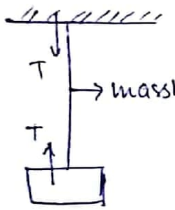
Engg. Mechanics

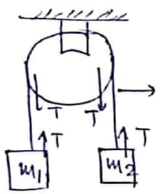
Study of motion of rigid bodies under the forces

Force: It is action of one body on another

To define a force
- Magnitude
- direction
- Point of application

Type of force
- Gravity [$wt = mg\downarrow$]
- Contact force (R) → Normal Reaction (N), Friction (f)
- Tension (T)
- spring force (F_s)





Newtons Ist law

For a particle

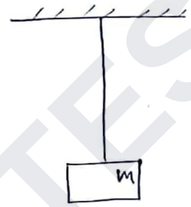
if $\Sigma \vec{F} = 0$ then $\vec{a} = 0$

rest / uniform linear velocity

For a rigid body

if $\underbrace{\Sigma \vec{F}_{ext} = 0}_{\text{on a rigid body}}$ then $\vec{a}_{CM} = 0$

Case 1



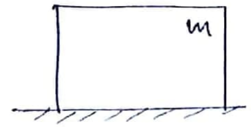
$\vec{a}_{CM} = 0$

$\Rightarrow \Sigma \vec{F}_{ext} = 0$ (newtons first law)

$T - mg = 0$

$T = mg$

Case 2



$\vec{a}_{CM} = 0$

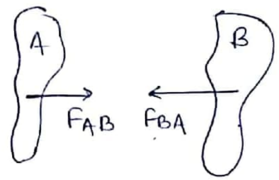
$\Rightarrow \Sigma \vec{F}_{ext} = 0$

$N - mg = 0$

$N = mg$

Above is not newtons 3rd law because for newton 3rd law 2 body is required.

Newtons 3rd law



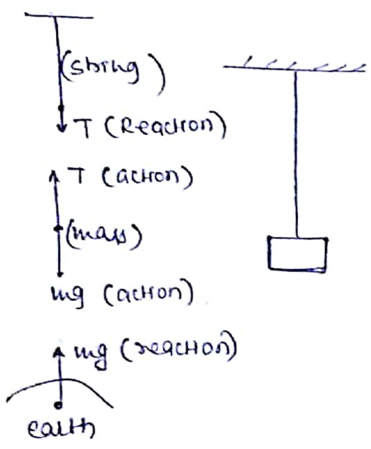
I) equal & opposite
II) Same nature (attractive or repulsive)
III) Co-linear

$F_{AB} = -F_{BA}$

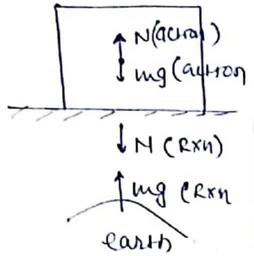
(action) (reaction)

(Reaction) (action)

Case 1



Case 2



Reading of weighing machine $= \frac{N}{g}$

FBD : It is representation of all the forces acting on the system by the surrounding.

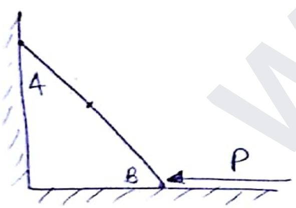
Equilibrium → Rest → uniform linear velocity

(i) $\Sigma \vec{F} = 0$

$\Sigma \vec{F_x} = \Sigma \vec{F_y} = \Sigma \vec{F_z} = 0$

(ii) $\Sigma T = 0$ ↳ about any point or line or space.

AS 2005



A uniform ledder of wt w and length l is held in equilibrium by a force P at B on ledder as shown find P in terms of wt.

(all the surfaces are smooth)

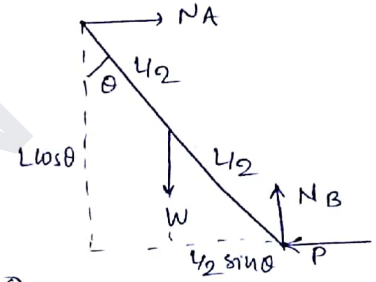
$\Sigma F_V = 0$

$N_B = W$

$\Sigma F_H = 0$

$N_A = P$

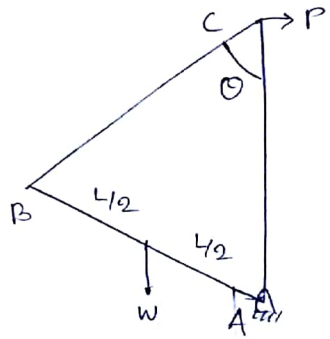
$\Sigma T_B = 0$

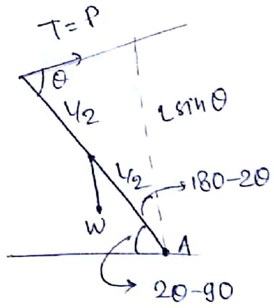


$N_A L\cos\theta - W \frac{L}{2} \sin\theta = 0$

$N_A = \boxed{\frac{W}{2} \tan\theta = P}$

A 2001 | A uniform rod AB of wt. W is movable in a vertical plane about a hinge at A and is sustained in equilibrium by a wt P attached to a string BC which is passing over a smooth surface C as shown in fig., AC being vertical if AC be equal to AB, then the wt P is





$\Sigma T_A = 0$

$W \cdot \frac{L}{2} \cos(2\theta - 90) = T L \sin\theta$

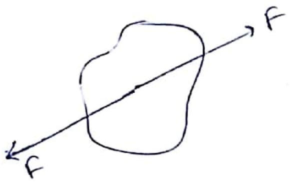
$\frac{W}{2}\, 2 \sin\theta \cos\theta = T \sin\theta$

$\boxed{W\cos\theta = T = P}$

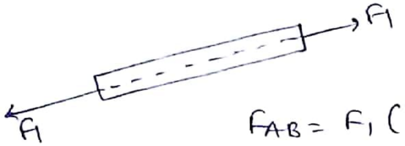
Two special case of equlibrium

Two-force system

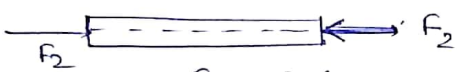
~~two~~ to keep two force in eqm they must be equal is magnitude, opposite in direction and co linear in action.



Application - Truss



$F_{AB} = F_1$ (tensile)



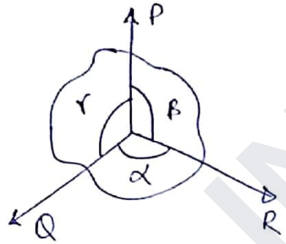
$F_{AB} = F_2$ (compressive)

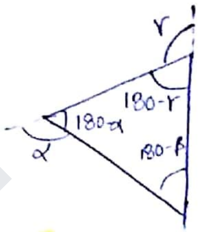
Three force system:

To keep 3 non-parallel forces in eqm they must be co-planer and con-current

(i) $\vec{P} + \vec{Q} + \vec{R} = 0 \longrightarrow$ Coplaner

(ii) $\Sigma T = 0 \longrightarrow$ concurrent



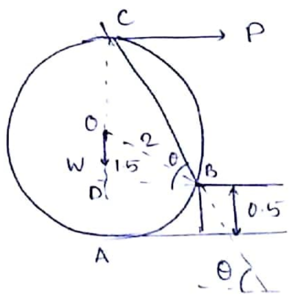


$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k}$$ Lami's theorem

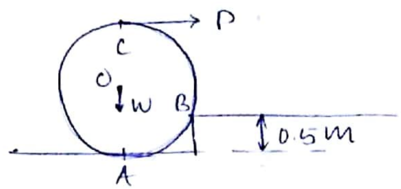
Limitation of lami's theorem

either converging or diverging only.

Que Find the horizontal force P required to move the cylinder out of the ditch (small pit/obstacle) as show in fig.



$W = 1000N$
$r = 2m$



Note (i) when the cylinder is about to move about the ditch it will louse it contact at point A the only contact ~~at~~ will be at B

(ii) To keep the cylinder in equilibrium under P, W and R_B they must be at same point i.e at C

In ΔODB

$$OD^2 + DB^2 = OB^2$$

$$DB = \sqrt{2^2 - 1.5^2}$$

$$= 1.3228\ m.$$

$$CD = 3.5$$

In ΔBCD.

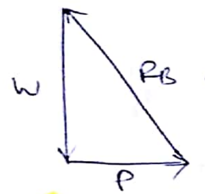
$$\tan\theta = \frac{CD}{DB} = \frac{3.5}{1.3228} \Rightarrow \theta = 69.29^\circ$$

$$\Sigma T_B = 0$$

$$P \times CD - W \times DB = 0$$

$$P_{min} = \frac{1000 \times 1.3228}{3.5} = 377.94N$$

$$\vec{W} + \vec{P} + \vec{R}_B = 0$$



(Force triangle)

$$\frac{W}{P} = \tan\theta = \frac{3.5}{1.3228}$$

$$P = \frac{1000 \times 1.3228}{3.5} = 377.94\ N$$

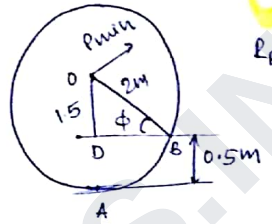
$$\frac{W}{R_B} = \sin 69.29^\circ$$

$$R_B = 1069.08N.$$

Que Find the min. force P required at center of cylinder so as to move it out of ditch as shown in fig.

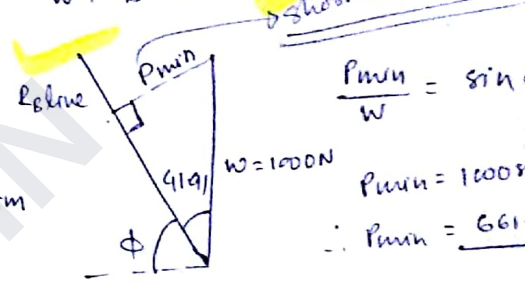
$$\vec{W} + \vec{R}_B + \vec{P}_{min} = 0$$

In ΔODB



$$\sin\phi = \frac{OD}{OB} = \frac{1.5}{2}$$

$$\phi = 48.59^\circ$$



$$\frac{P_{min}}{W} = \sin 41.41$$

$$P_{min} = 1000 \sin 41.41$$

$$\therefore P_{min} = 661.44\ N$$

Note: For P to be minimum its line of action should be $\perp^r$ to line OB i.e ~~RB~~ R_B

Plane Truss (2-D)

Truss: It is a rigid structure in which all the member are subjected to either axial tensile or axial compressive load only.

→ Bending moment is zero every where in the structure.

Condition of truss

1) Member should be pin jointed or hinged only
ii) loads should be applied on the joints only
iii) Only concentrated point load should be applied

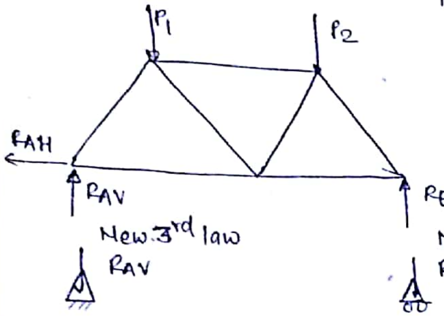
Note iv) A member should make joint at its end only.

Assumptions

1) wt of the members is neglected.

