It is Taxing to be Coherent

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Abstract

The presence of an occasionally binding constraint due to the effective lower bound (ELB) in New Keynesian models generally gives rise to multiple equilibria under active monetary policy. To restore uniqueness in the model with an active Taylor rule, we consider appropriate simple fiscal policy instruments. Without relaxing the assumptions of Ricardian equivalence, full information, and rational expectations, we show that appropriate fiscal targeting rules ensure that New Keynesian models subject to the ELB possess a unique solution.

Keywords: fiscal policy, uniqueness, multiplicity, rational expectations, effective lower bound
JEL Codes: C62, E4, E61, E62, E63

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1. Introduction

The canonical New Keynesian (NK) model with an occasionally binding constraint arising from the effective lower bound (ELB), with full information and rational expectations (FIRE) and an active Taylor rule[1] (TR), possesses multiple minimum state variable (MSV) solutions. Thus, it is termed as "incomplete". Furthermore, when subjected to significant shocks, an MSV solution may not exist, rendering it "incoherent". These crucial findings were demonstrated in seminal works by Ascari and Mavroeidis (2022) (AM) in a stochastic environment with rational expectations and Holden (2023) under perfect foresight.

This paper, maintaining FIRE, demonstrates that simple Ricardian fiscal policy (FP) ensures coherency and completeness. This holds robustly across shocks of varying sizes and supports. Our key finding reveals that persistent and reactive FP to inflation and output fluctuations guarantees a unique MSV solution, while also satisfying Blanchard-Kahn (BK) local determinacy conditions. This paper identifies two critical properties for achieving uniqueness of an MSV solution. Firstly, at the ELB, FP stabilises the economy when monetary policy is constrained, establishing an equilibrium path. Secondly, a countercyclical rule-based FP eliminates belief-driven equilibria when it follows adequate persistence.

To underscore the significance of this paper – and the relevance of model coherency and completeness – it is crucial to contextualise the primary contributions of AM and the literature. While previous studies often employed simplified approaches regarding shocks in models with the ELB, such as assuming a singular structural shock or imposing rigid assumptions on shock duration, AM consider multiple structural shocks and their serial occurrences within a forward-looking dynamic model with FIRE. Although this paper abstracts from multiple structural shocks, it examines the recurring structural shock scenario.

Building on the work of Gourieroux, Laffont, and Monfort (1980) (GLM),

1. An active Taylor rule is one which satisfies the Taylor principle.

AM derive two main results using a linearised equation system and endogenous regime switching. Firstly, they demonstrate that achieving coherency in ELB-constrained NK models poses a nontrivial challenge, particularly when the inflation coefficient in the TR exceeds unity or when optimal monetary policy under discretion does not ensure coherency. Additionally, AM identify conditions that restrict the support of stochastic shocks, necessary to ensure model coherency. However, these support restrictions prove cumbersome, dependent on model structural parameters and past realisations of state variables in backward-looking models. Secondly, even with support restrictions to ensure coherency, the model might still exhibit multiple MSV solutions, potentially up to 2^k solutions, where k represents the number of discrete shock states (Rouwenhorst, 1995).

This concern extends beyond the conventional scope of the ELB literature, which mainly examined sunspot shocks or belief-driven fluctuations between steady states.[2] However, general conditions to ensure model coherency and completeness in macroeconomic DSGE literature remain limited, although recent papers have provided sufficiency conditions for MSV equilibrium existence in NK models (Eggertsson, 2011; Christiano, Eichenbaum, and Johannsen, 2018; Nakata, 2018; Nakata and Schmidt, 2019). Compared to this strand of literature, this paper studies solution existence and uniqueness.

As highlighted in follow-up work,Ascari, Mavroeidis, and McClung (2023) show that multiplicity of MSV solutions emerges from the interplay between rational expectations and the inherently nonlinear nature of the ELB constraint. While they focus on relaxing FIRE assumptions, this paper maintains the FIRE framework and proposes alternative mechanisms, specifically emphasising the role of simple Ricardian FP, to address issues identified by AM.

Our objective is to offer qualitative results in resolving the problem of MSV solution multiplicity using appropriate FP. Thus, our paper adds to the

2. See, for example, Eggertsson and Woodford (2003), Guerrieri and Iacoviello (2015), Kulish, Morley, and Robinson (2017), Aruoba, Cuba-Borda, and Schorfheide (2018), Aruoba et al. (2021), and Angeletos and Lian (2023).

studies that explored fiscal policy, the ELB, and multiple equilibria interactions. Seminal work by Benhabib, Schmitt-Grohé, and Uribe (2001) examined how Ricardian FP with active monetary policy leads to unique convergence to a steady state equilibrium. However, convergence was not always to a unique steady state and could include an unintended liquidity trap steady state. Benhabib, Schmitt-Grohé, and Uribe (2002) extended this to establish convergence to a non-liquidity trap steady state. Both studies assumed perfect foresight environments, while this paper maintains FIRE.[3]

There is a vast literature exploring the interaction between FP and the ELB. Prominent theoretical contributions include Schmidt (2016), Tamanyu (2021), and Nakata and Schmidt (2022), which showcased how expectations-driven liquidity traps could be avoided with appropriate FP, emphasising fiscal rule variations. Meanwhile, an example of a more policy-focused contribution is Correia et al. (2013) which showed that distortionary tax policy can perfectly replicate the unique rational expectations equilibrium without the ELB constraint. While these results were quantitatively demonstrated in a perfect foresight environment with agents making expectation errors, our work – using a textbook New Keynesian setup – encompasses the mechanisms of their basic model as a special case.

It is notable that the aforementioned literature on the ELB and FP primarily focused on model completeness or the elimination of a liquidity trap steady state, often assuming restrictions on the shock process or stochastic environment. Our primary contribution is to simultaneously consider coherence, completeness, and local determinacy (BK conditions) concerning the ELB and FP instruments. Additionally, despite the paper delving into fiscal and monetary policy interactions,[4] it refrains from examining fiscal policy potency or fiscal multipliers at the ELB.

The paper proceeds as follows: Section 2 provides an overview of CC

3. See Definition 3 and Propositions 5 and 6 of Benhabib, Schmitt-Grohé, and Uribe (2001).

4. This literature is vast – see, for example, Galí, López-Salido, and Vallés (2007), Davig and Leeper (2011), Eggertsson and Krugman (2012), Billi and Walsh (2022), and Hills and Nakata (2018).

conditions within the context of an ELB-bound NK model and describes the methodology used to verify CC conditions. Section 3 demonstrates how Ricardian FP restores coherency and completeness in a purely forward-looking reference NK model constrained by the ELB. Section 4 assesses CC conditions for an NK model with FP featuring policy inertia. Finally, Section 5 concludes the paper.

2. Verifying Coherency and Completeness of the New Keynesian Model with the ELB

In this section, we provide a sketch of the AM's methodology to verify coherency and completeness of systems of linear equations, applying the methodology to the textbook NK model subject to the ELB. Further explanation and derivation can be found in AM or Appendix A.

General Verification for Linear Models. Let $\boldsymbol{Y}_t$ be a $n \times 1$ vector of endogenous variables, $\boldsymbol{X}_t$ be a $n_x \times 1$ vector of exogenous state variables, and $s_t \in \{0, 1\}$ be an indicator variable that is equal to 1 when some inequality constraint is slack and 0 otherwise.[5]

Coherency requires that there exist some function $f(\cdot)$ such that an MSV solution can be represented as $\boldsymbol{Y}_t = f(\boldsymbol{X}_t)$. Assume that the exogenous states $\boldsymbol{X}_t$ are k-state stationary first-order Markov processes with transition kernel $\boldsymbol{K}$. Stack the possible states of $\boldsymbol{X}_t$ for states $i = 1, .., k$ into a $n_x \times k$ matrix $\mathbf{X}$. Then, let $\boldsymbol{e}_i$ denote the i-th column of the $k \times k$ identity matrix $\boldsymbol{I}_k$, such that $\mathbf{X}\boldsymbol{e}_i$, the i-th column of $\mathbf{X}$, is the i-th state of $\boldsymbol{X}_t$.[6] Then define $\mathbf{Y}$ as an $n \times k$ matrix whose i-th column, $\mathbf{Y}\boldsymbol{e}_i$, corresponds to $\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i$ along an MSV solution. For Sections 2 and 3, we abstract from models that feature endogenous state

5. Additionally, let $\boldsymbol{\Omega}_t$ denote the information set, thus allowing us to write: $\boldsymbol{Y}_{t+1|t} = \mathbb{E}_t[\boldsymbol{Y}_{t+1}|\boldsymbol{\Omega}_t]$ and $\boldsymbol{X}_{t+1|t} = \mathbb{E}_t[\boldsymbol{X}_{t+1}|\boldsymbol{\Omega}_t]$.

6. The elements of the transition kernel $\boldsymbol{K}$ are $\boldsymbol{K}_{ij} = \Pr(\boldsymbol{X}_{t+1} = \mathbf{X}\boldsymbol{e}_j | \boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i)$ and hence, $\mathbb{E}_t[\boldsymbol{X}_{t+1}|\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i] = \mathbf{X}\boldsymbol{K}^\top \boldsymbol{e}_i$.

variables.[7] Thus, along an MSV solution we have:

$$\mathbb{E}[\boldsymbol{Y}_{t+1}|\boldsymbol{Y}_t = \mathbf{Y}\boldsymbol{e}_i] = \mathbb{E}_t[\boldsymbol{Y}_{t+1}|\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i] = \mathbf{Y}\boldsymbol{K}^\top \boldsymbol{e}_i. \tag{1}$$

This allows us to write state-space models, and thus DSGE models, in the form:

$$\begin{aligned} \mathbf{0} &= (\boldsymbol{A}_{s_i}\mathbf{Y} + \boldsymbol{B}_{s_i}\mathbf{Y}\boldsymbol{K}^\top + \boldsymbol{C}_{s_i}\mathbf{X} + \boldsymbol{D}_{s_i}\mathbf{X}\boldsymbol{K}^\top)\boldsymbol{e}_i, \\ s_i &= \mathbb{1}\left(\left[\boldsymbol{a}^\top\mathbf{Y} + \boldsymbol{b}^\top\mathbf{Y}\boldsymbol{K}^\top + \boldsymbol{c}^\top\mathbf{X} + \boldsymbol{d}^\top\mathbf{X}\boldsymbol{K}^\top\right]\boldsymbol{e}_i > 0\right),\ i = 1, \dots, k, \end{aligned} \tag{2}$$

where $\boldsymbol{A}_{s_t}$, $\boldsymbol{B}_{s_t}$, $\boldsymbol{C}_{s_t}$, and $\boldsymbol{D}_{s_t}$ are coefficient matrices with dimensions $n \times n$, $n \times n$, $n \times n_x$, and $n \times n_x$, respectively; $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are coefficient vectors, and $\mathbb{1}(\cdot)$ is an indicator function that is equal to 1 if its argument holds true and 0 otherwise.

The system (2) relates $\mathbf{Y}$ to $\mathbf{X}$, and can be expressed as $F(\mathbf{Y}) = \lambda(\mathbf{X})$, where $\lambda(\cdot)$ is some function of $\mathbf{X}$, and $F(\cdot)$ is a piecewise linear continuous function of $\mathbf{Y}$. If $J \subseteq \{1, \dots, k\}$, then the piecewise linear function $F(\mathbf{Y})$ can then be expressed as:

$$F(\mathbf{Y}) = \sum_J \mathcal{A}_J \boldsymbol{S}_{\mathfrak{C}_J}\mathrm{vec}(\mathbf{Y}), \tag{3}$$

where $\mathfrak{C}_J = \{\mathbf{Y} : \mathbf{Y} \in \mathbb{R}^{n \times k}, s_i = \mathbb{1}(i \in J)\}$ is given by a configuration of regimes over the k states given by J, $\boldsymbol{S}_{\mathfrak{C}_J}$ is a $nk \times nk$ matrix of indicator elements,[8] and $\mathrm{vec}(\cdot)$ is the vector operator function.[9] In words: $\mathcal{A}_J$ and $\mathfrak{C}_J$ are such that if $F(\mathbf{Y})$ in (3) is invertible, then the linear system is coherent and complete. Put another way, there exists a unique MSV solution, as stipulated in GLM, if all the determinants of $\mathcal{A}_J$, $J \subseteq \{1, \dots, k\}$ share the same sign. Failure of this requirement implies that the model is generally incoherent and/or incomplete:

7. We revisit CC conditions for models with endogenous states in Section 4, where we study the baseline NK model with persistent FP rules.

8. Note that when $\mathbf{Y}_{n \times k}$ is vectorised, and if $k = 2$, the first n elements correspond to state 1 and the last n elements correspond to state 2. Thus, in essence, the elements of $\boldsymbol{S}_{\mathfrak{C}_J}$ map the entries of $\mathcal{A}_J$ for the k states to the vectorised set of endogenous variables in $\mathbf{Y}$.

9. The transformation of (2) into (3) is generally non-trivial (in which the expressions of $\mathcal{A}_J$ require Kronecker product operations) as it presents a Sylvester equation in $\mathbf{Y}$. See, for example, Kolmogorov and Fomin (1957). However, there are two exceptions that allow straightforward computation of the $\mathcal{A}_J$: $n = 1$ and $n = k > 1$. We make use of this simplifying assumption both in this example and the analytical derivation in Appendix A.

THEOREM 1 (GLM). *Suppose that the mapping* $F(\cdot)$ *defined in* (3) *is continuous. A necessary and sufficient condition for* $F(\cdot)$ *to be invertible is that all the determinants* $\det \mathcal{A}_J, J \subseteq \{1, \ldots, k\}$ *have the same sign.*

An application of GLM Theorem 1 to the simple Fisherian model in Aruoba, Cuba-Borda, and Schorfheide (2018) can be found in Appendix A.1. Below we provide an application to a textbook NK model.

A Reference New Keynesian Model with the ELB. Consider the canonical NK model as set out in, for example, Galí (2015). The model in its log-linearised form with the ELB can be written in three equations, the dynamic IS equation (DISE), New Keynesian Phillips Curve (NKPC), and the TR:[10]

$$\text{DISE: } \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}) + \varepsilon_t, \tag{4a}$$

$$\text{NKPC: } \hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t, \tag{4b}$$

$$\text{TR: } \hat{i}_t = \max\left\{-\mu, \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right\}, \tag{4c}$$

and where ε_t is a demand shock. Furthermore, $\hat{y}_t$ is the output gap, $\hat{\pi}_t$ is inflation, and $\hat{i}_t$ is the nominal interest rate.[11] The parameters of interest in the model are: σ, the coefficient of relative risk aversion; β, the representative household's subjective discount factor; κ, the slope of the NKPC; $\mu = \log(r\pi^*)$, the ELB of the nominal interest rate in deviation from the steady state, where $r = 1/\beta$ is the steady state gross real interest rate and π^* is the gross inflation target of the monetary authority; ϕ_y, the monetary authority's response parameter to output fluctuations; and ϕ_π, the monetary authority's responsiveness to inflation.

When the constraint on $\hat{i}_t$ is binding, the system can be rewritten as

10. To keep the analysis simple, we omit cost-push shocks in the NKPC and monetary policy shocks in the TR.

11. Hatted variables denote a variable in terms of log deviations from steady state. In other words, for any generic variable, say, X, we have:

$$\hat{x}_t = \ln X_t - \ln \bar{X} \approx \frac{X_t - \bar{X}}{\bar{X}},$$

where $\bar{X}$ is the value of X_t in the non-stochastic steady state.

follows

$$\begin{pmatrix} 1 & -\kappa \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t \\ \hat{y}_t \end{pmatrix} + \begin{pmatrix} -\beta & 0 \\ -\frac{1}{\sigma} & -1 \end{pmatrix} \begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ \varepsilon_t \end{pmatrix} = 0 \tag{5}$$

Whilst when the constraint is slack the system is given by

$$\begin{pmatrix} 1 & -\kappa \\ \frac{\phi_\pi}{\sigma} & 1 + \frac{\phi_y}{\sigma} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t \\ \hat{y}_t \end{pmatrix} + \begin{pmatrix} -\beta & 0 \\ -\frac{1}{\sigma} & -1 \end{pmatrix} \begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ \varepsilon_t \end{pmatrix} = 0 \tag{6}$$

The model can then be cast in the canonical form as in (2). To check whether the model satisfies the CC conditions, it is sufficient to check the invertability of $F(\cdot)$ by ensuring that the signs of $\det \mathcal{A}_{J_1}$ and $\det \mathcal{A}_{J_4}$ are identical. Assuming $p = q = 1$ yields

$$\det \mathcal{A}_{J_1} = \det \begin{pmatrix} 1-\beta & -\kappa \\ \frac{\phi_\pi - 1}{\sigma} & \frac{\phi_y}{\sigma} \end{pmatrix} = \frac{(1-\beta)\phi_y + \kappa(\phi_\pi - 1)}{\sigma} > 0, \tag{7}$$

$$\det \mathcal{A}_{J_4} = \det \begin{pmatrix} 1-\beta & -\kappa \\ -\frac{1}{\sigma} & 0 \end{pmatrix} = -\frac{\kappa}{\sigma} < 0. \tag{8}$$

We observe that the signs of $|\mathcal{A}_{J_1}|$ and $|\mathcal{A}_{J_4}|$ differ, which implies that the model is not generally coherent under an active TR with $\phi_\pi > 1$ and $\phi_y \geq 0$.

We can graphically represent the CC conditions for the canonical NK model by first considering the absorbing state of the model for when $\varepsilon_t = 0$. In the absorbing state we have $\hat{\pi}_t = \hat{\pi}_{t+1} = \hat{\pi}$ and $\hat{y}_t = \hat{y}_{t+1} = \hat{y}$. Hence, the NKPC can be written as the following aggregate supply (AS) relation:

$$\hat{\pi} = \frac{\kappa}{1-\beta}\hat{y} \quad AS. \tag{9}$$

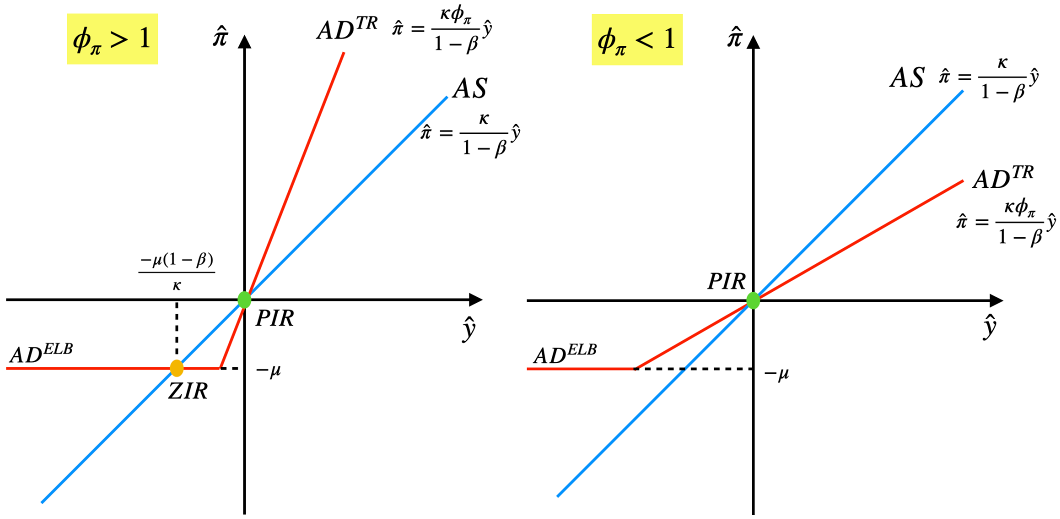
Meanwhile, the DISE can be written and rearranged to give a piecewise aggregate demand (AD) relation:

$$\hat{\pi} = \begin{cases} \frac{\kappa\phi_\pi}{1-\beta}\hat{y} & AD^{TR}, \\ -\mu & AD^{ELB}. \end{cases} \tag{10}$$

Clearly, the model admits two absorbing states: a PIR equilibrium, $\{\hat{\pi}, \hat{y}, \hat{i}\} = \{0, 0, 0\}$, and a ZIR equilibrium, $\{\hat{\pi}, \hat{y}, \hat{i}\} = \{-\mu, -\frac{\mu(1-\beta)}{\kappa}, -\mu\}$, which we can graphically see by plotting (9) and (10) as in Figure 1.

The left plot of Figure 1 shows the incompleteness problem when the

FIGURE 1. Absorbing State of the New Keynesian Model ($\varepsilon_t = 0$)



Note: Diagram on the left depicts equilibria when the Taylor principle is adhered to. Diagram on the right shows the PIR absorbing state for an interest rate rule that does not satisfy the Taylor principle.

NK model features an active TR; well studied in the literature. Absent of any shocks, the model implies two equilibria as the slope of AD^{TR} is steeper than that of AS. By contrast, when $\phi_\pi < 1$, as in the right plot of Figure 1, a unique equilibrium exists as the AD and AS curves intersect once. However, as is well known, a passive TR leads to issues with local model dynamics (Blanchard and Kahn, 1980).

Now consider the transitory state for when $\varepsilon_t = \frac{p}{\sigma}\hat{r}^T$. For simplicity, we assume that the shock is transitory and occurs once, in other words $q = 1$. As the model is completely forward looking, the economy remains in the transitory state for some indefinite period of time after which it jumps to an absorbing state – either a PIR or ZIR equilibrium.

