Mathematical Economics, Debt and the Sabbath Rest

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1. Introduction

This paper shows that the biblical laws concerning the Sabbath and debt forgiveness can be viewed as a rational answer to a technical economic problem. A geometric approach is used to mathematically model the debt cycle as a circular process. It is shown that there is a mathematical ratio of debt market activity to exchange market activity; between discrete and continuous economic activity, which is six-sevenths. Moreover, to facilitate this, agents must, on average, rest one seventh of the time given to economic activity. The observance of biblical commandments achieves this ratio. Hence, biblical laws can be interpreted as a means of managing the macroeconomic debt cycle.

The contribution to the field of mathematics is an economic interpretation, based on debt, of the relationship between Euclidean and spherical geometry. The difference between an economy without debt and one with debt can be interpreted as being equivalent to the difference between a linear line and a circular chord between two points: which is directly relates to the constant Pi.

This paper first shows that the biblical commands of God to the nation of Israel yield what we call a 'Fundamental Ratio'. This is a ratio of market exchange activity to debt market activity. It will then be shown that this 'scripturally derived' ratio of six-seventh can also be derived from a simple geometrically based mathematical economic model.

This paper takes up the challenge that Iannoconne (2010) implicitly sets when saying that:

'There really is no honest way to Christianize mathematical theorems, computer algorithms, or the laws of physics. Nor is there any efficient way to

[1] Please note that the all thoughts, views and opinions are entirely the author's and are not at all reflective of the official position of Her Majesty's Treasury (the UK's Economics and Finance Ministry)

Christianize microeconomic theory, econometrics, mathematical economics, and other mainstream standard economic topics.'

In contrast, it is shown that once sufficient light is shed on the true nature of the economic system, one discovers that there is no need to 'Christianize' any particular part of economics, but rather one will see an in-built mathematical economics, and indeed, it follows, a physics, already present in scripture: a natural and pre-existent integration.

The relation to physics is that this paper provides a mathematical explanation to the biblical message that the earth was made to be inhabited: the seven day week provides appropriate space for economic activity to occur harmoniously.

2. An Economic interpretation of the Laws of God

The two Judaeo-Christian biblical laws that are of relevance in this paper. They are found in the following scriptural references:

Exodus 20

[8]Remember the sabbath day, to keep it holy. [9]Six days shalt thou labour, and do all thy
work: [10]But the seventh day is the sabbath of the LORD thy God: in it thou shalt not
do any work,…'

Deuteronomy 15

[1]At the end of every seven years thou shalt make a release. [2]And this is the manner of
the release: Every creditor that lendeth ought unto his neighbour shall release it;…'

These verses dictate that the debt cycle was to occur over seven years but at the end of the seventh year the debt cycle was to cease. Concurrently, in this time period, six years of economic activity in the exchange market will have occurred i.e. work.

Hence, the ratio of the exchange market ‘work’ (or effort) to the debt cycle[2] was to be six-sevenths.

3. The Model

3.1 Definitions

In this framework, agents are economic actors. There are only two types of (representative) agent: creditors and debtors. Each individual can either be a creditor or a debtor in any distinct period. These roles naturally reverse in the eventuality of a debtor paying off their debts.

3.2 Output versus Production

Production is the units of goods made but not necessarily sold. However, production is either 'sold' for money or 'exchanged' for money, and subsequently lent. In contrast, output is that production which is sold in the goods exchange market.

3.3 The nature of Debt

Debt occurs when an agent consumes resources not funded by their own sold output. For every debt in the system there must be an equal and opposite credit, by definition.

3.4 Modelling money

Two of the roles of money are key in the paper. Firstly, it’s role as a means of exchange, but more importantly, it’s role as a store of value. It is this second property that permits money to facilitate a debt market. The Arrow-Debreu economy has no money. Hence, it has no debt. It follows that the debt cycle cannot be modelled within an Arrow-Debreu economy.

