

200

-: HAND WRITTEN NOTES:-

OF

200

MECHANICAL ENGINEERING

①

-: SUBJECT:-

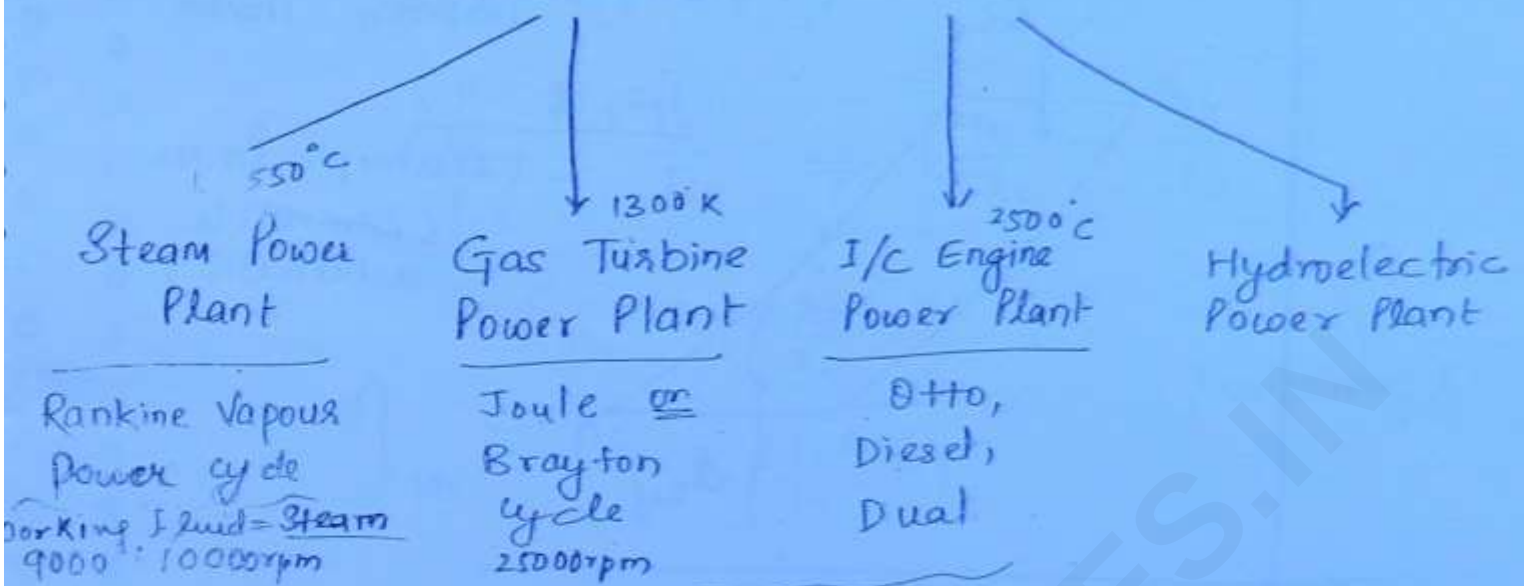
POWER PLANT ENGINEERING

005

2

WWW.GATENOTES.IN

POWER PLANT ENGG

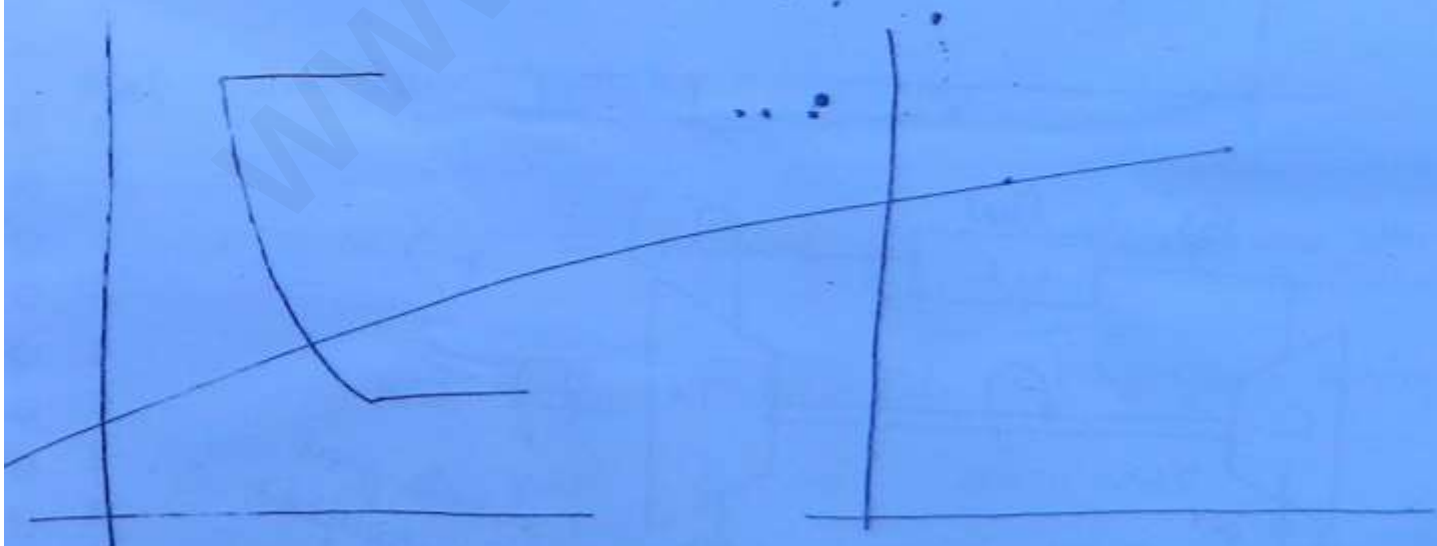


Working fluid
= GAS

③

GAS POWER CYCLE

BRAYTON CYCLE or IDEAL JOULE (Turbo jets) CYCLE



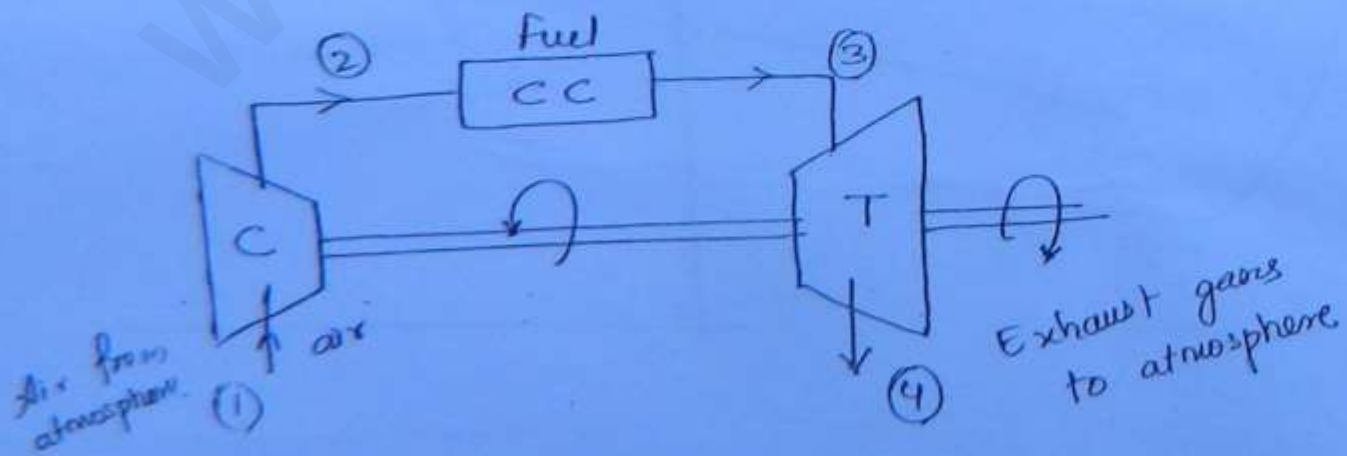
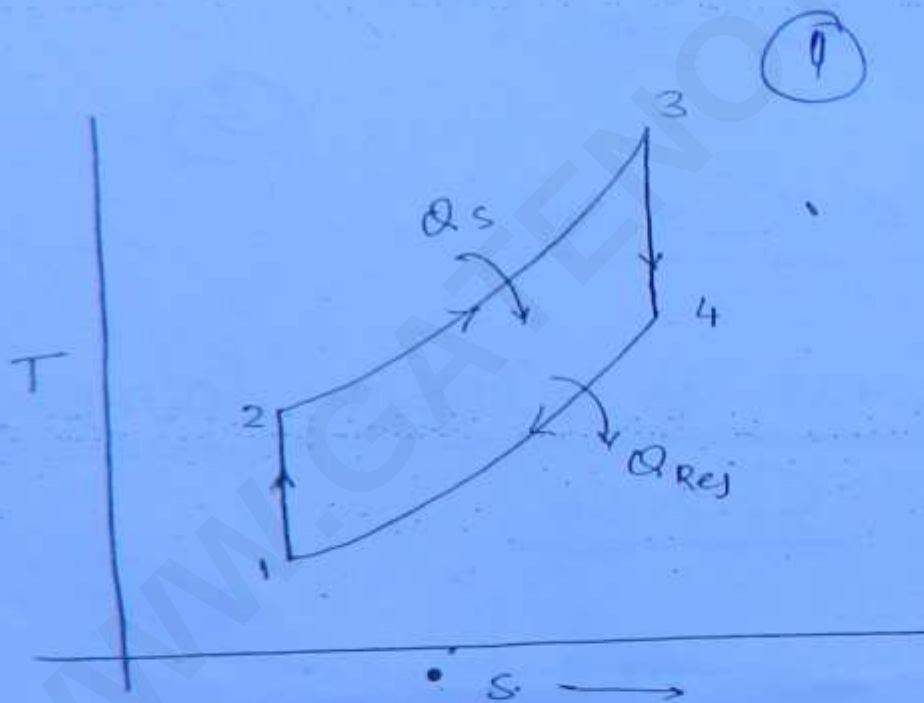
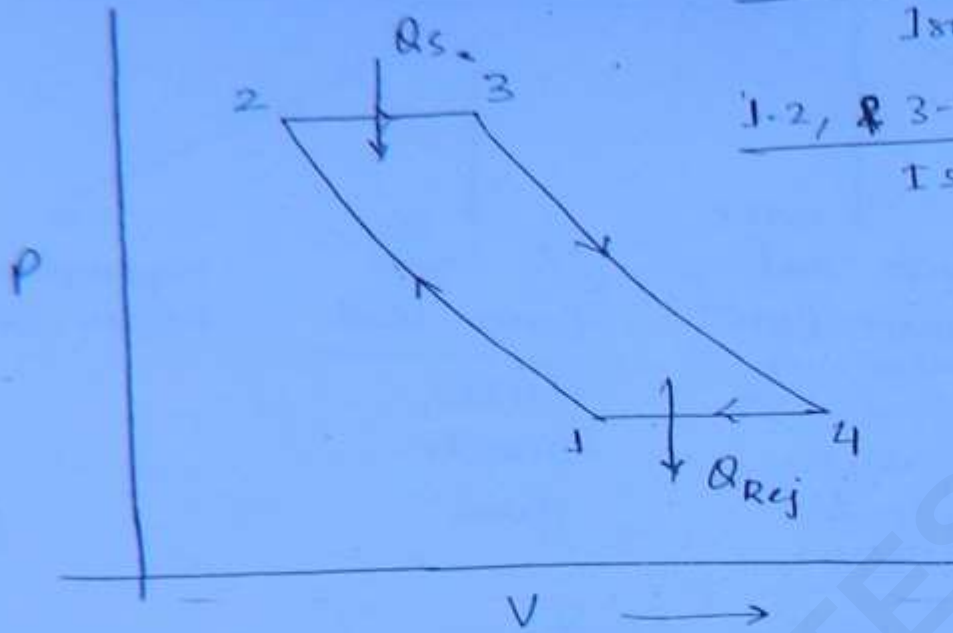
1-4 & 2-3,

Isobaric Process

1-2, & 3-4,

Isentropic Process

(Reversible
adiabatic)



from ① & ②

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \boxed{\frac{T_4}{T_1} = \frac{T_3}{T_2}}$$

$$\eta_{th} = 1 - \frac{T_1 \left(\frac{T_4}{T_1} - 1 \right)}{T_2 \left(\frac{T_3}{T_2} - 1 \right)} \quad (\text{as above})$$

$$\boxed{\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

Note:

As the pressure ratio of the Brayton Cycle, increases, the Ideal Air Standard Efficiency of the brayton cycle increases

Net work done per kg —

$$\boxed{\text{Net W.D./kg} = (\text{Turbine work}) - (\text{compressor work})}$$

$$\text{Turbine Work} = \Delta h = (h_3 - h_4) \text{ kJ/kg}$$

$$\left[\because T ds = dh - v dp, \text{ if } ds = 0 \quad dh = v dp \right]$$

$$\therefore \int dh = \int v dp \quad \left\{ \begin{array}{l} = \text{w.d. by a steady flow} \\ \text{open system in rev. process} \end{array} \right.$$

$$\text{Compressor Work} = \Delta h$$

For ideal gas, $\Delta h = C_p \cdot \Delta T$

$$\therefore \text{(iv) Turbine work} = \frac{C_{p_g} (T_3 - T_4)}{\quad} \text{ kJ/kg.}$$

$$\text{(cv) Compressor Work} = \frac{C_{p_a} (T_2 - T_1)}{\quad} \text{ kJ/kg}$$

$$\text{Net W.D./kg} = \text{TW} - \text{CW}$$

$$\boxed{\text{Net W.D./kg} = C_{p_g} (T_3 - T_4) - C_{p_a} (T_2 - T_1)}$$

$$C_{p_g} > C_{p_a} \Rightarrow 1005 \text{ kJ/kg K}$$

WORK RATIO (WR)

⑥

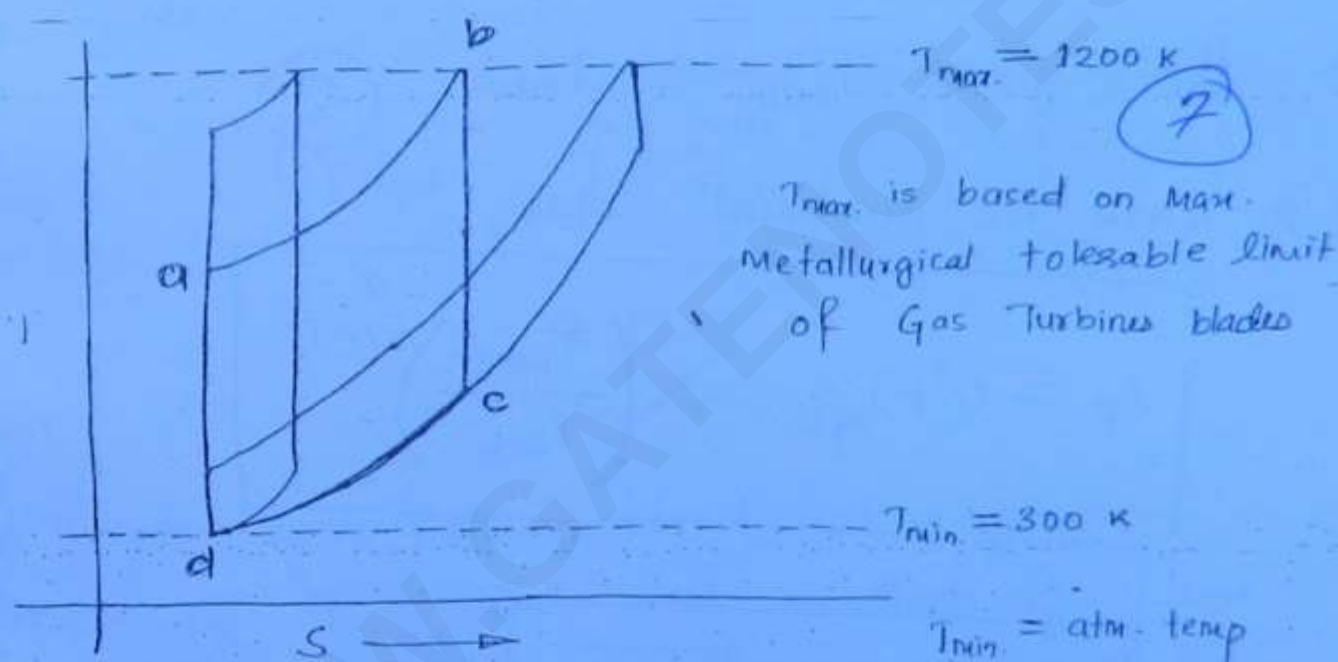
It is defined as net work done per kg and the turbine work done per kg.

$$\boxed{\text{WR} = \frac{\text{T.W.} - \text{C.W.}}{\text{T.W.}}}$$

$$\boxed{\text{WR} = \frac{C_{p_g} (T_3 - T_4) - C_{p_a} (T_2 - T_1)}{C_{p_g} (T_3 - T_4)}}$$

Work ratio also depends on pressure ratio (λ_p) of the cycle. As the pressure ratio increases, work ratio also increases, reaches a maximum and then decreases.

Effect of ^{Pressure} ~~Work~~ Ratio on net work done



For a given maximum and minimum temperatures of the cycle, there is a certain pressure ratio known as Optimum Pressure Ratio at which specific work output of cycle or its work ratio is maximum. Similarly, for a given T_{max} and T_{min} , the thermal efficiency of the Ideal

joule cycle, shall be maximum, when,

$$\boxed{\gamma_p = (\gamma_p)_{\max.}}$$

$$\boxed{\gamma_p = (\gamma_p)_{\max.} = \left(\frac{T_{\max}}{T_{\min.}} \right)^{\frac{\gamma}{\gamma-1}}}$$

But for a given $T_{\max.}$ & $T_{\min.}$ (8)

W.R. shall be maximum, when,

$$\boxed{\gamma_p = (\gamma_p)_{\text{opt.}} = \left(\frac{T_{\max.}}{T_{\min.}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$\boxed{(\gamma_p)_{\text{opt.}} = \sqrt{(\gamma_p)_{\max.}}}$$

Note: Value of ' γ ' —

for Helium — $\gamma = 1.67 //$

$\gamma_{\text{diatomic}} = 1.4 //$

$\gamma_{\text{monoatomic}} = 1.3 //$

if, $T_{max} = 1100$, $T_{min} = 300$, $\gamma = 1.4$

$$\therefore (\gamma_p)_{opt} = 9.439 //$$

$$\text{and } (\gamma_p)_{opt} = 9.71 //$$

Gate 2000

Q.) In an ideal air standard, Gas Turbine cycle, the minimum and maximum temperatures are respectively 310 K and 1100 K. Calc. optimal pressure ratio of the cycle.

Soln

$$(\gamma_p)_{opt} = \sqrt{\left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}}} \quad (\gamma = 1.4)$$

$$= 9.17 //$$

(9)

Gate 2001

Q.) A Brayton cycle (air standard) has a pressure ratio of 4 and inlet condition of 1 std. atm pressure and 27°C . Find the air flow rate required for 100 kW power output if the max. temp. in cycle is 1000°C . Assume $\gamma = 1.4$, $C_p = 1 \text{ kJ/kg K}$

Soln

Q2

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = 4 \quad \gamma_p = 4$$

$$\therefore T_2 = T_1 \cdot (\gamma_p)^{\frac{\gamma-1}{\gamma}} =$$

Solⁿ

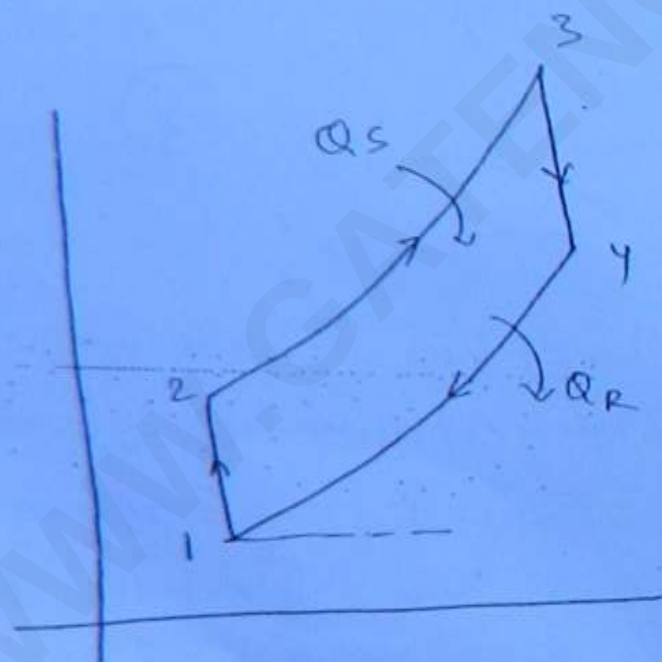
$$T_{\max} (T_3) = 1000^\circ \text{C} = 1273 \text{ K}$$

$$T_{\min} (T_1) = 27^\circ \text{C} = 300 \text{ K}$$

$$\begin{aligned} T_2 &= T_1 (r_p)^{\frac{\gamma-1}{\gamma}} \\ &= 300 (4)^{\frac{1.4-1}{1.4}} \\ &= 445.7 \text{ K} \end{aligned}$$

$$\text{and } T_4 = 856.67 \text{ K}$$

$$r_w = 416.3$$



(10)

$$\begin{aligned} \text{Turbine Work} &= C_{pa} (T_3 - T_4) \\ &= 416.32 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Compressor Work} &= C_{pg} (T_2 - T_1) \\ &= 145.7 \text{ kJ/kg} \end{aligned}$$

$$\therefore \text{Net WD/kg} = \text{TW} - \text{CW} \\ = 270.6 \text{ kJ/kg}$$

Mass flow rate

$$\text{Power Output} = \dot{m} \times \text{Net WD/kg}$$

$$\therefore \dot{m} = \frac{100}{270.6}$$

$$\boxed{\dot{m} = 0.369 \text{ kg/sec}}$$

(11)

GATE 1999

Q.) An isentropic air turbine is used to supplied 0.1 kg/sec. of air at 0.1 MN/m² and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m². Determine the temperature at turbine inlet and the power developed.

Soln

$$P_1 = P_4 = 400 \text{ Pa.},$$

$$P_2 = P_3 = 100 \text{ Pa.}, T_3 = 285 \text{ K}$$

$$\left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4} \therefore T_4 = 423.5 \text{ K}$$

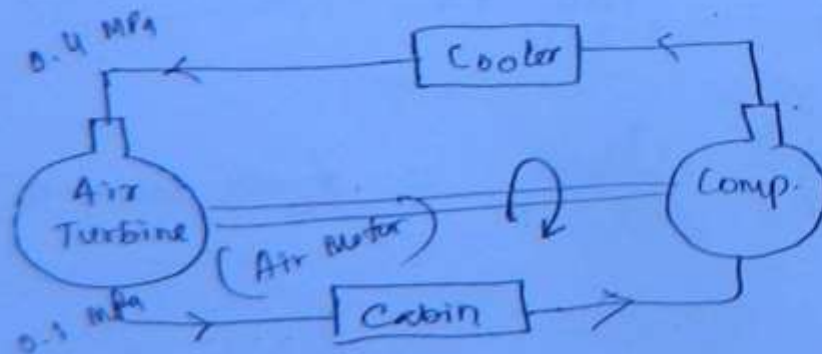
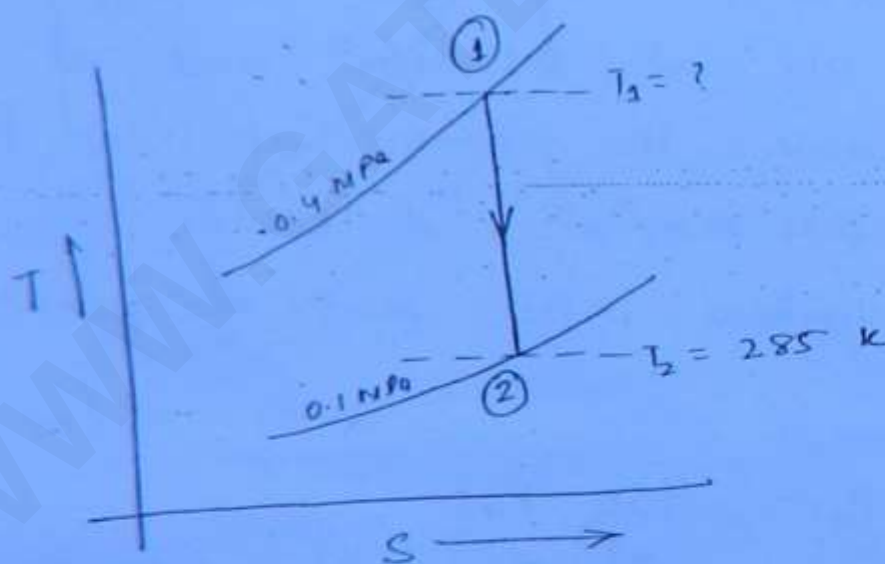
$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

The above given turbine is a part of air craft refrigeration system, running on Bell-Coleman Cycle (Reversed Brayton)

Now, $s_p = 0.25$

(12)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$



$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \quad (T_2 = 285 \text{ K})$$

$$T_1 = T_2 \left(\frac{400}{100} \right)^{\frac{1.4-1}{1.4}}$$

$$= 423.7 \text{ K}$$

(13)

$$\therefore \text{Net WD/sec} = \dot{m} \times \Delta h$$

$$= \dot{m} \times C_p (T_2 - T_1)$$

$$= 0.1 \times 1 (423.7 - 285)$$

$$= 13.85 \text{ kJ/sec.}$$

MACHINE EFFICIENCY



1-2 $\Rightarrow$ Actual compression with friction.

$$\eta_{(180) \text{ compressor}} = \frac{\text{Isentropic work}}{\text{Actual work}}$$

$$\eta_{\text{ISO (compressor)}} = \frac{c_p (T_2' - T_1)}{c_p (T_2 - T_1)} \quad (\approx 80-85\%)$$

Isentropic efficiency of the turbine is defined as the ratio between actual work output of the turbine with friction and the isentropic work output of the turbine without friction. (15)

$$\eta_{\text{(ISO) Turbine}} = \frac{\text{Actual work}}{\text{Isentropic work output}} = \frac{c_p (T_3 - T_4)}{c_p (T_3 - T_4')}$$

$$\eta_{\text{(ISO) turbine}} = \frac{T_3 - T_4}{T_3 - T_4'} \quad (\approx 80-85\%)$$

Note

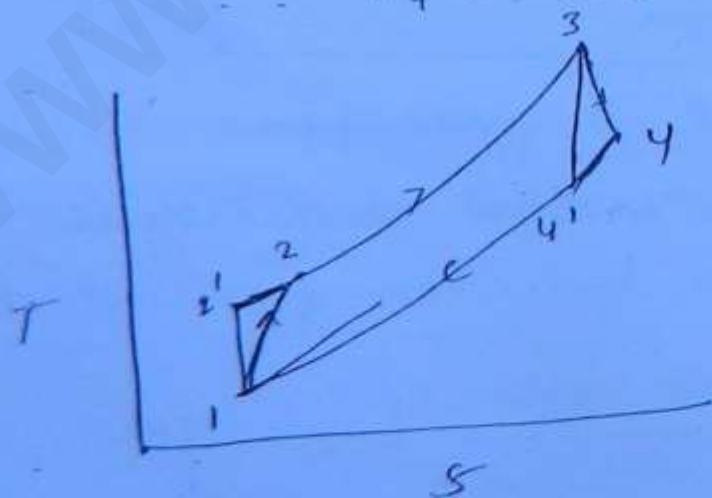
(Steam Engine running on modified Rankine cycle)
Oldest Engine

Q.) Find the required air fuel ratio in a gas turbine whose turbine and compressor efficiency are 85% and 80%. Max. cycle temp. is 875° . The working fluid can be taken as air. $C_p = 1 \text{ kJ/kg K}$, $\gamma = 1.4$, which enter the compressor at 1 atm and 27°C . The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg . There is a loss of 10% of calorific value in combustion chamber.

solⁿ $T_3 = 875^{\circ}$, $\eta_{\text{comp}} = 80\%$, $\eta_{\text{turb}} = 85\%$

$$\frac{T_3}{T_4} = \left(\gamma_p\right)^{\frac{\gamma-1}{\gamma}} \quad T_3 = 1148$$

$$T_4 = \frac{1148}{0.85} = 1350.5 \text{ K}$$



$$T_1 = 300 \text{ K}$$

$$\frac{T_2'}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_2' = 445.79 \text{ K}$$

$$\eta_{\text{comp}} = \frac{T_2' - T_1}{T_2 - T_1}$$

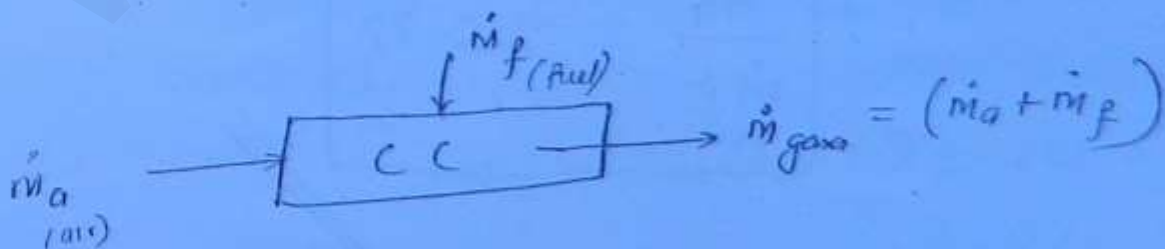
$$T_2 = 482.25 \text{ K}$$

$$\eta_{\text{turb}} = \frac{T_3 - T_4}{T_3 - T_4'}$$

17

$$T_4 = 828.83 \text{ K}$$

for combustion chamber (CC)



Energy balance

Heat liberated ^{due} to combustion of fuel

= Rate of change of Enthalpy
of gases in CC.

$$\dot{m}_f \times L.C.V = (\dot{m}_a + \dot{m}_f) C_{pg} (T_3 - T_2)$$

$$L.C.V = \left(\frac{\dot{m}_a}{\dot{m}_f} + 1 \right) C_{pg} (T_3 - T_2)$$

Efficiency of combustion is 90% (due to 10% loss)

(18)

$$\Rightarrow 0.9 \times L.C.V = \left(\frac{\dot{m}_a}{\dot{m}_f} + 1 \right) C_{pg} (T_3 - T_2)$$

$$\Rightarrow 0.9 \times 42000 = \left(\frac{\dot{m}_a}{\dot{m}_f} + 1 \right) 1 (1148 - 482.25)$$

$$\therefore \frac{\dot{m}_a}{\dot{m}_f} = 55.77$$

Q) Air enters the compressor of gas turbine plant operating on brayton cycle at 1 bar, 27°C . The pressure ratio in cycle is 6 if $w_t = 2.5 w_c$, where, w_t & w_c are turbine & compressor work respectively. Calc the max. temperature and the cycle efficiency.

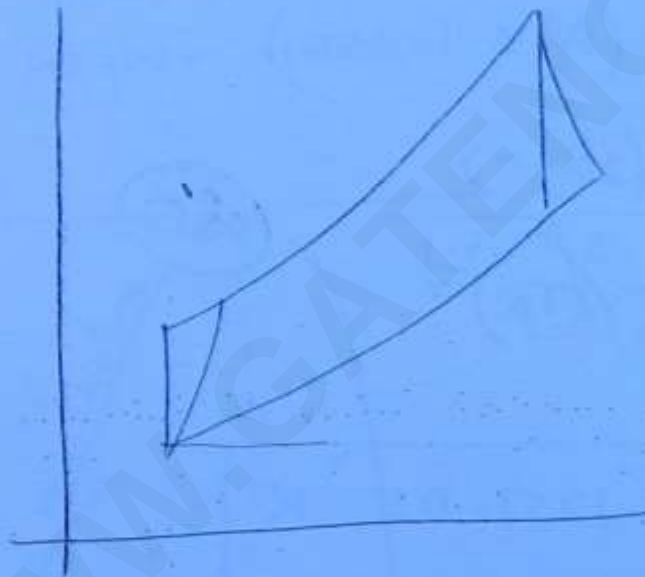
Solⁿ

$$w_t = 2.5 w_c$$

$$P_1 = 1 \text{ bar} \\ = 100 \text{ Pa}$$

$$T_1 = 300 \text{ K}$$

(19)



$$\frac{T_2}{T_1} = 6^{\frac{0.4}{1.4}} \Rightarrow T_2 = 500.55 \text{ K}$$

$$w_c = C_p (T_2 - T_1) = 200.55 \text{ kJ/kg}$$

$$\therefore w_t = 501.38 \text{ kJ/kg}$$

$$\eta_p = \left(\frac{T_{\max}}{T_{\min}} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\eta = 1 - \frac{T_1}{T_2} \quad \text{or} \quad 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \eta = 40 \%$$

$$W_T = 2.5 \text{ Wc}$$

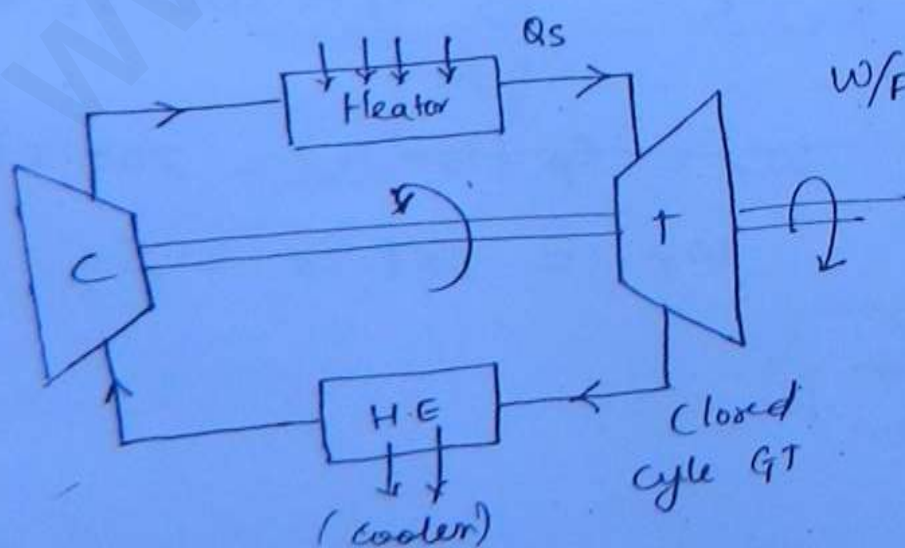
$$C_p(T_3 - T_4) = 2.5(T_2 - T_1)$$

$$T_3 \left(1 - \frac{T_4}{T_3}\right) = 2.5(500.5 - 300)$$

$$T_3 = \frac{2.5(200.5)}{1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}}$$

(20)

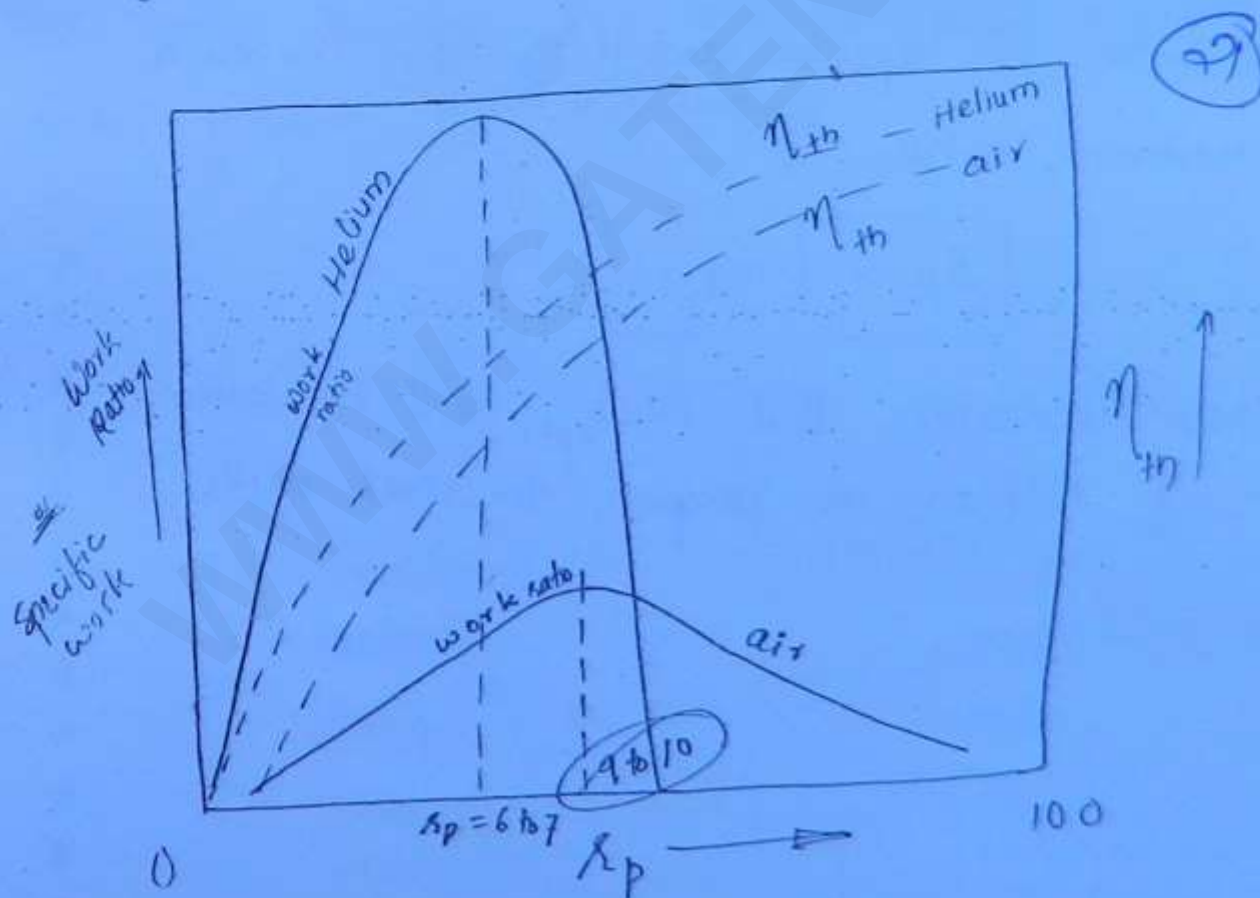
$$T_3 = 1251.05 \text{ K}$$



W/F = Helium
 $\gamma = 1.66$

In closed cycle gas turbine power plant, the chemical composition of working fluid will not change because heat is supplied to it from in the heater, after having combustion of the fuel outside the heater (which means fuel). Also that, working fluid may be circulating in the system @ with higher pressure.

Since helium has better thermodynamic properties than air i.e. higher value ' γ ', it can give higher thermal efficiencies.



(for given T_{max} & T_{min} .)
(1100 K & 300 K)

For a given T_{min} & T_{max} in the cycle, as the pressure ratio r_p increases, the thermal efficiency of the cycle increases either case of Helium or air, reaches a maximum, when,

$$r_p = (r_p)_{max.}$$

At any r_p (Pressure Ratio),

$$\eta_{He} > \eta_{air}$$

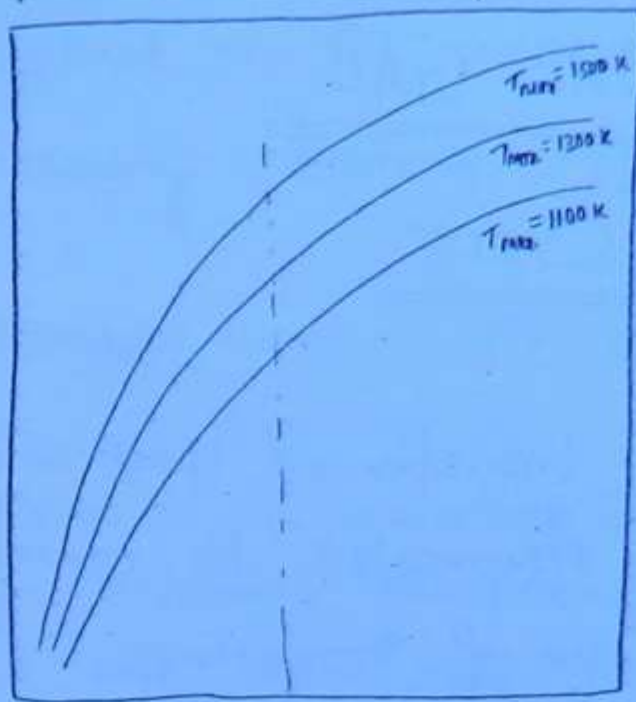
(22)

For work ratio, as r_p increases, it ~~also~~ also increases, reaches maximum, when,

$$r_p = (r_p)_{opt}$$

and then decreases. But $(r_p)_{opt}$ will be lesser in case of helium as compare to that with air.

η
thermal



$r_p \rightarrow$

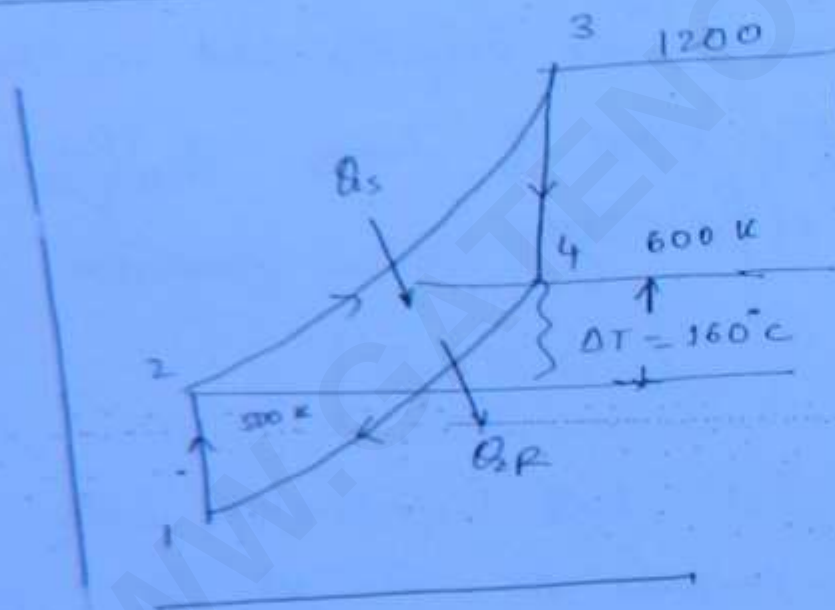
(23)

At any given pressure ratio r_p provided in the cycle, higher the value of T_{max} in the cycle, greater the efficiency.

REGENERATION IN GAS

TURBINE CYCLE

Regeneration means transfer of Heat within the cycle from Higher Temperature to Lower temperature for the sake of increasing Thermal Efficiency.



$$\gamma_p = 4$$

$$T_1 = 300 \text{ K}$$

$$T_3 = 1200 \text{ K}$$

$$T_2 = T_1 (4)^{\frac{\gamma-1}{\gamma}}$$

$$= 445.77 \text{ K}$$
$$= 500.55 \text{ K}$$

$$T_4 = \frac{T_3}{(\gamma_p)^{\frac{\gamma-1}{\gamma}}} = \frac{1200}{4^{\frac{\gamma-1}{\gamma}}} = 807 \text{ K}$$

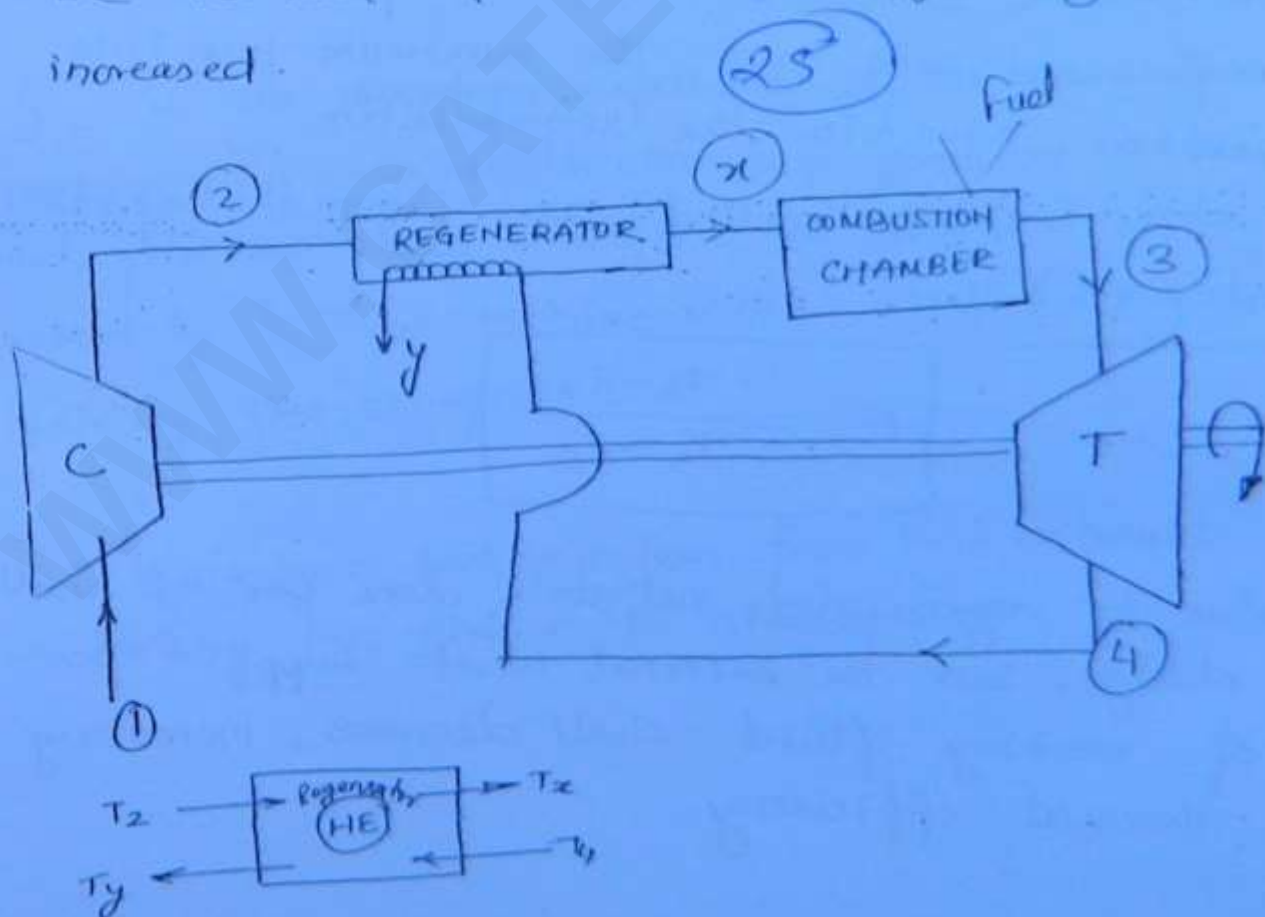
The Hot exhaust gases leaving the turbine are utilized for transferring the heat to the compressed air in a counter flow heat exchanger called Regenerator, there by reducing the external

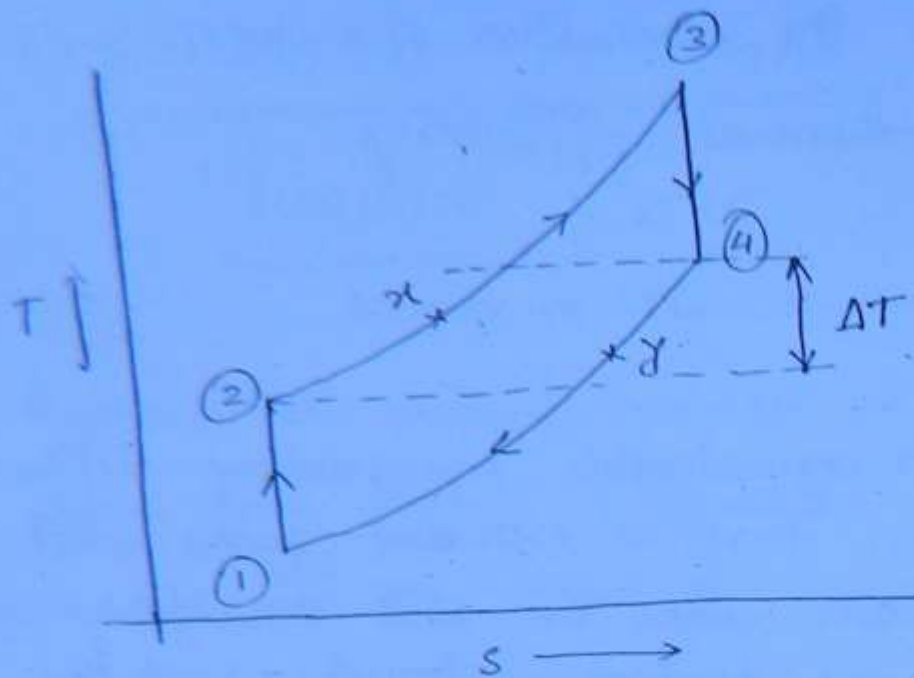
heat supplied in the combustion chamber,
hence increasing thermal efficiency.

notes

If $\%p$ decreases $\longrightarrow \Delta T$ increases

Since there is a considerable temperature difference existing between turbine exhaust gases and compressed air, there could be heat transfer possible between the turbine exhaust and the compressed air in a heat exchanger called REGENERATOR, thereby external heat supplied can be reduced & thermal efficiency can be increased.





(26)

T_x is the temperature of compressed air

The effectiveness of the regenerator is defined as the ratio between actual temperature rise of compressed air and the maximum possible temperature rise in the regenerator.

It is also called as Thermal Aspect Ratio (ϵ)

$$\epsilon = \frac{T_x - T_2}{T_4 - T_2} \cong 0.7$$

Due to regeneration, network done per kg shall not change, but the external heat supplied per kg of working fluid shall decrease, increasing the thermal efficiency.

The compressor

* 1-2, is the isentropic compression of air from atmospheric pressure to combustion chamber pressure increasing the pressure and temperature of air in the rotary compressor (centrifugal or axial)

* 2-3, is the isobaric heat addition to the air in the combustion chamber, due to the heat liberated by burning of fuel which is sprayed directly into the air, rising the temperature of the gases. (25)

* 3-4, is the isentropic expansion of the gases in the turbine from combustion chamber pressure to the exhaust pressure doing work on the turbine blade. Approximately $\frac{1}{3}$ rd of turbine work shall be used for driving the compressor.