PIR absorbing state. At time t the economy is in a transitory state. With probability p the economy remains in the transitory state $(\hat{y}^T, \hat{\pi}^T)$; with complimentary probability $1 - p$ the economy moves to the PIR absorbing state. Thus, the *AS* and *AD* relations can be written as

$$\hat{\pi}^T = \frac{\kappa}{1-p\beta}\hat{y}^T \quad AS, \tag{11a}$$

$$\pi^T = \begin{cases} \frac{\sigma(1-p)}{p-\phi_\pi}\hat{y}^T - \frac{p}{p-\phi_\pi}\hat{r}^T & AD^{TR} \text{ for } \hat{\pi}^T \geq -\frac{\mu}{\phi_\pi}, \\ \frac{\sigma(1-p)}{p}\hat{y}^T - \frac{\mu}{p} - \hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi_\pi}. \end{cases} \tag{11b}$$

ZIR absorbing state. Here we repeat the above exercise but for when the absorbing state is a ZIR equilibrium. As before, at time t the economy is in a transitory state, and with probability p it remains in the transitory state, and with probability $1-p$ it transitions to the ZIR absorbing state. As previously mentioned, the absorbing state here now differs in value from the PIR case, and as such, the *AS* and *AD* relations can be written as:

$$\hat{\pi}^T = \frac{\kappa}{1-p\beta}\hat{y}^T - \frac{\beta(1-p)}{1-p\beta}\mu \quad AS, \tag{12a}$$

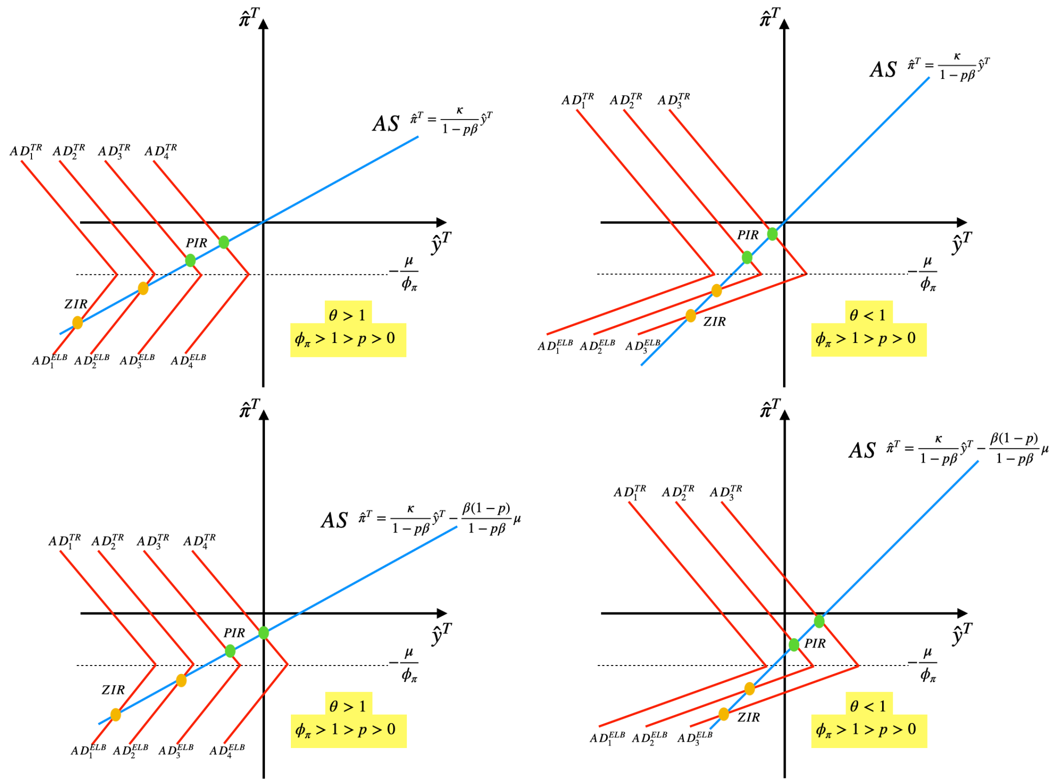
$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{p-\phi_\pi}\hat{y}^T + \frac{1-p}{p-\phi_\pi}\left[\frac{(1-\beta)}{\kappa}+1\right]\mu - \frac{p}{p-\phi_\pi}\hat{r}^T & AD^{TR} \text{ for } \hat{\pi}^T \geq -\frac{\mu}{\phi_\pi}, \\ \frac{\sigma(1-p)}{p}\hat{y}^T + \frac{1-p}{p}\left[\frac{(1-\beta)\sigma}{\kappa}+1\right]\mu - \frac{\mu}{p} - \hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi_\pi}, \end{cases} \tag{12b}$$

respectively. To complete the description of this simple example, we define θ as the ratio of the slopes of the AD^{ELB} and *AS* relations:

$$\theta = \frac{\sigma(1-p)(1-p\beta)}{p\kappa}. \tag{13}$$

Figure 2 then plots the *AS* and *AD* when monetary policy adheres to the Taylor principle ($\phi_\pi > 1$) for either a PIR or ZIR absorbing state when the economy is subject to the shock term ε_t. The plots on the left hand side of Figure 2 are for the case of $\theta < 1$, i.e., when *AD* is flatter relative to *AS*. The plots on the right in Figure 2 are for the case where $\theta > 1$, i.e., when *AD* is more steep than *AS*. The different values for θ correspond to different values of p. Namely, the plots shown in Subfigure 2B with $\theta < 1$ are for higher values of p than those generated for the case where $\theta > 1$. Additionally, the higher value of p corresponds to a higher probability that the model remains in a transitory state each period.

FIGURE 2. Transitory States of the New Keynesian Model ($\phi_\pi > 1$)



A. PIR (top) and ZIR (bottom) absorbing states ($\theta > 1$)

B. PIR (top) and ZIR (bottom) absorbing states ($\theta < 1$)

As discussed by AM, and shown in Figure 2, for the case where $\theta > 1$, the only support restriction necessary for an MSV solution to exist in the absorbing state is $(r\pi^*)^{-1} \leq 1$. But for when $\theta \leq 1$, the necessary support restriction becomes:

$$\frac{1}{r\pi^*} = 1 \text{ and } -\hat{r}^L \leq \log(r\pi^*)\left(\frac{\phi\pi - p}{\phi\pi\, p} + \frac{\theta}{\phi\pi}\right). \tag{14}$$

To put it simply, these support restrictions ensure that a negative shock to AD does not lead it shifting too far to the left or above of AS, as shown in $AD_1^{TR,ELB}$ of Subfigure 2B.

Derivations and further explanation can be found in Appendix A, or interested readers can refer to AM for more detail. We emphasise that non-uniqueness of equilibria in the baseline NK model is driven by exogenous uncertainty captured by p. We proceed with analysing how simple Ricardian FP can counteract the effects of uncertainty and restore coherency and completeness.

3. Fiscal Policy and Coherency and Completeness

In this section, we show how Ricardian fiscal policy that consists of government spending can render a baseline NK model subject to the ELB coherent and complete.

Model. We augment the baseline NK model with a simple FP setup following Woodford (2011). The model is otherwise standard, and derivation is given in Appendix B. In what follows, we show that under simple fiscal feedback rules, the model can generate a unique MSV solution in the presence of the ELB under certain restrictions on FP. The model is described by the DISE, NKPC, TR, government budget constraint, and the natural rate given by:

$$\hat{x}_t = \mathbb{E}_t\hat{x}_{t+1} - \frac{c}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}^n_t), \tag{15a}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa_y\hat{x}_t + \frac{\epsilon}{\Phi}\left(\Psi^w\hat{\tau}^w_t - \Psi^c\hat{\tau}^c_t - \frac{1}{\epsilon}\hat{\tau}^s_t - \frac{\sigma}{c}\hat{g}_t\right), \tag{15b}$$

$$\hat{i}_t = \max\{-\mu, \phi_\pi\hat{\pi}_t + \phi_y\hat{x}_t\}, \tag{15c}$$

$$g\hat{g}_t = \frac{T}{Y}\hat{\tau}_t + \tau^c c(\hat{\tau}^c_t + \hat{c}_t) + \tau^s(\hat{\tau}^s_t + \hat{x}_t) + \tau^w(\hat{\tau}^w_t + \hat{l}_t), \tag{15d}$$

$$\hat{r}^n_t = -\mathbb{E}_t\Delta\hat{\tau}^c_{t+1} - \frac{\sigma g}{c}\mathbb{E}_t\Delta\hat{g}_{t+1} - \frac{\sigma}{c}\mathbb{E}_t\Delta\hat{z}_{t+1}, \tag{15e}$$

where $c = C/Y$ is the steady-state consumption-output ratio, $g = G/Y$ is the steady-state government expenditure-output ratio, and κ_y and $\kappa_g = \frac{\epsilon\sigma}{c\Phi}$ denote slopes of the NKPC and coefficient on government expenditure, g_t, respectively. Additionally, consumption taxes are τ^c_t, labour income taxes are τ^w_t, production taxes are τ^s_t, and z_t are household preference shocks.

The model is closed with a rule for government expenditure of the form

$$\mathbb{E}_t\hat{g}_{t+1} = \rho_g\hat{g}_t + \psi_\pi\hat{\pi}_t + \psi_y\hat{x}_t, \tag{16}$$

where ψ_π and ψ_y denote the degree of reaction of taxes to deviations of in-

TABLE 1. Model Calibration

Parameter	Value	Description
σ	2	Coefficient of relative risk-aversion
φ	2/3	Frisch elasticity of labour supply
β	0.99	Discount factor
τ^c	1/4	Steady-state level of fiscal instrument
κ_y	0.23	Slope of NKPC
Ψ^c	1/3	Coefficient on fiscal instrument
γ	3/4	Calvo probability
ϵ	10	Elasticity of substitution between goods
c	3/4	Fraction of consumption in output
g	1/4	Fraction of government spending in output
ϕ_π	1.5	Weight on inflation, Taylor rule
ϕ_y	0.2	Weight on output gap, Taylor rule

flation and the output gap, respectively. Throughout this section, we assume that the rule is "fully inertial", that is $\rho_g = 1$.[12]

Calibration. In what follows, for all quantitative results the model is calibrated according to the values in Table 1 unless specified otherwise. These parameter values are standard in the NK DSGE literature.

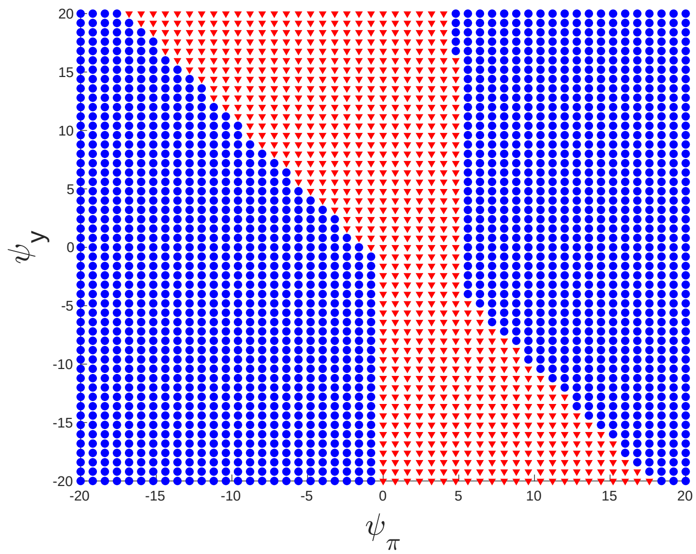
3.1. Permanent Fiscal Policy Change

Assume that fiscal expenditure is financed by a mix of labour income, output, and lump-sum taxes such that the effects of fiscal policy are offset in the NKPC: $\Psi^w\hat{\tau}^w_t - \frac{1}{\epsilon}\hat{\tau}^s_t = \frac{\sigma}{c}\hat{g}_t$ and $\hat{\tau}^c_t = 0$.[13] Thus, government spending only directly affects aggregate demand. We have the following proposition:

12. This assumption allows us to check if the model is coherent and complete analytically. We relax this assumption in Section 4.

13. Absence of direct supply-side effects allows for analytical derivation of CC conditions. Government expenditure in the NKPC could be offset using a different combination of taxes, for instance $\tau^c = \tau^w = 0$ and $-\hat{\tau}^s_t/\epsilon = (\sigma/c)\hat{g}_t$. Equivalently, this would also be the case under preferences as in Greenwood, Hercowitz, and Huffman (1988) or inelastic labour supply.

FIGURE 3. Coherency and Completeness Region for Inflation and Output Gap Fiscal Rule

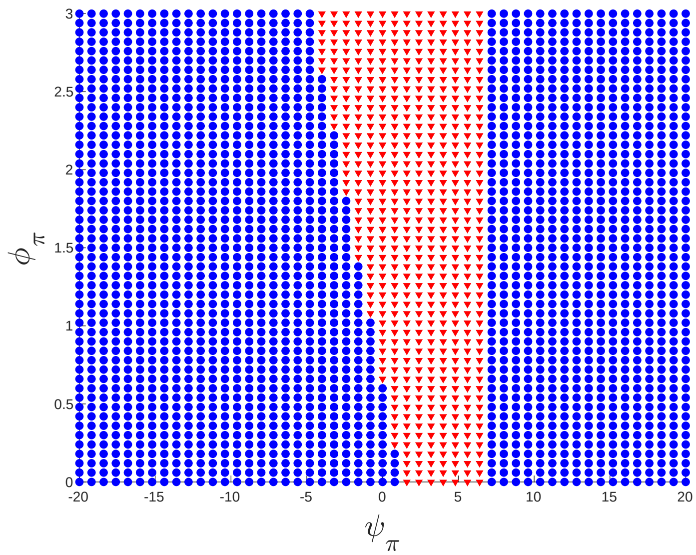


Blue circles denote regions where coherency and completeness conditions are satisfied. Red triangles denote region where the model is either incoherent or incomplete.

PROPOSITION 1. *A baseline New Keynesian model with fiscal policy that consists of government spending, output taxes, labour income taxes, and lump-sum taxes as defined in* (15)*, is generally coherent and complete when the reaction of fiscal policy to inflation and the output gap, ψ_π and ψ_y, respectively, as described in the fiscal rule Equation* (16)*, is sufficiently strong.*

Proof of Proposition 1 is provided in Appendix B.2. The region for which the model satisfies the CC conditions as a function of the fiscal authority's reaction parameters, ψ_π and ψ_y, are shown in Figure 3. Using our baseline calibration, we see that the model generally satisfies the CC conditions in the negative orthant of ψ_π and ψ_y space, $\mathbb{R}^2_-$, and when ψ_π is sufficiently large. The intuition for this is that a strong enough reaction on the part of the fiscal authority to inflation and output deviations leads to a unique MSV solution by ensuring an intersection between *AD* and *AS*. We illustrate this for a simple case in Figure 5. Furthermore, we note that the degree of reaction on the part of the fiscal authority to the output gap is largely irrelevant as to whether

FIGURE 4. Coherency and Completeness Region for Simple Inflation Targeting Fiscal Rule (Equation (17))



Blue circles denote regions where coherency and completeness conditions are satisfied. Red triangles denotes region where the model is either incoherent or incomplete.

or not the model satisfies the CC conditions.

As such, in order to clarify our intuition and understand the mechanism driving requirements of the CC conditions, we focus on a simplified version of (16) where the fiscal authority strictly targets inflation ($\psi_y = 0$) with the following rule:

$$\mathbb{E}_t \Delta \hat{g}_{t+1} = \psi_\pi \hat{\pi}_t. \tag{17}$$

We then plot the region for which the model satisfies the CC conditions, for parameters of ϕ_π and ψ_π as shown in the Figure 4. We emphasise two points: with the simple FP rule in Equation (17), the model satisfies the CC conditions despite the monetary authority adopting an active TR. This addresses one of the major concerns surrounding NK models subject to the ELB as raised by AM. Secondly, in order to satisfy the CC conditions, the fiscal authority must respond strongly to inflation.

Consider the simple FP rule, Equation (17), and the absorbing state case with $\varepsilon_t = 0$. The *AS* curve takes a similar form as in Equation (9), and the *AD* curve is piecewise linear, giving us the following system:

$$\hat{\pi} = \frac{\kappa_y}{1-\beta}\hat{x} \quad AS, \tag{18a}$$

$$\hat{\pi} = \begin{cases} (\phi_\pi + \psi^*)\frac{\kappa_y}{1-\beta}\hat{x} & AD^{TR} \text{ for } \hat{\pi} > -\mu, \\ \psi^* \frac{\kappa_y}{1-\beta}\hat{x} - \mu & AD^{ELB} \text{ for } \hat{\pi} < -\mu, \end{cases} \tag{18b}$$

where

$$\psi^* = \frac{\sigma\psi_\pi g}{c}. \tag{19}$$

The PIR equilibrium is trivial and is given, as before, by $\{\hat{\pi}, \hat{x}, \hat{i}\} = \{0, 0, 0\}$. The ZIR equilibrium here is $\{\hat{\pi}, \hat{x}, \hat{i}\} = \left\{\frac{\mu}{\psi^*-1}, \frac{(1-\beta)\mu}{\kappa_y(\psi^*-1)}, -\mu\right\}$.

PIR absorbing state with active FP ($\psi^* > 1$ and $\psi^* < -\phi_\pi$). [14] If $\psi^* > 1$ or $\psi^* < -\phi_\pi$, the economy cannot be at the ELB in the absorbing state. That is, for an inflation level that is higher than the lower bound on the nominal interest rate, $\hat{\pi} > -\mu$, the nominal interest rate as per the TR is unconstrained, and so the following equality – obtained by substituting (18a) into AD^{ELB} in (18b) – implies an inflation rate that is higher than $-\mu$:

$$\hat{\pi} = \frac{\mu}{\psi^* - 1}.$$

Thus, inflation cannot be at its ELB steady state level, and by implication the nominal interest rate cannot be at the ELB. Hence, no ZIR equilibrium can exist with an active FP rule as in Equation (17).

Absorbing states with passive FP ($\psi^* < 1$ and $0 < \psi^* + \phi_\pi < 1$). Here, the ZIR equilibrium is consistent with the ELB constraint on the nominal interest rate since implied inflation is less than or equal to $-\mu$. Passive FP also implies that the slope of AD^{TR} is flatter than that of *AS*, hence there are two absorbing – both the PIR and ZIR equilibria. Moreover, a passive FP rule implies that in

14. Our use of "active" and "passive" to describe FP should not be confused with the more conventional use of these terms established by Leeper (1991) to describe monetary and fiscal policy interactions.

a ZIR equilibrium the inflation rate is lower than that of the ZIR equilibrium with no fiscal rule. Figure 5 illustrates the two cases.

We proceed with analysing the transitory equilibria with $\varepsilon_t = \frac{p}{\sigma}\hat{r}^T$ in each of the cases. As before, the economy remains in a transitory state for an indefinite amount of time, before transitioning to an absorbing state thereafter.

Transitory states with active FP ($\psi^* \notin (-\phi_\pi, 1)$). With probability p, the economy remains in a transitory state and jumps to the PIR absorbing state with complimentary probability $1 - p$. AS and AD are given by

$$\hat{\pi}^T = \frac{\kappa_y}{1 - p\beta}\hat{x}^T \; AS \tag{20a}$$

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{c(p-\phi_\pi-\psi^*)}\hat{x}^T - \frac{p}{c(p-\phi_\pi-\psi^*)}\hat{r}^T & AD^{TR} \text{ for } \hat{\pi}^T \geq -\frac{\mu}{\phi_\pi}, \\ \frac{\sigma(1-p)}{c(p-\psi^*)}\hat{x}^T - \frac{\mu}{p-\psi^*} - \frac{p}{c(p-\psi^*)}\hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi_\pi}. \end{cases} \tag{20b}$$

When $\psi^* > 1$, AD^{TR} and AD^{ELB} have negative slope. The model thus satisfies the CC conditions. When $\psi^* < -\phi_\pi$, the slopes of AD^{TR} and AD^{ELB} are positive and AD^{ELB} is flatter than AS. This implies a unique transitory equilibrium.

Transitory states with passive FP ($\psi^* \in (-\phi_\pi, 1)$). In the case where $\psi^* \in (-\phi_\pi, 1)$, there are two potential absorbing states. In the PIR equilibrium, we have the system as above. The slope of AD^{TR} is negative, while the slope of AD^{ELB} can either be positive or negative. Hence, for some values of p, the slope of AD^{ELB} can be flatter than that of AS, which implies incoherency or incompleteness in absence of support restrictions. This is the case when

$$\theta = \frac{\partial AD^{ELB}/\partial\hat{x}}{\partial AS/\partial\hat{x}} = \frac{\sigma(1-p)(1-p\beta)}{\kappa_y c(p-\psi^*)} < 1. \tag{21}$$

If the ZIR equilibrium is the absorbing state, the system takes the form

$$\hat{\pi}^T = \frac{\kappa_y}{1 - p\beta}\hat{x}^T + \frac{\beta(1-p)}{1-p\beta}\frac{\mu}{\psi^*-1} AS, \tag{22a}$$

$$\hat{\pi}^T = \begin{cases} \frac{(1-p)\sigma}{c(p-\phi_\pi-\psi^*)}\hat{x}^T - \frac{(1-p)\mu}{(\psi^*-1)(p-\psi^*-\phi_\pi)}\left[\frac{(1-\beta)\sigma}{c\kappa_y}+1\right] - \frac{p}{c(p-\phi_\pi-\psi^*)}\hat{r}^T \; AD^{TR}, \\ \frac{(1-p)\sigma}{c(p-\psi^*)}\hat{x}^T - \frac{(1-p)\mu}{(\psi^*-1)(p-\psi^*)}\left[\frac{(1-\beta)\sigma}{c\kappa_y}+1\right] - \frac{\mu}{(p-\psi^*)} - \frac{p}{c(p-\psi^*)}\hat{r}^T \; AD^{ELB}. \end{cases} \tag{22b}$$

Since FP is passive, the slope of AD^{TR} is negative and the slope of AD^{ELB} can be positive. In this case, the model can be incoherent or incomplete due to AD^{ELB} being flatter than AS ($\theta < 1$).

The four cases related to active and passive FP in both absorbing states are illustrated in Figure 6.

Moreover, the rule in Equation (16) nests the special case where FP can fully replicate monetary policy as in the simple case considered in Correia et al. (2013), who termed this as "unconventional fiscal policy". This is the case if FP activates at the ELB, and its feedback coefficients are set such that they exactly mirror the effects of the counterfactual unconstrained monetary policy. Further details and derivations of this special case are provided in Appendix B.4. Analogous results hold when monetary policy is conducted optimally under discretion, which we provide analytical derivation for in Appendix B.5.

Relationship to baseline NK model and importance of commitment. For the baseline NK model considered in Section 2, Equation (13) summarises the conditions under which the model possesses a unique MSV solution. It is worth reiterating that unlike in the case of a model with FP, the baseline NK model implies that the slope of AD^{ELB} is determined only by exogenous uncertainty p and deep structural parameters.

Thus, one can argue that if the effects of uncertainty can be counteracted by FP, the model becomes coherent and complete. The condition on θ implies that the probability of shock persistence, p, must be low so that CC conditions are satisfied. An analogous condition for the model with permanent fiscal policy changes can be drawn from Equation (21); if $\theta > 1$, the model is coherent and complete. This condition shows that unlike in the baseline NK model, FP can alleviate the effects of exogenous uncertainty on the slope of

AD^{ELB} and, thus, ensures satisfaction of the CC conditions.

