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Mechanical ENGG

Mechanical Engg.

MACHINE

①

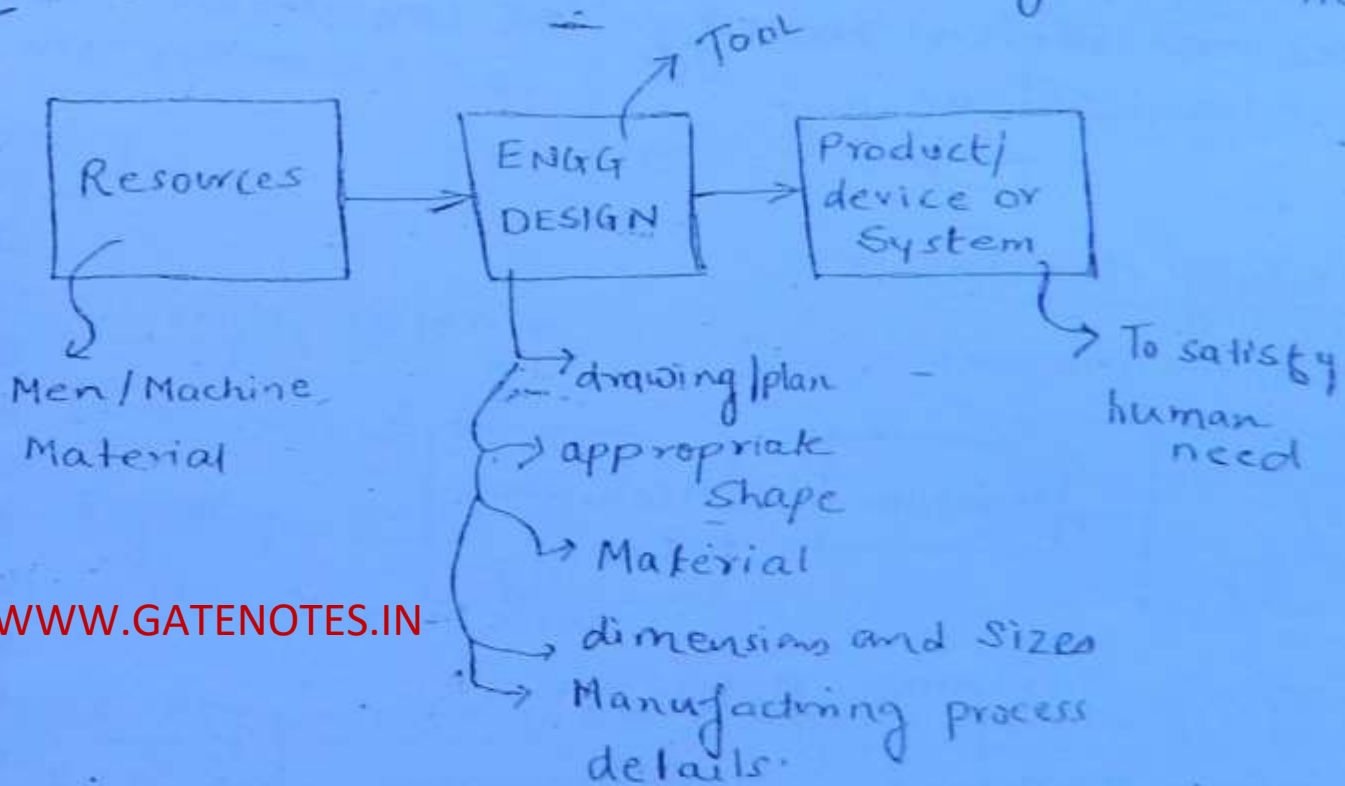
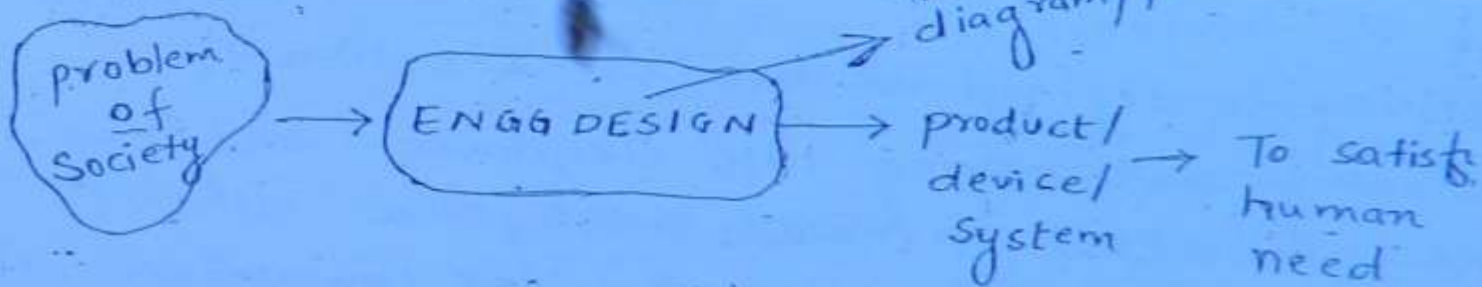
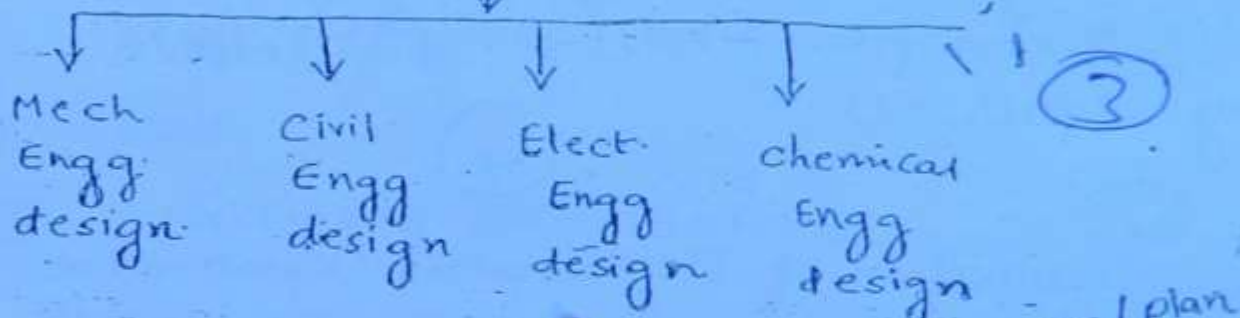
DESIGN

MIS
Mechanical Engineering

②

Introduction

ENGG. DESIGN



WWW.GATENOTES.IN

ENGG. Design: It is defined as an iterative decision making activities to produce a drawing or a plan, to convert resources optimally into a product or device or a system to satisfy the human need.

The ultimate aim of design is to select appropriate shape, material, size and manufacturing process details in such a way that the resulting m/c component should perform its given function satisfactorily (i.e., without any failure).

Machine

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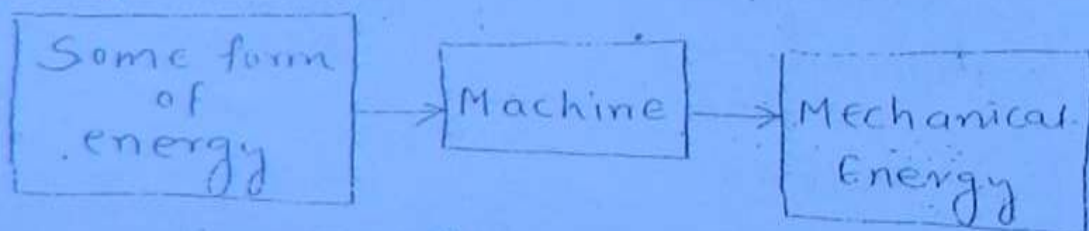
It is a combination of Mechanisms (Combination of M/c elements).

Machine is defined as the combination of stationary and moving m/c elements and they are assembled in such a way that either produce mechanical energy or convert or utilise mechanical energy.

Types

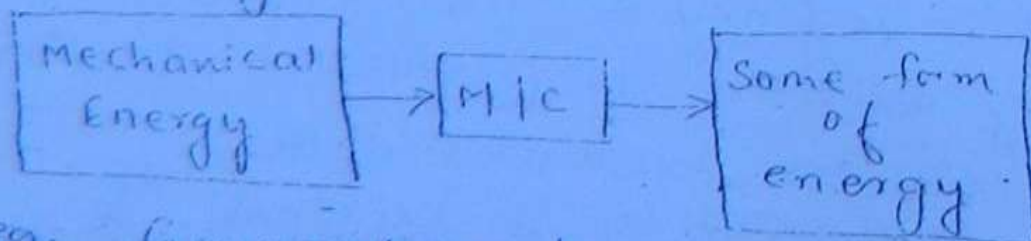
Generating Machine

eg Prime movers



eg: Engines, Turbines, Motors

Generating Machines



eg: Generators, hand pumps

Selection of appropriate Material

- ① List of properties required
- ② selecting group of Materials
- ③ Availability
- ④ Cost
- ⑤ selecting a best Material

⑤

Friction lining Material

⇒ 1	2	3	4	5
⇒ Strength ↑	X	More	Costlier	
⇒ μ ↑	Y			×
⇒ Wear resistance ↑	Z			
⇒ K ↑	W	less	cheap	

① Strength criterion

$$(\sigma_{\max})_{\text{ind.}} \leq \sigma_{\text{per}}$$

② Rigidity criterion

$$(\delta_{\max})_{\text{ind.}} \leq \delta_{\text{per}}$$

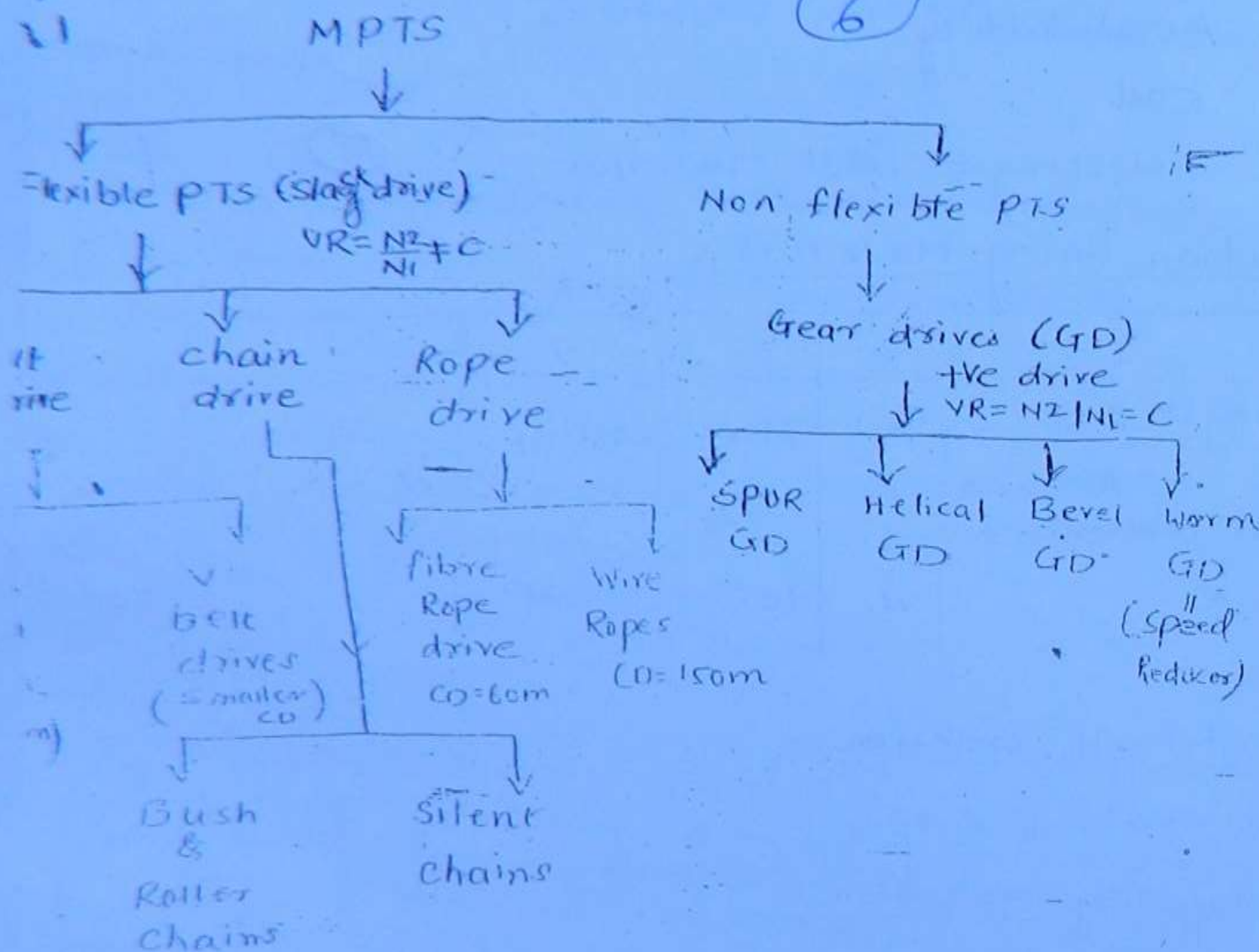
Basic Requirements for a Machine Elements

- ① high strength
- ② More rigidity
- ③ high service life
- ④ less cost
- ⑤ more wear resistant

D POWER TRANSMISSION SYSTEMS

Called as Mechanical power Transmission Systems (MPTS)

⑥



Factors to be considered in the selection of a proper MPTS

Centre distance (C.D)

Shaft layout

power to be transmitted

velocity Ratio

Advantage of Flexible PTS

- ① Larger centre distance
- ② cost is less
- ③ Centre distance can be achieved
- ④ damping capacity is more

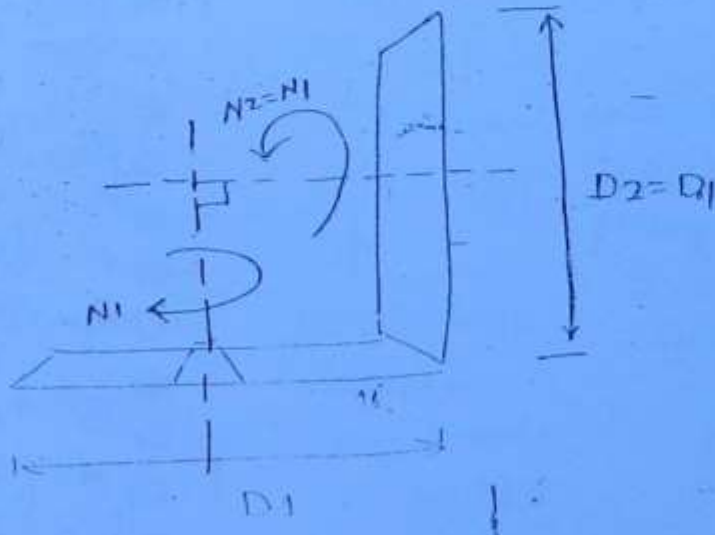
⑦

Disadvantages of flexible PTS

- ① velocity ratio is ^{not} constant due to slip
- ② efficiency is less
- ③ Service life is less

MITRE GEARS

Two equal sized gear mounted on two intersecting perpendicular shafts.



Spur Gear

F_r and F_t → thrust force

$F_a = 0$ (thrust force is zero)

$F_a = F_t \tan \beta$

$\beta = 0 \Rightarrow F_a = 0$

Steps used in Design of a Machine Element

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Specify function of a M/E Element

Determination of loads acting on a M/E element

Selection of an appropriate shape for a Machine element

→ expression for Geometrical properties of the selected shape

Selection of appropriate material for the Machine element

→ properties of that selected material
eg. $E, G, \nu, \sigma_s, \sigma_u, \epsilon_L$

Selection of Mode of failure

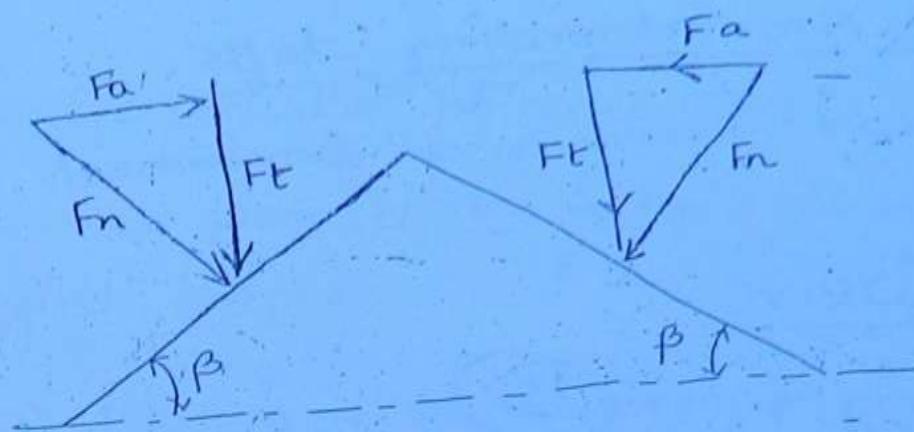
→ failure by Elastic deflection $\Rightarrow$ Elastic limit

→ failure by yielding $\Rightarrow \sigma_s$

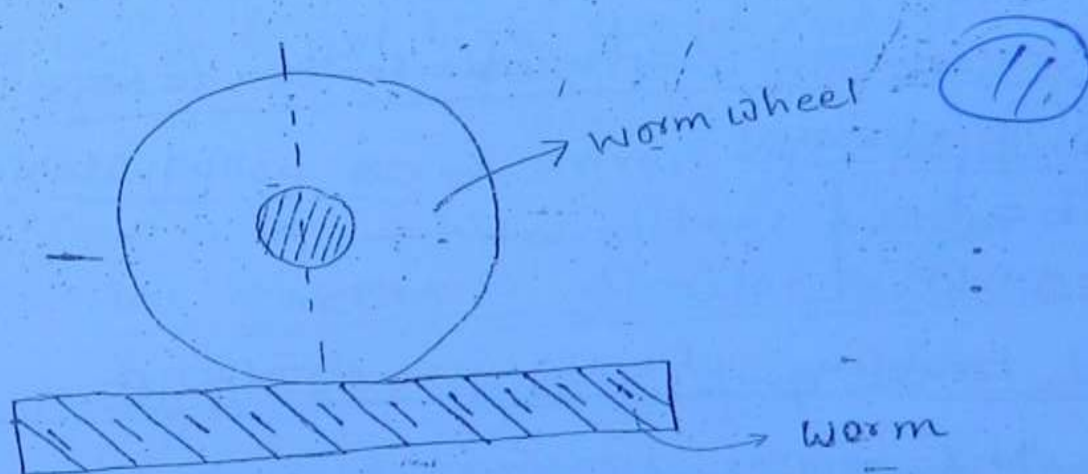
→ failure by fracture $\Rightarrow \sigma_u$

Determination of dimensions by using Strength of Material equations

Preparation of part drawing for the given machine element



→ axial thrust is eliminated ($F_a = 0$)



(multi start power screw)

$$\text{Lead} = np$$

Lead = axial distance travelled by the screw in one rotation

p = pitch

n = no. of starts

$L = p \Rightarrow$ single start power screw

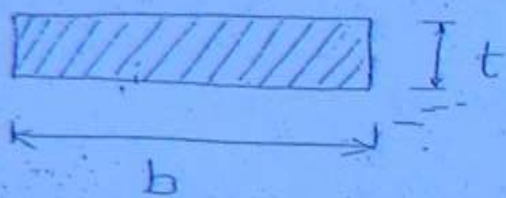
$L = 2p \Rightarrow$ double start power screw

In above case efficiency is less
but speed reduction ratio is high

for Non parallel, Non intersecting shafts

eg hypoid gears, worm and worm wheel

① (a) FLAT BELT DRIVE



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always preferable thin and wider belts to have less bending stresses

$$\text{as } \sigma_b = \frac{E \cdot t}{D}$$

Tensile and bending stresses are produced

Types of flat Belt drive

- ① open belt drive
- ② cross belt drive
- ③ compound belt drive
- ④ fast & loose pulley belt drive
- ⑤ stepped pulley drive
- ⑥ Jockey pulley drive (open belt drive with idler pulley)
- ⑦ Quarter turn belt drive
(Right angled belt drive)

Suitable for Medium Centre distance

⇒ 1-6 ⇒ are used for parallel shafts -

⇒ 7 ⇒ for non parallel non intersecting right angled shafts

Open BD ⇒ rotating in same direction

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Cross BD ⇒ rotating in opposite direction

Compound BD ⇒ to get high speed reduction

Fast and loose pulley BD ⇒ driven m/c requires intermittent motion

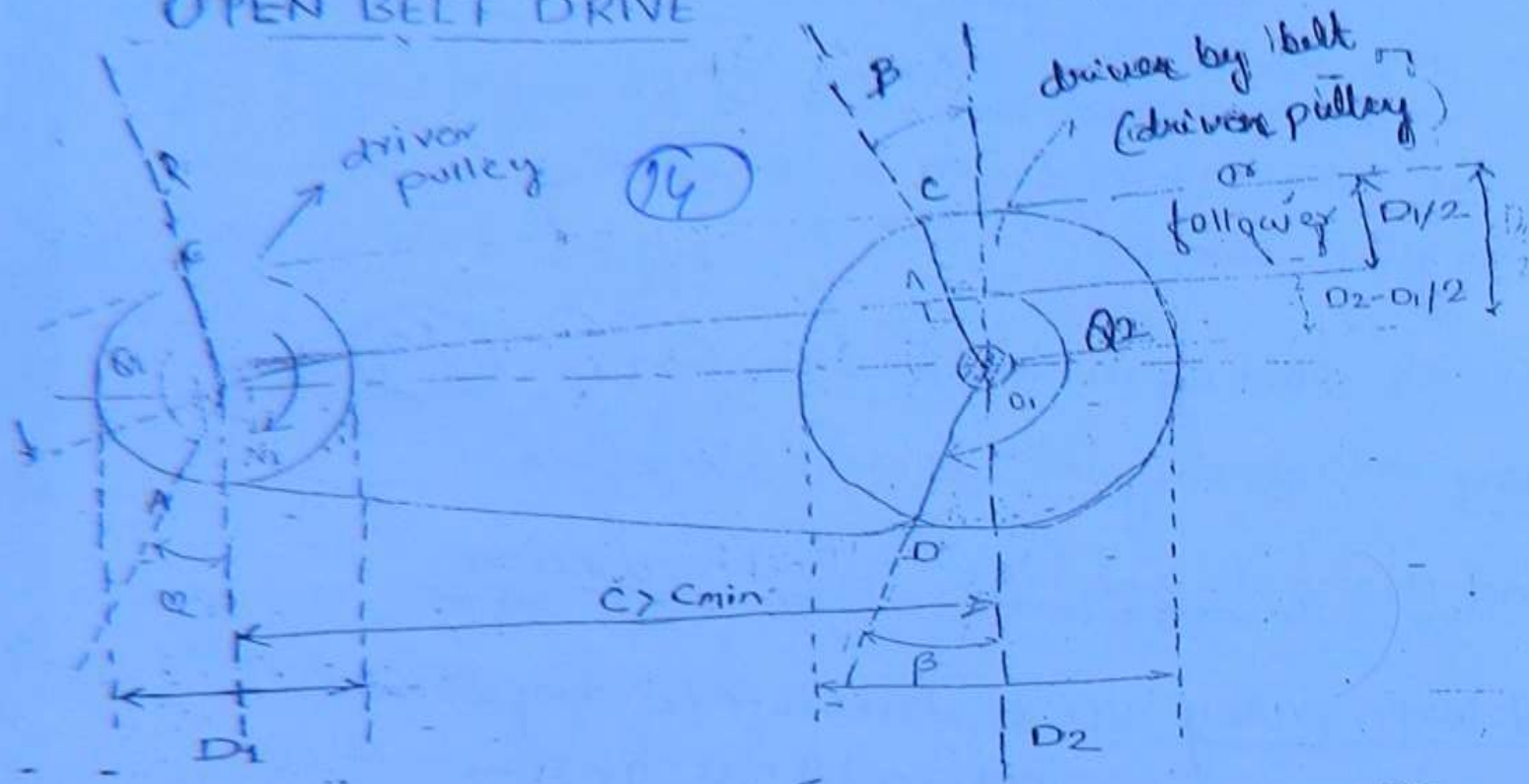
Its function is similar to clutches

Stepped pulley drive ⇒ variable speed drive

Jockey pulley drive ⇒ transmit power between two parallel shafts which are at smaller centre distance.

OPEN BELT DRIVE

OPEN BELT DRIVE



$$\theta_1 = \pi - 2\beta$$

$$\theta_2 = \pi + 2\beta$$

$$\theta_1 + \theta_2 = 2\pi$$

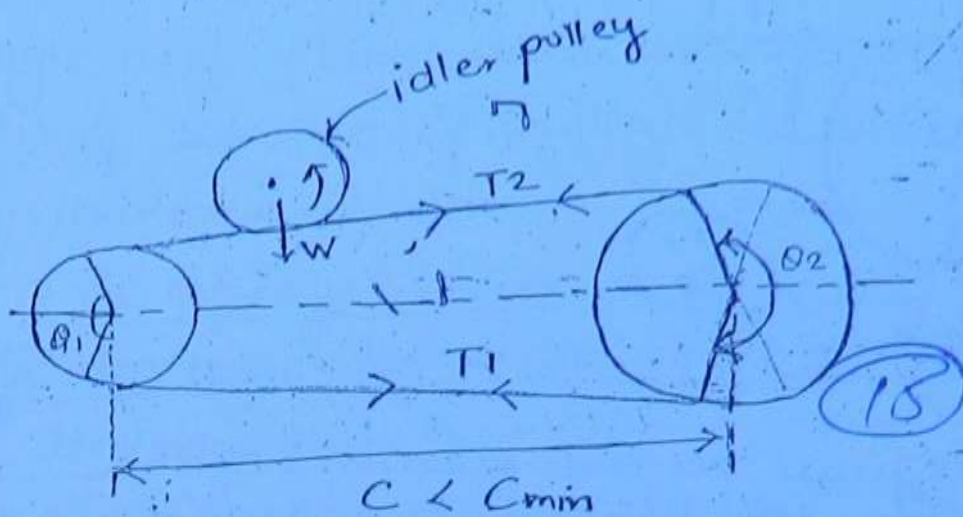
$$\beta = \sin^{-1} \left[\frac{D_2 - D_1}{2C} \right] \times \frac{\pi}{180}$$

$$\sin \beta = \frac{O_1 A}{O_1 O_2} = \frac{D_2 - D_1}{2C}$$

3. Centre distance ^{decrease} decrease, β increases

θ_1 decreases, θ_2 increases

Slip will occur at driver pulley

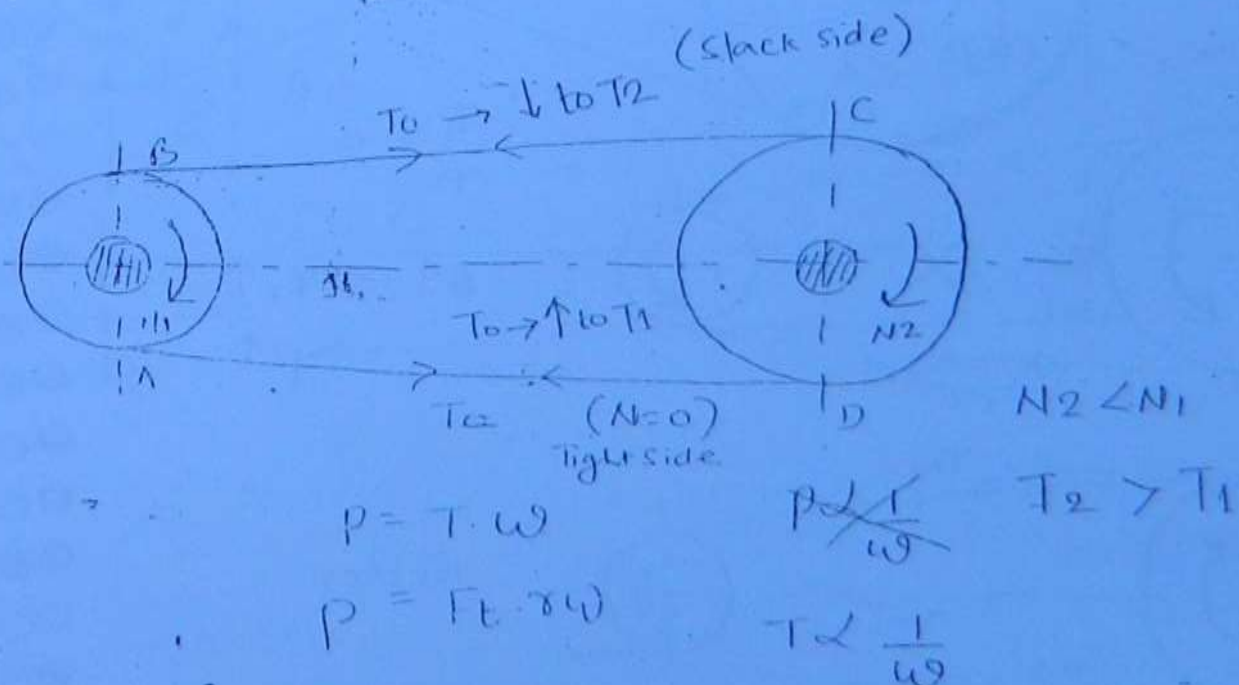


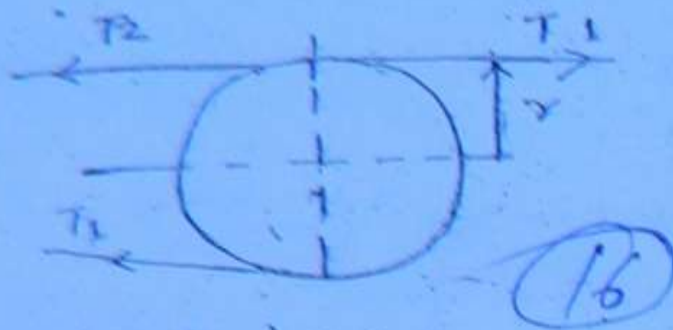
Length of the open belt drive

$$L_{OBD} = \text{Arc } AB + BC + \text{Arc } CD + DA$$

$$L_{OBD} = 2C + \frac{\pi}{2} (D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C} \quad **$$

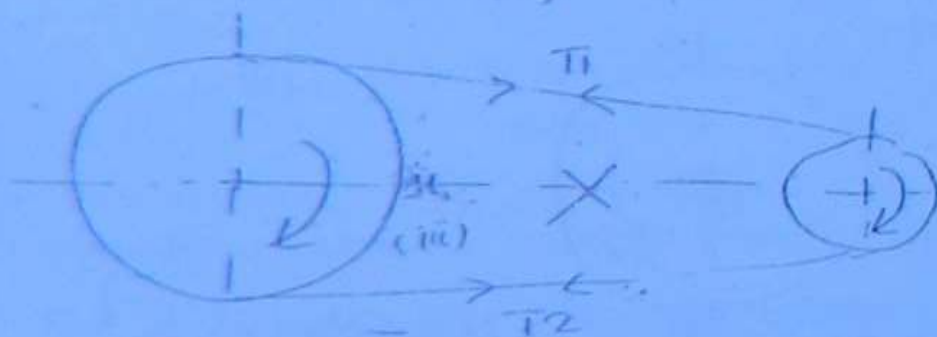
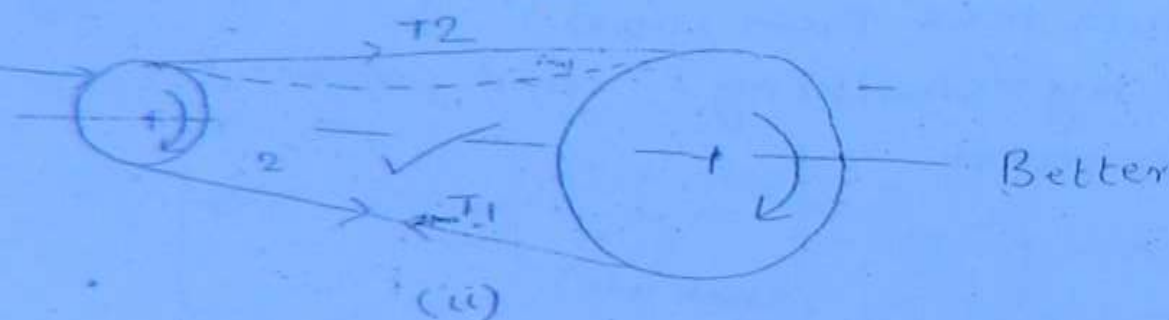
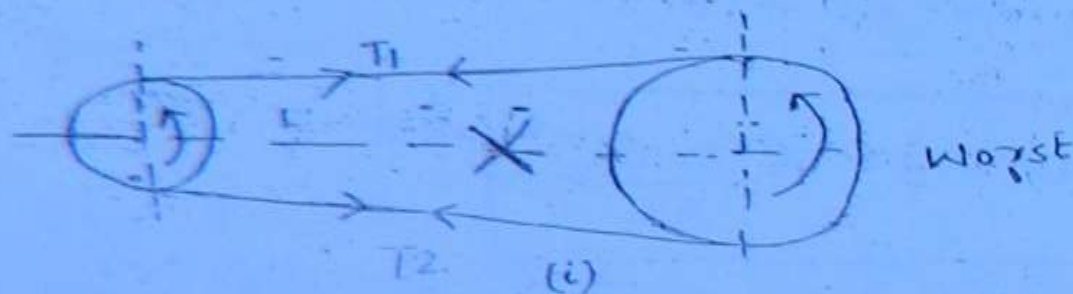
Length choosen is less than L_{OBD} . for Initial Tension (by stretching) and power Transmission increases





$$P = (T_1 - T_2)V$$

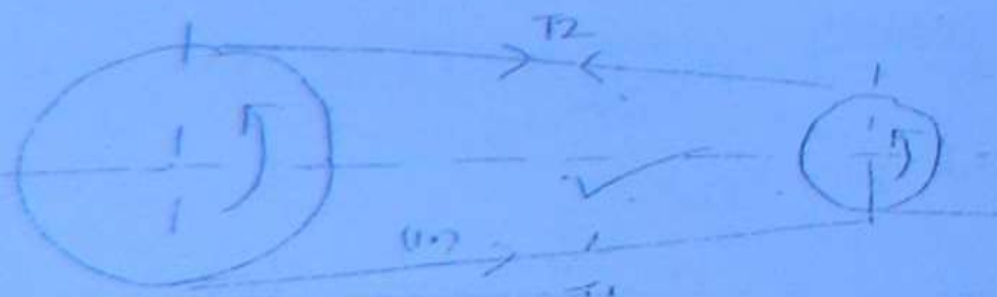
glt Side $\Rightarrow$ It is defined as the portion of the belt which is entering the driver pulley & leaving the driven pulley



$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\theta \uparrow \Rightarrow T_1 \uparrow$$

$$\Rightarrow P \uparrow$$



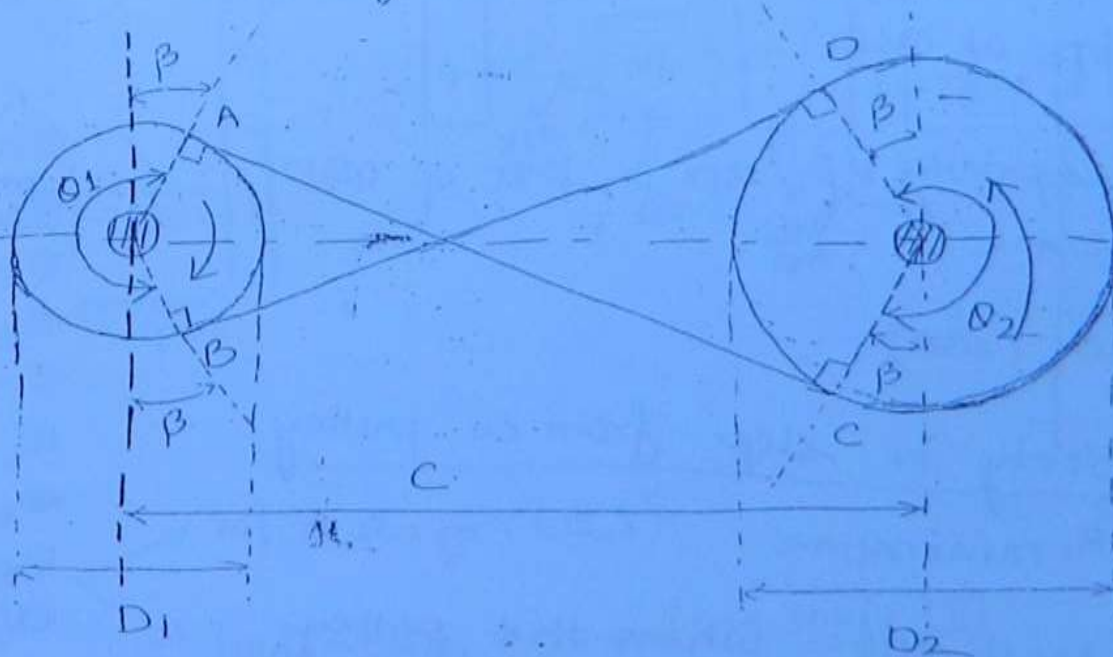
Better

The tight and slack sides are depends upon direction of rotation of pulley as well as position of the driver and driven pulley.

⇒ It is always better to have tight side at the bottom because of the sagging of the belt on the top side (Slack side) (17)
angle of contact increases at the smaller pulley (as angle of contact increases the T_1 increases and hence power will increase.

CROSS BELT DRIVE

To transmit power between parallel shafts which are running in opposite direction.



$$D_1 = D_2 = \pi + 2\beta$$

$$\beta = \sin^{-1} \left[\frac{D_2 + D_1}{2C} \right] \times \frac{\pi}{180} \quad *$$

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$$L_{CBD} = \text{Arc AB} + AC + \text{Arc CD} + DB$$

$$L_{CBD} = 2C + \frac{\pi}{2} (D_1 + D_2) + \frac{(D_2 + D_1)^2}{4C} \quad *$$

diff. of L_{OBD} and L_{CBD} is

$$\Delta L = \frac{D_1 D_2}{C} \quad **$$

fc of CBD < fc of OBD

Power Transmission Capacity of CBD > PTC of OBD
(PTC)

$$\Rightarrow P_1 \text{ CBD} > P_1 \text{ (OBD)}$$

Belt is always likely to slip from a pulley where μ is minimum

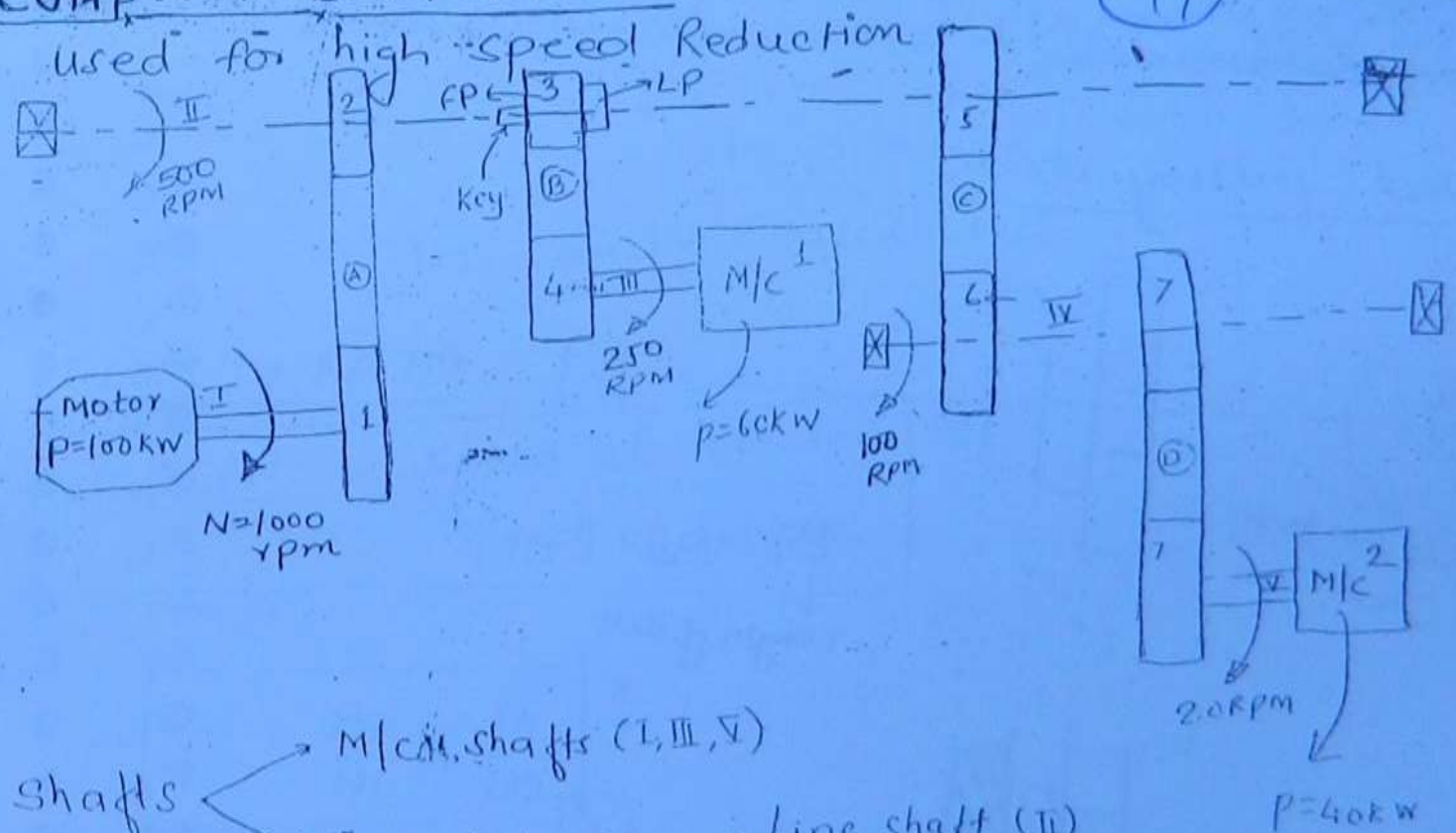
$\Rightarrow$ In an open belt drive when the pulleys are made of same material the belt is likely to slip from smaller pulley because $\mu_1 = \mu_2$ but $D_1 < D_2$

⇒ In a cross belt drive when the pulleys are made up of same material the belt is likely to slip from both the pulley simultaneously because $\mu_1 Q_1 = \mu_2 Q_2$ [$\mu_1 = \mu_2$, $Q_1 = Q_2$]

⇒ In a CBD when pulleys are made up of diff material the belt is likely to slip from pulley where μ is minimum because [$Q_1 = Q_2$]

⇒ Always we have to design with respect to pulley where the belt is likely to slip

COMPOUND BELT DRIVE



Shafts → M/C, shafts (I, III, V)

Transmission Shafts (II, IV) → Line shaft (II)

Counter shaft (IV)

above - fast and loose pulley also

Intermediate shaft It is a intermediate transmission shaft which is used for distributing the power among various machines.

Under shaft it is a transmission shaft which is used to get the higher speed reduction. $\therefore$

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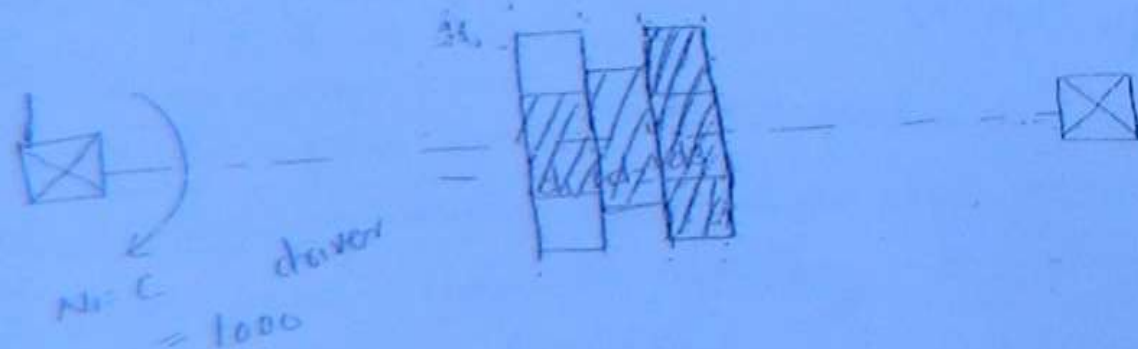
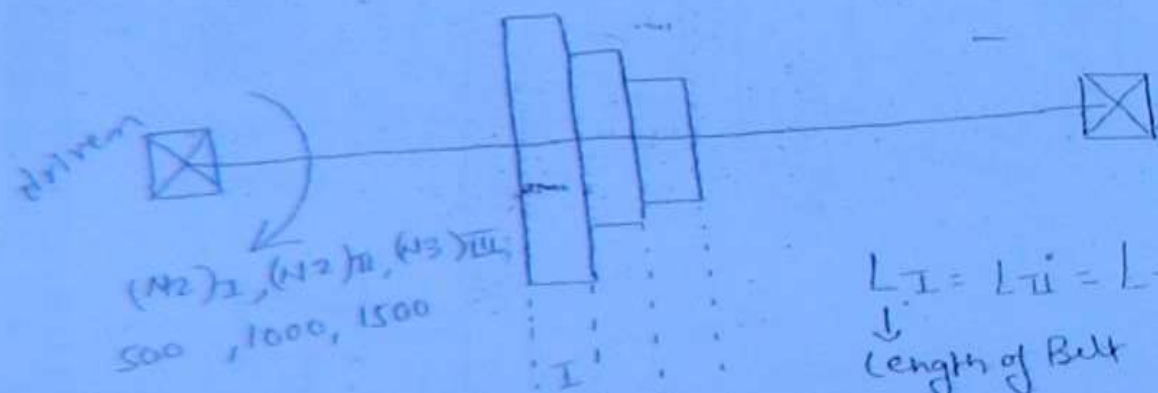
Fast pulley

Key connection
capable of
power transmission

Loose pulley

1. No Key connection
2. Incapable of power transmission G.M.O.

Stepped pulley drive



$$\Rightarrow L_I = L_{II} = L_{III}$$

$$\Rightarrow \frac{(N_1)_I}{N_1} = \frac{d_1}{D_1} \Rightarrow D_1 = ?$$

if d_1 is known

D_1 can be determined

$$\frac{(N_2)_{II}}{N_2} = \frac{d_2}{D_2} \quad d_2 = x \cdot D_2 \quad (2)$$

$$\text{Now } L_I = L_{II}$$

$$2C + \frac{\pi}{2} (D_1 + d_1) + \frac{(D_1 - d_1)^2}{4C} = 2C + \frac{\pi}{2} (D_2 + d_2) + \frac{(D_2 - d_2)^2}{4C}$$

$$d_2 \rightarrow x D_2$$

$\therefore D_2$ can be calculated

VELOCITY RATIO

$$VR = \frac{N_2}{N_1} = \frac{\text{Speed of follower}}{\text{Speed of driver}} \quad *$$

$$VR = \frac{N_2}{N_1} \times \frac{D_1}{D_2} \quad *$$

equation is valid by neglecting belt thickness effect and slip effect

Let V_1 = Linear velocity of driver pulley

V = Linear velocity of belt

V_2 = Linear velocity of follower

$$V_1 = V = V_2 \text{ [No slip]} \quad (22)$$

$$V_1 > V > V_2 \text{ [in presence of slip]}$$

Def! It is defined as the relative motion between belt and pulley surfaces due to insufficient frictional grip. (because of air layer present between pulley and belt surface)

$\Rightarrow$ belt velocity is less than driver pulley velocity but more than driven pulley velocity hence in presence of slip belt moves somewhat slower than driver pulley but more somewhat faster than the driven pulley.

$\Rightarrow$ In presence of slip speed of the follower decreases, hence velocity ratio of a belt drive and efficiency of belt drive decreases.

$$\text{as } V_1 = V_2$$

$$\frac{\pi D_1 N_1}{60} = \frac{\pi D_2 N_2}{60}$$

$$\boxed{\frac{D_1}{D_2} = \frac{N_2}{N_1}}$$

$$\boxed{\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}}$$

by neglecting effect of slip

In presence of slip

$$V = V_1 - V_1 \frac{S_1}{100}$$

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$$V = V_1 \left[1 - \frac{S_1}{100} \right] \rightarrow (i)$$

$\rightarrow S_1 =$ percentage of slip between driver pulley and belt.

$$V_2 = V - V \frac{S_2}{100}$$

$S_2 =$ % of slip between driven pulley and belt.

$$V_2 = V \left[1 - \frac{S_2}{100} \right] \rightarrow (ii)$$

Subst eq (i) in eq (ii) we get

$$V_2 = V_1 \left[1 - \frac{S_1}{100} \right] \left[1 - \frac{S_2}{100} \right]$$

$$\frac{\pi (D_2 + t) N_2}{60} = \frac{\pi (D_1 + t) N_1}{60} \left[1 - \left(\frac{S_1 + S_2}{100} \right) + \frac{S_1 S_2}{100^2} \right]$$

neglected

$$\boxed{\frac{N_2}{N_1} = \left(\frac{D_1 + t}{D_2 + t} \right) \left[1 - \frac{S}{100} \right]}$$

**

$S =$ percentage of total slip at belt drive

$$\boxed{S = S_1 + S_2}$$

VELOCITY RATIO IN A COMPOUND BELT DRIVE

$$VR = \frac{N_n}{N_1} = \left[\frac{(D_1+t)(D_3+t) \dots (D_{n-1}+t)}{(D_2+t)(D_4+t) \dots (D_n+t)} \right] \left[1 - \frac{S}{100} \right]^*$$

$$S = S_1 + S_2 + \dots + S_n$$

Q. 3 belt drives

$S = ?$

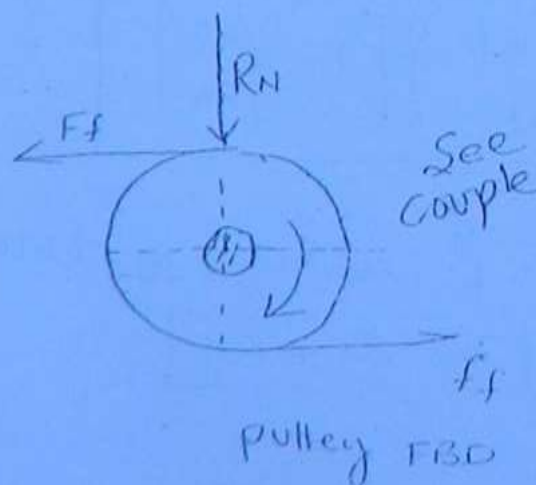
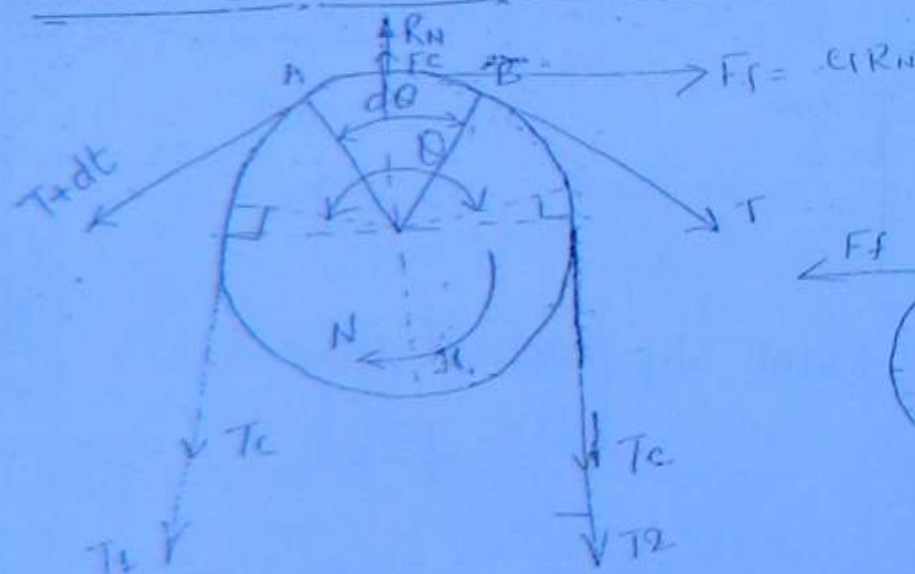
If slip at each belt drive = 2%

$$S = (S_1 + S_2) + (S_3 + S_4) + (S_5 + S_6)$$

2% 2% 2%

$$S = 6\%$$

RATIO OF BELT TENSIONS

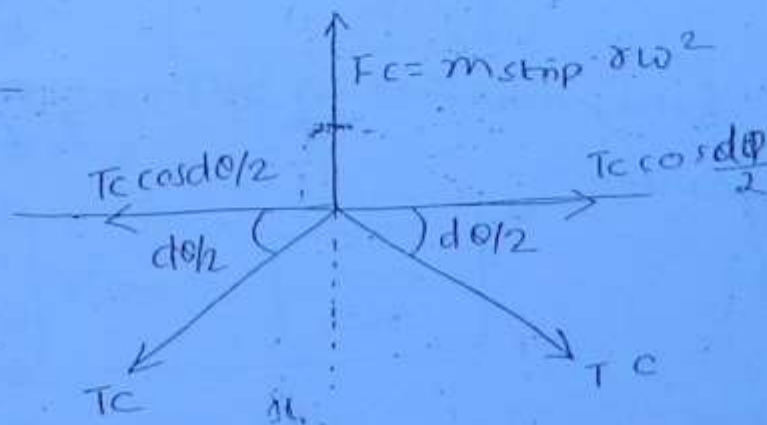
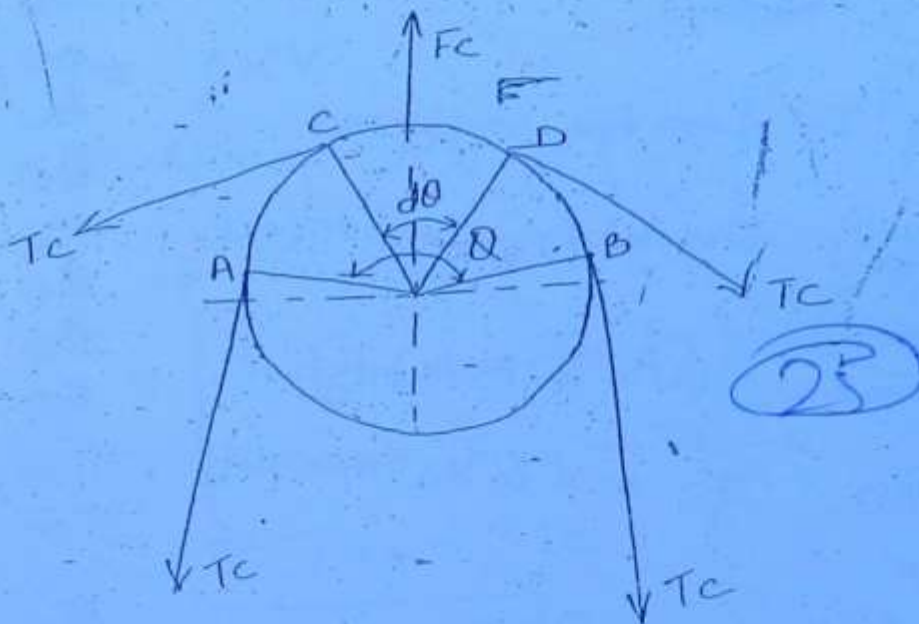


T_c = Centrifugal tension

Centrifugal Tension: additional tension induced in the belt in presence of centrifugal force

T_t = Total tension on tight side = $T_1 + T_c$

T_s = Total tension on slack side = $T_2 + T_c$



$$\sum V = 0$$

$$F_c - T_c \sin \frac{d\theta}{2} - T_c \sin \frac{d\theta}{2} = 0$$

$$m_{strip} \cdot r \omega^2 - T_c \cdot \frac{d\theta}{2} - T_c \cdot \frac{d\theta}{2} = 0$$

$$m_{strip} \cdot r \omega^2 - T_c d\theta = 0$$

let $m =$ mass of the belt per unit length in kg/m

$$m_{\text{strip}} = m \cdot r \cdot d\theta$$

$$0 = \sum \text{force} = m \cdot r \cdot d\theta \cdot \omega^2 - T_c \cdot d\theta = 0$$

$$m \cdot r^2 \omega^2 d\theta = T_c d\theta$$

$$\boxed{mv^2 = T_c}$$

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$v < 8 \text{ m/s} \Rightarrow$ effect of T_c can be neglected

$v \geq 8 \text{ m/s} \Rightarrow$ Effect of T_c should be considered

$$\begin{aligned} m &= \rho \cdot V \\ m &= 1000 \times A \times L \end{aligned} \quad \left\{ \begin{aligned} \rho_{\text{leather}} &= (950 \text{ to } 1050) \text{ kg/m}^3 \\ \sigma_{\text{per}} &= \frac{\sigma_{yt}}{N} = 2 \text{ to } 2.5 \text{ MPa} \end{aligned} \right\}$$

$$m = 1000 \times \frac{b}{1000} \times \frac{t}{1000} \times 1 \text{ m}$$

T_{max} - maximum Tensile force material of belt can withstand without failure

for safe design of belt

$$[(\sigma_t)_{\text{max}}]_{\text{ind}} \leq (\sigma_t)_{\text{per}}$$

$$\frac{T_1}{A} \leq (\sigma_t)_{\text{per}}$$

$$\text{or } \frac{T_1 \text{ or } T_2}{A} \leq (\sigma_t)_{\text{per}}$$

Condition for Maximum Power Transmission

$$P = (T_1 - T_2)V$$

$$P \propto V \text{ as } V \uparrow \Rightarrow P \uparrow$$

$$\text{but as } V \uparrow \Rightarrow T_c \uparrow \Rightarrow P \downarrow$$

$$\text{Now } P = T_1 K' V$$

$$P = (T_{\max} - T_c) K' V = T_{\max} K' V - k' m V^3$$

$$\frac{dP}{dV} = 0 \Rightarrow T_{\max} K' - 3 K' m V^2 = 0$$

$$K' [T_{\max} - 3T_c] = 0$$

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$$K' \neq 0 \Rightarrow T_{\max} - 3T_c = 0$$

$$\Rightarrow T_{\max} = 3T_c$$

$$\boxed{T_c = \frac{T_{\max}}{3}}^*$$

$$m V_{\max}^2 = \frac{T_{\max}}{3}$$

$$\boxed{V_{\max} = \sqrt{\frac{T_{\max}}{3m}}}^*$$

$P_{\max}$:

$$(1) T_{\max} = \text{6 per. b.t} = \frac{?}{2} N$$

$$(2) T_c = \frac{T_{\max}}{3} = \frac{?}{2} N$$

$$(3) T_1 = T_{\max} - T_c = 2T_c$$

$$1) \frac{T_1}{T_2} = \frac{(6.0) \text{ min}}{e} = K$$

$$5) m = \int \frac{B}{1000} \times \frac{t}{1000} \times 1 \text{ m}$$

$$T_2 = \frac{T_1}{K}$$

$$6) V_{\max} = \sqrt{\frac{T_{\max}}{3m}}$$

$$P_{\max} = (T_1 - T_2) V_{\max}$$

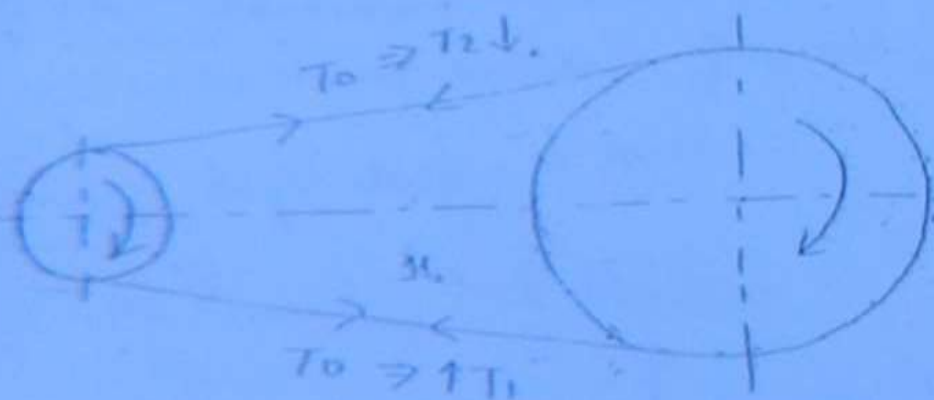
(28)

Expression for Initial Tension (T_0)

Initial Tension is the tension induced in the belt when it is in the stationary condition.