* 4-1, is isobaric heat rejection from hot exhaust gases leaving the turbine to atmosphere

$$\text{Pressure Ratio } (r_p) = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\eta_{th} = 1 - \frac{Q_R}{Q_S}$$

Ideal
Brayton

$$Q_{Rej} = C_p (T_4 - T_1) \text{ kJ/kg}$$

$$Q_S = C_p (T_3 - T_2) \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, 1-2 is, Isentropic Process,

$$\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{--- (1)}$$

3-4 is, Isentropic process,

$$\left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{--- (2)}$$

Actual thermal efficiency of regenerative cycle is,

$$\eta_{(th) \text{ Reg.}} = \frac{\text{Net. Work Output/kg}}{Q_s \text{ /kg}} = \frac{W_T - W_C}{Q_s}$$

$$\eta_{th} = \frac{C_{p_g}(T_3 - T_4) - C_{p_a}(T_2 - T_1)}{C_{p_g}(T_3 - T_x)}$$

Mostly $C_{p_g} \cong C_{p_a}$

(29)

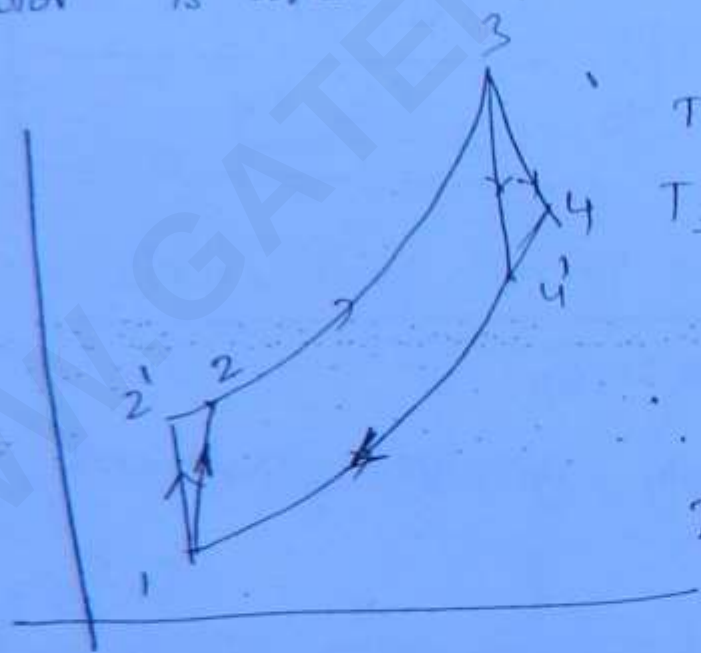
$$\eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_x)}$$

Hence, Regeneration in GT cycle results in,

- ① No change in $W.D / kg$
- ② Compressor heat supply / kg get reduced.
- ③ $\eta_{(th) \text{ cycle}}$ increases.
- ④ Specific fuel consumption gets reduced.
(Fuel Consumption required to produce 1 kW power)

Q.) A Gas Turbine Cycle operates with air having constt. specific heats. The inlet air is at 1 bar and 20°C . The max. cycle temp. is 1200 K. The compressor & turbine have same pressure ratio and have internal efficiency of 0.85. A regenerator with 85% effective ness is consider. Calc. all temp. around the cycle, η_{th} , heat added to produce 10MW power, cycle efficiency if no regenerator is used. Assume $r_p = 5$

Solⁿ



$$T_1 = 293 \text{ K}$$

$$T_3 = 1200 \text{ K}$$

$$T_2 = 464.06 \text{ K}$$

$$T_4 = 757.66 \text{ K}$$

$$\eta_{\text{cycle}} = 1 - \frac{T_1}{T_2} \therefore 0.85 = 1 - \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1}{1 - 0.85}$$

or

$$\cancel{T_2 = 713.62 \text{ K}}$$

$$\eta_{\text{comp}} = \frac{T_2' - T_1}{T_2 - T_1}$$

$$\cancel{I_2} \Rightarrow 0.85(T_2 - 293) = 464.06 - 293$$

$$\boxed{T_2 = 494.2 \text{ K}}$$

$$\cancel{\eta_T = \frac{T_3 - T_4}{T_2 - T_4}}$$

(3)

$$\eta_T = \frac{T_3 - T_4}{T_2 - T_4'}$$

$$\boxed{T_4 = 824 \text{ K}}$$

$$\epsilon = \frac{T_2 - T_1}{T_4 - T_2}$$

$$\boxed{T_2 = 774.53 \text{ K}}$$

$$\eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_2}$$

$$= 0.41 = 41\%$$

Without regeneration,

$$\eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_2}$$

$$= 24.7 \%$$

$$Q_s = \frac{\text{Power Supp.}}{\eta_{th}}$$

$$Q_s = \frac{10 \times 1000}{0.41} = 24.39 \text{ MW}$$

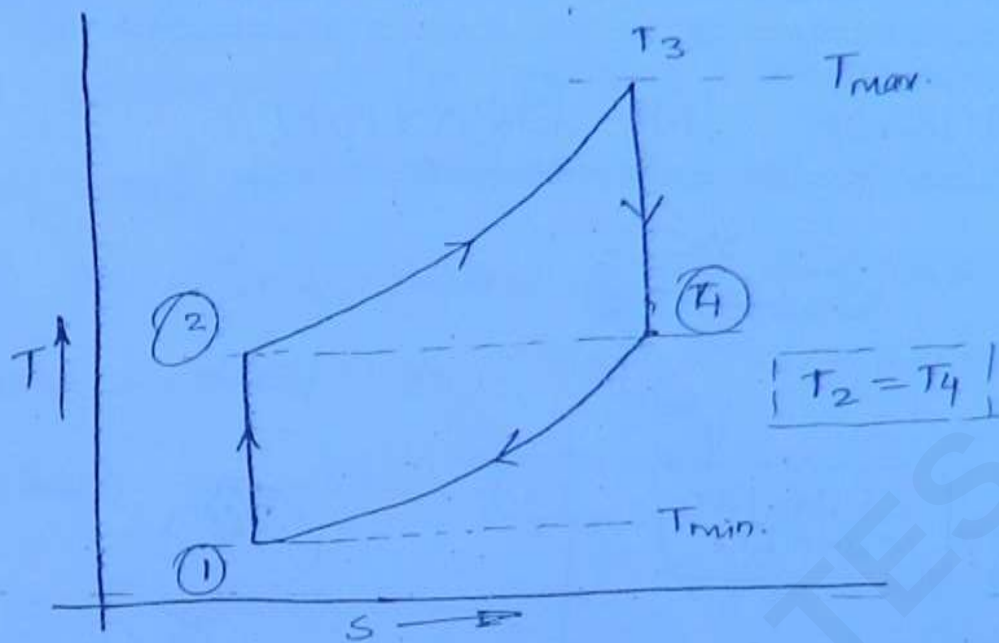
(32)

Note:

The effect of regeneration is increasing the efficiency is more pronounced at low air pressure ratios, since there could greater ΔT between turbine exhaust gases and compressed air.

There is one particular pressure ratio called Critical Pressure ratio where, there is no chance for regeneration because at these pressure ratio, compressed air temperature shall be equal to the turbine exhaust temperature.

For a given T_{min} and T_{max} . —



1120, $\frac{T_2}{T_1} = (r_p)_{critical}^{\frac{\gamma-1}{\gamma}}$ and $\frac{T_3}{T_4} = (r_p)_{critical}^{\frac{\gamma-1}{\gamma}}$

$$\boxed{\frac{T_2}{T_1} = \frac{T_3}{T_4}}$$

33

Put $T_2 = T_4 \Rightarrow T_2^2 = T_1 T_3 \Rightarrow \boxed{T_2 = \sqrt{T_1 T_3}}$

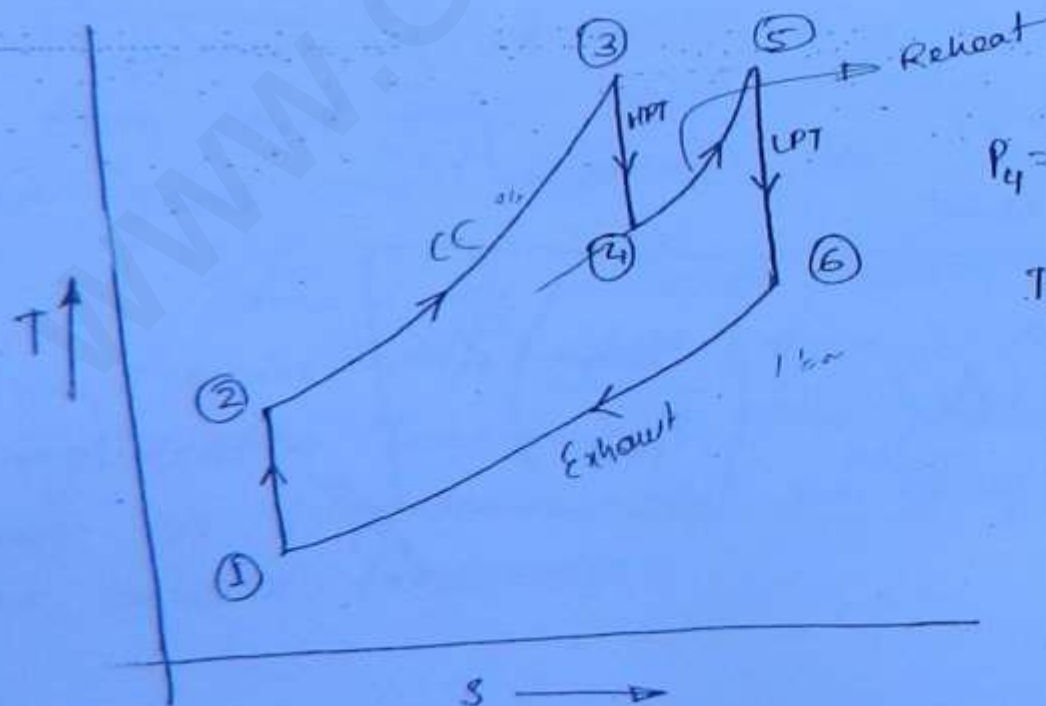
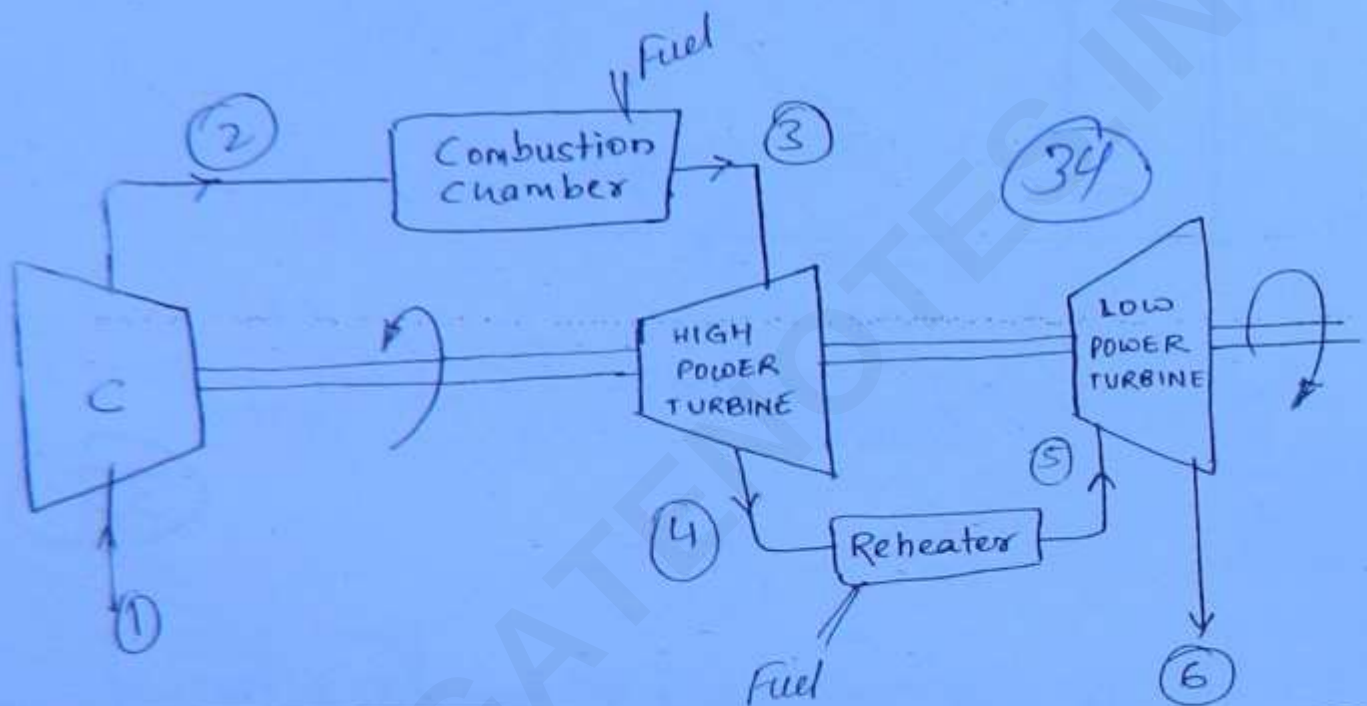
$$(r_p)_{critical}^{\frac{\gamma-1}{\gamma}} = \frac{\sqrt{T_1 T_3}}{T_1} = \sqrt{\frac{T_3}{T_1}}$$

$$\boxed{(r_p)_{critical} = \left(\frac{T_{max}}{T_{min}} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

PREHEATING WITH MULTISTAGE

EXPANSION IN BRAYTON

CYCLE



$$P_4 = P_5 = \text{Reheater Pressure}$$

$$T_2 = T_5$$

Gases generated in the combustion chamber, expand in the high pressure turbine from combustion chamber to some intermediate reheat temperature, where it is again reheated ~~to some~~ at constant pressure back to the same initial peak temperature. Then the gases expand in the low pressure turbine from reheat pressure to exhaust pressure.

For getting maximum total net work output from the cycle, when reheating is provided, the reheat pressure must be geometric mean of suction and delivery pressures i.e. pressure ratio across each stage of turbine, must be same.

$$\frac{P_3}{P_4} = \frac{P_5}{P_6} \quad \text{and} \quad P_4 = P_5 \quad (35)$$

$$\therefore P_5 = \boxed{P_4 = \sqrt{P_3 P_6}}$$

Effects of Reheating : —

- ① Net work output per kg of cycle increases.
- ② For a given mass flow rate of air, power output of the cycle increases.
- ③ For a given power output, overall size of the plant becomes smaller. $(\because P = \underset{\substack{\uparrow \\ \text{decrease}}}{m_a} \times \underset{\substack{\uparrow \\ \text{increase}}}{W.D./\text{sec}})$

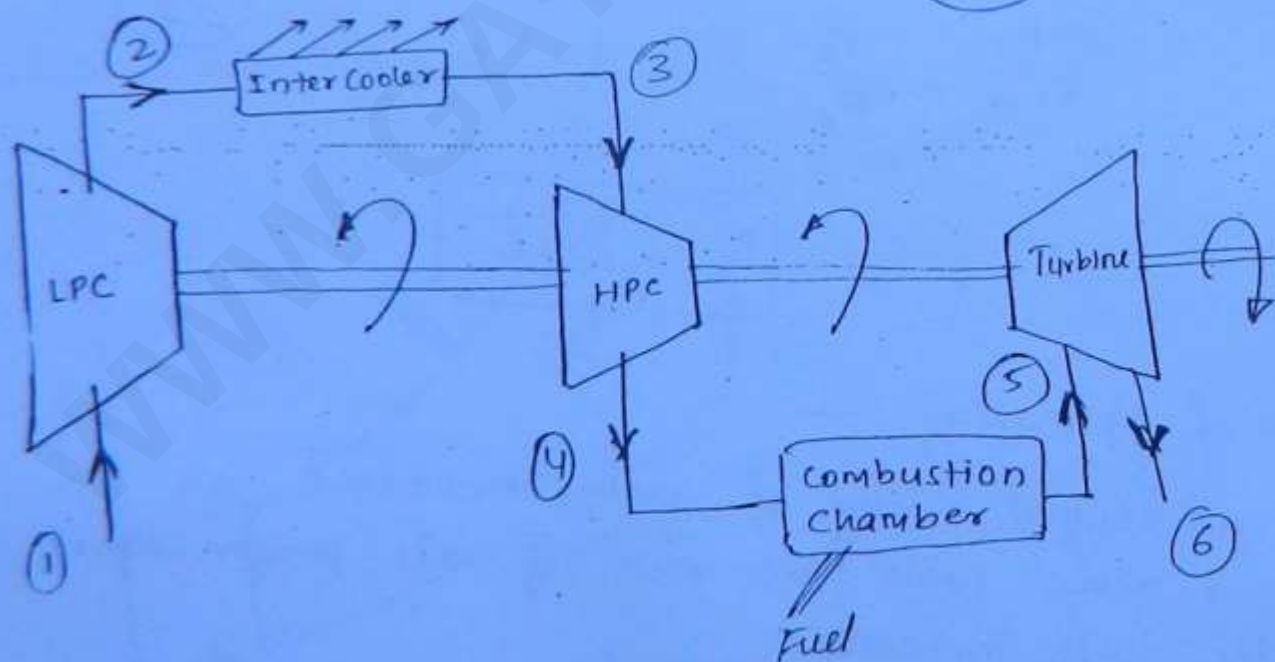
- ④ Thermal efficiency marginally increases.
- ⑤ Work Ratio increases.

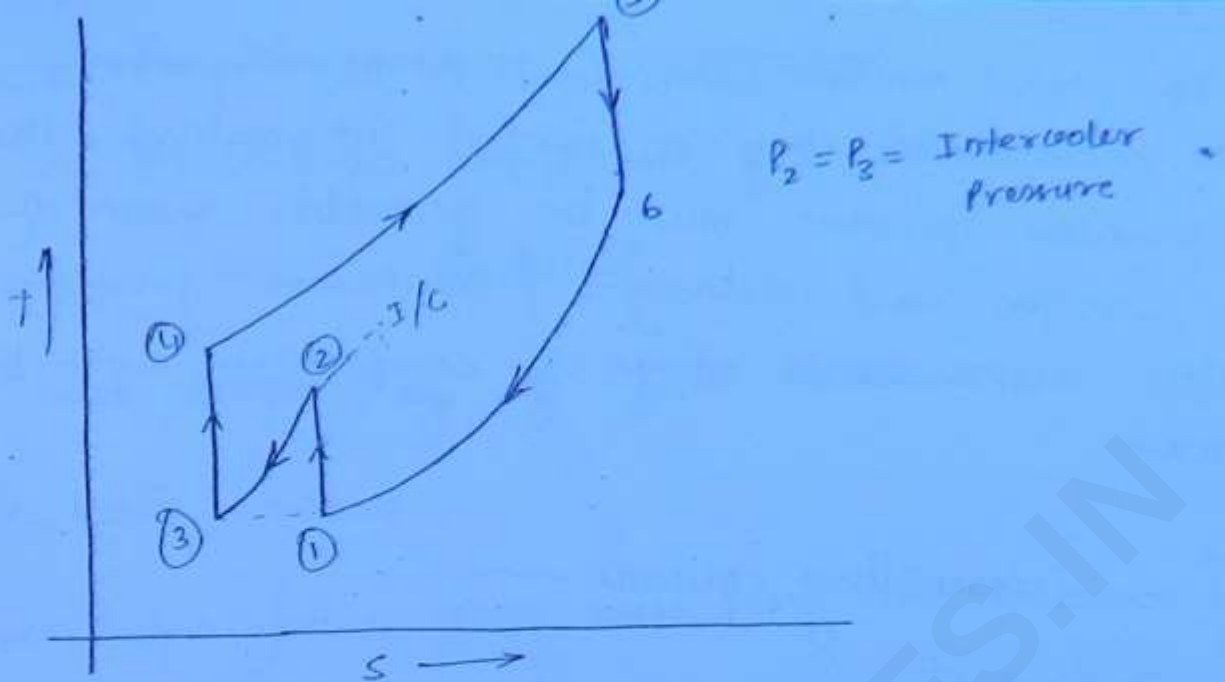
INTERCOOLING WITH MULTISTAGE COMPRESSION

IN BRAYTON CYCLE

For a given mass of air and for a given pressure ratio, work required in the multistage compression with intercooling shall always be lesser than work required for single stage compression.

③⑥





Air entering from the atmosphere into the compressor is compressed in the low pressure compressor from atmospheric pressure to some intermediate pressure called Intercooler pressure, where it is cooled at constant pressure (2-3) in the intercooler (37) back to the initial temperature, prior to L.P. compression. Then it is again compressed in the H.P. compressor, from intercooler pressure to combustion chamber pressure.

Ans More the no. of stages of compression, ^{with intercooling} lesser the work input required. If infinite no. of stages of compression are provided, then it ideal most compression, which requires least work input i.e. isothermal compression is obtained.

For total minimum work input to the compressor, with ideal intercooling or perfect intercooling, the intercooler pressure must be geometric mean of the suction and delivery pressures i.e. pressure ratio across each stage of compression must be same.

Perfect Intercooling means —

- ① $P_2 = P_3$ (No pressure drop in intercooler)
- ② $T_3 = T_1$ (Temp. of air after intercooling = Temp. of air prior to L.P. stage compression)

③ — then for total minimum work input,

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

(38)

$$\Rightarrow P_2 = P_3 = P_{i/c} = \sqrt{P_1 P_4}$$

All, 3 pressures (P_1, P_3, P_4) are in geometric progression

HPC < LPC ($\because$ there is constt $\dot{m}$)

$$\therefore fQ = C$$

$$\therefore V_3 < V_1$$

$$Q_3 < Q_1 //$$

EFFECTS OF MULTISTAGE COMPRESSION

Effects of Multistage Compressor with Intercooler —

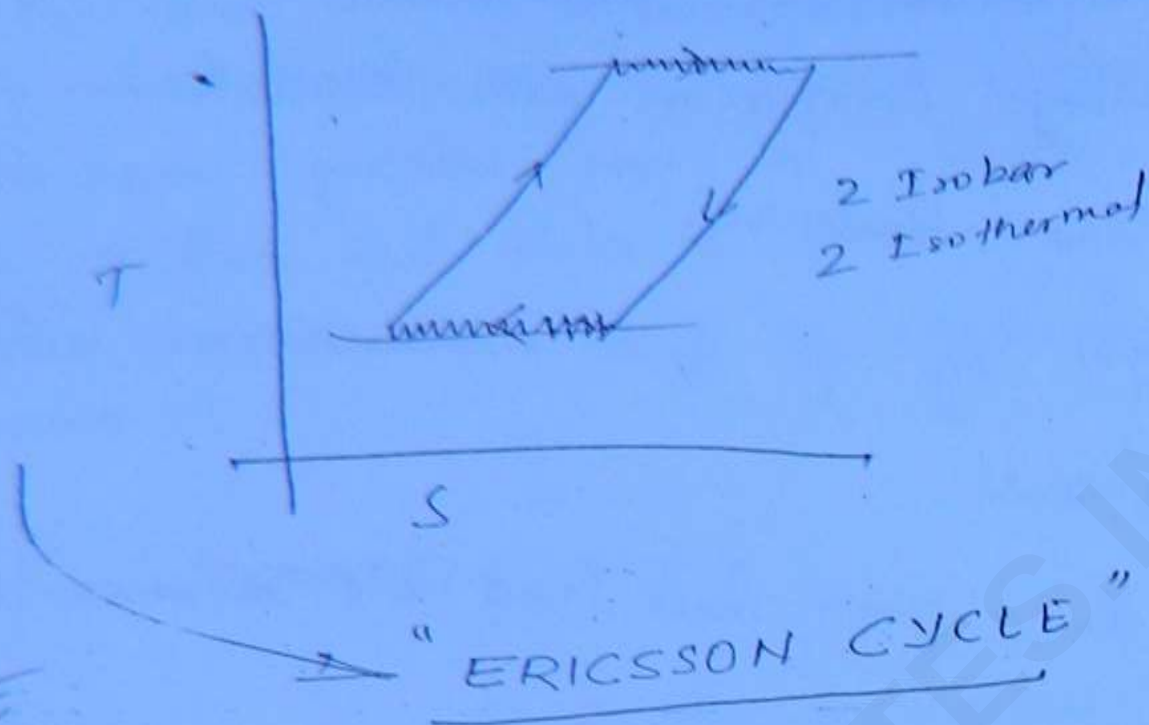
- (1) Compressor W_{input} Reduced.
- (2) Net work output / kg of cycle increases.
- (3) Work Ratio increases.
- (4) Power Output for a given mass flow rate increases.
- (5) η_{th} decreases.

(39)

The decrease in η_{th} because of multistage compression with intercooling, can be compensated by coupling the regenerator into these system, because, there is a greater scope for regeneration in these cycle because ΔT between turbine exhaust and compressed air is higher here.

Q.) A gas turbine cycle with infinite no. of stages of multistage compression with intercooling and infinite no. of stages of multistage expansion with reheating, shall result in what cycle —

P.T.O.



NOTE

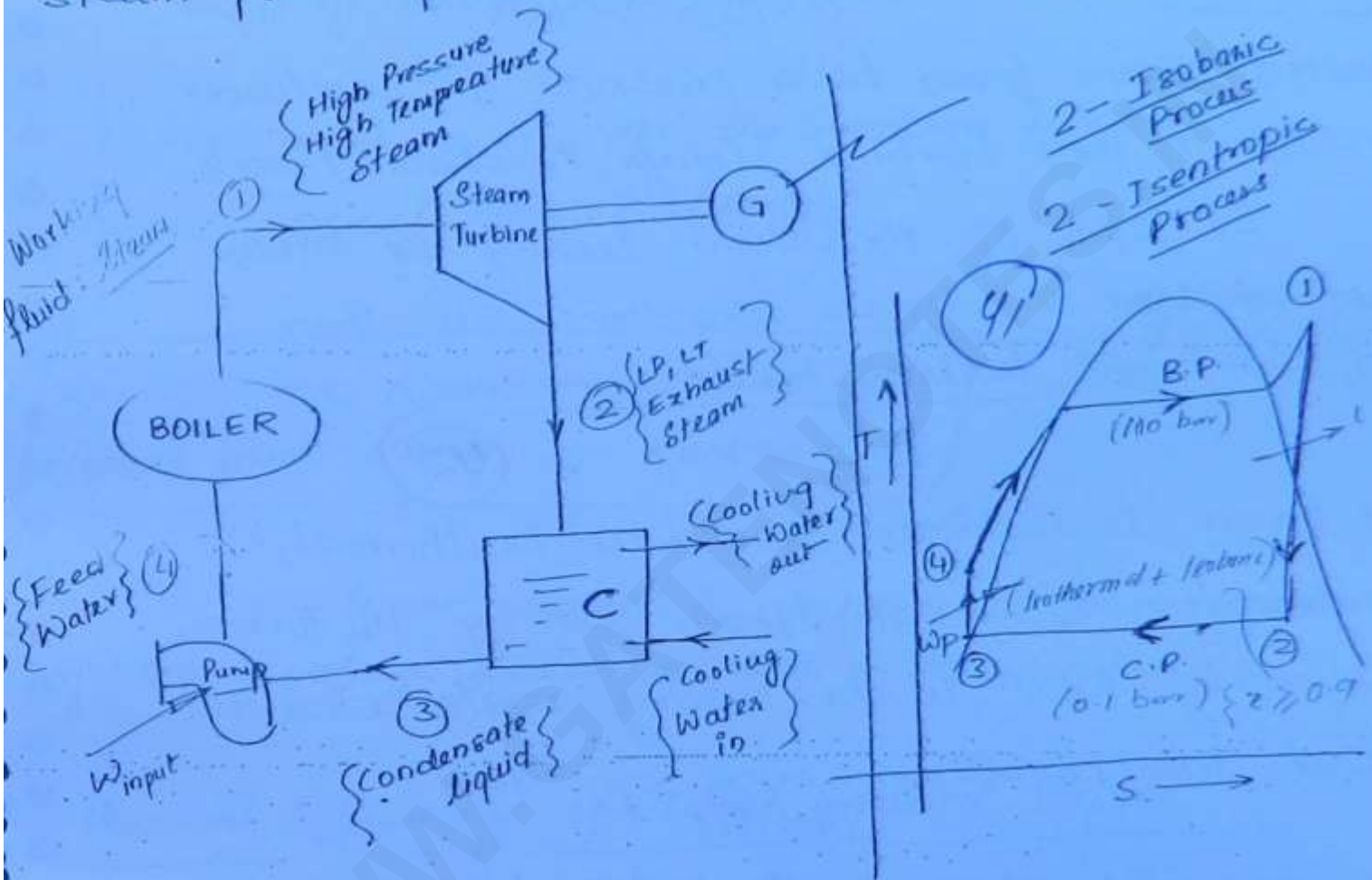
$$\eta_{\text{Stirling}} = \eta_{\text{Carnot}} = \eta_{\text{Ericsson}}$$

if regeneration
is provided,
then only

40

RANKINE VAPOUR POWER CYCLE

This is the basic thermodynamic cycle for steam power plant or thermal power plant



If $x \leq 0.9$ there will be erosion.
 $W_{Pump} \ll W_T$ (around 5% of W_T 200 times less)

4-1 indicates isobaric heating of feed water, its change of phase from liquid to vapour and then its superheating in the boiler due to the heat transfer from the hot combustion flue gases to the feed water.

The state of the steam leaving the boiler is high pressure, high temperature, super heated steam.

1-2 Process, is isentropic expansion of super-heated steam from boiler pressure to condenser pressure in the turbine doing mechanical work.

The state of the ^{exhaust} steam leaving the turbine should not have a dryness fraction less than 90%, to avoid turbine blade erosion

(42)

2-3 Process, is isobaric as well as isothermal, condensation of wet steam leaving the turbine, by rejecting heat to the cooling water, circulated around the condenser bank of tubes (bundle).

The state of the water leaving the condenser should be saturated liquid corresponding to condenser pressure i.e. state (3).

Any undercooling or subcooling in the is undesirable.

3-4 Process is the isentropic pumping of liquid condensate leaving the condenser, from condenser pressure to boiler pressure. The work required by the pump is supplied by turbine.

$$W_p \lll W_t$$

Work ratio of the Rankine cycle is almost equal to 1 (unity)

Note:

Lowest Work Ratio $\rightarrow$ Carnot cycle
Highest Work Ratio $\rightarrow$ Rankine cycle

43

The condenser pressure is decided by the temperature of the cooling water supplied to the condenser.

Thermal Efficiency of Ideal Rankine Vapour Power Cycle

$$\eta_{th} = \frac{\text{Net work done / cycle}}{Q_s}$$

$$= \frac{W_t - W_p}{(Q_s)_{\text{in boiler}}}$$

$$Tds = dh - vdp$$

$$s = c \Rightarrow ds = 0$$

$$dh = vdp \Rightarrow \int dh = \int vdp$$

= W.D. by Turbine

Note:

$\int vdp$ indicates reversible work done on open system

$$\therefore W_T = (h_1 - h_2) \text{ kJ/kg}$$

$$k)_{\text{input to pump}} = \int vdp \quad \text{But } v = c \text{ since liquid is incompressible}$$

$$= v_{\text{water}} (\text{Boiler pressure} - \text{Condensor pressure})$$

$$v_f = 0.001016 (\text{B.P.} - \text{C.P.}) \cong 2 \text{ to } 10 \text{ kJ/kg}$$

(B.P. = 100 bar
C.P. = 0.1 bar)

$$W_{\text{pump}} = (\Delta h) \text{ of water in pump}$$

$$= h_4 - h_3$$

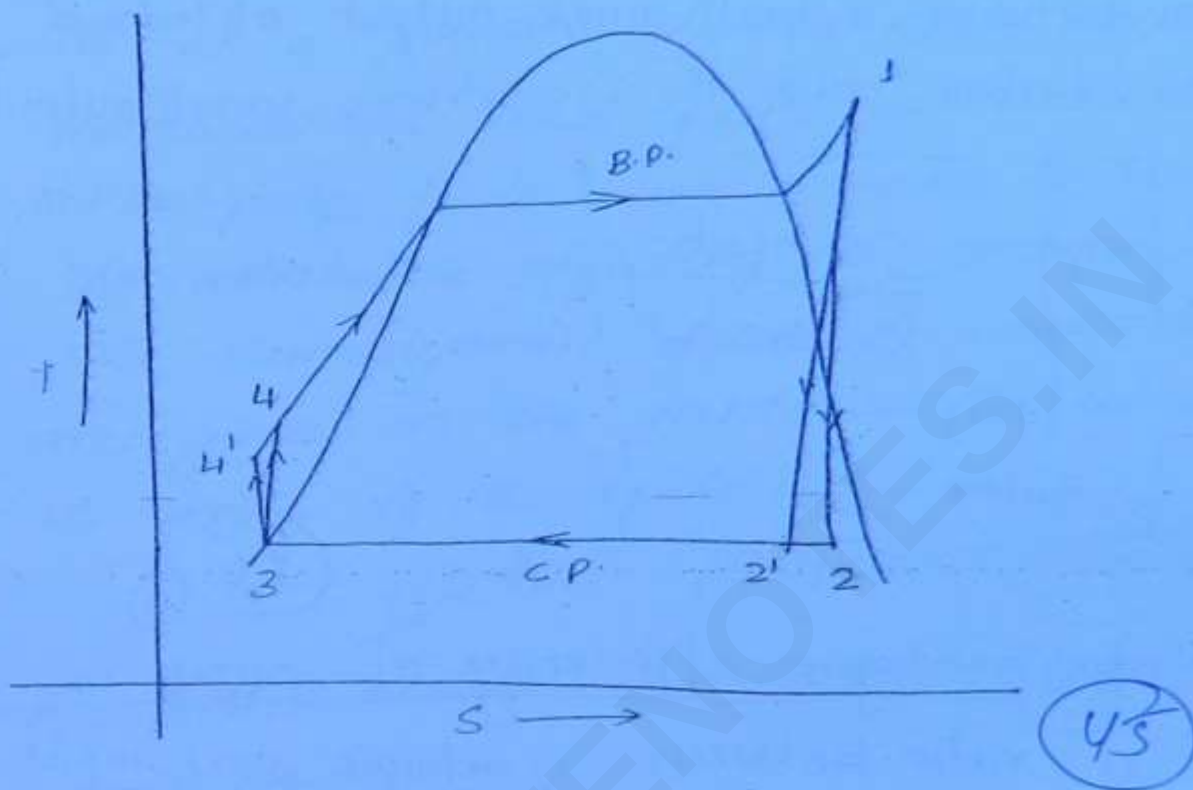
Since pump work is very small i.e. (2 to 6 kJ/kg)

$$\Rightarrow h_4 \cong h_3$$

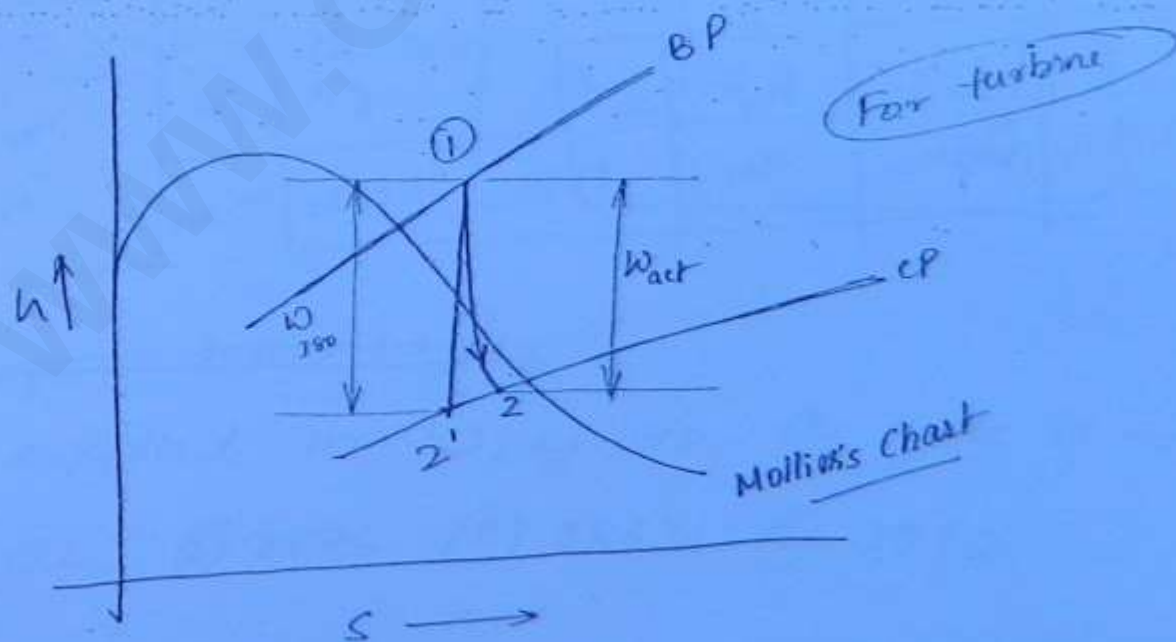
Neglecting Pump Work,

$$\eta_{\text{Ideal Rankine Cycle}} = \frac{W_T}{Q_s} = \frac{h_1 - h_2}{h_1 - h_4} = \frac{h_1 - h_2}{h_1 - h_3}$$

RANKINE CYCLES WITH MACHINE EFFICIENCY



1-2' is isentropic expansion of steam in turbine
 1-2 is actual expansion of steam with friction



Iso entropic efficiency of turbine is defined as the ratio between actual work output obtained from the turbine and the isentropic work output

$$\boxed{\eta_{(Iso) \text{ Turbine}} = \frac{h_1 - h_2}{h_1 - h_2'}} \quad \approx 75 \text{ to } 80\%$$

(46)

Similarly isentropic efficiency of pump is defined as ratio between isentropic work input to the pump and actual work input to the pump

$$\boxed{\eta_{(Iso) \text{ Pump}} = \frac{h_4' - h_3}{h_4 - h_3}} \quad \approx 85 \text{ to } 90\%$$

In a steam power plant operating on the Rankine cycle, steam enters the turbine at 4 MPa, 350°C and exits at a pressure of 15 kPa, then it enters the condenser and exits as saturated water. Next, the pump feeds back the water to the boiler. The adiabatic efficiency of turbine is 90%. The thermodynamic states of water and steam are given in the table. The net work output in kJ/kg of the cycle is —

- (a) 718 (b) 775 (c) 860 (d) 957

~~$P_1 = 4 \text{ MPa} = 4000 \text{ Pa}$~~ ~~$T_1 = 623 \text{ K}$~~ 07
 ~~$P_2 = 15 \text{ kPa}$~~

State	h (kJ/kg)		s (kJ/kg°C)		v m³/kg	
Steam: 4 MPa 350°C	3092.5		6.5821		0.06645	
Water at 15 kPa	h_f	h_g	s_f	s_g	v_f	v_g
	225.94	2599.1	0.7549	8.0085	0.001014	10.02

~~$h_1 = 3092.5 \text{ kJ/kg}$~~

Heat supplied in kJ/kg to the cycle is —

- (a) 2372 (b) 2576 (c) 2863 (d) 3092

$$\eta_{\text{turbine}} = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$0.90 = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$6.5821 = 0.7549 + x(8.0085 - 0.7549)$$

$$x = 0.803$$

48

$$h_2' = 2132.42$$

$$h_2 = 2228.43 \text{ kJ/kg}$$

$$w_{\text{turbine}} = h_1 - h_2'$$

$$\therefore \text{Actual W.D.}_{\text{turbine}} = \eta \times w_{\text{turbine}}$$

$$= 0.9 \times 960.08$$

$$= 864.072$$

$$\text{Net W.D./cycle} = 864.072 - \int v dp$$

$$= 864.072 - 0.001014(40 - 0.13) \times 100$$

$$= 864 - 4 = 860 \text{ kJ/kg}$$

$$Q_s = \frac{W_T + W_S}{\eta}$$

$$h_3 = 225.94$$

$$\cancel{h_4 - h_3}$$

$$\eta_{(th) \text{ cycle}} = \frac{h_1 - h_3}{h_1 - h_3}$$

$$\eta_{(th) \text{ cycle}}$$

(49)

Here, Pump Work is $\int V dP = 0.001014(40 - 0.15)$
 $= 4 \text{ kJ/kg}$

$$(Q_s) \text{ in boiler} = h_1 - h_4$$

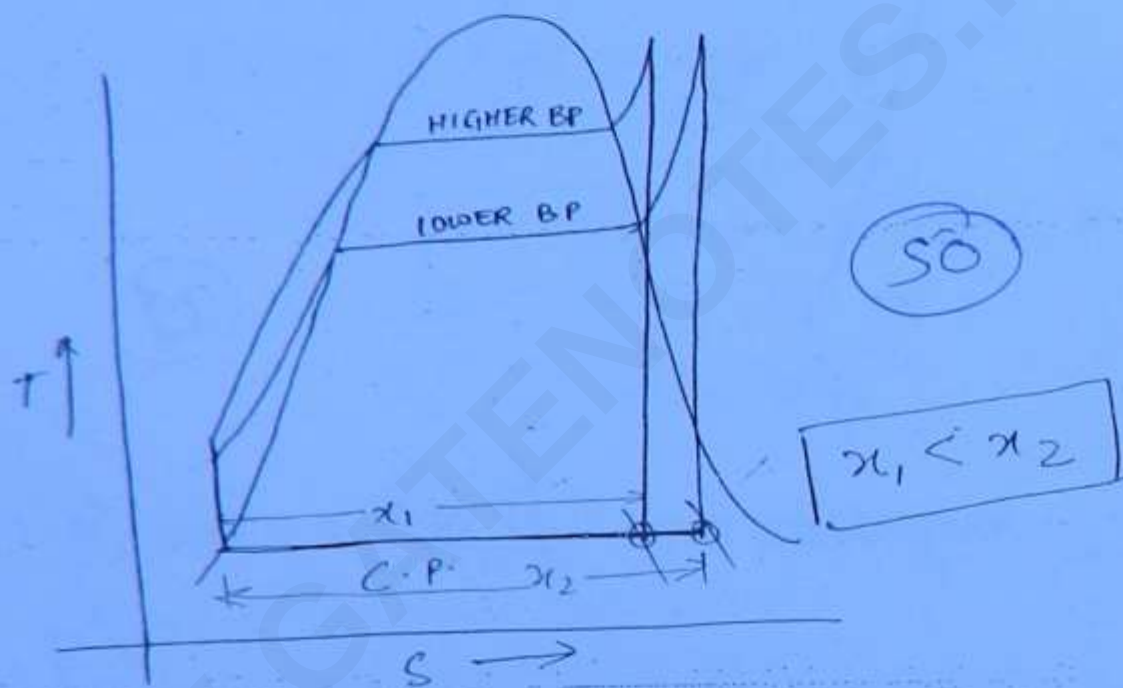
$$= h_1 - h_3$$

$$= 3092.5 - 225.94$$

$$= 2866.56 \text{ kJ/kg}$$

$$\eta_{th} = \frac{h_1 - h_2}{h_1 - h_3} = 0.301 = 30.1\%$$

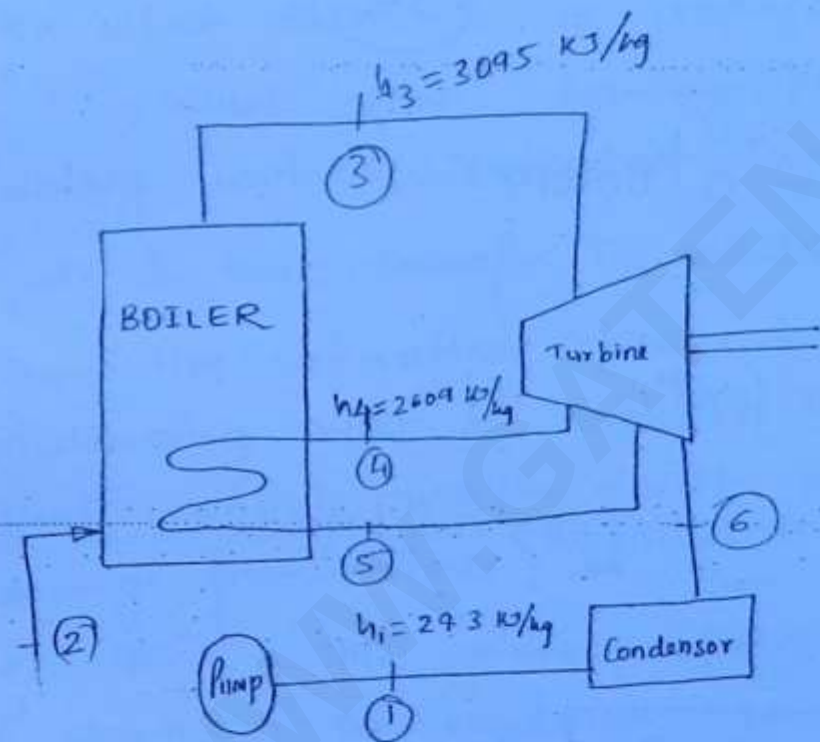
Note: Reheating in the steam power plant is basically provided for the sake of increasing dryness fraction, during of steam during lateral stages of expansion, particularly when higher boiler pressure are used.



When higher boiler pressure are employed, the dryness fraction values of steam are low during later stages of expansion, causing blade erosion of turbine.

Reheating takes care of these matter (ie. Reheating is done to avoid it)

Q.) Consider a steam power plant using a reheat cycle as shown. Steam ~~reaches~~^{leaves} the boiler and enters the turbine at 4 MPa, 250°C ($h_3 = 3095 \text{ kJ/kg}$). After expansion in the turbine, to 400 kPa ($h_4 = 2609 \text{ kJ/kg}$). The steam is reheated to 350°C ($h_5 = 3170 \text{ kJ/kg}$) and then expanded in a low pressure turbine to 10 kPa ($h_6 = 2165 \text{ kJ/kg}$). The specific volume of the liquid handled by the pump can be assumed



The thermal efficiency of the plant neglecting pump work is —

- (a) 15.8% (b) 41.1% (c) 48.5% (d) 58.6%

The enthalpy at the pump discharge (h_2) is —

- (a) 0.33 kJ/kg (b) 2.33 (c) 33.3 (d) 4.0

$$\eta_m = \frac{HPT + LPT - \text{Pump Work}}{Q_s} \quad \text{--- 0}$$

$$= \frac{(h_3 - h_4) + (h_5 - h_6)}{(h_3 - h_1) + (h_5 - h_4)}$$

$$= 41.1\%$$

$$\text{Pump work} = \int v \, dP$$

(52)

$$v_{\text{water}} \text{ is const.} = 0.001014$$

$$h_2 - h_1 = v \int dP$$

$$h_2 - h_1 = 0.001014 [40 - 0.1] \times 100$$

$$h_2 - 29.3 = 3.99$$

$$\therefore \boxed{h_2 = 33.3} \text{ kJ/kg}$$

Assertion:
Q) Condensor is an essential equipment in a steam power plant —

Reason: For same mass flow rate and for same pressure rise, a water pump requires substantially less power than a steam compressor.

Note: (S3)
Condensor is a shell and tube heat exchanger in which there is a transfer of heat from the exhaust steam leaving the turbine to the cooling water circulated in the condensor tubes (mode of heat transfer is conduction & convection).

Since the saturation temperature of steam while condensing must be at least 20°C greater than usual cooling water inlet temperature of ~~25~~ 25°C (coming from river) ~~is~~ T_{sat} of steam is chosen as 45°C . The corresponding saturation pressure of steam in the condensor is 14 kPa or 0.14 Bar .

So, to maintain these low pressure of condensor, a sufficient amount of cooling water supply must be ensured, also heat transfer must be appropriate.

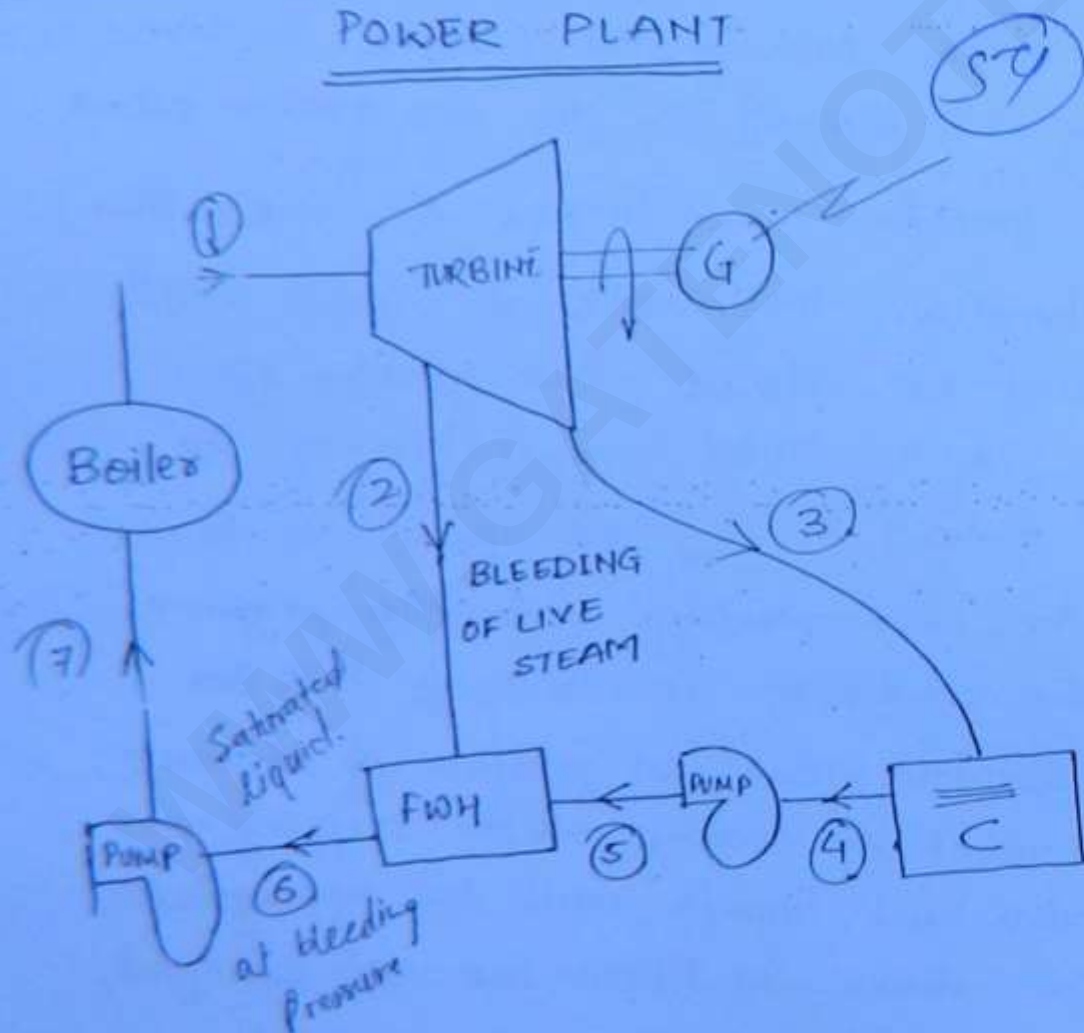
⊙ If the above conditions are not satisfied, condensor vacuum will ~~decrease~~ decrease, which increases the absolute pressure of steam in the condensor, thereby decreasing the overall efficiency.

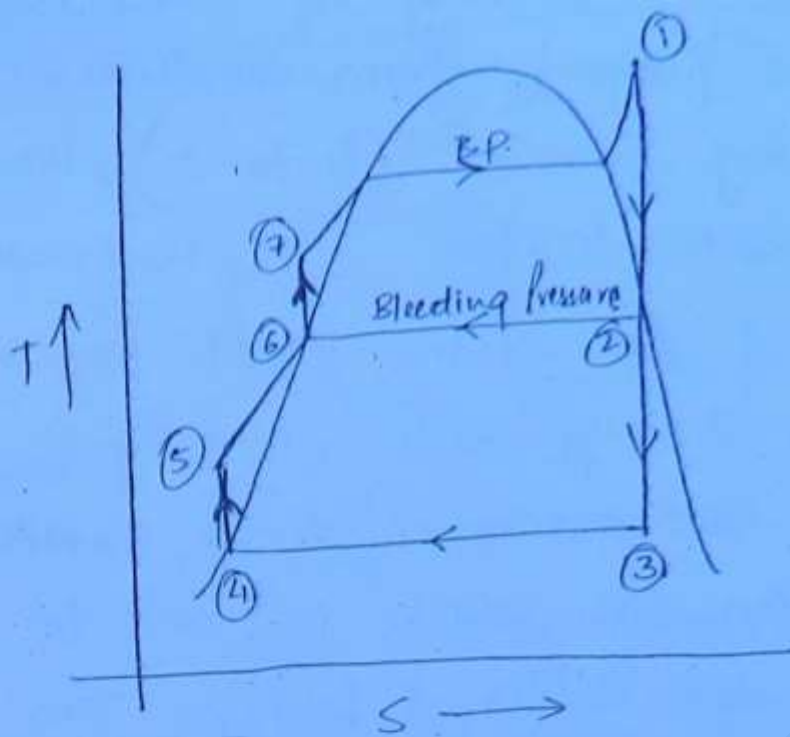
65 cm of Hg, vacuum is maintained, in condenser,

76 cm of Hg — 101.3 kPa

11 cm of Hg — $\frac{11}{76} \times 101.3 \text{ kPa}$

POWER PLANT





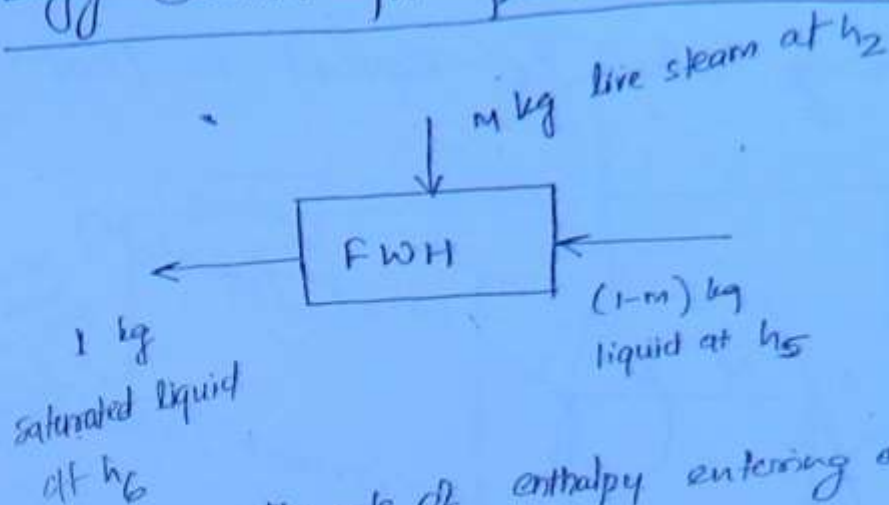
regeneration in rankine cycle shall result in increase in mean temperature of heat addition which in turn results in increase of thermal efficiency of cycle

Let 1 kg of steam enters the turbine from the boiler which will expand from boiler pressure to bleeding pressure, doing work, then m kg of ^{live} steam is bleed from the turbine and remaining $(1-m)$ kg continues to expand from bleeding pressure to condenser pressure doing work on the rotor. These $(1-m)$ kg of steam leaving the turbine condenses in the condenser at constant pressure becoming saturated liquid and at condenser pressure.

