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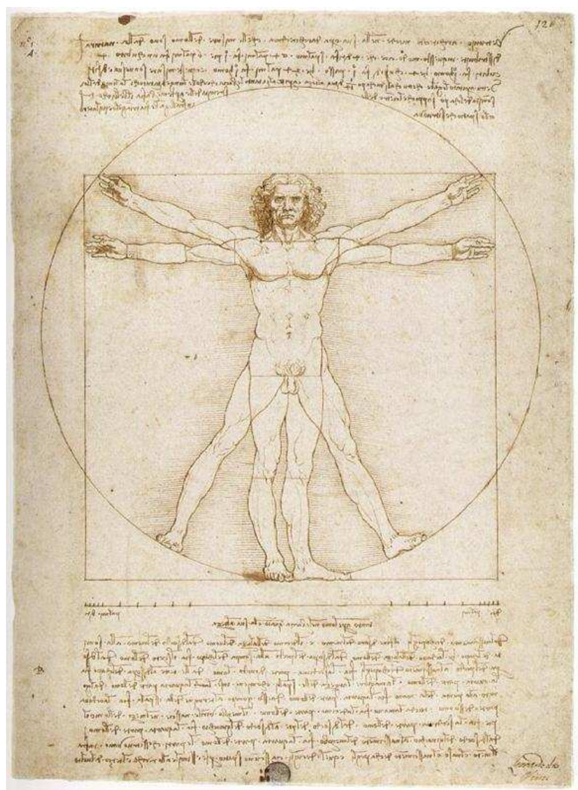
Lecture Notes in:

STRUCTURAL CONCEPTS AND SYSTEMS

FOR ARCHITECTS



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Chapter 1

INTRODUCTION

1.1 Science and Technology

1 "There is a fundamental difference between science and and technology. Engineering or technology is the making of things that did not previously exist, whereas science is the discovering of things that have long existed. Technological results are forms that exist only because people want to make them, whereas scientific results are informations of what exists independently of human intentions. Technology deals with the artificial, science with the natural." (Billington 1985)

1.2 Structural Engineering

2 Structural engineers are responsible for the detailed analysis and design of:

Architectural structures: Buildings, houses, factories. They must work in close cooperation with an architect who will ultimately be responsible for the design.

Civil Infrastructures: Bridges, dams, pipelines, offshore structures. They work with transportation, hydraulic, nuclear and other engineers. For those structures they play the leading role.

Aerospace, Mechanical, Naval structures: aeroplanes, spacecrafts, cars, ships, submarines to ensure the structural safety of those important structures.

1.3 Structures and their Surroundings

3 Structural design is affected by various environmental constraints:

1. Major movements: For example, elevator shafts are usually shear walls good at resisting lateral load (wind, earthquake).
2. Sound and structure interact:
   - A **dome** roof will concentrate the sound
   - A **dish** roof will diffuse the sound

1.6 Architectural Design

12 Architectural design must respect various constraints:

Functionality: Influence of the adopted structure on the purposes for which the structure was erected.

Aesthetics: The architect often imposes his aesthetic concerns on the engineer. This in turn can place severe limitations on the structural system.

Economy: It should be kept in mind that the two largest components of a structure are labors and materials. Design cost is comparatively negligible.

13 Buildings may have different functions:

Residential: housing, which includes low-rise (up tp 2-3 floors), mid-rise (up to 6-8 floors) and high rise buildings.

Commercial: Offices, retail stores, shopping centers, hotels, restaurants.

Industrial: warehouses, manufacturing.

Institutional: Schools, hospitals, prisons, chruch, government buildings.

Special: Towers, stadium, parking, airport, etc.

1.7 Structural Analysis

14 Given an **existing** structure subjected to a certain load determine internal forces (axial, shear, flexural, torsional; or stresses), deflections, and verify that no unstable failure can occur.

15 Thus the basic structural requirements are:

Strength: stresses should not exceed critical values: $\sigma < \sigma_f$

Stiffness: deflections should be controlled: $\Delta < \Delta_{max}$

Stability: buckling or cracking should also be prevented

1.8 Structural Design

16 Given a set of forces, **dimension** the structural element.

Steel/wood Structures Select appropriate section.

Reinforced Concrete: Determine dimensions of the element and internal reinforcement (number and sizes of reinforcing bars).

17 For **new structures**, **iterative** process between analysis and design. A preliminary design is made using **rules of thumbs** (best known to Engineers with design experience) and analyzed. Following design, we check for

Tension & Compression Structures: only, no shear, flexure, or torsion. Those are the **most efficient** types of structures.

Cable (tension only): The high strength of steel cables, combined with the efficiency of simple tension, makes cables ideal structural elements to span large distances such as bridges, and dish roofs, Fig. 1.2. A cable structure develops its load carrying

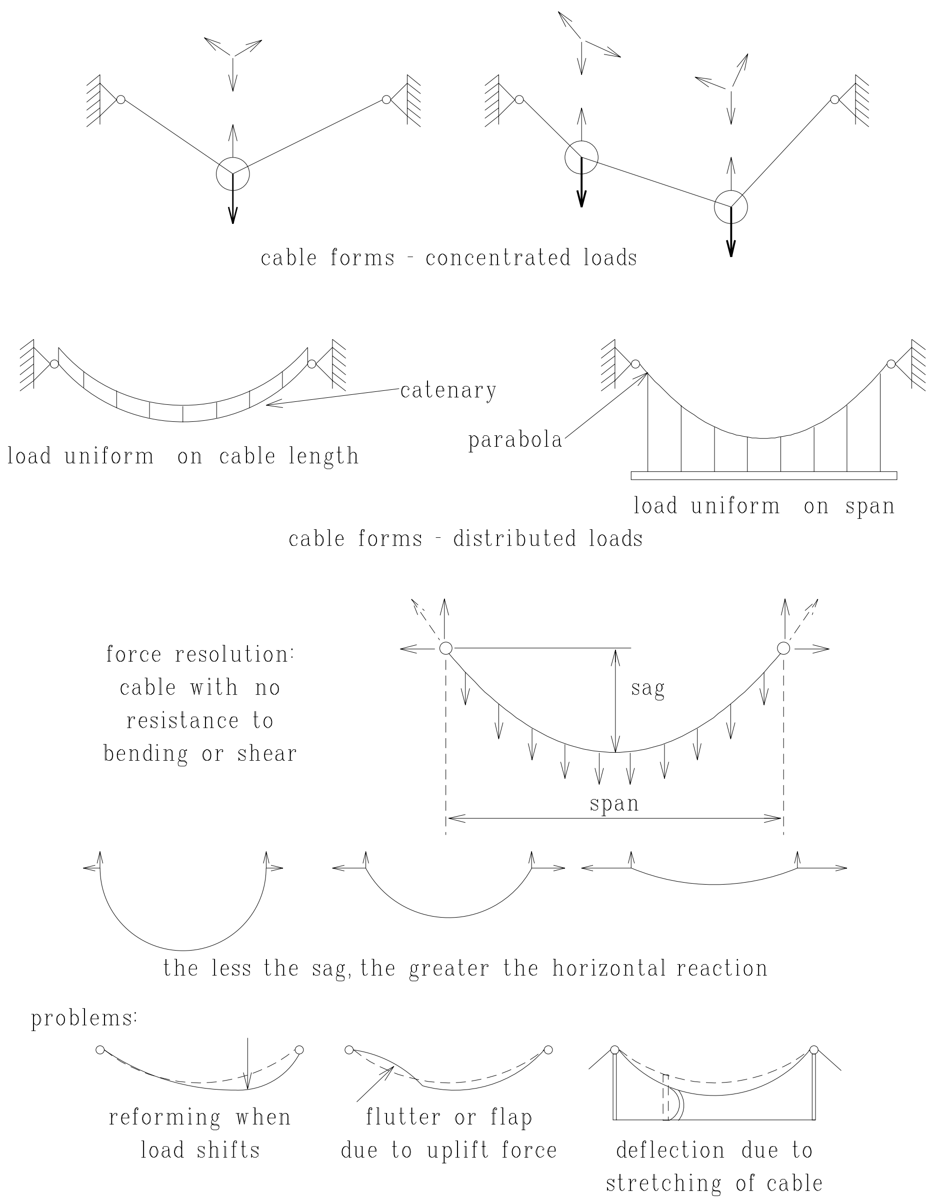

Figure 1.2: Basic Aspects of Cable Systems

capacity by adjusting its shape so as to provide maximum resistance (*form follows function*). Care should be exercised in minimizing large deflections and vibrations.

Arches (mostly compression) is a “reversed cable structure”. In an arch, flexure/shear is minimized and most of the load is transfered through axial forces only. Arches are

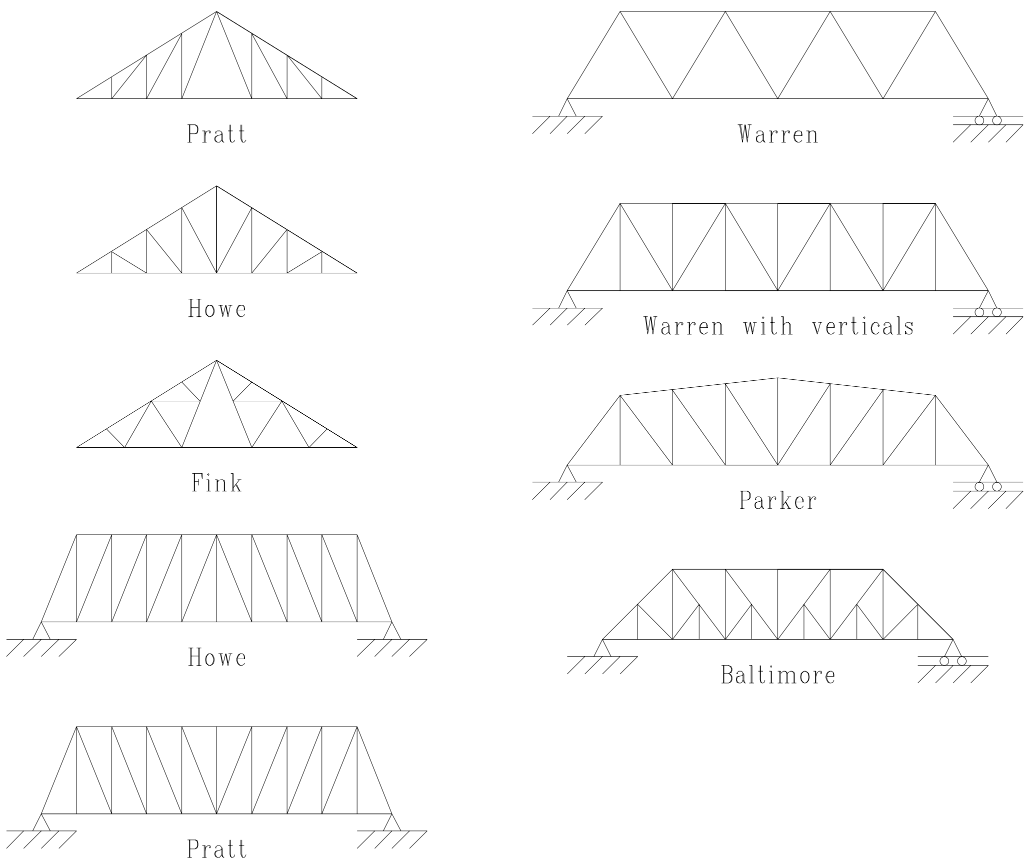

Figure 1.4: Types of Trusses

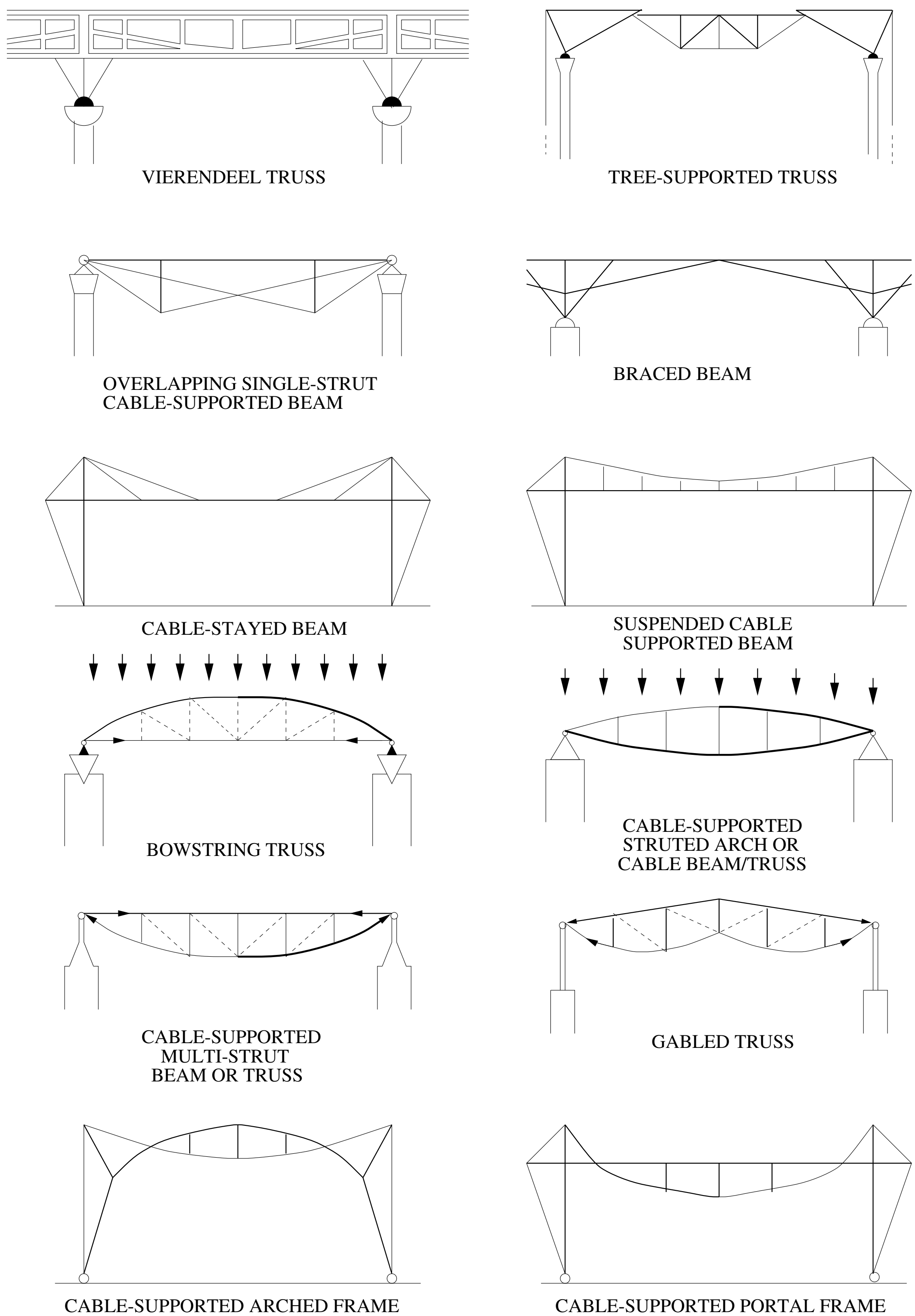

Figure 1.6: Different Beam Types

Folded plates are used mostly as long span roofs. However, they can also be used as vertical walls to support both vertical and horizontal loads.

Membranes: 3D structures composed of a flexible 2D surface resisting tension only. They are usually cable-supported and are used for tents and long span roofs Fig. 1.8.

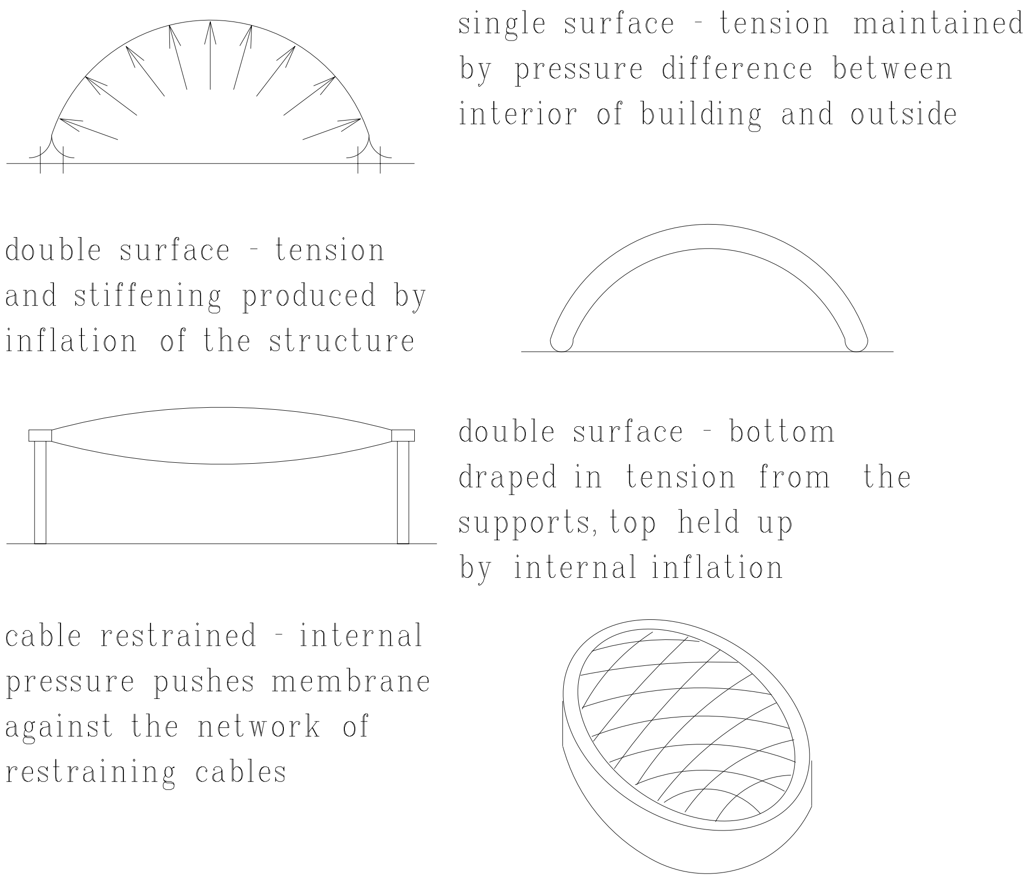

Figure 1.8: Examples of Air Supported Structures

Shells: 3D structures composed of a curved 2D surface, they are usually shaped to transmit compressive axial stresses only, Fig. 1.9.

Shells are classified in terms of their curvature.

1.11 Structural Engineering Courses

22 Structural engineering education can be approached from either one of two points of views, depending on the audience, **??**.

Architects: Start from overall design, and move toward detailed analysis. Emphasis on good understanding of overall structural behavior. Develop a good understanding of load transfer mechanism for most types of structures, cables, arches, beams, frames, shells, plates. Approximate analysis for most of them.

Engineers: Emphasis is on the individual structural elements and not always on the total system. Focus on beams, frames (mostly 2D) and trusses. Very seldom are arches covered. Plates and shells are not even mentioned.

1.12 References

23 Following are some useful references for structural engineering, those marked by † were consulted, and "borrowed from" in preparing the Lecture Notes or are particularly recommended.

Structures for Architect

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Structures for Engineers

Chapter 2

LOADS

2.1 Introduction

1 The main purpose of a structure is to transfer load from one point to another: bridge deck to pier; slab to beam; beam to girder; girder to column; column to foundation; foundation to soil.

2 There can also be secondary loads such as thermal (in restrained structures), differential settlement of foundations, P-Delta effects (additional moment caused by the product of the vertical force and the lateral displacement caused by lateral load in a high rise building).

3 Loads are generally subdivided into two categories

Vertical Loads or gravity load

1. **dead load** (DL)
2. **live load** (LL)

also included are snow loads.

Lateral Loads which act horizontally on the structure

1. **Wind load** (WL)
2. **Earthquake load** (EL)

this also includes hydrostatic and earth loads.

4 This distinction is helpful not only to compute a structure's load, but also to assign different factor of safety to each one.

5 For a detailed coverage of loads, refer to the Universal Building Code (UBC), (UBC 1995).

2.2 Vertical Loads

6 For closely spaced identical loads (such as joist loads), it is customary to treat them as a uniformly distributed load rather than as discrete loads, Fig. 2.1

Material	lb/ft^2
Ceilings	
Channel suspended system	1
Acoustical fiber tile	1
Floors	
Steel deck	2-10
Concrete-plain 1 in.	12
Linoleum 1/4 in.	1
Hardwood	4
Roofs	
Copper or tin	1-5
5 ply felt and gravel	6
Shingles asphalt	3
Clay tiles	9-14
Sheathing wood	3
Insulation 1 in. poured in place	2
Partitions	
Clay tile 3 in.	17
Clay tile 10 in.	40
Gypsum Block 5 in.	14
Wood studs 2x4 (12-16 in. o.c.)	2
Plaster 1 in. cement	10
Plaster 1 in. gypsum	5
Walls	
Bricks 4 in.	40
Bricks 12 in.	120
Hollow concrete block (heavy aggregate)	
4 in.	30
8 in.	55
12 in.	80
Hollow concrete block (light aggregate)	
4 in.	21
8 in.	38
12 in.	55

Table 2.2: Weights of Building Materials

Material	lb/ft^2
Timber	40-50
Steel	50-80
Reinforced concrete	100-150

Table 2.3: Average Gross Dead Load in Buildings

Floor	Roof	10	9	8	7	6	5	4	3	2	Total
Cumulative R (%)	8.48	16.96	25.44	33.92	42.4	51.32	59.8	60	60	60	
Cumulative LL	20	80	80	80	80	80	80	80	80	80	740
Cumulative R× LL	18.3	66.4	59.6	52.9	46.08	38.9	32.2	32	32	32	410

The resulting design live load for the bottom column has been reduced from 740 Kips to $\boxed{\text{410 Kips}}$.

5. The total dead load is $DL = (10)(60) = 600$ Kips, thus the total reduction in load is $\frac{740-410}{740+600} \times 100 = \boxed{25\%}$.

■

2.2.3 Snow

19 Roof snow load vary greatly depending on **geographic location and elevation**. They range from 20 to 45 psf, Fig. 2.2.

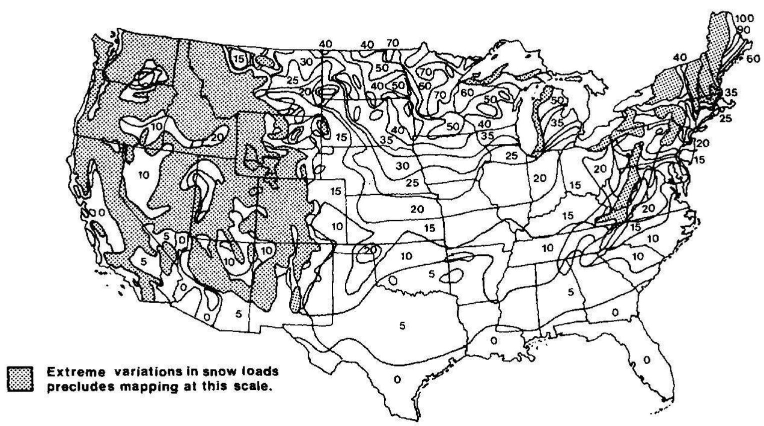


Figure 2.2: Snow Map of the United States, ubc

20 Snow loads are always given on the projected length or area on a slope, Fig. 2.3.

21 The steeper the roof, the lower the snow retention. For snow loads greater than 20 psf and roof pitches α more than 20° the snow load p may be reduced by

$$R = (\alpha - 20)\left(\frac{p}{40} - 0.5\right) \qquad \text{(psf)} \tag{2.2}$$

22 Other examples of loads acting on inclined surfaces are shown in Fig. 2.4.

2.3 Lateral Loads

2.3.1 Wind

23 Wind load depend on: **velocity of the wind, shape of the building, height, geographical location, texture of the building surface and stiffness of the structure**.

24 Wind loads are particularly significant on **tall buildings**[1].

25 When a *steady* streamline airflow of velocity V is completely stopped by a rigid body, the **stagnation pressure** (or velocity pressure) q_s was derived by Bernouilli (1700-1782)

$$q_s = \frac{1}{2}\rho V^2 \tag{2.3}$$

where the air mass density ρ is the air weight divided by the acceleration of gravity $g = 32.2$ ft/sec^2. At sea level and a temperature of 15^{o}C (59^{o}F), the air weighs 0.0765 lb/ft^3 this would yield a pressure of

$$q_s = \frac{1}{2}\frac{(0.0765)\text{lb/ft}^3}{(32.2)\text{ft/sec}^2}\left(\frac{(5280)\text{ft/mile}}{(3600)\text{sec/hr}}V\right)^2 \tag{2.4}$$

or

$$\boxed{q_s = 0.00256V^2} \tag{2.5}$$

where V is the maximum wind velocity (in miles per hour) and q_s is in psf. V can be obtained from wind maps (in the United States $70 \le V \le 110$), Fig. 2.5.

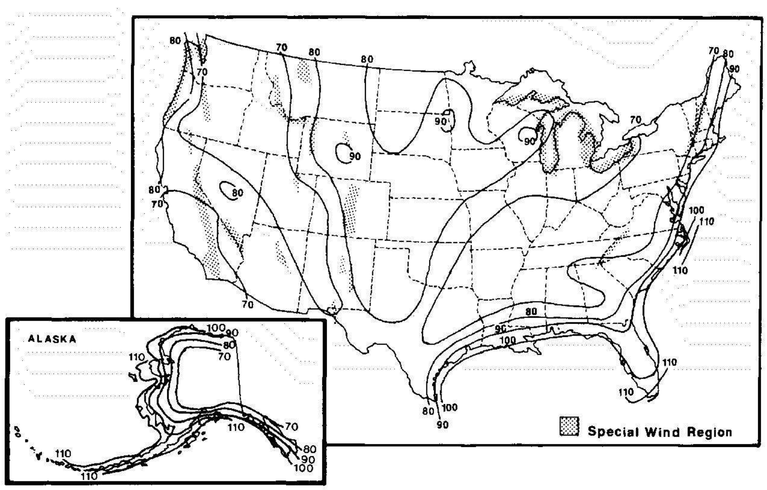


Figure 2.5: Wind Map of the United States, (UBC 1995)

26 During storms, wind velocities may reach values up to or greater than 150 miles per hour, which corresponds to a dynamic pressure q_s of about 60 psf (as high as the average vertical occupancy load in buildings).

27 Wind pressure **increases with height**, Table 2.5.

28 Wind load will cause suction on the leeward sides, Fig. 2.7

29 This magnitude must be modified to account for the shape and surroundings of the building.

[1]The primary design consideration for very high rise buildings is the excessive **drift** caused by lateral load (wind and possibly earthquakes).

Thus, the design pressure p (psf) is given by

$$\boxed{p = C_e C_q I q_s} \tag{2.6}$$

The pressure is assumed to be **normal** to all walls and roofs and

C_e **Velocity Pressure Coefficient** accounts for height, exposure and gust factor. It accounts for the fact that wind velocity increases with height and that dynamic character of the airflow (i.e the wind pressure is not steady), Table 2.6.

C_e	Exposure	
1.39-2.34	D	Open, flat terrain facing large bodies of water
1.06-2.19	C	Flat open terrain, extending one-half mile or open from the site in any full quadrant
0.62-1.80	B	Terrain with buildings, forest, or surface irregularities 20 ft or more in height

Table 2.6: C_e Coefficients for Wind Load, (UBC 1995)

C_q **Pressure Coefficient** is a shape factor which is given in Table 2.7 for gabled frames.

		Windward Side	Leeward Side
Gabled Frames (V:H)			
Roof Slope	<9:12	-0.7	-0.7
	9:12 to 12:12	0.4	-0.7
	>12:12	0.7	-0.7
Walls		0.8	-0.5
Buildings (height < 200 ft)			
Vertical Projections	height < 40 ft	1.3	-1.3
	height > 40 ft	1.4	-1.4
Horizontal Projections		-0.7	-0.7

Table 2.7: Wind Pressure Coefficients C_q, (UBC 1995)

I **Importance Factor** as given by Table 2.8. where

I Essential Facilities: Hospitals; Fire and police stations; Tanks; Emergency vehicle shelters, standby power-generating equipment; Structures and equipment in government. communication centers.

II Hazardous Facilities: Structures housing, supporting or containing sufficient quantities of toxic or explosive substances to be dangerous to the safety of the general public if released.

III Special occupancy structure: Covered structures whose primary occupancy is public assembly, capacity > 300 persons.

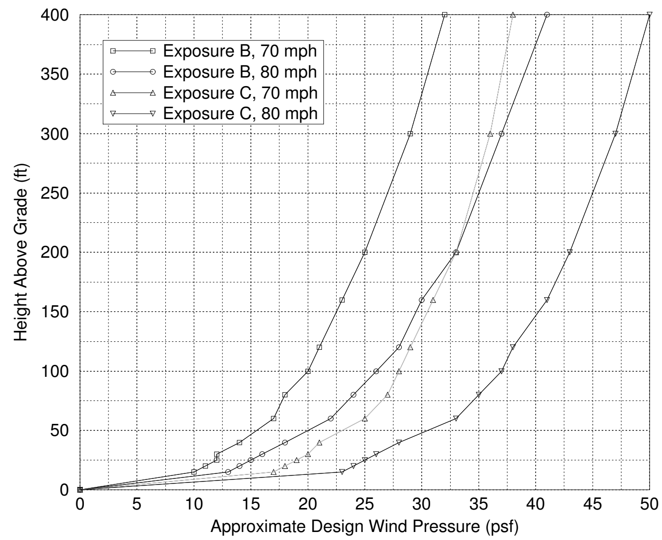
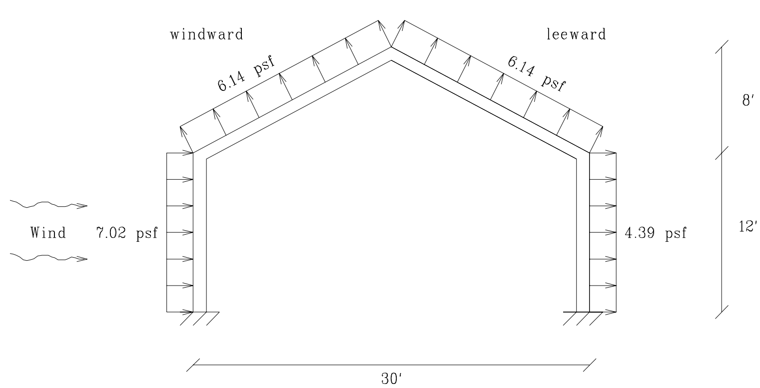


Figure 2.7: Approximate Design Wind Pressure p for Ordinary Wind Force Resisting Building Structures

■ Example 2-2: Wind Load

Determine the wind forces on the building shown on below which is built in St Louis and is surrounded by trees.
Solution:

1. From Fig. 2.5 the maximum wind velocity is St. Louis is 70 mph, since the building is protected we can take $C_e = 0.7$, $I = 1.$. The base wind pressure is $q_s = 0.00256 \times (70)^2 = 12.54$ psf.



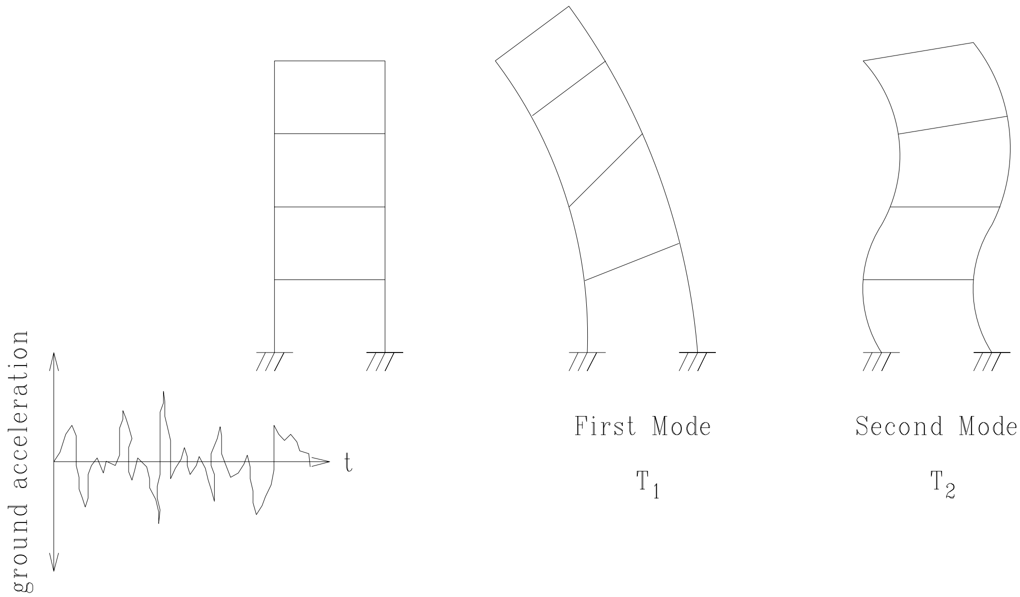


Figure 2.8: Vibrations of a Building

36 The horizontal force at each level is calculated as a portion of the base shear force V

$$\boxed{V = \frac{ZIC}{R_W} W} \tag{2.8}$$

where:

Z**: Zone Factor:** to be determined from Fig. 2.9 and Table 2.10.

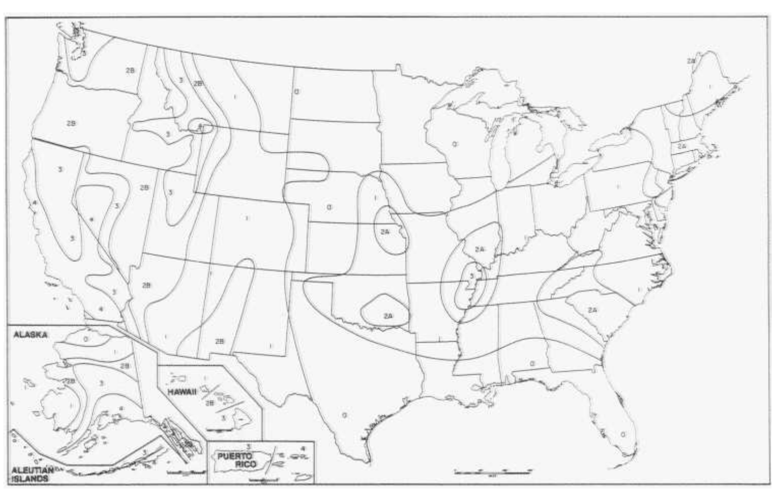


Figure 2.9: Seismic Zones of the United States, (UBC 1995)

I**: Importance Factor:** which was given by Table 2.8.

Structural System	R_W	**H** (ft)
Bearing wall system		
Light-framed walls with shear panels		
Plywood walls for structures three stories or less	8	65
All other light-framed walls	6	65
Shear walls		
Concrete	8	240
Masonry	8	160
Building frame system using trussing or shear walls)		
Steel eccentrically braced ductiel frame	10	240
Light-framed walls with shear panels		
Plywood walls for structures three stories or less	9	65
All other light-framed walls	7	65
Shear walls		
Concrete	8	240
Masonry	8	160
Concentrically braced frames		
Steel	8	160
Concrete (only for zones I and 2)	8	-
Heavy timber	8	65
Moment-resisting frame system		
Special moment-resisting frames (SMRF)		
Steel	12	N.L.
Concrete	12	N.L.
Concrete intermediate moment-resisting frames (IMRF)only for zones 1 and 2	8	-
Ordinary moment-resisting frames (OMRF)		
Steel	6	160
Concrete (only for zone 1)	5	-
Dual systems (selected cases are for ductile rigid frames only)		
Shear walls		
Concrete with SMRF	12	N.L.
Masonry with SMRF	8	160
Steel eccentrically braced ductile frame	6-12	160-N.L.
Concentrically braced frame	12	N. L.
Steel with steel SMRF	10	N.L.
Steel with steel OMRF	6	160
Concrete with concrete SMRF (only for zones 1 and 2)	9	-

Table 2.12: Partial List of R_W for Various Structure Systems, (UBC 1995)

5. The total vertical load is

$$W = 2\left((200 + 0.5(400)\right)(20) = 16000 \text{ lbs} \tag{2.17}$$

6. The total seismic base shear is

$$V = \frac{ZIC}{R_W} = \frac{(0.3)(1.25)(2.75)}{12} = 0.086W \tag{2.18-a}$$

$$= (0.086)(16000) = \boxed{1375 \text{ lbs}} \tag{2.18-b}$$

7. Since $T < 0.7$ sec. there is no whiplash.

8. The load on each floor is thus given by

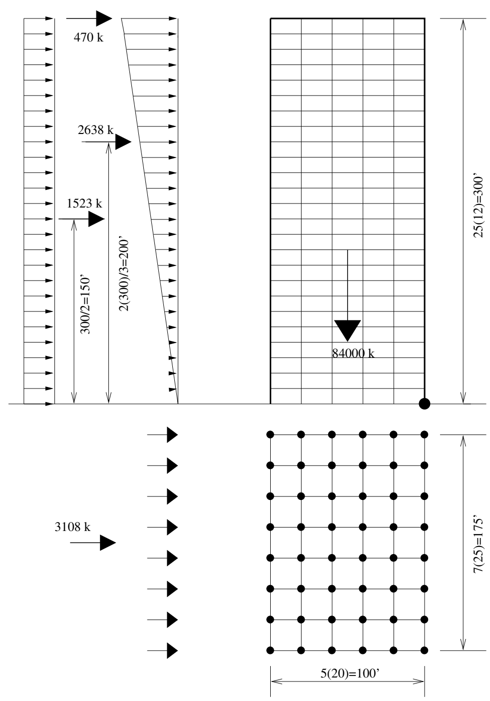
$$F_2 = \frac{(1375)(24)}{12+24} = \boxed{916.7 \text{ lbs}} \tag{2.19-a}$$

$$F_1 = \frac{(1375)(12)}{12+24} = \boxed{458.3 \text{ lbs}} \tag{2.19-b}$$

■

■ **Example 2-4: Earthquake Load on a Tall Building, (Schueller 1996)**

Determine the approximate critical lateral loading for a 25 storey, ductile, rigid space frame concrete structure in the short direction. The rigid frames are spaced 25 ft apart in the cross section and 20 ft in the longitudinal direction. The plan dimension of the building is 175x100 ft, and the structure is 25(12)=300 ft high. This office building is located in an urban environment with a wind velocity of 70 mph and in seismic zone 4. For this investigation, an average building total dead load of 192 psf is used. Soil conditions are unknown.



