(144)

-: HAND WRITTEN NOTES:-

OF

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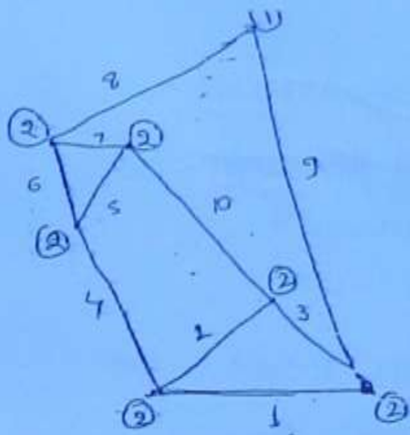
MECHANICAL ENGINEERING

(1)

-: SUBJECT:-

THEORY OF MACHINES

2

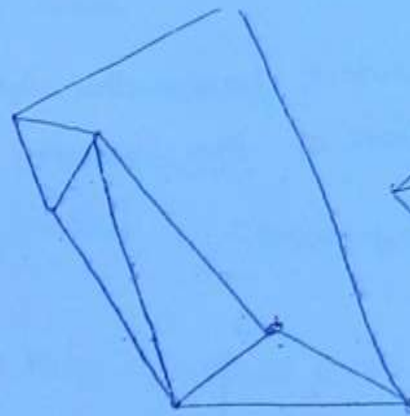


$e = 10$

$j = 13$

$\boxed{13 = 13}$

$\therefore$ kinematic chain (KC)

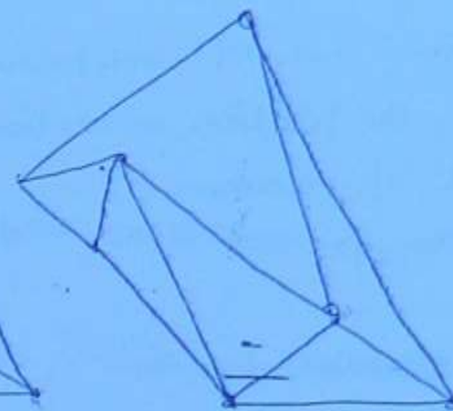


$\boxed{e = 11}$

$\boxed{j = 15}$

$15 > 14.5$

Frame



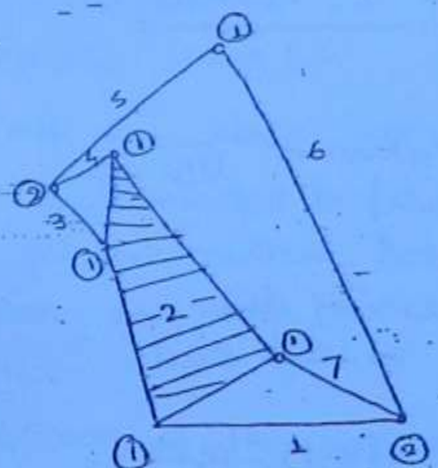
$e = 12$

$j = 17$

$17 > 16$

super structure
OR
Indeterminate structure

Problem :



3

$e = 7$

$j = 9$

$\boxed{9 > 8.5}$

Frame

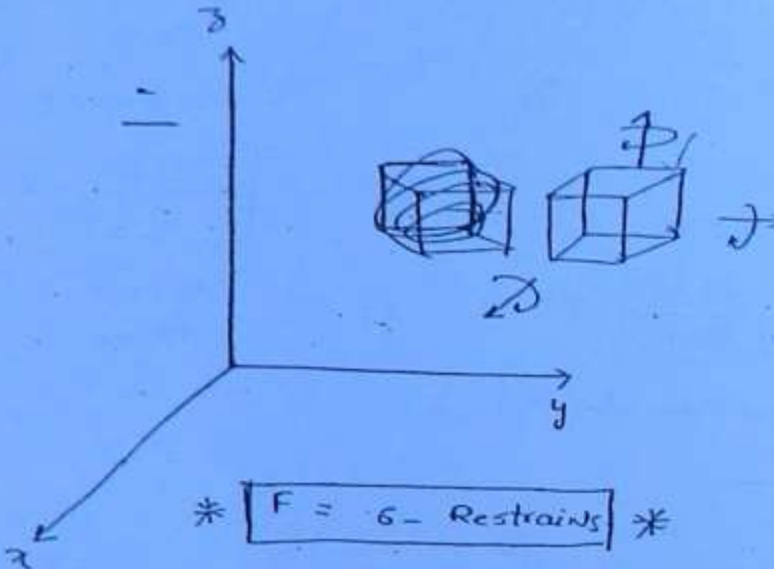
* Degrees of Freedom :-

The minimum no. of independent parameters require to define the the position or motion of the system is known as Degrees of Freedom

In TOM, restraind are in the form of pair

LP → 1 DOF.

HP → 2 DOF.



WWW.GATENOTES.IN

* $\boxed{F = 6 - \text{Restrains}}$ *

4

Pair	Restrains	DOF
	3T + 2R = 5	6-5 = 1
	0 1T	6-1 = 5
	1R + 1R = 2	6-2 = 4

Mechanism (3D)

↓

F

l = No. of Links

one Link fixed

$$F = 6(l-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

P_1 – No. of those pair whose DOF. = 1

P_2 ——— do ——— = 2

P_3 ——— do ——— = 3

P_4 ——— do ——— = 4

P_5 ——— do ——— = 5

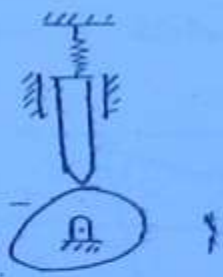
3> According to the type of clousre :

a> Self closed Pair (closed Pair) :-

Pamanent contact . ~~sliding pair~~ , Ball bearing, cylinder & piston

b> Forced closed Pair (Unclosed Pair) : - Forceful contact

e.g.



Cam and follower

e.g. automatic clutch operating system

~~...~~ doors closure

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* kinematic chain :-

If all the links are connected in such a way that first link is connected to the last link to form a closed chain and the relative motion Between any two links is the constrained motion and such a chain is known as kinematic chain.

When one of the link of the kinematic chain is fixed it Becomes a mechanism which can give desired output w.r.t some given input.

A mechanism or groups of mechanism when utilized and when ~~...~~ desired output is obtained it becomes a machine.

- Conditions for the kinematic chain :-

1>
$$l = 2p - 4$$

l = No. of links

p = No. of kinematic Pairs

$\boxed{j = \frac{3}{2}l - 2}$ $\longrightarrow$ $\boxed{j + \frac{h}{2} = \frac{3l}{2} - 2}$

where,

~~$J \cdot \left(\frac{3}{2}l - 2\right)$~~

h = No. of higher pair

where j = No. of Binary joints

l = No. of links

$\boxed{j = \frac{3}{2}l - 2}$

$l = 4$

$j = 4$

$4 = \frac{3}{2} \times 4 - 2$

$\boxed{4 = 4}$

Kinematic chain.

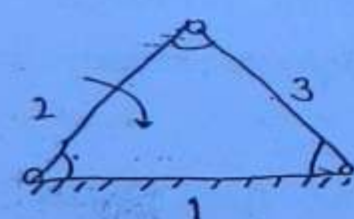
6

$\boxed{j > \frac{3}{2}l - 2}$

$\Downarrow$

No relative motion

Frame/ structure



$l = 3$

$j = 3$

$\boxed{3 > 2.5}$

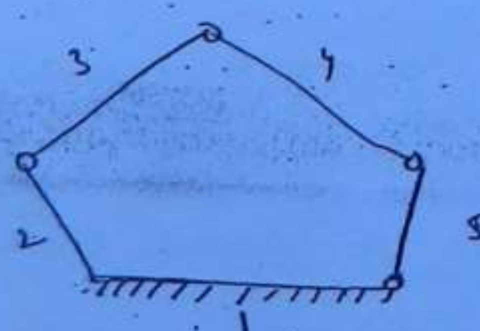
motion and Power can't be transmitted
only force can be transmitted.

$\boxed{j < \frac{3}{2}l - 2}$

$\Downarrow$

Relative motion

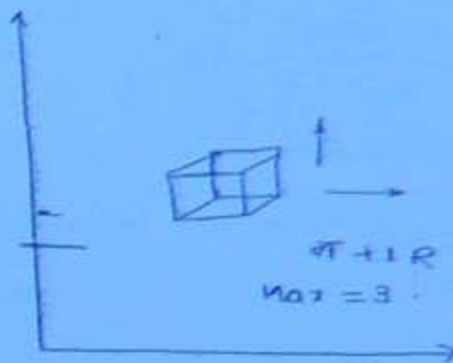
$\Rightarrow$ Unconstrained



$\boxed{5 < 5.5}$

★ Planner Mechanism

2-D



$$F = 3(l-1) - 2P_1 - 1P_2$$

P_1 = No. of LP or Binary joint (j)

P_2 = No. of ~~higher~~ H.P → h

$$\therefore \boxed{F = 3(l-1) - 2j - h}$$ → kutzback criterion Equation

$$\boxed{F' = [3(l-1) - 2j - h] - F_r}$$

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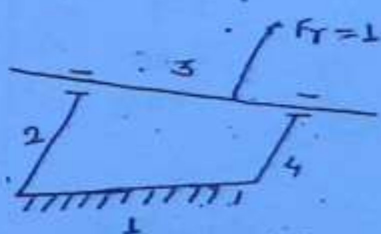
(Redundant degree of freedom)

⇓

Those independent motions which are not the part of the mechanism.

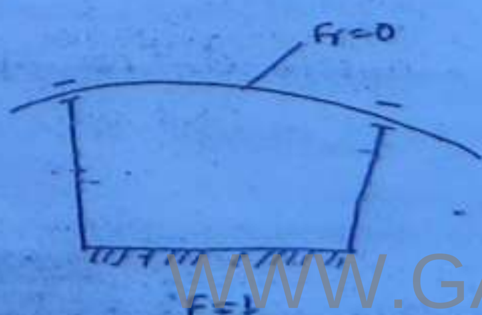
Pb



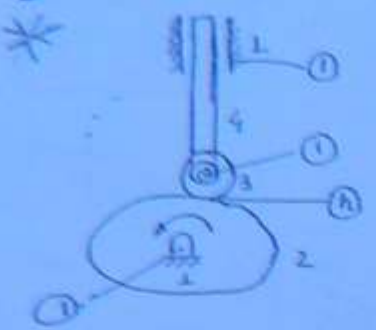
$l = 4 \quad j = 4 \quad h = 0$ — Ans $F = 0$

$$F = 3(l-1) - 2\times j - h = 3(4-1) - 2\times 4 - 0 = 1 - F_r = 0.$$

Pb.



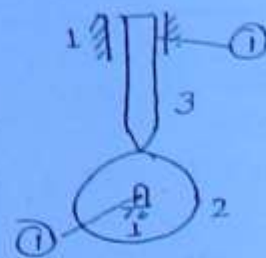
*



$l = 4$
$j = 3$
$h = 1$

$F = [3(4-1) - 2\times3 - 1] - 1$
$= 2 - 1$
$= 1$

pb.



$l = 3$
$j = 2$
$h = 1$

$F = 3(3-1) - 2\times2 - 1$
$= 1$

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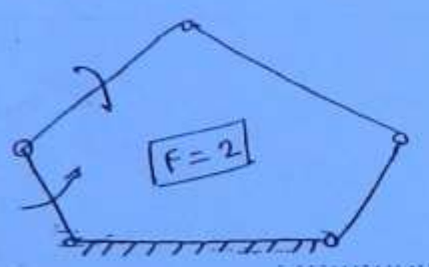
If $\boxed{F = 0}$ ⟶ Frame / structure

$\boxed{F = -1, = -2, = -3, \ldots}$ ⟶ Super structure

$\boxed{F = 1}$ ⟶ kinematic chain

$\boxed{F = 2, = 3, = 4, \ldots}$ ⟶ Unconstrained



$l = 5$
$j = 5$
$h = 0$
$F = 2$

Note :- DOF of a mechanism is equal to the no of input required to get a constrained output

Grubler's Equations :-

+

Kutzback eq^n :-

$\hookrightarrow F = [3(l-1) - 2j - h]$

Only those ~~Mechanical~~ Mechanism

$\boxed{F = 1}$ $\boxed{h = 0}$

$1 = 3(l-1) - 2j - 0$

$1 = 3l - 3 - 2j$

$\boxed{3l - (2j+4) = 0}$ → Grubler's Equation

$3l \rightarrow$ even

↓

$\boxed{L \rightarrow \text{even}}$ **

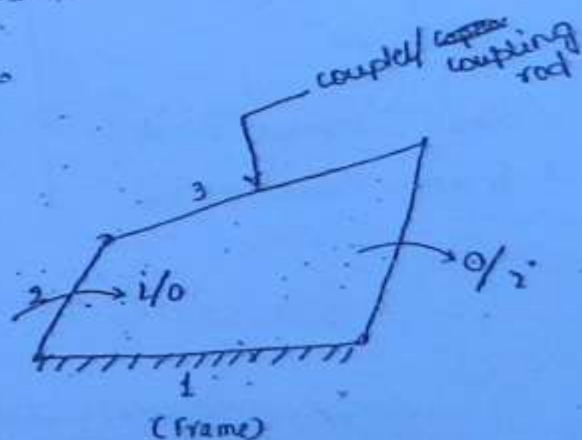
↓

$\boxed{L_{min} = 4}$

4- bar Mechanism

↓

$\boxed{\begin{matrix} \text{4 Links} \\ + \\ \text{4TP} \end{matrix}}$



i/o

↓

complete Rotation → Crank

Partial Rotation → Rocker/ Lever

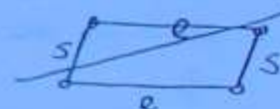
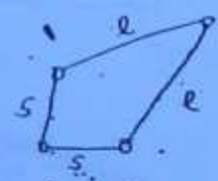
oscillation

1. Double crank - Mechanism
2. Crank - Rocker Mechanim
3. Double - Rocker Mechanism

* Grashof's law :-

For the continuous relative motion between the link and the mechanism the summation of shortest and longest link should not be ~~greater~~ less greater than other two links.

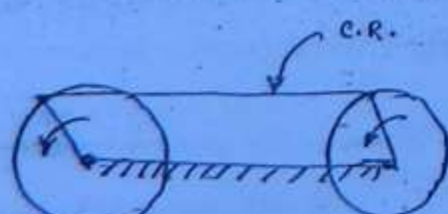
$$(s+l) \le (p+q)$$

$(s+l) < (p+q)$	$(s+l) = (p+q)$	$(s+l) = (p+q)$	If the law is not satisfied $(s+l) > (p+q)$ ⇓ Double rocker
	2, 5, 3, 4	having pairs of equal length	
	not having pairs of equal length	2, 2, 3, 3	
1. s-fixed — Double crank 2. s adjucent to fixed — Crank-rocker 3. s → Coupler - Double rocker	same	1. Parallelogram linkage  double crank in any case 2. Deltoid linkage  s' fixed – Double crank l fixed – Crank Rocker	

1. Coupling Rod of Locomotive :-

4 bar _

Double crank

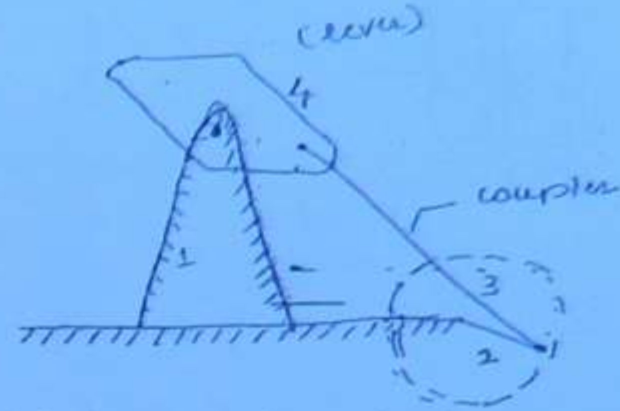


2> Beam Engine :-

4 bar

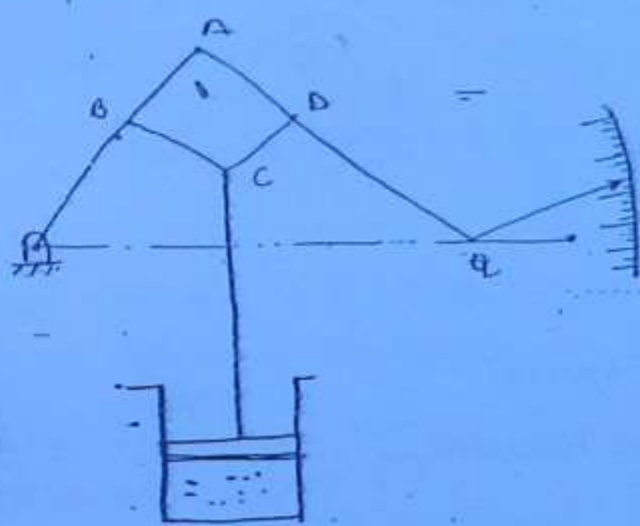
Crank - Rocker

Rot → oscillation



3> Watt's Indicator Mechanism : * *

oscillation → oscillation (Double Rocker Mechanism)



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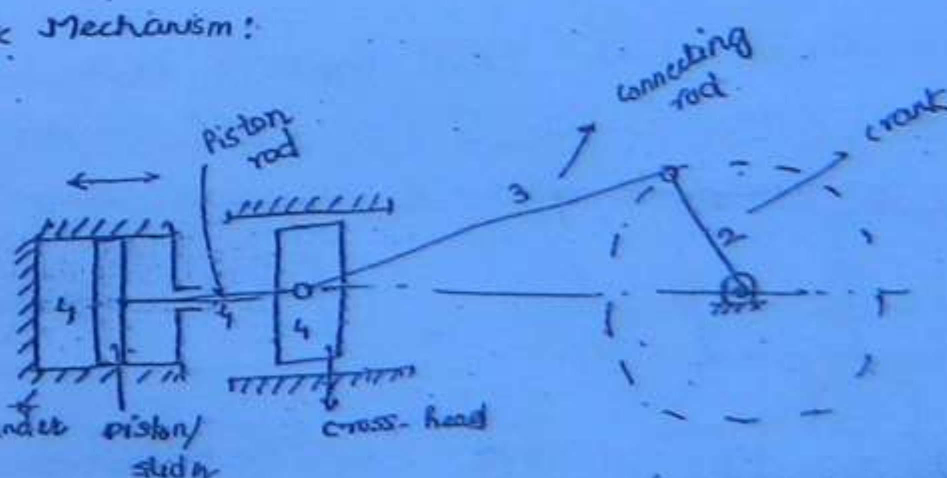
4> * Single slider Crank Mechanism :

4 links
+
3TP
+
1SP



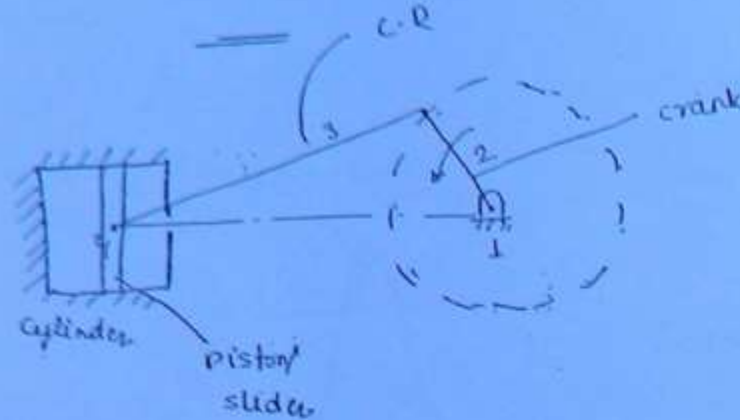


fig. I.C Engine

- Cylinder fixed :-

Rot $\longleftrightarrow$ Reciprocation

* 0 ⟵ (i) ⟶ Reciprocating Engine

(i) ⟶ (0) ⟶ " ~~compression~~ $comp^n$

(12)

- Crank fixed :-
  - Rotary I.C. - Engine (GNOME ENGINE)
  - Withworth QRM.

- connecting Rod fixed
  - Crank- slotted lever QRM
  - oscillating cy. Engine- Mechanism

- slider fixed :-

Hand Pump (Bull Engine) (Pendulum Pump).

$\boxed{F=1}$

$\boxed{F=0}$

* Crank and slotted lever QRMM :-

stroke :-

R_1R_2

$\Rightarrow C_1C_2$

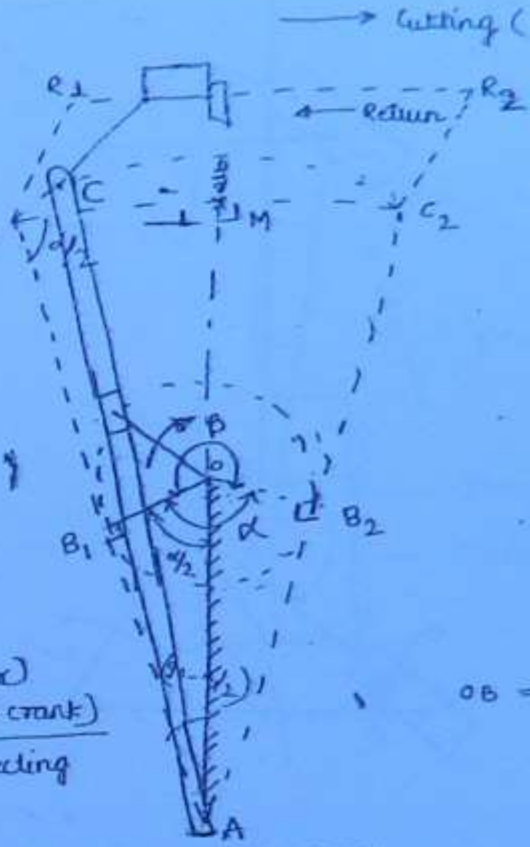
$\Rightarrow 2\,C_1M$

$\Rightarrow 2(AC_1)\cos\alpha/2$

$\Rightarrow 2\,\dfrac{AC_1 \cdot OB_1}{OA}$

$\Rightarrow \dfrac{2(AC)(OB)}{OA}$

stroke $= \dfrac{2\,(\text{length of slotted Bar}) \times (\text{length of crank})}{\text{length of connecting rod}}$



$\beta \rightarrow$ cutting angle

$\alpha \rightarrow$ Return stroke angle

$\alpha < \beta$

$\beta + \alpha = 360^\circ$

$\beta/\alpha \Rightarrow$ Quick Return Ratio (Always >1)

OB =

13

- single slider crank inversion

fig. Crank and slotted lever

- Motion is rotation to osculation

* Withworth QRMM :-
shortest link is fixed i.e., crank

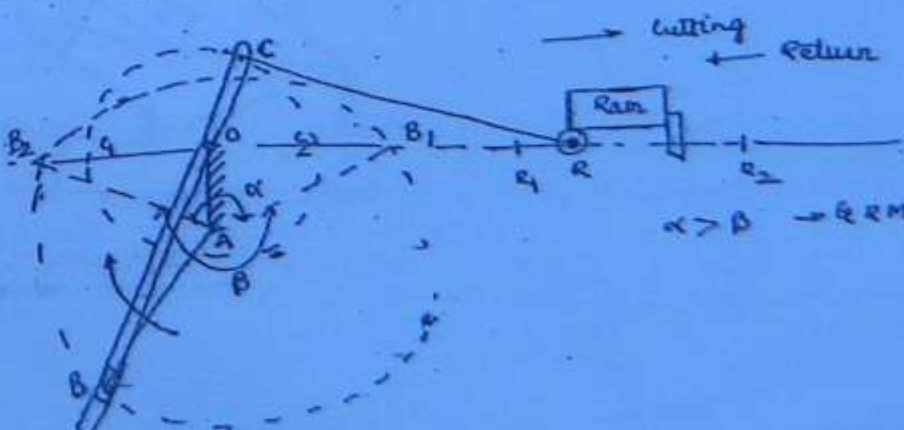
AB → Driving crank

stroke :
↓
R_1R_2

$\Rightarrow C_1C_2$

$\Rightarrow 2(OC)$

- Motion is rotation to rotation



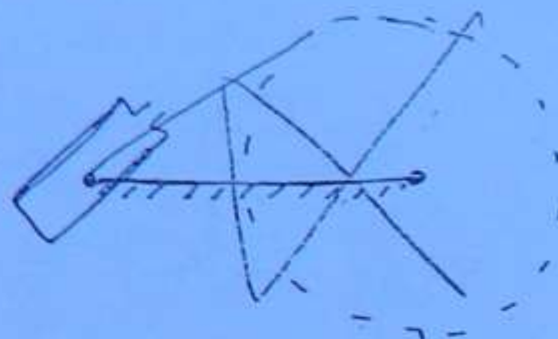
$\alpha > \beta \rightarrow$ QRMM

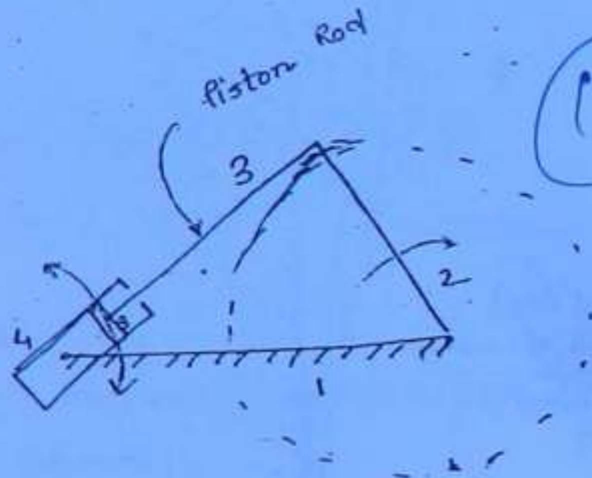
Crank and link QRMM

* oscillating cylinder Engine :-

[redacted]

- single slider crank mechanism
- connecting rod is fixed
- motion is rotation to oscillation.

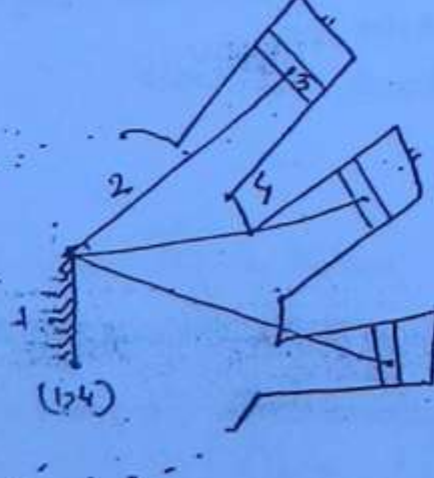




(19)

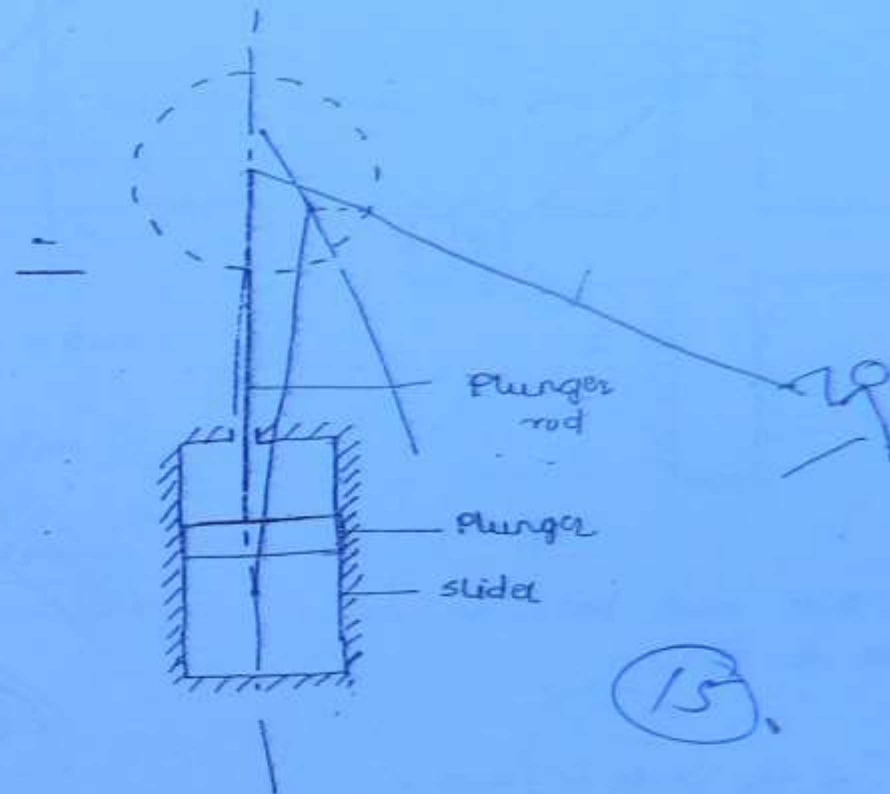
* Rotary Internal Combustion Engine (GNOME ENGINE):

- Input link is piston and o/p link is cylinder Block.



7 or 9 cylinders are mounted

* Hand Pump :-



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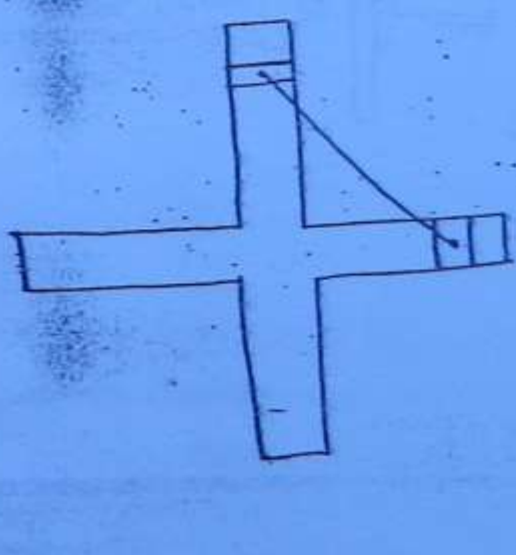
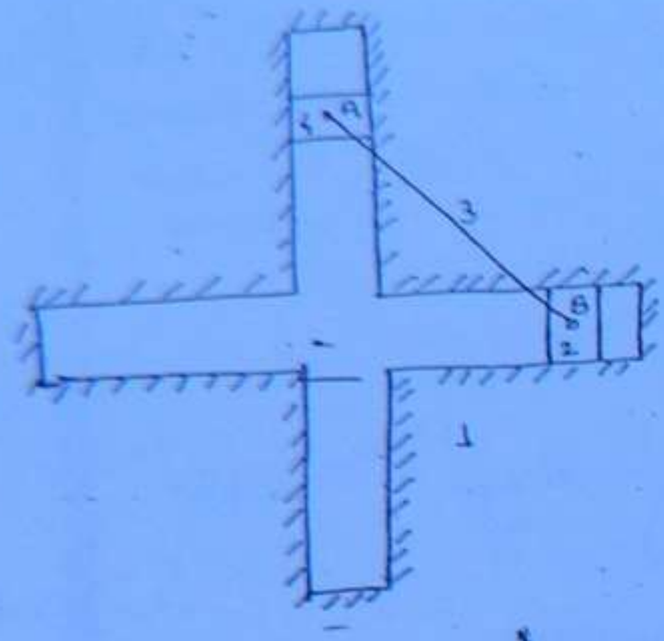
* Double slider Crank chain :-

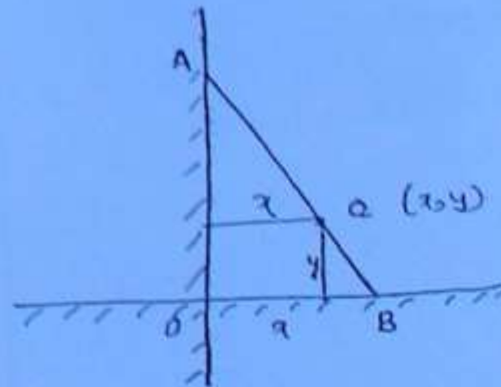


4 links
+
2TP
+
2SP

Three inversion can be obtained.

1. Slotted Plate Fixed :-
(Elliptical Trammels)





$\sin\theta = \frac{y}{BQ}$ $\cos\theta = \frac{x}{AB}$

$\therefore \sin^2\theta + \cos^2\theta = 1$

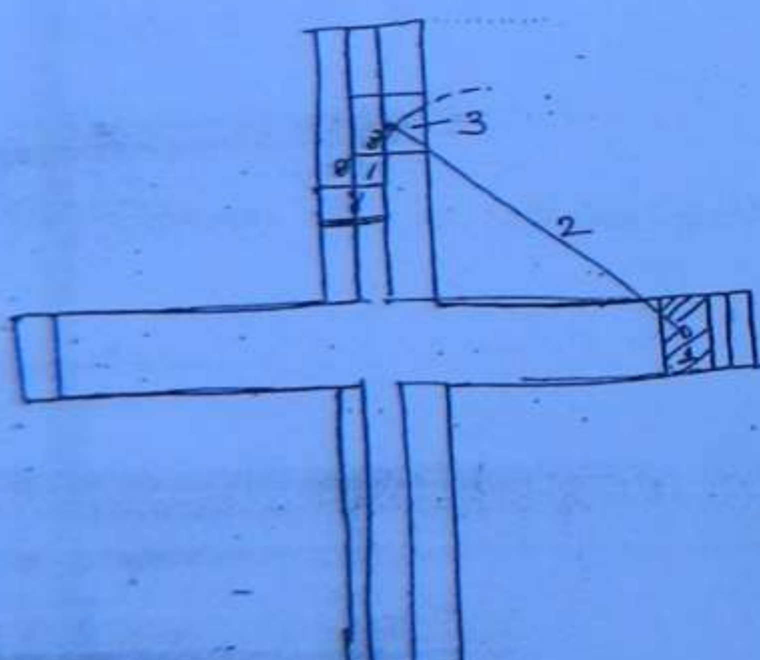
or, $\frac{y^2}{BQ^2} + \frac{x^2}{AB^2} = 1$

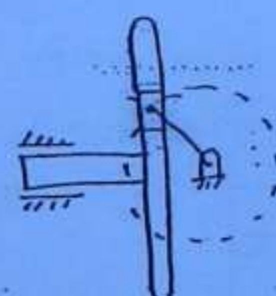
Ellipse

If Q is mid- point we can obtain a circle.

6

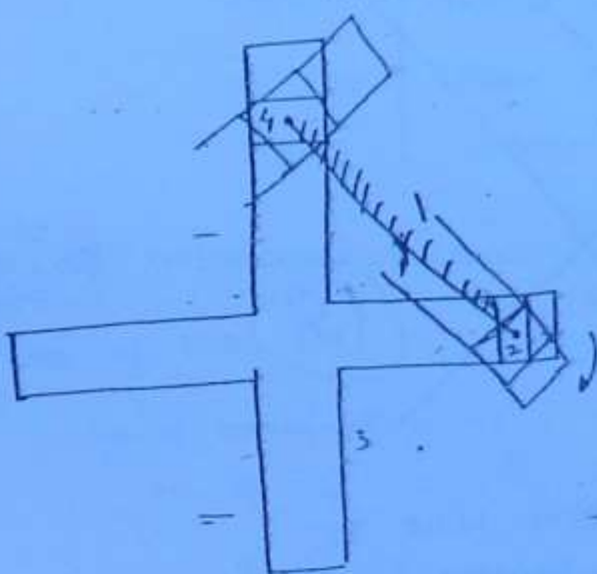
2. Any of the slider is fixed :
or, Scotch - Yoke Mechanism (Rot ⟷ Reciprocation)





3. link connecting ~~stator~~ rod is fixed:
(i.e. oldham's coupling)

" This coupling is basically used to connect the two shaft which has ~~[illegible]~~ Lateral misalignment"



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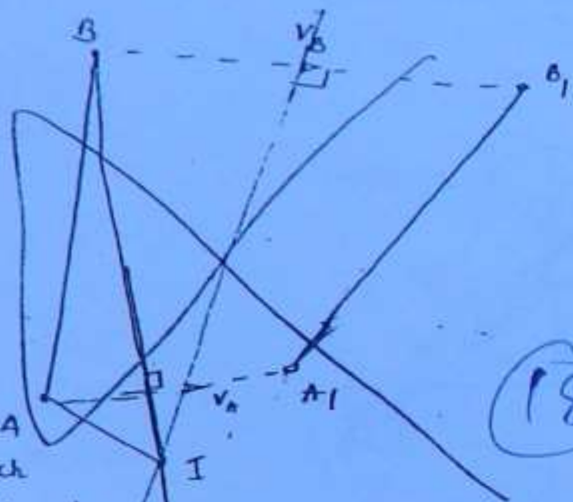
———o———

Velocity Analysis

1) Instantaneous Centre Method Approach :-

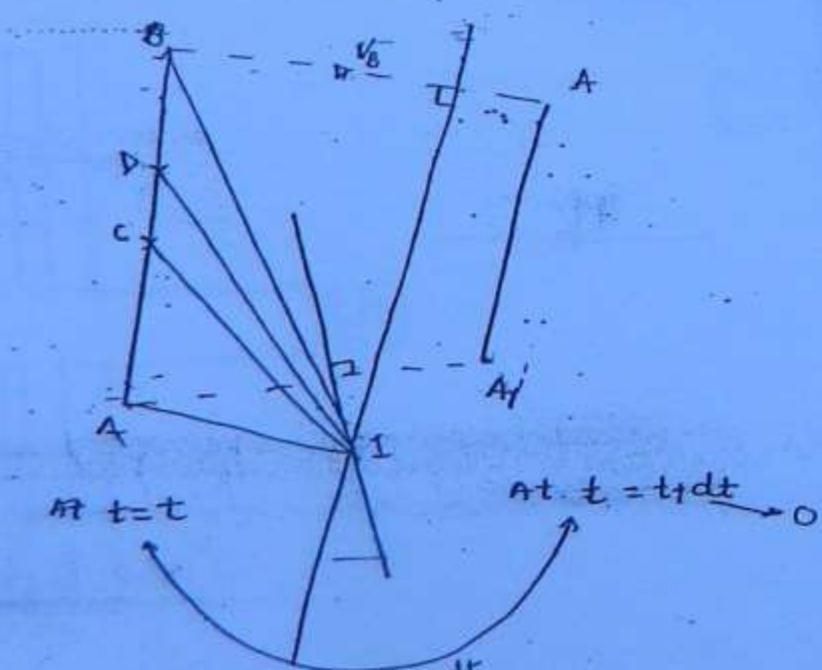
- Instantaneous Centre of Rotation :



In general a motion of a link in a mechanism is neither purely translational nor purely rotational it is the combination of translation and rotation, which is the general motion but the whole link at any movement can be assumed to be in perfect rotation w.r.t to a point in the space known as Instantaneous centre of rotation. This point is also known as Virtual Centre.

centrode

$$\omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \frac{V_D}{DI}$$



As the link is in motion its I-centre keeps on changing the locus of the I-centre for a particular link during its whole motion is known as centrode of the link.

The locus of instantaneous axis of rotation for a particular link during its whole motion is known as Axode of the link

Centrode	Axode
Curve	Curved surface
st. line	Plane surface
Point	line

No. of instantaneous centres in the Mechanism :-

No. of links $= l$

No. of combinations i.e.,

No. of IC $= {}^{l}C_{2} = \frac{l(l-1)}{2}$

if $l = 4$, $IC = 6$

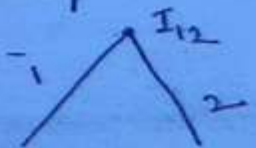
12 13 14
23 24
34

$l = 6$, $IC = 15$

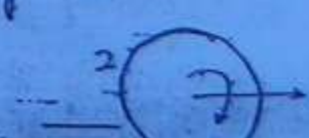
12 13 14 15 16
23 24 25 26
34 35 36
45 46
56

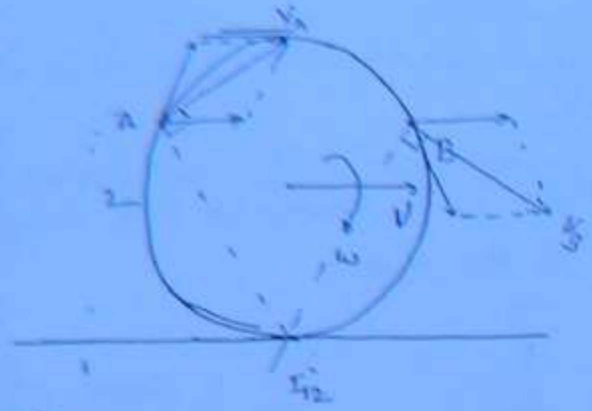
* Basic Instantaneous centre in the mechanism :-

1. Turning Pair



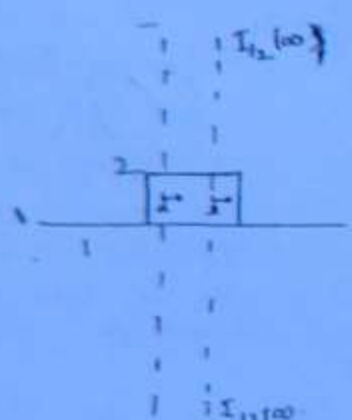
2. Rolling Pair :-

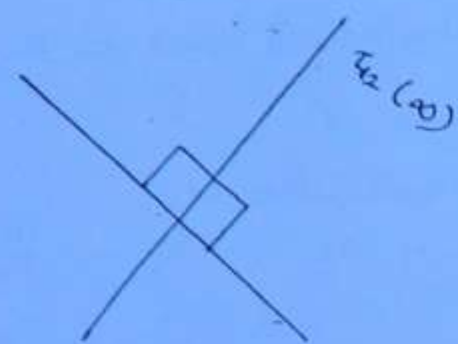




3. Sliding Pair

- Plane surface





Curve surface

- Concave

I centre is at the radius of curvature in sliding pair

convex

Note :- Any 3 IC ~~three~~ ~~in a mechanism~~ should be in a straight line for the relative motion in a mechanism → Kenedy theorem

Problem :-

Given

Link OA — 120 r.p.m (clock)

Find :-

$v_B = ?$ (3.2 m/s)

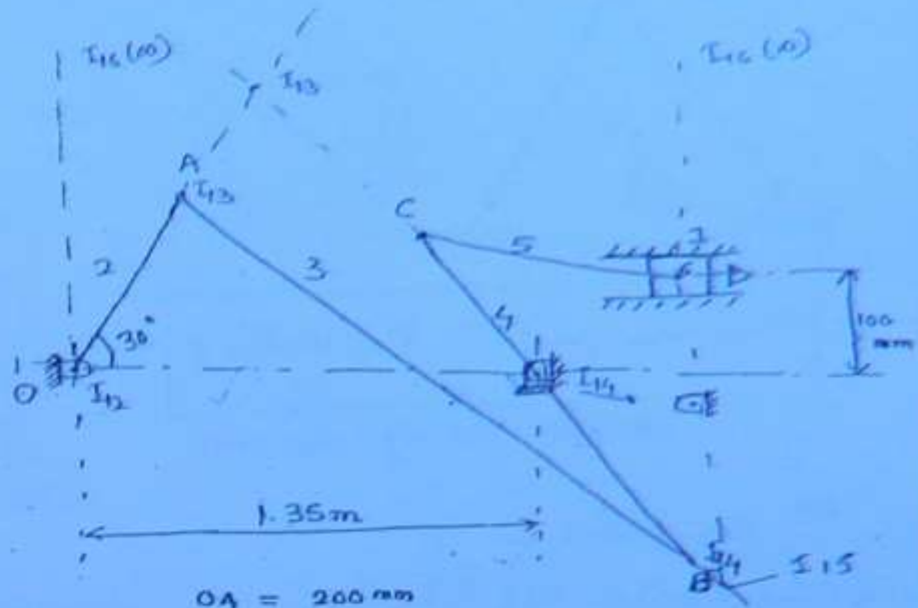
$v_C = ?$ (1.6 m/s)

$v_D = ?$ (1.08 m/s)

$\omega_{AB} = ?$ (2.99 rad/s)

$\omega_{BC} = ?$ (8 rad/s)

$\omega_{CD} =$ (2.16 rad/s).



OA = 200 mm

AB = 1.5 m

BC = 600 mm

CD = 500 mm

BE = 400 mm

fig. Configration diagram

$l = 6 \quad j = 7 \quad h = 0$

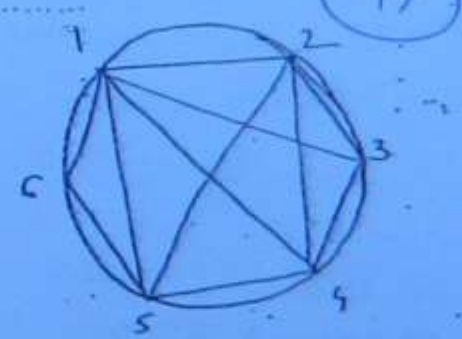
$3(6-1) - 2\times 7 - 0$

$= 1$

No. of links = 6

IC = 15

12	13	14	15	16
	23	24	25	26
		34	35	36
			45	46
				56



13 < 12, 23 ; 14, 43

35 < 34, 65 ; 32, 25 ; 31, 45

15 < 16, 65 ; 14, 45

24 < 23, 45 ; 21, 14

25 < 26, 15 ; 21, 15

26 < 25, 56

known IC should be given a line

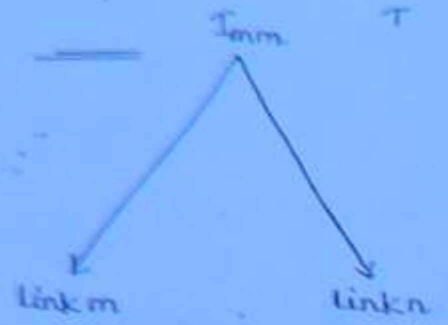
$v_A = OA \frac{(2\pi \times 120)}{60} = \frac{200}{1000} \times \frac{2\pi \times 120}{60}$

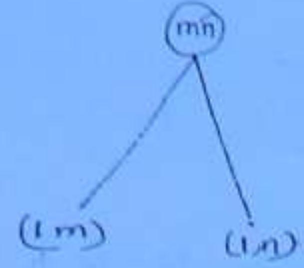
$= 2.5133 \text{ m/s}$

Link 3 : (I_{13}) (A, B)

$\omega_{AB} = \omega_3 = \frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}A}$

Link 4 - (B, C) - (I_{14}) $\quad \omega_{BC} = \omega_4 = \frac{v_B}{I_{14}B} = \frac{v_C}{I_{14}C}$ — ?





straight line
1,m,n

$$v_{I_{mn}} = \boxed{\omega_m (I_{mn} I_{1m}) = \omega_n (I_{mn} I_{1n})} = v_{I_{mn}}$$

Angular velocity theorem

22

25

$\omega_2 (I_{25} I_{12}) = \omega_5 (I_{25} I_{15})$

?

Note :-

If the I_c of a link is in the same side of fixed link (i.e., 1) the direction is same otherwise opposite

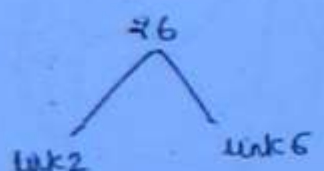
- i.e., If 1m and 1n lies on the same side of mn the direction is same otherwise opposite.

24

$\omega_2 (I_{24} I_{12}) = \omega_4 (I_{24} I_{14})$

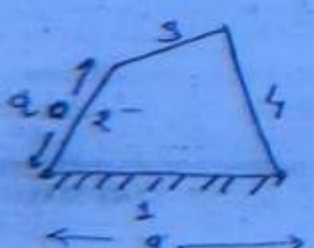
45

$\omega_4 (I_{45} I_{14}) = \omega_5 (I_{45} I_{15})$



$\omega_2 (I_{26} I_{12}) = v_D$

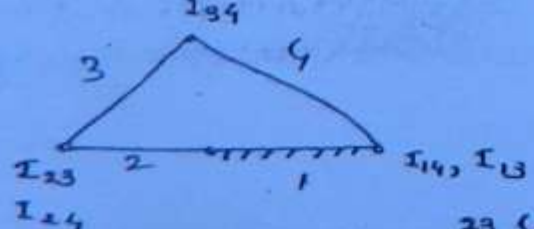
28|06|2011.



when $\theta = 180^\circ$

$\omega_2 = 2$ rad/s (clock)

$\omega_3 = ?$



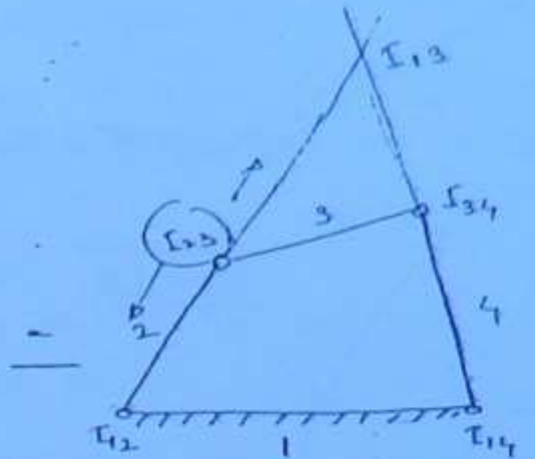
23 (angular velocity theorem)

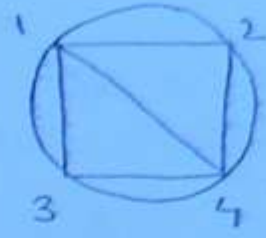
$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$

$2 \times a = \omega_3 (2a$

$\omega_3 = 1$ rad/s.