It is provided in the belt by taking a length of belt by taking a length of belt less than the actual required length.

$$\left\{ \begin{array}{l} \text{as } L \downarrow \Rightarrow T_0 \uparrow \Rightarrow F_f \uparrow \Rightarrow \frac{T_1}{T_2} \uparrow \\ \therefore T_1 \uparrow \text{ or } T_2 \downarrow \text{ and } P \uparrow \end{array} \right\}$$



Increase in length of belt = decrease in length of belt on slack side

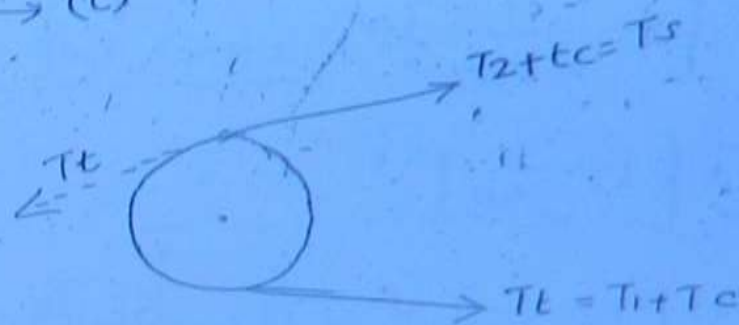
$$T_h \text{ or } T_c \leq A \cdot (Gt)_{\text{per}}$$

$$T_h \text{ or } T_c \leq (b \cdot t \cdot (Gt)_{\text{per}}) \rightarrow T_{\text{max}}$$

$$T_{\text{max}} = b \cdot t \cdot (Gt)_{\text{per}}$$

power transmission capacity (PTC) (29)

$$PTC = (T_h - T_c) V \rightarrow (i)$$



$$P = (T_h - T_c) V$$

$$P = [(T_h + T_c) - (T_2 + T_c)] V$$

$$P = (T_h - T_2) V \rightarrow (ii)$$

$$T_h \text{ or } T_c \leq T_{\text{max}}$$

$$\Rightarrow T_c = 0 \Rightarrow T_h = T_{\text{max}}$$

$$\Rightarrow T_c \neq 0 \Rightarrow T_c = T_{\text{max}}$$

$$\Rightarrow T_h + T_c = T_{\text{max}}$$

$$\Rightarrow T_h = T_{\text{max}} - T_c$$

Effect of T_c on PTC

$$T_c = 0 \Rightarrow T_1 = T_{\max}$$

$$T_c \neq 0 \Rightarrow T_1 = T_{\max} - T_c$$

$$P = (T_1 - T_2)V$$

$$P = T_1 \left[1 - \frac{T_2}{T_1} \right] V$$

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$$P = T_1 \left[1 - \frac{1}{T_1/T_2} \right] V$$

$$P = T_1 \left[1 - \frac{1}{e^{a_0}} \right] V$$

$$P = T_1 \left[1 - \frac{1}{K} \right] V \quad \left[\because K = \frac{T_1}{T_2} = e^{a_0} \right]$$

$$\boxed{P = T_1 \cdot K' V}$$

$$K' = \left(1 - \frac{1}{K} \right)$$

When $T_c = 0$

$$\boxed{P = T_{\max} \cdot K' V} \rightarrow (i)$$

$$T_c \neq 0 \Rightarrow \boxed{P = (T_{\max} - T_c) K' V} \rightarrow (ii)$$

from the above two eqⁿ we can conclude that presence of centrifugal tension power transmission capacity of belt drive decreases and hence with respect PTC of a belt drive centrifugal tension is harmful.

α = coefficient of change in length belt / unit force

$$(T_1 - T_0)\alpha = (T_0 - T_2)\alpha$$

$$\boxed{T_0 = \frac{T_1 + T_2}{2}} *$$

considering centrifugal Tension

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

Design procedure used in flat Belts

1) $VR = \frac{N_2}{N_1}$

2) Determination of Dia. of pulleys [either D_1 or D_2]

$$\frac{N_2}{N_1} = \left(\frac{D_1 + t}{D_2 + t} \right) \left[1 - \frac{S}{100} \right]$$

D_1 or D_2 can be determined

3) Thickness of belt

$$(S_b)_{\max} \leq (S_t)_{\text{per}}$$

$$\frac{Et}{D_1} \leq (S_t)_{\text{per}}$$

$$t \leq \text{--- mm}$$

4) belt velocity (v)

(a) In the absence of slip

$$v = v_1 \text{ or } v_2 = \frac{\pi (D_1 + t) N_1}{60 \times 1000} \text{ or } \frac{\pi (D_2 + t) N_2}{60 \times 1000} = \text{--- m/s}$$

i) In presence of slip

$$V = V_1 \left[1 - \frac{S_1}{100} \right] \text{ or } V_2 = V \left[1 - \frac{S_2}{100} \right]$$

$$V = \text{---} \text{ m/s}$$

(32)

$$T_{\max} = \underbrace{\sigma_{pa}}_{\substack{\text{N/mm}^2 \\ \text{m/ps}}} \cdot \underbrace{b}_{\text{mm}} \cdot \underbrace{t}_{\text{mm}} = \text{---} \text{ N} = b \text{ in N}$$

$$m = \underbrace{\rho}_{\text{kg/m}^3} \cdot \underbrace{b}_{1000} \cdot \underbrace{t}_{1000} \cdot 1 \text{ m} = \text{---} b \text{ in kg/m}$$

$$T_c = mv^2 = \text{---} b \text{ in N}$$

$$T_1 = T_{\max} - T_c = \text{---} b \text{ in N}$$

$$T_2 = \text{---}$$

$$\frac{T_1}{T_2} = e^{(\theta_1)_{\min} \text{ rad}}$$

$$\text{ABD, } \theta_1 = \pi - 2 \left[\sin^{-1} \left(\frac{D_2 - D_1}{2c} \right) \times \frac{\pi}{180} \right]$$

$$\theta_1 = \text{---} \text{ radian}$$

$$\theta_2 = 2\pi - \theta_1 = \text{---} \text{ rad}$$

$$L_1 \theta_1 = \text{---}$$

$$L_2 \theta_2 = \text{---}$$

$$(\theta)_{\min} = \min \text{ of } [L_1 \theta_1 \text{ \& } L_2 \theta_2]$$

In CBD

$$\theta_1 = \theta_2 = \pi + 2 \left[\sin^{-1} \left(\frac{D_2 + D_1}{2C} \right) \times \frac{\pi}{180} \right]$$

$$l_{11} \theta_1 = \text{---}$$

$$l_{p2} \theta_2 = \text{---}$$

$$(l_{10})_{\min} = \theta_1 \text{ or } \theta_2 \times [\min \text{ of } l_{11} \text{ and } l_{p2}]$$

$$\frac{T_1}{T_2} = e^{(l_{10})_{\min}} = K$$

(33)

$$\therefore T_2 = \frac{T_1}{K} = \text{---} \text{ bin N}$$

(10) b -

$$p = (T_1 - T_2) V$$

in watt $p = (\text{---} b) V$

$$b = 991 \text{ mm, can be calculated}$$

$$b = 100 \text{ mm}$$

(11)

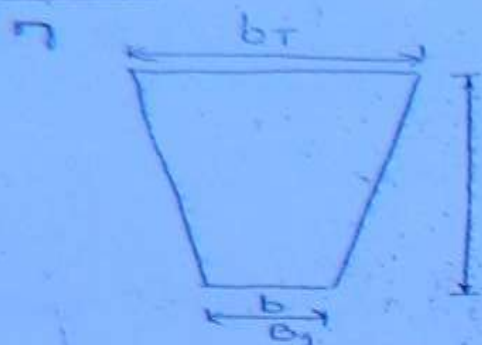
$$L_{OBD} = 2C + \frac{\pi}{2} (D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C} = \text{--- mm}$$

$$L_{CBD} = 2C + \frac{\pi}{2} (D_1 + D_2) + \frac{(D_2 + D_1)^2}{4C} = \text{--- mm}$$

S02

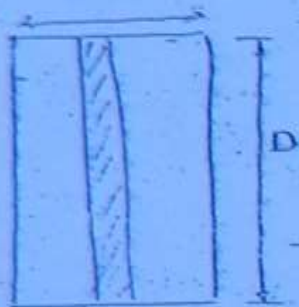
A Tra

V-BELTS

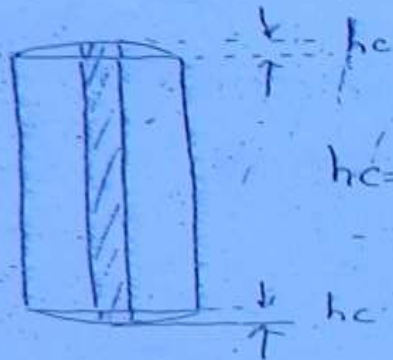


(34)

ROWNING

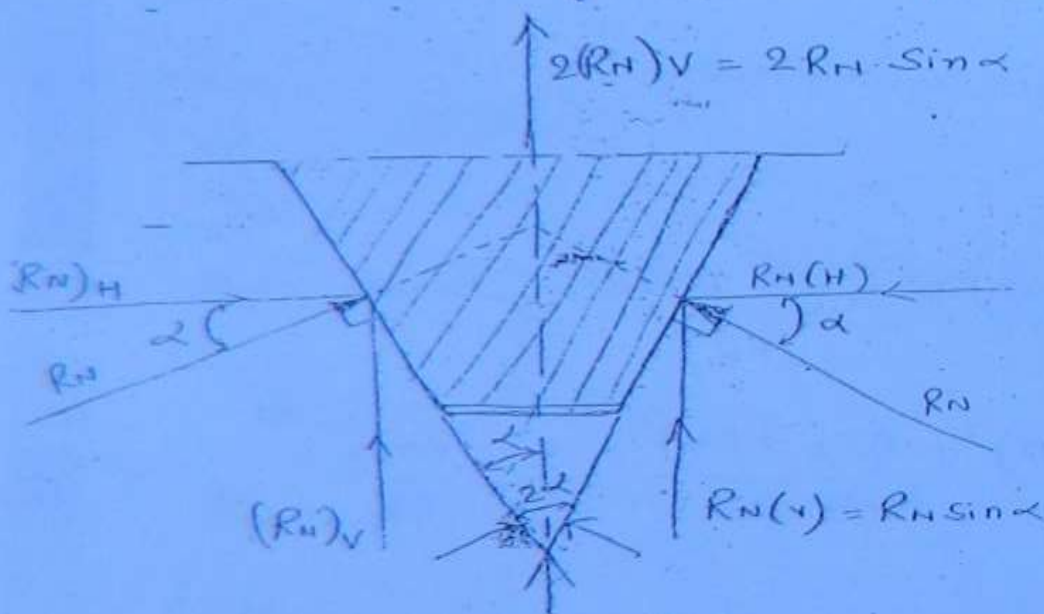


flat pulley



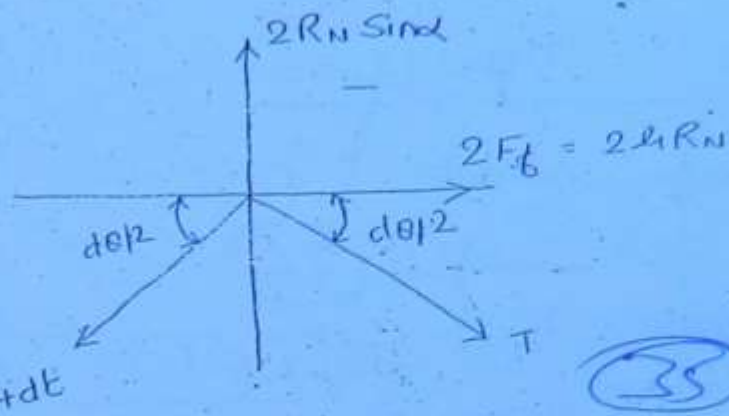
crown pulley

hc = crownhead or crown height



2α = groove angle
 = 36° to 42°
 = 40°

α = semi groove angle



$$\sum H = 0, \quad \sum V = 0$$

We get, $\boxed{\frac{T_1}{T_2} = e^{\frac{\mu \theta}{\sin \alpha}}}$ **

$$\alpha \approx 20^\circ \quad \sin 20^\circ < 1 \therefore \frac{\mu \theta}{\sin \alpha} > \mu \theta$$

$$\Rightarrow \left(\frac{T_1}{T_2} \right)_{V\text{Belt}} > \left(\frac{T_1}{T_2} \right)_{\text{flat belt}}$$

$$\Rightarrow PTC_{V\text{Belt}} > PTC_{\text{flat belt}}$$

$$\boxed{P_{\text{design}} = P_T \times K_a}$$

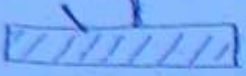
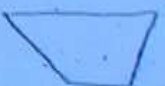
K_a = overload / service factor

P_T = power to be Transmitted

$\Rightarrow$ In case of Multiple V-belts even if a single belt gets damaged entire set of the V-belts has to be replaced by a complete new set of V-belts to ensure uniform tension in all the belts.

$$\text{No. of 'v' belts} = (n)$$

$$\boxed{n = \frac{P_{\text{Total}}}{P_{\text{each}}}}$$

Parameter	FLAT BELTS	V-BELTS
Centre distance	Medium	Short
Cross section		 (36)
T_1/T_2	less $T_1/T_2 = e^{\mu \theta}$	More $e^{\mu \sin \alpha}$ $T_1/T_2 = e$
Slip	occurs	are
m	less	more
Cost	less	more
pulley	flat pulley	grooved pulley
idler pulley		
No. of belt	one or Two	Multiple v belts
Noise	More, because of joint	Less, (endless belt) No joint

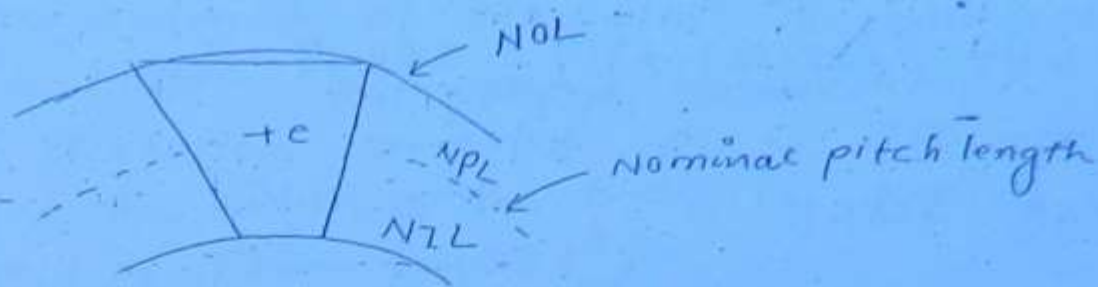
V Belt designation

B-3638 - Gr. 52

3 $\Rightarrow$ Type of v belt

638 $\Rightarrow$ nominal inside length (NIL)

52 $\Rightarrow$ Grade number
(oversized belt)



$$NPL = NIL + K$$

(37)

② B-3638 - Gr 52 → oversize

③ B-3638 - Gr 50 → Standard Size

④ B-3638 - Gr 46 → undersize

Manufacturing Length (ML) = NPL ⇒ std. size belt ⇒ Gr 50

ML > NPL ⇒ oversize belt ⇒ Gr > 50

ML < NPL ⇒ undersize belt ⇒ Gr < 50

⇒ 1. Grade Number is deviation from standard size (50) is equal to 2.5 mm variation

$$ML = NPL \pm [\text{Difference in Grade No.} \times 2.5]$$

$$ML = (NIL + K) \pm [\text{Diff in Grade No} \times 2.5]$$

Type of V belt

	dimensions	PTC	Cost	K
A				36
B				43
C	Increase			56
D				79
				92

$$\frac{ML}{A(2)} = (3638 + 43) + [(52 - 50) \times 2.5] = 3681 + 5 = 3686$$

$$\frac{ML}{3} = 3638 + 43 = 3681$$

$$\frac{ML}{4} = (3638 + 43) - ((50 - 46) \times 2.5) = 3671$$

Calculation of no. of belts

(28)

$n = \text{no. of } v \text{ belts}$

$$n = \frac{P_{\text{Total}} \times K_a}{P_{\text{each}}}$$

$$P_{\text{each}} = (T_1 - T_2) \times V$$

$$n = \frac{P_{\text{Total}} \times K_a}{P_{\text{each}} \times K_b \times K_c}$$

$K_b = \text{arc of contact factor}$

$K_c = \text{length correction factor}$

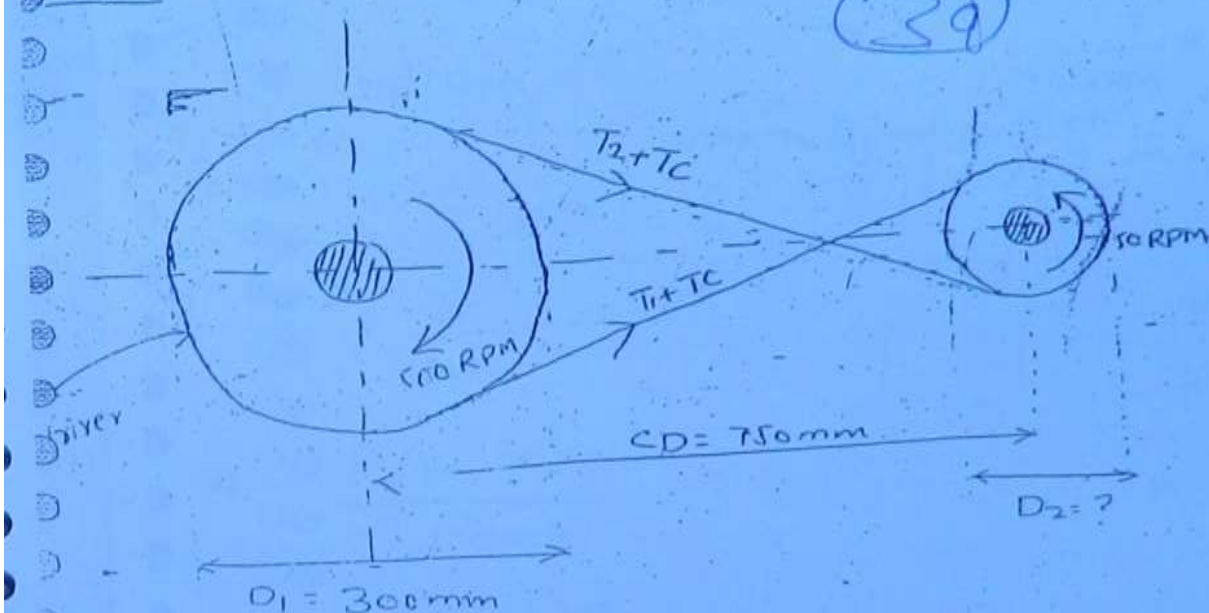
> P_{each} belt is calculated by taken $\theta = 180^\circ$ or π^c

11.

which has a density of 970 kg/m^3 , The allowable stress is 2 MPa . Two pulleys rotates in opposite direction and the CD is 750 mm . determine the width of the belt by taking coefficient of friction is 0.3 ?

Soln

(39)



$$(1) \quad VR = \frac{N_2}{N_1} = \frac{750}{500} = 1.5$$

$$(ii) \quad D_2 = ?$$

$$\frac{N_2}{N_1} = \left(\frac{D_1 + t}{D_2 + t} \right) \left[1 - \frac{s}{100} \right]$$

$$1.5 = \left[\frac{300 + 4.75}{D_2 + 4.75} \right] [1 - 0]$$

$$\therefore D_2 = 198.4 = 200 \text{ mm}$$

$$(3) \quad V = V_1 \text{ or } V_2$$

$$V = \frac{\pi (D_1 + t) N_1}{60 \times 1000} = \frac{\pi (300 + 4.75) \times 500}{60 \times 1000}$$

$$V = 7.98 \text{ m/s}$$

$$T_{max} = 6 p_{ex} \times b \times t$$

$$= 2 \times b \times 4.75$$

$$\therefore T_{max} = 9.5 b \text{ in } N$$

$$m = f \times \frac{b}{1000} \times \frac{t}{1000} \times 1 \text{ m}$$

(40)

$$= 970 \times \frac{4.75}{1000} \times \frac{t}{1000} \times 1$$

$$m = 4.607 \times 10^{-3} b \text{ in } \text{Kg/m}$$

$$T_c = m v^2$$

$$= 0.293 b \text{ in } \cdot N$$

$$T_1 = T_{max} - T_c$$

$$T_1 = 9.207 b \text{ in } N$$

$$\frac{T_1}{T_2} = \frac{(e_1 \theta)_{min}}{e}$$

$$\theta_1 = \theta_2 = \pi + 2 \left[\sin^{-1} \left(\frac{D_2 + D_1}{2c} \right) \times \frac{\pi}{180} \right]$$

$$\therefore \theta_1 = \theta_2 = 3.82 \text{ radians}$$

$$\frac{T_1}{T_2} = 3.15$$

$$T_2 = \frac{T_1}{3.15} = 2.933 b \text{ in } N$$

$$p = (T_1 - T_2) v$$

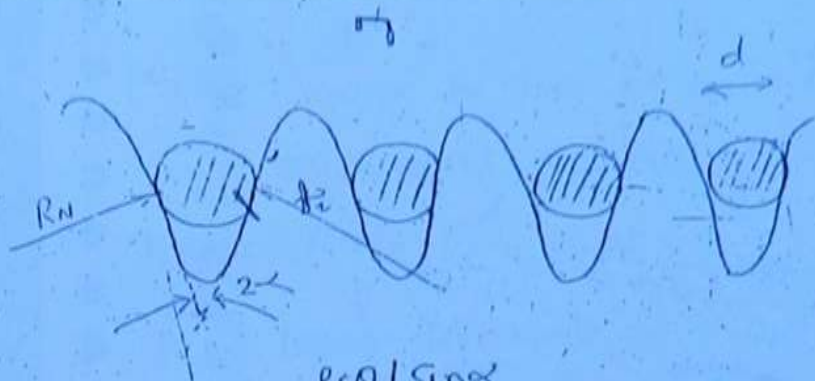
$$3.75 \times 10^3 = (9.207 b - 2.933 b) 7.98$$

$$\therefore b = 74.71 \text{ mm} \approx 75 \text{ mm}$$

$$L_{CBD} = 2c + \left(\frac{\pi}{2} (D_1 + D_2) \right) + \left(\frac{D_2 + D_1}{4c} \right)^2$$

=

DESIGN OF FIBRE ROPE



(41)

$$\Rightarrow \frac{T_1}{T_2} = e^{\frac{W}{T_2 \sin \alpha}} \Rightarrow T_{max} = 6 \text{ per} \times \frac{\pi}{4} d^2$$

$$\Rightarrow m = f \times \frac{\pi}{4} d^2 \times 1 \text{ m}$$

106

$$\Rightarrow \eta = \frac{P_{Total} \times K_a}{P_{each}}$$

$$\Rightarrow P_{each} = (T_1 - T_2) V$$

$$\Rightarrow T_1 = T_{max} - T_C$$

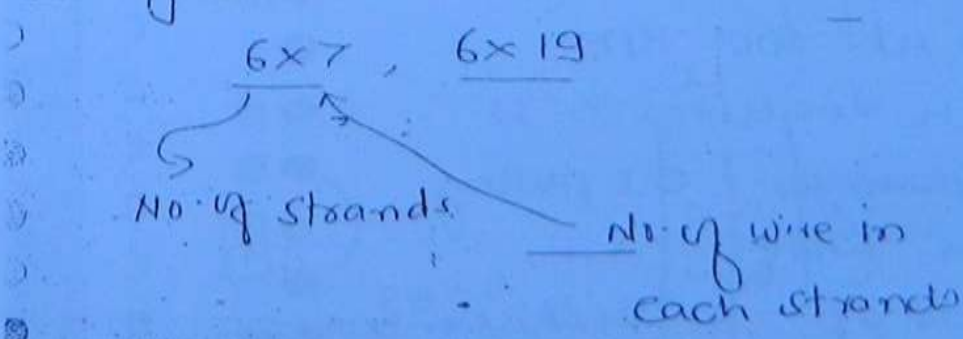
$$\Rightarrow T_C = mv^2$$

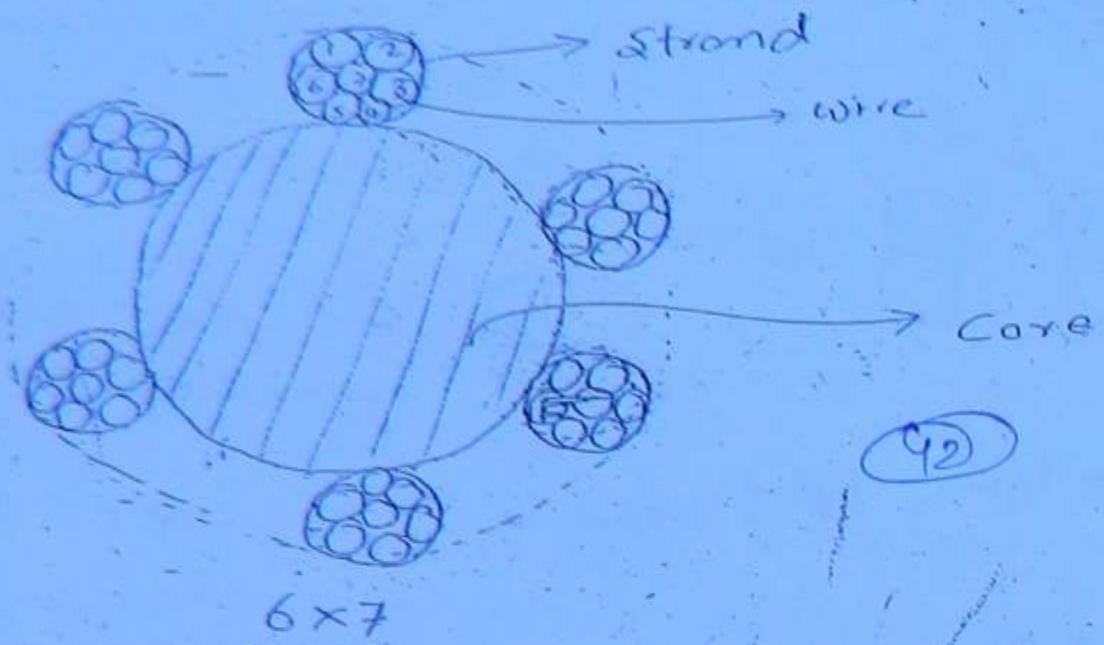
$$\Rightarrow T_2 = \frac{T_1}{(e^{\theta / \sin \alpha})}$$

WIRE ROPES

used in hoisting applications

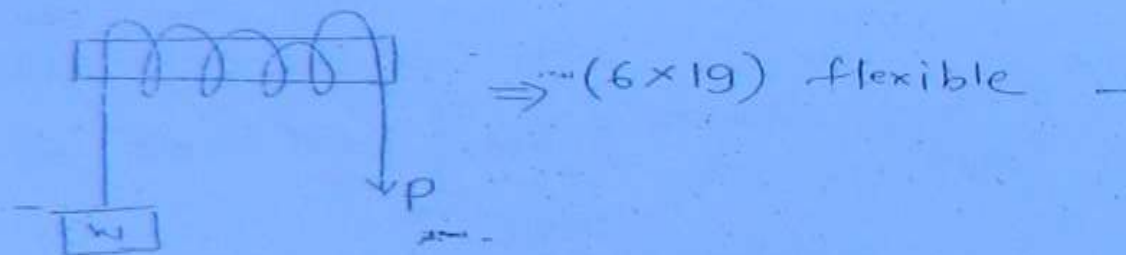
Designation





7 = Small number of large diameter wire
 → (high stiff)

9 = Large no. of small diameter wire
 → (more flexible)



A Transmission shaft rotating at 500 RPM
 is a milling machine which requires 3.75
 at 750 RPM, a 300 mm diameter CI pulley
 mounted on the Transmission shaft, an
 at design proposes, a belt of 4.75 thickness (mm)

IES-07

$$P_D = P_T \times K_a \times K_{FL}$$

P_T = Power to be transmitted

K_a = overload factor

K_{FL} = friction loss

Take less thickness (as bending stresses will be less)

$$\sigma_b = \frac{E t}{D}$$

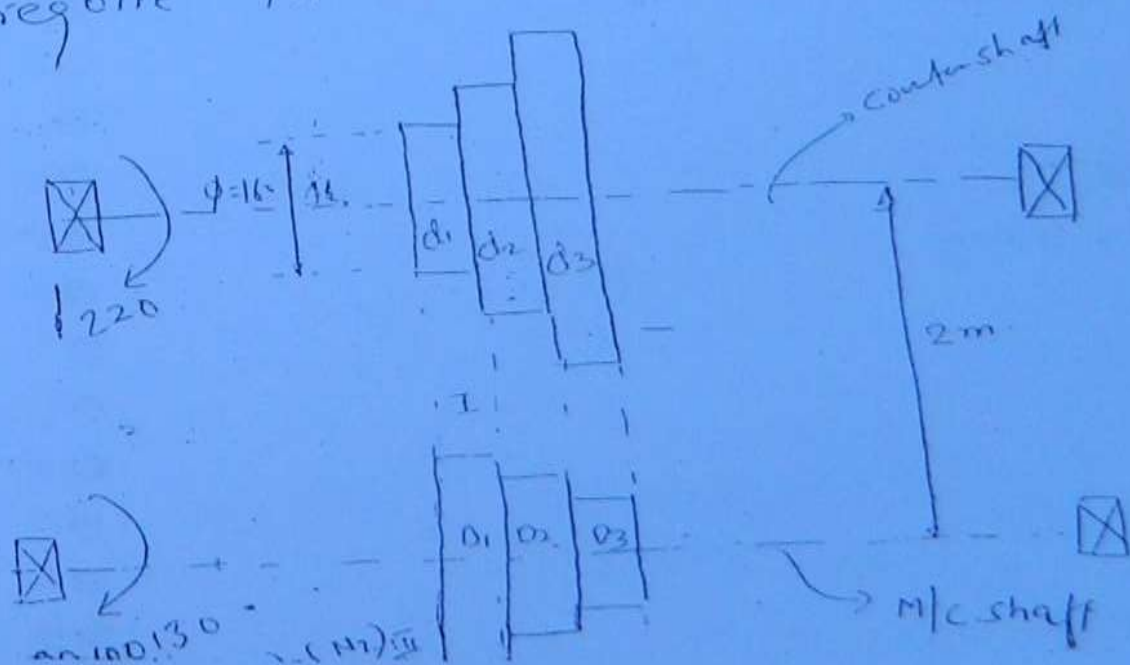
$$\text{IES-2007, } \left\{ \begin{array}{l} b = 323.3 \approx 325 \text{ mm} \\ L = 8.83 \text{ m} \end{array} \right\}$$

(43)

IES-01

Q.2 design a set of stepped pulley to drive a machine from a countershaft that runs at 220 RPM. The CD between 2 sets of pulleys is 2m the diameter of the smallest step of the countershaft is 160 mm. The m/c is to run at 80, 100 and 130 RPM and should be able to rotate in either direction. find the length of the belt require for both the cases?

80m



step

$$VR = \frac{(N_2)_I}{N_1} = \frac{d_1}{D_1}$$

$$\frac{80}{220} = \frac{160}{D_1} \quad \therefore D_1 = 440 \text{ mm}$$

$$L_{OBD} = L_I = L_{II} = L_{III}$$

$$= 2C + \frac{\pi}{2} (D_1 + d_1) + \frac{(D_1 - d_1)^2}{4C}$$

$$L_{OBD} = 4.952 \text{ m}$$

(44)

2nd step

$$VR = \frac{(N_2)_{II}}{N_1} = \frac{d_2}{D_2}$$

$$VR = \frac{100}{220} = \frac{d_2}{D_2}$$

$$\therefore D_2 = 2.2 d_2$$

$$L_{II} = 2C + \frac{\pi}{2} (D_2 + d_2) + \frac{(D_2 - d_2)^2}{4C} = 4.952 \text{ m}$$

$$\therefore d_2 = 189 \text{ mm}$$

$$D_2 = 417 \text{ mm}$$

3rd step

$$d_3 = 380 \text{ mm}$$

$$D_3 = 224.3 \text{ mm}$$

CREEP

When the belt is in running condition the tensions in the belt changes from T_1 to T_2 and T_2 to T_1 and so on, due to this varying tensions in the belt, the belt is subjected to ~~merit~~ ^{merit} extending and contractions, hence length ~~received~~ ^{received} by the belt ~~received~~ ^{received} and delivered by a pulley are unequal because of ~~length~~ ^{length} difference in length being ~~received~~ ^{received} and delivered by a pulley relative motion takes place between belt and pulley surfaces, this relative motion or length being ~~received~~ ^{received} and delivered by a pulley is called as a Creep.

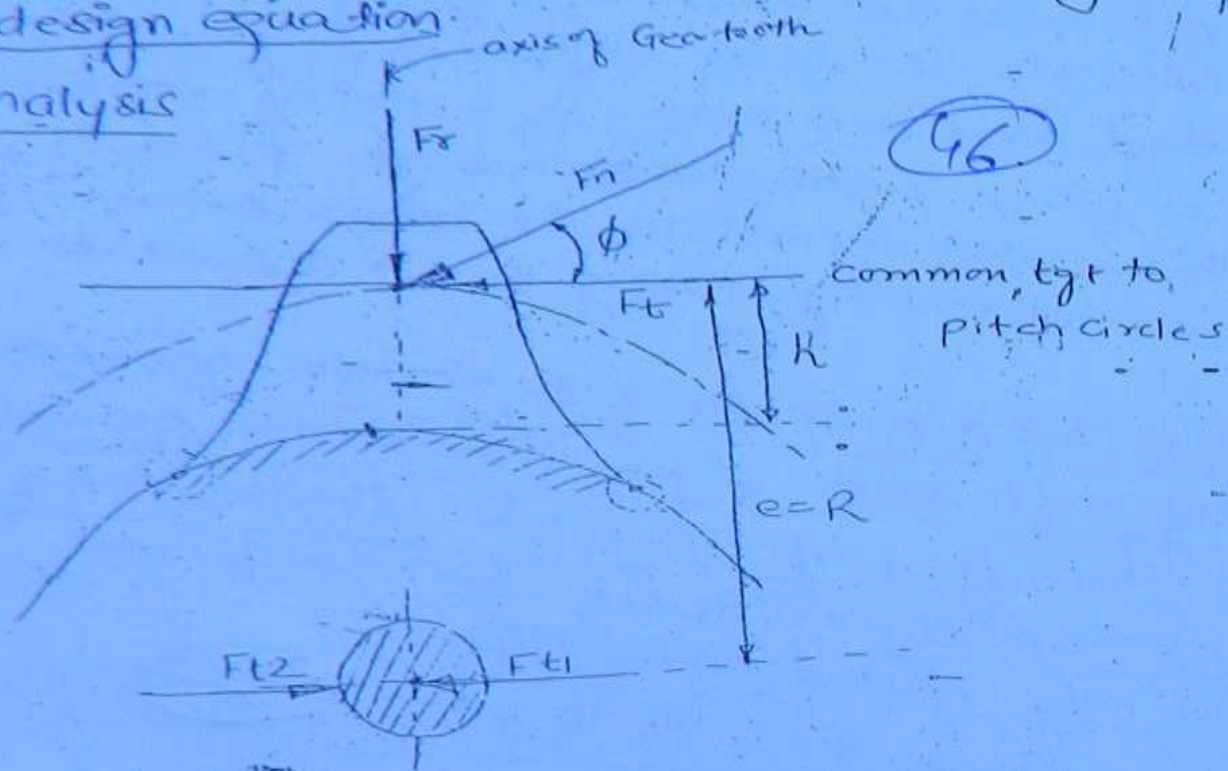
Due to creep speed of the follower decreases (i.e. velocity ratio and power transmission capacity reduces). The effect of creep is similar to the effect of creep slip. Hence the combined effect generally called as a slip generally it is $3\frac{1}{2}$ to 4%.

(45)

② DESIGN OF SPUR GEARS

- To determine the dimensions of a Gear Tooth & Gear (i.e., a , d , P_c , P_d , b , ^{backlash} ~~backlash~~, ϕ) clearance
- To determine the above dimensions module is required.
- To determine m $\Rightarrow$ Lewis or Beam strength eqn. or design equation.

FORCE ANALYSIS



$$F_r = F_n \cdot \sin \phi \Rightarrow F_r = F_t \tan \phi$$

$$F_t = F_n \cos \phi$$

$$\Rightarrow F_n = \frac{F_t}{\cos \phi}$$

For Gear, F_r is a axial compressive load

F_t is a TSL

$\Rightarrow$ due to $F_r \Rightarrow \sigma_{ac}$

$\Rightarrow$ due to $F_t \Rightarrow \sigma_b$ and τ_s

always $\cos \phi > \sin \phi$ [$\phi = 20^\circ, 14\frac{1}{2}^\circ$]

and hence effect of F_r is neglected

F_{ac} is neglected

Gear tooth is designed by considering bending stresses only

Shaft: F_r is TSL
 F_t is eccentric TSL

(47)

$$F_{t1} = F_{t2} = F_t$$

F_t and F_{t2} produces Twisting Moment (TM)

$$TM = F_t \cdot e = F_t \cdot R$$

Finally shaft is subjected to

⇒ (i) Twisting moment

⇒ (ii) BM in vertical plane

⇒ (iii) BM in horizontal plane

Design a GD, $P = x \text{ KW}$ at $y \text{ RPM}$? (default pinion RPM)

$$(i) \quad T_1 = \frac{P \times 60}{2\pi N_1} \times 10^6 \text{ N-mm}$$

$$(ii) \quad T = F_t \cdot R$$

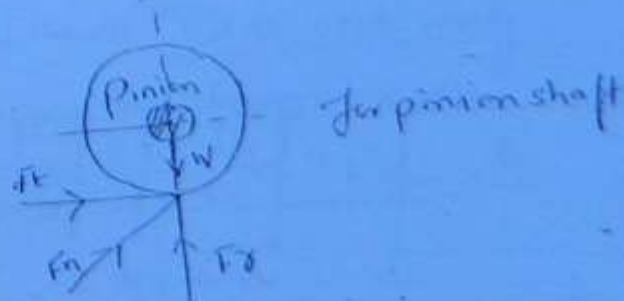
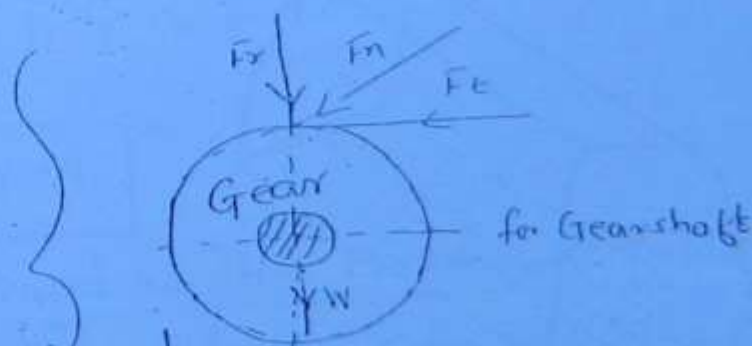
$$F_{t1} = \frac{T}{R} = \frac{2T}{D_1}$$

$$F_{t2} = \frac{2T_2}{D_2}$$

$$F_{t1} = F_{t2}$$

$$\frac{2T_1}{D_1} = \frac{2T_2}{D_2}$$

$$T_1 = \frac{D_1}{D_2} T_2$$



$$\frac{T_1}{T_2} = \frac{D_1}{D_2} = \frac{Z_1 m_f}{Z_2 m_2}$$

$\left\{ \begin{array}{l} Z_1, Z_2 = \text{Teeth on pinion} \\ \text{and Gear} \end{array} \right.$

$$\frac{T_1}{T_2} = \frac{D_1}{D_2} = \frac{Z_1}{Z_2}$$

Torque on Gear will be more as D is more

$$F_r = F_t \tan \phi$$

$$F_n = \sqrt{F_t^2 + F_r^2}$$

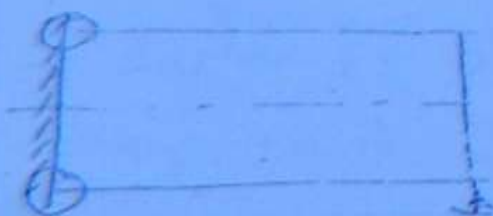
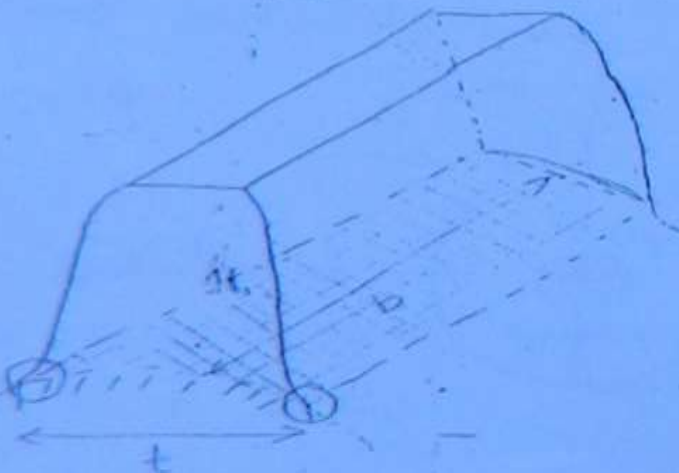
(48)

LEWIS (OR) BEAM STRENGTH EQUATION

Assumptions

- i) F_r is neglected
- ii) Cantilever beam is considered

$$\sigma_{b \max} = \frac{M_{\max}}{Z_{NA}} = \frac{F_t \times h}{bt^2/6}$$



$$\begin{aligned} Z_{NA} &= \frac{I_{NA}}{y_{\max}} \\ &= \frac{\frac{bt^3}{12}}{t/2} \end{aligned}$$

$$\sigma_b = \frac{6 F_t h}{b t^2}$$

$$F_t = \sigma_b \cdot b \cdot \frac{t^2}{6h} \quad \left\{ \text{tgt. force acts on Pitch point} \right\}$$

$$\left\{ \begin{array}{l} \omega \mid T \uparrow, F_t \uparrow \Rightarrow \sigma_b \uparrow (\text{limit}) \\ \sigma_b = [\sigma_b] \leftarrow \text{permissible stress} \\ \text{Then } F_t \Rightarrow F_{t \max} \end{array} \right\} *$$

$$(F_t)_{\max} = [\sigma_b] b \cdot Y \cdot m$$

Now $F_t = \sigma_b \cdot b \cdot \frac{t^2}{6h} \times \frac{m}{m}$

$$F_t = \sigma_b \cdot b \cdot \left(\frac{t^2}{6hm} \right) m$$

$$F_t = \sigma_b \cdot b \cdot Y \cdot m$$

$$Y = \frac{t^2}{6hm} = \text{Lewis form factor}$$

$$(F_t)_{\max} = [\sigma_b] \cdot b \cdot Y \cdot m \text{ or } F_s$$

$$F_s = [\sigma_b] \cdot b \cdot Y \cdot m$$

Beam strength is defined as the Maximum value of F_t that a given gear tooth can withstand without any bending failure (i.e. at any instant the load coming on the gear tooth should be less than or equal to F_s to avoid bending failure.

$$F_{S1} = 10 \text{ KN (Pinion)}$$

$$F_{S2} = 8 \text{ KN (Gear)}$$

Load coming on Gear tooth $\leq (F_S)_{\text{weaker gear}}$

Load coming on Gear tooth $\leq [\text{Min of } F_{S1} \text{ and } F_{S2}]$

Load coming on Gear tooth $\leq 8 \text{ KN}$

weaker Gear: (G.G)

$$Y_1 < Y_2$$

$$\Rightarrow Y_2 > Y_1 \quad [\because Z_2 > Z_1]$$

It is a gear which has minimum value of σ , always we have to design with respect to a weaker Gear.

When the Gear and pinion are made up of same material we have to design w.r.t to pinion because pinion is the weaker Gear.

$$[\sigma b_1] = [\sigma b_2], b_1 = b_2, m_1 = m_2 \text{ but } Y_1 < Y_2]$$

$$\therefore F_{S1} < F_{S2}$$

When Gears and pinion are made of different material we have to design with respect to gear which has minimum value for the product of $[\sigma b]$ and Y .

Assumptions Made in Lewis equation

Effect of F_r (i.e., axial compressive stresses) are neglected.

2) Each gear tooth is treated as a cantilever beam fixed at the root position and free at the tip of the tooth.

3) Effect of stress concentration at the root of the Gear tooth is neglected.

$\Rightarrow \sigma_{max} = K_t \text{ or } K_f [\sigma_n]$ (σ_n = nominal stress)
 by using some equations

K_t = theoretical stress concentration factor

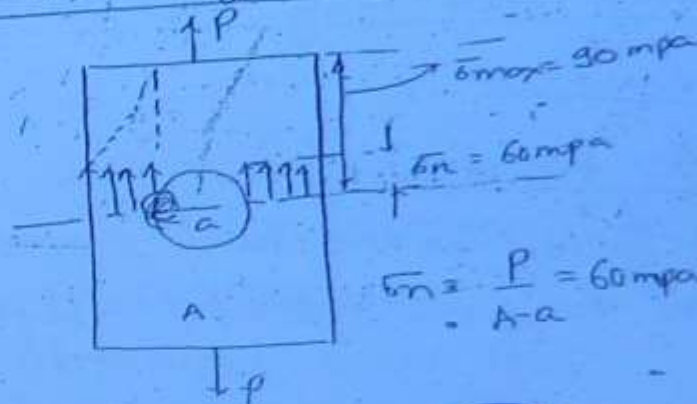
$$K_t = \frac{\sigma_{max}}{\sigma_n}$$

= Max stress near the discontinuity

Nominal stress

$$K_t = \frac{90}{60} = 1.5$$

used in static loading



$$\sigma_n = \frac{P}{A-a} = 60 \text{ mpa}$$

(5)

$$K_f = 1 + q (K_t - 1)$$

K_f = fatigue stress concentration factor

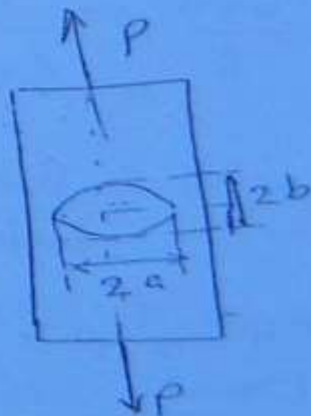
q = notch sensitivity factor Index

for elliptical hole:-

$$K_t = 1 + 2 \left(\frac{a}{b} \right)$$

a = Semi major axis

b = Semi minor axis



for Circular hole $\Rightarrow (K_t)_{max} = 3$

actual values lies between 1 to 3

Because of effect of stress concentration in static loading is less serious than fatigue loading.

i) Effect of errors in tooth profiles and tooth spacing are neglected

Effect of Manufacturing errors.

(52)

ii) contact ratio is assumed as 1. (i.e., it is assumed that only one pair of Gear tooth is in contact)

Dynamic Load (F_d)

- It is defined as the load coming on the gear tooth any instant under dynamic load

$$(F_d)_{\text{Lewis}} = F_t \cdot C_v \quad \text{or} \quad \frac{F_t}{C_v}$$

C_v = velocity factor

$$C_v = \frac{3+v}{3} \quad \text{or} \quad \frac{3}{3+v} \quad [v \leq 10 \text{ m/s}]$$

v = pitch line velocity

$$C_v = \frac{6+v}{6} \quad \text{or} \quad \frac{6}{6+v} \quad [10 \leq v \leq 20 \text{ m/s}]$$

$$v = v_1 \text{ or } v_2 = \frac{\pi D_1 N_1}{60 \times 1000} \quad \text{or} \quad \frac{\pi D_2 N_2}{60 \times 1000}$$

$$F_t = \frac{2T_1}{D_1} \quad \text{or} \quad \frac{2T_2}{D_2}$$

To avoid bend failure

$$(F_d)_L \leq (F_s)_{\text{weaker Gear}}$$

Buckingham Dynamic Load (F_d)

(53)

$$F_d = F_t + F_i$$

$$F_d = F_t + \frac{20.67 \sqrt{[bc + F_t]}}{20.67 \sqrt{1 + [bc + F_t]}}$$

b = face width

c = constant

$$c = \frac{e}{K \left[\frac{1}{E_1} + \frac{1}{E_2} \right]}$$

e = error in tooth action in mm

K = constant

Values of e & K are obtained from the tables of design data book.

$F_d \leq F_s \Rightarrow$ Design is safe w.r.t to bending

Reasons for dynamic Load

- 1) Deflection of tooth under load.
- 2) Inaccuracies in tooth profile.
- 3) Error in tooth spacing.
- 4) Misalignment between bearings.
- 5) Inertia of reciprocating parts.

WEAR STRENGTH : $[F_w]$

it is always calculated w.r.t to pinion because pinion is subject to more wear than gear

$$[\because N_1 > N_2]$$

$$F_w = D_1 \cdot Q \cdot K \cdot b \text{ in } N$$

$$Q = \frac{2G}{G \pm 1}$$

(54)

$$\Rightarrow G = \text{Gear Ratio} = \frac{N_1}{N_2} \quad \{\text{always more than 1}\}$$

$\Rightarrow + \Rightarrow$ for External Gears

$\Rightarrow - \Rightarrow$ for Internal Gears

$b =$ face width

$$K = \text{constant} = \frac{(\sigma_{es})^2 \sin \phi \left[\frac{1}{E_1} + \frac{1}{E_2} \right]}{1.4}$$

$\sigma_{es} =$ surface endurance limit or
Surface fatigue limit

$\phi =$ pressure angle

When $F_d \leq F_w \Rightarrow$ No wear failure
for safe designing gear

$$F_w > F_s > F_d$$

$F_w > F_s$ generally.

