

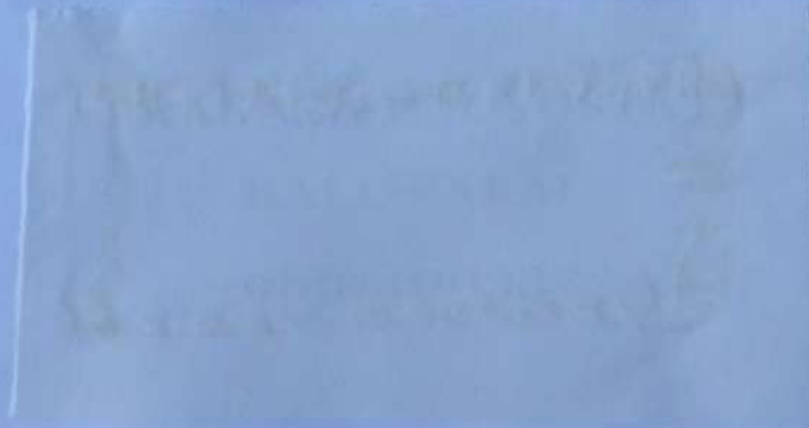
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-: HAND WRITTEN NOTES:-
OF
MECHANICAL ENGINEERING

-: SUBJECT:-
FLUID MECHANICS

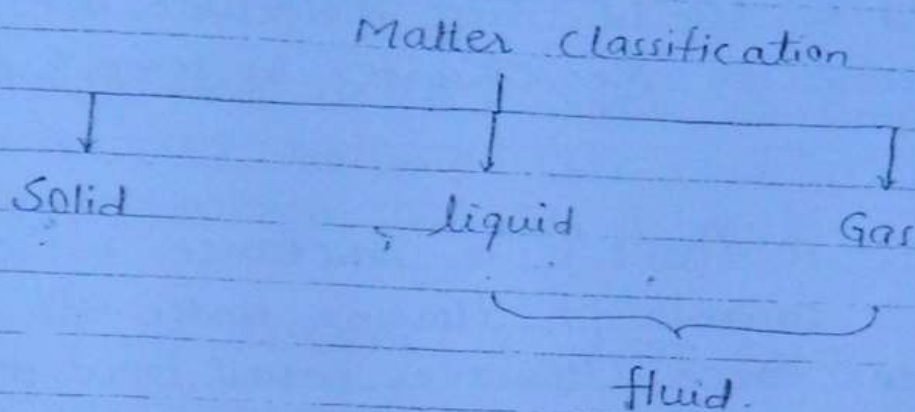


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Basic - Concept

"MADE EASY"
Hand Written Notes Available

Matter : Matter is anything that has mass & occupies space.



Physical significance :

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1. Solid : In solid state, the molecule's of a substance are tightly bound to each other as the molecule's have a fixed position in a space. A solid exhibits rigidity & possess a distinct shape. The intermolecular distance in a solid are the least while the intermolecular force of attraction are the strongest.
2. liquid : The force of attraction b/w the molecule's of liquid are less than that in a solid & greater than that in a gas. A substance in liquid state does not have a defined shape, rather its shape is defined by that of its container.
3. Gas : Gas molecule's are in a state of random motion. They exhibit the least intermolecular force of attraction & hence gases lack a definite shape. The intermolecular distances are very large.

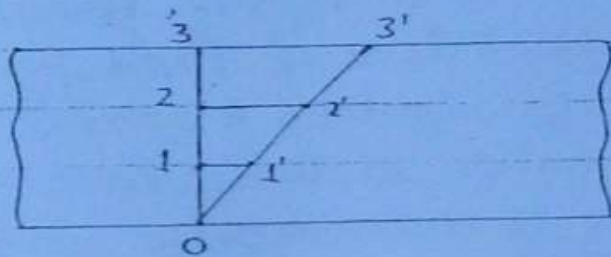
due to which the particles of gases are widely separated from each other.

The density & viscosity of gas is less as compare to solid & liquid. Gas molecules are highly vulnerable to the change's in temp & press.

Fluid : A fluid is a substance which is capable of flowing under the action of shear force (However small force may be).

Eg. - Liquid, Gases, Vapour.

For a static fluid shear force is zero.



Solids regain original shape if the shear force within elastic limit when the force is removed. But fluid never regain original shape after the removal of force.

Cohesive Force : Attraction b/w two similar molecules is known as cohesive force.

Adhesive Force : Attraction b/w two dissimilar molecules is known as adhesive force.

FLUID-PROPERTIES

Date _____
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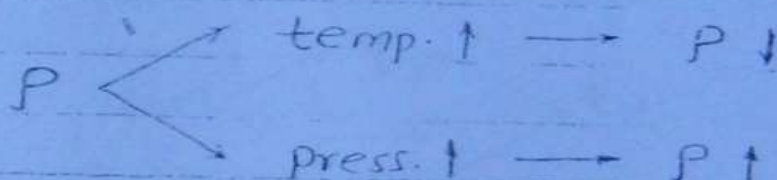
Properties are the characteristics of the fluid.

Density or mass density (ρ) :

It is defined as the ratio of mass of the fluid to its volume. Its unit is kg/m^3 & its dimensional formula is ML^{-3} .

Density of water is 1000 kg/m^3 .

Density depends on temp. & pressure.



Specific weight or weight density (ω)

It is defined as the ratio of weight of the fluid to its volume.

$$\omega = \frac{\text{weight of fluid}}{\text{Volume}}$$

$$\text{Unit} \rightarrow \omega = \frac{N}{\text{m}^3} = \frac{F}{\text{m}^3} = \frac{m \cdot a}{\text{m}^3} = \frac{\text{kg} \times \text{m}}{\text{m}^3 \cdot \text{sec}^2} = \frac{\text{ML}}{\text{L}^3 \text{T}^2} = \text{ML}^{-2} \text{T}^{-2}$$

Its unit is N/m^3 & its dimensional formula is $\text{ML}^{-2} \text{T}^{-2}$.

$$\omega = \frac{m \cdot g}{\text{volume}} = \rho \cdot g$$

$$\boxed{w = \rho \cdot g}$$

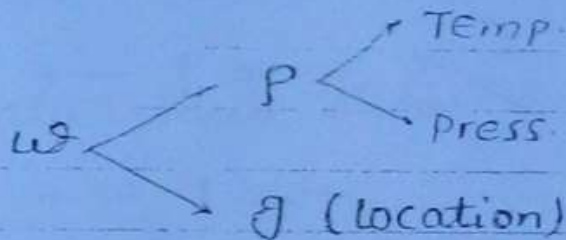
Specific weight of the water.

$$w_{H_2O} = \rho_{H_2O} \cdot g$$

$$= 1000 \times 9.81$$

$$\boxed{w_{H_2O} = 9810 \text{ N/m}^3}$$

Specific weight is depend's upon three parameter.



Specific gravity or Relative density :- (s)

It is defined as the ratio of density of fluid to the density of standard fluid.

or

It can also be defined as the ratio of specific weight of fluid to the specific weight of standard fluid.

It is unit less or dimension less.

For liquids the standard fluid is water.

For Gases the standard fluid is either H_2

or air at a given temp. & pressure

The specific gravity of water is 1.

If the specific gravity of liquid less than one, it is lighter than water. If the specific gravity of liquid is greater than one, it is heavier than water.

4. Compressibility $\therefore (\beta)$

It is the reciprocal of bulk modulus (K).

$$\beta = \frac{1}{K}$$

Where K = bulk modulus

Bulk modulus is defined as the ratio of compressive stress to volumetric strain.

$$K = \frac{\text{Increase of Press.}}{\text{Volumetric Strain}}$$

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$$K = \frac{dP}{\left(-\frac{dv}{v}\right)}$$

-ve sign means the volume decreases with increase of pressure

We know that,

$$\rho = \frac{m}{v}$$

" R = Characteristic gas constant.

{ T = Constant (Iso-thermal process).

Differentiating above equation.

$$\frac{dP}{dT} = RT$$

Now Iso-thermal bulk modulus.

$$K_T = P \cdot \frac{dP}{dP} = P \cdot RT = P \quad \text{from eqn. (1).}$$

$$\boxed{K_T = P}$$

Adiabatic bulk modulus of an Ideal gas.

For adiabatic process,

$$P V^\gamma = C$$

$$P = \left(\frac{C}{V} \right)^\gamma = C \quad \left\{ \because P = \frac{m}{V} \text{ \& } m = \text{const.} \right\}$$

$$P = C P^\gamma$$

Differentiate ~~from~~ above eqn.

$$dP = C \cdot \gamma P^{\gamma-1} \cdot dP + 0$$

$$\frac{dP}{dP} = \frac{C \cdot \gamma \cdot P^\gamma}{P}$$

$$P \frac{dP}{dP} = C \cdot P^\gamma \cdot \gamma$$

$$P \frac{dP}{dP} = \gamma P$$

Now adiabatic bulk modulus.

$$K_a = P \frac{dP}{dP} = \gamma P$$

$$\boxed{K_a = \gamma P}$$

Prob-1: The eqn. of state for liquid is $P = (3500 P^{1/2} + 2500)$ N/m^2 . Then find the bulk modulus at a pressure of 10^5 N/m^2 .

Soln: given, $P = 3500 P^{1/2} + 2500$

$$dP = 3500 \times \frac{1}{2} P^{1/2-1} \cdot dP + 0$$

$$\frac{dP}{dP} = \frac{3500}{2} P^{-1/2}$$

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But we know that bulk modulus,

$$K = P \frac{dP}{dP}$$

$$K = P \cdot \frac{3500}{2} P^{-1/2}$$

$$K = \frac{3500}{2} P^{1/2}$$

from eqn. (i), we get.

$$P = 3500 P^{1/2} + 2500$$

$$(P - 2500) = 3500 P^{1/2}$$

Put the value in eqn. (ii), we get.

$$K = \frac{1}{2} (P - 2500)$$

$$K = \frac{10^5 - 2500}{2}$$

$$\boxed{K = 48750 \text{ N/m}^2} \text{ Ans.}$$

Prob-2 An increase of pressure 2 bar decrease the volume of a liquid by 0.01%. then the bulk modulus is -

(a) $2 \times 10^5 \text{ N/m}^2$ (b) $2 \times 10^7 \text{ N/m}^2$ (c) $2 \times 10^9 \text{ N/m}^2$

(d) $2 \times 10^{11} \text{ N/m}^2$

Soln:- We know that bulk modulus,

$$K = \frac{dP}{(-dv/v)}$$

$$dP = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$$

$$\frac{-dv}{v} = \frac{0.01}{100}$$

Then, bulk modulus is,

$$K = \frac{2 \times 10^5}{\frac{0.01}{100}} = 2 \times 10^9$$

$$\boxed{K = 2 \times 10^9 \text{ N/m}^2} \text{ Ans.}$$

prob-3 When the pressure on a given mass of liquid is increases from 3 MPa to 3.5 MPa the density of liquid increases from 500 kg/m³ to 501 kg/m³. Then find the bulk modulus of liquid over given pressure range. -

- (a) 700 MPa. (b) 600 MPa. (c) 500 MPa. (d) 250 MPa.

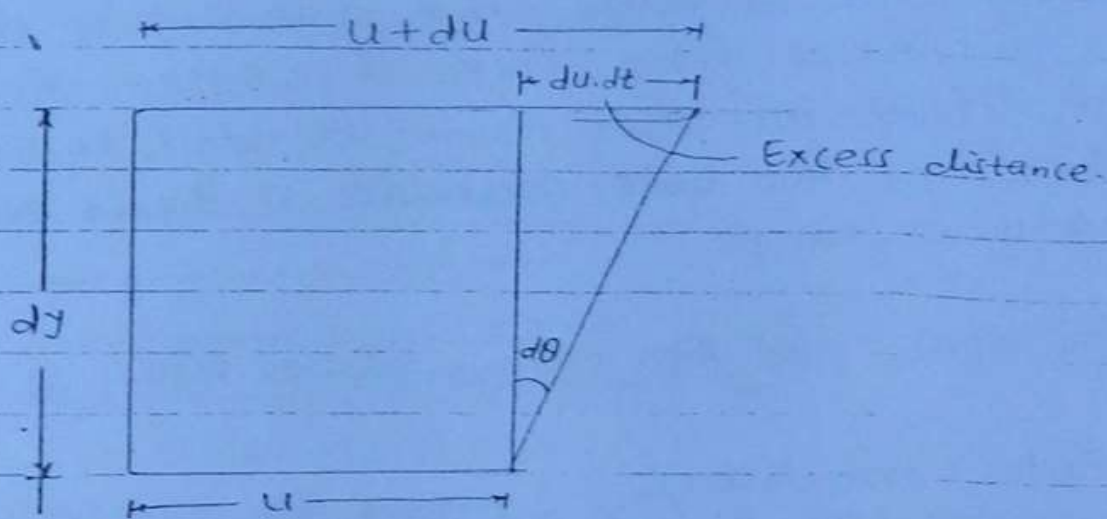
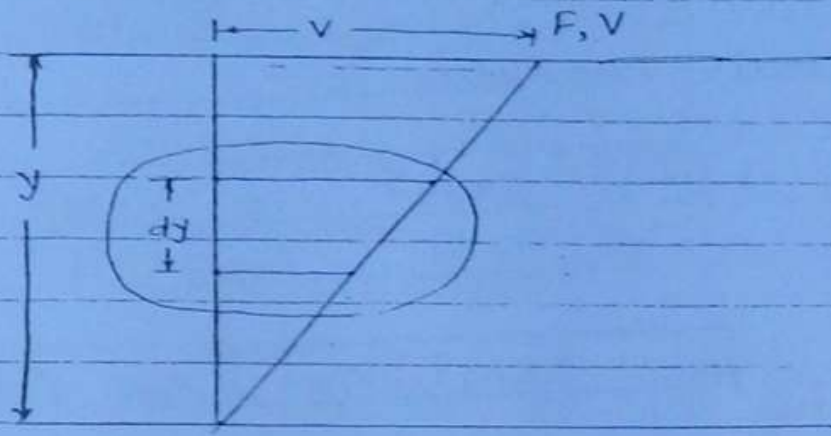
Solⁿ: Bulk modulus,

$$K = P \frac{dP}{dP}$$

$$K = \frac{dP}{\frac{dP}{P}} = \frac{3.5 - 3}{\frac{501 - 500}{500}} = 250$$

$$\boxed{K = 250 \text{ MPa}} \text{ Ans.}$$

5. Viscosity :- It is the internal resistance offered by one layer fluid to the other layer.



$$V = \frac{d}{t}$$

$$d = v \cdot t$$

$$d = du \cdot dt$$

$$\tan d\theta = \frac{du \cdot dt}{dy}$$

As the $\tan \theta$ is very less. then.

$$\tan \theta = d\theta.$$

then,
$$d\theta = \frac{du \cdot dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

we know that,

$$\text{shear stress} = \frac{\text{shear force}}{\text{shear Area.}}$$

$$\tau = \frac{F}{A}$$

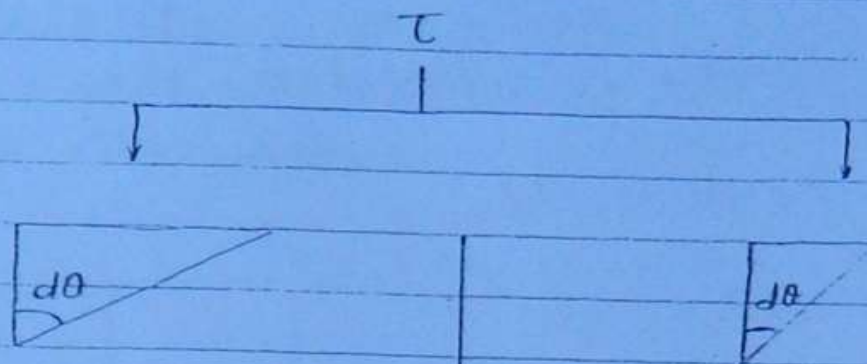
As the molecule's are taken into same area.

$$\uparrow \tau \propto F \uparrow$$

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt}$$

$$\mu = \frac{\tau}{\frac{d\theta}{dt}}$$



$\frac{d\theta}{dt}$ is large

flow is easy

Resistance is less

μ is less

$\frac{d\theta}{dt}$ is less

flow is not easy.

Resistance is more

μ is more

μ represents resistance to be flow this μ is known as coefficient of viscosity or absolute viscosity or dynamic viscosity.

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Rate of angular
deformation
or

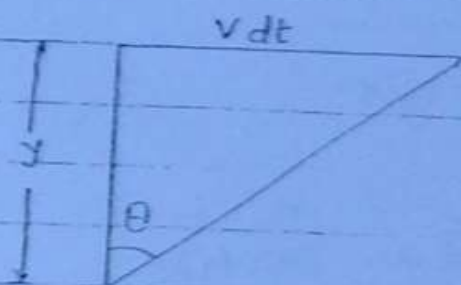
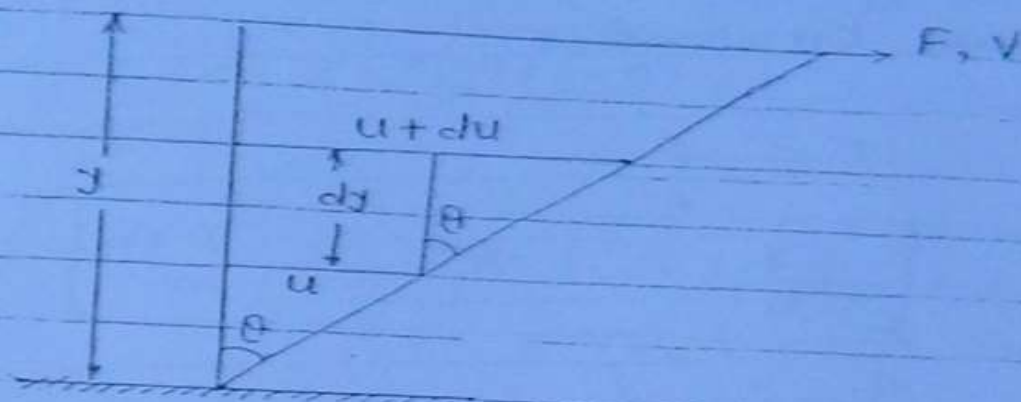
Velocity
gradient.

Rate of shear
strain.

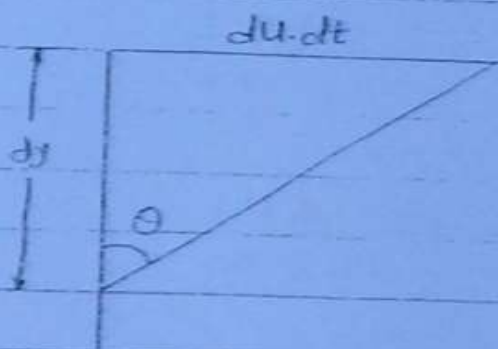
$$\tau = \mu \cdot \frac{d\theta}{dt}$$

$$\tau = \mu \cdot \frac{du}{dy}$$

Viscosity for the linear profile :-



$$\tan \theta = \frac{v \cdot dt}{y} \quad \text{--- (i)}$$



$$\tan \theta = \frac{du \cdot dt}{dy} \quad \text{--- (ii)}$$

from eqn. (i) & (ii) we get

$$\frac{v \cdot dt}{y} = \frac{du \cdot dt}{dy}$$

$$\frac{du}{dy} = \frac{v}{y}$$

$$\tau = \mu \frac{du}{dy} = \frac{\mu v}{y}$$

$$\tau = \frac{\mu v}{y}$$

But we know that.

$$\tau = \frac{F}{A}$$

$$\frac{F}{A} = \frac{\mu v}{y}$$

$$F = \frac{\mu A v}{y}$$

This is valid only for linear profile.

Unit of viscosity :-

We know that,

$$F = \frac{\mu A v}{y}$$

$$\mu = \frac{F y}{A v}$$

$$\mu = \frac{N \cdot m}{m^2 \times \frac{m}{sec}} = \frac{N \cdot s}{m^2} = Pa \cdot s$$

$$\text{but } N = F = m a = kg \cdot \frac{m}{sec^2}$$

then
$$\mu = \frac{\text{Kg-m}}{\text{sec}^2} \times \frac{\text{m}}{\text{m}^2 \times \frac{\text{m}}{\text{sec}}}$$

$$\mu = \text{Kg/m-s}$$

$\frac{\text{N-s}}{\text{m}^2} = \text{Pa-s} = \frac{\text{Kg}}{\text{m-s}}$
--

Unit of viscosity In C.G.S. system :-

$$\text{Kg/m-s}$$

$$\frac{1 \text{ gm}}{\text{cm-s}} = 1 \text{ poise.}$$

$$\frac{1 \text{ Kg}}{\text{m-s}} = \frac{10^3 \text{ gm}}{10^2 \text{ cm-s}}$$

$$\frac{1 \text{ Kg}}{\text{m-s}} = 10 \frac{\text{gm}}{\text{cm-s}}$$

$$\frac{1 \text{ Kg}}{\text{m-s}} = 10 \text{ poise.}$$

$1 \text{ poise} = 0.1 \frac{\text{Kg}}{\text{m-s}}$
--

Dimensional formula of viscosity is $\text{ML}^{-1}\text{T}^{-1}$

Kinematic viscosity (ν) :

In fluid mechanics the term μ/ρ appears frequently & for convenience this term is known as kinematic viscosity.

where μ = dynamic viscosity
& ρ = Density.

$$\boxed{\nu = \frac{\mu}{\rho}}$$

Its unit is m^2/sec & its dimensional formula is $L^2 T^{-1}$.

In C.G.S. system its unit is cm^2/sec .

$$\boxed{\frac{\text{cm}^2}{\text{sec}} = \text{stroke}}$$

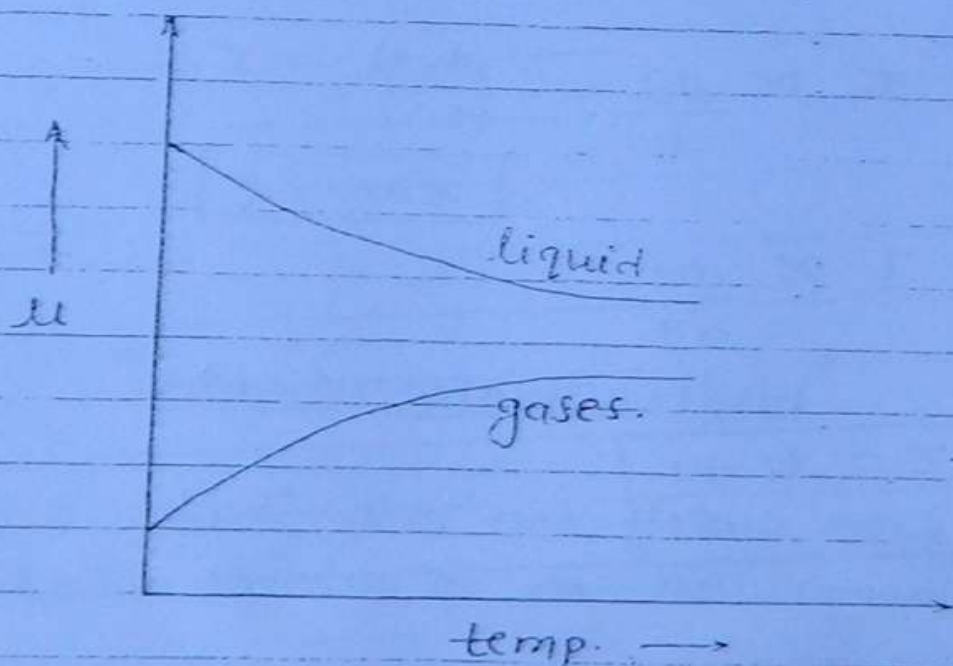
$$\text{TE} \rightarrow \boxed{\text{Centi stroke} = \frac{1}{100} \text{ stroke}}$$

NOTE- If in a problem not given viscosity profile then take linear profile & solve the problem.

Variation of viscosity with temperature:

In case of liquid, intermolecular distance is small & hence cohesive force are large with rise in temp. Cohesive force decreases & hence resistance to the flow also decreases & hence in case of liquid's with rise in temp. viscosity decreases.

In case of gases cohesive force are negligible & with rise in temp. molecular disturbance (agitation) increases & hence there resistance to flow also increases. i.e. in case of gases with rise in temp. viscosity increases.



NOTE - viscosity varies very little with pressure but at very-very high pressure viscosity increases because at high high pressure movement of fluid will be difficult.

• Type of fluid :

1. NEWTONIAN FLUID.
2. Non-newtonian fluid.
3. Real fluid.
4. Ideal fluid.

* Newtonian fluid *

A fluid is said to be newtonian fluid if shear stress is directly proportional to rate of shear strain or rate of angular deformation or velocity gradient.

For a newtonian fluid,

$$\tau \propto \frac{d\theta}{dt}$$

or,

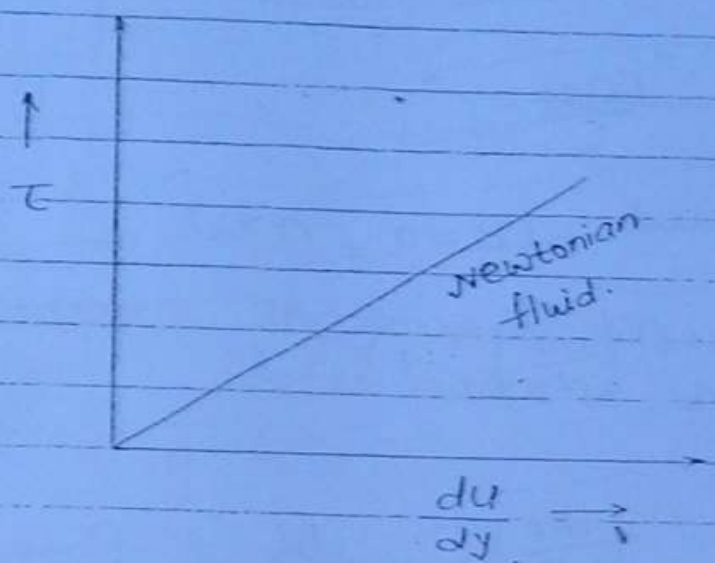
$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

e.g. - Air, water, petrol, diesel, mercury, Kerosin, & some oil.

Fluid which follow newton's law of viscosity are known as newtonian fluid.

Newton's law of viscosity is similar to Hooke's law for solid.



$$\tau = \mu \frac{du}{dy}$$

$$\boxed{y = mx}$$

* Non-Newtonian fluid *

A fluid which does not follow newton's law of viscosity is known as non-newtonian fluid.

Study of non-newtonian fluid is also known as Rheology.

The general relationship b/w shear stress (τ)

& velocity gradient (du/dy) is.

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

Where A , n & B are constant.

→ Case-I →

Dilatant fluid ($B=0$, $n>1$).

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

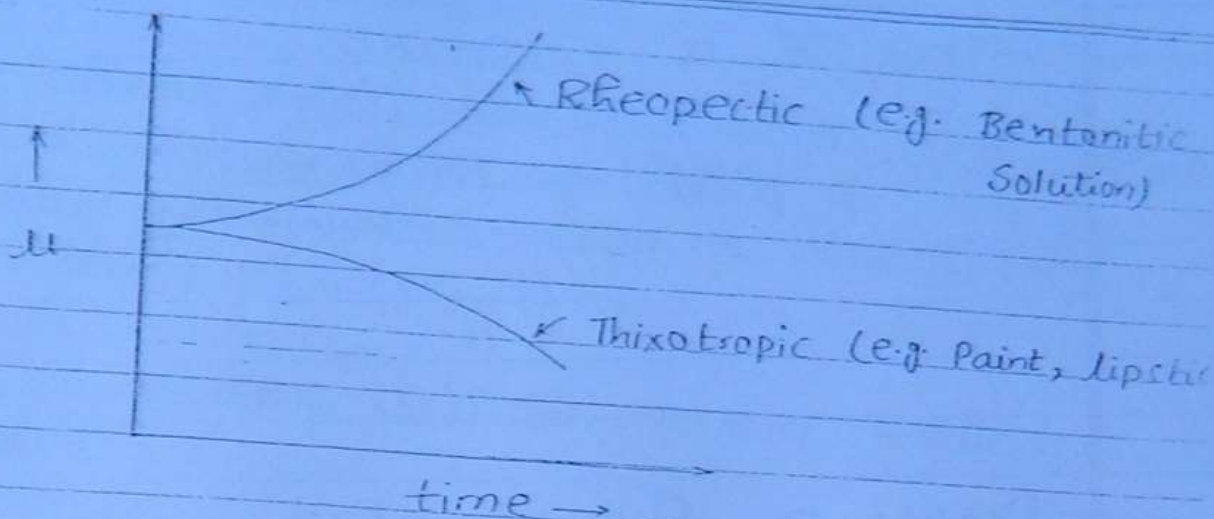
$$\tau = A \left(\frac{du}{dy} \right)^{n-1} \cdot \frac{du}{dy} + 0$$

$$\tau = A \left(\frac{du}{dy} \right)^{n-1} \cdot \frac{du}{dy}$$

$$\tau = \mu_{app} \cdot \frac{du}{dy}$$

A fluid is said to be dilatant fluid if its apparent viscosity increases with rate of deformation. These fluid's is also known as shear thickening fluid.

e.g. - Rice starch, sugar in water.



prob-4 Assertion:- In general viscosity in liquid increases & in gases it decreases with rise in temp.

Reason:- viscosity is caused by cohesion & molecular agitation.

Solⁿ: Assertion is wrong & Reason is right.

prob-5 Match the following.

LIST-I

LIST-II

1. sp. weight

A. L/T^2

2. Density

B. F/L^3

3. Shear stress

C. F/L^2

4. viscosity

D. FT^2/L^2

E. $\frac{FT^2}{L^4}$

Where F is Force (N).

Solⁿ: Specific weight = $\frac{\text{wt of fluid}}{\text{Volume}}$

$$= \frac{N}{m^3} = \frac{F}{l^3}$$

Density $\Rightarrow \rho = \frac{W}{V} = \frac{F/l^3}{L/T^2} = \frac{FT^2}{L^4}$

shear stress $\Rightarrow \tau = \frac{F}{A} = \frac{F}{m^2} = \frac{F}{l^2}$

Viscosity $\Rightarrow \mu = \frac{kg}{m-s} = \frac{N-s}{m^2} = \frac{FT}{l^2}$

1	2	3	4	}
B	E	C	D	

prob-6- The Kinematic viscosity of a gas at 20°C is $1.6 \times 10^{-5} \text{ m}^2/\text{sec}$. then its kinematic viscosity at 70° can be -

- (a) 1.6×10^{-5} (b) 1.2×10^{-5} (c) 1×10^{-5} (d) 2.2×10^{-5}

Solⁿ: For a gas If temp. is increases then molecular disturbance (agitation) increases & resistance to flow also increases. i.e. viscosity increases.

prob-7- The shear stress developed in a lubricating oil of viscosity 0.981 poise filled b/w two parallel plates 1 cm. apart & moving with a velocity of 2 m/s. is -

Solⁿ: let us assume that the viscosity profile is linear.

given, $\mu = 0.981 \text{ poise}$.

but $1 \text{ poise} = 0.1 \frac{\text{kg}}{\text{m-s}}$

$$\therefore 0.981 \text{ poise} = 0.1 \times 0.981 \text{ kg/m-s.}$$

$$= 0.0981 \text{ kg/m-s.}$$

& $V = 2 \text{ m/s.}$

$$y = 1 \text{ cm} = 0.01 \text{ m.}$$

we know that for linear profile.

$$\tau = \frac{\mu V}{y}$$

$$\tau = \frac{0.0981 \times 2}{0.01}$$

$$\boxed{\tau = 19.62 \text{ N/m}^2} \text{ Ans.}$$

prob-8 - The velocity distribution over a flat plate is given by ~~the~~

$$u = \frac{3}{4} y - y^2$$

The shear stress at a location 0.3m above the plate is K times shear stress at a location 0.2 m above the plate then the value of K is -

- (a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

Soln: given, $\tau_{0.3} = K \tau_{0.2}$

we know that,

$$\tau = \mu \frac{du}{dy}$$

$$\tau_{0.3} = K \tau_{0.2}$$

$$\left[\mu \frac{du}{dy} \right]_{0.3} = K \left[\mu \frac{du}{dy} \right]_{0.2} \quad \text{--- (1)}$$

given, $u = \frac{3}{4} y - y^2$

$$\frac{du}{dy} = \frac{3}{4} - 2y$$

$$\left. \frac{du}{dy} \right|_{0.1} = \frac{3}{4} - 2(0.3) = 0.15$$

$$\left. \frac{du}{dy} \right|_{0.2} = \frac{3}{4} - 2(0.2) = 0.35$$

then put the value in eqn. (1) we get

$$0.15 = K \times 0.35$$

$$K = \frac{3}{7} \quad \text{Ans.}$$

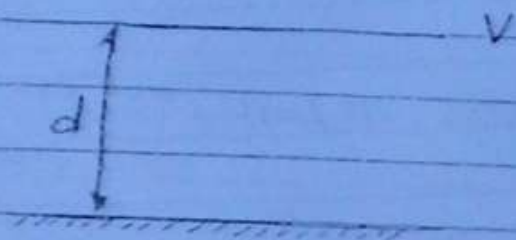
Imp.
prob-9

A fluid obeying the equation-

$$\tau = \tau_0 + K \left(\frac{du}{dy} \right)^{1/2}$$

Is held b/w two parallel plates. 'd' apart if the stress applied to the top plate is $3\tau_0$ then the velocity with which top plate moves is -

Soln:



On the top plate

$$\tau = 3\tau_0 \quad (\text{given})$$

then, $3T_0 = T_0 + K \left(\frac{du}{dy} \right)^{1/2}$

$$2T_0 = K \left(\frac{du}{dy} \right)^{1/2}$$

$$\frac{2T_0}{K} = \left(\frac{du}{dy} \right)^{1/2}$$

$$\frac{4T_0^2}{K^2} = \frac{du}{dy}$$

$$\frac{4T_0^2}{K^2} = \frac{V-0}{d}$$

$$\boxed{V = \frac{4dT_0^2}{K^2}} \quad \text{Ans.}$$

Qb-10 A piston of 60 mm dia. moves inside a cylinder of 60.1 mm dia. determine the % decrease in force necessary to move this piston when the lubricant is heated from 0°C to 120°C.

$$\mu_{0^\circ\text{C}} = 0.0182 \text{ Ns/m}^2$$

$$\mu_{120^\circ\text{C}} = 0.00206 \text{ Ns/m}^2$$

Soln:

$$F = \frac{\mu AV}{y}$$

$$F \propto \mu$$

$$\% \text{ decrease in force} = \frac{F_2 - F_1}{F_1} \times 100$$

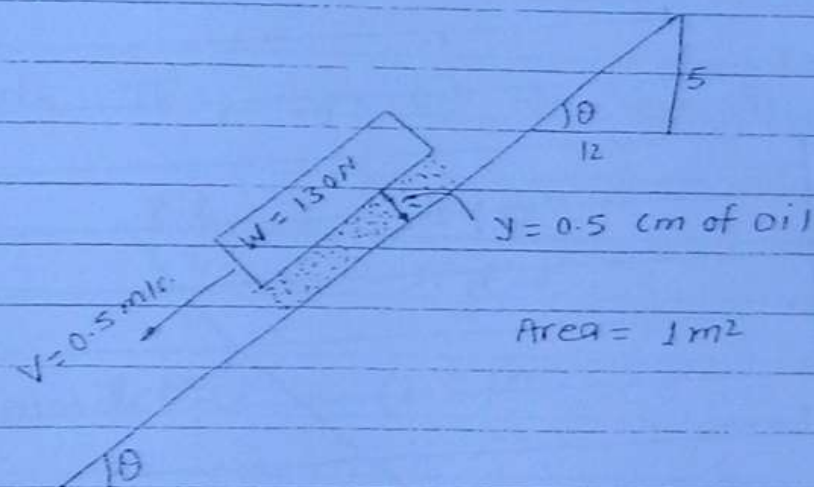
$$= \frac{\mu_2 - \mu_1}{\mu_1} \times 100$$

$$= \frac{0.00206 - 0.0182}{0.0182} \times 100$$

$$= 88.68 \% \text{ Ans.}$$

Imp

Prob-11 Find the viscosity of oil shown in figure.



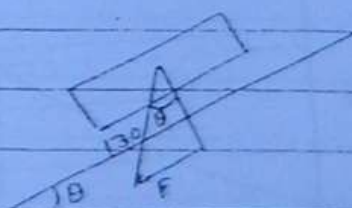
Soln:

$$F = \frac{\mu A V}{y}$$

$$y = 0.5 \times 10^{-2} \text{ m.}$$

$$A = 1 \text{ m}^2$$

$$V = 0.5 \text{ m/s.}$$



$$\sin \theta = \frac{F}{130}$$

$$\mu = \frac{50 \times 0.5 \times 10^{-2}}{1 \times 0.5}$$

$$\frac{5}{13} = \frac{F}{130}$$

$$\boxed{\mu = 0.5 \text{ N-s/m}^2} \text{ Ans.}$$

$$\boxed{F = 50 \text{ N.}}$$

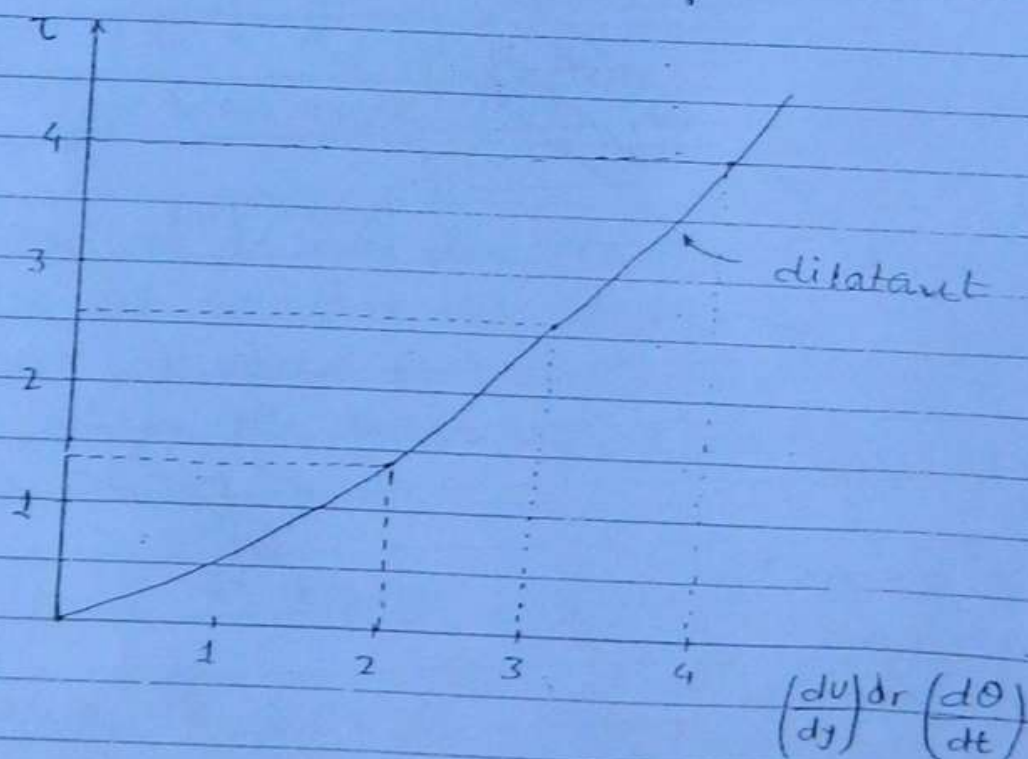
prob-12 In an Experiment the following shear strain & rate of shear strain value are obtained for a fluid.

$d\theta/dt$	0	2	3	4
τ	0	1.4	2.6	4

Then the fluid under consideration is

- (a) newtonian fluid (b) Bingham plastic
 (c) Pseudoplastic (d) Dilatant.

Solⁿ: Method-I.



Method-II -

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

$$\tau = 0 \quad \& \quad \frac{du}{dy} = 0.$$

then $B = 0.$

then, the eqn. become.

$$\tau = A \left(\frac{du}{dy} \right)^n$$

$$1.4 = A (2)^n \quad \text{--- ①}$$

$$2.6 = A (3)^n \quad \text{--- ②}$$

Divide the eqn. ② to eqn. ①.

$$\frac{2.6}{1.4} = \frac{A (3)^n}{A (2)^n}$$

$$1.857 = (1.5)^n$$

$$n = 1.52$$

$$\boxed{n > 1}$$

It is dilatant fluid.

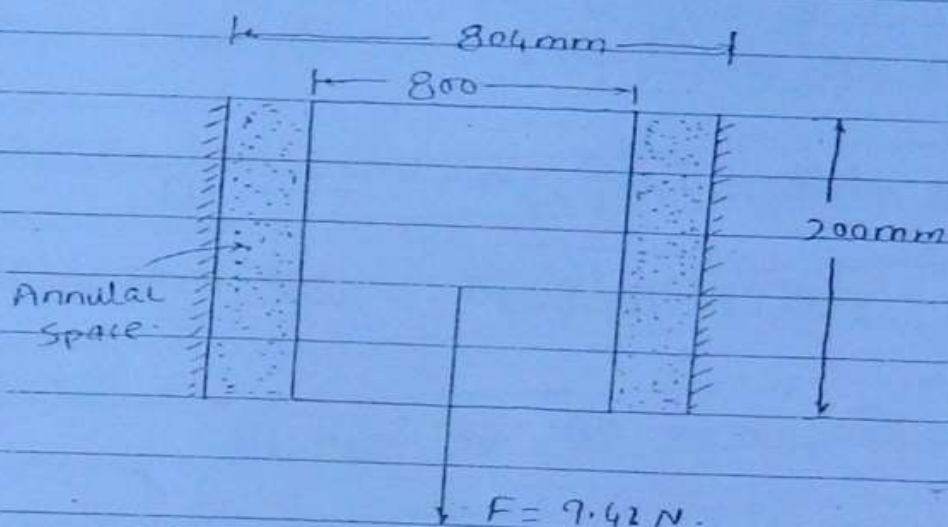
prob-13 The shear stress in a fluid may be expressed as $\tau = \mu \left(\frac{du}{dy} \right)^n$. Then the value of n for newtonian & non-newtonian fluid.

- Ⓐ $n=1 \& n>1$ Ⓑ $n<1 \& n>1$ Ⓒ $n=1 \& n<1$ Ⓓ $n=1 \& n \neq 1$

Prob-14.

A piston of 800 mm dia. & 200 mm long works inside a cylinder of 804 mm dia. If the annular space is filled with a lubricating oil of viscosity 5 centi poise then find the speed of piston in vertically downward direction, take weight of piston action load equal to 9.42 N.

Solⁿ:-



We know that,

$$F = \frac{\mu A V}{y}$$

$$V = \frac{F y}{\mu A}$$

Area of the piston,

$$A = \pi d l = \pi \times (0.8) \times 0.2 = 0.503 \text{ m}^2$$

$$y = 200 \text{ mm} = 2 \times 10^{-3} \text{ m.}$$

$$\mu = 5 \text{ Centi poise} = 5 \times 10^{-2} \times 0.1 \text{ kg/cm-s}$$

$$F = 9.42 \text{ N.}$$

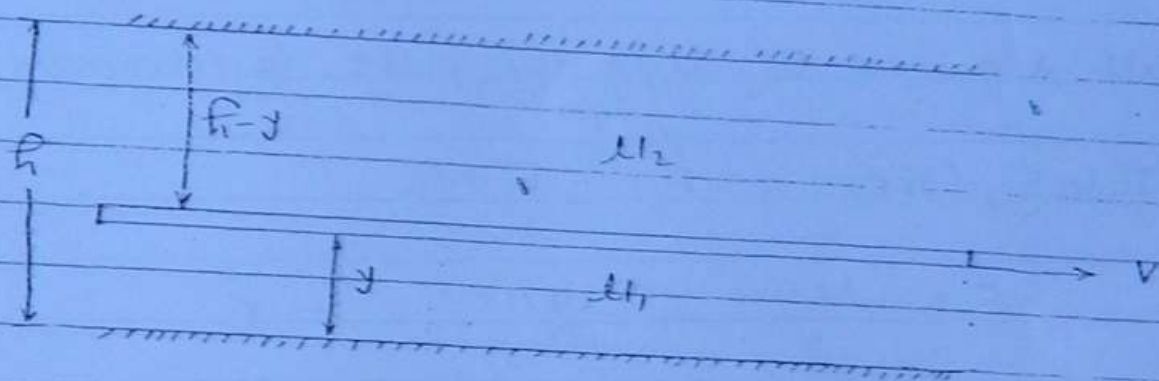
then,

$$V = \frac{9.42 \times 2 \times 10^{-3}}{5 \times 10^{-3} \times 0.503} = 7.5 \text{ m/s Ans.}$$

Through a very narrow gap of height 'h' a thin plate pulled at velocity v of on side of plate there oil of viscosity μ_1 & other side there is oil viscosity μ_2 as shown in figure.

Find the value of y - when -

- (i) Shear force on two side's of the plate is zero
- (ii) Pull required to drag the plate is minimum



Solⁿ:- We know that,

$$F = \frac{\mu A v}{y}$$

$$F_1 = \frac{\mu_1 A v}{y}$$

$$F_2 = \frac{\mu_2 A v}{h-y}$$

(i) $F_1 = F_2$ (given)

$$\frac{\mu_1 A v}{y} = \frac{\mu_2 A v}{h-y}$$

$$\frac{\mu_1}{y} = \frac{\mu_2}{R-y}$$

$$\mu_1(R-y) = \mu_2 y$$

$$\mu_1 R - \mu_1 y = \mu_2 y$$

$$(\mu_1 + \mu_2) y = \mu_1 R$$

$$\boxed{y = \frac{\mu_1 R}{\mu_1 + \mu_2}} \quad \text{Ans.}$$

Pull required to drag the plate is minimum.

$$\text{Total force} = F = F_1 + F_2$$

$$F = \frac{\mu_1 A v}{y} + \frac{\mu_2 A v}{(R-y)}$$

For minimum F ,

$$\frac{dF}{dy} = 0.$$

$$\frac{d}{dy} \left[\frac{\mu_1 A v}{y} + \frac{\mu_2 A v}{R-y} \right] = 0.$$

$$-\frac{\mu_1 A v}{y^2} + \frac{\mu_2 A v}{(R-y)^2} = 0.$$

$$\frac{\mu_1 A v}{y^2} = \frac{\mu_2 A v}{(R-y)^2}$$

$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(R-y)^2}$$

$$\frac{(R-y)^2}{y^2} = \frac{\mu_2}{\mu_1}$$

$$\frac{R-y}{y} = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$\frac{R}{y} - 1 = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$\frac{R}{y} = 1 + \sqrt{\frac{\mu_2}{\mu_1}}$$

$$y = \frac{R}{1 + \sqrt{\frac{\mu_2}{\mu_1}}}$$

$$\boxed{y = \frac{\sqrt{\mu_1} R}{\sqrt{\mu_1} + \sqrt{\mu_2}}}$$

Prob-10: A Circular disc of diameter 'D' is slowly rotated in a liquid of viscosity μ at a small distance R from fixed surface as shown in fig then find the torque necessary to maintain the angular velocity, ω .

$$\frac{\mu_1}{y} = \frac{\mu_2}{R-y}$$

$$\mu_1(R-y) = \mu_2 y$$

$$\mu_1 R - \mu_1 y = \mu_2 y$$

$$(\mu_1 + \mu_2) y = \mu_1 R$$

$$\boxed{y = \frac{\mu_1 R}{\mu_1 + \mu_2}} \quad \text{Ans.}$$

(ii) Pull required to drag the plate is minimum.

$$\text{Total force} = F = F_1 + F_2$$

$$F = \frac{\mu_1 A v}{y} + \frac{\mu_2 A v}{(R-y)}$$

For minimum F ,

$$\frac{dF}{dy} = 0.$$

$$\frac{d}{dy} \left[\frac{\mu_1 A v}{y} + \frac{\mu_2 A v}{R-y} \right] = 0.$$

$$-\frac{\mu_1 A v}{y^2} + \frac{\mu_2 A v}{(R-y)^2} = 0.$$

$$\frac{\mu_1 A v}{y^2} = \frac{\mu_2 A v}{(R-y)^2}$$

$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(h-y)^2}$$

$$\frac{(h-y)^2}{y^2} = \frac{\mu_2}{\mu_1}$$

$$\frac{h-y}{y} = \sqrt{\frac{\mu_2}{\mu_1}}$$

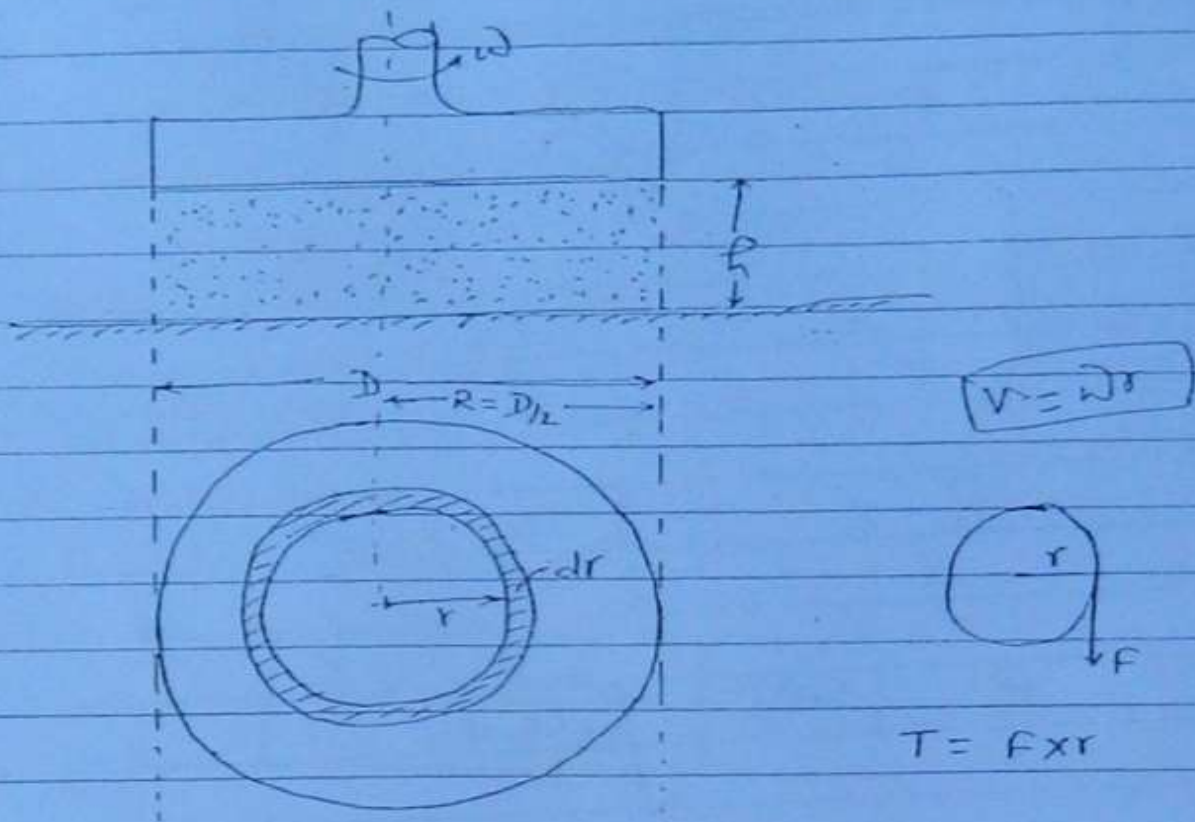
$$\frac{h}{y} - 1 = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$\frac{h}{y} = 1 + \sqrt{\frac{\mu_2}{\mu_1}}$$

$$y = \frac{h}{1 + \sqrt{\frac{\mu_2}{\mu_1}}}$$

$$y = \frac{\sqrt{\mu_1} h}{\sqrt{\mu_1} + \sqrt{\mu_2}}$$

Qb-16- A Circular disc of diameter 'D' is slowly rotated in a liquid of viscosity μ at a small distance h from fixed surface as shown in fig. then find the torque necessary to maintain the angular velocity ω .



As the velocity is vary with radius.
 Taken a small elementary area of dA .
 Then in small elementary area force;

$$dF = \frac{\mu dA v}{h}$$

$$dF = \frac{\mu \cdot 2\pi r \cdot dr \times r \omega}{h}$$

$$dT = dF \times r$$

$$dT = \frac{2\pi \mu \omega r^2 dr}{h} \times r$$

$$dT = \frac{2\pi \mu \omega r^3 dr}{h}$$

Integrate the above eqn. with in limit 0 to $D/2$

$$\int dT = \int_0^{D/2} \frac{2\pi\mu\omega}{R} \cdot r^3 \cdot dr$$

$$T = \frac{2\pi\mu\omega}{R} \left[\frac{r^4}{4} \right]_0^{D/2}$$

$$T = \frac{2\pi\mu\omega}{4R} \left[\left(\frac{D}{2} \right)^4 - 0 \right]$$

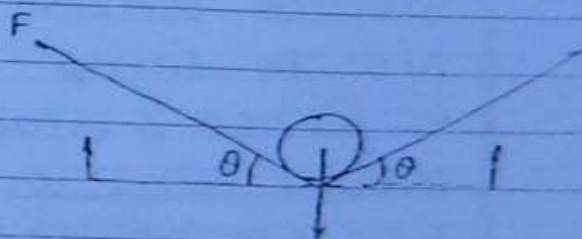
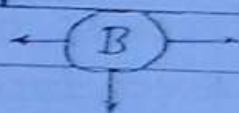
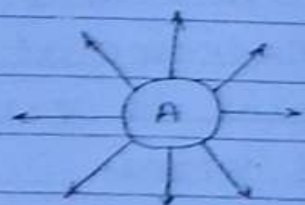
$$T = \frac{\pi\mu\omega}{2R} \cdot \frac{D^4}{16}$$

$$T = \frac{\pi\mu\omega D^4}{32R} \quad \text{Ans.}$$

* Surface - Tension *

Atmospheric
Air

Water



Consider a molecule 'A' which is below the free surface of liquid this molecule is surrounded by various corresponding molecule's &

hence under the influence of these forces it will be in equilibrium. now consider the molecule 'B' which is on the free surface of liquid this molecule is under the influence of net downward force (unbalanced cohesive force). because of these forces there seems to be a layer formed at the surface which can resist small tension load. & this phenomenon is known as surface tension.

It is a line force & it is expressed as force per unit length drawn on the surface & it acts normal to the line but in the plane of surface.

Its unit is N/m & dimensional formula is MT^{-2} .

Surface tension for water air interface at 20°C is 0.0736 N/m .

Surface tension is depends on temp. with rise in temp. cohesive forces decreases & hence surface tension also decreases at critical point surface tension equal to zero because at critical point liquid vapour interface vanishes.

liquid droplet's assume spherical shape due to surface tension. (sphere has minimum surface area for a given volume).

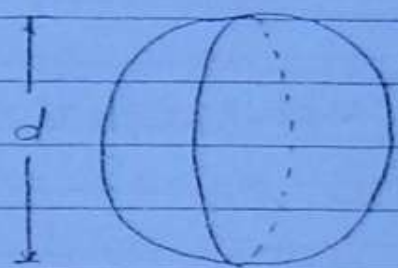
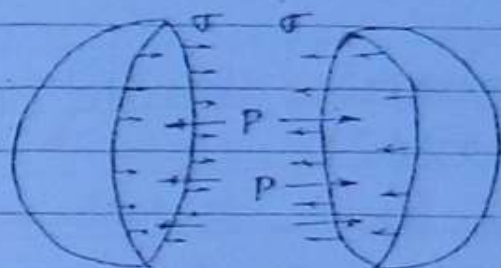


fig. droplet.



For, Equilibrium.

$$F_p = F_s \quad \text{--- (1)}$$

But pressure,

$$P = \frac{F_p}{A}$$

$$F_p = P \times A = P \times \frac{\pi d^2}{4}$$

Surface tension,

$$\sigma = \frac{F_s}{L}$$

$$F_s = \sigma \times L = \sigma \times \text{circumference}$$

$$F_s = \sigma \times \pi d$$

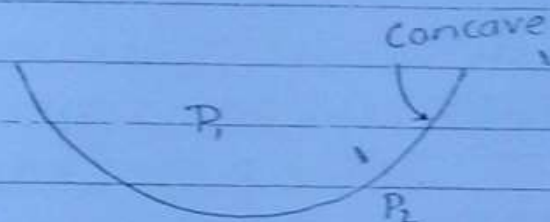
Now from eqn. (1) we get

$$P \times \frac{\pi}{4} \times d^2 = \sigma \times \pi d$$

$$P = \frac{4\sigma}{d}$$

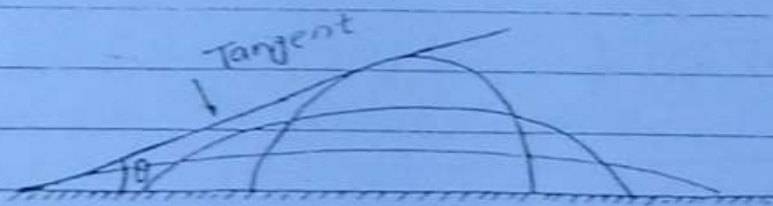
NOTE - (i) Pressure inside a Soap bubble equal to $\frac{8\sigma}{d}$.

(ii) Due to surface tension pressure on Concave side is more than press. on convex side.

