Graph $y = 2\sin(x) + 1$ Using The Tactile Graph

Steps to complete this problem:

1. Construct Graph
2. Find intercepts
3. Find and label coordinates
4. Draw the line

Now to begin the problem, start with Step 1:

1. Construct graph:
   (a) Choose origin type (3D Cartesian)
   (b) Count number of needed axis segments (4, 2 red $x, -x$ and 2 blue $y, -y$)
   (c) Insert axes into the origin (make a plus sign + with each line one color)
   (d) Put toppers the axis end (4)
   (e) Insert an axis label into each topper (4, 2 red $x, -x$ and 2 blue $y, -y$)
2. Find the a, b, c, d from

$$y = a\sin(bx - c) + d \quad \text{for} \quad y = 2\sin(x) + 1.$$

   (a) For $y = 2\sin(x) + 1$ we can see that 2 is in the place of a so $a = 2$. In this problem, a is defined to be the amplitude of our wave, and you can think about that like how tall the wave turns out to be. Each bump in our wave should be 2 units tall, regardless of it is a peak shaped like $\cap$ or a trough shaped like $\cup$.

   Therefore, $a = 2$.

   (b) For $y = 2\sin(x) + 1$ there is an implicit 1 inside our $\sin(x)$ function, so $\sin(x) = \sin(1x)$. Implicit just means it is implied so not explicit or directly shown to you. All numbers have an implicit 1 you could multiply them by since any number multiplied by 1 is itself.

   Therefore, $b = 1$. In this step, we are not quite done by finding b. We care about b as it relates to our period, or how wide our wave turns out to be. In this case,

$$\text{Period } = T = \frac{2\pi}{|b|} = \frac{2\pi}{|1|} = \frac{2\pi}{1} = 2\pi.$$

We are going to use this period to decide how to mark the scale of our x-axis, so it is important to think about what a period is. Picture a sine wave, and imagine you only have one peak $\cap$ and one trough $\cup$.

Then the length of the period is the length from the start of the peak $\cap$ to the end of the trough $\cup$. We want to graph at least one full period, but 2 would be even better so we can really see what our wave is supposed to look like.

Therefore, $b = 1$ and so $T = 2\pi$.

(c) For $y = 2\sin(x - 0) + 1$, I added an implicit 0 to this equation since any number minus 0 is still itself. This 0 just helps us see that c is not in our equation as I presented it to you initially.

If c were not equal to 0, we would see a shift in our graph to the left if $c < 0$ or to the right if $c > 0$. For this problem, we have no shift, but these shifts can impact your x-intercepts and all the other points in your graph.

Therefore, $c = 0$.

(d) Lastly, for $y = 2\sin(x) + 1$ we can see that there is a $+1$ at the end of this equation, which must be d.

This 1 tells us the vertical shift. If $d < 0$ then our graph would shift down the y-axis. If $d = 0$ then for a $\sin(x)$ function, the y-intercept would be 0. Since $d > 0$ for our problem, we see our wave jumps up to have a y-intercept of 1.

It is important to note that d is only equal to our y-intercept in this case because our function is a $\sin(x)$ function. If we were graphing $\cos(x)$ then we would still have a vertical shift of 1, but our y-intercept would be 2 since $\cos(0) = 1$, unlike the sine function which equals 0 at 0.

Therefore, $d = 1$.

3. Label Axes Scale:

Unlike our other problem where we cared a lot about the x and y intercepts, wavy problems don't typically have nice values because that π gets in the way of pretty numbers, so we found the period and vertical shift instead in step 2.

For this part of the problem, we need to consider a, b, c, d for this function to ensure we can fit our wave on our graph.

We need to use a and d to define our y-axis scale. Since $d = 1$, we know we need at least 1 on our y-axis to fit that part of the graph.

Since $a = 2$, we know that each peak $\cap$ and trough $\cup$ are going to be 2 units tall. If we take $d + a = 1 + 2 = 3$, then we know that will be the tallest needed point on our y-axis. If we take $d - a = 1 - 2 = -1$ then we know -1 is going to be the lowest needed point on our y-axis.

So we need the y-axis to extend at least to -1 and at most to 3. We cna have a longer axis than we need, so I chose to scale each hole or tick in my y-axis to be worth 0.5, but you can choose other numbers as long as each segment is worth the same value, and you have included the interval $[-1, 3]$.

