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198

-: HAND WRITTEN NOTES:-

OF 198

MECHANICAL ENGINEERING

1

-: SUBJECT:-

INDUSTRIAL ENGINEERING

11

2

By. Saurabh Pandey Sir

TOPICS

1. Introduction and B.E.A
2. Inventory*
3. Sequencing*
4. PERT/CPM *
5. Forecasting*
6. line Balancing
7. Queuing*
8. Work Study
9. SQC
10. Linear Programming
    - a. Graphical*
    - b. Simplex
    - c. Transportation
    - d. Assignment*
11. MRP and VE
12. Plant Layout + Wage Incentive Plan + M.H + P.P.C

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BOOKS

1. O.R : Kanti Swarup, Heera & Gupta, N D Vohra
2. I E : Mahajan, O.P Khanna

Newspaper

1. front
2. Economic
3. Sports page

General Science

IES { 1. Modern India
{ 2. NCERT - XII + Made Eas
IAS 3. Spectrum (GS)
(Medieval + Ancient)

English

English Easy : Chetan Anand. (B.S.C)

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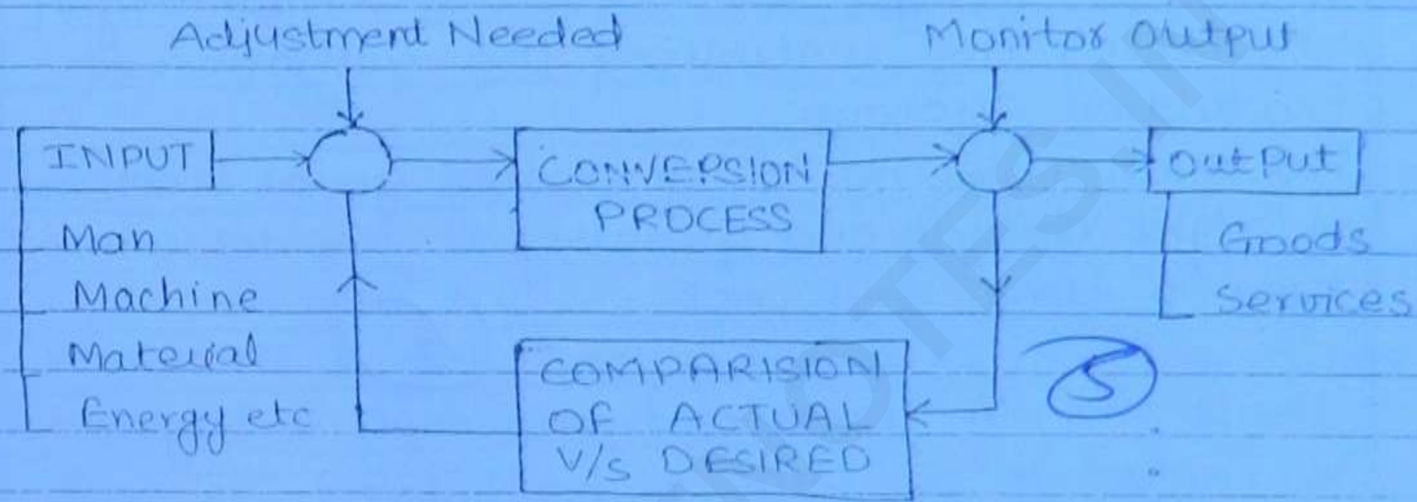
Production is step by step Value Addition process of converting one form of material into another form to enhance the utility of the product for the user. Input → Conversion Process → Output

Production System



It is an organised process of Conversion of Raw material into finished Goods through a Feedback loop.

Productivity $= \dfrac{\text{Output}}{\text{Input}}$ [Different from efficiency; Always greater one (100%)

- Output always more than input to make profit so, Productivity will always increase
- It is the Quantitative relation between what we produce and what we use as recources to produce them.
- It is the Continuous effort to apply new technique and Methods.

Difference Between
Industrial Engineer & Production Manager

Industrial Engineer is concerned with the Design, improvement and Installation of Production System. His main objective is to increase productivity by Eliminating unproductive operations and improving the effective utilisation of Man, Power & Resources.

Production Manager is concerned with planning, organising and controlling the activities of Production system.
His main objective is to produce goods & services of right quality and quantity at pre-determined ~~type~~ o time and cost

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He is mainly concerned with Day to Day working of Production System.

Name of Sciencists: Important from objective Point

1. F.W. Taylor: Father of Industrial Engineering and He is primarily concerned with the concept of Productivity

2. Adam Smith: Laid the foundation of Scientific Manufacturing and Introduce the concept of Division of Labour.

3. Henry L. Gantt: He developed a Wage Incentive Plan and Gantt chart for Production Scheduling.

4. Gilberth: (Therbligs): Advancement of Motion study is contributed to Gilberth.

Date :

5. L.H.C. Tippet : Developed the concept of Work Sampling (Time study)

6 George B. Dantzing: Regarded as Father of Linear programming

7. Henry Ford : Introduce the concept of Mass Production.

COST IN PRODUCTION

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1. Prime or Direct Cost := Direct Material + Direct Labour + Direct Expenses
   - Part of Final Product
2. Factory Overhead or Expenses = Indirect Material + Indirect Labour + Indirect Expenses.
   - (Not part of final Product but necessary for conversion Process)
   - Indirect Material includes Cotton, jute, lubricant, Cutting fluid
   - Indirect Labour : Watchman, Supervisor, Higher Officer
   - Indirect Expenses : Rent, Electricity bill, Telephone bills etc.

3. Factory Cost = Prime Cost + Factory O/H Cost (Cost at the Outlet of the Factory)

4. Total Cost = Factory Cost + Marketing, Advertising and Transportation Cost.

5. Selling Cost = Total Cost + Profit

5. Selling Cost = Total Cost + Profit

BREAK EVEN ANALYSIS (BEA)

Factors of Profit:

1. Total Cost
2. Selling Cost or Total Sales
3. Volume of Production.

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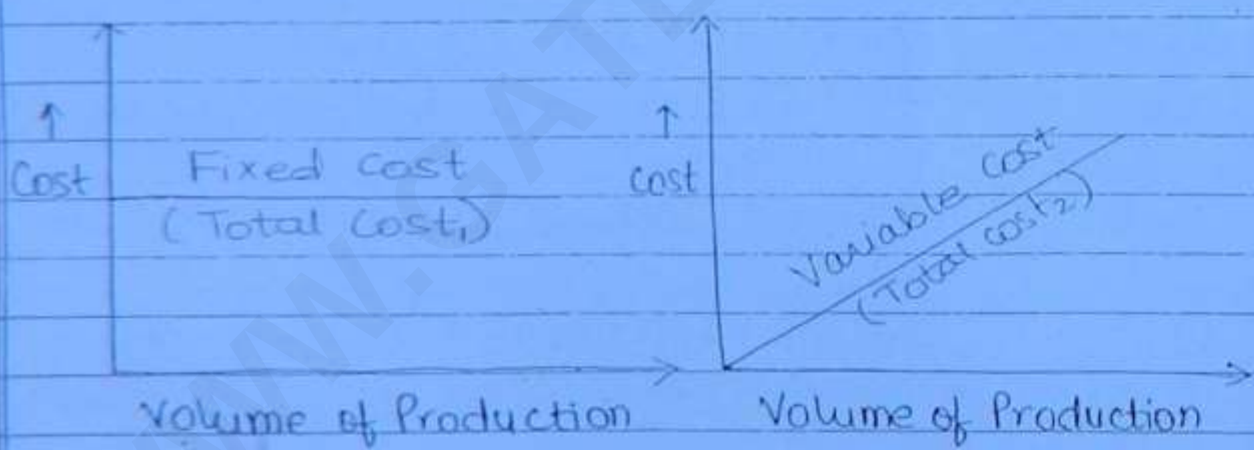
- BEA is the relation b/w the above three.

- It is a technique to study the relation between Total Sales, Total Cost and Volume of Production.

- Total Cost of a product consists of Fixed and Variable Cost



a) Fixed Cost : It remains Fixed or constant irrespective of volume of Production and it includes

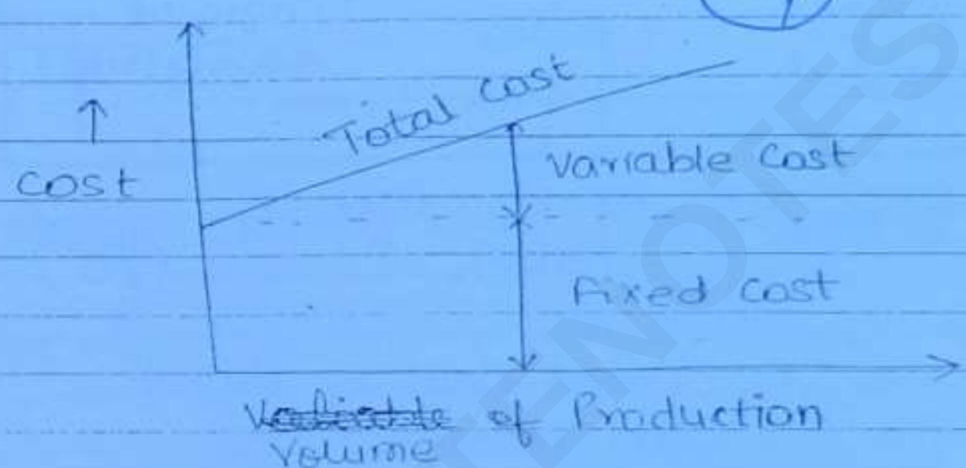
b) Fixed Cost : It includes Salaries of Higher officers, Supervisors, Rent of building, Insurance cost, cost

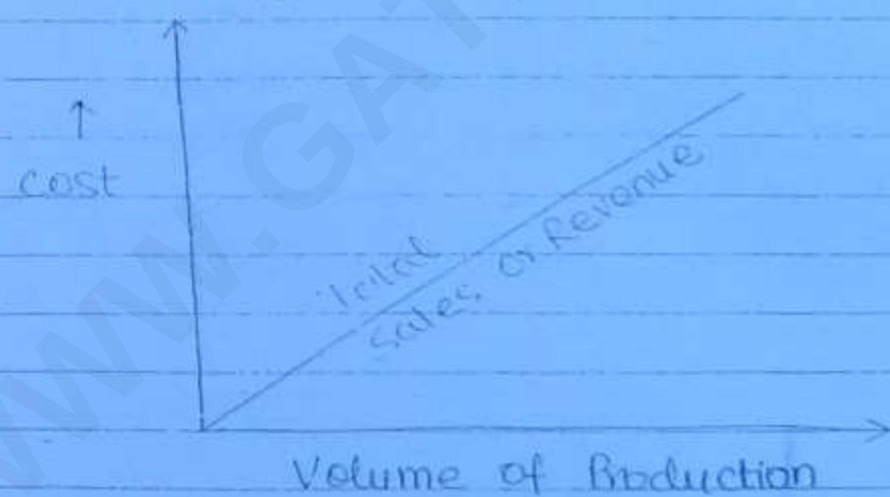
Date :

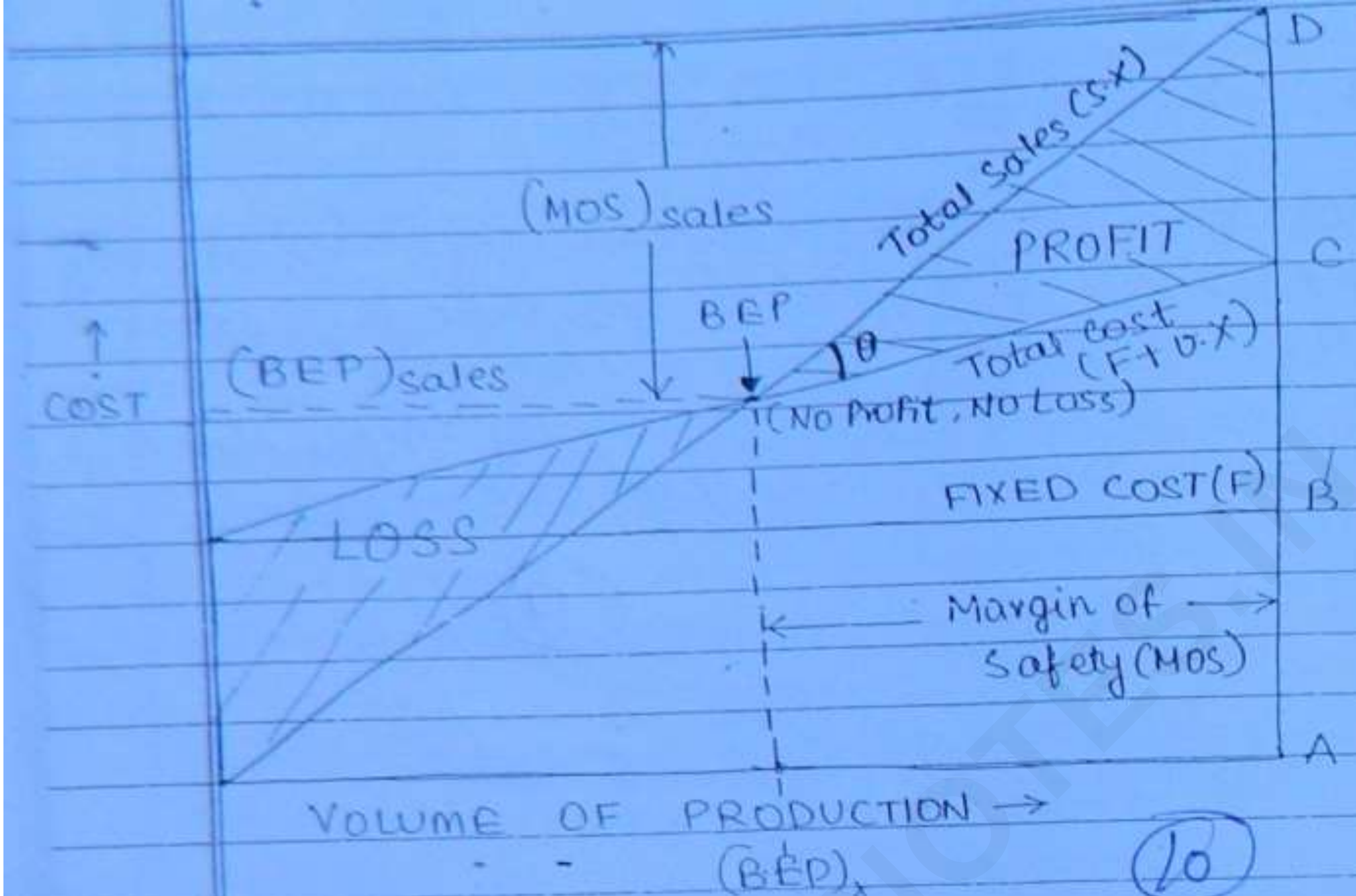
of Machine, taxes etc.

b) Variable Cost: These vary directly & Proportionally with Volume of production and there is a constant ratio between the change in cost and the change in the level of output normally Direct Material Cost and Direct Labour Cost are included in Variable Cost

(9)







Let 'F' : Fixed Cost in Rupess

X : No. of Units produced

v : Variable Cost per Unit (Rupees / Unit)

V : Total Variable Cost ($= v \times X$)

s : Selling Cost per Unit (Rupees/Unit)

S : Total Sales or Revenue ($= s \times X$)

P : Profit by producing Units 'X'.

Total Cost = F.C + V.C

$= F + v \times X$

Total Sales $= s \times X$.

Also Total Sales = Total Cost + Profit

$$* \boxed{s \times X = (F + v \times X) + P} *$$

$(s - v) X = F + P$

$$\Rightarrow \boxed{X = \frac{F+P}{s-v}}$$

For Profit

$\boxed{\text{Selling Cost} > v}$

θ: Angle of Incidence b/w Total Sales & Total cost
(Can't be more to compete other companies)

Break Even Point : may be defined as that level of Sales at which total sales = Total Cost and the organisation Neither Earns profit Nor Suffer from Loss.

At BEP, Profit, P=0

$$\boxed{(BEP)_x = \frac{F}{s-v}}$$

$(BEP)_{sales} = (BEP)_x \cdot S = F + v\,(BEP)_x$

$$\boxed{(BEP)_{sales} = \left(\frac{F}{s-v}\right) S}$$

Angle Of Incidence: It is the angle between the total sales line and the total cost line

- Larger the angle better the working conditions as profits are made at a faster rate

Contribution Margin: It is the difference between Total Sales − Total Variable cost

$$\boxed{CM = S - V = F + P}$$

Also, $S = F + V + P$

$S - V = F + P$

→ It is also known as Marginal Profit or Gross Margin.

From Fig: $CM = AD - BC = AB + CD$

PROFIT - VOLUME GRAPH

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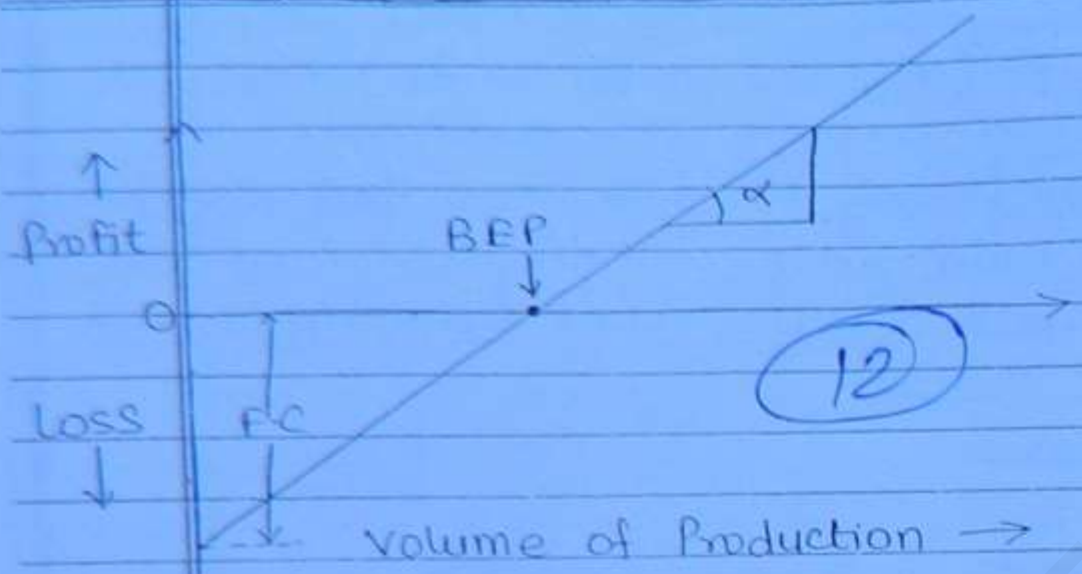

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$$s \cdot x = F + v \cdot x + P$$

$$\boxed{P = (s - v)x - F} \rightarrow (1)$$

Profit - Volume Ratio : (P/V ratio)

P/V Ratio gives relation between profit & Volume of Production and it measures the Profitability in relation to sales

$$P/V \text{ Ratio} = \frac{\text{Total Sales} - \text{Total Variable Cost}}{\text{Total Sales}}$$

$$\boxed{(P/V)_{Ratio} = \frac{S - V}{S} = \frac{CM}{S} = \frac{F + P}{S}}$$

$$(P/V)_{Ratio} = \frac{F}{(S)_{B.E.P}} \qquad [\because \text{At B.E.P, } P = 0]$$

$$\boxed{(P/V)_{Ratio} = \frac{F + P_x}{S_x}}$$

Margin Of Safety (M.O.S) :-

(A) Sales Wise

It is the difference of output at any point compare to output at BEP

$$(MOS)_{sales} = (Sales)_x - (Sales)_{BEP}$$

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$$= S\,x - S\left(\frac{F}{S-V}\right)$$

$$= S\left[\frac{(S-V)x - F}{(S-V)}\right]$$

From ①

$$(MOS)_{sales} = S\left[\frac{P}{S-V}\right]$$

$$\boxed{(MOS)_{sales} = \frac{P}{\left(\frac{S-V}{S}\right)} = \frac{P}{(P/V)_{Ratio}}}$$

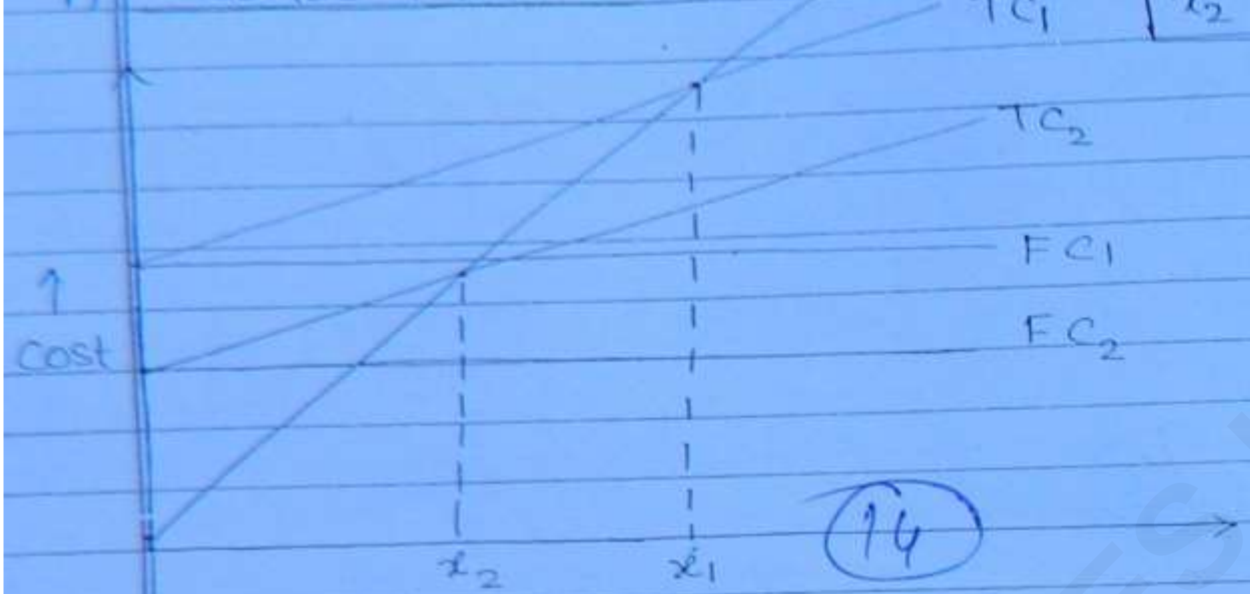
(Rupees wise)

(B) Percentage Wise

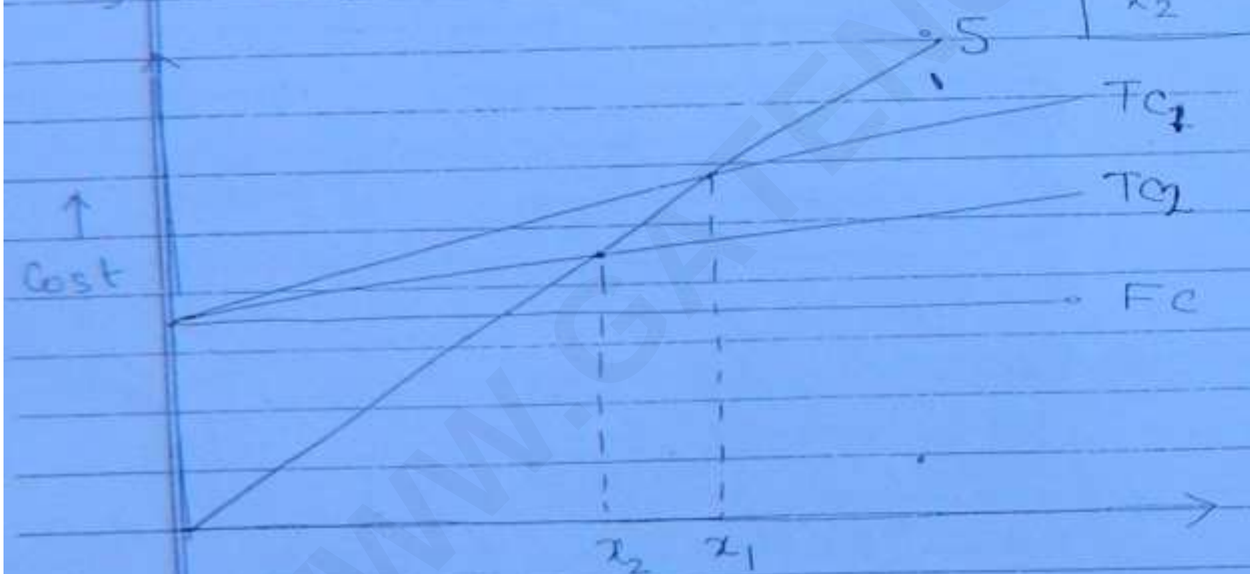
$$(MOS)\% = \frac{(Sales)_x - (Sales)_{BEP}}{(Sales)_x} \times 100$$

$$(MOS)\% = \frac{(MOS)_{sales}}{(Sales)_x} \times 100$$

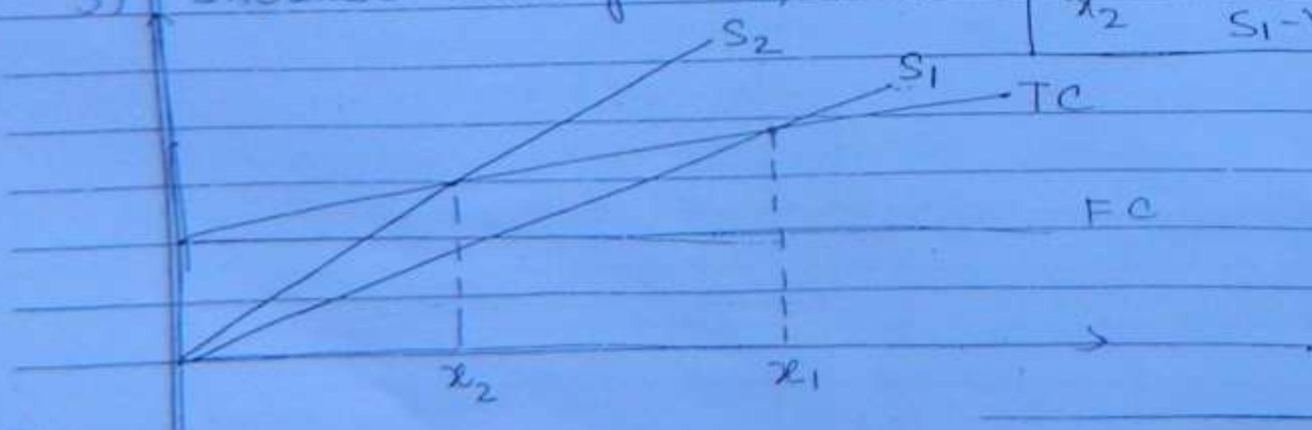
1) Reduce Fixed Cost: $\boxed{\dfrac{x_1}{x_2} = \dfrac{F_1}{F_2}}$



2) Reduce Variable Cost / unit: $\boxed{\dfrac{x_1}{x_2} = \dfrac{S-V_2}{S-V_1}}$



3) Increase selling cost / unit: $\boxed{\dfrac{x_1}{x_2} = \dfrac{S_2-V}{S_1-V}}$



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INVENTORY

Inventory is the stock of any item or resources that are used in any organisation It may be defined as stock in hand at a given time and it may be held for the purpose of later use or sale

It is a usable but idle resource having an economic value and it may include Raw material, work in process inventory, semi-finished inventory or finished goods

Imp * **Purpose Of Inventory** (15)

1. It helps in smooth and continuous running of Production system.
2. Helps to meet the variation in demand pattern.
3. It provides a safe gaurd against the variation in raw material delivery time.
4. Bulk purchasing may give quantity discount.
5. Gives better service level to the customers

Inventory Cost

1. Purchase or Nominal Cost
2. Ordering or Setup Cost: Bringing to the Producⁿ system
3. Holding or Carrying Cost : Storing & Holding
4. Shortage or Back Order Or Stock Out Cost

1 Purchase or Nominal Cost. (P.C)

It is the cost of purchasing Inventory item and it depends upon the quantity or Volume to be purchased

P.C = No. of Units purchased × Cost Per unit

4) Ordering or Set-up Cost (O.C)

→ It is the cost associated with bringing Inventory item to the production system and it includes cost associated with processing and chasing of Purchased order, tendering cost, quality inspection cost, transportation cost etc.

It is also Known as Procurement Cost.

→ When the Units are produced internally, then the cost associated with bringing production system again into play is called Set-Up Cost.

It includes schedule chart preparation cost, labour and raw material, arrangement cost, Maintenance cost of Machines etc.

$$\boxed{O.C = \text{No. of Orders} \times \text{cost / order}}$$

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5. Holding or Carrying Cost (H.C)

- It is the cost spend in storing and Keeping Inventory items within production system.

- It includes storage cost, Damage & Depriciation cost, Handling cost, Insuarance cost, Logged up capital interest etc.

- It depends upon the quantity to be stored and the period for which it is stored.

$$\boxed{H.C = \text{Average Inventory cost} \times \text{Holding cost/unit}}$$

4 Shortage Cost (S.C)

- Shortage simply means absence of Inventory items and the cost associated with not serving the customer is called shortage cost

$$\boxed{S.C = \text{No. of Units short} \times \text{shortage cost/Unit}}$$

- only used for Large cost (like Mercedees, Benz) and where Holding Cost is more

Inventory Classification 17

1. Transit or Pipeline or Movement Inventory

Those inventory items which are in transportation can not provide service and that inventory item is called Transit or Pipeline or Movement Inventory

2 Buffer or Safety Stock Inventory

It is the minimum amount of Inventory item i.e kept for protecting against the fluctuation in Demand and Lead time.

These are not used under normal working conditions and used only to prevent stock out during adverse situation.

Adverse Situation: Actual demand is more than the Average demand or Actual Lead time is more than the Average lead time

Lead Time :-

Lead time is the time gap between placing an order and the inventory item in hand so that it can be consume.

3 Seasonal Inventory :-

These inventories are build-up to meet the future demand of a product due to seasonal variation

4. Anticipation Inventory :- 18

These inventory items are build up due to Anticipated demand in future like vig selling forecast, Govt. Policy change, strike or plant shutdown, price high etc.

Characteristics of Inventory

A Dependent & Independent Demand Item

1. Dependent Demand Item

The need for any one item is a direct result of the lead for some other item, usually of a higher level of which it becomes a part

2 Independent Demand Item

These are those items whose demand is not directly linked or related to any other item.

B. Inventory Review System.

1. Q-System or Fixed Order System or Re-order Point system or Two bin system

→ In this system, a reorder point is fixed and as Inventory level reaches that point a fresh order for a fixed quantity is placed at that time. In this, size of order is fixed while period of order is not fixed.

2. P-System or Periodic Review System or Fixed Period System

→ In this system, Inventory is reviewed after a fixed period of time and fresh orders are placed at that time. In this, Period of order is fixed while size of order is not fixed.

3. S-S system: Combination of Q & P-system

→ Now-a-days S-S system are used.

* Q-system is best over S-system as it gives a warning before stock out

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C. Probalistic and Deterministic Models

1. Deterministic Model

In these models, Demand & Lead time are constant & so we need not to carry safety of buffer stock

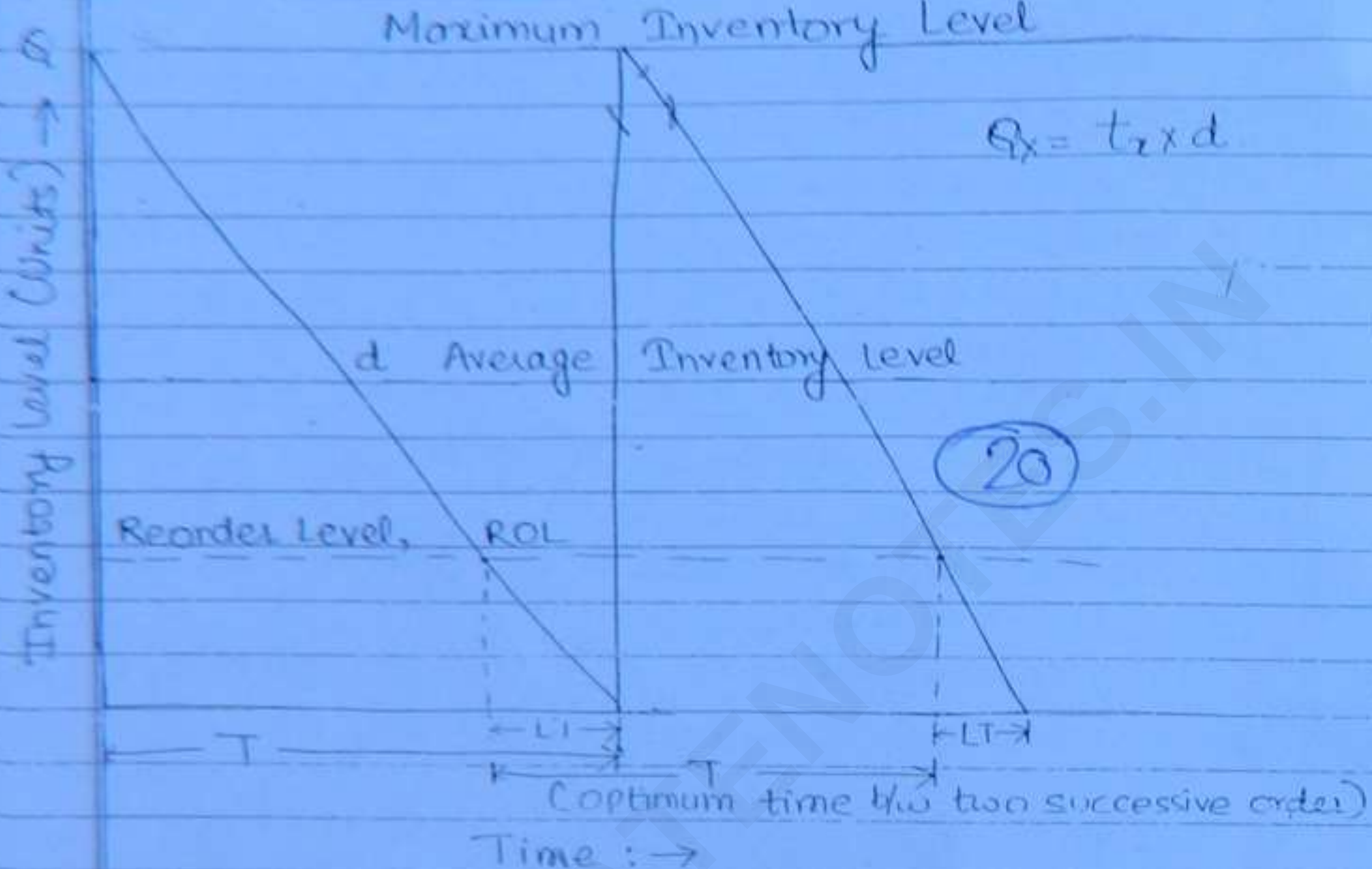
2. Probalistic Model

In these models, demand & Lead time are variable and they need not to be constant so, we need to carry safety or Buffer stock to prevent the stock out.

C.1* Deterministic Model

Economic Order Quantity, [EOQ] or Harris Wilson model

MODEL #1



D: Annual Demand of Inventory Item (Units/Year)

Q: Quantity to be ordered at each order point (Units/order)

C: Unit price of Inventory item (Rs/Unit)

C_0: Cost of placing an order (Rs/order)

C_h: Cost of Holding inventory of an item for 1 year (Rs/unit/Year)

Total Annual Cost: Purchase cost + ordering cost + Holding cost

$P.C = D \cdot C$

$O.C =$ No. of orders $\times$ Cost/order $= D/Q \times C_0$

$H.C =$ Average Inventory for the year $\times$ Holding cost/unit/Year $= \frac{Q}{2} \times C_h$

Date :

Total Annual Cost $= D \cdot C + \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h$

Q is only Variable, one ↑ and another dec ↓
so, the summation of O·C & H·C should be minimum so that Total Annual cost minimum.
Q → Order Size

Total Inventory Cost (TIC) = O·C + H·C
or Variable

$$\boxed{TIC(Q) = \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h}$$

By Graphical Method.

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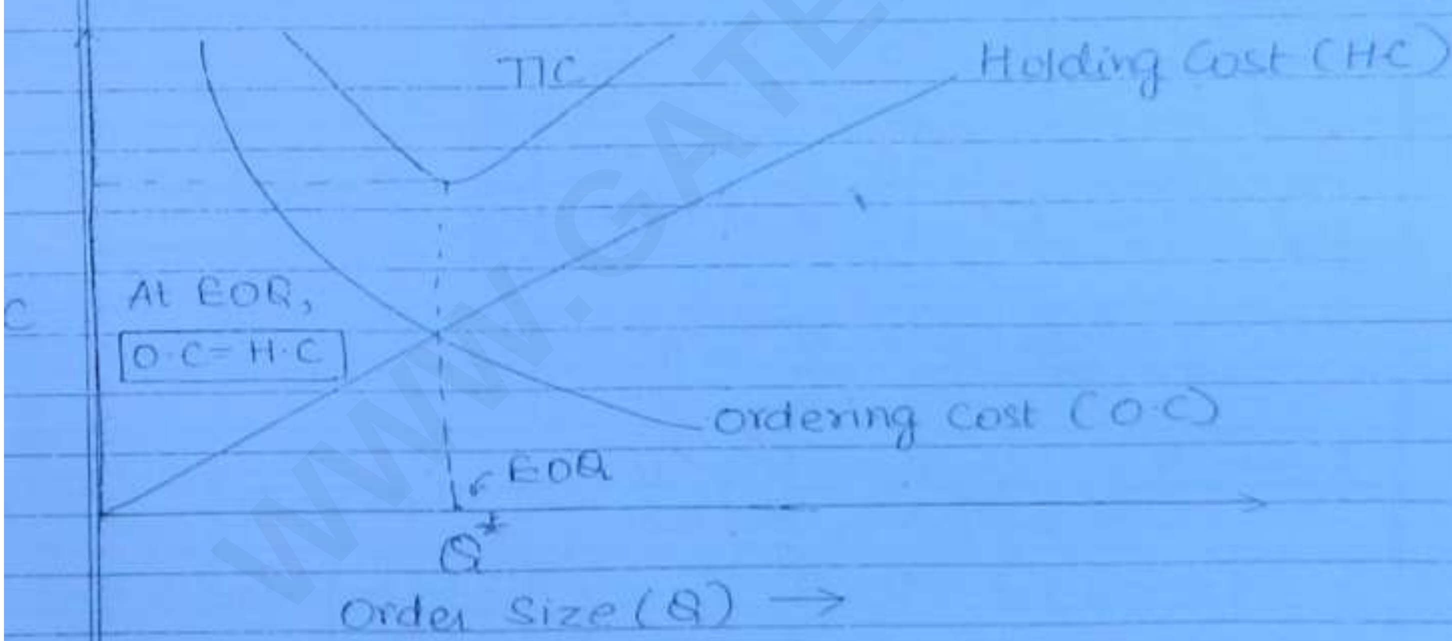

The Order quantity (Q^*) at which H·C = O·C and Total Inventory Cost is minimum is known as Economic Order Quantity.

At EOQ, O.C = H.C

$$\frac{D}{Q^*} C_o = \frac{Q^*}{2} \cdot C_h$$

$$Q^{*2} = \frac{2 D C_o}{C_h}$$

$$\boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}}}$$

2. By Analytical method. 22

$$TIC(Q) = \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h$$

$$\because \frac{d}{dQ}[TIC(Q)] = 0, \text{ minimum if } \frac{d^2}{dQ^2} TIC(Q) = +ve$$

$$\frac{C_h}{2} - \frac{D}{Q^{*2}} \cdot C_o = 0$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

$$TIC = \frac{D}{Q^*} \cdot C_o + \frac{Q^*}{2} C_h$$

$$\text{At EOQ} = \frac{D}{Q^*} C_o = \frac{Q^*}{2} \cdot C_h$$

$$TIC(Q^*) = 2 \cdot \frac{Q^*}{2} \cdot C_h = \left(\sqrt{\frac{2DC_o}{C_h}}\right) \cdot C_h$$

$$\boxed{TIC(Q^*) = \sqrt{2 D C_o C_h}}$$

Date :

C.1.b ROBUSTNESS OR MODEL SENSITIVITY

It is defined as the ratio of Total Inventory cost at any point to the minimum Inventory cost corresponding to EOQ $= \frac{TIC(Q)}{TIC(Q^*)}$

$\boxed{Q = K \cdot Q^*}$; K less than or greater than one.
(20% less, K = 0.8) (20% more, K = 1.2)

$$TIC(Q^*) = \frac{D}{Q^*} \cdot C_o + \frac{Q^*}{2} C_h$$

At EOQ, $\frac{D}{Q^*} \cdot C_o = \frac{Q^*}{2} \cdot C_h$

$$\boxed{TIC(Q^*) = 2\frac{D}{Q^*} \cdot C_o} \rightarrow (A)$$

At Any other Point, $TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} \cdot C_h$

Now, putting, $Q = K \cdot Q^*$

$$TIC(Q) = \frac{D}{K \cdot Q^*} C_o + \frac{K \cdot Q^*}{2} C_h$$

$$= \frac{D}{K \cdot Q^*} \cdot C_o + K \cdot \frac{D}{Q^*} C_o$$

$$\boxed{TIC(Q) = \frac{D}{Q^*} C_o \left[\frac{1}{K} + K\right]} \rightarrow (B)$$

$B \div A$: * $\boxed{\frac{TIC(Q)}{TIC(Q^*)} = \frac{1}{2}\left[\frac{1}{K} + K\right]}$ *

$TIC(Q) \geq TIC(Q^*)$ and it will be equal when $K = 1$ i.e. $Q = Q^*$

Date :

Formulae :-

1. Reorder Level (R.O.L)

ROL = Lead time × demand rate (Units same)

$$\boxed{ROL = LT \times d}$$

2. Optimum No. of order per year

$$\boxed{N^* = \frac{D}{Q^*}} \quad \frac{\text{Units/Year}}{\text{Units/order}}, \quad \frac{\text{Order}}{\text{Year}}$$

3. Optimum time between two successive order

$$\boxed{T^* = \frac{Q^*}{D} = \frac{1}{N^*}} \quad \frac{\text{Year}}{\text{order}}$$

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4. When holding cost is given in terms of Holding Rate of Unit purchase price.

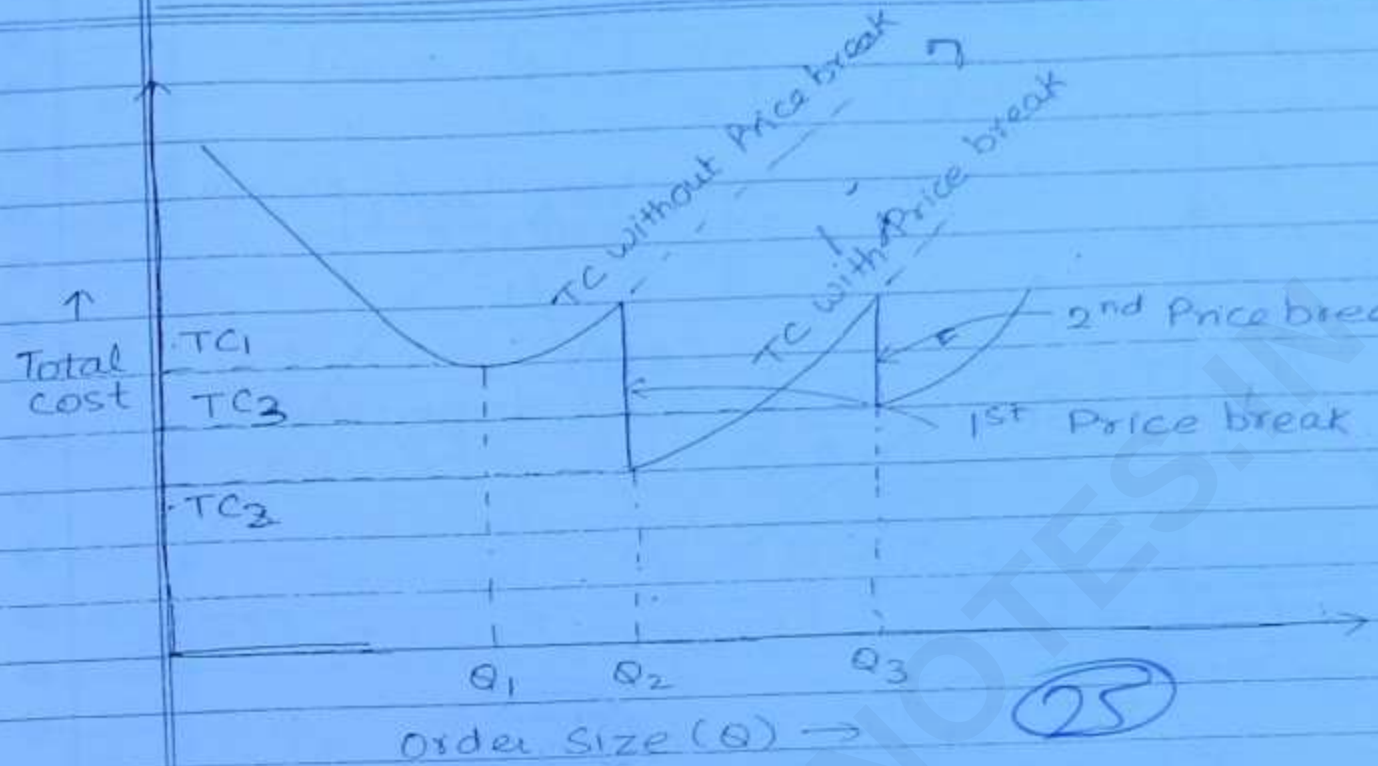
C = Rs 50/unit, C_h = 10% of C/annum.