Pb.





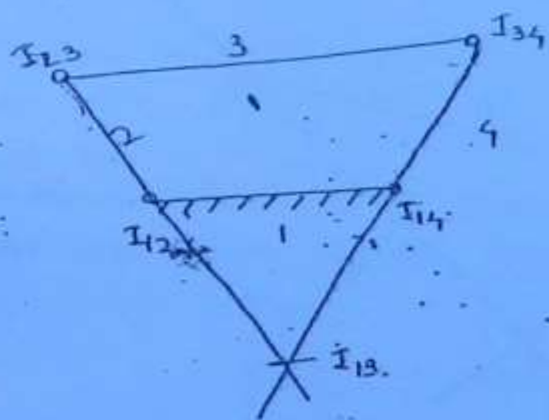
$\omega_2 =$ 5 rad/s (clockwise)

$\omega_3 :$ 14 rad/s

what will be the angular velocity of link 2 w.r.t link 3

$$\omega_{23} = \omega_2 - \omega_3$$
$$= (+5) - (-14)$$
$$= +19$$
$$= 19 \text{ (clockwise)}$$

Pb.

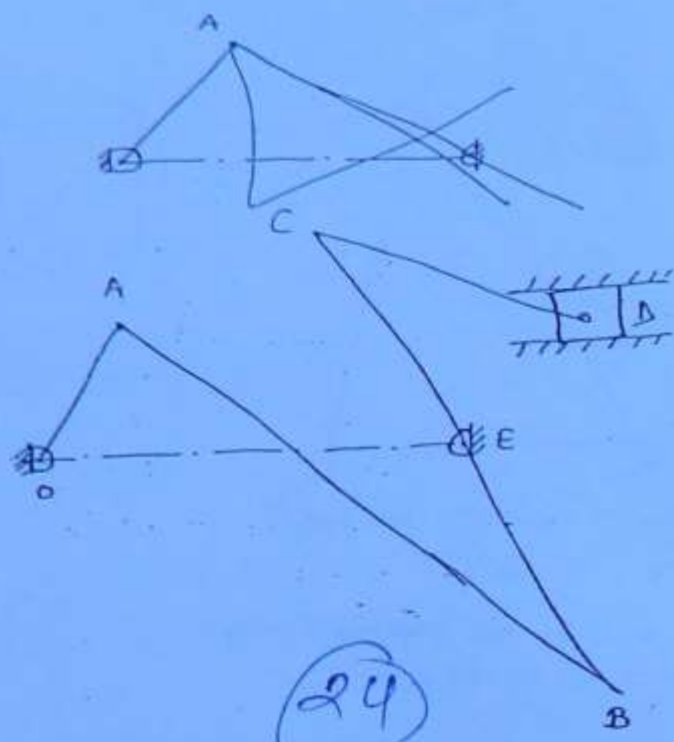


$$\omega_{23} = +5 - (+14)$$
$$= -9$$
$$= 9 \text{ (AC)}.$$

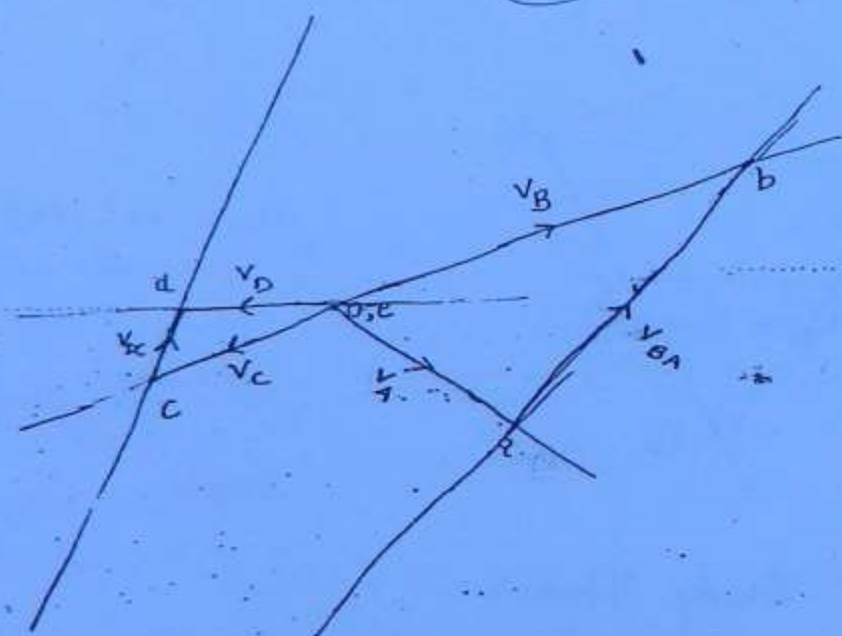
* Relative Velocity Approach :-

Velocity of point A w.r.t B will be in the direction $\perp^{ar}$ to the link AB

Point	w.r.t	Procedure
A	O	line $\perp^{ar}$ to OA
B	A	line $\perp^{ar}$ to AB
B	E	line $\perp^{ar}$ to EB
C	$\frac{BC}{BE}$ =	$\frac{bc}{be} \rightarrow ?$
D	C	line $\perp^{ar}$ to CD
D	fixed	line ∥ to the motion of slider



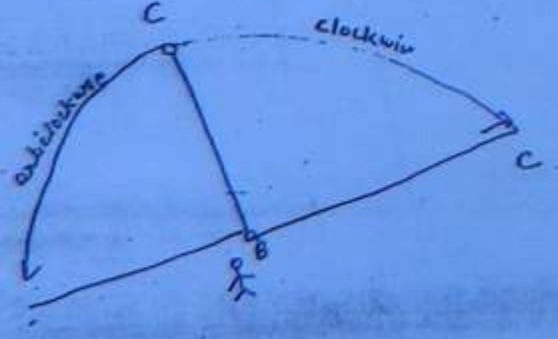
24



For direction.

- See the configuration diagram

· BC –



⇒ Direction of link BC is anti'clock wise

* Simple Mechanisms :-

. First harmonic motion → Simple Harmonic Motion
· all the first mechanism is simple mechanism

- Link :-

Every part of a machine which is having relative motion with respect to some other part is known as kinematic link or element

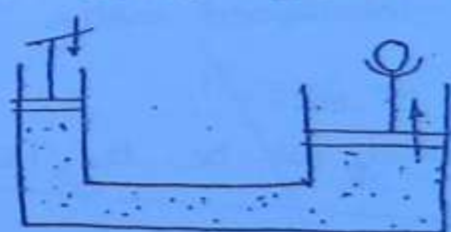
Note :- It is not necessary for the link to be rigid only but it is necessary for the link to be a resistant body so that it is capable of transmitting power from one body to the other ~~xxx~~ body.

rubber - flexibility /

25

Types of link :-

1> Rigid link : deformations are ~~negligible~~ negligible e.g., piston, connecting rod, piston etc
2> Flexible link : deformations are there but they are in permissible zone e.g. belt drive, rope drive etc.
3> Fluid link :
sometimes ~~fxx~~ power is transmitted because of the fluid pressure in that case fluid behaves like a link. eq. hydraulic break, coupling, Jack, press, crane, lift etc.



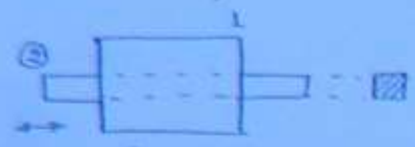
- Types of Relative motion :-

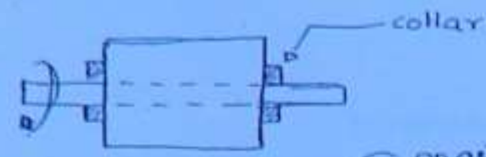
1> Completely constrained motion :- (self).
2> Successfully constrained motion :

constrained : ~~desig~~ desired (only one output w.r.t. input).

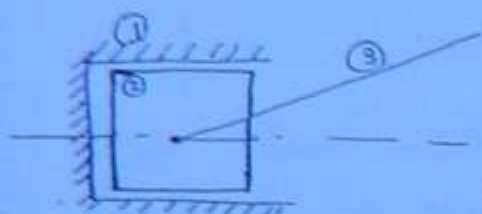
3> Incompletely constrained motion : unconstrained : (more than one output at same input)

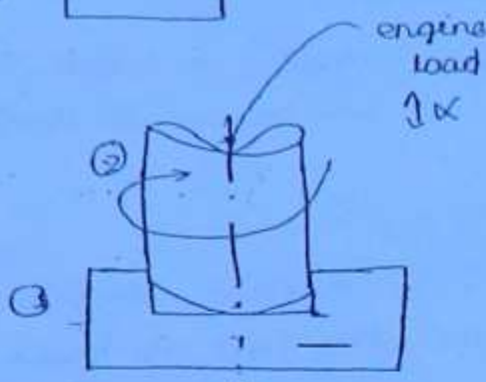
①





②





footstep Bearing

③

Incompletely constrained
motion

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* Kinematic ~~Link~~ & Pair :-

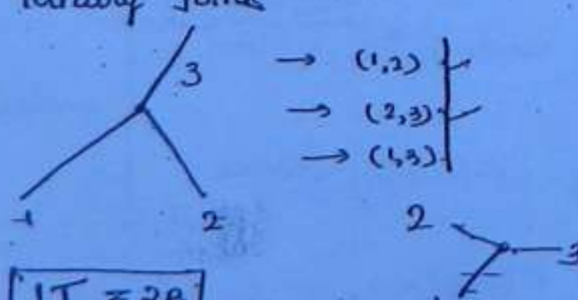
The connection between two links is ~~always~~ a joint or a pair but this pair is said to be kinematic pair if the relative motion between the links is the constrained motion.

- Every kinematic pair is, a pair or joint.
- But every pair or joint may or may not be the kinematic pair.

* Types of joints :-

Binary joints :

Ternary joints



$\boxed{1T = 2B}$

Pb. 2.

AB = 150 mm

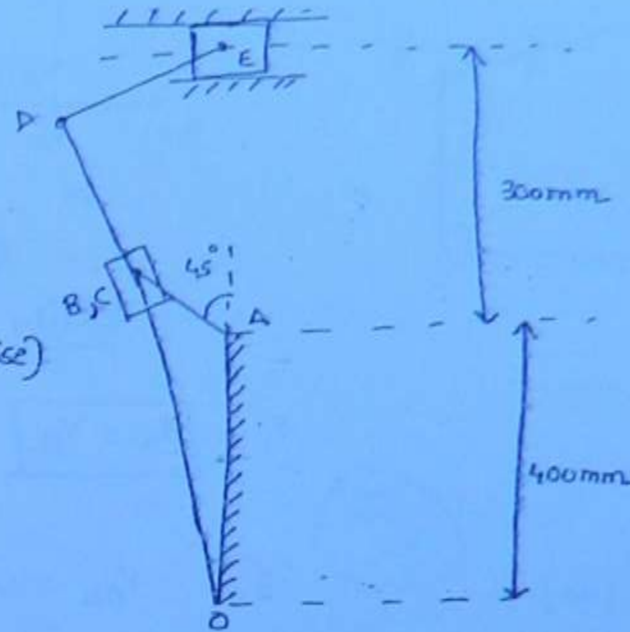
OD = 700 mm

DE = 200 mm

crank AB

↳ 120 r.p.m (anti-clockwise)

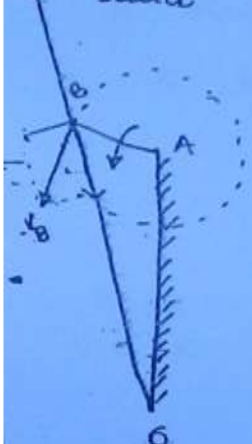
get v_E = ? (2.15 m/s).



Point B

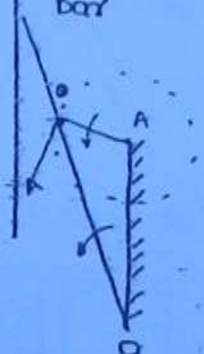
↓

slider



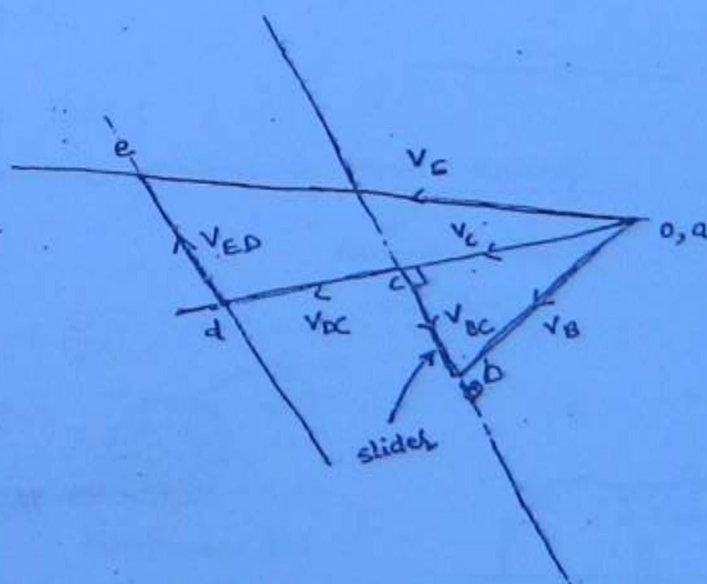
Point C

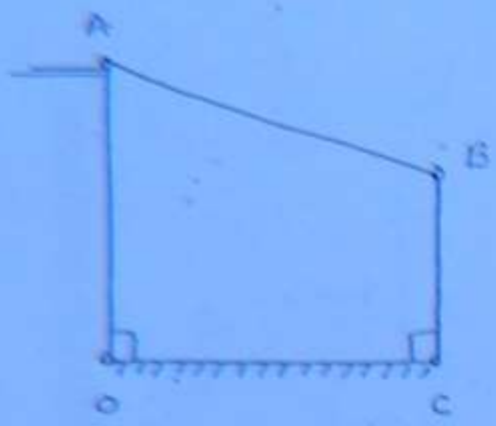
coincident point of point B but on slotted Bar



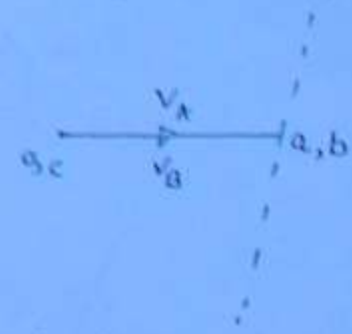
27

point	w.r.t	Procedure
C	B	line ∥ to slotted Bar
C	O	line $\perp^{ar}$ to slotted Bar





1. Velocity diagram:-



$\therefore$ Velocity diagram is a straight line

2. $\boxed{V_A = V_B}$

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3. $V_{AB} = 0$

$$\therefore \omega_{AB} = \frac{V_{AB}}{AB} = 0 \Rightarrow \text{pure translation}$$

4. $OA = 3\,cm$, $BC = 2\,cm$] $\omega_{OA} = 2\,rad/s$, $\omega_{BC} = ?$

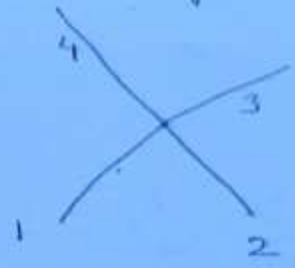
$$V_A = V_B$$

$$\omega_A \cdot r_A = \omega_B \cdot r_B$$

$$3 \times 2 = 2 \times \omega_B$$

$$\text{or, } \omega_B = 3\,rad/s.$$

Quaternary joints :



12 13 14
23 24
34

$\boxed{1q \equiv 3B}$

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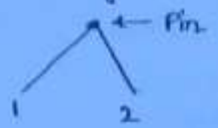
24/06/2011

* Classification of kinematic Pair :

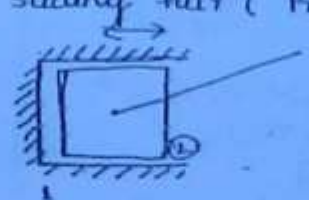
1> According to the types of Relative motion between the links :

a> Turning Pair (Revolute Pair) (Pin-joint) :-



crank pin

b> Sliding Pair (Prismatic pair) :



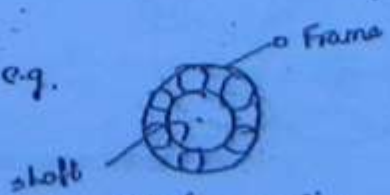
static friction is variable

c> Rolling Pair :

→ relative motion is pure Rolling

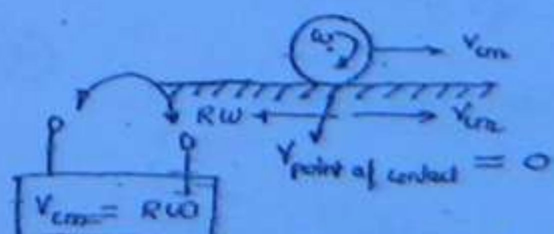
Rolling without slipping

e.g.



interference fit

fig. Ball Bearing



single start thread
linear movement = P(pitch)
double start = 2pitch
and so on.

d> Screw Pair

Nut and Bolt.

i) Spherical Pair (Ball and Socket Joint)

- Pen stand
- mirror of bike

→ 3-D Rotation
spherical rotation.

2> According to the type of contact :-

a> Lower Pair : → surface contact
e.g., TP, SP, screw Pair . spherical Pair

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b> Higher Pair : → Point / line contact :- Rolling Pair
e.g. cam and follower.

c> Wrapping Pair :- one link is wrapped over other
eq. Belt & pulley =. Rope and pulley

* $\boxed{1HP \equiv 2LP}$ *

* $\boxed{1HP \equiv 2LP}$ *

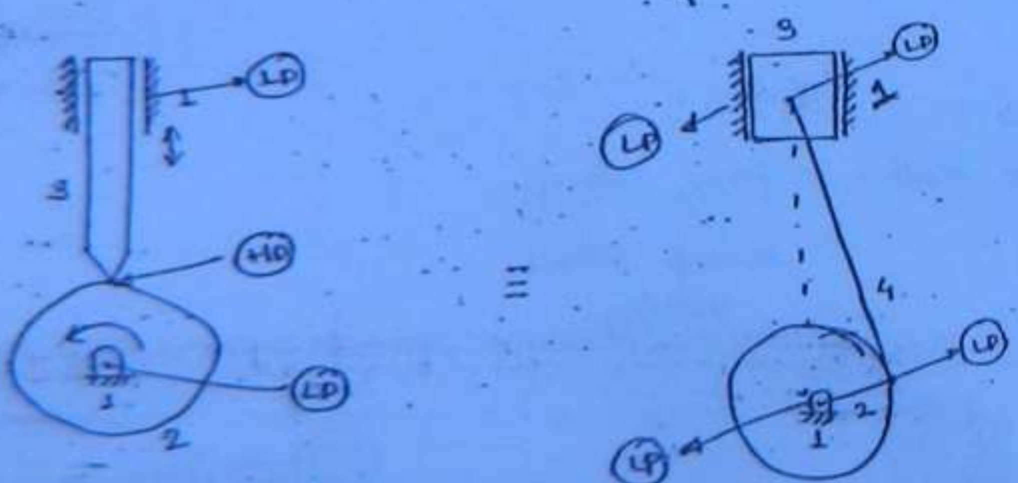
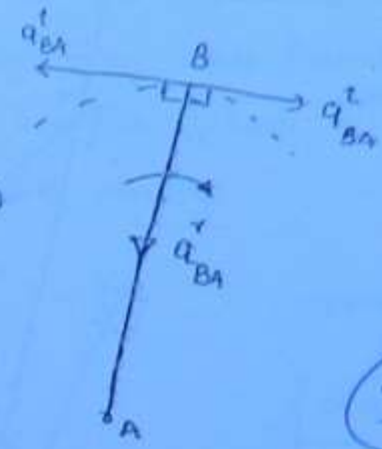


fig. Cam and follower

Acceleration Analysis

$$a_{BA} \begin{cases} a^r_{BA} = \dfrac{V^2_{BA}}{(BA)} \quad (B \to A) \quad \text{(All known)} \\ a^t = (BA)\,\alpha_{BA} \quad (\perp^{ar} \text{ to radial}) \end{cases}$$

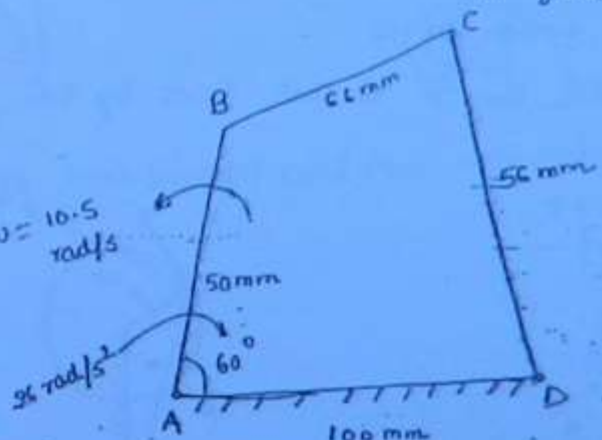


configuration diagram

and acceleration is not given take it as zero

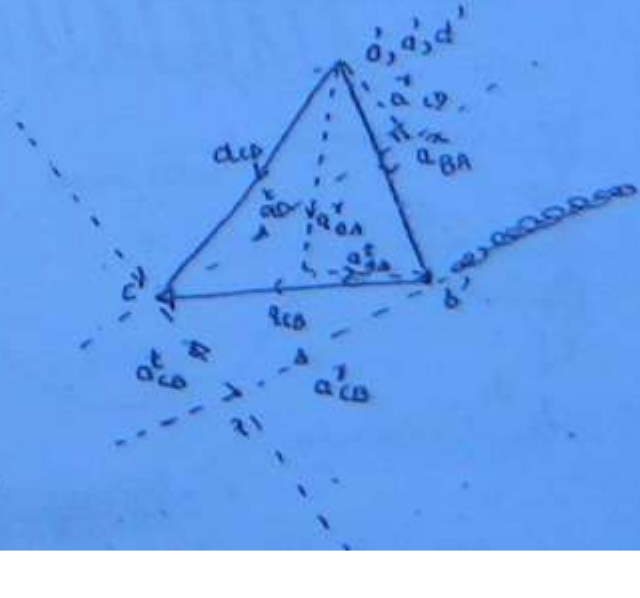
Pb :

Point	w.r.t	Procedure
B	A	$a^r_{BA} = \dfrac{V^2_{BA}}{AB}$ (given) $(B \to A)$ $\omega = 10.5$ rad/s $a^t_{BA} = (BA)\,\alpha_{BA}$ (given) ($\perp^{ar}$ to radial)
C	B	$a^r_{CB} = \dfrac{V_{CB}^2}{CB}$ $(C \to B)$ (known) $a^t_{CB} = (CB)\,\alpha_{CB}$ Unknown $\perp^{ar}$ to radial
C	D	$a^r_{CD} = \dfrac{V_{CD}^2}{CD}$ $(C \to D)$ known $a^t_{CD} = CD\,\alpha_{CD}$ (Unknown) ($\perp^{ar}$ to radial)



$\alpha_{BC} = ?$ (34.09 rad/s²)

$\alpha_{CD} = ?$ (79.11 rad/s²)



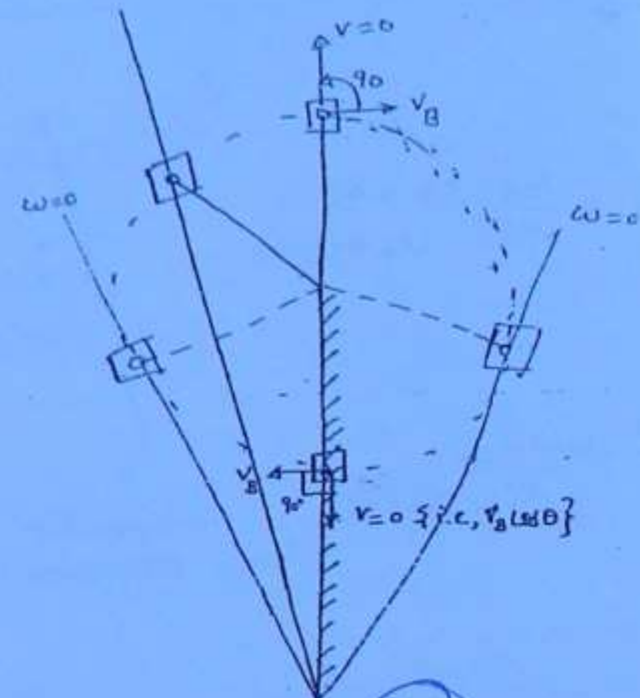
Note :- radial component of acceleration can never be zero. in a circular motion.

* ~~sss~~ Coriolis Acc^{n} :-

This acceleration is associated with the slider, if the slider is sliding on a rotating object.

$a^c = 2V\omega$ → ω of the body on which sliding is there

↑ sliding velocity of slider

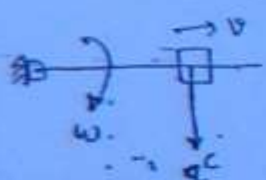


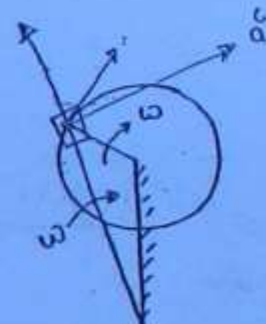
Direction of a^c : (Tangential).

i) Take the sense of ω

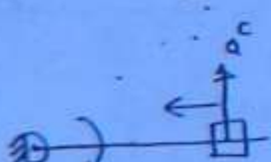
ii) ~~Rota~~ Rotate the $\vec{V}$ in that sense by 90°.

1.

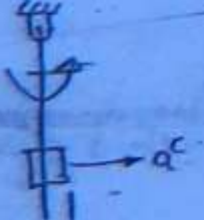


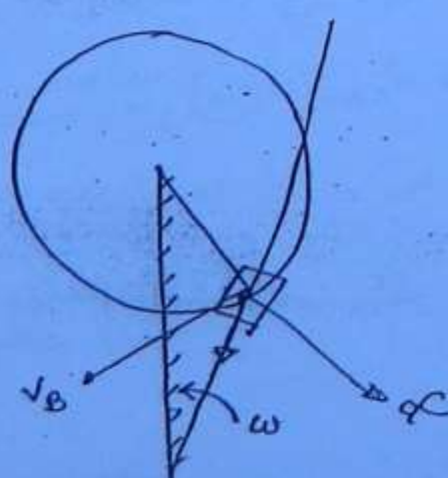


2.



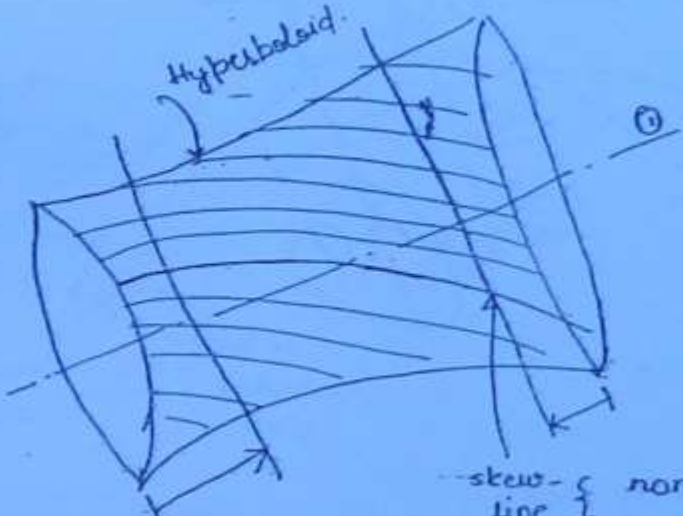
3.





iii> When the shafts are neither parallel nor intersecting :-

When the shafts which are non-parallel & non-intersecting are supposed to be connected, any kind pure rolling motion is impossible. Therefore, the motion which is possible is the rolling motion having some partial sliding.



spiral Gear
↓
Skew-Bevel Gear

when the end section of hyperboloid is used to form a spiral Gear → Hypoid Gear.

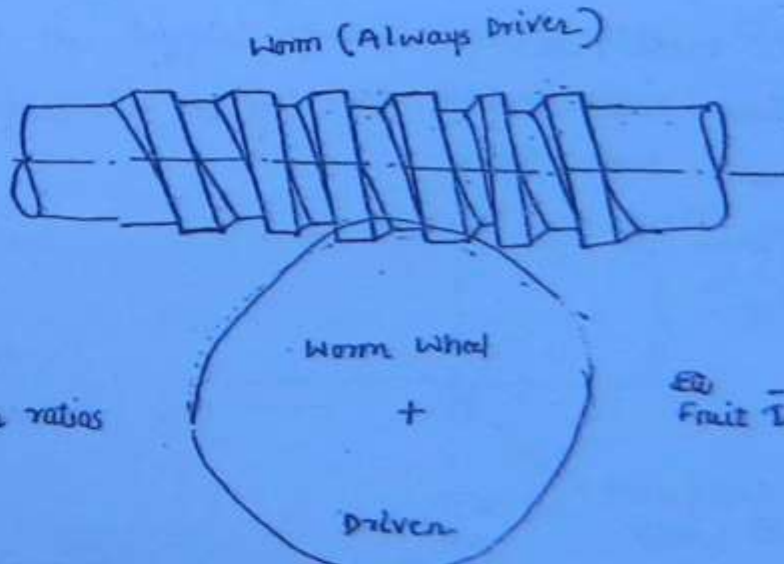
skew-line { non-parallel and non-intersecting }

- Worm & Worm wheel Gear

(*)

spiral

- D (less) — D (more)
- spiral angle ψ high — ψ less



Ex: Fruit Juicer

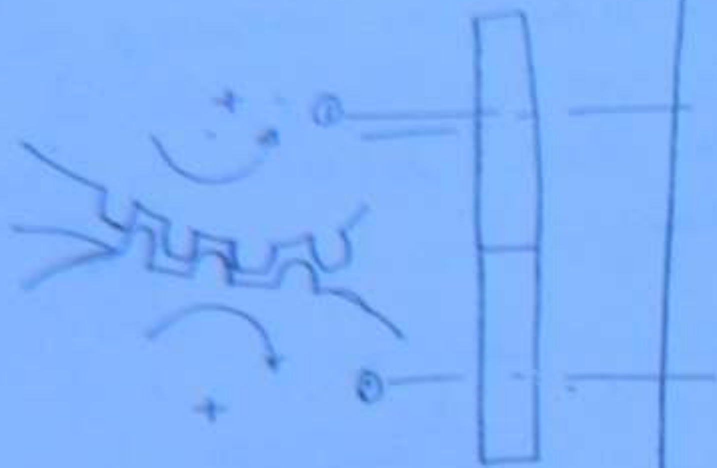
- Very high speed reduction ratios

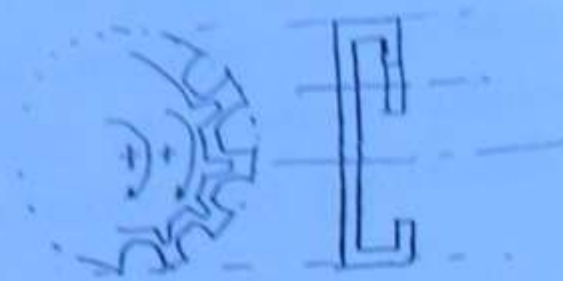
10 : 1
30 : 1
300 : 1
1000 : 1

2) According to the type of gearing :-

External Gearing

Internal Gearing





Bigger - Annular

Smaller - Pinion

Bigger → Gear or Spur

Smaller → Pinion

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3. According to the tangential speed :-

$v < 3 m/s$ → low velocity Gear

$3 \leq v \leq 15 m/s$ → Medium velocity Gear

$v > 15 m/s$ → High " "

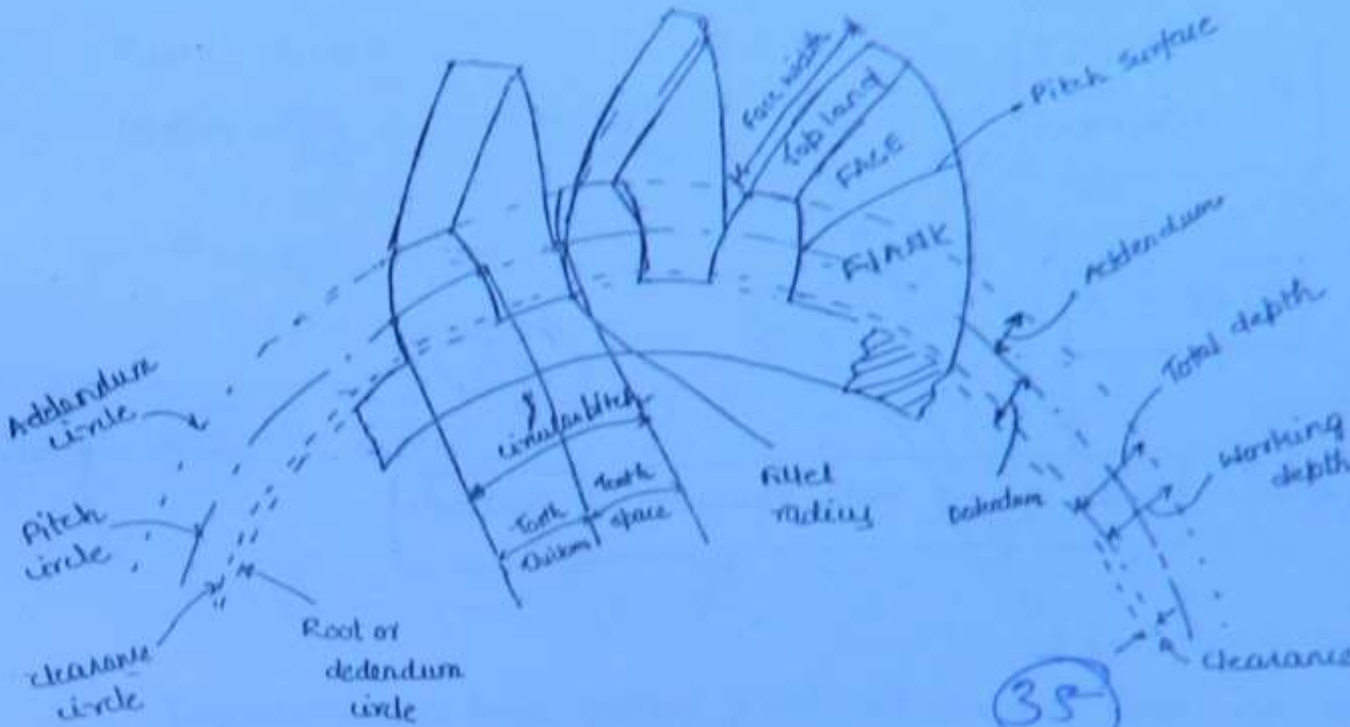
5) According to the type of teeth :-

st. teeth Gear

Inclined teeth Gear

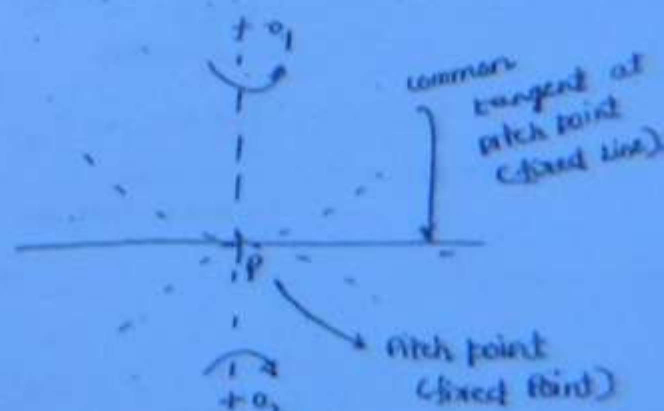
Curved teeth Gear.

* Gear Terminology :-



35

1> Pitch circle :-
It is an immaginary circle on which pure rolling motion is observed. Being an immaginary circle it can't Be a physical characteristic of the gear but being the most important circle of the gear it is the biggest specification of gear. The size of the gear is defined by the dia of pitch circle.



2> Circular Pitch (P_c):-

$$\boxed{P_c = \frac{\pi D}{T}}$$

For two mating Gears :-

$P_{c1} = P_{c2}$

$\Rightarrow \pi D_1 = \pi D_2 \Rightarrow \boxed{D_1 = D_2}$

3> Module ~~is~~ (m) :-

$$\boxed{m = \frac{D(mm)}{T}}$$

To avoid the stress concentration sharp corner is avoided and fillets are provided.

4> Dimetral Pitch (P_d) :-

$$\boxed{P_d = \frac{T}{D(inch)}}$$

$$\boxed{P_c \cdot P_d = \pi}$$

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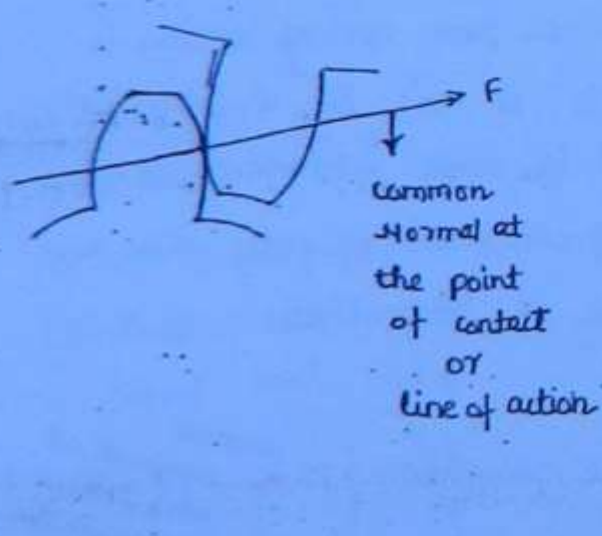
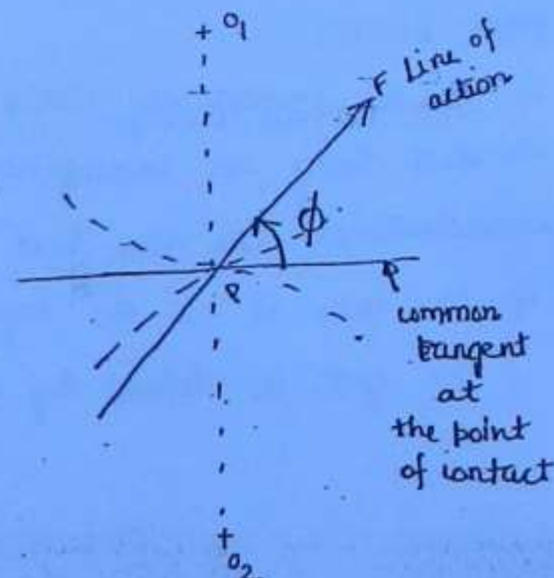
**** $\boxed{\text{Tooth space} - \text{Tooth thickness} = \text{Backlash}}$

5> Pressure Angle (ϕ) :-

It is an angle between the Line of action and common tangent at pitch point.





$P <$ O_1O_2 line / line of action

* Law of Gearing :-

For proper contact

$v_1 \cos\alpha = v_2 \cos\beta$

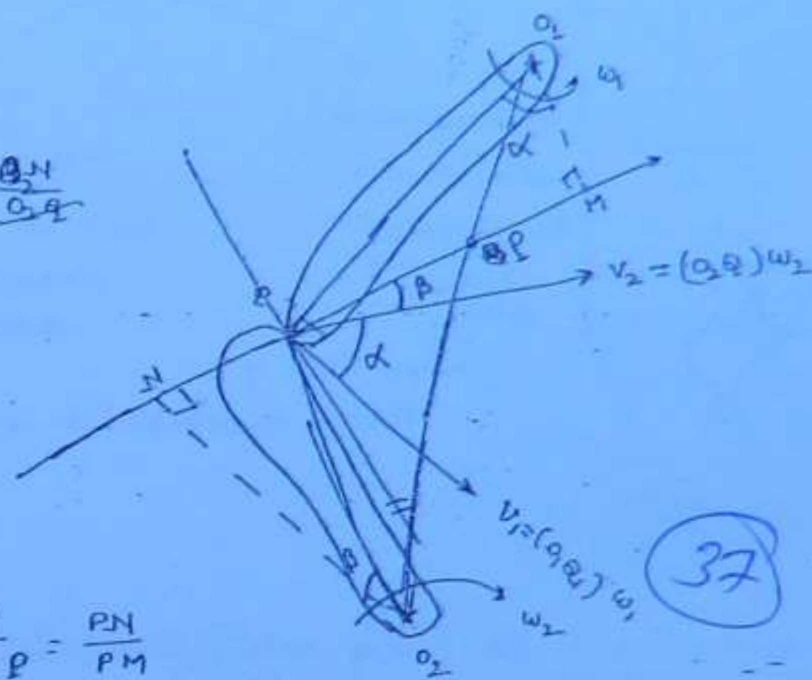
$(O_1Q)\omega_1 \cdot \frac{QM}{O_1Q} = (O_2Q)\,\omega_2 \cdot \frac{QN}{O_2Q}$

$\Rightarrow \omega_1 \cdot QM = \omega_2 \cdot QN$

$\Delta O_1PM \sim \Delta O_2PN$

$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$

$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}$



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If these bodies are gear

$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM} = \text{const.}$

$\frac{O_2P}{O_1P} = \text{const.} \Rightarrow P \rightarrow$ fixed point

$V_{sliding} = \left| v_1 \sin\alpha - v_2 \sin\beta \right|$

$= \left| O_1Q\,\omega_1 \cdot \frac{QM}{O_1Q} - O_2Q\,\omega_2 \cdot \frac{QN}{O_2Q} \right|$

$= \left| \omega_1 (QP + PM) - \omega_2 (PN - QP) \right|$

$= \left| \omega_1 QP + \cancel{\omega_1 PM} - \cancel{\omega_2 PN} + \omega_2 QP \right|$

$$\boxed{V_{sliding} = (\omega_1 + \omega_2)\,QP}$$

"Common normal at the point of contact between the two mating gear should always pass through a fixed point on the line joining the centres of rotation of the gears and this fixed point is known as pitch point."

*

Mating profile → (Conjugate profile
↓
when law of gearing satisfied

Line of action
↓
Common Normal at the ...] → should always pass through P

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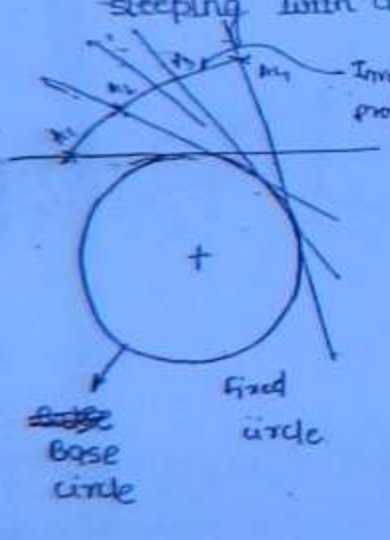
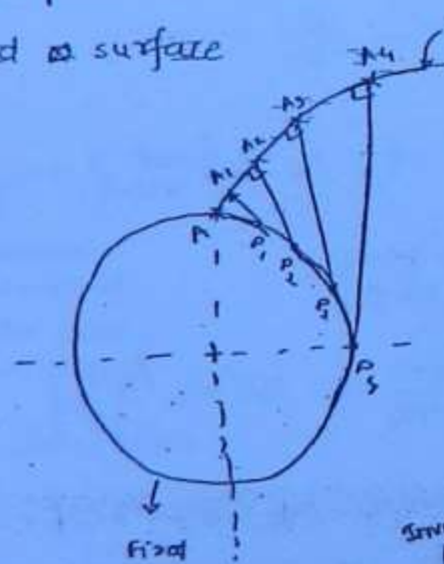
* Involute Profile (By nature conjugate, Pure mathematical curve)

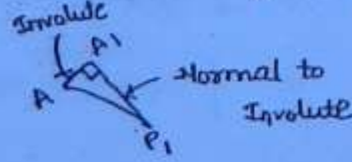
It is a locus of a point on a line which rolls without ~~sleeping~~ with a fixed surface





Arc $(AP_1) \Rightarrow P_1A_1$
Arc $(AP_2) \Rightarrow P_2A_2$
Arc $(P_1P_2) \Rightarrow P_3A_3$
Arc $(AP_4) \Rightarrow P_4A_4$
⋮

In Reality



* Normal drawn at any point on involute curve will Become tangent to its base circle
* Involute curve is the combination of very small arc of circles ~~of different~~ having different radius and different centre.

[redacted]

$1' = 14.6$

[redacted]

Mechanics
↓
study of motion

$\vec{v} = \frac{d}{dt}\vec{s}$ 1-D

$\vec{a} = \frac{d}{dt}\vec{v}$ 2-D (Projectile)

Kinematics — study of the motion without considering the cause of the motion i.e, Force

study of motion with considering the case of the motion i.e, Force — Dynamics or Kinetics

NSL :- Force $= \frac{d}{dt}(m\vec{v})$ — mass

moment of interial — mass distribution

Mass is the measurement of inertia property
Parameter

mass unit directly / Indirectly — dynamic quantity parameter
kg or N — not present — kinematic parameter

Theory of Machines

[redacted]

Theory of Machines

TOM

[redacted]

- Simple Mechanism (understanding)
- Velocity Analysis (A+G) [analytical & Graphical]
- Acceleration Analysis (A+G)
- GEARS & Gear Trains
- Governors
- Flywheel
- Balancing (A+G)
- Vibrations (A)
- Cams & Followers (A).

