

# **FINITE ELEMENT ANALYSIS OF ALVEOLAR LUNG TISSUE MICROSTRUCTURE:**

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## Abstract

The alveolar region of the lung is a thin-walled, highly porous cellular solid whose septal walls of elastin and collagen enclose the terminal air spaces where gas exchange occurs. Because the mechanical behavior of this microstructure governs how stress and strain are transmitted through the parenchyma during breathing, and because it underlies the constitutive laws used in continuum and whole-organ lung models, an accurate micromechanical description of the alveolar wall network is of direct interest to respiratory biomechanics, injury biomechanics, and the design of mechanically ventilated and pathological lung models. This thesis develops and compares two idealized finite element (FE) micro-models of a 1 mm x 1mm x 1mm representative volume element (RVE) of alveolar tissue: a regular, space-filling Kelvin-cell (truncated octahedron) tessellation, and an irregular tessellation generated from a three-dimensional Voronoi construction seeded with randomly distributed points. Both microstructures were built as thin-shell wall networks, meshed in Altair HyperMesh, and solved in Abaqus/Standard under quasi-static uniaxial compression to 5% nominal strain, representing a stretch ratio of  $\lambda = 0.95$  applied to the top face of the RVE with the base fully encastred.

The regular Kelvin-cell model was specified as a linear elastic wall material ( $E = 5$  kPa,  $\nu = 0.45$ ) taking estimated elasticity moduli of average alveolar wall from Bou Jawde et al. For the irregular Voronoi model that is built by 212,607 nodes and 428,810 shell element it was specified a stiffer linear elastic wall material ( $E = 12$  kPa,  $\nu=0.45$ ,  $\rho=1060$  kg/m<sup>3</sup>) derived from the same source. Under the identical compression range (5%), the regular Kelvin-cell model output a near uniform stress distribution field of von Mises stress, peaking at 1.55 kPa on the walls - with the stress focusing at the single pattern (dihedral with truncated octahedron). In contrast to this the irregular Voronoi model output stressed walls with a peak stress at 12.9 kPa - some eight times greater than the previous in overall value (or simply  $\sim 3.5$  times greater once the  $2.4 \times$  (disparity of wall modulus from two models) has been factored, at intersections between sections of the junctions on the walls that it was compressed. Compared the findings with published work on a cluster of tetrahedron alveoli, a calibration of those found curves to stretching was created to establish maximum stress boundaries from curves digitised from Fig (Singh, Slutsky & Cronin, 2026). 9(b) via WebPlotDigitizer to a single reference (for the stress-stretch relationship on the cellular scale).

Compared to the response of an ideally geometric repeating unit cell, an irregular Voronoi-based geometry more faithfully reproduces the local stress concentration behaviour of real alveolar septa; a dry-wall non-pre-strained micro-model without surface tension membrane is bound to under-predict the macroscopic stiffness of in vivo parenchyma by a factor of the order of two to three. This is consistent with surface-tension and pre-strain stiffening contributions reported in the literature. Drawing analogy from similar Voronoi-tessellation-based structural modelling studies involving porous materials from trabecular bone to additive-manufactured porous scaffolds to granular and polycrystalline rock mechanics to porous-media transport, this thesis attempts to place the two models presented in the lung microstructure in the bigger context of Voronoi as a geometrically general and increasingly validated tool for modelling irregular nature cellular solids; the alveolar wall network is just one of them. The thesis closes with a discussion on the limitations of the model - use of data from rodent not human; the lack of surface tension; the limitation in the number of nodes of our Abaqus/Standard license; and the range over which strain is applied corresponding to only the initial near-linear phase of the stress-stretch response curve of tissue. A list of recommendations that represent specific next step for the model to take are then put forward; these include modelling the surface tension explicitly with membrane elements, the pre-strain with initialisation to the state at functional residual capacity and the derivation of the reaction force history which is needed for producing a macroscopic stress-stretch curve for comparison with results previously published.

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# Chapter 1 Introduction

## 1.1 Motivation

The human lung is, mechanically speaking, an open-cell cellular solid: some 480 million alveoli, each on the order of 200; 300  $\mu$ m across, are packed into the parenchyma and separated by thin septal walls comprising largely of elastin and collagen septal wall fibres. Those walls are the skeletal support, it is what stretches and recoils as the diaphragm and intercostal muscles vary volume of the thorax and it is their statistically-averaged stiffness that macro-scale models report as the 'elastic modulus of lung tissue'. Yet nearly every practical model of the lung - applied to injury, mechanical breath and fibrotic or emphysematous disease - treats the parenchyma as a continuum and prescribes empirically-fit constitutive laws; it bypasses the question of how the continuum's properties actually emerge from the geometry and material properties of the septal wall network.

It is not just that from a modelling perspective, no one has really asked it, that the question of how septal geometry and wall mechanics contribute to lung parenchymal mechanics has not attracted the attention it deserves. Why should we care how it does it so long as it works? It appears, however, that this not also constitutes a practical oversight at a much less trivial mechanistic level. All the continuum mechanical models mentioned operate on the question of what rather than the level of how. That is to say, they do not and cannot, on the basis they are founded, explain why disease processes involving changes in alveolar geometry to the degree observed (the destruction and coalescence of alveolar walls in emphysema; the stiffening and thickening of septa in fibrosis) should change the macroscopic stiffness of the lung by the amount they do. A continuum law cannot be extrapolated with confidence outside the strain and loading mode in which it was fitted. A micro-mechanical model on the other hand, starting from the septal wall geometries and wall mechanics could, provided certain assumptions about geometrical changes to walls were made, predict the bulk response as an output and so be much more powerful for addressing the all-important 'what if questions' related to wall properties and configuration of parenchyma.

The notion of micro-mechanical finite element (FE) modeling of alveolar tissue is not new; since at least the 1990's micro-mechanical work of Stamenović of using idealized honeycomb and

truncated-octahedron (Kelvin-cell) representations of the alveolar sac in an attempt to relate septal wall properties to bulk lung elasticity [1], however, the great majority of the published micro-models (including among them, now widely accepted, work of Concha et al. and recent work of Singh Slutsky and Cronin [2,3]) used a repeating unit cell. A repeating unit cell is attractive since it is easy to model, to mesh and to interpret but it is, by design, not representative of the lung: no two alveoli are the same size or shape and walls do not meet at a single repeating dihedral angle. Whether this idealisation is of significance - would it alter stress distributions in a way that is relevant to injury or diseases (if any), which for this class of model - is a largely uncharted territory.

## **1.2 Problem Statement**

The present masters-thesis sets out to answer this question. Two Finite Element micro-models of the same 1 mm ×1 mm ×1 mm representative volume of alveolar parenchyma are built: the first with a regular space-filling geometry (Kelvin cell), the second having an irregular geometry generated by a three-dimensional Voronoi tessellation and are simultaneously subjected to an identical uniaxial compression load case. Each model is held as similar to the other as possible in every respect except the chosen geometry: representative-volume size, use of thin-shell-wall-idealization and boundary conditions - which are consistent between two case - with material properties derived from the same source of experiment. The FE-calculated stress distributions are considered to quantify the effect of the geometric irregularity in terms of the wall stress field, while the case of the irregular geometry is also tested qualitatively by comparing the prediction from an independent and verified alveolar cluster model with the one calculated hereof.

The project, in an earlier format, framed the loading condition of interest as an impact-related or a crash-related event. This came out of its eventual place within a human body model for injury biomechanics. This framing was, however, discarded on the advice of the thesis advisor: there was no impact being run, nor any car-crash being simulated, and referring hyperbolically to the 5% compression as car-crash problem statement served poorly to model the actual work. The modified objective below correctly describes the scope of work.

### 1.3 Objectives

- Construct a regular, space-filling Kelvin-cell (truncated octahedron) representation of the alveolar septal wall network within a  $1\text{mm}^3$  representative volume element.
- From the same representative volume create an irregular model based on a 3D Voronoi tessellation between randomly distributed points to ensure neighboring cells vary in size, shape and number of faces to be similar to living parenchyma.
- Meshing both wall networks as thin shells in Altair HyperMesh, giving each and every wall the same linear elastic properties based on the published experimental estimations for the stiffness of the alveolar wall and solving two of them in Abaqus in the same uniaxial compression under uniaxial condition.
- Make such comparisons between regular and irregular models in terms of von Mises stress field and try to characterise what effects the geometric irregularities have on the distribution and extent of stress in the septal wall.
- Digitise the compression branch of a similar, previously proven alveolar-cluster FE model from the literature and use this as a qualitative 'benchmark' to compare to the predicted response of the irregular model.
- Discuss the modeling assumptions such as material data source, absence of surface tension, absence of pre-strain, strain range and mesh size that restrict the conclusions that can be drawn from the analysis and identify the nature of extensions required to take the model from here to an in vivo lung.

### 1.4 Scope and Limitations of the Study

The scope of this thesis is deliberately narrow and stated here so that the results in later chapters are not over-read. Only uniaxial compression to 5% nominal strain is run; the tension and shear load cases given in the literature benchmark are not run. Both models use a linear elastic wall material than the nearly incompressible hyperelastic law which is more appropriate at larger strains; this can be justified in the small-strain near-linear 'toe' region of the pulmonary tissue response but would not be appropriate above roughly 10% strain. Surface tension at the air-medium interface, a dominant component of lung stiffness in vivo, is not presented here; both models have dry walls. Neither model is pretrained to FRC. Material properties are from (murine) rodent tissue instead of

human tissue (which is not available for experiment); the resulting numbers should be read as informative of trend than of exact human values. Finally, the Abaqus/Standard educational licence has a limit of 250,000 nodes placed on the model, setting an upper bound on mesh density and preventing the performance of a breathing cycle (cyclic loading) simulation prior at an earlier stage in the project. Each of these limitations is expanded upon in Chapter 7, with suggestions about the specific extensions to the models indicated below listed explicitly.

## **1.5 Significance and Downstream Applications**

Aside from the purposes of the specific comparison made in this thesis, an accurate micromechanical account of the alveolar wall network has several applications that justify studying the problem. In injury biomechanics, whole-body human body models used to predict blunt thoracic trauma (e.g. in automotive crash or a blast-overpressure scenario, for which context was originally chosen for the project framing before it was corrected to a quasi-static scope as given in Section 1.2) need to be given a lung constitutive law at organ scale and this law is only as well-grounded in physics as the micromechanical law from which it is derived; applying a continuum law which fits only macroscopic average behaviour without referring to the peaked local stress might systematically miss the loading condition when local alveolar wall rupture occurs. In researching about mechanical ventilation techniques, lung mechanics models are used by clinicians and biomedical engineers to understand ventilator induced lung injury (VILI), because mechanical ventilation using positive pressure to over-distend alveoli is believed to be the reason for tissue damages; because VILI is mainly a local than averaged phenomenon; the injury begins at the maximally strained alveoli than those with averaged strain of the entire lung, a micro-model of the kind used in this thesis which could resolve local stress concentration of an irregular geometry would be more relevant to the question than a regular unit-cell model alone. In mechanical ventilation research, genetic changes to the alveolar wall network like with emphysema (septae destruction and coalescence) or pulmonary fibrosis (septae thickening and stiffening) are best represented (with the method developed in this report) as irregular Voronoi-type geometry than a change from their regular Kevin-cell type since the disease processes may not uniformly and cyclically modify every alveolus.