Agents can only exchange money for goods and goods for money. Also, money is endogenous in this model as it is 'created' by 'exchanging' goods for money. This is

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[2] Or financial market

obtained from a 'central bank'[3] and so, in any given period of analysis, half production must be held at a central bank to fund the other half of production dedicated to purchases of output. Despite it being a 'monetary economy' it is a real economy in so far as there is no inflation.

3.5 Assumptions

1. There is one good and this is a durable good.
2. All debts are repaid in full.
3. There is only one currency and one good. One unit of currency is exchangeable for one unit of good.
4. The is no inflation.
5. Production technology is independent of an agents position as debtor or creditor.
6. There are no interest rates.
7. There is no seigneorage. Money is paper money and is only given value over a single period. It is created, given a unit value equivalent to goods and then destroyed at the end of the period.
8. It is a closed economy.
9. It takes one unit of effort to produce one unit goods.
10. There is a central bank, which is a part of government, which has sole power to create money. It is willing to exchange one unit of money for one unit of goods. The unit value of money is fixed.
11. There are not gifts or arbitrary transfers of income in the model. However, a gift can merely be interpreted as an expenditure of the giver and an earning of the receiver.

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[3] Here a central bank is a part of government: a literal monetary authority. The term central bank is used more for expository purposes given their ubiquitous role in the modern economy.

12. This model, does not consider the mechanisms by which an agent becomes indebted such as asymmetric stochastic negative shocks to production technology.

3.6 The model's dynamics

Economy demarcated into two sets of agents: net creditors and net debtors. These are represented by a representative creditor and a representative debtor respectively. There are two markets: the debt market (for lending and borrowing) and exchange market (for buying and selling).

There are two periods of analysis. This is split into equal halves in terms of overall joint cumulative output. The mid-point is such that the debt reaches its highest point, which marks the half-way point in cumulative output over the two periods.

At the start of period one there is no debt in the system. As such, output equals expenditure for each group for the cumulative period up until period one and hence there is no outstanding debt as every agent has funded their expenditure. At the end of period two, cumulative output over the two periods is equal so that there is no debt at the end of the debt cycle.

Over the two periods, by definition, the debt and credit positions of the two representative agents move in equal and opposite directions. More specifically, their shares of total cumulative output must move in equal and opposite directions throughout: debt is a zero sum game.

4. An Example

There is a representative net debtor and a representative net creditor

4.1 Scenario 1: No debt

Both agents produce 100 units and sell 50 units. They use the other 50 units to exchange for money (with a central bank) to make the purchases. The key constraint is that outputs must be equal at all times.

4.2 Scenario 2: Debt

The debtor is still restricted to producing 100 units. The creditor now wants to sell 10 more units than the debtor: 110 units. The creditor must now produce 120 units, then exchange 10 units of that 120 to obtain money, which is then lent to the debtor who can now purchase 10 more of the creditor's units of production. Hence, the creditor has produced 120 units and sold 110 units, whilst the debtor has produced 100 units and purchased 110 units.

So the creditor can sell more than it purchases but only by lending to the debtor. In the case of debt, only one agent, the creditor, produces the relative excess output. As such, combined output, over and above the non-debt scenario, increases at only half the previous joint rate vis-à-vis the production levels of any one agent.

4.3 The required rest

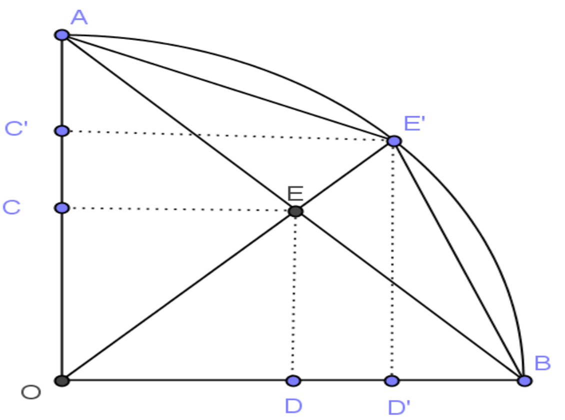
With debt, output can be higher than without it. This is achieved via the augmented rate of output. However, as the example has shown, the increase of the rate is limited to an extra half the initial rate of production. Essentially, the two sets of agents (debtors and creditors) can take turns to share this burden of producing extra output. This is achieved by an agent using produce to obtain money from the central bank, then lending this to the other representative agent to then purchase their own produce. Crucially, this means that, of the extra output achievable over and above the non-debt scenario, it is only the creditor that is doing the extra work. Hence, in the first period debtor rests whilst the creditor works. This is reversed in the second period when the debt is repaid.