40

* ~~Simple Mecha~~

9313467612
kakkar_amit@rediffmail.com

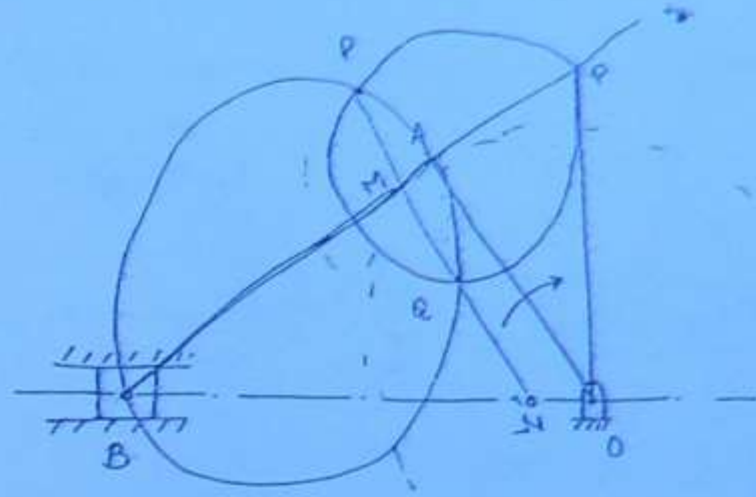
* Klein's Construction :-

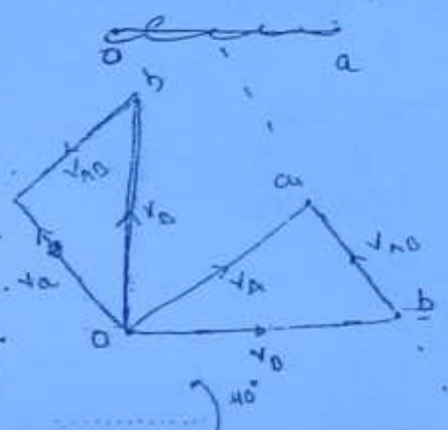
↓

only applied in single slider crank mechanism

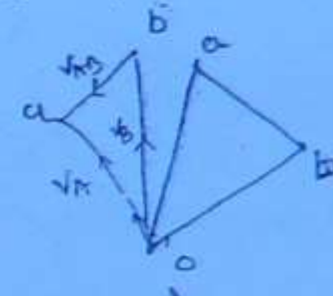
$\boxed{\alpha_{input} = 0}$

$\omega_{crank} \to$ given





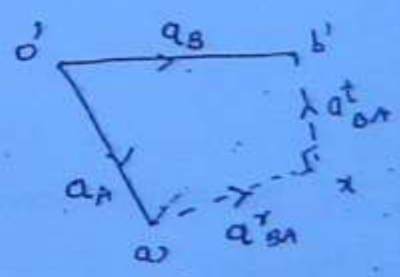
Velocity triangle (in rough) :-



Δ

$$\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{AB}}{AP} = \omega_{crank} \text{ given}$$

□ OAMN → Accⁿ ◇ :



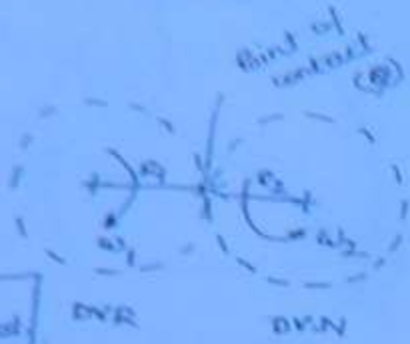
$$\frac{a_A}{OA} = \frac{a^t_{BA}}{MN} = \frac{a_B}{NO} = \frac{a^r_{BA}}{AM} = \omega^2_{crank}.$$

Gears

slipping is possible
↓
(Negative Drive)
1. Belt
2. Rope

No slipping → $\boxed{R_1\omega_1 = R_2\omega_2}$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \text{const}}$$



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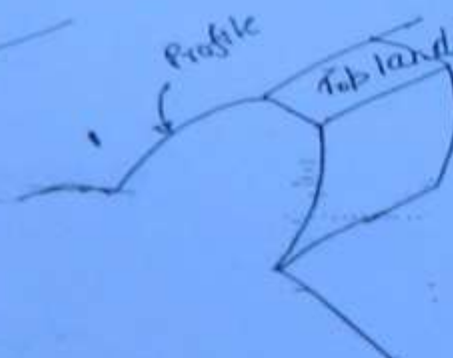
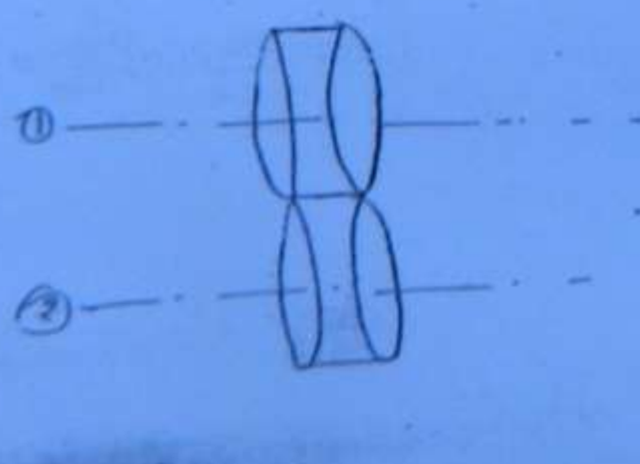
slip → Impossible
↓
Positive Drive
↓
Gear Drive



* Classification of Gears :- 29/06/2011

1) According to the axis of the shafts connected :-

i) When both Axes are Parallel:
⇓
Pure rolling motion b/w two cylinder in contact



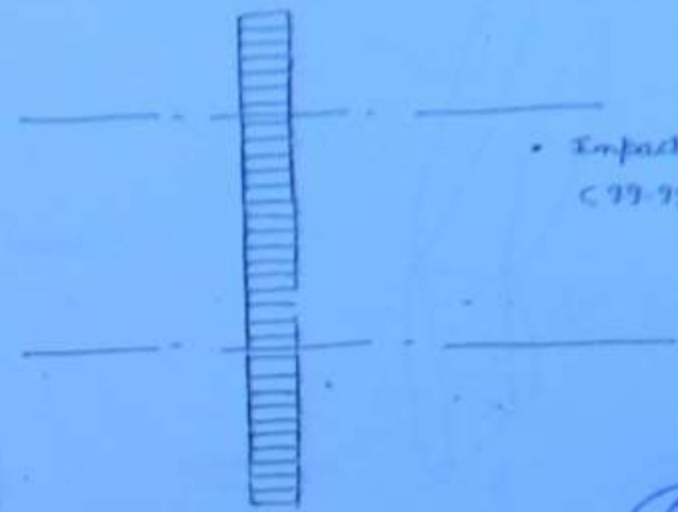
A> Teeth are straight and ∥ to the axis of rotation

⇓

spur gear

- No axial thrust

- Since impact stress is present (high), it is noisy.



- Impact stress (99.99% Failed)

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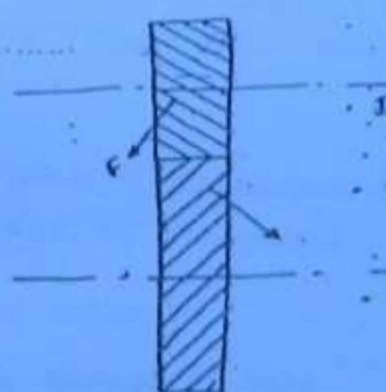
B> Teeth are straight but inclined to the axis of rotation

⇓

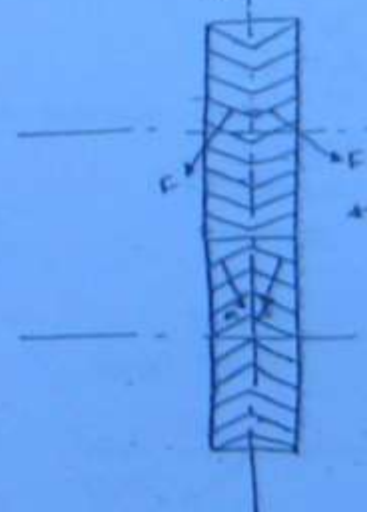
Helical gear.



- Impact is eliminate
- Axial thrust present
- gradual engagement

Double helical

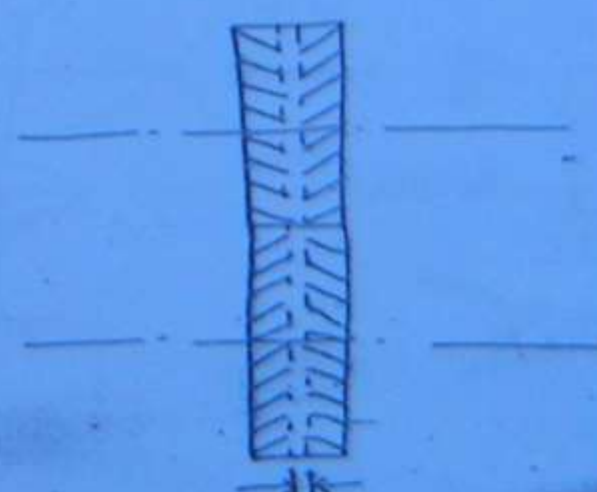
To minimise axial thrust



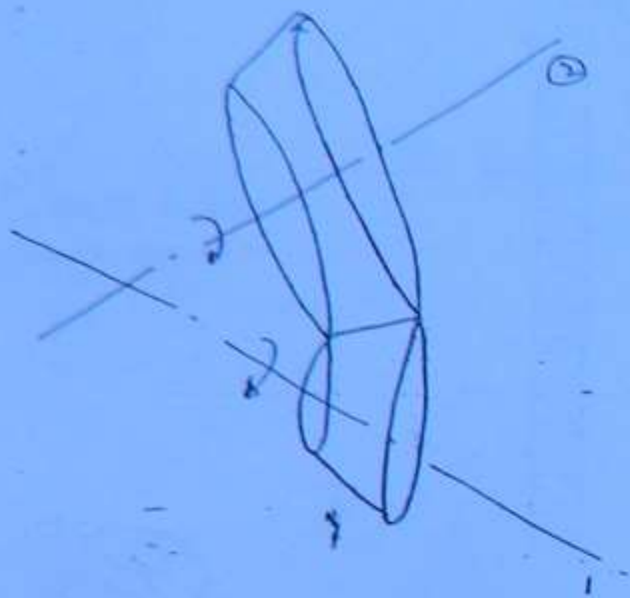
Herringbone gear

Herringbone gear

Applications :-



ii) When both axis are inclined (intersecting) :-



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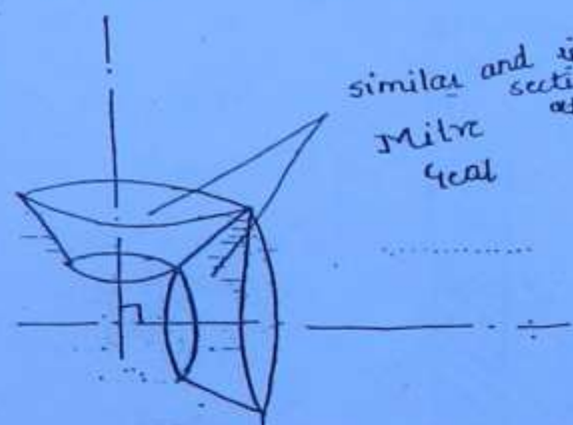
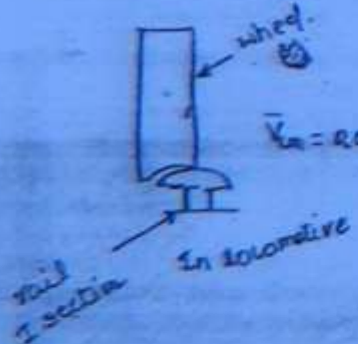
Bevel Gear

- Teeth are st. but ∥ to the axis of rotation
  ↓
  St. Bevel Gear
- Teeth are st. but Inclined to the axis of rotation.
  ↓
  Helical Bevel Gear

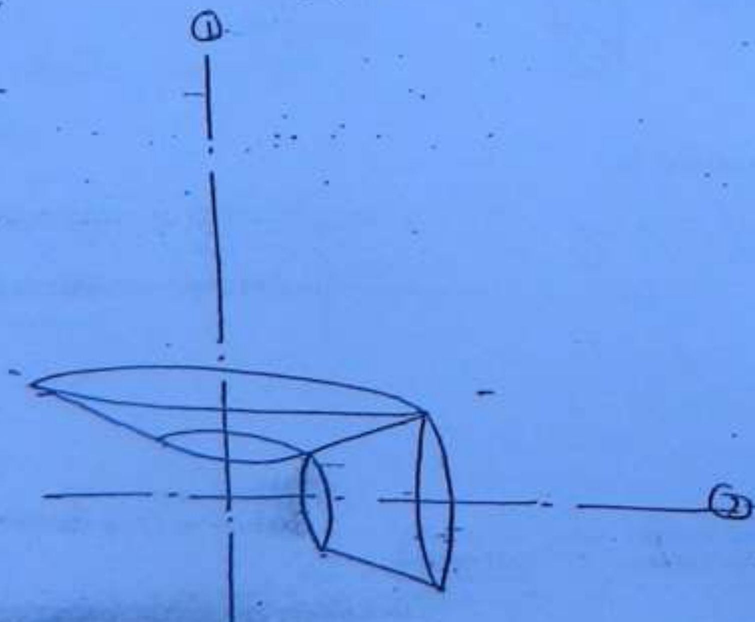


similar and intersecting at 90°

Mitre gear



$\bar{V}_m = R\omega$

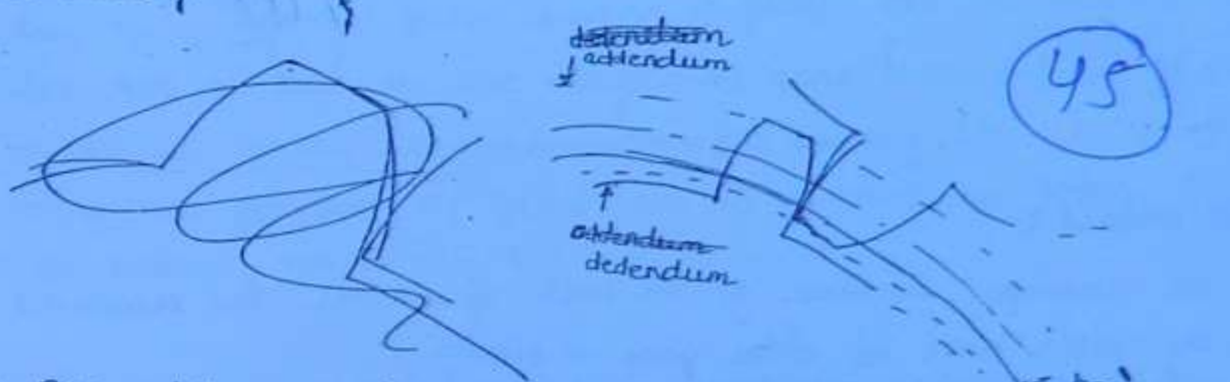


Hyperbola is a curve obtained when the centre is fixed but the radius is changing with negligible amount say dr.

1/07/2011

* Analysis of Involute Gear :-

undercutting



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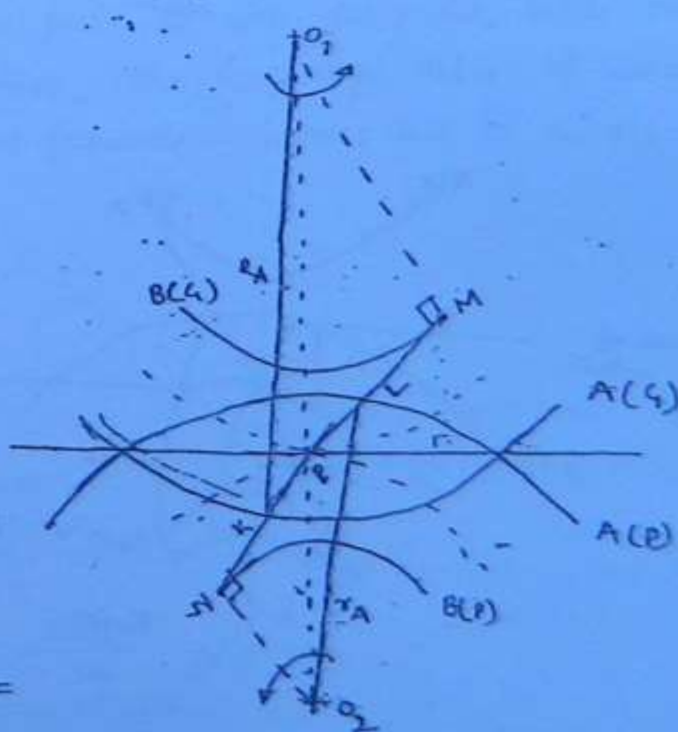
* Base circle can never be on dedendum circle.



K → point of Engagement
L → End of Engagement

Line of action :-

1. Pass through (P) Gear
2. Tangent to both the Base circle (Involute)

KL – Total distance travelled by (P) from start to end of the engagement.
→ (Path of contact) = KP + PL
KP ↓ Path of approach
PL ↓ Path of recess

$\Delta\ O_1KM$:

$$\left.\begin{aligned} O_1M &= R\cos\phi \\ PM &= R\sin\phi \end{aligned}\right] \qquad R_A^2 = R^2\cos^2\phi + (KP + R\sin\phi)^2$$

$$\boxed{\therefore KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi}$$

$$\boxed{PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi}$$

$$KL = KP + PL$$

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* Arc of contact :-

It is an analogus distance of the path of contact but measured along the pitch circle of either gear or pinion.

Arc of contact

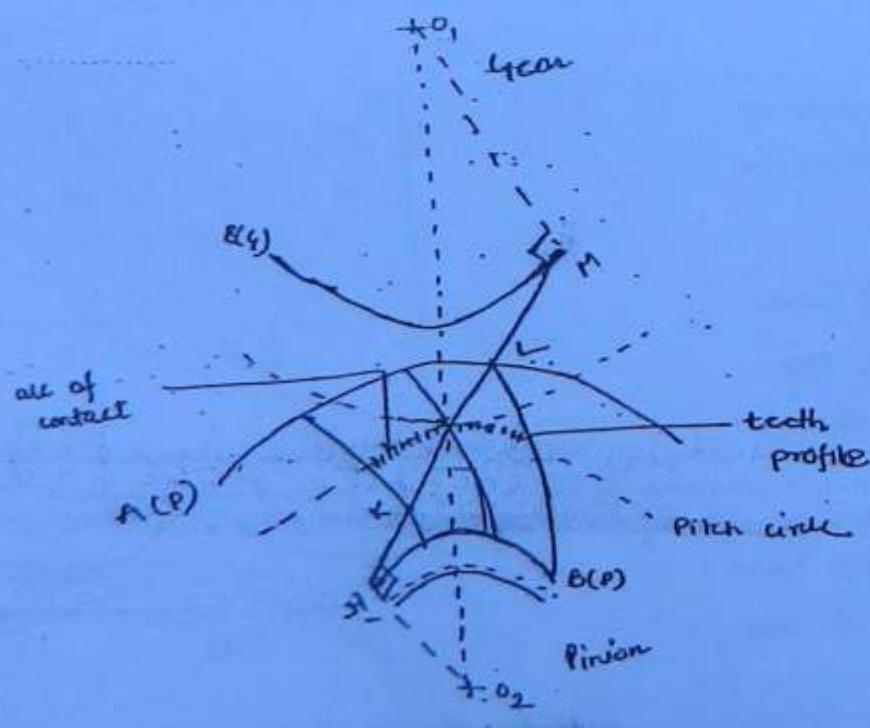
$$= \frac{\text{Path of contact}}{\cos\phi}$$

Arc of app.

$$= \frac{\text{Path of App}}{\cos\phi}$$

Arc of recess

$$= \frac{\text{Path of recess}}{\cos\phi}$$



For exam:

Arc of contact $\Rightarrow$ 1 pc, 2pc

$$\boxed{\text{Contact Ratio} = \frac{\text{Arc of contact}}{\text{pc}}}$$

$\Downarrow$

No. of pairs in contact $\rightarrow$ 1.3 — 1.8

min $\rightarrow$ 1.

* For high power transmission contact ratio ~~is more~~ should be more

if contact ratio = 1.4

justify answer

~~For~~ ~~[illegible]~~.

In one ~~enagement~~ zone 1 pair of gears are in contact But for 40% of time in one enagement zone 1 more pair of gears are in contact. ~~At this moment 2~~ During this 40% of time two ~~geo~~ pair of gears are in contact in total. So, the contact ratio is 1.4

"One pair is ~~enageg~~ engaged in complete engagement period but 40% of time of this engagement period is like that along with this pair 1 more pair i.e, total two pairs are engaged. Therefore, the average value of contact ratio in one engagement period is comes out to be 1.4."

Prob:

$t = 24$

$T = 36$

$m = 8mm$

Addendum = 7.5 mm

Gear

$$R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144\,mm$$

$$R_A = 144 + 7.5 = 151.5\,mm$$

Pinion:

$$r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96\,mm$$

$T_A = 96 + 7.5 = 103.5 mm$

Path of contact

$KL = KP + PL$

$= \left[\sqrt{R_A^2 - R^2\cos\phi} - R\sin\phi\right] + \left[\sqrt{r_A^2 - r^2\cos\phi} - r\sin\phi\right]$

$= 18.8806 \text{ mm} + 17.9037 \text{ mm}$

$= 36.7843 \text{ mm}$

Arc of contact $= \dfrac{36.7843}{\cos 20^\circ} = 39.1450 \text{ mm}$

(48)

i) $\dfrac{39.1450}{96} \times \dfrac{180}{\pi} \quad \gamma = 23.3630^\circ$

ii) $N_p = 450$ r.p.m

$\dfrac{N_G}{N_P} = \dfrac{r}{R} \qquad N_G = 300$ r.pm.

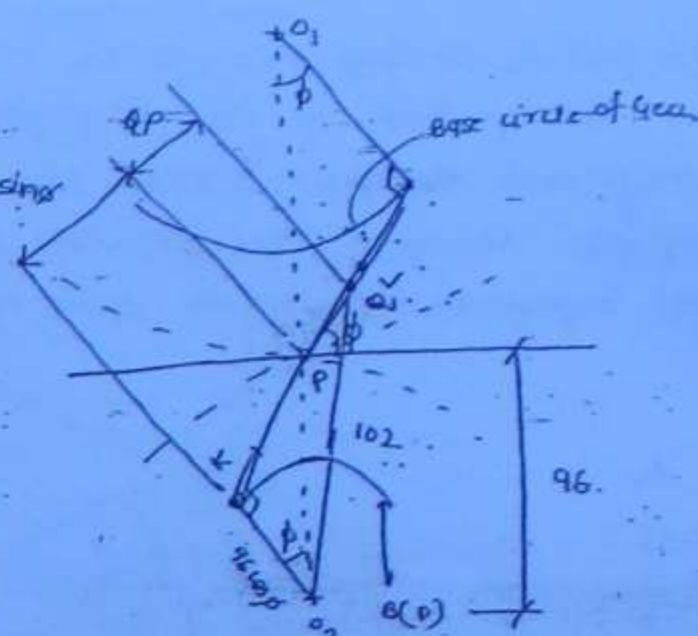
$(102)^2 = (96\cos 20)^2 + (96\sin 20 + QP)^2$

$QP = 14.7693$ mm

$V_{sliding} = (\omega_1 + \omega_2)\, QP$

$= \dfrac{2\pi}{60}(N_G + N_P)\, QP$

$= 1.16$ m/s



* Interference (Death of Involute Gear) :-

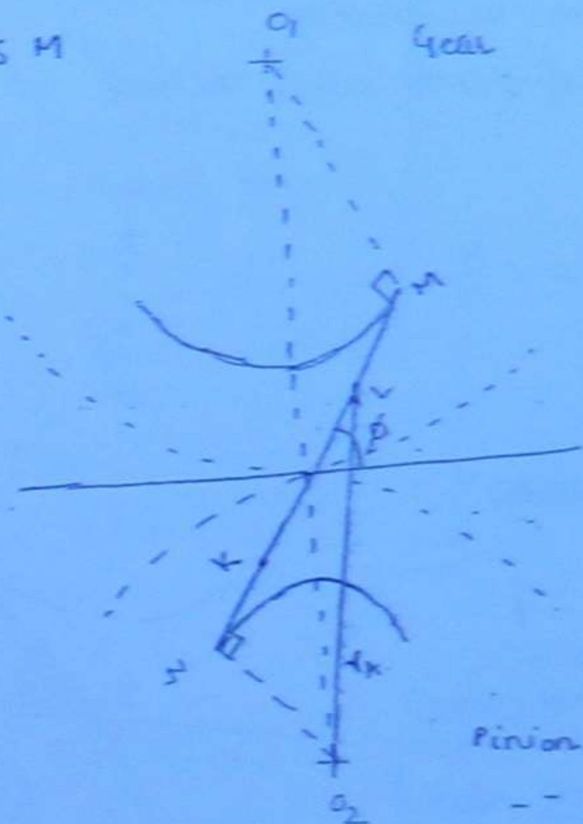
For pinion :

If $r_A \uparrow \Rightarrow$ L will shifts towards M

till M $\rightarrow$ No problem

If $r_A > O_2M$

undercutting will happen $\rightarrow$ results in interference



For gear :

same

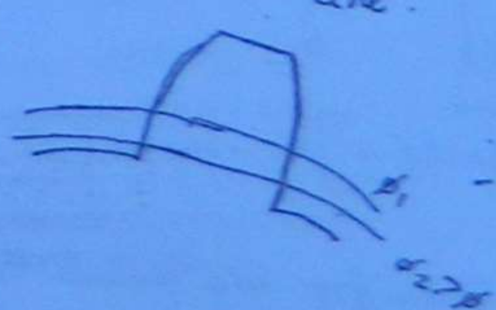
if $R_A > O_1N$.

M, N] critical point (interference point)

Methods to Prevent interference :

1. Undercut Gears :

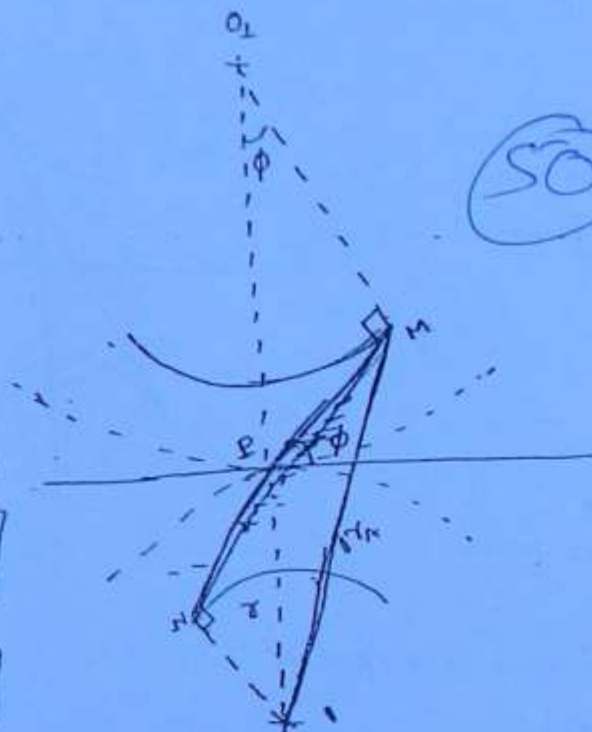
2. $\phi \uparrow \Rightarrow r_B \downarrow$ $\rightarrow$ radius of base circle.



3. ↑ the no. of Teeth :

If the minimum no of teeth are increased, the addendum circle radius will decrease.

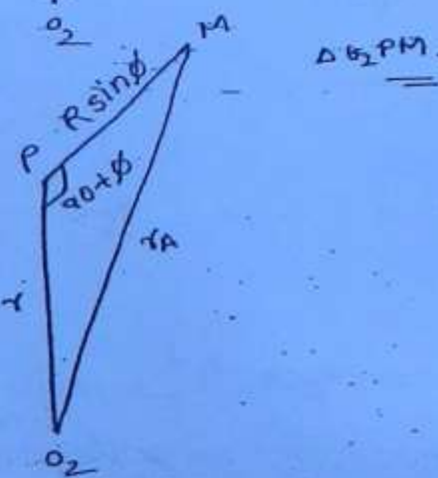
clearly ∵ r_A is decreasing
interference will decrease.



$$\frac{T}{t} = \text{Gear ratio} = \frac{R}{r}$$

$\frac{\text{Bigger}}{\text{Smaller}}$

Applying cos rule to ~~[illegible]~~
$\Delta\, O_2PM$



$$r_A^2 = r^2 + R^2\sin^2\phi - 2r(R\sin\phi)\cdot\cos(90+\phi)$$

$$= r^2\left[1 + \frac{R^2 + 2rR}{r^2}\cdot\sin^2\phi\right]$$

$$= r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$\therefore r_A^2 = r^2\left[1 + G(G+2)\sin^2\phi\right]$$

$$r_A = r\sqrt{1 + G(G+2)\sin^2\phi}$$

Addendum of pinion $= r_A - r$

$$= r\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$

$$= \frac{m t_{min}}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right] \quad \text{——— (1).}$$

A_p → Fractional Addendum of Pinion for one module in order to avoid interference

If module ⇒ m
↓
Addendum of pinion $= mA_p$ ←—— (2)

$m A_p = \frac{m t_{min}}{2} \left[\sqrt{1 + G(G+2)\sin^2\phi} - 1 \right]$

$$\boxed{\therefore \; t_{min} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$$

$$\boxed{t_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}}$$

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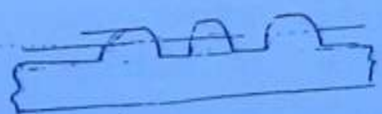
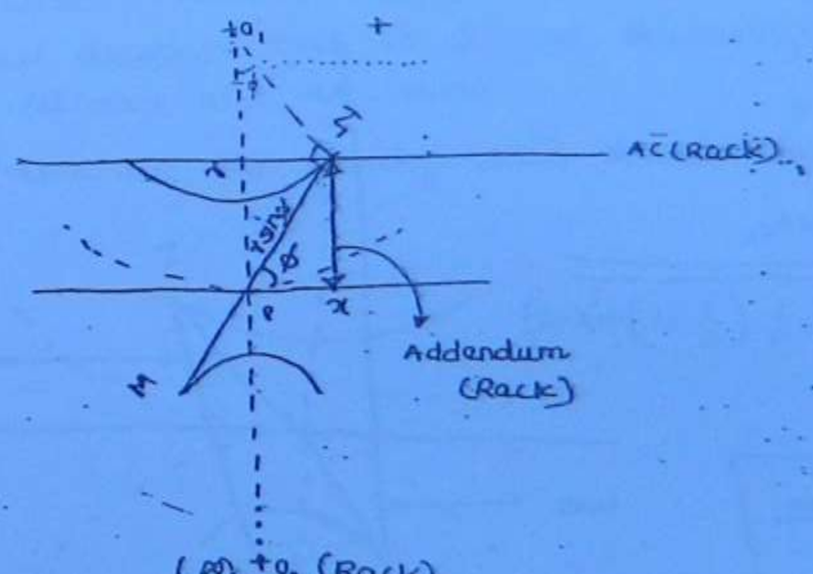
* Minimum No. of Teeth on Pinion to avoid interference in Involute Rack and Pinion Arrangement





Addendum rack

$Nx = r\sin^2\phi$

$= \frac{m t_{min}}{2} \sin^2\phi$ — (1)

A_R — Fractional addendum of Rack for one module in order to avoid interference

If module = m

Add (Rack) = $m A_R$ — (2)

$m A_R = \frac{m t_{min}}{2} \sin^2\phi$

$$\boxed{t_{min} = \frac{2A_R}{\sin^2\phi}}$$

$14\frac{1}{2}^\circ$, 20° Full depth $\rightarrow$ standard Addendum $\Rightarrow$ 1m

$$mA = 1m$$

$$\boxed{A = 1}$$

$\boxed{20^\circ}$, 22°, 25° stub Addendum < 1m

$$mA < 1m$$

$$\boxed{A < 1}$$

generally 0.8 - 0.75

52

20° stub is said to be Best due to following movement

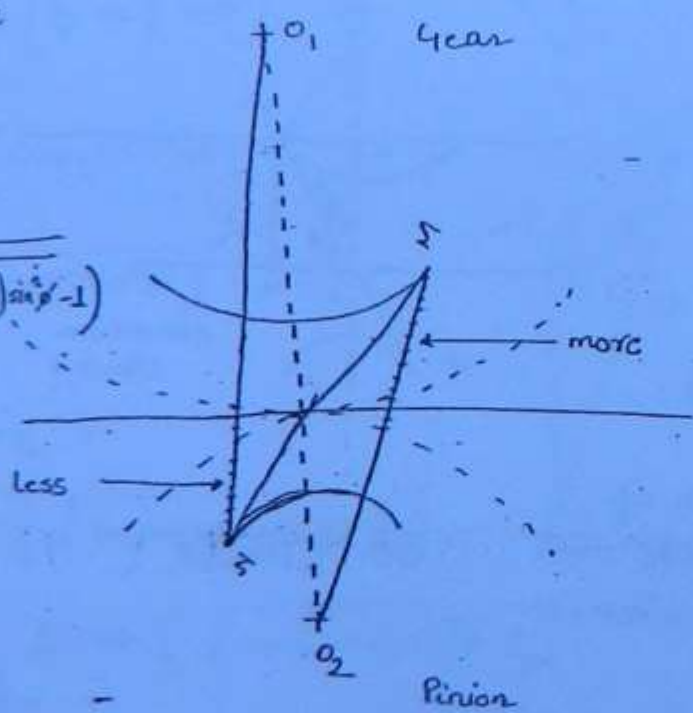
1. stronger Tooth
2. low interference
3. lesser no. of teeth (tmin) Tmin
4. cost is also less.

Addendum → same

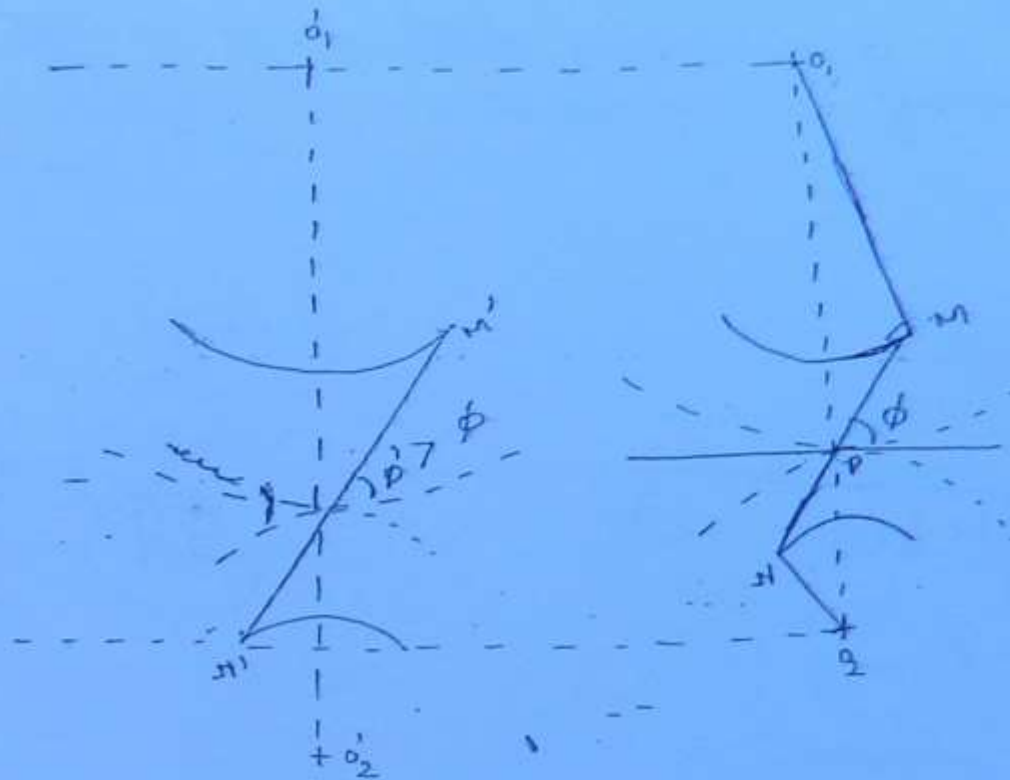
$t_{min} = ?$

$$t_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$\boxed{t_{min} = \frac{T_{min}}{G}}$$



* Effects of Vibration and Centre Distance on Involute Gear :-



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- Its centre distance increases pressure angle increases and its ~~centre~~ interference will decrease but it is not the correct method to reduce centre distance and vice-versa
- By changing the centre distance the velocity ratio doesn't change.

2007 - section B. - 3

Que: 3.

a) $G = 3$

$A_p, A_G = 1$

$\phi = 20^\circ$.

$$T_{min} = \frac{2A_G}{\left(\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right)} = 44.9426 \Rightarrow 45$$

$$t_{min} = \frac{45}{3} = 15$$

$$t_{min} = 12 \Rightarrow T_{min} = 36.$$

$$36 = \frac{2A_q}{\left(\sqrt{1+\frac{1}{q}\left(\frac{1}{q}+2\right)\sin^2\phi} - 1\right)}$$

$\Rightarrow 2A_q = 0.8$

$\Rightarrow$ 20% stubbing

Q. No. 3

$T_{min} = 36.$

$$36 = \frac{2A_q}{\left(\sqrt{1+\frac{1}{q}\left(\frac{1}{q}+2\right)} - 1\right)}$$

54

$\phi = 22.5^\circ$

Pb $A_p, A_q = 1$

$\left.\begin{array}{l} q = 4 \\ \phi = 20^\circ \end{array}\right]$ T_{min}

$$T_{min} = \frac{2A_q}{\left(\sqrt{1+\frac{1}{q}\left(\frac{1}{q}+2\right)\sin^2\phi} - 1\right)} \simeq 61.77$$

$\boxed{\simeq 62}$ — 64

$t_{min} = \frac{62}{4} = 15.5 \approx 16$

work book (page 34)

Q. No. 5.

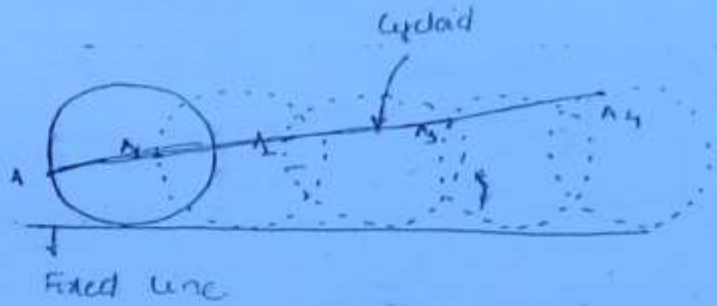
$\boxed{\phi = 20^\circ}$

$m = 10\,mm$

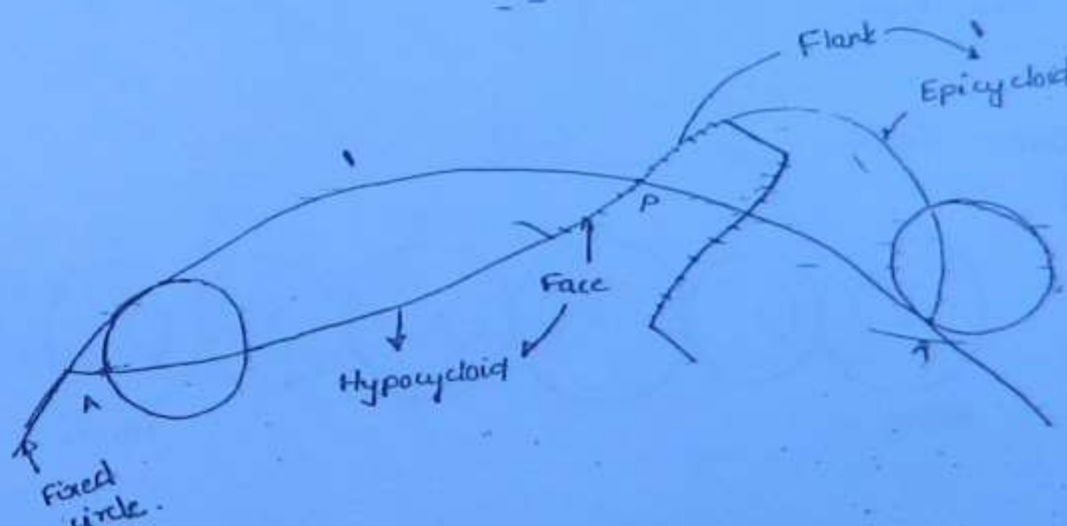
$- A_p$

* Cycloidal Profile :-

It is the locus of the point on the circumference of the circle which rolls without slipping on a fixed straight line. This profile is also by nature conjugate.



55



Advantages

1. Tooth cost is high But overall cost is low
2. Interference is absent Because automatic undercutting is provided due its nature of profile
3. Flank Wide → strength more
4. Life more → less wear

disadvantage

1. pressure ange (ϕ) is continuously changing [Max – 0 – Max] (Ecosϕ)
2. Sever effect of vibration → velocity ratio is changing

Gear Train

↓

combination of Gears.

why combination of Gears are required?

→ 1. Centre distance is Large

2. centre distance is less but velocity ratio is high i.e., $\frac{\omega_1}{\omega_2} = 10, \frac{1}{10}$] high/ low

* Three parts of Gear Train :-

1. Main DVR
2. Main DVN
3. Intermediate Gears.

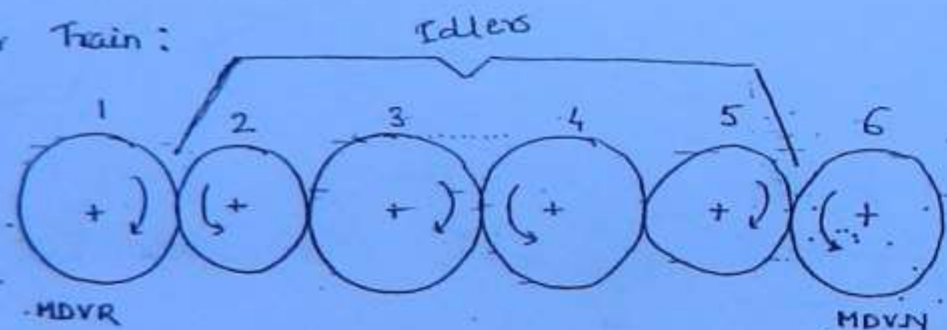
$$\boxed{\text{Velocity ratio of Gear} = \frac{\omega_{DVN}}{\omega_{DVR}}}$$

$$\boxed{\text{speed ratio (Velocity Ratio)} = \frac{\omega_{main\ DVR}}{\omega_{main\ DVN}}}$$

$$\boxed{\frac{1}{S.R.} = \text{Train Value}}$$

* Simple Gear Train :

module of each gear should be same



1, 2 :-

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

2, 3 :-

$$\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$$

3, 4 :-

$$\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$$

4, 5 :-

$$\frac{\omega_4}{\omega_5} = \frac{T_5}{T_4}$$

5, 6 :-

$$\frac{\omega_5}{\omega_6} = \frac{T_6}{T_5}$$

All eqⁿ (x)

$$\boxed{\frac{\omega_1}{\omega_6} = S.R. = \frac{T_6}{T_1}}$$

Clearly, from the equation it can be seen that S.R. depends only on MDVR and MDVN. It doesn't depends on intermediate gear. So, these are called Idlers.

No. of Idlers is odd → direction opposite

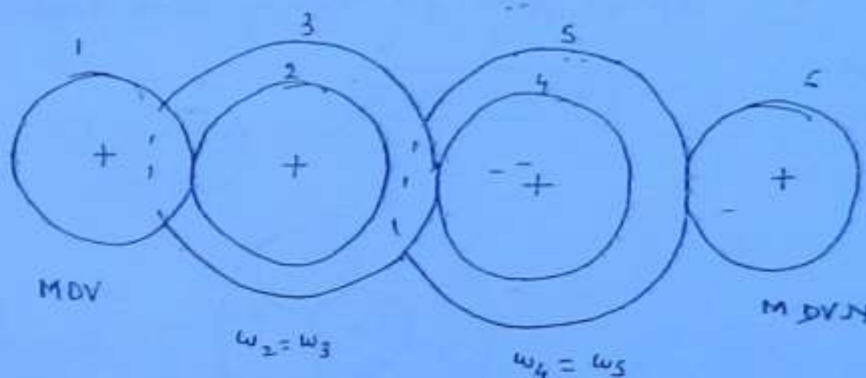
No. of Idlers is even → direction same

- If only one gear is mounted in a all shafts in a gear then it is called Simple Gear Train

* Compound Gear Train :-

- Any of the intermediate shafts if it is having more than one gear in use, such a gear train is called compound Gear Train

2-3 and 4-5 are compound gear



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DVR : (1, 3, 5)

DVN : (2, 4, 6)

$m_1 = m_2$

$m_3 = m_4$

$m_5 = m_6$

1, 2 :

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad — ①$$

3, 4 :

$$\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3} \quad — ②$$

5, 6 :

$$\frac{\omega_5}{\omega_6} = \frac{T_6}{T_5} \quad — 3$$

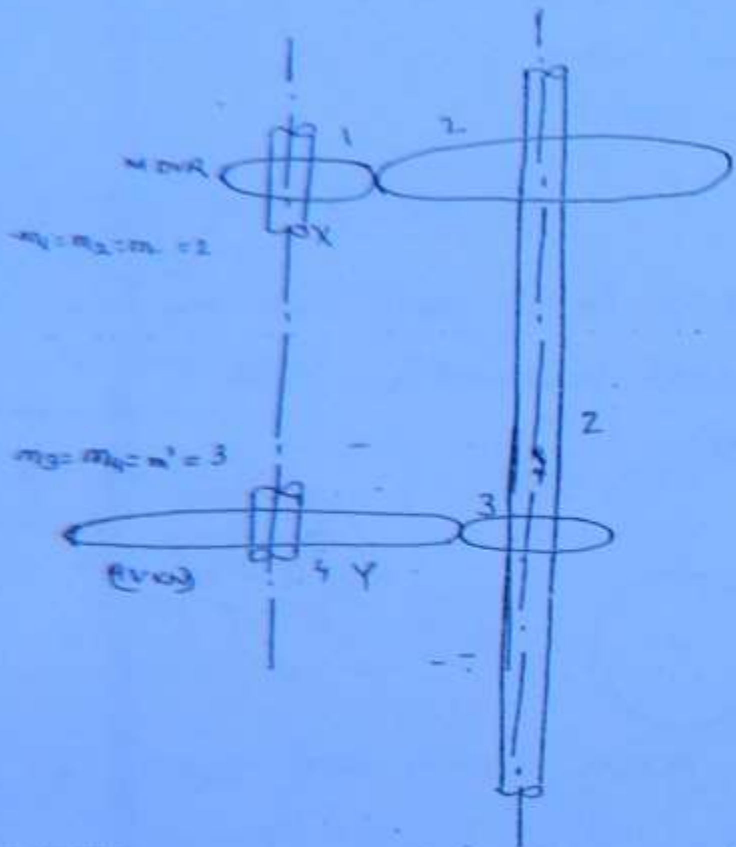
x

$$S.R : \frac{\omega_1}{\omega_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\therefore \boxed{S.R = \frac{\omega_1}{\omega_6} = \frac{\text{Product of No. of Teeth on DVN}}{\text{Product of No. of Teeth on DVR}}}$$

* Reverted Gear Train :

It is a compound gear train in which main driver and main driven shafts are collinear i.e, co-axial.



DVR (1,3)

DVN (2,4)

$$\frac{\omega_1}{\omega_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

$$r_1 + r_2 = r_3 + r_4$$

$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

If all gears are having same module

$$\boxed{T_1 + T_2 = T_3 + T_4}$$

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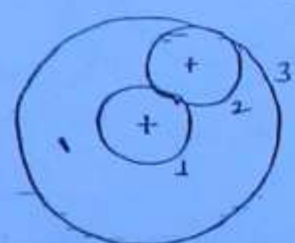


$$r_1 + 2r_2 = r_3$$

$$T_1 + 2T_2 = T_3$$

2006 (b)

Problem :-

$$\omega_4 = \frac{\omega_1}{12} \qquad \frac{\omega_1}{\omega_4} = 12 = \frac{T_2 \times T_4}{T_1 \times T_3}$$

$$\text{or,} \quad 12 = \frac{T_2 \times T_4}{24 \times 24} \qquad [T_1 = T_3 = 24]$$

$$T_2 \cdot T_4 = 12 \times 24 \times 24$$

$$2(24 + T_2) = 3(24 + T_4) \quad — \quad (2)$$

Solving eqn (1) & (2)

$$\left.\begin{matrix} T_4 = 64 \\ T_2 = 108 \end{matrix}\right]$$

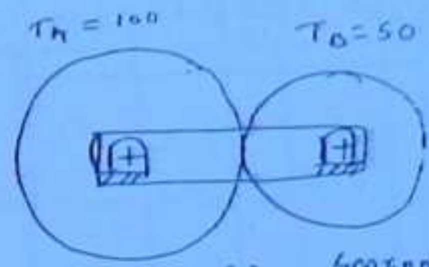
$$(r_1 + r_2) = \frac{m}{2}(T_1 + T_2)$$

$$= \frac{2}{2}(24 + 108)$$

$$= 132 \text{ mm}$$

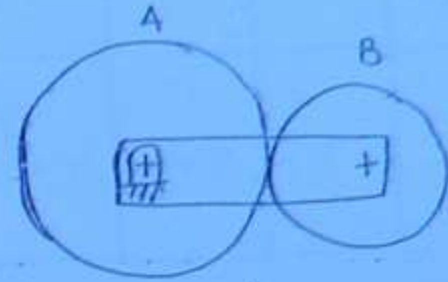
* Epicyclic Gear Train (Best)
(Space → less)

axis, rotation



200 r.p.m (C) 400 r.p.m (AC)

0 rpm 0 r.p.m



Arm free
↓
axis centre will rotate
and it is epi-cyclic.

$$\frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$N_A = \frac{T_B \cdot N_B}{T_A}$$

$3 \longrightarrow 4$

$-x \qquad \mp \frac{x \times T_3}{T_4}$

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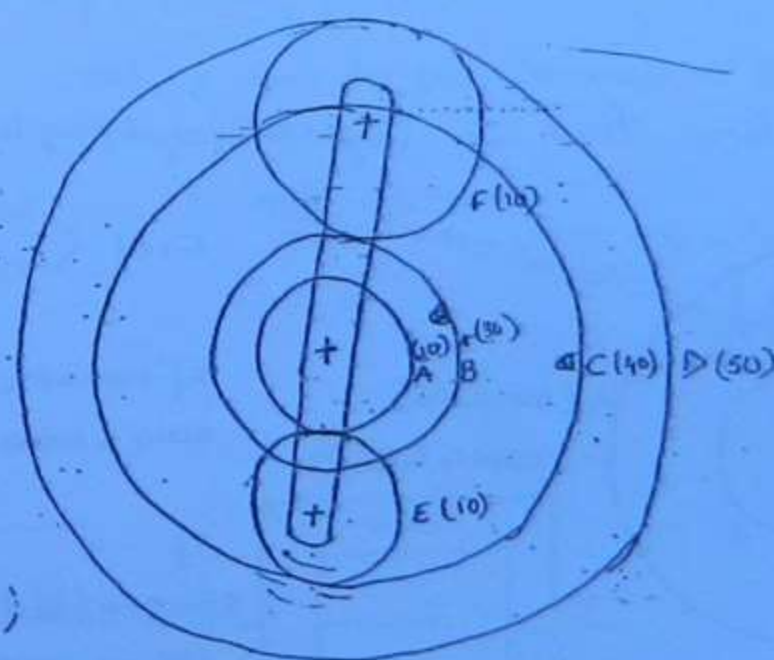
Problem :-

A - B
compound.