For our x-axis, we need to consider the values for b and c to determine the scale. Since $c = 0$ we don't have to worry about our graph shifting on the x-axis, so we will ignore that for this problem.

Since $b = 1$ which means our period $T = 2\pi$,then we know to get a full peak $\cap$ and trough $\cup$ in our graph, our x-axis needs to be at least 2π long. However, I recommend at least two periods in length to ensure you get a full picture of the wave with many peaks $\cap$ and troughs $\cup$. This full picture can help you identify key wavy features.

Since the period has a π in it, it is crucial to our problem that we include a π in our x-axis unit. I chose to increment each hole or tick by $\frac{\pi}{4}$ to make sure I have enough coordinates to graph my full period, and that my coordinates land on holes. Choosing a smaller number results in a skinny looking wave, and a larger unit would make our wave look fatter.

4. Find and label coordinates

   (a) Starting with the intercept we already found:

$$x = 0, y = 1 \Rightarrow (0, 1).$$

   (b) Then we increment x by 1 unit, and we choose each unit to be $\frac{\pi}{2}$ for easy graphing. Because this graph is wavy (we have a $\sin(x)$ term), we will want to increment from 0 in the positive as well as the negative like this:

$$x = \frac{\pi}{2}, y = 2\sin\left(\frac{\pi}{2}\right) + 1 = 2(1) + 1 = 3 \Rightarrow \left(\frac{\pi}{2}, 3\right).$$

$$x = -\frac{\pi}{2}, y = 2\sin\left(-\frac{\pi}{2}\right) = 2(-1) + 1 = -1 \Rightarrow \left(-\frac{\pi}{2}, -1\right).$$

   (c) And keep going:

$$x = \pi, y = 2\sin(\pi) + 1 = 2(0) + 1 = 1 \Rightarrow (\pi, 1).$$

$$x = -\pi, y = 2\sin(-\pi) + 1 = 2(0) + 1 = 1 \Rightarrow (-\pi, 1).$$

(d) And again:

$$x = \frac{3\pi}{2}, y = 2\sin\left(\frac{3\pi}{2}\right) + 1 = 2(-1) + 1 = -1 \Rightarrow \left(\frac{3\pi}{2}, -1\right).$$

$$x = -\frac{3\pi}{2}, y = 2\sin\left(-\frac{3\pi}{2}\right) + 1 = 2(1) + 1 = 3 \Rightarrow \left(-\frac{3\pi}{2}, 3\right).$$

Because when $x = -\frac{3\pi}{2}$, we have completed a full period, we will not worry about marking any further coordinates explicitly as the wave pattern will be cyclic throughout the period. I did add four extra coordinates on the Tactile Graph simply because I had the space and wanted to keep going as far as I could.

5. Now it is time to draw the wave. It is important to note that the function does not stop when we run out of x-axis holes, but repeats the pattern forever if we had an infinitely long x-axis.

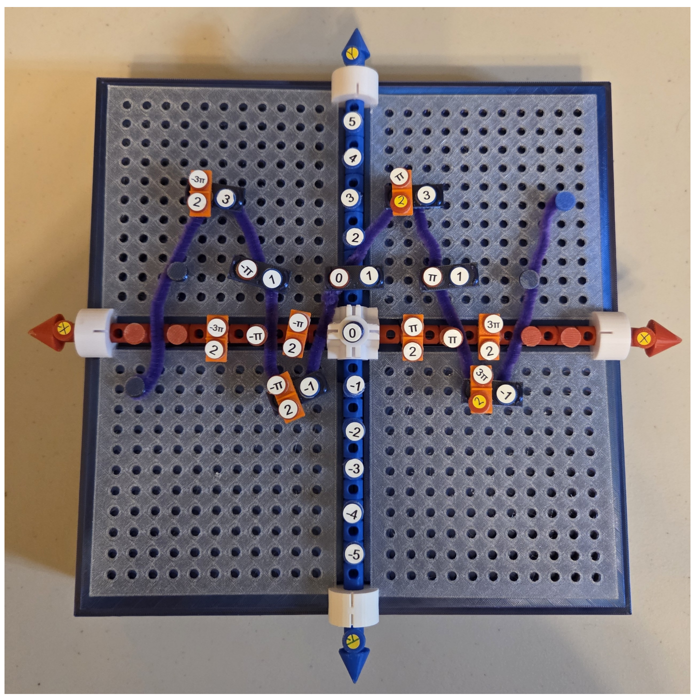

Figure 1: Final Graph Using the Tactile Graph