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2

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-: HAND WRITTEN NOTES:-

OF

MECHANICAL ENGINEERING

1

-: SUBJECT:-

HEAT & MASS TRANSFER

2

2

WWW.GATENOTES.IN

[redacted]

Modes of Heat Transfer.

i) Conduction

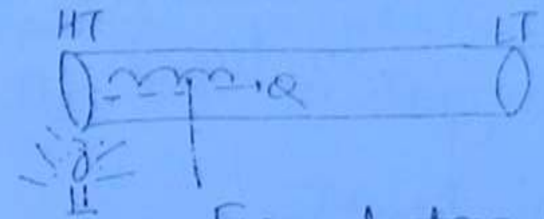
ii) Convection

iii) Radiation.

(3)

* Conduction

Molecular lattice vibrations energy transfer (30%)



k = 2300

, Free electron transfer (70%)

Diamond — Bad conductor of electricity

Silver, Copper, Aluminium, Steels

k = 405 W/mk   385   200   15 to 35

** Perfect crystalline lattice arrangement is there in diamond so that k is high. **

k = 0.2 W/mk

Insulators ⇒ Asbestos, Rockwool, Glasswool, Refractory brick, Saw dust.

Glass ⇒ Amorphous Material (k = 1.2 W/mK)

Defn ⇒ It is the mode of heat transfer which generally occurs in solids due to temperature difference by molecular lattice vibrational energy transfer (around 30%) and also by free electron transfer (around 70%).

All good electrical conductors are also good conductors of heat due to the presence of free

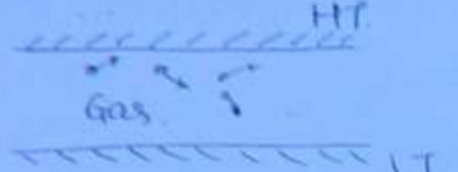
electrons. e.g. Silver, Copper, Aluminium, Steel.
The only exception being Diamond b'coz of its perfect crystalline order of lattice arrangement of molecules.

Gases are very bad conductor of heat.

$K_{air} \Rightarrow$ 0.026 W/mK

(4)

Conduction in gases



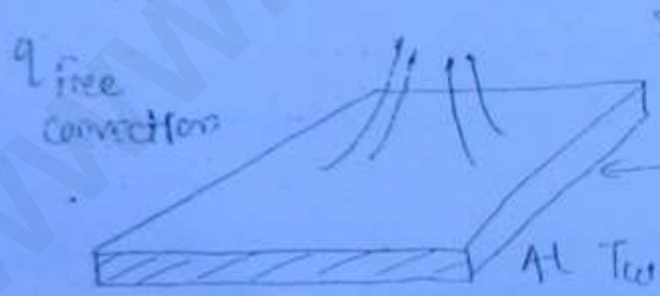
Conduction also occurs in liquids

$\bar{C} \propto \sqrt{T}$

Conduction also occurs in gases by molecular momentum transfer when high velocity high temp. molecules collide with the low velocity, low temp. molecules. (It is just similar phenomenon as viscosity)

– As the temp. of gases increase, their thermal conductivity also increase.

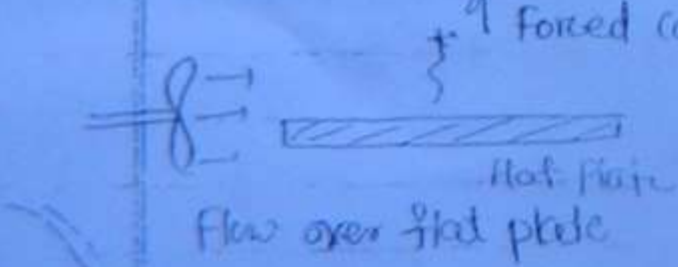
Convection :-



$PV = mRT$

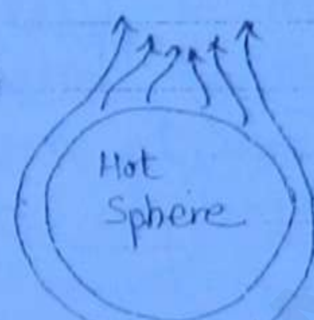
$\frac{P}{\rho} = RT$

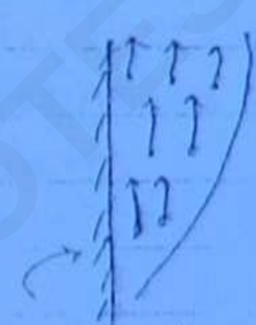
As $T \uparrow \rightarrow \rho \downarrow$



Flow over flat plate

Convection is mode of heat transfer which occurs b/w a hot solid surface & the surrounding cold fluid due to temp. difference associated with the a microscopic bulk displacement of the fluid over the solid surface which is provided by density changes and the resulting buoyancy forces in the case of free convection. or which is provided by an external agency like fan in the case of forced convection.

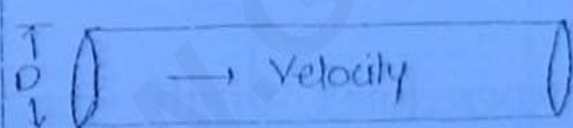




Hot vertical
Wall

(5)

Flow inside ducts

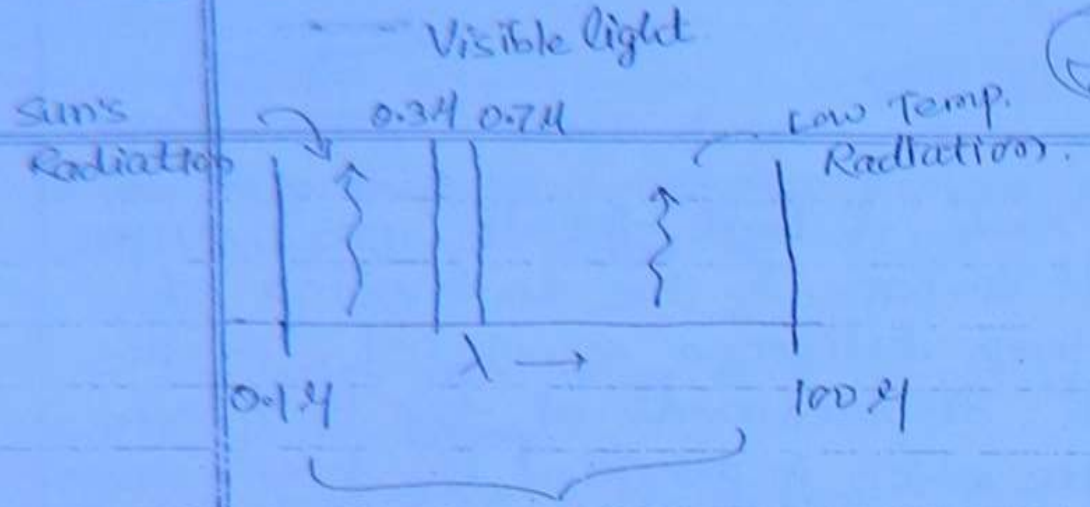


Hot Wall

Babcock Wilcox Boiler - free convection.

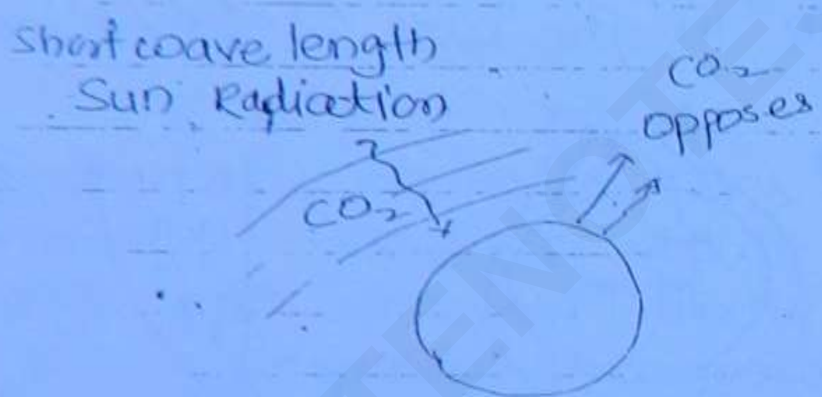
* Radiation.

Electromagnetic waves.



Thermal Radiation Spectrum.

(Global Warming) (6)



Defⁿ ⇒ Radiation is the mode of heat transfer which does not require any material medium & hence occurs by electromagnetic waves travelling with speed of light.

All bodies at all temperatures emitt thermal radiation except the body at 0 K. Also the rate of emission from any surface of a body per unit time and per unit area is directly directly proportional to the fourth power of absolute temp. of body. ⇒ (Stephen Boltzman Law)*

If the temp. diff. is quite high, thermal radiation completely predominates over conduction & convection.

eg. heat transfer b/w hot flue gases &

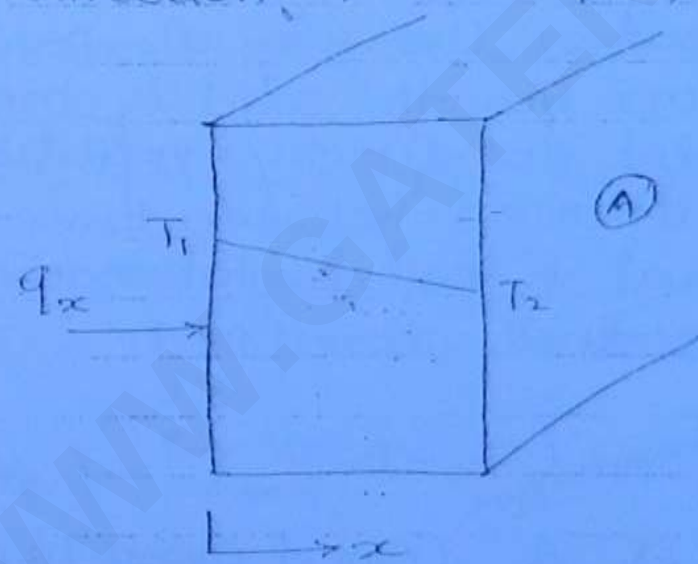
Benson boiler we required high level pump to create critical pressure of steam i.e. 221 bar.

the refractory brick wall in a large furnace of a steam generator/boiler is predominantly by radiation **

* Laws of Heat Transfer

i) Fourier's Law of Conduction :-

The law states that, "the rate of heat transfer by conduction in any given direction is directly proportional to the temperature gradient along that direction & is also directly proportional to the ~~heat~~ area of heat transfer lying ⊥^{ar} to the direction of heat transfer."



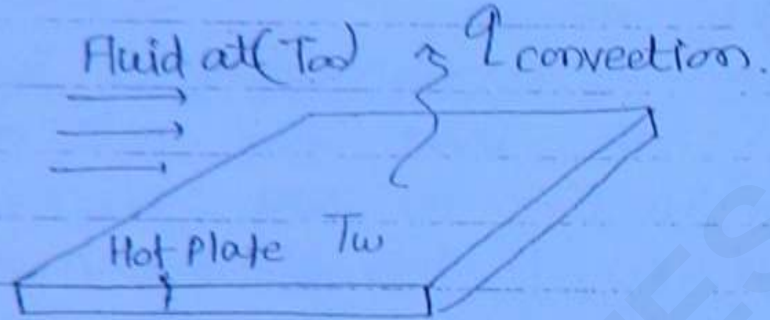
$$q_x \text{ (Rate of H.T.) } \propto -\left(\frac{dT}{dx}\right)$$

$$\propto A$$

$$\boxed{q_x = -KA\left(\frac{dT}{dx}\right)}$$ Joules/sec or Watts.

K = Thermal conductivity
(A property of material of the slab)

2] Newton's Law of Cooling
(For convection H.T.)



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T_w = Temp. of wall of plate

Law states that - The rate of heat transfer by convection b/w the solid & the surrounding cold fluid is directly proportional to the temperature difference between them & is also directly proportional to the area of exposure of the body with the fluid.

$$q_{convection} \propto (T_w - T_\infty)$$

$$\propto A$$ (Area of contact b/w solid & the fluid)

$$q_{conv.} = hA\Delta T$$

$$= hA(T_w - T_\infty) \text{ Watts.}$$

h = convection heat transfer coefficient in watts/m²K

[h isn't property of mtrl. but depends on ρ, μ, C_p, K]

Unlike K, h is not property of material but depends upon some of the thermo-physical properties of fluid like density (ρ), specific heat (C_p), dynamic viscosity (μ) & thermal conductivity.

ρ = kg/m^3 C_p = J/kg·K , μ = kg/m·s or Pascal Second
&
K = W/mK

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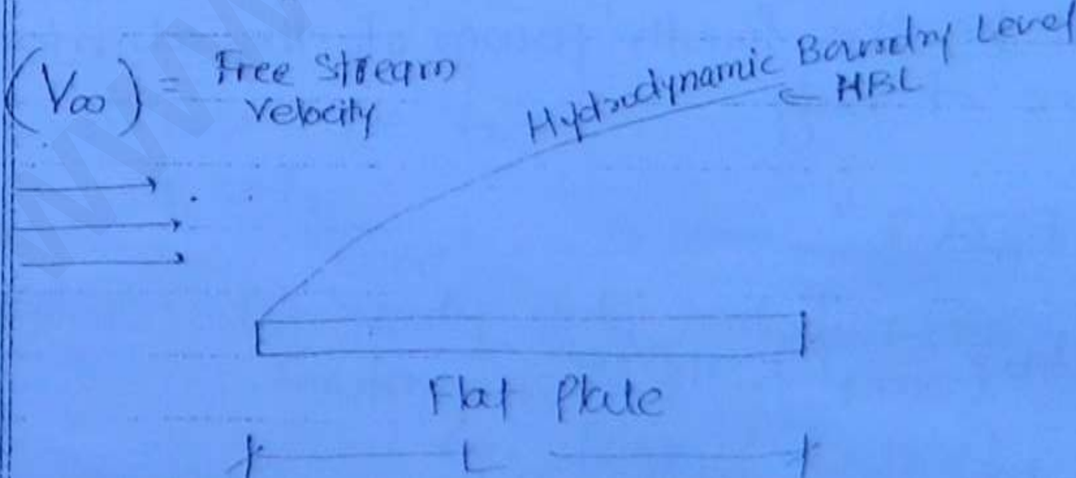
In forced convection heat transfer

$$h = f(\vec{V}, D, \rho, \mu, C_p, k)$$

$\vec{V}$ = Velocity of fluid
D = characteristic dimension of solid body.



or natural

In free convection H.T.

$$h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, c_p, k}_{\text{Properties of fluid}})$$

β = Isobaric volume expansion coeff.

$$= \left(\frac{\partial V}{\partial T}\right)_{P=const} \frac{1}{V} = \frac{1}{V}\left[\frac{\Delta V}{\Delta T}\right]_{p=const.}$$

unit is per kelvin /K. (10)

For Ideal gas like air $\beta = \frac{1}{T}$

~~B] Stefan~~

3] Stefan - Boltzman Law of Radiation.

The law states that - The total radiation energy emitted from the surface of a black body per unit time and per unit area is directly proportional to the fourth power of the absolute temperature of body.

$$E_b \propto T^4$$

$$E_b = \sigma T^4 \quad W/m^2 = \frac{J}{sec.m^2}$$

where σ = Stefan-Boltzman const

$= 5.67 \times 10^{-8}\ W/m^2K^4$

A black body is the body which absorbs all the incident radiation falling upon it. It is an also an ideal emitter.

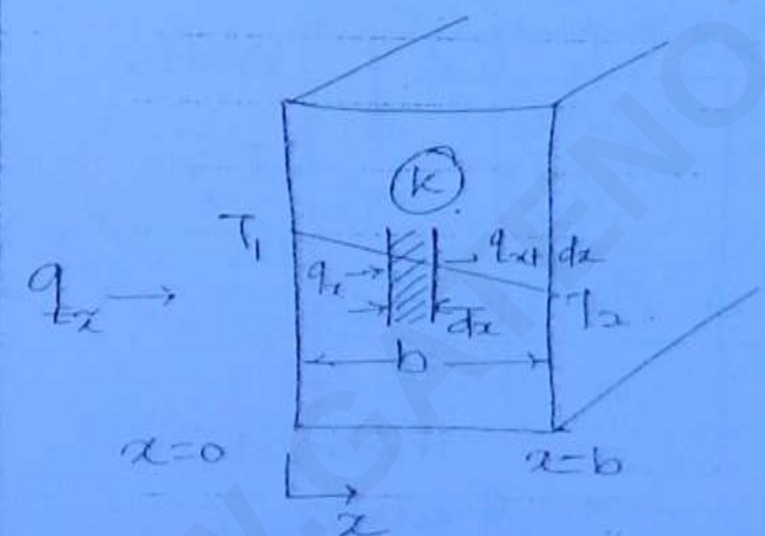
A thermally black body need not appear black in colour to the human eye.

e.g. Snow & Ice. Any good absorber is always a good emitter (Kirchoff's law)

*

(11)

* Conduction Heat Transfer through a slab.



At $x=0 \implies T=T_1$

At $x=b \implies T=T_2$

$T_1 > T_2$

Assumptions :- (i) Steady state heat transfer (Temp isn't f^n of time)

i.e. Temp $\neq f$(time)

(ii) One dimensional conduction.

(iii) Uniform thermal conductivity (k).

$k \neq f(x) \neq f$(temp)

$$q_x = -kA\frac{dT}{dx}$$

$$\int_0^b q_x \cdot dx = \int_{T_1}^{T_2} -kA\,dT$$

$$q_x = q_{x+dx}$$

$$q_x = C \neq f(x)$$

$$\therefore\ q_x \times (b) = kA(T_1 - T_2)$$

$$\therefore\ q_x = \left[\frac{kA(T_1 - T_2)}{b}\right] \text{ Watts}$$

Electrical Analogy of Conductance

Electrical		Thermal
i (Amp)	$\longrightarrow$	q watts
ΔV (emf)	$\longrightarrow$	ΔT (°C) $\Rightarrow$ Thermal Potential
R_{ele} (Ohm)	$\longrightarrow$	R_{Th}

$$R_{ele} = \left(\frac{\Delta V}{i}\right)\ \Omega \qquad R_{Th} = \left(\frac{\Delta T}{q}\right)\ °C/\text{Watt}$$

or K/Watt

$$R_{(Th)\,(Slab)} = \frac{b}{KA} = \left(\frac{T_1 - T_2}{q}\right)$$

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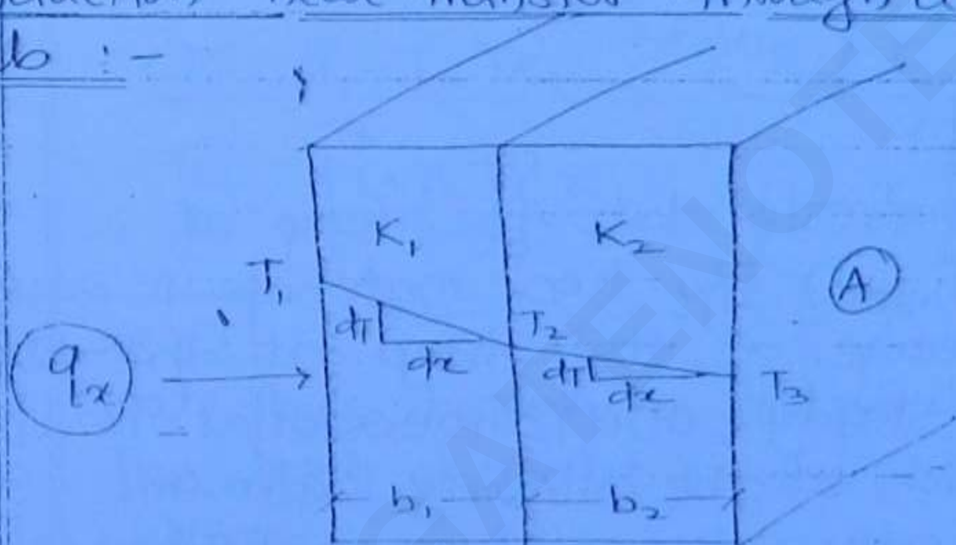
b = width of slab (Higher the width, more the resistance lower the H.T. rate)

Lower the k value, more the Resistance (R_{Th}).

* <u>Conduction heat transfer through a composite slab</u> :-

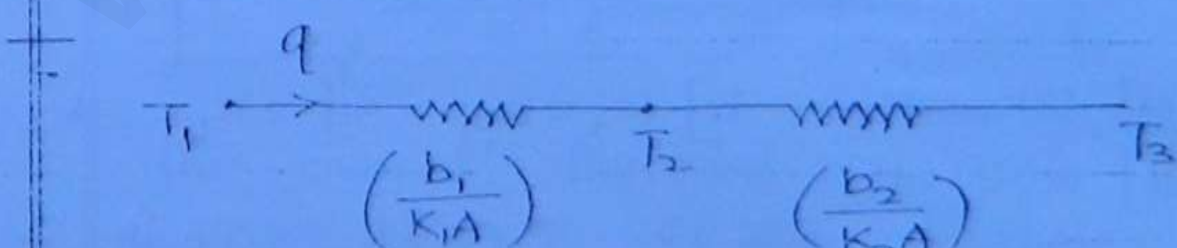


Let ⇒
Steady State
One dimensional
Uniform k value

$T_1 > T_2 > T_3$

T_2 = Junction / Interface temperature °c

Thermal circuit :-



$$q = \frac{(T_1 - T_3)}{(\Sigma R_{Th})} = \left[\frac{T_1 - T_3}{b_1/K_1A + b_2/K_2A}\right] \text{ Watts}$$

Heat transfer rate per unit area

$$= \text{Heat flux} = \frac{q}{A} = \left[\frac{T_1 - T_3}{b_1/k_1 + b_2/k_2}\right] W/m^2$$

$$q_x = -kA\left(\frac{dT}{dx}\right)$$

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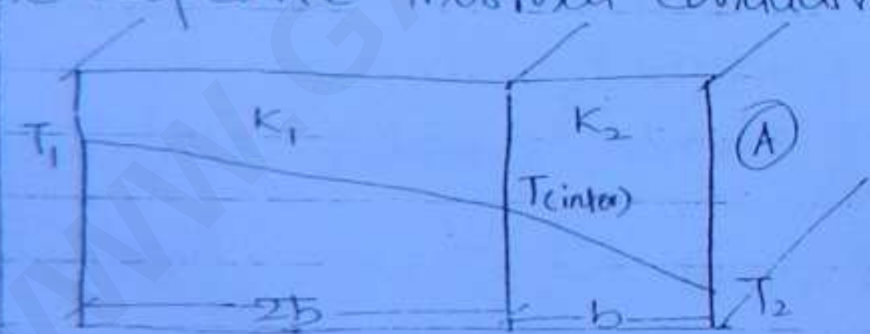
since $\left(\frac{dT}{dx}\right)_{in ①} > \left(\frac{dT}{dx}\right)_{in ②}$

$\Rightarrow k_2 > k_1$

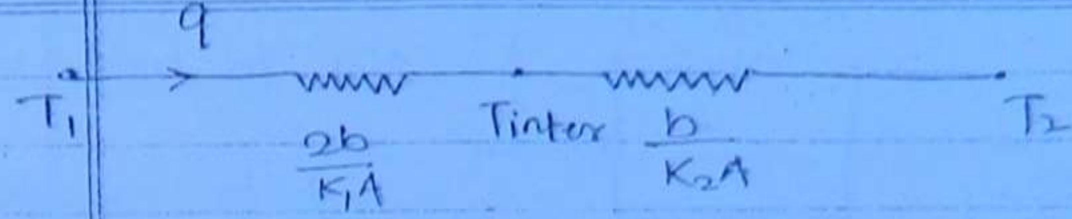
$\boxed{\text{GATE 2006}}$

In a composite slab the temp. at a inter-face (T_{inter}) b/w two materials is equal to the average of the temp. at the two ends. Assuming steady, one dimensional heat conduction, which of the following statement is true are about the respective thermal conductivities



a] $2k_1 = k_2$ b] $k_1 = k_2$ c] $2k_1 = 3k_2$

d] $k_1 = 2k_2$



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$$q = \frac{T_1 - T_{inter}}{2b/K_1A} = \frac{T_{inter} - T_2}{b/K_2A}$$

Given $T_{inter} = \left(\frac{T_1 + T_2}{2}\right)$

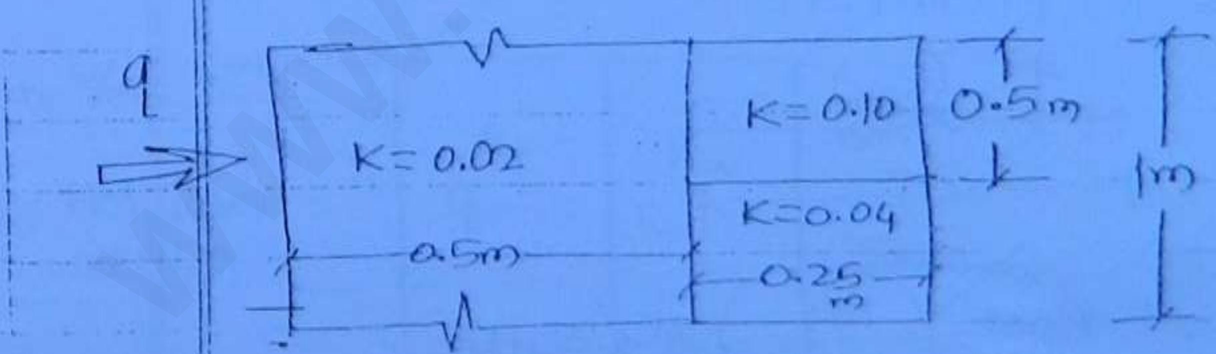
$$\cancel{\frac{(T_1 - (T_1 - T_2))K_1}{4}} = \cancel{\frac{(T_1 + T_2 - T_2)K_2}{2}}$$

$$\left\{T_1 - \left[\frac{T_1 + T_2}{2}\right]\right\}\frac{K_1}{2} = \left[\left(\frac{T_1 + T_2}{2}\right) - T_2\right]K_2$$

$$\left(\frac{T_1 - T_2}{2}\right)\frac{K_1}{2} = \left(\frac{T_1 - T_2}{2}\right)K_2 \quad \text{i.e. } \boxed{K_1 = 2K_2}$$

$\boxed{\text{GATE 2005}}$

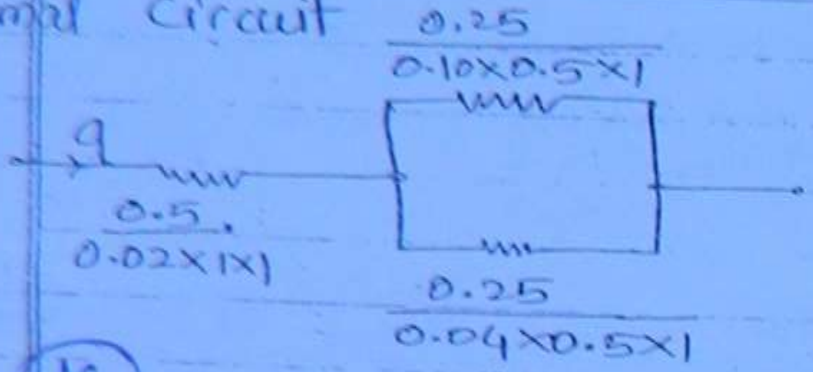
Heat flows through a composite as shown below. The depth of slab is 1 m. The k values are in W/m.K



The overall thermal resistance

a] 17.2 b] 21.9 c] 28.6 d] 39.2

Thermal Circuit



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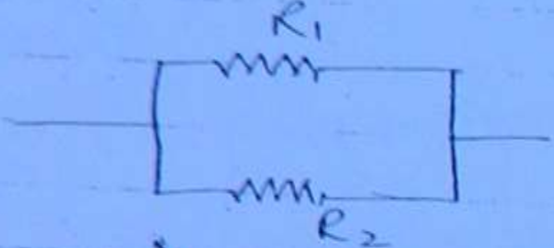


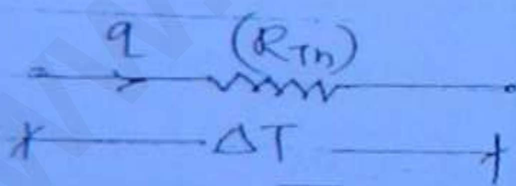
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Answer is (C) 28.6

Convection Thermal Resistance

From Newton's law of cooling

$$q = hA\Delta T$$



$$\therefore R_{Th} = \left(\frac{\Delta T}{q}\right) {}^\circ C/\text{Watt}$$

$$= \left(\frac{1}{hA}\right)$$

A = Area of contact between solid & fluid.

[Rate of H.T. by water is 25 times more than that of air.]

Ranges of h

i) Free convection in gases ⇒ 3 to 25 W/m^2K

ii) Forced ——"—— ⇒ 25 to 450 W/m^2K

iii) Free convection in liquids like water ⇒ 50 to 600 W/m^2K

iv) Forced ——"—— ⇒ 500 to 3000 W/m^2K

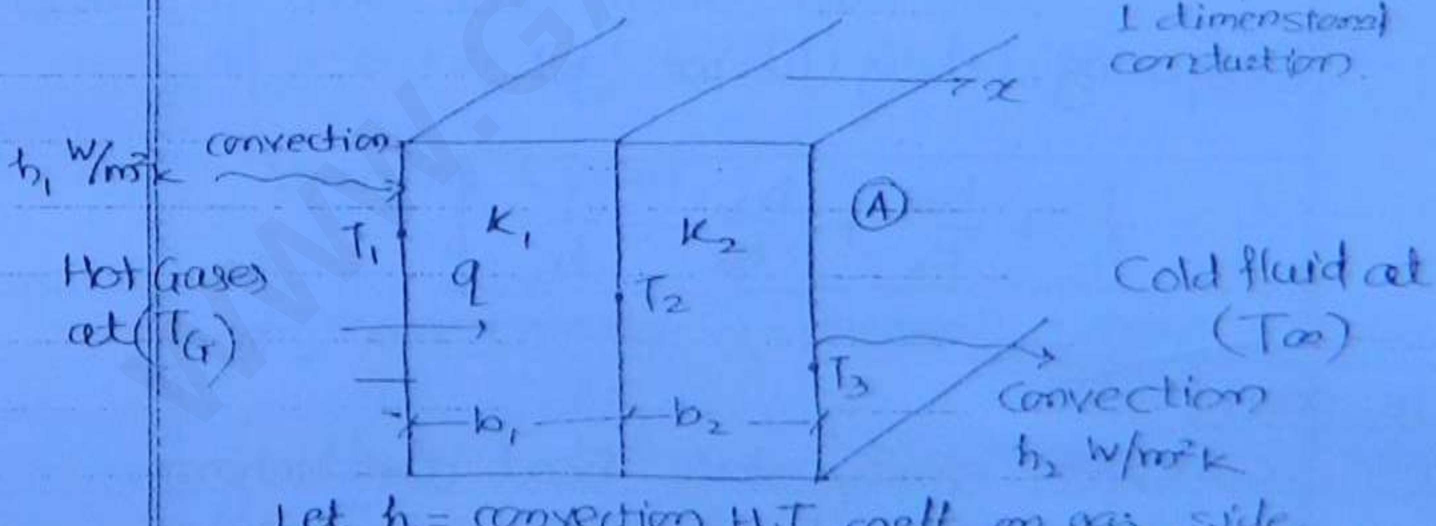
v) Condensation H.T. ⇒ 3000 to 25000 W/m^2K

vi) Boiling H.T. ⇒ 5000 - 50000 W/m^2K

[Condensation ⇒ change of phase of vapour to liquid
Boiling ⇒ liquid to vapour]

* Conduction - convection H.T. through a composite ~~cylinder~~ Slab

Let, Steady state, 1 dimensional conduction



Let h_1 = convection H.T. coeff. on gas side
h_2 = ——"—— cold fluid side

$$T_G \xrightarrow{q} \left(\frac{1}{h_1 A}\right) - T_1 - \left(\frac{b_1}{K_1 A}\right) - T_2 - \left(\frac{b_2}{K_2 A}\right) - T_3 - \left(\frac{1}{h_2 A}\right) - T_\infty$$

$$\therefore q = \frac{T_G - T_\infty}{\sum R_{th}}$$

(18) $$= \frac{T_G - T_\infty}{\frac{1}{h_1 A} + \frac{b_1}{k_1 A} + \frac{b_2}{K_2 A} + \frac{1}{h_2 A}}$$

Heat transfer rate per unit area.

$$(\text{Heat flux}) = \frac{q}{A} = \left[\frac{T_G - T_\infty}{\frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}}\right] \quad \text{—(i)}$$

Watt/m^2

* Overall Heat Transfer Coefficient - (U)

It takes into account all the modes of heat transfer & is defined from equation

$$\boxed{Q = UA\Delta T} \qquad U \Rightarrow W/m^2K \quad \text{—(ii)}$$

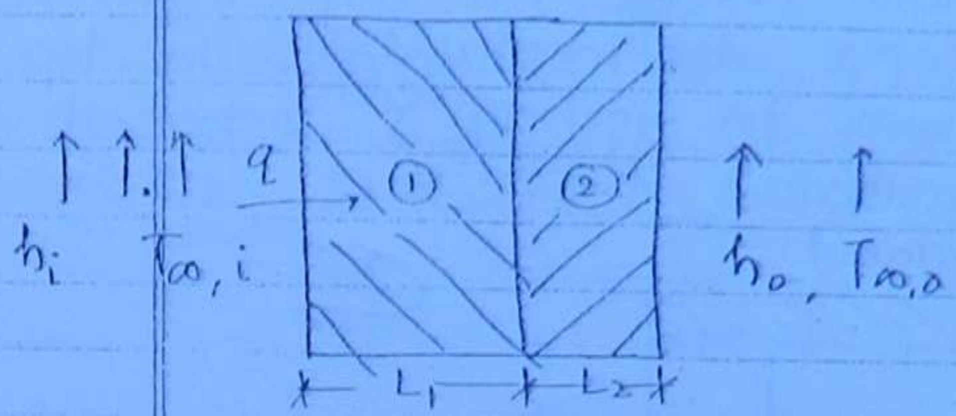
Comparing (i) & (ii) we get,

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{K_2} + \frac{1}{h_2}}$$

GATE 2009

Consider steady state heat conduction across the thickness in a plane composite wall (as shown in fig.) exposed to convection conditions on both sides.

(19)



Given $h_i = 20\ W/m^2k$

$h_o = 50\ W/m^2k$

$T_{\infty_i} = 20°C$ $\quad T_{\infty_o} = -2°C$

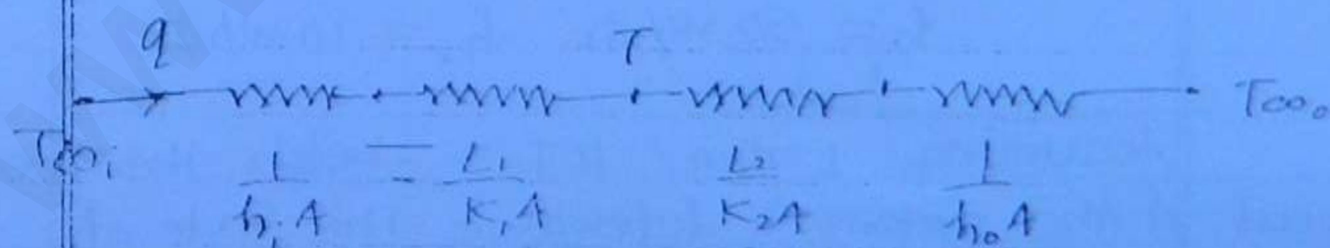
$K_1 = 20\ W/m^2k$ $\quad K_2 = 50\ W/mk$

$L_1 = 0.30\ m$ $\quad L_2 = 0.15\ m$

Assuming negligible contact resistance b/w wall surfaces, the interface temperature T in °C ~~after~~ of the two walls will be

a] −0.50 b] 2.75 c] 3.75 d] 4.50

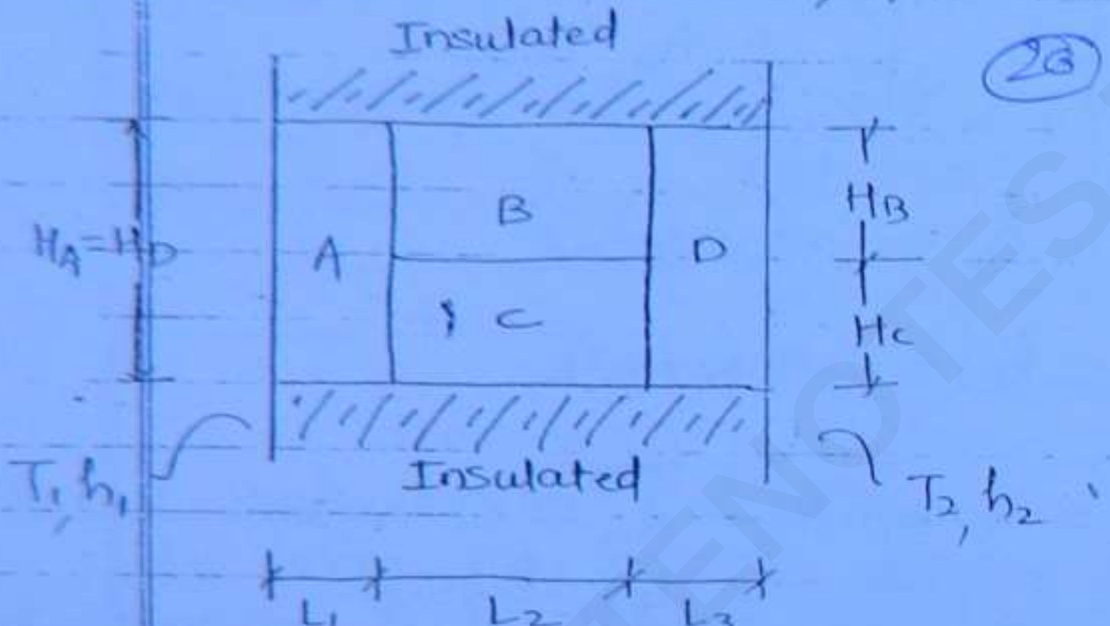
⟹ Thermal Circuit



$$\therefore q = \frac{T_{\infty_i} - T}{\frac{1}{h_i A} + \frac{L_1}{K_1 A}} = \frac{T - T_{\infty,o}}{\frac{L_2}{K_2 A} + \frac{1}{h_o A}}$$

GATE 2001

A composite wall having unit length normal to the plane of paper, is insulated at the top and bottom as shown in fig. It is comprised of four different materials A, B, C & D.



