

# GCSE MATHEMATICS

## Revision Guide

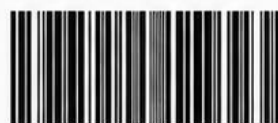
EXAM SERIES



**COMPLETE  
REVISION GUIDE  
YOU NEED  
TO KNOW**

**You must have:**

- Mathematical instruments
- A black ink pen

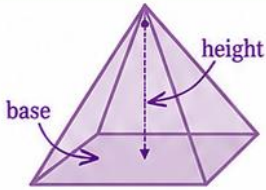


For exams from May/June

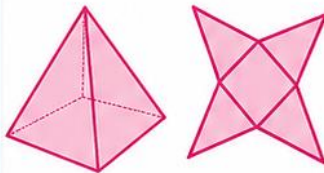
# EXAMINER REPORT

## WHERE MOST MARKS ARE LOST

### 1 USING THE WRONG FORMULA FOR AREA OR VOLUME



$$\text{Volume} = \frac{1}{3} \times \text{Area of base} \times \text{Height}$$



$$\text{Surface Area} = a^2 + 2a\sqrt{((a^2/4) + h^2)}$$

### 2 ROUNDING NUMBERS TOO SOON IN QUESTIONS

Calculate  $\frac{8.57 \times 3.5}{5.84}$

Incorrect:

$$\frac{8.57 \times 3.5}{5.84} = 8.6 \times 3.5$$

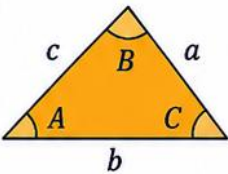
$$\frac{8.6 \times 3.5}{5.84} = 30.1$$

$$\frac{30.1}{5.84} = 5.15$$

### 3 CONFUSING COSINE & SINE RULES

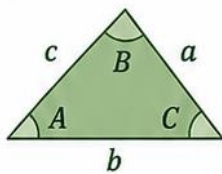
Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$



### 4 MISREADING MULTI-STEP QUESTIONS

Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as 10p coins
- £17.70 in total.

Work out total value of 2p coins : total value of 5p coins

Give your answer in its simplest form.

### 5 USING THE CORRECT METHOD BUT EITHER MAKING A MINOR ERROR OR NOT SHOWING ANY/ ENOUGH WORKINGS

The number of hedgehogs in England is expected to reduce by 4% each year.

Assume there are now 1000 000 hedgehogs in England.

Work out the expected number of hedgehogs in England after five years.

You must show your working.

[3 marks]

Answer

815373



# MATHS GCSE

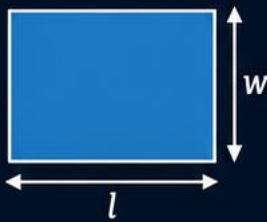
# KEY FORMULAS

EXAM SERIES



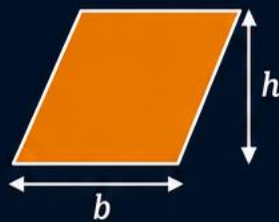
## AREA OF A RECTANGLE

$$A = l \times w$$



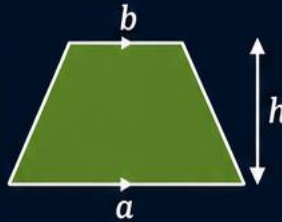
## AREA OF A PARALLELOGRAM

$$A = b \times h$$



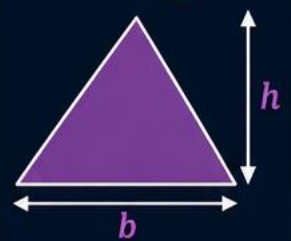
## AREA OF A TRAPEZIUM

$$A = \frac{1}{2}(a + b)h$$



## AREA OF A TRIANGLE

$$A = \frac{bh}{2}$$



## CIRCUMFERENCE OF A CIRCLE

$$C = \pi d$$



## AREA OF A CIRCLE

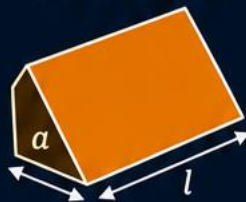
$$A = \pi r^2$$



## VOLUME OF A PRISM

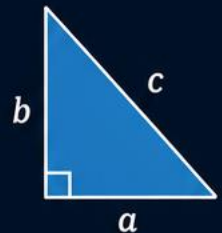
$$V = a \times l$$

a is the area of the cross-section



## PYTHAGORAS' THEOREM

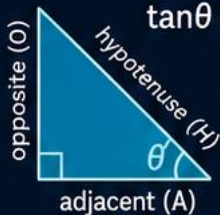
$$a^2 + b^2 = c^2$$



## TRIGONOMETRY FORMULAS

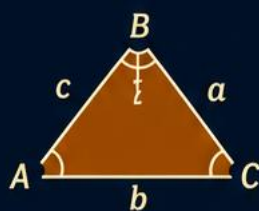
$$\sin \theta = \frac{O}{H} \quad \cos \theta = \frac{A}{H}$$

$$\tan \theta = \frac{O}{A}$$



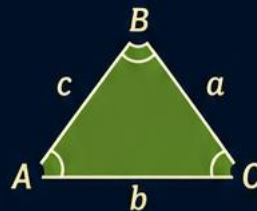
## SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



