# CBDCs, Financial Inclusion, and Optimal Monetary Policy*

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23 February 2024
(first version: 6 May 2023)

**Abstract**

In this paper we study the interaction between monetary policy and financial inclusion in an economy that introduces a central bank digital currency (CBDC). Using a New Keynesian two-agent framework with banked and unbanked households, we show that CBDCs provide a more efficient savings device for the unbanked to smooth consumption, increasing welfare. A Ramsey optimal policy exercise reveals that the CBDC rate is set at a constant spread to the policy rate. We observe a policy trade-off: a higher CBDC rate benefits the unbanked, but disintermediates banks and reduces welfare of banked households. Taken together, our findings highlight the role of tailoring CBDC design based on the level of financial inclusion in an economy.

**Keywords**: central bank digital currency, financial inclusion, optimal monetary policy, Taylor rules, welfare

**JEL Classifications**: E420, E440, E580

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The authors kindly thank Andrea Ferrero and Guido Ascari for their feedback and guidance. We also thank David Bounie, Abel Francois, Todd Keister, Michael Kumhoff, Jean-Baptiste Michau, Ines Goncalves Raposo, Linda Schilling, Antonella Trigari, Michael Wulfsohn, and seminar participants at Institut Polytechnique de Paris, Télécom Paris, Bocconi University, University of Pavia, University of Warwick, 2022 UWA Blockchain and Cryptocurrency Conference, 17th Dynare Conference, EEA-ESEM Conference 2023, and the 2023 Warsaw MMF Conference for helpful comments and feedback.

# 1 Introduction

Central bank digital currency (CBDC) is a form of digital money, denominated in the national unit of account, which is a direct liability of the central bank.[1] Central banks are actively studying the potential adoption of CBDCs; notable examples include Sweden's E-Krona and China's Digital Currency Electronic Payment. In this emerging macroeconomics literature there is a focus on the macroeconomic effects, and implications for banking and financial stability.

In this paper, we focus on the welfare implications of introducing a retail CBDC. We answer a number of macroeconomic questions on CBDC design: do CBDCs increase welfare of the unbanked through financial inclusion? Do they fundamentally change monetary policy transmission? Should a CBDC be interest bearing, and how should interest rates be optimally set?

We answer these questions using a two-agent New Keynesian (TANK) model with a financial intermediary that invests in firms' equity, and a central bank that sets interest rates on both deposits and the CBDC. Additionally, the two types of households in our model are referred to as the "banked" and "unbanked". Banked households are akin to "unconstrained" households as in, for example, Galí, López-Salido, and Vallés (2007), Bilbiie (2018), and Debortoli and Galí (2017), and operate on their Euler equation due to having access to a non-contingent asset, bank deposits. Conversely, the unbanked can only smooth their consumption through real money balances and are subject to a cash-in-advance constraint. We then relax this restriction by allowing both the banked and unbanked household access to an interest bearing CBDC.

Our analysis proceeds as follows. First, we address the effect of a CBDC on the transmission of monetary policy. We assume the central bank implements a standard Taylor rule, and the CBDC interest rate tracks the rate on deposits. When a CBDC is introduced, the most notable difference lies in the response of unbanked household

1. For more detail on the taxonomy of CBDC designs we refer readers to Auer and Böhme (2020). They discuss many aspects of CBDC design, such as whether the CBDC uses a distributed ledger technology (DLT), is account or token based or wholesale or retail. In this paper we focus solely on retail CBDCs.

consumption. These households can actively mitigate the shock by reducing their savings, thereby attenuating their decline in consumption. This moderates the overall impact on consumption and output, leading to a quicker dissipation of the shocks compared to an economy without a CBDC. In contrast, banked households show minimal changes in consumption responses to policy shocks with a CBDC due to having alternative savings in deposits. Consequently, the bank's balance sheet remains comparable to a scenario without a CBDC. Conditional on a monetary policy shock, we observe a trade-off between macroeconomic and financial stability: while output and consumption effects dissipate faster in a CBDC-equipped economy, there is a more persistent effect of monetary policy shocks on bank equity prices and net worth.

Second, we explore the distributional implications of CBDC rates relative to policy rates and their impact on welfare. Our findings reveal that unbanked households benefit when CBDC rates exceed policy rates, while banked households are worse off. The savings channel explains the positive effect on unbanked households, as CBDC yields a higher interest rate. Conversely, high CBDC rates adversely affect banked households through the disintermediation channel, causing a reduction in bank balance sheets, equilibrium lending, capital, and consumption. Our analysis highlights a crucial trade-off in setting CBDC interest rates: for economies with a high level of financial inclusion, lower CBDC rates are optimal, while economies with low financial inclusion may benefit from higher CBDC rates to encourage unbanked households to utilize CBDC for consumption smoothing.

Third, we conduct a Ramsey optimal policy exercise to evaluate the path of monetary policy that maximizes welfare of households. The social planner maximizes a weighted average of banked and unbanked household welfare using the two policy instruments: the central bank rate on bank deposits and the CBDC interest rate. Our framework allows to test alternative regimes for the CBDC monetary policy implementation, such as whether the CBDC rate should be adjustable or fixed. The results show that when CBDC are a near substitute to regular deposits, it is optimal for the CBDC rate to track a constant spread with the policy rate.

Additionally, we decompose the welfare effects of introducing a CBDC and implementing optimal monetary policy with one or two instruments. For economies with low financial inclusion, the welfare gains are primarily associated with the introduction of CBDC, whereas in economies with higher financial inclusion, the gains come from optimal monetary policy. Optimal monetary policy with one instrument, where the CBDC rate tracks the policy rate, yields quantitatively similar welfare outcomes to optimal policy with two instruments.

Our work relates to three literature on CBDCs. First, we contribute to a literature understanding the benefits of introducing a CBDC (Chen et al., 2022).[2] Our contribution is to show that the welfare effects depend crucially on the level of financial inclusion, with positive welfare effects on the unbanked through a CBDC increasing savings and acting as a consumption smoothing device.

Second, we contribute to the literature on the implications of CBDC adoption for financial stability (Brunnermeier and Niepelt, 2019; Fernández-Villaverde et al., 2021; Agur, Ari, and Dell'Ariccia, 2022; Andolfatto, 2021; Chiu et al., 2023; Keister and Sanches, 2021; Keister and Monnet, 2022). Financial stability considerations include studying the competition between bank deposits and CBDCs. For example, Keister and Sanches (2021) determine conditions in which the private sector is dis-intermediated with CBDC leading to welfare losses. We incorporate elements of the disintermediation channel in our framework through modeling the substitution between bank deposits and CBDC. We find that there are negative effects on banked agents consistent with a disintermediation channel.

Finally, we contribute to an growing literature that deals with the closed economy (Burlon et al., 2022; Davoodalhosseini, 2022; Das et al., 2023; Barrdear and Kumhof, 2022; Assenmacher, Bitter, and Ristiniemi, 2023; Abad, Nuño Barrau, and Thomas, 2023) and open economy macroeconomic implications of introducing a CBDC (Ikeda, 2020; Kumhof et al., 2021; Minesso, Mehl, and Stracca, 2022). This includes a discus-

2. These studies include the potential for CBDCs to address financial inclusion in emerging market economies such as India and Nigeria, which have a large unbanked population and increasing reliance on digital payments and private payment providers, and theoretical models of financial inclusion in an economy with competition between different types of payments.

sion of optimal monetary policy and transmission effects, the use of CBDC in a monetarist framework, and the introduction of CBDC on output and the ability to stabilize business cycle fluctuations. Our contribution is to show the transmission of monetary policy and derive the optimal path of interest rates when the central bank controls two instruments: the interest rate on deposits and the CBDC interest rate. The welfare effects on banked and unbanked agents depend crucially on whether the CBDC is interest bearing, and through a Ramsey optimal policy exercise we show that CBDC rates should target a constant spread with respect to the policy rate.

The remainder of the paper is structured as follows. In Section 2 we outline a simple two-period, two-agent endowment economy to clarify our intuition, and examine the welfare implications of introducing a CBDC. In Section 3, we setup the TANK model and state our modeling assumptions. Section 4 examines the effect of introducing a CBDC on monetary policy, including optimal policy exercises for when a social planner can set interest rates on deposits and the CBDC, and examines the welfare implications of alternative rules for targeting the interest rate on the CBDC. Section 5 concludes the paper.

## 2 Simple Endowment Economy

To highlight the key mechanisms through which digital currency can improve welfare, we consider a simplified two-agent model, where an agent can be of type $i = \{h, u\}$. In this setup, the banked household (BHH; $i = h$) has access to a first-best risk-free savings device $(D)$, while the unbanked household (UHH; $i = u$) can save in money balances $(M)$.[3] Each of the agents lives for two periods, receives an initial endowment $(y)$ in the first period, and maximizes lifetime utility,

$$u^i = \ln c_1^i + \beta \ln c_2^i,$$

subject to a set of budget constraints for each period.

3. We abstract from inflation in this simple setup as we do not discuss considerations in setting interest rates. We include inflation in our TANK framework in Section 3 where we also study optimal monetary policy.

**No digital currency.** For the banked, they face the following budget constraints:

$$c_1^h + D = y, \tag{1a}$$

$$c_2^h = RD + \epsilon, \tag{1b}$$

where $R > 1$ is the return on $D$, and $\epsilon$ is a shock that impacts resources in the second period.[4] Conversely, the unbanked face a set of budget constraints and a cash-in-advance (CIA) constraint on their consumption in the second period, and so their constraints are:

$$c_1^u + M = y, \tag{2a}$$

$$c_2^u \leq M + \epsilon, \tag{2b}$$

$$\alpha_M c_2^u \leq M, \tag{2c}$$

where $\alpha_M \in (0, 1]$ is the fraction of consumption that is subject to the cash-in-advance constraint. It is similar to the inverse of the velocity of money. In what follows, we assume $\alpha_M = 1$ for tractability.[5]

Solving for optimal consumption in periods 1 and 2[6] for both households yields the following lifetime consumption ratio:

$$\frac{c_1^h + c_2^h}{c_1^u + c_2^u} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y + \mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0 \end{cases} \tag{3}$$

Figure 1 plots the consumption ratio (3) with respect to the expected value of the shock.[7] As the Figure illustrates, the BHH have higher lifetime consumption than the UHH. These consumption gains are increasing in the magnitude of the income shock. Deposits of the BHH are countercyclical with respect to the income shock: the banked save in anticipation of a negative income shock, and reduce savings in anticipation of positive income shocks, enabling them to better smooth consumption. In contrast, the

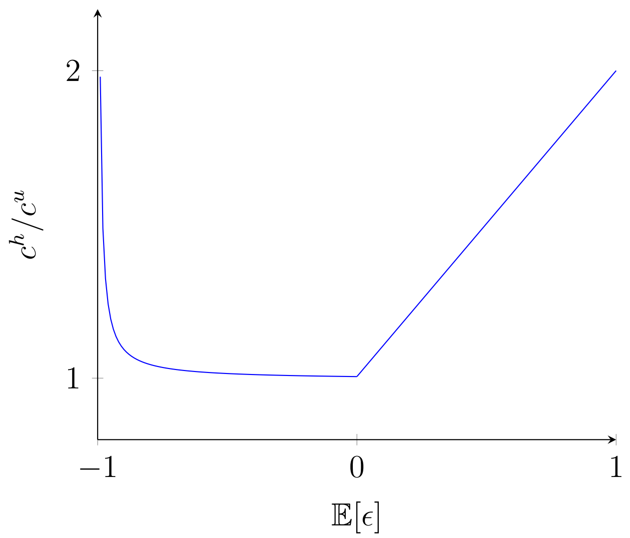
4. In this setup, for simplicity, we do not allow banked agents to hold CBDC as deposits are the first best savings device. We relax this assumption in the TANK model in section 3, where we extend the framework to allow banked agents to hold both deposits and CBDC.

5. We check that the results for consumption are qualitatively similar for different values of $\alpha_M < 1$.

6. See Appendix A.1 for details.

7. The expected value of the income shock can be written as $\mathbb{E}[\epsilon] = 1 - 2p$, where $p$ is the probability of a negative realization of the shock. Therefore for $p \in [0, 1]$, the range of our income shock is $[-1, 1]$.

Figure 1: **Consumption ratios: Banked to unbanked without digital currencies**




Note: Vertical axis: lifetime consumption ratios of the banked relative to the unbanked households. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R = 1/\beta$.

unbanked do not have access to an interest-bearing consumption smoothing device. This suggests that they are more adversely exposed by negative income shocks. For positive income shocks ($\mathbb{E}[\epsilon] > 0$), we note that the UHH cannot increase consumption in period 2 as they are bounded by the cash-in-advance constraint. Therefore the ratio of lifetime consumption of the banked to unbanked generates a linear relationship with respect to positive expected income shocks.

