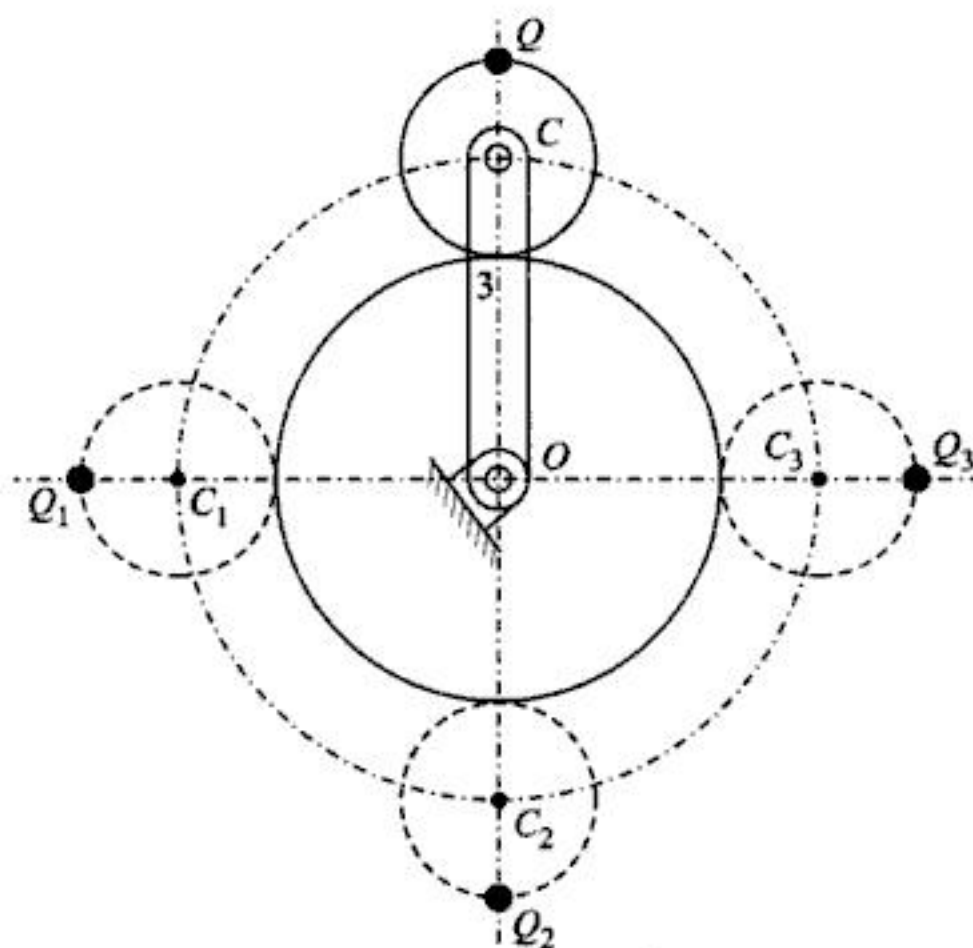


Fig. 9.12 Illustration for Tabulation method.

The step described above ensures position of arm in the 'final orientation'. But some of the other components (like planet wheels and other wheels) are still to assume their proper orientation. In this step therefore, arm is fixed and the inversion of the gear train is considered.

One of the locked wheels is given a full rotation and, treating the gear train as reverted one, corresponding rotations are established in step 1. In step 2 assuming that actual speed of one of the gear wheel in the (inverted) reverted train to be  $x$  times the speed so established, we multiply all the entries in the table by  $x$ . This step ensures proper orientation of all the gear wheels with respect to one another. Steps described in this para, followed by the steps in the preceding para ensures relative orientation for each member identical to what exists in the original epicyclic gear train.

The procedure can now be summarised in the following steps:

1. Lock the arm so as to obtain the kinematic inversion in the form of a simple reverted train. Except for the arm, all the wheels are now free to rotate.
2. Consider any convenient gear and wheel and turn it in clockwise direction through one revolution. Establish corresponding revolutions of all other gears and enter the information under step 1 and 2 in the first row of the table.
3. Multiply each entry of row 1 of the table by  $x$  and enter the values of product in second row of the table. Physically this implies that the chosen wheel is rotated through  $x$  revolutions.
4. Add  $y$  to all the entries of row 2 of the table and enter the result in row 3 of the table. Physically this implies that all the wheels are locked to the arm in the position corresponding to entries in row 2 and arm (along with the complete assembly) is turned through  $y$  revolutions.

In general  $x$  and  $y$  are unknown and the third row gives r.p.m. of each component in terms of  $x$  and  $y$ . By equating any two of these expressions for speed, in terms of  $x$  and  $y$  in row 3, to the given data, variables  $x$  and  $y$  can be established. With  $x$  and  $y$  so established, actual speeds of all other components of epicyclic gear train can be determined. This is explained through solved problems below.

**EXAMPLE 9.4** An epicyclic speed reduction gear is shown in Fig. 9.13. The driving shaft carries on the arm  $A$  a pin, on which the compound wheel  $B-C$  is free to revolve. Wheel  $C$

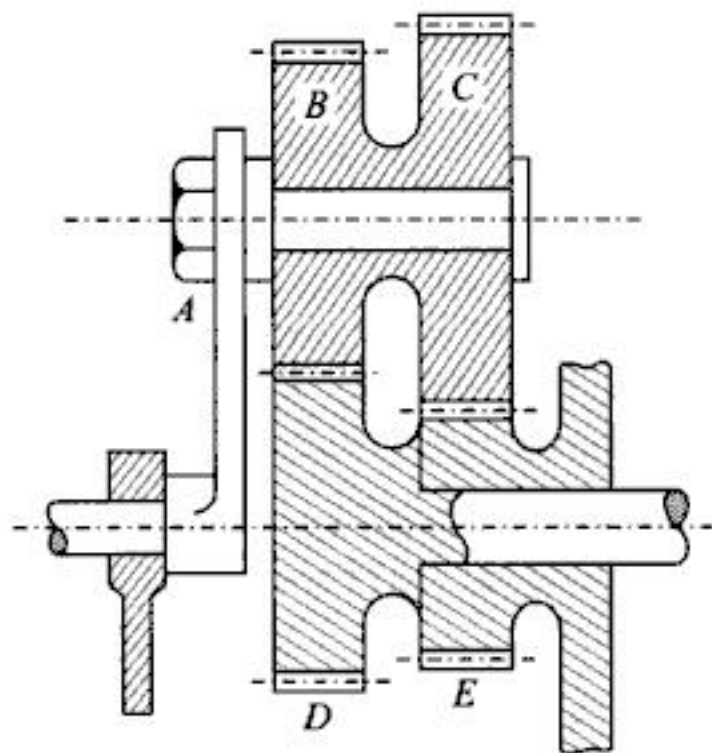


Fig. 9.13 An Epicyclic Speed Reduction Gear train.

meshes with the fixed wheel  $E$  and wheel  $B$  meshes with a wheel  $D$  keyed to the driven shaft. The numbers of teeth on the wheels are:

$T_b = 27$ ;  $T_c = 30$ ;  $T_d = 24$ ; and  $T_e = 21$ . Find the ratio of the speed of the driving shaft to the speed of the driven shaft. (DAVV Indore, 1989)

**Solution:** Given:  $T_b = 27$ ;  $T_c = 30$ ;  $T_d = 24$ ; and  $T_e = 21$  and, wheel  $E$  is fixed. The table is prepared as follows:

**Table 9.1** Solution by Tabulation Method.

S.No.	Operations	Arm A	Wheel E $T_e = 21$	Wheel B-C (compound) $T_b = 27$ $T_c = 30$	Wheel D (driven shaft) $T_d = 24$
1.	Fix the arm and rotate wheel $E$ by +1 revolution	0	+1	$\frac{-T_e}{T_c}$	$\frac{T_e}{T_c} \frac{T_b}{T_d}$
2.	Multiply by $x$	0	$x$	$\frac{-T_e}{T_c} x$	$\frac{T_e}{T_c} \frac{T_b}{T_d} x$
3.	Add $y$ to all	$y$	$x + y$	$-\left(\frac{T_e}{T_c}\right)x + y$	$\frac{T_e}{T_c} \frac{T_b}{T_d} x + y$

Since the gear wheel  $E$  is fixed,

$$x + y = 0 \quad \text{or} \quad x = -y$$

Hence, the output speed, as given by speed of  $D$

$$\begin{aligned} &= \left(\frac{T_e}{T_c}\right)\left(\frac{T_b}{T_d}\right)(-y) + y \\ &= \left[-\left(\frac{21}{30}\right)\left(\frac{27}{24}\right) + 1\right]y = \left(\frac{17}{80}\right)y \end{aligned}$$

Hence, the ratio of the speed of driving shaft to the driven shaft

$$\frac{N_A}{N_D} = y / \left(\frac{17}{80}\right)y = \frac{80}{17} = 4.71 \quad \text{Ans.}$$

**EXAMPLE 9.5** In the planetary gear train shown in Fig. 9.14, gear  $A$  is the driver and gears  $B$  and  $D$  are compounded. Gears  $E$  and  $C$  are internal gears with  $C$  as the fixed gear. Number of teeth on gears are:  $T_A = 20$ ;  $T_B = 60$ ;  $T_C = 140$ ;  $T_D = 40$ ;  $T_E = 120$ . If the driver gear rotates at 720 r.p.m. c.c.w. when seen from the right, find out the speed at which driven gear rotates.

**Solution:** Given  $T_A = 20$ ;  $T_B = 60$ ;  $T_C = 140$ ;  $T_D = 40$ ;  $T_E = 120$

But the input shaft (Gear  $A$ ) rotates at 720 r.p.m. and internal gear is fixed, i.e.  $N_A = 720$  and  $N_C = 0$

Thus

$$x + y = 0$$

and

$$-7x + y = 720$$

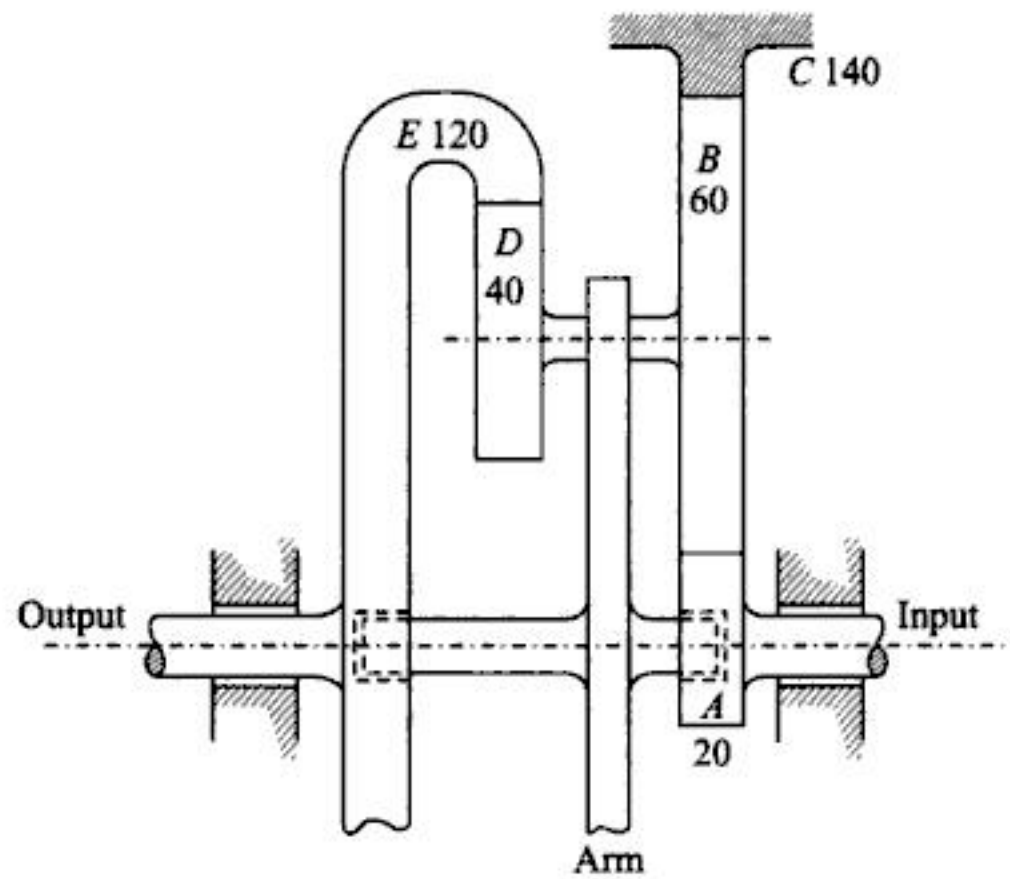


Fig. 9.14 Planetary gear train.