Over the entire period, in terms of the extra debt induced output, both agents put forth a level of effort equivalent to half the extra output produced. This is because, whilst the creditor is 'taking responsibility' to autonomously provide the extra output to lend, the debtor agent is effectively resting. If debts are repaid, as assumed, this will be reversed.

Hence if, as it will be shown later, debt increases output per agent from 0.5 to 0.7 units, only 0.6 units of this new production level is realised as output; and only 0.6 units of effort are put forth by the society as a whole at any one point in time.

The Sabbath commandment to rest on the seventh day of a seven day week can be interpreted as a representative rest in lieu of this rest that the debtor must observe in relation to creditor

Figure 1. Geometric depiction



In Figure 1 above, the axes measure output and production. The debtor's output and production levels are measured along the x axis and the creditor's along the y axis. Points A and B represent non-debt production levels normalised to one unit

Money is endogenous to the model and is obtained by putting 'potential' output with a central bank. Hence, with total productive potential normalised to one unit, output is limited to 0.5, at point E.

Point E, which is a non-debt scenario, represents an initial equilibrium. The line AB represents a 'rate of sharing' output; the levels of output achievable if total output could be shared between the two agents.

However, in a debt scenario, the circular cord AB represents the augmented trade-off achievable[4]. This constraint is established by the fixed rate of production. The ability to lend and borrow increases the production space but this is not due to any effect on the inherent rate of production. Allowing debt to occur, moves the equilibrium from E to E': (the output levels are to be read off the axes), where the 45 degree line crosses the circular cord AB. Hence, this increases the level of average output from 0.5 to approximately 0.7: from points C to C' and D to D'.

This yields the following result; the Fundamental Equation:

Fundamental Equation: $1 = \int_0^2 y_C = \int_0^2 y_D = Y_\Omega = \frac{6\,Y_\pi}{7}$

Where, the non-debt total output achievable is normalised to one. y_C is the cumulative output level of the creditor i.e. those who were creditors in the first period. y_D is the cumulative output level of the debtor i.e. those who were debtors in the first period. Y_Ω is the cumulative output level of each output. Y_π is the cumulative output levels when debt is allowable.

The reason the debt space is expanded is because, the use of debt markets an increased rate of output: an augmentation from an initial rate represented by the 45 degree AB line, to a rate of 67.5 degrees. This expansion occurs because, as the example above has shown, debt allows output levels to increase by an extra half over and above a non-debt scenario. Lines AE' and BE' represent this increased rate.

This occurs because they can lend to purchase their own output. The normal rate is only a function of sharing output (responsibility) between the two agents over time. This is effectively equivalent to a barter economy where money is merely used as a means of exchange. However, debt allows an augmentation of this rate by, not only sharing responsibility, but also taking on an extra burden of purchasing one's own output for the other agent (via lending) over time rather than just allowing the other agent to purchase one's output.

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[4] See Appendix 1 for a proof of the mathematical representation of debt as a circular function.

The constraint changes from the linear combination having to add to a normalised value of one at all times, to the new constraint, under a debt scenario, that the Euclidean space must remain one unit, i.e. the agents can now jointly move away from the origin by one unit in any direction in Euclidean space rather than only a maximum of one unit being permitted when summing their orthogonal output levels

This result can be obtained geometrically by two alternative steps. Firstly, by tilting the 'exchange market' line - the line AB - to make it consistent with a debt scenario. This solution then occurs there the new tilted line(s) crosses the 45 degree line OE’ (as output levels must be equal by the end of the second period). Secondly, this can be achieved by shifting the exchange market line, AB, outward until the debt market curve (the circular cord AB) is consistent with the exchange market curve. This occurs at point E'. Hence, at point E' the exchange market and debt market are consistent.