All gears are having same module.

$T_A = 20$
$T_B = 30$
$T_E = T_F = 10.$



i> $N_D = 0$

ii> $N_{arm} = 200$ r.pm (AC)

Final N of all other gears

motion	Arm	C 40	E 20	A/B 20/30	F 10	D 50.
1. Arm fixed Gear C rotates $+x$ rev (clock)	0	$+x$	$+\frac{x \times 40}{20}$	$-4x \times \frac{10}{20}$	$+2x \times \frac{30}{10}$	$+6x \times \frac{10}{5}$
2. Arm free	y	$y+x$	$y+4x$	$y-2x$	$y+6x$	$y+\frac{6x}{5}$

$T_A + 2T_E = T_C$

$T_B + 2T_F = T_D$ }

Now, $y + \frac{6x}{5} = 0$

$y = -200$ r.p.m

$x = \frac{200 \times 5}{6} = \frac{1000}{6} = 166.667$ rpm

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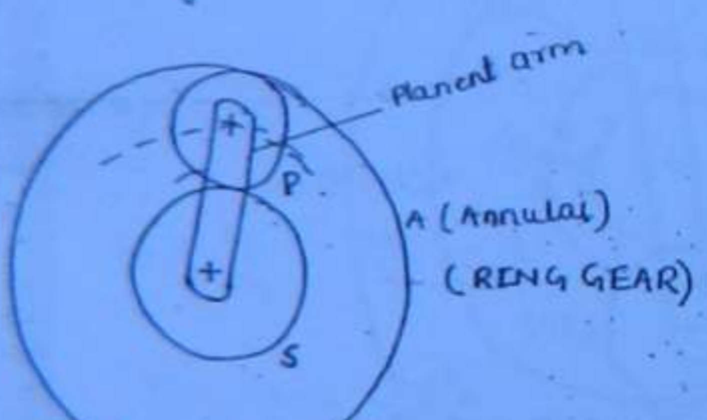
* Planetary Gear Train :- (By nature Epicyclic)



Fixed < Sun / Ring

If SUN → fixed
RING → Driver ← 2.

1. Arm (Input DVR)

If
Ring → fixed
SUN → Driver

Year 2000 (Page-47)

2b) $\frac{252}{T_D} = 3.5 \Rightarrow T_D = 72$

	(A) Arm	T_S S	T_P P	D
1.	0	$+x$	$-x\frac{T_S}{T_P}$	$-x \cdot \frac{T_S}{\cancel{T_P}} \cdot \frac{\cancel{T_P}}{T_D}$
2.	y	$y+x$	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{72}$

$y - x\frac{T_S}{72} = 0$ —— ①

$N_S = 5\,N_{arm}$

$y + x = 5y$

$x = 4y$

$y_0 - 4y\frac{T_S}{72} = 0$

$y\left(1 - \frac{4T_S}{72}\right) = 0$

But y can't be zero because it is an epicyclic gear

$1 - \frac{4T_S}{72} = 0$

$T_S = 18$

$T_S + 2T_P = T_D$

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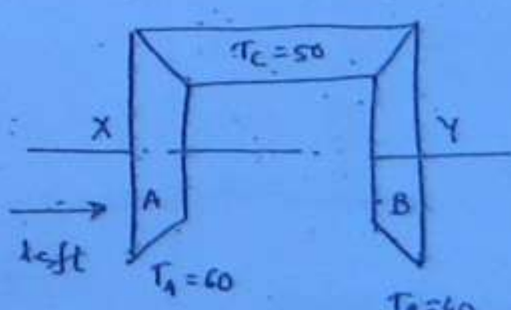
Page 123.

Arm	A	C	B
0	$+x$	$\pm x \cdot x\frac{60}{50}$	$-x\left(\frac{60}{50}\right) \times \frac{50}{40}$
y	$y+x$	$y \pm x\left(\frac{60}{50}\right)$	$y - x\left(\frac{60}{40}\right)$



$N_A = 120$ r.pm (clock)

$N_{arm} = 120$ rpm (AC)

Homepage Gear

* Fixing Torque in Epicyclic Gear Train :-

$$T_{input} + T_{output} + T_{Fixing} = 0$$

or, $$T_{Fixing} = -(T_{input} + T_{output}) \quad \text{—— (1)}$$

$$T_{input} \cdot \omega_{input} \pm T_{output} \cdot \omega_{output} = 0 \quad \text{— (2)}$$

Workbook Page No. 9.

Que 20.

$N_{input} = +100$

$N_{output} = +250$

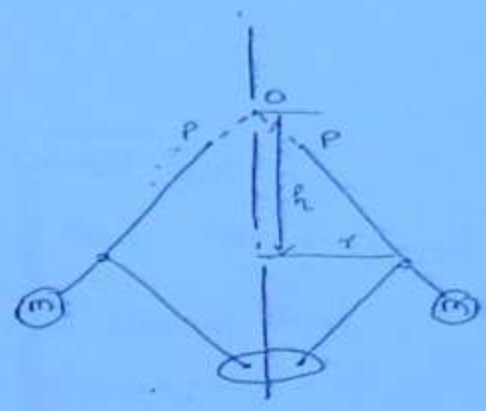
$T_{input} = +50$

$T_{Fixing} = ?$

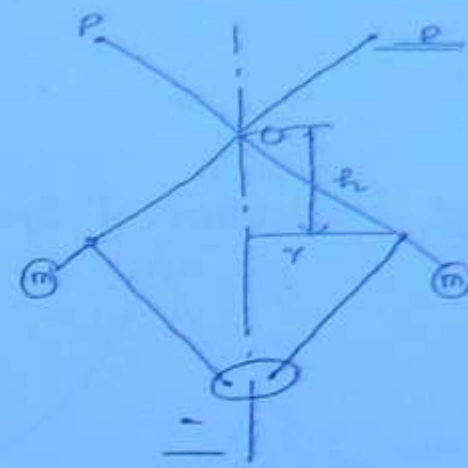
$$(50) \times (+100) + T_{output} \cdot (+250) = 0$$

$$\boxed{T_{output} = -20}$$

$$\begin{aligned} T_{fixing} &= -(T_{input} + T_{output}) \\ &= -(+50 - 20) \\ &= -(+30) \\ &= -30 \\ &= 30 \text{ kNm in anticlockwise direction.} \end{aligned}$$



Open Arm Tybe



Crossed Arm Type

- O is the point of intersection of the arm
- P is the point of pivot.

⇒

1> Pendulum Type of Governor (Watt Governor) :-

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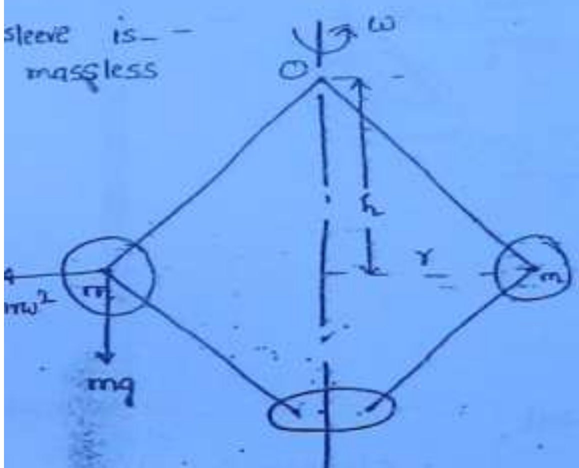

FBD Governor :-

Rot Eq^n :-

Point (O)

$(mr\omega^2)h = mg \cdot r$

$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}$

$N^2 = \left(\frac{60}{2\pi}\right)^2 \cdot \frac{g}{h}$ → 9.81

$\rightarrow \boxed{N^2 = \frac{895}{h}}$

Engine 1 : (30 r.p.m) → 35 r.pm

$h_1 = \frac{895}{(30)^2}$
$h_2 = \frac{895}{(35)^2}$
$h_1 - h_2$ → Bell crank lever

Engine 2 (80 r.pm) → 85 r.pm

$h_1 = \frac{895}{80^2}$

$h_2 = \frac{895}{85^2}$ $(h_1 - h_2) \rightarrow$ less

949 r.m

Beyond 60 r.pm

Failed

* Instantaneous Fluctuation of Speed Control Devices :-

[illegible] ← S
[illegible] ← C
[illegible] ← P
500rpm ← E

- Intra-cycle Fluctuation
  - Flywheel
- Inter-cycle Fluctuation
  - Governor
    - speed should be constant
    - discontinuous

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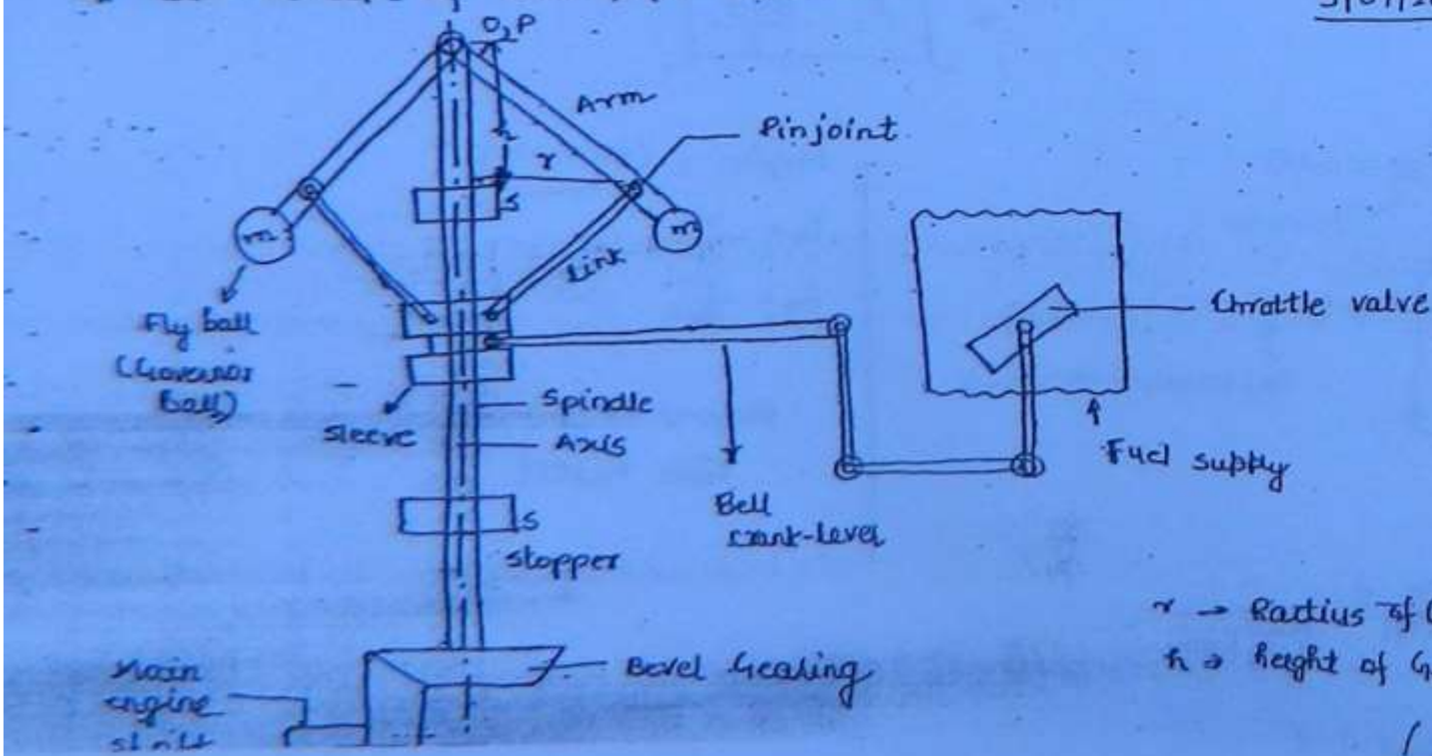
GOVERNOR

- Inertia Governor
- Centrifugal Governor
  - ⇓ Sir James Watt

* Basic Concept of Centrifugal Governor :-

5/07/2011



r → Radius of Gov.

h → height of Gov.

Loaded type :-

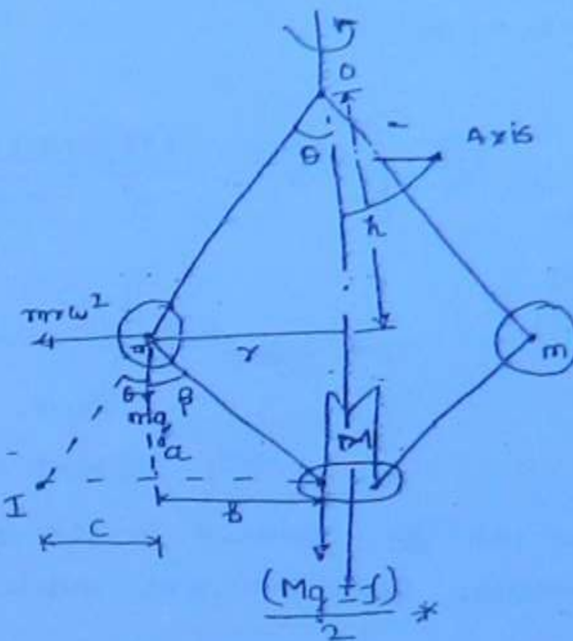
* Porter Gov :-

M → Mass of sleeve

$M >>>>>> m$

Taking moments w.r.t I :-

$$(mr\omega^2).a = mg\cdot c + \frac{(Mg \pm f)(b+c)}{2}$$

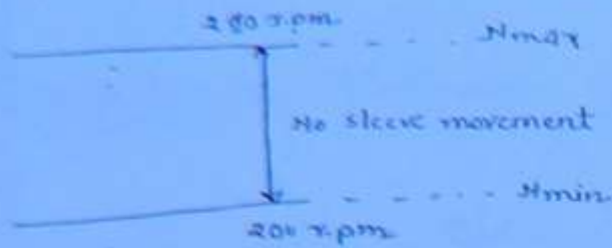


$$\boxed{N^2 = \frac{895}{h}\left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]}$$

$$K = \frac{\tan\beta}{\tan\theta}$$

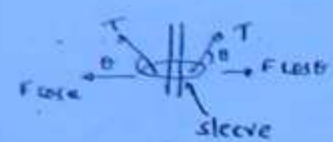
$$\left(1 + \frac{(Mg \pm f)(1+K)}{2mg}\right)$$

* Biggest Problem in Governor i.e,
Friction :-



$$N^2 = \frac{895}{h}\left[1 + \frac{(Mg \pm f)(1+k)}{2mg}\right]$$

$f_{max} = \mu N$



$\mu > 2$

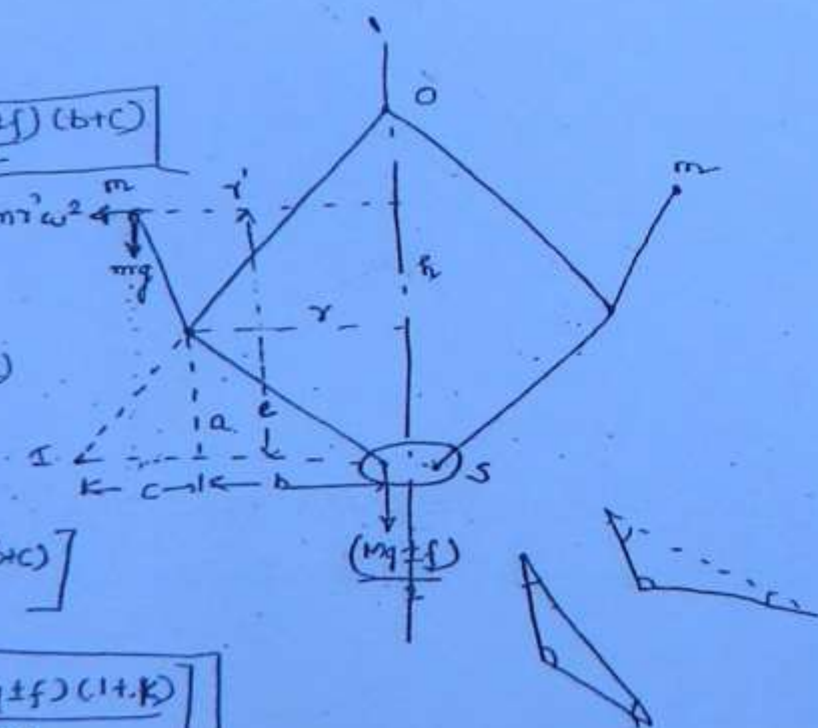
} Both element of F cosθ will cancel and normal reaction can be reduced to zero and friction will close to zero. This is reason governor's are made symetrical.

Loaded category (Dead Weight)

* Proell Gov :-

Moments :

$$\boxed{(mr'\omega^2)e = mg(c+r-r') + \frac{(Mg \pm f)}{2}(b+c)}$$



If $r' = r$

$$(mr\omega^2).e = mg.c + \frac{(Mg \pm f)}{2}(b+c)$$

$$(mr\omega^2)a = \frac{a}{e}\left[mgc + \frac{(Mg \pm f)}{2}(b+c)\right]$$

$$\boxed{N^2 = \frac{895}{h}\left(\frac{a}{e}\right)\left[\frac{2mg \pm (Mg \pm f)(1+k)}{2mg}\right]}$$

$\boxed{\frac{a}{e} < 1}$

Always

$1 + \frac{(Mg \pm f)(1+k)}{2mg}$ m↓ whole quantity increase $\times \left(\frac{a}{e}\right)$ will be same as Portall.

Total Inertia of the system will reduce

$$(mr'\omega^2)e = mg(c + r - r') + \frac{(Mg \pm f)}{2}(b+c)$$

sleeve is moving up

h↓ (r↑, r'↑)

4%	5% ↑	→ ω ↑
5% ↓	4% ↑	→ ω↓↓ → Unstability
4% ↑	4% ↑	→ ω → constant. (Isochronism)

$\frac{2\pi N}{60}$

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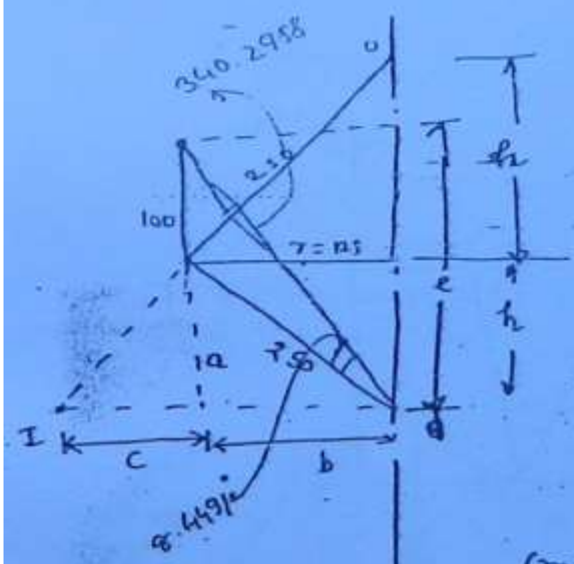


$m = 15$ kg

$M = 75$ kg

$h = 216.5063$

$a = 216.5063$

$r = 125$

$r' = .125$

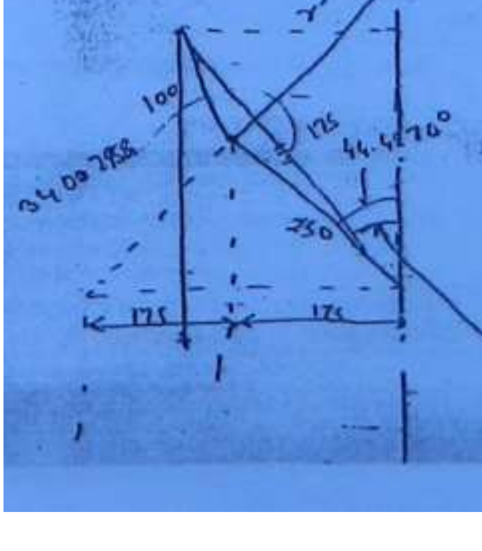
$b = 125$

$c = 125$

$e = 316.5063$

$$(mr'\omega^2).e = mg(c + r - r') + \frac{Mg}{2}(b+c)$$

$$\boxed{N = 130.225 \text{ r.pm}}$$



$b = 175$

$c = 175$

$r = 175$

$h = 178.5357$

$a = 178.5357$

35.9779°

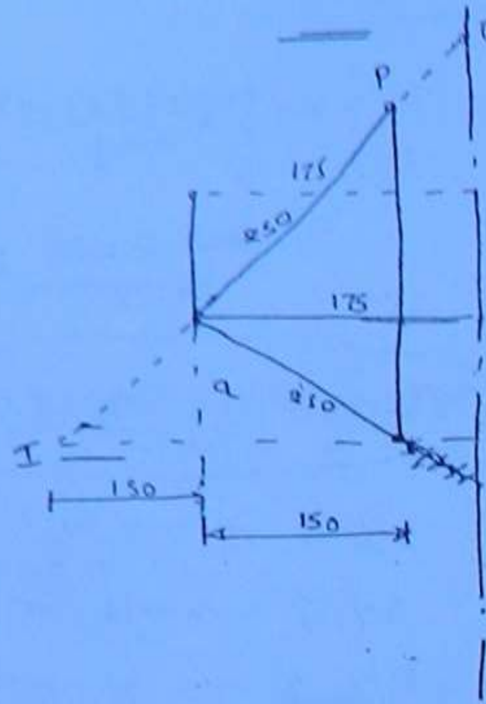
$r' = 199.9146$

$e = 275.3622$

$N = 129.0860$ rpm.

$b = 150$
$c = 150$
$a = 200$
$r = 175$
$r' = 175°$
$e = ?$

$m = 3.2$ kg
$M = 25$ kg
$N = 160$ r.p.m
$\rightarrow \omega = ?$



$(m r' \omega^2) e$

$= mg(c + r - r') + \frac{Mg}{2}(b + c)$

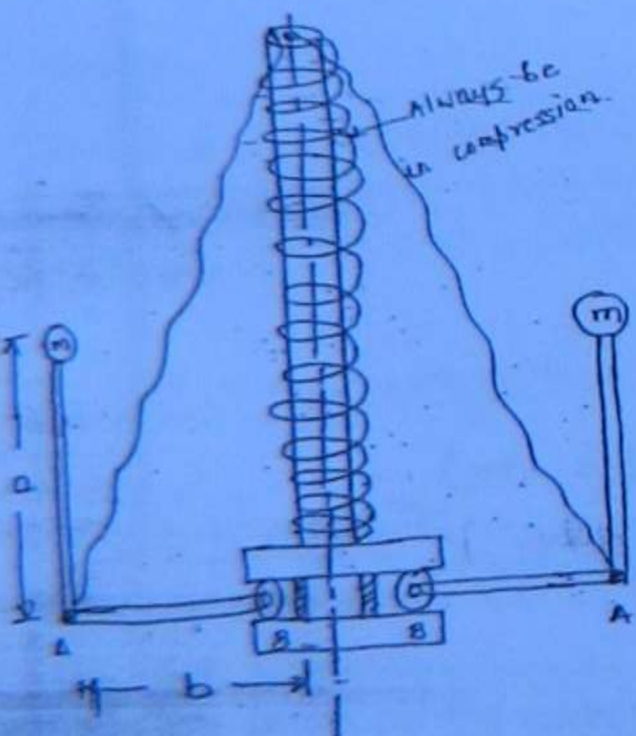
$e = 263.9594$ mm

307.9456.

06/07/11

* Spring Control Category :-

1> Hartnell Governor :-



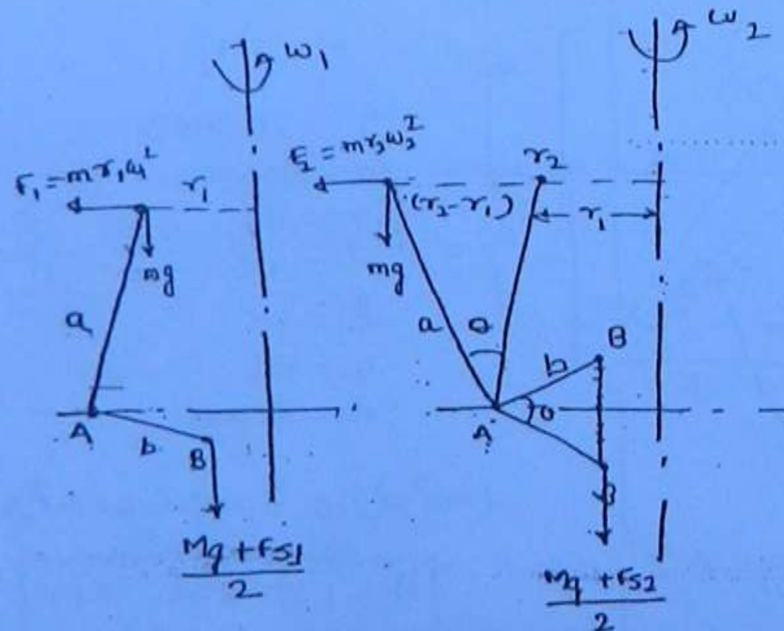


fig. representing two different situation

Moments w.r.t Point A :

(Neglect obliquity)

$$F_1 \cdot a = \frac{Mg + F_{S1}}{2} \cdot b \quad (1)$$

$$F_2 \cdot a = \frac{Mg + F_{S2}}{2} \cdot b \quad (2)$$

(2) − (1)

$$\frac{2a}{b}(F_2 - F_1) = F_{S2} - F_{S1} \quad \text{——} \quad (A)$$

Sleeve movement $= b \cdot \theta = \dfrac{b \cdot (r_2 - r_1)}{a}$

Additional compression in spring $= b \cdot \theta = \dfrac{b(r_2 - r_1)}{a}$

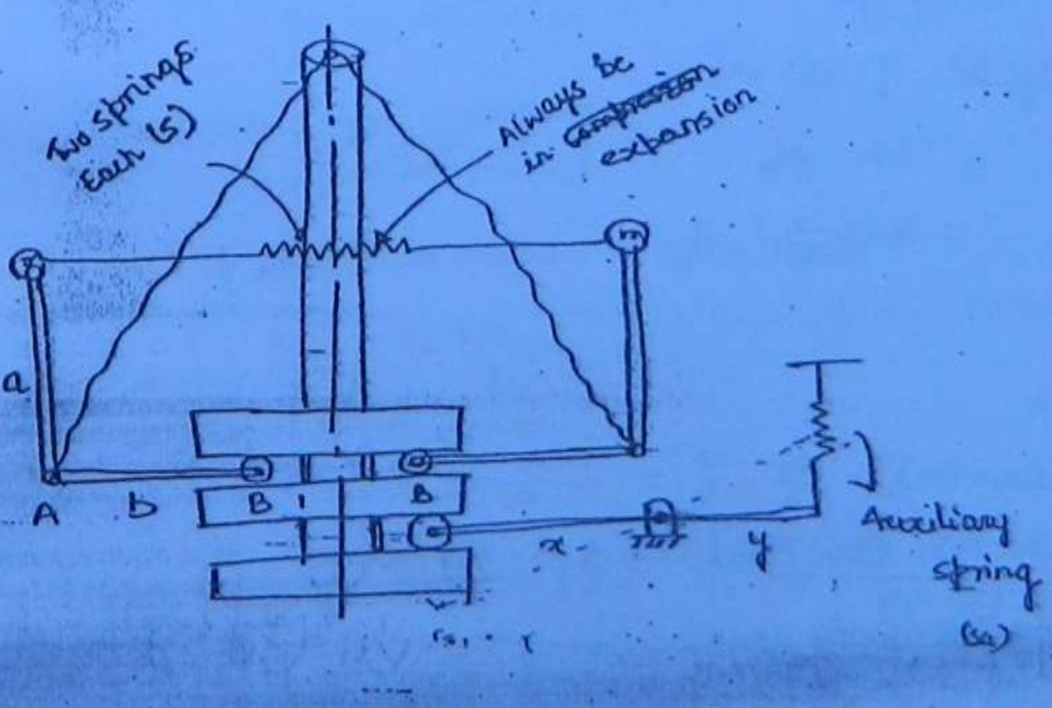
$$F_{S2} - F_{S1} = s \cdot \frac{b(r_2 - r_1)}{a} \rightarrow \text{obj}$$

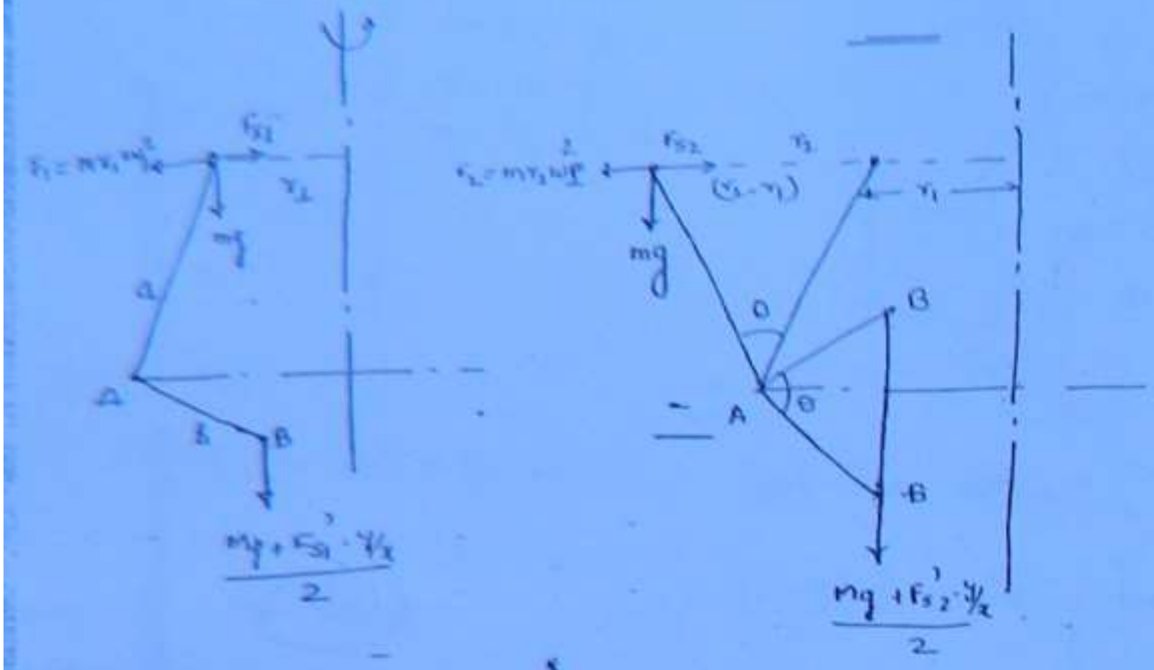
s = spring constant.

$$2\left(\frac{a}{b}\right)(F_2 - F_1) = s \cdot \frac{b}{a}(r_2 - r_1)$$

$$s = \frac{2(F_2 - F_1)}{(r_2 - r_1)}\left(\frac{a}{b}\right)^2 = \frac{2m(r_2\omega_2^2 - r_1\omega_1^2)}{(r_2 - r_1)} \cdot \left(\frac{a}{b}\right)^2$$

2> Wilson - Hartnell Governor :-





$F_1 = 266.7076$
F_2 $= 346.4743$
307.7395

Moment (A) :-

$(F_1 - F_{S1})a = \frac{Mg + F_{S1}' \cdot y/x}{2} \cdot b$ —— ① $(F_2 - F_{S2})a = \frac{(Mg + F_{S2}' \cdot y/x)}{2} \cdot b$ — ②

② - ①

$$\left[(F_2 - F_1) - (F_{S2} - F_{S1})\right] \cdot \frac{2a}{b} = (F_{S2}' - F_{S1}') \cdot \frac{y}{x} \quad \text{—— (A)}$$

~~Main~~ extension in Main spring

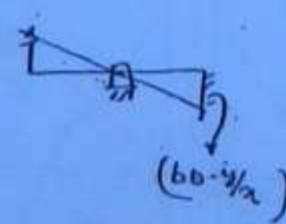
$= 2(r_2 - r_1)$ —— I obj.

$F_{S2} - F_{S1} = 2(r_2 - r_1) \times 5 \times 2 = 45(r_2 - r_1)$ —— II obj

Sleeve movement $= b\theta = \frac{b(r_2 - r_1)}{a}$ | III obj

More exp. in auxiliary sp. $= b\theta \cdot y/x$

$= b\frac{(r_2 - r_1)}{a} \cdot \frac{y}{x}$



$F_{S2}' - F_{S1}' = \frac{b(r_2 - r_1)}{a} \cdot \frac{y}{x} \cdot S_a$

Put the value ; $\left[(F_2 - F_1) - (F_{S2} - F_{S1})\right] \cdot \frac{2a}{b} \cdot \frac{x}{y} = \frac{b(r_2 - r_1)}{a} \cdot \frac{y}{x} \cdot S_a$

$$\Rightarrow \quad S_a = \frac{2\left[(F_2 - F_1) - (F_{S2} - F_{S1})\right]}{(r_2 - r_1)} \cdot \left(\frac{a}{b}\right)^2 \cdot \left(\frac{x}{y}\right)^2$$

$$= \left[\frac{2\left[m(r_2\omega_2^2 - r_1\omega_1^2) - 45(r_2 - r_1)\right]}{(r_2 - r_1)} \cdot (a/b)^2 (x/y)^2\right]$$

Workbook ...

Q.3 $\frac{B(r_2-r_1)}{a} = 3.0\ cm$ —①

$m = 1.5\ kg$

$B = 6.5cm$

$a = 7.5cm$

$\frac{r_1+r_2}{2} = 10.5$

$r_1+r_2 = 21\ cm$

$r_2 = 0.1223\ m$

$r_1 = 0.08769\ m$

$N_2 = 415\ r.p.m$

$N_1 = 430\ rpm$

$F_1 = m r_1 \left(\frac{2\pi \times 430}{60}\right)^2$

$F_2 = m r_2 \left(\frac{2\pi \times 415}{60}\right)^2$

$\therefore S = \frac{2(F_2-F_1)}{r_2-r_1}\left(\frac{a}{b}\right)^2$

$S = 6136.854$

$F_1 . a = \frac{(\cancel{Mg} + F_{S1})}{2} . b$

0

$F_{S1} = x_i \times S$

$x_i = 0.10028\ m$

$x_i = 10.028\ cm$

$F_2 . a = \frac{\cancel{Mg} + F_{S2}}{2} . b$

0

$F_S = ? = x_f \times S$

$x_f = 0.13026$

$= 13.028cm$

(ii)

(ii) $N_1 = 430\ r.p.m$, $N_2 = 440\ r.p.m$] x_i

$F_{S1} \rightarrow$ Same $\quad S \rightarrow$ New

$F_{S1} = x_i \times S$

Same ↑ ↓ New

(71)

* Stability of Governor :-

A governor is said to be in stable equilibrium if

i) every equilibrium speed is having its unique radius and every equilibrium radius is having its unique equilibrium speed.

Friction → zero

ii) the restoring forces should be quite dominent as compared to disturbing forces.

a) $r\uparrow \Rightarrow N\uparrow$
$r\downarrow \Rightarrow N\downarrow$] stable Eq^m

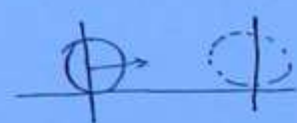
- Unstability of Governor

b) $r\uparrow \Rightarrow N\downarrow$
$r\downarrow \Rightarrow N\uparrow$ Unstable Eq^m

- Isochronism in Governor

c) $r\uparrow$, $r\downarrow$ $\Rightarrow N = \text{const.}$

Neutral Equilibrium



Isochronous Gov.

* Sensitivity of the Governor :-

For sleeve Movement

sensitivity $\alpha \dfrac{1}{N_1 - N_2}$

$$\boxed{\text{sensitivity} = \frac{N}{N_1 - N_2}} \quad \left(N = \frac{N_1 + N_2}{2}\right)$$

$$\boxed{\frac{N_1 - N_2}{N}}$$

- sensitivity of gov
- performance of engine

Note: Sensitivity and the stability Both are inverse property.

* Haunting :- (Governor and System both will die).

It is an extreme problematic situation with excessively high sensitive governor.

The very fast movement of the sleeve between the stoppers is known as Haunting

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If the governor ~~for~~ sensitivity is beyond - the certain limit for the slight movement of the load of the engine, then slight ~~incre~~ increase of speed of engine and governor. Immideately the sleeve is going to hit top stopper. Throttle valve fully closed. Fuel injection totally cut off and speed of the engine and gov. will dranstically decrease. Right at the same speed sleeve is going to sleeve ~~big~~ hit ~~lower~~ bottom stopper. ~~Throttle valve fully~~ open with faster speed and throttle will be fulley open fuel injection drastically increase and speed of engine will drastically increase. This phenomena will be repeated thereafter until and unless the stopper will be out of order. The system ~~is~~ will be out of control of governor.

Till the movement stoppers are there big vibration and ~~fo~~ fluctuation of speed will be introduced which damaged other parts of the system.

* Isochronism :-

A governor is said to be an Isochronous Governor excluding friction if the sleeve is moving and the radius of rotation ~~of~~ is changing but the equilibrium speed is not changing

$$\left.\begin{array}{l} r \uparrow \\ r \downarrow \end{array}\right\} \Rightarrow N_{eq}{}^{m} = \text{const.}$$

$$\boxed{\text{sensitivity} = \infty}$$

Pobs X

Hartnell :- $f = 0$

$$F_1 \cdot a = \frac{Mg + F_{s1}}{2} \cdot b \quad \text{— (1)}$$

$$F_2 \cdot a = \frac{Mg + F_{s2}}{2} \cdot b \quad \text{— (2)}$$

$$F_1 / F_2 = \frac{Mg + F_{s1}}{Mg + F_{s2}}$$

$$\frac{m r_1 \omega_1^2}{m r_2 \omega_2^2} = \frac{Mg + F_{s1}}{Mg + F_{s2}}$$

For Isochronism $\omega_1 = \omega_2$

$$\boxed{\frac{r_1}{r_2} = \frac{Mg + F_{s1}}{Mg + F_{s2}}}$$

$$\boxed{(\text{Hunting})_{\text{Isochronous}} = 0}$$

can't be used ($f \neq 0$).

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Isochronous gov. are not used for the practical purposes Because friction betn the sleeve and spindle can't be zero.

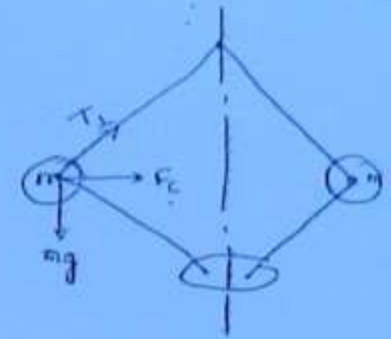
Q:- The condition of Isochronism can only be achieved at the expense of ~~sensitivity~~ stability. Yes

- leaving stability will always give Isochronism. No.

* Controlling Force Diagram :-

In every Governor the balls are in continuous rotation. There-fore, the force which is controlling the balls in rotation i.e., centripetal force ~~two~~ towards the centre along the radius is known as Controlling Force. Its value is $\frac{mv^2}{r}$ or $mr\omega^2$.

Watt



Protel / Proell

(mg) , (Mg)

Spring control

(mg) , (Mg) , (Fs)

$F_c = mr\omega^2$

$\omega^2 = \frac{F_c}{mr}$

$\left(\frac{2\pi N}{60}\right)^2 = \frac{F_c}{mr}$

$N^2 = \frac{60}{2\pi} \cdot \frac{1}{m} \cdot \frac{F_c}{r}$

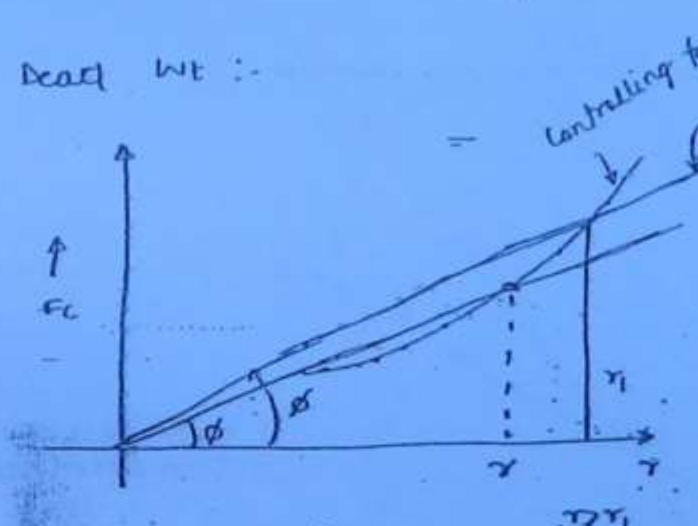
$N = \frac{60}{2\pi\sqrt{m}} \sqrt{\frac{F_c}{r}}$

$N = \text{const} \sqrt{\frac{F_c}{r}} = \text{cons} \sqrt{\tan\phi}$

$F_c = mr\omega^2$

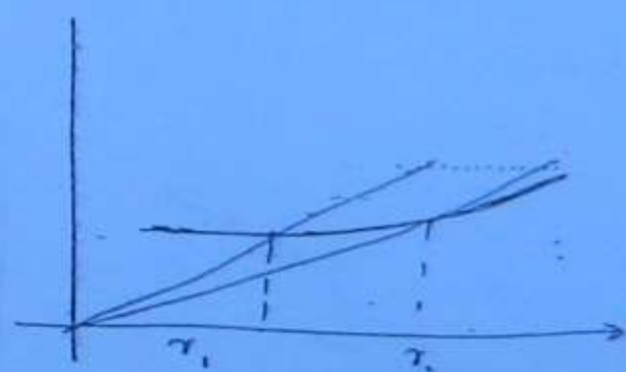
$(F_c - r) \rightarrow$ controlling force diagram

Dead Wt :-



r↑ N↑

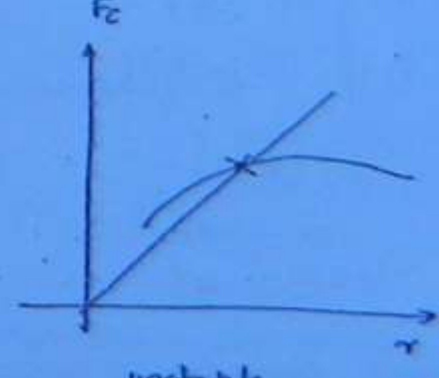
stable



r1 > r2   r↑ N↓

unstable

* The slope of controlling force diagram should be > the speed line slope for stability.

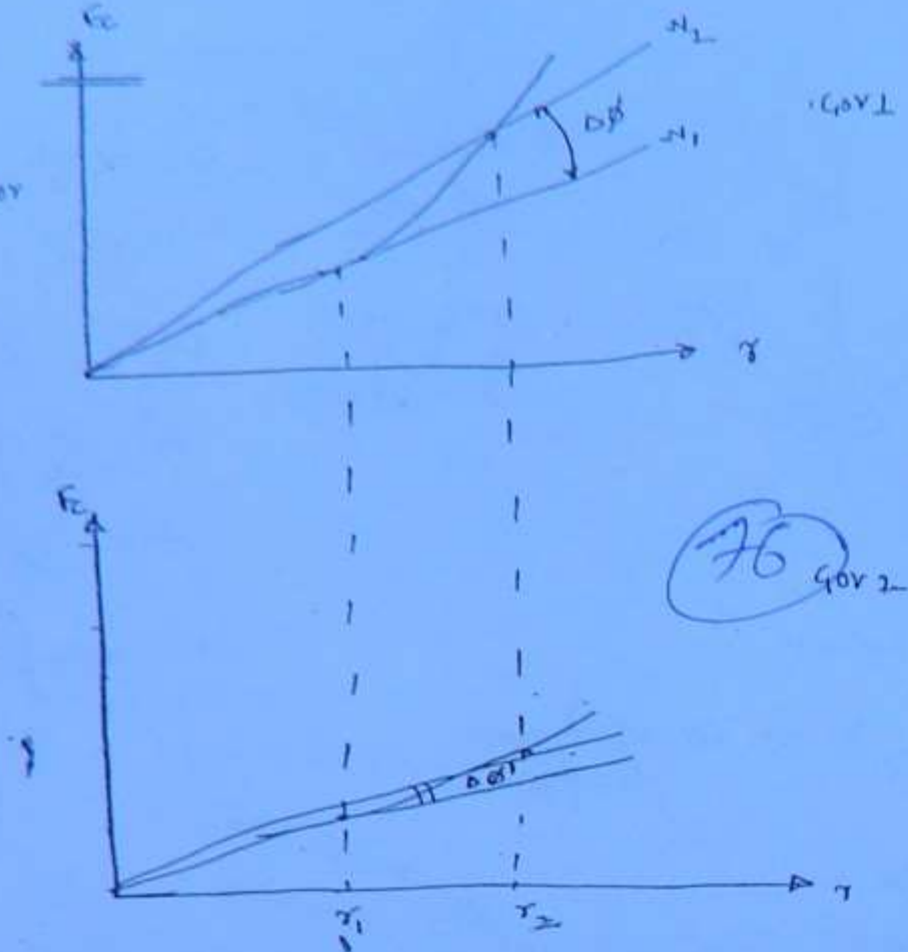


unstable

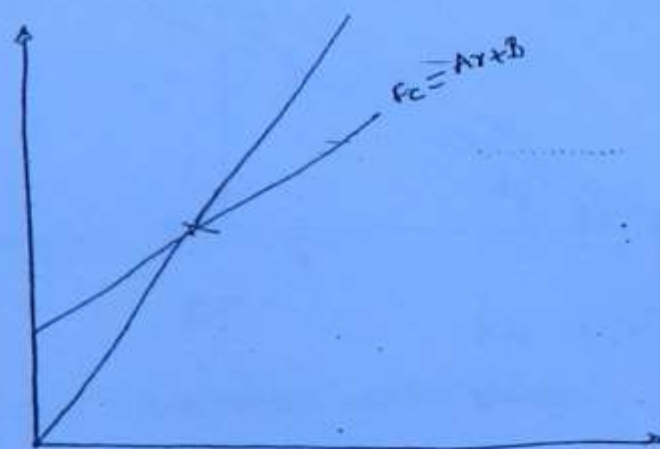
For the same sleeve movement, less speed is required the governor is sensitive.

If a governor is highly sensitive less stable

speed less → $\Delta\phi$ will be less.