Subtracting

$$8x = -720,$$

Therefore

$$x = -90$$

Substituting back,  $y = 90$

Table 9.2 Solution by Tabulation Method.

S.No.	Operations	Arm	Gear A $T_A = 20$	Compound Gear B-D $T_B = 60$ $T_D = 40$	Gear C $T_C = 140$	Gear E $T_E = 120$
1.	Fix the Arm and give 1 revolution to C c.c.w.	0	$-\frac{140}{60} \frac{60}{20}$	$+\frac{140}{60}$	+1	$+\frac{140}{60} \frac{40}{120}$
2.	Multiply by x	0	$-7x$	$\frac{14}{6}x$	x	$\frac{14}{18}x$
3.	Add y to all	y	$-7x + y$	$\frac{14}{6}x + y$	$x + y$	$\frac{14}{18}x + y$

Hence, output shaft speed

$$\begin{aligned} N_E &= \frac{14}{18}x + y \\ &= \frac{14}{18}(-90) + y \\ &= -70 + 90 = 20 \text{ r.p.m.} \end{aligned}$$

**Ans.**

Thus, there is a speed reduction by a ratio =  $\frac{720}{20} = 36$

## 9.6 TORQUES AND TOOTH LOADS IN EPICYCLIC GEAR TRAINS

Let us assume that all the parts of the epicyclic gear trains are moving at uniform speeds, avoiding any angular accelerations, and hence the inertia torques too. When such a geared system is used to transmit power, there are tooth loads and consequent torques transmitted from one gear to the other. Each gear wheel can be said to be in equilibrium under the action of external torques acting on it. Thus applying equilibrium condition to the gear train as a whole,

$$\Sigma(M) = 0$$

$$\text{or} \quad M_i + M_o + M_b = 0 \quad (9.32)$$

where

$M_i$  = input torque on driving member,

$M_o$  = output torque on driven member, and

$M_b$  = holding or braking torque on the fixed gear.

If  $F_i$ ,  $F_o$  and  $F_b$  are the tooth loads at radii  $r_i$ ,  $r_o$  and  $r_b$  respectively on driving, driven and fixed gears then, from equation (9.32),

$$(F_i)r_i + (F_o)r_o + F_b(r_b) \quad (9.33)$$

Finally, if there are no frictional losses within the gear train, either at the bearings or at the contact surfaces of teeth, the energy balance dictates,

$$\Sigma M(\omega) = 0$$

$$\text{or} \quad M_i\omega_i + M_o\omega_o + M_b\omega_b = 0 \quad (9.34)$$

Again, as the angular speed  $\omega_b = 0$  for fixed gear, equation (9.34) simplifies to

$$(M_i)\omega_i + (M_o)\omega_o = 0 \quad (9.35)$$

Substituting for  $M_o$  from equation (9.35) in equation (9.32),

$$M_b = M_i \left( \frac{\omega_i}{\omega_o} - 1 \right) \quad (9.36)$$

Equation (9.36) enables to obtain braking or fixing torque required to be applied to the fixed gear. Equation (9.33) can be used to evaluate tooth loads.

**EXAMPLE 9.6** In an epicyclic gear train of the sun and planet type, shown in Fig. 9.4, the pitch circle diameter of the annular wheel is to be nearly 216 mm and module 4 mm. When the annular wheel is stationary, the spider which carries three planet wheels of equal size, is to make one revolution for every five revolutions of the driving spindle carrying the sun wheel.

Determine suitable number of teeth for all the wheels and the exact diameter of the pitch circle of the annular wheel. If an input torque of 19.6 N·m is applied to the spindle carrying the sun wheel; determine the fixing torque on the annular ring. (AMIE, Summer 1983, DAVV Indore, 1983; J. Univ. Gwalior, 1983)

**Solution:** Referring to Fig. 9.4,

$$T_2 = \frac{P.C.D.}{\text{module}} = \frac{216}{4} = 54$$



For kinematic calculations, it is sufficient to consider one planet wheel instead of three. As given in the problem,

$$N_5/N_A = 5,$$

where  $N_5$  and  $N_A$  are speeds in **r.p.m.** of sun wheel and spider.

**Table 9.3** Solution by Tabulation Method.

S.No.	Operations	Spider/Arm A	Sun wheel (Input) $T_4$	Planet wheel $T_5$	Annulus $T_2 = 54$
1.	Fix the spider/arm and give +1 revolution to the annulus	0	$-\left(\frac{T_2}{T_5}\right)\left(\frac{T_5}{T_4}\right)$	$+\left(\frac{T_2}{T_5}\right)$	+1
2.	Multiply by $x$	0	$-\frac{T_2}{T_4} x$	$+\left(\frac{T_2}{T_5}\right)x$	+ $x$
3.	Add $y$ to all	$y$	$-\left(\frac{T_2}{T_4}\right)x+y$	$+\left(\frac{T_2}{T_4}\right)x+y$	$x+y$

When the annulus is stationary, sun wheel makes 5 revolutions for 1 revolutions of spider.

Hence  $y = 1$  (i)

$$x + y = 0 \quad \text{(ii)}$$

and  $-\left(\frac{T_2}{T_4}\right)x + y = 5$  (iii)

From (i) and (ii),  $x = -1$  and  $y = 1$

Hence from (iii)

$$\left(\frac{T_2}{T_4}\right) + 1 = 5$$

or  $\left(\frac{T_2}{T_4}\right) = 4$  (iv)

Since  $T_2 = 54$ ,  $T_4 = \frac{54}{4} = 13.5$

Selecting next higher integer value,  $T_4 = 14$  **Ans.**

And therefore  $T_2 = 14 \times 4 = 56$

Hence, pitch circle diameter of annulus,  $mT = 4 \times 56 = 224$  mm **Ans.**

Again from the geometry of Fig. 9.4,

$$\frac{D}{2} = \left(\frac{d_4}{2}\right) + d_5 \text{ and, dividing by common module,}$$



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- 11.21** 74.57 kw power is transmitted at 300 r.p.m. by a multiplate disc friction clutch. The plates run in oil and have the friction surfaces of steel and phosphor bronze alternately.  $\mu = 0.07$ , and the axial intensity of pressure is not to exceed  $14.7 \text{ N/cm}^2$ . External radius is 1.25 times the internal radius. The external radius is 12.5 cm. Determine the number of plates needed to transmit the required torque. Make suitable assumptions in solving the problem and derive the relations used.

(Ans. Required number = 14; However as  $n_i = n_o - 1$  Take  $n = 15$ ;  
 $n_o = 8$ ;  $n_i = 7$ )

- 11.22** Explain what is meant by friction circle. In a direct acting steam engine, the stroke is 60 cm and the diameter of the piston is 30 cm, the length of the connecting rod is 4 times the length of the crank. The diameter of the cross-head pin, crank pin, crank-shaft are 8.7 cm, 11.2 cm and 12.5 cm respectively. The coefficient of friction between the cross-head and guides is 0.08 and for the two pins and crank-shaft it is 0.005. When the crank has moved through an angle of  $45^\circ$ , from the i.d.c., the effective steam pressure on piston is  $39.24 \text{ N/cm}^2$ .

Draw a diagram to show the direction of forces acting in the link-work, and calculate the turning moment on the crank-shaft. (DAV Indore, 1917)

- 11.23** A multi-plate clutch is to transmit 3.73 kw at 1500 r.p.m. The inner and outer radii of the plates are to be 5 cm and 10 cm respectively. The maximum axial spring force is restricted to 981 N. calculate the necessary number of pairs of surfaces if  $\mu = 0.35$ , assuming constant wear. What will be the necessary axial force?

(Ans. Calculated value of  $n_a = 1$ )



# BELT, ROPE AND CHAIN DRIVES

## 12.1 INTRODUCTION

Belt, rope and chain drives are the examples of higher pair. As explained earlier, in Chapter 2, pairs of elements having relative motions of sliding, turning or screw motion are known as lower pairs. Relative motion between pairing elements of a higher pair is more complicated compared to that in lower pairs. In the case of belts if we neglect slip, the relative motion is that of rolling; the belt winding on to the pulley at one end and leaving the pulley at the other. The belt drive thus is not a positive drive due to slip associated with the rolling motion. Power is transmitted from one shaft to another through friction between belt and pulleys.

Ordinarily, a belt drive is used to transmit power between two parallel shafts separated by a certain minimum distance; the minimum distance being dependent on the type of belt used. The above condition of minimum distance is a desirable condition for the belts to work most efficiently. Flat belts are used to connect shafts with centre distance as large as 10 m, while V-belts are used to connect shafts with centre distance less than or equal to 5 m. V-belts permit a speed ratio upto a maximum of 7. As against this, a rope drive is used for connecting grooved pulleys or drums at a centre distance upto a maximum of 30 m.

When using flat belts, clutch action may be obtained by shifting the belt from a loose to a tight pulley. Step pulleys may be used to obtain a desired change in velocity ratio economically. Belts and ropes transmit power due to friction between them and the pulleys. *When power being transmitted exceeds the torque capacity, corresponding to limiting coefficient friction, the belt/rope slips over the pulley. A positive aspect of this is that a belt/rope drive may also be used as an overload protection device.*

## 12.2 VELOCITY RATIO

A set of parallel shafts may be connected by a flat belt so as to produce either—

- (i) an open belt drive or
- (ii) a crossed belt drive

These arrangements are shown in Figs. 12.1(a) and (b). One of the two connected pulleys is called driver while the other one is called the driven pulley. Note that unlike gears, it is not necessary for the driver pulley to be smaller than driven pulley as the belt drive may be used either as a speed-reducer or for increasing the speed.