DESIGN PROCEDURE USED IN SPUR GEAR

data: $P = x \text{ kW at } Y \text{ RPM}$

$G =$ Given [Gear ratio]

G or N_2 will be given

ϕ ; $[\sigma_{b1}]$, $[\sigma_{b2}]$, σ_{es} ,

$$(1) \quad G = \frac{N_1}{N_2} = \frac{\text{Speed of pinion}}{\text{Speed of Gear}} = ?$$

$$(2) \quad G = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{Z_2}{Z_1} \quad \text{SS}$$

$$Z_2 = G \cdot Z_1$$

Imp. (3) $Z_1 \geq (Z_1)_{\min}$

$(Z_1)_{\min}$ = minimum no. of teeth provided on the pinion to avoid interference.

$$(Z_1)_{\min} = \frac{2 \cdot a_w}{\sin^2 \phi}$$

a_w = addendum coefficient

$$a_w \times m = a$$

$$\Rightarrow \text{For full depth tooth} \Rightarrow \underline{a = m \Rightarrow a_w = 1}$$

$$\Rightarrow \text{for stub tooth} \Rightarrow \underline{a = 0.8m \Rightarrow a_w = 0.8}$$

$$\Rightarrow \text{for } \phi = 20^\circ \text{ (Full depth)}$$

$$(Z_1)_{\min} = \frac{2 \times 1}{\sin^2 20^\circ} = 17.09$$

$$Z_1 = 18$$

$$\Rightarrow \text{for } \phi = 20^\circ \text{ (Stub teeth)}$$

$$Z_1 \min = \frac{2 \times 0.8}{\sin^2 20^\circ} = 13.67$$

$$Z_1 = 14$$

$$Z_2 = G \cdot Z_1$$

$$T_1 = \frac{P \times 60}{2\pi N_1} \times 10^6 = \text{--- N-mm}$$

$$[T_1] = \text{design torque} = T_1 \cdot K_a = \text{--- N-mm}$$

K_a = overload / service factor

$$K_a = 1.25$$

(56)

(Module:-)

$$m = 1.26 \sqrt[3]{\frac{[T_1]}{([S_b] \cdot Y)_{w.G} \cdot \psi \cdot Z_1}}$$

$$\psi = \frac{b}{m} = 10 \Rightarrow \boxed{b = 10 \text{ mm}}$$

$$8 \leq \psi \leq 12$$

$$F_s = (F_t)_{\max} = ([S_b] \cdot Y)_{w.G} \cdot b \cdot m$$

$$\frac{2[T_1]}{D_1} = ([S_b] \cdot Y)_{w.G} \cdot \psi \cdot m$$

$$\frac{2[T_1]}{m \cdot Z_1} = ([S_b] \cdot Y)_{w.G} \cdot \psi \cdot m^2$$

$$2[T_1] = ([S_b] \cdot Y)_{w.G} \cdot \psi \cdot m^3 \cdot Z_1$$

$$m^3 = \frac{2[T_1]}{([S_b] \cdot Y)_{w.G} \cdot \psi \cdot Z_1}$$

$$m = \sqrt[3]{\frac{2[T_1]}{([S_b] \cdot Y)_{w.G} \cdot \psi \cdot Z_1}}$$

$$m = 1.26 \sqrt{\frac{[T_1]}{([S_b]Y)_{wg} \psi Z_1}}$$

⑥ Dimension of Gear tooth

$$D_1 = m \cdot Z_1$$

$$D_2 = m \cdot Z_2$$

$$C = \frac{D_1 + D_2}{2} = \frac{m}{2} [Z_1 + Z_2]$$

$$a = m$$

$$b = \psi m = 10m$$

$$d = 1157m$$

$$c = d \cdot m \cdot a$$

(57)

Q. A pair of spur gears having $14\frac{1}{2}^\circ$ involute full depth teeth is to transmit 12 kW at 300 RPM of the pinion the Gear ratio is 3:1 the static strength of CI Gear and steel pinion are 60 MPa and 105 MPa respectively design the Gear pair. checks for dynamic strength and wear strength by assuming following data

$$C_v \text{ is } C_v = \frac{4.5}{4.5 + v}, \quad \sigma_{el} = 600 \text{ MPa}, \quad E_{ms} = 200 \text{ GPa}$$

$$E_{CI} = 100 \text{ GPa}$$

Data : $\phi = 14\frac{1}{2}^\circ$, $P = 12 \text{ kW}$

$$N_1 = 300 \text{ RPM}, \quad G = 3:1$$

$$\sigma_{b1} = 105 \text{ MPa}, \quad \sigma_{b2} = 60 \text{ MPa}$$

$$G = \frac{N_1}{N_2} = \frac{Z_2}{Z_1} = 3$$

$$Z_2 = 3Z_1$$

$$z_1 \geq (z_1)_{\min}$$

$$(z_1)_{\min} = \frac{2a_w}{\sin^2 \phi}$$

for full depth pinion $a_w = 1$

$$(z_1)_{\min} = \frac{2 \times 1}{\sin^2 14.72} = 31.9$$

$$z_1 \geq 31.9$$

$$z_1 = 32, z_2 = 3 \times z_1 = 96$$

(58)

T_1 = torque to be transmitted by the pinion

$$T_1 = \frac{P \times 60}{2\pi N} \times 10^6$$

$$= \frac{12 \times 60}{2\pi \times 300} \times 10^6 = 381.97 \times 10^3 \text{ N-mm}$$

$$\text{Design Torque} = [T_1]$$

$$[T_1] = T_1 \times k_a$$

assuming overload as 25%

$$\therefore k_a = 1.25$$

$$[T_1] = 381.97 \times 10^3 \times 1.25$$

$$[T_1] = 477.46 \times 10^3 \text{ N-mm}$$

$$m \geq 1.26 \sqrt[3]{\frac{[T_1]}{([Sb_1] Y_1) \psi z_1}}$$

$$[Sb_1] Y_1 \text{ \& } [Sb_2] Y_2 = ?$$

$$\phi = 14.72^\circ, y = 0.124 - \frac{0.634}{z}$$

$$\phi = 20^\circ (\text{FD})$$

$$y = 0.154 - \frac{0.912}{Z}$$

$$\phi = 20^\circ (\text{Stob})$$

$$y = 0.175 - \frac{0.844}{Z}$$

$$y_1 = 0.124 - \frac{0.684}{Z_1} = 0.10265$$

(59)

$$y_2 = 0.124 - \frac{0.684}{Z_2} = 0.1168$$

$$Y_1 = \pi y_1 = \pi \times 0.10265 = 0.322$$

$$Y_2 = \pi y_2 = \pi \times 0.1168 = 0.367$$

$$[\sigma_b] Y_1 = 33.85 \text{ MPa}$$

$$[\sigma_b] Y_2 = 22.03 \text{ MPa}$$

$$[\sigma_b] Y_2 < [\sigma_b] Y_1$$

$\Rightarrow$ Gear is weaker

Hence we have to design w.r.t Gear

$$([\sigma_b] Y)_{\text{wg}} = [\sigma_b] Y_2 = 22.03 \text{ MPa}$$

$$\psi = \frac{b}{m} = 8 \text{ to } 12$$

$$b = 10 \text{ mm}$$

$$m \geq 126 \sqrt{\frac{4.7746 \times 10^3}{22.03 \times 10 \times 32}}$$

$$m \geq 5.136$$

$$\underline{m = 6 \text{ mm}}$$

1) Dimension of Gear pair

$$D_1 = m Z_1 = 192 \text{ mm}$$

$$D_2 = m Z_2 = 576 \text{ mm}$$

$$C = \frac{m}{2} [Z_1 + Z_2] = 384 \text{ mm}$$

$$b = 10m = 60 \text{ mm}$$

$$P_c = \pi \phi m = 18.85 \text{ mm}$$

$$a = m = 6 \text{ mm}$$

$$d = 1.157 m = 6.94 \text{ mm}$$

66

2) Beam strength

$$(F_s)_{WG} = F_s ([\sigma_b] Y)_{W_r} b m$$

$$= 22.03 \times 60 \times 6$$

$$= 13218 \text{ N} \quad 7930.8 \text{ N}$$

3) Check for dynamic load wrt to bending failure

$$(F_d)_L = F_d \times C_v^{2m} \quad \text{or} \quad \frac{F_t}{C_v}$$

$$C_v = \frac{4.5}{4.5 + V} = 0.5987$$

$$V = \frac{\pi D_1 N_1}{60 \times 1000} = \frac{\pi \times 192 \times 300}{60 \times 1000} = 3.016 \text{ m/s}$$

$$F_t = \frac{2 T_1}{D_1} \quad \text{or} \quad \frac{2 T_2}{D_2}$$

$$F_t = 4.97 \times 10^3 \text{ N} \quad 3978.85 \text{ N}$$

$$(F_d)_L = \frac{F_t}{C_v} = \frac{4.97 \times 10^3}{0.5987} = 6645.8 \text{ N}$$

Since $(F_d)_L < F_s$

(61)

design of Gear pair is safe with respect to bending failure

(9) check for wear failure or wear strength

wear strength is always calculate for pinion as pinion is subjected to more wear

$$F_w = D_1 \cdot Q \cdot K_b$$

$$Q = \frac{2G}{G+1} = \frac{2 \times 3}{3+1} = 1.5$$

$$K = \frac{C_{es}^2 \sin \phi}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2} \right] = \frac{(600)^2 \times \sin 14.5}{1.4} \left[\frac{1}{200 \times 10^9} + \frac{1}{200 \times 10^9} \right]$$

$$\therefore K = 161 \times 10^3 \times 0.965 = K$$

$$F_w = 192 \times 1.5 \times 0.965 \times 60$$

$$F_w = 16675.2 \text{ N}$$

$F_w > F_d \Rightarrow$ design is safe for Gear pair w.r.t to wear failure

③ DESIGN OF SHAFTS

axle: subjected to only bending

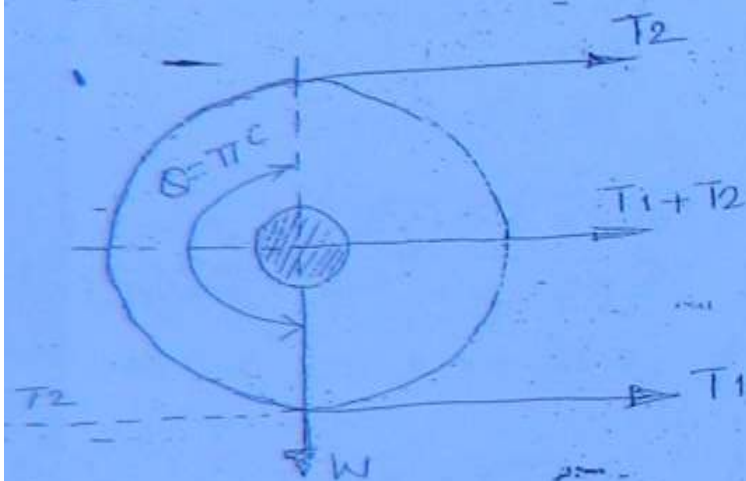
spindle: a short rotating shaft which supports tool or w/p

$$(1) \quad T = \frac{\overset{\text{KW}}{P} \times 60 \times 10^6}{2\pi \underset{\text{RPM}}{TFN}} = \frac{Z}{N} \text{ Nmm}$$

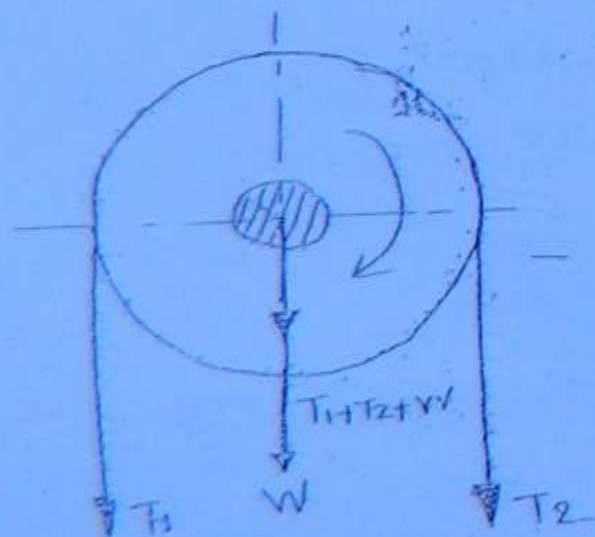
②

② shafts with pulleys:

(a) Horizontal belt drive



(b) Vertical belt drive



$T = (T_1 - T_2) R \rightarrow \text{V. Imp.}$

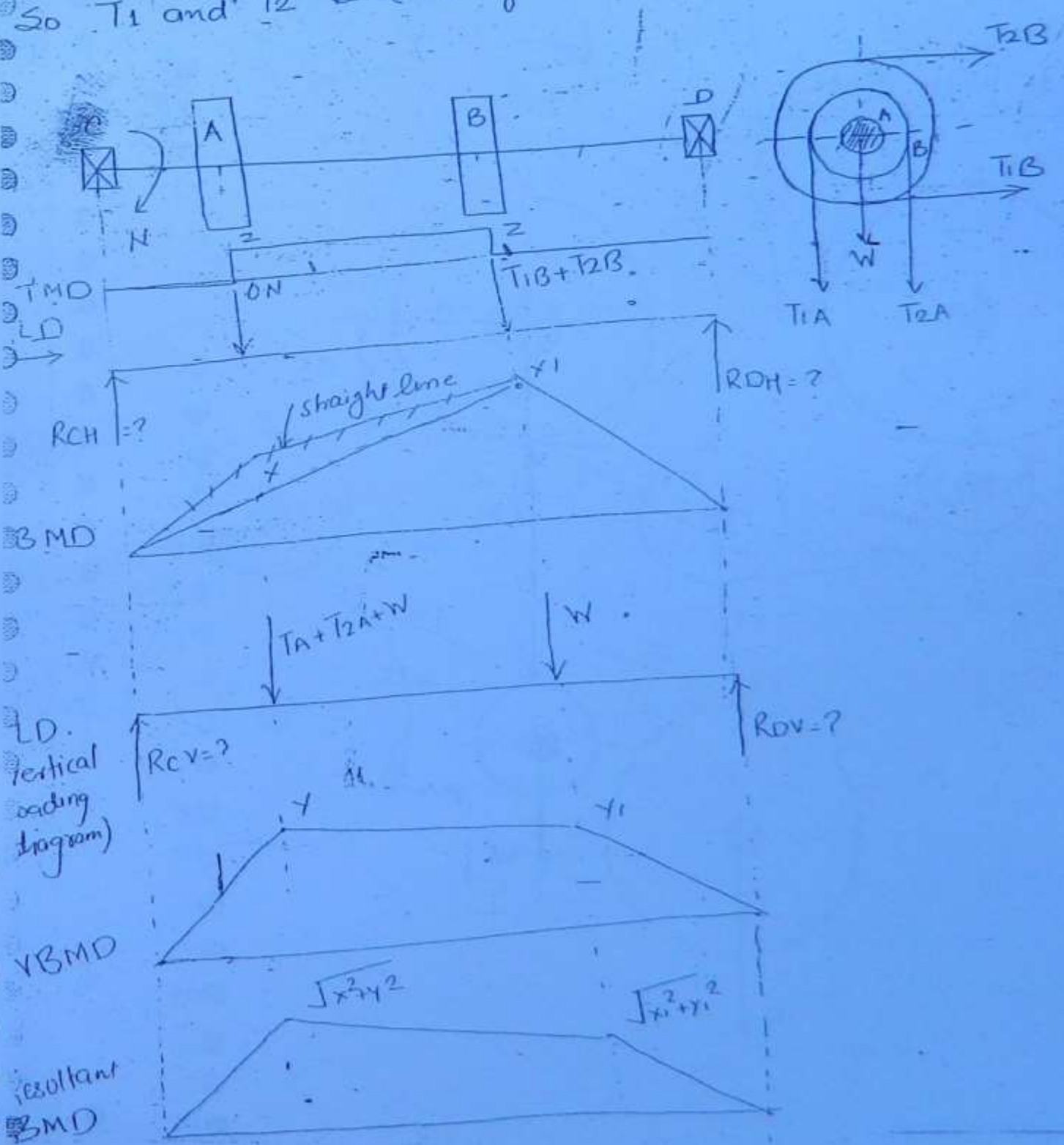
$T_1 - T_2 = \frac{?}{?} \text{ (I)}$

$\frac{T_1}{T_2} = e = v \text{ (II)}$

$T_{\max} = 5 \text{ per b.t.} = \text{--- (III)}$

63

So T_1 and T_2 can be found out.



Design of shafts

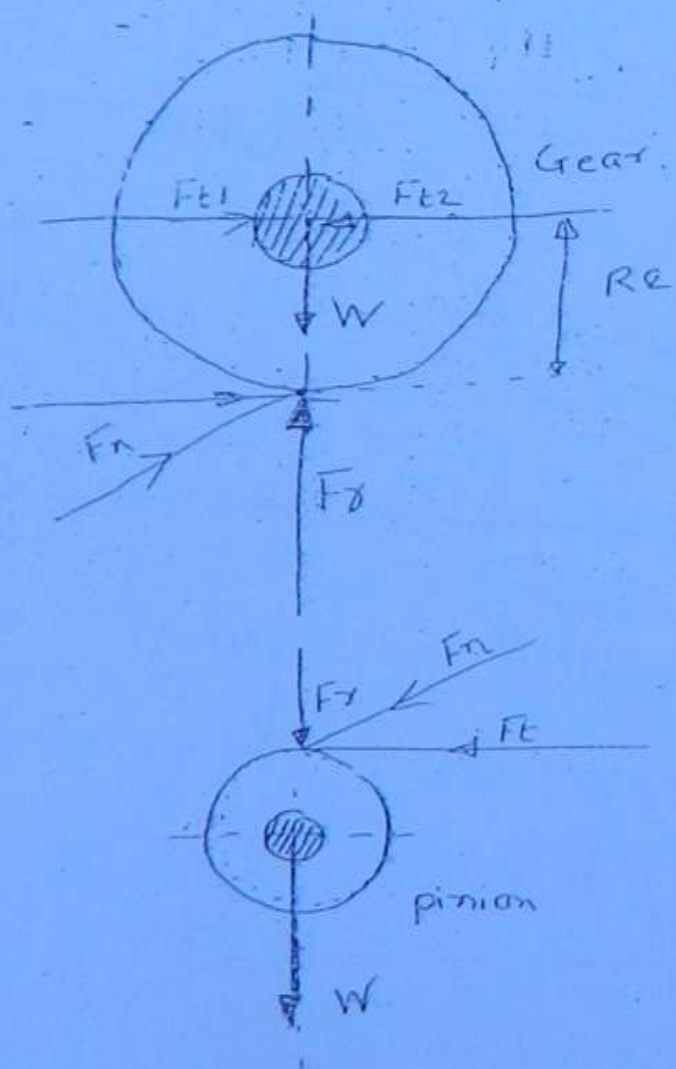
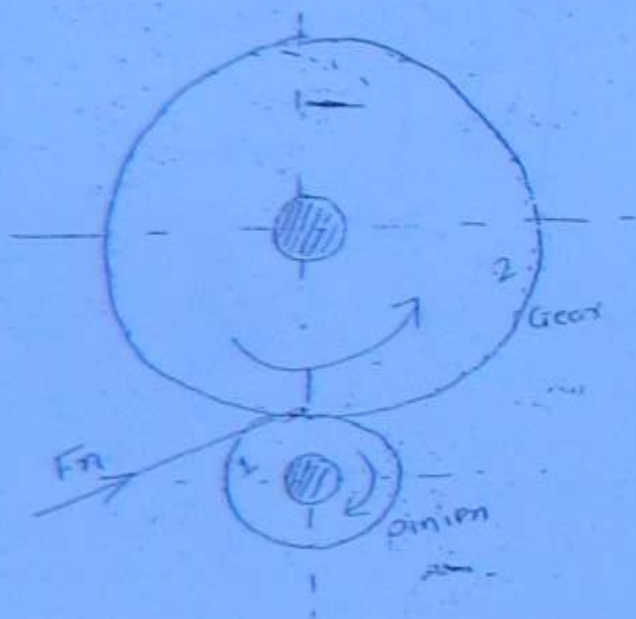
as shaft is subjected to both BM and TM we have to go for Theories of failure

$$T_e = \sqrt{(M_R)_A^2 + T_A^2} = \frac{\pi}{16} d^3 \tau_s$$

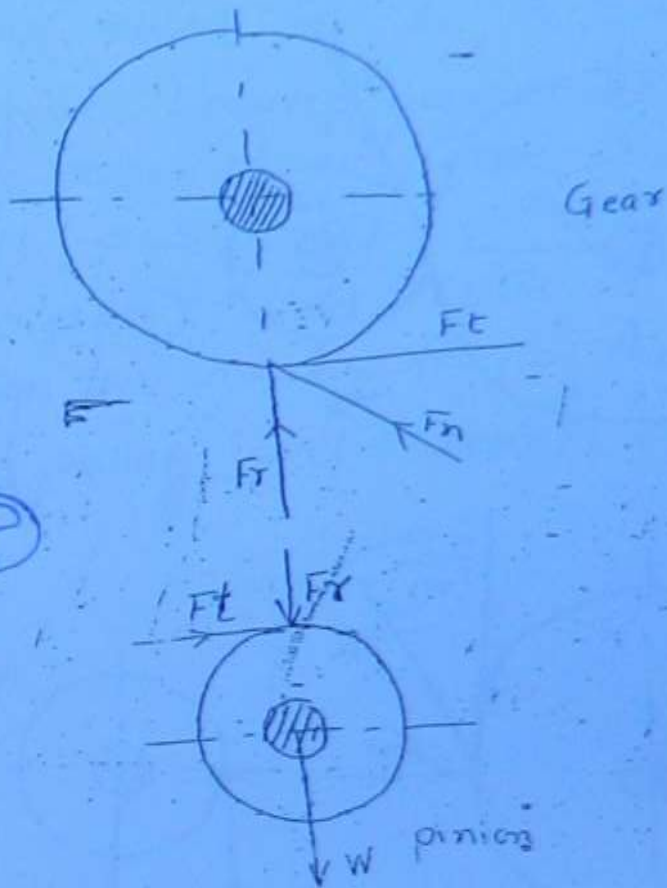
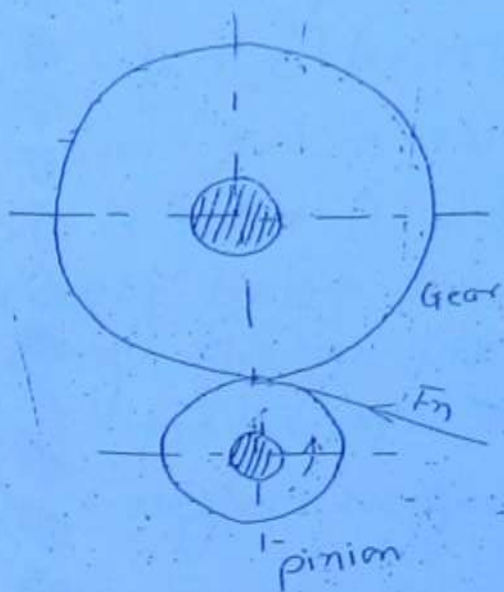
$d = ?$

(64)

Shafts with Gears

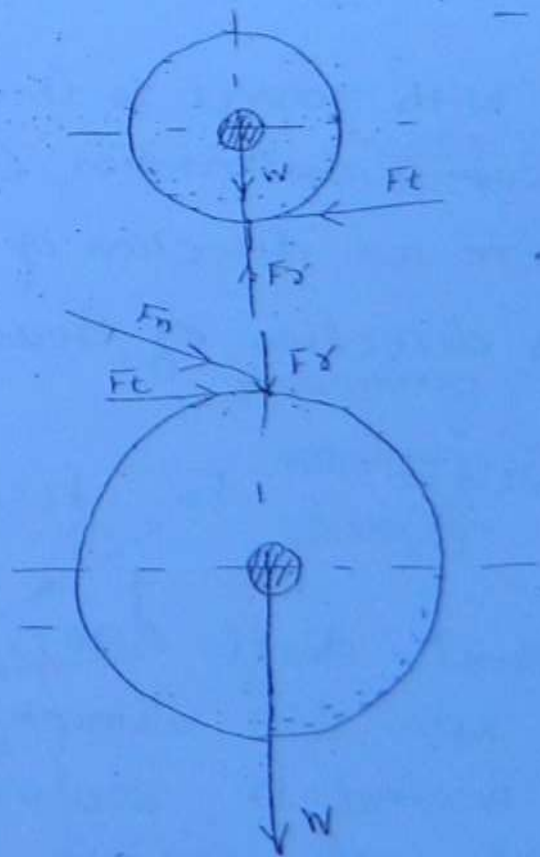
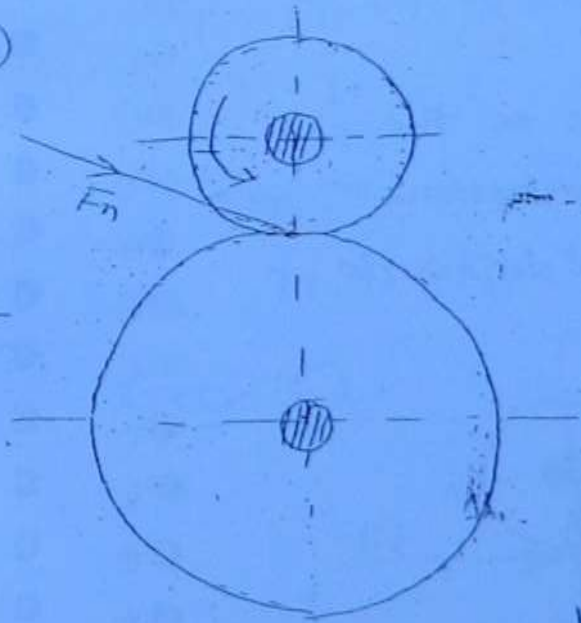


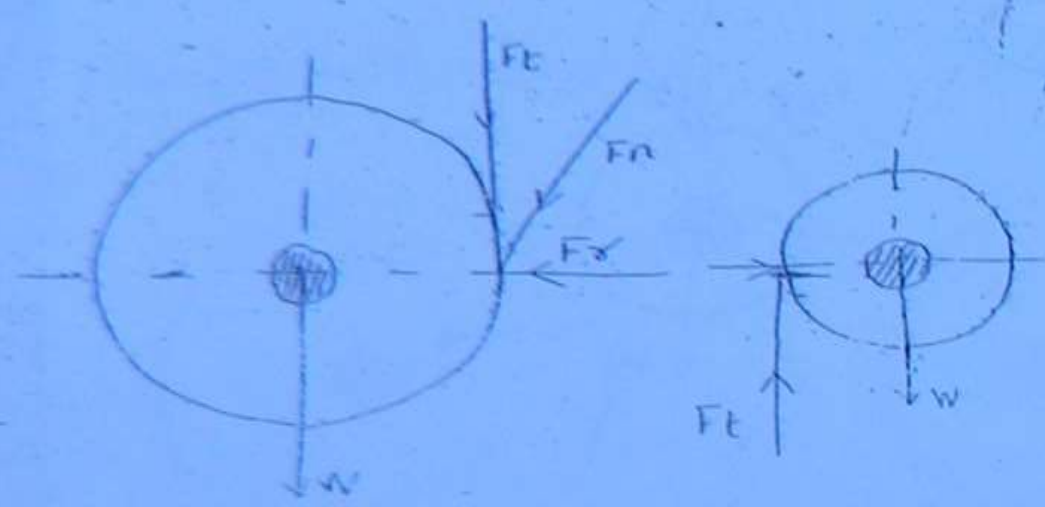
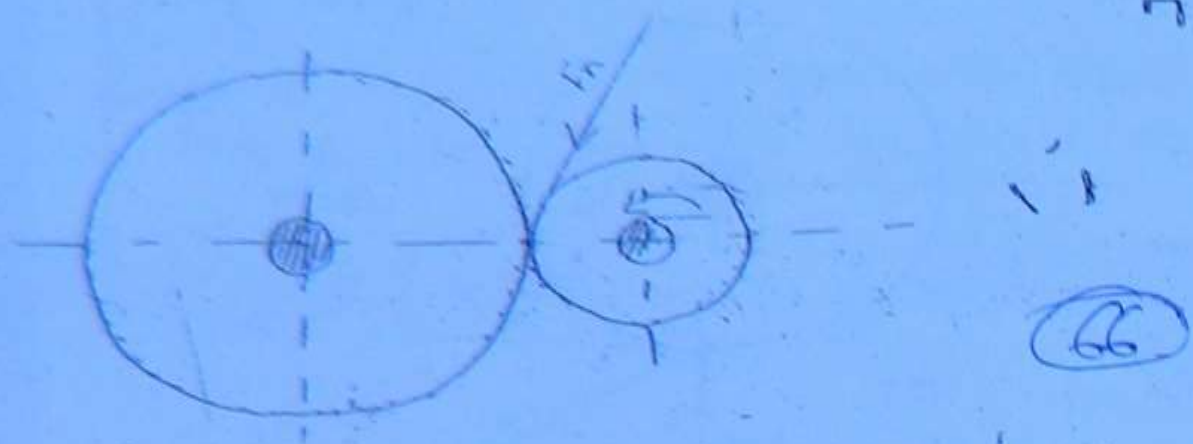
(2)



65

(3)





direction of F_r with respect to Gear is in the direction of power transmission (Top to bottom or left to right) where as direction of F_t depends on the rotation direction of Gear.

$$F_t = \frac{2T_1}{D_1} \text{ or } \frac{2T_2}{D_2} \quad F_r = F_t \cdot \tan \phi$$

In a Mild steel shaft ABCD Transmits 20 kW at 300 RPM, it is simply supported in bearings at A and D, 800 mm apart it carries a pulley of 500 mm diameter located at a point B (AB = 200 mm)

which receives power, by a horizontal belt drive with the belt tension ratio of 2, 200 mm diameter, 20° involute gear. Located at point 'c' (CD is 200 mm), delivers power to a gear directly below the shaft, assuming safe working stresses ($\sigma_t = 70 \text{ mpa}$) and τ_c is equal to 56 mpa, design the diameter of the shaft (Neglect weight of pulley and gear).

Soln (Pulley B)

$$P = 20 \text{ kW}, N = 300 \text{ RPM}$$

$$T = \frac{P \times 60 \times 10^6}{2\pi N}$$

$$T = 636619.77 \text{ N-mm}$$

$$T = (T_1 - T_2) \times 250$$

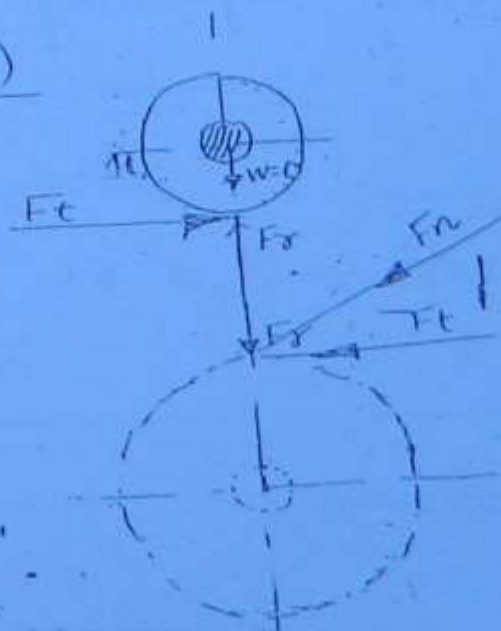
$$\frac{T_1}{T_2} = 2$$

$$T_1 = 509296 \text{ N}$$

$$T_2 = 254648 \text{ N}$$

$$T_1 + T_2 = 763944 \text{ N}$$

Gear (C)



Assuming shaft rotating in clockwise direction

$$T_C = T_B = 636619.17 = F_t \cdot R_c$$

$$F_t = 6366.19 \text{ N}$$

$$F_r = F_t \tan \phi = 2317.11 \text{ N}$$

(68)

$$(MR)_B = \sqrt{(HBM)_B^2 + (VBM)_B^2} = 1468775.97 \text{ N}\cdot\text{mm}$$

$$(MR)_C = \sqrt{(1336900)^2 + (347550)^2} = 1381337.25 \text{ N}\cdot\text{mm}$$

⇒ The critical point is B on shaft because MR is Max and Torque is maximum.

Design of shaft hence we have to design w.r.t 'B' on shaft.

ie. Shaft is design using Theories of failures.

(MSST and MDET) because it is subjected to both BM and TM.

$$\text{MSST } (T_e)_B = \sqrt{(k_b MR)_B^2 + (k_t T_B)^2} = \frac{\pi}{16} d^3 \tau_s$$

Where k_b and k_t combined shock and fatigue factor for bending, combined shock and fatigue factor for twisting respectively.

$k_b = 1.5$ and $k_t = 2 \leftarrow$ Assumption

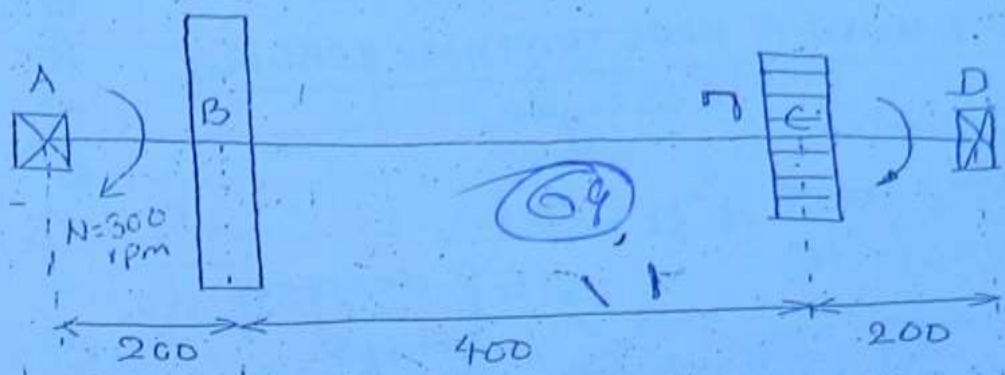
$$d = 458.5 \text{ mm}$$

$$\text{MDET } (M_e)_B = \sqrt{(k_b MR)_B^2 + \frac{3}{4}(k_t T_B)^2} = \frac{\pi}{32} d^3 \sigma_b$$

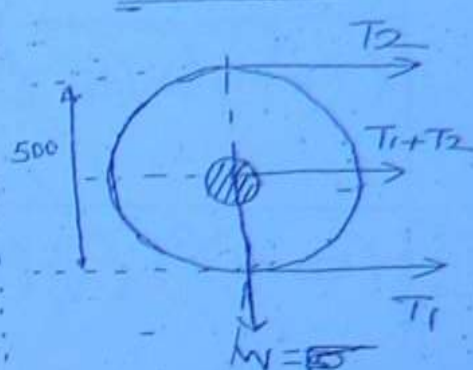
$$\therefore d = 71.03 \text{ mm}$$

$$\therefore \text{choose } d = 71.03 \text{ mm}$$

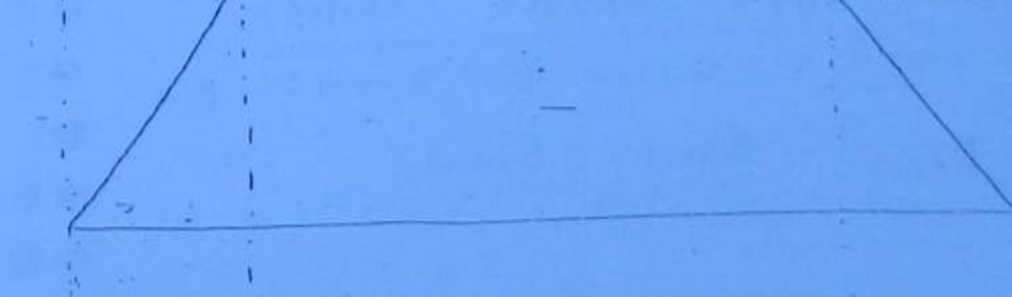
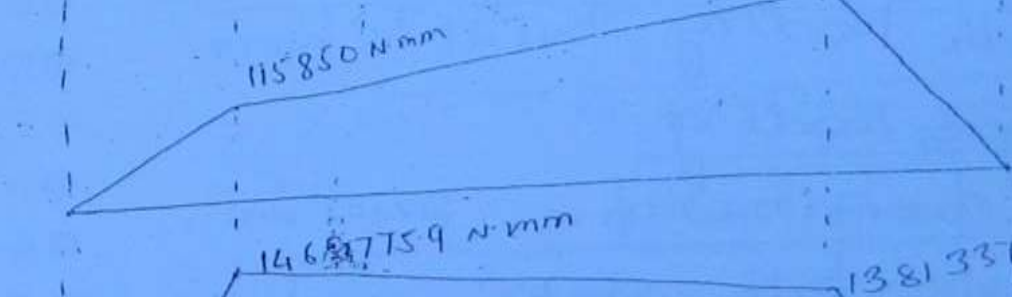
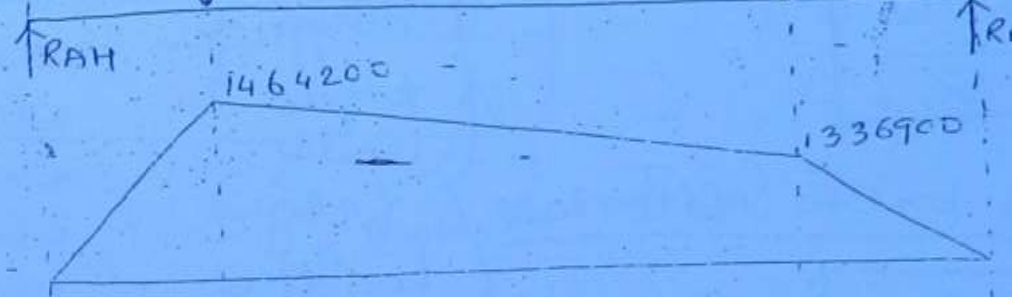
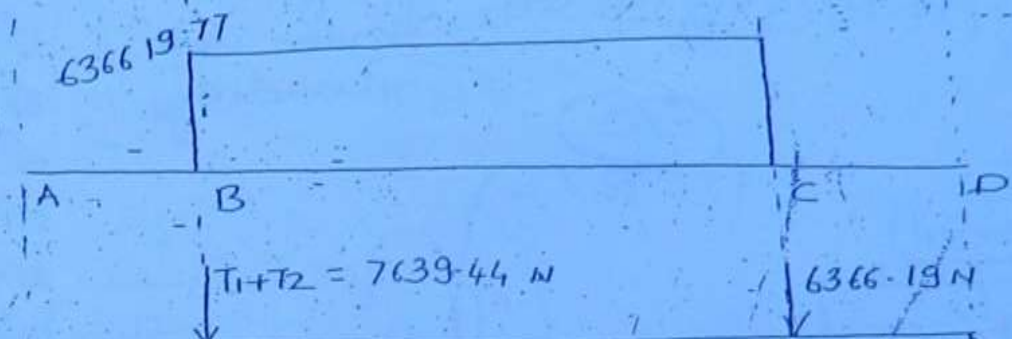
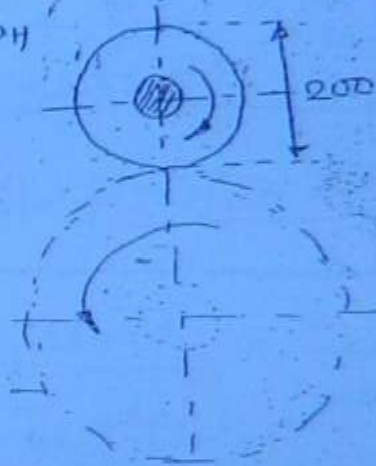
$$\therefore d = 75 \text{ mm}$$



(Pulley B)



(Gear C)



DESIGN OF SHAFT UNDER FLUCTUATING LOADING (fatigue)

$$T_e = \sqrt{(k_b \cdot M)^2 + (k_t \cdot T)^2} = \frac{\pi}{16} d^3 \tau_s$$

$$M_e = \sqrt{(k_b \cdot M)^2 + \left(\frac{3}{4} [k_t \cdot T]^2\right)} = \frac{\pi}{32} d^3 \tau_s$$

designing
shaft
under static
Loading

Soderberg equations:-
(for ductile Material)

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt} \cdot S_{ut}} + \frac{k_f \cdot \sigma_v}{\sigma_e}$$

Goodman Eqn → for brittle Materials

$$\frac{1}{N} = \frac{\sigma_m \cdot k_t}{\sigma_{ut} \cdot S_{ut}} + \frac{k_f \cdot \sigma_v}{\sigma_e}$$

designing
torque
under
fatigue
Loading

Stress Concentration is less serious in ductile materials under static loading but it is more serious under fatigue loading.

Effect of stress concentration is serious in brittle materials under both static and fatigue loadings

$$\sigma_m = \text{mean stress} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_v = \text{variable stress} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$



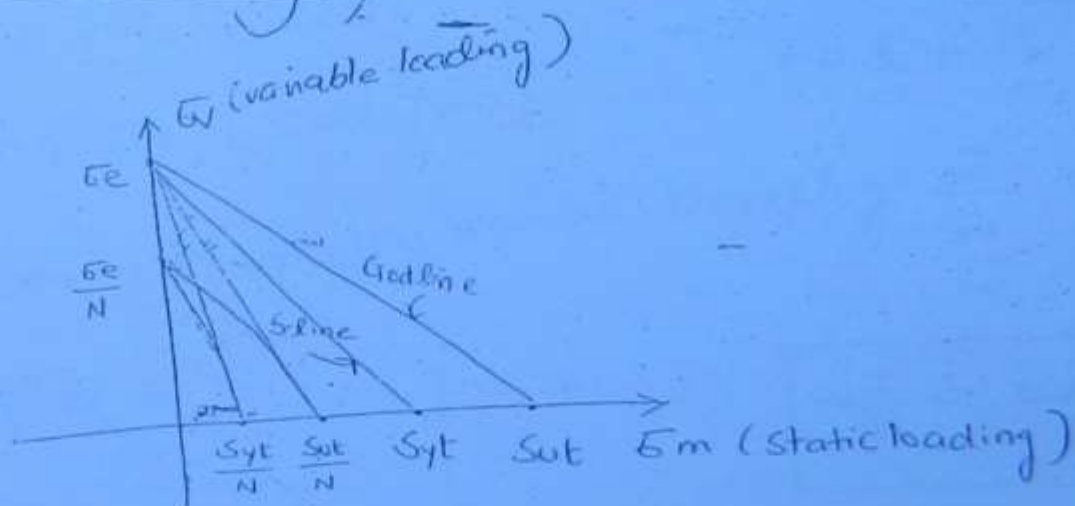
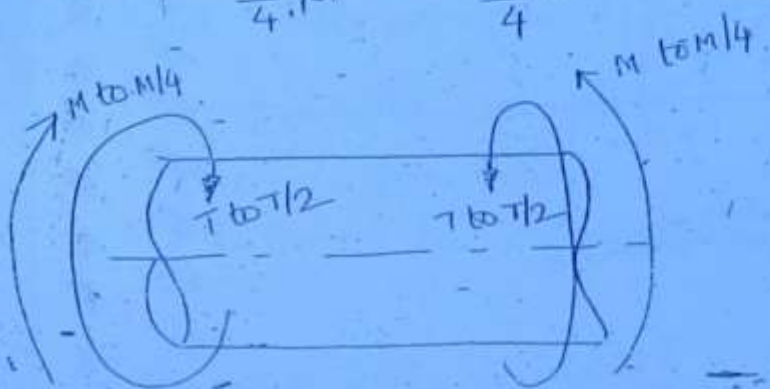
$\sigma_m \neq 0$
 $\sigma_v \neq 0$

$\left\{ \begin{array}{l} \nearrow \\ \searrow \end{array} \right\}$ alternating loads

$$\sigma_{max} = \frac{P_{max}}{\frac{\pi}{4} \cdot d^2} = \frac{6000}{\frac{\pi}{4} d^2}$$

$$\sigma_{min} = \frac{P_{min}}{\frac{\pi}{4} \cdot d^2} = \frac{4000}{\frac{\pi}{4} d^2}$$

(7)

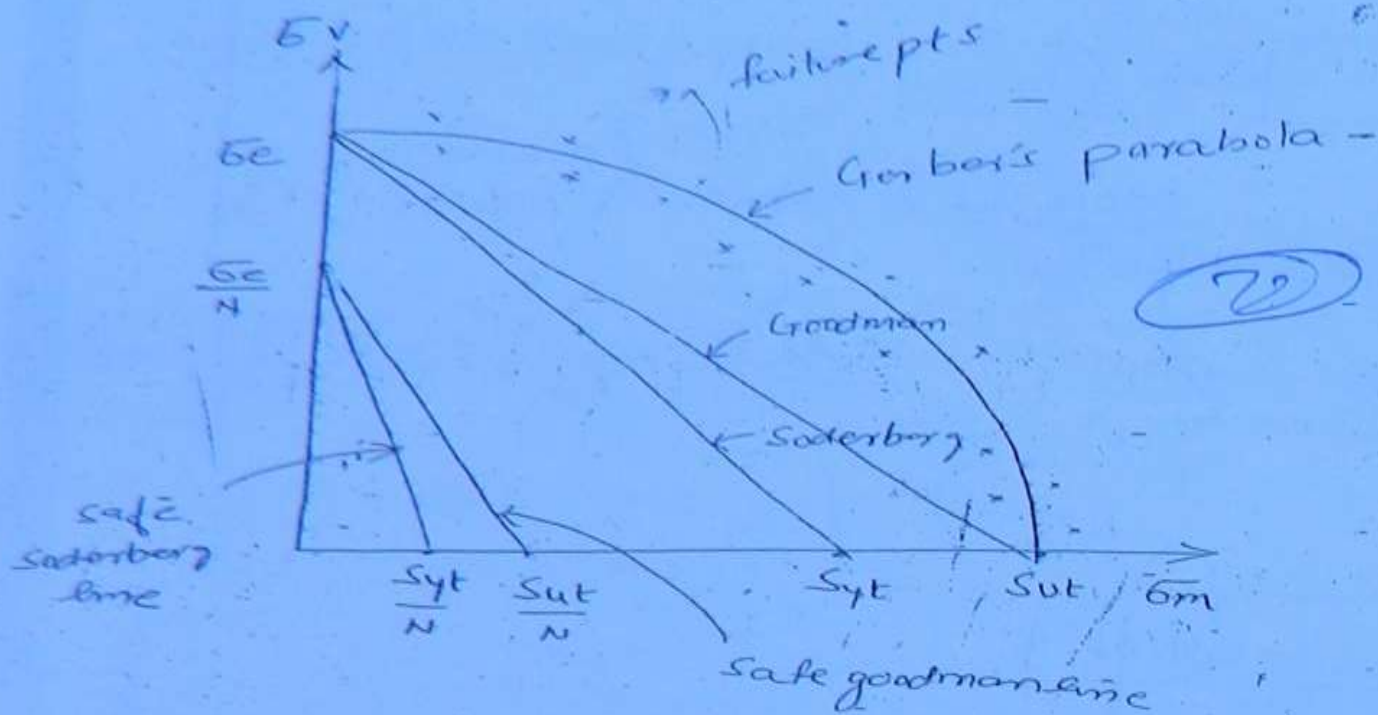


$\Rightarrow$ X-axis, $\Rightarrow \sigma_v = 0 = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\boxed{\sigma_{max} = \sigma_{min}}$ static loading

$\Rightarrow$ Y-axis, $\Rightarrow \sigma_m = 0 = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\boxed{\sigma_{max} = \sigma_{min}}$ fatigue loading



eqⁿ of line

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{\sigma_m}{\frac{\sigma_{yt}}{N}} + \frac{\sigma_v}{\frac{\sigma_e}{N}} = 1$$

$$\frac{\sigma_m}{\sigma_{yt}} + K \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$

Soderberg line - will give safest design

$$\sigma_e = \sigma_e^* \cdot k_a \cdot k_b \cdot k_c$$

σ_e^* = endurance limit of a standard specimen (from fatigue test)

σ_e = endurance limit of a Mechanical component

→ k_a = Size factor

→ k_b = Surface finish factor

Size $\uparrow \Rightarrow$ defects $\uparrow$
EL $\downarrow \Rightarrow$ Ka $\downarrow$

(7.3)

SF $\downarrow \Rightarrow$ Roughness $\uparrow$
EL $\downarrow \Rightarrow$ Kb $\downarrow$

$K_c = 1 \rightarrow$ for completely reverse bending
 $= 0.7 \rightarrow$ for completely reverse axial loading
 $= 0.6 \rightarrow$ for completely reverse torsion

$$\bar{\sigma}_e^* = 0.5 \bar{\sigma}_{ut} \text{ [Steel]}$$

$$\bar{\sigma}_e^* = 0.4 \bar{\sigma}_{ut} \text{ [Cast Iron]}$$

$\bar{\sigma}_e^* = EL$ under completely reverse bending

$$\frac{1}{N} = \frac{\bar{\sigma}_m}{\bar{\sigma}_{yt}} + \frac{K_f \bar{\sigma}_v}{\bar{\sigma}_e}$$

$$\frac{\bar{\sigma}_{yt}}{N} = \bar{\sigma}_m + \frac{K_f \bar{\sigma}_v \bar{\sigma}_{yt}}{\bar{\sigma}_e}$$

$$\bar{\sigma}_{eq} = \bar{\sigma}_m + \frac{K_f \bar{\sigma}_v \bar{\sigma}_{yt}}{\bar{\sigma}_e} \quad (1)$$

$$\bar{\sigma}_{eq} = \frac{\bar{\sigma}_{yt}}{N} = \frac{\bar{\sigma}_{yt}}{N}$$

used in Axial or Bending

Torsion

replace $\sigma \rightarrow \tau$

$$\frac{1}{N} = \frac{\tau_m}{\tau_{ys}} + \frac{k_f \cdot \tau_v}{\tau_e}$$

(74)

$$\tau_{ys} = S_{ys} = \frac{S_{yt}}{2}$$

$$\tau_e = \sigma_e^* \cdot k_a \cdot k_b \cdot k_c \quad \leftarrow 0.6$$

0.5 Sut

$$\frac{\tau_{ys}}{N} = \frac{\tau_m}{\tau_e} + \frac{k_f \cdot \tau_v \cdot \tau_{ys}}{\tau_e} \quad \tau_e = \frac{S_{ys}}{N^*}$$

$$\tau_{eq} = \tau_m + \frac{k_f \cdot \tau_v \cdot \tau_{ys}}{\tau_e} \quad (2)$$

MSST

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 4 \tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{(\sigma_{eq})^2 + 4 (\tau_{eq})^2}$$

d = ?

MDT

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 3 \tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\sigma_{eq}^2 + 3 (\tau_{eq})^2}$$

MSST

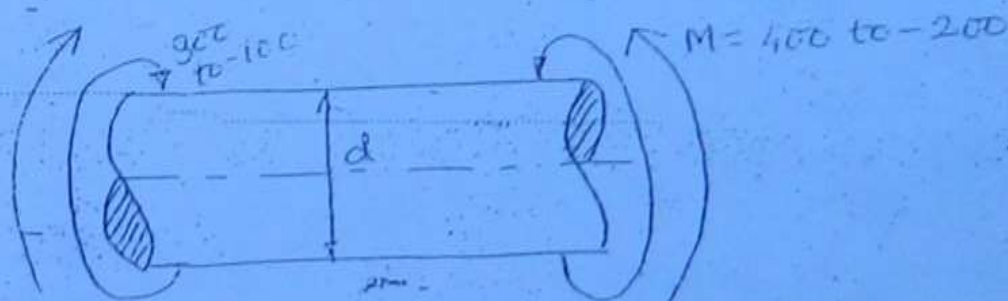
$$\tau_s = \frac{S_{ys}}{N} = \left(\frac{S_{yt}}{2N} \right) = \frac{1}{2} \sqrt{\sigma_x^2 + 12 \tau_{xy}^2}$$

$$\tau_s = \frac{S_y S}{N} = \frac{1}{2} \sqrt{6\tau^2 + 4\tau\tau_g^2}$$

(575)

IES-04

Q: A hot rolled steel shaft is subjected to a torsional load that varies from 300 kN-mm (clockwise) to 100 kN-mm (counterclockwise). As an applying bending moment at a critical section varies from 400 kN-mm to -200 kN-mm, the shaft is of uniform cross-section and no keyway is present at the critical section. determine the reqd. shaft diameter by taking factor of safety as 1.5, S_{ut} is 660 mpa, S_{yt} is 420 mpa design stress is 280 mpa also take the modification factor as 0.62, Size correction factor as 0.85, Load factor (K_c) as 1 and load factor of torsion 0.58.



Bending

$$\begin{aligned} \sigma_{max} &= \frac{M_{max}}{Z} = \frac{32 M_{max}}{\pi d^3} = \frac{32 \times 400 \times 10^3}{\pi d^3} \\ &= \frac{2}{d^3} \text{ mpa} \end{aligned}$$

$$\begin{aligned} \sigma_{min} &= \frac{32 M_{min}}{\pi d^3} = \frac{32 \times -200 \times 10^3}{\pi d^3} \\ &= \frac{4}{d^3} \text{ mpa} \end{aligned}$$

$$\sigma_m = \sigma_{max} + \sigma_{min} \quad \tau = \sigma_{max} - \sigma_{min} = \dots$$

$$K_f = 1$$

$$\sigma_{yt} = 420 \text{ MPa}, \quad \sigma_e = \sigma_e^* \cdot K_a \cdot K_b \cdot K_c \quad (76)$$

$$= 280 \times 0.85 \times 0.62 \times 1$$

$$\sigma_e = ? \text{ MPa}$$

by using Soderberg eqn

$$\sigma_{eq} = \frac{\sigma_m}{2} + \frac{K_f \cdot \sigma_e \cdot \sigma_{yt}}{\sigma_e} = \frac{\sigma_x^*}{d^3} \text{ MPa} \rightarrow (1)$$

Torsion Case

$$T_{max} = 800 \times 10^3 \text{ N-mm}$$

$$T_{min} = -100 \times 10^3 \text{ N-mm}$$

$$\tau_{max} = \frac{T_{max}}{Z_p} = \frac{16 T_{max}}{\pi d^3} = \frac{?}{d^3} \text{ MPa}$$

$$\tau_{min} = \frac{16 T_{min}}{\pi d^3} = \frac{?}{d^3} \text{ MPa}$$

$$\tau_m = \frac{T_{max} + T_{min}}{2} = \frac{?}{d^3} \text{ MPa}$$

$$\tau_v = \frac{T_{max} - T_{min}}{2} = \frac{?}{d^3} \text{ MPa}$$

by using Soderberg equation

$$K_f = 1, \quad \tau_{ys} = \frac{\sigma_{ys}}{2} = 210 \text{ MPa}$$

$$\tau_e = \sigma_e^* \cdot K_a \cdot K_b \cdot K_c =$$

$$= 280 \times 0.85 \times 0.62 \times 0.58$$

$$= ? \text{ MPa}$$

$$\tau_{eq} = \tau_m + K_f \tau_v \tau_{ys} = 2(?) \text{ MPa} \rightarrow (2)$$

by using

MDET

$$\bar{\sigma}_t = \frac{\sigma_{yt}}{N} = \sqrt{(\bar{\sigma}_{eq})^2 + 3(\bar{\tau}_{eq})^2}$$

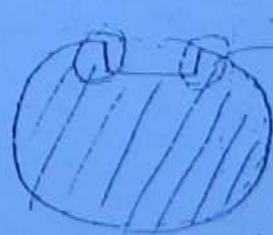
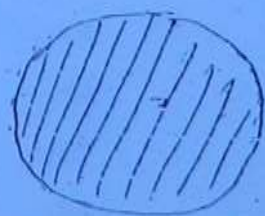
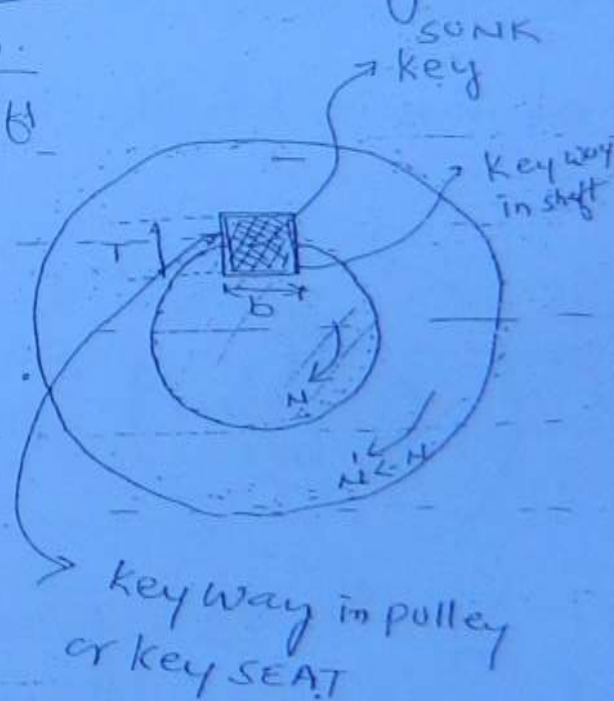
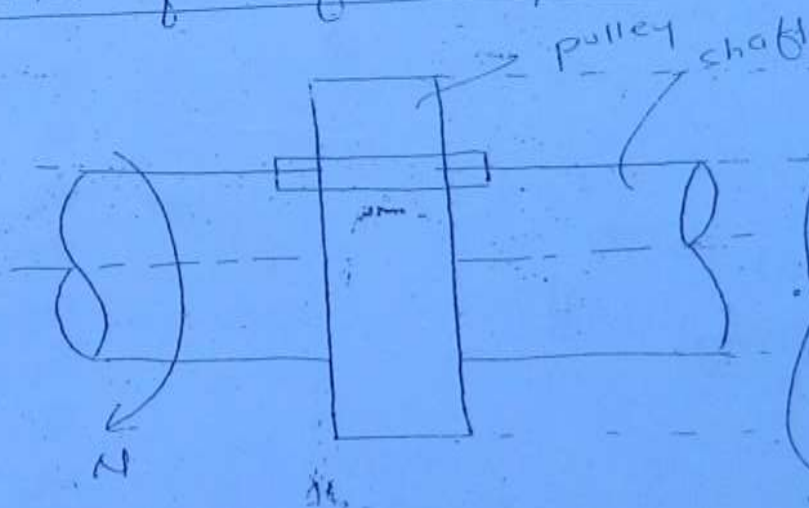
'd' can be found out

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④ KEYS

Key is defined as a metal piece which is inserted between shaft and its assembly to transmit power between them and to prevent relative motion between them.