C_h = 0.1×50 = Rs 5/unit/year

i = %/month (×12 /Year)

v.Imp * MODEL # 2 (IES:12)

EOQ with Price Break or Quantity Discount



$$\text{Total Cost} = \text{Purchase cost} + \text{Ordering cost} + \text{Holding cost}$$

$$= D \cdot C + \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

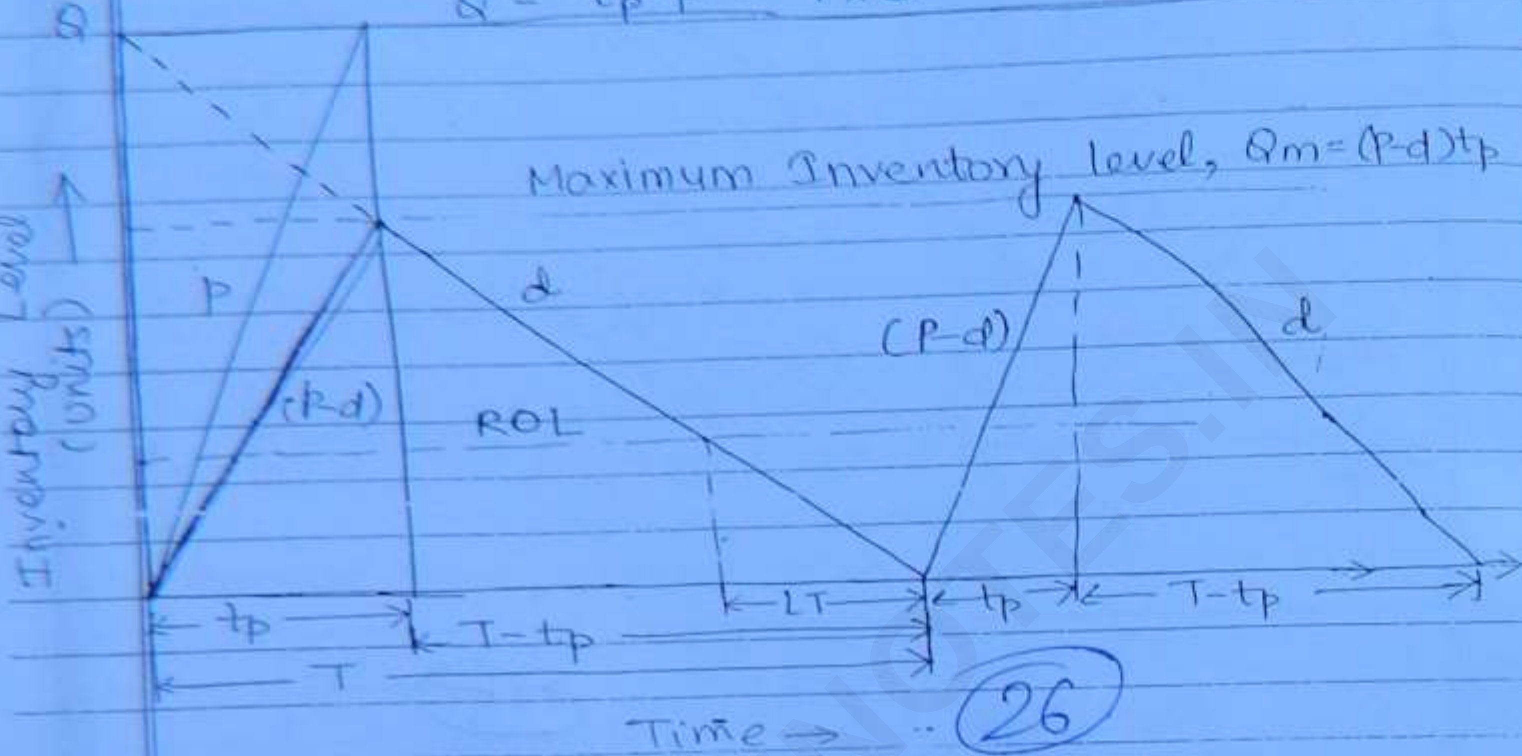
In some situations discount are offered on unit purchase price of inventory and these discounts take the form of price break. Discount is always offered on Unit Purchase Price of Inventory. So, in these Models, Purchasing cost is also considered along with Ordering and Holding cost in determining Best order size.

In this Model, First of all Feasible EOQ is computed and then total cost is calculated at Feasible EOQ and the next Price break points. The order size at which Total cost comes out to be Minimum is the Best Order Size.

MODEL # 3

PRODUCTION or BUILD-UP MODEL

$$Q = t_p \cdot P = T \times d$$



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This Model is similar to first Model EOQ, the only difference that Inventory build up is gradual rather than instantaneous.

if P: Production rate, d: consumption/demand rate

Then, Total Inventory cycle, T is divided into two parts :-

a) t_p (Production or Manufacturing time)
It is the time during which there is production and inventory builds up at the rate of (P-d) units per unit time

b) $T-t_p$ (I
It is the time during which there is no ~~consumpti~~ production and only consumption with a demand rate of 'd' units per unit time

Date

$$Q_m = t_p(P-d)$$

$$Q = t_p \cdot P \Rightarrow \boxed{t_p = Q/P}$$

$$\boxed{Q_m = \frac{Q(P-d)}{P}}$$

$$Q_{average} = \frac{\text{Area of } \Delta}{\text{Base of } \Delta} = \frac{\frac{1}{2} \times \cancel{T} \times Q_m}{\cancel{T}} = \frac{Q_m}{2}$$

$$\boxed{Q_{average} = \frac{Q_m}{2} = \frac{Q}{2}\left(\frac{P-d}{P}\right)}$$

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$$TIC(Q) = O.C + H.C$$

$$\boxed{TIC(Q) = \frac{D}{Q} \times C_o + \frac{Q}{2}\left(\frac{P-d}{P}\right) \times C_h}$$

$$\frac{dTIC(Q)}{dQ} = 0$$

$$\frac{1}{2}\left(\frac{P-d}{P}\right) \times C_h - \frac{D}{Q^2} C_o = 0$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{P}{P-d}\right)}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}} \sqrt{\frac{P}{P-d}}} \rightarrow \text{Production factor} > 1.$$

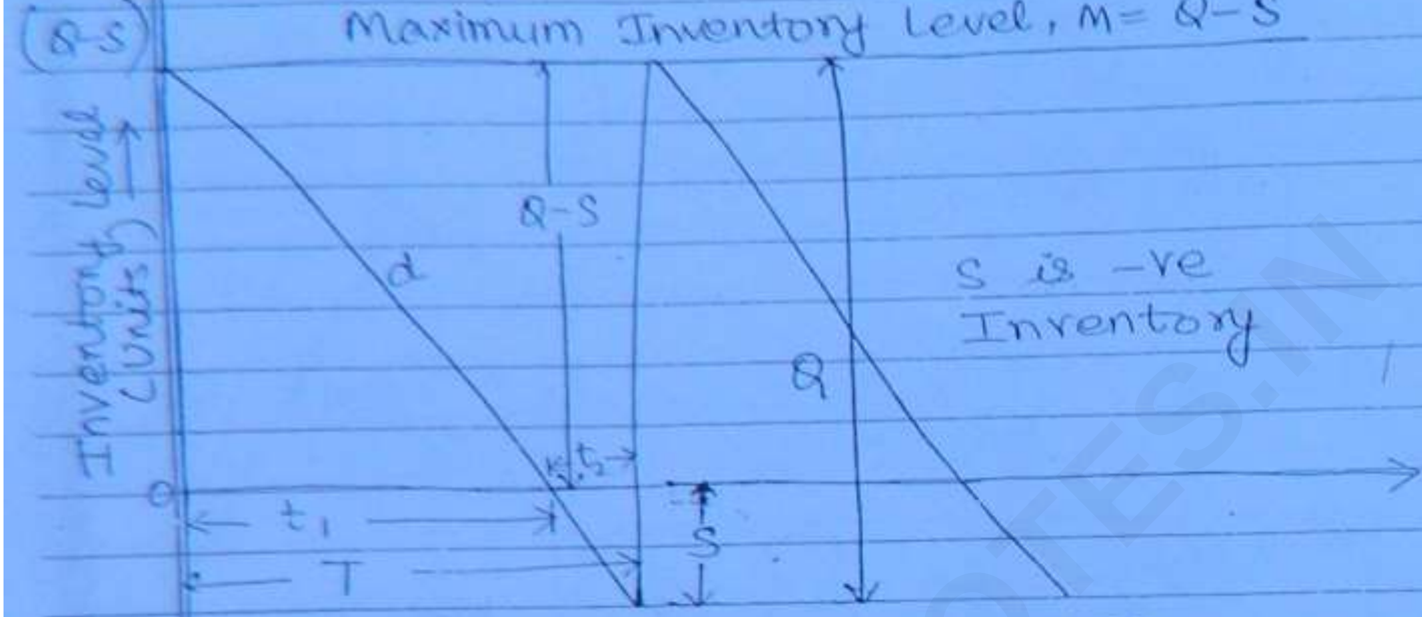
At EOQ, $\Rightarrow$ O.C = H.C

$$\boxed{TIC^* = \sqrt{2DC_oC_h\left(\frac{P-d}{P}\right)}} < 1 \text{ (compare to model EOQ \#1)}$$

MODEL # 4

Shortage or Back Order

Maximum Inventory Level, $M = Q - S$



Time →

This model is extension of First Model EOQ. The only difference that here shortages are allowed.

Planned Shortage or Back order is the situation in which a customer places an order and finds that inventory is out of stock. When the customer wait for the next shipment to arrive to get his order fullfilled.

Shortages also have certain advantages :-

1. shortage results in a decrease net stock in inventory Thus reducing holding cost
2. Shortages increases the cycle time & thus lowers the ordering cost

S: No. of Units back ordered

C_b: Back order or Shortage cost Per unit/bo/year (Rs /unit / Year)

$$TIC(Q) = O.C + H.C + \text{shortage Cost}$$

$$O.C = \frac{D}{Q} \times C_o$$

$$H.C = \frac{(Q-S)^2}{2Q} \times C_h$$

$$S.C = \frac{S^2}{2Q} \times C_b$$

$$\boxed{TIC(Q) = \frac{D}{Q} \times C_o + \frac{(Q-S)^2}{2Q} \times C_h + \frac{S^2}{2Q} \times C_b}$$

from Dia. $\left(\frac{Q-S}{2}\right) \times C_h \times t_1$ 29

$$Q - S = t_1 \times d$$

$$Q = Td$$

$$t_1 = \frac{(Q-S)}{Q} \times T$$

$$\frac{(Q-S)^2}{2Q} \times C_h \times T$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)}}$$

At EOQ, $\boxed{OC = H.C + S.C}$

Put, $C_b = \infty$, to make it EOQ Model # 1.
that's why we don't kept shortage coz
shortage cost is very high in model # 1.

$$\boxed{TIC^* = \sqrt{2DC_oC_h\left(\frac{C_b}{C_h + C_b}\right)}}$$

Date :

Optimum No. of Units backordered, $S^* = ?$

First differenciate & $= 0$

$$\left(\frac{Q^* - S^*}{2}\right) C_h = \frac{S^*}{2} \cdot C_b$$

$$\frac{Q^* - S^*}{S^*} = \frac{C_b}{C_h}$$

$$\frac{Q^*}{S^*} = \frac{C_b + C_h}{C_h}$$

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$$\boxed{S^* = Q^* \left(\frac{C_h}{C_b + C_h}\right)}$$

and Maximum Inventory Level, $M^* = Q^* - S^*$

$$\boxed{M^* = Q^* \left(\frac{C_b}{C_h + C_b}\right)}$$

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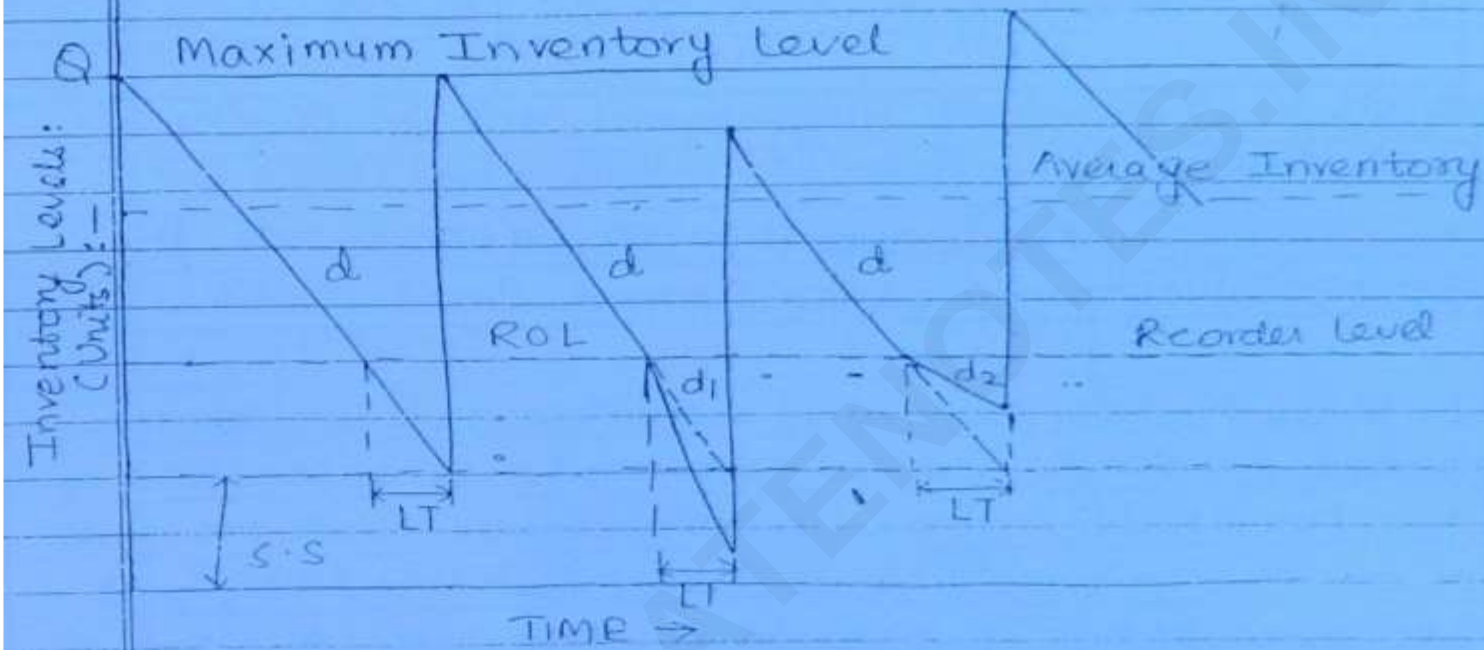
C.2 PROBALISTIC MODEL

In these Models, Demand & lead time fluctuates and they need not to be constant.

In these Models we carry safety or Buffer stock in order to prevent stock out during unfavourable conditions.



Factors Encouraging Higher Safety Stock

1. When demand rate variation is more
2. When lead time fluctuates
3. When Inventory Carrying Cost is less and is of not much concerned
4. When stock out cost or loss due to stock out is very high.
5. When No. of orders in a year are more.
6. To provide better service level to the customers at a short notice.

Reorder Level = Average demand during Lead Time + Safety stock

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$ROL = ADDLT + SS$

$$\boxed{ROL = LT \times d + SS}$$

$d_1 > d$

$$\boxed{\text{Average Inventory} = \frac{EOQ}{2} + S.S} \quad d_2 < d$$

safety stock Cost = $S*S \times C_h$

~~Technique~~ **MODEL #1**

Static Inventory Model or Demand Profit Model.

In these Models, Demand is uncertain and decision is based on single order. This model is applicable for perishable items like vegetables, fruits, flower etc or for those items which becomes outdated very fast like newspaper, faishon goods. items etc. Also known as Newspaper boy problem or Christmas Tree Problem.

In these models, Either replacement orders are not possible or become very expensive.

D: Demand of Inventory Item
S: Supply or quantity to be ordered.
p: Profit per unit
l: loss per unit

1. Overdemand / Under supply
   $D > S$ $\quad P(D-S)$

2) over supply / Under demand

$S > D$ $\qquad\qquad$ $l(S-D)$

Then choose the Order Quantity 's' such that

$$\boxed{P(S-1) < \frac{p}{p+l} \leq P(S)}$$

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$P(S-1)$ is the cummulative probability of the demand for $(S-1)$ Units and $P(S)$ is the cummulative probabilty of the demand for 'S' units.

$$\boxed{P(S-1) < \frac{C_b}{C_b+C_h} \leq P(S)}$$

MODEL # 2

Service Level or Safety Stock

- When stock out cost is not known

- In these Models, the amount of Safety stock is kept according to the level of Service, Management, aims to achieve

$$\boxed{\text{Service Level} = \frac{\text{No. of Units supplied w/o delay}}{\text{Total No. of Units demanded}}}$$

Service level = In number = 0 - 1 (in fraction)

In Percentage = 0 - 100%

95% Service level would mean that 95% of the customers order are fullfilled and 5% of the orders are rejected due to stock out during Lead Time.

When the demand during lead Time may be approximated by a normal distribution curve with a certain mean. ($\bar{X}$ or μ) and standard deviation (σ). Then, the Reorder level is given by:-

$$\boxed{ROL\,(x) = \bar{X} = Z \cdot \sigma}$$

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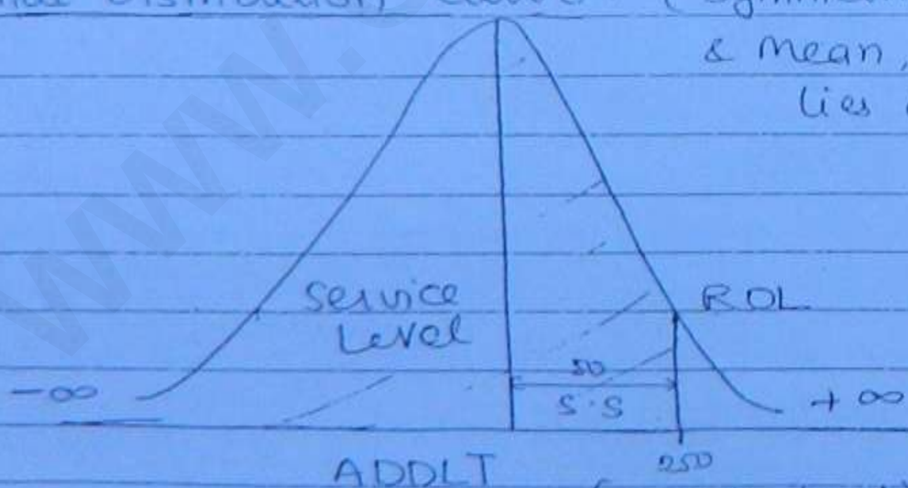
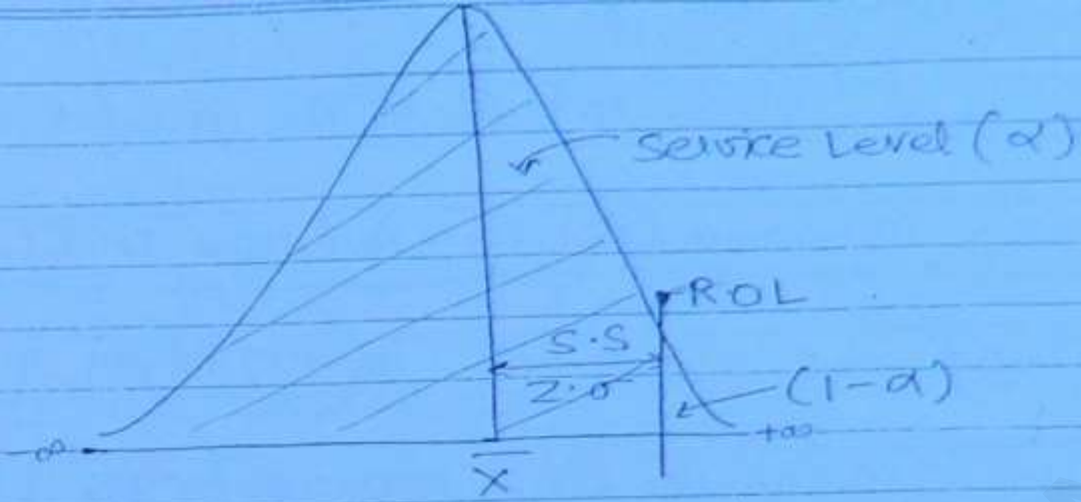
Where, Z is the Standard Normal Variante whose value depends upon the level of service, management wants to achieve.

Z	Service Level (%)
0·84	80%
1·28	90%
* 1·645	95% → Standard Value
2·33	99%

Normal Distribution Curve :- (Symmetric about Y axis & Mean, Mode & Meadian lies at one Point)



$\bar{X} =$



ADDLT = LT × d

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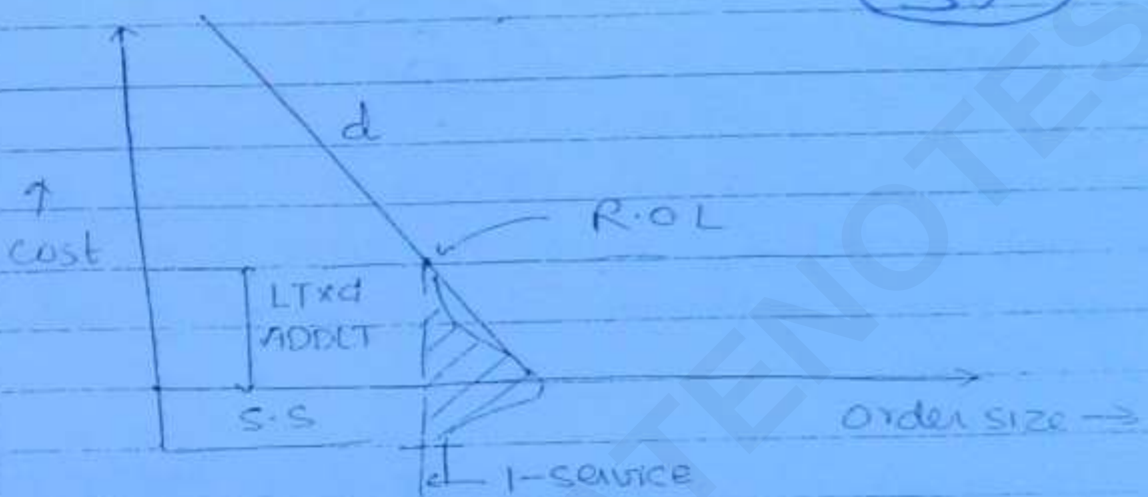

Service level shows in EOQ model # 1

(S.S) Safety stock = $Z \cdot \sigma$

$$\boxed{X = \bar{X} + Z \cdot \sigma}$$

$\bar{X} \rightarrow$ ADDLT = LT × d.

For a half cycle, standard deviation, σ_1
For next half, standard deviation, σ_2

$\sigma^2 = \sigma_1^2 + \sigma_2^2$ $\boxed{\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}}$

$$\boxed{S \cdot S = Z \cdot \sigma}$$

Date :

a) If Demands are given: 208, 242, 216, 233, 211

S.S = 242 − Average of the given demands

$\boxed{SS = \text{Maximum DDLT} - \text{Average DDLT}}$

b) If Lead time are given: 10, 14, 13, 19, 9

$\boxed{S.S = (\text{Maximum LT} - \text{Average LT}) \times d}$

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Inventory Controls

- Inventory items are given different priority for their control & management because some inventory items have more importance compared to other inventory items

Imp

1.) A B C Control (80-20 Law) [Always Better Control]
(based on Pareto Law or Pareto Curve)

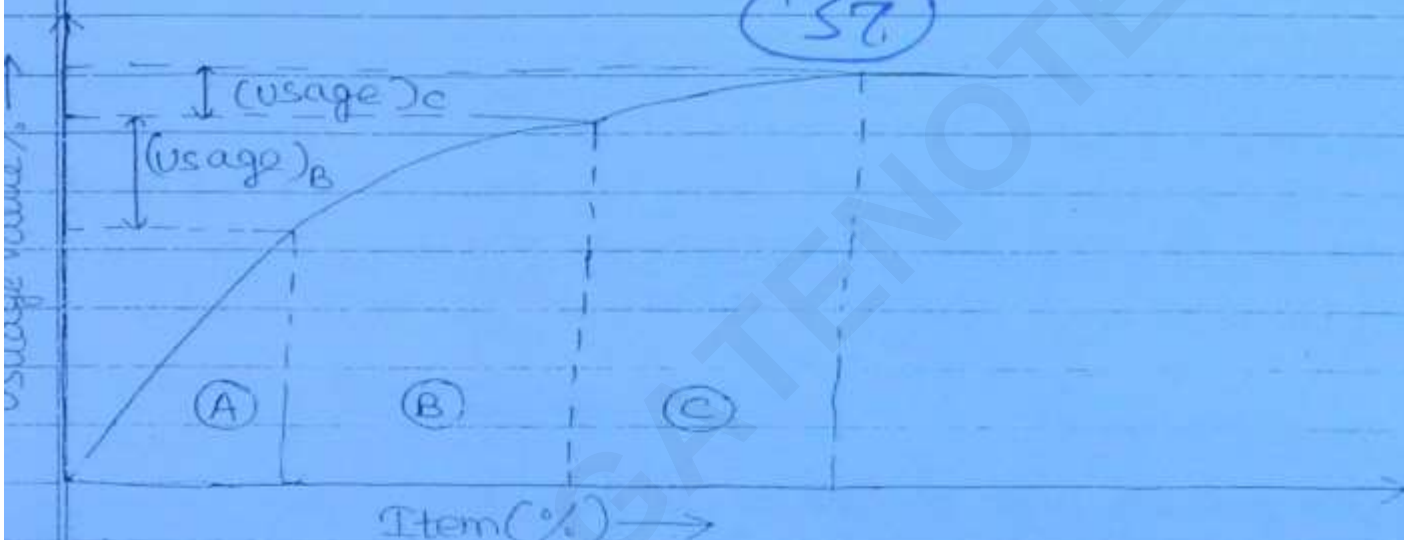
	Usage %	Item %
A	50-60%	10-20%
B	30-40%	30-40%
C	10-20%	50-60%

Item	Unit (₹/unit) Price	No. Required (units/year)	Usage Value (₹)	Usage %
	a	b	a×b	$\frac{a \times b}{c} \times 100$

$\Sigma a = c$

In this system, Inventory ~~sy~~ item are classified on the basis of usage value of Inventory Item in monetary terms,

Inventories are classified into A, B & C categories. For 'A' category inventory item, Inventory is kept almost nill and frequent review is done on the other hand For 'C' category item, Large inventory is kept & it is reviewed after long duration



VED Control (Vital, Essential & Desirable)
(depends on the criticality of items)

- Classified on the basis of Criticality of Inventory Items for production system

HML Control (High, Medium & Low)

- Classified on Unit purchase price of Inventory.

Date :

4) SDE control [Scarce, Difficult & Easily Available]

- Classified on the easiness or difficulty of availability of Inventory item.

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5) Seasonal - Offseasonal

- Classified on the availability of Inventory based on seasonal variation

SEQUENCING

Sequencing problem comes when we are concerned with situation where is a choice as to the order in which a no. of task on jobs can be performed so that utilisation of man, power and Machine can be optimised and their idle time is minimised.

Assumptions:-

1. One job on one machine at a time.
2. Given processing order for the machine will remain constant
3. Each job once started on a machine must be fully completed.
4. The processing time for the different jobs remains constant irrespective of their order.
5. Time taken by job from one machine to other is negligible.

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TERMINOLOGY

1. N job on 1 Machine

(39)

1) Job flow time: It is the time from some starting time untill that ~~complete~~ particular job is completed.

2) Make Span time: It is the time from when processing begins on the first job in the set untill the last job is completed.

3) Tardiness: It is the amount of time after its due date that a particular job is completed.

4) $\boxed{\text{Average No of Jobs in System} = \dfrac{\text{Total Job flow time}}{\text{Make Span time}}}$

2. Sequencing Rules for N Job on 1 machines

1) Shortest Processing Time (S.P.T) Rule

In this rule, Jobs are sequenced in order of their increasing processing time i.e least time first & longest time in the last.

2) Earliest Due Date (EDD) Rule

Jobs are sequenced in order of Increasing due date

E.g.

	Jobs	Processing Time	Due date		CR
4	1	11	34	3	34/11
1	2	7	39	5	39/7
5	3	14	31	2	31/14
3	4	9	36	4	36/9
2	5	8	29	1	29/8

Date :

3. Critical Ratio (C.R) Rule

$$\boxed{C.R = \frac{\text{Due Date}}{\text{Processing Time}}}$$

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Jobs are sequenced in order of increasing Critical Ratio.

4. Slack Time Remaining (STR) Rule

$$\boxed{\text{Slack Time} = \text{Due Date} - \text{Processing Time}}$$

Jobs are sequenced in order of Increasing Slack Time Remaining.

2. N Jobs on 2-Machines:-

Machine $\boxed{A}$ $\boxed{B}$ $i = (1, 2, 3 \ldots N)$

Ex:-

Jobs	A_i	B_i	
1	6	9	1, 3, 2
2	11	4	
3	13	6	
	6	13	3, 1, 2
	11	9	
	6	11	
N	13	6	2 1 3
	11	9	
	12	6	
	13	6	2 3 1 or
	11	9	2 1 3
	13	6	

In these problem, only 2 machines are inv
let there processing time are A_i and B
i is from 1, 2, 3 . . . . . N. The problem is
a sequencing order which minimises ideal time
machine and man power. This is done by
Johnson's Rule or Johnson's Algorithm. (only
2 machines)

The steps involved are :- 91

1. Find out the minimum of A_i & B_i.
2. If the minimum is for A_r i.e for the rth job on Machine A. Then perform the rth job first.
3. If the minimum is for B_s i.e for the sth job on Machine B. Then perform the sth job in the Last
4. In case of a tie between A_r & B_s, do the rth job first and the sth job in the Last.
5. Strike of the job, which is assigned so that it cannot be considered again.
6. Continue the process of assigning until all jobs are assigned.

3. Sequencing of N jobs on 3 - Machines :-

Machine → Jobs	A	B	C
	A_i	B_i	C_i
1			
2			
3			
⋮			
N			

Date :

i) $\boxed{\text{Min } A_i \geq \text{Max } B_i}$ ~~any Two~~

ii) $\boxed{\text{Min } C_i \geq \text{Max } B_i}$ any one is satisfied.

X Y

$X_i = A_i + B_i$

$Y_i = B_i + C_i$

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In these problems, there are 3 machines A, B & C with their respective processing time, A_i, B_i & C_i for applying Johnson's Rule, we need to convert these 3 machines into two imaginary machines X & Y. If out of the two conditions, Atleast one is satisfy. The two imaginary machines 'X' & 'Y' are such that $X_i = A_i + B_i$ & $Y_i = B_i + C_i$

We find the sequence of two imaginary machines, then after finding sequence make span time is calculated on the original 3 machines

N Jobs on M- Machines

Date [redacted]

Machine A_1, $\boxed{A_2 \; - - - \; B_i \; - - - \; A_{m-1}}$, A_m

A_{m-2}

Jobs

1

2

3

:

N

CONDITIONS:-

$$\boxed{\begin{array}{l} 1. \ \text{Min } A_1 \geq \max.[A_2, A_3 \ldots A_{m-1}] \\ 2. \ \text{Min } A_m \geq \max[A_2, A_3 \ldots A_{m-1}] \end{array}}$$

Any one out of 2 can satisfy

$$X = A_1 + A_2 + \ldots + A_{m-1}$$

$$Y = A_2 + A_3 + \ldots + A_{m-1} + A_m$$

PERT & CPM

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Project : Project is a group of Inter-related activities which must be executed in a certain order before the entire task can be completed. The activities are related in a logical and sequential order in the sense that some activities cannot start untill all the activities prior to them are completed. When all the activities are executed then only the project is completed.

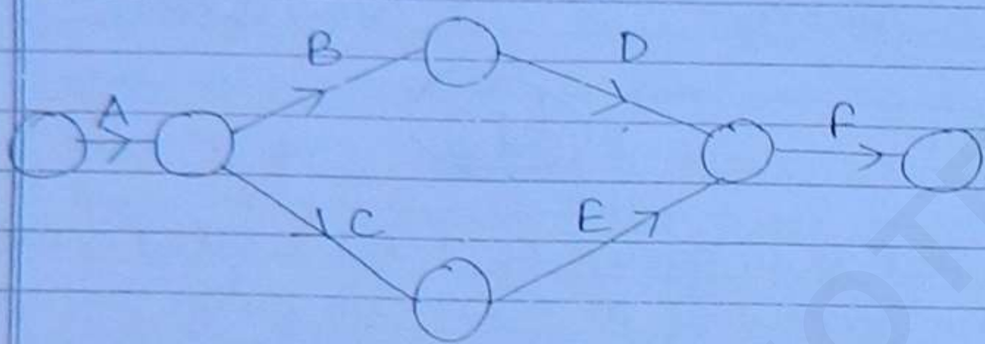
Event : The events are point in time & denotes the begining and the end point of an activity. An Event defines an accomplishment occuring at an instantaneous point of time which neither consumes any time nor resources for its completion.

Activity: It is a recognizable and identifiable part of a project which consumes time & resouces for its completion and may involve physical or mental work.

Network Diagram:- It represent the sequence of different activities that make a project.

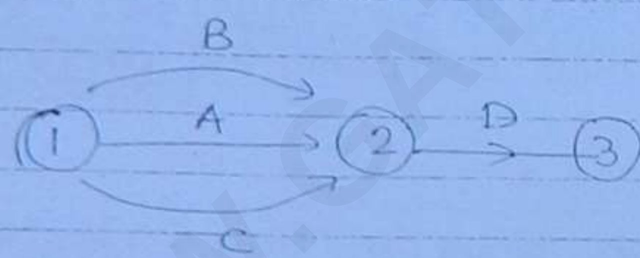
Rules for Network Diagram

1. No activity can be started untill all the activities prior to it has been completed.



2. No two or more activities may have the same tail & head event.

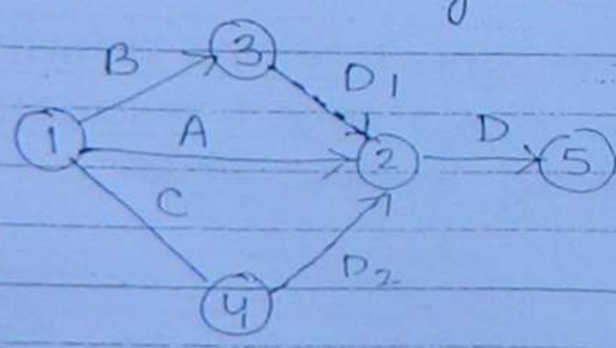
Wrong



A → 1 - 2
B → 1 - 2
C → 1 - 2

This representation is wrong & we have to use dummy.

- In these situation dummy activities are used



A - 1 - 2
B - 1 - 3
C - 1 - 4

Date :

Dummy activity : The activity which only show the dependency of one activity over the other but neither consume any time nor resouces for its completion is called Dummy Activity.

It is shown by dotted line arrow.

3. The time always ~~through~~ flow from Left to Right in the Network Diagram and the length & direc^{n} of arrow is indicative only

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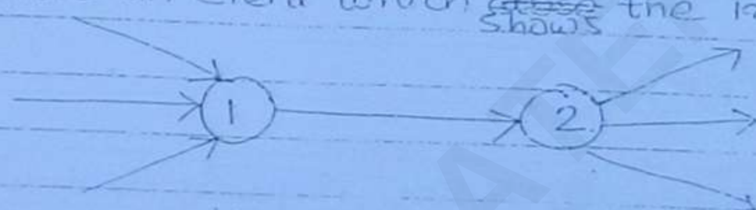
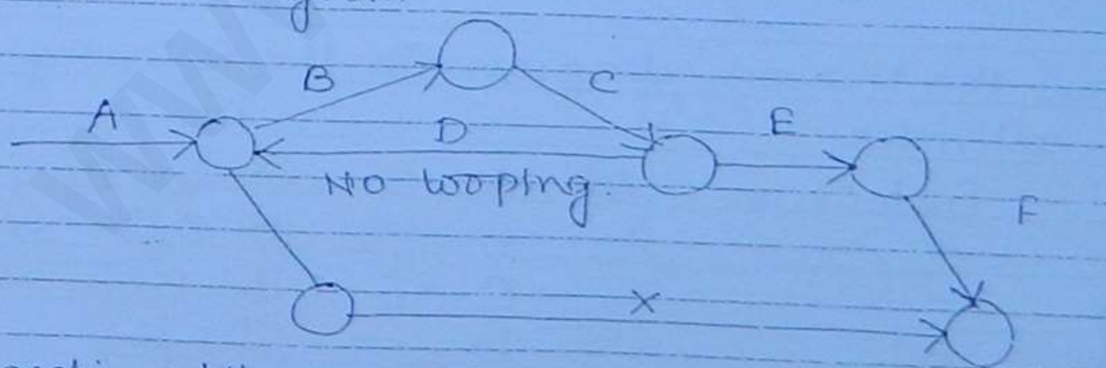
4. Dummy activities should only be used if it is very necessary

5. An event which represents the joint completion of more that one activity is known as Merge Event (1) while an event which ~~close~~ shows the initiation of more than



one activity is known as Burst Event (2)

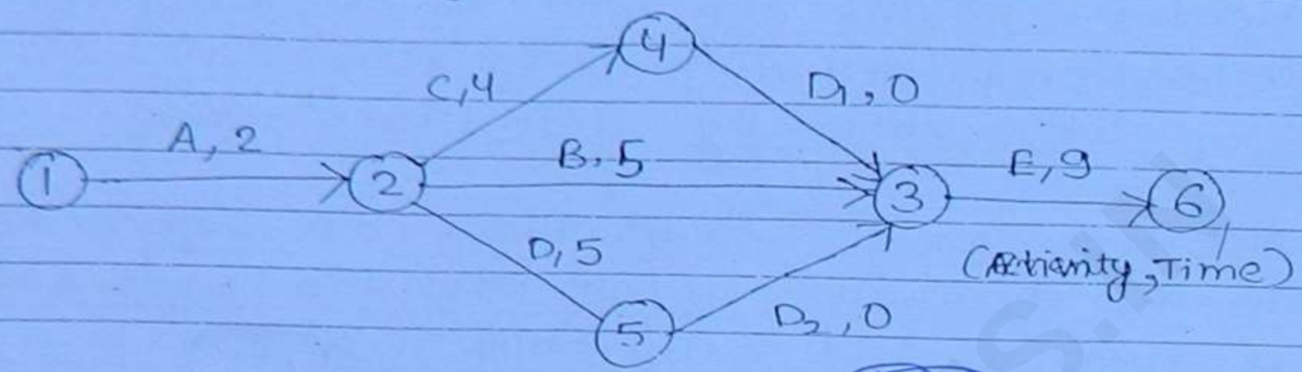
6. There should be No looping & dangling in the Network diagram



Dangling : When activities other than the final activity do not have any successor activity. The situation is called Dangling.

TWO TYPES OF NETWORK DIAGRAM

1. Event on Node (EON) or Activity on Arrow or Arrow Diagram



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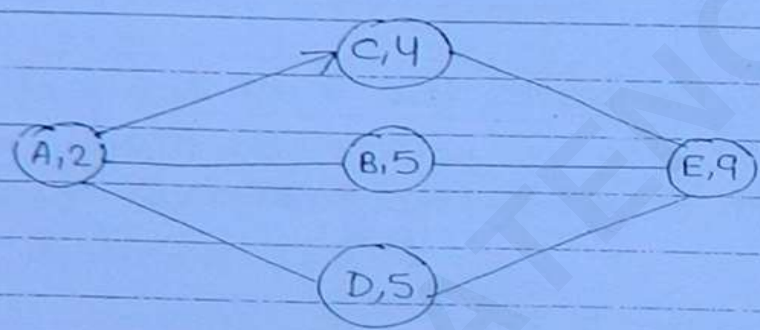
2 Activity on Node (AON)



2. AON: In AON dia. will Never require dummy activities and these are simple to draw and easy to explain. Irrespective of these advantages, EON dia. are more popular in PERT & CPM.

Draw the Network diagram for the following set of activities:

Q1

	Predecessor
A	-
B	-
C	B
D	A
E	C

Q3.

A	—	G	C,F M
B	—	H	D,G
C	—	I	E,G
D	—	J	H,I
E	A	K	D
F	B	L	J,K

F	C
G	F
H	F
I	H
J	I
K	D,E,G,J
L	I
M	K,L

Q2

A	–	I	E
B	–	J	G,E
C	A	K	G,E
D	A	L	H,K
E	B	M	L
F	B,D		
G	C,F		
H	G,E		

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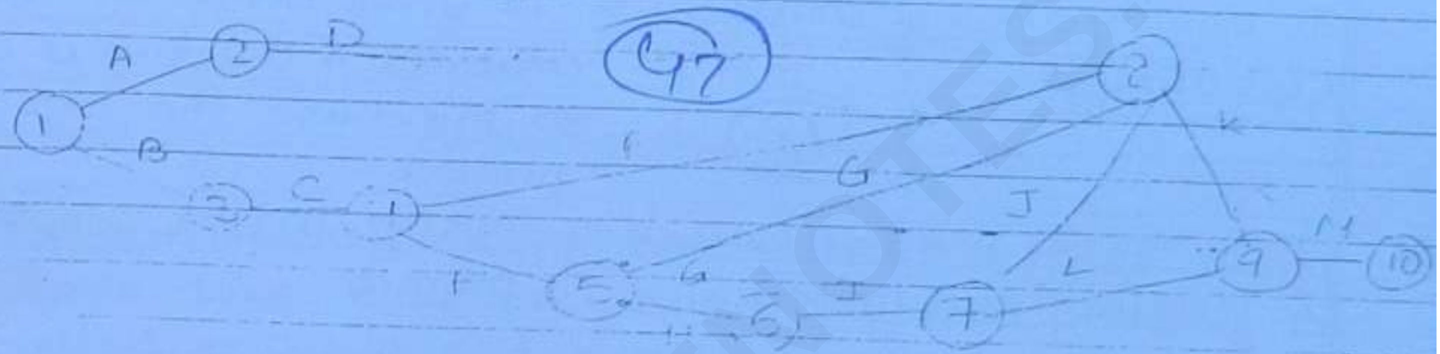
Q1



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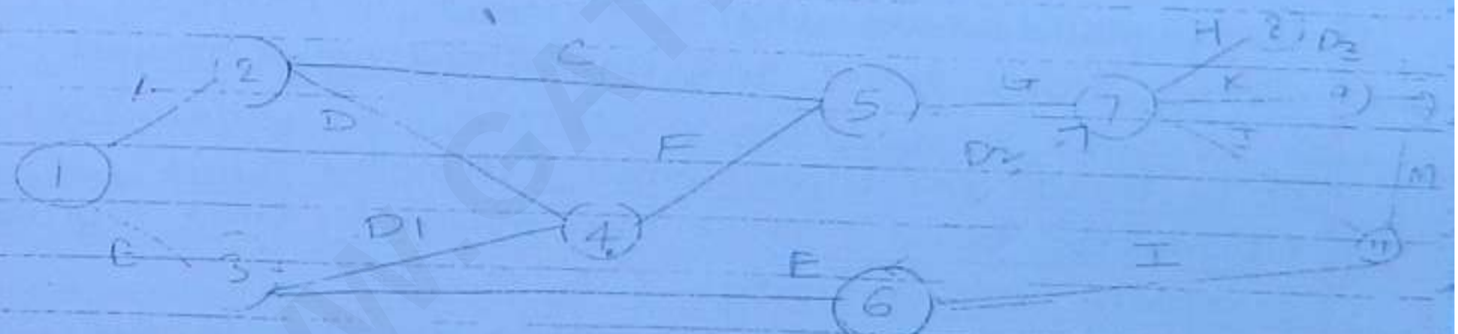

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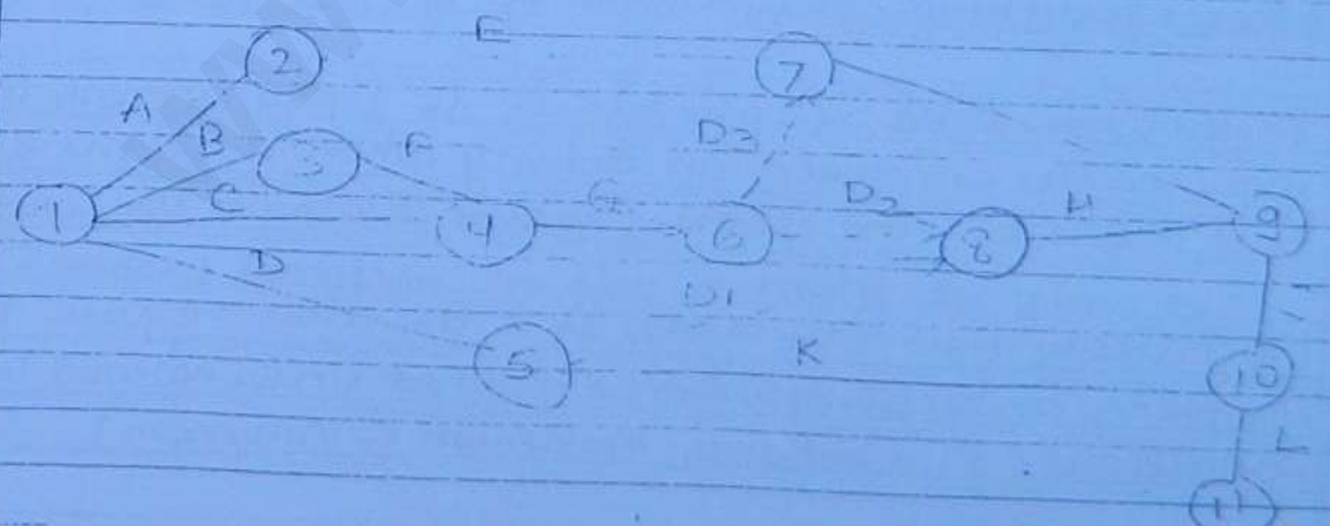

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Difference Between

	PERT	CPM.
1.	It is Event Oriented.	1. It is activity Oriented.
2.	It is based on Prabalistic activities	2. associated with Determini-tic activities.
3.	It is based on 3 times estimate of an activity	3. It is based on single time Estimate for an activity.
4.	It is used where time require to complete various activities is not certain.	4. It is used for repetitive work where one has prior experience of Handling similar project.
5.	It usually doesnot consider Cost analysis.	5. It gives stress on cost analysis & crashing is done to minimise the cost of project.
6	It is mostly used in Research & Development project.	6. Mostly used for construction project.