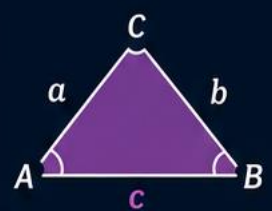
## COSINE RULE

$$a^2 = b^2 + c^2 - 2bccosA$$



## SINE AREA RULE

$$\text{Area} = \frac{1}{2} absinC$$



## COMPOUND INTEREST FORMULA

$$\text{Total} = P \times \left(1 + \frac{r}{100}\right)^n$$

P is the amount invested  
r is the interest rate  
n is the number of times interest is compounded



## QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It solves the quadratic equation:

$$ax^2 + bx + c = 0$$

## SUM OF INTERIOR ANGLES

$$(n - 2) \times 180$$

n is the number of sides the shape has



# INDEX LAWS

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MULTIPLYING

$$a^m \times a^n = a^{m+n}$$



DIVIDING

$$\frac{a^m}{a^n} = a^{m-n}$$



RAISING TO  
ANOTHER POWER

$$(a^m)^n = a^{mn}$$



ANYTHING TO  
THE POWER  
OF ZERO

$$n^0 = 1$$



NEGATIVE  
INDEX

$$a^{-m} = \frac{1}{a^m}$$



UNIT  
FRACTIONAL  
INDEX

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$



ANY  
FRACTIONAL  
INDEX

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

# SURDS

EXAM SERIES



**YOU NEED THESE SURD RULES!**



**PRODUCT  
RULE**

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$



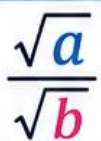
**SQUARE  
ROOT RULE**

$$\sqrt{a} \times \sqrt{a} = a$$



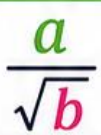
**QUOTIENT  
RULE**

$$\sqrt{a} \div \sqrt{b} = \sqrt{a} \div \sqrt{b}$$



**FRACTION  
RULE**

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$$



**RATIONALISE  
DENOMINATOR**

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$a\sqrt{b} \times c\sqrt{d}$$

**MULTIPLY  
SURDS**

$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$

$$a\sqrt{b} \div c\sqrt{d}$$

**DIVIDE  
SURDS**

$$a\sqrt{b} \div c\sqrt{d} = \frac{a\sqrt{b}}{c\sqrt{d}}$$

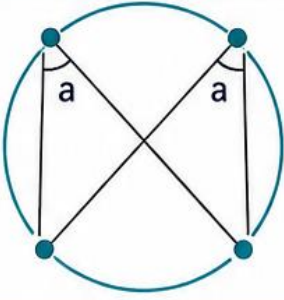


# CIRCLE THEOREMS

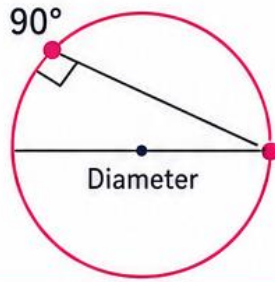
EXAM SERIES



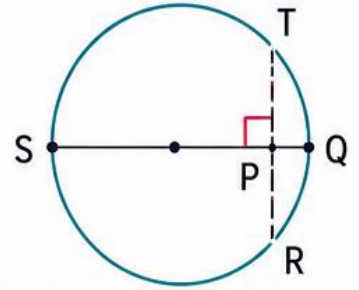
ANGLES IN THE SAME SEGMENT ARE EQUAL



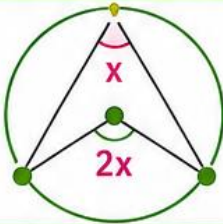
THE ANGLE FORMED IN A SEMI-CIRCLE IS A RIGHT ANGLE



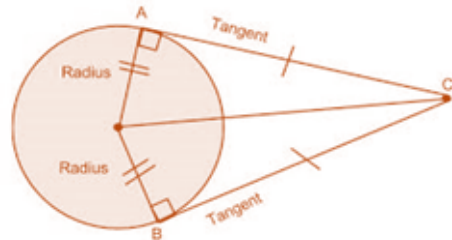
A DIAMETER BISECTS A CHORD AT A RIGHT ANGLE



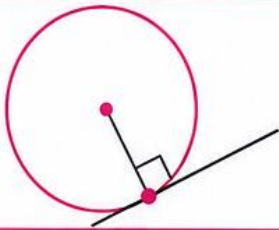
THE ANGLE FORMED AT THE CENTRE IS DOUBLE THE ANGLE FORMED AT THE CIRCUMFERENCE



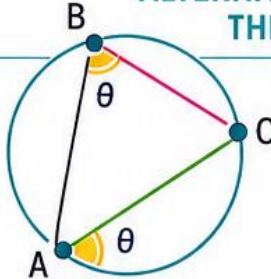
TWO TANGENTS THAT MEET AT THE SAME POINT HAVE EQUAL LENGTH



A TANGENT MEETS A RADIUS AT A RIGHT ANGLE

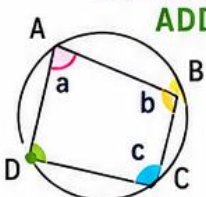


ALTERNATE SEGMENT THEOREM

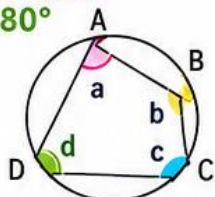


The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL ADD UP TO 180°



$$a + c = 180^\circ$$



$$b + d = 180^\circ$$



KEY IDEA



THESE THEOREMS COME UP ALL THE TIME IN EXAMS!