SPRING CONTOL :-



$F_c = Ar - B$

$\frac{F_c}{r} = A - \frac{B}{r}$

$r\uparrow \quad \frac{B}{r}\downarrow$

$\therefore \left(A - \frac{B}{r}\right)\uparrow$

$\left(\frac{F}{r}\right)\uparrow$

$\Rightarrow \tan\phi \uparrow$

$\Rightarrow N\uparrow$

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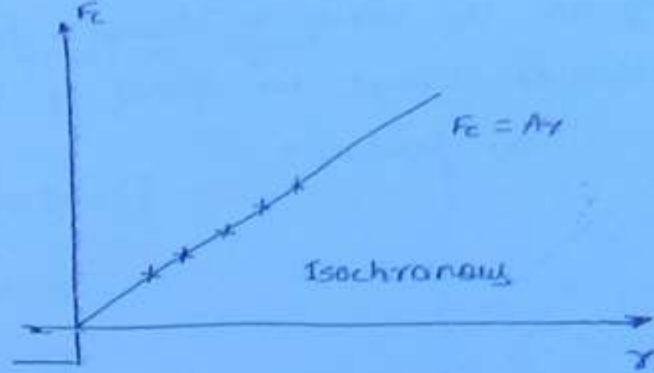
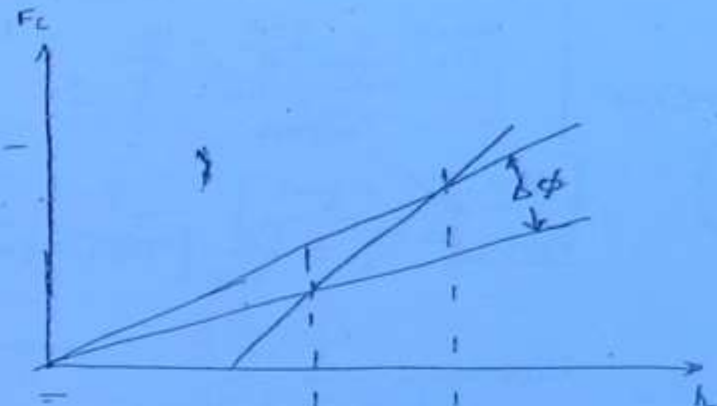
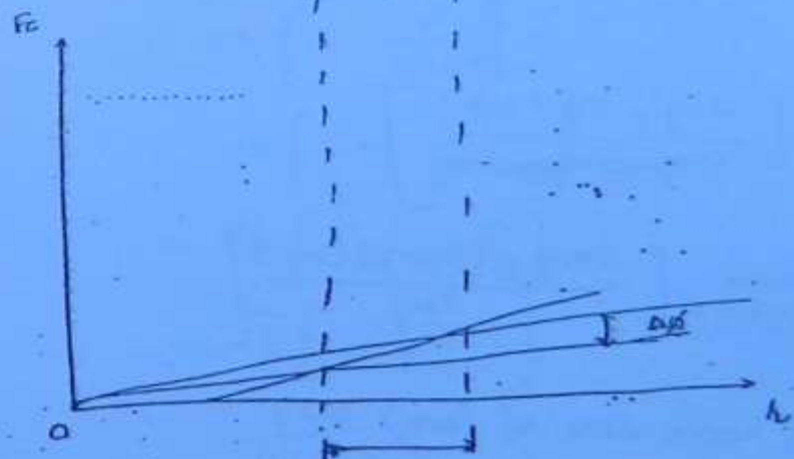
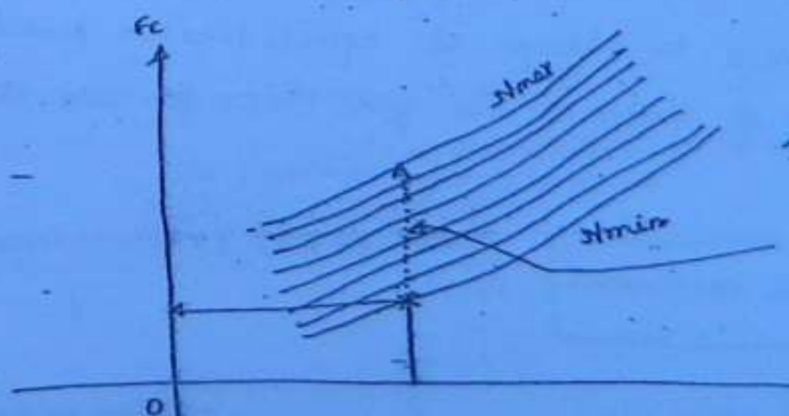








In practical

$$\frac{(N_{max} - N_{min})}{N}$$

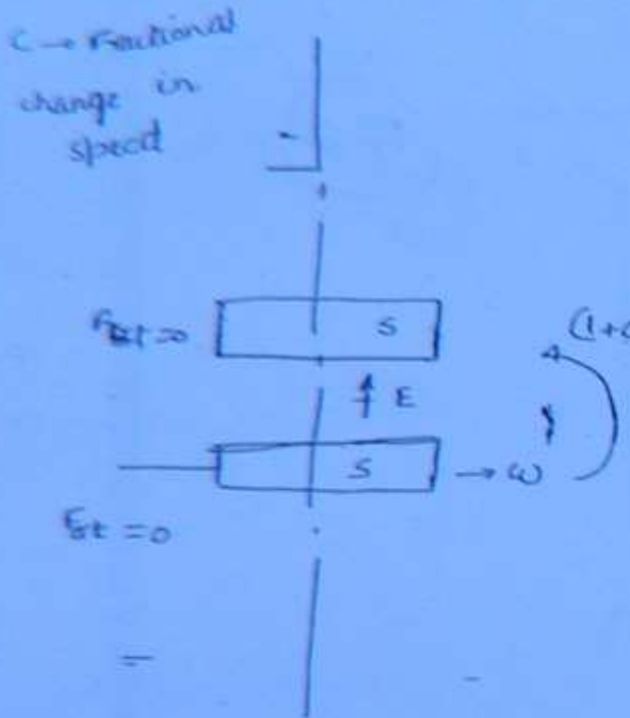
↓

co-efficient of Insensitiveness

$$N = \frac{N_{max} + N_{min}}{2}$$

* Effort of the Governor :-

The mean force acting on the sleeve to change its equilibrium position for the fractional change in speed of the governor is known as effort of the Governor



Mean Force on the sleeve $= \frac{0+E}{2} = \frac{E}{2} \rightarrow$ Eeffort

Eeffort.

Porter: $\frac{E}{2} = \frac{cg}{1+k}\left[2m + M(1+k)\right]$

Hartnell :-

$$\frac{E}{2} = c(Mg + F_s)$$

$$h = \frac{g}{\omega^2}\left[\frac{2mg + Mg(1+k)}{\cancel{2mg}\,2mg}\right]$$

$$h = \frac{g}{(1+c)^2\omega^2}\left[\frac{2mg + (Mg+E)(1+k)}{2mg}\right]$$

* Power of Governor :- (Work done of Gov)

The workdone at the sleeve to change its equilibrium position for the fractional change in speed of the gov. is known as Power of Governor.

$$\boxed{P = \frac{E}{2} \times \text{sleeve movement}}$$

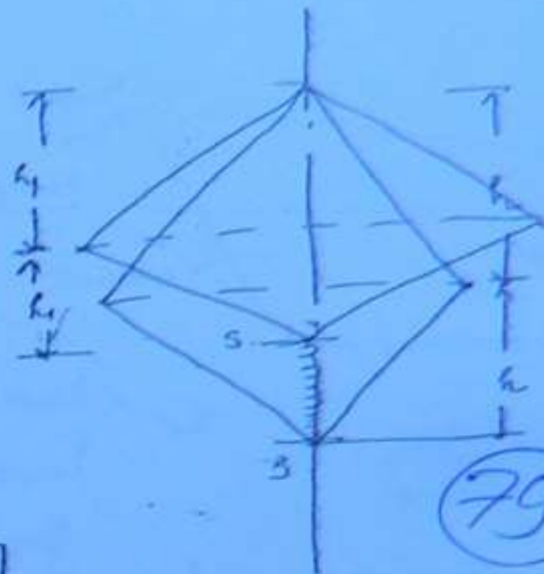
Porter Governor:- All arms are Equal

$\therefore K = 1$

$$\frac{E}{2} = \frac{cg}{1+1}\left[2m + M(1+1)\right]$$

$$= \frac{cg}{2} \times 2(m+M)$$

$$\boxed{\frac{E}{2} = cg(m+M)}$$



$$\omega \rightarrow h = \frac{g}{\omega^2}\left[\frac{2mg + Mg(1+1)}{2mg}\right]$$

$$(1+c)\omega \rightarrow h_1 = \frac{g}{(1+c)^2\omega^2}\left[\frac{2mg + Mg(1+1)}{2mg}\right]$$

$$\text{sleeve movement} = 2(h - h_1)$$

$$= 2h\left(1 - \frac{h_1}{h}\right)$$

$$= 2h\left[1 - \frac{1}{(1+c)^2}\right]$$

$$= 2h\left[1 - \frac{1}{1+c^2+2c}\right]$$

$$= 2h\left[1 - \frac{1}{1+2c}\right]$$

$$= 2h\left[\frac{\cancel{1}+2c-\cancel{1}}{1+2c}\right]$$

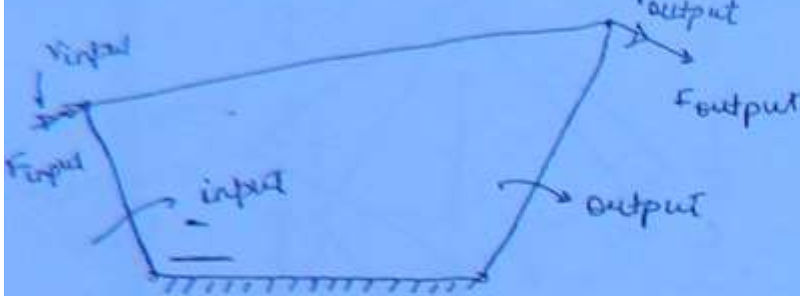
$$= \frac{4h \cdot c}{(1+2c)}$$

$$P = \frac{E}{2} \times \text{sleeve movement}$$

$$= cg(m+M) \cdot \frac{4h \cdot c}{1+2c}$$

$$= gh(m+M) \cdot \frac{4c^2}{1+2c}$$

* Mechanical Advantage of the Mechanism :-



Ideal :

$\eta_{mechanism} = 100\%$

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$$M.A. = \frac{F_{output}}{F_{input}}$$

$$= \frac{T_{output}}{T_{input}}$$

$$F_{inp} \cdot V_{inp} = F_{out} \cdot V_{out}$$

$$\frac{F_{out}}{F_{inp}} = \frac{V_{inp}}{V_{out}}$$

$$\therefore \cancel{M.A = \frac{V_{input}}{V_{output}} \times \eta_{mechanism}}$$

$$M.A = \frac{\omega_{input}}{\omega_{output}} \times \eta_{mechanism}$$

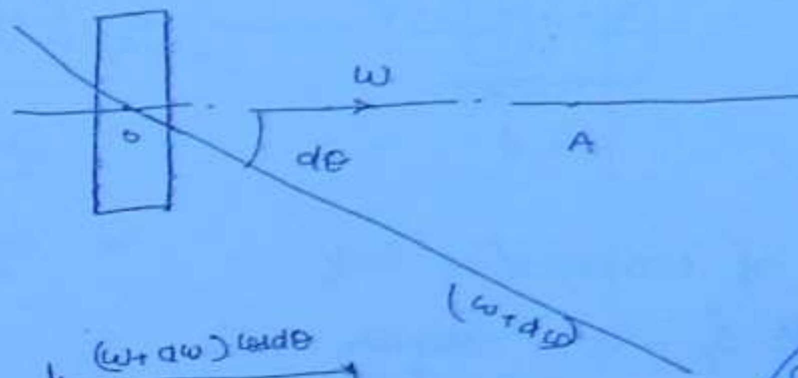
$\omega_{input} = \omega$

$\omega_{output} = 0$

$$M.A. = \frac{\omega}{0} = \infty$$

Along OA

which
acceleration is thue?



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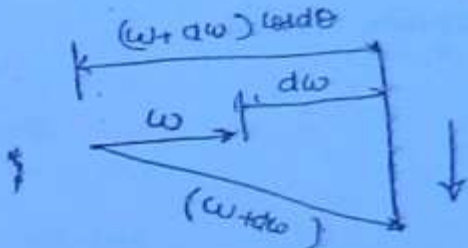
$\frac{d\omega}{d\theta}$



$\perp^r$ to OA

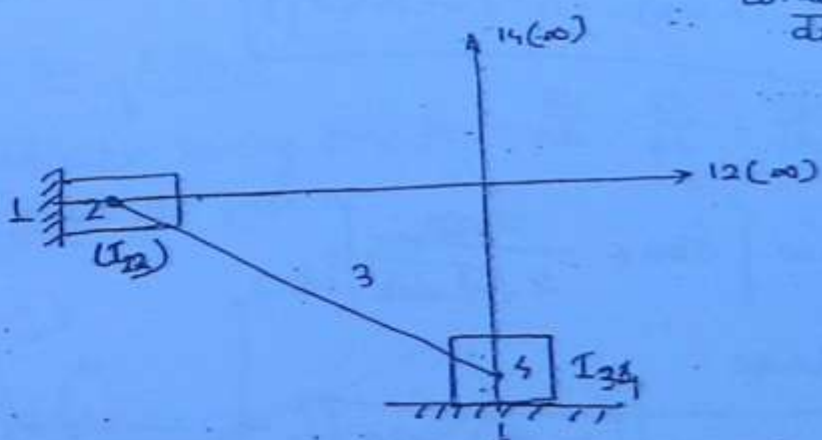
$(\omega+d\omega)\sin d\theta$ $\qquad \omega \cdot d\theta - 0$

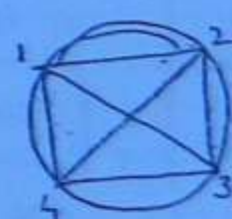
$= (\omega+d\omega)\cdot d\theta$ $\qquad = \omega \cdot d\theta$

$\omega \cdot d\theta + d\omega \cdot d\theta = \omega \cdot d\theta$

$\therefore \omega \cdot \frac{d\theta}{dt}$

*





Kinematic Analysis of Single Slider-Crank Mechanism :-

(Inertia of ~~single~~ C.R is not consider)

$m \rightarrow$ mass of reciprocating Parts

$\frac{l}{r} = n \rightarrow$ obliquity Ratio

$\omega \rightarrow$ crank speed

$\omega = \frac{d\theta}{dt}$

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Piston

$x = B_1B$

$= OB_1 - OB$

$= (r+l) - (BM + MO)$

$= (r+l) - (l\cos\beta + r\cos\theta)$

$n = \frac{l}{r} \Rightarrow l \Rightarrow nr$

$AM = l\sin\beta = r\sin\theta$

$\sin\beta = \frac{\sin\theta}{n}$

$\cos\beta = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$

$\cos\beta = \frac{\sqrt{n^2 - \sin^2\theta}}{n}$

n is very very large

$\boxed{a_{approx} = r\omega^2\cos\theta}$

$x = r + nr - \frac{nr\sqrt{n^2 - \sin^2\theta}}{n} - r\cos\theta$

$= r(1-\cos\theta) + r(n - \sqrt{n^2-\sin^2\theta})$

$$\boxed{x = r\left[(1-\cos\theta) + (n - \sqrt{n^2 - \sin^2\theta})\right]}$$

$v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$

$$v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

n is Large

$$\boxed{V_{approx.} = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]}$$

n is very-very Large

$V_{approx.} = r\omega\sin\theta$

Acc^n :-

$a_{approx} = \frac{dV_{approx}}{d\theta} \cdot \frac{d\theta}{dt}$

$a_{approx} = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

↓ large

C.R.

$$\omega_{CR} = \frac{d\beta}{dt}$$

$$\sin\beta = \frac{\sin\theta}{n}$$

$$\cos\beta \cdot \frac{d\beta}{dt} = \frac{\cos\theta}{n} \cdot \frac{d\theta}{dt}$$

$$\frac{\sqrt{n^2-\sin^2\theta}}{n} \cdot \omega_{CR} = \frac{\cos\theta}{n} \cdot \omega$$

$$\omega_{CR} = \frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}$$

n is large

$$\omega_{CR(approx)} = \frac{\omega\cos\theta}{n}$$

$$\alpha_{CR(approx)} = -\frac{\omega^2}{n}\sin\theta$$

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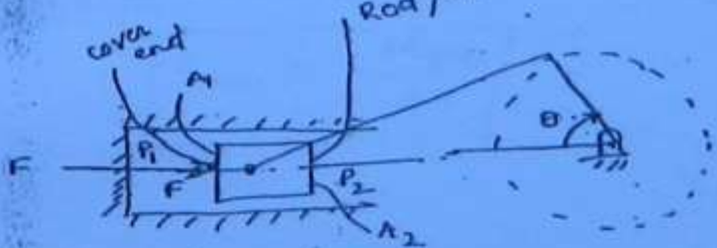
* Dynamic Analysis of Single Slider Crank Mechanism :-

1. Piston Effort
↓
Effective Driving force to drive the piston.



$$A_1 = \frac{\pi}{4} D^2$$

$$A_2 = \frac{\pi}{4}(D^2 - d^2)$$

$$\therefore F = (F_{gas} - F_I - f)$$

If it is a case of vertical engine

$$F = (F_{gas} - F_I - f) \pm mg.$$

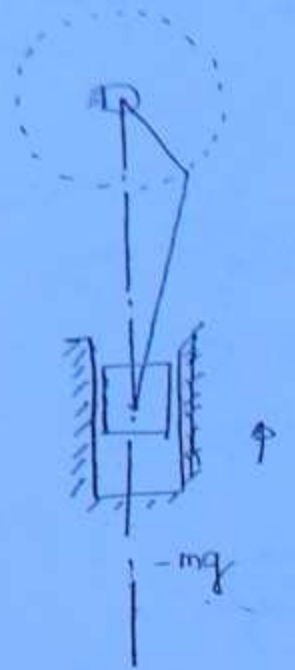
F

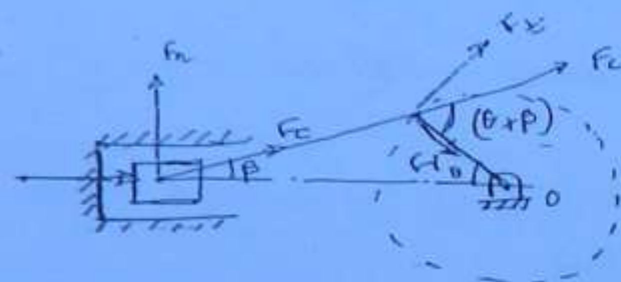
$$F_{gas} = (P_1A_1 - P_2A_2)$$

$$F_I = m.a$$

$$= m.r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

iii) f





2. Force Along C.R.

$F_c \cos\beta = F \Rightarrow F_c = \frac{F}{\cos\beta}$

3. Normal thrust to cy. walls: $\omega = \omega_0 + \alpha t$

$F_n = F_c \cdot \sin\beta \Rightarrow F \tan\beta$

4. Crank effort :

$F_t = F_c \sin(\theta+\beta) = \frac{F}{\cos\beta} \cdot \sin(\theta+\beta)$

5. Radial thrust to crank shaft Bearings :

$F_r = F_c \cos(\theta+\beta)$

$F_r = \frac{F}{\cos\beta} \cdot \cos(\theta+\beta)$

6. Turning Moment on Crank shaft :-

$T = F_t \cdot r = \frac{F}{\cos\beta} \cdot \sin(\theta+\beta) \cdot r$

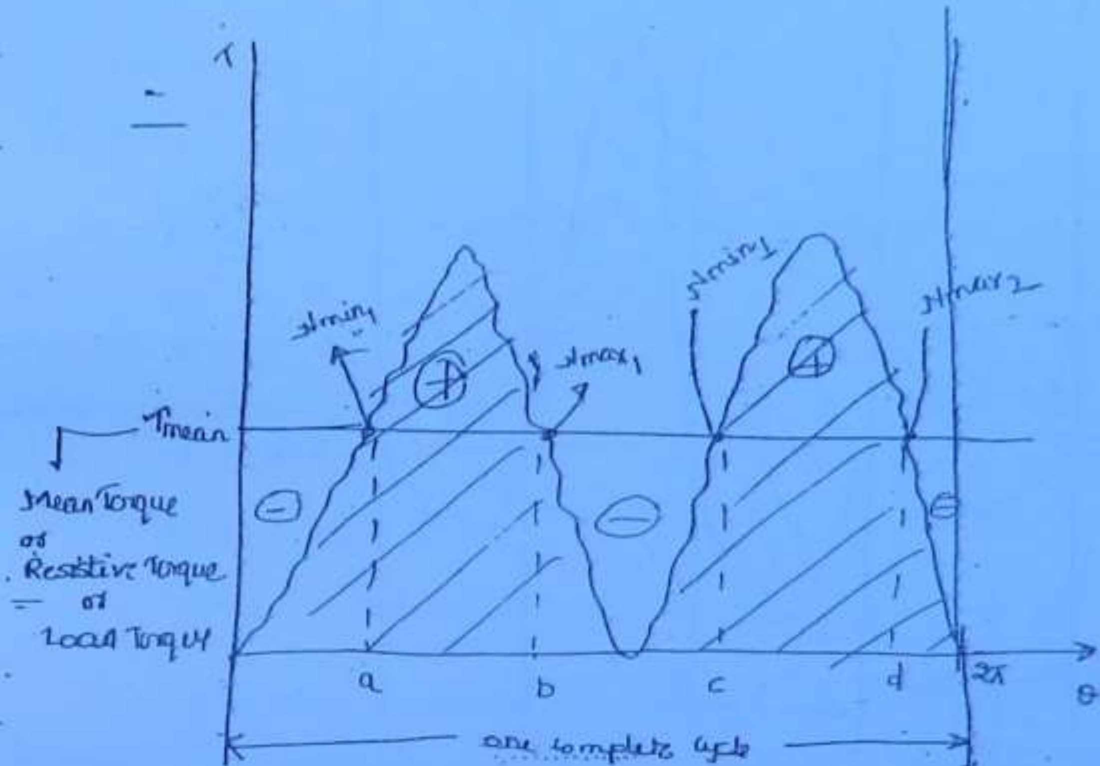
$T = f(\theta)$ i.e, $T = fn(time)$

$I\alpha = fn(time)$

$\alpha = f(time) \longrightarrow$ jerk.

Fly Wheel

* Turning Moment Diagram of Single Cylinder Double acting Steam Engine :



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W_{cycle} ⇒ area under (T-θ) Diagram

$T_{mean} \times 2\pi = W_{cycle}$

$$\boxed{T_{mean} = \frac{W_{cycle}}{2\pi}}$$

$\underbrace{\min(N_{min_1}, N_{min_2}, \ldots)}_{N_{min}}$

$\underbrace{\max(N_{max}, N_{max_2}, \ldots)}_{N_{max}}$

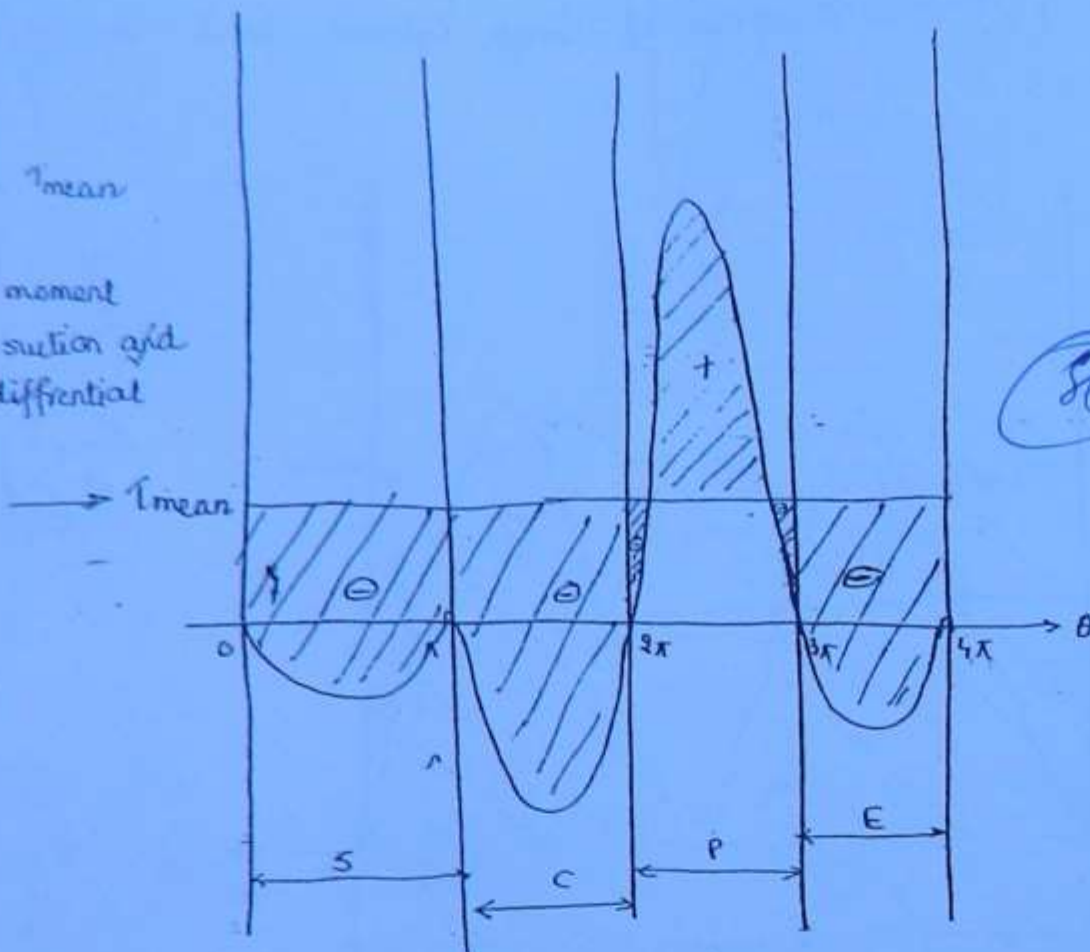
$(N_{max} - N_{min})$

Normally the flywheels are Bigger (heavier) in slow speed running engine and they are comparatively lighter in a high speed running engine.

* ~~Turning~~ Moment Diagram of Single Cylinder 4-stroke I.C. engine :-

$\frac{W_{cycle}}{4\pi} = T_{mean}$

The turning moment diagram of suction and exaust are diffrential positive due to valve action.



* Co-efficient of fluctuation of speed for the flywheel :-

$$\boxed{C_s = \frac{N_{max} - N_{min}}{N}} \qquad N = \frac{N_{max} + N_{min}}{2}$$

3% → 0.03

5% → 0.05

± 5% → 10% ⇒ 0.10

± 3% → 6% ⇒ 0.06

* Co-efficient of steadiness of speed for the flywheel.

$\frac{1}{C_s}$

* Co-efficient of fluctuation of energy for the flywheel :-

Maximum fluctuation = Variation

$$\boxed{C_E = \frac{E_{max} - E_{min}}{W_{cycle}}}$$

* Fundamental Equation of the flywheel :-

m → mass of flywheel

K → Radius of Gyration

$I = mk^2$

$E_{max} = \frac{1}{2} I \omega_{max}^2$

$E_{min} = \frac{1}{2} I \omega_{min}^2$

∴ Maximum fluctuation of Energy

Variation

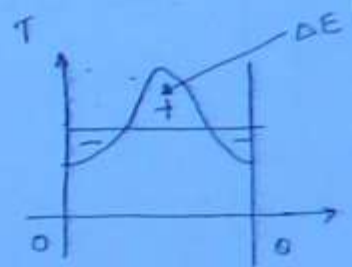
$\Delta E = E_{max} - E_{min}$

$\therefore \Delta E = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$

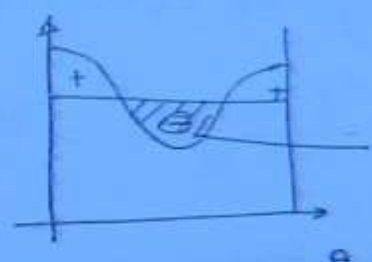
$= \frac{1}{2} I \frac{(\omega_{max} + \omega_{min})}{2} \frac{(\omega_{max} - \omega_{min})}{\omega} \times 2 \times \omega$

$$\boxed{\Delta E = I \omega^2 C_s}$$

1.



2.



Q. The turning moment diagram of an engine is represented as the equation $T = 2000 + 9500 \sin 2\theta - 5700 \cos 2\theta$, where θ is the angle turned by the crank from IDC. If the resisting torque is constant and the maximum fluctuation of speed w.r.t to mean speed which is 300 r.p.m. should not be more than 3%. Find

i) Power of the engine

ii) Mass of the flywheel required having the radius of Gyration 0.5m from the IDC

Harmonic function

$\rightarrow \quad T = 20{,}000 + 9500\sin 2\theta - 5700\cos 2\theta$

$C_s = 0.03$

$N = 300$ r.p.m

$\omega = \frac{2\pi N}{60} = 10\pi$ rad/s

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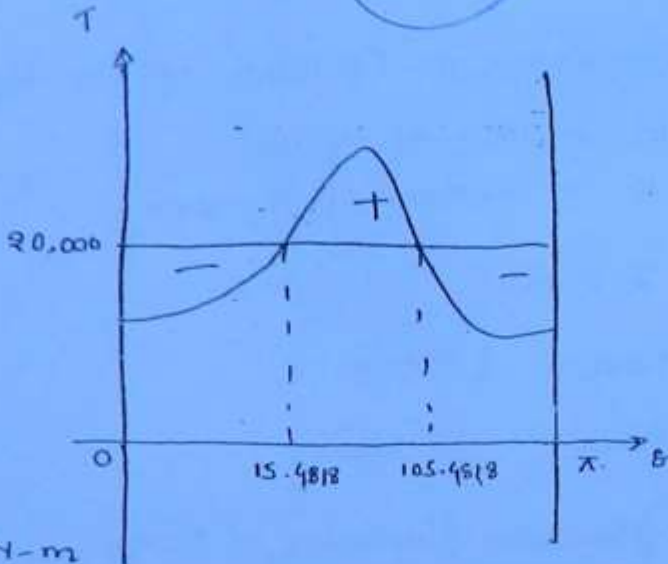
$\sin 2\theta \Rightarrow \frac{\pi}{2}$

$\cos 2\theta \Rightarrow \frac{\pi}{2}$

$\frac{\text{L.C.M of } N^r}{\text{H.C.F of } D^r} \Rightarrow \frac{\pi}{1} = \pi$



cycle $\rightarrow (0, \pi)$

$W_{cycle} = \int_0^{\pi} T \cdot d\theta = (20{,}000)\pi$

$T_{mean} = \frac{(20{,}000\,\pi)}{\pi} = 20{,}000$ N-m

i) $P = T_{mean} \cdot \omega_{mean}$

$= 20{,}000 \times 10\pi$

$= 20{,}0000\,\pi$ watt

$P = (200\pi)$ kwatt

Points where T-curve cuts Tmean line

at these point

$T = T_{mean}$

$\cancel{20{,}000} + 9500\sin 2\theta - 5700\cos 2\theta = \cancel{20{,}000}$

$\tan 2\theta = \frac{57\cancel{00}}{95\cancel{00}} = 0.6$

$2\theta = 30.9637^\circ, 210.9637^\circ, 390.9637^\circ, \ldots$

$\theta = 15.4818, 105.4818, 195.4818$

$$\Delta E = \int_{15.4918}^{105.4918} (T - T_{mean}) \cdot d\theta$$

$$= 11078.8086 \ N\text{-}m \ (or) \ J$$

$$\Delta E = I\omega^2 C_s$$

iii> $(T - T_{mean})_{\theta=45} = I \cdot \alpha$

$$9500 = I \cdot \alpha$$

$$\alpha = \frac{9500}{I} = 25.3893 \ rad/s^2$$

Que :-

$$\left.\begin{array}{ll} T = 5000 + 1500 \sin 3\theta & \longrightarrow \frac{2\pi}{3} \\ T_{mean} = 5000 + 600 \sin\theta & \longrightarrow \frac{2\pi}{1} \end{array}\right\} \frac{2\pi}{1} = 2\pi$$

Points where T curve cuts Tmean curve :

$$T = T_{mean}$$

$$5000 + 1500 \sin 3\theta = 5000 + 600 \sin\theta$$

$$5(3\sin\theta - 4\sin^3\theta) = 2\sin\theta$$

$$13\sin\theta - 20\sin^3\theta = 0$$

$$\sin\theta(13 - 20\sin^2\theta) = 0$$

when $\theta \equiv 0$

$$\theta = 0, \pi, 2\pi$$

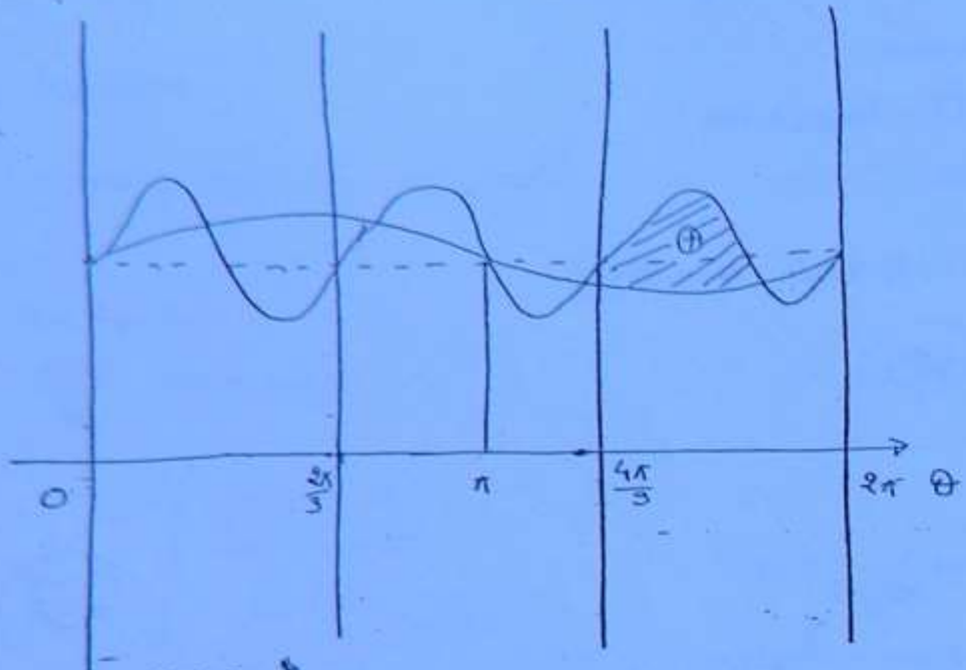
$$\sin\theta = \pm\sqrt{\frac{13}{20}} \begin{cases} +\sqrt{\frac{13}{20}} \\ -\sqrt{\frac{13}{20}} \end{cases}$$

$\theta = 53.7258°$

$126.2711°$

$233.7281°$

$306.2712°$



$$\Delta E = \int_{333.7288}^{306.2712} (T - T_{mean})\,d\theta$$

1656.502339.

* Requirement of Flywheel in Power Press :-

Que:- A punching press is used to punch 720 holes per/hour. The diameter of the whole is 20mm and the thickness of the sheet is 10mm. It requires 7 N-m of Energy per mm^2 of sheared area. If each punching operation requires 1/5 sec and the speed of the flywheel fluctuates from 120 to 100 r.p.m ~~during dura~~ during punching. What should be the mass of the flywheel require for this punching operation. If radius of gyration is 0.3m.

→ 120 r.p.m 100 r.p.m

$$N_{mean} = 110 \text{ r.p.m}$$

$$C_s = \frac{120-100}{110} = 0.1818$$

$$\omega = \frac{2\pi \times 110}{60} = 11.5192 \text{ rad/s}$$

720 holes / hr

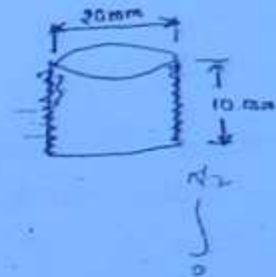
720 holes / 3600 hr

0.2 holes / sec

1 holes / 5 sec + cycle time

0.32

1 hole / $\frac{1}{5}$ sec → Exact punching time



r — 180°

$$A_{sheared} = \pi(20) \times 10$$
$$= (200\pi)\ mm^2$$

$$E_{hole} = 7 \times 200\pi$$

$$E_{hole} = 1400\pi \text{ Joules}$$

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Motor :-

$$P_{motor} = \text{Energy Req. / sec}$$
$$= E_{hole} \times \text{No. of holes / sec}$$
$$= 1400\pi \times \frac{720}{3600}$$
$$= (280)\pi \text{ watt. J/s.}$$

Punching : ($\frac{1}{5}$ sec)

$$E_{available} = 280\pi \times \frac{1}{5}$$
$$= (56\pi) \text{ Joules}$$

$$E_{hole} = 1400\pi \text{ Joules}$$

$$[1400\pi - 56\pi] = I\omega^2 C_s$$

mk^2

1944.7694 kg.

* cycle time → 5 sec

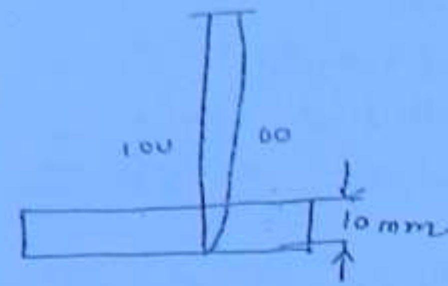
Exact punching time → ??

200 mm — 5 sec

10 mm — $\frac{5}{200} \times 10$ sec

→ Exact punching time

stroke Length → 100 mm



* Exact punching time ?

$\omega = ?$

cycle time → 5 sec

$c_s = \pm 3\%$

Exact-punching is done in 45° of crank rotation

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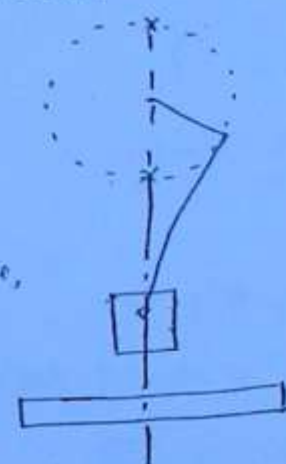
crank
↓
2π rotation for one cycle i.e, 5 sec



$\therefore \omega = \frac{2\pi}{5}$

360° ——— 5 sec

45° ——— $\frac{5}{360} \times 45°$ sec

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Q.3 Cycle time → 2 sec

$P_{motor} = 1500$ W

$1500 = E_{hole} \times \frac{1}{2}$

$E_{hole} = 3000$ Joules

$\omega = \pi$ rad/s

Exact Punching Time = $\frac{1}{6}$ sec

$3000 - 1500 \times \frac{1}{6} = I \omega^2 c_s$

30 holes — 1/2 hole

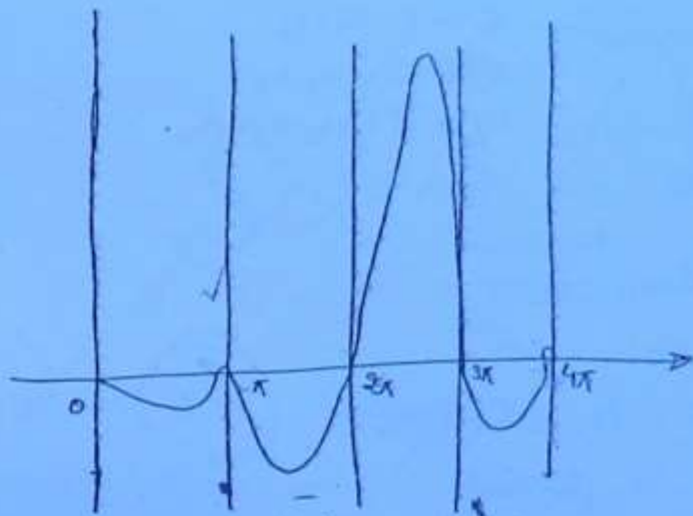
$\omega = \frac{\pi}{2}$

360° — 2 sec

$\frac{2}{360} \times 30 = \frac{1}{6}$ sec

$CS = 0.2$

* Concept of Multicylinders :-

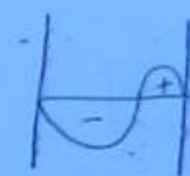


single ~~cyl~~ cylinder

2 - cy

cyl 1 — cy 2

P — S



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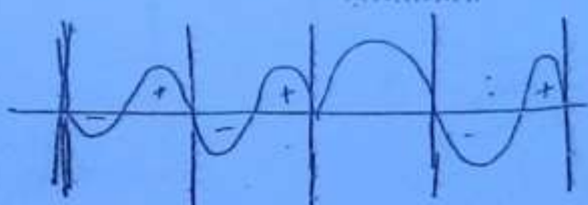
3 - cyl

c1 — c2 — c3

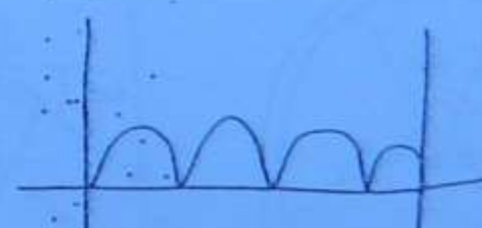
P — S — C



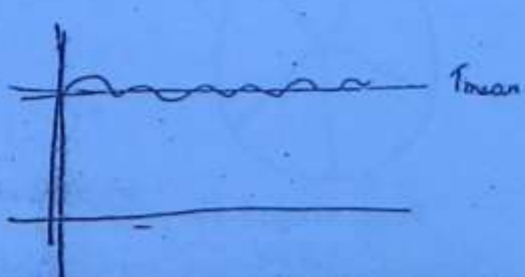
4 cylinder

1 — 2 — 3 — 4

P — S — C — E.

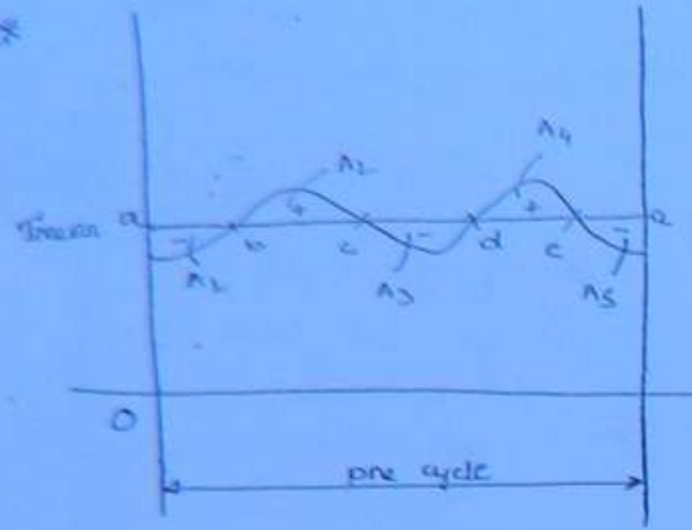


5/6



Concept of Multi-cylinder was introduced not to increase the power rather it is introduced to uniform power i.e, uniform turning moment.

*



A_1, A_2, A_3, A_4, A_5 — unit ⇓ mm^2

$$\boxed{A_2 + A_4 = A_1 + A_3 + A_5}$$

let us assume

$E_a = E$

$E_b = E - A_1$

$E_c = E - A_1 + A_2$

$E_d = E - A_1 + A_2 - A_3$

$E_e = E - A_1 + A_2 - A_3 + A_4$

$E_a = E$

$E_{max} = E + 30$

$E_{min} = E - 5$

$$\Delta E = (E + 30) - (E - 5)$$

$$= 35\ mm^2$$

$$= 35 \times \underbrace{mm}_{\theta\,scale} \times \underbrace{mm}_{T\,scale}$$

$$= 35 \times \frac{\pi}{3} \times 10 \text{ Joules} = I\omega^2 C_s$$

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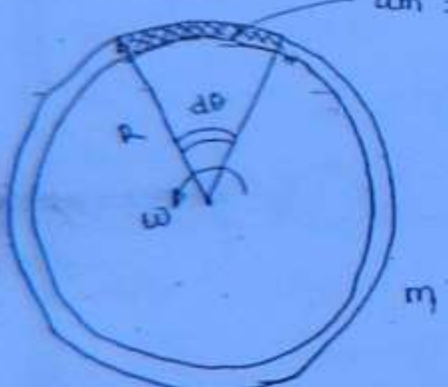
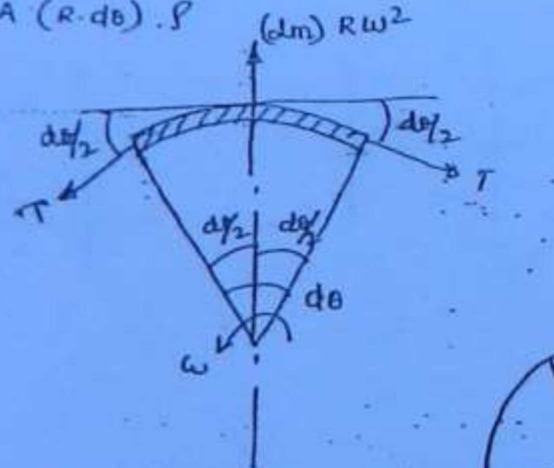
* Designing of flywheel :-

$dm = A\,(R \cdot d\theta) \cdot \rho$



Ring shaped flywheel



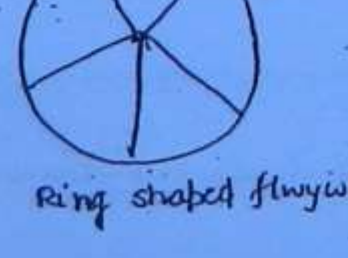
$$2T \sin\frac{d\theta}{2} = dm \cdot R\omega^2$$

$$2T \cdot \frac{d\theta}{2} = (A \cdot R\,d\theta) \cdot \rho \cdot R\omega^2$$

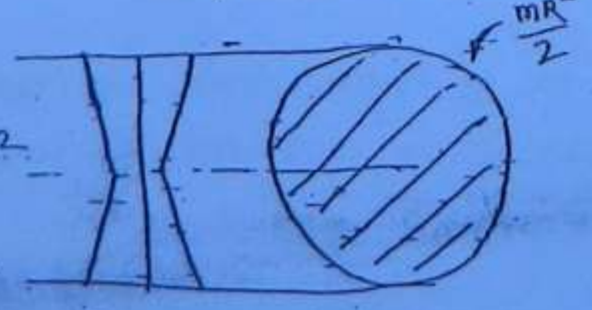
$$\frac{T}{A} = \rho\,(R\omega)^2 = \rho \cdot v^2$$

$$\boxed{\sigma = \rho v^2}$$

$$v_{max} = \sqrt{\frac{\sigma_b}{\rho}}$$



Ring shaped flwywheel



$$\frac{mR^2}{2} < I < mR^2$$

BALANCING

(one question sure*)

$F_{unbalance}$

- Rotational Unbalance
- Reciprocating Unbalance

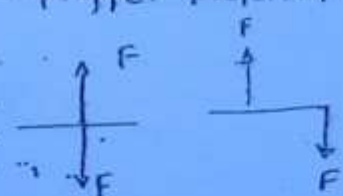
Balancing

- static Balancing
- Dynamic Balancing

static Balancing: $\Sigma F = 0$

Dynamic Balancing: $\Sigma F = 0$, $\Sigma M = 0$

if, All masses are rotating in the same plane, static balancing is ~~preffer~~ preferred

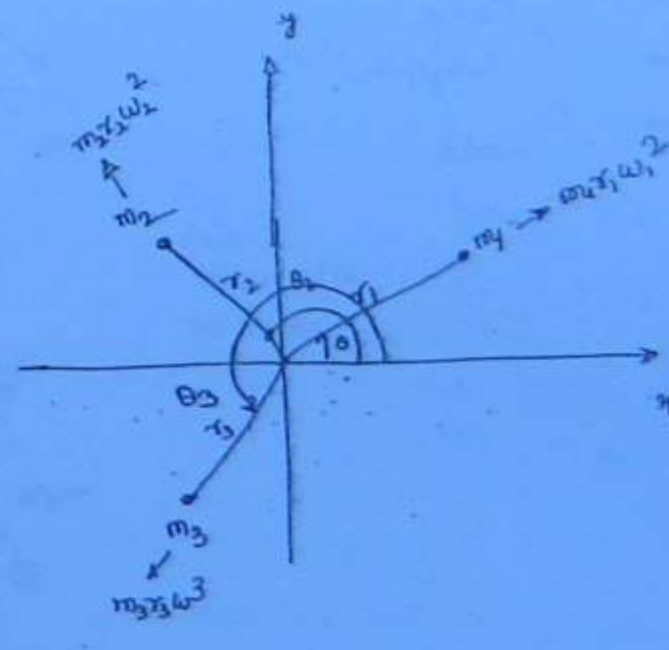


* Static Balancing :-

$m_B = ?$

$r_B = ?$

$\theta_B = ?$



$F_x = 0$

$$m_1 r_1 \omega^2 \cos\theta_1 + m_2 r_2^2 \omega^2 \cos\theta_2 + m_3 r_3 \omega^2 \cos\theta_3 + m_B r_B \omega^2 \cos\theta_B = 0$$

$$m_B r_B \cos\theta_B = -\Sigma(mr\cos\theta) \quad \text{—(1)}$$

$F_y = 0$

$$m_B r_B \sin\theta_B = -\Sigma mr\sin\theta \quad \text{—(2)}$$

$\sqrt{1^2+2^2}$

$$m_B r_B = \sqrt{\{-\Sigma mr\cos\theta\}^2 + \{-\Sigma mr\sin\theta\}^2}$$

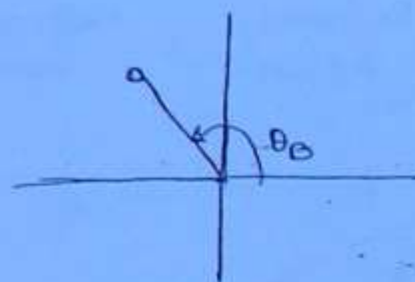
$$\boxed{\tan\theta_B = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta}}$$