Key acts as a safety device for shaft and its assembly in presence of overloads. Stress concentration factor and strength criteria of shaft is a problem.



Stress concentration region

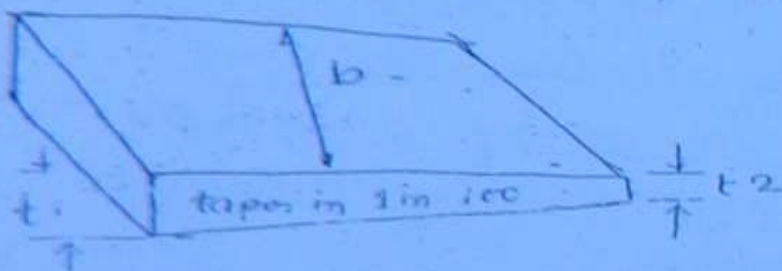
shaft

3D figure of Key

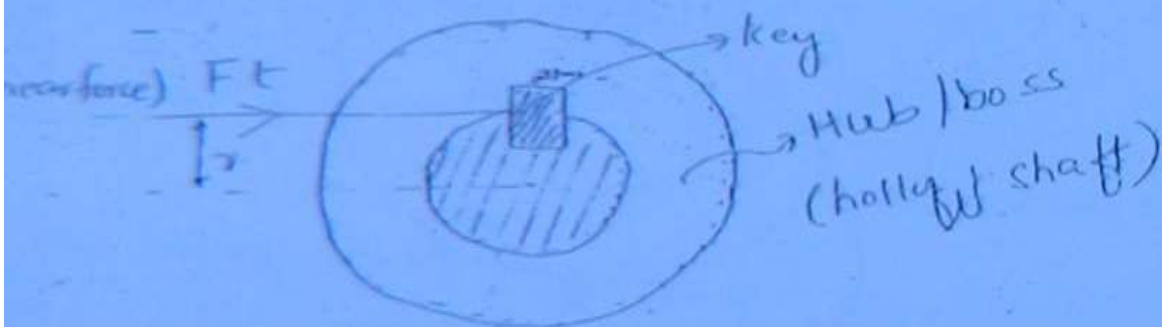


(i) parallel Key

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every 100 mm in length thickness reduces by 1mm



$$F_t = \frac{2T}{d} \quad \therefore \quad \tau_s = \frac{F_s}{A_s} = \frac{F_t}{L \times b} = \frac{2T}{d \cdot L \cdot b}$$

d = diameter of shaft

$$\tau_{ind} \leq \tau_{per}$$

$$\tau_s = \frac{2T}{d \cdot L \cdot b}$$

$$\frac{2T}{d \cdot l \cdot b} \leq \tau_{per}$$

$$b \geq \text{--- mm}$$

Crushing stress (σ_c)

$$\sigma_c = \frac{F_t}{A_c} = \frac{2T}{d \times \frac{l \cdot t}{2}}$$

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$$\sigma_c = \frac{4T}{d \cdot l \cdot t}$$

$$\sigma_c \leq (\sigma_c)_{permissible}$$

$$t \geq \text{--- mm}$$

Standard proportions of key

$$U = d + 13$$

$$L = 1.5 U$$

$$b = \frac{U}{4} \quad t = \frac{U}{6}$$

$$\rightarrow U = 4b$$

$$t = \frac{4b}{6} = \frac{2}{3} b$$

$$t = \frac{2}{3} b$$

d = diameter of shaft

check for safe design

$$\tau_{ind} = \frac{2T}{d \cdot l \cdot b} = ?$$

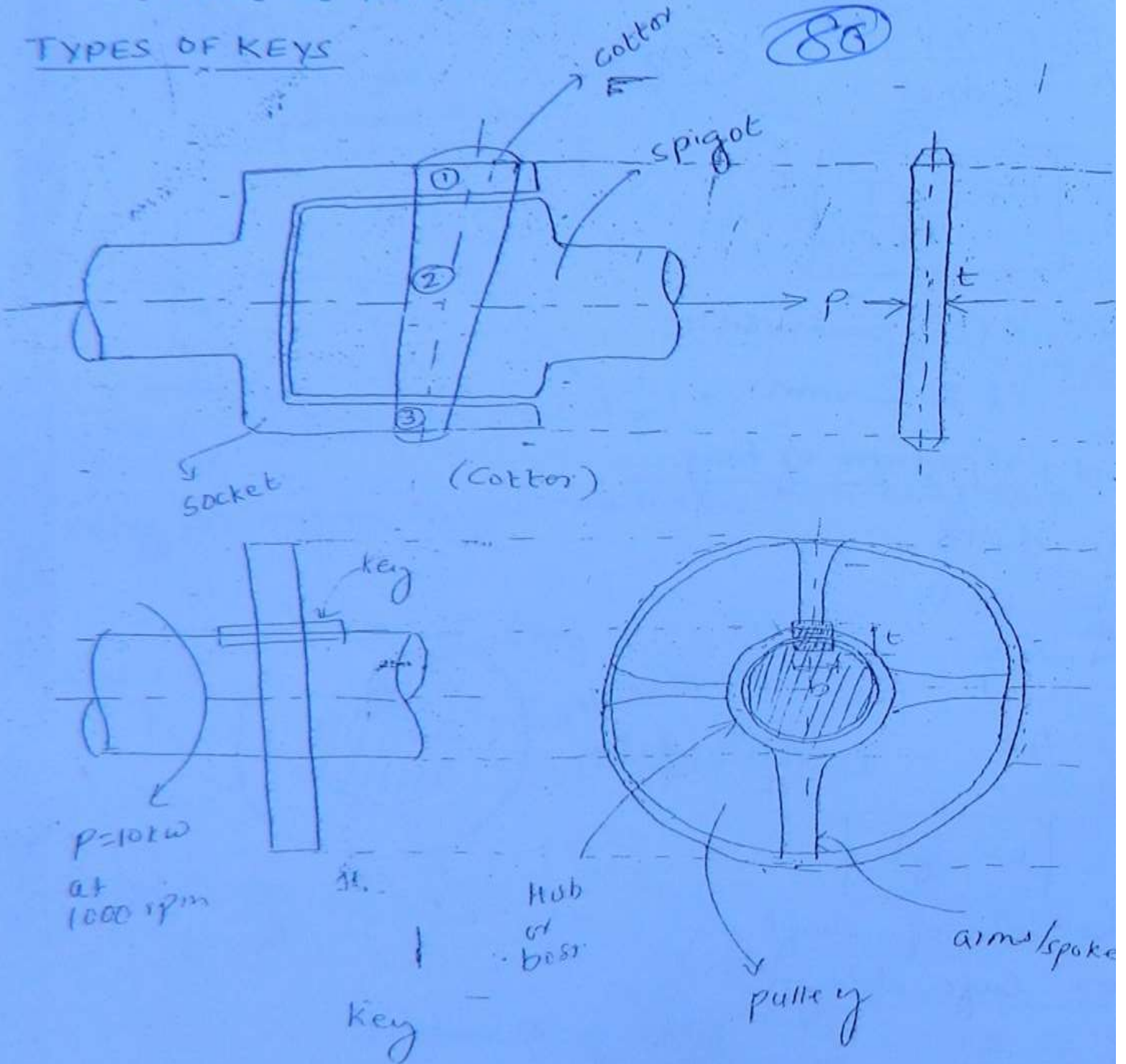
$$\tau_{ind} \leq \tau_{per} \text{ [No shear failure]}$$

$$(\sigma_c)_{ind} = \frac{4T}{d \cdot t} = ?$$

$$-(\sigma_c)_{ind} \leq (\sigma_c)_{permissible}$$

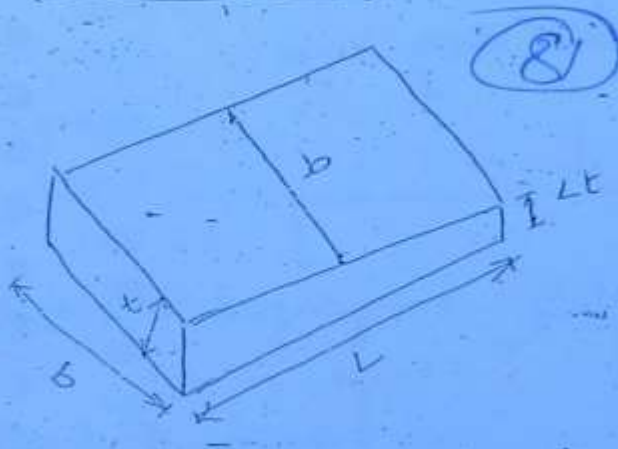
So increase thickness

TYPES OF KEYS



Key

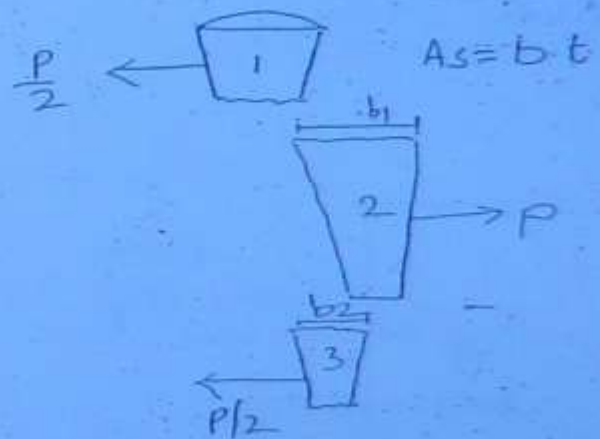
- 1) Keys are subjected to shear over a longitudinal section
- (2) parallel to axis of shaft
- (3) Temporary fastener used for power transmission



- (4) They are subjected to single shear
- (5) $A_s = L \times b$
- (6) Taper is provided only on top surface
- (7) Taper is provided on the thickness
- (8) $\tau_s = \frac{P_s}{A_s} = \frac{P/2}{b \times L} = \frac{P}{2bL}$

Cotter

- 1) cotter are subjected to shear over a transverse section
- 2) Per to axis of shaft
- 3) used to join two co-axial bar or rods



- (4) They are subjected to double shear
- (5) $A_s = b \times t$
- (6) Taper is provided on both side
- (7) Taper is provided on the width
- (8) $\tau_s = \frac{P_s}{2A_s} = \frac{P}{(b_1 + b_2)t} = \frac{P}{2bt}$

$$(T_{max})_{ind} \leq T_{per}$$

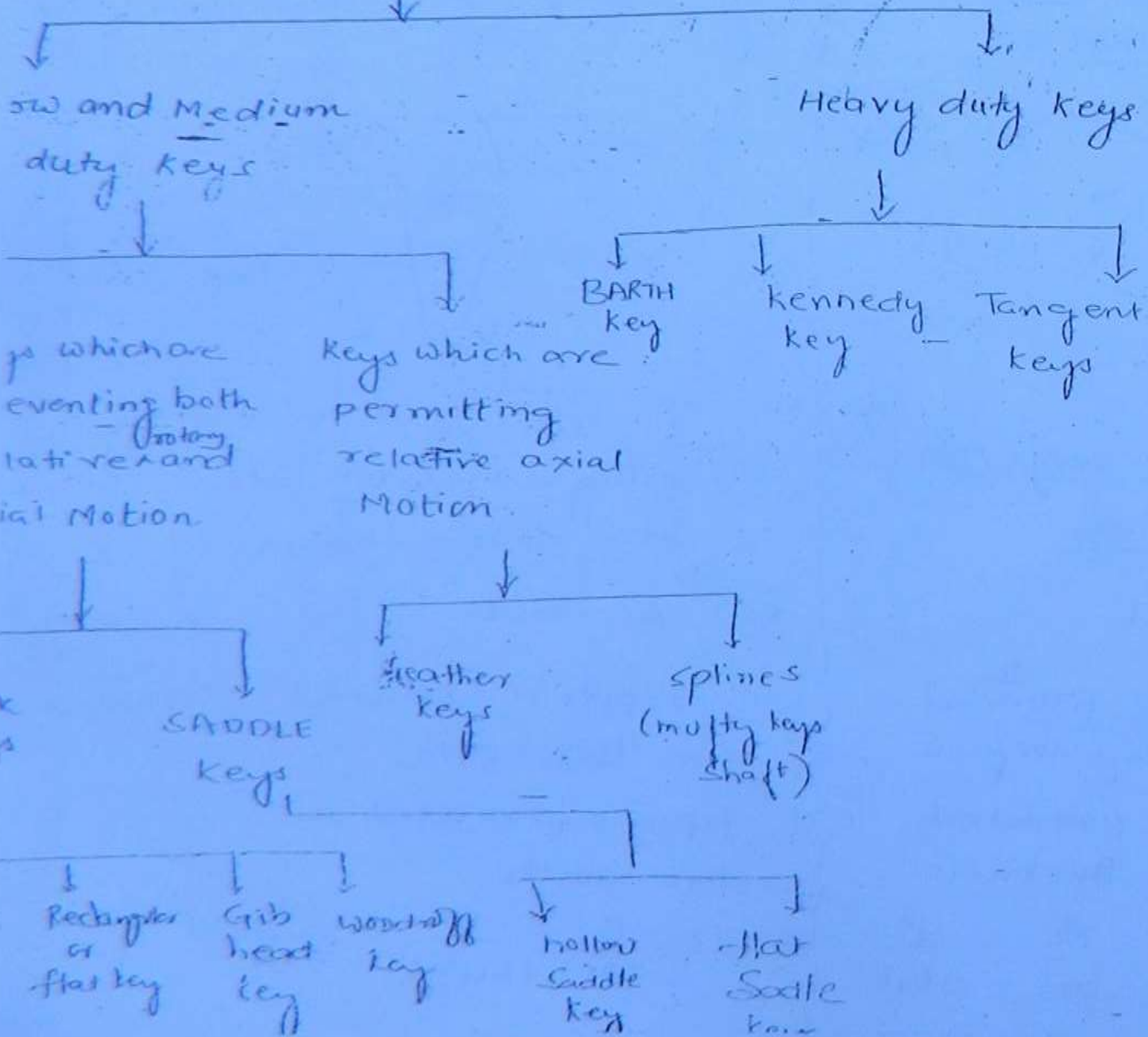
$$\frac{P}{2bt} \leq T_{per}$$

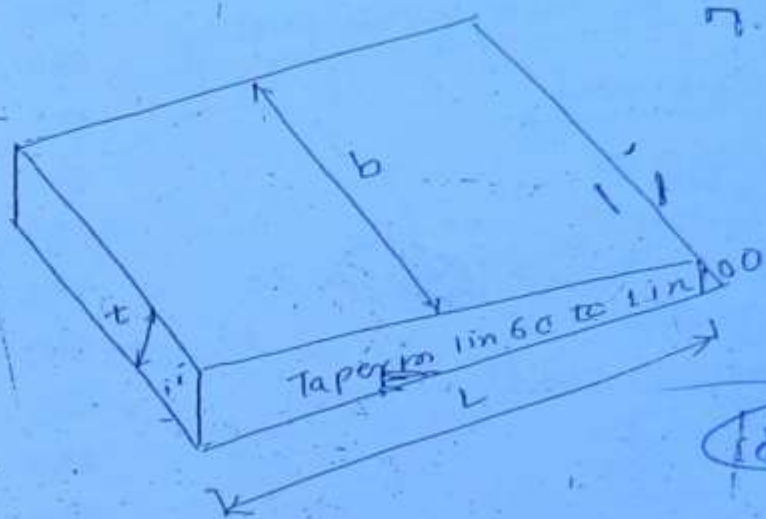
$$P \leq 2bt T_{per}$$

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shear strength of collar = $2 \cdot b \cdot t \cdot T_{per}$

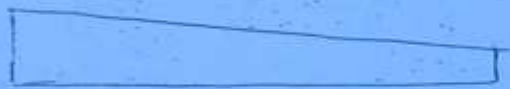
Types of Keys





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Taper key / Rect sunk key / Flat key



Fv

(i) Taper Key



Gib head Key

(ii) Gib head key

$\Rightarrow b = t \Rightarrow$ Square Key

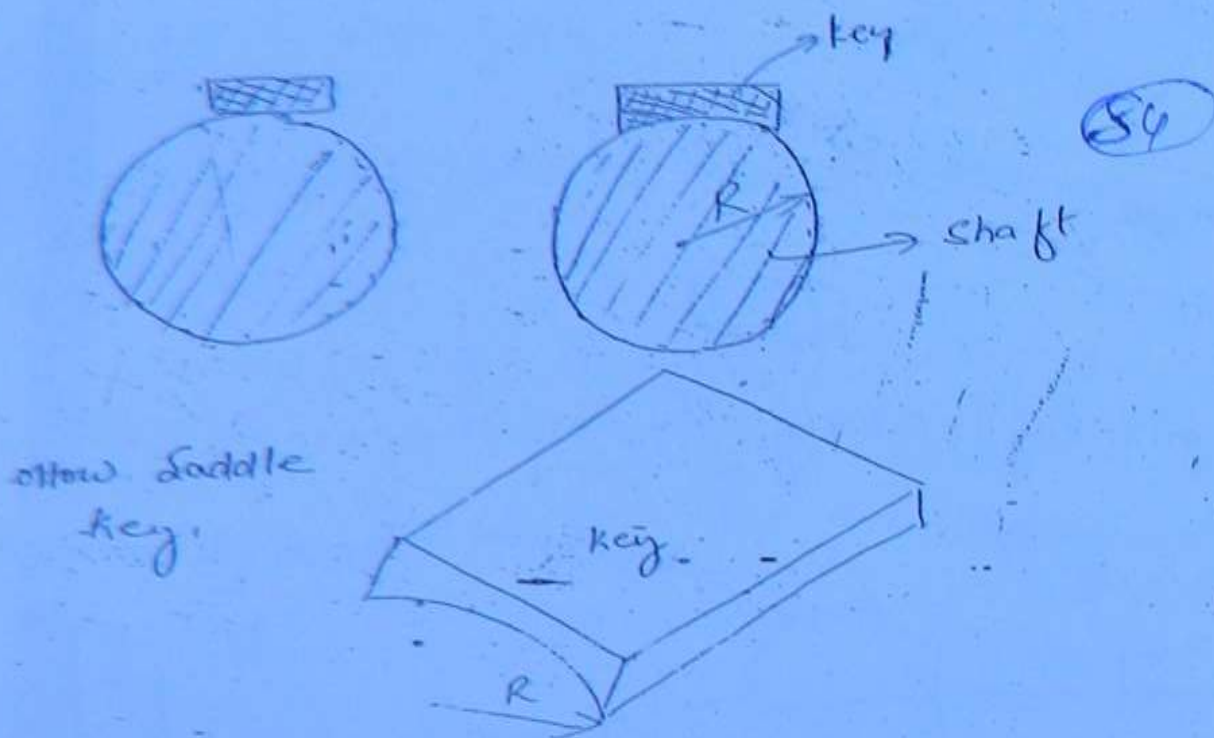
iii) WOODRUFF KEYS

used in tapered shafts, because of self aligning properties, form of semi-circular disk

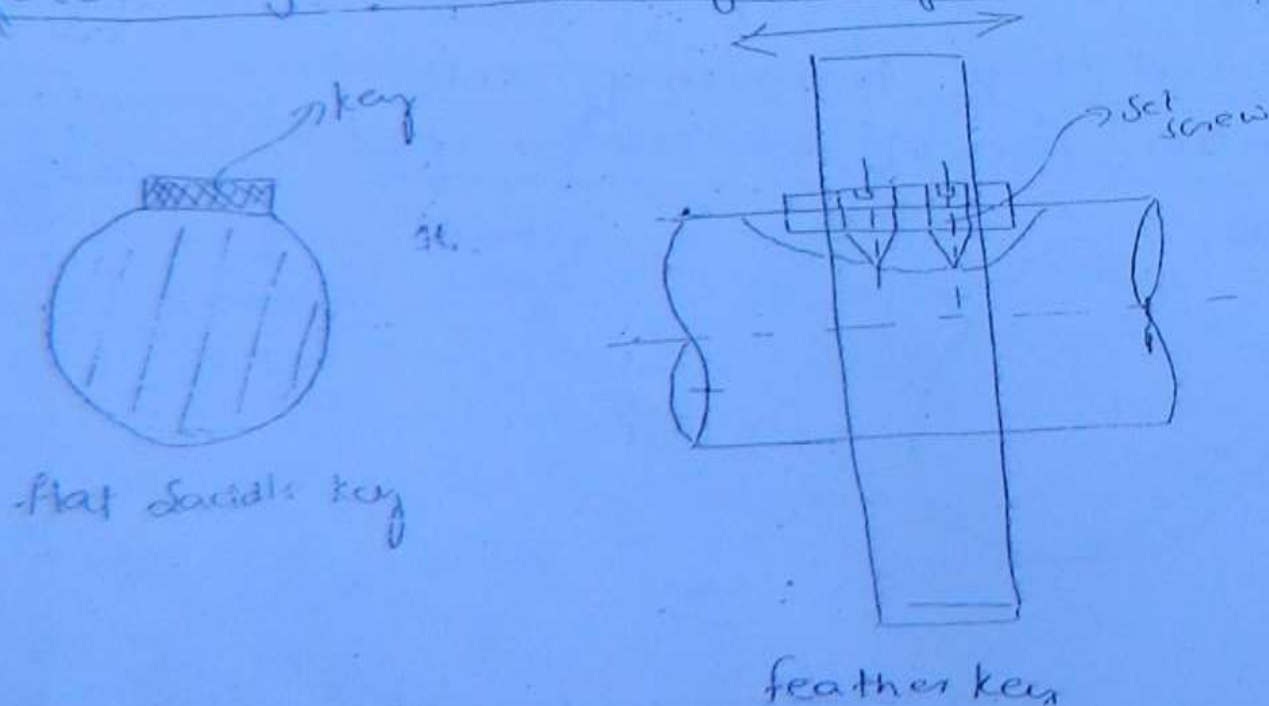


Saddle keys

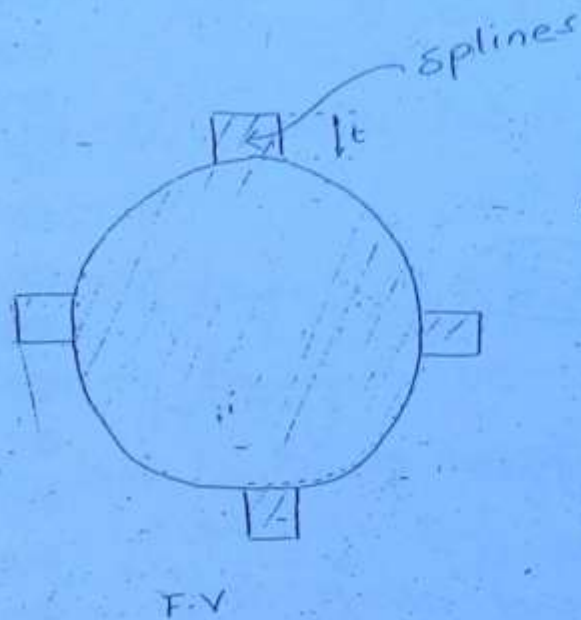
keyways is present only in the hub of the pulley
here only one keyway is required.



power is transmitted due to frictional forces
developed between shaft and key surfaces -
they comes under low duty key, here key
face is adjusted with shaft surface.



Splines



a key which is integral with the shaft are called splines.

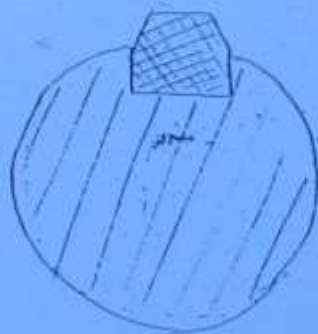
~~all the keys in a shaft are splines~~

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4-splined shaft (Multi keyed shaft)

Heavy duty keys

(i) Barth key



it is a modified rectangular key

(ii) Kennedy key



it is a modified square key

here shear area increases and Power transmitting capacity increases.

Tangent Key



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a square of side $\frac{d}{4}$ is to be fitted on a shaft of diameter 'd' and in the hub of the pulley. If the material of key and shaft are same and both are equally strong in shear what is the length of the key. —

(a) $\frac{\pi d}{2}$ (b) $\frac{3\pi d}{2}$ (c) $\frac{3\pi d}{4}$ (d) $\frac{4\pi d}{5}$

Ans

$$T_s = T_{key}$$

$$\frac{\pi}{16} d^3 \tau_s = \frac{b d L \tau_k}{2}$$

$$\frac{\pi}{16} d^3 = \frac{d}{4} \times \frac{d}{2} \times L$$

$$\therefore L = \frac{\pi d}{2}$$

a square key of side $d/4$ each and length 'L' is used to transmit torque 'T' from the shaft of diameter 'd' to the hub of a pulley. Assuming the length of the key is equal to the thickness of the pulley, the average shear stress developed in the key is given by

(a) $\frac{16T}{\pi d^3}$ (b) $\frac{16T}{\pi d^2}$ (c) $\frac{16T}{\pi d}$ (d) $\frac{16T}{\pi d^4}$

Soln $\tau_s = \frac{2T}{bde} = \frac{2 \times T}{\frac{d}{4} \times d \times l} = \frac{8T}{ld^2}$

Q: Match list I with list II

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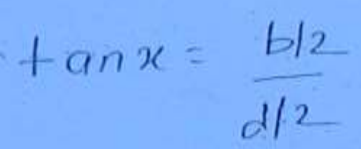
LIST-I

LIST-II

- | | |
|------------------|--------------------------------|
| (a) Woodruff key | → 1. Loose fitting, Light duty |
| (b) Kennedy key | → 2. heavy duty |
| (c) feather key | → 3. Self aligning |
| (d) flat key | → 4. Normal industrial use |

b-2, a-3, c-1, d-4

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$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots + A_n x_n}{A_1 + A_2 + A_3 + \dots + A_n}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\bar{x} = \frac{b+b+0+0}{4}$$

$$\boxed{\bar{x} = \frac{b}{2}}$$

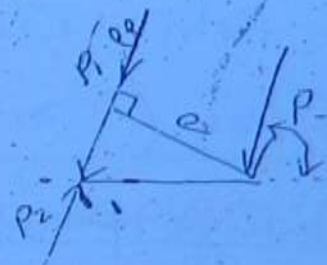
(29)

$$\bar{y} = \frac{y_1+y_2+y_3+y_4}{4} = \frac{d+0+0+d}{4} = \frac{d}{2}$$

(2) Introduce two equal and opposite forces P_1 and P_2 ^{through centroid} such that $P_1 = P_2 = P$ parallel to the applied load

$$P_1 = P_2 = P$$

(3) e



(4) Effect of P_1

effect of P_1 is to cause a primary shear force (P_p) of equal magnitude at each and every rivets.

$$P_p = \frac{P_1}{n} = \frac{P}{4}$$

(5) Effect of P and P_2

P and P_2 causes a twisting couple with respect of group of rivets.

$T.M = P \cdot e \rightarrow$ clockwise, due to this twisting couple rivets are subjected to a secondary shear force (P_s) and the P_s magnitude is directly proportional to 'r' (i.e., r = distance between C.G. of group of rivets and C.G. of each rivets).

$$P_s \propto r$$

$$\begin{array}{ccc}
 0^7 & 0^4 & 0^1 \\
 0^8 & 0^5 & 0^2 \\
 0^9 & 0^6 & 0^3
 \end{array}
 \quad (P_{s1} = P_{s3} = P_{s7} = P_3) > (P_{s2} = P_4 = P_6) = P_{s8}$$

$\textcircled{90}$
 $P_{s5} = 0$

hence P_s is maximum at a rivet which is far away from the CG of the rivet system. P_s is at each and every rivet are equal in magnitude when all the rivets are located at same distance from CG of rivet system.

$$P_{s1} = P_{s2} = P_{s3} = P_{s4} \quad (\because r_1 = r_2 = r_3 = r_4)$$

P_s direction is always perpendicular to the line joining CG of group of rivets and CG of each rivet.

Calculation of r_1, r_2, r_3 and r_4

$$r_1 = r_2 = r_3 = r_4 = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{d}{2}\right)^2}$$

Calculation of P_{s1}, P_{s2}, P_{s3} and P_{s4}

$$P_{s1} \propto r_1 \Rightarrow P_{s1} = k \cdot r_1$$

$$P_{s2} \propto r_2 \quad \therefore k = \frac{P_{s1}}{r_1}$$

$$P_{s4} \propto r_4 \Rightarrow P_{s2} = k \cdot r_2$$

$$P_{s2} = P_{s1} \left(\frac{r_2}{r_1} \right)$$

$$\Rightarrow P_{s4} = P_{s1} \left[\frac{r_4}{r_1} \right]$$

$$P_{Sn} = P_{s1} \left[\frac{\delta n}{\delta 1} \right]$$

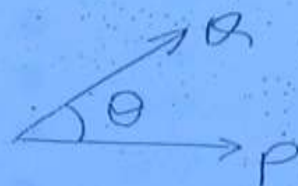
$$P_{s1} \cdot \delta_1 + P_{s2} \cdot \delta_2 + P_{s3} \cdot \delta_3 + P_{s4} \cdot \delta_4 = p \cdot e$$

$$\frac{P_{s1}}{\delta_1} [\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2] = p \cdot e \quad (9)$$

P_{s1} can be determined

3) Calculation of $\theta_1, \theta_2, \theta_3, \theta_4$

$$(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$$



$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$P = P_p, \quad Q = P_s$$

$$(R_1 = R_2) > (R_3 = R_4)$$

$$R_{\max} = R_1 \text{ or } R_2$$

hence worst rivets are 1 and 2.

$$R_{\max} = R_1 \text{ or } R_2 = \sqrt{P_p^2 + P_s^2 + 2P_p P_s \cos \theta_1}$$

4) diameter of rivet

condition for safe design

$$(T_{\max})_{\text{ind}} \leq T_{\text{per}}$$

$$\frac{R_{\max}}{\frac{\pi d^2}{4}} \leq T_{\text{per}}$$

$$R_1 \text{ or } R_2 \leq \frac{\pi d^2}{4} T_{\text{per}} \quad \therefore d \geq ?$$

shear strength

shear strength of rivet = $k \cdot \frac{\pi}{4} d^2 \tau_s$

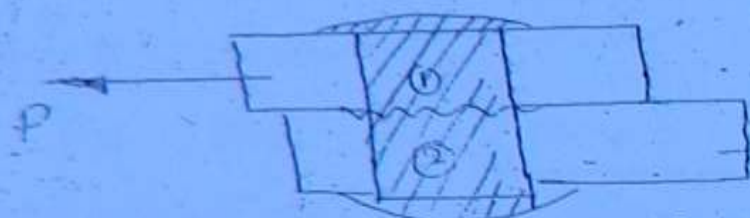
$k = 1 \Rightarrow$ single shear

eg. Lap joint single strap butt joint

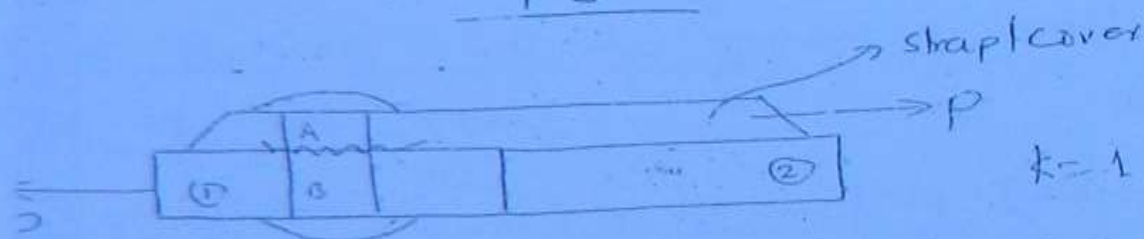
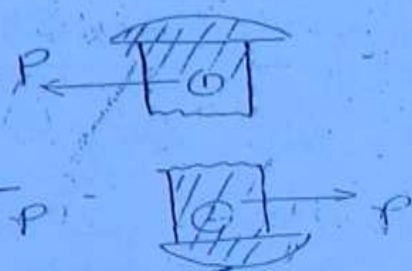
$k = 2 \Rightarrow$ double shear

eg. double strap butt joint

$k = 1.875$ by IBR

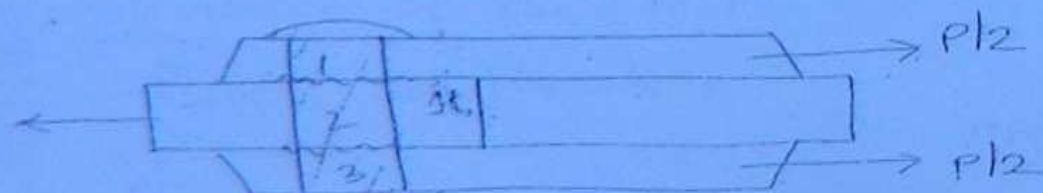


Lap joint



$k = 1$

butt joint (single strap)



$k = 2$

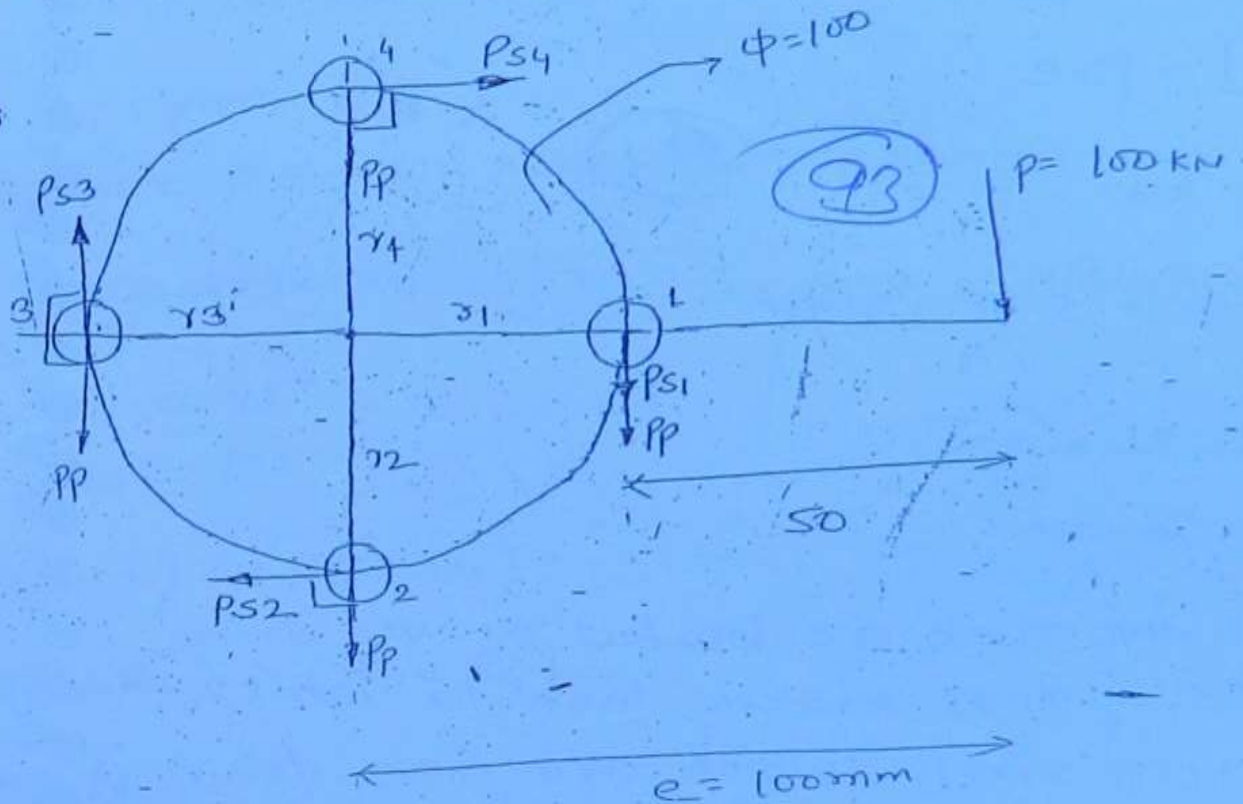
double strap butt joint



stress is same

$P = 1 \rightarrow P/2$

Q: design an eccentrically loaded riveted joint as shown in the figure. If the permissible shear stress for the rivet material is 75 mpa.



$$P_p = \frac{P}{n} = \frac{P}{4} = 25 \text{ kN}$$

$$T.M = P \cdot e = 100 \times 100 = 10^4 \text{ kN} \cdot \text{mm} = 10^7 \text{ N} \cdot \text{mm}$$

$$r_1 = r_2 = r_3 = r_4 = 50 \text{ mm}$$

$$P_{s1} = P_{s2} = P_{s3} = P_{s4} =$$

$$\theta_1 = 0^\circ, \theta_2 = 90^\circ, \theta_3 = 180^\circ, \theta_4 = 90^\circ$$

$$\theta_1 < (\theta_2 = \theta_4) < \theta_3$$

$$R_1 > (R_2 = R_4) > R_3$$

$$R_{\max} = R_1$$

$$P \rightarrow, \theta \rightarrow \therefore R = P + \theta$$

$$R_{max} = P_p + P_s = \frac{\pi d^2 \tau_s}{4}$$

$$\frac{P_s}{s_1} [1 s_1^2] = p \times e$$

$$P_{s1} = 50 \text{ kN}$$

$$25 \times 10^3 + 50 \times 10^3 \leq \frac{\pi d^2}{4} \times 75$$

$$\therefore d \geq 35.6 \text{ mm}$$

$$\therefore d = 36 \text{ mm}$$

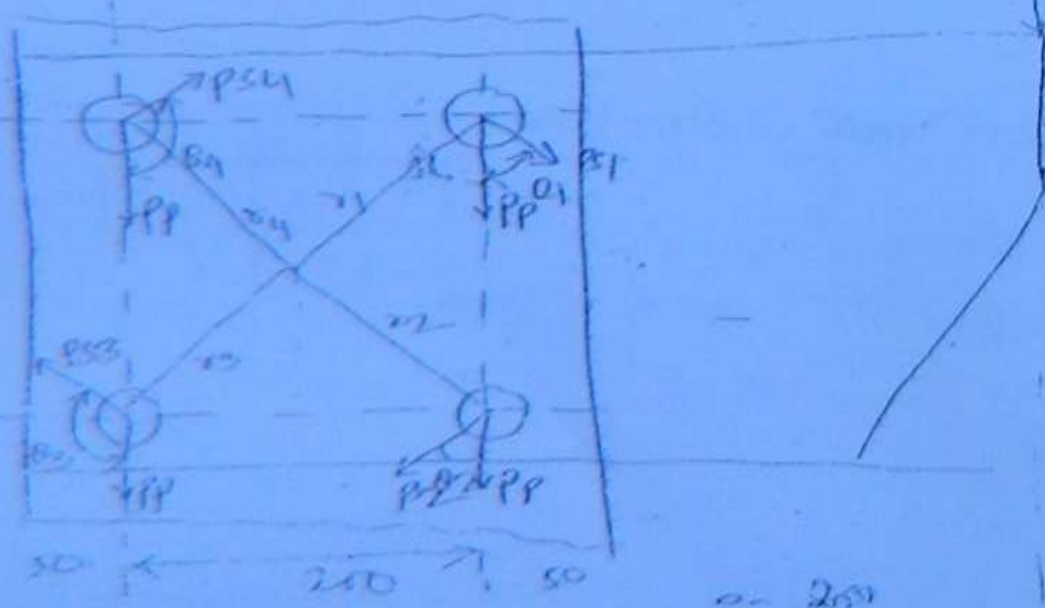
Ex: When all the rivets are located at the same from the C.G. of rivet system then the worst rivets are those rivets which are near to the line of action of applied load.

$$P = 100, b = 200, d = 200, e = 250$$

$$\tau_{per} = 60 \text{ MPa}, d = ? R_1, R_2, R_3, R_4 = ?$$

$$\theta_1 = 45^\circ, (\theta_3 = \theta_4) = 135^\circ$$

$$P = 100 \text{ kN}$$



$$\bar{x} = 110$$

$$y = 100$$

$$P_p = \frac{W_D}{4} = 25 \text{ kN}$$

$$e = 250 \text{ mm}$$

$$\text{Now } T_M = P \cdot e = 100 \times 250 = 25 \times 10^3 \text{ N-mm}$$

$$\text{Now, } r_1 = r_2 = r_3 = r_4 = \sqrt{(b/2)^2 + (d/2)^2} = 141.42 \text{ mm}$$

$$\theta_1 = \theta_2 = 45^\circ$$

$$\theta_3 = \theta_4 = 135^\circ$$

$$(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$$

$$(R_1 = R_2) > (R_3 = R_4)$$

$$R_{\max} = R_1 \text{ or } R_2$$

$$R_{\max} = \sqrt{P_p^2 + P_{s1}^2 + 2P_p P_{s1} \cos \theta_1}$$

$$= \sqrt{(25)^2 + (176.778)^2 + 2 \times 25 \times 176.778 \times \cos 45^\circ}$$

$$\therefore R_{\max} = 195.25 \text{ kN}$$

$$\text{also, } \frac{P_{s1}}{r_1} \times 4r_1^2 = P \cdot e$$

$$\therefore P_{s1} \times r_1 = P \cdot e$$

$$P_{s1} = \frac{P \cdot e}{r_1} = \frac{100 \times 10^3 \times 250}{141.42}$$

$$\therefore P_{s1} = 176.778 \text{ kN}$$

$$R_{max} = \frac{\pi d^2}{4} \tau_s$$

$$195.25 = \frac{\pi d^2}{4} \times 60$$

$$195.25 \times 10^3 = \frac{\pi d^2}{4} \times 60$$

$$d = 64.36$$

$$d \approx 66 \text{ mm}$$

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$$R_3 = R_4$$

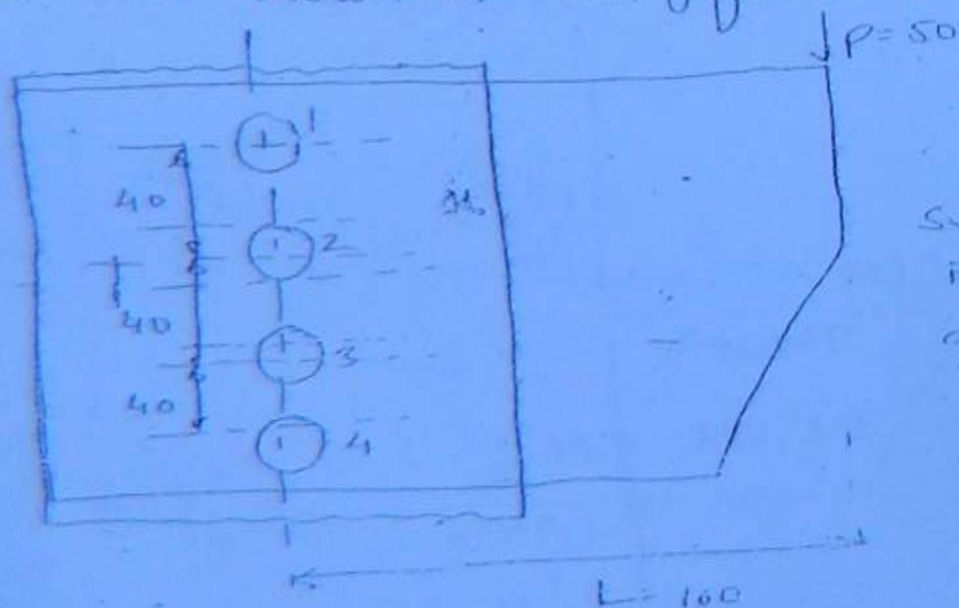
$$R_3 = R_4 = \sqrt{P^2 + P_{s3}^2 + 2PP_{s3} \cos \theta_3}$$

$$R_1 = P_{s2} = P_{s3} = P_{s4}$$

$$R_3 = R_4 = \sqrt{(25)^2 + (176.778)^2 + 2 \times 25 \times 176.778 \times \cos 135^\circ}$$

$$R_3 = R_4 = 160.079 \text{ kN}$$

determine which of the following rivets are
most loaded rivets in the eccentrically loaded
it as shown in the figure?



$$S_{yt} = 210 \text{ MPa}$$

$$F_s = 2$$

$$d = ?$$

(a) All the rivets

(b) 1 only

(c) 4 only

(d) 1 and 4 only

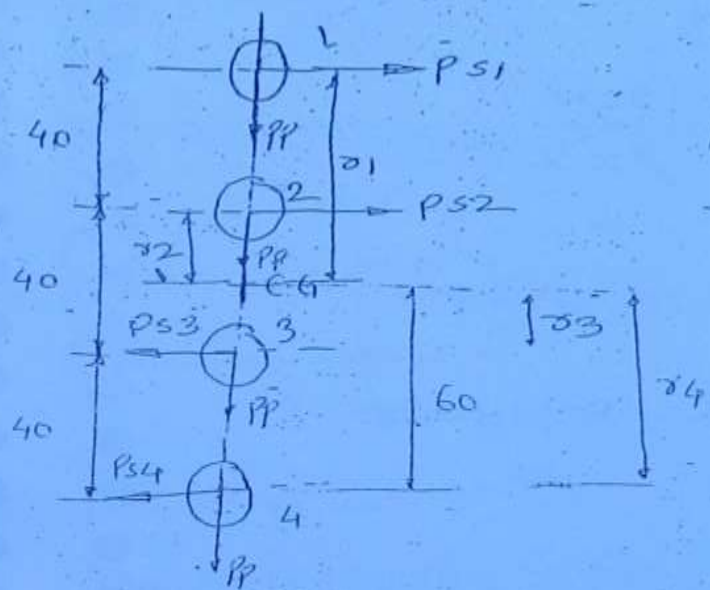
here, $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 90^\circ$

$$(P_{s1} = P_{s4}) > (P_{s3} = P_{s2})$$

$$(r_1 = r_4) > (r_3 = r_2)$$

$$(R_1 = R_4) > (R_2 = R_3)$$

$$R_{\max} = R_1 \text{ or } R_4$$



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When all the rivets are arranged in a single vertical row the worst rivets are those rivets which are far away from the C.G. of group of rivets.

$$R_{\max} = R_1 \text{ or } R_4 = \sqrt{P_P^2 + P_{s1}^2} = \frac{\pi}{4} d^2 \tau_s$$

$$= \frac{\pi}{4} d^2 \times \frac{S_{yt}}{N} @ \frac{S_{yt}}{2N}$$

$$P_P = \frac{P}{4} = 12.5 \text{ kN}$$

$$r_1 = r_4 = 60, r_2 = r_3 = 20 \text{ mm}$$

$$\frac{P_{S1}}{x_1} [2x_1^2 + 2x_2^2] = p \cdot e$$

$$\frac{P_{S1}}{60} [2 \times 60^2 + 2 \times 20^2] = 50 \times 100$$

$$P_{S1} = 37.5 \text{ kN}$$

$$3) R_{\max} = P_{S1} \text{ or } P_{S4}$$

(98)

$$R_{\max} = \sqrt{P_1^2 + P_2^2} = \frac{\pi}{4} d^2 \frac{S_{yt}}{2N}$$

$$\Rightarrow \sqrt{12.5 \times 10^3 + (37.5)^2} = \frac{\pi}{4} d^2 \times \frac{250}{2 \times 2}$$

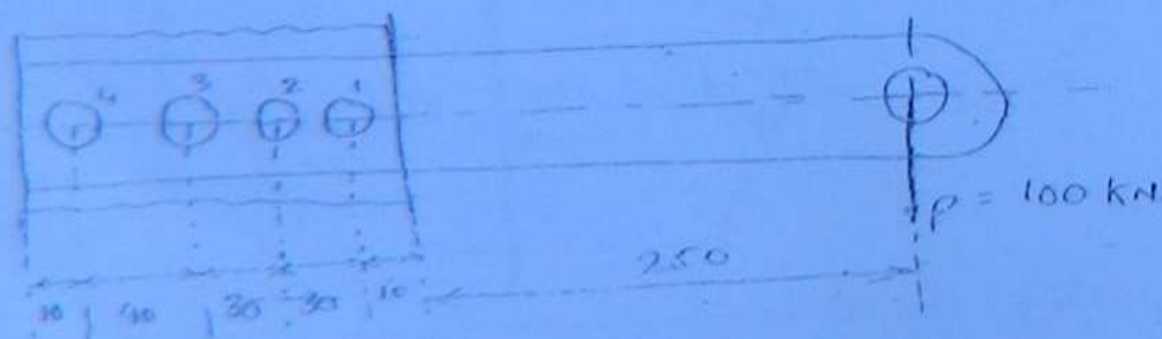
$$d = 31.7$$

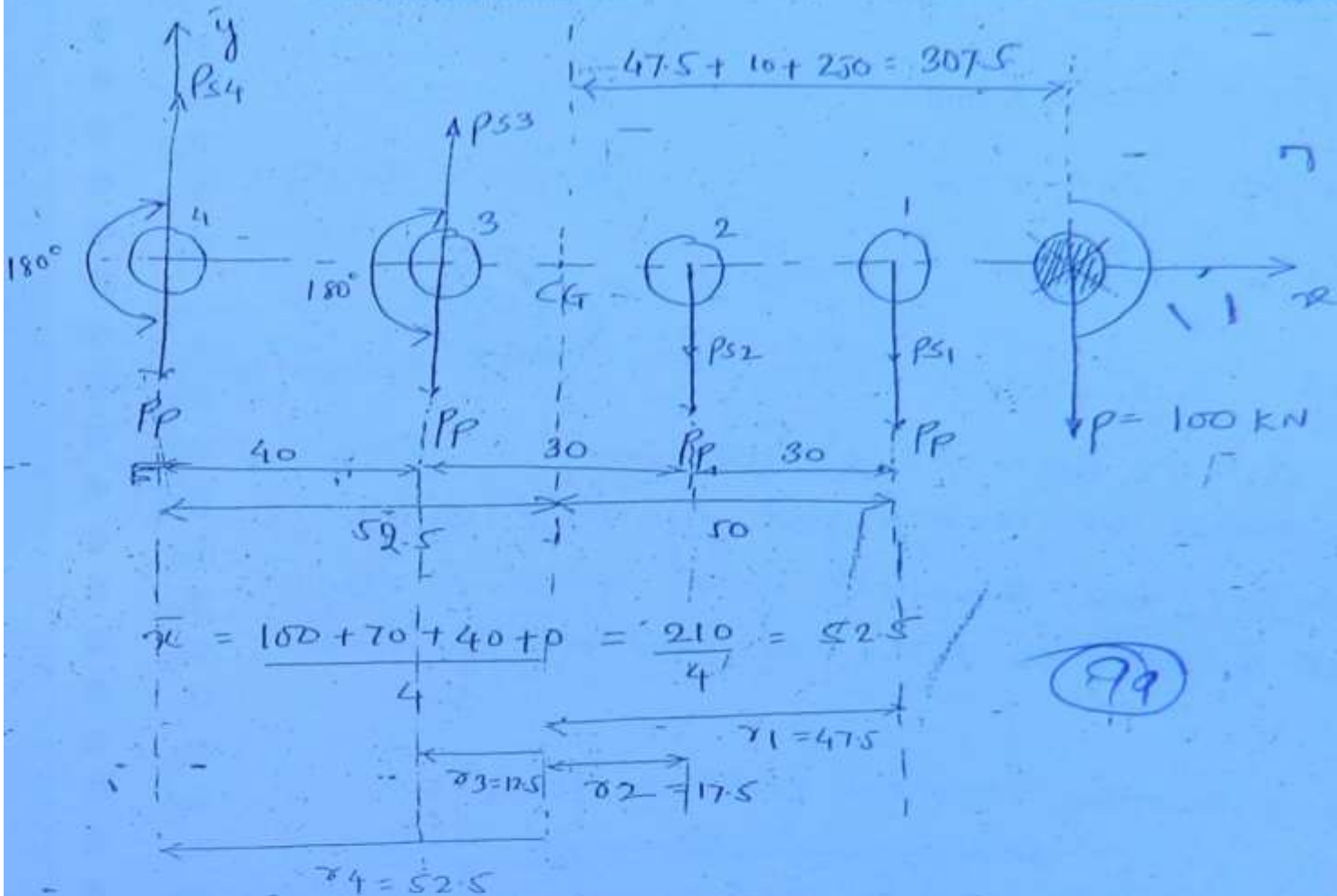
$$d \approx 32 \text{ mm}$$

Repeat the above question for the resultant force in all the rivets?

$$P_{S2} = P_{S1} \left(\frac{x_2}{x_1} \right)$$

for eccentrically loaded riveted joint as shown in the figure which of the following rivets is worst load rivet and also determine diameter of the rivets?