The comparison in this thesis thus has some wider applicability beyond its data-points: by its being the first controlled exploration into whether it is worth it for such applications to model the alveolus using an irregular Voronoi geometry (Section 6.5) than the standard Kelvin Cell on account of some appreciable difference in the two cases. The result in chapter 5, around 3.5 times difference in peak local wall stress on geometry alone, appears large enough compared to typical insult or damage boundaries discussed in literature of tissue mechanics, to suggest that it may be, as local than averaged wall tissue response, may be concerned. The result in chapter 5, around 3.5 times difference in peak local wall stress on account of geometry alone, appears large enough in comparison to the insult or damage boundary generally discussed in literature to suggest its feasibility when considering in terms of local. Rather than finding the added cost worthless, it does appear large enough in comparison to boundaries in literature to appear worth it. And so in considering local stresses and than ones averaged over the alveolus. To have a meaning when used to determine whether the cells in some instances where response local, than on average are interested into, worth it, in an effort towards having more reliable prediction into effects of disease progressions.

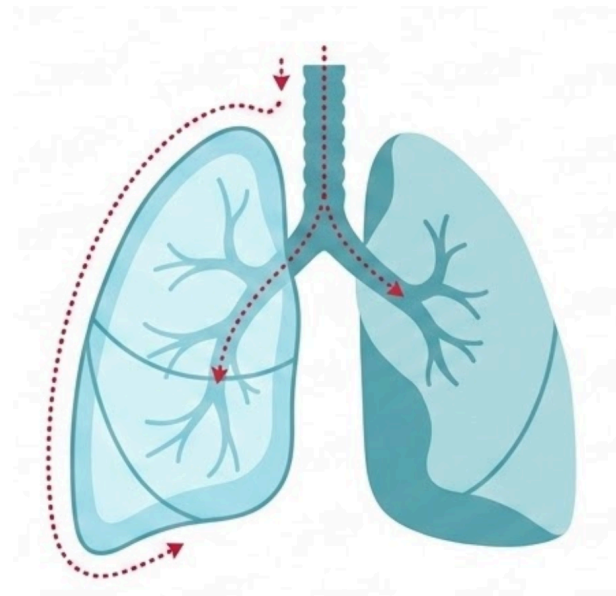
## **1.6 Thesis Organization**

In Chapter 2 I recap alveolar physiology and mechanics and summarise the basics of FEM analysis covered in this work. In Chapter 3 I place the models in relation to previous literature: first, a survey of micromechanical models of the alveolus; second, the analytical and experimental background of the material properties described here; third - as the irregular model here is based on Voronoi tessellation - previous micro-mechanical models that use Voronoi tessellation: in trabecular bone, AM- scaffolds; granular matter, polycrystals and porous-media transport etc; cases where the lung is NOT modelled at all but the suitability of using Voronoi models can be debated. In Chapter 4 I outline the method of geometry generation, meshing, material property assignment, boundary conditions and solver configuration of the models. In Chapter 5 the results are presented and in Chapter 6 they are put in the light of the benchmark literature presented above. Limitations of the project are the subject of Chapter 7 along with future work stemming from that. Chapter 8 serves to draw the conclusions.

## **Chapter 2 Background**

## 2.1 The Alveolar Microstructure

The lung parenchyma is the gas exchanging tissue of the lung: the alveolar sacs (the sites of gas exchange), the alveolar ducts leading to them, as well as the fine network of capillaries and connective tissue binding it all together. From the point of view of mechanical behaviour, pulmonary parenchyma can be thought of as an open cell polymer foam with the structural skeleton being a thin, continuous sheet of a septal wall a few micrometers thick - consisting of a network of elastin and collagen fibres in a ground substance covered by epithelial cells. Adjacent alveoli (sacs) share septal walls, hence in a topological sense, parenchyma is one single network of walls encompassing approximately 300 million closed air-spaces (about 480 million in recent morphometric and physiological measurements by Ochs et al.) within adult lungs.



*Figure 2.1. Schematic of the conducting airways feeding the alveolar gas-exchange region. The septal wall network within each alveolar sac is the mechanical structure represented by the finite element models in this thesis.*

Two particular points related to this wall network are critical for the modelling choices made in this thesis. First, the walls are pre-stressed in vivo due to surface tension at the air-tissue interface: a thin film of water coats the inside of every alveolus, whose surface tension (partially counteracted by pulmonary surfactant) pulls the wall network taut in a way that is utterly absent from excised, ex vivo tissue sample with dry walls. Second, the wall material is non-linear and at the scale of the wall

material close to incompressible - networks of collagen and elastin fibers strongly stiffen as they get recruited to carry load, such that small-strain ("toe region") stiffness of wall material is much lower than its large-strain stiffness. Both topics will be discussed in further detail and explicitly excluded from the model below, in Sections 2.2 and 4.3.

## **2.2 Mechanics of Lung Tissue and the Role of Surface Tension**

The classic demonstration of the importance of surface tension to the mechanics of lung tissue is the saline-inflation experiment, performed by John von Neill and Mead which eliminates lung surface tension. During the experiment the lung is inflated by the much more viscous saline solution, resulting in it being much easier to inflate, whilst the actual tissue properties have been unchanged. This observation and the physiological fact that premature infants may have problem inflating their lungs as the surface tension stiffening is not inhibited in them, form the basis for my decision to adopt surface tension as a first-order contributor to stiffness of the human lung - not some minor one. It should therefore be expected that all models neglecting the effect of surface tension and in particular the two which have been made, should give values that are lower in numerical magnitude for the stiffness of the human lung. The degree to which is neglected is one of the questions that will be tackled in Chapter 5 through the stiffening factors generated by Singh, Slutsky and Cronin. It is also of great importance within the field to model the changes and remodelling of connective tissues during the mechanical action as changes to these are frequently the first indicator of pathologies.

Another source of in vivo lung stiffening related to pre-tension of septal walls. At the resting volume (FRC), septal walls are at finite tensile pre-strain relative to their empty condition. Because soft biological tissues mostly stiffen with increasing strain, a wall network prestrained already is stiffer, for a given deformation applied on it, than this network unstrained. Prestrain is also not addressed to, like with surface tension, in this modelling study.

## **2.3 Fundamentals of the Finite Element Method**

Finite element analysis (FEA) refers to a particular numerical method used to address the differential equations governing a physical mechanism with a geometrically intricate domain that does not have a closed-form analytical solution. In this case, the analysis involves the state of

mechanical equilibrium with time-invariant loads. Instead, the area is subdivided to a finite number of tiny basic shaped elements that intersect at points called nodes, the governing equations on each element are simplified independently, element equations are assembled into a set of global matrix equations that are solved for the unknown nodal variables (displacements, in this case) and subsequently strain, stress and reaction forces may be obtained. Finite element model has gained popularity so far is that it provides a cheap/quick substitute to digital prototyping test in place of the expensive physical test and thus, enables many design or loading case tests with a low marginal cost on a one-time basis after the model is built.

There are three stages to any FEA study, both models in this thesis contain all three. In the preprocessing phase the geometry, mesh, material properties, loads and boundary conditions of interest are prescribed. The solution stage involves the composition and solution of the global equations; in the linear elastic, quasi-static problems considered in the presented models the overall set of solutions resolves to a direct linear solve at each load case. During the postprocessing stage insight is gleaned from the results of the FEA into the stress, strain, displacement fields and any derivable values such as reaction forces.

### ***2.3.1 Shell Elements for Thin-Walled Structures***

Considering that the septal wall is geometrically thin in relation to its in-plane extent - ratio on the order of 1:100 or greater for wall thicknesses and cell sizes used in this thesis - both micro-models represent the wall network with shell elements than solid (continuum) elements. A shell element has membrane and bending stiffness on a two dimensional reference surface of assigned thickness, which avoids the impossibly fine through-thickness mesh a solid-element representation of a wall of micrometer scale would require at the cost of assuming the wall is thin enough that through-thickness stress gradients are not of primary interest. This is the same modelling choice as was adopted in the alveolar-cluster benchmark model of Singh, Slutsky and Cronin (walls as shell membranes).

### ***2.3.2 Tools Used in This Project***

### FE tools used

The commercial FE pre-processor used was Altair HyperMesh, used to import and clean the Kelvin-cell and the Voronoi geometries, mesh the shell walls (for septa) and run sanity checks on the meshes (element aspect ratio, warpage, Jacobian) before export. The solver used was Abaqus/Standard, to run the non-linear equilibrium iterations under applied compressive Boundary Condition and report back resultant stress, strain and stiffness of the tissue. Regular Kelvin cell geometry was also generated in addition in MATLAB parametrically and subsequently carried in to HyperMesh, as stated in Section 4.2.

## **2.4 Historical Context of Alveolar Micro-Modeling**

Modelling of the alveolus as an idealized geometry predates finite element analysis itself: it was common clinical practice for early twentieth-century respiratory physiologists to assume the alveolus could be approximated as a simple sphere or polyhedron to derive estimations of surface area and of the distance of gas-diffusion, without however attempting to predict mechanical stress. As soon became recognized [throughout the work of the mid-to-late twentieth century literature on pulmonary mechanics and as formalized in the micromechanical framework of Stamenović reviewed in Section 3.1], the surface tension alone cannot account for the gross elastic recoil of the lung, a quantity relevant in diseases such as emphysema and pulmonary fibrosis and direct consideration is required for the load-bearing microstructure (alveolar wall network) of the lung tissue if its macroscopic elastic properties are to be modelled accurately: the alveolus needed to be modelled not only as a gas exchanger, but as a cellular solid subject to loading, too. The Kelvin cell was embraced in favour of any other primarily due to its popularity in the engineering literature of foam mechanics - it is a near-isotropic space-filling polyhedron already familiar to the field- than on the basis of being chosen independently for having an appropriate resemblance to the alveolus, as suggested by its imaging data (Section 2.5.2); and it maintains its relevance through its usage in the work of Concha et al. and the base geometry of Singh, Slutsky and Cronin not because it appears to correspond well to the *anatomical* my, but because, like a regular geometry, it presents several mathematical advantages in that respect (one of the reasons why the alternative of exploring the behaviour of an irregular model alveolus arose to begin with).

The tools used in this thesis: MATLAB for parametric geometry, Hypermesh for meshing and Abaqus for solution show the modeling pipeline to be typical throughout academic biomechanical research within approximately the past 15 years within academia, with purpose or bespoke meshing codes having been slowly replaced by commercial packages which can tackle the boundary cropping and quality element challenges that have come with dealing with irregular and natural geometries, as is touched upon with the Voronoi literature cited in Section 3.2 and discussed further in this thesis in Section 4.2.2.

## **2.5 Mechanics of Cellular Solids**

Since the micro-models examined in this thesis are open-celled cellular solids in terms of geometry (instead of a solid continuum solid), they can be viewed through the general lens of engineering cellular solids presented by Gibson and Ashby about foams, honeycombs and lattices, which is the same as the implied framework used when a lung micromodel's wall level properties are linked to a predicted bulk (macroscopic) stiffness, as done by Stamenovic' through his original micromechanical treatment of the lung (Section 3.1).