LEARN THEM. USE THEM. ACE IT!

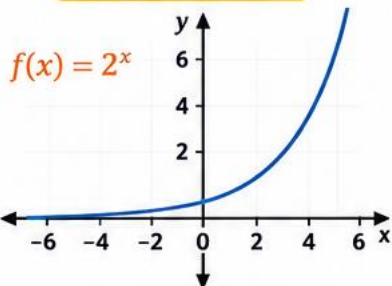
# MATHS

# GRAPHS & STATS



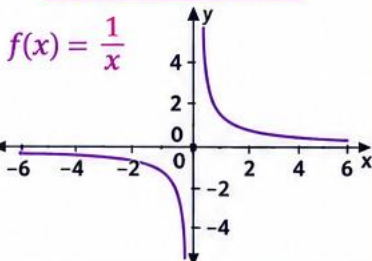
THE 12 MAIN GRAPHS YOU NEED TO KNOW!

## EXPONENTIAL



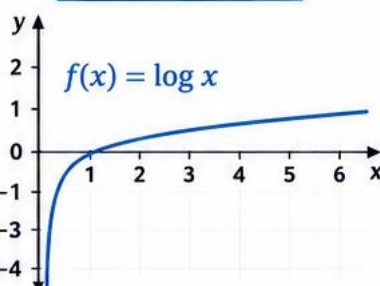
Rapid growth or decay.

## RATIONAL RECIPROCAL



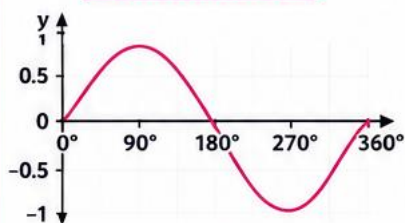
Two branches, never touch the axes.

## LOGARITHMIC



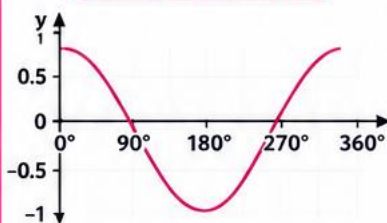
Slow growth, inverse of exponential.

## SINE GRAPH



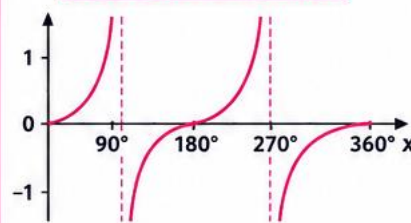
$y = \sin x$

## COSINE GRAPH



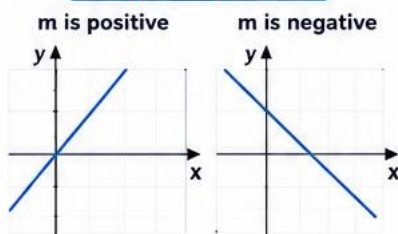
$y = \cos x$

## TANGENT GRAPH



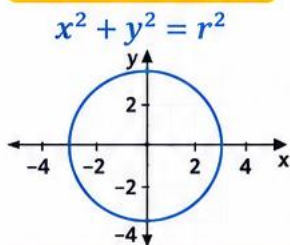
$y = \tan x$

## STRAIGHT-LINE



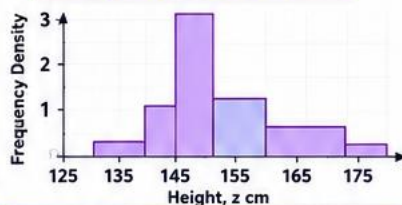
$y = mx + c$

## CIRCLE GRAPHS



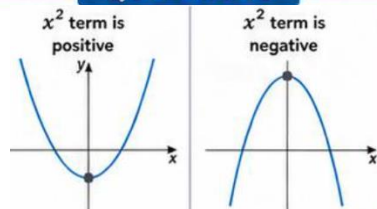
Centre (0, 0), radius r

## FREQUENCY DENSITY (HISTOGRAM)



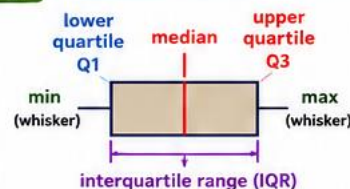
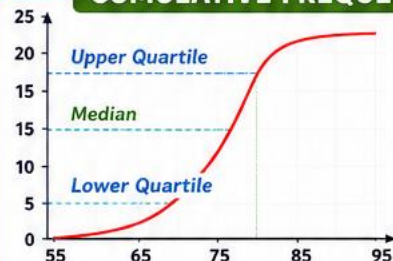
Bar width shows class width, height shows frequency density.