Truss system = member + Pins/joint

$= (m) \cdot \quad (j)$



$\Sigma \vec{F}_H = 0$

$R_{AH} = 0$

$\Sigma F_V = 0$

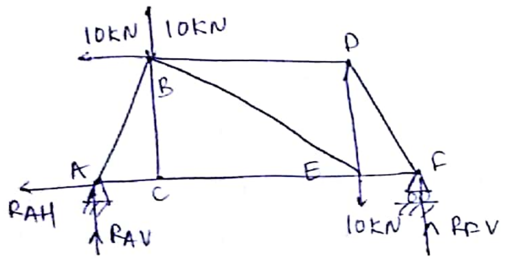
$R_{AV} + R_{EV} = P_1 + P_2$

$$\boxed{\text{for a stable/perfect/Determinate Truss} \Rightarrow m = 2j-3}$$

$$\underbrace{m+3}_{\text{no. of unknown}} = \underbrace{2j}_{\text{no. of equ}^m\ \text{eq}^{n}\text{s}}$$

Determination of Reaction

Reaction at supports are calculated by considering eq^m of the entire truss



$\Sigma \vec{F}_H = 0$

$\Rightarrow -R_{AH} - 10 = 0$

$R_{AH} = 10\,kN\,(\longrightarrow)$

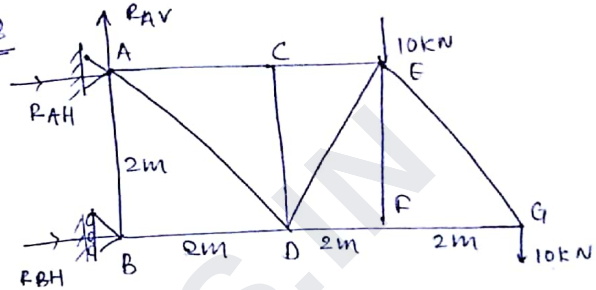
$\Sigma F_V = 0$

$R_{AV} + R_{FV} = 20\,kN \longrightarrow R_{AV} = 16.67\,kN\,(\uparrow)$

$\Sigma \vec{T}_A = 0 \qquad 10 \times 2 + 10 \times 4 - 10 \times 4 - R_{FV} \times 6 = 0$

$R_{FV} = 20/6 = 3.33\,kN\,(\uparrow)$

Que



$\Sigma \vec{F}_V = 0 \qquad R_{AV} - 10 - 10 = 0$

$R_{AV} = 20\,(\uparrow)$

$\Sigma F_H = 0 \qquad R_{AH} + R_{BH} = 0$

$R_{AH} = -R_{BH}$

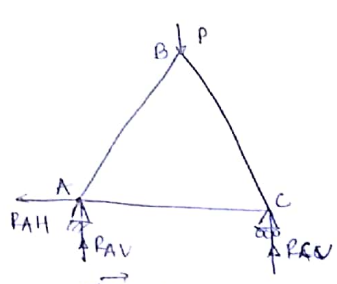
$\Sigma T_A = 0 \qquad 10 \times \cancel{4}^{2} + 10 \times \cancel{6}^{3} - R_{BH} \times \cancel{2} = 0$

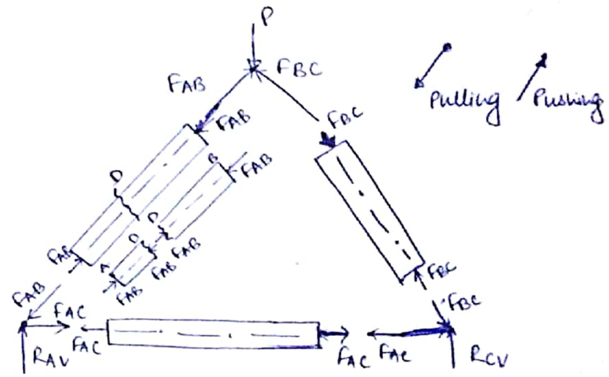
$20 + 30 - R_{BH} =$

$R_{BH} = 50\,kN\,(\longrightarrow)$

$R_{AH} = -50\,kN\,(\longleftarrow)$

Interaction of loads, Reactions & internal forces :





$\Sigma \vec{F}_H = 0$

$R_{AH} = 0$

$\Sigma F_V = 0$

$R_{AV} + R_{CV} = P$

Analysis of truses

→ Method of joint
→ Method of section

Method of joint:

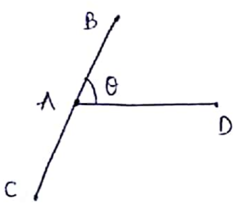
Eq^m of a joint is considered in method of joints

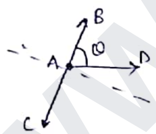
Procedure:

i) Find reaction at supports if required
ii) Consider eq^m of a joint where only 2 unknown member are meeting and used ~~sum~~ $\Sigma \vec{F_x} = \Sigma \vec{F_y} = 0$ to find the unknowns

Note:

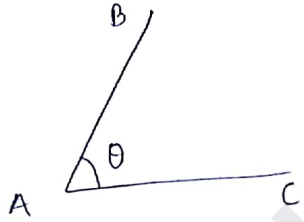
i) If a member pushes a joint then the member itself will be in compression with the same magnitude.
ii) If a member pulls a joint then the member itself will be in tension with the same magnitude.
iii) If at a joint three members are meeting and 2 are co-linear then force in third member will be zero (if there is no load & reaction at that joint)

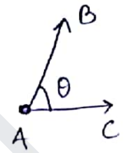




$F_{AD} \sin\theta = 0$
$\sin\theta \neq 0$
$F_{AD} = 0$

iv) If at a joint 2-members are meeting & they are non-colinear then force in both the members will be zero (if there is no load and Rxn at the joint)

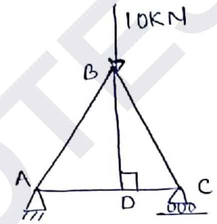




$\Sigma \vec{F_V} = 0$
$F_{AB} \sin\theta = 0$
$\sin\theta \neq 0$
$F_{AB} = 0$

$\Sigma \vec{F_H} = 0$
$F_{AC} + F_{AB}\cos\theta = 0$
$F_{AC} = 0$

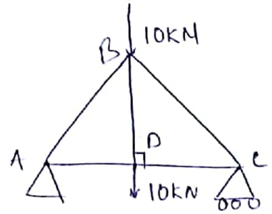
Que



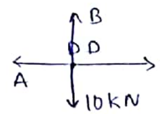
$F_{BD} = 0$?

$\Sigma \vec{F_V} = 0$
$F_{BD} = 0$
$F_{AD} = F_{DC}$

Que

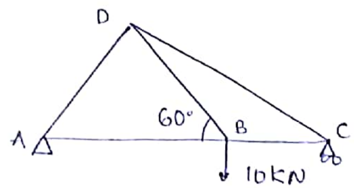


$F_{BD} = ?$

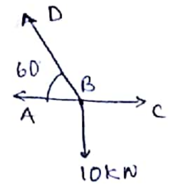


$F_{BD} = 10KN$ (tensile)
$F_{AD} = F_{DC}$

Que

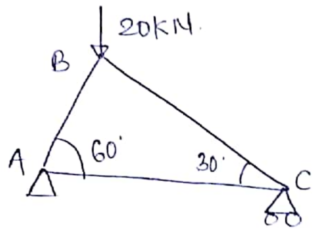


$F_{BD} = ?$

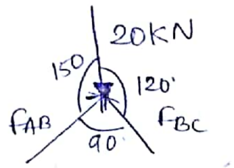


$10 = F_{BD} \sin 60°$
$10 = F_{BD} \cdot \frac{\sqrt{3}}{2}$
$\frac{20}{\sqrt{3}} = F_{BD}$
$F_{BD} = 11.54\ KN$ (tensile)

Que



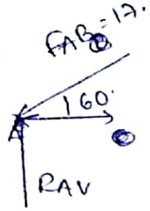
F_{AC}



$\frac{20}{\sin 90} = \frac{F_{BC}}{\sin 150} = \frac{F_{AB}}{\sin 120°}$

$F_{AB} = 17.32\ KN$ (comp.)
$F_{BC} = $ [illegible] KN (comp.

Joint A



$17.32\cos 60° = F_{AC}$

$8.66\,kN = F_{AC}$ (tensile)

$F_{AV} = F_{AB}\sin 60°$

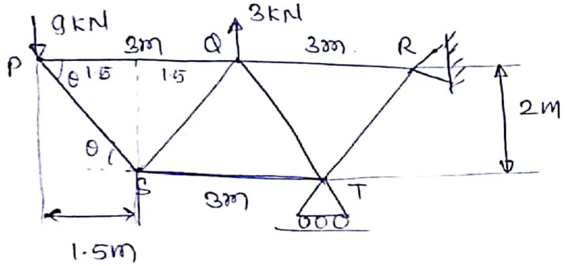
$F_{AV} = 17.32 \times \frac{\sqrt{3}}{2}$

$F_{AV} = 15\,kN\ (\uparrow)$

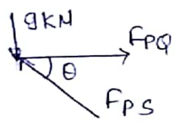
& $R_{CV} = 20 - 15 = 5\,kN (\uparrow)$

Que

$F_{SQ} = ?$



Joint P



$\tan\theta = \frac{2}{1.5} \Rightarrow \theta = 53.13°$

$F_{PS}\sin\theta = 9\,kN$

$F_{PS} = \frac{9}{\sin 53.13} = 11.25\,kN$ (comp)

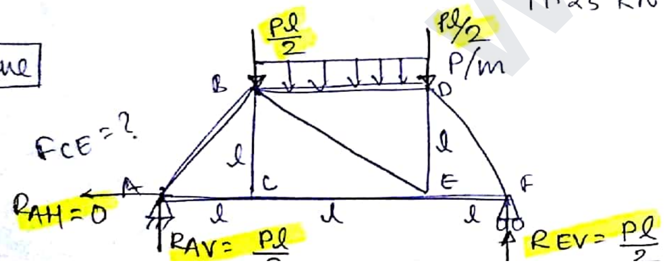
Joint S

$F_{PS}\cancel{\sin\theta} = F_{SQ}\cancel{\sin\theta}$

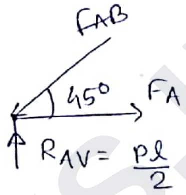
$11.25\,kN = F_{SQ}$ (tensile)

Que

$F_{CE} = ?$



Joint A



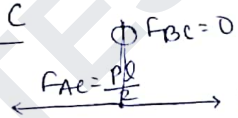
$F_{AB}\sin 45° = \frac{Pl}{2}$

$F_{AB} = \frac{Pl}{\sqrt{2}}$ (compressive)

$F_{AB}\cos\theta = F_{AB}\cos 45° = F_{AC}$

$\Rightarrow \frac{Pl}{2} = F_{AC}$ (tensile)

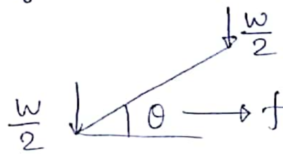
Joint C



$F_{CE} = \frac{Pl}{2}$ (tensile)

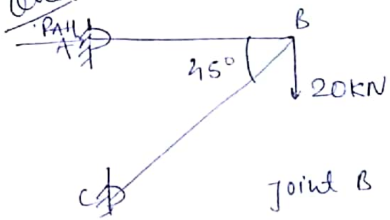
Note

If w = wt of member is given



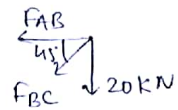
→ for any θ

Que



Find R_{AH} & R_{CH}

Joint B



$\Sigma V = 0$

$F_{BC}\sin 45 = -20$

$F_{BC} = -20\sqrt{2}$

$\Sigma H = 0$

$F_{AB} + F_{BC}\sin 45 = 0$

$F_{AB} = -F_{BC}\sin 45$

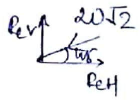
$= +20\sqrt{2} \times \frac{1}{\sqrt{2}} = 20$

~~R_{AH} +~~

$R_{AH} + 20 = 0$

$R_{AH} = -20$ &

$= 20$ (~~compressive~~ ←)



$\Sigma H = 0$

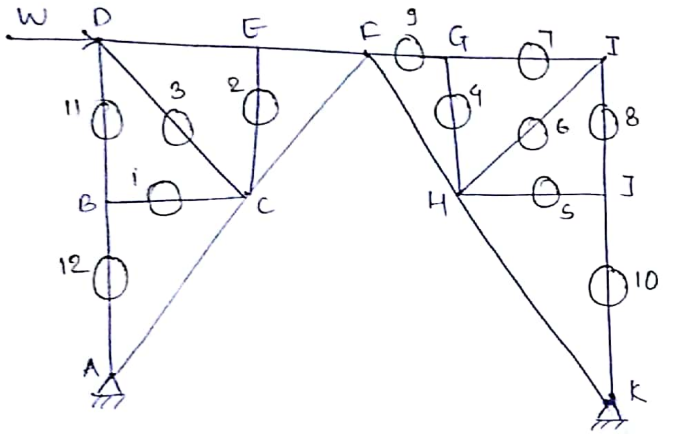
$20\sqrt{2}\cos 45 = R_{CH}$

$20 = R_{CH}$ (→)

$R_{CV} = 20\sqrt{2} \times \frac{1}{\sqrt{2}}$

$R_{CV} = 20\,kN (\uparrow)$

Que Find the no. of member having zero force in them



12

Method of section

Concept:

Eqm of a section of truss is considered in method of section

Procedure:

i) Find rxn at support if required

ii) Cut the member under consideration by a section ①-① and consider eqm of either L.H.S or RHS of section ①-① and use

$$\Sigma \vec{F_x} = \Sigma \vec{F_y} = 0$$

& $\Sigma \vec{M} = 0$ to find the unknowns

Note:

Advantage of method of section is that force in any intermediate can be found directly without finding force in any other member.

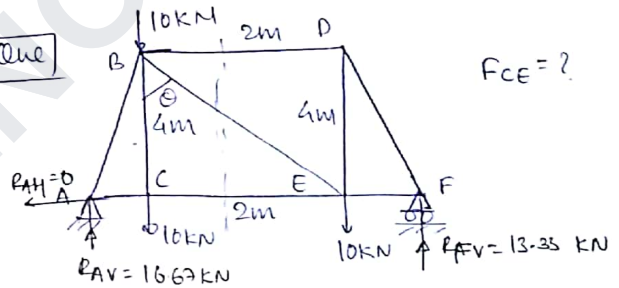
Points to remember

① Cut the member such that entire truss is devided into 2-separate part

(ii) Preferably donot cut more than 3-member as in method of section we have only 3-eqn of equilibrium.