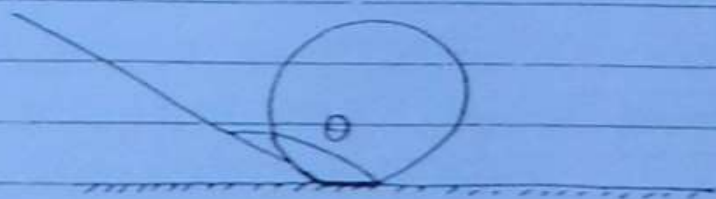


$$\text{i.e. } P_1 > P_2$$

NOTE -



- Adhesion is more
- Wetting liquid
- $\theta < 90^\circ$ (acute)

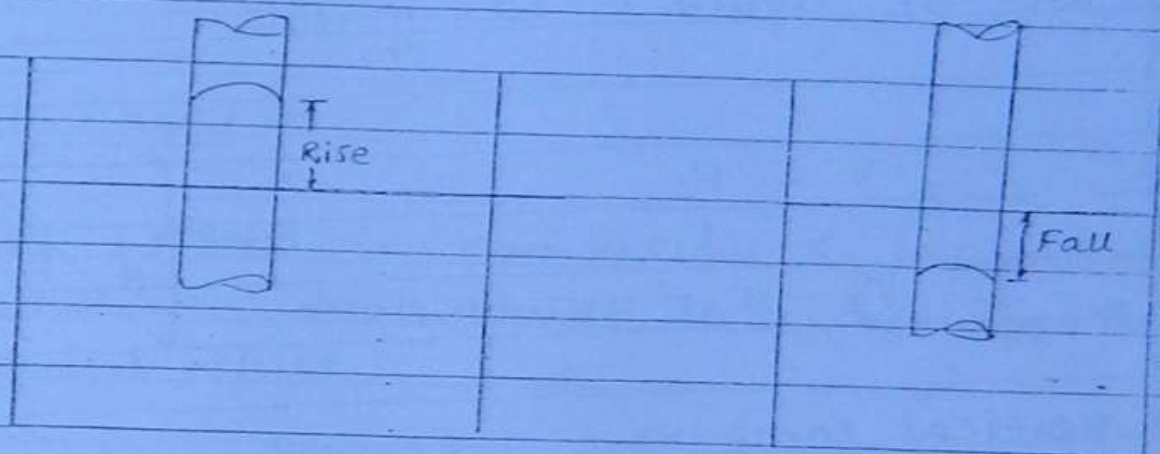


- Cohesion is more
- Non-wetting liquid
- $\theta > 90^\circ$ (obtuse)

Capillarity :-

The rise or fall of a liquid when a small tube is immersed in it is known as capillarity.

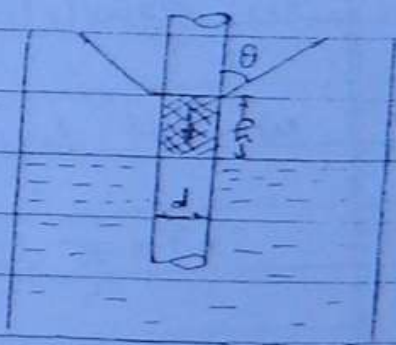
The rise is due to adhesion (e.g. water) & fall is due to cohesion (e.g. mercury) & hence capillarity is due to both cohesion & adhesion.



Expression for capillary rise or fall :-

Consider a glass tube of small dia. d , opened at both ends & is inserted in liquid.

let h = height of liquid in the tube.



Specific weight of fluid = $\frac{\text{Wt. of fluid}}{\text{Volume}}$

$$\text{Wt. of fluid} = W \times \text{Vol.} = W \times \frac{\pi d^2 h}{4}$$

Under a equilibrium state, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

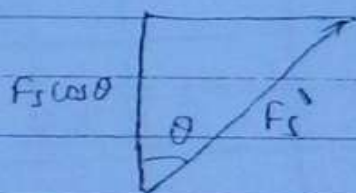
let σ = Surface tension of liquid.

θ = Angle of contact b/w liquid & glass tube.

$$\text{Wt. of liquid} = \omega \times \frac{\pi}{4} d^2 h$$

$$\sigma = \frac{F_s}{L}$$

$$F_s = \sigma \times L = \sigma \times \pi d$$



$$\begin{aligned} \text{vertical component} &= F_s \cos \theta \\ &= \pi d \times \sigma \times \cos \theta \end{aligned}$$

In equilibrium,

wt. of liquid = vertical component of surface tension

$$\omega \times \frac{\pi}{4} \times d^2 \times h = \sigma \cdot \pi d \cdot \cos \theta$$

$$h = \frac{4\sigma \cos \theta}{\omega d}$$

where σ = surface tension

ω = sp. weight of liquid.

d = dia. of glass tube.

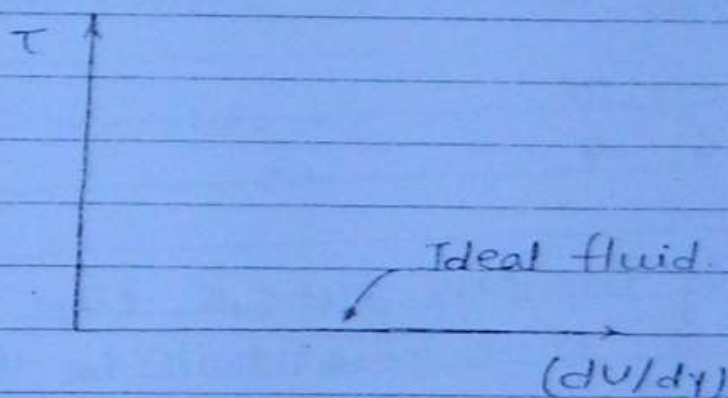
NOTE - The angle of contact θ for ordinary water in contact with glass tube is around 21° .

The angle of contact of mercury with glass tube is 132° .

Angle of contact of pure water in contact with clean glass tube is 0° .

:- Ideal Fluid :-

A fluid which is non-viscous & incompressible is known as ideal fluid. It is an hypothetical (Imaginary) fluid.



:- Real fluid :-

A fluid which possesses viscosity is known as Real fluid. All the fluid in actual practice are real fluid.

prob-17. Match the following -

LIST-I

LIST-II

- | | |
|-----------------------|--|
| A) Ideal fluid | 1. Viscosity does not change with rate of deformation. |
| B) Newtonian fluid | 2. Fluid of zero viscosity |
| C) $\frac{\mu}{\rho}$ | 3. Dynamic viscosity. |
| D) mercury in glass. | 4. Capillary Depression |
| | 5. Capillary Rise. |
| | 6. Kinematic viscosity. |

Soln:	A	B	C	D
	2	1	6	4

prob-18- If the dia. of capillary tube is 1mm then capillary rise is 3cm. What will be the capillary rise when the dia. is change it to 0.2mm.

Soln:
$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

$$h \propto \frac{1}{d}$$

$$\frac{h_2}{h_1} = \frac{d_1}{d_2}$$

$$\frac{h_2}{3 \text{ cm}} = \frac{1 \text{ mm}}{0.2 \text{ mm}}$$

$$h_2 = \frac{3 \text{ cm}}{0.2}$$

$$h_2 = 15 \text{ cm} \quad \text{Ans.}$$

b-19- Find the capillary rise in pure water when a clean glass tube of 1mm dia is immersed in it.

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

For pure water $\theta = 0^\circ \Rightarrow \cos 0^\circ = 1$.

$$h = \frac{4\sigma \times 1}{\rho g d}$$

$$h = \frac{4 \times 0.0736}{9810 \times 10^{-3}}$$

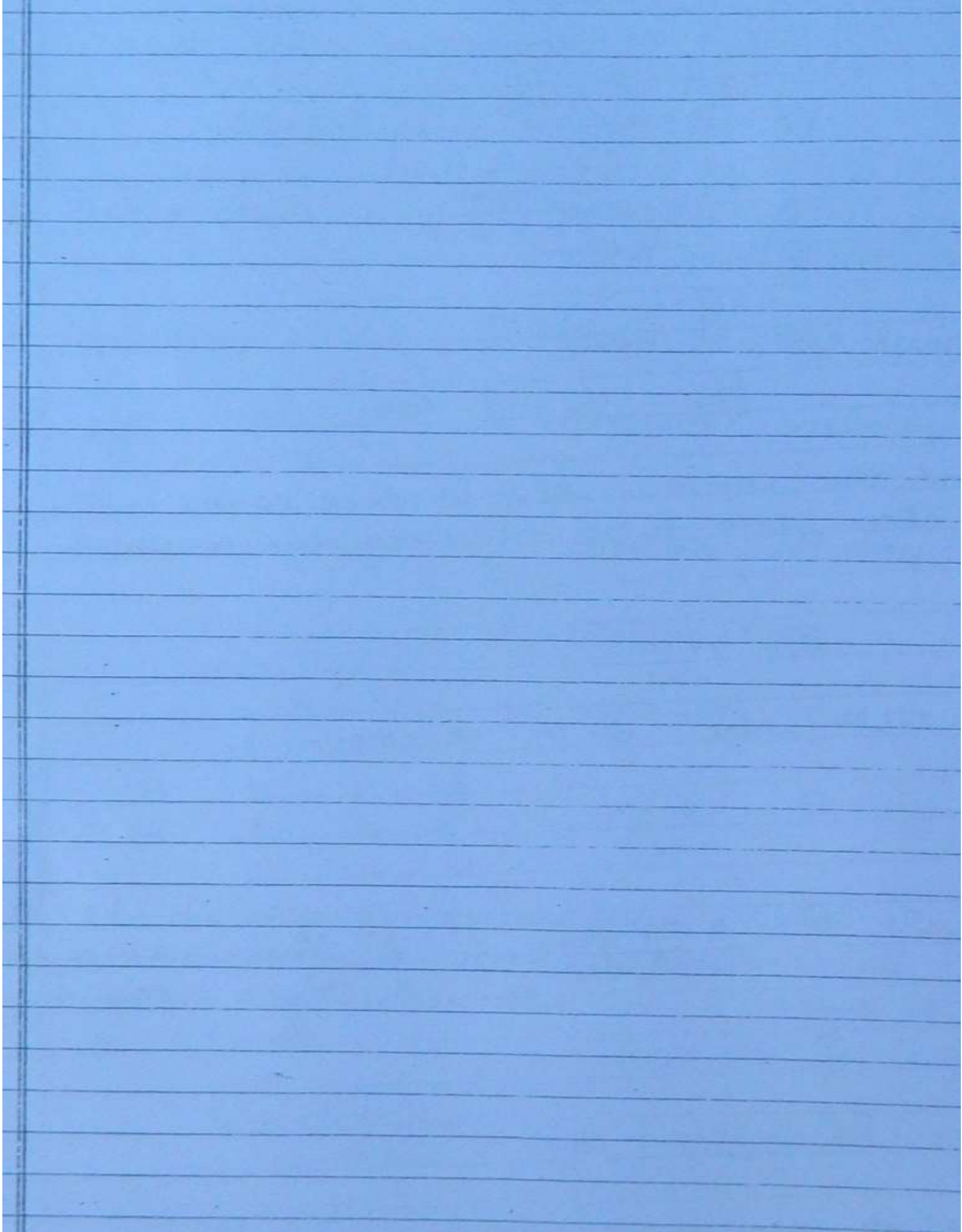
$$\left\{ \begin{array}{l} \because \sigma_{\text{water}} = 0.0736 \text{ N/m} \\ \rho_{\text{water}} = 9810 \text{ N/m}^3 \end{array} \right.$$

$$h = 0.03 \text{ m.}$$

$$h = 3 \text{ cm.} \quad \text{Ans.}$$

E- Viscosity of water at 20°C is 1 centi poise (10^{-3} kg/m-s).
Viscosity of mercury at 20°C is 0.0016

$$1 \text{ Centi poise} = 10^{-2} \text{ poise} = 10^{-2} \times 0.1 \text{ kg/m-s} = 10^{-3} \text{ kg/m-s.}$$



CHAPTER - II

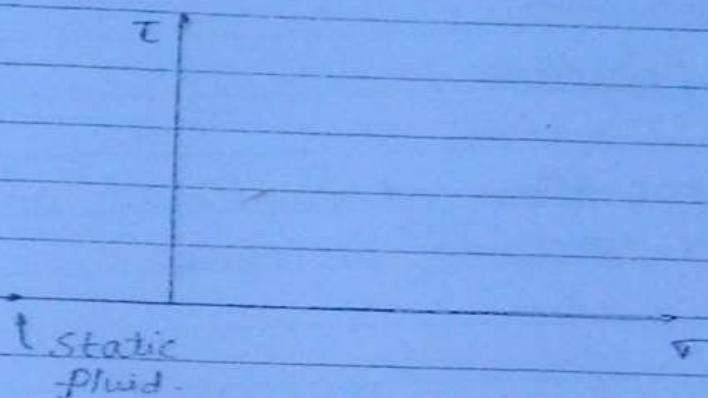
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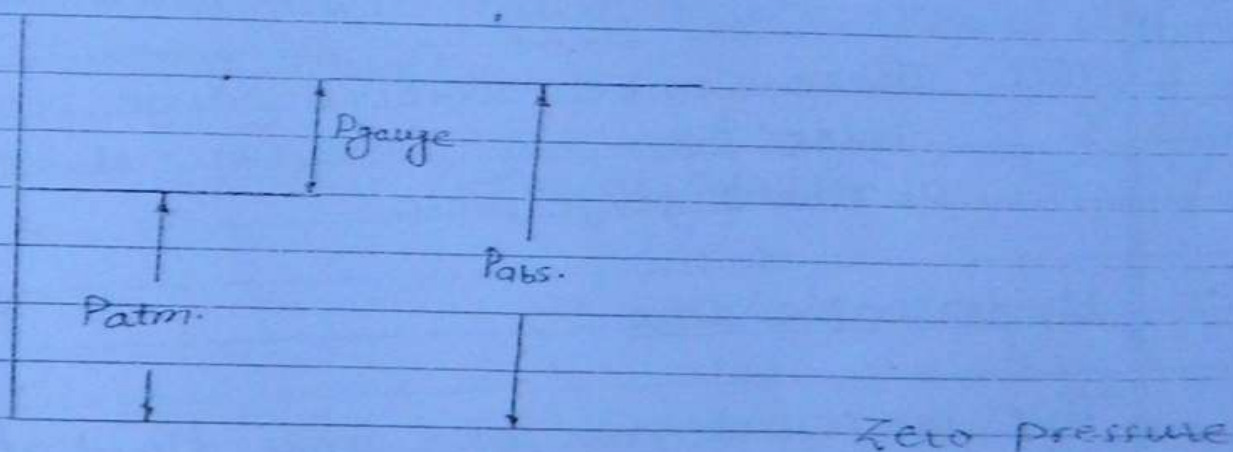
FLUID PRESSURE MEASUREMENT (MANOMETRY).

Fluid pressure is normal force per unit area & it is compressive in nature. For a static fluid as there is no shear stress Mohr circle is a point



Gauge pressure :-

The pressure measured relative to atmospheric pressure is known as gauge pressure.



$$P_{abs.} = P_{atm.} + P_{gauge}$$

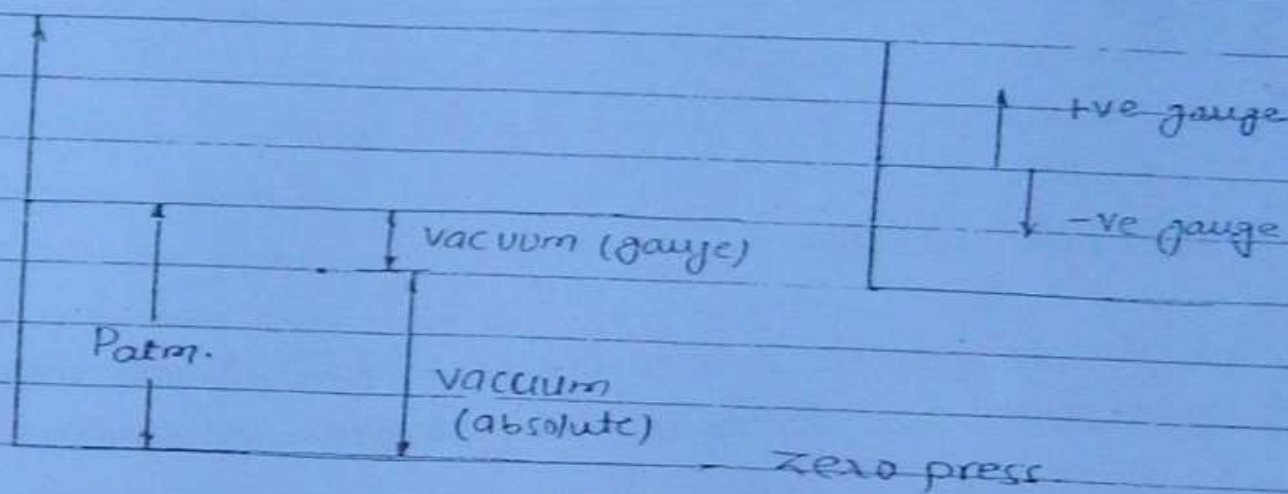
NOTE - For finding out absolute press. local atmospheric press. is taken into account.

Absolute pressure ($P_{abs.}$) :-

Pressure measured with respect to zero pressure is known as absolute pressure.

Vacuum pressure :-

Press. less than atmospheric pressure is known as vacuum pressure.



NOTE - There can be positive gauge press. negative gauge press. but there can not be negative absolute gauge press.

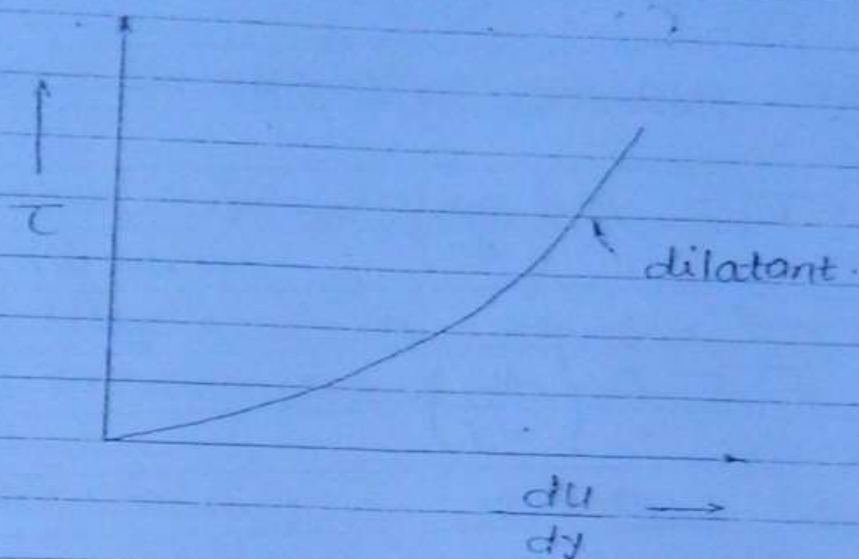
Hydrostatic law :-

The press. at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate of increase of press. in a vertically downward direction must be equal to the specific weight of the fluid at that point.

at $h = 0$ (free surface)

$$[P = P_{atm.}]$$



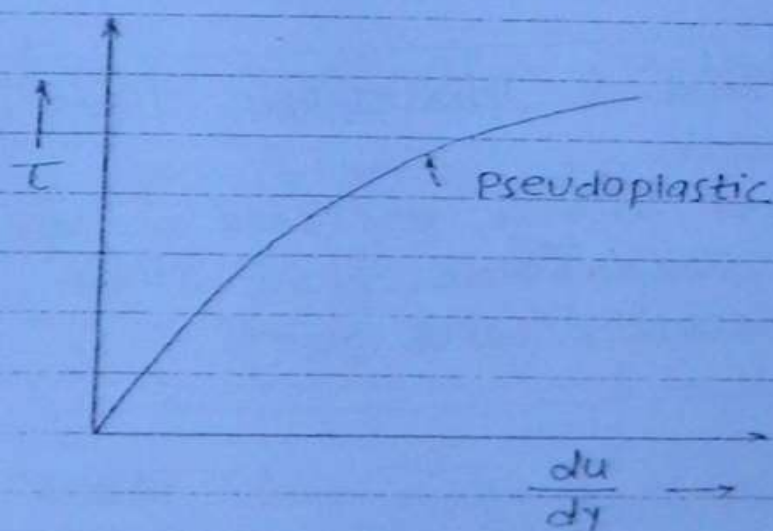


* Case - II *

Pseudoplastic fluid ($B=0$, $n < 1$).

A fluid is said to be a pseudoplastic fluid if the apparent viscosity decreases with rate of deformation these fluid is also known as shear thinning fluid's.

e.g. - Blood, milk, Colloidal solution.



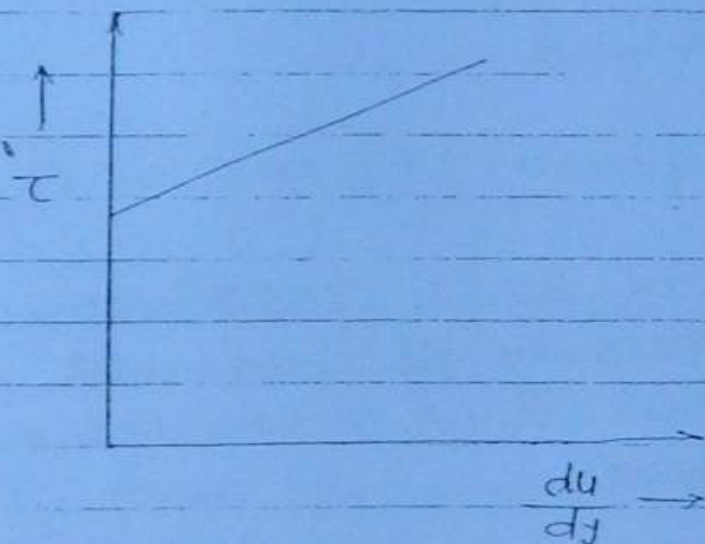
* Case - III *

Bingham plastic ($B \neq 0, n=1$).

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

$$\tau = A \left(\frac{du}{dy} \right) + B$$

$$y = mx + c$$



NOTE - This figure is known as Stress-strain curve for the fluid.

e.g. - Toothpaste, some gel also.

- Rheopectic fluid & Thixotropic fluid :-

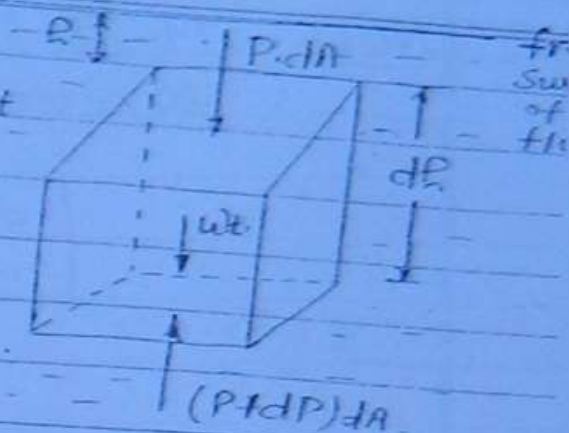
Rheopectic fluid shows increase in viscosity with time. & Thixotropic fluid shows decrease in viscosity with time.

Consider a small fluid element

let dA = Cross-sectional area of element.

dh = Height of fluid element.

P = pressure on upper face.



The force acting on the fluid element are -

fig. forces on fluid elem

1. Pressure force on upper ~~sur~~ face = $P \cdot dA$ & acting perpendicular to upper face in the downward direction.

2. Pressure force on lower face = $(P + dP) dA$, acting perpendicular to lower face, vertically upward direction.

weight of fluid element = weight density $\times$ Volume
 $= w \times dA \times dh$

For equilibrium of fluid element,

$$P \cdot dA + w \cdot dA \cdot dh = (P + dP) dA$$

$$P + w \cdot dh = P + dP$$

$$\boxed{\frac{dP}{dh} = w}$$

Hydrostatic law give's variation of press. in vertical direction.

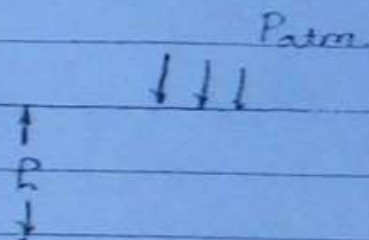
NOTE - If h is taken in vertically upward direction then.

$$\boxed{\frac{dP}{dh} = -w}$$

Pressure at any depth ' h ' :

at $h=0$ (free surface)

$$P = P_{atm.}$$



$$\frac{dP}{dh} = w$$

$$dP = w dh$$

Integrate the above eqn.,

$$P = w \cdot h + C \quad \text{--- ①}$$

$$P_{atm.} = w \cdot (0) + C$$

$$C = P_{atm.}$$

Now from eqn. ① we get.

$$P = w \cdot h + P_{atm.}$$

For gauge pressure, $P_{atm} = 0$.

$$P = w \cdot h + 0$$

$$P = w \cdot h \quad \left\{ \because w = \rho \cdot g \right\}$$

$$\boxed{P = \rho g h}$$

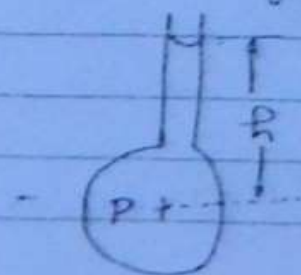
Where P is the press. above atmospheric pressure & h is the height of the point from free surface.

NOTE - Pressure is sometimes expressed in height of fluid column because ρ & g are almost constant & hence pressure depends on ' h '.

Pressure Measuring device:-

1. Piezometer:-

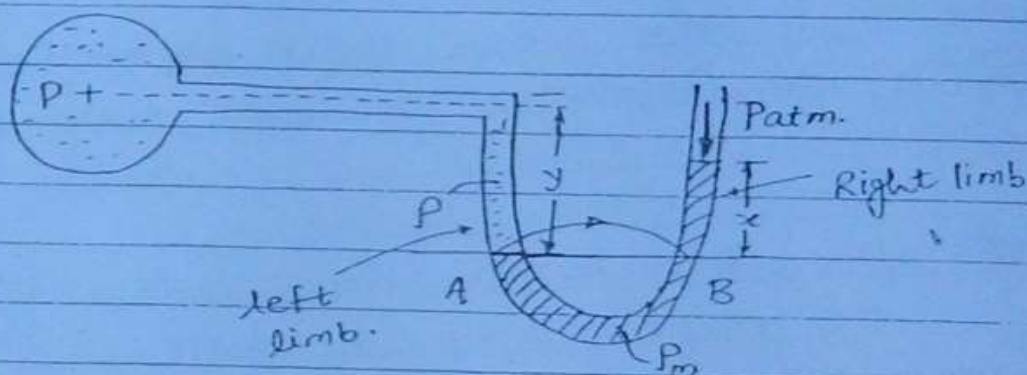
Piezometer is a device which is open at both the ends, one end is connected to the point where the pressure is to be found & other end is open to atmosphere. It is used for measuring moderate liquid pressure.



$$\begin{aligned} P_{atm} + \rho g h &= P \\ 0 + \rho g h &= P \\ \boxed{P = \rho g h} \end{aligned} \quad \left\{ \begin{array}{l} \because \text{For gauge} \\ \text{Pressure,} \\ P_{atm} = 0 \end{array} \right\}$$

2. Manometer:-

Manometers are used for finding out pressure, generally mercury is used as a manometric fluid because of its low vapour pressure & high density.
($\rho = 13.6 \times 10^3 \text{ kg/m}^3$).



For finding pressure P there are two techniques

(i) Jumping of fluid technique

$$P + \rho g y - \rho_m g x = P_{atm.}$$

$$P + \rho g y - \rho_m g x = 0 \quad [\text{for gauge } P_{atm} = 0]$$

$$\boxed{P = \rho_m g x - \rho g y}$$

(ii) Datum technique:-

$$P_A = P_B$$

$$\text{But } P_A = P + \rho g y$$

$$\therefore P_B = 0 + \rho_m g x$$

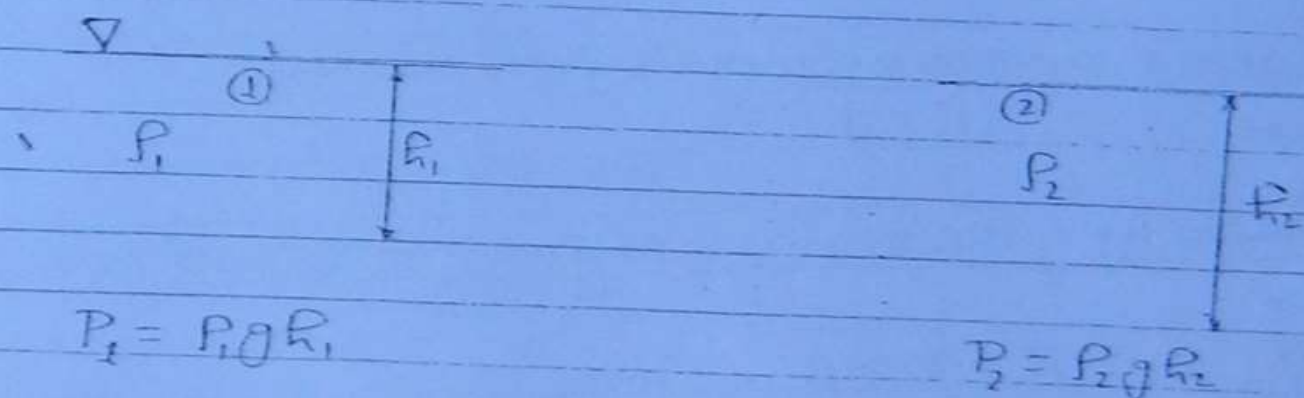
$$[\text{At gauge press. } P_{atm} = 0]$$

$$P + \rho g y = \rho_m g x.$$

$$P = \rho_m g x - \rho g y$$

Simple manometer are used for finding out press. at a point. whereas differential manometer's are used for finding out pressure difference b/w two point.

Conversion of one fluid column into the other column. :-



$$P_1 = \rho_1 g h_1$$

$$P_2 = \rho_2 g h_2$$

Since Both Column pressure is same.

$$P_1 = P_2$$

$$\rho_1 \cdot g \cdot h_1 = \rho_2 \cdot g \cdot h_2$$

$$\rho_1 h_1 = \rho_2 h_2$$

$$\frac{\rho_1 h_1}{\rho_{H_2O}} = \frac{\rho_2 h_2}{\rho_{H_2O}}$$

$$S_1 h_1 = S_2 h_2$$

Qb-20 Pressure's have been observed at 4 different point is different unit as follow's -

- (a) 150 kpa. (b) 1800 mili bar (c) 20m of H_2O
(d) 1240 mm of Hg.

Arrange the point's in descending order of magnitude of pressure.

Solⁿ: $150 \text{ kpa} = 150 \times 10^3 \text{ N/m}^2$.

$$1800 \text{ mili bar} = 1800 \times 10^{-3} \times 10^5 \text{ N/m}^2 = 180 \times 10^3 \text{ N/m}^2$$

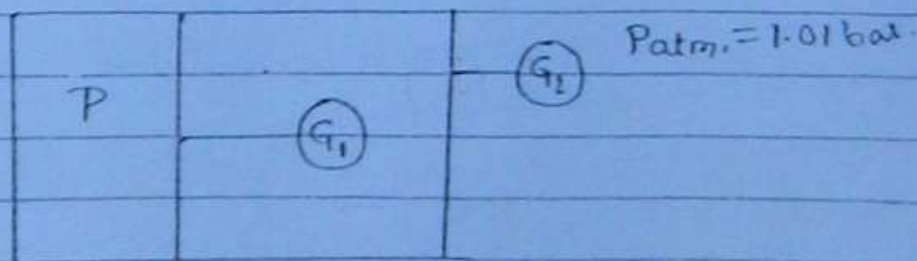
$$20 \text{ m of } H_2O = 1000 \times 9.81 \times 20 = 196.2 \times 10^3 \text{ N/m}^2$$

($P \times g \times R$)

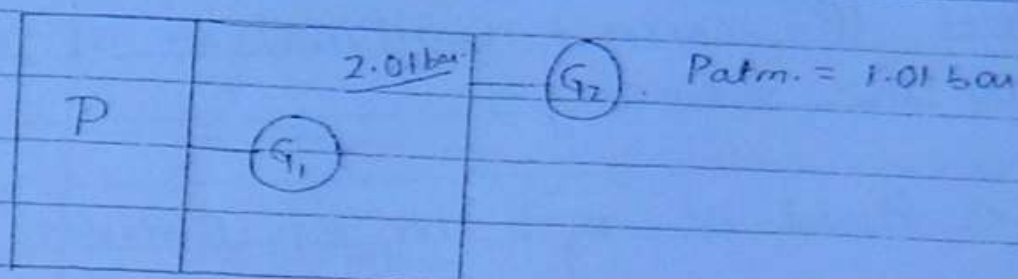
$$1240 \text{ mm of Hg} = 13.6 \times 10^3 \times 9.81 \times 1240 \times 10^{-3} = 165.4 \times 10^3 \text{ N/m}^2$$

~~Q~~ $C > B > D > A$ Ans.

Qb-21 The press. gauge G_1 & G_2 installed in a system show's press. on $P_{G_1} = 5 \text{ bar}$, $P_{G_2} = 1 \text{ bar}$. Then find the unknown press. P in absolute scale.



Soln:



$$P_{abs.} = P_{gauge} + P_{atm.}$$

$$P_{abs.} = P_{G_2} + P_{atm.}$$

$$= 1 + 1.01$$

$$P_{abs.} = 2.01 \text{ bar.}$$

Q. $P = P_{G_1} + P_{atm.}$ ($P_{atm.} = \text{local atm. press}$
 $= 2.01 \text{ bar.}$)

$$P = 5 + 2.01$$

$$P = 7.01 \text{ bar.} \quad \text{Ans.}$$

Prob-22

An open tank contains water to a depth of 1m. & oil over it to the depth of 1m. the density of oil is 800 kg/m^3 then find the press. at the interface of two fluid layers.

Soln:

oil	$\rho = 800 \text{ kg/m}^3$
H ₂ O	$\rho = 1000 \text{ kg/m}^3$

$$P = 0 + \rho_{oil} \times g \times h_{oil} = 800 \times 9.81 \times 1 = 7.848 \text{ N/m}^2 \text{ Ans.}$$

Qb-23- Select the correct combination of statement from the following-

For a fluid at rest in equilibrium.

- (a) Pressure must be same over any horizontal plane.
- (b) Density must be same over any horizontal plane.
- (c) Shear stress must have same non-zero value over any horizontal plane.
- (d) ~~a~~ $\frac{dP}{dh} = w$

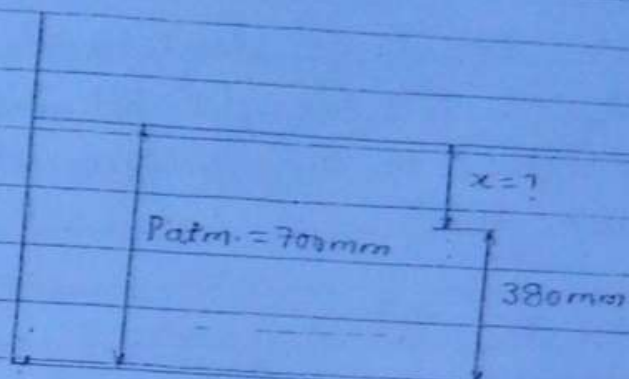
- (1) a, b, c (2) b, c, d (3) ~~a, b, d~~
- (4) a, c, d

mp.

Qb-24- The standard atm. press. is 762 mm of Hg at a specific location local atm. press. is 700 mm of Hg at this location what does an absolute press. of 380 mm of Hg correspond to-

- (a) 320 mm of Hg vacuum.
- (b) 382 mm of Hg vacuum.
- (c) 62 mm of Hg vacuum.
- (d) 60 mm of Hg vacuum.

soln:



Zero press.

$$x = 700 - 380 = 320 \text{ mm of Hg vacuum. Ans.}$$

prob-25- The specific weight of ocean water changes according to the equation $w = w_0 + C\sqrt{h}$ where w_0 is sp. wt. of water at free surface then find the press. as a function of depth.

(a) $w_0 h + \frac{2}{5} C h^{5/2}$

(b) $w_0 h + \frac{5}{2} C h^{2/5}$

(c) $w_0 h + \frac{2}{3} C h^{3/2}$

(d) $w_0 h + \frac{3}{2} C h^{2/3}$

soln: we know that Hydrostatic law.

$$\frac{dP}{dh} = w$$

$$\frac{dP}{dh} = w_0 + C\sqrt{h}$$

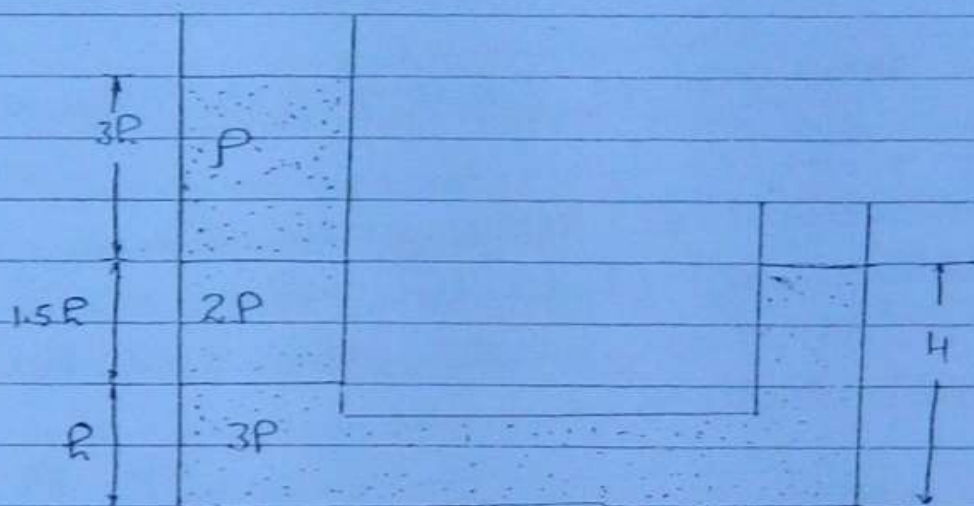
$$dP = (w_0 + C\sqrt{h}) dh$$

$$P = w_0 h + C \cdot \frac{2}{3} h^{3/2}$$

Qb-26 Three immiscible liquid of density P , $2P$ & $3P$ are kept in a jar the height of liquid in jar & piezometer fitted to the bottom of jar is given then find H/R .

- (a) 9 (b) 6 (c) 3 (d) 12

Soln;



For the gauge press. the atm. press. is taken as zero.

$$0 + P \cdot g \cdot 3R + 2P \cdot g \cdot (1.5)R - 3P \cdot g \cdot (H - R) = 0$$

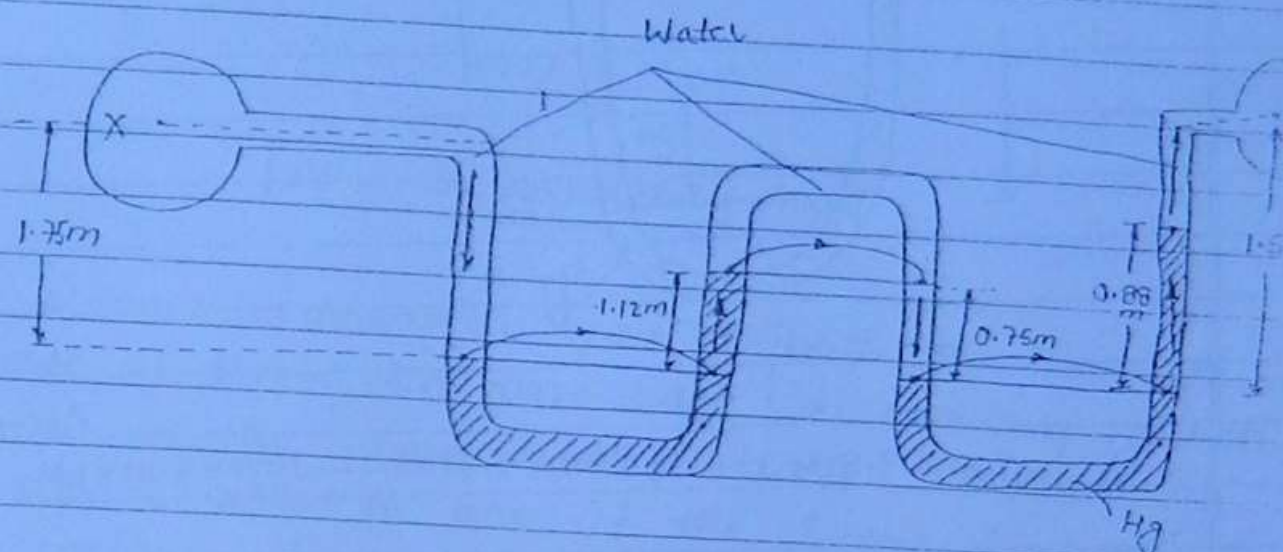
$$3PgR + 3PgR - 3PgH + 3PgR = 0$$

$$9PgR = 3PgH$$

$$\frac{H}{R} = \frac{9}{3} = 3$$

$$\boxed{\frac{H}{R} = 3} \text{ Ans.}$$

prob-27- Two - U-tube manometer connected in a series as shown in fig. then find the press. difference b/w x & y in kPa.



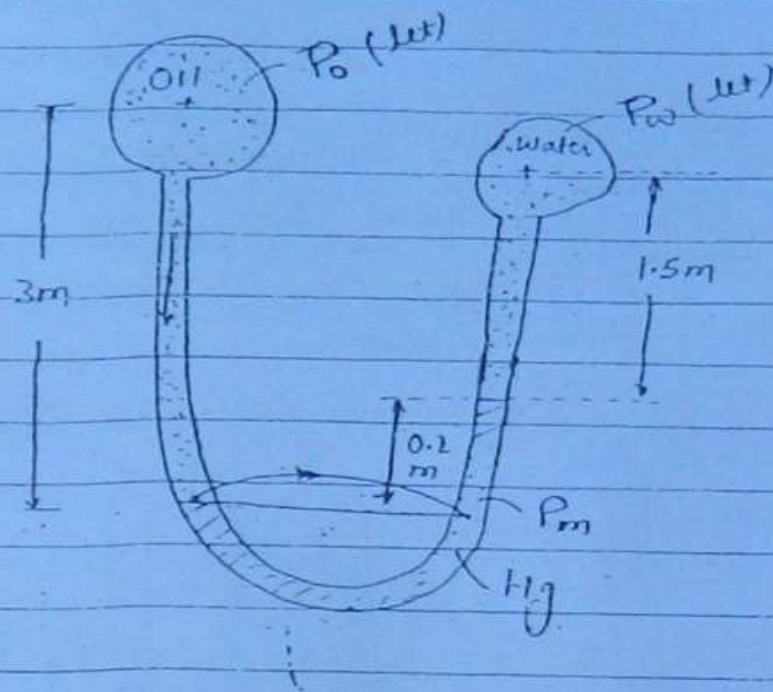
$$P_x + 1000 \times 9.81 \times 1.75 - 13.6 \times 10^3 \times 9.81 \times 1.12 + 1000 \times 9.81 \times 0.75 - 13.6 \times 10^3 \times 9.81 \times 0.88 - 1000 \times 9.81 \times (1.5 - 0.88) = P_y$$

$$P_x - 248.8 \times 10^3 = P_y$$

$$P_x - P_y = 248.8 \times 10^3 \text{ N/m}^2$$

$$P_x - P_y = 248.8 \text{ kN/m}^2 \text{ Ans.}$$

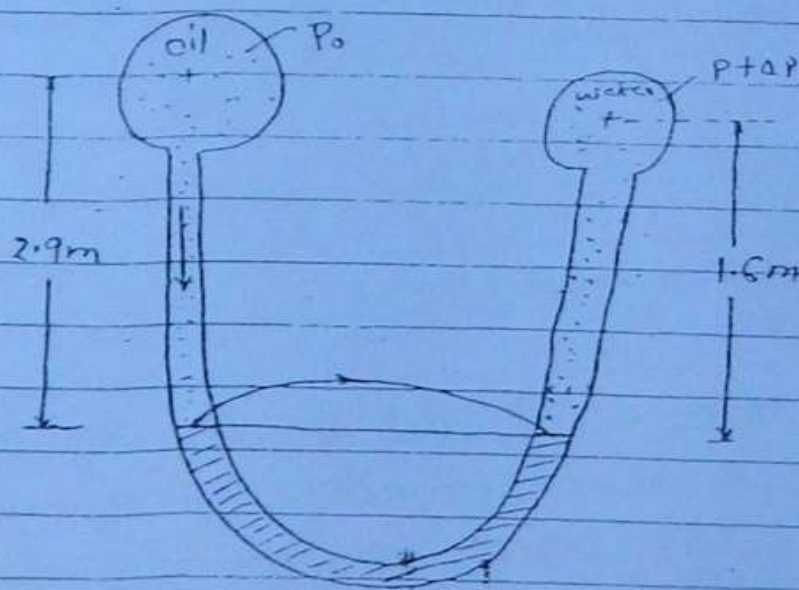
prob-28- Two pipe line's one with oil in density 900 kg/m^3 & other with water are connected to a manometer as shown in figure. By what amount pressure in water pipe should be increased without changing oil pressure, ^{show} that mercury level in both limbs of manometer become equal. Take density of mercury as 13550 kg/m^3 .



$$P_o + 900 \times 9.81 \times 3 - 13550 \times 9.81 \times 0.2 - 1000 \times 9.81 \times 1.5 = P_w$$

$$P_o - 14813.1 = P_w$$

let P_w increase to $P_w + \Delta P$.



$$P_o + 900 \times 9.81 \times 2.9 - 1000 \times 9.81 \times 1.6 = P_w + \Delta P$$

$$P_o + 9908.1 = P_w + \Delta P$$

$$P_0 + 9908.1 = P_0 - 14813.1 + \Delta P$$

$$\Delta P = 9908.1 + 14813.1$$

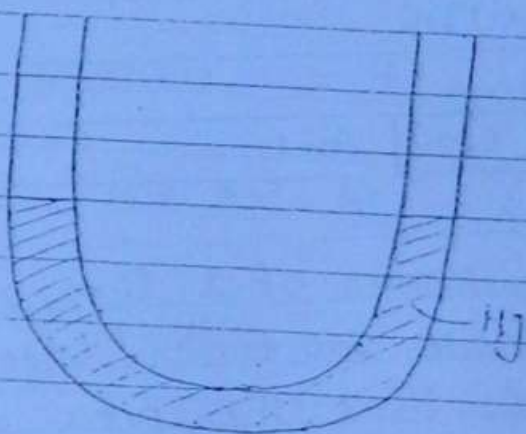
$$\Delta P = 24721.2 \text{ N/m}^2$$

$$\Delta P = 24.72 \text{ kN/m}^2 \text{ Ans.}$$

Imp.

Prob-29 A U-tube manometer open at both ends & made of 8mm dia. glass tube has mercury in both limbs to the same level initially. If 19 cm^3 of water is added to one of the limbs find the dia. in mercury level's in both limbs under equilibrium.

Soln:



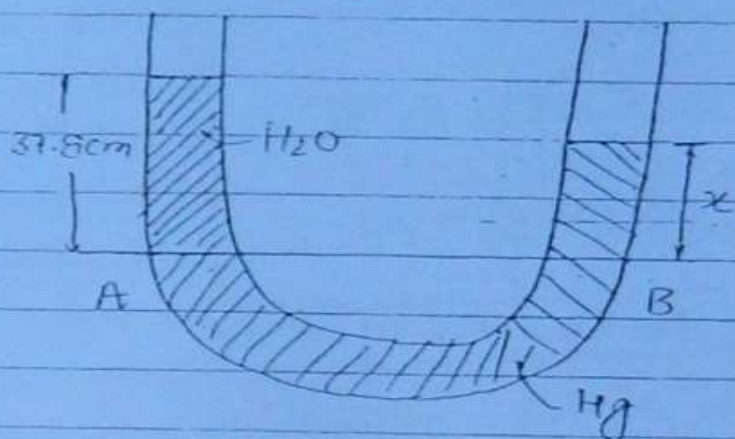
$$V = AL$$

$$V = \frac{\pi d^2}{4} \times L$$

$$19 = \frac{\pi}{4} \times (0.8)^2 \times L$$

$$L = 37.8 \text{ cm.}$$

When water is added.



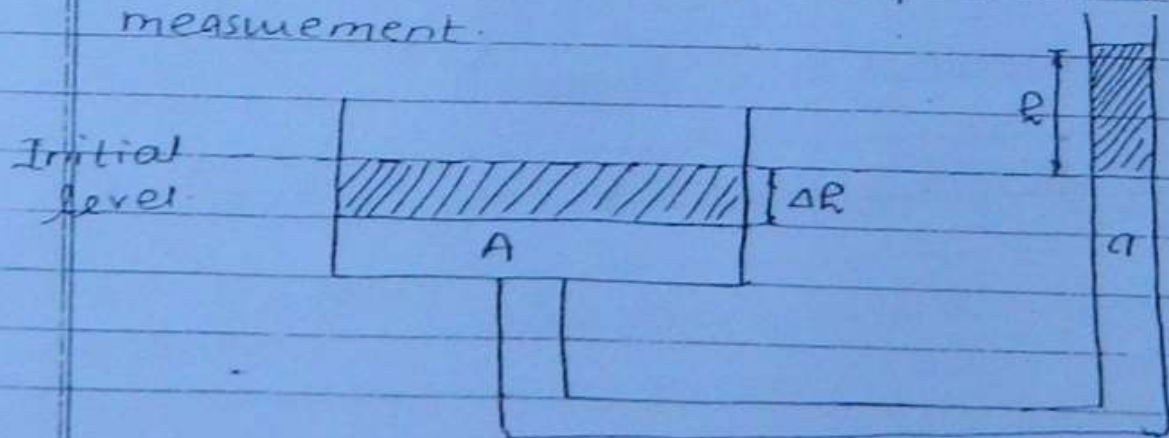
$$P_A = P_B$$

$$h_1 S_1 = h_2 S_2$$

$$37.8 \times 1 = x \times 13.6$$

$$x = 2.78 \text{ cm.} \quad \text{Ans.}$$

2-30 The cross-sectional area of 1 limb of a U-tube manometer is made 500 times larger than the other limb so that the press. difference b/w two limbs can be determined by measuring 'R' on one limb of the manometer then find the % error with respect to this press. measurement.



Page 65

This manometer is known a micro manometer & is used for measuring press. accurately.

Volume leaving from left limb = Volume entering to the right limb.

$$A \cdot \Delta h = a h$$

$$\frac{\Delta h}{h} = \frac{a}{A} = \frac{a}{500a} = \frac{1}{500}$$

$$\frac{\Delta h}{h} = 1/500$$

Actual pressure = $h + \Delta h$.

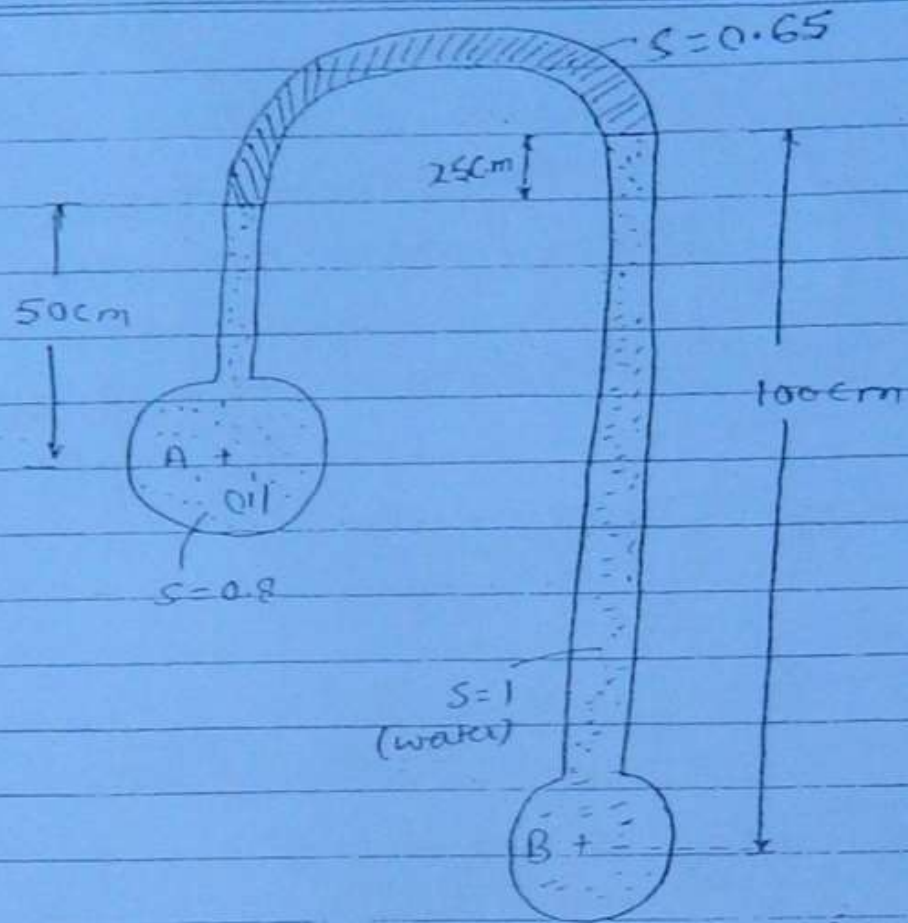
$$\text{Error} = h + \Delta h - h = \Delta h$$

$$\% \text{ Error} = \frac{\Delta h}{h} \times 100$$

$$\% \text{ Error} = \frac{1}{500} \times 100$$

$$\boxed{\% \text{ Error} = 0.2} \text{ Ans.}$$

Find the press. difference b/w point B & point A as shown in figure in cm of water.



Oil is converted in terms of water.

$$h_{oil} \times S_{oil} = h_w \times S_w$$

$$50 \times 0.8 = h_w \times 1$$

$$h_w = 50 \times 0.8$$

mercury is converted in terms of water.

$$h_m S_m = h_w S_w$$

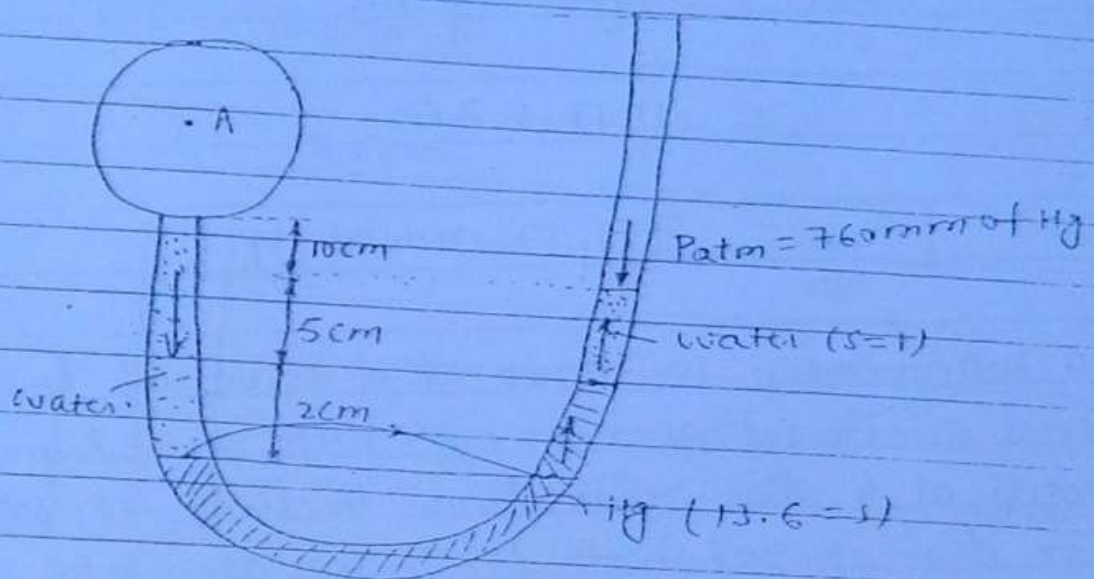
$$25 \times 0.65 = h_w \times 1$$

$$h_w = 25 \times 0.65$$

$$P_A - 50 \times 0.8 - 25 \times 0.65 + 100 = P_B$$

$$P_B - P_A = 43.75 \text{ cm of H}_2\text{O} \quad \text{Ans}$$

ob-32- Refer to the fig. & find the absolute pressure of gas a in mm of Hg.



left.

Right limb water is converted in terms of Hg.

$$h_w s_w = h_m \cdot s_m$$

$$170 \times 1 = h_m \cdot 13.6$$

$$h_m = \frac{170}{13.6}$$

in Right limb water is converted in terms of Hg.

$$h_w s_w = h_m \cdot s_m$$

$$50 \times 1 = h_m \times 13.6$$

$$h_m = \frac{50}{13.6}$$

$$P_A + \frac{170 \cdot 3}{13.6} - \frac{20 - 50}{13.6} = 0$$

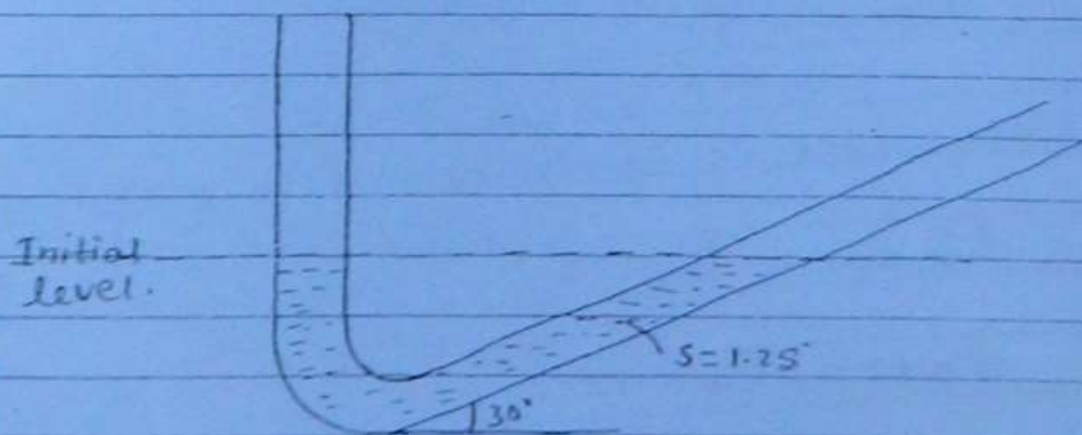
$$P_A = 11.17 \text{ mm of Hg (gauge)}$$

$$P_{abs.} = P_g + P_{atm.}$$

$$= 11.17 + 760$$

$$P_{abs.} = 771.17 \text{ mm of Hg}$$

Ex-33- A manometer is made of a tube of uniform cross-sectional area 0.5 cm^2 with one limb vertical & the other limb inclined at 30° to horizontal both limbs are open to atm. it is partly filled with a manometric liquid of specific gravity 1.25. If an additional volume of 7.5 cm^3 of water is added to inclined tube then find the rise of liquid in vertical tube from initial level.



