Monetary Tightening, Quantitative Easing, and Financial Stability

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Abstract

This paper analyses central bank balance sheet policies in a framework with banks facing occasionally-binding leverage constraints and endogenous disruptions in financial intermediation. Whilst balance sheet policies are effective in alleviating negative effects of financial stress episodes, they increase the probability of such episodes occurring and increase their duration. Moreover, balance sheet policies are found to be effective in mitigating negative implication of financial stress on economic activity in a tightening cycle but come at a cost to price stability.

Keywords: Quantitative easing, financial stability, monetary policy, financial crises.

JEL Codes: E52, E44, E58.

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1. Introduction

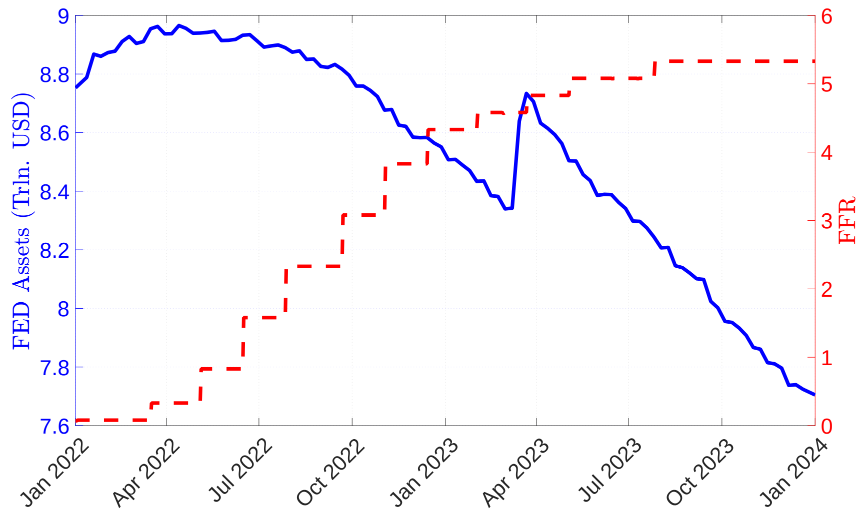
Central bank balance sheet policies have been widely used as a stabilisation tool in times of financial stress since the Great Financial Crisis of 2008 by major reserve banks, including US Federal Reserve, Bank of England, and the European Central Bank. While balance sheet interventions have been shown to be effective in mitigating recessionary effects of financial stress episodes (Del Negro et al. 2017), it is not clear whether such a policy contributes to the moral hazard problem of financial intermediaries inducing them to take on more risk which, in turn, results in higher likelihood of financial stress episodes.

Prior to 2022, balance sheet expansions (QE) were used to complement cuts to conventional policy rates. More recently, QE was paired with increases in policy rates. In 2022, developed countries saw an unprecedented increase in inflation rates, which prompted their central banks to undertake substantial interest rate hikes. Surge in interest rates has led to a decline in financial stability and triggered several instances of financial turmoil – Silicon Valley Bank and Credit Suisse collapse in March 2023, and UK Liability Driven Investment Crisis, amongst others. In the wake of financial turmoil, FED, Bank of England, and Swiss National Bank resorted to balance sheet expansions whilst continuing to raise their policy rates. Figure 1 illustrates an instance of unconventional pairing of QE and increase in policy rate in the US around March 2023.

Do balance sheet expansions increase the probability of financial stress events happening? If so, are such balance sheet interventions nonetheless welfare improving? Can balance sheet interventions in a tightening cycle address financial fragility without severely compromising price stability? This paper addresses these questions thought the lens of a macroeconomic model.

In the model, the central bank purchases long-term government bonds from households and issues reserves to financial intermediaries (banks) to finance the purchase. Banks intermediate funds between households and

Figure 1. Federal Reserve Assets and Policy Rate



Note: Federal Reserve Assets (left, solid blue line, Trillions of US Dollars), Effective Federal Funds Rate (right, dashed red line, percentage points).

non-financial firms. Banks are modelled in the spirit of Gertler and Karadi (2011) and Akinci and Queralto (2022); bankers can abscond with a fraction of assets, that consist of firm equity and safe central bank reserves, if their value is greater than bank's franchise value. It is more difficult for a bank to divert safe assets than firm equity. These assumptions translate into an incentive compatibility constraint, which is more likely to bind when safe asset to portfolio ratio of the bank is smaller. The constraint is assumed to be occasionally binding and, thus, frictions in financial intermediation are state-dependent. In tranquil times, when the constrain is not binding, financial intermediation is frictionless. In times of financial stress, however, when the constraint is binding, financial intermediation is frictional. Balance sheet expansions of the central bank have real effects in times of financial stress; QE increases reserves provision to financial intermediaries and alleviates the severity of their moral hazard problem.

I find that a rule-based QE triggered by financial stress is able to alleviate recessionary pressures of crippling credit frictions, but increases the probability of such episodes occurring as well as their duration. Two distinct channels drive this result. First, a QE rule alters bank's choice of leverage; banks pick higher leverage, and are thus closer to their leverage constraint, when the QE rule is in place than when banks anticipate no balance sheet intervention. Second, a balance sheet intervention reduces the banks' excess returns during financial stress episodes which does not allow banks to recapitalise as quickly as they otherwise would had the intervention not been used.

The fact that rule-based QE effectively stabilises the economy during a financial stress episode, but in tranquil times induces banks to pick higher leverage, which results in higher frequency of financial stress episodes, creates a non-trivial trade-off for the central bank. In an optimal policy exercise I find that **[add on OSR exercise results]**.

QE is also effective in stabilising the economy if financial stress is triggered by rising inflation and interest rates. I simulate a financial turmoil event making the model economy subject to an inflationary shock and find that QE is indeed able to mitigate the adverse implications of financial stress in a tightening cycle at a significant cost to price stability.

Related literature. First, this paper relates to the vast literature on central bank balance sheet policies for macroeconomic stabilisation that emerged past the Great Financial Crisis of 2008. Crucially, contributions of this literature break the irrelevance result described in Wallace (1981) along two dimensions. The balance sheet policies have been found to have real effects in environments with scarce liquidity and financial frictions. Seminal papers such as Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Cúrdia and Woodford (2011), Chen, Cúrdia, and Ferrero (2012), Harrison (2017), Del Negro et al. (2017), and Haas (2023) have found that balance sheet policies have significant real effects on macroeconomic stability. In contrast to this strand of literature, that focuses on the effects of balance sheet interventions in the frameworks where financial frictions are always present, I allow for the financial frictions to be state-dependent. This allows for analysing asset shifting behaviour of banks, driven by expectations of a QE intervention in times of financial stress.

Second, since the model economy endogenously switches between tranquil periods, when financial intermediation is frictionless, and financial stress times, when bank leverage constraint is binding, the paper relates to the literature on non-linearities in DSGE models. Seminal contributions include Bianchi (2010), Mendoza (2010), Akinci and Queralto (2022), Akinci et al. (2023) amongst others. The model framework used in the paper is close to the one used in Akinci et al. (2023), with the difference being that this paper uses a monetary general equilibrium framework, whereas Akinci et al. (2023) uses a real model where the interest rate is exogenous. Compared to this strand of literature, this paper emphasises central bank balance sheet interventions and changes in precautionary behaviour of banks that arises therefrom.

Third, this paper contributes to an emerging strand of literature on the optimal sequencing of central bank balance sheet interventions and interest rate policies. Benigno and Benigno (2022) examine the trade-offs linked to raising policy rates and reducing the central balance sheet. Airaudo (2023) studies the effects of quantitative tightening under passive monetary and active fiscal policy. Within this strand of literature, this paper is close to Haas (2023) as it also looks into the implications of pairing central bank balance sheet expansion with interest rate hikes. Haas (2023) finds that a balance sheet expansion can foster financial stability without compromising price stability, although does not consider implications of such a policy pairing for welfare. Similar to Haas (2023), this paper presents evidence that QE can indeed attenuate negative implications of financial stress on economic activity. This, however, comes at a cost to price stability, which contradicts the conclusions of Haas (2023).

The paper proceeds as follows. Section 2 outlines the model and calibration. Section 3 shows that the calibrated model produces empirically relevant financial crisis dynamics. Section 4 looks into stabilisation properties of QE and presents the policy counterfactuals in a crisis experiment. Section 5 presents optimal balance sheet rules and looks into their welfare implications. Section 6 concludes.

2. Model

The model framework comprises households, production sector, financial intermediaries, central bank, and treasury.

A representative household consumes final goods, supplies labour to non-financial firms, holds long-term treasury debt and deposits with financial intermediaries.