For a pb:

$$\Sigma mr\cos\theta = +4$$

$$\Sigma mr\sin\theta = -5$$

$$\tan\theta = \frac{-(-5)}{-(+4)} = \frac{+5}{-4}$$



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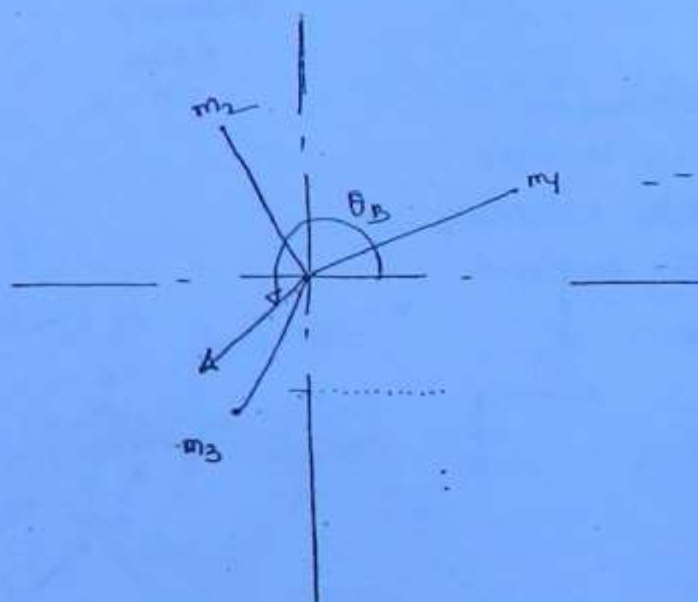
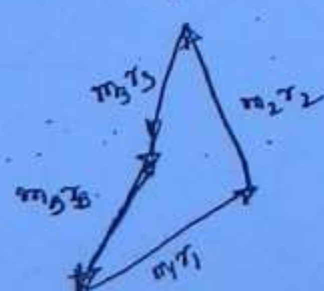


$m_1r_1 \longrightarrow$ Force

$m_2r_2 \longrightarrow$ Polygon

$m_3r_3 \longrightarrow$ Scale



[redacted]

Pb:-

$m_1 r_1 = 2$

$m_2 r_2 = 4$

$m_3 r_3 = 5$

What should be the position of mass 2 and 3 w.r.t. mass 1 in order to have complete Balancing

$F_x = 0$

$$m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 = 0$$

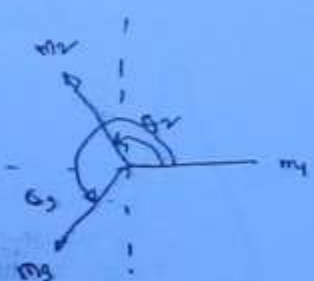
$$2\cos\theta_1 + 4\cos\theta_2 + 5\cos\theta_3 = 0$$

$$4\cos\theta_2 + 5\cos\theta_3 = -2$$

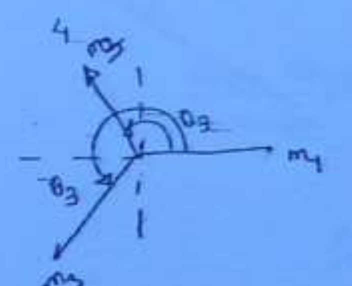
$$m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 = 0$$

$$4\sin\theta_2 + 5\sin\theta_3 = 0$$

$$\Rightarrow \quad 4\sin\theta_2 = -5\sin\theta_3$$



$\Rightarrow$



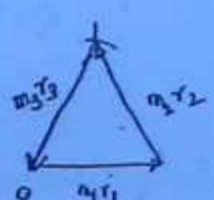
$\theta_2, \theta_3 = (\; , \;)$

$(\; , \;)$

$(\; , \;)$

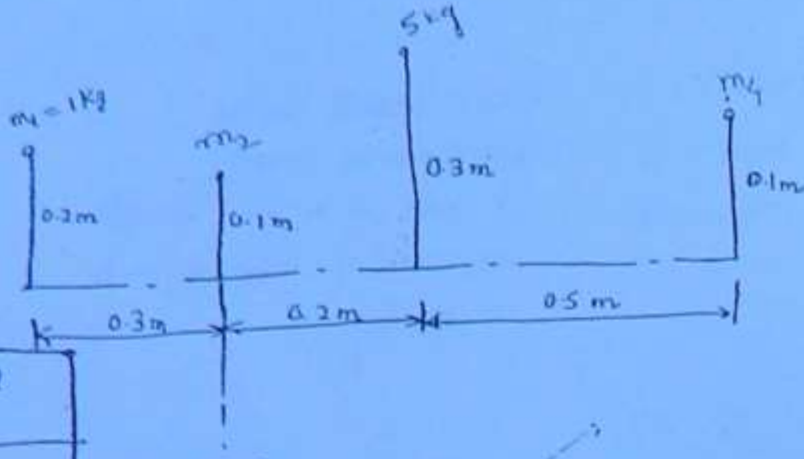
$(\; , \;)$

two value will be same

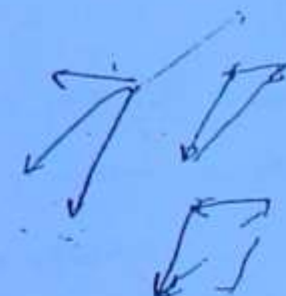


* Dynamic Balancing :- (all the masses are not rotating in the same plane)

[redacted]



Planes	m	r	m.r (Force)	l (Distance from R.P.)	m.r.l (moment)
1	4	0.2	0.8	−0.3	−0.24
2	m_2	0.1	$0.1m_2$	0	0
3	5	0.3	1.5	0.2	0.30
4	m_4	0.1	$0.1m_4$	0.7	$(0.07)m_4$



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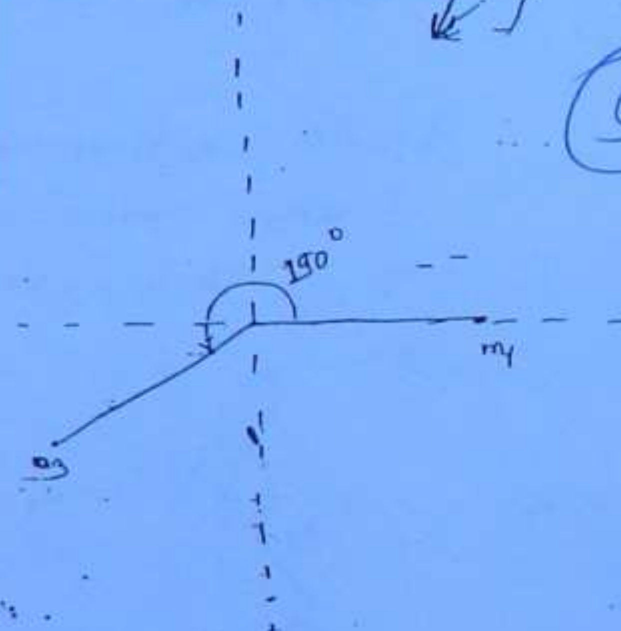
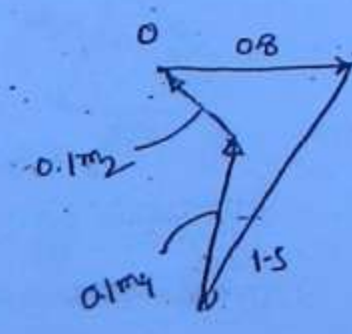


moment Polygon

scale

Force Polygon



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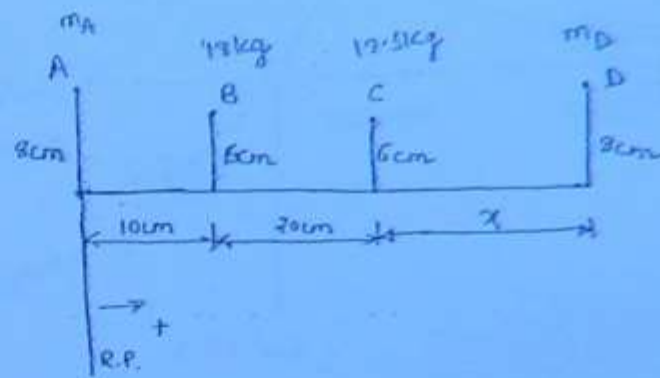
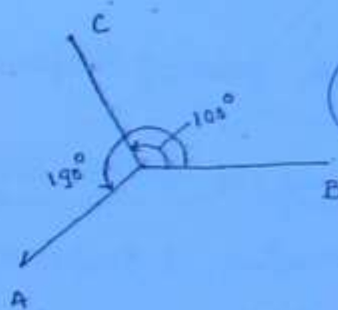
Que : 2

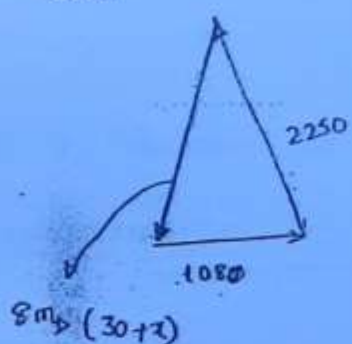
Planes	m	r	m.r	l	m.r.L
A	m_A	8	$8m_A$	0	0
B	18	6	108	10	1080
C	12.5	6	75.0	30	2250
D	m_D	8	$8m_D$	$30+x$	$8m_D(30+x)$





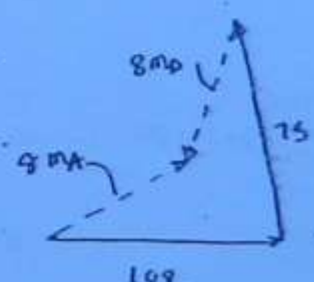
Moment Polygon

scale:



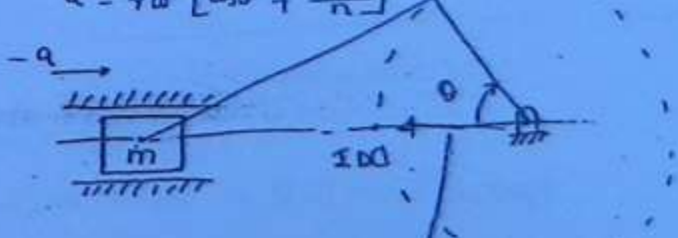
Force Polygon

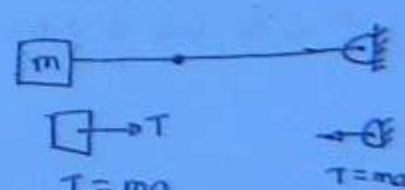
scale



* Balancing of Reciprocating masses :-

$$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$



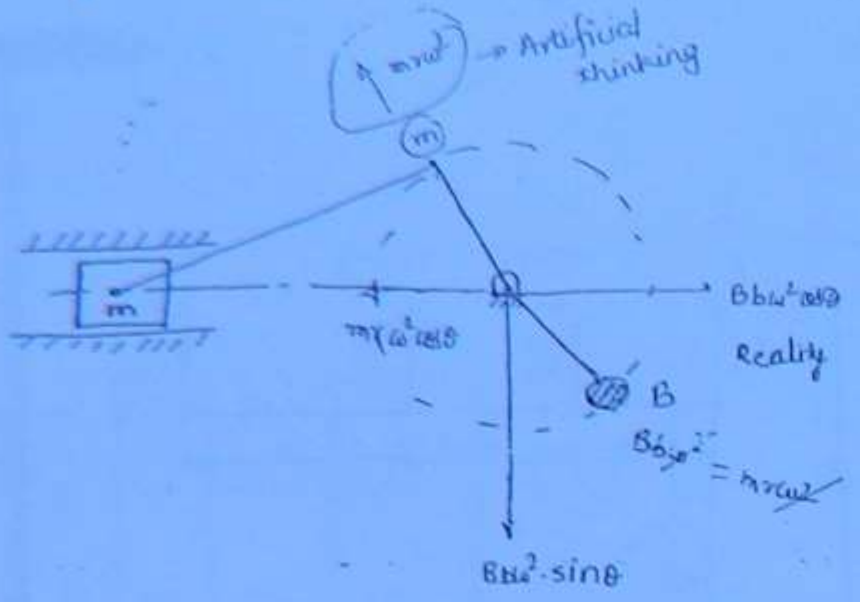


$F_{primary} >>>> F_{secondary}$

$$F_{un} = mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= mr\omega^2\cos\theta + mr\omega^2\cos 2\theta/n$$

* Primary Balancing of Reciprocating Masses :-



We will never balance Reciprocating masses completely. 160

Partial Balancing of Reciprocating masses :-

Let $c \rightarrow$ Fraction of Reciprocating mass to be balanced

$0 < c < 1$

$B \cdot b = c m \cdot r \Rightarrow < m r$

F_{UN} (along the line of stroke)

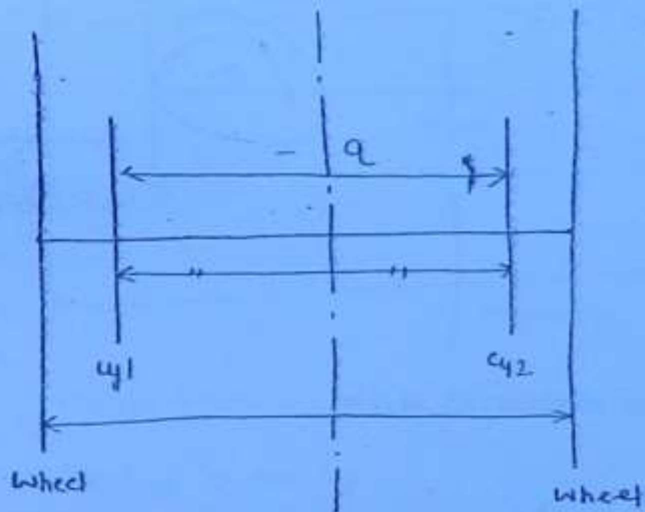
$$= m r \omega^2 \cos\theta - \underbrace{B \cdot b}_{c m \cdot r} \omega^2 \cos\theta$$

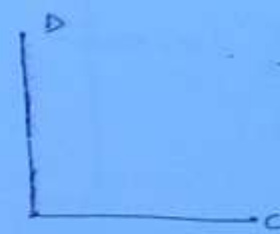
$$= (1-c) m r \omega^2 \cos\theta$$

$$F_{UN} (\perp^{ar} \text{ to the line of stroke}) = B b \omega^2 \sin\theta$$
$$= c m \cdot r \cdot \omega^2 \sin\theta$$

* Effect of Partial Balancing in Two-Cylinder Locomotive :-

$$\text{2-cylinder locomotive} \to \text{Inside cylinder}, \quad \text{2-cylinder locomotive} \to \text{outside cy.}$$





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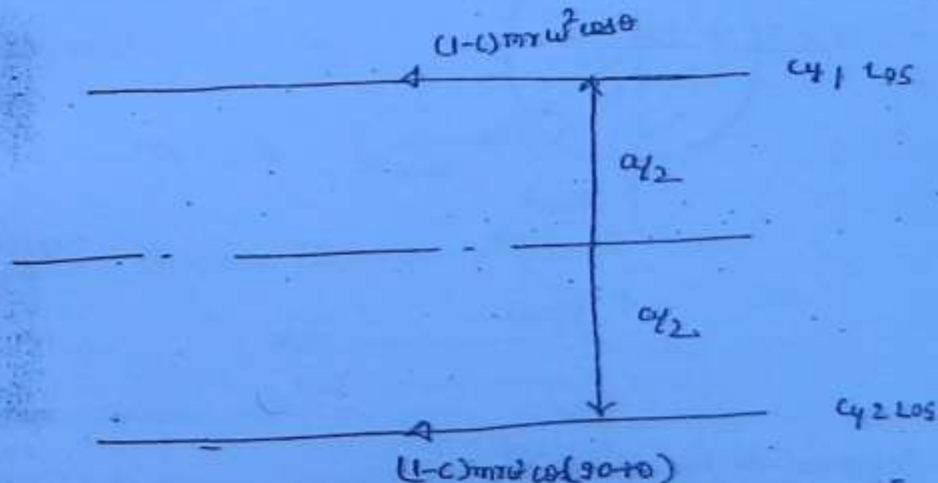
1. Variation in Tractive Force :-



$$F_{UN} = (1-c)\, m r \omega^2 \cos\theta + (1-c)\, m r \omega^2 \cos(90+\theta)$$

$$F_{UN} = (1-c)\, m r \omega^2 [\cos\theta - \sin\theta]$$

$\downarrow$ $\boxed{\text{Tractive force}}$

$\rightarrow$ max. $\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$

$\tan\theta = -1$

$F_{UN} = \pm \sqrt{2}\,(1-c)\, m r \omega^2$

max

+

variation

2 Variation in swaying couple :-

$$\text{couple} = \left\{(1-c)\, m r \omega^2 \cos\theta\right\}\frac{a}{2} - \left\{(1-c)\, m r \omega^2 \cos(90+\theta)\right\}\frac{a}{2}$$

$$= (1-c)\, m r \omega^2 \frac{a}{2}\left[\cos\theta + \sin\theta\right]$$

Swaying couple

max. $\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$

$\tan\theta = 1$

$\theta = 45^\circ, 225^\circ$

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$$\text{max} = \pm \frac{a}{\sqrt{2}}(1-c)\, m r \omega^2$$

variation

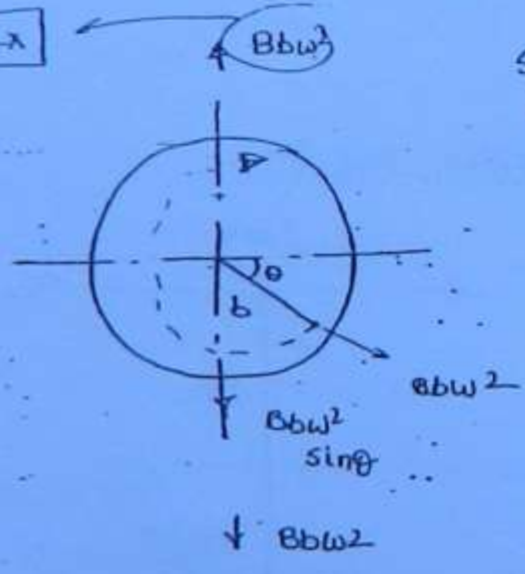
3. Hammer Blow :-

wheel → having the static load in the downward dir^{n}

$Bb \neq cmr$
mass are attached in different plane

$\boxed{\text{Max}}$ ← $Bb\omega^2$

$Bb\omega^2$ → Hammer Blow



Super Passenger Locomotive

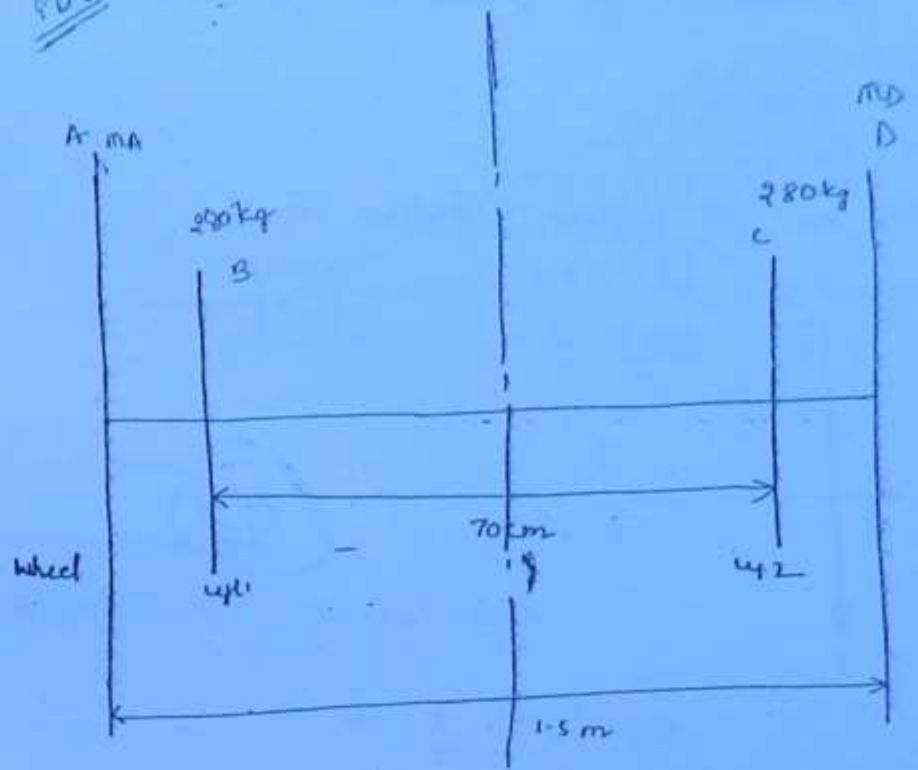
$Bb\omega^2 \leq$ per wheel static load

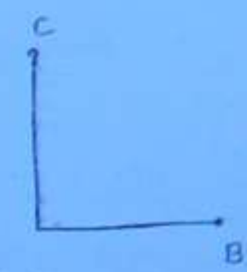
ω is restricted

Hammer Blow → $Bb\omega^2$

Balance mass required to be attached on the wheel in order to balance to balance C fraction of reciprocating mass of the cylinder

Pb. 1





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$N = 300$ r.p.m.

$\text{Rot}/\text{cy} = 160$ kg

$\text{Reci}/\text{cy} = 186$ kg

Total mass to be balance $=$ Rot $+ \frac{2}{3}$ Reciprocating

$= 160 + \frac{2}{3} \times 180 = 280$ kg.

$m_A = m_D$ [symmetric]

Hammer Blow $= B b \omega^2$

280 ———— 50

120 ———— $\frac{50}{280} \times 120$

Swaying couple $= \pm \sqrt{2}\,(1-c)\, m r \omega^2$

$\pm \frac{a}{\sqrt{2}} (1-c)\, m r$

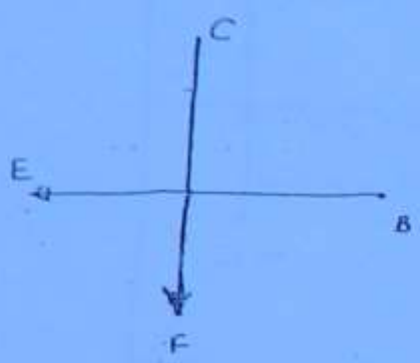
Problem No 1 continued .....

& mass of Each CR = 100 kg

Radius of rotation of C.R Pin = 0.6 m

Plane gaps for C.R.'s ⇒ 1.8 m

The ~~crank~~ coupling Rod are 180° to their Respective cranks.



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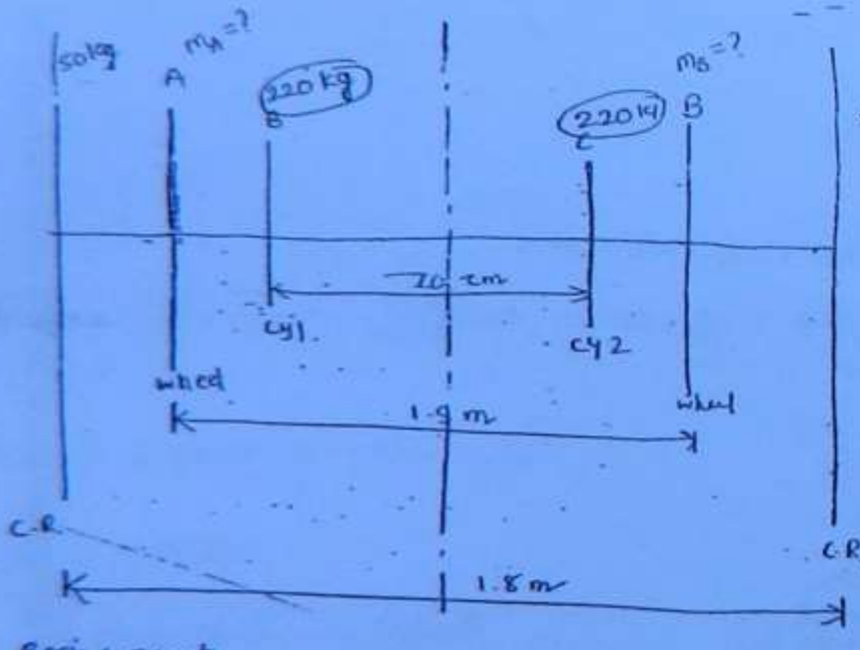


Pbr-1

50 kg Find the Balance masses on the driving wheel along with their position in order to have balancing

Trailing wheel

Hammer blow

Basic concept

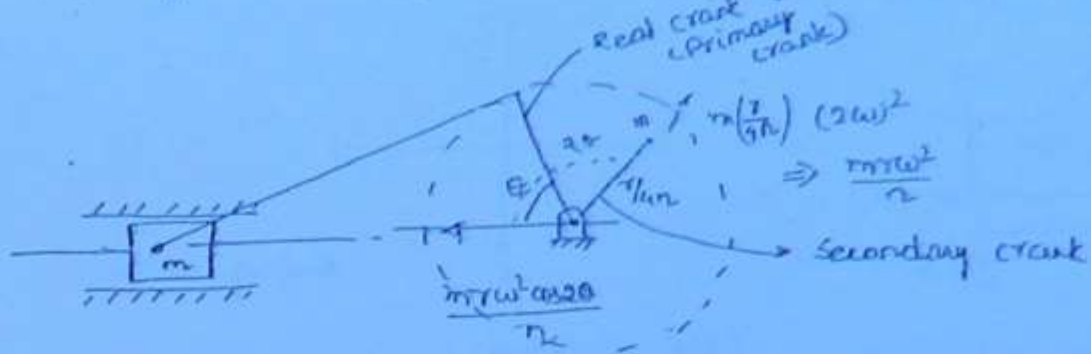
Rel → 160 kg — (D)

Rec → 180 kg

$\frac{2}{3} \times 180$ → 120 kg → 60 (D), 60 (T)

CR → 100 kg → 50 (D), 50 (T)

* Secondary Balancing of Reciprocating Masses :-

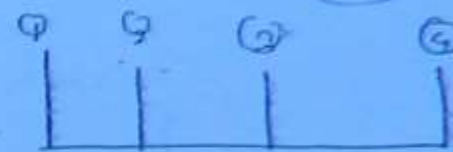


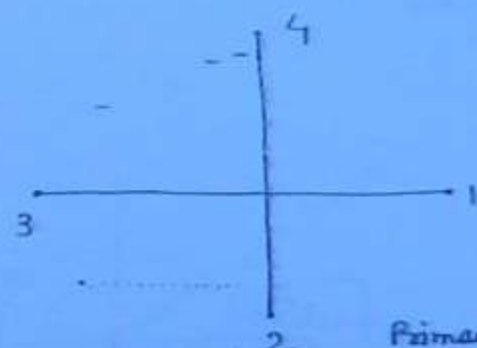
Primary

$m r \omega^2 \cos\theta$

$$m \cdot r\omega^2 \cdot \frac{\cos 2\theta}{n}$$

$$= m \cdot \frac{r}{4n} \cdot (2\omega)^2 \cos 2\theta$$

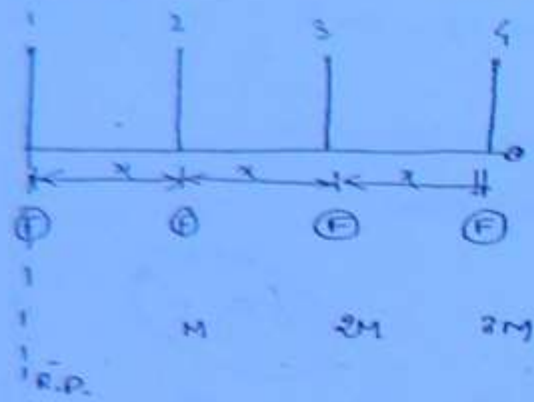




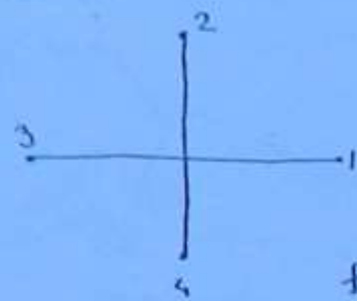
Primary configuration diagram

* Role of firing order in Balancing in Multicylinder Engines :-

- 4-cylinder inline engine :-

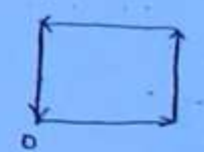


Case 1:



firing order

Force polygon:



Moment ~~order~~ polygon:

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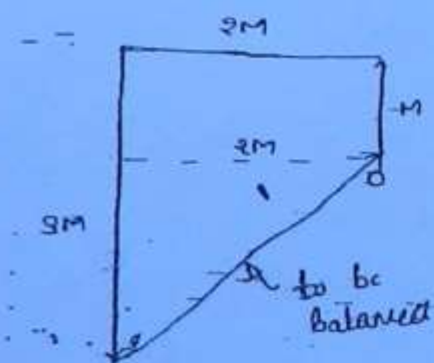
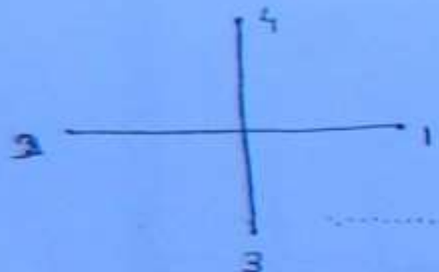


Resultant will be calculated from 2M and 2M.

* Balancing can be done By firing order.

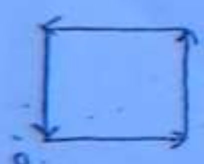
Case 2 :-

change in firing order

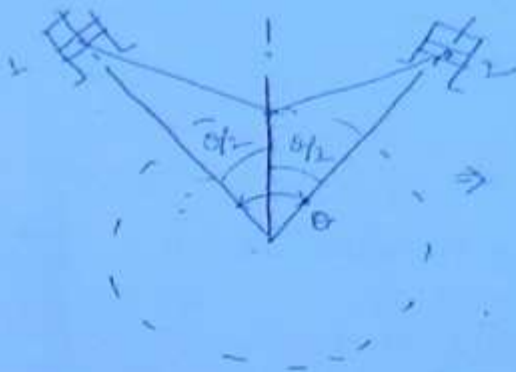


active and passive balancing

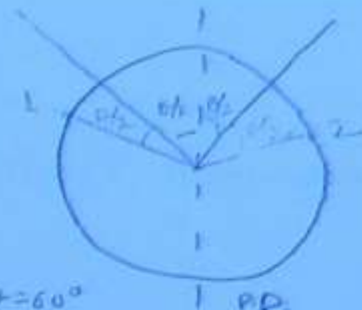
Force Polygon:



Moment Polygon

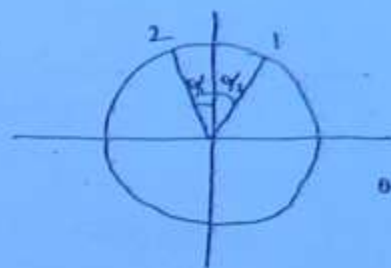


⇒ θ V- Engine



θ = 60°

60° V-Engine



θ = -60°

Pb

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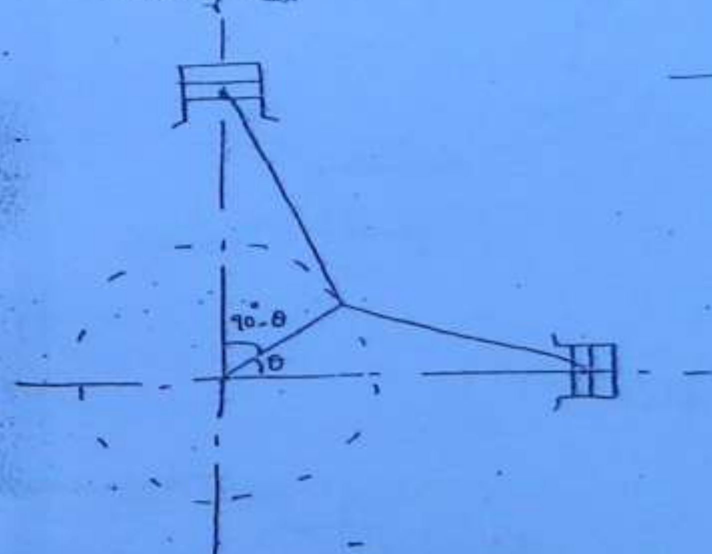
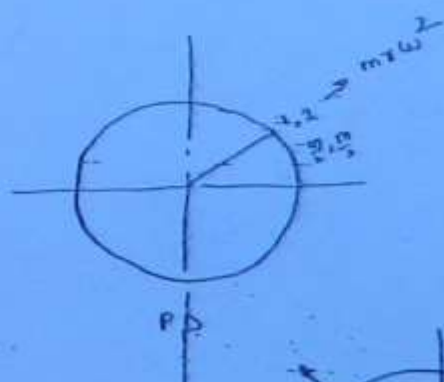
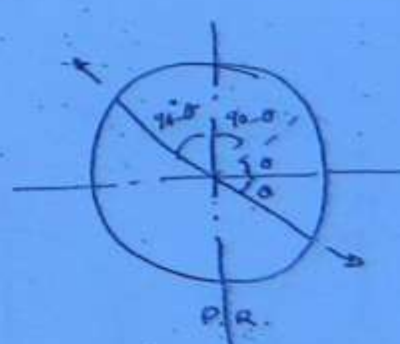
90° – V-engine

$\vec{F}$ - $primary_{max}$ = ?

$\vec{F}_{secondary\ max}$ = ?







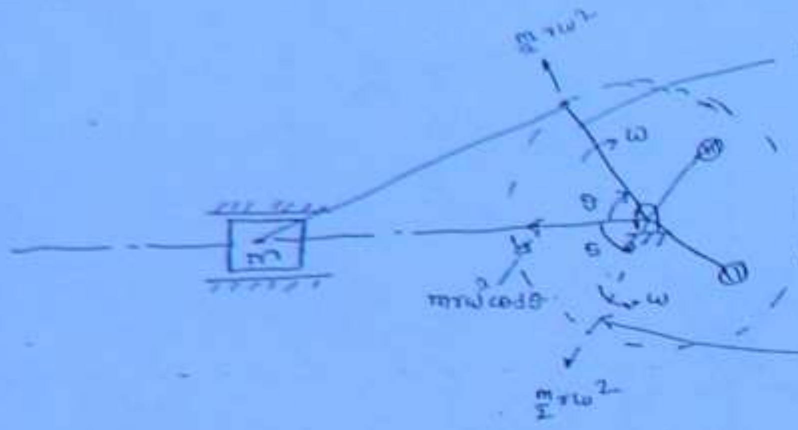
Net force zero

∴ Every moment $F_{primary}$

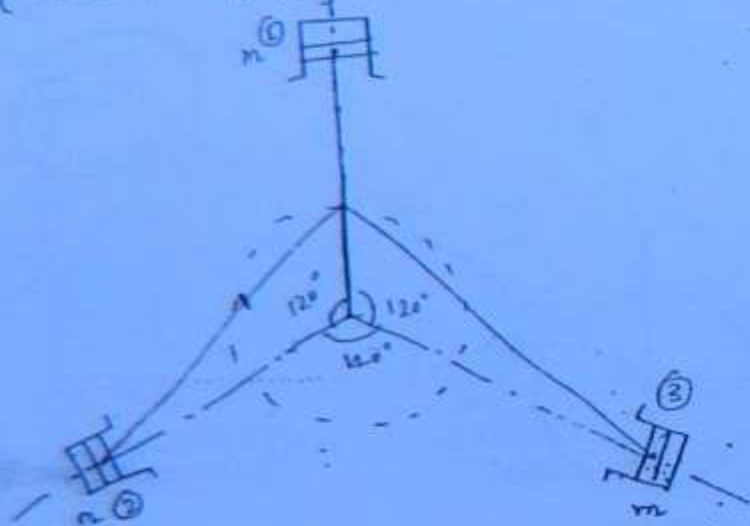
$= mr\omega^2$

∴ clearly, In a 90°-V engine $F_{primary}$ doesn't depend on crank position i.e, θ

* Direct and Reverse Crank Method :- [mostly applied in Radial Engine]



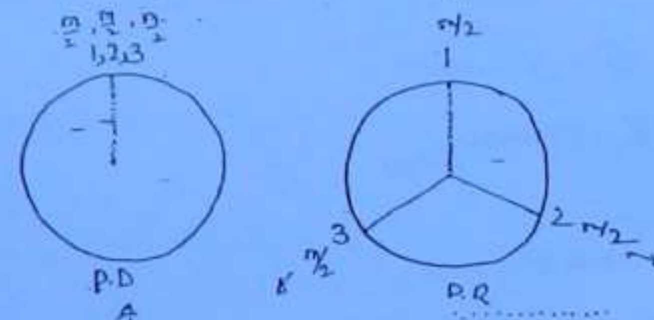
{ Radial Engine }



$3 \times \frac{m}{2} r\omega^2$

$F_{primary} = ?$

(10P)

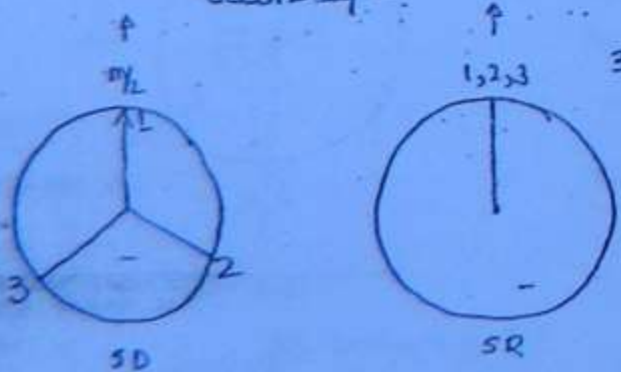


$\underbrace{\frac{3m}{2} r\omega^2 \qquad \text{Zero}}_{\text{summation}}$

$\frac{3}{2} m r\omega^2 = F_{primary}$

$F_{secondary} = ?$

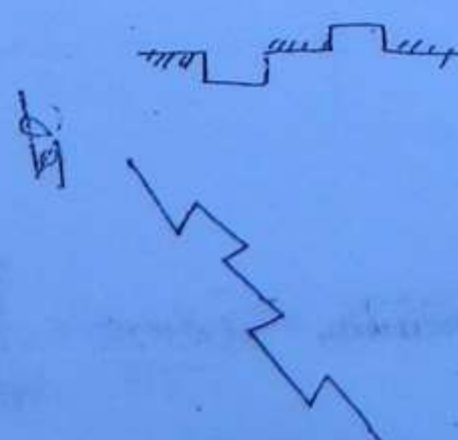
$3 \times \left[\frac{m}{2} \left(\frac{r}{4n} \right) (2\omega)^2 \right]$



$\frac{3m}{2} \frac{r\omega^2}{n}$

$\underbrace{\text{Zero} \qquad \qquad}_{\text{summation}}$

$\frac{3m}{} r\omega^2$



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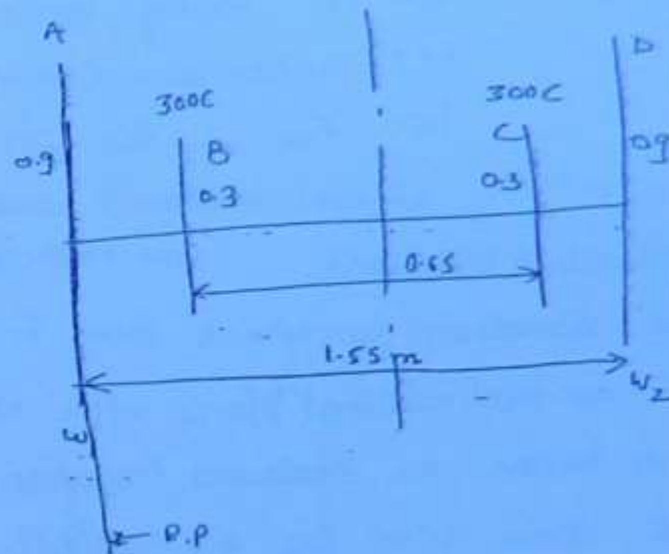
Q.4

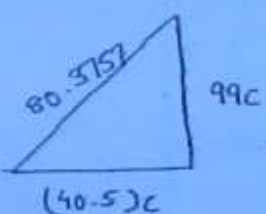
$$(46\times10^3) = 8\cdot(0.9)\left(\frac{96.5\times5/18}{0.9}\right)^2$$

$B = 57.6170$ kg.

Plane	m	r	mr	l	mrl
A	57.6170	0.9	51.8553	0	0
B	300C	0.3	90C	0.45	40.5C
C	300C	0.3	90C	1.1	99C
D	57.6170	0.9	51.8553	1.55	80.3757



Front View



$\rightarrow C = ?$

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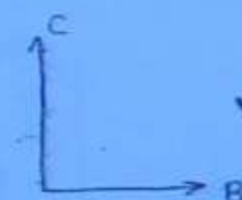

Vibrations

↓↓

Any vibrating system

⇓

i> K.E storing Device (mass)

ii> P.E. storing Device (Stiffness)

iii> Friction $\neq 0$ (Damping)

iv> F_{ext} + this will cause p vibration.

$\sin x$
$\sin^2 x$
$|\sin x|$
$\cos x$
$|\cos^2 x|$

* Natural Vibrations. (Force Method)

The vibration in which there is no friction at all as well as there is no ~~fid~~ external ~~f~~ force after the initial release of the system, are known as Natural Vibration. These vibration was seen at atomic level by sir Gallilio.

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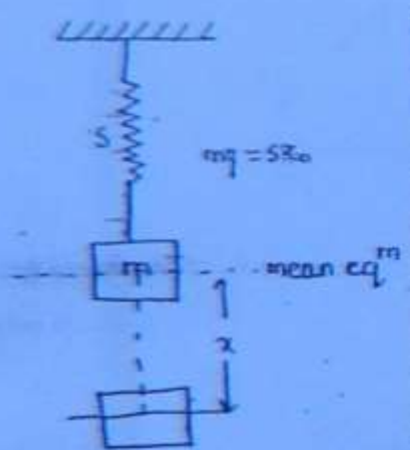
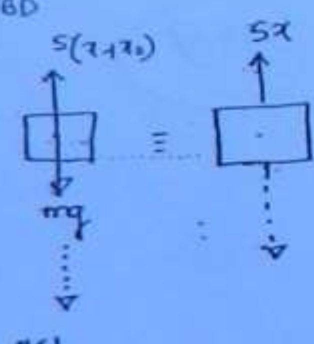


FBD



NSL

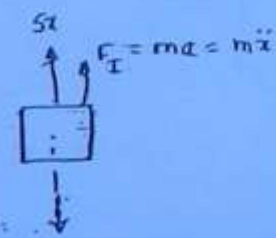
$(0 - Sx) = ma$

$ma + Sx = 0$

$\boxed{m\ddot{x} + Sx = 0}$

D'alambert

DAP



$\boxed{m\ddot{x} + Sx = 0}$

$m\ddot{x} + Sx = 0$

$\ddot{x} + \left(\frac{S}{m}\right)x = 0$

The solution of this eq^n

$$\boxed{x = R\sin\left(\sqrt{\frac{S}{m}}\,t + \phi\right)}$$

Amplitude ⇓ const.