These liquid is pumped from condenser pressure to bleeding pressure (4 to 5), then enters the feed water heater. In feed water heater, 'm' kg of live steam bleed from the turbine is mixed directly with $(1-m)$ kg of liquid water. The live steam condenses, liberating latent heat of condensation which is used to increase the temperature of liquid heater. The resulting mixture being saturated liquid water at bleeding pressure (State 6). These liquid is pumped from bleeding pressure to boiler pressure and then supplied to the boiler.

(56)

Energy balance for FWH



The rate of enthalpy entering the

FWH = Rate of enthalpy leaving FWH

$$(1-m)h_5 + (m \times h_2) = 1 \times h_6$$

(57)

$$h_5 - mh_5 + mh_2 = h_6 \quad (h_6 = h_f \text{ at bleeding pressure})$$

$$m = \left(\frac{h_6 - h_5}{h_2 - h_5} \right)$$

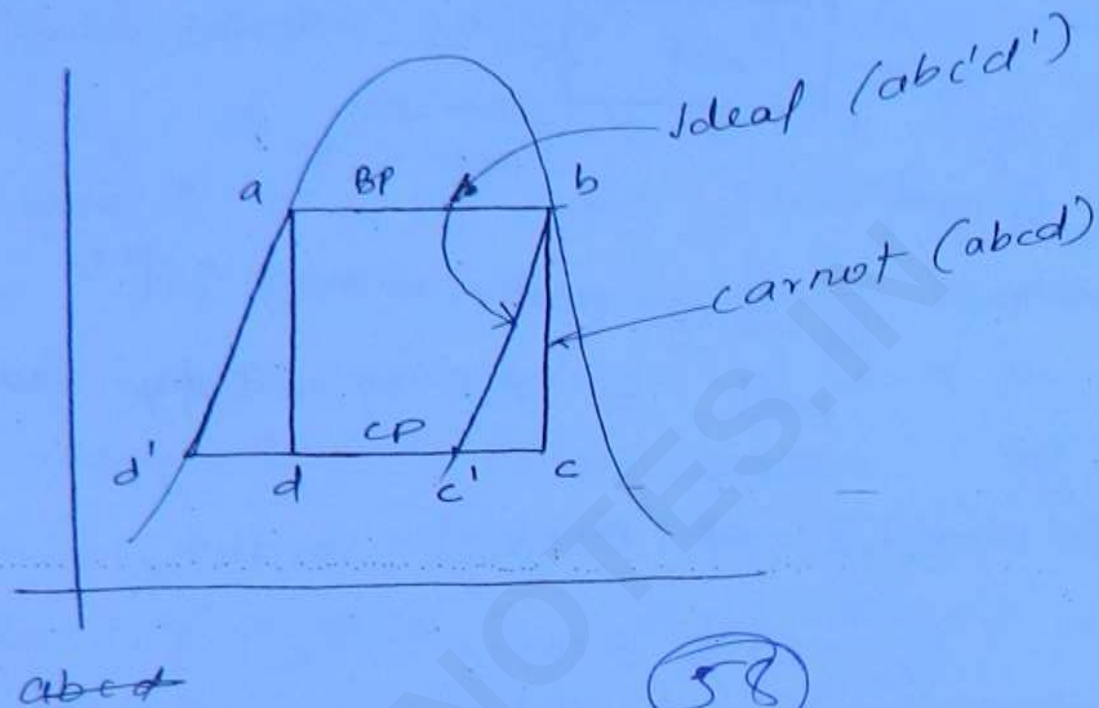
kg of live steam bleeds per 1 kg of steam entering turbine

Efficiency

$$\eta_{th} = \frac{(h_1 - h_2) + (1-m)(h_2 - h_3)}{(h_1 - h_7 = h_6)}$$

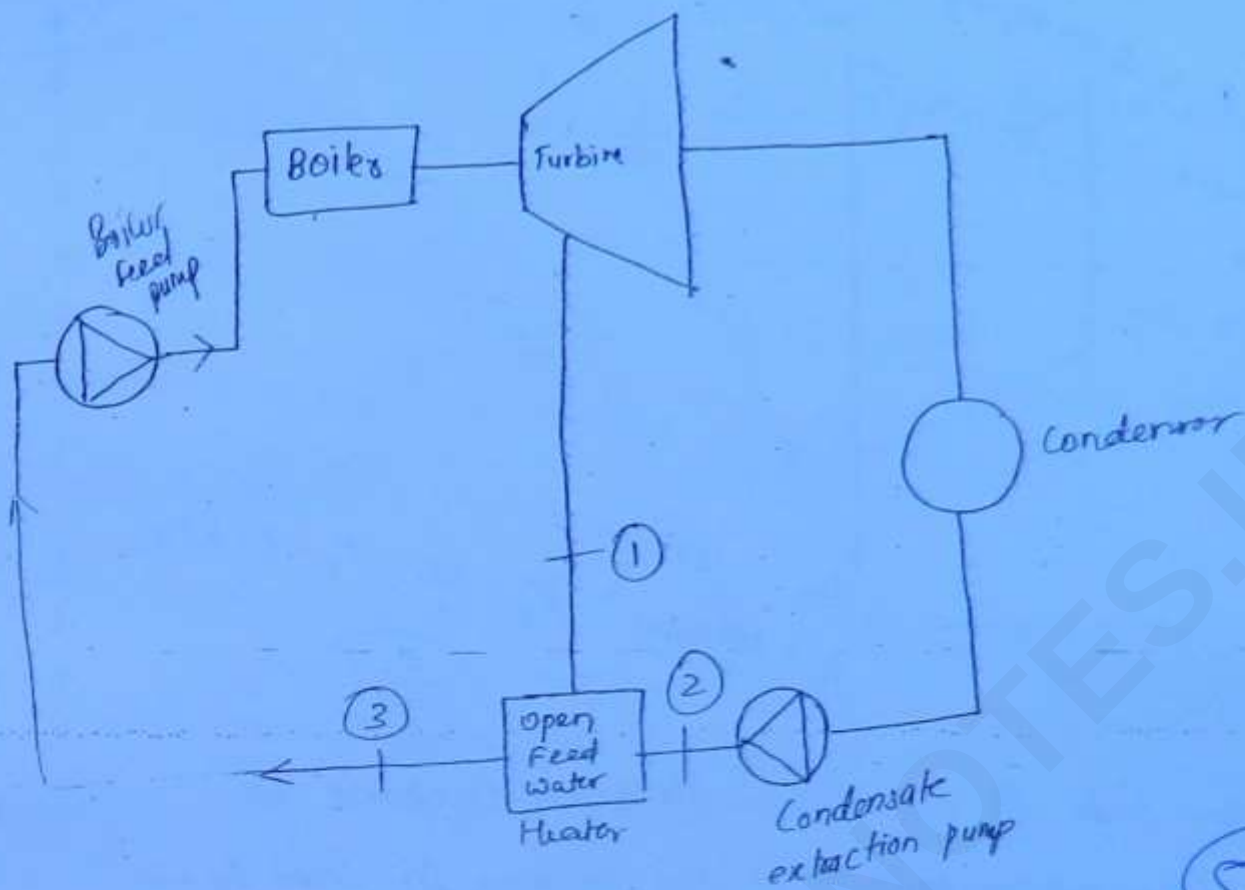
Higher the no. of bleeding points, with feed water heaters, higher is the increasing in thermal efficiency. If infinite bleeding points are provided with infinite feed water heaters, then the cycle obtained is called Ideal Regenerative Rankine Cycle.

whose thermal efficiency shall be equal to that of Carnot cycle.



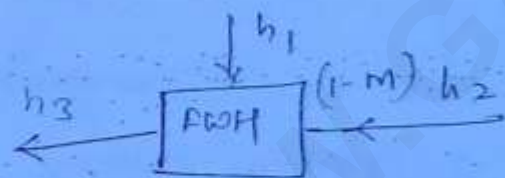
Q- A thermal power plant operates on a regenerative cycle, with a single open feed water heater as shown in the figure. For the state points shown, the specific enthalpies are, $h_1 = 2800 \text{ kJ/kg}$ and $h_2 = 200 \text{ kJ/kg}$. The bleed to the feed water heater is 20% of the boiler steam generation rate. The specific enthalpy at state 3 is

- ☒ (A) 720
 (B) 2280
 (C) 1500
 (D) 3000



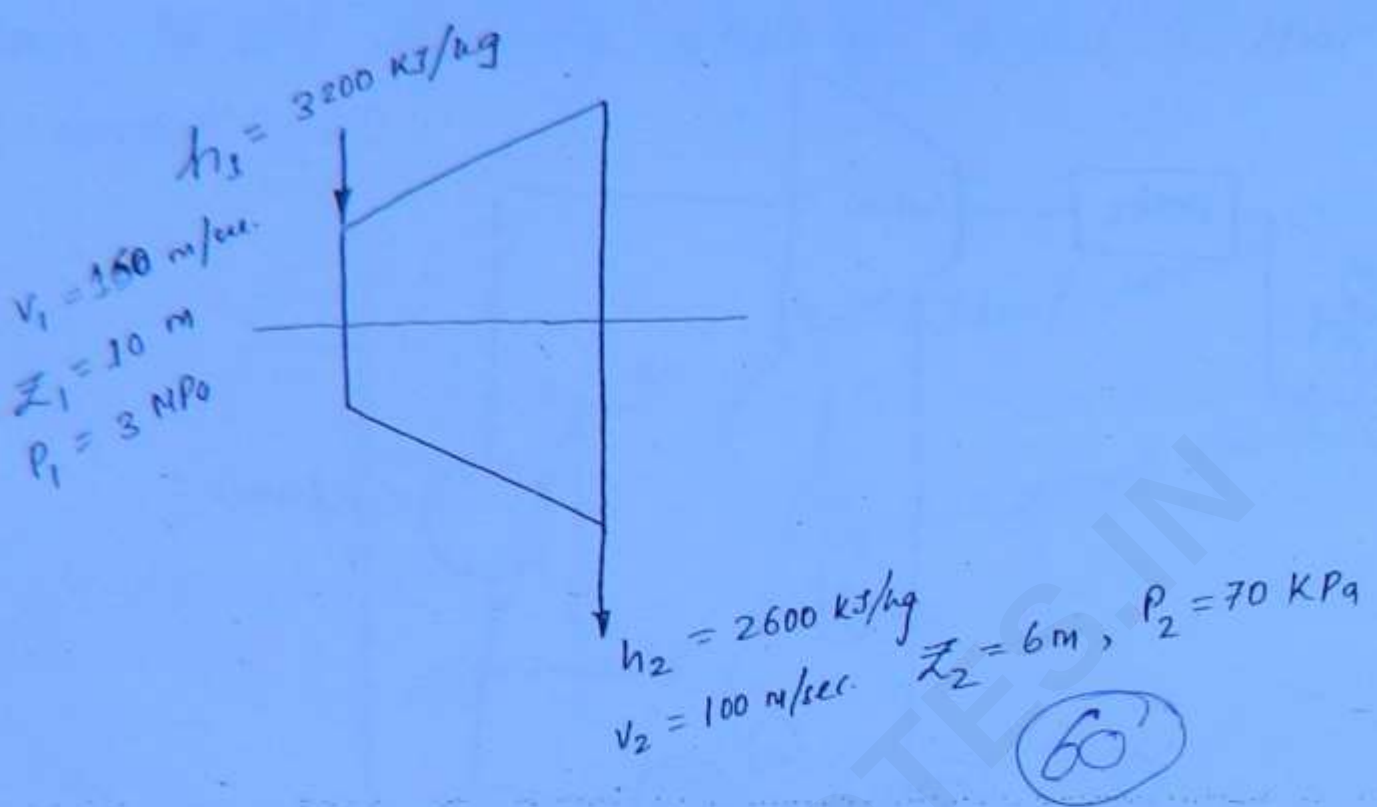
$$(1-0.2)h_2 + 0.2h_1 = 1 \times h_3$$

$$h_3 = 720 \text{ kJ/kg}$$



Gate 09

The inlet and outlet conditions of steam for an adiabatic steam turbine are as indicated in the figure.



If mass flow rate of the steam through turbine is 20 kg/sec . The power output of the turbine in MW is —

- ☒ (a) 12.157
 ☐ (b) 12.941
 ☐ (c) 168.001
 ☐ (d) 168.725

Assume the above turbine to be part of a simple rankine cycle. The density of water at inlet to the pump is 1000 kg/m^3 . Ignoring Kinetic & Potential energy effects, the specific work in kJ/kg supplied to the pump is —

- ☐ (a) 0.293
 ☐ (b) 0.251
 ☒ (c) 2.93
 ☐ (d) 3.51

Soln

$$P = \dot{m} \times WD/sec.$$

Turbine is Steady flow Open System.

SFEE:

$$\begin{aligned} \leftarrow \cancel{q} - \cancel{W} &= \Delta h + \Delta KE + \Delta PE \\ \text{(\because adiabatic)} &= (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2000} \right) + g \left(\frac{Z_2 - Z_1}{1000} \right) \\ &= 607 \text{ kJ/kg} \end{aligned}$$

$$P = \dot{m} \times WD/sec$$

$$= 20 \times 607 = 12156.7 \text{ kW}$$

$$= 12.157 \text{ MW}$$

BP

$$\text{Pump Work} = \int_{C.P.}^{B.P.} V dp = \frac{1}{1000} \left[\text{BP} - \text{C.P.} \right] \text{ in kPa}$$

$$= \frac{1}{1000} (3000 - 70)$$

$$= \frac{2930}{1000} = 2.93 \text{ kJ/kg}$$

Q) Which combination of following statements is correct: —

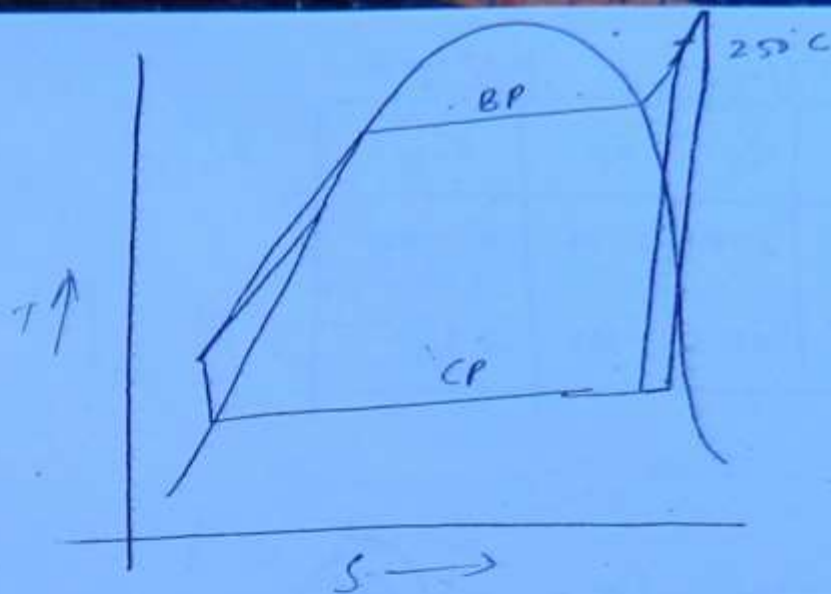
- P) The incorporation of reheater in a steam power plant always increases the thermal efficiency of the plant.
- Q) Always increases the dryness fraction of steam at condenser inlet.
- R) Always increases the mean temperature of heat addition.
- S) Always increases the specific work output.

(62)

- (A) P & S
- (B) Q & S
- (C) P, R & S
- (D) P, Q, R, S

Q) In rankine cycle, regeneration results in higher thermal efficiency because —

- (a) Pressure inside the boiler increases.
- (b) Heat is added before steam enters the low pressure turbine.
- (c) The average temperature of heat addition in the boiler increases.
- (d) The work delivered by the turbine increase



Gate 2002

Q) The efficiency of superheated rankine cycle is higher than that of simple rankine cycle because —

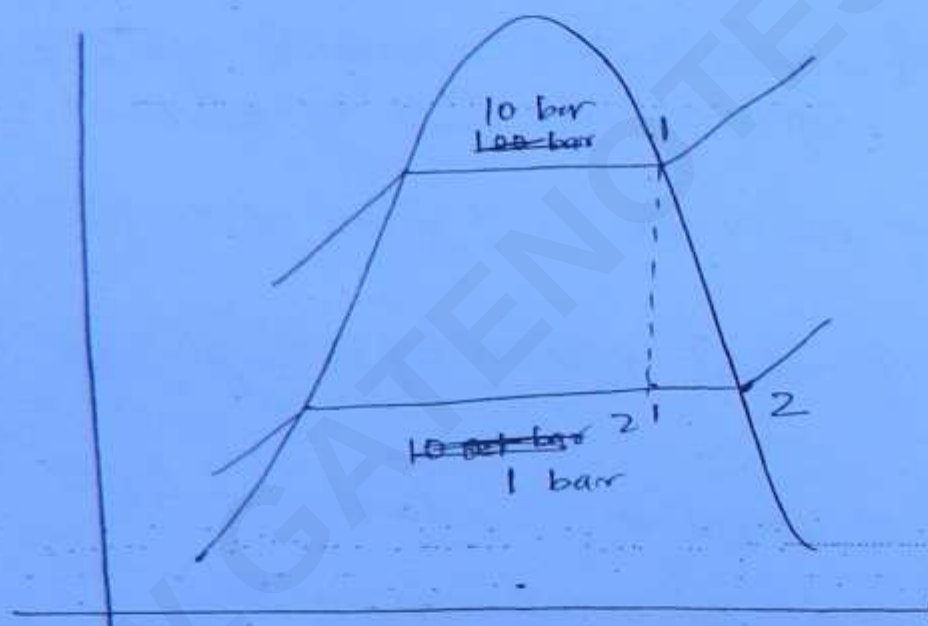
- (a) Enthalpy of main steam is higher for superheat cycle. (63)
- (b) Mean temp of reheat addition is higher for superheat cycle.
- (c) Temperature of steam in condensor is high.
- (d) Quality of steam in condensor is low.

Gate 1999

Q) An adiabatic steam turbine receives dry saturated steam at 1 MN/m^2 and discharges it at 0.1 MN/m^2 . The steam flow rate is 3 kg/sec and the moisture at exit is negligible. Calculate the power output and isentropic efficiency of turbine. The properties of steam are given as —

		KJ/kg		KJ/kg K	
$P \text{ MN/m}^2$	$T_{\text{sat.}}$	h_f	h_g	s_f	s_g
1.0	179.9	762.8	2778.1	2.139	6.586
0.1	99.6	417.5	2675.5	1.303	7.359

$$P.v = RT$$



$$1 \text{ MN/m}^2 = 10 \text{ bar}$$

$$6.586 = 1.303 + x(7.359 - 1.303)$$

$$x = 0.872 = 87.2\%$$

$$h_2' = 417.5 + 0.872(2675.5 - 417.5)$$

$$= 2386.47 \text{ KJ/kg}$$

$$h_1 = 2778.1 \text{ kJ/kg}$$

$$\eta_{th} = \frac{h_1 - h_2}{h_1 - h_2'} = 26.2 \%$$

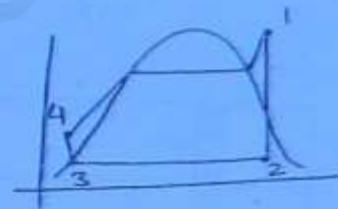
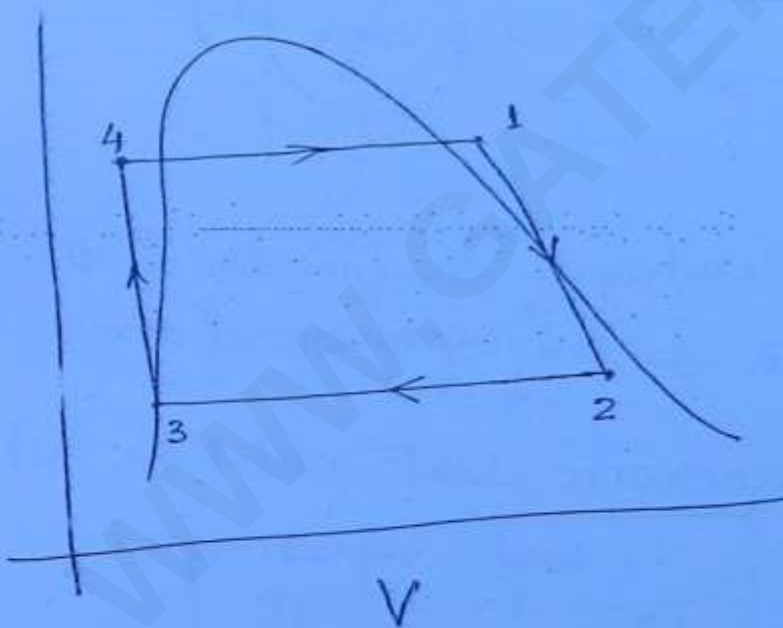
$$P = \dot{m} w_{ac} \rightarrow w_{act} = h_1 - h_2$$

$$= \dot{m} [h_1 - h_2'] = \cancel{1174.89 \text{ kW}}$$

$$= 307.8 \text{ kW} = \cancel{1.175 \text{ MW}}$$

(65)

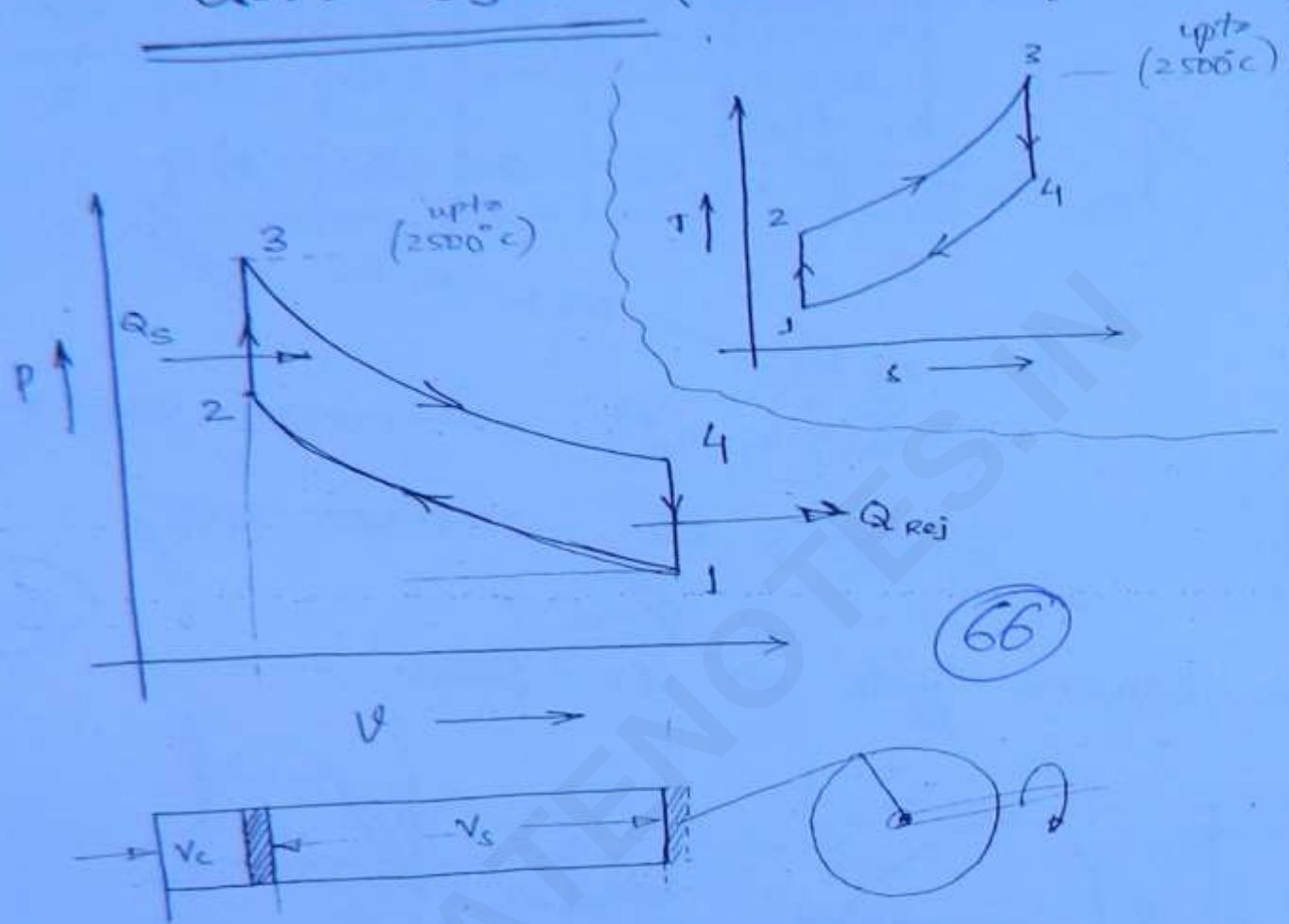
Draw Rankine cycle on PV-plane



$$P = 1000 \text{ kg/m}^3$$

P.T.O

Otto Cycle (Air Standard Cycle)



1-2 & 3-4 $\rightarrow$ are isentropic compression of air and expansion of air

2-3 & 4-1 $\rightarrow$ are isochoric heat addition and heat rejection

$$\eta_{th} = 1 - \frac{Q_R}{Q_s} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\eta_{th (otto)} = 1 - \frac{T_1 \left(\frac{T_1}{T_2} - 1 \right)}{T_2 \left(\frac{T_2}{T_1} - 1 \right)}$$

$$\eta_{th (otto)} = 1 - \frac{1}{\left(\frac{T_2}{T_1} \right)}$$

Defining compression ratio of the cycle as the ratio between volume of the air at the beginning of isentropic compression and the volume of air at the end of the isentropic compression i.e.

$$r_c = C.R. = \frac{V_1}{V_2}$$

(67)

For 1-2, it is isentropic,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = r_c^{\gamma-1}$$

$$\eta_{th (otto)} = 1 - \frac{1}{(r_c)^{\gamma-1}}$$

$$r_c = \frac{V_c + V_s}{V_c} = 1 + \left(\frac{V_s}{V_c} \right)$$

$$\therefore r_c = 1 + \frac{\frac{\pi D^2 L}{4}}{V_c}$$

Which one of the following is NOT a necessary assumption for the air standard Otto Cycle —

- ☒ (a) All processes are both internally as well as externally reversible.
- ☐ (b) Intake and exhaust processes are const. volume heat rejection processes.
- ☐ (c) The combustion process is a constant volume heat addition process. (68)
- ☐ (d) The working fluid is an ideal gas with constant specific heat.

2.) An engine working on Air Standard Otto Cycle has a cylinder diameter of 10 cm and stroke length of 15 cm. The ratio of specific heat for air 1.4. If clearance volume is 196.3 cc. and heat supplied per kg of air/cycle is 1800 kJ/kg, The work output per cycle/kg of air is —

- (a) 879.1 kJ (b) 890.2 (c) 895.3 (d) 973.5

Solⁿ

$$r = 1 + \frac{\frac{\pi}{4} D^2 L}{V_c} = 7$$

$$\eta_{th} = 1 - \frac{1}{r^{\gamma-1}} = 0.5408$$

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$\eta_{th} = \frac{WD/kg}{Q_S/kg}$$

$$\therefore 0.54 \times 1800 = WD/kg$$

$$\therefore WD/kg = 973.6 \text{ kJ}$$

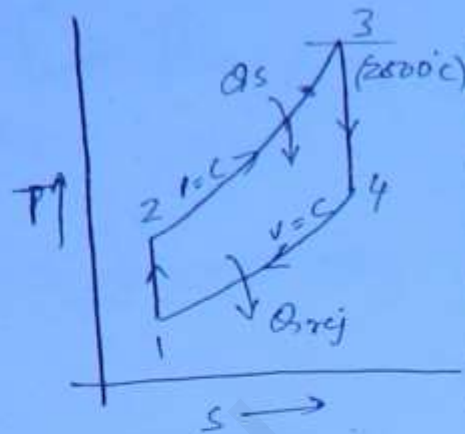
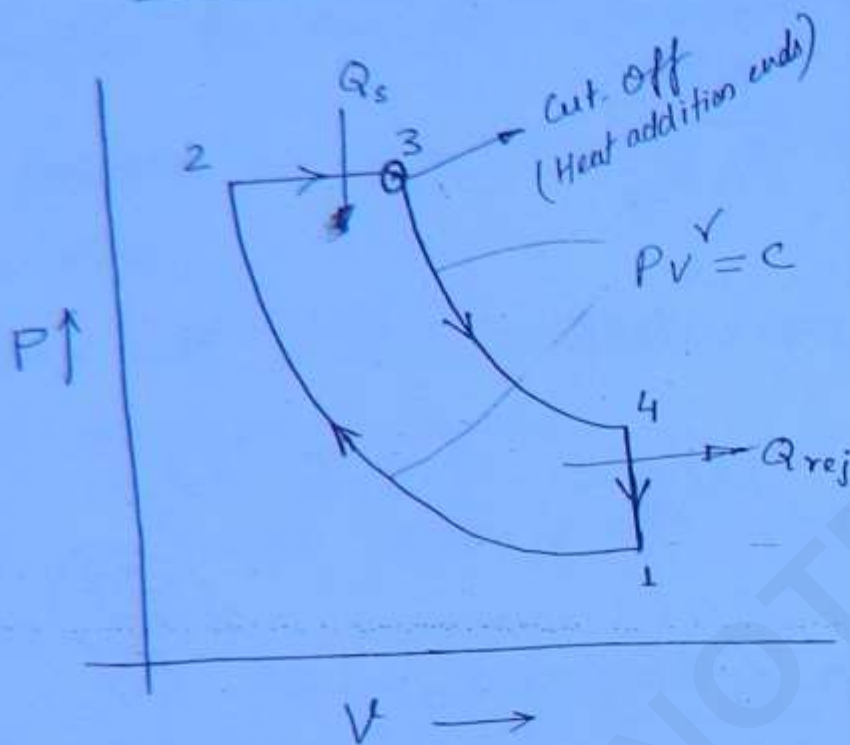
Note:

As the compression ratio (α) increases, Air Standard Thermal efficiency (η_{th}) of otto cycle also increases.

But in carburetted S.I. engine the max. compression ratio is limited to 10 $\pm$ 11 only.

{ Detonation or Knocking } tendency leads to increases ^{with} ~~to~~ & increases

DIESEL CYCLE



Compression Ratio $= r_c = \frac{V_1}{V_2}$

Cut off Ratio $= r_c = \frac{\text{Volume at which heat addition ceases}}{\text{Volume at which heat addition begins}}$

$$r_c = \frac{V_3}{V_2}$$

$$\eta_{th} = 1 - \frac{Q_R}{Q_S} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

(diesel)

$$\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} \left[\frac{r_c^{\gamma} - 1}{\gamma(\gamma - 1)} \right]$$

(diesel)

$$\text{As } r_c \uparrow \Rightarrow \eta_{th} \uparrow \quad (\text{diesel}) \quad (\text{S.I.})$$

$$\text{In C.I. Engines} \quad \boxed{r_c = 16 \text{ to } 22}$$

$$\text{As } r_c \uparrow \Rightarrow \eta_{th} \downarrow$$

When same compression ratio and same heat is supplied for both Otto & Diesel cycles, Thermal Efficiency of Otto Cycle is greater than that of diesel cycle.

$$\boxed{\eta_{th} (\text{otto}) > \eta_{th} (\text{diesel})}$$

For same r_c

(7)

the value $\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)} \right]$ for diesel is greater than 1

The more practical comparison is, for same peak temp and pressure in both Otto & Diesel cycles (ie for diff. compression ratios) and for the same heat supplied thermal efficiency of the diesel cycle is greater than that of Otto cycle.

Note: Practically C.I. engine are ^{thermally} more efficient than S.I. engine because they employ for greater compression ratio than S.I. Engines.

2S-CI

ought

, 4S-SI, 4S-CI, 2S-SI

Arrange in descending order of efficiency (thermal)

$$4S-CI > 4S-SI > 2S-CI > 2S-SI$$

due to compression ratio

72

POWER PLANT

73

* GAS TURBINE *

Advantage of Gas Turbine over I.C. engine →

1. High speed can be obtained
2. Easy Balancing.
3. They are compact. ($\frac{W}{P}$) [weight to power ratio is less]
4. simple mechanism.

Disadvantage →

- 1) As compressor is used in gas turbine system the work of compression is large and hence net work is less this results in lower efficiency.
- 2) As gas turbine system are subjected to higher temp. continuously. high heat resistant costly material is required.
- 3) Additional Reduction gear box is required as the speeds are very high.

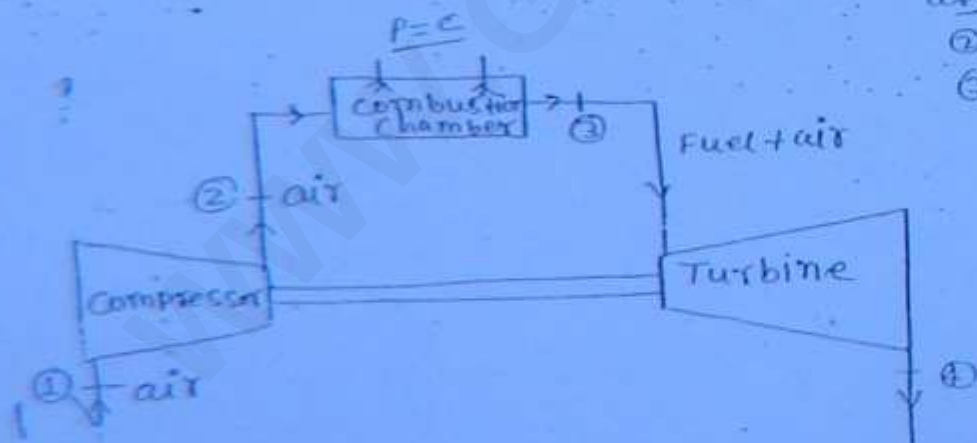
(74)

Open cycle gas turbine → ^{works on} ~~cycle~~ (Joule / Brayton cycle)

application → Aircraft.

② automotive (bus and truck)

③ industrial gas turbine installation.

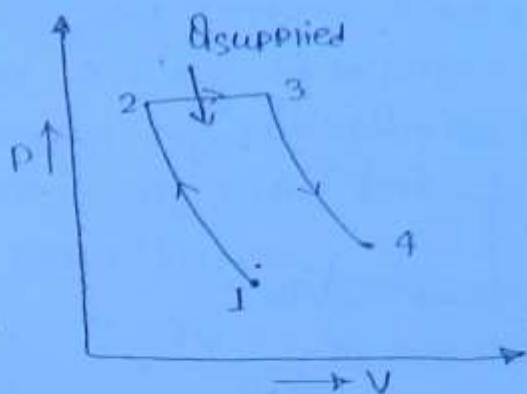


I.C. engine
Called

1-2 → Reversible adiabatic compression (Isentropic)

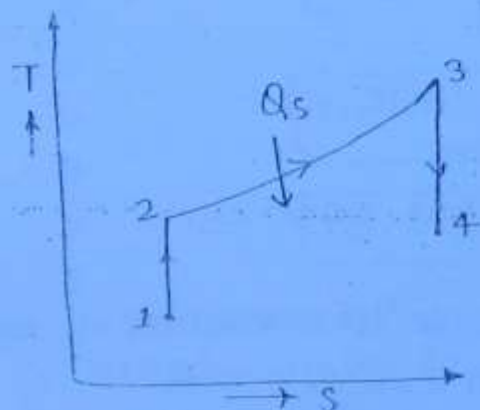
2-3 → Constant pressure heat addition.

3-4 → Rev. adiabatic expansion. (Isentropic)



$$pV = \gamma RT$$

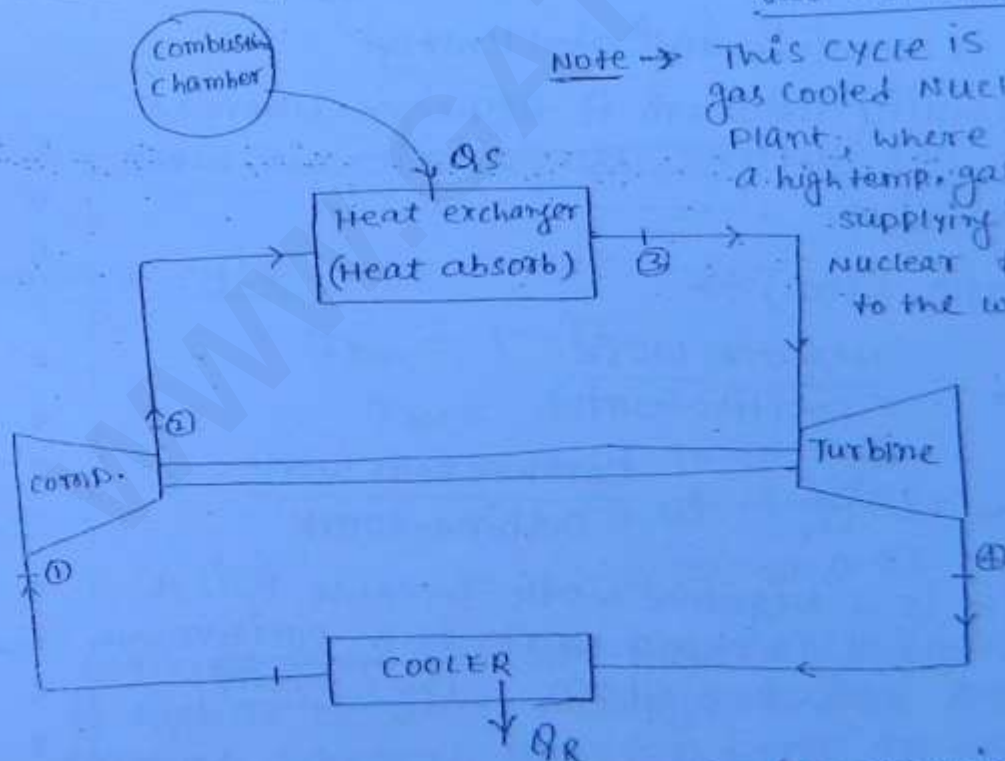
$$V \propto T$$



Note → The Brayton cycle is the air standard cycle for the gas turbine power plant.

(75)

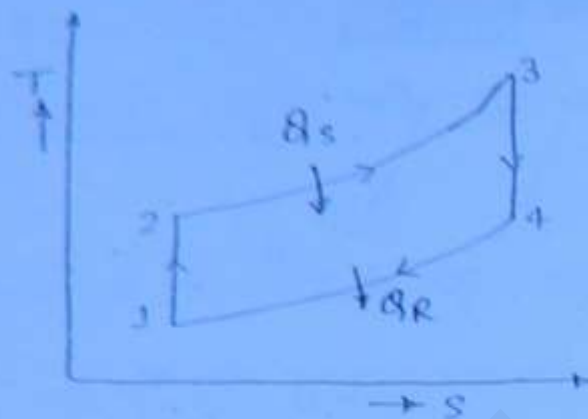
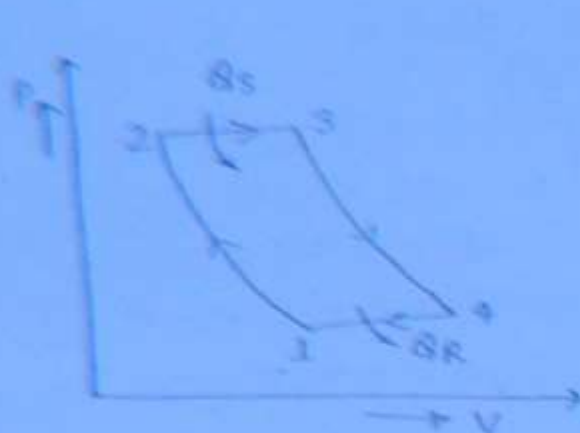
② closed cycle gas turbine → ② Two Reversible isobars and two reversible adiabatic



Note → This cycle is used in a gas cooled nuclear reactor plant, where the source is a high temp. gas cooled reactor supplying heat from nuclear fission directly to the working fluid (a gas).

⊗ External Combustion Engine

- 1-2 → Rev. adiabatic compression (Isentropic)
- 2-3 → Constant pressure heat addition (Isobar)
- 3-4 → Rev. adiabatic expansion (Isentropic)
- 4-1 → Constant pressure heat rejection.



(BRAYTON CYCLE)

1 Gas Turbine cycle work on Brayton cycle.

Advantage of closed cycle gas turbine $\Rightarrow$

- 1) It can work on any working fluid but open cycle gas turbine must work on air only.
- 2) Any cheaper fuel can be used because the product of combustion do not come directly in contact with turbine blade.

Disadvantages $\Rightarrow$

- ① The system is complicated and costly
- ② Additional cooling medium is required, whereas an open cycle gas turbine atmospheric air work as a cooling medium.

3 Back work Ratio (γ_{bw}) $\Rightarrow$

$$\gamma_{bw} = \frac{\text{Negative work}}{\text{Positive work}}$$

$$\gamma_{bw} = \frac{W_C}{W_T} \Rightarrow \frac{\text{Compressor work}}{\text{Turbine work}}$$

compressor work is a Negative work because it is a open system whereas Turbine work is a Positive work because this is a closed system $[W_C = -\int v dp]$

- 4 It is the ratio of Negative work to the Positive work.

In case of gas Turbine Power Plant the Back work ratio are around 40 to 60%. Whereas in the case of Steam Power Plant (vapour power cycle / Rankine cycle) the Back work ratio are 1 to 2%.

⑧ WORK Ratio (R_w) $\Rightarrow$

$$\gamma_w = \frac{\text{Net work}}{\text{Turbine work}} = \frac{W_{\text{net}}}{W_T} \Rightarrow \frac{W_{\text{net}}}{W_T}$$

$$\gamma_w = \frac{W_T - W_C}{W_T}$$

$$\gamma_w = 1 - \frac{W_C}{W_T}$$

$$\text{because } \gamma_{bw} = \frac{W_C}{W_T}$$

$$\gamma_w = 1 - \gamma_{bw}$$

⑨ The work ratio of Rankine cycle (vapour power cycle) is almost unity (1.)

⑩ Brayton cycle

$$\gamma_{bw} = 0.4 \text{ to } 0.6$$

$$\gamma_w = 1 - 0.4 \Rightarrow 0.6$$

$$= 1 - 0.6 \Rightarrow 0.4$$

$$\gamma_w = 0.4 \text{ to } 0.6$$

⑪ Rankine cycle

$$\gamma_{bw} = 1 - \gamma_w$$

$$\gamma_{bw} = 1 \text{ to } 2\%$$

$$= 0.01 \text{ to } 0.02$$

$$\gamma_w = 1 - 0.01 \Rightarrow 0.99$$

$$= 1 - 0.02 \Rightarrow 0.98$$

Note $\rightarrow$ Rankine cycle (vapour power cycle) work ratio is higher because pump are less work consume whereas Brayton cycle in compressor are more work is consume. (more power is consume)

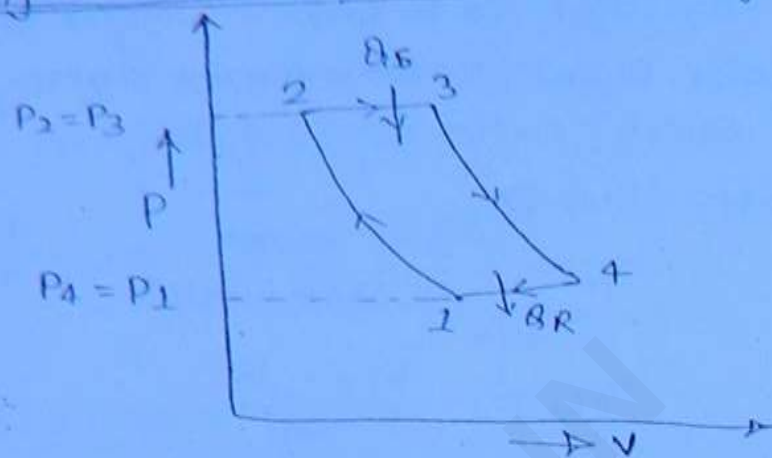
77.

Efficiency of a simple gas turbine cycle on p-v diagram

$$\eta_T = \frac{W_{net}}{Q_s \text{ (Heat supplied)}}$$

$$\eta = \frac{Q_s - Q_R}{Q_s}$$

$$\eta = 1 - \frac{Q_R}{Q_s}$$



where $Q_R = h_4 - h_1 \Rightarrow C_p (T_4 - T_1)$

$$Q_s = h_3 - h_2 \Rightarrow C_p (T_3 - T_2)$$

$$\eta = 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)} \quad (78)$$

$$\eta = 1 - \frac{T_1 \left(\frac{T_4}{T_1} - 1 \right)}{T_2 \left(\frac{T_3}{T_2} - 1 \right)} \quad (1)$$

Pressure Ratio (γ_p) = $\frac{P_2}{P_1} = \frac{P_3}{P_4}$

1-2 $\rightarrow$ Reversible adiabatic compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_1} = (\gamma_p)^{\frac{\gamma-1}{\gamma}} \quad (2)$$

3-4 $\rightarrow$ Reversible adiabatic expansion.

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_4} = (\gamma_p)^{\frac{\gamma-1}{\gamma}} \quad (3)$$

*** $\boxed{\frac{T_2}{T_1} = \frac{T_3}{T_4}}$

$$\boxed{\frac{T_4}{T_1} = \frac{T_3}{T_2}}$$

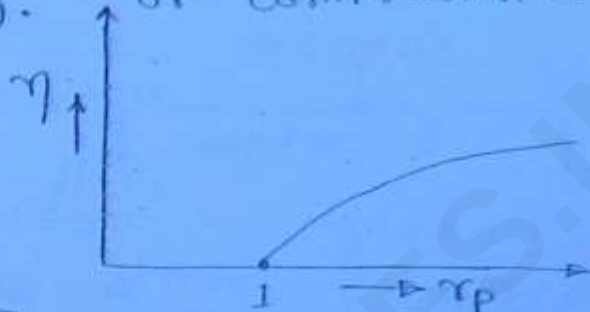
$$\eta = 1 - \frac{T_1}{T_2}$$

$$\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

Note → For the same compression ratio, the Brayton cycle efficiency is equal to the Otto cycle efficiency.

Note → The efficiency of Brayton cycle depends on the pressure ratio (r_p) or compression ratio.

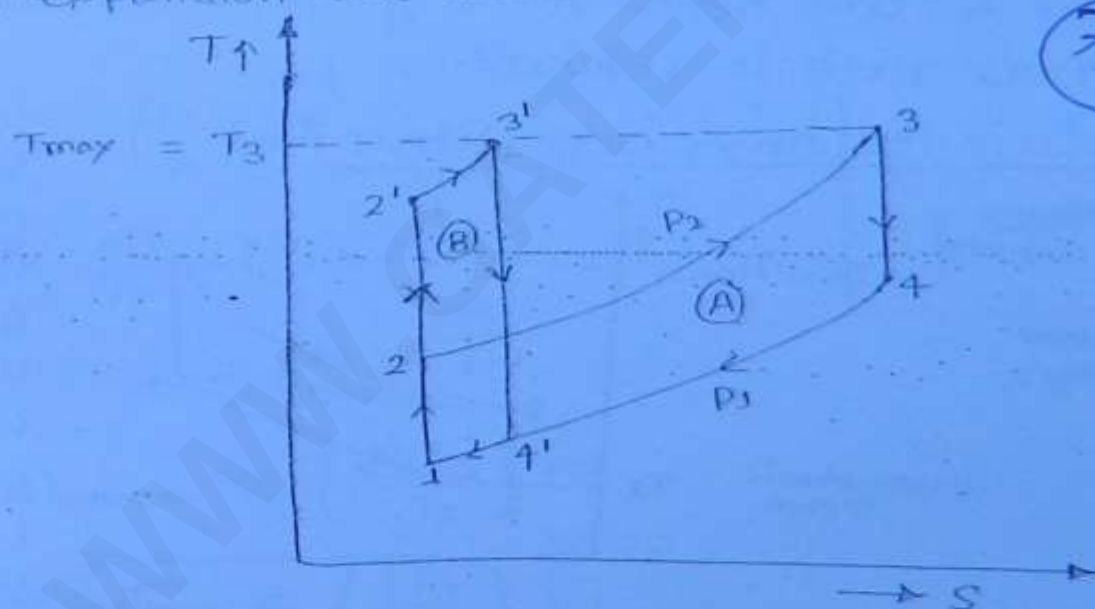
* When $r_p = 1$, then $\eta = 0$, and r_p increases then η also increases.



$$\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

Note →

This equation of η is valid only when compression and expansion are isentropic.



$$\eta_B = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

$$\eta_B > \eta_A$$

⇒ It means efficiency depends on pressure ratio because B cycle pressure ratio is more compared to cycle A.

$W_A > W_B \rightarrow$ Area of A cycle is more compare to cycle B.
 so ^{W_{net}} mass flow rate is more than area is
 much required then cost is high. so we are not
 take more pressure ratio.
case $\rightarrow$ SAME Capacity to take.

$$\text{Power} = \dot{m} \times \text{net work}$$

$$\begin{array}{l}
 \text{1500 kW} \\
 \downarrow \\
 P = \dot{m} W_{\text{net}} \\
 \swarrow \quad \searrow \\
 \text{A} \qquad \qquad \text{B} \\
 W_A = 150 \text{ kJ/kg} \quad W_B = 15 \text{ kJ/kg} \\
 1500 = \dot{m}_A \times 150 \quad P = \dot{m}_B \times 15 \\
 \dot{m}_A = 10 \text{ kg/kg} \quad 1500 = \dot{m}_B \times 15 \\
 \dot{m}_B = 100 \text{ kg/kg}
 \end{array}$$

(80)

⊛ Though the efficiency of $\eta_B > \eta_A$ the net work in B < cycle A and hence for a given capacity, greater mass flow rate is required.