$H_A = H_D = 3m$ $H_B = H_C = 1.5m$

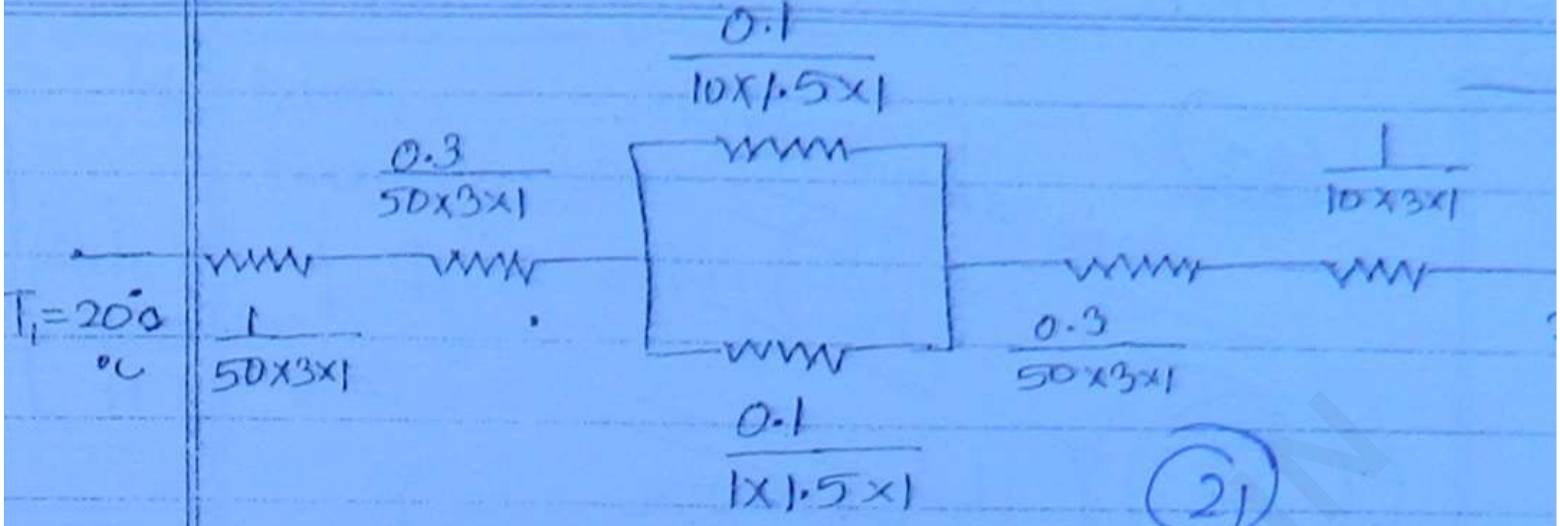
$L_1 = L_3 = 0.3m$ $L_2 = 0.1m$

$k_A = k_D = 50$ W/mK $k_B = 10$ W/mK

$k_C = 1$ W/mK. The fluid temps & heat transfer coeff. are. $T_1 = 200°C$, $T_2 = 25°C$

$h_1 = 50$ W/m²K $h_2 = 10$ W/m²K

Assuming 1 dim. H.T. sketch the thermal circuit of the system & determine the rate of H.T. ~~to~~ through the wall.



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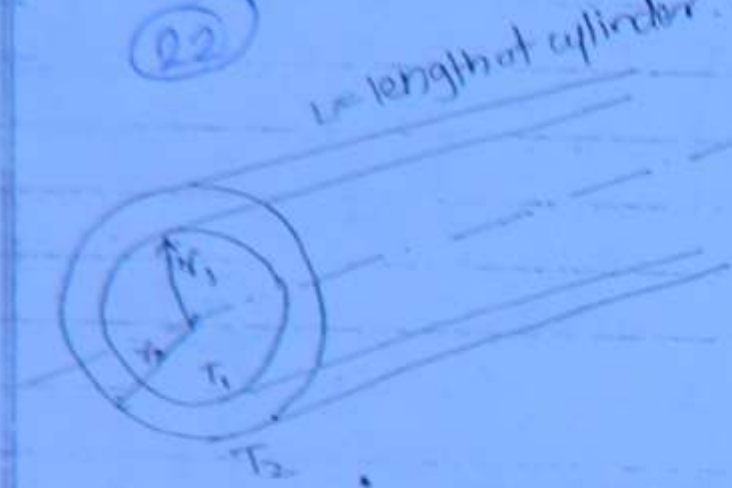
Text book

Incropera & Dewitt
J.P. Holman
Sukhatme
Ozisik
Cengel & Boles
D.S. Kumar → Numerical Problems.

* Conduction Heat Transfer through a hollow cylinder



Assumptions

i) Steady state H.T (ii) One dimensional radial conduction H.T. (iii) k is uniform $k \neq f(r)$

At ~~...~~, $r = r_1$ $T = T_1$ $(T_1 > T_2)$

At $r = r_2$ $T = T_2$

Conduction occur radically outwards from inside surface at r_1 & T_1 to outside surface at ~~T_2~~ r_2 & T_2.

Here the conduction area of H.T. is increasing from inside to the outside

At any radius r, area of conduction H.T

$= (2\pi r L)$

We have,

Fourier's Law

$$\begin{pmatrix}\text{Rate of} \\ \text{conduction} \\ \text{H.T.}\end{pmatrix} \quad q = -KA\left(\frac{dT}{dr}\right) \text{ Watts.}$$

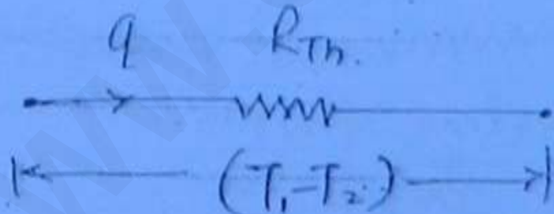
$$q = -k\,2\pi r L\left(\frac{dT}{dr}\right)$$

$$q \neq f(r)$$

$$\int_{r_1}^{r_2} q\,\frac{1}{r}\,dr = \int_{T_1}^{T_2} -k\,2\pi L\,dT$$

$$q = \frac{2\pi k L\,(T_1 - T_2)}{\log_e\left(\frac{r_2}{r_1}\right)}$$

Thermal Circuit.



$$(R_{Th})_{\text{cylinder}} = \frac{\Delta T}{q} = \frac{\ln(r_2/r_1)}{2\pi k L} \quad \text{K/watt}$$

* Conduction of H.T. through a composite cylinder.

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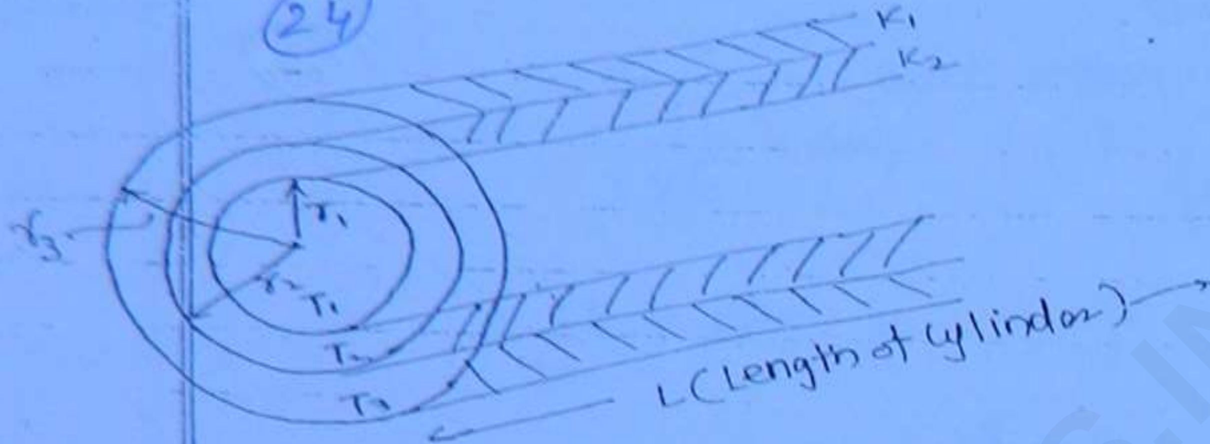


Assumptions

(i) Steady state ii) One dimensional radial conduction H.T. (iii) Uniform value of K.

At $r = r_1$: $T = T_1$, $r = r_2$: $T = T_2$

$r = r_3$: $T = T_3$.

Thermal Circuit

Junction/Interface Temp.

$$T_1 \xrightarrow{q} \underset{\frac{\ln(r_2/r_1)}{2\pi k_1 L}}{\text{wwww}} \; T_2 \; \underset{\frac{\ln(r_3/r_2)}{2\pi k_2 L}}{\text{wwww}} \; T_3$$

(Rate of H.T) $$q = \frac{(T_1 - T_3)}{\Sigma R_{Th}} \quad \text{Watts}$$

$$q = \frac{T_1 - T_3}{\frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L}}$$

GATE 2004 (25)

A stainless steel tube $K_S = 19$ W/mK of 2 cm inner dia. & 5 cm outer dia. is insulated with ~~three~~ 3 cm thick asbestos $K_A = 0.2$ W/mK. If the temperature diff. b/w the innermost & outer most surface is 600°C. The heat transfer rate per unit length is

(i) 0.94 W/m (ii) 944 W/m
(iii) 9447.2 W/m (iv) 94.7 W/m

Solution

$$T_1 \xrightarrow{q} \underset{\frac{\ln(5/2)}{2\pi \times 19 \times L}}{\text{---WWW---}} \; T_2 \; \underset{\frac{\ln(5.5/2.5)}{2\pi \times 0.2 \times L}}{\text{---WWW---}} \; T_3$$

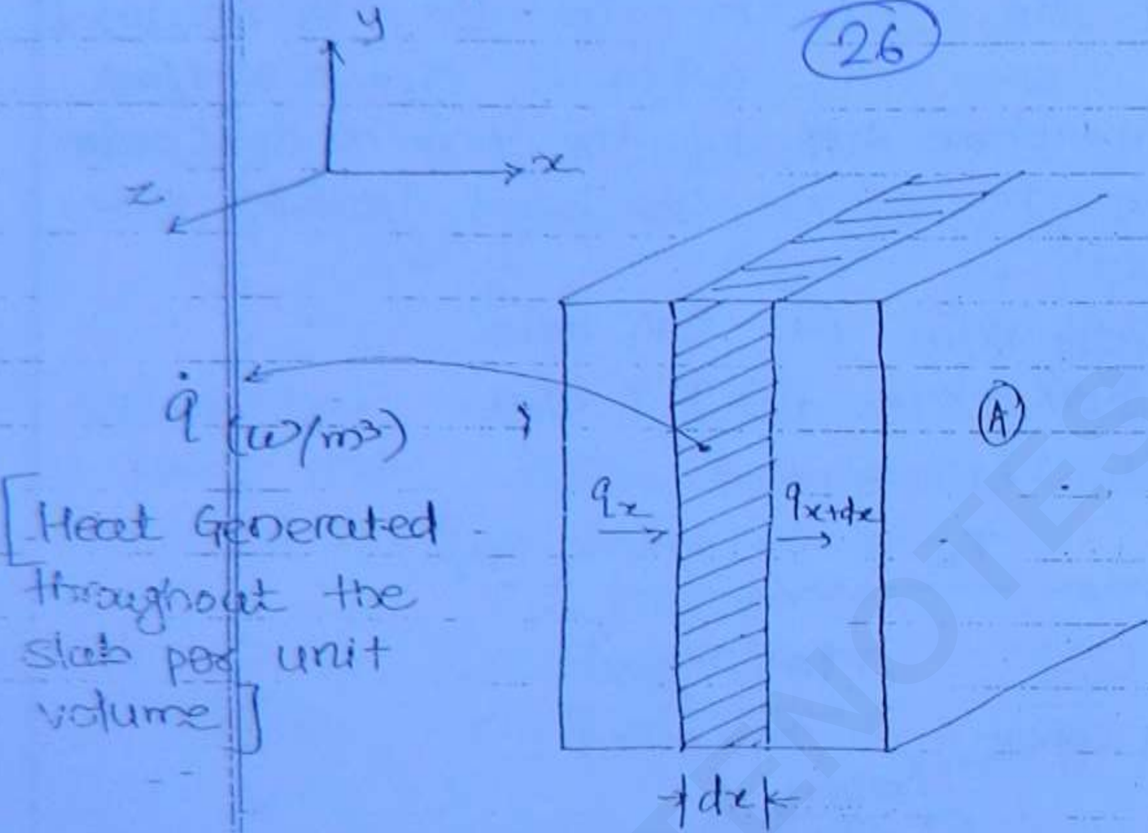
$$q = \frac{(T_1 - T_3)}{\frac{\ln(5/2)}{2\pi \times 19 \times L} + \frac{\ln(5.5/2.5)}{2\pi \times 0.2 \times L}}$$

$$\frac{q}{L} = \frac{600}{\frac{\ln(5/2)}{2\pi \times 19} + \frac{\ln(5.5/2.5)}{2\pi \times 0.2}}$$

$$=$$

* To derive generalised (3-dimensional, Steady or unsteady, with or without heat generation) conduction equation *

(26)



Consider a differential element in the slab of length dx

$$\dot{q} = f(x, y, z)$$

Let q_x = Heat conducted into the element

$$= -KA\left(\frac{dT}{dx}\right)$$

q_{x+dx} = Heat conducted out of element

$$= q_x + \frac{\partial}{\partial x}(q_x)\,dx$$

$$q_{\left(\substack{\text{Generated in}\\ \text{element}}\right)} = \dot{q}\,(A \cdot dx)$$

Writing the energy balance eqn for element along the x-direction i.e. heat conducted into element + heat generated = Heat conducted out of element + Rate of change of Internal energy of element w.r.t. time

$$q_x + \dot{q}\,A \cdot dx = q_{x+dx} + \frac{\partial}{\partial \tau}(m C_p T)$$

$$q_x + \dot{q}\,A \cdot dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x) \cdot dx + \frac{\partial}{\partial \tau}(m C_p T)$$

$$\dot{q}\,A\,dx = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)dx + \rho A \cdot dx\, C_p \frac{\partial T}{\partial \tau}$$

$$K\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial \tau} \quad \cdots \because T = f(x, y, z, \tau)$$

Similarly & simultaneously writing the ~~energy~~ balance eqn to the element for all the 3 directions x, y & z put together we get -

$$K\frac{\partial^2 T}{\partial x^2} + K\frac{\partial^2 T}{\partial y^2} + K\frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial \tau}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \left(\frac{\rho C_p}{k}\right)\frac{\partial T}{\partial \tau}}$$

* **Thermal Diffusivity (α)**

A property of material as the ratio b/w thermal conductivity (k) & thermal capacity.

$$\boxed{\alpha = \frac{k}{(\rho C_p)} \quad m^2/sec.}$$

Thermal diffusitivity (α) is the ability of material to allow, the heat energy to get diffused through the medium.

Prandtl Number (for fluids only)

$$P_r = \left(\frac{\nu}{\alpha}\right) \qquad \nu = \text{kinematic viscosity.}$$

If there is no heat generation,

$$\dot{q} = 0$$

If conditions are steady

$$\partial T / \partial \tau = 0$$

We get,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\boxed{\nabla^2 T = 0} \quad \text{Laplace eqn in } T$$

... h is not only associated with fluid ... as it is associated with contact material & fluid.

This is the three dimensional heat conduction eq^n without heat generation under steady state.

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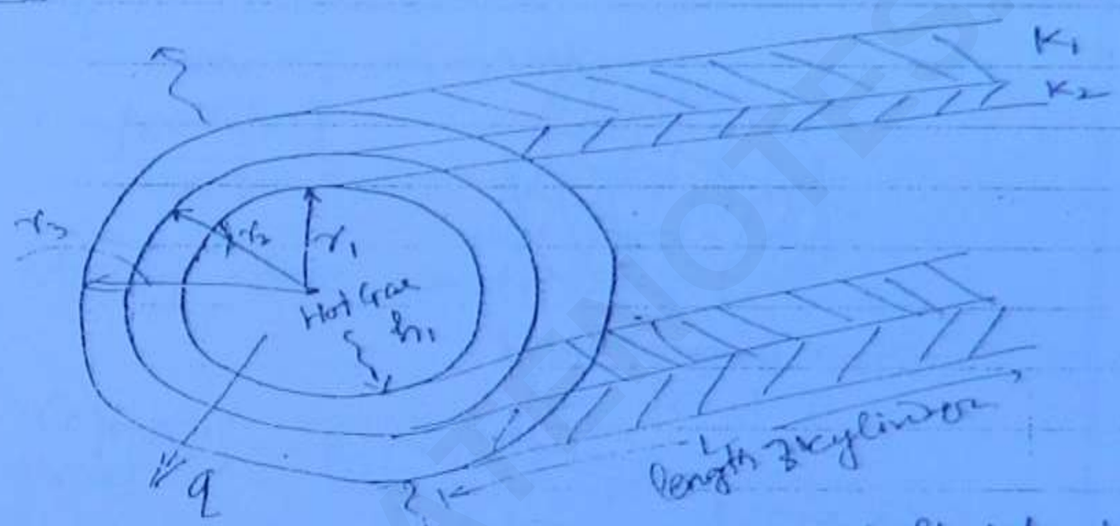
Q. * Convection – conduction H.T. through a composite cylinder.



Let,

T_G = Temperature of gas

T_∞ = Temperature of outside cold fluid

h_1 = convection H.T. coeff. on gas side (W/m^2K)

h_2 = convection —"— fluid side (W/m^2K)

Assumption ⟹ ③ Assumptions are followed.

Thermal Circuit

$$T_G \quad \frac{1}{h_1 A_1} \quad \frac{\ln(r_2/r_1)}{2\pi K_1 L} \quad \frac{\ln(r_3/r_2)}{2\pi K_2 L} \quad \frac{1}{h_2 A_2} \quad T_\infty$$

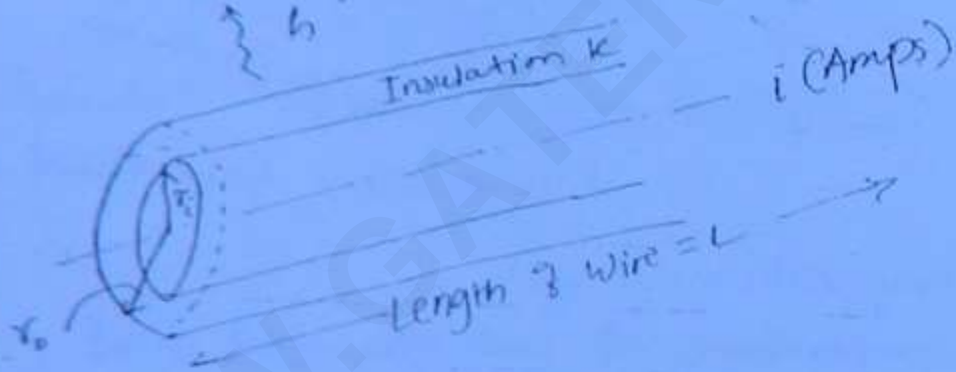
A_1 = Inside convection H.T. area $= 2\pi r_1 L$

A_2 = Outside —"— $= 2\pi r_3 L$

$$\therefore q = \frac{T_a - T_\infty}{\Sigma R_{th}}$$

$$\therefore q = \frac{T_a - T_\infty}{\frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L} + \frac{1}{h_2 A_2}}$$

* Critical Radius of Insulation



Consider a solid wire of radius r_i inside which there is heat generated due to the passage of electric current.

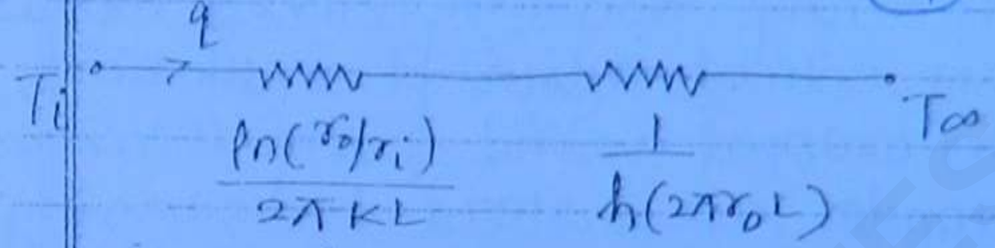
Let r_0 be the radius at upto which insulation is wrapped & k is the thermal conductivity of insulation.

T_i = Temperature of solid wire at r_i

T_∞ = Temp. of fluid ; h = convection H.T. coeff. b/w surface of ..

The heat generated in the wire is radially conducted through the Insulation & then convected to the surrounding fluid at T_∞ with a convection H.T. coeff. of h

Between T_i & T_∞, thermal circuit



Assume steady state conditions.

$$q = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{h(2\pi r_o L)}}$$

Rate of heat loss

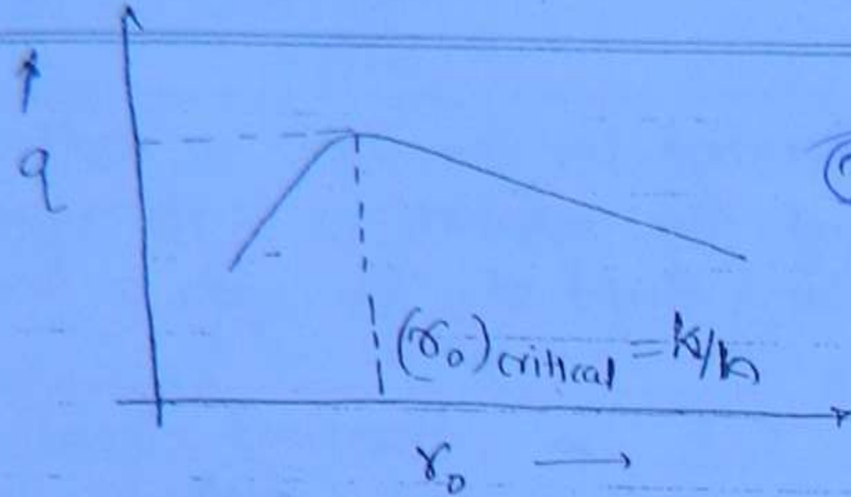
Keeping all other parameters const ; q is function r_o. Greater the insulation wrapped around the wire, higher the value of r_o.

For maximum heat transfer rate q,

$$\frac{dq}{dr_o} = 0$$

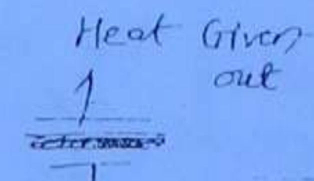
$$= \frac{d}{dr_o}\left[\frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{h(2\pi r_o L)}}\right] = 0$$

$$\Rightarrow \boxed{r_o = \left(\frac{k}{h}\right)} = \text{Critical radius of insulation.}$$



For sufficiently thin wires whose radius is lesser than critical radius of insulation, any insulation wrapped around it shall increase the heat transfer rate instead of decreasing it. The reason is — there is a rapid decrease in a convection thermal resistance as compared to the increase in conduction thermal resistance. The overall effect being the decrease in total thermal resistance & increase in H.T. rate. Such phenomenon is continuous to happen upto critical radius of insulation beyond which any insulation wrapped shall decrease the H.T. rate.

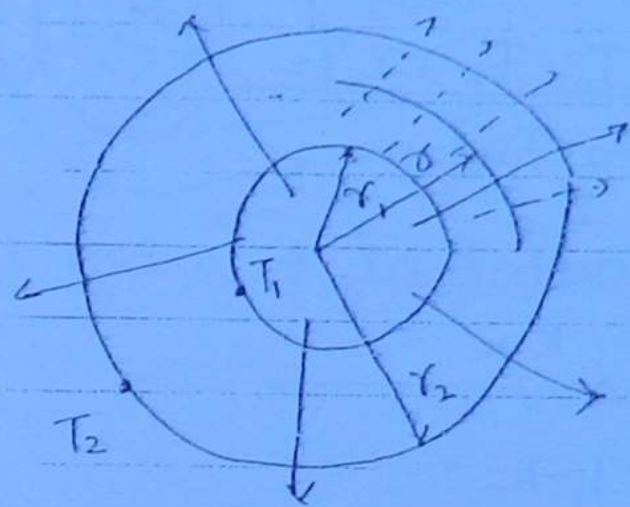
* Practical Application



(i) Electric power transmission cables
(ii) Electronic Devices

For spherical cases the critical radius of insulation is $\boxed{= \left(\frac{2k}{h}\right)}$

* Conduction H.T. through a hollow sphere.



(33)

The inside surface of hollow sphere is at T_1 i.e. at $r = r_1$

The outside surface of sphere is at T_2 i.e. at $r = r_2$

H.T. occurs radially outwards.

At any radius r, conduction area of HT $(A) = 4\pi r^2$

Fourier law of conduction.

$$q = -kA\frac{dT}{dr}$$

Assumptions :- (i) Steady state H.T. (ii) One dimensional radial H.T. (iii) K is uniform.

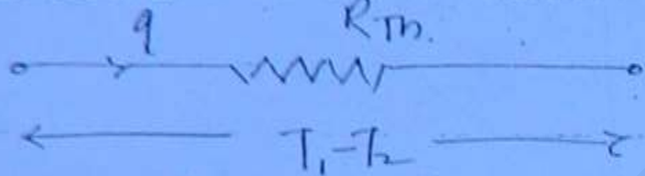
Now $q = -k\,4\pi r^2\left(\frac{dT}{dr}\right)$ $\quad T \neq f(\text{time})$

$$\int_{r_1}^{r_2} q\cdot\frac{1}{r^2}\,dr = \int_{T_1}^{T_2} -k4\pi\,dT \qquad q \neq f(r)$$

$$q \cdot \left[\frac{-1}{r}\right]_{r_1}^{r_2} = 4\pi k (T_1 - T_2)$$

$$\therefore q = \frac{4\pi k (T_1 - T_2) r_1 r_2}{(r_2 - r_1)}$$

Thermal Circuit –

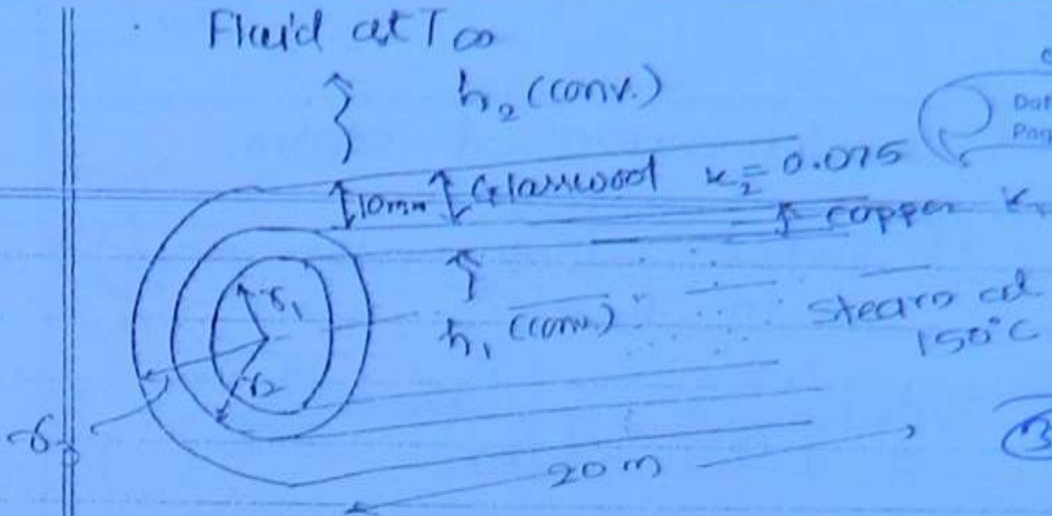


$$R_{Th.} = \frac{q}{T_1 - T_2}$$

$$= \left(\frac{r_2 - r_1}{4\pi k r_1 r_2}\right) \text{ K/Watt}$$

Problem

A copper tube of 20 mm outer dia., 1 mm thickness & 20 m long (thermal conductivity k = 400 W/mK) is carrying saturated steam at 150°C (Convective h.t. coeff. h = 150 W/m²K). The tube is exposed to an ambient temp. of 27°C. The convective h.t. coeff of air is 5 W/m²K. Glasswool is used for insulation. ($k_{Glasswool}$ = 0.075 W/mK). If the thickness of the insulation used is 5 mm higher than the critical thickness of insulation, calculate the rate of heat lost by the steam & the rate of steam condensation in kg/hr. The enthalpy of condensation of steam = 2230 kJ/kg



35

(i) Critical Radius of Insulation $= \dfrac{k}{h_{(outside)}}$

$$= \frac{0.075}{5} = 0.015\,m$$

$$= 15\ mm$$

Critical thickness of Insulation $= 15 - 10$
$= 5\ mm$

Actual thickness of Insulation $= 5 + 5$
$= 10\ mm$

$r_1 = 9\ mm \quad r_2 = 10\ mm$

$r_3 = r_2 + 10 = 20\ mm$

Thermal Circuit

$$T_{steam} = 150°C \quad \frac{1}{h_1 2\pi r_1 L} \quad \frac{\ln(r_2/r_1)}{2\pi k_1 L} \quad \frac{\ln(r_3/r_2)}{2\pi k_2 L} \quad \frac{1}{h_2 2\pi r_3 L} \quad T_{air} = 27°C$$

$$\therefore q = \frac{150 - 27}{\dfrac{1}{150 \times 2 \times \pi \times 9 \times 10^{-3} \times 20} + \dfrac{\ln(10/9)}{2\pi \times 400 \times 20} + \dfrac{\ln(20/10)}{2\pi \times 0.075 \times 20} + \dfrac{1}{5 \times 2\pi \times 20 \times 10^{-3} \times 20}}$$

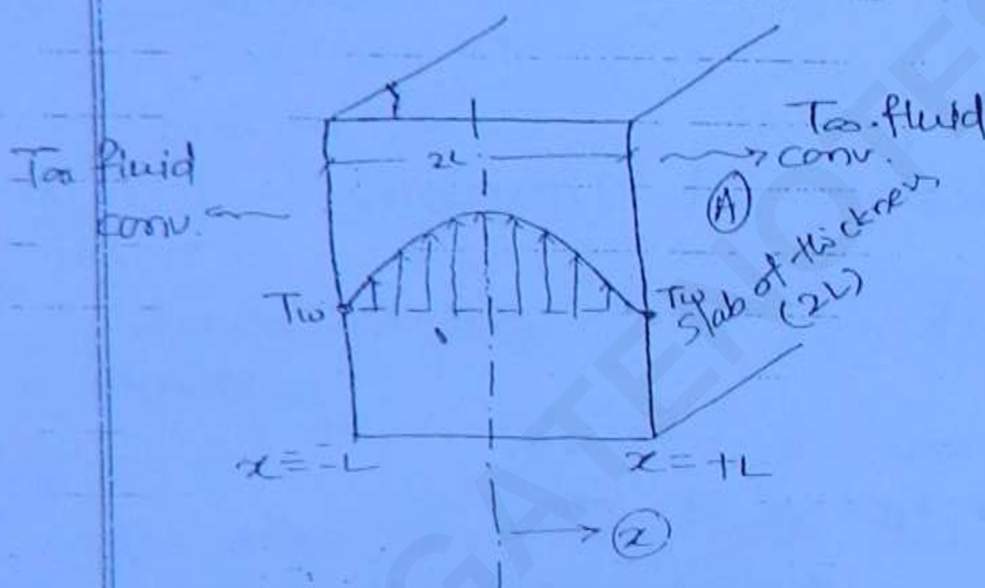
$\therefore q = 776$ Watts.

∴ Rate of condensation of steam

$$= \frac{q \text{ (Watts)}}{\text{Latent heat of condensation of steam in J/kg}}$$

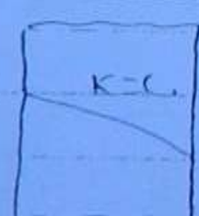
$$= 1.25 \text{ kg/hr.}$$

+ Conduction with Heat Generator in Slab.

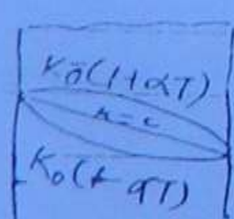


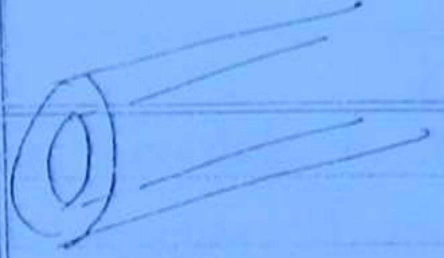
(i) Steady state H.T. Temp $\neq f$ (Time)
(ii) One dimensional conduction
(iii) Uniform heat generation $\dot{q} \neq f(x, y, z)$

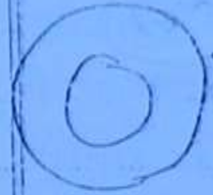
(i) K = Const.



(2) K not const



(3)  K = logarithmic

(4)  ← sphere K = hyperbolic 37

Now we have,

$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}_{\text{zero as one dimensional H.T.}} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\underbrace{\frac{\partial T}{\partial \tau}}_{\text{Steady state, so zero.}}$$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$$

Integrating, $\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$ — ①

Integrating, $T = -\frac{\dot{q}}{k}\cdot\frac{x^2}{2} + C_1 x + C_2$ — ②

C_1 & C_2 are const. of integration that are to be evaluated from boundary conditions.

One typical B.C. is at $x = \pm L$
$T = T_w$

This will be possible only when both sides of slab are subjected to the same fluid at the same temp. with the same ~~coe~~ h (W/m²K)

To satisfy this $\boxed{C_1 = 0}$

Let T_0 = centre line temp. of slab at $x=0$

When T is maximum $\frac{dT}{dx} = 0$ 38

$\therefore 0 = -\frac{\dot{q}}{k} x \Rightarrow x = 0$

$\therefore$ Max temp at the centre line.

Hence $T_0 = T_{max}$.

Put $x = 0$

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_2$$

$\therefore x = 0 \quad T = T_0 \quad ; \quad \boxed{C_2 = T_0}$

The final temperature distribution

$$T = -\frac{\dot{q}}{k}\cdot\frac{x^2}{2} + T_0$$

$$\therefore \boxed{T_0 - T = \frac{\dot{q}}{k}\frac{x^2}{2}} \quad \text{Eqn of Parabola} \quad —— (3)$$

Put $x = L$

$$\boxed{T_0 - T_w = \frac{\dot{q}}{k}\frac{L^2}{2}} \quad —— (4)$$

(3)/(4) We get

$$\boxed{\frac{T_0 - T}{T_0 - T_w} = \left(\frac{x}{L}\right)^2} \leftarrow \therefore \text{In non dimensional form}$$

[GATE 2007]

consider steady one dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MWatt/m^3, the left & right faces are kept at const temp of 160°C & 120°C respectively. The plate has const. k of 200 W/mK. The location of max. temp within plate from its left face is

(i) 15 mm (ii) 10 mm (iii) 5 mm (iv) 0 mm

The max temp. within the plate in °C is

(i) 160 (ii) 165 (iii) 200 (iv) 250

We have,

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$$

$\int$ ing

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$$

Again ∫

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1x + C_2$$

at $x=0$ $T=160$

~~at x=~~

(49)

$\therefore \quad 160 = 0 + 0 + C_2$

$\therefore C_2 = 160$

At $x = 0.02m$ $T = 120^\circ$

$$120 = -\frac{80\times10^6}{200}\left(\frac{0.02^2}{2}\right) + C_1(0.02) + 160$$

$\therefore C_1 = 2000$

$\therefore$ Now to get location for max temp

$$\frac{dT}{dx} = 0$$

$$0 = -\frac{80\times10^6}{200}x + 2000$$

$$x = \frac{-2000\times200}{80\times10^6}$$

$$x = 0.005\ m$$

$$\boxed{x = 5\ mm}$$

$$T_{max} = -\frac{80\times10^6}{200}\left(\frac{0.005^2}{2}\right) +$$

$$2000(0.005) + 160.$$

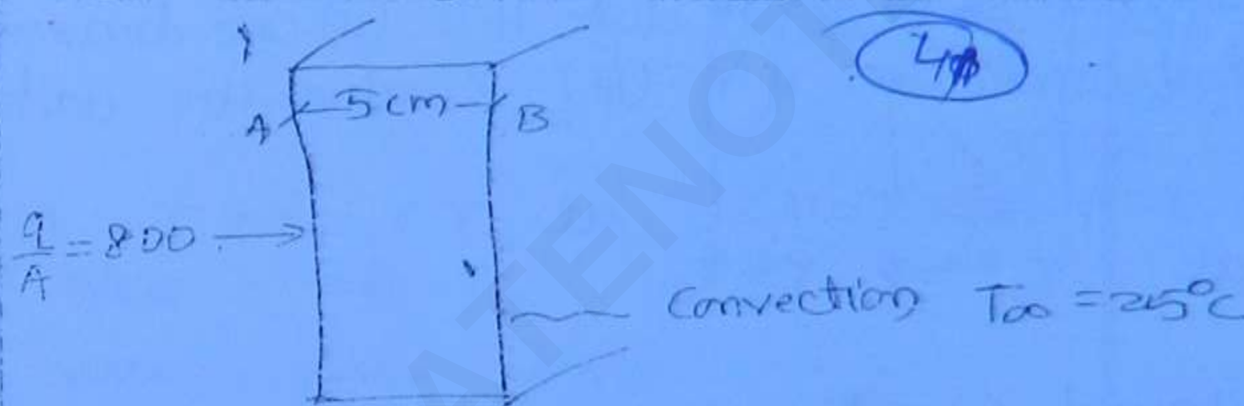
$$\therefore \boxed{T_{max} = 165^\circ C.}$$

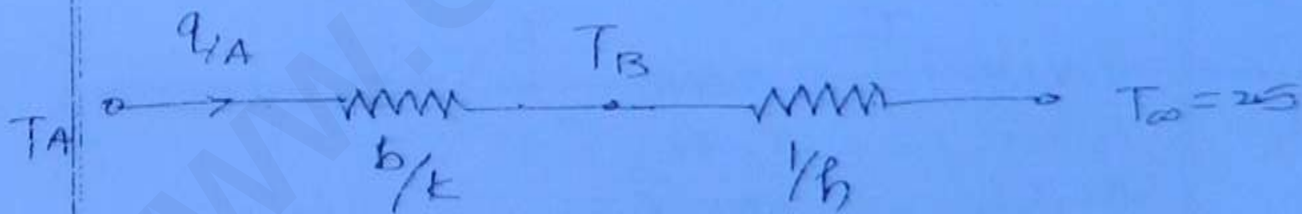
IES 99

A steel plate of thickness 5 cm & k = 20 W/mK is subjected to a uniform heat flux of 800 W/m² on one surface A and transfers heat by convection with a ~~h~~ h = 80 W/m²K from the other surface B into the ambient air at T_∞ = 25° C. The temperature of surface B transferring heat by convection is ⇒

(i) 25° C (ii) 31°C (iii) 45°C (iv) 55°C.