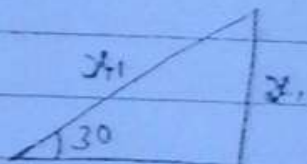
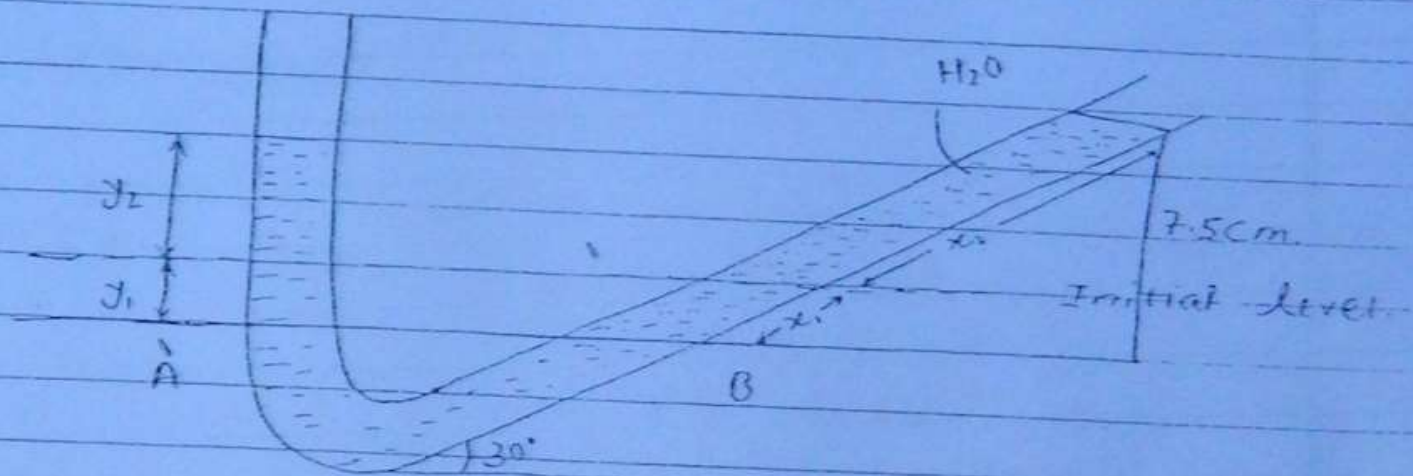
Area, $a = 0.5 \text{ cm}^2$

Additional volume $V = 7.5 \text{ cm}^3$

then, $V = a \cdot l$

$7.5 = 0.5 \times l$

$l = 15 \text{ cm}$



$\sin 30^\circ = \frac{x_1}{x_1}$

$\frac{1}{2} = \frac{x_1}{x_1}$

$x_1 = 2x_2$

$x_1 + x_2 = 15$

$P_A = P_B \Rightarrow (y_1 + y_2) \times 1.25 = 7.5 \times 1$

$y_1 + y_2 = 6$

$x_1 = y_2 = 2y_1$

$$y_1 + 2y_1 = 6$$

$$3y_1 = 6$$

$$y_1 = 2$$

$$4 \quad y_1 + y_2 = 6$$

$$2 + y_2 = 6$$

$$\boxed{y_2 = 4 \text{ cm.}} \text{ Ans.}$$

Hydraulic forces on surfaces

• Total pressure & centre pressure :-

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surface. This force always acts normal to the surface.

• Static fluid :-

A fluid is said to be static fluid when the fluid is at rest. This means that there will be no relative motion b/w adjacent fluid layers. The velocity gradient, which is equal to the change of velocity b/w two adjacent fluid layers divided by the distance b/w the layers, will be zero. or $\frac{du}{dy} = 0$. The shear stress which is equal

to $\mu \frac{du}{dy}$ will also be zero. Then the force

acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface.
2. due to gravity (or self-weight of fluid particles).

• Pascal's law :- It states that the intensity of press. in a static fluid is transmitted equally in all directions.

Static fluid pressure :-

The pressure exerted by fluid depends only upon the depth of the fluid, the density of the fluid & acceleration of gravity.

It is independent of total mass, weight, volume etc. of the fluid.

Centre pressure (C.P.) :-

~~It is defined as the force exerted by on static fluid on a surface~~

It is defined as the point of application of the total hydrostatic force on the surface.

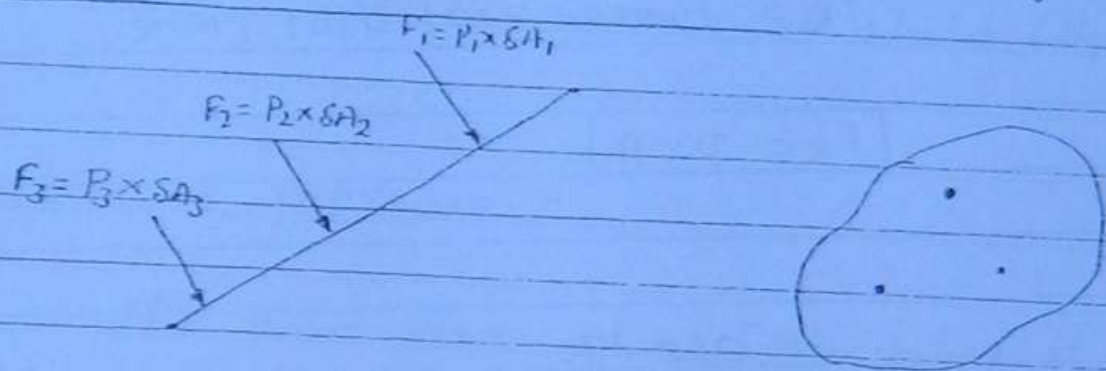
or

✓ It is the point from which the total hydrostatic force is suppose to be acting.

Hydrostatic forces on plane surface :

Basic Knowledge :- (General submerged plane)

Consider the plane surface shown in figure below



The total area is made up of many elemental area. The force on each elemental area is always normal to the surface but in general, each force is of different magnitude as the press. usually varies.

We can find the total or resultant force F_R on the plane by summing up all of the forces on the small elemental i.e.

$$F_R = P_1 SA_1 + P_2 SA_2 + \dots + P_n SA_n = \sum P SA$$

This Resultant force will act through the centre of press. Hence we can say If the surface is a plane, the force can be represented by a single resultant force, acting at right-angle to the plane through the centre pressure.

Horizontal Submerged plane :-

In a liquid (or a plane experiencing uniform press. over it's surface), the press. P will be equal at all points at the surface.

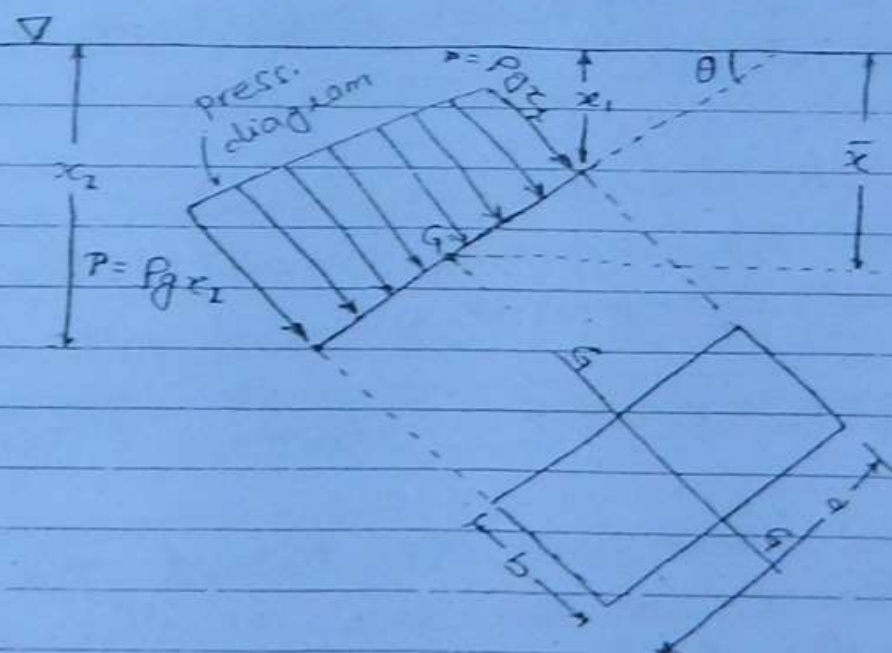
Thus the resultant force will be given by,

$$F_R = \text{Press.} \times \text{area of Plane}$$

$$[F_R = P \times A]$$

Case-I.

∴ Inclined surface :- (Plane)



As the pressure is varying with depth, let us take a small strip & find the force on small strip & integrate the small strip

force for finding the total force.

i.e.

$$F = W \cdot A \bar{x}$$

Where, $\bar{x}$ = distance $\perp$ C.G. from free surface.
 W = specific weight of fluid. (liquid).

Centre of pressure :-

It is the point from which the total hydrostatic force is suppose to be acting.

Centre of pressure is calculated by using the "Principle of Moment's", which states that the moment of the resultant force about an axis is equal to the sum of moment's of the component's about the same axis.

From principle of moment's we have centre of pressure.

$$h = \bar{x} + \frac{I_G \sin^2 \theta}{A \bar{x}}$$

Where θ = Angle made by plane with the free surface

I_G = Moment of Inertia of the plane surface about centroidal axis G-G.

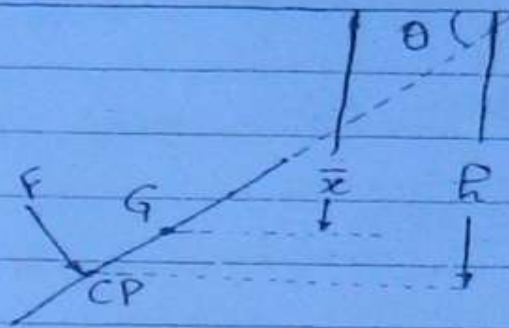


fig. Resultant diagram.

Case-II

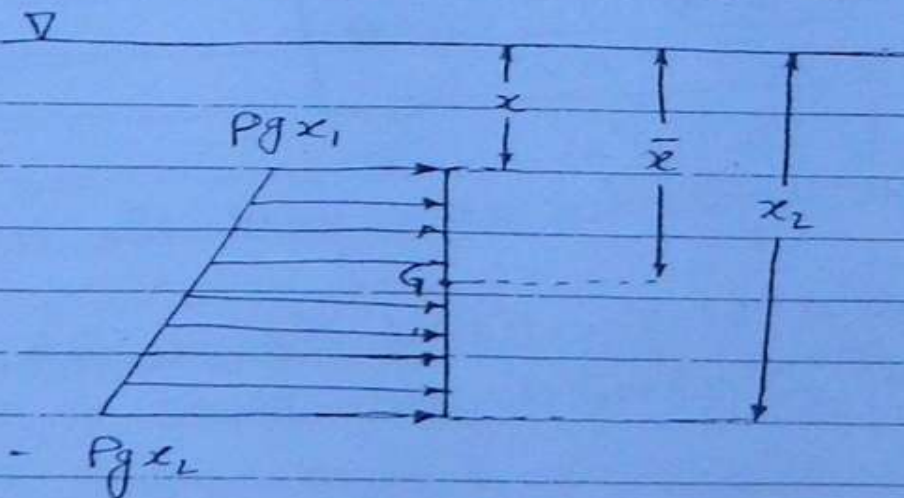
Plane vertical surface :-

put $\theta = 90^\circ$,

then, $F = \rho g \cdot A \cdot \bar{x}$

$$h = \bar{x} + \frac{I_G}{A \bar{x}} \sin^2 90^\circ$$

$$h = \bar{x} + \frac{I_G}{A \bar{x}}$$



V

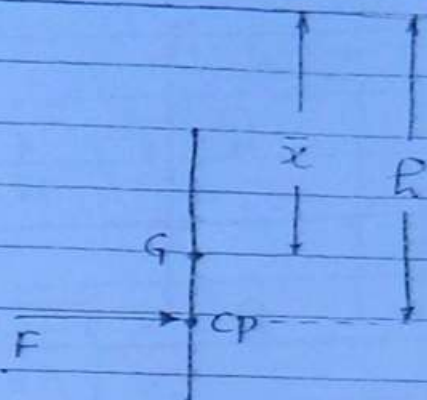


fig. - Resultant diagram.

Case - III.

Plane Horizontal surface :-

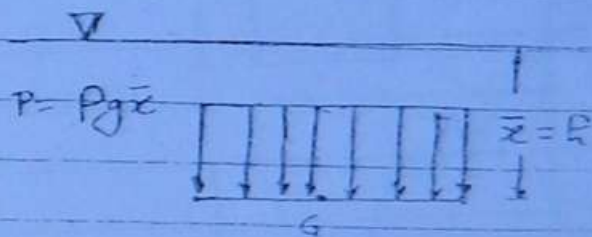
put $\theta = 0^\circ$.

then, $P = \frac{F}{A}$

$$F = P \times A$$

$$F = P_g \bar{x} A$$

$$F = W \bar{x} A$$



$$h = \bar{x} + \frac{I_G}{A \bar{x}} \sin^2 \theta$$

$$h = \bar{x}$$

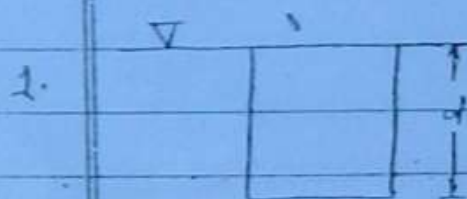
In case of plane vertical surface's C.P. & C.G. are almost same when the depth of immersion $\bar{x}$ is made very large.

Case	force	C.P.
Inclined	$W A \bar{x}$	$\bar{h} = \bar{x} + \frac{I_G}{A \bar{x}} \sin^2 \theta$
Vertical	$W A \bar{x}$	$\bar{h} = \bar{x} + \frac{I_G}{A \bar{x}}$
Horizontal	$W A \bar{x}$	$\bar{h} = \bar{x}$

prob-34 Match the following -

LIST-I
(surface)

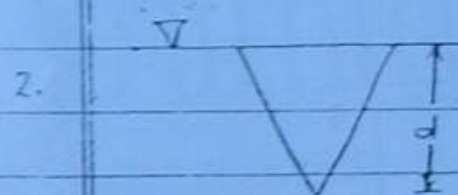
LIST-II
(C.P.)



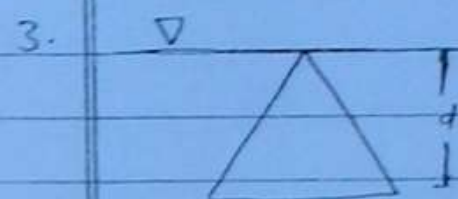
A. $5d/8$

B

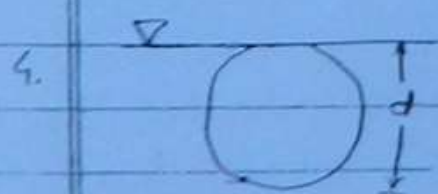
B. $3d/4$



C. $d/2$

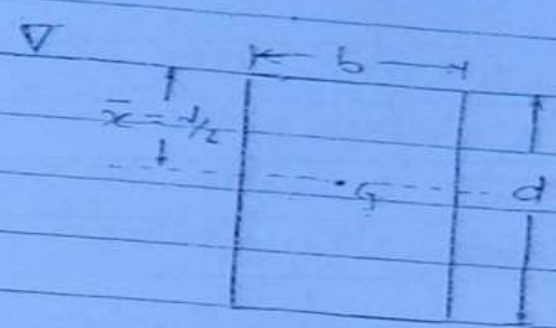


D. $2d/3$



All these plane vertical free surface.

Solⁿ: 1.



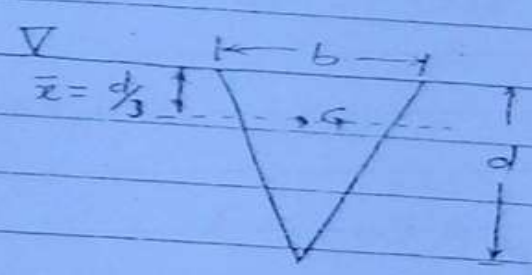
$$I_G = \frac{bd^3}{12} \quad \& \quad \bar{x} = \frac{d}{2}$$

then
$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$h = \frac{d}{2} + \frac{bd^3/12}{bd \times d/2} = \frac{2d}{3}$$

$$h = \frac{2d}{3}$$

2.

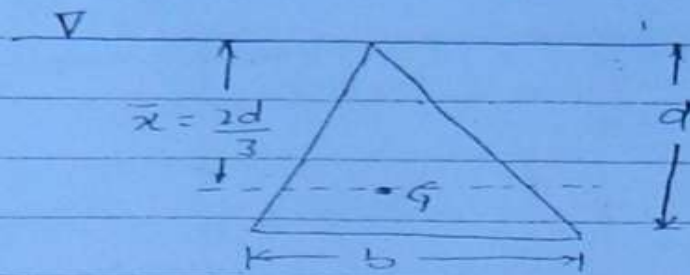


$$I_G = \frac{bd^3}{36} \quad \& \quad \bar{x} = \frac{d}{3}$$

then
$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$h = \frac{d}{3} + \frac{bd^3}{36} \times \frac{2}{bd} \times \frac{3}{d} = \frac{d}{2}$$

$$h = \frac{d}{2}$$

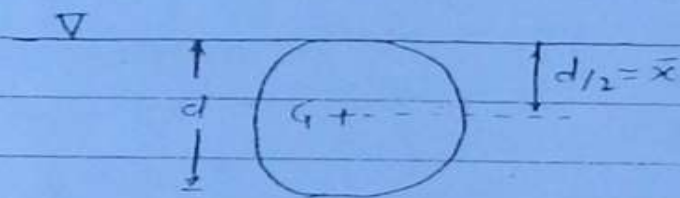


$$I_G = \frac{bd^3}{36} \quad \& \quad \bar{x} = \frac{2d}{3}$$

then,
$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$h = \frac{2d}{3} + \frac{bd^3/36}{\frac{1}{2}bd \times \frac{2d}{3}} = \frac{3d}{4}$$

$$\boxed{h = \frac{3d}{4}}$$



$$I_G = \frac{\pi d^4}{64} \quad \& \quad \bar{x} = \frac{d}{2}$$

then
$$h = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$h = \frac{d}{2} + \frac{\pi d^4/64}{\frac{\pi d^2}{4} \times \frac{d}{2}} = \frac{5d}{8}$$

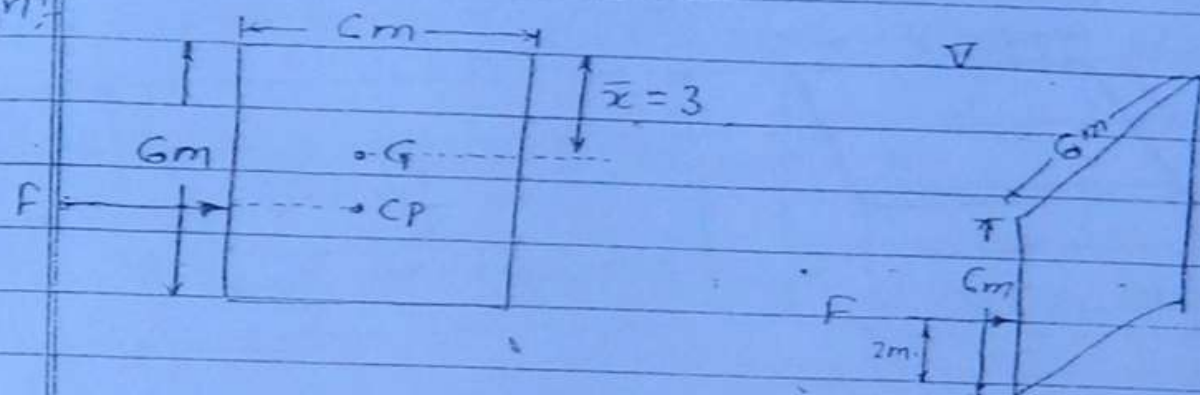
$$\boxed{h = \frac{5d}{8}}$$

Ans- 1 2 3 4
D C B A

prob-35- A vertical gate $6\text{m} \times 6\text{m}$ holds water on one side with free surface at its top then find the moment about the bottom edge of gate due to hydrostatic force ($\omega = \text{specific weight}$).

- (a) 36ω (b) 72ω (c) 108ω (d) 216ω .

Soln:-



We know that C.P. for vertical plane,

$$h = \frac{2d}{3} = \frac{2 \times 6}{3} = 4\text{m}.$$

Hydrostatic force,

$$F = \omega A \bar{x}$$

$$F = \omega \times 6 \times 6 \times 3$$

$$F = 108 \omega.$$

Moment about the bottom edge of gate due to hydrostatic force.

$$= 108 \omega \times 2$$

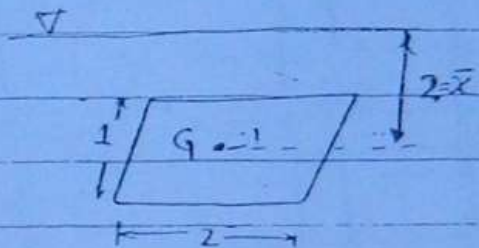
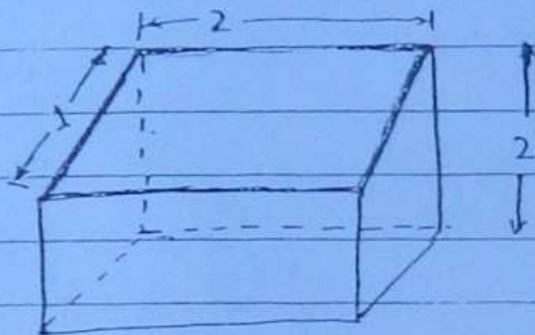
$$= 216 \omega \text{ Ans.}$$

mob-36

A rectangular container with length, width & height in the ratio 2:1:2 is completely filled with water what is the ratio of hydrostatic force at the bottom surface to that on any large vertical surface.

- (a) 2 (b) $\frac{1}{4}$ (c) 4 (d) 1.

Solⁿ:-



Force at the bottom,

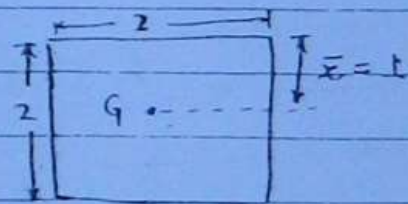
$$F_B = \rho \times A \times \bar{x}$$

$$F_B = \rho \times 2 \times 1 \times 2 = 4\rho$$

Force on larger vertical surface,

$$F_V = \rho \times 2 \times 1$$

$$F_V = 4\rho$$

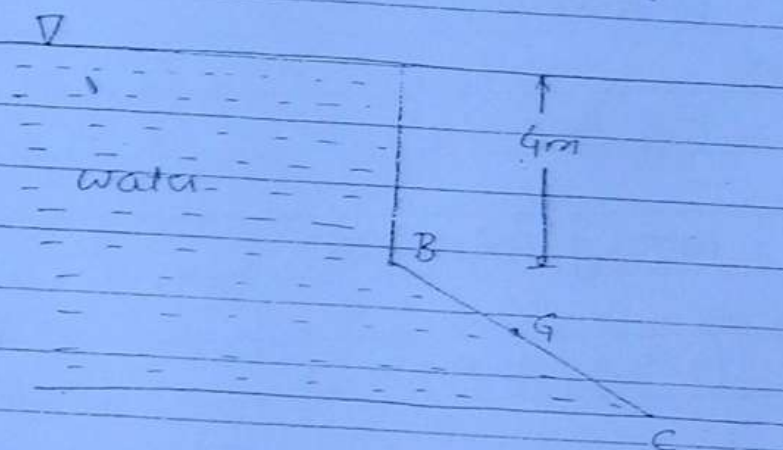


$$\frac{F_B}{F_V} = \frac{4\rho}{4\rho} = 1 \text{ Ans.}$$

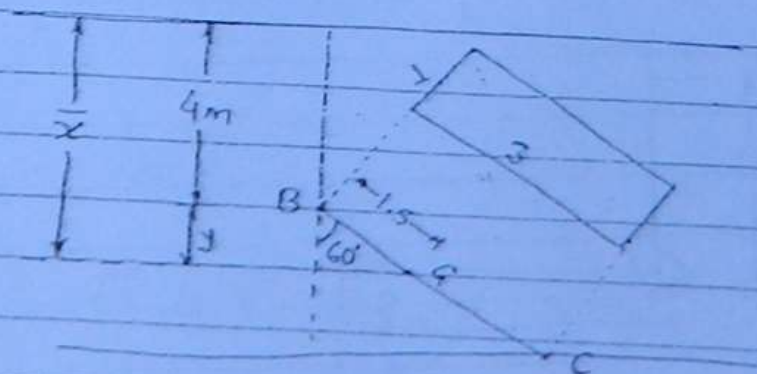
prob-37 In the situation shown in fig. the length BC is 3m. & G is the mid point of the BC the hydrostatic force on BC measured per unit width with g being acceleration due to gravity

- (a) 16500 g passing through G.
- (b) 16500 g passing through a point b/w G & C.
- (c) 14250 g passing through G.
- (d) 14250 g passing through a point b/w G & C.

Soln:



Hydrostatic force on a BC act's as below the C.G. (G)



from trigonometry of figure.

$$\cos 60 = \frac{y}{1.5}$$

$$y = 1.5 \cos 60 = 0.75$$

$$\bar{x} = 4 + y = 4 + 0.75 = 4.75$$

Area $A = b \times d = 3 \times 1 = 3$

Hydrostatic force,

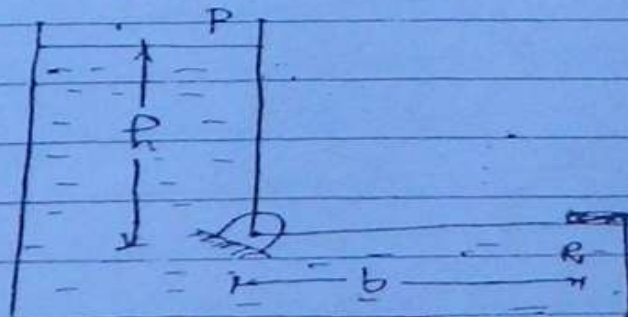
$$F = w \cdot A \cdot \bar{x}$$

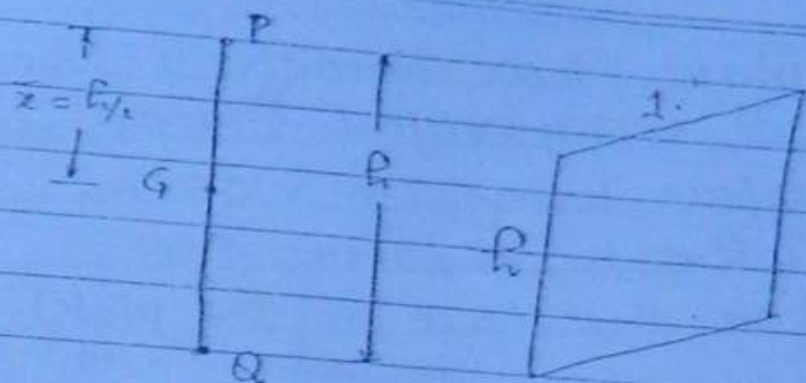
$$F = \rho g \cdot A \cdot \bar{x}$$

$$F = 1000 \cdot g \cdot 3 \times 1 \times 4.75$$

$$F = 1425g \text{ Ans.}$$

Prob-38- The figure shows a hydraulic gate PQR whose weight is very small compare to hydrostatic force's then find the value of 'h' for equilibrium.





width is not given
take width = 1

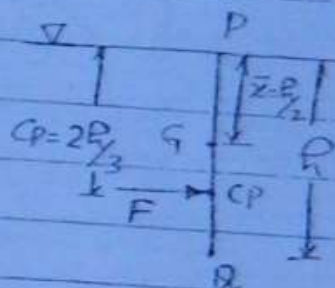
Hydrostatic force on PQ.

$$F_{PQ} = w \times R \times 1 \times \frac{R}{2}$$

$$F_{PQ} = \frac{wR^2}{2}$$

For the vertical plane surface

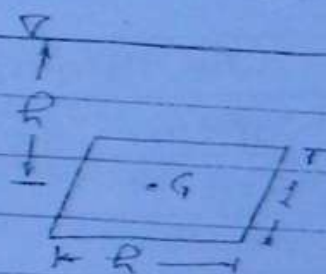
$$R = \frac{2R}{3}$$



$$A = b \times 1$$

$$F_{QR} = w A \bar{x}$$

$$= w \times b \times 1 \times R$$



$$F_{QR} \times \frac{R}{3} = F_{QR} \times \frac{b}{2}$$

$$\frac{wR^2}{2} \times \frac{R}{3} = w \times b \times R \times \frac{b}{2}$$

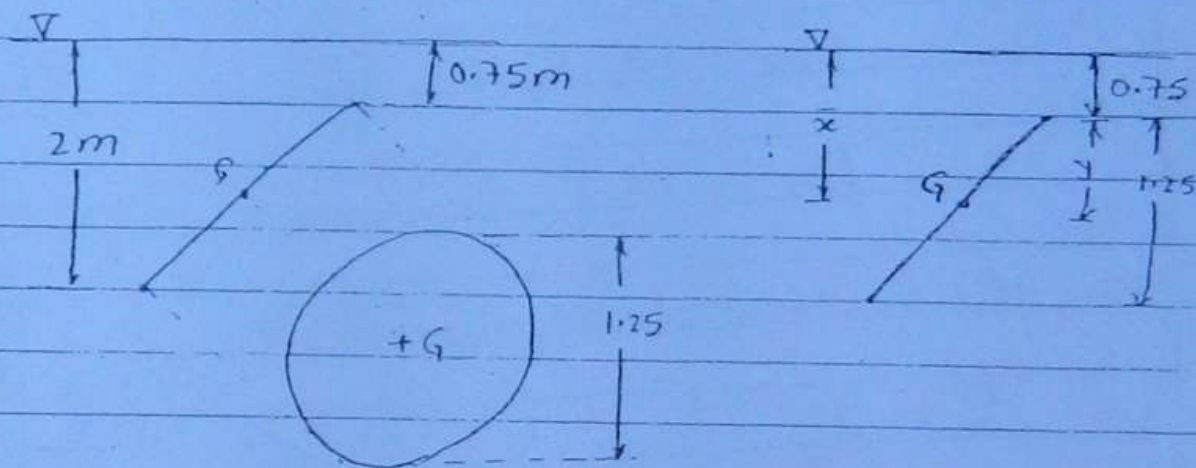
$$\frac{R^2}{3} = b^2$$

$$R = \sqrt{3} b \quad \text{Ans.}$$

Prob-39- A circular plate 1.5 m diameter is submerged in water with its greatest & least depth below the surface being 2 m & 0.75 m respectively. What is the total force. (approximately on one face of the plate).

- (a) 12 kN (b) 16 kN (c) 24 kN (d) None.

Soln:



$$F = \rho g A \bar{x}$$

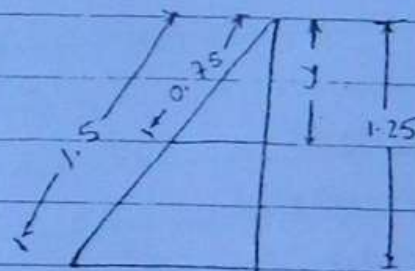
$$\rho = 9810 \text{ N/m}^3$$

$$A = \frac{\pi}{4} \times (1.5)^2$$

$$F = 9810 \times \frac{\pi}{4} \times (1.5)^2 \times 1.375$$

$$= 23.98 \times 10^3 \text{ N}$$

$$= 23.98 \text{ kN Ans.}$$



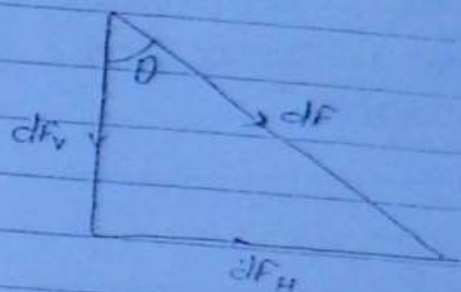
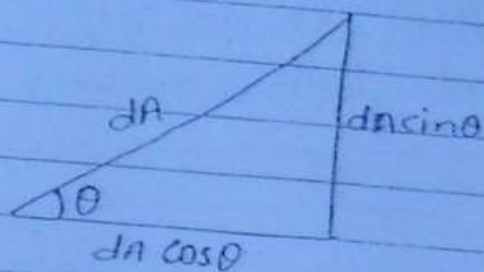
$$y = 1.25/2 = 0.625$$

$$x = 0.75 + y$$

$$= 0.75 + 0.625$$

$$= 1.375$$

$$dF = \rho g \times dA$$



Resolving the area into vertical & Horizontal component

• Resolving the force on Horizontal & Vertical Component.

$$dF_H = dF \sin \theta$$

$$dF_V = dF \cos \theta$$

then,

$$dF_H = dF \sin \theta$$

$$dF_H = \rho \cdot g \cdot x \cdot dA \cdot \sin \theta$$

Imp.

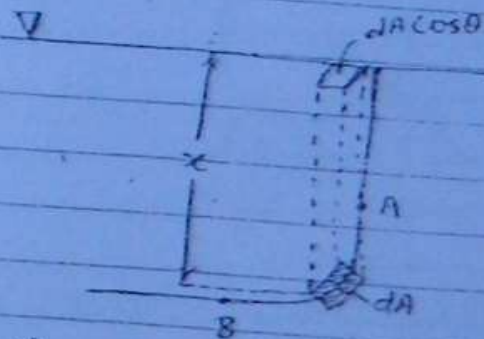
The horizontal component of force on curved surface is Hydrostatic force on vertical projection area & this force will act at the centre of pressure.

$$dF_V = dF \cos \theta$$

$$= \rho g x dA \cos \theta$$

$$= \rho \cdot g \cdot \text{Volume}$$

$$dF_V = \text{wt. of fluid}$$



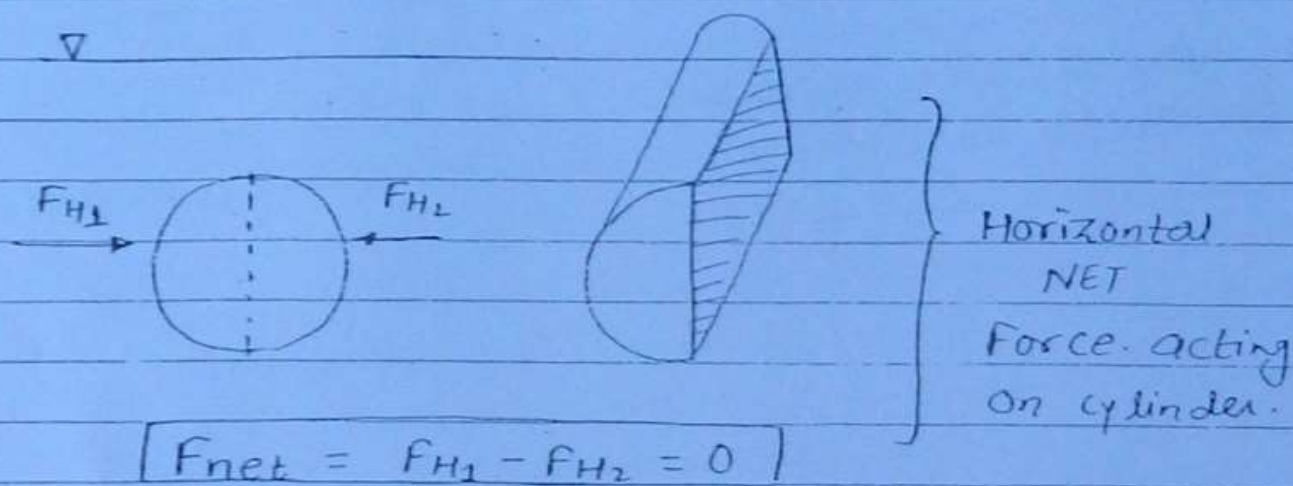
$$\therefore W = \text{wt. of fluid volume}$$

$$\text{wt. of fluid} = W \times \text{Vol.} = \rho g \times \text{Vol.}$$

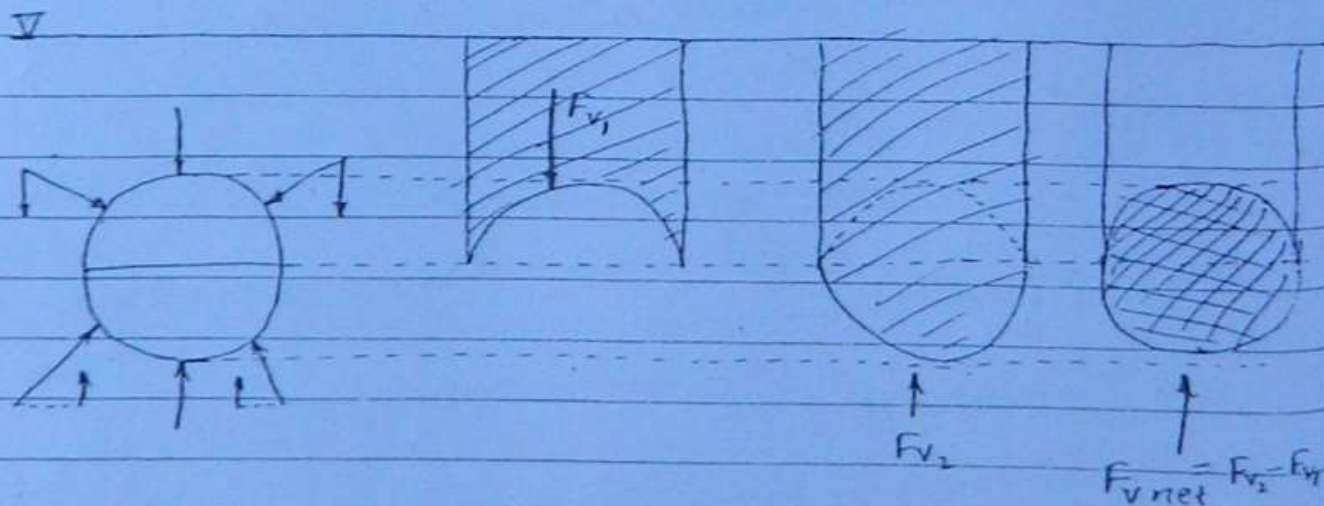
Imp:

Vertical component of force on curved surface is equal to weight of the fluid contained by the curved surface upto it's free surface & this weight will act at the centre of gravity of corresponding weight.*

NOTE -

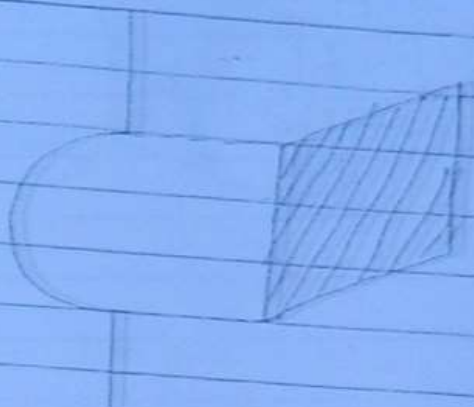


* vertical net force acting on cylinder.



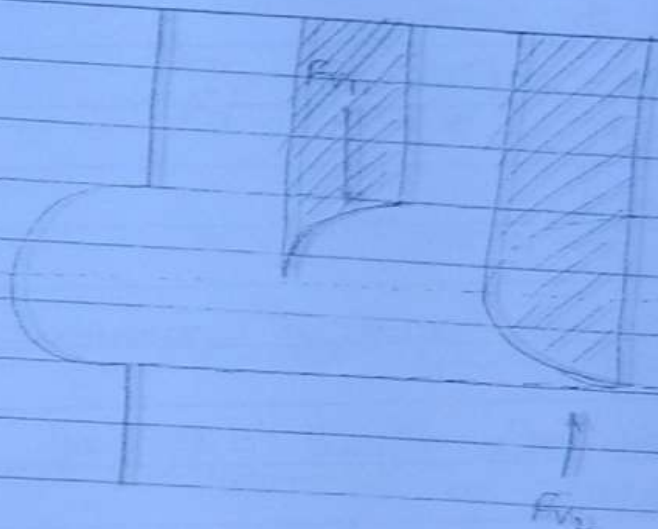
NET vertical force is equal to weight of the fluid corresponding to cylindrical volume.

7



Horizontal force

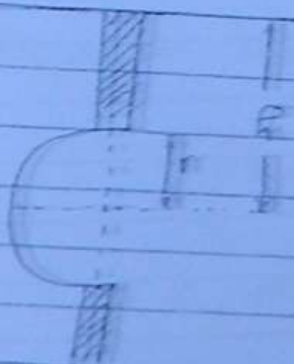
8



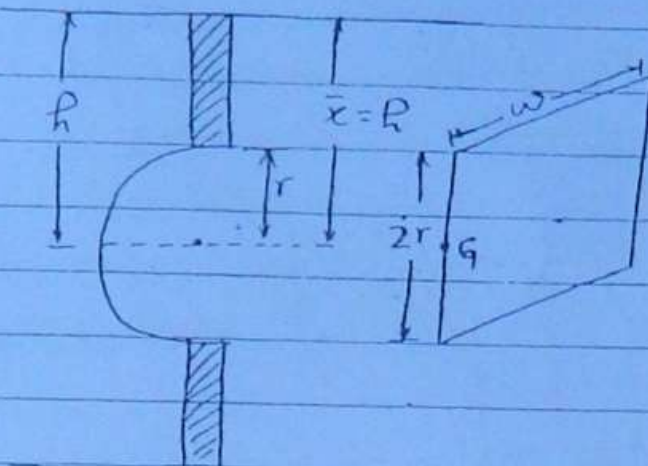
Vertical force

$$F_{\text{net}} = F_{y2} - F_{x1}$$

41- Find the horizontal & vertical component of hydrostatic force F_x & F_y on semi-circular gate having width w perpendicular to the plane of paper.



Solⁿ:-



Hydrostatic force on horizontal component,

$$F_x = w \cdot A \cdot \bar{x} = P_g \times 2rw \times R$$

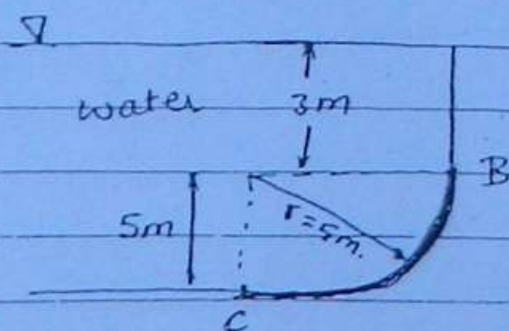
$$F_x = 2P_g R r w$$

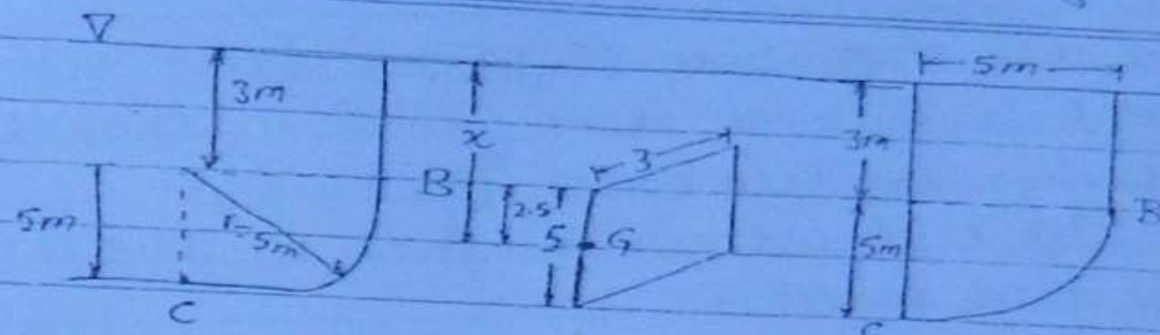
& Hydrostatic force on vertical component,

$$F_y = w \cdot t \cdot \text{of the fluid} = P_g \times \text{vol.}$$

$$F_y = P_g \times \frac{\pi r^2 \cdot w}{2}$$

Prob-42: The tank in the fig. is 3m wide calculate horizontal & vertical component of force on $\frac{1}{4}$ th of a circle BC.





$$\bar{x} = 3 + 2.5 = 5.5$$

Horizontal Component of force,

$$F_H = w A \bar{x} = 9810 \times 3 \times 5 \times 5.5$$

$$F_H = 809.3 \times 10^3 \text{ N}$$

$$F_H = 809.3 \text{ kN Ans.}$$

Vertical Component of force,

$$F_v = \text{Wt. of fluid} = \rho \cdot g \cdot \text{Volume}$$

$$F_v = \rho g v = 100 \times 9.81 \times 103.9$$

$$F_v = 1019.3 \text{ kN Ans.}$$

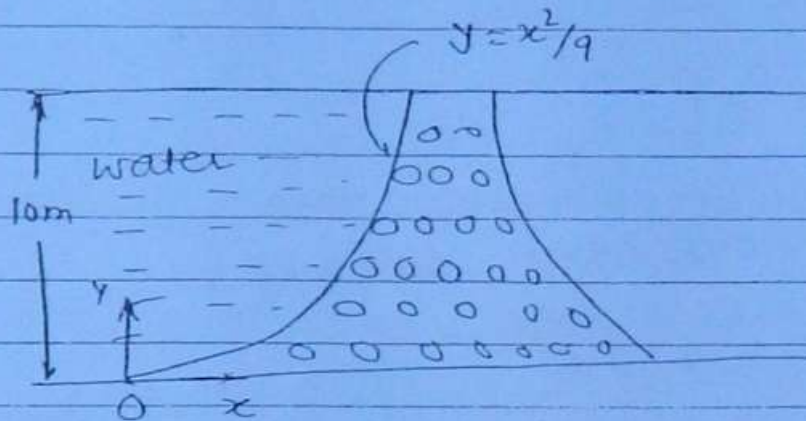
Volume of fluid = Area $\times$ length

$$= \left[5 \times 3 \times \frac{\pi (5)^2}{4} \right] \times 3$$

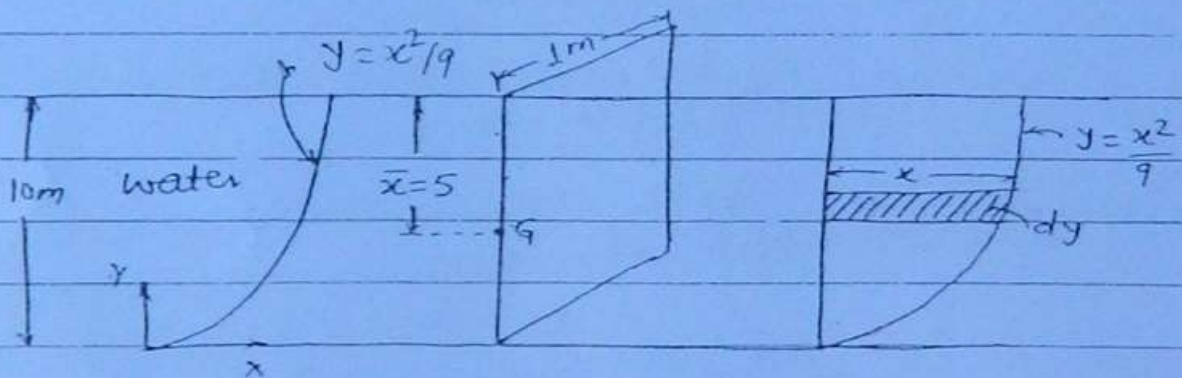
$$= 103.9$$

prob-43- Find the magnitude of water force acting on curved surface of dam which is shaped according to relationship. $y = x^2/9$. The height of water retained by dam is 10m. take width as 1m.

Soln:-



Soln:-



Horizontal force acting,

$$F_H = w \cdot A \cdot \bar{x} = 9810 \times 10 \times 1 \times 5 = 490.5 \text{ kN}$$

$$F_H = 490.5 \text{ kN.}$$

Vertical force acting,

$$F_V = \text{wt. of fluid} = \rho g v$$

$$dA = x dy$$

$$A = \int_0^{10} x dy$$

$$A = \int_0^{10} 3\sqrt{y} dy$$

$$A = 3 \int_0^{10} \sqrt{y} dy$$

$$A = 3 \left[\frac{2}{3} y^{3/2} \right]_0^{10}$$

$$A = 63.2 \text{ m}^2 \text{ Ans.}$$

$$\text{Volume} = \text{Area} \times \text{length}$$

$$\text{Vol.} = 63.2 \times 1 = 63.2$$

then vertical force,

$$F_v = \rho g V = 1000 \times 9.81 \times 63.2$$

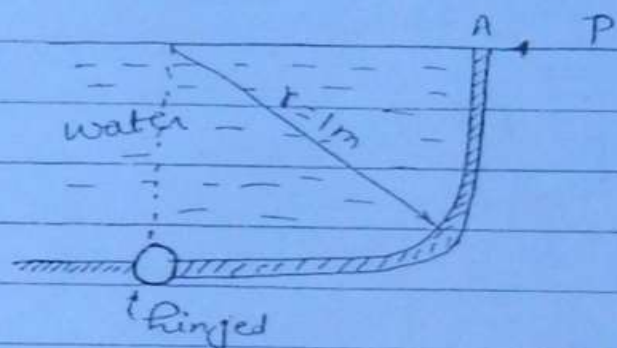
$$F_v = 620.4 \text{ kN.}$$

Magnitude of water force acting on curved surface

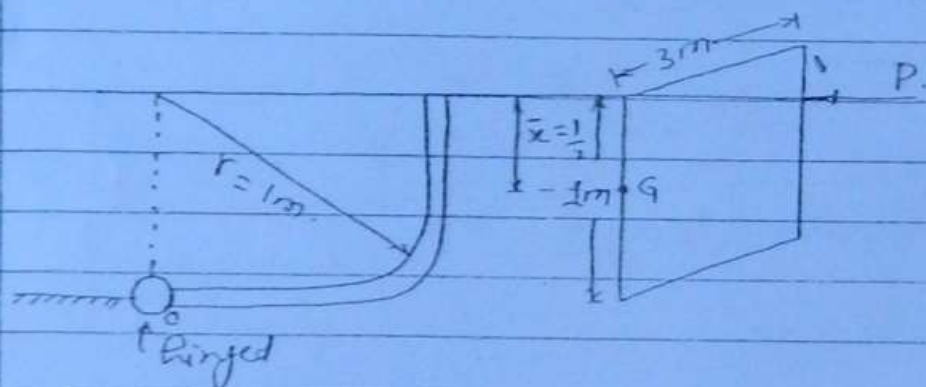
$$F = \sqrt{F_H^2 + F_v^2} = \sqrt{490.5^2 + 620.4^2}$$

$$\boxed{F = 790.9 \text{ kN.}} \text{ Ans.}$$

Prob-44- The gate OA shown in fig. is hinged at O & is in the form of $\frac{1}{4}$ th of a circle of radius 1m. It supports water on one side as shown if the width of the gate is 3m. find the force P required to hold the gate in position.



Soln:



Then the horizontal hydrostatic force,

$$F_H = \rho g A \bar{x} = 9810 \times 1 \times 3 \times \frac{1}{2} = 14715 \text{ N}$$

$$F_H = 14.7 \text{ kN.}$$

The force P required to hold the gate in position

$$P = F_H = 14.7 \text{ kN} \text{ Ans.}$$

Buoyancy & Floatation

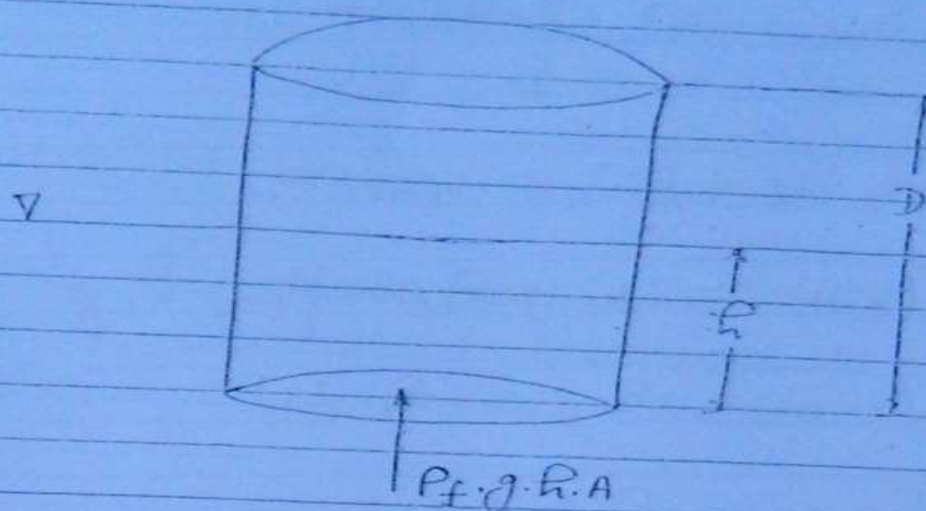


fig. Partially submerged or floating body.

Volume of fluid displaced = Volume of body submerged

$$V_{fd} = P_f \cdot g \cdot h \cdot A$$

Net vertical upward force.

$$F = P \times A$$

$$F_{vnet} = P_f \cdot g \cdot h \cdot A$$

$$F_{vnet} = P_f \cdot g \cdot V_{fd}$$

$$F_{vnet} = \text{Wt. of fluid displaced}$$

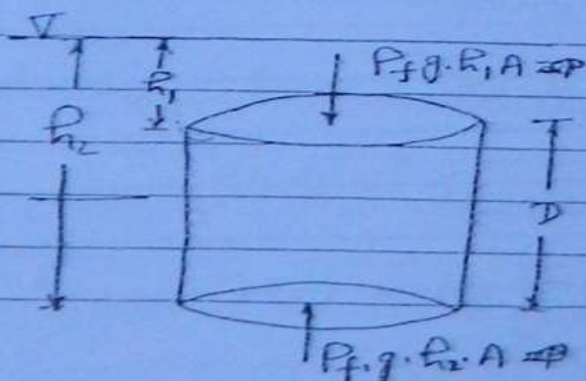


fig. Completely submerged body.

Vol. of ~~body~~ submerged = vol. of fluid displaced.

$$V_{fd} = A \cdot D$$

Net vertical upward force,

$$F_{net} = P_f \cdot g \cdot h_2 A - P_f \cdot g \cdot h_1 A$$

$$F_{net} = P_f \cdot g \cdot A (h_2 - h_1)$$

$$F_{net} = P_f \cdot g \cdot A \cdot (D)$$

$$F_{net} = P_f \cdot g \cdot V_{fd}$$

$$\boxed{F_{net} = \text{wt. of the fluid displaced}}$$

When a body is immersed either partially or completely the ~~result~~ net vertical upward force exerted by the fluid on the body is known as buoyancy force & this upward force equal to weight of the fluid displaced & this is known as Archimide's principle.

Buoyancy force is basically due to press. difference.

Principle of floating floatation :-

Centre of buoyancy (B) :-

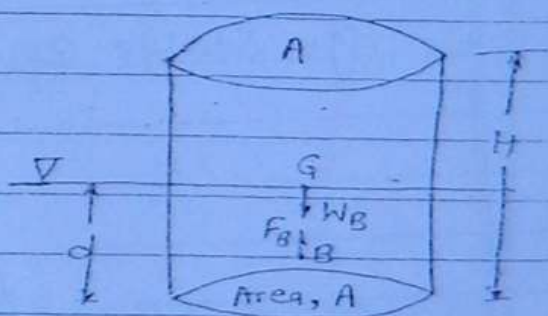
It is the point from which the buoyancy force acts & centre of buoyancy is equal to centroid of displaced volume.

According to principle of floating floatation when a body is immersed partially it will be in equilibrium when weight of the body equals to weight of the fluid displaced.

$$W_B = F_B$$

where W_B = wt. of the body

& F_B = Buoyancy force



But W_B = wt. of fluid displaced.

$$*** \boxed{W_B = W_{fd}} ***$$

$$\text{but, } W_B = \rho_B \cdot g \cdot V_B$$

$$W_B = \rho_B \cdot g \cdot A \cdot H$$

$$\& W_{fd} = \rho_f \cdot g \cdot A \cdot d$$

then, $P_B \cdot g \cdot A \cdot H = P_f \cdot g \cdot A \cdot d$

$$P_B \cdot H = P_f \cdot d$$

:- Stability of bodies:-

★ Types of equilibrium-

1. Stable equilibrium.
2. Unstable equilibrium.
3. Neutral equilibrium.

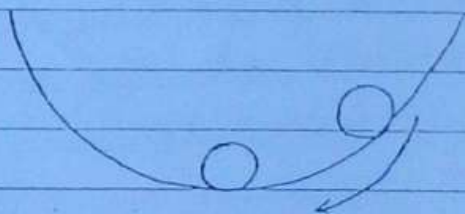


fig. Stable equilibrium.

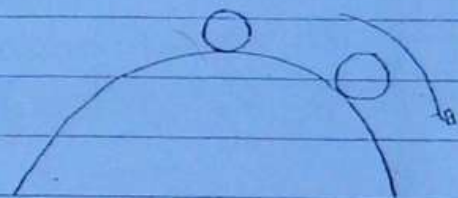


fig. - Unstable equilibrium.

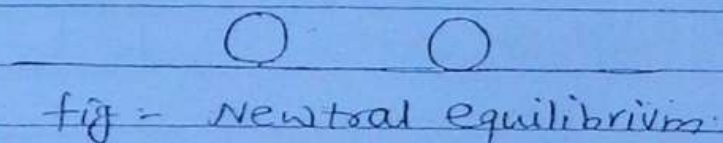
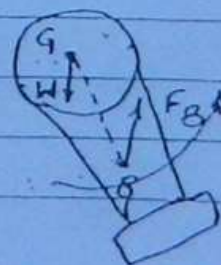
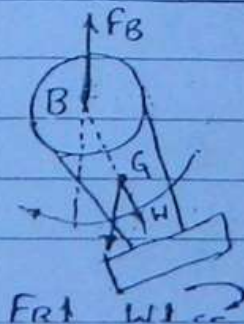
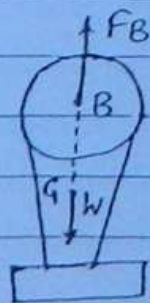


fig - Neutral equilibrium.