(iii) Cut the member such that all the cut members donot meet at same joint (if they meet at same joint it becomes method of joints only)

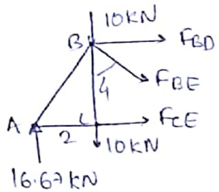
Que



$F_{CE} = ?$

$\tan\theta = \frac{2}{4}$

$\theta = 26.56$



$\Sigma M_B = 0$

$F_{CE} \times 4 = 16.67 \times 2$

$F_{CE} = 8.33$ kN (tensile)

$\boxed{F_{BD} = ?}$

$\Sigma M_E = 0$

$F_{BD} \times 4 - 10 \times 2 - 10 \times 2 + 16.67 \times 2 = 0$

$F_{BD} = -6.67$ KN

$= 6.67$ kN (compressive)

$\boxed{F_{BE} = ?}$

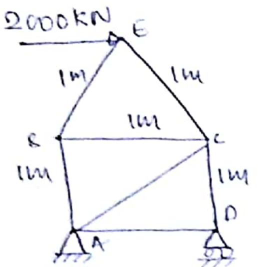
$\Sigma \vec{F_y} = 0$

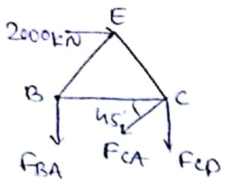
$16.67 - 10 - 10 - F_{BE} \cos 26.56 = 0$

$F_{BE} = -3.72$ kN (compres

Que | ISRO-15

$F_{CA} = ?$



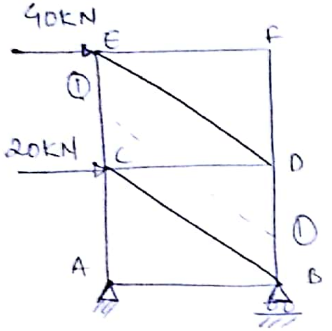


$\Sigma F_H = 0$

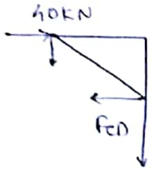
$F_{CA}\cos 45° - 2000 = 0$

$F_{CA} = 2000\sqrt{2}$ (tensile)

IAS



$F_{CD} = ?$

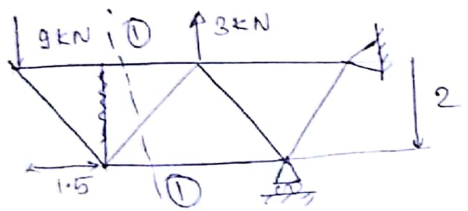


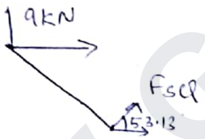
$\Sigma F_H = 0$

$F_{CD} - 40 = 0$

$F_{CD} = 40$ kN (tensile)

Que



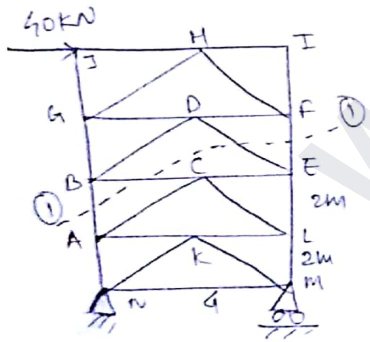


$F_{SQ}\sin\theta = 9$

$F_S = \frac{9}{\sin 53.13} =$

$= 11.25$ (tensile)

Que



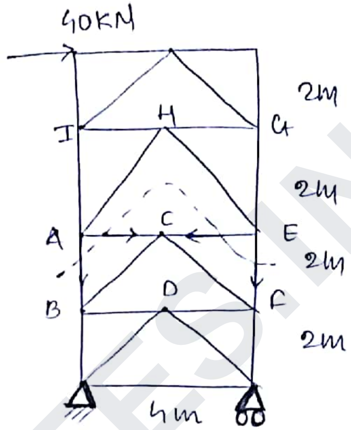
$F_{AB} = ?$

$\Sigma \vec{M_E} = 0$

$40\times4 - F_{AB}\times4 = 0$

$F_{AB} = 40$ kN (tensile)

Que



$\Sigma M_A = 0$

$40\times4 + F_{EF}\times4 = 0$

$F_{EF} = -40$ kN

$= 40$ kN (compressive)

Workdone

Workdone

$W_F = \vec{F}\cdot\vec{ds} = F\,ds\cos\theta$

→ displacement of point where force (F) is acting

Principle of virtual work

It state "If a system is in eq^m and virtually displaced then sum of virtual work by all the forces will be zero"

Virtual work (Imaginary) $= F(\partial s)$ → virtual displacement

Note:

i) Using POVW we can find forces necessary to keep a system in eq^m

ii) If using eq^m of equilibrium is difficult and time taking then use POVW to find unknown easily

iii) POVW is used in the system where no of inter-connected rigid body are capable of motion but are equlibrium.

iv) In POVW we don't need to consider Rxn at supports because their workdone is always zero.

Procedure:

i) Take any fix point in problem as origin, fixed co-ordinate axis, and find the co-ordinates of all the point where forces are acting.

ii) Find virtual displacements

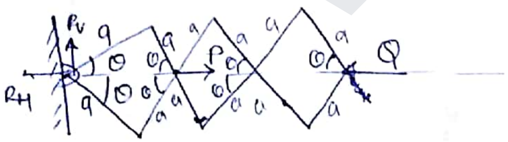
iii) Use POVW to find unknowns easily.

sign convention

<i> sign convention for co-ordinates is choosen based on the co-ordinate in which they are lying

<ii> If any force is acting along horizontal forward or vertical upward then take that force ⊕ and vice versa.

Que For the lazy tong mechanism shown in fig the relationship between P & Q to keep the system in eqm is



1. $x_A = +2a\cos\theta$

$x_B = +6a\cos\theta$

2. $\partial x_A = -2a\sin\theta\, d\theta$

$\partial x_B = -6a\sin\theta\, d\theta$

3. $(V.W)_P + (VW)_Q = 0$

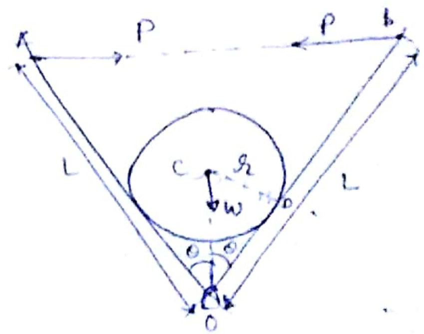
$(+P)(\partial x_A) + (-Q)(\partial x_B) = 0$

$P(-2a\sin\theta\, d\theta) - Q(-6a\sin\theta\, d\theta) = 0$

$-2P + 6Q = 0$

$\boxed{P = 3Q}$

Que A sphere of wt W and radius R is supported by two rods of length L as shown in fig to be find the value of P to keep the system in eqm



1. $x_A = -L\sin\theta$

$x_B = +L\sin\theta$

$y_c = +r\,\mathrm{cosec}\,\theta$

In ΔODC

$\sin\theta = \frac{CD}{OC} = \frac{r}{y_c}$

2. $dx_A = -L\cos\theta\, d\theta$

$dx_B = +L\cos\theta\, d\theta$

$dy_c = -r\cos\theta\cot\theta\, d\theta$

3. $(+P)(-L\cos\theta\, d\theta)+(-P)(L\cos\theta\, d\theta)+(-W)(-r\,\mathrm{cosec}\theta\cot\theta)d\theta=0$

$$-2PL\cos\theta = -Wr\frac{1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$\boxed{P=\frac{Wr}{2L\sin^2\theta}}$$

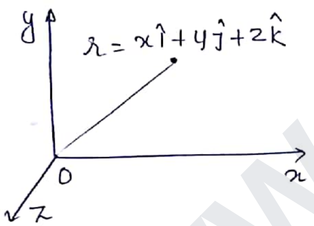
Dynamics

Kinematics $(\vec{r}, \vec{v}, \vec{a})$

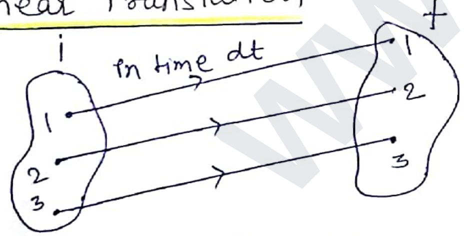
Kinetics $(\vec{r}, \vec{v}, \vec{a}, \vec{F})$

$$v=\frac{d\vec{r}}{dt}=\frac{d\vec{s}}{dt}$$

$$a=\frac{d\vec{v}}{dt}$$



Rectilinear Translation



$(d\vec{s})_1=(d\vec{s})_2=(d\vec{s})_3=(ds)_{cm}\rightarrow$ in time dt

$\left.\begin{array}{l}\vec{V}_1=\vec{V}_2=\vec{V}_3=\vec{V}_{cm}=\vec{V}\\ \vec{a}_1=\vec{a}_2=\vec{a}_3=\vec{a}_{cm}=\vec{a}\end{array}\right\}$ at any instant

for $\boxed{\text{uniform accelaration}}$

$\left.\begin{array}{ll}(i) & v=u+at\\ (ii) & s=ut+\frac{1}{2}at^2\\ (iii) & v^2=u^2+2as\end{array}\right\}$ $\boxed{\text{vector eq}^n}$

eg. bodies falling under gravity only

$$\vec{a}=g(-\hat{j})=-9.81(\hat{j})$$

G-15 Que) Initial velocity of an object is 40 m/s and acc^n, $\vec{a}=-0.1V$ where v is instentaneous velocity the velocity of the object after 3 sec is.

Sol Given => $u=40$ m/s at $t=0$

~~$V = 40 \quad 0.1\times3$~~

~~$= 40 =$~~

$a=-0.1V$

$V_3=?$ at $t=3$ sec

$$a=\frac{dv}{dt}=-0.1v$$

$$\int_{40}^{V_3}\frac{dv}{v}=\int_0^3-0.1\,dt$$

$$\ln v\Big|_{40}^{V_3}=-0.1(3)$$

$$\ln V_3-\ln 40=-0.3$$

$$V_3=29.63\ m/s$$

Que A stone is thrown vertically upward it reaches back to ground in 4 sec. what will be max hight it will go

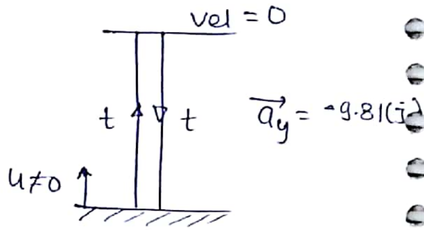
Sol

$$S = ut + \frac{1}{2}gt^2$$

$$S = \frac{1}{2} \times 9.81 \times 4 \times 4$$

$$= 9.81 \times 8$$

$$S = 78.48 \text{ m}$$



in y-direction

from max^n height to ground.

$2t = 4$ sec

$t = 2$ sec.

$$S_y = u_y t + \frac{1}{2} a_y t^2$$ → the above velocity

$$-h_{max} = 0 + \frac{1}{2}(-g) \times 2^2$$

$$h_{max} = 9.81 \times 2$$

$$h_{max} = 19.62 \text{ m}$$

If $a = F(s)$

$$v = \frac{ds}{dt} \quad ; \quad a = \frac{dv}{dt}$$

$$dt = \frac{ds}{v} \quad ; \quad dt = \frac{dv}{a}$$

equating $dt = dt$

$$\frac{ds}{v} = \frac{dv}{a}$$

$$\boxed{a\,ds = v\,dv}$$

$$\boxed{F(s)\,ds = v\,dv}$$

Que If acc^n $a = -8s^{-2}$ and motion starts with infinite displacement then the velocity of the object at $s = 16$ m is

Sol

$$F(s)\,ds = v\,dv$$

$$\int_{\infty}^{16} -8s^{-2}\,ds = \int_0^v v\,dv$$

$$\frac{-8s^{-1}}{-1}\Big|_{\infty}^{16} = \frac{v^2}{2}\Big|_0^v$$

$$8\left[\frac{1}{s}\right]_{\infty}^{16} = \frac{v^2}{2}$$

$$8\left[\frac{1}{16} - 0\right] = \frac{v^2}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{v^2}{2} \Rightarrow v = 1 \text{ m/s}$$

if a →
- constant → 3 eq^n
- fu^n (t, v) → $a = \frac{dv}{dt}$
- fu^n (s) → $a = v\frac{dv}{ds}$

} (imp)