**With digital currency.** Now assume that the unbanked have access to digital currency ($DC$) which is an interest bearing savings device that pays out $R^{DC} \leq R$ upon maturity. Their set of budget constraints are now:[8]

$$c_1^u + M + DC = y, \tag{4a}$$

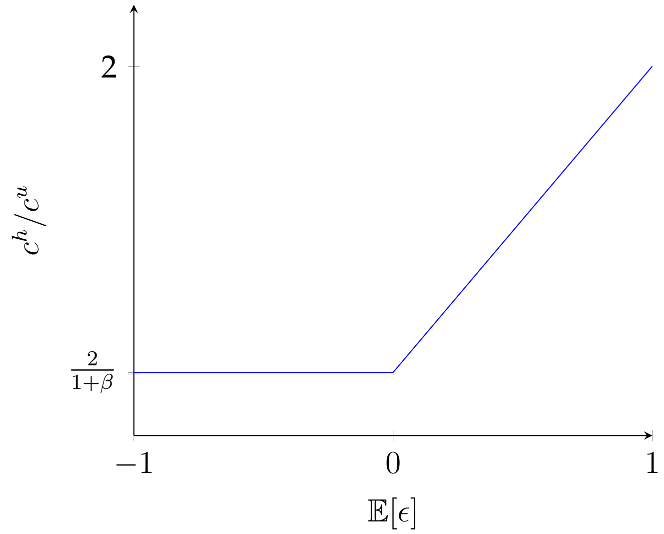
$$c_2^u \leq R^{DC} DC + M + \epsilon, \tag{4b}$$

$$\alpha_M c_2^u \leq M, \tag{4c}$$

We can analyze the implications of the introduction of digital currency for household lifetime consumption. Repeating the previous exercise, we solve for optimal con-

8. Technically, one can extend the access of $DC$ to the banked household. But so long as the returns to $D$ dominate the returns on $DC$, the banked will choose to hold no digital currency.

Figure 2: **Consumption ratios: Banked to unbanked with digital currencies**




Note: Vertical axis: lifetime consumption ratios of the banked relative to the unbanked households after introducing a digital currency. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R^{DC} = 1/\beta$

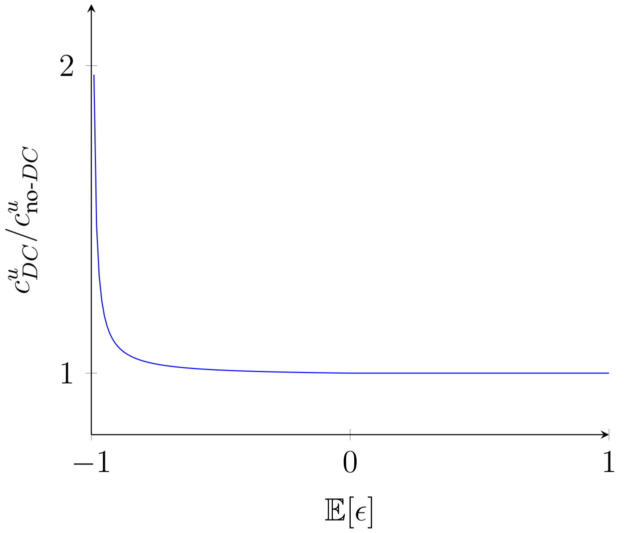
sumption quantities for the households to express the consumption of the banked to unbanked with digital currencies:

$$\frac{c_1^h + c_2^h}{c_{1,DC}^u + c_{2,DC}^u} = \begin{cases} \frac{2}{1+\beta} & \text{if } \mathbb{E}[\epsilon] < 0, \\ \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right)}{y} & \text{if } \mathbb{E}[\epsilon] \geq 0. \end{cases} \tag{5}$$

Figure 2 plots the consumption ratio (5) with respect to the expected value of the income shock. Introducing digital currency makes the unbanked more resilient with respect to the anticipation of a negative income shock – particularly for large expected negative shocks – as they now have access to a savings device and can better smooth consumption than with just holding money balances. The ratio of lifetime consumption between the two sets of households is constant with respect to expected negative income shocks ($\mathbb{E}[\epsilon] < 0$). However, for positive anticipated income shocks the digital currency cannot improve the welfare of the UHH. This is due to two factors: (i) the UHH's consumption in period 2 is limited by the CIA constraint, and (ii) we do not allow the unbanked to take a short-position on $DC$ (we require $DC \geq 0$).

Therefore, the ratio of lifetime consumption of the banked to unbanked is identical to the regime with no digital currency in Figure 1 for positive anticipated income

Figure 3: **Consumption ratios: Unbanked with and without digital currencies**




Note: Vertical axis: lifetime consumption ratios. Horizontal axis: Period 2 resource shock. For calibration, $\beta = 0.99$ and $y = 1$ and $R^{DC} = 1/\beta$.

shocks; but the unbanked are better off for the case of a large negative anticipated shock. This can be seen by plotting the ratio of lifetime consumption of the unbanked household under the two regimes – with and without digital currencies, illustrated in Figure 3.

$$\frac{c^u_{1,DC} + c^u_{2,DC}}{c^u_{1,no-DC} + c^u_{2,no-DC}} = \begin{cases} \frac{\frac{2}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R^{DC}}\right)}{y+\mathbb{E}[\epsilon]} & \text{if } \mathbb{E}[\epsilon] < 0, \\ 1 \quad \text{if } \mathbb{E}[\epsilon] \geq 0. \end{cases} \tag{6}$$

In summary, our analysis highlights one channel of welfare improvement associated with introduction of digital currency. If the digital currency is interest bearing, it is a more efficient savings device than money. It allows the unbanked to engage in more efficient consumption smoothing, particularly providing better insurance against anticipated negative income shocks.

While our simple model sheds light on the role of financial inclusion, this framework is limited as we cannot study: (i) the role of monetary policy, and (ii) whether it is optimal for the interest rate on digital currency to track movements in the policy rate. We now turn to these policy questions in Section 3 by embedding the two-agent framework in a New Keynesian model with digital currency access to both banked and unbanked households.

# 3 Two-Agent New Keynesian Model with Central Bank Digital Currency

In this section, we present a two-agent New Keynesian (TANK) model as in Debortoli and Galí (2017, 2022) and Bilbiie (2018). Notably, our model features a banking sector accompanied with credit frictions (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010). In this framework, a fixed fraction of the banked household are bankers, which allows us to maintain a representative setup of the household sector. Banked households hold claims on CBDC and deposits. Deposits are denominated in fiat currency and held at banks. Banked households may also directly invest in firms by purchasing equity holdings. Banks convert deposits into credit, facilitating loans to firms who acquire capital for the means of production, as in Gertler and Kiyotaki (2010, 2015). Unbanked households are still limited to money holdings and CBDCs.

## 3.1 Production

The supply side of the economy is standard. Final goods are produced by perfectly competitive firms that use labor and capital to produce their output.[9] They also have access to bank loans, and conditional on being able to take out a loan, they do not face any financial frictions. These firms pay back the crediting banks in full via profits. Meanwhile, capital goods are produced by perfectly competitive firms, which are owned by the collective household.

**Capital good firms.** We assume that capital goods are produced by perfectly competitive firms, and that the aggregate capital stock grows according to the following law of motion:

$$K_t = I_t + (1 - \delta)K_{t-1}, \tag{7}$$

where $I_t$ is investment and $\delta \in (0, 1)$ is the depreciation rate.

9. We relegate the discussion of final good firms to the Appendix A.2.1 as it is standard.

The objective of the capital good producing firm is to choose $I_t$ to maximize revenue, $Q_t I_t$. Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} \left\{ Q_t I_t - I_t - \Phi\left(\frac{I_t}{I}\right) I_t \right\},$$

where $\Phi(\cdot)$ are investment adjustment costs and are defined as:

$$\Phi\left(\frac{I_t}{I}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$.

**Intermediate goods producers.** The continuum of intermediate good producers are normalized to have a mass of unity. A typical intermediate firm $i$ produces output according to a constant returns to scale technology in capital and labor with a common productivity shock:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha}.$$

The problem for the $i$-th firm is to minimize costs,

$$\min_{K_{t-1}(i), L_t(i)} z_t^k K_{t-1}(i) + w_t L_t(i),$$

subject to their production constraint:

$$A_t K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha} \geq Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t.$$

This yields the minimized unit cost of production:[10]

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{9}$$

The price-setting problem of firm $i$ is set up à la Rotemberg (1982) where firm $i$ maximizes the net present value of profits,

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty} \Lambda_{t,t+s}^h \left\{ \left(\frac{P_{t+s}(i)}{P_{t+s}}(1-\tau) - MC_{t+s}\right) Y_{t+s}(i) - \frac{\kappa}{2}\left(\frac{P_{t+s}(i)}{P_{t-1+s}(i)} - 1\right)^2 Y_{t+s} \right\}\right],$$

by optimally choosing $P_t(i)$, and where $\kappa$ denotes a price adjustment cost parameter

10. Cost minimization implies:

$$\frac{z_t^k K_{t-1}}{w_t L_t} = \frac{\alpha}{1-\alpha}. \tag{8}$$

for the firms.[11]

Evaluating at the symmetric equilibrium where intermediate firms optimally price their output at $P_t(i) = P_t, \forall i$, yields the standard Phillips curve:

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa}\left(\mathcal{M}_t MC_t + \tau - 1\right) + \mathbb{E}_t\left[\Lambda^h_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right], \qquad (10)$$

where $\mathcal{M}_t$ is the representative intermediate firm's markup.[12]

Also, under the symmetric equilibrium we can express output as:

$$Y_t = A_t K^{\alpha}_{t-1} L^{1-\alpha}_t. \qquad (11)$$

As noted above, there is a distortion arising from monopolistic competition among intermediate firms. We assume that there is a lump-sum subsidy to offset this distortion, $\tau$. From Equation (10), we see that the policymaker chooses a subsidy such that the markup over marginal cost is offset in the deterministic steady state:[13]

$$\tau = -\frac{1}{\epsilon - 1}$$

which guarantees a non-distorted steady-state. Hereinafter, we abstract from distorted steady states and only consider the efficient steady state. Our choice to model nominal rigidity following Rotemberg pricing should not alter our welfare analysis in Section 4. As noted by Nisticò (2007) and Ascari and Rossi (2012), up to a second order approximation and provided that the steady state is efficient, models under both Calvo and Rotemberg pricing imply the same welfare costs of inflation. Therefore, a welfare-maximizing social planner would prescribe the same optimal policy across the two regimes.

11. We calibrate $\kappa$ to the following:

$$\kappa = \frac{\epsilon\theta}{(1-\theta)(1-\beta\theta)},$$

where $\theta$ is the probability of firm $i$ being unable to optimally adjust its price in any given period as in a model with Calvo (1983) pricing. For further details please refer to Appendix A.2.2.

12. In the deterministic steady state the markup is

$$\mathcal{M} = \frac{\epsilon}{\epsilon - 1}.$$

13. Note that this assumes that steady state inflation is net-zero, i.e., $\pi = 1$.

## 3.2 Households and Workers

The representative household contains a continuum of individuals, normalized to 1, each of which are of type $i \in \{h, u\}$. Bankers and banked workers $(i = h)$ share a perfect insurance scheme, such that they each consume the same amount of real output. However, unbanked workers $(i = u)$ are not part of this insurance scheme, and so their consumption volumes are different from bankers and workers. Similar to before in Section 2, we define $\Gamma_h$ as the proportion of the BHH and bankers, and the UHH are of proportion $\Gamma_u = 1 - \Gamma_h$.

We endogenize labor supply decisions on the part of households, and so the BHH maximizes the present value discounted sum of utility:[14]

$$\mathbb{V}_t^h = \max_{\{C_{t+s}^h, L_{t+s}^h, D_{t+s}, K_{t+s}^h, DC_{t+s}^h\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln \left( C_{t+s}^h - \zeta_0^h \frac{(L_{t+s}^h)^{1+\zeta}}{1+\zeta} \right), \tag{12}$$

subject to their period budget constraint:

$$\begin{aligned} &C_t^h + D_t + Q_t K_t^h + \chi_t^h + DC_t^h + \chi_t^{DC,h} + T_t^h \\ &\qquad = w_t L_t^h + \Pi_t + (z_t^k + (1-\delta)Q_t)K_{t-1}^h + \frac{R_{t-1}D_{t-1} + R_{t-1}^{DC} DC_{t-1}^h}{\pi_t}, \end{aligned} \tag{13}$$

where $w_t$ are real wages, $L_t^i, i \in \{h, u\}$, is labor supply, $\zeta$ is the inverse-Frisch elasticity of labor supply, $\zeta_0^i$ is a relative labor supply parameter, $K_t^h$ are equity holdings in firms by the BHH, $\chi_t^h$ are the costs of equity acquisitions incurred by the BHH, $\chi_t^{DC,i}$ are digital currency management costs,[15] $T_t^i$ are lump-sum taxes, $Q_t$ is the price of equity/capital, and $\Pi_t$ are distribution of profits due to the ownership of banks and firms. There is a

14. We make use of Greenwood–Hercowitz–Huffman preferences for both the BHH and UHH to eliminate the income effect on an agent's labor supply decision. Additionally, it allows us to develop a tractable analytical solution for the model steady state.