The importance of persistence implied by (17) cannot be overstated and is a key point of this paper. To highlight this consider the case where the fiscal targeting rule is given in deviations and not in growth rates; $\hat{g}_t = \psi_\pi \hat{\pi}_t$. This will imply the following AD^{ELB}/AS slope ratio that is required to be greater than unity to satisfy CC:

$$\theta = \frac{\sigma(1-p)(1-p\beta)}{p+\psi^*(1-p)} > 1,$$

which does not hold if p is sufficiently large for any bounded value of ψ^*.

The key intuition for permanent policy changes can be drawn from the fact that the FP instrument is present in the AD relationship in Equation (15a) in expectation. Thus, any rule that targets contemporaneous deviations would imply additional terms in expectation that add uncertainty to the system. This highlights the importance of commitment to future changes in policy that depend on contemporaneous deviations of endogenous variables as in, for example, (17). The assumption that a fiscal authority needs to apply a targeting rule to growth rates of the instrument and not to its deviations is rather restrictive, however. We relax this assumption in Section 4.1 allowing for an inertial rule with $\rho_g < 1$.

3.2. Equivalence of Simple Fiscal Policy Regimes

So far, we have considered a standard NK model augmented with a simple FP setup where government spending targets inflation and output gap. The key difference between this model, described by the system (15), and a standard NK model as in Equations (4a)-(4c) is the presence of a fiscal instrument in the DISE. As shown above, if FP uses this instrument to react to exogenous disturbances aggressively enough, the model satisfies the CC conditions. Existence of such an instrument in the DISE is, however, not exclusive to the fiscal setup we have discussed.

For example, consider the case where the fiscal authority levies consumption and wage taxes, τ^c and τ^w, respectively, and only redistributes the taxes as lump-sum transfers, τ_t. Additionally, assume that there are no production subsidies, $\tau_t^s = 0, \forall t$. Then, if the following condition holds, the effects of FP are offset in the NKPC in Equation (15b):

$$\Psi^w \hat{\tau}_t^w = \Psi^c \hat{\tau}_t^c. \tag{23}$$

The natural interest rate in the DISE can then be written as:

$$\hat{r}_t^n = -\mathbb{E}_t \Delta \hat{\tau}_{t+1}^c - \frac{\sigma}{c} \varepsilon_t \tag{24}$$

where $\Delta \hat{\tau}_{t+1}^c$ is the consumption tax growth rate.

Thus, FP can replace active monetary policy when the latter is constrained thus rendering the model linear and guaranteeing that the CC conditions are satisfied. This is in line with the results in Correia et al. (2013). Under this formulation, the strength of the fiscal instrument (in the DISE) may be higher or lower than in (15), depending on the values of the model's structural parameters. For example, under our calibration in Table 1, the coefficient on the fiscal instrument (government expenditure) is g, while on the former it is Ψ^c/σ. While qualitatively the role of the instruments in both cases is identical, the degree of reaction of $\Delta \hat{\tau}_{t+1}^c$ is required to be greater than that of $\Delta \hat{g}_{t+1}$ since the coefficient on the former is smaller.

Using this alternative setup, we show the CC regions in Figure 7 under: (i) simple inflation targeting in Figure 7A and (ii) inflation and output targeting in Figure 7B. The relevant coefficients of the canonical form are given in Appendix B.6. As before, if the degree of reaction of differentials of consumption tax to inflation is large enough, the model is coherent and complete. The intuition for this case is simple and mirrors that in Section 3.1.

FIGURE 5. Coherency and Completeness with Active and Passive Simple Fiscal Rules (Absorbing State; $\varepsilon_t = 0$)

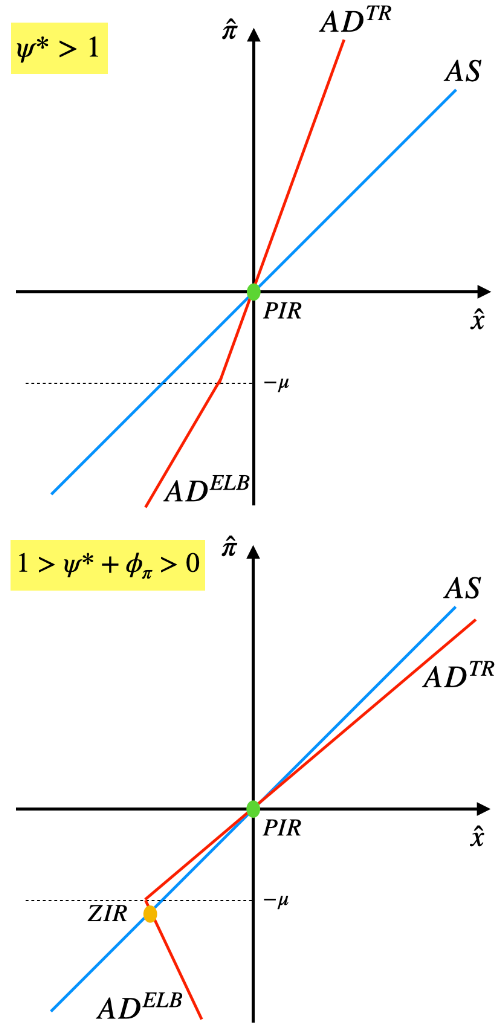
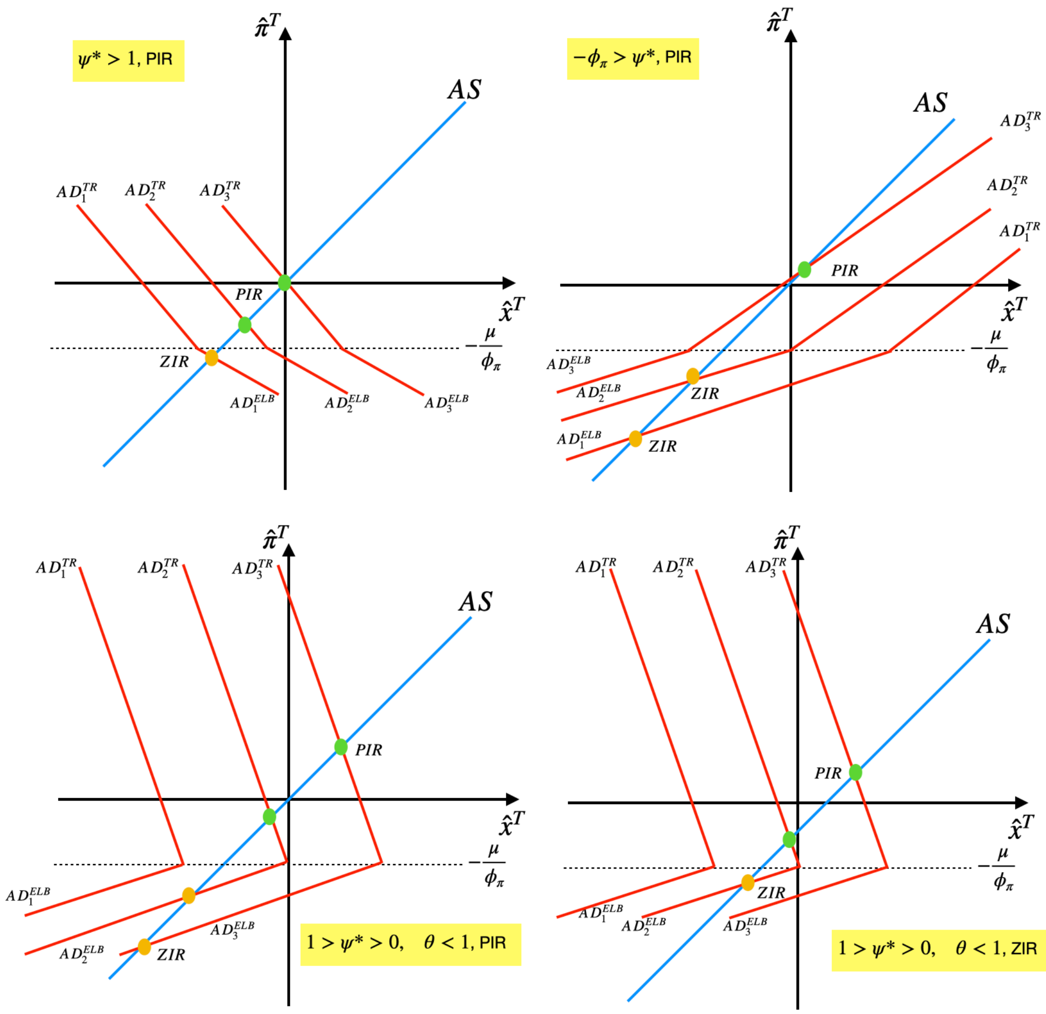


Figure illustrates uniqueness of the absorbing state under active fiscal policy (top-left and bottom-right). If the fiscal policy is insufficiently aggressive, there are multiple absorbing states (top-right and bottom-left).

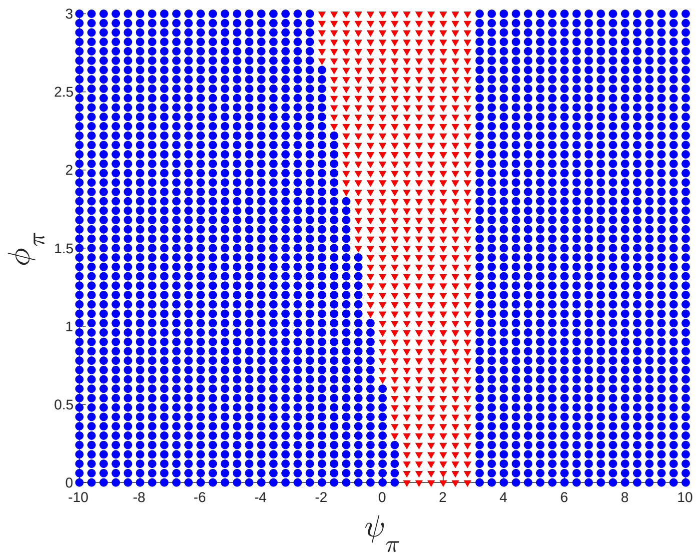
FIGURE 6. Transitory States under Active and Passive Fiscal Policy



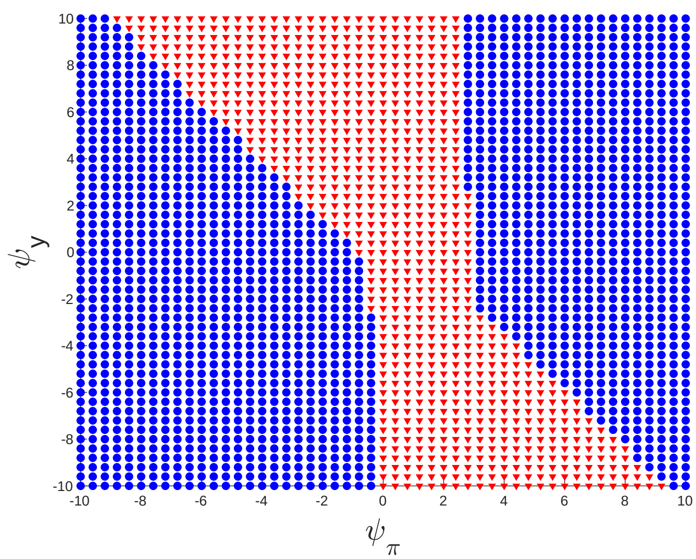
Top row shows transitory state with a positive interest rate absorbing state with active fiscal policy. Top left panel shows procyclical fiscal policy. Top right shows countercyclical fiscal policy. Bottom panels shows passive procyclical fiscal policy regime. Passive fiscal policy in general implies non-existence of solution or two solutions as a special case. Active fiscal policy implies existence of unique solution.

FIGURE 7. Coherency and Completeness Regions for Consumption Tax Fiscal Policy Regimes

A. Simple Inflation Targeting FP ($\psi_y = 0$)



B. Inflation and Output Gap Rule



Blue circles denote regions where coherency and completeness conditions are satisfied. Red triangles denote regions where the model is incoherent or incomplete. Consumption tax rule is given by $\Delta\hat{\tau}^c_{t+1} = \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t$.

3.3. Contemporaneous Rules

Now, we assume that the rule in Equation (16) is replaced with a contemporaneous or non-inertial fiscal feedback rule of the form

$$\hat{g}_t = \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t. \tag{25}$$

We also assume that only lump-sum taxes are levied, so there is a one-to-one mapping of lump-sum taxes to government spending. We then have the following proposition:

PROPOSITION 2. *A baseline New Keynesian model with a simple fiscal rule described by* (25) *and* (15)*, in which monetary policy adheres to strict inflation targeting (*$\phi_y = 0$*) and the Taylor principle is satisfied (*$\phi_\pi > 1$*), fails to satisfy the coherency and completeness conditions.*

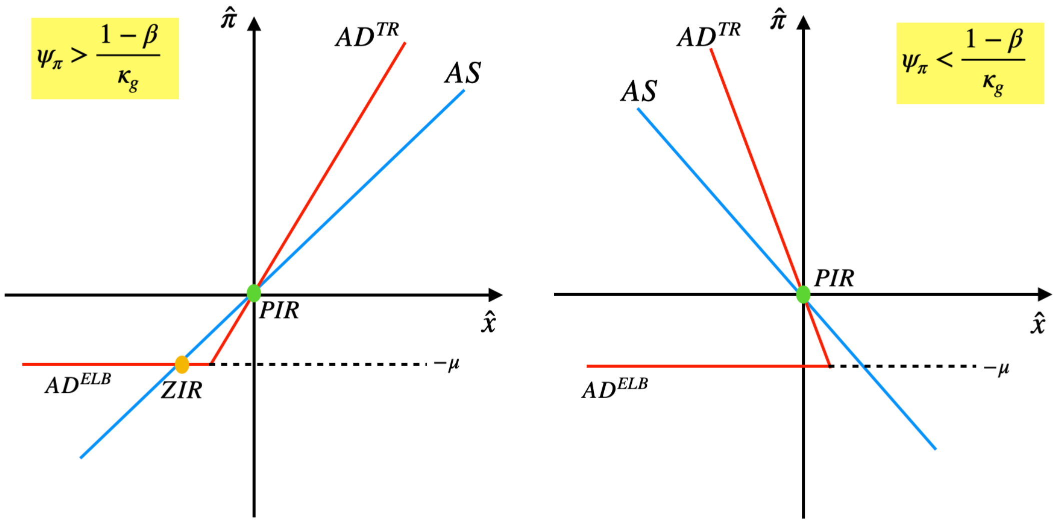
Rearranging the system of equations and obtaining the relevant matrices from the canonical form with relevant coefficients provided in Appendix B.3, CC conditions are satisfied if and only if the signs of $|\mathcal{A}_{J_1}|$ and $|\mathcal{A}_{J_4}|$ are identical. This is not the case since

$$\begin{aligned}
|\mathcal{A}_{J_1}| &= |\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{I}_2| \\
&= \begin{vmatrix} \frac{c}{\sigma}(1-\phi_\pi) & 0 \\ \beta - 1 - \kappa_g \psi_\pi & \kappa_y - \kappa_g \psi_y \end{vmatrix} \\
&= \frac{c(1-\phi_\pi)(\kappa_y - \kappa_g \psi_y)}{\sigma}, \\
|\mathcal{A}_{J_4}| &= \frac{c(\kappa_y - \kappa_g \psi_y)}{\sigma}.
\end{aligned} \tag{26}$$

To put simply, the above gives $|\mathcal{A}_{J_1}| = |\mathcal{A}_{J_4}|(1-\phi_\pi)$, which implies that under an active TR the NK model with FP rule (25) does not generally satisfy the CC conditions.

As in the baseline NK model without FP, this can be seen graphically in the case of the absorbing state ($\varepsilon_t = 0$) or the transitory state with a PIR absorbing state. We illustrate the case of the absorbing state with $\psi_y = 0$ by

FIGURE 8. Strict Inflation Targeting Monetary Policy: Absorbing State



The absorbing state is described as the permanent state of the economy when $\varepsilon_t = 0$. Monetary policy is conducted by adjusting the nominal interest rate to only close the inflation gap, implying that $\phi_y = 0$.

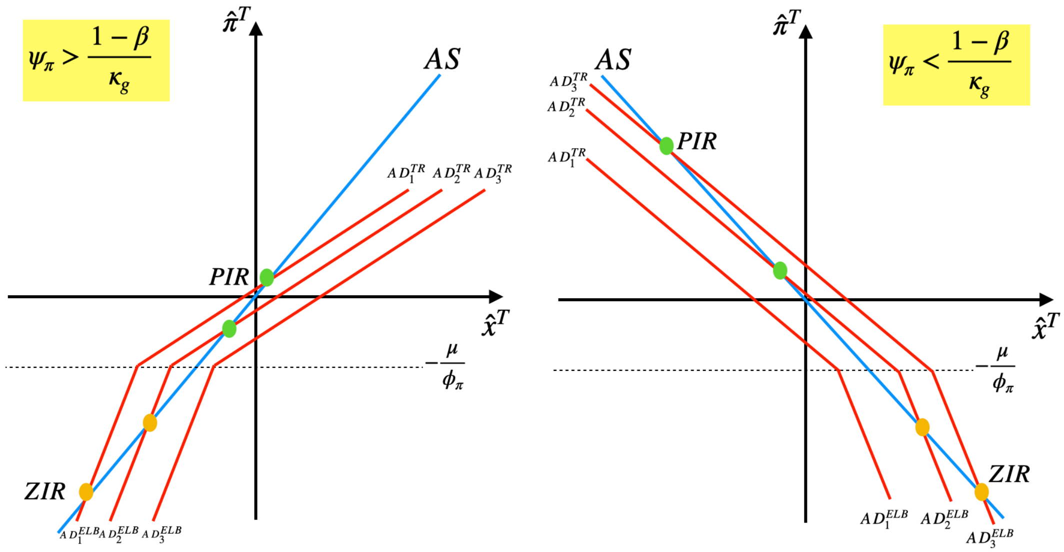
rearranging the system of equations into *AS* and *AD* schedules:

$$\hat{\pi} = \frac{\kappa_y}{1-\beta+\kappa_g\psi_\pi}\hat{x} \quad AS, \tag{27a}$$

$$\hat{\pi} = \begin{cases} \frac{\phi_\pi \kappa_y}{1-\beta+\kappa_g\psi_\pi}\hat{x} & AD^{TR}, \\ -\mu & AD^{ELB}. \end{cases} \tag{27b}$$

Notice that with the introduction of FP, both the *AS* and AD^{TR} are augmented and sensitive to two FP parameters: the NKPC coefficient of government expenditure, κ_g, and the FP rule reaction parameter to inflation, ψ_π. Furthermore, notice that the slopes of *AS* and AD^{TR} are dependent on the relative sizes of these parameters. Namely, if $\psi_\pi > (1-\beta)/\kappa_g$, then *AS* and *AD* are diagrammatically similar to the case of the baseline NK model with no FP (Figure 1), with two solutions – one PIR and one ZIR. This is illustrated in the left diagram of Figure 8. However, if the condition $\psi_\pi < (1-\beta)/\kappa_g$ holds, then both *AS* and AD^{TR} become downward sloping, and if AD^{TR} is steeper than *AS* then only one unique solution remains – the PIR equilibrium. This

FIGURE 9. Strict Inflation Targeting Monetary Policy: Transitory State with PIR Absorbing



In each period the economy is subject to a shock with probability p. With complementary probability $1-p$, the economy transitions to the PIR absorbing state. Monetary policy is conducted by adjusting the nominal interest rate to only close the inflation gap, implying that $\phi_y = 0$.

case is illustrated in the right diagram of Figure 8.

But is the condition $\psi_\pi < (1-\beta)/\kappa_g$ enough to ensure a unique solution once the economy is subject to shocks? No. To see this diagrammatically we consider the economy when it starts in the transitory state and is subject to shocks with probability p in each period, and with probability $1-p$ the economy transitions to its PIR absorbing state. *AS* and *AD* can then be written as follows:

$$\hat{\pi}^T = \frac{\kappa_y}{1-p\beta+\kappa_g\psi_\pi}\hat{x}^T \quad AS, \tag{28a}$$

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{pc+(1-p)\sigma g\psi_\pi - c\phi_\pi}\hat{x}^T - \frac{p}{cp+(1-p)g\sigma\psi_\pi - c\phi_\pi}\hat{r}^T & AD^{TR}, \\ \frac{\sigma(1-p)}{pc+(1-p)\sigma g\psi_\pi}\hat{x}^T - \frac{c}{pc+g\sigma\psi_\pi(1-p)}\mu - \frac{p}{cp+(1-p)g\sigma\psi_\pi}\hat{r}^T & AD^{ELB}. \end{cases} \tag{28b}$$

Figure 9 plots *AS* and *AD* from (28), where we can clearly see that regardless of the relative size of κ_g and ψ_π, the CC conditions are not satisfied and thus, in general, no solution exists or that two solutions exist. This result holds true even when the condition $\psi_\pi < (1-\beta)/\kappa_g$ holds, which is the condition needed to ensure a unique absorbing state.

Monetary policy targets inflation and output gap. Consider the case where the monetary authority targets both the inflation and output gap, ($\phi_y > 0$).