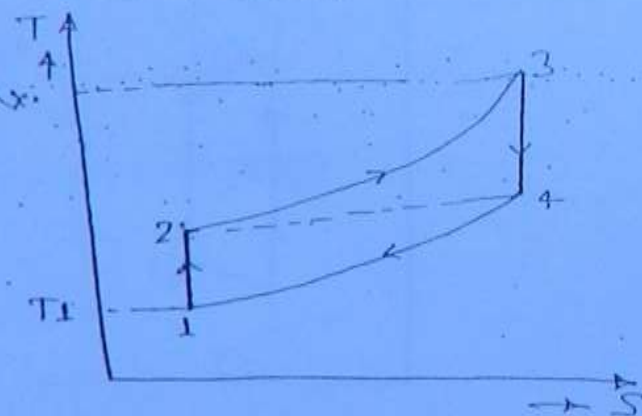
⊛ Optimum pressure for max. work output $\rightarrow$

where $T_3 \rightarrow$ Temp. Constant
 because material property.

$$T_3 = T_{\text{max}}$$

$T_1, T_4 \rightarrow$ Temp. Constant
 (not change)

atmospheric
Temp.



$$W_{\text{net}} = W_T - W_C$$

$$W_{\text{net}} = (h_3 - h_4) - (h_2 - h_1)$$

$$W_{\text{net}} = C_p (T_3 - T_4) - C_p (T_2 - T_1)$$

$$W_{\text{net}} = C_p (T_3 - T_4 - T_2 + T_1)$$

$$= \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$T_2 = \frac{T_3 T_1}{T_4}$$

$$W_{net} = C_p \left[T_3 - \frac{T_1 T_3}{T_2} - T_2 + T_1 \right]$$

for max. $W_{net} = \frac{dW_{net}}{dT_2} = 0$

$$\frac{dW_{net}}{dT_2} = C_p \left[0 - T_1 T_3 \left(-\frac{1}{T_2^2} \right) - 1 + 0 \right] = 0$$

$$\frac{T_1 T_3}{T_2^2} - 1 = 0$$

$$T_1 T_3 = T_2^2 \Rightarrow T_2 = \sqrt{T_1 T_3}$$

$$T_4 = \frac{T_1 T_3}{\sqrt{T_1 T_3}} \Rightarrow T_4 = \sqrt{T_1 T_3}$$

**

$$T_2 = T_4 = \sqrt{T_1 T_3}$$

Pressure ratio condition for max. work $\Rightarrow$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

then $\frac{P_2}{P_1} = (\gamma_p)^{\frac{\gamma-1}{\gamma}}$

$$= \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

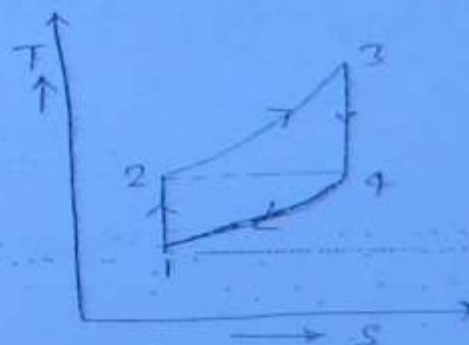
$$(\gamma_p)_{optimum} = \left(\frac{\sqrt{T_1 T_3}}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$(\gamma_p)_{optimum} = \left(\sqrt{\frac{T_3}{T_1}} \right)^{\frac{\gamma}{\gamma-1}}$$

**

$$(\gamma_p)_{optimum} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

Condition for max. work

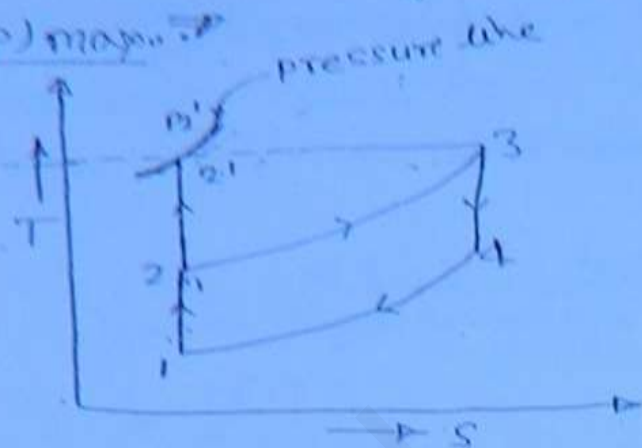


Maximum pressure ratio $(\gamma_p)_{max}$

$$W_c = W_T$$

$W_{compression} = W_{expansion}$
then net work is zero

$$W_{net} = 0$$



1-2 $\rightarrow$ Rev. adiabatic process

$$(\gamma_p)_{max} = \frac{P_2'}{P_1}$$

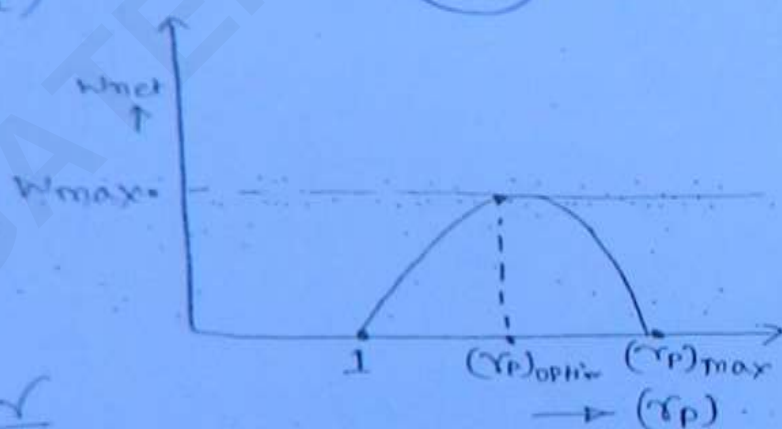
$$\frac{T_2'}{T_1} = \left(\frac{P_2'}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_2'}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

Because $T_2' = T_3$

$$\left(\frac{T_3}{T_1} \right) = (\gamma_p)_{max}^{\frac{\gamma-1}{\gamma}}$$

$$\frac{P_2'}{P_1} = (\gamma_p)_{max} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

(82)



$$(\gamma_p)_{max} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$(\gamma_p)_{optimum} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$(\gamma_p)_{optimum}^2 = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

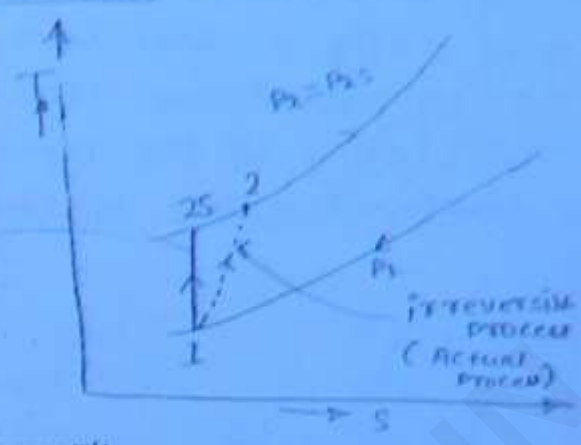
$$(\gamma_p)_{optimum}^2 = (\gamma_p)_{max}$$

Isentropic efficiency of a Compressor $\Rightarrow$

That means irreversible process in entropy increased.

(Isentropic line) Reversible adiabatic process

$$\eta_c = \frac{\text{Isentropic work}}{\text{Actual work}}$$



$$\eta_c = \frac{\text{Compressor work} \rightarrow \text{Less work}}{\text{Actual work} \rightarrow \text{more work}}$$

(Compressor work are less work because more power is required and turbine net work are more)

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{\text{Isentropic compression}}{\text{Actual compression}}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\left(\frac{T_{2s}}{T_1} \right) = \left(\gamma_p \right)^{\frac{\gamma-1}{\gamma}}$$

(83)

Isentropic efficiency of a Turbine $\Rightarrow$

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}}$$

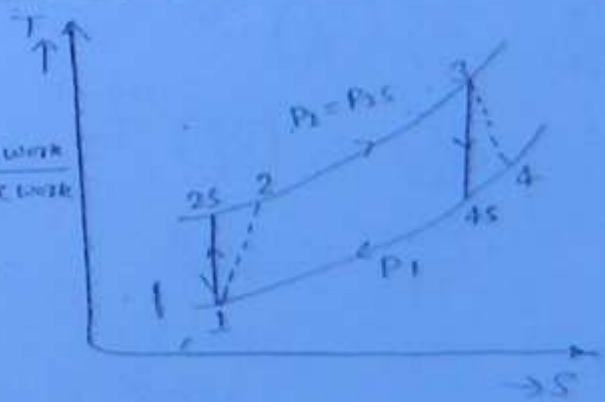
$$\eta_T = \frac{\text{Turbine actual work} \rightarrow \text{less work}}{\text{Turbine isentropic work} \rightarrow \text{more work}}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

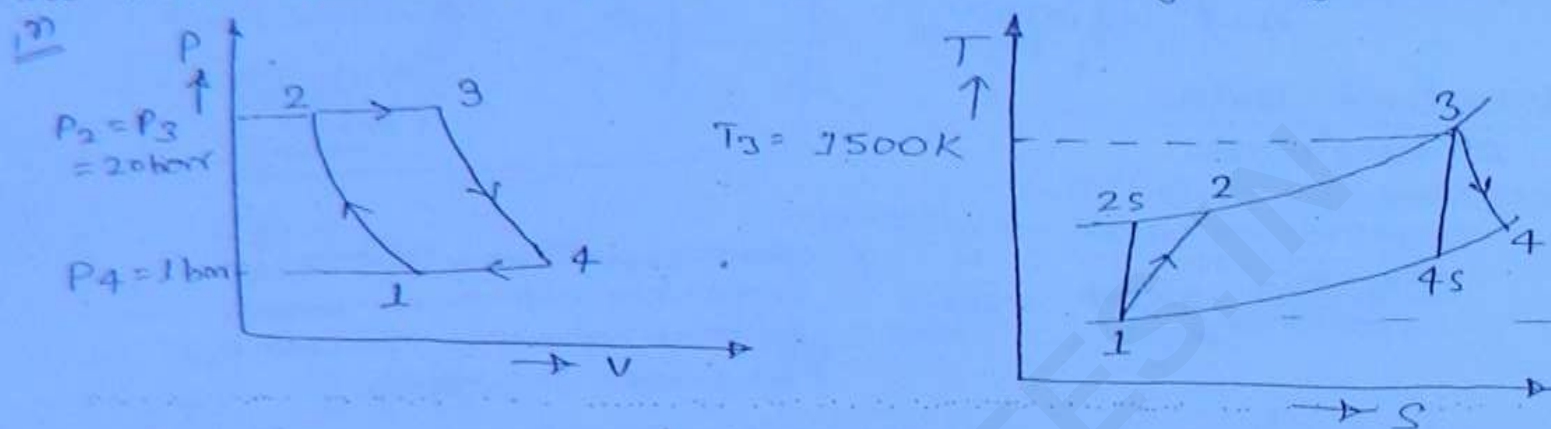
$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_{4s}} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\left[\frac{T_3}{T_4} = \left(\gamma_p \right)^{\frac{\gamma-1}{\gamma}} \right] **$$



Note $\Rightarrow$ Turbine work is opposite to compressor work.

Q. In a gas turbine hot combustion product with $c_p = 0.98 \text{ kJ/kg-K}$, and $c_v = 0.7538 \text{ kJ/kg-K}$ enter turbine at 20 bar, 1500K and leave at 1 bar. The isentropic efficiency of the turbine is 0.94 then find the work developed by the turbine per kg of gas.



$$c_p = 0.98 \text{ kJ/kg-K}$$

$$c_v = 0.7538 \text{ kJ/kg-K}$$

$$P_2 = P_3 = 20 \text{ bar} \quad T_3 = 1500 \text{ K}$$

$$P_4 = 1 \text{ bar} \quad \eta = 0.94 \%$$

84

$$W_T = h_3 - h_4$$

$$W_T = c_p (T_3 - T_4)$$

$$\frac{T_3}{T_{4s}} = (\gamma_P)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{20}{1} \right)^{\frac{1.3-1}{1.3}}$$

$$\frac{1500}{T_{4s}} = \frac{1.99}{1}$$

$$1.99 \times T_{4s} = 1500 \times 1$$

$$T_{4s} = \frac{1500}{1.99}$$

$$T_{4s} = 751.36 \text{ K}$$

$$\gamma = \frac{c_p}{c_v}$$

$$\gamma = \frac{0.98}{0.7538}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$\eta_T (T_3 - T_{4s}) = T_3 - T_4$$

$$W_T = C_p (T_3 - T_4)$$

$$W_T = C_p [\eta_T (T_3 - T_{4s})]$$

$$W_T = 0.98 [0.94 (1500 - 751.36)]$$

$$W_T = 689.8 \text{ kJ/kg}$$

PROB-2 air enter the compressor of a gas turbine operating on Brayton cycle at 27°C ($T_1 = 273 + 27 = 300\text{K}$). The pressure ratio is ($\frac{P_2}{P_1} = 6$). Calculate the max. Temp. in the cycle and cycle efficiency. Assume Turbine work = 2 Comp. work.
 $W_T = 2 W_C$, $\gamma = 1.4$

Soln $\gamma = 1.4$

$$W_T = 2.5 W_C$$

$$\gamma_p = \frac{P_2}{P_1} = 6$$

$$T_1 = 300\text{K}$$

$$\frac{T_2}{T_1} = (\gamma_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{300} = (6)^{\frac{1.4-1}{1.4}}$$

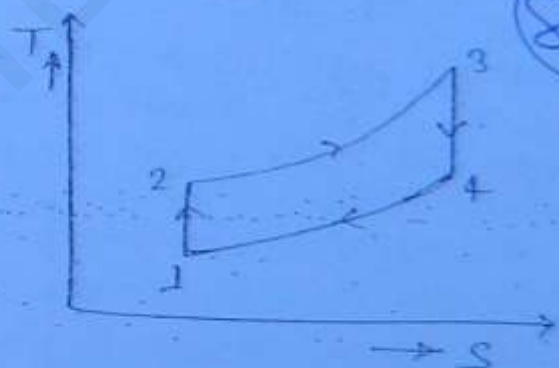
$$T_2 = 500.5\text{K}$$

$$W_T = 2.5 W_C$$

$$(h_3 - h_4) = 2.5 (h_2 - h_1)$$

$$C_p (T_3 - T_4) = 2.5 C_p (T_2 - T_1)$$

$$T_2 T_4 = \frac{T_1 T_3}{\gamma}$$



$$T_3 - \frac{T_1 T_2}{T_2} = 2.5 (T_2 - T_1)$$

$$T_3 \left(\frac{T_2 - T_1}{T_2} \right) = 2.5 (T_2 - T_1)$$

$$\frac{T_3}{T_2} = 2.5$$

$$T_3 = 2.5 \times T_2$$

$$T_3 = 2.5 \times 500.5$$

$$\boxed{T_3 = 1251.25 \text{ K}}$$

$$\eta_T = 1 - \frac{1}{(\gamma_p)^{\frac{\gamma-1}{\gamma}}}$$

$$\eta_T = 1 - \frac{1}{(6)^{\frac{1.4-1}{1.4}}}$$

$$\boxed{\eta_T = 0.4 \Rightarrow 40\%} \quad \text{Ans:}$$

86

Q8-3 In a simple gas Turbine power plant. T_1 is minimum Temp. and T_3 is the max. Temp. and what is the work ratio in terms of pressure ratio (γ_p).

$$\gamma_w = \frac{W_{net}}{W_T}$$

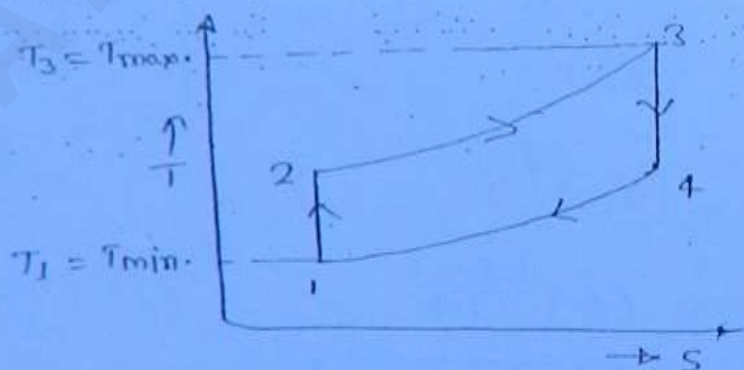
$$\gamma_w = \frac{W_T - W_C}{W_T}$$

$$\gamma_w = 1 - \frac{W_C}{W_T}$$

$$\gamma_w = 1 - \frac{(h_2 - h_1)}{(h_3 - h_4)}$$

$$\gamma_w = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)}$$

$$\gamma_w = 1 - \frac{T_1 \left(\frac{T_2}{T_1} - 1 \right)}{T_3 \left(1 - \frac{T_4}{T_3} \right)}$$



$$\eta_w = 1 - \left(\frac{T_1}{T_3} \left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}} \right] \right)$$

$$\boxed{\eta_w = 1 - \frac{T_1}{T_3} (r_p)^{\frac{\gamma-1}{\gamma}}} \quad \underline{\text{Ans:}}$$

PROB-④ A simple gas turbine power plant working on Brayton cycle. the power required to drive the compressor is 175 kW. Heat supplied during constant pressure heat addition is 675 kW. turbine output obtained during expansion is 425 kW. find the heat rejection.

Soln

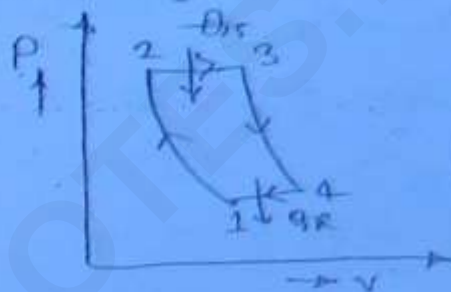
$$W_c = 175 \text{ kW}$$

$$W_{\text{net}} = Q_s - Q_R$$

$$W_t - W_c = Q_s - Q_R$$

$$425 - 175 = 675 - Q_R$$

$$\boxed{Q_R = 425} \text{ kW. } \underline{\text{Ans:}}$$



(82)

PROB-⑤ A closed cycle gas turbine unit develop a power of 1500 kW. the pressure and temp. at the inlet to the compressor are 1 bar and 20°C. Pressure ratio is 6. the max. temp. to the inlet to the turbine 800°C. find the isentropic efficiency of turbine and mass flow rate of air if the isentropic efficiency of the compressor is 85% and overall efficiency is 20%. , Cp take equal to 1.005 kJ/kg·K. $\gamma = 1.4$.

Soln.

$$P = 1500 \text{ kW}$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 273 + 20 = 293 \text{ K}$$

$$r_p = 6 = \frac{P_2}{P_1}$$

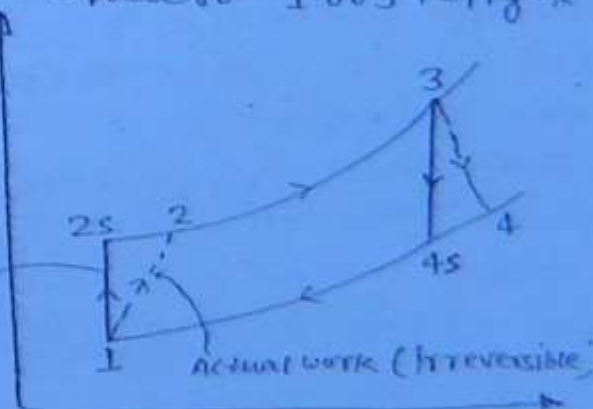
$$T_3 = 273 + 800 = 1073 \text{ K}$$

$$\eta_c = 0.85$$

$$\eta_o = 0.20$$

(Rev. adiabatic process)

Isentropic work



$$\eta_0 = \frac{W_{net}}{Q_s}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = 488.8 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \begin{array}{l} \text{isentrope work} \rightarrow \text{less} \\ \text{Actual work} \rightarrow \text{more compression} \end{array}$$

$$\eta_c = \frac{488.8 - 293}{T_2 - 293}$$

$$0.85 = \frac{488.8 - 293}{T_2 - 293}$$

$$T_2 = 523.4 \text{ K}$$

(88)

$$\eta_0 = \frac{W_{net}}{Q_s} = \frac{W_T - W_c}{Q_s}$$

$$\eta_0 = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

$$\eta_0 = \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_2)}$$

$$0.20 = \frac{(1073 - T_4) - (523.4 - 293)}{1073 - 523.4}$$

$$T_4 = 732.6 \text{ K}$$

$$\frac{T_3}{T_{4s}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1073}{T_{4s}} = (6)^{\frac{1.4-1}{1.4}}$$

$$T_{4s} = 643.1 \text{ K}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \rightarrow \text{less work} \quad \text{expansion} \\ \rightarrow \text{more work (because more } \cancel{\text{compression}} \text{ decreased.)}$$

$$\eta_T = \frac{1073 - 732.6}{1073 - 643.1}$$

$$\eta_T = 79\%$$

$$(3) P = m \times W_{\text{net}}$$

$$P = m (\eta_0 \times Q_s)$$

$$\text{where } Q_s = \frac{h_3 - h_2}{\eta_0}$$

$$Q_s = C_p (T_3 - T_2)$$

$$= 1.005 (1073 - 523.4)$$

$$Q_s = 552.31 \text{ KJ/kg}$$

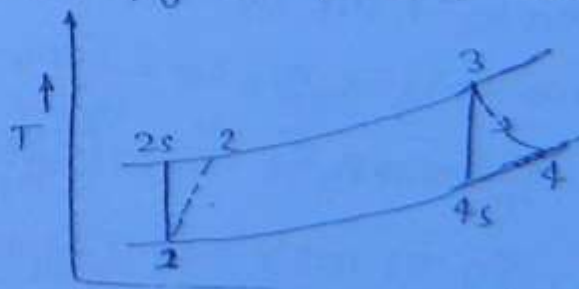
$$P = m \times W_{\text{net}}$$

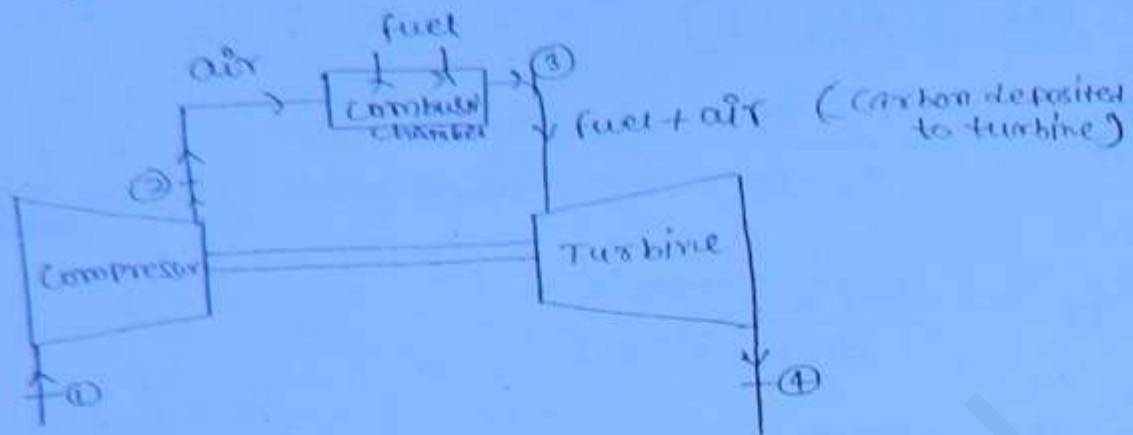
$$1500 = m (552.31 \times 0.20)$$

$$m = 13.5 \text{ kg/sec} \quad \text{Ans:}$$

PROB. (6) Air enters the compressor of a gas turbine at a bar and 293K the pressure after compression is 6 bar. Air fuel ratio 60 the isentropic efficiency of compressor and turbine are 0.84 and 0.88 respectively. Calculate the power developed by the units & its overall efficiency. Assume γ to be same for air and combustion product. take specific heat of gases = 1.1 KJ/kg-K, mass of air 5 kg/s $C_v = 40000 \text{ KJ/kg}$.

soln





$$T_1 = 293, \quad \gamma_p = 6$$

$$\eta_c = 0.84, \quad \eta_T = 0.88$$

$$\gamma = 1.4$$

$$\frac{T_{2s}}{T_1} = (\gamma_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

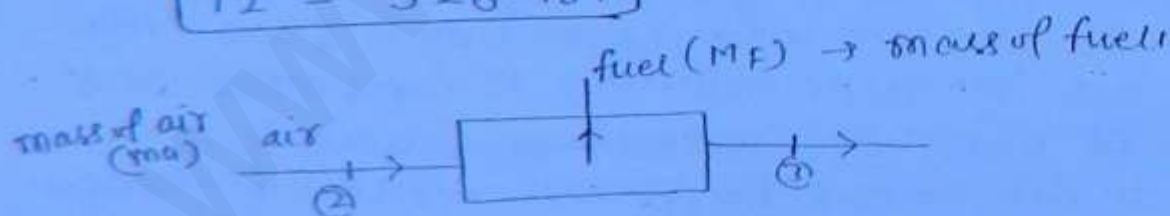
$$T_{2s} = 488.87 \text{ K}$$

90

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\text{Isentropic work} \rightarrow \text{less}}{\text{Actual work} \rightarrow \text{more}}$$

$$0.84 = \frac{488.87 - 293}{T_2 - 293}$$

$$T_2 = 526.18 \text{ K}$$



Energy eqn.

$$m_a h_2 + m_f C_v = (m_a + m_f) h_3$$

$$m_a C_p T_2 + m_f C_v = (m_a + m_f) C_p T_3$$

$$\therefore \frac{m_a}{m_f} C_p T_2 + C_v = \left(\frac{m_a}{m_f} + 1 \right) C_p T_3$$

$$60 \times 1.005 \times 526.18 + 40000 = (60 + 1) \times 1.1 \times T_3$$

$$T_3 = 1050.3 \text{ K}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_{4s}} \right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1059.3}{T_{4s}} = (6)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_{4s} = 635 \text{ K}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$0.88 = \frac{1059.3 - T_4}{1059.3 - 635}$$

$$\boxed{T_4 = 685.8 \text{ K}}$$

$$W_c = \dot{m}_a (h_2 - h_1) \rightarrow \text{Compressor work}$$

$$= \dot{m}_a \times C_p (T_2 - T_1)$$

$$= 5 \times 1.005 (526.18 - 293)$$

$$\boxed{W_c = 1171.72 \text{ kW}}$$

$$\text{Turbine work } W_T = (\dot{m}_a + \dot{m}_F) (h_3 - h_4)$$

$$= (\dot{m}_a + \dot{m}_F) C_{p_{\text{gas}}} (T_3 - T_4)$$

$$= \left(5 + \frac{5}{60} \right) \times 1.11 \times (1059.3 - 685.5)$$

$$\boxed{W_T = 2107 \text{ kW}}$$

$$W_{\text{net}} = W_T - W_c$$

$$= 2107 - 1171.72$$

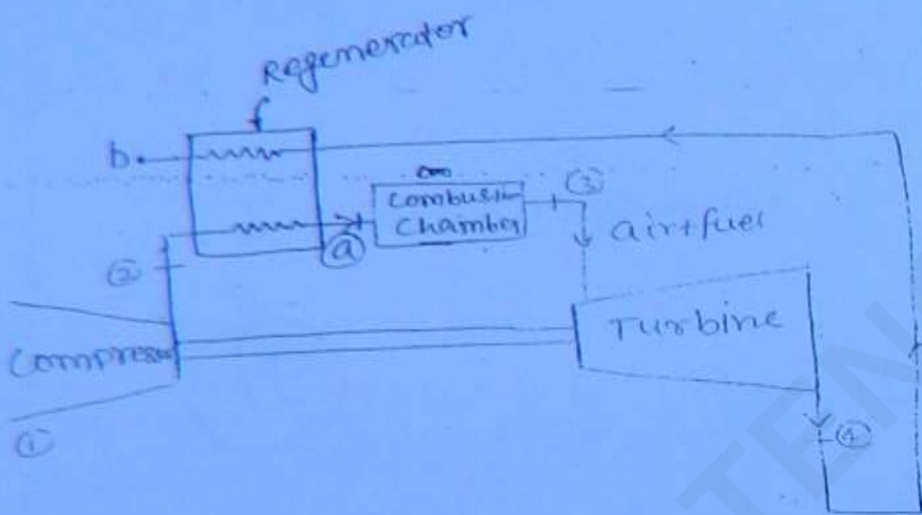
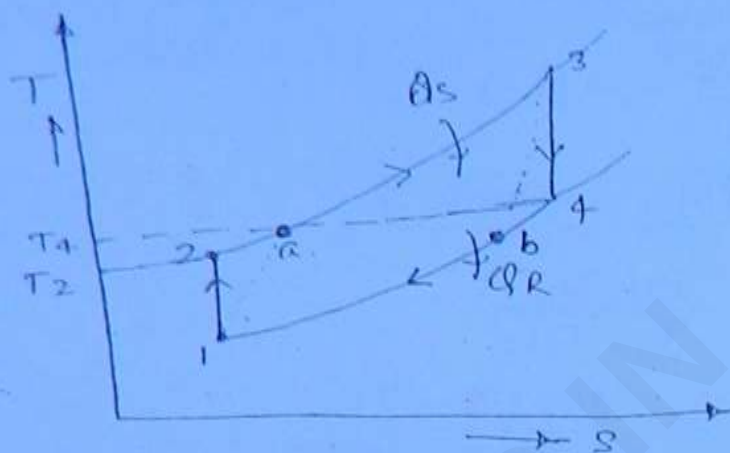
$$\boxed{W_{\text{net}} = 935.7}$$

$$Q_s = \dot{m}_F \times C_v$$

$$\eta_o = \frac{W_{\text{net}}}{Q_s} \Rightarrow \frac{W_{\text{net}}}{\dot{m}_F \times C_v} \Rightarrow \frac{935.7}{\frac{5}{60} \times 40000}$$

$$\eta_o = 0.2807 \quad (\text{Ans.})$$

USE OF REGENERATOR IN GAS TURBINE $\Rightarrow$



$$\eta_p = \frac{W_{net}}{Q_s} = \frac{W_T - W_C}{Q_s} \rightarrow \text{constant}$$

When Q_s is decreases then efficiency of gas turbine increases.

effect of regeneration $\Rightarrow$

- ① No change in compressor work.
- ② No change in Turbine work.
- ③ No change in Net work.
- ④ Reduction in heat supplied.
- ⑤ Increase in Turbine efficiency.

$$\eta = \frac{W_T - W_C}{Q_s} \Rightarrow \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_a}$$

$$\eta = \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_a)}$$

$$\eta = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_1}$$

⊕ effectiveness of Regenerator ⇒

It is the ratio of Actual rise in temp. to the ideal Rise in Temperature.

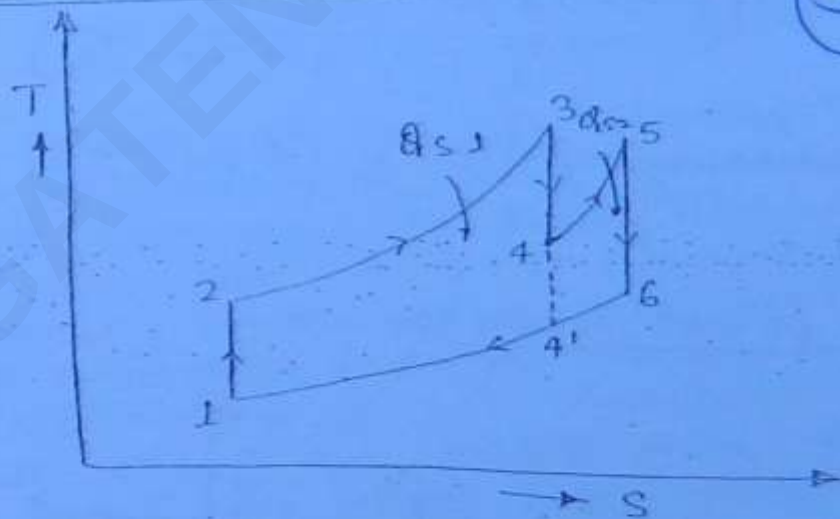
$$\epsilon = \frac{\text{Actual rise in temp.}}{\text{Ideal rise in temp.}}$$

$$\epsilon = \frac{t_a - t_2}{t_4 - t_2}$$

For effective utilisation of Regenerator the temp. difference between T_4 and T_2 should be large.

⊕ USE OF REHEATING IN GAS TURBINE ⇒

93



$$W_T (\text{simple cycle}) = h_3 - h_4' = (h_3 - h_4) + (h_4 - h_4')$$

$$W_T (\text{reheat cycle}) = (h_3 - h_4) + (h_5 - h_6)$$

$$W_{\text{net}} = W_T - W_C \quad \text{when } W_C = \text{constant}$$

then W_T increase the network increase

As constant pressure line diverge

$$h_5 - h_6 > h_4 - h_4'$$

$$\therefore W_T (\text{reheat cycle}) > W_T (\text{without reheat})$$

$$\eta = \frac{W_{net} \uparrow}{Q_s \uparrow}$$

then efficiency will be decreases.

→ with reheating the net work increases this does not mean an increase in efficiency of the cycle cause with reheating the reheat supplied is also increase th reheat because the temp $T_6 > T_4'$ and hence whenever reheating is used it is generically coupled th regeneration therefore regeneration from efficiency, increases and due to Reheating from W_{net} increases.

$$\eta = \frac{W_T - W_C}{Q_s}$$

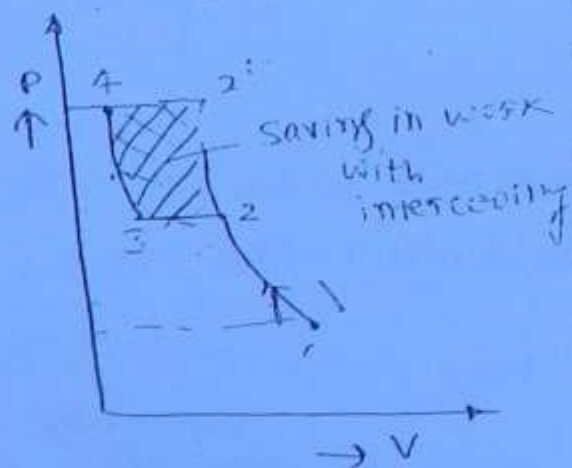
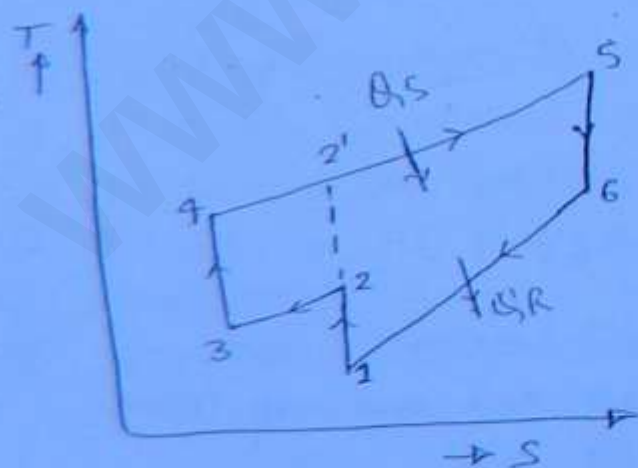
$$\eta = \frac{[(h_3 - h_4) + (h_5 - h_6)] - [h_2 - h_1]}{(h_3 - h_2) + (h_5 - h_4)}$$

$$\eta = \frac{[(T_3 - T_4) + (T_5 - T_6)] - [T_2 - T_1]}{(T_3 - T_2) + (T_5 - T_4)}$$

(94)

→ due to Reheating efficiency of the cycle decreases ↓

Intercooling in Gas Turbine →



$$W_{net} = W_T - W_C$$

$$\downarrow \eta = \frac{W_{net} \uparrow}{Q_r \uparrow}$$

⊕ Intercooling in gas turbine →

In Inter cooling the work of compression decreases and this result in increase in Net work ^{but} does not mean an increase in efficiency because the heat supplied also increases. but with intercooling the slope of for Regeneration increases because $T_4 < T_2'$ and hence intercooling is generally coupled with regeneration. →

$$\eta = \frac{W_T - W_C}{Q_S}$$

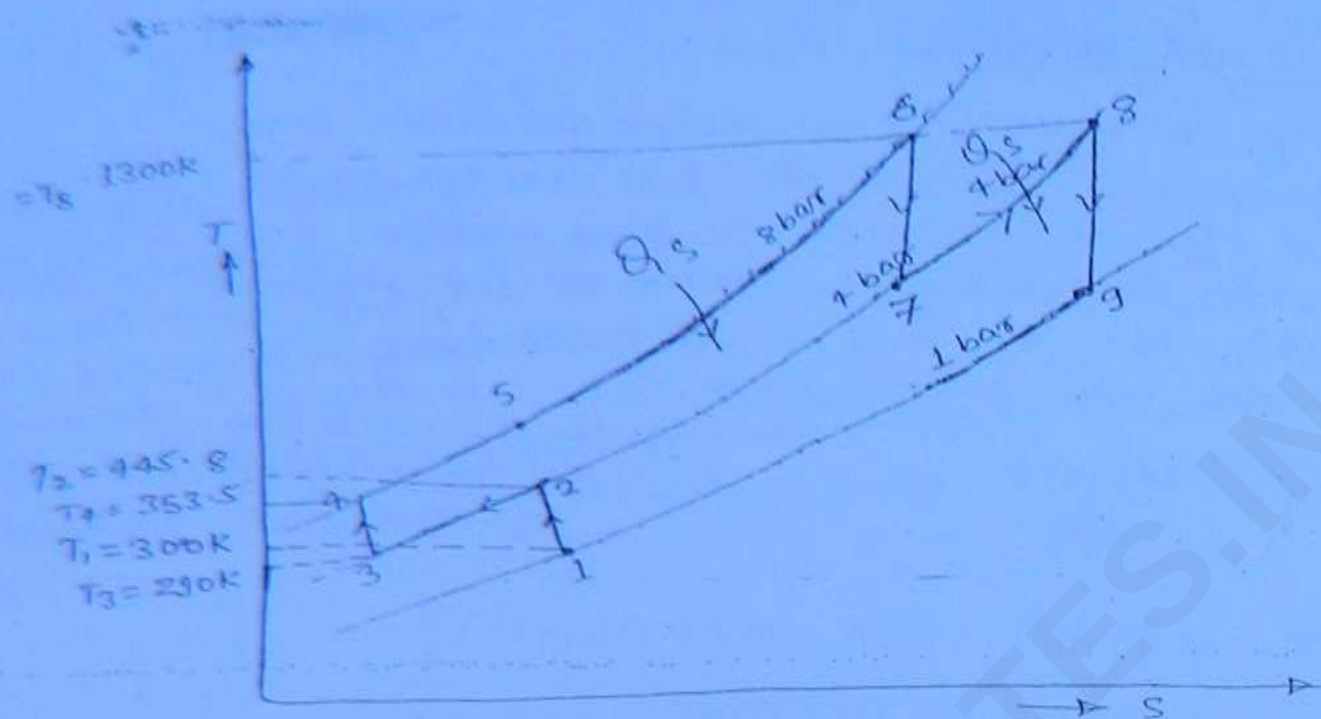
① due to intercooling efficiency of cycle decreases (↓)

$$\eta = \frac{(h_5 - h_6) - [(h_2 - h_1) + (h_4 - h_3)]}{h_5 - h_4}$$

$$\eta = \frac{(T_5 - T_6) - [(T_2 - T_1) + (T_4 - T_3)]}{T_5 - T_4}$$

(95)

PRCB-Q A Regenerative reheat cycle has air entering at 1 bar, 300K into the Compressor, having intercooling in between two stage of compression air leaving first stage of compression is cooled upto 290K and 4 bar pressure in intercooler and subsequently compressed to 9 bar pressure air leaving second stage compressor is passed through a regenerator having an effectiveness of 0.8. Subsequent combustion its 1300K at inlet to the turbine having expansion upto 4 bar and then heated upto 1300K before being expanded upto 1 bar. exhaust from turbine is passed to Regenerator. Find the total turbine work and efficiency consider compression and expansion to be isentropic and air as the working fluid throughout the cycle.



$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{300} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 445.8 \text{ K}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_4}{290} = \left(\frac{8}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$T_4 = 353.5 \text{ K}$$

$$\frac{T_6}{T_7} = \left(\frac{P_6}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1300}{T_7} = \left(\frac{8}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$T_7 = 1066.4 \text{ K}$$

$$\frac{T_8}{T_9} = \left(\frac{P_8}{P_9}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1300}{T_9} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$T_9 = 874.83 \text{ K}$$

$$\epsilon = \frac{T_5 - T_4}{T_9 - T_4}$$

$$0.8 = \frac{T_5 - 353.5}{874.83 - 353.5}$$

$$T_5 = 770.6 \text{ K}$$

96

$$W_T = (h_6 - h_7) + (h_8 - h_9)$$

$$W_T = C_p(T_6 - T_7) + C_p(T_8 - T_9)$$

$$= 1.005(1300 - 1066.4) + 1.005(1300 - 874.8)$$

$$\boxed{W_T = 662 \text{ KJ/kg}}$$

$$\eta = \frac{W_T - W_C}{Q_S}$$

$$W_C = (h_2 - h_1) + (h_4 - h_3)$$

$$= C_p(T_2 - T_1) + C_p(T_4 - T_3)$$

$$= 1.005(445.8 - 300) + 1.005(353.5 - 290)$$

$$\boxed{W_C = 210 \text{ KJ/kg}}$$

$$Q_S = (h_6 - h_5) + (h_8 - h_7)$$

$$Q_S = C_p(T_6 - T_5) + C_p(T_8 - T_7)$$

$$= 1.005(1300 - 770.5) + 1.005(1300 - 1066.4)$$

$$\boxed{Q_S = 766.8}$$

$$\eta = \frac{W_T - W_C}{Q_S} \Rightarrow \frac{662 - 210}{766.8}$$

$$\eta = 0.589$$

$$\boxed{\eta = 58.9\%}$$

Ans

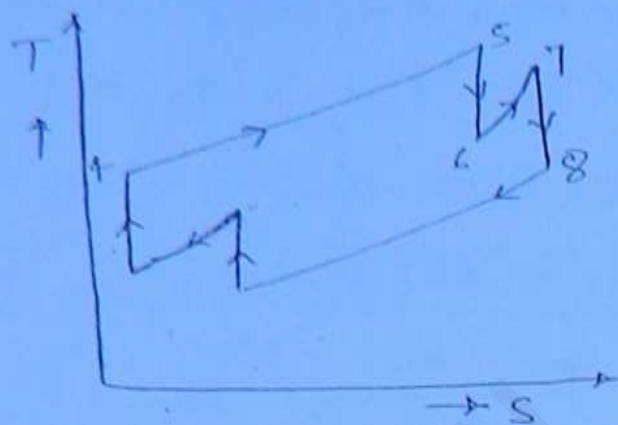
I.E.S

prob-2

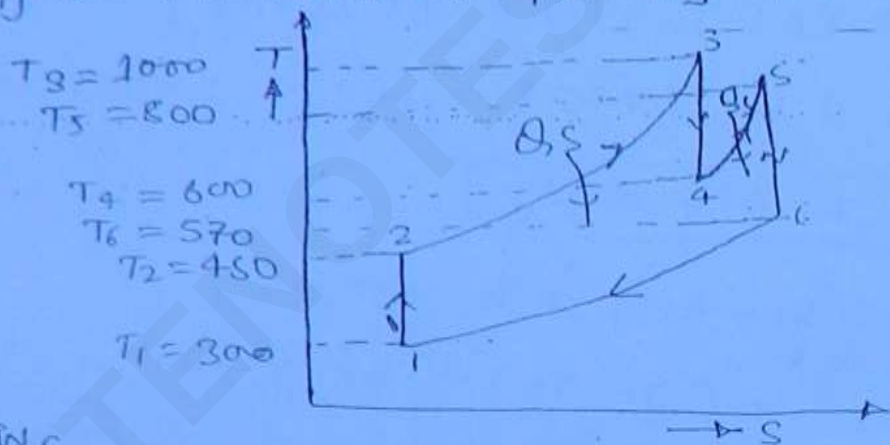
The cycle shown in figure represent a gas turbine cycle with intercooling and reheating on T-s diagram.

- | | | |
|---------------------------|---|-----|
| 1. Inter cooler | → | 2-3 |
| 2. combustor | → | 4-5 |
| 3. Reheater | → | 6-7 |
| 4. High press. compressor | → | 3-4 |

match list (2)



Ques 3 The given figure shows a gas turbine cycle employing reheating on T-s diagram then find the efficiency of the cycle.



Soln

$$\eta = \frac{W_T - W_C}{Q_S}$$

$$= \frac{[(h_3 - h_4) + (h_5 - h_6)] - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

$$= \frac{(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)}{(T_3 - T_2) + (T_5 - T_4)}$$

$$\eta = \frac{(1000 - 600) + (800 - 570) - (450 - 300)}{(1000 - 450) + (800 - 600)}$$

$$\eta = 0.64 = 64\% \text{ Ans}$$

98

Ques In a simple gas turbine power plant operating on standard Brayton cycle 175 kW is required to drive the compressor heat supplied during constant pressure heat addition process is 675 kW. The turbine power is 425 kW. What is the heat rejection.

Soln

$$Q_s = 675$$

$$W_T = 425$$

$$W_C = 175$$

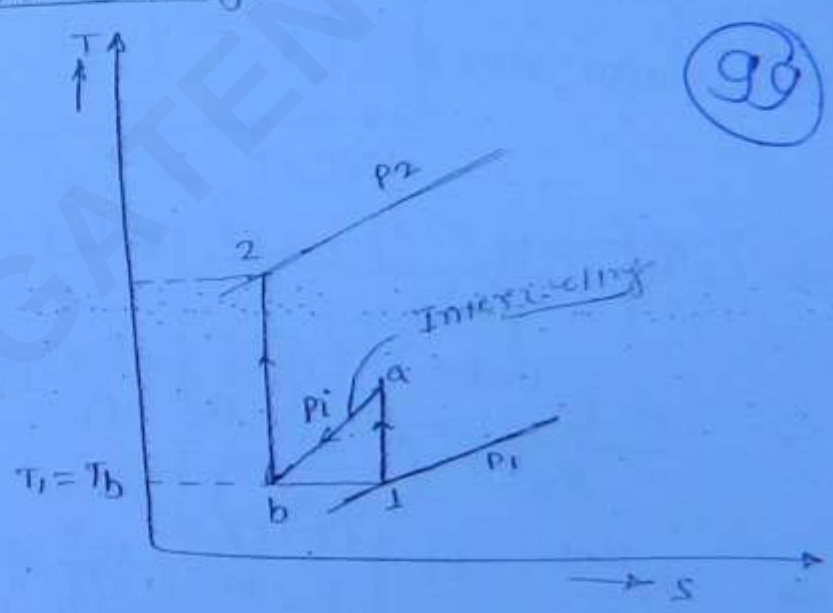
$$Q_R = ? \quad \Sigma Q = \Sigma W$$

$$Q_s - Q_R = W_T - W_C$$

$$675 - Q_R = 425 - 175$$

$$\boxed{Q_R = 425} \quad \underline{\text{Ans.}}$$

optimum pressure ratio for min work input
(with perfect inter cooling)



$$W_C = W_{C1} + W_{C2}$$

$$W_C = (h_a - h_1) + (h_2 - h_b)$$

$$W_C = C_p (T_a - T_1) + C_p (T_2 - T_b)$$

$$W_C = C_p [T_a - T_1 + T_2 - T_b]$$

$$W_C = C_p T_1 \left[\frac{T_a}{T_1} - 1 + \frac{T_2}{T_1} - \frac{T_b}{T_1} \right]$$

because
 $\boxed{T_b = T_1}$

$$W_c = C_p T_1 \left[\frac{T_2}{T_1} - 1 + \frac{T_2}{T_1} - 1 \right]$$

$$W_c = C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 2 + \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\therefore \frac{T_2}{T_1} = \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Let $\frac{\gamma-1}{\gamma} = K$ Constant

$$W_c = C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^K - 2 + \left(\frac{P_2}{P_1} \right)^K \right]$$

$$W_c = C_p T_1 \left[\frac{P_2^K}{P_1^K} - 2 + \frac{P_2^K}{P_1^K} \right]$$

P_1 and P_2 , C_p , T , K Constant only variable P_i

For minimum work

$$\frac{dW}{dP_i} = 0$$

$$\Rightarrow C_p T_1 \left[\frac{K P_i^{K-1}}{P_1^K} - 0 + P_2^K (-K P_i^{-K-1}) \right]$$

$$\Rightarrow \frac{K P_i^{K-1}}{P_1^K} = K P_2^K P_i^{-(K+1)}$$

$$\Rightarrow P_i^{K-1} \cdot P_i^{K+1} = P_2^K P_1^K$$

$$P_i^{2K} = (P_1 P_2)^K$$

$$P_i^2 = P_1 P_2$$

$$P_i = \sqrt{P_1 P_2}$$

Intermediate pressure = Geometric Mean of P_1 and P_2

$P_1 \rightarrow$ Inlet ~~compressor~~ ^{pressure} for compressor 1

$$W_{C1} = W_{C2}$$

$$W_C = W_{C1} + W_{C2} = 2W_{C1}$$

$$W_C = 2C_p T_1 \left[\left(\sqrt{\frac{P_2}{P_1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

For perfect intercooling equal work input for each compressor will be required.

$$W_{C1} = h_a - h_1$$

$$W_{C1} = C_p (T_a - T_1)$$

$$= C_p T_1 \left[\frac{T_a}{T_1} - 1 \right]$$

$$= C_p T_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\text{where } P_i = \sqrt{P_1 P_2}$$

$$W_{C1} = C_p T_1 \left[\left(\frac{\sqrt{P_1} \sqrt{P_2}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$W_{C1} = C_p T_1 \left[\left(\sqrt{\frac{P_2}{P_1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

(10)

$$W_{C2} = h_2 - h_b = C_p (T_2 - T_b)$$

$$= C_p T_b \left[\frac{T_2}{T_b} - 1 \right]$$

$$\text{where } T_b = T_1$$

$$= C_p T_1 \left[\left(\frac{P_2}{P_i} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= C_p T_1 \left[\left(\frac{P_2}{\sqrt{P_1 P_2}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$W_{C2} = C_p T_1 \left[\left(\sqrt{\frac{P_2}{P_1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$W_{C2} = W_{C1}$$

$$W_C = W_{C1} + W_{C2}$$

$$W_C = 2W_{C1}$$

$$W_C = 2C_p T_1 \left[\left(\sqrt{\frac{P_2}{P_1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

Vapour power cycles (Rankine cycle)

Reason for using water as a working fluid in vapour power cycle.

1) It is cheap.

2) It is chemically stable.

3) It is harmless substance.

4) Specific steam consumption $\Rightarrow$

$$SSC = \frac{\text{mass of steam}}{P_{net}} = \frac{m_s}{P_{net}} \rightarrow \frac{kg/s}{kW/sec}$$

$$SSC = \frac{m_s}{m_s \times W_{net}} \rightarrow \frac{kg/s}{kg/s \times kJ/kg}$$

$$SSC = \frac{1}{W_{net}} \frac{kJ}{kg} \Rightarrow \frac{kg}{W_{net} \cdot kJ}$$

$$SSC = \frac{kg}{W_{net} \frac{kJ \times sec}{sec}}$$

$$SSC = \frac{kg}{W_{net} \cdot kW \cdot sec}$$

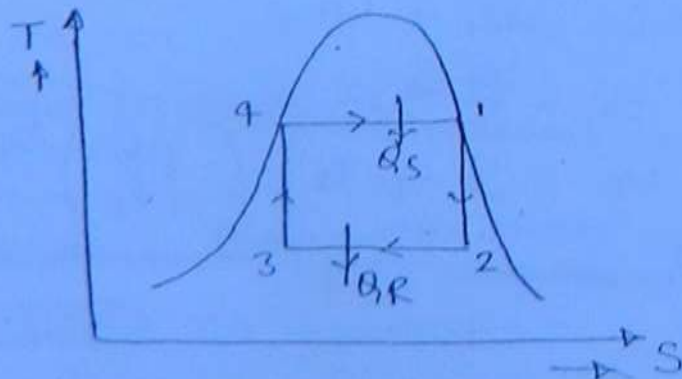
$$SSC = \frac{kg}{W_{net} \cdot kW \times \frac{1}{3600} hr} \quad \begin{matrix} 1 hr = 3600 sec \\ \frac{1 hr}{3600} = \frac{1}{3600} sec \end{matrix}$$

$$SSC = \frac{3600 kg}{W_{net} \cdot kW \cdot hr}$$

If the net work is more specific steam consumption is less and hence mass flow rate of steam will be less and hence the size of the plant will be small.