15. The digital currency management costs for household of type $i$ are:

$$\chi_t^{DC,i} = \frac{\varkappa^{DC}}{2} \left( \frac{DC_t^i}{\widetilde{DC}^i} \right)^2, \quad i \in \{h, u\},$$

where $\widetilde{DC}^i$ are target digital currency balances, calibrated in the baseline case such that aggregate holding of digital currencies is one-third of output. Alternatively, we could assume a non-pecuniary motive for holding digital currency that would manifest as an additional term of the same form in the household utility function. This setup would imply the same first-order conditions.

shock to agents' preferences, $\Xi_t$, and it is given by:

$$\Xi_{t+s} = \begin{cases} e^{\xi_1} e^{\xi_2} ... e^{\xi_s} & \text{for } s \geq 1, \\ 1 & \text{for } s = 0, \end{cases}$$

where $\xi_t$ is a preference (demand) shock given by an AR(1) process. We also note that $\Lambda^h_{t,t+s}$ is the BHH stochastic discount factor (SDF):

$$\Lambda^h_{t,t+s} \equiv \beta^s \mathbb{E}_t \left( \frac{\xi_{t+s} \lambda^h_{t+s}}{\lambda^h_t} \right), \tag{14}$$

where $\lambda^h_t$ is the marginal utility of consumption for the BHH.

One distinction between banked workers and bankers purchasing equity in firms is the assumption that the worker pays an efficiency cost, $\chi^h_t$, when they adjust their equity holdings. We assume the following functional form for $\chi^h_t$:

$$\chi^h_t = \frac{\varkappa^h}{2} \left( \frac{K^h_t}{K_t} \right)^2 \Gamma_h K_t. \tag{15}$$

Meanwhile, the UHH maximizes the present discounted sum of per-period utilities given by:

$$\mathbb{V}^u_t = \max_{\{C^u_{t+s}, L^u_{t+s}, M_{t+s}, DC^u_{t+s}\}^\infty_{s=0}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Xi_{t+s} \ln \left( C^u_t - \zeta^u_0 \frac{(L^u_t)^{1+\zeta}}{1+\zeta} \right), \tag{16}$$

subject to its budget constraint,

$$C^u_t + M_t + \chi^M_t + DC^u_t + \chi^{DC,u}_t + T^u_t = w_t L^u_t + \frac{M_{t-1} + R^{DC}_{t-1} DC^u_{t-1}}{\pi_t}, \tag{17}$$

and the CIA constraint,

$$\alpha_M C^u_t \leq \frac{M_{t-1}}{\pi_t}. \tag{18}$$

## 3.3 Banks

Bankers are indexed on the continuum $j \in [0, 1]$. Among the population of bankers, each $j$-th banker owns and operates their own bank which has a continuation probability given by $\sigma_b$. A banker will facilitate financial services between households and firms by providing loans to firms in the form of equity, $k^b_t$, funded by deposits, $d_t$, and their own net worth, $n_t$.

As is standard in the literature, bankers face a balance sheet constraint:

$$Q_t k_t^b = d_t + n_t, \tag{19}$$

and a flow of funds constraint:

$$n_t = [z_t^k + (1-\delta)Q_t]k_{t-1}^b - \frac{R_{t-1}}{\pi_t} d_{t-1}, \tag{20}$$

where net worth is the difference between gross return on assets and liabilities. Note that for the case of a new banker, the net worth is the startup fund given by the collective household by fraction $\gamma_b$:

$$n_t = \gamma_b [z_t^k + (1-\delta)Q_t]k_{t-1}.$$

The objective of a banker is to maximize franchise value, $\mathbb{V}_t^b$, which is the expected present discount value of terminal wealth:

$$\mathbb{V}_t^b = \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \Lambda_{t,t+s}^h \sigma_b^{s-1} (1-\sigma_b) n_{t+s} \right]. \tag{21}$$

A financial friction in line with Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) is used to limit the banker's ability to raise funds from depositors, whereby the banker faces a moral hazard problem: the banker can either abscond with the funds they have raised from depositors, or the banker can operate honestly and pay out their obligations. Absconding is costly, however, and so the banker can only divert a fraction $\theta^b > 0$ of assets they have accumulated.[16] Thus, bankers face the following incentive compatibility constraint:

$$\mathbb{V}_t^b \geq \theta^b Q_t k_t^b. \tag{22}$$

The problem of the banker consists of maximizing (21) subject to the balance sheet constraint (19), the evolution of net worth (20), and the incentive compatibility constraint (22).

Since $\mathbb{V}_t^b$ is the franchise value of the bank, which we can interpret as a "market value", we can divide $\mathbb{V}_t^b$ by the bank's net worth to obtain a Tobin's Q ratio for the

16. It is assumed that the depositors act rationally and that no rational depositor will supply funds to the bank if they clearly have an incentive to abscond.

bank denoted by $\psi_t$:

$$\psi_t \equiv \frac{\mathbb{V}_t^b}{n_t} = \mathbb{E}_t \left[ \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \frac{n_{t+1}}{n_t} \right]. \tag{23}$$

We define $\phi_t$ as the maximum feasible asset to net worth ratio, or, rather, the leverage ratio of a bank:

$$\phi_t = \frac{Q_t k_t^b}{n_t}. \tag{24}$$

Additionally, if we define $\Omega_{t,t+1}$ as the stochastic discount factor of the banker, $\mu_t$ as the excess return on capital over fiat currency deposits, and $\upsilon_t$ as the marginal cost of deposits, we can write the banker's problem as the following:

$$\psi_t = \max_{\phi_t} \left\{ \mu_t \phi_t + \upsilon_t \right\}, \tag{25}$$

subject to

$$\psi_t \geq \theta^b \phi_t.$$

Solving this problem yields:

$$\psi_t = \theta^b \phi_t, \tag{26}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{27}$$

where:

$$\mu_t = \mathbb{E}_t \left[ \Omega_{t,t+1} \left\{ \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right\} \right], \tag{28}$$

$$\upsilon_t = \mathbb{E}_t \left[ \Omega_{t,t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{29}$$

$$\Omega_{t,t+1} = \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}). \tag{30}$$

For the complete solution of the banker, please refer to Appendix A.2.4 and A.2.5.

## 3.4 Fiscal and Monetary Policy

We assume that the government operates a balanced budget:

$$\frac{R_{t-1}^{DC}}{\pi_t} DC_{t-1} + \frac{M_{t-1}}{\pi_t} = \tau Y_t + \Gamma_h T_t^h + \Gamma_u T_t^u + DC_t + M_t, \tag{31}$$

where it levies taxes to cover the producer subsidy to address the distortions arising from monopolistic competition, money balances, and digital currencies. Our budget constraint allows for money and digital currency to be a liability of the central bank, and is consistent with other studies that model the issuance of CBDC (Barrdear and Kumhof, 2022; Kumhof et al., 2021).

Meanwhile, the central bank is assumed to operate an inertial Taylor rule for the nominal interest rate:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{1-\rho_R} \exp(\varepsilon_t^R), \tag{32}$$

where variables without time subscripts denote steady state values. Additionally, we assume that the central bank sets the nominal return on digital currency one-for-one in line with the nominal interest rate on deposits:

$$R_t^{DC} = R_t. \tag{33}$$

We explore the implications of alternative rules on model dynamics and welfare in Section 4.

## 3.5 Market Equilibrium

Aggregate consumption, labor supply, and digital currency holdings by the BHH and UHH are given as:

$$C_t = \Gamma_h C_t^h + \Gamma_u C_t^u, \tag{34}$$

$$L_t = \Gamma_h L_t^h + \Gamma_u L_t^u, \tag{35}$$

$$DC_t = \Gamma_h DC_t^h + \Gamma_u DC_t^u. \tag{36}$$

Then define $\omega_t$ as the consumption inequality factor, as in Debortoli and Galí (2017), between the banked and unbanked households:

$$\omega_t = 1 - \frac{C_t^u}{C_t^h}. \tag{37}$$

This will allow us to track consumption inequality between the two types of household. Increases (decreases) in $\omega_t$ follow from banked households consuming a larger (smaller) share of aggregate consumption.

The aggregate resource constraint of the economy is:

$$Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{I}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t + \Gamma_h(\chi_t^h + \chi_t^{DC,h}) + \Gamma_u(\chi_t^M + \chi_t^{DC,u}), \quad (38)$$

with aggregate capital being given by:

$$K_t = \Gamma_h(K_t^h + K_t^b). \quad (39)$$

Aggregate net worth of the bank is given by:

$$N_t = \sigma_b \left[(z_t^k + (1-\delta)Q_t)K_{t-1}^b - \frac{R_{t-1}}{\pi_t} D_{t-1}\right] + \gamma_b(z_t^k + (1-\delta)Q_t)\frac{K_{t-1}}{\Gamma_h}, \quad (40)$$

and the aggregate balance sheet of the bank is given by the following equations:

$$Q_t K_t^b = \phi_t N_t, \quad (41)$$

$$Q_t K_t^b = D_t + N_t. \quad (42)$$

Finally, the stationary AR(1) processes for TFP, markup, and preference shocks are given by:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad (43)$$

$$\mathcal{M}_t = (1-\rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon_t^M, \quad (44)$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi \quad (45)$$

A competitive equilibrium is a set of seven prices, $\{ MC_t, R_t, R_t^{DC}, \pi_t, Q_t, w_t, z_t^k \}$, nineteen quantity variables, $\{ C_t, C_t^h, C_t^u, D_t, DC_t, DC_t^h, DC_t^u, I_t, K_t, K_t^b, K_t^h, L_t, L_t^h, L_t^u, M_t, N_t, T_t^h, T_t^u, Y_t \}$, four bank variables, $\{ \psi_t, \phi_t, \mu_t, \upsilon_t \}$, and three exogenous variables, $\{ A_t, \xi_t, \mathcal{M}_t \}$, that satisfies 33 equations. For a complete list of the equilibrium conditions please refer to Appendix A.2.6. Steady state solutions are provided in Appendix A.2.7 for the baseline TANK model.

### 3.6 Model Parameterization and Steady State Values

We set model parameters, which are found in standard New Keynesian models, in line with the literature. See, for example, Galí (2015), Walsh (2010), and Woodford (2003).

Table 1: **Parameter values**

| | | |
|---|---|---|
| $\theta^b$ | 0.399 | Banker absconding ratio |
| $\sigma_b$ | 0.940 | Survival probability |
| $\gamma^b$ | 0.005 | Fraction of total assets inherited by new banks |
| $DC/4Y$ | 1/3 | DC to Output |
| $\beta$ | 0.990 | Discount rate |
| $\zeta$ | 0.333 | Inverse-Frisch elasticity |
| $\zeta_0^h$ | 3.050 | Labor supply disutility |
| $\varkappa^h$ | 0.020 | Cost parameter of direct finance |
| $\Gamma_h$ | 0.500 | Proportion of BHH |
| $\alpha_M$ | 1 | Inverse velocity of money |
| $\phi_M$ | 0.010 | Money adjustment cost parameter |
| $\varkappa^{DC}$ | 0.001 | Digital currency adjustment cost parameter |
| $\alpha$ | 0.333 | Capital share of output |
| $\delta$ | 0.025 | Depreciation rate |
| $\epsilon$ | 10 | Elasticity of demand |
| $\kappa_I$ | 2/3 | Investment adjustment cost |
| $\theta$ | 0.750 | Calvo parameter |
| $\tau$ | 0.111 | Producer subsidy |
| $\mathcal{M}$ | 1.111 | Markup |
| $\phi_\pi$ | 2 | Taylor rule inflation coefficient |
| $\phi_Y$ | 0.100 | Taylor rule output coefficient |
| $\rho_b$ | 0.850 | AR(1) coefficient for demand shock |
| $\rho_A$ | 0.850 | AR(1) coefficient for TFP shock |
| $\rho_M$ | 0.850 | AR(1) coefficient for markup shock |
| $\rho_R$ | 0.550 | Taylor rule persistence |

Parameter values are provided in Table 1.

Model parameters that are not standard, particularly the bank parameters, are set based on Akinci and Queralto (2022). For example, a banker's survival rate, $\sigma_b$, is chosen so that the annual dividend payout is a share of $4 \times (1 - \sigma_b) = 0.24$ of net worth. The banker absconding ratio, $\theta^b$; the banker management cost of digital currencies, $\varkappa^b$; and the fraction of total assets inherited by new bankers, $\gamma^b$, are chosen so that in steady state the bank leverage ratio is approximately 4 and that the share of equity financed by bank finance is approximately 0.70. Furthermore, parameters pertaining to adjustment costs of money balances, $\phi_M$, and of CBDCs, $\varkappa^{DC}$, are calibrated such that digital currency is more easily adjustable than money balances and deposits are the first-best transactions and savings vehicle. Our results are robust to different calibrations of these parameters as long as $0 < \varkappa^{DC} < \phi_M$. We calibrate $\widetilde{DC}$ such that CBDC to output ratio is approximately one third, which is similar to the baseline calibration in (Barrdear and Kumhof, 2022; Kumhof et al., 2021), and implies the ratio of CBDC to the sum of CBDC and deposits of approximately 14%, similar to Assenmacher, Bitter, and Ristiniemi (2023).