Production sector comprises final goods firms, intermediate goods firms, and capital goods producers. Intermediate firms produce differentiated intermediate goods and are subject to price rigidities as in Calvo (1983). Competitive final good firms produce final goods using intermediate goods as inputs. Capital goods firms transform final goods into physical capital and are subject to investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005).

Financial sector is modelled following Gertler and Kiyotaki (2010) and Akinci and Queralto (2022). Bankers are part of the household and are experts in intermediation of funds from households to firms; they use deposits and their retained net-worth to purchase equity from non-financial firms and safe assets. Bankers can abscond with a fraction of their assets which results in a moral hazard problem and implies an incentive compatibility constraint (ICC) to ensure non-absconding in equilibrium. The severity of the moral hazard problem depends on the share of safe assets in bankers' portfolio. In contrast to Gertler and Kiyotaki (2010) and following Akinci and Queralto (2022), the ICC is assumed to be occasionally binding. When the ICC does not bind, financial intermediation is frictionless. If the constraint binds, however, financial intermediation becomes frictional and the economy enters a financial stress episode, which is triggered by the financial accelerator mechanism and characterised by volatile investment and spikes in equity spreads. In financial stress episodes, central bank asset purchase programmes have real effects as they increase the proportion of safe assets in banker's portfolio and render the moral hazard problem less severe.

Central bank sets short-term interest rate and effectuates balance sheet policy. It purchases long-term debt from households and provides reserves to financial intermediaries. Treasury issues short- and long-term debt inelastically and levies lump-sum taxes from households.

2.1. Households

The model economy is populated with representative households that consume final goods, C_t, supply labour, L_t, hold deposits with financial intermediaries, D_t, and purchase long-term treasury debt, $B^H_{L,t}$. The household maximises the following infinite stream of discounted instantaneous utilities

$$\max_{\{C_t,L_t,D_t,B^H_{L,t}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \zeta_t^h \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\nu}}{1+\nu} \right),$$

where ζ_t^h is preference shock, β is discount factor, ν is inverse-Frisch elasticity of labour supply, and σ is coefficient of relative risk aversion.

Per-period household budget constraint in real terms is given by

$$C_t + D_t + (1+\xi_{L,t})B^H_{L,t} = w_t L_t + \frac{R^d_{t-1}}{\pi_t} D_{t-1} + \frac{R_{L,t}}{\pi_t} B^H_{L,t-1} + \Xi_t,$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, D_t is deposits, w_t is real wage, $B^H_{L,t}$ is real market value of long-term debt belonging to the household, Ξ_t denotes proceeds from ownership of banks and producers, and $\xi_{L,t}$ is adjustment cost of long-term debt holdings given by

$$1+\xi_{L,t} = \bar{\xi}_L \left(\frac{B^H_{L,t}}{B^H_L} \right)^{\xi}, \tag{1}$$

where ξ denotes the elasticity of the adjustment cost with respect to long-term debt holdings and $\bar{\xi}_L$ is steady-state term premium.

2.2. Production

Production sector consists of capital goods producers, final goods producers, and intermediate goods firms. Capital goods producers transform final goods into investment goods and are subject to investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005). Final goods producers use intermediate inputs for production of a synthetic consumption good and are perfectly competitive. Intermediate goods producers use labour and capital to produce varieties of intermediate goods, are monopolistically competitive, and are subject to nominal rigidities as in Calvo (1983).

Capital goods producers. Capital goods are produced by perfectly competitive firms. Aggregate capital stock grows according to a standard law of motion:

$$K_t = I_t + (1 - \delta)K_{t-1}, \tag{2}$$

where I_t is investment and $\delta \in (0, 1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximise revenue, $Q_t I_t$. I assume that capital goods producing firm is subject to investment adjustment cost as in Christiano, Eichenbaum, and Evans (2005). Thus, the representative capital good producing firm's objective function is:

$$\max_{I_t} \Lambda_{t,t+s} \left\{ Q_{t+s} - 1 - \frac{\kappa_I}{2} \left(\frac{I_{t+s}}{I_{t+s-1}} - 1 \right)^2 \right\} I_{t+s},$$

where κ_I is scaling parameter of adjustment cost and $\Lambda_{t,t+s}$ is unrestricted households' stochastic discount factor given by

$$\Lambda_{t,t+s} = \beta \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma}$$

Final goods producers. Final goods producers are perfectly competitive and use differentiated inputs $y_t(i)$, produced by an individual intermediate good firm i, to produce final goods y_t. They maximise the following profit function

$$\max_{y_t(i)} \left(P_t y_t - \int_0^1 P_t(i) y_t(i) di \right)$$

subject to the production constraint

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}}, di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$$

where ϵ_t denotes time-varying elasticity of substitution between differentiated inputs.

Optimisation yields the condition for demand for intermediate goods

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} y_t \tag{3}$$

Intermediate goods producers. Intermediate goods producers use a constant returns to scale Cobb-Douglas production technology to produce differentiated inputs for final production. With an exogenous probability θ they cannot adjust their prices in a given period. Their objective is, thus, to choose prices and production inputs, labour $l_t(i)$ and capital $k_t(i)$ to maximise the following discounted stream of profits

$$\max_{P_t(i), l_t(i), k_t(i)} \mathbb{E}_0 \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \left\{ \left(\frac{P_t(i)}{P_{t+s}} - mc_{t+s}(i) \right) y_{t+s}(i) \right\},$$

subject to demand for intermediate goods (3) and the production technology constraint

$$y_t(i) = A_t k_{t-1}(i)^{\alpha} l_t(i)^{1-\alpha},$$

where α denotes capital share in output and $mc_t(i)$ denotes i'th firm's marginal cost.

Solution to the problem yields a standard New-Keynesian Phillips curve and demand schedules for labour and capital.

2.3. Financial intermediaries

There is a continuum of bankers who are specialists in intermediation of funds between households and non-financial firms. Bankers are part of unrestricted household whom they share a consumption insurance scheme with. An individual banker uses its net-worth, n_t, and deposits obtained from households, d_t, to issue loans to non-financial firms, k_t, and accumulate safe assets, $b^I_{S,t}$. Safe assets are composed of public short-term debt and central bank reserves; since these assets are assumed to have the same risk-return profile, they are aggregated in a single variable. Individual banker's balance sheet is thus given by

$$Q_t k_t + b^I_{S,t} = n_t + d_t \tag{4}$$

Each period, bankers stay in business with an exogenous probability σ_b and exit with a complimentary probability $1 - \sigma^b$. If they exit, they transfer their franchise value V_t to households. Every period, $1 - \sigma_b$ new bankers get a start-up fraction γ of total equity $Q_t K_t$.

Bankers can abscond with a fraction $\Theta(x_t)$ of their assets, $Q_t k_t + b^I_{S,t}$, and will only do so if this fraction of assets exceeds their franchise value. This gives rise to the agency problem. Bankers do not abscond if the following incentive compatibility constraint is satisfied

$$V_t \geq \Theta(x_t)(Q_t k_t + b^I_{S,t}), \tag{5}$$

where $\Theta(x_t)$ is proportion of divertible assets[1] and x_t is safe asset to portfolio ratio

$$x_t = \frac{b^I_t}{Q_t k_t + b^I_{S,t}}. \tag{6}$$

The function $\Theta(\cdot)$ that determines proportion of assets that can be diverted is decreasing, $\Theta(x_t)' < 0$, and convex, $\Theta(x_t)'' > 0$, indicating that a banker can divert a smaller portion of assets when the portfolio includes more safe assets. Nevertheless, when the share of safe assets is substantial, the

1. The following functional form for divertible assets is assumed, $\Theta(x_t) = \left(1 - \frac{\lambda_b}{\kappa} x_t^{\kappa}\right)$, following Akinci et al. (2023).

incremental increase in x_t leads to a smaller reduction in the divertible proportion. This assumption implies that the moral hazard problem of financial intermediaries in more severe when their safe asset holdings are low, and gives rise to the real effects of central bank balance sheet policies in financial stress episodes. In times of financial stress, central bank can increase its provision of safe assets to the financial system thus reducing the severity of the constrain of the financial sector.

The banker maximises the present discounted franchise value

$$\max_{k_t, b^I_{S,t}, d_t} V_t = \mathbb{E}_t \sum_{s=0}^{\infty} \sigma_b^s (1 - \sigma_b) \left(\Lambda_{t,t+s+1} n_{t+s+1} + \Lambda_{t,t+s+1} \zeta^b_{t+s} b^I_{s,t+s} \right),$$

where ζ^b_t denotes an exogenous shock process that governs banker's preference for safe assets.