4th





$$\frac{q}{A} = \frac{T_B - T_\infty}{1/h} =$$

$$800 = \frac{T_B - 25}{1/80} \qquad \therefore \boxed{T_B = 35°C}$$

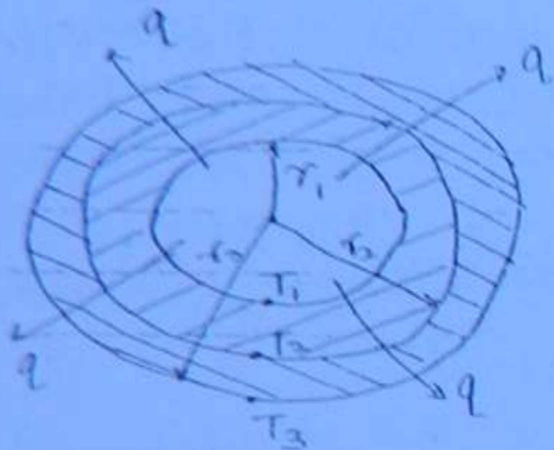
like

$$800 = \frac{T_A - T_B}{b/k} \qquad T_A = ?$$

Prandtl No. is property of fluid.

15th Jan '10, 8:30 AM.

* Conduction H.T. through a composite hollow sphere *



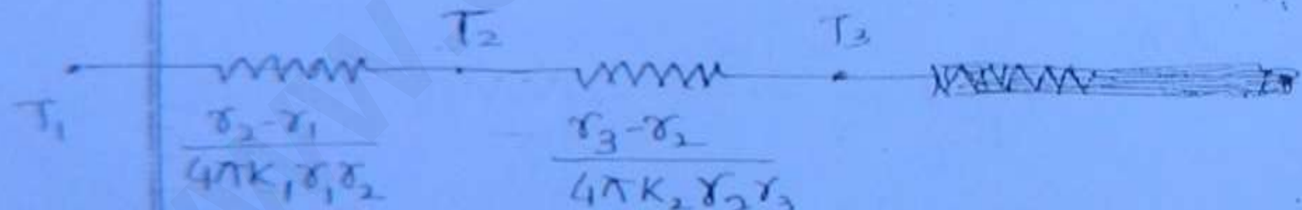
$T_1 > T_2 > T_3$

42

Assumptions ⇒ (i) steady state H.T. (ii) One dimensional Radial conduction H.T. (iii) K values are uniform.

At $r = r_1 \Rightarrow T = T_1$ at $r = r_2 \Rightarrow T = T_2$
& at $r = r_3 \Rightarrow T = T_3$

Thermal Circuit



$$q = \frac{T_1 - T_3}{\Sigma R_{Th}}$$

$$q = \left\{ \frac{T_1 - T_3}{\left[\frac{r_2 - r_1}{4\pi K_1 r_1 r_2} + \frac{r_3 - r_2}{4\pi K_2 r_2 r_3} \right]} \right\} \text{ Watts}$$

The Junction temp. i.e. T_2 is given.

$$q = \frac{T_1 - T_2}{r_2 - r_1 / \ldots}$$

$$\therefore T_2 = \cancel{\frac{q\,4\pi K_1 r_2 r_1}{}} \; -\frac{q(r_2-r_1)}{4\pi K r_1 r_2} + T_1$$

$$\therefore T_2 = T_1 - \frac{q(r_2-r_1)}{4\pi K r_1 r_2}$$

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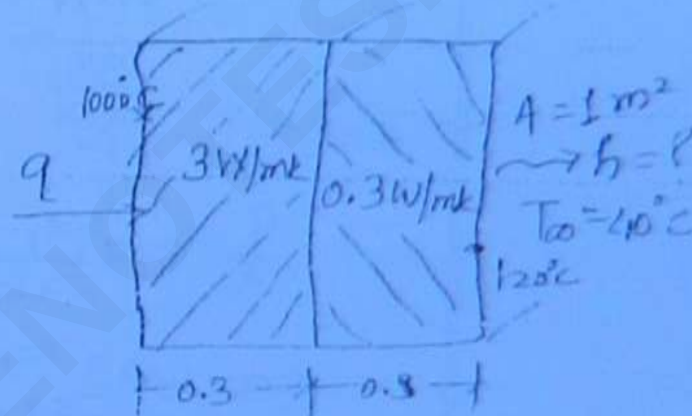
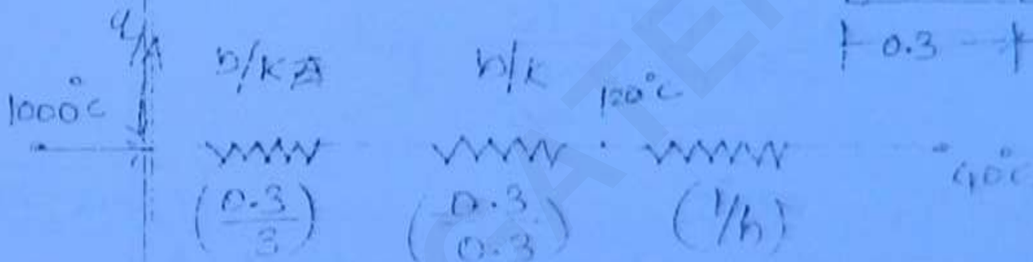
| IES Problem |

A furnace wall is constructed as shown in the given fig. The heat transfer coeff across the outer surface will be ⇒

(i)

(ii)





$$\therefore \frac{q}{A} = \frac{1000-120}{\left(\frac{0.3}{3}\right)+\left(\frac{0.3}{0.3}\right)} = \frac{120-40}{(1/h)}$$

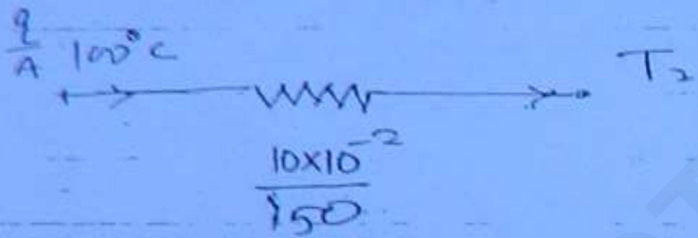
$$\therefore \boxed{h = 10 \quad W/m^2K}$$ (Free convection in air)

| IES 2009 |

A steel plate of $k = 50$ W/mk & thickness 10 cm passes a heat flux by conduction of 25 kW/m². If the temp of hot surface of plate is 100°C Then what is the temp of the cooler side of the plate.

$k = 50\ W/mK$

49



$$\therefore \frac{q}{A} = \frac{T_1 - T_2}{b/k}$$

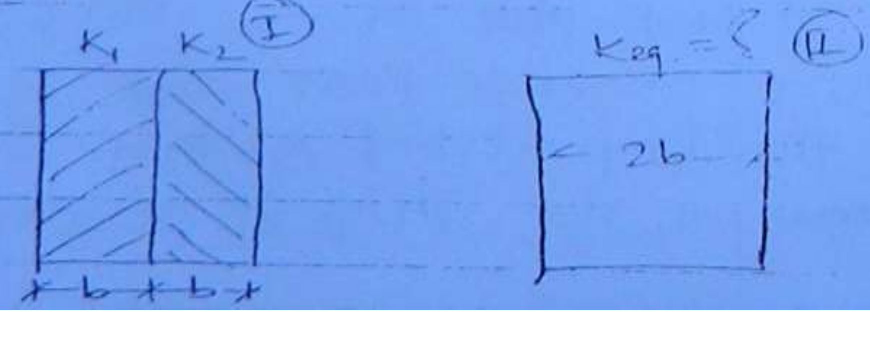
$$\therefore 25\times10^3 = \frac{100 - T_2}{\frac{10\times10^{-2}}{50}}$$

$$\therefore T_2 = 50^\circ C$$

$\boxed{\text{ES 2009}}$

A composite slab has two layers of diff. materials having thermal conductivities k_1 & k_2. If each layer has the same thickness, then what is the equivalent thermal conductivi of the slab.

(a) $\frac{K_1K_2}{2(K_1+K_2)}$ (b) $\frac{2K_1K_2}{(K_1+K_2)}$ ✓ (c) $\frac{(K_1+K_2)}{2K_1K_2}$ (d) $\frac{2(K_1+K_2)}{K_1K_2}$



In case I

$$R_{Th_{(I)}} = \frac{b}{K_1} + \frac{b}{K_2}$$

& In equivalent (II) case.

$$R_{Th_{(II)}} = \frac{2b}{K_{eq}}$$

$$\therefore R_{Th_{(I)}} = R_{Th_{(II)}}$$

$$\frac{b}{K_1} + \frac{b}{K_2} = \frac{2b}{K_{eq}}$$

$$\frac{K_1+K_2}{K_1 \cdot K_2} = 2/K_{eq}$$

$$\therefore K_{eq} = \frac{2K_1 \cdot K_2}{K_1+K_2}$$

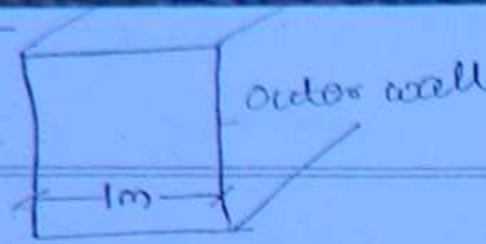
$\boxed{\text{ES 2009}}$

A large concrete slab 1 m thick has one dimensional temp distribution $T = 4 - 10x + 20x^2 + 10x^3$ where T is temp. & x is distance from one face towards the other face of wall. If the slab material has thermal diffusivity of (α) $2\times10^{-3}\ m^2/hr$ what is the rate of change of temp. at the outer face of the wall.

(a) 0.1 °C/hr (b) 0.2 °C/hr (c) 0.3 °C/hr (d) 0.4 °C/hr

→ We have

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$



(46)

We have for the condition.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

$$\therefore \frac{\partial T}{\partial x} = -10 + 40x + 30x^2$$

$$\therefore \frac{\partial^2 T}{\partial x^2} = 60x + 40$$

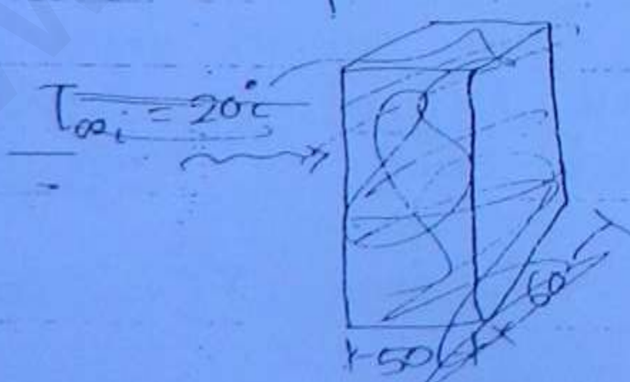
$$\therefore \left(\frac{\partial^2 T}{\partial x^2}\right)_{at\ x=1m} = \frac{1}{\alpha}\left(\frac{\partial^2 T}{\partial \tau^2}\right)_{at\ x=1.}$$

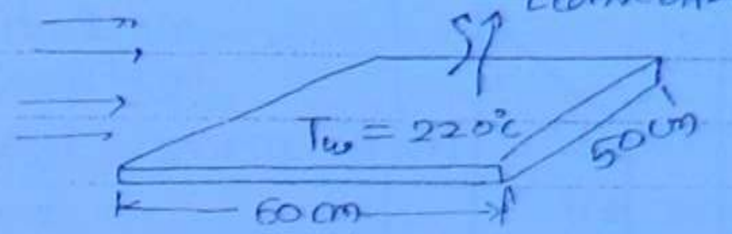
$$\therefore \left(\frac{\partial T}{\partial \tau}\right) = 2\times10^{-3}(60+40)$$

$$\therefore \frac{\partial T}{\partial \tau} = 0.2\ ^\circ C/hr.$$

$\boxed{\text{IES 09}}$

An air at 20°c ~~closed~~ blows over a hot plate of 50 x 60 cm, made of carbon steel maintained at 220 °C. The h = 25 w/m²K. What will be the heat loss from the plate



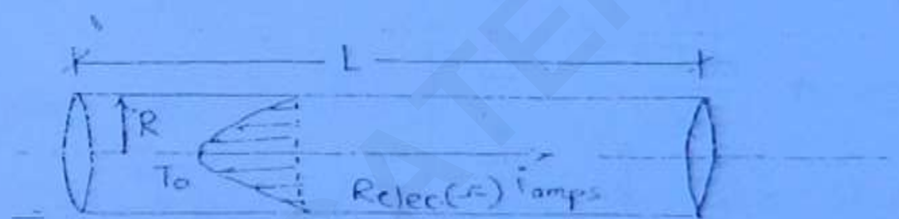


$\therefore$ Rate of H.T. (from top as well as bottom) $= hA\,(T_w - T_\infty)$

$$= 25 \times \left(\frac{50}{100} \times \frac{60}{100}\right) \times 2\,[220 - 20]$$

$$= 3000 \text{ Watts}$$

* Heat Generation in the cylinder :-



Heat generated per second $= Vi$
$= i^2 R_{ele}$ Watts

Assumptions $\Rightarrow$ i) Steady state heat transfer $T \neq f(\text{time})$
(ii) The heat generated in wire should be taken/given for which to the surrounding fluid at T_∞ by convection.
One dimensional radial conduction H.T. $(T = f(r))$

The generalised heat conduction eqn in cylindrical co-ordinates is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

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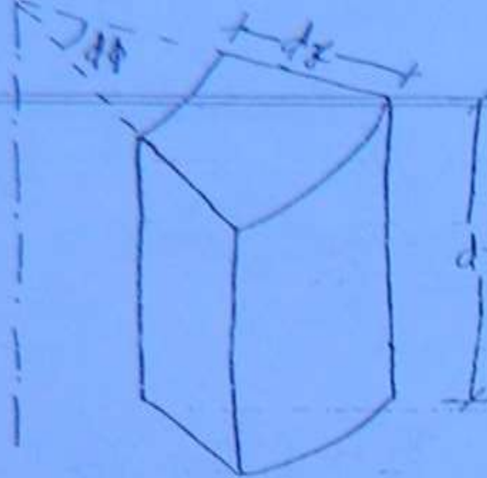

r = Radial
ϕ = Azimuthal
z = Axial

Since temperature is function of radius only
i.e. $T = f$ (Radial Dir^n Only)

$$\therefore \frac{\partial^2 T}{\partial \phi^2} = 0 \; ; \; \frac{\partial^2 T}{\partial z^2} = 0 \; ; \; \frac{\partial T}{\partial \tau} = 0 \text{ (Steady State)}$$

$$\Rightarrow \frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{k} = 0$$

$$r \cdot \frac{d^2 T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}}{k} r$$

As we have $\Rightarrow \frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = \left[r\frac{d^2 T}{dr^2} + \frac{dT}{dr}\frac{dr}{dr}\right]$

$$\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = -\frac{\dot{q}}{k} r$$

Integrating

$$r \cdot \frac{dT}{dr} = -\frac{\dot{q}}{2k} r^2 + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{2k} + \frac{C_1}{r} \quad \text{—— (I)}$$

Integrating

$$\boxed{T = -\frac{\dot{q} r^2}{4k} + C_1 \log r + C_2} \quad \text{—— (II)}$$

where C_1 & C_2 are constants of integration to be obtained from B.C.

should be ∞. $\therefore C_1 = 0$]

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(49)

Boundary condition
(i) The heat generated in the wire = Heat conducted at the surface radially.

$$\dot{q}(\pi r^2 L) = -k\, 2\pi r L \left(\frac{dT}{dr}\right)_{at\ r=R}$$

$$\therefore \left(\frac{dT}{dr}\right)_{at\ r=R} = -\frac{\dot{q}R}{2k} \quad \text{—— (iii)}$$

According to (i)

$$\left(\frac{dT}{dr}\right)_{r=R} = -\frac{\dot{q}R}{2k} + \frac{C_1}{R}. \quad \text{—— (iv)}$$

from (iii) & (iv)

$$-\frac{\dot{q}R}{2k} = -\frac{\dot{q}R}{2k} + \frac{C_1}{R}$$

$$\therefore \boxed{C_1 = 0}$$

(ii) At the centre line axis i.e. at $r = 0$, $T = T_0$
(T_0 = Centre line Temp.)

When T is maximum,

$$\frac{dT}{dr} = 0 \Rightarrow 0 = -\frac{\dot{q}r}{2k} + 0$$

$$\Rightarrow r = 0$$

$\therefore$ T_0 is T_{max} i.e. temp. at the axis

from eq^n (ii)

Now $T = -\frac{\dot{q}r^2}{4k} + C_2$

To get, Put $r=0 \Rightarrow T = T_0 = C_2$

$$\therefore \quad T = \frac{-\dot{q} r^2}{4k} + T_0$$

$$\boxed{T_0 - T = \frac{\dot{q} r^2}{4k}}$$

This is the eqn of temp. distribution which is of parabolic nature.

Let T_w = Surface temp of wire (i.e. at $r=R$)

Put $r=R \Rightarrow T_0 - T_w = \frac{\dot{q} R^2}{4k}$

$$\Rightarrow \frac{T_0 - T}{T_0 - T_w} = \left(\frac{r}{R}\right)^2$$

To get T_w from energy balance, for steady state conditions $\Rightarrow$

Total heat generated in the wire = Heat convected at the surface

$$\dot{q}(\cancel{\pi} R^{\cancel{2}} \cancel{L}) = h(2\cancel{\pi} \cancel{R} \cancel{L})(T_w - T_\infty)$$

$$\frac{\dot{q} R}{2h} = T_w - T_\infty$$

$$\therefore \quad T_w = T_\infty + \frac{\dot{q} R}{2h}$$

$$\boxed{\text{Centre line Temp} \quad T_0 = T_w + \frac{\dot{q} R^2}{4k}}$$

A current of 200 Amps is passed through a stainless steel wire 0.25 cm in dia. The resistivity of steel is 70 $\mu\Omega$ cm & length of wire is 1 m. If the outer surface temp. of wire is maintained at the 180°C. Calculate the centre temp. Assume K of steel as 30 W/mK.

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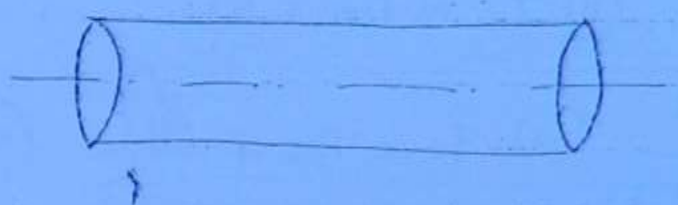


$i = 200$ Amps

$\rho = 70 \times 10^{-6}\ \Omega\,cm$

$$R_{elect.} = \rho \cdot \frac{L}{A} \Rightarrow R_{elec.} = \frac{70 \times 10^{-6}}{100} \times \frac{1}{\frac{\pi}{4}\left(\frac{0.25}{100}\right)^2} =$$

$$= \frac{70 \times 10^{-4}}{4.908 \times 10^{-6}}$$

$$= \cancel{14.262}\ \ 14.26 \times 10^{-2}\ \Omega$$

$$\cancel{1.426 \times 10^{-2}}\ \Omega$$

Heat Generated per unit volume

$$\dot{q} = \frac{i^2 R_{elect}}{(\pi R^2 L)} \text{ Watts}/m^3$$

$(\pi R^2 L) \leftarrow$ Volume of wire

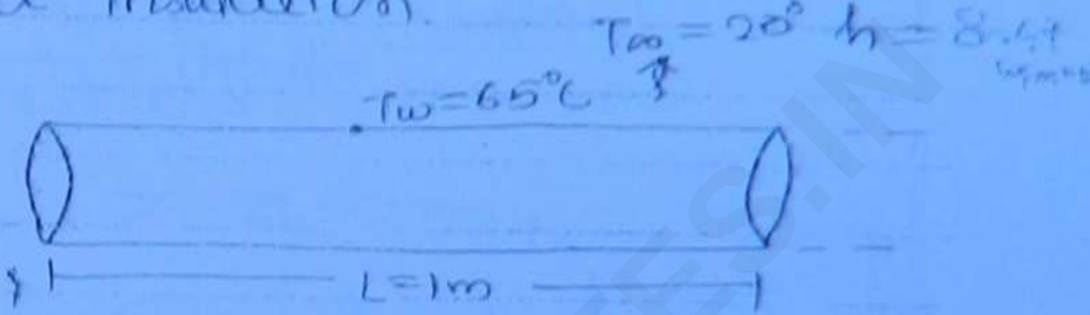
$$\therefore \dot{q} = \left[\frac{200^2 \times 0.143}{\pi \left[\frac{0.25}{2 \times 100}\right]^2 \times 1}\right] W/m^3$$

$$\therefore \dot{q} = 1.165 \times 10^{9}\ W/m^3$$

$$T_0 = T_w + \frac{\dot{q} R^2}{4K} = 180 + \frac{(\dot{q})\left(\frac{0.25}{2 \times 100}\right)^2}{4 \times 30}$$

$$T_0 = 195.2°C$$

A 10 mm cable is to be laid in a atmosphere of 20°c ($h = 8.49\ W/m^2 k$). The surface temp of the cable is likely to be 65°C due to heat generated within the wire. Find the rate of heat loss a) with critical radius of insulation
b) Without insulation



(53)

$$q_{\text{Without Insulation}} = h A (T_w - T_\infty)$$

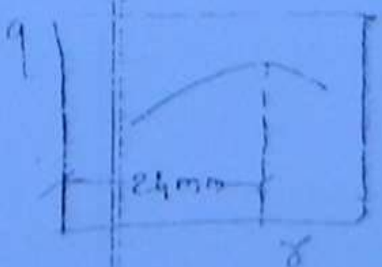
$$q = 8.49 \times \left(\pi \times \frac{10}{100} \times 1\right)(65 - 20)$$

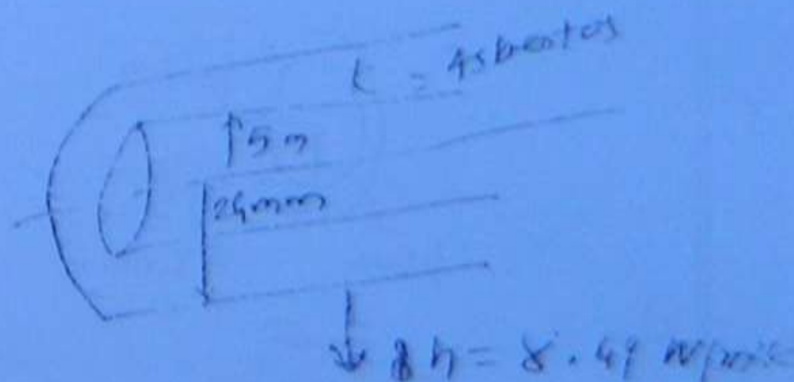
$$q = 12.09\ W/m$$

Critical radius of insulation (Asbestos) $= \dfrac{k}{h}$

$$= \frac{0.2}{8.49}$$

$$= 0.024\ m = 24\ mm$$





$$65^\circ C \longrightarrow \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} \longrightarrow \frac{1}{h\, 2\pi r_2 L} \longrightarrow 20^\circ C$$

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$$q_{\text{with Ins.}} = \frac{65-20}{\frac{\ln(24/5)}{2\pi \times 0.2 \times 1} + \frac{1}{8.49 \times 2 \times \pi \times \frac{24}{1000} \times 1}}$$

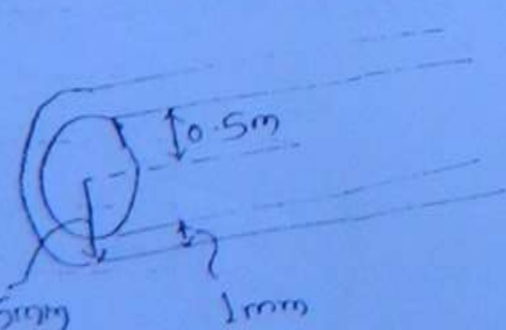
$$q_{\text{with Insu}} = 19\ W$$

A copper wire of radius 0.5 mm is insulated with a thickness of 1 mm having a thermal conductivity of 0.5 W/mK. The outside surface convective H.T. coeff is 10 W/m²K. If the thickness of insulation is raised by 10 mm, then the electrical current carrying capacity of wire will a

[a] Increase [b] Decrease [c] Remains Same

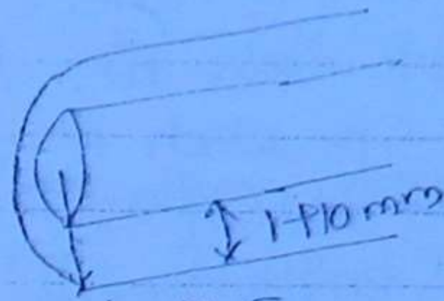
[d] depends upon the electrical conductivity of wire.

$\Longrightarrow$



$r_{critical} = \frac{k}{h} = \frac{0.5}{10} = 50 \text{ mm}$

(55)



$r_2' = 11.5$ mm

$r_{critical} = 50$ mm

$\therefore r_2 = 1.5$ mm &

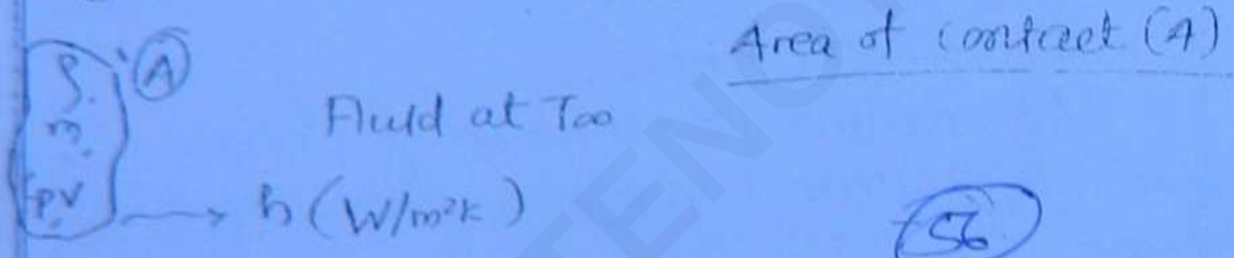
$r_2' = 11.5$ mm

$\therefore$ As radius is increasing heat loss will increase & current carrying capacity will also increase.

* Unsteady State or Transient heat conduction

In this type of conduction H.T., temp. of the body may change w.r.t time

Consider a body of mass m, density ρ, volume V, Specific heat C_p which is at an initial temp. of T_i (When taken out of the furnace) is suddenly exposed to an environment at T_∞. Since the body looses heat by convection to the fluid of environment with a convection heat transfer coeff. of h W/m²K) the temp of body keeps on decreasing as the time progresses.



(56)

Let T_i = Initial temp of body at an instante of time $\tau = 0$ sec.

Let T = Temp. of body at any instance of time τ sec, measured from the instance when the body is just exposed to environment

$T = f(\tau)$

Writing the energy balance at any instance of time τ sec.

The rate of convection heat loss from surface of body to fluid = Rate of decrease in internal energy of the body

Temp w.r.t. time

$$hA(T-T_\infty) = -mc_p\left(\frac{dT}{d\tau}\right)$$

$$= -\rho V c_p\left(\frac{dT}{d\tau}\right)$$

Keeping all other parameter const. & separating the variable i.e. Temp. (T) & Time (τ)

$$-\frac{hA}{\rho V c_p}\,d\tau = \frac{dT}{(T-T_\infty)}$$

$$-\left(\frac{hA}{\rho V c_p}\right)d\tau = \frac{dT}{(T-T_\infty)}$$

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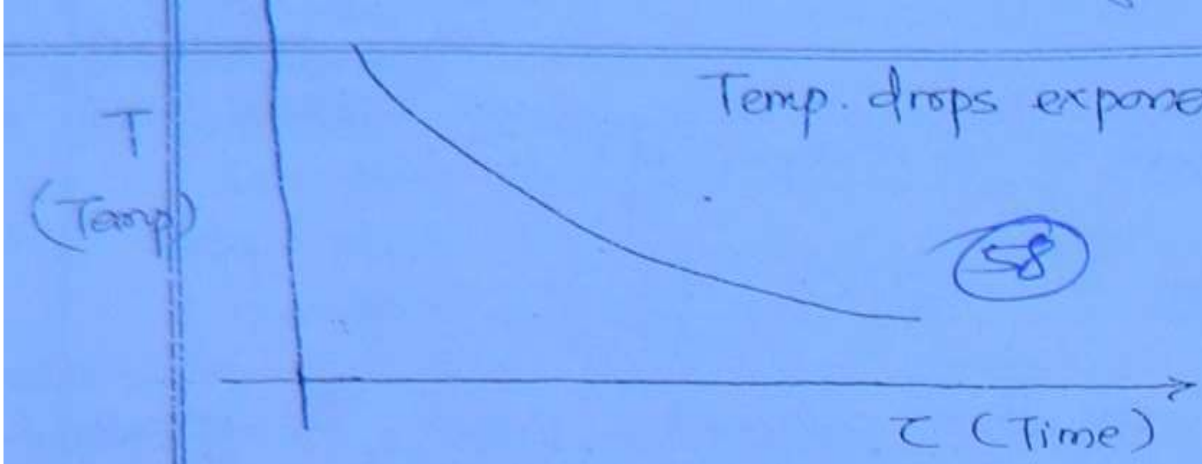
Integrating.

$$\int_0^\tau -\left(\frac{hA}{\rho V c_p}\right)d\tau = \int_{T_i}^{T}\frac{dT}{(T-T_\infty)}$$

$$\left(\frac{hA}{\rho V c_p}\right)\tau = \log_e\left(\frac{T_i-T_\infty}{T-T_\infty}\right)$$

$$\boxed{\left(\frac{T_i-T_\infty}{T-T_\infty}\right) = e^{\left(\frac{hA}{\rho V c_p}\right)\tau}}$$

As the time progresses the rate of cooling of body decreases. Since the temp diff b/w the body & fluid decreases.



Since $\left(\frac{\rho V C_p}{hA}\right)$ has the unit of second. It is called time constant.

(Among all the liquids K of Hg is very high as close to steel 8 W/mK ? Alcohol is also used as thermometric liquid).

Assumption ⇒

[At any instant of time τ sec body have same temp.] lumped h

In the above analysis it is assumed that internal temperature gradients within the body are neglected i.e. at any instance of time τ sec the entire body has uniform temp. Such ~~any~~ analysis is called lumped heat capacity analysis.

Criteria for lumped heat capacity analysis

⇒ Biot Number < 0.1

Where Biot No. = $\frac{h \cdot s}{K_{solid}}$

$s = \left(\frac{V}{A}\right)$ — V = Volume of body, A = Surface Area of body

For sphere $s = V/A = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$

Biot No. Resembles Nusselt No.

$Nu = \frac{hD}{k_{fluid}}$ $Nu > 1$

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$$\text{Biot No} = \frac{(S/KA)}{(1/hA)}$$

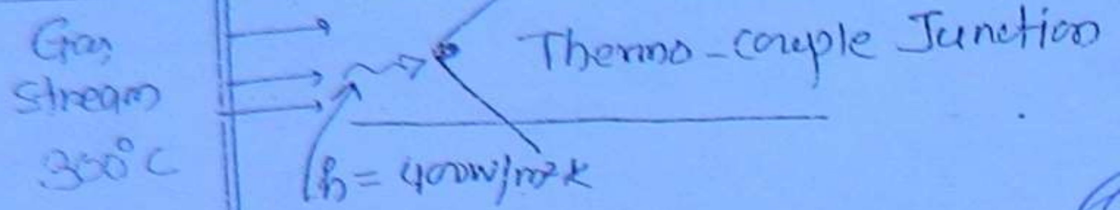
$$= \frac{\text{Internal conductive resistance}}{\text{External convective resistance}} = \frac{ICR}{ECR}$$

For Lumped heat capacity analysis is to be valid, the body is of very slender or thin shaped. And also thermal conductivity of the body is very high. e.g. Small copper sphere/ball

GATE 04

A spherical thermal couple junction of diameter 0.706 mm is to be used for the measurement of temp of a gas stream, the convective h.t. coeff. (h) on the bead surface is 400 W/m²k. Thermo-physical properties of thermo-couple material are. k = 20 W/mk, c = 400 J/kgK & ρ = 8500 kg/m³ If thermo couple is initially at the 30°C is placed in a hot stream of 300°c, the time taken by the bead to reach 298°c is ⇒

(i) 2.35 sec (ii) 4.9 sec (iii) 14.7 sec (iv) 29.6 ...



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Now we have

$$\text{Biot No.} = \frac{h \cdot s}{K_{\cancel{fluid}\ solid}}$$

$$= \frac{400 \times 0.706 \times 10^{-3}}{3 \times 20 \times 2}$$

$$= 0.000235 < 0.1$$

$$\frac{T_i - T_\infty}{T - T_\infty} = e^{(hA/\rho V c_p) \cdot \tau}$$

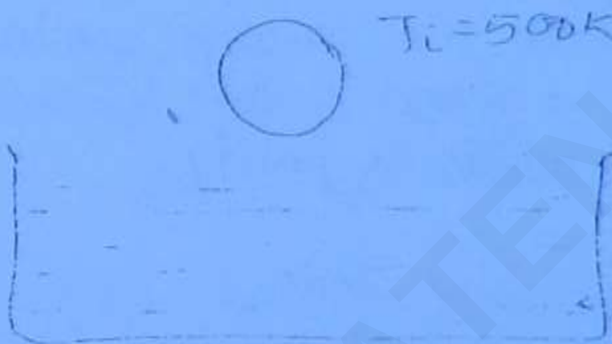
$$\frac{30 - 300}{298 - 300} = e^{\left(\frac{400 \times 3 \times 2}{8500 \times 0.706 \times 10^{-3} \times 400}\right) \tau}$$

$$\boxed{\tau = 4.9 \text{ seconds}}$$

GATE 2005

A small copper ball of 5 mm dia. at 500 K is dropped into an oil bath whose temp. is 300 K. The k of copper is 400 W/mK. Its density 9000 kg/m³ & its specific heat 385 J/kgK. If the heat transfer coeff. is 250 W/m²K. and the lumped analysis is assumed to be valid, the rate of fall of temperature of the ball at the beginning of cooling will be in K/s ⇒ (61)

a] 8.7 b] 13.9, c] 17.3 ✓ d] 27.7



$(dT/d\tau) = ?$

Writing energy balance eqn at the very beginning of dropping the ball.

$$hA(T_i - T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right)_{at\ \tau=0}$$

$$200 \times 4 \times \pi \times (2.5\times10^{-3})^2 (500-300) = -9000 \times \left(\frac{2.5}{1000}\right)^3 \times \frac{4}{3}\pi \times 385 \times \frac{dT}{d\tau}$$

$$\therefore (200 \times 200)(2.5\times10^{-3})^2 = -3000\left(\frac{2.5}{1000}\right)^3 \times 385 \times \frac{dT}{d\tau}$$

$$\therefore dT/d\tau = \underline{17.3}$$

As the time progresses, since the temperature diff. b/w the ball and the fluid keeps on decreasing the rate of cooling of the ball also decreasing i.e. the ball takes more time to get cool. for each °C.

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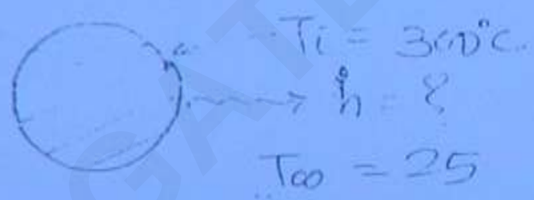
IES 2008

A copper sphere weighing 3 kg is heated in a furnace to a temp. of 300°C and is suddenly take out & allowed to cool in ambient air at 25°C. If it takes 60 min. for the copper sphere to cool down to 35°C, what is the average surface H.T. coeff. Take $\rho_{copper} = 8950 \ kg/m^3$ & $C_p = 0.383 \ kJ/kg$ State the assumptions made & justify.