$$\theta_4 > \theta_1 > \theta_2 > \theta_3$$

$$P_{s4} > P_{s1} > P_{s2} > P_{s3}$$

$$(P_s)_{\max} = P_{s4}$$

$$\theta_1 = \theta_2 = 0$$

$$\theta_3 = \theta_4 = 180^\circ$$

$$R_1 = P_p + P_{s1}$$

$$R_4 = P_{s4} - P_p$$

$$P_p = \frac{100}{4} = 25 \text{ kN}$$

$$\frac{P_{s1}}{\theta_1} [\theta_1^2 + \theta_2^2 + \theta_3^2 + \theta_4^2] = P_e$$

$$\therefore P_{s1} = 266.78 \text{ kN}$$

$$P_{s4} = P_{s1} \sqrt{r_4} = 294.8 \text{ kN}$$

$$R_1 = P_p + P_{s1} = 293.78 \text{ kN}$$

$$R_4 = P_{s4} - P_p = 269.8 \text{ kN}$$

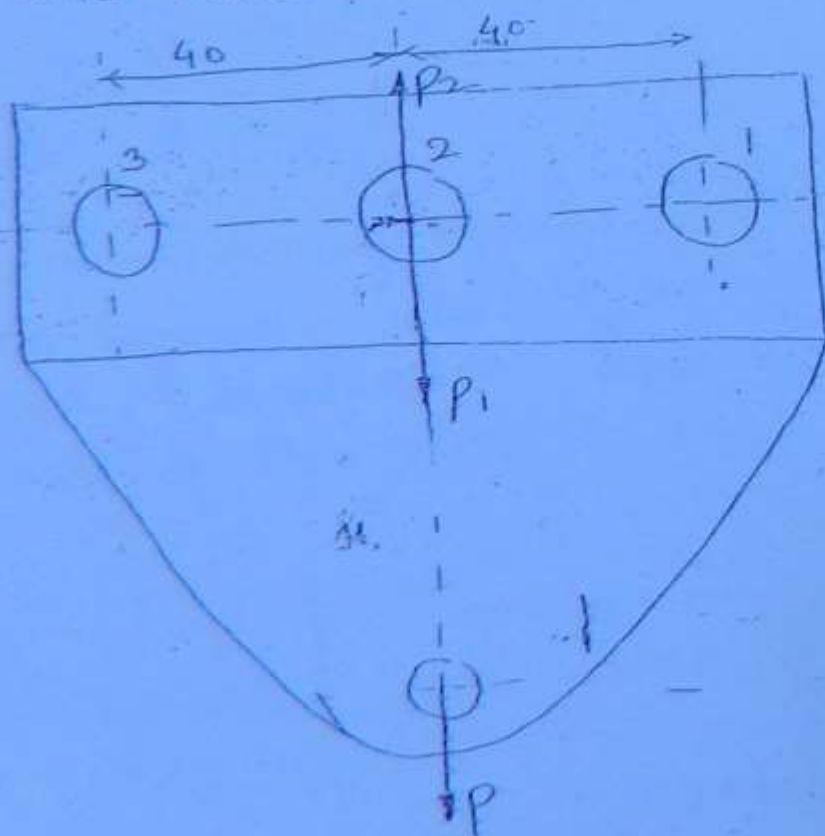
$$R_{\max} = R_1 = \frac{\pi}{4} d^2 \tau_s$$

$$293.78 = \frac{\pi}{4} d^2 \times 60$$

$$d = 78.68 \therefore d = 80 \text{ mm}$$

If all the rivets are arranged symmetrically
a single horizontal row then the worst rivet is the
rivet which is near to line of action of the
load.

for an eccentrically loaded riveted joint as
shown in the fig. determine diameter of the
rivets and worst loaded rivets?



$$T_M = 0 \rightarrow e = 0$$

$$P_{S1} = P_{S2} = P_{S3} = 0$$

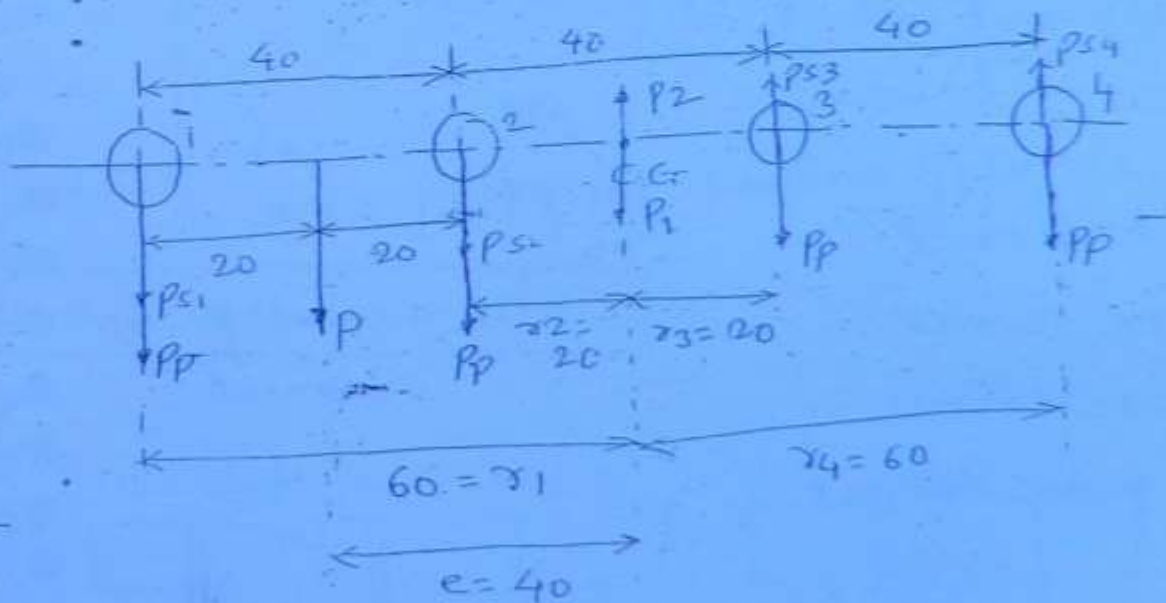
$$P_P = \frac{P}{2}$$

$$(T_{max})_{md} \leq T_{per}$$

$$\therefore \frac{P_S}{A_S} \leq T_S \Rightarrow \frac{P}{3 \times \frac{\pi}{4} d^2} \leq T_S$$

$$d \geq \sqrt{\frac{4P}{3\pi T_S}}$$

(10)

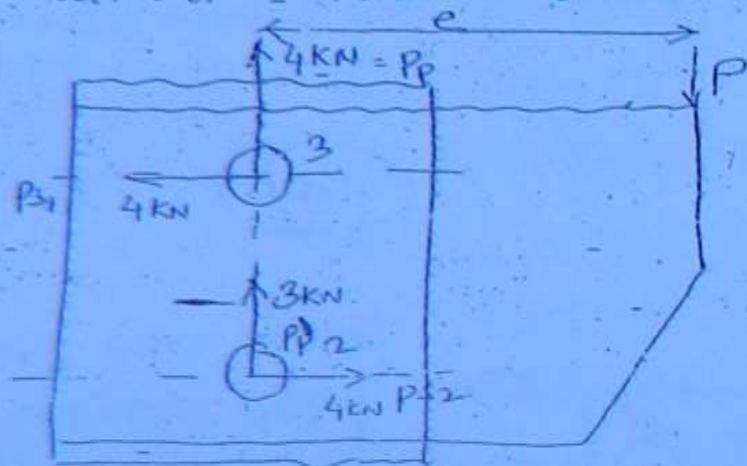


$$(P_S)_{max} = P_{S1} \text{ or } P_{S4}$$

$$D_{min} = D_1 \text{ or } D_2$$

$$R_{max} = R_1 - P_P + P_{S1} = \frac{\pi}{4} d^2 T_S$$

- for an eccentrically loaded riveted joint as shown in the fig. which of the following statements are correct
- 1. If area of rivets is 100 mm^2
 - 2. eccentricity of the load is 100 mm
 - 3. The maximum shear stress in all the rivets 50 MPa
 - 4. The total load applied on the joint is 6 kN
 - 5. resultant force in all the rivets is 5 kN



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- (a) 1, 3, 4 are correct
- (b) 1, 2, 4 are correct
- (c) 2, 3, 4 are correct
- (d) 2 and 4 are correct

$$\frac{P_1}{s_1} [2x_1^2] = P \cdot e$$

$\therefore e = \text{cannot be determined}$

$$P = \sum P_i = 2 \times 3 = 6 \text{ kN}$$

$$R_1 = R_2 = \sqrt{3^2 + 4^2} = 5 \text{ kN}$$

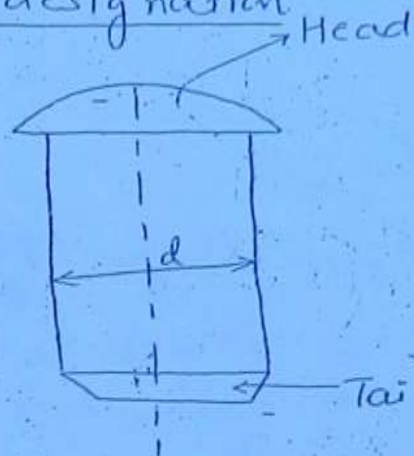
$$R_{\max} = R_1 \text{ or } R_2$$

$$\tau_{\max} = \frac{R_1 \text{ or } R_2}{A} = \frac{5000}{100} = 50 \text{ MPa}$$

6) ECCENTRICALLY LOADED BOLTED JOINT

a) Rivet designation

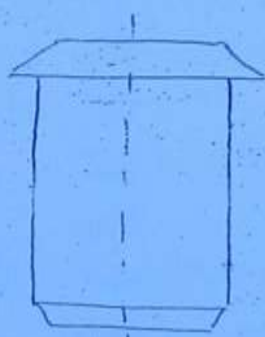
(Upset forging is used in making bolt heads)



Rivet is specified by its
(i) diameter of shank
(ii) Type of head

(i) Snap head rivet

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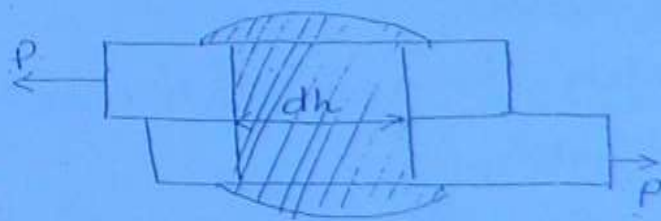
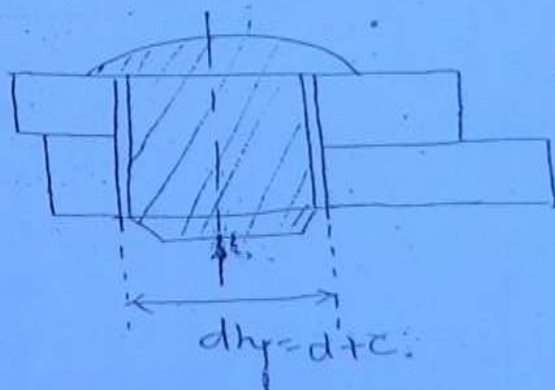
empirical formula for dia of rivet is

$$d = 6 \sqrt{t}$$

$$t \geq 8 \text{ mm}$$

above is Unwin's formula

$$\text{diameter of hole} = d + c$$

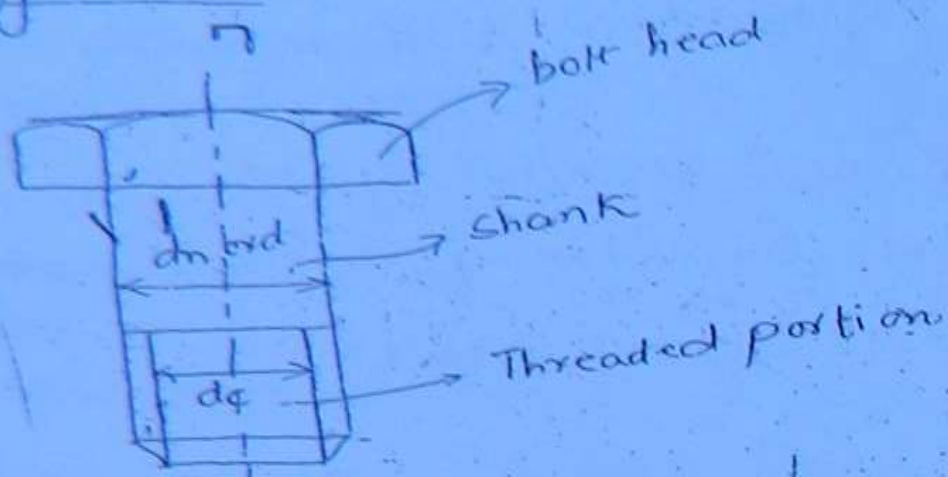


$$P_t = (p - d_h) \cdot t \cdot \sigma_t$$

$$P_s = K \cdot \frac{\pi}{4} d^2 \cdot L_s$$

during the calculation of strength of the rivet shank or rivet diameter is taken into consideration

alt designation



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d_c = core diameter

d_n = nominal diameter or major diameter

$\Rightarrow M 20$ ← nominal diameter

thread profile $\Rightarrow M 20 \times 2$ ← pitch or fine pitch

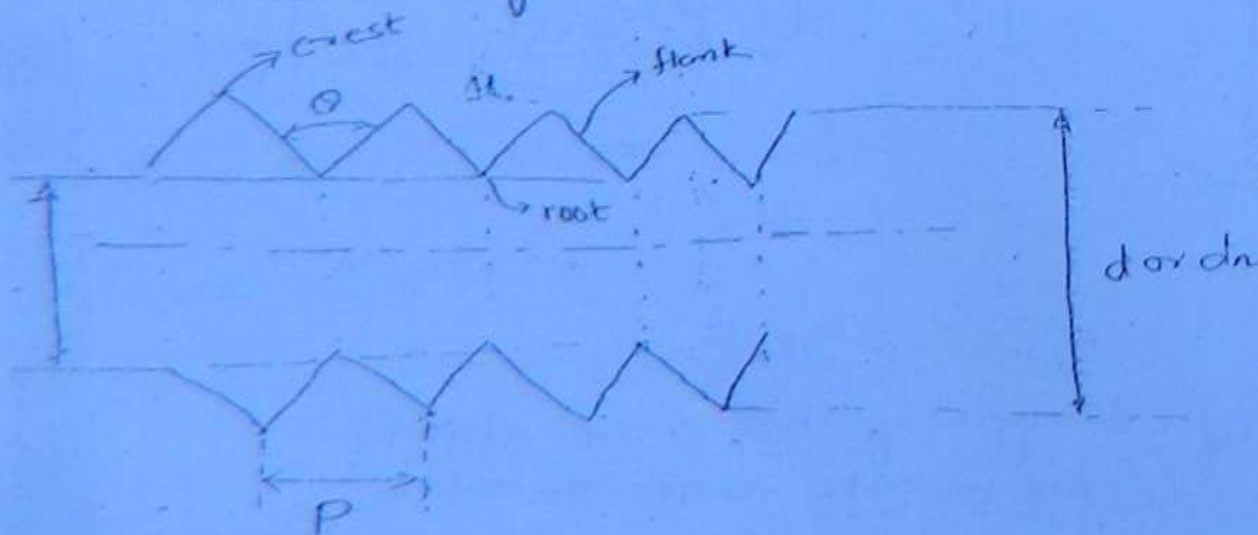
No pitch indicated means = Coarse pitch

$\Rightarrow S 20 \times 2$ (for power transmission)

↳ square thread

$\Rightarrow ACME 20 \times 2$

Coarse pitch = largest pitch available



Major or Nominal diameter: it is defined as the diameter of an imaginary circle passing through the crest of external thread or roots of an internal thread.

(105)

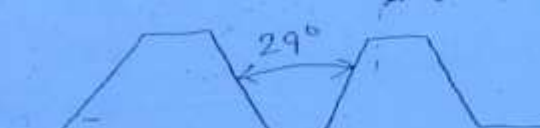
Minor diameter or Core diameter: of a thread is defined as the diameter of an imaginary circle passing through the roots of an external thread or crest of an internal thread.

Thread angle: The angle between the adjacent flank is ^{inclined} ~~included~~ angle.

$$L = n \cdot p \quad L = \text{lead}$$

①  → Square thread

⇒ in square thread, $\phi = 0^\circ$

②  → ACME thread

⇒ in acme thread, $\phi = 29^\circ$

③  → Buttress thread

⇒ angle in Buttress thread = 45°

Power Transmission in 2 direction

Transmit power in one direction eg presses

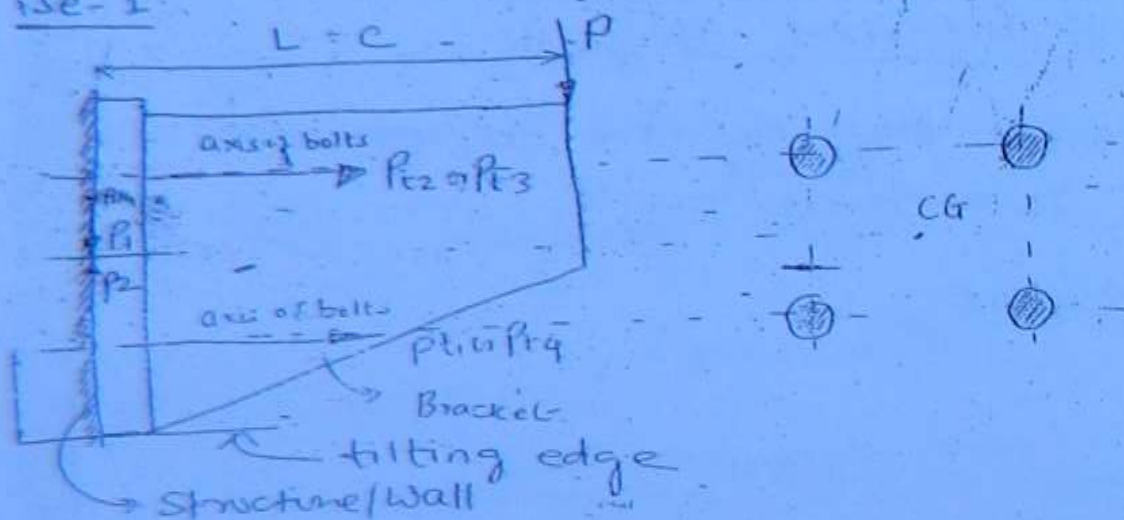
→ fine pitch

itches	2	4	6	8	10
dia					
20	✓	✓	✓	✗	✓
24	✓	✗	✗	✓	✗

→ Coarse pitch

(166)

Case-1



Load is acting in a plane $\parallel$ to plane of bolts
 Load is acting $\perp$ to axis of bolts
 bolts are subjected to shear and tensile stress.

Introduce $P_1 = P_2 = P$

$e = L$ sh.

effect of P_1

is to cause a shear force of equal magnitude at each and every bolt

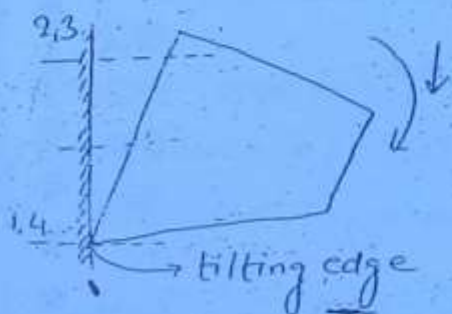
$$P_{\text{shear}} = \frac{P_1}{n} = \frac{P}{n} = \frac{P}{4} = P_s \text{ [direct shear]}$$

$$\tau_s = \frac{P_s}{\frac{\pi d_c^2}{4}} = \frac{4 P_s}{\pi d_c^2}$$

$$\tau_s = \frac{x}{d_c^2} \text{ mpa} \quad (1)$$

4] Effect of P & P_2'

$$\text{Couple} = P \cdot e = P \cdot L$$



due to this couple

as the bracket bends

bolts are elongated

due to a tensile force

The elongation of the bolts

is max which are far away from the tilting edge

$$P \propto L$$

and the tensile force in the bolts is directly

proportional to L

$$(P_{t2} = P_{t3}) > (P_{t1} = P_{t4})$$

because $(l_2 = l_3) > (l_1 = l_4)$

$$(P_t)_{\max} = P_{t2} \text{ or } P_{t3}$$

2 and 3 are worst bolts

$$(P_t)_n = P_{t1} \left(\frac{l_n}{l_1} \right) -$$

5] Calculation of $(P_t)_{\max}$

$$P_{t1} l_1 + P_{t2} l_2 + \dots = P \cdot e$$

$$\frac{P_{t1}}{l_1} [l_1^2 + l_2^2 + \dots] = P \cdot e$$

$$P_{t1} = ?$$

$$(P_t)_{\max} = P_{t1} \text{ or } P_{t2} = P_{t1} \left(\frac{l_2 \text{ or } l_3}{l_1} \right)$$

$$\begin{aligned} \underline{6] (\sigma_t)_{\max}} &= \frac{P_{t\max}}{\frac{\pi}{4} d_c^2} = \frac{4(P_t)_{\max}}{\pi d_c^2} \\ &= \frac{y}{d_c^2} \text{ mpa} \end{aligned}$$

(7) diameter of bolts (d or d_n)

here bolts are designed

by using either MSST or

MDET because they are subjected to combined stress and made up of ductile material

MSST

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{6\sigma_x^2 + 4\tau_{xy}^2}$$

$$\tau_s = \frac{S_{ys}}{N} = \frac{1}{2} \left(\frac{S_{yt}}{N} \right) = \frac{1}{2} \sqrt{(\sigma_{t\max})^2 + 4\tau_s^2}$$

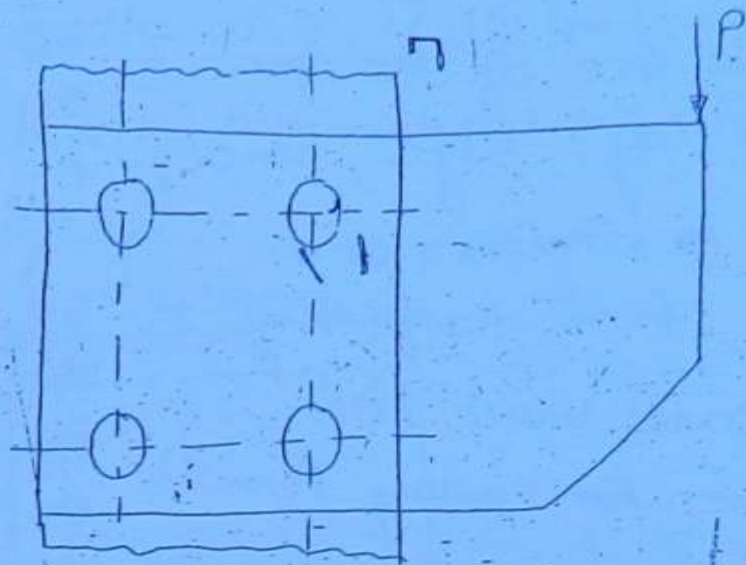
$$\tau_s = \frac{1}{2} \sqrt{\left(\frac{y}{d_c} \right)^2 + 4 \left(\frac{x}{d_c^2} \right)^2}, d_c = ?$$

MDET

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{6\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma_t = \frac{S_{yt}}{N} = \sqrt{\left(\frac{y}{d_c} \right)^2 + \left(3 \left(\frac{x}{d_c} \right)^2 \right)}, d_c = ?$$

Case-3



(109)

Load is acting in the plane but away from the plane of bolts.

⇒ subjected to primary and secondary shear stress

$$R_{\max} = \frac{\pi d_c^2 \tau_s}{4}$$

$$d_c = ?$$

$$d_m = \frac{d_c}{0.84}$$

secondary tensile force (P_s) magnitude is directly proportional to distance of the axis of the bolt from the tilting edge, hence secondary tensile force is maximum at a bolt which is far away from the tilting edge, hence in this case worst bolts are those bolts which are far away from the tilting edge.

$$P_s \propto l$$

1/6

$$(l_2 = l_3) > (l_1 = l_4)$$

$$(P_{s2} = P_{s3}) > (P_{s1} = P_{s4})$$

$$(P_s)_{\max} = P_{s2} \text{ or } P_{s3} = P_{s1} \left(\frac{l_2 \text{ or } l_3}{l_1} \right)$$

$$\frac{P_{s1}}{l_1} [l_1^2 + l_2^2 + l_3^2 + l_4^2] = P \cdot e$$

$$P_{s1} = ?$$

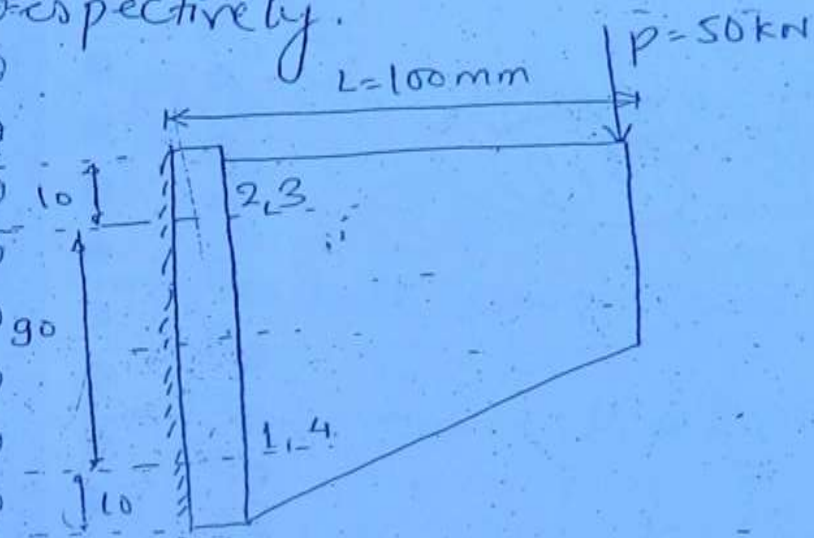
$$(P_s)_{\max} = ?$$

$$R_{\max} = R_2 \text{ or } R_3 = P_p + P_{s2} \text{ or } P_{s3} = \frac{\pi}{4} d_c^2 \sigma_t$$

$$d_c = ?$$

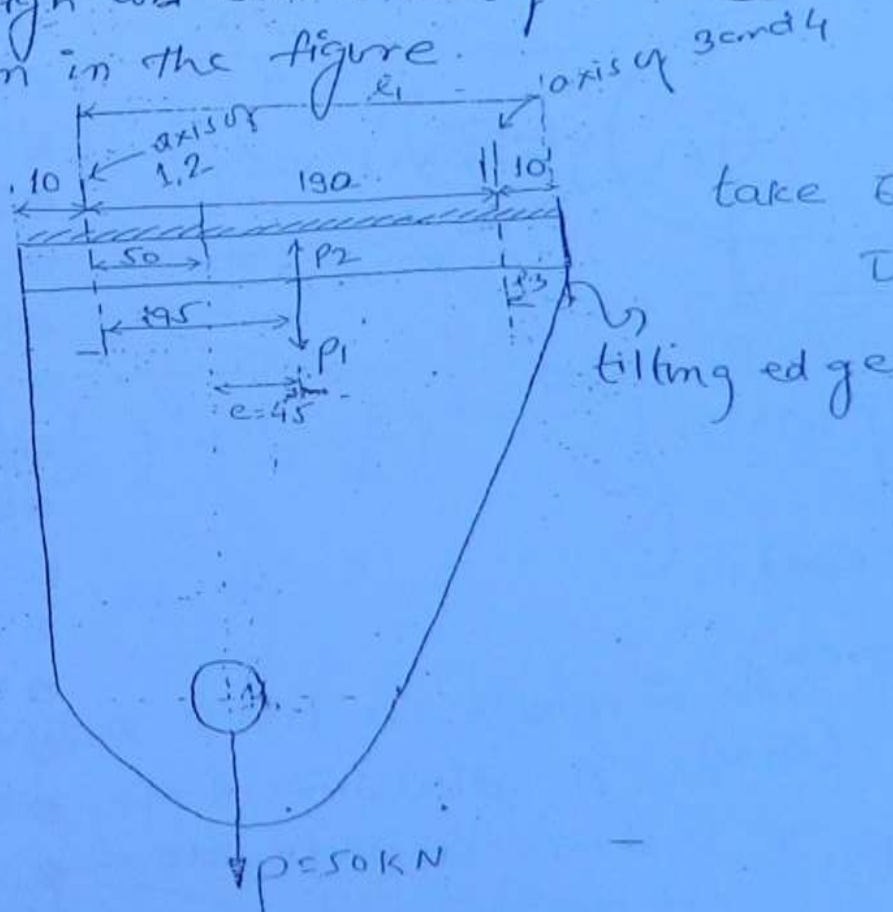
$$d_m = \frac{d_c}{0.84}$$

1. design an eccentrically loaded bolted joint as shown in the fig. if $(\sigma_t)_{per}$, tensile and shear stress of bolt material are 100 mpa and 60 mpa respectively.



111-

2. Design an eccentrically loaded bolted joint as shown in the figure.



take $\sigma_t = 100 \text{ mpa}$
 $\tau_s = 60 \text{ mpa}$

$$R_{max} = R_1 \propto R_2 = P_p + P_{s1} \propto P_{s2} = \frac{\pi}{4} d c^2 \cdot \sigma_t$$

$$P_p = \frac{P}{4} = 12.5 \times 10^3 \text{ N}$$

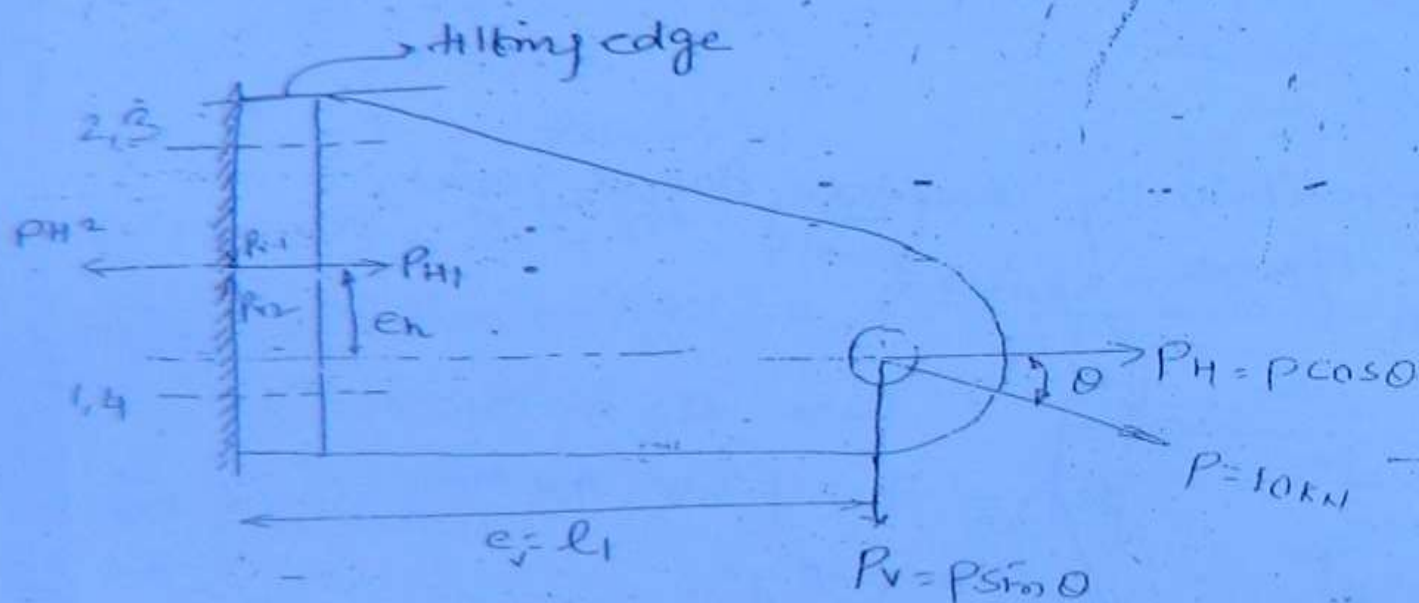
$$\frac{P_{s1}}{e_1} [2e_1^2 + 2e_3^2] = P \cdot e$$

$$P_{s1} = ?$$

$$dc = ?$$

$$dn = \frac{dc}{0.84}$$

112



P_H [-one tensile, force]

$$M_v = P_v \times l_1 \text{ (CW)}$$

$$M_H = P_H \cdot e_H \text{ (ACW)}$$

$$M_R = M_H - M_v \text{ (ACW)}$$

$$P_s = \frac{P_v}{n} \text{ [effect of } P_{v1}] -$$

$$\tau_s = \frac{4P_s}{\pi d c^2} = \frac{x}{dc^2} \rightarrow \text{① MPa}$$

(assuming M_H is more)

effect of P_H

$$P_p = \frac{P_H}{4}$$

(113)

effect of M_R :

$$P_{s1} = P_{s4} = (P_s)_{\max} = \frac{P_{s1}}{e_1} [2e_1^2 + 2le^2] = M_R$$

$$P_{s1} = ?$$

It involves both case 1 and case-2)

$$Resultant \text{ force } \max = R_1 \text{ or } R_4 = P_p + P_{s1}$$

$$\tau_{\max} = \frac{4R_{\max}}{\pi d c^2} = \frac{Y}{d c^2} \quad (2)$$

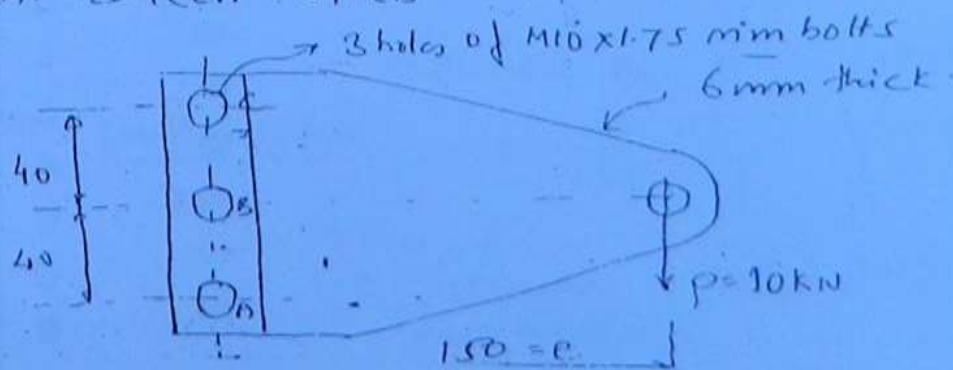
$$\tau_s = \frac{1}{2} \left[\left(\frac{Y}{d c} \right)^2 + 4 \left(\frac{x}{d c} \right)^2 \right] \quad \left. \vphantom{\tau_s} \right\} \text{MSST}$$

$$\therefore d c = ?$$

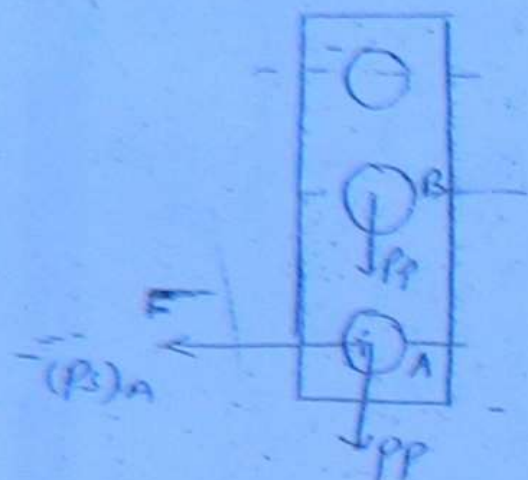
$$\tau_t = \sqrt{\left(\frac{Y}{d c} \right)^2 + 3 \left(\frac{x}{d c} \right)^2} \quad \left. \vphantom{\tau_t} \right\} \text{MDET}$$

$$\therefore d c = ?$$

for a bolted joint as shown in the fig. determine max. shear stress in bolts A and B



- a) 242.6, 42.5 (c) 42.5, 42.5
 b) 42.5, 242.6 (d) 242.6, 242.6



$$P_P = \frac{P}{3}$$

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$$R_B = P_P \quad [\because \tau_B = 0 \Rightarrow (P_r)_B = 0]$$

$$\tau_B = \frac{4R_B}{\pi d^3} = 42.5 \text{ mpa}$$

$$R_A = \sqrt{P_P^2 + (P_S)_A^2}$$

$$\frac{(P_S)_A}{\tau_A^2} [2\tau_A^2 + 0^2] = P \cdot e$$

$$\therefore (P_S)_A = ?$$

$$\tau_A = \frac{4R_A}{\pi (10)^3} = 242.6$$

⑦ WELDED JOINTS

→ Permanent Joint

Advantages

- ① 100% Leak proof joint
- ② 98 to 100% efficiency is possible

(115)

$$\eta = \frac{\text{strength of riveted joint}}{\text{strength of the solid plate}} \times 100$$

- ③ Weight of welded joint is less. (because of absence of no. of rivets and straps).
- ④ fatigue strength of welded joint is more
- ⑤ Production time is less.

ATM: To determine the dimension of the weld which are obtained by.

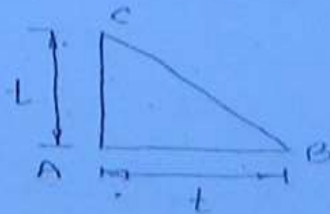
DISADVANTAGES

- ① Residual thermal stresses are developed
- ② grain structure is effected
- ③ Skilled labor is required

Types of welds used in fusion welding

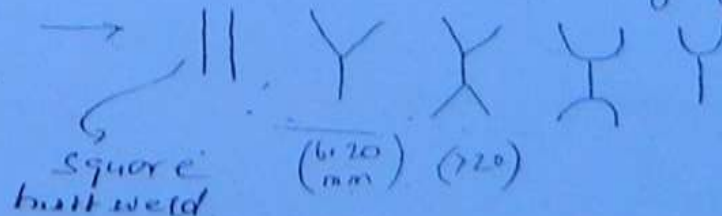
1) Tack welds →

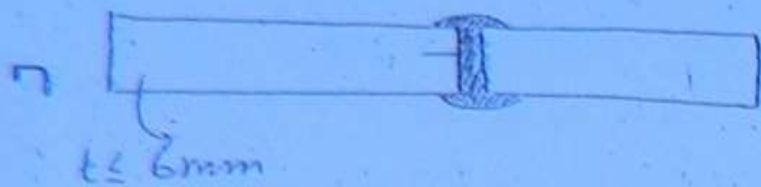
2) fillet welds →



AB and AC are called leg of fillet weld

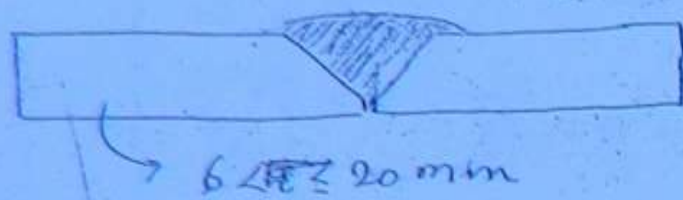
3) Butt welds →



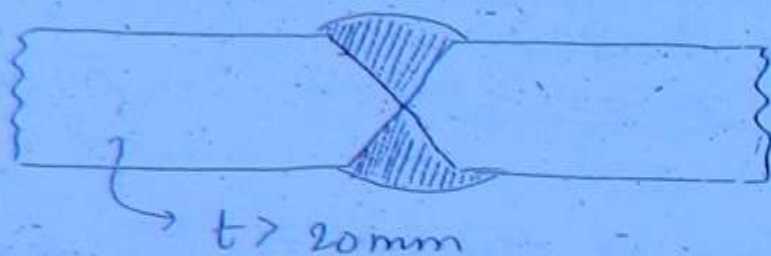


Square butt weld

116

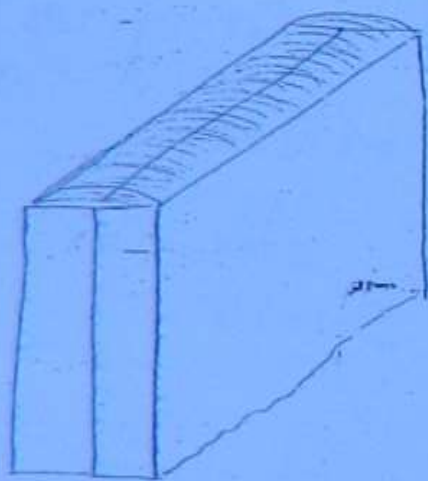


Single V butt weld

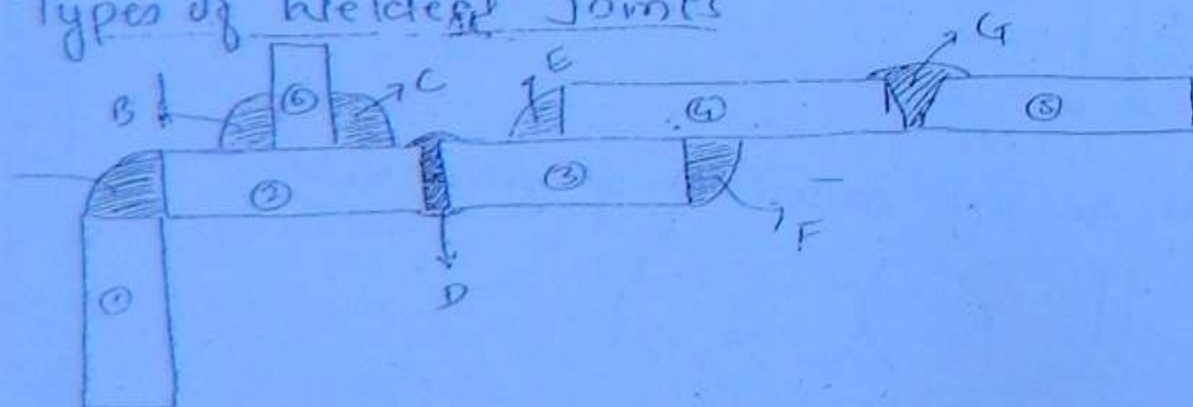


double V butt weld

Edge welds



Types of welded joints



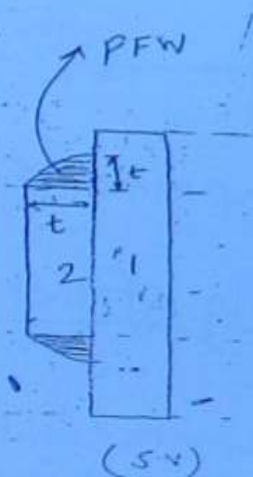
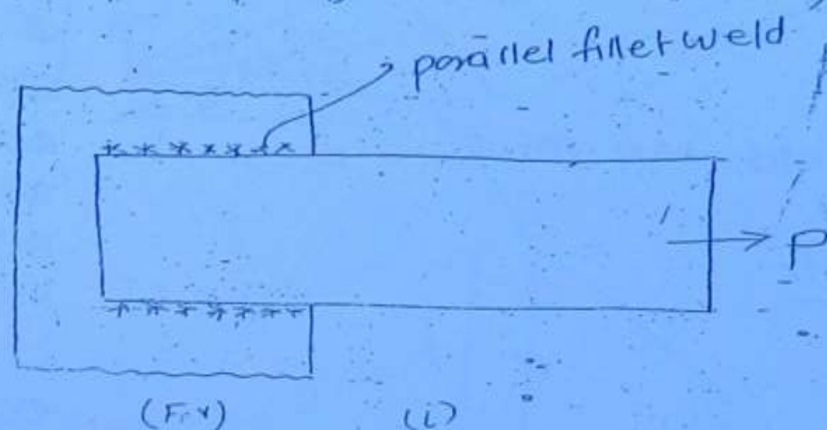
A, B, C, E, F $\Rightarrow$ Fillet welds

D and G $\Rightarrow$ Butt welds

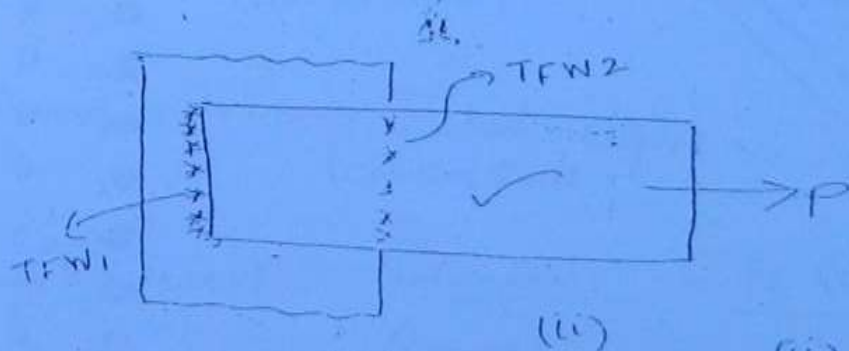
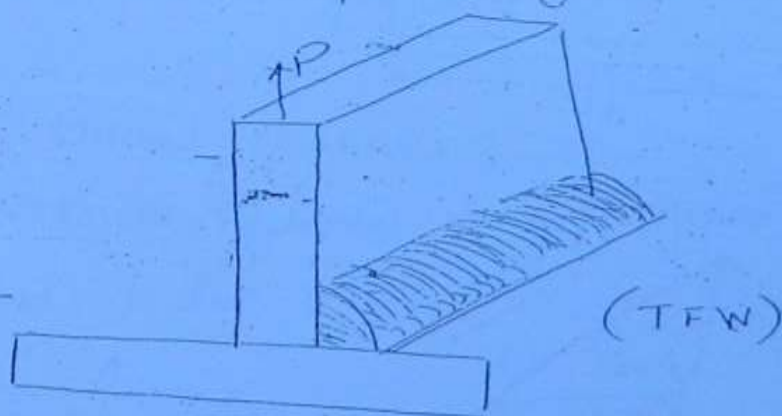
Types of fillet welds

- ① parallel fillet weld (PFW)
- ② Transverse fillet weld (Tav) (TFW)
- ③ Compound fillet weld (CFW)

(117)



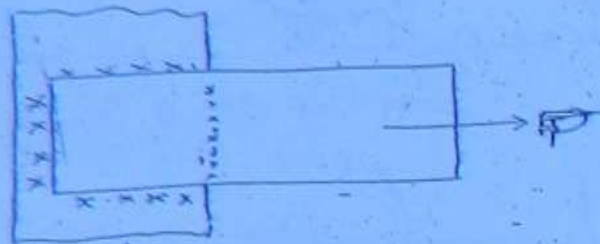
double parallel fillet weld



(ii) is better than (i)

$$P \leq P_{welds}$$

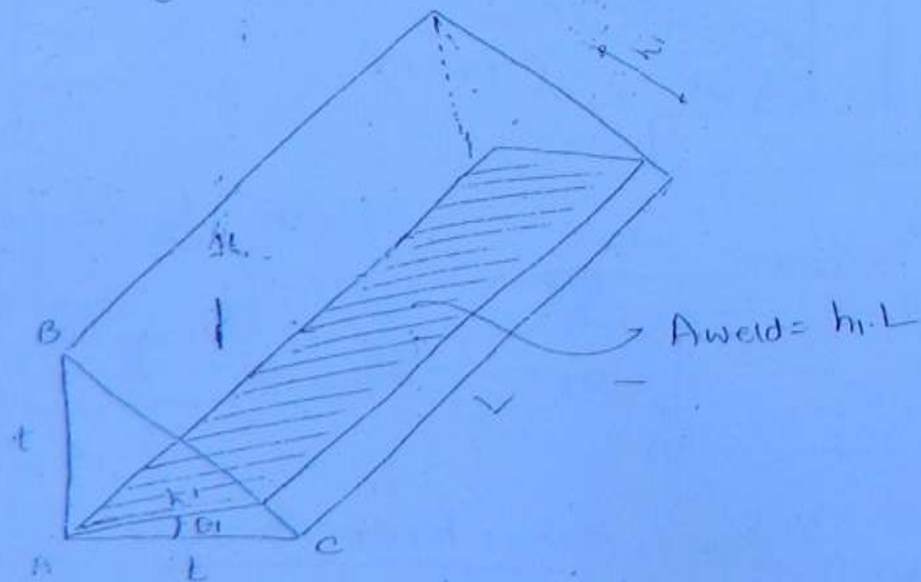
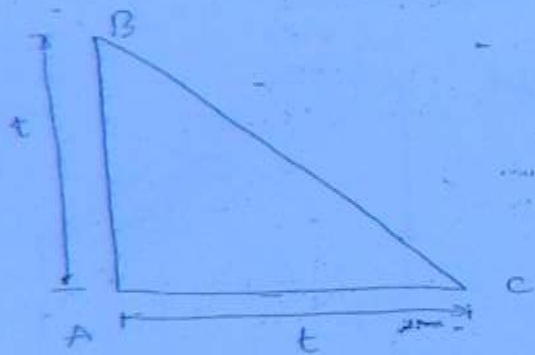
note: Strength of TFW > Strength of PFW
 hence always welding is done into direction
of load.

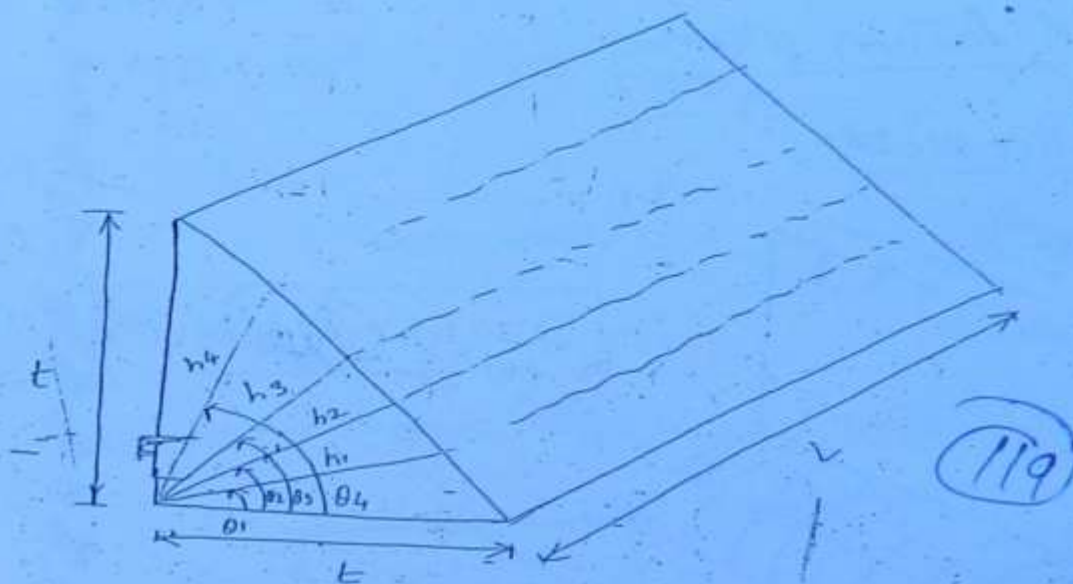


1/8

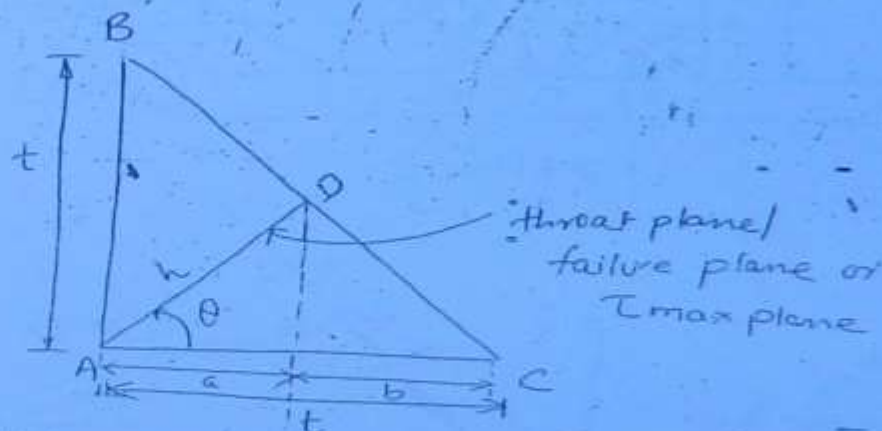
compound fillet weld. lap joint

Strength of Fillet Welds





(119)



h = throat thickness

thickness of weld along failure plane

$$A_{\text{weld}} = h \cdot L_e$$

$$h = \frac{t}{(\cos \theta + \sin \theta)}$$

$$t = a + b$$

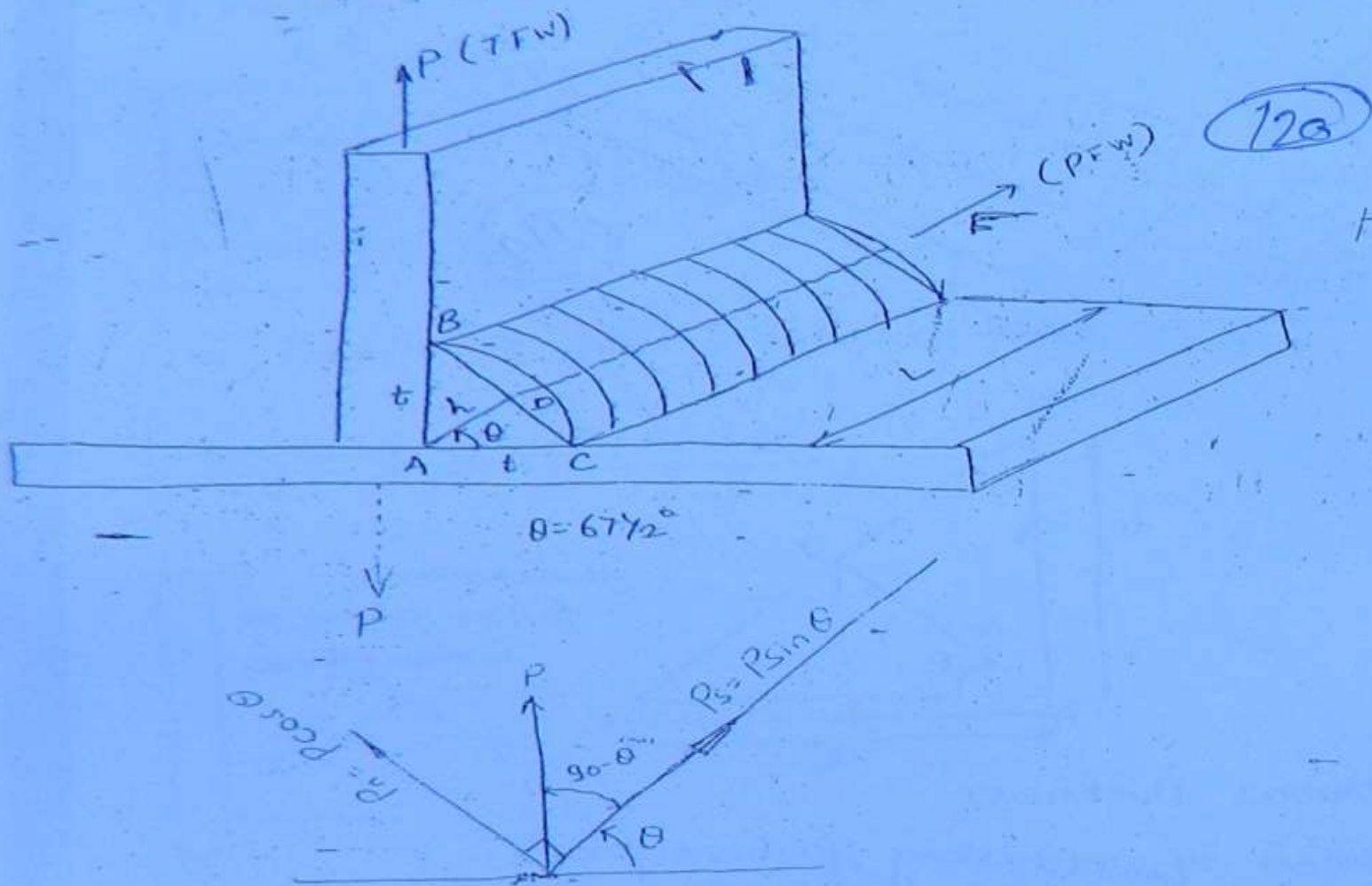
$$t = h \cos \theta + h \sin \theta$$

$$A_{\text{weld}} = h \cdot L_e = \left(\frac{t}{(\cos \theta + \sin \theta)} \right) L_e$$

$$(\sigma_s)_{\text{weld}} = \frac{P_{\text{shear force}}}{A_{\text{weld}}} = \frac{P_s (\cos \theta + \sin \theta)}{t \cdot L_e}$$

(Q) Location of failure plane

(a) Transverse fillet weld



$$P_s = P \sin \theta$$

$$T_s = \frac{P \sin \theta (\cos \theta + \sin \theta)}{t \cdot L}$$

$$\frac{dT_s}{d\theta} = 0$$

$$\frac{d}{d\theta} \left[\frac{P \sin \theta (\cos \theta + \sin \theta)}{t \cdot L} \right] = 0$$

$$\frac{d}{d\theta} [\sin \theta (\cos \theta + \sin \theta)] = 0$$

$$\tan 2\theta = -1$$

$$\therefore 2\theta = 135^\circ \therefore \theta = 67\frac{1}{2}^\circ$$

$$A_{TFW} = h \cdot L_e = \left(\frac{t}{\cos\theta + \sin\theta} \right) l_e$$

$$\therefore A_{TFW} = 0.765 t \cdot l_e$$

(121)

$l_e = L \rightarrow$ single fillet welded joint

$l_e = 2L \rightarrow$ double fillet welded joint

condition for safe design of Transverse fillet weld

$$(T_{max})_{ind} \leq T_{per}$$

$$\frac{P_s}{A_{TFW}} \leq T_{per}$$

$$\therefore \frac{P \sin\theta}{0.765 t \cdot l_e} \leq T_{per} \rightarrow \text{constant}$$

$$P \leq 0.828 t \cdot l_e \cdot T_s \Rightarrow \text{strength of Transverse fillet weld}$$

$$P_{TFW} = 0.828 t \cdot l_e \cdot T_s$$

As per AWS (American welding society)

$$P_{TFW} = 0.83 \frac{11}{2} t \cdot l_e \cdot T_s$$

→ parallel fillet weld (PFW)

$$P_s = P, P_n = 0$$

$$T_s = \frac{P_s}{t \cdot L_e} (\sin \theta + \cos \theta)$$

$$T_s = \frac{P}{t \cdot L_e} (\sin \theta + \cos \theta)$$

$$\frac{dT_s}{d\theta} = \frac{P}{t \cdot L_e} \frac{d}{d\theta} (\sin \theta + \cos \theta) = 0$$

$$\tan \theta = 1, \theta = 45^\circ$$

$$\theta_{PFW} = 45^\circ$$

$$h_{PFW} = \frac{t}{\cos \theta + \sin \theta} = \frac{t}{\sqrt{2}} = 0.707 t$$

$$A_{PFW} = 0.707 t \cdot L_e$$

for safe design of parallel fillet weld

$$(T_{max})_{ind} \leq T_{per}$$

$$\frac{P_s}{A_{weld}} \leq T_{per}$$

$$\frac{P}{0.707 t \cdot L_e} \leq T_{per}$$

$$P \leq 0.707 t \cdot L_e T_{per}$$

→ changing PFW

$$P_{PFW} = 0.707 t \cdot L_e T_s$$

$$0.825 t \cdot L_e T_s$$

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$$\frac{P_{TFW}}{P_{PFW}} = \frac{0.832 \cdot t \cdot L_e \cdot \tau_s}{0.707 \cdot t \cdot L_e \cdot \tau_s} = 1.18$$

123

$$P_{TFW} > P_{PFW}$$

⇒ for a given dimension of weld and given weld material the strength of transverse fillet weld is 18% more than the strength of parallel fillet weld.