### ***2.5.1 Relative Density and the Scaling of Bulk Stiffness***

The relative density ratio of the solid wall material to air of an open-cell foam of strut or wall thickness  $t$  and cell size  $\ell$  roughly scales as  $(t/\ell)^2$  for a wall-dominated cellular solid example and the classical Gibson-Ashby scaling result found in material science is that the modulus of the cellular solid,  $E^*$ , scales with the wall (or strut) material itself modulus,  $E_s$  and then to a power of something between one or two based on a debate over whether the individual wall material bends or stretches predominantly. In bending-dominated walls of open-cell foams  $E^*/E_s$  scales roughly with the power of two of the relative density, whilst for stretching-dominated closed-cell foams of walls that in fact behave as plates, such as those of the septal wall network, as they are not subject to a large amount of bending they scale at near zero (or at least close to linear in relative density). This scaling reason is where a change in wall thickness (or a change in relation to cell size) for an alveolar micron-model of the previous models will experience a large amount of increase in terms of its overall bulk stiffness because it takes relative density to a power of something similar to that for the stiffness

scaling - meaning increasing the wall thickness,  $t$ , has the effect of increasing the thickness ratio faster than can be accounted for.

In this way the scaling framework also helps to give, at a more fundamental level, some indication of why it is that the regular and irregular geometries of this thesis can vary mechanically even at matched relative density and matched wall material. While the Gibson-Ashby scaling laws are derived for idealized, statistically regular foams; for real, irregular foams of the same relative density but a wider distribution of cells sizes and wall orientations it is well known in the broader literature of cellular solids that there will be a wider distribution of local strut or wall stress at a given applied macro-scopic strain than for a regular foam of the same relative density, which is the same local stress concentration effect as studied directly by finite element results in this thesis in Chapter 5 and interpreted in 6.1 coming from general scaling theory than the specific topic of Voronoi meshes considered for review in Chapter 3.

### ***2.5.2 Anisotropy and the Kelvin Cell as an Idealization***

The Kelvin cell (truncated octahedron) used as the unit cell for the regular models in this thesis was a historically favoured choice as it is one of the few space-filling polyhedra with properties approaching geometrical isotropy - for a macroscopically isotropic material such as lung parenchyma is often assumed to be at the organ level, a unit cell with omnidirectional or approximately omnidirectional (which is true in this case as mechanical response is not perfectly isotropic) mechanical response is desirable; though the real alveolar wall network is not in the ideal sense statistically isotropic at the micron level, local anisotropy caused by airway branching, pleural attachment and change in lung inflation gradients under gravity is noted well in physiology and naturally irregular tessellations of space such as Voronoi tessellations create local variation in wall orientations, however a statistically isotropic seed-point distribution has been employed in this thesis to generate the tessellations as discussed in Section 4.2.2. The comparison undertaken in this thesis can thus cellular-solids wise be read as one between an ideal isotropic unit cell, one that exhibits both isotropic bending and stretching behavior upon affine deformation and a statistically erratic microstructure - where this erratic phenomenon in this case is due to local variability in cellular geometry than a locally imposed anisotropic trend.

## Chapter 3 Literature Review

In this chapter, the literature is reviewed in three parts. First, 3.1, considers previous micro-mechanical finite element models of the alveolus and the experimental/analytical work used in establishing the material properties employed in the current thesis.

### 3.1 Micro-mechanical Finite Element Models of the Alveolus

- The idea of modeling lung parenchyma as a periodic unit (repeating cellular solid rather than a continuum) goes back to Stamenović's micromechanical foundation of pulmonary elasticity that related the mechanical behavior of the lung to an idealized network of septal struts establishing the fundamental principle - relation of unit-cell wall properties to the bulk of tissue stiffness that still the bases of all subsequent alveoli models including the ones presented in the thesis presented. They then extended their work and developed a micromechanical model of hyperelasticity built on a periodic structure of polyhedral unit cell model with nonlinear (hyperelastic) wall model more representative of collagen and elastin fibers network than the previous assumption based on linear elasticity in this thesis.

The most direct precedent for the present work is the pair of studies by Singh, Slutsky and Cronin. Their 2024 study developed a finite element alveolar-cluster model, a tetrakaidecahedron (14-faced) unit representing 48 alveoli within a 1mm<sup>3</sup> volume, solved in LS-DYNA; and used it, together with experimental stress; stretch and pressure; volume data, to back out a hyperelastic material law for the alveolar wall. Their 2026 follow-up study, which is used directly as the benchmark in this thesis, enhanced that same alveolar-cluster geometry with an explicit finite element representation of the surface tension membrane at the air; liquid interface, calibrated against experimental surface-tension force data, and used the enhanced model to quantify how surface tension and tissue pre-strain each increase the macroscopic stiffness of lung parenchyma in tension, compression, and pure shear. Their reported finding; that surface tension increases tissue stiffness in non-volumetric (deviatoric) deformation modes in essentially the same way it is known to increase stiffness in volumetric (inflation) deformation; is the specific literature result invoked in

Chapter 6 to interpret why the dry-wall models built in this thesis are expected to read soft relative to an in vivo lung. Both Singh et al. studies use a single repeating unit cell rather than an irregular tessellation; the comparison between a regular and an irregular alveolar geometry, which this thesis performs directly, is not addressed by either study.

Bou Jawde, Takahashi, Bates and Suki, who developed an analytical model to back out the elastic modulus of the alveolar wall material from uniaxial measurements on lung tissue strips, provide the wall material properties for both models of this thesis. They found an elastic modulus that increased with age from 4.4 to 5.9 kPa ( $p = 0.043$ ) for the alveolar wall and an even stiffer modulus for collagen/elastin fibers (311 to 620 kPa,  $p = 0.001$ ), which is considered by the different models using 5 kPa as wall modulus for the regular geometry and a higher value that considers the difference between models, 12 kPa, for the irregular geometry in Section 4.3. The morphometric baseline in terms of alveolar count and packing density for sizing the representative volume element in the models are taken from Ochs et al. as up to today, Ochs's detailed serial section stereology is a standard reference for the number of alveoli in the human lung.

In Chapter 6, because values are retrieved from the published Figures instead of from a furnished data table, the compression curve of Figure Fig 9(b) of Singh, Slutsky and Cronin is digitized using WebPlotDigitizer, an open-source plot digitization tool developed by Rohatgi, whereby the axes gridlines become the calibration reference to resolve the pixel resolution into stress (i.e. force per unit area)-stretch (i.e. force per unit area) data.

### **3.2 Voronoi as a Tool for Representing Irregular Cellular**

The decomposition of a domain using a Voronoi tessellation creates regions, one surrounding each seed point; every location contained that part of the domain is closer to the cell's seed point than to any other. Where seed points are introduced with some degree of randomness, i.e. not ordered in a regular lattice pattern, the polyhedra produced are of varying size, shape and number of faces - precisely the kind of geometrical irregularity that cannot be reproduced using a regularly tessellating unit cell such as the Kelvin cell used as the unit cell for the regular tessellation model in this project. As such, Voronoi tessellation has been employed in numerous engineering and materials science applications as a means by which to design suitable and structurally realistic

representations of natural cellular and granular media. The microstructure is known to be irregular and a regular idealisation proves useful for analytical or computational studies. This literature is reviewed below by application.

### ***3.2.1 Trabecular Bone and Other Biomedical Porous Structures***

Looking beyond the lung literature, is trabecular bone (another natural, irregular, loadbearing cellular solid); the closest structural analogue in its problem formulation to the present model. Zhou et al [214] proposed an improved Voronoi tessellation bone model based on Boolean operation to merge together scaled Voronoi cells and produce both plate-like and rod-like geometries of the trabecular structures but also control both its porosity and anisotropy, an issue in some Voronoi based bone models which have resulted in simple, rod-like lattice structures to which the authors point out can not represent the porosity typically occurring in bone samples. Zhou et al's mechanical evaluating based on combination between analytical estimation, Finite Element simulation results (used in similar pipeline to that of geometry-mesh-solve based to irregular lung model) and on compression tests performed over 3-d printed samples are closely related to the method taken in this dissertation for estimating the mechanical properties of an irregular cellular biological material; although for a different tissue with validation against real sample possible and performed on their behalf, a possible experiment if any was of interest in the mechanical properties of irregular lung models, similar samples are not yet in disposal for mechanical testing for 3-d printing yet if available in near future.

A Voronoi microarchitecture is also used in the 3D-printing of bone scaffolds by Winarso et al., applying a Taguchi-grey relational analysis to improve printing parameters (layer height and line width, printing temperature and speed) for the compressive strength and elastic modulus of bone with polylactic acid. Although their application of optimizing bone scaffold manufacturing does not coincide with the loaded case examined in this thesis, their work showing that the actual mechanical response within a porous biological structure, of both the compressive strength and the modulus of elasticity of the structure, can indeed be adjusted systematically by merely adapting these Voronoi microarchitecture parameters supports reasoning of this thesis: that geometrical

parameters in a Voronoi cell formation method like the seed density, cell size and wall thickness that follow therefrom are more than simply a means of visually obtaining irregularity - for the mechanics of that structure, its mechanical response it forms a viable engineering control.

On a smaller length-scale, a three dimensional Voronoi tessellation was used to construct nearest neighbour lipid shells around membrane proteins in a study by Cheng et al. They classified lipid disruption in protein containing lipid membranes by their Voronoi neighbour relations 23. Although this is more geometric and statistical than structural- there are no simulating mechanical loading - it does show that Voronoi tessellation is being used in biomedical modelling as a tool to model the neighbour relations in irregular packed structures on length-scales of lipid membranes to organs 23.

Along similar biomedical-motivated lines, Zhang et al. designed personalized, thermally-comfortable orthopedic casts using a hollowed Voronoi tessellation pattern; here, optimization of the Voronoi cell pattern was done not for one of the competing mechanical objectives used in this thesis, namely stiffness and fluid flow resistance, but for a patient-specific thermal-sensitivity map derived from infrared imaging, with mechanical stiffness requirements being separately taken into account. This use of combining mechanical performance with another, non-mechanical performance criterion using the same Voronoi pattern with a single geometrically-tunable parameterization bears likeness to a lung-micro model where mechanical fidelity, the focus of this thesis, needs to be ultimately reconciled with computational budget (see max node count in the discussion §4.4 and §7) within the same Voronoi network.

### ***3.2.2 Granular, Polycrystalline, and Geological Media***

The other main area in which Voronoi tessellation is used is in the representation of grain and block structure within porous/granular materials or polycrystalline materials where - as for the case of alveolar septa - the physically important geometry of the material is irregular and the Voronoi cell boundary represents a real internal interface between areas of the material of different mechanical or transport properties. Ghazvinian, Diederichs and Quey created a three-dimensional random Voronoi grain-based model in distinct elements in which grain boundaries represents defects (or flaws) within an intact rock material along which the processes of crack initiation and propagation may occur and simulated this model: they note explicitly that difficulty of generating such models in

3D had hindered previous attempts to study such models (a difficulty that was noted earlier in modelling the alveoli in 3D in 4.4).