The flow budget constraint of a typical banker is given by

$$Q_t k_t + b^I_{s,t} + \frac{R^d_{t-1}}{\pi_t} d_{t-1} = R^k_t Q_{t-1} k_{t-1} + \frac{R_{t-1}}{\pi_t} b^I_{S,t} + d_t, \tag{7}$$

which, combined with Equation (4), yields the following expression for net-worth

$$n_t = \left(R^k_t - \frac{R^d_{t-1}}{\pi_t} \right) Q_{t-1} k_{t-1} + \left(\frac{R_{t-1}}{\pi_t} - \frac{R^d_{t-1}}{\pi_t} \right) b^I_{S,t} + \frac{R^d_{t-1}}{\pi_t} n_{t-1}$$

Defining leverage ratio as

$$\phi_t \equiv \frac{Q_t k_t + b^I_{S,t}}{n_t} \tag{8}$$

and franchise value to net-worth ratio, $\psi_t = V_t / n_t$, allows to rearrange the banker's problem such that the banker picks safe asset and leverage ratios[2]:

$$\psi_t = \max_{x_t, \phi_t} \left(\mu_t (1 - x_t) + (\mu^B_t + \zeta^b_t) x_t \right) \phi_t + \upsilon_t$$

subject to incentive compatibility constraint

$$\left(\mu_t (1 - x_t) + (\mu^B_t + \zeta_t) x_t \right) \phi_t + \upsilon_t \geq \Theta(x_t) \phi_t, \tag{9}$$

where the following definitions of banker's stochastic discount factor, dis-

2. Derivation is provided in Appendix C

counted equity spread, safe asset spread, and return on deposits are made use of

$$\Omega_{t,t+1} \equiv \mathbb{E}_t \Lambda_{t,t+1}(1 - \sigma_b + \sigma_b \psi_{t+1})$$
$$\mu_t \equiv \mathbb{E}_t \Omega_{t,t+1} \left(R^k_{t+1} - \frac{R^d_t}{\pi_{t+1}} \right)$$
$$\mu^B_t \equiv \mathbb{E}_t \Omega_{t,t+1} \left(\frac{R_t}{\pi_{t+1}} - \frac{R^d_t}{\pi_{t+1}} \right)$$
$$\upsilon_t \equiv \mathbb{E}_t \Omega_{t,t+1} \frac{R^d_t}{\pi_{t+1}}.$$

Optimisation yields the following FOC for x_t

$$\mu^B_t - \mu_t + \zeta^b_t = \frac{\bar{\lambda}_t}{1 + \bar{\lambda}_t} \Theta'(x_t), \tag{10}$$

where $\bar{\lambda}_t$ denotes the Lagrange multiplier on the constraint in Equation (9). Note that when the constraint is not binding, the condition collapses to

$$\mu_t \equiv \mathbb{E}_t \Omega_{t,t+1} \left(R^k_{t+1} - \frac{R_t}{\pi_{t+1}} \right) = \xi^b_t,$$

which pins down credit spread in equilibrium with no financial stress. Absent of bankers' preference for safe assets, i.e. $\xi^b_t = 0$, this condition implies that, in tranquil times, equity spread is zero.

Optimisation with respect to ϕ_t yields

$$\bar{\mu}_t \equiv \mu_t(1 - x_t) + (\mu^B_t + \zeta_t)x_t = \frac{\bar{\lambda}_t}{1 + \bar{\lambda}_t} \Theta(x_t), \tag{11}$$

where $\bar{\mu}_t$ denotes total excess returns of the financial sector.

Aggregating across bankers who continue in business and new bankers yields the following equation for evolution of net-worth

$$N_t = \sigma^b \left[\left(R^k_t - \frac{R^d_{t-1}}{\pi_t} \right) Q_{t-1} K_{t-1} + \frac{\left(R_{t-1} - R^d_{t-1} \right)}{\pi_t} B^I_{t-1} + \frac{R^d_{t-1} N_{t-1}}{\pi_t} \right] \tag{12}$$
$$+ (1 - \sigma^b)\gamma Q_{t-1} K_{t-1}$$

The incentive constraint (9) can be expressed to define the upper bound on leverage

$$\bar{\phi}_t = \frac{\upsilon_t}{\Theta(x_t) - \bar{\mu}_t}. \tag{13}$$

This condition highlights the mechanism through which Central Bank interventions alleviate the severity of a financial stress episode. Central bank balance sheet expansion directly affects the upper bound for leverage through safe asset ratio, x_t. When the central bank expands its balance sheet, it directly affects the amount of central bank reserves, thus increasing safe asset ratio of financial intermediaries.

Using the definition of $\bar{\mu}_t$, divide (11) by (10) to get

$$\mu_t^B - \mu_t + \zeta_t = \bar{\mu}_t \frac{\Theta'(x_t)}{\Theta(x_t)}. \tag{14}$$

When the constraint in (9) does not bind, i.e. $\bar{\lambda}_t = 0$, total excess returns of the banker are equal to zero, $\bar{\mu}_t = 0$. Financial intermediation is thus frictionless. By implication, $\bar{\phi}_t > \phi_t$. If the ICC binds, the excess returns are no longer zero, $\bar{\mu}_t > 0$, but realised leverage is equal to its upper bound, $\bar{\phi}_t = \phi_t$. Hence, the following regime determination condition holds

$$\bar{\mu}_t(\bar{\phi}_t - \phi_t) = 0. \tag{15}$$

2.4. Policy Authorities

Central Bank. Monetary authority sets the policy rate and effectuates asset purchases. Central bank can purchase both public and private debt. Policy rate is set according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\pi_t^{\phi_\pi} X_t^{\phi_y}\right)^{1-\rho_R} \exp\left(\varepsilon_t^R\right), \tag{16}$$

where ρ_R denotes policy rate inertia, ϕ_π and ϕ_y are denote coefficients of feedback to inflation and output gap deviations, respectively, ε_t^R is an exogenous disturbance, and X_t is defined as an output gap between the realised output Y_t and a counterfactual measure of real activity that would have otherwise occurred in the same economy with no price rigidities and frictions in financial intermediation.

Balance sheet policy consists of purchases of long-term government debt and provision of reserves to financial intermediaries. The budget constraint of the monetary authority is given by

$$R_{t-1}\frac{B^{cb}_{S,t-1}}{\pi_t} + R_{L,t}\frac{B^{cb}_{L,t-1}}{\pi_t} = B^{cb}_{L,t} + B^{cb}_{S,t} + \Lambda^{cb}_t, \tag{17}$$

where Λ^{cb}_t denotes the transfers from the central bank to Treasury. Note that $B^{cb}_{S,t}$ is a composite of public short-term debt holdings of the central bank and the central bank reserves, which, by assumption, have the same risk-return profile and, hence, are aggregated in a single variable.

When the central bank effectuates balance sheet policy, it adheres to the following revenue-neutrality constraint

$$B^{cb}_{L,t} + B^{cb}_{S,t} = 0, \tag{18}$$

which implies that if the central bank increases its holdings of long-term debt, $B^{cb}_{L,t}$, it increases the provision of reserves to the financial intermediaries.

Treasury. Treasury collects lump-sum taxes from households τ_t, receives transfers from the central bank Λ^{cb}_t, and issues short-term and long-term debt inelastically. The budget constraint of the government reads as

$$\tau_t + \bar{B}_L + \bar{B}_S + \Lambda^{cb}_t = \frac{R_{t-1}}{\pi_t}\bar{B}_S + \frac{R_{L,t}}{\pi_t}\bar{B}_L. \tag{19}$$

Issuance of public debt follows a constant maturity structure, $B_S = \rho B_L$, with ρ determining the ratio of short-term to long-term debt.

Public short-term debt issued by the Treasury is held by financial intermediaries or the central bank

$$\bar{B}_S = B^{cb}_{S,t} + B^{I}_{S,t}. \tag{20}$$

Long-term debt issued by the Treasury is held by the central bank or by households

$$\bar{B}_L = B^{cb}_{L,t} + B^{H}_{L,t}. \tag{21}$$

Combining budget constraint of the central bank (17), that of Treasury (19), and using market clearing conditions for short- and long-term debt, (20) and

(21), yields consolidated budget constraint of the government

$$\tau_t + B^H_{L,t} + B^I_{S,t} = \frac{R_{t-1}}{\pi_t} B^I_{S,t-1} + \frac{R_{L,t}}{\pi_t} B^H_{L,t-1}, \tag{22}$$

which is cast in terms of public debt held by the private sector.