As before, rearrange the model for the absorbing state where $\varepsilon_t = 0$, and express the model in two equations, *AS* and *AD*:

$$\hat{\pi} = \Theta\hat{x} \quad AS, \tag{29a}$$

$$\hat{\pi} = \begin{cases} \left(\phi_\pi\Theta + \phi_y\right)\hat{x} & AD^{TR}, \\ -\mu & AD^{ELB}, \end{cases} \tag{29b}$$

where

$$\Theta = \frac{\kappa_y - \kappa_g\psi_y}{1 - \beta + \kappa_g\psi_\pi}.$$

Note carefully that the slopes of *AS* and *AD* in (29) are potentially ambiguous. In line with the calibration in Table 1 and with ψ_y, ψ_π being sufficiently large in absolute value, the following assumptions on parameter values are made:

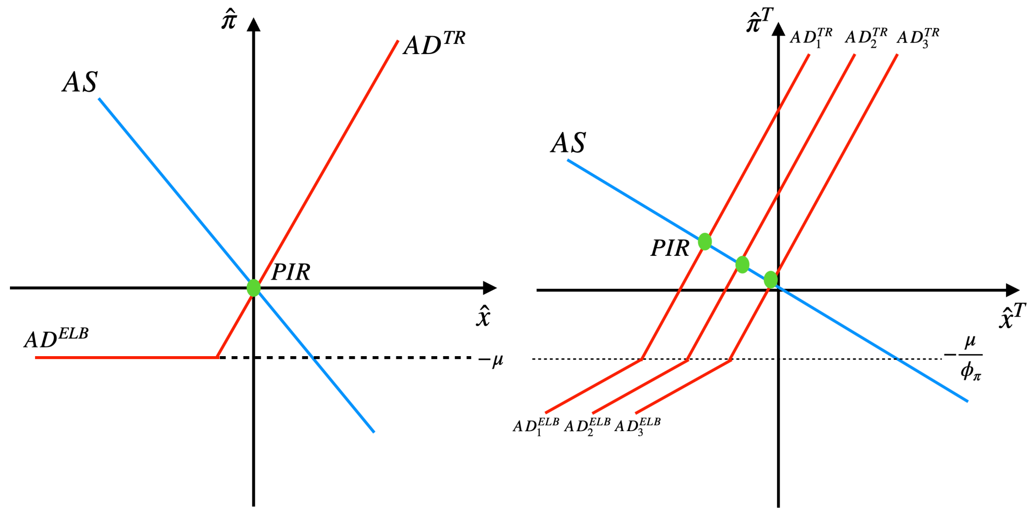
$$\Theta < 0, \quad \phi_\pi\Theta + \phi_y > \Theta,$$

which then implies

$$\Theta(1 - \phi_\pi) < \phi_y.$$

One can see that the LHS of the above inequality is always positive, and that ϕ_y must be sufficiently large for the inequality to hold. This inequality highlights the role of a TR that includes both inflation and the output gap. If monetary policy follows a strict inflation targeting regime, the inequality would never be satisfied and, hence, no matter the fiscal policy stance (captured by Θ), the existence of multiple absorbing states is never ruled out. Hence, multiplicity of steady states in this case is only ruled out under a particular configuration of fiscal-monetary mix. In other words, should the slope of *AS* be positive then under conventional restrictions on TR parameters, the slope of AD^{TR} would also be positive and greater than

FIGURE 10. Contemporaneous Fiscal Policy and Taylor rule with Inflation and the Outgap Gap



A. PIR Absorbing State

B. Transitory State with PIR Absorbing State

that of AS, creating the two – PIR and ZIR – absorbing states. With these assumptions, AS and AD in (29) is plotted in Figure 10A, and we have the following proposition:

PROPOSITION 3. *A baseline New Keynesian model with a simple fiscal rule described by* (25) *and* (15)*, is coherent and complete if fiscal policy responds aggressively enough to inflation and the output gap and monetary policy responds to both inflation and the output gap.*

Next we show analytical results for when the economy is in the transitory state. As mentioned above, as the ZIR absorbing state is eliminated, we restrict attention to the existence of a unique PIR absorbing state. Assume that initially the economy is in a transitory state with $\varepsilon_t \neq 0$ and it will remain in this transitory state with probability p. The system can then be written as follows

$$\hat{\pi}^T = \frac{\kappa_y - \kappa_g \psi_y}{1 - p\beta + \kappa_g \psi_\pi}\hat{x}^T \quad AS, \tag{30a}$$

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)(1-g\psi_y)+c\phi_y}{pc+(1-p)\sigma g\psi_\pi - c\phi_\pi}\hat{x}^T - \frac{p}{cp+(1-p)g\sigma\psi_\pi - c\phi_\pi}\hat{r}^T & AD^{TR}, \\ \frac{\sigma(1-p)(1-g\psi_y)}{pc+(1-p)\sigma g\psi_\pi}\hat{x}^T - \frac{c}{pc+g\sigma\psi_\pi(1-p)}\mu - \frac{p}{cp+(1-p)g\sigma\psi_\pi}\hat{r}^T & AD^{ELB}. \end{cases} \tag{30b}$$

With ψ_π sufficiently large and ψ_y positive but not too large, AS is downward sloping and AD^{TR} is upward sloping. Since AD^{ELB} is also upward-sloping ($p < 1$) or flat ($p = 1$), there is a unique solution for any realisation of $\hat{r}^T$. The system (30) is illustrated in Figure 10B.

Much like the case for the absorbing state, the downward sloping AS curve is central to the uniqueness result. This is predicated on: (i) direct influence of fiscal policy on aggregate supply, (ii) fiscal policy being procyclical, and (iii) the TR also being a function of the output gap. Absent of either of the aforementioned points, the model would imply a non-unique solution and, thus, the policy stance presented above merely presents a special case that is not applicable to a more general class of models. First, absent of direct fiscal policy effects, the AS curve is always upward sloping as in a baseline NK model. This is true, for example, if there is no income effect on the household's labour supply decision due to preferences such as in Greenwood, Hercowitz, and Huffman (1988) (GHH), or if labour is supplied inelastically. In such a case, fiscal policy would not directly affect AS and thus its slope would remain positive. Second, even if fiscal policy had direct effects on AS, it needs to react positively to deviations of inflation and output. If this were not to hold, AS would be upward sloping, which would generate multiple solutions.

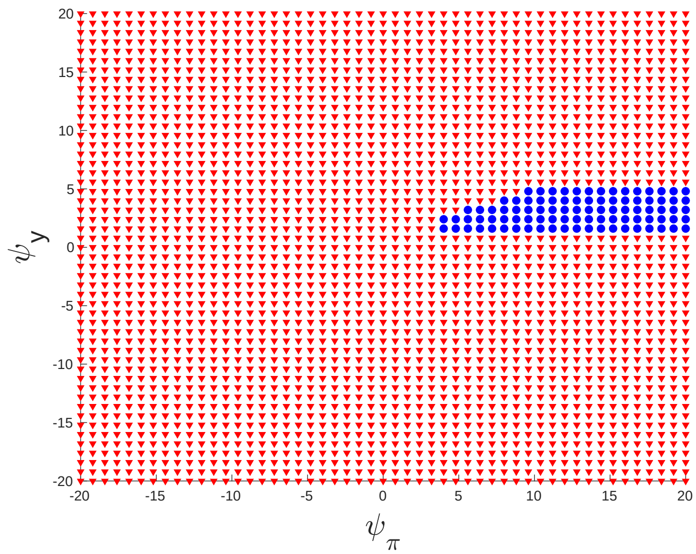
However, the result is robust to the calibration of the government expenditure share in output, g. To illustrate this, consider upper and lower bounds on ψ_y. The upper bound on ψ_y can be inferred from the restriction on AD^{ELB} being upward sloping or flat, which is the case if and only if

$$1 - g\psi_y \geq 0 \;\Rightarrow\; \psi_y \leq \frac{1}{g}.$$

The lower bound on ψ_y can be inferred from the restriction on the slope of AS which must be negative. This implies that

$$\kappa_y - \kappa_g\psi_y < 0 \;\Rightarrow\; \psi_y > \frac{\kappa_y}{\kappa_g}.$$

FIGURE 11. Coherency and Completeness Region for Inflation and Output Gap Targeting Monetary Policy



Blue circles denote regions where coherency and completeness conditions are satisfied. Red triangles denotes region where the model is either incoherent or incomplete.

These conditions imply that the model is coherent and complete only if

$$\frac{\kappa_y}{\kappa_g} > \frac{1}{g} \;\Rightarrow\; -\rho g^2 + (\rho + 1)g - 1 > 0,$$

where $\rho = \varphi/\sigma$. Thence, the relationship holds for $g < \min(1, 1/\rho)$, which is always the case if the coefficient of relative risk aversion is greater than inverse-Frisch elasticity of labour supply, $\sigma > \varphi$.

Finally, to visually see the need for procyclical fiscal policy – and the upper and lower limits on ψ_y – we numerically compute regions for which CC conditions are satisfied in $\{\psi_\pi, \psi_y\}$ space in Figure 11.

4. Coherency and Completeness with an Endogenous State

Thus far, we have only considered DSGE models that did not feature endogenous state variables. In other words, in the canonical form representation of

DSGE models,

$$\begin{aligned}\mathbf{0} &= \boldsymbol{A}_{s_t}\boldsymbol{Y}_t + \boldsymbol{B}_{s_t}\boldsymbol{Y}_{t+1|t} + \boldsymbol{C}_{s_t}\boldsymbol{X}_t + \boldsymbol{D}_{s_t}\boldsymbol{X}_{t+1|t} + \boldsymbol{H}_{s_t}\boldsymbol{Y}_{t-1},\\ s_t &= \mathbb{1}\left(\boldsymbol{a}^\top\boldsymbol{Y}_t + \boldsymbol{b}^\top\boldsymbol{Y}_{t+1|t} + \boldsymbol{c}^\top\boldsymbol{X}_t + \boldsymbol{d}^\top\boldsymbol{X}_{t+1|t} + \boldsymbol{h}^\top\boldsymbol{Y}_{t-1} > 0\right),\end{aligned}$$

we assumed that the coefficient matrix, $\boldsymbol{H}_{s_t} = \boldsymbol{O}$, and the coefficient matrix, $\boldsymbol{h} = \mathbf{0}$. This was to keep the computation and verification of the CC conditions analytically tractable by omitting endogenous state variables. With these assumptions, an MSV solution could be represented as $\boldsymbol{Y}_t = f(\boldsymbol{X}_t)$ by the time-invariant matrix $\mathbf{Y}$; and variables along an MSV solution satisfy the condition (1).

However, this assumption is restrictive and makes assessment of CC conditions in standard DSGE models limited, particularly in the literature exploring the monetary and fiscal policy mix and ELB. In this section, we loosen this assumption and consider a canonical NK-FP model with an endogenous state variable, namely government expenditure.

With endogenous states, along an MSV solution we have

$$\mathbb{E}_t\left[\boldsymbol{Y}_{t+1}|\boldsymbol{Y}_t = \mathbf{Y}_t\boldsymbol{e}_i, \boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i\right] = \mathbf{Y}^i_{t+1}\boldsymbol{K}^\top\boldsymbol{e}_i, \tag{31}$$

where $\mathbf{Y}^i_{t+1}$ gives the support of $\boldsymbol{Y}_{t+1}$ when $\boldsymbol{Y}_t$ is in the i-th state. However, this presents a non-trivial computational challenge: the support of $\boldsymbol{Y}_t$ is exponentially rising for a given initial condition $\boldsymbol{Y}_0$. Therefore, an MSV solution cannot be characterised by a finite-dimensional system of piecewise linear equations. This requires a different method of analysis to that of Section 3.

In the fashion of AM, we solve the model recursively from some terminal state $t = T$. For simplicity, assume that the endogenous state variable is a scalar, $\boldsymbol{H}_{s_t}\boldsymbol{Y}_{t-1} = \boldsymbol{h}_{s_t}y_{t-1}$, where $\boldsymbol{h}_{s_t}$ is $n \times 1$ and $y_t = \boldsymbol{g}^\top\boldsymbol{Y}_t$ is a linear combination of $\boldsymbol{Y}_t$ and where $\boldsymbol{g} = \begin{pmatrix} 0 & 0 & 0 & 1\end{pmatrix}^\top$.

For a date T whereby $t \geq T$, the MSV solution $f(y_{t-1}, \boldsymbol{X}_t)$ can be written as

$$\mathbf{Y}_t = \mathbf{G}y_{t-1} + \mathbf{Z},$$

where $\mathbf{G}$ and $\mathbf{Z}$ are $n \times k$ matrices. $\mathbf{Z}$ captures the portion of $\boldsymbol{Y}_t$ that depends on exogenous variables $\boldsymbol{X}_t$. In the case of no exogenous variables, we have that $\mathbf{G} = \boldsymbol{O}$ and so $\mathbf{Y}_t = \mathbf{Z}$, yielding the standard case with a time invariant $\mathbf{Y}$ and when the analysis of Section 3 applies. The columns of $\mathbf{G}$ return the coefficients of y_{t-1} in the MSV solution, mapping it to different states of $\boldsymbol{X}_t$. Assume, as before, that $k = 2$, whereby the "bad state" corresponds to $i = 1$, and the "good state" is given by $i = 2$. In other words, $i = 1$ is the ZIR state and $i = 2$ is a PIR state. Then the endogenous dynamics in the bad state can be different from the good state.

With no endogenous dynamics, where $\mathbf{Y} = \mathbf{Z}$, we can put the model in k-state canonical form as in (2). With endogenous dynamics, the equivalent expression is given by

$$\begin{aligned}\mathbf{0} = &\left(\boldsymbol{A}_{s_{t,i}}\mathbf{G}\boldsymbol{e}_i + \boldsymbol{h}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\mathbf{G}\boldsymbol{K}^\top \boldsymbol{e}_i\boldsymbol{g}^\top \mathbf{G}\boldsymbol{e}_i\right) y_{t-1} \\ &+ \left(\boldsymbol{A}_{s_{t,i}}\mathbf{Z} + \boldsymbol{B}_{s_{t,i}}\mathbf{G}\boldsymbol{K}^\top \boldsymbol{e}_i\boldsymbol{g}^\top \mathbf{Z} + \boldsymbol{B}_{s_{t,i}}\mathbf{Z}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}}\mathbf{X} + \boldsymbol{D}_{s_{t,i}}\mathbf{X}\boldsymbol{K}^\top\right) \boldsymbol{e}_i,\end{aligned} \tag{32}$$

for all $i = 1, \ldots, k$. For a given regime J corresponding to the k states and their transitions, $\{(PIR, PIR), (ZIR, PIR), (PIR, ZIR), (ZIR, ZIR)\}$ (see (65)), a slackness condition for the constraint $s_{t,i}$ is determined which gives a system of $2nk$ polynomial equations in the $2nk$ unknowns $\mathbf{G}$ and $\mathbf{Z}$ by equating the coefficients on y_{t-1} and the constant terms to zero, respectively. As these conditions are polynomial and not piecewise linear in $\mathbf{G}$ and $\mathbf{Z}$, the algorithm and theorem of Gourieroux, Laffont, and Monfort (1980) is no longer suitable to check coherency. Instead, we build on the algorithm and "brute force" numerical solution method of AM,[15] which essentially goes through all possible 2^k J regime configurations to check if there are any feasible solutions that satisfy the inequality constraints.

The model can be solved backwards starting from some terminal date T. We know that at T, the solution to the model takes the following form

$$\mathbf{Y}_T = \boldsymbol{G}_{J_0} y_{T-1} + \mathbf{Z}_{J_0}, \tag{33}$$

where $J_0 \in \mathcal{J}$, and where $\mathcal{J}$ defines the configuration of regimes in T. As we explain in Appendix A.4, in order for CC conditions of a DSGE model with an

15. See Appendix A.5.2 of their paper.

endogenous state variable to be satisfied, the determinants,

$$|\mathcal{A}_{J_0 J_1}| = \prod_i^k \det\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right), \quad \forall t \leq T, \tag{34}$$

must all have the same sign. If this indeed the case, then the model solution is given as:

$$\begin{aligned}\mathbf{Y}_{T-1}\boldsymbol{e}_i = &-\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1} \\ &\left[\left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\mathbf{X} + \boldsymbol{D}_{s_{T-1},i}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i}y_{T-2}\right],\end{aligned}$$

Iterating the solution backwards implies that all the determinants of $\mathcal{A}_{J_0,\ldots,J_{T-t}}$ must have the same sign. The recursive solution will be given by

$$\mathbf{Y}_t = \boldsymbol{G}_{J_0,\ldots,J_{T-t}}y_{t-1} + \boldsymbol{Z}_{J_0,\ldots,J_{T-t}},$$

where $\boldsymbol{G}_{J_0,\ldots,J_{T-t}}$ and $\boldsymbol{Z}_{J_0\ldots,J_{T-t}}$ can be computed recursively using

$$\begin{aligned}\boldsymbol{Z}_{J_0\ldots,J_{T-t},i} = &-\left(\boldsymbol{A}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1} \\ &\left(\boldsymbol{B}_{s_{t,i}}\boldsymbol{Z}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}}\mathbf{X} + \boldsymbol{D}_{s_{t,i}}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i,\end{aligned} \tag{35}$$

$$\boldsymbol{G}_{J_0,\ldots,J_{T-t},i} = -\left(\boldsymbol{A}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1}\boldsymbol{h}_{s_{t,i}}. \tag{36}$$

The recursive solution from terminal T solves the model backwards to $t = 1$ and implies up to $2^{(T-1)k}$ solution paths. Given some initial condition, y_0, and conditional on satisfaction of CC conditions, the recursive solution is unique. If the CC conditions are not satisfied, there can be either no or multiple solutions.

We thus apply the following algorithm to check CC conditions assuming that the shocks are two-state Markovian. First, calculate $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ from (63) and (64) for four possible regime configurations in J_0. For each of the four regime configurations in J_1 compute $|\mathcal{A}_{J_0 J_1}|$. If for some regime configuration J_0, $|\mathcal{A}_{J_0 J_1}|$ have the same sign, a unique solution is possible; or else we conclude that no unique solution exists. Second, for all configurations of J_0, where $|\mathcal{A}_{J_0 J_1}|$ have the same sign, compute $\boldsymbol{G}_{J_0 J_1}$ and $\boldsymbol{Z}_{J_0 J_1}$ using (35) and (36). Third, continue solving backwards for each J_{T-t} until $t = 1$. If $t = 1$ can be reached with: i) the condition on the signs of determinants being satisfied along the solution path; and ii) the model solution being consistent with the implied $s_{t,i}$, $\forall t$, then we can conclude that the model is coherent and complete.

4.1. New Keynesian Model with Persistent Fiscal Policy

We have shown in Section 3.1 how a permanent change in fiscal policy can ensure coherency and completeness. In this section, we generalise over the fiscal rule in Equation (16) and allow for inertia, i.e. $\rho_g < 1$, instead of a permanent policy change.

Consider the baseline NK-FP model described by (15). But assume that there are no distortionary taxes ($\hat{\tau}_t^s = \hat{\tau}_t^c = \hat{\tau}_t^w = 0, \forall t$). We close the model by augmenting the rule (25) to account for inertia in $\hat{g}_t$. Specifically, we replace (25) with:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t, \tag{37}$$

where $\rho_g \in (0, 1)$ is a persistence parameter. Thus, the system of equations considered is:

$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n\right), \tag{38a}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t - \kappa_g \hat{g}_t, \tag{38b}$$

$$\hat{i}_t = \max\{-\mu; \phi_\pi \hat{\pi}_t + \phi_y \hat{x}_t\}, \tag{38c}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t, \tag{38d}$$

with

$$\hat{r}_t^n = -\frac{\sigma}{c}(g \mathbb{E}_t \Delta \hat{g}_{t+1} + \varepsilon_t).$$

The system in (38) can be evaluated about two absorbing states, either PIR or ZIR. About the PIR steady state, it must be the case that $\{\hat{x}, \hat{\pi}, \hat{i}, \hat{g}\} = \{0, 0, 0, 0\}$. The existence of the PIR steady state is trivial to reconcile. When policy is effective, the inflation and output gaps are closed and the system of equations gives the solution in the PIR absorbing state. However, about the ZIR absorbing state we have that $\hat{i} = -\mu$, thus giving a solution of the form

$$\{\hat{x}, \hat{\pi}, \hat{i}, \hat{g}\} = \left\{-\frac{(1-\beta)(1-\rho_g) + \kappa_g \psi_\pi}{\kappa_y(1-\rho_g) - \kappa_g \psi_y}\mu, -\mu, -\mu, \frac{\psi_y \hat{x} + \psi_\pi \hat{\pi}}{1-\rho_g}\right\}. \tag{39}$$

One can observe that under certain fiscal policy rules, the above ZIR equilibrium is not consistent with the constraint on the TR and, thus, the ZIR equilibrium is ruled out. Specifically, we require that $\hat{x}$ be sufficiently large such that the ELB constraint on $\hat{i}_t$ is not binding:

$$-\mu < -\phi_\pi \mu + \phi_y \hat{x} \;\Rightarrow\; -\frac{\mu(1-\phi_\pi)}{\phi_y} < \hat{x}, \quad \phi_y \neq 0.$$

This implies

$$\frac{1-\phi_\pi}{\phi_y} > \frac{(1-\beta)(1-\rho_g)+\kappa_g\psi_\pi}{\kappa_y(1-\rho_g)-\kappa_g\psi_y},$$

with the LHS being negative under conventional restrictions on TR coefficients. As ρ_g tends to unity, we get

$$\lim_{\rho_g\to 1}\frac{(1-\beta)(1-\rho_g)+\kappa_g\psi_\pi}{\kappa_y(1-\rho_g)-\kappa_g\psi_y} = -\frac{\psi_\pi}{\psi_y} < \frac{1-\phi_\pi}{\phi_y},$$

which holds under countercyclical fiscal policy, $\psi_y < 0$, $\psi_\pi < 0$ with ψ_π sufficiently large in absolute value. Thus, under countercyclical fiscal policy, if ρ_g is sufficiently large there cannot exist a ZIR absorbing state. This analysis highlights two points made earlier in Section 3. First, if the monetary authority targets the output gap, a ZIR equilibrium can be ruled out. Second, if the fiscal rule is sufficiently persistent, the ZIR absorbing state is ruled out under countercyclical fiscal policy.

As the model contains an endogenous state, it cannot be represented as a finite-dimensional piecewise linear function and the GLM result does not apply. We are limited in using analytical expressions to exhibit intuition; the CC conditions cannot be verified analytically. Thus, we have the following proposition that we verify numerically using an algorithm that is based on the work by AM:[16]

PROPOSITION 4. *A New Keynesian model subject to an occasionally binding ELB constraint on interest rates and with fiscal policy as described in* (38) *satisfies the coherency and completeness conditions if fiscal policy is sufficiently persistent and*

16. AM developed an algorithm to verify the CC conditions for baseline New Keynesian model subject to the ELB on interest rates, active TR, and whereby the TR exhibited persistence.

countercyclical.

The system of equations in (38) can be cast in the canonical form (32) with relevant coefficients given in Appendix B.7.