In a pair of studies on the micromechanics of brittle rock, Lv, Li, Zhu and Liu modelled micro-fracturing and tension cracking in brittle rock including rocking-slope failure under earthquake loading or rock fragmentation by tunnel-boring-machine cutters using a discrete element method (DEM) approach with random Voronoi grain-based models; they generated the Voronoi tessellations using MATLAB's `mpt_voronoi` function, the same general approach to generation of the tessellation with MATLAB which was used to create the meshed regular Kelvin-cell model as described above in Section 4.2. On a more esoteric tack, Benedetti, Gulizzi and Milazzo use a modelled Voronoi-tessellated polycrystalline representation of grain boundaries, a boundary-element model of the (also, intergranular) micromechanics and a finite-element model of the diffusion of hydrogen therein in their models of hydrogen-assisted intergranular stress corrosion cracking; their use of the Voronoi grain boundary as the locus for both mechanical damage and a coupled diffusive process reminds us that even within the context of the lung, the alveolar wall is both the locus for the mechanical load path (which is the quantity examined here) and, within the intact organ, the coupled transports associated with gas exchange (a topic beyond the scope of this study).

In another attempt to relate the directly Voronoi-tessellation geometrical statistics to observed wave-propagation behaviour, Ryzy et al. looked at attenuation of ultrasonic waves, again in polycrystalline media, with varying grain morphology mapped as statistically equiaxed Voronoi structures. Finally Shentu, Lin, Jin and Yoon incorporated Voronoi tessellation into a grain-based discrete element model (for hydraulic fracture propagation in heterogeneous conglomerate rock) and supplemented the Voronoi model with a machine learning framework (extreme gradient boosting with Shapley additive explanations for explainable machine learning) to predict the resulting fracture behaviour from the generated Voronoi-based geometries and material properties: the geometry generated using Voronoi tessellation thus being not only a direct mesh for input to the simulation, but also providing structural features for an interpretable machine learning model.

### ***3.2.3 Porous Media Transport and Additive Manufacturing***

The third cluster of Voronoi-tessellation literature concerns porous media whose function is transport (fluid flow, heat conduction) than or in addition to, load-bearing structure - relevant to this thesis because the alveolar wall network is simultaneously a load-bearing structure and the boundary of a gas-transport pathway. Xiao and Yin had constructed granular, tubular and fibrous porous-medium geometries directly from Voronoi tessellations to solve fluid flow through them with a massively parallelized lattice Boltzmann method, producing porosity-permeability relations in so doing and finding that the granular and tubular cases could be unified by a modified Kozeny-Carman equation, while fracture networks and isolated large pores (vugs) each perturbed the relation in opposite directions. Using a constrained Voronoi tessellation coupled with a genetic-algorithm/least-squares optimisation technique, Shen et al. had obtained three-dimensional nanocrystalline grain models of a lithium-silicate ceramic with a specified log-normal grain-size distribution for molecular dynamics simulation of tritium breeding materials - an application entirely unconnected to biomechanics, but an illustration yet again that Voronoi tessellation is generally regarded as a suitable substrate for imposing a target statistical (than a regular single) grain- or cell-size distribution, by engineers of all stripes.

Additionally, Voronoi tessellations may be of direct relevance in the design of additively manufactured (3D-printed) porous structures, forming a manufacturing-oriented literature with relevance to the cell-merging and geometry-cleanup problems encountered later in Section 4.4 of this chapter. Alkner, Saktani and Arab produced light, strong, bio-inspired 3D Voronoi porous scaffolds of controlled porosity and cell elongation with stereolithography and tested them under compression, finding that elongated Voronoi structures gave higher Young modulus and energy absorption than normal (not elongated) Voronoi designs: a useful empirical indication that Voronoi cell aspect ratio forms a first-order control over compressive mechanical response, possibly relevant in Chapter 6's discussion of why the irregular alveolar model's junction geometry is such stress amplifiers. Also Anwajler produced a Voronoi diagram-design method for forming 3D porous thermal-insulation composites with functionally graded porosities from a biodegradable resin using vat polymerisation and tested their thermal conductivity, finding that both porosity and geometry (irregularity) independently affected the experimental effective thermal conductivity; evidence, from outside the biomechanical field, that Voronoi irregularity forms not just a cosmetic feature of a

natural-looking structure but changes the physical responses of the structure in a measurable and relevant way for structural design.

Liu, Feng, Chen, He and Fu used topology optimization and Voronoi porous solid design together, mapping an optimized density field to a spatially graded Voronoi seed-point distribution allowing local cell size (and therefore local stiffness) to follow the design, using the resulting method to design medical pillows with improved shape retention and reduced weight compared to those constructed with uniform Voronoi structure. Two other additive manufacturing applications by Lee, et al. and Choi, et al. use Voronoi diagrams of ellipses and ellipsoids respectively to tackle the problem of support-free internal hollowing during 3D printing - a geometric than mechanical loading problem compared to the systems studied in thesis, but another data point showing the versatility of the Voronoi formation block as a mesher and geometry creator and also illustrating the implementation hassles (isolated cells or degenerate cells due to proximity to boundary, boundary-crop artefacts, edge effects etc. ) involved in cropping a Voronoi diagram to a finite-sized cube (e.g. at the alveolar tissue samples for irregular model creation which are the same as cropping finite-sized segments in section 4.2.2).

### **3.3 Synthesis and Identified Gap**

There are two threads evident in the previous section. Firstly, the study of alveolar mechanics [3.1] shows that it is possible to model the network structure of alveolar wall by finite element simulation using repeating unit-cells. Stamenović, then Concha et al. and then Singh, Slutsky and Cronin (most convincingly following their work in 2009 with Cronin's group (2007)), show that when surface tension and pre-strain are included -the network can be shown to display a stiffness comparable to experiment. However, the models are all based on single, repeating unit-cells.

The second argument threads through the modelling of trabecular bone [3.2] through additively manufactured scaffolds, to granular and polycrystalline rock and then to porous media. In each, without reference to one and other, it is noted that geometric irregularities affect mechanical

properties e.g. irregular cells lead to redistribution of loading to local sites for Alknery *et al.*; the compressive strength and stiffness is found to change according to how uniform the design (Winarso *et al.* ; Liu *et al.*); transport coefficients also change from a regular-lattice prediction and it was noted further that irregular Voronoi meshes change transport coefficient values greatly. (Xiao & Yin; Anwajler)).

No work uncovered by this article applies a three-dimensional Voronoi tessellation to the lung wall network itself and compares the mechanical response of such to a repeating design under identical loading. Therefore to this specific, narrow extent it is claimed this work fills a gap within the contemporary literature: not to propose a new material law, nor a new claim about the absolute stiffness of the lung, but a controlled comparison, within the same representative volume under the same shell elements and boundary conditions and of the same wall characteristics, between a regular arrangement and a tessellation based, as per the above literature survey in related fields, on the mechanics of irregular micro-structures within shell networks.

### **3.4 Methodological Lessons Carried Forward from the Voronoi Literature**

Finally, in providing further to evidence that geometric irregularity is at play in granular and polycrystalline-media mechanics, the work referenced in section 3.2 also gave rise to some explicit methodological lessons that were adopted in the modelling pipeline presented in chapter 4. First, the observation often repeated in the literature review on granular and polycrystalline media by numerous authors (Ghazvinian et al, Lv et al, Benedetti et al) that three-dimensional voronoi generation and boundary cropping gives rise to mesh intensity problems and the likely formation of degenerated geometry, was the justification for carrying out mesh quality checking described in chapter 4 (section 4.4.1) and gave rise to the expectation, met in practise, that the irregular alveolar model would require more manual clean up than regular the equivalent regular Kelvin cell models. Secondly, the work by Alknery, Sktani and Arab demonstrating that that cell elongation and aspect ratio can have controlling effects on the compressive mechanical response, justified the use of an isotropic non-elongated seed point generation employed when giving rise to the irregular model described in chapter 4.2.2. If this were not employed then any stress concentration would be

attributeable to any arbitrary cell elongation direction, instead it may more be attributed simply to the irregularity.

Third, while this work uses Voronoi tessellation to provide a fixed representative geometry, Shen et al. 's use of constrained, log-normally distributed grain sizes and Liu et al. 's topology optimization driven, spatially graded seeding highlights that it can also be used as a way to input a distribution, than a particular shape. Again highlighted in Chapter 7, a natural extension to this thesis with the irregular alveolar model would use a spatially varying seed point density than a statistically uniform distribution within a single RVE. Regional variation in alveolar size might result during gravitational loading, for example, from apex to base of the lung. Fourth, Shentu et al use Voronoi-derived geometric features as inputs to an explainable, machine learning model used to predict the fracture behaviour of cellular materials directly from the statistics of the Voronoi grains, doing away with the need to solve a new finite element model every time. This highlights a potential longer-term extension to this comparative thesis between regular cells and irregular, alveolar cells. If for each candidate alveolar geometry, the full finite model did not need to be solved, then in principle the approach from Chapter 5 of generating solved results for a large number of regular and irregular cells to train a fast surrogate model that maps the seed statistics into, for instance, peak local wall stress without suffering from the problems encountered with meshing and convergence described in Section 4.4 and Section 5.2.

## **Chapter 4 Methodology**

This chapter describes, in the order they were carried out, the five steps which were common for the micro-models: they were generation of geometry, meshing, assignment of material, application of boundary conditions and load and solution. Where the regular (Kelvin-cell) and irregular (Voronoi) models differ, they are each described separately.

### **4.1 Overview of the Modeling Pipeline**

While both these structures model the same type of problem: a 1 mm ×1 mm ×1 mm representative volume of the alveolar parenchyma, in which the septal wall network is modelled as a network of thin shells compacted uniaxially by 5% along one axis. The pipeline used to go from idealized

geometric concept to solved Abaqus model has five steps for each structure: (1) produce the three-dimensional polyhedral tessellation defining the wall network; (2) convert that tessellation into a clean shell geometry ready for meshing; (3) run a shell mesh in HyperMesh with and check element qualità; (4) assign material properties, boundary conditions and compressive load to the model in Abaqus/CAE; (5) solve and post-process in Abaqus/Standard. For the regular-cell additionally begins with a MATLAB to construct geometric models step described in Section 4.2.1.

## **4.2 Geometry Generation**

### ***4.2.1 Regular Model: The Kelvin Cell***

The regular model represents the alveolar wall network as a tiling of identical Kelvin cells; truncated octahedra, the same space-filling polyhedron used in essentially all prior repeating-unit-cell alveolar models, including those of Concha et al. and the base geometry of Singh, Slutsky and Cronin. The Kelvin cell was constructed parametrically in MATLAB: each cell has a predefined, fixed arrangement of vertices, edges, and faces, and identical cells are tiled periodically to fill the 1 mm ×1 mm ×1 mm representative volume. Because every cell in the tiling is geometrically identical and meets its neighbors at the same fixed dihedral angle, the resulting wall network is maximally regular: cell size, wall orientation, and junction angle are constant throughout the domain. Adjacent cells share common faces, which represent the septal walls between neighboring alveoli and are the surfaces meshed with shell elements in Section 4.4.

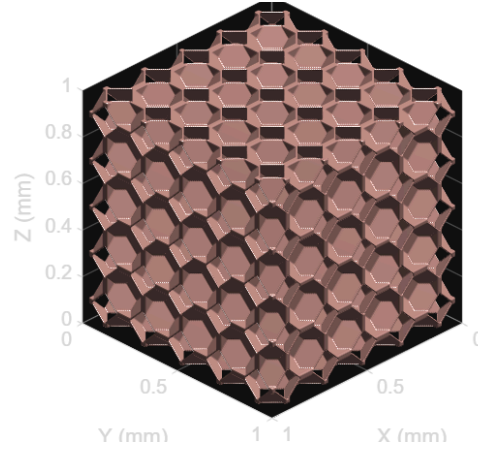


Figure 4.1. Regular Kelvin-cell tiling of the  $1\text{ mm} \times 1\text{ mm} \times 1\text{ mm}$  representative volume, generated parametrically in MATLAB. Every cell is geometrically identical.