CARNOT CYCLES $\Rightarrow$

$$W_{net} = W_T - W_C$$



④ Drawback of Carnot cycle $\Rightarrow$

- ① It is difficult to design and control partial Condenser.
- ② The design of a compressor which handles a mixture of liquid and vapour is difficult.
- ③ As compressor is used the compression work is large and hence the net work is less. $w_{net} = w_T - w_c$
- ④ The saturated steam that enters the turbine at point ① leaves at point ② which is in wet region. The liquid droplets in high-velocity steam cause erosive action on turbine blades.

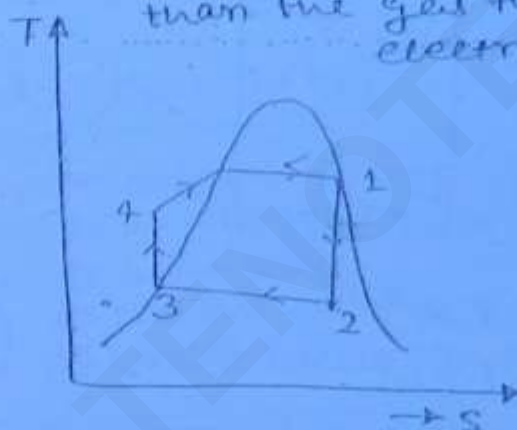
⑤ Rankine cycle $\Rightarrow$

Steam power plant are more popular than the gas turbine plant for electricity generation

$$w_T \gg w_c$$

$$w_{net} = w_T - w_p$$

$$w_p = - \int v dp$$

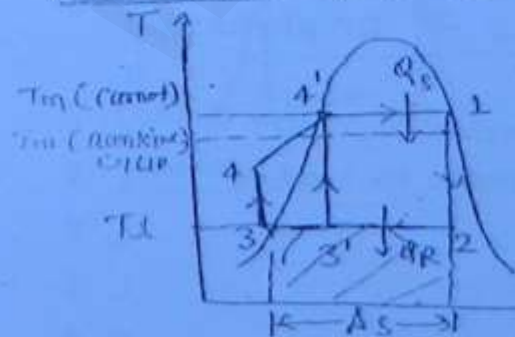


103

*** Carnot cycle $\rightarrow w_T - w_c \uparrow = w_{net} \downarrow$
Rankine cycle $\rightarrow w_T - w_p \downarrow = w_{net} \uparrow$

$$\eta = \frac{w_T - w_p}{Q_s} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

⑥ Reason for lower efficiency of Rankine cycle compare to Carnot cycle $\Rightarrow$



$$Q_R = T_c (\Delta s)$$



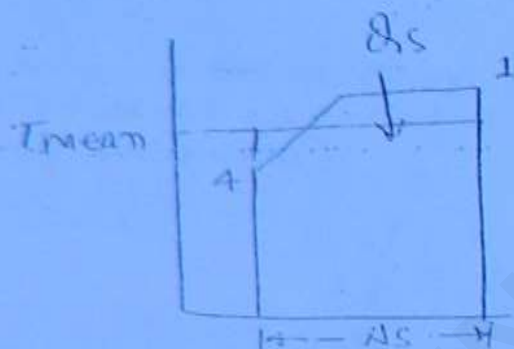
$$\eta = \frac{W_{net}}{Q_s}$$

$$\eta = \frac{Q_s - Q_R}{Q_s}$$

$$\eta = 1 - \frac{Q_R}{Q_s}$$

where $Q_R = T_L (\Delta S)$

and



$$Q_s = h_1 - h_4$$

$$= T_{mean} (\Delta S)$$

$$Q_R = T_L (\Delta S)$$

$$\eta = 1 - \frac{T_L (\Delta S)}{T_{mean} (\Delta S)}$$

(104)

$$\eta = 1 - \frac{T_L}{T_{mean}}$$

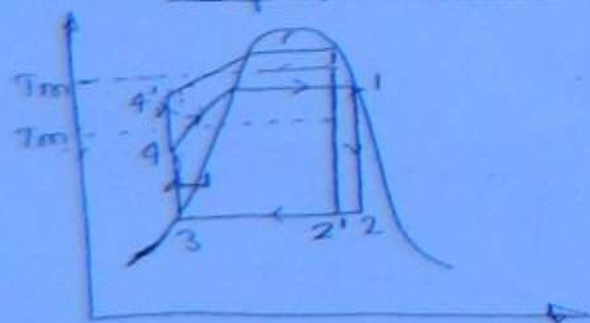
$$* (T_{mean})_{carnot} > (T_{mean})_{Rankine}$$

$$* \eta_{carnot} > \eta_{Rankine}$$

- Though the net work output of Rankine cycle is greater than the net work output of Carnot cycle the efficiency of Rankine cycle is less because the mean temp. of heat addition in Rankine cycle is lower than that in Carnot cycle.

Method of Improving the performance of Rankine cycle

→ ① Increase in Boiler Pressure →

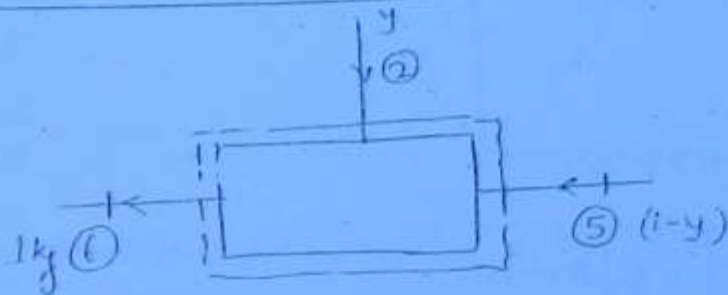


- ① $W_T \uparrow \rightarrow$ Increase in Turbine work
- ② $Q_R \downarrow \rightarrow$ decrease in Heat addition
- ③ $W_P \uparrow$
- ④ $T_{mean} \uparrow$
- ⑤ $\eta \uparrow$
- ⑥ $x \downarrow$ dryness fraction is

$$Q_s = 1(h_1 - h_7)$$

$$\eta = \frac{W_T - W_P}{Q_s}$$

$$\eta = \frac{[1(h_1 - h_2) + (1-y)(h_2 - h_3)] - [(1-y)(h_5 - h_4) + 1(h_7 - h_6)]}{1(h_1 - h_7)}$$



$$y \times h_2 + (1-y)h_5 = 1 \times h_6$$

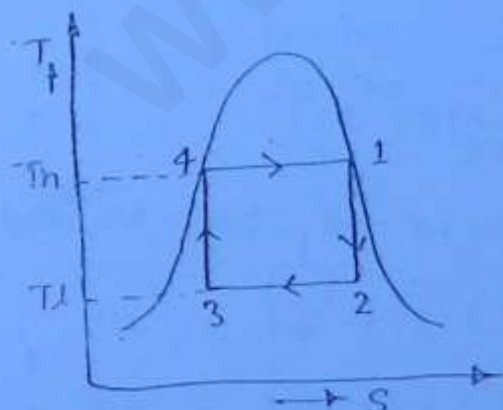
*** gmp. eqn. ***

- Prob-① A Carnot steam cycle operates between a source temp of 311.06°C equal to 10 MPa and sink temp. of 32.88°C ($P = 5 \text{ kPa}$). find
- ① work ratio.
 - ② Thermal efficiency.
 - ③ Specific steam consumption.

105

Pressure	$t_{\text{sat}}^\circ\text{C}$	$h_f \text{ kJ/kg}$	$h_g \text{ (kJ/kg)}$	$s_f \text{ (kJ/kg)}$	$s_g \text{ (kJ/kg)}$
10 MPa	311.06	1407.56	2724.7	3.3596	5.6141
5 MPa	32.88	137.82	2561.2	0.4764	8.3951

Soln



$$T_h = 311.06 + 273 = 584.06$$

$$T_l = 32.88 + 273 = 305.88$$

$$\eta = \frac{T_h - T_l}{T_h}$$

$$\eta = 1 - \frac{T_l}{T_h}$$

$$\eta = 1 - \frac{305.88}{584.06} = 47.67\%$$

$$\eta = \frac{W_{net}}{Q_s}$$

$$Q_s = h_1 - h_4$$

$$= 2724.7 - 1407.56$$

$$Q_s = 1317.14$$

$$W_{net} = \eta \times Q_s$$

$$W_{net} = 0.476 \times 1317.14 = 627.8$$

$$SSC = \frac{3600}{627.8} = 5.739 \text{ kg/kW-hr}$$

$$\text{work Ratio} = \frac{W_{net}}{W_T}$$

$$\text{where } W_T = h_1 - h_2$$

$$\left[\begin{array}{l} h_2 = h_f + x(h_g - h_f) \\ h_f = 137.82 \text{ at 5 mpa} \\ h_g = 2561.62 \text{ at 5 mpa} \end{array} \right]$$

$$S_1 = S_2$$

$$S_2 = S_f + x(S_g - S_f)$$

$$5.6141 = 0.4764 + x(8.3951 - 0.4764)$$

$$\boxed{x = 0.648}$$

$$h_2 = 137.82 + 0.648 \times (2561.62 - 137.82)$$

$$h_2 = 1708.3 \text{ kJ/kg}$$

$$W_T = 2724.7 - 1708.3 = 1016.3$$

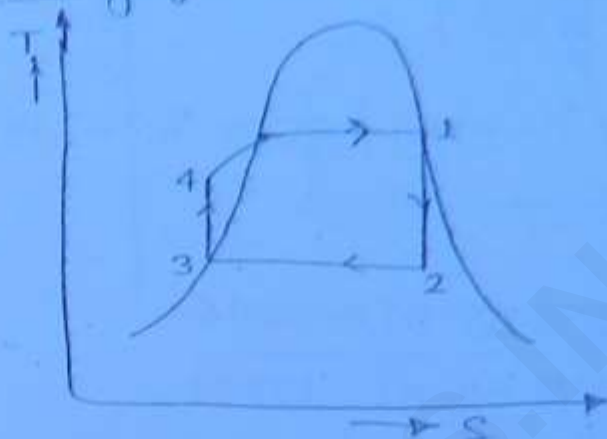
$$\gamma_w = \frac{627.8}{1016.3}$$

$$\boxed{\gamma_w = 0.618}$$

Ans:

100

PROB-2) water is the working fluid in Rankine cycle saturated vapour enters the turbine at 2 mpa the condenser pressure is 10 kpa the pump work is negligible.



Sol: Heat supplied $Q_s = h_1 - h_4$ (because pump work is negligible)

$$W_p = h_4 - h_3 \quad (\text{but pump work is negligible})$$

so that

$$W_p = h_4 = h_3$$

$$\eta = \frac{W_{net}}{Q_s}$$

102

P (kPa)	$t_{sat} ^\circ C$	$v_f (m^3/kg)$	$v_g (m^3/kg)$	$h_f (kJ/kg)$	$h_g (kJ/kg)$	$s_f (kJ/kg \cdot K)$
2000	212.42	0.001177	0.09963	908.77	2799.51	2.447
10	45.81	0.00101	14.6735	191.81	2584.63	0.6432

$s_g (kJ/kg \cdot K)$
6.3408
8.1501

$$\eta = \frac{W_T - W_p \rightarrow 0 \text{ (negligible)}}{Q_s}$$

$$\eta = \frac{h_1 - h_2}{h_1 - h_4} \quad \text{--- (1)}$$

$$h_1 = 2799.51, \quad h_g = 2584.63$$

10 bar at $h_f = h_g = h_3 = 191.81, \quad h_g = 2584.63$

$$h_2 = h_f' + x(h_g - h_f)$$

$$h_2 = 191.81 + x(2584.63 - 191.81)$$

but $s_1 = s_2 = 6.3408$ at 20 ~~bar~~ kPa

$$s_2 = s_f + x(s_g - s_f)$$

$$6.3408 = 0.6492 + x(8.15101 - 0.6492)$$

$$\boxed{x = 0.758}$$

$$h_2 = 191.81 + (0.758)[2584.63 - 191.81]$$

$$h_2 = 2006$$

now

$$\eta = \frac{2799.51 - 2006}{2799.51 - 191.81}$$

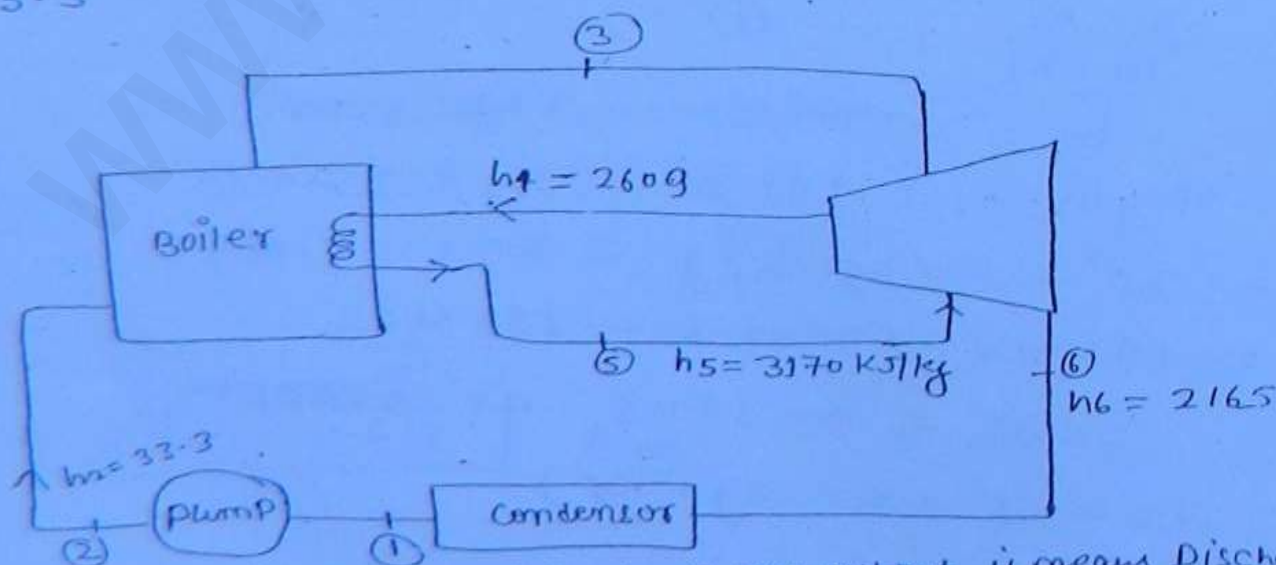
$$\eta = 30.4\% \text{ - Ans}$$

Q3-3 Consider a steam power plant using a Reheat cycle as shown in fig. Steam leaves the boiler enter the turbine at 4 mpa, 350°C ($h_3 = 3095 \text{ kJ/kg}$). After expansion in the turbine to 400 kPa ($h_4 = 2609 \text{ kJ/kg}$) the steam is Reheated to 350°C ($h_5 = 3170 \text{ kJ/kg}$) and then expanded in a pressure turbine to (100 kPa) ($h_6 = 2165 \text{ kJ/kg}$) find out the enthalpy at the pump discharge h_2 is. (ii) The efficiency of the plant is

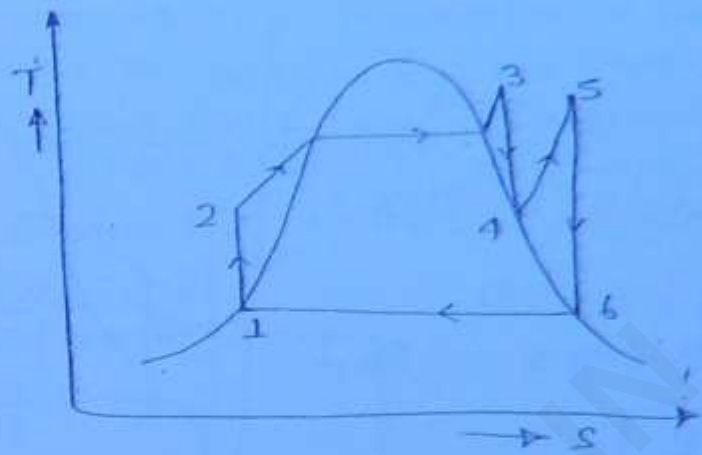
- (A) 19.3
- (B) 16.3
- (C) 22.3
- (D) 33.3

$$\begin{aligned} W_P &= 0 \\ h_2 - h_1 &= 0 \\ h_1 &= h_2 \end{aligned}$$

108



... and output it means Discharge



$$\eta = \frac{W_T - W_P}{Q_S}$$

$$\eta = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

$$\eta = \frac{(3095 - 2609) + (3170 - 2165)}{(3095 - 2931) + (3170 - 2609)}$$

109

$$\eta = 41.08\% \text{ Ans.}$$

prob. (4) steam is working fluid in Rankine cycle with superheating and reheat steam enters the first stage of Turbine at 5 mpa, 480°C. superheated state $h = 3548 \text{ kJ/kg}$, $s = 6.6586 \text{ kJ/kg-K}$ and expand to 0.7 mpa ($s_f = 1.9922 \text{ kJ/kg-K}$, $s_g = 6.768 \text{ kJ/kg-K}$, $h_f = 697.22$, $h_{fg} = 2066.3 \text{ kJ/kg}$) it is then reheated to 440°C before entering the second stage turbine which is in superheat state at 0.7 mpa, 440°C. ($h = 3353.3$, $s = 7.7591$) where it expand to condenser pressure of 0.008 mpa ($s_g = 0.5926$, $s_f = 8.2287$, $h_f = 173.88$, $h_{fg} = 2403.1$) the net power output is 100 mwatt. find the-

- ① efficiency of the cycle
- ② mass flow rate of steam in kg/hr
- ③ take specific volume of saturated liquid at 0.008 mpa is $1.0085 \times 10^{-3} \text{ m}^3/\text{kg}$

L

$$s_2 = s_1 = 6.6586 \text{ kJ/kg} \cdot \text{K}$$

$$= 3353.8 \quad s_3 = 7.7571$$

$$g = 6.708 \text{ KJ/kg K} \cdot 752$$

$$= 2741.88$$

$$= 2428 \cdot 56$$

$$5 = 173 \cdot 588$$

oint 2 in wet Region

$$S_2 = S_f + x_2 \cdot (S_g - S_f)$$

$$6586 = -1.9922 + x_2 (6.708 - 1.9922)$$

$$x_2 = 0.989$$

$$z_2 = 0.989$$

$$z_2 = h_f + x h_{fg} = 697.22 + 0.989 (2066.3)$$

$$h_2 = 2741.88$$

$$S_3 = S_4 = 7.7541 \text{ kJ/kg-K}$$

$\leq S_g (8.2287) \Rightarrow$ Point 4 in wet Region.

$$S_4 = S_f + x_4(S_g - S_f)$$

$$5.7571 = 0.5926 + 24(8.2287 - 0.5926)$$

$$x_4 = 0.438$$

$$I_4 = I_f + x_4 (I_{fg}) \Rightarrow 173588 + 0.938 (2403.1)$$

$$h_4 = 2498.56 \text{ KJ/kg}$$

$$W_p = - \int v dp \Rightarrow - 1.0085 \times 10^{-3} \frac{m^2}{kg} (8000 - 8) \frac{kg}{m^2} \\ \Rightarrow - 8.06 \text{ kJ/kg}$$

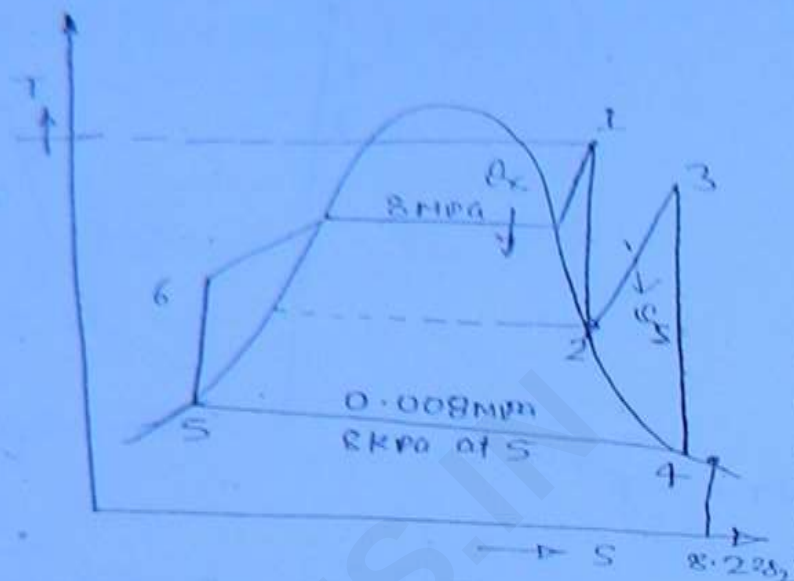
$$\begin{aligned} WP &= h_6 - h_5 = 8.06 \text{ kJ/kg} \\ &= h_6 - 173.28 = 8.06 \end{aligned}$$

$$h_6 = 181.94 \text{ kJ/kg}$$

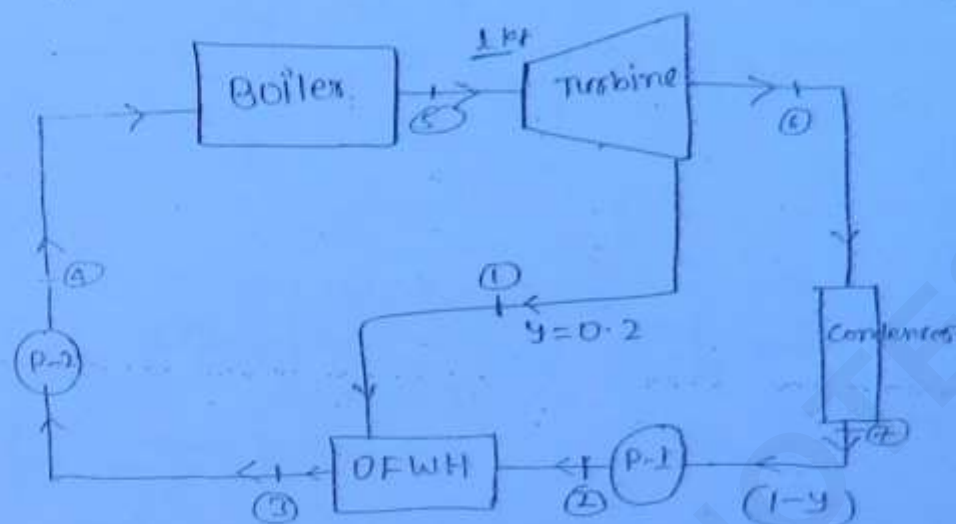
$$\eta = \frac{w_{net}}{Q_s} \Rightarrow \frac{(h_1 - h_2) + (h_3 - h_4) - (w_p)}{(h_1 - h_c) + (h_3 - h_2)}$$

$$\eta = 0.4052 \Rightarrow 40.52\%$$

① $Q_R = h_2 - h_5$ ② $w_{net} = (h_1 - h_2) + (h_3 - h_4) - W_T$



Ques 6: A thermal power plant operates on a regenerative cycle with open feed water heater. As shown in fig. for the state point shown enthalpy's at 1 and 2 are $h_1 = 2800 \text{ kJ/kg}$, $h_2 = 200 \text{ kJ/kg}$. the bleed of feed water heater is 20% of steam generation rate then find the enthalpy at state 3.



$$y \times h_1 + (1-y) h_2 = y \times h_3$$

$$0.2 \times 2800 + (1-0.2) \times 200 = h_3 \times 1$$

$$h_3 = 720 \text{ kJ/kg}$$

PROB 6

A steam Regenerative Rankine cycle at steam entering turbine 200 bar, 650°C superheated state and leaving at 0.05 bar considering the open feed water heater into open type determine the efficiency of the plant where the feed water heater is operating at 8-bar.

At 8 bar $h_f = 721.11$, $h_{fg} = 2048$, $v_f = 0.001115 \text{ m}^3/\text{kg}$

At 200 bar, 650°C $h_i = 3675.3$, $S_i = 6.6582$

At 8 bar $S_f = 2.0462 \text{ kJ/kg}\cdot\text{K}$

$S_g = 4.6468 \text{ kJ/kg}\cdot\text{K}$

At 0.05 bar $h_f = 137.82$

$h_{fg} = 2423.7$

$v_f = 0.001005 \text{ m}^3/\text{kg}$

$S_f = 0.4764$

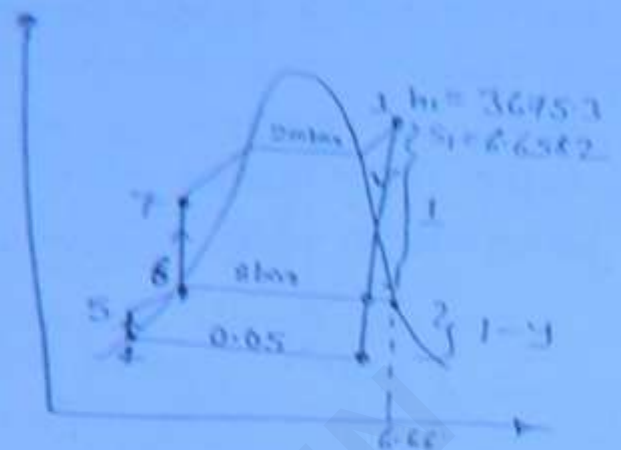
$S_{fg} = 7.9187$

$$\eta = 41.5\%$$

$$y = 0.204$$

$$\begin{aligned}
 Q - Q &= S_{fg} \\
 Q - Q &= 4.6166 \\
 Q &= Q + 4.6166 \\
 &= 20462 + 4.6166 \\
 Q &= 6.6608
 \end{aligned}$$

$$\begin{aligned}
 h_1 &= 2767.07 \text{ KJ/kg} \\
 h_2 &= 2029.9 \text{ KJ/kg} \\
 h_3 &= 137.82 \text{ KJ/kg} \\
 h_4 &= 721.11 \text{ KJ/kg}
 \end{aligned}$$



$$\begin{aligned}
 \text{Pump} \quad \frac{5}{4} \quad 8 \text{ bar} &= 8 \times 100 \text{ kPa} \\
 &= 0.05 \text{ bar} = 0.05 \times 100 \text{ kPa} \\
 Q &= 0.001005
 \end{aligned}$$

when $W_p = \int v dp$ because already negative work

$$\begin{aligned}
 W_p &= 0.001005 \times (8 - 0.05) \times 100 \\
 W_p &= 0.75375 \text{ KJ/kg}
 \end{aligned}$$

$$\begin{aligned}
 W_{p1} &= h_5 - h_4 \\
 0.75375 &= h_5 - 137.82
 \end{aligned}$$

(No)

$$h_5 = 138.62 \text{ KJ/kg}$$

assume after open feed water heater as saturated liquid

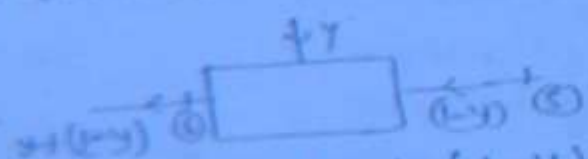
$$h_6 = 721.11 \text{ KJ/kg}$$

$$W_{p2} = -\int v dp \Rightarrow 0.001115 \frac{\text{m}^3}{\text{kg}} \times (200 - 8) \times 100 \text{ kN/m}^2$$

$$W_{p2} = 21.408 \text{ KJ/kg}$$

$$W_{p2} = h_7 - h_6$$

$$21.408 = h_7 - 721.11 \Rightarrow h_7 = 742.518 \text{ KJ/kg}$$



$$\begin{aligned}
 y \times h_2 + (1-y) \times h_5 &= 1 \times h_6 \\
 y &= 0.2216
 \end{aligned}$$

$$W_T = 1 \times (h_1 - h_2) + (1-y) \times (h_2 - h_3)$$

$$W_T = 1481 \text{ KJ/kg}$$

$$W_T = (1-y) W_{T1} + 1 W_{T2} \quad W_p = 22.07 \text{ KJ/kg}$$

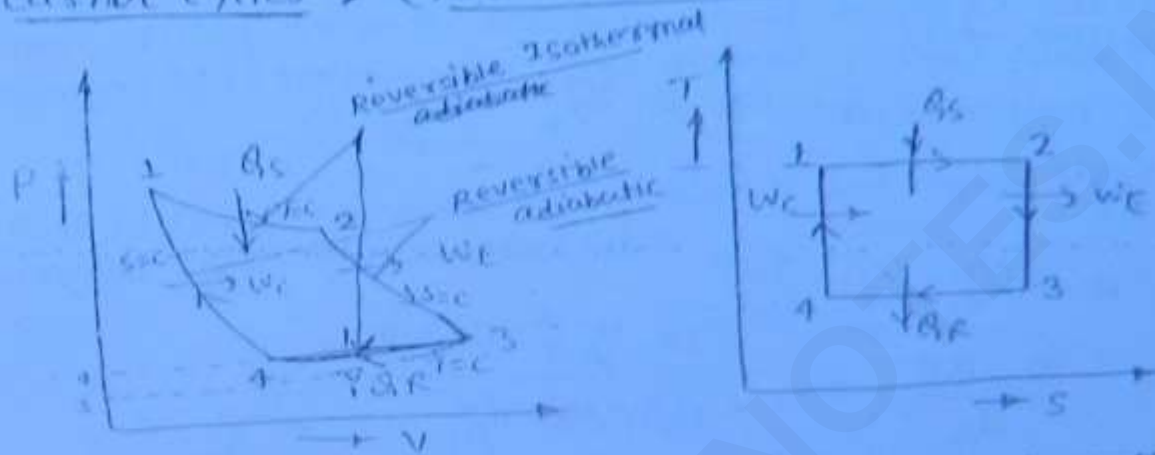
$$Q_s = h_1 - h_2 \Rightarrow 2932.782$$

Ans = 10.7 %

Gas Power Cycles

Gas power cycles are used in gas as a working fluid. It does not undergo any phase change. Engines operating on gas cycles may be either cyclic or non-cyclic. Hot air engine using air as the working fluid operate on a closed cycle. Internal combustion engine where the combustion of fuel takes place inside the engine cylinder or non-cyclic heat engine.

Carnot Cycle $\rightarrow$ (Reversible Cycles) (1824)



1-2 $\rightarrow$ Isothermal heat addition

2-3 $\rightarrow$ Adiabatic expansion

3-4 $\rightarrow$ Isothermal heat rejection

4-1 $\rightarrow$ Adiabatic compression.

(~~compression~~)

(~~expansion~~)

113

This cycle is two reversible isothermal and two reversible adiabatic.

If an ideal gas is assumed as the working fluid. Then for 1 mole of gas

$$Q_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = 0$$

$$Q_{3-4} = RT_2 \ln \frac{V_4}{V_3}$$

$$Q_{4-1} = 0$$

$$W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$W_{2-3} = 0 - C_v (T_3 - T_2)$$

$$W_{3-4} = RT_2 \ln \frac{V_4}{V_3}$$

$$W_{4-1} = -C_v (T_1 - T_4)$$

$$\sum_{\text{cycle}} dQ = \sum_{\text{cycle}} dW$$

$$\text{or } \frac{V_2}{V_3} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$$

$$\text{or } \frac{V_1}{V_4} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_2}{V_3} = \frac{V_1}{V_4} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Therefore

$$Q_S = \text{Heat added} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_R = \text{Heat rejected} = RT_2 \ln \frac{V_3}{V_4}$$

$$W_{\text{net}} = Q_S - Q_R$$

$$= R \ln \frac{V_2}{V_1} \cdot (T_1 - T_2)$$

$$\eta_{\text{cycle}} = \frac{Q_S - Q_R}{Q_S} = \frac{R \ln \frac{V_2}{V_1} (T_1 - T_2)}{RT_1 \ln \frac{V_2}{V_1}}$$

$$\boxed{\eta_{\text{cycle}} = \frac{T_1 - T_2}{T_1}}$$

$$\Rightarrow \boxed{\frac{T_h - T_l}{T_h}}$$

114

In a Carnot gas cycle is big drawback because W_{net} process compressor work is more so more power will consume then less power output.

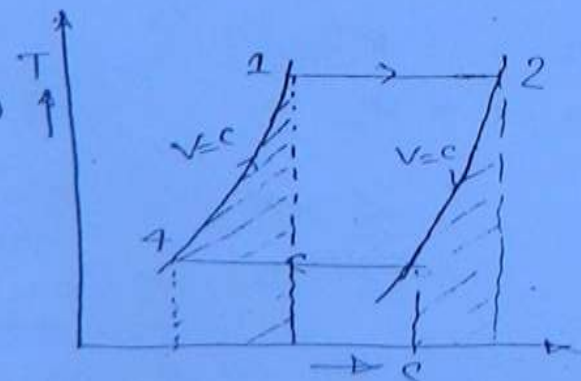
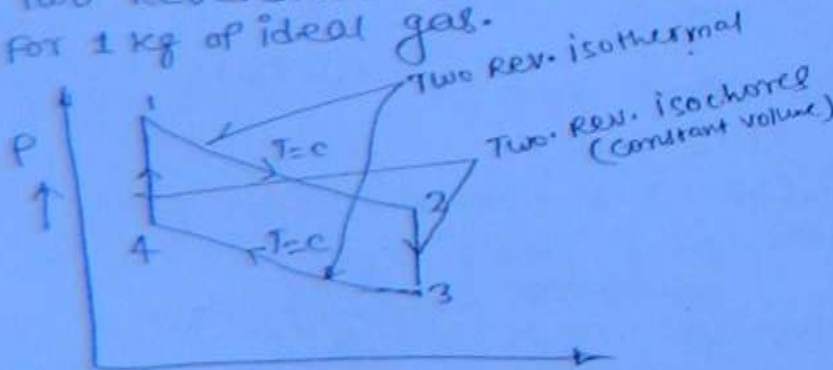
- 1) The efficiency of all the Reversible engine working between same temp. limit is same.
- 2) efficiency of a Reversible cycle depend only on Temp. limit.
- 3) efficiency of a Reversible cycle independent of working fluid.

STIRLING CYCLE (1827) →

(two constant volume process)

- 1) Two Reversible isothermal and two reversible isochores.

For 1 kg of ideal gas.



$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{v_2}{v_1}$$

$$Q_{2-3} = -C_v(T_2 - T_1); W_{2-3} = 0$$

$$Q_{3-4} = W_{3-4} = RT_2 \ln \frac{v_3}{v_4}$$

$$Q_{4-1} = C_v(T_1 - T_2); W_{4-1} = 0$$

Due to heat transfer at constant volume processes, the efficiency of the Stirling cycle is less than that of the Carnot cycle. However if a regenerative arrangement is used such that

$$Q_{2-3} = Q_{4-1}$$

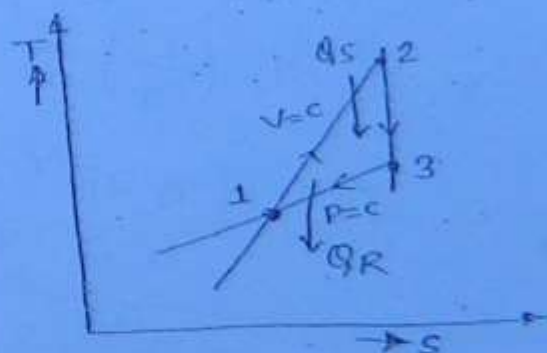
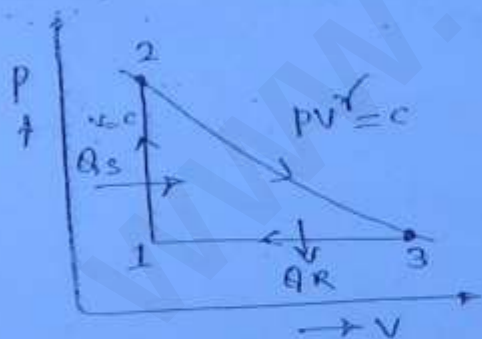
that is the area under 2-3 is equal to the area under 4-1, then the cycle efficiency becomes:

$$\eta = \frac{RT_1 \ln \frac{v_2}{v_1} - RT_2 \ln \frac{v_3}{v_4}}{RT_1 \ln \frac{v_2}{v_1}}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}} \quad \text{Ans.}$$

so the regenerative Stirling cycle has the same efficiency as the Carnot cycle.

⊕ LENOIR CYCLE → This cycle is applicable to pulse jet engine.



- 1-2 → Constant volume heat addition.
- 2-3 → Reversible adiabatic expansion.
- 3-1 → Constant pressure heat rejection.

115

$$Q_1 = C_v (T_2 - T_1)$$

$$Q_2 = C_p (T_3 - T_1)$$

$$\eta_{\text{cycle}} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{C_p (T_3 - T_1)}{C_v (T_2 - T_1)}$$

$$\text{where } \gamma = \frac{C_p}{C_v}$$

$$\eta_{\text{cycle}} = 1 - \frac{\gamma (T_3 - T_1)}{(T_2 - T_1)}$$

using $\gamma_p \rightarrow$ pressure ratio

$$\gamma_p = \frac{P_2}{P_1} \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore T_2 = T_1 \frac{P_2}{P_1}$$

$$T_2 = \gamma_p \cdot T_1$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}}$$

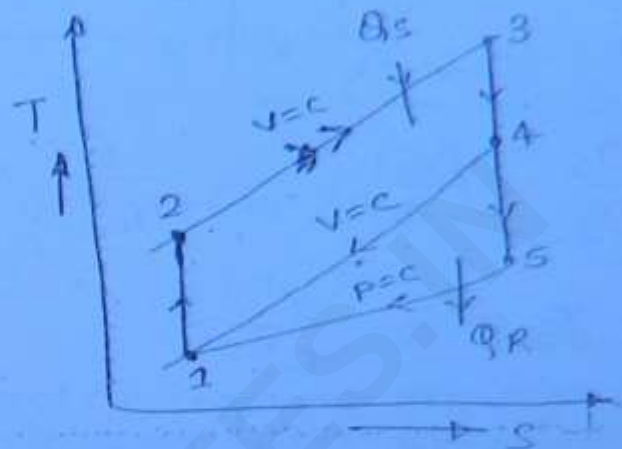
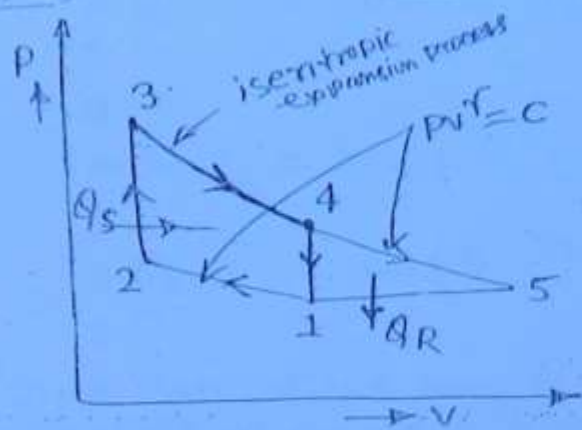
$$\therefore T_3 = T_2 \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_1 \gamma_p \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_{\text{Lenoir}} = 1 - \frac{\gamma [T_1 \gamma_p^{1/\gamma} - T_1]}{T_1 \gamma_p - T_1}$$

$$\eta_{\text{Lenoir}} = 1 - \gamma \left(\frac{\gamma_p^{1/\gamma} - 1}{\gamma_p - 1} \right)$$

Thus the cycle efficiency depends on the pressure ratio and the specific heat ratio.

(1) ATKINSON CYCLE → Atkinson cycle is an ideal cycle for an Otto engine exhausting to a gas turbine. In this cycle the isentropic expansion (3-4) process, of an Otto cycle is allowed to further expand to the lowest cycle pressure (3-5) so as to increase the work output.



1-2-3-4-1 - Otto cycle

1-2-3-5-1 - Atkinson cycle

For 1 kg gas, $Q_s = C_v(T_3 - T_2)$

$Q_R = C_p(T_5 - T_1)$

$$\eta_{\text{cycle}} = 1 - \frac{C_p(T_5 - T_1)}{C_v(T_3 - T_2)} \Rightarrow 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)} \quad \text{--- (a)}$$

Let $\gamma_k \rightarrow$ Compression ratio $= \frac{V_1}{V_2}$

$\gamma_e \rightarrow$ expansion ratio $= \frac{V_5}{V_3}$

$$\frac{T_2}{T_1} = \frac{V_1}{V_2} \quad \therefore T_2 = T_1 \gamma_k^{\gamma-1} \quad \text{--- (b)}$$

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = \frac{P_3}{P_5} \times \frac{P_5}{P_2} = \frac{P_3}{P_5} \times \frac{P_1}{P_2}$$

$$\frac{P_3}{P_5} = \left(\frac{V_5}{V_3}\right)^{\gamma} = \gamma_e^{\gamma}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{1}{\gamma_k^{\gamma}}$$

$$\therefore T_3 = T_2 \cdot r_e^{\gamma} \cdot \frac{1}{r_k^{\gamma}}$$

$$= T_1 \cdot r_k^{\gamma-1} \cdot r_e^{\gamma} \cdot \frac{1}{r_k^{\gamma}}$$

$$\boxed{T_3 = T_1 \cdot \frac{r_e^{\gamma}}{r_k^{\gamma}}} \quad \text{--- (c)}$$

$$\frac{T_5}{T_3} = \left(\frac{V_3}{V_5} \right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$T_5 = T_3 \cdot \frac{1}{r_e^{\gamma-1}}$$

$$= T_1 \cdot \frac{r_e^{\gamma}}{r_k^{\gamma}} \cdot \frac{1}{r_e^{\gamma-1}}$$

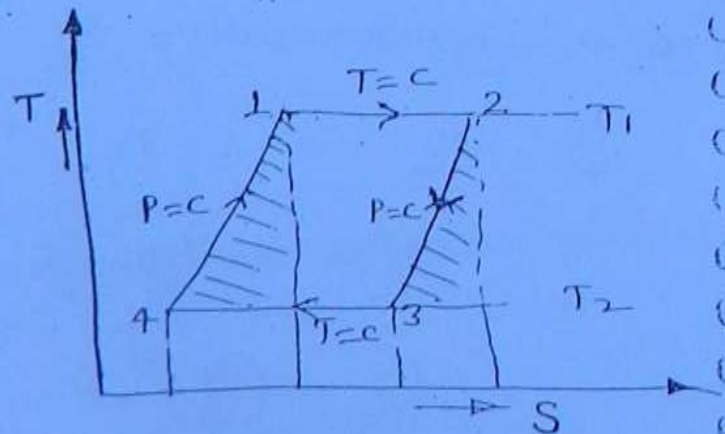
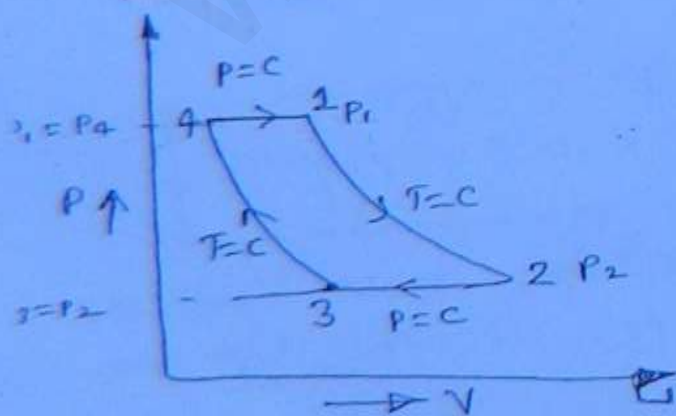
$$\boxed{T_5 = T_1 \cdot \frac{r_e}{r_k}} \quad \text{--- (d)}$$

Substituting T_2, T_3, T_5 from eqn (b), (c), (d)

$$\eta_{\text{Atkinson}} = 1 - \gamma \frac{T_1 \cdot \frac{r_e}{r_k} - T_1}{T_1 \cdot \frac{r_e^{\gamma}}{r_k^{\gamma}} - T_1 r_k^{\gamma-1}} \quad (11.8)$$

$$\boxed{\eta_{\text{Atkinson}} = 1 - \gamma \frac{r_e - r_k}{r_e^{\gamma} - r_k^{\gamma}}}$$

⊕ ERICSSON CYCLE →



Two Reversible isothermal and two Reversible isobars.

For 1 kg of ideal gas.

$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{P_1}{P_2}$$

$$Q_{2-3} = C_p (T_2 - T_1) \quad ; \quad W_{2-3} = P_2 (V_3 - V_2)$$

$$= R (T_2 - T_1)$$

because $T_2 = T_3$

$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{P_1}{P_2}$$

$$Q_{4-1} = C_p (T_1 - T_4) \quad ; \quad W_{4-1} = P_1 (V_1 - V_4)$$

$$= R (T_1 - T_2)$$

$T_4 = T_2$

since part of the heat is transferred at constant pressure and part at constant temp., the efficiency of the Ericsson cycle is less than that of the Carnot cycle.

But with ideal regeneration $Q_{2-3} = Q_{4-1}$ so that all the heat is added from the external source at T_1 and all the heat is rejected to an external sink at T_2 , the efficiency of the cycle becomes equal to the Carnot cycle efficiency.

$$\eta = 1 - \frac{Q_2}{Q_1} \Rightarrow 1 - \frac{RT_2 \ln \frac{P_1}{P_2}}{RT_1 \ln \frac{P_1}{P_2}}$$

(11g)

$$\boxed{\eta \Rightarrow 1 - \frac{T_2}{T_1}}$$

The regenerative, Stirling and Ericsson cycle have the same efficiency as the Carnot cycle, but much less back work. Hot air engine working on these cycle have been successfully operated. But it is difficult to transfer heat to a gas at high rates. Since the gas film has a very low thermal conductivity, so there has not been much progress in the development of hot air engines. However, since the cost of I.C. engine fuel is getting excessive, these may find a field of use in the near future.

Q. In a gas turbine plant air at pressure P_1 and Temp. T_1 is compressed to a pressure of $R P_1$ and then heated to a temp. T_3 at constant pressure. The air is then expanded in a two stage turbine such that pressure ratio is same in both stages. Air is reheated between turbine stage to Temp. T_3 . Assuming that working fluid is perfect gas having ratio of specific heat γ and at that compression and both expansion to be isentropic. show that net output per kg of working fluid will be maximum when R is equal to —

$$R = \left(\frac{T_3}{T_1} \right)^{\frac{2\gamma}{3(\gamma-1)}} = \left(\frac{P_2}{P_1} \right)$$

Soln

$$W_{net} = (h_b - h_d) + (h_3 - h_a) - (h_2 - h_1)$$

$$= C_p [(T_b - T_d) + (T_3 - T_a) - (T_2 - T_1)]$$

$$= C_p \left[T_4 \left(\frac{T_b}{T_4} - 1 \right) + T_3 \left(1 - \frac{T_a}{T_3} \right) - (T_2 - T_1) \right]$$

120

s = T

$$W_{net} = (h_5 - h_6) + (h_3 - h_4) - (h_2 - h_1)$$

$$= C_p (T_5 - T_6) + (T_3 - T_4) - (T_2 - T_1)$$

$$= C_p \left[T_5 \left(1 - \frac{T_6}{T_5} \right) + T_3 \left(1 - \frac{T_4}{T_3} \right) - T_1 \left(\frac{T_2}{T_1} - 1 \right) \right]$$

$$\frac{P_2}{P_1} = \frac{P_i}{P_1} \quad [\text{equal pressure ratio in expansion of both stages}]$$

$$\Rightarrow \frac{R P_1}{P_1} = \frac{P_i}{P_1}$$

$$\Rightarrow P_i^2 = R P_1^2 \Rightarrow \boxed{P_i = \sqrt{R} P_1}$$

$$\frac{T_3}{T_5} = T_5 \quad W_{net} = C_p \left[T_5 \left(1 - \left(\frac{P_1}{P_i} \right)^{\frac{\gamma-1}{\gamma}} \right) + T_3 \left\{ 1 - \left(\frac{P_i}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \right\} - T_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right]$$

Assume $\frac{\gamma-1}{\gamma} = K$

$$W_{net} = C_p \left[T_3 \left\{ 1 - \left(\frac{1}{\sqrt{R}} \right)^K \right\} + T_3 \left\{ 1 - \left(\frac{1}{\sqrt{R}} \right)^K \right\} - T_1 \left\{ R^K - 1 \right\} \right]$$

For max. work

$$\frac{dW_{net}}{dR} = 0$$

$$W_{net} = C_p \left[-2 T_3 \left(-\frac{K}{2} \right) R^{-\frac{K}{2}-1} - T_1 \cdot K R^{K-1} \right]$$

(12)

122

WWW.GATENOTES.IN

123

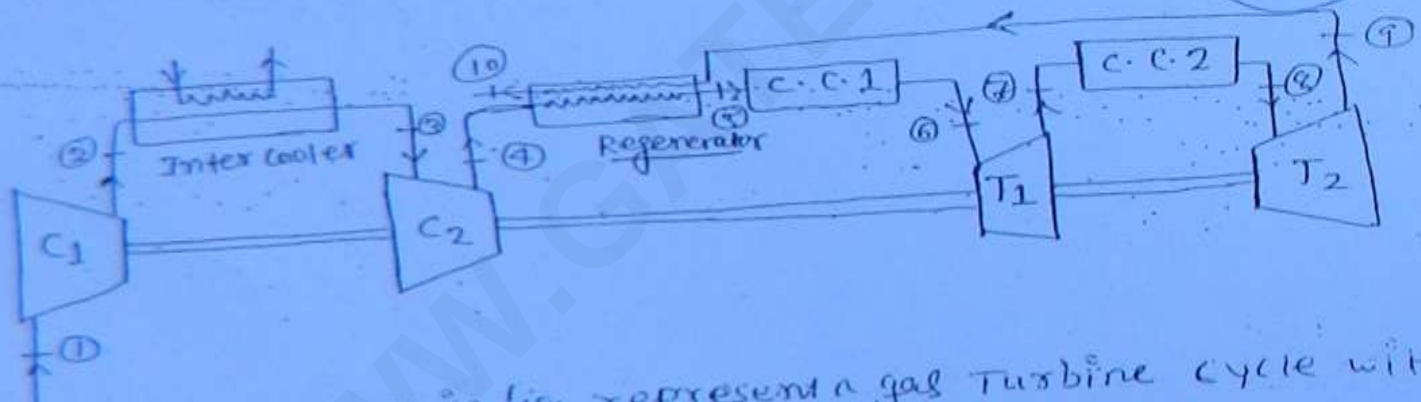
2. A Regenerative and Reheat cycle have air entering at 1 bar, 300K into compressor, having intercooling between 2 stage of compression. Air leaving first stage of compression is cooled to 290K, at 4 bar in intercooler and subsequently compressed to 8 bar. Compressed air leaving 2nd stage compressor is passed through a regenerator or having effectiveness of 0.8. Subsequent combustion field 1300K at inlet to the turbine having expansion up to 4 bar. and has reheated up to 1300K before being expanded up to 1 bar. Exhausted from Turbine is passed through regenerator before discharge out of cycle. For fuel having calorific value of 42000 KJ/kg. find

- 1) Fuel-Air ratio in each combustion chamber.
- 2) Total turbine work.
- 3) Thermal efficiency.

