Graph the Parametric Curve
$f(x(t), y(t), z(t)) = \langle \sin(t), \cos(t), \sin(2t) \rangle$
on the interval $t = [0, 2\pi]$

Steps to complete this problem:

1. Construct Graph
2. Find intercepts
3. Find and label coordinates
4. Draw the line

Now to begin the problem, start with Step 1:

1. Construct graph:
   (a) Choose origin type (3D Cartesian)
   (b) Count number of needed axis segments (6, 2 red $x, -x$ 2 blue $y, -y$, and 2 yellow $z, -z$)
   (c) Insert axes into each hole in the origin (make a 3D plus sign + with each line one color)
   (d) Put toppers the axis end (6)
   (e) Insert an axis label into each topper (6, 2 red $x, -x$ 2 blue $y, -y$, and 2 yellow $z, -z$)
2. You will need a unit circle handy if you don't have it memorized. Intercepts for this vector valued function will be at the points on the unit circle where either the x or the y coordinate of the circle is 0. On the interval $t = [0, 2\pi]$, we can see that zeros occur at $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π.

   We should also label the octants as we find points. This label will help us graph later. There are eight octants in a 3D graph, so let's iterate through them to label:

   (a) O1: (x, y, z)
   (b) O2: $(-x, y, z)$
   (c) O3: $(x, -y, z)$
   (d) O4: $(x, y, -z)$
   (e) O5: $(-x, -y, z)$

(f) O6: $(-x, y, -z)$

(g) O7: $(x, -y, -z)$

(h) O8: $(-x, -y, -z)$

Now with each coordinate point, we should also identify the octant. With the intercepts, since zero is neither positive nor negative, some of those points will live in many of our octants. We can find all octants that it lives in according to our iterative list.

(a) At $t = 0$, we see that

$$f(x(0), y(0), z(0)) = (\sin(0), \cos(0), \sin(2(0)) = (0, 1, 0).$$

We can say this coordinate lives in O1, O2, O4, and O6.

(b) At $t = \frac{\pi}{2}$, we see that

$$f\left(x\left(\frac{\pi}{2}\right), y\left(\frac{\pi}{2}\right), z\left(\frac{\pi}{2}\right)\right) = \left(\sin\left(\frac{\pi}{2}\right), \cos\left(\frac{\pi}{2}\right), \sin(\pi)\right) = (1, 0, 0).$$

This coordinate lives in O1, O3, O4, and O7.

(c) At $t = \pi$, we see that

$$f(x(\pi), y(\pi), z(\pi)) = (\sin(\pi), \cos(\pi), \sin(2(\pi)) = (0, -1, 0).$$

This coordinate lives in O3, O5, O7, and O8.

(d) At $t = \frac{3\pi}{2}$, we see that

$$f\left(x\left(\frac{3\pi}{2}\right), y\left(\frac{3\pi}{2}\right), z\left(\frac{3\pi}{2}\right)\right) = \left(\sin\left(\frac{3\pi}{2}\right), \cos\left(\frac{3\pi}{2}\right), \sin(3\pi)\right) = (-1, 0, 0).$$

This coordinate lives in O2, O5, O6, and O8.

(e) At $t = 2\pi$, we see that

$$f(x(2\pi), y(2\pi), z(2\pi)) = (\sin(2\pi), \cos(2\pi), \sin(4\pi) = (0, 1, 0).$$

This coordinate lives in O1, O2, O4, and O6.

We can see that point (a) and point (e) are the same, so we've completed a full cycle through our curve. We should expect to see a cyclic type curve with a parametric function like this since all vectors are sine and cosine functions.

3. Label Axes Scale:

For this problem, we can see from the intercepts that we have a lower bound of -1 and an upper bound of 1 for the x and y axes.

We will be using the unit circle coordinate values to find our remaining coordinate points, so we can conclude that all of our coordinates on all axes will not escape our intercept bounds. These bounds are known because all other unit circle coordinates,

$$\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}, \quad and \quad \pm\frac{\sqrt{2}}{2}$$

are all within the interval $[-1, 1]$.

Therefore, we can choose our scale to be 0.1 for each hole or tick mark on our axes. Since $\frac{\sqrt{2}}{2} \approx 0.7$ and $\frac{\sqrt{3}}{2} \approx 0.9$ then all of our significant points can be approximately graphed in 3D space with an axes scale of 0.1.