#### **4.2.2 Irregular Model: Three-Dimensional Voronoi Tessellation**

The irregular model represents the same nominal wall network using a three-dimensional Voronoi construction. A set of seed points was distributed randomly through an oversized domain surrounding the target  $1\text{ mm}^3$  volume; each seed point generates a polyhedral Voronoi cell consisting of every location closer to that seed than to any other, and the shared face between two neighboring cells — the Voronoi facet — is taken as a septal wall. Because the seed points are randomly, rather than periodically, distributed, the resulting cells differ from one another in size, shape, and number of faces, which is precisely the geometric variability observed in real alveolar tissue and precisely what a single repeating Kelvin cell cannot represent. The full tessellation was cropped to the  $1\text{ mm} \times 1\text{ mm} \times 1\text{ mm}$  representative volume used for the regular model, so that the two geometries occupy an identical domain and can be compared under identical boundary conditions.

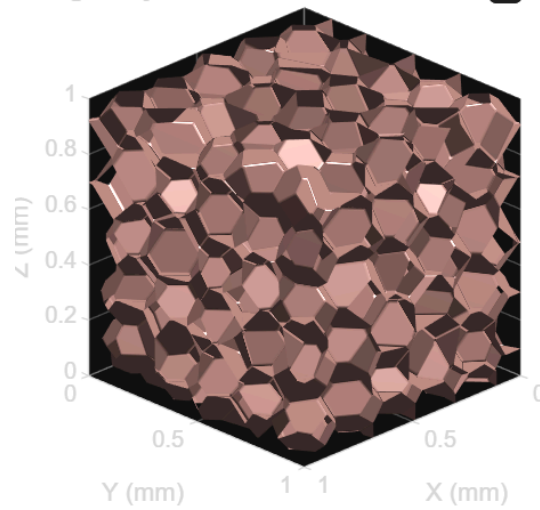


Figure 4.2. Irregular Voronoi-tessellation representation of the same 1 mm × 1 mm × 1 mm representative volume. Cell size, shape, and number of faces vary from cell to cell, unlike the regular model in Figure 4.1.

This was a nontrivial geometry-cleanup problem in its own right, and specifically mentioned by Ghazvinian, Diederichs and Quey as difficult for this type of problem; like the analogous issue in the granular- and polycrystalline-media literature in Section 3.2.2, of generating a clean three-dimensional (not two-dimensional) Voronoi tessellation and cropping it to a finite bounding volume without slivers, isolated fragments or open (non-manifold) faces at the crop boundary. The cell geometries, both regular- and irregular, were brought into Altair HyperMesh from MATLAB to be meshed.

### 4.3 Material Properties

A linear elastic, isotropic material law is chosen for the septal wall in both models representing only that small range of small strains, low stiffness near-linear 'toe' region of the response discussed earlier in section 1.4, which will later be seen quantified in section 6.4. The elastic values of the wall modulus are inspired by analysis of Bou Jawde, Takahashi, Suki and Bates, who predict that alveolar wall elastic modulus, increasing with age, lies between 4.4 and 5.9 kPa. The regular-cell model takes a value at the lower end of that reported range and a higher value represents the stiffer fibre reinforced response, which has been used in the irregular cell model. Table 4.1 shows typical settings implemented into Abaqus corresponding to the above material assignments.

Table 4.1. Septal wall material properties assigned in Abaqus for the regular and irregular models.

| Property               | Regular (Kelvin-cell) model | Irregular (Voronoi) model |
|------------------------|-----------------------------|---------------------------|
| Constitutive law       | Linear elastic, isotropic   | Linear elastic, isotropic |
| Young's modulus, E     | 5 kPa (0.005 MPa)           | 12 kPa (0.012 MPa)        |
| Poisson's ratio, $\nu$ | 0.45                        | 0.45                      |

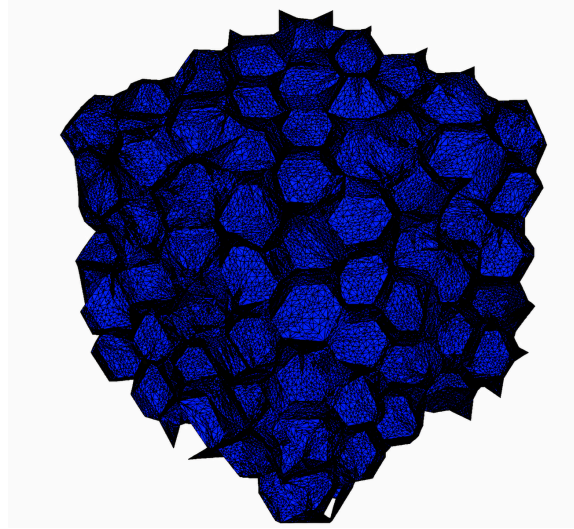
| Property            | Regular (Kelvin-cell) model                                     | Irregular (Voronoi) model |
|---------------------|-----------------------------------------------------------------|---------------------------|
| Density, $\rho$     | $1.0 \times 10^{-9}$ (flagged as an incorrect entry; see Ch. 7) | 1060 kg/m <sup>3</sup>    |
| Wall thickness, $t$ | — (see Section 4.4.1)                                           | 0.008 mm                  |

The density entry applied for this regular-cell model ( $1.0 \times 10^{-9}$  (and in the Abaqus consistent unit system used for that run) was discovered during the project to not be the same as the density used for the irregular-cell model and it is flagged here and again in Chapter 7, as a known input error rather than a deliberate modeling choice; as the analysis is quasi-static, density only affects the solution through the (small) inertial terms retained by the dynamic, quasi-static solution procedure described in Section 4.5, so the reported stress fields are not materially affected, but the inconsistency should be corrected before the model is extended to any rate-dependent or dynamic load case.

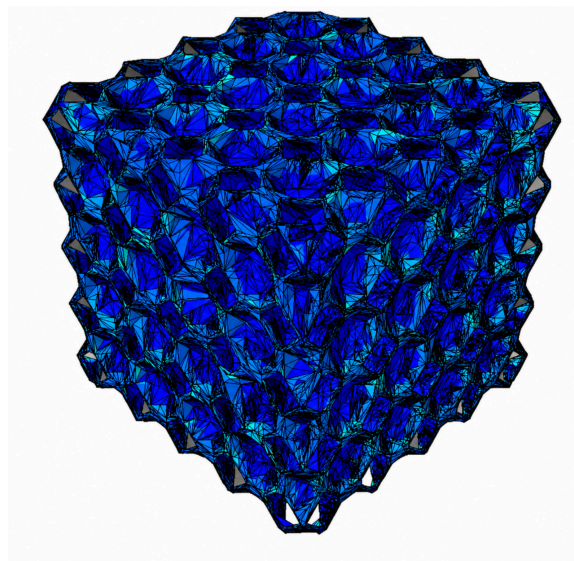
## 4.4 Meshing

### 4.4.1 Shell Discretization

The two septal wall networks were meshed in Altair HyperMesh, based on the assumption of modelling the walls as thin shells described in Section 2.3.1. The irregular Voronoi geometry was meshed with a single layer of S3 type (3 node triangular shell) elements and a thickness of 0.008 mm. The resulting full  $1\text{mm}^3$  (model volume) meshed geometry composed of 212,607 nodes and 428,810 elements, which was a sufficient resolution to resolve the curved, sharply intersecting junctions that are distinctive of this irregular geometry and close to the limitation posed by the usage of Abaqus, which is discussed below.



*Figure 4.3. HyperMesh shell discretization of the irregular Voronoi wall network (212,607 nodes, 428,810 S3 shell elements).*



*Figure 4.4. HyperMesh shell discretization of the regular Kelvin-cell wall network. Uniform cell geometry produces a correspondingly uniform mesh pattern.*

Running element-quality checks (aspect ratio, warpage and Jacobian) using HyperMesh on both meshes before exporting into the Abaqus input (.INP) format, it was found that the regular mesh required less clean-up work than the irregular one. At sharp, low-angle wall junctions created by Voronoi construction, automatic triangulation may create slivers with a high aspect ratio. This further confirms results presented within section 3.2.2 about meshing problems with cropped 3D Voronoi tessellations of granular media.

#### ***4.4.2 Mesh Size Constraint***

This study used Abaqus/Standard with academic license. It strictly imposes a limit of 250,000 nodes/model. The irregular-cell resultant mesh had 212,607 nodes and was deliberately created under the limit. Although previous attempted meshes which failed were denser and surpassed the ceiling, thus could not be solved. For example, cyclic breathing-load simulation was attempted to be performed on a mesh finer than the one used for the quasi-static results in Chapter 5. But it failed. Refer Chapter 7 for more details about this. In the chapter further, the limit placed for the number of nodes per model is considered due to license restriction on the mesh refinement and hence the smallest feature size at cell wall junctions achieved with the academic software used for this study.

### **4.5 Boundary Conditions, Loading, and Solution Procedure**

The identical quasi-static uniaxial compression boundary condition was applied to both 1mm by 1mm by 1mm representative volumes. A fully encastred bottom face was enforced on the RVE, meaning all translational and rotational degrees of freedom were fixed. The top face received a predetermined displacement of  $-0.05$  mm along the loading (Z) axis. This constitutes a 5% nominal compressive strain, equating to a stretch ratio of  $\lambda=0.95$  and this displacement was applied to every node along the top face. The four lateral faces of the RVE had no boundary conditions imposed, allowing the model lateral deformation in response to the applied compression, thereby simulating a Poisson effect; periodic boundary conditions, which would more accurately represent an RVE located within a larger tissue mass, were deliberately not utilized but are flagged as a potential avenue for model improvement in Chapter 7.

Abaqus/Standard's \*Dynamic application=QUASI-STATIC procedure has now been used in combination with irregular-cell model in which automatic time increment is used and damping can be tuned for slowly applied loads (a dynamic-solver with a small inertial term only to provide some stability). As Chapter 5 mentions, in the case of irregular-cell, final and largest 212 607 node mesh, the software did not reach fully convergence at the target load magnitude after which an error message about too many solution attempts given by Abaqus occurred: results in Chapter 5 about irregular-cell model are based on the load after which convergence stopped. The convergence issue has been studied within Chapter 7 and may be caused by the sharp high curvature cell junctions

created during the method of choice used to generate irregular-cells (based on random points method described by Voronoi diagrams).

## 4.6 Governing Equations

While stress and load quantities in Chapter 5 derived from Abaqus/Standard should ideally be referred to by a rigid mathematical definition, brief governing equations from Abaqus/Standard for quasi-static shell models can still be stated to remove any opaqueness to the solver's outputs.