PERT

- Programme Evaluation & Review Techniques
- When first time we are starting a project

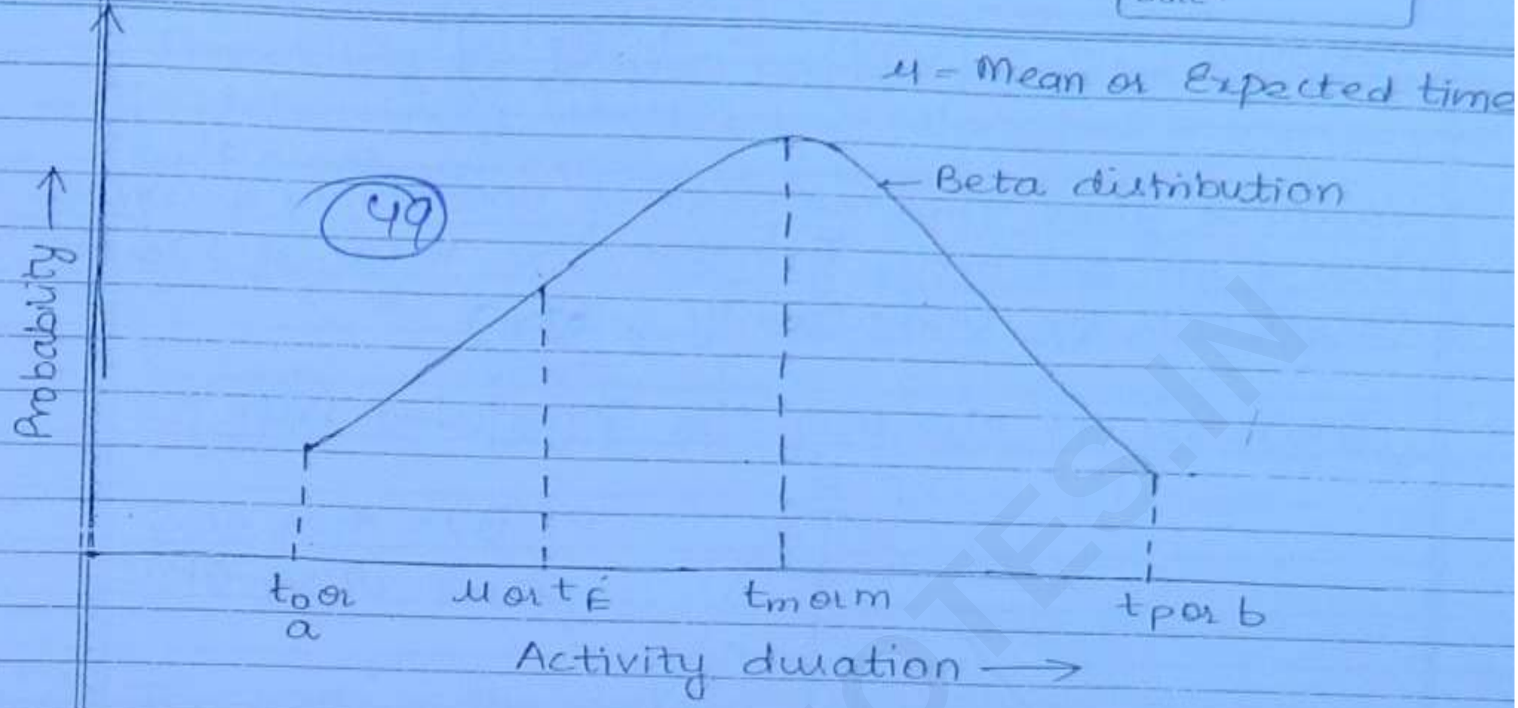
It is based on three - time Estimates :-

a) Optimistic time : It is the minimum time required to complete an activity. (t_o or a)
- When everything goes according to the plan.

b) Pessimistic time : (t_p or b) : It is the maximum time required to complete an activity when everything goes bad, against the plan. [Natural Calamities Not Consider]

c) Most likely time (t_m or m): It is the time require to complete an activity when executed under Normal Working conditions. (in between maximum & minimum)

Activity : Beta distribution
Project : Normal distribution



The Fundamental assumption in PERT is that the Three-time Estimate make the End point of Distribution curve and activity is assume to follow Beta Distribution.

It is also assume that the probability of completing activity in time 'a' & 'b' is equal while the probability of completing activity in time 'm' is Four times of 'a' or 'b'.

Expected Time or Average Time (μ or t_E) of an Activity

$$\boxed{\mu \text{ or } t_E = \frac{a + 4m + b}{6} = \frac{t_o + 4t_m + t_p}{6}}$$

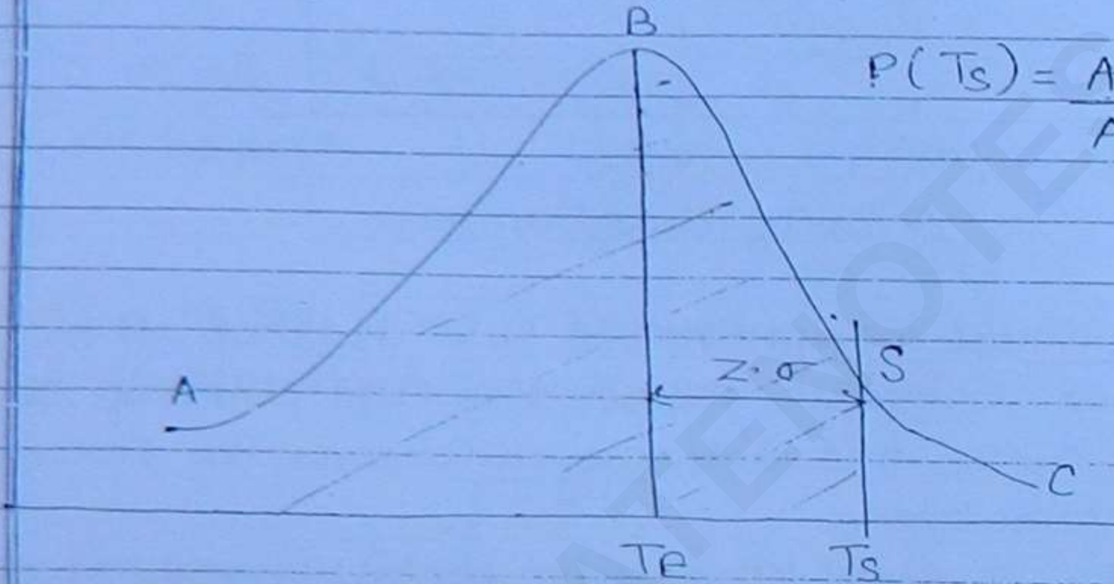
Standard Deviation for the activity is given by:

$$\boxed{\sigma = \left(\frac{b - a}{6}\right) = \left(\frac{t_p - t_o}{6}\right)}$$

$$\text{Variance} = \sigma^2 = \left(\frac{b-a}{6}\right)^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Variance gives the measure of Uncertainty of Completion of an activity. Greater the Value of Variance Larger will be Uncertainty.

Probability of Meeting the Scheduled Date



$$P(T_S) = \frac{\text{Area ABS}}{\text{Area ABC}}$$

Where, T_E is Expected project completion time and is calculated along the critical path.

T_S : Schedule date to complete the project

σ : Standard Deviation along Critical Path.

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

$$\sigma = \sqrt{\text{Sum of Variance along Critical Path}}$$

$$T_S = T_E + Z \cdot \sigma$$

$$\boxed{Z = \frac{T_S - T_E}{\sigma}}$$

Probability of Project completion helps the management in determining whether the allocated resources are sufficient, in-excess or required more.

Probability	upto 0·3 →	Replan the Project
	0·3 - 0·5 →	Close examination
51	0·5 - 0·65 →	Satisfactory
	above 0·65 →	Excess Resources, Replan & Reallocate.

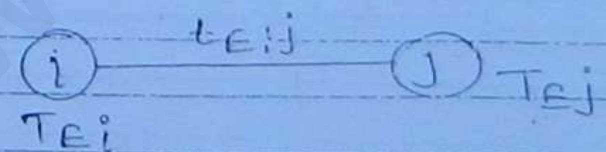
CRITICAL PATH

The path in the network which take the maximum of time is called critical Path and the activities on critical path are called critical activities.

The time taken along the critical path gives the expected project completion time

The procedure for identifying Critical Path is similar both in CPM & PERT and it Consist of Two Phases:-

1) Forward Pass Computation: The time at which an Event is Expected to occur at the earliest is computed in forward Pass computation.



T_E^j = Maximum of all $[T_E^i + t_E^{ij}]$

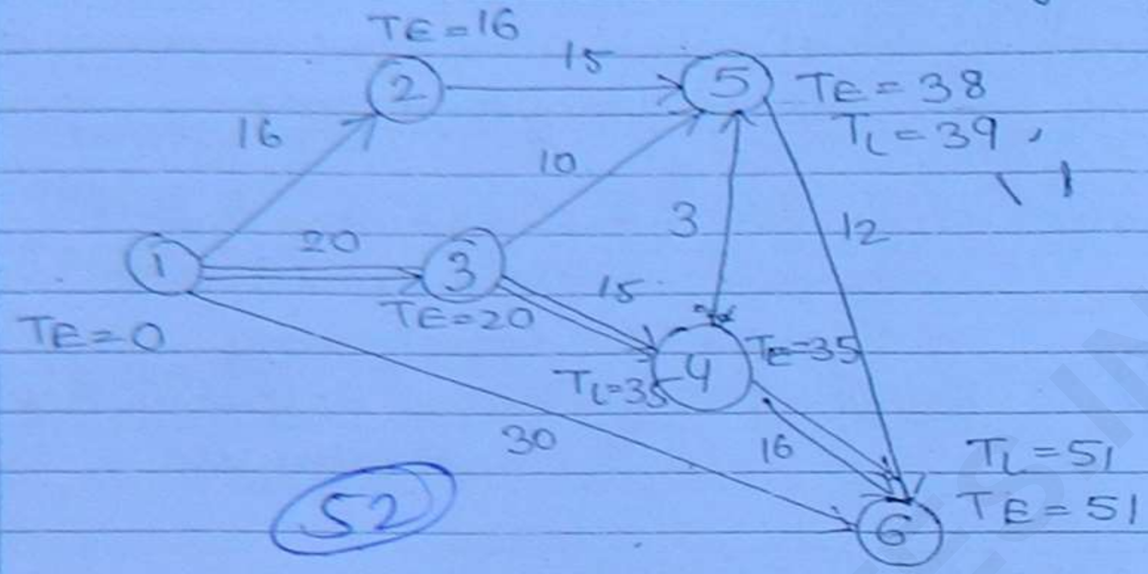
T_E^j is Earliest Expected Time for Event j

T_E^i is Earliest Expected Time for Event i

Date :

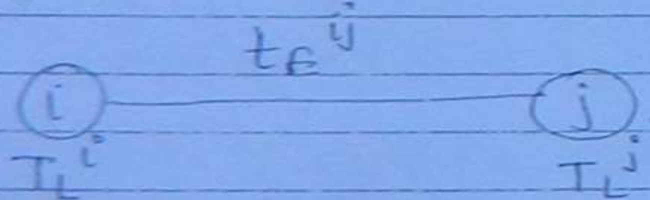
and t_E^{ij} is Expected Time for activity ij

Eg.



Event	Earliest Time.
1	0
2	16
3	20
4	35
5	Max [31, 30, 38] 38
6	Max [50, 51, 30], 51

2) Backward Pass Computation: The time at which a particular Event must occur at the latest is computed by backward pass computation



T_L^i = Minimum of all $[T_L^j - t_E^{ij}]$

T_L^i is Latest Allowable Time for event i
T_L^j is Latest Allowable Time for event j

and t_E^{ij} is Expected Time for activity ij.

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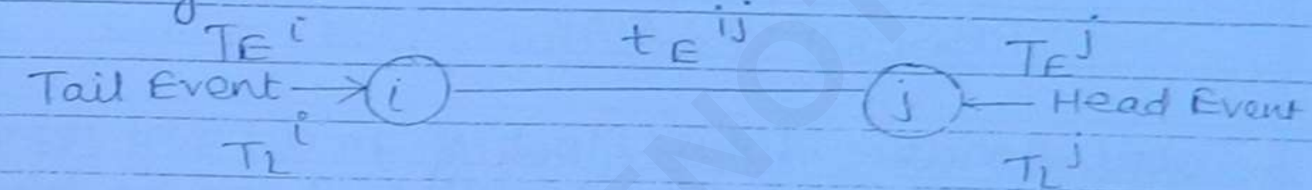
Event	Latest Time
6	51
5	39
4	[51-16, 38-3]; [36, 35], 35
3	[35-15, 39-10], [20, 29], 20
2	[39-15] = 24
1	min [8, 0, 21], 0

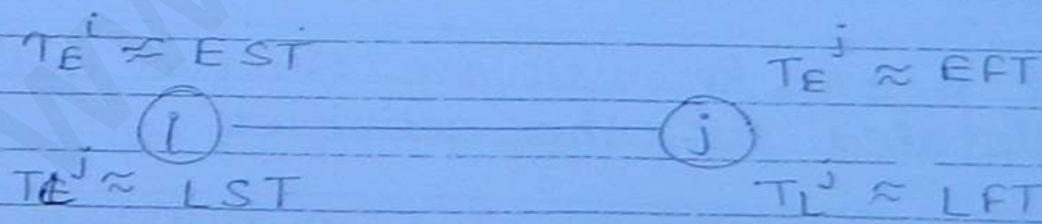
The three conditions for any activity to be Critical Activity is :-



1. Head Event Slack, $T_L^j - T_E^j = 0$.
2. Tail Event Slack, $T_L^i - T_E^i = 0$
3. $T_L^j - T_L^i = T_E^j - T_E^i = t_E^{ij}$

In above Example : 1-3-4-6 is Critical Path

SLACK AND FLOAT



$T_E^i + t_E^{ij} = T_E^j$ $\qquad$ $T_L^j - t_E^{ij} = T_L^i$

$EST + t_E^{ij} = EFT$ $\qquad$ $LFT - t_E^{ij} = LST$

(1) $LFT - EFT = 0$ $\quad$ (2) $LST - EST = 0$

(3) $EFT - EST = LFT - LST = t_E^{ij}$

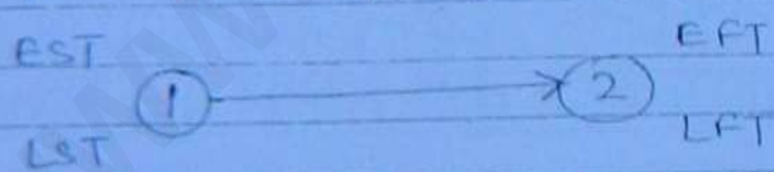
Slack in PERT and Float in CPM both correspond to the amount of time by which a particular event or activity can be delayed w/o delaying the expected project completion time.

The terms like Earliest Expected Time, Latest Allowable Time and slack Corresponds to Event in PERT while the terms like Earliest Start Time and Earliest Finish Time, Latest Start Time, Latest Finish Time and Float correspond to activities in CPM.

TYPES OF FLOAT

1. Total Float :- Total Float of an activity represents the amount of Time by which it can be delayed without delaying the project Completion time. Thus, it is Extra Time Available for an activity w/o affecting the completion of project.

$$TF = LFT - EFT = LST - EST$$



If TF is +ve, Resources are Surplus and can be allocated for other activities

If TF is zero, Resources are just sufficient to complete the activity on time.

If TF is -ve, Resources are not sufficient & Activity may not complete on time.

Date :

2. Free Float : It is that part of the total float which can be utilised w/o affecting the Float of succeeding activities. Thus, it is that value of Float which can be consumed so that succeeding activities starts at their Earliest Start Time.

F.F = T.F − Head Event Slack

3. Independent Float : It is the amount of Float Time which can be used w/o affecting either the Head or Tail Event

I.F = F.F − Tail Event Slack

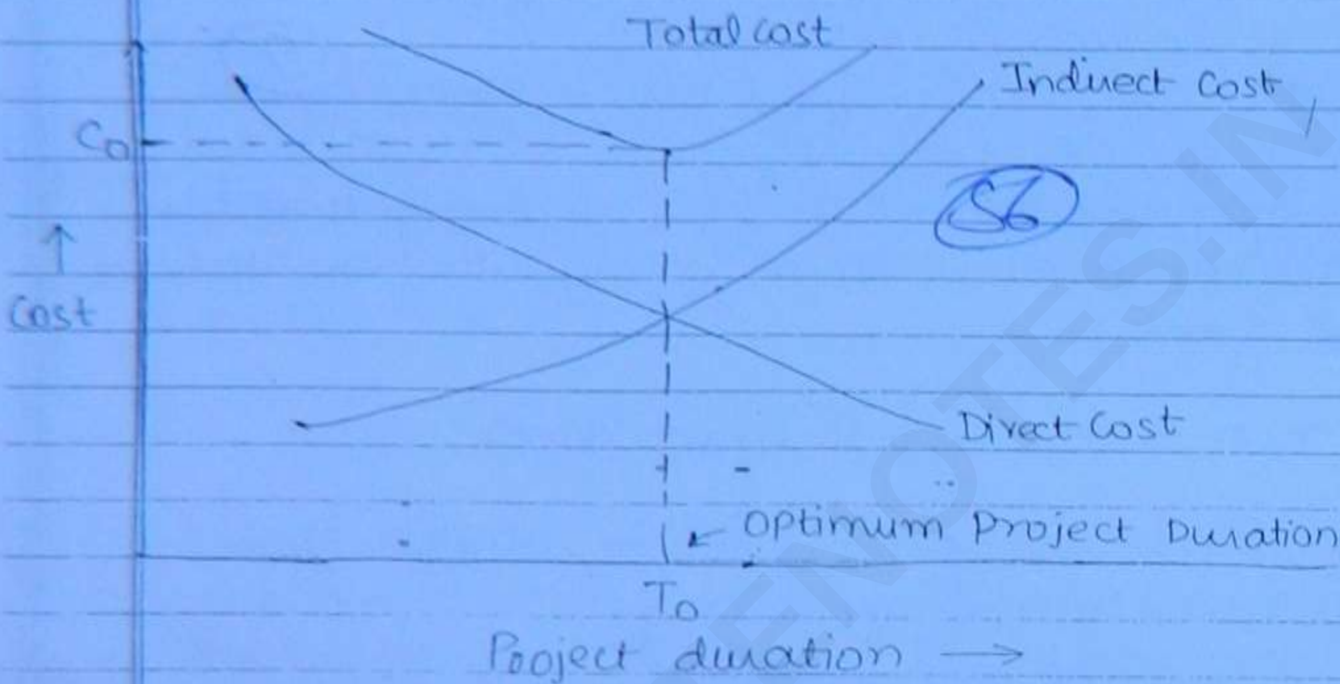
→ If F.F or I.F values come −ve, then take zero coz it doesnot make any sense.

$$\boxed{T.F \geq F.F \geq I.F}$$

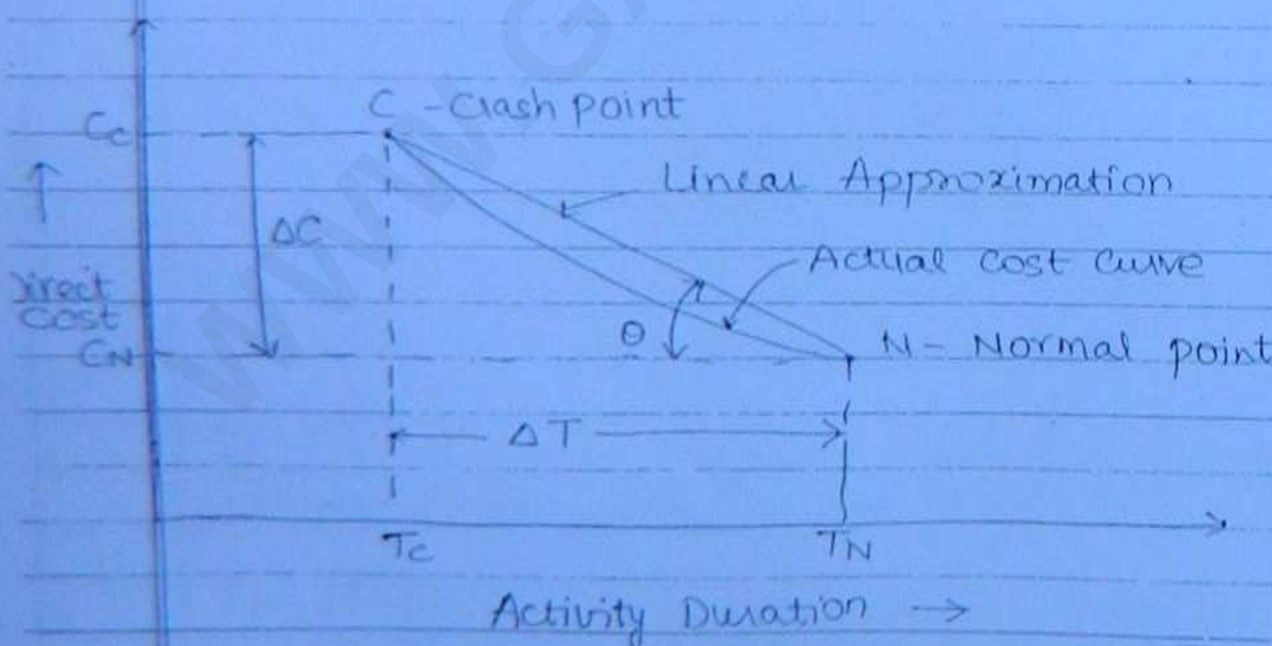
Crashing or Time-Cost Model

Date : [redacted]

It is an extension of critical path Model that consider the compromise between the cost and time required to complete a project(where). The total cost of any project consist of the direct and indirect cost involve in its execution



Direct Cost:



Crash Time is the minimum activity duration to which an activity can be compress by increasing the resources and hence, by increasing the direct cost. The slope of line gives the increase in direct cost per unit time for crashing the activity

$$\text{Cost Slope} = \frac{\Delta C}{\Delta T} = \tan\theta = \frac{C_C - C_N}{T_N - T_C}$$

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Steps for Crashing a Network

The Objective of crashing a Network is to obtain optimum project duration where the total cost is minimum.

The steps involved are:-

1. Draw the Network diagram & determine Critical Path
2. Select the critical activity that has got the Minimum cost slope.
3. Reduce the duration of this activity by one time unit
4. Revise the network by adjusting the time & cost of the crash activity and again calculate critical path
5. If the optimum project duration is achieved then stop otherwise go back to step 1

* FORECASTING *

Date :

Forecasting is the prediction of Future sales or Demand for a product. It is defined as the Estimation of future activities i.e the estimation of time, quality, quantity of Future work. These estimate provide the basis for determining the demand of man power, machines and material in future.

It is not a guess work but a projection based on passed sales figure and human judgement.

Need of forecasting

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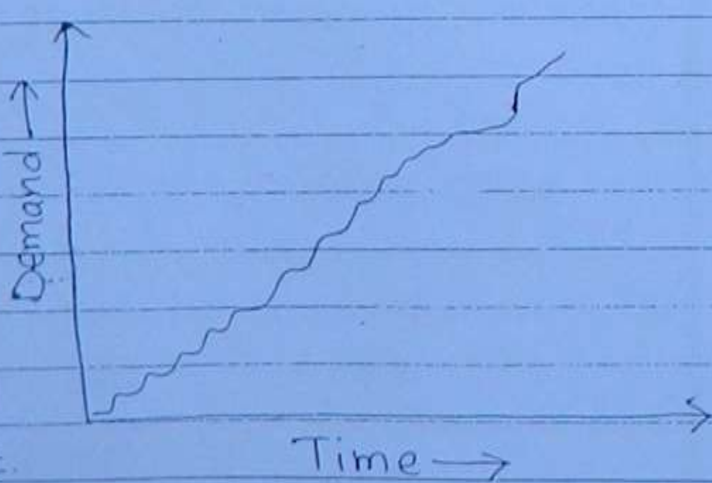
1. It helps in determining the volume of production & production rate.
2. It forms the basis for production budget, Labour budget, material budget etc.
3. It suggest the need for plant expansion.
4. It helps in product design & development.
5. It helps in determining price policies.
6. Helps in determining the extent of marketing, advertisement and distribution required

Types of Demand Variation

1. Trend Variation:

It shows a long term upward or downward movement in the demand or sales of a product. It shows a regular pattern.

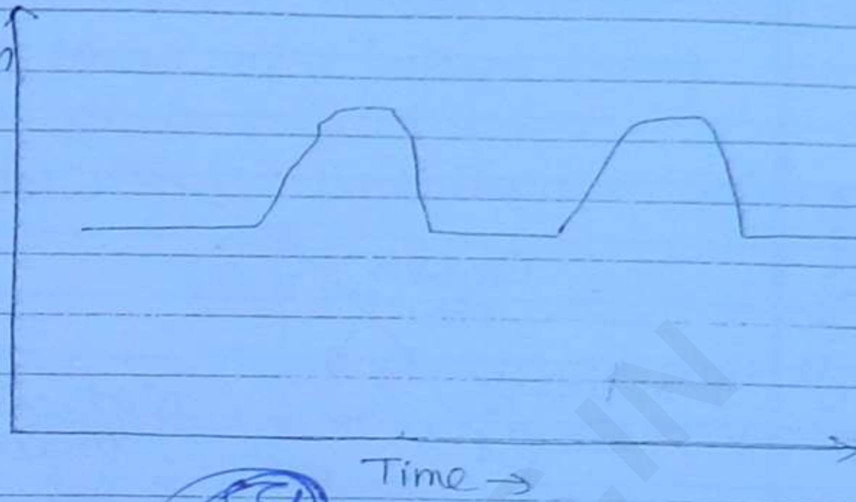


eg Newspaper, Cellphones etc

Date :

2) Seasonal Variation:

It shows a 'short-term regular variation related to the time of a day or day of a week



e.g. Electricity Bill (daily)
movie Tickects etc

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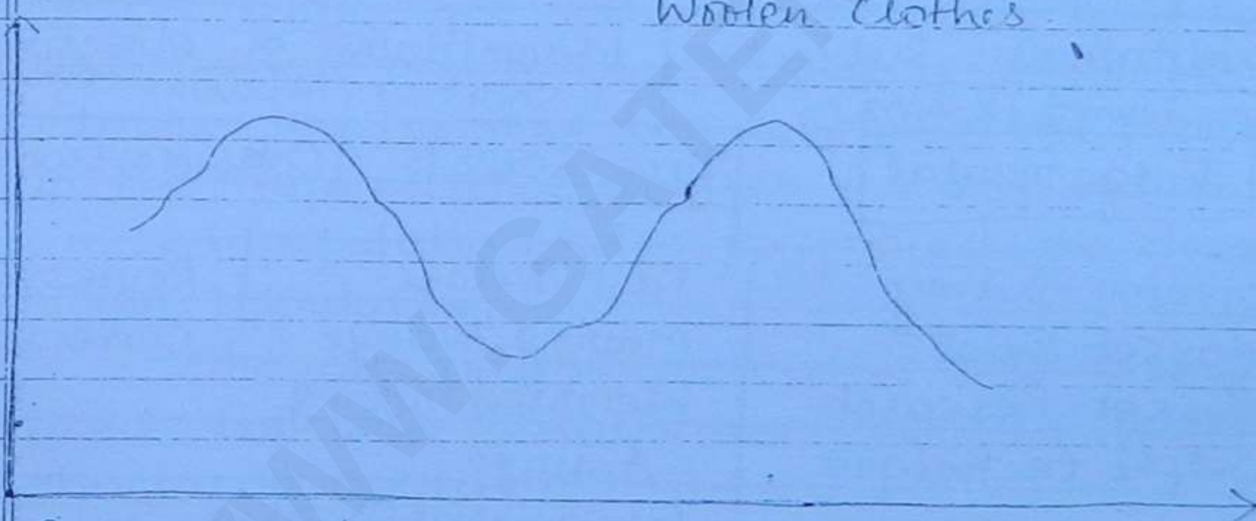
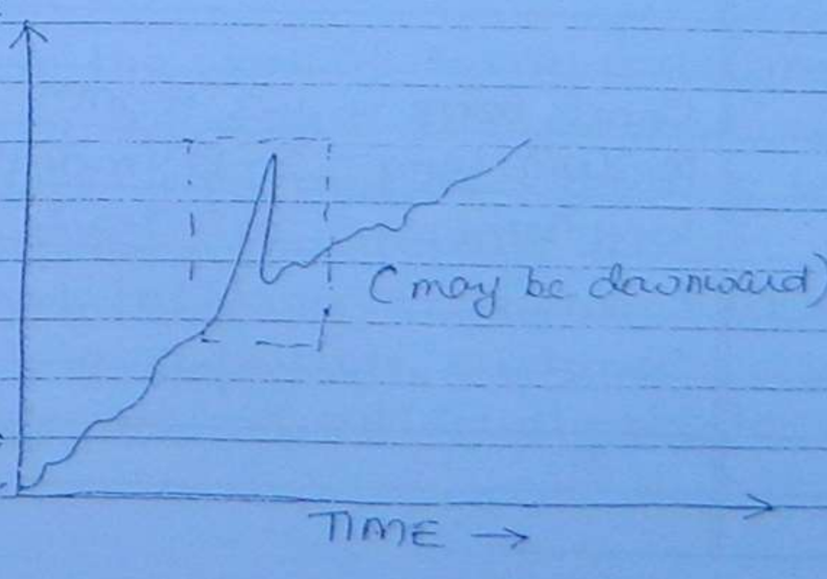
3) Cyclic Variation: Wave like variation Lasting more than ~~the earlier~~ a year & it is repeated after a certain period. e.g. Yearly Electricity Bill, Refrigerator, Woolen Clothes.



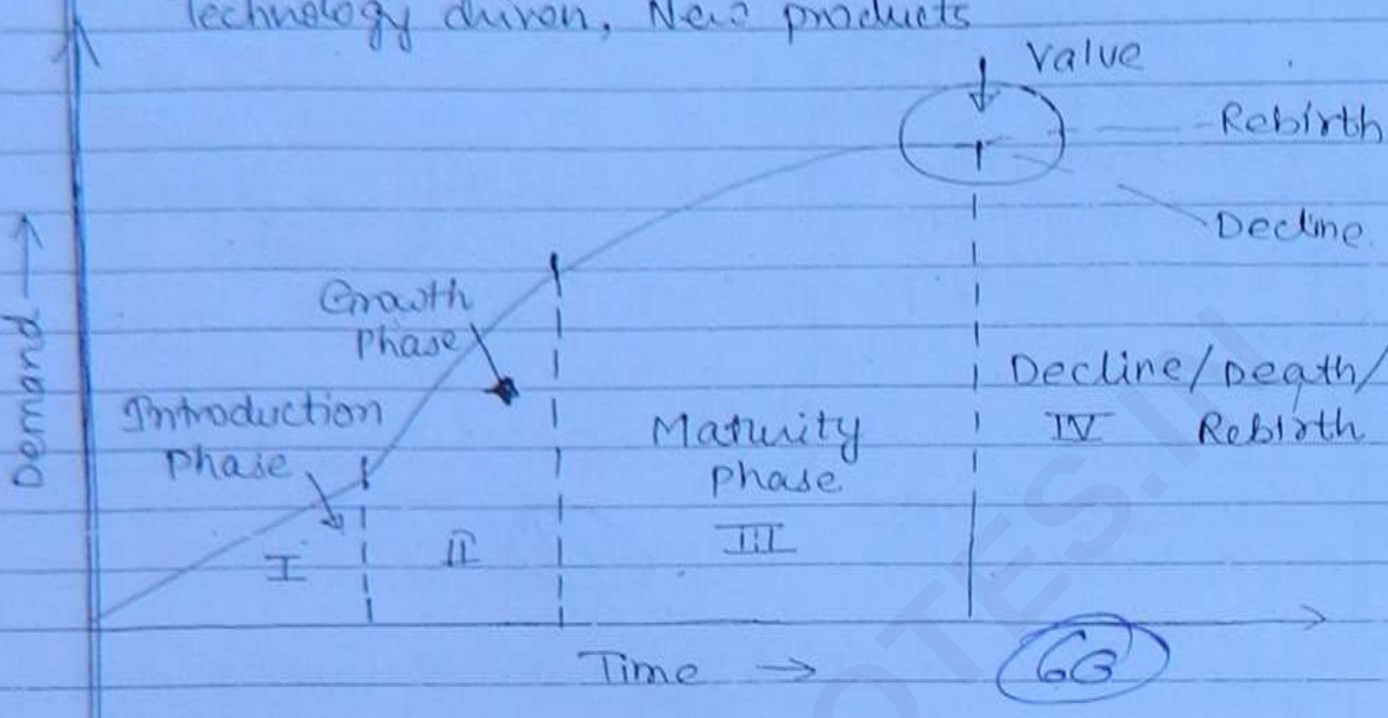
4) Irregular Variation:

These are caused by unusual circumstances which are not reflected of typical behaviour/ Normal behaviour
These may be due to severe weather conditions, Strike, Plant Shutdown etc.



Product life cycle :-

Technology driven, New products



TYPES OF FORECASTING :

- Qualitative or Subjective
  - Long Term (2-5 Yrs)
  - Judgemental
    - opinion survey
    - Market Trial
    - Market Research
    - Delphi technique
- Quantitative or Objective
  - (Short or Medium term)
  - Time Series ↔ Causal or Econometric
  - Time Series
    - Past Average
    - Moving Average
    - Weighted moving Average
    - Exponential Smoothing
  - Causal or Econometric
    - Regression
    - Correlation

Short term = 1-3 month
Medium term = 3-12 month
Long Term = 2-5 Years

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A Qualitative or Subjection.

Judgementae: This method is based on the art of human judgement ie How well a human being can predict a demand of product in future.

This method doesnot require past data a sales figure

(61)

1. Opinion Survey: In this method opinions are collected from the customers, retailers and distributor regarding the demand pattern for a product.

They give information regarding why they buy a particular product, what cost they are willing to pay, additional features required in the product, their margin in profit etc.

2 Market Trial: This method is normally used for a New product and in that case it is advisable to introduce the product between a limited population in the form of free samples The response from the limited population helps to project the demand from bigger population.

The cost of this method is High and it is used for low cost consumables like Toothpaste, choclates, Colddrinks etc.

3. Market Research: Here the work of survey is assigned to external marketting agencies & the purpose of research is to collect information regarding the consumption of a product, the details about various factor which influence the demand like location, customer's occupation, customer's income quantity & quality etc are scientifically related to get the Forecast

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4. Delphi Technique: In this method, a panel of experts are asked a series of questions in which the response to one question is used to produce next question. The information available to some experts are made available to other experts. It is a step by step procedure in which opinion's are collected from the experts to arrive at a reliable forecast.

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B Quantitative or Objective

(i) Time Series :- It refers to the past data arranged in a chronological order as dependent variable & Time as independent variable (Year / Demand)

1. Past average: In this method forecast is equal to average sells or demand of a product for previous period.

2 Moving Average: This method uses past data & calculates a rolling average for a constant period (n). Fresh average is computed at the end of each period by adding the demand of the most recent period and deleting the demand for the older period. Since data in this method changes from period to period, it is called moving average method

$$F_{2008} = \frac{99+116+124+94}{4} \quad (4\text{-Time}$$

$$F_{2008} = 0.25 \times 99 + 0.25 \times 116 + 0.25 \times 124 + 0.25 \times 94$$

Year	Demand
2004	94
5	124
6	116
7	99
8	136
9	114
10	148

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3. Weighted Moving average:

$$F_{2008} = 0.4 \times 99 + 0.3 + 116 + 0.2 \times 124 + 0.1 \times 94. \quad (n=4)$$

$$(0.4 + 0.3 + 0.2 + 0.1 = 1 \text{ should})$$

In this method, Unequal weights are assigned to the demand data such that sum of all weights equals to one. The most recent data is given the highest weight and the weight assigned to the oldest data is the least.

Here, n = no. of period.

1. Sum of 'n' natural no's $= \Sigma n = \frac{n(n+1)}{2}$

2. Arrange in decreasing order of

$$\frac{n}{\Sigma n}, \frac{n-1}{\Sigma n}, \frac{n-2}{\Sigma n} \cdots \cdots \frac{1}{\Sigma n}$$

4. Exponential Smoothing: In this method, we neednot to carry large data base as this method requires only the current demand & the forecasted value for the current period to give the next forecast. This method is preferred as it assigns weight to all previous data & the pattern of weight assign are

Date :

of exponential form. The most recent data is given the highest weight & the weight assigned to the older data decreases exponentially.

New Forecast = Forecast for the last Period + α [Demand for the last period − Forecast for the last period]

$$F_t = F_{t-1} + \alpha \left[D_{t-1} - F_{t-1} \right]$$

$$\boxed{F_t = F_{t-1} + \alpha \cdot \text{Error}}$$

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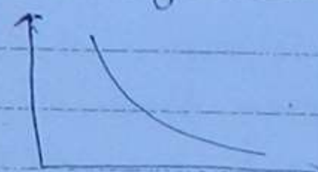
$$\text{Error} = \Delta = D_{t-1} - F_{t-1}$$

where, α is known as smoothing constant and is equivalent of 'n' period, moving average method & is given by

$$\boxed{\alpha = \frac{2}{n+1}}$$



General Equation for Exponential Smoothing:-

$$F_t = \alpha D_{t-1} + \alpha(1-\alpha) D_{t-2} + \alpha(1-\alpha)^2 D_{t-3} + \cdots$$

$$F_t = \alpha D_{t-1} + (1-\alpha) \left[\alpha . D_{t-2} + \alpha(1-\alpha) D_{t-3} + \cdots \infty \right]$$

$$\boxed{F_t = \alpha D_{t-1} + (1-\alpha) \cdot F_{t-1}}$$

$$\boxed{F_t = F_{t-1} + \alpha \left[D_{t-1} - F_{t-1} \right]}$$

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If for initial period forecasted value is not given Then :

1. Take the demand for the I period equals to the Forecasted value for the I period. i.e $D_1 = F_1$ Naive Method.

2. Take the average of the demand values as the Forecasted Value for the I period and proceed.

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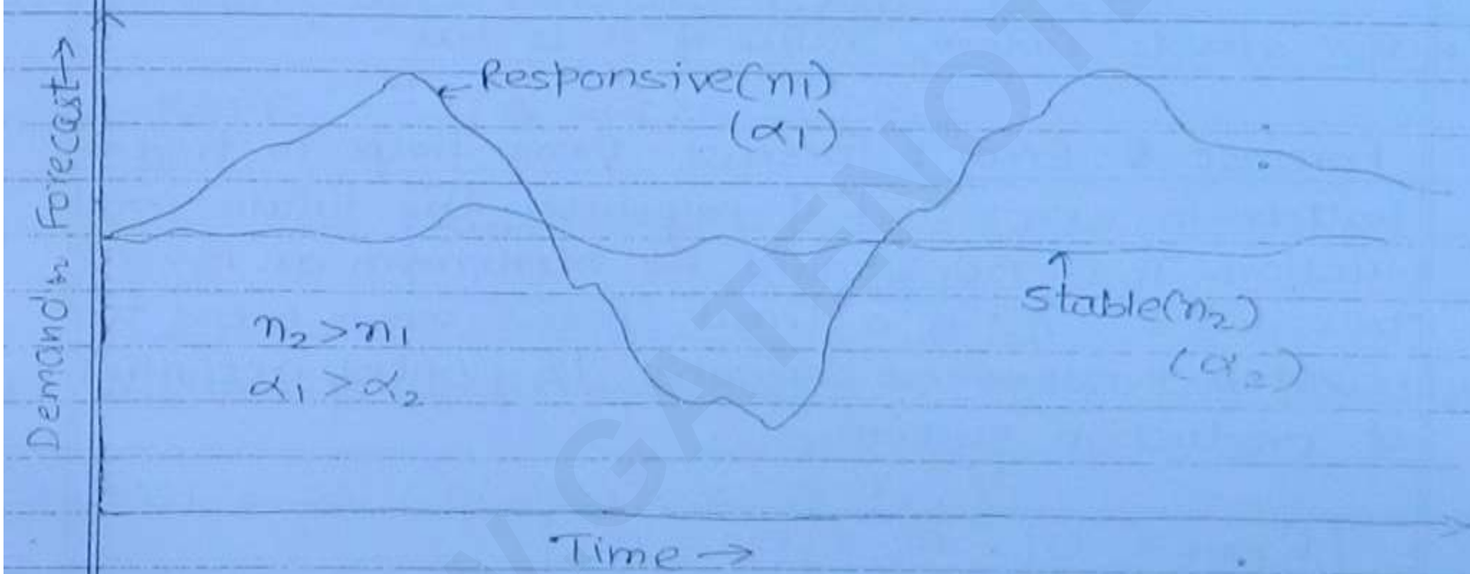
Responsiveness & Stability



1. Responsiveness: It incidates that Forecast have a Fluctuating or swinging pattern. These are preferred for new product & for that number of period is kept low.

2. Stability : It indicates that Forecast pattern have levelled or Flat pattern or has less fluctuation. It indicates ideal condition & is used for old existing product & for that no. of period is kept Large.

In case of exponential smoothing, $\alpha_1 > \alpha_2$.

$\alpha = \frac{2}{n+1}$, $\therefore$ For α_2, n is more.

$$F_t = F_{t-1} + \alpha\left[D_{t-1} - F_{t-1}\right]$$

1. If $\alpha = 0$, $n \to \infty$.

   $\boxed{F_t = F_{t-1}}$ limit of stability

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2. If $\alpha = 1$,

   $\boxed{F_t = D_{t-1}}$ limit of Responsiveness.

* For stable curve, value of α is less.

Forecast Error: Forecast Error helps to find the pattern in error and it regulates the future production. The error should be minimum as far as possible & within a limit. These error helps to find a pattern or sequence to control activities of production system

$$\boxed{\text{Error} = D_i - F_i = e_i = \Delta i}$$

1. Mean Absolute Deviation (MAD)

$$MAD = \sum_{i=1}^{n} \frac{|D_i - F_i|}{n} = \sum_{i=1}^{n} \frac{|e_i|}{n}$$

It gives the magnitude of error, w/o considering sign. It is calculated as the summation of absolute value of error for given period divided by the no. of periods.