4. Find and label coordinates

(a) At $t = \frac{\pi}{6}$, we see that

$$f\left(x\left(\frac{\pi}{6}\right), y\left(\frac{\pi}{6}\right), z\left(\frac{\pi}{6}\right)\right) = \left(\sin\left(\frac{\pi}{6}\right), \cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{3}\right)\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O1.

(b) At $t = \frac{\pi}{4}$, we see that

$$f\left(x\left(\frac{\pi}{4}\right), y\left(\frac{\pi}{4}\right), z\left(\frac{\pi}{4}\right)\right) = \left(\sin\left(\frac{\pi}{4}\right), \cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{2}\right)\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right).$$

This coordinate lives in O1.

(c) At $t = \frac{\pi}{3}$, we see that

$$f\left(x\left(\frac{\pi}{3}\right), y\left(\frac{\pi}{3}\right), z\left(\frac{\pi}{3}\right)\right) = \left(\sin\left(\frac{\pi}{3}\right), \cos\left(\frac{\pi}{3}\right), \sin\left(\frac{2\pi}{3}\right)\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O1.

(d) At $t = \frac{2\pi}{3}$, we see that

$$f\left(x\left(\frac{2\pi}{3}\right), y\left(\frac{2\pi}{3}\right), z\left(\frac{2\pi}{3}\right)\right) = \left(\sin\left(\frac{2\pi}{3}\right), \cos\left(\frac{2\pi}{3}\right), \sin\left(\frac{4\pi}{3}\right)\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O7.

(e) At $t = \frac{3\pi}{4}$, we see that

$$f\left(x\left(\frac{3\pi}{4}\right), y\left(\frac{3\pi}{4}\right), z\left(\frac{3\pi}{4}\right)\right) = \left(\sin\left(\frac{3\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{2}\right)\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -1\right).$$

This coordinate lives in O7.

(f) At $t = \frac{5\pi}{6}$, we see that

$$f\left(x\left(\frac{5\pi}{6}\right), y\left(\frac{5\pi}{6}\right), z\left(\frac{5\pi}{6}\right)\right) = \left(\sin\left(\frac{5\pi}{6}\right), \cos\left(\frac{5\pi}{6}\right), \sin\left(\frac{5\pi}{3}\right)\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O7.

(g) At $t = \frac{7\pi}{6}$, we see that

$$f\left(x\left(\frac{7\pi}{6}\right), y\left(\frac{7\pi}{6}\right), z\left(\frac{7\pi}{6}\right)\right) = \left(\sin\left(\frac{7\pi}{6}\right), \cos\left(\frac{7\pi}{6}\right), \sin\left(\frac{7\pi}{3}\right)\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O5.

(h) At $t = \frac{5\pi}{4}$, we see that

$$f\left(x\left(\frac{5\pi}{4}\right), y\left(\frac{5\pi}{4}\right), z\left(\frac{5\pi}{4}\right)\right) = \left(\sin\left(\frac{5\pi}{4}\right), \cos\left(\frac{5\pi}{4}\right), \sin\left(\frac{5\pi}{2}\right)\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1\right).$$

This coordinate lives in O5.

(i) At $t = \frac{4\pi}{3}$, we see that

$$f\left(x\left(\frac{4\pi}{3}\right), y\left(\frac{4\pi}{3}\right), z\left(\frac{4\pi}{3}\right)\right) = \left(\sin\left(\frac{4\pi}{3}\right), \cos\left(\frac{4\pi}{3}\right), \sin\left(\frac{8\pi}{3}\right)\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O5.

(j) At $t = \frac{5\pi}{3}$, we see that

$$f\left(x\left(\frac{5\pi}{3}\right), y\left(\frac{5\pi}{3}\right), z\left(\frac{5\pi}{3}\right)\right) = \left(\sin\left(\frac{5\pi}{3}\right), \cos\left(\frac{5\pi}{3}\right), \sin\left(\frac{10\pi}{3}\right)\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}, -\frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O6.

(k) At $t = \frac{7\pi}{4}$, we see that

$$f\left(x\left(\frac{7\pi}{4}\right), y\left(\frac{7\pi}{4}\right), z\left(\frac{7\pi}{4}\right)\right) = \left(\sin\left(\frac{7\pi}{4}\right), \cos\left(\frac{7\pi}{4}\right), \sin\left(\frac{7\pi}{2}\right)\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1\right).$$

This coordinate lives in O6.