### 4.6.1 Static Equilibrium and the Quasi-Static Procedure

For a body in static equilibrium under prescribed boundary displacements and negligible body forces, the finite element method solves the discretized form of the principle of virtual work,  $\int_V \sigma_{ij} \delta \epsilon_{ij} dV = \int_{\Gamma_t} t_i \delta u_i dS$ , where  $\sigma_{ij}$  is the Cauchy stress tensor,  $\delta \epsilon_{ij}$  is a kinematically admissible virtual strain,  $t_i$  is the traction on the loaded boundary, and  $\delta u_i$  is the corresponding virtual displacement. Because the wall junctions in the irregular model undergo locally large rotations even under a globally small (5%) applied strain, Abaqus/Standard's *Dynamic, application=QUASI-STATIC procedure was used for that model rather than a purely static (Static) step*: this procedure retains a small inertial term,  $M\ddot{u} + Ku = F(t)$ , with the mass matrix  $M$  scaled and the loading rate chosen so that inertial forces remain negligible relative to the internal elastic forces, while gaining the numerical robustness of a dynamic time-stepping scheme for the locally large-rotation, geometrically nonlinear equilibrium iterations at the sharp wall junctions. This is also the reason mass density, flagged as a known input inconsistency in Table 4.1, enters the solution at all for what is conceptually a static problem.

### 4.6.2 Shell Kinematics and the Membrane, Bending Stress Resultants

Each S3 shell element carries both membrane (in-plane stretching) and bending (out-of-plane curvature) stiffness on a reference mid-surface with assigned thickness  $t$ . The through-thickness stress at a material point offset by a normal distance  $z$  from the mid-surface is recovered from the membrane strain  $\epsilon_0$  and curvature  $\kappa$  of the shell reference surface as  $\epsilon(z) = \epsilon_0 + z\kappa$ , and the corresponding stress follows from the linear elastic constitutive law  $\sigma = D\epsilon(z)$ , where  $D$  is the plane-stress elastic stiffness matrix built from the wall's Young's modulus and Poisson's ratio (Table 4.1). This shell kinematic assumption, that strain varies linearly through the thickness, is standard

for thin-shell elements and is the formal statement of the thin-wall idealization introduced qualitatively in Section 2.3.1.

#### **4.6.3 Von Mises Equivalent Stress**

The scalar stress quantity reported throughout Chapter 5 is the von Mises equivalent stress,  $\sigma_v = \sqrt{[\frac{1}{2}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)]}$ , where  $\sigma_1, \sigma_2, \sigma_3$  are the principal stresses at a given integration point. Von Mises stress was chosen, consistent with its use in the benchmark study of Singh, Slutsky and Cronin and in the broader biomechanics literature, because it collapses the full stress tensor into a single scalar measure of distortional (shape-changing) stress intensity that correlates well with ductile and soft-tissue yielding behavior, and because it allows the regular- and irregular-model stress fields, which have different local principal-stress orientations at every wall junction, to be compared on a single common, orientation-independent scale.

#### **4.6.4 Reaction Force and the Macroscopic Stress Conversion**

The macroscopic engineering stress used for the literature-benchmark comparison in Section 6.2 is not a direct Abaqus output; it must be computed as  $\sigma_{aCr0} = \Sigma RF3 / A_0$ , where  $\Sigma RF3$  is the sum of the Z-component reaction force over every node in the top-face node set (the set on which the  $-0.05$  mm displacement boundary condition was applied) and  $A_0 = 1 \text{ mm} \times 1 \text{ mm}$  is the undeformed cross-sectional area of the representative volume. This is the specific calculation identified as outstanding work in Section 7.5: because it requires summing a nodal output variable over a defined node set rather than reading a single field value, it had not been performed on either the regular- or irregular-model output at the time of writing.

### **4.7 Verification and Validation Approach**

It proves enlightening to discern between two facets which, in standard computational mechanics, frequently necessitate verification procedures: verification (the model's correctness being assuredly solved). Validation (the model's correctly applied being ensured) also falls under scrutiny. Specific identification of which amongst these is achieved by this thesis can add further clarity. Concerning verification (1), the HyperMesh-generated element-quality checks already mentioned in section 4.4.1 (which included aspect ratio, warpage and Jacobian, performed on both meshes prior to

solving), the correct imposition of boundary conditions/load to create the regular near-uniform stress-field as shown under validation subsection (2) below (expected of a geometrically regular cellular solid using Gibson\_Ashby type scaling argument (refer 2.5 of section) and, subsequently verification through plotting and inspecting the stress contour map) and the WebPlotDigitizer test explained in section 4.6 (where the digitised points where seen to trace with high accuracy on the printed graph-paper, as shown in figure 5.1(a)) serve as instances.

Formal validation; comparing the model's predictions against an independent physical or experimental measurement of the same 1mm by 1mm by 1mm

block of lung parenchyma in the controlled way either model idealizes. In its absence, this thesis instead relies on the qualitative literature-benchmark comparison developed in Chapter 6, which checks the present models' expected position relative to a published, independently developed alveolar-cluster model (Singh, Slutsky and Cronin) rather than against a direct physical measurement. This is a real limitation, restated in Chapter 7, and is the reason the conclusions of Chapter 8 are stated in terms of relative comparison (regular versus irregular geometry) and expected trend (softer than an in vivo lung, by an estimated factor) rather than as validated absolute predictions.

## **4.8 Post-Processing and the Literature Benchmark**

Von Mises stress contours have been extracted to be compared in chapter 5. From both solved models in Abaqus/CAE. To provide an external check of result plausibility the Figure 9(b), compression branch, of Singh, Slutsky and Cronin showing macroscopic stress-stretch response of his model under four combinations of tension force and prestrain from alveolar cluster were digitised via WebPlotDigitizer: its pixel coordinate were calibrated to the data coordinate with aid to printed axis gridlines and each of the four graphs presented were tracing base on their colours and type of line. This digitised paper is use to provide chapter 6 the idea of scaling the presented model at dry wall cases without any pret strain under the same macroscopic stress, about what was used in 20/ cluster model at the existence of surface tension force and also to estimate actual stress that models under dry wall case will reach if 5% compression was applied which, is currently represented

as stress in the walls, not one unique macroscopic stress requiring conversion via its reaction force, which is left as discussion and outstanding work in chapter 7.5.

## Chapter 5 Results

In this Chapter, after an introduction, the finite element results for the regular (Kelvin-cell) and irregular (Voronoi) models for the same 5% uniaxial compression boundary condition as set up in Section 4.5, will be presented first, to be followed by the digitized literature benchmark that is used as a qualitative comparison that will be developed further in Chapter 6.

### 5.1 Regular Kelvin-Cell Model

The peak wall von Mises stresses for 5% compression (top face displaced -0.05mm, base fixed) under the regular Kelvin-cell model was 1.55 kPa. The stress contour looks close to spatially uniform: essentially every septal wall in the tessellation carries its similar share of the applied load with minimal local variation. This is essentially geometry dictated; since the Kelvin cell is the regular space-filling polyhedron, every wall junction in the tessellation meets neighbours at the regular dihedral angle and therefore no particular geometrical feature - an especially sharp corner or wall thin, for example - on which stresses may concentrate.

**Table 5.1. Regular (Kelvin-cell) model — summary of applied load and resulting stress.**

| Quantity                      | Value                                                       |
|-------------------------------|-------------------------------------------------------------|
| Applied top-face displacement | -0.05 mm (5% nominal compressive strain, $\lambda = 0.95$ ) |
| Boundary condition            | Base fully encastred; top face displaced in Z               |
| Wall material                 | Linear elastic, $E = 5 \text{ kPa}$ , $\nu = 0.45$          |
| Peak wall von Mises stress    | 1.55 kPa                                                    |
| Stress distribution           | Approximately uniform across all septal walls               |

### 5.2 Irregular Voronoi Model

Under the identical 5% compression same  $1 \text{ mm}^3$  domain, same boundary conditions, same nominal strain the irregular Voronoi model produced a peak wall von Mises stress of 12.9 kPa:

roughly eight times the peak stress of the regular model in absolute terms, or approximately 3.5 times higher once the 2.4× difference in assigned wall modulus between the two models (12 kPa versus 5 kPa) is factored out. Unlike the regular model, the stress field in the irregular model is markedly non-uniform: stress concentrates sharply at the junctions where three or more walls meet at acute angles, while the flat wall faces between junctions remain comparatively lightly loaded. This pattern load routed preferentially through a minority of geometric “hot spots” rather than distributed evenly is qualitatively consistent with the stress-concentration behavior reported for irregular Voronoi structures in other structural domains reviewed in Section 3.2, including the compression response of elongated versus regular Voronoi scaffolds reported by Alknery, Sktani and Arab.

**Table 5.2. Irregular (Voronoi) model — summary of applied load and resulting stress.**

| Quantity                      | Value                                                                                                             |
|-------------------------------|-------------------------------------------------------------------------------------------------------------------|
| Mesh                          | 212,607 nodes, 428,810 S3 shell elements                                                                          |
| Applied top-face displacement | −0.05 mm (5% nominal compressive strain, $\lambda = 0.95$ )                                                       |
| Boundary condition            | Base fully encastred; top face displaced in Z                                                                     |
| Wall material                 | Linear elastic, $E = 12$ kPa, $\nu = 0.45$ , $\rho = 1060$ kg/m <sup>3</sup>                                      |
| Peak wall von Mises stress    | 12.9 kPa                                                                                                          |
| Stress distribution           | Concentrated at sharp wall junctions; wall faces lightly loaded                                                   |
| Solver note                   | Run terminated before full target-load convergence (“too many attempts made for this increment”); see Section 7.3 |

As noted in Section 4.5, the \*Dynamic application=QUASI-STATIC solution for the irregular model did not reach full convergence at the target displacement and terminated with an Abaqus increment-convergence error. The stress values reported above reflect the converged solution up to the point of termination. The practical implication of this non-convergence, and the mesh- and geometry-related factors likely responsible for it, are discussed in Section 7.3.

### 5.3 Direct Comparison of the Two Geometries

**Table 5.3. Side-by-side comparison of the regular and irregular model results under identical 5% compressive loading.**

| Metric        | Regular (Kelvin-cell)                       | Irregular (Voronoi)                        |
|---------------|---------------------------------------------|--------------------------------------------|
| Cell geometry | Single repeating unit, fixed dihedral angle | Randomly seeded, variable size/shape/faces |

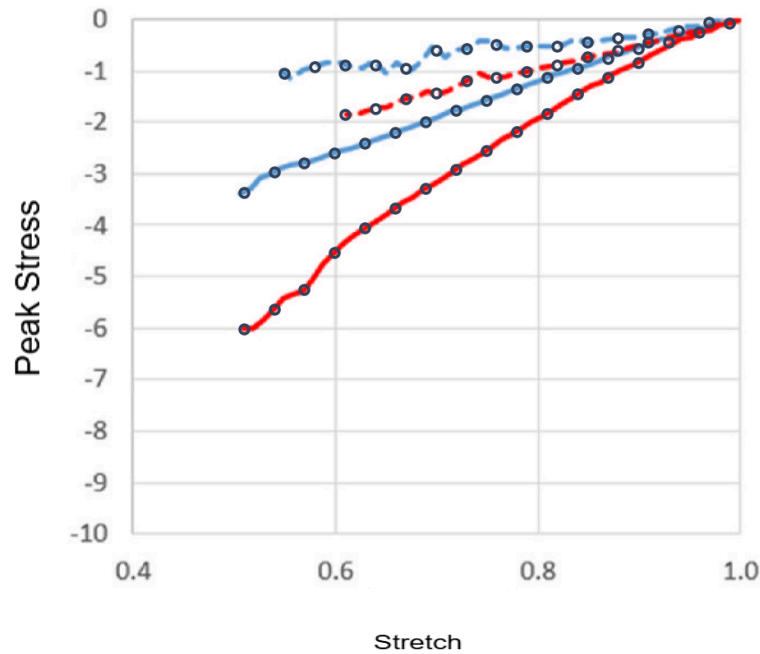
| Metric                        | Regular (Kelvin-cell)         | Irregular (Voronoi)             |
|-------------------------------|-------------------------------|---------------------------------|
| Wall modulus, E               | 5 kPa                         | 12 kPa                          |
| Peak von Mises stress         | 1.55 kPa                      | 12.9 kPa                        |
| Ratio to regular model (raw)  | 1.0×                          | ≈ 8.3×                          |
| Ratio after normalizing for E | 1.0×                          | ≈ 3.5×                          |
| Stress field character        | Near-uniform across all walls | Concentrated at sharp junctions |

The two right-hand columns of Table 5.3 clearly state the centre piece of this thesis's empirical findings: even after correcting for the stiffer wall material of the irregular model, switching from a regular to an irregular layout of the same nominal wall network increases the peak local wall stress by a factor of about 3.5 in response to otherwise identical loading. Because peak local stress (than average or macroscopic stress) is the key determinant of local tissue failure or injury, this forms the basis of the interpretation further built upon in Chapter 6 that geometric irregularity is not merely a secondary feature to be omitted from most alveolar micro-modelling but a first order determinant of local wall stresses exerted onto tissue.