## QUADRATIC



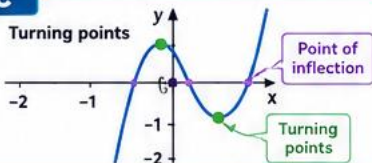
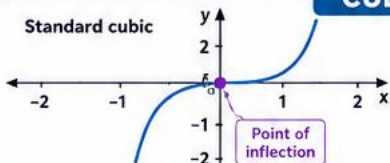
Parabola shape.

## CUMULATIVE FREQUENCY



Shows how data accumulates. Useful for medians & quartiles.

## CUBIC



## TOP TIP!

Learn the shape, key features & what each graph is used for!

# MATHS SEQUENCES



## THE 7 MAIN TYPES YOU NEED TO KNOW!

### 1 LINEAR SEQUENCES

Add or subtract by the same amount.



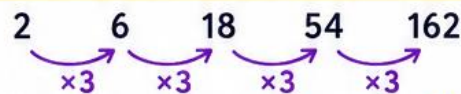
RULE: +1



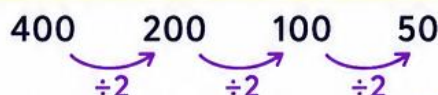
RULE: -3

### 2 GEOMETRIC SEQUENCES

Each term is obtained by multiplying or dividing the previous term by a constant,  $r$ , called the **common ratio**.



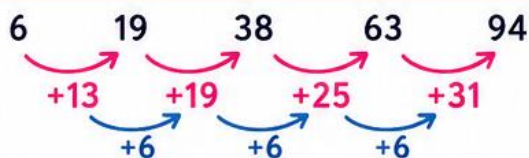
The next term in the sequence is **486**



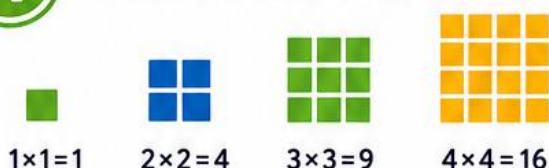
The next term in the sequence is **25**

### 3 QUADRATIC SEQUENCES

The difference between the terms changes. However, the difference between the differences is always the same.



### 4 SQUARE NUMBERS



The difference between each number in the sequence is an **odd number** that **increases by 2**.

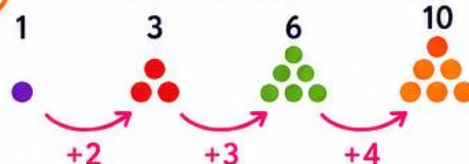
### 5 FIBONACCI SEQUENCE

1, 1, 2, 3, 5, 8, 13, 21, 34, 55...

Add together to get the next number.

Every term is the **sum** of the last two preceding terms.

### 6 TRIANGLE NUMBERS

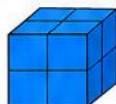


The difference between each number in the sequence **increases by 1**.

### 7 CUBE NUMBERS



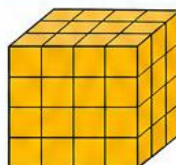
$1^3 = 1$



$2^3 = 8$



$3^3 = 27$



$4^3 = 64$

CUBING CONSECUTIVE INTEGERS

# ANGLE RULES

EXAM SERIES

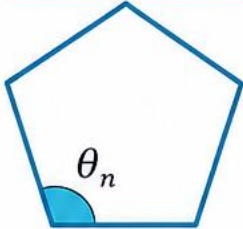
Graham Waller  
TUITION



EVERYTHING YOU NEED TO KNOW!