Finally, we set the parameters pertaining to monetary policy, namely the sensitivity of nominal interest rates to inflation, $\phi_\pi$, the sensitivity of nominal interest rates to the output gap, $\phi_Y$, and the interest rate smoothing parameter, $\rho_R$, in line with Guerrieri and Iacoviello (2015).

We assume the persistence of our exogenous AR(1) processes to be 0.85 per quarter. Standard deviations of shocks are set to be 0.5% per quarter for TFP, and 0.1% for the cost-push shock, 0.1% for the preference shock, and 0.1% for the monetary policy shock unless otherwise stated – for instance, innovations to shocks are 1% when plotting the impulse response functions.

# 4 Dynamics and Welfare Implications

## 4.1 Impulse Responses to a Monetary Policy shock

Figure 4 presents impulse responses to a 1% (annualized) monetary policy shock with the Taylor rule (32) and $R_t = R_t^{DC}$. We plot impulse responses for two alternative regimes: a CBDC-equipped economy as described in Section 3 (red dashed line) and an economy with no CBDCs (blue line).

Upon impact, the tightening of monetary policy has standard responses for the real economy: output, consumption, and the marginal cost decline in response to the increase in the real interest rate. Here consumption inequality actually improves as the unbanked household benefits from the deflationary pressure in the economy, increasing real money balances.[17] The impact on financial variables are also in line with standard models equipped with banking (see for example Gertler and Karadi (2011)): A

17. For brevity, we avoid plotting banked wages, the banked household labor supply, and the banked household consumption as they are highly correlated with output due to the specification of GHH preferences and operating on a standard consumption Euler equation.

small decline in the price of equity leads to a decline in bank intermediation as bank equity, deposits, and net worth shrinks, affecting the real economy via the financial accelerator mechanism (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999).

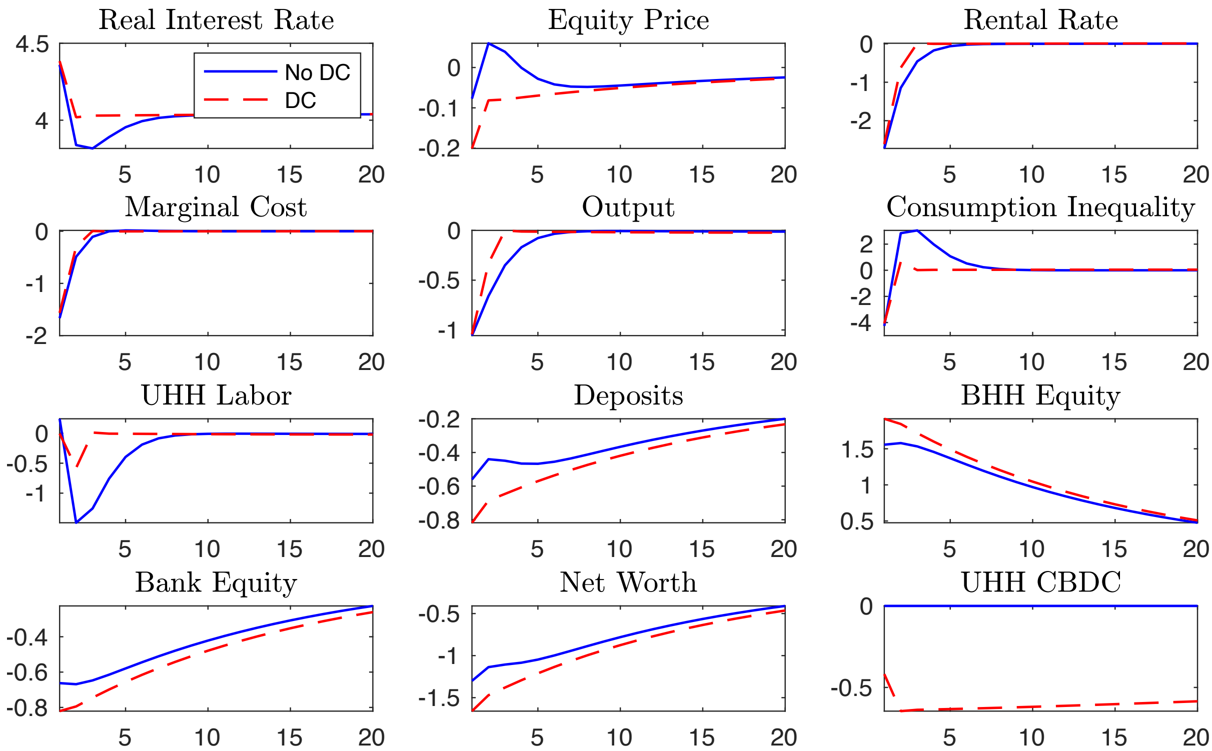
However, there are differences in the dynamics of the CBDC and no-CBDC economies, arising due to the CIA constraint on the unbanked. The intuition is as follows. In the no-CBDC economy, the initial increase in the real interest rate and decline in consumption inequality is reversed in the following period as inflation re-emerges and the real interest rate falls below its steady-state level. This is because the central bank reacts to close the inflation and output gap. But higher inflation erodes the purchasing power of the unbanked as they base their consumption decisions on inherited real money balances (see Equation (18)). This can be interpreted as the CIA constraint binding more severely. Then, by rearranging the unbanked intratemporal Euler equation (labor supply condition), we can write:

$$L^u = \left[ \frac{\lambda^u}{\zeta_0^u(\lambda^u + \mu^u)} w \right]^{1/\zeta},$$

where for simplicity we abstract from time, and where $\lambda^u$ is the Lagrange multiplier on the budget constraint, $\mu^u$ is the CIA constraint Lagrange multiplier, and $\lambda_u + \mu^u$ is the marginal utility of consumption for the unbanked. The CIA constraint binding more severely ($\mu^u \uparrow$), combined with the decrease in wages ($\lambda^u \uparrow$) leads to an increase in the marginal utility of consumption, as well as the "wage multiplier term" $\lambda^u/(\lambda^u + \mu^u)$. The result is a substantial fall in labor and consumption of the unbanked, leading to a reversal and worsening of consumption inequality in the no-CBDC economy once inflation re-emerges and erodes the purchasing power of real money balances.

Put simply, with the provision of a CBDC, the key difference is the response of unbanked consumption and labor due to the CIA constraint – which is a proxy for lack of financial inclusion. The unbanked cut consumption drastically when they only have access to cash as a savings vehicle. Access to a savings device through the CBDC allows these households to buffer against the shock by reducing their savings (the decline in

Figure 4: **IRFs to a 1% annualized monetary policy shock**




Note: Figure plots impulse responses of model variables with respect to a 1% annualized innovation to the Nominal Interest Rate. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation ($\pi$) and Nominal Interest Rates ($R$) which are expressed as annualized net rates.

$DC^u$), dampening the decline in their consumption – these mechanisms were highlighted in our simple endowment economy in Section 2. Access to an effective savings device for the unbanked mutes the aggregate response of consumption and output, and the effects of the monetary shocks dissipate quicker than in an economy without CBDCs. For the banked household, in contrast, there is little difference in the response of consumption to monetary policy shocks upon introducing a CBDC. This is because they have access to a first best savings device, bank deposits, and do not adjust their holdings of CBDC in response to the shock. Monetary policy transmission to unbanked consumption is mitigated with the introduction of the CBDC.[18]

However, the decline in real interest rates in the no-CBDC economy stimulates investment and equity acquisitions relative to the CBDC-equipped economy. This presents an interesting trade-off for a policymaker (conditional on innovations to the policy rate

18. IRFs to a fundamental TFP, cost-push, and demand shock are provided in Appendix A.2.8.

Table 2: Model simulated standard deviations (%)

| | no-CBDC | CBDC |
|---|---|---|
| Output | 2.35 | 1.89 |
| Inflation | 1.60 | 1.44 |
| Nominal Interest Rate | 2.21 | 1.90 |
| Net Worth | 6.87 | 5.83 |
| Bank Leverage | 1.92 | 1.71 |

Note: Standard deviations are conditional standard deviations based on model simulations. The model is solved and simulated via second-order perturbation about the deterministic steady state. Inflation and the Nominal Interest Rate are annualized.

being the only source of fluctuations): macroeconomic stability and financial stability. The CBDC-equipped economy returns back to steady state for variables such as output, consumption, and the return on capital, however bank variables – such as net worth and bank equity – declines due to the path of equity prices and the aforementioned financial accelerator.

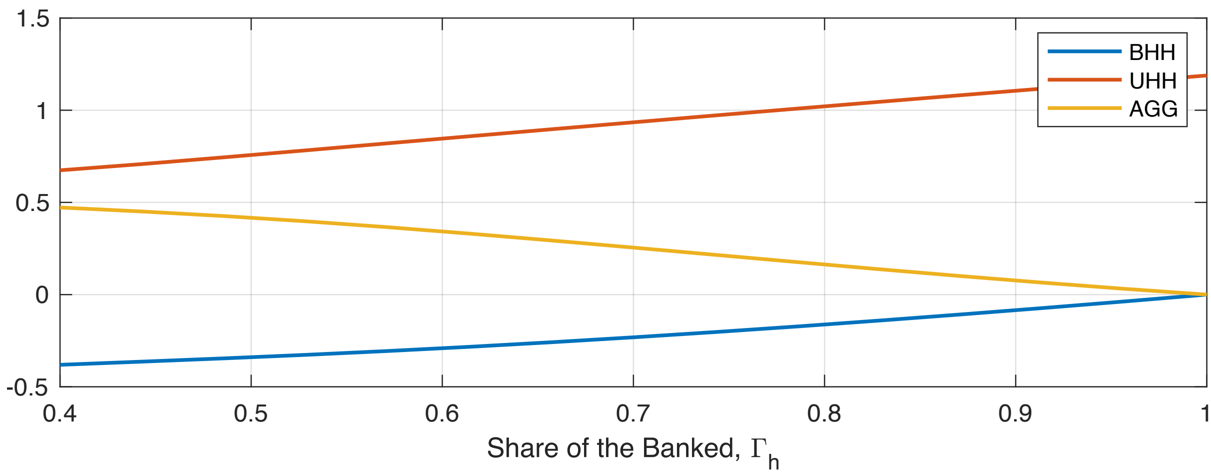
To explore this further, we simulate the two economies to capture the conditional standard deviations of select macroeconomic and financial variables when the model is subject to all shocks. Table 2 summarizes these results. The stability trade-off is eliminated for the economies with all four shocks present: the CBDC-equipped economy features less volatility for both macroeconomic and financial variables. Smaller fluctuations of variables are due to unbanked households gaining access to an effective consumption smoothing device. We further highlight these effects in our section on estimating welfare.

## 4.2 Welfare Effects

### 4.2.1 CBDC introduction

Figure 5 evaluates the welfare effects of introducing a CBDC, when the economy is subject to TFP, cost-push, demand, and monetary policy shocks, and monetary policy is conducted according to the Taylor rule. We find that the unbanked experience welfare gains in the CBDC-equipped economy. This is due to CBDC offering a rate of remuneration and it being a more efficient savings device than money balances, allowing the

Figure 5: **Welfare comparison (CBDC regime %ch. over no-CBDC regime)**




Note: Figure plots welfare for BHH, UHH and aggregate households as a function for the share of the banked population, $\Gamma_h$. The welfare is calculated as a per cent change from the regime with no digital currency.

unbanked to better insure against adverse shocks.

Turning to the banked households, we find that they experience net negative welfare benefits after introduction of the CBDC. To explain this, we note that the banked household face management costs in holding a CBDC relative to bank deposits, and therefore do not gain directly from access to a CBDC as they already have bank deposits – which are a first best transaction and savings device. Second, banked households experience net negative welfare losses from a disintermediation channel: as their holdings of CBDC increase, the bank loses deposit funding (Keister and Sanches, 2021). The decline in bank funding leads to lower intermediation, amplifying the response of capital and production through bank balance sheets via a financial accelerator mechanism (Bernanke, Gertler, and Gilchrist, 1999; Kiyotaki and Moore, 1997, 2019). Both of these channels can explain why banked households experience net negative welfare losses relative to an economy with no CBDC. Turning to aggregate welfare, we observe net welfare benefits are highest when the economy is primarily unbanked. As the ex-ante proportion of the unbanked population declines, the welfare benefits of introducing CBDC tend to zero which suggests a stronger use case of CBDCs in emerging markets with lower degrees of financial inclusion.