2.5. Market clearing and equilibrium

Output of final goods is either consumed or invested

$$Y_t = C_t + \left(1 + \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t. \tag{23}$$

This completes the description of the model. The competitive equilibrium is a set of 36 variables: 14 quantities { C_t, L_t, K_t, I_t, Y_t, N_t, $B^{cb}_{S,t}$, $B^I_{S,t}$, $B_{S,t}$, $B^{cb}_{L,t}$, $B^H_{L,t}$, $B_{L,t}$, G_t, τ_t }, 9 prices { mc_t, z^k_t, w_t, π_t, Q_t, R^k_t, R^d_t, R_t, $R_{L,t}$ }, 9 banker variables { $\Omega_{t,t+1}$, μ_t, μ^B_t, υ_t, ψ_t, ϕ_t, x_t, $\bar{\mu}_t$, $\bar{\phi}_t$ }, and 4 exogenous processes { ζ^h_t, A_t, ϵ_t, ζ^b_t } that satisfy the equilibrium conditions outlined in Appendix A.

2.6. Calibration

Certain parameters are calibrated to match first moments in the data. β is set to match an average interest rate of 2%, short-term debt to GDP is set to 15%, while ρ is set such that long-term debt to GDP is around 100%. Assets of the central bank to GDP are set to 45%. Steady state term premium matches the average of 1% consistent with the data.

Other parameters are calibrated to the values that are standard in the literature. Constant relative risk aversion coefficient σ is set to 2. Inverse Frisch elasticity of labour supply is set to 1/4; since the model does not feature nominal rigidities in wage setting, I use a rather low value for inverse-Frisch elasticity. Relative disutility of labour is set such that agents work one third of their time endowment. Elasticity of substitution across intermediate inputs is set to match 10% markup. Capital depreciation δ is standard and is set to 0.025, the probability of not being able to adjust the price in a given period, θ is set to 0.8. Feedback coefficients to inflation and output gap deviations are assumed to be equal to 2 and 0.05, respectively, consistent with estimates in Bianchi, Faccini, and Melosi (2022). Taylor rule inertia is set equal to 0.55. Elasticity of long-term yield to long-term debt holdings is set equal to the estimate in Chen, Cúrdia, and Ferrero (2012).

Parameters pertaining to the banking sector are calibrated as follows. Parameters that govern the severity of incentive compatibility constraint, θ^b, κ, and λ^b, are set to match average occurrence of financial stress of 3% and such that $\Theta(x_t)$ is decreasing and convex. Other banker parameters, γ and σ^b, are calibrated to match steady state leverage of approximately 6.

Table 1. Parameter values

Symbol	Value	Description	Source/Target
Households			
β	0.9928	Discount factor	Interest rate 3%
σ	2	Relative risk aversion	Standard
χ	8.3	Relative disutility of labour	Labour 1/3 of time
ν	1/3	Inverse Frisch	Gertler and Kiyotaki (2010)
ξ	0.01	Elasticity of LTD adj. cost	Chen, Cúrdia, and Ferrero (2012)
$\bar{\xi}_L$	0.025	s.s. term premium	2% term premium
Production			
ϵ	10	Elasticity of sub. across int. inputs	10% Markup
δ	2.5%	Capital depreciation	Standard
α	1/3	Capital share	Standard
κ_I	2/3	Investment adjustment cost	–
θ	0.9265	Calvo probability	–
Bankers			
θ_b	0.724	Fraction of divertible funds	3% frequency of fin. crises
κ	0.124		–
λ_b	0.117		–
σ_b	0.925	Continuation probability	Av. bank survival 3.5y.
γ	0.2		Leverage 6
x	0.2	Safe asset to portfolio	Data
ζ^b	0.00125	Safe asset preference	1% equity spread
Monetary policy			
ρ_R	0.55	Policy rate inertia	-
ϕ_π	2	Inflation feedback coefficient	-
ϕ_y	0.125	output feedback coefficient	-
$B_L^{cb}/4Y$	45%	SS value of LTD holdings	Data
Fiscal policy			
G/Y	20%	Gov. spending to GDP	Data
$B_S/4Y$	15%	ST Gov. debt to GDP	Data
ρ	1/8	Maturity structure of public debt	Data
Exogenous Processes			
ρ_A	0.85	TFP persistence	
ρ_h	0.85	Preference shock persistence	
ρ_ϵ	0.85	Markup shock persistence	
ρ_B	0.85	Safe asset preference persistence	
σ_A	0.44%	TFP std. deviation	
σ_R	0.05%	MP shock std. deviation	
σ_h	0.25%	Preference shock std. deviation	
σ_ϵ	5%	Markup shock std. deviation	
σ_B	0.0202%	Safe asset preference std. deviation	

3. Quantitative properties

In this section, I show that the model can well account for the stylised empirical facts that characterise financial stress episodes presented in Akinci and Queralto (2022).

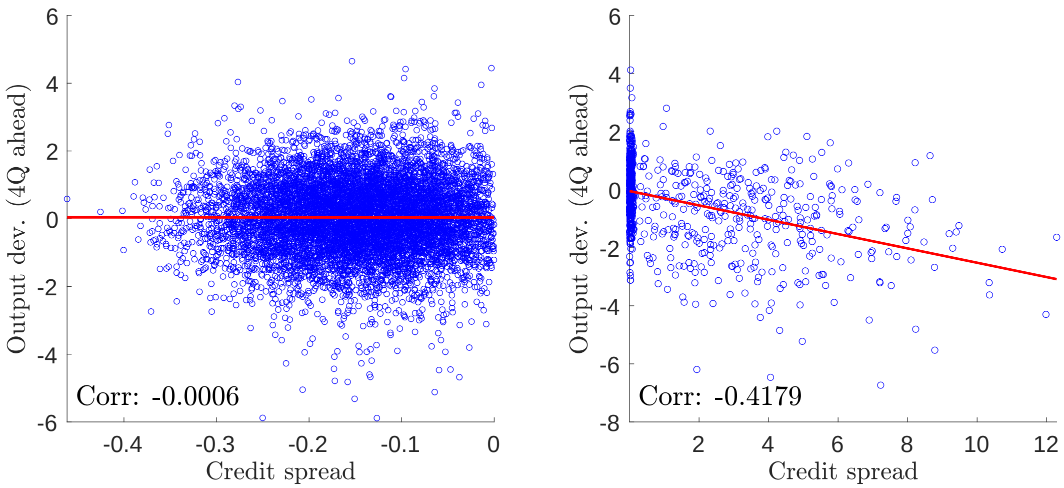
First, as is empirically established, credit spreads are countercyclical and are even more so in episodes of financial stress suggesting a strong non-linearity in the financial friction. I show that the model indeed generates an asymmetric relationship between credit spreads and output deviations. The model is able to account for this non-linearity due to the presence of the occasionally binding incentive constraint in the financial intermediaries' problem. Second, the model generates a realistic average financial stress episode that is characterised by depressed output, sharp decline in investment and bank net-worth, and credit spread spikes. Third, the model generates a right-skewed distribution of credit spreads with a fat right tail arising from rare occurrence of financial stress episodes.

First, the model gives rise to an asymmetric relationship between credit spread and economic activity. When spreads are elevated, they tend to be more strongly associated with negative deviations of output. When spreads are low, they tend to be weakly negatively correlated with output.

This relationship is due to the inherent non-linearity in financial intermediation. In normal times, when the economy is far from the constrained region, financial intermediation is frictionless. On the contrary, when financial intermediaries are subject to stress and are in the constrained region, credit spreads demonstrate occasional spikes. As financial intermediation becomes frictional, banks are no longer able to effectively intermediate funds between households and non-financial firms. This leads to depressed investment in physical capital, which, in turn, triggers a decline in price of equity and leads to a credit crunch. Simultaneously, a binding incentive constraint implies a high and volatile credit spread. Figure 2 illustrates this property of the model.