★ Stability Condition for completely submerged body:-

▽

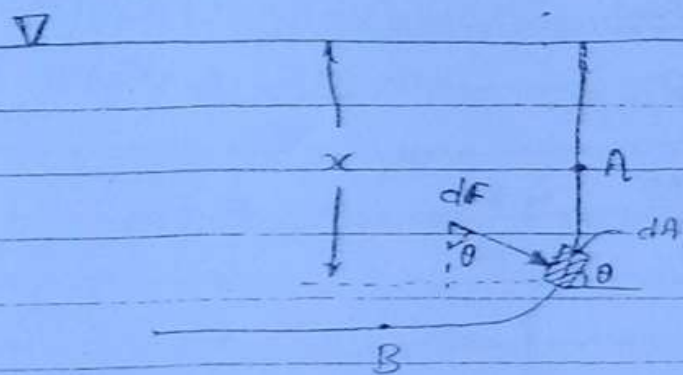


W ↓ F_B ↑
USE

Hydrostatic force on curved surface :-

Basic Knowledge (Curved surface submerged)

If the surface is curved, each elemental force will be of different magnitude & in different direction but still normal to the surface of that element's. The resultant force can be found by resolving all force's into orthogonal co-ordinate direction's to obtain it's magnitude & direction. This will always be less than the sum of the individual force.



Consider a curved surface AB, submerged in a static fluid as shown in fig.

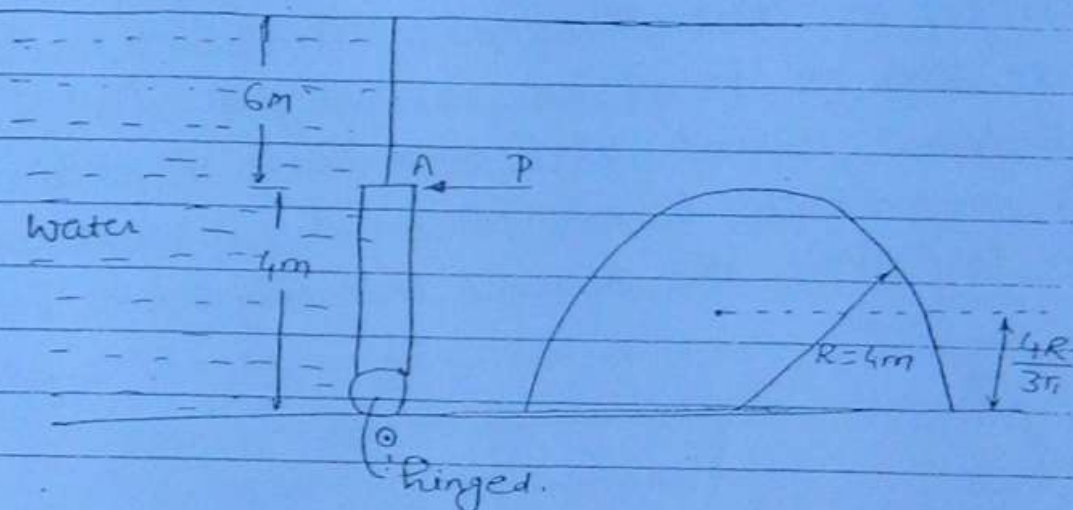
Let dA is the area of a small strip at a depth of x from water surface.

Then the pressure intensity on the area $dA = Pgh$.

pressure force,

$$dF = P \times Area$$

40. A semi-circular gate hinged at 'O' is held in place by a horizontal force P acting at A as shown in fig. Calculate the force P required for equilibrium for semicircular plate the distance of centre of gravity from straight h is $\frac{4r}{3\pi}$. & moment of inertia $I_G = 0.035r^4$.



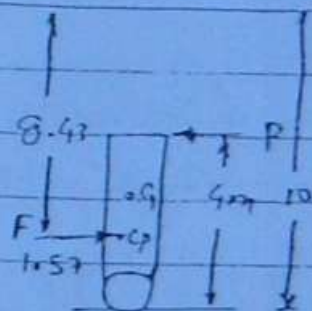
$$\bar{x} = 10 - \frac{4r}{3\pi} = 10 - \frac{4 \times 4}{3\pi} = 8.3m.$$

Hydrostatic force,

$$F = w \cdot A \cdot \bar{x}$$

$$F = 1000 \times 9.81 \times \frac{\pi}{8} \times 8^2 \times 8.3$$

$$F = 2846.3 \text{ kN.}$$

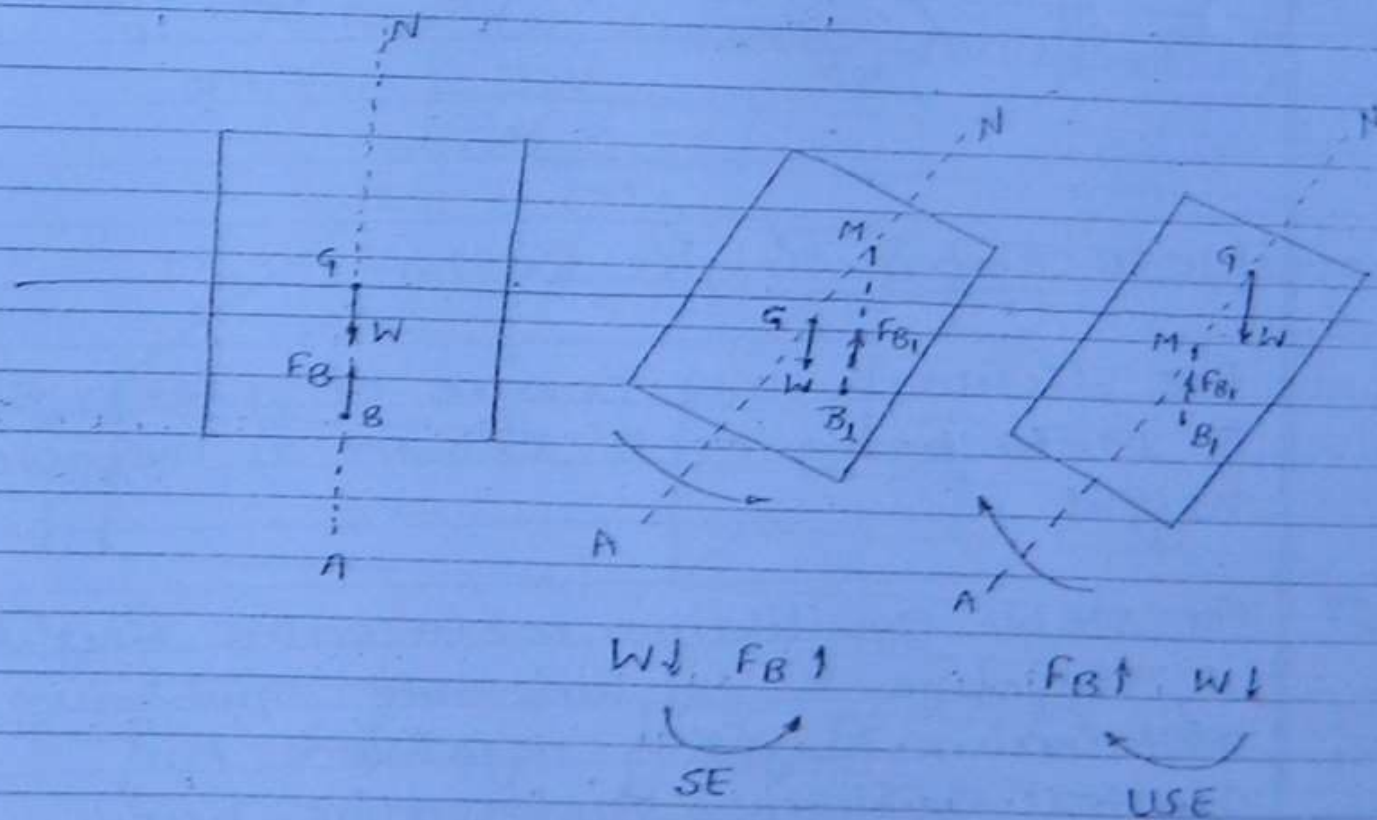


$$h = \bar{x} + \frac{I_G}{A\bar{x}} = 8.3 + \frac{0.035\pi(4)^4}{\frac{\pi(4)^2}{2} \times 8.3} = 8.43m. \text{ Ans.}$$

$$F \times 1.57 = P \times 4 \Rightarrow [P = 803 \text{ kN Ans.}]$$

- * A Completely submerged body will be stable equilibrium when the centre of buoyancy is above the centre of gravity.
- * A Completely submerged body will be unstable equilibrium when the centre of ~~low~~ buoyancy is below the centre of gravity.
- * A Completely submerged body will be neutral equilibrium when centre of gravity & centre of buoyancy coincide.

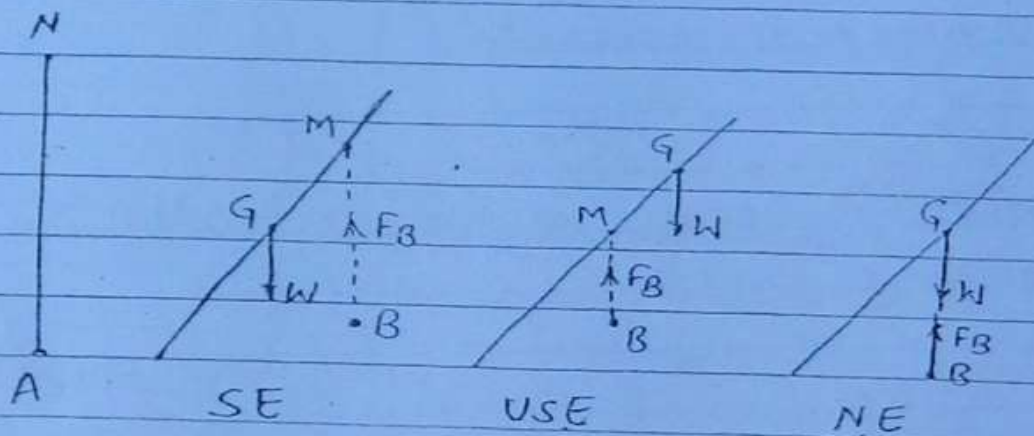
* Stability Condition for partially submerged or floating body.



A floating body will be in stable equilibrium when meta-centre is above the centre of gravity.

A floating body will be in unstable equilibrium when meta-centre is below the centre of gravity.

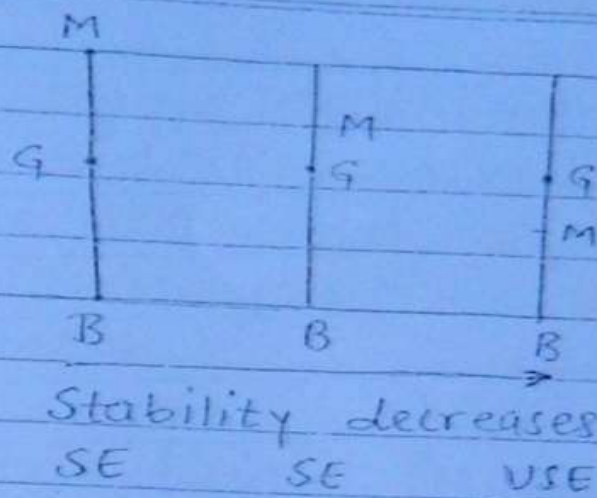
A floating body will be in neutral equilibrium when meta-centre & centre of gravity co-incide.



Metacentric Height (GM) :-

The distance b/w centre of gravity G & meta-centre M is known as metacentric height.

For stable equilibrium metacentric height is positive & for unstable equilibrium metacentric height is negative & For neutral equilibrium metacentric height is zero.



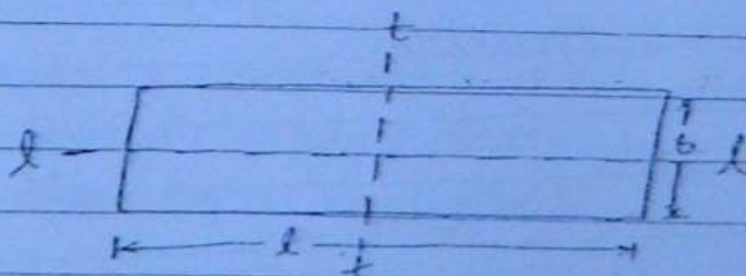
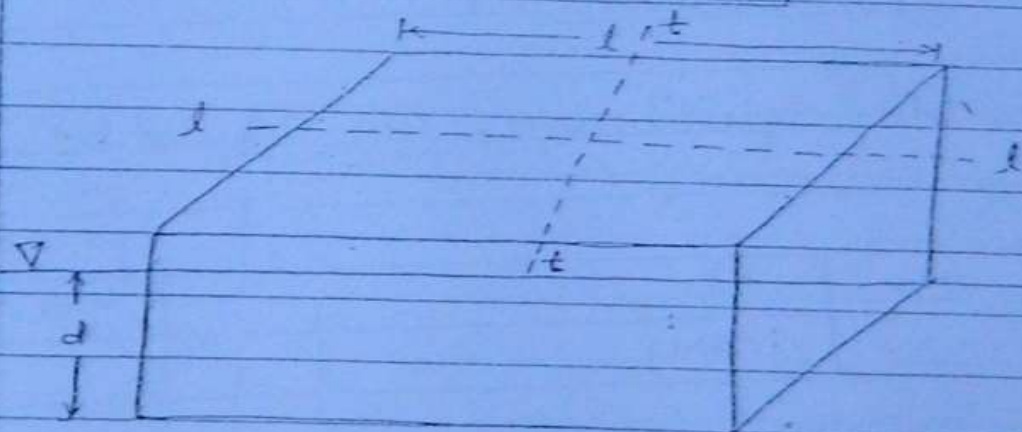
For more stable equilibrium condition BM should be large.

The condition for stable equilibrium is

$$BM > BG$$

The condition for unstable equilibrium is,

$$BM < BG$$



$$I_t = \frac{bl^3}{12} \quad \& \quad I_x = \frac{b^3l}{12}$$

$$I_t > I_x$$

If v = vol. of fluid displaced.

then $BM = \frac{I}{V}$

$$BM_t = \frac{I_t}{V}$$

$$\hookrightarrow BM_l = \frac{I_l}{V}$$

$$I_{t_t} > I_l$$

$$\boxed{BM_t > BM_l}$$

from design point of view least moment of inertia is taken that is about longitudinal axis.

Time period of oscillation:

For more stable equilibrium, GM should be large.

$$T = 2\pi \sqrt{\frac{K_g^2}{g(GM)}}$$

Where K_g = Radius of gyration

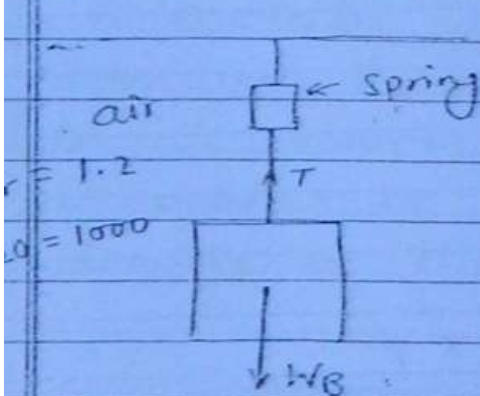
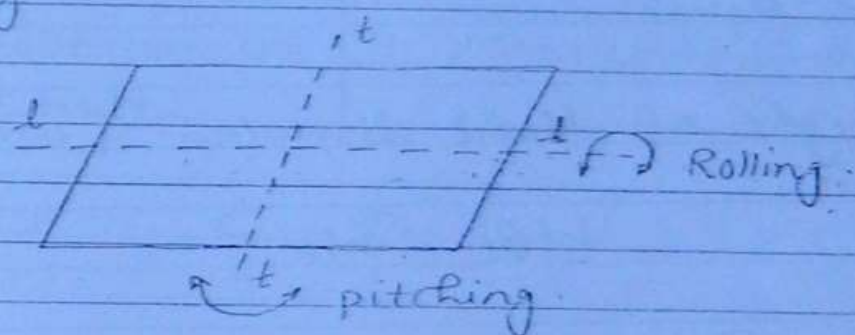
$$I = A K_g^2$$

$$K_g = \sqrt{I/A}$$

For more stable equilibrium condition GM m be large but this results in smaller time period of oscillation i.e. frequent oscillations & hence passengers are not comfortable in such situation.

Metacentric height for merchant ships is 0.5 to 1m. & for warships 1m to 1.5m.

Oscillation about longitudinal axis is known as Rolling & oscillation about transverse axis is known as pitching.

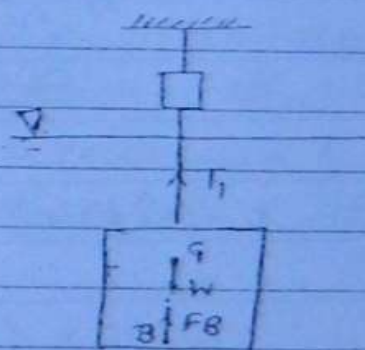


Body is immersed in air.
& P of air is very less
& then F_B is negligible.

Then $W_B = T$

$F_B = W - T$

$F_B = \rho \cdot g \cdot V_d$



Body is completely immersed in water. & P of H₂O is large then we will take

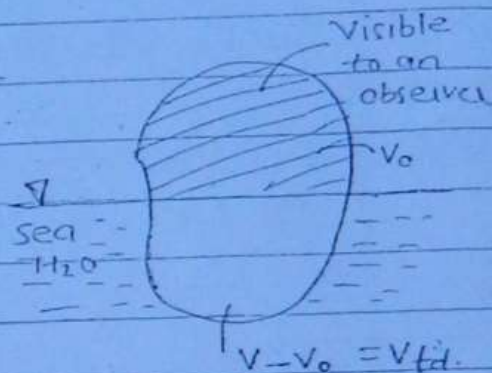
$W_B = T_1 + F_B$

$F_B = W_B - T_1$

$W - \text{loss} = \text{Buoyancy force}$

prob-45- A body of specific weight 8976 N/m^3 extends above the surface of sea water of specific weight 10104 N/m^3 . Then find the % of total volume of body visible to an observer.

Soln: let the total volume of the body is V . & V_0 is the volume of body visible to an observer.



given $W_B = 8976 \text{ N/m}^3$
 $W_f = 10104 \text{ N/m}^3$

For equilibrium,

Weight of fluid displaced = weight of the body

$$W_{fd} = W_B$$

$$\rho_f \cdot g \cdot V_{fd} = \rho_B \cdot g \cdot V_B$$

~~$$\rho_f \cdot V_{fd} = \rho_B \cdot V_B$$~~

$$W_f V_{fd} = W_B \cdot V_B$$

$$10104 \times (V - V_0) = 8976 \times V$$

$$10104V - 10104V_0 = 8976 \times V$$

$$\frac{V_0}{V} \times 100 = 11.16 \% \text{ Ans.}$$

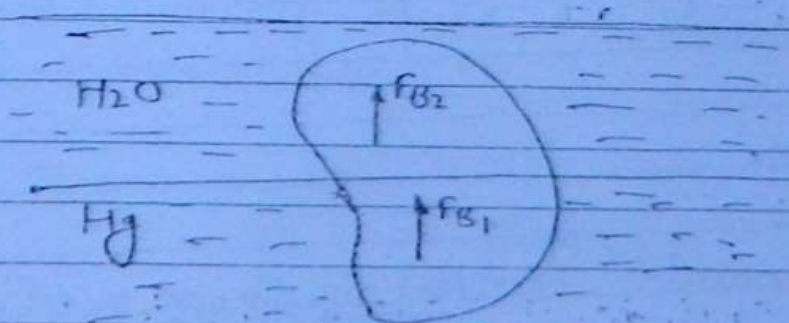
prob-45 Match the following -

LIST-I		LIST-II	
A)	Stable equilibrium of a floating body	1.	Centre of buoyancy below C.G.
B)	Stable equilibrium of submerged body.	2.	Metacentre above C.G.
C)	Unstable equilibrium of a floating body.	3.	Centre of buoyancy above C.G.
D)	Unstable equilibrium of a submerged body.	4.	Meta-Centre below C.G.

Soln:

A	B	C	D.
2	3	4	1

prob-47- A metallic body floats at the interface of mercury & water in such a way that 40% of its volume is submerged in mercury & 60% in water as shown in fig. then find the density of the body.



Soln: In equilibrium,

weight of the fluid displaced = weight of the body

$$F_B = W_B \quad [\because F_B = W_{fd} = \rho_{fd} \cdot V_{fd}]$$

$$F_{B_1} + F_{B_2} = W_B$$

$$\rho_{Hg} \cdot g \times 0.4V + \rho_{H_2O} \cdot g \times 0.6V = \rho_B \cdot g \cdot V$$

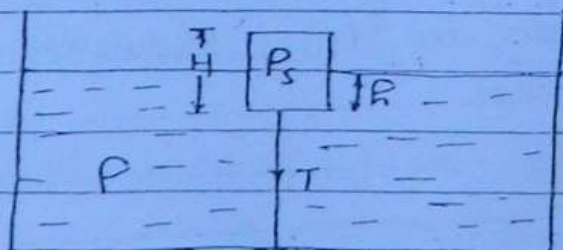
$$\rho_{Hg} \times 0.4 + \rho_{H_2O} \times 0.6 = \rho_B$$

$$13.6 \times 10^3 \times 0.4 + 10^3 \times 0.6 = \rho_B$$

$$\rho_B = 6040 \text{ kg/m}^3 \text{ Ans.}$$

Prob-48 A cylindrical body of cross-sectional area of A . Height H & density ρ_s is immersed to a depth h in a liquid of density ρ & tied down to the bottom with string then find the tension on in the string.

Soln:



In equilibrium,

weight of cylinder + Tension in string = F_B

$$Wt = F_B$$

$$W - 30 = \rho_f g V_{fd}$$

$$W - 30 = 800g \cdot V \quad \text{--- (1)}$$

$$W - 15 = F_B$$

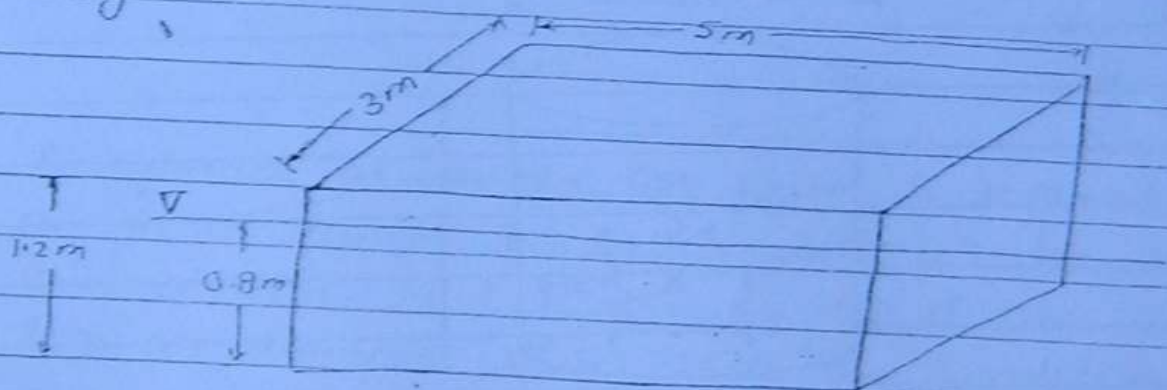
$$W - 15 = \rho_f \cdot g \cdot V_{fd}$$

$$W - 15 = 1200V \quad \text{--- (2)}$$

Solving the eqn. (1) & (2) we get.

$$V = 0.00382 \text{ m}^3 \text{ Ans.}$$

Prob-51- A rectangular body 5m. long 3m wide & 1.2 m high is immersed in water as shown in fig. The depth of immersion is 0.8m. density of water is 1025 Kg/m^3 . Then find metacentric height.



$$BG = 0.6 - 0.4 = 0.2 \text{ m.}$$

$$BM = \frac{I}{V}$$

$$V = \text{Vol. of fluid displaced.}$$

$$= 5 \times 3 \times 0.8$$

$$\text{then, } BM = \frac{5 \times 3^3}{12} \times \frac{1}{5 \times 3 \times 0.8}$$

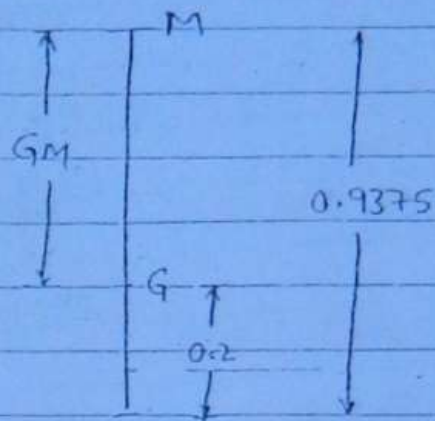
$$I = \frac{5 \times 3^3}{12}$$

$$BM = 0.9375 \text{ m.}$$

$$GM = BM - BG$$

$$= 0.9375 - 0.2$$

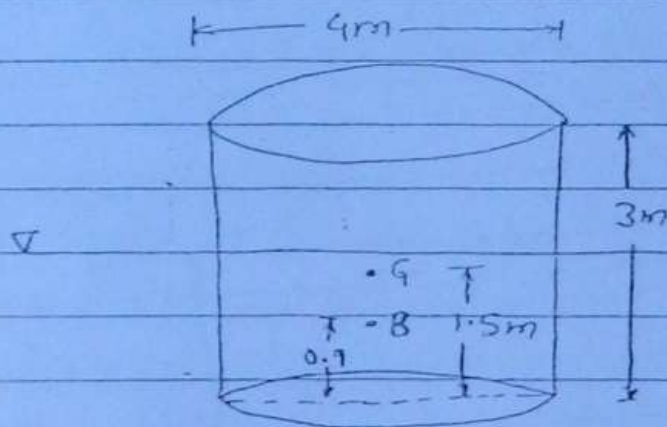
$$= 0.7375 \text{ m. Ans.}$$



Prob-52

A solid cylinder of 3m height has a dia. of 4m. it floats in water such that its longitudinal axis is vertical test whether the cylinder is in stable equilibrium or otherwise. take density of cylinder as 600 kg/m^3 . & density of water as 1000 kg/m^3 .

Soln:



For equilibrium,

Weight of fluid displaced = Weight of body.

$$W_{fd} = W_B$$

$$\rho_f \cdot g \cdot V_{fd} = \rho_B \cdot g \cdot V_B$$

$$1000 \times 9.81 \times \frac{\pi}{4} \times 4^2 \times 1.5 = 600 \times 9.81 \times \frac{\pi}{4} \times 4^2 \times 3$$

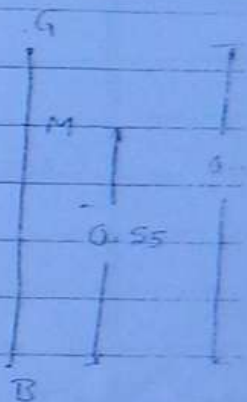
$$h = \frac{1800}{1000} = 1.8$$

$$BG = 1.5 - 0.9 = 0.6 \text{ m.}$$

$$BM = \frac{I}{V} = \frac{\frac{\pi}{64} \times 4^4}{\frac{\pi}{64} \times 4^2 \times 1.8} = 0.55$$

As $BM < BG$

then unstable equilibrium.



prob-53- A cylinder of density 600 kg/m^3 is required to float in an oil of density 900 kg/m^3 with its longitudinal axis vertical. If L is the height of the cylinder & D its diameter show that the condition for stable equilibrium is -

$$\frac{L}{D} < \frac{3}{4}$$

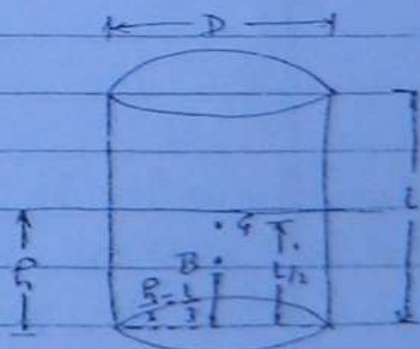
Soln:-

$$W_B = W_d$$

$$\rho_{\text{cyl}} \cdot g \cdot V_{\text{cyl}} = \rho_{\text{oil}} \cdot g \cdot V_{\text{oil displaced}}$$

$$600 \times g \times \frac{\pi}{4} \times D^2 \times L = 900 \times g \times \frac{\pi}{4} \times D^2 \times h$$

$$600L = 900h$$



$$h = \frac{600}{900} L = \frac{2}{3} L$$

$$h = 0.67 L$$

$$BG = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$$

$$BM = \frac{I}{V} = \frac{\pi/64 \times D^4}{\pi/64 \times D^4 \times \frac{2}{3} L} = \frac{3D^2}{32L}$$

Condition for stable equilibrium,

$$BM > BG$$

$$\frac{3D^2}{32L} > \frac{L}{6}$$

$$\frac{9D^2}{16L} > \frac{L^2}{D^2}$$

$$\boxed{\frac{L}{D} < \frac{3}{4}}$$

Fluid-Kinematics

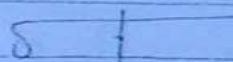
Kinematics deals with motion of fluid with any reference to force.

• Fluid flow analysed by using -

1. Lagrangian method.



2. Eulerian method



Type of fluid flow:-

1. Steady & Unsteady flow:-

A flow is said to be steady if the fluid properties do not vary with respect to time at any given section. Otherwise the flow is unsteady.

For steady flow,

$$V = \text{Const} \Rightarrow \frac{dV}{dt} = 0.$$

$$P = \text{Const} \Rightarrow \frac{dP}{dt} = 0.$$

2. Uniform & Non-uniform flow:-

A flow is said to be uniform if the velocity remains constant at different sections at any given instant of time. Otherwise the flow is non-uniform.

For Uniform flow,

$$\left[\frac{dv}{ds} = 0 \right]$$

3. Laminar & Turbulent flow :-

When fluid particles move in the form of layers which one layer sliding over the other then that flow is known as laminar flow. Laminar flow is generally occurs at low velocities.

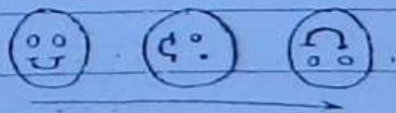
In turbulent flow fluid particles move in highly disorganised manner leading to rapid mixing of particles. Turbulent flow generally occurs at high velocity.

4. Rotational & Irrotational flow :-

A ~~fluid~~ flow is said to be a rotational when fluid particles rotate about their mass centres during motion. Otherwise the flow is irrotational.



Irrotational



Rotational

NOTE-

Irrotation $\rightarrow$ no rotation

$$\downarrow$$

$$T = 0$$

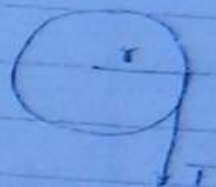
$\downarrow$

$$F = 0$$

$\downarrow$

$$F = \frac{\mu A V}{y}$$

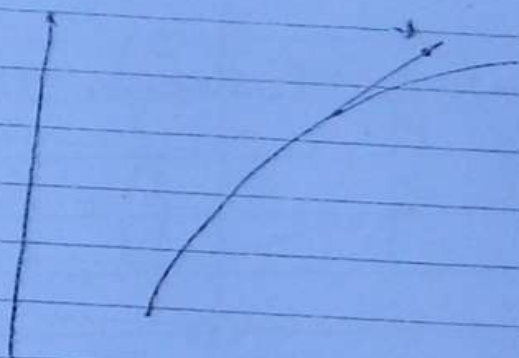
$$\boxed{\mu = 0} \quad \text{non-viscous}$$



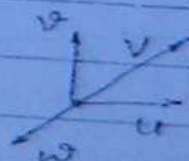
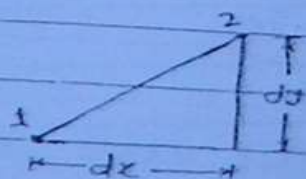
$$\boxed{T = F \times r}$$

Stream line :-

It is imaginary line or curve drawn in space such that a tangent drawn to it at any point gives velocity vector. There is flow along the stream line & there is no flow across the stream line.



Equation of stream line :-



Component

A fluid particle move from point 1 to 2. in (x, y, z) direction at time dt . let the horizontal distance travel from 1 to 2 dx & vertical dy . (u, v, w) are velocity

then, $u = \frac{dx}{dt}$ & $v = \frac{dy}{dt}$

$$\Rightarrow dt = \frac{dx}{u} \quad \& \quad dt = \frac{dy}{v}$$

then, $\frac{dx}{u} = \frac{dy}{v}$

$$v dx = u dy$$

$$\boxed{v dx - u dy = 0}$$

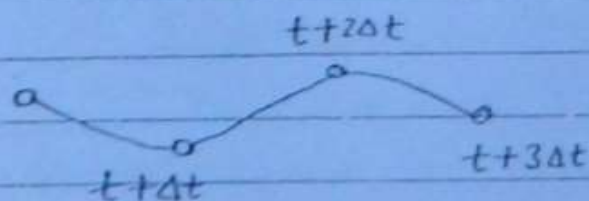
This is the equation of stream line in 2d

& $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$

This is the equation of stream line in 3d.

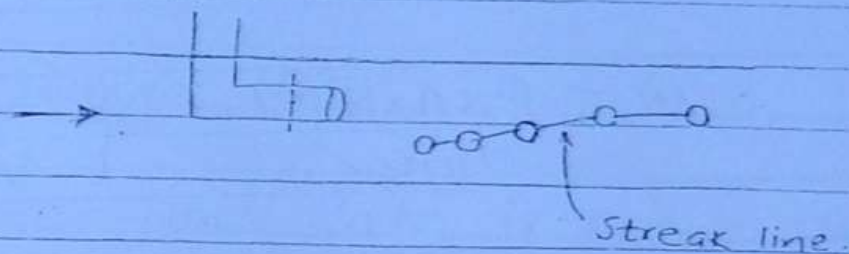
Path line :-

It is the path traced by a single fluid particle at different instant's of time.



!- Streak line :-

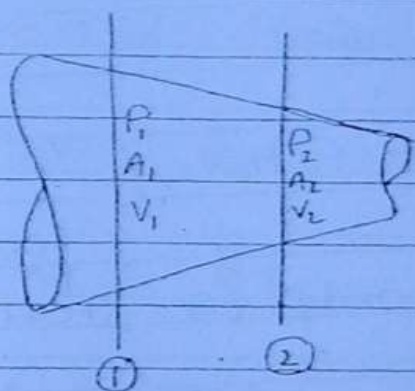
It is the locus of various fluid particle passing through a fixed point.



!- Continuity equation (Conservation of mass) :-

The equation based on the principle of conservation of mass is called continuity equation.

I. Steady one dimensional flow :-



We know that, mass flow rate,

$$\dot{m} = \frac{\text{mass}}{\text{time}} \quad \text{--- (i)}$$

↳ we also know that,

$$\rho = \frac{\text{mass}}{\text{vol.}}$$

then $m = P \times \text{Vol.}$

then from eqn. (1), we get,

$$\dot{m} = \frac{P \times \text{Vol.}}{\text{time}}$$

$$\dot{m} = \frac{P \times A \times L}{\text{time}}$$

$$\left\{ \begin{array}{l} \because \text{Vol.} = \text{Area} \times \text{length} \\ \text{Vol.} = A \times L \end{array} \right.$$

But distance covered by per unit time is velocity

$$\dot{m} = P \times A \times v$$

$$\dot{m} = P A v$$

For steady flow, the

$$\dot{m}_1 = \dot{m}_2$$

$$[P_1 A_1 v_1 = P_2 A_2 v_2]$$

This is valid only for steady one dimensional flow.

i.e. $[P A v = \text{Const.}]$

For Incompressible fluid,

$$P = \text{Const.}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$A_1 V_1 = A_2 V_2$$

This equation is valid for steady, one-dimensional & incompressible flow.

Discharge (Q) :-

Volume Flow Rate is known as discharge. It is also known as Rate of flow.

$$Q = \frac{\text{Vol.}}{\text{time}} = \frac{A \times L}{\text{time}} = A \times V$$

$$Q = AV \quad \text{where } V = \text{velocity.}$$

We know that for steady, one-dimensional & incompressible flow.

$$A_1 V_1 = A_2 V_2$$

$$Q_1 = Q_2$$

$$Q = \text{const.}$$

Discharge remains constant for steady one-dimensional incompressible flow.

Generalised Continuity equation:

$$\frac{\partial P}{\partial t} + \frac{\partial (Pu)}{\partial x} + \frac{\partial (Pv)}{\partial y} + \frac{\partial (Pw)}{\partial z} = 0$$

This equation is applicable for any type of fluid flow.

Case - a

Steady flow:-

$$\frac{dP}{dt} = 0$$

then,

$$\frac{\partial (Pu)}{\partial x} + \frac{\partial (Pv)}{\partial y} + \frac{\partial (Pw)}{\partial z} = 0$$

Case - b

Incompressible flow:-

$$P = \text{const.}$$

$$\frac{dP}{dt} = 0.$$

then,

$$0 + P \frac{\partial u}{\partial x} + P \frac{\partial v}{\partial y} + P \frac{\partial w}{\partial z} = 0$$

$$\rho \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] = 0$$

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \right]$$

This is the continuity eqn. for incompressible flow in 3-dimensional.

E-1. Continuity eqn. for incompressible flow in 2d is

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \right]$$

2. Every fluid flow must satisfy its corresponding continuity equation. If the continuity equation is violated then such a flow is impossible.

3. Acceleration of a fluid particle:-

$$a = \frac{dv}{dt}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

The velocity component is the function of space (x, y, z) & time

$$u = f(x, y, z, t)$$

Acceleration in x-direction is,

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt} + \frac{\partial u}{\partial t} \frac{dt}{dt}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$\therefore \frac{\partial x}{\partial t} = u, \quad \frac{\partial y}{\partial t} = v \quad \& \quad \frac{\partial z}{\partial t} = w$$

Similarly,

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$\& \quad a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Local Acceleration & Convective acceleration:-

The acceleration due to change of velocity with time is known as the temporal or local acceleration.

The acceleration due to change of velocity with space is known as Convective acceleration.

$$W_{cyl} + T = F_B$$

$$\rho_{cyl} \cdot g \cdot V_{cyl} + T = \rho_f \cdot g \cdot V_{fd}$$

$$\rho_s \cdot g \cdot A \cdot H + T = \rho \cdot g \cdot A \cdot R$$

$$T = \rho \cdot g \cdot A \cdot R - \rho_s \cdot g \cdot A \cdot H$$

$$\boxed{T = gA [\rho R - \rho_s H]} \text{ Ans.}$$

49- A body weighs 100N in air & 80N in water then find the density of body.

Ans: We know that the true weight of body is in air.

given, True weight of body in air $W = 100\text{N}$.

& Weight = mass $\times$ gravity.

$$W = m \times g$$

$$m = \frac{W}{g} = \frac{100}{9.81}$$

$$\text{Wt. loss} = F_B$$

$$100 - 80 = F_B$$

$$F_B = 20$$

$$\rho_f \cdot g \cdot V_{fd} = 20$$

$$1000 \times 9.81 \times V_{fd} = 20$$

$$V = \frac{20}{1000 \times 9.81}$$

we know that,

$$\text{Density} = \frac{\text{mass}}{\text{Vol.}}$$

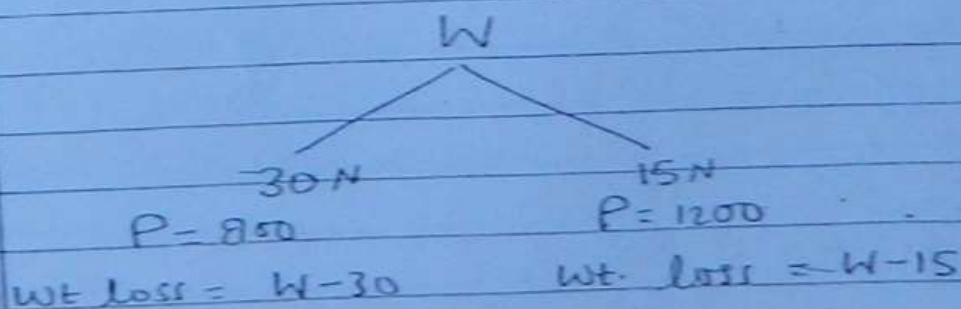
$$\rho = \frac{m}{V} = \frac{100}{9.81} \div \frac{20}{1000 \times 9.81}$$

$$\rho = \frac{100}{9.81} \times \frac{9.81 \times 1000}{20}$$

$$\rho = 5000 \text{ kg/m}^3 \text{ Ans.}$$

Qb-50 A body weighs 30N & 15N when immersed in liquids of density 800 kg/m³ & 1200 kg/m³ respectively. then find the volume of the body.

Solⁿ: let the total or true weight = W.



$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

↓
Convective acceleration

↓
Temporal
or
local
acceleration

Total acceleration = Convective + Temporal

Type of flow	Convective	Temporal
1. Steady & Uniform	0	0
2. Steady & non-uniform	Exist	0
3. Unsteady & Uniform	0	Exist
4. Unsteady & non-uniform	Exist	Exist

Q4. The flow is represented by,

$$\vec{V} = a x \hat{i} + a y \hat{j}$$

where a is constant then find the eqn of stream line passing through (1, 2)

The fundamental eqn. is.

$$\vec{V} = u \hat{i} + v \hat{j} + w \hat{k}$$

Compare the eqn.

$$u = ax \quad \& \quad v = ay.$$

We know that the eqn. of stream line in 2-d.

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \frac{dx}{ax} = \frac{dy}{ay}$$

$$\Rightarrow \frac{dx}{x} = \frac{dy}{y}$$

Integrate the above eqn, we get.

$$\ln x = \ln y + \ln C$$

$$\ln x = \ln yC$$

$$\boxed{x = yC} \quad \text{--- (1)}$$

Since the stream line is passing through (1,2)

$$\text{then } 1 = 2 \times C$$

$$C = \frac{1}{2}$$

then from eqn. (1),

$$x = \frac{y}{2}$$

$$2x = y$$

$$\boxed{2x - y = 0} \text{ Ans.}$$

35- Velocity component in x & y direction are given by,

$$\vec{u} = \lambda x y^3 - x^2 y$$

$$\vec{v} = x y^2 - \frac{3}{4} y^4$$

then find the value of λ for incompressible flow.

We know that, for incompressible flow continuity equation for 2-dimensional is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad \text{--- (1)}$$

given $u = \lambda x y^3 - x^2 y$

$$\Rightarrow \frac{\partial u}{\partial x} = \lambda y^3 - 2xy$$

& $v = x y^2 - \frac{3}{4} y^4$

$$\frac{\partial v}{\partial y} = 2xy - \frac{3 \times 4}{4} y^3 = 2xy - 3y^3$$

put the value in eqn. (1) we get.

$$\lambda y^3 - 2xy + 2xy - 3y^3 = 0.$$

$$\boxed{\lambda = 3} \text{ Ans.}$$

Prob-56 The velocity for a flow is given by,

$$\vec{V} = (5x+6y+7z)\hat{i} + (6x+5y+9z)\hat{j} + (3x+2y+\lambda z)\hat{k}$$

& the density varies as -

$$\rho = \rho_0 e^{-2t}$$

in order that mass is conserved find the value of λ .

Soln:- The density is vary with respect to time then flow is unsteady.

given, $\rho = \rho_0 e^{-2t}$ ——— ①

$$\frac{\partial \rho}{\partial t} = \rho_0 \cdot e^{-2t} [-2 \cdot (1)]$$

$$\frac{\partial \rho}{\partial t} = -2\rho_0 e^{-2t}$$

$$\frac{\partial \rho}{\partial t} = -2\rho \quad [\text{from eqn. ①}]$$

Here, $u = 5x+6y+7z \Rightarrow \frac{\partial u}{\partial x} = 5$

$$v = 6x+5y+9z \Rightarrow \frac{\partial v}{\partial y} = 5$$

$$w = 3x+2y+\lambda z \Rightarrow \frac{\partial w}{\partial z} = \lambda$$

Then the continuity eqn.

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x} (Pu) + \frac{\partial}{\partial y} (Pv) + \frac{\partial}{\partial z} (Pw) = 0$$

here P is vary with time not vary with space (x, y, z) .

then,
$$\frac{\partial P}{\partial t} + P \frac{\partial u}{\partial x} + P \frac{\partial v}{\partial y} + P \frac{\partial w}{\partial z} = 0.$$

$$-2P + P(5) + P(5) + P(\lambda) = 0.$$

$$8P + \lambda P = 0.$$

$$\boxed{\lambda = -8} \text{ Ans.}$$

Qb-57 A compressible fluid is ~~flowing~~ flowing steady through a pipe whose area decreases by 40% from section 1 to section 2. It is further known that the corresponding reduction in density is 15%. Compare to the velocity of fluid at section one. The velocity at section 2 is increased by a factor of -

- (a) 2.96 (b) 1.61 (c) 1.96 (d) 2.61

Q17: The Continuity Eqn. for compressible flow, steady.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

given, $A_2 = 0.6 A_1$ & $P_2 = 0.85 P_1$

then, $P_1 A_1 V_1 = 0.85 P_1 \times 0.6 A_1 \times V_2$

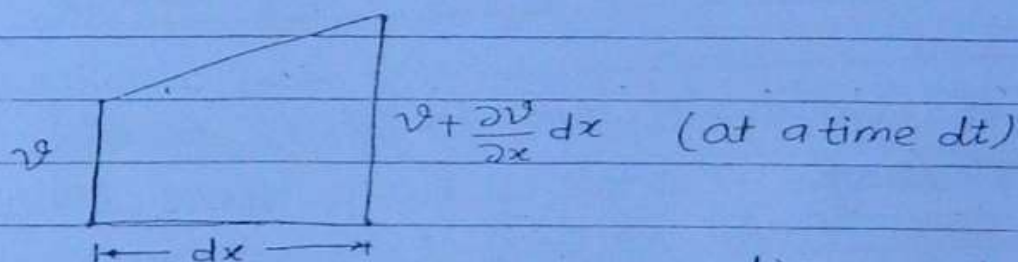
$$V_1 = 0.85 \times 0.6 \times V_2$$

$$V_2 = \frac{V_1}{0.85 \times 0.6}$$

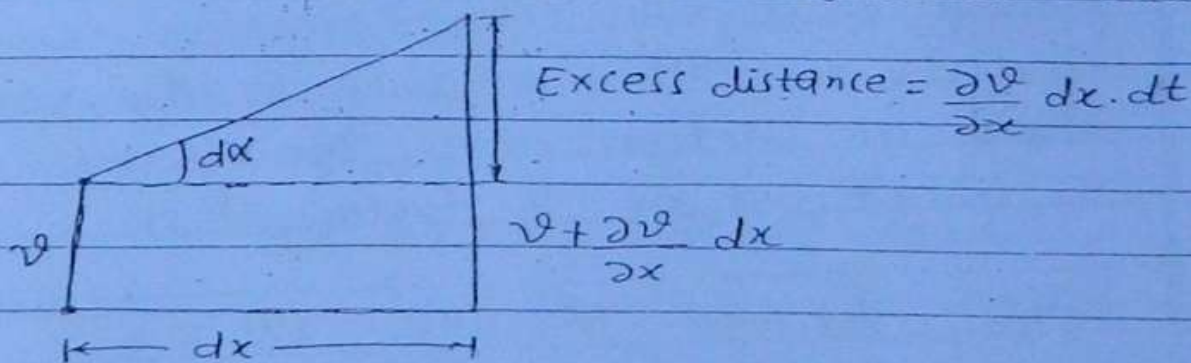
$$\boxed{V_2 = 1.96 V_1} \text{ Ans.}$$

Rotational Component's:

TE Euler's Expansion serie's.



distance = velocity $\times$ time



$$\tan \alpha = \frac{\frac{\partial v}{\partial x} \cdot dx \cdot dt}{dx}$$

$$\tan d\alpha = \frac{\partial v}{\partial x} dt$$

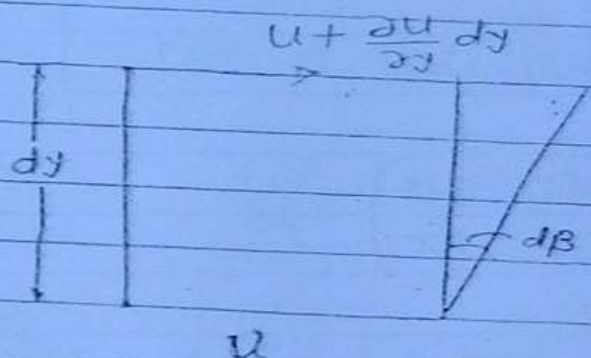
As the $d\alpha$ is very small then $\tan d\alpha = d\alpha$.

$$d\alpha = \frac{\partial v}{\partial x} dt$$

$$\boxed{\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}}$$

where $\frac{d\alpha}{dt}$ = Angular velocity.

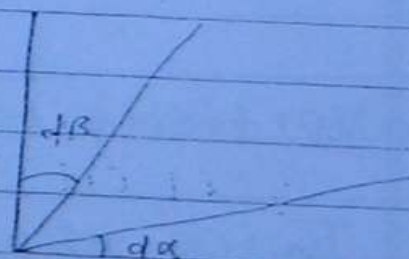
Similarly,



$$\boxed{\frac{d\beta}{dt} = -\frac{\partial u}{\partial y}}$$

(clock wise)

$$\omega_z = \frac{1}{2} \left(\frac{d\alpha}{dt} + \frac{d\beta}{dt} \right)$$



$$\boxed{\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)}$$

In fluid mechanics ω is defined as average

Angular velocity of initially two perpendicular line segment:

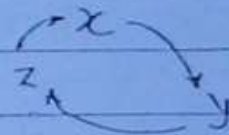
$$\omega_z = \frac{1}{2} \begin{vmatrix} \frac{\partial v}{\partial x} & \frac{\partial u}{\partial y} \end{vmatrix}$$

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$\omega = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\omega_x = \frac{1}{2} \begin{vmatrix} \frac{\partial w}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} & \frac{\partial w}{\partial x} \end{vmatrix}$$

$$\omega_y = \frac{1}{2} \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} & \frac{\partial u}{\partial y} \end{vmatrix}$$



$$\omega_z = \frac{1}{2} \begin{vmatrix} \frac{\partial v}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial w}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix}$$

Vorticity:

Twice the rotation is known as vorticity.

$$\omega = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Continuity eqn. for two-dimensional incompressible flow.

$$\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \right]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial x} \right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right]$$

Case-I- If ϕ satisfied Laplace Equation.

then, $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$

So that, $\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \right]$

Continuity eqn. is satisfied i.e. flow is possible.

Case-II- If ϕ does not satisfy Laplace Equation.

then $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \neq 0$

So that $\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0 \right]$

Continuity eqn. is not satisfied i.e. flow is not possible

Rotational Component,

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left[\frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial x} \right) \right]$$

$$\omega_z = \frac{1}{2} \left[\frac{-\partial^2 \phi}{\partial x \cdot \partial y} + \frac{\partial^2 \phi}{\partial y \cdot \partial x} \right]$$

$$\omega_z = \frac{1}{2} (0) \quad [\text{partial differentiation is equal \& opposite}]$$

$$\boxed{\omega_z = 0} \Rightarrow \text{flow is irrotational.}$$

NOTE- Velocity potential function exist only for irrotational flow. i.e. the existence of velocity potential function implies the flow is irrotational.

Qb-60- The two dimensional flow with

$$\vec{V} = (x+2y+2)\hat{i} + (4-y)\hat{j}$$

is -

- (a) Incompressible & Irrotational
- (b) Compressible & Irrotational
- (c) Compressible & not Irrotational
- (d) Incompressible & not Irrotational.

Q:- given, $\vec{V} = (x+2y+2)\hat{i} + (4-y)\hat{j}$

Compare with $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$u = x+2y+2 \Rightarrow \frac{\partial u}{\partial x} = 1 \quad \& \quad \frac{\partial u}{\partial y} = 2$$

$$v = 4-y \Rightarrow \frac{\partial v}{\partial y} = -1 \quad \& \quad \frac{\partial v}{\partial x} = 0$$

here $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + (-1) = 0$

then the fluid is incompressible fluid.

Rotational Component:

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$= \frac{1}{2} [0 - 2]$$

$$= -1$$

here $\omega_z \neq 0$.

then the flow is rotational.

Q:- Find the circulation around the circle if radius is 2 unit's for a velocity field.

$$u = 2x + 3y$$

$$v = -2y$$

Solⁿ: Circulation = Vorticity $\times$ Area.

$$\text{But Vorticity} = 2\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = (0-3) = -3$$

$$\therefore \text{Area} = \pi R^2 = \pi (2)^2 = 4\pi$$

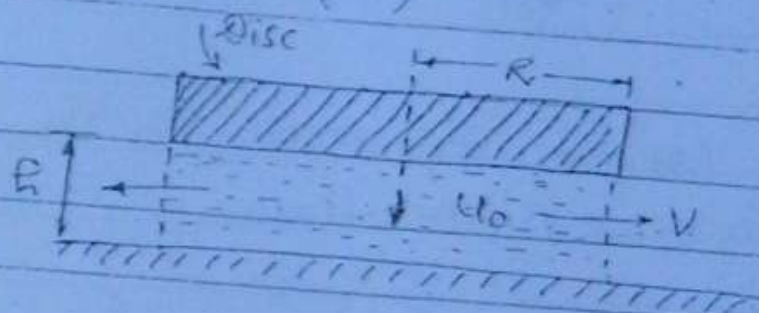
$$\text{then Circulation} = -3 \times 4\pi = -12\pi \text{ Aw.}$$

Prob-62 The x component of velocity in an in-compressible two dimensional flow is given by $u = 1.5x$ it is found that at a point (1,0) the y component velocity is zero then find y component velocity.

Solⁿ:

prob-63 - A circular disc with radius R is pushed with velocity u_0 towards & infinitely large plate with the gap ' h ' b/w two being occupied by a fluid as shown in fig. Assuming ideal flow the velocity v at the edge of disc.

- (a) $u_0 \left(\frac{R}{h} \right)$ (b) $u_0 \left(\frac{h}{R} \right)$ (c) $u_0 \left(\frac{R}{2h} \right)$ (d) $u_0 \left(\frac{h}{2} \right)$



Stream function (ψ) :-

It's a function of space & time defined in such a manner that it satisfied Continuity Equation.

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

The continuity eqn. for two dimensional incompressible flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= \frac{\partial}{\partial x} \left(-\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial x} \right) \\ &= -\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x} \\ &= 0 \end{aligned}$$

IE- Velocity potential function can be defined for three dimensional flow. but stream function is defined only for two dimensional flow.

The rotational component ω_z is given by

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$v = \frac{\partial \psi}{\partial x}, \quad u = -\frac{\partial \psi}{\partial y}$$

$$\omega_z = \frac{1}{2} \left[\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) - \frac{\partial}{\partial y} \left(-\frac{\partial \psi}{\partial y} \right) \right]$$

$$\omega_z = \frac{1}{2} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right]$$

Case-I - If ψ satisfied Laplace eqn.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

i.e. $\omega_z = \frac{1}{2} (0) = 0.$

$$\boxed{\omega_z = 0} \Rightarrow \text{flow is irrotational.}$$

Case-II - If ψ does not satisfy Laplace eqn.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \neq 0$$

i.e. $\boxed{\omega_z \neq 0} \Rightarrow \text{flow is rotational.}$

Velocity potential function exist only for irrotational flow but stream function exist both for rotational as well as irrotational.

If stream function satisfied Laplace eqn. the flow is irrotational.

∴ We know that from Stream function.

$$u = \frac{-\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

∴ we also know that eqn. of stream line.

$$\frac{dx}{u} = \frac{dy}{v}$$

$$v dx = u dy$$

$$v dx - u dy = 0$$

put the value of v & u .

$$\frac{\partial \psi}{\partial x} dx - \left(-\frac{\partial \psi}{\partial y} \right) dy = 0.$$

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad \text{--- (i)}$$

Since $\psi = f(x, y, t)$

for any instant time,

$$\psi = f(x, y)$$

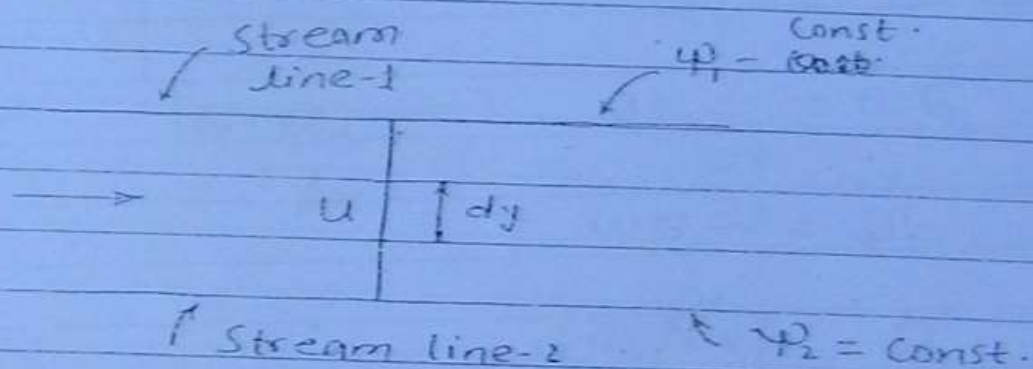
$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad \text{--- (ii)}$$

Comparing the eqn. ① & ②, we get.

$$d\psi = 0.$$

$$\boxed{\psi = \text{const.}}$$

For a particular stream line, stream function remains constant.



$$u = -\frac{\partial \psi}{\partial y} \quad (\text{for stream function})$$

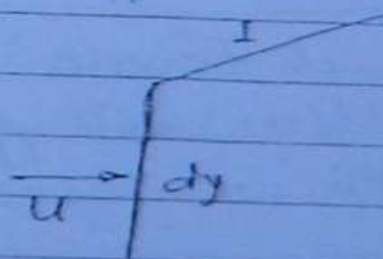
Discharge = Area $\times$ Velocity.

$$dQ = dx \times 1 \times u$$

$$dQ = u dy$$

$$dQ = -\frac{\partial \psi}{\partial y} dy$$

$$-dQ = \frac{\partial \psi}{\partial y} dy \quad \text{--- ①}$$

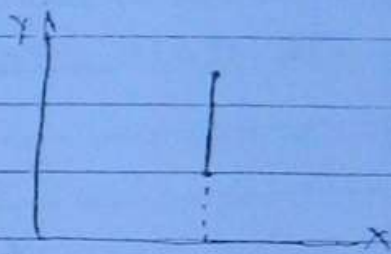


The stream function at any instant of time.

$$\psi = f(x, y)$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial y} dy$$



$$\partial \psi = -dQ \quad (\text{from eqn. ①})$$

$$x_1 = x_2 = dx = 0$$

$$dQ = -\partial \psi$$

$$Q = \int_{\psi_1}^{\psi_2} -\partial \psi$$

$$Q = -[\psi]_{\psi_1}^{\psi_2}$$

$$Q = -[\psi_2 - \psi_1]$$

$$Q = \psi_1 - \psi_2$$

* The difference b/w any two stream function will give discharge per unit width.