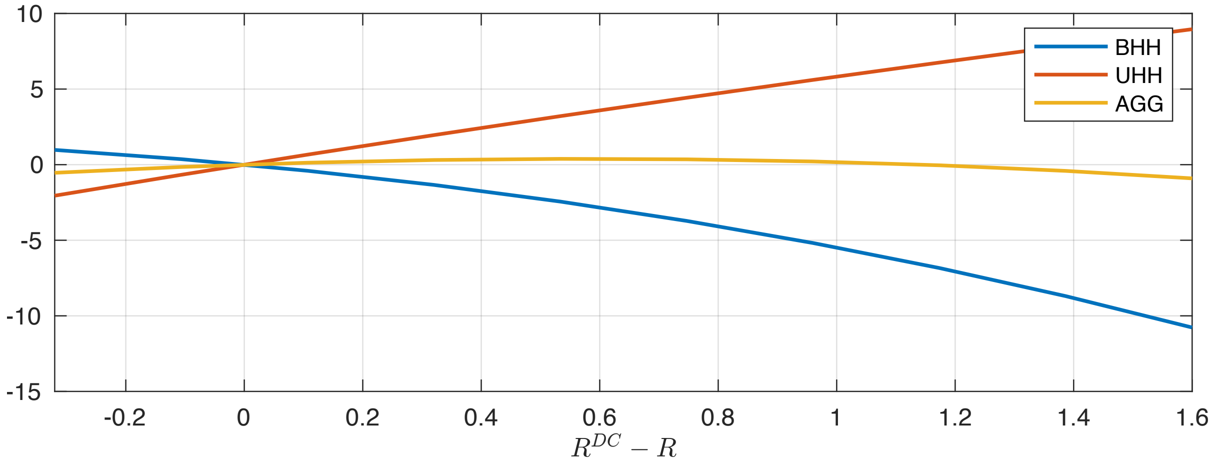
### 4.2.2 Constant Spread Rules

To further illustrate the savings and disintermediation channels of welfare, we explore the implications for different levels of CBDC interest rates, which is measured by the spread $\mathcal{S}^{DC} = R_t^{DC} - R_t$. We hold constant the baseline degree of financial inclusion ($\Gamma_h = 0.5$), and the economy is subject to TFP, cost-push, and preference shocks, and monetary policy is conducted according to the Taylor rule. Figure 6 plots the relative welfare gains and losses of agents in a CBDC-equipped economy against a benchmark zero spread between the CBDC and policy rate ($R_t^{DC} = R_t$). The spread is quoted in annualized percent levels.

Our results show that the unbanked are better off when CBDC rates are higher than the policy rate. This is consistent with the unbanked benefiting through the savings channel, where CBDC receive a higher rate of interest, providing a buffer against adverse shocks. In contrast, the banked are worse off as the CBDC rate is higher than the policy rate through the disintermediation channel. As CBDC rates increase, banked agents substitute away from holding bank deposits to holding CBDC. Therefore the deposit base of bank balance sheets shrinks. This in turn leads to a lower equilibrium levels of bank lending, capital, and consumption of banked households.

In summary, setting the optimal spread between the CBDC and policy rate depends on the level of financial inclusion. All else equal, our model suggests that economies with lower levels of financial inclusion and a higher share of the unbanked population should find it optimal to set a higher spread of digital currency rates to policy rates. In contrast, developed economies with a predominantly banked population should set rates on the CBDC lower than the policy rate. This is consistent with pilot studies of advanced economies with high level of financial inclusion, such as the Swedish E-Krona, which typically propose a non-interest bearing currency.

Figure 6: **CBDC economy welfare comparison (% ch.)**




Note: Figure plots relative welfare gains for BHH, UHH, and aggregate households as a function of the spread between the policy rate and the CBDC rate. Note that $\Gamma_h = 0.5$.

## 4.3 CBDC Design and Optimal Monetary Policy

### 4.3.1 Steady state analysis

We now explore the implications for optimal policy, assuming that a policymaker has access to two instruments in order to maximize welfare: nominal interest rates on deposits, $R$, and nominal interest rates on digital currency, $R^{DC}$. More formally, let us state the problem for the welfare maximizing policymaker as:

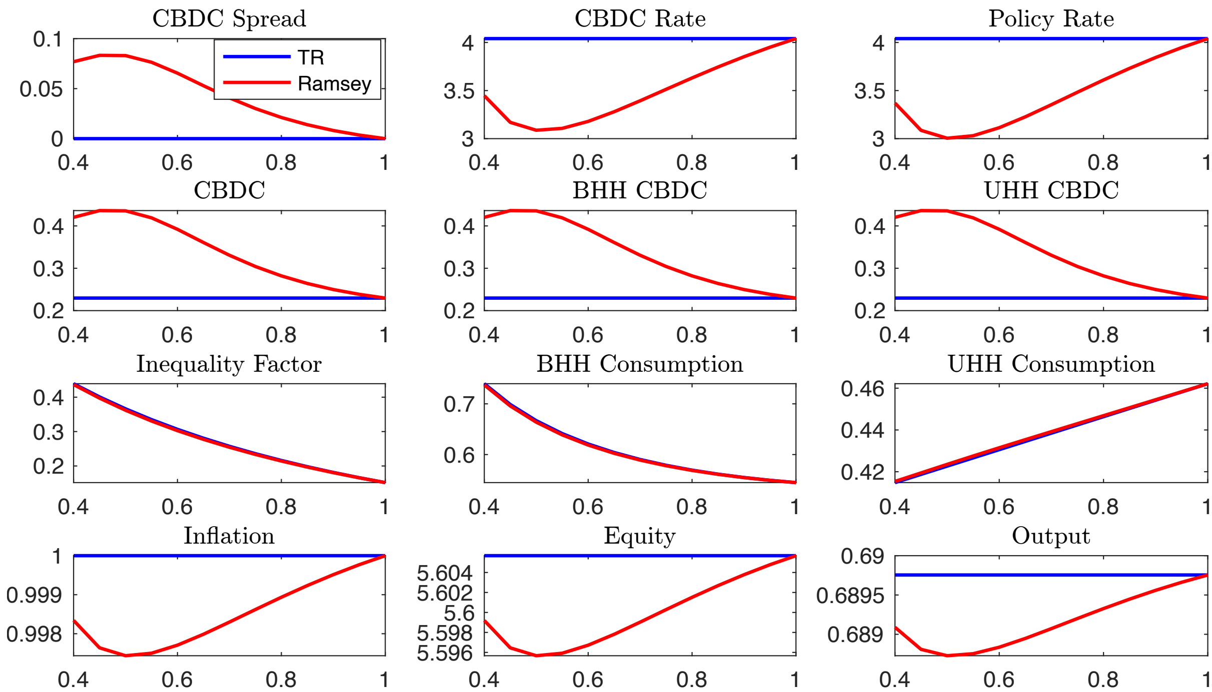
$$\max_{\{R_{t+s}, R^{DC}_{t+s}\}_{s=0}^{\infty}} \mathbb{V}_t = \Gamma_h \mathbb{V}^h_t + \Gamma_u \mathbb{V}^u_t, \tag{46}$$

subject to the entire set of structural equations as set out in Section 3. As CBDC and deposits are imperfect substitutes, the instruments available to the policymaker are not collinear, allowing us to conduct the optimal policy exercise.[19]

The steady state values implied by the solution of the social planner problem are shown in Figure 7. The choice of instruments by the Ramsey policymaker implies a steady state that is generally different to the one under the baseline configuration with a Taylor rule. The presence of unbanked households subject to a CIA constraint leads

19. We argue that $R^{DC}$ is different to $R$ as a Ramsey-instrument in two distinct ways. First, $DC$ are a sub-optimal savings and consumption smoothing instruments compared to $D$ due to the presence of convex adjustment costs. Secondly, the existence of $DC$ in the economy potentially induces disintermediation. Thus, $R^{DC}$ is set to balance positive welfare effects that $DC$ brings in for the unbanked against the broader effects of bank deposit disintermediation.

Figure 7: **Steady state values and financial inclusion**




Note: Vertical axis indicates absolute values of variables in steady state, except for $\pi$, $R$, and $R^{DC}$: these variables are represented as annualized rates. The horizontal axes are values of financial inclusion parameter $\Gamma_h$.

the social planner to pick a deflationary steady state. This is a result well covered in, for example, Chari, Christiano, and Kehoe (1991) and Schmitt-Grohé and Uribe (2010). Deflation is, however, costly through inefficient price adjustments; thus the policymaker induces a relatively low level of deflation. As the share of unbanked households converges to zero (greater financial inclusion), the model becomes a standard representative agent setup and the optimal net inflation rate converges to zero, $\pi \to 1$. Moreover, for relatively low ex-ante financial inclusion, the social planner picks higher values of steady state CBDC holdings by picking a higher spread between $R^{DC}$ and $R$. This is due to the fact that while maximizing the aggregate welfare of the economy as in (46), the social planner wishes to redistribute resources from the otherwise wealthier banked household to the unbanked household, and is able to do so only via interest-bearing CBDC holdings.

#### 4.3.2 Welfare decomposition: optimal policy and CBDC introduction

The prior welfare exercises in section 4.2 conducted monetary policy with a Taylor rule. Figure 8 shows the decomposition of welfare gains associated with both the introduction of the CBDC and optimal monetary policy. For different levels of the banked population share, we decompose welfare improvements associated with the transition from the no-CBDC economy and a standard Taylor rule, to the CBDC-equipped economy and a Ramsey-optimal monetary policy (two instruments). The model economy is subject to TFP, markup and preference shocks. These welfare gains are associated with: (i) the introduction of CBDCs, (ii) optimal conventional monetary policy, and (iii) optimal $R_t^{DC}$ setting.[20]

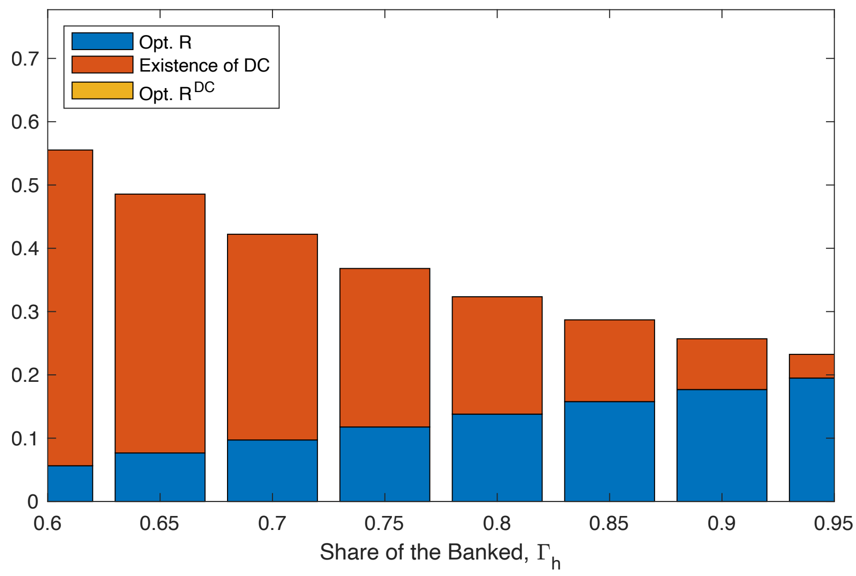
We observe that for the economy with low financial inclusion, welfare improvements are mainly associated with the introduction of a CBDC. This is consistent with our earlier findings on how a larger share of the unbanked population increases the welfare gains due to using CBDC as a savings device.

For economies with a higher level of financial inclusion ($\Gamma_h$ increasing), welfare improvements are due primarily to optimal monetary policy, with the interest rate on CBDC tracking the policy rate. This is intuitive, as a higher share of the banked population means there is a natural amplification of monetary transmission, by changing the response of capital and production through bank balance sheets via a financial accelerator mechanism. The increased importance of monetary policy to stabilize macroeconomic fluctuations increases the gains from conducting optimal monetary policy relative to a benchmark Taylor rule.

Evaluating optimal policy design, we observe negligible welfare improvements from optimal policy with one instrument, in which the CBDC rate tracks the policy rate ($R^{DC} = R$), to optimal policy with two instruments, in which the policy rate and the CBDC rate are set independently. This suggests that while an optimal spread is typi-

20. We compare welfare under the three policy changes to the baseline Taylor-rule regime and no CBDCs. The welfare improvements associated with each regime change do not include cross effects, which are small in magnitude. We approximate all the models around the Ramsey-optimal steady state to ensure that welfare rankings are not spurious, following Benigno and Woodford (2012). This implies steady-state deflation and a spread between $R^{DC}$ and $R$.

Figure 8: **Welfare improvement decomposition**




Note: Vertical axis indicates percent increase in welfare compared to baseline specification without digital currency access.

cally non-zero, according to the steady state values of the optimal spread in Figure 7, it leads to quantitatively similar welfare to a rule where the CBDC rate tracks the policy rate.

In summary, the welfare decomposition suggests that gains from introducing a CBDC diminish as financial inclusion increases. Optimal monetary policy is quantitatively similar to a rule in which the rate on CBDC tracks the policy rate. Deviating from this rule results in negligible welfare improvements and is an order of numerical approximation error.