## 5.4 Digitized Literature Benchmark

The compression-branch curves of Singh, Slutsky and Cronin's Figure 9(b), which were recovered after using WebPlotDigitizer (as explained in Section 4.6), to trace the four model conditions reported using in this publication, along the compression part of the stress-stretch curves. X-axis is stretch ratio  $\lambda$  (where 1.0 refers to unstrained state and decreasing values means that compression increases); y-axis macroscopic (engineering) stress in kPa. Close tracking between digitized points (open circles) and the three printed (three curves on the right side) curves, implying that calibration against reference grid lines was with accuracy (about what is reported possible visually).

### Extracted points (○) over the published compression curves



**Blue dashed** — bare wall tissue, relaxed. This is the one that matches my model.

**Red dashed** — add surface tension.

**Blue solid** — no surface tension, but pre-stretched.

**Red solid** — both. That's a real lung in a real person, and it's the stiffest by a long way.

*Figure 5.1. Digitized compression-branch data (open circles) overlaid on the published compression curves of Singh, Slutsky and Cronin (2026), Fig. 9(b), recovered with WebPlotDigitizer. Curves correspond to different combinations of surface tension and tissue pre-strain reported in the source study.*

These digitized curves are used in Section 6.2 as the reference envelope against which the present, dry-wall models are interpreted. The macroscopic-stress conversion needed to plot the regular- and irregular-cell model results from this thesis on the same axes — which requires the total reaction force on the top-face node set of each model — had not been extracted at the time of writing and is identified as the highest-priority piece of outstanding work in Section 7.5.

## **Chapter 6 Discussion**

### **6.1 Why Geometric Irregularity Concentrates Stress**

There is a relatively simple geometrical/morphological reason for the 3.5 fold increase in peak wall stress observed between regular and irregular models in 5.3 that is consistent with the broader body of literature on Voronoi-tessellations introduced in Chapter 3; In the regular tessellation all junctions are identical and therefore all share the applied compressive load equally as opposed to irregular where the junctions change; the load now travels down the sharpest junctions while the rest do little. This is the same reason found by Aleknery, Oskani and Arab for the difference in stiffness between more elongated and irregular scaffolds with the more regular in Section 3 and consistent to the finding in Sections 3.2.2 that when deformed, the stresses in a grain will localise preferably in the grain boundaries - the area that is by virtue of geometry the more inhomogeneous.

The implications for alveolar biomechanics are that the use the regular repeating unit-cell model for - a model adopted by most of the the prior micro-mechanical literature reviewed of section 3.1 including a base geometry model used by Singh, Slutsky and Cronin that of Concha- the use of this sort of model may under-predict peak local wall stress undergone even if it predicts the macroscopic average stiffness of tissue well. Peak local stress not average stress is the more relevant quantity for local injury considerations such as alveolar wall rupture at high volumes or under blast loading - hence the irregular model has been stressed as more appropriate despite the relative ease of building the repeating one.

### **6.2 Placing the Results on the Literature Benchmark**

Read against the digitized compression curves of Singh, Slutsky and Cronin (Figure 5.1), the two models built in this thesis — which have dry walls and no tissue pre-strain — are expected to sit closest to the benchmark study's Condition (1): alveolar wall tissue alone, evaluated at the stress-free, 30% total-lung-capacity (TLC) reference state, without surface tension or pre-strain. At the 5% compression level modeled in this thesis ( $\lambda = 0.95$ ), the digitized Condition (1) curve gives a macroscopic compressive stress of approximately  $-0.14$  kPa. This is the specific, single number against which the reaction-force output of the present Abaqus models should ultimately be

checked, once that output is extracted (Section 7.5); it is not yet possible to state whether either model's macroscopic response matches this value, because the wall-stress fields reported in Chapter 5 are local (element) quantities, not the volume-averaged macroscopic stress the benchmark curve reports.

What can be stated without the reaction-force conversion is the expected direction and rough magnitude of the gap between the present models and an in vivo lung. The source study's own Table 2 reports that, in compression, surface tension alone stiffens the macroscopic response by a factor of about 1.8×, pre-strain to functional residual capacity alone stiffens it by about 2.1×, and the two effects together stiffen it by about 3.3×, relative to the dry-wall, non-pre-strained baseline. Because neither surface tension nor pre-strain is represented in either model of this thesis, both models should be expected to read soft — by roughly that 2– to 3-fold combined margin — relative to an in vivo human lung at the same nominal strain, independent of any error in the wall material properties themselves. This is a testable, quantitative prediction that the outstanding reaction-force extraction (Section 7.5) can check directly.

### **6.3 Regular versus Irregular Geometry as Complementary, Not Competing, Models**

The results in this thesis should not be interpreted as simply showing that the regular Kelvin-cell model is plain wrong and the irregular Voronoi irregular model plainly right. The regular model, by displaying a nigh-uniform stress field and comprising solely of identical, symmetrical unit cells makes relating of a single well-defined local stress to prediction for the overall stiffness easier. Such an analysis is performed analytically by Stamenović and numerically by Concha et al. and the base model used by Singh, Slutsky and Cronin. When performing parameter sweeps, e.g. how a given change in wall modulus or wall thickness propagates to overall stiffness it is nice to have minimal variance, 'clean' distribution of 'local' stresses: such conditions make the relationships between local and global parameters easily established and results easily interpreted. In a study concerned by the local, worst-case environment or, for instance, a question whether a given inflation causes local rupturing of a wall, using the irregular model that recapitulates the load concentration of a real, non-uniform alveolar is the right approach. Therefore a complete approach would likely use both model types: parameter sweeps undertaken using the regular model and, at the load case of

interest, checking if such peak, local stress as could be measured by or calculated from, regular model is systematically underestimated.

## **6.4 Validity of the Linear Elastic, Small-Strain Assumption**

The models developed within this thesis employ a linear elastic material for the alveolar walls; however, physiological evidence and the hyperelastic model proposed by Concha et al. indicate that this material stiffens at higher strain levels. The decision to model it as linearly elastic, specifically at the single strain level of 5% compression examined here, is supported by the digitized benchmark curves presented in Figure 5.1. Across the stretch range from  $\lambda = 1.0$  to  $\lambda \approx 0.90$ , all three digitized compression curves exhibit a nearly linear behavior. Pronounced curvature, indicative of stiffening, only emerges at compression strains exceeding approximately 10%. Consequently, the 5% compression level falls within the near-linear "toe" region of the tissue response, as described in Section 1.4, where a linear elastic wall model serves as a reasonable local approximation. Nevertheless, this same figure also underpins the explicit warning, reiterated in Chapter 7, that the linear elastic assumption should not be extrapolated to strain levels beyond 10%. Doing so would necessitate the adoption of the hyperelastic wall law, as utilized by Concha et al. and by Singh, Slutsky and Cronin, because the tissue's response becomes distinctly nonlinear past that point.

## **6.5 Computational Cost of the Two Geometries**

Aside from the purely mechanical difference of the stress results, note a practical difference between the two models. In contrast with the regular Kelvin-cell model which had repetitive, uniform geometry and meshed cleanly with minimal manual intervention and fitting with the trend observed in cellular solids/Voronoi literature (Section 3.2), irregular Voronoi models can be expensive to make due to their non-uniform geometry being expensive to mesh and to solve, in spite of being an irregular tessellation of the same space. An irregular Voronoi model at 212,607 nodes and 428,810 elements, required more meshing time owing to the regular clear-up manual of sliver elements with sharp wall angle as noted in Section 4.4.1. Furthermore, we saw in Section 5.2 that where the regular model solved cleanly, the irregular model did not reach full solver convergence within the scope of this thesis. Cost models differ; when there are cost implications

such as this, then to decide if the added fidelity of the irregular cell geometry noted in Section 6.1 compared to the regular cell model in any given applications, becomes a worthwhile factor to consider before commencing modelling; what the deciding factor is noted in Section 6.3 as being dependent on whether the applications are concerned about macroscopic behavior or the peaks and lows in results.

## **6.6 Sensitivity of the Comparison to the Wall Modulus Assignment**

Because the regular and irregular models in this thesis were assigned different wall moduli (5 kPa and 12 kPa respectively, Table 4.1) rather than a strictly matched material, the raw 8.3× peak-stress ratio reported in Table 5.3 conflates two effects: the geometric effect of irregularity, and the material effect of the stiffer wall assigned to the irregular model. The 3.5× ratio obtained after linearly rescaling for the 2.4× modulus difference (Table 5.3) is the more defensible estimate of the geometric effect alone, under the linear elastic assumption justified in Section 6.4, since a linear elastic model's stress field scales exactly linearly with the assigned modulus for a fixed geometry and fixed applied displacement. This rescaling is exact only within the linear elastic regime; it would not hold if either model were extended to a hyperelastic wall law, since hyperelastic stress does not scale linearly with a single modulus parameter. A cleaner test of the pure geometric effect — assigning both models the same wall modulus and re-running the comparison — is a low-cost, high-value confirmatory analysis that follows directly from the existing model files and is recommended alongside the extensions listed in Chapter 7.

## **6.7 Implications for Whole-Lung Continuum Models**

The comparison made in this chapter has direct significance to the calibration of the continuum-scale lung models actually used for simulations of ventilation and injury of organs, than the micro-models investigated in this thesis. Traditionally such a model is fitted by reproduction of a large-scale stress-strain curve such as that of Singh, Slutsky and Cronin (Figure 5.1 in digitized form) with no consideration of the underlying stress distribution. The stress-strain curve was averaged over larger region. Hence it may come from either the quasi-regular stress or irregular stress. However the ratio of peak stress to average stress in these two kinds of structure is different, leading to difference of when and where the local tissue injury may occur. In principle, there is

nothing to distinguish between the two models in the kind of bulk behaviour, one of which could show quasi-uniform local stress and the other show stress concentrations. The irregular Voronoi model built in the thesis is a start to a tool box for solving the problem of how stress inhomogeneities may vary to injury onset and its extent. By giving the estimate of ratio of the peak to the average stress as approximately 3.5, one may make prediction out of continuum model using a stress concentration factor without resolving the geometry.