Relationship b/w Equipotential line & Constant Stream function line:

A line along which the velocity potential is constant, is called equipotential line.

$$\phi(x, y) = \text{Const.}$$

then,
$$\partial\phi = \frac{\partial\phi}{\partial x} dx + \frac{\partial\phi}{\partial y} dy = 0.$$

$$-\frac{\partial\phi}{\partial x} dx = -\frac{\partial\phi}{\partial y} dy$$

$$\frac{dy}{dx} = \frac{(\partial\phi/\partial x)}{-(\partial\phi/\partial y)}$$

$$\frac{dy}{dx} = -\frac{u}{v}$$

$$\begin{cases} u = -\frac{\partial\phi}{\partial y} \\ v = \frac{\partial\phi}{\partial x} \end{cases}$$

Slope of equipotential line = $-\frac{u}{v}$
--

Line of constant stream function,

$$\psi(x, y) = \text{Const.}$$

$$\partial\psi = \frac{\partial\psi}{\partial x} dx + \frac{\partial\psi}{\partial y} dy = 0$$

$$\frac{\partial\psi}{\partial x} dx = -\frac{\partial\psi}{\partial y} dy$$

$$\frac{dy}{dx} = \frac{(\partial\psi/\partial x)}{(-\partial\psi/\partial y)}$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\boxed{\frac{dy}{dx} = \frac{v}{u}}$$

Slope of constant stream ~~line~~ function line = $\frac{v}{u}$

$$\text{product of slope} = \frac{-u}{v} \times \frac{v}{u} = -1$$

Equipotential line & constant stream function line are orthogonal to each other.

Cauchy - Reiman (C-R) :-

$$u = -\frac{\partial\phi}{\partial x}, \quad v = -\frac{\partial\phi}{\partial y}$$

$$\& \quad u = -\frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}$$

$$\text{then, } \frac{-\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y}$$

$$\text{Vorticity} = 2\vec{\omega} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

For,

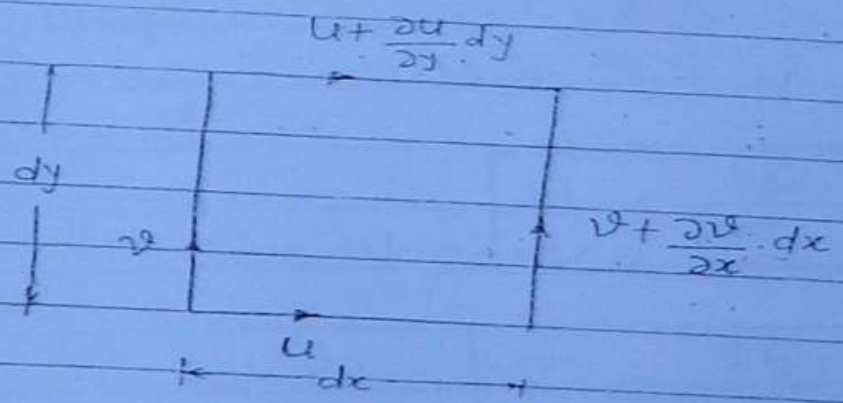
$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

This is the vorticity of z component.

Circulation $\therefore (\Gamma) :-$

It is the line integral of tangential component of velocity taken around a closed curve.



$$\Gamma = \int u dx + \left(v + \frac{\partial v}{\partial x} dx \right) dy - \left(u + \frac{\partial u}{\partial y} dy \right) dx - v dy$$

$$\Gamma = u dx + v dy + \frac{\partial v}{\partial x} dx dy - u dx - \frac{\partial u}{\partial y} dx dy - v dy$$

$$\Gamma = \int \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx dy$$

$$\boxed{\text{Circulation} = \text{Vorticity} \times \text{Area.}}$$

$$\therefore \omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$2\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

↙
vorticity.

Velocity potential function (ϕ) :-

It's a function of space & time defined in such a manner that its negative derivative with respect to space gives velocity in that direction.

$$\boxed{-\frac{\partial \phi}{\partial x} = u}$$

$$\boxed{-\frac{\partial \phi}{\partial y} = v}$$

$$\boxed{-\frac{\partial \phi}{\partial z} = w}$$

$$\left. \begin{aligned} \frac{\partial \phi}{\partial x} &= \frac{\partial \psi}{\partial y} \\ -\frac{\partial \phi}{\partial y} &= \frac{\partial \psi}{\partial x} \end{aligned} \right\} \text{Cauchy-Riemann}$$

Qb-64 - The velocity field is given by -

$$\vec{V} = 2y \hat{i} + 3x \hat{j}$$

x & y in meter then find the acceleration of the particle at (1,1) in x -direction.

Soln: given, $\vec{V} = 2y \hat{i} + 3x \hat{j}$

Compare with $\vec{V} = u \hat{i} + v \hat{j} + w \hat{k}$

$$u = 2y \Rightarrow \frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = 2, \frac{\partial u}{\partial z} = 0$$

$$\& \quad v = 3x \Rightarrow$$

Acceleration of particle in x -direction,

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 2y(0) + 3x(2) + 0 + 0$$

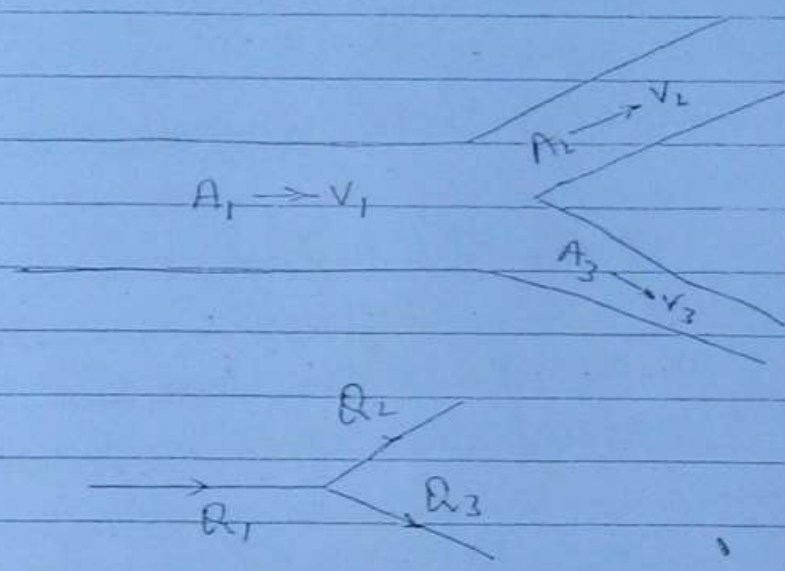
$$= 6x$$

$$\text{at } (1,1) \Rightarrow a_x = 6 \times 1 = 6 \text{ m/s}^2. \text{ Ans.}$$

prob 65

Water enters a pipe of cross-sectional area A_1 that branches out into a section of equal area A_2 & A_3 at one instant flow velocity are $V_1 = 2 \text{ m/s}$, $V_2 = 3 \text{ m/s}$ & $V_3 = 5 \text{ m/s}$ at another instant $V_1 = 3 \text{ m/s}$, $V_2 = 4 \text{ m/s}$. find V_3 .

Soln:



$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

Initially, $V_1 = 2 \text{ m/s}$, $V_2 = 3 \text{ m/s}$, $V_3 = 5 \text{ m/s}$

then, $A_1 (2) = A_2 (3) + A_3 (5)$

$$2A_1 = 3A_2 + 5A_3$$

$$2A_1 = 8A_2 \quad [\because A_2 = A_3 \text{ given}]$$

$$A_1 = 4A_2$$

Page _____
finally, $V_1 = 3 \text{ m/s}$, $V_2 = 4 \text{ m/s}$, $V_3 = ?$

$$\therefore Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$A_1 (3) = A_2 (4) + A_3 8 \times V_3$$

$$3A_1 = 4A_2 + A_3 V_3$$

$$3(4A_2) = 4A_2 + A_3 V_3 \quad [\because A_1 = 4A_2]$$

$$12A_2 = 4A_2 + A_3 V_3$$

$$A_3 V_3 = 8A_2$$

$$V_3 = 8 \text{ m/s} \quad [\because A_2 = A_3]$$

b-66- The stream function for 2 dimensional incompressible flow is given by,

$$\psi = Px^2 + Qy^2$$

where P & Q are constant. the velocity potential function for this flow exist only when -

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

$\frac{\partial \psi}{\partial x} = 2Px$	$\& \frac{\partial \psi}{\partial y} = 2Qy$
$\frac{\partial^2 \psi}{\partial x^2} = 2P$	$\frac{\partial^2 \psi}{\partial y^2} = 2Q$

then $2P + 2Q = 0$

$$[P = -Q]$$

A velocity potential function exist then the flow is irrotational & for irrotational flow stream function must satisfy's Laplace eqn.

prob-67 In steady flow through a nozzle the flow velocity at nozzle axis is given by,

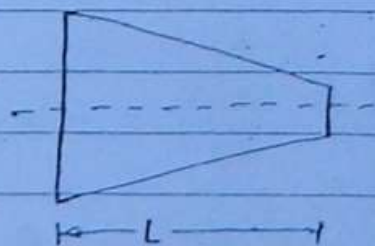
$$\vec{V} = u_0 \left(1 + \frac{3x}{L} \right) \hat{i}$$

where x is the distance along the axis of nozzle from inlet & L is the length of nozzle. then find the time required for the fluid particle on the axis to travel from inlet to exit.

soln: given, $\vec{V} = u_0 \left(1 + \frac{3x}{L} \right) \hat{i}$

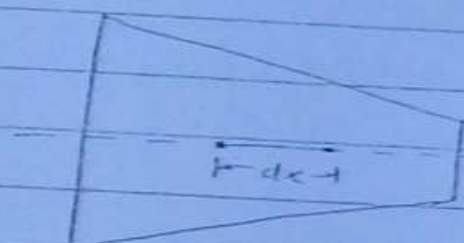
Compare with $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$u = u_0 \left(1 + \frac{3x}{L} \right)$$



$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{velocity}}$$



time taken by small particle from inlet to exit in small section.

$$dt = \frac{dx}{u_0 \left(1 + \frac{3x}{L}\right)}$$

$$\therefore t = \int_0^L \frac{dx}{u_0 \left(1 + \frac{3x}{L}\right)}$$

$$t = \frac{1}{u_0} \int_0^L \frac{dx}{\left(1 + \frac{3x}{L}\right)}$$

$$t = \frac{1}{u_0} \left[\ln \left(1 + \frac{3x}{L}\right) \right]_0^L$$

$$t = \frac{1}{3u_0} \left[\ln \left(1 + \frac{3L}{L}\right) - \ln \left(1 + \frac{3(0)}{L}\right) \right]$$

$$t = \frac{1}{3u_0} (\ln 4) \text{ Ans.}$$

prob-68- For a 2-d irrotational flow the velocity potential function is defined as

$$\phi = \ln(x^2 + y^2)$$

then which one of the following is a possible stream function.

(a) $2 \sin^{-1}\left(\frac{y}{x}\right)$ (b) $2 \tan^{-1}(y)$

(c) $2 \tan^{-1}\left(\frac{y}{x}\right)$ (d) $2 \cos^{-1}\left(\frac{y}{x}\right)$

Soln:-

$$\phi = \ln(x^2 + y^2)$$

$$\frac{\partial \phi}{\partial x} = \frac{1}{x^2 + y^2} \cdot (2x + 0)$$

$$\frac{\partial \phi}{\partial x} = \frac{2x}{x^2 + y^2}$$

From Cauchy Reiman Eqn.

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{2x}{x^2 + y^2} = \frac{\partial \psi}{\partial y}$$

$$\partial \psi = \frac{2x}{x^2 + y^2} \partial y$$

$$\frac{\partial \phi}{\partial x} = \frac{2x}{x^2 \left(1 + \frac{y^2}{x^2}\right)} \frac{\partial y}{\partial y}$$

$$\frac{\partial \phi}{\partial y} = \frac{2}{x \left(1 + \frac{y^2}{x^2}\right)} \frac{\partial y}{\partial y}$$

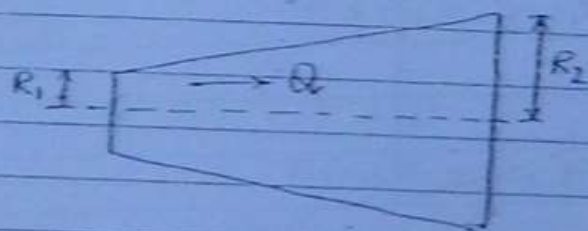
put $\frac{y}{x} = t \Rightarrow y = xt$

$$\boxed{\phi = 2 \tan^{-1} \left(\frac{y}{x} \right)} \quad \text{Ans.}$$

Q- For a liquid flow through a divergent passage of length L having inlet & outlet radii of R_1 & R_2 & a constant flow rate of Q find the acceleration at the exit of pipe. assume steady one dimensional incompressible flow.

For steady one dimensional compressible fluid

$$\boxed{Q = \text{const.}}$$



$$a = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$a = a_x \hat{i} \quad (\text{one-dimensional})$$

$$Q = Q_x$$

$$\& Q_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$\text{Steady flow} \rightarrow \frac{\partial u}{\partial t} = 0$$

$$\text{One dimensional flow} \rightarrow v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 0$$

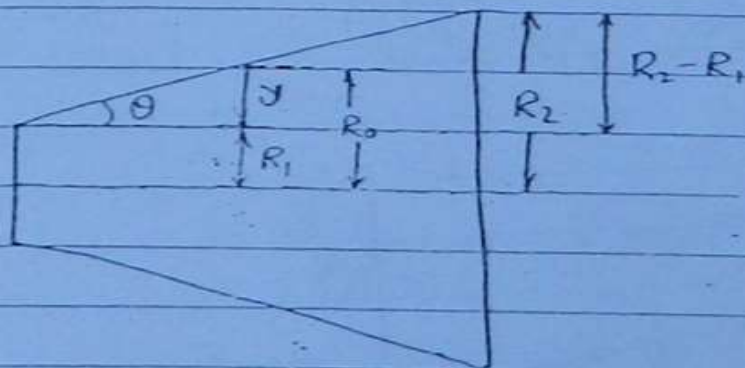
$$\text{then, } Q_x = u \frac{\partial u}{\partial x}$$

$$Q = Q_x = u \frac{\partial u}{\partial x}$$

$$\because Q = A \times u \Rightarrow u = \frac{Q}{A}$$

$$Q = \frac{Q}{A} \cdot \frac{\partial}{\partial x} \left(\frac{Q}{A} \right)$$

$$Q = \frac{Q^2}{A} \cdot \frac{\partial}{\partial x} \left(\frac{1}{A} \right) \quad [\because Q = \text{const}]$$



from geometry of figure.

$$\tan \theta = \frac{y}{x} = k$$

$$y = x \tan \theta$$

$$R = R_1 + y$$

$$R = R_1 + kx$$

$$A = \pi R^2$$

$$A = \pi (R_1 + kx)^2$$

then, $Q = \frac{Q^2}{\pi (R_1 + kx)^2} \cdot \frac{\partial}{\partial x} \left[\frac{1}{\pi (R_1 + kx)^2} \right]$

$$Q = \frac{Q^2}{\pi^2 (R_1 + kx)^2} \cdot \frac{\partial}{\partial x} \left[\frac{1}{(R_1 + kx)^2} \right]$$

$$Q = \frac{Q^2}{\pi^2 (R_1 + kx)^2} \cdot \left[\frac{-2k}{(R_1 + kx)^3} \right]$$

$$Q = \frac{-2Q^2 k}{\pi^2 (R_1 + kx)^5}$$

$$Q = \frac{-2Q^2}{\pi^2 (R_1 + kx)^5} \cdot \left(\frac{R_2 - R_1}{L} \right)$$

$$\frac{R_2 - R_1}{L} = k \Rightarrow R_2 = R_1 + kL$$

At exit, $x = L$

then, $Q = \frac{-2Q^2}{\pi^2(R_1 + KL)^5} \left(\frac{R_2 - R_1}{L} \right)$

$$\boxed{Q = \frac{-2Q^2}{\pi^2 \times R_2^5} \left(\frac{R_2 - R_1}{L} \right)} \text{ Ans.}$$

Q-70 The stream function is given by,

$$\psi = 3xy$$

then find the velocity at (2,3).

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$|\vec{V}| = \sqrt{u^2 + v^2}$$

$$u = -\frac{\partial \psi}{\partial y} = -3x$$

$$v = \frac{\partial \psi}{\partial x} = 3y$$

At point (2,3).

$$u = -3x = -3(2) = -6$$

$$v = 3y = 3(3) = 9$$

then, $V = \sqrt{(-6)^2 + 9^2}$

$$\boxed{V = 10.82 \text{ m/s.}} \text{ Ans.}$$

Fluid-Dynamic's

In General force's acting on fluid element's are pressure force F_p , gravity force F_g & viscous force F_v .

If all these three force's are taken into account then that analysis is known as Navier Stokes's Analysis.

If viscous force are neglected then the force's acting on fluid element are pressure force F_p & gravity force F_g .

In Euler's Analysis pressure & gravity force's are taken into account.

Bernoulli's Equation (Conservation of Energy)

Assumption's :-

Non-viscous.

Steady flow.

Incompressible flow.

Euler's Equation of motion :-

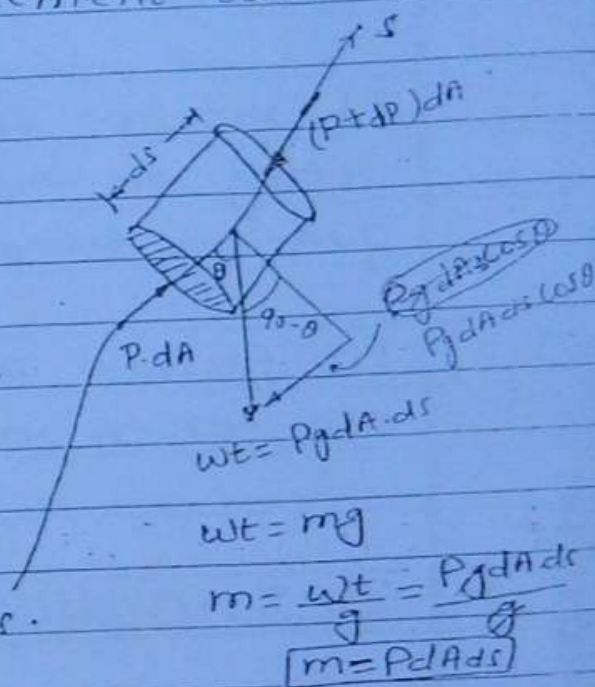
This is the Equation of motion in which the force due to gravity & pressure are taken into consideration. This is deriving by considering

the motion of fluid element along a stream line as-

Consider a stream line in which flow is taking place in s -direction.

Consider a cylindrical fluid element of cross-section dA & length ds . The force acting on the cylindrical element are:

1. Pressure force $P \cdot dA$ in the direction of flow.
2. pressure force $(P + dP)A$ in opposite to the direction of flow. ($dP = +ve$ or $-ve$).
3. weight of element $Pg \cdot dA \cdot ds$.



Let θ is the angle b/w the direction of flow & the line of action of the weight of element

From newton 2nd law,

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element $\times$ acceleration in the direction of s .

$$P \cdot dA - (P + dP) \cdot dA - P \cdot g \cdot dA \cdot ds \cos \theta = P \cdot dA \cdot ds \left(\frac{v \cdot dv}{ds} \right)$$

$$P - (P + dP) - \rho g ds \cos \theta = \rho ds \left[v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right]$$

$$-dP - \rho g ds \cos \theta = \rho ds \left[v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right]$$

$$\boxed{-dP - \rho g dz = \rho ds \left[v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right]} \leftarrow \text{Euler's eqn. for unsteady flow.}$$

If flow is steady then $\frac{\partial v}{\partial t} = 0$.

$$-dP = \rho g dz = \rho ds \left[v \frac{\partial v}{\partial s} \right]$$

$$-dP - \rho g dz = \rho ds \left[v \frac{dv}{ds} \right]$$

$$-dP - \rho g dz = \rho v dv$$

$$\boxed{dP + \rho g dz + \rho v dv = 0} \leftarrow \text{Euler eqn. for steady flow.}$$

$$\frac{dP}{\rho} + \frac{\rho g dz}{\rho} + \frac{\rho v dv}{\rho} = 0$$

$$\frac{dP}{\rho} + g dz + v dv = 0$$

Integrate the above eqn.

$$\int \frac{dP}{\rho} + \int g dz + \int v dv = \text{const.}$$

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Const.}$$

$$\left| \frac{P}{\rho} + \frac{v^2}{2} + gz = \text{const.} \right|$$

In the above equation each term represent energy of the fluid per unit mass.

i.e. In steady incompressible, non-viscous, flow along stream line sum of press. energy, kinetic energy & potential energy is constant.

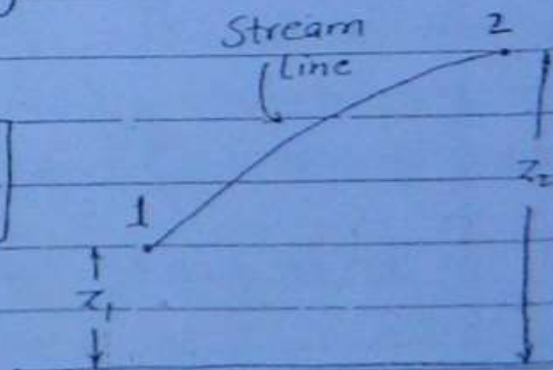
$$\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{Const.}$$

$$\left| \frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{const.} \right|$$

This is Bernoulli's equation.

In this eqn. each term's represent Energy of the fluid per unit weight.

$$\left| \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 \right|$$



Relationship b/w first law of thermodynamics & Bernoulli's equation:-

From 1st law of thermodynamics in SFEE we know that,

$$h_1 + \frac{v_1^2}{2} + z_1 g + Q = h_2 + \frac{v_2^2}{2} + z_2 g + W$$

$$[\because h = u + Pv]$$

$$u_1 + P_1 v_1 + \frac{v_1^2}{2} + z_1 g + Q = u_2 + P_2 v_2 + \frac{v_2^2}{2} + z_2 g + W$$

Assumption:-

1. $u_1 = u_2$ i.e. Internal Energy remains constant

2. $Q = 0$, i.e. No Heat transfer.

3. $W = 0$, i.e. No work transfer.

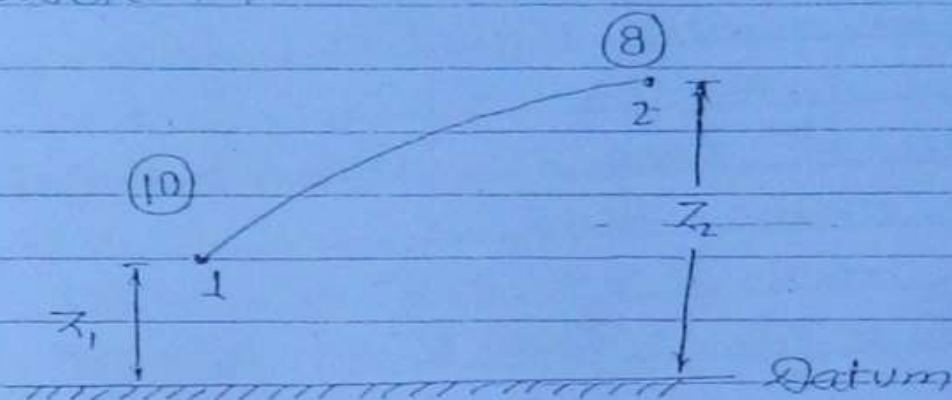
4. Incompressible

$$\text{then, } P_1 v_1 + \frac{v_1^2}{2} + z_1 g = P_2 v_2 + \frac{v_2^2}{2} + z_2 g$$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g = \frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g$$

$$\boxed{\frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g = \frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g}$$

Bernoulli's Equation for a real fluid flow problem :-



If Here, $\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = 10$

& $\frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 = 8$

then $10 = 8 + 2$

$$\left[\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L \right]$$

$\frac{P}{w} =$ Pressure head.

$\frac{V^2}{2g} =$ K.E. head or velocity head.

$z =$ Datum or P.E. head.

$\left(\frac{P}{w} + z \right) =$ piezometric head.

V.IMP

NOTE-

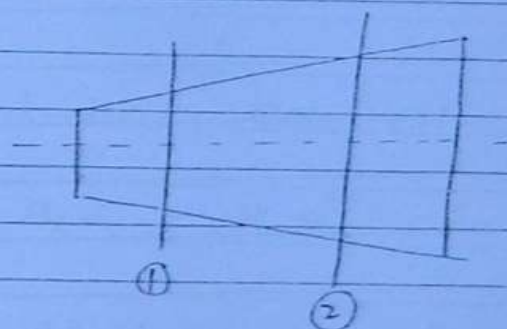
In Case of irrotation flow bernoulli's equation is applicable through out the flow field (not necessarily along stream line) because the value of constant is same for different stream line's.

In Case of rotational flow bernoulli's equation is applicable along stream line because the value of constant is different for different stream line.

Prob-71- Consider ideal flow through a horizontal nozzle. the area ratio at two section 1 & 2 is $A_1/A_2 = 2$. Then find the press. coefficient

$$\frac{P_1}{P_2} = \frac{1}{2} \frac{P V_1^2}{P V_2^2}$$

Soln:-



Apply bernoulli's equation b/w 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

$$\therefore Z_1 = Z_2$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2g}$$

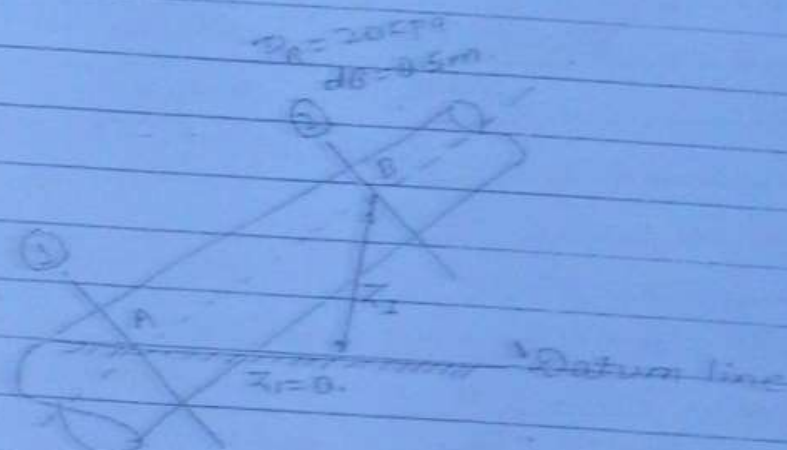
$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{P_1 - P_2}{\rho} = \frac{V_1^2}{2} \left[\left(\frac{V_2}{V_1} \right)^2 - 1 \right]$$

$$\frac{P_1 - P_2}{\frac{1}{2} \rho V_1^2} = \frac{V_2^2}{V_1^2} - 1$$

Prob 72- At a point A in a pipe line carrying water the diameter is 1m, press is 98 kPa. & the velocity is 1 m/s. at a point B, 2m higher than A the dia. is 0.5m & the press. is 20 kPa. the direction of flow would be -

- (a) A to B (b) B to A (c) Can not say (d) None



$$P_A = 98 \text{ kPa}$$

$$V_A = 1 \text{ m/s}$$

$$d_A = 1 \text{ m}$$

Apply Continuity eqn. at section 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

$$d_1^2 V_1 = d_2^2 V_2$$

$$d_A^2 V_A = d_B^2 V_B$$

$$1 \times 1 = 0.5^2 \times V_B$$

$$V_B = 4 \text{ m/s.}$$

Total Energy at point A.

$$TE_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$$

$$= \frac{98 \times 10^3}{9810} + \frac{1^2}{2 \times 9.81} + 0$$

$$= 10.04$$

Total Energy at point B.

$$TE_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$= \frac{20 \times 10^3}{9810} + \frac{4^2}{2 \times 9.81} + 2$$

$$= 4.85$$

Then the flow direction is A to B.

Application's of Bernoulli's Equation :-

1. Venturimeter

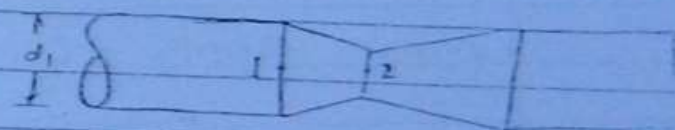
A venturimeter is a device used for measuring the rate of flow of fluid flowing through pipe.

It contains of three parts -

- (i) A short converging part.
- (ii) Throat &
- (iii) Diverging part.

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing.



let d_1 = diameter at inlet or section 1.

P_1 = Press. at section 1.

V_1 = velocity of fluid at section 1.

& d_2, P_2, V_2 = Corresponding value at section 2.

Now apply Bernoulli's equation at section 1 & 2

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$[\rho g = w]$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ = difference of press. head at section 1 & 2
 $= h$

then, $h = \frac{v_2^2 - v_1^2}{2g}$

$$v_2^2 - v_1^2 = 2gh \quad \text{--- ①}$$

Now apply continuity eqn. at section 1 & 2.

$$Q_1 v_1 = Q_2 v_2$$

Discharge, $Q = a_1 v_1$, $Q = a_2 v_2$

$$v_1 = \frac{Q}{a_1} \quad , \quad v_2 = \frac{Q}{a_2}$$

put the value in eqn. ①,

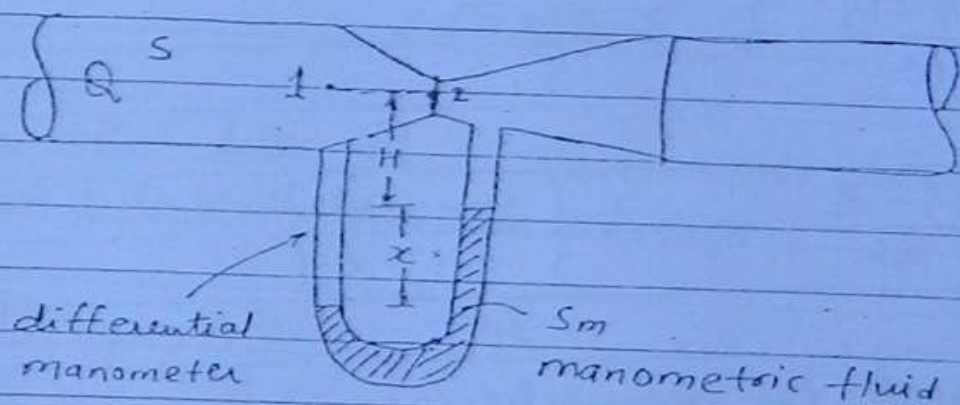
$$\left(\frac{Q}{a_2}\right)^2 - \left(\frac{Q}{a_1}\right)^2 = 2gh$$

$$Q^2 \left[\frac{a_1^2 - a_2^2}{a_1^2 a_2^2} \right] = 2gh$$

$$Q^2 = \frac{a_1^2 a_2^2 2gh}{(a_1^2 - a_2^2)}$$

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

∴ Value of 'h' given by differential U-tube manometer



x = manometric fluid deflection.

From continuity eqn. at section 1 & 2.

$$a_1 V_1 = a_2 V_2$$

↓
small

$$a_2 < a_1$$

$$V_2 > V_1$$

Apply bernoulli's eqn. b/w section 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\text{As } V_2 > V_1$$

$$\therefore P_2 < P_1$$

from manometric principle,

$$\frac{P_1}{w} + H + x - x \frac{S_m}{S} - H = \frac{P_2}{w}$$

$$\frac{P_1 - P_2}{w} = x \frac{S_m}{S} - x$$

$$\boxed{\frac{P_1 - P_2}{w} = x \left(\frac{S_m}{S} - 1 \right)}$$

Principle of venturimeter:

By decreasing area in a steady one dimension incompressible flow velocity increases & this increase & this increase in velocity result's in decrease in press., this press. difference result's in manometric fluid deflection when connected by finding manometric fluid deflection discharge can be found

Coefficient of discharge (C_d):

It is defined as the ratio of actual discharge to theoretical discharge.

As venturimeter is gradually converging & diverging device losses are less & hence C_d is equal to 0.94 to 0.98.

$$C_d = \frac{Q_{act.}}{Q_{theor.}}$$

$$Q_{act.} = C_d Q_{th.}$$

$$Q_{act.} = C_d \times a_1 a_2 \sqrt{2gH} \sqrt{a_1^2 - a_2^2}$$

Consider the loss h_L , then apply Bernoulli's eqn. at section 1 & 2 in horizontal pipe.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + h_L$$

$$\left(\frac{P_1}{\rho} - \frac{P_2}{\rho} \right) - h_L = \frac{V_2^2 - V_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho} = h$

then, $h - h_L = \frac{V_2^2 - V_1^2}{2g}$

but $Q_{act} = a_1 v_1$; $Q_{act} = a_2 v_2$

$$v_1 = \frac{Q_{act}}{a_1} ; v_2 = \frac{Q_{act}}{a_2}$$

then, $h - h_L = \frac{1}{2g} \left[\frac{Q_{act}^2}{a_2^2} - \frac{Q_{act}^2}{a_1^2} \right]$

$$2g(h - h_L) = Q_{act}^2 \left[\frac{a_1^2 - a_2^2}{a_1^2 a_2^2} \right]$$

$$Q_{act}^2 = \frac{a_1^2 a_2^2 \cdot 2g(h - h_L)}{a_1^2 - a_2^2}$$

$$Q_{act} = \frac{a_1 a_2 \sqrt{2g(h - h_L)}}{\sqrt{a_1^2 - a_2^2}} \quad \text{--- (i)}$$

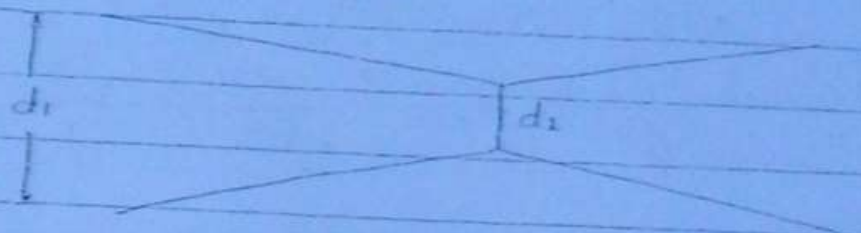
also $Q_{act} = \frac{C_d a_1 a_2 \sqrt{2gH}}{\sqrt{a_1^2 - a_2^2}} \quad \text{--- (ii)}$

then from eqn. (i) & (ii),

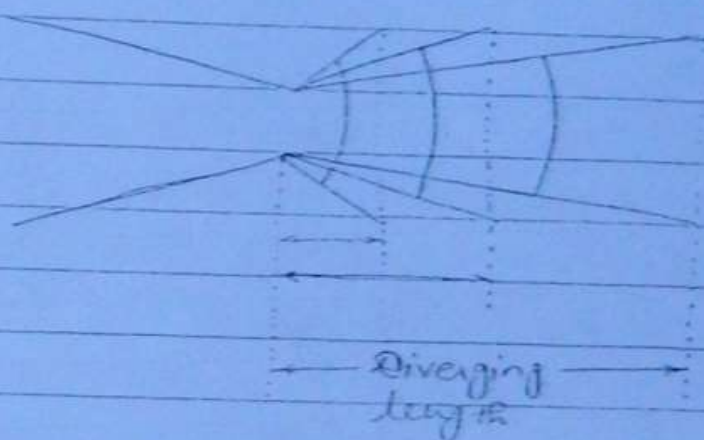
$$\frac{C_d a_1 a_2 \sqrt{2gH}}{\sqrt{a_1^2 - a_2^2}} = \frac{a_1 a_2 \sqrt{2g(h - h_L)}}{\sqrt{a_1^2 - a_2^2}}$$

$$C_d = \sqrt{\frac{h - h_L}{H}}$$

General proportions of venturimeter:

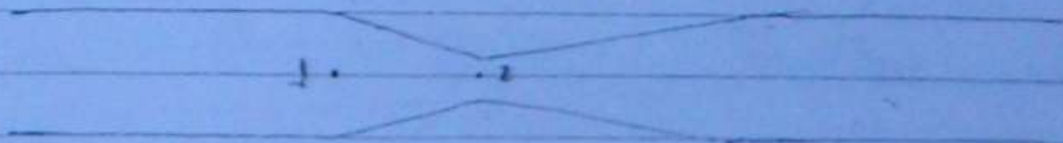


$$d_2 = \left(\frac{1}{3} \text{ to } \frac{1}{2} \right) d_1$$



By increasing the angle of divergent then the length of divergent is decreases.

- 73- A venturimeter of 20 mm throat dia. is fitted in a horizontal pipe of 40 mm. dia. the press. difference b/w pipe & throat section is 30 Kpa. Water is flowing through pipe. then find velocity in pipe.



given $d_1 = 40\text{mm}$ & $d_2 = 20\text{mm}$.

Apply Bernoulli's equation at section 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

As the pipe is horizontal $Z_1 = Z_2$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{30 \times 10^3}{10^3} = \frac{V_2^2 - V_1^2}{2}$$

$$V_2^2 - V_1^2 = 60 \quad \text{--- (1)}$$

Apply continuity eqn. b/w section 1 & 2.

$$Q_1 V_1 = Q_2 V_2$$

$$\frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} d_2^2 \times V_2$$

$$40^2 \times V_1 = 20^2 \times V_2$$

$$4V_1 = V_2$$

then from eqn. (1), we get.

$$(4V_1)^2 - V_1^2 = 60$$

$$16V_1^2 - V_1^2 = 60$$

$$15V_1^2 = 60$$

$$V_1 = 2 \text{ m/s. Ans.}$$

14. Water flow through a horizontal pipe of 5 cm^2 area to measure discharge a venturimeter of 3 cm^2 throat area is connected in pipe line. If the pressure head difference b/w pipe & throat section is 10 cm of water find the discharge in cm^3/sec .

given, $a_1 = 5 \text{ cm}^2$, $a_2 = 3 \text{ cm}^2$

$h = 10 \text{ cm.}$ Take $C_d = 1.$

The discharge,

$$Q = \frac{C_d a_1 a_2 \sqrt{2gH}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = \frac{1 \times 5 \times 3 \times \sqrt{2 \times 981 \times 10}}{\sqrt{5^2 - 3^2}} = 525 \text{ cm}^3/\text{sec}$$

Ans.

prob-75

Two venturimeter of different area ratio are connected at different location of same pipe line similar manometer's are used across two venturimeter's to measure discharge, The first venturimeter of area ratio 2. Resister's a press. head difference 'R' & the second venturimeter resister's '5R' then find the area ratio of 2nd venturimeter.

Soln:

The discharge of ^{1st} venturimeter,

$$Q = \frac{a_1 a_2 \sqrt{2gR}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = \frac{a_1 a_2 \sqrt{2gR}}{a_2 \sqrt{\left(\frac{a_1}{a_2}\right)^2 - 1}}$$

$$Q = \frac{a_1 \sqrt{2gR}}{\sqrt{a_1^2 - 1}}$$

Similarly discharge of 2nd venturimeter.

$$Q = \frac{a_1 \sqrt{2g(5R)}}{\sqrt{a_1^2 - 1}}$$

$$\frac{1}{\sqrt{a_{r_1}^2 - 1}} = \frac{\sqrt{5}}{\sqrt{a_{r_2}^2 - 1}}$$

$$\frac{1}{a_{r_1}^2 - 1} = \frac{5}{a_{r_2}^2 - 1}$$

$$\frac{1}{2^2 - 1} = \frac{5}{a_{r_2}^2 - 1} \quad [\because a_{r_1} = 2] \text{ give}$$

$$\frac{1}{3} = \frac{5}{a_{r_2}^2 - 1}$$

$$a_{r_2}^2 - 1 = 15$$

$$a_{r_2}^2 = 16$$

$$a_{r_2} = 4 \text{ Ans.}$$

Qb-7C- A venturimeter is installed in a horizontal pipe line of 400 mm dia. the throat dia. is $\frac{1}{3}$ of pipe dia. the press. in the pipe line is 1.405 kgf/cm² & vacume in throat is 37.5 cm of Hg. water flows through pipe & if 4% of differential head is lost b/w gauge find the discharge in pipe.

Solⁿ:- pipe dia. $d_1 = 400 \text{ mm} = 0.4 \text{ m.}$

throat dia. $d_2 = \frac{1}{3} d_1 = \frac{1}{3} \times 400 = 133.33 \text{ mm} = 0.133 \text{ m.}$

then $a_1 = \frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$

$$\& \quad a_2 = \frac{\pi}{4} \times 0.133^2 = 0.0138 \text{ m}^2$$

$$P_1 = 1.405 \text{ kgf/cm}^2$$

$$= \frac{1.405 \times 9.81}{(10^{-2})^2} \text{ N/m}^2 \quad [\because 1 \text{ kgf} = 9.81 \text{ N}]$$

$$= 1.375 \times 10^5 \text{ N/m}^2.$$

$$\& \quad P_2 = 37.5 \text{ cm of Hg.}$$

$$= -P_g h$$

$$= -13.6 \times 10^3 \times 9.81 \times 37.5 \times 10^{-2} \text{ (-ve for vacume)}$$

$$= -50031 \text{ N/m}$$

$$\& \quad h_L = 0.4 h \quad (\text{given})$$

In case of horizontal pipe the press. head difference is,

$$h = \frac{P_1 - P_2}{w}$$

$$= \frac{1.375 \times 10^5 + 50031}{9810}$$

$$= 19.14 \text{ m.}$$

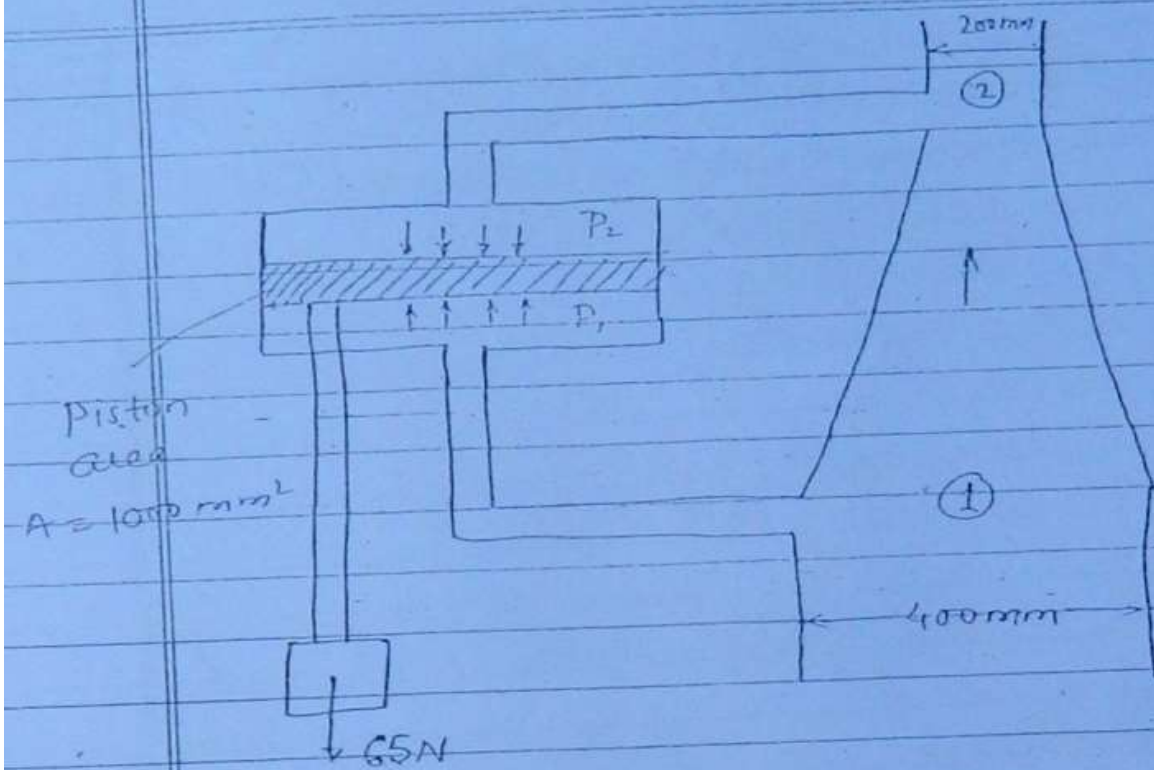
Then Co-efficient of discharge, (C_d):

$$\begin{aligned} C_d &= \sqrt{\frac{h - h_c}{h}} \\ &= \sqrt{\frac{19.14 - 0.04}{19.14}} \\ &= 0.979 \end{aligned}$$

Then discharge,

$$\begin{aligned} Q &= C_d a_1 a_2 \sqrt{2gH} \\ &\quad \sqrt{a_1^2 - a_2^2} \\ &= \frac{0.979 \times 0.1256 \times 0.0138 \times \sqrt{2 \times 9.81 \times 19.14}}{\sqrt{(0.1256)^2 - (0.0138)^2}} \\ &= 0.266 \text{ m}^3/\text{sec. Ans.} \end{aligned}$$

Oil of specific gravity of 0.8 flows upward to a vertical venturimeter fitted in a 400mm dia. pipe line the throat dia. is 200 mm calculate the discharge through a venturimeter if a total weight of 55N is required to maintain the piston in equilibrium neglect potential energy change, frictional losses & cross-sectional area of piston rod.



Soln: given, pipe line dia. = $d_1 = 400 \text{ mm} = 0.4 \text{ m}$.

Throat dia. = $d_2 = 200 \text{ mm} = 0.2 \text{ m}$.

$$\therefore Q_1 = \frac{\pi}{4} \times 0.4^2 = 0.1256 \text{ m}^2$$

$$Q_2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

If the potential energy is neglected then the press. head difference.

$$h = \frac{P_1 - P_2}{w} \quad \text{--- (1)}$$

$$\text{where } w = \rho g = 800 \times 9.81 = 7848 \text{ N/m}^3$$

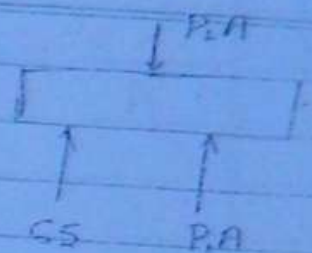
Net force acting on the piston,

$$P_2 A + GS = P_1 A$$

$$GS = (P_1 - P_2) A$$

$$P_1 - P_2 = \frac{GS}{A_{\text{piston}}} = \frac{65}{1000 \times (10^{-3})^2}$$

$$P_1 - P_2 = 65 \times 10^3$$



then from eqn. ①, we get.

$$h = \frac{P_1 - P_2}{\rho g} = \frac{65 \times 10^3}{7848}$$

$$h = 8.28 \text{ m.}$$

discharge,

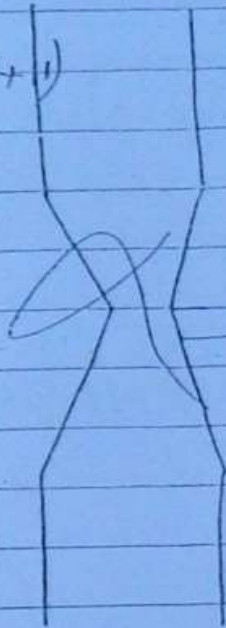
$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.1256 \times 0.0314 \sqrt{2 \times 9.81 \times 8.28}}{\sqrt{(0.1256)^2 - (0.0314)^2}}$$

$$= 0.413 \text{ m}^3/\text{sec. Aw.}$$

The press. difference across a venturimeter as shown in fig. is measured with the help of a mercury manometer. Then find the expression for velocity of water at throat.

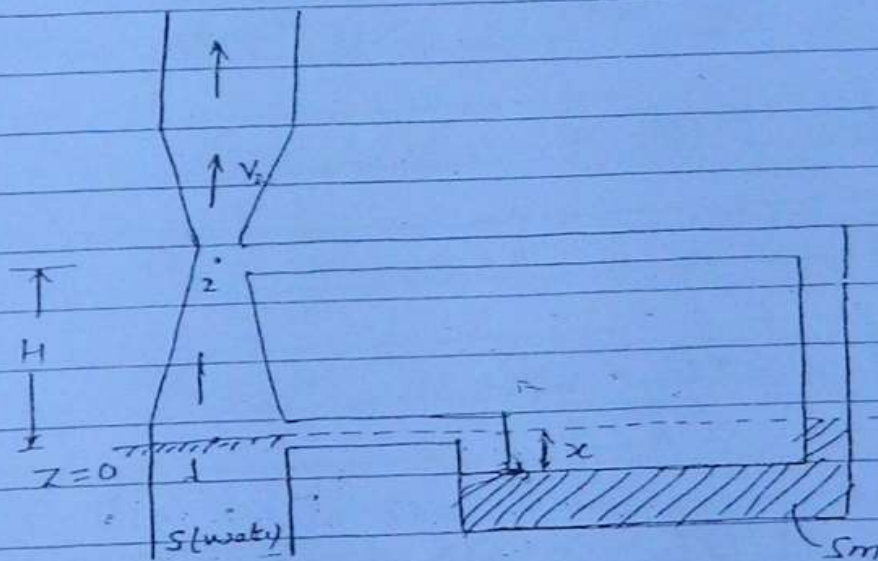
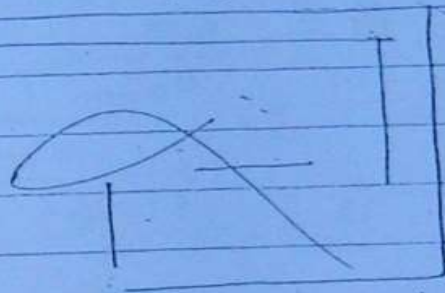
$$d) \frac{v_2^2 - v_1^2}{2g} = x \left(\frac{S_m}{S} + 1 \right)$$



$$a) \frac{v_2^2 - v_1^2}{2g} = x \left(\frac{S_m}{S} - 1 \right)$$

$$b) \frac{v_2^2 - v_1^2}{2g} = x \left(\frac{S_m}{S} \right)$$

$$c) \frac{v_2^2}{2g} = H + x \left(\frac{S_m}{S} - 1 \right)$$



Soln:- Apply bernoulli's eqn. at section 1 & 2.

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} + 0 = \frac{P_2}{\rho} + \frac{v_2^2}{2g} + H$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} - H = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} - H = \frac{V_2^2 - V_1^2}{2g} \quad \text{--- (I)}$$

From manometric principle.

$$\frac{P_1}{\rho g} + x - x \frac{S_m}{S} - H = \frac{P_2}{\rho g}$$

$$[\because x S_m = h S]$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} - H = \frac{x S_m}{S} - x$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} - H = x \left(\frac{S_m}{S} - 1 \right) \quad \text{--- (II)}$$

from eqn. (I) & (II), we get

$$\frac{V_2^2 - V_1^2}{2g} = x \left(\frac{S_m}{S} - 1 \right)$$

Discharge equation remain's same irrespective of positioning of venturimeter.

In horizontal,

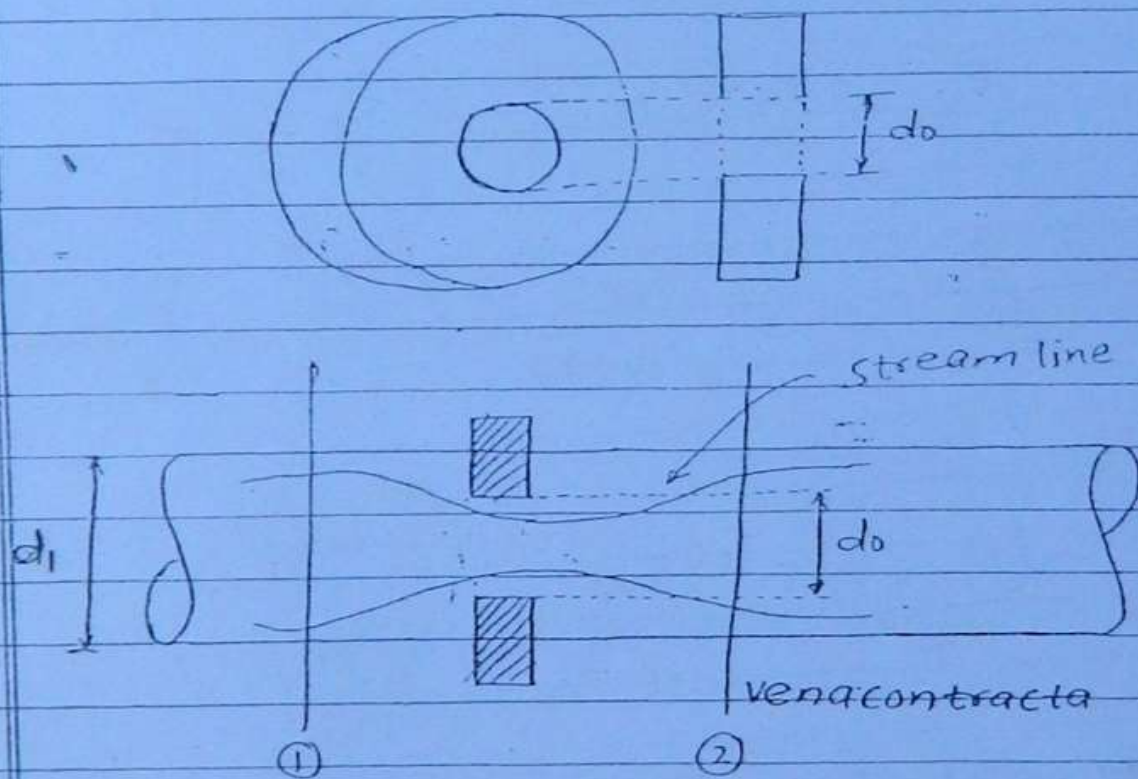
$$h = \frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$h = x \left(\frac{S_m}{S} - 1 \right)$$

2. Orificemeter

It is a device used for measuring the rate of flow of a fluid (discharge) through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter.

~~It~~ It consists of a flat circular plate which has a circular sharp edge hole called orifice, which is concentric with the pipe.



Coefficient of contraction :- (C_c)

$$C_c = \frac{\text{Area of vena contracta}}{\text{Area of orifice}} = \frac{a_2}{a_o}$$

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$$C_c = \frac{Q_2}{Q_0}$$

$$Q_2 = C_c \times Q_0 \quad \text{--- (1)}$$

Apply bernoulli's eqn. at section 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

As the pipe is horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2g} = h \text{ (Differential head)}$$

$$\Rightarrow V_2^2 - V_1^2 = 2gh \quad \text{--- (11)}$$

Now apply continuity eqn. b/w ① & 2.

$$Q_1 V_1 = Q_2 V_2$$

$$V_1 = \frac{Q_2 V_2}{Q_1}$$

$$V_1 = \frac{C_c \times Q_0 \times V_2}{Q_1} \quad \text{[from eqn. (1)]}$$

Now put the value in eqn. (11) we get.

$$V_2^2 - \frac{C_c^2 a_0^2 V_2^2}{a_1^2} = 2gh$$

$$V_2^2 \left[1 - \frac{C_c^2 a_0^2}{a_1^2} \right] = 2gh$$

$$V_2^2 = \frac{2gh}{1 - \frac{C_c^2 a_0^2}{a_1^2}}$$

$$V_2 = \sqrt{\frac{2gh}{1 - \frac{C_c^2 a_0^2}{a_1^2}}}$$

Discharge,

$$Q = a_2 V_2$$

$$= C_c a_0 \sqrt{2gh} \sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}$$

$$= \frac{C_c a_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}} \times \frac{\sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{a_0^2}{a_1^2}}}$$

$$C_d = \frac{C_c \sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}} \quad C_v$$

$$C_d = C_c \cdot C_v$$

then, $Q = \frac{C_d \cdot a_0 \sqrt{2gH}}{\sqrt{1 - \frac{a_0^2}{a_1^2}}}$

$$Q = \frac{C_d \cdot a_1 a_0 \sqrt{2gH}}{\sqrt{a_1^2 - a_0^2}}$$

where a_0 = orifice area.
 a_1 = Pipe area.

7- As area reduction is sudden in orificemeter, losses are more & hence C_d is equal to 0.68 to 0.76.

80- An orificemeter is being used for measuring flow rate of a liquid in a pipeline shows a press. head difference of x m. of water column when the flow rate is Q . If the flow rate is doubled then what is the press. head difference in m of water column.

$$Q = \frac{a_1 a_0 \sqrt{2gH}}{\sqrt{a_1^2 - a_0^2}}$$

$$Q \propto \sqrt{H}$$

$$\frac{Q_2}{Q_1} = \frac{\sqrt{H_2}}{\sqrt{H_1}}$$

$$\text{given } Q_2 = 2Q_1$$

$$\frac{2Q_1}{Q_1} = \frac{\sqrt{H_2}}{\sqrt{x}}$$

$$[h_L = 4x] \text{ Ans.}$$

Prob-81- An orificemeter with $C_d = 0.61$ is substituted by venturimeter with $C_d = 0.98$ in a pipeline carrying oil having same dia. as that of orifice meter dia. for the same flow rate the ratio of press. drop for venturimeter to that of orificemeter.