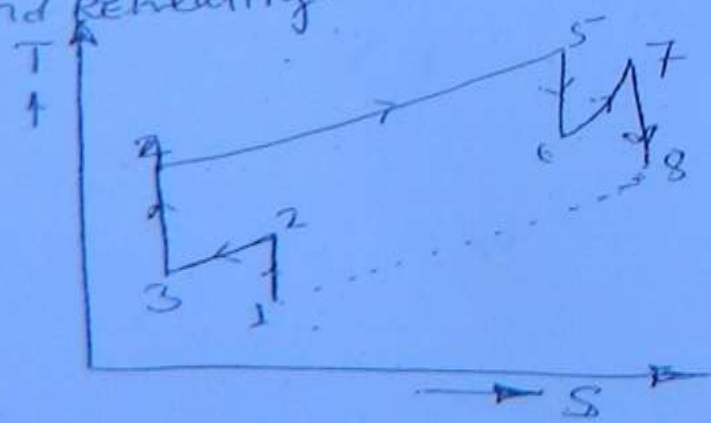
Consider compression and expansion to be and air is working fluid throughout the cycle. $C_{p,air} = 1.005 \text{ KJ/kg-K}$.

At Ready Solved

124



3. The cycle shown in fig. represent a gas turbine cycle with intercooling and reheating.



list I

list II

① Inter cooler



2-3

② combustor



4-5

③ Reheater



6-7

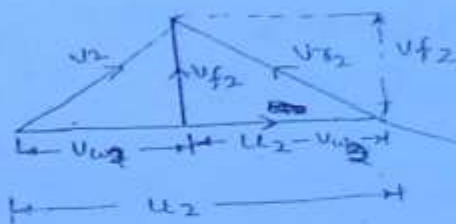
④ High pressure compressor



3-4

125

126



$$v_2^2 = v_{f2}^2 + v_{w2}^2$$

$$\Rightarrow v_{f2}^2 = v_2^2 - v_{w2}^2$$

$$v_{r2}^2 = v_{f2}^2 + (u_2 - v_{w2})^2$$

$$v_{f2}^2 = v_{r2}^2 - (u_2 - v_{w2})^2$$

$$\begin{aligned} \therefore v_2^2 - v_{w2}^2 &= v_{r2}^2 - (u_2 - v_{w2})^2 \\ &= v_{r2}^2 - [u_2^2 + v_{w2}^2 - 2u_2 v_{w2}] \\ &= v_{r2}^2 - u_2^2 - v_{w2}^2 + 2u_2 v_{w2} \end{aligned}$$

$$\Rightarrow v_2^2 - v_{r2}^2 + u_2^2 = 2u_2 v_{w2}$$

$$\Rightarrow v_{w2} u_2 = \frac{1}{2} [v_2^2 - v_{r2}^2 + u_2^2]$$

similarly

$$v_{w1} u_1 = \frac{1}{2} [v_1^2 - v_{r1}^2 + u_1^2]$$

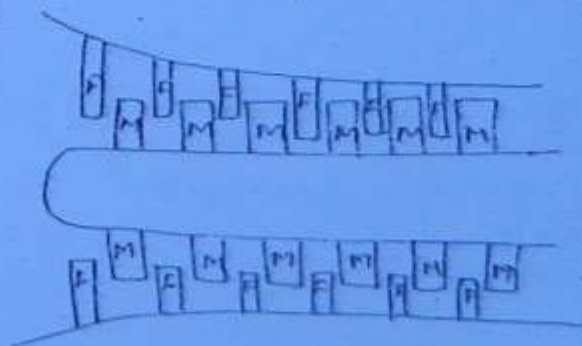
$$\therefore W = v_{w2} u_1 - v_{w1} u_2$$

$$W = \frac{1}{2} [(v_2^2 - v_1^2) + (v_{r1}^2 - v_{r2}^2) + (u_2^2 - u_1^2)]$$

etc. $\Rightarrow$

If the fluid is entering and leaving at the same diameter

$$u_1 = u_2$$



127

centrifugal compressor are not suitable for multi staging but axial flow compressor are suitable for multi staging.

^{centrifugal comp.}
~~supercharging~~ are used in supercharging. Axial flow compressor are used in gas turbine plants. In supercharging.

Gas entry and exit is occurring at the same diameter

$$u_1 = u_2 \quad \therefore W = V_{w2} u_2 - V_{w1} u_1$$

$$W = (V_{w2} - V_{w1}) u_1 \quad [u_1 = u_2 = u]$$

③ To Reduce the axial thrust on Bearings V_{f1} is taken as equal to V_{f2}

$$\text{work/stage} \times W = \frac{1}{2} \left[(V_2^2 - V_1^2) + \frac{1}{2} (V_{r1}^2 - V_{r2}^2) \right]$$

fixed Blade (Absolute Velocity)

$$(V_{w2} - V_{w1}) u = \frac{1}{2} (V_2^2 - V_1^2) + \frac{1}{2} (V_{r1}^2 - V_{r2}^2)$$

$\frac{m+F}{2}$
work/stage

moving Blade
then (Relative velocity)

④ Degree of Reaction (R) $\Rightarrow$

It is the ratio of enthalpy rise in moving blade to the enthalpy rise in stage.

$$R = \frac{(\Delta h)_m}{(\Delta h)_{\text{stage}}}$$

$$R = \frac{(\Delta h)_m}{(\Delta h)_m + (\Delta h)_f}$$

$$R = \frac{\frac{V_{r1}^2 - V_{r2}^2}{2}}{(V_{w2} - V_{w1})U} \Rightarrow \frac{V_{r1}^2 - V_{r2}^2}{2U(V_{w2} - V_{w1})}$$

2.4.3559
2.4

$$R = \frac{V_{r1}^2 - V_{r2}^2}{2U(V_{w2} - V_{w1})}$$

⑧ Construct a velocity Triangle $\Rightarrow$

$$W = (V_{w2} - V_{w1})U$$

$$\tan \beta_1 = \frac{U - V_{w1}}{V_f}$$

$$V_f \tan \beta_1 = U - V_{w1}$$

$$V_{w1} = U - V_f \tan \beta_1$$

$$V_{w2} = U - V_f \tan \beta_2$$

$$W / \text{stage} = [(U - V_f \tan \beta_2) - (U - V_f \tan \beta_1)] U$$

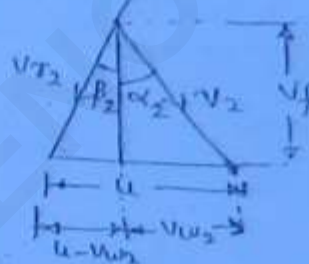
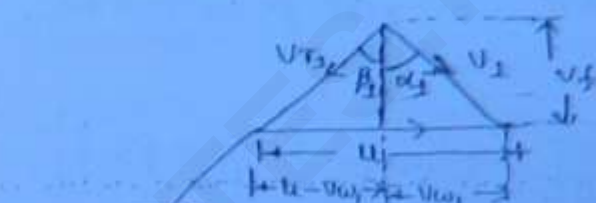
$$W / \text{stage} = [V_f \tan \beta_1 - V_f \tan \beta_2] U$$

$$W / \text{stage} = U V_f [\tan \beta_1 - \tan \beta_2]$$

where β_1 and β_2 are blade angle at inlet and exit.

$$R = \frac{V_{r1}^2 - V_{r2}^2}{2U(V_{w2} - V_{w1})}$$

$\underbrace{2U(V_{w2} - V_{w1})}_{U V_f (\tan \beta_1 - \tan \beta_2)}$



(129)

$$\cos \beta_1 = \frac{V_f}{V_{r1}}$$

$$V_{r1} = \frac{V_f}{\cos \beta_1}$$

$$V_{r1} = V_f \sec \beta_1$$

$$V_{r2} = V_f \sec \beta_2$$

Similarly

$$R = \frac{V_f^2 \sec^2 \beta_1 - V_f^2 \sec^2 \beta_2}{2u V_f (\tan \beta_1 - \tan \beta_2)}$$

$$R = \frac{V_f^2 (\sec^2 \beta_1 - \sec^2 \beta_2)}{2u V_f (\tan \beta_1 - \tan \beta_2)}$$

$$R = \frac{V_f}{2u} \frac{[1 + \tan^2 \beta_1 - 1 - \tan^2 \beta_2]}{\tan \beta_1 - \tan \beta_2}$$

W.G.M.P

$$R = \frac{V_f}{2u} [\tan \beta_1 + \tan \beta_2]$$

Condition for 50% Reaction $\Rightarrow$

$$\tan \beta_1 = \frac{u - V_{w1}}{V_f}$$

$$\tan \alpha_1 = \frac{V_{w1}}{V_f}$$

$$\tan \alpha_1 + \tan \beta_1 = \frac{u - V_{w1}}{V_f} + \frac{V_{w1}}{V_f}$$

$$\tan \alpha_1 + \tan \beta_1 = \frac{u}{V_f}$$

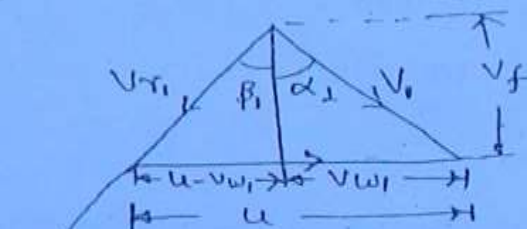
then

$$R = \frac{V_f}{2u} [\tan \beta_1 + \tan \beta_2]$$

$$R = \frac{V_f}{2u} [\tan \beta_1 + \tan \beta_2 + \tan \alpha_1 - \tan \alpha_1]$$

$$R = \frac{V_f}{2u} \left[\frac{u}{V_f} + \tan \beta_2 - \tan \alpha_1 \right]$$

$$R = \frac{1}{2} + \frac{V_f}{2u} (\tan \beta_2 - \tan \alpha_1)$$



130

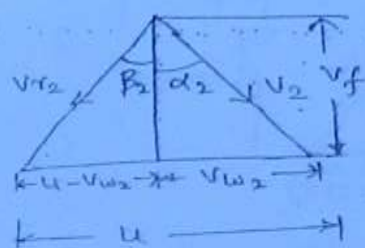
i) $R = 50\% = 0.5 = \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{2} + \frac{V_f}{2u} (\tan \beta_2 - \tan \alpha_1)$$

$$0 = \frac{V_f}{2u} (\tan \beta_2 - \tan \alpha_1)$$

$$\tan \beta_2 = \tan \alpha_1$$

Similarly $\left. \begin{array}{l} \boxed{\beta_2 = \alpha_1} \\ \boxed{\beta_1 = \alpha_2} \end{array} \right\} \text{50\% Reaction.}$



$$\alpha_1 = \beta_2$$

$$\cos \alpha_1 = \cos \beta_2$$

$$\frac{V_f}{V_1} = \frac{V_f}{V_{f2}}$$

v.v. smd $\boxed{V_1 = V_{f2}}$

$$\alpha_2 = \beta_1$$

$$\cos \alpha_2 = \cos \beta_1$$

$$\frac{V_f}{V_2} = \frac{V_f}{V_{f1}}$$

v.v. smd $\boxed{V_2 = V_{f1}}$

(13)

Equation to be Remembered $\Rightarrow$

- ① $u_1 = u_2 = u$
- ② $V_{f1} = V_{f2} = V_f$
- ③ $W/\text{stage} = u V_f (\tan \beta_1 - \tan \beta_2)$
- ④ $R = \frac{V_f}{2u} (\tan \beta_1 + \tan \beta_2)$

⑤ For 50% Reaction

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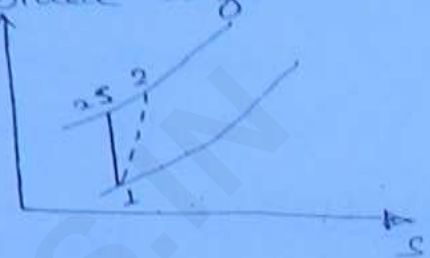
- $$\left\{ \begin{array}{l} \alpha_1 = \beta_2 \\ \beta_2 = \alpha_1 \\ V_1 = V_{f2} \\ V_2 = V_{f1} \end{array} \right.$$

13. A 10 stage Axial flow compressor provides an overall pressure ratio of 5 with an overall isentropic efficiency of 87%. The temp. of air inlet is 15°C . The work is equally divided between all stages. A 50% Reaction is used with a blade speed of 210 m/sec. and axial velocity of flow 170 m/sec. Find the blade angle at inlet and exit. $T_1 = 273 + 15 = 288\text{K}$

$$\eta_{\text{isentropic}} = 87\% = 0.87$$

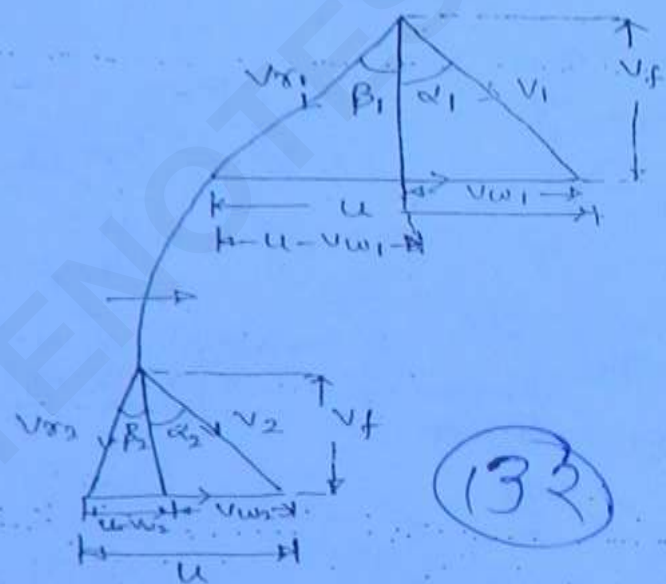
$$u_1 = 210 \text{ m/sec.}$$

$$v_f = 170 \text{ m/sec.}$$



$$W = (V_{w1} - V_{w2}) \cdot u$$

$$= (V_1 \sin \alpha_1 - V_2 \sin \alpha_2) \cdot u$$



$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{288} = (5)^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = 456.1 \text{ K}$$

$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.87 = \frac{456.1 - 288}{T_2 - 288}$$

$$T_2 = 481.26 \text{ K}$$

(133)

$$\begin{aligned}
 W &= h_2 - h_1 \\
 &= C_p (T_2 - T_1) \\
 &= 1.005 (481.26 - 288)
 \end{aligned}$$

$$W = 194.2 \text{ kJ/kg}$$

$$\therefore W/\text{stage} = \frac{194.2}{10} = 19.42 \text{ kJ/kg}$$

$$W/\text{stage} = \frac{U V_f (\tan \beta_1 - \tan \beta_2)}{1000}$$

$$19.42 = 210 \times 170 (\tan \beta_1 - \tan \beta_2)$$

$$\tan \beta_1 - \tan \beta_2 = 0.544 \quad \text{--- (1)}$$

$$R = \frac{V_f}{2U} (\tan \beta_1 + \tan \beta_2)$$

$$\frac{1}{2} = \frac{V_f}{2U} (\tan \beta_1 + \tan \beta_2)$$

$$\text{or } \frac{V_f}{2U} = \tan \beta_1 + \tan \beta_2$$

$$\frac{210}{2 \times 170} = \tan \beta_1 + \tan \beta_2$$

$$\tan \beta_1 + \tan \beta_2 = 1.235 \quad \text{--- (2)}$$

$$\beta_1 = 41.65^\circ$$

$$\beta_2 = 19^\circ$$

Ans:

133

Ex. 2

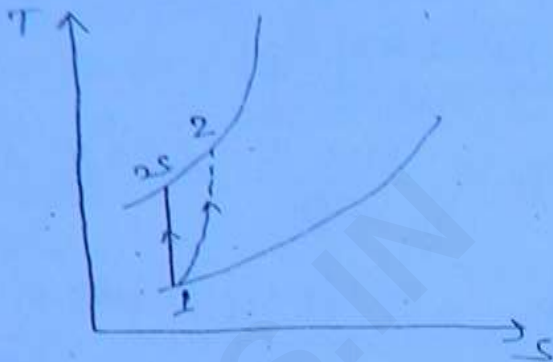
An axial flow air compressor of 50% reaction has blades with inlet and outlet angles of 45° and 10° respectively. Compressor is to produce an overall pressure ratio of 6 with an overall isentropic efficiency of 85%. The inlet temp. is 37°C . The blade speed is 200 m/sec. Then find the number of stages.

$\beta_1 = 45^\circ$, $\beta_2 = 10^\circ$, $\frac{P_2}{P_1} = 6$, $T_1 = 273 + 37$
 $\eta = 85\% \Rightarrow 0.85$, $T_1 = 310 \text{ K}$

$$\frac{T_{2s}}{T_1} = (6)^{\frac{1.4-1}{1.4}}$$

$$\frac{T_{2s}}{310} = (6)^{\frac{0.4}{1.4}}$$

$$T_{2s} = 517 \text{ K}$$



$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.85 = \frac{517 - 310}{T_2 - 310}$$

$$T_2 = 553.8 \text{ K}$$

Total work, $W_T = c_p (T_2 - T_1)$
 $= 1.005 (553.8 - 310)$

$$W_T = 245 \text{ kJ/kg}$$

$$W_{\text{stage}} = u V_f (\tan \beta_1 - \tan \beta_2)$$

$$R = \frac{V_f}{2u} (\tan \beta_1 + \tan \beta_2)$$

$$\frac{1}{2} = \frac{V_f}{2 \times 200} (\tan 45^\circ + \tan 10^\circ)$$

$$V_f = 170 \text{ m/sec}$$

$$W_{\text{stage}} = \frac{200 \times 170 (\tan 45^\circ - \tan 10^\circ)}{1000}$$

$$W_{\text{stage}} = 28.025 \text{ kJ/kg}$$

(39)

$$\text{No. of stage} = \frac{W_T}{W/\text{stage}} \Rightarrow \frac{245}{28.025}$$

$$\boxed{\text{No. of stage} = 8.71 \approx 9 \text{ stage}} \quad \text{Ans.}$$

3. In an axial flow compressor producing a pressure ratio of 6 with air entering at 20°C , the Blade velocity is 200 m/sec . The inlet and exit angle of moving blade are 45° and 10° . The no. of stage equal to 12. For a 50% Reaction find the isentropic efficiency of a compressor. Ans: 58.98%

Soln

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = 488.86 \text{ K}$$

$$u = 200 \text{ m/sec}$$

$$\beta_1 = 45^\circ$$

$$\beta_2 = 10^\circ$$

$$N = 12$$

$$R = \frac{V_f}{2u} (\tan \beta_1 + \tan \beta_2)$$

$$\frac{1}{2} = \frac{V_f}{2 \times 200} [\tan 45^\circ + \tan 10^\circ]$$

$$\frac{1}{2} = \frac{V_f \times 1.176}{400}$$

$$V_f = 170 \text{ m/sec}$$

$$W/\text{stage} = \frac{u V_f (\tan \beta_1 - \tan \beta_2)}{1000} = \frac{200 \times 170 (\tan 45^\circ - \tan 10^\circ)}{1000}$$

$$W/\text{stage} = 28 \text{ kW}$$

$$W_{\text{total}} = 28 \times 12 = 336$$

$$W_{\text{total}} = C_p (T_2 - T_1)$$

$$336 = 1.005 (T_2 - 293)$$

$$\text{Ans: } 58.98\%$$

$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1} \quad (135)$$

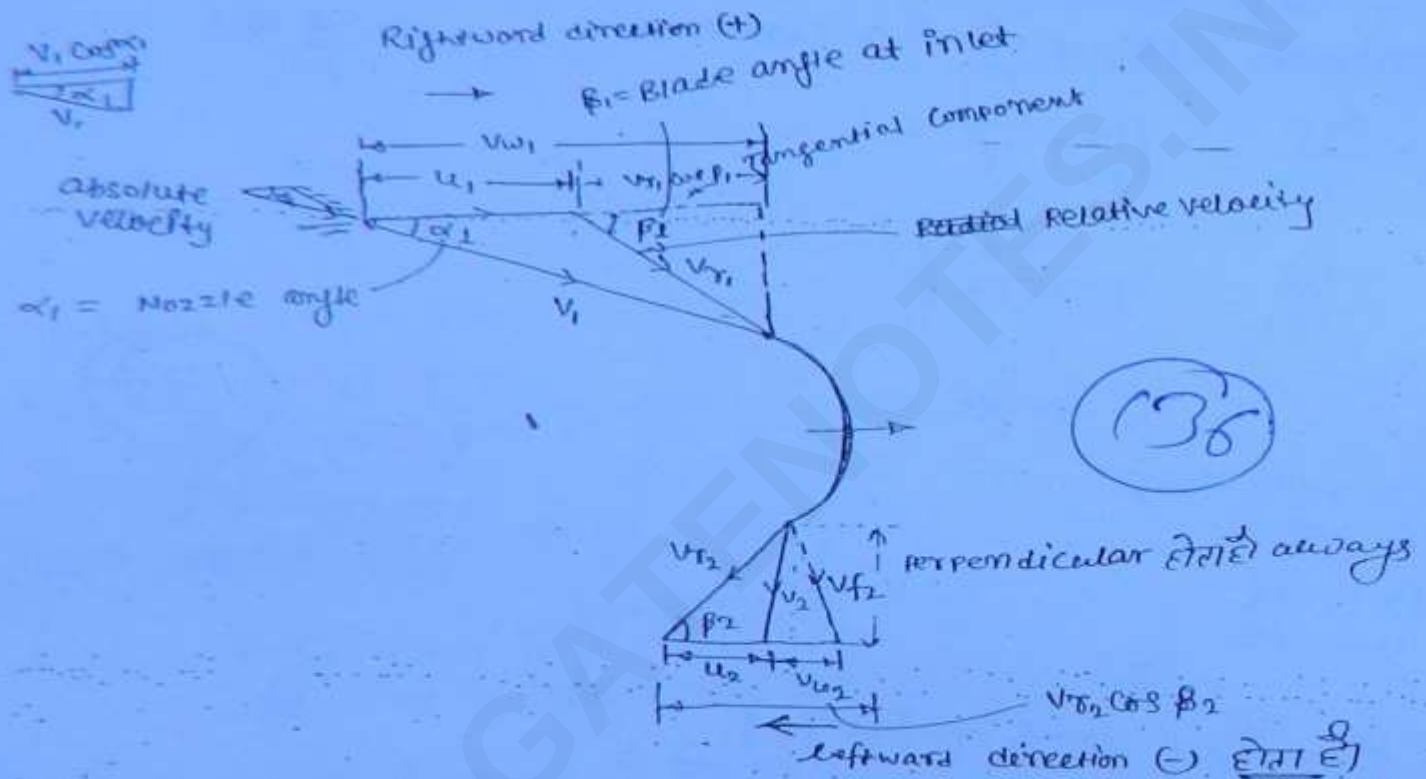
$$= \frac{488.86 - 293}{627.38 - 293}$$

$$\eta = 0.5857\%$$

Ans

1) IMPULSE TURBINE $\Rightarrow$ In impulse turbine the fluid enters the turbine blade with K.E. and while moving in the turbine blade, the static pressure remains constant and kinetic energy is used in developing power.

De Laval is a simple impulse turbine.



$$W = Vw_1 u_1 - Vw_2 u_2$$

$$W = (Vw_1 - Vw_2) u$$

$$\therefore [u_1 = u_2 = u]$$

$$W = [Vw_1 - (-Vw_2)] u$$

$$W = (Vw_1 + Vw_2) u$$

$$\eta_d = \frac{W}{\frac{1}{2} \dot{m} V_1^2} = \frac{\dot{m} (Vw_1 + Vw_2) u}{\frac{1}{2} \dot{m} V_1^2}$$

$\eta_d \rightarrow$ diagram efficiency
 or
 blade efficiency.

$$\boxed{\eta_d = \frac{2 (Vw_1 + Vw_2) u}{V_1^2}}$$

④ Condition for max. Efficiency $\Rightarrow$ (Diagram or Blade efficiency)

① The Blade are assumed to be symmetrical i.e. $\beta_1 = \beta_2$

② The flow is assumed to be frictionless i.e. $v_{r1} = v_{r2}$

$$v_{r1} = v_{r2}$$

Blade velocity coefficient $k = \frac{v_{r2}}{v_{r1}}$

$$v_{w1} = v_1 \cos \alpha_1$$

$$v_{w2} = v_{r2} \cos \beta_2 - u$$

$$v_{w2} = v_{r1} \cos \beta_1 - u$$

$$v_{w2} = (v_{w1} - u) - u$$

$$v_{w2} = v_{w1} - 2u$$

$$\Rightarrow v_{w2} = v_1 \cos \alpha_1 - 2u$$

$$\therefore \eta_d = \frac{2 [v_1 \cos \alpha_1 + v_1 \cos \alpha_1 - 2u] u}{v_1^2}$$

$$\eta_d = \frac{2 [2v_1 \cos \alpha_1 - 2u] u}{v_1^2}$$

$$\eta_d = \frac{4 [v_1 u \cos \alpha_1 - u^2]}{v_1^2}$$

$$\eta_d = 4 \left[\frac{u}{v_1} \cos \alpha_1 - \frac{u^2}{v_1^2} \right]$$

Let $\frac{u_0}{v_1} = p = \text{Blade speed ratio}$

$$\eta_d = 4 [p \cos \alpha_1 - p^2]$$

For max. $\eta_d = \frac{d\eta_d}{dp} = 0$

$$= 4 [\cos \alpha_1 - 2p] = 0$$

$$= \cos \alpha_1 = 2p$$

$$\Rightarrow \left[p = \frac{\cos \alpha_1}{2} \right] \text{ Blade speed ratio for max. efficiency}$$

$$p = \text{Blade speed} = \frac{u_0}{v_1}$$

$$p = \frac{u_0}{v_1} = \frac{\cos \alpha_1}{2}$$

$$u_0 = 0.5 v_1 \cos \alpha_1$$

Blade speed

Efficiency

137

$$\eta_{\max} = 4 \left[\frac{\cos^2 \alpha_1}{2} - \frac{\cos^2 \alpha_1}{4} \right]$$

u.v. grad

$$\eta_{\max} = \cos^2 \alpha_1$$

where $\alpha_1 \rightarrow$ Nozzle angle

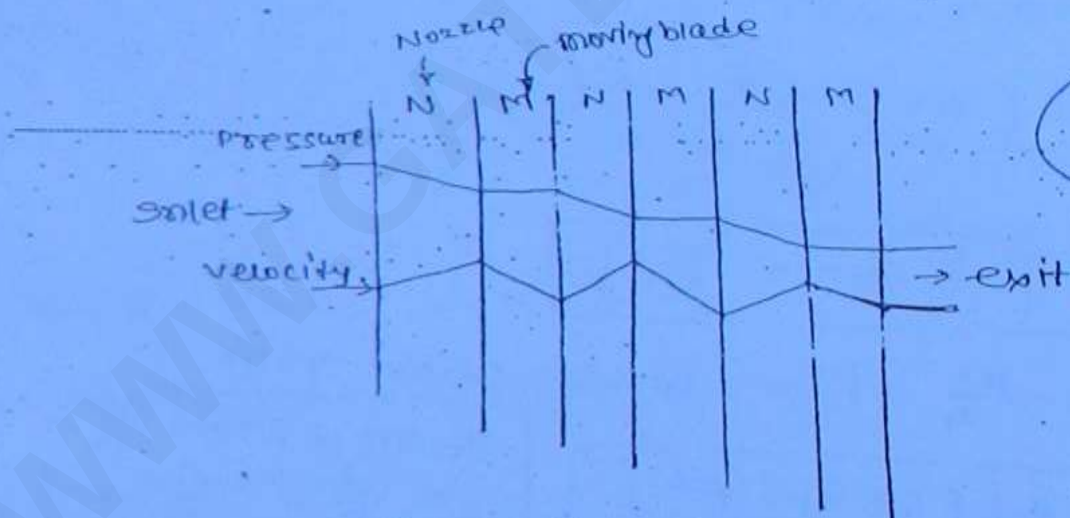
$\Rightarrow$ max. efficiency of an impulse Turbine.

This equation is valid when the blade are symmetrical and frictionless

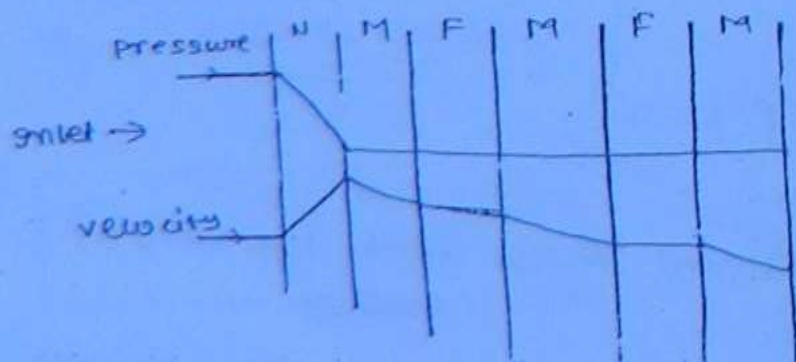
3) Compounding in impulse Turbine $\Rightarrow$

In steam Turbine the steam enters the turbine at a pressure of about 170 bar and leaves at a condenser pressure of about 0.035 bar thus the pressure ratio ^{are} about 1800. If steam is allowed to expand in a single stage the Rotor speed are very high (about 20,000 rpm). Therefore the speeds must be limited and to Reduce the Rotor speed compounding is used.

1. Pressure Compounding $\Rightarrow$ Pressure compounding to example
Ex = (Rateau Turbine)



2. velocity Compounding $\Rightarrow$ (Curtis Turbine) $\leftarrow$ Example



In velocity compounding velocity in fixed blade slightly decreases.

Driving force on wheel

$$F = \frac{m(v-u)}{t}$$

$$F = \dot{m}(v-u)$$

$$F = \dot{m}[v_{w1} - (-v_{w2})]$$

$$F = \dot{m}[v_{w1} + v_{w2}] \leftarrow \text{Driving force}$$

$$P = F \times u$$

$$P = \dot{m}[v_{w1} + v_{w2}] u \leftarrow \text{Power}$$

$$\text{Axial thrust} = \dot{m}[v_{f1} - v_{f2}]$$

$$K = \frac{v_{r2}}{v_{r1}} \Rightarrow \text{Blade velocity coefficient}$$

Prob. 1

The velocity of steam, leaving the nozzle of an impulse turbine is 900 m/sec, and nozzle angle is 20° . The blade velocity is 300 m/sec (u) and blade velocity coefficient is 0.7. Calculate for a mass flow rate of 1 kg/sec and symmetric blading.

- ① Blade angle at inlet
- ② Driving force on wheel
- ③ Axial thrust
- ④ Power developed
- ⑤ Diagram efficiency.

139

Soln.

Given

$$v_1 = 900 \text{ m/sec}$$

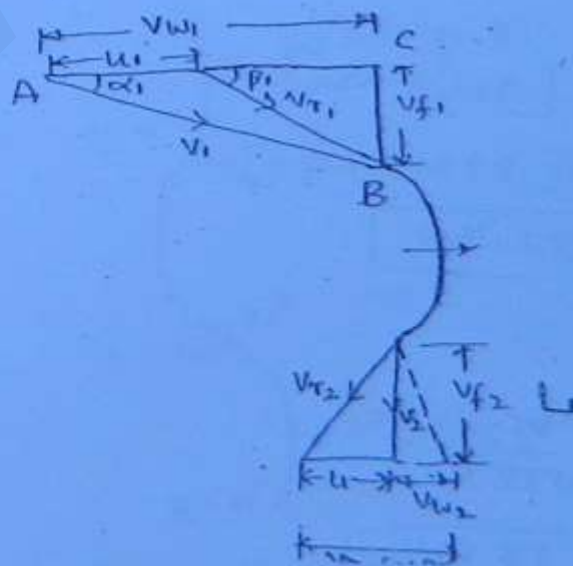
$$\alpha_1 = 20^\circ$$

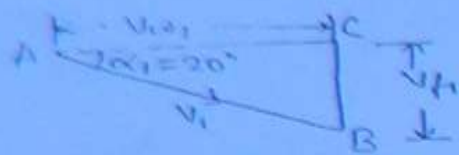
$$u = 300 \text{ m/sec}$$

$$K = 0.7$$

$$\dot{m} = 1 \text{ kg/s}$$

$$\beta_1 = \beta_2$$





$$V_{w1} = V_1 \cos \alpha_1$$

$$V_{w1} = 900 \cos 20^\circ$$

$$V_{w1} = 845.7 \text{ m/sec}$$

$$V_{f1} = V_1 \sin \alpha_1$$

$$= 900 \sin 20^\circ$$

$$V_{f1} = 307.8 \text{ m/sec}$$

$$\tan \beta_1 = \frac{V_{f1}}{V_{w1} - u}$$

$$\tan \beta_1 = \frac{307.8}{845.7 - 300} \Rightarrow$$

$$\boxed{\beta_1 = 29.4^\circ = \beta_2}$$

$$\sin \beta_1 = \frac{V_{f1}}{V_{r1}}$$

$$\sin 29.4^\circ = \frac{307.8}{V_{r1}}$$

$$\boxed{V_{r1} = 626.5 \text{ m/sec}}$$

$$k = \frac{V_{r2}}{V_{r1}} \Rightarrow 0.7 = \frac{V_{r2}}{626.5}$$

$$\boxed{V_{r2} = 438.5 \text{ m/sec}}$$

140

$$V_{w2} = V_{r2} \cos \beta_2 - u$$

$$= 438.5 \cos 29.4^\circ - 300$$

$$\boxed{V_{w2} = 82 \text{ m/sec}}$$

$$\text{Driving force (F)} = \dot{m} [V_{w1} + V_{w2}]$$

$$= 1 \times [845.7 + 82]$$

$$\boxed{F = 927.7 \text{ N}}$$

$$\sin \beta_2 = \frac{V_{f2}}{V_{r2}}$$

$$\sin 29.4 = \frac{V_{f2}}{438.5}$$

$$\boxed{V_{f2} = 215.47 \text{ m/sec}}$$

$$\begin{aligned} \text{Thrust} &= m (V_{f1} - V_{f2}) \\ &= 1 (307.8 - 215.26) \\ &= 92.34 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Power developed (P)} &= m (V_{w1} + V_{w2}) u \\ &= 1 \times (845.7 + 82) \times 4300 \\ &= 378.3 \text{ kW} \end{aligned}$$

$\frac{1.8}{3} \times \frac{30}{3} \times \frac{10}{3}$
 $\text{m/s} \times \text{m/s} \times \text{m/s}$

$$\begin{aligned} \eta_f &= \frac{2 (V_{w1} + V_{w2}) u}{V_1^2} \Rightarrow 0.687 \quad \text{Ans:} \\ &= \frac{2 (845.7 + 82) \times 300}{300^2} \Rightarrow 0.687 \end{aligned}$$

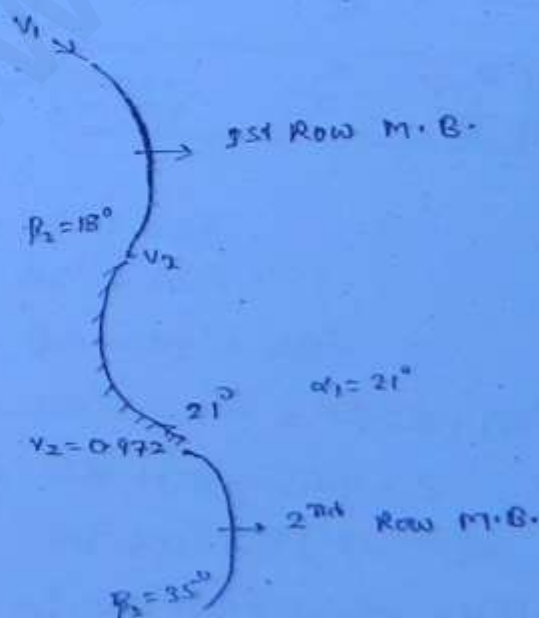
*** Example ***

PROB. 2 The first stage of a turbine is a two Row velocity compounded impulse turbine. The velocity of steam at inlet is 600 m/sec. The blade speed is 120 m/sec. The blade velocity coefficient is 0.9 for all blades. The nozzle angle is 16° . The exit angles for first Row of moving blades, fixed blades and second Row of moving blade are 18° , 21° and 35° respectively. Calculate (1) Blade inlet angle for each Row (2) Driving force on each Row of moving Blade for a mass flow rate of 1 kg/sec.

(14)

Ans: 1st Row
 $\beta_1 = 19.5^\circ$
2nd Row
 $\beta_1 = 34.38^\circ$
Driving force on 1st
 $F = 872.63 \text{ N}$
2nd Row
 $F = 290.27 \text{ N}$

Sol:
 $u = 120 \text{ m/sec}$
 $\alpha_1 = 16^\circ$
 $k = 0.9$
 $V_1 = 600 \text{ m/sec}$



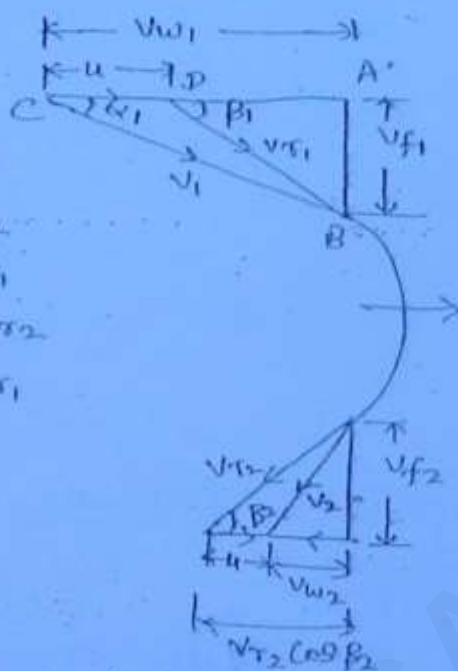
142

Reaction Turbine (R)

In case of Reaction Turbine the fluid enters with both kinetic and pressure energy therefore input energy is equal to

$$\text{Input energy/mass} = \underbrace{\frac{V_1^2}{2}}_{\text{K.E.}} + \underbrace{\frac{V_{r2}^2 - V_{r1}^2}{2}}_{\text{Pressure energy}}$$

Max. efficiency of Parson's Reaction Turbine $\Rightarrow$ 50%



$$\begin{aligned}\alpha_1 &= \beta_2 \\ \alpha_2 &= \beta_1 \\ V_1 &= V_{r2} \\ V_2 &= V_{r1}\end{aligned}$$

$$W = V_{w1} u_1 - V_{w2} u_2$$

$$W = (V_{w1} + V_{w2}) u$$

$$\eta_d = \frac{\text{Output}}{\text{Input}}$$

$$\eta_d = \frac{(V_{w1} + V_{w2}) u}{\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}}$$

Let $u = V_1 \cos \alpha_1$

$$V_{w1} = V_1 \cos \alpha_1$$

$$V_{w2} = V_{r2} \cos \beta_2 - u$$

$$V_{w2} = V_1 \cos \alpha_1 - u$$

$$\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2} \Rightarrow \frac{V_1^2}{2} + \frac{V_1^2 - V_{r1}^2}{2}$$

Input For Reaction Turbine

$$\text{Input energy} = \frac{V_1^2}{2} - \frac{V_{r1}^2}{2}$$

From $\triangle ABD$

$$BD^2 = AD^2 + AB^2$$

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = (AD + CD)^2 + AB^2$$

$$BC^2 = AD^2 + CD^2 + 2AD \cdot CD + AB^2$$

$$BC^2 = BD^2 + CD^2 + 2AD \cdot CD$$

$$V_1^2 = V_{r1}^2 + u^2 + 2(V_{r1} - u)u$$

$$V_1^2 = V_{r1}^2 + u^2 + 2[V_1 \cos \alpha_1 - u]u$$

143

$$V_1^2 = V_{r1}^2 + u^2 + 2uV_1 \cos \alpha - 2u^2$$

$$V_1^2 = V_{r1}^2 - u^2 + 2uV_1 \cos \alpha_1$$

$$\boxed{V_{r1}^2 = V_1^2 + u^2 - 2uV_1 \cos \alpha_1}$$

$$\eta = \frac{(V_{w1} + V_{w2})u}{V_1^2 - \frac{V_{r1}^2}{2}}$$

$$\eta = \frac{(V_1 \cos \alpha_1 + V_1 \cos \alpha_1 - u)u}{V_1^2 - \frac{(V_1^2 + u^2 - 2uV_1 \cos \alpha_1)}{2}}$$

$$\eta = \frac{(2V_1 \cos \alpha_1 - u)u}{\frac{V_1^2}{2} - \frac{u^2}{2} + uV_1 \cos \alpha_1}$$

$$\eta = \frac{(2uV_1 \cos \alpha_1 - u^2)2}{V_1^2 - u^2 + 2uV_1 \cos \alpha_1}$$

$$\boxed{P = \frac{u}{V_1}} \rightarrow \text{Blade speed ratio}$$

$$\eta = \frac{2[2uV_1 \cos \alpha_1 - u^2]}{V_1^2 - u^2 + 2uV_1 \cos \alpha_1}$$

194

$$\boxed{\eta = \frac{2[2P \cos \alpha_1 - P^2]}{1 - P^2 + 2P \cos \alpha_1}}$$

$$\eta = \frac{2[1 + 2P \cos \alpha_1 - P^2 - 1]}{1 - P^2 + 2P \cos \alpha_1}$$

$$\eta = 2 \left[1 - \frac{1}{1 - P^2 + 2P \cos \alpha_1} \right]$$

for max. efficiency η , $\frac{d\eta}{d\beta} = 0$

$$\frac{d\eta}{d\beta} = 2 \left[0 - \frac{(-1)}{1 - \beta^2 + 2\beta \cos \alpha_1} 2[0 - 2\beta + 2 \cos \alpha_1] = 0 \right]$$

$$= -2\beta + 2 \cos \alpha_1 = 0$$

$$\boxed{\beta = \cos \alpha_1}$$

$$\eta_{\max} = \frac{2 [2 \cos \alpha_1 \cdot \cos \alpha_1 - \cos^2 \alpha_1]}{1 - \cos^2 \alpha_1 + 2 \times \cos \alpha_1 \times \cos \alpha_1}$$

$$\boxed{\eta_{\max} = \frac{2 [\cos^2 \alpha_1]}{1 + \cos^2 \alpha_1}}$$

145

Impulse	Reaction
$\beta_1 = \beta_2$	50% Reaction
$V_{r1} = V_{r2}$	$\alpha_1 = \beta_2, \alpha_2 = \beta_1$
$\beta = \frac{\cos \alpha_1}{2}$	$V_1 = V_{r2}, V_2 = V_{r1}$
$\eta_{\max} = \cos^2 \alpha_1$	$\beta = \cos \alpha_1 = \frac{u}{V_1} \rightarrow \text{Blade speed ratio}$
	$\eta_{\max} = \frac{2 \cos^2 \alpha_1}{1 + \cos^2 \alpha_1}$

PROBLEM Note $\Rightarrow$

$$W = V_{w1} u_1 - V_{w2} u_2 = \frac{V_1^2 - V_2^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}$$

① IN Reaction Turbine enthalpy drop in moving blade is equal to $\Rightarrow \frac{V_{r2}^2 - V_{r1}^2}{2}$

$$\textcircled{2} \quad R = \frac{1}{2} = \frac{(\Delta h)_m}{(\Delta h)_m + (\Delta h)_f}$$

$$(\Delta h)_m + \Delta h_f = 2 \Delta h_m$$

$$\Delta h_f = 2 \Delta h_m - \Delta h_m$$

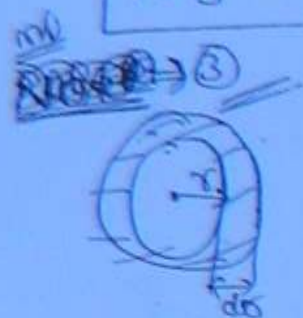
$$\Delta h_f = \Delta h_m$$

$$\Delta h_s = \Delta h_m + \Delta h_f$$

$$\Delta h_s = \Delta h_m + \Delta h_m$$

$$\Delta h_s = 2 \Delta h_m \Rightarrow 2 \left(\frac{v_{r2}^2 - v_{r1}^2}{2} \right)$$

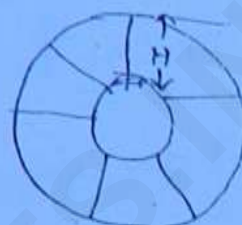
$$\Delta h_s = 2 \Delta h_m = v_{r2}^2 - v_{r1}^2$$



$$2\pi r dr$$

$$\pi D \cdot dr$$

where $H \rightarrow$ Height of Blade.



Area of flow

$$A = \pi D H$$

$$\dot{m} = \rho A V$$

$$\dot{m} = \rho \times \pi D H \times V_f$$

$$\dot{m} = \frac{1}{v} \cdot \pi D H V_f$$

(146)

Because $\rho = \frac{1}{v}$

ROB-1 A stage of a steam turbine with Parson's Blading delivers a dry saturated steam at 2.71 bar from fixed blades at 90 m/sec (v_1). The Blade height is 40 mm and the moving blade exit angle is $\beta_2 = 20^\circ$. The flow component of velocity is $\frac{3}{4}$ th Blade velocity, ($V_f = \frac{3}{4} u$). steam is supplied at the rate of 9000 kg/hr. The effect of Blade thickness can be neglected.

Calculate (1) Rotational speed (N).

(2) Power developed.

(3) efficiency.

(4) enthalpy drop of steam in this stage.

Take specific volume of steam at 2.7 bar dry saturated = $0.6686 \text{ m}^3/\text{kg}$.

$$V_1 = 90 \text{ m/sec}$$

$\beta_2 \rightarrow$ Blade exit angle $= \alpha_1$

$$\beta_2 = 20^\circ = \alpha_1$$

$$\sin \alpha_1 = \frac{V_{f1}}{V_1}$$

$$V_{f1} = V_1 \sin \alpha_1$$

$$V_{f1} = 90 \sin 20^\circ$$

$$V_{f1} = 30.78 \text{ m/sec}$$

$$V_f = \frac{3}{4} u$$

$$30.78 = \frac{3}{4} u$$

$$u = 41.04 \text{ m/sec}$$

$$u = \frac{\pi D N}{60}$$

$$\omega = \frac{\pi D H V_f}{V}$$

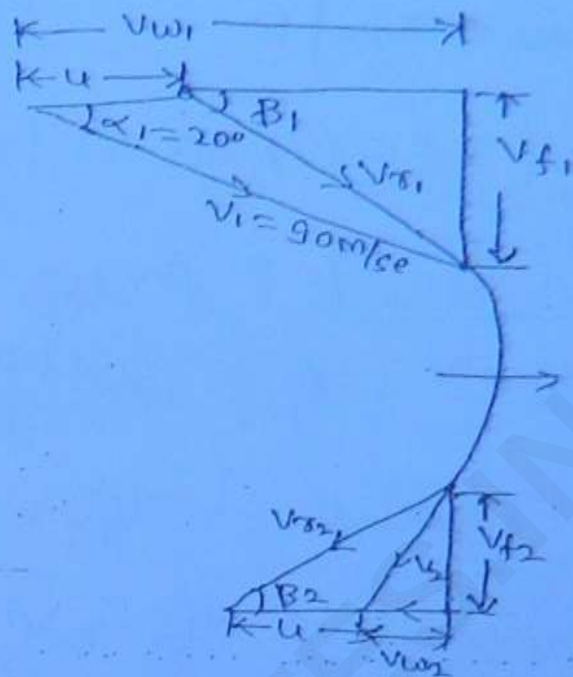
$$\frac{9000}{3600} = \frac{\pi D \times 40 \times 10^{-3} \times 30.78}{0.6686}$$

$$D = 0.432 \text{ m}$$

$$u = \frac{\pi D N}{60}$$

$$41.04 = \frac{\pi \times 0.432 \times N}{60}$$

$$N = 1813 \text{ rpm}$$



147

② power developed $\Rightarrow V_{w1} = V_1 \cos \alpha_1$
 $= 90 \cos 20^\circ$
 $V_{w1} = 84.57 \text{ m/sec}$

$$V_{w2} = V_{r2} \cos \beta_2$$

$$V_{w2} = (V_{r2} \cos \beta_2 - u)$$

$$V_{w2} = V_1 \cos \alpha_1 - u$$

$$= V_{w1} - u$$

$$= 84.57 - 41.04$$

$$P = \dot{m} [V_{w1} + V_{w2}] u$$

$$\frac{\text{kg}}{\text{s}} \times \frac{\text{m}}{\text{s}} \times \frac{\text{m}}{\text{s}}$$

$$P = \frac{9000}{3600} [84.57 + 43.57] \times 41.04$$

$$P = 13.14 \text{ kW}$$

Ans.

③ efficiency $\Rightarrow \eta = \frac{\text{output}}{\text{input}}$

$$g_{\text{input}} = \frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}$$

$$\frac{V_1^2}{2} + \frac{V_1^2}{2} - \frac{V_{r1}^2}{2}$$

$$g_{\text{input}} = V_1^2 - \frac{V_{r1}^2}{2}$$

(148)

$$g_{\text{input}} = \dot{m} \left[V_1^2 - \frac{V_{r1}^2}{2} \right]$$

$$V_{r1}^2 = u^2 + V_1^2 - 2uV_1 \cos \alpha_1$$

$$V_{r1}^2 = 41.04^2 + 90^2 - 2 \times 41.04 \times 90 \cdot \cos 20^\circ$$

$$V_{r1} = 53.3 \text{ m/sec}$$

$$g_{\text{input}} = \frac{9000 \text{ kg}}{3600 \text{ sec}} \left[90^2 - \frac{53.3^2}{2} \right] \frac{\text{kg}}{\text{s}} \times \frac{\text{m}^2}{\text{s}^2}$$

$$g_{\text{input}} = 16.69 \text{ kW}$$

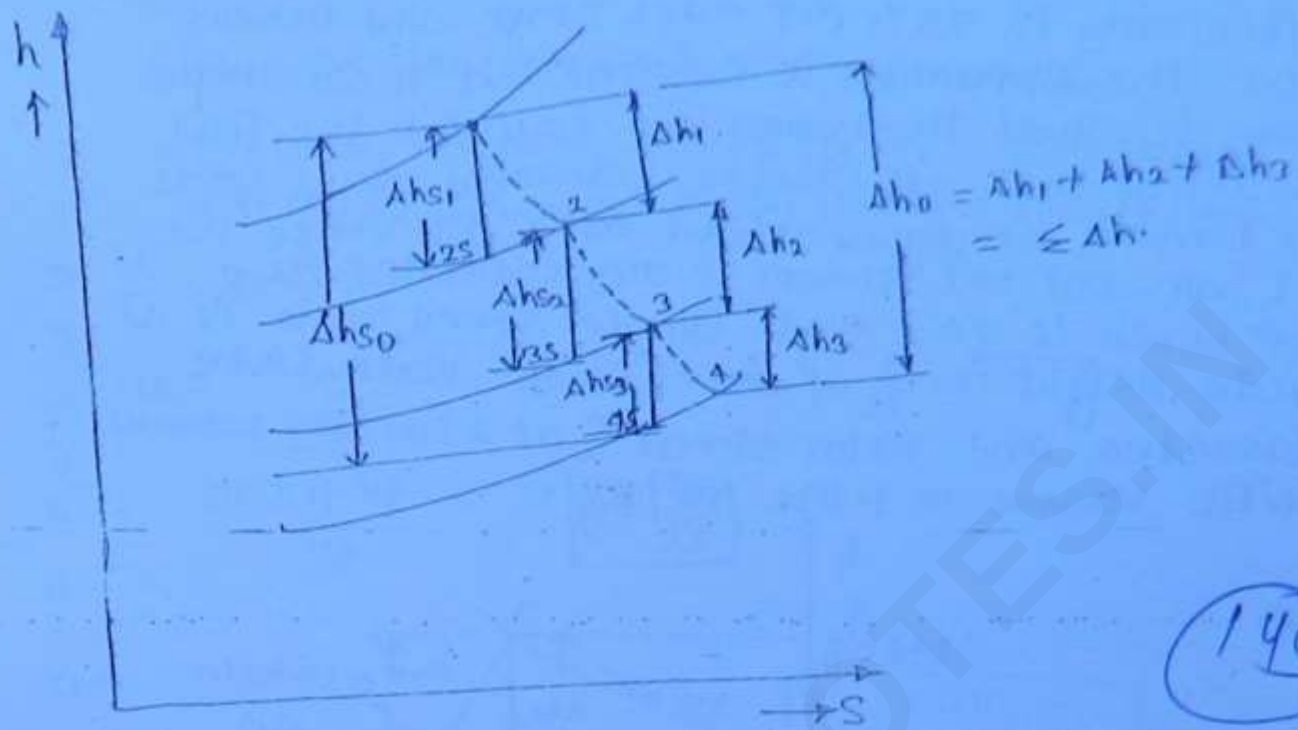
$$\eta = \frac{13.14}{16.69} = 0.786 \Rightarrow 78.6\%$$

④ $= V_{r2}^2 - V_{r1}^2$
 $= V_1^2 - 53.3^2$
 $= 90^2 - 53.3^2$

$$h) = 5259 \text{ J/kg}$$

$$(h)_{\text{eff}} = 5.259 \text{ kJ/kg}$$

REHEAT FACTOR $\Rightarrow$ (RF)



140

$$\eta_{s1} = \frac{\Delta h_1}{\Delta h_{s1}}$$

$$\eta_{s1} = \eta_{s2} = \eta_{s3} = \eta_s$$

$$\Delta h_1 = \eta_{s1} \Delta h_{s1}$$

$$\Delta h_2 = \eta_{s2} \Delta h_{s2}$$

$$\Delta h_3 = \eta_{s3} \Delta h_{s3}$$

$$\Delta h_1 + \Delta h_2 + \Delta h_3 = \eta_{s1} \Delta h_{s1} + \eta_{s2} \Delta h_{s2} + \eta_{s3} \Delta h_{s3}$$

$$\Delta h_0 = \eta_s [\Delta h_{s1} + \Delta h_{s2} + \Delta h_{s3}]$$

$$\frac{\Delta h_0}{\Delta h_{s0}} = \frac{\eta_s [\Delta h_{s1} + \Delta h_{s2} + \Delta h_{s3}]}{\Delta h_{s0}}$$

$$\eta_o = \eta_s [RF]$$

$$R.F > 1$$

It is the ratio of summation of isentropic enthalpy drop in each stage to the overall isentropic enthalpy drop.