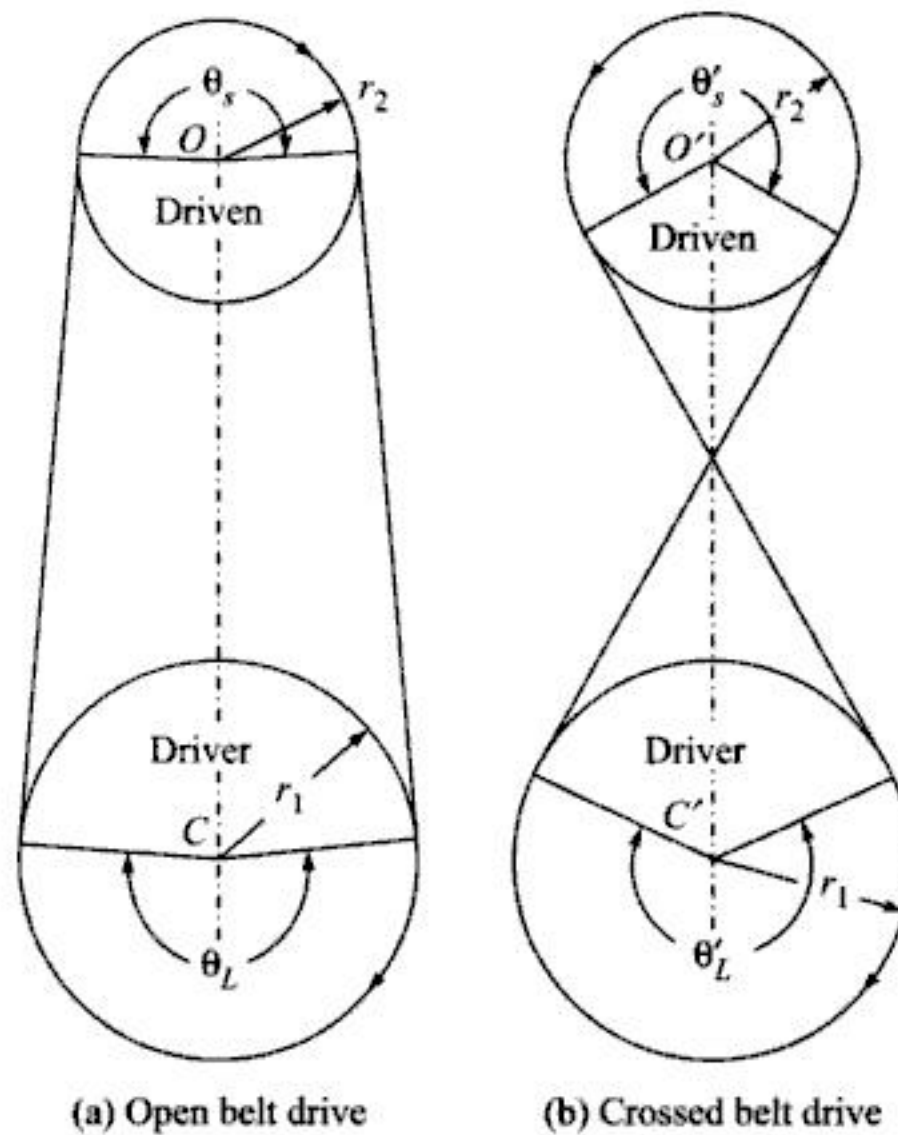


Fig. 12.1

Assuming the driver pulleys to rotate in the clockwise sense, it is easy to see that driver and driven pulleys rotate in the same sense in open belt drive, but in crossed belt drive, they rotate in opposite sense. Let us first neglect belt thickness.

Let  $r_1$  = the radius of driver pulley in either case,  
 $r_2$  = the radius of driven pulley in both the cases,  
 $\omega_1$  = angular speed of driver pulley, and  
 $\omega_2$  = angular speed of driven pulley.

Assuming no slip to occur between belt and pulleys, the peripheral speed of rotation of pulleys must equal the belt speed. Thus

$$V = r_1\omega_1 = r_2\omega_2$$

or 
$$\frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} \quad (12.1)$$

Thus, if  $N_2$  be the speed in r.p.m. of driven pulley and  $N_1$  be the speed in r.p.m. of driver pulley, we have

$$\frac{\omega_2}{\omega_1} \equiv \frac{N_2}{N_1}$$

Hence, from equation (12.1),

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \quad (12.2)$$

In other words, the velocity ratio of belt drive varies inversely as their radii.

### **Effect of Belt Thickness $t$**

Assume the belt thickness to be uniform. The effective radii at the neutral surface of belt, which is neither compressed nor elongated as the belt is subjected to flexure (while passing over the pulleys), are

$$R_1 = \left( r_1 + \frac{t}{2} \right) \quad \text{and} \quad R_2 = \left( r_2 + \frac{t}{2} \right)$$

Hence, for no slip,

$$V = R_1\omega_1 = R_2\omega_2 \quad \text{or} \quad (\omega_2/\omega_1) = R_1/R_2$$

Substituting for  $R_1$  and  $R_2$ ,

$$(\omega_2/\omega_1) = (r_1 + t/2)/(r_2 + t/2) \quad (12.3)$$

And in terms of speeds in r.p.m.,

$$(N_2/N_1) = (r_1 + t/2)/(r_2 + t/2) \quad (12.4)$$

As  $r_1 = d_1/2$  and  $r_2 = d_2/2$ , this leads to the expression

$$(N_2/N_1) = (d_1 + t)/(d_2 + t) \quad (12.5)$$

### **Effect of Slip on Velocity Ratio**

On the driver pulley, the pulley drives the belt and on driven pulley, the belt drives the pulley. When there is no slip between belt and pulleys, the peripheral speed of pulleys and the belt speed are equal. The slip is thus measured by relative velocity between the belt and the pulley.

Let  $s_1$  = percentage slip between driver pulley and belt, and  
 $s_2$  = percentage slip between belt and driven pulley.

Clearly for a peripheral speed  $r_1\omega_1$  of the driver pulley, the belt speed will be less than the peripheral speed of pulley and is given by

$$\text{belt speed} = (r_1\omega_1) \left( \frac{100 - s_1}{100} \right) \quad (12.6)$$

As the belt drives driven pulley, when slip occurs, the belt speed must be larger than the peripheral speed ( $r_2\omega_2$ ) of the driven pulley. Thus

$$r_2\omega_2 = (\text{belt speed}) \times \left( \frac{100 - s_2}{100} \right)$$

Substituting for belt speed from equation (12.6),

$$r_2\omega_2 = r_1\omega_1 \left( \frac{100 - s_1}{100} \right) \left( \frac{100 - s_2}{100} \right) \quad (12.7)$$

If  $s$  be the total percentage slip between the driver and the driven pulleys, then

$$r_2\omega_2 = r_1\omega_1\left(\frac{100-s}{100}\right) \quad (12.8)$$

Equating R.H.S. of (12.7) and (12.8),

$$\left(\frac{100-s}{100}\right) = \left(\frac{100-s_1}{100}\right)\left(\frac{100-s_2}{100}\right)$$

or  $(1 - 0.01s) = (1 - 0.01s_1)(1 - 0.01s_2)$

or  $(1 - 0.01s) = 1 - (s_1 + s_2)0.01 + (0.0001)s_1s_2$

or  $s = (s_1 + s_2) - (0.01)s_1s_2 \quad (12.9)$

The last term in equation 12.9 is generally quite small compared to other terms, and can therefore be neglected. Hence,

$$s = s_1 + s_2 \quad (12.10)$$

And therefore,

$$\omega_2r_2 = (\omega_1r_1)\left[1 - \frac{s_1 + s_2}{100}\right] \quad (12.11)$$

**EXAMPLE 12.1** A prime mover running at 400 r.p.m. drives a D.C. generator at 600 r.p.m. through a belt drive. Diameter of pulley on the output shaft of the prime mover is 60 cm. Assuming a total slip of 3%, determine the diameter of the generator pulley, if the belt thickness is 5 mm.

**Solution:** Given  $s_1 + s_2 = 3$ ;  $d_1 = 60$  cm;  $t = 0.5$  cm and  $N_1 = 400$ ;  $N_2' = 600$  r.p.m.

Let  $d_2$  and  $N_2$  be the diameter and speed in r.p.m. of driven pulley and  $d_1$ ,  $N_1$  be the diameter and speed of driver pulley. Then considering effect of belt thickness alone,

$$\frac{N_2}{N_1} = \left(\frac{d_1 + t}{d_2 + t}\right) \quad \text{or} \quad N_2 = N_1\left(\frac{d_1 + t}{d_2 + t}\right)$$

Incorporating effect of slip,

$$N_2' = N_2\left(1 - \frac{s}{100}\right)$$

Therefore  $N_2' = N_1\left(\frac{d_1 + t}{d_2 + t}\right)\left(1 - \frac{s}{100}\right)$

or,  $600 = 400\left(\frac{60 + 0.5}{d_2 + 0.5}\right)\left(1 - \frac{3}{100}\right)$

or  $\frac{60.5}{(d_2 + 0.5)} = \left(\frac{600}{400}\right)\frac{100}{97}$

Therefore 
$$d_2 + 0.5 = \frac{400}{600} \times \frac{97}{100} \times 60.5 = 39.123$$

Therefore 
$$d_2 = 39.123 - 0.5 = 38.623 \text{ cm} \approx 38.62 \text{ cm} \quad \text{Ans.}$$

## 12.3 BELT LENGTH

### For Open Belt Drive

The angle of lap  $\theta$  (also called angle of wrap or contact) is the angle subtended by segment of belt, in contact with smaller pulley, at the centre of that pulley. Let  $P, Q$  and  $M, N$  be the points at which belt is tangent to driver and driven pulley respectively (see Fig. 12.2). Let  $O$  and  $C$  be the centres of the driver and driven pulleys.

Join  $OP, OQ$  and  $CM, CN$ . Then, as  $MP$  is a common tangent to the two pulleys,

$$\angle OPM = \angle CMP = 90^\circ \text{ and thus, } PO \text{ is parallel to } MC.$$

From  $O$  drop a perpendicular  $OL$  on  $CM$  and let  $\alpha$  be the angle made by  $OL$  with the line of centres. Then, from right angled triangle  $OLC$ ,

$$\sin \alpha = (LC/OC)$$

Thus, if  $L' = OC$  be the centre distance, then with  $LC = (r_2 - r_1)$

$$\sin \alpha = \frac{(r_2 - r_1)}{L'}$$

Further, from right angled triangle  $OLC$ ,

$$OL = OC \cos \alpha = L' \cos \alpha$$

Hence, by symmetry of Figure 12.2 about centre lines  $OC$ ,

$$NQ = PM = OL = L' \cos \alpha$$

Again

$$\theta = (\pi - 2\alpha)$$

The belt length  $l$  is given by,

$$l = 2PM + \text{arc } PKQ + \text{arc } MRN$$

or 
$$l = 2L' \cos \alpha + r_1(\pi - 2\alpha) + r_2[2\pi - (\pi - 2\alpha)]$$

or 
$$l = 2L' \cos \alpha + r_1(\pi - 2\alpha) + r_2(\pi + 2\alpha)$$

or 
$$l = 2L' \cos \alpha + (r_1 + r_2)\pi + (r_2 - r_1)2\alpha \quad (12.12)$$

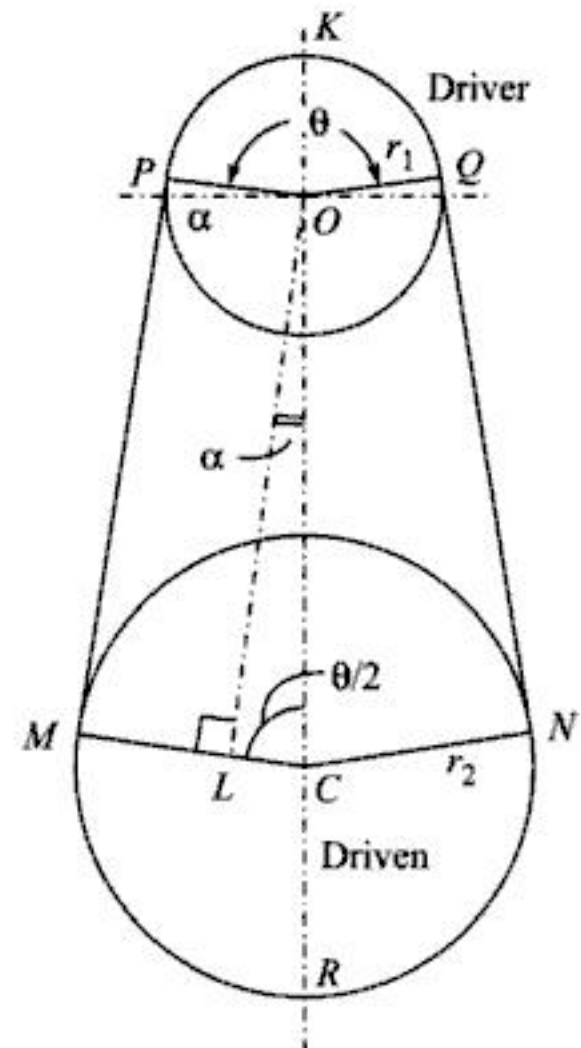


Fig. 12.2 Flat-Belt length.