Vibration

Angular frequency

$\boxed{\omega_n = \sqrt{\frac{S}{m}}}$

$T_n = \frac{2\pi}{\omega_n}$

$\boxed{\frac{1}{T_n} = f_n}$ ⇒ th

R, ϕ will be found by initial condition

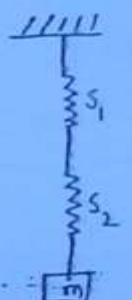
1. $t=0$ $\begin{cases} x = x_i \\ \dot{x} = 0 \end{cases}$ 2. at $t=0$ $\begin{cases} x = 0 \\ \dot{x} = v_i \end{cases}$ R, ϕ

3. $t=0$ $\begin{cases} x = x_i \\ \dot{x} = v_i \end{cases}$

∴ General Equation of natural Vibration can be written as

$$\boxed{\ddot{x} + (\omega_n^2) x = 0}$$

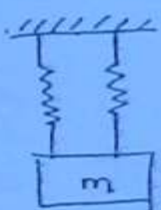
$\omega_n = \sqrt{\frac{s}{m}}$

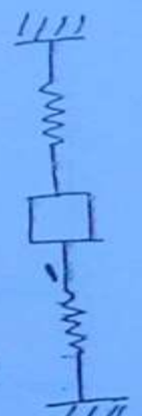


s

Series $\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$

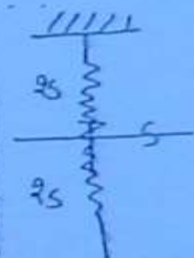
Parallel

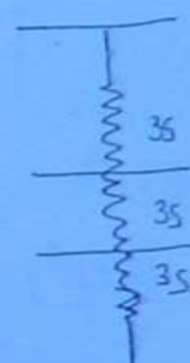




Parallel

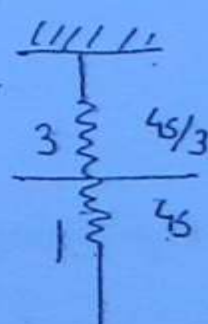
∴ $s = s_1 + s_2$

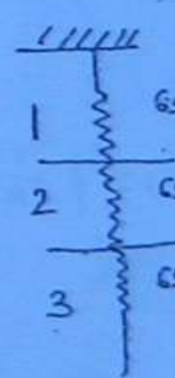




$\frac{(m+n)s}{m}$

$\frac{(m+n)s}{n}$

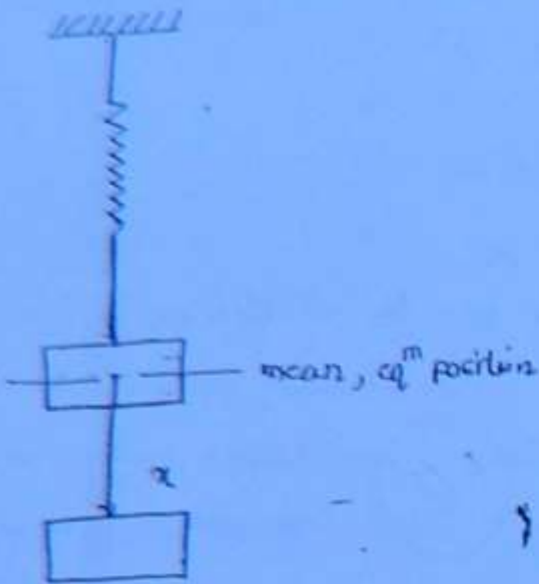




* Energy Method :-

Energy is conserved

$\therefore \frac{dE}{dt} = 0$



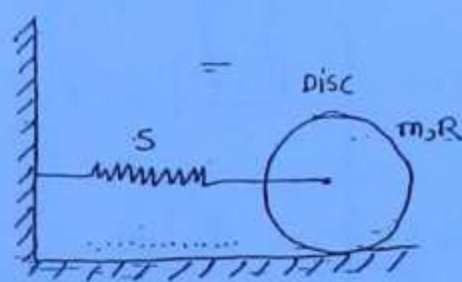
$$E = \frac{1}{2}mv^2 + \frac{1}{2}sx^2$$

$$\frac{dE}{dt} = 0$$

$$\frac{1}{2}m \cdot 2v \cdot \frac{dv}{dt} + \frac{1}{2}s \cdot 2x \cdot \frac{dx}{dt} = 0$$

$$m\ddot{x} + sx = 0$$

$$\boxed{\ddot{x} + (\omega_n^2)x = 0}$$



$$E = \frac{1}{2}sx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\bar{\omega}^2$$

$$= \frac{1}{2}sx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\cdot\left(\frac{v^2}{R^2}\right)$$

$$= \frac{1}{2}sx^2 + \frac{1}{2}\left(\frac{3m}{2}\right)\left(v^2\right)$$

$$\therefore \boxed{\omega_n = \sqrt{\frac{2s}{3m}}}$$ natural frequency

At the point of contact

$$E = \frac{1}{2}sx^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}sx^2 + \frac{1}{2}\left(\frac{mR^2}{2} + R^2\right)\cdot\frac{v^2}{R^2}$$

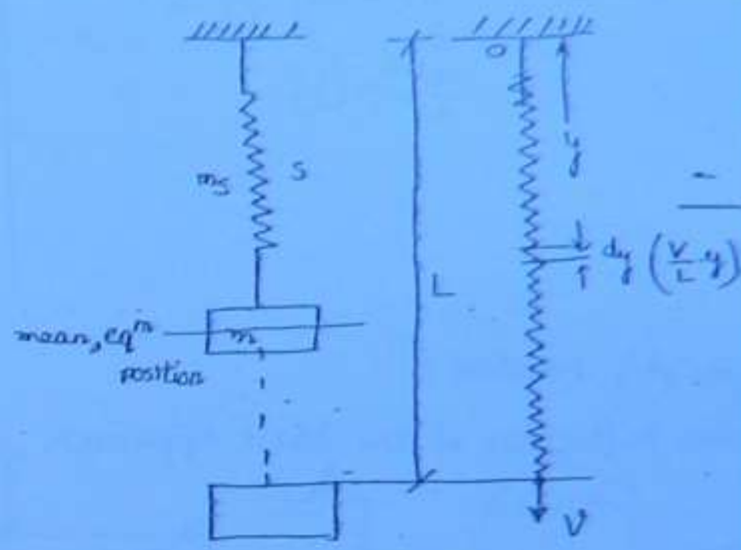
$$= \frac{1}{2}sx^2 + \frac{1}{2}\left(\frac{3m}{2}\right)\cdot v^2$$

* If spring is having mass :-

$$K.E_{spring} = \int_0^L \frac{1}{2}\left(\frac{m_s}{L}\cdot dy\right)\left(\frac{v}{L}\cdot y\right)^2$$

$$= \frac{1}{2}\,\frac{m_s}{L}\cdot\frac{v^2}{L^2}\cdot\frac{L^3}{3}$$

$$= \frac{1}{6}\,m_s v^2$$



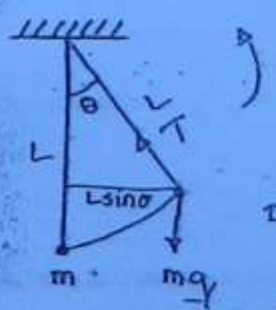
$$E = \frac{1}{2}sx^2 + \frac{1}{2}mv^2 + \frac{1}{6}m_s v^2$$

$$= \frac{1}{2}sx^2 + \frac{1}{2}\left(m + \frac{m_s}{3}\right)v^2$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m + \frac{m_s}{3}}}}$$

$R\alpha = \frac{FR^2}{I}$ $\qquad \omega = \sqrt{\frac{f}{m}}$

* Torque Method :-



$I\alpha = I\ddot{\theta}$ $\qquad mg\,L\sin\theta \approx mgL\theta$

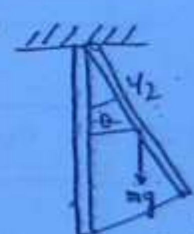
$$I\ddot{\theta} + mg\cdot L\cdot\theta = 0$$

$$(mL^2)\ddot{\theta} + mgL\theta = 0$$

$$\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0$$

$$\therefore \omega_n = \sqrt{\frac{g}{L}}$$

Pb



θ $\qquad I\ddot{\theta}$

$Mg\cdot\frac{L}{2}\theta$

$$I\ddot{\theta} + Mg\cdot\frac{L}{2}\cdot\theta = 0$$

$$\frac{mL^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$$

$$\frac{ML^2}{3}\ddot{\theta} + mg\frac{L}{2}\theta = 0$$

$$\ddot{\theta} + \frac{3g}{2L}\theta = 0$$

Pb

$I\ddot{\theta}$

$mg\frac{L}{4}\theta$

$I\ddot{\theta} + mg\cdot\frac{L}{4}\theta = 0$

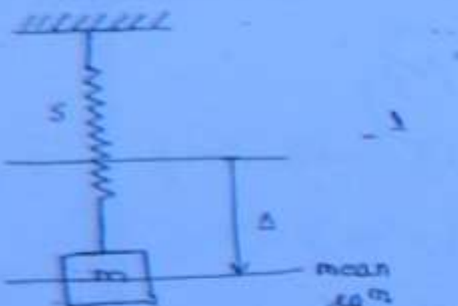
$\frac{mL^2}{12} + m\left(\frac{L}{4}\right)^2$

$\omega_n = 0$

$T_n = \infty$

* Rayleigh's Method:

or, Static Deflection of the Mass Approach

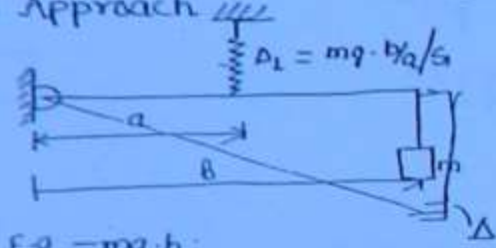


$\Delta = \frac{mg}{s}$

$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{mg/s}} = \sqrt{\frac{s}{m}} = \omega_n$

$\therefore \boxed{\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{s}{m}}}$

Pb 1

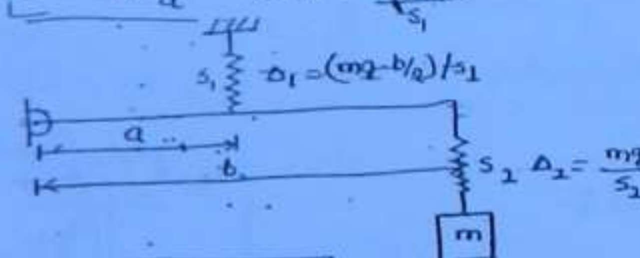


$F\cdot a = mg\cdot b$

$F = mg\cdot b/a$

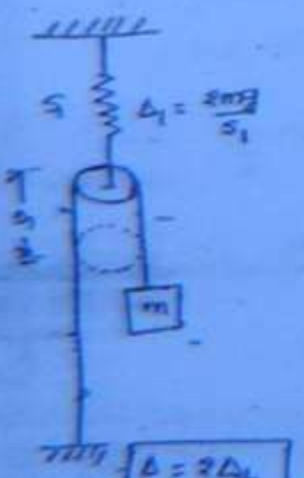
$\boxed{\Delta = \Delta_1\cdot\frac{b}{a}}$ $\left(\Delta_1 = \frac{mg\cdot b/a}{s_1}\right)$

Pb2



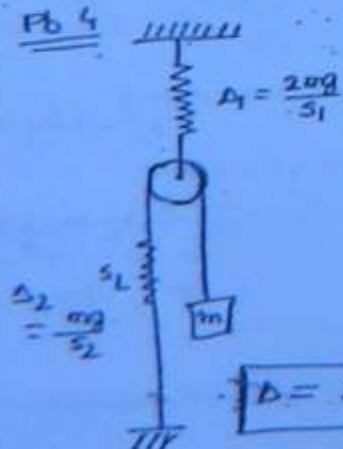
$\therefore \boxed{\Delta = \Delta_1\cdot\frac{b}{a} + \Delta_2}$

Pb3



$\boxed{\Delta = 2\Delta_1}$

Pb 4



$\boxed{\Delta = 2\Delta_1 + \Delta_2}$

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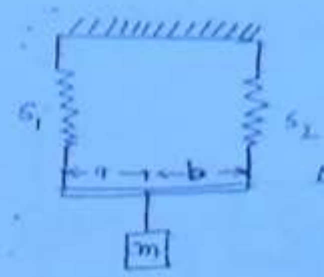
Pb. 5

$\Delta_1 = \frac{F_1}{S_1}$



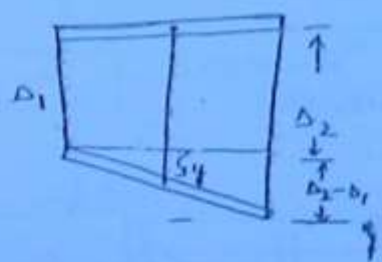
$\Delta_2 = \frac{F_2}{S_2}$

$F_1 + F_2 = mg$ — (i)

$F_1 \cdot a = F_2 \cdot b$ — (ii)

$F_1 = ?$

$F_2 = ?$



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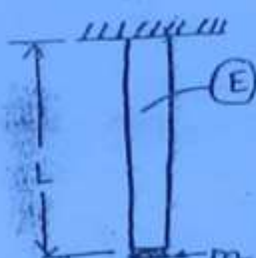
$$\frac{\Delta_2 - \Delta_1}{a+b} = \frac{y}{a}$$

$$y = (\Delta_2 - \Delta_1)\left(\frac{a}{a+b}\right)$$

* Longitudinal Vibration of Beams :-



(E) Young's modulus

$$\text{stress} = \frac{mg}{A} \quad \left[A = \frac{\pi}{4}D^2\right]$$

$$\text{strain} = \frac{\Delta L}{L}$$

$$E = \frac{(mg/A)}{(\Delta L/L)} \Rightarrow$$

$$\boxed{\Delta L = \frac{(mg)L}{AE} = \frac{FL}{AE}}$$

$$\therefore \ \omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{FL/AE}}$$

$$\text{or,} \ \omega_n = \sqrt{\frac{AE \cdot g}{mg\,L}}$$

$$\text{or,} \ \boxed{\omega_n = \sqrt{\frac{AE}{mL}}}$$

$$\Delta L = \frac{FL}{AE}$$

If $\Delta L = 1m$

$$F = \frac{AE}{L} \rightarrow S \text{ stiffness}$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{AE}{L.m}}$$

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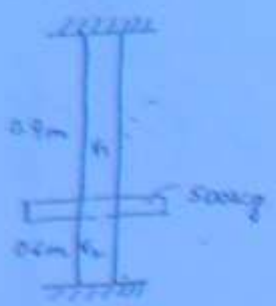
Q1



$$\left.\begin{aligned} s_1 &= \frac{AE}{L_1} \\ s_2 &= \frac{AE}{L_2} \end{aligned}\right\} s = s_1 + s_2$$

$$\omega_n = \sqrt{\frac{s}{500}}$$

$$n = \frac{\omega_n}{2\pi}$$

2nd Method :-

$$mg = F_1 + F_2$$

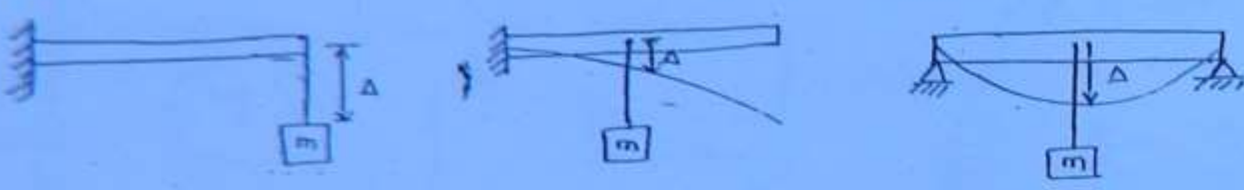
$$\Delta L = \frac{F_1 L_1}{AE} = \frac{F_2 L_2}{AE}$$

$\downarrow \Delta$

$$\Delta W = W$$

$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

* Transverse Vibrations of the Beam :- (Across the length)



$$\omega_n = \sqrt{\frac{g}{\Delta}}$$

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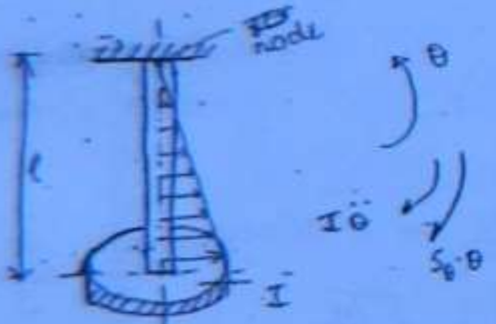
* Torsional Vibration :-



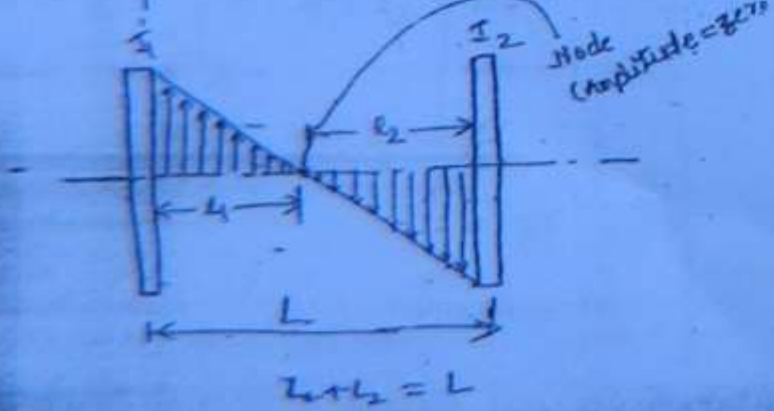
$$I\ddot{\theta} + S_\theta \cdot \theta = 0$$

$$\ddot{\theta} + \left(\frac{S_\theta}{I}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{S_\theta}{I}}$$

if shafts is having I_{shaft}

$$\omega_n = \sqrt{\frac{S_\theta}{I + \frac{I_{shaft}}{3}}}$$



$$l_1 + l_2 = L$$

Rotors : n

No. of node Points = (n−1).

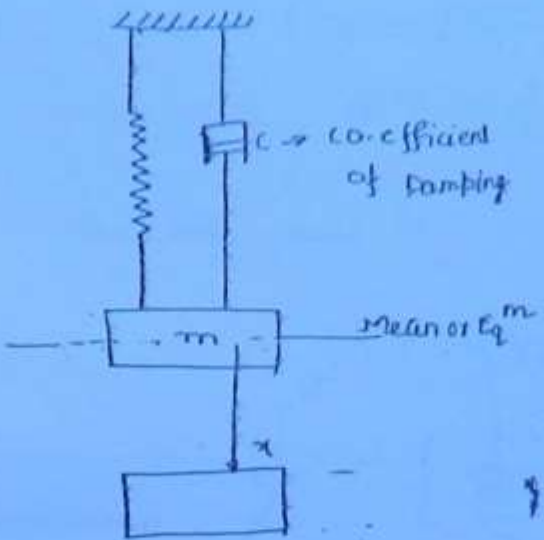
$$\sqrt{\frac{S_{\theta_1}}{I_1}} = \sqrt{\frac{S_{\theta_2}}{I_2}}$$

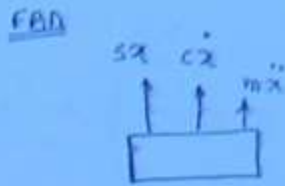
$$\frac{S_{\theta_1}}{I_1} = \frac{S_{\theta_2}}{I_2}$$

$$\frac{GJ}{l_1 I_1} = \frac{GJ}{l_2 I_2}$$

$$\boxed{\frac{l_1}{l_2} = \frac{I_2}{I_1}} \quad \text{— (2)}$$

* Damped Vibration : (Friction $\neq 0$)





DAP :-

$$m\ddot{x} + c\dot{x} + sx = 0$$

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + (\omega_n^2)x = 0 \qquad \text{— (1)}$$

The solution of this eq^n :-

$$\boxed{x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}} \quad (\alpha_1 \neq \alpha_2)$$

$$\boxed{x = (A+Bt)e^{\alpha t}} \quad (\alpha_1 = \alpha_2 = \alpha)$$

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$\therefore$ Eq^n ① can be written as

$$\boxed{\alpha^2 + \frac{c}{m}\alpha + \omega_n^2 = 0} \rightarrow \text{Auxiliary Eq}^n$$

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}}{2}$$

$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left[\frac{c}{2m}\right]^2 - \omega_n^2}$$

$\frac{c}{2m} \cdot \frac{1}{2} \cdot$ ... 2ξ

$$\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2} \Rightarrow \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2}} = \text{Damping factor or Damping Ratio } (\xi)$$

$$\xi = \sqrt{\frac{\frac{c^2}{4m^2}}{s/m}} = \frac{c}{2\sqrt{sm}}$$

$$2\xi\omega_n = 2 \times \frac{c}{2\sqrt{sm}} \times \sqrt{\frac{s}{m}} = \frac{c}{m}$$

$$\therefore \boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + \omega_n^2 x = 0} \qquad \therefore \alpha_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$$

Malky Kaski

Exponential function = Non - harmonic function

If $\xi > 1 \Rightarrow$ over damped system No Vib.

If $\xi = 1 \Rightarrow$ critically damped system No Vib.

If $\xi < 1 \Rightarrow$ Under damped system Vib

1. Over damped system : ($\xi > 1$)

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

No Vib. →

$$\boxed{x = Ae^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + Be^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t}}$$

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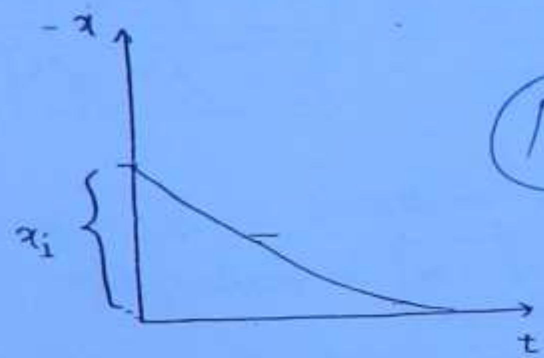
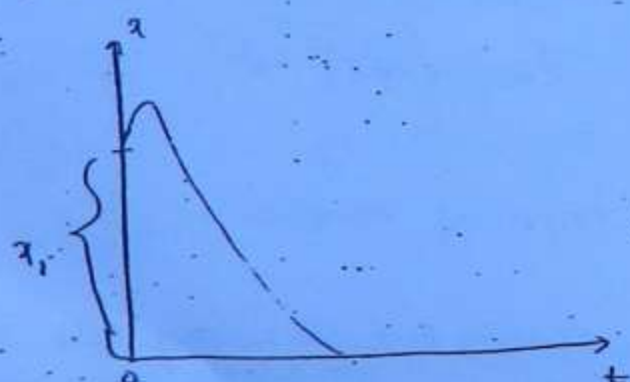


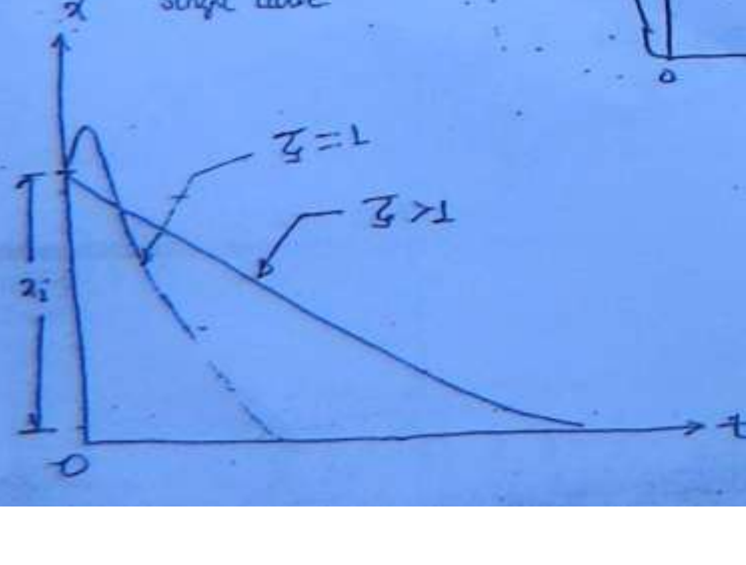
2. Critically damped System : ($\xi = 1$)

$$\alpha_1 = \alpha_2 = \alpha = -\omega_n$$

$$x = (A + Bt)e^{-\omega t}$$



Both are compare in single curve



3. Under - Damping $(\xi < 1)$

$$\alpha_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1})\,\omega_n$$

$$= -\xi\omega_n \pm i\,\boxed{\sqrt{1-\xi^2}\cdot\omega_n} \qquad \omega_d = \text{const} < \omega_n$$

The sol^n will be :-

$$x = A e^{(-\xi\omega_n + i\omega_d)t} + B e^{(-\xi\omega_n - i\omega_d)t}$$

$$= e^{-\xi\omega_n t}\left[\underbrace{(A+B)\cos\omega_d t}_{X\sin\phi} + \underbrace{i(A-B)}_{X\cos\phi}\sin\omega_d t\right]$$

$$\boxed{x = X e^{-\xi\omega_n t}\,\sin(\omega_d t + \phi)}$$

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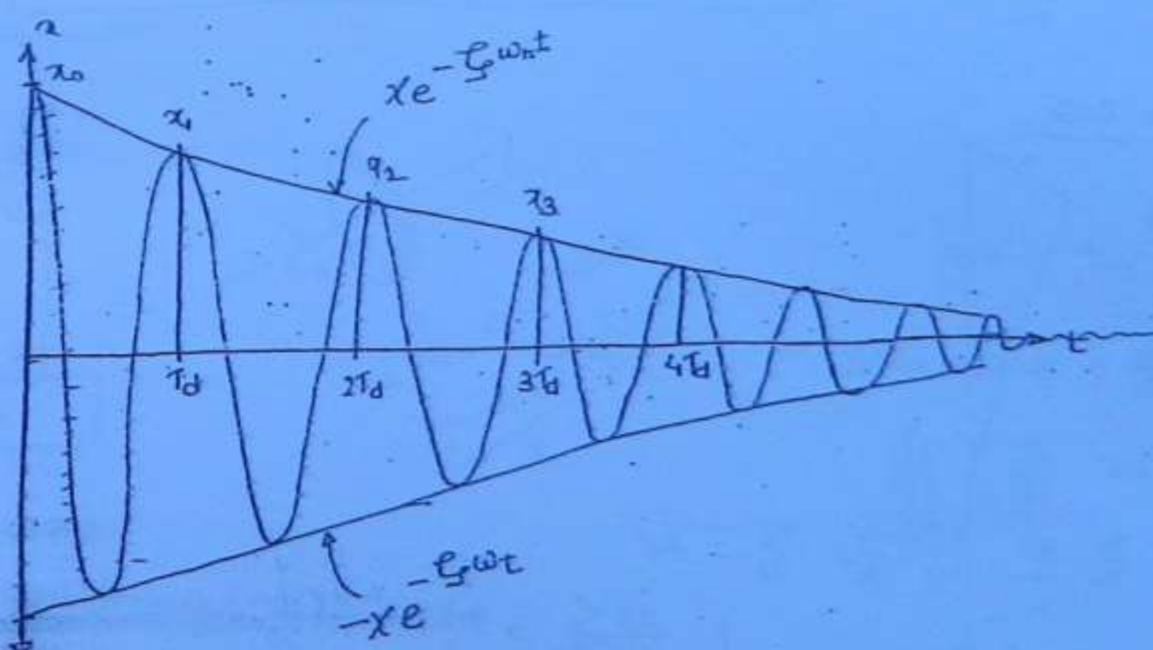
Amplitude fn. of time

VIBRATION with Freq. $\omega_d = (\sqrt{1-\xi^2})\,\omega_n$

$\therefore T_d = \dfrac{2\pi}{\omega_d} = \text{const.}$



Clearly :

at $t = 0$: $\quad x_0 = X\sin\phi$

at $t = T_d$ $\quad x_1 = X e^{-\xi\omega_n T_d}\sin\phi$

at $t = 2T_d$ $\quad x_2 = X e^{-\xi\omega_n (2T_d)}\sin\phi$

$$\boxed{\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \cdots\cdots = e^{\zeta\omega_n T_d} = \text{const}}$$

Decrement Ratio

$\therefore e^{\frac{\delta}{\omega}}$

Logarithmic Decrement (δ):

$$\delta = \ln e^{\zeta\omega_n T_d}$$

$$= \zeta\omega_n \cdot \frac{2\pi}{\omega_d} = \frac{\zeta \cdot \omega_n \cdot 2\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$

$$\boxed{\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$$

from 1st cycle to 3rd cycle

$\frac{x_0}{x_3}$

from 2nd cycle to 4th cycle

$\frac{x_1}{x_4}$

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Critical damping Co-efficient :-

$$2\zeta\omega_n = \frac{c}{m}$$

$$2\times 1\,\omega_n = \frac{c_c}{m}$$

$$\boxed{\zeta = \frac{c}{c_c} = \frac{\text{actual Damping Co-efficient}}{\text{Critical Damping Co-efficient}}}$$

Page No. 70 Work Book.

Q.5. $m = 7.5$ kg

$T_d = \frac{35}{60}$

$\omega_d = \frac{2\pi}{T_d} = 10.7711$ rad/s

$\frac{x_0}{x_7} = 2.5$

$\frac{x_0}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} \cdot \frac{x_3}{x_4} \cdot \frac{x_4}{x_5} \cdot \frac{x_5}{x_6} \cdot \frac{x_6}{x_7} = 2.5$

$\Rightarrow (e^{\delta})^7 = 2.5$

$\Rightarrow e^{7\delta} = 2.5$

$7\delta = \ln 2.5$

$\delta = \frac{\ln 2.5}{7}$

$$\frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{\ln 2.7}{7}$$

$$\boxed{\zeta = 0.0208280}$$

$\omega_d = 0.020828$

$\omega_d = \sqrt{1-\zeta^2} \cdot \omega_n$

$\omega_n = 10.7734 \text{ rad/s}$

i> $\omega_n = \sqrt{\frac{s}{m}}$

$s = 670.507 \text{ N/m}$

ii> $2\zeta\omega_n = \frac{c}{m}$

$c = 3.3658 \text{ N/(m/s)}$

iii> $\zeta = \frac{c}{c_c}$

$c_c = 161.661 \text{ N/(m/s)}$

$$x = X e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$$

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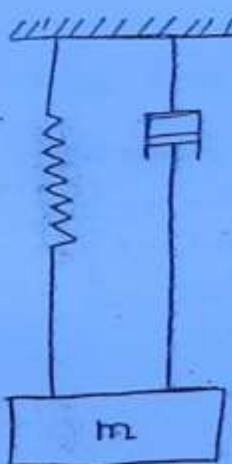
* Forced ~~Damed~~ Damped Oscillations / Vibration :-

(Perfect Rebility)

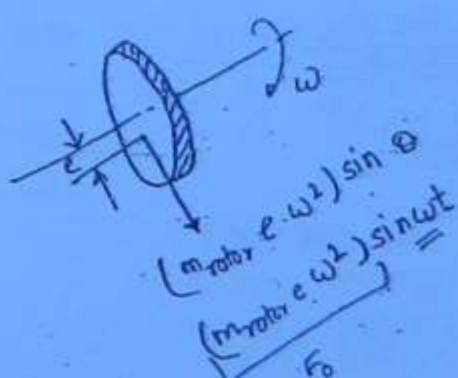


$F_0 \sin\omega t$

$\omega \rightarrow$ force frequency

↓

unbalanced force



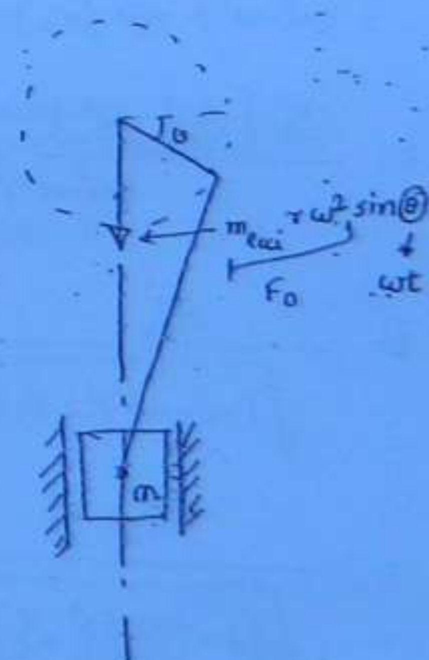
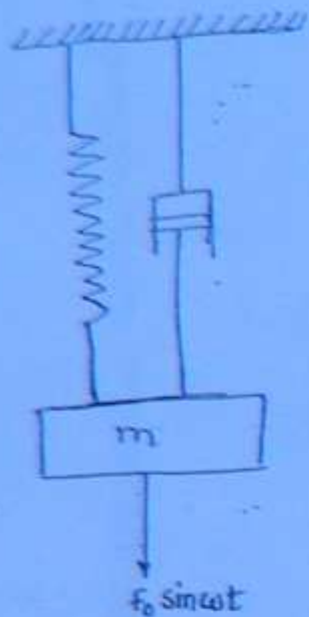
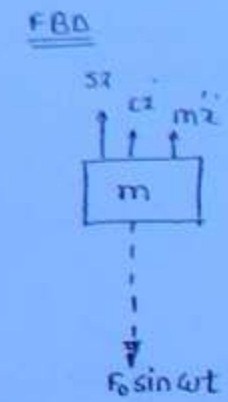


fig. Vertical engine

Forced Damped Vibration (Perfect Reality)





$$m\ddot{x} + c\dot{x} + sx - F_0\sin\omega t = 0$$

$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n)^2 x = \frac{F_0}{m}\sin\omega t$$

The final sol^n is $x = CF + P.I.$

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$$CF \left\langle \begin{matrix} \zeta > 1 \\ \zeta = 1 \\ \zeta < 1 \end{matrix} \right] \text{ after sometimes} \rightarrow \underline{\underline{0}}$$

$$\left[\begin{matrix} D \rightarrow \text{Differential operator} \\ D^2 = -\omega^2 \end{matrix} \right]$$

$$P.I = \frac{(F_0/m)\sin\omega t}{D^2 + (2\zeta\omega_n)D + \omega_n^2}$$

$$= \frac{F_0/m \sin\omega t}{(\omega_n^2 - \omega^2) + (2\zeta\omega_n)D} \times \frac{(\omega_n^2 - \omega^2) - (2\zeta\omega_n)D}{(\omega_n^2 - \omega^2) - (2\zeta\omega_n)D}$$

$$= \frac{\frac{F_0}{m}\left[\underbrace{(\omega_n^2 - \omega^2)}_{R\cos\phi}\sin\omega t - \underbrace{(2\zeta\omega\cdot\omega_n)}_{R\sin\phi}\cos\omega t\right]}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\cdot\omega_n)^2}$$

$$= \frac{\frac{F_0}{m}}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega\omega_n)^2}} \sin(\omega t - \phi)$$

$$= \frac{F_0/s}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}} \sin(\omega t - \phi)$$

∴ Final sol^n :-

$$x = CF + PI$$

$$= \underbrace{CF}_{\text{Damped free Response}} + \underbrace{\underbrace{\frac{F_0/S}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}}_{\text{Amplitude = const.}} \underbrace{\sin}_{\text{vibration}}(\omega t - \phi)}_{\text{steady state response}}$$

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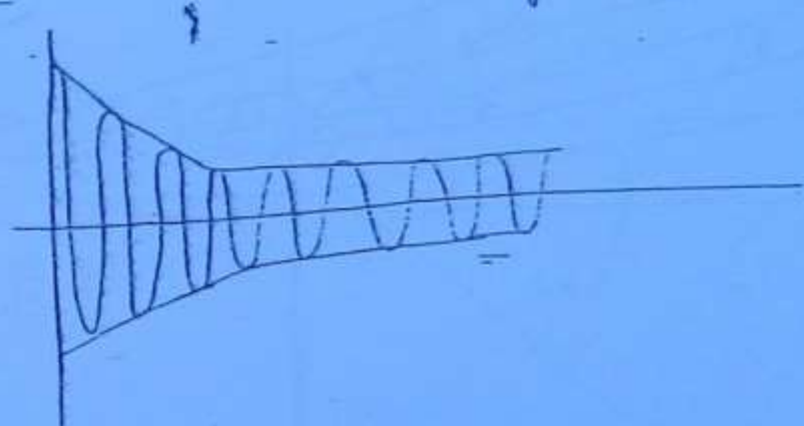


After some time when CF becomes zero

the final sol^n will be

$$\boxed{x = A\sin(\omega t - \phi)}$$

where, A → Amplitude of steady-state Vib.

Amplitude of forced Vib.

$$A = \frac{F_0/S}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}} \Rightarrow \text{const}$$

∵ the amplitude is const.

∴ System should have RUNNING LIFE

$$MF = \frac{A}{F_0/S} = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

depends upon

i) ω/ω_n

ii) ξ

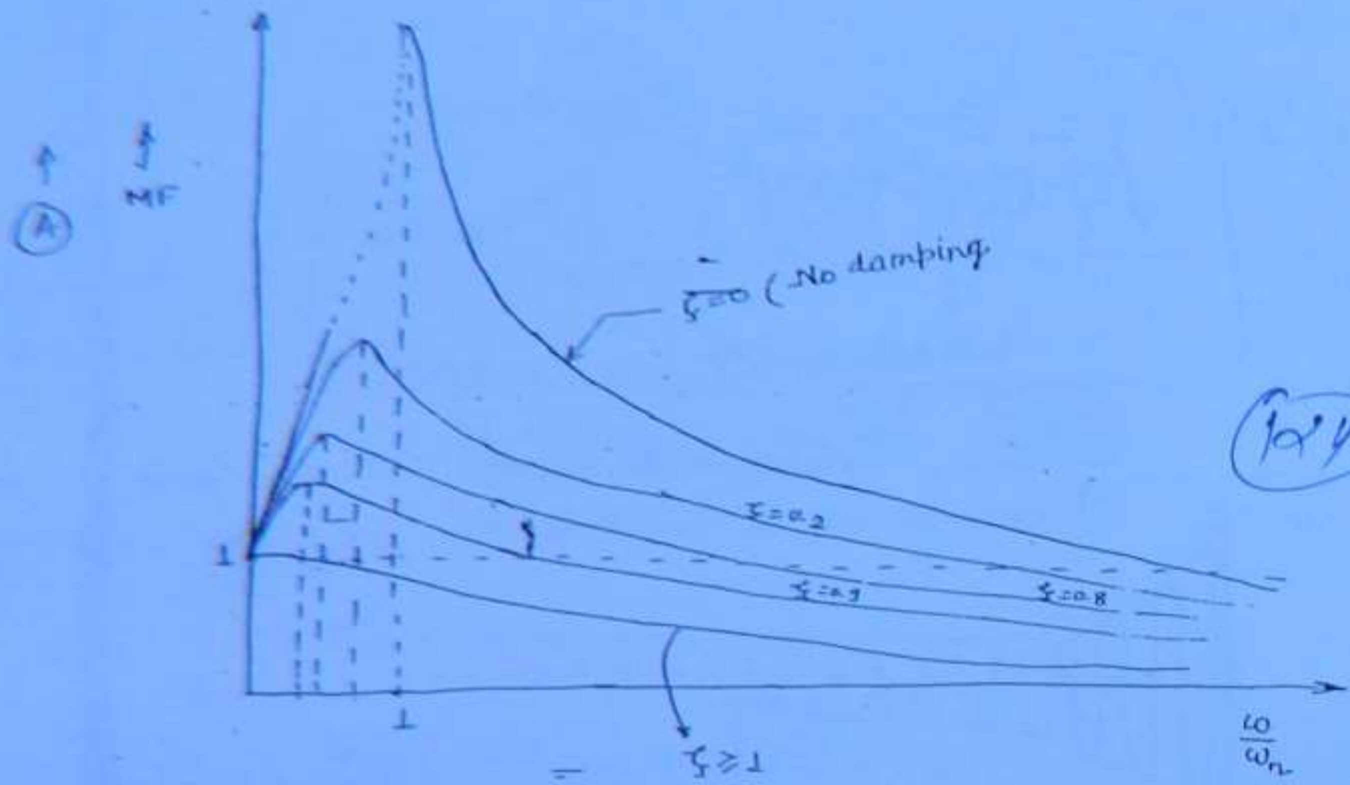
MF ↓ Magnification Factor

⇓

strength of A

Maximum value of Force = F_0

↑ damping ⇒ ξ ↑ ; ↑ Underdamping



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A_{max} at

i) $\omega = \omega_n$

ii) $\omega < \omega_n$

iii) $\omega > \omega_n$

iv) N.O.T

$\frac{\omega}{\omega_n} < 1$ — Is resonance is dangerous in under-damping case

$\Rightarrow \omega < \omega_n$ ✓

→ No ✓

* Vibration Isolation :-

$$x = A\sin(\omega t - \phi)$$

$$\dot{x} = A\omega\cos(\omega t - \phi) = A\omega\sin\left[\frac{\pi}{2} + (\omega t - \phi)\right]$$

$$\ddot{x} = -A\omega^2\sin(\omega t - \phi)$$

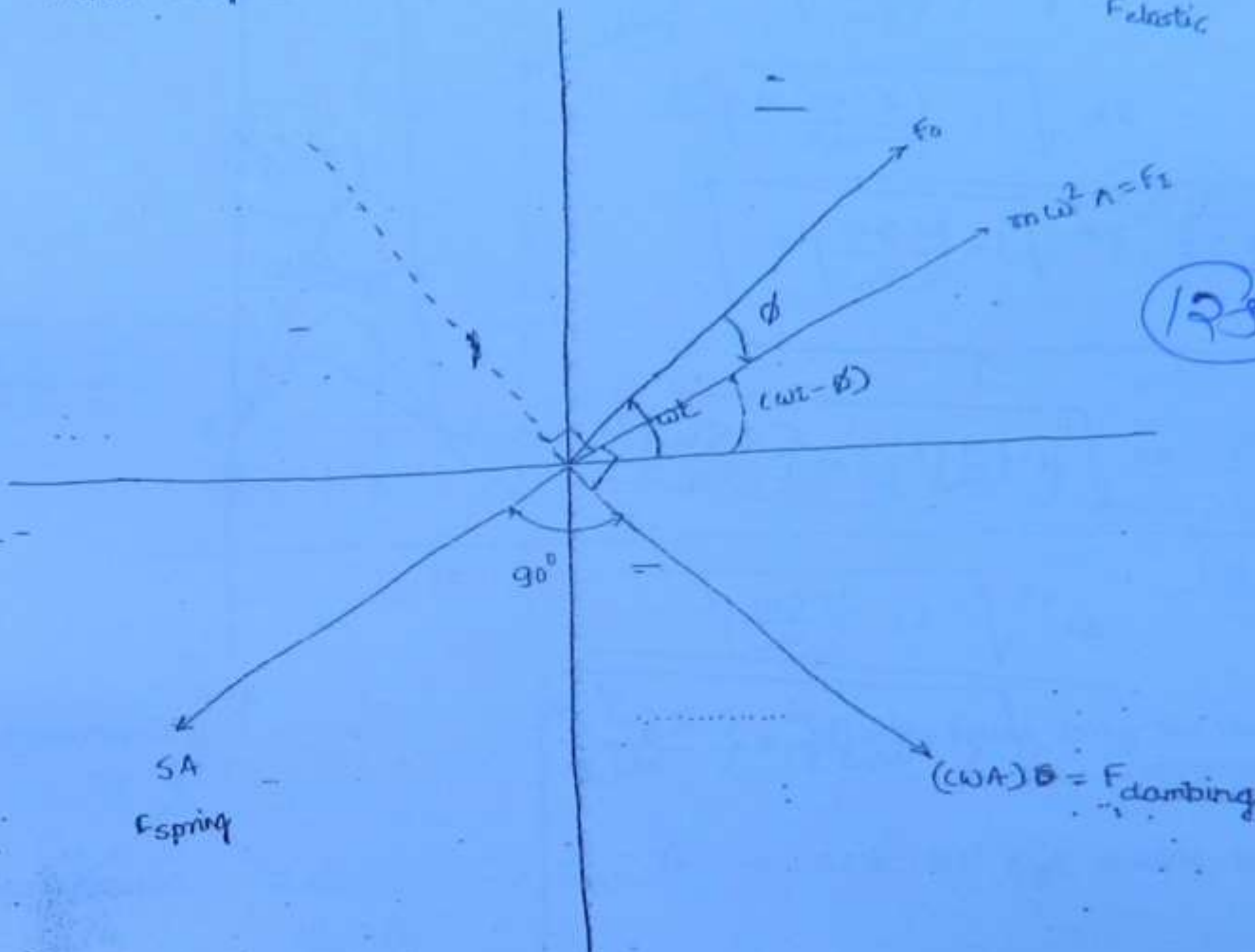
$$m\ddot{x} + c\dot{x} + s\dot{x} = F_0\sin\omega t$$

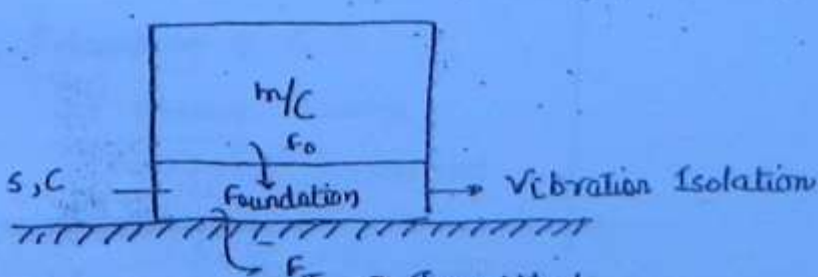
$$\text{or, } F_0\sin\omega t - m\ddot{x} - c\dot{x} - s\dot{x} = 0$$

or, $\underbrace{F_0 \sin\omega t}_{} + \underbrace{m\omega^2 A}_{F_I} \sin(\omega t - \phi) - \underbrace{C\omega A}_{F_{damping}} \sin\left[\frac{\pi}{2} + (\omega t - \phi)\right] - \underbrace{SA \sin(\omega t - \phi)}_{F_{spring}} = 0$

$F_{elastic}$

Phasor diagram





$F_T <<<<<<<<<<<<< F_0$

$\epsilon = \frac{F_T}{F_0}$

→ Transmissibility

$\epsilon \to 0$

$$\therefore F_T = \sqrt{(SA)^2 + (c\omega A)^2}$$

$$= SA\sqrt{1 + \left(\frac{c\omega A}{SA}\right)^2}$$

$$= SA\sqrt{1 + \left(\frac{c/m\,\omega}{S/m}\right)^2}$$

$$= SA\sqrt{1 + \left(\frac{2\zeta\omega_n\cdot\omega}{\omega_n^2}\right)^2}$$

$$\boxed{F_T = SA\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

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$$\cancel{A = \frac{F_0/S}{}}$$

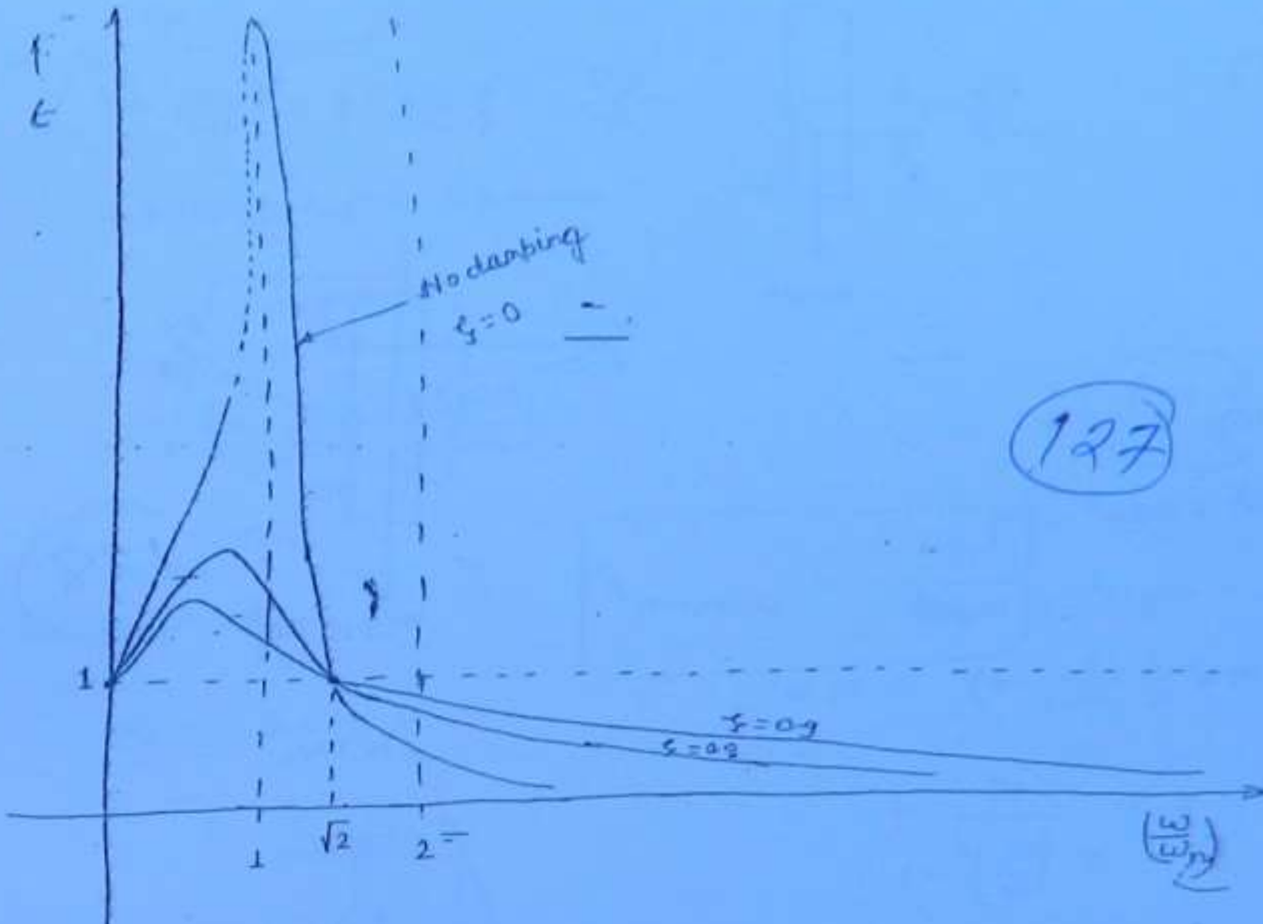
$$F_0 = SA\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}$$

$$\therefore \epsilon = \frac{\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}$$

depends on

i) $\frac{\omega}{\omega_n}$

ii) ζ



underdamping ↑

$\Rightarrow \xi \downarrow$

$\xi > 1$ — $\frac{\omega}{\omega_n} < \sqrt{2}$

$\xi < 1$ — $\frac{\omega}{\omega_n} > \sqrt{2}$

↑ damping $\Rightarrow \xi \uparrow$

Damping becomes $\boxed{\text{harmful}}$ → detrimental

If $\frac{\omega}{\omega_n} > \sqrt{2}$

If

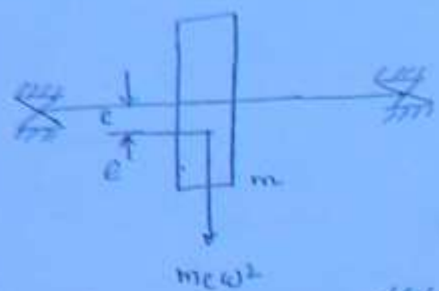
$\frac{\omega}{\omega_n} > \sqrt{2}$ → spring → Favour (very less damping is required)

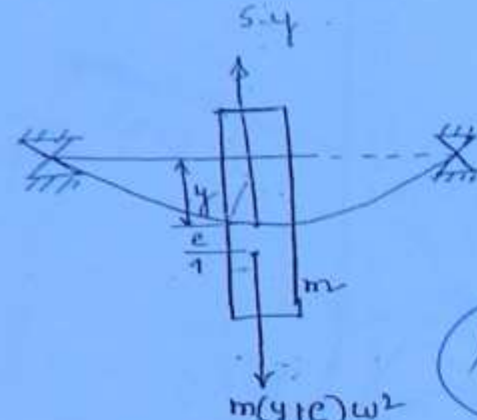
$\frac{\omega}{\omega_n} = \sqrt{2}$ → little bit high damping is req.

$\frac{\omega}{\omega_n} < \sqrt{2}$ → (very - very high damping is required).

* Whirling of shafts :
Death
↓
Critical
whipping

Sychronous Motor is used to start and stop the turbine.





(128)

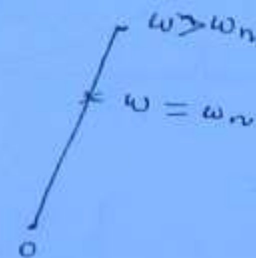
$$m(y+e)\omega^2 = Sy$$

$$my\omega^2 + me\omega^2 = Sy$$

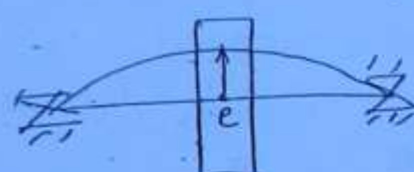
$$Sy - my\omega^2 = me\omega^2$$

$$y\, m\omega^2\left(\frac{S}{m\omega^2} - 1\right) = me\,\omega^2$$

$$y = \frac{e}{=\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$



critical speed when $\omega_n = \omega$



Work Book. 72

Q-10

$m = 17\,kg$

$S = 1{,}000\ N/m$

$\omega = 52.3595$ rad/s

$F_0 = 205.6161\ N$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{1000}{17}}$$

$$= 7.6696 \text{ rad/s}$$

$$\frac{\omega}{\omega_n} = 6.8269$$

$\boxed{\zeta = 0.20}$ — given

$m_{piston} = 2\,kg$

$$r = \frac{75}{2000}\ m$$

$$\omega = \frac{2\pi \times 500}{60} \text{ rad/s}$$

$\omega = 52.3598$ rad/s

$$F_0 = (m_{piston} \cdot r \cdot \omega^2) = 2 \times \frac{75}{2000} \times (52.3598)^2$$

$$= 205.6161\ N$$

$$A = \frac{F_0/s}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}$$

$$= 4.5223 \times 10^{-3}\,m = 4.5223\,mm$$

ii>
$$\epsilon = \frac{\sqrt{1+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

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$$\epsilon = 0.0636$$

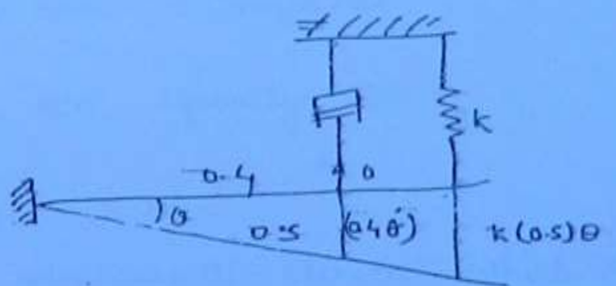
$$\boxed{A_{resonance} = \frac{F_0/s}{2\zeta}}$$

$$\epsilon = \frac{F_T}{F_0}$$

$$F_T = 13.0772\,N$$

Dry

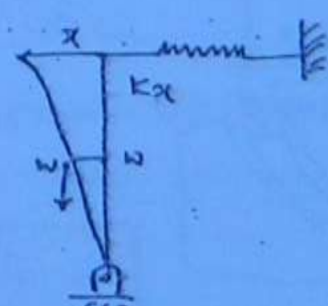
18/19



$$I\ddot{\theta} + c(0.4)^2\dot{\theta} + \{k(0.5)^2 + k_0)\}\theta = 0$$

$$I = \frac{ml^2}{3}$$

21

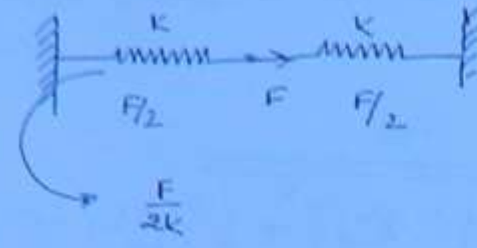


$$\frac{kx \times 300}{1000} = wx\,\frac{1}{2}$$

$$= \frac{k \times 300}{1000} = \frac{300}{2}$$

$$k = 500\,N/m$$

$E_1 = \frac{1}{2}k\left(\frac{F}{2k}\right)^2 = \frac{F^2}{8k}$



①

41 = $v = n\lambda$

$72 \times \frac{5}{18} = n \times 5$

$\frac{20}{5} = n \Rightarrow 4\,Hz$

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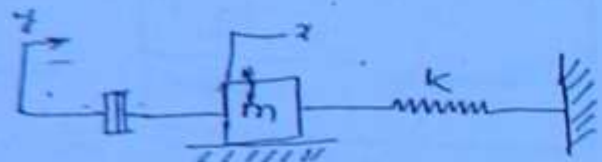
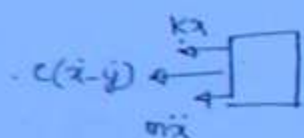
24.