Newton's 2nd law (NSL):

for a particle

If $\sum \vec{F} \neq 0$ then $\vec{a} \neq 0$

$$\boxed{\vec{a} = \frac{\Sigma \vec{F}}{m}} \longrightarrow \text{NSL}$$

$$\boxed{\Sigma \vec{F} = m\vec{a}} \longrightarrow \text{NSL}$$

result of actual force on particle

effect/response

For a rigid body

if $\Sigma F_{ext} \neq 0$ then $\vec{a}_{cm} \neq 0$

$$\vec{a}_{cm} = \frac{\Sigma \vec{F}_{ext}}{m}$$

$$\boxed{\Sigma F_{ext} = m\vec{a}_{cm}} \longrightarrow \text{NSL}$$

result of actual ext. force on rigid body

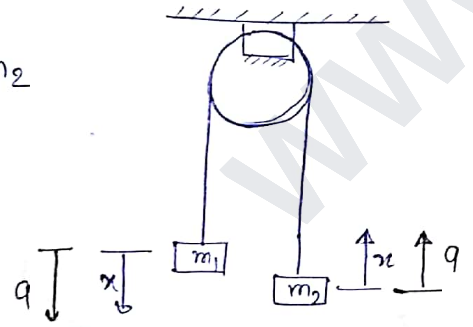
effect/response

Rect. Translation

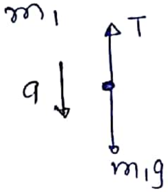
kinetics

Case 1

$m_1 \neq m_2$

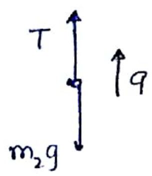


m_1



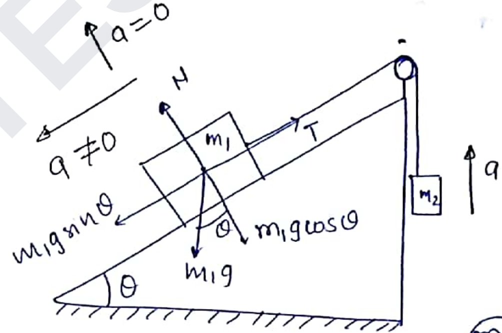
$m_1 g - T = m_1 a \rightarrow \text{NSL}$

m_2



$T - m_2 g = m_2 a$
$\rightarrow \text{NSL}$

Case-II

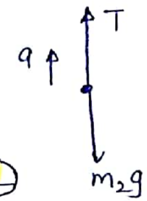


m_1

$N - m_1 g\cos\theta = 0 \rightarrow \text{NFL}$

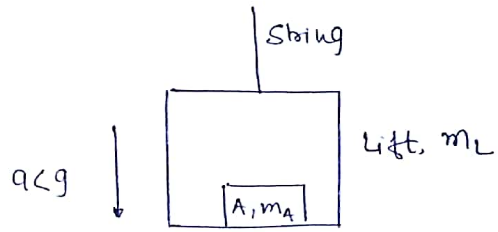
$m_1 g \sin\theta - T = m_1 a \rightarrow \text{NSL}$

m_2



$T - m_2 g = m_2 a$
$\rightarrow \text{NSL}$

Case III



A

$m_A g - N_{AL} = m_A a$
$\rightarrow \text{NSL}$
———— (1)

Lift

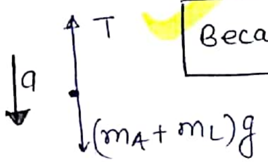
$m_L g + N_{LA} - T = m_L a$
$\rightarrow \text{NSL}$
———— (2)

① + ②

$$m_A g - \cancel{N_{AL}} + m_L g + \cancel{N_{LA}} - T = (m_A + m_L)a$$

$$(m_A + m_L)g - T = (m_A + m_L)a$$

A + lift



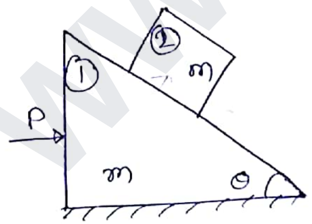
$\boxed{\text{Because } \vec{a}_A = \vec{a}_{lift} = \vec{a}}$

$$(m_A + m_L)g - T = (m_A + m_L)a \longrightarrow \text{NSL}$$

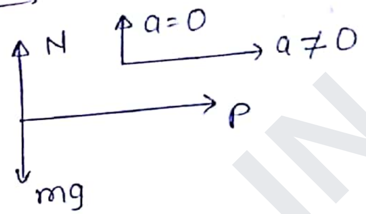
Note: We can consider more than one rigid body in a single system to apply NSL provided their acc^n must be same in magnitude as well as direction.

IAS Que Find the horizontal force P on the block 1 so as to prevent slipping of block 2 over the block 1 both the block have same mass 'm'. assume all the surface is to be smooth.

Sol



① + ②

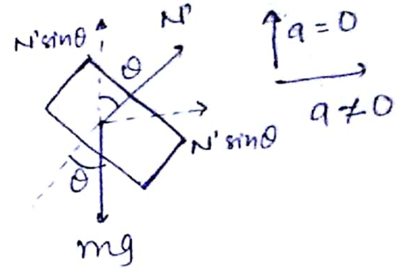


$N - mg = 0$ NFL

$P = 2ma \longrightarrow$ NSL

$P = 2ma$

$P = 2mg\tan\theta$



~~$\tan\theta = 0$~~

$N'\sin\theta = ma \longrightarrow$ NSL

$N'\cos\theta = mg \longrightarrow$ NFL

$\tan\theta = \dfrac{a}{g}$

$a = g\tan\theta$

—— x ——

Friction (f)

Static (f_s)
(rest + tendency of motion)

$0 < f_s \le (f_s)_{max} = \mu_s N$

↑ Normal Rxn

$(f_s)_{max} \propto N$

$(f_s)_{max} = \cancel{f}\cdot \mu_s N.$

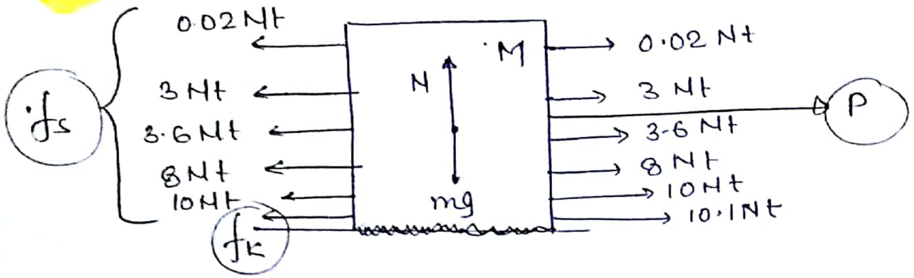
↑ coefficient of static friction

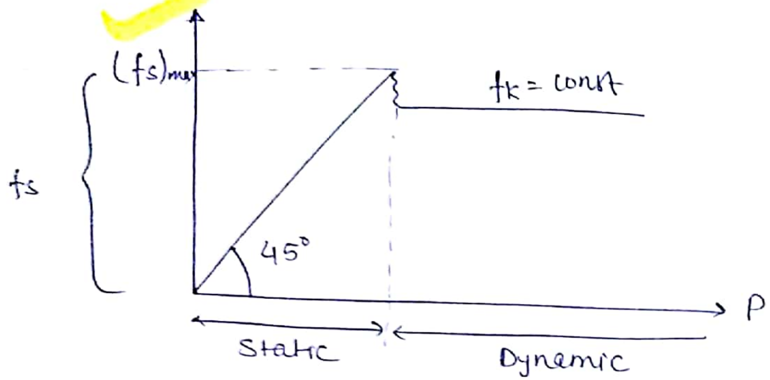
Kinematic (f_k)
(motion)

$f_k = \text{const} = \mu_k N$

↑ coefficient of kinetic friction

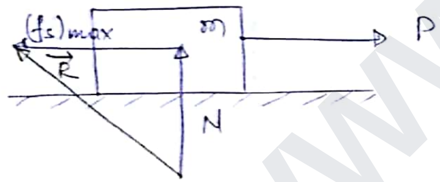
$$\boxed{\begin{array}{c} (f_s)_{max} > f_k \\ \hline \mu_s > \mu_k \end{array}}$$





Angle of static friction (ϕ_s) ⇒

It is the angle made by contact force with normal rxn, when the body is at verge of motion



$$\tan\phi_s = \frac{(f_s)_{max}}{N}$$

$$= \mu_s N / N$$

$$\tan\phi_s = \mu_s$$

Angle of kinetic friction, ϕ_k

$$\boxed{\tan\phi_k = \mu_k}$$

→ It is the angle made by contact force with normal reaction when the body is in motion

if μ_s and μ_k is not given separately

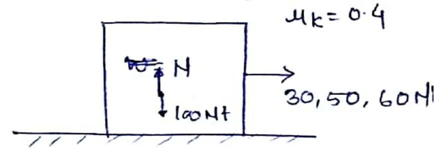
i) $\mu_s = \mu = \mu_k$

ii) $0 \le f_s \le (f_s)_{max} = \mu N = f_k$

iii) $\tan\phi = \mu$

Que Find the frictional force developed in the system shown in fig. for P = 30, 50, 60 Nt

$W = 100$ Nt
$\mu_s = 0.5$
$\mu_k = 0.4$



Sol $N = 100$ Nt ⟶ NFL

$(f_s)_{max} = \mu_s N = 0.5 \times 100 = 50$ Nt

$f_k = \mu_k N = 0.4 \times 100 = 40$ Nt

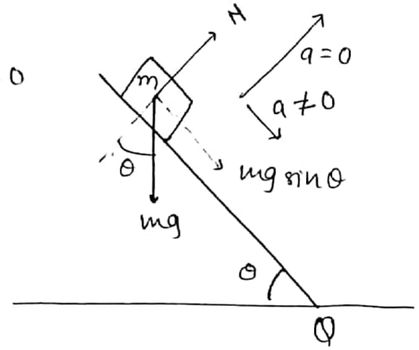
(i) P = 30 Nt	(ii) P = 50 Nt	(iii) P = 60 Nt → motion
$P < (f_s)_{max}$	$P = (f_s)_{max}$	$P > (f_s)_{max}$
$f = f_s = P = 30$ Nt (rest)	$f = (f_s)_{max} = 50$ Nt (verge of motion)	$f = f_s = 40$ Nt

Scanned by CamScanner

Que A block of mass M is released from point P on a rough inclined plane with inclination angle θ as shown in figure the coeff of friction is μ if $\mu < \tan\theta$, then the time taken by the block to reach another point Q on the inclined plane, where PQ = s is

Sol

$u = 0$



$N = mg\cos\theta$ NFL

for motion: $mg\sin\theta > (f_s)_{max} = \mu N$

$\cancel{m}g\sin\theta > \mu \cancel{m}g\cos\theta$

$\tan\theta > \mu \xrightarrow{\text{for motion}}$

$\tan\theta > \tan\phi$

$\boxed{\theta > \phi} \longrightarrow$ For motion

$mg\sin\theta = (f_s)_{max} \longrightarrow$ <u>verge of motion</u>

$\tan\theta = \mu = \tan\phi$

$\boxed{\theta = \phi} \longrightarrow$ verge of motion

↑ Angle of repose

$mg\sin\theta < (f_s)_{max} \longrightarrow$ <u>no motion</u>

$\boxed{\theta < \phi} \longrightarrow$ no motion

$mg\sin\theta - f_k = ma \longrightarrow$ NSL

$mg\sin\theta - \mu mg\cos\theta = ma$

$a = g\cos\theta(\tan\theta - \mu) = \underline{\text{const}}$

$s = ut + \frac{1}{2}at^2$, $u = 0$

$s = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}$

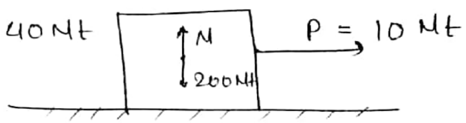
Que A block weighing 200 N. is in contact with level plane whose coeff of static & μ_k are 0.4 & 0.2 respectively the block B acted upon by a horizontal force (in Nt) P = 10 t, where t denotes the time in seconds, the velocity in m/s of the block attained after 10 sec is

Sol

$W = 200$ Nt
$\mu_s = 0.4$
$\mu_k = 0.2$

$(f_s)_{max} = \mu_s N = 0.4 \times 200 = 80$ Nt

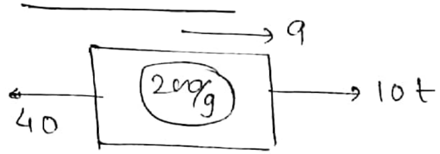
$f_k = \mu_k N = 0.2 \times 200 = 40$ Nt



At t = 8 s block will be at verge of motion

$V_{t=8s} = 0$

from <u>t = 8 at 10 s</u>



$\frac{3.14}{3.7}$

$$a = \frac{10t-40}{100/g} = \frac{dv}{dt}$$

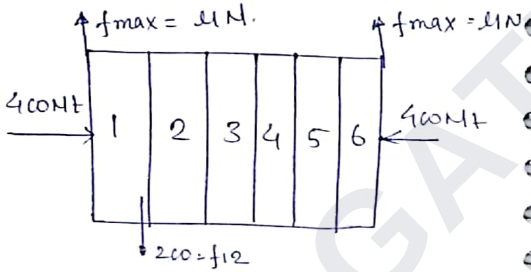
$$\int_8^{10} (10t-40)\,dt = \frac{200}{g}\int_0^{v_{10}} dv$$

$$v_{10} = 4.905\ m/s$$

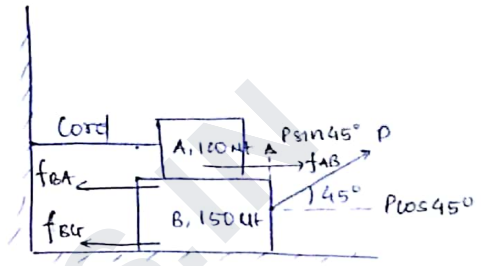
Que 6 identical books each weighing 100N are supported by applying a compressive force of 400N by hands as shown in figure if the books are on the verge of motion from the hand then the coeff. of friction between books and hand is

Sol

$$2f_{max} = 600$$
$$2\times 400 \times \mu = 600$$
$$800 \times \mu = 100 \times 6$$
$$\mu = \frac{6}{8} = \frac{3}{4}$$
$$\mu = 0.75$$

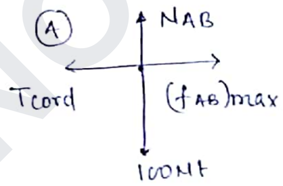


Que A 100 N block A rest on 150N block B, which rest rough horizontal plane the block A is tight with weightless horizontal chord to a wall. A force P is applied to the block B at 45° to the horizontal as shown in fig. if $\mu = 0.25$ b/w the blocks & 0.3 between block B and the floor determine the tension T in the chord and value of force P show that ~~point~~ block Bat the point of sliding.