**For Crossed Belt Drive**

For the crossed belt drive shown in Fig. 12.3, let

$r_1$  = radius of driver pulley

$r_2$  = radius of driven pulley

$OC = L' =$  centre distance of pulleys

$\theta$  = angle of lap which can be shown to be equal on both the pulleys

Let  $PN$  and  $MQ$  be the common tangents to the circles. Extend radius  $CM$  and drop perpendicular  $OL$  on it from  $O$  and let  $\angle COL = \alpha$ ,

Then  $CL = CM + ML = CM + OQ = (r_1 + r_2)$

Also, from right angled triangle  $OLC$ ,

$$\beta = \angle OCL = \left( \frac{\pi}{2} - \alpha \right)$$

Also  $(2\pi - \theta) = \angle MCN = 2\beta = 2\left( \frac{\pi}{2} - \alpha \right)$

or  $\theta = (\pi + 2\alpha)$  (12.13)

Hence belt length

$$l = MQ + PN + r_1\theta + r_2\theta$$

or  $l = 2(OL) + (r_1 + r_2)\theta$

or  $l = 2\sqrt{\{L'^2 - (r_1 + r_2)^2\}} + (r_1 + r_2)(\pi + 2\alpha)$  (12.14)

where  $\alpha = \sin^{-1}[(r_1 + r_2)/L']$

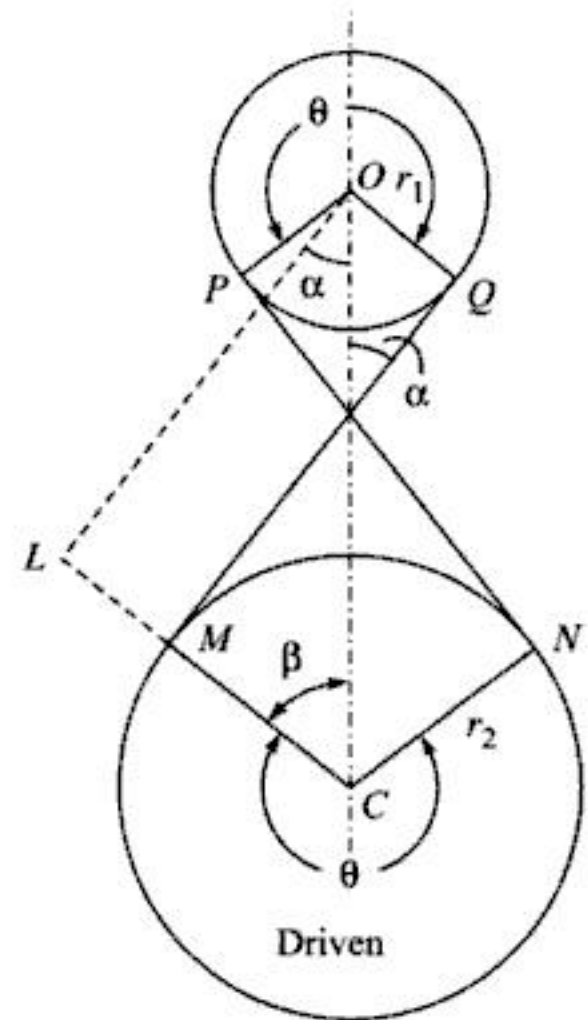


Fig. 12.3 Crossed-Belt length.

**EXAMPLE 12.2** Two parallel shafts are to be connected by a suitable drive with pulleys of 320 mm and 640 mm diameters. Determine the required length of belt if the centre distance is 3.2 m, and

- the pulleys must rotate in same sense, and
- when the pulleys must rotate in opposite sense.

**Solution:** Given  $r_1 = 16$  cm;  $r_2 = 32$  cm;  $L = 320$  cm

If the pulleys are to rotate in same sense, they must be connected through an open belt drive and for securing rotations in opposite sense, they must be connected through a crossed belt drive.

(a) Belt length for open belt drive

$$\sin\alpha = \frac{r_2 - r_1}{L} = (32 - 16)/320 = 0.05 \quad \text{Hence } \alpha = 2.867 \text{ degrees}$$

$$\begin{aligned} \text{Belt length} \quad l &= 2(320)\cos 2.866 + (32 + 16)\pi + (32 - 16) \frac{2.866 \times \pi}{180} \times 2 \\ &= 639.199 + 150.796 + 1.6 \\ &= 791.595 = 791.6 \text{ cm} \end{aligned}$$

Ans.

In the case of crossed belt drive, angle of lap is equal on both the pulleys and therefore, limiting ratio of tension can be established for any of the two pulleys.

### V-Belt and Rope Drives

V-belt and rope drive arrangements are shown in Fig. 12.5. Figures at (a) and (b) show grooved cross-sections used for wedge section (V-section) and circular section belts or ropes. The most striking feature being that the lower side of belt and rope do not touch bottom of the V-groove, but are wedged between the side faces of the groove. The moment belt or rope touches bottom of the groove, the wedge action become inoperative, and the drive is considered to have failed. Flat rim section may be treated as the limiting case of grooved section with groove angle  $2\alpha$  approaching  $180^\circ$ .

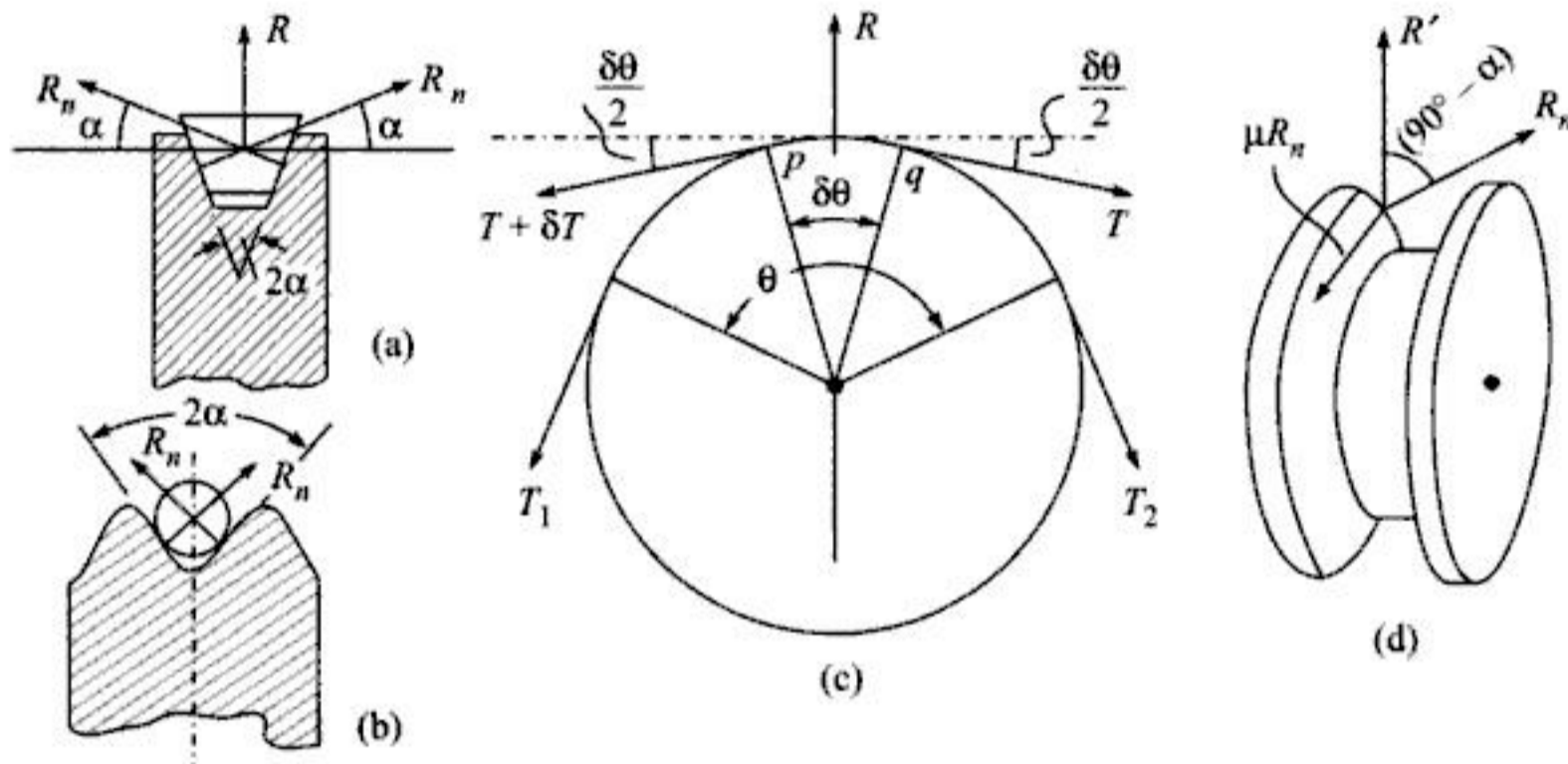


Fig. 12.5 Tension in Belts of V and circular cross-section.

Let the groove angle of the wedge-portion of belt/pulley be  $2\alpha$ . Also, let  $T_1$  and  $T_2$  be the tensions on tight and slack side respectively of the belt and rope.

The resultant reaction  $R$  in this case is contributed by normal reactions  $R_n$  acting on side faces of the V-groove (Figs. 12.5a, b and d) and belt/rope. The frictional forces act tangential to the periphery of circle on the side faces as shown at Fig. 12.5(d). Thus, from Fig. 12.5(a),

$$R = 2R_n \sin \alpha \quad (12.22)$$

And for equilibrium of element  $pq$  from Fig. 12.5(c),

$$(T + \delta T) \cos \frac{\delta\theta}{2} - T \cos \frac{\delta\theta}{2} = 2(\mu R_n) \quad (12.23)$$

and

$$(T + \delta T) \sin \frac{\delta\theta}{2} + T \sin \frac{\delta\theta}{2} = R \quad (12.24)$$



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**Solution:** Given:  $d_1 = 30$  cm;  $N_1 = 200$  r.p.m.;  $\theta = 160^\circ$ ;  $\mu = 0.25$ ; Power = 2.61 kW  
Limiting ratio of tensions

$$T_1/T_2 = e^{0.25(160\pi/180)} = e^{0.698} = 2.01$$

Belt speed

$$V = \frac{\pi \times 30 \times 200}{60 \times 100} = 3.142 \text{ m/s}$$

Thus power transmitted

$$= \frac{T_1(1 - e^{-\mu\theta})V}{1000} \text{ kW}$$

Therefore

$$2.61 = \frac{T_1(1 - 1/2.01) 3.142}{1000}$$

or

$$T_1 = \frac{1000 \times 2.61}{3.142 \times 0.5025} = 1653.1 \text{ N}$$

Therefore

$$T_2 = 1653.1/2.01 = 822.44 \text{ N}$$

Therefore

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1653.1 + 822.44}{2} = 1237.8 \text{ N}$$

Also

$$e^{-\mu\theta} = \frac{1}{2.01} = 0.4975$$

*First Alternative:*

Increasing initial tension by 10% means new value of initial tension  $T'_0 = 1237.8 \times 1.10$

or

$$T_0 = 1361.58 \text{ N}$$

New value of power transmitted

$$= \frac{2T'_0(1 - e^{-\mu\theta})V}{75(1 + e^{-\mu\theta})}$$

or

$$P' = \frac{2 \times 1361.58 (1 - 0.4975) \times 3.142}{1000 (1 + 0.4975)} = 2.871 \text{ kW}$$

Therefore, percentage of increase in power =  $\frac{2.871 - 2.61}{2.61} \times 100 = 10.0\%$

**Ans.**

*Second Alternative:*

Increasing coeff. of friction  $\mu$  by 10% means,

New value of coeff. of friction

$$\mu' = 1.10 \times 0.25 = 0.275$$

Therefore

$$e^{+\mu'\theta} = e^{0.275(160\pi/180)} = 2.155$$

Hence

$$e^{-\mu'\theta} = \frac{1}{2.155} = 0.464$$

Therefore power transmitted

$$P'' = \frac{2T_0(1 - e^{-\mu'\theta})V}{1000(1 + e^{-\mu'\theta})}$$

or

$$P'' = \frac{2 \times 1237.8 (1 - 0.464) \times 3.142}{1000 (1 + 0.464)} = 2.8478$$

**EXAMPLE 12.9** An open belt drive connects two pulleys 1.2 and 0.5 m diameter on parallel shafts 3.6 m apart. The belt has a mass of 0.9 kg/m length, and the maximum tension in it is not to exceed 2.0 kN.