5. Conclusion

It has been shown that society must rest for one seventh of this time dedicated to exchange market activity to facilitate debt. This is due to the mathematical ratio between debt market and exchange market activity being six-sevenths. This can be interpreted as a relationship between continuous economic activity (debt) and discrete economic activity (market exchange). As such, biblical commands can be interpreted as a method of facilitation a harmonious macroeconomic debt cycle and reflecting the required rest that society must take on average.

The novelty of the method of modelling relates to three key areas: the use of trigonometry [5]rather than using algebraic tools[6]; the modelling of the economic system rather than individual choice[7]; and the centrality of money as a 'real' rather than 'nominal' element of the model.

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[5] Here trigonometry is viewed as a branch of geometry

[6] See Appendix two

[7] Rather than taking the individual (rational maximizer) as the point of reference, this paper takes the viewpoint of a non-partisan observer. The goal can be still interpreted

Appendix 1: **Mathematical proofs of the model**

Proof One

Based on Figure 1, we let:

The y axis measure creditor's share of total cumulative output. The x axis measure debtors share of total cumulative output. In both cases the hypotenuse is measured by the radius of normalised value one, which is the non-debt output cumulative output over two periods.

Noting that:

y_C is the creditors' level of output.

y_D is the debtors' level of output.

y_Ω is the non-debt output, and is normalised to one. It is also the length of the radius of the circular cord AB.

α is the angle from point O to the point consistent with any given creditor and debtor relative output positions.

$$\frac{y_C}{y_\Omega} = \sin\alpha \quad (1)$$

And

$$\frac{y_D}{y_\Omega} = \cos\alpha \quad (2)$$

Over a debt cycle the relative debt positions of the debtors and creditors (from the first period) imply the following:

$$\partial\,\frac{y_C}{Y_\Omega} = -\,\partial\,\frac{y_D}{Y_\Omega} \quad (3)$$

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as maximizing output or utility, but that of the all individuals rather than a given individual maximizing on their own behalf.

Equation (3) says that the changes in the levels of output of each representative agent (when debt is permissible), in relation to the non-debt output, are equal and opposite throughout the period: debt equilibrates divergences of output of the two sets. The mean of share of the creditors output in each period, is an equal share of the total cumulative output over the two periods. Hence, they contribute an equal share to total cumulative output throughout the period of constant mean total production (joint production).

Hence, $\sin\alpha = -\cos\alpha$, then differentiate so $\cos\propto = \sin\propto$

A unit circle is defined mathematically as:

$$\sin^2\propto + \cos^2\alpha = 1 \quad (4)$$

Differentiating and simplifying:

$$\cos\propto = \sin\propto \quad (5)$$

Hence, comparing equations (3) and (5), we see that the macroeconomic debt process can be modelled as a circle. This proves that the circular cord represents a debt constraint on economic activity. Hence, the intersection of this and the forty-five degree line represents the maximum output achievable.

To find the optimal solution, we take a unit circle. Points A' and B', from Figure 1, are obtained as follows:

$$\max_{\propto} \cos^2\propto + \sin^2\propto = 1 \quad (6)$$

Differentiating

$$2\cos\propto - 2\sin\propto = 0 \quad (7)$$

Hence,

$$\propto = \frac{\pi}{4} \quad (8)$$

This implies that the orthogonal counterparts i.e. the x and y axis are $\sqrt{0.5} \cong 0.7$

Proof Two

An Exchange economy versus a Debt economy

Let C be the creditor's output and x be the debtor's output. Non-debt output is normalised to one. We have the following constraints:

For an Exchange Economy:

$$D = C \quad (9)$$

$$D + C = 1 \quad (10)$$

The solution is $D = C = 1$

For a Debt Economy

$$D_1 = C_2 \quad (11)$$

$$D_2 = C_1 \quad (12)$$

Where subscript i represents the time period. Conditions (11) and (12) say that over the two periods the outputs of the creditor and debtor must be equal.