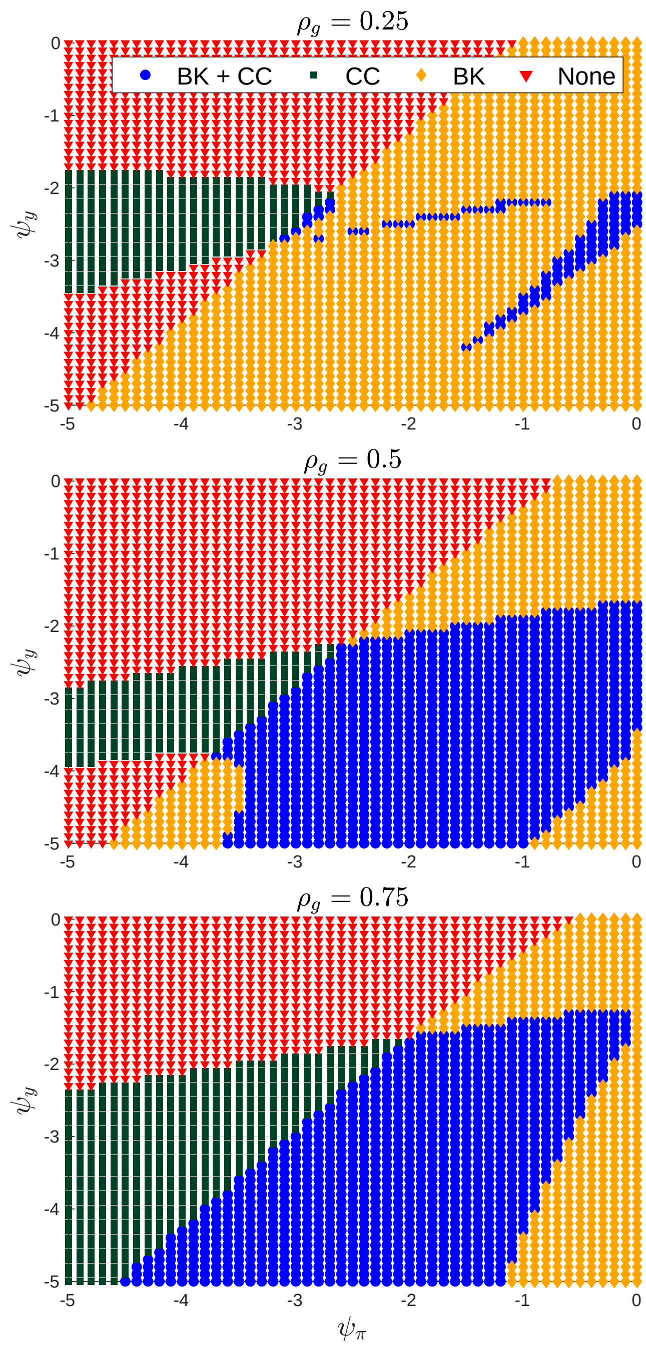
We plot regions where CC and BK local determinacy conditions are satisfied in Figure 12 for different values of ρ_g. As fiscal policy becomes more persistent, the CC region becomes larger. Moreover, the figure shows that the parameter space where CC conditions are satisfied largely corresponds to regions where the model satisfies BK conditions (blue circles).

To satisfy CC, the fiscal policy is required to be sufficiently persistent and aggressive. First, the intuition for persistence can be drawn from the model considered in Subsection 3.1. There, we considered the special case where $\rho_g = 1$, which implied that the fiscal authority commits to a permanent policy change in reaction to deviations of inflation and the output gap. Committing to the policy change in this case can be seen as a form of expectations management as the fiscal authority promises to change spending permanently in response to low inflation. This promise can be seen as a factor that reduces fundamental uncertainty in the system. Second, the policy is required to be sufficiently aggressive to guarantee that its effect on the system is sufficiently large to eliminate multiplicity of equilibria. Much like the Taylor principle requires the monetary authority to react by more than one-to-one to inflation, we require that fiscal policy is sufficiently aggressive to guarantee a unique solution.

Other approaches in the literature (price level targeting (PLT) in Holden (2023) and unconventional monetary policy (UMP) in AM and Ikeda et al. (2021)) rely on a similar mechanism to guarantee uniqueness. As argued in Holden (2023), PLT rules can restore uniqueness in the presence of an occasionally binding ELB constraint as such a policy implies a promise about future inflation given inflation today. If monetary policy is committed to a given price level path, the monetary authority promises that a period of low inflation today will be followed by a period of high inflation in the future. Thus, agents expecting high prices in the future increase their consumption in periods of low inflation and, by implication, the system has a unique solution around the PIR absorbing state. The commitment to higher inflation in the future delivers sufficient information about the expected dynamics of the system that alleviates uncertainty that would otherwise engender multiplicity and, by implication, pins down the unique solution much like persistent fiscal policy.

As shown in AM, the baseline NK model with UMP as in Chen, Cúrdia, and Ferrero (2012) satisfies the CC conditions if UMP is effective enough. This result is consistent with the logic presented above. When the ELB is binding, the effect of UMP on model dynamics needs to be sufficiently strong to pin down a unique solution. In this case, UMP is used to alleviate the effects of exogenous uncertainty and ensure that the solution is unique around the PIR absorbing state. This is in line with the restrictions we establish for FP such that it guarantees a unique solution.

FIGURE 12. Coherency and Completeness Region with Persistent Fiscal Rule



Note: Figure shows regions where both coherency and completeness and Blanchard-Kahn conditions are satisfied (blue circles) for different values of policy inertia, ρ_g.

5. Conclusion

This paper explores whether fiscal policy can restore coherency and completeness in a baseline NK model subject to an occasionally binding constraint on the interest rate that is generally incoherent in absence of FP. Our findings suggest that simple Ricardian FP can restore coherency and completeness thus guaranteeing a unique solution that is also locally determinate. We establish that, to guarantee MSV solution uniqueness and local determinacy, FP needs to be sufficiently persistent and aggressive.

First, we analytically verify that if the fiscal authority is able to credibly commit to a sufficiently strong countercyclical permanent policy change in response to an exogenous disturbance, coherency and completeness of the model is restored. This conclusion is rationalised by the fact that fiscal policy is not constrained by the ELB and provides an active policy response when monetary policy is constrained. Moreover, by committing to a permanent policy change, the fiscal authority is able to alleviate the fundamental uncertainty that engenders multiplicity of equilibria in the baseline NK model.

Second, we find that the fiscal response need not imply a permanent policy change but rather it has to be sufficiently persistent to guarantee existence and uniqueness of an MSV solution. The persistence property of the policy rule, coupled with it being sufficiently countercyclical, are needed to eliminate belief-driven equilibria and pin down a unique solution. By showing this, we address the main concerns raised by Ascari and Mavroeidis (2022) about NK models featuring occasionally binding constraints.

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A. Verification of Model Coherency and Completeness

This appendix provides an overview of the canonical NK model subject to the ELB, and conditions required to adhere to the CC conditions. In other words, we illustrate the conditions that the model must satisfy to have a unique MSV solution in the presence of occasionally binding constraints.

A.1. Methodology of Ascari and Mavroeidis (2022)

As stated in AM, many solution methods of log-linearised models which feature the ELB, such as Eggertsson and Woodford (2003), Guerrieri and Iacoviello (2015), Kulish, Morley, and Robinson (2017), Eggertsson and Singh (2019), and Holden (2023), can be verified for coherency in a simple manner. DSGE models can be written in the following canonical form:[17]

$$\begin{aligned} \mathbf{0} &= \boldsymbol{A}_{s_t}\boldsymbol{Y}_t + \boldsymbol{B}_{s_t}\boldsymbol{Y}_{t+1|t} + \boldsymbol{C}_{s_t}\boldsymbol{X}_t + \boldsymbol{D}_{s_t}\boldsymbol{X}_{t+1|t} + \boldsymbol{H}_{s_t}\boldsymbol{Y}_{t-1}, \\ s_t &= \mathbb{1}\left(\boldsymbol{a}^\top\boldsymbol{Y}_t + \boldsymbol{b}^\top\boldsymbol{Y}_{t+1|t} + \boldsymbol{c}^\top\boldsymbol{X}_t + \boldsymbol{d}^\top\boldsymbol{X}_{t+1|t} + \boldsymbol{h}^\top\boldsymbol{Y}_{t-1} > 0\right), \end{aligned} \tag{40}$$

The key contribution of the paper by AM is that it analyses (40) with rational expectations and Markovian shocks with discrete support. This was as opposed to GLM which analysed the coherency of a system like (40) when $\boldsymbol{B}_{s_t} = \boldsymbol{H}_{s_t} = \boldsymbol{O}$ and $\boldsymbol{b} = \boldsymbol{h} = \mathbf{0}$. In other words, with no endogenous state variables and no expectations on future realisations of the endogenous variables. If the model features endogenous state variables, $\boldsymbol{H}_{s_t} \neq \boldsymbol{O}, \boldsymbol{h} \neq \mathbf{0}$, the canonical form (40) is not piecewise linear and, thus, the standard approach presented by GLM does not apply.

Coherency requires that for the system (40) there exists some function $f(\cdot)$ such that an MSV solution can be represented as $\boldsymbol{Y}_t = f(\boldsymbol{X}_t)$. Assume that the exogenous states $\boldsymbol{X}_t$ are k-state stationary first-order Markov processes with transition kernel $\boldsymbol{K}$. Stack the possible states of $\boldsymbol{X}_t$ for states $i = 1, .., k$ into a $n_x \times k$ matrix $\mathbf{X}$. Then, let $\boldsymbol{e}_i$ denote the i-th column of the $k \times k$ identity matrix $\boldsymbol{I}_k$, such that $\mathbf{X}\boldsymbol{e}_i$, the i-th column of $\mathbf{X}$, is the i-th state of $\boldsymbol{X}_t$.[18] Then define $\mathbf{Y}$ as an $n \times k$ matrix whose i-th column, $\mathbf{Y}\boldsymbol{e}_i$, corresponds to $\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i$ along an MSV solution. Thus, along an MSV solution we have:

$$\mathbb{E}[\boldsymbol{Y}_{t+1}|\boldsymbol{Y}_t = \mathbf{Y}\boldsymbol{e}_i] = \mathbb{E}_t[\boldsymbol{Y}_{t+1}|\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i] = \mathbf{Y}\boldsymbol{K}^\top\boldsymbol{e}_i.$$

Substituting this into (40), yields Equation (2) in the main text.

17. Here $\boldsymbol{H}_{s_t}$ is an $n \times n$ coefficient matrix and $\boldsymbol{h}$ is a coefficient vector.
18. The elements of the transition kernel $\boldsymbol{K}$ are $\boldsymbol{K}_{ij} = \Pr(\boldsymbol{X}_{t+1} = \mathbf{X}\boldsymbol{e}_j|\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i)$ and hence, $\mathbb{E}_t[\boldsymbol{X}_{t+1}|\boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i] = \mathbf{X}\boldsymbol{K}^\top\boldsymbol{e}_i$.

Example: Simple Fisherian Model. To demonstrate the methods of AM and GLM, consider the simple model taken from Section 2 of Aruoba, Cuba-Borda, and Schorfheide (2018), which consists of the Euler equation,

$$1 = \mathbb{E}_t \left[M_{t,t+1} \frac{1+i_t}{\pi_{t+1}} \right], \tag{41}$$

and a TR that only targets inflation,

$$1 + i_t = \max \left\{ 1, r\pi^* \left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \right\}, \quad \phi_\pi > 1, \tag{42}$$

where $M_{t,t+1}$ is the stochastic discount factor and the steady value of the gross real interest rate is given by $r = 1/M = (1+i)/\pi$. The law of motion of $M_{t,t+1}$ is given by a 2-state Markov process with a transitory state $r^{-1}\exp(-r^L) > r^{-1}$ and an absorbing state r^{-1} with transition probabilities p and q, respectively.[19]

Combine the two equations above and log-linearise about the non-stochastic steady state to get

$$\mathbb{E}_t \hat{\pi}_{t+1} = \mathbb{E}_t \hat{M}_{t,t+1} + \max \{-\mu, \phi_\pi \hat{\pi}_t\},$$

which can be cast in canonical form (2) as follows:

$$0 = \left(-\phi_\pi \begin{pmatrix} \hat{\pi}_t^1 & \hat{\pi}_t^2 \end{pmatrix} + \begin{pmatrix} \hat{\pi}_{t+1}^1 & \hat{\pi}_{t+1}^2 \end{pmatrix} \boldsymbol{K}^\top + \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\exp(-r^L)}{r} & 0 \\ 1 & 1 \end{pmatrix} \boldsymbol{K}^\top \right) \boldsymbol{e}_i,$$

if $s_t = \mathbb{1}(\{\phi_\pi \hat{\pi}_t + \mu > 0\})$. However, if $s_t = \mathbb{1}(\{\phi_\pi \hat{\pi}_t + \mu < 0\})$ then we have

$$0 = \left(\begin{pmatrix} \hat{\pi}_{t+1}^1 & \hat{\pi}_{t+1}^2 \end{pmatrix} \boldsymbol{K}^\top + \begin{pmatrix} 0 & \mu \end{pmatrix} \begin{pmatrix} \frac{\exp(-r^L)}{r} & 0 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\exp(-r^L)}{r} & 0 \\ 1 & 1 \end{pmatrix} \boldsymbol{K}^\top \right) \boldsymbol{e}_i,$$

where the transition matrix $\boldsymbol{K}$ is

$$\boldsymbol{K} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{43}$$

and where $\mathbf{Y}_t = \hat{\pi}_t$ and $\boldsymbol{X}_{t+1} = \begin{pmatrix} \hat{M}_{t,t+1} & 1 \end{pmatrix}^\top$. The coefficient matrices[20] are given as $\boldsymbol{A}_0 = 0$, $\boldsymbol{A}_1 = -\phi_\pi$, $\boldsymbol{B}_0 = \boldsymbol{B}_1 = 1$, $\boldsymbol{C}_0 = \begin{pmatrix} 0 & \mu \end{pmatrix}$, $\boldsymbol{C}_1 = \begin{pmatrix} 0 & 0 \end{pmatrix}$, and $\boldsymbol{D}_0 = \boldsymbol{D}_1 = \begin{pmatrix} -1 & 0 \end{pmatrix}$. The coefficient vectors are given as $\boldsymbol{a} = \phi_\pi$, $\boldsymbol{b} = 0$, $\boldsymbol{c} = \begin{pmatrix} 0 & \mu \end{pmatrix}^\top$, and $\boldsymbol{d} = \begin{pmatrix} 0 & 0 \end{pmatrix}^\top$.

19. $r^L < 0$ is a simple negative real interest rate shock, representing a temporary liquidity trap.
20. In this example, since $n = 1$, some of the coefficient matrices are scalars.

The coefficient matrices $\mathcal{A}_J$ in (3) are given by

$$\begin{aligned}
\mathcal{A}_{J_1} &= \boldsymbol{A}_1\boldsymbol{I}_2 + \boldsymbol{B}_1\boldsymbol{K}, \quad J_1 = \{1,2\},\\
\mathcal{A}_{J_2} &= \boldsymbol{e}_1\boldsymbol{e}_1^\top\mathcal{A}_{J_4} + \boldsymbol{e}_2\boldsymbol{e}_2^\top\mathcal{A}_{J_1}, \quad J_2 = \{2\},\\
\mathcal{A}_{J_3} &= \boldsymbol{e}_2\boldsymbol{e}_2^\top\mathcal{A}_{J_4} + \boldsymbol{e}_1\boldsymbol{e}_1^\top\mathcal{A}_{J_1}, \quad J_3 = \{1\},\\
\mathcal{A}_{J_4} &= \boldsymbol{A}_0\boldsymbol{I}_2 + \boldsymbol{B}_0\boldsymbol{K}, \quad J_4 = \emptyset.
\end{aligned} \tag{44}$$

As $J \subseteq \{1,\ldots,k\}$ and $k = 2$ – and J contains all configurations of combinations of the k states – we can think of the above equations as transitions between positive and negative states. Specifically, positive (PIR) and zero interest rate (ZIR) states: $\mathcal{A}_{J_1}$ and $J_1 = \{1,2\}$ is associated with being in the PIR state and remaining in the PIR state, $\mathcal{A}_{J_2}$ and $J_2 = \{2\}$ is associated with being the ZIR state and transitioning to the PIR state, $\mathcal{A}_{J_3}$ and $J_3 = \{1\}$ is associated with being in the PIR state and transitioning to the ZIR state, and $\mathcal{A}_{J_4}$ and $J_4 = \{\emptyset\}$ is associated with being in the ZIR state and remaining in the ZIR state.

Substituting the coefficient matrices into (47), the determinants of the $\mathcal{A}_J$ are:

$$\begin{aligned}
|\mathcal{A}_{J_1}| &= (\phi_\pi - 1)(1 - p - q + \phi_\pi),\\
|\mathcal{A}_{J_2}| &= p(1-\phi_\pi) + q - 1,\\
|\mathcal{A}_{J_3}| &= p - 1 + q(1-\phi_\pi),\\
|\mathcal{A}_{J_4}| &= p + q - 1.
\end{aligned} \tag{45}$$

Since $\phi_\pi > 0$ (satisfaction of the Taylor principle), and $0 \leq p, q \leq 1$, it is straightforward to see that $|\mathcal{A}_{J_1}| > 0$ and $|\mathcal{A}_{J_2}|, |\mathcal{A}_{J_3}| < 0$, and so this is a violation of the CC conditions according to Theorem 1 of GLM.

A.2. Coherency and Completeness of the Canonical New Keynesian Model

Below, we provide a sketch of the insight of AM as applied to the canonical NK model, (4), but for simplicity $\phi_y = 0$. Then assume as before that $k = 2$ and the transition kernel is given by

$$\boldsymbol{K} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}. \tag{46}$$

The coefficient matrices $\mathcal{A}_J$ in (3) are given by

$$\begin{aligned}
\mathcal{A}_{J_1} &= \boldsymbol{A}_1\boldsymbol{I}_2 + \boldsymbol{B}_1\boldsymbol{K}, & J_1 &= \{1,2\}\ \text{(PIR,PIR)},\\
\mathcal{A}_{J_2} &= \boldsymbol{e}_1\boldsymbol{e}_1^\top\mathcal{A}_{J_4} + \boldsymbol{e}_2\boldsymbol{e}_2^\top\mathcal{A}_{J_1}, & J_2 &= \{2\}\ \text{(ZIR,PIR)},\\
\mathcal{A}_{J_3} &= \boldsymbol{e}_2\boldsymbol{e}_2^\top\mathcal{A}_{J_4} + \boldsymbol{e}_1\boldsymbol{e}_1^\top\mathcal{A}_{J_1}, & J_3 &= \{1\}\ \text{(PIR,ZIR)},\\
\mathcal{A}_{J_4} &= \boldsymbol{A}_0\boldsymbol{I}_2 + \boldsymbol{B}_0\boldsymbol{K}, & J_4 &= \emptyset\ \text{(ZIR,ZIR)}.
\end{aligned} \tag{47}$$

As $J \subseteq \{1, \ldots, k\}$ and $k = 2$ – and J contains all configurations of combinations of the k states – we can think of the above equations as transitions between positive and negative states. Specifically, positive (PIR) and zero interest rate (ZIR) states: $\mathcal{A}_{J_1}$ and $J_1 = \{1, 2\}$ is associated with being in the PIR state and remaining in the PIR state, $\mathcal{A}_{J_2}$ and $J_2 = \{2\}$ is associated with being the ZIR state and transitioning to the PIR state, $\mathcal{A}_{J_3}$ and $J_3 = \{1\}$ is associated with being in the PIR state and transitioning to the ZIR state, and $\mathcal{A}_{J_4}$ and $J_4 = \{\emptyset\}$ is associated with being in the ZIR state and remaining in the ZIR state. The relevant coefficient matrices of the canonical form (2) are given by

$$\boldsymbol{A}_0 = \begin{pmatrix} 1 & -\kappa \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{A}_1 = \begin{pmatrix} 1 & -\kappa \\ \frac{\phi_\pi}{\sigma} & 1 \end{pmatrix}, \quad \boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{pmatrix} -\beta & 0 \\ -\frac{1}{\sigma} & -1 \end{pmatrix}.$$

Observe that, in the special case where $p = q = 1$, the determinants of $\mathcal{A}_{J_1}$ and $\mathcal{A}_{J_4}$ are:

$$|\mathcal{A}_{J_1}| = \begin{vmatrix} 1-\beta & -\kappa \\ \frac{\phi_\pi - 1}{\sigma} & 0 \end{vmatrix} = \frac{\kappa(\phi_\pi - 1)}{\sigma} > 0, \quad |\mathcal{A}_{J_4}| = \begin{vmatrix} 1-\beta & -\kappa \\ -\frac{1}{\sigma} & 0 \end{vmatrix} = -\frac{\kappa}{\sigma} < 0. \tag{48}$$

Thus, we observe that with an active TR ($\phi_\pi > 1$) the function $F(\mathbf{Y})$ is not invertible and, hence, the model is generally incomplete. Additionally, in Appendix A.3 we show an analytical derivation of the CC conditions. Denoting ϕ_π^* as

$$\phi_\pi^* = p + q - 1 - \frac{\sigma}{\kappa}[1 - \beta(p + q - 1)](2 - p - q),$$

then the model satisfies the CC conditions when

$$\phi_\pi < \phi_\pi^*, \quad \text{if } \phi_\pi^* > 0, \tag{49a}$$

$$\phi_\pi < 1, \quad \text{if } \phi_\pi^* < 0. \tag{49b}$$

A.3. Analytical Derivation of CC Conditions

To attain an analytical expression for the CC conditions for the baseline New Keynesian model, we first look for a solution of the form $\hat{\pi}_t = f_\pi(\varepsilon_t)$ and $\hat{y}_t = f_y(\varepsilon_t)$. Let ε_t denote the vector k states of the shock and similarly for the solutions π and y. Denote $\boldsymbol{K}$ as the transition kernel of the Markov chain for ε_t. Then, with some abuse of notation, define $\mathbb{E}_t\hat{\pi}_{t+1} = \boldsymbol{K\pi}$ and $\mathbb{E}_t\hat{y}_{t+1} = \boldsymbol{K y}$, then rewrite the model equations (4a)-(4c) as the following:

$$\boldsymbol{K y} = \boldsymbol{y} + \frac{1}{\sigma}(\boldsymbol{i} - \boldsymbol{K\pi}) - \varepsilon,$$

$$\boldsymbol{\pi} = \beta\boldsymbol{K\pi} + \kappa\boldsymbol{y},$$

$$\boldsymbol{i} = \max\{-\mu\iota, \phi_\pi\boldsymbol{\pi}\},$$

where for ease of exposition we have assumed that $u_t = 0$ and $\phi_y = 0$. To clarify the notation: symbols in bold are either vectors or matrices, and ι is a k-length unit vector.