Second, as documented in Akinci and Queralto (2022), financial crisis

Figure 2. Output and Credit Spreads

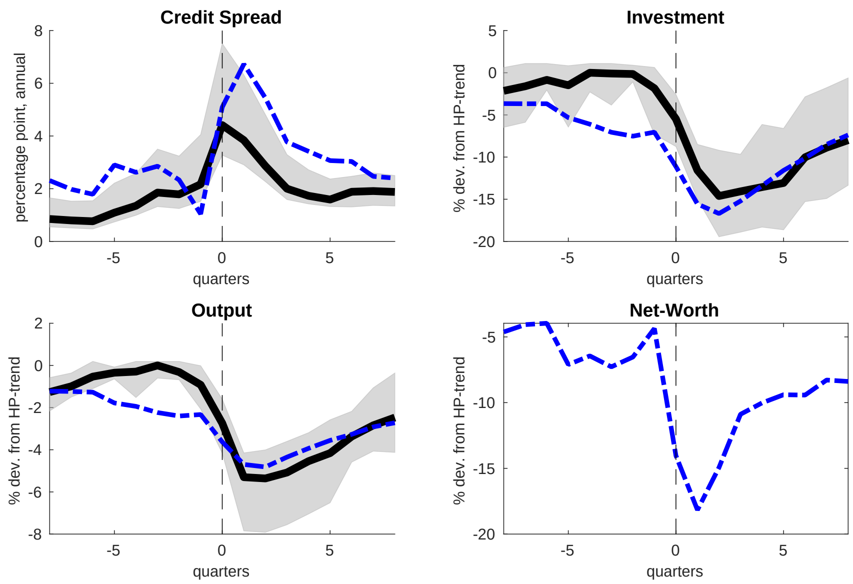


Note: The model is simulated for 10'000 periods. Financial crisis episodes occur around 3% of the simulated sample. Left panel plots the relationship between the cyclical component of output four periods ahead and the credit spread when credit spread is below sample mean. Right panel plots the same relationship when credit spread is above mean.

episodes are characterised by a severe decline in output, investment, and bank net-worth, and spikes in credit spreads. Figure 3 shows that an average financial crisis episode is the model is consistent with the empirical evidence. In an average financial crisis episode, output declines by around 4%, investment and bank net-worth fall sharply by around 15%, and credit spreads demonstrate a spike of around 6%.

The model generates an empirically relevant distribution of credit spreads. The data features a right-skewed distribution of credit spreads. In normal times, spreads are low and not volatile. In times of financial stress, however, credit spreads are high and volatile. The model generates a right-skewed credit spread distribution as in the data. Skewness in the credit spread distribution is caused by the presence of the occasionally binding leverage constraint. When the constraint becomes binding, banks' ability to intermediate funds is constrained, which depresses investment and triggers and sharp increase in return on equity.

Figure 3. Average financial crisis



Note: Credit spread, policy rate, and inflation are percentage points. Other variables are in percent deviations from HP-trend. Solid black line - data mean, shaded regions - min. and max. values of variables in the data. Average financial crisis episode is defined as a period where the leverage constraint of the banking sector binds for at least 4 consecutive periods. Financial crisis starts in period zero. The plot shows dynamics of aggregate variables 20 periods prior to and after the first period where the leverage constraint starts to bind. Sources: Akinci and Queralto (2022), FRED, Bank of England, author's calculations.

4. Policy and Financial Stress

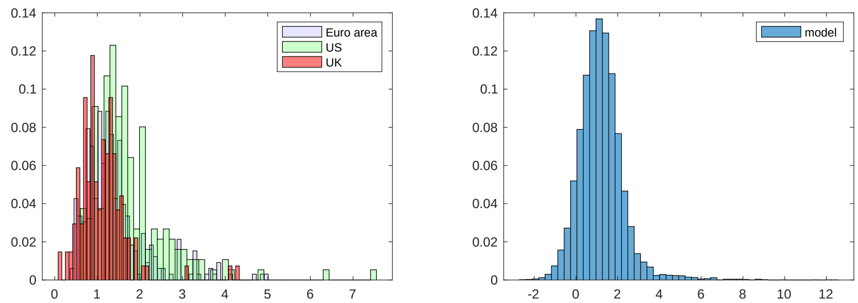
In this section, I analyse stabilisation properties of balance sheet policies that target credit spreads. As credit spreads demonstrate spikes in times of financial stress, they serve as a natural target for a rule-based balance sheet intervention. Further, I conduct a financial stress experiment where I make the model economy subject to an inflationary shock that leads to an interest hike, which endogenously brings banks to their leverage constraint.

Balance sheet intervention is governed by the following policy rule.

$$\frac{B^{cb}_{L,t}}{B^{L,cb}} = \left(\frac{S_t}{S}\right)^{\phi_{QE}} \exp\left(\varepsilon^{QE}_t\right), \tag{24}$$

where $\phi_{QE} > 0$ is the feedback coefficient that governs the magnitude of

Figure 4. Credit spreads: data and model



Note: data sourced from BoE, FRED, Akinci and Queralto (2022). Model is simulated for 10'000 periods.

long-term debt purchases made by the central bank with respect to the credit spread, $S_t \equiv \mathbb{E}_t\{R^K_{t+1} - R_t/\pi_{t+1}\}$; if the credit spread increases, indicating a financial stress episode, the central bank increases its purchases of long-term debt which, in turn, leads to the reserves provision to financial intermediaries via condition (18).

Balance sheet rule stabilisation properties. To explore stabilisation properties of balance sheet rules, I simulate the model for 10'000 periods under the three calibrations of the policy rule presented above. Using the simulated data, I calculate standard deviations of key variables, frequency of financial stress episodes, and welfare improvements compared to baseline case, $\phi_{QE} = 0$. The results are presented in Table 2.

One can observe that endogenous balance sheet expansions significantly decrease standard deviations of output, investment, net-worth, and leverage of financial intermediaries. This is rationalised by their direct effect on financial intermediaries' net-worth and the severity of the moral hazard problem. Moreover, balance sheet interventions are found to be generally welfare improving. Balance sheet expansions in financial crisis episodes, however, adversely impact standard deviation of inflation as highlighted above and increase the likelihood of a financial stress episode occurring. The frequency of financial stress episodes increases from 4.8% under no intervention to almost 5.9% under a balance sheet intervention scenario.

Table 2. Standard deviations, welfare, and crisis frequency

	$\phi_{QE} = 0$	$\phi_{QE} = 10$	$\phi_{QE} = 100$
Output, Y	2.12	2.05	1.94
Inflation, π	0.46	0.48	0.51
Policy Rate, R	0.68	0.65	0.67
Investment, I	7.16	6.83	6.38
Net-Worth, N	5.22	4.62	3.75
Leverage, ϕ	3.59	3.21	2.57
Credit Spread, $\mathcal{S}$	1.03	0.74	0.37
Welfare Imp.	-	0.56%	0.79%
Stress Frequency	4.82%	5.4%	5.88%

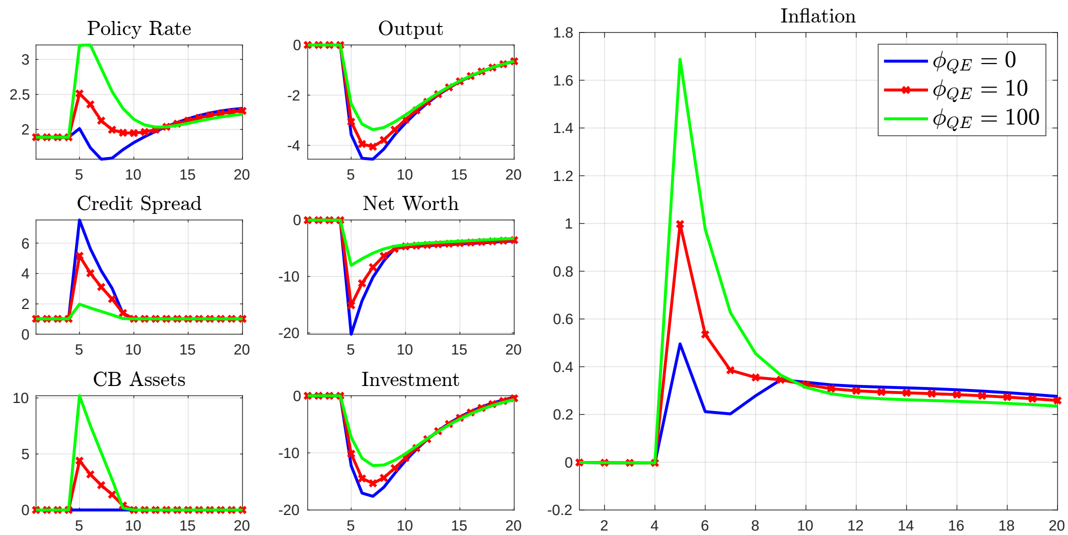
Note: standard deviations in % from simulated quarterly mean, except π, R, and $\mathcal{S}$, which are annualised. Welfare is relative to no intervention case. Crisis frequency: number of periods where ICC is binding.