Hence,

$$D_1 - D_2 = C_2 - C_1 \quad (13)$$

At the optimum: $D_1^2 + C_1^2 - 2D_1C_1 = 0 \quad (14)$

This is because, at the optimum:

$$D_1^2 + D_2^2 - 2D_1D_2 = Q \quad (15)$$

$$\max_{D_1} \ D_1^2 + D_2^2 - 2D_1D_2 = Q \qquad (16)$$

Hence,

$$2D_1 - 2D_2 = 0 \qquad (17)$$

Therefore, $D_1 = D_2$ (18)

And $C_1 = C_2$ (19)

Hence,

$$\frac{\sqrt{D^2+C^2}}{\sqrt{DC}} = \sqrt{2} \qquad (20)$$

This solution shows that debt can be modelled as a unit circles where $\frac{\sqrt{2}}{\sqrt{DC}} = 1$, for a unit circle.

This has a solution at 0.5 (the exchange economy solution) as well as at (approximately) 0.7. Hence, it is a necessary but not sufficient solution for modelling a debt economy which is consistent with an exchange economy.

Appendix 2: Geometry versus Algebra

Atiya (2002) describes geometry and algebra as the ‘two formal pillars of mathematics’. Mathematical economics as practiced conventionally derives from the algebraic branch of mathematics. One could trace a line from Leibniz, through to Hilbert and then the Bourbaki school, of whom Debreu and hence where the Arrow-Debreu General Equilibrium model sits. However, this paper derives from the geometric side of mathematics, in particular trigonometry. Newton was ‘fundamentally a geometer, Leibniz was fundamentally an algebraist..’ . ‘For Newton, geometry, or the calculus as he developed it, was the mathematical attempt to describe the laws of nature. He was concerned with physics in a broad sense, and physics took place in the world of geometry.’

This papers takes the geometric approach to mathematically explain the laws of God and as a by-product expose the laws of economics.

Atiya (2002) expresses that ‘Algebra is concerned with manipulation in time and geometry is concerned with space’. However, in this paper, the geometric picture explained describes the economic process in a unified time-space framework. In particular, it provides a foundation for the seven day weekly cycle in the context of macroeconomic debt management.

Economic activity space is portrayed as relative time units: how distinct economic activities (namely buying and selling versus lending and borrowing) occur naturally in time periods of a specified ratio. This ratio is described mathematically in Euclidean space but the interpretation is that of relative time periods.

Atiya (2002) says: ‘..spatial intuition or spatial perception is an enormously powerful tool, and that is why geometry is actually such a powerful part of mathematics – not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition. Our intuition is our most powerful tool…..’ Seeing is synonymous with understanding, and we use the word ‘perception’ to mean both things as well.

In economics is often separate mathematics from intuition which makes sense in the context of the points made by Atiya above. Most mathematics used in economics is algebraic in nature. However, geometry is a more useful branch of mathematics use as it is so closely aligned with intuition which is what economic theory is fundamentally about.

Atiya (2002) says:

‘One way to put the dichotomy in a more philosophical or literary framework is to say that algebra is to the geometer what you might call the ‘Faustian offer’. As you know, Faust in Goethe’s story was offered whatever he wanted (in his case the love of a beautiful woman), by the devil, in return for selling his soul. Algebra is the offer made by the devil to the mathematician. The devil says: ‘I will give you this powerful machine, it will answer any question you like. All you need to do is give me your soul: give up geometry and you will have this marvellous machine.’ (Nowadays you can think of it as a computer!) Of course we like to have things both ways; we would probably cheat on the devil, pretend we are selling our soul, and not give it away. Nevertheless, the danger to our soul is there, because when you pass over into algebraic calculation, essentially you stop thinking; you stop thinking geometrically, you stop thinking about the meaning.”

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