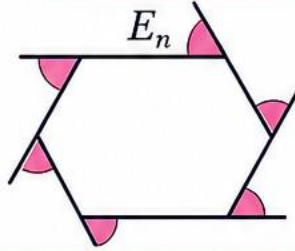
## 1 SUM OF INTERIOR ANGLES

$$(n-2) \times 180^\circ$$



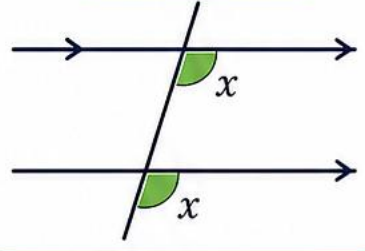
## 2 SUM OF EXTERIOR ANGLES

$$360^\circ$$



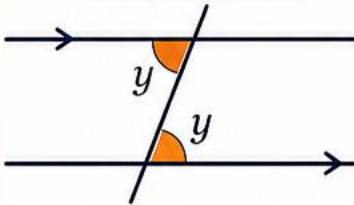
## 3 CORRESPONDING ANGLES

$$x = x$$



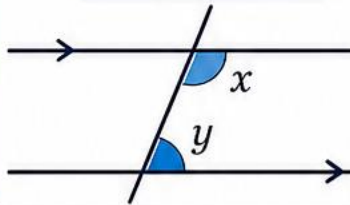
## 4 ALTERNATE ANGLES

$$y = y$$



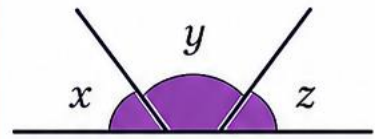
## 5 SUM OF CO-INTERIOR ANGLES

$$x + y = 180^\circ$$



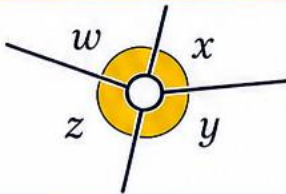
## 6 ANGLES ON A STRAIGHT LINE

$$x + y + z = 180^\circ$$



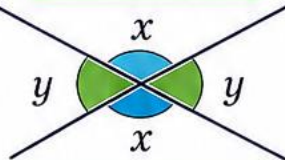
## 7 ANGLES AT A POINT

$$w + x + y + z = 360^\circ$$



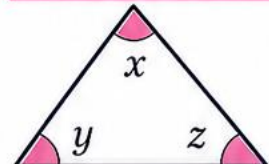
## 8 VERTICALLY OPPOSITE ANGLES

$$x = x, y = y$$



## 9 ANGLES IN A TRIANGLE

$$x + y + z = 180^\circ$$



## 10 COMPLIMENTARY ANGLES

Total  $90^\circ$

These angles do not need to be together and form a right angle



## 11 SUPPLEMENTARY ANGLES

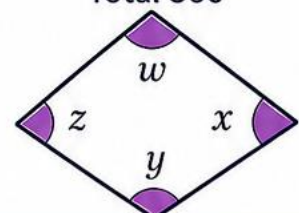
Total  $180^\circ$

These angles do not need to be together on a straight-line



## 12 ANGLES IN A QUADRILATERAL

Total  $360^\circ$



# TRIGONOMETRY



**DOES THE TRIANGLE HAVE A RIGHT ANGLE?**

**YES**

**NO**

**DOES THE QUESTION INVOLVE ANY OTHER ANGLES?**

**DO YOU KNOW A SIDE AND ITS OPPOSITE ANGLE?**

**YES**

**NO**

**YES**

**NO**

**SOH CAH TOA**

**PYTHAGORAS' THEOREM**

**SINE RULE**

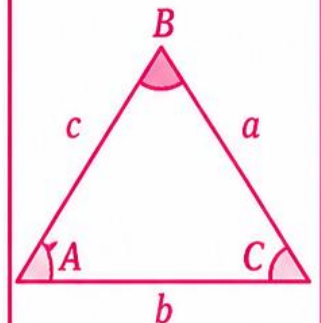
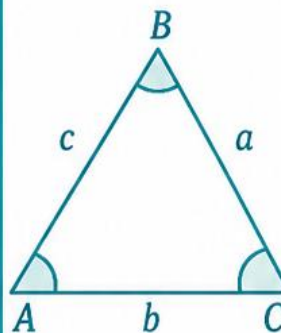
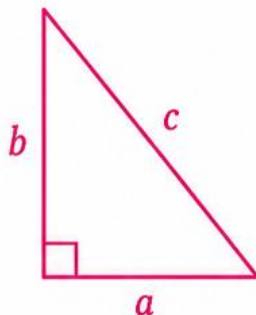
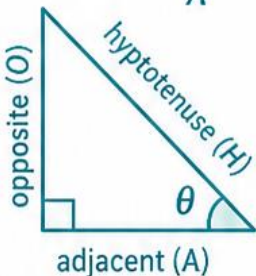
**COSINE RULE**

$$\sin \theta = \frac{O}{H}$$
$$\cos \theta = \frac{A}{H}$$
$$\tan \theta = \frac{O}{A}$$

$$a^2 + b^2 = c^2$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

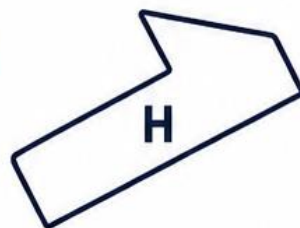
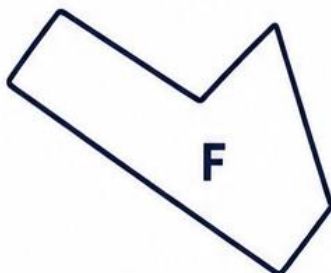
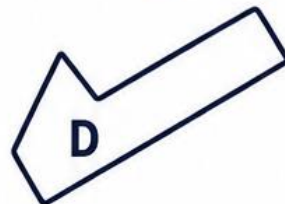
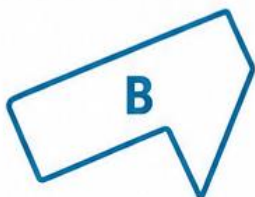


# CONGRUENT SHAPES



ARE EXACTLY THE SAME SHAPE  
AND THE SAME SIZE.

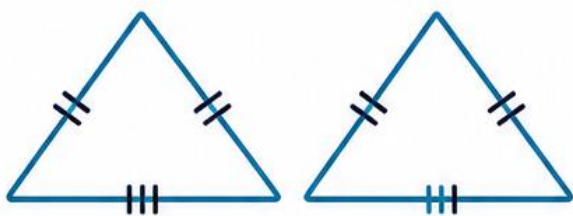
Two shapes that are the same size and the same shape are **congruent**.



Shapes A, B, E and G are congruent. They are identical in size and shape.