Higher frequency of financial stress episodes can be rationalised through two distinct channels. First, since balance sheet expansions are governed by a policy rule and are anticipated by banks, they alter financial intermediaries portfolio choice. Banks anticipate that the central bank will intervene should the financial stress episode emerge and pick higher leverage. Thus, they are closer to their leverage constrained, all else equal. When adverse shock materialise, the banks are more likely to hit the leverage constraint since their leverage is closer to the constraint under the intervention scenario. Second, balance sheet interventions directly affect the ability of banks to recapitalise themselves in times of stress through excess returns. As credit spreads increase during financial stress episodes, banks earn an excess return on their equity holdings. Under a balance sheet expansion, bank excess returns are naturally lower as their leverage constraint becomes less tight. This leads to lower excess returns over the course of the financial stress episode, which does not allow the banks to increase net-worth and decrease leverage below its upper bound.

Deploying a balance sheet expansion in a financial crisis episode leads to better stabilisation of bank variables allowing for more effective intermediation of funds to non-financial firms. Mitigation of adverse effects of financial stress on output and better financial stability, however, comes at a cost to price stability. This presents a non-trivial trade-off for a central bank and naturally presents an avenue for an optimal policy exercise that I conduct below. I explore how balance sheet and interest rate policy should be jointly conducted in the following section.

Financial stress experiment. The economy is initially close to the bankers' incentive constraint and is hit by a 5% markup shock that materialises in period 5. Figure 5 plots the simulated paths of key variables under different values of the elasticity of the central bank balance sheet size with respect to credit spreads, ϕ_{QE}. The central bank is either does not expand its balance sheet, $\phi_{QE} = 0$ (solid blue line) or expands its balance sheet endogenously with feedback coefficient equal to 10 or 100, red and green lines, respectively.

Figure 5. Crisis experiment: cost-push shock



Note: Simulation based on three consecutive markup shocks materialising in period 5 onwards calibrated such that the leverage constraint binds for four periods. All variables are in deviations from stochastic steady state except inflation, interest rates, and spreads, which are annualised rates, net-worth subsidy, which is a quarterly rate, and tax to GDP ratio. Solid blue line – path of variables absent of intervention. Solid red line – rule-based QE intervention. Solid green line – rule-based macroprudential intervention.

First, consider the scenario where the central bank does not expand its balance sheet. The markup shock triggers a mild increase in inflation which leads to an initial increase in the policy rate. As the economy enters a financial stress episode, intermediation of funds between households and non-financial firms is not longer frictionless. Banks, being at their incentive constraint, engage in fire sale dynamics of firm equity, which leads to a decline in investment, further decline in equity price, and another round of fire sales. A sharp decline in investment increases return on equity and, by implication, the credit spread, which attains around 7%. As output drops, central bank decreases its policy rate to stimulate the economy and counteract the adverse implications of the financial stress episode on output.

Second, consider the case where the central bank endogenously expands its balance sheet in response to the financial stress episode. The balance sheet intervention is able to attenuate the negative effects of financial stress as it has direct implications on the severity of the banker incentive constraint. As the central bank purchases long-term debt, it does so by issuing reserves to financial intermediaries, which, in turn, leads to higher proportion of safe assets in their portfolio and reduces the severity of their moral hazard problem. Under either calibration of the central bank elasticity coefficient with respect to credit spread, the balance sheet expansion mitigates the adverse implications of financial stress on output. The balance sheet expansion directly improves net-worth of financial intermediaries thus allowing for better intermediation of funds to non-financial firms, which, in turn, leads to higher output compared to the no-intervention case. The intervention also allows for more scope for policy rate tightening to quench inflationary pressures.

Under balance sheet intervention, however, inflation attains higher levels than under no intervention. The level of inflation on impact is inversely related to the size of the balance sheet intervention of the central bank. This showcases the fundamental trade-off between price and financial stability that cannot be resolved if the central bank resorts to expanding its balance sheet in the tightening cycle. Although the balance sheet expansion allows to mitigate the adverse implications of financial stress on output, this stabilisation comes at a cost to price stability.

As noted above, central bank balance sheet interventions effectively mitigate the adverse implications of financial stress on economic activity. The, however, comes at a cost to price stability if the financial turmoil occurs in a tightening cycle. Furthermore, rule based balance sheet expansions increase the frequency of financial stress episodes. In other words, balance sheet interventions imply that financial stress episodes happen more often but are less severe. This creates a non-trivial trade-off for the central bank. In the next section, I conduct an optimal policy exercise that sheds light on how to navigate this trade-off.

5. Optimal Simple Rules and Welfare

[discussion and results to be completed]

6. Conclusion

Central bank balance sheet policies have been a widely used stabilisation tool since the Great Financial Crisis of 2008 to complement expansionary interest rate policy when the latter was constrained. More recently, they have been deployed in a tightening cycle to quench financial stability concerns. This paper has built a monetary model that generates empirically realistic endogenous financial stress episodes and analysed the implications of balance sheet policies on financial and price stability.

First, whilst central bank balance sheet expansions can effectively mitigate the adverse implications of financial stress on economic activity, they result in higher frequency of such episodes. This result is driven by two distinct channels. One, banks, anticipating a central bank intervention if a financial turmoil occurs, over-leverage and are, thus, closer to their leverage constraint. In other words, banks are willing to take on more risk if they expect the central bank to intervene should a financial stress episode occur. Two, if the central bank deploys a balance sheet expansion in a financial stress episode, credit spreads are compressed compared to the non-intervention case, and, thus, banks earn lower excess returns which does not allow them to recapitalise as quickly as they otherwise would in the no-intervention case.

Second, if a financial stress episode is triggered by inflationary pressures and subsequent interest rate hikes, balance sheet interventions still have a benign impact on economic activity, however, this comes at a cost to price stability.

The fact that balance sheet expansions lead to less severe yet more frequent financial stress episodes creates a non-trivial trade-off for the central bank. This paper conducts an optimal policy exercise: **[discussion on optimal policy exercise tbd]**.

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A. Equilibrium conditions

Households. Household optimisation implies the conditions for labour supply

$$\chi(L_t)^{\nu}(C_t)^{\sigma} = w_t \tag{A1}$$

Euler equation for long-term debt

$$\mathbb{E}_t\Lambda_{t,t+1}\left(\frac{R_{L,t+1}}{\pi_{t+1}}\right) = 1 + \xi_{L,t} \tag{A2}$$

Euler equation for deposits

$$\mathbb{E}_t\Lambda_{t,t+1}\left(\frac{R_t^d}{\pi_{t+1}}\right) = 1, \tag{A3}$$

where $\Lambda_{t,t+s}$ is households stochastic discount factor given by

$$\Lambda_{t,t+s} = \mathbb{E}_t\beta^s\left(\frac{C_{t+s}}{C_t}\right)^{-\sigma}$$

Capital goods producers. Law of motion for capital

$$K_t = I_t + (1-\delta)K_{t-1}, \tag{A4}$$

Price of equity

$$Q_t = 1 + \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \kappa_I\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} - \kappa_I\mathbb{E}_t\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t} - 1\right)\frac{I_{t+1}^2}{I_t^2} \tag{A5}$$

Intermediate goods producers. Producer optimisation implies the following conditions for capital-labour ratio, capital demand, and output

$$Y_t = \frac{A_t K_{t-1}^{\alpha} L_t^{1-\alpha}}{\vartheta_t} \tag{A6}$$

$$mc_t = \frac{1}{A_t}\left(\frac{z_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{A7}$$

$$\frac{1-\alpha}{\alpha} = \frac{w_t L_t}{z_t^k K_{t-1}}. \tag{A8}$$

Inflation determination. As indicated in the main text, proportion θ of firms cannot adjust their prices and a complimentary proportion $1-\theta$ can do so, hence inflation is given by

$$\pi_t^{1-\epsilon_t} = (1-\theta)(\pi_t^*)^{1-\epsilon_t} + \theta, \tag{A9}$$

where π_t^* is growth rate of optimal price given by

$$\pi_t^* = \frac{\epsilon_t}{\epsilon_t - 1}\frac{X_{1,t}}{X_{2,t}}$$

$$X_{1,t} = (C_t)^{-\sigma} mc_t Y_t + \beta\theta\pi_{t+1}^{\epsilon_t} X_{1,t+1}$$
$$X_{2,t} = (C_t)^{-\sigma} Y_t + \beta\theta\pi_{t+1}^{\epsilon_t - 1} X_{2,t+1}$$

Price dispersion is given by

$$\vartheta_t = (1-\theta)\left(\frac{\pi_t}{\pi_t^*}\right)^{\epsilon_t} + \theta\pi_t^{\epsilon_t}\vartheta_{t-1}$$