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2. Mean Forecast Error : (MFE) or (Bias)

$$MFE \text{ or } Bias = \sum_{i=1}^{n} \frac{(D_i - F_i)}{n} = \sum_{i=1}^{n} \frac{(e_i)}{n}$$

It tells the direction of error & here signs are considered. This Error tells us any chances of over estimated & Under Estimated. Positive value for bias Indicates Under estimated forecasting. Negative value for bias indicates Over estimated forecasting

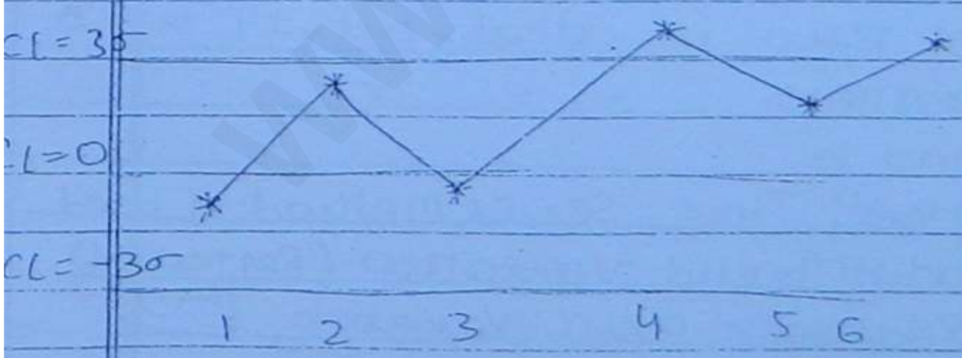
3. Running Sum Forecast Error (RSFE)

$$RSFE = \sum_{i=1}^{n} (D_i - F_i) = \sum_{i=1}^{n} (e_i)$$

$$\boxed{Bias = \frac{RSFE}{n}}$$

4. Mean Square Error (MSE) * Mostly used Nowadays.

$$MSE = \sum_{i=1}^{n} \frac{(D_i - F_i)^2}{n} = \sum_{i=1}^{n} \frac{(e_i)^2}{n}$$



Upper Control limit

$\sigma = \sqrt{MSE}$

Lower Control limit)

We make a range of limits & we want our Forecast value within this limits otherwise our forecasting is out of control.

* MSE > MAD > MFE or bias ' Give priority if not mention. *

Now-a-days it is the most used one. It is used for plotting control chart for Forecast Error. They magnify the ~~magnitude~~ Error of Larger Magnitude

5 Mean Absolute Percentage Error (MAPE) :-

$$MAPE = \frac{\sum_{i=1}^{n} \left| \left(\frac{Di - Fi}{Di} \right) \times 100 \right|}{n}$$

$$\boxed{MAPE = \frac{\sum_{i=1}^{n} \left| \left(\frac{e_i}{Di} \right) \times 100 \right|}{n}}$$

It gives the forecasted error in proportion to actual demand in absolute terms. It pulls error in perspective because there is difference between 10 errors out of 100 & 100 out of 1000.

6 Tracking Signal (T.S) = $\frac{RSFE}{MAD}$

It tells how well the forecast is predicting actual values. A value of zero is ideal, but ±4 or ±5 is the acceptable range.

* a) for Cyclic Variations, Time Series method is used
b) for Causal Method : Trend Variation (Regression)

Eg. a) 108/92/116/98/112 cyclic Variation
b) 108/112/118/124/142 Trend
a) Exponential is frequently asked

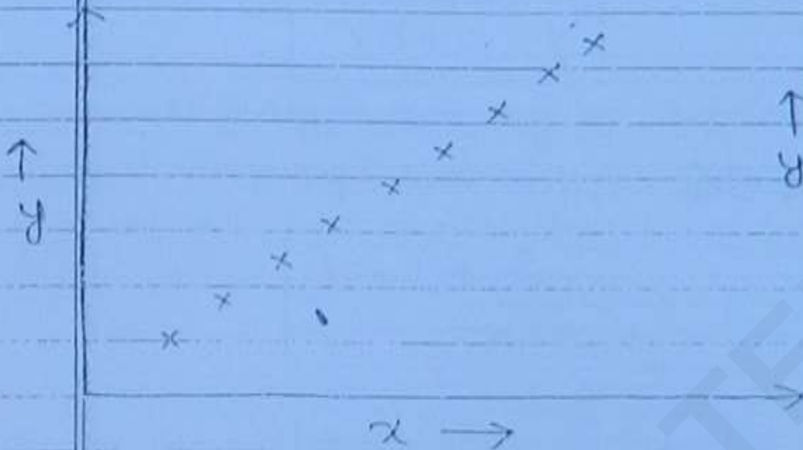
(ii) Causal or Econometric Method:

In this method Forecasted price to establish cause & effect relation between demand & some other parameter that are, related to the demand of product.

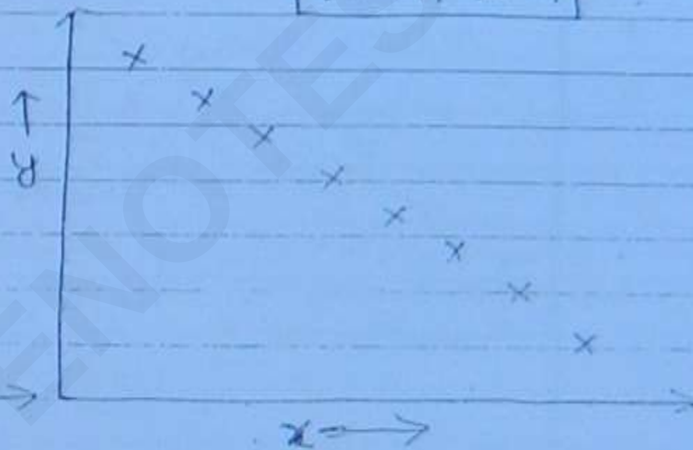
The objective is to establish a relation b/w changes in Demand Value with corresponding change in other Variable.

I Correlation Analysis: Value ranges from +1 to -1.

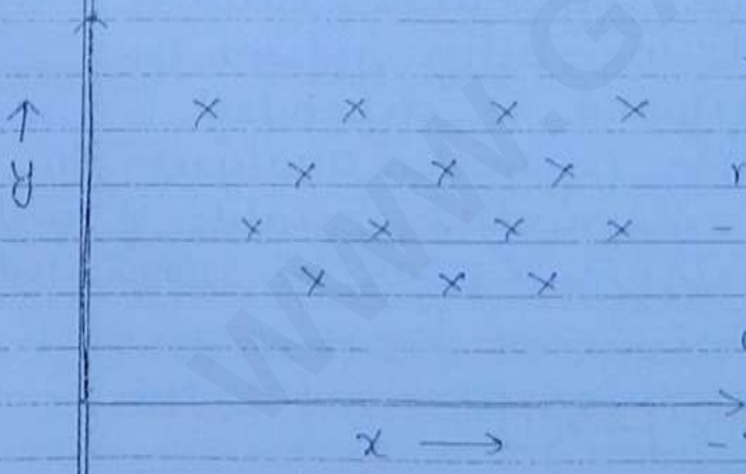
$\boxed{r = +1 \text{ to } -1}$



$r = +1$



$r = -1$



$r = 0$

No-relation

- It is used in determining the degree of closeness or relationship b/w 2 variable.
- It is an indication of the extent to which the knowledge of one variable is useful in the prediction of other.
- The correlation b/w 2 variables x & y is given by:-

$$r = \frac{\sum (x-\bar{x}) \cdot (y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \cdot \sum (y-\bar{y})^2}} \longrightarrow 1$$

$$\boxed{r = \frac{\sum (x-\bar{x})\cdot(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \cdot \sum (y-\bar{y})^2}}}$$

Where $\bar{x}$ & $\bar{y}$ are the mean Value of individual 'x' & 'y' Values.

Linear Regression Analysis

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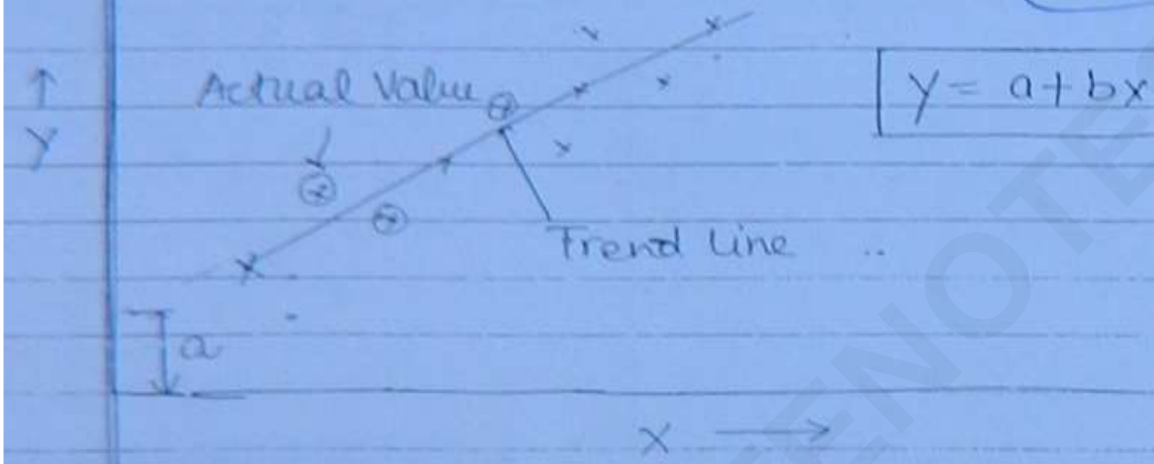

$$\boxed{Y = a + bx}$$

This is mathematical technique of obtaining the line of best fit b/w the dependent variable which is usually demand & some other independent variable on which demand is dependent.

In a simple regression analysis, the relationship b/w the dependent variable 'y' and some independent variable 'x' can be represented by a straight line:-

$$\boxed{Y = a + bx} \longrightarrow (1)$$

Where 'a' is the intercept on 'y' axis and 'b' is the slope of line.

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Now taking summation (Σ) both side for 'n' Period.

$$\boxed{\Sigma y = a \cdot n + b \Sigma x} \rightarrow (2)$$

Then (1) × (2), $xy = ax + bx^2$

Taking Σ both sides -

$$\boxed{\Sigma xy = a \cdot \Sigma x + b \Sigma x^2} \rightarrow (3)$$

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Eq. (3) × n — (2) × Σx, we get

$$n \Sigma xy - \Sigma x \cdot \Sigma y = \cancel{an \Sigma x} + bn \Sigma x^2 - \cancel{\Sigma x\, a \cdot n} - b \Sigma x^2$$

$$n \Sigma xy - \Sigma x \cdot \Sigma y = nb \Sigma x^2 - b(\Sigma x)^2$$

$$\boxed{b = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \cdot \Sigma x^2 - (\Sigma x)^2}}$$

∵ When x is independent & y depends on x

$$\boxed{a = \frac{\Sigma y - b \Sigma x}{n}}$$

Now, put the values of 'a' & 'b' in Regression line or Trend line (1)

$Y = a + bx$

Dependent Independent

* Least Square Method * When independent variable 'x' is linear & Uniform and it can be modified such that Σx becomes = 0. Then the method is called Least Square method.

When, $\Sigma x = 0$, then, $b = \frac{\Sigma xy}{\Sigma x^2}$ & $a = \frac{\Sigma y}{n}$

ODD	X	EVEN		
2004	-2	2004	-2.5	-5
2005	-1	2005	-1.5	-3
2006	0	2006	-0.5	-1
2007	1	2007	0.5	+1
2008	2	2008	1.5	+3
	$\Sigma x = 0$	2009	2.5	+5

LINE BALANCING (Assembling)

It aims at grouping the task or workers. In an effective manner in order to obtain optimum utilisation of man-power & machine & to minimise Ideal time.

Task are grouped so that there total time is preferrably equal to or a little lesser than the time available at each work station. This reduces the ideal time.

Advantages :-

1. Uniform rate of production.
2. less material handling.
3. Less work in process inventory.
4. Effective Utilisation of man power & machine.
5. Easy production control.
6. less ~~tran~~ conjestion within production system.

Terminology :-

1. Work Element : → (M) → (T) → (D) → (G) →

Element

2min
Task Time.

* Cycle time $\geq$ max. Workstation Time.

Each job is completed by a set of operations & each operation which is to be performed on the job is called Work Element or ~~simply~~ simply Element

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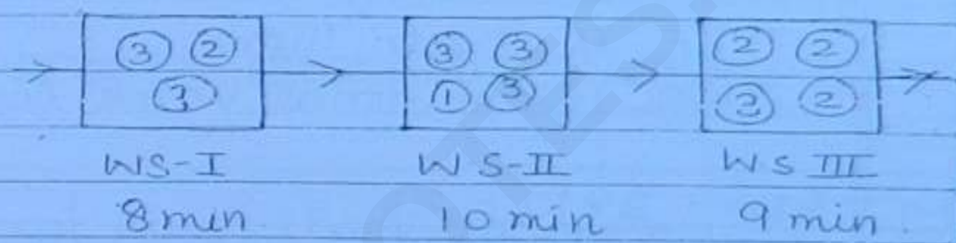
2. Task Time : It is the standard time require to complete elemental task. (Ti)

3. Work Station :
(2) ⇒ time (2 mins)



It is a specific location on the assembly line where given amount of elemental task are performed and the summation of all elemental task time is known as Work station time.

4. Total Work Content : It is ~~th~~ equal to the sum of processing time of each operation that is to be performed on the job

5. Cycle Time : It is the amount of time for which a given job remains with an operator on the assembly line or it may be defined as the time between two successive product coming out of assembly line.

NOTE : Minimum possible cycle time can be equal to maximum work station time.

$$\boxed{T_c \geq (T_{si})_{max}}$$

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BALANCE DELAY (B.D)

It is the measure of line ineffectiveness. It is the ratio of total ideal time over the total time spend by the job on the assembly line.

$$\boxed{B.D = \left(\frac{n\,T_c - TWC}{n \cdot T_c}\right) \times 100}$$

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Where, n is no. of workstations
T_c is cycle Time.
TWC is the total work content.

e.g. $BD = \frac{3 \times 10 - 27}{3 \times 10} = \frac{3}{30} = \frac{1}{10} \Rightarrow 10\%$

LINE EFFICIENCY

$$\text{Line } \eta = \left(\frac{T.W.C}{n \cdot T_c}\right) \times 100 = 100\% - BD\%$$

e.g. 90%

SMOOTHNESS INDEX (S.I)

$$S.I = \sqrt{\sum_{i=1}^{n} (\text{Maximum station time} - \text{Station time})^2}$$

$$S.I = \sqrt{\sum_{i=1}^{n} \left((T_{si})_{max} - T_{si} \right)^2}$$

It tells how the load is distributed between diff. work station.

e.g. $S.I = \sqrt{(10-8)^2 + (10-10)^2 + (10-9)^2} = \sqrt{5}$

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THEORITICALLY MIN NO. OF W.S REQUIRE

Theoretically, minimum No. of Work Station Required

$$n_{min} = \frac{TWC}{T_c}$$

e.g $n_{min.} = \frac{27}{10} = 2.7$

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Methods of Line Balancing :-

Under Heuristic approach, there are 2 methods of Line balancing :-

*1. Largest Candidate Rule: (Prefered one)
2. Rank positioned Weighted (RPWM) (Helgeson & Bernie) Method

1. Largest Candidate Rule: In this method, the work elements are assigned to the workstation on the basis of their elemental time values.

The steps involved are:-

(i) Draw the precedence diagram of given elements.
(ii) List all the elements in decreasing order of their elemental time values.
(iii) Starting from the first element in the list moving downwards search first feasible element which can be placed in a work station. For a feasible element, there are 2 conditions :-
a) All the elements prior to it must be completed.
b) When that element is placed in the work station, the summation of all elemental task time in the station must not exceed the cycle time.
iv) The element which is assign to a work station, strike it off so that it cannot be considered again

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v) Repeat the above steps untill all the elements are assigned to the workstation.

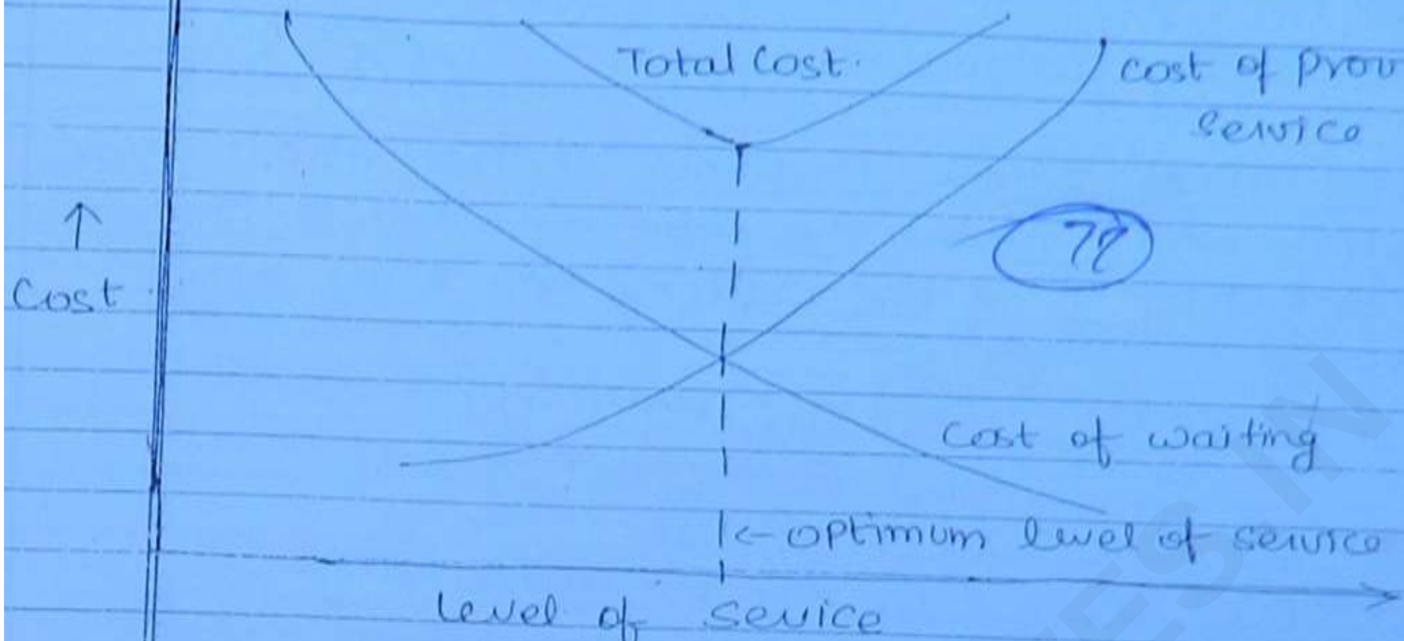
\ RANK POSITION WEIGHTED METHOD 76

In this method Elements are assign in the work station acc. to their positional rate. The positional weight of an element correspond to the time of the longest path ~~to the remai~~ from the begining of element through the remainder of network.

The elements are arrange in decreasing order of their positional weight. The procedure of assignment to the workstation is similar to Largest Candidate Rule, the only difference the skipping of positional weight is not allowed.

QUEUING THEORY

The aim of Queuing Model is achievement of an economic balance b/w the cost of providing service & the cost associated with the wait required for that service. These models are used in service oriented organisation, m/c repair shop, Food chain restaurant etc.



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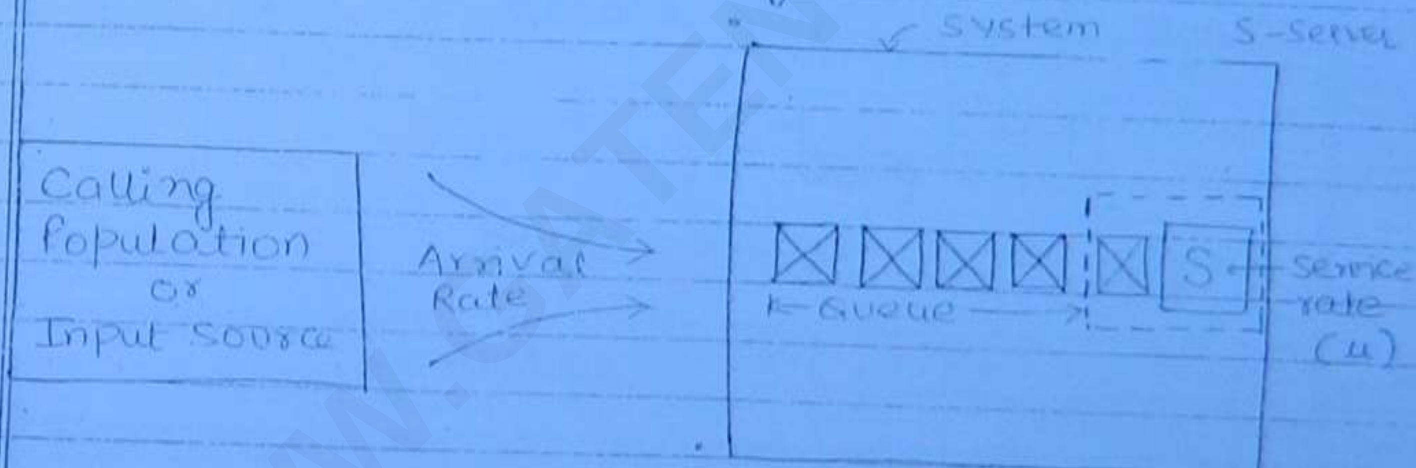
Characteristics of Queuing Model



1. Arrival pattern or rate: The no. of customers arriving the system per unit time is called arrival rate & Arrival is random & randomness is best described by Poisson's distribution

   Queuing models are based on the assumption that arrival pattern is Poisson's distributed

2. Service pattern or rate: The no. of customers being served per unit time is called service rate. & Queuing

Models are based on the assumption that service rate is exponentially distributed.

3 Service rule or Queue Discipline: It tells about Queue Discipline which means the order by which the customers are picked up from the waiting line & are being served.

FIFO/FCFS : First In First Out
First Come First serve

LIFO: Last In First Out

SIRO: Service In Random order

VIP treatment: like Senior citizenship etc

service channel & server:

i. Single server, single channel

$\rightarrow \boxed{S_1} \rightarrow$

2. Single server, Multiple channel

$\rightarrow \boxed{S_1} \rightarrow$ eg Railway

$\rightarrow \boxed{S_1} \rightarrow$

$\rightarrow \boxed{S_1} \rightarrow$

3. Single channel, Multiple server

$\rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow \boxed{S_3} \rightarrow$

4. Multiple server, Multiple channel
eg Automobile

$\rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow \boxed{S_3} \rightarrow$

$\rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow \boxed{S_3} \rightarrow$

$\rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow \boxed{S_3} \rightarrow$

5. System and Calling population : The service facility where customer arrives in order to get service is called system and the size of system may be finite or infinite.

The samples of customers from which only a few visit the service facility is known as calling population or input source. The size of calling population may be finite or infinite.

When calling population is infinite, then the arrival pattern of future customers remain constant.

6. Customer's attitude:

- ATTITUDE
  - Patient
  - Impatient
    - Jockeying
    - Balking
    - Reneging
    - Cheater

a Patient : Customer join the Queue and wait in the service station patiently for his turn.

b Impatient: These are the customers who are trying to get service at the faster rate.

(i) Jockeying: Customer keeps on changing queue in hope to get service at a faster rate.

(ii) Balking: Customer meets the system w/o joining the queue as Queue length is very long.

(iii) Reneging: Customer joins the Queue for sometime and then leaves the system as Queue is moving very slowly.

(iv) Cheater: Customer takes illegal means like bribing, Fighting etc. in order to get service faster.

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Representation of Queuing Model

Queuing models are represented by Kendall & Lee notations whose general form is $(a/b/c):(d/e/f)$

Where, $a \rightarrow$ probability distribution of arrival pattern

$b \rightarrow$ probability distribution of service pattern

$c \rightarrow$ No. of servers.

$d \rightarrow$ service rule or Queue discipline

$e \rightarrow$ maximum no of customers allowed in the system

$f \rightarrow$ size of calling population or input source.

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i) a & b : M = Markovian (Poisson).

For arrival pattern or exponential service pattern

E = Erlangian (Gamma distribution)

GI = General Independent arrival pattern or service pattern

D = Deterministic arrival or service time

ii) c : 1, 2, 3, 4 . . . (no of servers)

iii) d : FIFO/ FCFS, LIFO, SIRO, GD (General Discipline)

iv) e/f : N = Finite , ∞ = infinite

e.g $(M/E/2):(SIRO/N/\infty)$

Single Server Queuing Model

$(M/M/1):(FIFO/\infty/\infty)$: This notation denotes a Queuing Model with poisson arrival & exponential service pattern with a single server, The service rule is FIFO & the no. of customers allowed in the system is infinite.

$\lambda \rightarrow$ arrival rate (Poisson) : $15 \frac{\text{customer}}{\text{hr}}$ (Probability)

$\frac{1}{\lambda} \rightarrow$ Inter arrival time $= \frac{1}{15} \frac{\text{hr}}{\text{customer}} = 4 \frac{\text{min}}{\text{Cust}}$

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$\mu \rightarrow$ service rate (exponential) : $20 \frac{cust}{hr}$

$\frac{1}{\mu} \rightarrow$ Inter service time $= \frac{1}{20} \frac{hr}{cust} = \frac{3 min}{cust}$

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1. If $\lambda > \mu$ [25 cust/hr > 20 cust/hr]
The queue length will kept on increasing & ultimately system will fail.
2. If $\lambda \leq \mu$ – system works.

$$\boxed{\rho = \frac{\lambda}{\mu}}$$

The ratio between arrival rate and service rate is called system utilisation and is denoted by ρ. It is also called as utilisation pattern, channel efficiency, clearing ratio, percentage time server is busy or probability that ~~th~~ a customer has to wait.

FORMULAE

1. Probability of zero customer in the system or the probability that the system is ideal or the probability that a customer doesnot have to wait in the queue.

$$\boxed{P_0 = 1 - \rho}$$

2. Probability of having exactly 'n' customers in the system.

$$\boxed{P_n = \rho^n \cdot P_0 = \rho^n (1 - \rho)}$$

$P_0 + P_1 + P_2 + P_3 \ldots = 1$

$P_2 + P_3 + \ldots = 1 - (P_0 + P_1) = 1 - (P_0 + \rho P_0)$

3. Average no of customers in the system, L_s

$$L_s = \sum_{n=0}^{\infty} n \cdot P_n$$

$$\boxed{L_s = \frac{\rho}{1-\rho}}$$

It includes the average No. of customers being served along with those waiting in the Queue

4. Average no of customers in the queue, L_q

$L_q = L_s - \rho$ or $\boxed{L_q = \frac{\rho^2}{1-\rho}}$

$L_q = \rho L_s$

5 Average length of Non-empty Queue.

$$\boxed{L_q' = \frac{1}{1-\rho}}$$

Little's Law

For a stable system, the average no of customer in Queue or in the system is equal to the Average customer arrival rate × Average time in Queue or in system.

$$\boxed{\begin{array}{c} L_s = \lambda W_s \\ \hline L_q = \lambda W_q \end{array}} \Rightarrow W_s = L_s/\lambda$$

$$L_s > L_q \quad [\because \rho < 1]$$

Where,

W_s : Average waiting time of customer in the system

W_q : Average waiting time of customer in the Queue.

$$\boxed{W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu}}$$

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* Probability of atleast 2 in Queue means atleast 3 in system.

$$P_3 + P_4 \ldots = 1 - (P_0 + P_1 + P_2)$$
$$= 1 - (P_0 + \rho P_0 + \rho^2 P_0)$$
$$= 1 - P_0(1 + \rho + \rho^2)$$
$$= \cancel{1-0}$$

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* Probability of 'n' arrival in the system during period $\boxed{P(n,T) = \frac{(\exp)^{-\lambda T}(\lambda T)^n}{n!}}$

* Probability that more than 'T' time period needed to service a customer, $\boxed{P = (\exp)^{-\mu T}}$

* Probability that the waiting time in the Queue is greater than T,

$$\boxed{P(W_q > T) = \rho \cdot (\exp)^{-T/W_s}}$$

* Probability that the waiting time in the system is greater than T,

$$\boxed{P(W_s > T) = (\exp)^{-T/W_s}}$$

WORK STUDY + Theory

BOOKS(NE)

a) Method or Motion b) Time or Work Measurement

Work study analyse the work into smaller parts in order to rearrange these parts to increase productivity & to find a standard time for each job.

It is divided into 2 parts:-

1. **Method or Motion study**: It is a set of techniques developed to create new alternate methods of doing the job at a lesser effort & more effectiveness

Steps of Method Study:- 84

SELECT → a Job

RECORD → all the data related to the job

EXAMINE → Recorded data

DEVELOP → few alternate methods

INSTALL → best alternate

MAINTAIN → the installed alternate

→ **Recording Techniques**:- These are designed to simplify & standardize, the recorded work. The most generally used recording techniques are:-

1 Process Charts (P.C)

a) outline P.C b) Flow P.C c) Two handed P.C

2. Time Scale

a) Multiple activity chart

b)(i) SIMO } micro motion

(ii) PMTS

3 Diagrams

a) Flow c) Travel chart e) Chromocycle graph

b) String d) cycle graph f) 2-D & 3-D Models & Templates.

1. <u>Process Chart (P.C)</u>: It indicates the sequence of operation & the process chart symbols use are

○ - Operation △ - Storage

□ - Inspection ⇨ - Transportation

D - Delay or Temporary storage.

a) <u>Outline P.C</u>: These charts are used to get a little bit of information about the process & it uses only 2 symbols i.e Operation & Inspection

b)* Flow P.C: These are of 3 types: Man, Material & Machine types. These chart are much detailed & carry additional informations. All the five symbols are used. These charts are normally used alongwith corresponding flow diagrams to give complete picture.

c) <u>Two handed</u> P.C: In this chart, activities of Left Hand & the Right hand of an operator are recorded on a common

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time scale related to each other. All the Five symbols are used and these are prefered for short cycle. repetitive setting jobs which are repeated many times a day

2 Time Scale:

a) MULTIPLE ACTIVITY CHART (86)

Time	Operator	Machine	Remark.
1.5 min	▨		Job Loading
5 min		▨	M/C working
0.5 min	▨		Job Unloading

▨ working ☐ Idle

Operator utilisation = $\frac{2}{7} \times 100$, Cycle Time = 7 min

M/C Utilisation = $\frac{5}{7} \times 100$

In these Charts, the activities of more than 1 item are arranged on a common time scale to show their inter-relationship, the study of these charts makes it possible to rearrange these activities so that their utilisation is optimised.

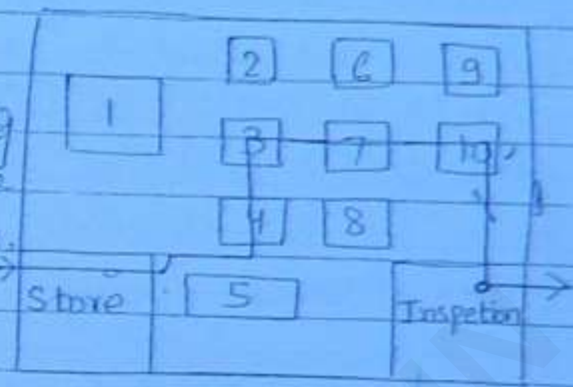
Gang Process chart: It is the form of multiple activity chart in which the activities of Gang or group of workers is synchronised on a common time scale while performing a similar task.

eg Formula-1 Racing car, after 1 lap.

3 Diagrams

a) Flow Diagram

These are upto scale drawing of working area showing the different facilities, Machines, equipments etc. with there numbered symbols.



Scale 1 cm = 2m

These diagram are used along with associated flow process chart to give complete picture of working area.

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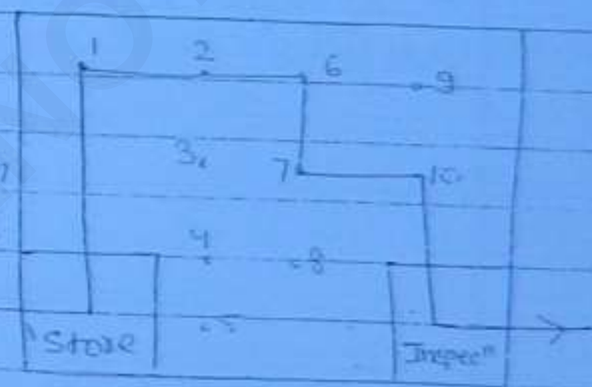
b) String Diagram

In SD are used to trace path of Minimum distance travelled on a upto scale drawing of working area with the help of thread or string.



Scale 1 cm = 2m

It also helps to avoid back-tracking

c) Travel Chart

4–2–5–3–1

It is a tabular movement record for presenting Quantitative data about the movements of workes & Material. The travel chart is always square & Each square represents a work station.

Movement from →

to ↓	1	2	3	4	5
1			4		
2				1	
3					3
4					
5		2			

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2. Time Scale :

b) MicroMotion Study :
These are used to breakdown a job into smaller parts in order to rearrange those parts to increase the effectiveness of operator.
This is done by Film analysis to facilitate micromotion study.

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Gilbesth divided all the basic human movement into 17 fundamental Hand & Eye movement to which one was added later on. So, Now there are 18 Therbligs and each therbligs has a specific symbol, letter & Colour for recording purpose b.1) SIMO CHART

Simultaneous Motion Cycle Chart (Assembly work)

ink unter	Left Hand			Right Hand		
	Activity	therblig	Time	Activity	therblig	Time
	$\boxed{1 \text{ wink} = \frac{1}{2000} \text{ minute}}$					

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It is micromotion form of 2handed process chart in which the activities of two-hands are related to each other. All the 18 symbols of therbligs are used & are prefered for short duration, repetitive, sitting jobs. The time is measured in Wink Counter.

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3 Diagram

d) Cyclegraph: In this method a continuous source of light like a bulb is attached to the hand of an operator and the movement of light is recorded by a camera. The study is performed in a dark room.

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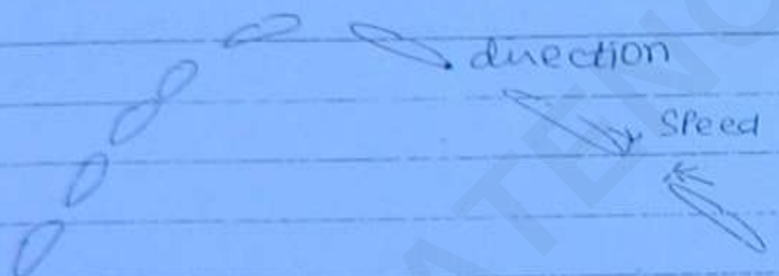
e) Cronocyclegraph: In these graph the light source is interrupted so that the path appears as a series of dots. The pointed end indicates the direction of motion (movement) and the spacing b/w the dots indicates the speed of movement.



TIME STUDY

Date :

- DIRECT
  - StopWatch
  - Work Sampling
- Synthetic or Standard data
- INDIRECT
  - Analytical Estimation
  - PMTS
    - WF
    - MTM

I. DIRECT

1. StopWatch:

a) Observed or Elemental time: It is the time measured or observed by an observer using some measuring device like stopwatch.

b) Normal time: It is the time required to complete a job by a normal average worker under normal working conditions.

$$\boxed{\text{Normal Time} = \text{Observed Time} \times \text{Performance Rating factor}}$$

<u>NOTE</u>: Rating factor is always applied to manual controlled operation & is never applied on Machine Element.

c) Standard time: It is the time in which a job is completed after taking all uncertainties & allowances into consideration.
It is the time in which a job is completed when the operator works Continuously for long duration

$$\boxed{\text{Standard Time} = \text{Normal Time} + \text{Allowances}}$$

If nothing is given for allowances, Then always take it equal to Normal Time.

d) Allowances: These are the extra-time provided to qualified workers when they work for long duration. Few of the allowances are rest & personal allowance, contingency allowance, policy allowance, fatigue allowance etc. Delay allowance.
Contigency: Power cut, Failure in system

2. Work sampling

It is also called as Activity sampling or Ratio Delay study. It is a work measurement technique in which large no of observations are taken at random intervals over a specified period of time on a group of workers or machines.
Larger the no of observations better will be the results as it is based on probability theory and theory of sampling.

It is the best technique for finding the allowances needed for operators.

a. No of observations, $\boxed{N = \frac{Z^2 P(1-P)}{L^2}}$

$$\boxed{N = \frac{Z^2 P\% (100 - P\%)}{(L\%)^2}}$$

where,
P is % or fraction of occurance of an activity
L is limit of accuracy and
Z is standard normal variate

Z	Confidence level
1.96	95%
2	95.45%
3	99.74%

b. Confidence precision factor = Z/L

take $z \approx 2$ for 95% [for objective]

$$\boxed{N = \frac{4P(1-P)}{L^2}}$$

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31/8/11
Happy EID.

Quality Control

Quality is defined as fitness for use where fitness is defined by the customer who is using that product.

Quality of a product refers to the degree to which the product meet the customer's expectation. Quality has no specific meaning unless related to a specific function or object.

Cost of Quality or Quality Cost

- Failure cost
  - Internal
  - External (customers who are using) (retailers, distributers)
- Appraisal cost
- Preventive cost

All the Expenditure that is associated with preventing a production of effective product & the cost associated with companies quality functions like finding, repairing and replacing defective product can be termed as Cost of Quality.

1. Failure Cost: It is the cost of producing defective product in the production system.

Johnson Taguchi
Jason(+ve) (-ve)

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2. Appraisal Cost : It is the cost associated with evaluating, measuring and finding out defective product within the production system.

3. Preventive cost : All the cost that is associated with minimising failure & appraisal cost can be termed as preventive cost.

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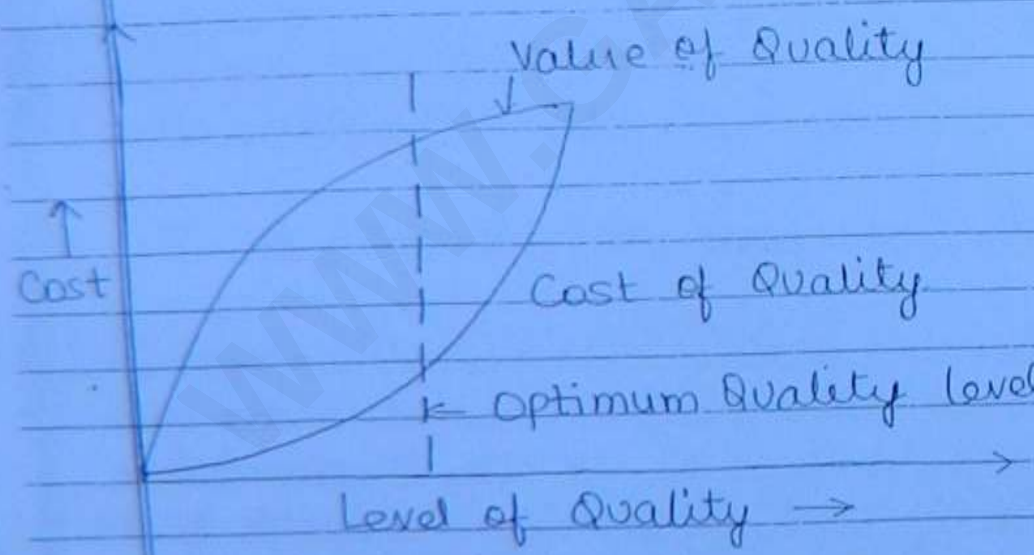
Value of Quality

The return direct or indirect gained by an organisation due to quality control mission is called Value of Quality.

Good quality can earn Market share, firm price good response from the customer, firm price policy, higher percentage of successful bids and other benefits to the income of organisation.



Quality Control & Inspection

Inspection is an act of checking & sorting out defective product whereas Q.C is a broad term which include many factors including inspection and regulates the quality of future production.

Quality control find out the reasons behind a defective product & also include steps that may be taken so that defective parts may not be repeated in future.

TYPES OF VARIATION

i. Chance or Common or Random: They are difficult to control & difficult to trace even under best conditions of production. These variations are of lower magnitude & each component show different variation [This is desirable (in limit)].

2. Assignable Variation: In these variations, Variations are of greater magnitude as compared to common or chance variation and these can be easily traced & detected. They may be due to M/c setting change, tool wear, M/c play, improper training of worker or any other factor.
[This is undesirable, out of limits].

TYPES OF ERROR

1. TYPE I ERROR: If there is no assignable cost of variation and still we predict that there is some assignable cost, it is called type I error.

2. TYPE II ERROR: If we conclude that the Universe has not changed, but it really has change

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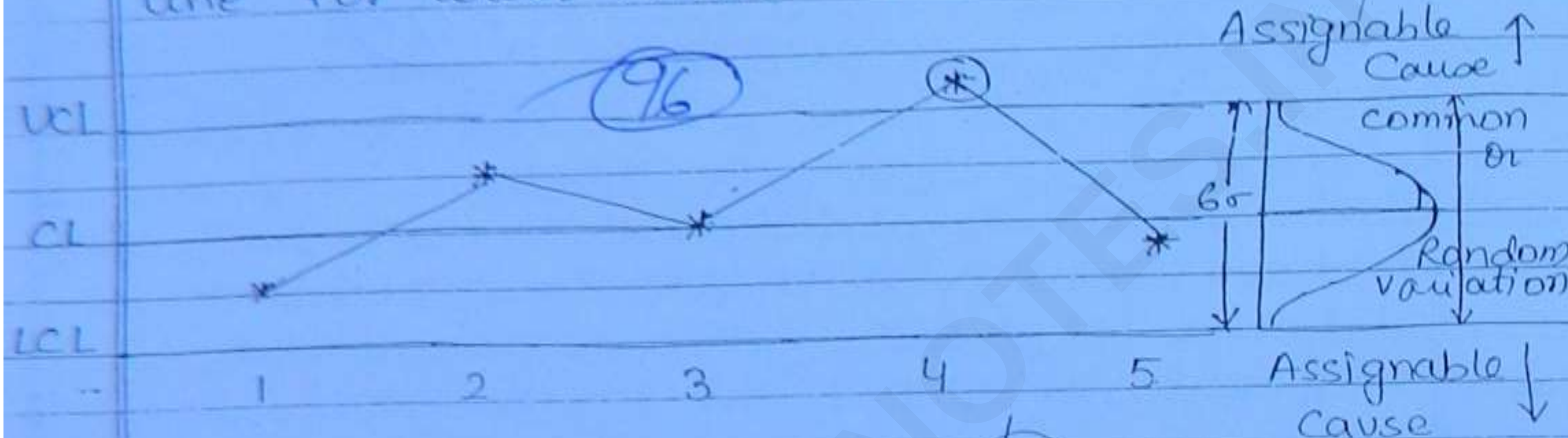
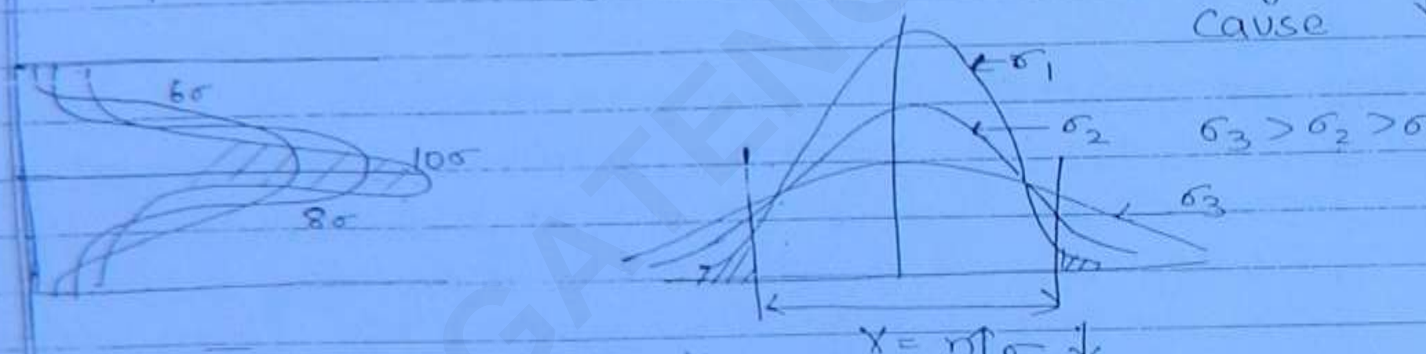
CONTROL CHART

Control Chart is a graph used to study how a process changes over time. Data are plotted in time order a control chart has a central line for the average an upper line for upper control limit and a lower line for lower control limit





e.g $\bar{X}$, x_1, x_2, x_3, x_4

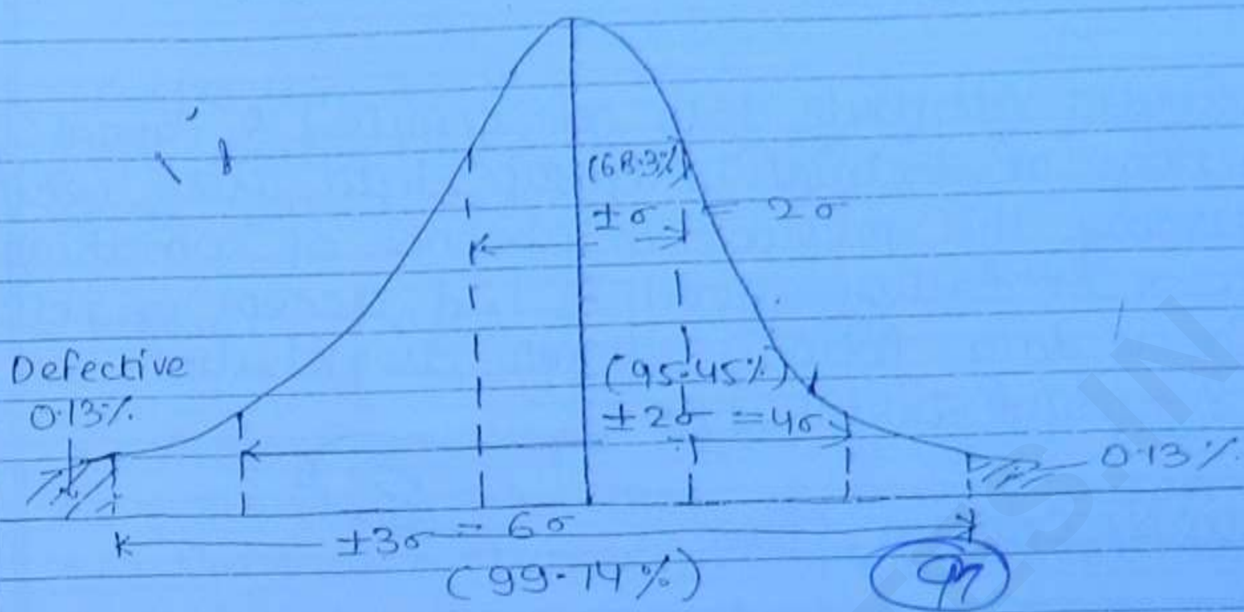
$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+(x_4-\bar{x})^2}{n=4}}$$

$\sigma \uparrow$, Non- defective $\downarrow$ $30 = 6\sigma$, $\sigma = 5$

$\boxed{\sigma \downarrow, \text{ Defective } \downarrow \quad \text{N.Defective } \uparrow}$ $30 = 8\sigma$, $\sigma = 3.75$

$30 = 10\sigma$, $\sigma = 3$

CONTROL LIMITS σ is same



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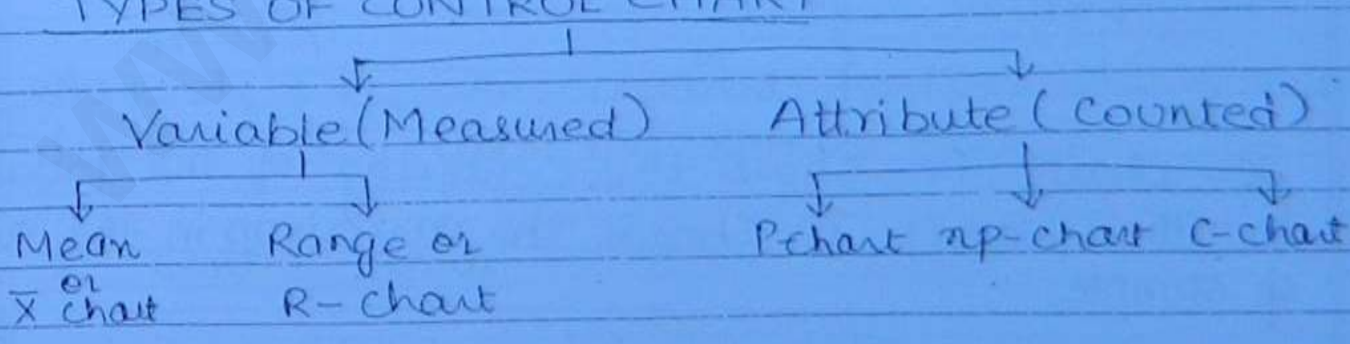
Normally, $\pm 3\sigma$ control limits are selected most of the time for plotting control charts Therefore, such charts are called 3σ Control Charts. Since, 99.74% of items fall within this region. So, only 0.26% defective items are produced on a longer run (3 out of 1000)

3.2 for 6σ units, means 3.2 out of 1 million units

TYPES OF CONTROL CHART



1. Variable: These charts are applied to data that follow a continuous distribution. Variable data are measured on a continuous scale & it can be

measured in fractions or decimals. It follows Normal distribution.