(l) At $t = \frac{11\pi}{6}$, we see that

$$f\left(x\left(\frac{11\pi}{6}\right), y\left(\frac{11\pi}{6}\right), z\left(\frac{11\pi}{6}\right)\right)$$

$$= \left(\sin\left(\frac{11\pi}{6}\right), \cos\left(\frac{11\pi}{6}\right), \sin\left(\frac{11\pi}{3}\right)\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right).$$

This coordinate lives in O6.

We can see that we are really only working in quadrants 1, 5, 6, and 7.

5. Now it is time to draw the function. It is important to note that the function is cyclic. If the interval for t went on forever, we would see that the function values just repeat and we get the same loop no matter what interval of length 2π that we went.
This is best photographed per octant since the 3D image makes looking at the whole graph difficult:

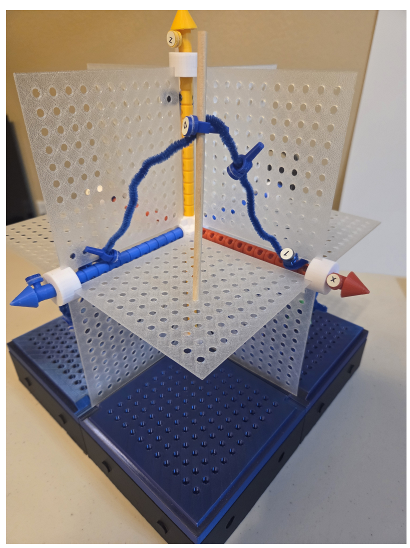

Figure 1: Octant 1

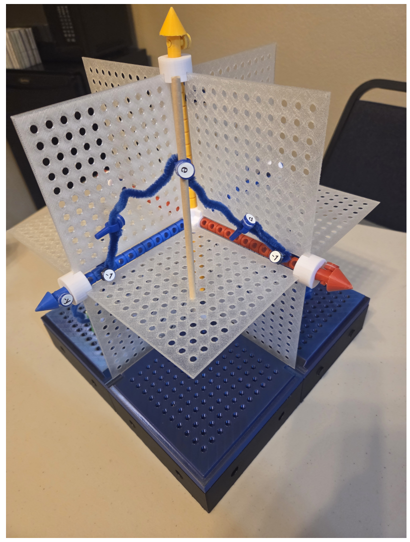

Figure 2: Octant 5

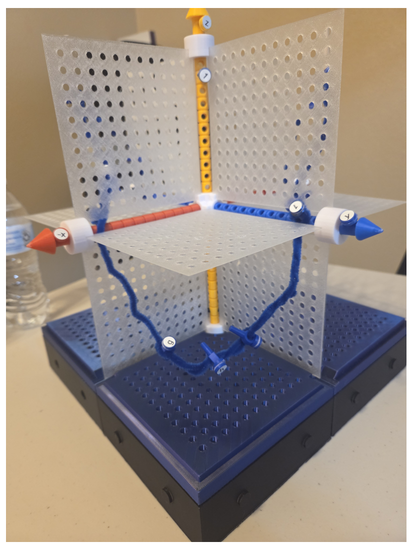

Figure 3: Octant 6

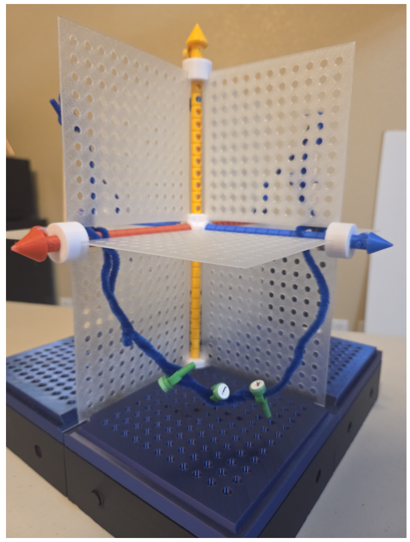

Figure 4: Octant 7

Putting those four images together builds a solid graph for this parametric curve. Be sure to check out the YouTube tutorial to see the curve on video for a full picture of this 4D graph.