The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Due to belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, and the power lost in friction.  $\mu = 0.3$ .

What is the efficiency of the drive?

(AMIE, Winter 1979)

**Solution:** The angle of lap is given by  $\theta = (\pi - 2\alpha)$

where,

$$\alpha = \sin^{-1} \left[ \frac{(1.2/2) - (0.5/2)}{3.6} \right]$$

$$= \sin^{-1}(0.09722) = 5.579^\circ$$

$$\theta = 180 - (5.579) \cdot 2 = 168.84^\circ$$

Hence

$$T_1/T_2 = e^{\mu\theta} = e^{0.3(168.84\pi/180)}$$

Therefore

$$\frac{T_1}{T_2} = e^{0.884} = 2.42 \text{ Therefore } e^{-\mu\theta} = 0.413$$

Also, belt speed

$$= \frac{\pi d N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566 \text{ m/s}$$

Hence the centrifugal tension,

$$T_c = \frac{w}{g} V^2 = mv^2 = (0.9) \times (12.566)^2 = 142.11 \text{ N}$$

The maximum tension in the belt

$$T_t = T_1 + T_c = 2000 \text{ N}$$

Also

$$\left( \frac{T_t - T_c}{T_s - T_c} \right) = \frac{T_1}{T_2} = e^{\mu\theta} = 2.42$$

Therefore

$$\frac{2000 - 142.11}{T_s - 142.11} = 2.42$$

Therefore

$$T_s = 142.11 + \frac{2000 - 142.11}{2.42} = 909.83 \text{ N}$$

Hence torque on driving pulley =  $(2000 - 909.83) \times 0.6 = 654.1 \text{ N}\cdot\text{m}$

**Ans.**

Torque on the driven pulley,

$$M = (T_t - T_s) \times r'$$

or

$$M = (2000 - 909.83) \times 0.25 = 272.54 \text{ N}\cdot\text{m}$$

**Ans.**

Also, as

$$T_0 = 3000 \text{ N, equating (i) \& (ii),}$$

$$p \left( \frac{1.05}{2} \right) \times 0.15 \times \delta\theta = 3000 \times \delta\theta$$

Therefore

$$p = 38095 \text{ N/m}^2$$

**Ans.**

*Case (b):* When the shaft rotates at 400 r.p.m., the belt is subjected to centrifugal force as it passes over the pulley. This is balanced by a part of initial tension which we call as centrifugal tension. Note that there is no resisting torque and as such the belt length remains the same as in case (a) above.

Thus,

$$\begin{aligned} T_0' + T_c &= 3000 \text{ N} \quad \text{But, } T_c = mv^2 \\ &= (1.5) \left( \frac{\pi \times 1.05 \times 400}{60} \right)^2 = 725.4 \text{ N} \end{aligned}$$

Therefore

$$T_0' = 3000 - 725.4 = 2274.6 \text{ N}$$

Hence by proportion, the pressure between belt and pulley

$$p' = \frac{2274.6}{3000} \times 38095 = 28883.6 \text{ N/m}^2$$

**Ans.**

*Case (c):* With upper shaft rotating at speed same as that in case (b)

$$T_c = 725.4 \text{ N}$$

When the power is being transmitted, let

$$T_t = \text{tension on tight side} \quad \text{and} \quad T_s = \text{tension on slack side}$$

Then

$$\frac{T_1}{T_2} = \frac{T_t - 725.4}{T_s - 725.4} = e^{0.3(\theta)}$$

Again, angle of lap,

$$\theta = \pi - 2\alpha \quad \text{where} \quad \alpha = \sin^{-1} \left( \frac{15 - 1.05}{2 \times 4.8} \right) = 2.687^\circ$$

Therefore

$$\theta = 180 - 2(2.687) = 174.6^\circ;$$

But

$$\frac{T_1}{T_2} = e^{0.3(174.6\pi/180)} = 2.495$$

Therefore

$$\frac{T_t - 725.4}{T_s - 725.4} = 2.495$$

or

$$T_t - 2.495 T_s = 1084.47 \text{ N}$$

Again,

$$T_t + T_s = 2T_0 = 6000 \text{ N}$$

Solving simultaneously,

$$(3.495)T_s = 4915.53$$

Therefore

$$T_s = 1406.45 \text{ N} \quad \text{and} \quad T_t = 4593.55 \text{ N}$$

**Ans.**



## 12.9 CONDITION FOR MAXIMUM POWER TRANSMISSION

A belt drive transmits maximum power when

- the maximum tension on tight side equals maximum permissible value of the tension for the belt, and
- condition of limiting ratio of tension is reached, *i.e.*, when the limiting frictional force acts between belt and pulley surface.

Based on expressions (12.32) and (12.34) for belt drive, power transmitted by a flat belt drive can be expressed as

$$P = kT_1(1 - e^{-\mu\theta})v \quad (12.50)$$

where  $k$  is a constant, value of which depends on units being followed. Thus, let

$$T_1 = \text{Tension on tight side} \quad (12.51)$$

and

$$v = \text{velocity of belt in m/s}$$

Now

$$T_1 = (T_t - T_c) = \left( T_t - \frac{w}{g} v^2 \right)$$

Substituting for  $T_1$  from above in equation (12.50),

$$P = k \left( T_t - \frac{wv^2}{g} \right) (1 - e^{-\mu\theta})v$$

Note that in the above expression, value of maximum permissible (total) tension  $T_t$  on tight side depends on material properties of belt and can, therefore, be treated as constant for a given belt. Again factors  $k$  and  $e^{-\mu\theta}$  are assumed invariant with velocity  $v$ . Writing above expression as

$$P = k(1 - e^{-\mu\theta}) \left( T_t v - \frac{wv^3}{g} \right) \quad (12.52)$$

Differentiating w.r. to velocity for maximum power and equating to zero,

$$\frac{dP}{dv} = 0 = k(1 - e^{-\mu\theta}) \frac{d}{dv} \left[ T_t v - \frac{wv^3}{g} \right]$$

or

$$k(1 - e^{-\mu\theta}) \left[ T_t - \frac{3wv^2}{g} \right] = 0 \quad (12.53)$$

Since  $k \neq 0$  and also as

$$(1 - e^{-\mu\theta}) \neq 0$$

it follows that,

$$T_t - \frac{3wv^2}{g} = 0 \quad (12.54)$$

or

$$T_t = \frac{3wv^2}{g} \equiv 3T_c \quad (12.55)$$

## 12.12 TIMING BELT

Moulded, endless and flat belts with regularly spaced teeth, formed on one side, have a comparatively recent origin. They incorporate the positive action of a chain drive with most of the advantages of the other types of belt drives (Fig. 12.13).

Actual constructional details vary among manufacturers, but the tooth shear strength always exceeds the tensile strength of belt, provided that six or more teeth are in mesh. There is a positive tooth and groove engagement of the belt without slip, creep or speed variations. Using this drive a precise synchronization between the driver and driven elements is possible. This can be used for belt speeds of 4800 m/min and for h.p. from sub-fraction to 60 h.p.

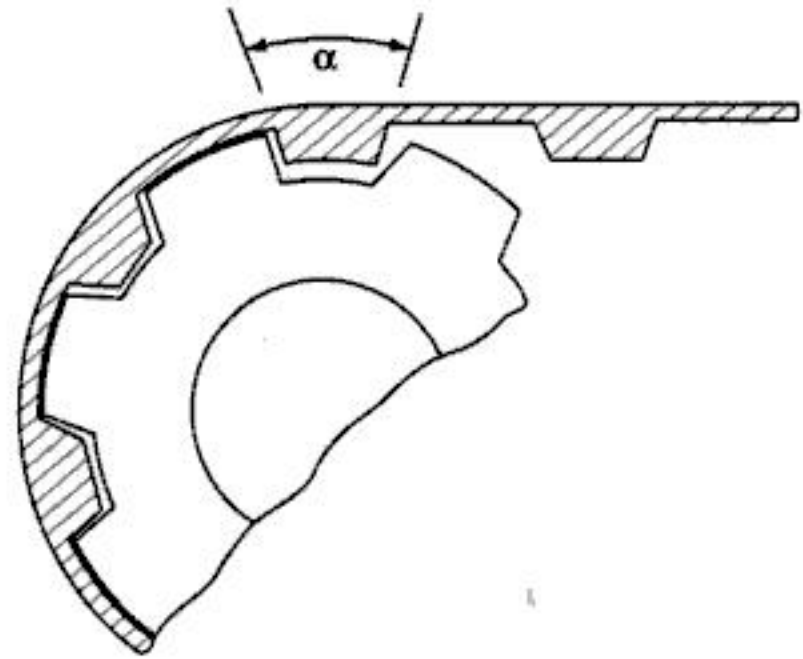


Fig. 12.13

This drive is deliberately designed as a positive drive and will not slip in conditions of sudden overloads or starts. Applications include business machines, sewing machines, timing drives, portable wood working equipment and power transmission units. Besides being positive, the drive is compact, light, quiet, versatile and low in maintenance.

## 12.13 ROPE DRIVE

Expression for limiting ratio of tension, derived in equation (12.27) for circular cross-section and V-shaped groove namely,

$$T_1/T_2 = e^{\mu'\theta}$$

holds good in the case of a rope drive when slip is just to begin. By coiling rope for number of turns  $n$  round a rotating cylinder or capstan, a large value of limiting ratio  $T_1/T_2$  can be obtained. Physically this means that by applying a small force  $T_2$  at one end of the rope, leaving the capstan, comparatively much larger resistance (equal to  $T_1$ ) can be overcome at the other end of rope approaching the capstan.

If  $n$  = number of coils/turns (integral or fraction) of the rope around cylinder,

$T_1$  = tension on tight side, corresponding to limiting ratio of tension,

$T_2$  = tension on slack side at the point of slipping,

Then the total angle of lap,

$$\theta = (2\pi n)\text{rad} \quad (12.69)$$

Thus, assuming  $\mu' = 0.25$ , the limiting ratio of tension is,

$$T_1/T_2 = e^{0.25(2\pi)n} = e^{(1.57)n} \quad (12.70)$$

It is interesting to note the effect of number of turns/coils of rope around capstan on the limiting ratio of tension. This feature is reflected in Table 12.2

Table 12.2

No. of coils $n$ :	0	1	2	3	4	5	6
Limiting ratio of Tension $T_1/T_2$ :	1.0	4.8066	23.1038	111.0522	533.788	2565.734	12332.58

It is seen that limiting ratio of tension increases rapidly as  $n$  increases. Note that above results are true for a cylindrical drum. The ordinary capstan has a concave profile rather than the plane surface of a cylinder. Hence the pressure exerted by capstan surface on the rope is not everywhere in the transverse plane of the rope.

**EXAMPLE 12.11** A leather belt of 10 mm thickness transmits 37.3 kw from a pulley 120 cm in diameter running at 240 r.p.m. The angle of lap is  $170^\circ$  and coeff. of friction is 0.3. The weight of 1 cu cm leather is  $9.316 \times 10^{-3}$  N. What width of belt is required if the stress is limited to 245.2 N per centimetre of width?