⇒



$$\left.\begin{array}{l} T_0 = 300°C \\ T = 35°C \end{array}\right\} \text{Takes } \tau = 60 \times 60 = \text{sec}$$

$m = 3 \ kg \qquad \rho = 8950 \ kg/m^3$

$$\rho = \frac{m}{V}$$

$$8950 = \frac{3}{\frac{4\pi}{3}R^3}$$

$$\boxed{R = 0.043 \ m}$$

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Page 4

$$\frac{T_i - T_\infty}{T - T_\infty} = e^{+\left(\frac{hA}{\rho V C_p}\right)\tau}$$

Assume $\Rightarrow$ (i) Lumped heat capacity analysis is valid

$$\text{Biot No} = \frac{hs}{k} < 0.1$$

Assuming highest probable value of convection heat transfer coefficient in air (forced conv.) as 400 W/m²K.

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$$\therefore \text{Biot No} = \frac{400 \times 0.043}{3 \times 385} \quad \text{— k of copper —}$$

$$= 0.0148 < 0.1$$

$$\therefore \frac{300 - 25}{35 - 25} = e^{+\left(\frac{h \cdot 3}{8950 \times 0.043 \times 0.383}\right) 60 \times 60}$$

$$27.5 = e^{\left(\frac{h \times 3}{8950 \times 0.043 \times 0.383}\right) 3600}$$

$$\therefore \boxed{h = 45.1} \text{ W/m}^2\text{K}$$

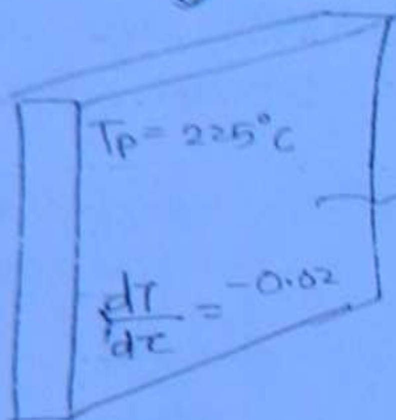
** Forced convection **

$\boxed{\text{GATE 2007}}$

The average H.T. coeff. on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temp with time as it cools. Assume the plate temp to be uniform at any instance of time & radiation heat exchange with the surrounding negligible. The ambient temp. is 25°C. The plate has a total surface area of 0.1 m² & mass of 4 kg. The specific heat of plate mtrl. 2.5 KJ/KgK. The h = in W/m²K at the instant when the plate temp. is 225°c & change in plate temp. with time $\frac{dT}{d\tau}$ = −0.02 K/sec is

(64)

(i) 200 (ii) 20 (iii) 15 (iv) 10



$T_\infty = 25°c$

(Still Air)

At the instance of time when $\left(\frac{dT}{d\tau}\right)$ is $= -0.02$ K/sec.

Energy Balance eqⁿ

Rate of convection heat = Rate of decrease in I.E. of plate

$$h A (T_p - T_\infty) = -m c_p \left(\frac{dT}{d\tau}\right)$$

$$h \times 0.1 (225 - 25) = -4 \times 2.5 \times 10^3 \times (-0.02)$$

$$\therefore h = 10 \ W/m^2 K$$

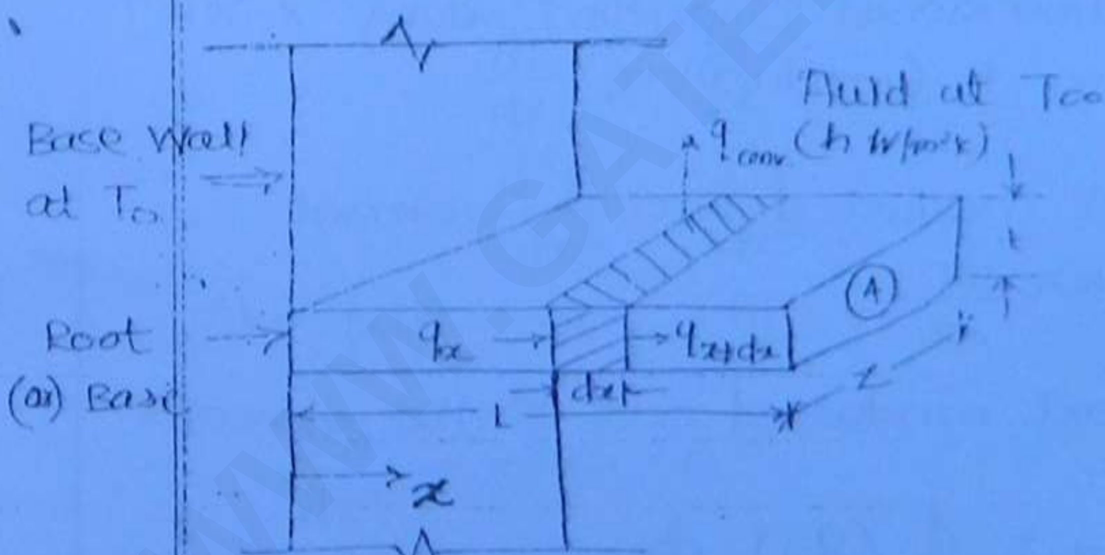
* FINS (Extended Surfaces)

Fins are meant for increasing the H.T. rate from any surface to the fluid by increasing surface area of heat transfer. They are the extended surfaces projecting from the base wall. (65)

Fins are generally prefered only when the convective heat transfer coefficient (h) values are relatively low i.e. with air.

e.g. Air-cooled I.C Engine, Reciprocating air compressors, Car radiators, electric motors & transformers, Condenser of a Refrigerating Unit.

* Analysis of a Rectangular Fin



L = Length of fin, A = Cross-sectional area of fin $= z \times t$
z = Width of fin
t = Thickness of fin

Heat is conducted into the fin at its root or base & then while conducting along the length of fin, it is simultaneously convected to the surrounding fluid at T_∞

Objectives of analysis

i) What is temp. of fin along x dirn.

$$T = f(x) = ?$$

ii) Heat transferred through the fin

$$q_{fin} = ?$$

Consider a differential element in the fin of length dx & let h be the convective H-T. coeff b/w the surface of fin & the fluid at T_∞

Assumptions :-

i) Steady state conditions.

ii) One dimensional conduction along x-dirn

iii) Uniform K (conductivity)

Let, q_x = Heat conducted into the element

$$= -KA\frac{dT}{dx}$$

q_{x+dx} = Heat conducted out of the element

$$= q_x + \frac{d}{dx}(q_x)\cdot dx$$

$q_{convection}$ = Heat convected from the surface of element.

$$= h(P.dx)(T - T_\infty)$$

where P = Perimeter of fin

$$= (2z + 2t)$$

Writing the energy balance for steady state condi^n of element.

$$q_x = (q_{x+dx}) + q_{conv.} \qquad (67)$$

$-KA$

$$\cancel{q_x} = \cancel{q_x} + \frac{d}{dx}(q_x)\,dx + h(P.dx)(T-T_\infty)$$

$$0 = \frac{d}{dx}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + h(P.\cancel{dx})(T-T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T-T_\infty) = 0$$

Put $(T-T_\infty) = \theta$

$$\therefore \frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\therefore \frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

Also put $m^2 = \left(\frac{hP}{KA}\right)$

$$\therefore \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

This 2nd order D.E. in θ.

Solution to this is $\Rightarrow$

$$\theta_1 = C_1 e^{-mx} + C_2 e^{mx}$$

— where $m = \sqrt{\frac{hP}{KA}}$ (/meter)

C_1 & C_2 are const of integration to be found from boundary conditions.

One common B.C. is
at $x=0$ $T=T_0$ & $\theta=\theta_0=T_0-T_\infty$
(At the base). * The other B.C. depends upon
three diff. cases.

⇒ Case (i) Fin is infinitely long (very long)
then the temperature at the tip of the fin
will be essentially that of the fluid.
i.e. at $x=\infty$ (Infinity)

$$T=T_\infty$$
$$\& \quad \theta \to 0$$

Then the solution will be :-

$$\boxed{\frac{\theta}{\theta_0}=\left(\frac{T-T_\infty}{T_0-T_\infty}\right)=e^{-mx}}$$

Also heat transfer rate through the fin

$$q_{fin}=q_{\substack{\text{conducted}\\ \text{at the root}}}=-KA\left(\frac{dT}{dx}\right)_{x=0}$$

$$=\sqrt{hPKA}\,\theta_0$$

$$\boxed{q_{fin}=\sqrt{hPKA}\,(T_0-T_\infty)} \text{ Watts.}$$

Temp. drops exponentially along the length of fin when the fin is very long.

Case (II) Fin's Tip is Insulated.

Then

$$q_{\substack{conducted \\ at\, x=L}} = 0$$

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$$\therefore -KA\left(\frac{dT}{dx}\right)_{x=L} = 0$$

$$\therefore \left(\frac{dT}{dx}\right)_{x=L} = 0$$

Then the solution is

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh mL}}$$

Also H.T. rate through fin $= q = \sqrt{hPKA}\,\theta_0 \tanh mL$

$$\boxed{q = \sqrt{hPKA}\,\theta_0 \tanh(mL)}$$

Case (III) Fin is finite in length & also looses heat by convection from its tip

$\Rightarrow$ Then the soln is

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}}$$

& H.T. Rate through fin $\boxed{q = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL_c)}$

where L_c = corrected Length $= (L + t/2)$

Among all the three cases, most commonly used case is " fin's tip ~~with~~ insulated."

* Fin Efficiency *

It is defined as the ratio b/w actual H.T. rate through the fin and the max. possible H.T. rate through the fin. i.e. when the entire fin is at it's base temperature.

$$\eta_{fin} = \frac{q_{actual}}{q_{max.\ Possible}}$$

~~For fin's tip insulated case.~~

Entire fin will be at its root temp. only when the mtrl of fin has infinite thermal conductivity (k)

For fin's tip insulated case :-

$$q_{fin} = \sqrt{hPKA}\ \theta_0 \tanh(mL)$$

$$q_{max.} = h(PL)(T_0 - T_\infty) \text{ Watts}$$

$$\eta_{fin} = \frac{\sqrt{hPKA}\ \cancel{\theta_0} \tanh(mL)}{hPL\ \cancel{(T_0 - T_\infty)}}$$

$$\boxed{\eta_{fin} = \frac{\tanh(mL)}{mL}}$$

$$\boxed{\eta_e \propto \sqrt{k}}$$

* Fin Effectiveness

It is ratio b/w heat transfer rate with fin and the heat transfer rate without fin.

$$\epsilon_{fin} = \frac{q_{with\ fin}}{q_{without\ fin}} \qquad 7$$

This parameter highlights the usefulness of fin i.e. whether the fins are really worth keeping or not.

$$\therefore \epsilon_{fin} = \frac{\sqrt{hPKA}\ \theta_0 \tanh(mL)}{hA(T_0 - T_\infty)}$$

$$\boxed{\epsilon_{fin} = \frac{\tanh(mL)}{\sqrt{hA/KP}}}$$

$$\boxed{\epsilon_{fin} \propto \sqrt{k}}$$

Hence the material of the fin should be of very high thermal conductivity. e.g. aluminium

$$\boxed{\epsilon_{fin} \propto 1/\sqrt{h}}$$

Hence fins do not really help when h value is very high. e.g. with water. So fins are kept when air only.

[IES 2004]

A electronic semiconductor device generates heat equal to 480×10^{-3} Watts. In order to keep the surface temp. at the upper safe limit of 70°c. The generated heat has to be dissipated to the surrounding which is at 30°c. To accomplish this task, aluminium fins of 0.7 mm square & 12 mm long are attached to the surface. The $K_{Aluminium} = 170$ W/mk. If the H.T. coeff. is 12 W/m²K. Calculate the no. of fins required. Assume no heat loss from the tip of fins.

⟹

(72)

$q = 480 \times 10^{-3}$ Watts.

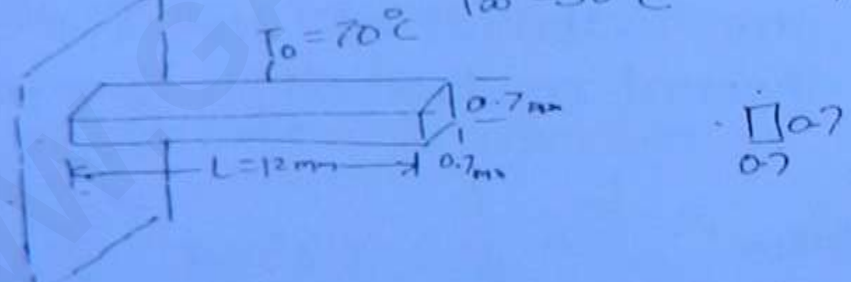
$T = 70°c \qquad T_\infty = 30°c$

$A_{fin} = 0.7\ mm^2 \qquad L = 12\ mm$

$K_{Aluminium} = 170\ W/mk \qquad h = 12\ W/m^2 K$

(Free convection in air.)

$T_\infty = 30°c$



$$q = \sqrt{hPKA}\ \theta_0 \tanh(mL)$$

$$m = \sqrt{hP/KA} = 12 \times$$

$P = 2 \times z + 2 \times t$

$= 2 \times 0.7 + 2 \times 0.7$

$=$

$$A = \left(\frac{0.7}{1000} * \frac{0.7}{1000} \right) m^2$$

$=$

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$\therefore \quad m = 20 \, /m$

Now Heat transfer rate through each fin (Insulated Tip)

$$q = \sqrt{hPKA} \; \theta_0 \tanh(mL)$$

$$q = \sqrt{12 \times P \times 170 \times A} \; (70 - 30) \tanh(20L)$$

$$\therefore q = 16 \times 10^{-3} \text{ Watts}$$

$$\text{No. of finss} = \frac{480 \times 10^{-3}}{16 \times 10^{-3}}$$

$= 30$ Fins are required.

IES 1993

Two long rods of same diameter one made of brass $k = 85 W/mK$ & other made of copper have one of their ends inserted in furnace. Both rods are exposed to the same environment at a section 10.5 cm away from the furnace end, the temp. of the brass rod is 120°C at what distance from the furnace end, the same temp would be reached → in the copper rod.

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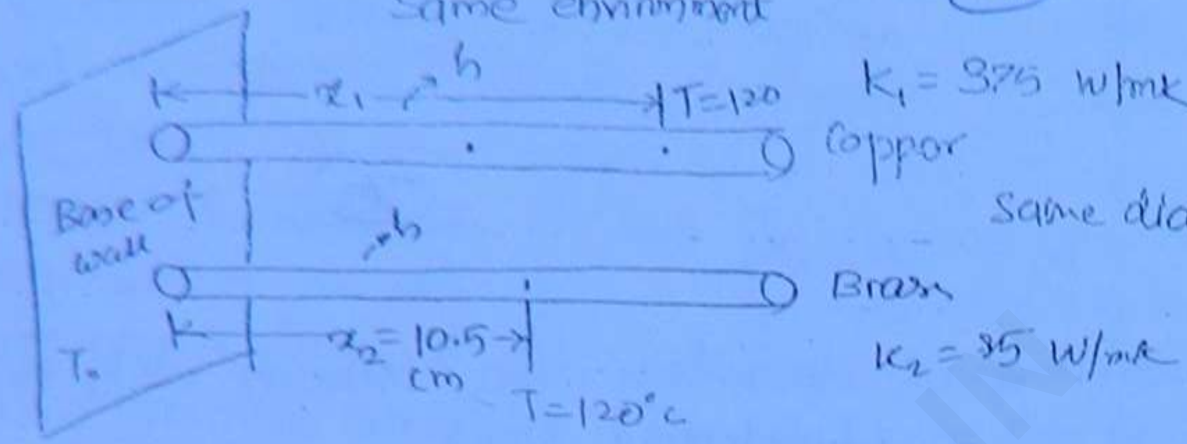


$x_1 = ?$

For very long rod

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

This factor remains same for both rods.

For copper rod

$$\frac{\theta}{\theta_0} = e^{-mx_1}$$

For brass rod

$$\frac{\theta}{\theta_0} = e^{-mx_2}$$

$$\Rightarrow 1 = \frac{e^{-mx_1}}{e^{-mx_2}} = e^{-mx_1 + m_2 x_2}$$

$$\log_e 1 = \log\left(e^{-mx_1 + m_2 x_2}\right)$$

$$0 = -m_1 x_1 + m_2 x_2$$

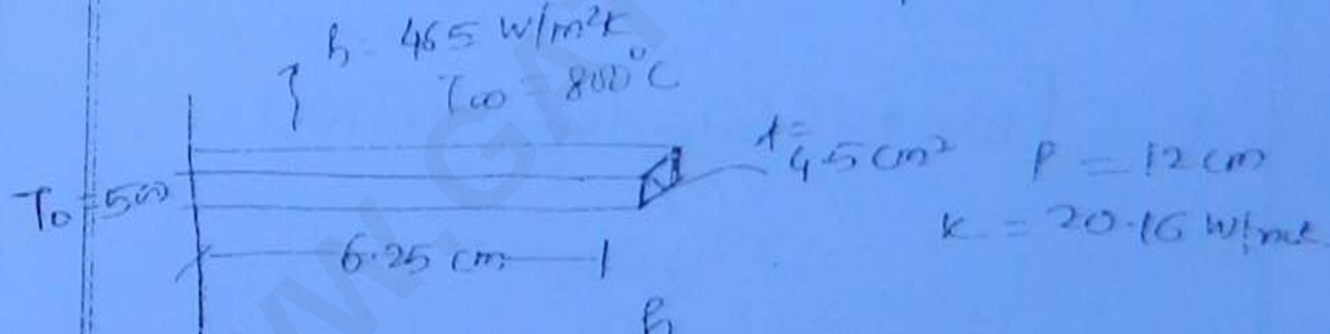
$$m_1 x_1 = m_2 x_2$$

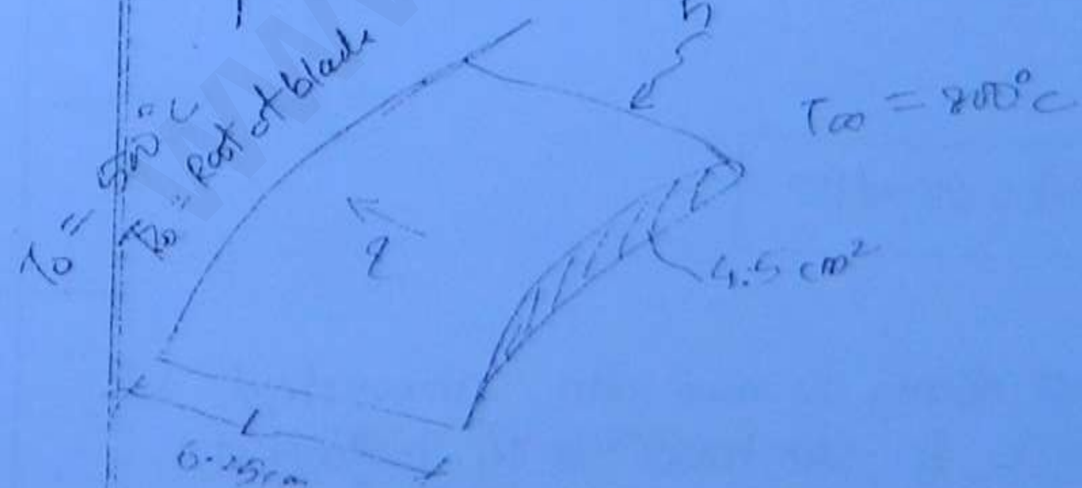
$$\frac{m_1}{m_2} = \frac{x_2}{x_1} = \sqrt{\frac{k_2}{k_1}}$$

75

$$\therefore \boxed{x_2 = 22 \text{ cm}}$$

A turbine blade is 6.25 cms long, c/s area 4.5 cm², Perimeter 12 cm is made of stainless steel $k = 26.16$ W/mk. The temp. of root is 500°c. The blade is exposed to a hot gas at 800°c & the average heat transfer coeff. is 0.465 kW/m²k. Determine the temp. at the tip & the rate of heat flow at the root of the plate. Assume that tip is insulated.





Heat Transfer is from gases to the plate

) To get temp. at tip of fin. (76)

$$\frac{\theta}{\theta_0} = \frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

Put $x = L$ $\quad m = \sqrt{\frac{hP}{kA}} = 68.84/m$

$$\frac{T_{x=L}-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-L)]}{\cosh(mL)}$$

$$\frac{T_{x=L}-800}{500-800} = \frac{\cosh(0)}{\cosh(mL)}$$

$$T_{x=L} = 791.7^\circ C$$

$q = 243$ Watts

at $L = 0$

$$q = \sqrt{hPKA}\,(T_0-T_\infty)\tanh(0)$$

$$\boxed{q = 243 \text{ Watts}}$$

IES 2000

Steel ball ~~diam~~ 12 mm dia. annealed by heated to 800°C & ~~st~~ then slowly cooling to 127°C in air at 50°C. The $h_{air} = 20\ W/m^2K$

Calculate the time required for cooling process. The properties of steel are taken as $K = 45 W/mK$

$\rho = 7830\ kg/m^3$ & $C_p = 600\ J/kgK$

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$\longrightarrow$

$$hA(T_0 - T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right)$$

$$\text{Biot No} = \frac{hs}{k} = \frac{hR}{3K} = 0.0008 < 0.1$$

Lumped heat capacity analysis can be done.

To get time of cooling

$$\frac{T_0 - T_\infty}{T - T_\infty} = e^{\left(\frac{hA}{\rho V C_p}\right)\tau}$$

$$\frac{800 - 50}{127 - 50} = e^{\left(\frac{20 \times 3}{7830 \times 6 \times 10^{-3} \times 600}\right)\tau}$$

$$\frac{750}{77} = e^{(60/783 \times 6 \times 6)\tau}$$

$$\frac{750}{77} = e^{(10/783 \times 6)\tau}$$

Taking $\log_e$

$$\left(\frac{10}{783 \times 6}\right)\tau =$$

$$\therefore \tau = 17.8\ min.$$

IES 2000

A steel pipe having 10 cm bore & 12 cm outside diameter carries hot water at 80 °C, when the surrounding temp. is 15 °C. The thermal conductivity of pipe material is 54 W/mK & the inner & outer H.T. coeff. are 1 kW/m²K & 9 W/m²K. Calculate the heat loss per meter length of the coil & surface temp. Also calculate the heat loss & the surface temp. when the pipe is covered with a 4 cm thick insulation having thermal conductivity of 0.048 W/mK. Outer h reduced to 7 W/m²K.

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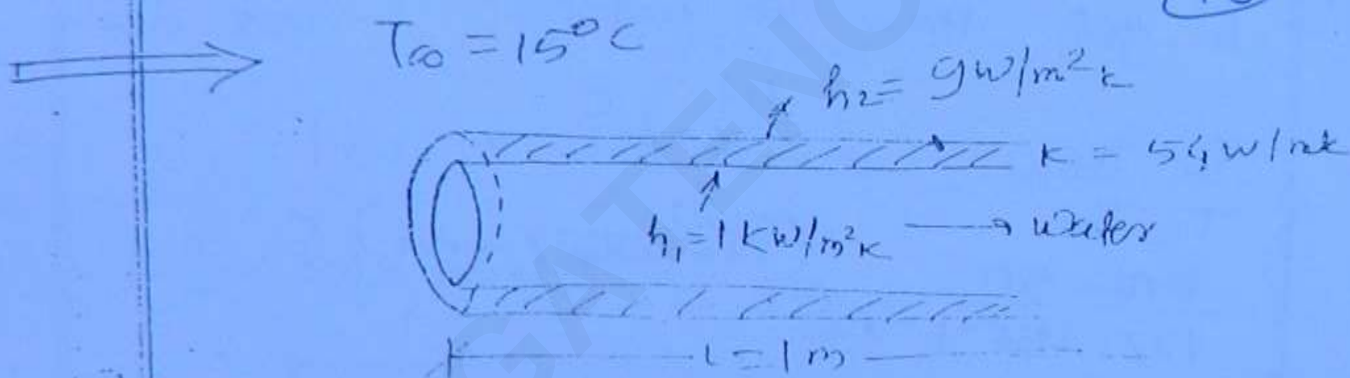

$$\frac{1}{h_1 2\pi r_1 L} \qquad \frac{\ln(r_2/r_1)}{2\pi K_1 L} \qquad \frac{1}{h_2 2\pi r_2 L}$$

$r_1 = 5$ cm

$r_2 = 6$ cm

$$\frac{q}{L} = \frac{80-15}{\frac{1}{h_1 2\pi r_1} + \frac{\ln(r_2/r_1)}{2\pi K_1} + \frac{1}{h_2 2\pi r_2}}$$

$$\therefore q/L = \frac{80-15}{\frac{1}{1000 \times 2 \times \pi \times 5/100} + \frac{\ln(6/5)}{2\pi \times 54} + \frac{1}{9 \times 2 \times \pi \times 6/100}}$$

$$\therefore \frac{q}{L} =$$

$$\therefore \frac{q}{L} = 217.79 \text{ W/m}$$

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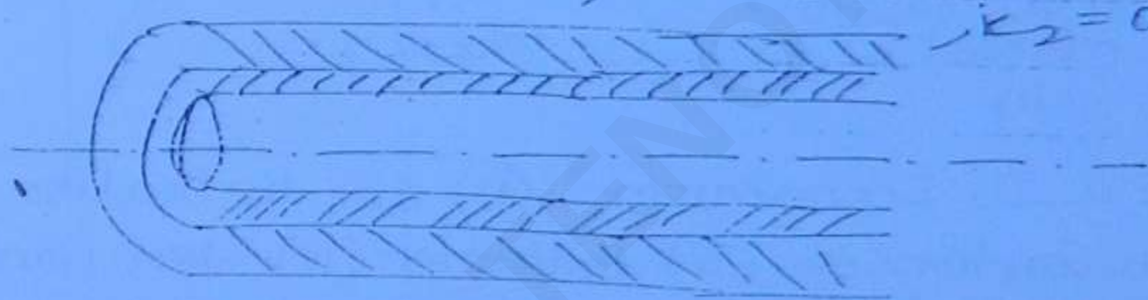
Now with Insulation, $h = 7 \text{ W/m}^2\text{K}$, $k_2 = 0.048$ W/mK



$80 \xrightarrow{q}$ $\quad \frac{1}{h_1 2\pi r_1 L} \quad \frac{\ln(r_2/r_1)}{2\pi k_1 L} \quad \frac{\ln(r_3/r_2)}{2\pi k_2 L} \quad T = ? \quad \frac{1}{h_2 2\pi r_3 L} \quad T_\infty = 15°C$

$$\therefore \frac{q}{L} = \frac{80-15}{\frac{1}{1000 \times 2\pi \times \frac{5}{100}} + \frac{\ln(6/5)}{2\pi \times 54} + \frac{\ln(10/6)}{2\pi \times 0.048} + \frac{1}{7 \times 2\pi \times \frac{10}{100}}}$$

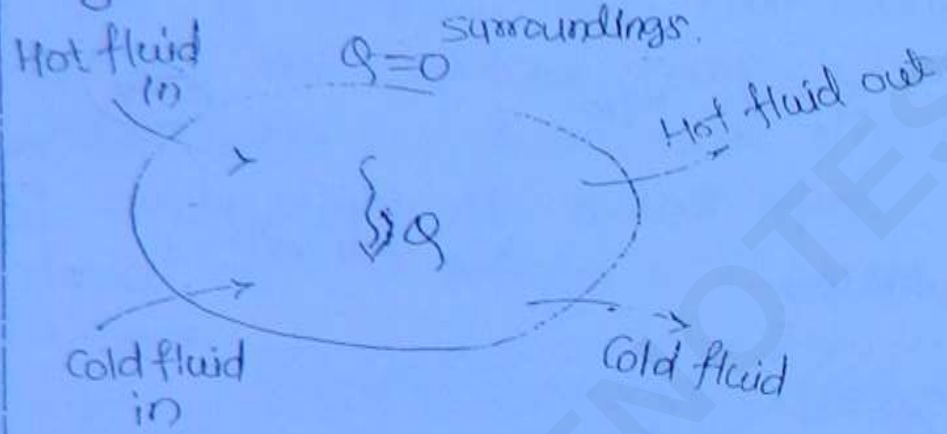
$$\therefore \frac{q}{L} =$$

$$\frac{q}{L} =$$

Heat Exchangers.

* **Heat Exchanger** —

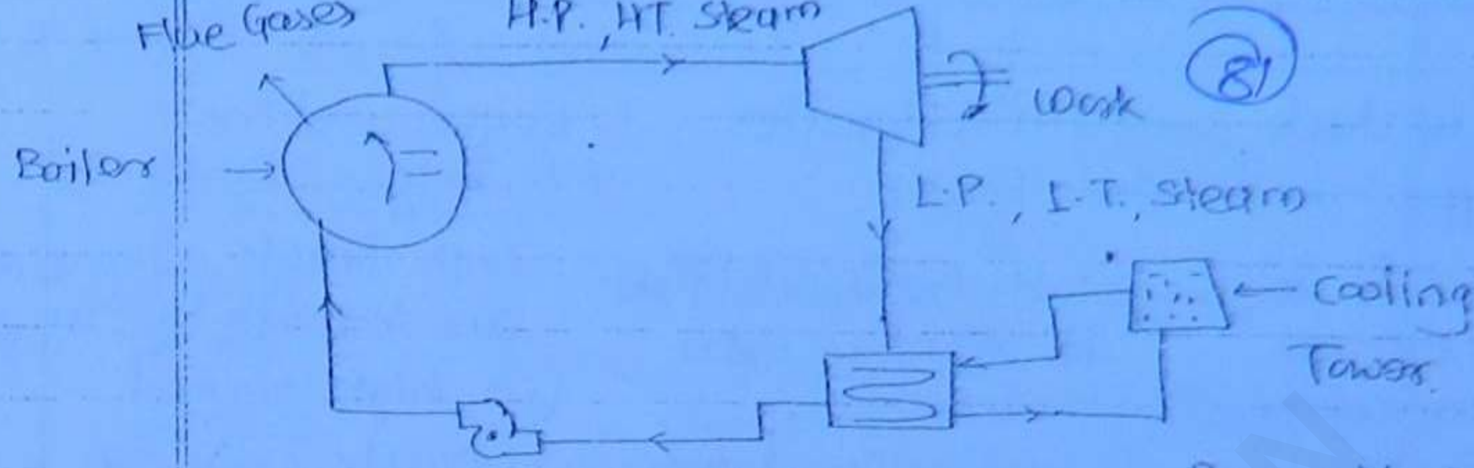
Heat exchanger is an adiabatic device in which two flowing streams of fluids exchange heat between themselves due to temperature difference without loosing or gaining any heat from the surroundings.



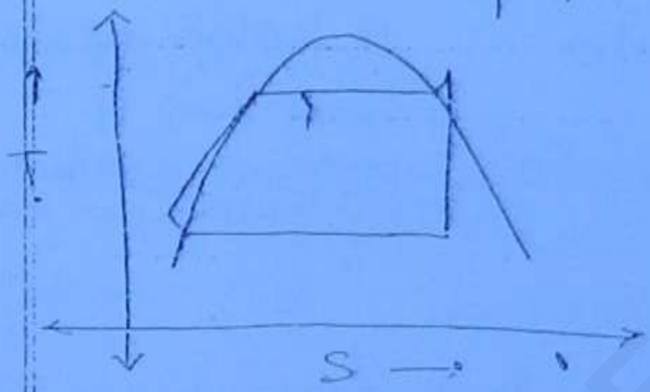
E.g. (i) Economiser (ii) Air Preheater (iii) Steam Condenser (iv) Radiator (iv) Steam Generators (v) Cooling Towers (vi) Evaporator.

(i) Economiser — Hot flue gases → Feed water.
(ii) Air Preheater — " → Air going for combustion
(iii) Steam Condenser — Steam → Cooling water
(iv) Radiator — Hot water → Atm. air
(v) Steam Generator — Hot flue gases → Steam or sat. water
(vi) Cooling Tower — Hot water → Atm. air.
(vii) Evaporator — Atm. air → Refrigerant.

16-Jan'18

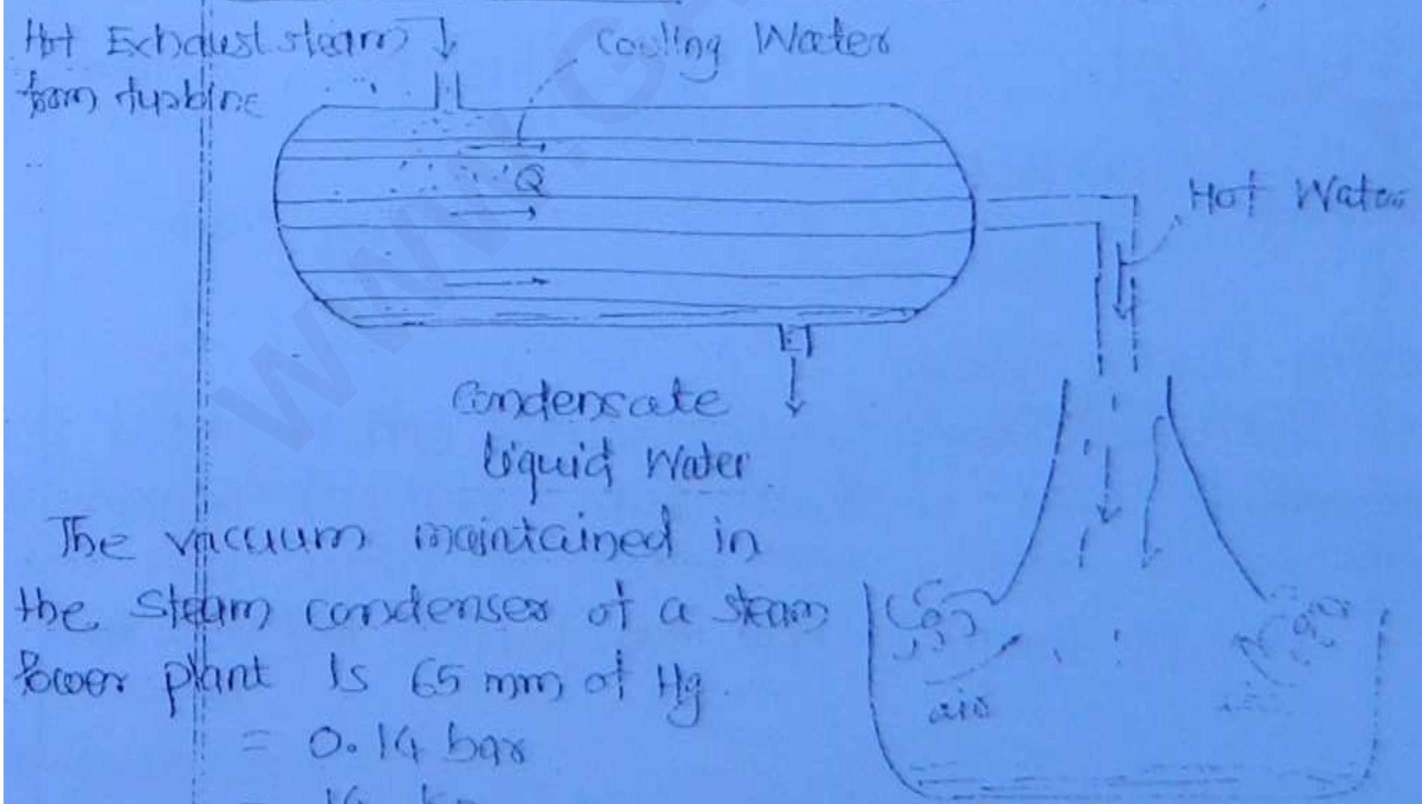


Power Plant



Ideal Rankine cycle

* Steam Condenser (Surface condenser)



The vacuum maintained in the steam condenser of a steam power plant is 65 mm of Hg

= 0.14 bar

= 14 Kpa

* Classification of Heat Exchangers

Direct contact Type

Both hot & cold fluids mix-up with each-other

i) Cooling Tower

ii) Jet condenser

Direct Transfer Type

Both hot & Cold fluids do not come into contact.

(i) Surface Condenser

(ii) Economiser

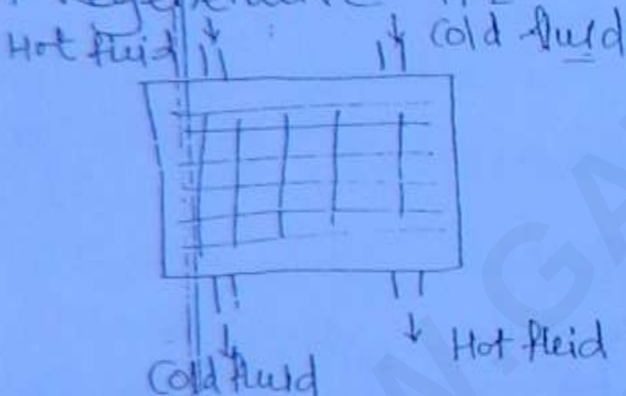
(iii) Air Preheater

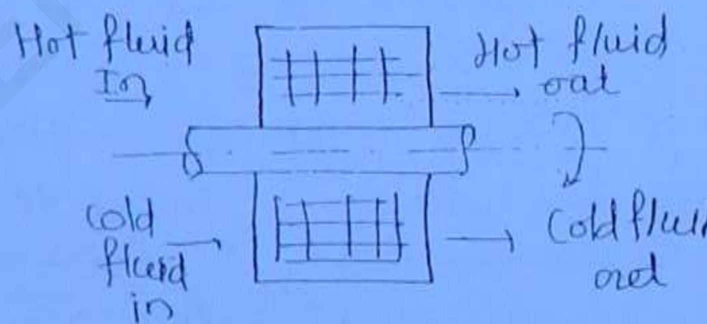
Regenerative Type.

Both fluids alternatively pass through the HE (i.e. high thermal capacity cellulose matrix) One heating it & other picking up heat from it.

e.g. Jungstorm air preheater used in gas turbine power plant.

* Regenerative H.E.