The DISE can be written as:

$$(\boldsymbol{I}-\boldsymbol{K})\,\boldsymbol{y} = -\frac{1}{\sigma}\left(\max\{-\mu\iota, \phi_\pi\boldsymbol{\pi}\} - \boldsymbol{K}\boldsymbol{\pi}\right) + \varepsilon.$$

Then, premultiply the NKPC by $(\boldsymbol{I}-\boldsymbol{K})$ to get

$$(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{\pi} = \kappa(\boldsymbol{I}-\boldsymbol{K})\,\boldsymbol{y} + \beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\boldsymbol{\pi}.$$

Then, substitute the expression for $(\boldsymbol{I}-\boldsymbol{K})\,\boldsymbol{y}$ from the DISE and do some slight rearranging to write:

$$\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\boldsymbol{\pi} = -\frac{\kappa}{\sigma}\max\{-\mu\iota, \phi_\pi\boldsymbol{\pi}\} + \kappa\varepsilon.$$

Continue rearranging this expression:

$$\begin{aligned}
\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\boldsymbol{\pi} &= \frac{\kappa\mu}{\sigma}\iota - \frac{\kappa}{\sigma}\max\{\boldsymbol{0}, \phi_\pi\boldsymbol{\pi}+\mu\iota\} + \kappa\varepsilon \\
\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\boldsymbol{\pi} &= (1-1)\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\frac{\mu}{\phi_\pi}\iota \\
&\quad + \frac{\kappa\mu}{\sigma}\iota - \frac{\kappa}{\sigma}\max\left\{\boldsymbol{0}, \phi_\pi\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right)\right\} + \kappa\varepsilon \\
\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right) &= \left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\frac{\mu}{\phi_\pi}\iota \\
&\quad + \frac{\kappa\mu}{\sigma}\iota - \frac{\kappa}{\sigma}\max\left\{\boldsymbol{0}, \phi_\pi\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right)\right\} + \kappa\varepsilon.
\end{aligned}$$

But since $\left[(\boldsymbol{I}-\boldsymbol{K}) - a\boldsymbol{K} - b(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\iota = -a\iota$ for generic scalars a and b, we can write:

$$\begin{aligned}
\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right) &= -\frac{\kappa}{\sigma}\frac{\mu}{\phi_\pi}\iota + \frac{\kappa\mu}{\sigma}\iota + \kappa\varepsilon \\
&\quad - \max\left\{\boldsymbol{0}, \frac{\kappa\phi_\pi}{\sigma}\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right)\right\} \\
\left[\boldsymbol{I}-\boldsymbol{K}-\frac{\kappa}{\sigma}\boldsymbol{K}-\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right) &= \frac{\kappa\mu}{\sigma}\left(\frac{\phi_\pi-1}{\phi_\pi}\right)\iota + \kappa\varepsilon \\
&\quad - \max\left\{\boldsymbol{0}, \frac{\kappa\phi_\pi}{\sigma}\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right)\right\} \\
\left[\boldsymbol{K}-\boldsymbol{I}+\frac{\kappa}{\sigma}\boldsymbol{K}+\beta(\boldsymbol{I}-\boldsymbol{K})\boldsymbol{K}\right]\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right) &= \frac{\kappa\mu}{\sigma}\left(\frac{1-\phi_\pi}{\phi_\pi}\right)\iota - \kappa\varepsilon \\
&\quad + \max\left\{\boldsymbol{0}, \frac{\kappa\phi_\pi}{\sigma}\left(\boldsymbol{\pi}+\frac{\mu}{\phi_\pi}\iota\right)\right\}.
\end{aligned}$$

The above system can be generically written as

$$\boldsymbol{B}\boldsymbol{v} = \boldsymbol{b} + \max\{\boldsymbol{0}, \boldsymbol{D}\boldsymbol{v}\}, \tag{50}$$

where

$$\boldsymbol{B} = \boldsymbol{K} - \boldsymbol{I} + \frac{\kappa}{\sigma}\boldsymbol{K} + \beta(\boldsymbol{I} - \boldsymbol{K})\boldsymbol{K},$$
$$\boldsymbol{\nu} = \boldsymbol{\pi} + \frac{\mu}{\phi_\pi}\iota,$$
$$\boldsymbol{b} = \frac{\kappa\mu}{\sigma}\left(\frac{1-\phi_\pi}{\phi_\pi}\right)\iota - \kappa\varepsilon,$$
$$\boldsymbol{D} = \frac{\kappa\phi_\pi}{\sigma}\boldsymbol{I}.$$

The CC conditions can be analytical derived in the case that $k = 2$. We thus have a piecewise linear system in four orthants that can be written as:

$$\begin{cases} \mathbb{R}_1 & = \{(\nu_1, \nu_2) : \nu_1 \geq 0, \nu_2 \geq 0\}, \\ \boldsymbol{A}_1 & = \boldsymbol{B} - \boldsymbol{D}, \end{cases}$$
$$\begin{cases} \mathbb{R}_2 & = \{(\nu_1, \nu_2) : \nu_1 \geq 0, \nu_2 < 0\}, \\ \boldsymbol{A}_2 & = \boldsymbol{B} - \begin{pmatrix} \frac{\kappa\phi_\pi}{\sigma} & 0 \\ 0 & 0 \end{pmatrix}, \end{cases}$$
$$\begin{cases} \mathbb{R}_3 & = \{(\nu_1, \nu_2) : \nu_1 < 0, \nu_2 < 0\}, \\ \boldsymbol{A}_3 & = \boldsymbol{B}, \end{cases}$$
$$\begin{cases} \mathbb{R}_4 & = \{(\nu_1, \nu_2) : \nu_1 < 0, \nu_2 \geq 0\}, \\ \boldsymbol{A}_4 & = \boldsymbol{B} - \begin{pmatrix} 0 & 0 \\ 0 & \frac{\kappa\phi_\pi}{\sigma} \end{pmatrix}. \end{cases}$$

Let $\boldsymbol{K}$ be defined as:

$$\boldsymbol{K} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}.$$

Theorem 1 of Gourieroux, Laffont, and Monfort (1980) states that the system of equations (50) is coherent and complete if and only if all the determinants of the matrices below have the same sign:

$$\det \boldsymbol{A}_1 = \frac{\kappa(1-\phi_\pi)}{\sigma}\left[(p+q-1)\frac{\kappa}{\sigma} + a(p+q-2) - \frac{\kappa\phi_\pi}{\sigma}\right],$$
$$\det \boldsymbol{A}_2 = \frac{\kappa}{\sigma}\left[a(p-2) + (a\sigma - \kappa\phi_\pi)\frac{q}{\sigma} - a(q-1)\phi_\pi + (p+q-1)\frac{\kappa}{\sigma}\right],$$
$$\det \boldsymbol{A}_3 = \frac{\kappa}{\sigma}\left[(p+q-1)\frac{\kappa}{\sigma} + a(p+q-2)\right],$$
$$\det \boldsymbol{A}_4 = \frac{\kappa}{\sigma}\left[a(q-2) + (a\sigma - \kappa\phi_\pi)\frac{p}{\sigma} - a(p-1)\phi_\pi + (p+q-1)\frac{\kappa}{\sigma}\right],$$

where $a = (1 - \beta(p + q - 1))$. Alternatively, we could write the above matrices more compactly as:

$$\det \boldsymbol{A}_1 = (1 - \phi_\pi)\left(\det \boldsymbol{B} - \frac{\kappa^2\phi_\pi}{\sigma^2}\right), \tag{51}$$

$$\det \boldsymbol{A}_2 = \det \boldsymbol{B} + \frac{\kappa\phi_\pi}{\sigma}\left[(1-q)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}q\right], \tag{52}$$

$$\det \boldsymbol{A}_3 = \det \boldsymbol{B}, \tag{53}$$

$$\det \boldsymbol{A}_4 = \det \boldsymbol{B} + \frac{\kappa\phi_\pi}{\sigma}\left[(1-p)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}p\right]. \tag{54}$$

It is evident that the CC conditions will crucially depend on the sign of $\det \boldsymbol{B}$, which we can write as:

$$\det \boldsymbol{B} = \frac{\kappa}{\sigma}\left[(p+q-1)\frac{\kappa}{\sigma} - (1-\beta(p+q-1))(2-p-q)\right]. \tag{55}$$

Observing this quantity, we know that $\frac{\kappa}{\sigma} > 0$ and that $(1-\beta(p+q-1))(2-p-q) > 0$. Thus, we need to check the relative value of $(p+q-1)\frac{\kappa}{\sigma}$ to see if $\det \boldsymbol{B}$ is greater or less than zero. We thus need to check two cases:

Case 1: $(p+q-1)\frac{\kappa}{\sigma} > (1-\beta(p+q-1))(2-p-q)$. The RHS of the inequality is always greater than 0, thus implying:

$$p + q - 1 > 0 \implies \det \boldsymbol{B} > 0.$$

Given this, what does it mean for $\det \boldsymbol{A}_i$ for $i = \{1, 2, 4\}$? First, rearrange the quantities for $\det \boldsymbol{A}_1 > 0$ in Equation (51) to write:

$$\phi_\pi < \frac{\sigma^2}{\kappa^2}\det \boldsymbol{B},$$

as in $\det \boldsymbol{A}_1 > 0$ if and only if

$$0 < (1-\phi_\pi)(\det \boldsymbol{B} - \frac{\kappa^2}{\sigma^2}\phi_\pi),$$

$$\therefore \phi_\pi < \min\left\{1, \frac{\sigma^2}{\kappa^2}\det \boldsymbol{B}\right\} \Leftrightarrow \phi_\pi > \max\left\{1, \frac{\sigma^2}{\kappa^2}\det \boldsymbol{B}\right\}.$$

For $\det \boldsymbol{A}_2$ we need

$$\frac{\kappa\phi_\pi}{\sigma}\left[(1-q)(1-\beta(p+q-1)) - \frac{\kappa}{\sigma}q\right] + \det \boldsymbol{B} > 0.$$

If the term in the square brackets is greater than zero, then $\det \boldsymbol{A}_2 > 0, \forall \phi_\pi \geq 0$, so long as $\det \boldsymbol{B} > 0$ (which was shown above). But what if this bracketed quantity is less than zero? Then, write $\det \boldsymbol{A}_2$ as:

$$0 < \frac{\kappa\phi_\pi}{\sigma}\left[\underbrace{(1-q)(1-\beta(p+q-1)) - \frac{\kappa}{\sigma}q}_{<0}\right] + \frac{\kappa}{\sigma}\left[(p+q-1)\frac{\kappa}{\sigma} - (1-\beta(p+q-1))(2-p-q)\right],$$

where we can write:

$$\phi_\pi < \frac{(p+q-1)\frac{\kappa}{\sigma} - (1-\beta(p+q-1))(2-p-q)}{\underbrace{\frac{\kappa}{\sigma}q - (1-q)(1-\beta(p+q-1))}_{>0}} < 1.$$

Use (A.3) to then write:

$$\begin{aligned}\phi_\pi &< \frac{\sigma^2}{\kappa^2}\det \boldsymbol{B} \\ &< p+q-1-\frac{\sigma}{\kappa}((1-\beta(p+q-1))(2-p-q).\end{aligned}$$

A symmetric argument holds for $\det \boldsymbol{A}_4$.

Case 2: $(p+q-1)\frac{\kappa}{\sigma} < (1-\beta(p+q-1))(2-p-q)$. This case now assumes that $\det \boldsymbol{B} < 0$, so we need all the other determinants to be negative too.

For $\det \boldsymbol{A}_1$, clearly for any $0 \leq \phi_\pi < 1$, $\det \boldsymbol{A}_1 < 0$.

$\det \boldsymbol{A}_2$ is negative $\forall \phi_\pi \geq 0$ if and only if

$$(1-q)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}q < 0.$$

But if

$$(1-q)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}q > 0,$$

then we have

$$\begin{aligned}0 &> \det \boldsymbol{B} + \frac{\kappa\phi_\pi}{\sigma}\left[(1-q)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}q\right] \\ \phi_\pi &< -\frac{\sigma}{\kappa}\frac{\det \boldsymbol{B}}{\left[(1-q)(1+\beta(1-p-q)) - \frac{\kappa}{\sigma}q\right]} < 0.\end{aligned}$$

Denote ϕ_π^* as

$$\phi_\pi^* = p+q-1-\frac{\sigma}{\kappa}(1-\beta(p+q-1))(2-p-q),$$

then we can write the region for which the baseline NK model satisfies the CC conditions as:

$$\begin{cases}\phi_\pi < \phi_\pi^*, & \text{if } \phi_\pi^* > 0, \\ \phi_\pi < 1, & \text{if } \phi_\pi^* < 0.\end{cases}$$

These are the conditions in (49).

A.3.1. Derivation for Graphical Representation

First consider the absorbing state when $\varepsilon = 0$, and also when $\phi_y = 0$, $u_t = 0$. From (4a)-(4c), we can write *AS* as:

$$\hat{\pi} = \frac{\kappa}{1-\beta}\hat{y}. \tag{56}$$

Meanwhile, from the DISE we have:

$$\hat{\pi} = \begin{cases} \phi_\pi \hat{\pi} & \text{TR}, \\ -\mu & \text{ELB}. \end{cases}$$

Substituting *AS* into the above expression gives *AD*:

$$\hat{\pi} = \begin{cases} \frac{\kappa\phi_\pi}{1-\beta}\hat{y} & AD^{TR}, \\ -\mu & AD^{ELB}. \end{cases} \tag{57}$$

We plot *AS* and *AD* in Figure 1.

Next, consider the transitory state when $\varepsilon = \frac{\rho}{\sigma}\hat{r}^T \neq 0$.

PIR absorbing state. At time t the economy is in a transitory state. With probability p the economy remains in the transitory state, and with $(1-p)$ the economy moves to the PIR absorbing state. From (4b), we can write:

$$\hat{\pi}^T = \kappa\hat{y}^T + p\beta\hat{\pi}^T,$$

where the second term on the RHS comes from the fact that in period $t+1$ you may be in a transitory state where $\pi \neq 0$. Thus, *AS* is:

$$\hat{\pi}^T = \frac{\kappa}{1-p\beta}\hat{y}^T. \tag{58}$$

For *AD*, begin by writing the DISE as:

$$\hat{y}^T = p\hat{y}^T - \frac{1}{\sigma}(\hat{i} - p\hat{\pi}^T) + \varepsilon.$$

Rearrange and substitute in (4c) and $\varepsilon = \frac{p}{\sigma}\hat{r}^T$ to get *AD*:

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{p-\phi_\pi^T}\hat{y}^T - \frac{p}{p-\phi_\pi}\hat{r}^T & AD^{TR} \text{ for } \hat{\pi}^T \geq -\frac{\mu}{\phi_\pi}, \\ \frac{\sigma(1-p)}{p}\hat{y}^T - \frac{\mu}{p} - \hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi_\pi}. \end{cases} \tag{59}$$

ZIR absorbing state. Here in period t the economy is in a transitory state. With probably p the economy can remain in a transitory state, and with $(1-p)$ it can move to a ZIR absorbing state. Therefore, from (4b), *AS* can be written as:

$$\begin{aligned}\hat{\pi}^T &= \beta[p\hat{\pi}^T + (1-p)(-\mu)] + \kappa\hat{y}^T \\ &= \frac{\kappa}{1-p\beta}\hat{y}^T - \frac{\beta(1-p)}{1-p\beta}\mu.\end{aligned} \tag{60}$$

To find *AD*, first begin by writing the DISE as:

$$\hat{y}^T = \left[p\hat{y}^T + (1-p)\left(\frac{-\mu(1-\beta)}{\kappa}\right)\right] - \frac{1}{\sigma}\left[\hat{i} - \left(p\hat{\pi}^T + (1-p)(-\mu)\right)\right] + \varepsilon,$$

then substitute in (4c) and the ε to get *AD*:

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{p-\phi\pi}\hat{y}^T + \frac{1-p}{p-\phi\pi}\left[\frac{(1-\beta)}{\kappa}+1\right]\mu - \frac{p}{p-\phi\pi}\hat{r}^T & AD^{TR} \text{ for } \pi^T \geq -\frac{\mu}{\phi\pi}, \\ \frac{\sigma(1-p)}{p}\hat{y}^T + \frac{1-p}{p}\left[\frac{(1-\beta)\sigma}{\kappa}+1\right]\mu - \frac{\mu}{p} - \hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi\pi}. \end{cases} \tag{61}$$

To find θ simply divide the slop of AD^{TR} by the slope of *AS*:

$$\begin{aligned}\theta &= \frac{\partial AD^{ELB}/\partial\hat{y}^T}{\partial AS/\partial\hat{y}^T} \\ &= \frac{\frac{1}{p}-\sigma}{\frac{\kappa}{1-p\beta}} \\ &= \frac{\sigma(1-p)(1-p\beta)}{p\kappa}.\end{aligned}$$

We plot *AD* and *AS* for when the economy is in the transitory state and with PIR and ZIR absorbing states for $\theta > 1$ and $\theta < 1$ in Figure 2.

A.3.2. Proof of Support Restrictions

For the case of $\theta > 1$, the two solutions imply that

$$\frac{1}{r\pi^*} \leq 1$$

is a necessary support restrictions. Why? If $r\pi^* < 1$, then either the gross real interest rate, the gross target rate of inflation, or their product is less than one. But this cannot be the case as we define $\mu = \log(r\pi^*)$.

For the case of $\theta < 1$, we need further support restrictions to ensure coherency. This can be found by finding the point at which *AD* and *AS* intersect at the kink of *AD*. The case with a PIR absorbing state is analytically more tractable, so we focus on that.

$AD = AS$ when $\pi^T = -\frac{\mu}{\phi\pi}$, and when we wish to find shock size $\hat{r}^T = \bar{r}^T$ such that the equations have a solution for all $-\hat{r}^T \leq -\bar{r}^T$. Hence, the cutoff can be found by setting *AS* and AD^{TR} equal:

$$AS: \ \hat{y}^T = \frac{(1-p\beta)}{\kappa}\hat{\pi}^T$$
$$AD^{TR} \ \hat{y}^T = \frac{(\phi_\pi - p)}{\sigma(1-p)}\hat{\pi}^T + \frac{p}{\sigma(1-p)}\hat{r}^T.$$

Substitute in $\hat{\pi}^T = -\frac{\mu}{\phi_\pi}$ and rearrange to get:

$$-\hat{r}^T = \mu\left(\frac{\theta}{\phi_\pi} + \frac{\phi_\pi - p}{p\phi_\pi}\right). \tag{62}$$

A.4. Coherency and Completeness with an Endogenous State

We no longer assume that $\boldsymbol{H}_{s_t} = \boldsymbol{O}$ and $\boldsymbol{h} = \mathbf{0}$ in (40), but maintain the assumption that $\boldsymbol{X}_t$ follows a k-state stationary Markov process. This implies that, as before, the i-th column of $\mathbf{X}$ gives the value of $\boldsymbol{X}_t$ for a given state i. However, as stipulated by AM, with endogenous states the support of $\boldsymbol{Y}_t$ will vary endogenously over time along the MSV solution given by $\boldsymbol{Y}_t = f(\boldsymbol{Y}_{t-1}, \boldsymbol{X}_t)$. This implies that the solution can no longer be characterised by a time invariant matrix $\mathbf{Y}$. In other words, despite the variables $\boldsymbol{X}_t$ being time invariant (by definition as they are purely forward looking), the support of $\boldsymbol{Y}_t$ must now be a function of $\boldsymbol{Y}_{t-1}$, too. With endogenous states, along an MSV solution we have:

$$\mathbb{E}_t\left[\boldsymbol{Y}_{t+1} | \boldsymbol{Y}_t = \mathbf{Y}_t \boldsymbol{e}_i, \boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i\right] = \mathbf{Y}^i_{t+1}\boldsymbol{K}^\top \boldsymbol{e}_i,$$

Starting from terminal date, T, the model solution is:

$$\mathbf{Y}_T = \boldsymbol{G}_{J_0} y_{T-1} + \boldsymbol{Z}_{J_0},$$

where $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ can be solved from (32):

$$\mathbf{0} = \boldsymbol{A}_{s_{t,i}}\mathbf{G}\boldsymbol{e}_i + \boldsymbol{h}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\mathbf{G}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \mathbf{G}\boldsymbol{e}_i, \tag{63}$$
$$\mathbf{0} = \left(\boldsymbol{A}_{s_{t,1}}\mathbf{Z} + \boldsymbol{B}_{s_{t,i}}\mathbf{G}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \mathbf{Z} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{Z}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}}\mathbf{X} + \boldsymbol{D}_{s_{t,i}}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i, \tag{64}$$

$\forall i = 1, \ldots, k$.

$\mathbf{Y}_T$ is a function of $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$, which are both treated as known.[21] Thus, $\mathbf{Y}_T$ is known and we can solve for $\mathbf{Y}_{T-1}$ from

$$\mathbf{0} = \left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \mathbf{Y}_{T-1}\boldsymbol{e}_i\right)$$
$$+ \left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\mathbf{X} + \boldsymbol{D}_{s_{T-1},i}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i} y_{T-2}.$$

21. In practice $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ are precalculated as they are not time-varying per-se but are state dependent. For example, if J_0 always corresponds to the PIR case, then the ELB is never binding and $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ can easily be obtained from the model policy function (Blanchard and Kahn, 1980).

For every $t \leq T$ the determinants relevant for CC conditions are given by

$$|\mathcal{A}_{J_0 J_1}| = \prod_{i}^{k} \det\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right).$$

If $k = 2$, the determinants can be rewritten as

$$\begin{aligned}
|\mathcal{A}_{J_0 J_1}| &= \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \quad J_1 = \{1, 2\} \text{ (PIR,PIR)},\\
|\mathcal{A}_{J_0 J_1}| &= \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \quad J_1 = \{2\} \text{ (ZIR,PIR)},\\
|\mathcal{A}_{J_0 J_1}| &= \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \quad J_1 = \{1\} \text{ (PIR,ZIR)},\\
|\mathcal{A}_{J_0 J_1}| &= \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \quad J_1 = \{\emptyset\} \text{ (ZIR,ZIR)}.
\end{aligned} \tag{65}$$

If the model is coherent and complete, use (33), with (63) and (64), to solve for $\mathbf{Y}_{T-1}$ as a function of y_{T-2}:

$$\begin{aligned}
\mathbf{Y}_{T-1}\boldsymbol{e}_i = &-\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1}\\
&\left[\left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\mathbf{X} + \boldsymbol{D}_{s_{T-1},i}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i}\, y_{T-2}\right],
\end{aligned}$$

$\forall i = 1, \ldots, k$.