2. **Attribute**: Attribute data are counted & cannot have fractions or decimals. Attribute data arise while determining the presence or absence of something like success or failure, good or bad, accept or reject etc. These data follow Discrete distribution. (Binomial Distribution)

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1. **Variable Control Chart**

n	n	- - -	n
1	2	- - -	N
$\overline{X}_1$	$\overline{X}_2$	- - -	$\overline{X}_N$
R_1	R_2	- - -	R_N

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \dots + \overline{X}_N}{N} = \sum_{i=1}^{N} \frac{\overline{X}_i}{N}$$

$$\overline{R} = \frac{R_1 + R_2 + \dots + R_N}{N} = \sum_{i=1}^{N} \frac{R_i}{N}$$

a) **$\overline{X}$ - Chart**: It shows the centering of the process or in other words it shows the variation in the average of sample.

b) **R - Chart**: It shows the variation in the range of sample. This chart is a measure of spread of the samples.

a) $\bar{X}$ - control limits:

Centre line $= \bar{\bar{X}}$

Upper control line $= \bar{\bar{X}} + 3 \cdot \sigma_{\bar{X}} = \bar{\bar{X}} + \frac{3 \cdot \sigma}{\sqrt{n}}$

Lower control line $= \bar{\bar{X}} - 3 \cdot \sigma_{\bar{X}} = \bar{\bar{X}} - \frac{3 \cdot \sigma}{\sqrt{n}}$

Where $\boxed{\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}}$

$\bar{\bar{X}}$ = Average of sample mean or ~~Grant~~ grand average

$\sigma_{\bar{X}}$ = Standard deviation of sample mean.

σ = Universe or process standard deviation

n = Sample size or No. of observation in each sample

$$\bar{R} = \sigma d_2 \Rightarrow \sigma = \frac{\bar{R}}{d_2}$$

$$\boxed{\begin{aligned} U.C.L &= \bar{\bar{X}} + \frac{3 \cdot \bar{R}}{d_2\sqrt{n}} \\ L.C.L &= \bar{\bar{X}} - \frac{3 \cdot \bar{R}}{d_2\sqrt{n}} \end{aligned}}$$

$U.C.L = \bar{\bar{X}} + A_2 \bar{R}$

$L.C.L = \bar{\bar{X}} - A_2 \bar{R}$, where $\boxed{A_2 = \frac{3}{d_2\sqrt{n}}}$

Where, d_2 & A_2 are constant factors whose value depends only upon the sample size 'n'

b) R - Control limits

$C.L = \bar{R} = \sigma d_2$

$U.C.L = \sigma \cdot d_2 + 3 \cdot \sigma d_3$

$L.C.L = \sigma d_2 - 3 \sigma d_3$

Now as $\boxed{\bar{R} = \sigma d_2}$ or $\boxed{\sigma = \bar{R}/d_2}$

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$C.L = \bar{R} = \sigma \cdot d_2$

$U.C.L = \bar{R} + \frac{3 \cdot \bar{R} \cdot d_3}{d_2}$

$L.C.L = \bar{R} - \frac{3 \cdot R d_3}{d_2}$

$U.C.L = \bar{R}\left[1 + \frac{3d_3}{d_2}\right]$ 100

$L.C.L = \bar{R}\left[1 - \frac{3d_3}{d_2}\right]$

$\boxed{U.C.L = D_4 \cdot \bar{R}}$ - $\boxed{L.C.L = D_3 \cdot \bar{R}}$

Where $D_4 = \left[1 + \frac{3d_3}{d_2}\right]$ & $D_3 = \left[1 - \frac{3d_3}{d_2}\right]$

where, $\left[\begin{matrix} d_2 \;\&\; d_3, \\ D_3 \;\&\; d_4 \end{matrix}\right]$ are the constant factors whose value depends upon sample size 'n'.
and for $n < 7$, $D_3 = 0$

2. Attribute Control Chart

a) P- Chart : It is also known as fraction or proportion defective chart. It is used for the situation where sample size is not constant.

Sample No.	sample size	No. of Defective	Proportion defective $\left(P_i = d_i/n_i\right)$
1	n_1	d_1	$P_1 = d_1/n_1$
2	n_2	d_2	$P_2 = d_2/n_2$
|	|	|	
N	n_N	d_N	$P_N = d_N/n_N$

$$\bar{P} = \frac{P_1 + P_2 + \ldots + P_N}{N} = \sum_{i=1}^{N} \frac{P_i}{N}$$

$$\bar{n} = \frac{n_1 + n_2 + \ldots n_N}{N} = \sum_{i=1}^{N} \frac{n_i}{N}$$

where. $\sigma_{\bar{P}} = \sqrt{\frac{\bar{P}(1-\bar{P})}{\bar{n}}}$

$$UCL = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{\bar{n}}}, \quad LCL = \bar{P} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{\bar{n}}}$$

(or)

$CL = \bar{P}$

$UCL = \bar{P} + 3\sigma_{\bar{P}}$

$LCL = \bar{P} - 3\sigma_{\bar{P}}$

Where,

$\bar{P}$ = Average proportion defective

$\sigma_{\bar{P}}$ = standard deviation of average proportion defective

$\bar{n}$ = Average sample size.

) np- chart: It is also known as No of defective chart & is used for the situation where sample size is constant.

Sample No.	Sample size	No. of defective	Proportion defective ($P_i = d_i/n$)
1	n	d_1	$P_1 = d_1/n$
2	n	d_2	$P_2 = d_2/n$
N	n	d_n	$P_N = d_N/n$

$CL = n\bar{P}$ $\qquad \sigma_p = \sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$

$UCL = n\bar{P} + 3\sqrt{n\bar{P}(1-\bar{P})}$

$LCL = np - 3\sqrt{n\bar{P}(1-\bar{P})}$

$U.CL = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}} \qquad L.C.L = \bar{P} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$

p, np → defective (Binomial)
c → defect (Poisson)

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c) C-chart: It is also known as Doubt of Defect chart and it is used when we can compute only the no of defects but cannot compute the proportion that is defective.

It follows poisson's distribution & as in Poisson's distribution variance equals to mean

i.e. Variance = Mean.

$$\sigma^2 = \bar{c}$$

$$\boxed{\sigma = \sqrt{\bar{c}}}$$

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$CL = \bar{c}$

$UCL = \bar{c} + 3\sqrt{\bar{c}}$

$LCL = \bar{c} - 3\sqrt{\bar{c}}$

ACCEPTANCE SAMPLING

It refers to the process of randomly inspecting certain no of items from a lot in order to decide whether to accept or reject the entire lot. It is used when inspecting every item is either physically not possible or would be very expensive.

It is the only method of inspection where testing is done through destuctive manner.

Sampling Plan

1. Single Sampling Plan: In this plan 'n' sample are taken randomly once and if no. of defective is equal to or less than acceptance no. (c). Then the entire lot is accepted otherwise rejected.

$$\text{Max. } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$$
$$x_1, x_2 \cdots x_n \ge 0$$

variables = n, C equations = m 103

If there 'm' equality constraints & 'n' is the no. of variable. ~~Then~~, & $n > m$, Then we need to put '$n-m$' variable = 0 and solve the remaining 'm' variables with 'm' equality constraints.

The $n-m$ variables which are put equal to zero are called Non-basic variables and the remaining 'm' variables are called Basic variables which gives basic solution.

This step reduces the no of alternatives for optimal solution whose maximum limit is given by $\boxed{{}^nC_m = \dfrac{n!}{m!(n-m)!}}$

$n = 5$, $m = 3$, $n-m = 2 \Rightarrow$ Non basic, $m = 3$ – basic

Definitions :-

Feasible Solution : A solution which satisfy the ggiven constraints along with non-negative condition is called feasible solution

Basic solution : The solution of 'm' basic variable when each of '$n-m$' Non-basic variable is put equal to zero is called basic solution

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8 Basic Feasible Solution : It is the basic sol^n which also satisfy the non-negative condition.

3 BIG M- METHOD

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Big M Method is used in those situation where an Identity matrix is not operate initially. In those situation we add artificial ~~met~~ variable (A) into the model to get an initial working matrix. This method is always used for greater than or equal to ($\geq$ or $=$) sign irrespective of whether the given problem is minimisation or $maximisa^n$ type. These artificial variables must not apper in the final solution & this is ensured by Assigning extremely negative value to their profit coefficient in the objective function

$$Max = -MA_1$$

$$Min = +MA_1$$

where, M is a number higher than any Finite No.

Special Cases

1. Minimization Problem :- These problems can be solved by 2 method

a. By changing Minimisation problem into $Maximisa^n$ by multiplying objective function by -1 and solving as before.

b. It can be solved as minimisation by selecting the most $-ve$ Δj row value as key Column & optimal sol^n is achived when all the Δj values are either Positive or zero.

For Basic Variables : $\Delta_j = 0$

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2. Infinite No. of solutions :- When a non-basic variable in an optimal solution has 'zero' value for Δ_j row. Then, the solⁿ is not unique & the problem has infinite no. of solutions

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3. Unbounded ~~Variable~~ (solution) :- In simplex method to determine outgoing variable, we select the row which has the minimum positive replacement ratio (θ_i). If in a case there are all θ_i either -ve or infinite then the solⁿ terminates and this indicates that the problem has Unbounded solution

4 No solution or Infeasibility : When in the Final solution an artificial solⁿ remains in the basis, Then, there is no feasible solⁿ to the problem.

INfeasibility if present is detected in phase I, if Two Phase Method is apply.

5. Degeneracy :- When one more of the basic variable assumes a value of zero, it is called degeneracy & the solution is called degenerate solution.

4. DUALITY : The initial problem is known as Primal & another problem obtained after transposing Row & column & having the same optimal solution is called DUAL Problem

Constraints

1. $\leq$ for Maximisation problem
2. $\geq$ for Minimisation problem

Primal ⟷ Dual

Maxim. ⟷ min.

No. of Constraints ⟷ No. of Variables

$\leq$ type of constraint ⟷ Non-Negative Variable

= type of constraint ⟷ Unrestricted sign variable.

Profit coefficient ⟷ Resource value

Resource value ⟷ Profit coefficient

5 TRANSPORTATION MODEL (+20 marks)

Transportation problem are used for meeting the supply and demand requirements under the given condition in such a manner that total transportation cost is minimised.

- It is a special case of simplex method

Factories or source or Origin / Destination

	D1	D2	---	Dn	Supply ↓
F1	x_{11} $\boxed{c_{11}}$	x_{12} $\boxed{c_{12}}$	---	x_{1n} $\boxed{c_{1n}}$	a_1
F2	x_{21} $\boxed{c_{21}}$	x_{22} $\boxed{c_{22}}$	---	x_{2n} $\boxed{c_{2n}}$	a_2
⋮	⋮	⋮			⋮
Fm	x_{m1} $\boxed{c_{m1}}$	x_{m2} $\boxed{c_{m2}}$	---	x_{mn} $\boxed{c_{mn}}$	a_m
Demand →	b_1	b_2	--	b_n	

Demand = supply $\sum_{i=1}^{m} a_i = \sum_{j=1}^{m} b_j$ } simplex method

$$\boxed{\min.\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}$$ → Canonical Form

c_{ij} : cost to transport from i to j destination.

1. Feasible Solution : A set of non-negative individual allocation which satisfies the given constraint is called feasible solution.

$$\sum_{j=1}^{n} x_{ij} = a_i , \quad (i = 1, 2, \ldots n)$$

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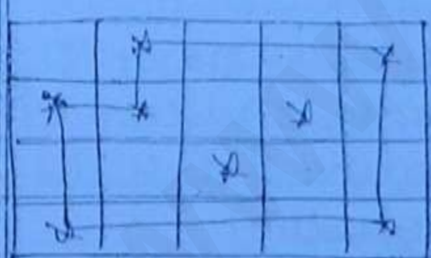
$$\sum_{j=1}^{m} x_{ij} = b_j , \quad (j = 1, 2, \ldots n).$$

2. Basic Feasible Solution : A feasible solution of $m \times n$ transportation problem is called basic feasible solution if the total no of allocation is exactly equal to $(m+n-1)$

3. Non-degenerate basic feasible solution : A feasible solution of $m \times n$ transportation problem is Non-degenerate if :-

(i) Total No of allocation is exactly equal to $(m+n-1)$ &

(ii) These $(m+n-1)$ allocations must be at independent positions. (No looping)



Dependent $\quad$ 4x5

Allocations $= 4+5-1 = 8$

Here by Independent Position we mean that, it is always impossible to form a closed loop by joining these allocations by a series of horizontal & Vertical lines from one occupied cell to another.

NOTE: Optimality test can only be performed if the initial solution is non-degenerate.

BALANCED & UNBALANCED T. Model

If total supply from all the sources equals to the total demand in all the destination, the problem is called balanced otherwise Unbalanced.
If the given problem is unbalanced, balance it by adding dummy source or destination

1. North-West Corner Rule
2. Row minima method.
3. Column Minima method
4. Least Cost method (Method of Matrix minima)
5. Vogel's Approximation Method's optimality test :-

8(i) Stepping stone method
7(ii) MODI- Method or U-V Method

Special Case

1. Degeneracy: When an no. of allocations in $m \times n$ transportation model is less than $m+n-1$. Then optimality test cannot be performed & such a sol^n is called degenerate solution.

2. Maximisation problem: Sometimes, T.M is used for Maximisation of profit or sales & it can be sold by 2 methods:-

i) By converting Maximisation problem into Min. problem by subtracting from the highest element. All the elements of the matrix & solving as before.

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ii) It can be solved as maximisation & here we write the difference of highest and the second highest profit element in each row & column. below the respective row & column During allocation, these we allocate to highest profit cost cell first.

5/09/11.
Happy Teacher's day

Gate'13

ASSIGNMENT

Assignment problem are special case of Transportation problem & it differs from Transportation Model on basically two grounds:- 109

1. The matrix must be a square matrix
2. The optimal solution must always be such that there will be one allocation in each row and column $x_{ij} = 0$ or 1.

$$\boxed{\text{Min } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}}$$

Hungarian Method is only applicable for minimisation problem. So, maximisation problem needs to be converted into minimisation before hungarian method may be applied.

It is done by subtracting from the highest element. All the elements of matrix

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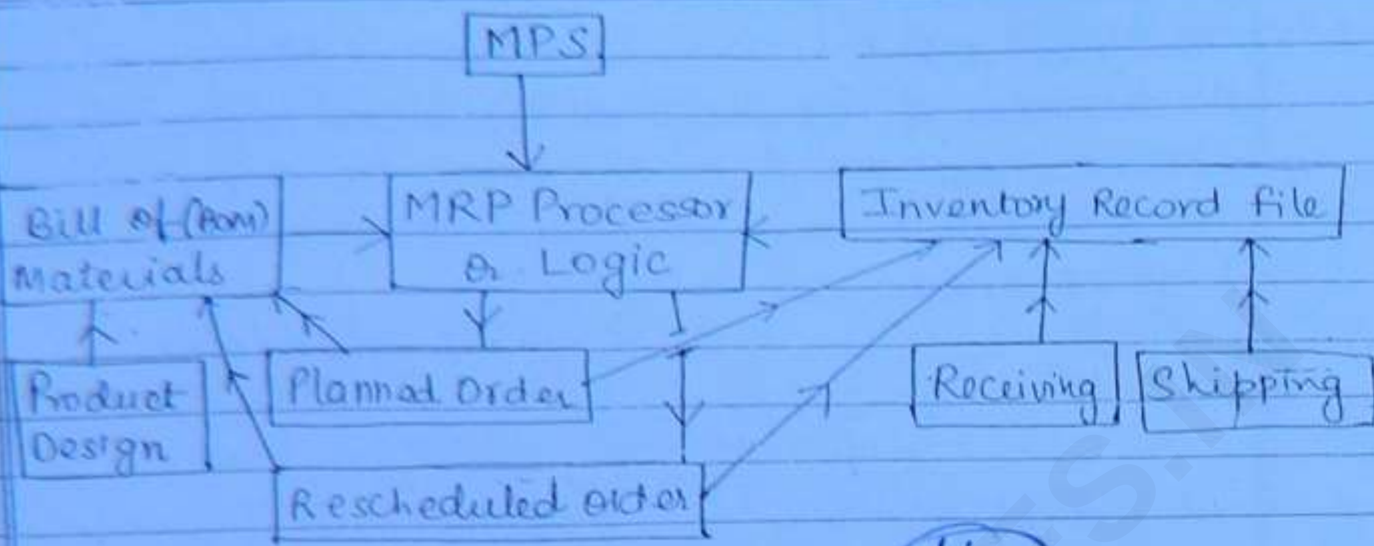
MATERIAL REQUIREMENT PLANNING (M.R.P)



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Independent : Forecasting
Dependent : MRP

MRP is a method of working out the production plan in a multi-stage production system which requires many product, sub-assemblies & raw material so that all things needed are available at an appropriate time.

Today, MRP is a computer based information system for scheduling, production and purchasing of dependent demand items.

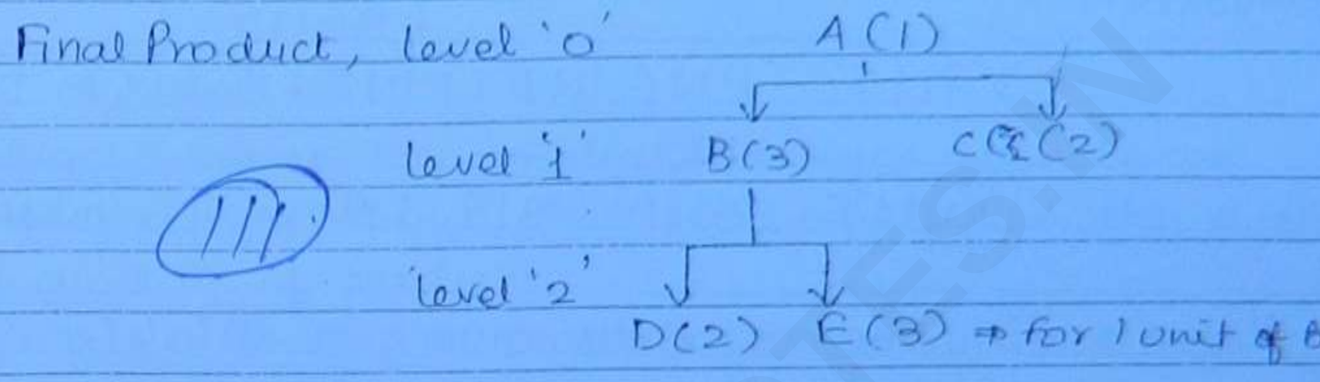
1. Master Production Schedule (MPS): It contains information about what product is to be produced, when it is to be produced & in what quantity.

2. Inventory Record or status file: This file gives complete & upto date information on the in hand inventory, gross requirement, receiving & scheduled receipt for an item.

Net Requirement = Gross Req − [Inventory in Hand + Scheduled Receipt]

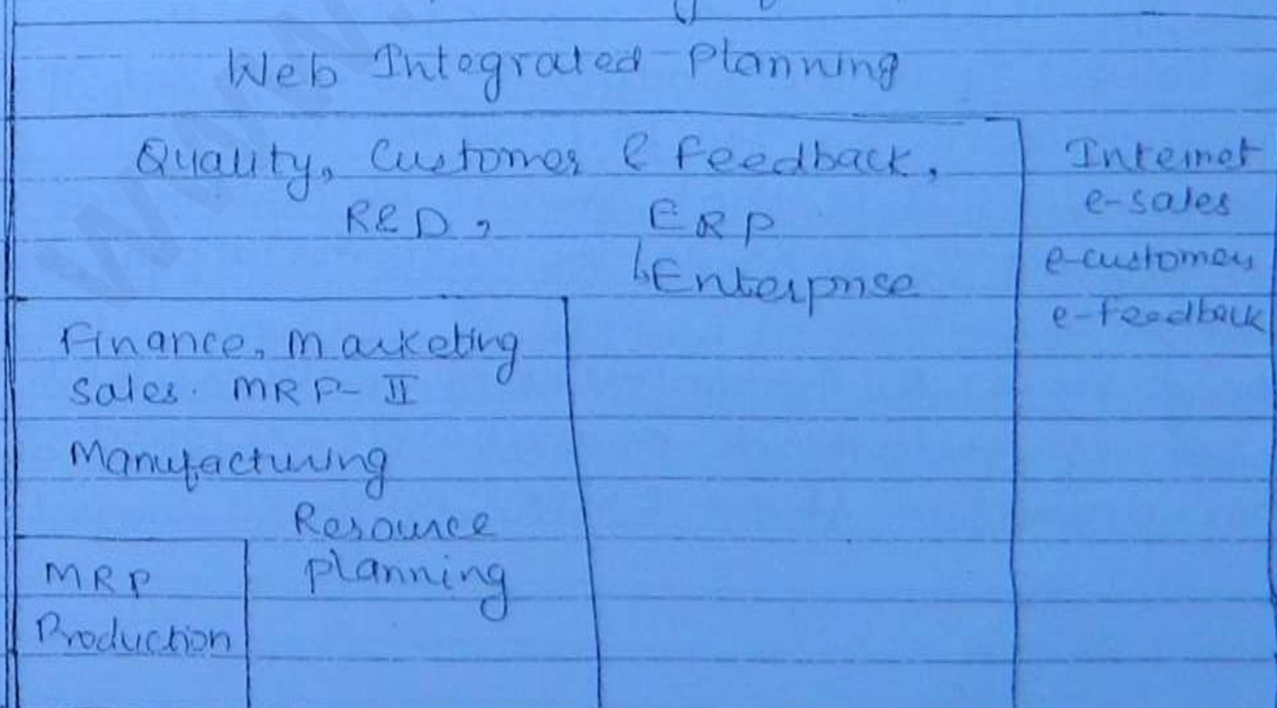
3. Bill of Material (BOM): It tells how each ~~end~~ product is produced, specifying ~~all som~~ sub-component items and their sequence of build-up in the final product

4. Product structure:



Advantages of MRP

1. It helps to know when & how much to order.
2. helps in inventory reduction.
3. helps to avoid delay in production
4. It gives marketing, timely information about likely delivery time.
5. eliminates the possibility of duplicate ordering



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	U.S.A (America)	Japan (Just in Time)
	MRP	JIT
1)	Push	Pull
2) Surplus inventory.	✓	✗
3) Good relation with vendors	✗	✓
4) Safety stock	✓	✗
5) Handle Adverse situation	✓	✗

VALUE ENGINEERING : Product in Design.

OR

VALUE ANALYSIS : Already in market.

$$\text{Value} = \frac{\text{Function or Performance or Quality}}{\text{Cost}}$$

Value means different things to different people & it may be mainly classified into 4 types:-

1. Cost Value : It is the cost of manufacturing a product & it is a summation of raw material cost, tooling cost, labour cost & other overheads required to produce that product.
2. Use Value : It is the amount of cost included to make a product work. eg Tye pin [Gold and Aluminium]
3. Esteem Value : It is that amount of cost included into the product to enhance its customer appeal that is desired to own.
4. Exchange Value : All the properties or features which make it possible ~~in~~ to trade or exchange a product with other product = Use + Esteem Value.

Steps in Value Analysis :-

BLAST
↓
CREATE
↓
REFINE

Blast: Select the product & collect all relevant information about that product. Define the function & quality of product
Create: Develop or create alternative functions or methods which decreases the cost
Refine: Install the best alternate that increases the value of product.

Advantages:-

1. Reduction in the no. of useless parts.
2. Reduction in the amount of scrap.
3. Overall cost reduction.
4. Higher customer satisfaction.

Value Analysis & Value Engineering are similar except the stage at which they are applied. Value Analysis is applied to the existing product in order to increase its value while V. Engineering is applied to the product at the design phase before being introduced into the market.

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Problems:-

Q1. A component can be produce by Four processes to produce 100 units which process should be used

Processes	Fixed cost	v (Rs/unit)
I	20	3
II	50	1
III	40	2
IV	10	4

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Given: $x = 100$

Total Cost $= F + v.x$

I $(20 + (3) \times 100) = 320$

II $(50 + (1) \times 100) = 150$ (min.) Best Process

III $(40 + (2) \times 100) = 240$

IV $(10 + (4) \times 100) = 410$

Q2 For a Production System a certain component 'x' is required for which there are 3 options:-

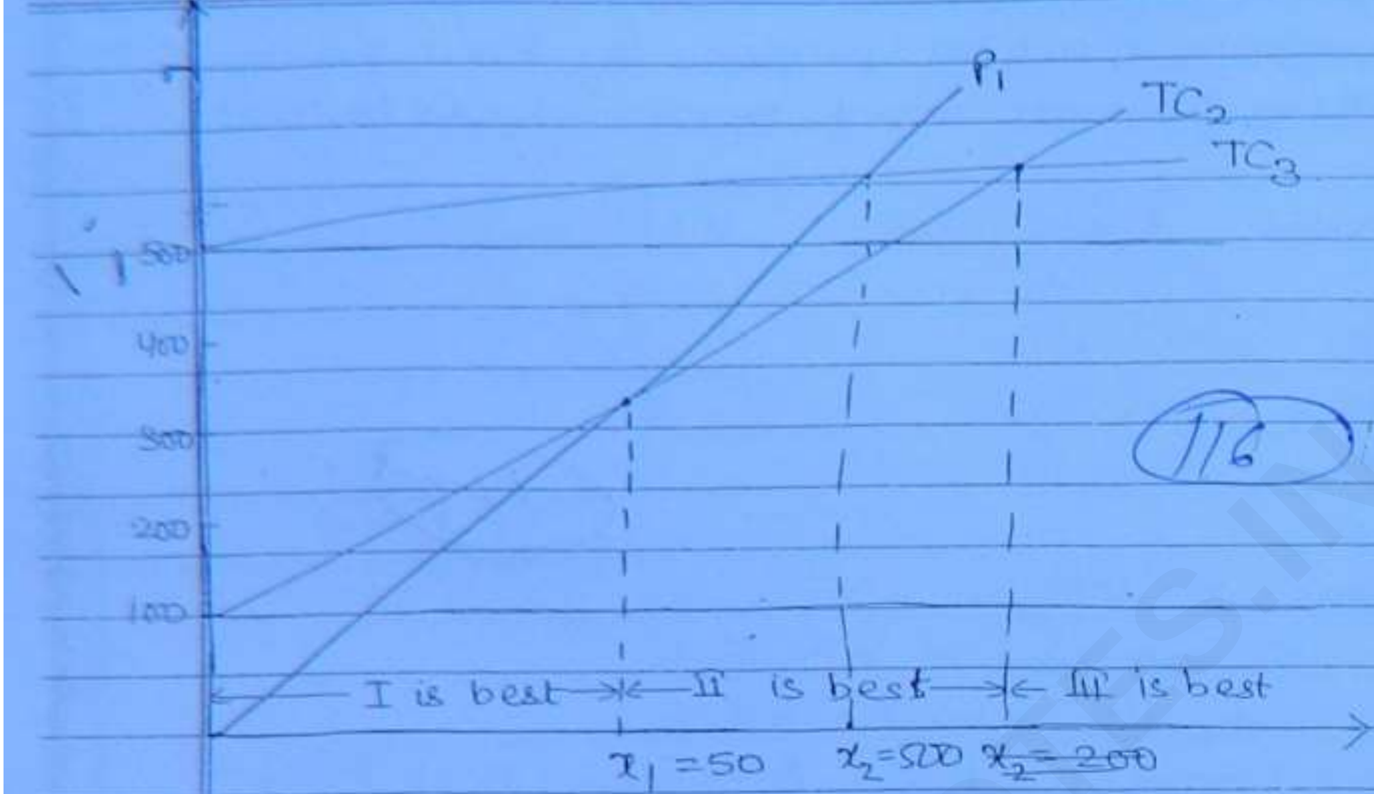
1) It can be purchased directly @ Rs 6/unit
2) It can be produced internally by a semi-Automatic machine with fixed cost of Rs 100 & Variable cost Rs 4 Per unit.
3) It can be produced by an Automatic Machine with a Fixed cost of Rs 500 and Variable cost Rs 2/unit

Find the Decision rule?

1) P.C = Rs 6/unit $= 6x$
2) F = Rs 100, V = Rs 4/unit $= 100 + 4x$
3) F = Rs 500, v = Rs 2/unit $= 500 + 2x$

$x_1 \Rightarrow 6x_1 = 100 + 4x_1 \Rightarrow 2x_1 = 100 \Rightarrow x_1 = 50$

$x_2 \Rightarrow 100 + 4x_2 = 500 + 2x_2 \Rightarrow 2x_2 = 400 \Rightarrow x_2 = 200$



IAS-06

83. Fixed cost = Rs 4500 = F

Total Variable cost = Rs 7500 = V

Total Sales = Rs 15,000 = S and

Unit Sold = 5000 = x

Find out :-

a) B.E.P in Units

b) Margin of safety

c) Profit

d) volume of sales to earn profit of 6000 Rs.

a) $B.E.P = \frac{F}{S-V} = \frac{4500}{15,000-7500} = \frac{\cancel{4500}^{\,\cancel{9}\,3}}{\cancel{7500}_{\,\cancel{15}\,5}} = \frac{3}{5} \times \overset{1000}{\cancel{5000}}$

$= \underline{3000 \text{ units}}$

b) $(M.O.S)_{sales} = S\left[\frac{(S-V)x - F}{(S-V)}\right] = 15,000\left[\frac{7500 \times 5000 - 4500}{7500}\right]$

$= \frac{\cancel{150}^{\,5\,\cancel{10}\,\cancel{30}}}{\cancel{75}_{\,2\,\cancel{5}\,\cancel{15}}}\left[\frac{37500000 - 4500}{4500}\right]$

$= \frac{5}{2}\left[37495500\right] = 5x$

Date :

$$s = \frac{S}{X} = \frac{15{,}000}{5000} = 3 \text{ Rs/unit}$$

$$v = \frac{V}{X} = \frac{7500}{5000} = 1.5 \text{ Rs/unit}$$

1) $B.E.P = \dfrac{F}{s-v} = \dfrac{4500}{3-1.5} = 3000$ Units

2) a) $(MOS)_{sales} = (Sales)_x - (Sales)_{BEP}$

$= 15000 - 3000 \times 3$

$=$ Rs 6000

b) $(MOS)\% = \dfrac{(MOS)_{sales} \times 100}{(Sales)_x} = \dfrac{6000}{15000} \times 100 = 40\%$

3) $S = V + F + P$

$P = 15000 - 7500 - 4500 = 3000$ Rs

4) $Q = \dfrac{F+P}{(s-v)} = \dfrac{4500+6000}{3-1.5} = 7000$ Units

IES 06 Q4 For a product S.P per unit is Rs 100 and Variable cost per Unit is Rs 60, and fixed cost is Rs 10lakh due to inflation the Variable cost have increased by 10% while fixed cost have increased by 5%. If Break Even quantity is constant. By what % should the sales price to be raised?

Given: $s =$ Rs 100, $F = 10\times10^5$ Rs, $F_1 = 10^6 + 10^2\times10^4 \times \frac{5}{100} =$

$v =$ Rs 60 $\quad v_1 = 60 + \frac{60\times10}{100} = 66$

$F_1 = 10^4[100+5] = 105\times10^4$

$B.E.P = \dfrac{F}{s-v} = \dfrac{10^6}{100-60} = \dfrac{10^6}{40} = \dfrac{10^5}{4} = \dfrac{100\times10^3}{4}$

$= 25000$ units.

$(B.E.P)_{sales} = \sum S \times (B.E.P)_x =$

$$\text{\&}\ B.E.P = \frac{F'}{S' - V'} = 25000$$

$$= \frac{105 \times 10^4}{S' - 66} = 25000$$

$$= 1050 = 25(S' - 66) \Rightarrow 1050 = 25S' - 25 \times 66$$

$$S' = Rs\ 108$$

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$$\%\ \uparrow = \frac{(S' - S)}{S} \times 100 = 8\%$$

23/8/11

Gate-04
Q1. There are two products 'P' and 'Q' with the following characteristics. Find the EOQ of product P & Q in ratio.

Product	D(Units/Year)	Co(Rs/order)	Ch(Rs/Unit/year)
P	100	50	4
Q	400	50	1

$$(EOQ)_P = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 100 \times 50}{4}} = \sqrt{2500} = 50$$

$$(EOQ)_Q = \sqrt{\frac{2 \times 400 \times 50}{1}} = \sqrt{40000} = 200$$

$$\frac{(EOQ)_P}{(EOQ)_Q} = \frac{50}{200} = \frac{1}{4} = 0.25$$

Gate-07
Q2. The maximum level of Inventory is 100 Units & it is achieved with infinite rate of Replenishment. The inventory becomes zero over $1\frac{1}{2}$ month due to consumption at a Uniform rate. This cycle continues

throughout the year. O.C = Rs 100/order and Inventory carrying cost = Rs 10 /item/month find the annual Inventory cost neglecting purchase cost. (Ans: 6800)

$$T = \frac{3}{2} \times \frac{\cancel{12}}{\cancel{12}} = 1/8 \text{ years}, \quad Q = 100 \text{ Units}$$
$$\left(\frac{12}{3} \times 2\right)$$

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$C_0 = 100$/order

$C_h = $ I.C = Rs 10/item/month = Rs 120 /unit /year

$$T.I.C = \frac{\cancel{10}^{5} \times 100 \times 1}{\cancel{4}8} = \frac{\cancel{500}^{250}}{\cancel{4}2} = 125$$

$$8 = \frac{D}{Q}$$
$$8 = \frac{D}{100}$$

$$TIC = \frac{D}{Q} \cdot C_0 + \frac{Q}{2} C_h$$

$$= 8 \times 100 + \frac{100}{2} \times 120 = \text{Rs } 6800$$

$$EOQ, \quad \frac{D}{Q} C_0 = \frac{Q}{2} \cdot C_h$$

$$Q = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{\cancel{2} \times \cancel{800}^{400} \times 100}{\cancel{6}_3\cancel{120}}} = \sqrt{\frac{4000}{3}} = 36.51$$

$$TIC = 8 \times 100 + \frac{36.51}{2} \times 120$$

$$TIC = \sqrt{2DC_0C_h} = 4381.4 \text{ Rs}$$

Q3 A companies annual demand is 18000 units & Cost Price per unit is Rs 4. O.C is Rs 120/order and inventory carrying cost is 1 %/month of Unit purchase price. Further, if it is Known that lead time is 10 working days & total working days in a year are 300. Determine :- a) EOQ b) optimum No of order/year c) length of Inventory cycle. d) TIC at EOQ e) Reorder level

f) No of days of stock at Re-order point.
g) Amount of saving with EOQ against earlier practise of 4 orders per year.
h) The increase in total cost associated with ordering :—
(i) 25% more than EOQ. , K = 1.25
(ii) 40% less than EOQ K = 0.60 (120)

D = 18000 units
C = Rs 4
O.C = Rs 120/order
$C_h = \frac{1\%}{\text{month}}$ of C $= .01 \times 12 \times 4 = 0.12 \times 4 = 0.48$
LT = 10 working days
Total working days = 300/year

1 Year = 300 days.
1 Year = 12 months
12 mon = 300 days
$1m = \frac{300}{12}$ days
=

a) $EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 18000 \times 12000}{0.48}} = 3000$

b) $N^* = \frac{D}{Q^*} = \frac{18000}{3000} = 6$

c) Length, $T = \frac{1}{N^*} = \frac{1}{6} \times 300 = 50$ days

d) $TIC = \sqrt{2DC_oC_h} = \sqrt{2 \times 18000 \times 120 \times 0.48} = 1440$.

e) Reorder level $= LT \times d = 10 \times \frac{18000}{300} = 600$

f) No. of days of stock at Reorder Point = Lead Time = 10 days

g) No. of order = 4, $Q = \sqrt{2 \times 18000 \times}$, $Q = \frac{18000}{4} = 4500$

Date :

g) $TIC(Q) = \frac{D}{Q} \times C_o + \frac{Q}{2} \cdot C_h = 4 \times 120 + \frac{4500}{2} \times (0.48) = 1560$ Rs.

Saving $= 1560 - 1440 = $ Rs 120

h) (i) $Q = KQ^* = 1.25 \times 3000 = 3750$

(ii) $Q = KQ^* = 0.60 \times 3000 = 1200$

i) $\frac{3750 - 3000}{3000} = 0.25 = 25\%$

ii) $\frac{3000 - 1200}{3000} = 0.6 = 60\%$

(121)

i) $\frac{TIC(Q)}{TIC(Q^*)} = \frac{1}{2} \left[K + \frac{1}{K} \right]$

$\Rightarrow TIC(Q) = \frac{\cancel{3750}\ 1440}{2} \left[1.25 + \frac{1}{1.25} \right] = $ Rs 1476

Increase $= 1476 - 1440 = $ Rs 36 ↑

i) $TIC(Q) = \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h = \frac{18000}{3750} \times 120 + \frac{3750}{2} \times 0.48$

$= $ Rs 1476

ii) $TIC(Q) = $ Rs 1632. ~~C 16~~

↑ = Rs 192 ↑ (1632 − 1440)

IES 04 8

Demand = 2400 units/year

Unit cost, C = Rs 6/unit

Ordering cost, O.C = Rs 32/order

Holding cost, $H.C = \frac{16\%}{100}$ per annum of Unit Purchase Price

5% Discount is given, if ordersize ≥ 500

Is it advisable to accept the discount.

$\cancel{Q} = D\cancel{Q} \times 2400 = 14400$, $C_o = 32$

$C_h = 0.16 \times \cancel{14400}\ 6 = 0.96$

$EOQ = \sqrt{\frac{2 \times 2400 \times 32 \times \cancel{3400}}{0.16 \times \cancel{14400}\ 6}} = 400$

$T.C = D \cdot C + \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h$ 7

$= 2400$

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$1 - 499 \rightarrow$ Rs 6/Unit

500 & above $\rightarrow$ Rs 5·7/Unit $= 6 - \frac{6 \times 5}{100} = 6 + \frac{3}{10}$

Starting from the lowest Unit Price,
searching feasible EOQ., $C = 5·7$/unit

$$EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 2400 \times 32}{0.16 \times 5.7}} = 410.39 \text{ units/order}$$

$\therefore$ 5·7/units corresponds to 410·39 units/order < 500

So, it is Not feasible, as Rs 5·7/Unit is applicable for order size $Q \geq 500$.

Moving to next higher Price level of $C =$ Rs 6/Unit
and of Rs 6/Unit & searching feasible EOQ.

$$EOQ = \sqrt{\frac{2 \times 2400 \times 32}{0.16 \times 6}} = 400 \text{ Units/order}$$

It is feasible as for Units price of Rs 6/unit
Order size of $Q = 400$ is applicable.

Now Calculating Total cost at feasible EOQ and the next Price break point of Q = 500.

$$TC(400) = D \cdot C + \frac{D}{Q} \cdot C_o + \frac{Q}{2} \cdot C_h$$

$$TC(400) = 2400 \times 6 + \frac{2400}{400} \times 32 + \frac{400}{2} \times \cancel{7.2}\ 6 \times 0.16$$

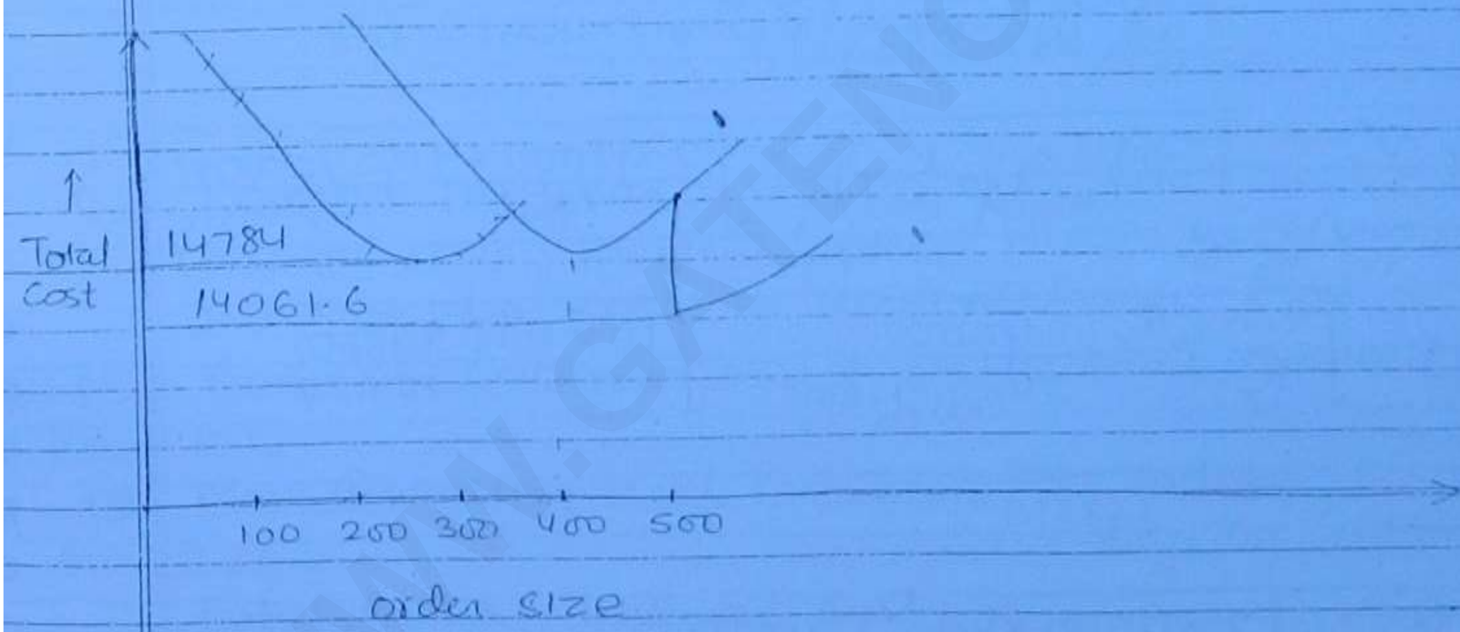
$$= 14784 \text{ Rs}$$

$$TC(500) = 2400 \times 5.7 + \frac{2400}{500} \times 32 + \frac{500}{2} \times (5.7 \times 0.16)$$

$$= 14061.6 \text{ Rs}$$

$$TC(600) = 14081.6$$

$$TC(450) = 14786.65$$



IES-08 Q

Expected annual sales was 8000 Units. O.C = Rs 18/order & H.C = 10% of Unit Purchase Price. The items can be purchase -

Determine the best order size.