Rotating Matrix.

* Regarding Cooling Tower.

The lowest temp. to which water can be cooled in any cooling tower is wet bulb temp. of ingoing air.

Dew pt. Temp < Wet Bulb temp < Dry Bulb Temp.

* Direct Transfer type H.E.

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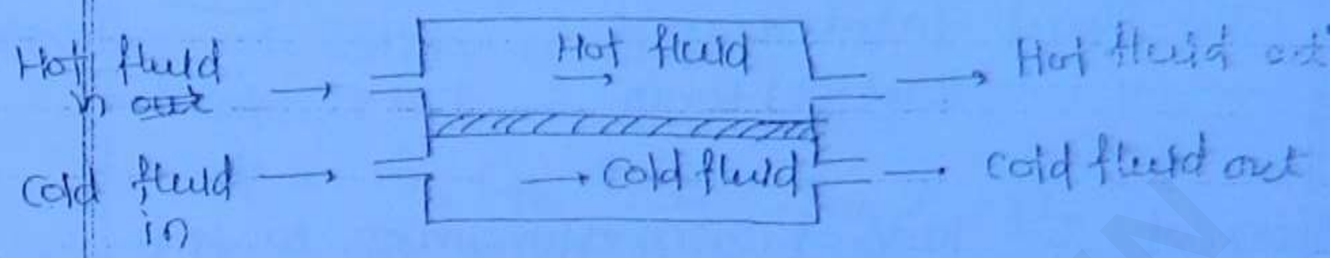
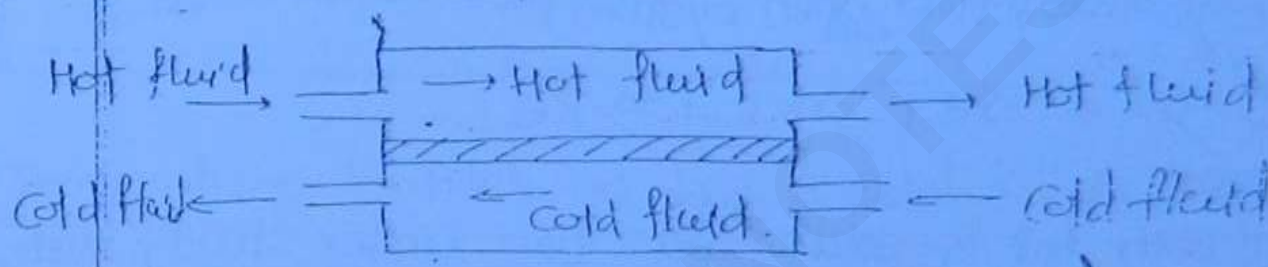
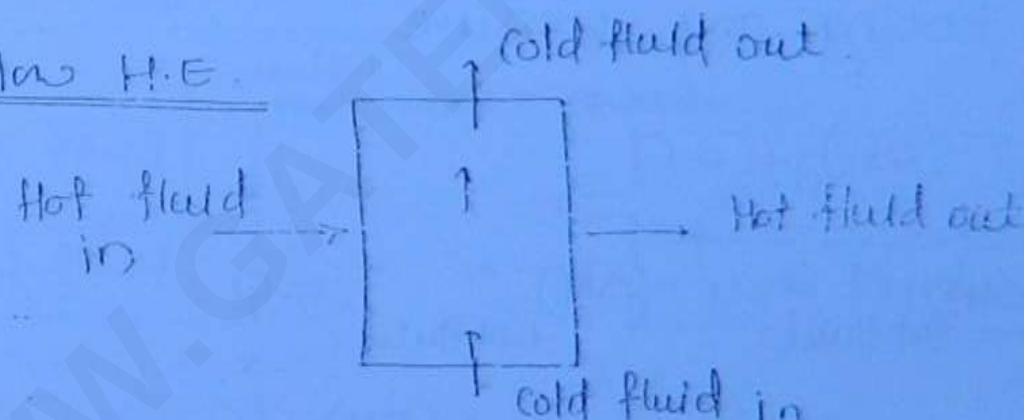
(i) Parallel Flow H.E.



(ii) Counter flow H.E.



(iii) Cross flow H.E.



The counter flow H.E. could transfer more heat as compared to the parallel flow H.E. for a given H.E. area (Generally among three counter flow has more heat transfer rate).

* Notations :-

$\dot{m}_h$ = Mass flow rate of hot fluid (kg/min)

$\dot{m}_c$ = ——— " ——— cold ——— " ———

C_{p_h} = Sp. heat of Hot fluid in KJ/kg·K

C_{p_c} = —— " —— cold —— " ——

T_{hi} = Hot fluid inlet temp.

T_{he} = —— " —— outlet (exit) —— " ——

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T_{ci} = Cold fluid inlet temp.

T_{ce} = —— " —— (outlet) (exit) temp.

Application of Ist law of thermodynamics to H·E.

* SFEE (Steady flow Energy Equation)

(Steady flow open system)

$$\cancel{Q}^{0} - \cancel{W}^{0} = \Delta H + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$$

Assumptions

(i) Both hot & cold fluids flow under steady state without loosing any pressure.

$$(\Delta H)_{H.E.} = 0$$

$$(\Delta H)_{\text{Hot fluid}} + (\Delta H)_{\text{cold fluid}} = 0$$

$$\Longrightarrow \quad -(\Delta H)_{\text{Hot fluid}} = +(\Delta H)_{\text{cold fluid}}$$

The rate of enthalpy decrease of hot fluid = Rate of enthalpy increase of cold fluid

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

** Also known as energy balance eqn or heat balance eqn.

From thermodynamic rule of

$$Q_{P=C} = \Delta H$$
(Isobaric)

We can say that the rate of H.T. b/w hot & cold fluids is equal to the rate of change of enthalpy of either of fluids.

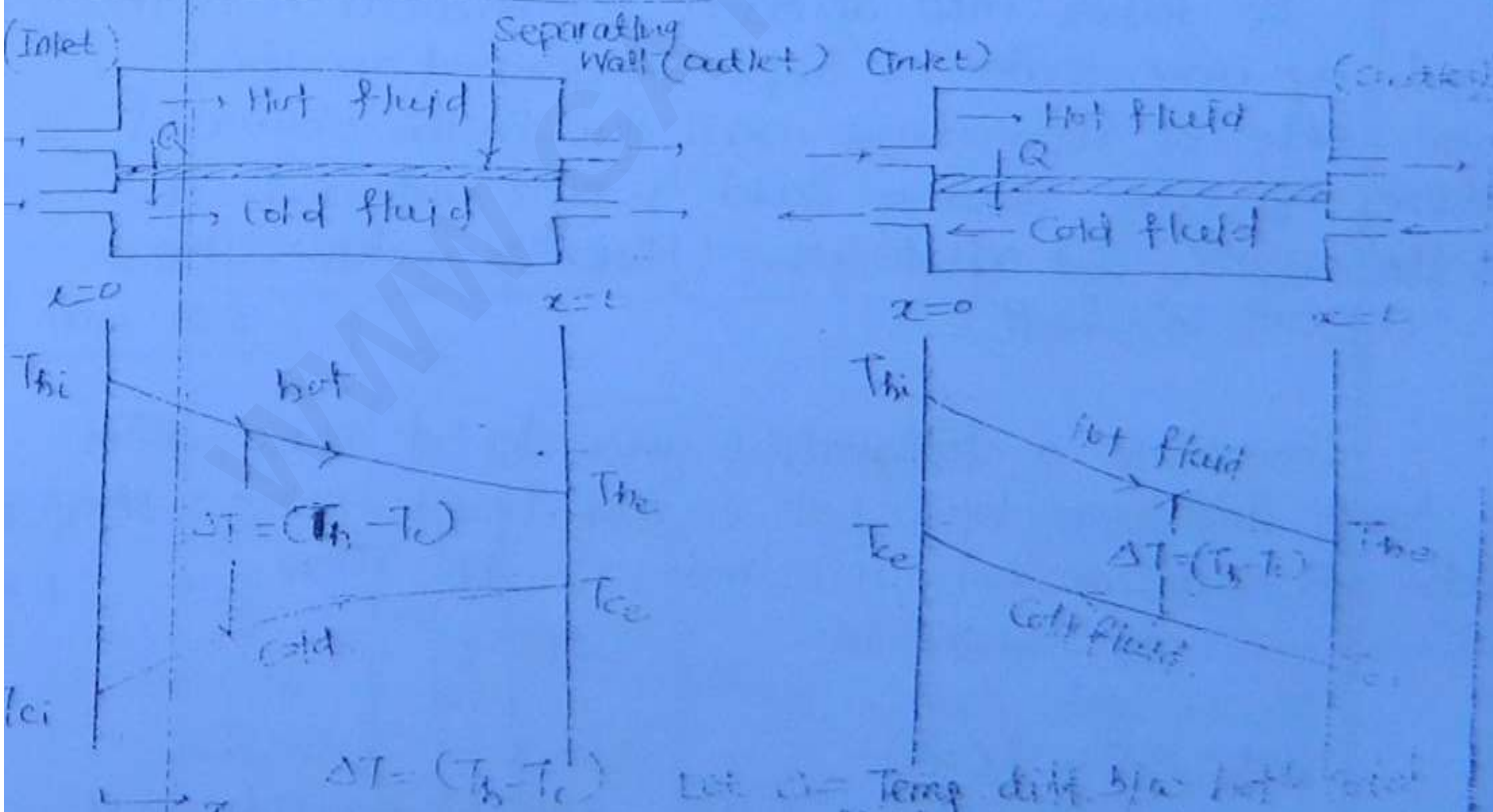
Rate of H.T.

$$\therefore Q = \dot{m}_h C_{p_h} (T_{hi} - T_{he}) \text{ or}$$

$$Q = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

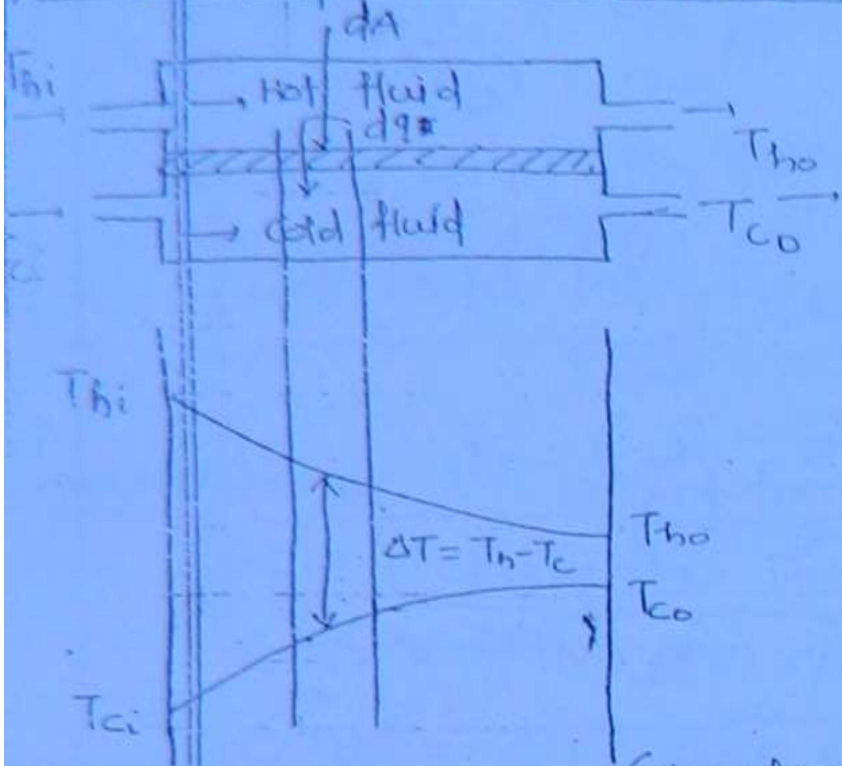
$$\boxed{C_{p_{water}} = 4.187 \text{ kJ/kg°C} \quad C_{p_{air}} = 1.005 \text{ kJ/kg°C}}$$

* Temperature Profiles of Hot & Cold Fluids in Parallel & Counter Flow H.E



Let ΔT = Temp diff b/w hot & cold fluid at any section at a dist x from inlet ΔT = (Th - Tc)

Mean Temperature Difference (MTD) [ΔT_m]



Let h_1 = Convection H.T coeff. on hot side.
h_2 = convection H.T. coeff on cold side.
Overall H.T coeff

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + F_1 + F_2$$

(Neglecting conducting resistance of pipe or wall)

where F_1 & F_2 are Fouling Factors.

Fouling Factor (F)

It takes into account the thermal resistance offered by any scale or deposite formed on the either side of separating wall due to the chemical reaction b/w the flowing fluid & pipe material. It has the units of $m^2K/Watt$. (Usually values are $F = 0.0002\ m^2K/watt$)

Consider a differential area dA of H.E, where the temp. difference b/w hot & cold fluids is ΔT through which the differential H.T. rate is dq. Then,

$$dq = U \Delta T \, dA$$

But $\Delta T = f(dA)$

$dA = B \cdot dx$ (B = width of separating wall $\perp^{r}$ to plane of fig.)

$dq = U B\,dx\ \Delta T$

By integrating, (87)

$$\int_{Inlet}^{Exit} dq = Q = \text{Total H.T. rate in the H.E.} = \int_{Inlet}^{Exit} U \Delta T\,dA$$

$$= U \int_{Inlet}^{Exit} \Delta T\,dA \quad \text{—— (i)}$$

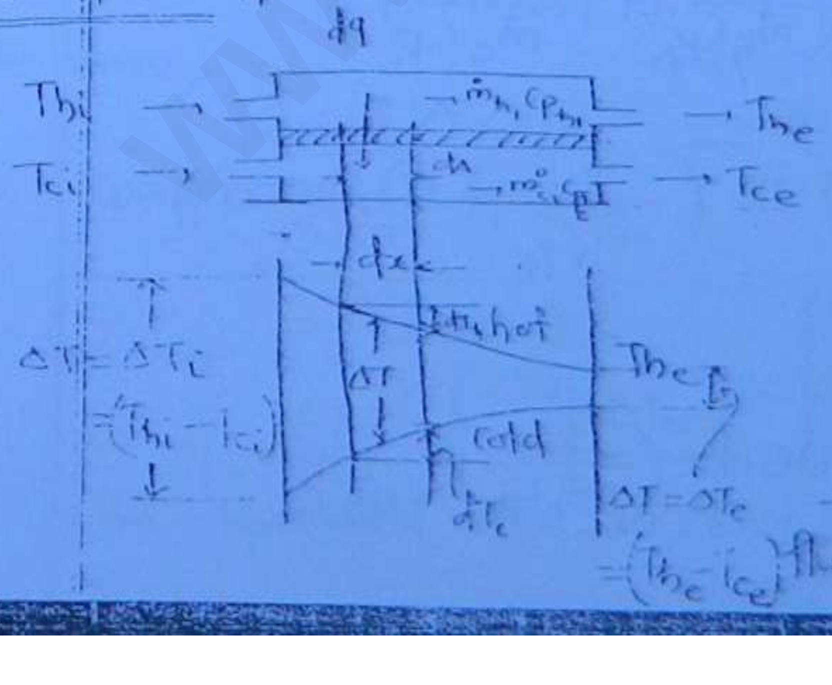
Now defining ΔT_m from the eqn

$Q = UA\,\Delta T_m$ —(ii), where A = Total H.T. area of H.E.

Comparing (i) & (ii)

$$\Delta T_m = \frac{1}{A}\int_{Inlet}^{Exit} \Delta T\,dA$$

** To derive Mean Temp. Diff. for a parallel flow H.E. *



Consider a diff. area of length dx & length dx through which H.T. rate is dq

Let, $\Delta T = T_h - T_c$

Temp diff. b/w hot & cold fluids at dA

$dq = U\Delta T\, dA$

$= U\Delta T\, B.dx$

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Also

$dq = -\dot{m}_h C_{p_h}\, dT_h$

$dq = +\dot{m}_c C_{p_c}\, dT_c$

At $x=0$ $\Delta T = \Delta T_i$

$= T_{hi} - T_{ci}$

At $x=L$ $\Delta T = \Delta T_e$

$= T_{he} - T_{ce}$

let $\Delta T = T_h - T_c$

$d(\Delta T) = dT_h - dT_c$

$$= -\frac{dq}{\dot{m}_h C_{p_h}} - \frac{dq}{\dot{m}_c C_{p_c}}$$

$$= -dq\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$d(\Delta T) = -U\Delta T\, B\, dx\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$\frac{d(\Delta T)}{\Delta T} = -U B\, dx\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L -UB\, dx\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$\therefore -\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L UB\, dx\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UB\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)L$$

But $(B \times L)$ = Total H.T. area of H.E.

$$\therefore \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

Energy Balance Eq^n

$$Q = \dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$$\frac{1}{\dot{m}_h C_{p_h}} = \left(\frac{T_{hi} - T_{he}}{Q}\right), \quad \frac{1}{\dot{m}_c C_{p_c}} = \left(\frac{T_{ce} - T_{ci}}{Q}\right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q}\right)$$

$$= \frac{UA}{Q}\left[(T_{hi} - T_{ci}) - (T_{he} - T_{ce})\right]$$

$$\therefore Q = \frac{UA\left[\Delta T_i - \Delta T_e\right]}{\ln(\Delta T_i / \Delta T_e)} \quad \text{——(i)}$$

We have,

$$Q = UA\,\Delta T_m \quad \text{——(ii)}$$

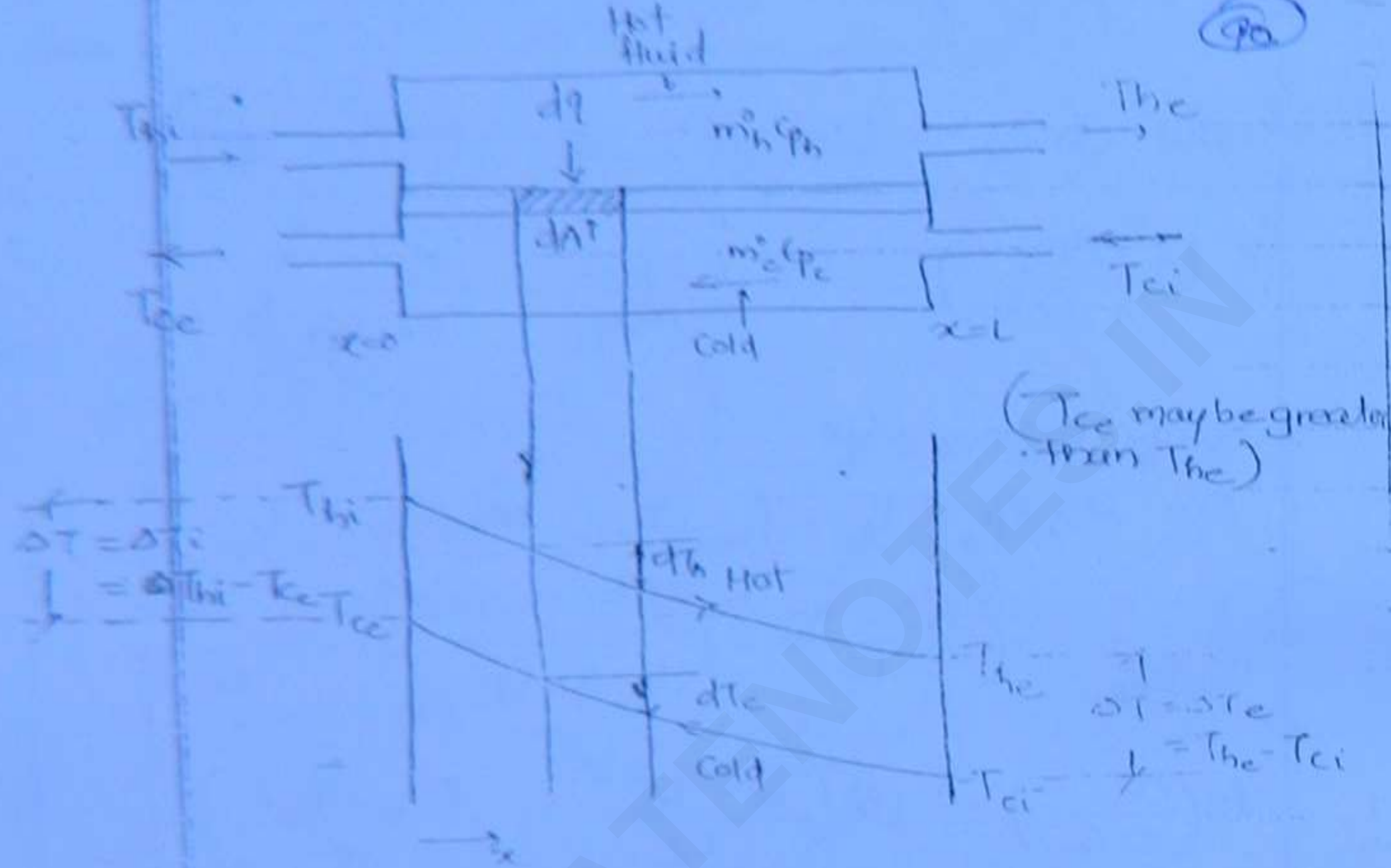
Comparing (i) & (ii)

$$\boxed{\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}}$$

This is called as LMTD (Log mean temp. diff.)

LMTD For Counter Flow Heat Exchangers

(90)



Consider a differential area dA of length dx through which the differential H.T. rate b/w hot & cold fluid is dq

Let $\Delta T = T_h - T_c = f(x)$

At $x=0$ $\Delta T = \Delta T_i = T_{hi} - T_{ce}$

At $x=L$ $\Delta T = \Delta T_e = T_{he} - T_{ci}$

Then $dq = U \Delta T dA$

$= U \Delta T\, B\, dx$

Also $dq = -\dot{m}_h c_{p_h} dT_h$

$" \quad " = -\dot{m}_c c_{p_c} dT_c$

Let $\Delta T = T_h - T_c$

$$d(\Delta T) = dT_h - dT_c$$

$$= -\frac{dq}{\dot{m}_h C_{p_h}} + \frac{dq}{\dot{m}_c C_{p_c}} \qquad (9)$$

$$= -dq\left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$d(\Delta T) = -U\Delta T\, B.dx \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$-\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L UB\,dx \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}}\right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UB\left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}}\right)L$$

$\because$ $B \times L = A$ (Area of H.T)

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}}\right) \quad \text{--- (i)}$$

We have,

From Energy Balance eq^n.

$$Q = \dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$$\frac{1}{\dot{m}_h C_{p_h}} = \left(\frac{T_{hi} - T_{he}}{Q}\right) \; ; \; \frac{1}{\dot{m}_c C_{p_c}} = \left(\frac{T_{ce} - T_{ci}}{Q}\right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left[\left(\frac{T_{hi}-T_{he}}{Q}\right) - \left(\frac{T_{ce}-T_{ci}}{Q}\right)\right]$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = \frac{1}{Q}(UA)\left[(T_{hi}-T_{ce})-(T_{he}-T_{ci})\right]$$

$$Q = \frac{UA\left[\Delta T_i - \Delta T_e\right]}{\ln(\Delta T_i/\Delta T_e)} \quad \text{——(ii)}$$

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We have $Q = UA\Delta T_m$ (iii) Comparing (ii) & (iii)

$$\boxed{\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i/\Delta T_e)}}$$

~~ΔTe~~ This is eqn of LMTD

Eventhough the formula for LMTD is same in both parallel flow & counter flow H.E. The defn of ΔT_i & ΔT_e are different b/w them

For the same inlet & exit temp of hot & cold fluids and for the same mass flow rate of the same fluid, the counter flow H.E. would occupy lesser Heat Transfer Area. (It is more compact as compared to parallel flow H.E.)

For the same H.T. area provided in both parallel flow & counter flow modes, the counter flow H.E. can transfer higher H.T. rate than parallel ~~flow~~ H.E. i.e. hot fluid exit temp lesser in counter flow

Two Special Cases.

(Case-I) ⇒ When one of the fluids is undergoing change of phase like in steam condenser or evaporator ; then temperature profile will like ⇒

Example ⇒ Steam Condenser.

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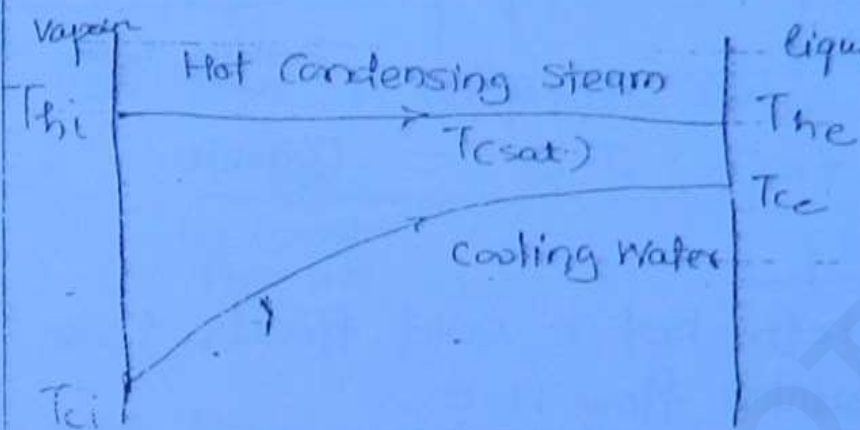
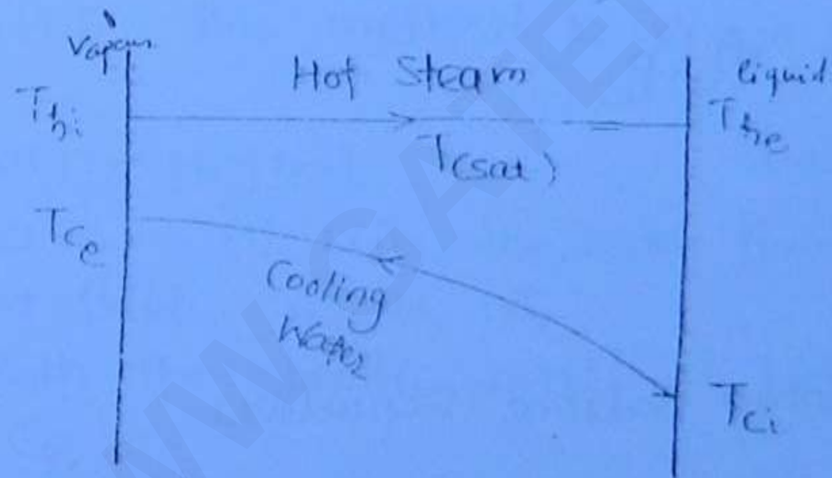


(Parallel flow H.E.)

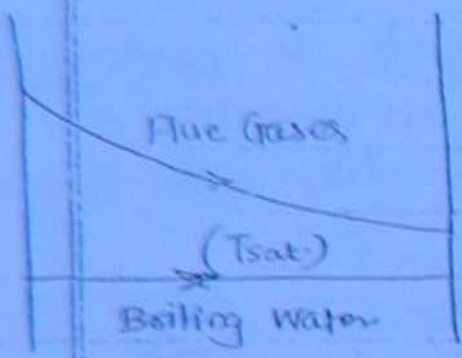


(Counter flow H.E.)

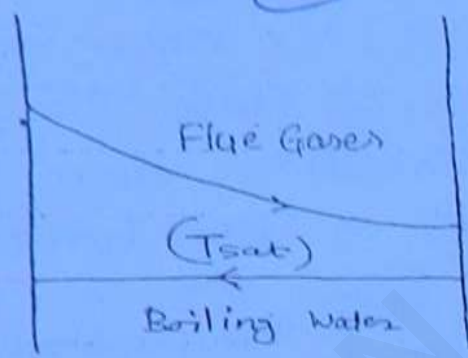
In this case $(\Delta T_m)_{\substack{\parallel el \\ H.E}} = (\Delta T_m)_{\substack{\text{Counter flow} \\ H.E}}$

Hence it is immaterial, what type of H.E is to be designed.

In case of (evaporator)/Steam Generator



Parallel



Counter

(Case II) ⇒ When both hot & cold fluids have equal capacity rate in counter flow H.E

Capacity rate $= (\dot{m} \times C_p)$

i.e.

$$\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$$

(kg/s) (J/kg K)

⇓

(Watt/K)

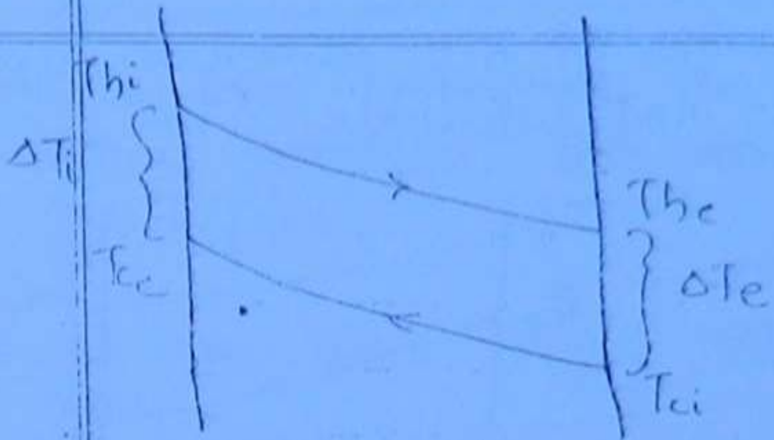
Then from Energy Balance Equation

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$T_{hi} - T_{he} = T_{ce} - T_{ci}$$

$$T_{hi} - T_{ce} = T_{he} - T_{ci}$$

$$\boxed{\Delta T_i = \Delta T_e}$$



But from L' Hospital's Rule

$$\Delta T_m = \Delta T_i \ (\text{or}) \ \Delta T_e$$

For counter flow H.E &

al

Design of Heat Exchangers

Here the objective is to calculate the H.T area of H.E. This method is known as LMTD method

LMTD Method

Given data ⇒ (i) Both the mass flow rates of hot and cold fluids $\dot{m}_h$ & $\dot{m}_c$

(ii) Both the specific heats of hot & cold fluids c_{p_h} & c_{p_c}

(iii) Overall Heat Transfer coeff. (U) in W/m²K

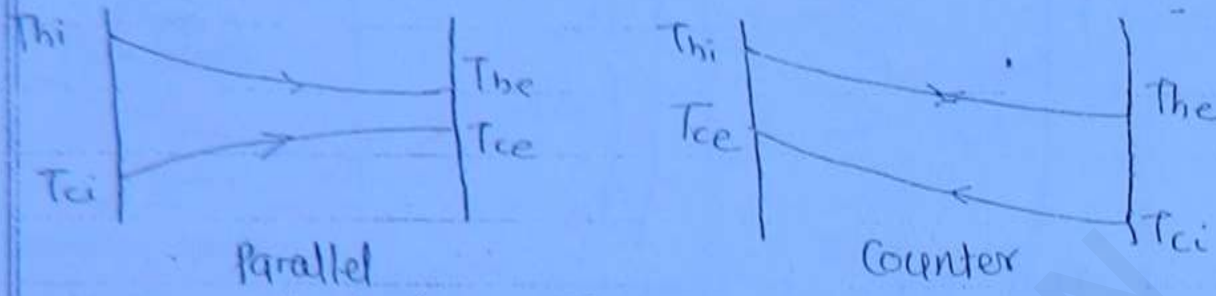
(iv) Only 3 Temps among four temperatures (T_{hi}, T_{ci}, T_{he}, T_{ce})

To find ⇒ Area (A) of H.T. in H.E. = ?

Step (i) Calculate the 4th unknown temperature from energy balance equation

i.e. $\dot{m}_h c_{p_h} (T_{hi} - T_{he}) = \dot{m}_c c_{p_c} (T_{ce} - T_{ci})$

Step (ii) Draw the temp. profiles of H.E. based on type of H.E to be designed. And calculate ΔT_m



Hence calculate ΔT_m

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

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Step (iii) obtain H.T. rate Q b/w hot & cold fluid

$$Q = \dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

J/kg k

Watts

Step (iv) Then obtain the area of H.t from

$$Q = UA\Delta T_m$$

$$\therefore \left(A = \frac{Q}{U \Delta T_m} \right) m^2$$

IES1999

Light lubricant oil $C_p = 2090$ J/kg K. is cooled by allowing it to exchange energy to with water in a small H.E The oil enters & leaves the H.E at 375 K & 350 K respectively. And flows at rate of 0.5 kg/s. Water at 280 K is available in sufficient quantity to allow 0.201 kg/s. to be used for cooling surface. Determine the require H.T. area for counterflow operation. The overall H.T. coeff. is taken as 250 W/m²K.

(97)

⟹ We have energy balance eqⁿ.

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$(0.5)(2090)(375-350) = (0.201)(4.187\times10^3)(T_{ce} - 280)$$

$$26125 = 841.587\,(T_{ce} - 280)$$

$$\therefore\ T_{ce} = 311.043\ ^\circ K$$

Now

$$\Delta T_m = \frac{63.957 - 70}{\ln(63.957/70)}$$

$$\Delta T_m = 67\ K$$

$$Q = (0.5)(2090)(375-350)$$

$$= 26125\ \text{Watts}$$

$$A = \frac{26125}{250 \times 67} = 1.56\ m^2$$

In a condenser water enters at 30°c & flows at the rate of 1500 kg/hr. The condensing steam at temp. of 120°c & cooling water leaves the condenser at 80°c $C_{p_{water}} = 4.187$ KJ/kgK. If $U = 2000\ W/m^2K$. The H.T. area is

(i) $0.707\ m^2$ (ii) $7.07\ m^2$ (iii) $70.7\ m^2$ (iv) $141.0\ m^2$

$\Longrightarrow$

$$\frac{1500}{3600} \times 4.187 \times 10^3 \times (80 - 30) = Q$$

87.229 KW

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)} = \frac{90 - 40}{\log_e(90/40)}$$

$\Delta T_m = 61.657\ °c$

$\therefore\ 87.229 = U A \Delta T_m$

$\therefore\ 87.229 \times 10^3 = 2000 \times A \times 61.657$

$\therefore\ \boxed{A = 0.707\ m^2}$

GATE 2008

Counterflow

The LMTD of H.E. is 20°C. The cold fluid enters at 20°C and hot fluid enters at 100°C. Mass flow rate of cold fluid is twice that of hot fluid, sp. heat at const. pressure of hot fluid is twice that of cold fluid. The exit temp. of cold fluid is ⟹

a] 40°C b] 60°C c] 80°C d] Can't be determined

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$\Delta T_m = 20°$

$T_{ci} = 20°C$

$T_{hi} = 100°C$

$\dot{m}_c = 2\,\dot{m}_h$

$C_{p_h} = 2\,C_{p_c}$

$\dot{m}_c C_{p_c} = \dot{m}_h C_{p_h}$

Equal capacity rates on hot side & cold side

$\therefore \Delta T_m = \Delta T_i$ or ΔT_e

We have,

$$\dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$$\therefore \cancel{100 - T_{he} = T_{ce} - 20}$$

~~Mass flow rate~~

By applying L' Hospital's Rule

$$\Delta T_m = \Delta T_i \text{ or } \Delta T_e$$

$$\therefore 20 = T_{hi} - T_{ce}$$

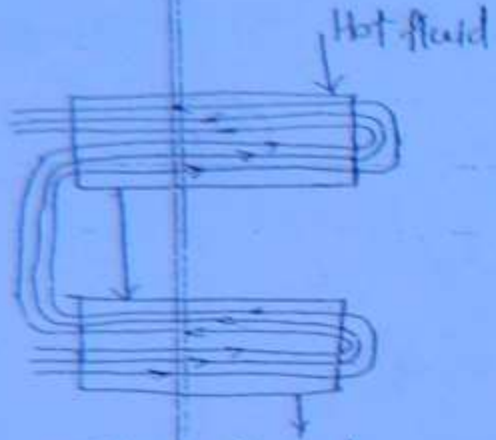
$$\therefore 20 = 100 - T_{ce}$$

$$\therefore \boxed{T_{ce} = 80°C}$$

* Shell & Tube Heat Exchanger

Single Shell Pass & Single Tube Pass

2 Tube Pass & single shell Pass H.E.



2 Shell Pass & 4 Tube Pass H.E.

100

In this cases

$$\Delta T_m = \Delta T_m \times F$$

(LMTD) $\quad$ Counterflow

F = Correction Factor.