(a) $\frac{0.61}{0.98}$ (b) $\frac{0.98}{0.61}$ (c) $\frac{0.61^2}{0.98^2}$ (d) $\frac{0.98^2}{0.61^2}$

Soln: Discharge is same for ~~pipeline~~ orifice & venturimeter.

$$Q_v = Q_o$$

$$\frac{C_{dv} a_1 a_2 \sqrt{2g h_v}}{\sqrt{a_1^2 - a_2^2}} = \frac{C_{do} a_1 a_0 \sqrt{2g h_o}}{\sqrt{a_1^2 - a_0^2}}$$

but $d_2 = d_0 \Rightarrow a_2 = a_0$

$$C_{dv} \sqrt{h_v} = C_{do} \sqrt{h_o}$$

$$C_{dv} \times \sqrt{\frac{\Delta P_v}{w}} = C_{do} \times \sqrt{\frac{\Delta P_o}{w}}$$

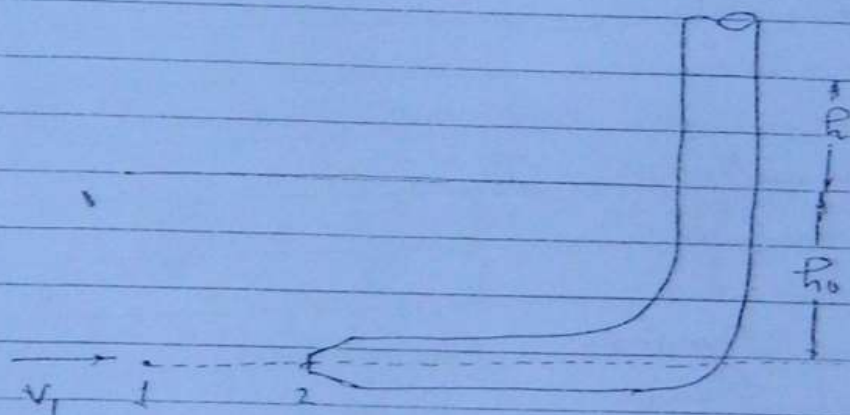
$$C_{dv}^2 \times \Delta P_v = C_{do}^2 \times \Delta P_o$$

$$\frac{\Delta P_v}{\Delta P_o} = \frac{C_{do}^2}{C_{dv}^2} = \frac{0.61^2}{0.98^2} \text{ Ans.}$$

3. Pitot tube

It is a device used for measuring ~~velocity~~ the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity of flow at a point become zero, the press. there ~~is~~ is increased due to conversion of the kinetic energy into press. energy.



$$\rho g h + \rho g h_0 = P_2$$

$$(h + h_0) \rho g = P_2$$

$$\frac{P_2}{\rho g} = h + h_0$$

Apply bernoulli's eqn b/w 1 & 2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2$$

As $z_1 = z_2$ & $v_2 = 0$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} = \frac{P_2}{\rho}$$

$$h_0 + \frac{v_1^2}{2g} = (h + h_d)$$

$$\frac{v_1^2}{2g} = h \leftarrow \text{Dynamic Head.}$$

$$\left\{ \begin{array}{l} \because P_1 = P_2 = P_0 \\ \frac{P_1}{\rho g} = h_0 \\ \frac{P_1}{\rho} = h_0 \end{array} \right.$$

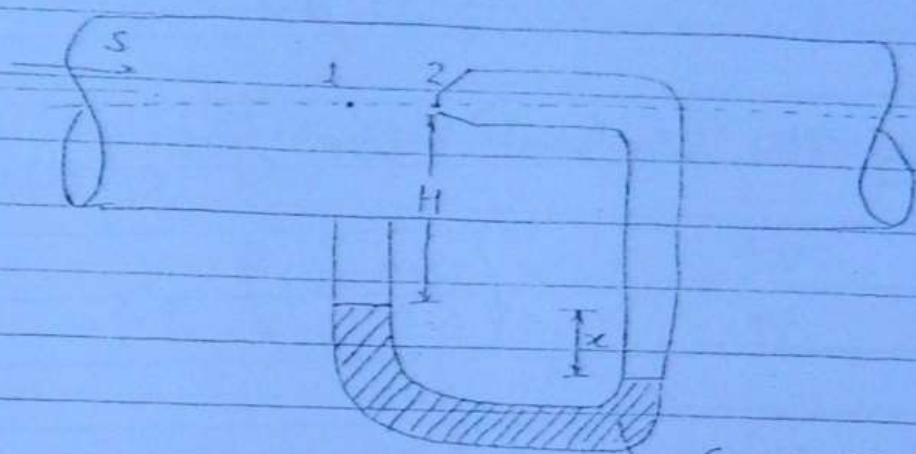
$$\boxed{v_1 = \sqrt{2gh}}$$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} = \frac{P_2}{\rho}$$

$\downarrow$ $\downarrow$ $\downarrow$
 Static Head Dynamic Head Stagnation Head

$$\boxed{\text{Static Head} + \text{Dynamic Head} = \text{Stagnation Head}}$$

velocity of flow in a pipe by pitot-static probe



S_m = manometric fluid.

Apply bernoulli equation b/w 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

As $Z_1 = Z_2$ & $V_2 = 0$.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho}$$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho} \quad \text{--- (1)}$$

Apply manometric principle,

$$\frac{P_1}{\rho} + H - x \frac{S_m}{S} - x - H = \frac{P_2}{\rho}$$

$$x \frac{S m}{S} - x = \frac{P_2}{w} - \frac{P_1}{w}$$

$$x \left(\frac{S m}{S} - 1 \right) = \frac{P_2 - P_1}{w} \quad \text{--- (11)}$$

from eqn. (1) & (11) we get.

$$\frac{v_1^2}{2g} = x \left(\frac{S m}{S} - 1 \right)$$

$$v_1^2 = 2g x \left(\frac{S m}{S} - 1 \right)$$

$$v_1 = \sqrt{2g x \left(\frac{S m}{S} - 1 \right)}$$

4. Force on pipe - Bend

Momentum equation:-

It is based on the law of conservation of momentum or on the momentum principle, which state that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

The force acting on a fluid mass m is given by the newton's second law of motion.

$$\Sigma F = m \times a$$

where a is the acceleration acting in the same direction as F .

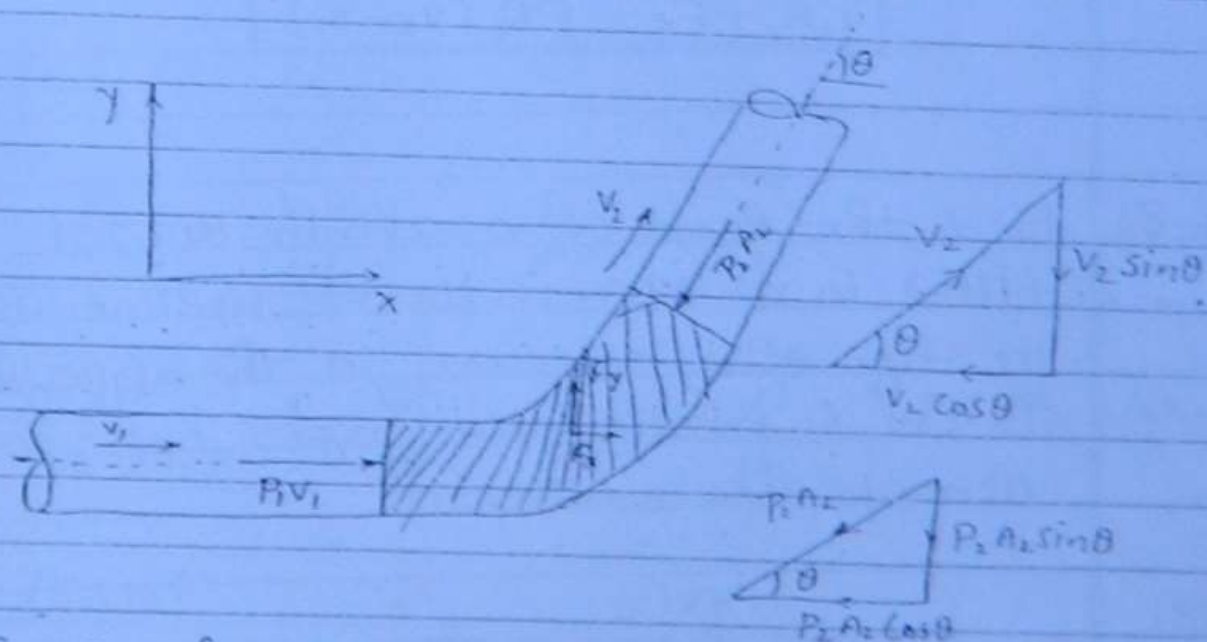
$$\Sigma F = m \left(\frac{v_2 - v_1}{t} \right)$$

$$\Sigma F = \dot{m} (v_2 - v_1)$$

$$\boxed{\Sigma F = P Q (v_2 - v_1)}$$

$$\left\{ \begin{array}{l} \because \dot{m} = P A V \\ \text{but } Q = A V \\ \dot{m} = P \times Q \end{array} \right.$$

Force exerted by a flowing fluid on a pipe-bend



F_x & F_y is force exerted by pipe or bend on the fluid.

Momentum equation in x -direction;

$$P_1 A_1 + F_x - P_2 A_2 \cos \theta = \rho Q (v_2 \cos \theta - v_1)$$

↳ Momentum equation in y -direction.

$$F_y - P_2 A_2 \sin \theta = \rho Q (v_2 \sin \theta - 0)$$

$$F_y - P_2 A_2 \sin \theta = \rho Q v_2 \sin \theta$$

NOTE-

Horizontal pipe when discharging fluid at atmospheric conditions then $\theta = 0^\circ$ & $P_2 = 0$ (atm.) then $F_y = 0$ (because horizontal).

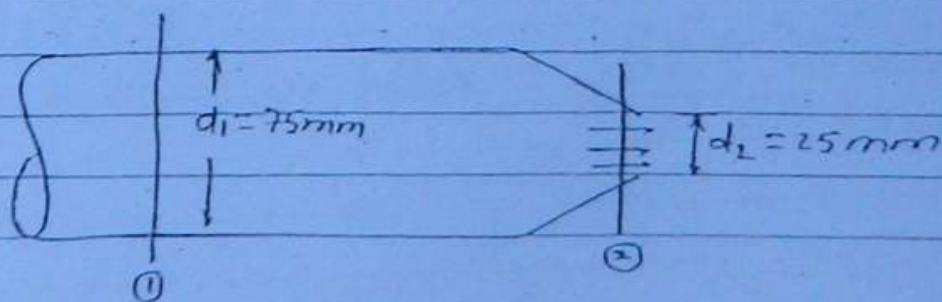
then from eqn. (1),

$$P_1 A_1 + F_x - 0 = \rho Q (v_2 \cos 0 - v_1)$$

$$P_1 A_1 + F_x = \rho Q (v_2 - v_1)$$

Prob-82

Find the force on a 25mm dia. of nozzle fitted in a 75mm dia of pipeline as shown in fig. the pressure in the pipe line is 75 kPa & nozzle discharges out-water at atmospheric conditions.



Solⁿ:- Apply continuity eqn. b/w 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times d_2^2 \times V_2$$

$$d_1^2 \times V_1 = d_2^2 \times V_2$$

$$75^2 \times V_1 = 25^2 \times V_2$$

$$\boxed{9V_1 = V_2}$$

Apply bernoulli's eqn. b/w 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

As $Z_1 = Z_2$.

then, $\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$

$$\frac{75 \times 10^3}{10^3 \times 9.81} + \frac{V_1^2}{2g} = 0 + \frac{V_2^2}{2g} \quad [\because P_2 = 0 \text{ atm}]$$

$$\frac{75 \times 10^3}{10^3 \times 9.81} + \frac{V_1^2}{2 \times 9.81} = \frac{(9V_1)^2}{2 \times 9.81}$$

$$\begin{aligned} 75 \times 10^3 + 80V_1^2 &= 80V_1^2 \\ 80V_1^2 &= 75 \times 10^3 \end{aligned}$$

$$75 \times 10^3 + \frac{V_1^2}{2} = \frac{81 V_1^2}{2}$$

$$V_1 = \sqrt{\frac{75 \times 10^3 \times 2}{80}}$$

$$V_1 = 1.369 \text{ m/s.}$$

$$V_2 = 9V_1 = 9 \times 1.369 = 12.321 \text{ m/s.}$$

discharge,

$$Q = A_1 V_1$$

$$= \frac{\pi}{4} \times d_1^2 \times V_1$$

$$= \frac{\pi}{4} \times (75 \times 10^{-3})^2 \times 1.369$$

$$= 6.04 \times 10^{-3} \text{ m}^3/\text{sec.}$$

Force :-

Apply momentum eqn.

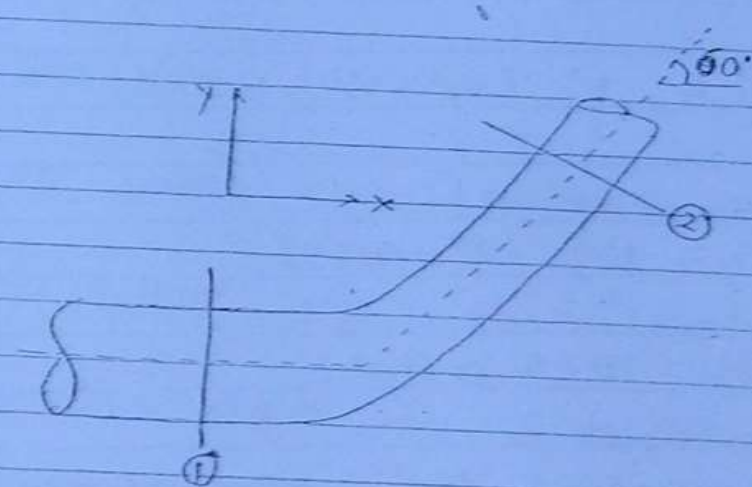
$$P_1 A_1 + F_x = P_2 Q (V_2 - V_1)$$

$$75 \times 10^3 \times \frac{\pi}{4} \times (75 \times 10^{-3})^2 + F_x = 10^3 \times 6.04 \times 10^{-3} \times (12.321 - 1.369)$$

$$\boxed{F_x = -265 \text{ N.}} \quad \text{Ans.}$$

prob-83 - A stream of water enters a pipe bend in horizontal plane through section 1. at the rate of $0.5 \text{ m}^3/\text{sec}$. as shown in fig. water is discharge to atmosphere at section 2 the area of cross-section 1 & 2 are 1 m^2 & 0.25 m^2 the central axis of the pipe makes an angle of 60° with x-direction neglect friction through out the pipe calculate -

- (i) velocity V_2 (ii) pressure P_1
 (iii) Force required to hold the pipe in x-direction



given, $Q = 0.5 \text{ m}^3/\text{sec}$
 $A_1 = 1 \text{ m}^2$
 $A_2 = 0.25 \text{ m}^2$

discharge,

$$Q = A_1 V_1 \quad \& \quad Q_2 = A_2 V_2$$

$$0.5 = 1 \times V_1 \quad 0.5 = 0.25 \times V_2$$

$$\boxed{V_1 = 0.5} \quad V_2 = 2 \text{ m/s.}$$

Apply Bernoulli equation b/w 1 & 2.

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + \frac{v_2^2}{2g}$$

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + \frac{v_2^2}{2g} \quad [P_2 = 0 \text{ atm. condition}]$$

$$\frac{P_1}{\rho} = \frac{v_2^2 - v_1^2}{2g}$$

$$\frac{P_1}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$P_1 = \frac{\rho}{2} (v_2^2 - v_1^2)$$

$$P_1 = \frac{10^3}{2} (2^2 - 0.5^2)$$

$$P_1 = 1875 \text{ N/m}^2$$

Apply momentum eqn. ~~between 1 & 2~~ in x-direction

$$P_1 A_1 + F_x - P_2 A_2 \cos \theta = \rho Q (v_2 \cos \theta - v_1)$$

$$P_1 A_1 + F_x = \rho Q (v_2 \cos \theta - v_1) \quad [\because P_2 = 0 \text{ atm.}]$$

$$1875 \times 1 + F_x = 10^3 \times 0.5 (2 \times \cos 60 - 0.5)$$

$$\boxed{F_x = -1625 \text{ N}} \quad \text{Ans.}$$

VISCOUS FLOW OF INCOMPRESSIBLE FLOW

(LAMINAR FLOW)

• Reynold's Number (Re) :

It is defined as the ratio of Inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

$$Re = \frac{V L}{\nu}$$

For pipe flow,

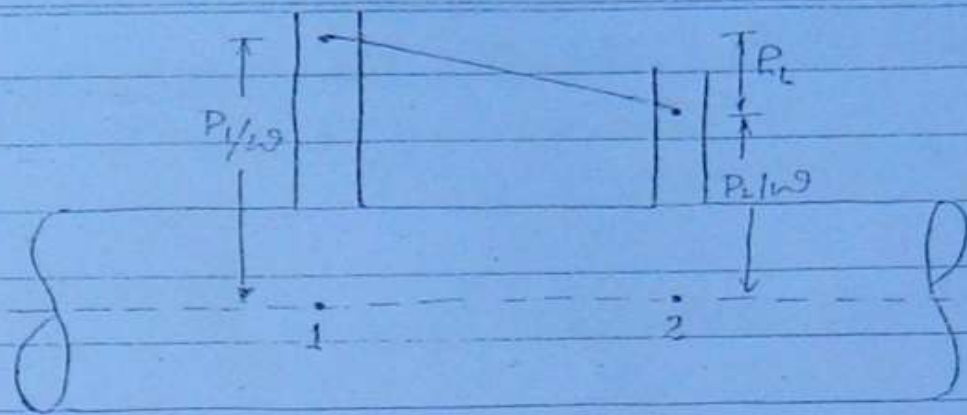
$Re < 2000 \rightarrow$ Laminar (or viscous) flow.

$Re > 4000 \rightarrow$ Turbulent flow.

$2000 < Re < 4000 \rightarrow$ Transition flow.

For pipe flow the characteristic length L is taken as dia. of the pipe.

$$Re = \frac{V D}{\nu}$$



Apply continuity eqn. b/w 1 & 2.

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = V_2$$

$$[\because A_1 = A_2]$$

Apply bernoulli eqn. b/w 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_L$$

$$h_L = \frac{P_1 - P_2}{\rho g}$$

In the direction of flow press. decreases in overcome to losses because press. gradient in the direction of flow is negative.

Darcy-Weisbach Equation:-

This equation is used for finding out head loss due to friction in steady laminar & turbulent flow.

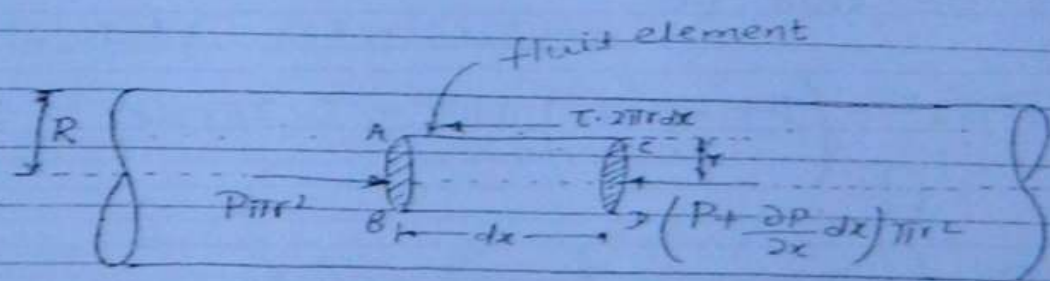
$$h_L = \frac{4f'LV^2}{2gD} \quad \leftarrow \text{oldest formula}$$

where f' = friction co-efficient

$$\boxed{h_L = \frac{fLV^2}{2gD}} \quad \leftarrow \text{New formula (use this)}$$

where f = friction factor = $4f'$

Laminar flow through circular pipe (Hagen-poiseuille flow):-



Consider a horizontal pipe of radius R . The viscous fluid is flowing from left to right in the pipe as shown in fig.

Consider a fluid element of radius r , sliding in

a cylindrical fluid element of radius $(r+dr)$.
let the length of the fluid element be dx .

If 'P' is the intensity of pressure on the face AB, then intensity of pressure on face CD will be $(P + \frac{\partial P}{\partial x} dx)$. [from Euler's Expansion series]

Then the force acting on fluid element -

1. The press. force, $P \times \pi r^2$ on face AB.
2. The press. force, $(P + \frac{\partial P}{\partial x} dx) \pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r dx$ on the surface of fluid element.

As there is no acceleration, hence the summation of all forces in the direction of flow must be zero i.e.

$$P \pi r^2 - \left(P + \frac{\partial P}{\partial x} dx \right) \pi r^2 - \tau \cdot 2\pi r dx = 0.$$

$$Pr - \left(P + \frac{\partial P}{\partial x} dx \right) r = 2\tau \cdot dx$$

$$Pr - Pr - \frac{\partial P}{\partial x} dx \cdot r = 2\tau \cdot dx$$

$$-\frac{\partial P}{\partial x} dx \cdot r = 2\tau dx$$

$$\tau = \left(\frac{-\partial P}{\partial x} \right) \cdot \frac{r}{2}$$

NOTE- The shear stress τ across a section varies with r as $\frac{\partial P}{\partial x}$ across the section is constant. Hence shear stress distribution across a section is linear.

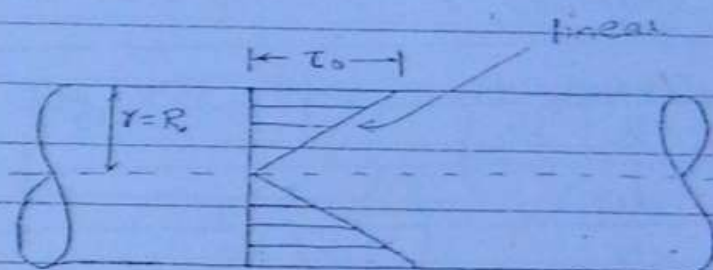
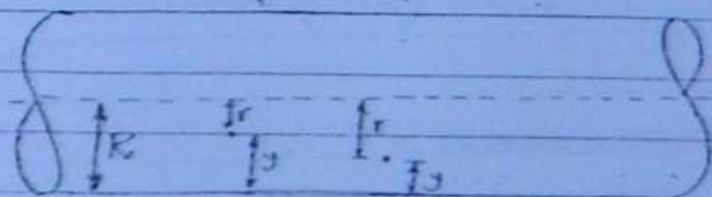


fig. shear stress distribution across a section

maximum shear stress τ_0 at the pipe wall.

• velocity distribution:-



$$R = r + y$$

$$0 = dr + dy$$

$$\Rightarrow dy = -dr$$

[$\because R$ is const. differentiating the eq. we get]

but we know that,

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\tau = \mu \frac{du}{(-dr)}$$

$$\tau = -\mu \frac{du}{dr}$$

& also $\tau = \left(-\frac{\partial P}{\partial x} \right) \frac{r}{2}$

Equating above two eqn, we get.

$$-\mu \frac{du}{dr} = \left(-\frac{\partial P}{\partial x} \right) \frac{r}{2}$$

$$du = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) \cdot r \cdot dr$$

Integrate the above eqn. we find.

$$u = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) \frac{r^2}{2} + C$$

At the pipe wall,

$$r = R \quad \& \quad u = 0.$$

then, $0 = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) \frac{R^2}{2} + C$

$$C = \frac{-1}{4\mu} \left(\frac{\partial P}{\partial x} \right) R^2$$

then, $U = \frac{1}{4\mu} \left(\frac{\partial P}{\partial x} \right) \frac{r^2}{2} = \frac{1}{4\mu} \left(\frac{\partial P}{\partial x} \right) R^2$

$$U = \frac{-1}{4\mu} \left(\frac{\partial P}{\partial x} \right) R^2 \left[\frac{1-r^2}{R^2} \right]$$

At the centre,

$$r=0 \quad \& \quad U = U_{\max}$$

$$U_{\max} = \frac{-1}{4\mu} \left(\frac{\partial P}{\partial x} \right) R^2 \quad \text{*****}$$

& $U = U_{\max} \left(\frac{1-r^2}{R^2} \right) \quad \text{*****}$

value of μ , $\frac{\partial P}{\partial x}$ & R are constant, which mean the velocity, U varies with the square of r . Thus the eqn. is parabola eqn. This shows the velocity distribution across the section of a pipe is parabolic.

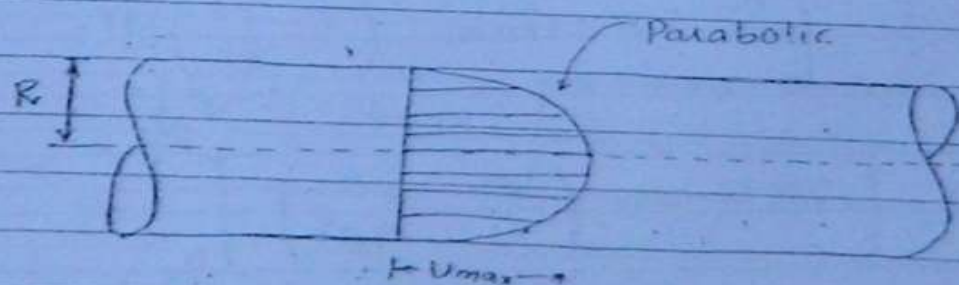
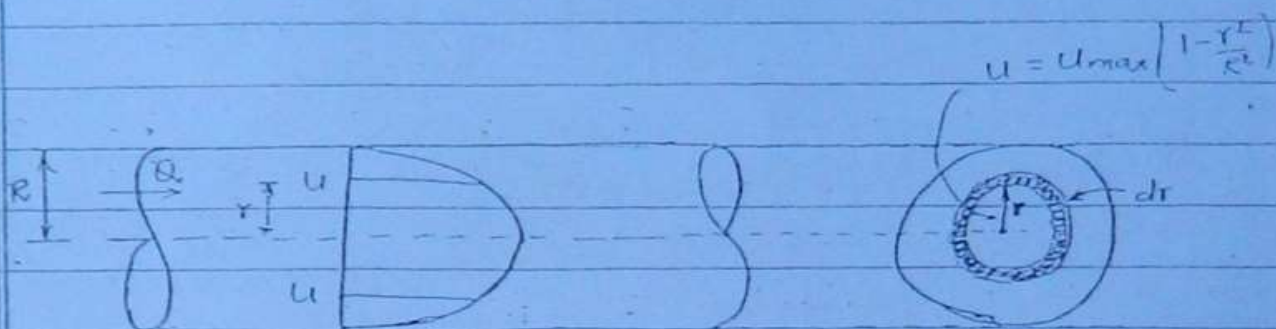


fig- velocity distribution across a section

Discharge (Q) :-



$$u = u_{max} \left(1 - \frac{r^2}{R^2}\right) \quad \text{u vary with } r$$

local velocity

discharge through circular ring element of radius r & thickness dr is,

$dQ = \text{velocity at radius } r \times \text{area of ring element}$

$$dQ = u \times 2\pi r dr$$

$$= u_{max} \left(1 - \frac{r^2}{R^2}\right) \times 2\pi r dr$$

Integrate the above eqn. we get,

then, $Q = \int_0^R 2\pi u_{max} \left(r - \frac{r^3}{R^2}\right) dr.$

$$Q = \left[2\pi u_{max} \left(\frac{r^2}{2} - \frac{r^4}{4R^2} \right) \right]_0^R$$

$$Q = 2\pi U_{max} \left[\frac{R^2}{2} - \frac{R^4}{4R^2} \right]$$

$$Q = 2\pi U_{max} \left[\frac{R^2}{4} \right]$$

$$Q = \frac{\pi U_{max} R^2}{2}$$

$$Q = \frac{\pi R^2}{2} \times \frac{1}{4\mu} \left(\frac{-\partial P}{\partial x} \right) R^2$$

$$Q = \frac{\pi}{8\mu} \left(\frac{-\partial P}{\partial x} \right) R^4 \quad \text{*****}$$

• Average velocity :-

$$Q = AV$$

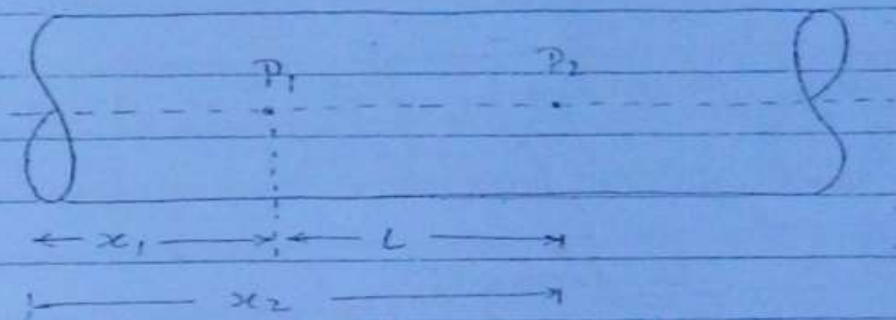
$$V = \frac{Q}{A}$$

$$V = \frac{\pi U_{max} R^2}{2} \div \pi R^2$$

$$V = \frac{U_{max}}{2} \quad \text{*****}$$

It is defined as the ratio of total discharge to the total area of flow.

Pressure drop in a given length (L) :-



$$V = \frac{U_{\max}}{2}$$

$$V = \frac{1}{2} \left[\frac{1}{4\mu} \left(\frac{-\partial P}{\partial x} \right) R^2 \right]$$

$$\frac{8\mu V}{R^2} \frac{\partial x}{\partial x} = -\partial P$$

Integrate the above eqn.

$$\int_{x_1}^{x_2} \frac{8\mu V}{R^2} \frac{\partial x}{\partial x} = \int_{P_1}^{P_2} -\partial P$$

$$\frac{8\mu V}{R^2} (x_2 - x_1) = (P_2 - P_1)$$

$$\frac{8\mu V L}{R^2} = P_1 - P_2$$

$$P_1 - P_2 = \frac{8\mu V L}{R^2}$$

$$P_1 - P_2 = \frac{8 \mu V L}{(D/2)^2}$$

$$\boxed{P_1 - P_2 = \frac{32 \mu V L}{D^2}} \quad \text{*****}$$

Head of the laminar flow eqn.

We know that,

$$\frac{P_1 - P_2}{\omega} = h_L$$

$$P_1 - P_2 = \omega \times h_L$$

$$P_1 - P_2 = \rho g \times h_L$$

$$\frac{32 \mu V L}{D^2} = \rho g \times \frac{f V L}{2 g D}$$

$$\frac{32 \mu}{D} = \frac{\rho f V}{2}$$

$$\frac{64 \mu}{\rho V D} = f$$

$$\frac{64}{\left(\frac{\rho V D}{\mu}\right)} = f$$

$$\boxed{f = \frac{64}{Re}} \quad \text{*****}$$

Imp.

NOTE - Friction factor in laminar flow through circular pipe depends upon Reynold number only.

∴ shear velocity (v^*) :-

We know that,

$$\tau = \left(\frac{-\partial P}{\partial x} \right) \frac{r}{2}$$

At the wall of the pipe,

$$\tau = \tau_0 \quad \text{at } r = R$$

then, $\tau_0 = \left(\frac{-\partial P}{\partial x} \right) \frac{R}{2}$

$$\tau_0 = - \frac{(P_2 - P_1)}{x_2 - x_1} \cdot \frac{R}{2}$$

$$\tau_0 = \frac{P_1 - P_2}{x_2 - x_1} \cdot \frac{D}{2} \cdot \frac{1}{2}$$

$$\tau_0 = \frac{P_1 - P_2}{L} \cdot \frac{D}{4} \quad [\because x_2 - x_1 = L]$$

$$\tau_0 = \frac{(P_1 - P_2) \cdot D}{4L}$$

But we know that,

$$\frac{P_1 - P_2}{\omega} = h_L$$

$$\Rightarrow P_1 - P_2 = \omega h_L$$

then, $T_0 = \omega h_L \cdot \frac{D}{4L}$

$$T_0 = \frac{\rho g \times f L V^2 \cdot D}{2gD \cdot 4L}$$

$$\boxed{T_0 = \frac{\rho f V^2}{8}} \quad \text{*****}$$

$$\Rightarrow \frac{T_0}{\rho} = \frac{f}{8} V^2$$

$$\sqrt{\frac{T_0}{\rho}} = \sqrt{\frac{f}{8}} V$$

$$\boxed{V^* = \sqrt{\frac{f}{8}} V} \quad \text{*****}$$

Where $\sqrt{\frac{T_0}{\rho}} = V^* = \text{shear velocity.}$

! Equation to be remember from objective point of view

1.
$$\tau = \left(\frac{-\partial P}{\partial x} \right) \frac{r}{2}$$

2.
$$U_{\max} = -\frac{1}{4\mu} \left(\frac{\partial P}{\partial x} \right) R^2$$

3.
$$U = U_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

4.
$$Q = \frac{\pi U_{\max} R^2}{2}$$

5.
$$V = \frac{U_{\max}}{2}$$

6.
$$P_1 - P_2 = \frac{32\mu VL}{D^2}$$

7.
$$f = \frac{64}{Re}$$

8.
$$T_0 = \frac{P f V^2}{8}$$

prob-84

A velocity profile in a laminar flow through pipe of dia. D is given by

$$u = u_0 \left(1 - \frac{4r^2}{D^2} \right)$$

where r is the radial distance from the center. if the viscosity is μ then the press. drop a length L is.

- (a) $\frac{32\mu u_0 L}{D^2}$ (b) $\frac{\mu u_0 L}{D^2}$ (c) $\frac{16\mu u_0 L}{D^2}$ (d) $\frac{8\mu u_0 L}{D^2}$

Soln:-

We know that,

$$u = u_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

$$u = u_{\max} \left(1 - \frac{4r^2}{D^2} \right)$$

Compare with the given eqn. we find.

$$u_{\max} = u_0$$

but $V = \frac{u_{\max}}{2} = \frac{u_0}{2}$

Then press. drop in a length L is,

$$P_1 - P_2 = \frac{32\mu V L}{D^2} = \frac{32\mu u_0}{2} \frac{L}{D^2} = \frac{16\mu u_0 L}{D^2}$$

prob-85

Assertion:- For a laminar flow through a circular pipe the velocity distribution across any section is parabolic in nature.

Reason:- The shear stress distribution from the centre line of the pipe upto pipe surface increases linearly.

Solⁿ:-

$$\tau = -\mu \frac{du}{dr} = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$du = \frac{1}{2\mu} \frac{\partial P}{\partial x} r dr$$

Assertion is right & Reason Statement

Support Assertion because parabolic distribution of velocity is obtained from linear shear stress distribution.

prob-86

Find the min. value of friction factor that can occur in laminar flow through circular pipe.

Solⁿ:-

Friction factor for laminar flow through circular pipe.

$$f = \frac{64}{Re}$$

$$f_{min} = \frac{64}{(Re)_{max}}$$

For laminar flow max $Re = 2000$

$$f_{min} = \frac{64}{2000} = 0.032 \text{ Ans.}$$

6-87 In a laminar flow through a pipe of 10 cm. radius the average velocity is 5 m/s then find the velocity at 5 cm radius.

Ans: we know that,

$$U = U_{max} \left(1 - \frac{r^2}{R^2} \right)$$

given $V = 5 \text{ m/s}$, $R = 10 \text{ cm}$, $r = 5 \text{ cm}$.

& we know that,

$$V = \frac{U_{max}}{2} = 5$$

then, $U_{max} = 10 \text{ m/s}$.

$$U = 10 \left(1 - \frac{r^2}{R^2} \right)$$

$$U = 10 \left(1 - \frac{5^2}{10^2} \right)$$

$$U = 10 \left(1 - \frac{25}{100} \right)$$

$$\boxed{U = 7.5 \text{ m/s}} \text{ Ans.}$$

prob-88

In a laminar flow through a pipe of dia. D , the average velocity is indicated as local velocity at a radial distance's measured from the centre of.

- (a) $0.707 D$ (b) $0.223 D$ (c) $0.576 D$ (d) $0.354 D$.

Solⁿ: We know that,

$$u = u_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

$$V = \frac{u_{\max}}{2}$$

$$\Rightarrow u_{\max} = 2V$$

$$\text{then, } u = 2V \left(1 - \frac{r^2}{R^2} \right)$$

$$\frac{1}{2} = 1 - \frac{r^2}{R^2}$$

$$\frac{r^2}{R^2} = 1 - \frac{1}{2}$$

$$\frac{r^2}{R^2} = \frac{1}{2}$$

$$\frac{r}{R} = \frac{1}{\sqrt{2}}$$

$$\gamma = \frac{R}{\sqrt{2}}$$

$$\gamma = \frac{D}{2} \cdot \frac{1}{\sqrt{2}} = 0.354 D \text{ Ans.}$$

Imp.

prob-89

show that the max wall shear stress for laminar flow through a given pipe of dia. D & with given fluid properties μ & P is

$$\tau_0 = \frac{16000 \mu L}{P D^2}$$

Soln:

we know that,

$$\tau = \left(\frac{-\partial P}{\partial x} \right) \cdot \frac{r}{2}$$

At the wall of the pipe,

$$\tau = \tau_0 \quad \& \quad r = R$$

$$\tau_0 = \left(\frac{-\partial P}{\partial x} \right) \cdot \frac{R}{2}$$

$$\tau_0 = - \frac{(P_2 - P_1)}{x_2 - x_1} \cdot \frac{D}{2} \cdot \frac{1}{2}$$

$$\tau_0 = \frac{P_1 - P_2}{L} \cdot \frac{D}{4}$$

T_0 is max., when $\frac{P_1 - P_2}{L}$ is max.

$$\text{But, } P_1 - P_2 = \frac{32 \mu V L}{D^2}$$

$$\frac{P_1 - P_2}{L} = \frac{32 \mu V}{D^2}$$

$\frac{P_1 - P_2}{L}$ is max. when $\frac{32 \mu V}{D^2}$ is max.

$\frac{32 \mu V}{D^2}$ is max. when V is max.

$$\frac{PVD}{\mu} = Re$$

V is max. when Re is max.

But for laminar flow the max. value of Re is 2000

$$\text{then, } \frac{PVD}{\mu} = 2000$$

$$V = \frac{2000 \mu}{PD}$$

$$\text{then, } \frac{P_1 - P_2}{L} = \frac{32 \mu}{D^2} \cdot \frac{2000 \mu}{PD}$$

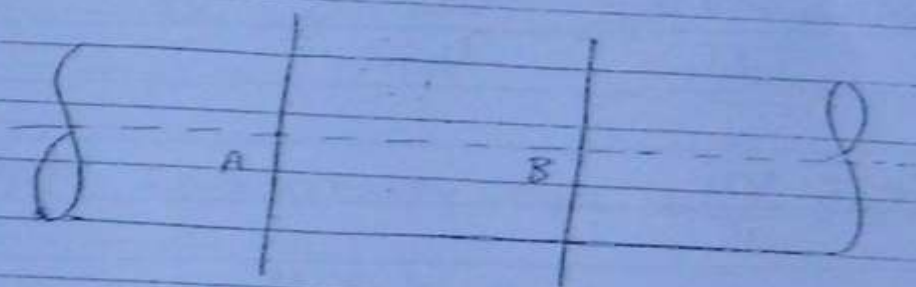
$$\frac{P_1 - P_2}{L} = \frac{64000 \mu^2}{PD^3}$$

∴ then, $T_0 = \frac{P_1 - P_2}{L} \cdot \frac{D}{4}$

$$T_0 = \frac{64000 \mu^2}{PD^3 D^2} \times \frac{D}{4}$$

$$T_0 = \frac{1600 \mu^2}{PD^2}$$

10- Oil flows from A to B through a 100m. long horizontal steel pipe of 150mm dia. the pressure at A is 1.08 MPa & at B is 0.95 MPa the kinematic viscosity is $412.5 \times 10^{-6} \text{ m}^2/\text{sec}$, $P = 918 \text{ kg/m}^3$. Find the Reynold number & discharge if the flow is laminar.



given,

$$P_A = 1.08 \text{ MPa} = 1.08 \times 10^6 \text{ N/m}^2$$

$$L = 100 \text{ m.}$$

$$P_B = 0.95 \text{ MPa} = 0.95 \times 10^6 \text{ N/m}^2$$

$$\nu = 412.5 \times 10^{-6} \text{ m}^2/\text{sec.}$$

$$P = 918 \text{ kg/m}^3$$

$$\mu = P \times \nu = 918 \times 412.5 \times 10^{-6} = 0.3786$$

$$D = 150 \text{ mm} = 0.15 \text{ m}$$

We know that for laminar flow,

$$P_A - P_B = \frac{32 \mu V L}{D^2}$$

$$1.08 \times 10^6 - 0.95 \times 10^6 = \frac{32 \times 0.3786 \times V \times 100}{(0.15)^2}$$

$$V = 2.413 \text{ m/s.}$$

The discharge,

$$Q = A \times V$$

$$\Rightarrow Q = \frac{\pi}{4} \times (0.15)^2 \times 2.413$$

$$\Rightarrow Q = 0.042 \text{ m}^3/\text{sec.}$$

We know that Reynold number,

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\mu / \rho} = \frac{V D}{\nu}$$

$$Re = \frac{V D}{\nu}$$

$$Re = \frac{2.413 \times 0.15}{412.5 \times 10^{-6}}$$

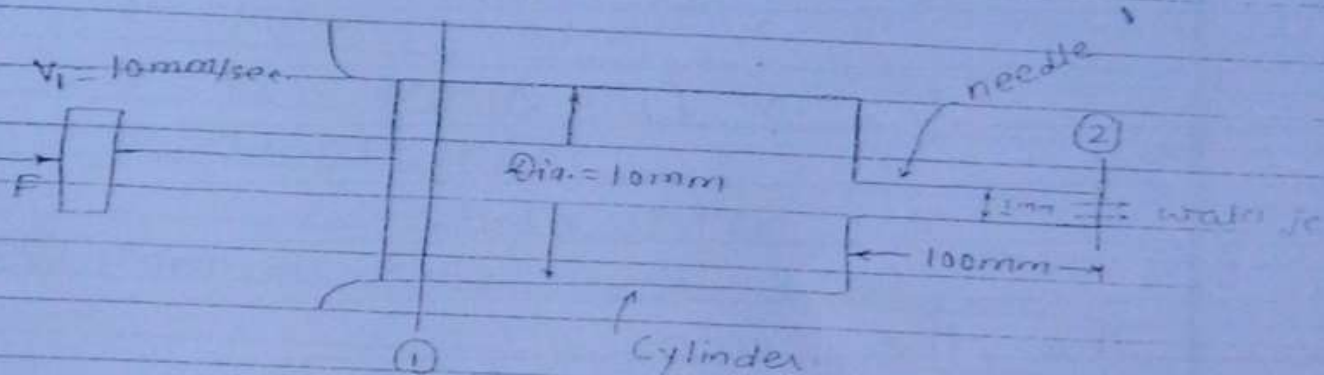
$$Re = 877. \text{ Ans.}$$

prob-91

A Spring with a frictionless plunger contains water & has at its end a 100 mm long needle of 1 mm dia the internal dia. of Spring is 10 mm. water density is 1000 kg/m^3 the plunger is pushed in at 10 mm/sec & water comes out as a jet to atmosphere -

- i) Assuming ideal flow find the force F required to push the plunger to push out water.
- ii) Neglect losses in the cylinder & assume laminar flow through out the needle. Find the force F . take $\mu = 10^{-3} \text{ N-s/m}^2$

Soln:-



Apply continuity eqn. b/w section 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times d_2^2 \times V_2$$

$$10^2 \times 10^{-2} = 1^2 \times V_2$$

$$V_2 = 1 \text{ m/s.}$$

ii) Apply Bernoulli's equation b/w 1 & 2.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \quad [\because P_2 = 0 \text{ at } \text{atm. condition}]$$

$$\frac{P_1}{\rho} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{P_1}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$P_1 = \frac{\rho}{2} (V_2^2 - V_1^2)$$

$$P_1 = \frac{10^3}{2} [1^2 - (10^{-2})^2]$$

$$P_1 = 499.95 \text{ N/m}^2$$

The the Force,

$$F = P_1 A_1$$

$$= 499.95 \times \frac{\pi}{4} \times (10 \times 10^{-3})^2$$

$$\boxed{F = 0.039 \text{ N}} \text{ Ans.}$$

(ii) Apply Bernoulli's eqn. b/w ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + h_L$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + h_L \quad (\because P_2 = 0 \text{ atm and } V_1 = 0)$$

$$\frac{P_1}{\rho} = \frac{V_2^2 - V_1^2}{2g} + h_L$$

But $h_L = \frac{f L V^2}{2g D}$

$$L = 100 \text{ mm} = 0.1 \text{ m}$$

$$V = V_2 = 1 \text{ m/s}$$

$$D = 1 \text{ mm} = 10^{-3} \text{ m}$$

For laminar flow friction factor (f)

$$f = \frac{64}{Re}$$

$$\therefore Re = \frac{\rho V D}{\mu} = \frac{10^3 \times 1 \times 10^{-3}}{10^{-3}} = 1000$$

then $f = \frac{64}{1000} = 0.064$

then $h_L = \frac{0.064 \times 0.1 \times 1^2}{2 \times 9.81 \times 10^{-3}} = 0.326 \text{ m}$

Now,
$$\frac{P_1}{9810} = \frac{1^2 - (10^{-2})^2}{2 \times 9.81} + 0.326$$

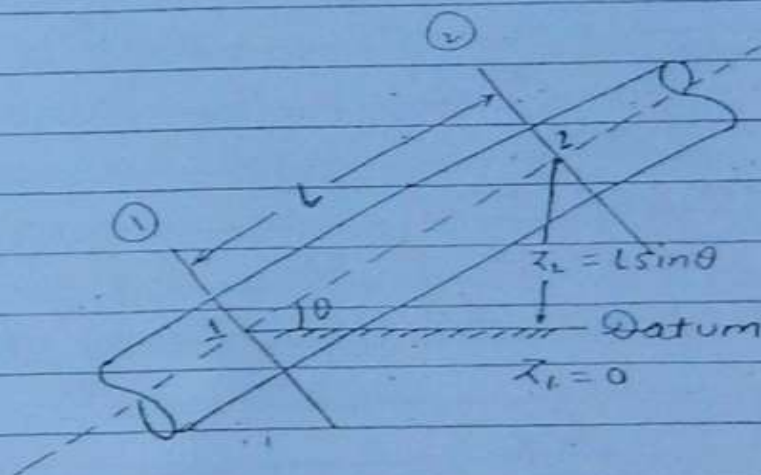
$$P_1 = 3697 \text{ N/m}^2$$

The force,

$$F = P_1 A_1 = 3697 \times \frac{\pi}{4} \times (10 \times 10^{-3})^2$$

$$\boxed{F = 0.29 \text{ N.}} \text{ Ans.}$$

* Inclined pipe's *



Continuity equation b/w 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$\therefore$ Area of the pipe at section 1 & 2 are Same.

then $\boxed{V_1 = V_2}$

Apply Bernoulli's eqn. b/w 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{fr}$$

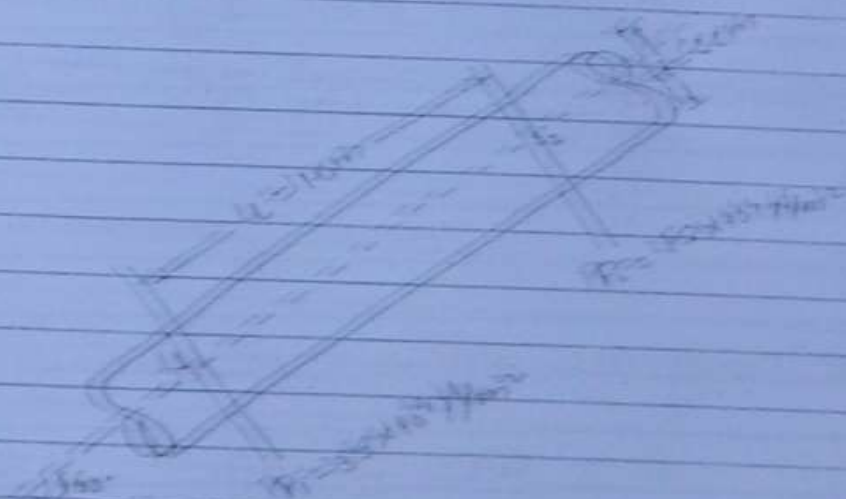
$$[V_1 = V_2 = V]$$

$$\left[\left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = h_{fr} \right]$$

Prob 12:

An oil with density 900 kg/m^3 & kinematic viscosity $0.0002 \text{ m}^2/\text{sec}$ flows upwards through an inclined pipe as shown in figure. The press. & elevation are known at section 1 & 2 which are 10 m apart. Assume steady, laminar flow.

- Head loss b/w 1 & 2.
- Discharge.
- Velocity.
- Reynold's number.



$$Z_2 = L \sin \theta = 10 \sin 40^\circ = 6.42 \text{ m.}$$

Apply bernoulli's equation b/w ① & ②.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

from continuity eqn. b/w ① & ②.

$$A_1 V_1 = A_2 V_2$$

$$V_1 = V_2 \quad [\because A_1 = A_2]$$

$$\text{then, } \frac{P_1}{\rho g} + Z_1 = \frac{P_2}{\rho g} + Z_2 + h_L$$

$$\frac{35 \times 10^4}{900 \times 9.81} + 0 = \frac{25 \times 10^4}{900 \times 9.81} + 6.42 + h_L$$

$$h_L = 4.89 \text{ m.}$$

from Darcy-Weisbach eqn.

$$h_L = \frac{f L V^2}{2g \times D}$$

for laminar flow, we know that friction factor

$$f = \frac{64}{Re} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64 \mu}{\rho V D} = \frac{64 \nu}{V D}$$

then
$$h_L = \frac{f L V^2}{2g \times D}$$

$$h_L = \frac{64 \nu}{V D} \times \frac{L V^2}{2g \times D}$$

$$h_L = \frac{32 \nu L V}{g \times D^2}$$

$$4.89 = \frac{32 \times 0.0002 \times 1000 \times V}{9.81 \times (6 \times 10^{-2})^2}$$

$$\boxed{V = 2.7 \text{ m/s.}} \quad \text{Ans.}$$

Discharge,

$$Q = A \times V$$

$$= \frac{\pi}{4} \times D^2 \times V$$

$$= \frac{\pi}{4} \times (6 \times 10^{-2})^2 \times 2.7$$

$$\boxed{Q = 7.63 \times 10^{-3} \text{ m}^3/\text{s.}} \quad \text{Ans.}$$

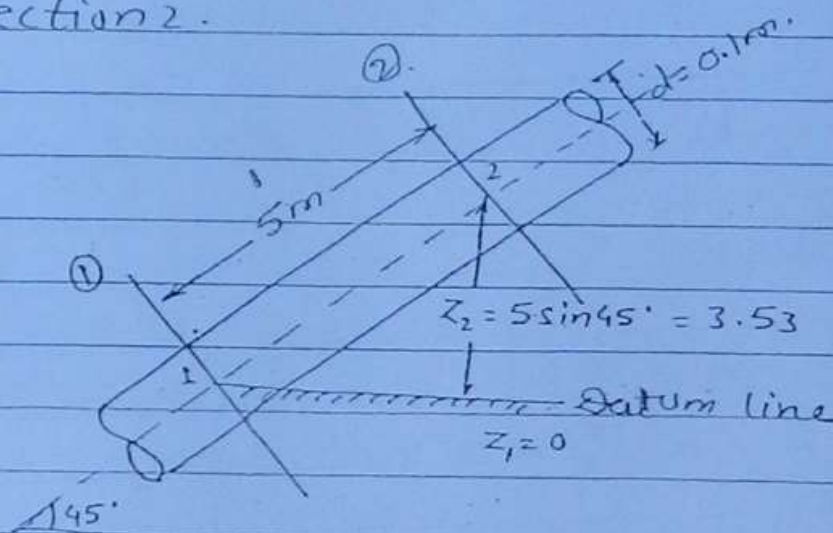
Reynold's number,

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\mu / \rho} = \frac{V D}{\nu}$$

$$Re = \frac{2.7 \times 6 \times 10^{-2}}{0.0002} = 810 \quad \text{Ans.}$$

Prob-93 An Upward flow of oil of density 800 kg/m^3 , $\mu = 0.8 \text{ N-s/m}^2$. Take's place under laminar condition's in an inclined pipe of 0.1 m dia. as shown in fig. The press. at section 1 & 2 are 435 kN/m^2 & 200 kN/m^2 find-

- (i) Discharge in pipe.
- (ii) If the flow direction is reversed keeping the same discharge & press. at section 1 is maintained as 435 kN/m^2 find the press. at section 2.



or: (i) $z_2 = 5 \sin 45 = 3.53 \text{ m}$.

Apply bernoulli's eqn. b/w section ① & 2. ∴

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

Apply continuity eqn. b/w 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$$V_1 = V_2$$

$$[\because A_1 = A_2 \text{ for pipe}]$$

then,

$$\frac{P_1}{\rho g} + z_1 = \frac{P_2}{\rho g} + z_2 + h_L$$

$$\frac{435 \times 10^3}{800 \times 9.81} + 0 = \frac{200 \times 10^3}{800 \times 9.81} + 3.53 + h_L$$

$$h_L = 26.47 \text{ m.}$$

from Darcy-Weisbach eqn.

$$h_L = \frac{f L V^2}{2g \times D}$$

for laminar flow, we know that friction factor

$$f = \frac{64}{Re} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64 \mu}{\rho V D}$$

then,

$$h_L = \frac{64 \mu}{\rho V D} \times \frac{L V^2}{2g \times D}$$

$$h_L = \frac{32 \mu V L}{\rho g D^2}$$

$$26.47 = \frac{32 \times 0.8 \times V \times 5}{800 \times 9.81 \times 0.12}$$

$$V = 16.18 \text{ m/s.}$$

Then discharge,

$$Q = A \times V$$

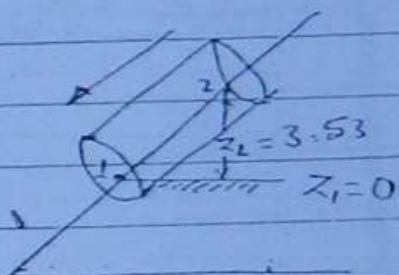
$$= \frac{\pi}{4} \times (0.1)^2 \times 16.1$$

$$= 0.127 \text{ m}^3/\text{sec. Ans.}$$

i) If flow direction is reversed,

Apply the bernoulli's eqn. b/w 1 & 2.

$$\frac{P_1}{\rho} + Z_2 = \frac{P_2}{\rho} + Z_1 + h_L$$



Q = discharge is same then
head loss is also same.

$$\frac{P_2}{800 \times 9.81} + 3.53 = \frac{435 \times 10^3}{800 \times 9.81} + 0 + 26.47$$

$$P_2 = 614 \times 10^3 \text{ N/m}^2 \text{ Ans.}$$

E- In turbulent flow rapid & regular mixing is occur.

The deviation is more in laminar flow but in turbulent flow deviation is less.

Momentum Correction factor (β):

It is defined as the ratio of momentum based upon, momentum based on actual velocity to the momentum based on average velocity.

Momentum, $P_{avg} = \text{mass} \times \text{velocity} = \rho A V \times V$

$$P_{avg} = \rho A V^2$$

$$\& \quad P_{act} = \rho \int u^2 dA$$

$$\beta = \frac{\int u^2 dA}{A V^2}$$

Kinetic Energy Correction factor (α):

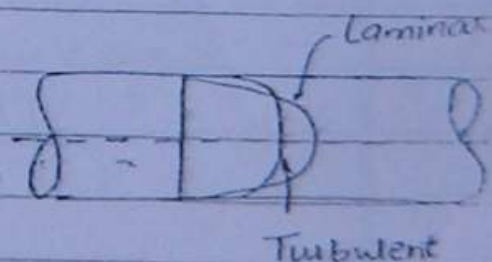
It is the ratio of kinetic energy based on actual velocity to the kinetic energy based on average velocity.

$$\alpha = \frac{\int u^3 dA}{A V^3}$$

NOTE

$$\beta \begin{cases} \text{Laminar} \rightarrow 4/3 \\ \text{Turbulent} \rightarrow 1.2 \end{cases}$$

$$\alpha \begin{cases} \text{Laminar} \rightarrow 2 \\ \text{Turbulent} \rightarrow 1.33 \end{cases}$$



Prob-

Find the momentum correction factor for laminar flow through circular pipe.

Soln:-

$$\beta = \frac{\int U^2 dA}{AV^2}$$

$$U = U_{max} \cdot \left(1 - \frac{r^2}{R^2}\right)$$

$$U^2 = U_{max}^2 \left(1 - \frac{r^2}{R^2}\right)^2$$

$$U^2 = U_{max}^2 \left(1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right)$$

$$\int U^2 dA = \int U_{max}^2 \left(1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right) \cdot 2\pi r dr$$

$$= 2\pi U_{max}^2 \int_0^R \left(r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right) dr$$

$$= 2\pi U_{max}^2 \left[\frac{r^2}{2} + \frac{r^6}{6R^4} - \frac{2r^4}{4R^2} \right]_0^R$$

$$= 2\pi U_{max}^2 \left[\frac{R^2}{2} + \frac{R^6}{6R^4} - \frac{2R^4}{4R^2} \right]$$

$$= \frac{\pi R^2 U_{max}^2}{3}$$

$$\beta = \frac{\int U^2 dA}{AV^2} = \frac{\pi R^2 U_{max}^2}{3} \bigg/ \frac{\pi R^2 \times U_{max}^2}{4}$$

$$\boxed{\beta = \frac{4}{3}} \text{ Ans.}$$

Turbulent flow

~~The~~ In laminar flow, the fluid particles move along straight parallel paths in layers or laminae such that the paths of individual fluid particles do not cross those of neighbouring particles. Laminar flow is possible only at low velocity & when the fluid is highly viscous. But when the velocity is increased or fluid is less viscous, the fluid particles do not move in straight paths. The fluid particles move in random manner resulting in general mixing of the particles. This type of flow is called turbulent flow.

A laminar flow changes to turbulent flow when

- i) Velocity is increased or
- ii) dia. of pipe is increased or
- iii) The viscosity of fluid is decreased.

Boussinesq developing equation for turbulent shear stress as

$$\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy}$$

η is known as eddy viscosity & it is of flow characteristic's, as it is difficult to find the value of η this equation is not used.

Reynold developed equation for turbulent shear stress.

$$\tau = \rho u'v'$$

where u' & v' are fluctuating component in x & y -direction, respectively.

Prandtl developed equation for u' & v' as

$$u' = v' = l \frac{du}{dy}$$

where l is ~~the~~ known as ~~prad~~ prandtl mixing length "It is the length in which the particle lose excess momentum & reach that of surrounding"

$$\tau = \rho u'v'$$

$$\tau = \rho \cdot l \frac{du}{dy} \cdot l \frac{du}{dy}$$

$$\tau = \mu l^2 \left(\frac{du}{dy} \right)^2 \quad \text{****}$$

$$l = 0.4 y \quad \leftarrow \text{Imperical Relationship}$$

Where y is distance from the pipe wall.