$D = 8000$, $C_o = \text{Rs } 18$, $C_h = 0.10 \times 18 = 0.18$

Q. For a Company, Annual Demand is 12,000 Units Production rate per month is 2000 Units and Holding cost per unit per month is Rs 0.15 If the Unit cost is Rs 4/unit & the set up cost is Rs 400 Per setup. Then find :-

$$d = \frac{D}{12} = \frac{12000}{12} = 1000 \frac{\text{Units}}{\text{month}}$$

a) Optimum Lot size
b) Total Cost per year
c) Maximum Inventory Level.
d) Production time.
e) Total Cycle time.

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Given : $D = 12{,}000$, $P = 2000$, $H.C = 0.15$

$C = Rs\ 4$ ∴ $C_o = 400$

$C_h = 0.15 \times 12 = 0.15 \times 12 = 1.8$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{P}{P-d}\right)} = \sqrt{\frac{2\times 12000 \times 400}{0.6}\left(\frac{2000\times 12}{2000\times 12 - 12000}\right)}$$

$$= \sqrt{\frac{96\times 10^5}{6} \times \frac{24000}{24000-12000}} = \sqrt{16\times 10^6 \times \frac{24000}{12000}}$$

$$= \sqrt{32\times 10^6}$$

$$Q^* = \sqrt{\frac{2\times 12000 \times 400}{1.8}\left(\frac{2000\times 12}{2000\times 12 - 12000}\right)}$$

$$= 3265.98 \text{ Units/setup}$$

$$TIC = \cancel{DC} + \frac{D}{Q}\times C_o + \frac{Q}{2}\left(\frac{P-d}{P}\right)C_h$$

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b) $TIC = D\cdot C + \sqrt{2DC_oC_h\left(\frac{P-d}{P}\right)}$

$= 12{,}000 \times 400 + \sqrt{2 \times 12000 \times 400 \times 4.8\left(\frac{1000}{2000}\right)}$

$= 50939$ Rs

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c) $Q = Q^*\left(\frac{P-d}{P}\right)$

$Q = 3266\left(\frac{1000}{2000}\right) = 1633$ units / setup

d) Production time, $t_p = \frac{Q^*}{P} = \frac{3266}{2000} = 1.633\ \frac{\text{Month}}{\text{Setup}}$

e) Total cycle time, $\frac{Q^*}{d} = T = \frac{\cancel{12000}}{\cancel{1000}} = \frac{3266}{1000}$

$= 3.266$ month/setup

Q. Demand per year = 6000 Units. Unit cost = Rs 40
Ordering cost Rs 500/order. H.C = Rs 8/unit/year
and shortage cost Rs 20/unit/year. Find :-
(i) EOQ (ii) Optimum No of units back ordered
Ans = 1025 = 293
(iii) Max. Inventory level (iv) Time b/w orders
Ans = 732 62.3 days/or 0.25 month
Total Annual Inventory cost
5855

Ans:- $D = 6000$ Units, $C = $ Rs 40, $C_o = $ Rs 500/order
$C_h = $ Rs 8, $C_b = $ Rs 20.

i) $EOQ = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h}{C_b}+1\right)} = \sqrt{\frac{\cancel{2}\times \cancel{6000}^{3000} \times 500}{\cancel{8}\,4_2}\left(\frac{8}{20}+1\right)}$

$= 1024.69 \approx 1025$

Date :

(ii) Optimum No. of Units Back ordered.

$$S^* = Q^*\left(\frac{C_h}{C_b+C_h}\right)$$

$$= 1025\left(\frac{8}{8+20}\right) = 1025\left[\frac{8}{28}\right] = 292.85 \simeq 293$$

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(iii) Max. Inventory level

$$M^* = Q^*\left(\frac{C_b}{C_h+C_b}\right) = 1025\left(\frac{20}{20+8}\right) = 1025 \times \frac{20}{28}$$

$$= 732$$

(iv) Time between Orders. ; $T = \frac{Q}{d} = \frac{1025}{}$

(v) Total Annual Inventory cost,

$$TC = \sqrt{2DC_oC_h\left(\frac{C_b}{C_h+C_b}\right)} = \sqrt{2 \times 6000 \times 500 \times 8\left(\frac{20}{20+8}\right)}$$

$$= 5855.400$$

24/8/11

Gate-06 Q A shopkeeper wants to optimize his profit when the demand distribution for perishable item is as given below. The shopkeeper pays Rs 70 for each ~~ea~~ item & sells it at Rs 90 each. If the stock is left unsold he can return the item at Rs 50 each. To maximize the expected profit. Find the optimum stock level.

Demand (Units)	2	3	4	5
Probability	0·10	0·35	0·35	0·20
Cumm. Prob.	0·10	0·45	0·80	1·0

$P = Rs\ 20$

$l = Rs\ 20$

$$\frac{P}{P+l} = \frac{20}{20} = 0{\cdot}5 \quad \underset{P(S-1)}{[0{\cdot}45} < 0{\cdot}5 < \underset{P(S)}{0{\cdot}80]}$$

$P(S) = 0{\cdot}80$. Demand = 4. [Best optimum Size]

Q. What is the shortage cost range, if H·C is Rs 4000 and the demand & Probability is as given below. It is given that optimum Inventory level is 3 units.

	Demand	Probability	Cum
	0	0·65	0·65
	1	0·18	0·83
S−1	2	0·05	0·88
S =	3	0·03	0·91
	4	0·05	0·96
	5	0·04	1·00

$$S^* = Q^*\left(\frac{C_h}{C_b+C_h}\right) -$$

$$P(S-1) = 0{\cdot}88 < \frac{C_b}{C_b+C_h} \leq P(S)$$

$PS = 0{\cdot}88$

$= 0{\cdot}91 \qquad 0{\cdot}88 < \dfrac{C_b}{C_b+4000} \leq 0{\cdot}91$

$C_b \leq 0{\cdot}91 C_b + 3640 \qquad 0{\cdot}88 C_b + 3520 < C_b$

$C_b \leq 3640 \div {\cdot}09 \qquad 3520 < 0{\cdot}12 C_b$

$C_b \leq 40444{\cdot}44 \qquad C_b > 29333{\cdot}33$

$$29333{\cdot}33 < C_b \leq 40444{\cdot}44 \text{ Rs}$$

Q. Average weekly demand is 400 units and weekly standard deviation is 312.5 Units. Cost/Unit is Rs 4 & Lead time is of 2 weeks. If holding is Rs 0.01 per unit per week. Then for 95% S.L, Find :-

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a) level of safety stock
b) Re-order point
c) Annual Cost of Maintaining Safety stock.

Given, $D = 400$ units.
$\sigma = 312.5$, $\sigma' = 441.94$
$Z = 1.645$, $S.L = 95\%$.
$C = Rs\ 4$ $C_h = Rs\ 0.52/\text{unit}/\text{year}$
$C_h = 0.01/\text{week}$
$LT = 2$ weeks

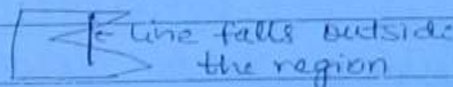
i) Level of Safety stock, $S.S = Z\ \sigma'$
$= 1.645 \times \sqrt{312.5^2 + 312.5^2}$
$= \cancel{514.06}\ 726.99 \simeq 727$ units

ii) Reorder Point, $X = \bar{X} + Z \cdot \sigma$
$\bar{X} = LT \times d = 2 \times 400 + S.S$
$= 800 + 726.99$
$= 1526.99 \simeq 1527$ Units

iii) Annual Cost, $S.S = S.S \times C_h$
$= 727 \times 0.52 = 378$ Rs/annum

Q For a production system, Annual Consumption is 20,000 units. Cost of 1 Unit is Rs 4. C_o = Rs 60/per order. Inventory carrying cost = Rs 0.6 per Unit/year. Past lead time are 15, 20, 18, 12, 22, 27 days. If there are 330 working days in a year Calculate, EOQ, 3 Find out the point on the feasible region that optimises the objective function That point gives the optimal solution.

Feasible Region: When all the constraints of LP are plotted on a graph they make a common space or region within which all the constraints are satisfied This region is known as feasible region.
This region must be a convex set otherwise LP cannot be apply

Non-Convex 

Convex 

Binding Constraint :- When the optimal solution is put into the constraint equation & L.H.S = R.H.S Then the constraint is known as binding and the binding constraint gives the optimal solution and L.H.S ≠ R.H.S Ie Not binding.

Special Cases:

1. Infinite or ~~Multiple~~ optimal Solution: We get Unique solution when the slope of objective function is different from the slope of constraint equation but if the slope of one of the constraint equation becomes equal to objective function slope then infinite solution exist if the following 2 conditions are satisfied :-

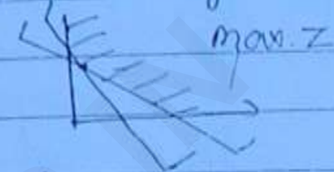
a. The constraint eq^n whose slope equals to objective function must make a part of feasible region i.e constraint must be Non-Redundant

b. The objective function must optimised in the same direc^n

Date :

Where constraint eqn makes the boundary of feasible region i.e constraint be a binding constraint.

2. Unbounded Solution: The greatest value of objective function in these situation lies at infinity. It's simply means at the common feasible region is not bounded by the limits.



(Infinite feasible Point)

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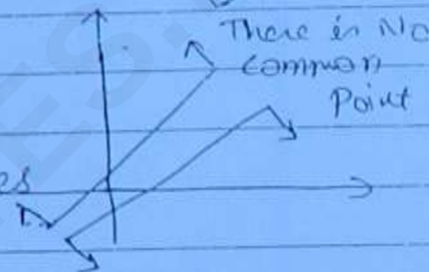
3. No solution or Infeasibility

In some situation constraints may be inconsistent i.e it is not possible to find a feasible solution which satisfies all the given constraints.



(No feasible point)

2/09/11

2 SIMPLEX METHOD:-

It is a step by step procedure in which we proceed in systematic manner from an initial feasible soln. We then improve upon this initial solution in consecutive steps so that after finite no. of steps, we reach optimal solution.

Standard form for simplex

1. All the resource value should be non-negative.
2. All the inequalities of the constraint equation should be converted into equalities.

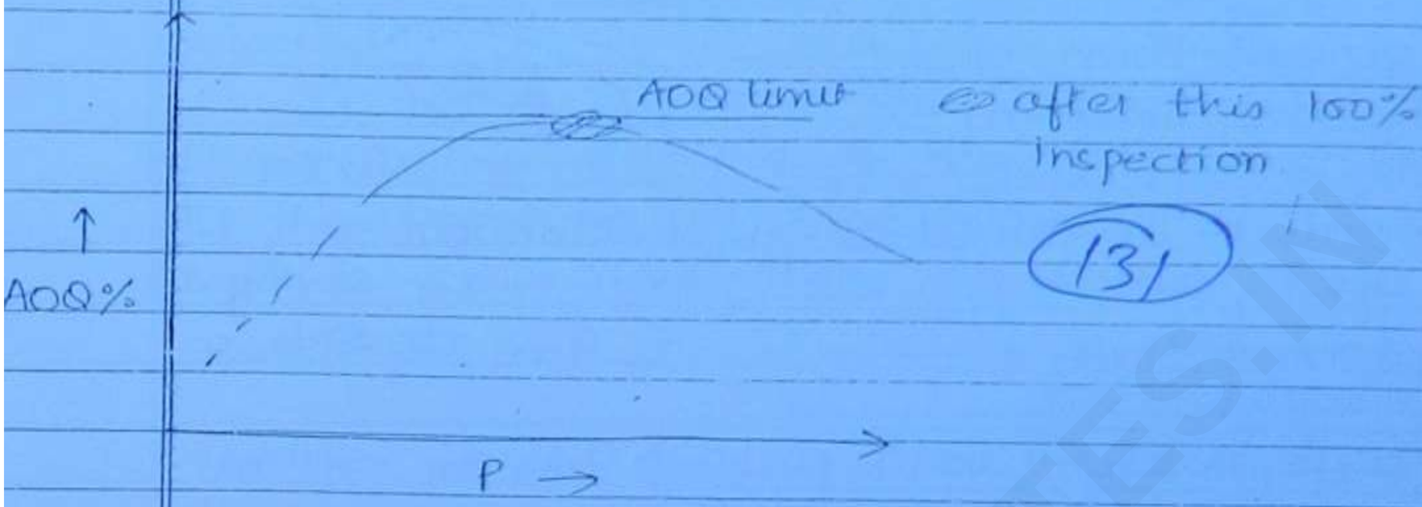
eg $2x_1 - 3x_2 \le -65$

1. $-2x_1 + 3x_2 \ge 65$ [S_2 = Surplus variable]
2. $-2x_1 + 3x_2 + S_2 = 65$ [S_1 = slack variable]

eg $3x_1 + 5x_2 \le 80 \Rightarrow 3x_1 + 5x_2 + S_1 = 80$

3. Each one of the decision variable should be Non-negative ($x_j \ge 0$)

where, P is percent defective & Pa is the Probability of acceptance.
N = lot size, n = Sample size



LINEAR PROGRAMMING

-: George B Dangling

L.P provides the method for optimization of resources when there are no. of possible solution or decisions. It is a mathematical technique & the term linear simply means that the variables are in search a form that their mathematical relation can be represented in the form of straight line.

Requirements of L.P:

1. Objective function: It is the function which need to be optimised and it should be clearly identifiable & measurable in quantitative term like Maximisation of profit, sales or minimisation of cost.

2. Constraint: Or: condition or Restriction: There are the limited resources within which we need to optimize

~~as~~ objective function.

§ objective function & Constraint Equation should be linear

General Form of LP

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objective function:-

$$\text{Max } Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

constraint or Condition or Restriction

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \leq b_2$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \leq b_m$$

$$x_1, x_2, \ldots x_n \geq 0$$

c_j, a_{ij} & b_i - constant

x_j - Variable

$i = (1, 2, \ldots m)$

$j = (1, 2, \ldots n)$

c_j = profit coefficient, a_{ij} = Technological coefficient or substitution, b_i - Resorce value and x_j is decision variable or choice variable.

These problem can be solved by graphical or Algebric method (Simplex).

*Es 1. Graphical method has the restriction that it can be used only when 2 variables are involved.

Steps for G M:

1. Identify the problem & define the decision variable, objective function & the constraints equation.
2. Draw a graph that includes all constraints ~~the~~ & identify the feasible region.

Reliability:
Single S < Double < Triple - -

Page No.
Date :

2 **Double Sampling Plan:** A sample of 'n_1' units is taken & if the no of defective item is C_1 or less then the entire lot is accepted & if no of defective is C_2 or more than the entire lot is rejected.

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If no of defective are between C_1 & C_2 then another sample of 'n_2' items is taken. If the total no. of defectives from the 2 samples together C_3 or less then the entire lot is accepted.

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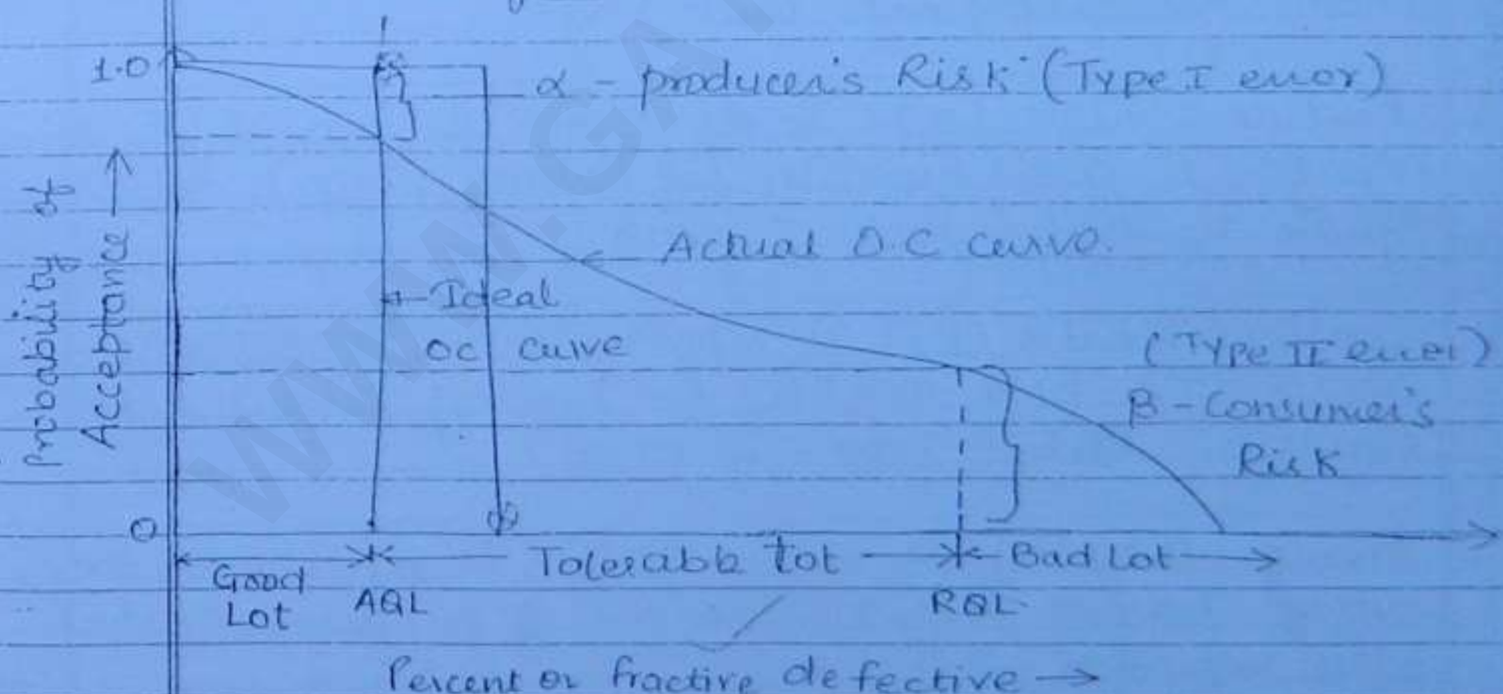
As the no. of sampling plan increases the probability of finding out the exact situation increases & the average no. of units expected decreases

Operating characteristic curve (O C curve)



O.C curve is a graph of fraction defective in a lot against the probability of acceptance & we can show the discriminating power of a sampling plan on a graph by O.C curve.

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The terms associated with O.C curve are:-

1. Acceptable Quality Level (AQL): There is a small percent of defective that consumers are willing to accept. AQL represents that level.

2. Rejectable Quality Level (RQL): Also known as Lot Tolerance Percent Defective (LTPD). Consumers normally tolerate a few more defect above AQL but then no of defective reaches a level beyond which consumers will not tolerate them. RQL represents that level.

3. Producer's Risk (α): It represents the probability of rejecting a good quality lot having percent defect equals to AQL (Type-I error).

4. Consumer's Risk (β): It represents the probability of accepting a bad quality lot having percent defect equal to RQL (Type-II error)

5. Average Outgoing Quality (AOQ): It represents the average percent defective in the outgoing products after inspection

$$\boxed{AOQ = P \cdot P_a\left(\frac{N-n}{N}\right)} \quad \text{if } N >> n, \quad \boxed{AOQ = P\, P_a}$$

S·S, ROL, Average stock in inventory.

$D = 20{,}000$, $C = Rs\ 4$, $C_o = Rs\ 60/\text{order}$, $TIC = Rs\ 0.6/\text{unit/year}$

$$EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 20{,}000 \times 60}{0.6 \times 300}} = \cancel{774.59 \approx 774.6} = 2000$$

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~~S.S~~ = Average LT = 19 days

= 485

$SS = (\text{Max LT} - \text{Averag LT})\,d = 27 - 19 =$ [illegible]

$ROL = LT \times d + SS = 19 \times \frac{20000}{300} + \cancel{8}\ 485$

$= \cancel{1.5\ldots} = 1637$ Units

Average stock in inventory $= \frac{EOQ}{2} + SS$

$= \frac{2000}{2} + \cancel{8}\ 485 = \cancel{1008\ldots} = 1485$

Q. Daily demand for a product is Normally distributed with mean of 50 units & $\sigma = 5$ units. $C_o = Rs\ 25/\text{order}$, $C_h = 20\%$ of Unit price of Rs 15. The Lead Time is consi at four working days for a 95% working level. Determine the Order quantity, S.S at Reorder Level Assuming that there are 300 working days in a year.

$D = d \times \text{working days} = 50 \times 300 = 15000$ Units

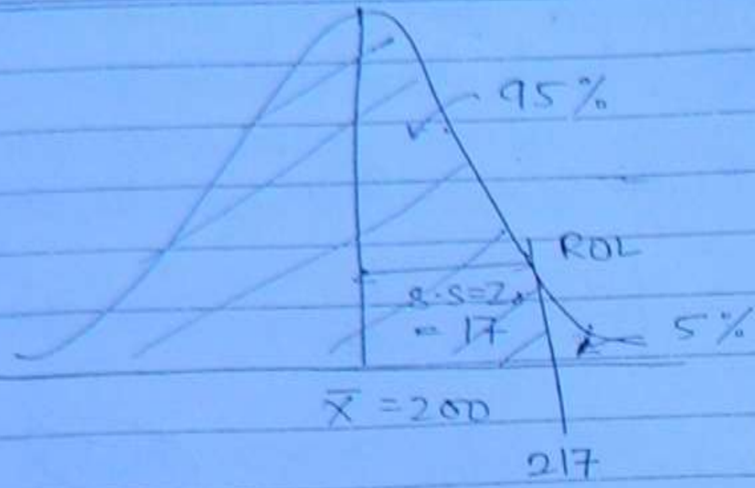
$\sigma' = \sqrt{4\sigma^2} = 2\sigma = 10$ units

ii) $S.S = Z\,\sigma' = 1.645 \times (5 \times 2) = 8.225 \times 2 = 16.45 \approx 17$ units

$\bar{x} = LT \times d = 50 \text{ units} \times 4 = 200$ Units

iii) $ROL = (LT \times d) + SS = 200 + 17 =$ Units $= 217$

i) $EOQ = \sqrt{\frac{2 \times 50 \times 25 \times 300}{0.2 \times 15}} = 500$ Units/order



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Q. A set of job are to be machined on a single machine obtain a sequence using SPT and EDD rule.
Also calculate Make Span Time, Average flow time per job, Average Tardiness per job, Average No. of jobs in system and No. of Tardy jobs.

Jobs	Proc. Time	Due Date	SPT	EDD
A	10	34	3	
B	8	32	2	
C	16	46	5	
D	6	36	1	
E	18	64	6	
F	12	41	4	

SPT

Jobs	P.T	DD	Job flow Time	Tardiness
D	6	36	0+6 = 6	0
B	8	32	6+8 = 14	0
A	10	34	14+10 = 24	0
F	12	41	24+12 = 36	0
C	16	46	36+16 = 52	6
E	18	64	52+18 = 70	6
				12

70 Make Span Time = 70

ΣJFT = 202

Tardiness = 12.

Total Job Flow time = 202

i) Make Span Time = 70

ii) Average FJob Flow time $= \frac{202}{6} = 33.67$

iii) Average Tardiness $= \frac{12}{6} = 2$

iv) Average No. of Jobs $= \frac{\text{Total Job Flow time}}{\text{Make Span Time}} = \frac{202}{70}$

$= 2.88$

$= 6 \times 6 + 5 \times 8 + 4 \times 10 + 3 \times 12 + 2 \times 16 + 1 \times 18 = 202$

v) No. of Tardy jobs = Two jobs (C & E are Tardy)

EDD

Jobs	PT	Due Date	Job Flow Time	Tardiness
B	8	32	0+8 = 8	0
A	10	34	8+10 = 18	0
D	6	36	18+6 = 24	0
F	12	41	24+12 = 36	0
C	16	46	36+16 = 52	6
E	18	64	52+18 = 70	6
			208	

i)* Total job flow time = 208

* Average J.F T = 308/6 = 34.67

ii) make Span time = 70

iii) Average Tardiness = 12.

(iv)* Average NO. of Jobs $= \frac{208}{70} = 2.97$

These values are always minimum in SPT.
So Preferred

Q. 4 jobs are to be processed on a machine as per data given below:-

i) If the earliest due date, Rule is to sequenced to job. Find the ~~ju~~ no. of jobs delayed.

ii) Using the SPT Rule. Find the Total Tardiness.

Jobs	P.T	D.D
1	4	6
2	7	9
3	2	19
4	8	17

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SPT ~~EDD~~

Jobs	P.T	D.D	J.F.T	Tardiness
3	2	19	0+2=2	0
1	4	6	6	0
2	7	9	13	4
~~EDD~~ 4	8	17	21	4
			42	8

Average Job flow Time
~~No of jobs delayed~~ = 10.5

Average No. of Jobs = 2

EDD ~~SPT~~

Jobs	P.T	D.D	JFT	Tardiness
1	4	6	0+4=4	0
2	7	9	11	2
4	8	17	19	2
3	2	19	21	2
			55	8

No. of Jobs delayed = (2,4,3) i.e 3

Total Tardiness = 8

Average Tardiness = 2

Average Job Flow Time = 13.75

Average No. of Jobs = 2.62

Q. Sequence the following set of jobs. Find make span time. Idle time for each machine and % utilisation.

Jobs	M/c I	M/c II
A	3	7
B	6	9
C	5	8
D	10	5
E	7	4
F	9	10
G	8	4.
H	6	9
I	5	12

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A C I B/H F D ~~G~~ E G

Jobs	I	II	IN	OUT		
A	3	7	0	3	③ 3	10
C	5	8	3	8	10	18
I	5	12	8	13	18	30
B	6	9	13	19	30	39
H	6	9	19	25	39	48
F	9	10	25	34	48	58
D	10	5	34	44	58	63
G	8	4	44	52	63	67
E	7	4	52	59	67	71
	59	68				

1) Make Span time = 71

2) Ideal time =

M/c I = 71 − 59 = 12

M/c II = 71 − 68 = 3

3) % Utilisation

I = $\frac{59}{71} \times 100 = 83.098$

II = $\frac{68}{71} \times 100 = 95.77$

Q. Obtain a sequence for the following set of jobs. Find the Make Span Time & the Ideal time for each machine.

Jobs	A	B	C
1	3	4	6
2	8	3	7
3	7	2	5
4	4	5	11
5	9	1	5
6	8	4	6
7	7	3	12
	46	22	52

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$3 \not\geq 5$ $\quad$ $5 \geq 5$

	Xi	Yi
1	~~7~~	~~10~~
2	11	10
3	~~9~~	~~7~~
4	~~9~~	~~16~~
5	~~10~~	~~6~~
6	12	10.
7	10	15

Seq. 1 4 7 ~~2~~ 6 2 3 5

	I	II	III	I in	I out	II in	II out	III in	III out
1	3	4	6	0	3	3	7	7	13
4	4	5	11	3	7	7	12	13	24
7	7	3	12	7	14	14	17	24	36
6	8	4	6	14	22	22	26	36	42
2	8	3	7	22	30	30	33	42	49
3	7	2	5	30	37	37	39	49	54
5	9	1	5	37	46	46	47	54	59

Make Span time = 59

Ideal time I = 13

II = 37

III = 7

% Utilisation, I = $\frac{46}{59} \times 100 = 76.966$

II = $\frac{22}{59} \times 100 = 37.288$

III = $\frac{52}{59} \times 100 = 88.1355$

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Q23

Jobs	M1	M2
A	~~4~~	~~4~~
B	7	7
C	~~3~~	~~10~~
D	~~12~~	~~8~~
E	~~11~~	~~10~~
F	~~9~~	~~13~~

C A F ___ E D B.

Q33 $X = 3000/\text{quarter} = 4 \times 3000/\text{year}$

$D = 12{,}000$ Units

Q36 $40{,}000 + 9x = 16000 + 24x$

$24000 = 15x \Rightarrow x = \frac{\cancel{24000}\ 4800}{\cancel{15}\ 3} = 1600$

Q37 $z = 2.335$, $\sigma = 99.7\%$

Date [redacted]

Q Find an optimal sequence for the following sequencing problems of 4 Jobs on five machines when passing is not allowed & machining time is given in hours.

Machine / Jobs	A	B	C	D	E
1	6	4	1	2	8
2	5	5	3	4	9
3	4	3	4	5	7
4	7	2	2	1	5
	22	14	10	12	29
% utilisation	51.16%	32.56%	23.255	27.90	67.44%

Min A, 4 $\geq$ Min of (B,C,D) 1

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	X_1		Y_1
1	6	~~13~~	~~15~~
2	5	17	21
3	9	~~16~~	~~19~~
4		~~12~~	~~10~~

Idle time I = 21
II = 29
III = 33
IV = 31
V = 14

Sequence $\boxed{1 \quad 3 \quad 2 \quad 4}$

	IN OUT	IN				
~~9~~	~~0 - 7~~	~~7 - 9~~	~~9 - 11~~	~~11 - 12~~	~~12 - 17~~	
1	0 - 6	6 - 10	10 - 11	11 - 13	13 - 21	
3	6 - 10	10 - 13	13 - 17	17 - 22	22 - 32	29
2	10 - 15	15 - 20	20 - 23	23 - 27	29 - 38	
4	15 - 22	22 - 24	24 - 26	27 - 28	38 - 43	

(43-24) = 19+6+2+2 = 29

Make Span time = 43

Q. For the following set of activities. Draw the Network Diagram and Determine:-

1. The critical Path & the expected project completion time.

2. Determine the probability of completing project in 50 days

3. If a company agrees to complete the project in 50 days, failing which it would pay Rs 200 per day as fine. Find the probability that the fine must be paid but not exceeding Rs 1000

Activity	Predecessor	Time (Days) a	m	b	Expected Time	Standard Deviation σ
A	—	4	6	8	6	0.667 (4/6)
B	A	5	7	15	8	1.667 (10/6)
C	A	4	8	12	8	1.33 (8/6)
D	B	15	20	25	20	1.667 (10/6)
E	B	10	18	26	18	2.67 (16/6)
F	C	8	9	16	10	8/6
G	E	4	8	12	8	8/6
H	D, F	1	2	3	2	2/6 = 1/3
I	G H	6	7	8	7	2/6 = 1/3
					87	

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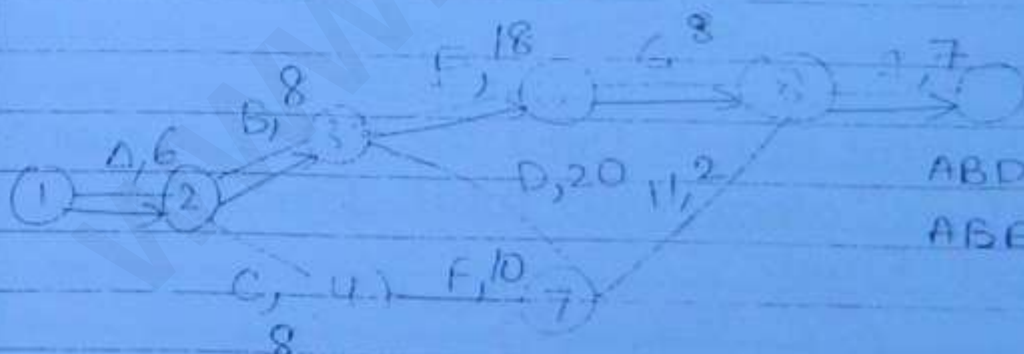


ABDHI 43

ABEGJ 47

1. 1–2 Critical Path: ABEGI = 47 days

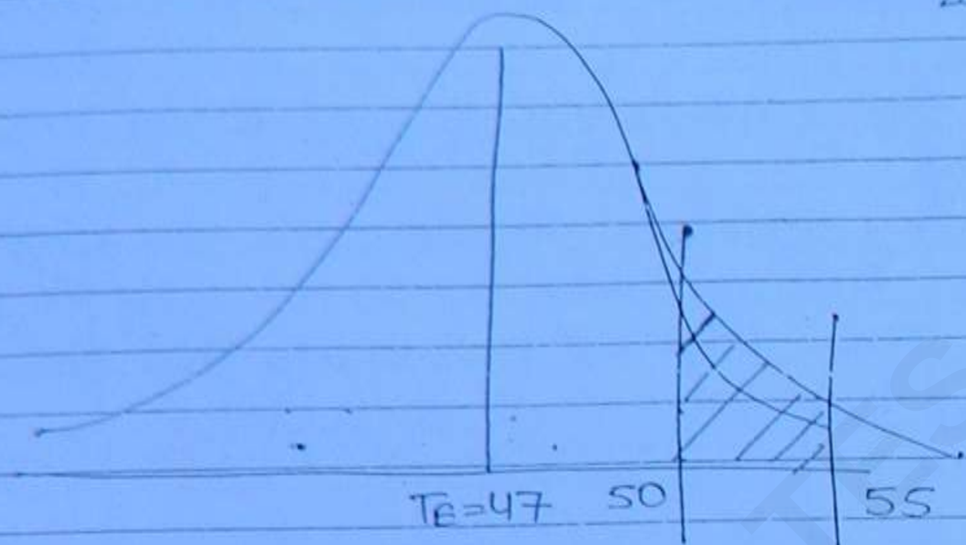
2. $\sigma = \sqrt{12.22} = 3.496$

$Z_1 = \dfrac{T_S - T_E}{\sigma} = \dfrac{50-47}{3.496} = 0.8581$

$P = 0.80455$ (By Calculator)

3. $Z_2 = \frac{T_S - T_E}{\sigma} = \frac{55-47}{3.946} = \cancel{1.43}$ $\cancel{0.9889}$ 2.288

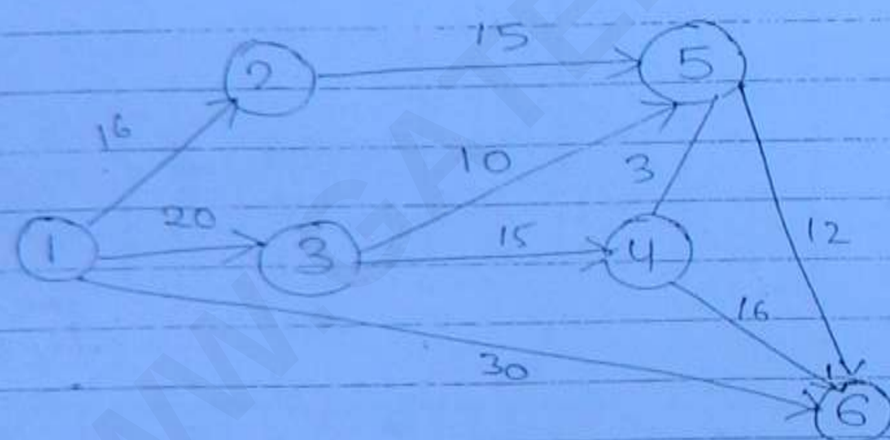
$P = 0.\cancel{92364}\ 0.9889$ $\frac{1080}{288} = 5$



Probability = $Z_2 - Z_1 = \cancel{0.9889 - 0.8581 = 0.1308}$

$= 0.9889 - 0.8045 = 0.1844$

Q.



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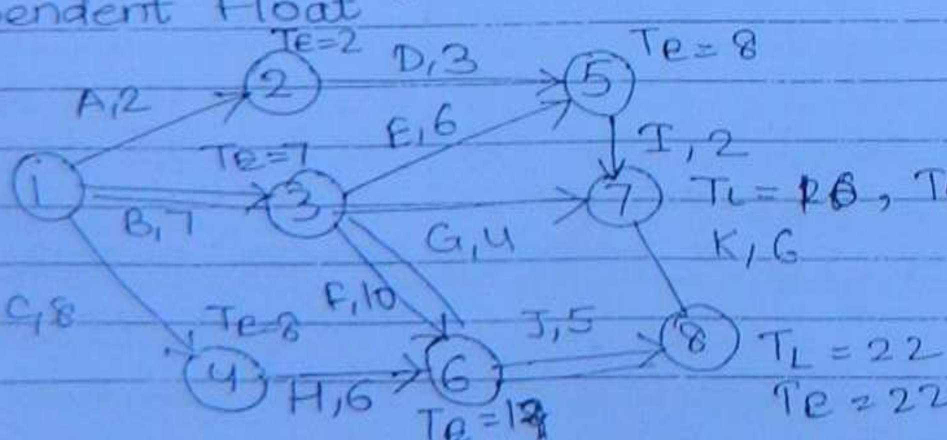
Q. Draw the critical Path & Draw the table showing all the details along with total Free and Independent Float



Date :

Event	Earlies Time
1	0
2	2
3	~~[5, 8]~~, ~~8~~ 7
4	8
5	[5, 8], 8 13
6	[17, 18], 18 17
7	[~~10, 11~~], ~~14~~ 15
8	~~22~~ 17+5 = 22

Event	Latest Time
8	~~26~~ 22
7	~~20~~ 16
6	17
5	16-2 = 14
4	11
3	7
2	11
1	0

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Critical Path : 1-3-6-8

[illegible]

Activity		te	Earliest		Latest				
			EST	EFT	LST	LFT	Total	Free	Ind.
A	1-2	2	*0	2	*9	11	9	0	0
B	1-3	7	0	7	0	7	0	0	0
C	1-4	8	0	8	3	11	3	0	0
D	2-5	3	2	5	11	14	98	8	1 0
E	3-5	6	7	13	8	14	18	0	0
F	3-6	10	7	17	7	17	0	0	0
G	3-7	4	7	11	12	16	5	4	4
H	4-6	6	8	14	11	17	3	3	0
I	5-7	2	13	15	14	16	1	0	1 0
J	6-8	5	17	22	17	22	0	0	0
K	7-8	6	15	21	16	22	1	1	0
								16	4

Free Float = T.F − Head slack

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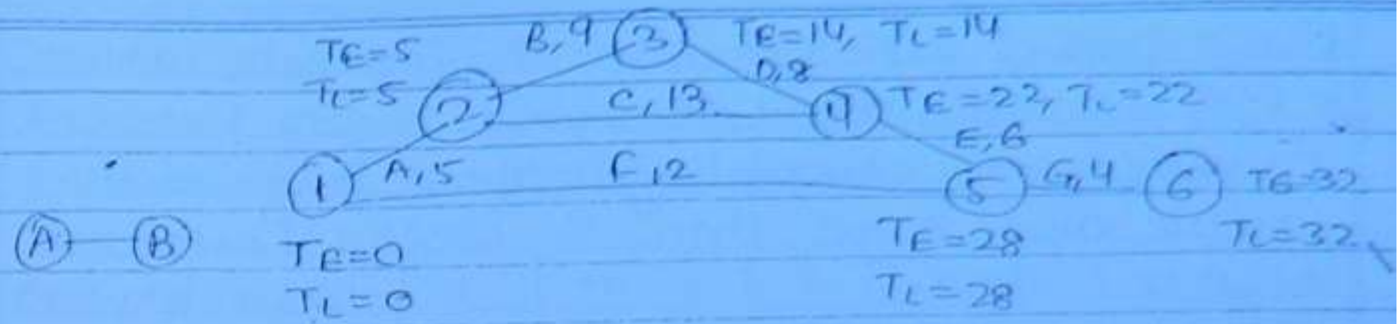
Q Draw the Network Diagram from the Critical Path & the expected project completion time. Along with Total & free Float. Also, find the probability of completing project in Expected Time.

Activity	Predecessor Activity	a	m	b	T_E	σ
		Time in Days				
A	—	3	5	7	5	4/6
B	A	7	9	11	9	4/6
C	A	4	14	18	13	9/6
D	B	4	8	12	8	8/6
E	C, D	4	6	8	6	4/6
F	—	5	12	19	12	14/6
G	E, F	2	4	6	4	4/6

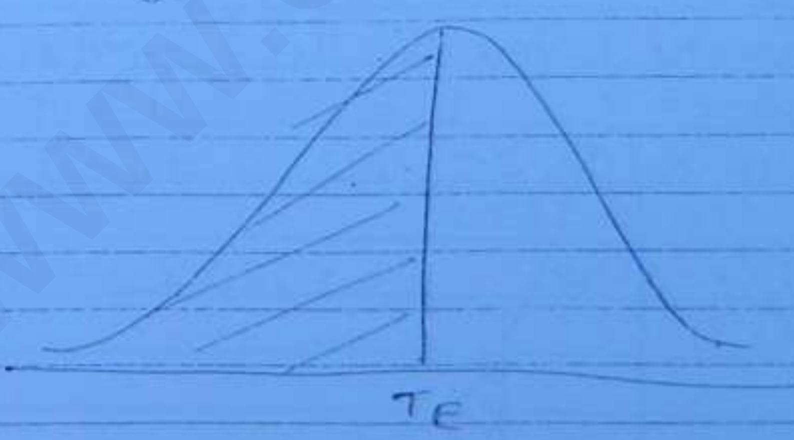


$\sigma = 3.354$ critical path = abdeg = 32

		E.S	E.F	L.S	L.F	Total	F.F	I.F
1–2	A	0	5	0	5	0	0	0
2–3	B	5	14	58	14	0	0	0
2–4	C	5	18	9	22	4	4	4
3–4	D	14	22	14	22	0	0	0
4–5	E	22	28	22	28	0	0	0
1–5	F	0	12	16	28	16	16	16
5–6	G	28	32	28	32	0	0	0

Here, $T_S = T_E$

$Z = \frac{T_S - T_E}{\sigma} = 0$ $P(0) = 0.5 = 50\%$



1 week = 7 $\quad \sigma = 21$ days

19 w = 7×17 days

Page No.

Date :

Q. for a project expected time is 17 weeks and σ is 3 weeks. Find :-

1. Probability of completing project Two weeks earlier and more than 2 weeks later the expected time.
2. If the project manager wants to be 99% sure that the project is completed on the scheduled date. Then How many weeks before the scheduled date, he should start the project work

$T_E = 17, \quad \sigma = 3.$

1- $Z_1 = \frac{T_S - T_E}{\sigma} = \frac{15-17}{3} = \frac{-2}{3} = 0.667$, $P = 0.25239$
$= 25.239\%$ -

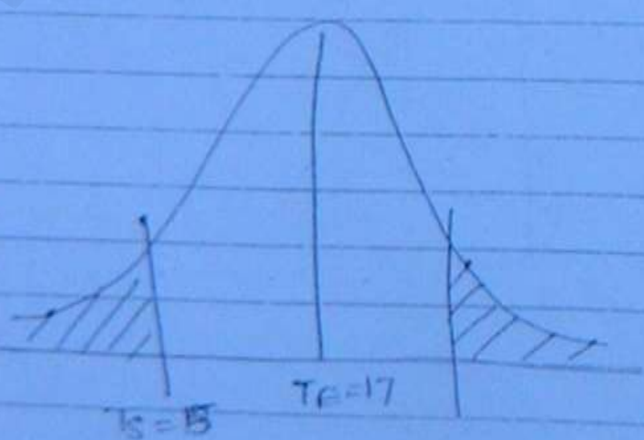
$Z_2 = \frac{19-17}{3} = \frac{2}{3} = 0.667$, $P = 0.7476$
$= 74.76\%$

Probability of more than 19 week $= 1 - 0.7474$
$= 0.2524$

2. $P(0.99) = \frac{T_S - 17}{3} = 2.33$

$T_S - 17 = 6.99$, $\quad T_S =$ ~~19.97~~ $23.99 \simeq 24$ week

How many weeks before the T_E = ~~20.97~~ $6.99 + 17$



Q. Construct the project Network. Find critical Path & Expected project completion time and also find the probability of completing project in 25 weeks.