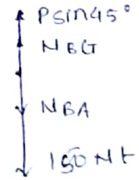
$$P\cos 45° = [(f_s)_{max}]_{BG} + [(f_s)_{max}]_{BA}$$
$$= \mu_{BA} N_{BA} + \mu_{BG} N_{BG} \quad \text{--- (1)}$$

(A)



$N_{AB} = 100\ N \longrightarrow$ NFL

(B)



$$P\sin 45° + N_{BG} = N_{BA} + 150$$
$$N_{BG} = 250 - P\sin 45°$$

from (1)

$$P\cos 45° = 0.25(100) + 0.3[250 - P\sin 45°]$$

$$P = 108.78\ N$$

Tension in the cord at the point of sliding.

$$T_{cord} = (f_{AB})_{max} = \mu_{AB} N_{AB}$$
$$= 0.25 \times 100$$
$$= 25\ N$$

Curvilinear Motion

→ It is summation two or three rectilinear motion

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$a = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

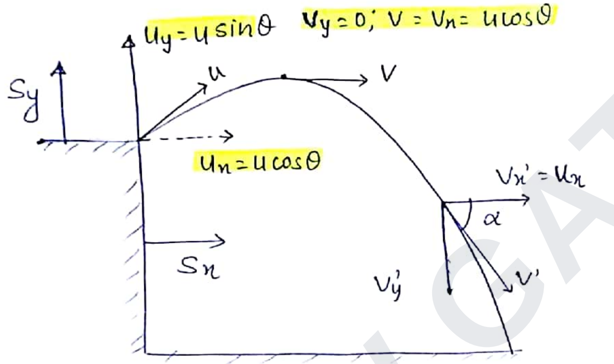
Projectile motion

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$a = a_x\hat{i} + a_y\hat{j}$$

$$\vec{a_x} = 0$$

$$a_y = g(-\hat{j})$$



$$\boxed{V_x' = v\cos\alpha = u\cos\theta}$$

in y- direction

1. $V_y = u_y + a_y t$
2. $s_y = u_y t + \frac{1}{2} a_y t^2$
3. $v_y^2 = u_y^2 + 2a_y s_y$

in x- direction

$$\boxed{a_x = 0 \Rightarrow V_x = \text{const} = U_x = u\cos\theta}$$

velocity in x-direc^n throughout ~~Ux~~ remains const

$$\boxed{S_x = U_x t}$$

$\cancel{V_x = 0}$

Que A projectile is fired horizontaly with a velocity of 6m/s from a point ~~foo~~ of height 'h' meter above and 12 meter away from an object what is the value of h required so that projectile hits the object

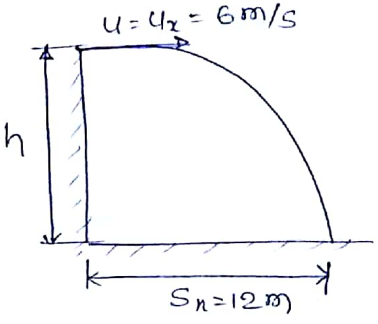
$$12 = 6 \times t$$

$$t = 2 \text{ sec}$$

$$h = \frac{1}{2}(9.81) \times 2^2$$

$$= 9.81 \times 2$$

$$= 19.62 \text{ m}$$



Que A stone is thrown with the velocity of 10m/s by making an angle of 60° with the horizontal. the to max height it has gone up is.

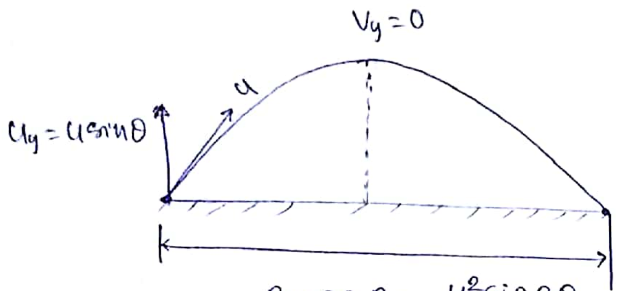
~~$S_x = 10 \times \frac{1}{2} \times$~~

$$\frac{(10\sin 60^\circ)^2}{2 \times 9.81} = S_y = h_{max}$$

$$\frac{100 \times 3}{4 \times 2 \times 9.81} = h_{max}$$

$$\frac{15}{4} = h_{max}$$

$$3.75 \text{ m} = h_{max}$$



Range $R = \frac{u^2 \sin 2\theta}{g}$

$(\text{Range})_{max} = \frac{u^2}{g}$ at $\theta = 45°$

$h_{max} = \frac{u^2 \sin^2\theta}{2g}$

T = Time of flight = $\frac{2u\sin\theta}{2g}$.

———x———

Circular motion

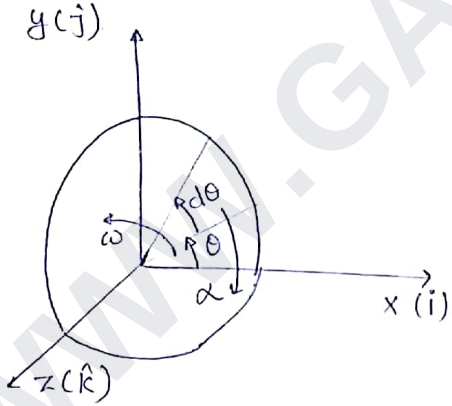
Kinematics:

angular velocity:

$\omega = \frac{d\theta}{dt}$ (rad/s)

ang. acc^n

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (rad/s^2)



$\vec{\omega}$ & $\vec{\alpha}$

direction: will be $\perp^r$ to plane of circle passes through centre & decided by right hand thumb rule

$\vec{\omega} = \frac{d\theta}{dt}(\hat{k})$

$\vec{\alpha} = \frac{d\omega}{dt}(-\hat{k})$

If $\vec{\alpha}$ is uniform

(i) $\omega = \omega_0 + \alpha t$

(ii) $d\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\, d\theta$

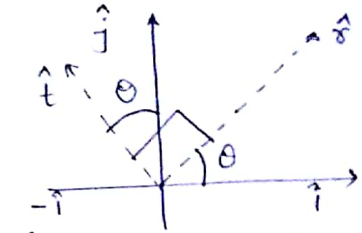
In 1 rev.

$d\theta = 2\pi$

$\omega = \frac{2\pi N}{60}$ (rad/s)

$\vec{\omega} = \cancel{\frac{d\theta}{dt}}(\hat{k})$

$\vec{\alpha} = \cancel{\frac{d\omega}{dt}}(-\hat{k})$



$\hat{r} = \cos\theta\, \hat{i} + \sin\theta\, \hat{j}$

$\hat{t} = \sin\theta(-\hat{i}) + \cos\theta\, \hat{j}$

v (linear speed) $= \frac{\text{distance}}{\text{time}}$

$= \frac{r\,d\theta}{dt}$

$\boxed{V = r\omega \text{ (m/s)}}$

$\vec{r}, \vec{v}, \vec{a}$

$\vec{r} = |\vec{r}|\,\hat{r}$

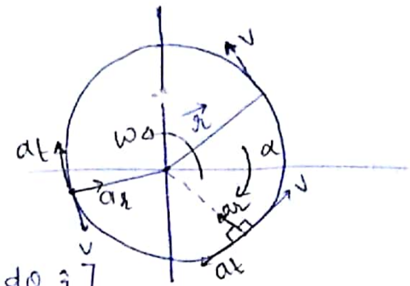
$\vec{r} = r[\cos\theta\, \hat{i} + \sin\theta\, \hat{j}]$

$\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{v} = r\left[-\sin\theta \frac{d\theta}{dt}\hat{i} + \cos\theta \frac{d\theta}{dt}\hat{j}\right]$

$\vec{v} = r\omega(-\sin\theta\, \hat{i} + \cos\theta\, \hat{j})$

$\vec{v} = r\omega(\hat{t})$



$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = r\left[\omega\left(-\cos\theta \frac{d\theta}{dt}\hat{i} - \sin\theta\frac{d\theta}{dt}\hat{j}\right) + (-\sin\theta\hat{i} + \cos\theta\hat{j})\frac{d\omega}{dt}\right]$$

$$\vec{a} = r\omega^2(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + r\alpha(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$= r\omega^2(-\hat{r}) + r\alpha(\hat{t})$$

$$\boxed{\vec{a} = \vec{a}_r + \vec{a}_t}$$

$\vec{a}_t \longrightarrow$ tangential accⁿ

$\vec{a}_r \longrightarrow$ radial / normal / centrifugal

$$\vec{a} = \vec{a}_n + \vec{a}_t$$

$\vec{a}_n$: $\perp^r$ to $\vec{v}$ — Cause change in dirⁿ of $\vec{v}$

$\vec{a}_t$: $\parallel$ to $\vec{v}$ — Cause change in mag. of $\vec{v}$

$a_n = \dfrac{v^2}{r} \longleftarrow \boxed{\begin{array}{l} a_n = r\omega^2 \\ a_t = r\alpha \end{array}}$ $\quad v = r\omega \Rightarrow \omega = v/r$

Uniform circular motion:

ω = const $\Rightarrow \alpha = 0 \Rightarrow \vec{a}_t = 0$

$\boxed{a = \vec{a}_r \neq 0 \Rightarrow \Sigma\vec{F} \neq 0} \longrightarrow$ NSL

Kinetics

$\vec{a}_r \neq 0 \Rightarrow \Sigma\vec{F}_r \neq 0 \rightarrow$ NSL

$\Sigma F_r = m\vec{a}_r \longrightarrow$ NSL

$\underbrace{\Sigma\vec{F}_r}_{\text{resultant of actual force in centripetal dir}^n} = \underbrace{mr\omega^2(-\hat{r})}_{\text{effect}} \rightarrow$ NSL

~~resultant radius~~ of actual force in centripetal dirⁿ (centripetal force) (~~centrifugal force~~)

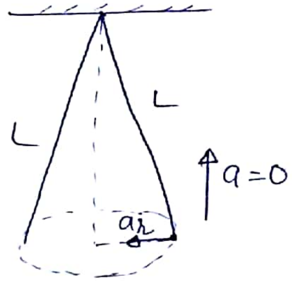
tangential

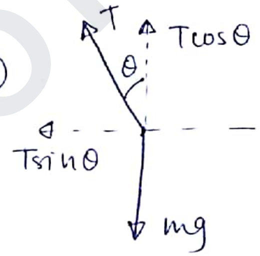
$$\Sigma\vec{F}_t = m\vec{a}_t \longrightarrow \text{NSL}$$

Que A particle of mass m is suspended from a ceiling through string of 'L' moves in horizontal circle of radius 'r' find

(1) Tension in string

(2) speed of particle.





$a_r \neq 0$

$T\cos\theta = mg.$

$T\sin\theta = ma_r \longrightarrow$ NSL

$$\underbrace{T\sin\theta}_{\text{centripetal force.}} = \frac{mv^2}{r}$$

(i) $T = \dfrac{mg}{\cos\theta}$

(ii) $\tan\theta = \dfrac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$

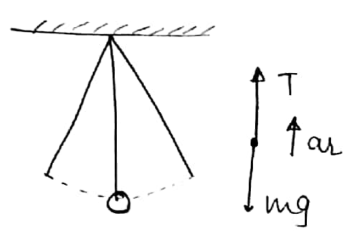
Que a simple pendulum of length 5 m with a bob of of mass one kg is in simple harmonic motion when it passes through its mean posⁿ the bob has a speed of 5 m/s the net force on the bob at the mean posⁿ is.

$F_{net} = m r \omega^2$

$= 1 \times 5 \times \frac{v^2}{r^2}$

$= 5 \times \frac{25}{25} = 5\ Nt$

Sir solution



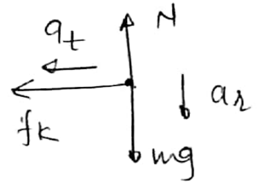
$\underbrace{T - mg}_{\text{centripetal force}} = m a_r \rightarrow NSL$

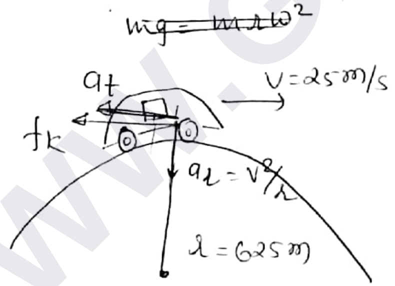
$= mv^2/r$

$F_{net} = \frac{mv^2}{r} = \frac{1 \times 5^2}{5} = 5\ Nt$

Que A automobile travelling at 25 m/s crests hill of radius 625 m just as the driver slams on the brake and skids to a stop. coeff of friction (μ) between car and road is 0.7 find the car retardation at the instant the brakes are applied ($g = 10 m/s^2$)

Sol.