⇒ If unless otherwise mentioned it is better to assume the fillet weld as parallel fillet weld because it is the worst weld (i.e., the shear stress induced in PFW is more than the TFW)

$$\text{or, } (\tau_{PFW} > \tau_{TFW})$$

S.No.	parameter	PFW	TFW
1	direction of load	to direction of load	⊥ to direction of load
2	P_s	P	$P \sin \theta$
3	P_n	0	$P \cos \theta$
4	θ	45°	67½°
5	h	0.707 t	0.765 t
6	A	0.707 t L _e	0.765 t L _e
7	strength	0.707 t L _e τ_s	0.832 t L _e τ_s

ES

1. The permissible stress in a fillet weld is 100 MPa . A fillet weld has equal leg lengths of 15 mm each. The allowable shear load on weld per cm length weld is

- (a) 22.5 kN (b) 15 kN (c) 7.5 kN (d) 10.6 kN ✓

1. defaut PFW

$$\begin{aligned} P_{FW} &= 0.707 \cdot t \cdot L_e \cdot \tau_s \\ &= 0.707 \times 15 \times 10 \times 100 \\ &= 10.6 \times 10^3 \text{ N} \end{aligned}$$

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2. A double fillet welded joint with parallel fillet weld of length l and leg b is subjected to tensile force P assuming uniform stress distribution the shear stress in the weld is given by

- (a) $\frac{\sqrt{2}P}{bl}$ (b) $\frac{2P}{bl}$ (c) $\frac{P}{2bl}$ (d) $\frac{P}{\sqrt{2}bl}$ ✓

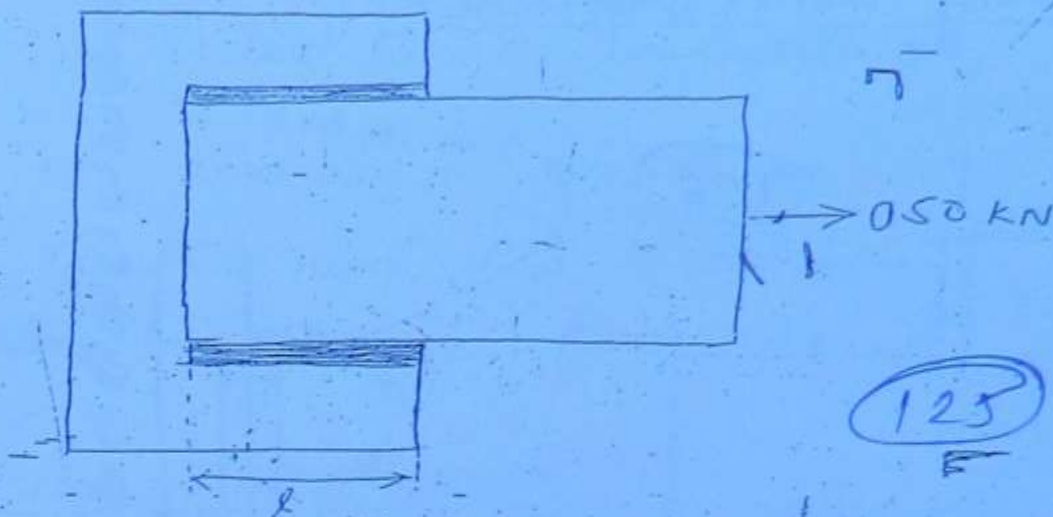
$$\text{in } P = 0.707 \cdot b \cdot 2l \cdot \tau_s$$

$$= \frac{1}{\sqrt{2}} \cdot b \cdot 2l \cdot \tau_s$$

$$\therefore \tau_s = \frac{P}{\sqrt{2}bl}$$

The two plates are joined by means of 1 fillet welds as shown in figure the size of the fillet weld is 10 mm and the allowable shear stress is 75 MPa the length of the weld is

- (a) 47 mm (b) 55 mm (c) 45 mm (d) 100 mm



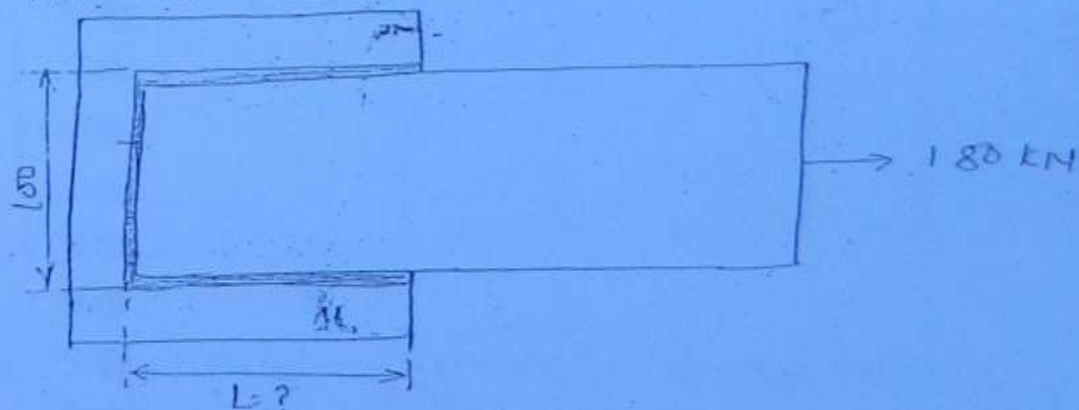
Soln $p < p_{\text{weld}}$

$$50 \times 10^3 \leq 0.707 \times 10 \times 2L \times 75$$

$$L \geq 47.14$$

Ques
Two plates are joined together by means of single transverse and double parallel fillet welds as shown in the figure the size of fillet weld is 6 mm and allowable shear load per mm of weld is 400 N find the length of each parallel fillet weld?

- (a) 170 (b) 175 (c) 185 (d) 225

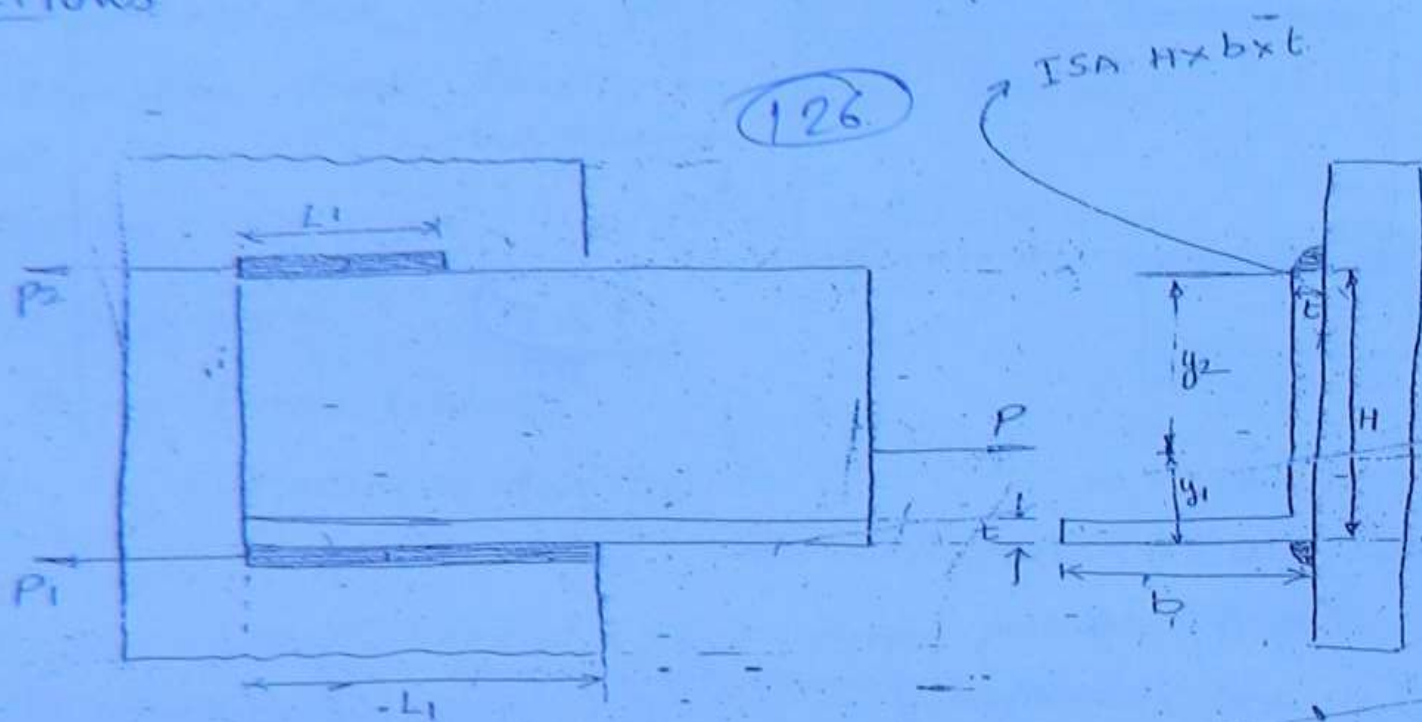


Soln Total length of weld = $2L + 100 = \frac{\text{Total Load}}{\text{Allowable shear load/mm}}$

$$2L + 100 = \frac{180 \times 10^3}{400}$$

$$\therefore L = 175$$

Fillet welding of axially loaded unsymmetrical sections



$$P = P_1 + P_2$$

$$P = 0.707 \times t \times L_1 \times \tau_s + 0.707 \times t \times L_2 \times \tau_s$$

$$P = 0.707 t \tau_s [L_1 + L_2]$$

$$L_1 + L_2 = \frac{P}{0.707 t \tau_s} \rightarrow \text{--- (I)}$$

$$\sum M = 0$$

$$P_1 y_1 - P_2 y_2 = 0$$

$$P_1 y_1 = P_2 y_2$$

$$0.707 \times t \times L_1 \times \tau_{s1} \times y_1 = 0.707 \times t \times L_2 \times \tau_{s2} \times y_2$$

$$L_1 y_1 = L_2 y_2$$

$$\frac{L_1}{L_2} = \frac{y_2}{y_1} \rightarrow \text{--- (II)}$$

Solving I and II we can get
 L_1 and L_2 can be calculated.

Q: ISA $200 \times 100 \times 10$, $P = 150$ kN

$\tau_{\text{weld}} = 75$ mpa find L_1 and L_2

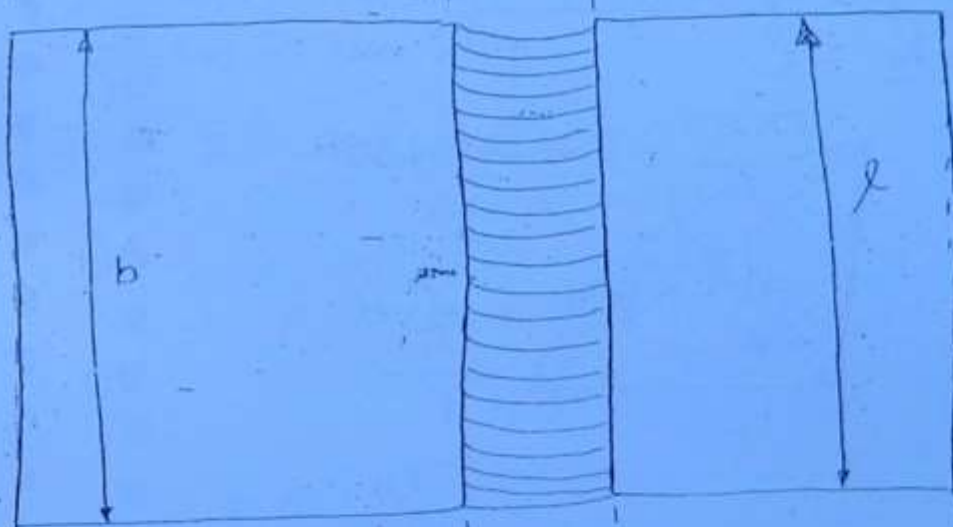
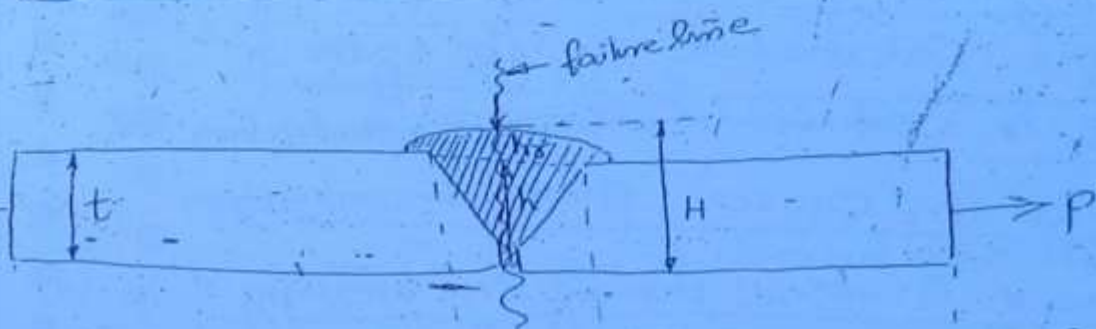
also, $y_1 = 71.8$, $y_2 = (200 - 71.8)$

$$L_1 = 191.78 \text{ mm}$$

$$L_2 = 108.81 \text{ mm}$$

(127)

DESIGN OF BUTT WELDS



for safe design of welds

$$(\sigma_{\text{max}})_{\text{ind}} \leq (\sigma_{\text{per}})_{\text{weld material}}$$

$$\frac{P}{A_{\text{weld}}} \leq (\sigma_t)_{\text{per}}$$

$$\frac{P}{h \cdot e} \leq (5t)_{\text{per}}$$

$$P \leq (5t)_{\text{per}} \times h \times l$$

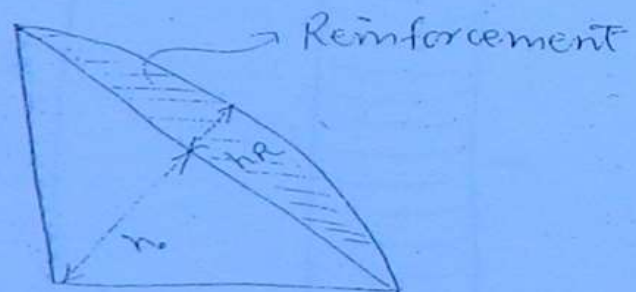
128

Now in fig. $h=t$, $l=b$

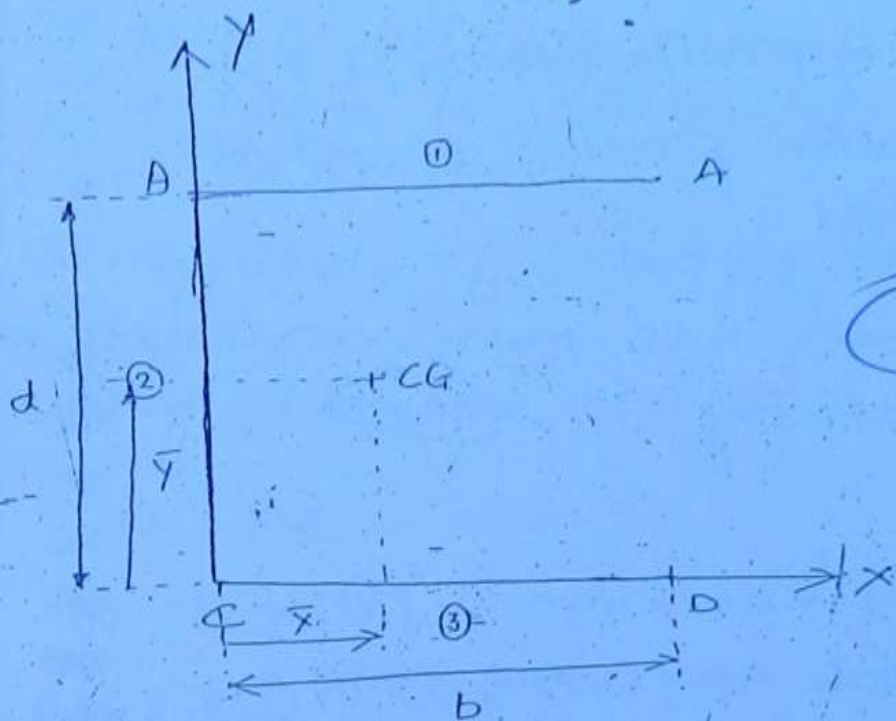
h_R = height of Reinforcement of welds

Reinforcement height is not taken into the consideration in the calculation of strength of weld because it causes stress concentration and is grinded off as it causes stress concentration.

Reinforcement is done during the welding to compensate strength of the welds in presence of weld defects.



$$h = \min [t_1 \text{ and } t_2]$$



Let CG of weld system is located at a distance of $\bar{x}$ and $\bar{y}$ from y and x axis respectively as shown in the fig

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{b \times \frac{b}{2} + d(0) + b/2}{b + d + b}$$

$$\bar{x} = \frac{b^2}{2b + d}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{b \times d + d \times \frac{d}{2} + b(0)}{b + d + b}$$

$$\bar{y} = d$$

Introduce two equal and opposite forces P_1 & P_2 through C.G in a direction parallel to applied load in such a way that

$$P_1 = P_2 = P$$

determination of eccentricity

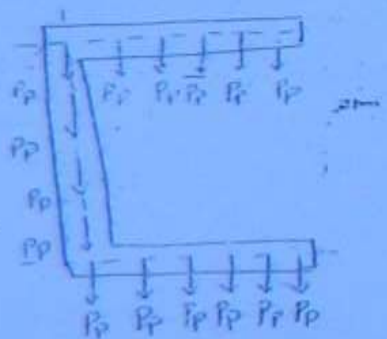
1.30

- Effect of P_1

is to cause a primary shear force (P_p) of same magnitude at each and every point on the weld system.

$$P_p = \frac{P_1}{\sum L_{\text{weld}}} = \frac{P}{2b+d} \quad \text{N/mm of weld}$$

$$\tau_p = \frac{P_p}{A_{\text{weld}}} = \frac{P_p}{0.707 \times t \times L \text{ mm}} \quad \text{MPa}$$



$$\text{or, } \tau_p = \frac{P_p}{A_{\text{weld}}} = \frac{P}{0.707 \cdot t \times (2b+d)} \quad \text{MPa}$$

5) effect of P_2

It causes a twisting moment

$$TM = P \cdot e$$

due to this twisting moment each point on the weld system is subjected to a secondary torsional shear stress.

The magnitude of secondary shear torsional shear stress is maximum at a point which is far away from the C.G. of weld system.

$$T_M = P \cdot e$$

$$R_s \tau_s \propto r$$

$$(\tau_A = \tau_D) > (\tau_B = \tau_C)$$

$$[(\tau_s)_A = (\tau_s)_D] > [(\tau_s)_B = (\tau_s)_C]$$

$$(\tau_s)_{\max} = (\tau_s)_A \text{ or } (\tau_s)_D$$

$$\tau_s = \frac{T}{Z_p} = \frac{T}{J/r} = \frac{T \cdot r}{J_{\text{weld}}}$$

$$(\tau_s)_{\max} = \frac{T (\tau_A \text{ or } \tau_D)}{J_{\text{weld}}} = ?$$

Where J_{weld} polar moment of inertia of the entire weld system about the C.G. of weld system

$$J_{\text{weld}} = J_{G1} + J_{G2} + J_{G3} + \dots$$

$$(6) (\theta_A = \theta_D) < (\theta_C = \theta_B)$$

$$(\tau_r)_{\max} \text{ where } \theta \text{ is minimum}$$

and τ_s is max

(13)

$$(TR)_{\max} = (TR)_A \text{ or } (TR)_D$$

using parallelogram law.

$$TR = \sqrt{\tau_p^2 + \tau_s^2 + 2\tau_p \tau_s \cos \theta}$$

$$(TR)_{\max} = \sqrt{(\tau_p)^2 + (\tau_s)_{\max}^2 + 2\tau_p (\tau_s)_{\max} \cos \theta_A}$$

$$= \frac{\tau (\alpha)}{t} \text{ mpa}$$

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3) Design of fillet welds

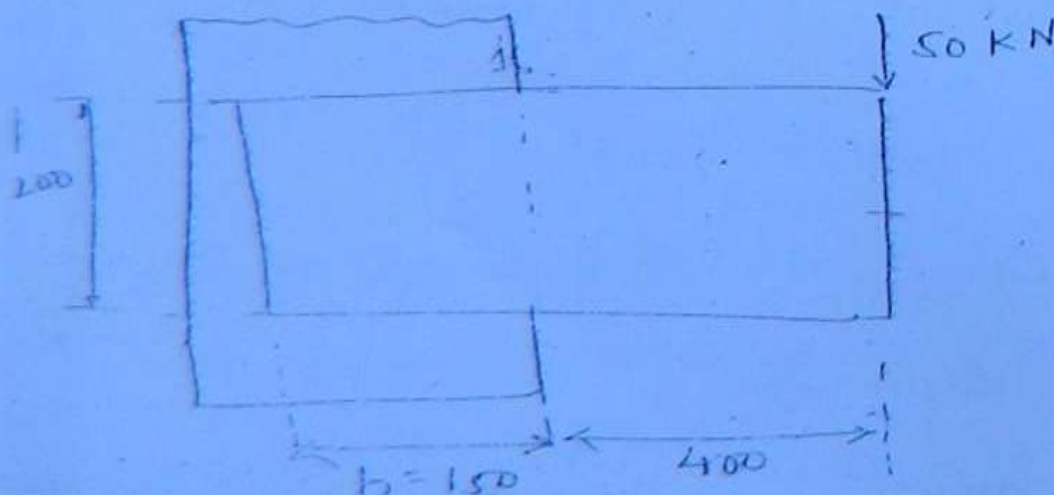
Safe design

$$(TR)_{\max} \leq \tau_{\text{per}}$$

$$\frac{\tau}{t} \leq (\tau_{\text{per}})_{\text{weld}}$$

$$t \geq \text{--- mm}$$

fig shows an eccentrically loaded welded joint
determine the fillet size if $(\sigma_t)_{\text{per}}$ is 80 MPa



$$J_w = \left[\frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{2b+d} \right] h \sim 0.707 t \text{ mm}^4$$

$$\bar{x} = \frac{b^2}{2b+d} = 45 \text{ mm}$$

$$\bar{y} = 100 \text{ mm}$$

$$\bar{e} = 505 \text{ mm}$$

$$\tau_p = \frac{P}{0.707 \times t \times l_e} = \frac{50 \times 10^3}{0.707 \times t \times 500} = \frac{141.4}{t} \text{ mpa}$$

$$\gamma_{\max} = \gamma_A \text{ or } \gamma_D = 145 \text{ mm}$$

$$J_w = 34.68.3 \times 10^3 t \text{ mm}^4$$

$$(\tau_s)_{\max} = (\tau_s)_A \text{ or } (\tau_s)_D = \frac{T \times \gamma_{\max}}{J_{\text{weld}}}$$

$$= \frac{105.56}{t} \text{ mpa}$$

$$\theta_{\min} = \theta_A \text{ or } \theta_D$$

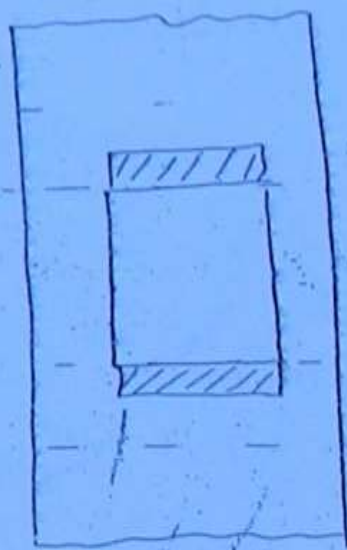
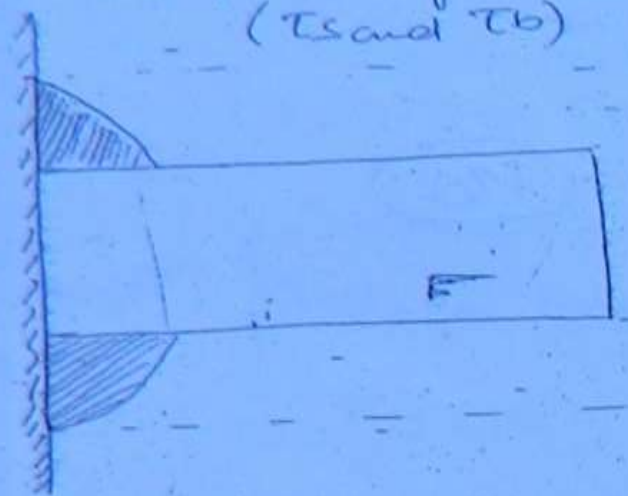
$$\cos \theta_{\min} = 0.724$$

$$(\tau_R)_{\max} = \frac{1162.1}{t} \text{ mpa} \leq 80$$

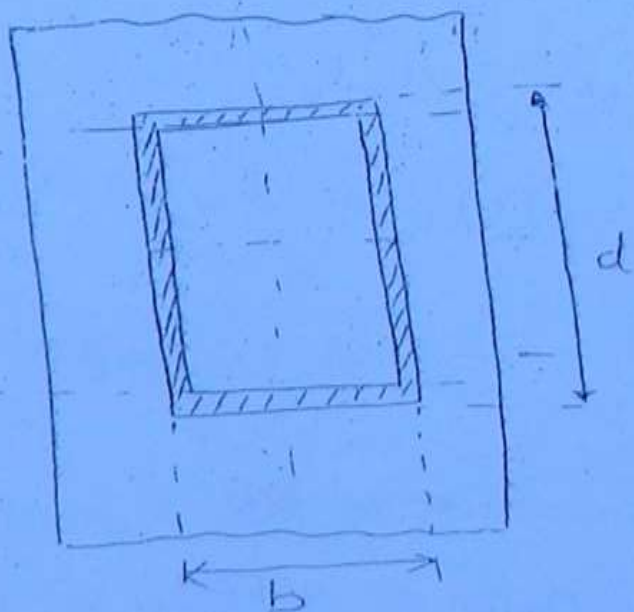
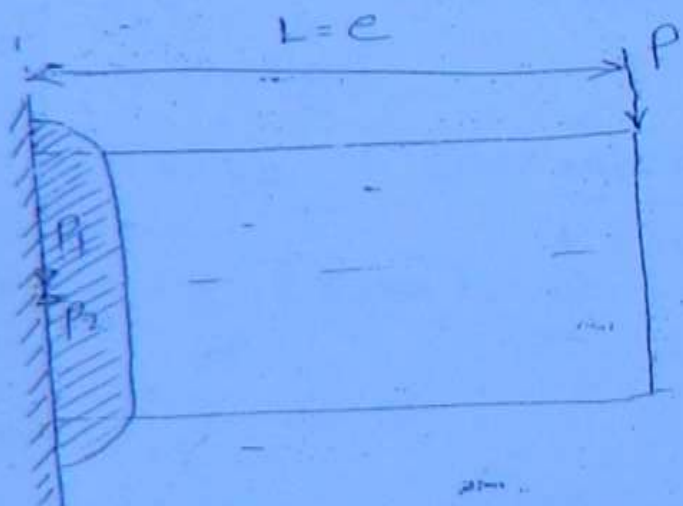
$$t > 14.5 \text{ mm}$$

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Sec II (Moment is acting in a plane $\perp$ or to the plane of welds) (T_s and T_b)



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Effect of P_1

is to cause a shear stress of same magnitude at each and every point on the weld

$$T_s = \frac{P_1}{A_w} = \frac{P}{0.707 L d} = \frac{P}{0.707 t \times (2b + 2d)} \quad \text{MPa}$$

3) Effect of p and p_2

$$M = p \cdot e = p \times L$$

due to this bending moment as the bar is subjected to bending the welds also subjected to bending stresses.

$$\sigma_b = \frac{M}{Z_{\text{weld}}} = \frac{-Y}{t} \text{ MPa} \quad (2)$$

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where

$$Z_w = \frac{I_w}{Y_{\text{max}}} = \frac{t}{12} \text{ MPa/mm}^3$$

(iii) design of fillet weld

Here fillet welds are designed by using maximum shear stress theory or maximum Distortion energy theory as fillet welds are subjected to combined stresses.

$$\begin{aligned} \text{MSST} \Rightarrow \tau_s &= \frac{\sigma_1}{2} = \frac{1}{2} \sqrt{\sigma_b^2 + 4\tau_s^2} \\ &= \frac{1}{2} \sqrt{\left(\frac{Y}{t}\right)^2 + 4\left(\frac{X}{t}\right)^2} \\ \therefore t &\geq \text{mm} \end{aligned}$$

$$\begin{aligned} \text{MDET} \Rightarrow \sigma_t &= \frac{\sigma_1}{2} = \sqrt{\sigma_b^2 + 3\tau_s^2} \\ &= \sqrt{\left(\frac{Y}{t}\right)^2 + 3\left(\frac{X}{t}\right)^2} \\ \therefore t &\geq \text{mm} \end{aligned}$$

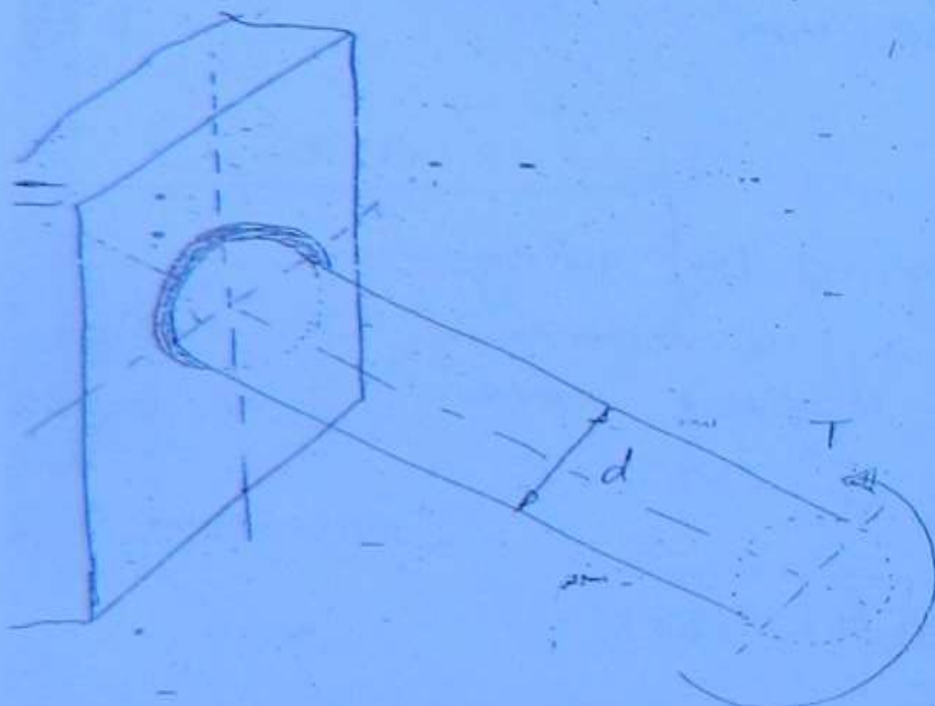
Ass-3



$$Z_w = \frac{\pi d^3}{4} \times h$$

$$J_w = \frac{\pi d^4}{4} \times h$$

Fillet welds under pure Torsion



$$\tau_s = \frac{T}{Z_p} = \frac{T \cdot r}{J_w}$$

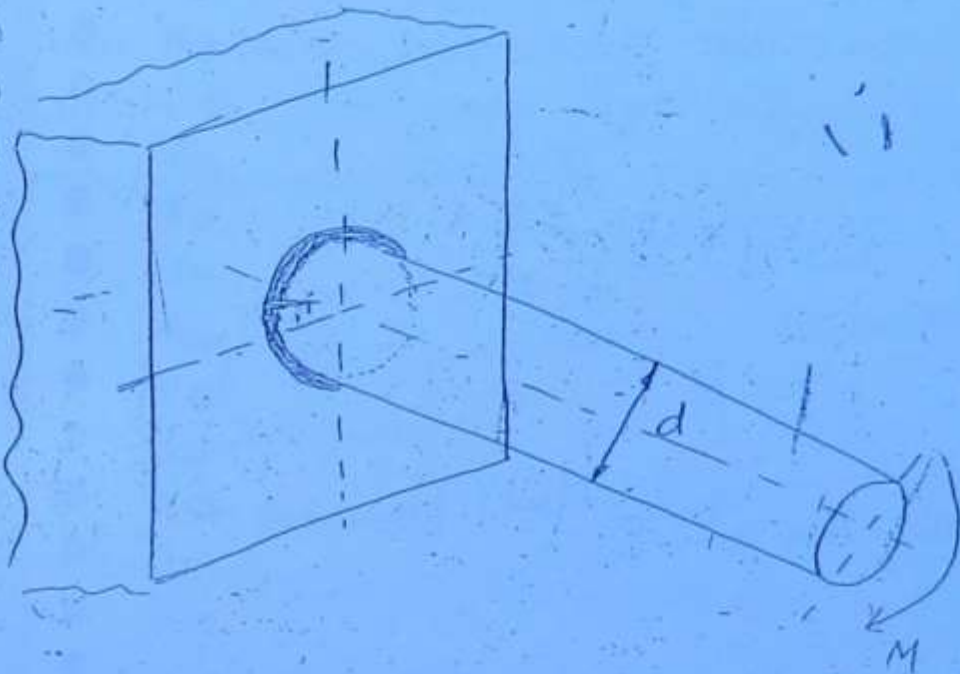
$$= \frac{T \cdot d/2}{\frac{\pi d^3}{4} \times \frac{t}{\sqrt{2}}} = \frac{1}{4} \frac{\sqrt{2} T \cdot d}{\pi d^3 t}$$

$$\tau_s = \frac{2.83 T}{\pi d^2 t}$$

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Case-4

fillet weld under pure bending

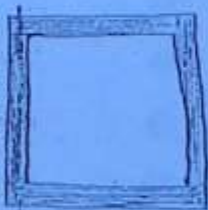


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$$\sigma_b = \frac{M}{Z}$$

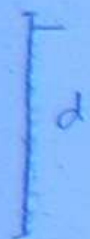
$$= \frac{M}{\frac{\pi d^2}{4} \times \frac{t}{\sqrt{2}}} = \frac{4\sqrt{2} M}{\pi d^2 t}$$

$$\sigma_b = \frac{5.66 M}{\pi d^2 t}$$

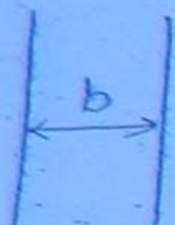


$$Z_w = \left(b d + \frac{d^2}{3} \right) h$$

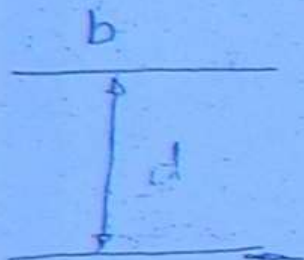
$$I_w = \frac{(b+d)^3}{6} h$$



$$Z_w = \frac{d^2}{6} h, \quad I_w = \frac{d^3}{12} h$$



$$Z_w = \frac{d^2}{2} h, \quad I_w = \frac{d(3b + d^2)}{6} h$$

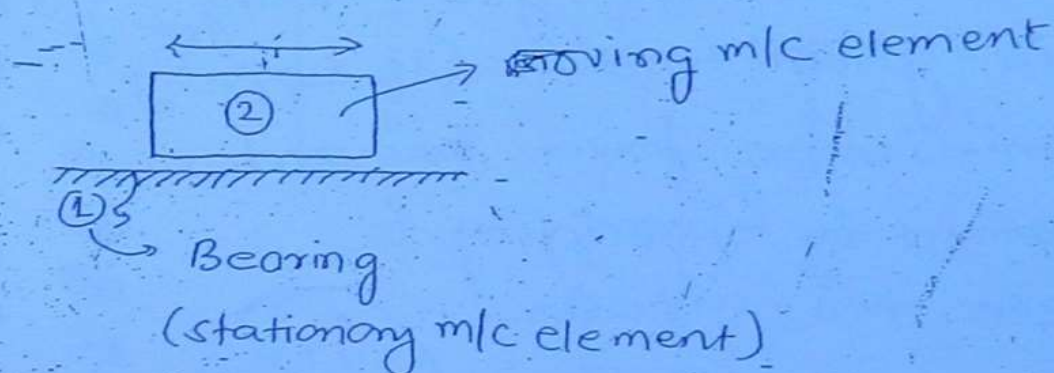


$$Z_w = bdh, \quad I_w = \left(\frac{b^3}{6} + \frac{3bd^2}{6} \right) h$$

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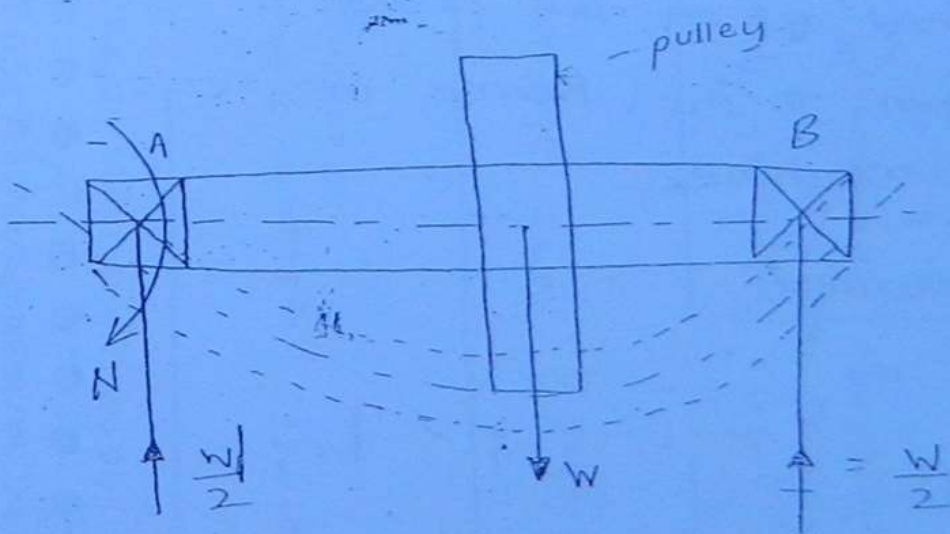
⑧ BEARINGS

- * Whenever relative motion takes place between
- Two machine elements the machine element
- which is stationary and supporting the moving
- machine element is called as a bearing



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- ⇒ Bearing is defined as a machine element
- whose function is to support a rotating
- machine element (i.e., a shaft) and to guide
- or confines its motion, while preventing its
- motion in the direction of applied load



because of relative motion between the shaft and bearing surfaces, always some amount of power loss takes place in overcoming the frictional resistance and wear of the surfaces takes place due to metal to metal contact hence a bearing is said to be a good bearing which performs its given function (i.e., supporting the shaft) with minimum power loss and wear, this is obtained by providing lubrication between two surfaces. (140)

Functions of bearing -

To support the shaft and axle and holds in its correct position

It ensures free rotation of the shaft and axle with minimum friction

Takes up loads that act on the shaft and transmits them to the frame or foundations of the machine.

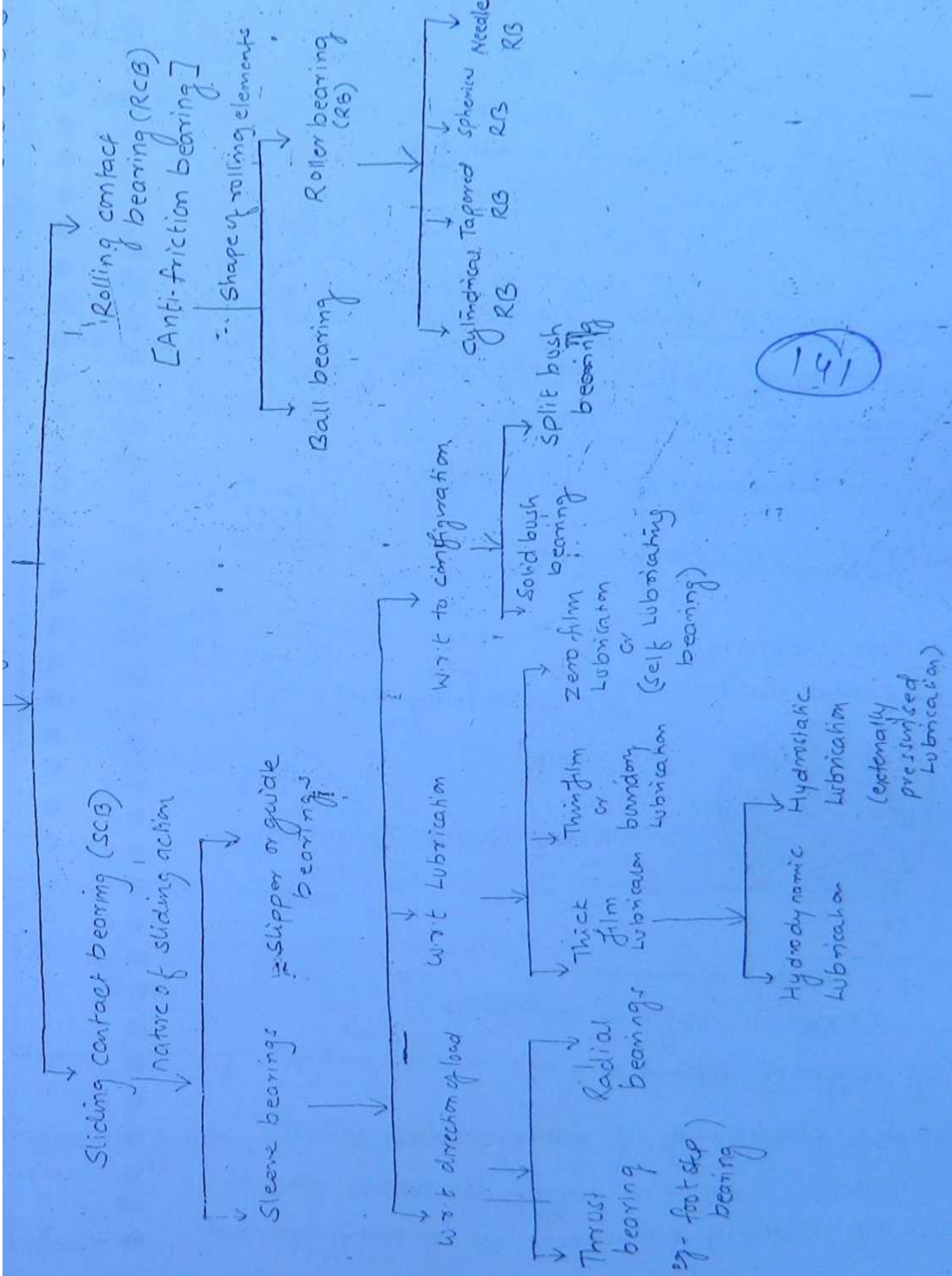
Classification of bearings

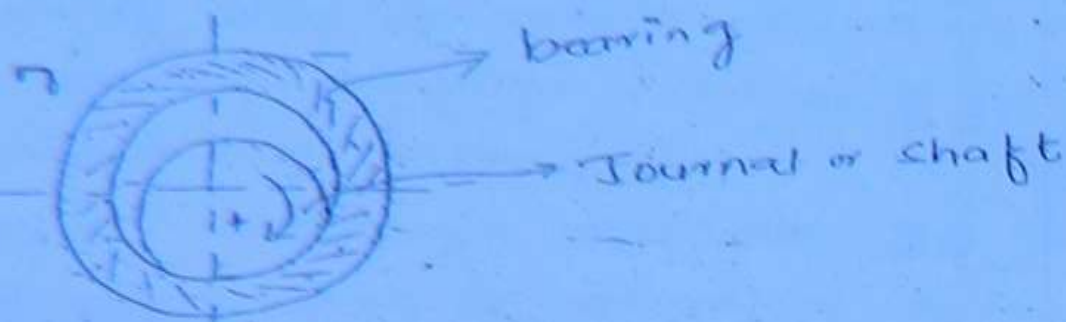
Press-fit WU

Rolling & Sliding

(Press) roller & (Press) screw

and hence called as anti friction bearing.





Sliding contact bearing (liquid Lubricants) -

Journal = a portion of shaft which is inside the bearing is called Journal

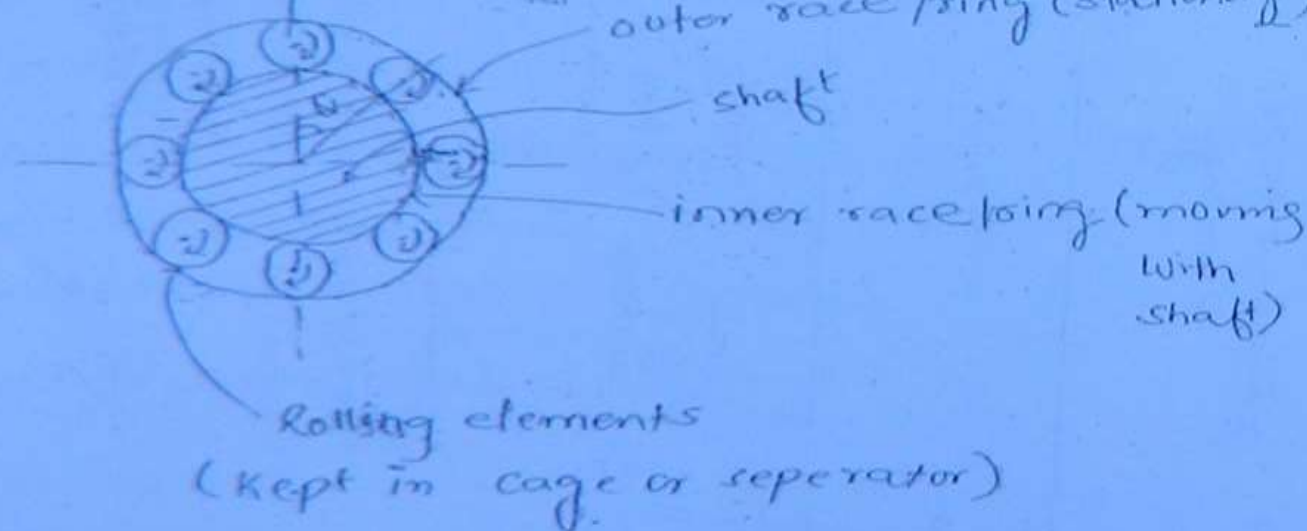
$$\{ L_{\text{Journal}} = L_{\text{bearing}} \} \quad L = \text{Length}$$

$$(\text{Diameter})_{\text{Journal}} = (\text{Diameter})_{\text{bearing}}$$

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Rolling contact bearing

where semi solid Lubricants are used.



Function of Cage or Separator -

To prevent clustering of rolling elements

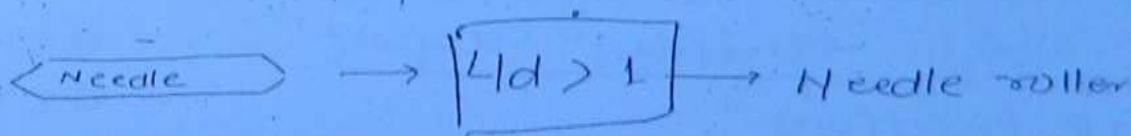
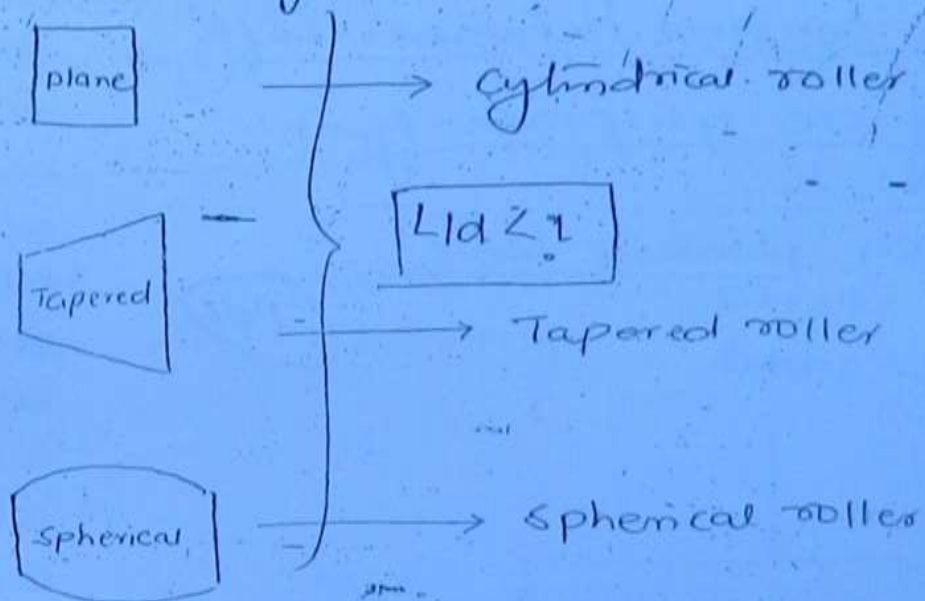
To maintain relative angle between the adjacent rolling elements or evenly spaced

3. To avoid contact or to separate the adjacent rolling elements.



⇒ In case of hollow shaft outer race will be moving and inner race will be stationary.

Roller bearings



It is used when radial space is constrained.

There is no cage in Needle roller bearing.

Sleeve bearing: sliding action along an arc of circle.

Slipper or guide bearing: sliding action along a straight line.
eg (lathe).

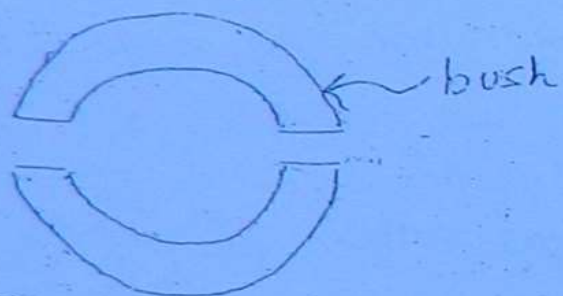
Radial bearings: They are used to support the radial loads (ie, loads per to the shaft axis).

Thrust bearings: Support a shaft where load is acting along the shaft axis, eg footstep bearing.

Zero film lubrication - materials are having property of Self Lubricating, eg CI and graphite.

generally bushes are provided between shaft and bearing (they are split piece-type).

Split bush bearing - eg plummer block.