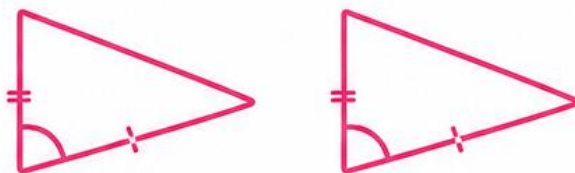
## SSS

(side-side-side)



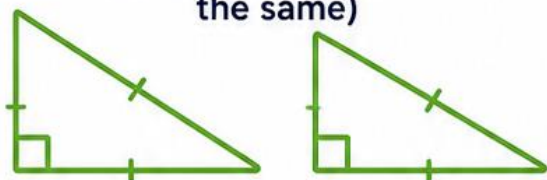
## SAS

(side-angle-side,  
two sides and the included angle)



## RHS

(right-angled triangle,  
hypotenuse and a side  
the same)



## ASA

(angle-side-angle or  
angle-angle-side)

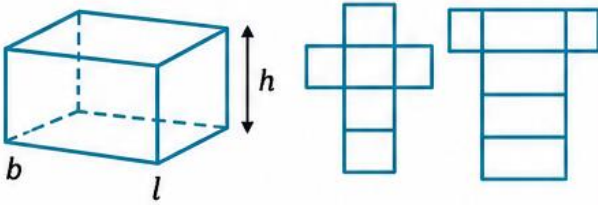


# 3D SHAPES

EXAM SERIES

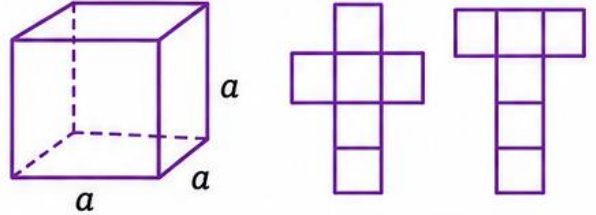


## RECTANGULAR PRISM



$$\text{SURFACE AREA} = 2(l \times b + b \times h + l \times h)$$

## CUBE



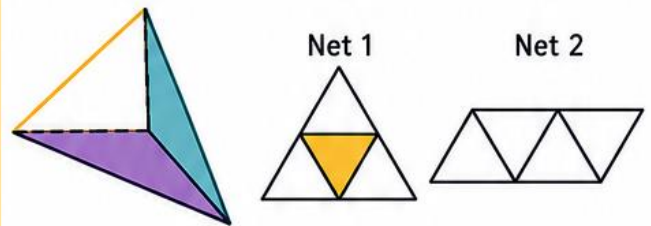
$$\text{SURFACE AREA} = 6a^2$$

## CONE



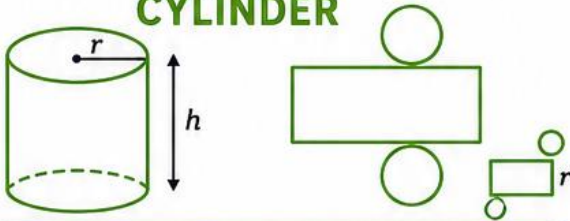
$$\begin{aligned} \text{CURVED SURFACE AREA} &= \pi r l \\ \text{TOTAL SURFACE AREA} &= \pi r l + \pi r^2 \\ &= \pi r(l + r) \\ (l &= \sqrt{h^2 + r^2}) \end{aligned}$$

## REGULAR TETRAHEDRON



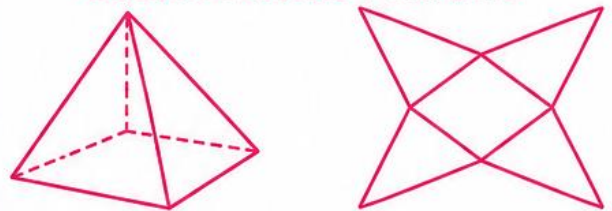
$$\text{TOTAL SURFACE AREA} = \sqrt{3} a^2$$

## CYLINDER



$$\begin{aligned} \text{CURVED SURFACE AREA} &= 2\pi r h \\ \text{TOTAL SURFACE AREA} &= 2\pi r h + 2\pi r^2 \\ &= 2\pi r(h + r) \end{aligned}$$

## SQUARE BASED PYRAMID



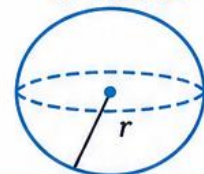
$$\text{SURFACE AREA} = a^2 + 2a\sqrt{(a^2/4) + h^2}$$

## TRIANGULAR PRISM



$$\begin{aligned} \text{TOTAL SURFACE AREA} &= (\text{PERIMETER} \times \text{LENGTH}) + (2 \times \text{BASE AREA}) \\ &= (s_1 + s_2 + s_3)l + bh \end{aligned}$$