IES 2009

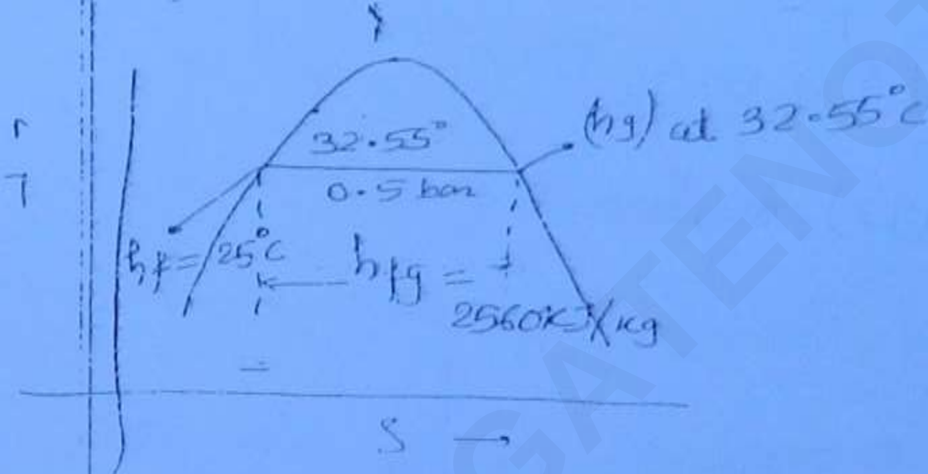
Find the surface area required, for a surface condenser dealing with a 25000 kg of saturated steam per hour at a pressure of 0.5 bar. Temperature of condensate water is 25°c. Cooling water is heated from 15°C to 25°C while passing through the condenser. Assume H.T. coeff of 10 KW/m²K. The condenser has two water passes with a tubes of 19 mm o.d. & 1.2 mm thickness. Find the length & no. of tubes per pass. Assume velocity of water is 1 m/s. Assume correction factor for 2 tube pass exchanger as 0.86. At 0.5 bar, saturation temp is 32.55°c and latent heat is 2560 kJ/kg. $C_{p_{water}} = 4.187$ kJ/kg K $\rho = 1000$ kg/m³

$\Longrightarrow$ $\dot{m}_h = 25000/3600 = 6.94$ kg/s

$T_{c_i} = 15°c$ $T_{c_e} = 25°c$

$U = 10 \times 10^3$ W/m²K

(101)



for 1 kg of steam, the heat rejected by steam

$= (\Delta h)$ kJ/kg $= (h_g)_{at\ 32.55°c} - (h_f)_{at\ 25°c}$

$T_{(sat)}$	h_f (kJ/kg)
32.55°c	4.187 × 32.55
25°c	4.187 × 25

$\Delta h = \cancel{136.29} - \cancel{104.68}$

$\therefore \Delta h = \cancel{31.335}$ kJ/kg

$(h_g)_{32.55} = (h_f + h_{fg})$

$= 2696$

When there is change of phase, then

$$\boxed{Q = \dot{m}_{steam} \times \text{Heat Rejected per kg}}$$

102

$\therefore$ Heat Rejected per kg

$$(\Delta h) = (2696 - 104.6)$$
$$= 2591.4 \text{ KJ/kg}$$

$$\therefore Q = 2591.4 \times 6.94$$
$$= (17997) \text{ KJ/sec}$$

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

$$=$$

$$= 8.75°C$$

$17997 \times 10^3 = 10 \times 10^3 (A)(8.75) \times 0.86$

$\therefore \quad A = 240 \ m^2$

$\therefore \quad A = \pi d L \times n \times p$ (103)

P = No. of Pass
n = no. of tubes per pass
d = dia. of tube
L = length of tube

$\dot{m}_\omega = \rho A v n$ $\quad$ A = c/s area

$= \rho \frac{\pi}{4} d_i^2 \times \bar{V} \times n$

$\dot{m}_\omega = 1000 \times \frac{\pi}{4} \times (16.6)^2 \times 1 \times n$

We have $\quad Q = \dot{m}_\omega \times 4.187 \times 10^3 \times (25-15)$

$17997 = \dot{m}_\omega \times$

$\dot{m}_\omega = 429.830 \ kg/sec$

$429.83 = 1000 \times \frac{\pi}{4} (16.6)^2 \times 1 \times n$

$n = 1986$

$240 = \pi \times 19 \times 10^{-3} \times L \times 1986 \times 2$

$L = 1.01 \ m.$

Effectiveness of H.E. (ϵ)

It is defined as the actual H.T. rate b/w hot & cold fluids & the maximum possible H.T. rate b/w them.

(104)

$$\epsilon_{HE} = \frac{Q_{actual}}{Q_{max}}$$

$$Q_{actual} = \dot{m}_h C_{p_h} (T_{hi} - T_{he})$$
$$= \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

Q_{max} = Maximum possible H.T. rate

$$= (\dot{m} C_p)_{small} \times (T_{hi} - T_{ci})$$

where $(\dot{m} C_p)_{small}$ is smaller capacity rate b/w $\dot{m}_h C_{p_h}$ & $\dot{m}_c C_{p_c}$.

If ~~$\dot{m}_h$~~ $\dot{m}_h C_{p_h} < \dot{m}_c C_{p_c}$

Then, $$\epsilon_{H.E.} = \frac{\dot{m}_h C_{p_h} (T_{hi} - T_{he})}{\dot{m}_h C_{p_h} (T_{hi} - T_{ci})}$$

$$\boxed{\epsilon_{H.E.} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}}$$

& If $\dot{m}_c C_{p_c} < \dot{m}_h C_{p_h}$

Then, $$\epsilon_{H.E.} = \frac{\dot{m}_c C_{p_c} (T_{ce} - T_{ci})}{\dot{m}_c C_{p_c} (T_{hi} - T_{ci})}$$

$$\boxed{\epsilon_{H.E.} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}}$$

Number of Transfer Units (NTU)

It is defined as the ratio b/w product of UA and the smaller capacity rate b/w hot & cold fluid.

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}}$$

$U \Rightarrow W/m^2K$ 105

$C_p \Rightarrow J/kg.K$

= No units.

Since NTU being proportional to the area of H.E., it signifies the overall size of H.E.

* Capacity Rate Ratio (C)

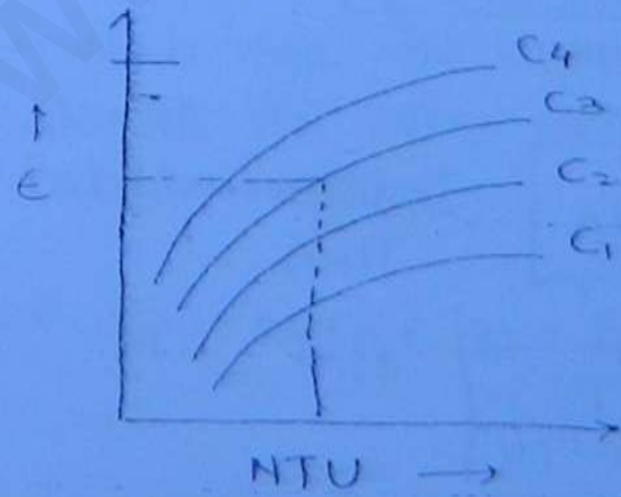
$$C = \frac{(\dot{m}C_p)_{small}}{(\dot{m}C_p)_{Big}}$$

$\epsilon < 1$ $\quad 0 \leq C \leq 1$ $\quad$ NTU > 1 or NTU < 1

C = 0 if one of the fluids change its phase like steam condenser or evaporator.

For any heat exchanger

$$\epsilon = f(NTU, C)$$



Various capacity rate ratio

For parallel flow heat exchanger

$$\epsilon_{Parallel} = \frac{1-e^{-(1+C)NTU}}{1+C}$$

For counter flow heat exchanger

$$\epsilon = \frac{1-e^{-(1-C)NTU}}{1-Ce^{-(1-C)NTU}}$$

When $C=0$ (When one of the fluids undergoes phase change)

$$\boxed{\epsilon_{Parallel} = 1-e^{-NTU}}$$

$$\boxed{\epsilon_{counter} = 1-e^{-NTU}}$$

When $C=1$ (Equal capacity rate)
i.e. $\dot{m}_c c_{p_c} = \dot{m}_h c_{p_h}$

$$\boxed{\epsilon_{Parallel} = \frac{1-e^{-2NTU}}{2}}$$

$$\boxed{\epsilon_{counter} = 0/0}$$ Indeterminate

Then

$$_{counter} = \frac{NTU}{1+NTU}$$

Effectiveness - NTU Method.

This method is adopted whenever both the exit temperatures of hot & cold fluids T_{he} & T_{ce} are unknown for a given designed heat exchanger area A.

Given data (i) Both the mass flow rates of hot & cold fluids ($\dot{m}_h$ & $\dot{m}_c$)

(ii) Both the specific heats C_{p_h} & C_{p_c}

(iii) Overall H.T. coefficient (U) in W/m²K

(iv) Only two inlet temperatures (T_{hi} & T_{ci})

(v) Area of heat exchanger (A)

To find T_{he} & T_{c_e} = ?

Step(i) Calculate both the capacity rates of hot & cold fluid i.e. $\dot{m}_h C_{p_h}$ & $\dot{m}_c C_{p_c}$. Hence get capacity rate ratio (C).

Step(ii) Calculate NTU from UA & $(\dot{m}C_p)$small. Keep C_p in J/kg.K

Step(iii) Calculate the effectiveness of heat exchanger since it is a function of NTU & C

$$\epsilon = f^n(NTU, C)$$

Step(iv) Calculate only one exit temp. of the H.E. based on which side the capacity rate is smaller.

Step(v) Calculate the other exit temp. based upon energy balance eqn.

i.e. $Q = \dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{c_e} - T_{ci})$

** Effectiveness - NTU method can be used for evaluating the area of heat exchanger with the same data given in the LMTD method. But LMTD method can't be used for evaluating both the exit temperatures with the data given in ϵ-NTU method. **

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IES 2005

A process energy industry employs a counter flow heat exchanger to cool 0.8 kg/sec of oil ($C_p = 2.5$ kJ/kg.K) from 140°C to 40°C by the use of water entering at 20°C. The overall H.T coeff. is estimated to be 1600 W/m²K. It is assumed that the exit temp. of water will not exceed 80°C. Using NTU method & NTU = 4 in this case. Calculate the following

(i) Mass flow rate of water.
(ii) Surface area required.
(iii) Effectiveness of Heat Exchanger.

$\longrightarrow$

$C_{p_h} = 2.5 \times 10^3$ J/kg.K

$\dot{m}_h = 0.8$ kg/sec

$T_{h_i} = 140$°C $\quad T_{h_e} = 40$°C

$T_{c_i} = 20$°C $\quad U = 1600$ W/m²K

$T_{c_e} = 80$°C $\quad NTU = 4 \quad C_{p_c} = 4.187$ kJ/kg.K

capacity heat ratio

$$\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$$

Energy balance eqn

(109)

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$0.8 \times 2.5 \times 10^3 (120 - 40) = \dot{m}_c \, 4.187 \times 10^3 (80 - 20)$$

$$\therefore \dot{m}_c = \cancel{0.78 \text{ kg/s}} \quad 0.64 \text{ kg/s}$$

& $\dot{m}_h C_{p_h} (140 - 40) = \dot{m}_c C_{p_c} (80 - 20)$

$$\therefore \frac{\dot{m}_h C_{p_h}}{\dot{m}_c C_{p_c}} = \frac{60}{100} = \frac{3}{4} < 1$$

$$\dot{m}_h C_{p_h} < \dot{m}_c C_{p_c}$$

capacity rate is smaller on hot side

$$\therefore \epsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

$$= \frac{120 - 40}{120 - 20}$$

$$= 0.8$$

$$NTU = 4 = \frac{UA}{(\dot{m} C_p)_{small}} = \frac{UA}{C_{p_h} \dot{m}_h}$$

$$4 = \frac{1600 \times A}{2.5 \times 10^3 \times 0.8} \quad \therefore A = 5 \text{ m}^2$$

Date
Page

GATE 08

In a parallel flow H.E operating under steady state. The heat capacity rates of hot & cold fluids are equal. The hot fluid flowing at 1 kg/s with $C_p = 4$ kJ/kg.K enters the exchanger at the 102°C while the cold fluid has inlet temp. of 15°C. The overall H.T. coeff. for the H.E. is estimated to be 1 kW/m²K & the corresponding heat transfer surface area is 5 m². Neglect H.T. b/w H.E & the ambient. The exit temp in °C for the cold fluid, is

a] 45°C b] 55 c] 65 d] 75

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$C = 1$ Capacity rate ratio.

For parallel flow H.E.

$$\epsilon = \frac{1 - e^{-2(NTU)}}{2}$$

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}}$$

$$= \frac{1000 \times 5}{1 \times 4 \times 10^3}$$

$$= 1.25$$

$$\epsilon = \frac{1 - e^{-2(1.25)}}{2}$$

$$\therefore \epsilon = 0.458$$

$$0.458 = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} = \frac{T_{ce} - 15}{102 - 15} \quad \boxed{T_{ce} = 54.84}$$

|IES| 1998 | (25 Marks)

In a counterflow double pipe H.E. water flows through a copper tube (19 mm outer dia. & 16 mm inner dia.). At a flow rate of 1.48 m/s the oil flows through the annulus formed by inner copper tube & outer steel tube (30 mm O.D. & 26 mm I.D.). The steel tube is insulated from outside. The oil enters at the 0.4 kg/sec & is cooled from 65°C to 50°C whereas water enters at 32°C. Neglecting the resistance of copper tube, calculate the length of the tube required.

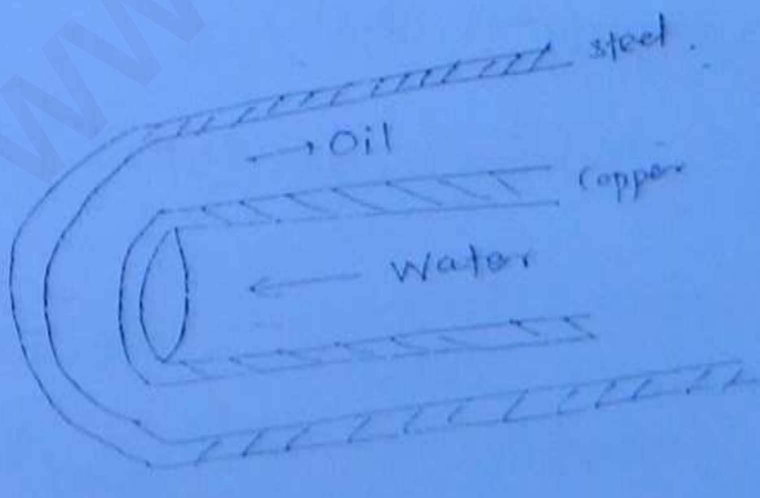
Data given ⇒ Fouling factor on water side = 0.0005 m²K/W

fouling factor on oil side = 0.0008 m²K/W

$Nu = 0.023\, Re^{0.8}\, Pr^{0.3}$ ⇒ Mc Adam's Eqn For turbulent flow through pipe

Property	Oil	Water
ρ (kg/m³)	850	995
C_p (kJ/kgK)	1.89	4.187
K (W/mK)	0.138	0.615
ν (m²/s)	7.44×10^{-6}	4.18×10^{-7}

1/1



$$Nu = 0.023\, Re^{0.8}\, P_r^{0.3}$$

$$\frac{hD}{k} = (0.023)\left(\frac{VD}{\nu}\right)^{0.8}\left(\frac{\mu C_p}{k}\right)^{0.3}$$

* To calculate $\left(h_{waterside}\right)$ (112)

$$Re_{water} = \frac{1.48 \times 16 \times 10^{-3}}{4.18 \times 10^{-7}}$$

$$= 56650.717 \text{ (Flow is turbulent)}$$

$$P_{r\,water} = \frac{\mu C_p}{k}$$

$$= \frac{\nu \times \rho \times C_p}{k} \quad \left(\frac{\frac{kg}{m\,sec} \times J/kgK}{W/mK}\right)$$

$$= \frac{995 \times 4.18 \times 10^{-7} \times 4.187 \times 10^{3}}{0.615}$$

$$= 2.83$$

(P_r no. is property of fluid)

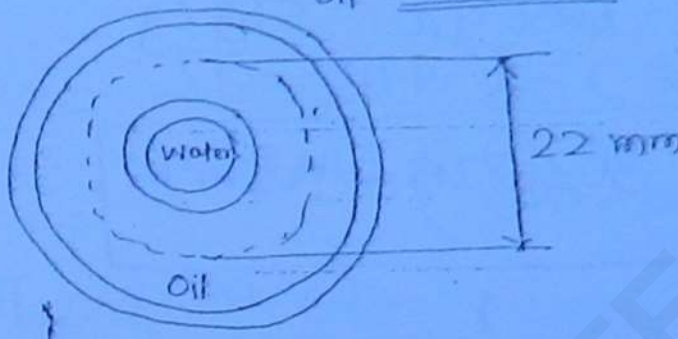
$$\frac{h \times 16 \times 10^{-3}}{0.615} = (0.023)(56650.717)^{0.8}(2.83)^{0.3}$$

$$h_{waterside} = 7666.176 \ W/m^2K$$

* To calculate ($h_{oilside}$)

$$Re_{oil} = \frac{VD}{\nu} \qquad \vec{V}_{oil} = \frac{\dot{m}_{oil}}{\frac{\pi}{4}\left[\left(\frac{26}{1000}\right)^2 - \left(\frac{19}{1000}\right)^2\right] \times 850}$$

$$\vec{V}_{oil} = \underline{1.9 \text{ m/s}}$$



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$$\therefore Re_{oil} = \frac{\vec{V} D}{\nu} = \frac{1.9 \times 22 \times 10^{-3}}{7.44 \times 10^{-6}}$$

$$= 5618 \quad (\text{Turbulent flow})$$

$$\boxed{Re = \frac{\text{Inertia force}}{\text{Viscous force}}} \qquad (Re < 2000 \text{ laminar})$$

$$Pr_{oil} = \frac{\mu C_p}{k}$$

$$= \frac{\nu \times \rho \times C_p}{k} = \frac{7.44 \times 10^{-6} \times 850 \times 1.89 \times 10^3}{0.138}$$

$$= 86.611$$

$$\frac{h \times 22 \times 10^{-3}}{0.138} = (0.023)(5618)^{0.8}(86.611)^{0.3}$$

$$\therefore h_{oilside} = 549.69 \text{ W/m}^2\text{K}$$

Now

$$\frac{1}{U} = \frac{1}{h_{waterside}} + \frac{1}{h_{oilside}} + F_{waterside} + F_{oilside}$$

$$= \frac{1}{7666.176} + \frac{1}{549.69} + 0.0005 + 0.0008$$

$$\boxed{U = 307.725 \ W/m^2K}$$

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Energy balance eq^n

$$\dot{m}_h Cp_h (T_{hi} - T_{he}) = \dot{m}_c Cp_c (T_{ce} - T_{ci})$$

$$0.4 \times 1.89 \times (65 - 50) = \left[\rho \frac{\pi}{4} (d_i)^2_{copper} \times V \right] \times 4.187 \times 10^3 (T_{ce} - 32)$$

$$11.34 \times 10^3 = 1239.702 (T_{ce} - 32)$$

$$\boxed{T_{ce} = 41.15^\circ C}$$

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

$$= \frac{(T_{hi} - T_{ce}) - (T_{he} - T_{ci})}{\ln\left(\frac{T_{hi} - T_{ce}}{T_{he} - T_{ci}}\right)}$$

$$= \frac{(65 - 41.15) - (50 - 32)}{\ln\left(\frac{65 - 41.15}{50 - 32}\right)}$$

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$$\Delta T_m = 20.7^\circ C$$

$$Q = UA\,\Delta T_m$$

$\cancel{Q = \dot m}$ $\quad Q = \dot m_h C_{ph} (T_{hi} - T_{he})$

$$= 11.34 \times 10^3$$

$$\therefore 11.34 \times 10^3 = 307.725 \times A \times 20.7$$

$$\therefore A = 1.78 \ m^2$$

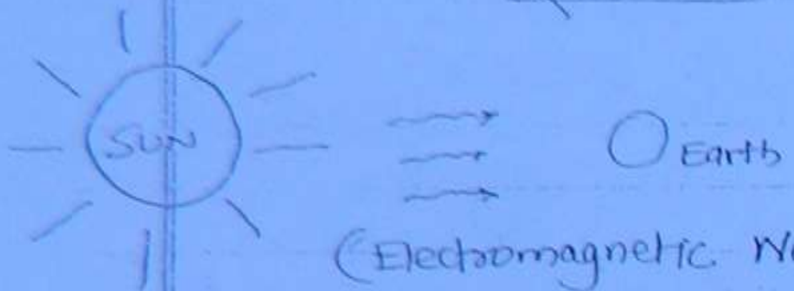
$$\pi D_o L = 1.78 \ m^2$$

(D_o ↑ copper)

$$L = \frac{1.78}{\pi \times 19 \times 10^{-3}} = 29.820 \ m$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{k_1}\left(\ln r_o / r_i\right) + \frac{r_i}{r_o}\frac{1}{h_o}$$

Radiation



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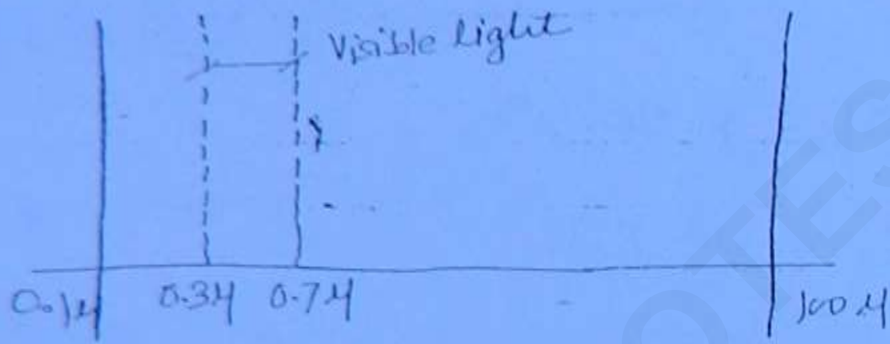
(Electromagnetic Waves)



Spectrum of Thermal Radiation

All bodies at all temperatures emit thermal radiation, the rate of emission or the radiation energy coming from the surface of a body per unit time & per unit area being directly proportional to the fourth power of the absolute temp. of body. Hence radiation completely pre-dominates over conduction & convection particularly when, the temp. diff. is quite high. e.g. Heat transfer from hot flue gases to the refractory wall in a large furnace is predominantly by radiation.

All high temperature bodies like sun emit radiation at the short wavelengths, whereas any earth bound body at long wavelengths.

Basic Defn

(1) Total hemispherical Emissive Power (E)

It is defined as the radiation energy emitted from a surface of the body per unit time per unit area in all hemispherical directions at all wavelength

$$E \Rightarrow \frac{J}{sec \cdot m^2} = \frac{W}{m^2}$$

(117)

(2) Total emissivity (ϵ)

It is defined as the ratio b/w total hemispherical emissive power of a non-black surface and total hemispherical emissive power of black surface both being at the same temperature.

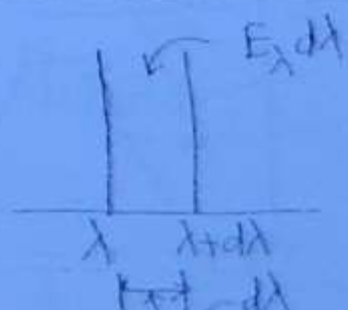
$$\epsilon \Rightarrow \frac{E}{E_b}$$

Black body is the body which absorbs all the incident radiation falling upon it.

A thermally black body need not appear black in colour to the human eye e.g. snow, ice.

A thermally black body is not only good absorber & also an ideal emitter.

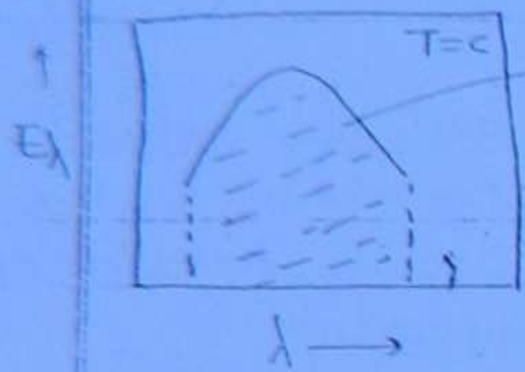
(3) Monochromatic (Spectral) Emissive Power (E_λ)



Unit $\frac{J}{sec \cdot m^2 \cdot \mu m}$

It is defined as the quantity which when multiplied by $d\lambda$, shall give the radiation energy emitted from surface of body per unit time & per unit area in the wavelength region λ to $\lambda + d\lambda$

$$E = \int_0^\infty E_\lambda \, d\lambda$$



Area $E = \int_0^\infty E_\lambda . d\lambda$

4) Monochromatic Emissivity (ϵ_λ).

It is defined as the ratio b/w monochromatic emissive power of a non-black surface & monochromatic emissive power of black surface. both being at the same temperature and the wavelength.

$$\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

$$\epsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda} = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} . d\lambda}$$

If $\epsilon_\lambda \neq f(\lambda)$, then

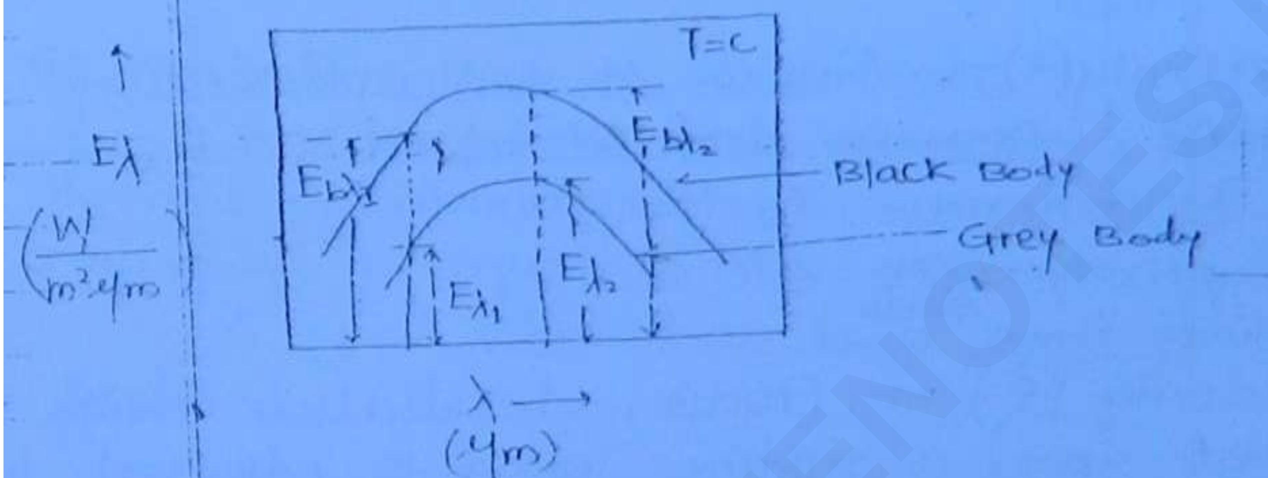
$$\epsilon = \frac{\epsilon_\lambda \int_0^\infty E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda} = \epsilon_\lambda$$

$$\boxed{\epsilon = \epsilon_\lambda}$$

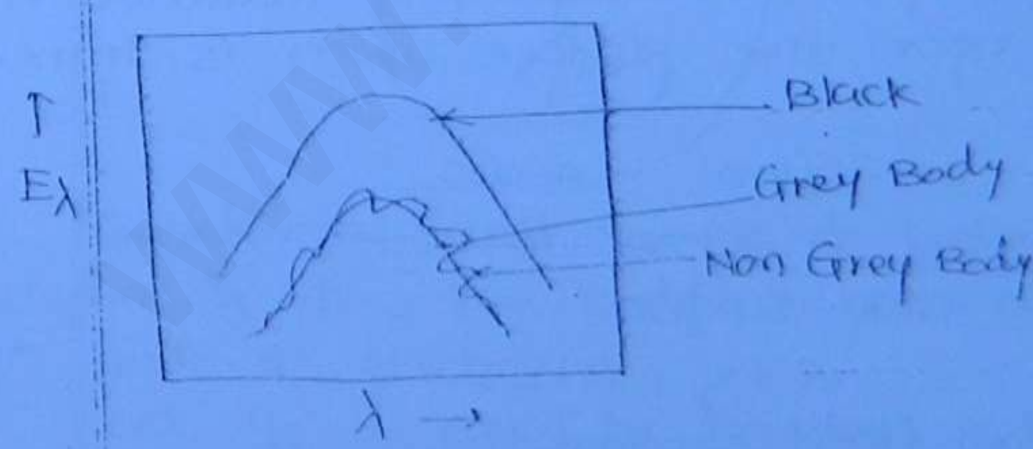
Grey surface.

The Grey body is a body whose monochromat emissivity ϵ_λ is independent of wavelength i.e.

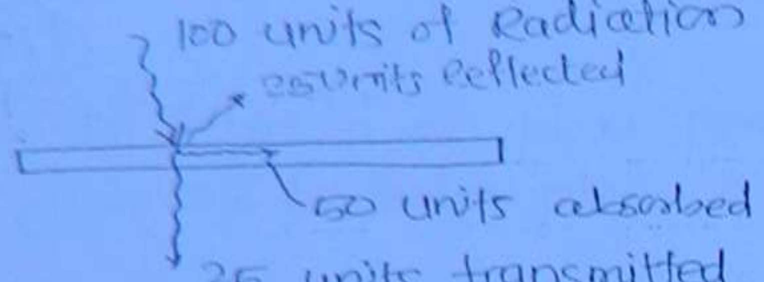
$$\epsilon_\lambda = \text{const.}$$



$$\epsilon_{\lambda_1} = \epsilon_{\lambda_2} \qquad \frac{E_{\lambda_1}}{E_{b\lambda_1}} = \frac{E_{\lambda_2}}{E_{b\lambda_2}}$$



* Absorptivity (α), Reflectivity (ρ) & Transmissivity (τ)



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Then,

Absorptivity (α) → Fraction of Radiation energy incident upon a surface which is absorbed by it.

$$= \frac{50}{100} = 0.5$$

Reflectivity (ρ) ⇒ Fraction of radiation energy incident upon a surface which is reflected by it

$$= \frac{25}{100} = 0.25$$

Transmissivity (τ) ⇒ Fraction of radiation energy incident upon the surface which is transmitted through it.

$$= 25/100 = 0.25.$$

For any surface

$$\alpha + \rho + \tau = 1$$

For black surface

$$\alpha_b = 1 \quad ; \quad \epsilon_b = 1$$

For opaque surface which does not transmit any energy.

$$\tau = 0 \quad , \quad \alpha + \rho = 1 .$$

The above properties defined are functions of wavelength λ. Example :- Window glass of a car (Transmissivity (τ) is more for sun's radiation (short wave) & τ is very low for long wavelength re-radiation.)

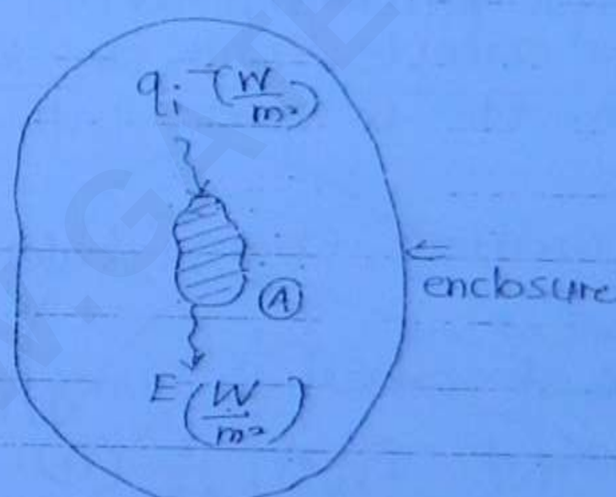
(121)

* Laws of Thermal Radiation.

(i) Kirchoff's Law of Radiation.

The law states that whenever a body is in thermal equilibrium with its surroundings. Its emissivity is equal to its absorptivity. OR
A good absorber is always a good emitter

Proof :-



Consider a small sample of body kept in enclosure & allow the body to come into thermal eq^m with its enclosure.

Let q_i is equal to incident radiation upon surface of body in $\frac{J}{sec \cdot m^2} = \frac{W}{m^2}$

A = Surface area of body

Total Incident Radiation upon the surface of body $= (q_i A)$ Watts.

Let α = Absorptivity of body

$\therefore$ Energy absorbed by body $= (q_i A \alpha)$ Watts

If E is the emissive power of body then total energy emitted by the body $= (EA)$ Watts.

For steady state thermal eq^m of body Energy balance is $\Rightarrow$

Energy absorbed = Energy emitted

$$q_i A \alpha = E A \quad \text{---(i)}$$

Replace the sample with an identical black body and allow it to come into eq^m with its enclosure.

Again writing energy balance for black body :-

Energy absorbed = Energy emitted

$$q_i A \alpha_b = E_b A \quad \text{---(ii)}$$

Eq^n (i) / Eq^n (ii)

$$\frac{q_i \cancel{A} \alpha}{q_i \cancel{A} \alpha_b} = \frac{E \cancel{A}}{E_b \cancel{A}}$$

$$\therefore \underline{\alpha} = \frac{E}{E_b} = \epsilon \qquad \because \alpha_b = 1$$

$\therefore$ $\boxed{\alpha = \epsilon}$ -- Kirchoff's Law.

(ii) Plank's Law of Radiation (Thermal) (Only for black body)

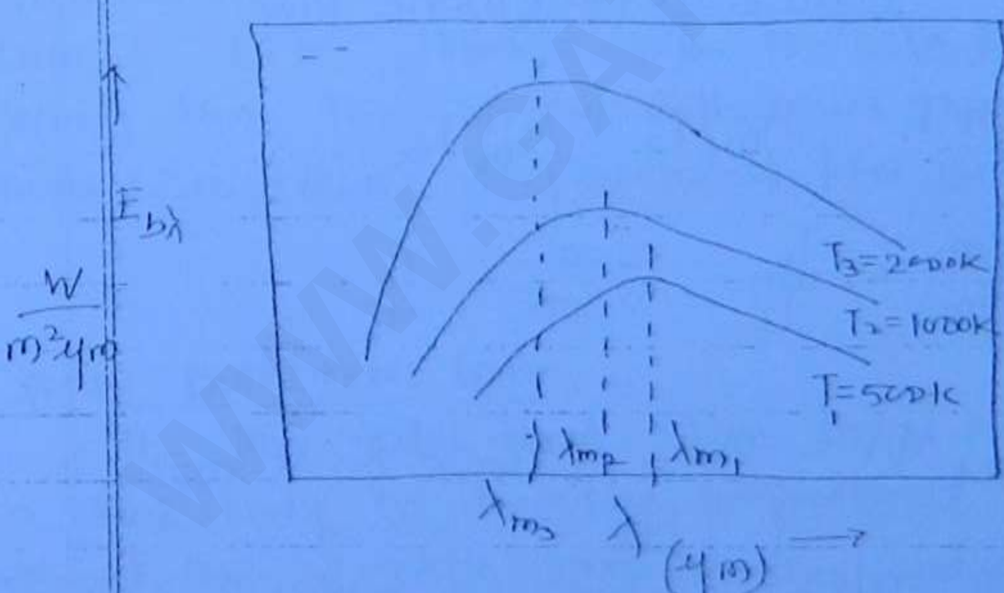
The law states that - the monochromatic emissive power of a black body is dependent upon the absolute temperature of black body & also on the wavelength of emission.

i.e. $E_{b\lambda} = f(\lambda, T)$

$$E_{b\lambda} = \frac{2\pi C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \quad \frac{\text{Watts}}{m^2 \mu m}$$

where C_1 & C_2 are constants.

The law can be graphically represented as -



Let λ_m is the wavelength at which $E_{b\lambda}$ is maximum at a given temperature of the black body

From the graph it can be deduced that

at a given temperature of the black body as the wavelength λ increases $E_{b\lambda}$ also increase reaches a maximum and then decreases.

Also as the temperature of the black body increases, most of the radiation will be shifted to smaller wavelengths i.e. as T increases λ_m decreases.

i.e. As $T \uparrow \Rightarrow \lambda_m \downarrow$

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$$\lambda_m \propto \frac{1}{T}$$

Hence λ_m is inversely proportional to the absolute temperature of body.

$$\boxed{\lambda_m T = \text{constant}}$$

Wein's displacement law.

$$\lambda_{m_1} T_1 = \lambda_{m_2} T_2 = \lambda_{m_3} T_3 \;\ldots$$

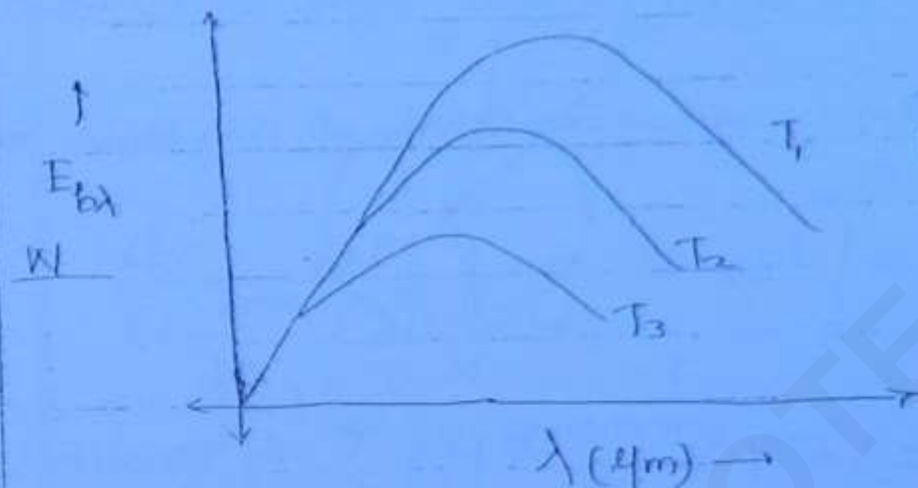
$$\frac{\lambda_{m_3}}{\lambda_{m_1}} = \frac{T_1}{T_3} = \frac{500}{2000} = \frac{1}{4}$$

$$\int E_{b\lambda} \cdot d\lambda = \text{Area of curve on X-axis}$$
$$= E_b$$

$$\frac{\text{Area under } T_2 \text{ curve on X-axis}}{\text{Area under } T_1 \text{ curve on X-axis}} = \left(\frac{2000}{500}\right)^4$$
$$= (4)^4$$

GATE 2005

The following fig was generated from experimental data relating spectral black body emissive power two wavelength at three temperature T_1, T_2 & T_3 & $T_1 > T_2 > T_3$



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The conclusion is that, the measurements are

a] Correct b'coz the maximum in $E_{b\lambda}$ show the corrected Trend.

b] Correct b'coz Plank's Law is satisfied.

c] Wrong b'coz the Stefan-Boltzman law isn't satisfied

d] Wrong b'coz Wein's displacement law isn't satisfied

(iii) Stefan Boltzman Law

The law states that - The total emissive power of a black body is directly proportional to the fourth power of the absolute temp. of the body.