2.4 Other Loads

2.4.1 Hydrostatic and Earth

39 Structures below ground must resist **lateral earth pressure**.

$$\boxed{q = K\gamma h} \tag{2.28}$$

where γ is the soil density, $K = \frac{1-\sin\Phi}{1+\sin\Phi}$ is the pressure coefficient, h is the height.

40 For sand and gravel $\gamma = 120$ lb/ ft^3, and $\Phi \approx 30°$.

41 If the structure is partially submerged, it must also resist **hydrostatic pressure** of water, Fig. 2.10.

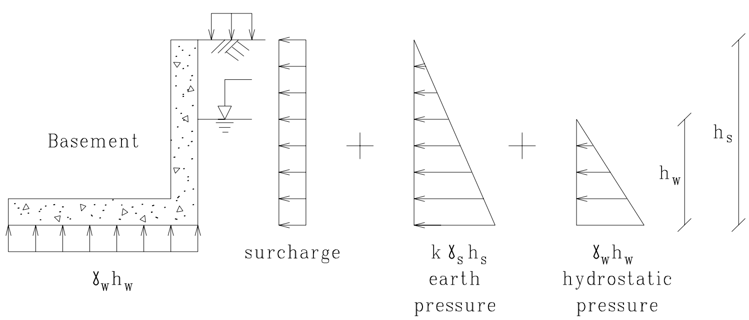


Figure 2.10: Earth and Hydrostatic Loads on Structures

$$\boxed{q = \gamma_W h} \tag{2.29}$$

where $\gamma_W = 62.4$ lbs/ft^3.

■ **Example 2-5: Hydrostatic Load**

The basement of a building is 12 ft below grade. Ground water is located 9 ft below grade, what thickness concrete slab is required to exactly balance the hydrostatic uplift?

Solution:

The hydrostatic pressure must be countered by the pressure caused by the weight of concrete. Since $p = \gamma h$ we equate the two pressures and solve for h the height of the concrete slab $\underbrace{(62.4) \text{ lbs/ft}^3 \times (12-9) \text{ ft}}_{\text{water}} = \underbrace{(150) \text{ lbs/ft}^3 \times h}_{\text{concrete}} \Rightarrow h = \frac{(62.4) \text{ lbs/ft}^3}{(150) \text{ lbs/ft}^3}(3) \text{ ft}(12) \text{ in/ft} = 14.976 \text{ in} \simeq \boxed{15.0 \text{ inch}}$ ■

2.4.3 Bridge Loads

For highway bridges, design loads are given by the AASHTO (Association of American State Highway Transportation Officials). The HS-20 truck is used for the design of bridges on main highways, Fig. 6.3. Either the design truck with specified axle loads and spacing must be used **or** the equivalent uniform load and concentrated load. This loading must be placed such that maximum stresses are produced.

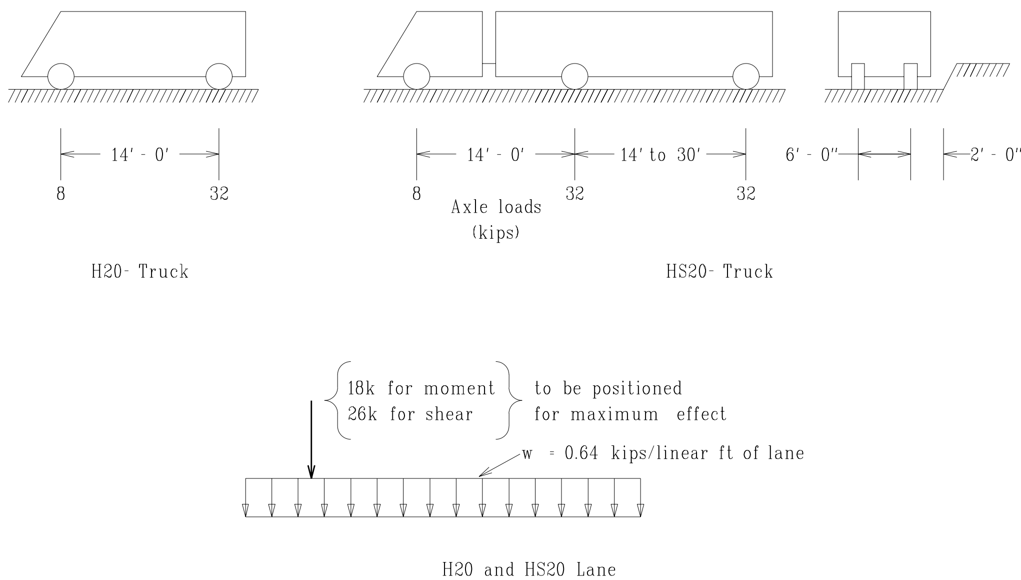


Figure 2.11: Truck Load

2.4.4 Impact Load

2.5 Other Important Considerations

2.5.1 Load Combinations

45 Live loads specified by codes represent the maximum possible loads.

46 The likelihood of all these loads occurring simultaneously is remote. Hence, building codes allow certain reduction when certain loads are **combined** together.

47 Furthermore, structures should be designed to resist a **combination of loads**.

48 Denoting D= dead; L= live; Lr= roof live; W= wind; E= earthquake; S= snow; T= temperature; H= soil:

49 For the load and resistance factor design (LRFD) method of concrete structures, the American Concrete Institute (ACI) Building design code (318) (318 n.d.) requires that the following load combinations be considered:

1. 1.4D+1.7L

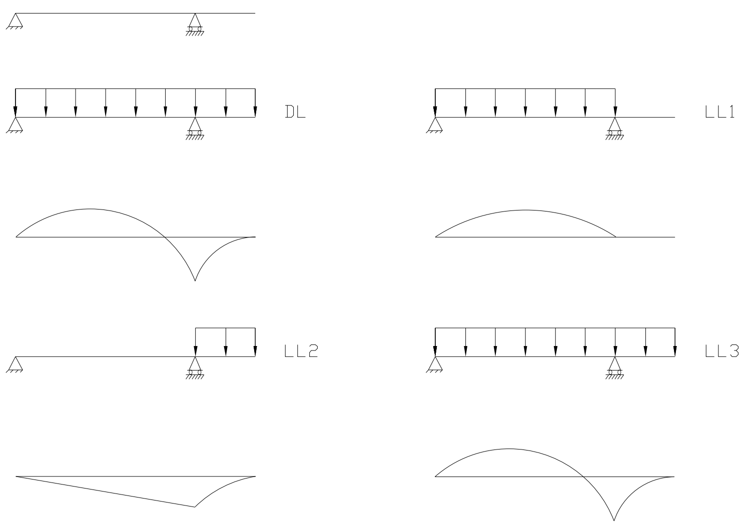


Figure 2.12: Load Placement to Maximize Moments

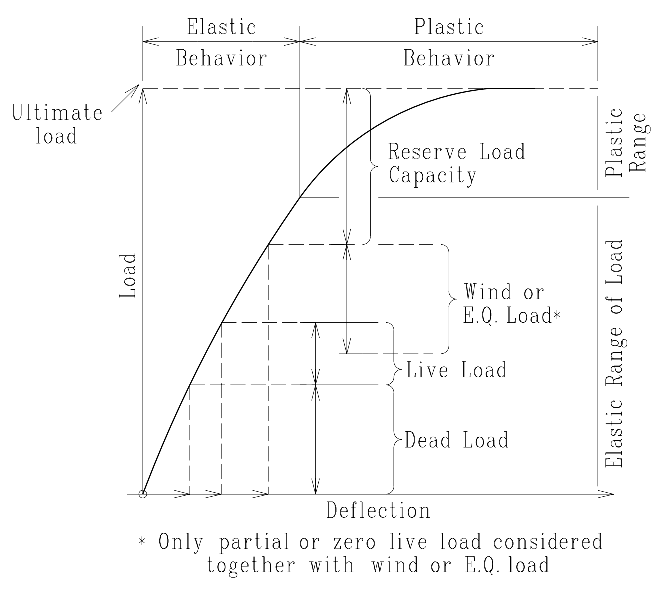


Figure 2.13: Load Life of a Structure, (Lin and Stotesbury 1981)

1. The section is part of a beam or girder.
2. The beam or girder is really part of a three dimensional structure in which load is transmitted from any point in the structure to the foundation through any one of various structural forms.

57 **Load transfer** in a structure is accomplished through a "hierarchy" of simple flexural elements which are then connected to the columns, Fig. 2.16 or by two way slabs as illustrated in Fig. 2.17.

58 An example of load transfer mechanism is shown in Fig. 2.18.

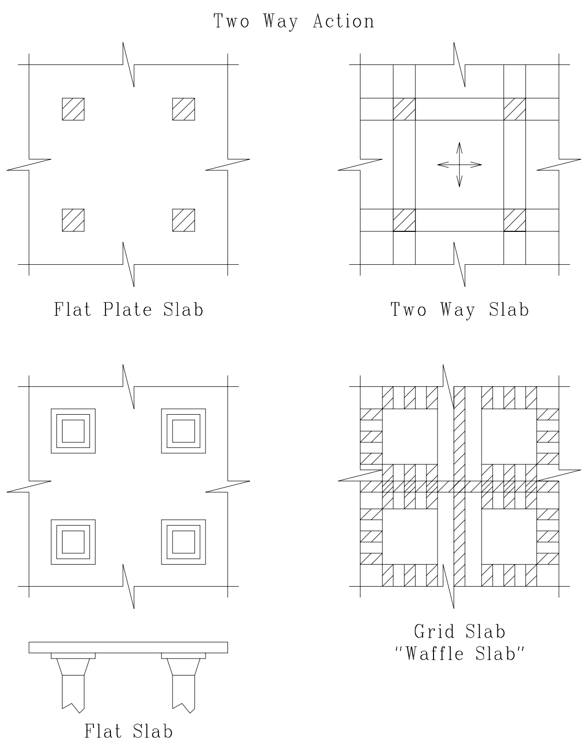


Figure 2.17: Two Way Actions

Chapter 3

STRUCTURAL MATERIALS

1 Proper understanding of structural materials is essential to both structural analysis and to structural design.

2 Characteristics of the most commonly used structural materials will be highlighted.

3.1 Steel

3.1.1 Structural Steel

3 Steel is an **alloy** of iron and carbon. Its properties can be greatly varied by altering the carbon content (always less than 0.5%) or by adding other elements such as silicon, nickle, manganese and copper.

4 Practically all grades of steel have a Young Modulus equal to **29,000 ksi**, density of 490 lb/cu ft, and a coefficient of thermal expansion equal to 0.65×10^{-5} /deg F.

5 The yield stress of steel can vary from 40 ksi to 250 ksi. Most commonly used structural steel are **A36** ($\sigma_{\rm yld} = 36$ ksi) and A572 ($\sigma_{\rm yld} = 50$ ksi), Fig. 3.1

6 Structural steel can be rolled into a wide variety of shapes and sizes. Usually the most desirable members are those which have a large section moduli (S) in proportion to their area (A), Fig. 3.2.

7 Steel can be bolted, riveted or welded.

8 Sections are designated by the shape of their cross section, their depth and their weight. For example W 27× 114 is a W section, 27 in. deep weighing 114 lb/ft.

9 Common sections are:

S sections were the first ones rolled in America and have a slope on their inside flange surfaces of 1 to 6.

W or wide flange sections have a much smaller inner slope which facilitates connections and rivetting. W sections constitute about 50% of the tonnage of rolled structural steel.

C are channel sections

MC Miscellaneous channel which can not be classified as a C shape by dimensions.

HP is a bearing pile section.

M is a miscellaneous section.

L are angle sections which may have equal or unequal sides.

WT is a T section cut from a W section in two.

10 The section modulus S_x of a W section can be roughly approximated by the following formula

$$\boxed{S_x \approx wd/10 \quad \text{or} \quad I_x \approx S_x \frac{d}{2} \approx wd^2/20} \tag{3.1}$$

and the plastic modulus can be approximated by

$$\boxed{Z_x \approx wd/9} \tag{3.2}$$

11 Properties of structural steel are tabulated in Table 3.1.

ASTM Desig.	**Shapes Available**	**Use**	σ_y **(kksi)**	σ_u **(kksi)**
A36	Shapes and bars	Riveted, bolted, welded; Buildings and bridges	36 up through 8 in. (32 above 8.)	
A500	Cold formed welded and seamless sections;	General structural purpose Riveted, welded or bolted;	Grade A: 33; Grade B: 42; Grade C: 46	
A501	Hot formed welded and seamless sections;	Bolted and welded	36	
A529	Plates and bars $\frac{1}{2}$ in and less thick;	Building frames and trusses; Bolted and welded	42	
A606	Hot and cold rolled sheets;	Atmospheric corrosion resistant	45-50	
A611	Cold rolled sheet in cut lengths	Cold formed sections	Grade C 33; Grade D 40; Grade E 80	
A 709	Structural shapes, plates and bars	Bridges	Grade 36: 36 (to 4 in.); Grade 50: 50; Grade 100: 100 (to 2.5in.) and 90 (over 2.5 to 4 in.)	

Table 3.1: Properties of Major Structural Steels

12 Rolled sections, Fig. 3.3 and welded ones, Fig3.4 have **residual stresses**. Those originate during the rolling or fabrication of a member. The member is hot just after rolling or welding, it cools unevenly because of varying exposure. The area that cool first become stiffer, resist contraction, and develop compressive stresses. The remaining regions continue to cool and contract in the plastic condition and develop tensile stresses.

13 Due to those residual stresses, the stress-strain curve of a rolled section exhibits a non-linear segment prior to the theoretical yielding, Fig. 3.5. This would have important implications on the flexural and axial strength of beams and columns.

3.1.2 Reinforcing Steel

14 Steel is also used as reinforcing bars in concrete, Table 3.2. Those bars have a deformation on their surface to increase the bond with concrete, and usually have a yield stress of 60 ksi[1].

Bar Designation	Diameter (in.)	Area (in^2)	Perimeter in	Weight lb/ft
No. 2	2/8=0.250	0.05	0.79	0.167
No. 3	3/8=0.375	0.11	1.18	0.376
No. 4	4/8=0.500	0.20	1.57	0.668
No. 5	5/8=0.625	0.31	1.96	1.043
No. 6	6/8=0.750	0.44	2.36	1.5202
No. 7	7/8=0.875	0.60	2.75	2.044
No. 8	8/8=1.000	0.79	3.14	2.670
No. 9	9/8=1.128	1.00	3.54	3.400
No. 10	10/8=1.270	1.27	3.99	4.303
No. 11	11/8=1.410	1.56	4.43	5.313
No. 14	14/8=1.693	2.25	5.32	7.650
No. 18	18/8=2.257	4.00	7.09	13.60

Table 3.2: Properties of Reinforcing Bars

15 Steel loses its strength rapidly above 700 deg. F (and thus must be properly protected from fire), and becomes brittle at −30 deg. F

16 Steel is also used as wire strands and ropes for suspended roofs, cable-stayed bridges, fabric roofs and other structural applications. A **strand** is a helical arrangement of wires around a central wire. A **rope** consists of multiple strands helically wound around a central plastic core, and a modulus of elasticity of 20,000 ksi, and an ultimate strength of 220 ksi.

17 Prestressing Steel cables have an ultimate strength up to 270 ksi.

3.2 Aluminum

18 Aluminum is used whenever **light weight** combined with strength is an important factor. Those properties, along with its **resistance to corrosion** have made it the material of choice for airplane structures, light roof framing.

19 Aluminum members can be connected by riveting, bolting and to a lesser extent by welding.

20 Aluminum has a **modulus of elasticity equal to 10,000 ksi** (about three times lower than steel), a coefficient of thermal expansion of 2.4×10^{-5} and a density of 173 lbs/ft^3.

21 The ultimate strength of pure aluminum is low (13,000 psi) but with the addition of alloys it can go up.

[1]Stirrups which are used as vertical reinforcement to resist shear usually have a yield stress of only 40 ksi.

30 Density of normal weight concrete is 145 lbs/ft^3 and 100 lbs/ft^3 for lightweight concrete.

31 Coefficient of thermal expansion is 0.65×10^{-5} /deg F for normal weight concrete.

32 When concrete is poured (or rather placed), the free water not needed for the hydration process evaporates over a period of time and the concrete will **shrink**. This shrinkage is about 0.05% after one year (strain). Thus if the concrete is restrained, then cracking will occur[3].

33 Concrete will also deform with time due to the applied load, this is called **creep**. This should be taken into consideration when computing the deflections (which can be up to three times the instantaneous elastic deflection).

3.4 Masonry

34 Masonry consists of either natural materials, such as stones, or of manufactured products such as bricks and concrete blocks[4], stacked and bonded together with mortar.

35 As for concrete, all modern structural masonry blocks are essentially compression members with low tensile resistance.

36 The mortar used is a mixture of sand, masonry cement, and either Portland cement or hydrated lime.

3.5 Timber

37 Timber is one of the earliest construction materials, and one of the few natural materials with good tensile properties.

38 The properties of timber vary greatly, and the strength is time dependent.

39 Timber is a good shock absorber (many wood structures in Japan have resisted repeated earthquakes).

40 The most commonly used species of timber in construction are Douglas fir, southern pine, hemlock and larch.

41 Members can be laminated together under good quality control, and flexural strengths as high as 2,500 psi can be achieved.

3.6 Steel Section Properties

42 Dimensions and properties of rolled sections are tabulated in the following pages, Fig. 3.7. ==============

[3]For this reason a minimum amount of reinforcement is always necessary in concrete, and a 2% reinforcement, can reduce the shrinkage by 75%.

[4]Mud bricks were used by the Babylonians, stones by the Egyptians, and ice blocks by the Eskimos...

Designation	A in^2	d in	$\frac{b_f}{2t_f}$	$\frac{h_c}{t_w}$	I_x in^4	S_x in^3	I_y in^4	S_y in^3	Z_x in^3	Z_y in^3
W 27x539	158.0	32.52	2.2	12.3	25500	1570	2110	277	1880.0	437.0
W 27x494	145.0	31.97	2.3	13.4	22900	1440	1890	250	1710.0	394.0
W 27x448	131.0	31.42	2.5	14.7	20400	1300	1670	224	1530.0	351.0
W 27x407	119.0	30.87	2.7	15.9	18100	1170	1480	200	1380.0	313.0
W 27x368	108.0	30.39	3.0	17.6	16100	1060	1310	179	1240.0	279.0
W 27x336	98.7	30.00	3.2	19.2	14500	970	1170	161	1130.0	252.0
W 27x307	90.2	29.61	3.5	20.9	13100	884	1050	146	1020.0	227.0
W 27x281	82.6	29.29	3.7	22.9	11900	811	953	133	933.0	206.0
W 27x258	75.7	28.98	4.0	24.7	10800	742	859	120	850.0	187.0
W 27x235	69.1	28.66	4.4	26.6	9660	674	768	108	769.0	168.0
W 27x217	63.8	28.43	4.7	29.2	8870	624	704	100	708.0	154.0
W 27x194	57.0	28.11	5.2	32.3	7820	556	618	88	628.0	136.0
W 27x178	52.3	27.81	5.9	33.4	6990	502	555	79	567.0	122.0
W 27x161	47.4	27.59	6.5	36.7	6280	455	497	71	512.0	109.0
W 27x146	42.9	27.38	7.2	40.0	5630	411	443	64	461.0	97.5
W 27x129	37.8	27.63	4.5	39.7	4760	345	184	37	395.0	57.6
W 27x114	33.5	27.29	5.4	42.5	4090	299	159	32	343.0	49.3
W 27x102	30.0	27.09	6.0	47.0	3620	267	139	28	305.0	43.4
W 27x 94	27.7	26.92	6.7	49.4	3270	243	124	25	278.0	38.8
W 27x 84	24.8	26.71	7.8	52.7	2850	213	106	21	244.0	33.2
W 24x492	144.0	29.65	2.0	10.9	19100	1290	1670	237	1550.0	375.0
W 24x450	132.0	29.09	2.1	11.9	17100	1170	1490	214	1410.0	337.0
W 24x408	119.0	28.54	2.3	13.1	15100	1060	1320	191	1250.0	300.0
W 24x370	108.0	27.99	2.5	14.2	13400	957	1160	170	1120.0	267.0
W 24x335	98.4	27.52	2.7	15.6	11900	864	1030	152	1020.0	238.0
W 24x306	89.8	27.13	2.9	17.1	10700	789	919	137	922.0	214.0
W 24x279	82.0	26.73	3.2	18.6	9600	718	823	124	835.0	193.0
W 24x250	73.5	26.34	3.5	20.7	8490	644	724	110	744.0	171.0
W 24x229	67.2	26.02	3.8	22.5	7650	588	651	99	676.0	154.0
W 24x207	60.7	25.71	4.1	24.8	6820	531	578	89	606.0	137.0
W 24x192	56.3	25.47	4.4	26.6	6260	491	530	82	559.0	126.0
W 24x176	51.7	25.24	4.8	28.7	5680	450	479	74	511.0	115.0
W 24x162	47.7	25.00	5.3	30.6	5170	414	443	68	468.0	105.0
W 24x146	43.0	24.74	5.9	33.2	4580	371	391	60	418.0	93.2
W 24x131	38.5	24.48	6.7	35.6	4020	329	340	53	370.0	81.5
W 24x117	34.4	24.26	7.5	39.2	3540	291	297	46	327.0	71.4
W 24x104	30.6	24.06	8.5	43.1	3100	258	259	41	289.0	62.4
W 24x103	30.3	24.53	4.6	39.2	3000	245	119	26	280.0	41.5
W 24x 94	27.7	24.31	5.2	41.9	2700	222	109	24	254.0	37.5
W 24x 84	24.7	24.10	5.9	45.9	2370	196	94	21	224.0	32.6
W 24x 76	22.4	23.92	6.6	49.0	2100	176	82	18	200.0	28.6
W 24x 68	20.1	23.73	7.7	52.0	1830	154	70	16	177.0	24.5
W 24x 62	18.2	23.74	6.0	50.1	1550	131	34	10	153.0	15.7
W 24x 55	16.2	23.57	6.9	54.6	1350	114	29	8	134.0	13.3
W 21x402	118.0	26.02	2.1	10.8	12200	937	1270	189	1130.0	296.0
W 21x364	107.0	25.47	2.3	11.8	10800	846	1120	168	1010.0	263.0
W 21x333	97.9	25.00	2.5	12.8	9610	769	994	151	915.0	237.0
W 21x300	88.2	24.53	2.7	14.2	8480	692	873	134	816.0	210.0
W 21x275	80.8	24.13	2.9	15.4	7620	632	785	122	741.0	189.0
W 21x248	72.8	23.74	3.2	17.1	6760	569	694	109	663.0	169.0
W 21x223	65.4	23.35	3.5	18.8	5950	510	609	96	589.0	149.0
W 21x201	59.2	23.03	3.9	20.6	5310	461	542	86	530.0	133.0
W 21x182	53.6	22.72	4.2	22.6	4730	417	483	77	476.0	119.0
W 21x166	48.8	22.48	4.6	24.9	4280	380	435	70	432.0	108.0
W 21x147	43.2	22.06	5.4	26.1	3630	329	376	60	373.0	92.6
W 21x132	38.8	21.83	6.0	28.9	3220	295	333	54	333.0	82.3
W 21x122	35.9	21.68	6.5	31.3	2960	273	305	49	307.0	75.6
W 21x111	32.7	21.51	7.1	34.1	2670	249	274	44	279.0	68.2
W 21x101	29.8	21.36	7.7	37.5	2420	227	248	40	253.0	61.7
W 21x 93	27.3	21.62	4.5	32.3	2070	192	93	22	221.0	34.7
W 21x 83	24.3	21.43	5.0	36.4	1830	171	81	20	196.0	30.5
W 21x 73	21.5	21.24	5.6	41.2	1600	151	71	17	172.0	26.6

Designation	A in^2	d in	$\frac{b_f}{2t_f}$	$\frac{h_c}{t_w}$	I_x in^4	S_x in^3	I_y in^4	S_y in^3	Z_x in^3	Z_y in^3
W 14x 74	21.8	14.17	6.4	25.3	796	112	134	27	126.0	40.6
W 14x 68	20.0	14.04	7.0	27.5	723	103	121	24	115.0	36.9
W 14x 61	17.9	13.89	7.7	30.4	640	92	107	22	102.0	32.8
W 14x 53	15.6	13.92	6.1	30.8	541	78	58	14	87.1	22.0
W 14x 48	14.1	13.79	6.7	33.5	485	70	51	13	78.4	19.6
W 14x 43	12.6	13.66	7.5	37.4	428	63	45	11	69.6	17.3
W 14x 38	11.2	14.10	6.6	39.6	385	55	27	8	61.5	12.1
W 14x 34	10.0	13.98	7.4	43.1	340	49	23	7	54.6	10.6
W 14x 30	8.9	13.84	8.7	45.4	291	42	20	6	47.3	9.0
W 14x 26	7.7	13.91	6.0	48.1	245	35	9	4	40.2	5.5
W 14x 22	6.5	13.74	7.5	53.3	199	29	7	3	33.2	4.4
W 12x336	98.8	16.82	2.3	5.5	4060	483	1190	177	603.0	274.0
W 12x305	89.6	16.32	2.4	6.0	3550	435	1050	159	537.0	244.0
W 12x279	81.9	15.85	2.7	6.3	3110	393	937	143	481.0	220.0
W 12x252	74.1	15.41	2.9	7.0	2720	353	828	127	428.0	196.0
W 12x230	67.7	15.05	3.1	7.6	2420	321	742	115	386.0	177.0
W 12x210	61.8	14.71	3.4	8.2	2140	292	664	104	348.0	159.0
W 12x190	55.8	14.38	3.7	9.2	1890	263	589	93	311.0	143.0
W 12x170	50.0	14.03	4.0	10.1	1650	235	517	82	275.0	126.0
W 12x152	44.7	13.71	4.5	11.2	1430	209	454	73	243.0	111.0
W 12x136	39.9	13.41	5.0	12.3	1240	186	398	64	214.0	98.0
W 12x120	35.3	13.12	5.6	13.7	1070	163	345	56	186.0	85.4
W 12x106	31.2	12.89	6.2	15.9	933	145	301	49	164.0	75.1
W 12x 96	28.2	12.71	6.8	17.7	833	131	270	44	147.0	67.5
W 12x 87	25.6	12.53	7.5	18.9	740	118	241	40	132.0	60.4
W 12x 79	23.2	12.38	8.2	20.7	662	107	216	36	119.0	54.3
W 12x 72	21.1	12.25	9.0	22.6	597	97	195	32	108.0	49.2
W 12x 65	19.1	12.12	9.9	24.9	533	88	174	29	96.8	44.1
W 12x 58	17.0	12.19	7.8	27.0	475	78	107	21	86.4	32.5
W 12x 53	15.6	12.06	8.7	28.1	425	71	96	19	77.9	29.1
W 12x 50	14.7	12.19	6.3	26.2	394	65	56	14	72.4	21.4
W 12x 45	13.2	12.06	7.0	29.0	350	58	50	12	64.7	19.0
W 12x 40	11.8	11.94	7.8	32.9	310	52	44	11	57.5	16.8
W 12x 35	10.3	12.50	6.3	36.2	285	46	24	7	51.2	11.5
W 12x 30	8.8	12.34	7.4	41.8	238	39	20	6	43.1	9.6
W 12x 26	7.7	12.22	8.5	47.2	204	33	17	5	37.2	8.2
W 12x 22	6.5	12.31	4.7	41.8	156	25	5	2	29.3	3.7
W 12x 19	5.6	12.16	5.7	46.2	130	21	4	2	24.7	3.0
W 12x 16	4.7	11.99	7.5	49.4	103	17	3	1	20.1	2.3
W 12x 14	4.2	11.91	8.8	54.3	89	15	2	1	17.4	1.9
W 10x112	32.9	11.36	4.2	10.4	716	126	236	45	147.0	69.2
W 10x100	29.4	11.10	4.6	11.6	623	112	207	40	130.0	61.0
W 10x 88	25.9	10.84	5.2	13.0	534	98	179	35	113.0	53.1
W 10x 77	22.6	10.60	5.9	14.8	455	86	154	30	97.6	45.9
W 10x 68	20.0	10.40	6.6	16.7	394	76	134	26	85.3	40.1
W 10x 60	17.6	10.22	7.4	18.7	341	67	116	23	74.6	35.0
W 10x 54	15.8	10.09	8.2	21.2	303	60	103	21	66.6	31.3
W 10x 49	14.4	9.98	8.9	23.1	272	55	93	19	60.4	28.3
W 10x 45	13.3	10.10	6.5	22.5	248	49	53	13	54.9	20.3
W 10x 39	11.5	9.92	7.5	25.0	209	42	45	11	46.8	17.2
W 10x 33	9.7	9.73	9.1	27.1	170	35	37	9	38.8	14.0
W 10x 30	8.8	10.47	5.7	29.5	170	32	17	6	36.6	8.8
W 10x 26	7.6	10.33	6.6	34.0	144	28	14	5	31.3	7.5
W 10x 22	6.5	10.17	8.0	36.9	118	23	11	4	26.0	6.1
W 10x 19	5.6	10.24	5.1	35.4	96	19	4	2	21.6	3.3
W 10x 17	5.0	10.11	6.1	36.9	82	16	4	2	18.7	2.8
W 10x 15	4.4	9.99	7.4	38.5	69	14	3	1	16.0	2.3
W 10x 12	3.5	9.87	9.4	46.6	54	11	2	1	12.6	1.7

Designation	A in^2	d in	$\frac{b_f}{2t_f}$	$\frac{h_c}{t_w}$	I_x in^4	S_x in^3	I_y in^4	S_y in^3	Z_x in^3	Z_y in^3
C 15.x 50	14.7	15.	0	0	404.0	53.8	11.	3.78	8.20	8.17
C 15.x 40	11.8	15.	0	0	349.0	46.5	9.23	3.37	57.20	6.87
C 15.x 34	10.0	15.	0	0	315.0	42.0	8.13	3.11	50.40	6.23
C 12.x 30	8.8	12.	0	0	162.0	27.0	5.14	2.06	33.60	4.33
C 12.x 25	7.3	12.	0	0	144.0	24.1	4.47	1.88	29.20	3.84
C 12.x 21	6.1	12.	0	0	129.0	21.5	3.88	1.73	25.40	3.49
C 10.x 30	8.8	10.	0	0	103.0	20.7	3.94	1.65	26.60	3.78
C 10.x 25	7.3	10.	0	0	91.2	18.2	3.36	1.48	23.	3.19
C 10.x 20	5.9	10.	0	0	78.9	15.8	2.81	1.32	19.30	2.71
C 10.x 15	4.5	10.	0	0	67.4	13.5	2.28	1.16	15.80	2.35
C 9.x 20	5.9	9.	0	0	60.9	13.5	2.42	1.17	16.80	2.47
C 9.x 15	4.4	9.	0	0	51.0	11.3	1.93	1.01	13.50	2.05
C 9.x 13	3.9	9.	0	0	47.9	10.6	1.76	0.96	12.50	1.95
C 8.x 19	5.5	8.	0	0	44.0	11.0	1.98	1.01	13.80	2.17
C 8.x 14	4.0	8.	0	0	36.1	9.0	1.53	0.85	10.90	1.73
C 8.x 12	3.4	8.	0	0	32.6	8.1	1.32	0.78	9.55	1.58
C 7.x 15	4.3	7.	0	0	27.2	7.8	1.38	0.78	9.68	1.64
C 7.x 12	3.6	7.	0	0	24.2	6.9	1.17	0.70	8.40	1.43
C 7.x 10	2.9	7.	0	0	21.3	6.1	0.97	0.63	7.12	1.26
C 6.x 13	3.8	6.	0	0	17.4	5.8	1.05	0.64	7.26	1.36
C 6.x 11	3.1	6.	0	0	15.2	5.1	0.87	0.56	6.15	1.15
C 6.x 8	2.4	6.	0	0	13.1	4.4	0.69	0.49	5.13	0.99
C 5.x 9	2.6	5.	0	0	8.9	3.6	0.63	0.45	4.36	0.92
C 5.x 7	2.0	5.	0	0	7.5	3.0	0.48	0.38	3.51	0.76
C 4.x 7	2.1	4.	0	0	4.6	2.3	0.43	0.34	2.81	0.70
C 4.x 5	1.6	4.	0	0	3.8	1.9	0.32	0.28	2.26	0.57
C 3.x 6	1.8	3.	0	0	2.1	1.4	0.31	0.27	1.72	0.54
C 3.x 5	1.5	3.	0	0	1.9	1.2	0.25	0.23	1.50	0.47
C 3.x 4	1.2	3.	0	0	1.7	1.1	0.20	0.20	1.30	0.40

Designation	A in^2	wgt k/ft	I_x in^4	S_x in^3	I_y in^4	S_y in^3	Z_x in^3	Z_y in^3
L 5.0x5.0x0.875	7.98	27.20	17.8	5.2	17.80	5.17	9.33	9.33
L 5.0x5.0x0.750	6.94	23.60	15.7	4.5	15.70	4.53	8.16	8.16
L 5.0x5.0x0.625	5.86	20.00	13.6	3.9	13.60	3.86	6.95	6.95
L 5.0x5.0x0.500	4.75	16.20	11.3	3.2	11.30	3.16	5.68	5.68
L 5.0x5.0x0.438	4.18	14.30	10.0	2.8	10.00	2.79	5.03	5.03
L 5.0x5.0x0.375	3.61	12.30	8.7	2.4	8.74	2.42	4.36	4.36
L 5.0x5.0x0.313	3.03	10.30	7.4	2.0	7.42	2.04	3.68	3.68
L 5.0x3.5x0.750	5.81	19.80	13.9	4.3	5.55	2.22	7.65	4.10
L 5.0x3.5x0.625	4.92	16.80	12.0	3.7	4.83	1.90	6.55	3.47
L 5.0x3.5x0.500	4.00	13.60	10.0	3.0	4.05	1.56	5.38	2.83
L 5.0x3.5x0.438	3.53	12.00	8.9	2.6	3.63	1.39	4.77	2.49
L 5.0x3.5x0.375	3.05	10.40	7.8	2.3	3.18	1.21	4.14	2.16
L 5.0x3.5x0.313	2.56	8.70	6.6	1.9	2.72	1.02	3.49	1.82
L 5.0x3.5x0.250	2.06	7.00	5.4	1.6	2.23	0.83	2.83	1.47
L 5.0x3.0x0.625	4.61	15.70	11.4	3.5	3.06	1.39	6.27	2.61
L 5.0x3.0x0.500	3.75	12.80	9.4	2.9	2.58	1.15	5.16	2.11
L 5.0x3.0x0.438	3.31	11.30	8.4	2.6	2.32	1.02	4.57	1.86
L 5.0x3.0x0.375	2.86	9.80	7.4	2.2	2.04	0.89	3.97	1.60
L 5.0x3.0x0.313	2.40	8.20	6.3	1.9	1.75	0.75	3.36	1.35
L 5.0x3.0x0.250	1.94	6.60	5.1	1.5	1.44	0.61	2.72	1.09
L 4.0x4.0x0.750	5.44	18.50	7.7	2.8	7.67	2.81	5.07	5.07
L 4.0x4.0x0.625	4.61	15.70	6.7	2.4	6.66	2.40	4.33	4.33
L 4.0x4.0x0.500	3.75	12.80	5.6	2.0	5.56	1.97	3.56	3.56
L 4.0x4.0x0.438	3.31	11.30	5.0	1.8	4.97	1.75	3.16	3.16
L 4.0x4.0x0.375	2.86	9.80	4.4	1.5	4.36	1.52	2.74	2.74
L 4.0x4.0x0.313	2.40	8.20	3.7	1.3	3.71	1.29	2.32	2.32
L 4.0x4.0x0.250	1.94	6.60	3.0	1.0	3.04	1.05	1.88	1.88
L 4.0x3.5x0.500	3.50	11.90	5.3	1.9	3.79	1.52	3.50	2.73
L 4.0x3.5x0.438	3.09	10.60	4.8	1.7	3.40	1.35	3.11	2.42
L 4.0x3.5x0.375	2.67	9.10	4.2	1.5	2.95	1.16	2.71	2.11
L 4.0x3.5x0.313	2.25	7.70	3.6	1.3	2.55	0.99	2.29	1.78
L 4.0x3.5x0.250	1.81	6.20	2.9	1.0	2.09	0.81	1.86	1.44
L 4.0x3.0x0.500	3.25	11.10	5.1	1.9	2.42	1.12	3.41	2.03
L 4.0x3.0x0.438	2.87	9.80	4.5	1.7	2.18	0.99	3.03	1.79

3.7 Joists

43 Steel joists, Fig. 3.8 look like shallow trusses (warren type) and are designed as simply supported uniformly loaded beams assuming that they are laterally supported on the top (to prevent lateral torsional buckling). The lateral support is often profided by the concrete slab it suppors.

44 The standard open-web joist designation consists of the depth, the series designation and the chord type. Three series are available for floor/roof construction, Table 3.3

Series	Depth (in)	Span (ft)
K	8-30	8-60
LH	18-48	25-96
DLH	52-72	89-120

Table 3.3: Joist Series Characteristics

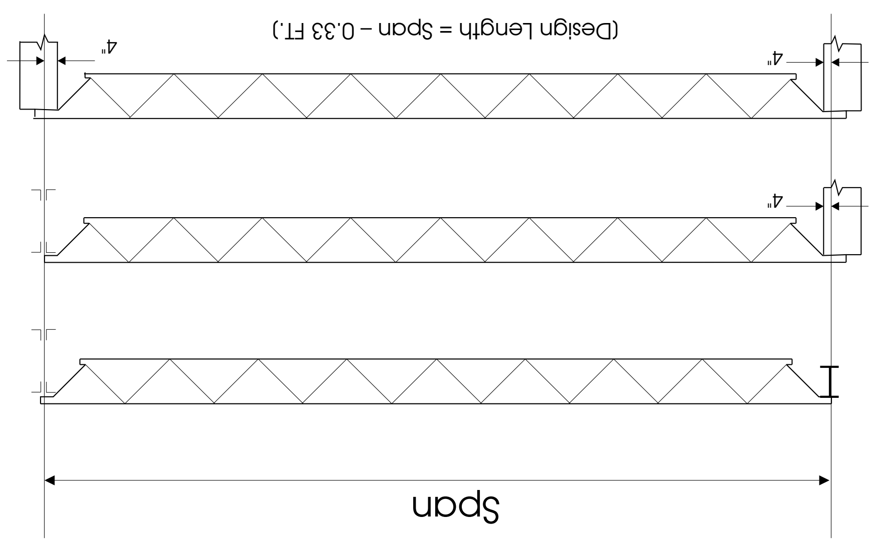


Figure 3.8: prefabricated Steel Joists

45 Typical joist spacing ranges from 2 to 4 ft, and provides an efficient use of the corrugated steel deck which itself supports the concrete slab.