$f_k = m a_t \longrightarrow NSL$

$\mu N = m a_t$ —— (1)

$mg - N = m a_r \longrightarrow NSL$

$\underbrace{mg - N}_{\text{centripetal}} = \frac{mv^2}{r}$

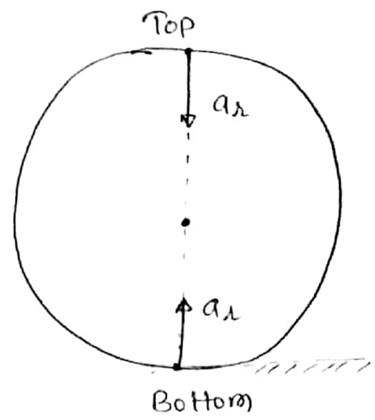
$N = mg - \frac{mv^2}{r}$ —— (2)

from (1) & (2)

$\cancel{\mu m}\left(g - \frac{v^2}{r}\right) = a_t$

$a_t = 6.3 m/s$

Que A stone of mass 'm' is tide to the string of length ~~of~~ 'L' and whirl in verticle circle find the min. velocity to complete the verticle circle



Top

T, mg

$T + mg = m a_r \rightarrow NSL$

$T + mg = \frac{mv^2}{L}$

for vel. to be minimum $T_{min} = 0$

$\cancel{m}g = \frac{\cancel{m} V_{min}^2}{L}$

$$\boxed{[V_{min}]_{Top} = \sqrt{gL}}$$

Bottom

No air drag

energy conservation

$\frac{1}{2}\cancel{m}v_T^2 + \cancel{m}(2L)g = \frac{1}{2}\cancel{m}v_B^2 + 0$

$v_T^2 + 4gL = v_B^2$

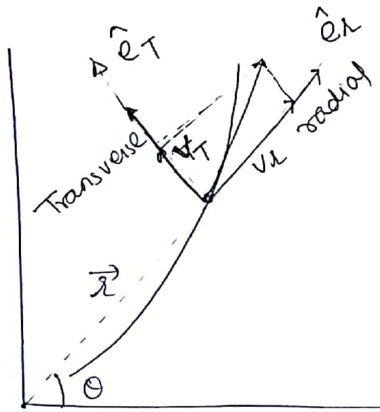
for v_B to be min $(v_T)_{min} = \sqrt{gL}$

$gL + 4gL = (V_B^2)_{min}$

$$\boxed{\sqrt{5gL} = (V_B)_{min}}$$

Radial & Transverse component of polar co-ordinates

r & $\theta \rightarrow$ fun^n of time



$r = |\vec{r}|\,\hat{e}_r$

$= r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}(\hat{e}_r) + r\omega(\hat{e}_t)$

$$\boxed{\vec{V} = \vec{V_r} + \vec{V_T}}$$

$|\vec{V}| = \sqrt{V_r^2 + V_T^2}$

$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\frac{d\hat{e}_r}{dt} = \omega\hat{e}_t$

$\hat{e}_t = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\frac{d\hat{e}_t}{dt} = -\omega\hat{e}_r$

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2r}{dt^2}\hat{e}_r + \frac{dr}{dt}\omega\hat{e}_T + \frac{dr}{dt}\omega(\hat{e}_T)$

$+ r\frac{d\omega}{dt}(\hat{e}_T) + r\omega^2(-\hat{e}_r)$

$$\vec{a} = \frac{d^2r}{dt^2}(\hat{e}_r) + \underbrace{2\frac{dr}{dt}\omega(\hat{e}_T)}_{\text{coriolis's acc}^n} + r\alpha(\hat{e}_T) - r\omega^2(\hat{e}_r)$$

Case 1 if $r = $ const. & $\theta = f(t)$ circular motion

Case 2 if $\theta = $ const & $r = f(t)$ rectilinear motion

Que If the path of a particle is defined by $r = 2t^2 + t$, $\theta = t^2 + 1$ then velocity at $t = 2$ sec is

Sol: $|\vec{V}| = \sqrt{V_r^2 + V_T^2}$

$V_r = \frac{dr}{dt} = (4t+1)_{t=2} = 9$ m/s

$V_T = r\omega = r\frac{d\theta}{dt} = \left[(2t^2+t)(2t)\right]_{t=2}$

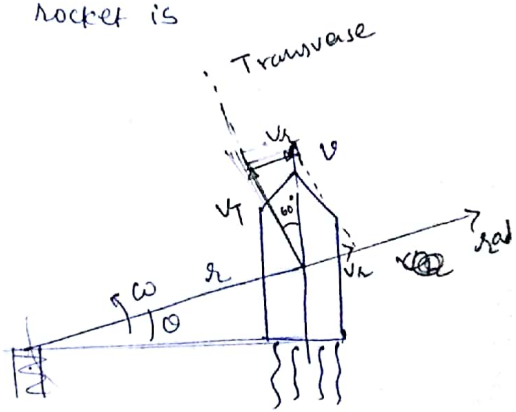
$= 10 \times 4$

$= 40$

$|\vec{V}| = \sqrt{9^2 + 40^2}$

$= \sqrt{81 + 1600} = 41$ m/s

Que A rocket is fired vertically and traced by a radar as shown in figure. at an instant $\theta = 60°$ it is known that $r = 10$ km $\dot{\theta}(\omega) = 0.02$ rad/s, then the velocity of the rocket is



Sol $V_T = r\omega$

$= 10000 \times 0.02$

$= 200.00$ m/s

~~$V_r = \frac{dr}{dt}$, $r\frac{d\theta}{dt}$~~

$\frac{V_T}{V} = \cos 60°$

$\frac{200}{V} = \frac{1}{2} \Rightarrow V = 400$ m/s

Linear momentum ($\vec{P}$)

$$\boxed{\vec{P} = m\vec{V}_{cm}}$$

$$\frac{d\vec{P}}{dt} = \frac{m\,dV_{cm}}{dt} = m a_{cm} = \Sigma \vec{F}_{ext}$$

NSL

$$\boxed{\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{ext}} \longrightarrow \text{NSL}$$

Conservation of momentum

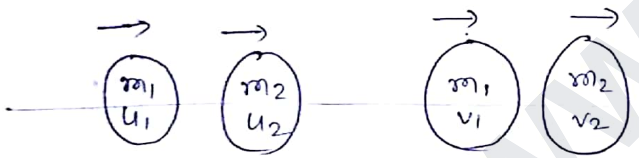
$\vec{P} = \text{const} \Rightarrow d\vec{P} = 0$

only when $\Sigma F_{ext} = 0$

if $\Sigma F_{ext,x} = 0$ $\quad\vec{P}_x = \text{const}$

if $\Sigma F_{ext,y} = 0$ $\quad\vec{P}_y = \text{const.}$

Collision



$u_1 > u_2$ $\qquad$ $V_2 > V_1$

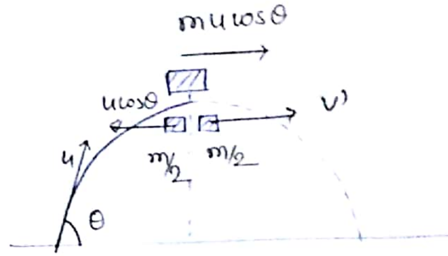
Conservation when considering $(m_1 + m_2)$

so,

$$\boxed{m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2} \longrightarrow \text{vector eq}^n$$

Que A shell is fired from a canon with a velocity u making an angle θ with the horizontal at the highest point in its path it explots into two pieces of equal mass one of the pieces retraces its path back to the canon, then the velocity of other piece immedidly after the impact is.

Sol: ~~$2m\,u\cos\theta = m$~~



Conservation for mass 'm'

$\Sigma F_x = 0 \Rightarrow \vec{P}_x = \text{const.}$

$(P_x)_{\text{before explosion}} = (P_x)_{\text{after explosion}}$

$$m u\cos\theta = \frac{m}{2}(-u\cos\theta) + \frac{m}{2}V'$$

$$\boxed{3u\cos\theta = V'}$$

When one body rest

$$mu\cos\theta = \frac{m}{2}(0) + \frac{m}{2}V'$$

$$\boxed{2u\cos\theta = V'}$$

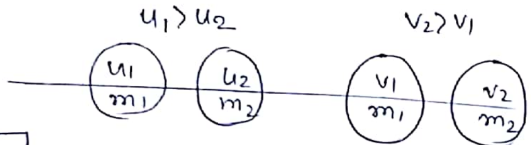
Collision / Impact: It is short time phenomenon in which momentum of individual body under going collision changes significantly.

(i) Perfectly elastic $(e=1)$

(ii) Perfectly inelastic / plastic $(e=0)$

(iii) Partially elastic $(0<e<1)$

(1) **Perfectly elastic collision:** When the initial kinetic energy is equal to final kinetic energy such a collision is called perfectly elastic collision.



$\boxed{m_1+m_2}$

$$m_1u_1+m_2u_2 = m_1v_1+m_2v_2 \quad \text{—— (I)}$$

$$\tfrac{1}{2}m_1u_1^2+\tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2 \quad \text{—— (II)}$$

from eqn (I) & (II)

$$\boxed{u_1-u_2 = v_2-v_1} \quad \text{—— (III)}$$

Velocity of approach = velocity of separation

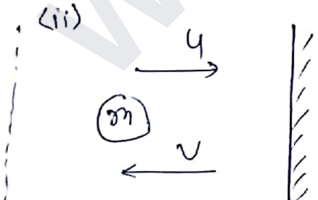
Special case

(i) if $m_1=m_2$

$\vec{u_1} = \vec{v_2}$

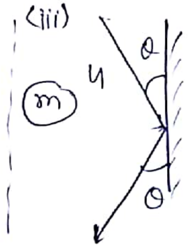
$\vec{u_2} = \vec{v_1}$

(ii)

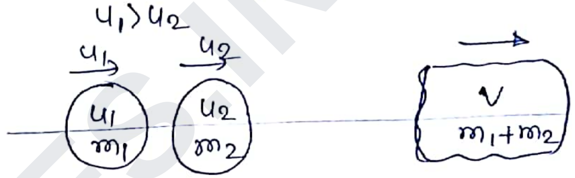


$v=-u$

(iii)



Perfectly inelastic $(e=0)$ / plastic: When two perfectly inelastic bodies moving along the same line collide they stick to each other.

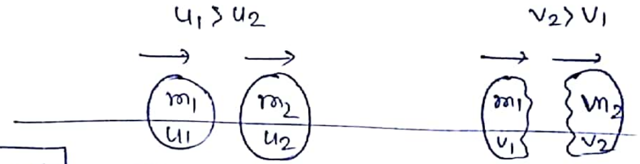


$\boxed{m_1+m_2}$

$$m_1u_1+m_2u_2 = (m_1+m_2)v$$

$$(K.E.)_{loss} = (K.E)_{initial} - (KE)_{final}.$$

Partially elastic $(0<e<1)$:



$\boxed{m_1+m_2}$

$$m_1u_1+m_2u_2 = m_1v_1+m_2v_2 \quad \text{—— (1)}$$

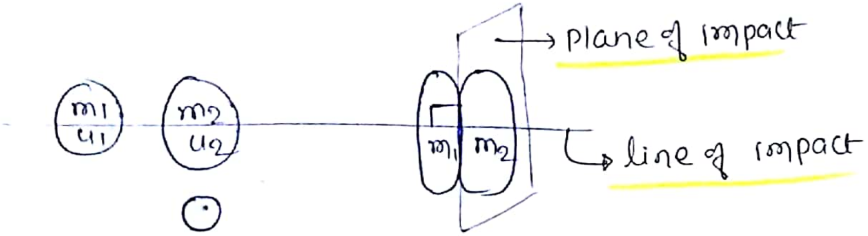
Vel. of separation = e (vel. of approach)

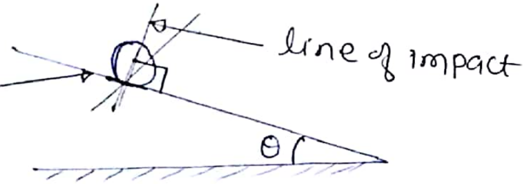
$$v_2-v_1 = e(u_1-u_2) \quad \text{—— (2)}$$

$$e \text{ (coeff of restitution)} = \frac{\text{Vel. of separation along line of impact}}{\text{Vel. of approach along line of impact}}$$

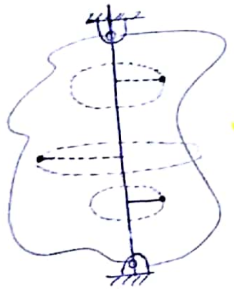
↓ represents recovery of shape

Plane of impact & line of impact:





Rotation: in rotation all the particles are in circular motion [except which are lying on the axis of rotation] & their centers will lie on a line which should remain fixed called axis of rotation.