**Solution:** Given  $P = 37.3$  kw;  $d_2 = 120$  cm;  $N_2 = 240$  r.p.m.;  $\theta = 170^\circ$ ;  $\mu = 0.3$ ;  
weight density =  $9.316 \times 10^{-3}$  N/cm<sup>3</sup>;  $T_t = 245.2$  N/cm of width

Let  $b$  be the width of belt in cm

Then weight per m length of belt,

$$w = (b \times 1.0) \times 100 \times 9.316 \times 10^{-3} = (0.9316)b \text{ N/m}$$

The maximum tension on tight side,

$$T_t = (245.2)b \text{ N}$$

Peripheral belt speed assuming no slip,

$$v = \frac{\pi \times 1.2 \times 240}{60} = 15.0796 \text{ m/s}$$

Therefore from,  $P = \frac{(T_1 - T_2)V}{1000}$ , we have

$$(T_1 - T_2) = \frac{37.3 \times 1000}{15.0796} = 2473.5 \text{ N} \quad (\text{a})$$

Also, for limiting ratio of tension,

$$T_1/T_2 = e^{0.3 \times (170\pi/180)} = e^{0.890} = 2.435$$

But,  $(T_1 - T_2) \equiv T_1(1 - e^{-\mu\theta})$

Therefore from (a),  $T_1 = \frac{2473.5}{(1 - 0.41067)} = 4197.1 \text{ N}$

Also 
$$T_c = \frac{w}{g} V^2 = \frac{(0.9316)b}{9.81} \times (15.0796)^2 = (21.594)b \text{ N}$$

Hence from

$$T_t = T_1 + T_c$$

We have 
$$(245.2)b = 4197.2 + (21.594)b$$

or 
$$b = 18.77 \text{ cm}$$

**Ans.**

**EXAMPLE 12.12** A belt transmitting power from a motor to a machine weighs 23.54 N/m and the maximum permissible tension for it is 5395 N. The angle of contact of belt with pulley is  $200^\circ$  and coeff. of friction is 0.28. If the belt runs under maximum power conditions, determine the maximum power transmitted and initial tension in the belt so that drive can fulfil the above conditions.

**Solution:** Given:  $w = 23.54 \text{ N/m}$ ;  $T_t = 5395 \text{ N}$ ;  $\theta = 220^\circ$ ;  $\mu = 0.28$ ;  
Limiting ratio of tensions,

$$(T_1/T_2) = e^{\mu\theta} = e^{(0.28)(200\pi/180)} = e^{0.97738} = 2.6575$$

For maximum h.p. transmitted,

$$T_c = \frac{1}{3}(T_t) = \frac{5395}{3} = 1798.33 \text{ N}$$

Hence 
$$T_1 = T_t - T_c = 5395 - 1798.33 = 3596.67 \text{ N}$$

Therefore the tension on slack side,

$$T_2 = T_1/e^{\mu\theta} = 3596.67/2.6575 = 1353.4 \text{ N}$$

Hence initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = 2475 \text{ N}$$

**Ans.**

Max power transmitted

$$P = \frac{(T_1 - T_2)V}{1000}$$

where 
$$V = v_{\text{opt}} = \sqrt{\frac{gT_t}{3w}} = \sqrt{\frac{9.81 \times 5395}{3 \times 23.54}} = 27.38 \text{ m/s}$$

Hence, power transmitted

$$P = \frac{(3596.67 - 1353.4) \times 27.38}{1000} = 61.42 \text{ kw}$$

**Ans.**

**EXAMPLE 12.13** A belt embraces the shorter pulley by an angle of  $165^\circ$  and runs at a speed of 1700 m/min. Dimensions of the belt are: width = 20 cm and thickness = 8 mm. It weighs  $0.00981 \text{ N/cm}^3$ . Determine the maximum power that can be transmitted at the above speed if the



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$$\text{The maximum safe stress in belt} = \frac{7 \times 10^6}{(100)^2} = 700 \text{ N/cm}^2$$

$$\text{Thus } T_t = 700 \times 7.5 = 5250 \text{ N}$$

$$\text{Therefore } T_1 = T_t - T_c = 5250 - 499.65 = 4750.35$$

$$\text{Therefore } T_2 = T_1 / e^{\mu'\theta} = 4750.35 / 4.29 = 1107.3 \text{ N}$$

$$\text{Thus } T_s = T_2 + T_c = 1107.3 + 499.65 = 1606.95$$

$$\text{Therefore, power transmitted} = \frac{(T_t - T_s)V}{1000} \text{ for one belt}$$

$$\text{Thus Power/belt} = \frac{(5250 - 1606.95) \times 23.562}{1000} = 85.837 \text{ kw}$$

$$\text{Therefore total power transmitted by 2 belts} = 85.867 \times 2 = 171.67 \text{ kw} \quad \text{Ans.}$$

Speed at which power transmitted is maximum, is given by

$$V = \sqrt{\frac{gT_t}{3w}} = \sqrt{\frac{T_t}{3m}} = \sqrt{\frac{5250}{3 \times 0.9}} = 44.095 \text{ m/s}$$

The shaft speed in r.p.m. for this speed,

$$N = \frac{44.095}{\pi d} = \frac{44.095 \times 100 \times 60}{\pi \times 30} = 2807 \text{ r.p.m.} \quad \text{Ans.}$$

**EXAMPLE 12.16** An electric motor drives a compressor through V-belt. The following data are known:

	motor pulley	compressor pulley
Pitch diameters cm	27.5	120
Angle of contact radians	2.0	4.28
Coeff. of friction	0.3	0.3
Groove angle	34°	34°
Power transmitted kw	26.82 kw	
Speed r.p.m.	1450	

Each belt weighs 2.943 N/m and the maximum allowable tension is 441.5 N/belt. Calculate the number of belts required for the drive.

**Solution:** The small pulley is the motor pulley. Hence ratio of limiting tensions,

$$(T_1/T_2) = e^{0.3(2)/\sin 17} = e^{2.052} = 7.785$$

$$\text{Therefore } e^{-\mu'\theta} = 0.12845$$

$$\text{Also belt speed } V = \frac{\pi \times 27.5 \times 1450}{60 \times 100} = 20.8785 \text{ m/s}$$

$$\text{Also Power} = \frac{(T_1 - T_2)V \times n}{1000} \text{ kw}$$

$$n(T_1 - T_2) = \frac{1000 \times 26.82}{20.8785} = 1284.6 \text{ N}$$

where  $n$  = number of belts operating the compressor. Maximum tension in each belt,

$$T_t = 441.5 \text{ N}$$

$$T_c = \frac{2.943 \times (20.8785)^2}{9.81} = 130.77 \text{ N}$$

Therefore

$$T_1 = 441.5 - 130.77 = 310.73 \text{ N}$$

Therefore

$$T_2 = T_1 / e^{\mu\theta} = \frac{310.73}{7.785} = 39.91 \text{ N}$$

Substituting for  $(T_1 - T_2)$  above,

$$n = \frac{1284.6}{(310.77 - 39.91)} = 4.74 \approx 5$$

**Ans.**

**EXAMPLE 12.17** The maximum power that can be transmitted by a belt is 55.93 kw. The belt is 25 cm wide and 10.5 mm in thickness and weights 0.00883 N/cm<sup>3</sup>. If the ratio of tension is 2, find value of maximum stress induced in the belt at this condition, for the belt.

**Solution:** Given belt cross-sectional area =  $a = 25 \times 1.05 = 26.25 \text{ cm}^2$

And

$$T_1/T_2 = e^{\mu\theta} = 2.0$$

Therefore, weight of belt per metre length =  $(26.25 \times 100 \times 0.00883) \text{ N}$

$$\text{or } w = \frac{26.25 \times 100 \times 0.00883}{1} = 23.18 \text{ N/m}$$

For maximum power,

$$T_c = \frac{1}{3} T_t$$

Also

$$v_{\text{opt}} = \sqrt{\frac{gT_t}{3w}} = \sqrt{\frac{9.81 \times T_t}{3 \times 23.18}} = 0.376 \sqrt{T_t}$$

Again, from

$$\text{Power} = \frac{T_t(1 - e^{-\mu\theta})v}{1000} \text{ kw}$$

or

$$\text{Power} = \frac{(T_t - T_c)(1 - e^{-\mu\theta})v}{1000} = \frac{(2/3)T_t(1 - 1/2)v_{\text{opt}}}{1000}$$

Substituting for  $V_{\text{opt}}$ ,

$$55.93 = \frac{\left(\frac{2}{3}\right)T_t\left(1 - \frac{1}{2}\right)0.376\sqrt{T_t}}{1000}$$



Therefore 
$$(T_t)^{\frac{3}{2}} = \frac{55.93 \times 1000}{\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)(0.376)} = 446250 \text{ Thus, } T_t = 5840 \text{ N}$$

Thus if  $\sigma$  be the permissible stress in the belt, then

$$\sigma(b \times t) = T_t$$

Therefore 
$$\sigma = \frac{5840}{(25 \times 1.05)} = 222.5 \text{ N/cm}^2 \quad \text{Ans.}$$

**EXAMPLE 12.18** A rope, 4 cm in diameter, drives a grooved pulley at a speed of 20 m/s. The angle lap of rope on the pulley is  $200^\circ$ , the angle of groove for the pulley is  $60^\circ$ ,  $\mu = 0.3$  and weight per m length of rope = 8.83 N. If permissible tension in the rope is  $147.15 \text{ N/cm}^2$ , calculate power that can be transmitted at this speed.

**Solution:** Given  $\sigma_{per} = 147.15 \text{ N/cm}^2$ ;  $d = \text{rope dia.} = 4 \text{ cm}$ ;

$$v = 20 \text{ m/s}; \theta = 200^\circ; \mu = 0.3; \alpha = \frac{60}{2} = 30^\circ; w = 8.83 \text{ N/m}$$

We have, 
$$T_t = \sigma_{per} \times \frac{\pi}{4} d^2 = 147.15 \times \frac{\pi}{4} (4)^2 = 1849.14$$

Also, 
$$T_c = \frac{8.83}{9.81} \times (20)^2 = 360.0 \text{ N}$$

Therefore 
$$T_1 = 1849.14 - 360 = 1489.14 \text{ N}$$

Also 
$$\mu'\theta = \frac{0.3}{\sin 30} \times \frac{200 \pi}{180} = 2.094$$

Therefore 
$$T_1/T_2 = e^{2.094} = 8.12$$

Therefore 
$$e^{-\mu'\theta} = \frac{1}{8.12} = 0.12315$$

Therefore power transmitted = 
$$\frac{T_1(1 - e^{-\mu'\theta})v}{1000}$$

$$= \frac{1489.14(1 - 0.12315)20}{1000} = 26.11 \text{ kw} \quad \text{Ans.}$$

**EXAMPLE 12.19** A pulley used to transmit power by means of ropes has a diameter of 3.6 m and has 15 grooves of  $45^\circ$  angle. The angle of contact is  $170^\circ$  and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 941.8 N and the weight of the rope is 14.72 N per m length. What is the speed of the pulley in r.p.m. and the power transmitted if the condition of maximum power prevail?