46 For preliminary estimates of the joist depth, a depth to span ratio of 24 can be assumed, therefore

$$\boxed{d \approx L/2} \tag{3.5}$$

where d is in inches, and L in ft.

47 Table 3.4 list the load carrying capacity of open web, K-series steel joists based on a amximum allowable stress of 30 ksi. For each span, the first line indicates the total safe uniformly

Joint Desig.	8K1	10K1	12K1	12K3	12K5	14K1	14K3	14K4	14K6	16K2	16K3	16K4	16K5	16K6	16K7	16K9
Depth (in.)	8	10	12	12	12	14	14	14	14	16	16	16	16	16	16	16
≈ W (lbs/ft)	5.1	5	5	5.7	7.1	5.2	6	6.7	7.7	5.5	6.3	7	7.5	8.1	8.6	10.0
Span (ft.)																
8	550 550															
9	550 550															
10	550 480	550 550														
11	532 377	550 542														
12	444 288	550 455	550 550	550 550	550 550											
13	377 225	479 363	550 510	550 510	550 510											
14	324 179	412 289	500 425	550 463	550 463	550 550	550 550	550 550	550 550							
15	281 145	358 234	434 344	543 428	550 434	511 475	550 507	550 507	550 507							
16	246 119	313 192	380 282	476 351	550 396	448 390	550 467	550 467	550 467	550 550	550 550	550 550	550 550	550 550	550 550	550 550
17		277 159	336 234	420 291	550 366	395 324	495 404	550 443	550 443	512 488	550 526	550 526	550 526	550 526	550 526	550 526
18		246 134	299 197	374 245	507 317	352 272	441 339	530 397	550 408	456 409	508 456	550 490	550 490	550 490	550 490	550 490
19		221 113	268 167	335 207	454 269	315 230	395 287	475 336	550 383	408 347	455 386	547 452	550 455	550 455	550 455	550 455
20		199 97	241 142	302 177	409 230	284 197	356 246	428 287	525 347	368 297	410 330	493 386	550 426	550 426	550 426	550 426
21			218 123	273 153	370 198	257 170	322 212	388 248	475 299	333 255	371 285	447 333	503 373	548 405	550 406	550 406
22			199 106	249 132	337 172	234 147	293 184	353 215	432 259	303 222	337 247	406 289	458 323	498 351	550 385	550 385
23			181 93	227 116	308 150	214 128	268 160	322 188	395 226	277 194	308 216	371 252	418 282	455 307	507 339	550 363
24			166 81	208 101	282 132	196 113	245 141	295 165	362 199	254 170	283 189	340 221	384 248	418 269	465 298	550 346
25						180 100	226 124	272 145	334 175	234 150	260 167	313 195	353 219	384 238	428 263	514 311
26						166 88	209 110	251 129	308 56	216 133	240 148	289 173	326 194	355 211	395 233	474 276
27						154 79	193 98	233 115	285 139	200 119	223 132	268 155	302 173	329 188	366 208	439 246
28						143 70	180 88	216 103	265 124	186 106	207 118	249 138	281 155	306 168	340 186	408 220
29										173 95	193 106	232 124	261 139	285 151	317 167	380 198
30										161 86	180 96	216 112	244 126	266 137	296 151	355 178
31										151 78	168 87	203 101	228 114	249 124	277 137	332 161
32										142 71	158 79	190 92	214 103	233 112	259 124	311 147

Table 3.4: Joist Properties

Chapter 4

Case Study I: EIFFEL TOWER

`Adapted from (Billington and Mark 1983)`

4.1 Materials, & Geometry

1 The tower was built out of wrought iron, less expensive than steel,and Eiffel had more expereince with this material, Fig. 4.1

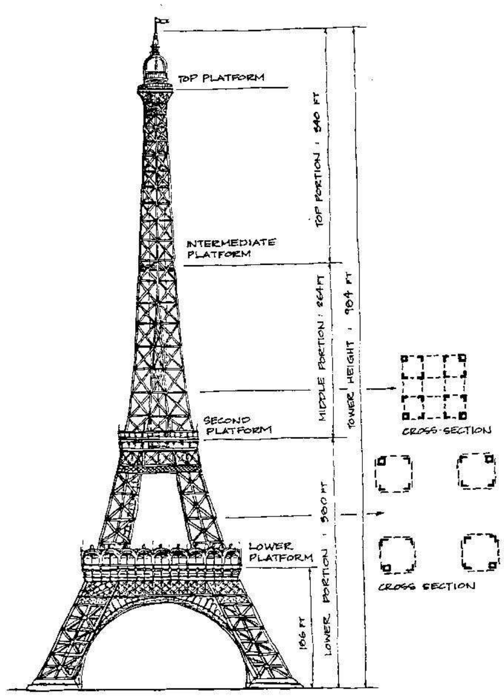


Figure 4.1: Eiffel Tower (Billington and Mark 1983)

Location	Height	Width/2	Width Estimated	Width Actual	$\frac{dy}{dx}$	β
Support	0	164	328		.333	18.4^o
First platform	186	108	216	240	.270	15.1^o
second platform	380	62	123	110	.205	11.6^o
Intermediate platform	644	20	40		.115	6.6^o
Top platform	906	1	2		.0264	1.5^o
Top	984	0	0		0.000	0^o

4 The tower is supported by four inclined supports, each with a cross section of 800 in^2. An idealization of the tower is shown in Fig. 4.2.

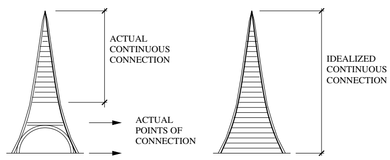


Figure 4.2: Eiffel Tower Idealization, (Billington and Mark 1983)

4.2 Loads

5 The total weight of the tower is 18, 800 k.

6 The dead load is not uniformly distributed, and is approximated as follows, Fig. 4.3:

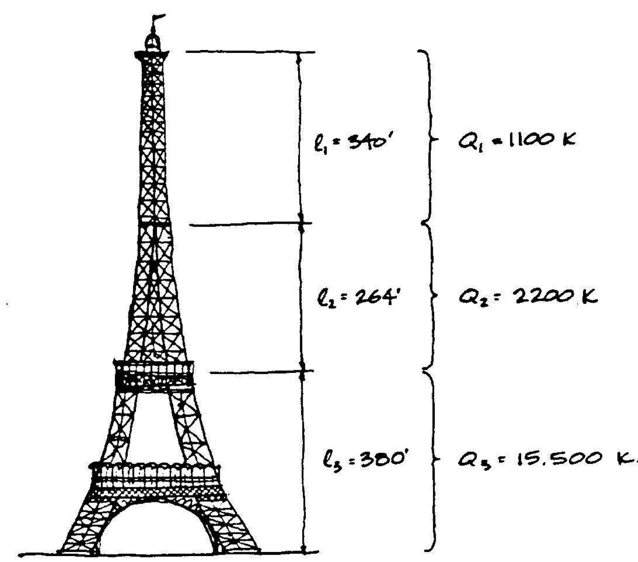


Figure 4.3: Eiffel Tower, Dead Load Idealization; (Billington and Mark 1983)

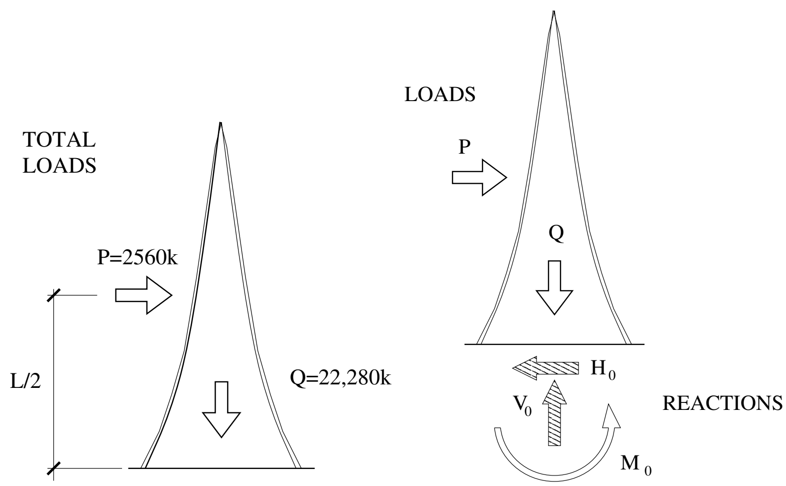


Figure 4.5: Eiffel Tower, Wind Loads, (Billington and Mark 1983)

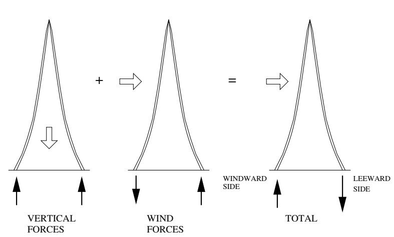


Figure 4.6: Eiffel Tower, Reactions; (Billington and Mark 1983)

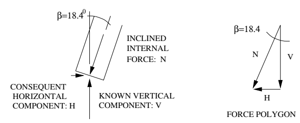


Figure 4.7: Eiffel Tower, Internal Gravity Forces; (Billington and Mark 1983)

12 **Gravity load** are first considered, remember those are caused by the dead load and the live load, Fig. 4.7:

$$\cos\beta = \frac{V}{N} \Rightarrow N = \frac{V}{\cos\beta} \tag{4.12-a}$$

$$N = \frac{11,140 \text{ k}}{\cos 18.4^o} = \boxed{11{,}730 \text{ kip}} \tag{4.12-b}$$

$$\tan\beta = \frac{H}{V} \Rightarrow H = V\tan\beta \tag{4.12-c}$$

$$H = 11,140 \text{ k}(\tan 18.4^o) = \boxed{3{,}700 \text{ kip}} \tag{4.12-d}$$

The horizontal forces which must be resisted by the foundations, Fig. 4.8.

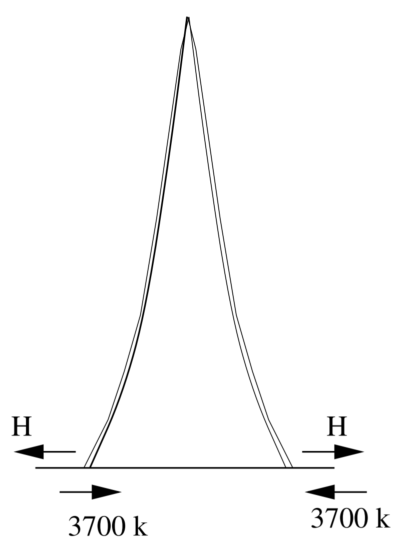


Figure 4.8: Eiffel Tower, Horizontal Reactions; (Billington and Mark 1983)

13 Because the vertical load decreases with height, the axial force will also decrease with height.

14 At the second platform, the total vertical load is $Q = 1,100 + 2,200 = 3,300$ k and at that height the angle is 11.6^o thus the axial force (per pair of columns) will be

$$N_{\text{vert}} = \frac{\frac{3{,}300 \text{ k}}{2}}{\cos 11.6^o} = 1,685 \text{ k} \tag{4.13-a}$$

$$H_{\text{vert}} = \frac{3,300 \text{ k}}{2}(\tan 11.6^o) = 339 \text{ k} \tag{4.13-b}$$

Note that this is about seven times smaller than the axial force at the base, which for a given axial strength, would lead the designer to reduce (or taper) the cross-section.

Chapter 5

REVIEW of STATICS

To every action there is an equal and opposite reaction.

Newton's third law of motion

5.1 Reactions

1 In the analysis of structures (hand calculations), it is often easier (but not always necessary) to start by determining the reactions.

2 Once the reactions are determined, internal forces are determined next; finally, internal stresses and/or deformations (deflections and rotations) are determined last[1].

3 Reactions are necessary to determine **foundation load**.

4 Depending on the type of structures, there can be different types of support conditions, Fig. 5.1.

Roller: provides a restraint in only one direction in a 2D structure, in 3D structures a roller may provide restraint in one or two directions. A roller will allow rotation.

Hinge: allows rotation but no displacements.

Fixed Support: will prevent rotation and displacements in all directions.

5.1.1 Equilibrium

5 Reactions are determined from the appropriate equations of static equilibrium.

6 Summation of forces and moments, **in a static system** must be equal to zero[2].

[1] This is the sequence of operations in the **flexibility** method which lends itself to hand calculation. In the **stiffness** method, we determine displacements firsts, then internal forces and reactions. This method is most suitable to computer implementation.

[2] In a dynamic system $\Sigma F = ma$ where m is the mass and a is the acceleration.

Structure Type	**Equations**					
Beam, no axial forces		ΣF_y				ΣM_z
2D Truss, Frame, Beam	ΣF_x	ΣF_y				ΣM_z
Grid			ΣF_z	ΣM_x	ΣM_y	
3D Truss, Frame	ΣF_x	ΣF_y	ΣF_z	ΣM_x	ΣM_y	ΣM_z
Alternate Set						
Beams, no axial Force	ΣM_z^A	ΣM_z^B				
2 D Truss, Frame, Beam	ΣF_x	ΣM_z^A	ΣM_z^B			
	ΣM_z^A	ΣM_z^B	ΣM_z^C			

Table 5.1: Equations of Equilibrium

2. Assume a direction for the unknown quantities

3. The right hand side of the equation should be zero

If your reaction is negative, then it will be in a direction opposite from the one assumed.

16 Summation of external forces is equal and **opposite** to the internal ones. Thus the net force/moment is equal to zero.

17 The external forces give rise to the (non-zero) shear and moment diagram.

5.1.2 Equations of Conditions

18 If a structure has an **internal hinge** (which may connect two or more substructures), then this will provide an additional equation ($\Sigma M = 0$ at the hinge) which can be exploited to determine the reactions.

19 Those equations are often exploited in trusses (where each connection is a hinge) to determine reactions.

20 In an **inclined roller** support with S_x and S_y horizontal and vertical projection, then the reaction R would have, Fig. 5.2.

$$\boxed{\frac{R_x}{R_y} = \frac{S_y}{S_x}} \tag{5.3}$$

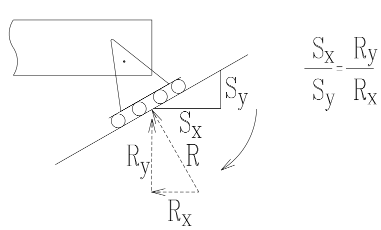


Figure 5.2: Inclined Roller Support

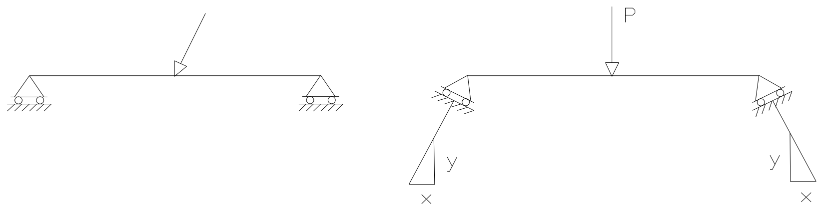


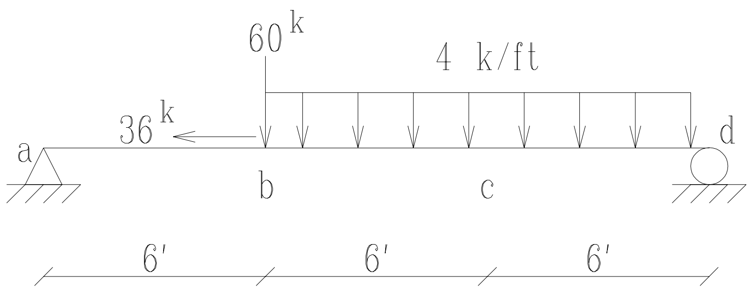
Figure 5.4: Geometric Instability Caused by Concurrent Reactions

5.1.5 Examples

29 Examples of reaction calculation will be shown next. Each example has been carefully selected as it brings a different "twist" from the preceding one. Some of those same problems will be revisited later for the determination of the internal forces and/or deflections. Many of those problems are taken from Prof. Gerstle textbok *Basic Structural Analysis*.

■ **Example 5-1: Simply Supported Beam**

Determine the reactions of the simply supported beam shown below.



Solution:
The beam has 3 reactions, we have 3 equations of static equilibrium, hence it is statically determinate.

$$
\begin{aligned}
(+\rightarrow)\ \Sigma F_x = 0; &\quad \Rightarrow \quad R_{ax} - 36 \text{ k} = 0 \\
(+\uparrow)\ \Sigma F_y = 0; &\quad \Rightarrow \quad R_{ay} + R_{dy} - 60 \text{ k} - (4) \text{ k/ft}(12) \text{ ft} = 0 \\
(+\circlearrowleft)\ \Sigma M_z^c = 0; &\quad \Rightarrow \quad 12R_{ay} - 6R_{dy} - (60)(6) = 0
\end{aligned}
$$

or through matrix inversion (on your calculator)

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 12 & -6 \end{bmatrix} \begin{Bmatrix} R_{ax} \\ R_{ay} \\ R_{dy} \end{Bmatrix} = \begin{Bmatrix} 36 \\ 108 \\ 360 \end{Bmatrix} \Rightarrow \begin{Bmatrix} R_{ax} \\ R_{ay} \\ R_{dy} \end{Bmatrix} = \begin{Bmatrix} 36 \text{ k} \\ 56 \text{ k} \\ 52 \text{ k} \end{Bmatrix}
$$

Alternatively we could have used another set of equations:

$$
\begin{aligned}
(+\circlearrowleft)\ \Sigma M_z^a = 0; &\quad (60)(6) + (48)(12) - (R_{dy})(18) = 0 \quad \Rightarrow \quad R_{dy} = \boxed{52 \text{ k} \uparrow} \\
(+\circlearrowleft)\ \Sigma M_z^d = 0; &\quad (R_{ay})(18) - (60)(12) - (48)(6) = 0 \quad \Rightarrow \quad R_{ay} = \boxed{56 \text{ k} \uparrow}
\end{aligned}
$$

2. Isolating bd:

$$(+\circlearrowleft)\ \Sigma M_d = 0; \quad -(17.7)(18) - (40)(15) - (4)(8)(8) - (30)(2) + R_{cy}(12) = 0$$
$$\Rightarrow R_{cy} = \frac{1{,}236}{12} = \boxed{103 \text{ k} \uparrow}$$
$$(+\circlearrowleft)\ \Sigma M_c = 0; \quad -(17.7)(6) - (40)(3) + (4)(8)(4) + (30)(10) - R_{dy}(12) = 0$$
$$\Rightarrow R_{dy} = \frac{201.3}{12} = \boxed{16.7 \text{ k} \uparrow}$$

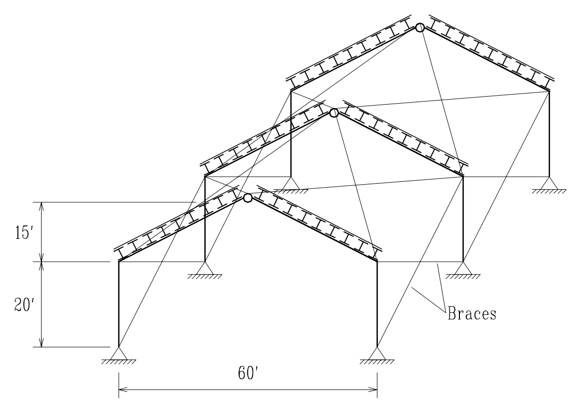
3. Check

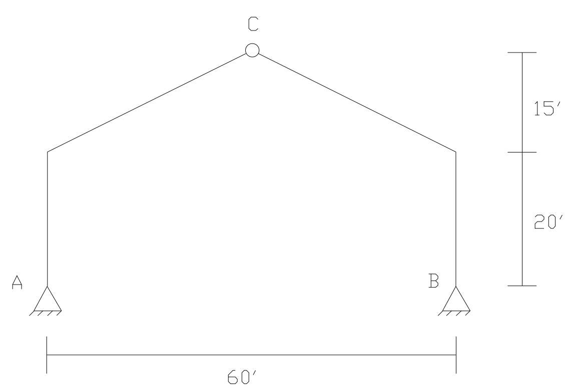
$$\Sigma F_y = 0; \uparrow; 22.2 - 40 - 40 + 103 - 32 - 30 + 16.7 = 0\surd$$

■

■ **Example 5-3: Three Hinged Gable Frame**

The three-hinged gable frames spaced at 30 ft. on center. Determine the reactions components on the frame due to: 1) Roof dead load, of 20 psf of roof area; 2) Snow load, of 30 psf of horizontal projection; 3) Wind load of 15 psf of vertical projection. Determine the critical design values for the vertical and horizontal reactions.





Solution:

1. Due to symmetry, we will consider only the dead load on one side of the frame.

5.2 Trusses

5.2.1 Assumptions

30 Cables and trusses are 2D or 3D structures composed of an assemblage of simple one dimensional components which transfer only **axial** forces along their axis.

31 Trusses are extensively used for bridges, long span roofs, electric tower, space structures.

32 For trusses, it is assumed that

1. Bars are **pin-connected**
2. Joints are frictionless hinges[4].
3. Loads are applied at the **joints only**.

33 A truss would typically be composed of triangular elements with the bars on the **upper chord** under compression and those along the **lower chord** under tension. Depending on the **orientation of the diagonals**, they can be under either tension or compression.

34 In a truss analysis or design, we seek to determine the internal force along each member, Fig. 5.5

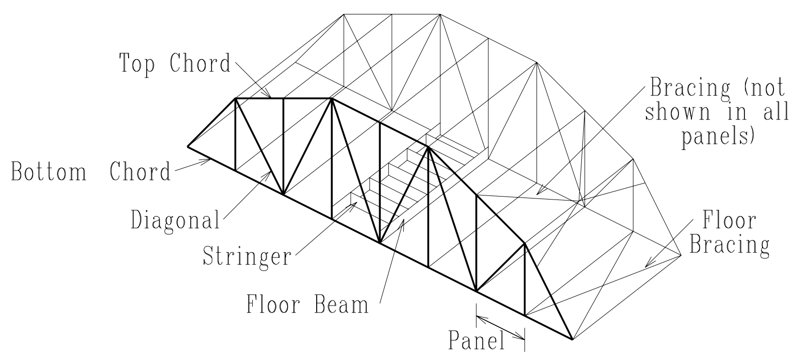


Figure 5.5: Bridge Truss

5.2.2 Basic Relations

Sign Convention: Tension positive, compression negative. On a truss the axial forces are indicated as forces acting on the joints.

Stress-Force: $\sigma = \frac{P}{A}$

Stress-Strain: $\sigma = E\varepsilon$

Force-Displacement: $\varepsilon = \frac{\Delta L}{L}$

[4]In practice the bars are riveted, bolted, or welded directly to each other or to gusset plates, thus the bars are not free to rotate and so-called **secondary bending moments** are developed at the bars. Another source of secondary moments is the dead weight of the element.

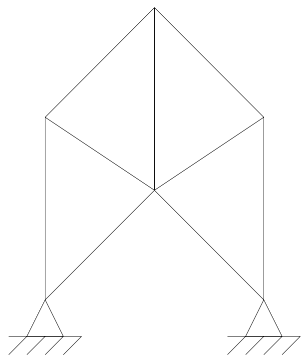


Figure 5.6: A Statically Indeterminate Truss

3. Sketch a free body diagram showing all joint loads (including reactions)
4. For each joint, and starting with the loaded ones, apply the appropriate equations of equilibrium (ΣF_x and ΣF_y in 2D; ΣF_x, ΣF_y and ΣF_z in 3D).
5. Because truss elements can only carry axial forces, the resultant force ($\vec{F} = \vec{F_x} + \vec{F_y}$) must be **along** the member, Fig. 5.7.

$$\boxed{\frac{F}{l} = \frac{F_x}{l_x} = \frac{F_y}{l_y}} \tag{5.4}$$

44 Always keep track of the x and y components of a member force (F_x, F_y), as those might be needed later on when considering the force equilibrium at another joint to which the member is connected.

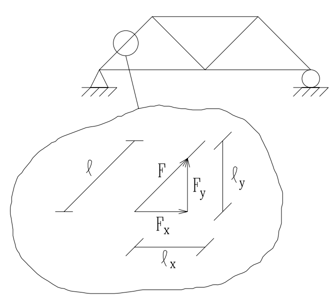
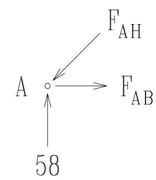


Figure 5.7: X and Y Components of Truss Forces

45 This method should be used when **all** member forces should be determined.

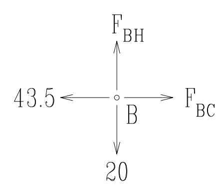
46 In truss analysis, there is **no sign convention**. A member is **assumed** to be under tension (or compression). If after analysis, the force is found to be negative, then this would imply that the wrong assumption was made, and that the member should have been under compression (or tension).

Node A: Clearly AH is under compression, and AB under tension.



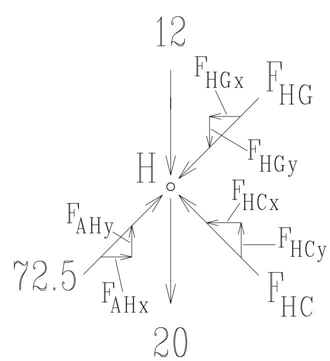
$$
\begin{array}{rll}
(+\uparrow)\,\Sigma F_y = 0; & \Rightarrow & F_{AH_y} - 58 = 0 \\
 & & F_{AH} = \frac{l}{l_y}(F_{AH_y}) \\
 & & l_y = 32 \qquad\qquad\qquad l = \sqrt{32^2+24^2} = 40 \\
 & \Rightarrow & F_{AH} = \frac{40}{32}(58) = 72.5 \qquad \text{Compression} \\
(+\rightarrow)\,\Sigma F_x = 0; & \Rightarrow & -F_{AH_x} + F_{AB} = 0 \\
 & & F_{AB} = \frac{l_x}{l_y}(F_{AH_y}) = \frac{24}{32}(58) = 43.5 \quad \text{Tension}
\end{array}
$$

Node B:



$$
\begin{array}{rllll}
(+\rightarrow)\,\Sigma F_x = 0; & \Rightarrow & F_{BC} & = & 43.5 \quad \text{Tension} \\
(+\uparrow)\,\Sigma F_y = 0; & \Rightarrow & F_{BH} & = & 20 \quad \text{Tension}
\end{array}
$$

Node H:



$$
\begin{array}{rll}
(+\rightarrow)\,\Sigma F_x = 0; & \Rightarrow & F_{AH_x} - F_{HC_x} - F_{HG_x} = 0 \\
 & & 43.5 - \frac{24}{\sqrt{24^2+32^2}}(F_{HC}) - \frac{24}{\sqrt{24^2+10^2}}(F_{HG}) = 0 \qquad \text{(I)} \\
(+\uparrow)\,\Sigma F_y = 0; & \Rightarrow & F_{AH_y} + F_{HC_y} - 12 - F_{HG_y} - 20 = 0 \\
 & & 58 + \frac{32}{\sqrt{24^2+32^2}}(F_{HC}) - 12 - \frac{10}{\sqrt{24^2+10^2}}(F_{HG}) - 20 = 0 \qquad \text{(II)}
\end{array}
$$

Solving for I and II we obtain

$$
\begin{array}{lll}
F_{HC} & = & -7.5 \quad \text{Tension} \\
F_{HG} & = & 52 \quad \text{Compression}
\end{array}
$$

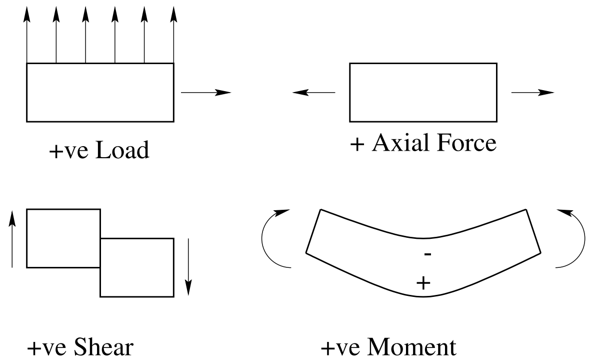


Figure 5.9: Shear and Moment Sign Conventions for Design

Load Positive along the beam's local y axis (assuming a right hand side convention), that is positive upward.

Axial: tension positive.

Flexure A positive moment is one which causes tension in the lower fibers, and compression in the upper ones. For frame members, a positive moment is one which causes tension along the inner side.

Shear A positive shear force is one which is "up" on a negative face, or "down" on a positive one. Alternatively, a pair of positive shear forces will cause clockwise rotation.

Torsion Counterclockwise positive

3D: Use double arrow vectors (and NOT curved arrows). Forces and moments (including torsions) are defined with respect to a right hand side coordinate system, Fig. 5.10.

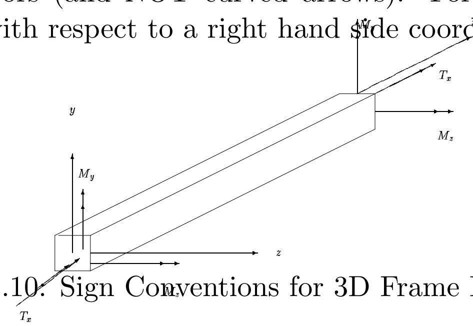


Figure 5.10: Sign Conventions for 3D Frame Elements

5.3.1.2 Load, Shear, Moment Relations

50 Let us (re)derive the basic relations between load, shear and moment. Considering an infinitesimal length dx of a beam subjected to a positive load[6] $w(x)$, Fig. 5.11. The infinitesimal section must also be in equilibrium.

51 There are no axial forces, thus we only have two equations of equilibrium to satisfy $\Sigma F_y = 0$ and $\Sigma M_z = 0$.

52 Since dx is infinitesimally small, the small variation in load along it can be neglected, therefore we assume $w(x)$ to be constant along dx.

[6]In this derivation, as in all other ones we should assume all quantities to be positive.

The change in moment between 1 and 2, ΔM_{21}, is equal to the area under the shear curve between x_1 and x_2.

57 Note that we still need to have V_1 and M_1 in order to obtain V_2 and M_2 respectively.

58 Fig. 5.12 and 5.13 further illustrates the variation in internal shear and moment under uniform and concentrated forces/moment.

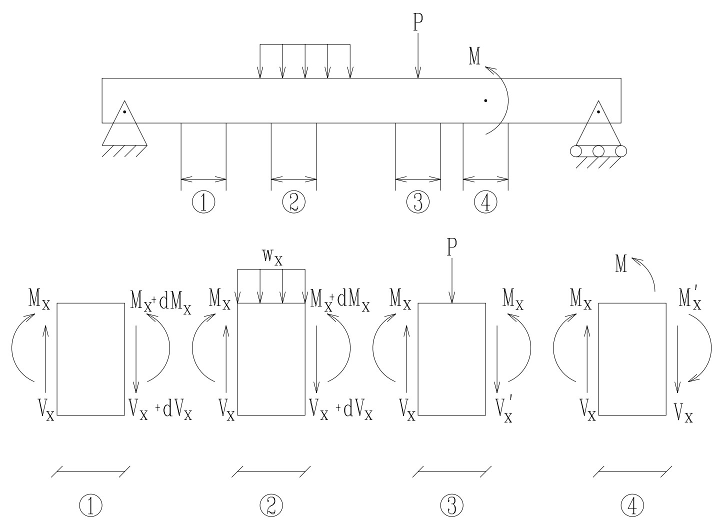


Figure 5.12: Shear and Moment Forces at Different Sections of a Loaded Beam

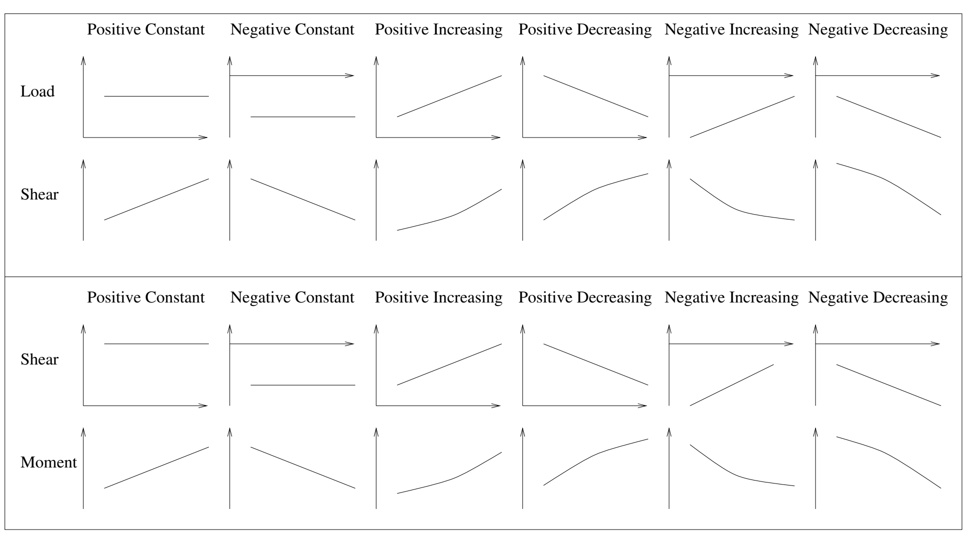
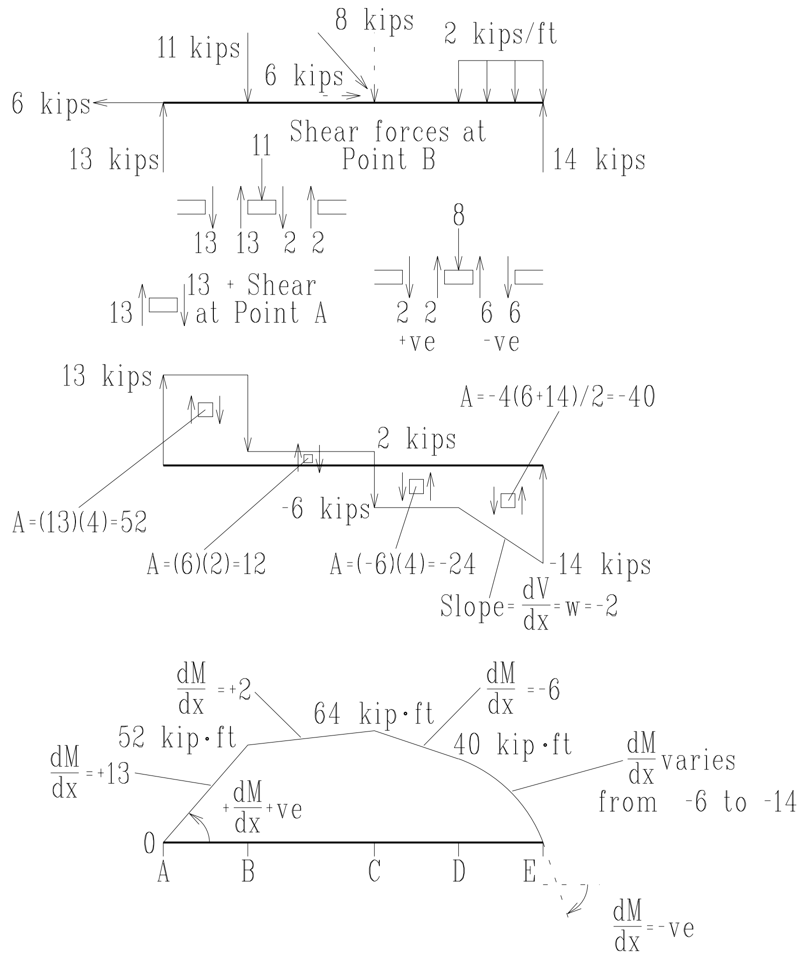


Figure 5.13: Slope Relations Between Load Intensity and Shear, or Between Shear and Moment



Reactions are determined from the equilibrium equations

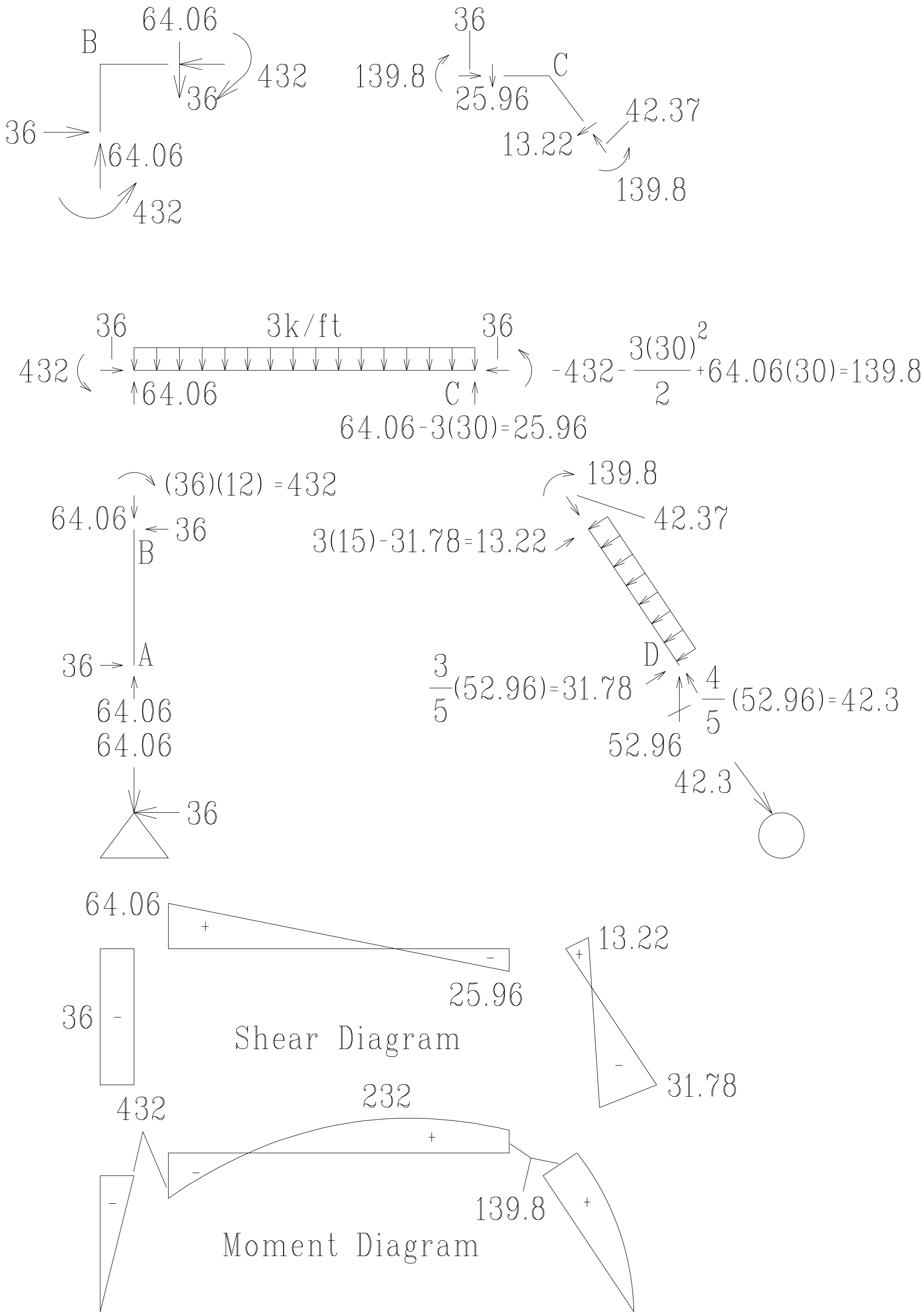
$$
\begin{array}{rl}
(+ \rightarrow) \Sigma F_x = 0; & \Rightarrow \quad -R_{A_x} + 6 = 0 \Rightarrow R_{A_x} = 6 \text{ k} \\
(+ \curvearrowleft) \Sigma M_A = 0; & \Rightarrow \quad (11)(4) + (8)(10) + (4)(2)(14+2) - R_{F_y}(18) = 0 \Rightarrow R_{F_y} = 14 \text{ k} \\
(+ \uparrow) \Sigma F_y = 0; & \Rightarrow \quad R_{A_y} - 11 - 8 - (4)(2) + 14 = 0 \Rightarrow R_{A_y} = 13 \text{ k}
\end{array}
$$

Shear are determined next.