$E_b \propto T^4$ (T in K)

$\boxed{E_b = \sigma T^4}$ W/m²

σ = Stefan-Boltzman Const. = 5.67×10^{-8} Watt/m²K⁴

$\because \epsilon = E/E_b$

E = Emissive power of non-black body

$= \epsilon E_b$

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$$\boxed{E = \epsilon \sigma T^4}$$

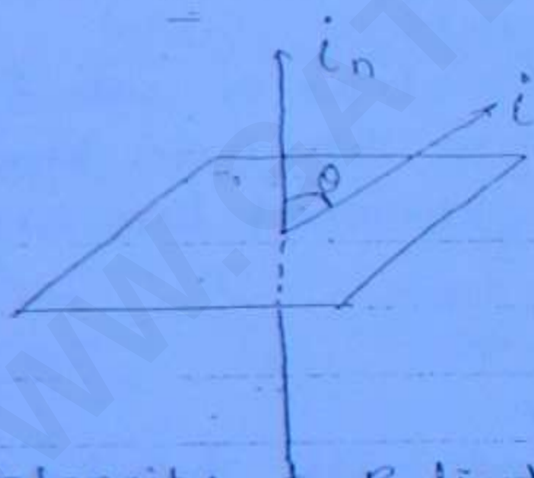
ϵ = Emissivity of non-black body.

Proof

$$E = \int_0^\infty E_{b\lambda}\, d\lambda = \int_0^\infty \frac{2\pi C_1 \, d\lambda}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

$= \sigma T^4$ | Keeping T as constant

iv) Lambert's Cosine Law

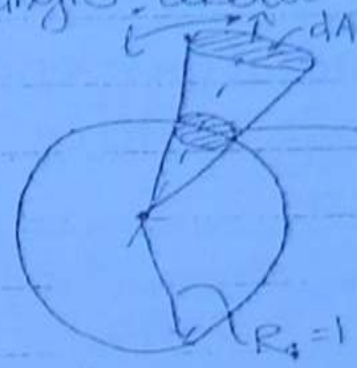


Let i = Intensity of Radiation along the direction making an angle θ with respect to Normal.

i_n = Normal Intensity of radiation.

According to law $\boxed{i = i_n \cos\theta}$

Intensity of radiation (i) is defined as the radiation energy emitted from a surface of the body per unit time per unit area along a given direction per unit solid angle about that direction.



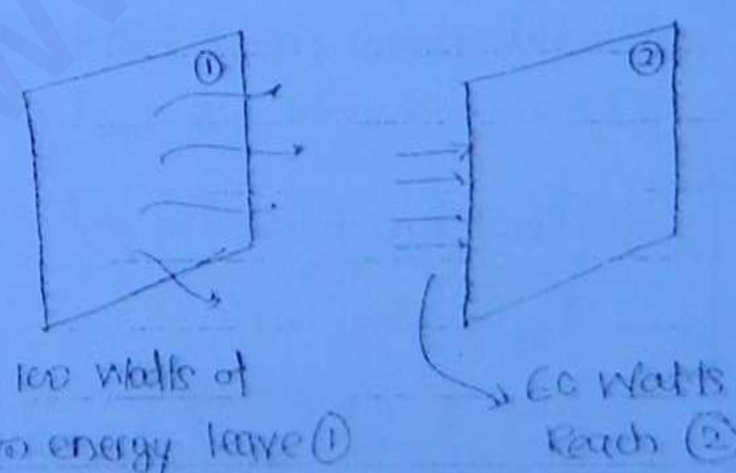
$$i = \frac{dE}{d\omega},\ \frac{J}{sec \cdot m^2\ steradians} = \frac{W}{m^2\ steradians}$$

↑ Solid angle

A diffuse surface will have uniform intensity of radiation along all the direction

Intensity is ⇒ Integrated for all the wavelengths & not monochromatic

* <u>Shape factor or Configuration Factor or View Factor</u>



If out of 100 watts of radiation energy that are surface ①, 60 ~~watts~~ units are reaching surface ②, then from shape factor ① w.r.t ②

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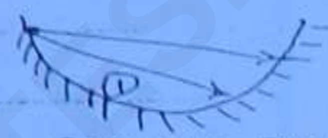
$$F_{12} = \frac{60}{100} = 0.6$$

Fraction of radiation energy leaving surface 'm' that reaches surface 'n'.

$F_{11} = 0$

$F_{11} = 0$

$F_{11} > 0$

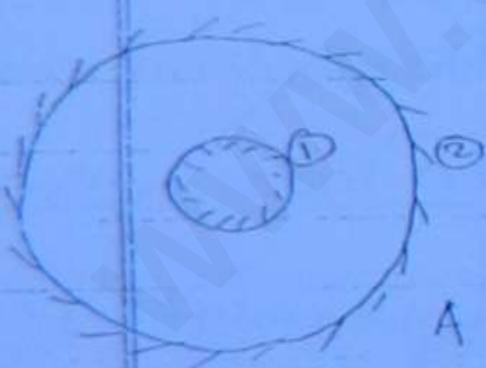


concave surface

When one body is completely surrounded by another body, the shape factor of the inner body w.r.t ~~anoth~~ outer is equal to 1.

$F_{12} = 1$

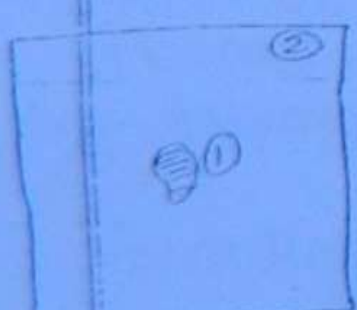
$F_{21} < 1$



$F_{12} = 1$

$F_{21} < 1$

A solid sphere in a hollow sphere.

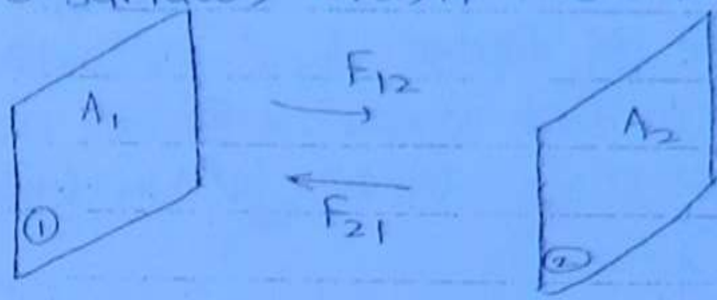


$F_{12} = 1$

$F_{21} < 1$

A small body kept in a large enclosure or Room

* Reciprocity Relation. – This relation holds good b/w any two surfaces even if there are more than two no. of surfaces involved to radiation exchange.



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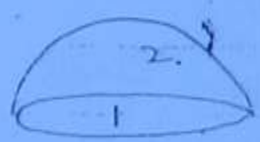
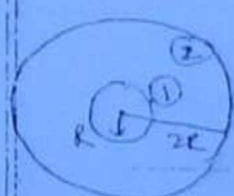
Then, $\boxed{A_1 F_{12} = A_2 F_{21}}$



$F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} = \frac{\pi R^2}{2\pi R^2} = \frac{1}{2}$

Sphere in sphere



$F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} = \frac{4\pi R^2}{4\pi (2R)^2} = \frac{1}{4}$

When there are 'n' no. of surfaces involve in a radiation H.E. problem then

$F_{11} + F_{12} + F_{13} + \cdots + F_{1n} = 1$

$F_{21} + F_{22} + F_{23} + \cdots + F_{2n} = 1$

$\vdots$

$F_{n-1} + F_{n-2} + F_{n-3} + \cdots + F_{n-n} = 1$

GATE 2001

From the circular tube of equal length & diameter shown below, the view factor F_{13} is 0.17, the view factor F_{12} in this case will be

(i) 0.17 (ii) 0.21 (iii) 0.79 (iv) 0.83

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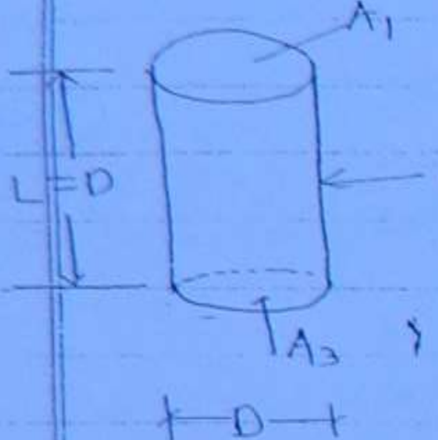


$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{13} = 0.17$$

$$F_{12} = 0.83$$

$$F_{21} + F_{22} + F_{23} = 1$$

$$F_{21} = F_{23}$$

Then $F_{22} = 1 - (F_{21} \times 2)$

$$A_2 F_{21} = A_1 F_{12}$$

$$F_{21} = \frac{A_1}{A_2} \times 0.83$$

$$= \frac{\pi/4\, D^2}{\pi D \times D} \times 0.83$$

$$= \frac{1}{4} \times 0.83$$

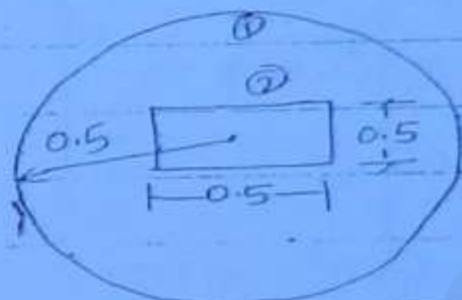
$$\therefore F_{22} = 1 - \left[\frac{1}{2}(0.83)\right]$$

$$= 0.585$$

[GATE 2005]

A solid cylinder (surface 2) is located at the centre of a hollow sphere (surface 1). The dia. of the sphere is 1 m while the cylinder has the diameter & a length of 0.5 m each. The radiation configuration factor F_{11} is

a] 0.375 b) 0.75 c) 0.625 d) 1.



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$F_{11} + F_{12} = 1$

$F_{21} = 1$

$A_1 F_{12} = A_2 F_{21}$

$$F_{12} = \frac{0.5 \times \pi \times 0.5 + \frac{\pi}{4}(0.5)^2 \times 2}{4\pi(0.5)^2} (1)$$

$F_{12} = \frac{1}{4}$

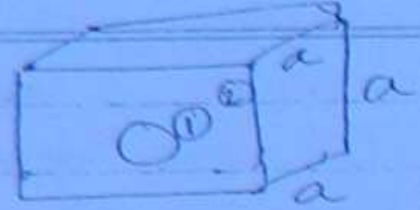
$\therefore F_{11} = 1 - \frac{1}{4}$

$\cancel{F_{11} = 3/4}$

[IES 2004]

A small sphere of outer area $0.6\,m^2$ is totally enclosed by a large cubical hall. The shape factor of hall w.r.t. sphere is 0.004. What is the measure of the internal side of the cubical hall.

a] 5 m b] 4 m c] 6 m d] 10 m



$F_{12} = 1$

$A_1 = 0.6$ $\qquad F_{21} = 0.004$

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$A_1 F_{12} = A_2 F_{21}$

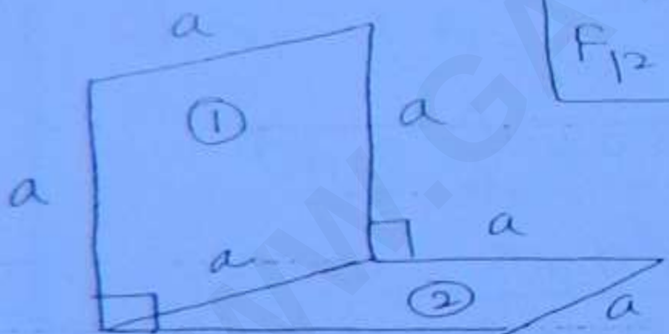
$0.6 = A_2 \, 0.004$

$A_2 = 150$

$\Rightarrow 6 \times a^2 = 150$

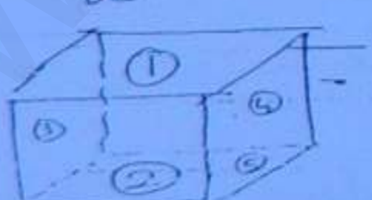
$a^2 = 25$

$\boxed{a = 5 \text{ m}}$



$\boxed{F_{12} = 0.2}$

F Hence in a cube the shape factor b/w any two surfaces will be 0.2



$\cancel{F_{11}}^{\,0} + F_{12} + F_{13} + F_{14} + F_{15} + F_{16} = 1$

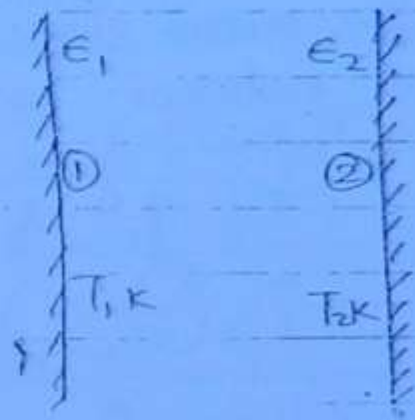
$F_{12} = 1 - 4(F_{13})$

$= 1 - 4(0.2) = 0.2$

* Radiation H.E. between two infinitely large parallel Grey surfaces (diffuse) *

⟹ Assume (1) Steady state H.T. conditions

(2) Isothermal plane surfaces



$\boxed{T_1 > T_2}$

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$F_{12} = 1$

$F_{21} = 1$

Also $A_1 = A_2 = 1 m^2$

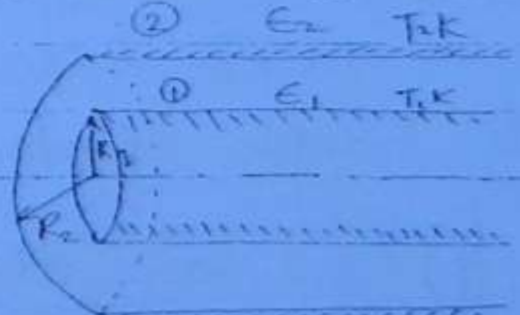
∴ Net Radiation H.E. between two surfaces per unit area

$$\frac{(q_{1-2})_{net}}{A} = \frac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}$$

where ϵ_1 & ϵ_2 are emissivities of Grey surfaces

** (Proof will be given based on Network method)

* Radiation H.E. between two infinitely long concentric cylindrical Grey (diffuse) surfaces



$T_1 > T_2$

Assume :- (i) Steady state conditions.
(ii) Isothermal, diffuse & Grey surface.

The outer surface of the inner cylinder at T_1 is exchanging heat by radiation with the inner surface of the outer ~~surface~~ cylinder at T_2. Then net radiation H.E. between two surfaces is

$$\boxed{(q_{1-2})_{net} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} \text{ Watts.}}$$

$F_{12} = 1 \cdot F_{21} < 1$

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The above formula put up is applicable for all the radiation H.E. cases whenever one shape factor is equal to 1 & other shape factor is less than 1.

Ex ①

Two concentric Spherical surfaces

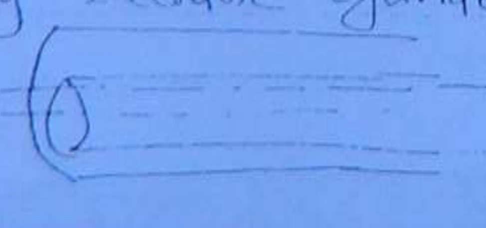
$F_{12} = 1 \quad F_{21} < 1$

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^2$$

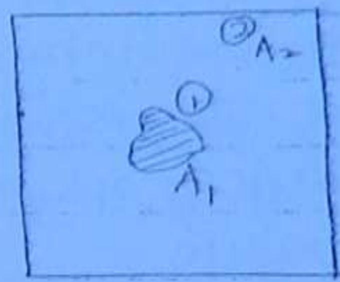
For length of L, $\frac{A_1}{A_2} = \frac{2\pi R_1 L}{2\pi R_2 L}$

$$= \frac{R_1}{R_2}$$

Ex ② Very long Eccentric cylinder



Ex (3) Small body kept in a large enclosure or [illegible]



$F_{12} = 1$
$F_{21} < 1$

Hence

$$q_{12} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$ (135)

since $A_1 <<< A_2$

$$\frac{A_1}{A_2} \rightarrow 0$$

$$\boxed{\therefore (q_{1-2})_{Net} = \sigma (T_1^4 - T_2^4) A_1 \epsilon_1} \text{ Watts.}$$

Ambient (2)

T_∞ = Ambient Atm. temp in K

$F_{Filament-ambient} = 1$

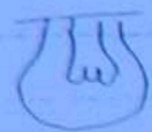
$$\frac{A_{filament}}{A_{ambient}} \rightarrow 0$$

Under steady state conditions of filament,
The wattage of bulb = Net radiation H.E. b/w filament & ambient atm. (neglecting glass)

$$100 \text{ Watts} = \sigma (T_{filament}^4 - T_\infty^4) \epsilon_{filament} \times A_{filament}$$

IES 1998

The filament of a 75W light bulb may be considered a black body radiating into a black enclosure at 70°C. The filament dia. is 0.10 mm & the length is 5 cm. Considering only radiation determine the filament temperature.



Under steady state conditions

Wattage = 75 W

$$= \sigma\left[T_{fil.}^4 - T_{\infty}^4\right]\epsilon_{fil.} \times A_{fil}$$

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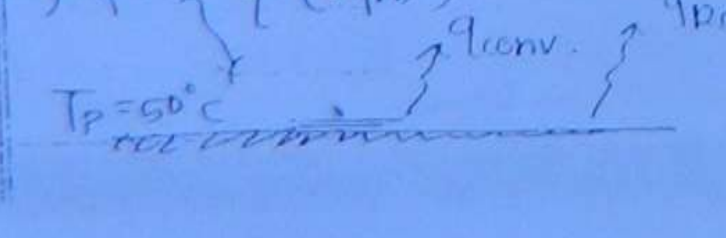
$$75 = 5.67\times10^{-8}\left[T_{fil}^4 - (70+273)^4\right]\times 1\times \pi\left(\frac{0.1}{1000}\right)\times\frac{5}{100}$$

$$T_{fil.} = 3029\ K$$

GATE 2000

Calculate incident solar radiation heat flux falling upon a thin flat horizontal plate whose eq^m steady state temp is 50°C. The plate is having an absorptivity of 0.9 at the solar radiation & emissivity of 0.1 at long wave length radiation. The h∞ b/w the plate & the surrounding ambient atm. is 17.4 W/m²K where T∞ = 27°C. Assume the bottom of plate is insulated



Solar

Let q_i = Incident Radiation in W/m^2

Total Incident radiation $= (q_i \times A)$ Watts

The radiation absorbed by plate

$$= (q_i \times A \times \alpha_p) \text{ Watts} \qquad 1.37$$

$$= (q_i \times A \times 0.9) \text{ Watts}$$

The rate of heat ~~exchange~~ transfer by convection between plate & fluid

$$= hA(T_p - T_\infty)$$

$$= 17.4 \times A (50-27) \text{ Watts}$$

Rate of H.E. by radiation between plate & ambient atm.

$$q_{Net} = \sigma (T_p^4 - T_\infty^4) \epsilon_{plate} \times A$$

between plate & ambient (a small body kept in large environment)

$$= \cancel{5.67 \times 10^{-8} [(50^4 - 27^4)(273)^4}$$

$$= 5.67 \times 10^{-8} [(50+273)^4 - 300^4] \times 0.1 \times A$$

Energy balance for steady state conditions of plate

$$q_i \times \cancel{A} \times 0.9 = 17.4 \times \cancel{A} \times (50-27) + 5.67 \times 10^{-8} [(50+273)^4 - 300^4] \times 0.1 \times \cancel{A}$$

$$\therefore \boxed{q_i = 462.20 \text{ Watt}/m^2}$$

IES 1997

138 During an experiment to measure the temp of hot gas flowing through a large pipe with a small thermocouple located centrally in the pipe, the following data were obtained under steady state conditions.

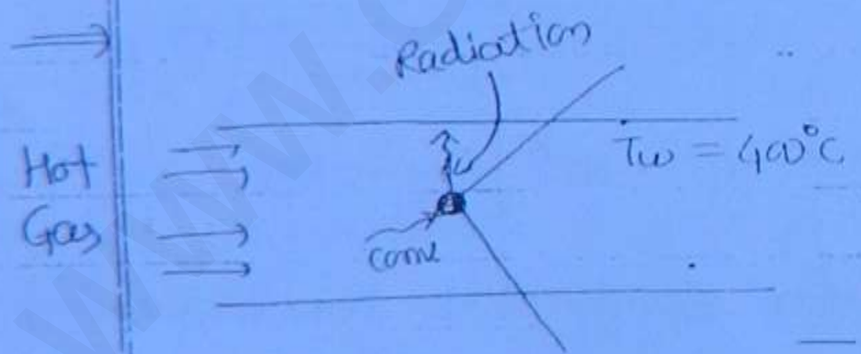
Velocity of gas in pipe flow = 4 m/s
Temp. of pipe wall = 400°C
Temp. indicated by Thermo-couple = 580°C
Dia. of thermocouple Junction = 1.5 mm
Emissivity of thermo-couple = 0.3

Calculate the gas temperature. The h can be calculated from $Nu_d = 0.5 Re_d^{1/2}$

The property values may be used at 400°C & are

$k = 49.72 \times 10^{-3}$ W/m°C

$\mu = 32.68 \times 10^{-6}$ kg/ms & $\rho = 0.05224$ kg/m³



Under steady state conditions of thermocouple junction; the rate of H.T. by convection from Gas to temp = Rate of Radiation H.E. b/w thermocouple & duct wall.

$$hA(T_G - T_{T/c}) = \sigma (T_{TK}^4 - T_w^4) A \epsilon_{TK}$$

To get h

$$Nu = 0.5 \times Re^{1/2}$$

$$\frac{hD}{k} = 0.5 \times \left(\frac{VD\rho}{\mu}\right)^{1/2}$$

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$$\frac{VD\rho}{\mu} = \frac{4 \times 1.5 \times 10^{-3} \times 0.05224}{32.68 \times 10^{-6}}$$

$$\frac{h \times 1.5 \times 10^{-3}}{49.72 \times 10^{-3}} = 0.5 \times (\quad)^{1/2}$$

$$h = 1623 \ W/m^2K$$

$$162.3(T_G - 580) = 5.67 \times 10^{-8} \left[(580 + 273)^4 - 67 \right. \times 0.3$$

$$\therefore \boxed{T_G = \cancel{1314.4}\ ^\circ C} \ 614.04\ ^\circ C$$

* Radiation Networks

(i) Radiosity & Irradiation.

Irradiation (G)

Irradiation (G) is defined as total incident radiation falling upon a surface per unit time & per unit area.

$G\ (J/sm^2) = W/m^2$

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Radiosity (J)

It is defined as the total radiation leaving a surface per unit time & per unit area

$J\ \frac{Watt}{m^2}$

J(leaving) = Emitted Energy + Reflected Energy.

$$J = E + \rho G$$

$$= \epsilon E_b + \rho G$$

for any surface $\alpha + \tau + \rho = 1$

For opaque surface $\tau = 0$

$$\alpha + \rho = 1$$

$$\rho = 1 - \alpha$$

But According to Kirchoff's law

$$\alpha = \epsilon$$

$$\therefore \boxed{J = \epsilon E_b + (1-\epsilon) G} \quad W/m^2$$

Hence the net radiation exchange b/w surface & it's surroundings $= (J-G)A$ Watts

$$q_{\text{surface - surrounding}} = (J-G)A \qquad (141)$$

$$= [\epsilon E_b + (1-\epsilon)G - G]A$$

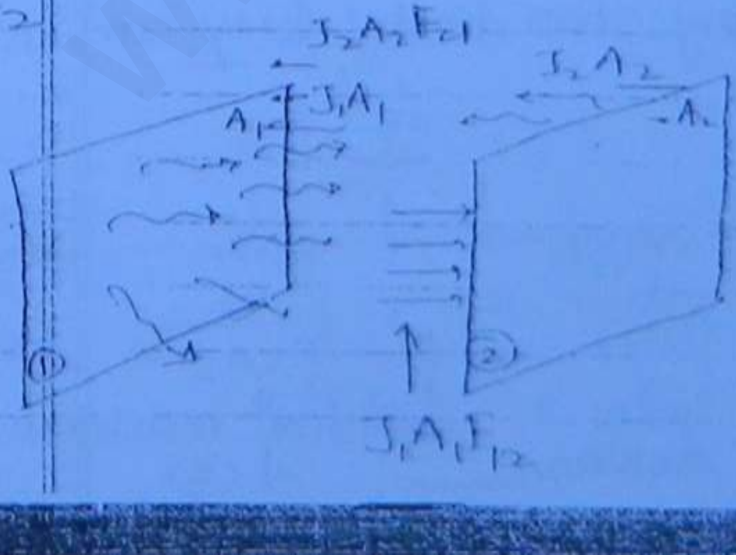
Eliminating G,

$$q_{\text{net b/w surface \& surrounding}} = \frac{E_b - J}{(1-\epsilon/\epsilon A)}.$$

The equivalent Radiation Circuit

$$E_b \xrightarrow{q_{net(surface - surrounding)}} \underbrace{\left(\frac{1-\epsilon}{\epsilon A}\right)}_{\text{Surface Resistance}} \longrightarrow J$$

Considering two finite Grey surfaces of areas A_1 & A_2



Out of total energy leaving surface ①, the energy that reaches surface ② will be $= J_1 A_1 F_{12}$

Out of total energy leaving surface ②, the energy that reaches surface ① will be $= J_2 A_2 F_{21}$

Net radiation H.E. b/w surface ① & surface ② is $q_{1-2} = (J_1 A_1 F_{12} - J_2 A_2 F_{21})$ Watts.

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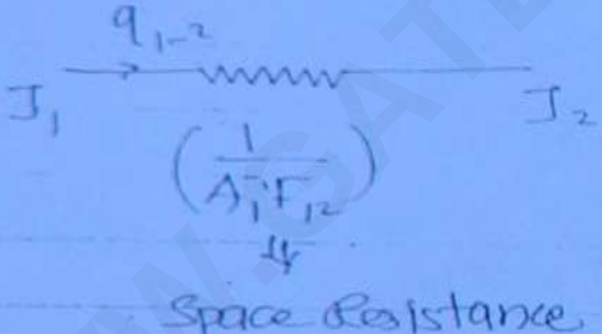
But $A_1 F_{12} = A_2 F_{21}$ --- Reciprocity rule

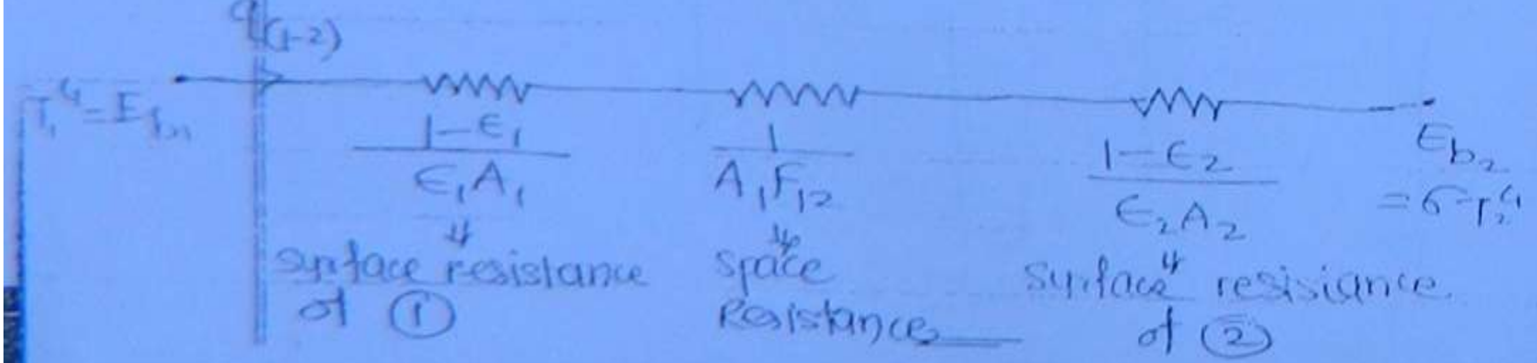
$$\therefore q_{12} = (J_1 - J_2) A_1 F_{12}$$

$$= \frac{J_1 - J_2}{(1/A_1 F_{12})} \leftarrow \text{Space Resistance}$$

The equivalent radiation circuit is



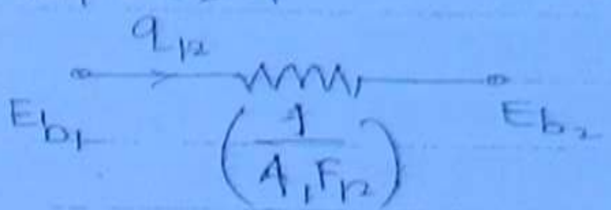
$\therefore$ The complete radiation network for radiation heat exchange b/w any two finite Grey surfaces is given by :-



If the surface is black, surface resistance becomes zero.

If both the surfaces are black, then

$\epsilon_1 = \epsilon_2 = 1$



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q_{1-2} Net by radiation $= \dfrac{E_{b_1} - E_{b_2}}{\Sigma R_{Th}}$

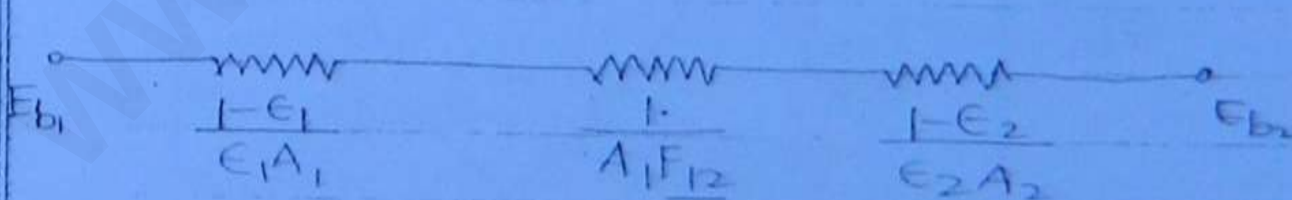
$= \dfrac{\sigma (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$ Watts.

* Surfaces are infinitely large, parallel, grey & diffuse

$F_{12} = 1$

$F_{21} = 1$

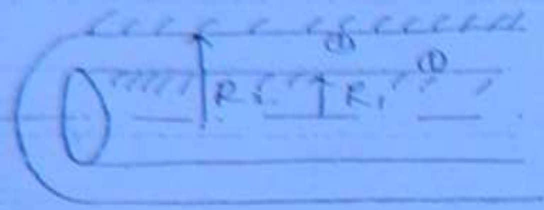
$A_1 = 1$, $A_2 = 1$



q without shield $= \dfrac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1 \times 1} - \frac{1}{1} + \frac{1}{1\times 1} + \frac{1}{\epsilon_2 \times 1} - \frac{1}{1}}$

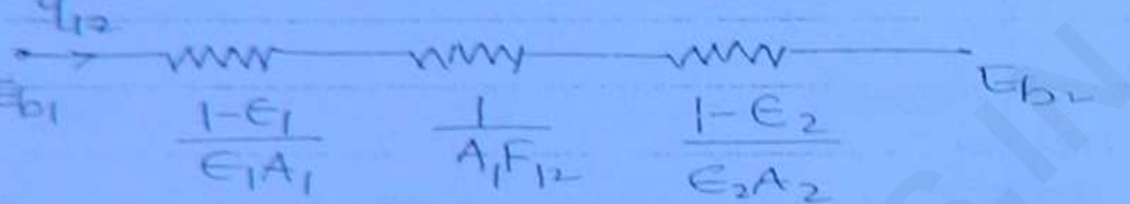
$= \dfrac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$ Watts/m²

* Very long concentric cylinders. (Grey & diffuse)



(744)

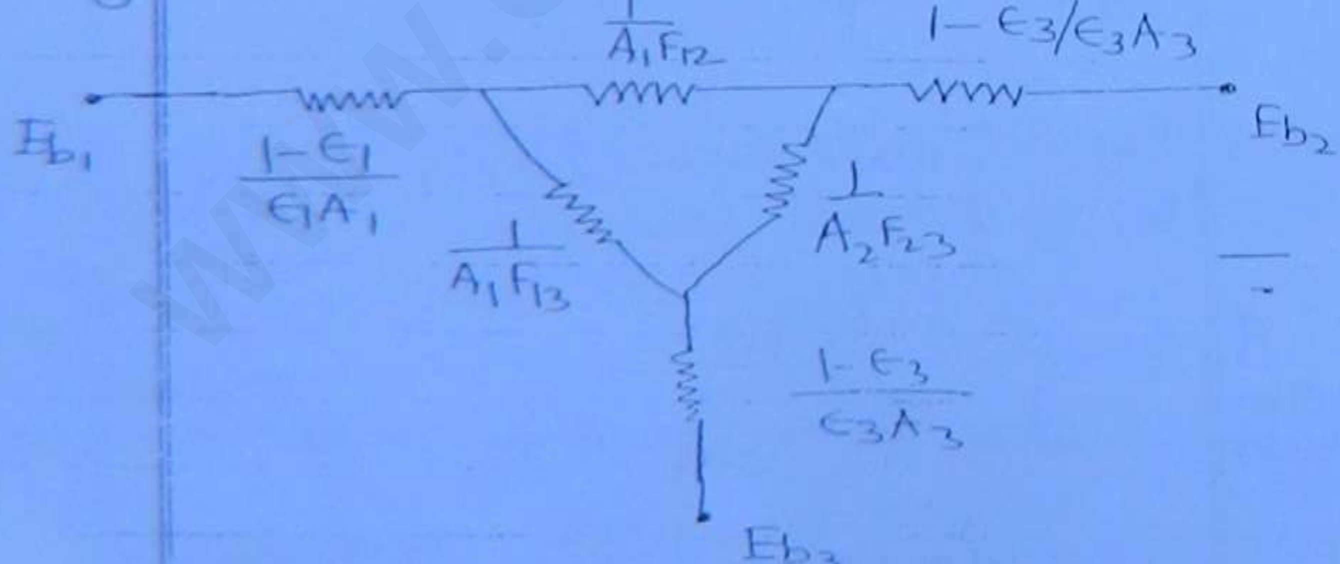
$F_{12} = 1$



$$q_{1-2} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1 A_1} - \cancel{\frac{1}{A_1}} + \cancel{\frac{1}{A_1(\times 1)}} + \frac{1}{\epsilon_2 A_2} - \frac{1}{A_2}}$$

$$q_{1-2} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} \quad \text{Watts.}$$

The network for three surfaces exchanging heat among themselves



GATE 09

Radiative H.T. is intended b/w the inner surfaces of two very large isothermal, parallel metal plates. While the upper plate (designated as plate 1) is a black surface & is the warmer one being maintained at 727°C. The lower plate (plate 2) is diffuse & grey surface with an emissivity of 0.7 & is kept at 227°C. Assume that the surfaces are sufficiently large to form a two surface enclosure & steady state conditions to exists. $\sigma = 5.678 \times 10^{-8}$

The irradiation in kW/m^2 for (plate 1) upper plate is ⟹

a] 2.5 b] 3.6 c] 17.0 d] 19.5 ✓

(145)

If plate 1 is also a diffuse & grey surface with an emissivity value of 0.8, the net radiation H.E. in kW/m^2 b/w plate 1 & plate 2 is

a] 17.0 b] 19.0 c] 23.0 d] 31.7 ✓

I^st

① Black $T_1 = (727+273) = 1000\,K$

E_{b_1}

$(1-\alpha_2)E_{b_1} = (1-\epsilon_2)E_{b_1}$ $E_2 = \epsilon_2 E_{b_2}$

② Grey $T_2 = (227+273) = 500\,K$.

(opaque by default)

$\alpha_2 E_{b_1}$ $\alpha_2 = \epsilon_2$ (Kirchoffs)

II^nd

$\epsilon_1 = 0.8$ $T_1 = 1000\,K$

$\epsilon_2 = 0.7$ $T_2 = 500\,K$

$$q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.678 \times 10^{-8}(1000^4 - 500^4)}{\frac{1}{.80} + \frac{1}{.70} - 1}$$

$= 31.7\,kW/m^2$

Case Ist.

Irradiation (Total Incident Radiation on 1)

$$= (1-\epsilon_2) E_{b_1} + \epsilon_2 E_{b_2}$$

$$= \left[(1-0.7) 5.67 \times 10^{-8} \times 1000^4\right] + \left[0.7 + 5.67 \times 10^{-8} \times 500^4\right]$$

$$= 19.5 \text{ kW/m}^2.$$

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Network with Shield.

$A_1 = A_2 = A_3 = 1 \text{ m}^2$

$F_{13} = 1$

$F_{32} = 1$

1-2

q/A

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} \quad \frac{1}{A_1 F_{13}} \quad \frac{1-\epsilon_3}{\epsilon_3 A_3} \quad \frac{1-\epsilon_3}{\epsilon_3 A_3} \quad \frac{1}{A_3 F_{32}} \quad \frac{1-\epsilon_2}{\epsilon_2 A_2}$$

$$q_{1-2} = \frac{E_{b_1} - E_{b_2}}{\sum R_{Th}}$$

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$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{13}} + 2\left(\frac{1-\epsilon_3}{\epsilon_3 A_3}\right) + \frac{1}{A_3 F_{32}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{2(1-\epsilon_3)}{\epsilon_3} + \frac{1-\epsilon_2}{\epsilon_2} + 2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} - 1 + \frac{1}{\epsilon_3} - 1 + \frac{1}{\epsilon_3} - 1 + \frac{1}{\epsilon_2} - 1 + 1 + 1}$$

$$\boxed{q_{1-2 \text{ with shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{1}{\epsilon_3} - 2}}$$

$$q_{\text{without shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

If $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$ then,

$$q_{\text{with one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{4}{\epsilon} - 2\right)}$$ ✓ Higher Resistance

$$q_{\text{without any shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{2}{\epsilon} - 1\right)}$$

To reduce h.t. rate low emissivity & high reflectivity matl is used.