(AMIE, Summer 1983)



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# INDEX

- Absolute motion, [28](#)
- Absorption dynamometer, 604, 605
- Acceleration, 87
  - absolute, 95
  - angular, 87–90, 92–99, 103, 106, 406, 410
  - centripetal, 89
  - Coriolis, 94, 103
  - equation, 91
  - gyroscopic, 409, 410
  - image, 90, 91
  - normal, 88
  - of piston, 96, 112, 115
    - analytical, 115–117
  - polygon, 94, 95, 103
  - relative, 89–92
  - rigid link, 88
  - tangential, 88, 90, 92
- Acceleration analysis, [61](#), 89–94, 135
  - analytical approach, 773
  - cam and follower, 287–293
  - complex mechanism, 118
  - of direct contact mechanisms, 232, 300
  - four-bar linkage, 99
  - Raven's method, 135
  - return mechanism, 103–104
  - slider-crank mechanism, 101, 136
  - slotted lever, 104
- Accuracy points, 205–208
- Ackerman steering gear, [32](#), 181–183
- Addendum of tooth, 306
  - modification, 328
  - standard, 327
- Aft, 428
- Amplitude of vibration, 920
  - damped vibration, 868
  - forced vibration, 851, 877
- Angle of approach
  - friction, 446
  - heel, 424
  - lap, [517](#), [518](#), 519, 520
  - obliquity, 308
  - repose, 446
- Approximate straight line motion mechanisms, 162–163
  - Grasshopper, 168
  - Robert's, 171
  - Tchebicheff, 169
- Arnhold-Kennedy's theorem, [82](#)
- Arc of action/contact, 320
- Atlas, Hrones-Nelson, 228, 230
- Automotive differential, [362](#)
- Axial pitch, 356
  
- Back cone bevel gear, [360](#)
- Backlash, 307
- Balancing by four-observations, 796
- Balancing, definition, 767
  - dynamic, 768–769
  - partial, 804
  - static, 768
- Balancing of cranks, 806
  - locomotives, 807–812
  - multicylinder in-line engines, 822–823
  - reciprocating masses, 801
  - revolving masses, 772, 775
  - rotors, 789
  - secondary forces, 839
  - V-engines, 841
- Ball bearing, 507
- Band brake, 583
- Band and block brake, 587

- Band brake, differential, 584
  - simple, 584
- Base circle of cam, 247, 270
- Base circle of involute gear, [316](#), [317](#)
- Beam engine mechanism, [32](#), [33](#)
- Bearing
  - ball, 507
  - horse shoe-shaped, 470, 471
  - journal, 504, 505
  - Michel thrust, 505
  - Needle roller, 507–508
  - roller, 507–508
- Belt drive
  - catenary effect, 538, 540
  - centrifugal tension, 525
  - creep, 539
  - flat, 519
  - initial tension, 526
  - law of (belting), 519–522
  - length of, [518–519](#)
  - limiting ratio of tensions, 519–522
  - material, 533
  - maximum HP transmitted, 522
  - maximum tension, 526, [532](#)
  - power transmitted, 526, [535](#)
  - slip, [515–516](#)
  - timer belt, 553
  - V-belt, 455, [521](#)
  - velocity ratio, [513–515](#)
- Belt transmission dynamometer, 609
- Bevel gears, 303, [362](#), [372](#)
  - miter, [362](#)
  - pitch angle, [362](#)
  - pitch cone, [360](#), [362](#)
  - pitch surface, [360](#), [362](#)
  - shaft angle, [361](#), [362](#)
  - tooth thickness, 307
  - Tredgold's approximation, [360](#)
  - Zerol, 304, [362](#)
- Bevis Gibson torsion dynamometer, 604, 612
- Binary links, [22](#), [54](#)
  - minimum number in chain, [53](#)
- Block brake, 560–561
- Boundary friction, 444, 501
- Bow of a ship, 430
- Brakes, band, 559, 583
  - band and block, 587
  - block, 560, 561
  - external, 559, 566
  - internal, 573
  - long shoe, 566–570
  - pivoted shoe, 560
  - self energization, 561–563
    - self locking, 565
    - short shoe, 561–563
- Braking of vehicles, 596
- Bush roller chain, 553, 554
- Cams, advantages, [241](#)
- Cams, analysis, 287
  - analytical design, [280](#)
  - angle of action, [244](#), 290
  - angle of return/descent, [265](#), [271](#), 274
  - angle of rise/assent/outstroke, [265](#), [271](#), 274
  - angle of dwell, [271](#), [272](#)
  - base circle, 273, 274
  - circular arc, 287
  - cylindrical, [243](#)
  - disc, [242](#), [244](#), 247
  - displacement diagram, 248, 249, [251](#), [253](#)
  - flat, 245
  - follower, oscillating, [243](#), 284, 285
  - jerk, 287
  - layout of, 270
  - master, 287
  - pitch circle, 248
  - pitch curve, 247
  - pitch point, 248
  - plate cam, [242](#), [253](#)
  - pressure angle, [244](#), 260
  - prime circle, 248
  - profile, 248, [263](#), [266](#)
  - radial, [242](#), [243](#)
  - specified contours, 270, 287
  - specified follower motion, [269–287](#)
  - tangent, 287, 292
  - types, [244](#)
  - undercutting, [269](#)
- Centre of percussion, 636
- Centre distance
  - spiral gears, 347, 350
  - worm gears, 355
- Chain
  - constrained, [40](#)
  - double slider, [35](#)
  - drive, 553–554
  - four-bar, [29](#)
  - kinematic, [21](#)
  - pitch, 553–554
  - pitch quadric cycle, [28](#)
  - roller, 554
  - silent, 555
  - slider-crank, [33](#)
  - sprocket, 554
- Chebyshev spacing, 206, 218

- Circular frequency, 893  
 Circular pitch, 306  
 Clearance, 307  
 Closed chain, 21  
 Closed pair, 27, 242  
 Closure, loop equation, 131  
 Clutches, cone, 475  
   disc, 479  
   multi-disc, 481  
   single plate, 479  
 Coefficient of rolling friction, 461–462  
   sliding friction, 443  
 Coefficient of energy fluctuation, 680  
 Coefficient insensitiveness, 755  
 Coefficient of speed fluctuation, 680  
 Cognate linkages, 232  
 Collar friction, 463  
 Collars and pivots, 463  
 Complex algebra, 135  
 Complex mechanisms, 118  
 Complex polar notations, 9  
 Compound gear train, 368  
 Conical clutch, 475  
 Conjugate tooth profiles, 312–314  
 Constrained mechanism, 38  
 Constraint, 38–40  
   motion, 38, 59  
 Contact, arc of, 320  
   path of, 318  
 Contact ratio, 307, 321  
 Coriolis acceleration, 103, 104, 126  
 Correction couple, 640, 643  
 Coupler, 29  
 Coupler curves, 228–230  
   equation of, 229  
   for 5-bar linkage, 235–236  
 Couple, gyroscopic, 410, 411  
   reaction, 411  
   representation vectorial, 405  
   swaying, 809  
 Cradle balancing machine, 792  
 Cramer's rule, 878  
 Crank effort, 657, 675  
 Cranks, direct and reverse, 836  
 Crank-pin effort, 657  
 Crank-rocker mechanisms, 30, 31, 32  
 Crank-shaft balancing, 806  
 Crank-slotted lever mechanism, 33  
 Critical damping coefficient, 869, 870, 876  
 Critical speed in rope dynamometer, 604, 606  
 Crosby indicator, 172  
 Crossed belt, 514, 518  
 Curvilinear motion of translation, 3  
 Cycloidal follower motion, 258, 268  
 Cycloidal gear teeth, 314  
 Cycloidal and involute tooth comparison, 342  
 Cylindrical cams, 243, 244  
   pairs, 25  
  
 D'Alembert's principle, 618–620  
 Dalby's method, 779–781  
 Damping factor/ratio coefficient, 869  
 Damped vibrations, 868  
   critical, 870  
   overdamped, 870  
   underdamped, 871  
 Davis steering gear, 178  
 Dedendum, 306  
 Degrees of freedom, 4, 23, 26, 38, 45  
   of chains, 38, 39  
   effect of  
     multiple joint, 41  
     spring connection, 41  
     even/odd number of links, 50  
   of mechanisms, 38, 39  
   of pairs, 25, 26, 45  
   redundant, 43  
 Diametral pitch, 306  
 Diagrams, free body, 620  
   schematic, 21  
 Differential mechanism, 398  
 Dimensional synthesis, 204, 208  
 Direct contact mechanism, 48  
 Direct and reverse cranks, 836  
 Displacement, finite and infinitesimal, 79  
 Displacement of  
   particle, 11  
   rigid body, 11, 12  
 Displacement, relative, 12, 13  
 Displacement-time curve  
   in follower motion, 250–254  
   in quick return motion mechanism, 158, 159  
   in slider crank mechanism, 654, 659, 663  
 Displacement vector, 11  
 Dobbie-McInnes indicator mechanism, 175  
 Double block brake, 563  
 Double-crank mechanism, 29  
 Double helical gear, 305  
 Double Hooke's joint, 192, 193  
 Double lever mechanism, 29  
 Drag link mechanism, 29  
 Drive, belt, 518, 529, 531, 532  
   chain, 513

- cam, [242](#)
- clutch, [475](#)
- friction, [474](#), [511](#)
- rope, [300](#), [513](#), [521](#), [526](#)
- Dry friction, [444](#)
- Dunkerley's method, [911](#)
- Dwell period in cams, [245](#), [246](#)
- Dynamic balancing, [768–769](#)
- Dynamic machines, [790](#)
- Dynamics, defined, [623](#), [679](#), [681](#)
- Dynamically equivalent two-mass system, [634](#), [639](#)
- Dynamometers, [604](#)
  - absorption, [604–605](#)
  - belt transmission, [609](#)
  - epicyclic gear train, [608](#)
  - Prony brake, [605](#)
  - rope brake, [606](#)
  - torsion, [611](#)
  - transmission, [604](#), [608](#)
- Effect of friction on governors, [717](#), [724](#), [754](#)
- Effect, gyroscopic, [404](#), [405](#), [406](#), [414](#), [427](#)
- Effect of partial balancing of locomotives, [807–810](#)
- Effective force, [619](#), [644](#)
  - tension, [522](#)
- Efficiency of inclined plane
  - helical gears, [344](#), [350](#)
  - spiral gears, [347](#), [350](#)
  - worm gears, [355](#)
- Effort at crank, [657](#)
- Effort of governor, [752](#)
  - piston, [657](#)
- Element, definition of, [20](#)
- Elliptical trammel, [35](#)
- Energy fluctuation, flywheel, [680](#), [681](#), [682](#), [684](#)
- Enumeration of chains, [53–57](#)
- Epicyclic dynamometer, [608](#)
  - gear train, [366](#)
  - torque and tooth loads, [384](#)
- Epicycloid, [342](#)
- Equilibrium, dynamic, [415](#)
- Equilibrium, static, [618–619](#)
- Equivalent linkage, [48](#)
- Equivalent two-mass system dynamic, [634–635](#)
- Error in function generator structural, [204–205](#)
- Euler's theorem, [4](#)
- Exact straight line motion
  - condition for, [163](#)
  - Hart, [165](#)
  - Peaucellier, [164](#)
  - Scott-Russel, [166](#)
- Face of gear tooth, [307](#)
  - width of helical gear, [345](#)
- Face cam, [243](#)
- Ferguson's paradox, [386](#)
- Field balancing, [794](#)
- Fillet radius, [307](#)
- Film lubrication, [502](#), [504](#)
- Firing order, [827](#)
- Flat face follower, [280](#)
- Flat pair, [24](#)
- Fluctuation of crank shaft speed, [678](#)
- Fluctuation of energy, [680](#)
- Flywheel, [682](#)
  - energy, [683](#), [684](#)
  - speed, [683](#), [684](#)
- Foettinger torsion dynamometer, [613](#)
- Follower, classification, [244–246](#)
- Follower, flat faced, [247](#), [273](#), [283](#)
  - cycloidal, [248](#), [258](#)
  - knife edge, [246](#)
  - lift, [248](#)
  - modified constant velocity, [249](#)
  - motions types, [244](#)
  - mushroom, [246](#)
  - offset, [244](#)
  - oscillating, [244](#)
  - parabolic, [248](#), [254](#)
  - radial, [244](#)
  - roller, [246](#)
  - simple harmonic, [250–251](#)
  - spherically seated, [246](#)
  - stroke, [248](#)
  - uniform velocity, [249](#), [250](#)
- Force analysis
  - dynamic, [646](#)
  - static, [62](#)
- Force analysis of
  - four-bar mechanism slider, [648](#)
  - crank mechanism, [659](#), [663](#)
- Force closure in cams, [242](#)
- Force, controlling
  - effective, [644](#)
  - inertia, [644](#), [646](#)
- Fore of ship, [428](#)
  - acceleration polygon, [94](#)
  - instantaneous centre, [78–79](#)
  - inversions, [29](#)
  - mechanical advantage, [36](#)
  - pressure angle, [37](#)
  - transmission angle, [36](#)
- Free body diagram, [620](#)
- Freedom, degrees of, [4](#)