B. A New Keynesian Model with Fiscal Policy

Households.. The economy is populated with households indexed with i on a continuum of measure one. The households gain utility from consumption, dislike labour, and have access to one-period risk free bonds. The optimisation problem of the households is thus:

$$\max_{\{C_t,L_t,B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right) Z_t,$$

subject to the [nominal] period budget constraint is given by

$$(1-\tau_t^c)P_tC_t + B_t = (1-\tau_t^w)W_tL_t + R_{t-1}B_{t-1} + T_t^h,$$

where C_t is consumption, L_t is labour supply, B_t denotes bonds, R_t is nominal interest rate, P_t is the price level, τ_t^c is the consumption tax rate, τ_t^w is the wage tax rate, and T_t^h are lump-sum taxes.

The consumption bundle C_t consists of a continuum of differentiated goods, and is bundled by a CES aggregator of the form:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}.$$

The utility maximisation problem of the household results in the following intertemporal Euler equation:

$$\beta \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} = \mathbb{E}_t \frac{1-\tau_{t+1}^c}{1-\tau_t^c}.$$

The labour supply condition gives the following intratemporal Euler equation:

$$\frac{1-\tau_t^w}{1-\tau_t^c} w_t C_t^{-\sigma} = L_t^{\varphi}.$$

The intratemporal household problem of choosing a consumption bundle results in the following demand for good j:

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t.$$

Production.. Producers use labour as an input to produce differentiated consumption goods according to the following production technology:

$$Y_t(j) = A_tL_t(j).$$

The price-setting problem of an individual firm j follows Rotemberg (1982) where firm j maximises the discounted value of profits,

$$\max_{\{P_t(i)\}} \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \left[(1-\tau_t^s) P_t(j) Y_{t,T}(j) - w_T L_T(j) - \frac{\Phi}{2} \left(\frac{P_{t,T}(j)}{P_{t-1,T}(j)} - 1 \right)^2 Y_{t,T} \right],$$

subject to:

$$Y_{t,T}(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t,$$

where Φ denotes a price adjustment cost parameter for the firms.[22] $Y_{t,T}(j)$ denotes demand at time T conditional on the price unchanged since period t. The firm maximises infinite discounted stream of profits, with revenues given by the first term and costs given by the second term. The revenues of the firm are taxed with tax level denoted by τ_t^s. Households own firms, thus their revenues are discounted with the households' discount factor, $Q_{t,T}$:

$$Q_{t,T} = \beta \frac{P_t}{P_T} \left(\frac{C_T}{C_t} \right)^{-\sigma} \frac{Z_T}{Z_t}.$$

The solution to the firm problem results in the following equation for inflation:[23]

$$\Pi_t(\Pi_t - 1) = \frac{1}{\kappa} \left[\epsilon mc_t + 1 - \epsilon + \tau_t^s \epsilon - \tau_t^s \right] + \mathbb{E}_t \left[Q_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right].$$

Monetary authority.. The monetary authority uses the [gross] nominal interest rate, R_t, as its policy instrument and sets it according to a TR of the form:

$$\frac{R_t}{\bar{R}} = \max \left\{ 1, \left(\frac{\Pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right\},$$

where ϕ_π and ϕ_y is the degree of reaction to contemporaneous inflation and the output deviations, respectively.

Fiscal authority.. The nominal flow budget constraint for the government is

$$\tau_t^c C_t + \tau_t^s Y_t + \tau_t^w w_t L_t + T_t = G_t. \tag{66}$$

We make clear which taxes are enabled or disabled in each section of the paper, as we explore different tax regimes. An additional equation – a rule fiscal policy rule – is

22. We calibrate Φ to the following:

$$\Phi = \frac{\epsilon\gamma}{(1-\gamma)(1-\beta\gamma)},$$

where γ is the probability of firm j being unable to optimally adjust its price in any given period as in a model with Calvo (1983) pricing.

23. Gross inflation is defined as $\Pi_t = P_t/P_{t-1}$

needed to close the model. In what follows we explore different specifications of such rules.

Market clearing.. Markets clear, hence all output is consumed or used for government expenditure,

$$Y_t = C_t + G_t + \frac{\Phi}{2}\left(\Pi_t - 1\right)^2 Y_t.$$

Note that as we assume Rotemberg adjustment costs, the natural level of output, Y_t^n, coincides with headline output when $\Phi = 0$.

B.1. Log Linearised Equilibrium Conditions

Log linearising the non-linear model equations about a non-inflation deterministic steady state yields the following: Intertemporal Euler equation:[24]

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t + \mathbb{E}_t\left[\Delta\hat{z}_{t+1} + \Psi^c\Delta\hat{\tau}^c_{t+1} - \hat{\pi}_{t+1}\right]\right). \tag{67}$$

Labour supply condition:[25]

$$\hat{w}_t = \sigma\hat{c}_t + \varphi\hat{l}_t + \hat{\tau}^w_t\Psi^w - \Psi^c\hat{\tau}^c_t. \tag{68}$$

Output:

$$\hat{y}_t = \hat{a}_t + \hat{l}_t. \tag{69}$$

Inflation:

$$\hat{\pi}_t = \frac{1}{\Phi}(\epsilon\hat{mc}_t - \hat{\tau}^s_t) + \beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{70}$$

Marginal cost:

$$\hat{mc}_t = \hat{w}_t - \hat{a}_t. \tag{71}$$

Taylor rule:

$$i_t = \max\left\{-\mu, \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t\right\}. \tag{72}$$

Government budget constraint

$$g\hat{g}_t = \frac{T}{Y}\hat{t}_t + \tau^c c(\hat{\tau}^c_t + \hat{c}_t) + \tau^s(\hat{\tau}^s_t + \hat{y}_t) + \tau^w\frac{wL}{Y}(\hat{\tau}^w_t + \hat{w}_t + \hat{l}_t).$$

Aggregate resource constraint:

$$\hat{y}_t = c\hat{c}_t + g\hat{g}_t. \tag{73}$$

To get an expression for $\hat{y}^n_t$ in terms of $\hat{a}_t$, start by noting that wages are equal to the

24. We define $\Psi^c = \frac{\bar{\tau}^c}{1-\bar{\tau}^c}$ and $\Delta\hat{\tau}^c_{t+1} = \hat{\tau}^c_{t+1} - \hat{\tau}^c_t$.
25. We define $\Psi^w = \frac{\bar{\tau}^w}{1-\bar{\tau}^w}$.

marginal product of labour:

$$\hat{w}_t^n = \hat{a}_t, \tag{74}$$

and then combine with (68) to write:

$$\hat{a}_t = \sigma\hat{c}_t^n + \varphi\hat{l}_t^n.$$

Note that we assume that in the natural allocation, taxes and government spending do not fluctuate. Then use (73) to substitute in for $\hat{c}_t^n$, and use (69) to substitute in for $\hat{l}_t^n$:

$$\hat{a}_t = \frac{\sigma}{c}\hat{y}_t^n + \varphi(\hat{y}_t^n - \hat{a}_t).$$

Rearrange the above to write:

$$\hat{y}_t^n = \psi_{ya}\hat{a}_t, \tag{75}$$

where $\psi_{ya} = \frac{1+\varphi}{\varphi+\frac{\sigma}{c}}$.

Then, using (67) and (73), the DISE is given by

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{c}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\Delta\hat{z}_{t+1} + \mathbb{E}_t\Delta\hat{\tau}_{t+1}^c) - g\mathbb{E}_t\Delta\hat{g}_{t+1}.$$

We now turn to the NKPC. From (71), (68), and (73), marginal cost is given by

$$\hat{mc}_t = \frac{\sigma}{c}(\hat{y}_t - g\hat{g}_t) + \varphi\hat{y}_t + \hat{\tau}_t^w\Psi^w - \Psi^c\hat{\tau}_t^c - \hat{a}_t(1+\varphi).$$

Plug this into the relationship for inflation implied by Rotemberg pricing (70)

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\epsilon}{\Phi}\left(\frac{\sigma}{c}(\hat{y}_t - g\hat{g}_t) + \varphi\hat{y}_t + \Psi^w\hat{\tau}_t^w - \Psi^c\hat{\tau}_t^c - \frac{1}{\epsilon}\tau_t^s - (1+\varphi)\hat{a}_t\right),$$

to then yield the NKPC:

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa_y\hat{y}_t + \frac{\epsilon}{\Phi}\left(\Psi^w\hat{\tau}_t^w - \Psi^c\hat{\tau}_t^c - \frac{1}{\epsilon}\hat{\tau}_t^s - \frac{\sigma}{c}\hat{g}_t - (1+\varphi)\hat{a}_t\right),$$

where $\kappa_y = \frac{\epsilon}{\Phi}(\frac{\sigma}{c} + \varphi)$.

NKPC can be rewritten in terms of output gap, $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$ as follows

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa_y\hat{x}_t + \frac{\epsilon}{\Phi}\left(\Psi^w\hat{\tau}_t^w - \Psi^c\hat{\tau}_t^c - \frac{1}{\epsilon}\hat{\tau}_t^s - \frac{\sigma}{c}\hat{g}_t\right).$$

The model is thus given by

$$\hat{x}_t = \mathbb{E}_t\hat{x}_{t+1} - \frac{c}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \hat{r}_t^n), \tag{76}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa_y\hat{x}_t + \frac{\epsilon}{\Phi}\left(\Psi^w\hat{\tau}_t^w - \Psi^c\hat{\tau}_t^c - \frac{1}{\epsilon}\hat{\tau}_t^s - \frac{\sigma}{c}\hat{g}_t\right), \tag{77}$$

$$\hat{i}_t = \max\{-\mu, \phi_\pi\hat{\pi}_t + \phi_y\hat{x}_t\}, \tag{78}$$

$$g\hat{g}_t = \frac{T}{Y}\hat{\tau}_t + \tau^c c(\hat{\tau}_t^c + \hat{c}_t) + \tau^s(\hat{\tau}_t^s + \hat{x}_t + \hat{y}_t^n) + \tau^w(\hat{\tau}_t^w + \hat{w}_t + \hat{l}_t), \tag{79}$$

$$\hat{r}_t^n = \frac{\sigma\psi_{ya}}{c}\Delta\mathbb{E}_t\hat{a}_t - \mathbb{E}_t\Delta\hat{\tau}_{t+1}^c - \frac{\sigma g}{c}\mathbb{E}_t\Delta\hat{g}_{t+1} - \frac{\sigma}{c}\mathbb{E}_t\Delta\hat{z}_{t+1}, \tag{80}$$

which are equations (15) in the main text.

As mentioned, in order to close the model, tax rules and fiscal policy rules for $\hat{g}_t$, is required. In the text and subsequent appendix chapters, we describe the set of rules and assumptions we make to close the model.

B.2. Proof of Proposition 1

Use the NK-FP system in (15) with the following government spending rule:

$$\mathbb{E}_t \Delta \hat{g}_{t+1} = \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t.$$

The constraint on $\hat{i}_t$ can be either binding (ZIR) or slack (PIR). Substitute and rearrange the above system of equations to get the following:

$$\hat{x}_t = \begin{cases} \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{x}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) - g\psi_\pi \pi_t + \varepsilon_t, \\ \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(-\mu - \mathbb{E}_t \hat{\pi}_{t+1}\right) - g\psi_\pi \pi_t + \varepsilon_t, \end{cases}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t.$$

The vector of exogenous disturbances is denoted by $\boldsymbol{X}_t = \begin{pmatrix} \varepsilon_t & 0 \end{pmatrix}^\top$ and is assumed to follow a two-state first-order Markov process with transition kernel $\boldsymbol{K}$ defined as

$$\boldsymbol{K} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix},$$

where $p, q \in [0,1]$ are transition probabilities. This allows to write the model in the canonical form as in (2), and can be cast in the form $F(\mathbf{Y}) = \lambda(\mathbf{X})$. Following Gourieroux, Laffont, and Monfort (1980), it is sufficient to check that the mapping $F(\mathbf{X})$ is invertible for model coherency. The mapping is as in (3) and (47). The relevant coefficient matrices are given by

$$\boldsymbol{A}_1 = \begin{pmatrix} 1 + \frac{c}{\sigma}\phi_y + g\psi_y & \frac{c}{\sigma}\phi_\pi + g\psi_\pi \\ \kappa_y & -1 \end{pmatrix}, \quad \boldsymbol{A}_0 = \begin{pmatrix} 1 + g\psi_y & g\psi_\pi \\ \kappa_y & -1 \end{pmatrix},$$

$$\boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{pmatrix} -1 & -\frac{c}{\sigma} \\ 0 & \beta \end{pmatrix}.$$

$\mathcal{A}_{J_1}, \mathcal{A}_{J_2}, \mathcal{A}_{J_3}$, and $\mathcal{A}_{J_4}$ are then given by:

$$\mathcal{A}_{J_1} = \mathcal{A}_{J_3} = \begin{pmatrix} 1 + \frac{c}{\sigma}\phi_y + g\psi_y - p - \frac{c}{\sigma}(1-q) & \frac{c}{\sigma}\phi_\pi + g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p) \\ \kappa_y + \beta(1-q) & \beta q - 1 \end{pmatrix},$$

$$\mathcal{A}_{J_2} = \mathcal{A}_{J_4} = \begin{pmatrix} g\psi_y + 1 - p - \frac{c}{\sigma}(1-q) & g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p) \\ \kappa_y + \beta(1-q) & \beta q - 1 \end{pmatrix}.$$

Below we show that when $\psi_\pi \to \infty$, $\text{sign}(\mathcal{A}_{J_1}) = \text{sign}(\mathcal{A}_{J_2}) = \text{sign}(\mathcal{A}_{J_3}) = \text{sign}(\mathcal{A}_{J_4})$ and the model thus satisfies the CC conditions. We start with $\mathcal{A}_{J_1}$, determinant of which is given by

$$\begin{aligned} |\mathcal{A}_{J_1}| = |\mathcal{A}_{J_3}| = (\beta q - 1) \left[1 + \frac{c}{\sigma}\phi_y + g\psi_y - p - \frac{c}{\sigma}(1-q)\right] \\ - (\kappa_y + \beta(1-q)) \left[\frac{c}{\sigma}\phi_\pi + g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right] \end{aligned}$$

If ψ_π tends to infinity and ψ_y is bounded, the second term on the RHS is positive and, thus, $|\mathcal{A}_{J_1}| < 0$. We have that

$$\lim_{\psi_\pi \to \infty} |\mathcal{A}_{J_1}| = \lim_{\psi_\pi \to \infty} |\mathcal{A}_{J_3}| = -\infty \tag{81}$$

We proceed with $|\mathcal{A}_{J_2}| = |\mathcal{A}_{J_4}|$, which is nothing but

$$\begin{aligned} |\mathcal{A}_{J_2}| = |\mathcal{A}_{J_4}| = \left[g\psi_y + 1 - p - \frac{c}{\sigma}(1-q)\right] (\beta q - 1) \\ - (\kappa_y + \beta(1-q)) \left[g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right] \end{aligned} \tag{82}$$

As previously, the second term on the RHS is positive if ψ_π tends to infinity and ψ_y is bounded. Hence, we have that

$$\lim_{\psi_\pi \to \infty} |\mathcal{A}_{J_2}| = \lim_{\psi_\pi \to \infty} |\mathcal{A}_{J_4}| = -\infty \tag{83}$$

Thus, we have that if $\psi_\pi \to \infty$, the determinants of $\mathcal{A}_{J_j}$, $j \in \{1, 2, 3, 4\}$, are negative. If $\psi_\pi \to -\infty$, the same logic applies. In this case, the determinants of $\mathcal{A}_{J_j}$, $j \in \{1, 2, 3, 4\}$, are positive. This completes the proof. ■

Lower bound for $\boldsymbol{\psi_\pi}$. We now find the lower bound for ψ_π that guarantees the satisfaction of the CC conditions. For ease of exposition, we assume that $\psi_y = 0$. The model is coherent and complete, when $|\mathcal{A}_{J_k}| < 0$. Hence ψ_π must satisfy:

$$\begin{cases} \left[1 + \frac{c}{\sigma}\phi_y - p - \frac{c}{\sigma}(1-q)\right] (\beta q - 1) - (\kappa_y + \beta(1-q)) \left[\frac{c}{\sigma}\phi_\pi + g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right] < 0, \\ \left[1 - p - \frac{c}{\sigma}(1-q)\right] (\beta q - 1) - (\kappa_y + \beta(1-q)) \left[g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right] < 0. \end{cases}$$

Rearrange to get

$$\begin{cases} \left[1 + \frac{c}{\sigma}\phi_y - p - \frac{c}{\sigma}(1-q)\right] (\beta q - 1) < (\kappa_y + \beta(1-q)) \left[\frac{c}{\sigma}\phi_\pi + g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right], \\ \left[1 - p - \frac{c}{\sigma}(1-q)\right] (\beta q - 1) < (\kappa_y + \beta(1-q)) \left[g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p)\right]. \end{cases}$$

Rearrange to get the system that ψ_π must satisfy. Note that depending on the values of monetary policy feedback parameters, ϕ_π and ϕ_y, one of the conditions is redundant

$$\begin{cases} \frac{\left[1+\frac{c}{\sigma}\phi_y - p - \frac{c}{\sigma}(1-q)\right](\beta q-1)}{(\kappa_y+\beta(1-q))} < \frac{c}{\sigma}\phi_\pi + g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p), \\ \frac{\left[1-p-\frac{c}{\sigma}(1-q)\right](\beta q-1)}{(\kappa_y+\beta(1-q))} < g\psi_\pi - 1 + p - \frac{c}{\sigma}(1-p). \end{cases}$$

If the monetary authority follows strict inflation targeting ($\phi_y = 0$), given the second inequality, the first one is redundant with respect to ψ_π. Thus inspecting the second inequality we have:

$$\left\{ \frac{\left[1-p-\frac{c}{\sigma}(1-q)\right](\beta q-1)}{(\kappa_y+\beta(1-q))} + 1 - p + \frac{c}{\sigma}(1-p) \right\} \frac{1}{g} = \underline{\psi_\pi} < \psi_\pi,$$

where $\underline{\psi_\pi}$ denotes the lower bound for ψ_π.

B.3. Canonical form coefficients, Proposition 2

The relevant coefficient matrices for the proof of proposition 2 are:

$$\boldsymbol{A}_1 = \begin{pmatrix} -c\sigma^{-1}\phi_\pi + g\psi_\pi & -1 + g\psi_y - c\sigma^{-1}\phi_y \\ -1 - \kappa_g\psi_\pi & \kappa_y - \kappa_g\psi_y \end{pmatrix},$$

$$\boldsymbol{A}_0 = \begin{pmatrix} g\psi_\pi & -1 + g\psi_y \\ -1 - \kappa_g\psi_\pi & \kappa_y - \kappa_g\psi_y \end{pmatrix},$$

and

$$\boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{pmatrix} c\sigma^{-1} - g\psi_\pi & 1 - g\psi_y \\ \beta & 0 \end{pmatrix}.$$

B.4. The Unconventional Fiscal Policy Case

The first fiscal rule we inspect is what we term the "unconventional fiscal policy" (UFP) rule that replicates monetary policy at the ELB and mirrors the approach in Correia et al. (2013). Assume that the government expenditure growth rate, $\mathbb{E}_t\Delta\hat{g}_{t+1}$, responds to contemporaneous inflation and the output gap when the interest rate is at the ELB:

$$\mathbb{E}_t\Delta\hat{g}_{t+1} = \mathbb{1}\left(\{\hat{i}_t = -\mu\}\right)(\psi^u_\pi\hat{\pi}_t + \psi^u_y\hat{x}_t), \tag{84}$$

where ψ^u_π and ψ^u_y denote the coefficients of reaction to inflation and the output gap, respectively.

The presence of the FP instrument in the DISE allows the piecewise linear system to satisfy the CC conditions, despite the presence of the ELB constraint on $\hat{i}_t$ and an active

TR. The instrument $\mathbb{E}_t \Delta \hat{g}_{t+1}$ has the same effect in the NK model as the monetary policy instrument and, hence, it governs the linearity of the DISE (15a). The CC conditions are satisfied so long as:

$$\psi_\pi^u = \frac{c}{g\sigma}\phi_\pi, \quad \psi_y^u = \frac{c}{g\sigma}\phi_y, \tag{85}$$

which also allows (15c) to follow an active TR ($\phi_\pi > 1$). It is straightforward to see that since the model is now linear, it is generally coherent and complete. The UFP rule embeds the mechanism of the simple model in Correia et al. (2013), which showed that a set of tax instruments can replicate monetary policy when the interest rate subject to the ELB constraint. This rule also applies to models where monetary policy is strictly inflation targeting, whereby if $\phi_y = 0$ then $\psi_y = 0$. Thus, we have the following proposition:

PROPOSITION 5. *A baseline New Keynesian model with fiscal policy that consists of government spending, lump-sum, and output taxes as defined in* (15)*, is generally coherent and complete when the sensitivity parameters of the fiscal instrument,* ψ_π^u *and* ψ_y^u*, allow fiscal policy to replicate monetary policy at the ELB as described in the "unconventional fiscal rule in Equation* (84).

Coherency and completeness in this case is illustrated in Figure 13 for the special case where $\phi_y = \psi_y^u = 0$. We plot *AD* and *AS* for both the absorbing (steady state) case where $\varepsilon_t = 0$ (Subfigure 13A) and the transitory state with a PIR absorbing state (Subfigure 13B).