Prandtl mixing length at the pipe wall is equal to zero.

$$\text{Now,} \quad \frac{\tau}{\mu} = l^2 \left(\frac{du}{dy} \right)^2$$

$$\sqrt{\frac{\tau}{\mu}} = l \left(\frac{du}{dy} \right)$$

$$V_* = 0.4 y \frac{du}{dy}$$

$$\frac{V_*}{0.4} \frac{dy}{y} = du$$

Integrate the above eqn.

$$2.5 V_* \ln y = u + c \quad \text{****} \quad \text{--- (I)}$$

The velocity distribution in turbulent flow through pipe is logarithmic in nature.

∴ Hydrodynamically smooth & Rough boundary:

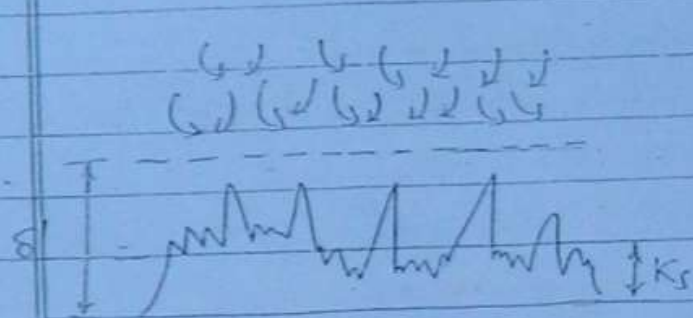


fig. Smooth boundary

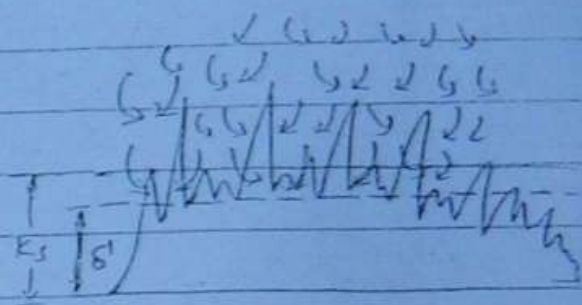


fig. Rough boundary

K_s = average height of Roughness

δ' = Laminar sublayer thickness

If $\frac{K_s}{\delta'} \leq 0.25 \rightarrow$ Smooth boundary

$\frac{K_s}{\delta'} > 6 \rightarrow$ Rough boundary

$0.25 < \frac{K_s}{\delta'} < 6 \rightarrow$ Transition boundary

Imperical
Relation-
ship.
by experi-
ment.

Now, $2.5 V_* \ln y = u + C$

At $y = y'$ (Close to pipe), $u = 0$.

then, $2.5 V_* \ln y' = 0 + C$

$$C = 2.5 V_* \ln y'$$

Then, $2.5 V_* \ln y = u + 2.5 V_* \ln y'$

$$2.5 V_* \ln y - 2.5 V_* \ln y' = u$$

$$2.5 V_* \left[\ln \frac{y}{y'} \right] = u$$

$$\frac{2.5 \ln y}{y'} = \frac{u}{V_*} \quad \left[\log_{10} = 2.303 \times \ln \right]$$

$$\boxed{5.75 \log_{10} \left(\frac{y}{y'} \right) = \frac{u}{V_*}} \quad \text{****}$$

This equation is valid smooth & rough surface.

$$\boxed{\frac{\delta'}{V_*} = 11.6 \frac{\nu}{V_*}} \quad \leftarrow \text{Empirical Relation-ship from Experiment.}$$

where δ' = laminar sublayer thickness.

ν = kinematic viscosity.

V_* = shear velocity.

For smooth pipe,

$$\boxed{\frac{y'}{107} = \frac{\delta'}{107}}$$

then, $y' = \frac{1}{107} \times \frac{11.6 \nu}{V_*}$

$$\boxed{\frac{y'}{107} = \frac{11.6 \nu}{V_*}}$$

For Rough pipe's :-

$$y' = \frac{K_s}{30}$$

Where K_s = Average height of Roughness.

Now, $\frac{U}{V_*} = 5.75 \log_{10} \left(\frac{y}{y'} \right)$

But for smooth pipe,

$$y' = \frac{\delta'}{107} = \frac{11.6}{107} \frac{D}{V_*}$$

then, $\frac{U}{V_*} = 5.75 \log_{10} \left(\frac{y}{\frac{11.6 D}{107 V_*}} \right)$

$$\frac{U}{V_*} = 5.75 \log_{10} \left(\frac{107}{11.6} \cdot \frac{V_* y}{D} \right)$$

$$\frac{U}{V_*} = 5.75 \left[\log_{10} \left(\frac{107}{11.6} \right) + \log_{10} \left(\frac{V_* y}{D} \right) \right]$$

$$\frac{U}{V_*} = 5.75 \log_{10} \left(\frac{107}{11.6} \right) + 5.75 \log_{10} \left(\frac{V_* y}{D} \right)$$

$$\frac{U}{V_*} = 5.5 + 5.75 \log_{10} \left(\frac{V_* y}{D} \right)$$

This is valid for smooth pipe only.

For Rough pipe,

$$y' = \frac{k_s}{30}$$

then, $\frac{u}{V_*} = 5.75 \log_{10} \left(\frac{y/k_s}{30} \right)$

$$\frac{u}{V_*} = 5.75 \log_{10} \left(30 \cdot \frac{y}{k_s} \right)$$

$$\frac{u}{V_*} = 5.75 \left[\log_{10} 30 + \log_{10} \frac{y}{k_s} \right]$$

$$\frac{u}{V_*} = 5.75 \log_{10} 30 + 5.75 \log_{10} \frac{y}{k_s}$$

$$\boxed{\frac{u}{V_*} = 8.5 + 5.75 \log_{10} \left(\frac{y}{k_s} \right)} \quad \text{where } u = \text{local velocity}$$

This is valid for Rough pipe only.

!:- Average velocity:-

NOTE- $V = \frac{Q}{A}$ & $Q = \int u dr$
As it is from laminar flow

$$\boxed{\frac{V}{V_*} = 5.75 \log_{10} \left(\frac{V_* R}{\nu} \right) + 1.75}$$

This is for the smooth pipe & $V = \text{avg. velocity}$.

$$\boxed{\frac{V}{V_*} = 5.75 \log_{10} \left(\frac{R}{k_s} \right) + 4.75}$$

This is valid for Rough pipe & $V = \text{avg. velocity}$

For the Rough pipe,

$$\frac{U}{V_*} - \frac{V}{V_*} = 8.5 + 5.75 \log_{10} \left(\frac{y}{K_s} \right) - 5.75 \log_{10} \left(\frac{R}{K_s} \right) - 4.75$$

$$\left[\frac{U}{V_*} - \frac{V}{V_*} = 3.75 + 5.75 \log_{10} \left(\frac{y}{R} \right) \right]$$

for smooth pipe.

$$\frac{U}{V_*} - \frac{V}{V_*} = 5.5 + 5.75 \log_{10} \left(\frac{V_* y}{\nu} \right) - 5.75 \log_{10} \left(\frac{V_* R}{\nu} \right) - 1.75$$

$$\left[\frac{U}{V_*} - \frac{V}{V_*} = 3.75 + 5.75 \log_{10} \left(\frac{y}{R} \right) \right] \text{****}$$

There for this is valid for both smooth as well as Rough pipe. $\&$ R = Radius of pipe.

Friction factor in turbulent flow:-

For smooth pipe,

$$f = \frac{0.3164}{(Re)^{1/4}} \quad \text{upto} \quad Re = 10^5$$

$$f = \frac{0.0032 + 0.221}{(Re)^{0.237}} \quad \text{for} \quad Re = 10^5 \text{ to } 4 \times 10^7$$

For Rough pipe,

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{R}{K_s} \right) + 1.74$$

1. Imp.

NOTE-

In laminar flow friction factor depends upon Reynold's number only but in turbulent flow friction factor depends on Reynold's number & average height of roughness (K_s)

Prob-94- Find the distance from pipe wall at which local velocity is equal to average velocity in turbulent flow.

Solⁿ: ~~locally~~ we know that,

$$\frac{u-v}{V_x} = 3.75 + 5.75 \log_{10} \left(\frac{y}{R} \right)$$

local velocity is equal to average velocity for,

$$0 = 3.75 + 5.75 \log_{10} \left(\frac{y}{R} \right)$$

$$-3.75 = 5.75 \log_{10} \left(\frac{y}{R} \right)$$

$$\frac{-3.75}{5.75} = \log_{10} \left(\frac{y}{R} \right)$$

$$\frac{y}{R} = \frac{-3.75}{10^{5.75}} = 0.223$$

$$\boxed{y = 0.223 R} \text{ Ans.}$$

prob-95- show that for turbulent flow in pipe's the ratio of max. velocity to the avg. velocity is.

$$\frac{U_{\max}}{V} = 1 + 1.33 \sqrt{f}$$

Solⁿ: We know that,

$$\frac{U - V}{V_*} = 3.75 + 5.75 \log_{10} \left(\frac{y}{R} \right)$$

$$U = U_{\max} \text{ at } y = R$$

$$\frac{U_{\max} - V}{V_*} = 3.75 + 5.75 \log_{10} \left(\frac{R}{R} \right)$$

$$\frac{U_{\max} - V}{V_*} = 3.75 + 5.75 \log_{10} 1$$

$$\frac{U_{\max} - V}{V_*} = 3.75 + 0$$

$$U_{\max} - V = 3.75 V_*$$

$$\& \quad \tau_0 = \frac{\rho f V^2}{8}$$

$$\frac{\tau_0}{\rho} = \frac{f V^2}{8}$$

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} V$$

$$V_* = \sqrt{\frac{f}{8}} V$$

$$U_{\max} - V = 3.75 \sqrt{\frac{f}{8}} V$$

$$U_{\max} - V = 1.33 \sqrt{f} V$$

$$\frac{U_{\max} - V}{V} = 1.33 \sqrt{f}$$

$$\frac{U_{\max}}{V} - 1 = 1.33 \sqrt{f}$$

$$\boxed{\frac{U_{\max}}{V} = 1 + 1.33 \sqrt{f}}$$

IE- First find the Reynold's number & check the flow (laminar or turbulent) then solve the problem.

Ex-96 A Rough pipe of 0.1m dia. carries water at the rate of 50 liter/sec. The average height of Roughness is $k_s = 0.15 \text{ mm}$ find -

- (i) Friction factor (ii) shear stress on the pipe.
 (iii) Max. velocity (iv) shear velocity.
 Take kinematic viscosity $\nu = 10^{-6} \text{ m}^2/\text{s}$ & $\rho = 10^3 \text{ kg/m}^3$.

Soln: given, $d = 0.1 \text{ m.}$

$$Q = 50 \text{ liter/sec.} = 50 \times 10^{-3} \text{ m}^3$$

Then the velocity, (average).

$$Q = A \times V$$

$$50 \times 10^{-3} = \frac{\pi}{4} \times d^2 \times V$$

$$50 \times 10^{-3} = \frac{\pi}{4} \times (0.1)^2 \times V$$

$$V = 6.36 \text{ m/s.}$$

Reynold's number,

$$Re = \frac{\rho V D}{\mu} = \frac{10^3 \times 6.36 \times 0.1}{10^{-6} \times 10^3}$$

$$Re = 6.3 \times 10^5$$

Since $Re > 4000$ then the flow is turbulent.

(i) For turbulent flow, Rough pipe, friction factor

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{R}{K_s} \right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{0.05}{0.15 \times 10^{-3}} \right) + 1.74$$

$$\boxed{f = 0.0216} \quad \text{Ans.}$$

(ii) $\hookrightarrow \tau_0 = \frac{P f v^2}{8}$ Shear stress on the pipe

$$\tau_0 = \frac{10^3 \times 0.0216 \times (6.36)^2}{8}$$

$$\boxed{\tau_0 = 109.4 \text{ N/m}^2} \quad \text{Ans.}$$

(iii) Maximum velocity,

$$\frac{V_{\max}}{v} = 1 + 1.33 \sqrt{f}$$

$$\frac{V_{\max}}{6.36} = 1 + 1.33 \sqrt{0.0216}$$

$$\boxed{V_{\max} = 7.6 \text{ m/s.}} \quad \text{Ans.}$$

(iv) Shear velocity,

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \cdot v$$

$$V_* = \sqrt{\frac{f}{8}} \cdot v = \sqrt{\frac{0.0216}{8}} \times 6.36$$

$$\boxed{V_* = 0.33 \text{ m/s.}} \quad \text{Ans.}$$

FLOW THROUGH PIPE

Page _____

When fluid flows through pipe it has to overcome various losses & these losses are termed as major & minor losses.

† Major losses :-

Head loss due to friction is known as major loss. It is found by -

① Darcy-Weisbach Formula (Equation) :-

$$h_f = \frac{f L V^2}{2g \times D} \quad \text{--- (1)}$$

where h_f = head loss due to friction,

L = length of pipe

V = mean velocity of flow.

D = dia. of pipe.

We know that,

$$Q = A \times V$$

$$Q = \frac{\pi}{4} \times D^2 \times V$$

$$V = \frac{4Q}{\pi D^2}$$

$$\therefore \text{then, } h_f = \frac{f L}{2g \times D} \left(\frac{4Q}{\pi D^2} \right)^2$$

$$h_L = \frac{16 f L Q^2}{2 \pi^2 D^5} = \frac{f L Q^2}{\left(\frac{2 \pi^2}{16}\right) D^5} = \frac{f L Q^2}{12 D^5}$$

$$\boxed{h_L = \frac{f L Q^2}{12 D^5}} \quad \text{--- (ii)}$$

(ii) Chezy's formula :-

$$\boxed{V = C \sqrt{m i}}$$

where, V = average velocity,

C = Chezy's constant,

m = Hydraulic mean depth

$$= \frac{\text{Area of flow}}{\text{Perimeter (wetted)}} = \frac{A}{P} = \frac{\pi/4 D^2}{\pi D}$$

D = dia. of the pipe.

i = Hydraulic slope = $\frac{h_L}{L}$

L = length of the pipe.

Then,
$$V = C \sqrt{\frac{D}{4} \times \frac{h_L}{L}}$$

$$V^2 = \frac{C^2 D h_L}{4 L}$$

$$\boxed{h_L = \frac{4 L V^2}{C^2 D}} \quad \text{--- (iii)}$$

Equating eqn. ① & ② we get,

$$\frac{f L V^2}{2g \times D} = \frac{4 f L V^2}{C^2 D}$$

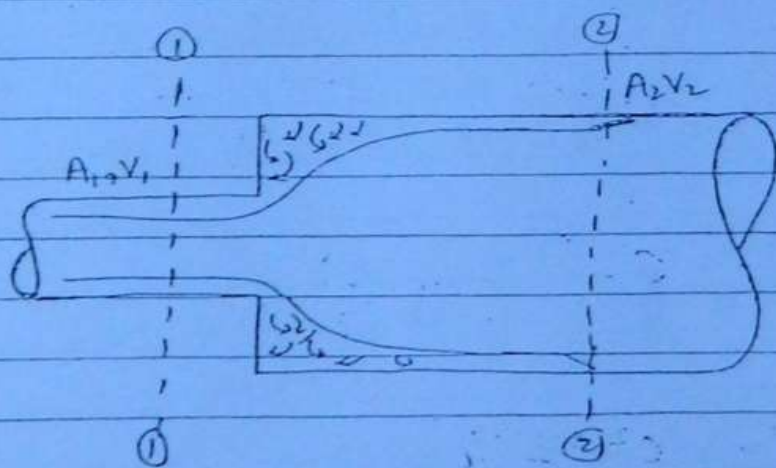
$$\frac{f}{2g} = \frac{4}{C^2}$$

$$C^2 = \frac{8g}{f}$$

$$C = \sqrt{\frac{8g}{f}}$$

✚ Minor Losses :-

① Losses due to sudden expansion :-



Due to a sudden change of dia. of the pipe from D_1 to D_2 , the liquid flowing from the smaller pipe is not able to ~~flow~~ follow abrupt change of

the boundary. Thus the flow separates from the boundary & turbulent eddies are formed.

The loss of head (or energy) takes place due to the formation of these eddies.

By Applying Bernoulli's eqn., continuity eqn & momentum eqn. we get,

$$h_{L \text{ Exp.}} = \frac{(V_1 - V_2)^2}{2g}$$

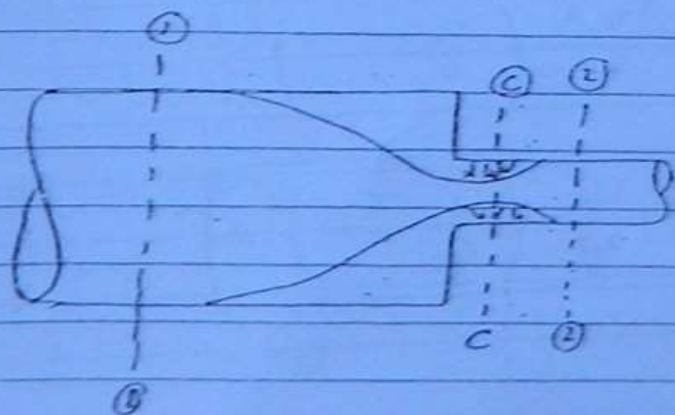
$$h_{L \text{ Exp.}} = \frac{V_1^2}{2g} \left(1 - \frac{V_2^2}{V_1^2} \right)^2$$

from continuity $A_1 V_1 = A_2 V_2$

$$\frac{V_2}{V_1} = \frac{A_1}{A_2}$$

then
$$h_{L \text{ Exp.}} = \frac{V_1^2}{2g} \left(1 - \frac{A_1}{A_2} \right)^2$$

Losses due to sudden contraction:-



As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing & becomes minimum at a section C-C.

This section C-C is called vena-contracta. After section C-C a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

Co-efficient of contraction (C_c)

$$C_c = \frac{\text{Area of vena-contracta}}{\text{area of orifice}}$$

$$C_c = \frac{A_c}{A_2}$$

But from continuity,

$$A_c V_c = A_2 V_2$$

$$\frac{A_c}{A_2} = \frac{V_2}{V_c} = C_c$$

$$h_{\text{contraction}} = \frac{(V_c - V_2)^2}{2g}$$

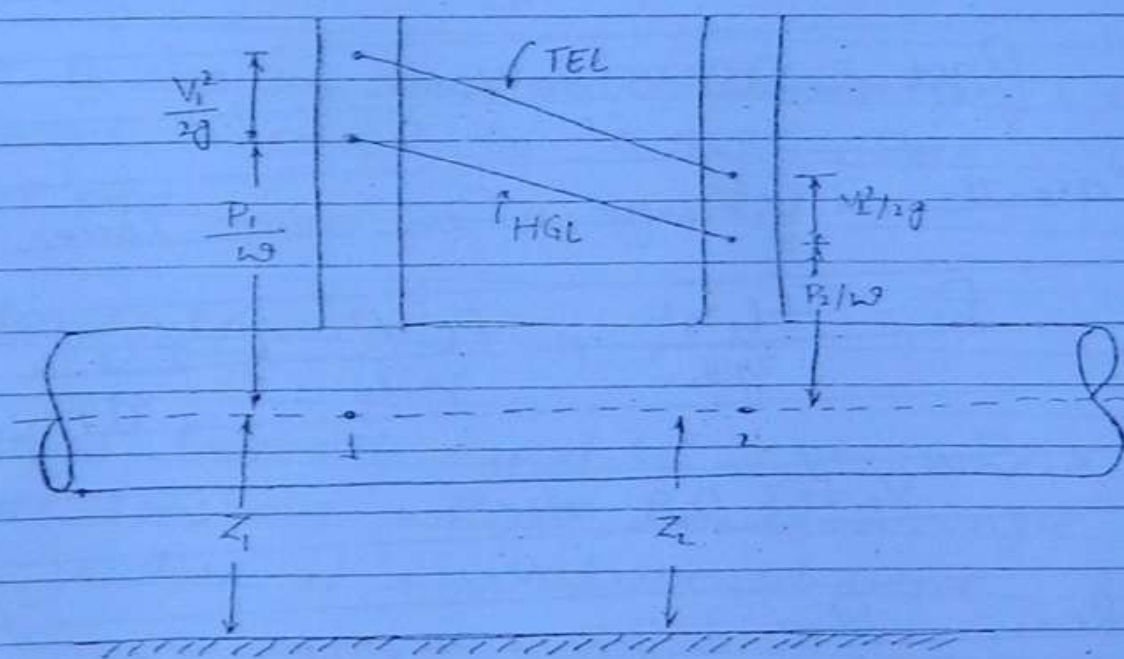
$$= \frac{V_2^2}{2g} \left(\frac{V_c}{V_2} - 1 \right)^2$$

$$\boxed{h_{\text{contraction}} = \frac{V_2^2}{2g} \left(\frac{1}{C_c} - 1 \right)^2}$$

Hydraulic gradient line (HGL) & total energy line (TEL) :-

The line which joins piezometric head $\left(\frac{P}{\rho} + z\right)$ at various points is known as hydraulic gradient line.

The line which joins the total energy $\left(\frac{P}{\rho} + \frac{V^2}{2g} + z\right)$ at various points is known as total energy line. The distance b/w HGL & TEL gives kinetic energy head.



$$HGL = \frac{P}{\rho} + z$$

$$TEL = \frac{P}{\rho} + \frac{V^2}{2g} + z$$

$$0.1^2 V_1 = 0.2^2 V_2 = 0.1^2 V_3$$

$$V_1 = 4V_2 = V_3$$

$$h_L = \frac{0.5V_1^2}{2g} + \frac{KV_1^2}{2g} + \frac{fL_1V_1^2}{2gd_1} + \frac{KV_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} \\ + \frac{fL_2V_2^2}{2gd_2} + \frac{0.5V_3^2}{2g} + \frac{fL_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

$$\therefore h_L = 20\text{m}, \quad K=1, \quad f=0.02$$

$$V_1 = 4V_2 = V_3$$

$$V_1 = 2.92 \text{ m/s.}$$

$$Q_{act} = Q_1 V_1 = \frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} (0.1)^2 \times 2.92$$

$$Q_{act} = 0.0229 \text{ m}^3/\text{sec.}$$

Case-II- Discharge when minor losses is neglected

$$h_L = \frac{fL_1V_1^2}{2gd_1} + \frac{fL_2V_2^2}{2gd_2} + \frac{fL_3V_3^2}{2gd_3}$$

$$V_1 = 3.08 \text{ m/s.}$$

$$\text{then, } Q_{th} = Q_1 V_1 = \frac{\pi}{4} \times (0.1)^2 \times 3.08 = 0.0242 \text{ m}^3/\text{s}$$

$$\% \text{ Error} = \frac{Q_{th} - Q_{act}}{Q_{act}} = \frac{0.0242 - 0.0229}{0.0229} \times 100$$

$$= 5.67\% \text{ Ans.}$$

NOTE- If C_c is not given then losses due to sudden contraction are taken as $\frac{0.5V_1^2}{2g}$ where V_1 is velocity in small pipe.

(iii) Losses at the exit of pipe:-

It is similar to sudden expansion with $A_2 = \infty$ i.e. exit loss is equal to $\frac{V^2}{2g}$ where V is velocity in pipe.

$$h_L = \frac{V^2}{2g}$$

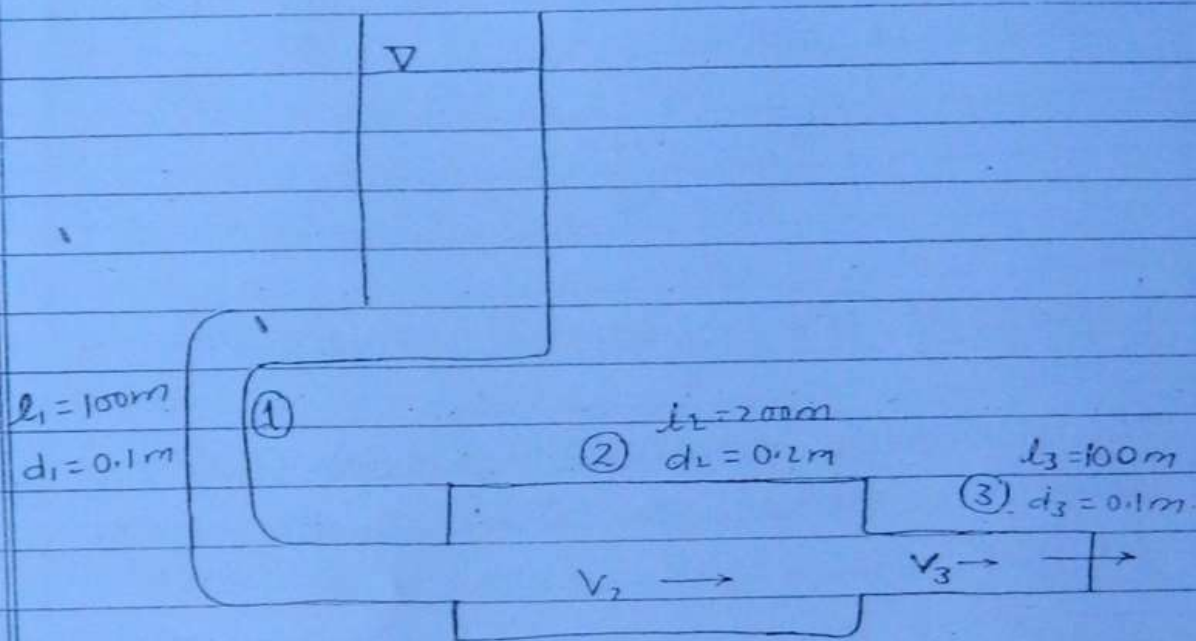
(iv) Losses at the entrance of pipe:-

It is similar to sudden contraction i.e. entrance losses is equal to $\frac{0.5V^2}{2g}$ where V is velocity in pipe.

(v) Bend losses:-

Bend losses is equal to $\frac{K_b V^2}{2g}$ where K_b is the constant which depends on angle of bend & radius of curvature of bend.

Prob-97- Water flows from a reservoir through a series of pipe joint as shown in fig. Find the percentage error in discharge if minor losses are neglected. assume $k=1$ for bends & friction factor $f=0.02$ for all pipes & the available head of 20m. is used to overcome losses.



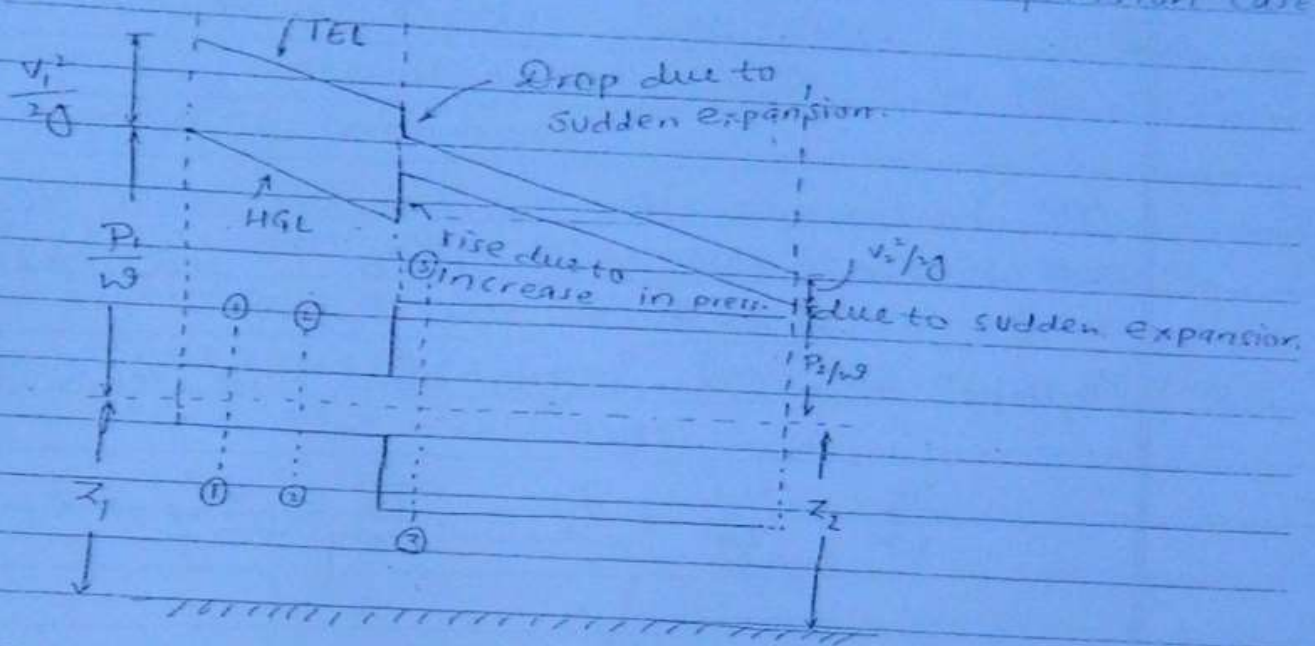
Soln:- Case-I- Discharge when all losses are taken into account.

Apply continuity eqn. b/w (1), (2) & (3).

$$A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4} d_3^2 V_3$$

Draw HGL & TEL line for sudden expansion case



$HGL = \frac{P}{\rho g} + Z$ but in horizontal pipe $Z_1 = Z_2$
then $P/\rho g$ decide the HGL line

At section ① & ② from continuity eqn.

$$A_1 V_1 = A_2 V_2$$

$$V_1 = V_2$$

$$[\because A_1 = A_2]$$

Then Apply bernoulli's eqn. b/w 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_L$$

To overcome the losses pressure decreases from section 1 to section 2.

At section 2 & 3 Area are increases from 2 to 3. Apply continuity equation b/w 2 & 3.

$$A_2 V_2 = A_3 V_3$$

To balanced the continuity eqn. Velocity decreases because area is increases at section 3

Apply the bernoulli's eqn. b/w ② & ③.

$$\frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2g} + Z_3$$

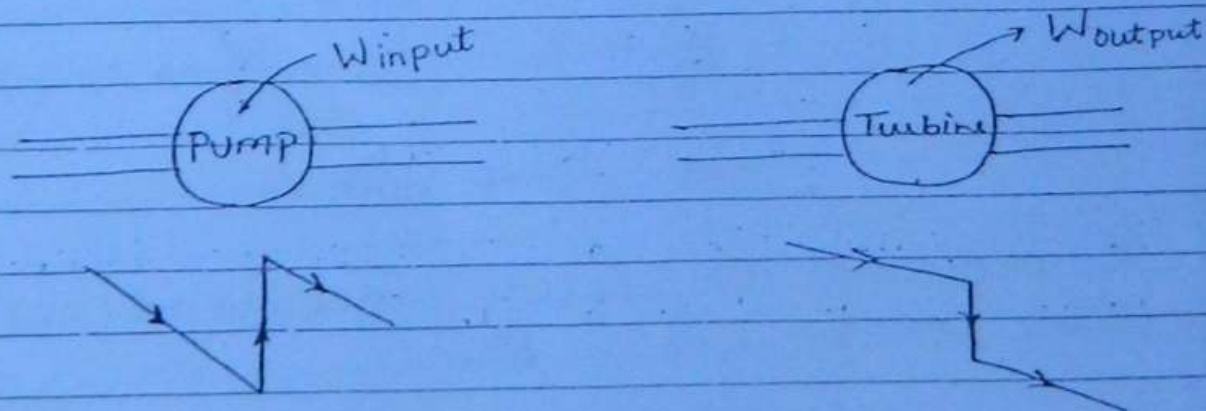
$Z_2 = Z_3$ As horizontal pipe.

$$\frac{P_2}{\rho} + \frac{V_2^2}{2g} = \frac{P_3}{\rho} + \frac{V_3^2}{2g}$$

∴ At section 3 the velocity ~~increases~~ decreases means kinetic energy decreases then the pressure energy increases.

head

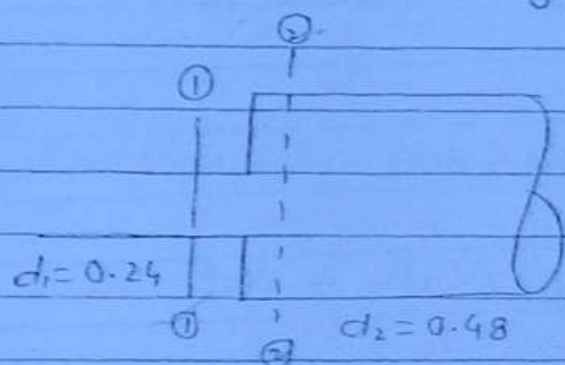
& then pressure decreases due to friction.



Hydraulic gradient line may rise or fall but the total energy line rises only when there is external energy input or in the case of pumps & compressor.

Prob-98- At a sudden enlargement of a water pipeline from a dia. of 0.24m to 0.48m. the hydraulic gradient line rises by 10mm. then find the discharge.

Soln:-



Apply bernoulli's eqn. b/w ① & ②.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2 + h_{Lc}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2 + \frac{(V_1 - V_2)^2}{2g}$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g} = \left(\frac{P_2}{\rho} + Z_2 \right) - \left(\frac{P_1}{\rho} + Z_1 \right)$$

$$\frac{V_1^2 - V_2^2 - (V_1^2 + V_2^2 - 2V_1V_2)}{2g} = \frac{10}{1000}$$

$$\text{Where } \left(\frac{P_2}{\rho} + Z_2 \right) - \left(\frac{P_1}{\rho} + Z_1 \right) = \text{rise of HGL} = 0.01$$

$$\frac{2V_1V_2 - 2V_2^2}{2g} = 0.01$$

$$V_1V_2 - V_2^2 = 0.01 \times g$$

Apply continuity eqn. b/w ① & ②

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times d_2^2 \times V_2$$

$$0.24V_1 = 0.48V_2$$

$$V_1 = 4V_2$$

then $4V_2 \times V_2 - V_2^2 = 0.01 \times 9.81$

$$3V_2^2 = 0.01 \times 9.81$$

$$V_2 = 0.18 \text{ m/s.}$$

discharge,

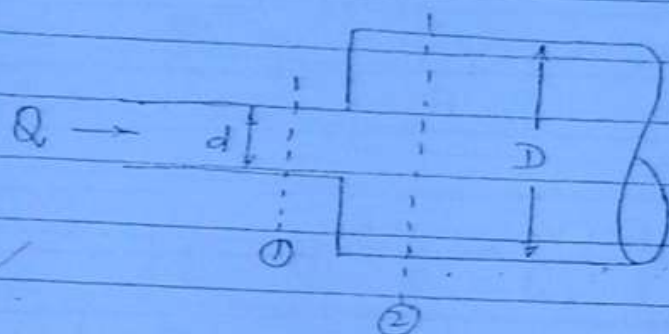
$$Q = A_2V_2 = \frac{\pi}{4} d_2^2 V_2$$

$$= \frac{\pi}{4} \times (0.48)^2 \times 0.18$$

$$= 0.0327 \text{ m}^3/\text{sec. Ans.}$$

Prob-99- A horizontal pipe of a given dia. d suddenly enlarges to dia D find the ratio D/d such that the rise in press. for a given discharge past the enlargement shall be maximum.

Soln:-



let $d_1 = d$ (gk)
& $d_2 = D$.

Apply continuity eqn. b/w 1 & 2.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

$$[d_1^2 V_1 = d_2^2 V_2]$$

Apply Bernoulli's eqn. b/w 1 & 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g} = \frac{P_2}{\rho g} - \frac{P_1}{\rho g}$$

$$\frac{V_1^2 - V_2^2 - (V_1^2 + V_2^2 - 2V_1 V_2)}{2g} = \frac{P_2 - P_1}{\rho g}$$

$$\frac{V_1^2 - V_2^2 - (V_1^2 + V_2^2 - 2V_1V_2)}{2} = \frac{P_2 - P_1}{\rho g}$$

$$\frac{2V_1V_2 - 2V_2^2}{2} = \frac{\Delta P}{\rho}$$

$$V_1V_2 - V_2^2 = \frac{\Delta P}{\rho}$$

$$\Delta P = \rho (V_1V_2 - V_2^2)$$

For max. press. rise.

$$\frac{d(\Delta P)}{dV_2} = 0.$$

$$\frac{d(\Delta P)}{dV_2} = \rho [V_1 - 2V_2] = 0.$$

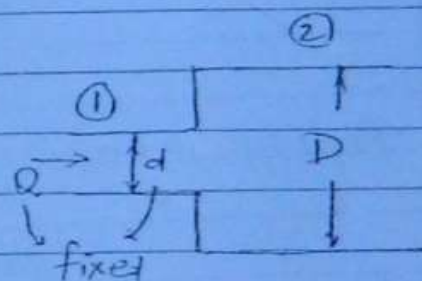
$$V_1 - 2V_2 = 0.$$

$$V_1 = 2V_2$$

then $d_1^2 V_1 = D^2 V_2$

$$d^2 \times 2V_2 = D^2 V_2$$

$$\boxed{\frac{D}{d} = \sqrt{2}} \quad \text{Ans.}$$



$$Q = A_1 V_1 \quad \& \quad Q = A_2 V_2$$

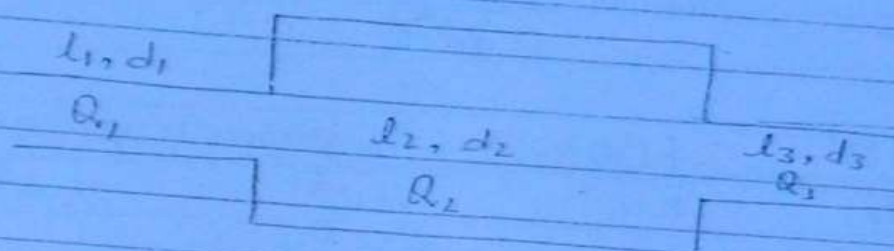
$$Q = \frac{\pi}{4} d^2 V_1 \quad \& \quad Q = \frac{\pi}{4} D^2 V_2$$

$V_1 = \text{fixed}$

$V_2 = \text{variable}$

$D = \text{variable}$

Pipes in series :-



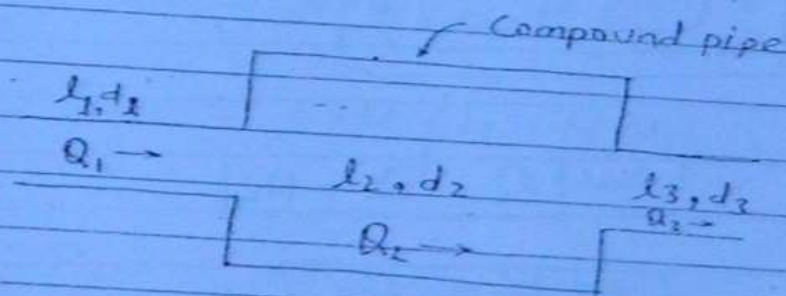
$$Q = Q_1 = Q_2 = Q_3$$

$$h_L = h_{L1} + h_{L2} + h_{L3}$$

Neglect minor loss

Equivalent pipe :-

A pipe of uniform diameter is said to be equivalent to a compound pipe when discharge & losses are same in both pipe.



$$h_L = h_{L1} + h_{L2} + h_{L3}$$

$$h_L = \frac{f L_1 Q^2}{12 d_1^5} + \frac{f L_2 Q^2}{12 d_2^5} + \frac{f L_3 Q^2}{12 d_3^5}$$

$Q \rightarrow$

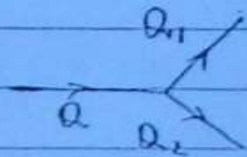
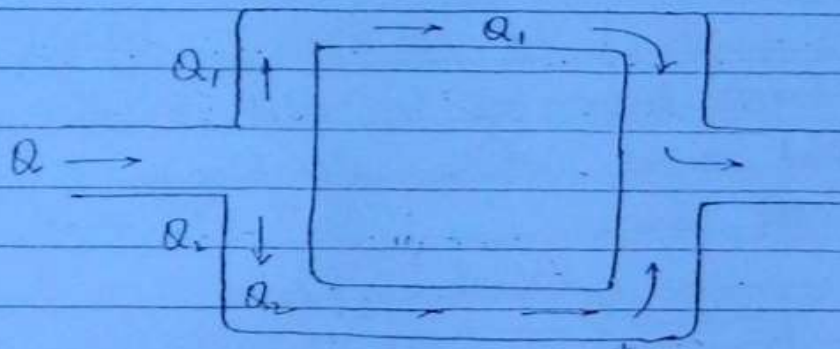
le, de

$$h_{fe} = \frac{f l_e Q^2}{12 d_e^5}$$

$$\frac{f l_e Q^2}{12 d_e^5} = \frac{f l_1 Q^2}{12 d_1^5} + \frac{f l_2 Q^2}{12 d_2^5} + \frac{f l_3 Q^2}{12 d_3^5}$$

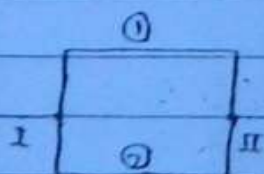
$$\left[\frac{le}{de^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} \right] \checkmark \text{ Dupuit's Eqn. minor losses are neglected.}$$

Pipes in parallel:-



$$Q = Q_1 + Q_2$$

$$h_{L1} = h_{L2}$$



$$P_I + \frac{V_I^2}{2g} + z_I = P_{II} + \frac{V_{II}^2}{2g} + z_{II} + h_{L1}$$

$$\frac{P_I}{\omega} + \frac{V_I^2}{2g} + Z_I = \frac{P_{II}}{\omega} + \frac{V_{II}^2}{2g} + Z_{II} + h_{L2}$$

$$[h_{L1} = h_{L2}]$$

Power transmission through pipe:-

$$P_{th} = \omega Q H$$

$$P_{act} = \omega Q (H - h_L)$$

$$\eta = \frac{P_{act}}{P_{th}} = \frac{\omega Q (H - h_L)}{\omega Q H}$$

$$\eta = \frac{H - h_L}{H}$$

Condition for max. power-

$$P_{act} = \omega Q (H - h_L)$$

$$P_{act} = \omega Q \left(H - \frac{f L Q^2}{12 d^5} \right)$$

$$P_{act} = \omega \left(Q H - \frac{f L Q^3}{12 d^5} \right)$$

At the given head & pipe.

For max. power,

$$\frac{dP_{act}}{dQ} = 0$$

$$\frac{dP_{act}}{dQ} = \omega \left[1(H) - \frac{f(3Q^2)}{12d^5} \right] = 0$$

$$\frac{H - \frac{3f(3Q^2)}{12d^5}}{12d^5} = 0$$

$$H = \frac{3f(3Q^2)}{12d^5}$$

$$H = 3h_L$$

then the efficiency,

$$\eta = \frac{H - h_L}{H}$$

$$\eta = 1 - \frac{h_L}{H}$$

$$\eta_{max} = 1 - \frac{1}{3}$$

$$\eta_{max} = \frac{2}{3}$$

$$\eta_{max} = 0.6667$$

$$\boxed{\eta_{max} = 66.67\%}$$

prob-100 - Two water carrying pipe's are connected in parallel the pipe length l , dia d , friction factor f , for the first pipe are 200m, 0.5m, 0.025m & for the second pipe are 100m, 1m, 0.02m respectively then find v_2/v_1 .

- (a) 5 (b) 7 (c) $\sqrt{7}$ (d) $\sqrt{5}$.

Solⁿ: The pipes are in parallel, then

$$h_{L1} = h_{L2}$$

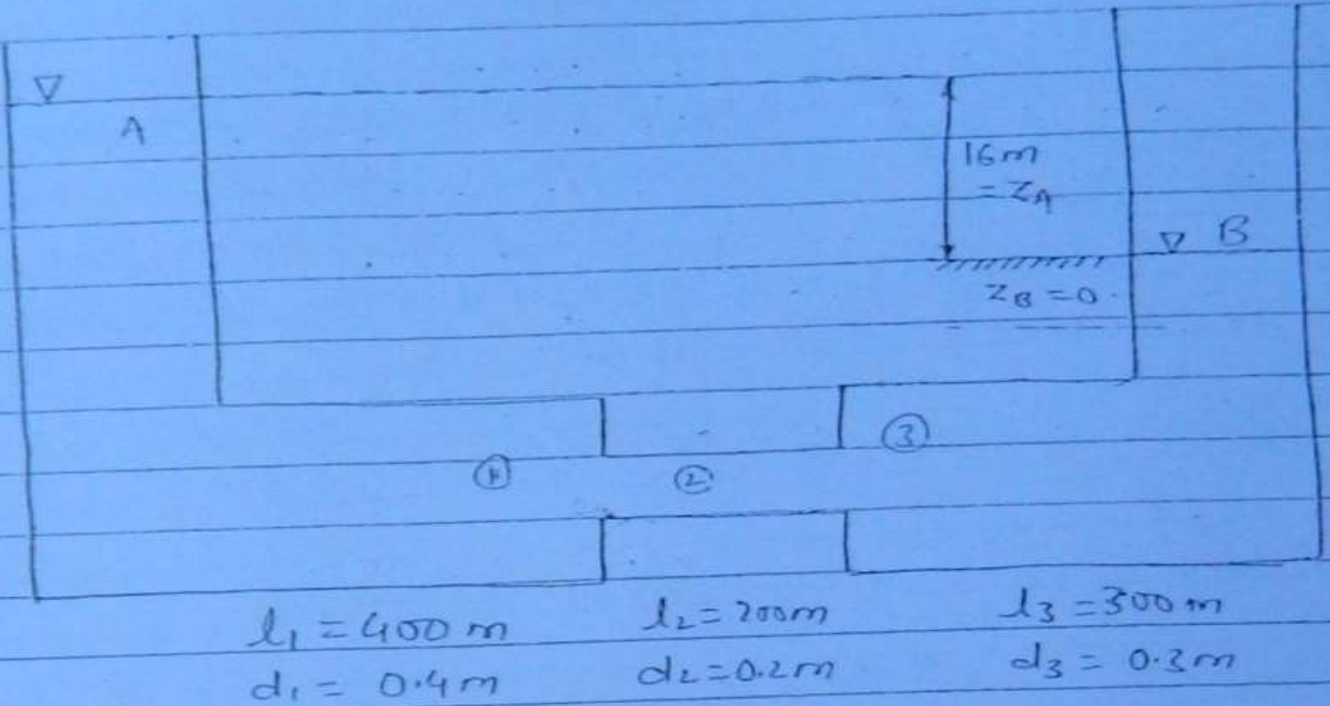
$$\frac{f_1 l_1 v_1^2}{2g \times d_1} = \frac{f_2 l_2 v_2^2}{2g \times d_2}$$

$$\frac{v_2^2}{v_1^2} = \frac{f_1 l_1 d_2}{f_2 l_2 d_1}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{f_1 l_1 d_2}{f_2 l_2 d_1}}$$

$$\frac{v_2}{v_1} = \sqrt{5} \quad \text{Ans.}$$

prob-101 - Three pipe's are connected in series as shown in fig. The difference in water level b/w two tank's is 16 If the friction factor for all pipe's is same & is equal to 0.02 find the discharge (neglect minor losses).



Apply bernoullis eqn. b/w A & B.

$$\frac{P_A}{\rho} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho} + \frac{V_B^2}{2g} + Z_B + h_L$$

$$Z_A = Z_B + h_L$$

$$16 = 0 + h_L$$

$$h_L = 16$$

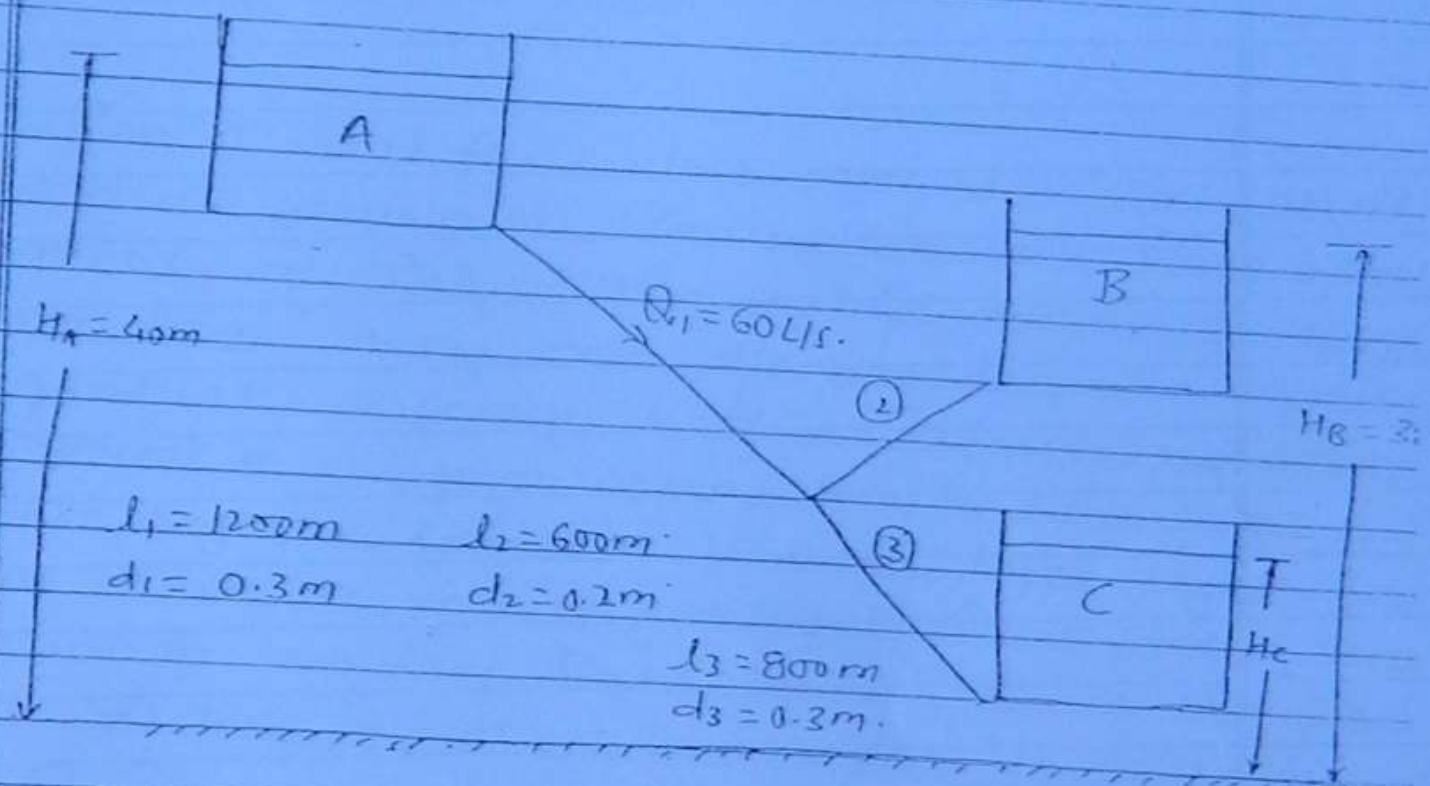
$$h_L = h_{L1} + h_{L2} + h_{L3}$$

$$h_L = \frac{f L_1 Q^2}{12 d_1^5} + \frac{f L_2 Q^2}{12 d_2^5} + \frac{f L_3 Q^2}{12 d_3^5}$$

$$16 = \frac{f Q^2}{12} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right]$$

$$Q = 0.11 \text{ m}^3/\text{se. Ans.}$$

06-102 - Three reservoirs A, B, C are connected in a pipe system as shown in fig. find the discharge into or from reservoirs B & C the flow rate from reservoir A is 60 liter/sec. also find the height of water level in reservoir C take friction factor $f = 0.024$ for all pipes.

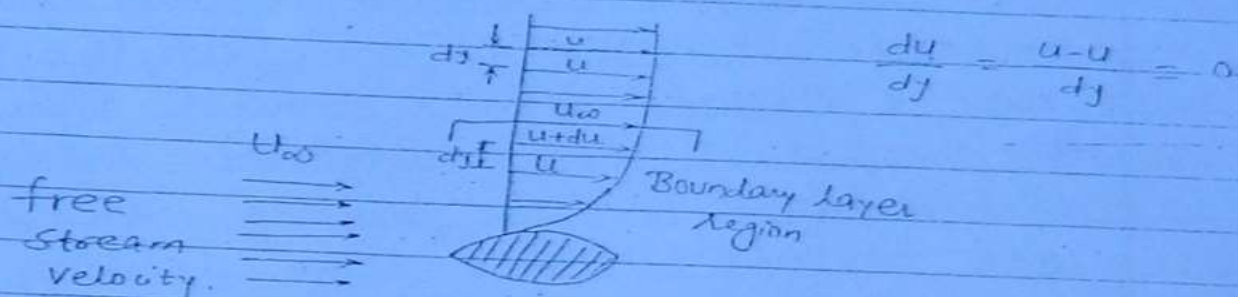


Ans: For Reservoir A,

$$\frac{P_A}{\rho} + \frac{V_A^2}{2g} + Z_A = H_A$$

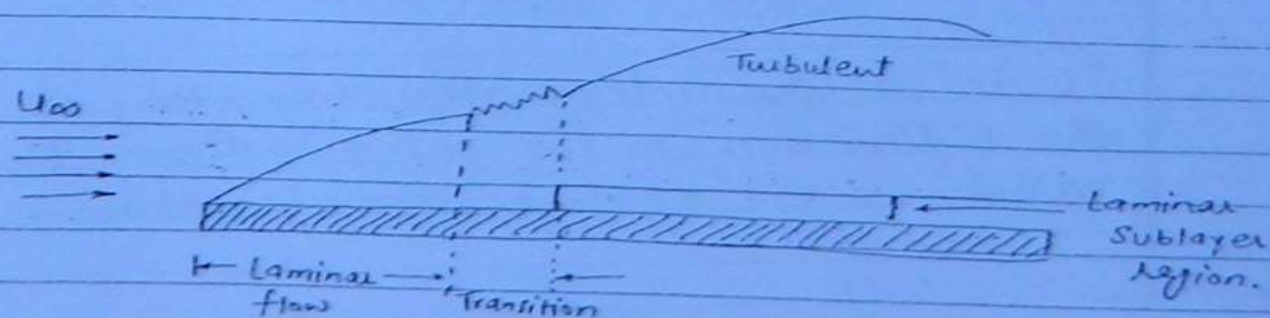
Boundary Layer Theory

It was introduced by prandtl in 1904.



When a real fluid flow is passed a solid object a fluid particle on the surface will have same velocity as that of the surface due to no-slip condition. Away from the boundary the velocity gradually increases & hence velocity gradient (du/dy) exist in a region close to boundary. This region is known as boundary layer region. As there are velocity gradient, shear stress exist in boundary layer region & hence the flow is viscous in nature in boundary layer region.

Development of boundary layer over a flat plate



When a real fluid flow passed a flat plate the viscosity velocity at the leading edge is zero. & the retardation of fluid particle's increases as more & more of the plate area is exposed to the flow & hence the boundary layer thickness increases at the distance of the leading edge increases upto certain distance from leading edge flow in boundary layer is found to be laminar & as the laminar boundary layer grows instability occurs & flow changes from laminar to turbulent through transition it is found that even in turbulent boundary layer region closed to the plate the flow is laminar & this region is known as laminar sublayer region. & This exist in turbulent boundary layer region.

Imp.

NOTE-

It is found that the velocity profile is parabolic in laminar boundary layer region, logarithmic in turbulent boundary layer region & linear in laminar sub layer region.

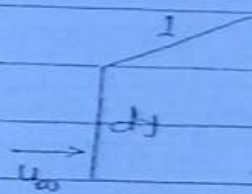
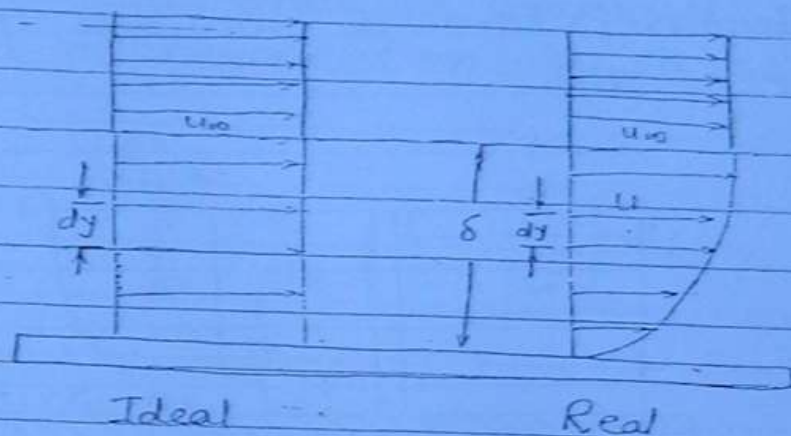
:-

Boundary layer thickness or nominal boundary layer thickness (δ):

It is distance from the boundary in the y -direction to the point where the velocity is 99% of free stream velocity.

NOTE- For all calculation purpose u is taken as u_∞ at ∞

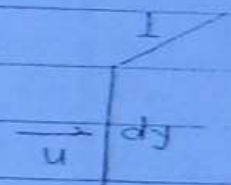
Displacement thickness



$$\dot{m} = \rho A V$$

$$\dot{m}_{\text{Ideal}} = \rho \times dy \times 1 \times u_\infty$$

$$\dot{m}_{\text{Ideal}} = \rho u_\infty dy$$



$$\dot{m} = \rho A V$$

$$\dot{m}_{\text{Real}} = \rho \times dy \times 1 \times u$$

$$\dot{m}_{\text{Real}} = \rho u dy$$

Reduction in mass flow rate

$$= \dot{m}_{\text{Ideal}} - \dot{m}_{\text{Real}}$$

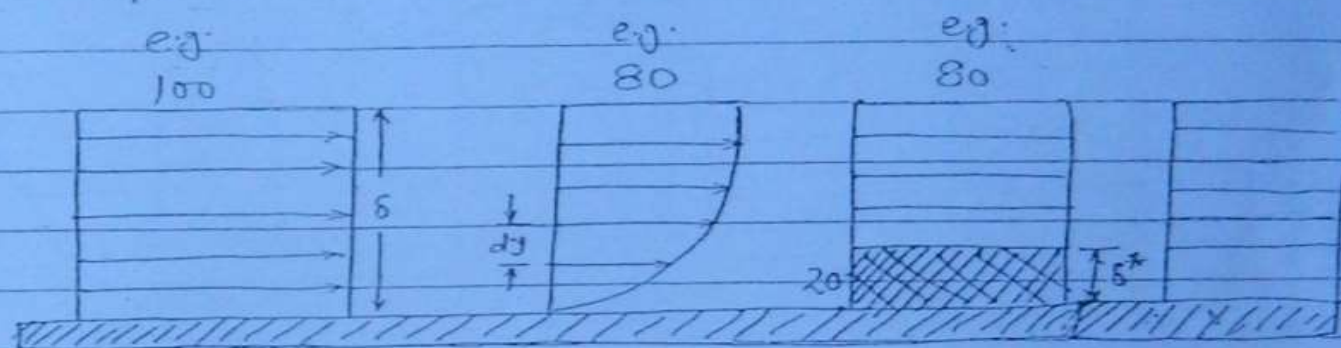
$$= \rho u_\infty dy - \rho u dy$$

$$= \rho (u_\infty - u) dy$$

Total Reduction in mass flow rate

$$= \int_0^{\delta} P (U_{\infty} - U) dy.$$

Boundary layer is form due to viscosity (because of no slip).



$$\dot{m} = \rho \times \delta^* \times 1 \times U_{\infty}$$

$$\dot{m} = \rho U_{\infty} \delta^*$$

then $\rho U_{\infty} \delta^* = \int_0^{\delta} \rho (U_{\infty} - U) dy.$

$$\delta^* = \int_0^{\delta} \left(\frac{U_{\infty} - U}{U_{\infty}} \right) dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{U}{U_{\infty}} \right) dy$$

It is the distance by which the solid boundary should be displaced in order to compensate for reduction in mass flow rate due to boundary

layer growth.