- Frequency of vibration  
 cyclic, 677  
 damped, 768  
 fundamental, 907, 910  
 natural, 906
- Freundenstein's equation, 217
- Friction, 443  
 angle, 446  
 axis, 495–496, 499  
 boundary, 444, 501  
 circle, 495  
 clutches, 474  
 coefficient, 445  
 dry, 443, 444  
 film, 444, 502  
 greasy, 444, 501  
 rolling, 444, 461  
 skin, 444, 501  
 solid, 443, 444  
 moment, 464, 496  
 viscous, 444, 502
- Friction in governors, 603, 664  
 pivot and collar, 463  
 rolling, 444, 461
- Full depth tooth, 327
- Function generation, 217, 219  
 algebraic method, 219, 373  
 graphical method, 208
- Function generator, 204, 205
- Gear, definition, 300
- Gear ratio, 307
- Gears, 301  
 bevel, 304  
 classification, 301  
 helical, 302, 344  
 Herringbone, 303  
 hypoid, 304  
 rack and pinion, 303  
 spur, 301, 302, 305  
 spiral, 304, 305, 347  
 worm, 305
- Gear teeth  
 base circle, 316  
 conjugate, 312  
 cycloidal profile, 314  
 face, 307  
 flank, 307  
 involute, 315  
 module, 306, 327  
 proportions, 327
- Gear trains, 366  
 algebraic method, 373  
 bevel gear differential, 397  
 compound, 368  
 epicyclic, 371, 372  
 formula (algebraic) method, 376  
 ordinary (simple), 366  
 reverted, 370  
 tabulation method, 379  
 train value, 368
- Globular pair, 24
- Governors, 705  
 centrifugal, 707  
 controlling force, 707  
 definitions, 708  
 effort, 752  
 emergency, 707  
 friction, 717, 754  
 gravity control, 708  
 Hartnell, 729  
 Hartung, 744  
 isochronous, 757  
 inertia, 708, 747  
 insensitiveness, 754–757  
 pendulum, 709  
 pickering, 746  
 porter, 713  
 power, 752  
 Proell, 724  
 quality, 748  
 sensitiveness, 752  
 spring controlled, 729  
 stability, 749  
 watt, 709  
 Wilson Hartnell, 729, 732
- Graphical cam design, 275
- Graphical differentiation, 64–66
- Grashof's chain, 29
- Grashoff's law, 29
- Grubler's criterion inconsistencies, 43
- Gyroscope, definition, 404
- Gyroscopic acceleration, 407
- Gyroscopic action, 439
- Gyroscopic action in grinding mill, 437
- Gyroscopic couple, 410
- Gyroscopic effects on  
 air planes, 414  
 four wheeler, 415  
 naval ship, 427  
 two wheeler, 423
- Gyroscopic ship stabilisation, 432

- Hammer blow, 807  
 Hand pump mechanism, 34  
 Hart mechanism, 165  
 Harmonic motion, simple, 625  
 Hartnell governor, 729  
 Helical gear, 302, 344  
     normal circular pitch, 346–347  
 Helix angle, 303  
 Herringbone gear, 303  
 Higher pair, 25, 28, 241  
     effect on d.o.f., 40  
 Hooke's joint, 185  
     double, 192  
 HP transmitted by belt and rope, 522  
 Hrones and Nelson atlas, 228  
 Hunting governor, 752  
 Hypoide gearing, 304
- Idler pulleys, 540  
 Images, velocity and acceleration, 68, 69, 88, 89  
 Inclined planes, 446  
     efficiency of, 447–448  
     friction of, 446–448  
 Indicator diagrams for engines, 171, 675  
 Inertia effects of  
     crank and connecting rod, 802  
     reciprocating masses, 801  
 Inertia force, 646  
     determination, 639  
 Inertia torque, 619, 620, 649  
 Insensitiveness, governor, 754–757  
 Instantaneous centres, 78  
     Aronhold Kennedy's theorem, 82  
     locations of, 80, 83  
     method for velocity, 85  
     notation, 81  
     number of, 82  
     properties, 79  
 Interference in involute teeth, 324  
     methods of elimination, 343  
     minimum number of teeth to avoid, 328, 336  
 Internally expanding shoe brakes, 573  
 Inversion, 28  
     of four-bar chain, 212  
     of double-slider chain, 35  
     of slider-crank chain, 33  
 Inversion, importance of, 28  
     properties of, 28  
 Inverted tooth chain, 555  
 Involute teeth, 312–315  
 Isochronous governors, 757
- Jaw clutches, positive, 475  
 Jerk, 248, 253, 254, 255  
 Jockey pulley's, 540  
 Joints, simple, 41  
     multiple, 41  
 Journal bearing, 503–504  
     friction in, 495–496  
     pressure distribution, 503  
     Tower's experiment, 502
- Kennedy's theorem, 82  
 Kinematics, 19  
 Kinematic  
     chain, 21  
     enumeration, 53  
     pairs, 23  
     synthesis, 203  
 Klein's construction, 109  
 Kutzbach's criterion, 40
- Law of gearing, 310  
 Laws of motion, 618  
 Laws of solid friction, 444  
 Length of arc of contacts, 320  
 Length of path of contact, 318  
     approach, 318  
     recess, 318  
 Limiting angle of friction, 445, 446  
 Limiting coefficient of friction, 445  
 Linear motion lower pairs, 40  
 Links, classification, 22  
     conventional representation, 22  
 Linkage, 28  
 Locomotives,  
     effects in, 807  
     partial balancing of, 807  
 Logarithmic decrement, 872  
 Long shoe brakes, 566  
     shorter method, 570  
 Longitudinal vibrations, 905  
 Loop closure equation, 39, 131  
 Lower pairs, 25  
     equivalent, 45  
 Lubrication, 444, 502, 504  
     film, 502  
     hydrostatic, 506  
     of journal bearing, 503–504  
     of plane surfaces, 500–501  
     viscous, 502



- Machine, definition, [19](#), [27](#)  
 magnification factor, [880](#)
- Manipulators, [58](#)
- Master cam, [287](#)
- Materials for belt and rope, [552](#)
- Mechanical advantage, [36](#), [77](#)
- Mechanism, definition, [22](#), [27](#)
- Michel thrust bearing, [505](#)
- Minimum number of pinion teeth, [336](#)
- Miter gear, [362](#)
- Mobility, [38](#)
- Mobility equation, [40](#)
- Mode of vibration, [907](#)
- Module, standard values, [306](#), [308](#)
- Motor car steering gear, [177](#)  
 Ackerman, [181](#)  
 Davis, [178](#)  
 condition of correct steering, [177](#)
- Motion generation, [204](#), [205](#), [208](#)
- Motion of the follower, [244](#), [245](#)
- Motion of translation, [3](#)  
 curvilinear, [3](#)  
 rectilinear, [3](#)
- Motion transfer link, [52](#)
- Motion, simple harmonic, [625](#)
- Movability, [40](#), [50](#)
- Multile joint, [41](#)
- Notation, static forces, [621](#)
- Number of instantaneous centres, [79](#)
- Number synthesis, [49](#), [203](#)
- Offset follower motion, [244](#), [263](#)
- Offset slider-crank mechanism, [158](#)  
 condition of rotatability of crank, [160](#)
- Oldham's coupling, [36](#)
- Open chain, [21](#)
- Oscillating cylinder engine mechanism, [33](#)
- Oscillating follower with flat face, [245](#)
- Oscillating follower with roller, [245](#), [284](#)
- Overdamped vibration, [870](#)
- Pairs, classification, [23](#)
- Pairing elements, [20](#)
- Pairs,  
 cylindrical, [23](#)  
 definition, [20](#)  
 flat, [24](#)  
 globular, [24](#)  
 higher, [25](#)  
 lower, [25](#)  
 prismatic, [23](#)  
 redundant, [43](#)  
 revolute, [23](#)  
 rolling, [25](#), [45](#)  
 screw, [23](#)  
 turning, [23](#)
- Pantograph, [32](#), [160–162](#)
- Parabolic follower motion, [254](#)
- Parallel, helical gears, [302](#), [344](#)
- Partial balancing of locomotives, [807–812](#)
- Particle, motion of, [3](#)
- Path generation, [204–205](#), [230–241](#)
- Peaucellier mechanism, [164](#)
- Pendulum, compound, [628](#), [631](#), [636](#)  
 simple, [626](#)  
 torsional, [629](#), [633](#)
- Percussion, centre, [636](#)
- Periodic time, [850](#)
- Phase angle, [793](#), [809](#), [853](#)
- Pickering governor, [746](#)
- Pinion, [303](#), [306](#)
- Piston effort, [657](#)
- Pitch circle, [248](#), [302](#)  
 cone, [360–363](#)  
 line, [336](#)
- Pitching motion of ships, [428](#)
- Pivot bearing, friction, [463](#)
- Plane of gyrocouple, [405](#)  
 precession, [406](#)  
 spin, [406](#)
- Planar motion, [24](#)
- Planar pair, [24](#)
- Planetary gear trains, [371](#)
- Plate clutch, [479](#)
- Poles, [208](#), [212](#)
- Port, [428](#)
- Position vector, [10](#)
- Precessional motion, [406](#)
- Precision points, [205](#)
- Pressure angle of,  
 cams, [244](#), [260–263](#)  
 four-bar mechanism, [37](#)  
 spur gears, [305](#)
- Primary and secondary unbalanced forces, [801–833](#)
- Primary balancing of multicylinder in-line engines,  
[822](#), [824](#)
- Prismatic pairs, [23](#)
- Properties of I.C., [79–80](#)
- Pulleys, idler and Jockey, [540](#)
- Pure rolling, [45](#), [177](#), [300](#)

- Quadric cycle chain, 28
- Quaternary link, 22
- Quick return mechanism, 31
  - drag-link type, 31
  - offset slider crank type, 158
  - slotted lever, 33, 34
  - Whit-Worth mechanism, 33, 106
  
- Rack, 326
- Radial flat faced cam follower, 244
- Radial roller cam follower, 244
- Raven's method, 135
- Rayleigh's method, 912
- Rectilinear motion, 2
- Rectilinear translation, 3
- Relative displacement, 12
- Relative velocity, 67, 68
  - acceleration, 88
- Relative poles, 208, 212
- Reverted gear trains, 370
- Revolute pair, 23
- Rigid body displacement, 11
  - guidance, 205
  - motion, 2
- Robert's Chebychev theorem, 232
- Rolling contact, 300, 506, 508
- Rolling friction, 461
- Rolling of ships, 428
- Rolling contact, bearing, 506, 508
- Rotation, motion, 4
- Roto-centres, 212
- Rubbing velocity, 75
  
- Scott Russel's mechanism, 166
- Screw friction, 450
  - jack, 450
  - pair, 23
- Secondary force balancing of multi-cylinder in-line engines, 823
- Secondary couples, 829
- Self actuating brake, 562
- Self-closed pairs, 27
- Self energization, 562
- Self locking, 454
- Sensitiveness of governors, 752
- Shaking force, 650
- Simple harmonic motion, 625
- Simple pendulum, 626
- Simplex indicator, 171
- Singular link, 21
  
- Skeleton diagram, 20
- Slider-crank mechanism, 34, 58, 132
  - acceleration, 654
  - displacement, 655
  - force analysis, 646, 659
  - inversions, 655
  - velocity, 655
- Sliding pair, 23
- Slotted lever quick return motion mechanism, 33, 34
- Spacing of accuracy points, 205
- Spatial mechanism, 57
- Specified cam profiles, 287
  - follower motion, 248, 263
- Speed regulation, 706
- Spherical pair, 24
- Spiral bevel gear, 304
- Spring constant, 891, 892
- Sprocket, 553
- Spur gears, 302, 303, 305
  - addendum, 306
  - arc, of contact, 307
  - backlash, 307
  - base circle, 315
  - circular pitch, 306
  - clearance, 307
  - conjugate action, 310
  - contact ratio, 307
  - dedendum, 306
  - face width, 307
  - fillet radius, 307
  - full-depth tooth, 327
  - interchangeable, 327
  - interference, 324
  - length of path of contact, 318
  - minimum number of teeth for pinion, 328, 336
  - pitch circle, 306
  - pitch circle diameter, 306
  - pitch line, 336
  - pressure angle, 308
  - stub tooth, 327
  - tooth face, 307
  - tooth proportions 327
  - tooth thickness, 307
  - undercutting, 324
  - working depth, 307
- Stability of
  - 2-wheeler, 423
  - 4-wheeler, 415
  - governors, 748
- Stabilization, gyroscopic, 432
- Star board, 427
- Static balance, 768
- Static balancing machines, 790

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