# 5 Conclusion

In this paper, we address research questions investigating the welfare implications of introducing a retail CBDC on financial inclusion, monetary policy transmission, and the optimal path of interest rates.

To motivate our analysis, we start with a simple endowment economy model featuring two types of agents. The introduction of an interest-bearing digital currency increases the resilience of unbanked households to negative income shocks, promoting financial inclusion. We then build a two-agent New Keynesian model with financial intermediaries. We use the model to assess monetary policy transmission and the welfare effects of optimal monetary policy.

First, we find the unbanked household consumption response to monetary shocks is notably different with a CBDC. Access to the digital currency enables the unbanked to counter the shock by reducing their savings, mitigating the decline in consumption and leading to a faster transition to the steady state. In contrast, the banked household consumption response remains largely unchanged with the introduction of a CBDC. Second, we show distributional effects of welfare; unbanked households benefit when CBDC rates are at a positive spread with respect to the rate on deposits, while banked households are worse off. Third, we conduct a Ramsey optimal policy exercise to determine the optimal path of interest rates on both the central bank and digital currency deposits. Our results indicate that when CBDC is a close substitute to bank deposits, it is optimal for the CBDC rate to track a constant spread with the policy rate.

In summary, our paper contributes to the design and macroeconomic effects of CBDCs. Our findings have salient policy implications on how the effectiveness of a CBDC can vary depending on the level of financial inclusion and whether it should be interest bearing.

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# A Appendix

## A.1 Simple Endowment Economy

**No digital currency.** Assuming that the banked and unbanked face constraints (1) and (2), respectively, and make the simplifying assumption that:

$$\epsilon = \begin{cases} -1 & \text{w.p.} \quad p, \\ 1 & \text{w.p.} \quad 1-p, \end{cases}$$

where $p \in (0,1)$.

Solving the BHH problem for optimal consumption across periods yields

$$c_1^h = \frac{1}{1+\beta}\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{47}$$

$$c_2^h = \frac{\beta}{1+\beta}R\left(y + \frac{\mathbb{E}[\epsilon]}{R}\right), \tag{48}$$

$$D = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{(1+\beta)R}, \tag{49}$$

with the standard consumption Euler equation:

$$c_2^h = \beta R c_1^h.$$

As expected, $c_1^h$ and $c_2^h$ are decreasing in $p$, while $D$ is increasing in $p$, highlighting the role of consumption smoothing for the banked household.

For the UHH, it is clear that the CIA constraint is not binding if $p > \frac{1}{2}$, which yields the following solutions:

$$c_1^u = \frac{1}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{50}$$

$$c_2^u = \frac{\beta}{1+\beta}\left(y + \mathbb{E}[\epsilon]\right), \tag{51}$$

$$M = \frac{\beta}{1+\beta}y - \frac{\mathbb{E}[\epsilon]}{1+\beta}, \tag{52}$$

and where their Euler equation is:

$$c_1^u = \beta c_2^u.$$

In the case where $p < \frac{1}{2}$ we have:

$$c_1^u = \frac{1}{1+\beta}y, \tag{53}$$

$$c_2^u = \frac{\beta}{1+\beta}y, \tag{54}$$

$$M = \alpha_M c_2^u. \tag{55}$$

**With digital currency.** The banked problem remains the same as without digital currency. The unbanked now face constraints in (4), and solving their problem yields the following FOCs:

$$\frac{1}{c_1^u} = \lambda_1,$$

$$\frac{1}{c_2^u} = \lambda_2 + \alpha_M \mu,$$
$$\lambda_1 = \beta \lambda_2 R^{DC},$$
$$\lambda_1 = \beta \lambda_2 + \beta \mu,$$

where $\lambda_t$ is the period-$t$ marginal utility of consumption and $\mu$ is the CIA constraint Lagrangian multiplier. Rearrange the above FOCs, and combine with the fact that for $R^{DC} > 1$ (4c) binds with equality, to get:

$$\frac{1}{c_2^u} = \lambda_2 \left[1 + \alpha_M (R^{DC} - 1)\right],$$
$$c_2^u = \mathcal{S} c_1^u,$$
$$M = \alpha_M c_2^u,$$

where $\mathcal{S} = \beta R^{DC} / \left[1 + \alpha_M (R^{DC} - 1)\right]$ is the marginal rate of transformation of $c_1^u$ and $c_2^u$ – the discounted return on deferring consumption using $M$ and $DC$. Then write the optimal quantities for the unbanked as:

$$c_1^u = \frac{1}{1+\beta} \left( y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right), \tag{56}$$
$$c_2^u = \frac{\mathcal{S}}{1+\beta} \left( y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right), \tag{57}$$
$$M = \alpha_M c_2^u, \tag{58}$$
$$DC = \frac{\mathcal{S}(1-\alpha_M)}{(1+\beta) R^{DC}} \left( y + \frac{\mathbb{E}[\epsilon]}{R^{DC}} \right) - \frac{\mathbb{E}[\epsilon]}{R^{DC}}. \tag{59}$$

There is a second case to the problem of the unbanked: when the second period budget constraint does not bind with equality but the CIA does. This yields the following expressions for consumption and digital currency holdings:

$$c_1^u = \frac{\alpha_M}{\alpha_M + \beta} y, \tag{60}$$
$$c_2^u = \frac{\beta}{\alpha_M + \beta} y, \tag{61}$$
$$DC = 0. \tag{62}$$

To understand the two cases, assume for simplicity that $\alpha_M = 1$. This means that (59) simplifies to

$$DC = -\frac{\mathbb{E}[\varepsilon]}{R^{DC}} \tag{63}$$

Since there is a non-negativity constraint on $DC$, it would imply that the above expression yields a positive balance of $DC$ if and only if $p > \frac{1}{2}$. In other words, if the expected value of the income shock is negative, then an unbanked household will attempt to save in $DC$ in order to fund its consumption in the second period. If the expected value of the income shock is positive, then the unbanked household would attempt to take a short position to increase period 2 consumption – which would violate the non-negativity constraint we placed on $DC$. Hence, in the simplifying case where $\alpha_M = 1$, expected lifetime consumption of the unbanked with and without $DC$ is given by

$$c^u_{\text{w/ } DC} = \begin{cases} y - \frac{1}{R^{DC}} & \text{w.p. } p, \\ y & \text{w.p. } 1-p, \end{cases} \tag{64}$$

$$c^u_{\text{w/o }DC} = \begin{cases} y-1 & \text{w.p. } p, \\ y & \text{w.p. } 1-p. \end{cases} \tag{65}$$

## A.2 TANK model with Central Bank Digital Currency

### A.2.1 Final Good Firms

There is a representative competitive final good producing firm which aggregates a continuum of differentiated intermediate inputs according to a Dixit-Stiglitz aggregator:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{66}$$

Final good firms maximize their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t Y_t(i) di.$$

Solving for the FOC for a typical intermediate good $i$ is:

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} Y_t. \tag{67}$$

The relative demand for intermediate good $i$ is dependent of $i$'s relative price with $\epsilon$, the price elasticity of demand, and is proportional to aggregate output, $Y_t$.

From Blanchard and Kiyotaki (1987), we can derive a price index for the aggregate economy:

$$P_t Y_t \equiv \int_0^1 P_t(i) Y_t(i) di.$$

Then, plugging in the demand for good $i$ from (67) we have:

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

### A.2.2 The New Keynesian Phillips Curve

If we log linearize Equation (10) about the non-inflationary steady state, we yield the NKPC. First start by totally differentiating (10):

$$(2\pi - 1) d\pi_t = \frac{(\epsilon-1)\mathcal{M}}{\kappa} dMC_t + \frac{(\epsilon-1)MC}{\kappa} d\mathcal{M}_t + \beta(2\pi - 1)\mathbb{E}_t d\pi_{t+1},$$

where $\pi = \frac{\mathcal{M}}{1-\tau} MC = 1$. Substitute these values in and assume that $dMC_t = MC_t - MC$ to get the log-linearized NKPC:

$$\hat{\pi}_t = \frac{(\epsilon-1)(1-\tau)}{\kappa} \hat{MC}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} + \hat{u}_t, \tag{68}$$

where hatted variables denote log-deviations from steady state values, $\hat{u}_t = \frac{(\epsilon-1)(1-\tau)}{\kappa} \hat{\mathcal{M}}_t$ is a cost-push shock, and where we calibrate $\kappa$ to a standard value as in Blanchard and Galí (2007):

$$\kappa = \frac{\epsilon\theta}{(1-\theta)(1-\beta\theta)}.$$

### A.2.3 Household Optimization Problem

The FOCs to the BHH problem are:

$$\lambda_t^h = \frac{1}{C_t^h - \zeta_0^h \frac{(L_t^h)^{1+\zeta}}{1+\zeta}}, \tag{69}$$

$$w_t = \zeta_0^h (L_t^h)^{\zeta}, \tag{70}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}}, \tag{71}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \left( \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left( \frac{K_t^h}{K_t} \right)} \right), \tag{72}$$

$$1 + \varkappa^{DC} \frac{DC_t^h}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}}. \tag{73}$$

The FOCs to the UHH problem are:

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}}, \tag{74}$$

$$\lambda_t^u w_t = \frac{\zeta_0^u}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\zeta}}{1+\zeta}} (L_t^u)^{\zeta}, \tag{75}$$

$$\lambda_t^u \left[ 1 + \phi_M (M_t - M) \right] = \beta \mathbb{E}_t \xi_{t+1} \left[ \frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}} \right], \tag{76}$$

$$1 + \varkappa^{DC} \frac{DC_t^u}{\widetilde{DC}^u} = \beta \mathbb{E}_t \xi_{t+1} \frac{\lambda_{t+1}^u}{\lambda_t^u} \frac{R_t^{DC}}{\pi_{t+1}}. \tag{77}$$

### A.2.4 Rewriting the Banker's Problem

To setup the problem of the banker as in Section 3.3, first iterate the banker's flow of funds constraint (20) forward by one period, and then divide through by $n_t$ to yield:

$$\frac{n_{t+1}}{n_t} = \frac{\left( z_{t+1}^k + (1-\delta) Q_{t+1} \right)}{Q_t} \frac{Q_t k_t^b}{n_t} - \frac{R_t}{\pi_{t+1}} \frac{d_t}{n_t}.$$

Rearrange the balance sheet constraint (19) to yield the following:

$$\frac{d_t}{n_t} = \phi_t - 1.$$

Substitute this value for $d_t/n_t$ into the expression for $n_{t+1}/n_t$, and we get:

$$\frac{n_{t+1}}{n_t} = \left( \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t + \mathbb{E}_t \frac{R_t}{\pi_{t+1}}.$$

Substituting this expression into (23), yields the following:

$$\begin{aligned} \psi_t &= \mathbb{E}_t \Lambda_{t,t+1}^h (1 - \sigma_b + \sigma_b \psi_{t+1}) \left[ \left( \frac{z_{t+1}^k + (1-\delta) Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}} \right) \phi_t + \frac{R_t}{\pi_{t+1}} \right] \\ &= \mu_t \phi_t + \upsilon_t, \end{aligned}$$

which is (25) in the text.

### A.2.5 Solving the Banker's Problem

With $\{\mu_t\} > 0$, the banker's incentive compatibility constraint binds with equality, and so we can write the Lagrangian as:

$$\mathcal{L} = \mu_t \phi_t + \upsilon_t + \lambda_t(\psi_t - \theta^b \phi_t),$$

where $\lambda_t$ is the Lagrangian multiplier. The FOCs are:

$$(1 + \lambda_t)\mu_t = \lambda_t \theta^b, \tag{78}$$

$$\psi_t = \theta^b \phi_t. \tag{79}$$

Substitute (79) into the banker's objective function to yield:

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t}, \tag{80}$$

which is (27) in the text.