1. At A the shear is equal to the reaction and is positive.
2. At B the shear drops (negative load) by 11 k to 2 k.
3. At C it drops again by 8 k to -6 k.
4. It stays constant up to D and then it decreases (constant negative slope since the load is uniform and negative) by 2 k per linear foot up to -14 k.
5. As a check, -14 k is also the reaction previously determined at F.

Moment is determined last:

1. The moment at A is zero (hinge support).
2. The change in moment between A and B is equal to the area under the corresponding shear diagram, or $\Delta M_{B-A} = (13)(4) = 52$.


B
64.06
432
36
36
64.06
432
36
139.8
25.96
C
42.37
13.22
139.8
36
3k/ft
36
432
64.06
C
-432-3(30)^2/2+64.06(30)=139.8
64.06-3(30)=25.96
(36)(12) =432
64.06
36
B
36
A
64.06
64.06
36
139.8
42.37
3(15)-31.78=13.22
D
3/5(52.96)=31.78
4/5(52.96)=42.3
52.96
42.3
64.06
36
25.96
Shear Diagram
13.22
31.78
432
232
139.8
Moment Diagram

3. If we need to determine the maximum moment along $B-C$, we know that $\frac{dM_{B-C}}{dx} = 0$ at the point where $V_{B-C} = 0$, that is $V_{B-C}(x) = 64.06 - 3x = 0 \Rightarrow x = \frac{64.06}{3} =$ 25.0 ft. In other words, maximum moment occurs where the shear is zero.
   Thus $M_{B-C}^{max} = -432 + 64.06(25.0) - 3\frac{(25.0)^2}{2} = -432 + 1,601.5 - 937.5 = 232$ k.ft
4. Finally along $C - D$, the moment varies quadratically (since we had a linear shear), the moment first increases (positive shear), and then decreases (negative shear). The moment along $C - D$ is given by

$$\begin{aligned} M_{C-D} &= M_C + \int_0^x V_{C-D}(x)dx = 139.8 + \int_0^x (13.22 - 3x)dx \\ &= 139.8 + 13.22x - 3\frac{x^2}{2} \end{aligned}$$

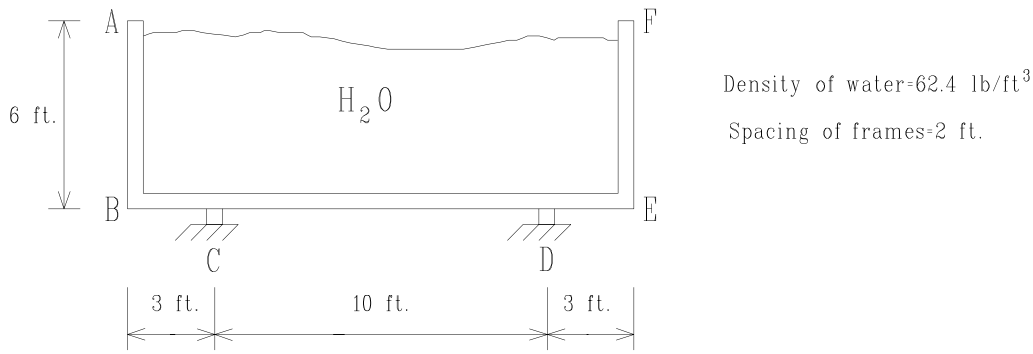
   which is a parabola.
   Substituting for $x = 15$, we obtain at node C $M_C = 139.8 + 13.22(15) - 3\frac{15^2}{2} = 139.8 + 198.3 - 337.5 = 0\surd$

■

■ **Example 5-7: Frame Shear and Moment Diagram; Hydrostatic Load**

The frame shown below is the structural support of a flume. Assuming that the frames are spaced 2 ft apart along the length of the flume,

1. Determine all internal member end actions
2. Draw the shear and moment diagrams
3. Locate and compute maximum internal bending moments
4. If this is a reinforced concrete frame, show the location of the reinforcement.



Solution:

The hydrostatic pressure causes lateral forces on the vertical members which can be treated as cantilevers fixed at the lower end.

1. Base at B the shear force was determined earlier and was equal to 2.246 k. Based on the orientation of the $x-y$ axis, this is a negative shear.
2. The vertical shear at B is zero (neglecting the weight of $A-B$)
3. The shear to the left of C is $V = 0 + (-.749)(3) = -2.246$ k.
4. The shear to the right of C is $V = -2.246 + 5.99 = 3.744$ k

Moment diagrams

1. At the base: B $M = 4.493$ k.ft as determined above.
2. At the support C, $M_c = -4.493 + (-.749)(3)(\frac{3}{2}) = -7.864$ k.ft
3. The maximum moment is equal to $M_{max} = -7.864 + (.749)(5)(\frac{5}{2}) = 1.50$ k.ft

Design: Reinforcement should be placed along the fibers which are under tension, that is on the side of the negative moment[7]. The figure below schematically illustrates the location of the flexural[8] reinforcement.

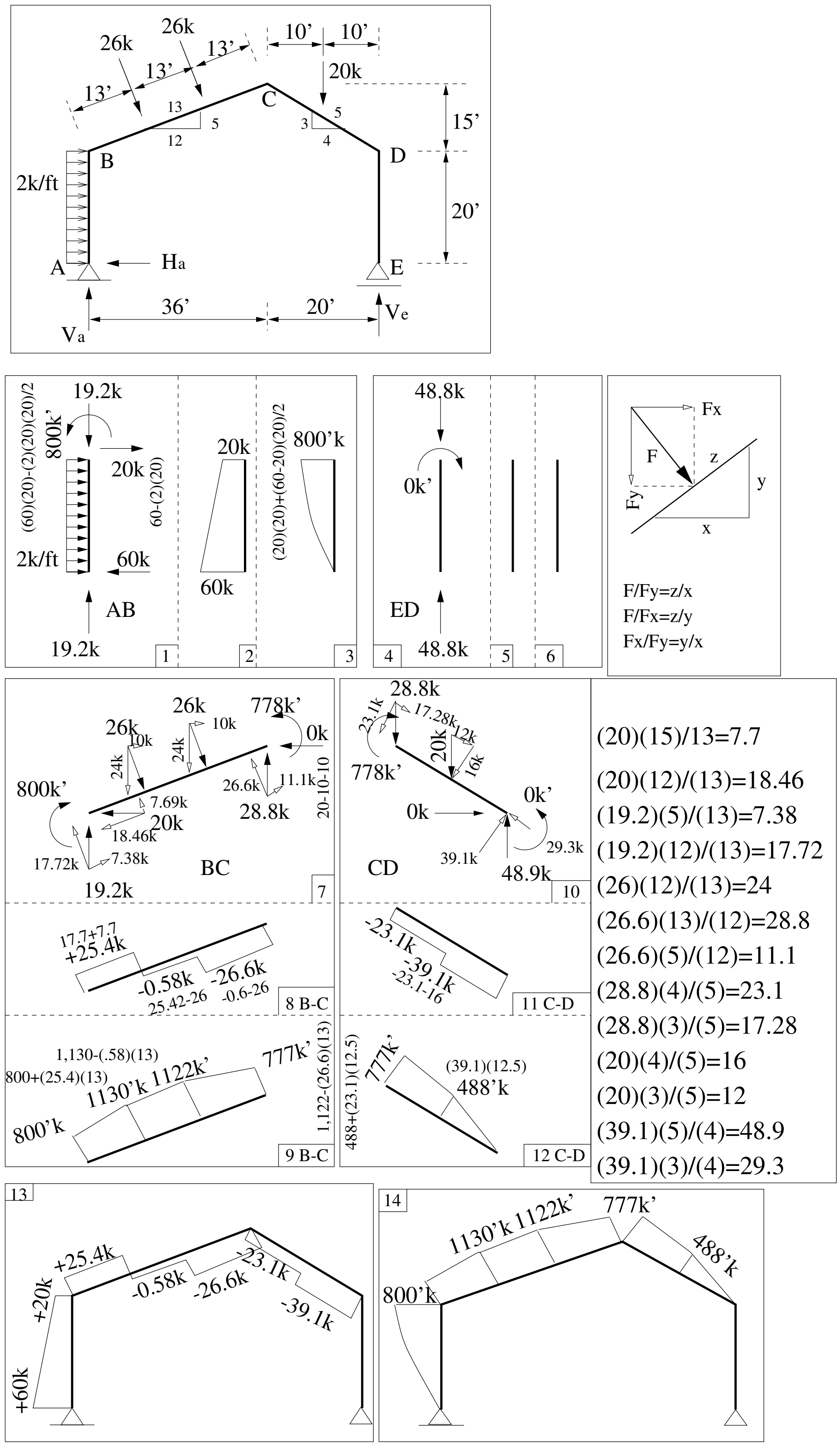
■

■ **Example 5-8: Shear Moment Diagrams for Frame**

[7]That is why in most European countries, the sign convention for design moments is the opposite of the one commonly used in the U.S.A.; Reinforcement should be placed where the moment is “postive”.

[8]Shear reinforcement is made of a series of vertical stirrups.

■ **Example 5-9: Shear Moment Diagrams for Inclined Frame**



■

where y is measured from the axis of rotation (neutral axis). Thus strains are proportional to the distance from the neutral axis.

64 ρ (Greek letter *rho*) is the **radius of curvature**. In some textbook, the **curvature** κ (Greek letter *kappa*) is also used where

$$\kappa = \frac{1}{\rho} \tag{5.14}$$

thus,

$$\boxed{\varepsilon_x = -\kappa y} \tag{5.15}$$

5.4.2 Stress-Strain Relations

65 So far we considered the kinematic of the beam, yet later on we will need to consider equilibrium in terms of the stresses. Hence we need to relate strain to stress.

66 For linear elastic material Hooke's law states

$$\boxed{\sigma_x = E\varepsilon_x} \tag{5.16}$$

where E is **Young's Modulus**.

67 Combining Eq. with equation 5.15 we obtain

$$\boxed{\sigma_x = -E\kappa y} \tag{5.17}$$

5.4.3 Internal Equilibrium; Section Properties

68 Just as external forces acting on a structure must be in equilibrium, the internal forces must also satisfy the equilibrium equations.

69 The internal forces are determined by *slicing* the beam. The internal forces on the "cut" section must be in equilibrium with the external forces.

5.4.3.1 $\Sigma F_x = 0$; Neutral Axis

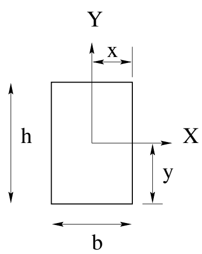
70 The first equation we consider is the summation of axial forces.

71 Since there are no external axial forces (unlike a column or a beam-column), the internal axial forces must be in equilibrium.

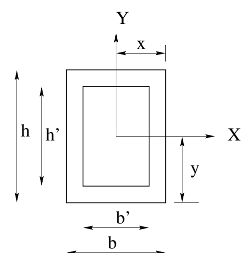
$$\Sigma F_x = 0 \Rightarrow \int_A \sigma_x dA = 0 \tag{5.18}$$

where σ_x was given by Eq. 5.17, substituting we obtain

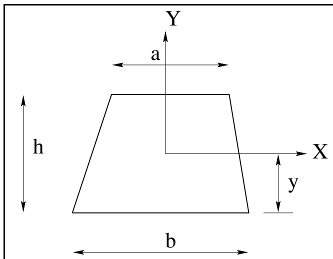
$$\int_A \sigma_x dA = -\int_A E\kappa y dA = 0 \tag{5.19-a}$$



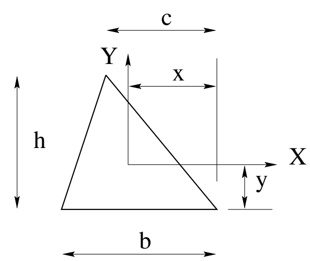
$$A = bh \quad x = \frac{b}{2} \quad y = \frac{h}{2} \quad I_x = \frac{bh^3}{12} \quad I_y = \frac{hb^3}{12}$$



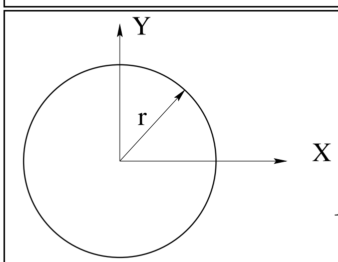
$$A = bh - b'h' \quad x = \frac{b}{2} \quad y = \frac{h}{2} \quad I_x = \frac{bh^3 - b'h'^3}{12} \quad I_y = \frac{hb^3 - h'b'^3}{12}$$



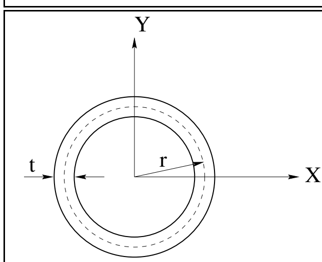
$$A = \frac{h(a+b)}{2} \quad y = \frac{h(2a+b)}{3(a+b)} \quad I_x = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$$



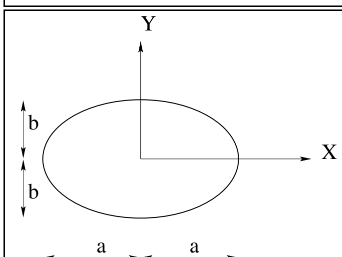
$$A = \frac{bh}{2} \quad x = \frac{b+c}{3} \quad y = \frac{h}{3} \quad I_x = \frac{bh^3}{36} \quad I_y = \frac{bh}{36}(b^2 - bc + c^2)$$



$$A = \pi r^2 = \frac{\pi d^2}{4} \quad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$



$$A = 2\pi r t = \pi d t \quad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$



$$A = \pi ab \quad I_x = \frac{\pi ab^3}{3} \quad I_y = \frac{\pi ba^3}{4}$$

Table 5.3: Section Properties

5. We now set those two values equal to their respective maximum

$$\Delta_{max} = \frac{L}{360} = \frac{(20)\text{ ft}(12)\text{ in/ft}}{360} = 0.67\text{ in} = \frac{65.65}{r^3} \Rightarrow r = \sqrt[3]{\frac{65.65}{0.67}} = 4.61\text{ in} \quad (5.31\text{-a})$$

$$\sigma_{max} = (18)\text{ ksi} = \frac{764}{r^2} \Rightarrow r = \sqrt{\frac{764}{18}} = \boxed{6.51\text{ in}} \quad (5.31\text{-b})$$

■

5.4.5 Approximate Analysis

80 From Fig. 5.14, and Eq. 5.25 ($\frac{M}{EI} = \kappa = \frac{1}{\rho}$), we recall that that the moment is directly proportional to the curvature κ.

81 Thus,

1. A positive and negative moment would correspond to positive and negative curvature respectively (adopting the sign convention shown in Fig. 5.14).
2. A zero moment correspnds to an inflection point in the deflected shape.

82 Hence, for

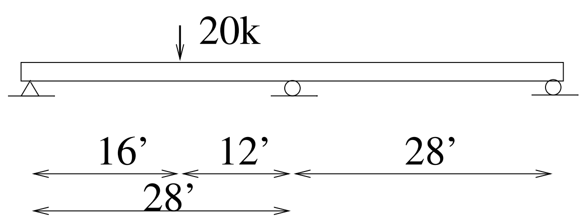
Statically determinate structure, we can determine the deflected shape from the moment diagram, Fig. 5.15.

Statically indeterminate structure, we can:

1. Plot the deflected shape.
2. Identify inflection points, approximate their location.
3. Locate those inflection points on the structure, which will then become statically determinate.
4. Perform an approximate analysis.

■ **Example 5-11: Approximate Analysis of a Statically Indeterminate beam**

Perform an approximate analysis of the following beam, and compare your results with the exact solution.



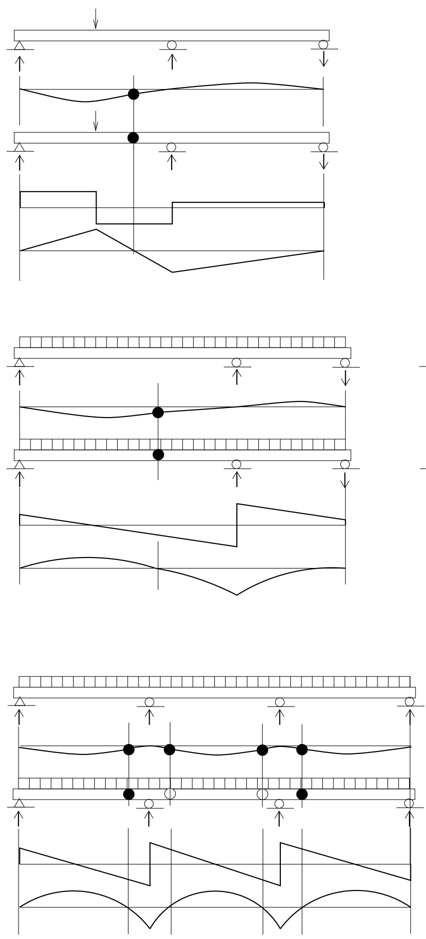


Figure 5.16: Approximate Analysis of Beams

6. Check

$$(+\curvearrowleft)\, \Sigma M_A = 0; \quad (20)(16) - (R_C)(28) + (R_D)(28+28) = \\ 320 - (17.67)(28) + (3.12)(56) = \\ 320 - 494.76 + 174.72 = 0\surd \tag{5.34-a}$$

7. The moments are determined next

$$M_{max} = R_A a = (5.45)(16) = \boxed{87.2} \tag{5.35-a}$$

$$M_1 = R_D L = (3.12)(28) = \boxed{87.36} \tag{5.35-b}$$

8. We now compare with the exact solution from Section **??**, solution 21 where: $L = 28$, $a = 16$, $b = 12$, and $P = 20$

$$R_1 = R_A = \frac{Pb}{4L^3}\left[4L^2 - a(L+a)\right]$$

$$= \frac{(20)(12)}{4(28)^3}\left[4(28)^2 - (16)(28+16)\right] = \boxed{6.64} \tag{5.36-a}$$

$$R_2 = R_B = \frac{Pa}{2L^3}\left[2L^2 + b(L+a)\right] \tag{5.36-b}$$

$$= \frac{(20)(16)}{2(28)^3}\left[2(28)^2 + 12(28+16)\right] = \boxed{15.28} \tag{5.36-c}$$

$$R_3 = R_D = -\frac{Pab}{4L^3}(L+a) \tag{5.36-d}$$

$$= -\frac{(20)(16)(12)}{4(28)^3}(28+16) = \boxed{-1.92} \tag{5.36-e}$$

$$M_{max} = R_1 a = (6.64)(16) = \boxed{106.2} \tag{5.36-f}$$

$$M_1 = R_3 L = (1.92)(28) = \boxed{53.8} \tag{5.36-g}$$

9. If we tabulate the results we have

Value	Approximate	Exact	% Error
R_A	5.45	6.64	18
R_C	17.67	15.28	-16
R_D	3.12	1.92	63
M_1	87.36	53.8	62
M_{max}	87.2	106.2	18

10. Whereas the correlation between the approximate and exact results is quite poor, one should not underestimate the simplicity of this method keeping in mind (an exact analysis of this structure would have been computationally much more involved). Furthermore, often one only needs a rough order of magnitude of the moments.

■

Chapter 6

Case Study II: GEORGE WASHINGTON BRIDGE

6.1 Theory

1 Whereas the forces in a cable can be determined from statics alone, its configuration must be derived from its deformation. Let us consider a cable with distributed load $p(x)$ **per unit horizontal projection** of the cable length (thus neglecting the weight of the cable). An infinitesimal portion of that cable can be assumed to be a straight line, Fig. 6.1 and in the absence of any horizontal load we have H =constant. Summation of the vertical forces yields

$$(+\downarrow)\,\Sigma F_y = 0 \Rightarrow -V + wdx + (V + dV) = 0 \tag{6.1-a}$$
$$dV + wdx = 0 \tag{6.1-b}$$

where V is the vertical component of the cable tension at x (Note that if the cable was subjected to its own weight then we would have wds instead of wdx). Because the cable must be tangent to T, we have

$$\tan\theta = \frac{V}{H} \tag{6.2}$$

Substituting into Eq. 6.1-b yields

$$d(H\tan\theta) + wdx = 0 \Rightarrow -\frac{d}{dx}(H\tan\theta) = w \tag{6.3}$$

2 But H is constant (no horizontal load is applied), thus, this last equation can be rewritten as

$$-H\frac{d}{dx}(\tan\theta) = w \tag{6.4}$$

3 Written in terms of the vertical displacement v, $\tan\theta = \frac{dv}{dx}$ which when substituted in Eq. 6.4 yields the **governing equation for cables**

$$-Hv'' = w \tag{6.5}$$

4 For a cable subjected to a uniform load w, we can determine its shape by double integration of Eq. 6.5

$$-Hv' = wx + C_1 \tag{6.6-a}$$

$$-Hv = \frac{wx^2}{2} + C_1x + C_2 \tag{6.6-b}$$

and the constants of integrations C_1 and C_2 can be obtained from the boundary conditions: $v = 0$ at $x = 0$ and at $x = L \Rightarrow C_2 = 0$ and $C_1 = -\frac{wL}{2}$. Thus

$$v = \frac{w}{2H}x(L-x) \tag{6.7}$$

This equation gives the shape $v(x)$ in terms of the horizontal force H,

5 Since the maximum sag h occurs at midspan ($x = \frac{L}{2}$) we can solve for the horizontal force

$$\boxed{H = \frac{wL^2}{8h}} \tag{6.8}$$

we note the analogy with the maximum moment in a simply supported uniformly loaded beam $M = Hh = \frac{wL^2}{8}$. Furthermore, this relation clearly shows that the horizontal force is inversely proportional to the sag h, as $h \searrow H \nearrow$. Finally, we can rewrite this equation as

$$r \stackrel{\text{def}}{=} \frac{h}{L} \tag{6.9-a}$$

$$\frac{wL}{H} = 8r \tag{6.9-b}$$

6 Eliminating H from Eq. 6.7 and 6.8 we obtain

$$\boxed{v = 4h\left(-\frac{x^2}{L^2} + \frac{x}{L}\right)} \tag{6.10}$$

Thus the cable assumes a parabolic shape (as the moment diagram of the applied load).

7 Whereas the horizontal force H is constant throughout the cable, the tension T is not. The maximum tension occurs at the support where the vertical component is equal to $V = \frac{wL}{2}$ and the horizontal one to H, thus

$$T_{\max} = \sqrt{V^2 + H^2} = \sqrt{\left(\frac{wL}{2}\right)^2 + H^2} = H\sqrt{1 + \left(\frac{wL/2}{H}\right)^2} \tag{6.11}$$

Combining this with Eq. 6.8 we obtain[1].

$$\boxed{T_{\max} = H\sqrt{1 + 16r^2} \approx H(1 + 8r^2)} \tag{6.12}$$

8 Had we assumed a uniform load w **per length of cable** (rather than horizontal projection), the equation would have been one of a catenary[2].

$$\boxed{v = \frac{H}{w}\cosh\left[\frac{w}{H}\left(\frac{L}{2} - x\right)\right] + h} \tag{6.13}$$

The cable between transmission towers is a good example of a catenary.

[1] Recalling that $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdot$ or $(1+b)^n = 1 + nb + \frac{n(n-1)b^2}{2!} + \frac{n(n-1)(n-2)b^3}{3!} + \cdots$; Thus for $b^2 << 1$, $\sqrt{1+b} = (1+b)^{\frac{1}{2}} \approx 1 + \frac{b}{2}$

[2] Derivation of this equation is beyond the scope of this course.

tower supports and are firmly anchored in both banks by huge blocks of concrete, the anchors.

14 Because the cables are much longer than they are thick (large $\frac{I}{L}$), they can be idealized a perfectly flexible members with no shear/bending resistance but with high axial strength.

15 The towers are 578 ft tall and rest on concrete caissons in the river. Because of our assumption regarding the roller support for the cables, the towers will be subjected only to axial forces.

6.2.2 Loads

16 The dead load is composed of the weight of the deck and the cables and is estimated at 390 and 400 psf respectively for the central and side spans respectively. Assuming an average width of 100 ft, this would be equivalent to

$$DL = (390) \text{ psf}(100) \text{ ft}\frac{\text{k}}{(1,000) \text{ lbs}} = 39 \text{ k/ft} \tag{6.14}$$

for the main span and 40 k/ft for the side ones.

17 For highway bridges, design loads are given by the AASHTO (Association of American State Highway Transportation Officials). The HS-20 truck is often used for the design of bridges on main highways, Fig. 6.3. Either the design truck with specified axle loads and spacing must be used **or** the equivalent uniform load and concentrated load. This loading must be placed such that maximum stresses are produced.

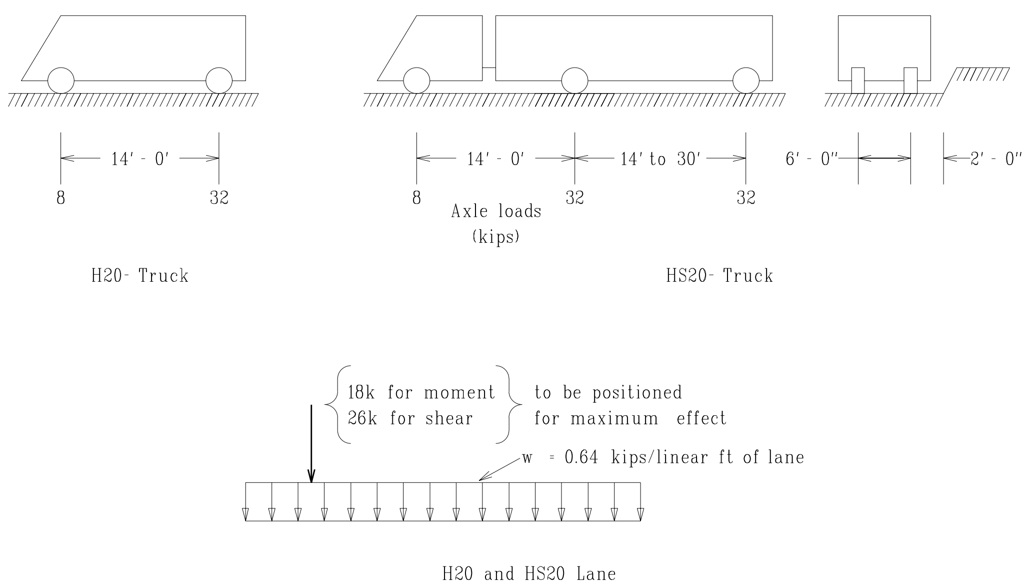


Figure 6.3: Truck Load

18 With two decks, we estimate that there is a total of 12 lanes or

$$LL = (12)\text{Lanes}(.64) \text{ k/ ft/Lane} = 7.68 \text{ k/ft} \approx 8 \text{ k/ft} \tag{6.15}$$

We do not consider earthquake, or wind loads in this analysis.

19 Final DL and LL are, Fig. 6.4: $TL = 39 + 8 = 47$ k/ft

6.2.3 Cable Forces

20 The thrust H (which is the horizontal component of the cable force) is determined from Eq. 6.8

$$
\begin{aligned}
H &= \frac{wL_{\text{CS}}^2}{8h} \\
&= \frac{(47)\ \text{k/ft}(3{,}500)^2\ \text{ft}^2}{(8)(327)\ \text{ft}} \\
&= 220,000\ \text{k}
\end{aligned}
$$

From Eq. 6.12 the maximum tension is

$$
\begin{aligned}
r &= \frac{h}{L_{\text{CS}}} = \frac{327}{3{,}500} = 0.0934 \\
T_{\text{max}} &= H\sqrt{1 + 16r^2} \\
&= (2,200)\ \text{k}\sqrt{1 + (16)(0.0934)^2} \\
&= (2,200)\ \text{k}(1.0675) = \boxed{235{,}000\ \ \text{k}}
\end{aligned}
$$

6.2.4 Reactions

21 Cable reactions are shown in Fig. 6.5.

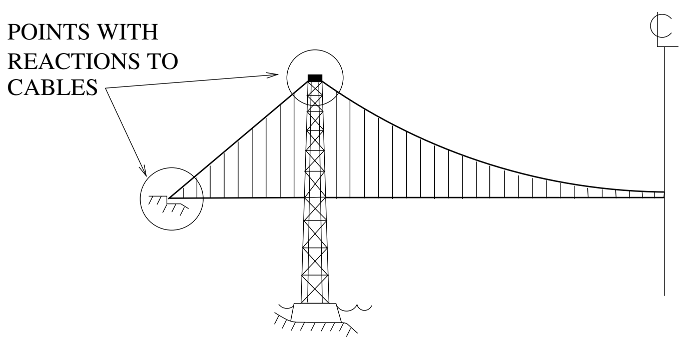


Figure 6.5: Location of Cable Reactions

22 The vertical force in the columns due to the central span (cs) is simply the support reaction, 6.6

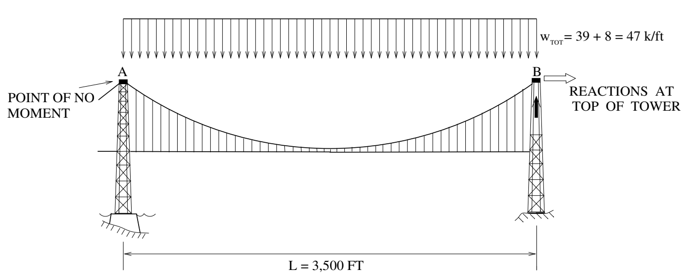


Figure 6.6: Vertical Reactions in Columns Due to Central Span Load

$$
V_{\text{CS}} = \frac{1}{2}wL_{\text{CS}} = \frac{1}{2}(47)\ \text{k/ft}(3,500)\ \text{ft} = 82,250\ \text{k} \tag{6.16}
$$

27 The cable stresses are determined last, Fig. 6.8:

$$A_{wire} = \frac{\pi D^2}{4} = \frac{(3.14)(0.196)^2}{4} = 0.03017 \text{ in}^2 \tag{6.22-a}$$

$$A_{total} = (4)\text{cables}(26,474)\text{wires/cable}(0.03017) \text{ in}^2/\text{wire} = 3,200 \text{ in}^2 \tag{6.22-b}$$

$$\text{Central Span } \sigma = \frac{H}{A} = \frac{(220,000) \text{ k}}{(3,200) \text{ in}^2} = 68.75 \text{ ksi} \tag{6.22-c}$$

$$\text{Side Span Tower } \sigma^{\text{ss}}_{\text{tower}} = \frac{T^{\text{ss}}_{\text{tower}}}{A} = \frac{(262,500) \text{ in}^2}{(3,200) \text{ in}^2} = \boxed{82 \text{ ksi}} \tag{6.22-d}$$

$$\text{Side Span Anchor } \sigma^{\text{ss}}_{\text{tower}} = \frac{T^{\text{ss}}_{\text{anchor}}}{A} = \frac{(247,000) \text{ in}^2}{(3,200) \text{ in}^2} = 77.2 \text{ ksi} \tag{6.22-e}$$

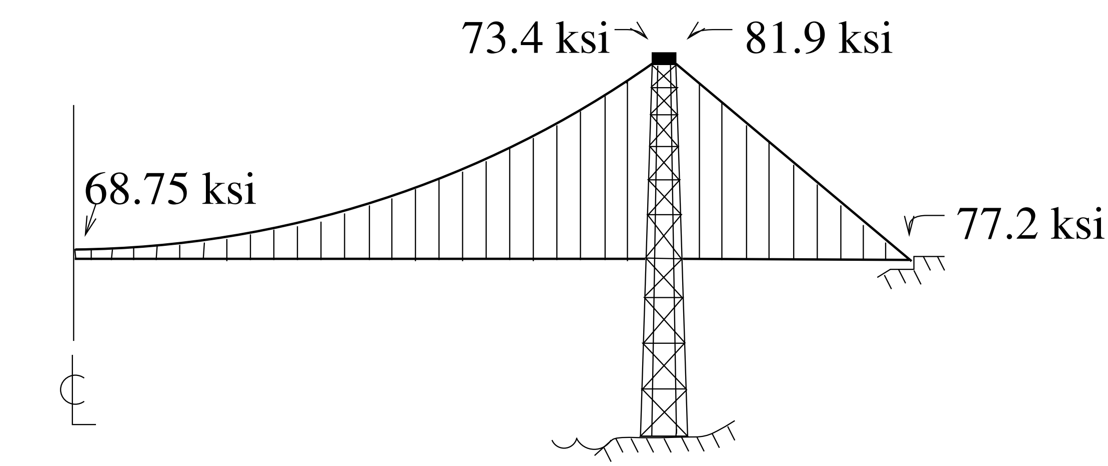


Figure 6.8: Cable Stresses

28 If the cables were to be anchored to a concrete block, the volume of the block should be at least equal to $V = \frac{(112,000) \text{ k}(1,000) \text{ lbs/ k}}{150 \text{ lbs/ft}^3} = 747,000 \text{ ft}^3$ or a cube of approximately 91 ft

29 The deck, for all practical purposes can be treated as a continuous beam supported by elastic springs with stiffness $K = AL/E$ (where L is the length of the supporting cable). This is often idealized as a beam on elastic foundations, and the resulting shear and moment diagrams for this idealization are shown in Fig. 6.9.

Chapter 7

A BRIEF HISTORY OF STRUCTURAL ARCHITECTURE

> *If I have been able to see a little farther than some others, it was because I stood on the shoulders of giants.*
>
> Sir Isaac Newton

1 More than any other engineering discipline, Architecture/Mechanics/Structures is the proud outcome of a of a long and distinguished history. Our profession, second oldest, would be better appreciated if we were to develop a sense of our evolution.

7.1 Before the Greeks

2 Throughout antiquity, structural engineering existing as an art rather than a science. No record exists of any rational consideration, either as to the strength of structural members or as to the behavior of structural materials. The builders were guided by rules of thumbs and experience, which were passed from generation to generation, guarded by secrets of the guild, and seldom supplemented by new knowledge. Despite this, structures erected before Galileo are by modern standards quite phenomenal (pyramids, Via Appia, aqueducs, Colisseums, Gothic cathedrals to name a few).

3 The first structural engineer in history seems to have been **Imhotep**, one of only two commoners to be deified. He was the builder of the step pyramid of Sakkara about 3,000 B.C., and yielded great influence over ancient Egypt.

4 Hamurrabi's code in Babylonia (1750 BC) included among its 282 laws penalties for those "architects" whose houses collapsed, Fig. 7.1.

7.2 Greeks

5 The greek philosopher **Pythagoras** (born around 582 B.C.) founded his famous school, which was primarily a secret religious society, at Crotona in southern Italy. At his school he allowed

Figure 7.2: Archimed

conqueror of Syracuse.

7.3 Romans

10 Science made much less progress under the Romans than under the Greeks. The Romans apparently were more practical, and were not as interested in abstract thinking though they were excellent fighters and builders.

11 As the roman empire expanded, the Romans built great roads (some of them still in use) such as the Via Appia, Cassia, Aurelia; Also they built great bridges (such as the third of a mile bridge over the Rhine built by Caesars), and stadium (Colliseum).

12 One of the most notable Roman construction was the **Pantheon**, Fig. 7.3. It is the best-

Figure 7.3: Pantheon

preserved major edifice of ancient Rome and one of the most significant buildings in architectural history. In shape it is an immense cylinder concealing eight piers, topped with a dome and fronted by a rectangular colonnaded porch. The great vaulted dome is 43 m (142 ft) in diameter, and the entire structure is lighted through one aperture, called an *oculus*, in the center of the dome. The Pantheon was erected by the Roman emperor Hadrian between AD 118 and 128.