Activity	Predecessor	a	m	b	T_E	σ
A	–	3	5	8	5.167	5/6
B	–	6	7	9	7.166	* 3/6
C	A	4	5	9	5.5	5/6
D	B	3	5	8	5.167	* 5/6
E	A	4	6	9	6.167	5/6
F	CD	5	8	11	8	* 6/6 = 1
G	CDE	3	6	9	6	6/6 = 1
H	F	1	2	9	3	* 8/6

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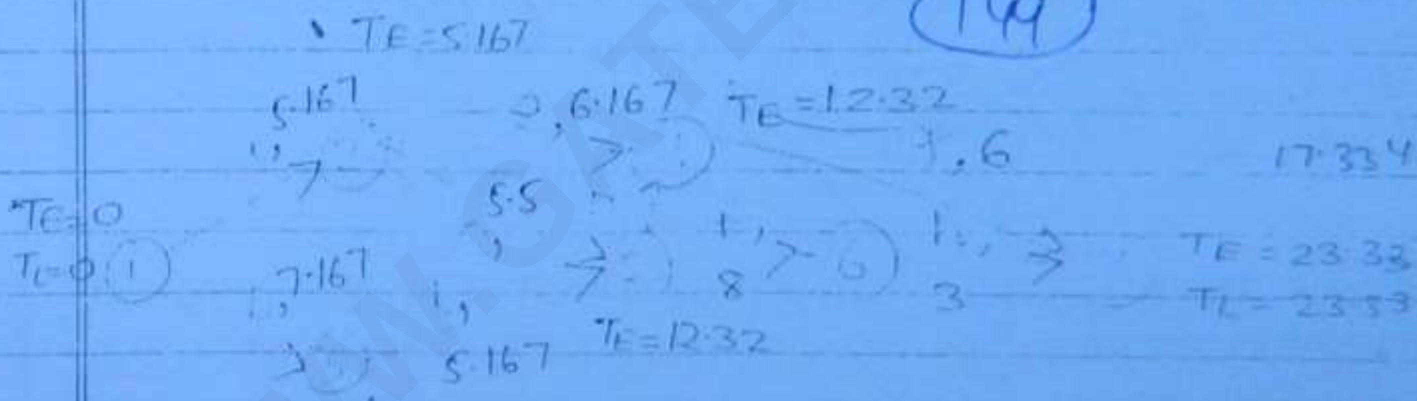


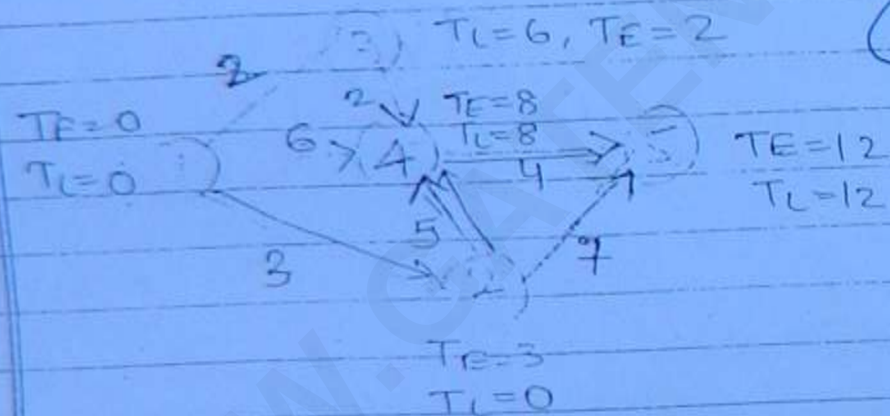
$b-d-f-h = 23.334, \quad \sigma = \cancel{2.61}\ 1.929$

$$Z = \frac{T_S - T_E}{\sigma} = \frac{25 - 23.33}{\cancel{2.61}\ 1.929} = \cancel{0.6398} = 0.8657$$

$P = .80667 = 80.67\ \%$

Q. The analysis of a network in relation to time & cost are as given below: Crash the Network to the optimum point when the indirect cost is Rs 900/day

Jobs	Normal Time (Day)	Normal Cost (Rs)	Crash Time (Day)	Crash Cost (Rs)	ΔC/ΔT
1-2	3	500	2	1000	500/1 *
1-3	2	750	1	1500 ~~2600~~	750
1-4	6	1400	4	2600 ~~1800~~	600
2-4	5	1000	3	1800 ~~1450~~	400 *
2-5	7	1150	6	1450	300
3-4	2	800	2	800	0/–
4-5	4	1000	2	2400	700 *
		6600			



Under normal working condition = 12 days
& critical path 1-2-4-5

So, Total cost of Project = D.C + I.D.C

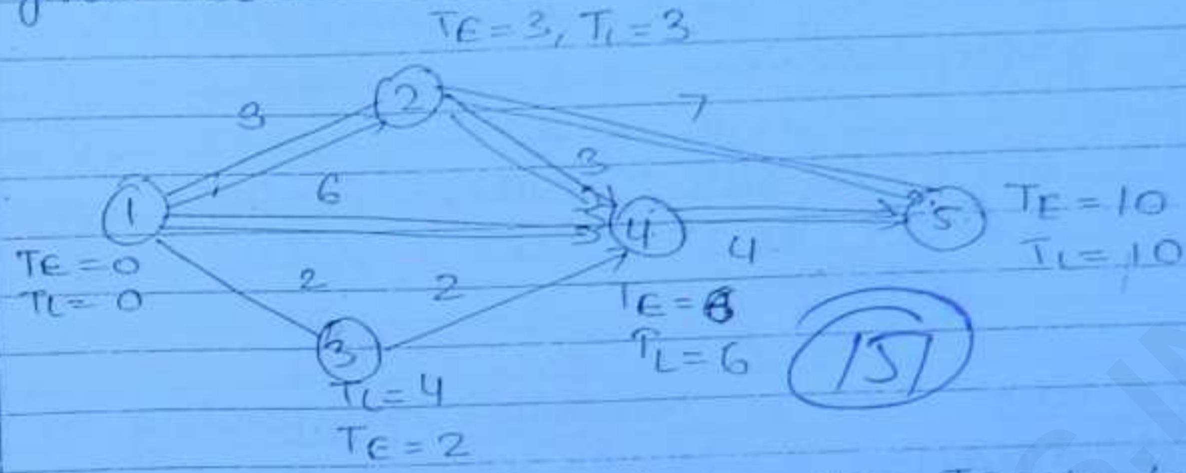
⇒ 6600 = D.C

⇒ I.D.C = 12×900 = 10800 Rs

Total cost = Rs 17400

Now crashing the minimum cost slope activity on the critical path. Here it is activity 2-4.
(Minimum among 3)

crashing it by 2 days. the revised network diagram is given below:



we got 3 critical path., Now, TE = 10 days

Total Cost = D.C + I.D.C

D.C = 6600 + 400 × 2 = 7400

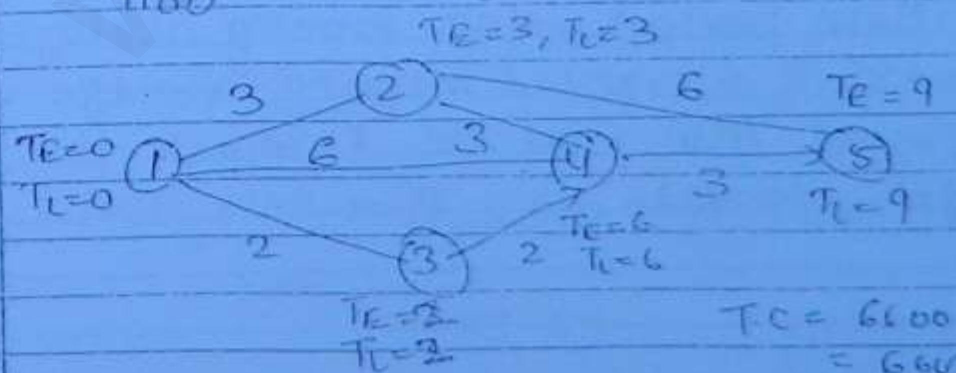
I.D.C = 10 × 900 = 9000

Total cost = Rs 16400

1-2 & 4-5, 1-2 & 1-4, 2-5 & 4-5

The revised network has Three critical path and by crashing any one activity the project duration remains the same. so we need to crash atleast 2 activities simultaneously. This gives 3 options:—

1. 1-2 & 1-4 = 500 + 600 = 1100
2. 2-5 & 4-5 = 300 + 700 = *1000
3. 1-2 & 4-5 = 500 + 700 = 1200



1-4-5 = 9

1-2-4-5 = 9

1-2-5 = 9

T.C = 6600 + 900 × 9

= 6600 + 8100

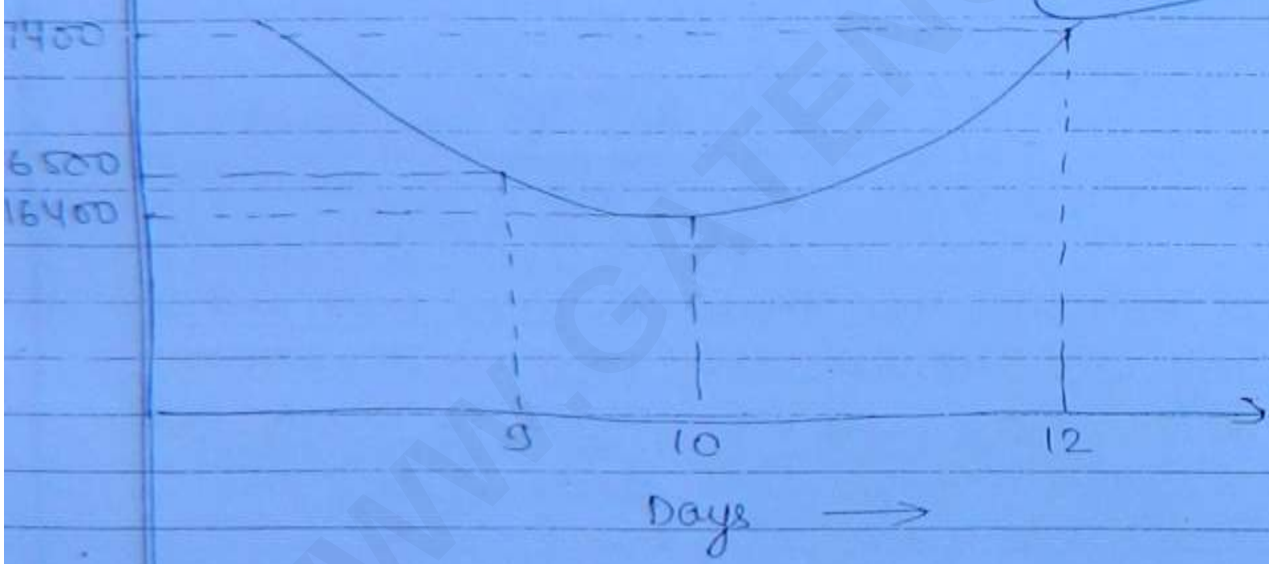
= 14700

Date :

Selecting the II option as the summation of cost slope is minimum, thus crashing activity 2-5 & 4-5 by one day each, the revised network is a given below

$$T.C = 6600 + (800 + 900 \times 9) + 1000$$
$$= 6600 + 800 + 8100 + 1000$$
$$= 16500.$$

As the total cost is minimum corresponding to project duration of 10 days. 10 Days is optimum project duration & Rs 16,400 optimum project cost.



Q For the given data, Generate the forecast for each of the time period using moving average for 'n' = 3 period & weighted moving average for 'n' = 4 period. Also, find the forecast for the 9, 10 & 11th period.

Date :

Days	Demand
1	150
2	120
3	110
4	130
5	140
6	160
7	150
8	140

5 ⇒ 119
6 ⇒ 129
7 ⇒ 143
8 ⇒ 149
9 ⇒ 147

153

1) Moving average, 4 ⇒ $\frac{110+120+150}{3} = 110$

5 ⇒ $\frac{130+110+120}{3} = 120$

6 ⇒ $\frac{140+130+110}{3} = 126.67$

7 ⇒ $\frac{160+140+130}{3} = 143.33$

8 ⇒ $\frac{150+160+140}{3} = 150$

9 ⇒ $\frac{140+150+160}{3} = 150$

10 ⇒ 150

11 ⇒ 150

2) Weighted Average, 5, ⇒ $0.4\times130+0.3\times110+0.2\times120+0.1\times150$
$= 119$

6 ⇒ $n=6$ ⇒ $\frac{6(7)}{2} = 21$

5 ⇒ $(0.4)\frac{5}{21}\times140+(0.3)\frac{4}{21}\times130+(0.2)\frac{3}{21}\times110+(0.1)\frac{2}{21}\times120$
$= 85.24$

7 ⇒ $\frac{7(8)}{2} = 28$

$= (0.4)\frac{6}{28}\times160+(0.3)\frac{5}{28}\times140+(0.2)\frac{4}{28}\times130+(0.1)\frac{3}{28}\times110$

$= 89.64$

Q. The sale of car in a showroom in 4 consecutive month is 70, 68, 82 & 95 resp. with a smoothing const = 0.4. Find the next month of forecast of car sales.

D	F
70	70
68	
82	
95	

$\alpha = 0.4$

$F_2 = F_1 + \alpha [D_1 - F_1]$

$= 70 + 0.4 [70 - 70] = 70$

$F_3 = 70 + 0.4 [68 - 70] =$

$= 70 - 0.8 = 69.2$

$F_4 = 69.2 + 0.4 [\cancel{69.2}\ 82 - \cancel{70}\ 69.2]$

$= \cancel{69.2} \quad\quad = 68.88$

$= \cancel{69.08}\ 74.32$

$F_5 = F_4 + \alpha [D_4 - F_4]$ 154

$= \cancel{68.88 + 0.4 [}$

$= \cancel{64.08 + 0.4 [95 - 64.08]}$

$= 74.32 + 0.4 [95 - 74.32]$

29/8/11

5-'08 Q The Demand for a newly introduced luxury car has been shown below. The experts forecast the Demand for 100 cars for the month of March with the smoothing constant of 0.15. Find the forecast for the month of August. Also, calculate MAD, MSE, MAPE & bias

Month	Demand	Forecast
March	150	100
April	200	175
May	100	137.5
June	50	93.75
July	150	178.125
Aug		
	650	734.375

$150 + 0.5 [200 - 150]$

$150 + 0.5 \times 50 = 150 + 25.0$

$=$

$175 + 0.5 [100 - 175]$

$= 175 - 0.5 (75)$

$= 175 - 37.5$

$= \quad .5$

1) $MAD = \sum_{i=1}^{n} \left| \frac{e_i}{n} \right| = \sum_{i=1}^{5} \left| \frac{274.103}{5} \right| = 54.82$

Forecast	e_i	e_i^2	$\lvert e_i/O_i \rvert \times 100$
100	50	2500	33.33
107.5	92.5	8556.25	46.25
121.375	−21.375	456.89	21.375
118.168	−68.168	4646.876	136.336
107.94	42.06	1769.04	28.04
114.25	$\Sigma e_i = 95.017$	17929.056	265.38
	$\Sigma \lvert e_i \rvert = 274.103$		

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3) $MSE = \frac{|e_i^2|}{5} = \frac{17929.056}{5} = 3585.8112$

4) $MAPE = \frac{\left| \left(\frac{e_i}{O_i}\right) \times 100 \right|}{n} = \frac{265.38}{5} = 53.076$

5) $Bias = \frac{\Sigma e_i}{n} = \frac{95.017}{5} = 19$

Q. A company manufacturing a T.V set. Establish a
ation that its sales is related to population of a city. The market research follows the following ~~The~~ linear regression & estimate the demand for the city with a population of 50 million.

5
7
15

Date :

Population (Million) x	No. of TV sets('1000) y	xy	x^2
5	28 (30.66)	140	25
7	40 (36.88)	280	49
15	65 (61.76)	975	225
22	80 (83.53)	1760	484
27	96 (99.08)	2592	729
36	130 (127.07)	4680	1296
112	439	10427	2808

$\Sigma x \cdot \Sigma y = 49168$

$$b = \frac{62562 - 49168}{6 \times 2808 - 112^2} = \frac{13394}{156}$$

$= 3.1119$

$a = 15.1133$

$y = a + bx = 15.11 + 3.11x$

$x = 50$

$Y = 170.61 = 170{,}610$ Units Required.

Q The annual sales of a company are as given below by the method of Least square method. Find the Trend value of each of the 5 years & Also Estimate the annual sales for the years 2011.

Year	Sales (Rs. Crore y)	x	xy	Σx^2
2006	50	-2	-100	4
2007	65	-1	-65	1
2008	75 →	0	0	0
2009	52	1	52	1
2010	72	2	144	4
2011	314	$\Sigma x = 0$	31	10

$b = \frac{\Sigma xy}{\Sigma x^2}$ $a = \frac{\Sigma y}{n}$

$b = \frac{361}{36} = 10.023$; $\frac{161}{10} = \frac{31}{10} = 3.1$

$a = 62.8$

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$y = 62.8 + 3.1\,x$

$y = 62.8 + 3.1 \times 3 \Rightarrow y = \cancel{75.2}\ 72.1$ crore

Q The sale of a company is given below Forecast the demand for the next 2 years by using least square method.

Year	Sales (y)	x	xy	x^2
2000	30	-9	-270	81
2001	33	-7	-231	49
2002	37	-5	-185	25
2003	39	-3	-117	9
2004	42	-1	-42	1
2005	46	1	46	1
2006	48	3	144	9
2007	50	5	250	25
2008	55	7	385	49
2009	58	9	522	81
	438		502	~~8336~~ 330

$b = \frac{502}{\cancel{83036}\ 330} = \cancel{0.06}\ 1.52$, $a = \frac{438}{10} = 43.8$

~~2010 $y = 43 + 0.06 \times x$, $x = 10 \Rightarrow y = 43.6$~~

2010 $y = 43 + \cancel{0.06}\ 1.52 \times x$ $x = 11 \Rightarrow y = \cancel{43.66}$

2011 y

2010 = 43.8 + 1.52 x , x = 11 ⇒ y = 60.53

2011 ⇒ x = 13, y = 63.56.

For the given elements, calculate suitable no. of work stations from a precedence diagram, balance the line & calculate Balance Delay, Line η & Smoothness index. Take Cycle Time, T_c = 1 min.

Element	Time (T_i) (min)	Preceded by
1	0.2	–
2	0.4	–
3	0.7	1
4	0.1	1, 2
5	0.3	2
6	0.11	3
7	0.32	3
8	0.6	3, 4
9	0.27	6, 7, 8
10	0.38	5, 8
11	0.5	9, 10
12	0.12	11

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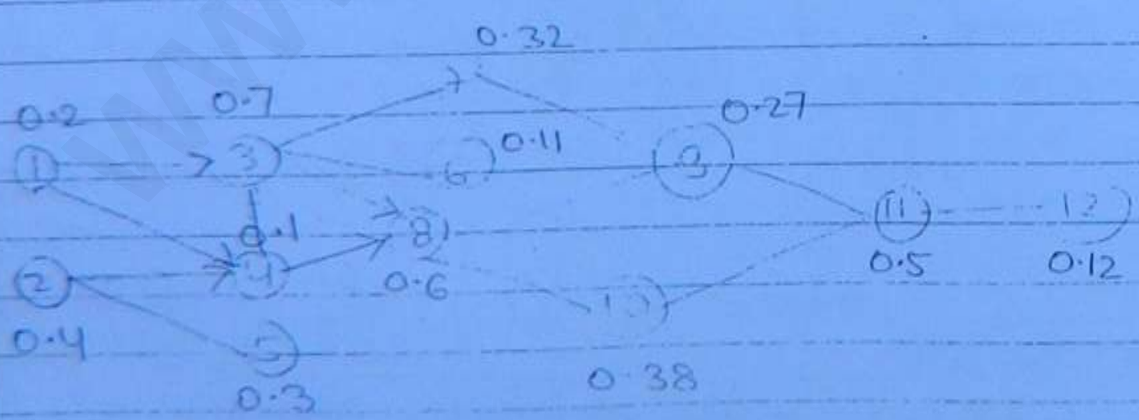

i) Min no of W.S required, $n_{min} = \frac{T_{wc}}{T_c}$

$= 5.557$

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Element	Ti(min)	Precod.
3	0.7	1
8	0.6	3,4
11	0.5	9,10
2	0.4	—
10	0.38	5,8
7	0.32	3
5	0.3	2
9	0.27	6,7,8
1	0.2	—
12	0.12	11
6	0.11	3
	3.89	
4	0.1	1,2
	3.99	

WS	Element	Ti	Tsi	Idle time $T_c - T_{si}$
I	2	0.4		
	5	0.3	1	1−1 = 0
	1	0.2		
	4	0.1		
II	3	0.7	0.81	1−0.81 = 0.19
	6	0.11		
III	8	0.6	0.98	0.02
	10	0.38		
IV	7	0.32	0.59	0.41
	9	0.27		
V	11	0.5	0.62	0.38
	12	0.12		

iii) Balance Delay, $\frac{nT_c - T_{WC}}{nT_c} = \frac{5\times1 - 3.99}{5\times1}$

$= \frac{5-4}{5} = 0.2 = 20\%$

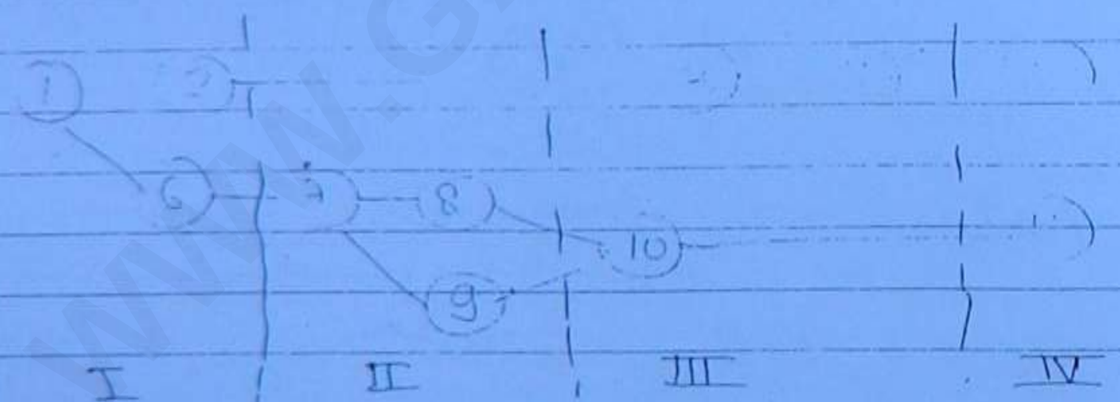
iv) Line $\eta = 100 - 20\% = 80\%$

v) Smoothness index $= \sqrt{((T_{si})_{max} - T_{si})^2} = \sqrt{0.349}$

$= 0.59$

Q. For the given elements make the precedence dia., Balance the line, & show it on Network dia. Also find balance delay, line η & smoothness index. Take cycle time, $T_c = 12$ min.

Element	T_i (min)	Preceded
1	4	–
2	3	1
3	6	2
4	3	3
5	5	4
6	4	1
7	7	6
8	2	7
9	3	7
10	3	8, 9
11	6	10
	46	



8	2	7	7	7	6		1	4		7
2	3	1	3	6	2	I	6	4	11	12-11 = 1
4	3	3	11	6	10		2	3		
9	3	7	5	5	4		7	7		
10	3	8,9	1	4	–	II	9	3	12	12-12 = 0
1	4	–	6	4	1		8	2		
6	4	1	2	3	1		3	6		
5	5	4	4	3	3	III	4	3	12	12-12 = 0
3	6	2	9	3	7		10	3		
11	6	10	10	3	8,9	IV	11	6	11	12-11 = 1
7	7	6	8	2	7		5	5	16	

7	7	6
11	6	10
13	6	2
8	5	4
6	4	1
1	4	–
10	3	8,9
9	3	7
4	3	3
2	3	

$B.D = \frac{4\times12-46}{4\times12}\times100 = 0.042\times100 = 4.16\%$

Line η = $100 - 4.16 = 95.84\%$

$SI = \sqrt{1+1} = \sqrt{2} = 1.414$

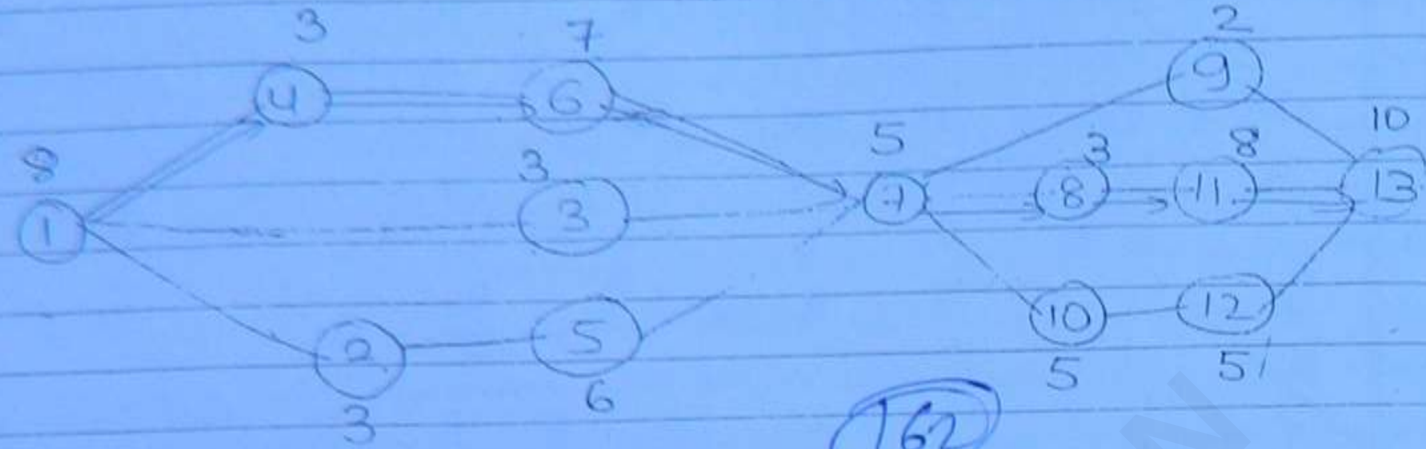
Q Design an assembly line for the following element by the Rank Positione weighted method Also find the balance delay, line η & SI

$T_c = 18$ min

Element

1	8	–	10	5	7
2	3	1	11	8	8
3	3	1	12	5	10
4	3	1	13	10	9, 11, 12
5	6	2			
6	7	4			
7	5	3,5,6			
8	3	7			
9	2	7			

$T_c = 18$ min



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1	8 1–4–6–7–8–11–13 = 44			I	8
~~2~~	~~35~~	4	36		3 18−14 = 4
~~3~~	~~29~~	2	35		3
~~4~~	~~36~~	6	33	II	7
~~5~~	~~32~~	5	32		6 18−16 = 2
~~6~~	~~33~~	3	29		3
~~7~~	~~26~~	7	26	III	5
~~8~~	~~21~~	8	21		3 18−13 = 5
9	12	10	20		5
~~10~~	~~20~~	11	18		8
~~11~~	~~18~~	12	15	IV	5 18−15 = 3
~~12~~	~~15~~	9	12		2
13	10	13	10	V	10 18−10 = 8

$BD = \frac{5\times18-48}{5\times18} = 0.4667 \quad = 0.244$

$= 46.67\% \quad = 24.44\%$

Line $\eta = 53.33\% \quad = 75.56$

$SI = \sqrt{54} = \cancel{7.348}\ 10.86$

$SI = \sqrt{(16-14)^2 + (16-13)^2 + (16-15)^2 + (16-10)^2}$

$= \cancel{5.477}\ \sqrt{50} = 7.07$

Q The no. of persons arriving at a service centre is 8 customers/hour & service provider takes 5 mins/customer on an average. Find :-

1) Average no. of customers in the Queue & system

ii) Waiting time of the customer in the Queue & system.

iii) Average length of Non-empty queue.

$$\lambda = 8 \frac{\text{cust}}{\text{hr}} \quad ; \quad \mu = \frac{1 \text{ custome}}{5 \text{ min}} = \frac{\overset{12}{\cancel{60}} \text{ custom}}{\cancel{5} \text{ hr}}$$

i) $$L_s = \frac{\rho}{1-\rho} = \frac{8/5}{1-8/5} = \frac{8/5}{\frac{5-8}{5}} = \frac{8}{5} \times \frac{5}{-3} = \frac{40}{1-40} = \frac{40}{39}$$

$$\rho = \frac{\lambda}{\mu} = \frac{8 \times \cancel{5}}{\cancel{5}\ 12} = \cancel{.40} = 0.66 \qquad = 1.98$$

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ii) $$L_q = \frac{\rho^2}{1-\rho} = \frac{(\cancel{40})^2}{-\cancel{39}} = 1.28 = \frac{(0.66)^2}{1-0.66}$$

iii) $$W_s = \frac{L_s}{\lambda} = \frac{1.98}{8} = 0.2475 \text{ hr} = 14.85 \text{ min}$$

iv) $$W_q = \frac{L_q}{\lambda} = \frac{1.28}{8} = 0.16 \text{ hr} = 9.6 \text{ min}$$

v) $$L_q' = \frac{1}{1-\rho} = 2.94 \simeq 3 \text{ customers}$$

Q. A. train reservation facility has 5 counters each capable of handling 20 request per hour. The persons arrive at a mean rate of 90/hr. Calculate :-

i) Average no of persons at any time at this facility 45

ii) The mean time a person spends at the facility

iii) Average length of Queue at each counter

iv) What would happen to Queue if the arrival rate reaches 105/hr

Date :

'04

8) In a m/c shop certain types of m/c break down at an average rate of 6/hr. The estimated cost of Ideal m/c is Rs 15/hr, two repairman A & B with different skills are considered to be hired as repairment. Repairmen of A takes 6 mins on an average to repair a m/c & his wages are Rs 8/hr whereas B takes 5 mins to repair and the wages are Rs 10/hr.
Which repairmen service should be used & why? Assume the work shift of 8 hours?

Soln. For A → , $\lambda = 6$ m/c/hr

$\frac{1}{\mu_A} = 6$ min/m/c

$\mu_A = \frac{1}{6} \times 60$ m/c/hr

$= 10$ m/c/hr

Rs 8/hr

For B → $\frac{1}{\mu_B} = 5$ min/m/c

$\mu_B = 12$ m/c/hr

Rs 10/hr

$\rho_B = \frac{6}{12} = \frac{1}{2}$

$L_B = \frac{0.5}{1-0.5} = 1$

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Total cost during A = wage of A + cost of Idle m/c during A $= 8 \times 8 + (L_s)_A \times$ Rs 15/hr $\times 8$

$= 64 + (L_s)_A \times 15 \times 8$

$(L_s)_A = \frac{\rho_A}{1-\rho_A} = \frac{0.6}{1-0.6} = 1.5$ m/c

A = $(T.C)_A = 64 + 120 \times 1.5 = 244$ Rs

$(L_s)_B =$

$(TC)_B$ = wage of B + cost of Idle m/c during B

$= 10 \times 8 + (L_s)_B \times 15 \times 8$

$= 80 + 120 =$ Rs 200

Preferred

~~Q Probability of N arrival in a system during T~~

Q An operator rated at 125% took 10 mins to complete & an observed job if a total 10% allowances are required for the job. Then, what is the expected no. of job produced in a day of 8 hr duration.

RF = 0.125 × ~~NT~~ O.T = 10 min =

ST = NT + 0.10 NT = 1.1 NT

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NT = 10 × 0.125 = 12.5 min

ST = 1.1 × 12.5 = 13.75 min = $\frac{13.75}{60}$ = 0.229/hr

= 13.75/60

In 8 hr = $\frac{60 \times 8}{13.75}$ = 34.9 ≃ 34

Q The observed time in min for 4 cycles of an operation using stopwatch are as given below. Calculate ST for the operation if element 2 & 4 are machine element for other elements operator is rated at 110% & the total allowances are 15% of ST

Element	Cycle time in mins 1	2	3	4	Average O.T	
1	1.5	1.5	1.3	1.4	1.425	0.15675
* 2	2.6	2.7	2.4	2.6	2.575	2.575
3	3.3	3.2	3.4	3.4	3.325	0.36575
* 4	1.2	1.2	1.1	1.2	1.175	1.175
5	0.51	0.51	0.52	0.49	0.5075	0.5582
						9.5322

1) NT = ~~110 × 1.5 + 2.6 + 0.110 × 3.3 + 1.2 + .110 × .57~~

= 9.5322

$ST = NT + \text{Allowances}$

$ST = NT + 0.15\,ST \Rightarrow 0.85\,ST = NT$

$$ST = \frac{9.5322}{0.85} = 11.21 \text{ min}$$

Q Find No of observations for 14 activities out of 20 samples for 95% confidence level & 5% accuracy

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$Z = 1.96, \quad P = \frac{14}{20} \quad N = 14, \ L = 5\%$

$$14 = \frac{(1.96)^2 \times 0.7\,(1-0.7)}{(0.05)^2} = 322.69 \simeq 323$$

Q A work study was conducted in a M/c shop. The data recorded are total no of observations 2000. No activity observation 500. The ratio b/w manual to machine is 3:1. Portion of activities RF = 85%. Total no of pieces produced during the study is 120 & duration of study is 60 hrs. Calculate the ST/piece assuming 15% allowance.

Sol^n. $N = 2000, \ RF = 0.85, \ N' = 500$

$N_{eff} = 1500$

$\% \text{ time} = \frac{1500}{2000} = \frac{3}{4} \times 100 = 75\%$

Total study duration = 60 hr

Thus, production time $= 60 \times 0.75 = 45$ hr

$$\frac{O.T}{P_c} = \frac{45 \times 60}{120} = 22.5$$

Manual $O.T = \frac{3}{4} \times 22.5 = 16.875$

M/c O.T = $\frac{1}{4} \times 22.5 = 5.625$ min.

NT = (Manual OT) × R.F + (M/c . O.T)
= $16.875 \times 0.85 + 5.625 = 19.968$ min.

ST = NT + allowances = NT + 0.15 NT
= 1.15 NT = 22.96 min.

Q The following observations are made during work sampling study conducted during 2 weeks.

M/c controlled ob. 448, Manual - 228, Personal need = 42 Contingency = 75, Fatigue = 52 & Total observations made are 845. No. of pieces produced is 768. Rating index = 110%. The firm is company 8 hrs a day & 5 days a week. Determine the ST per piece.

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Sol^n. - Rf = 1.10, N = 845
Allowances = 169
Manual OT 228 × ~~R.F~~ = 228 × ~~1.1~~ = 250.8
M/c × O.T = 448 = 448
Total O.T = 676, Total observation = 845
~~N.O.T = 698.8, = 0.909~~
~~ST = 867.8~~
% time production = $\frac{676}{845} \times 100 = 80\%$.

Duration of study = 2 × 5 × 8 = 80 hr
Production time = 80 × 0.8 = 64 hours
$\frac{O.T}{P_c} = \frac{64 \times 60}{768} = 5$ min

M/c O.T = $\frac{448}{676} \times 5 = 3.313$

Date :

7 Manual O.T = $\frac{228}{676} \times 5 = 1.686$.

$NT = (O.T)_{manual} \times RF + (O.T)_{M/c}$

$= 3.313 \times 1.1 + 1.686$

$= 5.17$ min.

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Total allowance = 169

Allowance % = $\frac{\text{total allowance}}{\text{Production activity}} = \frac{169}{676} \times 100 = 25\%$.

ST = NT + Allowances = $1.25 \times NT$

= 6.46 min

31/8/11

9. The following data give readings for 10 samples of size 8 each in the production of a certain component. Draw the control chart for mean & Range & find out which sample are out of limit & comments on the result

Sample	1	2	3	4	5	6	7	8	9	10	
mean	5.4	5.1	5.4	4.9	5.2	4.7	5.1	5.0	5	5.2	51
Range	0.4	0.7	0.7	0.8	0.9	0.6	0.5	0.6	0.7	0.6	6.5

$\bar{\bar{X}} = \frac{51}{10} = 5.1$, $\bar{R} = \frac{6.5}{10} = 0.65$

For $n = 8$, $d_2 = 2.847$, $D_3 = 0.136$, $D_4 = 1.864$

for $n = 10$, $d_2 = 2.916$, $D_3 = 0.227$, $D_4 = 2.013$

$U.C.L = D_4 \bar{R} = 1.864 \times 0.65 = 1.2116$

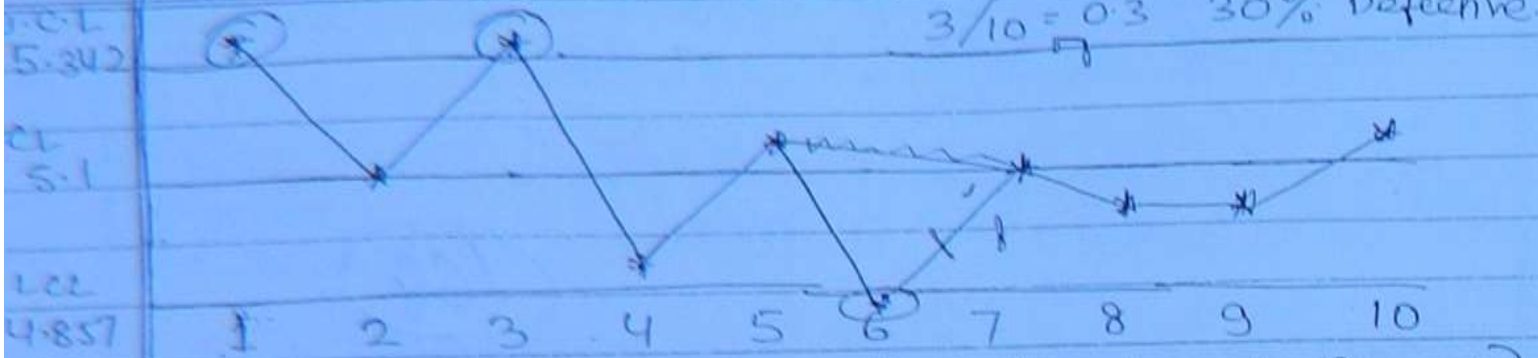
$L.C.L = D_3 \bar{R} = 0.136 \times 0.65 = 0.0884$

$\bar{R} = \sigma d_2 \Rightarrow \sigma = \bar{R}/d_2 = 0.65/2.847 = 0.2283$

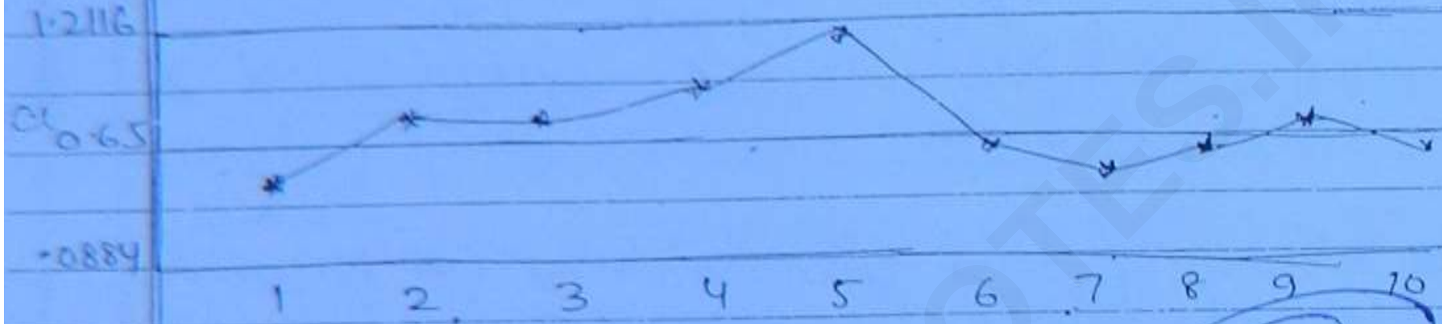
$UCL = \bar{\bar{X}} + \frac{3\sigma}{\sqrt{n}} = 5.342$, $LCL = \bar{\bar{X}} - \frac{3\sigma}{\sqrt{n}} = 4.857$

$\bar{X}$ - chart

3/10 = 0.3 30% Defective



30% due to Machine setting (Mention all Reasons)



R - chart

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comment :-

The process is out of Control & the reason for process being out of control may be due to Machine setting change, improper training of worker, tool wear, machine play or any other reasons. The process may be stopped & the assignable ~~cost~~ cause may be rectify.

Q. A manufacturer finds from his experience that on an average one out of 10 items produced by a machine is defective on a particular day, he selects a lot of 100 items randomly & finds that 18 of them are defective. Is the process under control.

$L.C.L = 0.1 - 3 \times 0.1 = 0.1 - 0.3 = -0.2$

$\bar{P} = \frac{18}{100} = 0.18$ $\bar{P} = \frac{1}{10} = 0.1$

$UCL = \bar{P} + 3\bar{P} = 0.18 + 3 \times 0.18 = 0.72$

$= 0.1 + 3 \times 0.1 = 0.1 + .3 = 0.4$

$\bar{P} = \frac{1}{10} = 0.1$, $n = 100$

$CL = n\bar{P} = 100 \times 0.1 = 10$

$UCL = n\bar{P} + 3\sqrt{n\bar{P}(1-\bar{P})} = 10 + 3\sqrt{100 \times 0.1 \times 0.9}$

$= 19$

$LCL = 1$ unit.

Q. In the following table,
the no of defective found on 24 consecutive production days in daily samples of 200 items draw np chart and find out the points which fall outside the control limits.

Day	1	2	3	4	5	6	7	8	9*	10*	11*	12
No. of Defective	10	5	10	12	11	9	19	4	12	27	25	9
Day	13	14	15	16	17	18	19	20	21*	22	23	24
No. of Defective	12	15	8	14	10	4	11	11	26	3	10	11

$n = 200$, Total No. of Defects $= 288$

$P_N = \frac{10}{200} + \frac{5}{200} + \ldots = \frac{288}{200} = 1.44$

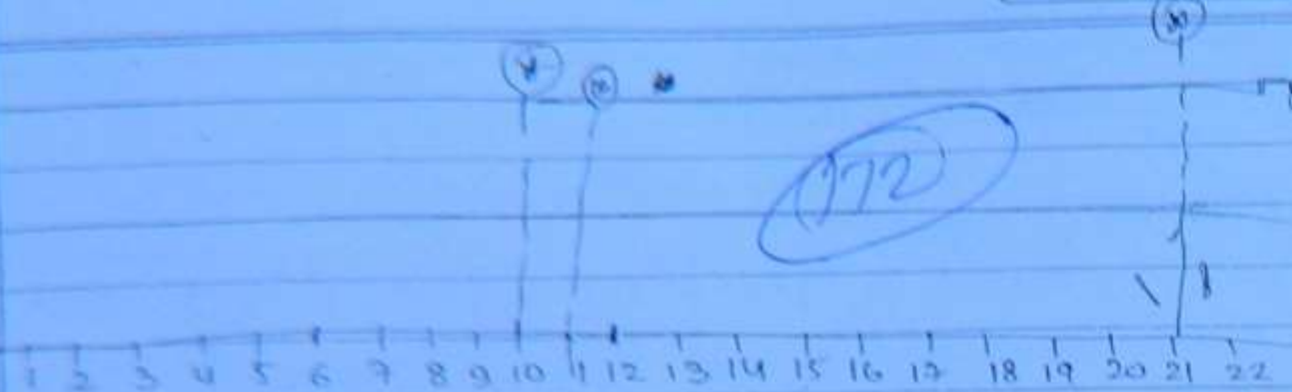
$\bar{p} = \frac{288}{200 \times 24} = 0.06$

$CL = n\bar{P} = 200 \times 0.06 = 12$ Units

$UCL = n\bar{P} + 3\sqrt{n\bar{P}(1-\bar{P})} = 22.07$

$LCL = n\bar{P} - 3\sqrt{n\bar{P}(1-\bar{P})} = 1.92$

10, 11 & 21 are defective



1/09/11.