Banks. I use the following auxiliary definitions for banker SDF, discounted equity spread, discounted safe asset spread, and discounted real deposit rate:

$$\Omega_{t,t+1} = \mathbb{E}_t\Lambda_{t,t+1}(1-\sigma^b+\sigma^b\psi_{t+1}) \tag{A10}$$
$$\mu_t = \mathbb{E}_t\Omega_{t,t+1}\left(R_{t+1}^k - \frac{R_t^d}{\pi_{t+1}}\right) \tag{A11}$$
$$\mu_t^B = \mathbb{E}_t\Omega_{t,t+1}\left(\frac{R_t}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}}\right) \tag{A12}$$
$$\upsilon_t = \mathbb{E}_t\Omega_{t,t+1}\frac{R_t^d}{\pi_{t+1}} \tag{A13}$$

Safe asset to portfolio ratio

$$x_t = \frac{B_{S,t}^I}{Q_t K_t + B_{S,t}^I} \tag{A14}$$

Franchise value to net-worth

$$\psi_t = \upsilon_t + \bar{\mu}_t\phi_t \tag{A15}$$

Maximum leverage ratio

$$\bar{\phi}_t = \frac{\upsilon_t}{\theta(1-\frac{\lambda}{\kappa}x_t^\kappa) - \bar{\mu}_t} \tag{A16}$$

Total excess returns

$$\bar{\mu}_t = \mu_t(1-x_t) + (\mu_t^B + \zeta_t^B)x_t \tag{A17}$$

Realised leverage

$$\phi_t = \frac{Q_t K_t + B_{S,t}^I}{N_t} \tag{A18}$$

Banker optimality condition

$$\mu_t^B - \mu_t + \zeta_t^b = -\bar{\mu}_t\frac{\lambda x_t^{\kappa-1}}{(1-\frac{\lambda}{\kappa}x_t^\kappa)} \tag{A19}$$

Net-worth evolution

$$N_t = \sigma_b\left[\left(R_t^k - \frac{R_{t-1}^d}{\pi_t}\right)Q_{t-1}K_{t-1} + \frac{(R_{t-1}-R_{t-1}^d)}{\pi_t}B_{S,t-1}^I + \frac{R_{t-1}^d}{\pi_t}N_{t-1}\right] + (1-\sigma_b)\gamma Q_{t-1}K_{t-1} \tag{A20}$$

Regime determination equation

$$\bar{\mu}_t(\bar{\phi}_t - \phi_t) = 0 \tag{A21}$$

Return on equity

$$R_t^k = \frac{z_t^k + (1-\delta)Q_t}{Q_{t-1}} \tag{A22}$$

Wage bill loan rate

$$R_t^w = R_t^d + \frac{\mu_t}{\mathbb{E}_t \Omega_{t,t+1}} \tag{A23}$$

Monetary authority. Conventional monetary policy is governed by a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\pi_t^{\phi_\pi} X_t^{\phi_y}\right)^{1-\rho_R} \exp\left(\varepsilon_t^R\right) \tag{A24}$$

Balance sheet policy is governed by the following rule

$$\frac{B_{L,t}^{cb}}{B^{L,cb}} = \left(\frac{S_t}{S}\right)^{\phi_{QE}} \exp\left(\varepsilon_t^{QE}\right) \tag{A25}$$

Reserves provision is given by the following revenue neutrality condition

$$B_{L,t}^{cb} + B_{S,t}^{cb} = 0 \tag{A26}$$

Fiscal authority. Consolidated budget constraint

$$\tau_t + B_{L,t}^H + B_{S,t}^I + T^r = \frac{R_{t-1}}{\pi_t} B_{S,t-1}^I + \frac{R_{L,t}}{\pi_t} B_{L,t-1}^H + G_t + \tau_t^n N_t \tag{A27}$$

Constant maturity structure condition

$$B_{S,t} = \rho B_{L,t} \tag{A28}$$

Short-term debt issuance

$$B_{S,t} = \bar{B}_S \tag{A29}$$

Market clearing and equilibrium. Resource constraint is given by

$$Y_t = C_t + I_t \left(1 + \frac{\kappa^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) \tag{A30}$$

Short-term bond markets clear

$$B_{S,t} = B_{S,t}^{cb} + B_{S,t}^I \tag{A31}$$

Long-term bond markets clear

$$B_{L,t} = B_{L,t}^{cb} + B_{L,t}^H \tag{A32}$$

Exogenous processes. Preference shock

$$\zeta_t^h = 1 - \rho^h + \rho^h \zeta_{t-1}^h + \varepsilon_t^h \tag{A33}$$

Total Factor Productivity

$$A_t = 1 - \rho^A + \rho^A A_{t-1} + \varepsilon_t^A \tag{A34}$$

Markup shock

$$\epsilon_t = \bar{\epsilon}(1 - \rho^{\epsilon}) + \rho^{\epsilon} \epsilon_{t-1} + \varepsilon_t^{\epsilon} \tag{A35}$$

Banker's safe asset preference

$$\xi_t^b = \bar{\xi}^b(1 - \rho^b) + \rho^b \xi_{t-1}^b + \varepsilon_t^b \tag{A36}$$

B. Unconstrained static equilibrium

Wider economy. I drop time sub-indices for variables in steady state. In a non-inflationary steady state $\pi = 1$, $R^d = 1/\beta$. Cost of capital is equal to unity, $Q = 1$. Since the leverage constraint is not binding, $R^k = R^d = R^w = 1/\beta$, and $R = (R^k - \xi^b)/\beta$. It follows that

$$z^k = R^K - 1 + \delta \tag{B1}$$

Marginal cost is equal to inverse of markup, $\mathcal{M} = \frac{\epsilon}{\epsilon-1} = mc^{-1}$. **[Set $\Upsilon = 0$ and delete]**

The definition of marginal cost implies

$$mc = \left(\frac{z^k}{\alpha}\right)^{\alpha} \left(\frac{w\left(1 + \Upsilon\left(R^w - 1\right)\right)}{1-\alpha}\right)^{1-\alpha} \Rightarrow w = \frac{(1-\alpha)}{(1+\Upsilon\,(R^w - 1))}\left[\frac{mc^{\frac{1}{\alpha}}\alpha}{z_k}\right]^{\frac{1}{\alpha}} \tag{B2}$$

I assume that there is a redistribution scheme that equalises consumption across households in steady state, $C^r = C^u$. This implies that labour supply is identical across households in steady state.

Using the condition for labour supply yields labour

$$L = \left(wC^{-\sigma}/\chi\right)^{\frac{1}{\nu}} \tag{B3}$$

I assume that there is a redistribution scheme that equalises consumption across households in steady state, $C^r = C^u$. This implies that labour supply is identical across households in steady state.

Output is then given by

$$Y = \frac{wL\left(1 + \Upsilon\left(R^w - 1\right)\right)}{(1-\alpha)mc} \tag{B4}$$

Capital is given by

$$K = \frac{\alpha mc Y}{z^k} \tag{B5}$$

Investment is given by

$$I = \delta K \tag{B6}$$

It is straightforward to solve for consumption given market clearing

$$C = Y - I - G \Rightarrow C = \left[\frac{w^{\frac{\nu+1}{\nu}}\left(1 + \Upsilon\left(R^w - 1\right)\right)}{(1-\alpha)mc\chi^{\frac{1}{\nu}}}\left\{1 - \frac{\alpha\delta mc}{z_k} - S_g\right\}\right]^{\frac{\nu}{\nu+\sigma}} \tag{B7}$$

Government. Steady-state values of G, B_S, and B_L^{cb} are calibrated.

$$B_S^{cb} = -B_L^{cb} \tag{B8}$$

Safe assets and central bank reserves are given by

$$B_S^I = B_S - B_S^{cb} \tag{B9}$$

Long-term government debt

$$B_L = \rho^{-1} B_S \tag{B10}$$

Private holdings of long-term debt

$$B_L^H = B_L - B_L^{cb} \tag{B11}$$

To determine the amount redistributed to restricted households, I use their budget constraint and assume that long-term per-capita holdings of long term debt are equivalent across households, $B_L^{H,r} = B_L^{H,u} = B_L^H$

$$T^r = C + B_L^H(1 - R_L) - wL \tag{B12}$$

[τ^n no longer relevant – delete]

Consolidated government budget constraint yields

$$\tau = G + (R_L - 1)B_L^H + (R - 1)B_S^I + \lambda_u T^r + \tau^n N \tag{B13}$$

Financial sector. Bank reserves and short-term assets B^I is determined residually. Thus, safe asset ratio is given by

$$x = \frac{B^I}{QK + B^I} \tag{B14}$$

From evolution of net-worth

$$N = \left(\frac{(1 - \sigma^b)\gamma QK - \xi^b \sigma^b B_S^I}{1 - \sigma^b(1 + \tau^n)R^d} \right) \tag{B15}$$