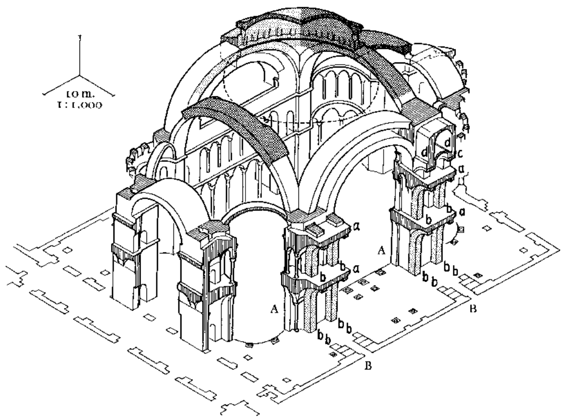


Figure 7.5: Hagia Sophia

dieval masons' efforts to solve the problems associated with supporting heavy masonry ceiling vaults over wide spans. The problem was that the heavy stonework of the traditional arched barrel vault and the groin vault exerted a tremendous downward and outward pressure that tended to push the walls upon which the vault rested outward, thus collapsing them. A building's vertical supporting walls thus had to be made extremely thick and heavy in order to contain the barrel vault's outward thrust.

Medieval masons solved this difficult problem about 1120 with a number of brilliant innovations. First and foremost they developed a ribbed vault, in which arching and intersecting stone ribs support a vaulted ceiling surface that is composed of mere thin stone panels. This greatly reduced the weight (and thus the outward thrust) of the ceiling vault, and since the vault's weight was now carried at discrete points (the ribs) rather than along a continuous wall edge, separate widely spaced vertical piers to support the ribs could replace the continuous thick walls. The round arches of the barrel vault were replaced by pointed (Gothic) arches which distributed thrust in more directions downward from the topmost point of the arch.

Since the combination of ribs and piers relieved the intervening vertical wall spaces of their supportive function, these walls could be built thinner and could even be opened up with large windows or other glazing. A crucial point was that the outward thrust of the ribbed ceiling vaults was carried across the outside walls of the nave, first to an attached outer buttress and then to a freestanding pier by means of a half arch known as a flying buttress. The flying buttress leaned against the upper exterior of the nave (thus counteracting the vault's outward thrust), crossed over the low side aisles of the nave, and terminated in the freestanding buttress pier, which ultimately absorbed the ceiling vault's thrust.

These elements enabled Gothic masons to build much larger and taller buildings than their Romanesque predecessors and to give their structures more complicated ground plans. The skillful use of flying buttresses made it possible to build extremely tall, thin-walled buildings whose interior structural system of columnar piers and ribs reinforced an impression of soaring verticality.

19 Vilet-Le-Duc classical book, (le Duc 1977) provided an in depth study of Gothic architecture.

31 Unfortunatly, these important findings, were buried in his notes, and engineers in the fifteenth and sixteenth centuries continued, as in the Roman era, to fix dimensions of structural elements by relying on experience and judgment.

7.5.2 Brunelleschi 1377-1446

32 **Brunelleschi** was a Florentine architect and one of the initiators of the Italian Renaissance. His revival of classical forms and his championing of an architecture based on mathematics, proportion, and perspective make him a key artistic figure in the transition from the Middle Ages to the modern era.

33 He was born in Florence in 1377 and received his early training as an artisan in silver and gold. In 1401 he entered, and lost, the famous design competition for the bronze doors of the Florence Baptistery. He then turned to architecture and in 1418 received the commission to execute the dome of the unfinished Gothic Cathedral of Florence, also called the **Duomo**. The dome, Fig. 7.6 a great innovation both artistically and technically, consists of two octagonal vaults, one

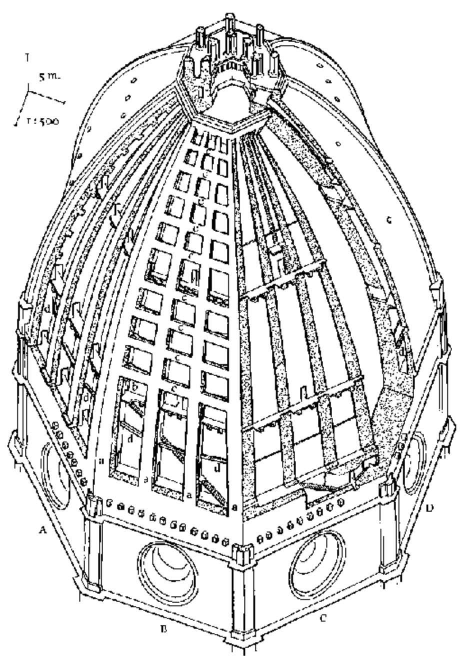


Figure 7.6: Florence's Cathedral Dome

inside the other. Its shape was dictated by its structural needs one of the first examples of **architectural functionalism**. Brunelleschi made a design feature of the necessary eight ribs of the vault, carrying them over to the exterior of the dome, where they provide the framework for the dome's decorative elements, which also include architectural reliefs, circular windows, and a beautifully proportioned cupola. This was the first time that a dome created the same strong effect on the exterior as it did on the interior.

34 Completely different from the emotional, elaborate Gothic mode that still prevailed in his time, Brunelleschi's style emphasized **mathematical rigor** in its use of straight lines, flat

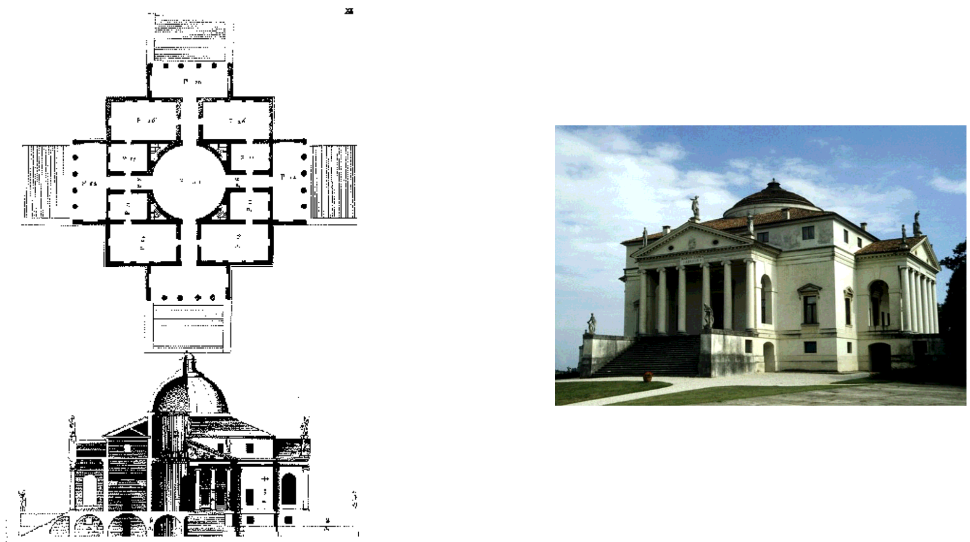


Figure 7.7: Palladio's Villa Rotunda

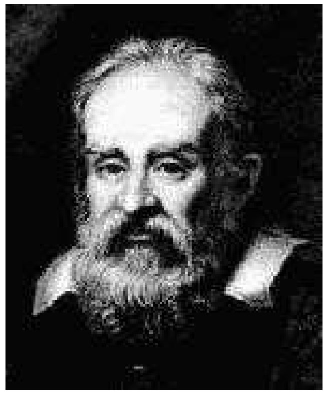


Figure 7.9: Galileo

was born. His contract was not renewed in 1592, probably because he contradicted Aristotelian professors. The same year, he was appointed to the chair of mathematics at the University of Padua, where he remained until 1610.

50 In Padua he achieved great fame, and lecture halls capable of containing 2,000 students from all over Europe were used. In 1592 he wrote *Della Scienza Meccanica* in which various problems of statics were treated using the **principle of virtual displacement**. He subsequently became interested in astronomy and built one of the first telescope through which he saw Jupiter and became an ardent proponent of the Copernican theory (which stated that the planets circle the sun as opposed to the Aristotelian and Ptolemaic assumptions that it was the sun which was circling Earth). This theory being condemned by the church, he received a semiofficial warning to avoid theology and limit himself to physical reasoning. When he published his books dealing with the two ways of regarding the universe (which clearly favored the Copernican theory) he was called to Rome by the Inquisition, condemned and had to read his recantation (At the end of his process he murmured the famous *e pur se muove*).

51 When he was almost seventy years old, his life shattered by the Inquisition, he retired to his villa near Florence and wrote his final book, *Discourses Concerning Two New Sciences*, (Galilei 1974), Fig. 7.10. His first science was the study of the forces that hold objects together

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE
intorno à due nuove scienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con una Appendice del centro di gravità d'alcuni Solidi.

IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII

Figure 7.10: *Discourses Concerning Two New Sciences*, Cover Page

appointed Gresham Professor of Geometry at Oxford in 1665. After the Great Fire of London in 1666, he was appointed surveyor of London, and he designed many buildings.

57 Hooke anticipated some of the most important discoveries and inventions of his time but failed to carry many of them through to completion. He formulated the theory of planetary motion as a problem in mechanics, and grasped, but did not develop mathematically, the fundamental theory on which Newton formulated the law of gravitation.

58 His most important contribution was published in 1678 in the paper *De Potentia Restitutiva*. It contained results of his experiments with elastic bodies, and was the first paper in which the elastic properties of material was discused, Fig. 7.12.

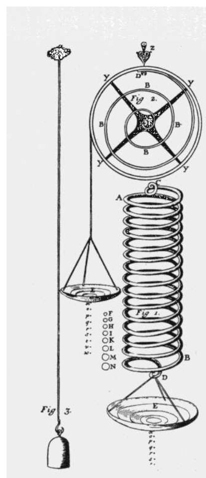


Figure 7.12: Experimental Set Up Used by Hooke

> "*Take a wire string of 20, or 30, or 40 ft long, and fasten the upper part thereof to a nail, and to the other end fasten a Scale to receive the weights: Then with a pair of compasses take the distance of the bottom of the scale from the ground or floor underneath, and set down the said distance,then put inweights into the said scale and measure the several stretchings of the said string, and set them down. Then compare the several strtchings of the said string, and you will find that they will always bear the same proportions one to the other that the weights do that made them*".

This became **Hooke's Law** $\sigma = E\varepsilon$.

59 Because he was concerned about patent rights to his invention, he did not publish his law when first discovered it in 1660. Instead he published it in the form of an anagram "*ceiinossssttuu*" in 1676 and the solution was given in 1678. *Ut tensio sic vis* (at the time the two symbos *u* and *v* were employed interchangeably to denote either the vowel *u* or the consonant *v*), i.e. *extension varies directly with force*.

7.6.2 Newton, 1642-1727

60 Born on christmas day in the year of Galileo's death, Newton, Fig. 7.13 was Professor of

63 The *Principia's* appearance also involved Newton in an unpleasant episode with the English philosopher and physicist Robert Hooke. In 1687 Hooke claimed that Newton had stolen from him a central idea of the book: that bodies attract each other with a force that varies inversely as the square of their distance. However, most historians do not accept Hooke's charge of plagiarism.

64 Newton also engaged in a violent dispute with Leibniz over priority in the invention of calculus. Newton used his position as president of the Royal Society to have a committee of that body investigate the question, and he secretly wrote the committee's report, which charged Leibniz with deliberate plagiarism. Newton also compiled the book of evidence that the society published. The effects of the quarrel lingered nearly until his death in 1727.

65 In addition to science, Newton also showed an interest in alchemy, mysticism, and theology. Many pages of his notes and writings particularly from the later years of his career are devoted to these topics. However, historians have found little connection between these interests and Newton's scientific work.

7.6.3 Bernoulli Family 1654-1782

66 The Bernouilli family originally lived in Antwerp, but because of religious persecution, they left Holland and settled in Basel. Near the end of the seventeenth century this family produced outstanding mathematicians for more than a hundred years. Jacob and John were brothers. John was the father of Daniel, and Euler his pupil.

67 Whereas Galileo (and Mariotte) investigated the strength of beams (Strength), Jacob Bernoulli (1654-1705) made calculation of their deflection (Stiffness) and did not contribute to our knowledge of physical properties. Jacob Bernouilli is also credited in being the first to to have assumed that a bf plane section of a beam remains plane during bending, but assumed rotation to be with respect to the lower fiber (as Galileo did) and this resulted in an erroneous solution (where is the exact location of the axis of rotation?). He also showed that the curvature at any point along a beam is proportional to the curvature of the deflection curve.

68 Bernoulli made the first analytical contribution to the problem of elastic flexure of a beam. In 1691 he published a logogriph *Qrzumubapt dxqopddbbp ...* whose secret was revealed in 1694. A letter is replaced by the next in the Latin alphabet, the second by the letter three away, and the third by the letter six away, so that *aaaaa* would be encoded as *bdgbd*. The logogriph reads *Portio axis applicatem...* and the decoded is that the radius of curvature at any point of an initially straight beam in inversely proportional to the value of the bending moment at that point.

69 Daniel Bernoulli (1700-1782) first postulated that a force can be decomposed into its equivalent (*"Potentiis quibuscunque possunt substitui earundem aequivalentes"*. Another hypothesis defined the sum of two "conspiring" forces applied to the same point. According to Bernoulli, this "necessary truth" follows from the metaphysical principle that the whole equalts the sum of its parts, (Penvenuto 1991).

7.6.4 Euler 1707-1783

70 **Leonhard Euler** was born in Basel and early on caught the attention of John Bernoulli whose teaching was attracting young mathematicians from all over Europe, Fig. 7.15. He

7.7 The pre-Modern Period; Coulomb and Navier

75 Coulomb (1736-1806) was a French military engineer, Fig. 7.16, as was the first to publish

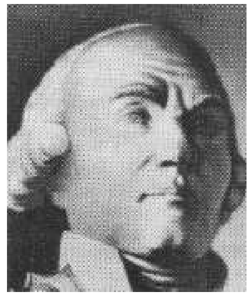


Figure 7.16: Coulomb

the correct analysis of the fiber stresses in flexed beam with rectangular cross section (*Sur une Application des Régles de maximis et minimis à quelques problèmes de statique relatifs à l'architecture* in 1773). He used Hooke's law, placed the neutral axis in its exact position, developed the equilibrium of forces on the cross section with external forces, and then correcly determined the stresses. He also worked on friction ("Coulomb friction") and on earth pressure.

76 Coulomb did also research on magnetism, friction, and electricity. In 1777 he invented the torsion balance for measuring the force of magnetic and electrical attraction. With this invention, Coulomb was able to formulate the principle, now known as Coulomb's law, governing the interaction between electric charges. In 1779 Coulomb published the treatise *Theorie des machines simples* (Theory of Simple Machines), an analysis of friction in machinery. After the war Coulomb came out of retirement and assisted the new government in devising a metric system of weights and measures. The unit of quantity used to measure electrical charges, the coulomb, was named for him.

77 Navier (1785-1836) Navier was educated at the Ecole Polytechnique and became a professor there in 1831. Whereas the famous memoir of Coulomb (1773) contained the correct solution to numerous important problems in mechanics of materials, it took engineers more than forty years to understand them correctly and to use them in practical application

78 In 1826 he published his *Leçons* (lecture notes) which is considered the first great textbook in mechanics for engineering. In it he developed the first general theory of elastic solids as well as the first systematic treatment of the theory of structures.

79 It should be noted that no clear division existed between the theory of elasticity and the theory of structures until about the middle of the nineteenth century (Coulomb and Navier would today be considered professional structural engineers).

80 Three other structural engineers who pioneered the development of the theory of elasticity from that point on were Lamé, Clapeyron and de Saint-Venant. Lam'e published the first book on elasticity in 1852, and credited Clapeyron for the theorem of equality between external and internal work. de Saint-Venant was perhaps the greatest elasticians who according to Southwell "... combined with high mathematical ability an essentially practical outlook which gave direction to all his work". In 1855-6 he published his classical work on torsion, flexure,

88 His famous axiom, **Form follows function** became the touchstone for many in his profession. Sullivan, however, did not apply it literally. He meant that an architect should consider the purpose of the building as a starting point, not as a rigidly limiting stricture.

89 He also had tremendous respect for the natural world which played an enormous role in forging his theories about architecture (he spent all of his first summers on his grandparents' farm in Massachusetts where he developed this love and respect for nature) expressed in his *Autobiography of an Idea)*, 1924).

7.8.4 Roebling, 1806-1869

90 John Augustus Roebling was an American **civil engineer**, who was one of the pioneers in the construction of suspension bridges. He was born in Germany, educated at the Royal Polytechnic School of Berlin and immigrated to the States in 1831.

91 In his first job he was employed by the Pennsylvania Railroad Corp. to survey its route across the Allegheny Mountains between Harrisburg and Pittsburgh. He then demonstrated the practicability of steel cables in bridge construction and in 1841 established at Saxonburg the first factory to manufacture steel-wire rope in the U.S.

92 Roebling utilized steel cables in the construction of numerous suspension bridges and is generally considered one of the pioneers in the field of suspension-bridge construction. He built railroad suspension bridges over the Ohio and Niagara rivers and completed plans for the Brooklyn Bridge shortly before his death. Roebling was the author of Long and Short Span Railway Bridges (1869).

7.8.5 Maillart

`From (Billington 1973)`

93 Robert Maillart was born on February 6, 1872, in Bern, Switzerland, where his father, a Belgian citizen, was a banker. He studied civil engineering at the Federal Institute of Technology in Zurich and graduated in 1894. Ironically, one of his lowest grades was in bridge design, even though he is regarded today as one of the half dozen greatest bridge designers of the twentieth century.

94 For eight years following his graduation, he worked with different civil engineering organizations. In 1902, he founded his own firm for design and construction; thereafter, his business grew rapidly and expanded as far as Russia and Spain. In the summer of 1914, he took his wife and three children to Russia. Since the World War prevented their return to Switzerland, Maillart stayed and worked in Russia until 1919, when his business was liquidated by the Revolution. Forced to flee, he returned to Switzerland penniless and lonely, his wife having died in Russia.

95 Because of these misfortunes Maillart felt unable to take up the construction business again and henceforth concentrated on design alone. He opened an office in Geneva in 1919 and branches in Bern and Zurich in 1924.

96 During the twenties he began to develop and modify his ideas of bridge design; and from 1930, when the Salginatobel and Landquart Bridges were completed, until his death in 1940,

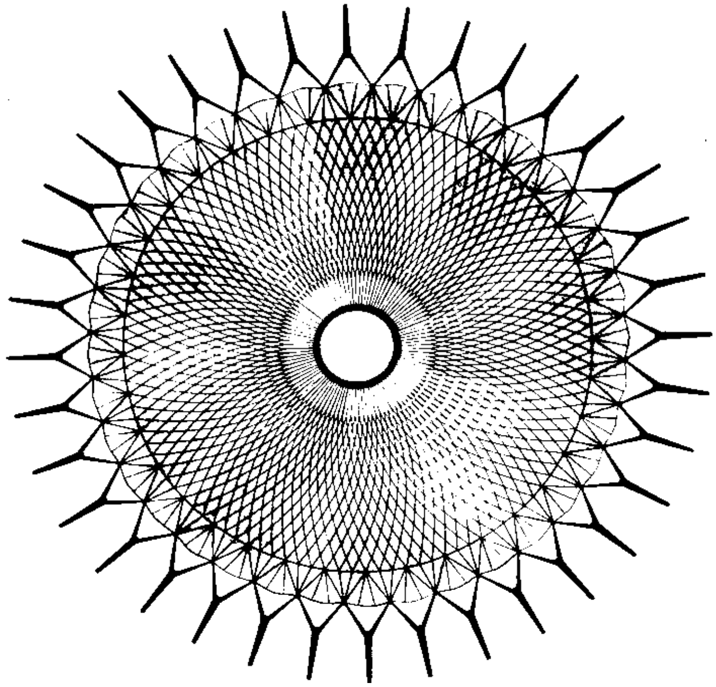


Figure 7.17: Nervi's Palazetto Dello Sport

needed for high towers, eliminated the need for internal wind bracing (since the perimeter columns carried the wind loadings), and permitted freer organization of the interior space.

His later projects included the strikingly different Haj Terminal of the King Abdul Aziz International Airport, Jiddah, Saudi Arabia (1976-81), and King Abdul Aziz University, also in Jiddah (1977-78).

7.8.8 *et al.*

102 To name just a few of the most influential Architects/Engineers: Menn, Isler, Candella, Torroja, Johnson, Pei, Calatrava, ...

than my bare hands and no further addition to my academic background. After several years of general practice in Mexico, as draftsman, designer and contractor, I recalled my old fancy with shells and began to collect again papers on the subject. Whatever I learned from then on was to be the hard way, working alone, with no direct help from any university or engineering office. But I am indebted to many people who did help me through their writings and Maillart was one of the foremost.

I discovered him in Giedion's Space, Time and Architecture; and then I got Max Bill's book with its invaluable collection of Maillart's essays. I devoured his articles about "Reinforced Concrete Design and Calculation" (he was very careful to differentiate the meaning of such words and to avoid the more than semantic confusion prevalent nowadays in English-speaking countries)., "The Engineer and the Authorities" which expresses his position in front of the establishment and "Mass and Quality in Reinforced Concrete Structures." Very short papers, indeed, but well provided with opinions, something I could rarely find in other engineering articles. I learned later that to express personal opinions is considered bad taste among technical writers. Any discussion should be restricted to insignificant details, but never touch fundamental dogmas, in a fashion curiously similar to what could be expected of the councils of the Church or the meetings of any Politbureau.

But my attitude with respect to calculations of reinforced concrete structures was becoming unorthodox, being tired perhaps of performing long and tedious routines whose results were not always meaningful. Therefore, I found Maillart's thoughts delightfully sympathetic and encouraging. If a rebel was able to produce such beautiful and sound structures there could not be anything wrong with becoming also a rebel, which was besides, my only way to break the mystery surrounding shell analysis.

Thus, I started to follow the bibliographic tread and met, through their writings, with Freudenthal, Johansen, Van der Broek, Kist, Saliger, Kacinczy and so many others who showed me there was more than a single and infallible manner to approach structural analysis. The discovery of rupture methods, with their emphasis on simple statics and their bearing on the actual properties of construction materials and their behavior in the plastic range, allowed me to trust in simplified procedures to understand and analyze the distribution of stresses in shell structures. It also helped me to get out of my naive belief in the indisputable truth of the printed word and to start reading with a new critical outlook. No longer did I need to believe whatever was in print, no matter how high-sounding the name of the author. I could make my own judgements about what methods of stress analysis were better suited for my practice.

Since I was working practically alone, I could not afford nor had time for complex calculations and did welcome Maillart's advice that simpler calculations are more reliable than complex ones, especially for somebody who builds his own structures. This was exactly my case and, since most structures I was building were of modest scale, I could control what was happening, check the results and confirm the accuracy of my judgement or correct my mistakes. In a way, I was working with full scale models. I understand that this was also true of Maillart who in many cases was the actual builder of his designs.

Following the general trend to mess up issues, there has been a lot of speculation about the engineer as an artist and in some instances, like in the case of Nervi, about the engineer as an architect (as if the title of architect could confer, per se, artistic ability to its holder); but few people realize that the only way to be an artist in this difficult specialty of building is to be your own contractor. in countries like this, where the building industry has been thoroughly and irreversibly fragmented and the responsibility diluted among so many trades, it may be shocking to think of a contractor as an artist; but it is indeed the only way to have in your hands the whole set of tools or instruments to perform the forgotten art of building, to produce

Chapter 8

Case Study III: MAGAZINI GENERALI

`Adapted from (Billington and Mark 1983)`

8.1 Geometry

1 This sotrage house, built by Maillart in Chiasso in 1924, provides a good example of the mariage between aesthetic and engineering.

2 The most strking feature of the Magazini Generali is not the structure itself, but rather the shape of its internal supporting frames, Fig. 8.1.

3 The frame can be idealized as a simply supported beam hung from two cantilever column supports. Whereas the beam itself is a simple structural idealization, the overhang is designed in such a way as to minimize the net moment to be transmitted to the supports (foundations), Fig. 8.2.

8.2 Loads

4 The load applied on the frame is from the weights of the roof slab, and the frame itself. Given the space between adjacent frames is 14.7 ft, and that the roof load is 98 psf, and that the total frame weight is 13.6 kips, the total uniform load becomes, Fig. 8.3:

$$q_{roof} = (98) \text{ psf}(14.7) \text{ ft} = 1.4 \text{ k/ft} \tag{8.1-a}$$

$$q_{frame} = \frac{(13.6) \text{ k}}{(63.6) \text{ ft}} = 0.2 \text{ k/ft} \tag{8.1-b}$$

$$q_{total} = 1.4 + 0.2 = \boxed{1.6 \text{ k/ft}} \tag{8.1-c}$$

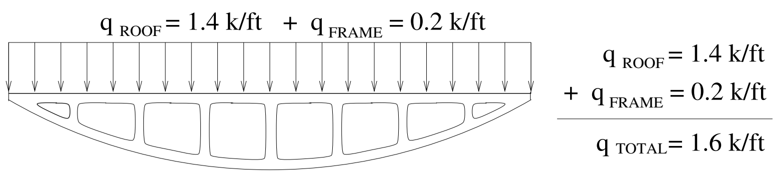


Figure 8.3: Magazzini Generali; Loads (Billington and Mark 1983)

8.3 Reactions

5 Reactions for the beam are determined first taking advantage of symmetry, Fig. 8.4:

$$W = (1.6)\text{ k/ft}(63.6)\text{ ft} = 102\text{ k} \quad (8.2\text{-a})$$

$$R = \frac{W}{2} = \frac{102}{2} = \boxed{51\ \text{ k}} \quad (8.2\text{-b})$$

We note that these reactions are provided by the internal shear forces.

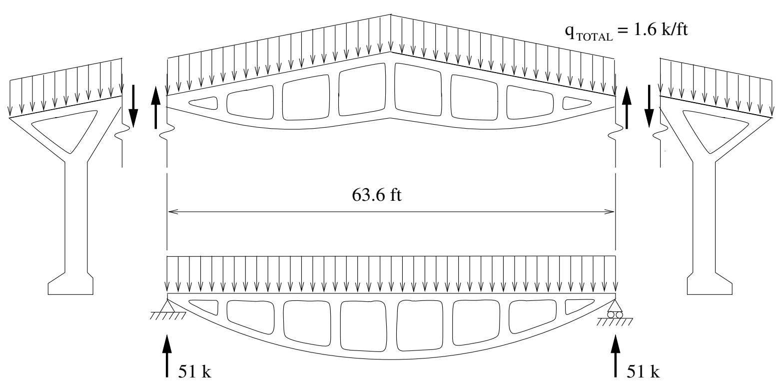


Figure 8.4: Magazzini Generali; Beam Reactions, (Billington and Mark 1983)

8.4 Forces

6 The internal forces are pimarily the shear and moments. Those can be easily determined for a simply supported uniformly loaded beam. The shear varies linearly from 51 kip to -51 kip with zero at the center, and the moment diagram is parabolic with the maximum moment at the center, Fig. 8.5, equal to:

$$M_{max} = \frac{qL^2}{8} = \frac{(1.6)\text{ k/ft}(63.6)\text{ ft}^2}{8} = \boxed{808\text{ k.ft}} \quad (8.3)$$

7 The externally induced moment at midspan must be resisted by an equal and opposite internal moment. This can be achieved through a combination of compressive force on the upper fibers, and tensile ones on the lower. Thus the net axial force is zero, however there is a net internal couple, Fig. 8.6.

$$M_{ext} = Cd \Rightarrow C = \frac{M_{ext}}{d} \quad (8.4\text{-a})$$

$$T = C = \frac{(808)\text{ k.ft}}{(9.2)\text{ ft}} = \boxed{\pm 88\ \text{ k}} \quad (8.4\text{-b})$$

8 Because the frame shape (and thus $d(x)$) is approximately parabolic, and the moment is also parabolic, then the axial forces are constants along the entire frame, Fig. 8.7.

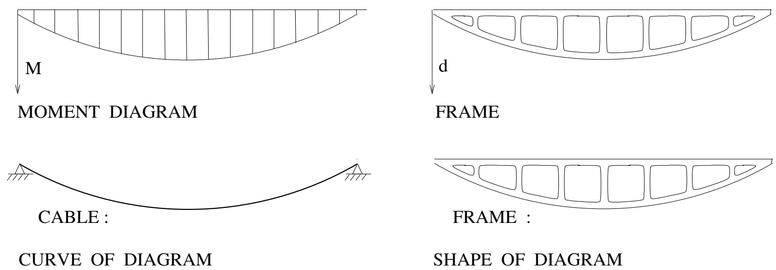


Figure 8.7: Magazzini Generali; Similarities Between The Frame Shape and its Moment Diagram, (Billington and Mark 1983)

9 The axial force at the end of the beam is not balanced, and the 88 kip compression must be transmitted to the lower chord, Fig. 8.8. Fig. 8.9 This is analogous to the forces transmiited

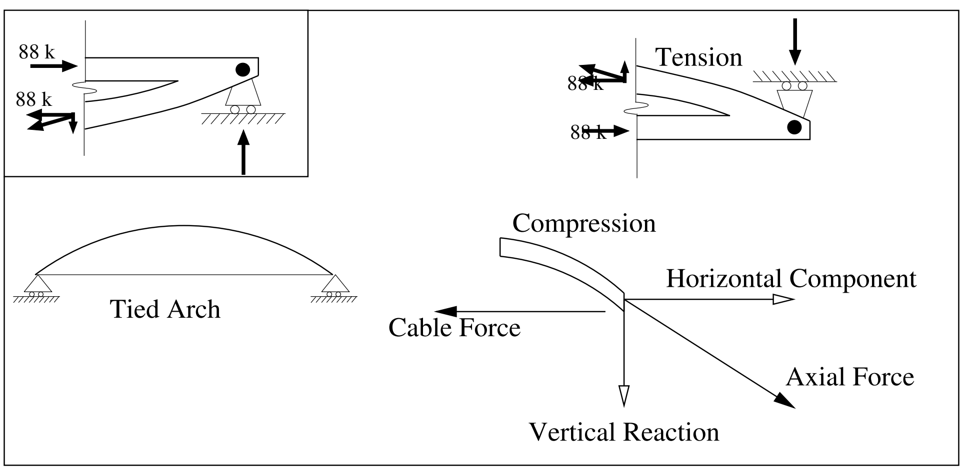


Figure 8.8: Magazzini Generali; Equilibrium of Forces at the Beam Support, (Billington and Mark 1983)

to the support by a tied arch.

10 It should be mentioned that when a rigorous computer analysis was performed, it was determined that the supports are contributing a compression force of about 8 kips which needs to be superimposed over the central values, Fig. 8.9.

Chapter 9

DESIGN PHILOSOPHIES of ACI and AISC CODES

9.1 Safety Provisions

1 Structures and structural members must always be designed to carry some reserve load above what is expected under normal use. This is to account for

Variability in Resistance: The **actual** strengths (resistance) of structural elements will differ from those **assumed** by the designer due to:

1. Variability in the strength of the material (greater variability in concrete strength than in steel strength).
2. Differences between the actual dimensions and those specified (mostly in placement of steel rebars in R/C).
3. Effect of simplifying assumptions made in the derivation of certain formulas.

Variability in Loadings: All loadings are variable. There is a greater variation in the live loads than in the dead loads. Some types of loadings are very difficult to quantify (wind, earthquakes).

Consequences of Failure: The consequence of a structural component failure must be carefully assessed. The collapse of a beam is likely to cause a localized failure. Alternatively the failure of a column is likely to trigger the failure of the whole structure. Alternatively, the failure of certain components can be preceded by warnings (such as excessive deformation), whereas other are sudden and catastrophic. Finally, if no redistribution of load is possible (as would be the case in a statically determinate structure), a higher safety factor must be adopted.

2 The purpose of safety provisions is to limit the **probability of failure** and yet permit economical structures.

3 The following items must be considered in determining safety provisions:

1. Seriousness of a failure, either to humans or goods.

$$\boxed{\sigma < \sigma_{all} = \frac{\sigma_{yld}}{F.S.}} \tag{9.1}$$

where $F.S.$ is the factor of safety.

10 Major limitations of this method

1. An elastic analysis can not easily account for creep and shrinkage of concrete.
2. For concrete structures, stresses are not linearly proportional to strain beyond $0.45f'_c$.
3. Safety factors are not rigorously determined from a probabilistic approach, but are the result of experience and judgment.

11 Allowable strengths are given in Table 9.1.

Steel, AISC/ASD	
Tension, Gross Area	$F_t = 0.6F_y$
Tension, Effective Net Area*	$F_t = 0.5F_u$
Bending	$F_b = 0.66F_y$
Shear	$F_v = 0.40F_y$
Concrete, ACI/WSD	
Tension	0
Compression	$0.45f'_c$

* Effective net area will be defined in section **??**.

Table 9.1: Allowable Stresses for Steel and Concrete

9.3 Ultimate Strength Method

9.3.1 The Normal Distribution

12 The normal distribution has been found to be an excellent approximation to a large class of distributions, and has some very desirable mathematical properties:

1. $f(x)$ is symmetric with respect to the mean μ.
2. $f(x)$ is a "bell curve" with inflection points at $x = \mu \pm \sigma$.
3. $f(x)$ is a valid *probability distribution function* as:
$$\int_{-\infty}^{\infty} f(x) = 1 \tag{9.2}$$
4. The *probability* that $x_{min} < x < x_{max}$ is given by:
$$P(x_{min} < x < x_{max}) = \int_{x_{min}}^{x_{max}} f(x)dx \tag{9.3}$$

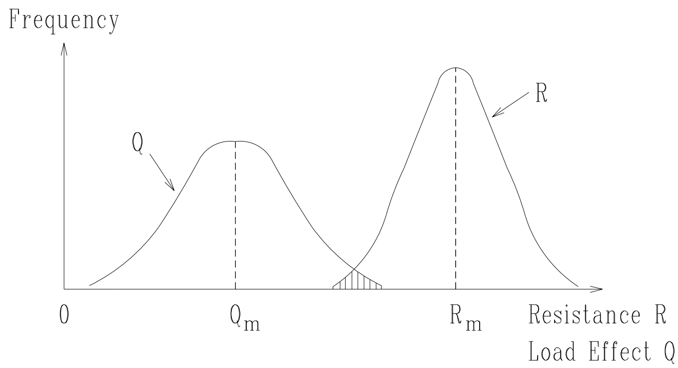


Figure 9.3: Frequency Distributions of Load Q and Resistance R

Failure would occur for negative values of X

19 The **probability of failure** P_f is equal to the ratio of the shaded area to the total area under the curve in Fig. 9.4.

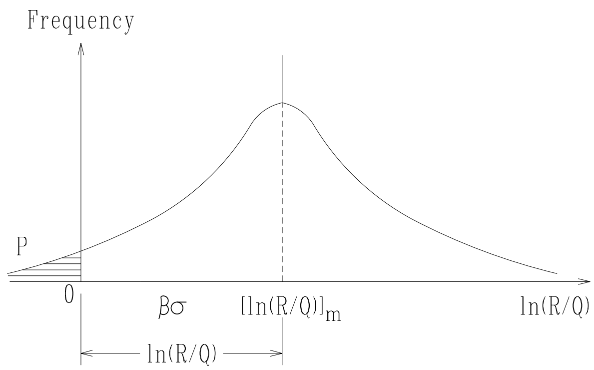


Figure 9.4: Definition of Reliability Index

20 If X is assumed to follow a **Normal Distribution** than it has a mean value $\overline{X} = \left(\ln \frac{R}{Q}\right)_m$ and a standard deviation σ.

21 We define the **safety index** (or **reliability index**) as $\beta = \frac{\overline{X}}{\sigma}$

22 For standard distributions and for $\beta = 3.5$, it can be shown that the probability of failure is $P_f = \frac{1}{9{,}091}$ or 1.1×10^{-4}. That is 1 in every 10,000 structural members designed with $\beta = 3.5$ will fail because of either excessive load or understrength sometime in its lifetime.

23 Reliability indices are a relative measure of the current condition and provide a qualitative estimate of the structural performance.

24 Structures with relatively high reliable indices will be expected to perform well. If the value is too low, then the structure may be classified as a hazard.

25 Target values for β are shown in Table 9.2, and in Fig. 9.5

Type of Load/Member	β
AISC	
DL + LL; Members	3.0
DL + LL; Connections	4.5
DL + LL + WL; Members	3.5
DL + LL +EL; Members	1.75
ACI	
Ductile Failure	3-3.5
Sudden Failures	3.5-4

Table 9.2: Selected β values for Steel and Concrete Structures

Φ is a **strength reduction factor**, less than 1, and must account for the type of structural element, Table 9.3.

Type of Member	Φ
ACI	
Axial Tension	0.9
Flexure	0.9
Axial Compression, spiral reinforcement	0.75
Axial Compression, other	0.70
Shear and Torsion	0.85
Bearing on concrete	0.70
AISC	
Tension, yielding	0.9
Tension, fracture	0.75
Compression	0.85
Beams	0.9
Fasteners, Tension	0.75
Fasteners, Shear	0.65

Table 9.3: Strength Reduction Factors, Φ

R_n is the **nominal resistance** (or strength).

ΦR_n is the **design strength**.

α_i is the **load factor** corresponding to Q_i and is greater than 1.

$\Sigma\alpha_i Q_i$ is the **required strength** based on the **factored load**:

i is the type of load

32 The various factored load combinations which must be considered are

AISC

9.4 Example

■ **Example 9-1: LRFD vs ASD**

To illustrate the differences between the two design approaches, let us consider the design of an axial member, subjected to a dead load of 100 k and live load of 80 k. Use A36 steel.