## SPHERE



$$\text{CURVED SURFACE AREA} = 4\pi r^2$$

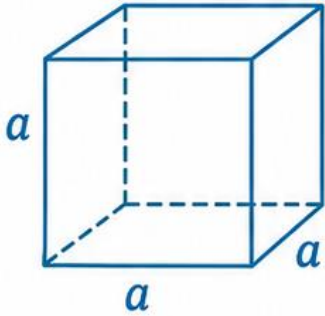
# 3D SHAPE

## VOLUME FORMULAS

EXAM SERIES

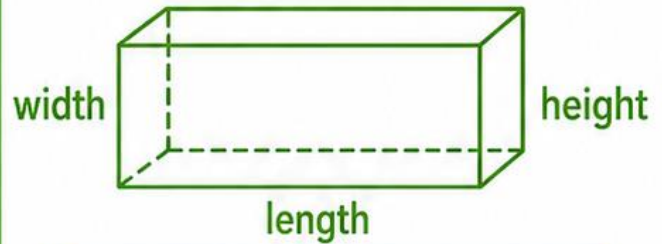


### CUBE



$$\text{VOLUME} = a^3$$

### CUBOID



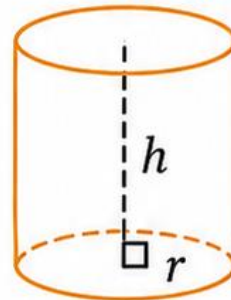
$$\text{VOLUME} = l \times w \times h$$

### PRISM



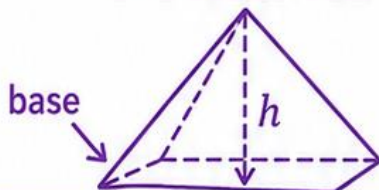
$$\text{VOLUME} = \text{Area of cross section} \times \text{Length}$$