Q. If the probability of acceptance of 1% defective lot is 0.95 Then find the AOQ

Soln:- $AOQ = 95 \times 0.01 = 95 \times 10^{-4} = 0.95\%$

Q. Formulate a L.P problem when a person want to maximize his daily profit by manufacturing 4 types of product whose manufacturing time in minute & profit per unit is as given below:

Product	Machining time in mins M_1	M_2	M_3	Profit/unit (Rs.)
A	4	3	2	45
B	3	5	3	60
C	2	3	4	48
D	2	2	3	59
maximum mins/day	560	640	620	

~~Max z~~ STEPS:-

1. Key decision is to determine no of units produced per day of product A, B, C & D. Let these are x_1, x_2, x_3 & x_4 respectively.
2. Feasible alternatives are all the values of x_1, x_2, x_3 & $x_4 \geq 0$
3. Objective is to Optimize daily profit when profit/unit is

given. so, the objective function :-

$$\text{Max. } Z = 45x_1 + 60x_2 + 48x_3 + 59x_4$$

4. The constraints are on the machining time available for the 3 m/cs per day :-

M1 $4x_1 + 3x_2 + 2x_3 + 2x_4 \leq 560$

M2 $3x_1 + 5x_2 + 3x_3 + 2x_4 \leq 640$

M3 $2x_1 + 3x_2 + 4x_3 + 3x_4 \leq 620$

Q. $\text{Max } Z = 30x_1 + 20x_2$

$2x_1 + x_2 \leq 90$ $\quad x_1/45 + x_2/90 = 1 \rightarrow (1)$

$x_1 + 2x_2 \leq 80$ $\quad x_1/80 + x_2/40 = 1 \rightarrow (2)$

$x_1 + x_2 \leq 50$ $\quad x_1/50 + x_2/50 = 1 \rightarrow (3)$

$x_1, x_2 \geq 0$

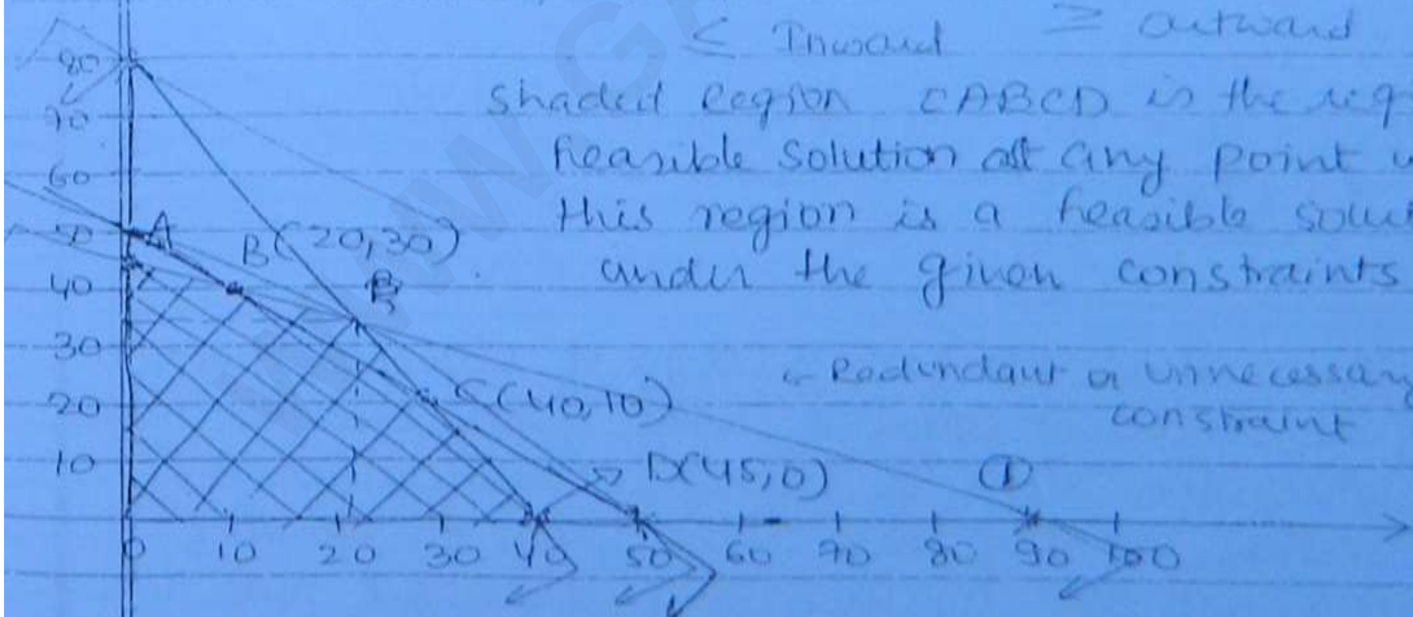
$x_1 = 0,\ x_2 = 90,\ x_1 = 1 \Rightarrow x_2 = 88$

$x_1 = 0,\ x_2 = 40,\ x_2 = 0,\ x_1 = 80$

$x_1 = 0,\ x_2 = 50,\ x_2 = 0,\ x_1 = 50$

$\leq$ Inward $\quad \geq$ outward

Shaded region OABCD is the region of feasible solution at any point within this region is a feasible solution under the given constraints.



1. We

Date :

Optimality :-

1. we check the vertex or corner point of feasible region by putting its value in the objective function. The point which optimises the objective function gives the optimal solution.

$z(A) = 30x_1 + 20x_1 = 0 \times 30 + 20 \times 40 = 800$
$(0, 40)$

$z(B)(20, 30) = 20 \times 30 + 30 \times 20 = 1200$

* $z(C)[40, 10] = 40 \times 30 + 10 \times 200 = 1400$

$z(D)[45, 0] = 30 \times 45 + 0 \times 20 = 1350$

$z(C)$ gives max optimum solution i.e C.

$x_1 = 40, \quad x_2 = 10.$

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II- method:- SLOPE METHOD :-

$M_z = -30/20 = -1.5$

$M_1 = -2$ (−1.5 lies between −1 & −2)

$M_2 = -0.5$ i.e 1 & 3 line

$M_3 = -1$

$2x_1 + \cancel{x_2} = 90$ $\quad x_2 = 50 - 40 = 10$

$x_1 + \cancel{x_2} = 50$

$x_1 = 40$

In this method, We take the slope of objective function and the constraint equation. ~~With~~ we them find the lines b/w which the slope of objective function lies. The Intersection point of these lines gives the optimal solution But this method has a drawback that it can give wrong solution if there are Redundant constraints in the problem.

Date :

Non-Binding

Q. Min. $Z = 6x_1 + 4x_2$ $= -6/4 = -3/2 = -1.5$

48 > 40 (12,4) $3x_1 + 3x_2 \geq 40$ $= -1$ $x_1 = 0,\ x_2 = 40/3$

40 = 40 $3x_1 + x_2 \geq 40$ $= -3/1 = -3$

44 = 44 $2x_1 + 5x_2 \geq 44$ $= -2/5 = -0.4$

Binding soln $x_1, x_2 \geq 0$

$$3x_1 + 3x_2 = 40$$
$$3x_1 + x_2 = 40$$
$$2x_2 = 0$$
$$x_1 = 40/3 = 13.33$$

$$\frac{x_1}{(40/3)} + \frac{x_2}{(40/3)} = 1 \quad (40/3) = 13.33$$

$$\frac{x_1}{(40/3)} + \frac{x_2}{40} = 1$$

$$\frac{x_1}{22} + \frac{x_2}{(44/5)} = 1 \quad (44/5) = 8.8$$

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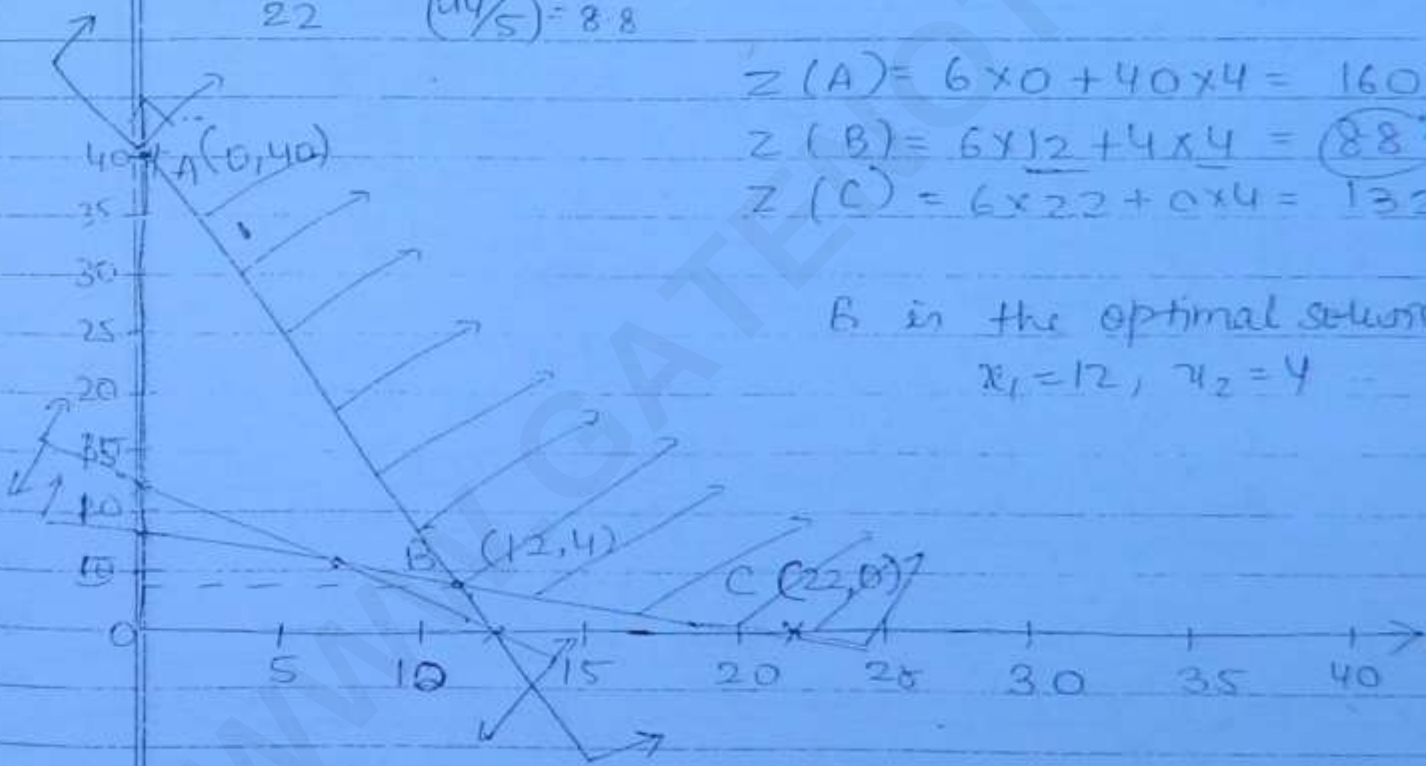

$Z(A) = 6 \times 0 + 40 \times 4 = 160$

$Z(B) = 6 \times 12 + 4 \times 4 = 88$

$Z(C) = 6 \times 22 + 0 \times 4 = 132$

B is the optimal solution $x_1 = 12,\ x_2 = 4$

Redundant Constraint, when a constraint doesnot become part of the boundary making the feasible region of the problem. It is called Redundant or Unnecessary constraint. The incluser or excluser of redundant constraint doesnot effect the optimal solution of the problem.

Q. 2 products A & B are machined on 3 machines as given below. Find the optimum product mix that maximises the profit

Product	P	Q	R	Profit/unit (Rs)
A	10	6	5	60
B	7.5	9	13	70
	75	54	65	Hrs/week

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Max $Z = 60x_1 + 70x_2$ = 60

$10x_1 + 7.5x_2 \le 75$ $\qquad \frac{x_1}{(75/10)} + \frac{x_2}{\frac{75}{7.5}} = 1$

$6x_1 + 9x_2 \le 54$

$5x_1 + 13x_2 \le 65$ $\qquad \frac{x_1}{7.5} + \frac{x_2}{10} = 1$

$x_1, x_2 \ge 0$

$\frac{x_1}{65/5 = 13} + \frac{x_2}{5} = 1$ $\qquad \frac{x_1}{54/6 = 9} + \frac{x_2}{54/9 = 6} = 1$

$Z(A) = 420$

$Z(B) = 455$

$Z(C) = 500$

$Z(D) = 350$

$-60/70 = -0.857$

$-10/7.5 = -1.33$

$-6/9 = -0.66$

$-5/13 = -0.385$

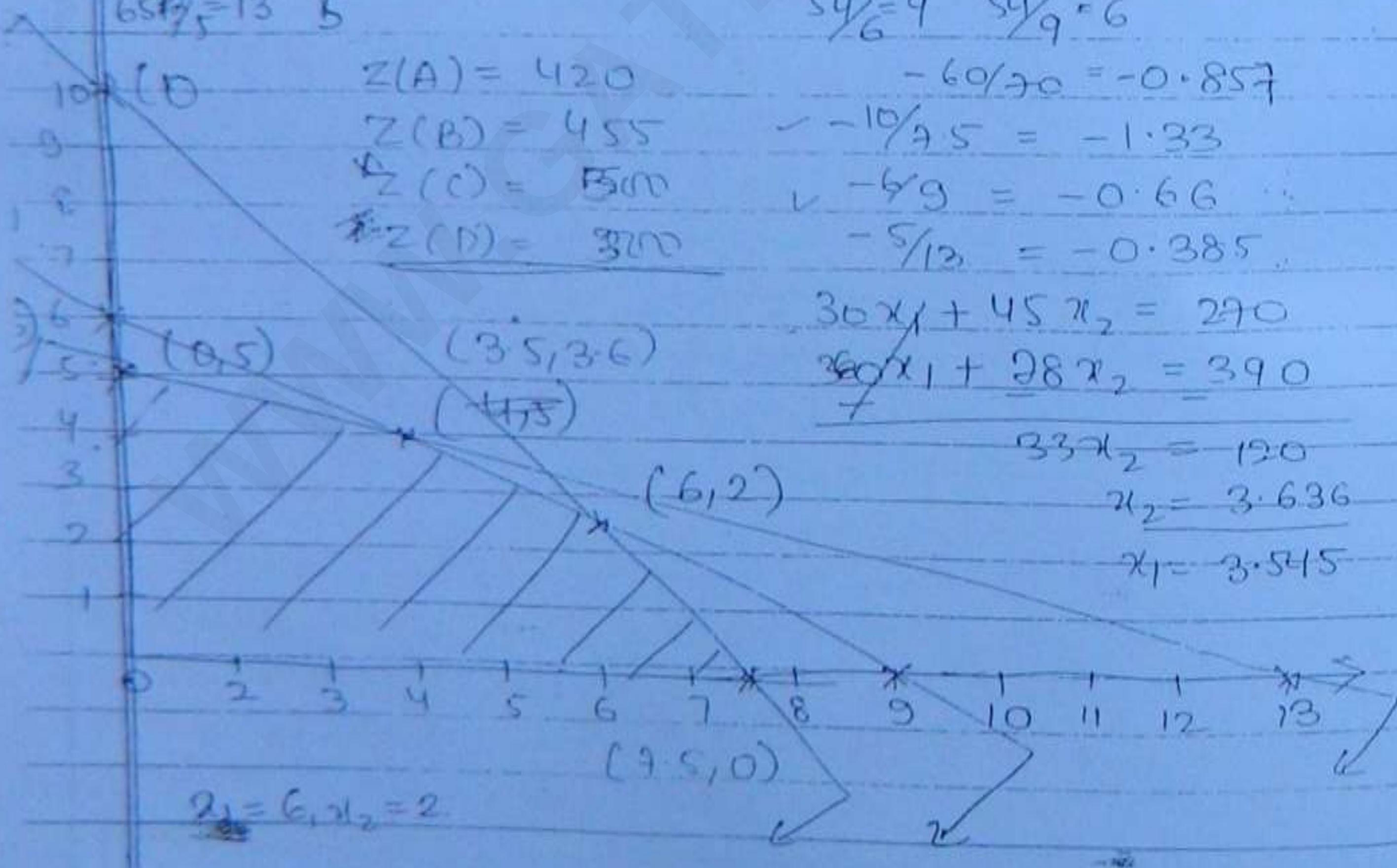
$30x_1 + 45x_2 = 270$

$30x_1 + 78x_2 = 390$

$33x_2 = 120$

$x_2 = 3.636$

$x_1 = 3.545$



$x_1 = 6, x_2 = 2$

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'1990

Q A plant makes 2 types of parts it purchases casting that are machined, ~~bold~~ bored & polished casting for part A & part B are purch-ased at Rs 2 & Rs 3 per unit respectively & they are sold for Rs 5 & Rs Rs 6 per unit resp. The 3 m/cs have running cost of Rs 20, Rs 14 and Rs 17.5 per hour & their capacities are as given below:- assuming that any combination of parts 'A' & 'B' can be sold find the product mixed that maximises the profit?

	Part A	Part B	
Machining	25/hr	40/hr	20
Capacity	0.8	0.5	
Boring	28/hr	35/hr	14
capacity	0.5	0.4	
Polishing	35/hr	25/hr	17.5
capacity	0.5	0.7	
	2	3	
Total Cost	3.8	4.6	
Selling cost	5	6.	
Profit/unit	1.2	1.4	

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$x_1 = 16.43$

$x_2 = 12.9$

Max $Z = 1.2x_1 + 1.4x_2$

constraints $\frac{x_1}{25} + \frac{x_2}{40} \leq 1$, $\frac{x_1}{28} + \frac{x_2}{35} \leq 1$

2/9/11.

Q. Find how many units of product A & B should be made per week by simplex method. Product 'A' requires 2 kg of raw material and 4 hrs of labour & B requires 3 kg of raw material & 3 hrs of labour every week availability of raw material is 60 kg & 96 hrs of labour. profit per unit of 'A' & 'B' is Rs 40 & Rs 35 respectively.

$$\text{Max } Z = 40x_1 + 35x_2$$

Constraints = $2x_1 + 3x_2 \le 60$

$4x_1 + 3x_2 \le 96$

$x_1, x_2 \ge 0$

x_1 & x_2 : are the no. of units of A & B respectively produced per week.

resource (60, 96)

1) All values are positive

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2) Remove inequalities

Constraint : $2x_1 + 3x_2 + S_1 = 60$

$4x_1 + 3x_2 + S_2 = 96$

$\Rightarrow$ Max $Z = 40x_1 + 35x_2 + 0 \cdot S_1 + 0 \cdot S_2$

x_1 & $x_2 \Rightarrow$ Non basic Variables, | S_1 & $S_2 \Rightarrow$ Basic Variable Always gives Identity Matrix

so take $x_1 = x_2 = 0$

$S_1 = 60$, $S_2 = 96$

which gives initial solⁿ (feasible)

$Z = 0 \cancel{\times 0} + 0 \cdot 60 + 96 \times 0 = 0.$

		Incoming Variable					Replacement Ratio
c_i	Basis	x_1	x_2	S_1	S_2	b_i	$\theta_i = b_i / a_{ij}$
0	S_1	2	3	1	0	60	30
0	S_2	(4)	3	0	1	96	24 ← Key Row
	c_j	40	35	0	0		(Always take minimum Positive one, if satisfies all constraints)
	$Z_j = \Sigma c_i a_j$	0	0	0	0		
	NER → $\Delta_j = c_j - Z_j$	40	35	0	0		

↑ Key column (Maximize problem, Max key)

NER → Net Evaluation row

Date :

Calculate Δj value and it is called Net Evaluation Row or Net oppurtunity cost row. The value of Δj row give the amount of increase in the objective function that could occur if one unit of the variable represented by the column head were introduced into the current Solution

A simplex table gives optimal solution if all the values in the delta Δj - row are :-

i) 0 or -ve when L.P is of Maximization type

ii) 0 or +ve when L.P is of Minimization type.

Here the problem is of Maximisation so we select the largest Δj value and the variable x_1 is known as Incoming variable & the column selected is called - Key Column.

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Now divide the bi values from the corresponding elements of key column which provide Replacement Ratio column. In this column the minimum +ve ratio is marked as Key row & the variable in the Key Row is called Outgoing variable. The element lying at intersection of key column & Key Row is called Key Element.

Key element is made unity by multiplying or dividing the key row by a common multiplying factor. Now all the element in the key column are made zero except the key element which will be Unity. This is done by adding or subtracting the proper multiples of Key row with other rows. This new table will replace the outgoing variable by the incoming variable.

$R_2 \to R_2/4$

c_i	Basis	x_1	x_2	s_1	s_2	b_i	
0	S_1	2	3	1	0	60	
0	S_2	1	3/4	0	1/4	24	←

$R_1 \to R_1 - 2R_2$

c_i	Basis	x_1	x_2	s_1	s_2	b_i	$\theta_i = b_i/a_{ij}$
0	S_1	0	$(3/2)^*$	1	$-1/2$	12	8 ← key row
40	x_1	1	3/4	0	1/4	24	32
C_j		40	35	0	0		

II feasible solⁿ $x_1 = 24$, $x_2 = 0$
$S_1 = 12$, $S_2 = 0$
$Z = 960$ Rs.

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$Z_j =$ 40 30 0 10
$\Delta_j =$ 0 5 0 -10
↑ Key column

$R_1 \to 2/3 \times R_1$

35	x_2	0	$(1)^*$	2/3	$-1/3$	8
40	x_1	1	0	$-1/2$	1/2	18

III feasible solution
$x_1 = 18$, $x_2 = 8$, $S_1 = 8$, $S_2 = 0$
$Z = 40 \times 18 + 8 \times 35$ $720 + 280 = 1000$

C_j	40	35	0	0
Z_j	40	35	$-10/3$	25/3
Δ_j	0	0	$-10/3$	$-25/3$

All the values of Δ_j are '0' or '$-$ve' Hence, the solⁿ is optimal feasible solution

$-1/4$
$\frac{1}{4}\left(-\frac{1}{4}\right)$
$\frac{1}{4} + \frac{1}{4}$
6
$+2 \times \frac{1}{2}$
24
-6
18
$2 - 8 \times 3/4$
$\frac{-70}{3} + 20$
$-\frac{35}{3} + 20$
$\frac{-35 + 60}{3}$
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Q Constraints Dual of Minimize $z = 3x_1 - 2x_2 + 7x_3$.

are :- $(\geq)$

y_1 [$2x_1 + 5x_2 - 3x_3 \geq 11$

y_2 $4x_1 - 3x_2 + 3x_3 \geq 6$

y_3 $2x_1 + 2x_2 - 7x_3 \leq 8 \Rightarrow -2x_1 - 2x_2 + 7x_3 \geq -8$

y_4 $3x_1 + 5x_2 + 4x_3 \geq 9$ ~~$-3x_1 - 5x_2 - 4x_3 \leq -9$~~

y_5 $2x_1 + 2x_2 + x_3 \geq 14$

$x_1, x_2, x_3 \geq 0$.

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Dual: Max $H = 11y_1 + 6y_2 - 8y_3 + 9y_4 + 14y_5$

const

$2y_1 + 4y_2 - 2y_3 + 3y_4 + 2y_5 \leq 3$

$5y_1 - 3y_2 - 2y_3 + 5y_4 + 2y_5 \leq -2$

$-3y_1 + 3y_2 + 7y_3 + 4y_4 + y_5 \leq 7$

$y_1, y_2, y_3, y_4, y_5 \geq 0$

Q. Max $Z = 8x_1 + 10x_2 + 5x_3$

① Primal:- $x_1 - x_3 \leq 4$

② $2x_1 + 4x_2 \leq 12$

③ $x_1 + x_2 - x_3 \geq 2 \Rightarrow -x_1 - x_2 + x_3 \leq -2$

④ $3x_1 + 2x_2 - x_3 = 8$ [break into two]

$x_1, x_2, x_3 \geq 0$

4 ④ $3x_1 + 2x_2 - x_3 \leq 8$

⑤ $3x_1 + 2x_2 - x_3 \geq 8 \Rightarrow -3x_1 - 2x_2 + x_3 \leq -8$

Dual :- Min. $H = 4y_1 + 12y_2 - 2y_3 + 8y_4 - 8y_5$

$1y_1 + 2y_2 - y_3 + 3y_4 - 3y_5 \geq 8$

$4y_2 - y_3 + 2y_4 - 2y_5 \geq 10$

$-y_1 + y_3 - y_4 + y_5 \geq 5$

$y_5, y_4, y_1, y_2, y_3 \geq 0$, $y_4 - y_5 = y_6$

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$\text{Min } H = \cancel{4}y_1 + 12y_2 - 2y_3 + 8y_6$

$y_1 + 2y_2 - y_3 + 3y_5 \geq 8$

$4y_2 + y_3 + 2(y_6) \geq 10$

$-y_1 + y_3 - y_6 \geq 5$

$y_1, y_2, y_3, \cancel{y_5} \geq 0$

& y_6 is unrestricted in sign.

Q A company is spend Rs 1200 on transportation of Units from 3 plants to four destinations. Find the max. saving possible by optimum allocation.

	1	2	3	4	supply/Produced
P_1	20 (5)	30 (2)	50	17	7/2/0
P_2	70	35 (6)	40 (4)	60	10/4/0
P_3	40	12	60 (3)	25 (15)	18/15/0
Demand	5/0	8/6/0	7/3/0	15/0	35 \ 35 Balanced

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1. North West Corner rule:

$Z = 20 \times 5 + 30 \times 2 + 35 \times 6 + 40 \times 4 + 25 \times 15 + 60 \times 3$

$= 1085$

Saving $= 1200 - 1085 = 115$ Rs

2. Row Minima Method:

$Z = 17 \times 7 + 35 \times 8 + 40 \times 2$

$+ 60 \times 5 + 25 \times 8 + 40 \times 5$

$= 1179$

saving = Rs 21

20	30	50	17 (7)	7/0
70	35 (8)	40 (2)	60	10/2/
40 (5)	12	60 (5)	25 (8)	18/10/
5/0	8/0	7/5/0	15/8/0	

3. Column Minima Method:

20 (5)	30	50	17 (2)	7/2/0
70	35	40 (7)	60 (3)	10/3/0
40	12 (8)	60	25 (10)	18/10/0
5/0	8/0	7/0	15/13/3/0	

Z = 940

Saving = 1200 − 940 = 260

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4. Least Cost Method (Method of Matrix Minima)

20	30	50	17/7	7/0
70 (3)	35	40 (7)	60	10/3/0
40 (2)	12/8	60	25/8	18/10/2/0
5/3/0	8/0	7/0	15/8/0	

Z = 985

Saving = 215

(If not mention, Use U-V M)

Page No.

Date :

Best

(5) Vogel's Approximation Method (~~U-V Method~~)
(Unit Cost Penalty Method)

	1	2	3	4			
P_1	20 (min) (5)	30	50	10 (2)	7/7/0	3 [20-17]	[30-17 = 13]
P_2	70	35	40 (7)	60 (3)	10/3/0	5	33 20
P_3	40	12 (8)	60	25 (10)	18/10/0	13	35
	5/0	8/0	7/0	15/5/3/0			
	20 ↑ [40-20] ↑ (max)	18 ↑	10 10 10	8 8 43 ↑			

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$Z = Rs\ 940$

Saving = Rs 260

In this method, we write the difference of smaller & the second smallest element in each row & column below the respective row & column. Then we choose the highest individual diff. & allocate the maximum possible amount to the cell with the lowest cost in that particular row or column.

→ Optimality Test : ~~5×4 3 → m = 5, n = 4,3~~

i) As both the conditions are satisfied, optimality test can be performed.

a) 4+3-1 = 6 allocations.

b) No looping

5.1) Stepping stone Method

1)

20 (5)	30	50	17 (2)
70	35 (+3)	40 (7)	(-3) 60 (3)
40 (2)	(-3) 12 (8)	60 (1)	25 (+3) (10)

(3) is min

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60 = +60 − 25 + 60 − 40 = 55
40 = + 40 − 25 + 17 − 20 = +12
50 = +50 − 40 + 60 − 17 = 53
35 = 35 − 12 + 25 − 60 = −12 ✓ Best

It is an trial and error method in which an arbitrary empty cell is selected and +1 allocation is done in that cell & the total effect in the cost of matrix is computed. The chances of making error in this method is more because we may miss the loop which gives optimal solution

5.2)
(i) Modified Distribution (MODI method) / U-V Method

This method is prefered over a stepping stone method as it provides considerable time saving & there no chances of making error. The steps involved are :-

(i) Developed cost Matrix: for allocated cells only
(ii) Calculating u_i & v_j value for the matrix & taking $v_1 = 0$
(iii) Fill the vacant cells of the matrix by summation of u_i & v_j values

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(i)

		V_1	V_2	V_3	V_4
		0	−16	−23	−3
u_1	20	20			17
u_2	63			40	60
u_3	28		12		25

of

$u_1 + v_1 = 20$ $\quad$ $u_1 + v_4 = 17$

$u_2 + v_3 = 40$ $\quad$ $u_2 + v_4 = 60$

$u_3 + v_2 = 12$ $\quad$ $u_3 + v_4 = 25$

$u_i + v_j$ matrix for unallocated cells.

(ii)

	20+(−16) = 4	20+(−23) −3	
63+0 = 63	63−16 = 47		
28		5	

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iii) Subtract the cell values of $U_i + V_j$ matrix for unallocated cell from the original cost matrix. This will provide cell evaluation matrix

	80−4 = 26	50−(−3) = 53	
70−63 = 7	35−47 = −12		
40−28 = 12		60−5 = 55	

−ve

Cell evaluation matrix

We minimise the cost.

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If any of the cells value in cell evaluation matrix is -ve. Then, current basic feasible solution is not optimal.

iv) In the cell Evaluation Matrix, identify the cell with the most negative value, mark it and it is called identified cell. (-12)

v) Trace a path in this matrix consisting of a series of alternate horizontal & vertical line. The path begins & terminates in the identified cell & the corner of path lies in the cells for which allocations have already been made.

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	9	3
8		10

	26	53	
7	12 +3		-3
12	-3	55	+3

		0
5		13

Cell Evaluation Matrix

vi) Make identified cell as +ve & each other cell at the corner of path alternatively -ve / +ve and so on. Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a -ve sign.
The basic cell whose allocation has been made zero leaves the solution.

$Z = Rs\ 904$

Saving = 1200 - 904 = Rs 296.

20 5	30	50	17 2
70	35 3	40 7	60
40	12 5	60	25 13

$-3+x=$

$-16+x=35$

51

	0	-16	-11	-3
20	20			17
51		35	40	
28		12		25

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	0	-16	-11	-3
20		-4	9	
51	51			48
28	28		17	

$u_i + v_j$ for Non-allocated cells

	30-4 =26	50-9 =41	
70-51 =19			60-48 =12
40-28 =12		60-17 =43	

Cell Evaluation matrix

Ex 05 Determine the initial ~~test~~ feasible solution using Vogel's approximation & find the optimum distribu^n for this company. Then the unit Transportation cost in Rs is as given below

	D	E	F	G	
A	44	50	40	39	180
B	42	51	54	53	170
C	41	40	42	45	200
	90	100	120	180	490

550 unbalanced

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44 (80)	50 (100)	40	39	0	180/80/0	6 6 6 544
42	51	54 120	53 50	0	170/50/0	1 2 0 042
41 10	40	42	45 (130)	0 (60)	200/70/10/0	3 4 4 44
90/10/0	100/0	120/0	180/130/0	60/0		
2 3	10	12↑	8	0		
2 3	1	0	8↑	0		
2 3	10↑	0	6	0		
3	0	0	6↑	0		
3	0	0	0	0		

		180	
90		20	60
	100	100	

Vogel's APP.

Z = 20080

In this problem as Total allocations are 6, ~~&~~ $m+n-1 = 7$. So, the current solution is degenerate.
Now allocating infinitely small but the value ϵ [Epsilon] at vacant minimum cost cell such that all allocations remains in independ-ent position.

When the optimal sol^n is achieved. Then the value of ϵ is put equal to zero.
(Min, 0 & NO looping).

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	-	ϵ	180	ϵ
90	ϵ 20	20 0		60
	80	120		
	160	100		

$Z = 20060$

5/9/11

Q Solve the following assignment problem for min of cost

	A	B	C	D
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

where assignment is made.

These problems are solved by using Hungarian Method which is also known as Flood technique. The steps involved are as below:-

1. Develop oppurtunity cost Matrix:-

a. Subtracting the smallest element of each row from every element of corresponding row

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

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b. Subtracting the smallest element of each column from every element of corresponding column

$\boxed{0}$	14	9	3
9	20	$\boxed{0}$	22
23	$\boxed{0}$	3	~~0~~
9	12	14	$\boxed{0}$

we get atleast one zero in each row & each column called Oppurtunity cost matrix

2. Make assignment in the oppurtunity cost matrix.

Note: First assignment in the First row with single zero. In assigned row, if there is one more zero in column, cross it.

a) Starting from the First row of matrix starts searching single zero make an assignment (square box $\square$) and cross all other zero in the assigned column. Continue in this manner untill all the rows are examined.

b) After examine all rows completely, proceeds similarly examine the columns

3 a Continue, step $\underline{a}$ & $\underline{b}$ untill all the zero's are either assigned or crossed.

b The remaining unmark zero lies atleast to 2 in each row & column (solve it by trial & error method).

4 If the no. of assignment is exactly equal to size of matrix. Then the current solution is optimal otherwise perform optimality.

No of Assignment = No. of square matrix.

$z = 12+9+25+14 = 60.$

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Q Solve the following problem for minimisation of cost

2	3	4	5
4	5	6	7
7	8	9	8
3	5	8	4

0	1	2	3
0	1	2	3
0	1	2	1
0	2	5	1

$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2
$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	1	3	$\cancel{0}$

opportunity cost matrix

$z = 3+6+8+3$

$= 6+14 = \underline{20}$.

Q Solve the following assignment problem for min. of cost.

9	22	58	11	19	27
43	78	72	50	63	48
41	28	91	37	45	33
74	42	27	49	39	32
36	11	57	22	25	18
3	56	53	31	17	28

0	13	49	2	10	18
0	35	29	7	20	5
13	0	63	9	17	5
47	15	0	22	12	5
25	0	46	11	14	7
0	53	50	28	14	25

Matrix size = 6 = order.

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~~0~~	13	49	$\boxed{0}$	~~8~~	13	5 < 6
~~0~~	35	29	5	10	$\boxed{0}$	✓ (4)
13	~~0~~	63	7	7	~~0~~	✓ (1)
47	15	$\boxed{0}$	20	2	~~0~~	
25	$\boxed{0}$	46	9	4	2	✓ (5)
$\boxed{0}$	53	50	26	4	20	✓ (7)
✓ (6)	✓ (2)				✓ (3)	

Since Third row & fifth column have no assignment & total assignment is five which is less than the matrix size or order (n = 6). So we proceed to find minimum no of horizontal & vertical lines required to cover all zeros atleast once. The steps involved are :-

1. Mark all rows for which assignment have not been made. (3 row)
2. Mark all columns for which have unassigned zero in the marked row. (2 & 6 column)
3. Mark all rows which have assignment in the marked column (2 & 5th row)
4. Continue step 2 & 3 until chain of marking end.

Column → Assigned
Row – Unassigned

5. Draw Min no. of lines through unmark row & through marked column to covers all zero's atleast once.

4	17	53	0	0	17
4	35	25	1	6	0
13	0	59	3	3	0
51	19	0	20	2	4
25	0	42	5	0	2
90	53	46	22	0	20

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6. Select the smallest element that do not have line to them subtract ~~th~~ it from all the elements that do not have a line through them & add it to every element that lies at the intersecⁿ of 2 lines & leave the remaining elements of the matrix unchanged.

Make new assignment to newly obtained matrix

4	17	53	$\boxed{0}$	$\cancel{0}$	17
0	35	25	1	6	$\boxed{0}$
13	$\boxed{0}$	59	3	3	$\cancel{0}$
51	19	$\boxed{0}$	20	2	4
25	$\cancel{0}$	42	5	$\boxed{0}$	2
$\boxed{0}$	53	46	22	$\cancel{0}$	20

$11 + 48 + 28 + 27 + 25 + 3$
$= 59 + 55 + 28$
$= 142.$

4	17	49	$\boxed{0}$	$\cancel{0}$	17
$\boxed{0}$	35	25	1	6	$\cancel{0}$
13	$\cancel{0}$	59	3	3	0
51	19	$\boxed{0}$	20	2	4
25	$\boxed{0}$	42	5	$\cancel{0}$	2
$\cancel{0}$	53	46	22	$\boxed{0}$	20

$z = 43 + 11 + 27 + 11 + 17$
$= 54 + 38 + 17$
$= 0$

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Q. Maximization Problem :
Find out Max profit possible through optimal allignment

30	37	40	28	40
40	24	27	21	36
40	32	33	30	35
25	38	40	36	36
29	62	41	34	39

2 9 12 0 12

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32	25	22	34	22
22	38	35	41	26
22	30	29	32	27
37	24	22	26	36
33	0	21	28	23

13	3	0	12	0
0	16	13	19	14
0	8	7	10	5
15	2	0	4	14
33	0	21	28	23

13	3	0	8	$\boxed{0}$
$\boxed{0}$	16	13	15	14
0	8	7	6	5
15	2	0	$\boxed{0}$	14
33	$\boxed{0}$	21	24	23

3
1
2

19	3	$\boxed{0}$	8	0
0	12	9	11	$\boxed{0}$
$\boxed{0}$	4	3	2	1
17	2	0	$\boxed{0}$	14
37	$\boxed{0}$	21	24	23

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19	3	$\boxed{0}$	8	0
$\boxed{0}$	10	8	10	9
0	3	2	1	$\boxed{0}$
21	2	0	$\boxed{0}$	14
39	$\boxed{0}$	21	24	23

$40 + 40 + 36 + 36 + 62$

$80 + 72 + 62$

$152 + 62$

$= 214$

Q. Given that product A, C, G & H have 40, 30, 70 & 90 Units in the inventory. Then, what is net requirement for producing 400 units of product 'x'. Ne

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X (1)

A (1) = 400 − 40 = 360

B (2) = 800. = 400 × 2

C (1) 400 − 30 = 370

D (2) 360 × 2 = 720

E (3) 360 × 3 = 1080

F (1) 370

G (1.) 370 − 70 = 300

H (4) 300 × 4 = 1200

~~1200~~ − 90 = 1110

4 H = 1 G

F + G = C

~~2x 40 + 3~~

$\cancel{400 = A + 2B + [1C]}$

$\cancel{= 400 + 2B + [F + G]}$

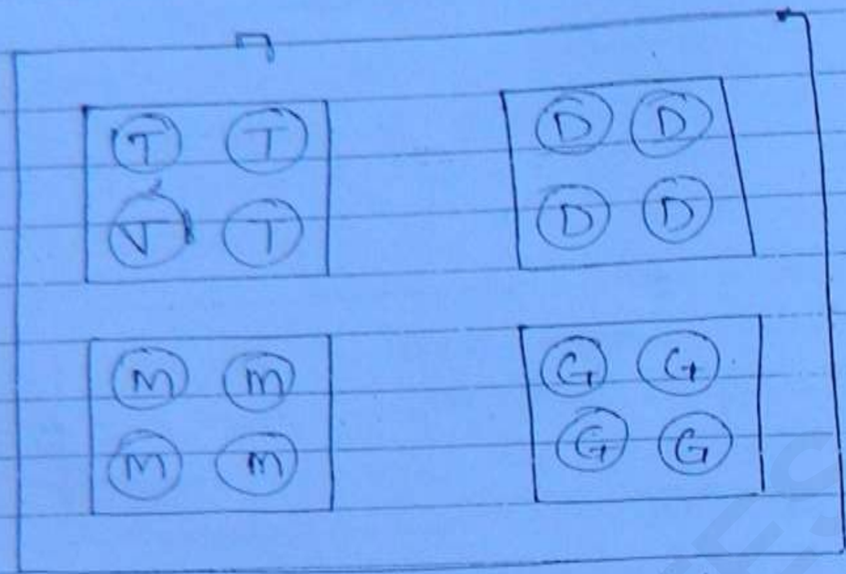
$\cancel{40 = 2x + 3y}$

$\cancel{30 = x +}$

D = 720 E = 1080, B = 800, F = 370, H = 1110

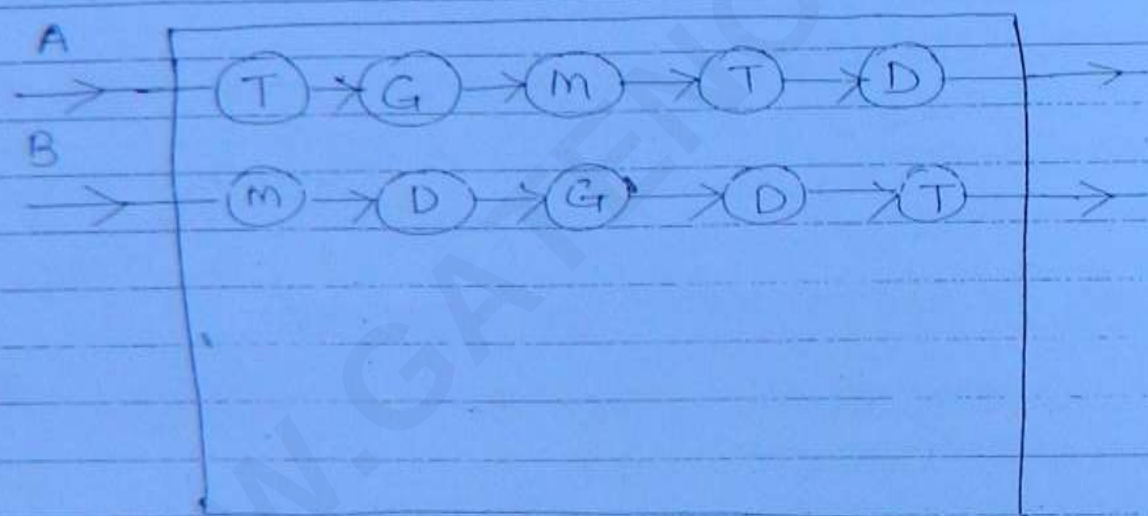
PLANT LAYOUT

(1)



Process or Functional layout

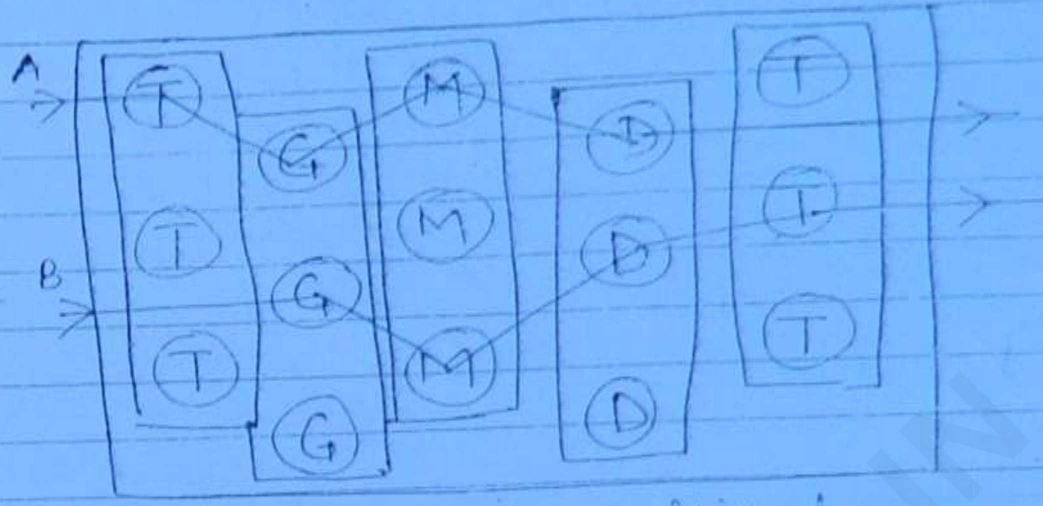
(2)



Product or Line Layout : Nut & Bolt

		(2)	(1)
1.	Work in process inventory	↑	↓
2.	output	↑	↓
3.	Quality of work	↓	↑
4.	Initial Investment	↑	↓
5	Prepare schedule Chart	Easy	Difficult.
6	Quantity	↑	↓
7.	Variety	→ less preferred	more preferred
8	mass production	→ more preferred	less preferred.
9	Job or batch type production	less.	more
	Production cost / Unit	↓	↑

Process Layout ↓



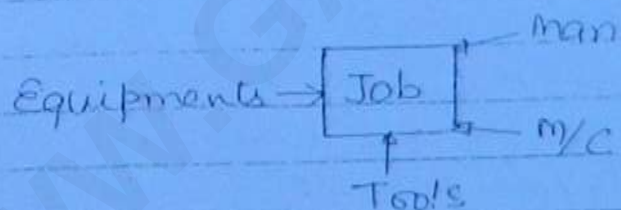
Combination or Mixed

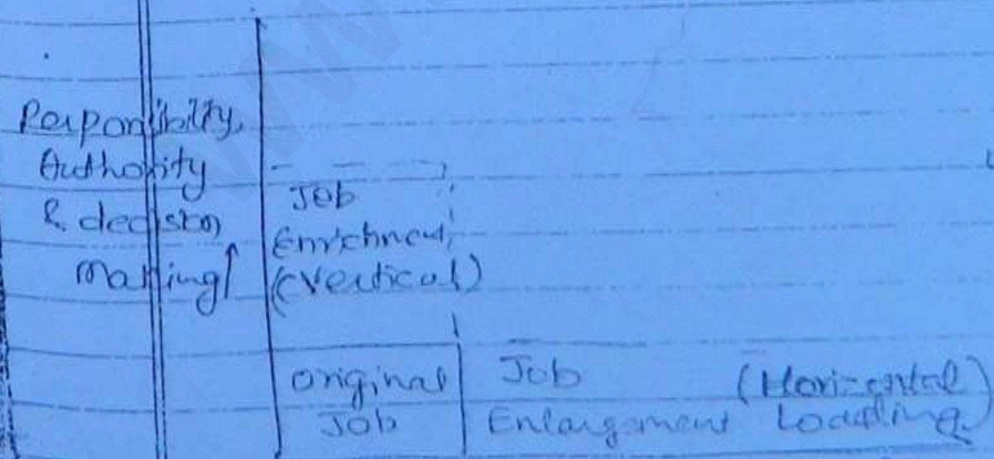
- Initial Investment is very High
- No back tracking
- Very high production rate
- Boilers, Rocket, Aircraft   man m/c material towards
- Fixed job

Fixed Position layout:

→ Used in Boilers, ~~Turbine~~, Rockets, Aircrafts

→ man, m/c & material moves towards fixed job





work study

More Jobs of same level