Ques Steam at 15 bar and 350°C is expanded through 50% Reaction turbine through a pressure of 0.14 bar. The stage efficiency is 75% for each stage and Reheat factor is 1.04. The expansion is carried out in 20 stages. The power is equal to 12000 Kw. Calculate the flow of steam assuming that all stages developed equal work.

In the turbine mentioned about at 1 ~~one~~ stage the pressure is 1 bar and the steam is dry saturated. The exit angle of blade is 20° and Blade speed ratio is 0.7. The Blade Height is $\frac{1}{12}$ of dia. of Blade. Calculate
 (i) Blade diameter and Rotor speed (at 1 bar dry saturated steam specific volume = $1.694 \text{ m}^3/\text{kg}$).

Ans.

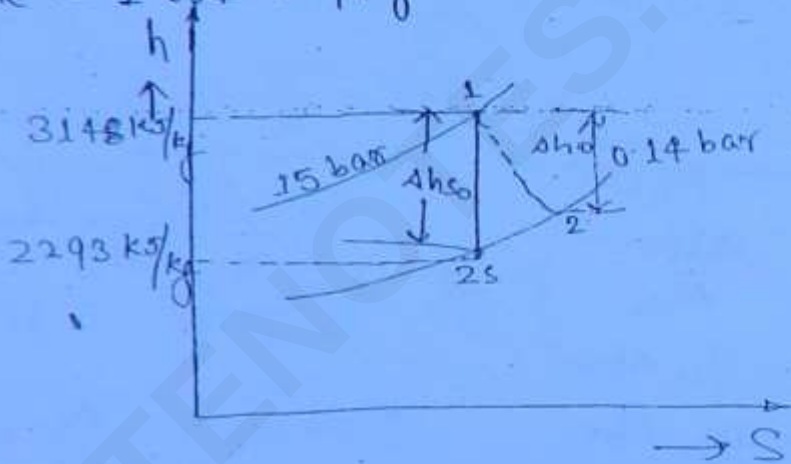
$$\beta_2 = \alpha_1$$

$$\beta_2 = 20^\circ = \alpha_1$$

$$\eta_0 = \eta_s \times R.F$$

$$\eta_0 = 0.75 \times 1.04$$

$$\eta_0 = 0.78$$



$$\eta_0 = \frac{\Delta h_0}{\Delta h_{so}}$$

$$0.78 = \frac{\Delta h_0}{3148 - 2293}$$

$$\Delta h_0 = 666.9 \text{ kJ/kg}$$

$$\Delta h/\text{stage} = \frac{666.9 \text{ kJ/kg}}{\text{No. of stage}} \Rightarrow \frac{666.9}{20} \Rightarrow 33.35 \text{ kJ/kg}$$

$$P = \dot{m} \times W$$

$$W = h_1 - h_2$$

$$12000 \text{ Kw} = \dot{m} \times 666.9 \text{ kJ/kg}$$

$$\dot{m} = 17.9 \text{ kg/sec}$$

150

$$w_{\text{stage}} = \frac{(V_{w1} + V_{w2}) u}{1000} = \underline{\underline{33.35}}$$

$$= (V_1 \cos \alpha_1 + V_1 \cos \alpha_1 - u) u = 33.35 \times 1000$$

$$= (V_1 \cos \alpha_1 + V_1 \cos \alpha_1 - 0.7 V_1) 0.7 V_1 = 33.35 \times 1000$$

$$= (V_1 \cos 20^\circ + V_1 \cos 20^\circ - 0.7 V_1) 0.7 V_1 = 33.35 \times 1000$$

$$\frac{u}{V_1} = 0.7$$

$$V_1 =$$

$$u =$$

$$H = \frac{\pi D H V_f}{2}$$

$$H = \frac{D}{12}$$

$$H = \frac{\pi D N}{60}$$

Ans:

$$D = 1.3 \text{ m}$$

$$N = 2067 \text{ rpm}$$

151

(H) Compressible flow (H fluid)

Stagnation state $\Rightarrow$ when a fluid is brought to zero velocity isentropically then that state is known as stagnation state.

$$h_1 + \frac{V_1^2}{2} = h_{01}$$

$$c = \sqrt{\gamma R T}$$

$$M = \frac{V}{c}$$

Mach number

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_{01}$$

$$T_1 + \frac{V_1^2}{2 C_p} = T_{01}$$

$$T_1 + \frac{V_1^2}{2 \times \frac{\gamma R}{\gamma - 1}} = T_{01}$$

$$T_1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{\gamma R} = T_{01} \Rightarrow T_1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2 \times T_1}{\gamma R T_1} = T_{01}$$

$$T_1 \left[1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{C_1^2} \right] = T_{01}$$

$$T_1 \left[1 + \frac{\gamma - 1}{2} M_1^2 \right] = T_{01}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\begin{matrix} 1 & 01 \\ P_1, T_1 & P_{01}, T_{01} \end{matrix}$$

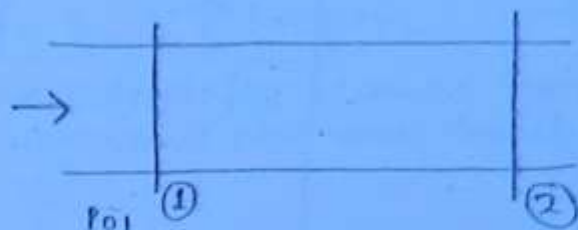
$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{P_{01}}{P_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2 \right]^{\frac{\gamma}{\gamma - 1}}$$

Prob-1 Air flow steadily through a pipe from 350 kPa, 60°C and 183 m/sec. at the inlet state. The outlet condition are Mach No. = 1.3, Stagnation Pressure 385 kPa, Stagnation Temp. 350 K, find the Stagnation Pressure and Stagnation Temp. at inlet and static pressure and static temp. at outlet.

Soln



P_{01}
 T_{01}

$P_1 = 350 \text{ kPa}$

$T_1 = 333 \text{ K}$

$V_1 = 183 \text{ m/sec}$

$P_{02} = 385$

$T_{02} = 350$

$M_2 = 1.3$

$P_2 = ?$ $T_2 = ?$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$M_1 = \frac{V_1}{C_1}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$C = \sqrt{\gamma R T_1}$$

$$C^2 = \gamma R T_1 \cdot \frac{\text{m}^2/\text{sec}^2 \cdot \text{kg}}{\text{kg}} \cdot \frac{\text{J}}{\text{kg}}$$

$$M_1 = \frac{183}{\sqrt{1.4 \times 287 \times 333}}$$

$$M_1 = 0.5$$

1.53

$$\frac{T_{01}}{333} = 1 + \frac{1.4-1}{2} \times 0.5^2$$

$$T_{01} = 349.66 \text{ K}$$

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{349.66}{333} = \left(\frac{P_{01}}{350} \right)^{\frac{1.4-1}{1.4}}$$

$$P_{01} = 415.17 \text{ kPa}$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$$

$$\frac{250}{T_2} = 1 + \frac{1.4-1}{2} (1.3)^2$$

$$T_2 = 261.58 \text{ K}$$

$$\frac{T_{02}}{T_2} = \left(\frac{P_{02}}{P_2} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{385}{P_2} = \left(\frac{350}{261.58} \right)^{\frac{1.4}{1.4-1}}$$

$$P_2 = 138.9 \text{ KPa}$$

Ans:

154

(Reversible flow)

3) FANNO FLOW $\Rightarrow$ Flow in constant area pipe with friction without Heat transfer is known as Fanno flow.

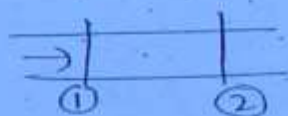
3) Governing equation $\Rightarrow$

① Continuity equation $\Rightarrow$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$A_1 = A_2$$

$$\rho_1 V_1 = \rho_2 V_2$$



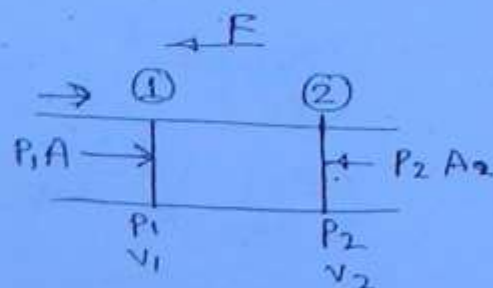
② Momentum equation $\Rightarrow$

$$F_{\text{net}} = m \cdot a$$

$$P_1 A - P_2 A - F = m \cdot a$$

$$F_{\text{net}} = m \left(\frac{V-u}{t} \right)$$

$$F_{\text{net}} = m(V-u)$$



$$P_1 A - P_2 A = F = \dot{m} (V_2 - V_1)$$

$$\boxed{(P_1 - P_2) A = F = \dot{m} (V_2 - V_1)} \quad \text{Momentum equation}$$

③ First Law of thermodynamics $\Rightarrow$

$$h_1 + \frac{V_1^2}{2} + Z_1 g + \cancel{q} = h_2 + \frac{V_2^2}{2} + Z_2 g + \cancel{q}$$

No heat transfer because farrow flow.

IN Horizontal pipe work transfer is zero.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_{01} = h_{02}$$

$$C_p T_{01} = C_p T_{02}$$

$$\text{Stagnation Temp. } \boxed{T_{01} = T_{02}}$$

155

④ Stagnation Temp. Remains constant in farrow flow.

④ second Law of thermodynamics $\Rightarrow$

$$\boxed{T ds = dh - V dp}$$

⑤ Equation of a state $\Rightarrow$

$$PV = mRT$$

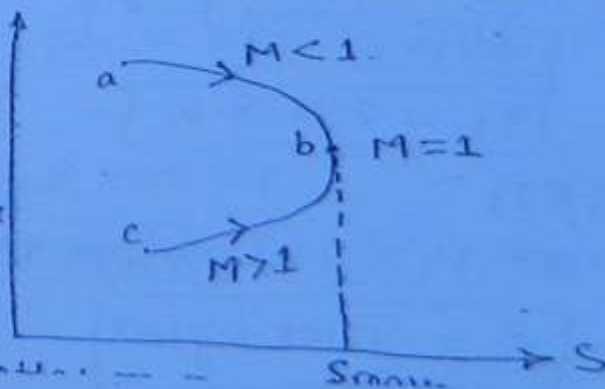
$$P = \frac{m}{V} RT$$

$$\boxed{P = \rho RT}$$

$$\text{⑥ } \boxed{dh = C_p dT}$$

There are seven unknown (P, V, T, ρ, h, s, F) and there are six equation and hence there are infinite No. of solutions. if one

Random Temp. is assumed then there are 6 unknown and there are 6 equations and we can find all these unknown corresponding to that temp. and this process is repeated for



According to second law of thermodynamics
 $(\Delta S)_{\text{universe}} > 0$

$$(\Delta S)_{\text{universe}} = (\Delta S)_{\text{sys}} + (\Delta S)_{\text{surrounding}} > 0$$

$$(\Delta S)_{\text{sys}} > 0$$

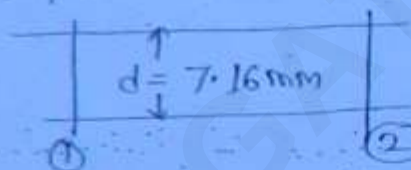
Because Fanno flow is irreversible flow so no heat transfer in surrounding so it is zero.

Fanno flow is possible either from A to B or C to B but not from B to A or B to C. Because this violates second law of thermodynamics.

208-2 Air flow in an insulated pipe of 7.16 mm dia. the stagnation pressure and stagnation temp. at inlet are 101 kPa and 23°C respectively. The static pressure is 98.5 kPa at a section (2) located at some distance downstream in constant area pipe the temp. is 14°C. find

- ① mass flow rate
- ② ~~stagnation~~ static pressure at section (2)
- ③ friction force on the wall (1) and (2)

Soln.



156

$$P_{01} = 101 \text{ kPa}$$

$$T_2 = 287$$

$$T_{01} = 296 \text{ K}$$

$$T_{02} = 296 \text{ K}$$

$$P_1 = 98.5 \text{ kPa}$$

$$T_1 = 293.8 \text{ K}$$

$$\dot{m} = \rho A V_1$$

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{296}{293.8} = \left(\frac{101}{98.5} \right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_1 = 293.8 \text{ K}}$$

$$P_1 = \rho_1 R T_1$$

$$98.5 = \cancel{287} \rho_1 \times 0.287 \times 293.8$$

$$\boxed{\rho_1 = 1.168 \text{ kg/m}^3}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{296}{293.8} = 1 + \frac{1.4-1}{2} M_1^2$$

$$M_1 = 0.189$$

$$M_1 = \frac{V_1}{C_1}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$0.189 = \frac{V_1}{\sqrt{1.4 \times 287 \times 293.8}}$$

$$V_1 = 64.9 \text{ m/sec}$$

① mass flow rate $\Rightarrow$

$$\dot{m} = \rho_1 A_1 V_1$$

$$= 1.162 \times \frac{\pi}{4} (7.16 \times 10^{-3})^2 \times 64.9$$

$$\dot{m} = 3.05 \times 10^{-3} \text{ kg/sec}$$

② Static pressure at section 2

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$$

$$\frac{296}{287} = 1 + \frac{1.4-1}{2} M_2^2$$

$$M_2 = 0.396$$

$$M_2 = \frac{V_2}{C_2}$$

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}}$$

$$\Rightarrow 0.396 = \frac{V_2}{\sqrt{1.4 \times 287 \times 287}}$$

$$V_2 = 134.47 \text{ m/sec}$$

$$\rho_1 V_1 = \rho_2 V_2$$

$$1.162 \times 64.9 = \rho_2 \times 134.47$$

$$\rho_2 = 0.565 \text{ kg/m}^3$$

157

$$P_2 = \rho_2 R T_2$$

$$P_2 = 0.56 \times 0.287 \times 287$$

$$\boxed{P_2 = 46.58 \text{ kPa}}$$

③ Friction force on the walls ① and ②

$$(P_1 - P_2)A - F = \dot{m}(V_2 - V_1)$$

$$\frac{\text{kN}}{\text{m}^2} \times \text{m}^2$$

$$\frac{\text{kg}}{\text{s}} \cdot \left(\frac{\text{m}}{\text{s}}\right)$$

$$\begin{aligned} [98.5 \times 10^3 - 46.58 \times 10^3] \frac{\pi}{4} (7.16 \times 10^{-3})^2 - F \\ = 3.08 \times 10^{-3} (134.4 - 64) \end{aligned}$$

$$\boxed{F = 1.88 \text{ N}} \quad \text{Ans:}$$

Expte 14/3/10

⊕ Rayleigh flow ⇒

Flow in constant area pipe with heat transfer without friction is known as Rayleigh flow.

⊕ Governing equations ⇒

① Continuity equation ⇒ $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\boxed{\rho_1 V_1 = \rho_2 V_2}$$

② Momentum equation ⇒ Put $F=0$ in Fanno flow momentum eqn:

$$\boxed{(P_1 - P_2)A = \dot{m}(V_2 - V_1)}$$

③ First Law of thermodynamics ⇒

$$h_1 + \frac{V_1^2}{2} + Q = h_2 + \frac{V_2^2}{2}$$

$$h_{01} + Q = h_{02}$$

$$Q = h_{02} - h_{01}$$

$$\boxed{Q = C_p(T_{02} - T_{01})}$$

④ Second Law of thermodynamics →

$$\boxed{T ds = dh - v dp}$$

⑤ Equation of state ⇒

$$PV = mRT$$

$$P = \frac{m}{V} RT$$

$$\boxed{P = \rho RT}$$

$$\boxed{\rho = \frac{m}{V}}$$

(6) $dh = C_p dT$

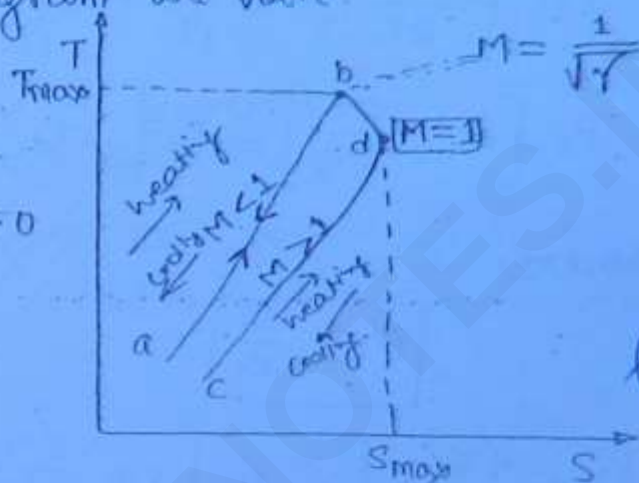
There are seven unknown (P, V, S, T, h, P, Q) and six Equation. Therefore there are infinite No. of solutions.

Let us assume one value of temp. Now there are six unknown and six equation and hence find the six values. If this process is repeated for other temp. and plotted on T-S diagram we have.

Rayleigh: $Q \neq 0$

$$(AS)_{\text{system}} + (AS)_{\text{surroundings}} > 0$$

$$T_{\text{max}} = M \Rightarrow \frac{1}{\sqrt{\gamma}}$$



In Rayleigh flow as there is heat transfer there is entropy change for surroundings and hence for deciding the direction of flow system entropy and surrounding entropy ~~must~~ must be taken into account. therefore in Rayleigh flow the flow is possible either from a to b or b to a or c to d or d to c.

⊕ PROPERTY variation in Rayleigh flow $\Rightarrow$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$(P_1 - P_2) A = \dot{m} (V_2 - V_1)$$

$$(P_1 - P_2) A = \dot{m} V_2 - \dot{m} V_1$$

$$(P_1 - P_2) A = \rho_2 A_2 V_2 V_2 - \rho_1 A_1 V_1 V_1$$

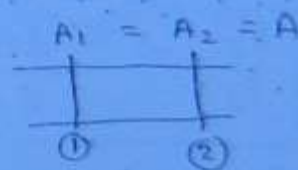
$$(P_1 - P_2) A = A [\rho_2 V_2^2 - \rho_1 V_1^2]$$

$$P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$P_1 + \rho V^2 \rightarrow \text{impulse function (F)}$$

$F_1 = F_2 \rightarrow$ in Rayleigh flow, impulse function Remains Constant.



1) PRESSURE RATIO $\Rightarrow$

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

$$P_1 = \rho_1 R T_1 \Rightarrow \rho_1 = \frac{P_1}{R T_1}$$

$$\rho_2 = \frac{P_2}{R T_2}$$

$$P_1 + \frac{P_1}{R T_1} \cdot v_1^2 = P_2 + \frac{P_2}{R T_2} \cdot v_2^2$$

$$P_1 \left[1 + \frac{v_1^2}{R T_1} \right] = P_2 \left[1 + \frac{v_2^2}{R T_2} \right]$$

where $c_1 = \sqrt{\gamma R T}$

$$M_1 = \frac{v_1}{c_1}$$

$$P_1 \left[1 + \frac{\gamma v_1^2}{\gamma R T_1} \right] = P_2 \left[1 + \frac{\gamma v_2^2}{\gamma R T_2} \right]$$

160

$$P_1 \left[1 + \frac{\gamma v_1^2}{c_1^2} \right] = P_2 \left[1 + \frac{\gamma v_2^2}{c_2^2} \right]$$

$$P_1 \left[1 + \gamma M_1^2 \right] = P_2 \left[1 + \gamma M_2^2 \right]$$

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

2) Temperature ratio $\Rightarrow$

$$\rho_1 v_1 = \rho_2 v_2$$

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} \quad \text{--- (1)}$$

where $M_1 = \frac{v_1}{c_1}$

$$v_1 = M_1 c_1$$

$$v_1 = M_1 \sqrt{\gamma R T_1}$$

$$v_2 = M_2 \sqrt{\gamma R T_2}$$

$$\frac{v_1}{v_2} = \frac{M_1 \sqrt{\gamma R T_1}}{M_2 \sqrt{\gamma R T_2}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{M_1}{M_2} \sqrt{\frac{T_1}{T_2}} \quad \text{--- (4)}$$

(2)

$$\frac{\rho_2}{\rho_1} = \frac{M_1 \sqrt{T_1}}{M_2 \sqrt{T_2}} \quad \text{--- (3)}$$

$$P_1 = \rho_1 R T_1 \rightarrow \rho_1 = \frac{P_1}{R T_1}$$

$$P_2 = \rho_2 R T_2 \rightarrow \rho_2 = \frac{P_2}{R T_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\frac{P_2}{R T_2}}{\frac{P_1}{R T_1}} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \quad \text{--- (4)}$$

from eqn (3) ~~and~~ ^{not equal to} (4)

$$\frac{M_1}{M_2} \cdot \sqrt{\frac{T_1}{T_2}} = \frac{P_2}{P_1} \times \frac{T_1 \sqrt{T_1}}{T_2 \sqrt{T_2}}$$

$$\frac{M_1}{M_2} = \frac{P_2}{P_1} \cdot \sqrt{\frac{T_1}{T_2}}$$

$$\frac{M_1^2}{M_2^2} = \frac{P_2^2}{P_1^2} \cdot \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^2 \cdot \frac{M_2^2}{M_1^2}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \cdot \frac{M_2^2}{M_1^2}} \rightarrow \text{Temperature Ratio}$$

Density Ratio $\Rightarrow$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \sqrt{\frac{T_1}{T_2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \cdot \frac{M_1}{M_2}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_1^2 (1 + \gamma M_2^2)}{M_2^2 (1 + \gamma M_1^2)}} \rightarrow \text{Density Ratio}$$

161

Velocity Ratio $\Rightarrow P_1 V_1 = P_2 V_2$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{M_2^2 (1 + \gamma M_1^2)}{M_1^2 (1 + \gamma M_2^2)}}$$

1) Stagnation Temperature Ratio $\Rightarrow$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}}{T_{01}} \cdot \frac{T_2}{T_1} \cdot \frac{T_1}{T_{01}}$$

Where $\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{T_{02}}{T_2} \right) \left(\frac{T_2}{T_1} \right) \frac{1}{\left(\frac{T_{01}}{T_1} \right)}$$

$$\boxed{\frac{T_{02}}{T_{01}} = \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \times \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \cdot \frac{M_2^2}{M_1^2}}$$

2) ① Air flows with negligible friction in a constant area pipe at section ① the properties are $t_1 = 60^\circ\text{C}$, $P_1 = 135 \text{ kPa}$, $V_1 = 732 \text{ m/sec}$. Heat is added b/w ① and ② and $M_2 = 1.2$. Find P_2 , t_2 , V_2 , ρ_2 , Heat added (Q), entropy change and also sketch the process on T-s diagram.

Q.11 $T_1 = 273 + 60 = 333 \text{ K}$ | $M_2 = 1.2$

$\gamma = 1.4$

$P_1 = 135 \text{ kPa}$

$V_1 = 732 \text{ m/sec}$

$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$

$M_1 = \frac{732}{\sqrt{1.4 \times 287 \times 333}}$

$\boxed{M_1 = 2}$

$$P_1 = \rho_1 R T_1$$

$$135 = \rho_1 \times 287 \times 333$$

$$\boxed{\rho_1 = 1.413 \text{ kg/m}^3}$$

$$\frac{P_2}{P_1} = \frac{M_1^2 (1 + \gamma M_2^2)}{M_2^2 (1 + \gamma M_1^2)}$$

$$\frac{P_2}{1.413} = \frac{2^2 (1 + 1.4 \times 1.2^2)}{1.2^2 (1 + 1.4 \times 2^2)}$$

$$P_2 = 1.799 \text{ kg/m}^3$$

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

$$\frac{P_2}{135} = \frac{1 + 1.4 \times 2^2}{1 + 1.4 \times 1.2^2}$$

$$P_2 = 295.42 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \cdot \frac{M_2^2}{M_1^2}$$

$$\frac{T_2}{333} = \left(\frac{1 + 1.4 \times 2^2}{1 + 1.4 \times 1.2^2} \right)^2 \cdot \frac{1.2^2}{2^2}$$

$$\boxed{T_2 = 574 \text{ K}}$$

$$\frac{V_2}{V_1} = \frac{M_2^2 (1 + \gamma M_1^2)}{M_1^2 (1 + \gamma M_2^2)}$$

$$\frac{V_2}{732} = \frac{1.2^2 (1 + 1.4 \times 2^2)}{2^2 (1 + 1.4 \times 1.2^2)}$$

$$\boxed{V_2 = 576 \text{ m/s}}$$

168

$$Q = C_p (T_{02} - T_{01})$$

$$\frac{T_{02}}{T_1} = 1 + \frac{\gamma-1}{2} M_2^2$$

$$\frac{T_{02}}{574} = 1 + \frac{1.4-1}{2} 2.2^2$$

$$T_{02} = 739.41 \text{ K}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

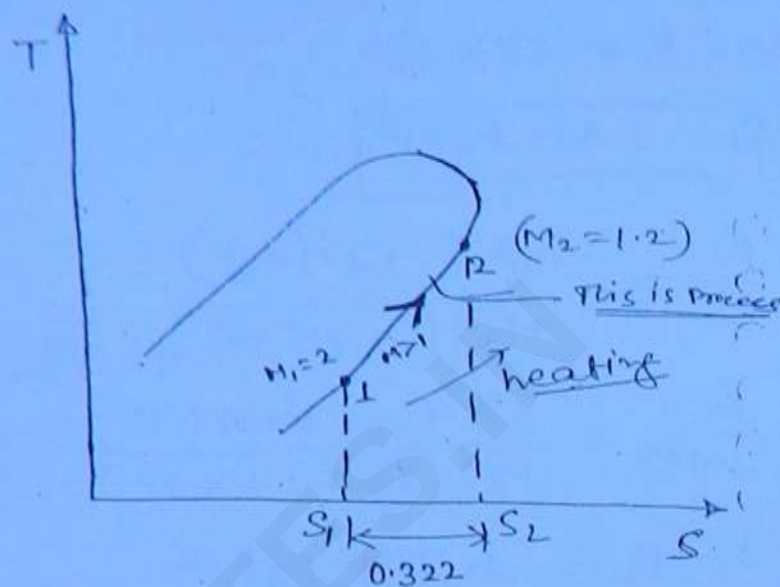
$$\frac{T_{01}}{333} = 1 + \frac{1.4-1}{2} 2^2$$

$$T_{01} = 599.4 \text{ K}$$

$$Q = C_p (T_{02} - T_{01})$$

$$Q = 1.005 (739.41 - 599.4)$$

$$Q = 140.6 \text{ kJ/kg}$$



169

$$T ds = dh - v dp$$

$$ds \Rightarrow \frac{dh}{T} - \frac{v}{T} dp$$

$$ds = C_p \frac{dT}{T} - \frac{R}{P} dp$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = 1.005 \ln \left(\frac{574}{333} \right) - 0.287 \ln \left(\frac{295.42}{135} \right)$$

$$S_2 - S_1 = 0.322 \text{ kJ/kg-K}$$

Ⓐ NORMAL SHOCK WAVES

These are highly irreversible discontinuity form in supersonic flows. The thickness of shock wave is around $0.2 \mu\text{m}$ (0.2×10^{-6} m) as shock waves are formed in a faster manner $q \approx 0$.

As the thickness of shock wave is very small friction

$$F = 0$$

Ⓐ Governing equation for shock waves $\Rightarrow$

① continuity equation $\Rightarrow$ $\boxed{\rho_1 V_1 = \rho_2 V_2}$

② Momentum eqn $\Rightarrow$

Thrust function are same on both side of normal shock $\Rightarrow$ $\boxed{(P_1 - P_2) A = \dot{m} (V_2 - V_1)}$

③ First Law of thermodynamics $\Rightarrow$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Stagnation enthalpy remains constant both side of shock wave

$$\boxed{h_{01} = h_{02}}$$

$$C_p T_{01} = C_p T_{02}$$

Stagnation Temp.

$$\boxed{T_{01} = T_{02}}$$

v.g.m.p

Stagnation Temp. Remains constant before and after the shock.

④ second law of thermodynamics

$$\boxed{T ds = dh - v dp}$$

⑤ equation of state $\Rightarrow$

$$\boxed{P = \rho R T}$$

⑥ $\boxed{dh = C_p dT}$

The no. of unknown are six (6) and no. of equation are six (6) and hence there is a unique solution.

Ⓐ Property variation in shock waves $\Rightarrow$

① Temperature ratio $\Rightarrow$ $\left(\frac{T_2}{T_1} \right)$

$$\frac{T_2}{T_1} = \frac{T_{02}}{T_{01}} \cdot \frac{T_{01}}{T_1} \cdot \frac{T_1}{T_2}$$

165

$$T_{01} = T_{02}$$

$$\frac{T_2}{T_1} = \frac{T_2}{T_{02}} \times \frac{T_{01}}{T_1}$$

$$\frac{T_2}{T_1} = \frac{\frac{T_{01}}{T_1}}{\frac{T_{02}}{T_2}} \Rightarrow \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$$

$$\boxed{\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}}$$

2) Density Ratio $\Rightarrow$ We know that

$$\rho_1 V_1 = \rho_2 V_2$$

$$M_1 = \frac{V_1}{C_1} \Rightarrow V_1 = M_1 C_1$$

$$V_1 = M_1 \sqrt{\gamma R T_1}$$

similarly

$$V_2 = M_2 \sqrt{\gamma R T_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{M_1 \sqrt{\gamma R T_1}}{M_2 \sqrt{\gamma R T_2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \sqrt{\frac{T_1}{T_2}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right)^{1/2}}$$

3) velocity Ratio $\Rightarrow$ $\rho_1 V_1 = \rho_2 V_2$

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{M_2}{M_1} \cdot \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right)^{1/2}}$$

166

④ PRESSURE RATIO $\Rightarrow$ we know that

$$P_1 = \rho_1 R T_1$$

$$P_2 = \rho_2 R T_2$$

$$\frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1}$$

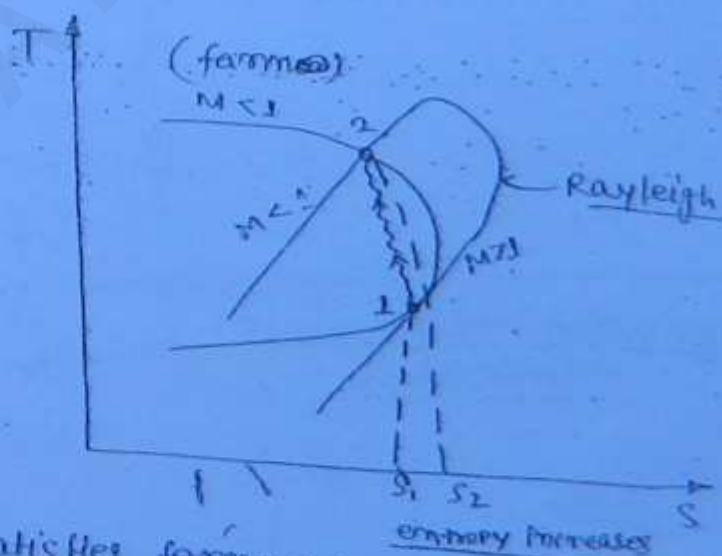
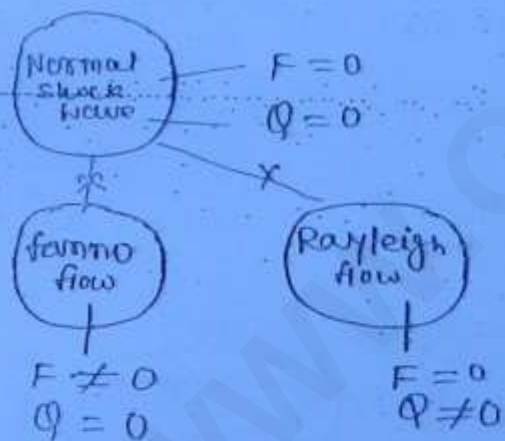
$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_2}{T_1}$$

$$\frac{P_2}{P_1} = \frac{\rho_1}{\rho_2} \cdot \left[\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right]^{1/2} \times \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$$

$$\boxed{\frac{P_2}{P_1} = \frac{\rho_1}{\rho_2} \cdot \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right)^{1/2}}$$

163

from the momentum eqn. the pressure ratio can also be derived as $\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$ (Refer Rayleigh flow)



**** Normal shock wave must satisfy Fanno and Rayleigh flow and hence it should lie on the intersection of Fanno or Rayleigh flow.**

Shock waves are formed from 1 to 2 but not from 2 to 1 because if shock wave were from 2 to 1, universe entropy decreases and hence it violates 2nd law of thermodynamics. therefore

After the shock waves, one flow change from supersonic to subsonic.

variation of properties $\Rightarrow$

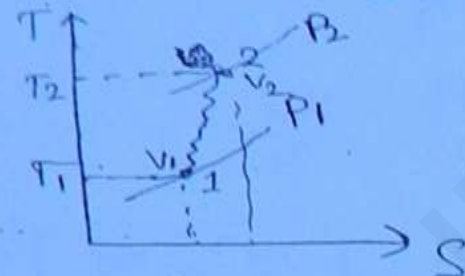
stagnation Temp. Remains constant i.e.

$$T_{01} = T_{02}$$

increase in static temp.

static pressure is increases

Decrease in velocity $\Rightarrow$



$$(P_1 - P_2)A = \dot{m}(V_2 - V_1)$$

$$P_2 > P_1$$

$$-ve = V_2 - V_1$$

Then velocity decreases

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$V_1 = M_1 \sqrt{\gamma R T_1}$$

$$V_2 = M_2 \sqrt{\gamma R T_2}$$

$$\frac{V_2}{V_1} = \frac{M_2}{M_1} \frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}}$$

$$M_2 < 1$$

$$M_1 > 1$$

$$\frac{M_2}{M_1} < 1$$

$$\frac{V_2}{V_1} = \frac{M_2}{M_1} \sqrt{\frac{T_2}{T_1}}$$

$\Rightarrow$ Density is increase $\Rightarrow$

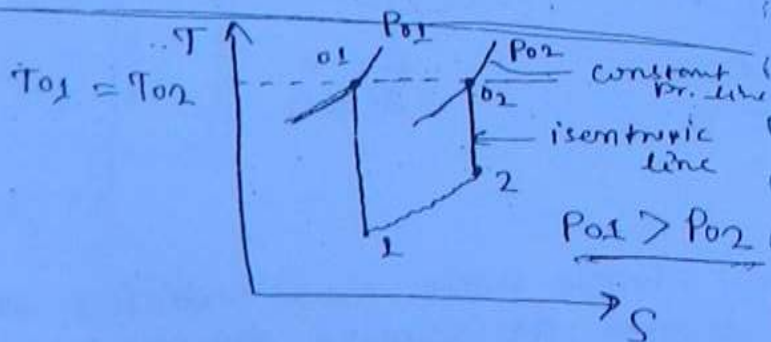
$$\rho_1 V_1 = \rho_2 V_2$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$$

$$\frac{\rho_2}{\rho_1} > 1$$

$$\rho_2 > \rho_1$$

$\Rightarrow$ Stagnation pressure is decreases.



* NOZZLE And DIFFUSER *

Nozzle is a device which is used for increasing velocity at the expense of pressure energy. (Pressure energy decreases)

Diffuser is used for increasing pressure at the expense of kinetic energy. (velocity decrease)

Application of Nozzles:

- ① Steam turbine
- ② Gas turbine
- ③ Jet propulsion system.

~~Internal check~~

Application of Diffuser $\Rightarrow$ Diffuser are used in Centrifugal Compressor.

$$\left(\frac{\partial p}{\partial \rho}\right)_s = c^2$$

where $c \rightarrow$ sonic velocity or velocity of sound.

Mach No. $M = \frac{V}{c}$

$M < 1 \Rightarrow$ subsonic.

$M = 1 \Rightarrow$ Sonic.

$M > 1 \Rightarrow$ supersonic.

Nozzle flow Analysis $\Rightarrow$

- Assumptions:
1. Steady flow.
 2. one dimensional flow.
 3. Isentropic flow.

$$\boxed{\rho A V = \text{Constant}} \quad \text{Continuity equation}$$

$$d\rho \cdot AV + \rho V \cdot dA + \rho A dv = 0$$

$$\Rightarrow \frac{d\rho AV + \rho V dA + \rho A dv}{\rho AV} = \frac{0}{\rho AV}$$

$$\boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0} \quad \text{--- ①}$$

For Bernoulli's and Euler's equation, we have

$$-\frac{dp}{\rho} + v dv + g dz = 0$$

169

IF pressure energy changes negligible, $dz=0$
 hence $\frac{dP}{\rho} + v dv = 0$

$$\Rightarrow \frac{dP}{\rho} = -v dv$$

$$dP = -\rho v^2 \frac{dv}{v}$$

$$\Rightarrow \boxed{\frac{dP}{\rho v^2} = -\frac{dv}{v}} \rightarrow (2)$$

from eqn (1) $\frac{dP}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0$

$$\frac{dA}{A} = -\frac{dP}{\rho} - \frac{dv}{v}$$

$$\Rightarrow \frac{dA}{A} = -\frac{dP}{\rho} + \frac{dP}{\rho v^2}$$

$$\Rightarrow \frac{dA}{A} = \frac{dP}{\rho v^2} \left[-\frac{v^2 dP}{dP} + 1 \right]$$

$$= \frac{dP}{\rho v^2} \left[1 - \frac{v^2}{\frac{dP}{d\rho}} \right]$$

$$= \frac{dP}{\rho v^2} \left[1 - \frac{v^2}{c^2} \right]$$

$$= \frac{dP}{\rho v^2} [1 - M^2]$$

where $M = \frac{v}{c}$

$$M^2 = \frac{v^2}{c^2}$$

$$= -\frac{dv}{v} [1 - M^2]$$

$$\boxed{\frac{dA}{A} = (M^2 - 1) \frac{dv}{v}} \text{ --- velocity}$$

For nozzle dv is always positive.

Case (1) $M < 1$ (subsonic)

$$\frac{dA}{A} = -\frac{dv}{v}$$

$$\frac{dA}{A} = -ve \text{ (negative)}$$

i.e. area is decreasing.

$$M < 1$$

Converging Nozzle

Case-I $M > 1$ (supersonic)

$$\frac{dA}{A} = +ve \text{ (positive)}$$

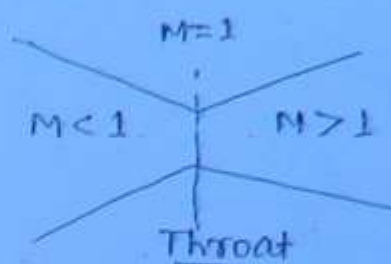
i.e. area is increasing.

$M > 1$

Divergent Nozzle

Case-III $M = 1$ (sonic)

Convergent - Divergent Nozzle



⊕ Diffuser →

$$\frac{dA}{A} = (M^2 - 1) \frac{dv}{v}$$

For diffuser (dv is always negative (-ve))

Case-1 When $M < 1$ (subsonic)

$$\frac{dA}{A} = +ve$$

∴ i.e. area is increasing

$M < 1$

Divergent diffuser

Case-2 When $M > 1$ (supersonic)

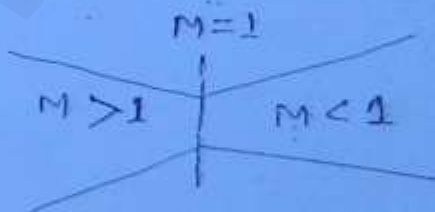
$$\frac{dA}{A} = -ve$$

∴ i.e. Area is decreasing.

$M > 1$

Convergent diffuser

Case-3 When $M = 1$ (sonic)



Convergent - Divergent diffuser

(12)

OR REVERSIBLE ADIABATIC

$$PV^\gamma = K \rightarrow \text{constant}$$

$$\Rightarrow \frac{P}{P^\gamma} = K \quad \text{because } P = \frac{m}{V}, \quad V \propto \frac{1}{P}$$

$$\Rightarrow P = K \cdot P^\gamma$$

$$\frac{dP}{dP} = K \cdot \gamma \cdot P^{\gamma-1}$$

$$= \gamma K \cdot \frac{P^\gamma}{P}$$

$$= \frac{\gamma}{P} \times P \quad \therefore P = \gamma RT$$

$$= \gamma \times RT$$

$$C^2 = \gamma RT$$

$$C = \sqrt{\gamma RT}$$

(172)

NOZZLE $\Rightarrow$

$$h_1 + \frac{V_1^2}{2} + \cancel{gZ_1} + \cancel{A} = h_2 + \frac{V_2^2}{2} + \cancel{gZ_2} + \cancel{A}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$V_1 \ll V_2$$

$$\Rightarrow h_1 = h_2 + \frac{V_2^2}{2}$$

$$\Rightarrow h_1 - h_2 = \frac{V_2^2}{2}$$

$$\Rightarrow V_2 = \sqrt{2(h_1 - h_2)}$$

$$V_2 = \sqrt{2 \times C_p (T_1 - T_2)}$$

$$V_2 = \sqrt{2 C_p T_1 \left(1 - \frac{T_2}{T_1}\right)}$$

for ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore V_2 = \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\text{As } \frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma}$$

$$\Rightarrow \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \frac{P_2}{P_1}$$

$$\rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\dot{m} = \rho_2 A_2 V_2$$

$$\Rightarrow \dot{m} = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} \cdot A_2 \cdot \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

critical pressure ratio : $\Rightarrow$

$$\dot{m} = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} \cdot A_2 \cdot \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\dot{m} = \rho_1 A_2 \cdot \sqrt{2 C_p T_1 \left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{\dot{m}}{A_2} = \rho_1 \sqrt{2 C_p T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}} \right]}$$

for max. mass flow rate per unit area.

$$\frac{d\left(\frac{\dot{m}}{A_2}\right)}{d\left(\frac{P_2}{P_1}\right)} = 0$$

$$\Rightarrow \frac{2}{\gamma} \left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma} \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}-1} = 0$$

$$\Rightarrow \frac{2}{\gamma} \left(\frac{P_2}{P_1}\right)^{\frac{2-\gamma}{\gamma}} = \frac{\gamma+1}{\gamma} \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\Rightarrow \frac{2}{\gamma+1} = \frac{\left(\frac{P_2}{P_1}\right)^{1/\gamma}}{\left(\frac{P_2}{P_1}\right)^{\frac{2-\gamma}{\gamma}}}$$

$$\Rightarrow \frac{2}{\gamma+1} = \left(\frac{P_2}{P_1}\right)^{1/\gamma - \frac{2-\gamma}{\gamma}}$$

173

$$\Rightarrow \frac{2}{\gamma+1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}}$$

critical pressure ratio

$\frac{\dot{m}}{A_2}$ will be maximum, when A_2 is minimum. i.e. this occurs at the throat section.

case-1 critical pressure ratio for air $\Rightarrow$

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(\frac{2}{1.4+1} \right)^{\frac{1.4}{1.4-1}}$$

(174)

$$\boxed{\frac{P_2}{P_1} = 0.528}$$

*** : PSU'S

case-2 IF superheated steam expand then $\gamma = 1.3$

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

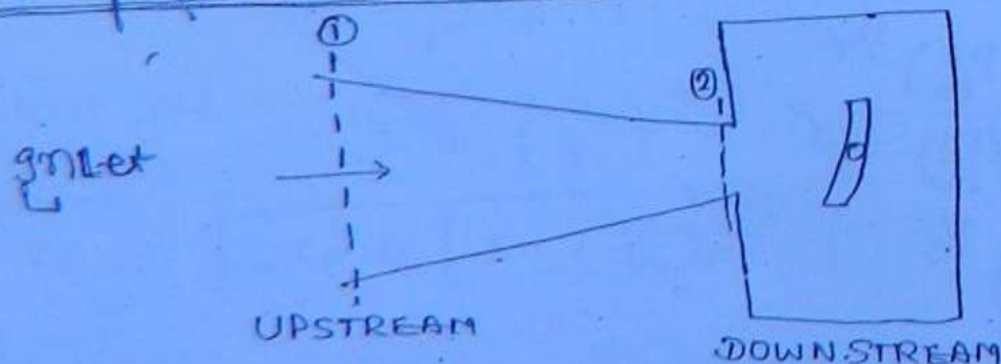
$$= \left(\frac{2}{1.3+1} \right)^{\frac{1.3}{1.3-1}} \Rightarrow 0.545 \quad \text{***}$$

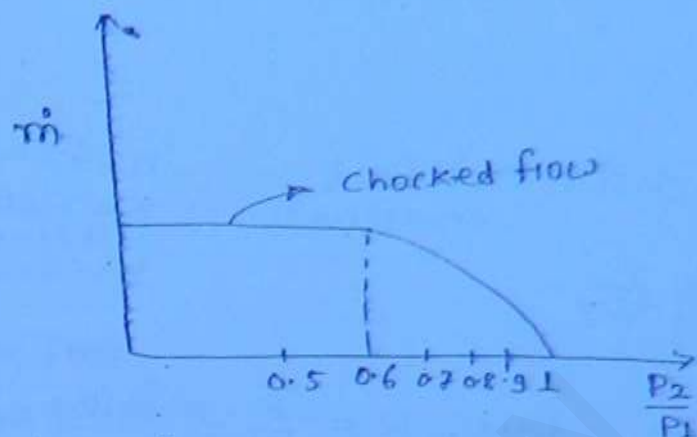
case-3 IF wet steam then $\gamma = 1.135$

$$\frac{P_2}{P_1} = \left(\frac{2}{1.135+1} \right)^{\frac{1.135}{1.135-1}}$$

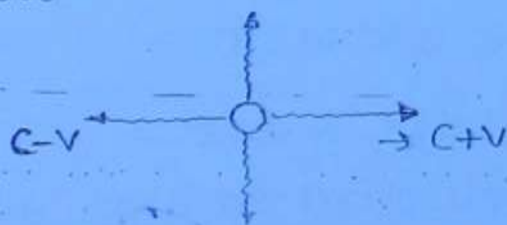
$$\frac{P_2}{P_1} = 0.577 \quad \text{***}$$

⊞ CHOKED FLOW FOR NOZZLES $\Rightarrow$





maximum mass flow irrespective of pressure ratio is known as choked flow.



$$M < 1$$

$$\frac{V}{C} < 1$$

$$V < C$$

$$M = 1$$

$$V = C$$

(175)

once the critical condition are reached even by decrease the pressure there will not be any change in mass flow rate. Because disturbance will not propagate in upstream direction and hence there will not be any change in mass flow rate, once critical condition are reached.

PROB 6 A fluid at 6.9 bar and 93°C enters a convergent nozzle with negligible velocity. and expand isentropically into a space at 3.6 bar. calculate mass flow per m² of exit area.

① when fluid is He (Helium)

$$C_p = 5.19 \text{ kJ/kg-K}, \text{ molecular weight} = 4$$

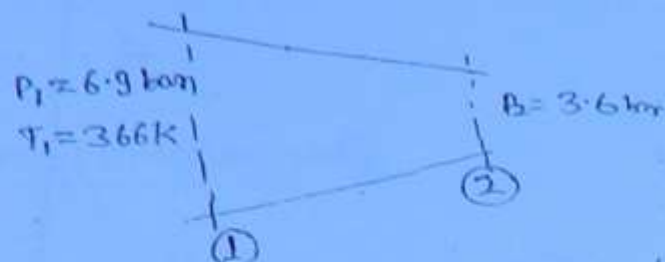
② when fluid is ethane

$$C_p = 1.88 \text{ kJ/kg-K}, \text{ molecular weight} = 30$$

Assume perfect gas behaviour in both cases.

Qⁿ Case 1:

First of all check critical pressure:-



$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$C_p = \frac{\gamma \cdot R}{\gamma-1} \cdot \text{but } R = \frac{\bar{R}}{M} = \frac{8.314}{4} = 2.078$$

$$\Rightarrow 5.19 = \frac{\gamma \times 2.078}{\gamma-1}$$

$$\Rightarrow \gamma = 1.668$$

$$\frac{P_2}{P_1} = \left(\frac{2}{1.668+1} \right)^{\frac{1.668}{1.668-1}}$$

(176)

$$\frac{P_2}{P_1} = 0.486$$

$$\frac{P_2}{6.9} = 0.486$$

Critical pressure $P_2 = 3.35 \text{ bar}$ Ans:

As the exit pressure is greater than critical pressure, critical condition are not reached and hence the flow is normal flow.

$$\dot{m} = \rho_2 A_2 V_2$$

$$\Rightarrow \frac{\dot{m}}{A_2} = \rho_2 V_2$$

$$\text{but } h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000 (h_1 - h_2)}$$

$$= \sqrt{2000 C_p T_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

$$V_2 = \sqrt{2000 \times 5.19 \times 366 \left[1 - \left(\frac{3.6}{6.9} \right)^{\frac{1.668-1}{1.668}} \right]}$$

$$V_2 = 322.2 \text{ m/s} \text{ Ans}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{T_2}{366} = \left(\frac{3.6}{6.9}\right)^{\frac{1.668-1}{1.668}}$$

$$\boxed{T_2 = 282.11 \text{ K}}$$

$$\text{But } P_2 = \rho_2 R T_2$$

$$\rho_2 = \frac{P_2}{R T_2} = \frac{3.6 \times 100}{2.078 \times 282.11}$$

$$\rho_2 = 0.614 \text{ kg/m}^3$$

$$\therefore \frac{\dot{m}}{A_2} = \rho_2 V_2 = 0.614 \times 933.8$$

$$= 573.4 \text{ kg/s/m}^2$$

Case-B Check for critical pressure

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{30} \Rightarrow 0.277$$

$$C_p = \frac{\gamma R}{\gamma-1} \Rightarrow 1.88 = \frac{\gamma \times 0.277}{\gamma-1}$$

$$\gamma = 1.175$$

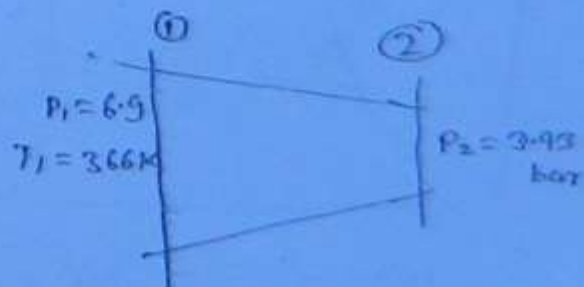
$$\frac{P_2}{P_1} = \left(\frac{2}{1.175+1}\right)^{\frac{1.175}{1.175-1}} = 0.569$$

$$\text{Critical pressure } P_2 = 0.569 \times 6.9 =$$

$$\boxed{P_2 = 3.93 \text{ bar}}$$

As the exit pressure is less than critical pressure, critical condition are reached and hence the pressure at the exit of nozzle is 3.93 bar.

$$\frac{\dot{m}}{A_2} = \rho_2 V_2$$



$$V_2 = \sqrt{2000 \text{ Cp T}_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]} = \sqrt{2000 \times 1.88 \times 366 \left[1 - \left(\frac{3.93}{6.9} \right)^{\frac{1.175-1}{1.175}} \right]}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_2}{366} = \left(\frac{3.93}{6.9} \right)^{\frac{1.175-1}{1.175}}$$

$$V_2 =$$

$$T_2 =$$

$$f_2 = \frac{P_2}{R T_2} = \frac{0.3.93}{0.277 \times 10^3}$$

$$f_2 = ?$$

$$\frac{\dot{m}}{A_2} = f_2 V_2 =$$

178

$$= 1394 \text{ kg/s/m}^2$$

Ex 10.2

10.2

Steam at 10 bar and 250°C enters a nozzle and is discharged at a pressure of 2 bar (wet condition). Initially the steam is in superheated state, find the final velocity of steam neglecting initially velocity and assume the flow to be isentropic.