Date
Page

$$q_{\text{with one shield}} = \frac{1}{2}\, q_{\text{without any shield}}$$

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$$\boxed{q_{\text{with } n \text{ shields}} = \frac{1}{(n+1)}\, q_{\text{without any shield}}}$$

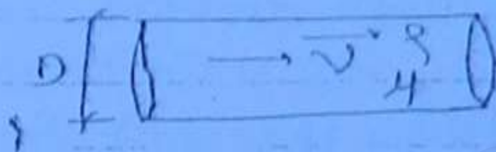
To reduce the H.T. rate to the max. extent, the shield ~~shew~~ used should have very low emissivity or very high reflectivity for which highly polished aluminium sheild with very low ϵ values are used.

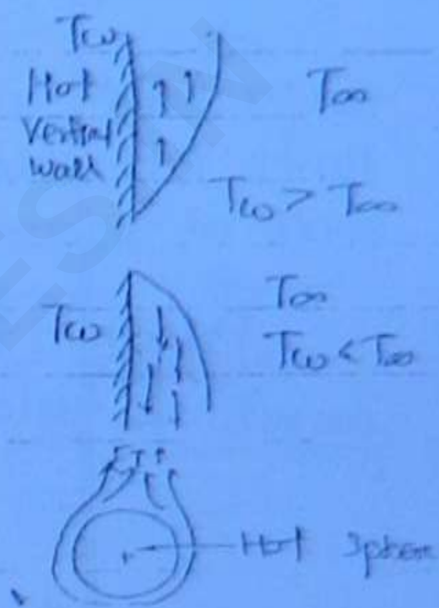
Convection Heat Transfer.

(18/19)

Convection H.T.

- Forced Convection H.T.
  - Flow over flat Plate
  - Flow Inside Circular Duct.
- Free Convection H.T. (Natural convection H.T.)





The velocity is given as parameter in forced convection H.T. only, whereas in free convection β (Isobaric volume expand coeff) is given

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P=\text{const}}.$$

In any forced convection heat transfer

$$h = f(\vec{V}, D, \underbrace{\rho, \mu, C_p, K}_{\text{Thermo-physical properties of fluid.}})$$

$\vec{V}$ = Velocity of fluid

D = Characteristic dimension of body

Total no. of variables in the function = n = 7

Total no of fundamental dimensions $= m = 4$

(M, L, T, θ)
Mass, length, Time, Temp

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$h = \frac{W}{m^2 K} = \frac{J}{Sec\, m^2 K}$

$= \frac{N\text{-}m}{sec \cdot m^2 K} = \frac{MLT^{-2}}{TL\theta}$

$= [MT^{-3}\theta^{-1}]$

Dynamic viscosity of fluid $\mu = Pa\text{-}Sec.$

$= \frac{N.sec.}{m^2}$

$= \frac{MLT^{-2}T}{L^2}$

$= [ML^{-1}T^{-1}]$

$K \rightarrow \frac{W}{mK} = \frac{J}{sec \cdot mK} = \frac{N\text{-}m}{sec \cdot m \cdot K}$

$= \frac{MLT^{-2}}{T\theta} = [MLT^{-3}\theta^{-1}]$

$C_p = \frac{J}{kg \cdot K} = \frac{N \cdot m}{kg \cdot k} = \frac{MLT^{-2}L}{M\theta}$

$= [L^2T^{-2}\theta^{-1}]$

$\rho = \frac{kg}{m^3} = [ML^{-3}]$

(157)

* Buckingham - π Theorem. *

Which states that, if there are total 'n' no. of variables in a functional relationship and if all the variables put together can be expressed in terms of 'm' no. of fundamentals dimension's, then the functional relationship among the variables can be expressed in terms of (n-m) no. of dimensionless π - terms.

$n = 7$ ⇒ No. of π - Terms will be 3

$m = 4$ Let the π terms will be $\pi_1, \pi_2 \, \& \, \pi_3$

Then $\pi_1 = [M^0 L^0 T^0 \theta^0]$ (Choose repeating variables as $h \, v \, D \, \rho$)

$= [h^{a_1} v^{b_1} D^{c_1} \rho^{d_1}] \times \mu$

$\pi_2 = [M^0 L^0 T^0 \theta^0] = [h^{a_2} v^{b_2} D^{c_2} \rho^{d_2}] \times C_p$

$\pi_3 = [M^0 L^0 T^0 \theta^0] = [h^{a_3} v^{b_3} D^{c_3} \rho^{d_3}] \times K$

While choosing the repeating variables two conditions must be satisfied (i) They themselves should not form the dimensionless group

(ii) All the repeating variables put together should contain fundamental dimensions.

$$\left[M^0L^0T^0\theta^0\right] \pi_1 = \left[MT^{-3}\theta^{-1}\right]^{a_1}\left[LT^{-1}\right]^{b_1}\left[L\right]^{c_1}\left[ML^{-3}\right]^{d_1} \times \left[ML^{-1}T^{-1}\right]$$

For M :- $0 = a_1 + d_1 + 1$

— " — L $\quad 0 = b_1 + c_1 - 3d_1 - 1$

— " — T $\quad 0 = -3a_1 - b_1 - 1$

— " — θ $\quad 0 = -a_1$

$b_1 = -1, \quad c_1 = -1, \quad d_1 = -1$

$$\Rightarrow \quad \pi_1 = \left(\frac{\mu}{VD\rho}\right)$$

$$\pi_2 = \left[M^0L^0T^0\theta^0\right] = \left[MT^{-3}\theta^{-1}\right]^{a_2}\left[LT^{-1}\right]^{b_2}\left[L\right]^{c_2}\left[ML^{-3}\right]^{d_2} \times \left[L^2T^{-2}\theta^{-1}\right]$$

For M :- $0 = a_2 + d_2$

$L \quad 0 = b_2 + c_2 - 3d_2 + 2$

$T \quad 0 = -3a_2 - b_2 - 2$

$\theta \quad 0 = -a_2 - 1$

$\therefore \quad a_2 = -1 \quad d_2 = 1 \quad b_2 = 1 \quad c_2 = 0.$

$$\Rightarrow \quad \pi_2 = \frac{\rho V C_p}{h}$$

$$\pi_3 = \left[M^0L^0T^0\theta^0\right] = \left[MT^{-3}\theta^{-1}\right]^{a_3}\left[LT^{-1}\right]^{b_2}\left[L\right]^{c_3}\left[ML^{-3}\right]^{d_3} \times \left[MLT^{-3}\theta^{-1}\right]$$

For $M \quad 0 = a_3 + d_3 + 1$

$L \quad 0 = b_3 + c_3 - 3d_3 + 1$

$T \quad 0 = -3a_3 - b_2 - 3$

$\theta \quad 0 = -a_3 - 1$

$$\boxed{\pi_3 = K/hD}$$

$a_3 = -1, \quad c_3 = -1, \quad b_3 = 0$

$$\frac{1}{\pi_1} = \frac{VD\rho}{\mu} = \text{Reynold's No. (Re)}$$

Only forced convection.

$$\frac{1}{\pi_3} = \frac{hD}{k_{fluid}} = \text{Nusselt No.} = Nu$$

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$$\frac{\pi_1 \pi_2}{\pi_3} = \frac{\mu}{\cancel{V}\cancel{D}\cancel{\rho}} \times \frac{\cancel{\rho}\cancel{V}C_p}{\cancel{h}} \times \frac{\cancel{h}\cancel{D}}{k}$$

$$= \frac{\mu C_p}{k} = \text{Prandtl Number (Pr.)}$$

Prandtl No. is only no. which is property of fluid

$$Pr. = \left(\frac{\nu}{\alpha}\right) = \frac{K.V.}{T.D.}$$

$$= \frac{\mu/\rho}{k/\rho C_p} = \frac{\mu C_p}{k}$$

Pr of air = 0.7

— " — water = 2 to 6

— " — Hg = 0.023

— " — oil = upto 100

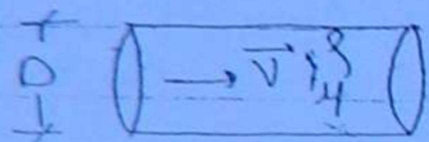
1) Significance of dimensionless-π thm. in forced conv. heat transfer.

(1) Reynold's Number – It is defined as the ratio b/w inertia force & viscous force of a fluid.

$$R_e = \frac{I.F.}{V.F.} = \frac{VD\rho}{\mu} = \frac{VD}{(\mu/\rho)} = \frac{VD}{\nu}$$

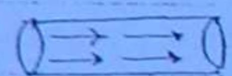
where ν = Kinematic viscosity of fluid.

* For flow through pipes (Poiseuelle flows) (Internal flows)



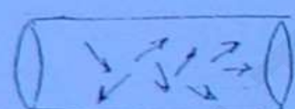
If $R_e < 2000$

↳ Flow is laminar flow



(Lower Critical Re)

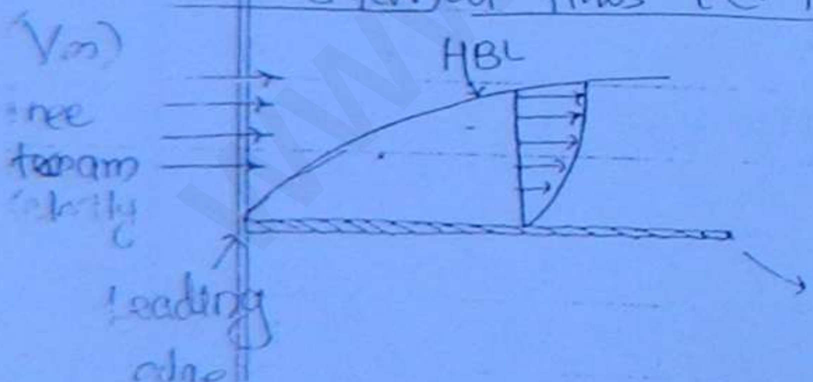
If, $R_e > 4000$ ⇒ Flow is turbulent



Turbulent flow.

High viscous oils flow are laminar flow

* For external flows i.e. flow over flat plates.



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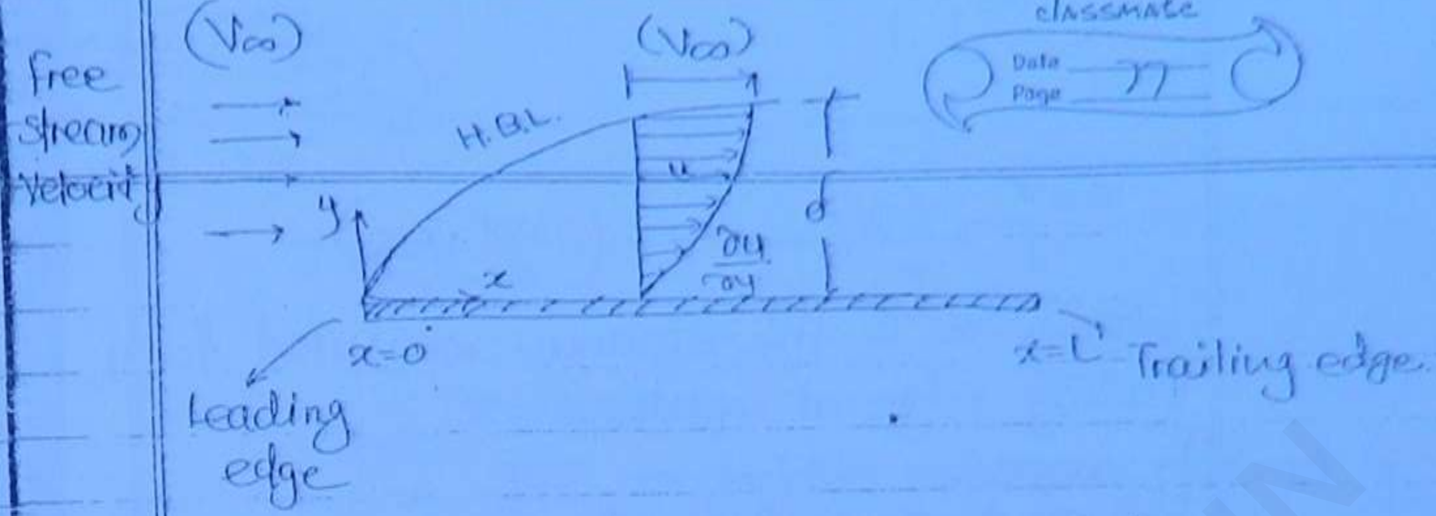
HBL ⇒ Hydrodynamic boundary level.



H.B.L. is a thin region over the Flat plate inside which there are shear stresses (viscous stresses) act b/w the fluid layers, resulting in velocity gradient developing in the normal dir^n of plate.

$\frac{\partial u}{\partial y}$ = Velocity gradient

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Newton's law of viscosity

$$\boxed{\tau = \mu\left(\frac{\partial u}{\partial y}\right)}$$

τ → Viscous stress

μ → Dynamic Viscosity.

δ = H.B.L. Thickness.

Put $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ = Velocity gradient at the stagnant layer.

Wall shear stress $= (\tau)_{wall} = \mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0}$

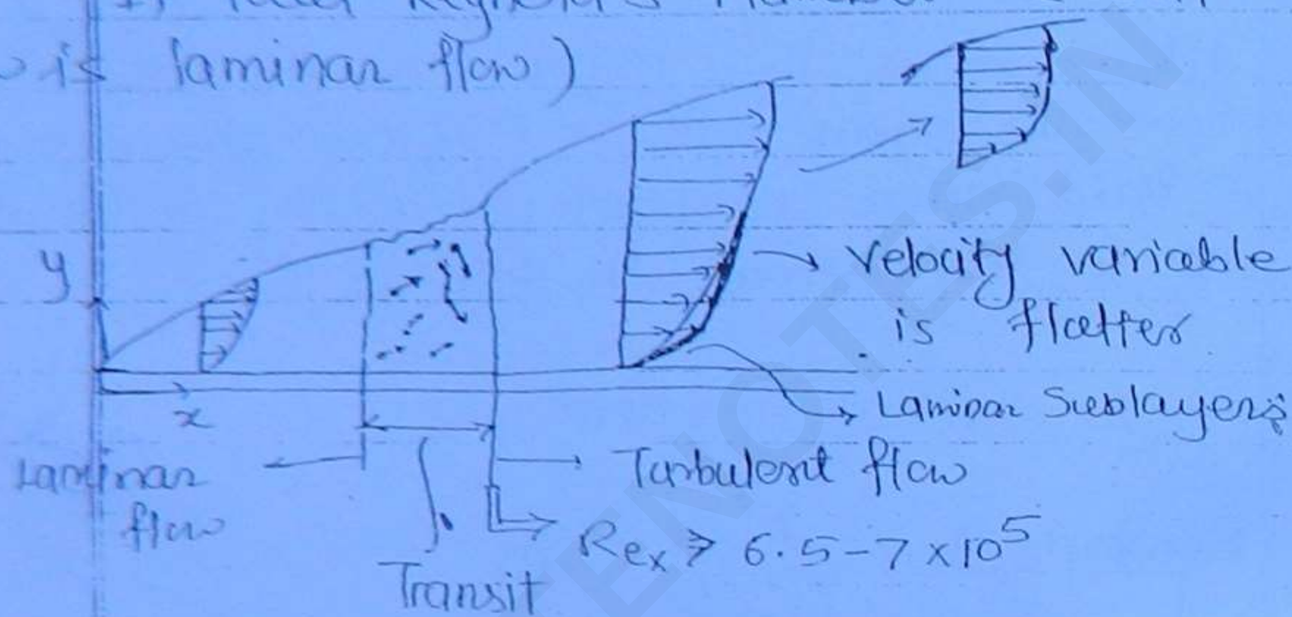
If Re_x = Local Reynold's Number

$$= \frac{V_\infty x \rho}{\mu}$$

$$= \frac{V_\infty x}{\nu}$$

where x is the distance measured from leading edge of plate.

If local Reynold's Number $< 5 \times 10^5$
(Flow is laminar flow)



Prandtl Number

Prandtl no. is property of fluid which is defined as the ratio b/w kinematic viscosity & thermal diffusivity of fluid.

$$Pr.\ No. = \frac{\nu}{\alpha} = \frac{K.V}{T.D.}$$

$$= \nu / \frac{K}{\rho C_p} = \frac{\mu C_p}{K}$$

$Pr_{air} = 0.69,$

$Pr_{water} = 2{\cdot}06$

Prandtl No. signifies the relative magnitudes of momentum diffusion rate & energy diffusion rate through the respective boundary layers.

Nusselt No. (Nu)

$$Nu = \left(\frac{hD}{k}\right) = \frac{(D/kA)}{(1/hA)}$$

= ~~The condition~~

= $\dfrac{\text{The conduction resistance offered by the fluid of it were stationary}}{\text{Convective Resistance}}$

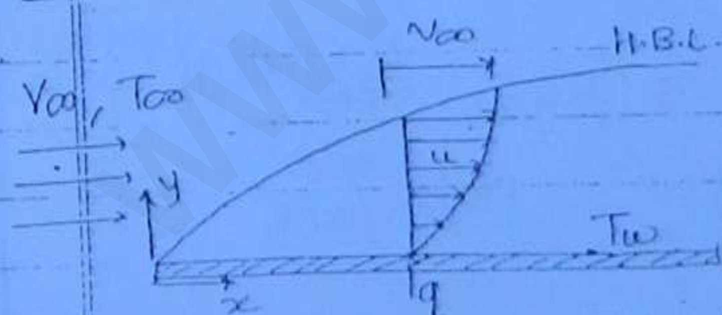
Nu Resembles Biot No.

$$\text{Biot No} = \frac{hs}{K_{solid}} < 0.1 \text{ (Lumped Analysis)}$$

** $\boxed{Nu > 1}$ Since conduction Resistance of fluid is always greater than ~~[illegible]~~ convective resistance } In any state of forced convection

$Nu = \{$

** Local Convection H.T. coefficient (h_x) **



Let T_w = Temperature of flat plate.

T_∞ = Free stream temp. of fluid.

The mode of H.T. taking place through the stagnant layer of (b/w plate & fluid) fluid i.e.

at $y=0$ is only by conduction.

Consider a differential area $dA = (B.dx)$ on the flat plate through which the differential H.T rate b/w plate & fluid is dq.

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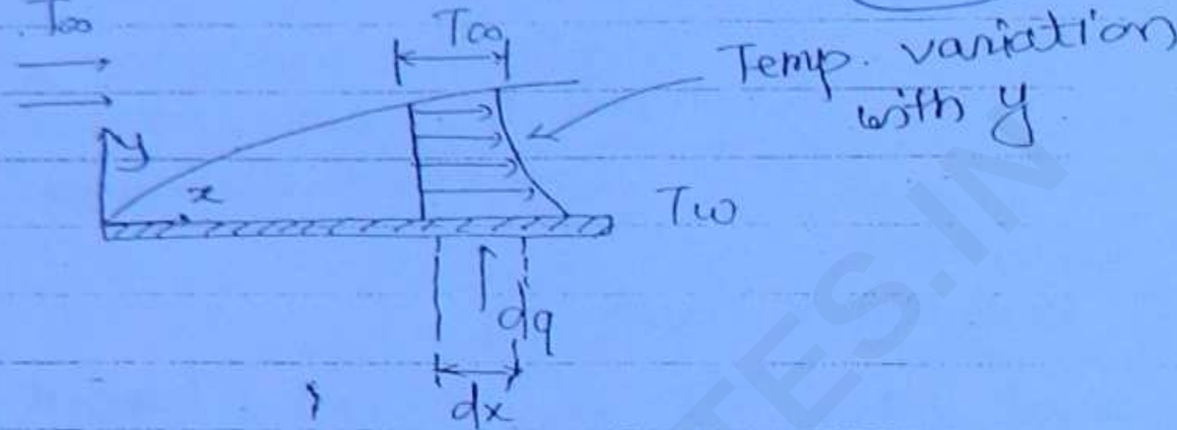


According to Fourier's law of conduction.

$$dq = -k_f (B.dx) \left(\frac{\partial T}{\partial y}\right)_{at\, y=0}$$

& According to Newton's law

$$dq = h_x dA (T_w - T_\infty)$$

$\therefore h_x$ = local convective H.T. coeff at x.

$$-k_f (B.\cancel{dx}) \left(\frac{\partial T}{\partial y}\right)_{at\, y=0} = h_x (B.\cancel{dx})(T_w - T_\infty)$$

$$\therefore h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)} \quad \text{Watts}/m^2 k$$

since $\left(\frac{\partial T}{\partial y}\right)_{y=0}$ = spatial gradient of temp. wi[th] the fluid at $y=0$

$= f(x)$

$\Rightarrow$ ~~f(x)~~ $h_x = f(x) \Rightarrow h_x \propto \frac{1}{\sqrt{x}}$

As h_x is defined at $y=0$ so there is no change in h_x as y increases.

The average convective H.T. coeff. can be calculated from ($\bar{h}$)

$$\bar{h} = \text{Avg. conv. H.T. coeff.} \quad 154$$

$$= \frac{1}{L}\int_0^L h_x \cdot dx$$

where L = Length of plate.

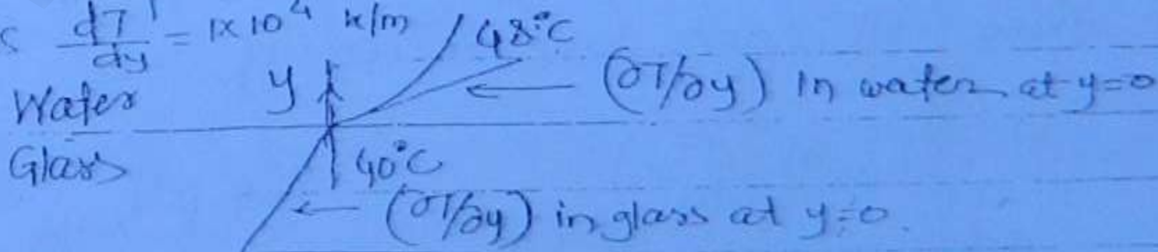
q = conv. H.T. rate b/w plate & fluid

$$= \bar{h} \cdot A\,(T_w - T_\infty) \text{ Watts}$$

where A = Area of Plate.

GATE 2003

Heat is being transferred by convection from water at 48°C to a glass plate whose surface i.e. exposed to the water is at 40°C. The thermal conductivity of water 0.6 W/mK & $K_{Glass} = 1.2$ W/mK. The spatial gr. of temp. in the water at the water-glass interface is $\frac{dT}{dy} = 1\times10^4$ K/m



The value of the temp. grad. in the glass at the water glass interface in K/m is

a] -2×10^4 b] 0.0 c] 0.5×10^4 d] 2×10^4

The H.T. coeff. in W/m²k is –

b] 4.8 c] 6 d] 750 ✓

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⟹ from Fourier's law of conduction

q = H.T. rate b/w water & glass

$$= K_{water} A \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}$$

(At the Interface within the water)

,

$$= K_{Glass} A \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}$$

(At the interface with in the glass)

$$\Rightarrow \boxed{\left(\frac{dT}{dy}\right)_{at\ y=0} = 0.5\times10^4}$$

= Spatial gradient of temp. within glass.

From Newton's law of cooling

$$\underset{0.6}{K_w} A\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = hA\left(T_{water} - T_{Glass}\right)$$

within water

$$h = \frac{0.6\ \cancel{(48-40)}\ 1\times10^4}{\cancel{1\times10^4}\ (48-40)}$$

$$\boxed{h = 750\ W/m^2k}$$

GATE 2009

A coolant fluid at 30°C flows over a heated flat plate maintained at a const. temp. of 100°C. The boundary layer temp. distribution at a given location on the plate may be approximated as -

$$T = 30 + 70\,\exp(-y)$$

where y in meters is the dist. normal to the plate & T is in °C. If the thermal conductivity of fluid is 1 W/mK. The local conv. H.T. coeff. in W/m²K at that location will be -

a) 0.5 b) 0.2 c) 1 d) 10

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$T_\infty = 30°c$

$T_w = 100$

$k_f = 0.1$ W/mK $\quad h_x = ?$

$$T = 30 + 70\,\exp(-y)$$

$$-k_f A \left(\frac{\partial T}{\partial y}\right)_{y=0} = h A (T_w - T_\infty)$$

$$-\left[(1)(-70)\right] = h(100 - 30) \qquad -\left[1 \times (-70)\right] = h(70)$$

$$\boxed{h = 1} \qquad \boxed{h = 1}$$

→ Conservation of Mass and conservation of momentum for laminar boundary layer over a flat plate *

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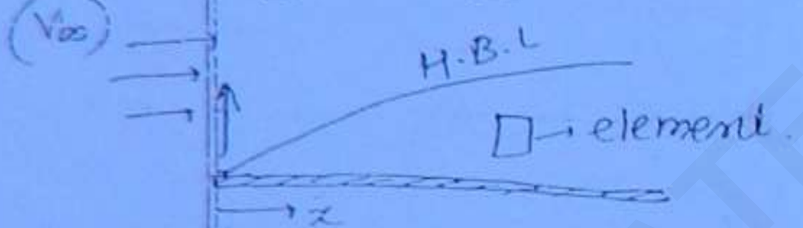
* Assumptions

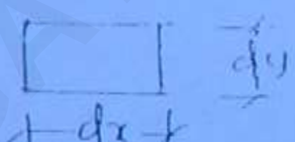
(1) Flow is steady
(2) Fluid is incompressible ($\rho = c$)
(3) Flow is Two dimensional (2D flows)

$\cancel{u = f(x)}$ $u = f(x, y)$

& $v = f(x, y)$

where u, v are velocity components of fluid along x & y directions.

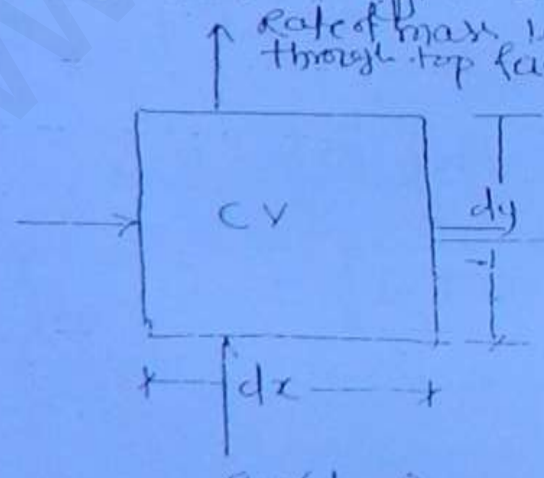




(4) Assume unit length $\perp^{ar}$ to plane of fig.

Rate of mass leaving through top face $= \rho\left(v + \frac{\partial v}{\partial y} \cdot dy\right)(dx \times 1)$

Mass entering the C.V. from left face $= \rho(dy \times 1)u$ kg/s



$\rho\left(u + \frac{\partial u}{\partial x} \cdot dx\right)(dy \times 1)$ = Rate of mass leaving C.V. through right face.

$\rho v(dx \times 1)$

Rate of Mass entering through bottom face.

For steady flow,

$$\left\{\begin{matrix}\text{The total mass flow rate} \\ \text{entering C.V.}\end{matrix}\right\} = \left\{\begin{matrix}\text{Total mass flow rate} \\ \text{leaving C.V.}\end{matrix}\right\}$$

$$\Rightarrow \rho \cdot u\,dy + \rho v\,dx = \rho\left(u + \frac{\partial u}{\partial x}dx\right)dy + \rho\left(v + \frac{\partial v}{\partial y}dy\right)dx$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$ Eqn of Continuity. 163

{In generalised way. (steady 3D. incompressible flow)

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

$$\boxed{\nabla \cdot \vec{V} = 0}$$

* Conservation of Momentum (Linear)

Principle

$\Rightarrow$ Newton's IInd law of Motion.

$$\sum F_x = m a_x$$

$$\left.\begin{matrix}\text{Net algebric sum of} \\ \text{all the force acting on} \\ \text{the fluid control volume}\end{matrix}\right\} = \begin{matrix}\text{The rate of momentum of} \\ \text{fluid leaving control volume} \\ - \text{ The Rate of momentum} \\ \text{of fluid entering control volume}\end{matrix}$$

[Impulse-Momentum Eqn]

Momentum = Mass × Velocity

Rate of momentum = $\frac{\text{Mass} \times \text{Velocity}}{\text{Time}}$

But $\frac{\text{Mass}}{\text{Time}}$ = Mass flow rate

$\Rightarrow$ Rate of momentum $= \dot{m} \times \text{velocity}$

$= \frac{kg\text{-}m}{sec^2}$

= Newton

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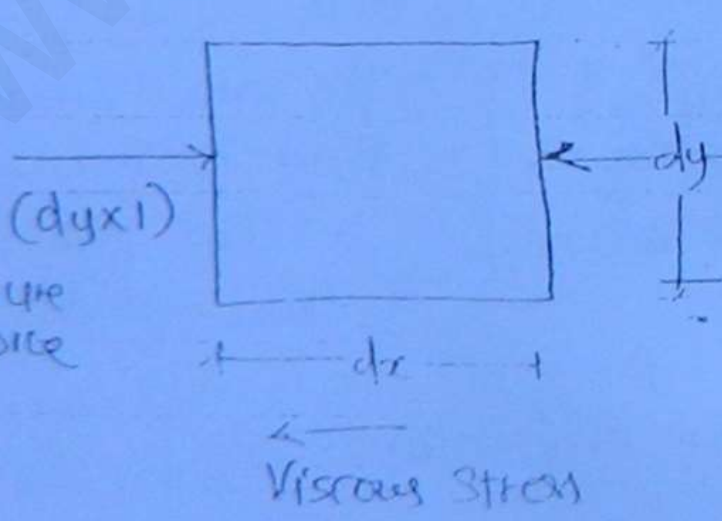
Assumptions.

(1) Flow is steady
(2) Incompressible flow
(3) Pressure variations along y-dir^n neglected
(4) No viscous forces along y-dir^n
(5) 2-D-flow $\left.\begin{matrix} u = f(x,y) \\ v = f(x,y) \end{matrix}\right\}$
(6) Unit length along z-dir^n

Laminar flow over flat plate

$= \left[\frac{\partial u}{\partial y} + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) \cdot d\right.$

Viscous force $= (\tau \times \text{Area}) = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)\right] \times (dx \times 1)$



$P\,(dy \times 1)$ = Pressure force

Pressure force on Right side $= \left(P + \frac{\partial P}{\partial x} \cdot dx\right)(dy \times 1)$

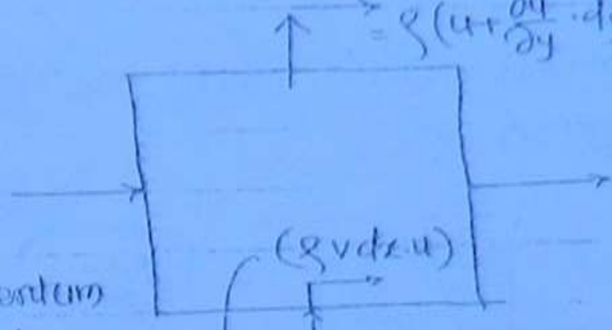
Viscous Stress

$= \tau \times \text{Area}$

$= \mu\left(\frac{\partial u}{\partial y}\right)(dx \times 1)$

Rate of momentum leaving top face along x-dir^n

$= \rho\left(u+\frac{\partial u}{\partial y}\,dy\right)\left(v+\frac{\partial v}{\partial y}\,dy\right)dx$



$\rho\left(u+\frac{\partial u}{\partial x}\,dx\right)^2 dy$ = Rate of momentum leaving through Right face.

$(\rho\, u\, dy)\cdot u$ = Rate of momentum entering through left face

$(\rho v\, dx\, u)$

$\rho v\, dx$

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Rate of momentum entering through bottom face along x-dir^n

Hence Impulse Moment Eq^n along x-direction for the element

$+P\,dy - \left(P+\frac{\partial P}{\partial x}\,dx\right)dy - \mu\left(\frac{\partial u}{\partial y}\right)dx$

$+\mu\left[\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right]dx$ = Total Rate of Momentum leaving CV − Total rate of momentum entering C.V.

$$\Rightarrow \boxed{\rho\left[u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)\right] = \mu\frac{\partial^2 u}{\partial y^2} - \frac{dp}{dx}}$$

Momentum Eq^n

Navier Stokes eq^n of motion along x-Direction

$$\left(u\cdot\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)\Big] \quad \nu\frac{\partial^2 u}{\partial y^2} \quad \frac{1}{\rho}\frac{dP}{dx}$$

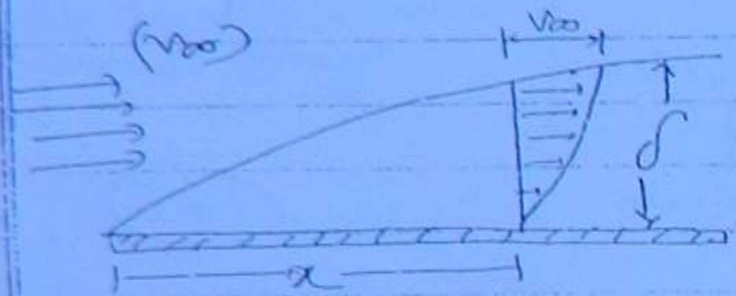
Conv. Accl^n along x dir^n

K.V. of fluid $= \mu/\rho$

Second order Non-Linear D.E.

Put $\vartheta = 0$ we get, Euler's eq^n of motion along x-dir^n

The sol^n to this D.E. could be given from the B.C which are given as



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At any x - ~~along~~ measured from leading edge,

At $y = 0 \Rightarrow u = 0$

Boundary condi^{ns} ? H.B.L

At $y = \delta \Rightarrow u = V_\infty$

At $y = \delta \Rightarrow \frac{\partial u}{\partial y} = 0$

At $y = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$ (Neglecting Pressure Gradient along x)

The solution for u is

$$\frac{u}{V_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

where δ = H.B.L. thickness at any given x

$= f(x)$

$$\boxed{\frac{\delta}{x} = \frac{4.64}{\sqrt{R_{ex}}}} \Rightarrow \delta = \frac{4.64x}{\sqrt{V_\infty x/\nu}}$$

As x increases δ also Increases

$$\boxed{\delta \propto x^{1/2}}$$

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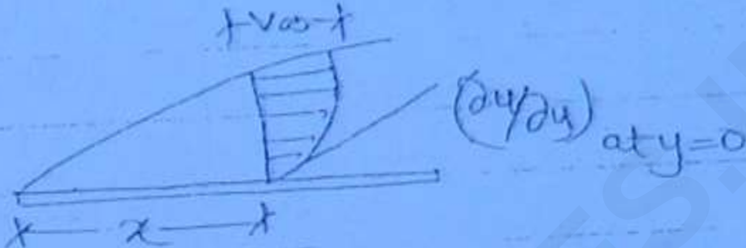
Conservation



Local wall shear stress (τ_x) at any given x

$$= \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu\frac{\partial}{\partial y}\left[V_\infty\left\{\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\}\right]_{y=0}$$

$\Longrightarrow$ As x increases τ_x decreases

$$\tau_x = f(x)$$

$$\tau_x = \mu V_\infty \frac{3}{2\delta}$$

As $x\uparrow \Rightarrow \tau_x \downarrow$

$$\bar{\tau} = \frac{1}{L}\int_0^L \tau_x \, dx$$

(Average Shear Stress)

* Conservation of Energy for laminar boundary layer

u, pv } $h = u + pv$

$\frac{V^2}{2} \to 0$

$gz \to 0$

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Assumptions.

(i) Flow is steady.

(ii) Fluid properties are constant

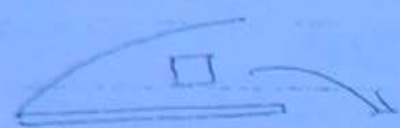
(iii) Conduction H.T. along x direction neglected.

(iv) Unit depth along z-dirn.

For Ideal gas like air

$h = f(T)$ only

$h = C_p T \Rightarrow$ Energy carried by fluid stream at gas per kg



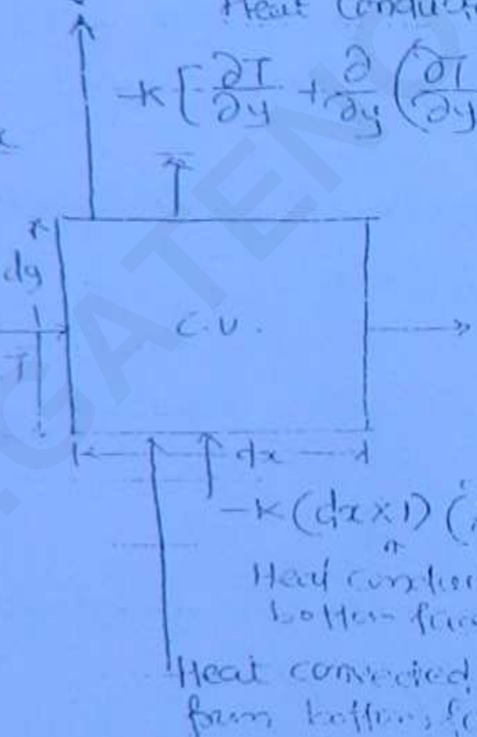
Heat convected out through top face

$= \rho\left[v + \frac{\partial v}{\partial y} \cdot dy\right]\left[T + \frac{\partial T}{\partial y} \cdot dy\right] C_p\, dx$

Heat conducted out from top face

$-k\left[\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) \cdot dy\right] \cdot dx$

Viscous heat dissipated $= \mu\, dx \left(\frac{\partial u}{\partial y}\right)^2 dy$



$(\rho u\, dy) C_p T$

Heat convected into a element through left face

Heat convected out right face

$\rho\left[u + \frac{\partial u}{\partial x} \cdot dx\right]\left[T + \frac{\partial T}{\partial x} \cdot dx\right] \times C_p\, dy$

$-k(dx \times 1)\left(\frac{\partial T}{\partial y}\right)$

Heat conducted from bottom face

Heat convected from bottom face $(\rho v\, dx) C_p T$

Total energy associated with moving fluid $= \dot{m} C_p T$