ASD: We consider the total load $P = 100 + 80 = 180$ k. From Table 9.1, the allowable stress is $0.6\sigma_{\rm yld} = 0.6 * 36 = 21.6$ ksi. Thus the required cross sectional area is

$$A = \frac{180}{21.6} = 8.33 \ \ \text{in}^2$$

USD we consider the largest of the two load combinations

$$\begin{array}{llcll} \Sigma\alpha_i Q_i: & 1.4D & = & 1.4(100) & = 140 \ \ \text{k} \\ & 1.2D + 1.6L & = & 1.2(100) + 1.6(80) & = 248 \ \ \text{k} \ \leftarrow \end{array}$$

From Table 9.3 $\Phi = 0.9$, and $\Phi R_n = (0.9)A\sigma_{\rm yld}$. Hence, applying Eq. 9.9 the cross sectional area should be

$$A = \frac{\Sigma\alpha_i Q_i}{\Phi\sigma_{\rm yld}} = \frac{248}{(0.9)(36)} = 7.65 \ \ \text{in}^2$$

Note that whereas in this particular case the USD design required a smaller area, this may not be the case for different ratios of dead to live loads. ■

Chapter 10

BRACED ROLLED STEEL BEAMS

1 This chapter deals with the behavior and design of **laterally supported** steel beams according to the LRFD provisions.

2 A laterally stable beam is one which is braced laterally in the direction perpendicular to the plane of the web. Thus overall buckling of the compression flange as a column cannot occur prior to its full participation to develop the moment strength of the section.

3 If a beam is not laterally supported, Fig. 10.1, we will have a failure mode governed by lateral

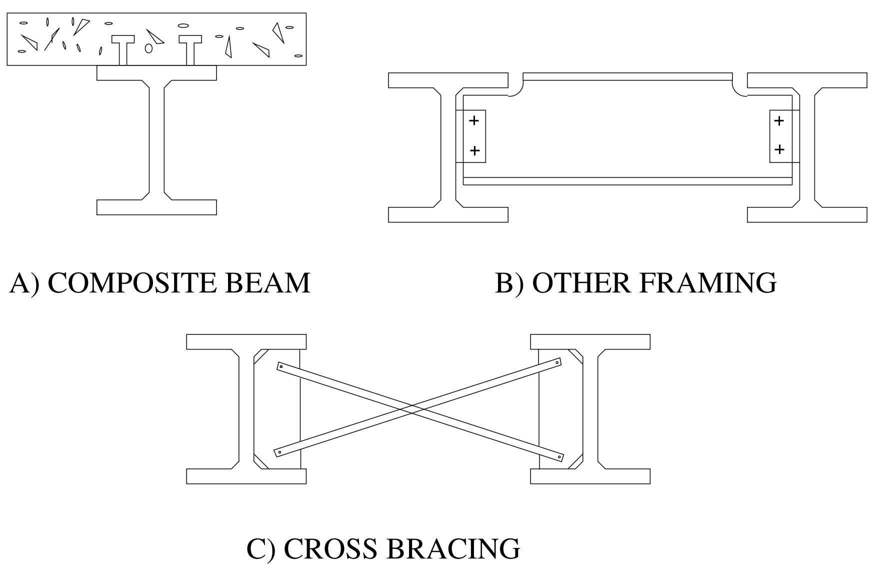


Figure 10.1: Lateral Bracing for Steel Beams

torsional buckling.

4 By the end of this lecture you should be able to select the most efficient section (light weight with adequate strength) for a given bending moment and also be able to determine the flexural strength of a given beam.

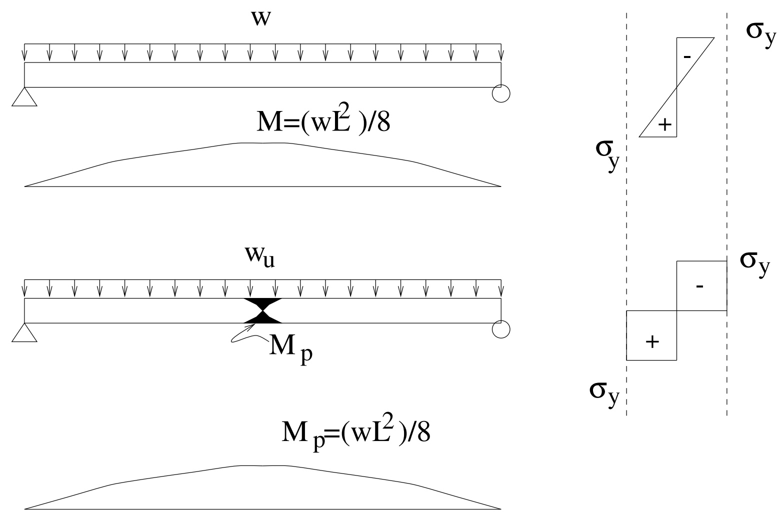


Figure 10.2: Failure of Steel beam; Plastic Hinges

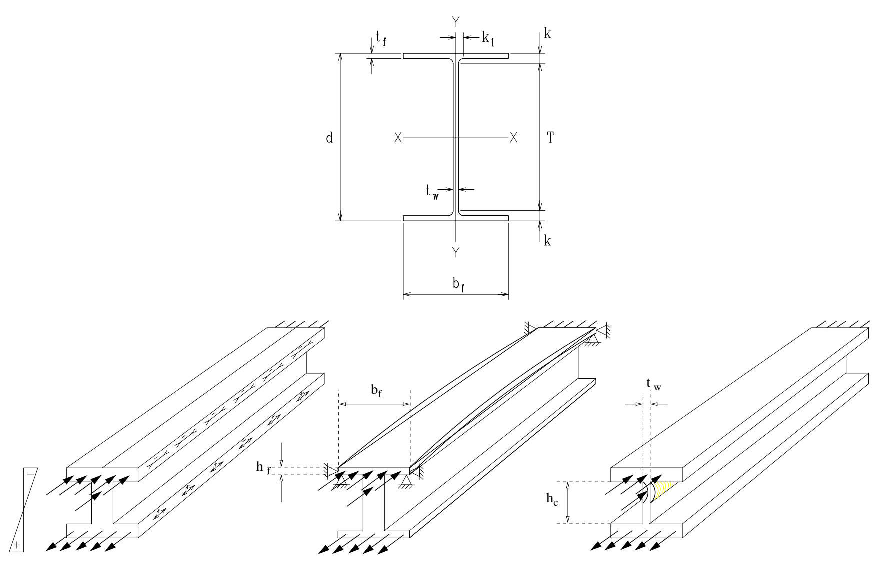


Figure 10.3: Failure of Steel beam; Local Buckling

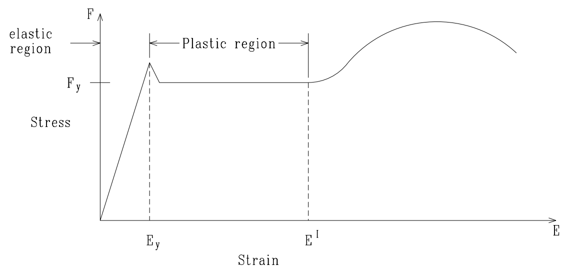


Figure 10.6: Stress-strain diagram for most structural steels

13 When the yield stress is reached at the extreme fiber, the nominal moment strength M_n, is referred to as the **yield moment** M_y and is computed as

$$\boxed{M_n = M_y = S_x F_y} \tag{10.3}$$

(assuming that bending is occurring with respect to the $x-x$ axis).

14 When across the entire section, the strain is equal or larger than the yield strain ($\varepsilon \geq \varepsilon_y = F_y/E_s$) then the section is fully plastified, and the nominal moment strength M_n is therefore referred to as the **plastic moment** M_p and is determined from

$$\boxed{M_p = F_y \int_A y dA = F_y Z} \tag{10.4}$$

where

$$\boxed{Z \stackrel{\text{def}}{=} \int y dA} \tag{10.5}$$

is the **Plastic Section Modulus**.

15 The plastic section modulus Z should not be confused with the **elastic** section modulus S defined, Eq. 5.23 as

$$S = \frac{I}{d/2} \tag{10.6-a}$$

$$I \stackrel{\text{def}}{=} \int_A y^2 dA \tag{10.6-b}$$

16 The section modulus S_x of a W section can be roughly approximated by the following formula

$$\boxed{S_x \approx wd/10 \quad \text{or} \quad I_x \approx S_x \frac{d}{2} \approx wd^2/20} \tag{10.7}$$

and the plastic modulus can be approximated by

$$\boxed{Z_x \approx wd/9} \tag{10.8}$$

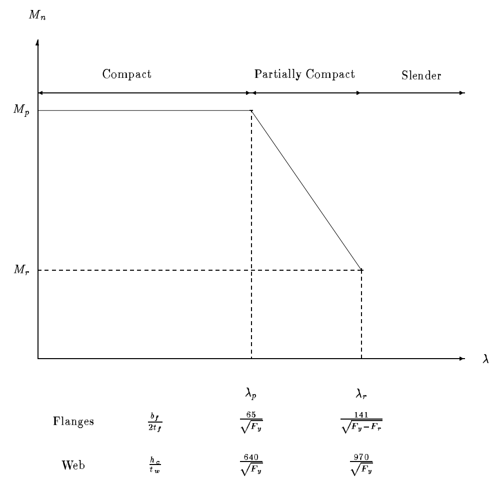


Figure 10.7: Nominal Moments for Compact and Partially Compact Sections

where:

M_r	Residual Moment equal to $(F_y - F_r)S$
λ	$b_f/2t_f$ for I-shaped member flanges and
	h_c/t_w for beam webs.

22 All other quantities are as defined earlier. Note that we use the λ associated with the one being violated (or the lower of the two if both are).

10.5 Slender Section

23 If the width to thickness ratio exceeds λ_r values of flange and web, the element is referred to as slender compression element. Since the slender sections involve a different treatment, it will not be dealt here.

10.6 Examples

■ **Example 10-1: Z for Rectangular Section**

Determine the plastic section modulus for a rectangular section, width b and depth d.

2. Compute the factored load moment M_u. For a simply supported beam carrying uniformly distributed load,

$$M_u = w_u L^2/8 = (1.52)(20)^2/8 = 76 \text{ k.ft}$$

Assuming compact section, since a vast majority of rolled sections satisfy $\lambda \le \lambda_p$ for both the flange and the web. The design strength $\phi_b M_n$ is

$$\phi_b M_n = \phi_b M_p = \phi_b Z_x F_y$$

The design requirement is

$$\phi_b M_n = M_u$$

or, combing those two equations we have:

$$\phi_b Z_x F_y = M_u$$

3. Required Z_x is

$$Z_x = \frac{M_u}{\phi_b F_y} = \frac{76(12)}{0.90(36)} = \boxed{28.1 \text{ in}^3}$$

From the notes on Structural Materials, we select a W12X22 section which has a $Z_x = 29.3$ in^3
Note that Z_x is approximated by $\frac{wd}{9} = \frac{(22)(12)}{9} = 29.3$.

4. Check compact section limits λ_p for the flanges from the table

$$\begin{aligned} \lambda = \tfrac{b_f}{2t_f} &= 4.7 \\ \lambda_p &= \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{36}} = 10.8 > \lambda\surd \end{aligned}$$

and for the web:

$$\begin{aligned} \lambda = \tfrac{h_c}{t_w} &= 41.8 \\ \lambda_p &= \frac{640}{\sqrt{F_y}} = \frac{640}{\sqrt{36}} = 107\surd \end{aligned}$$

5. Check the Strength by correcting the factored moment M_u to include the self weight. Self weight of the beam W12X22 is 22 lb./ft. or 0.022 kip/ft

$$\begin{aligned} w_D &= 0.2 + 0.022 = 0.222 \text{ k/ft} \\ w_u &= 1.2(0.222) + 1.6(0.8) = 1.55 \text{ k/ft} \\ M_u &= (1.55)(20)^2/8 = 77.3 \text{ k.ft} \\ M_n &= M_p = Z_x F_y = \frac{(29.3) \text{ in}^3 (36) \text{ ksi}}{(12) \text{ in/ft}} = 87.9 \text{ k.ft} \\ \phi_b M_n &= 0.90(87.9) = 79.1 \text{ k.ft} > M_u\surd \end{aligned}$$

Therefore use $\boxed{\text{W12X22}}$ section.

6. We finally check for the maximum distance between supports.

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{5}{6.5}} = 0.88 \text{ in} \quad (10.16\text{-a})$$

$$L_p = \frac{300}{\sqrt{F_y}} r_y \quad (10.16\text{-b})$$

$$= \frac{300}{\sqrt{36}} 0.88 = \boxed{43 \text{ ft}} \quad (10.16\text{-c})$$

Chapter 11

REINFORCED CONCRETE BEAMS

11.1 Introduction

1 Recalling that concrete has a tensile strength (f'_t) about one tenth its compressive strength (f'_c), concrete by itself is a very poor material for flexural members.

2 To provide tensile resistance to concrete beams, a reinforcement must be added. Steel is almost universally used as reinforcement (longitudinal or as fibers), but in poorer countries other indigenous materials have been used (such as bamboos).

3 The following lectures will focus exclusively on the flexural design and analysis of reinforced concrete rectangular sections. Other concerns, such as shear, torsion, cracking, and deflections are left for subsequent ones.

4 Design of reinforced concrete structures is governed in most cases by the *Building Code Requirements for Reinforced Concrete*, of the American Concrete Institute (ACI-318). Some of the most relevant provisions of this code are enclosed in this set of notes.

5 We will focus on determining the amount of flexural (that is longitudinal) reinforcement required at a **given section**. For that section, the moment which should be considered for design is the one obtained from the **moment envelope** at that particular point.

11.1.1 Notation

6 In R/C design, it is customary to use the following notation

11.1.3 Analysis vs Design

11 In R/C we always consider one of the following problems:

Analysis: Given a certain design, determine what is the maximum moment which can be applied.

Design: Given an external moment to be resisted, determine cross sectional dimensions (b and h) as well as reinforcement (A_s). Note that in many cases the external dimensions of the beam (b and h) are fixed by the architect.

12 We often consider the maximum moment along a member, and design accordingly.

11.1.4 Basic Relations and Assumptions

13 In developing a design/analysis method for reinforced concrete, the following **basic relations** will be used, Fig. **??**:

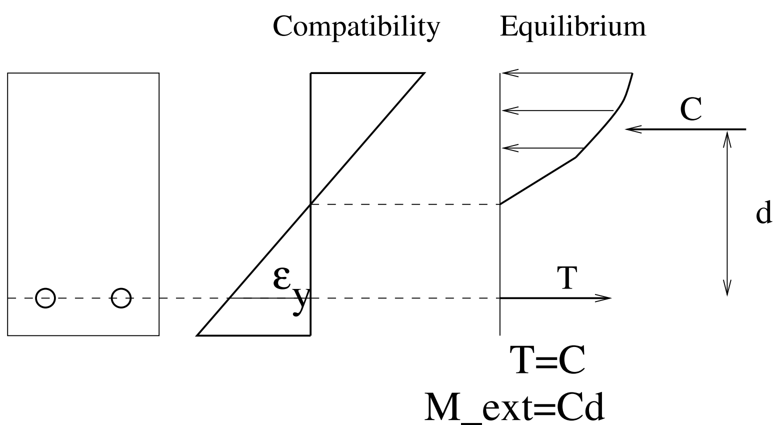


Figure 11.2: Internal Equilibrium in a R/C Beam

1. Equilibrium: of forces and moment at the cross section. 1) $\Sigma F_x = 0$ or Tension in the reinforcement = Compression in concrete; and 2) $\Sigma M = 0$ or external moment (that is the one obtained from the moment envelope) equal and opposite to the internal one (tension in steel and compression of the concrete).
2. Material Stress Strain: We recall that all normal strength concrete have a failure strain $\epsilon_u = .003$ in compression irrespective of f'_c.

14 Basic **assumptions** used:

Compatibility of Displacements: Perfect bond between steel and concrete (no slip). Note that those two materials do also have very close coefficients of thermal expansion under normal temperature.

Plane section remain plane ⇒ strain is proportional to distance from neutral axis.

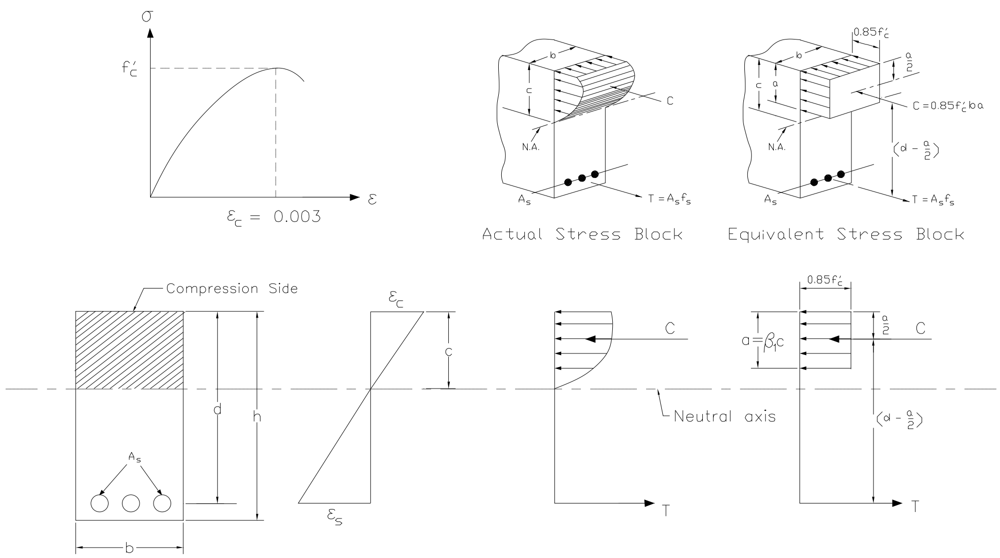


Figure 11.3: Cracked Section, Limit State

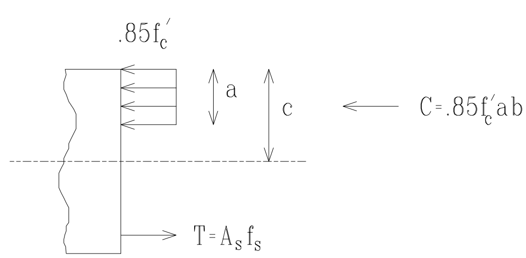


Figure 11.4: Whitney Stress Block

11.2.3 Analysis

Given A_s, b, d, f'_c, and f_y determine the design moment:

1. $\rho_{act} = \frac{A_s}{bd}$
2. $\rho_b = (.85)\beta_1 \frac{f'_c}{f_y}\frac{87}{87+f_y}$
3. If $\rho_{act} < \rho_b$ (that is failure is triggered by yielding of the steel, $f_s = f_y$)

$$\left.\begin{array}{rcl} a & = & \frac{A_s f_y}{.85 f'_c b} \text{ From Equilibrium} \\ M_D & = & \Phi A_s f_y \left(d - \frac{a}{2}\right) \end{array}\right\} M_D = \Phi \underbrace{A_s f_y \left(d - 0.59\frac{A_s f_y}{f'_c b}\right)}_{M_n}$$

   Combining this last equation with $\rho = \frac{A_s}{bd}$ yields

$$\boxed{M_D = \Phi \rho f_y b d^2 \left(1 - .59\rho\frac{f_y}{f'_c}\right)} \tag{11.9}$$

4. † If $\rho_{act} > \rho_b$ is not allowed by the code as this would be an over-reinforced section which would fail with no prior warning. However, if such a section exists, and we need to determine its moment carrying capacity, then we have two unknowns:

   (a) Steel strain ε_s (which was equal to ε_y in the previous case)

   (b) Location of the neutral axis c.

   We have two equations to solve this problem

   Equilibrium: of forces

$$c = \frac{A_s f_s}{.85 f'_c b \beta_1} \tag{11.10}$$

   Strain compatibility: since we know that at failure the maximum compressive strain ε_c is equal to 0.003. Thus from similar triangles we have

$$\frac{c}{d} = \frac{.003}{.003 + \varepsilon_s} \tag{11.11}$$

   Those two equations can be solved by either one of two methods:

   (a) Substitute into one single equation

   (b) By iteration

   Once c and $f_s = E\varepsilon_s$ are determined then

$$\boxed{M_D = \Phi A_s f_s \left(d - \frac{\beta_1 c}{2}\right)} \tag{11.12}$$

5. Check equilibrium of forces in the x direction ($\Sigma F_x = 0$)

$$a = \frac{A_s f_s}{.85 f'_c b} \tag{11.16}$$

6. Check assumption of f_s from the strain diagram

$$\frac{\varepsilon_s}{d-c} = \frac{.003}{c} \Rightarrow f_s = E_s \frac{d-c}{c}.003 < f_y \tag{11.17}$$

where $c = \frac{a}{\beta_1}$.

7. Iterate until convergence is reached.

■ Example 11-1: Ultimate Strength Capacity

Determine the ultimate Strength of a beam with the following properties: $b = 10$ in, $d = 23$ in, $A_s = 2.35$ in^2, $f'_c = 4,000$ psi and $f_y = 60$ ksi.

Solution:

$$\begin{aligned}
\rho_{act} &= \tfrac{A_s}{bd} = \tfrac{2.35}{(10)(23)} = .0102 \\
\rho_b &= .85\beta_1 \tfrac{f'_c}{f_y}\tfrac{87}{87+f_y} = (.85)(.85)\tfrac{4}{60}\tfrac{87}{87+60} = .02885 > \rho_{act}\surd \\
a &= \tfrac{A_s f_y}{.85 f'_c b} = \tfrac{(2.35)(60)}{(.85)(4)(10)} = 4.147 \text{ in} \\
M_n &= (2.35)(60)(23 - \tfrac{4.147}{2}) = 2,950 \text{ k.in} \\
M_D &= \Phi M_n = (.9)(2,950) = \boxed{2,660 \text{ k.in}}
\end{aligned}$$

Note that from the strain diagram

$$c = \frac{a}{0.85} = \frac{4.414}{0.85} = 4.87 \text{ in}$$

Alternative solution

$$\begin{aligned}
M_n &= \rho_{act} f_y b d^2 \left(1 - .59\rho_{act}\tfrac{f_y}{f'_c}\right) \\
&= A_s f_y d \left(1 - .59\rho_{act}\tfrac{f_y}{f'_c}\right) \\
&= (2.35)(60)(23)\left[1 - (.59)\tfrac{60}{4}(.01021)\right] = 2,950 \text{ k.in} = 245 \text{ k.ft} \\
M_D &= \Phi M_n = (.9)(2,950) = \boxed{2,660 \text{ k.in}}
\end{aligned}$$

■

■ Example 11-2: Beam Design I

6. Check equilibrium of forces:

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{(2.42)\ \text{in}^2(40)\ \text{ksi}}{(.85)(3)\ \text{ksi}(11.5)\ \text{in}} = 3.3\ \text{in}\surd$$

7. we have converged on a.
8. Actual ρ is $\rho_{act} = \frac{2.42}{(11.5)(20)} = .011$
9. ρ_b is equal to

$$\rho_b = .85\beta_1 \frac{f'_c}{f_y}\frac{87}{87+f_y} = (.85)(.85)\frac{3}{40}\frac{87}{87+40} = .037$$

10. $\rho_{max} = .75\rho = (0.75)(0.037) = .0278 > 0.011\surd$ thus $f_s = f_y$ and we use $\boxed{A_s = 2.42\ \text{in}^2}$

■

11.3 Continuous Beams

28 Whereas coverage of continuous reinforced concrete beams is beyond the scope of this course, Fig. **??** illustrates a typical reinforcement in such a beam.

11.4 ACI Code

Attached is an *unauthorized* copy of some of the most relevant ACI-318-89 design code provisions.

8.1.1 - In design of reinforced concrete structures, members shall be proportioned for adequate strength in accordance with provisions of this code, using load factors and strength reduction factors Φ specified in Chapter 9.

8.3.1 - All members of frames or continuous construction shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis, except as modified according to Section 8.4. Simplifying assumptions of Section 8.6 through 8.9 may be used.

8.5.1 - Modulus of elasticity E_c for concrete may be taken as $W_c^{1.5} 33\sqrt{f'_c}$ (psi) for values of W_c between 90 and 155 lb per cu ft. For normal weight concrete, E_c may be taken as $57,000\sqrt{f'_c}$.

8.5.2 - Modulus of elasticity E_s for non-prestressed reinforcement may be taken as 29,000 psi.

9.1.1 - Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this code.

9.2 - Required Strength

9.2.1 - Required strength U to resist dead load D and live load L shall be at least equal to

$$U = 1.4D + 1.7L$$

9.2.2 - If resistance to structural effects of a specified wind load W are included in design, the following combinations of D, L, and W shall be investigated to determine the greatest required strength U

$$U = 0.75(1.4D + 1.7L + 1.7W)$$

where load combinations shall include both full value and zero value of L to determine the more severe condition, and

$$U = 0.9D + 1.3W$$

but for any combination of D, L, and W, required strength U shall not be less than Eq. (9-1).

9.3.1 - Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this code, multiplied by a strength reduction factor Φ.

9.3.2 - Strength reduction factor Φ shall be as follows:

9.3.2.1 - Flexure, without axial load 0.90

9.4 - Design strength for reinforcement Designs shall not be based on a yield strength of reinforcement f_y in excess of 80,000 psi, except for prestressing tendons.

10.2.2 - Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except, for deep flexural members with overall depth to clear span ratios greater than 2/5 for continuous spans and 4/5 for simple spans, a non-linear distribution of strain shall be considered. See Section 10.7.

10.2.3 - Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003.

10.2.4 - Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

10.2.5 - Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete, except when meeting requirements of Section 18.4.

10.2.6 - Relationship between concrete compressive stress distribution and concrete strain may be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

10.2.7 - Requirements of Section 10.2.5 may be considered satisfied by an equivalent rectangular concrete stress distribution defined by the following:

10.2.7.1 - Concrete stress of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $(a = \beta_1 c)$ from the fiber of maximum compressive strain.

10.2.7.2 - Distance c from fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis.

10.2.7.3 - Factor β_1 shall be taken as 0.85 for concrete strengths f'_c up to and including 4,000 psi. For strengths above 4,000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4,000 psi, but β_1 shall not be taken less than 0.65.

10.3.2 - Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.3 - For flexural members, and for members subject to combined flexure and compressive axial load when the design axial load strength (ΦP_n) is less than the smaller of $(0.10f'_c A_g)$ or (ΦP_b), the ratio of reinforcement p provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of ρ_b equalized by compression reinforcement need not be reduced by the 0.75 factor.

10.3.4 - Compression reinforcement in conjunction with additional tension reinforcement may be used to increase the strength of flexural members.

Chapter 12

PRESTRESSED CONCRETE

12.1 Introduction

1 Beams with longer spans are architecturally more appealing than those with short ones. However, for a reinforced concrete beam to span long distances, it would have to have to be relatively deep (and at some point the self weight may become too large relative to the live load), or higher grade steel and concrete must be used.

2 However, if we were to use a steel with f_y much higher than $\approx$ 60 ksi in reinforced concrete (R/C), then to take full advantage of this higher yield stress while maintaining full bond between concrete and steel, will result in unacceptably wide crack widths. Large crack widths will in turn result in corrosion of the rebars and poor protection against fire.

3 One way to control the concrete cracking and reduce the tensile stresses in a beam is to prestress the beam by applying an initial state of stress which is opposite to the one which will be induced by the load.

4 For a simply supported beam, we would then seek to apply an initial tensile stress at the top and compressive stress at the bottom. In prestressed concrete (P/C) this can be achieved through prestressing of a tendon placed below the elastic neutral axis.

5 Main advantages of P/C: Economy, deflection & crack control, durability, fatigue strength, longer spans.

6 There two type of Prestressed Concrete beams:

Pretensioning: Steel is first stressed, concrete is then poured around the stressed bars. When enough concrete strength has been reached the steel restraints are released, Fig. 12.1.

Postensioning: Concrete is first poured, then when enough strength has been reached a steel cable is passed thru a hollow core inside and stressed, Fig. 12.2.

12.1.1 Materials

7 P/C beams usually have higher compressive strength than R/C. Prestressed beams can have f'_c as high as 8,000 psi.

8 The importance of high yield stress for the steel is illustrated by the following simple example.

If we consider the following:

1. An unstressed steel cable of length l_s
2. A concrete beam of length l_c
3. Prestress the beam with the cable, resulting in a stressed length of concrete and steel equal to $l'_s = l'_c$.
4. Due to shrinkage and creep, there will be a change in length

$$\Delta l_c = (\varepsilon_{sh} + \varepsilon_{cr}) l_c \tag{12.1}$$

   we want to make sure that this amout of deformation is substantially smaller than the stretch of the steel (for prestressing to be effective).
5. Assuming ordinary steel: $f_s = 30$ ksi, $E_s = 29,000$ ksi, $\varepsilon_s = \frac{30}{29,000} = 1.03 \times 10^{-3}$ in/ in
6. The total steel elongation is $\varepsilon_s l_s = 1.03 \times 10^{-3} l_s$
7. The creep and shrinkage strains are about $\varepsilon_{cr} + \varepsilon_{sh} \simeq .9 \times 10^{-3}$
8. The residual stres which is left in the steel after creep and shrinkage took place is thus

$$(1.03 - .90) \times 10^{-3} (29 \times 10^3) = 4 \text{ ksi} \tag{12.2}$$

   Thus the total loss is $\frac{30-4}{30} = 87\%$ which is unacceptably too high.
9. Alternatively if initial stress was 150 ksi after losses we would be left with 124 ksi or a 17% loss.
10. Note that the actual loss is $(.90 \times 10^{-3})(29 \times 10^3) = 26$ ksiin each case

9 Having shown that losses would be too high for low strength steel, we will use

Strands usually composed of 7 wires. Grade 250 or 270 ksi, Fig. 12.3.

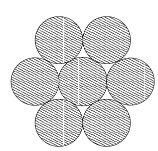


Figure 12.3: 7 Wire Prestressing Tendon

Tendon have diameters ranging from 1/2 to 1 3/8 of an inch. Grade 145 or 160 ksi.

Wires come in bundles of 8 to 52.

Note that yield stress is not well defined for steel used in prestressed concrete, usually we take 1% strain as effective yield.

10 Steel relaxation is the reduction in stress at constant strain (as opposed to creep which is reduction of strain at constant stress) occrs. Relaxation occurs indefinitely and produces

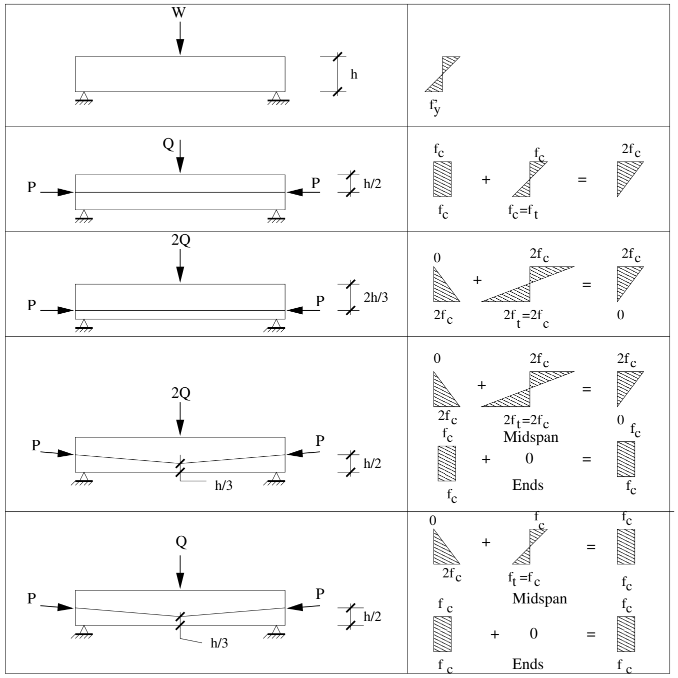


Figure 12.4: Alternative Schemes for Prestressing a Rectangular Concrete Beam, (Nilson 1978)

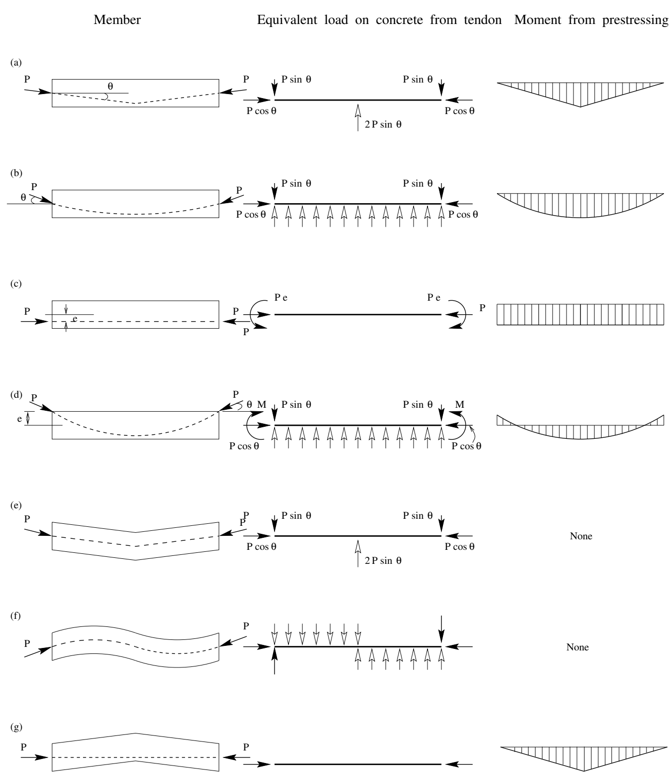


Figure 12.5: Determination of Equivalent Loads

4. P_e and $M_0 + M_{DL} + M_{LL}$

$$\boxed{\begin{array}{lcl} f_1 & = & -\frac{P_e}{A_c}\left(1-\frac{ec_1}{r^2}\right) - \frac{M_0+M_{DL}+M_{LL}}{S_1} \\ f_2 & = & -\frac{P_e}{A_c}\left(1+\frac{ec_2}{r^2}\right) + \frac{M_0+M_{DL}+M_{LL}}{S_2} \end{array}} \tag{12.7}$$

The internal stress distribution at each one of those four stages is illustrated by Fig. 12.7.

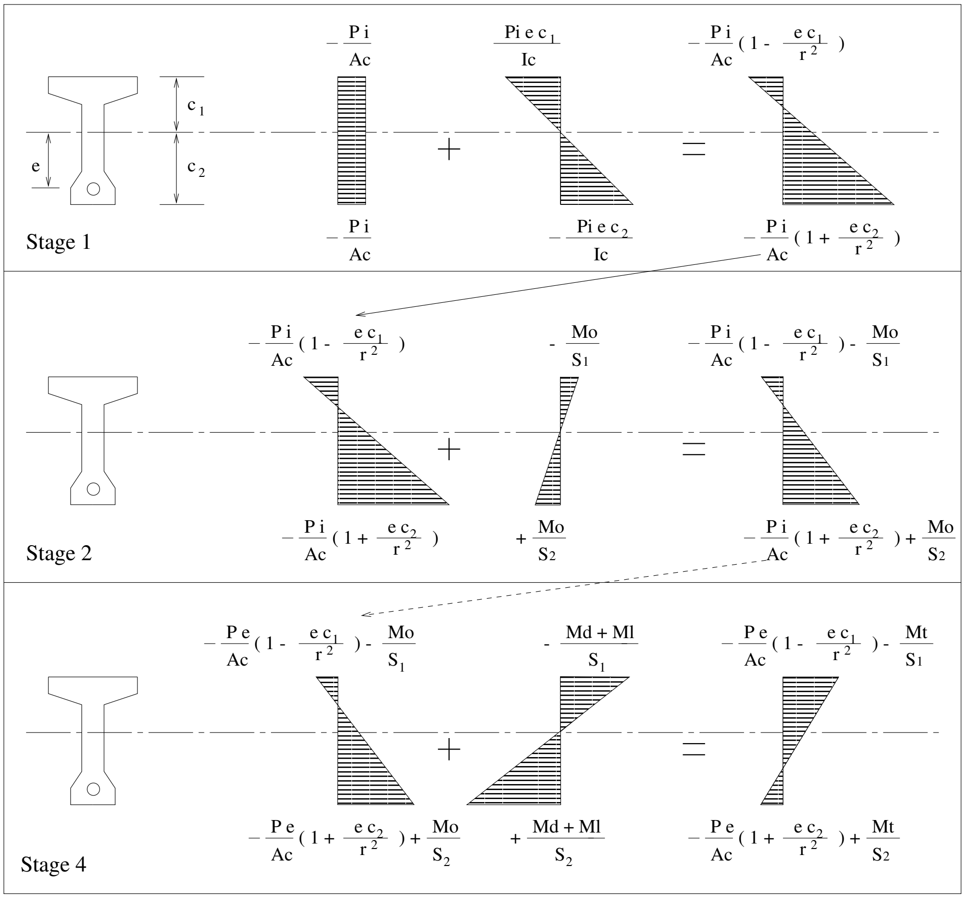


Figure 12.7: Flexural Stress Distribution for a Beam with Variable Eccentricity; Maximum Moment Section and Support Section, (Nilson 1978)

17 Those (service) flexural stresses must be below those specified by the ACI code (where the subscripts c, t, i and s refer to compression, tension, initial and service respectively):

f_{ci}	permitted concrete compression stress at initial stage	$.60f'_{ci}$
f_{ti}	permitted concrete tensile stress at initial stage	$< 3\sqrt{f'_{ci}}$
f_{cs}	permitted concrete compressive stress at service stage	$.45f'_c$
f_{ts}	permitted concrete tensile stress at initial stage	$6\sqrt{f'_c}$ or $12\sqrt{f'_c}$

Note that f_{ts} can reach $12\sqrt{f'_c}$ only if appropriate deflection analysis is done, because section would be cracked.