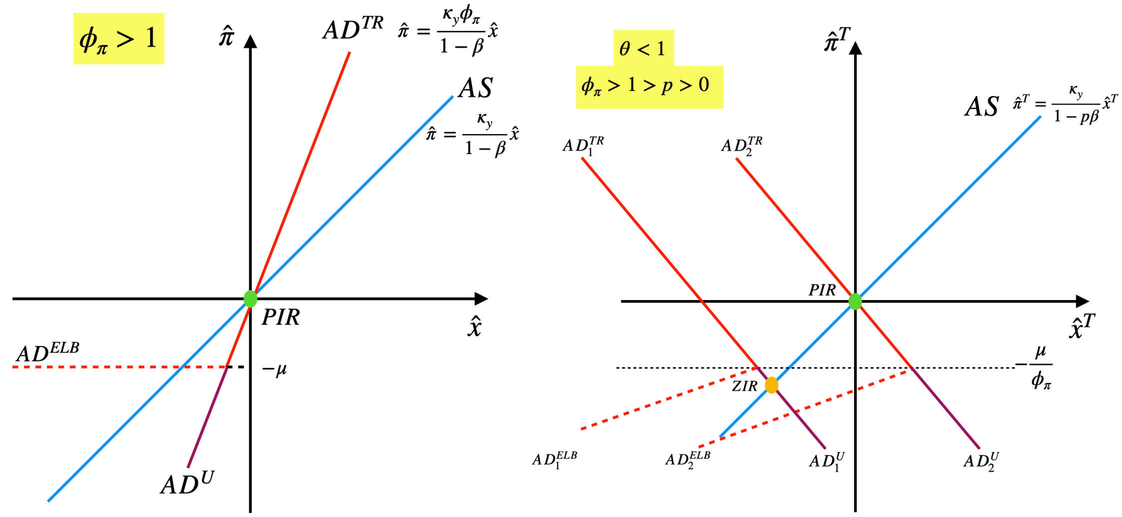
In the absence of active FP, the *AD* curve is illustrated, as before, with a piecewise red line, which may not intersect *AS* as shown with $AD^{ELB,TR}$ in Subfigure 13A and AD_1 in Subfigure 13B. Once FP is activate at the ELB, as in the UFP fiscal rule (84), it fully mimics monetary policy as if the latter were unconstrained. Thus, *AD* is a linear relation composed of the red AD^{TR} line and the purple AD^u line. In other words, in the presence of active FP stemming from the UFP rule, the model always has a unique solution.

B.5. Optimal Monetary Policy with Discretion

We now consider the case where the monetary authority operates optimal monetary policy under discretion (OP), as in Nakata (2018) and Nakata and Schmidt (2019). The optimal policy condition, when $\hat{i}_t$ is unconstrained, is:

$$\alpha_y \hat{y}_t + \kappa_y \hat{\pi}_t = 0, \tag{86}$$

FIGURE 13. Coherency and Completeness with Unconventional Fiscal Policy Rule



A. Absorbing State B. Transitory State with PIR Absorbing

Note: Left panel illustrates the steady-state equilibrium. Right panel illustrates the transitory state equilibrium with a PIR absorbing state.

where α_y is the relative weight that the policy maker attaches to the output gap in its loss function. When the ELB is non-binding, the model is given by condition (86), together with the NKPC given by

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \kappa_y \hat{x}_t, \tag{87}$$

and the fiscal rule is as before given by Equation (16):

$$\mathbb{E}_t \Delta \hat{g}_{t+1} = \psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t.$$

When the ELB is binding the model is given by the following set of equations:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{c}{\sigma}(-\mu - \mathbb{E}_t \hat{\pi}_{t+1}) - g\mathbb{E}_t \Delta \hat{g}_{t+1} + \varepsilon_t, \tag{88}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t. \tag{89}$$

We thus have the following proposition

PROPOSITION 6. *A baseline New-Keynesian model with fiscal policy that consists of government spending, lump-sum taxes, and output taxes as defined in* (86)-(89) *is generally coherent and complete when the reaction of fiscal policy to deviations of inflation is sufficiently strong.*

Proof of proposition. The model can be cast in the canonical form with the relevant matrices given by

$$\boldsymbol{A}_1 = \begin{pmatrix} \alpha_y & \kappa_y \\ \kappa_y & -1 \end{pmatrix}, \quad \boldsymbol{A}_0 = \begin{pmatrix} 1 & g\psi_\pi \\ \kappa_y & -1 \end{pmatrix},$$

$$\boldsymbol{B}_1 = \begin{pmatrix} 0 & 0 \\ 0 & \beta \end{pmatrix}, \quad \boldsymbol{B}_0 = \begin{pmatrix} -1 & -\frac{c}{\sigma} \\ 0 & \beta \end{pmatrix}.$$

$\mathcal{A}_{J_1}, \mathcal{A}_{J_2}, \mathcal{A}_{J_3}, \mathcal{A}_{J_4}$ are given by

$$\mathcal{A}_{J_1} = \mathcal{A}_{J_3} = \begin{pmatrix} \alpha_y & \kappa_y \\ \kappa_y + \beta(1-q) & \beta q - 1 \end{pmatrix},$$

$$\mathcal{A}_{J_2} = \mathcal{A}_{J_4} = \begin{pmatrix} 1 - p - \frac{c}{\sigma}(1-q) & g\psi_\pi - 1 + p - \frac{c}{\sigma}q \\ \kappa_y + \beta(1-q) & \beta q - 1 \end{pmatrix}.$$

We start with $|\mathcal{A}_{J_1}| = |\mathcal{A}_{J_3}|$:

$$|\mathcal{A}_{J_1}| = |\mathcal{A}_{J_3}| = \alpha_y(\beta q - 1) - \kappa_y(\kappa_y + \beta(1-q)) < 0.$$

Since $|\mathcal{A}_{J_1}| = |\mathcal{A}_{J_3}| < 0$, we require that ψ_π is such that $|\mathcal{A}_{J_2}| = |\mathcal{A}_{J_4}| < 0$

$$\begin{aligned} |\mathcal{A}_{J_2}| = |\mathcal{A}_{J_4}| = {} & (1 - p - \frac{c}{\sigma}(1-q))(\beta q - 1) \\ & - (\kappa_y + \beta(1-q))(g\psi_\pi - 1 + p - \frac{c}{\sigma}q) \end{aligned}$$

If $\psi_\pi \to \infty$, the determinants are negative. This completes the proof. ■

Lower bound for $\boldsymbol{\psi_\pi}$. ψ_π must satisfy that $|\mathcal{A}_{J_2}| = |\mathcal{A}_{J_4}| < 0$

$$(1 - p - \frac{c}{\sigma}(1-q))(\beta q - 1) < (\kappa_y + \beta(1-q))(g\psi_\pi - 1 + p - \frac{c}{\sigma}q)$$

which yields the lower bound for ψ_π

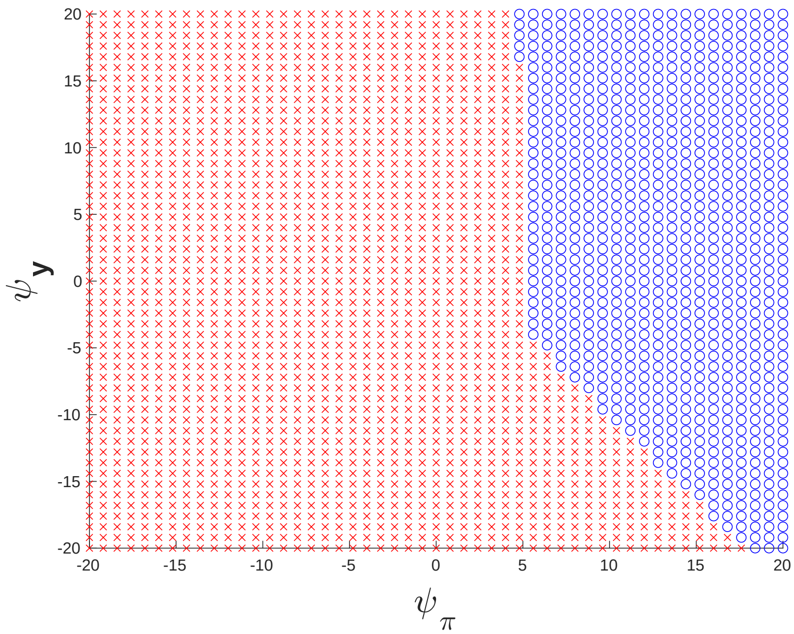
$$\left\{ \frac{(1 - p - \frac{c}{\sigma}(1-q))(\beta q - 1)}{(\kappa_y + \beta(1-q))} + 1 - p + \frac{c}{\sigma}q \right\} g^{-1} = \underline{\psi_\pi} < \psi_\pi$$

We plot the region for where the model satisfies the CC conditions as a function of ψ_π and ψ_y in Figure 14.

To illustrate the intuition of our findings, consider the absorbing state of the model with $\varepsilon_t = 0$.

$$\hat{\pi} = \frac{\kappa_y}{1-\beta}\hat{y} \quad AS, \tag{90a}$$

FIGURE 14. Coherency and Completeness Region for Optimal Monetary Policy and Inflation and Output Gap Fiscal Rule



Blue denotes regions where coherency and completeness conditions are satisfied. Red denotes regions where the model is incoherent or incomplete.

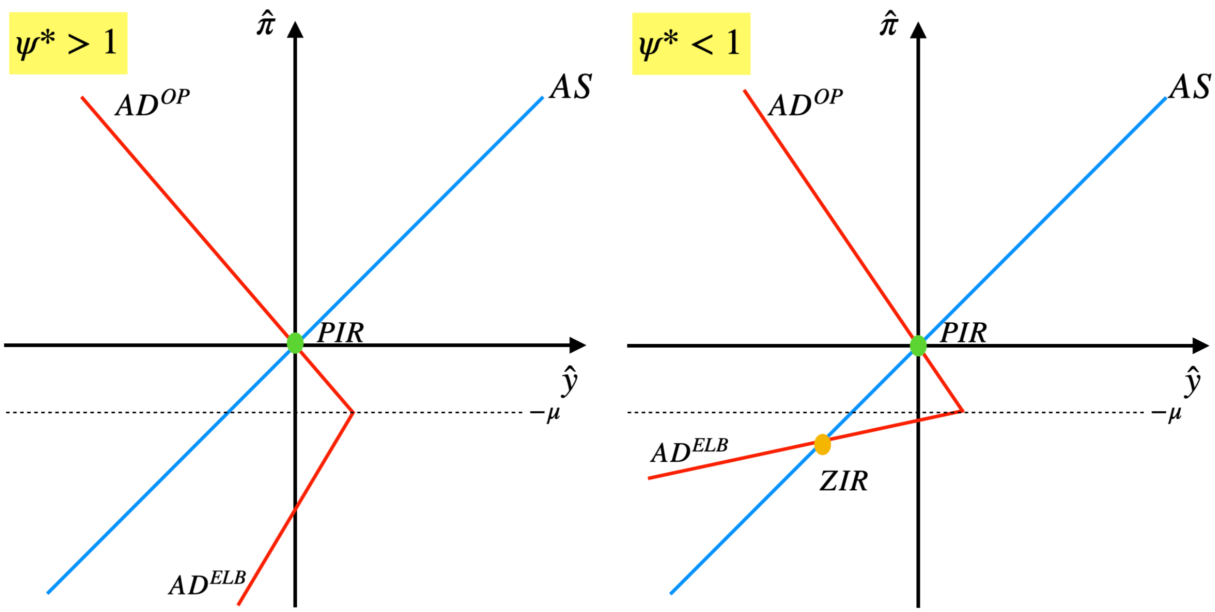
$$\hat{\pi} = \begin{cases} -\frac{\alpha_y}{\kappa_y}\hat{y} & AD^{OP} \text{ for } \hat{\pi} \geq -\mu, \\ \psi^* \frac{\kappa_y}{1-\beta}\hat{y} - \mu & AD^{ELB} \text{ for } \hat{\pi} \leq -\mu. \end{cases} \tag{90b}$$

We plot this system of equations for the case of passive and active FP in Figure 15.

We underline the following when observing Equation (90). First, note that AD^{ELB} in this regime is identical to AD^{ELB} in Equation (18). This makes intuitive sense as when facing the ELB constraint, the monetary authority is no longer able to conduct optimal monetary policy. Secondly, as seen in Figure 15, AD^{OP} has a negative slope which implies that there always exists a PIR equilibrium. The ZIR equilibrium can only exist below the AD^{OP} line when FP is passive ($\psi^* < 1$). Additionally, in the case where $\psi^* < 1$, AD^{ELB} is bound from above by the ELB on the interest rate, $-\mu$, whereby $\psi^* = 0$. Hence, we can rule out multiple PIR equilibria, and the system in Equation (90) nests the NK-OP system as described in Ascari and Mavroeidis (2022).

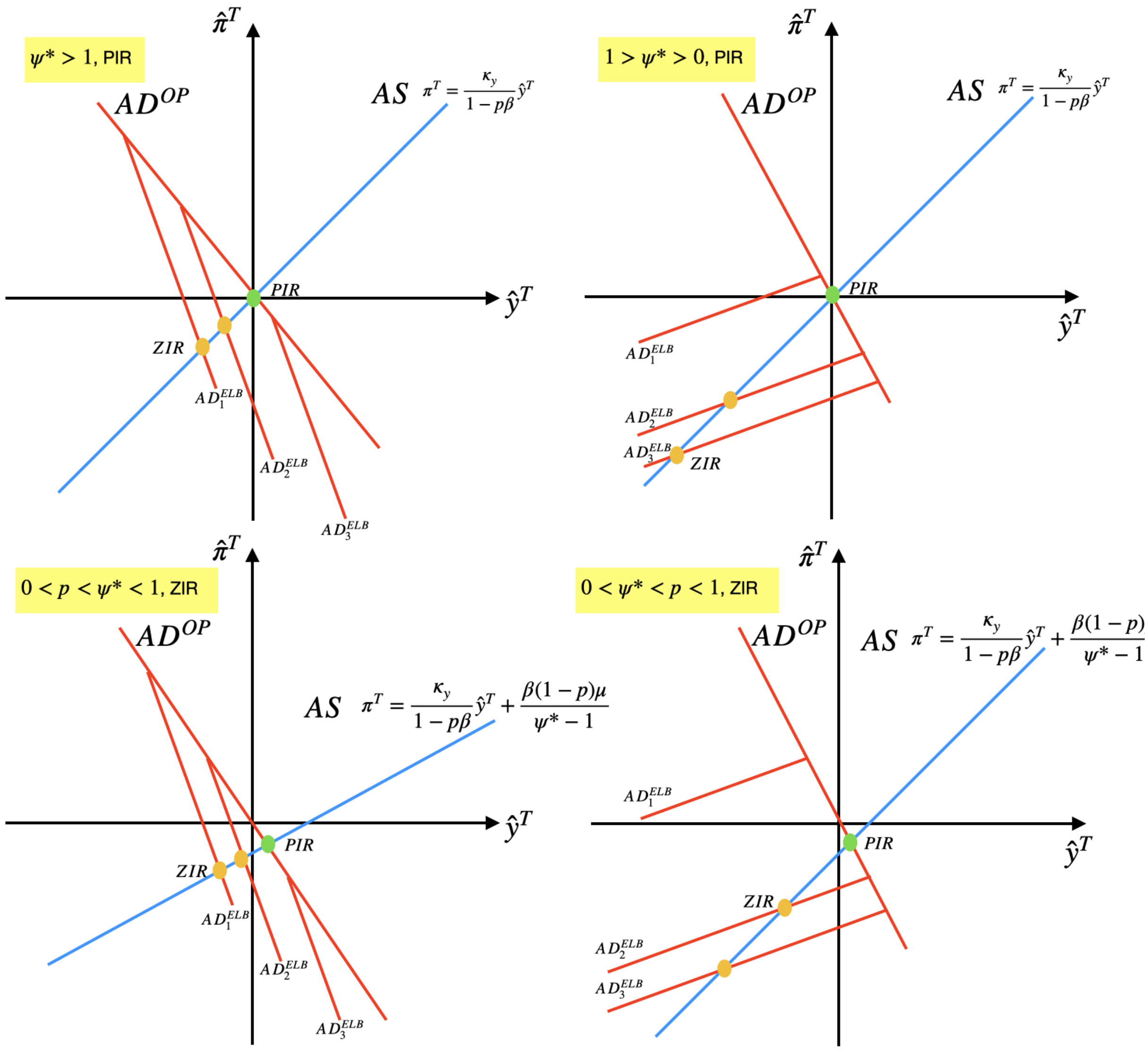
Next, consider the transitory state with $\varepsilon_t = \frac{p}{\sigma}\hat{r}^T < 0$. Here the economy starts off in a transitory state for an indefinite period of time before jumping to an absorbing state. Below we describe the MSV for both PIR and ZIR absorbing states, which we plot in Figure 16.

FIGURE 15. Absorbing State with Optimal Discretionary Monetary Policy ($\varepsilon_t = 0$)



Plot on the left depicts the positive interest rate absorbing state with an active fiscal policy regime. Plot on the right shows the equilibria for a passive fiscal policy regime.

FIGURE 16. Transitory States with Optimal Discretionary Monetary Policy under Active or Passive Fiscal Policy



Top row plots are with a positive interest rate absorbing state. Bottom row plots are with a zero interest rate absorbing state. Top left plot is with an active fiscal policy regime ($\psi^* > 1$). Top right and bottom plots are with passive fiscal policy regimes ($\psi^* < 1$).

OP transitory state with PIR absorbing. This implies that the system takes the following form:

$$\hat{\pi}^T = \frac{\kappa_y}{1-\beta p}\hat{y}^T \quad AS, \tag{91a}$$

$$\hat{\pi}^T = \begin{cases} -\frac{\alpha_y}{\kappa_y}\hat{y}^T & AD^{OP}, \\ \frac{\sigma(1-p)}{c(p-\psi^*)}\hat{y}^T - \frac{\mu}{p-\psi^*} - \frac{p}{c(p-\psi^*)}\hat{r}^T & AD^{ELB}. \end{cases} \tag{91b}$$

With active FP, AD^{OP} and AD^{ELB} are both downward sloping. Analogously, as in the absorbing state where $\varepsilon_t = 0$, AD^{ELB} in Equations (20) and (91) are identical. Thus, the model is coherent and complete. However, this is not true for passive FP, whereby the acutely kinked *AD*-curve implies the presence of a ZIR absorbing state. Thus, we conclude that when $\psi^* < 1$, the NK-OP model fails to satisfy the CC conditions.

OP transitory state with ZIR absorbing. A ZIR absorbing state is unfeasible with an active FP regime ($\psi^* > 1$), as shown in Equation (90) and Figure 15. Thus, for $\psi^* > 1$, a system in a transitory state will eventually move to a PIR absorbing state as described above.

As mentioned, when $\psi^* < 1$ the model does not satisfy CC conditions and there exists a ZIR absorbing state, as the the slope of AD^{ELB} can be upward sloping and flatter than that of *AS*. In such a case, the system takes the following form:

$$\hat{\pi}^T = \frac{\kappa_y}{1-\beta p}\hat{y}^T + \beta\frac{(1-p)\mu}{\psi^*-1} \quad AS, \tag{92a}$$

$$\hat{\pi}^T = \begin{cases} -\frac{\alpha_y}{\kappa_y}\hat{y}^T & AD^{OP}, \\ \frac{(1-p)\sigma}{c(p-\psi^*)}\hat{y}^T - \frac{(1-p)\mu}{(p-\psi^*)(\psi^*-1)}\left[\frac{(1-\beta)\sigma}{c\kappa_y}+1\right] - \frac{\mu}{p-\psi^*} - \frac{p}{c(p-\psi^*)}\hat{r}^T & AD^{ELB}. \end{cases} \tag{92b}$$

Here too AD^{ELB} in Equations (22) and (92) are identical, following the previously explained logic.

B.6. Canonical Form Coefficients under Consumption Tax Rules

A contemporaneous inflation targeting rule implies:

$$\mathbb{E}_t \Delta\hat{\tau}^c_{t+1} = \psi_\pi \hat{\pi}_t. \tag{93}$$

The relevant coefficient matrices are given by:

$$\boldsymbol{A}_1 = \begin{pmatrix} 1+\phi_y\sigma^{-1} & (\phi_\pi+\psi_\pi)\sigma^{-1} \\ \kappa_y & -1 \end{pmatrix}, \quad \boldsymbol{A}_0 = \begin{pmatrix} 1 & \psi_\pi\sigma^{-1} \\ \kappa_y & -1 \end{pmatrix},$$

and

$$\boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{pmatrix} -1 & -\sigma^{-1} \\ 0 & \beta \end{pmatrix}.$$

A contemporaneous inflation and output targeting rule implies:

$$\mathbb{E}_t \Delta \hat{\tau}^c_{t+1} = \psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t \tag{94}$$

The relevant coefficient matrices are given by:

$$\boldsymbol{A}_1 = \begin{pmatrix} 1 + (\phi_y + \psi_y)\sigma^{-1} & \sigma^{-1}(\phi_\pi + \psi_\pi) \\ \kappa_y & -1 \end{pmatrix}, \quad \boldsymbol{A}_0 = \begin{pmatrix} 1 + \psi_y \sigma^{-1} & \sigma^{-1}\psi_\pi \\ \kappa_y & -1 \end{pmatrix},$$

and

$$\boldsymbol{B}_1 = \begin{pmatrix} -1 & -\sigma^{-1} \\ 0 & \beta \end{pmatrix}, \quad \boldsymbol{B}_0 = \boldsymbol{B}_1.$$

B.7. NK-FP Model with Government Spending Inertia

The canonical form coefficients are given by

$$\boldsymbol{A}_1 = \begin{pmatrix} \kappa_y & -1 & 0 & \kappa_g \\ -1 & 0 & -\frac{c}{\sigma} & g \\ \phi_y & \phi_\pi & -1 & 0 \\ \psi_y & \psi_\pi & 0 & -1 \end{pmatrix}, \quad \boldsymbol{A}_0 = \begin{pmatrix} \kappa_y & -1 & 0 & \kappa_g \\ -1 & 0 & 0 & g \\ \phi_y & \phi_\pi & -1 & 0 \\ \psi_y & \psi_\pi & 0 & -1 \end{pmatrix},$$

$$\boldsymbol{B}_0 = \boldsymbol{B}_1 = \begin{pmatrix} 0 & \beta & 0 & 0 \\ 1 & \frac{c}{\sigma} & 0 & -g \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{h}_0 = \boldsymbol{h}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho_g \end{pmatrix},$$

$$\boldsymbol{C}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{C}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{c\mu}{\sigma} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{Y}_t = \begin{pmatrix} \hat{x}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{g}_t \end{pmatrix}, \quad \boldsymbol{Y}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{g}_{t-1} \end{pmatrix}, \quad \boldsymbol{X}_t = \begin{pmatrix} u_t \\ \varepsilon_t \\ 0 \\ 1 \end{pmatrix}.$$