At 10 bar, 250°C

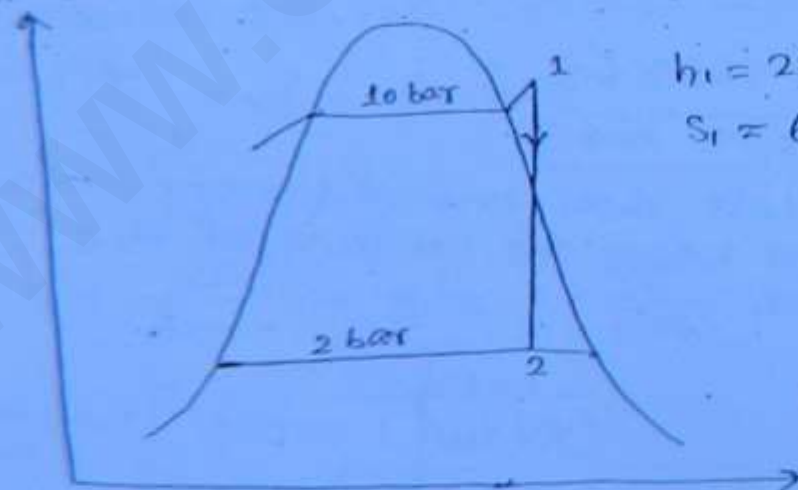
$$h = 2943 \text{ kJ/kg}$$

$$s = 6.926 \text{ kJ/kg-K}$$

At 2 bar $h_f = 504.7 \text{ kJ/kg}$

$$h_{fg} = 2201.6, \quad t_{\text{sat}} = 120.2^\circ\text{C}, \quad s_f = 1.53 \text{ kJ/kg-K}$$

$\eta =$



$$h_1 = 2943 \text{ kJ/kg}$$

$$s_1 = 6.926$$

$$h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

$$h_2 = h_f + x \cdot h_{fg}$$

$$s_1 = s_2 = 6.926$$

$$\text{but } s_2 = s_f + x \cdot s_{fg}$$

$$\text{but } s_{fg} = \frac{h_{fg}}{T} = \frac{2201.6}{273+120.2} \Rightarrow 5.599$$

$$s_2 = 1.53 + x \cdot 5.599 = 6.926$$

$$6.926 = 1.53 + x \cdot 5.599$$

$$x = 0.963$$

$$h_2 = 504.7 + 0.963 \times 2201.6$$

$$h_2 = 2625 \text{ kJ/kg}$$

$$v_2 = \sqrt{2000 (h_1 - h_2)}$$

$$v_2 = \sqrt{2000 (2943 - 2625)}$$

$$v_2 = 795.73 \text{ m/s} \quad \text{Ans}$$

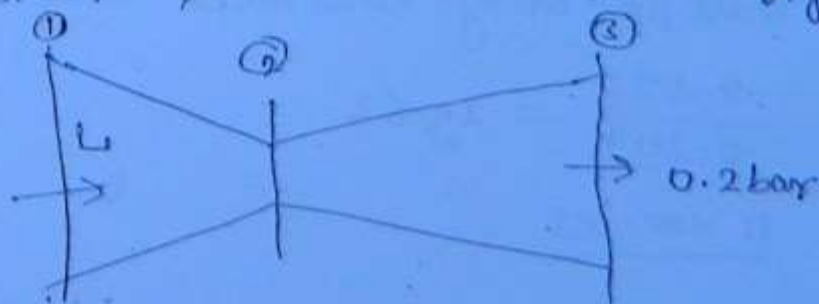
PROB ② A nozzle of an impulse turbine of 1 MW of capacity has steam entering at 20 bar, 300°C (superheated state) and steam consumption is 8 kg/kWh. Steam leaves at 0.2 bar and 10% of heat drop is lost in overcoming friction in diverging portion of nozzle. If throat dia. of each nozzle is 1 cm. find.

① No. of nozzles required.

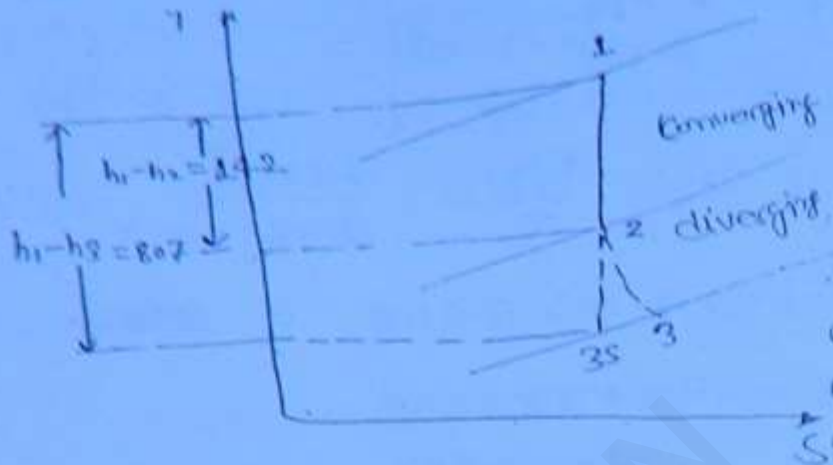
② Exit dia. of each nozzle, take $\eta = 1.3$.

Isentropic enthalpy drop upto throat is equal to 142 kJ/kg
Specific volume of steam at throat is 0.2 m³/kg. Total isentropic enthalpy drop is equal to 807 kJ/kg.
Specific volume at exit = 7.2 m³/kg.

Soln



$$SSC = \frac{\dot{m}s}{\text{Power}}$$



$$\Rightarrow \frac{8 \text{ kg}}{\text{Kwh}} = \frac{\dot{m}s}{10^3 \text{ Kw}}$$

$$\dot{m}s = \frac{10^3 \times 8 \text{ kg}}{3600 \text{ sec}}$$

$$\dot{m}s = 2.22 \text{ kg/sec}$$

mass flow rate to each Nozzle :-

$$\dot{m} = f_2 A_2 V_2$$

$$\text{But } f = \frac{m}{V} = \frac{1}{V/m} = \frac{1}{2}$$

$$\dot{m} = \frac{A_2 V_2}{V_2}$$

$$h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000 (h_1 - h_2)}$$

$$V_2 = \sqrt{2000 \times 14.2} = 532.9 \text{ m/s}$$

$$\text{Now } \dot{m} = \frac{1}{0.2} \times \frac{\pi}{4} \times (0.01)^2 \times 532.9$$

$$\dot{m} = 0.209 \text{ kg/sec}$$

$$\text{No. of Nozzles} = \frac{\text{Total mass flow rate}}{\text{mass flow through each Nozzle}}$$

$$= \frac{2.22}{0.209} = 10.62$$

$$\approx \underline{\underline{11 \text{ Nozzles}}}$$

180

For exit diameter $\Rightarrow$

$$\dot{m} = \rho_3 A_3 V_3 = \frac{A_3 V_3}{V_3}$$

but $\dot{m} = 0.209$, $V_3 = 7.2$

$$h_1 + \frac{V_1^2}{2000} = h_3 + \frac{V_3^2}{2000}$$

$$V_3 = \sqrt{2000 (h_1 - h_3)}$$

$$h_2 - h_{3s} = 807 - 142 = 665 \text{ KJ/Kg}$$

$$h_2 - h_3 = 0.9 (h_2 - h_{3s}) = 0.9 \times 665$$

$$h_2 - h_3 = 598.5 \text{ KJ/Kg}$$

$$\therefore h_1 - h_3 = (h_1 - h_2) + (h_2 - h_3)$$
$$= 142 + 598.5$$

$$h_1 - h_3 = 740.5 \text{ KJ/Kg}$$

$$V_3 = \sqrt{2000 \times 740.5}$$

$$V_3 = 1216.9 \text{ m/s}$$

$$\dot{m} = \rho_3 A_3 V_3 = \frac{A_3 V_3}{V_3}$$

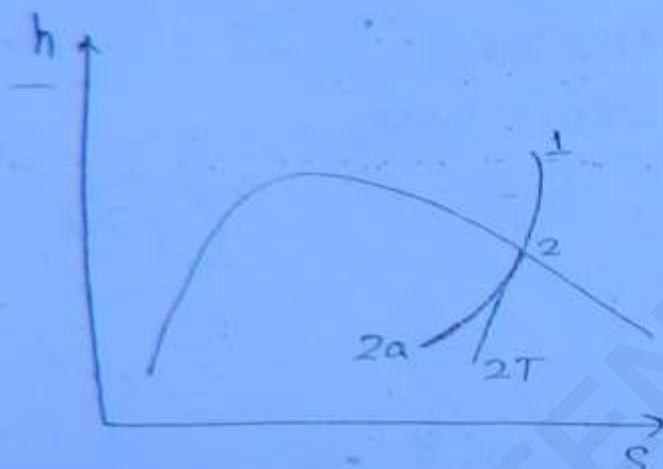
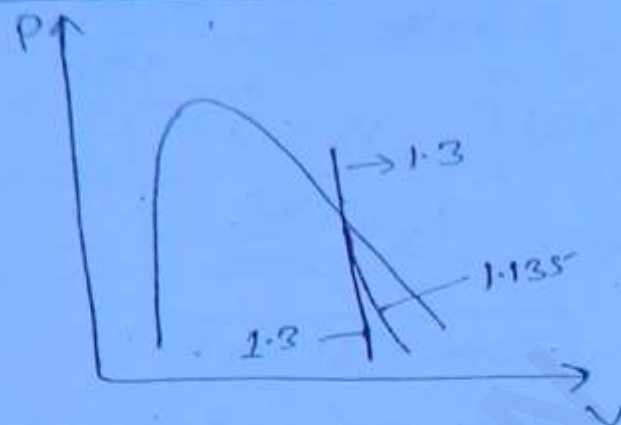
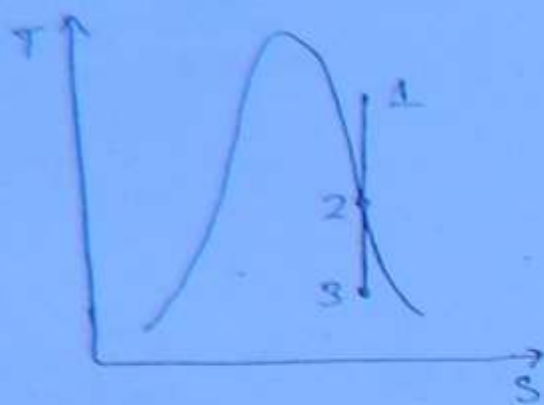
$$0.209 = \frac{\frac{\pi}{4} (D_3^2) \times 1216.9}{7.2}$$

$$D_3 = 0.0396 \text{ m}$$

$$\boxed{D_3 = 3.96 \text{ cm}} \quad \underline{\text{Ans}}$$

(181)

Supersaturated flow / Metastable flow in steam nozzle $\Rightarrow$



182

In steam nozzle as steam expand from superheated condition condensation should begin once it reaches saturation vapour state. But due to very high velocities steam does not undergo condensation and condensation is delayed.

This type of flow is known as supersaturated flow or metastable flow in nozzles.

⊕ Effect of supersaturated flow $\Rightarrow$

- ① Reduction in specific volume.
- ② decrease in Enthalpy drop
- ③ Decrease in exit velocity (but effect is small)
- ④ Increase in mass flow rate.

⊕ Reciprocating Compressor ⊕

Work input to the compressor without clearance volume.

$$W = \frac{n}{n-1} (P_1 V_1 - P_2 V_2)$$

$$= - \frac{n}{n-1} (P_2 V_2 - P_1 V_1)$$

$$W_{\text{input}} = \frac{n}{n-1} (P_2 V_2 - P_1 V_1)$$

$$= \frac{n}{n-1} P_1 V_1 \left(\frac{P_2 V_2}{P_1 V_1} - 1 \right)$$

But $P_1 V_1^n = P_2 V_2^n$

$$\Rightarrow \frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{1/n} = \left(\frac{P_2}{P_1} \right)^{-1/n}$$

183

$$\therefore W_{\text{input}} = \frac{n}{n-1} P_1 V_1 \left[\frac{P_2}{P_1} \times \left(\frac{P_2}{P_1} \right)^{-1/n} - 1 \right]$$

$$W_{\text{input}} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{\text{input}} = \frac{n}{n-1} P_1 V_{\text{act}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{\text{input}} = \frac{n}{n-1} m R T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

⊕ Work input to the compressor with clearance volume ⇒

$$W_{\text{input}} = W_c - W_e$$

$$W_c = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] L$$

$$W_e = \frac{n}{n-1} (P_3 V_3 - P_4 V_4)$$

$$= \frac{\eta}{\eta-1} P_4 V_4 \left(\frac{P_3 V_3}{P_4 V_4} - 1 \right)$$

$$\text{As } P_3 V_3^\eta = P_4 V_4^\eta$$

$$\frac{V_3}{V_4} = \left(\frac{P_4}{P_3} \right)^{1/\eta} = \left(\frac{P_1}{P_2} \right)^{1/\eta} = \left(\frac{P_2}{P_1} \right)^{-1/\eta}$$

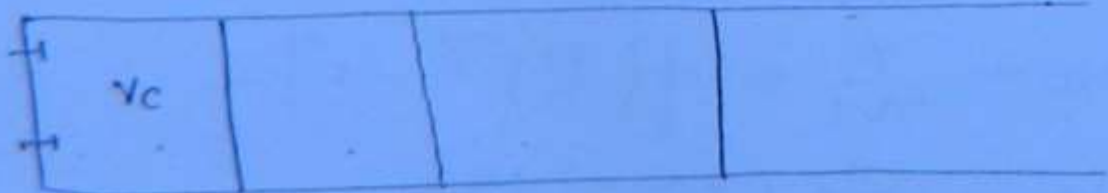
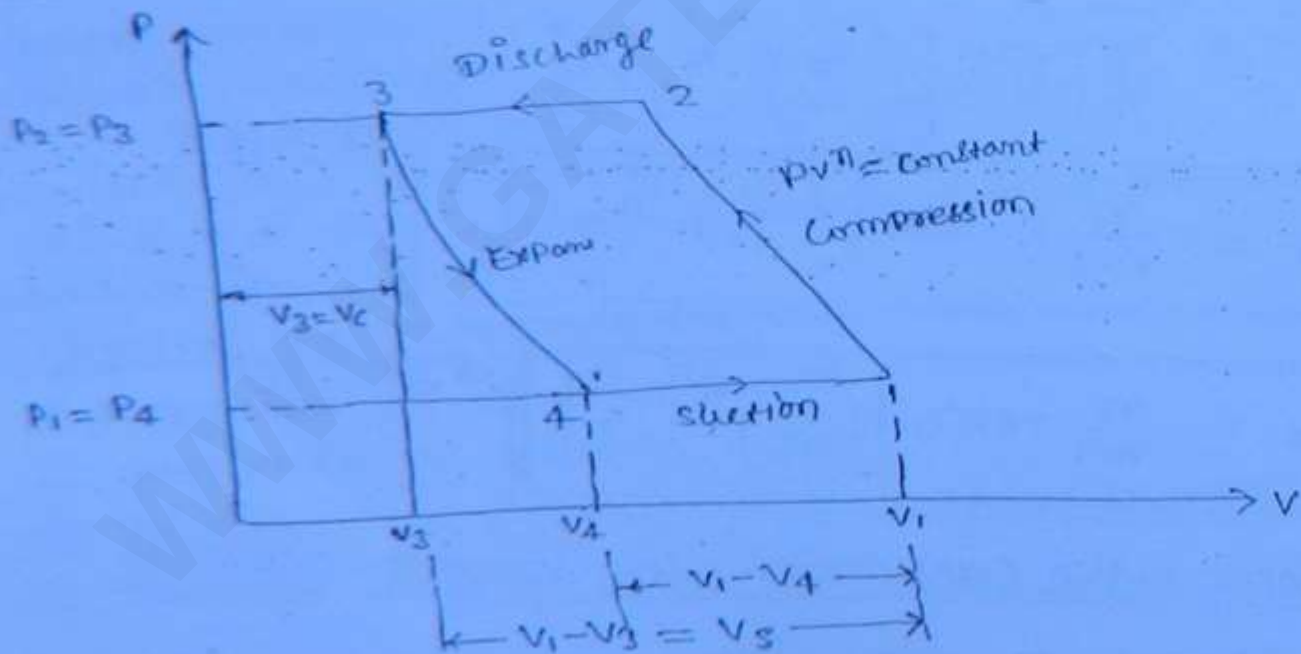
$$W_e = \frac{\eta}{\eta-1} P_1 V_4 \left[\frac{P_2}{P_1} \left(\frac{P_2}{P_1} \right)^{-\frac{\eta-1}{\eta}} - 1 \right]$$

$$= \frac{\eta}{\eta-1} P_1 V_4 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\eta-1}{\eta}} - 1 \right]$$

$$W_{\text{input}} = \frac{\eta}{\eta-1} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1} \right)^{\frac{\eta-1}{\eta}} - 1 \right]$$

$$W_{\text{input}} = \frac{\eta}{\eta-1} P_1 V_{\text{act}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\eta-1}{\eta}} - 1 \right]$$

80

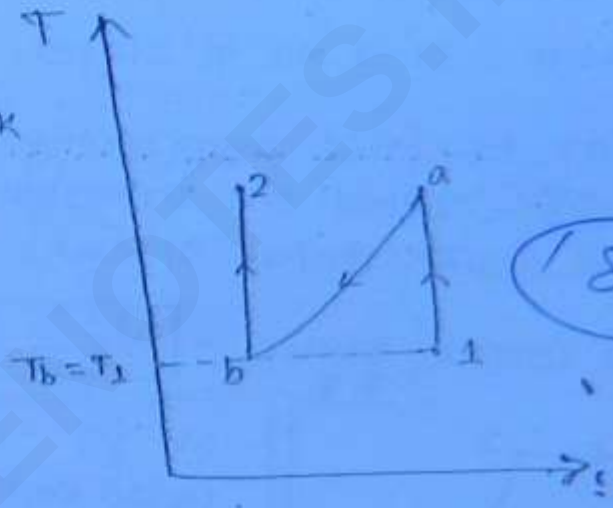
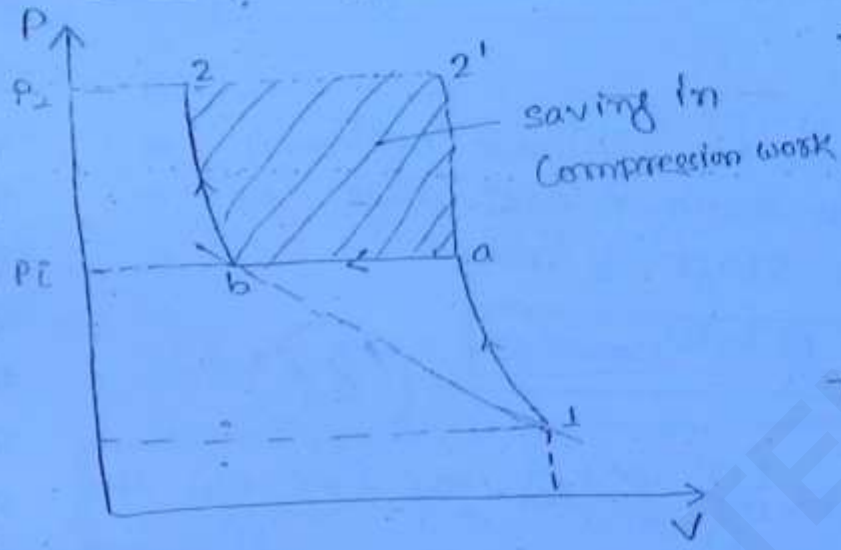


Note $\Rightarrow$ clearance volume has no effect on work input

$$\eta_{vol} = 1 + C - C \left(\frac{P_2}{P_1} \right)^{1/n}$$

Where $C = \text{clearance factor} = \frac{V_c}{V_s}$

⊕ Intercooling in Reciprocating Compressor $\Rightarrow$



185

Condition for min. work input with perfect intercooling $\Rightarrow$

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_1 = \frac{n}{n-1} P_i V_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_2 = \frac{n}{n-1} P_b V_b \left[\left(\frac{P_2}{P_i} \right)^{\frac{n-1}{n}} - 1 \right]$$

But for perfect intercooling
 $P_i V_1 = P_b V_b$

$$\therefore W_2 = \frac{n}{n-1} P_i V_1 \left[\left(\frac{P_2}{P_i} \right)^{\frac{n-1}{n}} - 1 \right]$$

for min. work input

$$\frac{P_i}{P_1} = \frac{\sqrt{P_1 P_2}}{P_1} = \sqrt{\frac{P_2}{P_1}} \quad , \quad P_i = \sqrt{P_1 P_2}$$

$$\frac{P_2}{P_i} = \frac{P_2}{\sqrt{P_1 P_2}} = \sqrt{\frac{P_2}{P_1}}$$

$$\text{overall ratio} = \frac{P_2}{P_1}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{P_2}{P_i} \times \frac{P_i}{P_1}$$

$$= \left(\frac{P_i}{P_1} \right)^2$$

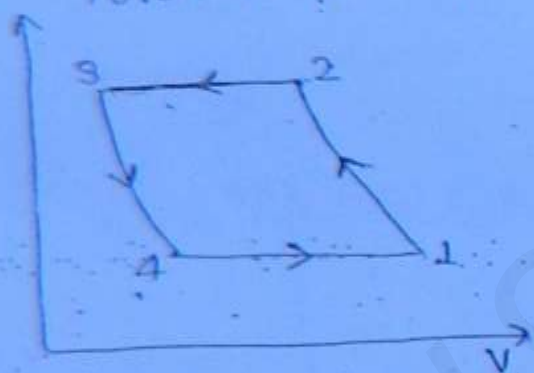
$$\text{overall pressure ratio} = (\text{pressure ratio in each stage})^N$$

where $N \rightarrow$ No. of stages.

etc $\Rightarrow$ For perfect intercooling with minimum (optimum) work input the pressure ratio in each stage is same and work input in each stage is more.

$$\text{Total work input} \rightarrow W_T = N \times W_{\text{each stage}}$$

186



$$\text{mass entering} = m_1 - m_4$$

$$\text{mass leaving} = m_2 - m_3$$

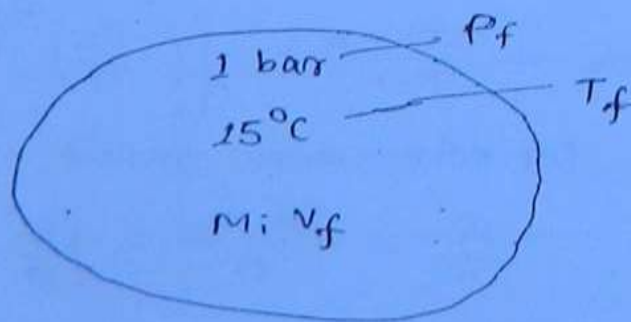
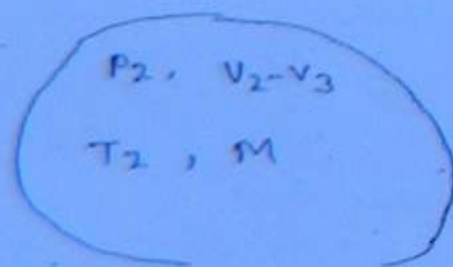
$$\text{but } m_1 = m_2, \quad m_3 = m_4$$

$$\therefore \text{mass entering} = \text{mass leaving}$$

Because during compression and expansion no mass is entering or leaving.

FREE AIR DELIVERY (FAD) $\Rightarrow$

The volume that is delivered i.e. $(V_2 - V_3)$, when reduced to 1 bar pressure, 15°C temp. is known as free air delivery.



$$P_2 (V_2 - V_3) = m R T_2$$

$$m = \frac{P_2 (V_2 - V_3)}{R T_2}$$

$$P_f V_f = m R T_f$$

$$m = \frac{P_f V_f}{R T_f}$$

$$\Rightarrow \frac{P_2 (V_2 - V_3)}{R T_2} = \frac{P_f V_f}{R T_f}$$

$$\Rightarrow \boxed{\frac{P_2 (V_2 - V_3)}{T_2} = \frac{P_f V_f}{T_f}} \quad \underline{\text{FAD}}$$

At entry,

$$P_1 (V_1 - V_4) = m R T_1$$

$$m = \frac{P_1 (V_1 - V_4)}{R T_1} = \frac{P_2 (V_2 - V_3)}{R T_2}$$

$$\Rightarrow \frac{P_1 (V_1 - V_4)}{R T_1} = \frac{P_2 (V_2 - V_3)}{R T_2}$$

$$\boxed{\frac{P_1 (V_1 - V_4)}{T_1} = \frac{P_2 (V_2 - V_3)}{T_2} = \frac{P_f V_f}{T_f}}$$

182

PROB ① Determine the minimum No. of Stages Required in an air compressor, which takes air at 1 bar, 27°C and deliver at 180 bar. The max. discharge temp. is limited to 150°C. Consider the index of polytropic compression as 1.25 and perfect and optimum intercooling in b/w stages.

Soln

$$P_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$P_2 = 180 \text{ bar} = 18000 \text{ kPa}$$

$$T_1 = 300 \text{ K}, T_2 = 423 = T_i$$

$$T_1 = 273 + 27 = 300 \text{ K}$$

$$T_2 = 273 + 150 = 423 \text{ K}$$

$$\text{overall pressure ratio } \frac{P_2}{P_1} = \frac{180}{1} \Rightarrow 180$$

$$\frac{T_i}{T_1} = \left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}}$$

$$\frac{423}{300} = \left(\frac{P_i}{P_1} \right)^{\frac{1.25-1}{1.25}}$$

$$\frac{P_i}{P_1} = 5.57$$

$$\text{As } \frac{P_2}{P_1} = \left(\frac{P_i}{P_1} \right)^N$$

$$180 = (5.57)^N$$

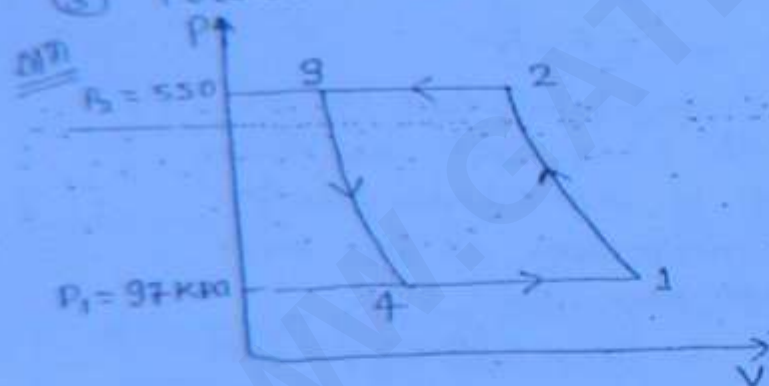
$$N = \text{No. of stages} = 3.024$$

$$\boxed{N \approx 3} \quad \underline{\text{Ans}}$$

QOB-2

A single stage reciprocating has a bore of 200 mm and stroke of 300 mm. It runs at a speed of 500 rpm. clearance volume is 5% of swept volume and $n = 1.3$. Intake pressure and temp. are 97 kPa and 20°C . Delivery pressure is 550 kPa. If free air condition are 101.325 kPa and 15°C . Then find:

- ① Free air delivery in m^3/min .
- ② volumetric efficiency.
- ③ Power required.



$$T_1 = 273 + 20 = 293\text{K}$$

$$D_2 = 0.2\text{m}, L = 0.3\text{m}$$

$$N = 500\text{ rpm}$$

$$P_f = 101.325\text{ kPa}$$

$$T_f = 273 + 15 = 288\text{K}$$

$$V_c = 0.05 V_s$$

$$n = 1.3$$

$$\eta_{\text{Vol}} = 1 + C - C \left(\frac{P_2}{P_1} \right)^{1/n}$$

$$\text{where } C = \frac{V_c}{V_s} = \frac{0.05 V_s}{V_s} = 0.05$$

$$\begin{aligned} \therefore \eta_{\text{Vol}} &= 1 + 0.05 - 0.05 \left(\frac{550}{97} \right)^{1/1.3} \\ &= 0.86 \\ &= 86\% \end{aligned}$$

(188)

$$\frac{P_1(V_1 - V_4)}{T_1} = \frac{P_2(V_2 - V_3)}{T_2} = \frac{P_f V_f}{T_f}$$

$$\text{but } \eta_{vol} = \frac{V_1 - V_4}{V_s} = \frac{V_1 - V_4}{\frac{\pi}{4} D^2 L}$$

$$= 0.86 = \frac{V_1 - V_4}{\frac{\pi}{4} (0.2)^2 \times 0.3}$$

$$V_1 - V_4 = 8.1 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow \frac{97 \times (8.1 \times 10^{-3})}{293} = \frac{101.325 \times V_f}{288}$$

$$V_f = 7.62 \times 10^{-3} \text{ m}^3/\text{cycle}$$

This is the free air delivery per cycle

$$1 \text{ cycle} = 1 \text{ rev.}$$

$$1 \text{ cycle/min} = 1 \text{ rpm.}$$

$$V_f = 7.62 \times 10^{-3} \frac{\text{m}^3}{\text{cycle}} \times 500 \frac{\text{cycle}}{\text{min}}$$

$$\boxed{V_f = 3.81 \text{ m}^3/\text{min}} \quad \text{Ans:}$$

② Net work input:

$$W = \frac{n}{n-1} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{1.3}{1.3-1} \times 97 \times (8.1 \times 10^{-3}) \left[\left(\frac{550}{97} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$W = 1.67 \text{ kJ/cycle}$$

$$W = 1.67 \frac{\text{kJ}}{\text{cycle}} \times 500 \frac{\text{cycle}}{\text{min}}$$

$$W = 835 \text{ kJ/min}$$

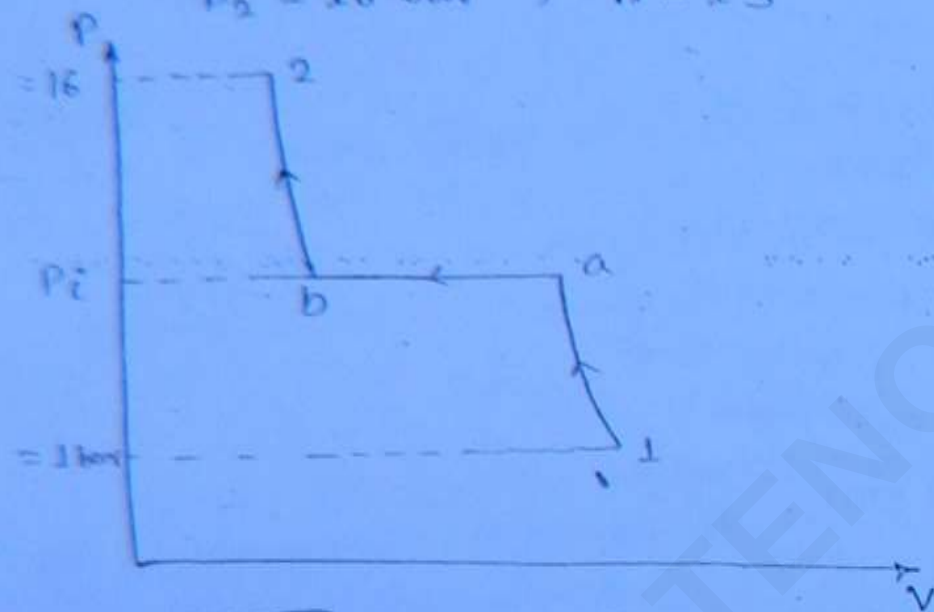
$$\boxed{W = 13.92 \text{ kJ/sec.}} \quad \text{Ans:}$$

(189)

Q.2

A two stage compressor with perfect intercooling delivers 5 kg/min of air at 16 bar. The intake condition are 1 bar and 300K. $\gamma = 1.3$ the calculate power required, isothermal efficiency, heat transferred in each cylinder and also heat transfer in intercooler.

$P_1 = 1 \text{ bar} = 100 \text{ KPa}$, $T_1 = 300 \text{ K}$
 $P_2 = 16 \text{ bar}$, $\gamma = 1.3$



190

$$P_i = \sqrt{P_1 P_2} = \sqrt{1 \times 16} = 4$$

$$W_1 = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{\gamma}{\gamma-1} m R T_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times \frac{5}{60} \times 0.287 \times 300 \left[\left(\frac{4}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$W_1 = 11.7 \text{ kW}$$

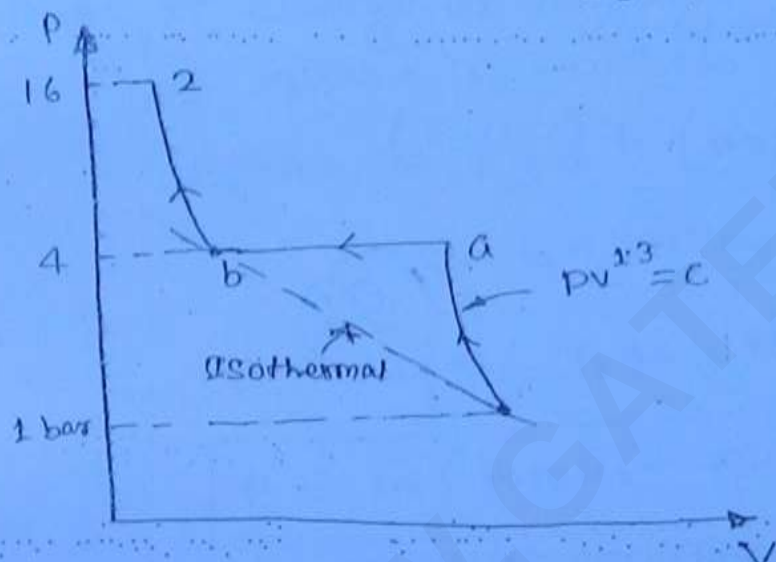
$$W_T = N \times W_1 = 2 \times 11.7$$

$$W_T = 23.4 \text{ kW}$$

The work input in isothermal process is minimum and hence,
 Isothermal efficiency = $\frac{\text{Isothermal work}}{\text{Actual work.}}$

$$\begin{aligned} W_{\text{isothermal}} &= mRT_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{P_1}{P_2} \\ &= \frac{5}{60} \times 0.287 \times 300 \ln \left(\frac{1}{16} \right) \\ &= -19.89 \text{ kW} \end{aligned}$$

$$\eta_{\text{isothermal}} = \frac{19.89}{23.4} \times 100 = 85\%$$



$$\frac{T_a}{T_1} = \left(\frac{P_a}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_a}{300} = \left(\frac{4}{1} \right)^{\frac{1.3-1}{1.3}} \Rightarrow \boxed{T_a = 413.1 \text{ K}}$$

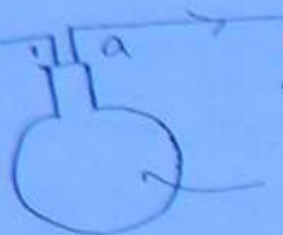
Heat rejected in intercooler = $m c_p (T_a - T_b)$
 but $T_b = T_1 = 300 \text{ K}$

$$Q = \frac{5}{60} \times 1005 (413.1 - 300)$$

$$\boxed{Q = 9.47 \text{ kJ/sec.}}$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$



$$P_a = 4$$

$$T_a = 413.1 \text{ K}$$

$$W = -11.7 \text{ kJ/sec.}$$

$$h_1 + \frac{v_1^2}{2} + gz_1 + Q = h_a + \frac{v_a^2}{2} + gz_a + W$$

$$h_1 + Q = h_a + W$$

$$Q = h_a - h_1 + W$$

$$Q = \dot{m} (h_a - h_1) + W$$

$$Q = \dot{m} C_p (T_a - T_1) + W$$

$$= \frac{5}{60} \times 1.005 (413.1 - 300) + (-11.7)$$

$$Q = 2.2 \text{ kJ/sec}$$

Ans.

192

Ⓐ Principle of Rotating machines Ⓐ

Let the fluid enters the rotor at a radius r_1 , then the rate of angular momentum at inlet is $\dot{m} v_{w1} r_1$, where v_{w1} is whirl component or tangential component of absolute velocity at inlet. Similarly, if the fluid leaves the rotor at a radius r_2 then rate of angular momentum at exit is $\dot{m} v_{w2} r_2$.

$$\begin{aligned} \text{Rate of change of Angular momentum} \\ = \dot{m} v_{w2} r_2 - \dot{m} v_{w1} r_1 \end{aligned}$$

But we know that

$$\text{Rate of change of angular momentum} = \text{Torque}$$

$$\Rightarrow \dot{m} v_{w2} r_2 - \dot{m} v_{w1} r_1 = T$$

$$\text{Power} = T \times \omega$$

$$P = (\dot{m} v_{w2} r_2 - \dot{m} v_{w1} r_1) \omega$$

$$P = \dot{m} [v_{w2} r_2 \omega - v_{w1} r_1 \omega]$$

$$P = \dot{m} [v_{w2} u_2 - v_{w1} u_1]$$

enter's equation for Compressor and Pump

$$P = \dot{m} [v_{w1} u_1 - v_{w2} u_2]$$

For Turbine

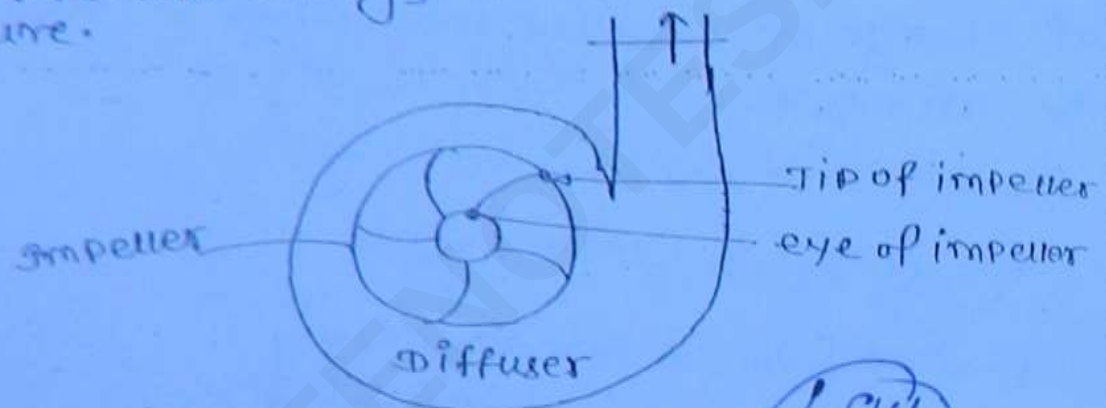
When energy is transferred from fluid to rotor, then the device is turbine.

When energy is transferred from rotor to fluid then the device is compressor or pump.

Centrifugal Compressor III

Principle of operation of a centrifugal compressor

In centrifugal compressor, the fluid enters axially at the eye of impeller and leaves radially at the tip of impeller. When the fluid moves radially outwards, i.e. to centrifugal force the pressure increases. The impeller also imparts kinetic energy to the fluid and hence the fluid leaves the impeller at high kinetic energy and high pressure energy. The fluid after leaving the impeller enters diffuser, where K.E. of fluid is converted into pressure energy. Thus the fluid leaves at high pressure.



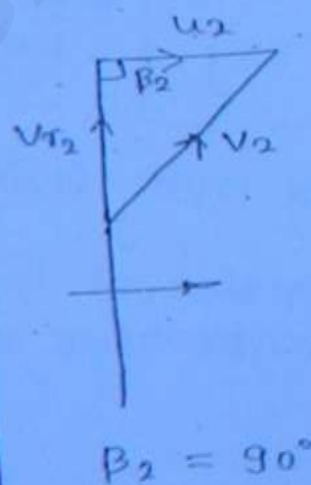
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Blade Angle Triangles

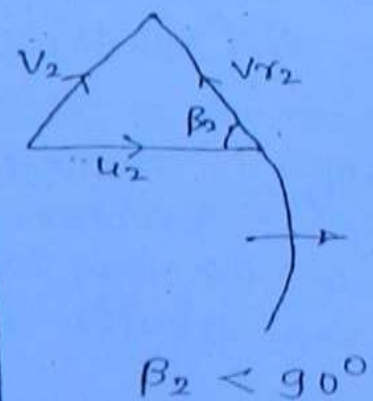
Forward Angle Blade



Radial Blade

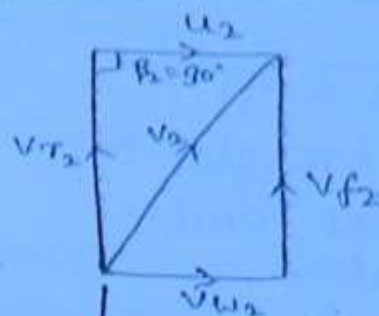


Backward angle blade



Ques. If nothing is mentioned then assume radial blade.

Ⓐ Power input for Radial Blade $\Rightarrow$



$$P = \dot{m} [Vw_2 u_2 - Vw_1 u_1]$$

But $Vw_1 = 0$

$$P = \dot{m} [Vw_2 u_2]$$

But $Vw_2 = u_2$

$$\therefore \boxed{P = \dot{m} u_2^2}$$

$V_{f1} = V_1$

$Vw_1 = 0$

$\alpha_1 = 90^\circ$

Ⓑ Slip in centrifugal compressor $\Rightarrow$

Due to Inertia the fluid particles are reluctant to move around the blade passage and because of this there is reduction in Vw_2 .

$$\text{Slip} = u_2 - Vw_2$$

$$\text{Slip factor } \sigma = \frac{Vw_2}{u_2}$$

$$P = \dot{m} [Vw_2 u_2 - Vw_1 u_1]$$

for radial blade $Vw_1 = 0$

$$P = \dot{m} [Vw_2 u_2]$$

but $Vw_2 = \sigma u_2$

$$\therefore \boxed{P = \dot{m} \sigma u_2^2}$$

Ⓒ Power input factor (ψ) $\Rightarrow$

To account for frictional losses power input factor is introduced and it is the ratio of actual power input to the theoretical power input

$$\boxed{P = \dot{m} \sigma \psi u_2^2}$$

$$\psi = 1.035 \text{ to } 1.05$$

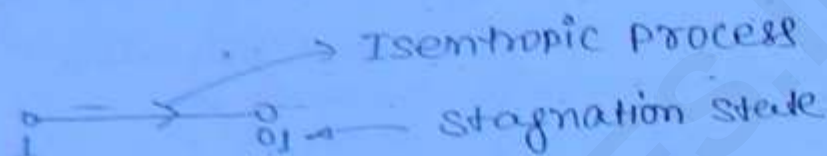
195

⇒ If ψ and σ are not given in the problem, then take them unity.

• USE OF SFEE IN Centrifugal compressor ⇒

Stagnation State: When the fluid is brought to zero velocity isentropically then that state reached is known as stagnation state.

$$h_{01} = h_1 + \frac{V_1^2}{2}$$



$$h_1 + \frac{V_1^2}{2} + z_1 g + \phi = h_2 + \frac{V_2^2}{2} + gz_2 + W$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + W$$

$$h_{01} = h_{02} + W$$

$$W = h_{01} - h_{02}$$

$$W = - (h_{02} - h_{01})$$

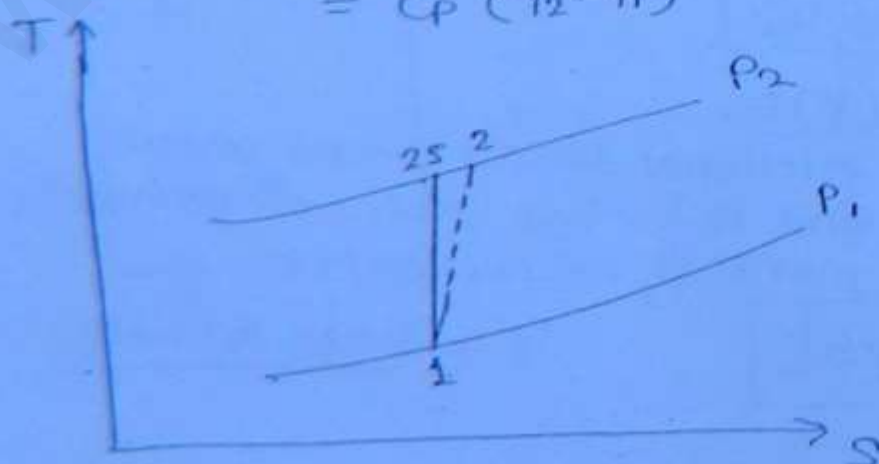
$$W_{\text{input}} = h_{02} - h_{01}$$

$$= C_p (T_{02} - T_{01})$$

If kinetic energy is neglected then

$$W_{\text{input}} = h_2 - h_1$$

$$= C_p (T_2 - T_1)$$



198

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

**

We know:

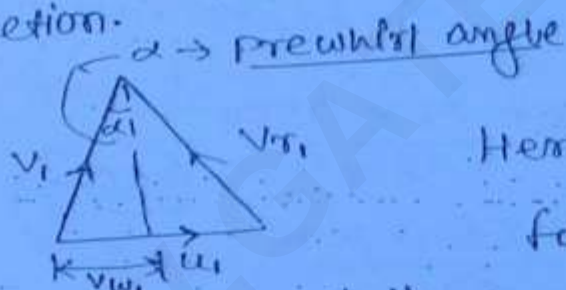
$$W = \sigma \psi u_2^2 \rightarrow \frac{m^2}{s^2} \rightarrow J/kg$$

$$W = C_p (T_2 - T_1) \rightarrow kJ/kg$$

$$\text{But } \frac{J}{kg} = \frac{Nm}{kg} = \frac{kg \cdot m \cdot m}{s^2 \cdot kg} \Rightarrow \frac{m^2}{s^2}$$

Ⓐ Prewirl in centrifugal compressor $\Rightarrow$

To Reduce the mach No. at the inlet prewhirl is given. Generally this prewhirl is mentioned with respect to axial direction.



$$\text{Here } M = \frac{V_{r1}}{c}$$

for same u_1

** As α increases $\Rightarrow V_{r1} \downarrow$

PROB: ①

A centrifugal compressor has an impeller dia of 0.5 m at the tip and running at 7000 rpm. Assume zero whirl at inlet and $T_1 = 290K$, $\sigma = 1$, $\psi = 1$, and process of compression is isentropic. Determine the pressure ratio developed and specific work input. Take $C_p = 1.005 kJ/kg \cdot K$

Soln.

$$D_2 = 0.5 \text{ m}, \quad N = 7000 \text{ rpm}$$

$$\alpha = 0, \quad T_1 = 290K, \quad \sigma = 1, \quad \psi = 1$$

$$u_2 = \frac{\pi D_2 N}{60} = 183.25 \text{ m/s}$$

$$W = \sigma \psi u_1^2 = 1 \times 1 \times (183.25)^2$$

$$= 33584 \text{ J/kg}$$

$$\boxed{W = 33.584 \text{ kJ/kg}}$$

$$\text{Also } W = h_2 - h_1 = C_p (T_2 - T_1)$$

$$33.584 = 1.005 (T_2 - 290)$$

$$\boxed{T_2 = 323.4 \text{ K}}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{323.4}{290}\right)^{\frac{1.4}{1.4-1}}$$

$$\text{Pressure ratio} = 1.465$$

Prob. 2 Determine the isentropic efficiency of compressor which under test gave following result $N = 11500 \text{ rpm}$, $T_1 = 21^\circ\text{C} = 273 + 21 = 294 \text{ K}$, pressure ratio = 4, Impeller tip dia. $D_2 = 0.75 \text{ m}$, slip factor $\sigma = 0.92$

$$\omega = u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.75 \times 11500}{60}$$

$$\boxed{u_2 = 451.6 \text{ m/s}}$$

$$W = \sigma \psi u_2^2 = 0.92 \times 1 \times (451.6)^2$$

$$= 187630 \text{ J/kg}$$

$$= 187.63 \text{ kJ/kg}$$

$$\text{Also } W = C_p (T_2 - T_1)$$

$$187.63 = 1.005 (T_2 - 294)$$

$$\boxed{T_2 = 480.7 \text{ K}}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{294} = (4)^{\frac{1.4-1}{1.4}}$$

(198)

$$T_{2s} = 436.9 \text{ K}$$

$$\text{Isentropic efficiency } \eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \parallel$$

$$\eta_c = \frac{436.9 - 294}{480.7 - 294}$$

$$\boxed{\eta_c = 76.5\%}$$

I.E.S. 2009

PROB. (3)

A Centrifugal Compressor running at $N = 16000 \text{ rpm}$ takes air at 17°C and compresses it to a pressure ratio of 4 with an isentropic efficiency of 82%. At the exit the blade are radially inclined and slip factor is 0.85. Guide vanes at inlet give air a prewhirl angle of 20° to the axial direction. The dia of impeller eye is 200 mm and absolute velocity of air at inlet is 120 m/sec. Find impeller tip dia.

Soln

$$N = 16000 \text{ rpm}, \quad T_1 = 17^\circ\text{C} = 273 + 17 = 290 \text{ K},$$

$$\frac{P_2}{P_1} = 4, \quad \eta_c = 82\%, \quad \sigma = 0.85, \quad \alpha = 20^\circ$$

$$D_1 = 200 \text{ mm} = 0.2 \text{ m}, \quad V_1 = 120 \text{ m/sec.}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60} = 167.55 \text{ m/s}$$

$$V_{w1} = V_1 \sin \alpha = 120 \times \sin 20^\circ$$

$$V_{w1} = 41.04 \text{ m/s}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{2s}}{290} = 1.486$$

$$\boxed{T_{2s} = 430.9 \text{ K}}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.82 = \frac{430.9 - 290}{T_2 - 290}$$

$$\boxed{T_2 = 461.8 \text{ K}}$$

$$C_p (T_2 - T_1) \\ = 1.005 (461.8 - 290)$$

$$W = 172.688 \text{ kJ/kg}$$

$$\text{But also } W = (V_{w2} u_2 - V_{w1} u_1)$$

$$\text{But } V_{w2} = \sigma u_2$$

$$10^3 \times 172.688 = [0.85 \times u_2^2 - 41.04 \times 167.55]$$

$$u_2^2 = 211252.6 \text{ m}^2/\text{s}^2$$

$$\boxed{u_2 = 459.6} \text{ m/s}$$

$$\text{But } u_2 = \frac{\pi D_2 N}{60}$$

$$459.6 = \frac{\pi \times D_2 \times 16000}{60}$$

$$D_2 = 0.549 \text{ m}$$

Imp dia.

$$\boxed{D_2 = 549 \text{ mm}}$$

Ans.

~~Imp dia~~

200

Power P_{ANT} - 