Momentum thickness (θ) :-

It is the distance by which the boundary should be displaced in order to compensate for the reduction of momentum due to boundary layer growth.

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}} \right) dy$$

Energy thickness (δ_E) :-

It is the distance by which the boundary should be displaced in order to compensate for the reduction in kinetic energy due to boundary layer growth.

$$\delta_E = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u^2}{u_{\infty}^2} \right) dy$$

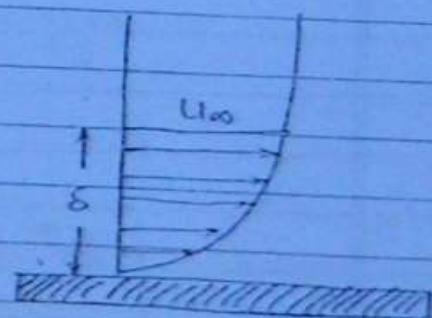
Boundary conditions :-

At $y=0$, $u=0$

At $y=\delta$, $u=u_{\infty}$

At $y=\delta$, $\frac{du}{dy} = 0$

At $x=0$, $\delta=0$.



prob-103- The velocity distribution in a boundary layer is given by $u/u_\infty = y/\delta$ then find displacement thickness δ^* & momentum thickness θ .

Soln:- give, $\frac{u}{u_\infty} = \frac{y}{\delta}$ (velocity profile).

We know that displacement thickness,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$\delta^* = \int_0^\delta dy - \frac{y}{\delta} dy$$

$$\delta^* = \left[y - \frac{y^2}{2\delta} \right]_0^\delta$$

$$\delta^* = \delta - \frac{\delta^2}{2\delta}$$

$$\delta^* = \delta - \frac{\delta}{2}$$

$$\boxed{\delta^* = \frac{\delta}{2}} \quad \text{Ans.}$$

We know that momentum thickness θ ,

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}} \right) dy$$

$$\theta = \int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right) dy$$

$$\theta = \int_0^{\delta} \frac{y}{\delta} dy - \frac{y^2}{\delta^2} dy$$

$$\theta = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} \right]_0^{\delta}$$

$$\theta = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}$$

$$\theta = \frac{\delta}{2} - \frac{\delta}{3}$$

$$\theta = \frac{\delta}{6} \quad \text{Ans.}$$

IE- The safe factor of boundary layer is

$$H = \frac{\delta^*}{\theta}$$

layer the safe factor greater the ^{chance} ~~change~~ of boundary layer separation.

Prob-104 - The shear stress distribution in a laminar boundary layer region is given by

$$\tau = \tau_0 \left(1 - \frac{y}{\delta} \right)$$

Find displacement thickness & momentum thickness.

Soln: given, $\tau = \tau_0 \left(1 - \frac{y}{\delta} \right) \quad \text{--- (1)}$

we know that,

$$\tau = \mu \frac{du}{dy} \quad \text{--- (11)}$$

from eqn. (1) & (11).

$$\tau_0 \left(1 - \frac{y}{\delta} \right) = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{\tau_0}{\mu} \left(1 - \frac{y}{\delta} \right)$$

$$du = \frac{\tau_0}{\mu} \left(1 - \frac{y}{\delta} \right) dy$$

Integrate the above eqn.

$$u = \frac{\tau_0}{\mu} \left(y - \frac{y^2}{2\delta} \right) + C$$

At $y=0$, $u=0 \Rightarrow C=0$

$$u = \frac{\tau_0}{\mu} \left(y - \frac{y^2}{2\delta} \right) \quad \text{--- (I)}$$

At $y=\delta$, $u=u_\infty$

$$u_\infty = \frac{\tau_0}{\mu} \left(\delta - \frac{\delta^2}{2\delta} \right)$$

$$u_\infty = \frac{\tau_0}{\mu} \left(\frac{\delta}{2} \right) \quad \text{--- (II)}$$

divide eqn. (I) with (II).

$$\frac{u}{u_\infty} = \frac{\tau_0}{\mu} \left(y - \frac{y^2}{2\delta} \right) \cdot \frac{\mu}{\tau_0 \delta}$$

$$\frac{u}{u_\infty} = \left(y - \frac{y^2}{2\delta} \right) \cdot \frac{2}{\delta}$$

$$\frac{u}{u_\infty} = \left(\frac{2y}{\delta} - \frac{2y^2}{2\delta^2} \right)$$

$$\frac{u}{u_\infty} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty} \right) dy$$

from displacen
thickness we k

then $\delta^* = \int_0^{\delta} \left[1 - \left\{ \frac{2y}{\delta} - \frac{y^2}{\delta^2} \right\} \right] dy$

$$\delta^* = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2} \right]_0^{\delta}$$

$$\delta^* = \left[\frac{\delta - \frac{2\delta^2}{2\delta} + \frac{\delta^3}{3\delta^2}}{\quad} \right]$$

$$\delta^* = \left[\frac{\delta - \delta + \frac{\delta}{3}}{\quad} \right]$$

$$\delta^* = \frac{\delta}{3} \quad \text{Ans.}$$

Momentum thickness,

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}} \right) dy$$

$$= \int_0^{\delta} \left\{ \frac{2y}{\delta} - \frac{y^2}{\delta^2} \right\} \left\{ 1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2} \right\} dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4} \right]_0^{\delta}$$

$$= \frac{2\delta^2}{2\delta} - \frac{5\delta^3}{3\delta^2} + \frac{4\delta^4}{4\delta^3} - \frac{\delta^5}{5\delta^4}$$

$$\theta = \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$\theta = 2\delta - \frac{5\delta}{3} - \frac{\delta}{5}$$

$$\theta = \frac{30\delta - 25\delta - 3\delta}{15}$$

$$\boxed{\theta = \frac{2\delta}{15}} \quad \text{Ans.}$$

Von Karman momentum Integral equation:-

Assumption:-

Steady flow.

Incompressible flow.

Two-dimensional flow.

pressure gradient in the direction of motion
 $\left(\frac{\partial p}{\partial x} = 0\right)$

$$\boxed{\frac{T_0}{\rho U_\infty^2} = \frac{d\theta}{dx}}$$

where T_0 = shear stress on the surface of plate.

θ = momentum thickness.

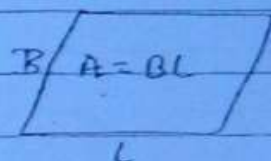
Drag force :- (F_D)

The force exerted by fluid on the surface of plate in a direction parallel to the relative motion is known as drag force.

At zero Angle of incident drag force is due to shear force.

Average drag co-efficient (C_D) :-

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U_{\infty}^2}$$



local drag coefficient or local skin friction coefficient (C_{fx}) :-

$$C_{fx} = \frac{\tau_0}{\frac{1}{2} \rho U_{\infty}^2}$$

Reynold's Number (Re) :-

$$Re_x = \frac{\rho U_{\infty} x}{\mu}$$

This Reynold's number is known as local Reynold's number.

The flow in the boundary layer over flat plate is laminar if Re is less than 5×10^5

NOTE- Reynold's number in non-circular pipes.

$$Re = \frac{PV D_{eq}}{\mu}$$

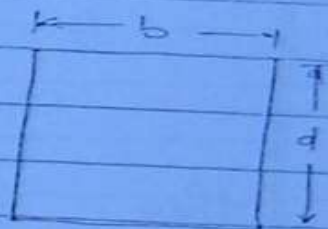
where $D_{equivalent} = \frac{4A}{P}$

A = Area.

P = Perimeter.

e.g. $D_{eq} = \frac{4(bd)}{2(b+d)}$

$$D_{eq} = \frac{2bd}{b+d}$$



Q-105 For a velocity profile in laminar boundary layer -

$$\frac{u}{u_{\infty}} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

then find -

- (i) Boundary layer thickness.
- (ii) Shear stress on the surface of the plate.
- (iii) Drag force.
- (iv) Average drag coefficient in terms of Reynold's number.

$$\frac{T_o}{\rho u_{\infty}^2} = \frac{d\theta}{dx}$$

$$T_o = \rho u_{\infty}^2 \frac{d\theta}{dx}$$

Momentum thickness,

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}} \right) dy$$

$$\theta = \int_0^{\delta} \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3} \right) \left\{ 1 - \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3} \right) \right\} dy$$

$$\theta = \frac{39\delta}{280}$$

then $T_0 = \rho u_{\infty}^2 \times \frac{d}{dx} \left(\frac{39\delta}{280} \right)$

$$T_0 = \frac{39}{280} \cdot \rho u_{\infty}^2 \frac{d\delta}{dx} \quad \text{--- (1)}$$

we know that,

$$\tau = u \frac{du}{dy}$$

$$\frac{u}{u_{\infty}} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

$$u = u_{\infty} \left[\frac{3y}{2\delta} - \frac{y^3}{2\delta^3} \right]$$

$$\frac{du}{dy} = u_{\infty} \left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3} \right]$$

$$\frac{du}{dy} = u_{\infty} \left[\frac{3}{2\delta} \quad \frac{3y^2}{2\delta^2} \right]$$

$$\left. \frac{du}{dy} \right|_{y=0} = \frac{3u_{\infty}}{2\delta}$$

$$\tau_0 = \mu \left. \frac{du}{dy} \right|_{y=0}$$

$$\tau_0 = \mu \times \frac{3u_{\infty}}{2\delta}$$

$$\tau_0 = \frac{3\mu u_{\infty}}{2\delta} \quad \text{--- (II)}$$

from eqn. (I) & (II),

$$\frac{39}{280} \rho u_{\infty}^2 \frac{d\delta}{dx} = \frac{3\mu u_{\infty}}{2\delta}$$

$$\frac{13}{140} \rho u_{\infty} \frac{d\delta}{dx} = \frac{\mu}{\delta}$$

$$\frac{13}{140} \frac{\rho u_{\infty}}{\mu} \delta \cdot d\delta = dx$$

Integrate above eqn.

$$\frac{13}{140} \frac{\rho u_{\infty}}{\mu} \cdot \frac{\delta^2}{2} = x + C$$

At $x=0$, $\delta=0 \Rightarrow C=0$.

$$\frac{13}{280} \frac{P U_{\infty} \delta^2}{\mu} = x$$

multiplied by x in both side.

$$\frac{13}{280} \frac{P U_{\infty} x \delta^2}{\mu} = x^2$$

$$\frac{13}{280} x R e_x \cdot \delta^2 = x^2$$

$$\delta^2 = \frac{280 x^2}{13 R e_x}$$

$$\delta = \sqrt{\frac{280}{13}} \frac{x}{\sqrt{R e_x}}$$

$$\delta = \frac{4.64 x}{\sqrt{R e_x}}$$

$$\delta = \frac{4.64 x}{\sqrt{\frac{P U_{\infty} x}{\mu}}}$$

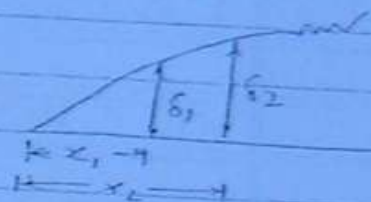
$$\delta \propto \frac{x}{\sqrt{x}}$$

$$\boxed{\delta \propto x^{1/2}}$$

δ vary with
parabolically.

$$\delta \propto \sqrt{x}$$

$$\frac{\delta_2}{\delta_1} = \frac{\sqrt{x_2}}{\sqrt{x_1}}$$



This is valid only for laminar boundary layer.

from eqn. (ii),

$$\tau_0 = \frac{3\mu U_\infty}{2\delta}$$

$$\tau_0 = \frac{3\mu U_\infty}{2 \times 4.64x \sqrt{Re_x}}$$

$$\tau_0 = \frac{3}{2 \times 4.64} \frac{\mu U_\infty \sqrt{Re_x}}{x}$$

$$\tau_0 = \frac{0.323 \mu U_\infty \sqrt{Re_x}}{x}$$

$$\tau_0 \propto \frac{1}{x} \cdot \sqrt{x}$$

$$\tau_0 \propto \frac{1}{x^{1/2}}$$

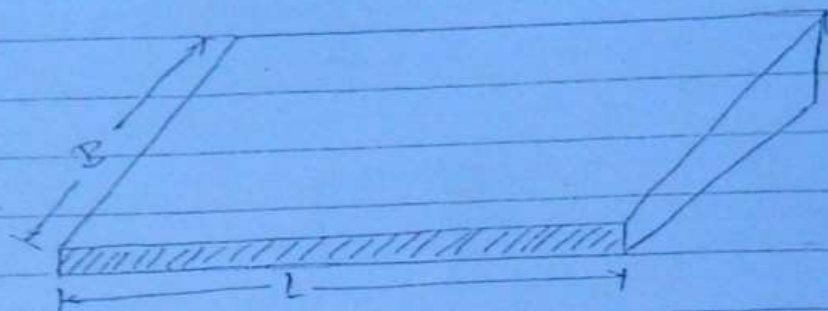
$$\tau_0 \propto x^{-1/2}$$



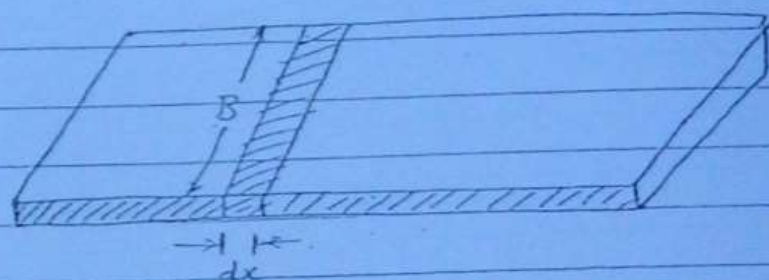
Shear stresses decrease along length.

$$\frac{\tau_{01}}{\tau_{02}} = \frac{\sqrt{x_2}}{\sqrt{x_1}}$$

Shear stress varies with inverse proportional.



As the shear stress T_0 vary with length then take small element and find the drag force in small element & then integrate for finding total drag force.



The drag force on small element.

$$dF_D = T_0 \times B dx$$

Total drag force.

$$F_D = \int_0^L T_0 \times B dx$$

$$F_D = \int_0^L 0.323 \frac{\mu U_\infty \sqrt{Re_x}}{x} B dx$$

$$F_D = \int_0^L 0.323 \frac{\mu U_\infty}{x} \sqrt{\frac{\rho U_\infty x}{\mu}} B dx$$

$$F_D = 0.323 \mu U_\infty B \frac{\sqrt{\rho U_\infty}}{\sqrt{\mu}} \int_0^L \frac{1}{\sqrt{x}} dx$$

$$F_D = 0.323 \mu U_{\infty} B \frac{\sqrt{P U_{\infty}}}{\sqrt{\mu}} [2\sqrt{L}]$$

$$F_D = 0.646 \mu U_{\infty} B \cdot \frac{\sqrt{P}}{\sqrt{\mu}} \sqrt{U_{\infty}} \sqrt{L}$$

$$F_D = 0.646 \mu U_{\infty} B \sqrt{\frac{P U_{\infty} L}{\mu}}$$

$$F_D = 0.646 \mu U_{\infty} B \cdot \sqrt{Re_L}$$

$$C_D = \frac{F_D}{\frac{1}{2} P A U_{\infty}^2}$$

$$C_D = \frac{0.646 \mu U_{\infty} B \sqrt{Re_L}}{\frac{1}{2} P A U_{\infty}^2}$$

$$C_D = \frac{1.292 \mu \sqrt{Re_L}}{P L U_{\infty}}$$

$$C_D = \frac{1.292 \sqrt{Re_L}}{\frac{P U_{\infty} L}{\mu}}$$

$$C_D = \frac{1.292 \sqrt{Re_L}}{Re_L}$$

$$C_D = \frac{1.292}{\sqrt{Re_L}}$$

∴ When velocity profile is not given use the following eqn. (Blausius equation)

Laminar	Turbulent.
$\delta = \frac{5x}{\sqrt{Re_x}}$	$\delta = \frac{0.576x}{(Re)^{1/5}}$
$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.059}{(Re)^{1/5}}$
$C_D = \frac{1.328}{\sqrt{Re_L}}$	$C_D = \frac{0.074}{(Re)^{1/5}}$

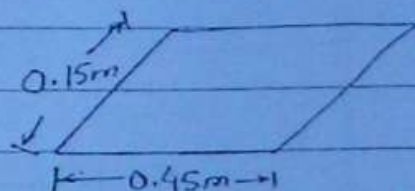
prob-106- Calculate the drag force on the plate 0.15m wide & 0.45m long placed in a stream of oil having a free stream velocity of 6m/s. also find the thickness of boundary layer & shear stress at the trailing edge. The density of oil has 925 kg/m^3 & kinematic viscosity is equal to $0.9 \times 10^{-4} \text{ m}^2/\text{s}$.

Solⁿ:

$$Re_L = \frac{\rho U_{\infty} L}{\mu}$$

$$Re_L = \frac{U_{\infty} L}{\frac{\mu}{\rho}} = \frac{U_{\infty} L}{\nu}$$

$$Re_L = \frac{6 \times 0.45}{0.9 \times 10^{-4}} = 3 \times 10^4$$



As $Re_L < 10^5$

flow in the boundary layer is laminar.

$$(i) \quad C_D = \frac{F_D}{\frac{1}{2} \rho A U_\infty^2} = \frac{1.328}{\sqrt{Re_L}}$$

$$\frac{F_D}{\frac{1}{2} \times 925 \times 0.15 \times 0.45 \times 6^2} = \frac{1.328}{\sqrt{3 \times 10^4}}$$

$$\boxed{F_D = 8.6 \text{ N}} \text{ Ans.}$$

This is the drag force on one side of the plate.

(ii) Shear stress at the trailing edge.

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{\tau_0}{\frac{1}{2} \rho U_\infty^2}$$

$$= \frac{0.664}{\sqrt{3 \times 10^4}} = \frac{\tau_0}{\frac{1}{2} \times 925 \times 6^2}$$

$$\boxed{\tau_0 = 63.8 \text{ N/m}^2} \text{ Ans.}$$

(iii) Boundary layer thickness.

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.45}{\sqrt{3 \times 10^4}}$$

$$\boxed{\delta = 0.0129 \text{ m.}} \text{ Ans.}$$

prob-107 - For an air flow over a flat plate the velocity u & boundary layer thickness δ can be expressed as-

$$\frac{u}{u_{\infty}} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

& $\delta = \frac{4.64x}{\sqrt{Re_x}}$ If $u_{\infty} = 2 \text{ m/s}$.

$$\nu_{\text{air}} = 1.5 \times 10^{-5} \text{ m}^2/\text{sec}.$$

$$\rho = 1.32 \text{ kg/m}^3.$$

find the shear stress on the surface of plate at $x = 1 \text{ m}$.

Soln: given,

$$\frac{u}{u_{\infty}} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

$$u = u_{\infty} \left[\frac{3y}{2\delta} - \frac{y^3}{2\delta^3} \right]$$

$$\frac{du}{dy} = u_{\infty} \left[\frac{3(1)}{2\delta} - \frac{3y^2}{2\delta^3} \right]$$

$$\left. \frac{du}{dy} \right|_{y=0} = u_{\infty} \left[\frac{3}{2\delta} \right]$$

& $\tau_0 = \mu \left. \frac{du}{dy} \right|_{y=0}$

$$\tau_0 = \mu \times \frac{3U_{\infty}}{2s} \quad \text{--- ①}$$

given $s = \frac{4.64x}{\sqrt{Re_x}}$

but $Re_x = \frac{U_{\infty} \cdot x}{\mu/\rho}$

$$Re_x = \frac{U_{\infty} \cdot x}{\nu}$$

$$Re_x = \frac{2x}{1.5 \times 10^{-5}} = 1.33 \times 10^5$$

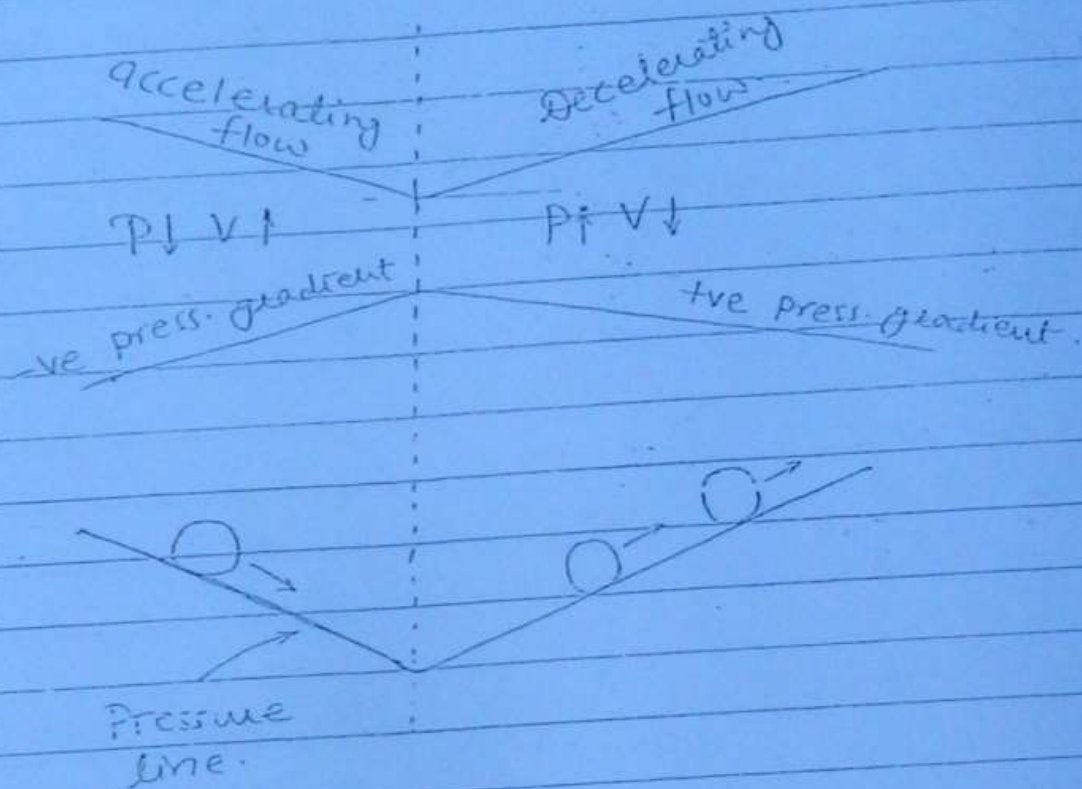
then $s = \frac{4.64 \times 1}{\sqrt{1.33 \times 10^5}} = 0.0127 \text{ m.}$

Now put the value in eqn. ①.

$$\tau_0 = \frac{3 \times 1.5 \times 10^{-5} \times 1.23 \times 10^3}{2 \times 0.0127}$$

$$\tau_0 = 4.35 \times 10^{-3} \text{ N/m}^2 \text{ Ans.}$$

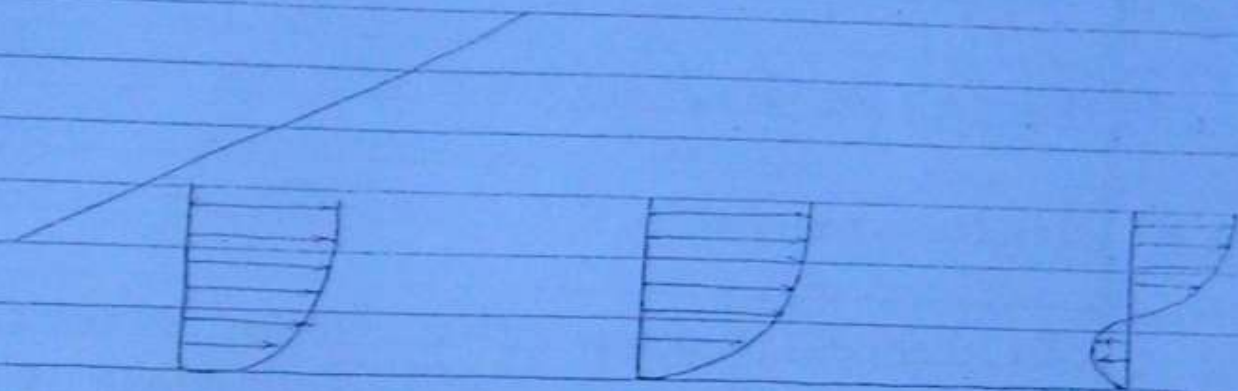
1- Boundary layer separation:-



In case of convergent flow the velocity increases & pressure decreases & the fluid slides down the press. line when fluid flow in divergent passage the velocity decreases & pressure increases. i.e. The fluid moves under positive pressure gradient & hence it has to climb press. hill if the velocity reduction is more momentum of particles may not be support to climb a pressure hill & hence the flow may reverse its direction from boundary & this is known as boundary layer separation & reversal of flow & this occurs under +ve press. gradients & These press. gradient is also known as

adverse press. gradient.

NOTE- For a fully developed flow the velocity profile remains same.



$$\left. \frac{du}{dy} \right|_{y=0} = +ve$$

No separation.
(Attached flow)

$$\left. \frac{du}{dy} \right|_{y=0} = 0$$

About to separate
Separation point.

$$[\tau = 0]$$

$$\left. \frac{du}{dy} \right|_{y=0} = -ve$$

Separated
detached

NOTE- The max. Thickness of boundary layer in pipe flow is equal to radius of pipe.

Vortex Motion

The motion of fluid in a curved path is known as vortex motion.

Vortex motion is of two types -

1. Force vortex.
2. Free vortex.

In forced vortex the fluid moves in curved path under the influence of external torque as there is a continuous expenditure of energy bernoulli's equation is not applicable.

e.g. ① Liquid in a container when rotated.

② Motion of the fluid in the impeller of a centrifugal pump.

For Force vortex $V = r\omega$ is applicable.

In case of free vortex the fluid moves in curved path due to motion given to it's earlier bernoulli's equation is applicable to free vortex.

e.g. ① Flow of fluid in a pipe bend.

② Motion of liquid in diffuser of the centrifugal pump.

- ③ Flow of liquid in wash basin.
 ④ Whirlpool in river.

Angular momentum = mvr (Moment of momentum)

Change in momentum is known as Torque.

$$\frac{d(mvr)}{dt} = T = 0.$$

$$\frac{d(mvr)}{dt} = 0.$$

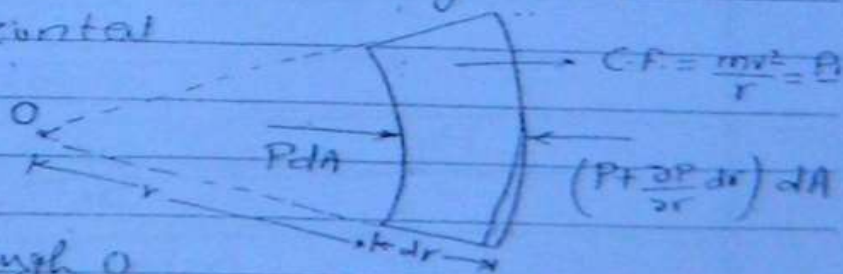
$$mvr = \text{const.}$$

$$vr = \frac{\text{const.}}{m}$$

$$vr = \text{const.}$$

Equation of motion For vortex flow:-

Consider a fluid element rotating at uniform velocity in a horizontal plane about an axis perpendicular to the plane of paper & passing through O.



let r = Radius of the element from O.

dr = Radial thickness of the element.
 dA = Area of cross-section of the element.

The force acting on the element.

- (i) Press. force $P dA$, on the face AB.
- (ii) Press. force $\left(P + \frac{\partial P}{\partial r} dr\right) dA$ on the face CD.
- (iii) Centrifugal force, $\frac{mv^2}{r}$ acting in the direction away from the centre O.

Now, the mass of the element

$$\begin{aligned} &= \text{mass density} \times \text{Volume} \\ &= \rho \cdot dA \cdot dr. \end{aligned}$$

$$\text{Centrifugal force} = \rho \cdot dA \cdot dr \cdot \frac{v^2}{r}$$

Now equating the force in the radial direction

$$P dA + \frac{\rho dA dr v^2}{r} = \left(P + \frac{\partial P}{\partial r} dr\right) dA$$

$$P + \frac{\rho v^2}{r} dr = P + \frac{\partial P}{\partial r} dr$$

$$\frac{\rho v^2}{r} dr = \frac{\partial P}{\partial r} dr$$

$$\left[\frac{\partial P}{\partial r} = \frac{\rho v^2}{r} \right] \leftarrow \text{valid for both free \& forced vortex.}$$

This shows the variation of press. in radial direction

We know that,

The press. variation in the vertical plane is given by the hydrostatic law i.e.

$$\frac{\partial P}{\partial z} = -w$$

$$\frac{\partial P}{\partial z} = -Pg.$$

Then

$$P = f(r, z)$$

$$dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$$

$$dP = \frac{\rho v^2}{r} dr + (-Pg) dz$$

$$dP = \frac{\rho v^2}{r} dr - Pg dz$$

This eqn. is valid for both free & forced vortex

For free vortex flow :-

$$vr = \text{const.}$$

$$v = \frac{C}{r}$$

$$\text{then, } dP = \frac{\rho}{r} \left(\frac{C}{r} \right)^2 dr - Pg dz$$

$$\int_{P_1}^{P_2} dP = \int_{r_1}^{r_2} \rho C^2 \frac{dr}{r^3} - \int_{z_1}^{z_2} \rho g dz$$

$$P_2 - P_1 = -\frac{\rho C^2}{2} \left[\frac{1}{r^2} \right]_{r_1}^{r_2} - \rho g (z_2 - z_1)$$

$$P_2 - P_1 = -\frac{\rho C^2}{2} \left[\frac{1}{r_2^2} - \frac{1}{r_1^2} \right] - \rho g z_2 + \rho g z_1$$

$$P_2 - P_1 = -\frac{\rho C^2}{2 r_2^2} + \frac{\rho C^2}{2 r_1^2} - \rho g z_2 + \rho g z_1$$

$$P_2 - P_1 = -\frac{\rho}{2} v_2^2 + \frac{\rho}{2} v_1^2 - \rho g z_2 + \rho g z_1$$

$$P_2 + \frac{\rho}{2} v_1^2 + \rho g z_1 = P_1 + \frac{\rho}{2} v_2^2 + \rho g z_2$$

$$\boxed{\frac{P_2}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_1}{\rho g} + \frac{v_2^2}{2g} + z_2}$$

This is bernoulli's eqn. find from free vortex.

• For Forced vortex flow:-

$$dP = \frac{\rho v^2}{r} dr - \rho g dz$$

for forced vortex,

$$v = r\omega$$

$$dP = \frac{\rho}{r} r^2 \omega^2 dr - \rho g dz$$

$$dP = \rho \omega^2 r dr - \rho g dz$$

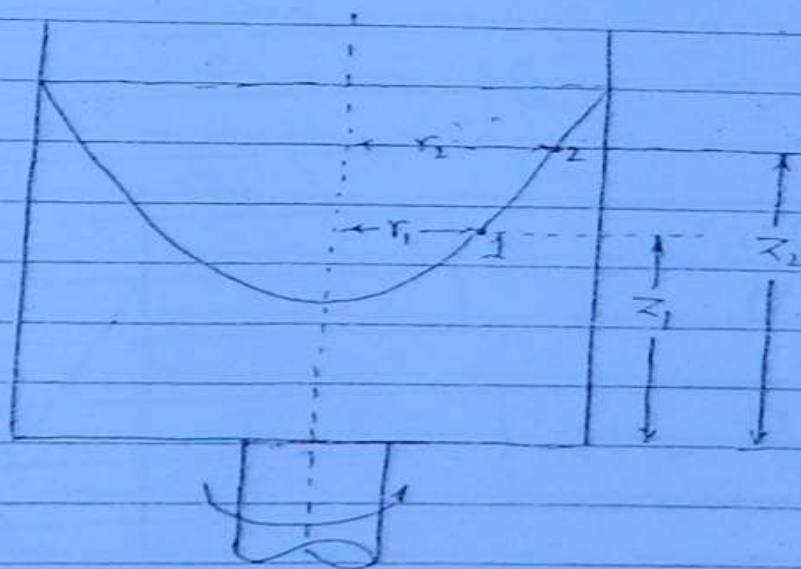
Integrate above eqn.

$$\int_{P_1}^{P_2} dP = \int_{r_1}^{r_2} \rho \omega^2 r dr - \int_{z_1}^{z_2} \rho g dz$$

$$P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho g (z_2 - z_1)$$

let us select point 1 & 2 on the surface of liquid.

then, $P_1 = P_2 \Rightarrow P_1 - P_2 = 0$.

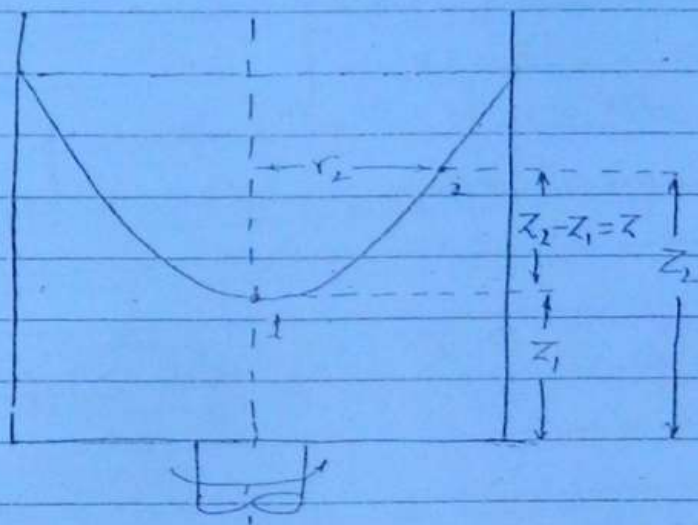


then, $0 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) - \rho g (z_2 - z_1)$

$$\frac{\rho \omega^2}{2} (r_2^2 - r_1^2) = \rho g (z_2 - z_1)$$

$$\frac{\omega^2}{2} (r_2^2 - r_1^2) = g (z_2 - z_1)$$

let us select point 1 on the axis of rotation.



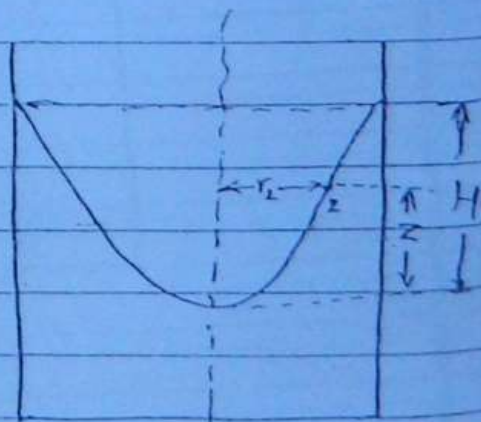
then $r_1 = 0$.

$$\frac{\omega^2 r_2^2}{2} = g (z_2 - z_1)$$

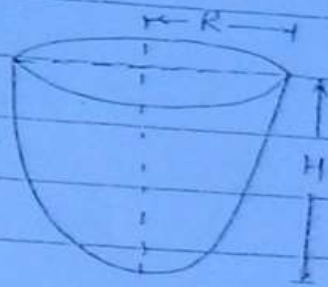
$$\frac{\omega^2 r_2^2}{2} = gz$$

$$\frac{\omega^2 r_2^2}{2g} = z$$

$$\frac{\omega^2 r_2^2}{2g} = H$$

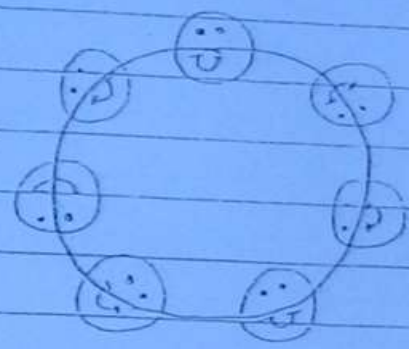


NOTE-

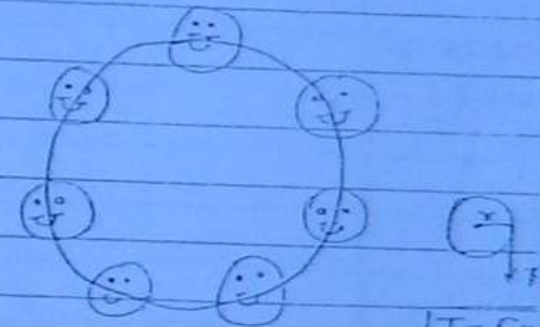


Vol. of paraboloid = $\frac{1}{2} \pi R^2 H$

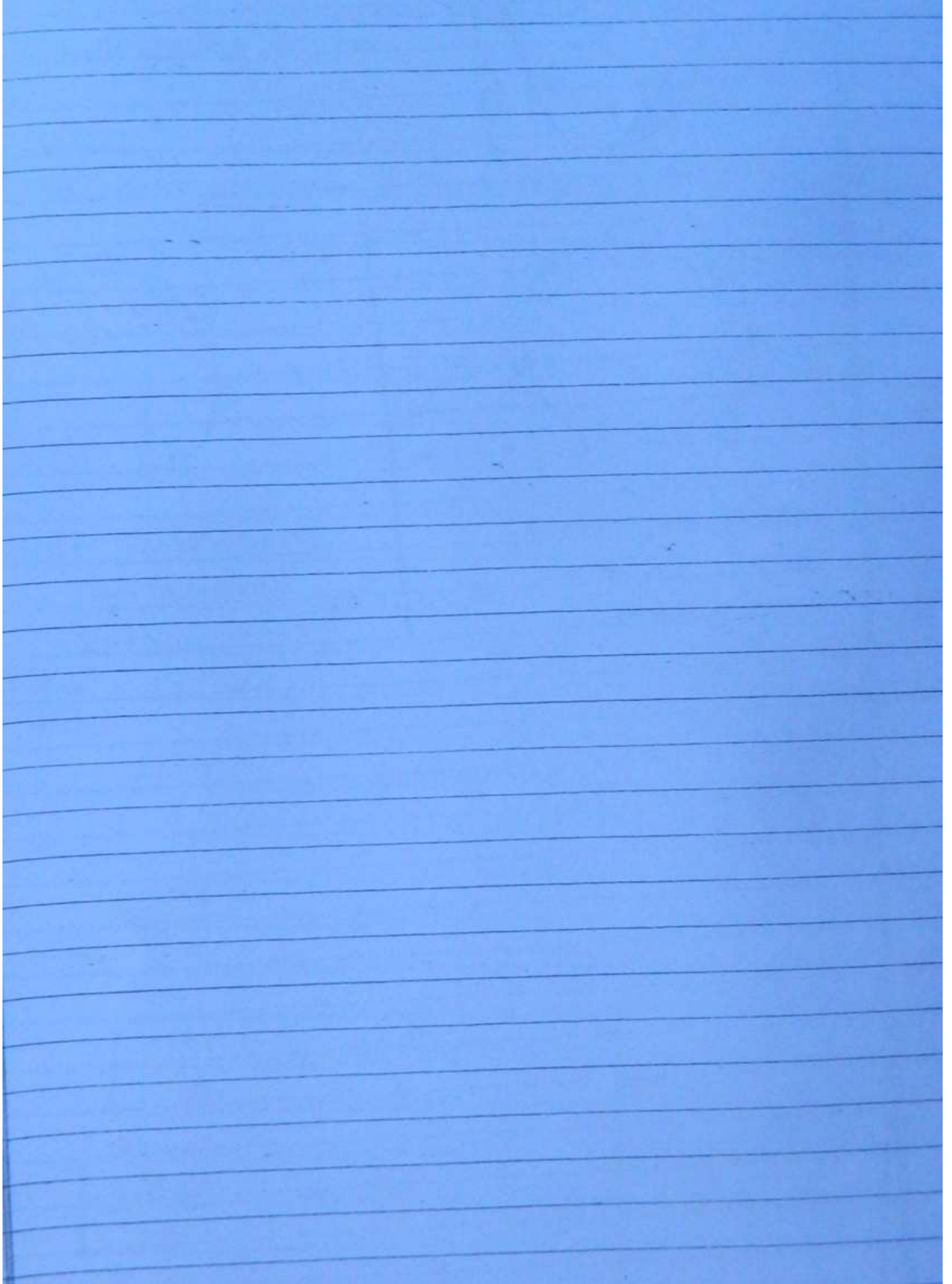
Imp.



Rotational flow.
Torque $\rightarrow$ $\checkmark$
forced vortex.



Irrotational flow
Torque $\rightarrow$ $\times$
Free vortex.



Dimensional Analysis

Rayleigh's method of grouping -

Prob- For a laminar flow in a pipe the press. drop ΔP is a function of pipe length L , dia. D , velocity V & viscosity μ using Rayleigh's method developed an expression for ΔP .

Soln:- $\Delta P = f(L, D, V, \mu)$

for laminar
 $\Delta P = \frac{32 \mu V L}{D^2}$

$$\Delta P = K L^a D^b V^c \mu^d$$

$$M L^{-1} T^{-2} = M^0 L^0 T^0 L^a (L)^b (L T^{-1})^c (M L^{-1} T^{-1})^d$$

$$M^1 L^{-1} T^{-2} = M^d L^{a+b+c-d} T^{-c-d}$$

$$d = 1$$

$$a+b+c-d = -1 \Rightarrow a+b+1-1 = -1$$

$$-c-d = -2$$

$$a+b = -1$$

$$-2 = -c-d$$

$$c+d = 2$$

$$c+1 = 2$$

$$c = 1$$

$$\Delta P = K L^a D^{-1-a} V^1 \mu^1$$

$$\Delta P = \frac{K L^a}{D^{1+a}} \mu V$$

$$\Delta P = \frac{K L^a}{D^{1+a}} \mu V \Rightarrow \boxed{\Delta P = K \frac{\mu V}{D} \left(\frac{L}{D} \right)^a}$$

↓ Buckingham's π -theorem :- (method of grouping)

$$Z = \phi(a, b, c, d, e)$$

$$n = 6 \quad (\text{Total variable})$$

Z -dependent variable

a, b, c, d, e are independent

m = no. of fundamental quantities.

Then the given system can be grouped into $(m-n)$ π terms.

Prob-

The resistance force F of a ship is a function of its length L , velocity v , acceleration due to gravity g & fluid property like density ρ & viscosity μ . Write this relationship in dimensional form using Buckingham π -theorem.

Soln:-

$$F = \phi(L, v, g, \rho, \mu)$$

$$F \rightarrow MLT^{-2}$$

$$L \rightarrow L$$

$$v \rightarrow LT^{-1}$$

$$g \rightarrow LT^{-2}$$

$$\rho \rightarrow ML^{-3}$$

$$\mu \rightarrow ML^{-1}T^{-1}$$

then here $m = 3$ & $n = 6$

Page _____

The given system can be grouped into $(m-n) = 3$ Π -term's.

Rule's for selecting Repeated variable -

1. NO. of Repeated variable is equal to No. of fundamental quantities.
2. Repeated variable must be selected from independent variable.
3. Repeated variable must have dimensions.
4. Repeated variable group must contain all fundamental quantities.
5. More fundamental quantity must be selected as repeated variable (e.g. - If v & g are independent variables then v must be taken as repeated variable because acceleration is derived from velocity).

$$F = \phi(L; v, g, \rho, \mu)$$

$$\Pi_1 = F(L^{a_1}, v^{b_1}, \rho^{c_1})$$

$$\Pi_2 = g[L^{a_2}, v^{b_2}, \rho^{c_2}]$$

$$\Pi_3 = \mu[L^{a_3}, v^{b_3}, \rho^{c_3}]$$

$$\pi_1 = F [L^{a_1} V^{b_1} P^{c_1}]$$

$$M^0 L^0 T^0 = M L T^{-2} [L^{a_1} (L T^{-1})^{b_1} (M L^{-3})^{c_1}]$$

$$M^0 L^0 T^0 = M^{1+c_1} L^{1+a_1+b_1+3c_1} T^{-2-b_1}$$

$$1+c_1 = 0$$

$$1+a_1+b_1-3c_1 = 0$$

$$-2-b_1 = 0$$

$$b_1 = -2$$

$$a_1 = -2$$

$$c_1 = -1$$

$$\pi_1 = F [L^{-2} V^{-2} P^{-1}]$$

$$\pi_1 = \frac{F}{\rho L^2 V^2}$$

$$\pi_2 = g [L^{a_2}, V^{b_2}, P^{c_2}]$$

$$M^0 L^0 T^0 = L T^{-2} [L^{a_2} (L T^{-1})^{b_2} (M L^{-3})^{c_2}]$$

$$M^0 L^0 T^0 = L^{1+a_2+b_2-3c_2} T^{-2-b_2} M^{c_2}$$

$$1+a_2+b_2-3c_2 = 0 \Rightarrow a_2 = 1$$

$$-2-b_2 = 0 \Rightarrow b_2 = -2$$

$$c_2 = 0$$

$$\pi_2 = g [L V^{-2} P^0]$$

$$\pi_2 = \frac{g L}{v^2}$$

$$L_4 \quad \pi_3 = \frac{\mu}{\rho v L}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

$$\frac{F}{\rho L^2 v^2} = \phi\left(\frac{g L}{v^2}, \frac{\mu}{\rho v L}\right)$$

$$\boxed{F = \rho v^2 L^2 \phi\left(\frac{g L}{v^2}, \frac{\mu}{\rho v L}\right)} \quad \text{Ans.}$$

prob- The pressure generated by the pump ΔP is the function of impeller diameter D , speed N , discharge Q , & fluid properties like density ρ & viscosity μ then write this relationship in dimensionless form using Buckingham π -theorem.

$$\text{Soln:} \quad \Delta P = \phi(D, N, Q, \rho, \mu)$$

$$\frac{F L}{m^2} = \frac{\rho \times Q}{m^2} = \frac{\rho \times m}{m^2 \times \text{sec}} = \frac{M L}{m^2 \text{ sec}}$$

$$\Delta P \rightarrow M L^{-1} T^{-2}$$

$$\& \mu = M L^{-1} T^{-1}$$

$$D \rightarrow L$$

$$N \rightarrow T^{-1}$$

$$Q \rightarrow L^3 T^{-1}$$

$$\rho \rightarrow M L^{-3}$$

$$m = 3 \quad \& \quad n = 6 - 3 = 3.$$

$$\Delta P = \Phi [D, N, Q, P, \mu]$$

$$\pi_1 = \Delta P [D^{a_1} N^{b_1} P^{c_1}]$$

$$\pi_2 = Q [D^{a_2} N^{b_2} P^{c_2}]$$

$$\pi_3 = \mu [D^{a_3} N^{b_3} P^{c_3}]$$

$$\pi_1 = \Phi (D^{a_1} N^{b_1} P^{c_1})$$

$$M^0 L^0 T^0 = M L^{-1} T^{-2} [L^{a_1} (T^{-1})^{b_1} (M L^{-3})^{c_1}]$$

$$M^0 L^0 T^0 = M^{1+c_1} L^{-1+a_1-3c_1} T^{-2-b_1}$$

$$1+c_1 = 0 \Rightarrow c_1 = -1$$

$$-1+a_1-3c_1 = 0 \Rightarrow a_1 = -2$$

$$-2-b_1 = 0 \Rightarrow b_1 = -2$$

$$\therefore \pi_1 = \Delta P [D^{-2} N^{-2} P^{-1}]$$

$$\pi_1 = \frac{\Delta P}{D^2 N^2 P}$$

$$\pi_2 = \frac{Q}{D^3 N} \quad \& \quad \pi_3 = \frac{\mu}{N D^2}$$

! - Various dimensionless number's :-

1. Inertia force :- (F_i)

$$F_i = ma$$

$$F_i = m \times \frac{v}{t}$$

$$F_i = \rho L^3 \cdot \frac{v}{t}$$

$$F_i = \rho L^2 \cdot L \cdot \frac{v}{t}$$

$$[F_i = \rho L^2 v^2]$$

2. Viscous force (F_v) :-

$$F_v = \frac{\mu A v}{y}$$

$$F_v = \frac{\mu L^2 v}{L}$$

$$[F_v = \mu v L]$$

3. Pressure force (F_p) :-

$$P = \frac{F_p}{A}$$

$$F_p = PA$$

$$[F_p = P L^2]$$

4. Gravity force (F_g):-

$$W_t = m \cdot g$$

$$F_g = mg$$

$$\boxed{F_g = \rho L^3 g}$$

$$\rho = \frac{m}{V} = \frac{m}{L^3}$$

$$m = \rho L^3$$

5. Surface tension force (F_s):-

$$\sigma = \frac{F_s}{L}$$

$$\boxed{F_s = \sigma L}$$

6. Elastic force (F_e):-

$$\boxed{F_e \propto K}$$

$$\frac{dP}{-\frac{dv}{v}} = K$$

$$\boxed{F_e = KL^2}$$

$$\Delta P \rightarrow K$$

$\uparrow$
 F_e then.

When a fluid is compressed there is a rise in press. this rise in press. is proportional to bulk modulus K & this rise in press will give rise in force is known as elastic force.

1. Reynold's Number:-

$$Re = \frac{F_i}{F_v} = \frac{\rho L^2 v^2}{\mu v L} = \frac{\rho v L}{\mu}$$

$$\boxed{Re = \frac{\rho v L}{\mu}}$$

When ever viscous force is coming use Reyn number.

2. Froude Number (F_r):-

$$F_r = \frac{F_i}{F_g} = \frac{\rho L^2 v^2}{\rho L^3 g} = \frac{v^2}{g L}$$

$$\boxed{F_r = \frac{v^2}{g L}}$$

When ever gravity force is coming use Froude number

3. Euler Number:-

$$\text{Euler No.} = \frac{F_i}{F_p} = \frac{\rho L^2 v^2}{\rho L^2} = \frac{\rho v^2}{p}$$

$$\boxed{EU = \frac{\rho v^2}{p}}$$

When ever press. force is coming use Euler N

4. Weber number:-

$$We = \frac{F_i}{F_s} = \frac{\rho L^2 v^2}{\sigma L} = \frac{\rho L v^2}{\sigma}$$

When ever surface tension is coming use Weber no.

5. Mach number:-

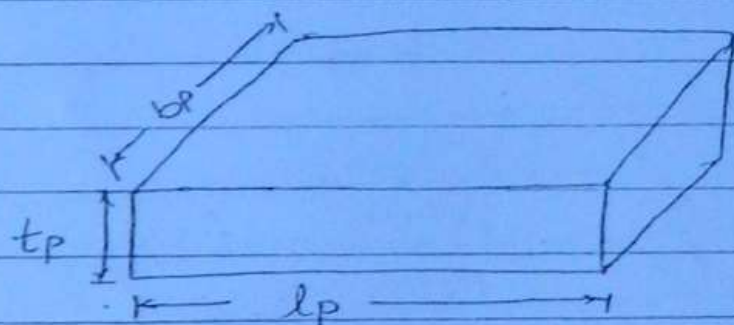
$$Ma = \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\rho L^2 v^2}{k L^2}} = \sqrt{\frac{v^2}{k/\rho}}$$

$$Ma = \frac{V}{\sqrt{\frac{k}{\rho}}} = \frac{V}{C}$$

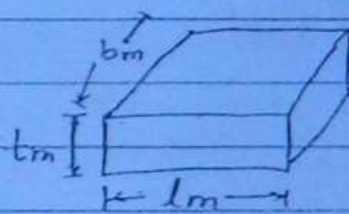
Whenever elastic force is coming use mach number.

Geometric Similarity

Model & prototype is said to be in geometric similarity if ratio of dimension's in model & prototype are same.



Prototype



Model

$$\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{t_m}{t_p} = l_r \text{ (scale ratio)}$$

$$Ar = \frac{A_m}{A_p} = \frac{l_m b_m}{l_p b_p}$$

$$Ar = l_r \cdot l_r$$

$$\boxed{Ar = l_r^2}$$

$$V_r = \frac{L_m b_m t_m}{L_p b_p t_p} = \frac{V_m}{V_p}$$

$$V_r = L_r \cdot L_r \cdot L_r$$

$$\boxed{V_r = L_r^3}$$

Kinematic Similarity

Model & prototype is said to be in kinematic similarity when ratio of velocity, acceleration & discharge at corresponding point in model & prototype is same.

For kinematic similarity geometric similarity is must.

Dynamic Similarity

Model & prototype are said to be in dynamic similarity when ratio of forces at corresponding points in model & prototype are same.

For dynamic similarity various dimensionless number are equated :-

—•—
1P

Prototype

—•—
1m

model

$$\frac{(F_i)_{1P}}{(F_i)_{1m}} = \frac{(F_v)_{1P}}{(F_v)_{1m}}$$

$$\frac{(F_i)_{IP}}{(F_i)_{Im}} = \frac{(F_v)_{IP}}{(F_v)_{Im}}$$

$$\left(\frac{F_i}{F_v} \right)_{IP} = \left(\frac{F_i}{F_v} \right)_{Im}$$

$$\boxed{(Re)_{IP} = (Re)_{Im}}$$

Q- A fluid flow phenomenon is to be studied in a model which is to be constructed by using Reynold's model law find expression's for model to prototype ratio of velocity & discharge.

$$Re = \frac{\rho V L}{\mu} = \frac{\rho V L}{\mu_p} = \frac{V L}{\nu}$$

$$(Re)_m = (Re)_p$$

$$\left(\frac{V L}{\nu} \right)_m = \left(\frac{V L}{\nu} \right)_p$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p}$$

$$\frac{V_m}{V_p} = \frac{1}{L_r} \times \nu_r \quad \left\{ \frac{L_m}{L_p} = L_r \right\}$$

$$\frac{V_r}{V_p} = \frac{V_m}{V_p} = \frac{V_r}{L_r}$$

$$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = L_r^2 \times \frac{V_r}{L_r}$$

$$\boxed{\frac{Q_m}{Q_p} = V_r L_r}$$

Qb- For Froude model law find the ratio of velocity & discharge.

$$Fr = \frac{V^2}{gL}$$

$$(Fr)_m = (Fr)_p$$

$$\left(\frac{V^2}{gL} \right)_m = \left(\frac{V^2}{gL} \right)_p$$

$$\frac{V_m^2}{g L_m} = \frac{V_p^2}{g L_p}$$

$$\left(\frac{V_m}{V_p} \right)^2 = \frac{L_m}{L_p}$$

$$\boxed{\frac{V_m}{V_p} = \sqrt{L_r}}$$

$$\frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = L_r^2 \times \sqrt{L_r} = L_r^{5/2}$$

Prob- A pipe of 1.5 m dia. is required to transport an oil of density 900 kg/m^3 & kinematic viscosity equal to 3×10^{-2} stoke at the rate of $3 \text{ m}^3/\text{sec}$. If a 15 cm dia. of pipe with water having a kinematic viscosity of 0.01 stoke is used to model the above flow find the velocity & discharge of the model.

Soln:

given, $D_p = 1.5 \text{ m}$
 $\rho_{\text{oil}} = 900 \text{ kg/m}^3$
 $\nu_p = 3 \times 10^{-2} \text{ stoke}$
 $Q_p = 3 \text{ m}^3/\text{sec}$

oil	water
$\nu = 3 \times 10^{-2} \text{ stoke}$	$\nu = 0.01 \text{ stoke}$
$d = 1.5 \text{ m}$	$d = 15 \text{ cm}$
$\rho = 900 \text{ kg/m}^3$	$\rho = 1000 \text{ kg/m}^3$
$Q = 3 \text{ m}^3/\text{s}$	
$Q = \frac{\pi}{4} \times d^2 \times V$	
$3 = \frac{\pi}{4} \times (1.5)^2 \times V$	
$V = 1.698 \text{ m/s}$	

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\frac{\mu}{\rho}} = \frac{V D}{\nu}$$

$$(Re)_o = (Re)_w$$

$$\left(\frac{VD}{\nu}\right)_o = \left(\frac{VD}{\nu}\right)_w$$

$$\frac{1.698 \times 1.5}{3 \times 10^{-2}} = \frac{V_w \times 0.15}{0.01}$$

$$V_w = 5.65 \text{ m/s.}$$

$$Q_w = A_w V_w$$

$$= \frac{\pi}{4} (0.15)^2 \times 5.65$$

$$= 0.1 \text{ m}^3/\text{sec. } A_w$$

obtained the expression for model to prototype scale ratio which has to satisfy both Reynold model law & Froud model law
(Express for answer in terms of viscosity.

From Reynold's model law.

$$\frac{V_m}{V_p} = \frac{\nu_r}{\nu_p}$$

From Froud model law.

$$\frac{V_m}{V_p} = \sqrt{L_r}$$

For satisfied by both law.

$$\frac{\tau_r}{L_r} = \sqrt{L_r}$$

$$\tau_r = L_r^{3/2}$$

$$[L_r = \tau_r^{2/3}] \text{ Ans.}$$

Turbulent flow

Boussinesq developing eqn. for turbulent shear stress as

$$\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy}$$

η is known as eddy viscosity & it is of flow characteristics as it is difficult to find the value of η this eqn. is not used.

Reynold developed eqn. for ~~shear~~ turbulent shear stress

$$\tau = \rho u'v'$$

where u' & v' are fluctuating component in x & y -directions respectively.