### CYLINDER



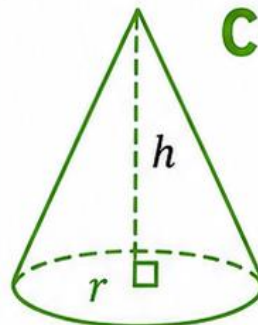
$$\text{VOLUME} = \pi r^2 h$$

### PYRAMID



$$\text{VOLUME} = \frac{1}{3} \times \text{Area of base} \times \text{Height}$$

### CONE



$$\text{VOLUME} = \frac{1}{3} \pi r^2 h$$

### SPHERE



$$\text{VOLUME} = \frac{4}{3} \pi r^3$$

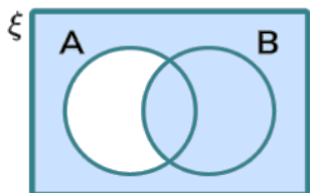
# SET THEORY

EXAM SERIES

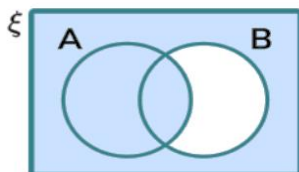


## VENN DIAGRAMS & NOTATION

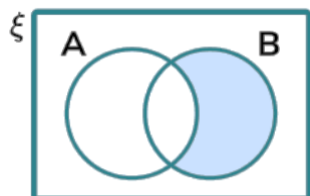
$A' \cup B$



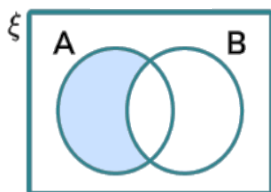
$A \cup B'$



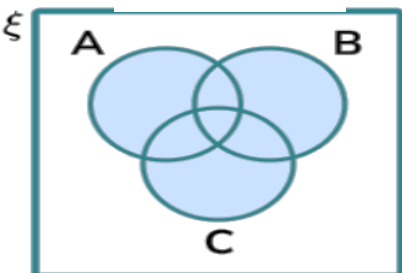
$B \cap A'$



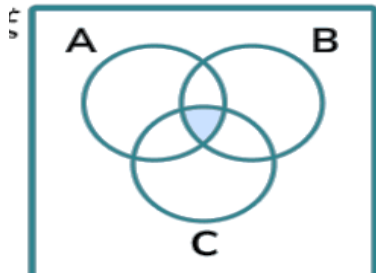
$A \cap B'$



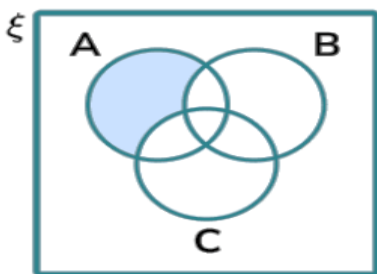
$A \cup B \cup C$



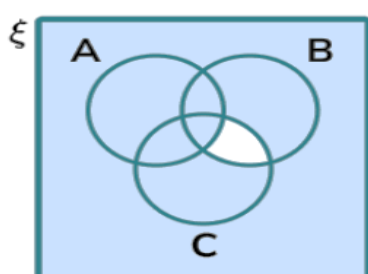
$A \cap B \cap C$



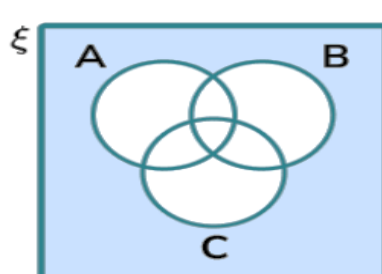
$A \cap (B \cup C)'$



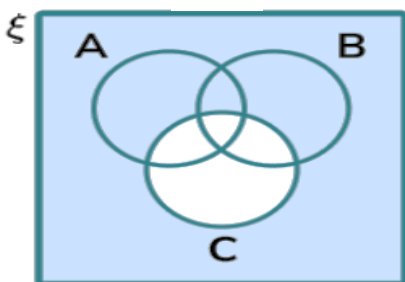
$(A \cup (B \cap C))'$



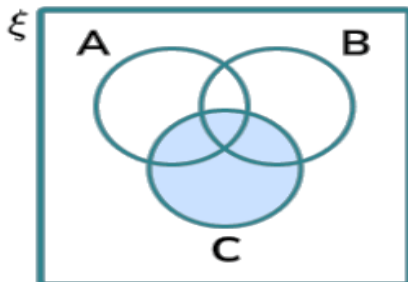
$(A \cup B \cup C)'$



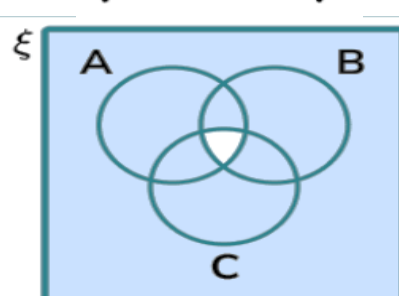
$C'$



$C$



$(A \cap B \cap C)'$

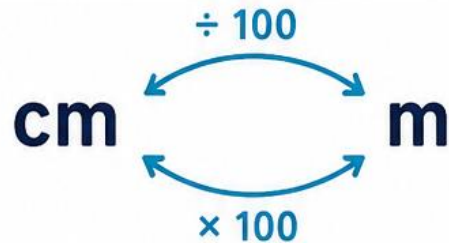


| Set Notation  | Description                                         |
|---------------|-----------------------------------------------------|
| $\xi$         | Universal set                                       |
| $A$           | Set A                                               |
| $A'$          | Not Set A<br>(the complement of Set A)              |
| $B$           | Set B                                               |
| $B'$          | Not Set B<br>(the complement of Set B)              |
| $A \cap B$    | A and B<br>(A intersection B)                       |
| $(A \cap B)'$ | Not A and B<br>(the complement of A intersection B) |
| $A \cup B$    | A or B<br>(A union B)                               |
| $(A \cup B)'$ | Not A or B<br>(the complement of A union B)         |

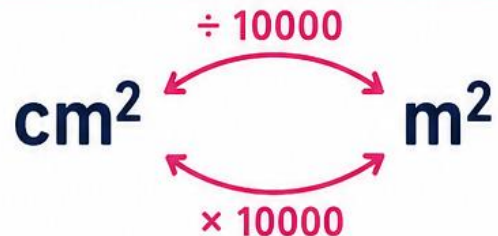
# COMPOUND MEASURES & UNIT CONVERSION



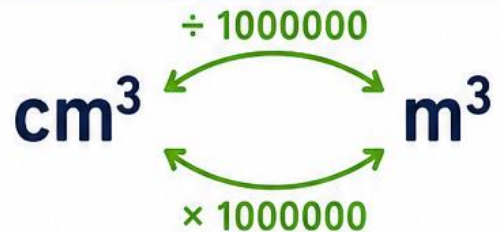
CONVERTING  
BETWEEN  
LENGTHS:  
cm AND m



CONVERTING  
BETWEEN  
AREAS:  
cm<sup>2</sup> AND m<sup>2</sup>



CONVERTING  
BETWEEN  
VOLUMES:  
cm<sup>3</sup> AND m<sup>3</sup>



## SPEED

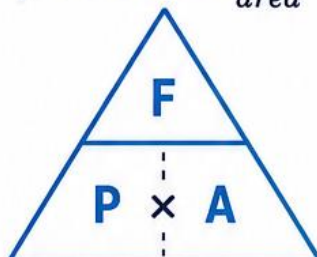
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$



$$D = S \times T$$

## PRESSURE

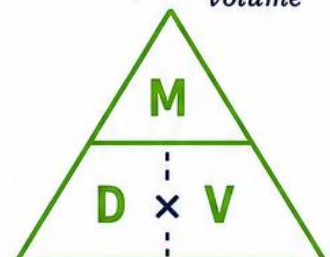
$$\text{pressure} = \frac{\text{force}}{\text{area}}$$



$$F = P \times A$$

## DENSITY

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$



$$M = D \times V$$

# VECTORS



### DIRECTION & SIGNS

### VECTOR NOTATION

$$\vec{AB} = a = a$$

$$\vec{BA} = -a = -a$$

$$-a = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

### VECTOR MAGNITUDE (ARROW LENGTH)

$$|a| = \sqrt{x^2 + y^2}$$

$$|a| = \sqrt{x^2 + y^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$$

### VECTOR ADDITION

$$\vec{XY} + \vec{YZ} = \vec{XZ}$$

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 4+1 \\ 2+(-4) \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

### VECTOR MULTIPLICATION

$$a = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$2a = 2 \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \times 5 \\ 2 \times 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

### VECTOR SUBTRACTION

$$\vec{YX} = -\vec{XY} \quad -\vec{XY} + \vec{XZ} = \vec{YZ}$$

$$-\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} -4+5 \\ -2+(-2) \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$