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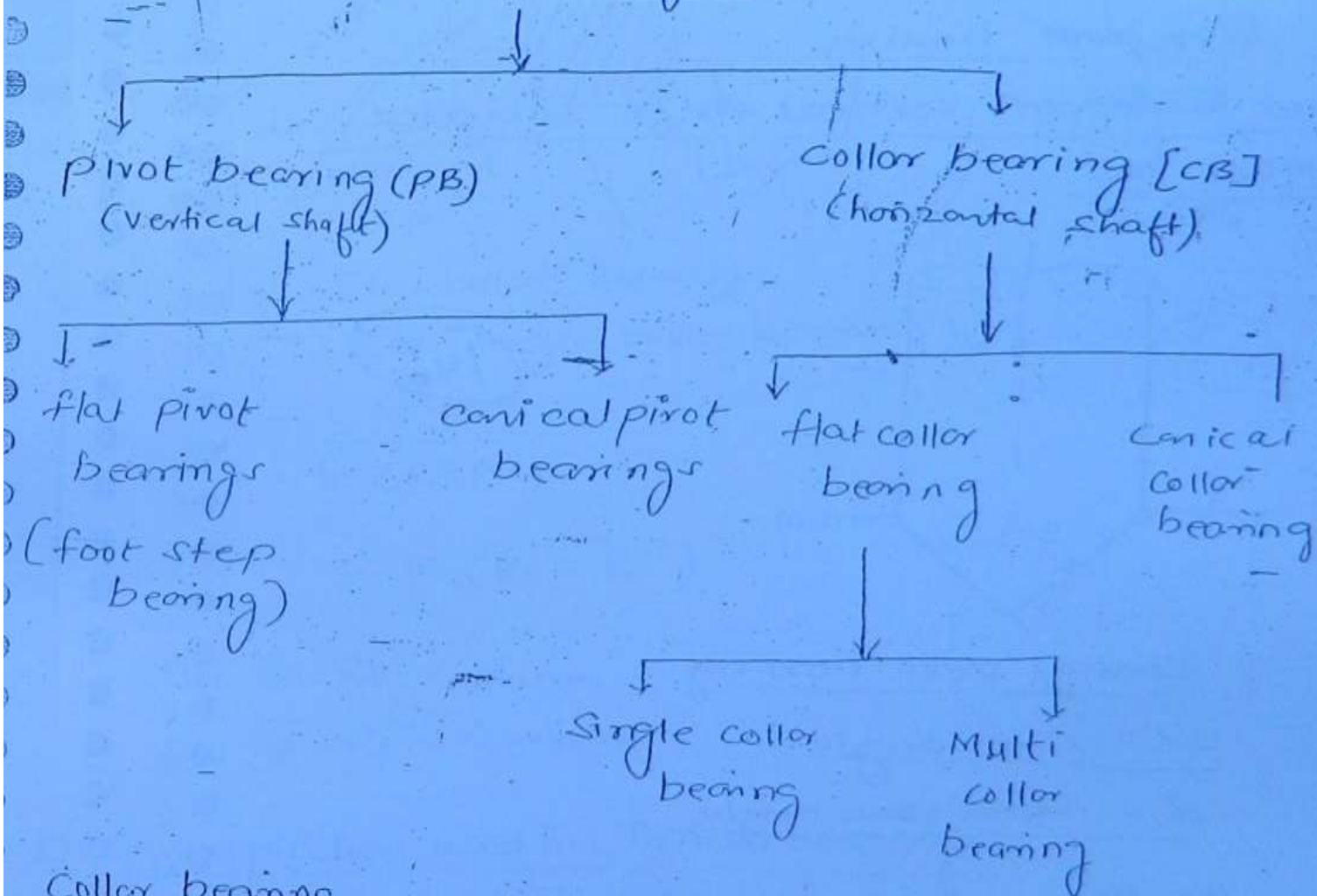
⇒ plummer block is used to support a lengthy shaft which requires support at intermediate locations.

Thrust bearings (TB)

They are used to support a shaft which is subjected to thrust loads i.e., loads acting on along the shaft axis.

Thrust bearings

(145)

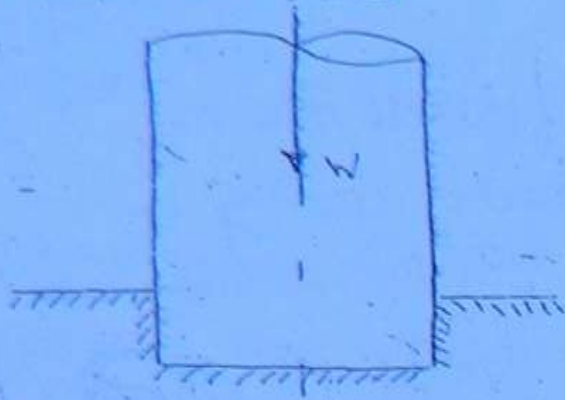


Collar bearing

R_i = shaft radius

To get equation of pivot bearing substitute $R_i = 0$ and $R_o = R$ in the equation of collar bearings.

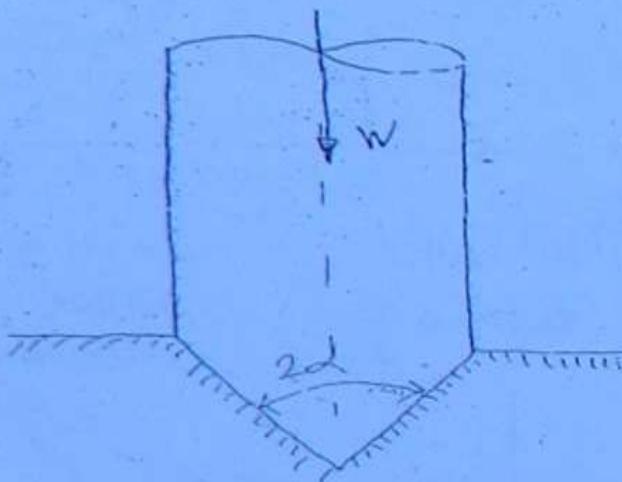
pivot bearing



foot step bearing

flat pivot bearing

used to support vertical shaft subjected to thrust load.



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- Conical pivot bearing

$2\alpha =$ cone angle

$\alpha =$ semi cone angle

When $\alpha = 90^\circ$ conical pivot bearing
becomes as flat pivot bearing

Collar bearing

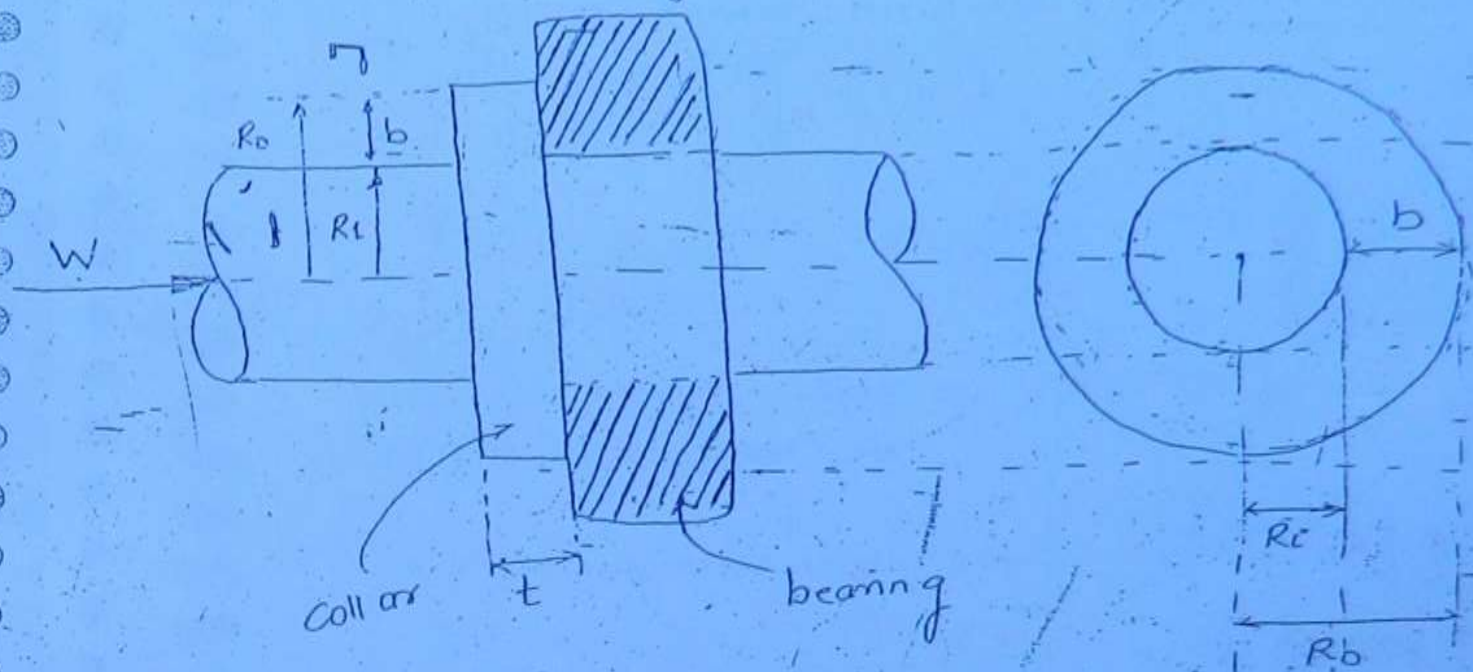


Fig: Flat collar bearing
(or single collar bearing)

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$$P = \frac{W}{\pi (R_o^2 - R_i^2)}$$

$$W = P \pi (R_o^2 - R_i^2)$$

$$W \Rightarrow R_o = \text{mm}$$

$$2W \Rightarrow R_o = 2 \times \text{mm}$$

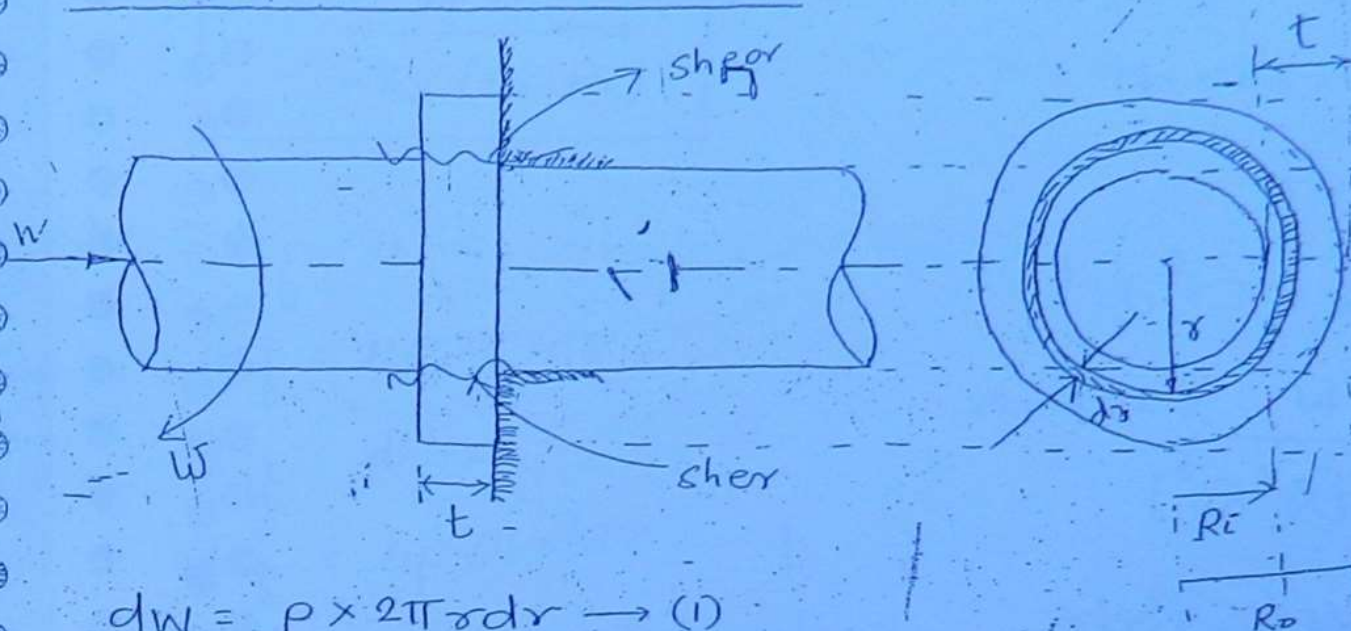
Design equation used in Thrust bearings

[P, W and T_f]

Uniform pressure theory [Pressure = constant]
(UPT)

and 2nd uniform wear theory
(UWT)

SINGLE COLLAR BEARING



$$dw = p \times 2\pi r dr \rightarrow (1)$$

$$W = \int_{R_i}^{R_o} dw = \int_{R_i}^{R_o} p \times 2\pi r dr \rightarrow (2)$$

as per uniform pressure theory ($p=c$)

$$W = p \times 2\pi \int_{R_i}^{R_o} r dr$$

$$W = p \times 2\pi \times \frac{R_o^2 - R_i^2}{2}$$

$$W = \pi \times p (R_o^2 - R_i^2) \rightarrow (3)$$

as per UPT

$$P_{upt} = \frac{W}{\pi (R_o^2 - R_i^2)} \rightarrow (4)$$

$$dF_f = \dot{u} \cdot dw$$

$$= u \cdot p \times 2\pi r \, dr$$

$$dT_f = dF_f \times r$$

$$= u \cdot p \cdot 2\pi r^2 \, dr$$

$$dT_f = \frac{u \times W}{\pi [R_o^2 - R_i^2]} \times 2\pi r^2 \, dr$$

$$dT_f = \frac{2 u W}{R_o^2 - R_i^2} r^2 \, dr$$

$$T_f = \int_{R_i}^{R_o} \frac{2 u W}{R_o^2 - R_i^2} r^2 \, dr$$

$$T_f = \frac{2 u W}{R_o^2 - R_i^2} \times \frac{R_o^3 - R_i^3}{3}$$

$$T_f = \frac{2}{3} u W \left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]$$

for uniform wear theory

$$Pr = \text{constant}$$

$$W = 2\pi Pr \int_{R_i}^{R_o} dr$$

$$W = 2\pi p \cdot r (R_o - R_i)$$

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$$p = \frac{W}{2\pi r (R_o - R_i)}$$

Now, $dF_f = \mu dw$

$$dF_f = \mu p 2\pi r dr$$

$$dT_f = dF_f \times r$$

$$dT_f = \mu p 2\pi r^2 dr$$

$$dT_f = \frac{\mu W \times 2\pi r^2 dr}{2\pi r (R_o - R_i)}$$

$$dT_f = \frac{\mu W}{R_o - R_i} r dr$$

$$T_f = \frac{\mu W}{R_o - R_i} \int_{R_i}^{R_o} r dr$$

$$T_f = \frac{\mu W}{R_o - R_i} \times \frac{R_o^2 - R_i^2}{2}$$

$$T_f = \frac{\mu W}{2} \left[\frac{R_o + R_i}{1} \right]$$

Thickness of collar is determined by considering shear failure of collar at inner radius

for safe design

$$(L_{max})_{ind} \leq T_{per}$$

$$\frac{F_s}{A_s} \leq \tau_{per}$$

$$\frac{W}{\pi D t} \leq \tau_{per}$$

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$$t \geq \text{--- mm}$$

frictional torque equation for flat Pivot bearing
or footstep bearing :-

The equations for the flat pivot bearing is
 putting $R_i = 0$ and $R_o = R$ in the equations
 of Single collar bearing.

$$(T_f)_{upt} = \frac{2}{3} \mu W \left(\frac{R^3 - 0}{R^2 - 0} \right)$$

$$\boxed{(T_f)_{upt} = \frac{2}{3} \mu W R} \quad \text{--- (i)}$$

$$(T_f)_{unt} = \frac{1}{2} \mu W (R + 0)$$

$$\boxed{(T_f)_{unt} = \frac{1}{2} \mu W R} \quad \text{--- (ii)}$$

$$\frac{(T_f)_{upt}}{(T_f)_{unt}} = \frac{4}{3} = 1.33$$

∴ upt gives more power loss ✓
and hence always design for upt default

⇒ From above two equations we can conclude that frictional torque or power loss as per uniform pressure theory is more than the power loss or frictional loss as per UWT, hence for the safe design of bearing, if unless otherwise mentioned it is better to use uniform pressure theory because always power loss takes place in bearing due to frictional forces.

(153)

For the safe design of clutches (old or worn out clutches) it is better to use uniform wear theory because clutches are used to transmit power by utilising frictional forces and when the clutches come into service pressure is not uniformly distributed.

For the safe design of new clutches it is better to use uniform pressure theory because pressure is uniformly distributed when the clutch surfaces are new in condition.

Multi Collar Bearing

Let n = no. of collars

$$P_{ind} \leq P_{per}$$

$$\frac{W_{each}}{\pi (R_o^2 - R_i^2)} \leq P_{per}$$

$$\frac{W}{n(R_o^2 - R_i^2)} \leq P_{per}$$

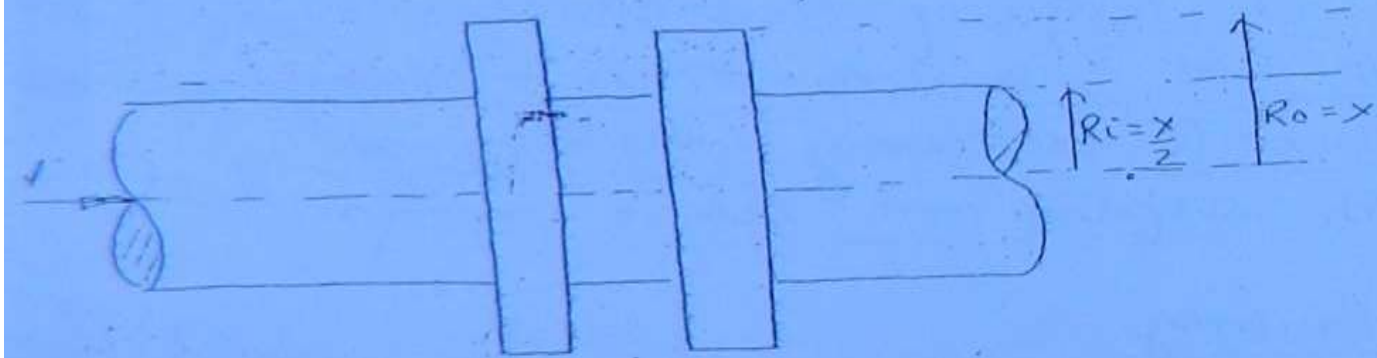
$$n = \frac{W_{each}}{W}$$

W = total load on the shaft

$$n \geq \text{--- No.}$$

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∴ no. of collars increases, Load coming on each collar decreases and pressure induced decreases which leads to the safety against the failure of the clutch or collar.



$$(T_f)_{MCB} = \eta (T_f)_{SCB}$$

$$= n \times \frac{2}{3} (\mu W) \left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right] \text{ each}$$

$$n = \frac{W}{W_{each}}$$

$$(T_f)_{MCB} = \frac{2}{3} \mu W \left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]$$

$$(T_f)_{MCB} = (T_f)_{SCB}$$

and hence frictional Torque is independent

$\Rightarrow$ For a given load and given dimensions of the collars The frictional Torque in single collar bearing and multicollar bearings remains same i.e., (frictional torque is independent of no. of collars) but the pressure induced at each collar in a multicollar bearing is less than the pressure induced at the collar of a single collar bearing.

CONICAL COLLAR BEARING (CCB)

(155)

$$P_{ind} = \frac{W}{\pi [R_o^2 - R_i^2]}$$

$$(T_f)_{CCB} = \frac{1}{\sin \alpha} (T_f)_{SCB}$$

There is no effect of cone angle on the pressure induced or load carrying capacity of the bearing but frictional torque or power loss is inversely proportional to $\sin \alpha$ (where α is the semicone angle).

$\alpha = 90^\circ \Rightarrow$ CCB becomes SCB

as $\alpha \uparrow \Rightarrow \sin \alpha \uparrow \Rightarrow T_f \downarrow \Rightarrow P_{loss} \downarrow \rightarrow$ CCB

$\alpha \downarrow \Rightarrow \sin \alpha \downarrow \Rightarrow T_f \uparrow \Rightarrow P_T \uparrow$] cone clutch

In conical collar bearing

$$2\alpha = 120^\circ \text{ to } 160^\circ$$

In

In case of cone clutch

$$\alpha = 72\frac{1}{2} \text{ to } 15^\circ$$

156

So, as to avoid self engagement of clutch when the intensity of pressure is uniform in a flat pivot bearing of radius r . The friction force is assumed to act at

1) r (ii) $\frac{r}{2}$ (iii) $\frac{2}{3}r$ (iv) $\frac{r}{3}$ ✓

Which of the following statement valid for a multi-collar thrust bearing carrying an axial thrust of W units.

1. Friction moment is independent of no. of collars.
2. Coefficient of friction of bearing surface is effected by the no. of collars.
3. Intensity of pressure is effected by number of collars.

- (a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2, and 3

A multi-collar thrust bearing having 300 mm and 400 mm as inner and outer diameters respectively determine, no. of collars reqd for the bearing if permissible pressure is 7 kN/mm^2 and W is 1750 MN?

Soln

$$n = \frac{W}{W_{\text{each}}}$$

$$= \frac{1750 \times 10^6}{P \times \pi \cdot (R_o^2 - R_i^2)}$$

$$= \frac{1750 \times 10^6}{7000 \times \pi \times (200^2 - 150^2)}$$

(157)

$$= 4.47$$

$$\therefore n = 5$$

Q: Repeat the above for thickness of collar if permissible shear stress is 60 mpa?

Soln

$$\frac{W_{\text{each}}}{n \cdot \pi D_i t} \leq \tau_{\text{per}}$$

$$\Rightarrow \frac{1750 \times 10^6}{5 \times \pi \times 300 \times t} \leq 60$$

$$\Rightarrow t \geq 6.8 \text{ mm}$$

$$t = 7 \text{ mm}$$

JOURNAL BEARINGS

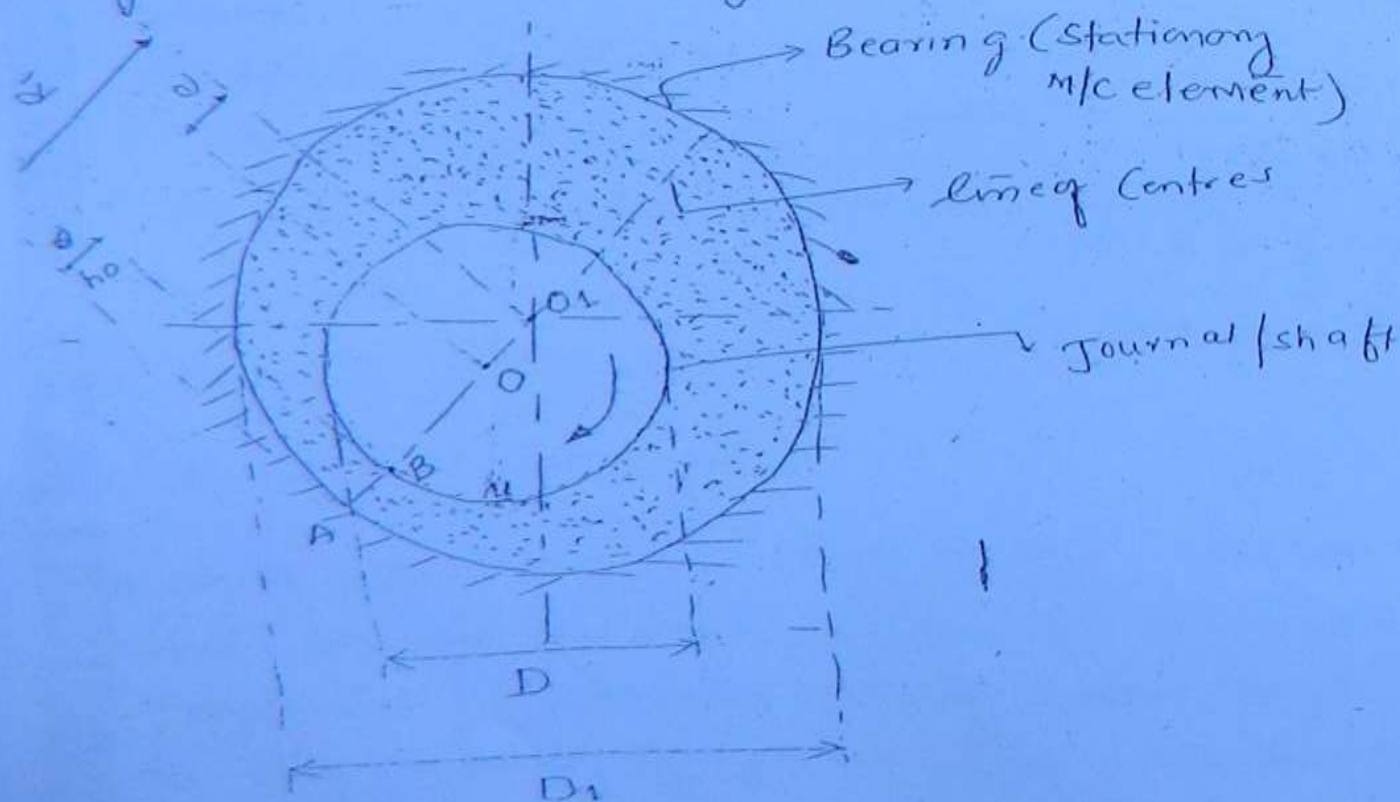
or (Radial bearings)

They are used to support a shaft which is subjected to radial loads (ie, loads acting perpendicular shaft axis)
eg weight of pulley on horizontal shaft, belt tensions, weight of gears. (F_r & F_t)

Terminology used in Journal bearing (158)

Journal bearing is defined as a sliding contact radial bearing which is operating with hydrodynamic lubrication.

They are suitable for high speed condition



position of Journal/shaft in Journal bearing at high speed

D_1 = diameter of bearing

D = diameter of Journal/shaft

h_0 = minimum film of thickness

e = eccentricity

C = diametral clearance

$$C = D_1 - D$$

$$\Rightarrow D_1 = D + C$$

C_1 = Radial clearance

$$C_1 = R_1 - R = \frac{C}{2}$$

$$C = 2C_1$$

$$\Rightarrow \frac{C}{D} = \text{diametral clearance ratio}$$

If $C \downarrow \Rightarrow \frac{C}{D} \downarrow \Rightarrow$ heat generated or power loss increases (undesirable)
and load carrying capacity increases (desirable)

If $C \uparrow \Rightarrow \frac{C}{D} \uparrow \Rightarrow$ heat generated or power loss decreases (desirable)

and load carrying capacity decreases (undesirable)

For a good bearing optimum value of $\frac{C}{D}$ is
 $\frac{C}{D}$ is 0.001 to 0.002.

⇒ $\frac{L}{D}$ ratio

L = length of bearing or Length of Journal

D = diameter of bearing

If $\frac{L}{D} = 1 \Rightarrow$ Square bearing

$\frac{L}{D} < 1 \Rightarrow$ short bearing

$\frac{L}{D} > 1 \Rightarrow$ Long bearing

⇒ $L \downarrow \Rightarrow \frac{L}{D} \downarrow \Rightarrow W \downarrow$ (undesirable)

⇒ and side leakage of lubricant is more

⇒ and effective lubrication decreases

⇒ and heat generated or power loss increases

⇒ $L \uparrow \Rightarrow \frac{L}{D} \uparrow \Rightarrow W \uparrow$

⇒ side leakage is less

⇒ effective lubrication increases

⇒ Heat generated or power loss decreases

Here $\frac{L}{D} > 1 \Rightarrow \boxed{\frac{L}{D} = 1 \text{ to } 2}^*$

shafts are available upto 6 to 7 m.

When $\frac{L}{D}$ is too large there is a alignment problems for shaft and bearing

bearing pressure (P_b)

$$P_b = \frac{W}{LD} \leq P_{\text{permissible}}$$

Where LD is the projected area

$\therefore L \geq \text{--- mm}$ can be calculated:

W = Load coming on shaft

$$W \leq (P_{\text{per}} \times L \times D)$$

(161)

→ Maximum load carrying capacity
of Journal bearing

$$W = P_{\text{per}} \times L \times D$$

So on increasing length, Load carrying capacity increases

Eccentricity (e)

$$R_1 = e + R + h_o$$

$$e = R_1 - R - h_o$$

$$e = C_1 - h_o$$

$$e = \frac{C}{2} - h_o$$

eccentricity ratio or altitude of bearing (ϵ)

$$\epsilon = \frac{\text{eccentricity}}{\text{Radial clearance}}$$

$$\epsilon = \frac{2e}{c}$$

$$\epsilon = \frac{2}{c} \left[\frac{c}{2} - h_0 \right]$$

$$\epsilon = 1 - \frac{2h_0}{c}$$

(62)

HYDRODYNAMIC LUBRICATION



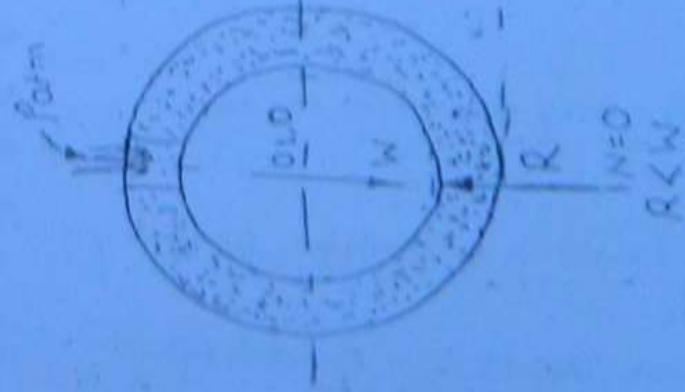
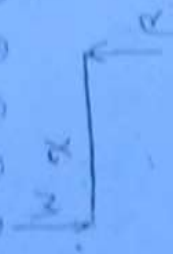
$$T_f = Wx = 10 \text{ N-m [CCW]}$$

$$T_{\text{apply}} = \frac{Px60}{2\pi N} = 100 \text{ N-m [CW]}$$

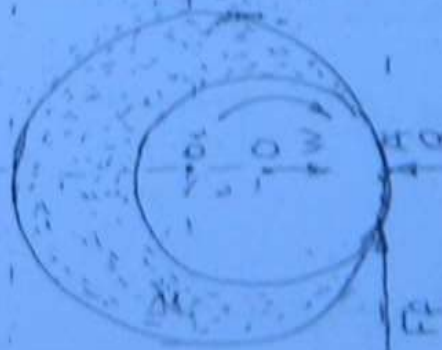
→ and hence 10 N-m power loss occurs
is arising in overcoming frictional
torque

3. $N \uparrow$ $P \uparrow \rightarrow$ film or layer (sticky)
and hence wedging action takes place
and due to this pressure increases

$$T_f = W \cdot x$$



(1)



(2)



(3)

(63)

(N is high)

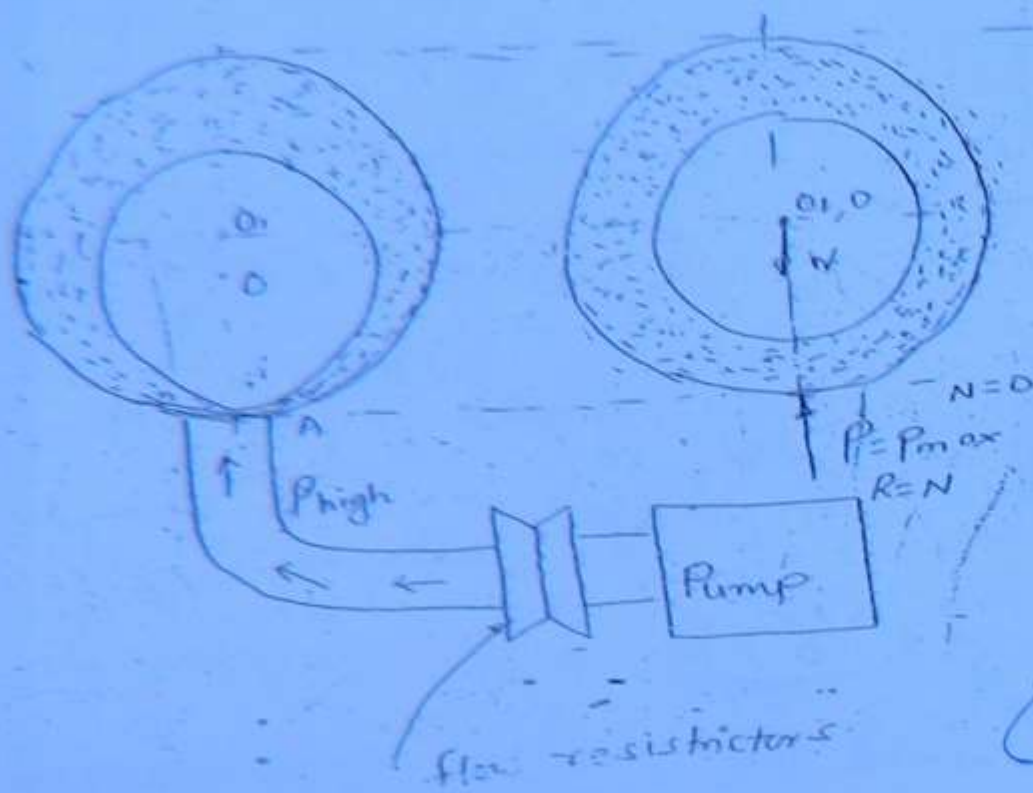
(4)



(5)

hydrodynamic lubrication

Hydrostatic Lubrication



Hydrodynamic Lubrication

Hydrostatic Lubrication

Wedge action

High speed application

Metal to metal can be avoided at

high speed

high starting torque is required

Cost is less

1) Ext device like pump

2) at any speed (ie, even at stationary)

3) No metal to metal contact

4) Low starting torque ie, min

5) Initial and maintenance cost is high

6) As load increases shaft is concentric

6) They are used when shaft are subjected to less load at the stationary condition.

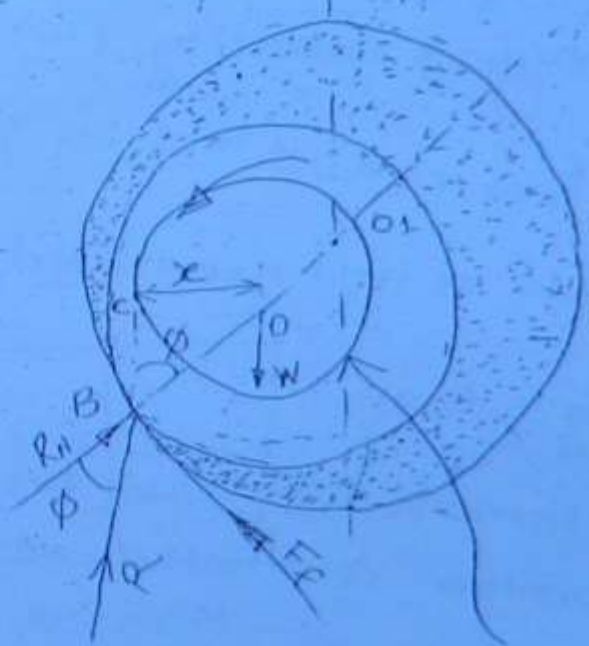
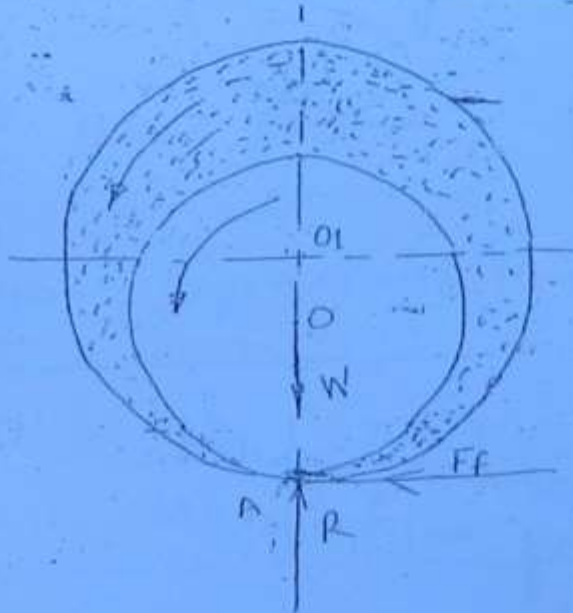
7) IC engine crank shaft

6) Shaft is subjected to high loads at stationary condition.

7) Vertical turbo generators and centrifugal pump Ball mills

FRICTION CIRCLE

(165)



Friction Circle

$$\sin \phi = \frac{x}{r}$$

$$x = r \sin \phi$$

$$x \approx r \tan \phi \quad [\because \phi \text{ is small} \Rightarrow \sin \phi \approx \tan \phi]$$

$$x \approx l \sin \phi \quad [\because l = r \tan \phi]$$

$$T_f = W l \sin \phi$$

$$T_f = l W x$$

When shaft is in stationary condition the resultant force is in line with the line of action of the load acting on the shaft but when shaft is running condition, due to frictional force, resultant force get displaced from the line of action of load acting on the shaft by the distance which is equal to friction circle radius ($OC = r_f$).

Design criteria used in Journal bearing

• Load carrying capacity (W)

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$$W = P_{per} \times L \times D$$

• Power loss or heat generated (H_g)

$$P \text{ or } H_g = \mu W V$$

Now to determine μ Two brothers called Mc-Kees brother conducted no. of experiments on Journal bearing and based on their experiment they conclude that μ is a function of

$$(1) \mu \propto f \left[\left(\frac{Z \eta}{P} \right), \left(\frac{D}{C} \right) \& \frac{L}{D} \right]$$

value of $\frac{C}{D}$ range from 500 to 1000

Range of $\frac{L}{D} = 1 \text{ to } 2$

bearing characteristic Number = $\frac{Z \cdot n}{P}$

where Z = absolute viscosity
of Lubricant at operating temp.
as (to). in $\text{pa} \cdot \text{s}$ or $\frac{\text{kg}}{\text{m} \cdot \text{s}}$ or Ns/m^2

(167)

$$1 \text{ cP} = 0.001 \text{ pa} \cdot \text{s}$$

N = speed of Journal in RPS

p = bearing pressure = $\frac{W}{LD}$ in pa

$$BNO = \frac{\text{pa} \cdot \cancel{\text{s}}}{\cancel{\text{pa}}} \times \frac{1}{\cancel{\text{s}}} \quad (\text{No unit})$$

They gave expression for ' μ ' as

$$\mu = \frac{33}{10^8} \left[\left(\frac{Z n'}{p'} \right) \left(\frac{D}{C} \right) \right] + K$$

where n' = speed in RPM

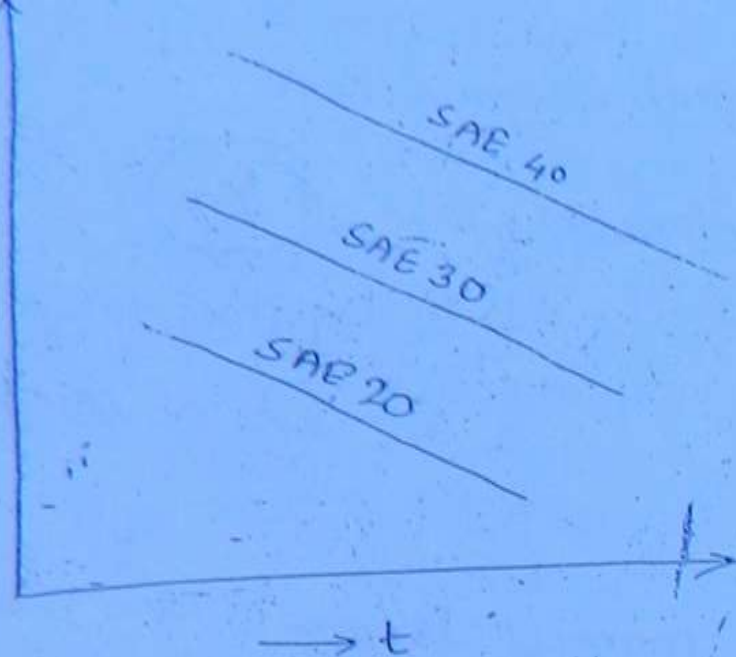
p' = pressure in MPa

Z = $\text{pa} \cdot \text{s}$ or $\frac{\text{kg}}{\text{m} \cdot \text{s}}$ or Ns/m^2

μ = coefficient of friction

K is constant depend on $\frac{L}{D}$ ratio.

$z \uparrow$



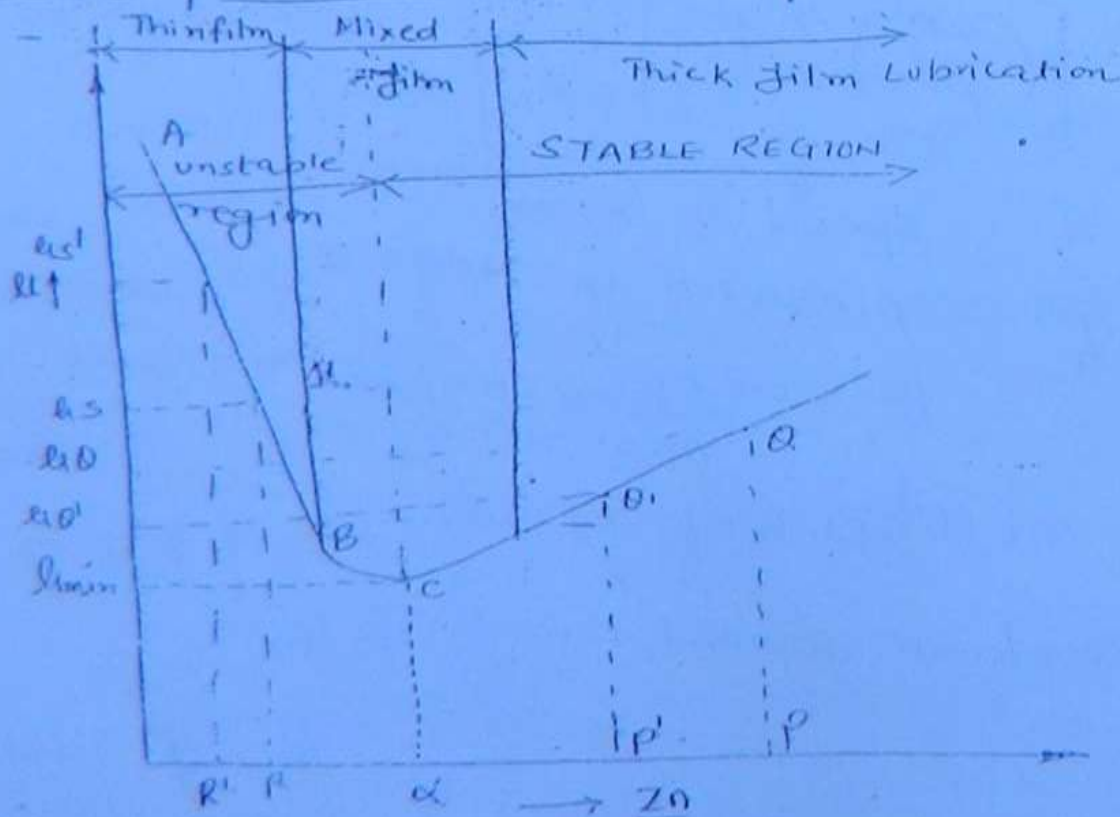
168

$\Rightarrow K = 0.002 \Rightarrow$ if $\frac{L}{D}$ has value

$$0.75 \leq \frac{L}{D} \leq 2.8$$

$\Rightarrow K = 0.003 \Rightarrow$ if $\frac{L}{D} > 2.8$

Effect of $\frac{z\eta}{P}$ on coefficient of friction (μ)



Bearing modulus (Z)

It is the value of bearing characteristic number corresponding to minimum coefficient of friction

always $\frac{Zn}{P}$ taken greater than 2:

as $T \uparrow$, $Z \downarrow$

(169)

and $\frac{Zn}{P} \downarrow$ and due to this $l \uparrow$

and hence heat generated is $\downarrow$

and due to this $T \uparrow$, $Z \uparrow$ and $\frac{Zn}{P} \uparrow$

Also

$W \uparrow$, $\frac{Zn}{P} \downarrow$, $l \downarrow$, $H_g \downarrow$, $T \downarrow$, $Z \uparrow$ and

finally $\frac{Zn}{P} \uparrow$ and stable condition is achieved.

For unstable region

$T \uparrow$, $Z \downarrow \Rightarrow \frac{Zn}{P} \downarrow$, $l \uparrow \Rightarrow H_g \uparrow$, $T \uparrow$, $Z \downarrow$

and due to this $\frac{Zn}{P} \downarrow$

generally $\frac{Zn}{P} \geq 3 \times$ (steady condition)

Sometime $\frac{Zn}{P} \geq 15 \times$ (under highly fluctuating Load condition)

viscosity index (VI)

It is a measure of change of viscosity with change in temperature

$$VI = \frac{dZ}{dT} = \frac{Z_2 - Z_1}{T_2 - T_1}$$

Sommerfeld Number (S)

170

$$S = \left(\frac{Zn}{P} \right) \left(\frac{D}{C} \right)^2$$

n is in rpm

P is Pa, or N/m^2

Z is in Pa.s

Sommerfeld number remains constant for given journal bearing (ie, for a given L & D) hence it is use to correlate the working condition of different machine which are operating with the same journal bearing.

M/c 1

$$L_1 = 500 \text{ mm}$$

$$D_1 = 250 \text{ mm}$$

$$W_1 = 10 \text{ kN}$$

$$N_1 = 1000 \text{ rpm}$$

(17)

Now New Machine 2

$$L_2 = 500 \text{ mm}$$

$$D_2 = 250 \text{ mm}$$

$$W_2 = 20 \text{ kN}$$

$$N_2 = ?$$

on equating 'S'

$$\text{i.e., } S_1 = S_2$$

$$\left(\frac{Z_1 n_1}{P_1} \right) \left(\frac{D_1}{L_1} \right)^2 = \left(\frac{Z_2 n_2}{P_2} \right) \left(\frac{D_2}{L_2} \right)^2$$

$$\frac{n_1}{P_1} = \frac{n_2}{P_2}$$

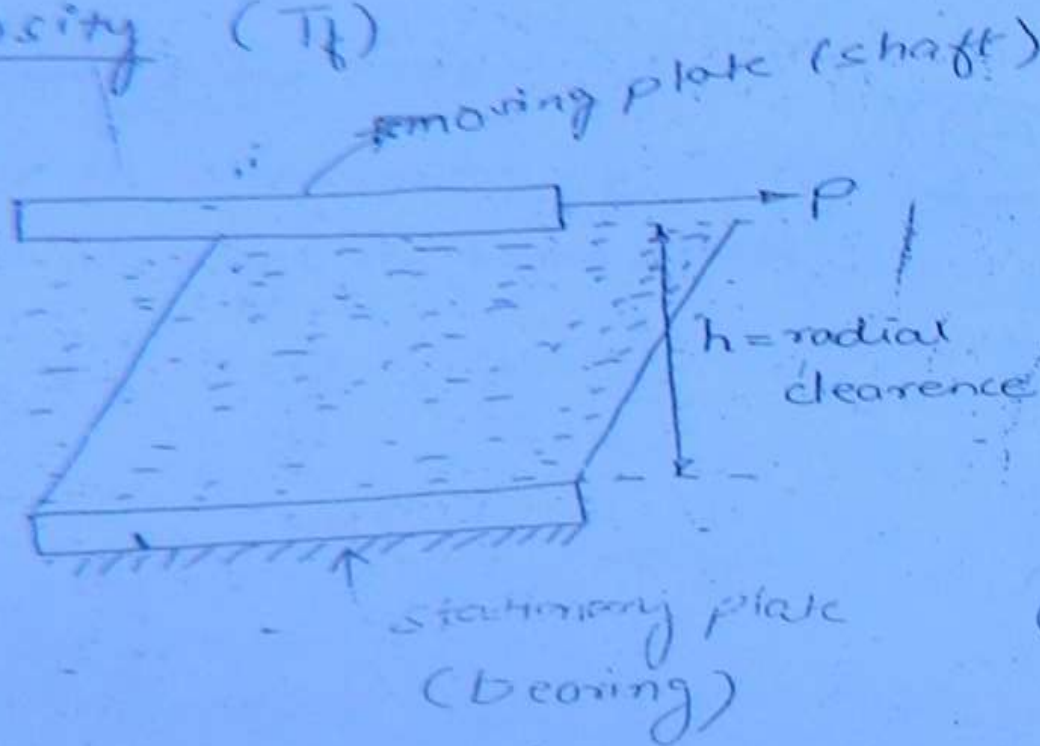
} Same Lubrication)

$$\frac{1000}{P_1} = \frac{n_2}{2P_1}$$

$$\therefore n_2 = 2000 \text{ RPM}$$

$$P_{loss} = T_f \cdot \omega \quad , \quad T_f = \int r \cdot \tau = \ell \cdot \omega \cdot \tau$$

Expression for frictional Torque in terms of viscosity (T_f)



(172)

according to Newton's Law of viscosity

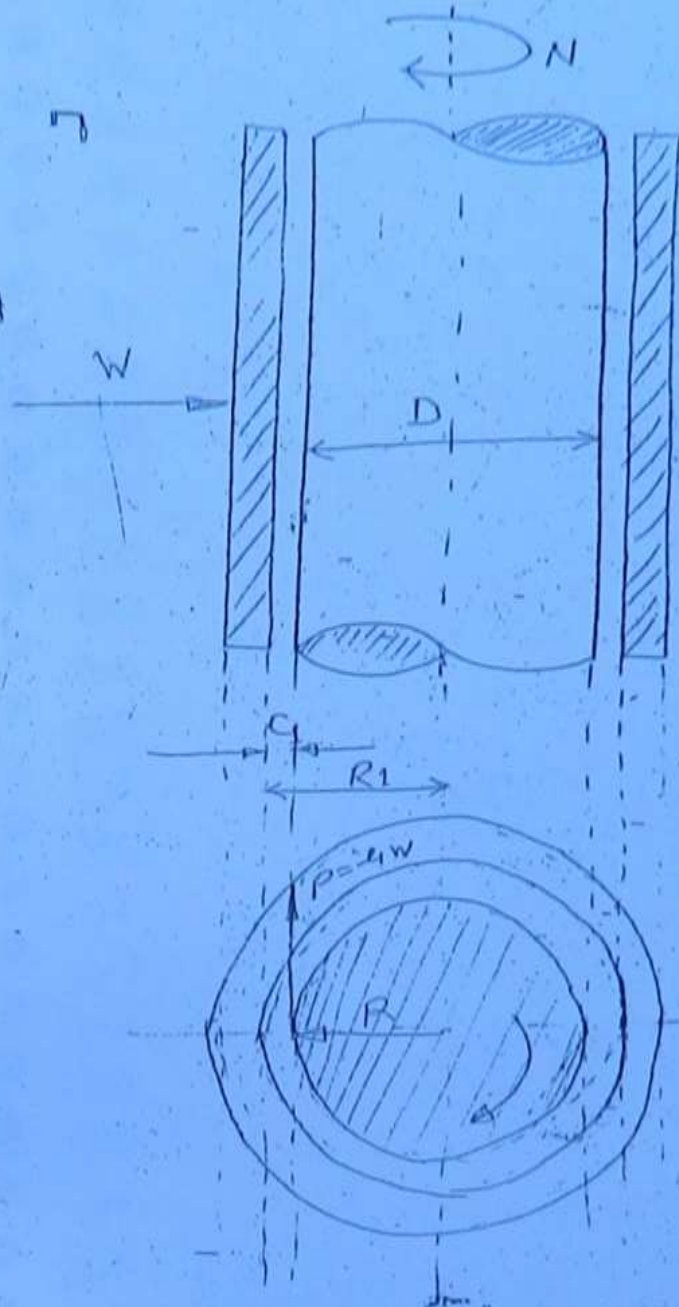
$$\tau \propto \frac{V}{h}$$

$$\tau = \frac{ZV}{h} \rightarrow (1)$$

$$\tau = \frac{P}{A} \rightarrow (2)$$

$$\therefore \frac{P}{A} = \frac{ZV}{h}$$

Now,
$$P = \frac{ZV}{h} \times A$$



$$P = \frac{Z \times \pi D N'}{60} \times \frac{\pi D L}{C}$$

$$= \frac{Z \pi^2 D^2 L N' \times 2}{60 \times C}$$

$$T_f = 4W R$$

$$= \frac{Z \pi^2 D^2 L N' \times 2}{60 \times C} \times \frac{D}{2}$$

$$T_f = \frac{Z \pi^2 D^3 L n'}{60 \times C} \rightarrow (I)$$

$$\text{Now } T_f = \mu W \frac{D}{2} \rightarrow (II)$$

Equating 1 and 2.

$$\mu \cdot W \cdot \frac{D}{2} = \frac{Z \pi^2 D^3 L n'}{60 \times C}$$

$$\frac{\mu \times P \times L \times D^2}{2} = \frac{Z \times \pi^2 D^3 \times L \times n'}{60 \times C}$$

$$\mu = \frac{2}{30} \left(\frac{Z n'}{P} \right) \frac{D \pi^2}{C} = \frac{\pi D^2}{30 C} \left(\frac{Z n'}{P} \right)$$

∴ Petroff's equation hence is

$$\mu = \frac{\pi^2}{30} \left(\frac{Z n'}{P} \right) \left(\frac{D}{C} \right) \quad \text{remember directly}$$

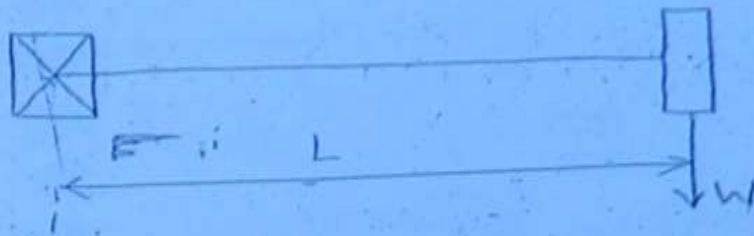
Design procedure used in Journal bearing

1) Shaft diameter or journal diameter [D]
by MSST

$$T_e = \sqrt{(K_b m)^2 + (K_L T)^2} = \frac{\pi}{16} D^3 \tau_s$$

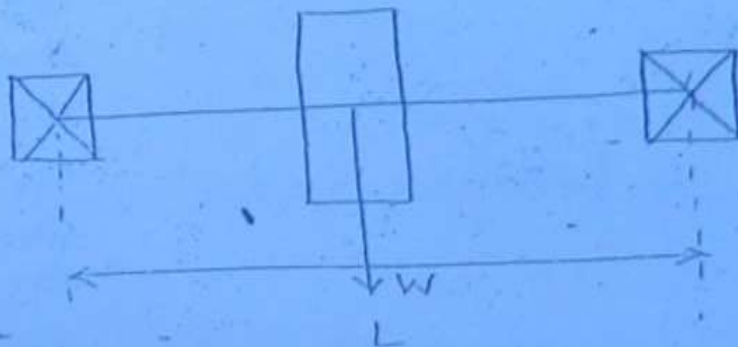
$$T = \frac{P \times 60}{2\pi N} \times 10^6 = \text{--- Nmm}$$

P is in kW
1 1



$$M = W \cdot L$$

(75)



$$M = \frac{WL}{4}$$

by MDET

$$M_e = \sqrt{(K_b \cdot M)^2 + \frac{3}{4} (K_t \cdot T)^2} = \frac{\pi}{32} D^3 (\sigma_b \text{ or } \sigma_t)$$

$$\therefore D = ?$$

2) Bearing Diameter (D)

$$D_1 = D + C$$

$$\frac{C}{D} = 0.001 \text{ to } 0.002$$

3) Length of bearing

$$P_{ind} \leq P_{per}$$

$$\frac{W}{LD} \leq P_{per}$$

$$\therefore L \geq \text{mm}$$

calculated $\frac{L}{D}$ should be less or equal to

$(\frac{L}{D})_{\text{given}}$

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4) Power Loss or (Hg)

$$P_L \text{ or } Hg = 4WV$$

$$= T_f \times \frac{LD}{8}$$

Now according to McKee's equation

$$u = \frac{33}{10^8} \left[\left(\frac{Zn'}{P'} \right) \left(\frac{D}{C} \right) \right] + K$$

$$n' \frac{dl}{dt} = m \text{ rpm}$$

$$P' = \text{mpa}$$

$$K \sim \frac{L}{D} \text{ ratio}$$

$$V = \frac{\pi D N \overset{\text{mm}}{\text{rpm}}}{60 \times 1000} = \text{m/s}$$

$$P_{\text{Loss}} = \underset{\substack{\rightarrow \text{mls} \\ \rightarrow \text{Newton}}}{\ell \cdot w \cdot v} = \frac{1}{\pi} \text{ watts}$$

$$P_{\text{Loss}} = T_f \cdot w$$

$$T_f = \frac{2\pi^2 D^3 L n'}{60 \cdot C}$$

(177)

$$w' = \frac{2\pi n'}{60} = \dots \text{ r/s}$$

5) Heat dissipated (Hd)

$$Hd = C_d \cdot (t_b - t_a) \times L \times D^m$$

= ——— watts

C_d = heat dissipation coefficient

$C_d \Rightarrow$ unit is $\text{W/m}^2 \cdot ^\circ\text{C}$

$$\Delta t = t_b - t_a = \frac{1}{2} [t_o - t_a]$$

t_o = operating temperature

t_a = atmospheric temperature

$\therefore$ Hd can be calculated in watts

t_b = temperature of bearing surface

nd method "Lasche's equations"

$$H_d = \frac{(\Delta t + 18)^2}{K} L \cdot D \rightarrow m$$

K = heat dissipation constant

✓ $K = 0.273 \Rightarrow$ well ventilated bearing

✓ $K = 0.484 \Rightarrow$ still air conditioning

$\therefore H_d$ can be calculated.