## **Chapter 7 Limitations and Future Work**

Here each limitation below is named alongside the model extension it motivates, so as to act as a concrete roadmap for continuing the work than only a list of caveats.

### **7.1 Species and Source of the Material Data**

Wall modulus used in models (Section 4.3) are taken from Bou Jawde et al., obtained from uniaxial stress-strain measurements performed on a rodent (mouse/rat) lung tissue and not human. Access to fresh human lung cannot be obtained and as a result rodent is a common input for models such as this and similar to the work done on modelling human body by authors such as Singh, Slutsky and Cronin. As such absolute magnitudes of the stresses reported in Chapter 5 can be viewed as indicator of the overall trend and for relative comparison only (regular versus irregular geometry); they cannot be explicitly assumed to represent the true situation for human alveolar tissues. Subsequent work will be to wait for estimates of alveolar wall properties as a result of using advanced human ex vivo imaging techniques and rerun models using updated stress-strain inputs, thereby directly testing its sensitivity.

### **7.2 Absence of Surface Tension and Tissue Pre-Strain**

As detailed in Sections 2.2 and 6.2, both the models under study exhibit no wall wetness and thus explicit modelling of air-liquid surface tension membrane, while neither of them has been pre-strained to the in vivo case corresponding to functional residual capacity. Singh, Slutsky and Cronin report surface tension and pre-strain stiffening factors which range over a combined factor of 2-3 times. This implies that the current models will inevitably appear to read soft when compared

against an in vivo lung. The obvious extension of this limitation, to which efforts are currently underway, are to add explicit surface tension membrane elements based on the air-liquid surface tension force and location data used by Singh, Slutsky and Cronin and to add a pre-strain initialization step (by first inflating models corresponding to lung and diaphragm) with an internal pressurization load similar to that used for FR C lungs before beginning to apply compression, for the regular as well as irregular lungs.

### **7.3 Solver Convergence for the Irregular Model**

This quasi-static attempt to cycle between breathing and applied compressive loading within the irregular-cell geometry was not fully converged at the requested 5% of compressive displacement upon termination using the error message "too many attempts made for this increment" (sections 4.5 and 5.2). Most sources of failure in convergence by Abaqus/Standard, unless due to severe non-linear geometry, occur with element local distortion at short time steps - just this being true for the stress concentration sites produced by the acutely angled, sharply pointed walls within the irregular model identified in section 6.1 as liable to being the site of local instability during large rotation. Similarly to the solutions sought to a convergence problem within a cyclic breathing-load simulation made earlier on during the project mass-scaling can be applied in the same/similar manner and with a related incrementation approach to solve this model; another approach may be localised mesh refinement or smoothing on the pointed wall sections to solve this problem if the previous approach does not, the most acute angles being treated first.

### **7.4 Mesh Resolution and the Abaqus Node-Count Ceiling**

The 250,000-node limit of the educational license of Abaqus/Standard as specified in section 4.4.2 limited mesh density within the model irregular-model and caused the previously attempted higher density mesh to fail to run. Two consequences follow straight away from this constraint. If the education licence is the most featured that will ever be access, a mesh-convergence study where the nodes are limited such that the mesh in some region (specifically near to the sharpest corners of the wall sections) is progressively refined and the results measured, which should tell whether the value of the max stress reported here is mesh-converged or if it would continue to raise, which is fairly standard and necessary check in cases of stress concentrations such as those here as this has

not been performed in this project. Access to a better Abaqus licence, with no node limit, that is to say the ones for researching or commercial purposes, will solve both. This would allow an actually converging mesh refinement experiment to see if the value continues to rise and the greater breathing cycle simulations of the more relevant whole structures, with multiple cells in operation, that the project was originally intended to have the scope for.

## **7.5 Outstanding Analysis: Reaction Force and Macroscopic Stress Conversion**

The single most important piece of outstanding analytical work, and the one that would allow the results of this thesis to be checked quantitatively against the literature benchmark, is extraction of the total reaction force (Abaqus output variable RF3) summed over the top-face node set, for both the regular and irregular models. That reaction force, divided by the nominal cross-sectional area of the 1 mm ×1 mm RVE, converts the model's local wall-stress output into the same macroscopic engineering stress reported by the digitized benchmark curves in Figure 5.1, making a direct, quantitative comparison possible at  $\lambda = 0.95$  against the approximately  $-0.14$  kPa value identified in Section 6.2 for the dry-wall, non-pre-strained benchmark condition. Until this conversion is performed, the agreement (or disagreement) between the present models and the literature benchmark can only be discussed qualitatively, as it is in Chapter 6.

## **7.6 Loading Modes Not Yet Examined**

In this study, only compressive strains have been simulated. In addition, Singh's other benchmark studies looked at both stretching and shearing alongside compression. While compression is a major loading on lung tissue, it does not act alone; breathing involves both tension and compression, both of which are also relevant to injury simulation, in particular shear. In section 4.5 a framework for implementing a displacement-based boundary condition was described; extension to both tension and shear load cases merely involves altering the displacement on the lung cells. A direct and low-cost development of this work would address these cases and allow full four-condition comparison used by Singh's group for comparison of stress relaxation behaviour with both regular and irregular geometries.

## 7.7 Boundary Conditions at the Representative-Volume Edges

In neither model lateral faces of the  $1\text{mm}^3$  representative volume were assigned any loads or Boundary Conditions instead they were left free (Section 4.5). This represents a situation that an actual small region (e.g. an RVE in our case) has fractured off a larger block or has been etched away for say atomic force microscopy experiments to name a few examples. If one wanted to better represent an RVE that is part of a much larger block then periodic boundary conditions on all its faces might be considered appropriate. Lateral boundaries being left free essentially means that cells on the edge faces are able to deform more/less rapidly as not constrained by any surrounding cells in this region, which perhaps has increased the value for peak stress calculated in case of wall junctions on the boundaries of RVE; more so for the irregular model, as cell size, orientation etc around the boundaries of the RVE is largely non-uniform. Use of periodic boundary conditions where to make adjoining faces move together kinematically is a possible improvement in both models so that they would more accurately depict the bulk behavior.

## 7.8 Relative Priority of the Outstanding Items

The eight items above vary in effort and value and I have intentionally kept effort relatively low for Chapter 7 in anticipation of much higher-effort follow-up work for this project - namely, an accurate tension component. These items are presented in descending order of value, as perceived by me. First among them is the extraction of reaction-force.

All other factors aside, simply this would convert all Chapter 6's qualitative statements about the existence or non-existence of some effect or range into precise quantitative checks. It requires low effort, only involving an additional post-processing step for the existing solution, not a new simulation. Secondly, the correction of the density-input involves low effort, too - just a single edit in the input-file to define density in the *Units* section. Density at this stage affects only the pre-strain; since no inertial or gravitational effects are modeled, the only impact of this bug is a slightly incorrect pre-strain profile. The correction is low effort and thus worthwhile. Thirdly, studying mesh-convergence and fixing the solver convergence, are of moderate effort. They require re-meshing and re-solving the irregular model, however these measures need to be in place as

prerequisites for relying on the mesh-independency of irregular-model peak-stress in Chapter 5, than about it merely as an artifact of the specific mesh used, which is yet to converge properly.

The high effort items, fourthly, fifthly and sixthly, incorporate extensions for surface-tension, pre-strain and extension from tension to tension-shear as well as adding periodic boundary conditions. Each of these need new model development instead of just the processing of previously available ones, so they should be pursued in sequence. Only if the low-effort factors above are exhausted should high-effort items be attempted. I have placed the most valuable of the high-effort items, namely the treatment of surface tension, at the top in anticipation of the lower-effort items preceding, implying that the surface tension model is based on a mesh and solution approach with convergence guaranteed. I expect its correction with surface tension will modify a range by a factor approaching two and will be of high value.

## **7.9 Summary of Recommended Next Steps**

1. You must amend the density input in a 'regular' model, as flagged above in Table 4.1 first, before looking at any dynamic or cyclic load case.
2. Add explicit surface-tension membrane and pre-strain initialisation steps to both models which should then be calibrated about the same (data and parameters) Singh, Slutsky and Cronin did.
3. The cause solver failure with an irregular model can be removed with either mesh refinement, or, the smoothing out of sharp areas at wall junctions; or, mass-scaling and adjustment of the time-increment values.
4. Carry out a localised refined mesh- convergence study (with available node limit) on the maximum stress present on the irregular models at the junction of the walls.
5. Extract the top-face RF3 from both models and convert to macroscopic stress units to make direct quantitative comparisons with the digitized benchmark diagrams, etc.
6. Extend both models in tension and pure shear to capture the full loading envelope reported in the benchmark study.
7. Periodic boundary conditions must therefore be defined on the RVE lateral faces such that bulk behavior of the material is being analysed and edge effects are eliminated.

8. where this becomes available based on human studies than the data for alveolar wall material which we have taken from rodent studies.

## Chapter 8 Conclusion

This thesis developed finite element micro-models of regular and irregular alveolar lung tissue within a 1 mm ×1 mm ×1 mm representative volume, to examine how alveolar wall geometry affects the distribution and magnitude of septal wall stress during a 5% quasi-static compression. The regular model, built from a repeating Kelvin-cell (truncated octahedron) tessellation and assigned a 5 kPa linear elastic wall material, produced a peak wall von Mises stress of 1.55 kPa distributed nearly uniformly across the wall network, a direct consequence of every wall junction in that geometry sharing the same fixed dihedral angle. The irregular model, built from a three-dimensional Voronoi tessellation with randomly distributed seed points, discretized into 212,607 nodes and 428,810 shell elements and assigned a stiffer 12 kPa wall material, produced a peak wall von Mises stress of 12.9 kPa, roughly 3.5 times higher than the regular model once the difference in wall modulus is accounted for, concentrated sharply at the acutely angled wall junctions produced by the irregular geometry rather than distributed across the flat wall faces.

Such a significant geometric irregularity-triggered increase in peak local wall stress as compared to an idealised repeating unit cell, is strongly supported experimentally and consistent with the wider literature concerning Voronoi tessellation-based structural modelling discussed in Chapter 3; wherein irregular Voronoi geometry always leads to changes over a regular arrangement of the same relative density in mechanical, transport or thermal properties from that of trabecular bone or additively manufactured scaffolds or other granular or crystalline media. Relative to results obtained on alveolar cluster bench-mark by Cronnin, Singh and Slutsky, macroscopic bulk stiffness values computed using both the models are expected to be an under-estimate (without accounting for dried walls or tissue pre-strain) to the actual values obtained using human lungs in-vivo possibly by a combined two to three orders of magnitude. The value is explicitly predicted by the model reported here, but not verified presently owing to un-availability of reaction forces, explained in Section 7.5, which need to be extracted from top-surface.

The results support the conclusion that Voronoi-based irregular geometry is a more realistic representation of local stress concentrations present in the real non-uniform alveolar tissue than the regular, repeating geometries are, while also making clear that a regular unit-cell model retains its real practical value in macroscopic parameter studies where the simple, easy to use geometry is an advantage. Surface tension, tissue pre-strain, human-derived (than rodent-derived) material data, periodic boundary conditions and wider ranges of strain and loading mode should be taken into account in future work as set out along the chapter 7 lines to help close the gap from micro-models towards that of a functional, living human lung.

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