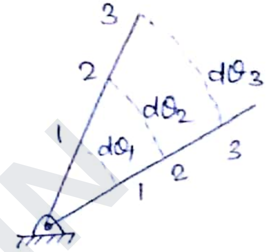
$I = \sum m_i r_i^2$ → mass moment of inertia

scalar quantity

- $I_{ring/hollow\ cy./rim} = mR^2$
- $I_{disc\ or\ solid\ cyl} = mR^2/2$
- $I_{rod/cm} = \dfrac{mL^2}{12}$
- $I_{rod/end} = \dfrac{mL^2}{3}$

Kinematics

$\vec{\omega}$ & $\vec{\alpha}$

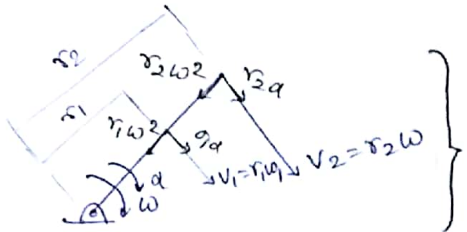


$d\theta_1 = d\theta_2 = d\theta_3 = d\theta$ → in time dt

$\Rightarrow$ $\left.\begin{array}{l} \omega_1 = \omega_2 = \omega_3 = \omega \\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha \end{array}\right\}$ at any instant

dirn: along axis of rotation & decide by right hand thumb rule

$r_2 \neq r_1$ $\quad V = r\omega \Rightarrow V \propto r$
$V_2 \neq V_1$



Kinetics

if $\sum \vec{T} = 0$ then $\vec{\alpha} = 0$

if $\sum \vec{T} \neq 0$ then $\vec{\alpha} \neq 0$

$$\boxed{\vec{\alpha} = \frac{\sum \vec{T}}{I}}$$

$$\boxed{\sum \vec{T} = I\vec{\alpha}}$$

$\left.\begin{array}{l} \sum \vec{T}_{axis} = I_{axis}\vec{\alpha} \\ \sum \vec{T}_{cm} = I_{cm}\vec{\alpha} \end{array}\right]$ → α will remain same

Angular momentum

In general, $\boxed{L = \vec{r} \times \vec{P}}$ → moment of momentum ; $P = m\vec{V}_{cm}$

$\boxed{\vec{L} = I\vec{\omega}}$ → for rotation

$\vec{L}$ - along axis of rotation & decided by right hand thumb rule

$$\frac{d\vec{L}}{dt} = \Sigma\vec{T}$$

Conservation of $\vec{L}$ ⇒ $\vec{L}$ = const ⇒ $d\vec{L} = 0$

only when $\Sigma\vec{T} = 0$

$\Rightarrow \boxed{I_1\omega_1 = I_2\omega_2}$

Kinetic energy

$$(K.E.)_i = \frac{1}{2} m_i v_i^2$$

$$= \frac{1}{2} m_i r_i^2 \omega^2$$

$$(K.E)_{body} = \Sigma \frac{1}{2} m_i r_i^2 \omega^2$$

$$\boxed{(K.E)_{body} = \frac{1}{2} I\omega^2}$$

Work done $= \vec{F}\cdot d\vec{s}$

Rotation W.D $=$ ~~F.ds~~ $T d\theta$

Power

Rate of doing work

$$P = \frac{F\cdot d\vec{s}}{dt} = F\cdot\vec{V}$$

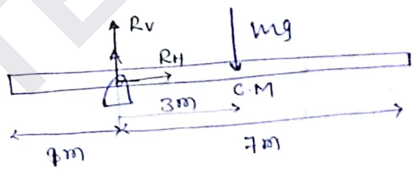
in case of rotation

$$P = \frac{T\cdot d\theta}{dt} = T\omega$$

$$P = \frac{2\pi NT}{60}$$

Mechanics

Que A uniform rod of mass 3 kg & length 8 m rotates in a vertical plane about a horizontal axis one meter from one of its end as shown in fig. Find angular accⁿ of the rod at the instant shown.



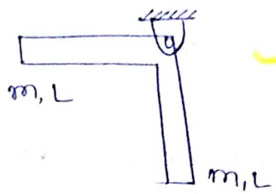
$$\Sigma T_A = I_A \alpha$$

$$mg(3) = \left[\frac{mL^2}{12} + m(3)^2\right]\alpha$$

$$9.81\times 3 = \left(\frac{8^2}{12} + 9\right)\alpha = \left(\frac{64}{12}+9\right)\alpha$$

$$\Rightarrow \alpha = 2.05 \text{ rad/s}$$

Que A uniform L-shaped member with each of its limb has mass 'm' and length L as shown in fig find α of the member at the position shown



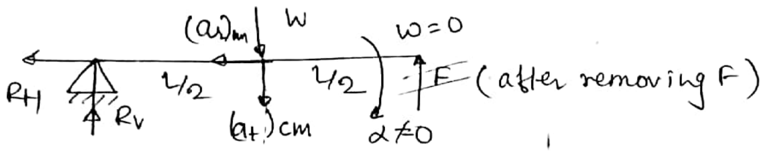
[जितने mass है उतना I लेना है]

$\Sigma T_A = I_A \alpha$

$mg\frac{L}{2} = \cancel{\left[\frac{mL^2}{12}\right]} \left[\frac{mL^2}{3} + \frac{mL^2}{3}\right] \alpha$

$\boxed{\frac{3g}{4L} = \alpha}$

Que A uniform rod of wt. 'W' and length L is supported as shown in figure. The ~~mo~~ instant force F is removed the rxn at hinge is



$\Sigma T_A = I_A \alpha$

$W\frac{L}{2} = \left(\frac{W}{g} \times \frac{L^2}{3}\right) \alpha$

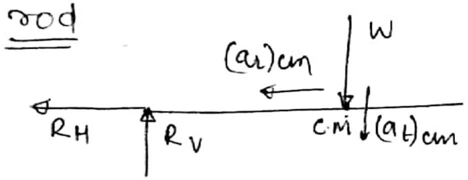
$\frac{3g}{2L} = \alpha$

$\Rightarrow R_V \times \cancel{L}/\cancel{2} = \frac{WL^2}{g\,12_6} \cdot \alpha$

$= \frac{W}{g}\left(\frac{L}{6}\right) \cdot \frac{3g}{2L}$

$R_V = W/4$

rod



$\Sigma F_{ext} = m\vec{a}_{cm} \longrightarrow$ NSL

in y. direction

$W - R_V = \frac{W}{g} a_{cm}(-y) \longrightarrow$ NSL

$W - R_V = W/g \, (a_t)_{cm} = W/g \times r_{cm}\alpha$

$W - R_V = \frac{W}{g} \times \frac{L}{2} \times \frac{3g}{2L}$

$R_V = \frac{W}{4} (\uparrow)$

in x- direction

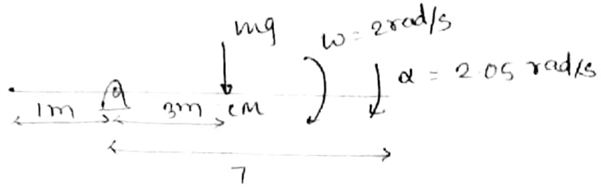
$R_H = \frac{W}{g} a_{cm}(-x) \longrightarrow$ NSL

$R_H = \frac{W}{g} r_{cm}\omega^2 = 0 \quad [\omega = 0]$

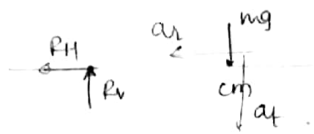
So, Reaction - $\sqrt{R_V^2 + R_H^2}$

$= R_V = W/4 \, (\uparrow)$

Que Find rxn at hinge for the system given in figure at the position shown $m = 3$ kg.



~~from~~



$mg - R_v = m(a_{cm})$ ——— NSL

$mg - R_v = m \cdot (a_t)_{cm}$

$R_v = m(r\alpha)_{cm}$

$R_v = -(3 \times 3 \times 2.05 - 3 \times 9.81)$

$R_v = 10.98$ N.

In x-dir^n

$$R_H = m(a_r)_{CM}\,\omega^2$$
$$= 3 \times 3 \times 2^2$$
$$= 36\,N$$

$$(R)_{total} = \sqrt{36^2 + 10.98^2}$$
$$= 37.637\,N$$

While solving Rotation problem:

use

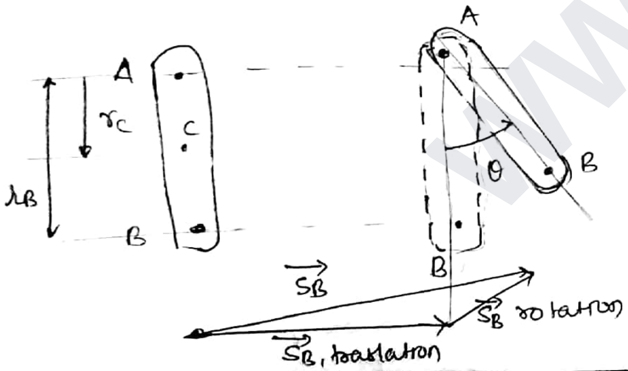
$$\boxed{\begin{array}{l} \sum T = I\alpha \\ \sum \vec{F}_{ext} = m\vec{a}_{cm} \end{array}}$$

General Motion:

If a rigid body rotates as well as translate simultaneously such a motion is called general motion

eg. motion of connecting rod, motion of ladder, Rolling wheel etc

Kinematics

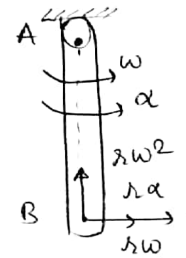


$$[\vec{s}, \vec{v}, \vec{a}]_{\text{general motion}} = [\vec{s}, \vec{v}, \vec{a}]_{\text{translation}} + [\vec{s}, \vec{v}, \vec{a}]_{\text{rotation}}$$

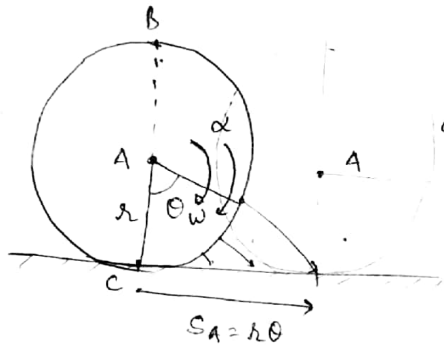
(B)
$$\vec{S}_B = \vec{S}_A + \vec{S}_{B/A} \qquad [\vec{S}_{B/A} = \vec{r_B\theta}]$$
$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} \qquad [\vec{V}_{B/A} = \vec{r_B\omega}]$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad [\vec{a}_{B/A} = \vec{r_B\omega^2} + \vec{r_B\alpha}]$$

(C)
$$\vec{S}_C = \vec{S}_A + \vec{S}_{C/A} \qquad [\vec{S}_{C/A} = \vec{r_c\theta}]$$
$$\vec{V}_C = \vec{V}_A + \vec{V}_{C/A} \qquad [\vec{V}_{C/A} = \vec{r_c\omega}]$$
$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} \qquad [\vec{a}_{C/A} = \vec{r_c\omega^2} + \vec{r_c\alpha}]$$

B w.r.t A



Pure Rolling:



(i) If $S_A = r\theta \rightarrow$ pure rolling [DOF = 1]

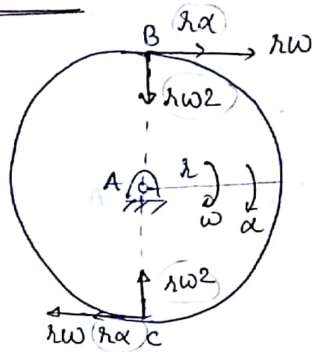
in 1 rev. $\theta = 2\pi \Rightarrow S_A = 2\pi r$

(ii) If $S_A > r\theta \rightarrow$ skidding

(iii) If $S_A < r\theta \rightarrow$ slipping

Pure Rolling

w.r.t. A



$\vec{S_A} = r\theta\ \hat{i}$

$\vec{V_A} = r\dfrac{d\theta}{dt}\ \hat{i}$

$\vec{V_A} = r\omega\ \hat{i}$

$\vec{a_A} = r\dfrac{d\omega}{dt}\ \hat{i}$

$\vec{a_A} = r\alpha\ \hat{i}$

Prop

$\vec{V_C} = \vec{V_A} + \vec{V_{C/A}} = r\omega\,\hat{i} + r\omega(-\hat{i}) = 0$

$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}} = r\omega\,\hat{i} + r\omega(\hat{i}) = 2r\omega\,\hat{i}$

$\vec{a_C} = \vec{a_A} + \vec{a_{C/A}} = \cancel{r\alpha\,\hat{i}} + r\omega^2(\hat{j}) + \cancel{r\alpha(-\hat{i})}$

$= r\omega^2(\hat{j}) \neq 0$

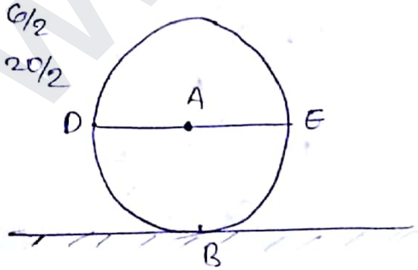
$a_B = \vec{a_B} + \vec{a_{B/A}} = r\alpha\,\hat{i} + r\omega^2(-\hat{j}) + r\alpha(\hat{i})$