(176)

1) check for artificial cooling is required or not

(i) If $H_g = H_d \Rightarrow$ no artificial cooling is required

(ii) If $H_g > H_d \Rightarrow$ artificial cooling is required

2) volume flow rate of coolant

Heat to be carried away by coolant (H_c)

$$H_c = H_g - H_d = \text{--- watts.}$$

$$H_c = m \cdot s \cdot (t_o - t_i)$$

kg/s

specific heat

t_o and t_i are inlet and outlet temp of cooling medium.

$$m = \text{--- kg/s}$$

$$S = 1840 - 2100 \text{ kJ/kg}^\circ\text{C}$$

-- $V = ?$ by $\dot{Q} = \frac{m}{V}$

$$V = \text{--- m/s}$$
$$= \text{Lit/hr}$$

(179)

$$1 \text{ litre} = 1000 \text{ cc}$$

Q: a full journal bearing having clearance to radius ratio of $1/100$ using a Lubricant with μ is equal to $28 \times 10^{-3} \text{ pa-s}$, supports the Journal running at 2400 RPM if bearing pressure 1.4 mpa , the Sommerfeld Number is ?

Soln

$$\frac{\text{clearance}}{\text{radius}} = \frac{1}{100} = \frac{C_1}{r}$$

$$\mu = 28 \times 10^{-3} \text{ pa-s}$$

$$n' = 2400 \text{ rpm}$$

$$p' = 1.4 \text{ mpa}$$

$$S = \left(\frac{Zn}{P} \right) \left(\frac{D}{C} \right)^2$$

$$= \left(\frac{2.8 \times 10^{-3} \times 2400}{1.4 \times 10^6 \times 60} \right) (100)^2$$

$$S = 0.008$$

$$\left\{ \begin{array}{l} \frac{C_1}{R} = \frac{1}{100} \\ \frac{C/2}{D/2} = \frac{1}{100} \\ \frac{D}{C} = 100 \end{array} \right.$$

∴ A journal bearing of diameter 50 mm and the length 50 mm, operating at 20 rps, carries a load of 2 kN, the lubricant used has a viscosity of 20 mpa-s, the radial clearance is 50 μm ($1 \mu\text{m} = 10^{-6}\text{m}$). The Sommerfeld number for this bearing is ?

dm $D = 50 \text{ mm}$

$L = 50 \text{ mm}$

$n = 20$

$W = 2 \text{ kN}$

$Z = 200 \times 10^{-3} \text{ pa-s}$

$C_1 = 50 \times 10^{-6} \text{ m}$

$S = ?$

$$S = \left(\frac{Zn}{P} \right) \left(\frac{D}{C} \right)^2$$

$$P = \frac{W}{LD} = \frac{2 \times 10^3}{50 \times 10^{-3} \times 50 \times 10^{-3}}$$

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$$S = \left[\frac{20 \times 10^{-3} \times 20}{-2000} \right] \left[\frac{50}{2 \times 50 \times 10^6 \times 10^3} \right]^2$$

$$S = 0.125$$

Q: a journal bearing

$$D = 40 \text{ mm}$$

$$L = 40 \text{ mm}$$

$$n = 20 \text{ rad/s}$$

$$Z = 20 \text{ mpa-s}$$

$$c = 0.02 \text{ mm}$$

(18)

determine Loss of torque due to viscosity of the Lubrication.

Som

$$T_f = \frac{Z \pi^2 D^3 n' L}{60 \times c}$$

$$= \frac{20 \times 10^{-3} \times \pi^2 \times (40 \times 10^{-3})^3 \times 40 \times 10^{-3}}{60 \times 0.02 \times 10^{-3}}$$

$$T_f = 0.08$$

$$V = \frac{\pi D n'}{60} = \frac{D \omega'}{2}$$

n' can be calculated.

a journal bearing of 50 mm diameter and 80 mm long has a bearing pressure of 6 MPa. is used to support a journal running at 1000 RPM. The bearing is lubricated with oil whose absolute viscosity at the operating temp. of 75°C may be taken as 0.015 kg/ms , room temperature 25°C (t_a), $(\frac{D}{C} = 1000)$.

determine the following

182

amount of artificial cooling required

- 1) mass of coolant oil required if specific heat of the oil $1900 \text{ J/kg}^{\circ}\text{C}$ and difference of inlet and outlet temp of coolant oil is 2°C and heat dissipation coefficient $500 \text{ W/m}^2^{\circ}\text{C}$.

$$\underline{m} \quad \mu = 0.00283$$

$$H_g = \mu W V = 177.95 \text{ W}$$

$$V = 2.62 \text{ m/s}$$

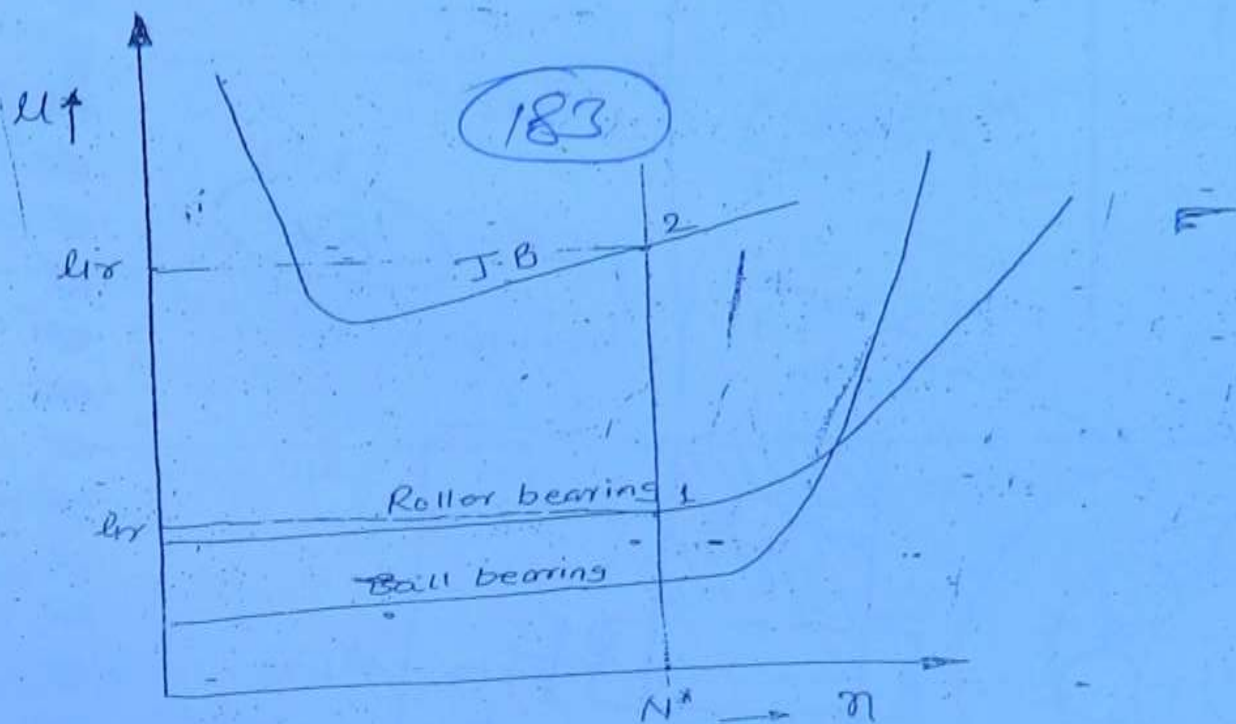
$$H_d = 50 \text{ W}$$

$$H_c = H_g - H_d = 127.95 \text{ W}$$

$$m = 0.0067 \text{ kg/s}$$

Antifriction Bearing *

Rolling Contact Bearing * [RCB]



→ Used for low or medium speed range.

$N^* \Rightarrow$ running condition (RPM)

parameters	Journal bearing	Antifriction bearing
1. Load	F_r	F_r and F_a
2. speed	high speed application	Low and medium speed application
3. Machines (application)	used in m/c where there is continuous application	intermittent application

Parameters

Journal bearing

Antifiction bearing

Cost

Less costly

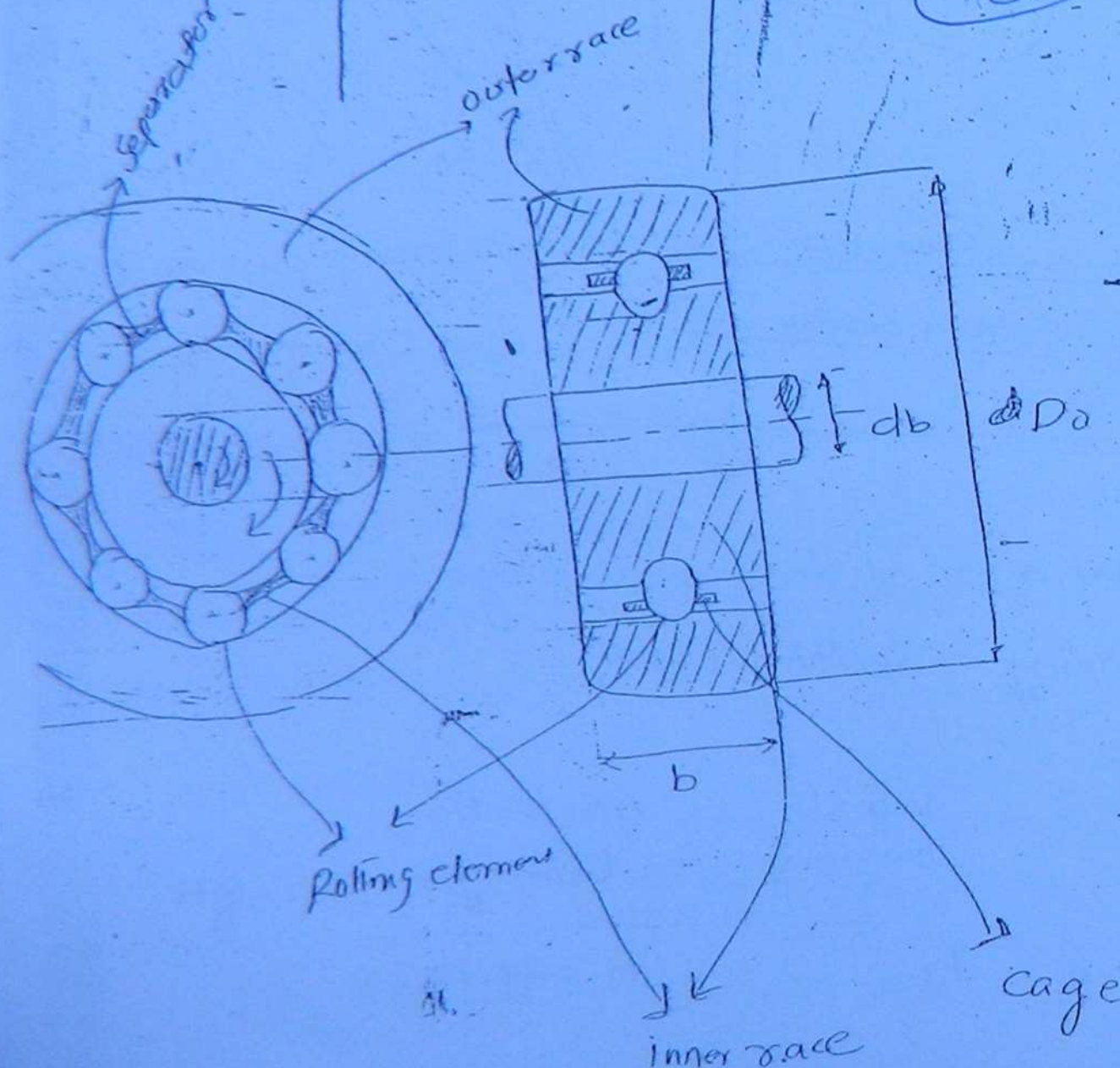
more costly

Axial space

More

less

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parameter	Journal bearing	Antifiction bearing
damping capacity	Less	more
Radial space	Less	185 more
precise alignment	not required	precise alignment
Starting torque	more	less
Lubrication	continuous lubrication is reqd.	is not reqd
Lubricant	Liquid	Semi solid
Noise	less	more
Life	more	finite

Antifiction designation

SKF 6308

BIS

40 BC 03

3) - SKF 6308x5

Type of series
Type of Anti friction bearing

6 → DGBB

(deep groove ball bearing)

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08x5 → diameter of bore or diameter of shaft

every AFB is manufactured in 5 different series.

- ① 6108 → 100 series → extra light series
- ② 6208 → 200 series → light series ✓
- ③ 6308 → 300 series → medium series ✓
- ④ 6408 → 400 series → heavy series ✓
- ⑤ 6508 → 500 series → Extra heavy series

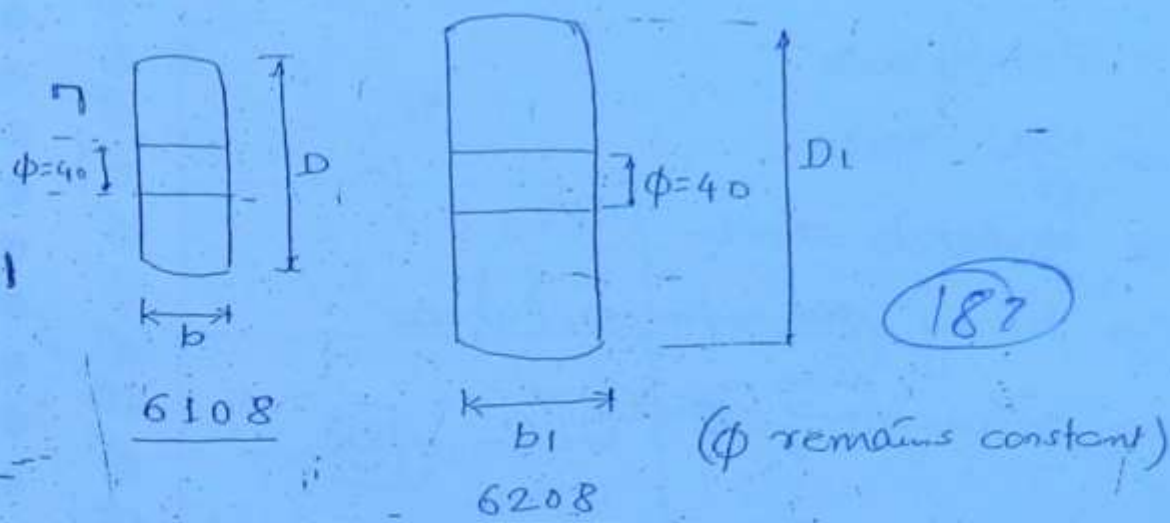
going from Top to bottom

→ Do and b increases.

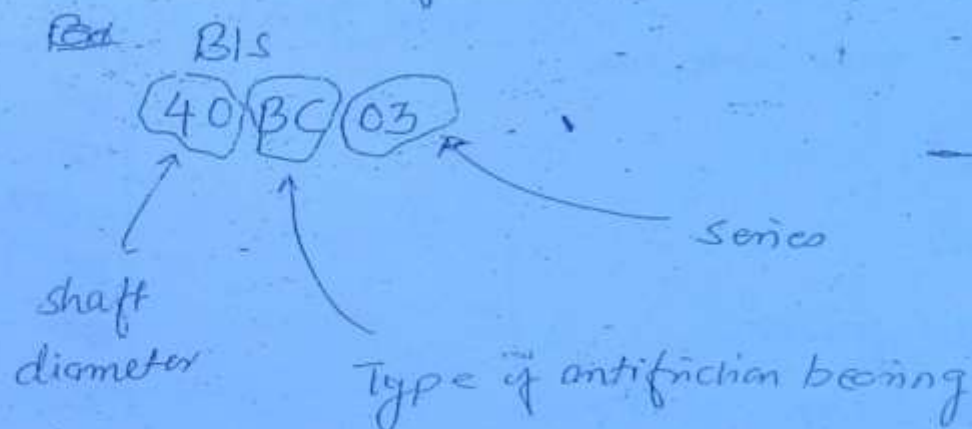
→ Cost increases.

→ Load carrying capacity increases.

1
2
3
4
5
↓



BIS (Bureau of Indian Standard)



03 $\Rightarrow$ 300 Series

Terms used in the selection of series of an AFB.

① Equivalent load $[P_e]$

Antifriction bearing manufacturing association (AFBMA)
 given P_e as

$$P_e = S [X V F_1 + Y F_a]$$

S = Service or shock factor

$\Rightarrow S = 1 \Rightarrow$ steady loads

1.5 $\Rightarrow$ light shocks

2 $\Rightarrow$ moderate shocks

2.5 $\Rightarrow$ heavy shocks / impact shocks

3 $\Rightarrow$ extra heavy shock

(188)

$\Rightarrow V =$ race proportion factor
rotation

$= 1 \Rightarrow$ inner race rotation

1.2 $\Rightarrow$ outer race rotation

$F_r =$ radial load

$F_a =$ axial load

$x =$ radial load factor

$y =$ Axial load factor

or Thrust ball bearing / Thrust roller bearing, withstand
only axial load $[F_r = 0]$

$\therefore x = 0$ and $y = 1$

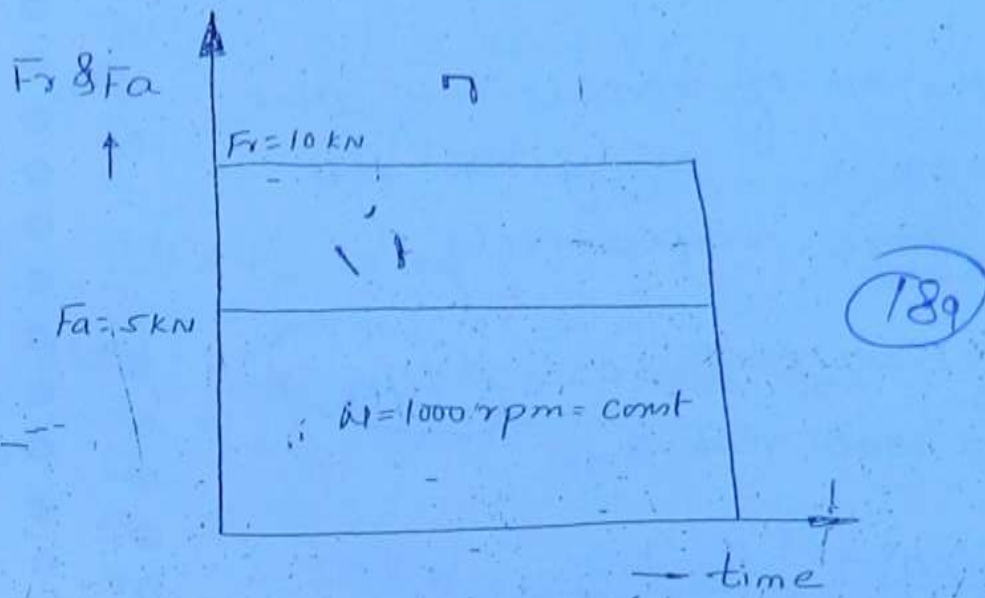
or

Cylindrical roller bearing

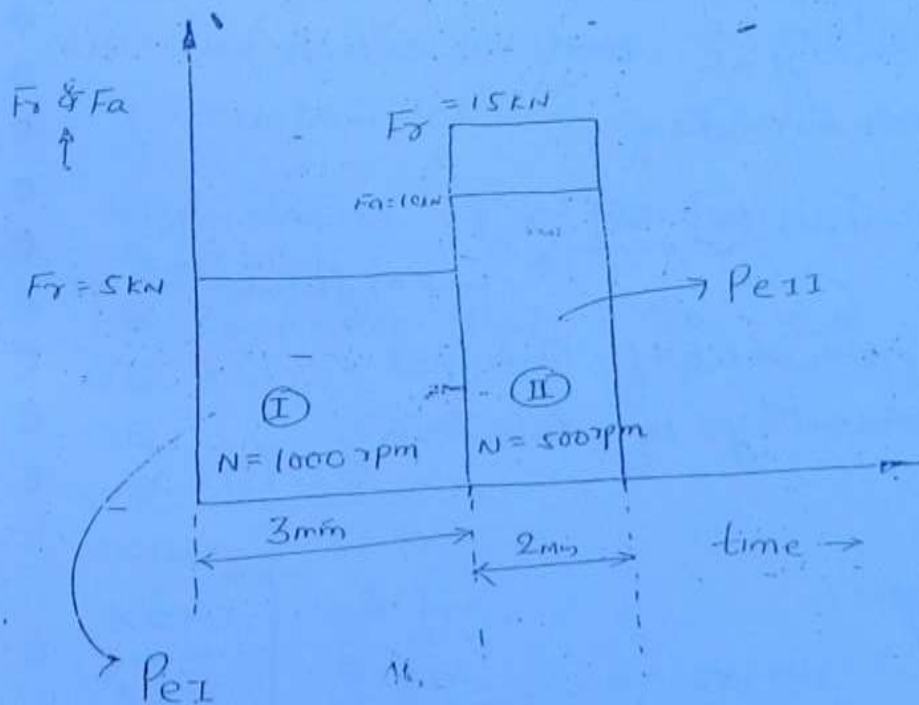
withstand only radial load $[F_a = 0]$

$\therefore x = 1, y = 0$

P_e formula is valid only if F_r and F_a
are constant and N is constant



for varying Load and RPM



using Cubic Mean load formulae

$$P_m = \sqrt[3]{\frac{P_{eI}^3 \eta_I + P_{eII}^3 \eta_{II}}{\eta_I + \eta_{II}}}$$

η_I and η_{II} are no. of revolution i.e., shaft or
hose ...

where

n_1 and n_2 are the no. of revolutions that a bearing has undergone during the first stage and second stage respectively.

$$n_1 = 1000 \times 3 = 3000 \text{ revs.}$$

$$n_2 = 500 \times 2 = 1000 \text{ revs.}$$

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3) Life of an Antifriction bearing

It is defined as the no. of revolution that a bearing has undergone before the evidence of first fatigue crack either in races or rolling elements.

Nominal or Rated Life $[L_{90}]$ or $[L_{10}]$

prob. of failure.
probability of survival or
percentage of reliability

SKF 6308

→ 100 bearings

$L_{90} = 100 \text{ million revolutions.}$

90% of bearings Life $\geq 100 \text{ MR}$

10% of bearings life $\leq 100 \text{ MR}$

$$P_e = S [\kappa V F_r + Y F_a]$$

$$= 1 [1 \times 10 + 0]$$

$$P_e = 10 \text{ kN}$$

(191)

Now

$$L_{90} = \frac{3.65 \times 24 \times 60 \times 1000}{10^6} = 525.6 \text{ mR}$$

$$525.6 = \left(\frac{C}{L_0} \right)^{10/3}$$

$$C = 65.49 \text{ kN}$$

Now see the catalogue to see which series can sustained C of 65.49 kN.

Cylindrical roller bearing (x)

$$\text{Last two digit} = \frac{100}{5} = 20$$

CRB	C
X120	10 kN
X220	25 kN
X320	50 kN
X420	70 kN
X520	80 kN

Selected

determine the condition for the load acting on a ball bearing if its life is to be halved? by how much life is to be increased?

$P_1 \Rightarrow$ Life is L_1

$P_2 \Rightarrow L_2 = \frac{L_1}{2}$

($k=3$) for ball bearing

$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_2} \right)^3$$

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$$\frac{L_2}{L_1} = \left(\frac{P_1}{P_2} \right)^3$$

$$\left(\frac{L_1}{2L_1} \right)^{\frac{1}{3}} = \frac{P_1}{P_2}$$

$$P_2 = 2^{\frac{1}{3}} P_1$$

$$P_2 = 1.26 P_1$$

What is the life of a ball bearing if load acting on the ball bearing is halved?

em

$P_1 \Rightarrow$ Life is L_1

$\frac{P_1}{2} \Rightarrow$ Life is L_2

$$\boxed{\frac{L_2}{L_1} = \left(\frac{P_1}{P_2}\right)^3}$$

$$\frac{L_2}{L_1} = \left(\frac{P}{P/2}\right)^3$$

$$\therefore L_2 = 2^3 L_1$$

(193)

$$L_2 = 8 L_1$$

$\therefore$ Life of the ball bearing is 8 times.

Q: What is the condition for the bearing if the life of the ball bearing is doubled when the load acting on the ball bearing is doubled?

$$\begin{aligned} \text{Soln} \quad \frac{L_2}{L_1} &= \left(\frac{C_2/P_2}{C_1/P_1}\right)^3 \\ &= \left(\frac{C_2}{C_1}\right)^3 \left(\frac{P_1}{P_2}\right)^3 \\ 2 &= \left(\frac{C_2}{C_1}\right)^3 \times \left(\frac{1}{2}\right)^3 \end{aligned}$$

$$C_2 = 2.52 C_1$$

Select a new bearing whose life, if the life to be doubled when load also becomes doubled a new bearing is to be selected whose dynamic capacity should be 2.52 times the original bearing dynamic capacity.

Q. SKF 6306 ball bearing with inner ring rotation have a 10 seconds work cycle as follows

Parameters	for 2 Sec	for 8 seconds
F_r	3640 N	2730 N
$F_a(N)$	1820 N	0 N
RPM	900	1200
Type of Load	Light shock	steady load
x and y values	$x = 0.56$ $y = 1.4$	$x = 1$ $y = 0$

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$$C = 22 \text{ kN}$$

find out expected average life in hrs (Larg) hrs

slm

I stage

II stage

$$n_I = ?$$

$$60 \text{ s} \rightarrow 900 \text{ rev}^n$$

$$2 \text{ s} \rightarrow ?$$

$$n_I = 80 \text{ rev}$$

$$60 \text{ s} \rightarrow 1200 \text{ rev}$$

$$8 \text{ s} \rightarrow ?$$

$$n_{II} = 160 \text{ rev}$$

$$P_{eI} = 5 [x F_r + y F_a]$$

$$P_{eI} = 6.8796 \text{ kN}$$

$$P_{eII} = 2.73 \text{ kN}$$

$$P_m = \sqrt[3]{\frac{P_{eI}^3 \eta_I + P_{eII}^3 \eta_{II}}{\eta_I + \eta_{II}}}$$

$$P_m = \sqrt[3]{\frac{(6.8796)^3 \times 30 + (2.73)^3 \times 160}{30 + 160}}$$

$$P_m = 4092.53 \text{ N}$$

$$L_{90} = \left(\frac{C}{P_m} \right)^3 = \left(\frac{22}{4092.53} \right)^3 = 155.34 \text{ million revolution}$$

$$L_{50} = 5 L_{90} = 776.7 \text{ million revolution}$$

$$\eta_I + \eta_{II} = 190 \text{ revs} \rightarrow 10.5$$

$$776.7 \times 10^6 \text{ rev}^n \rightarrow ?$$

$$L_{50} = 11355.25 \text{ hrs}$$

ACBB = angular contact ball bearings

Characteristic of an Antifiction Bearing

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Ball Bearing

DG BB $\rightarrow$ $F_r \uparrow, F_a \downarrow$
ie, $\frac{F_r}{F_a} > 1$

• Noise is less

Self aligning BB

They permit some amount of angular misalignment between shaft and bearing axes due to its self aligning properties

ACBB (angular contact)
 $F_r \downarrow, F_a \uparrow$

1) Single row ACBB

They withstand (F_a) thrust loads only in one direction

2) double row ACBB
 F_a in both direction

Thrust bearing

They withstand F_a

Roller bearing

(i) Cylindrical roller bearing
only F_r ie, $(F_a \approx 0)$

(ii) Tapered roller bearing
It withstand high radial loads, and high thrust loads.

(iii) Spherical roller bearing
It has self aligning properties, both F_r and F_a

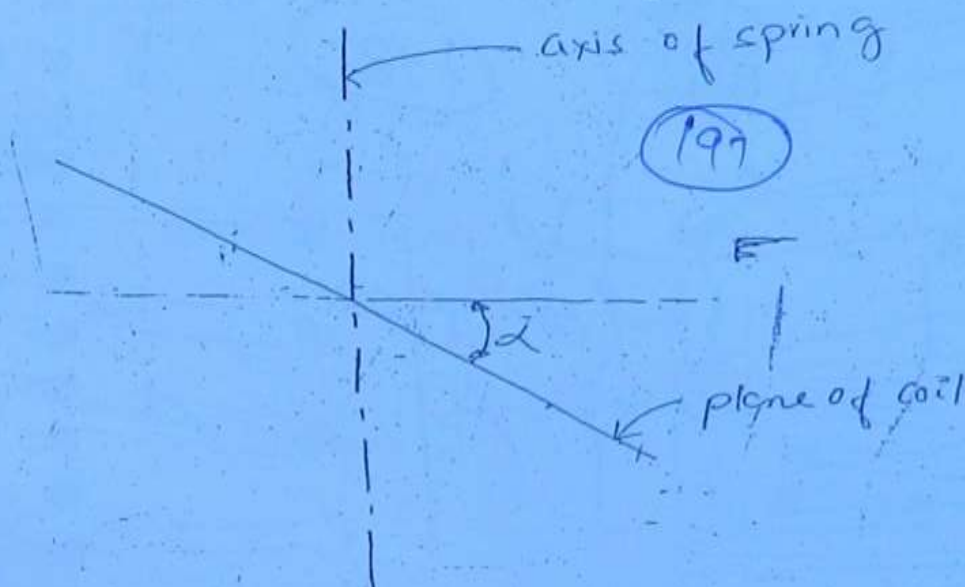
(iv) Needle roller bearing
• Where radial space is a constraint.

• No cages
• used in oscillating motions

(v) Thrust roller bearing
• They withstand only axial thrust (F_a) .

* (9) SPRINGS

Helical compression springs [close coiled]



$\alpha = 90^\circ \Rightarrow$ plane of coil is at $\perp$ or to axis of spring -

close coiled $\Rightarrow \alpha = 7^\circ$ to 8°

ie, plane of coil are almost at right-angle to axis of the spring.

$$C = 1 \rightarrow d = 50$$

$$D = 50$$

$$C = 4 \Rightarrow d = 50$$

$$D = 200$$

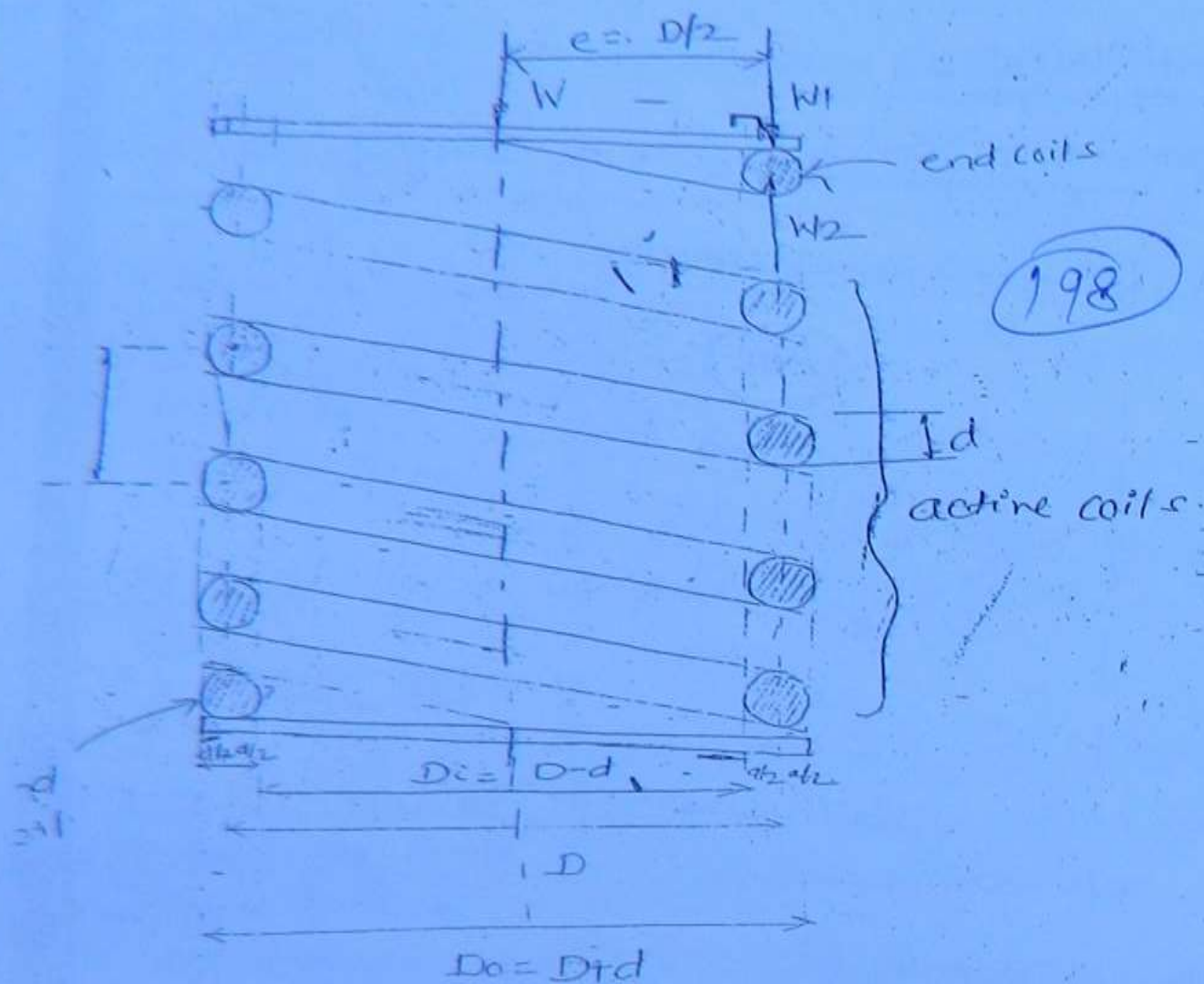
$$C = 20 \Rightarrow d = 50$$

$$D = 1000$$

$$C = \frac{D}{d}$$

$$\therefore C \neq 3$$

$$> 12$$



d = dia of spring wire

D = Dia of Spring

= mean coil diameter

D_o = outer dia of spring = $D + d$

D_i = inner dia of spring = $D - d$

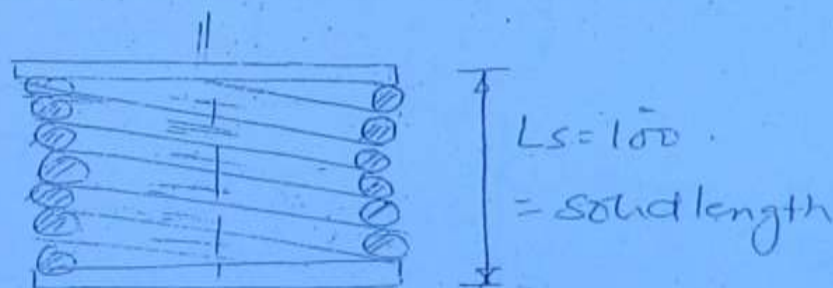
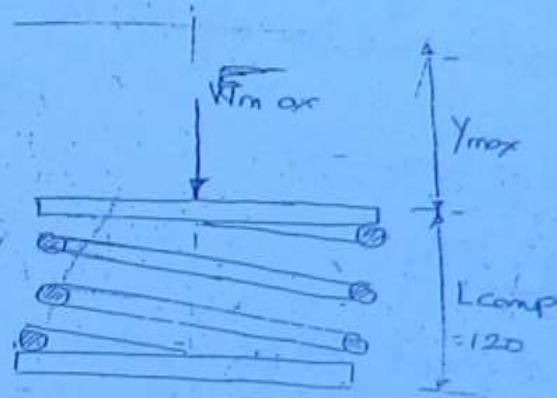
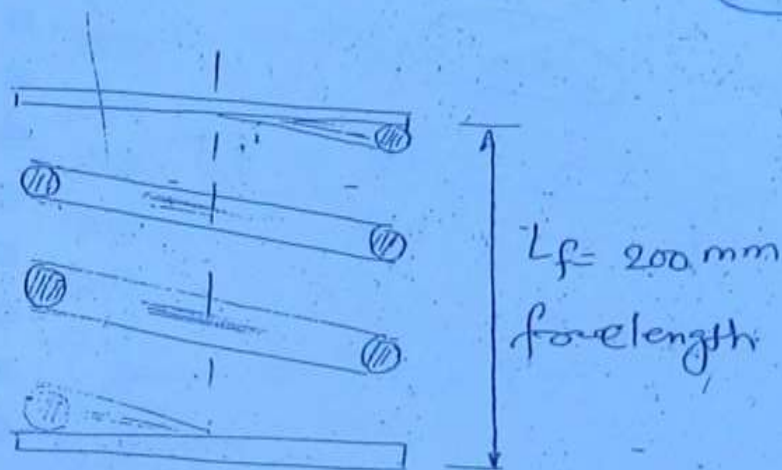
$$C = \text{Spring Index} = \frac{D}{d}$$

$$\Rightarrow C = 4 \text{ to } 12$$

$k = \text{Spring stiffness} = \text{spring rate}$

$$k = \frac{-W}{y} \quad \text{or} \quad \frac{\Delta W}{\Delta y}$$

(199)



$L_{comp} = L_s + \text{Total gap between coils under maximum deflected position}$

$$L_f = L_{comp} + Y_{max}$$

$= L_s + \text{total gap between coil under maximum deflection position} + Y_{max}$

$$= L_s + 1.5\% \text{ of } Y_{max} + Y_{max}$$

$$L_f = L_s + 1.15 Y_{max}$$

$$\text{Solid Length} = L_f = n \cdot d + 1.5 y_{\max}$$

$$L_s = n \cdot d$$

n = no. of active coils

d = diameter of wire

200

$$\text{Now } W_1 = W_2 = W$$

Effect of W_1 :

τ_1 : direct shear stress

$$\tau_1 = \frac{W_1}{\frac{\pi}{4} d^2}$$

$$\tau_1 = \frac{4W}{\pi d^2} \rightarrow (1)$$

Effect of W and W_2

It causes twisting moment

$$TM = W \times \frac{D}{2}$$

$$\tau_2 = \frac{T}{Z_p} = \frac{16T}{\pi d^3} = \frac{16^8 \times WD}{\pi d^3 \times 2}$$

$$\tau_2 = \frac{8WD}{\pi d^3} \rightarrow (2)$$

Now

$$T_1 = \frac{4W}{\pi d^2} \times \frac{2D}{d} \times \frac{d}{2D}$$

$$= \frac{8WD}{\pi d^3} \times \frac{d}{2D}$$

(2a)

$$T_1 = \frac{8WD}{\pi d^3} \times \frac{0.5}{C} \rightarrow (B)$$

Now

$$T_{\text{max}} \text{ or Resultant} = T_1 + T_2$$

$$= \frac{8WD}{\pi d^3} \left[1 + \frac{0.5}{C} \right]$$

$$T_{\text{max}} = T_R = \frac{8WD}{\pi d^3} \times k_{sh}$$

k_{sh} = shear stress correction factor

$$k_{sh} = 1 + \frac{0.5}{C}$$

$$T_{\text{max}} = T_R \times k_c$$

$$T_{\text{max}} = \frac{8WD}{\pi d^3} \times k_{sh} \times k_c$$

w = Wahl's factor

$$K_w = K_{sh} \cdot K_c \leftarrow \text{curvature effect}$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

2.02

$$(\tau_{max})_{ind} = \frac{8WD}{\pi d^3} \times K_w$$

$$\text{or } (\tau_{max})_{ind} = \frac{8WIC}{\pi d^2} \times K_w \leq \tau_{per}$$

$d \geq \text{min}$ can be found.

$$(\tau_{max})_{ind} = \frac{8W_{max} \cdot C}{\pi d^2} K_w \leq \tau_{per}$$

$$D = Cd$$

$$D_o = D + d$$

$$D_i = D - d$$

above parameters can be calculated
by calculating 'd'

Expression of y_{max}

Using the strain energy stored in the spring due to twisting we can find y_{max} .

$U =$ SE stored in spring

$$U = \frac{1}{2} T \cdot \theta$$

(203)

$$U = \frac{1}{2} \times T \times \frac{TL}{GJ} = \frac{T^2 L}{2GJ}$$

$$T = WD/2, L = \pi Dn$$

$$U = \left(\frac{WD}{2}\right)^2 \times \frac{\pi Dn}{2 \times G \times \frac{\pi d^4}{32}}$$

$$U = \frac{W^2 D^3 n}{8 G d^4}$$

$$U = \frac{4 W^2 D^3 n}{G d^4}$$

by using Castigliano's theorem

$$y_{max} = \frac{\partial U}{\partial P}$$

$$y_{max} = \frac{8 W D^3 n}{G d^4}$$

$$y_{\max} = \frac{8W_{\max} C^3 n}{Gd}$$

∴ from above eqn 'n' can be determined.

low $k = \frac{W_{\max}}{y_{\max}}$

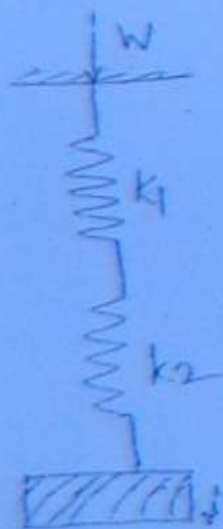
(204)

$$k = \frac{Gd}{8C^3 n}$$

$$\Rightarrow k \propto \frac{1}{n}$$

when the spring is cut into 'n' equal parts the stiffness of the spring increases by 'n' times.

Series



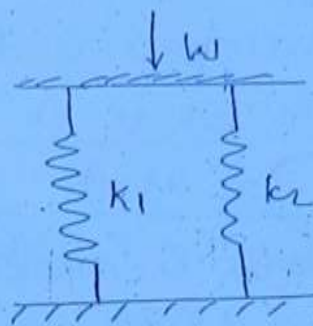
$$W_1 = W_2 = W$$

$$y = y_1 + y_2$$

$$\frac{W}{k} = \frac{W_1}{k_1} + \frac{W_2}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Parallel connection



(205)

$$y_1 = y_2 = y$$

$$W = W_1 + W_2$$

$$k_y = k_1 y_1 + k_2 y_2$$

$$\boxed{k_{eq} = k_1 + k_2}$$

① diameter of spring wire (d)

$$(\tau_{max})_{ind} \leq \tau_{per}$$

$$\frac{8 \cdot W_{max} \cdot C}{\pi d^3} \leq \tau_{per}$$

$$\therefore d \geq 2.7 \text{ mm}$$

② dimensions of springs

$$D = Cd$$

$$D_o = D + d$$

$$D_i = D - d$$

③ No. of active coils (n)

$$y_{ind} = \frac{8 W C^3 n}{G d} \leq y_{per}$$

or

$$\geq y_{per}$$

$$\therefore n \leq \frac{?}{?} \text{ can be found out}$$

4) Find out $y_{\max} = ?$

$$y_{\max} = \frac{8 W_{\max} C^3 n}{G d}$$

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(5) Solid length (L_s)

$$L_s = n \cdot d$$

(6) free length (L_f)

$$L_f = L_s + 1.15 y_{\max}$$

(7) check for buckling

$$\frac{L_f}{D} \leq 3.5 \Rightarrow \text{No buckling}$$

$$\frac{L_f}{D} > 3.5 \Rightarrow \text{buckling occurs}$$

~~when~~ ~~buckling occurs then the~~ of the springs, the spring should be placed in a sleeve or a hollow cylinder.

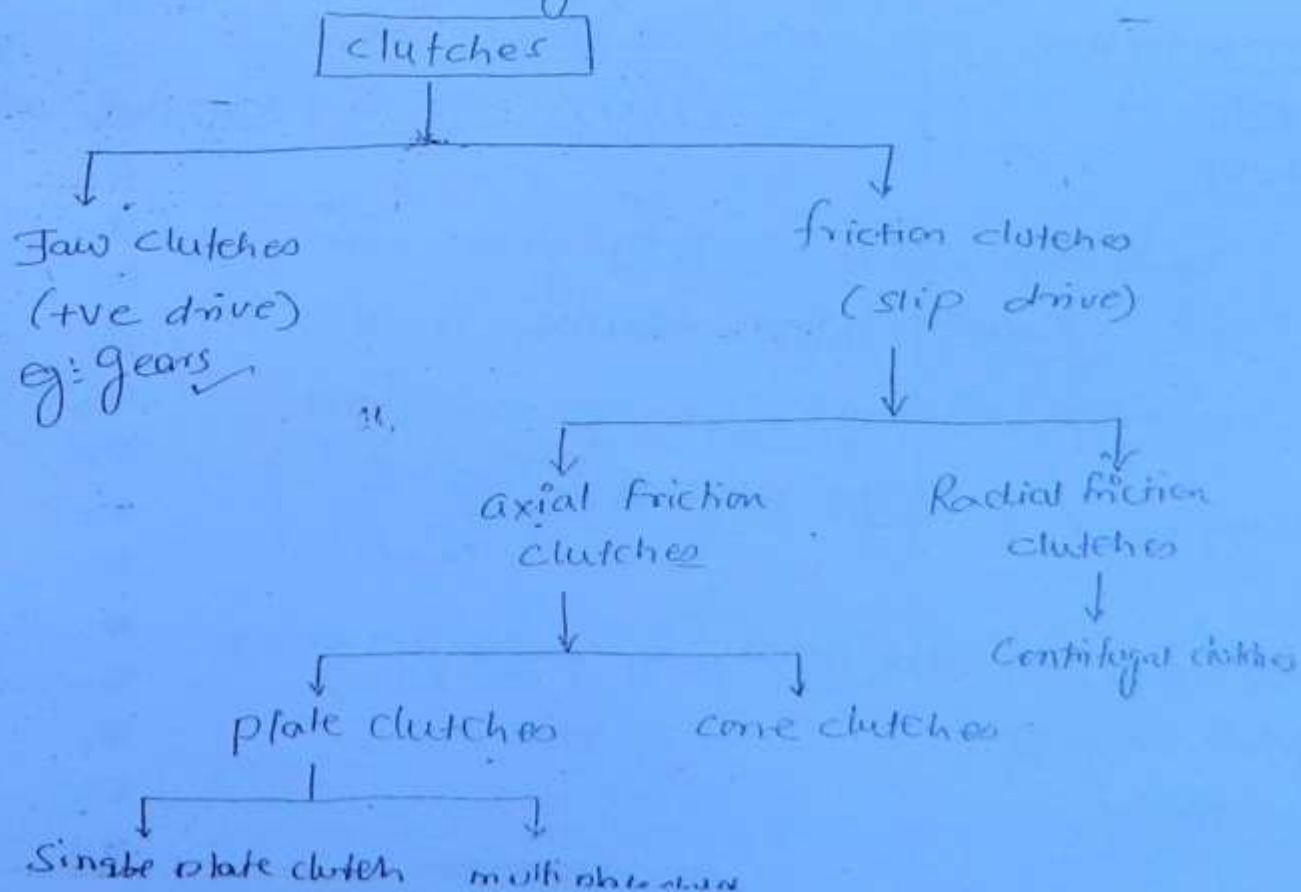
(10) CLUTCHES

(207)

clutch is defined as a mechanical device which is used to transmit power from a driver shaft to a driven shaft.

⇒ clutch is also defined as a mechanical device which is used to engage/disengage the driven shaft to/from the driver shaft, at the will of the operator.

⇒ The main function of clutch is to avoid frequent stopping or starting of the prime mover i.e., by means of clutch the vehicle can be stop or started any number of times, without stopping the prime mover or engine.



axial friction clutches: force/pressure is applied along the shaft axis. (208)

radial friction clutches: forces is applied in the radial direction (per to the shaft axis)

axial used in 4 wheeler like Tractors, Trucks!

radial used where space is a problem like bikes, scooters, autos.

Centrifugal clutches (Automatic clutches) used in mopeds.

Jaw clutches: used in M/C tools & Rolling mills.

old or worn out clutches
(UWT)

$$T = \frac{P \times 60}{2\pi N} \times 10^6 = \frac{?}{?} \text{ Nm}$$

$$W = \pi \frac{1}{2} L W [R_o + R_i]$$

W = axial force reqd to engage the clutch.

New clutches
(UPT)

Same

$$T_f = \pi \times \frac{2}{3} L W \left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} \right]$$

$$W = P \times \pi (R_o^2 - R_i^2)$$

Old or worn out
clutches
(UWT)

New clutches
(UPT)

(209)

$$W = p \times 2\pi R_i (R_o - R_i)$$

$$T_f = n \times \frac{1}{2} \mu \times p \times 2\pi R_i (R_o^2 - R_i^2)$$

$$\Rightarrow T_f = n \cdot \mu \cdot p \cdot \pi R_i (R_o^2 - R_i^2)$$

So, $R_o = ?$

$\Rightarrow n =$ no. of pair of
contact surfaces

$$n = 1 \Rightarrow \text{SPC}$$

$$n = 2 \Rightarrow \text{SPC effective on either side}$$

$$\Rightarrow n = n_1 + n_2 - 1 \Rightarrow \text{MPC}$$

$$T_f = n \times \frac{2}{3} \mu p \pi (R_o^3 - R_i^3)$$

$$T_f = \frac{2}{3} n \mu p (R_o^3 - R_i^3)$$

$R_o = ?$

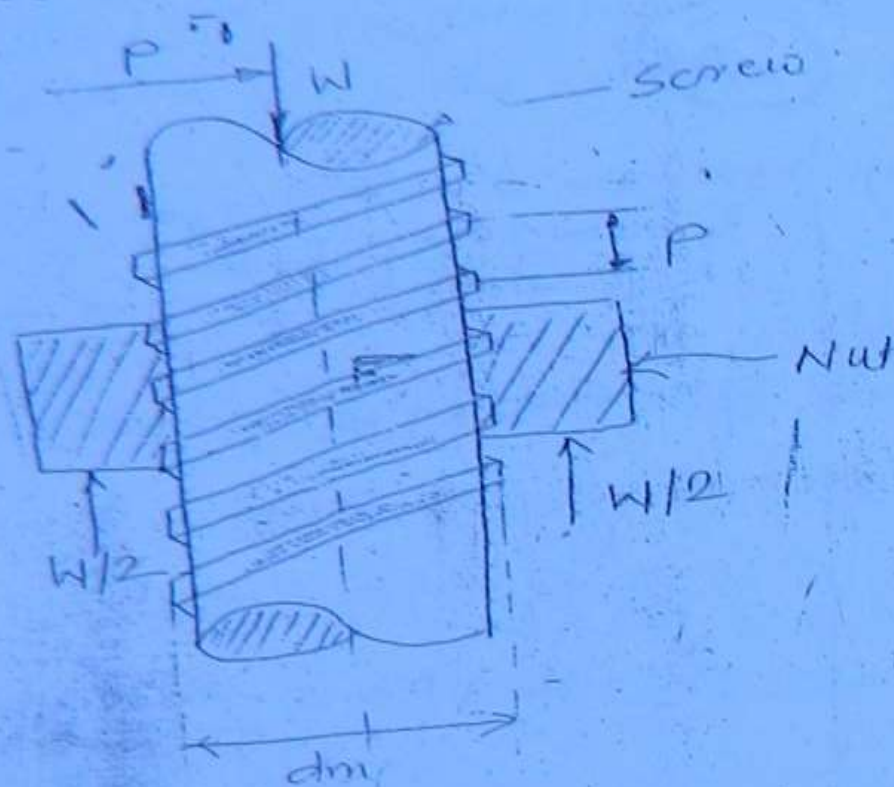
Can be
calculated

Where $n_1 =$ no. of plates or discs attached to a driver shaft

$n_2 =$ no. of plates or disc attached to a driven shaft

$n =$ even number in multiplate plate clutch

(11) POWER SCREWS



(210) -

Load applied on screw through horizontal effort
 Load to lifted vertically = W
 Mean diameter = d_m

effort applied is one revolution and load
 is lifted axially by pitch P of the thread,
 for single start threads and by lead
 of thread for multi start threads.

$$\text{Helix angle } \alpha = \tan^{-1} \left(\frac{P}{\pi d_m} \right) \rightarrow \text{Single start}$$

$$\alpha = \tan^{-1} \left(\frac{\text{Lead}}{\pi d_m} \right) \rightarrow \text{for multi-start threads}$$

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⇒ Let $\mu =$ coefficient of friction between Screw and Nut.

Let $\phi =$ angle of friction.

We have $[\mu = \tan \phi]$

Effort to raise the load (P_r)

$$P_r = W \tan(\alpha + \phi)$$

$$= W \left[\frac{\frac{\text{lead}}{\pi d_m} + \mu}{1 - \frac{\text{lead}}{\pi d_m} \times \mu} \right]$$

Turning moment applied on screw to raise the load $= T_r$

$$T_r = W \cdot \frac{d_m}{2} \tan(\alpha + \phi)$$

and 1. If $\alpha > \phi$

then, after the removal of the effort 'P' Load W, will come down without applying any rotational moment on the nut.

ex.

$$\boxed{\phi < \alpha} \quad \begin{matrix} \times \times \\ \times \end{matrix}$$

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When after the removal of the effort 'P' load 'W' will remain in position that is locked in the position without applying any brake and The screw is said to be Self Locked.

~~Now~~ Now, for $\phi > \alpha$, then effort will be required to lower the load.

$$P_{\text{lower}} = W \tan(\phi - \alpha)$$

$$\text{Turning moment } TL = W \frac{dm}{2} \tan(\phi - \alpha)$$

Efficiency of power screw

Ideal effort = It is that effort required to raise the load when $(\phi = 0)$

$$P_{\text{ri}} = W \tan \alpha$$

$$\therefore \eta_{\text{power screw}} = \frac{\text{Ideal effort}}{\text{actual effort}}$$

$$= \frac{P_{\text{ri}}}{P_{\text{ra}}}$$

$$\eta_{ps} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

$$\eta_{ps} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

⇒ for self locking screw i.e., $(\phi > \alpha)$

$$\text{We get } \eta_{ps} < 50\% \quad \text{*** (for any p.s.v exams)}$$

$$\Rightarrow \eta_{ps}(\text{Trapezoidal thread}) < \eta_{ps}(\text{square thread})$$

Condition for Maximum efficiency :-

i.e., To determine α

$$\frac{d\eta}{d\alpha} = 0$$

$$\alpha = \frac{\pi}{4} - \frac{\phi}{2} \quad \text{***}$$

→ Condⁿ for maximum efficiency

RAVE