### A.2.6 Full Set of Equilibrium Conditions

**Households.**

$$w_t = \zeta_0^h L_t^h \tag{81}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t}{\pi_{t+1}} \tag{82}$$

$$1 = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{z_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t + \varkappa^h \Gamma_h \left(\frac{K_t^h}{K_t}\right)} \tag{83}$$

$$1 + \varkappa^{DC} \frac{DC_t^h}{\widetilde{DC}^h} = \mathbb{E}_t \Lambda_{t,t+1}^h \frac{R_t^{DC}}{\pi_{t+1}} \tag{84}$$

$$C_t^u + M_t + \chi_t^M + DC_t^u + \chi_t^{DC,u} + T_t^u = w_t L_t^u + \frac{M_{t-1}}{\pi_t} + \frac{R_{t-1}^{DC}}{\pi_t} DC_{t-1}^u \tag{85}$$

$$\frac{\lambda_t^u}{\lambda_t^u + \alpha_M \mu_t^u} w_t = \zeta_0^u (L_t^u)^\varsigma \tag{86}$$

$$\lambda_t^u + \alpha_M \mu_t^u = \frac{1}{C_t^u - \zeta_0^u \frac{(L_t^u)^{1+\varsigma}}{1+\varsigma}} \tag{87}$$

$$\beta \mathbb{E}_t \xi_{t+1} \frac{\lambda_{t+1}^u + \mu_{t+1}^u}{\pi_{t+1}} = \lambda_t^u \left[1 + \phi_M (M_t - M)\right] \tag{88}$$

$$\lambda_t^u \left(1 + \varkappa^{DC} \frac{DC_t^u}{\widetilde{DC}^u}\right) = \beta \mathbb{E}_t \xi_{t+1} \lambda_{t+1}^u \frac{R_t^{DC}}{\pi_{t+1}} \tag{89}$$

$$\alpha_M C_t^u = \frac{M_{t-1}}{\pi_t} \tag{90}$$

**Production.**

$$Q_t = 1 + \frac{\kappa_I}{2}\left(\frac{I_t}{I} - 1\right)^2 - \frac{I_t}{I}\kappa_I \left(\frac{I_t}{I} - 1\right) \tag{91}$$

$$K_t = (1-\delta)K_{t-1} + I_t \tag{92}$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha} \tag{93}$$

$$\frac{z_t^k K_{t-1}}{w_t L_t} = \frac{\alpha}{1-\alpha} \tag{94}$$

$$MC_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{95}$$

$$\pi_t(\pi_t - 1) = \frac{\epsilon - 1}{\kappa}\left(\mathcal{M}_t MC_t + \tau - 1\right) + \mathbb{E}_t\left[\Lambda^h_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{96}$$

**Banks.**

$$\psi_t = \theta^b \phi_t \tag{97}$$

$$\phi_t = \frac{\upsilon_t}{\theta^b - \mu_t} \tag{98}$$

$$\mu_t = \mathbb{E}_t\left[\Omega_{t,t+1}\left\{\frac{z^k_{t+1} + (1-\delta)Q_{t+1}}{Q_t} - \frac{R_t}{\pi_{t+1}}\right\}\right] \tag{99}$$

$$\upsilon_t = \mathbb{E}_t\left[\Omega_{t,t+1}\frac{R_t}{\pi_{t+1}}\right] \tag{100}$$

$$\Omega_{t,t+1} = \Lambda^h_{t,t+1}(1 - \sigma_b + \sigma_b\psi_{t+1}) \tag{101}$$

**Monetary and fiscal policy.**

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{1-\rho_R}\exp(\varepsilon^R_t) \tag{102}$$

$$\frac{R^{DC}_{t-1}}{\Pi_t}DC_{t-1} + \frac{M_{t-1}}{\Pi_t} = \tau Y_t + \Gamma_h T^h_t + \Gamma_u T^u_t + DC_t + M_t \tag{103}$$

$$R^{DC}_t = R_t \tag{104}$$

**Market clearing.**

$$C_t = \Gamma_h C^h_t + \Gamma_u C^u_t \tag{105}$$

$$L_t = \Gamma_h L^h_t + \Gamma_u L^u_t \tag{106}$$

$$DC_t = \Gamma_h DC^h_t + \Gamma_u DC^u_t \tag{107}$$

$$\omega_t = 1 - \frac{C^u_t}{C^h_t} \tag{108}$$

$$\begin{aligned} Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{I}\right)\right] I_t + \frac{\kappa}{2}(\pi_t - 1)^2 Y_t \\ + \Gamma_h(\chi^h_t + \chi^{DC,h}_t) + \Gamma_u(\chi^M_t + \chi^{DC,u}_t) \end{aligned} \tag{109}$$

$$K_t = \Gamma_h(K^h_t + K^b_t) \tag{110}$$

$$\begin{aligned} N_t = \sigma_b\left[(z^k_t + (1-\delta)Q_t)K^b_{t-1} - \frac{R_{t-1}}{\pi_t}D_{t-1}\right] \\ + \gamma_b(z^k_t + (1-\delta)Q_t)\frac{K_{t-1}}{\Gamma_h} \end{aligned} \tag{111}$$

$$Q_t K^b_t = \phi_t N_t \tag{112}$$

$$Q_t K^b_t = D_t + N_t \tag{113}$$

**Exogenous processes.**

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon^A_t \tag{114}$$

$$\mathcal{M}_t = (1-\rho_M)\mathcal{M} + \rho_M \mathcal{M}_{t-1} + \varepsilon_t^M \tag{115}$$
$$\xi_t = \rho_b \xi_{t-1} + \varepsilon_t^\xi \tag{116}$$

### A.2.7 Model Steady State

In the non-stochastic steady state, we have the following:

$$Q = 1,$$
$$\pi = 1,$$
$$R = \frac{1}{\beta},$$
$$R^{DC} = R.$$

We define the discounted spreads on equity as:

$$s = \beta[z^k + (1-\delta)] - 1, \tag{117}$$

which we consider to be endogenous.

From the BHH's FOC with respect to equity, (72), we have:

$$\begin{aligned}
1 &= \beta \left[ \frac{z^k + (1-\delta)}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}} \right] \\
1 + \varkappa^h \Gamma_h \frac{K^h}{K} &= \beta \left[ z + (1-\delta) \right] \\
\Gamma_h \frac{K^h}{K} &= \frac{s}{\varkappa^h}.
\end{aligned} \tag{118}$$

Additionally, in steady state we have:

$$\Omega = \beta(1 - \sigma_b + \sigma_b \psi),$$
$$\upsilon = \frac{\Omega}{\beta},$$
$$\mu = \Omega \left[ z^k + (1-\delta) - \frac{1}{\beta} \right],$$

and so, using (117) we can write:

$$\frac{\mu}{\upsilon} = s.$$

Next, define $J$ as:

$$J = \frac{n_{t+1}}{n_t} = \left[ z^k + (1-\delta) \right] \frac{K^b}{N} - R \frac{D}{N},$$

and use the following:

$$\frac{D}{N} = \phi - 1,$$
$$\phi = \frac{K^b}{N},$$

to write $J$ as:

$$\begin{aligned}
J &= (z^k + (1-\delta) - R)\phi + R \\
&= \frac{1}{\beta} \left[ s\phi + 1 \right].
\end{aligned}$$

Then, from (40) we have:

$$
\begin{aligned}
N &= \sigma_b \left\{ \left[ z^k + (1-\delta) \right] K^b - RD \right\} + \gamma_b \left[ z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
\frac{N}{N} &= \sigma_b \left\{ \left[ z^k + (1-\delta) \right] \frac{K^b}{N} - R\frac{D}{N} \right\} + \frac{\gamma_b}{N} \left[ z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
\beta &= \sigma_b \beta J + \frac{\gamma_b}{N} \beta \left[ z^k + (1-\delta) \right] \frac{K}{\Gamma} \\
&= \sigma_b \beta J + \frac{\gamma_b K^b}{N} \left( 1 + \varkappa^h \Gamma \frac{K^h}{K} \right) \frac{K}{\Gamma K^b} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{\Gamma K^b}{K}} \\
&= \sigma_b \beta J + \gamma_b (1+s) \phi \frac{1}{\frac{K - \Gamma K^h}{K}} \\
&= \sigma_b \left[ s\phi + 1 \right] + \gamma_b (1+s) \phi \frac{1}{1 - \frac{s}{\varkappa^h}} \\
\beta &= \sigma_b + \left[ \sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}} \right] \phi,
\end{aligned}
$$

or

$$
\phi = \frac{\beta - \sigma_b}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}}
$$

Equation (23) in steady state gives us:

$$
\begin{aligned}
\psi &= \beta (1 - \sigma_b + \sigma_b \psi) J \\
&= \beta J - \beta \sigma_b J + \beta \sigma_b \psi J \\
&= \beta (1 - \sigma_b) J + \beta \sigma_b \psi J \\
&= \frac{\beta (1 - \sigma_b) J}{1 - \beta \sigma_b J} \\
&= \frac{(1 - \sigma_b) \left[ s\phi + 1 \right]}{1 - \sigma_b \left[ s\phi + 1 \right]} \\
&= \frac{(1 - \sigma_b) \left[ s\phi + 1 \right]}{1 - \sigma_b - \sigma_b s \phi},
\end{aligned}
$$

and from (79) we have

$$
\psi = \theta^b \phi.
$$

Combine the expressions for $\phi$ and $\psi$ to get:

$$
\frac{\theta^b (\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} = \frac{(1 - \sigma_b) \left[ \frac{s(\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} + 1 \right]}{1 - \sigma_b - \sigma_b \left[ \frac{s(\beta - \sigma_b)}{\sigma_b s + \gamma_b \frac{1+s}{1 - \frac{s}{\varkappa^h}}} \right]},
$$

then rearrange:

$$H(s) = (1-\sigma_b)\left[s\beta + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right]\left[s\sigma_b + \gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right] - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s + (1-\sigma_b)\gamma_b \frac{1+s}{1-\frac{s}{\varkappa^h}}\right].$$

We can observe that as $\gamma_b \to 0$,

$$H(s) = (1-\sigma_b)s^2\beta\sigma_b - \theta^b(\beta-\sigma_b)\left[\sigma_b(1-\beta)s\right]$$
$$\implies s \to \theta^b \frac{(\beta-\sigma_b)(1-\beta)}{(1-\sigma_b)\beta}.$$

Thus, there exists a unique steady state equilibrium with positive spread $s > 0$ for a small enough $\gamma_b$.

Given $s$, we then yield:

$$z^k = \frac{1}{\beta}(1+s) - (1-\delta),$$

and from (10) in the steady state:

$$MC = \frac{1-\tau}{\mathcal{M}},$$

and with (8), (9), and (11) we get:

$$MC = \frac{z^k}{\alpha}\frac{K}{Y},$$

or

$$\frac{K}{Y} = MC\frac{\alpha}{z^k}.$$

From the FOCs of the BHH and UHH problem, we have:

$$w = \zeta_0^h (L^h)^\zeta,$$
$$w = \frac{\zeta_0^u (L^u)^\zeta (1 + \frac{\alpha_M}{\beta} - \alpha_M)}{\left[C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta}\right]}.$$

But since we have that $\zeta_0^u = \frac{\zeta_0^h}{(1+\frac{\alpha_M}{\beta}-\alpha_M)}$, we can write:

$$w = \zeta_0^h L^\zeta.$$

We can then use our previous expression for $w$ to express $L$ as a function of $z^k$:

$$L = \left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1}{\zeta}}.$$

Since we know that

$$w = (1-\alpha)\frac{Y}{L},$$

we yield:

$$Y = \frac{\zeta_0^h}{\alpha}\left[\frac{1-\alpha}{\zeta_0^h}\left(\frac{z^k}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\right]^{\frac{1+\zeta}{\zeta}}.$$

Additionally, we have:

$$\frac{I}{K} = \delta,$$

and

$$\frac{1}{\beta} = \frac{\alpha \frac{Y}{K} + 1 - \delta}{1 + \varkappa^h \Gamma_h \frac{K^h}{K}}$$
$$\Leftrightarrow \frac{Y}{K} = \frac{\beta^{-1}(1+s) + \delta - 1}{\alpha},$$

from (118), and

$$\frac{I}{Y} = \frac{I/K}{Y/K} = \frac{\alpha\delta}{\beta^{-1}(1+s) + \delta - 1}.$$

These of course imply:

$$K = \left[ \frac{1-\alpha}{\zeta_0^h} \left( \frac{z^k}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{1+\zeta}{\zeta}} \frac{\zeta_0^h}{\beta^{-1}(1+s) + \delta - 1}$$

With $K$ and $s$ in hand, we can then turn back to the BHH's FOC wrt to equity, (72), to find $K^h$:

$$K^h = \frac{s}{\varkappa^h} \frac{K}{\Gamma_h},$$

and also get $K^b$:

$$K^b = \frac{K}{\Gamma_h} - K^h.$$

This then gives us $N$ as we already solved $\phi$:

$$N = \frac{K^b}{\phi}.$$

Then $D$ is also solved as a residual from (19):

$$D = K^b - N.$$

Given $Y$, $I$, and $K$, we can get $C$:

$$\frac{C}{Y} = 1 - \frac{I}{Y} - \frac{\varkappa^h}{2} (\Gamma_h K^h)^2 \left( \frac{K}{Y} \right)^{-1}.$$

From the UHH's FOC with respect to $M$, we have:

$$\mu^u = \lambda^u \left( \frac{1}{\beta} - 1 \right),$$

and the FOC with respect to consumption gives us an expression for the marginal utility from consumption:

$$\left( C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta} \right)^{-1} = \lambda^u \left( 1 + \frac{\alpha_M}{\beta} - \alpha_M \right).$$