Leverage is given by

$$\phi = \frac{QK + B_S^I}{N} \tag{B16}$$

and deposits are determined residually via balance sheet

$$D = QK + B_S^I - (1 + \tau^n)N \tag{B17}$$

C. Rewriting the Banker's problem

Rearrange budget constraint (7) to get

$$\overbrace{Q_t k_t + b_t^I - d_t}^{n_t} = R_t^k Q_{t-1} k_{t-1} + R_{t-1} b_{t-1}^I - R_{t-1}^d d_{t-1} \tag{C18}$$

Balance sheet identity (4) can be rolled back one period and rearranged for d_{t-1}

$$d_{t-1} = Q_{t-1} k_{t-1} + b_{t-1}^I - n_{t-1} \tag{C19}$$

Plug the above for d_{t-1} in the recursive expression for n_t (C18)

$$n_t = R_t^k Q_{t-1} k_{t-1} + R_{t-1} b_{t-1}^I - R_{t-1}^d (Q_{t-1} k_{t-1} + b_{t-1}^I - n_{t-1})$$

Or, equivalently

$$n_t = (R_t^k - R_{t-1}^d) Q_{t-1} k_{t-1} + (R_{t-1} - R_{t-1}^d) b_{t-1}^I + R_{t-1}^d n_{t-1}$$

Now, turn to the objective function. Observe that (C18) can be rewritten as

$$(R_{t+i+1}^k Q_{t+i} k_{t+i} + R_{t+i} b_{t+i}^I - R_{t+i}^d d_{t+i}) = n_{t+i+1}$$

Use that to rewrite the objective function

$$V_t = \sum_{i=0}^{\infty} (\sigma^b)^i \left\{ \Lambda_{t,t+i+1} (1 - \sigma^b) n_{t+i+1} + \Lambda_{t,t+i} \zeta_{t+i} b_{t+i}^I \right\}$$

Divide through by n_t

$$\frac{V_t}{n_t} = \sum_{i=0}^{\infty} (\sigma^b)^i \left\{ \Lambda_{t,t+i+1} (1 - \sigma^b) \frac{n_{t+i+1}}{n_{t+i}} + \Lambda_{t,t+i} \zeta_{t+i} \frac{b_{t+i}^I}{n_{t+i}} \right\}$$

or alternatively express recursively

$$\frac{V_t}{n_t} = \Lambda_{t,t+1} (1 - \sigma) \frac{n_{t+1}}{n_t} + \Lambda_{t,t+1} \zeta_t \frac{b_t^I}{n_t} + \Lambda_{t,t+1} \sigma^b \frac{V_{t+1}}{n_{t+1}} \frac{n_{t+1}}{n_t}$$

factor out n_{t+1}/n_t

$$\frac{V_t}{n_t} = \Lambda_{t,t+1} (1 - \sigma^b + \sigma^b \frac{V_{t+1}}{n_{t+1}}) \frac{n_{t+1}}{n_t} + \Lambda_{t,t+1} \zeta_t \frac{b_t^I}{n_t} \tag{C20}$$

Define leverage ratio

$$\phi_t = \frac{Q_t k_t + b_t^I}{n_t} \tag{C21}$$

Turn to the last term on the RHS of (C20), use (6) and (C21)

$$\frac{b_t^I}{n_t} = \frac{b_t^I}{n_t} \left(\frac{Q_t k_t + b_t^I}{b_t} \right) \frac{b_t^I}{Q_t k_t + b_t^I} = x_t \phi_t$$

Now, turn to n_{t+1}/n_t:

$$\frac{n_{t+1}}{n_t} = (R_{t+1}^k - R_t^d) \frac{Q_t k_t}{n_t} + (R_t - R_t^d) \frac{b_t^I}{n_t} + R_t^d$$

Using $Q_t k_t / n_t = (1 - x_t)\phi_t$

$$\frac{n_{t+1}}{n_t} = (R^k_{t+1} - R^d_t)(1 - x_t)\phi_t + (R_t - R^d_t)x_t\phi_t + R^d_t$$

Define $V_t/n_t = \psi_t$ and rewrite th objective as follows

$$\psi_t = \Lambda_{t,t+1}(1-\sigma^b+\sigma^b\psi_{t+1})\left((R^k_{t+1} - R^d_t)(1 - x_t)\phi_t + (R_t - R^d_t)x_t\phi_t + R^d_t\right)+\Lambda_{t,t+1}\zeta_t x_t\phi_t \quad \text{(C22)}$$

Define

$$\Omega_{t,t+1} = 1 - \sigma^b + \sigma^b\psi_{t+1} \quad \text{(C23)}$$

$$\mu_t = \Lambda_{t,t+1}\Omega_{t,t+1}(R^K_{t+1} - R^d_t) \quad \text{(C24)}$$

$$\mu^B_t = \Lambda_{t,t+1}\Omega_{t,t+1}(R_t - R^d_t) \quad \text{(C25)}$$

$$\upsilon_t = \Lambda_{t,t+1}\Omega_{t,t+1}R^d_t \quad \text{(C26)}$$

Banker optimisation thence collapses to

$$\psi_t = \max_{x_t,\phi_t}\left(\mu_t(1 - x_t) + (\mu^B_t + \zeta_t)x_t\right)\phi_t + \upsilon_t$$

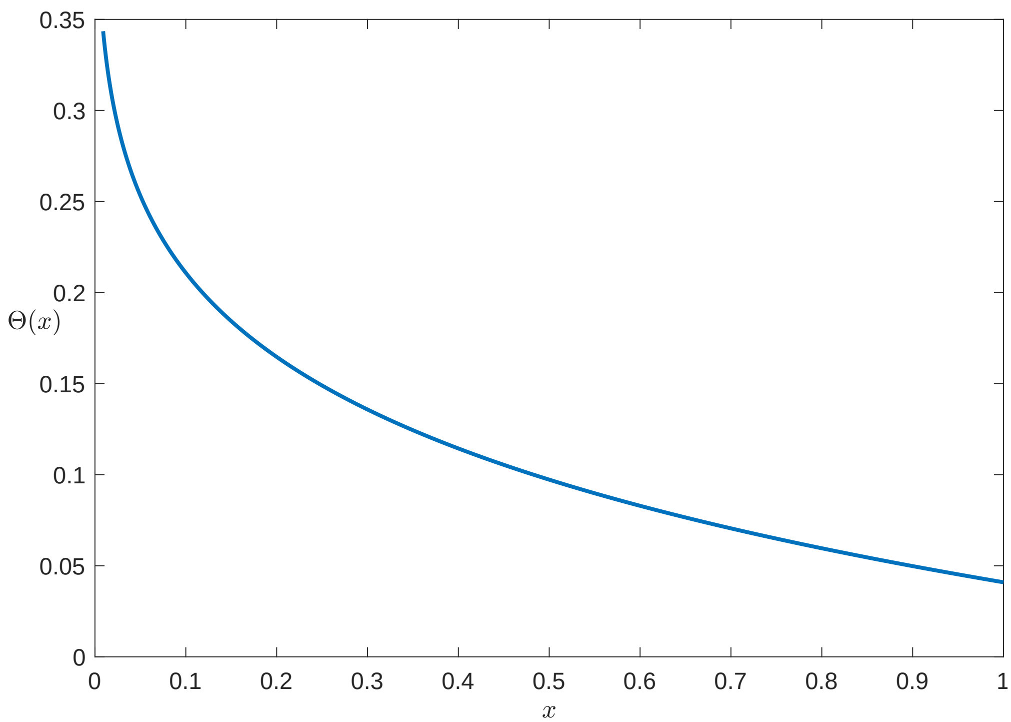
subject to incentive compatibility constraint

$$\left(\mu_t(1 - x_t) + (\mu^B_t + \zeta_t)x_t\right)\phi_t + \upsilon_t \geq \Theta(x_t)\phi_t$$

D. Functional form of divertible asset proportion

Functional form of $\Theta(x_t)$ is crucial for analysis of financial stress episodes as it governs the severity of the ICC in Equation (13). $\Theta(x_t)$ is assumed to be decreasing and convex in safe asset ratio x_t. These assumptions imply that if the banker's portfolio consists mostly of safe assets, the proportion of divertible funds is low and, by implication, the ICC is less severe. If the proportion of safe assets in portfolio, x_t, is high, however, increasing it further does not render the constraint a lot less severe.

Figure 6. Functional form of $\Theta(x)$



Note: Functional form of $\Theta(x_t)$. Horizontal axis shows values of safe asset proportion in bankers' portfolio. Vertical axis shows the proportion of divertible funds.