18 Based on the above, we identify two types of prestressing:

$$M_0 = \frac{(.183)(40)^2}{8} = 36.6 \text{ k.ft} \quad (12.9\text{-b})$$

The flexural stresses will thus be equal to:

$$f_{1,2}^{w_0} = \mp\frac{M_0}{S_{1,2}} = \mp\frac{(36.6)(12,000)}{1,000} = \mp 439 \text{ psi} \quad (12.10)$$

$$\begin{aligned}
f_1 &= -\frac{P_i}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{M_0}{S_1} && (12.11\text{-a})\\
&= -83-439 = \boxed{-522 \text{ psi}} && (12.11\text{-b})\\
f_{ti} &= 3\sqrt{f'_c} = +190\surd && (12.11\text{-c})\\
f_2 &= -\frac{P_i}{A_c}\left(1+\frac{ec_2}{r^2}\right)+\frac{M_0}{S_2} && (12.11\text{-d})\\
&= -1,837+439 = \boxed{-1,398 \text{ psi}} && (12.11\text{-e})\\
f_{ci} &= .6f'_c = -2,400\surd && (12.11\text{-f})
\end{aligned}$$

3. P_e and M_0. If we have 15% losses, then the effective force P_e is equal to $(1-0.15)169 = 144$ k

$$\begin{aligned}
f_1 &= -\frac{P_e}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{M_0}{S_1} && (12.12\text{-a})\\
&= -\frac{144,000}{176}\left(1-\frac{(5.19)(12)}{68.2}\right)-439 && (12.12\text{-b})\\
&= -71-439 = \boxed{-510 \text{ psi}} && (12.12\text{-c})\\
f_2 &= -\frac{P_e}{A_c}\left(1+\frac{ec_2}{r^2}\right)+\frac{M_0}{S_2} && (12.12\text{-d})\\
&= -\frac{144,000}{176}\left(1+\frac{(5.19)(12)}{68.2}\right)+439 && (12.12\text{-e})\\
&= -1,561+439 = \boxed{-1,122 \text{ psi}} && (12.12\text{-f})
\end{aligned}$$

note that -71 and $-1,561$ are respectively equal to $(0.85)(-83)$ and $(0.85)(-1,837)$ respectively.

4. P_e and $M_0 + M_{DL} + M_{LL}$

$$M_{DL} + M_{LL} = \frac{(0.55)(40)^2}{8} = 110 \text{ k.ft} \quad (12.13)$$

and corresponding stresses

$$f_{1,2} = \mp\frac{(110)(12,000)}{1,000} = \mp 1,320 \text{ psi} \quad (12.14)$$

Thus,

$$f_1 = -\frac{P_e}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{M_0+M_{DL}+M_{LL}}{S_1} \quad (12.15\text{-a})$$

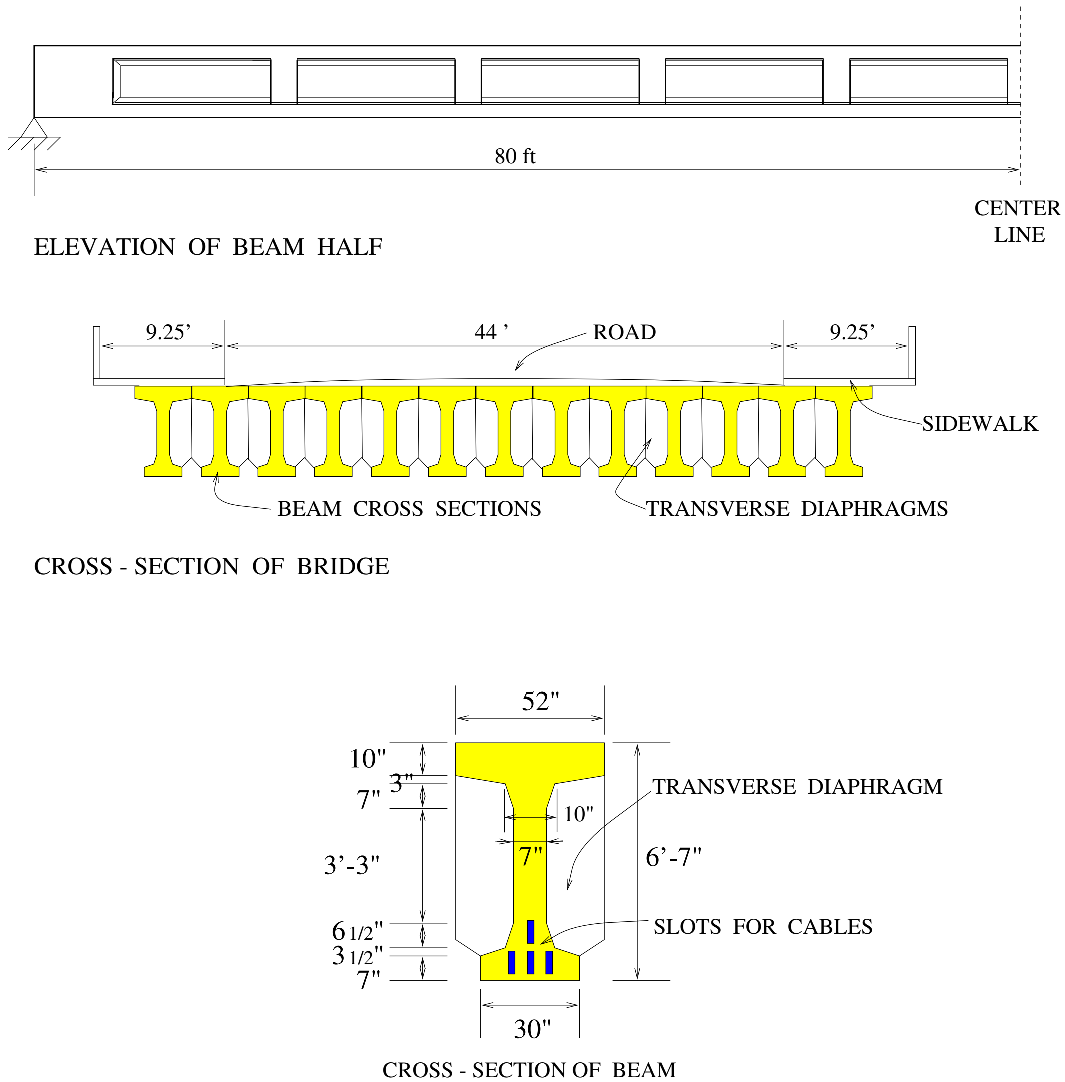


Figure 12.8: Walnut Lane Bridge, Plan View

12.3.3 Loads

26 The self weight of the beam is $q_0 = 1.72$ k/ft.

27 The concrete (density=.15 k/ ft^3) road has a thickness of 0.45 feet. Thus for a 44 foot width, the total load over one single beam is

$$q_{r,tot} = \frac{1}{13}(44)\text{ ft}(0.45)\text{ ft}(0.15)\text{ k/ ft}^3 = 0.23\text{ k/ft} \tag{12.20}$$

28 Similarly for the sidewalks which are 9.25 feet wide and 0.6 feet thick:

$$q_{s,tot} = \frac{1}{13}(2)(9.25)\text{ ft}(0.60)\text{ ft}(0.15)\text{ k/ ft}^3 = 0.13\text{ k/ft} \tag{12.21}$$

We note that the weight can be evenly spread over the 13 beams beacause of the lateral diaphragms.

29 The total dead load is

$$q_{DL} = 0.23 + 0.13 = 0.36\text{ k/ft} \tag{12.22}$$

30 The live load is created by the traffic, and is estimated to be 94 psf, thus over a width of 62.5 feet this gives a uniform live load of

$$w_{LL} = \frac{1}{13}(0.094)\text{ k/}ft^2(62.5)\text{ ft} = 0.45\text{ k/ft} \tag{12.23}$$

31 Finally, the combined dead and live load per beam is

$$w_{DL+LL} = 0.36 + 0.45 = 0.81\text{ k/ft} \tag{12.24}$$

12.3.4 Flexural Stresses

1. Prestressing force, P_i only

$$f_1 = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) \tag{12.25-a}$$

$$= -\frac{(2\times 10^6)}{1,354}\left(1 - \frac{(31.8)(39.5)}{943.}\right) = \boxed{490.\text{ psi}} \tag{12.25-b}$$

$$f_2 = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) \tag{12.25-c}$$

$$= -\frac{(2\times 10^6)}{1,354}\left(1 + \frac{(31.8)(39.5)}{943.}\right) = \boxed{-3,445.\text{ psi}} \tag{12.25-d}$$

2. P_i and the self weight of the beam M_0 (which has to be acconted for the moment the beam cambers due to prestressing)

$$M_0 = \frac{(1.72)(160)^2}{8} = 5,504\text{ k.ft} \tag{12.26}$$

The flexural stresses will thus be equal to:

$$f_{1,2}^{w_0} = \mp\frac{M_0}{S_{1,2}} = \mp\frac{(5,50.4)(12,000)}{943.} = \mp 2,043\text{ psi} \tag{12.27}$$

Chapter 13

ARCHES and CURVED STRUCTURES

1 This chapter will concentrate on the analysis of arches.

2 The concepts used are identical to the ones previously seen, however the major (and only) difference is that equations will be written in polar coordinates.

3 Like cables, arches can be used to reduce the bending moment in long span structures. Essentially, an arch can be considered as an inverted cable, and is transmits the load primarily through axial compression, but can also resist flexure through its flexural rigidity.

4 A parabolic arch uniformly loaded will be loaded in compression only.

5 A semi-circular arch unifirmly loaded will have some flexural stresses in addition to the compressive ones.

13.1 Arches

6 In order to optimize dead-load efficiency, long span structures should have their shapes approximate the coresponding moment diagram, hence an arch, suspended cable, or tendon configuration in a prestressed concrete beam all are nearly parabolic, Fig. 13.1.

7 Long span structures can be built using flat construction such as girders or trusses. However, for spans in excess of 100 ft, it is often more economical to build a curved structure such as an arch, suspended cable or thin shells.

8 Since the dawn of history, mankind has tried to span distances using arch construction. Essentially this was because an arch required materials to resist compression only (such as stone, masonary, bricks), and labour was not an issue.

9 The basic issues of static in arch design are illustrated in Fig. 13.2 where the vertical load is per unit horizontal projection (such as an external load but not a self-weight). Due to symmetry, the vertical reaction is simply $V = \frac{wL}{2}$, and there is no shear across the midspan of the arch (nor a moment). Taking moment about the crown,

$$M = Hh - \frac{wL}{2}\left(\frac{L}{2} - \frac{L}{4}\right) = 0 \quad (13.1)$$

Solving for H

$$H = \frac{wL^2}{8h} \quad (13.2)$$

We recall that a similar equation was derived for arches., and H is analogous to the $C-T$ forces in a beam, and h is the overall height of the arch, Since h is much larger than d, H will be much smaller than $C-T$ in a beam.

10 Since equilibrium requires H to remain constant across thee arch, a parabolic curve would theoretically result in no moment on the arch section.

11 Three-hinged arches are statically determinate structures which shape can acomodate support settlements and thermal expansion without secondary internal stresses. They are also easy to analyse through statics.

12 An arch carries the vertical load across the span through a combination of axial forces and flexural ones. A well dimensioned arch will have a small to negligible moment, and relatively high normal compressive stresses.

13 An arch is far more efficient than a beam, and possibly more economical and aesthetic than a truss in carrying loads over long spans.

14 If the arch has only two hinges, Fig. 13.3, or if it has no hinges, then bending moments may exist either at the crown or at the supports or at both places.

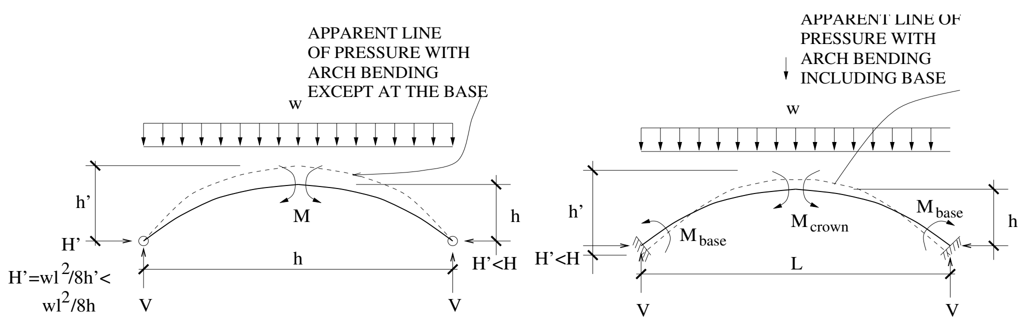


Figure 13.3: Two Hinged Arch, (Lin and Stotesbury 1981)

15 Since H varies inversely to the rise h, it is obvious that one should use as high a rise as possible. For a combination of aesthetic and practical considerations, a span/rise ratio ranging from 5 to 8 or perhaps as much as 12, is frequently used. However, as the ratio goes higher, we may have buckling problems, and the section would then have a higher section depth, and the arch advantage diminishes.

16 In a parabolic arch subjected to a uniform horizontal load there is no moment. However, in practice an arch is not subjected to uniform horizontal load. First, the depth (and thus the weight) of an arch is not usually constant, then due to the inclination of the arch the actual self weight is not constant. Finally, live loads may act on portion of the arch, thus the line of action will not necessarily follow the arch centroid. This last effect can be neglected if the live load is small in comparison with the dead load.

Solving those four equations simultaneously we have:

$$\begin{bmatrix} 140 & 26.25 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 80 & 60 & 0 & 0 \end{bmatrix} \begin{Bmatrix} R_{Ay} \\ R_{Ax} \\ R_{Cy} \\ R_{Cx} \end{Bmatrix} = \begin{Bmatrix} 2,900 \\ 80 \\ 50 \\ 3,000 \end{Bmatrix} \Rightarrow \begin{Bmatrix} R_{Ay} \\ R_{Ax} \\ R_{Cy} \\ R_{Cx} \end{Bmatrix} = \begin{Bmatrix} 15.1 \text{ k} \\ 29.8 \text{ k} \\ 34.9 \text{ k} \\ 50.2 \text{ k} \end{Bmatrix} \quad (13.4)$$

We can check our results by considering the summation with respect to b from the right:

$$(+\circlearrowleft)\ \Sigma M_z^B = 0; -(20)(20) - (50.2)(33.75) + (34.9)(60) = 0\surd \quad (13.5)$$

■

■ **Example 13-2: Semi-Circular Arch, (Gerstle 1974)**

Determine the reactions of the three hinged statically determined semi-circular arch under its own dead weight w (per unit arc length s, where $ds = rd\theta$). 13.6

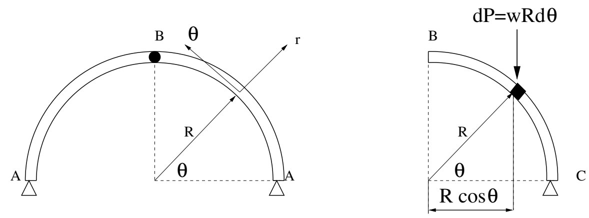


Figure 13.6: Semi-Circular three hinged arch

Solution:

I Reactions The reactions can be determined by **integrating** the load over the entire structure

1. **Vertical Reaction** is determined first:

$$(+\circlearrowright)\ \Sigma M_A = 0; -(C_y)(2R) + \int_{\theta=0}^{\theta=\pi} \underbrace{wRd\theta}_{dP} \underbrace{R(1+\cos\theta)}_{\text{moment arm}} = 0 \quad (13.6\text{-a})$$

$$\Rightarrow C_y = \frac{wR}{2}\int_{\theta=0}^{\theta=\pi}(1+\cos\theta)d\theta = \frac{wR}{2}[\theta - \sin\theta]\ |_{\theta=0}^{\theta=\pi}$$

$$= \frac{wR}{2}[(\pi - \sin\pi) - (0 - \sin 0)]$$

$$= \boxed{\frac{\pi}{2}wR} \quad (13.6\text{-b})$$

2. **Horizontal Reactions** are determined next

$$(+\circlearrowleft)\ \Sigma M_B = 0; -(C_x)(R) + (C_y)(R) - \int_{\theta=0}^{\theta=\frac{\pi}{2}} \underbrace{wRd\theta}_{dP} \underbrace{R\cos\theta}_{\text{moment arm}} = 0 \quad (13.7\text{-a})$$

$$\int_{\alpha=0}^{\theta} wRd\alpha \cdot R(\cos\alpha - \cos\theta) + M = 0 \tag{13.12}$$

$$\Rightarrow \boxed{M = wR^2\left[\frac{\pi}{2}(1-\sin\theta) + (\theta - \frac{\pi}{2})\cos\theta\right]} \tag{13.13}$$

III Deflection are determined last

1. The real curvature ϕ is obtained by dividing the moment by EI

$$\phi = \frac{M}{EI} = \frac{wR^2}{EI}\left[\frac{\pi}{2}(1-\sin\theta) + (\theta - \frac{\pi}{2})\cos\theta\right] \tag{13.14}$$

1. The virtual force $\delta\overline{P}$ will be a unit vertical point in the direction of the desired deflection, causing a virtual internal moment

$$\delta\overline{M} = \frac{R}{2}[1 - \cos\theta - \sin\theta] \qquad 0 \le \theta \le \frac{\pi}{2} \tag{13.15}$$

p

2. Hence, application of the virtual work equation yields:

$$\begin{aligned}\underbrace{1}_{\delta\overline{P}} \cdot \Delta &= 2\int_{\theta=0}^{\frac{\pi}{2}} \underbrace{\frac{wR^2}{EI}\left[\frac{\pi}{2}(1-\sin\theta) + (\theta - \frac{\pi}{2})\cos\theta\right]}_{\frac{M}{EI}=\phi} \cdot \underbrace{\frac{R}{2}\cdot[1-\cos\theta-\sin\theta]}_{\delta\overline{M}} \underbrace{Rd\theta}_{dx} \\ &= \frac{wR^4}{16EI}\left[7\pi^2 - 18\pi - 12\right] \\ &= \boxed{.0337\frac{wR^4}{EI}} \end{aligned} \tag{13.16-a}$$

■

13.1.2 Statically Indeterminate

■ Example 13-3: Statically Indeterminate Arch, (Kinney 1957)

Determine the value of the horizontal reaction component of the indicated two-hinged solid rib arch, Fig. 13.8 as caused by a concentrated vertical load of 10 k at the center line of the span. Consider shearing, axial, and flexural strains. Assume that the rib is a W24x130 with a total area of 38.21 in^2, that it has a web area of 13.70 in^2, a moment of inertia equal to 4,000 in^4, E of 30,000 k/in^2, and a shearing modulus G of 13,000 k/in^2.

Solution:

1. Consider that end C is placed on rollers, as shown in Fig. **??** A unit fictitious horizontal force is applied at C. The axial and shearing components of this fictitious force and of the vertical reaction at C, acting on any section θ in the right half of the rib, are shown at the right end of the rib in Fig. 13-7.

2. The expression for the horizontal displacement of C is

$$\underbrace{1}_{\delta\overline{P}}\Delta_{Ch} = 2\int_C^B \delta\overline{M}\frac{M}{EI}ds + 2\int_C^B \delta\overline{V}\frac{V}{A_wG}ds + 2\int_C^B \delta\overline{N}\frac{N}{AE}ds \quad (13.17)$$

3. From Fig. 13.9, for the rib from C to B,

$$M = \frac{P}{2}(100 - R\cos\theta) \quad (13.18\text{-a})$$
$$\delta\overline{M} = 1(R\sin\theta - 125.36) \quad (13.18\text{-b})$$
$$V = \frac{P}{2}\sin\theta \quad (13.18\text{-c})$$
$$\delta\overline{V} = \cos\theta \quad (13.18\text{-d})$$
$$N = \frac{P}{2}\cos\theta \quad (13.18\text{-e})$$
$$\delta\overline{N} = -\sin\theta \quad (13.18\text{-f})$$
$$ds = Rd\theta \quad (13.18\text{-g})$$

4. If the above values are substituted in Eq. 13.17 and integrated between the limits of 0.898 and $\pi/2$, the result will be

$$\Delta_{Ch} = 22.55 + 0.023 - 0.003 = 22.57 \quad (13.19)$$

5. The load P is now assumed to be removed from the rib, and a real horizontal force of 1 k is assumed to act toward the right at C in conjunction with the fictitious horizontal force of 1 k acting to the right at the same point. The horizontal displacement of C will be given by

$$\delta_{ChCh} = 2\int_C^B \delta\overline{M}\frac{\overline{M}}{EI}ds + 2\int_C^B \delta\overline{V}\frac{\overline{V}}{A_wG}ds + 2\int_C^B \delta\overline{N}\frac{\overline{N}}{AE}ds \quad (13.20\text{-a})$$
$$= 2.309 + 0.002 + 0.002 = 2.313 \text{ in} \quad (13.20\text{-b})$$

6. The value of the horizontal reaction component will be

$$H_C = \frac{\Delta_{Ch}}{\delta_{ChCh}} = \frac{22.57}{2.313} = \boxed{9.75 \text{ k}} \quad (13.21)$$

7. If only flexural strains are considered, the result would be

$$H_C = \frac{22.55}{2.309} = \boxed{9.76 \text{ k}} \quad (13.22)$$

Comments

1. For the given rib and the single concentrated load at the center of the span it is obvious that the effects of shearing and axial strains are insignificant and can be disregarded.

with respect to the x and y axis are BP and AB respectively. Applying three equations of equilibrium we obtain

$$F_z^A - \int_{\theta=0}^{\theta=\pi} wRd\theta = 0 \quad \Rightarrow \quad \boxed{F_z^A = wR\pi} \tag{13.23-a}$$

$$M_x^A - \int_{\theta=0}^{\theta=\pi} (wRd\theta)(R\sin\theta) = 0 \quad \Rightarrow \quad \boxed{M_x^A = 2wR^2} \tag{13.23-b}$$

$$M_y^A - \int_{\theta=0}^{\theta=\pi} (wRd\theta)R(1-\cos\theta) = 0 \quad \Rightarrow \quad \boxed{M_y^A = -wR^2\pi} \tag{13.23-c}$$

II Internal Forces are determined next

1. Shear Force:

$$(+\uparrow)\ \Sigma F_z = 0 \Rightarrow V - \int_0^\theta wRd\alpha = 0 \Rightarrow \boxed{V = wr\theta} \tag{13.24}$$

2. Bending Moment:

$$\Sigma M_R = 0 \Rightarrow M - \int_0^\theta (wRd\alpha)(R\sin\alpha) = 0 \Rightarrow \boxed{M = wR^2(1-\cos\theta)} \tag{13.25}$$

3. Torsion:

$$\Sigma M_T = 0 \Rightarrow + \int_0^\theta (wRd\alpha)R(1-\cos\alpha) = 0 \Rightarrow \boxed{T = -wR^2(\theta - \sin\theta)} \tag{13.26}$$

III Deflection are determined last we assume a rectangular cross-section of width b and height $d = 2b$ and a Poisson's ratio $\nu = 0.3$.

1. Noting that the member will be subjected to both flexural and torsional deformations, we seek to determine the two stiffnesses.
2. The flexural stiffness EI is given by $EI = E\frac{bd^3}{12} = E\frac{b(2b)^3}{12} = \frac{2Eb^4}{3} = .667Eb^4$.
3. The torsional stiffness of solid rectangular sections $J = kb^3d$ where b is the shorter side of the section, d the longer, and k a factor equal to .229 for $\frac{d}{b} = 2$. Hence $G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+.3)} = .385E$, and $GJ = (.385E)(.229b^4) = .176Eb^4$.
4. Considering both flexural and torsional deformations, and replacing dx by $rd\theta$:

$$\underbrace{\delta\overline{P}\Delta}_{\delta\overline{W}^*} = \underbrace{\underbrace{\int_0^\pi \delta\overline{M}\frac{M}{EI_z}Rd\theta}_{\text{Flexure}} + \underbrace{\int_0^\pi \delta\overline{T}\frac{T}{GJ}Rd\theta}_{\text{Torsion}}}_{\delta\overline{U}^*} \tag{13.27}$$

   where the real moments were given above.
5. Assuming a unit virtual downward force $\delta\overline{P} = 1$, we have

$$\delta\overline{M} = R\sin\theta \tag{13.28-a}$$

$$\delta\overline{T} = -R(1-\cos\theta) \tag{13.28-b}$$

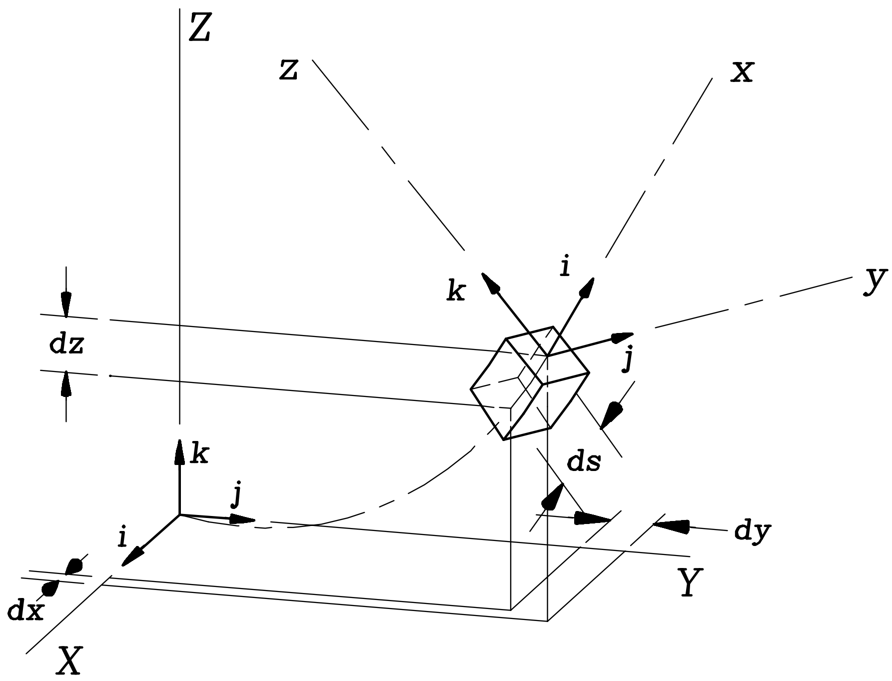


Figure 13.11: Geometry of Curved Structure in Space

24 The weak bending axis is normal to both N and S, and thus its unit vector is determined from

$$\mathbf{w} = \mathbf{n} \times \mathbf{s} \tag{13.33}$$

13.2.1.2 Equilibrium

25 For the equilibrium equations, we consider the free body diagram of Fig. 13.12 an applied load $\mathbf{P}$ is acting at point A. The resultant force vector $\mathbf{F}$ and resultant moment vector $\mathbf{M}$ acting on the cut section B are determined from equilibrium

$$\Sigma \mathbf{F} = 0; \quad \mathbf{P} + \mathbf{F} = 0; \quad \mathbf{F} = -\mathbf{P} \tag{13.34-a}$$

$$\Sigma \mathbf{M}^B = 0; \quad \mathbf{L} \times \mathbf{P} + \mathbf{M} = 0; \quad \mathbf{M} = -\mathbf{L} \times \mathbf{P} \tag{13.34-b}$$

where $\mathbf{L}$ is the lever arm vector from B to A.

26 The axial and shear forces N, V_s and V_w are all three components of the force vector $\mathbf{F}$ along the N, S, and W axes and can be found by dot product with the appropriate unit vectors:

$$N = \mathbf{F} \cdot \mathbf{n} \tag{13.35-a}$$

$$V_s = \mathbf{F} \cdot \mathbf{s} \tag{13.35-b}$$

$$V_w = \mathbf{F} \cdot \mathbf{w} \tag{13.35-c}$$

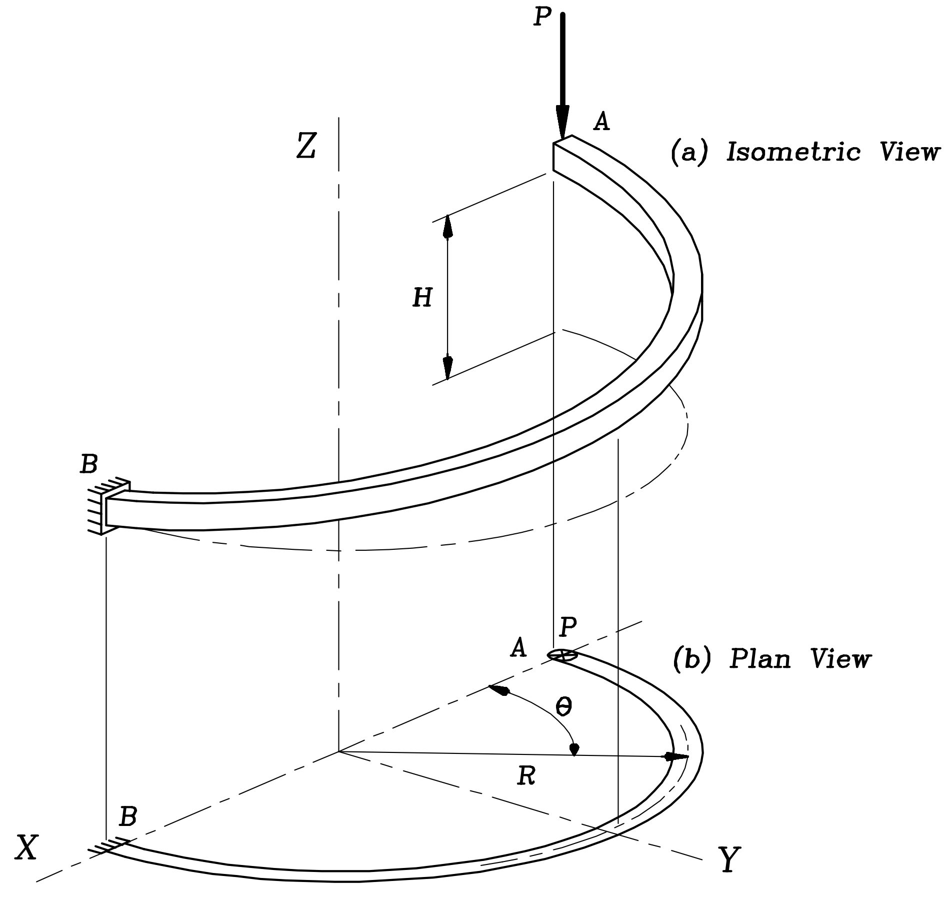


Figure 13.13: Helicoidal Cantilevered Girder

6. Finally, the components of the force $\mathbf{F} = -P\mathbf{k}$ and the moment $\mathbf{M}$ are obtained by appropriate dot products with the unit vectors

$$N = \mathbf{F}\cdot\mathbf{n} = \boxed{-\frac{1}{K}P\frac{H}{\pi R}} \quad (13.47\text{-a})$$

$$V_s = \mathbf{F}\cdot\mathbf{s} = \boxed{0} \quad (13.47\text{-b})$$

$$V_w = \mathbf{F}\cdot\mathbf{w} = \boxed{-\frac{1}{K}P} \quad (13.47\text{-c})$$

$$T = \mathbf{M}\cdot\mathbf{n} = \boxed{-\frac{PR}{K}(1-\cos\theta)} \quad (13.47\text{-d})$$

$$M_s = \mathbf{M}\cdot\mathbf{s} = \boxed{PR\sin\theta} \quad (13.47\text{-e})$$

$$M_w = \mathbf{M}\cdot\mathbf{w} = \boxed{\frac{PH}{\pi K}(1-\cos\theta)} \quad (13.47\text{-f})$$

■

Chapter 14

BUILDING STRUCTURES

14.1 Introduction

14.1.1 Beam Column Connections

1 The connection between the beam and the column can be, Fig. 14.1:

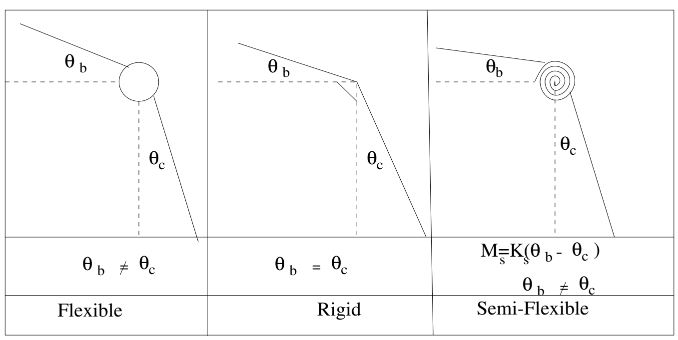


Figure 14.1: Flexible, Rigid, and Semi-Flexible Joints

Flexible that is a hinge which can transfer forces only. In this case we really have cantiliver action only. In a flexible connection the column and beam end moments are both equal to zero, $M_{\text{col}} = M_{\text{beam}} = 0$. The end rotation are not equal, $\theta_{\text{col}} \neq \theta_{\text{beam}}$.

Rigid: The connection is such that $\theta_{\text{beam}} = \theta_{\text{col}}$ and moment can be transmitted through the connection. In a rigid connection, the end moments and rotations are equal (unless there is an externally applied moment at the node), $M_{\text{col}} = M_{\text{beam}} \neq 0$, $\theta_{\text{col}} = \theta_{\text{beam}}$.

Semi-Rigid: The end moments are equal and not equal to zero, but the rotation are different. $\theta_{\text{beam}} \neq \theta_{\text{col}}$, $M_{\text{col}} = M_{\text{beam}} \neq 0$. Furthermore, the difference in rotation is resisted by the spring $M_{\text{spring}} = K_{\text{spring}}(\theta_{\text{col}} - \theta_{\text{beam}})$.

14.1.2 Behavior of Simple Frames

2 For vertical load across the beam rigid connection will reduce the maximum moment in the beam (at the expense of a negative moment at the ends which will in turn be transferred to

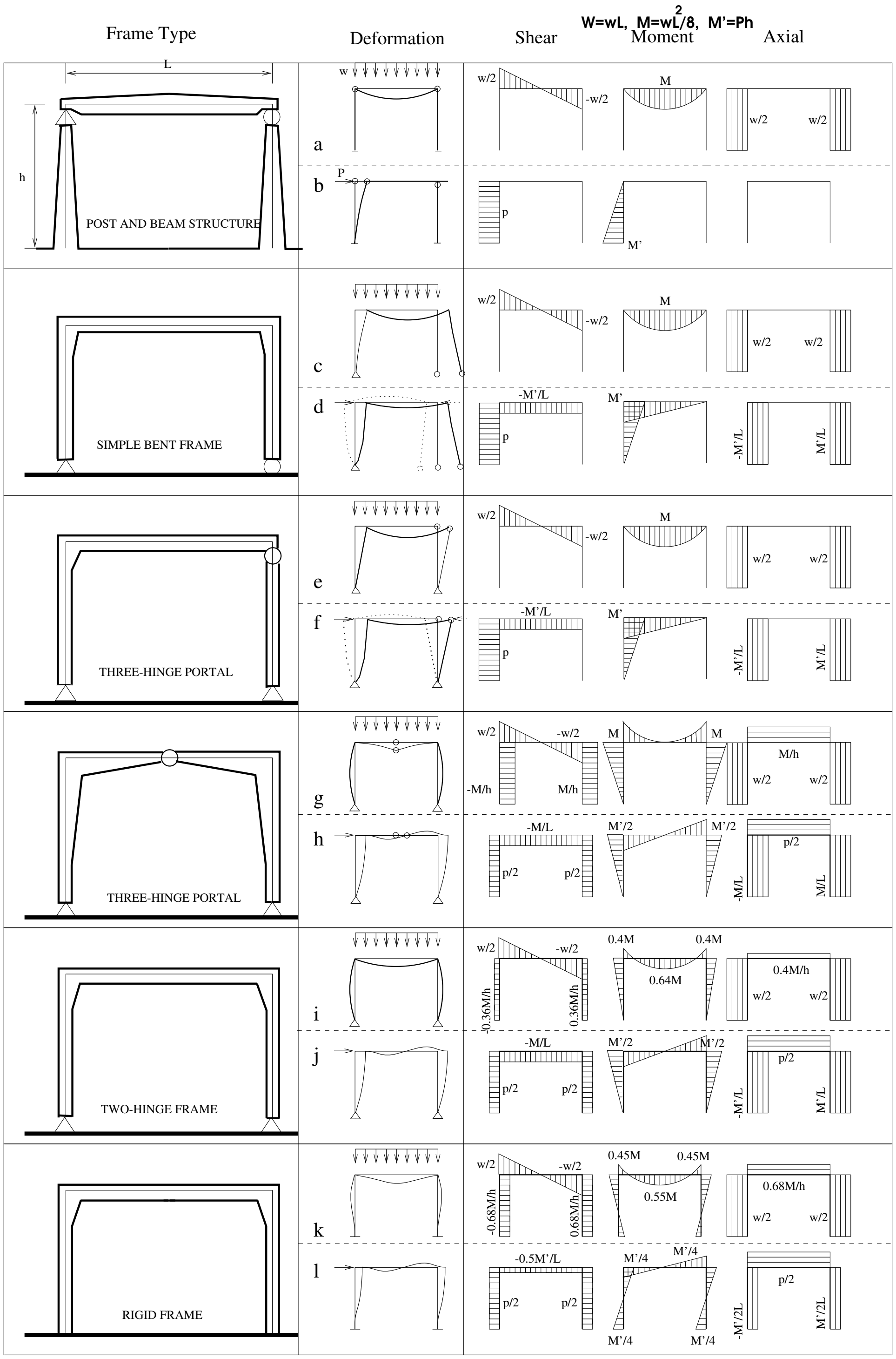


Figure 14.3: Deformation, Shear, Moment, and Axial Diagrams for Various Types of Portal Frames Subjected to Vertical and Horizontal Loads

14.2 Buildings Structures

11 There are three primary types of building systems:

Wall Subsytem: in which very rigid walls made up of solid masonry, paneled or braced timber, or steel trusses constitute a rigid subsystem. This is only adequate for small rise buildings.

Vertical Shafts: made up of four solid or trussed walls forming a tubular space structure. The tubular structure may be interior (housing elevators, staircases) and/or exterior. Most efficient for very high rise buildings.

Rigid Frame: which consists of linear vertical components (columns) rigidly connected to stiff horizontal ones (beams and girders). This is not a very efficient structural form to resist lateral (wind/earthquake) loads.

14.2.1 Wall Subsystems

12 Whereas exterior wall provide enclosure and interior ones separation, both of them can also have a structural role in trnsfering vertical and horizontal loads.

13 Walls are constructed out of masonry, timber concrete or steel.

14 If the wall is braced by floors, then it can provide an excellent resitance to horizontal load in the plane of the wall (but not orthogonal to it).

15 When shear-walls subsytems are used, it is best if the center of orthogonal shear resistance is close to the centroid of lateral loads as applied. If this is not the case, then there will be torsional design problems.