Thus, we can express $\lambda^u$ as a function of the marginal utility of consumption:

$$\frac{1}{\lambda^u} = \left( 1 + \frac{\alpha_M}{\beta} - \alpha_M \right) \left( C^u - \zeta_0^u \frac{(L^u)^{1+\zeta}}{1+\zeta} \right),$$

noting that because of the values of $\zeta_0^h$ and $\zeta_0^u$, we have:

$$L^u = \left( \frac{w}{\zeta_0^h} \right)^{\frac{1}{\zeta}}.$$

Finally, much like aggregate digital currency holdings, the BHH will not hold any digital currency holdings in steady state due to the presence of management costs. This means that in steady state:

$$DC^h = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^h$$

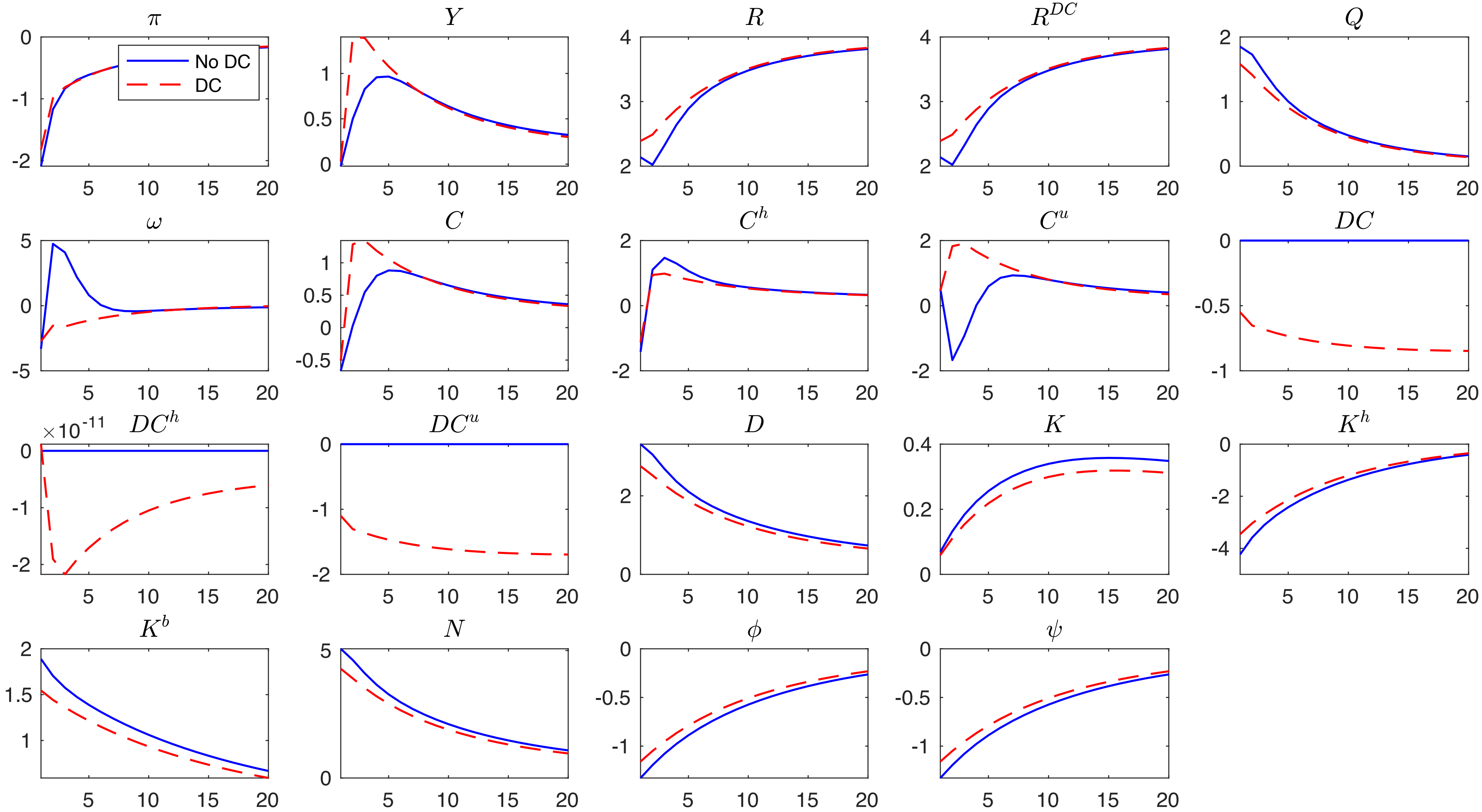
which, of course, implies:

$$DC^u = \frac{\beta R^{DC} - 1}{\varkappa^{DC}} + \widetilde{DC}^u.$$

### A.2.8 Additional Impulse Responses to Shocks

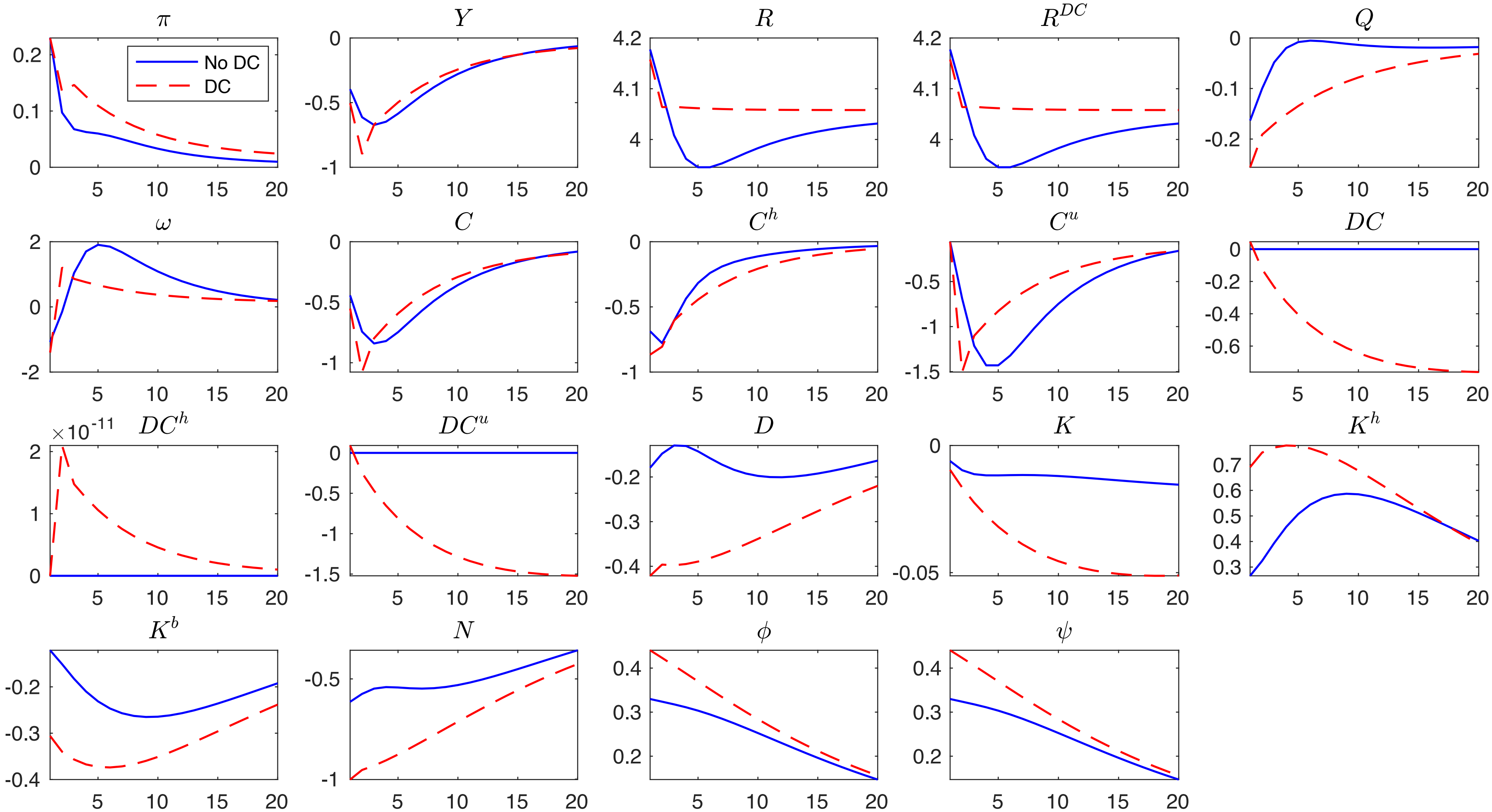
Figures 9, 10, and 11 present results in response to an annualized 1% orthogonal innovation to TFP, cost-push, and preference shocks, respectively. The figures compare IRFs for a no-CBDC economy and to a CBDC-equipped economy.

Figure 9: **IRFs to a 1% TFP shock**



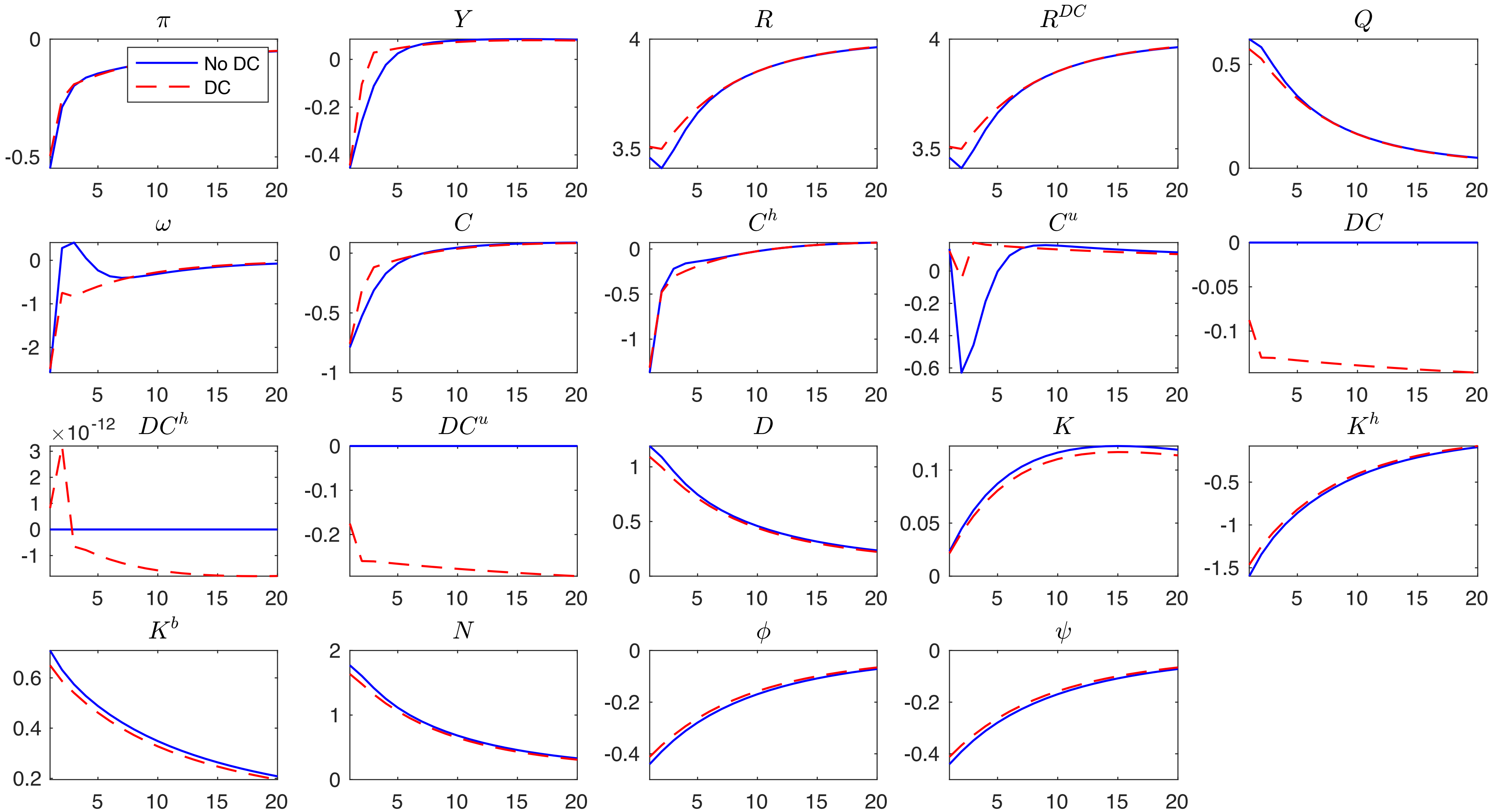

Note: Figure plots impulse responses of model variables with respect to a 1 % annualized innovation to TFP. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation ($\pi$), Nominal Interest Rates ($R$), and Digital Currency Returns ($R^{DC}$) which are expressed as annualized net rates.

Figure 10: **IRFs to a 1% cost-push shock**




Note: Figure plots impulse responses of model variables with respect to a 1 % annualized innovation to markups. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation ($\pi$), Nominal Interest Rates ($R$), and Digital Currency Returns ($R^{DC}$) which are expressed as annualized net rates.

Figure 11: **IRFs to a 1% annualized demand shock**




Note: Figure plots impulse responses of model variables with respect to a 1% annualized preference shock. Time periods are measured in quarters, and responses are measured as a percent deviation from steady state except for Inflation ($\pi$), Nominal Interest Rates ($R$) and Digital Currency Returns ($R^{DC}$) which are expressed as annualized net rates.