$= 2r\alpha\,\hat{i} + r\omega^2(-\hat{j})$

Que A wheel of radius 2m rolls freely on the surface as shown in fig. if $V_A = 6\,m^2$ & $a_A = 20\,m/s^2$ then find the velocity of of point D & E

$\omega = 3\ rad/s = 6/2$

$\alpha = 10\ rad/s^2 = 20/2$



$\vec{V_D} = \vec{V_A} + V_{D/A}$

$= r\omega\,\hat{i} + r\omega\,\hat{j}$

$= 6\hat{i} + 6\hat{j}$

$|\vec{V_D}| = 6\sqrt{2}\ m/s$

$\vec{V_E} = \vec{V_A} + V_{E/A}$

$= r\omega\,\hat{i} + r\omega(-\hat{j})$

$= 6\hat{i} + 6(-\hat{j})$

$|\vec{V_E}| = 6\sqrt{2}\ m/s$

$\vec{a_D} = \vec{a_A} + \vec{a_{D/A}}$

$= r\alpha\,\hat{i} + r\omega^2(\hat{i}) + r\alpha(\hat{j})$

$= 20\hat{i} + 2\times3^2(\hat{i}) + 20(\hat{j})$

$= 38\hat{i} + 20\hat{j}$

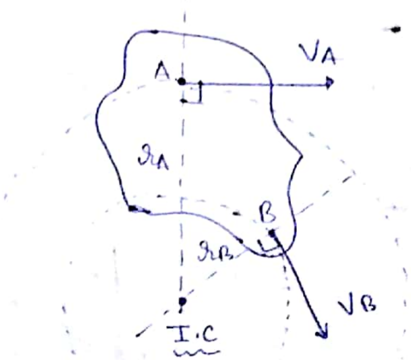
$|\vec{a_D}| = \sqrt{38^2 + 20^2}$

Instantaneous/center/Axis of rotation:

it is a point/line in space about which a body is in general motion can be assumed as in pure rotation to find the velocities

Locations of I-center

Case 1

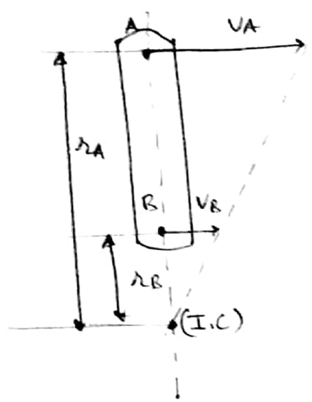


$V_A = r_A\omega$

$V_B = r_B\omega$

$\dfrac{V_A}{r_A} = \dfrac{V_B}{r_B}$

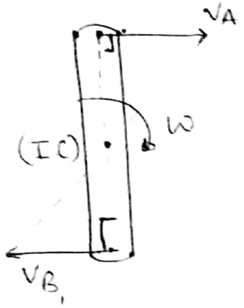
Case 2



$$\frac{V_A}{V_B} = \frac{r_A}{r_B}$$

$AB = r_A - r_B$

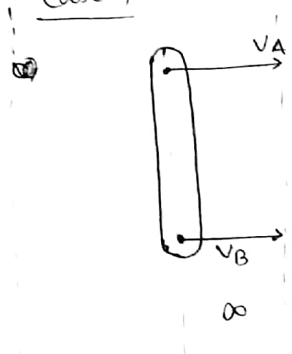
Case 3



$$\frac{V_A}{V_B} = \frac{r_A}{r_B}$$

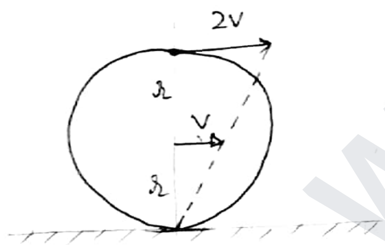
$AB = r_A + r_B$

Case 4

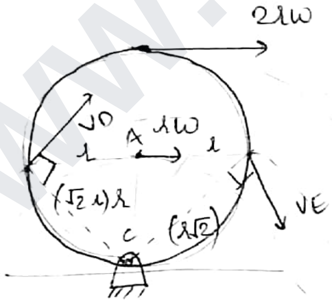


$V_A = V_B$

Application



$\vec{V} = 0, \ \vec{a} \neq 0$



$$\boxed{\begin{aligned} \Sigma T_A &= I_A \alpha \\ \Sigma T_C &= I_C \alpha \end{aligned}}$$

$$(K.E.)_{rolling} = (K.E)_{trans.} + (K.E)_{Rotation}$$

$$= \tfrac{1}{2} m v^2 + \tfrac{1}{2} I_A \omega^2$$

$$= \tfrac{1}{2} m r^2 \omega^2 + \tfrac{1}{2} I_A \omega^2$$

$$= \tfrac{1}{2} \omega^2 \left[m r^2 + I_A \right]$$

$$= \tfrac{1}{2} \omega^2 I_C$$

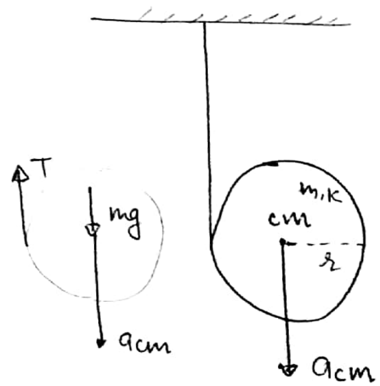
angular momentum

$L_C = I_C \omega$

Que A reel of mass m, radius r, and radius of gyration K, is rolling down smoothly from rest with one end of the thread wound on it is held in the ceiling as shown in fig find.

(a) linear acc of the real

(b) Tension in the thread.



$mg - T = m a_{cm} \longrightarrow$ NSL

$mg - T = m r \alpha \longrightarrow (1)$

$[\text{Torque}]_{cm} = I_{cm} \alpha$

$T r = m K^2 \alpha$

$\alpha = \dfrac{T r}{m K^2} \quad \text{——} \ (2)$

from (1) & (2)

$$mg - T = \frac{m r^2 T}{m K^2}$$

$$T = \frac{mg K^2}{r^2 + K^2}$$

$$a_{cm} = r\alpha = \frac{Tr^2}{mk^2} = \frac{mgk^2}{r^2+k^2}\cdot\frac{r^2}{mk^2}$$

$$a_{cm} = \frac{gr^2}{r^2+k^2}$$

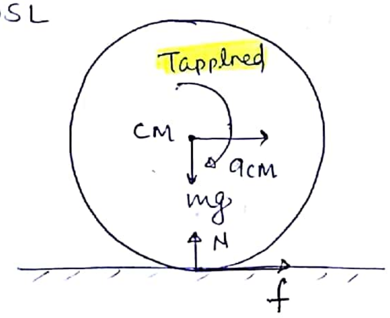
Direction of friction in rolling:

$f = m\,a_{cm}\,(+x) \longrightarrow$ NSL

$$\boxed{f = mr\alpha}$$

$N - mg = 0 \longrightarrow$ NFL

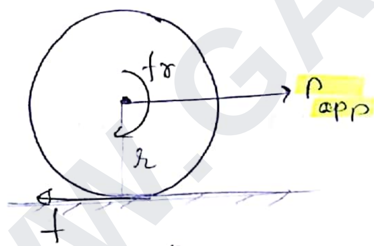
$$\boxed{T_{app} - fr = I_{cm}\alpha}$$



$\boxed{P_{app} - f = m a_{cm}} \longrightarrow$ NSL

$P_{app} - f = mr\alpha$ —— ①

$\boxed{fr = I_{cm}\alpha}$ —— ②



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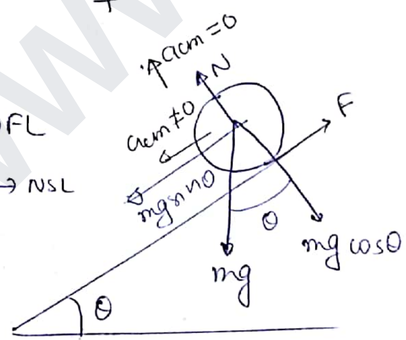
$N = mg\cos\theta \longrightarrow$ NFL

$mg\sin\theta - F = m a_{cm} \longrightarrow$ NSL

$T_c = I_c\alpha$



$$mg\sin\theta\,(r) = [I_{cm} + mr^2]\,\alpha \qquad I_{cm} = mk^2 = m\left(\frac{r}{2}\right)^2 = \frac{mr^2}{4}$$

$$= \left[\frac{mr^2}{4} + mr^2\right]\alpha$$

$$mg\sin\theta\,\cancel{\alpha} = \frac{5mr^2}{4}\,\alpha$$

$$\frac{4g\sin\theta}{5r} = \alpha$$

$$mg\sin\theta - F = m\cancel{r}\left(\frac{4g\sin\theta}{5\cancel{r}}\right)$$

$$F = \frac{1}{5}\,mg\sin\theta$$

—x—

Impluse momentum eqⁿ

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$\underbrace{\vec{F}\,dt}_{} = \underbrace{d\vec{P}} \longrightarrow$ change in momentum $= P_f - P_i$

[linear impulse]
(vector quantity)

$$\vec{T} = \frac{d\vec{L}}{dt}$$

$\underbrace{\vec{T}\,dt} = d\vec{L} \longrightarrow$ change in $\vec{L}$

(angular impulse)

Work energy theorem

Total W.D. on a system = change in KE

$$\boxed{TWD = (K.E)_f - (K.E)_i}$$

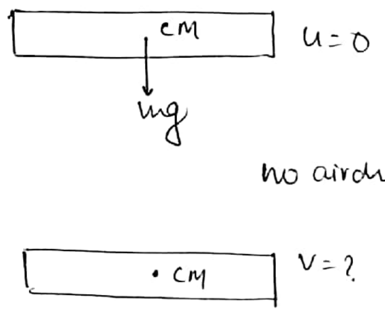
Que

(m)

(1st) TWD = ΔKE

$W_{mg} = mgh = \frac{1}{2}mv^2 - 0$

$v = \sqrt{2gh}$



$(WD)_{drag} = 0$ $\quad$ $[Energy]_m$ = conserved

(2nd) $(G.P.E.)_i + (K.E)_i = (GPE)_f + (K.E)_f$

$mgh + 0 = 0 + \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

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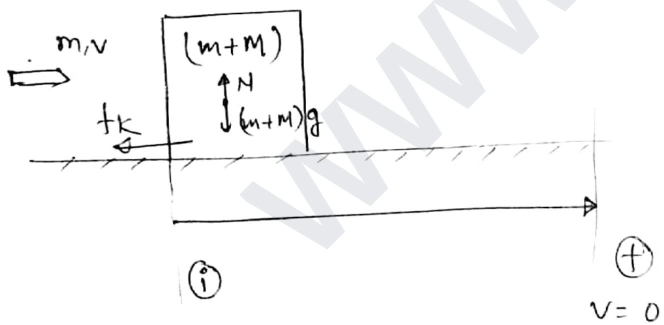
Q.9



$\boxed{m+M}$

$mv + 0 = (M+m)v'$

$\cancel{v' = \frac{mv}{(M+m)}} \Rightarrow v = \frac{(M+m)v'}{m}$

TWD = $\Delta K.E.$

$TWD = (W)_{m\cancel{g}}^{0} + \cancel{W_N}^{0} + W_{fk}$

$TWD = W_{fk} = -f_k S = -\mu N S = -\mu(m+M)gS$

$-\mu(M+m)g.s = 0 - \frac{1}{2}(m+M)v'^2$

$v' = \sqrt{2\mu g s}$

$v = \frac{(M+m)}{m}\sqrt{2\mu g s}$

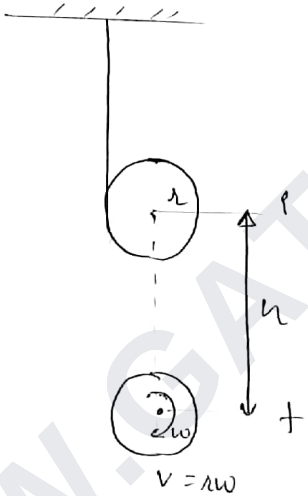
Note:

1. if workdone by friction is zero then use ~~con~~ conserve energy of system instead work energy theorem to get the ans easily & quickly
2. if workdone by ~~system~~ friction in no zero then use work energy theorem
3. (imp) We can conserve energy of the system in case of pure rolling because workdone by static friction is always zero

(iv) friction in rolling in is static friction b'coz the point where it is acting has zero velocity.

Que. A chord is wrapped around the solid cylinder of radius r and mass m as shown in fig. If the cylinder is realeased from rest the velocity of cylinder after it has moved through a distance h will be

$(GPE)_i + (KE)_i = (GPE)_f + (K.E)_f$



$$mgh = \frac{1}{2} I_c \omega^2$$

$$= \frac{1}{2} I_c \frac{v^2}{r^2}$$

$$= \frac{1}{2}\left(\frac{mr^2}{2} + mr^2\right)\frac{v^2}{r^2}$$

$$\cancel{m}gh = \frac{1}{2}\left[\frac{3}{2}\cancel{m}\right]v^2$$

$$\sqrt{\frac{4}{3}gh} = v$$