14.2.1.1 Example: Concrete Shear Wall

`From (Lin and Stotesbury 1981)`

16 We consider a reinforced concrete wall 20 ft wide, 1 ft thick, and 120 ft high with a vertical load of 400 k acting on it at the base. As a result of wind, we assume a uniform horizontal force of 0.8 kip/ft of vertical height acting on the wall. It is required to compute the flexural stresses and the shearing stresses in the wall to resist the wind load, Fig. 14.5.

1. Maximum shear force and bending moment at the base

$$V_{max} = wL = (0.8)\text{ k.ft}(120)\text{ ft} = 96\text{ k} \tag{14.8-a}$$

$$M_{max} = \frac{wL^2}{2} = \frac{(0.8)\text{ k.ft}(120)^2\text{ ft}^2}{2} = 5,760\text{ k.ft} \tag{14.8-b}$$

2. The resulting eccentricity is

$$e_{\text{Actual}} = \frac{M}{P} = \frac{(5,760)\text{ k.ft}}{(400)\text{ k}} = 14.4\text{ ft} \tag{14.9}$$

3. The critical eccentricity is

$$e_{cr} = \frac{L}{6} = \frac{(20)\text{ ft}}{6} = 3.3\text{ ft} < e_{\text{Actual}} N.G. \tag{14.10}$$

thus there will be tension at the base.

9. The compressive stress of 740 psi can easily be sustained by concrete, as to the tensile stress of 460 psi, it would have to be resisted by some steel reinforcement.

10. Given that those stresses are *service* stresses and not factored ones, we adopt the WSD approach, and use an allowable stress of 20 ksi, which in turn will be increased by 4/3 for seismic and wind load,

$$\sigma_{all} = \frac{4}{3}(20) = 26.7 \text{ ksi} \tag{14.16}$$

11. The stress distribution is linear, compression at one end, and tension at the other. The length of the tension area is given by (similar triangles)

$$\frac{x}{460} = \frac{20}{460+740} \Rightarrow x = \frac{460}{460+740}(20) = 7.7 \text{ ft} \tag{14.17}$$

12. The total tensile force inside this triangular stress block is

$$T = \frac{1}{2}(460) \text{ ksi}(7.7 \times 12) \text{ in} \underbrace{(12) \text{ in}}_{\text{width}} = 250 \text{ k} \tag{14.18}$$

13. The total amount of steel reinforcement needed is

$$A_s = \frac{(250) \text{ k}}{(26.7) \text{ ksi}} = \boxed{9.4 \text{ in}^2} \tag{14.19}$$

This amount of reinforcement should be provided at both ends of the wall since the wind or eartquake can act in any direction. In addition, the foundations should be designed to resist tensile uplift forces (possibly using piles).

14.2.1.2 Example: Trussed Shear Wall

`From (Lin and Stotesbury 1981)`

17 We consider the same problem previously analysed, but use a trussed shear wall instead of a concrete one, Fig. 14.6.

1. Using the maximum moment of 5,760 kip-ft (Eq. 14.8-b), we can compute the compression and tension in the columns for a lever arm of 20 ft.

$$F = \pm\frac{(5,760) \text{ k.ft}}{(20) \text{ ft}} = \pm 288 \text{ k} \tag{14.20}$$

2. If we now add the effect of the 400 kip vertical load, the forces would be

$$C = -\frac{(400) \text{ k}}{2} - 288 = \boxed{-488 \text{ k}} \tag{14.21-a}$$

$$T = -\frac{(400) \text{ k}}{2} + 288 = \boxed{88 \text{ k}} \tag{14.21-b}$$

3. The force in the diagonal which must resist a base shear of 96 kip is (similar triangles)

$$\frac{F}{96} = \frac{\sqrt{(20)^2+(24)^2}}{20} \Rightarrow F = \frac{\sqrt{(20)^2+(24)^2}}{20}(96) = \boxed{154 \text{ k}} \tag{14.22}$$

4. The design could be modified to have no tensile forces in the columns by increasing the width of the base (currently at 20 ft).

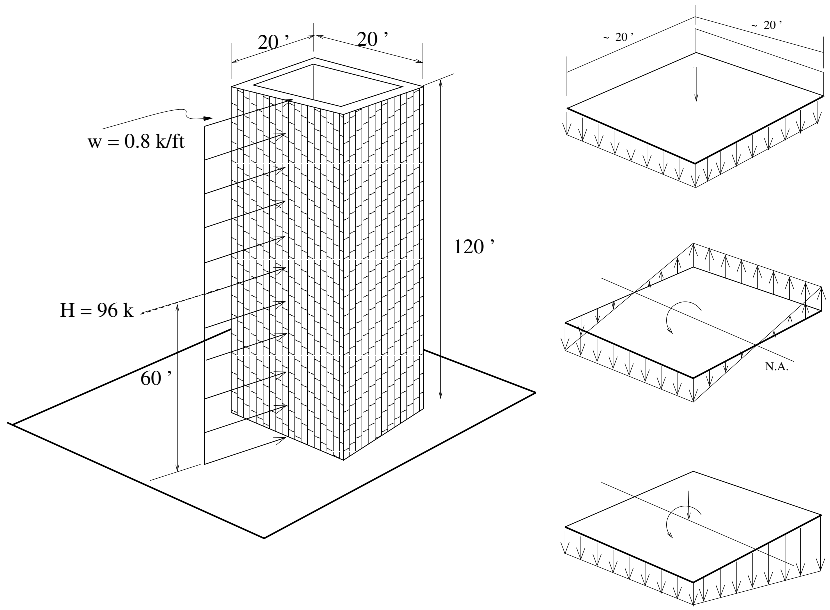


Figure 14.7: Design Example of a Tubular Structure, (Lin and Stotesbury 1981)

4. The maximum flexural stresses:

$$\sigma_{fl} = \pm\frac{MC}{I} = \pm\frac{(5,760)\text{ k.ft}(20/2)\text{ ft}}{(4,600)\text{ ft}^4} = \pm 12.5\text{ ksf} = \pm 87\text{ psi} \tag{14.25}$$

5. The average shear stress is

$$\tau = \frac{V}{A} = \frac{(96)\text{ k}}{2(20)(1)\text{ ft}^2} = 2.4\text{ ksf} = \boxed{17\text{ psi}} \tag{14.26}$$

6. The vertical load of 1,600 k produces an axial stress of

$$\sigma_{ax} = \frac{P}{A} = \frac{-(1,600)\text{ k}}{(4(20)(1)\text{ ft}^2} = -20\text{ ksf} = -140\text{ psi} \tag{14.27}$$

7. The total stresses are thus

$$\sigma = \sigma_{ax} + \sigma_{fl} \tag{14.28-a}$$
$$\sigma_1 = -140 + 87 = \boxed{-53\text{ psi}} \tag{14.28-b}$$
$$\sigma_2 = -140 - 87 = \boxed{-227\text{ psi}} \tag{14.28-c}$$

thus we do not have any tensile stresses, and those stresses are much better than those obtained from a single shear wall.

14.2.3 Rigid Frames

21 Rigid frames can carry both vertical and horizontal loads, however their analysis is more complex than for tubes.

14.3.1 Vertical Loads

30 The girders at each floor are assumed to be continuous beams, and columns are assumed to resist the resulting unbalanced moments from the girders.

31 Basic assumptions

1. Girders at each floor act as continous beams supporting a uniform load.
2. Inflection points are assumed to be at
   (a) One tenth the span from both ends of each girder.
   (b) Mid-height of the columns
3. Axial forces and deformation in the girder are negligibly small.
4. Unbalanced end moments from the girders at each joint is distributed to the columns above and below the floor.

32 Based on the first assumption, all beams are statically determinate and have a span, L_s equal to 0.8 the original length of the girder, L. (Note that for a rigidly connected member, the inflection point is at 0.211 L, and at the support for a simply supported beam; hence, depending on the nature of the connection one could consider those values as upper and lower bounds for the approximate location of the hinge).

33 End forces are given by

Maximum positive moment at the center of each beam is, Fig. 14.9

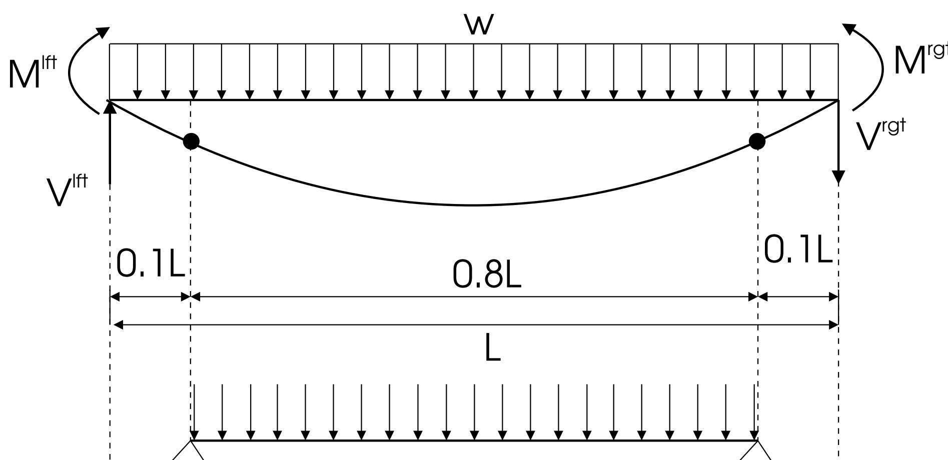


Figure 14.9: Approximate Analysis of Frames Subjected to Vertical Loads; Girder Moments

$$M^+ = \frac{1}{8}wL_s^2 = w\frac{1}{8}(0.8)^2L^2 = 0.08wL^2 \tag{14.29}$$

Maximum negative moment at each end of the girder is given by, Fig. 14.9

$$M^{left} = M^{\rm rgt} = -\frac{w}{2}(0.1L)^2 - \frac{w}{2}(0.8L)(0.1L) = -0.045wL^2 \tag{14.30}$$

Column Shear Points of inflection are at mid-height, with possible exception when the columns on the first floor are hinged at the base, Fig. 14.11

$$\boxed{V = \frac{M^{\rm top}}{\frac{h}{2}}} \tag{14.34}$$

Girder axial forces are assumed to be negligible eventhough the unbalanced column shears above and below a floor will be resisted by girders at the floor.

14.3.2 Horizontal Loads

34 We must differentiate between low and high rise buildings.

Low rise buidlings, where the height is at least samller than the hrizontal dimension, the deflected shape is characterized by shear deformations.

High rise buildings, where the height is several times greater than its least horizontal dimension, the deflected shape is dominated by overall flexural deformation.

14.3.2.1 Portal Method

35 Low rise buildings under lateral loads, have predominantly shear deformations. Thus, the approximate analysis of this type of structure is based on

1. Distribution of horizontal shear forces.
2. Location of inflection points.

36 The *portal method* is based on the following assumptions

1. Inflection points are located at
   (a) Mid-height of all columns above the second floor.
   (b) Mid-height of floor columns if rigid support, or at the base if hinged.
   (c) At the center of each girder.
2. Total horizontal shear at the mid-height of all columns at any floor level will be distributed among these columns so that each of the two exterior columns carry half as much horizontal shear as each interior columns of the frame.

37 Forces are obtained from

Column Shear is obtained by passing a horizontal section through the mid-height of the columns at each floor and summing the lateral forces above it, then Fig. 14.12

$$\boxed{V^{ext} = \frac{\sum F^{lateral}}{2\text{No. of bays}} \qquad V^{int} = 2V^{ext}} \tag{14.35}$$

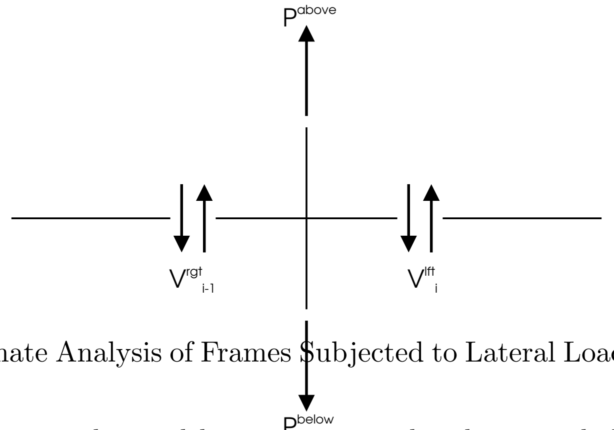


Figure 14.14: Approximate Analysis of Frames Subjected to Lateral Loads; Column Axial Force

Column Axial Forces are obtained by summing girder shears and the axial force from the column above, Fig. **??**

$$P = P^{\rm above} + P^{\rm rgt} + P^{\rm lft} \tag{14.39}$$

■ Example 14-1: Approximate Analysis of a Frame subjected to Vertical and Horizontal Loads

Draw the shear, and moment diagram for the following frame. **Solution:**

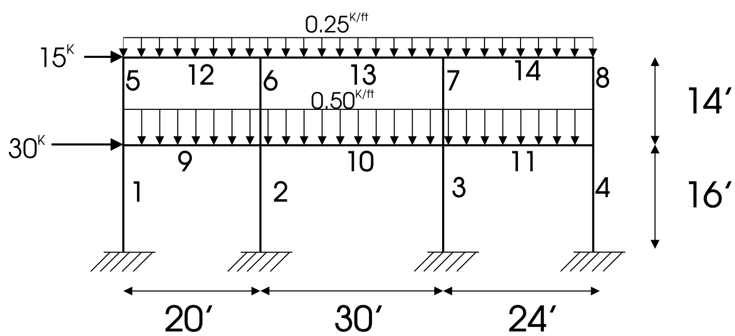


Figure 14.15: Example; Approximate Analysis of a Building

Vertical Loads

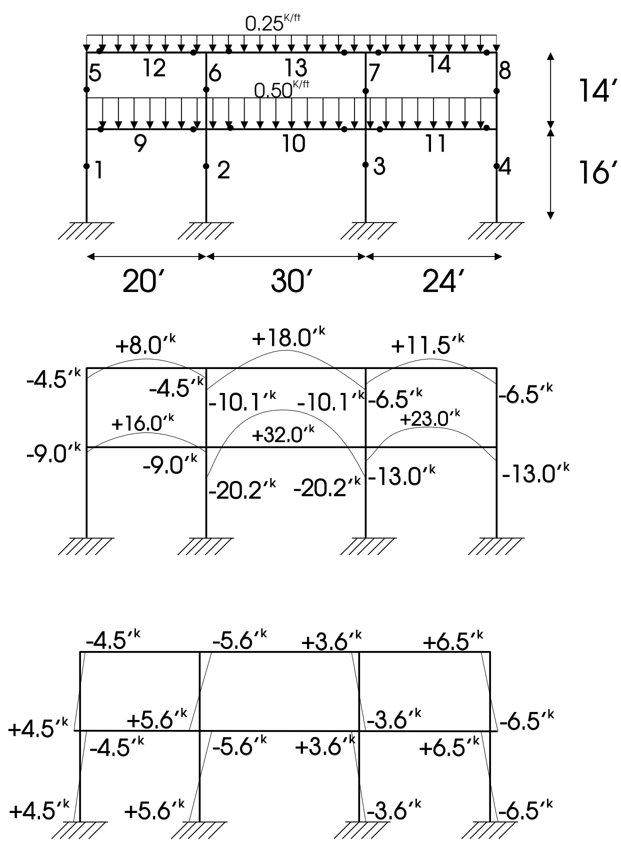


Figure 14.16: Approximate Analysis of a Building; Moments Due to Vertical Loads

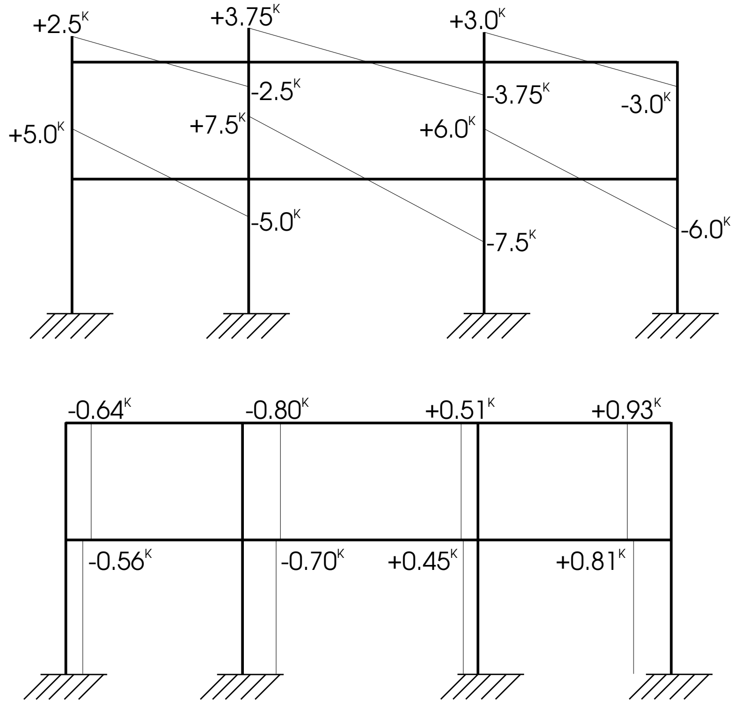


Figure 14.17: Approximate Analysis of a Building; Shears Due to Vertical Loads

Horizontal Loads, Portal Method

1. Column Shears

$$
\begin{aligned}
V_5 &= \tfrac{15}{(2)(3)} &&= 2.5 \text{ k}\\
V_6 &= 2(V_5) = (2)(2.5) &&= 5 \text{ k}\\
V_7 &= 2(V_5) = (2)(2.5) &&= 5 \text{ k}\\
V_8 &= V_5 &&= 2.5 \text{ k}\\
V_1 &= \tfrac{15+30}{(2)(3)} &&= 7.5 \text{ k}\\
V_2 &= 2(V_1) = (2)(7.5) &&= 15 \text{ k}\\
V_3 &= 2(V_1) = (2)(2.5) &&= 15 \text{ k}\\
V_4 &= V_1 &&= 7.5 \text{ k}
\end{aligned}
$$

2. Top Column Moments

$$
\begin{aligned}
M_5^{\text{top}} &= \tfrac{V_1 H_5}{2} = \tfrac{(2.5)(14)}{2} &&= 17.5 \text{ k.ft}\\
M_5^{\text{bot}} &= -M_5^{\text{top}} &&= -17.5 \text{ k.ft}\\
M_6^{\text{top}} &= \tfrac{V_6 H_6}{2} = \tfrac{(5)(14)}{2} &&= 35.0 \text{ k.ft}\\
M_6^{\text{bot}} &= -M_6^{\text{top}} &&= -35.0 \text{ k.ft}\\
M_7^{\text{top}} &= \tfrac{V_7^{\text{up}} H_7}{2} = \tfrac{(5)(14)}{2} &&= 35.0 \text{ k.ft}\\
M_7^{\text{bot}} &= -M_7^{\text{top}} &&= -35.0 \text{ k.ft}\\
M_8^{\text{top}} &= \tfrac{V_8^{\text{up}} H_8}{2} = \tfrac{(2.5)(14)}{2} &&= 17.5 \text{ k.ft}\\
M_8^{\text{bot}} &= -M_8^{\text{top}} &&= -17.5 \text{ k.ft}
\end{aligned}
$$

Approximate Analysis Vertical Loads APROXVER.XLS Victor E. Saouma

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1																	
2				**L1**					**L2**					**L3**			
3	**Height**	**Span**		**20**					**30**					**24**			
4	**14**	**Load**		**0.25**					**0.25**					**0.25**			
5	**16**	**Load**		**0.5**					**0.5**					**0.5**			
6			**MOMENTS**														
7			**Bay 1**					**Bay 2**					**Bay 3**				
8			Col	Beam			Column		Beam			Column		Beam			Col
9				Lft	Cnt	Rgt			Lft	Cnr	Rgt			Lft	Cnt	Rgt	
10				=-0.045*D4*D3^2	=0.08*D4*D3*D3	=+D10			=-0.045*I4*I3^2	=0.08*I4*I3*I3	=+I10			=-0.045*N4*N3^2	=0.08*N4*N3*N3	=N10	
11			=+D10				=-F10+I10					=-K10+N10					=-P10
12			=-C11				=-G11					=-L11					=-Q11
13				=-0.045*D5*D3^2	=0.08*D5*D3*D3	=+D13			=-0.045*I5*I3^2	=0.08*I5*I3*I3	=+I13			=-0.045*N5*N3^2	=0.08*N5*N3*N3	=+N13	
14			=+D13+C12				=-F13+I13+G12					=-K13+N13+L12					=-P13+Q12
15			=-C14				=-G14					=-L14					=-Q14
16			**SHEAR**														
17			**Bay 1**					**Bay 2**					**Bay 3**				
18			Col	Beam			Column		Beam			Column		Beam			Col
19				Lft		Rgt			Lft		Rgt			Lft		Rgt	
20				=+D3*D4/2		=-D20			=+I3*I4/2		=-I20			=+N3*N4/2		=-N20	
21			=2*C11/A4				=2*G11/A4					=2*L11/A4					=2*Q11/A4
22				=+D3*D5/2		=-D22			=+I3*I5/2		=-I22			=+N3*N5/2		=-N22	
23			=2*C14/A5				=2*G14/A5					=2*L14/A5					=2*Q14/A5
24			**AXIAL FORCE**														
25			**Bay 1**					**Bay 2**					**Bay 3**				
26			Col	Beam			Column		Beam			Column		Beam			Col
27				0					0					0			
28			=+D20				=-F20+I20					=-K20+N20					=-P20
29				0					0					0			
30			=+C28+D22				=+G28-F22+I22					=+L28-K22+N22					=+Q28-P22

Figure 14.19: Approximate Analysis for Vertical Loads; Equations in Spread-Sheet

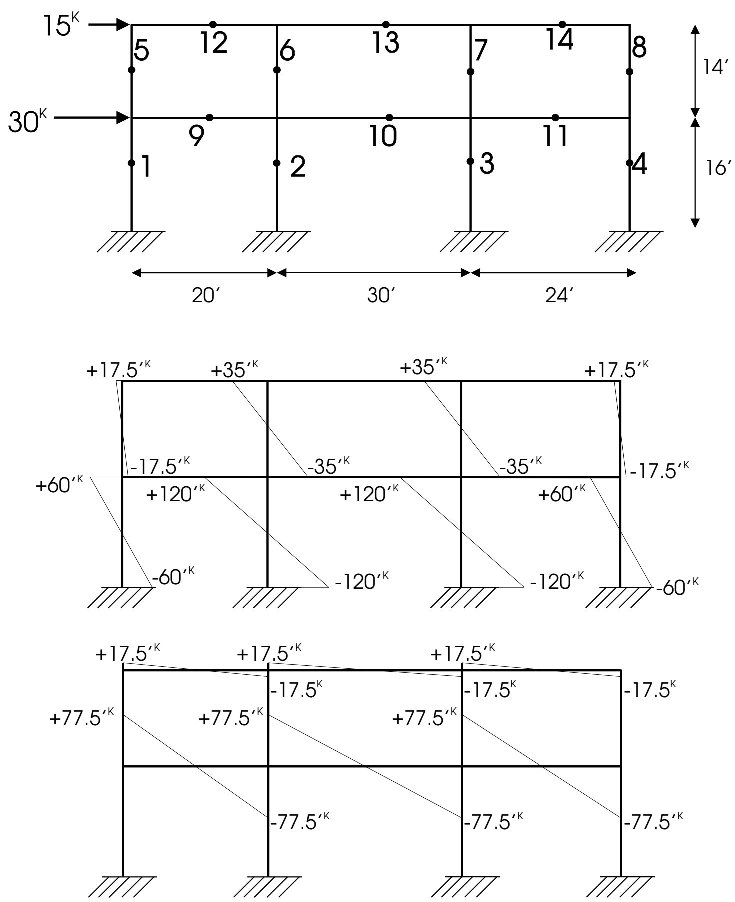


Figure 14.20: Approximate Analysis of a Building; Moments Due to Lateral Loads

Portal Method PORTAL.XLS Victor E. Saouma

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	**PORTAL METHOD**																		
2	**# of Bays**					**3**			**L1**				**L2**				**L3**		
3									**20**				**30**				**24**		
4								**MOMENTS**											
5	**# of Storeys**					**2**		**Bay 1**				**Bay 2**				**Bay 3**			
6			Force		Shear			Col	Beam		Column		Beam		Column		Beam		Col
7		**H**	**Lat.**	Tot	Ext	Int			Lft	Rgt			Lft	Rgt			Lft	Rgt	
8									=+H9	=-I8			=+J8+K9	=-M8			=+N8+O9	=-Q8	
9	**H1**	**14**	**15**	=+C9	=+D9/(2*F2)	=2*E9		=+E9*B9/2			=+F9*B9/2				=+K9				=+H9
10								=-H9			=-K9				=+K10				=+H10
11									=+H12-H10	=-I11			=+K12-K10+J11	=-M11			=+O12-O10+N11	=-Q11	
12	**H2**	**16**	**30**	=SUM(C9:C12)	=+D12/(2*F2)	=2*E12		=+E12*B12/2			=+F12*B12/2				=+K12				=+H12
13								=-H12			=-K12				=+K13				=+H13
14								**SHEAR**											
15								**Bay 1**				**Bay 2**				**Bay 3**			
16								Col	Beam		Column		Beam		Column		Beam		Col
17									Lft	Rgt			Lft	Rgt			Lft	Rgt	
18									=-2*I8/I$3	=+I18			=-2*M8/M$3	=+M18			=-2*Q8/Q$3	=+Q18	
19								=+E9			=+F9				=+F9				=+E9
20								=+H19			=+K19				=+O19				=+S19
21									=-2*I11/I$3	=+I21			=-2*M11/M$3	=+M21			=-2*Q11/Q$3	=+Q21	
22								=+E12			=+F12				=+F12				=+E12
23								=+H22			=+K22				=+O22				=+S22
24								**AXIAL FORCE**											
25								**Bay 1**				**Bay 2**				**Bay 3**			
26								Col	Beam		Column		Beam		Column		Beam		Col
27									0				0				0		
28								=-I18			=+J18-M18				=+N18-Q18				=+R18
29									0				0				0		
30								=+H28-I21			=+K28+J21-M21				=+O28+N21-Q21				=+S28+R21

Figure 14.22: Portal Method; Equations in Spread-Sheet

Design Parameters On the basis of the two approximate analyses, vertical and lateral load, we now seek the design parameters for the frame, Table 14.2.

■

14.4 Lateral Deflections

38 Even at schematic or preliminary stages of design, it is important to estimate the lateral deflections of tall buildings for the following reasons

1. Lateral deflections are often limited by code requirements, for example $\Delta < h/500$ where h is the height of the story or of the building. This is important because occupants should not experience uncomfortable horizontal movements.

2. A building that deflects severly under lateral forces may have damage problems associated with vibration (as with vertical defelctions of beams).

3. Through the evaluation of deflection, one may also get some idea of the relative horizontal load carried by the various vertical subsystems in a building (i.e. how much is carried by the shaft compared to the frames). Since all systems are connected, they must move together and through their stiffness (deformation per unit load) we can determine the contribution of each subsystem.

Mem.		Vert.	Hor.	**Design Values**
9	-ve Moment	9.00	77.50	**86.50**
	+ve Moment	16.00	0.00	**16.00**
	Shear	5.00	7.75	**12.75**
10	-ve Moment	20.20	77.50	**97.70**
	+ve Moment	36.00	0.00	**36.00**
	Shear	7.50	5.17	**12.67**
11	-ve Moment	13.0	77.50	**90.50**
	+ve Moment	23.00	0.00	**23.00**
	Shear	6.00	6.46	**12.46**
12	-ve Moment	4.50	17.50	**22.00**
	+ve Moment	8.00	0.00	**8.00**
	Shear	2.50	1.75	**4.25**
13	-ve Moment	10.10	17.50	**27.60**
	+ve Moment	18.00	0.00	**18.00**
	Shear	3.75	1.17	**4.92**
14	-ve Moment	6.50	17.50	**24.00**
	+ve Moment	11.50	0.00	**11.50**
	Shear	3.00	1.46	**4.46**

Table 14.2: Girders Combined Approximate Vertical and Horizontal Loads

14.4.1 Short Wall

39 In short structures (as with short beams), shear deflections, Fig. 14.23 dominates. For a

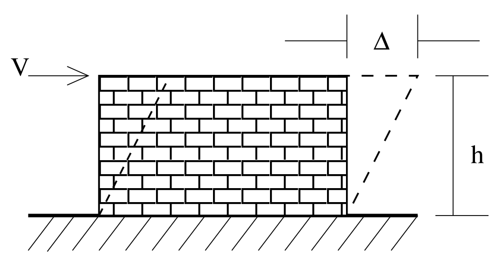


Figure 14.23: Shear Deformation in a Short Building, (Lin and Stotesbury 1981)

concentrated load

$$\boxed{\Delta \approx \frac{1.2Vh}{GA}} \quad (14.40)$$

where for concrete and steel $G \approx \frac{2}{5}E$.

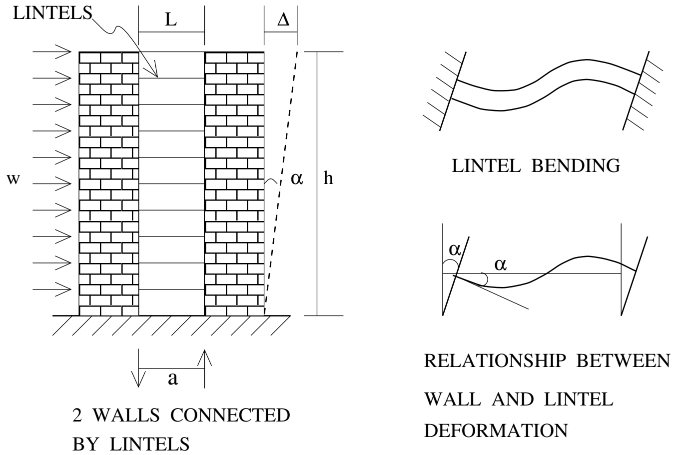


Figure 14.25: Deflection in a Building Structure Composed of Two Slender Walls and Lintels, (Lin and Stotesbury 1981)

and

$$\boxed{\delta \approx \alpha h} \tag{14.43}$$

14.4.4 Frames

43 Deflection of a rigid frame is essentially caused by shear between stories which produces vertical shears in the girders. From the portal method we can estimate those deformations, Fig. 14.26.

44 The deformation for the first story at the exterior joint can be approximated from

$$\Delta_{col} = \frac{V_{col_E} h^3}{12EI_{col_E}} \tag{14.44-a}$$

$$\Delta_{gdr} = \frac{V_{gdr} L^2 h}{12EI_{gdr}} = \frac{2V_{col_E} L h^2}{12EI_{gdr}} \tag{14.44-b}$$

$$\Delta_{tot_E} = \Delta_{col_E} + \Delta_{gdr} = \frac{V_{col_E} h^2}{12E}\left[\frac{h}{I_{col_E}} + \frac{2L}{I_g dr}\right] \tag{14.44-c}$$

45 For the interior joint:

$$\Delta_{col} = \frac{V_{col_I} h^3}{12EI_{col_I}} \tag{14.45-a}$$

$$\Delta_{gdr} = \frac{V_{gdr} L^2 h}{12EI_{gdr}} = \frac{2V_{col_I} L h^2}{12EI_{gdr}} \tag{14.45-b}$$

$$\Delta_{tot_I} = \Delta_{col_I} + \Delta_{gdr} = \frac{V_{col_I} h^2}{12E}\left[\frac{h}{I_{col_I}} + \frac{L}{I_g dr}\right] \tag{14.45-c}$$

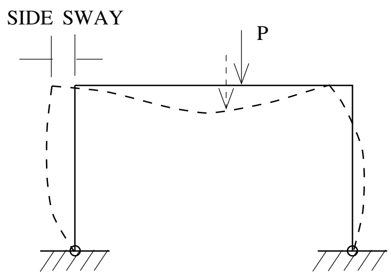


Figure 14.28: Side-Sway Deflection from Unsymmetrical Vertical Load, (Lin and Stotesbury 1981)

14.4.5 Trussed Frame

48 The cantilever deflection due to column shortening and lengthening (produced by overturning moment) is usually of secondary importance until the building is some 40 stories or higher,Fig. ??.

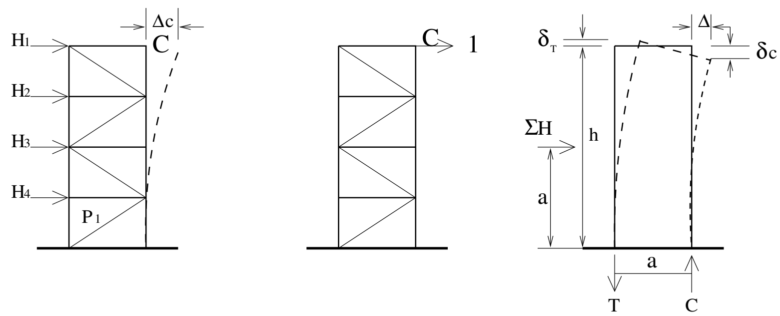


Figure 14.29: Axial Elongation and Shortening of a Truss Frame, (Lin and Stotesbury 1981)

49 The total deflection Δ at C is given by

$$\boxed{\Delta = \Sigma \frac{\overline{P}PL}{AE}} \tag{14.47}$$

where: P is the force in any member due to loading on the whole system, L is the length of the member, A and E the corresponding cross sectional area and modulus of elasticity, $\overline{P}$ the force in the same member due to a unit (1) force applied in the direction of the deflection sought, and at the point in question.

50 Alternatively, we can neglect the web deformation and consider only the axial deformations in the colums:

$$\boxed{\begin{array}{rcl} \Delta & \approx & \frac{\delta_t+\delta_c}{a}h \\ \delta_t+\delta_c & = & 2\frac{Th}{AE} \end{array}} \tag{14.48}$$

$$E = 3 \times 10^6 \text{ psi} = 432,000 \text{ ksf} \quad (14.49\text{-d})$$

$$\Delta = \frac{(4.8)\text{ k.ft}(156)^4\text{ ft}^4}{8(432,000)\text{ ksf}(9,400)\text{ ft}^4} = 0.087\text{ ft} \quad (14.49\text{-e})$$

$$\frac{\Delta}{h} = \frac{0.087}{156} = \frac{1}{1,800}\surd \quad (14.49\text{-f})$$

The $\frac{\Delta}{h}$ ratio is much less than 1/500 as permitted in most building codes, and s within the usual index for concrete buildings, which ranges between 1/1,000 and 1/2,500.

If the wall thickness is reduced, and if door openings are considered, the deflection will be correspondingly smaller.

The deflection due to moment increases rapidly at the top, the value of 1/1,800 indicates only the *average* drift index for the entire building, whereas the *story* drift index may be higher, especially for the top floor.

2. We next consider the deflection of the top of the frame. Assuming that each frame takes 1/9 of the total wind load and shear, and neglecting column shortening, then:

$$\Delta = \frac{V_{col_E}h^2}{12E}\left[\frac{h}{I_{col_E}} + \frac{2L}{I_gdr}\right] \quad (14.50\text{-a})$$

$$I_{col} = \frac{bh^3}{12} = \frac{(20/12)(20/12)^3}{12} = 0.64\text{ ft}^4 \quad (14.50\text{-b})$$

$$I_{gdr} = 3.64\text{ ft}^4 \quad (14.50\text{-c})$$

$$V_{col_I}^{\text{ground}} = \frac{(4.8)\text{ k.ft}(156)\text{ ft}}{(2)(9)} = 41.7\text{ k/col} \quad (14.50\text{-d})$$

$$\Delta = \frac{(41.7)\text{ k}(12)^2}{12(432,000)\text{ ksf}}\left[\frac{(12)\text{ ft}}{(0.64)\text{ ft}^4} + \frac{2(60)\text{ ft}}{(3.46)\text{ ft}^4}\right] \quad (14.50\text{-e})$$

$$= 0.00116(18.8 + 34.7) = 0.062\text{ ft} \quad (14.50\text{-f})$$

3. Since the story drift varies with the shear in the story, which decreases linearly to the top, the average drift will be $0.062/2 = 0.31$ ft per story and the deflection at top of the building is approximately

$$\Delta = (13)(0.031) = \boxed{0.40\text{ ft}} \quad (14.51)$$

which indicates a drift ratio of

$$\text{Drift Ratio for Building} = \frac{(0.4)\text{ ft}}{(156)\text{ ft}} = \boxed{\tfrac{1}{400}} \quad (14.52\text{-a})$$

$$\text{Drift Ratio for Ground Floor} = \frac{(0.062)\text{ ft}}{(12)\text{ ft}} = \boxed{\tfrac{1}{194}} \quad (14.52\text{-b})$$

4. Comparing the frame deflection of 0.40 ft with the shaft deflection of 0.087 ft, it is seen that the frame is about five times more flexible than the shaft. Furthermore, the frame would not be stiff enough to carry all the lateral load by itlself. Proportioning the lateral load to the relative stiffnesses, the frame would carry about 1/6 of the load, and the remaining 5/6 will be carried by the shaft.

Increasing the column size will stiffen the frame, but in order to be really effective, the girder stiffness will also need to be increased, since thegirders contribute about 2/3 of the

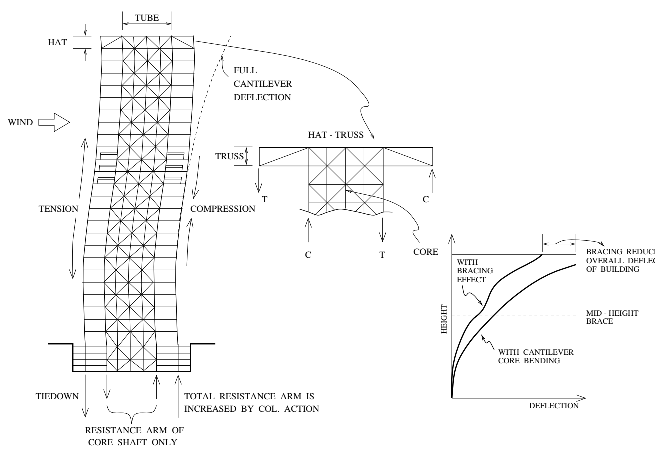


Figure 14.32: Effect of Exterior Column Bracing in Buildings, (Lin and Stotesbury 1981)

Draft

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