Aperiodic

$\zeta \geq 1$



case $\Rightarrow \theta = 0$

29.

$k\left(\frac{1}{2}\right)\theta = 1500$

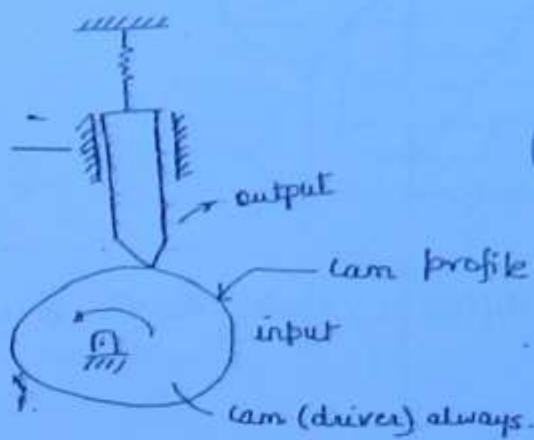
$1 \times 1^2\ddot{\theta} + 1500\theta \times \frac{1}{2}$

$\ddot{\theta} + 7000\theta = 0$

CAMS & FOLLOWERS

Higher Pair Mechanism

Biggest disadvantage
- if profile will wear



→ dwell period is that period of cam in which cam is rotating but follower will not ~~m~~ move.

Use :- Power Press, valve operating system mechanism

→ These mechanism are cheap
→ less space is required

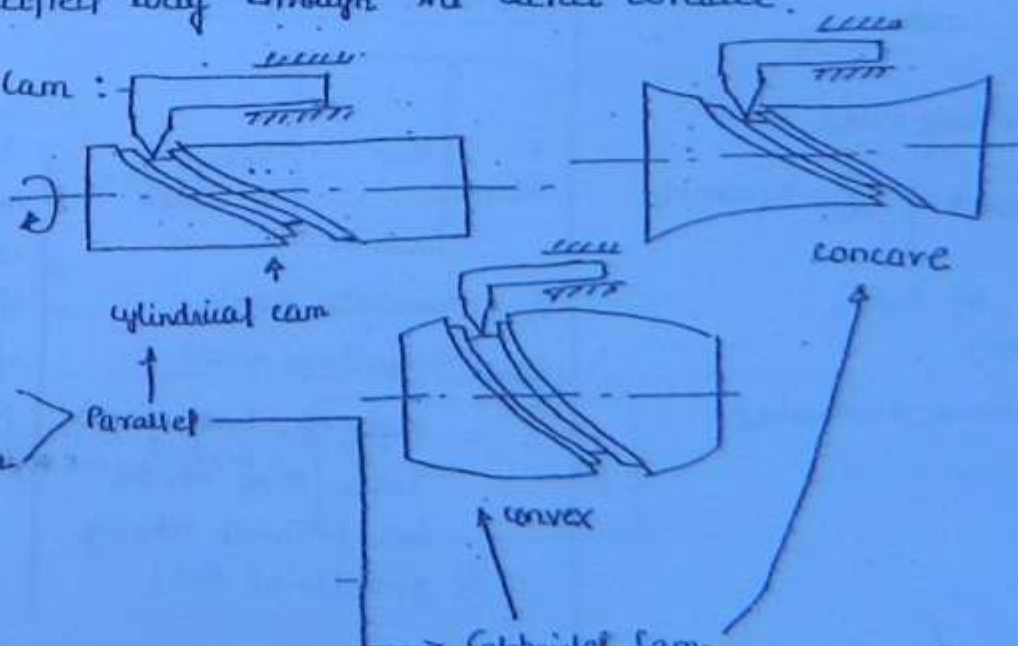
* Both are equally good.

* CAM :-
Its a mechanical element which drives another element known as follower in a specified way through the direct contact.

- Classification of Cam :-

Radial / Plate / Disc — CAM

when
{ Axis of rotation of cam
{ Line of motion of follower
→ Parallel
→ Both are ⊥ar



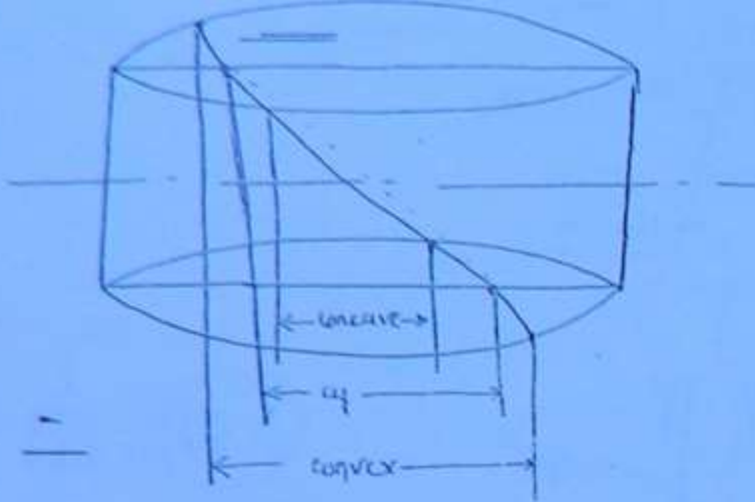
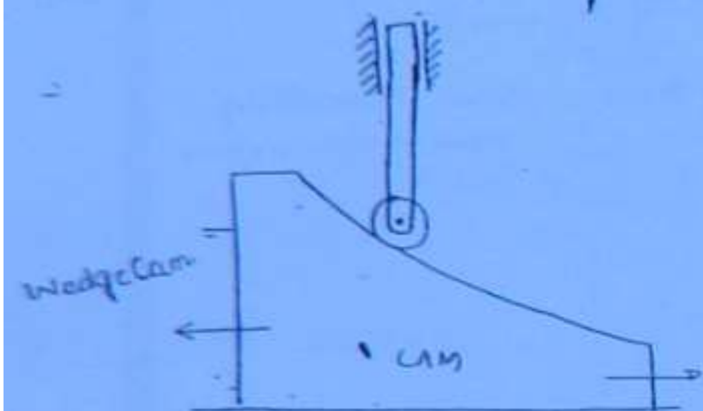


Fig. representing different stroke length in same spiral angle.



1. Cam may be
   - Rot.
   - Reci
2. Follower may be
   - Reci.
   - oscillatory.

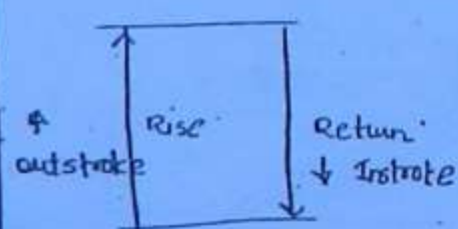
CAM ← motor
↓
Angular velocity = const = ω

If cam Rot θ | $\frac{d\theta}{dt} = \omega = \text{const}$.

One rot of cam: $(0-2\pi)$

$x \rightarrow$ Follower Displacement

In one rot of cam $(0-2\pi)$:-



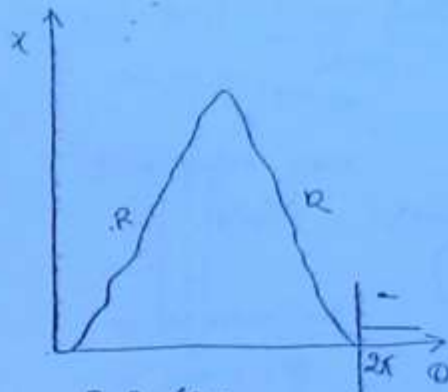
θ_0 = outstroke angle

θ_R = Returnstroke angle

δ = Angle of Dwell in which cam rotates. But Follower velocity is zero (is at Rest.)

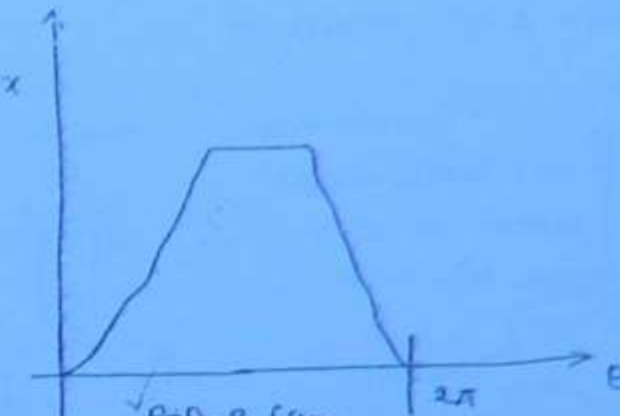
Angle of action:- Angle of rot of cam from begining of Rise to end of return

1.



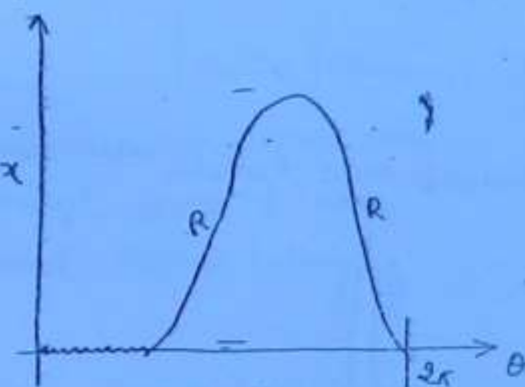
R-R Cam

3.



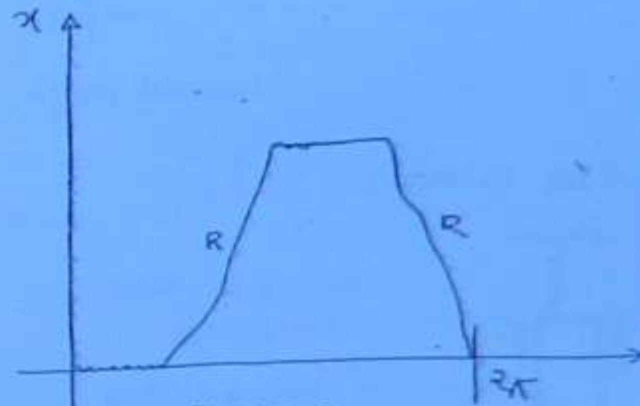
R-D-R Cam

2.

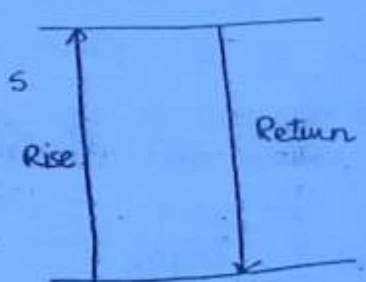


D-R-R
Cam

4.



D-R-D-R
Cam



$S \rightarrow$ stroke length of Follower

$t_o = \frac{\theta_o}{\omega}$ | $t_R = \frac{\theta_R}{\omega}$

$V_{o\,mean} = \frac{S}{t_o}$

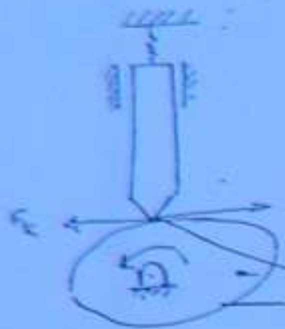
$= \frac{S}{\theta_o/\omega}$

$$\boxed{V_{o\,mean} = \frac{\omega S}{\theta_o}}$$

Similarly, $\boxed{V_{R,mean} = \frac{\omega \cdot S}{\theta_R}}$

* Classification of Follower :-

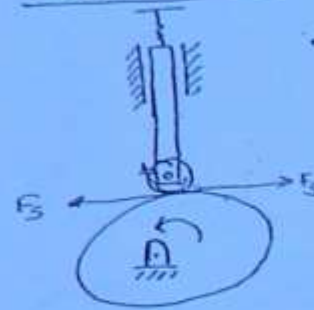
1. Knief - edge follower :-



Area = 0

- Excessive wear
- side thrust

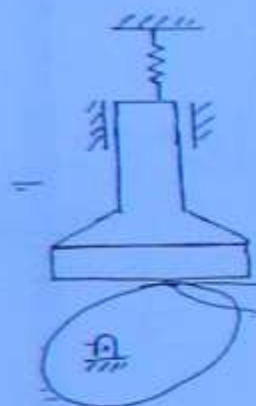
2. Roller Follower :



- wear absent
- space requirement high
- Air crafts engine gas engine

Best category

3. Flat - Face Follower :

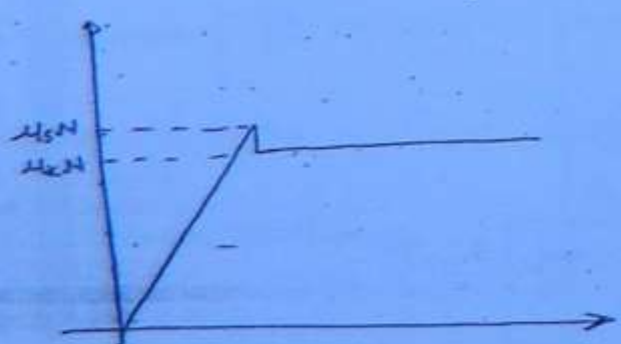


F_k

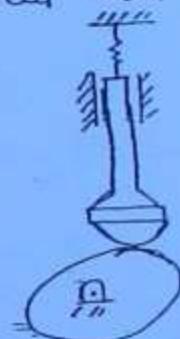
$dA \to 0$

- wear is less than knief - edge
- Surface stresses developed

- if Flat face is circular disc it is called Mushroom Follower

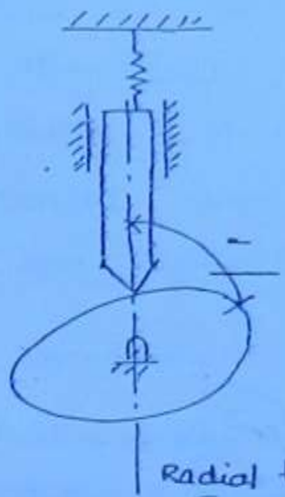


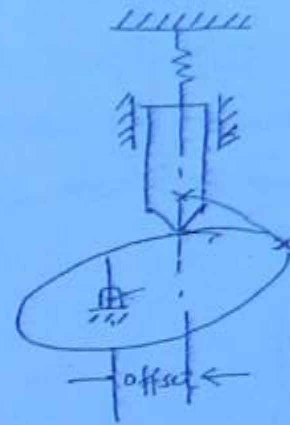
4. Spherical Faced Follower



- wear is less
- surface stresses are less
- used in I.C engine — (Valve operating mechanism)

* Radial and offset follower :-





offset was given to reduce slight wear.

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- when line of action pass through centre of cam, it is called Radial follower.

* Cam Terminology :-

1 Base circle :-

Minimum radius circle from the cam centre which touches the cam profile. It is also known as minimum radius of cam.

- size of cam is always defined by size of base circle.

2. Trace Point :-

- It is a point on follower which is required to trace the cam profile. And the curve traced by traced point is known as Pitch curve which will always be parallel to the cam profile.

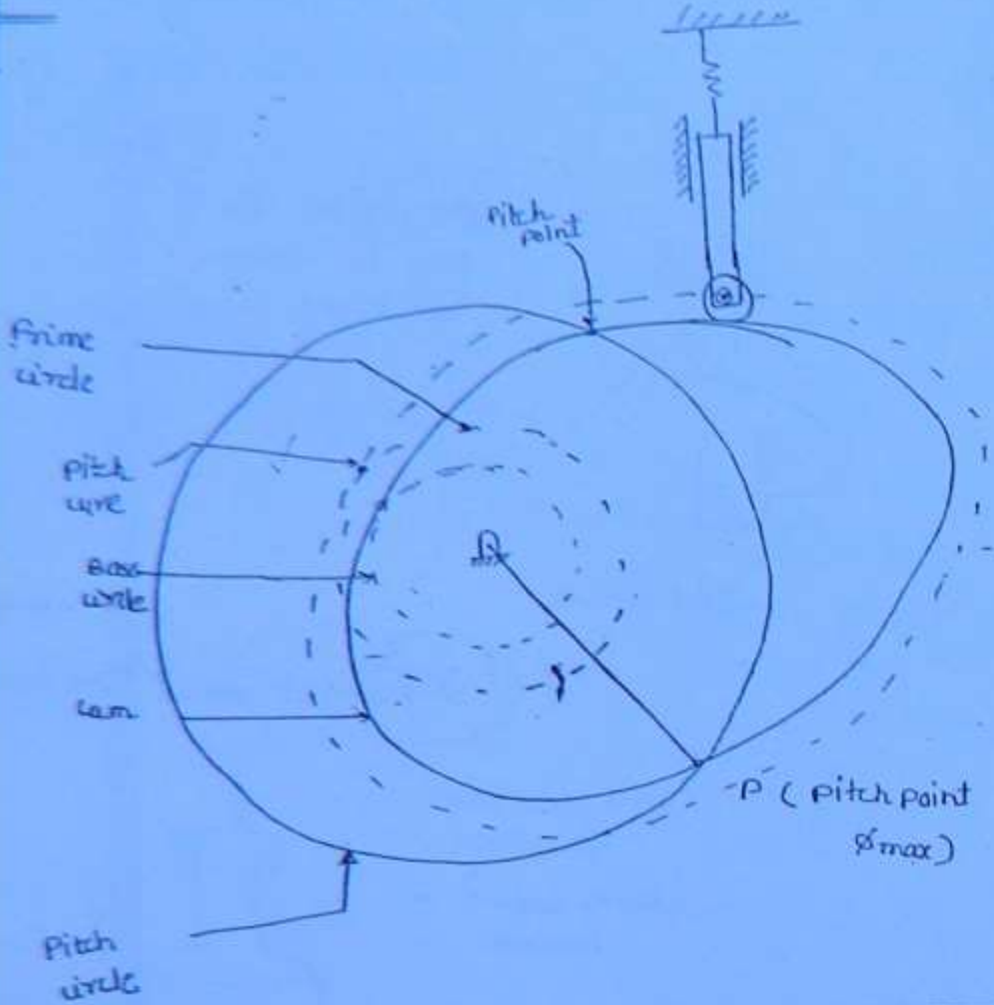
3. Prime circle :-

Minimum radius circle of the pitch curve which touches the pitch curve is called Prime circle

4. Pitch Point :-

The point on ~~which~~ pitch curve where pressure angle is maximum.

5. Pressure angle :-


Pitch point
Prime circle
Pitch curve
Base circle
cam
P (pitch point
ϕmax)
Pitch circle

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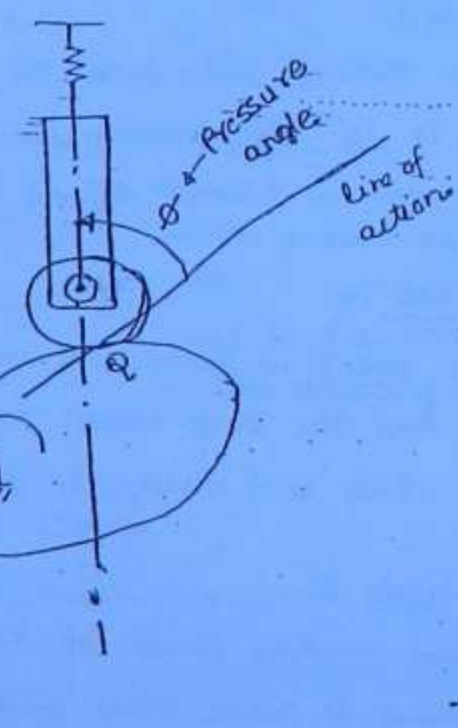

ϕ ← Pressure angle
line of action
Q

* Circular arc Cam with Flat face Follower :-

When the flanks of the cam are tangential to base circle and nose circle and is of convex circular arc such a cam is known as circular arc cam. There are six basic dimensions of the cam and are symmetrical cam

Basic Dimensions :

r_1 – Radius of Base Circle

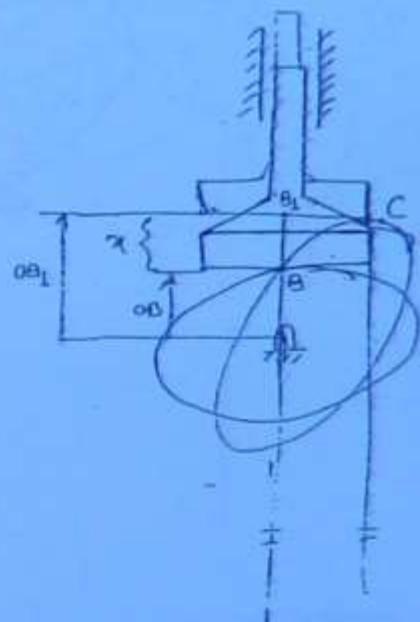
$r_2 \rightarrow$ Radius of Nose Circle

$R \rightarrow$ Rad of Flank

$\alpha \rightarrow$ Semi - Angle of action

$\phi \rightarrow$ Semi-Angle of Action on Flank

$OQ \Rightarrow L$ (Centre distance from Base & Nose Circle centre



Follower on Flank :-

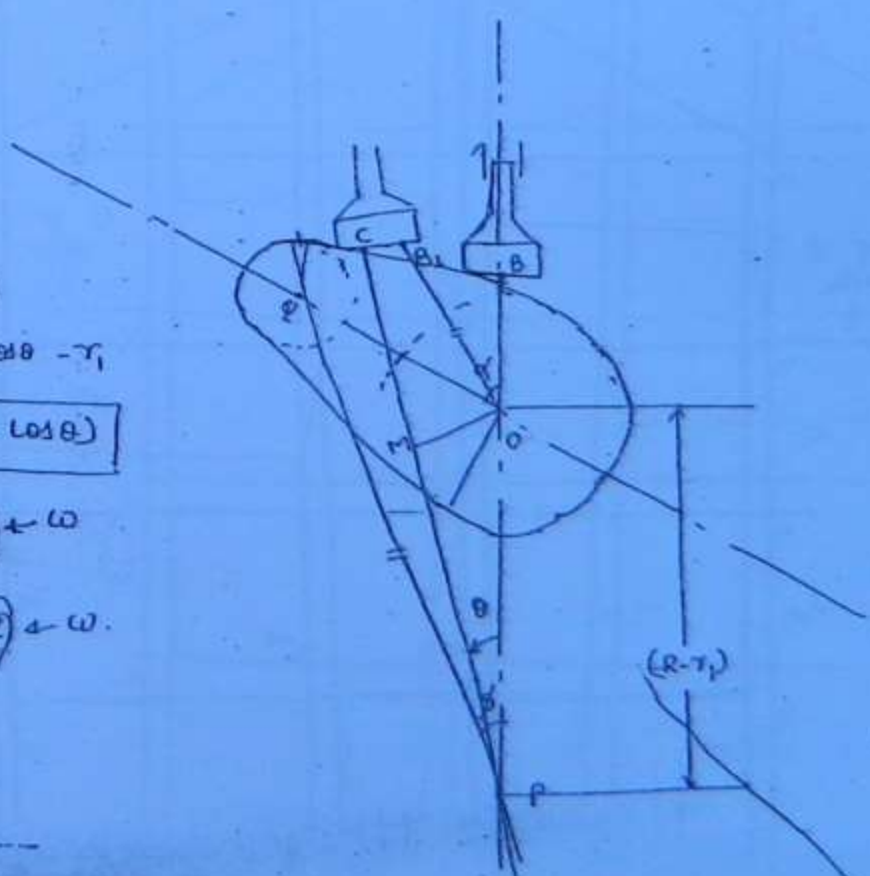
$\theta \in [0, \phi]$

$$x = (OB_1 - OB)$$
$$= (CM - OB)$$
$$= (PC - PM) - r_1$$
$$= R - (R - r_1)\cos\theta - r_1$$

$$\boxed{x = (R - r_1)(1 - \cos\theta)}$$

$$v = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} \leftarrow \omega$$

$$a = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} \leftarrow \omega .$$



Follower on Nose :-

$x = OB_1 - OB$

$= CM - OB$

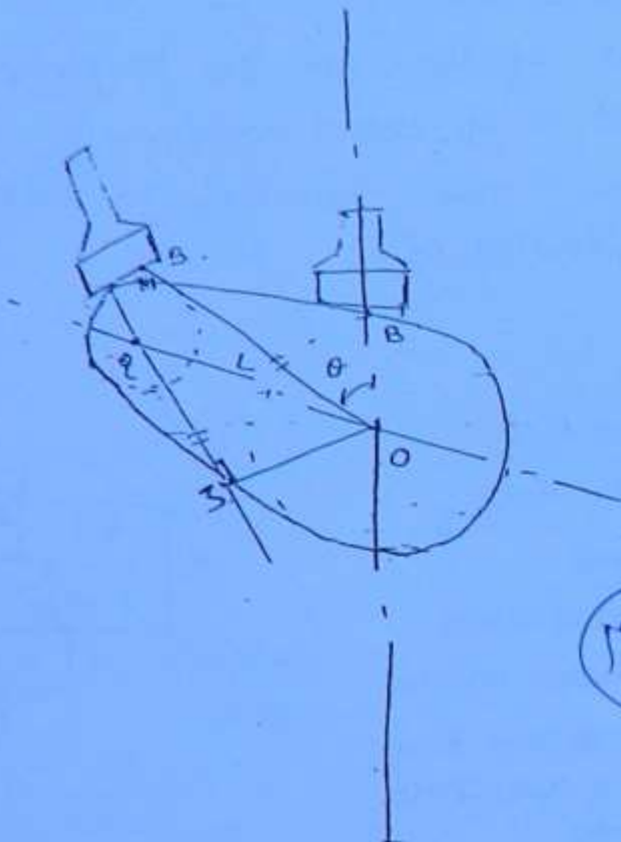
$= OQ + QM - r_1$

$= (r_2 - r_1) + QM$

$x = (r_2 - r_1) + L\cos(\alpha - \theta)$

Lift $\Rightarrow$ $\theta = \alpha$

$(r_2 - r_1) + L$



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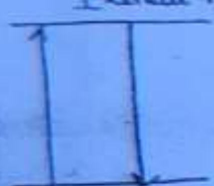
1. Uniform Velocity motion :-
(Linear motion)

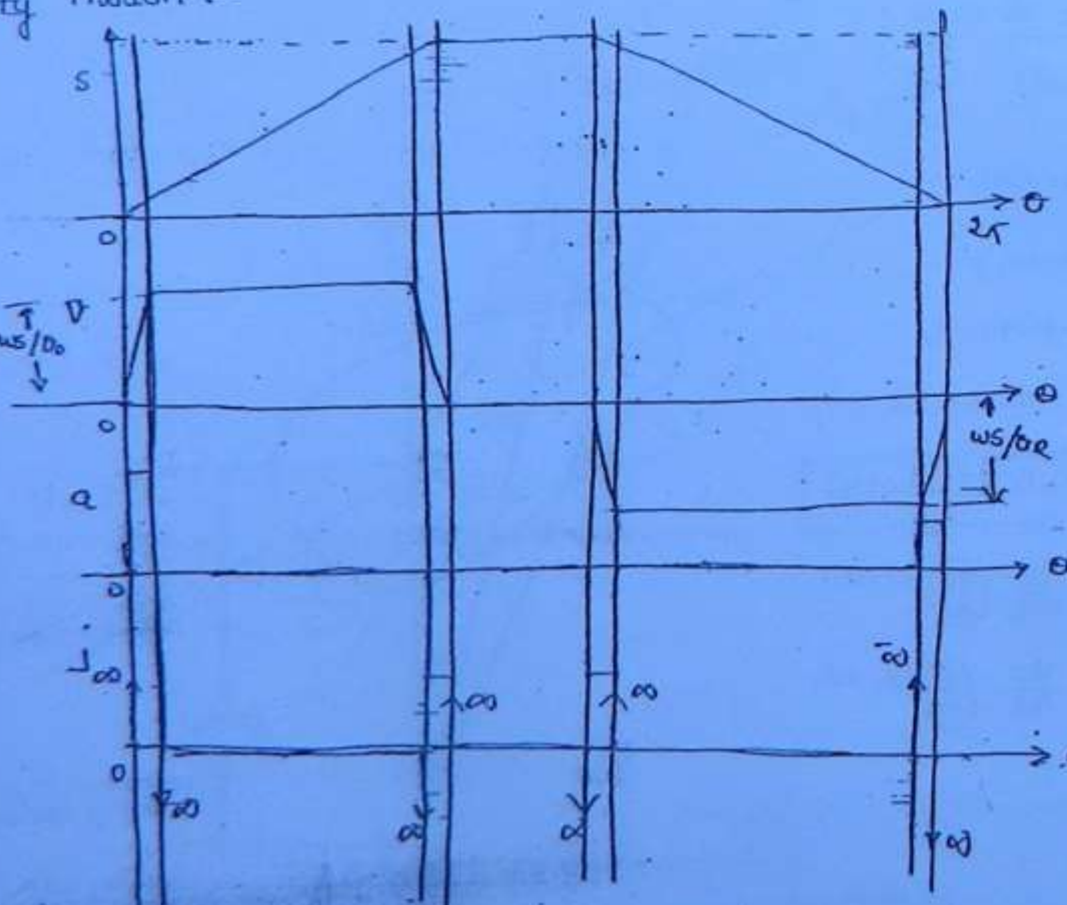


$v_0 = v_{omean} = v_{omax}$

$= \frac{\omega S}{\theta_0}$

$v_R = \frac{\omega S}{\theta_R}$

can be used in very very slow engines



Follower on Nose :-

$x = OB_1 - OB$

$= CM - OB$

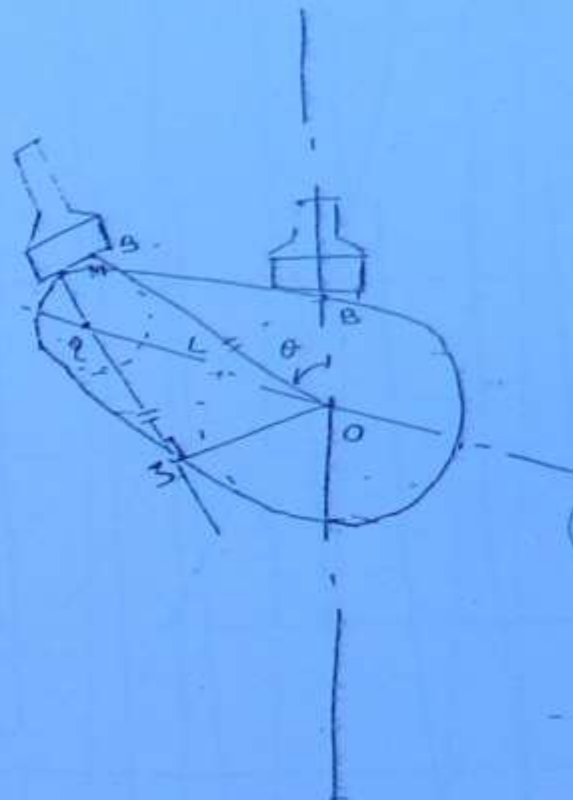
$= (Q + QM - r_1)$

$= (r_2 - r_1) + QM$

$x = (r_2 - r_1) + L\cos(\alpha - \theta)$

lift $\Rightarrow$ $\theta = \alpha$

$(r_2 - r_1) + L$



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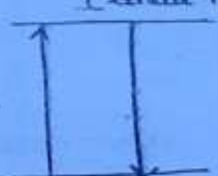
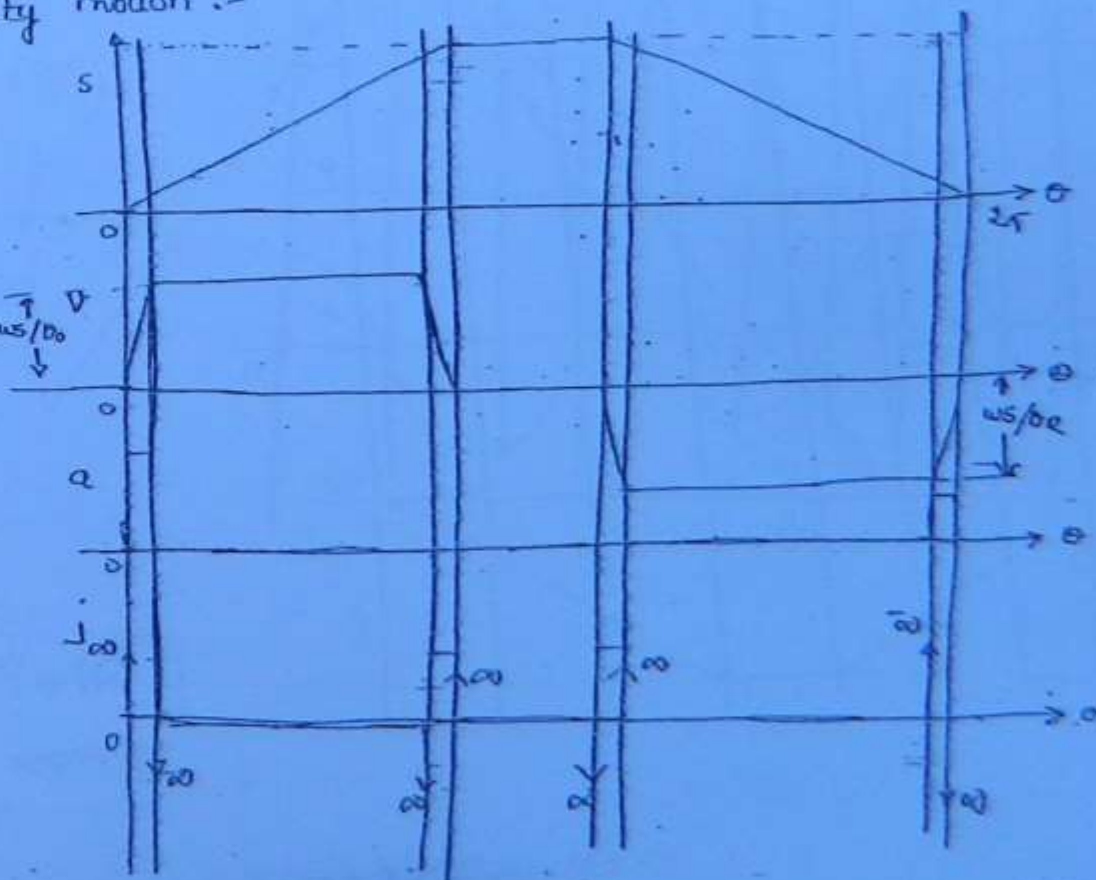
1. Uniform Velocity motion :-
(linear motion)





$V_0 = V_{omean} = V_{omax}$

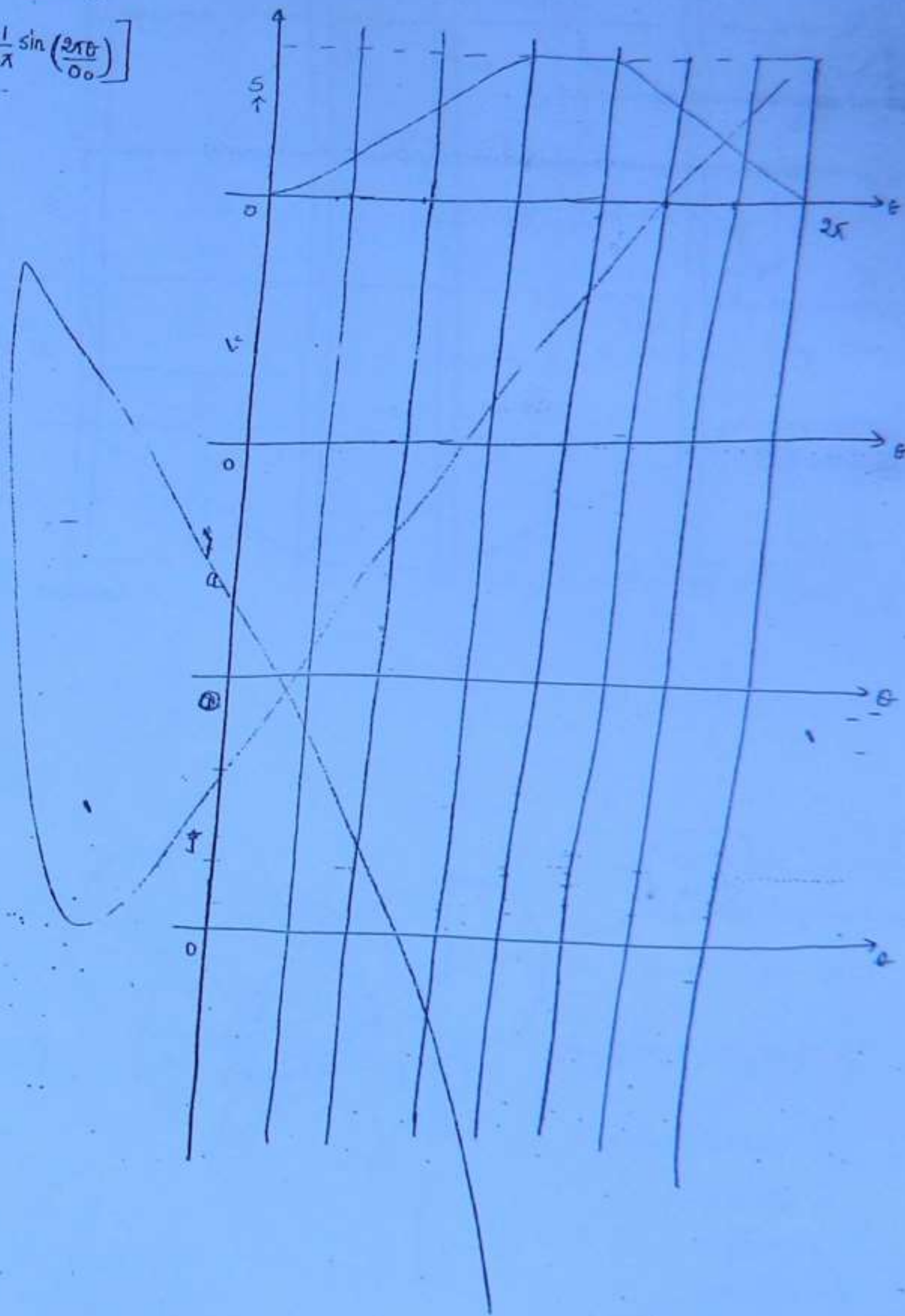
$= \frac{\omega s}{\theta_0}$

$V_R = \frac{\omega s}{\theta_R}$

can be used in very very slow engine

4. Cycloidal Motion :-

$$x_o = s\left[\frac{\theta}{\theta_o} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_o}\right)\right]$$



Cycloidal Motion :-

$$x_o = s\left[\frac{\theta}{\theta_o} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_o}\right)\right]$$

$$v_o = \frac{dx_o}{d\theta}\cdot\frac{d\theta}{dt}$$

$$= s\omega\left[\frac{1}{\theta_o} - \frac{1}{2\pi}\cos\left(\frac{2\pi\theta}{\theta_o}\right)\cdot\frac{2\pi}{\theta_o}\right]$$

$$= \frac{s\omega}{\theta_o}\left[1 - \cos\frac{2\pi\theta}{\theta_o}\right]$$

$\hookrightarrow f'(\sin^2\theta)$

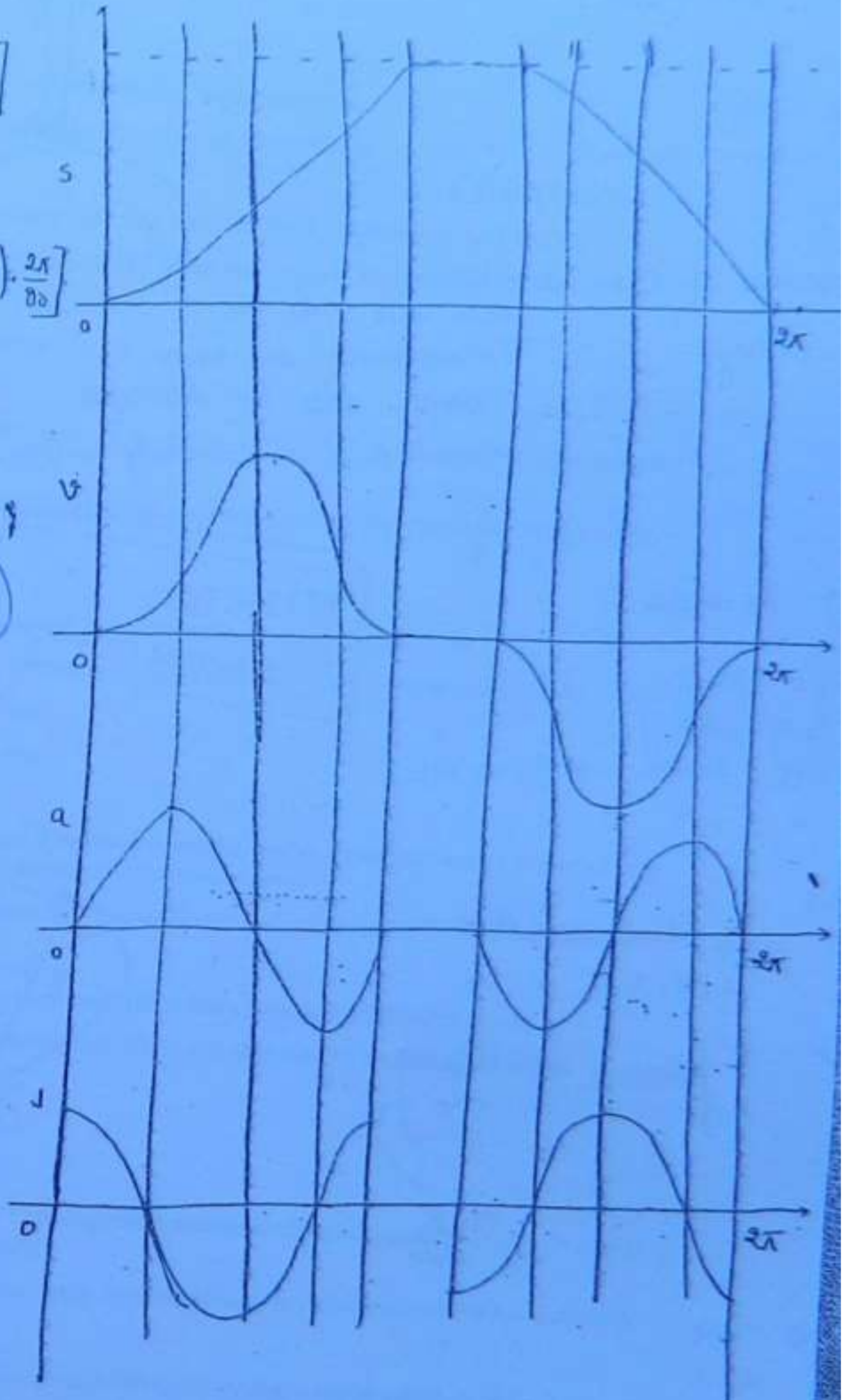
$$a_o = \frac{dv_o}{d\theta}\cdot\frac{d\theta}{dt}$$

$$= \frac{s\omega^2}{\theta_o}\left[\sin\frac{2\pi\theta}{\theta_o}\cdot\frac{2\pi}{\theta_o}\right]$$

$$= \frac{2\pi s\omega^2}{\theta_o^2}\cdot\sin\left(\frac{2\pi\theta}{\theta_o}\right)$$

$$J = \frac{4\pi^2\omega^3 s}{\theta_o^3}\cdot\cos\left(\frac{2\pi\theta}{\theta_o}\right)$$

can be used at very high speed - [I.C engine]



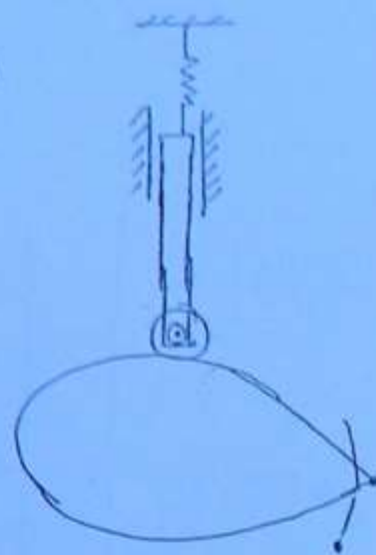
→ [illegible] — very dangerous

$$\frac{1}{R} = \frac{d^2y/dx^2 \to \infty}{\left\{1 + \left(dy/dx\right)^2\right\}^{3/2}}$$

If $a \to \infty$

$\quad \hookrightarrow R \to 0$

(sharp point).



- interference of cam · Follower
- undercutting

cam will rub the sharp point and cam profile will be damaged.

and this is something called interference of cam

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kakkar_amit@rediffmail.com

Problem :- 25 marks :-

$r_1 = 25mm$

$r_2 = 5mm$

$Lift = 20mm \rightarrow (r_2 - r_1) + L$

$\alpha = 75°$

Find

i) Basic dimensions of cam

L, ϕ, R

ii) $\left.\begin{matrix} V \\ a \end{matrix}\right]$ Follower at $\left.\begin{matrix} \text{begin} \\ \text{end} \end{matrix}\right]$

Flank

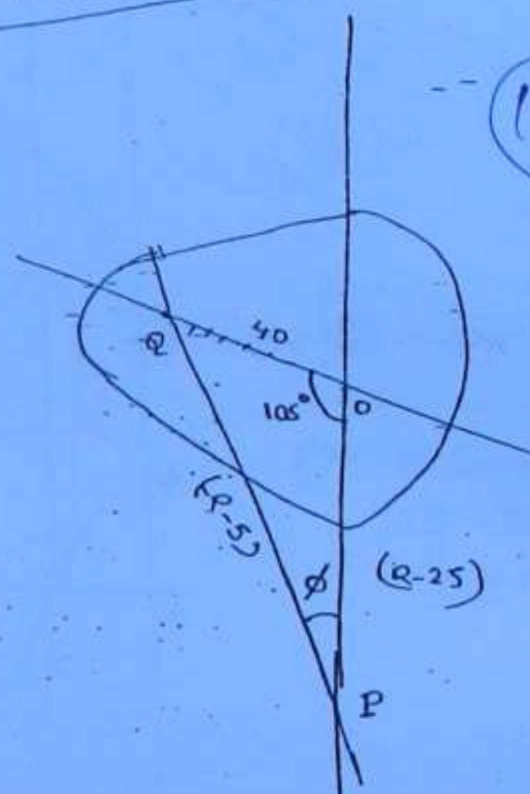
Nose

cos rule

$R = ?$

Sine rule

$\phi = ?$



(193)

24. Assertion (A) : UPSC is an independent organisation.

Reason (R) : UPSC is created by an act of Parliament.

Codes :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

25. **Assertion (A) :** Lok Sabha cannot make any change in the taxation proposals submitted to it.

Reasoning (R) : All taxation proposals are prepared in the executive organ of the government.

Codes :

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) Both (A) and (R) are correct, and (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

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26. **Assertion (A) :** The position of council of ministers in a state is similar to that of the council of ministers at the union level.

Reason (R) : The position of the Chief Minister is similar to that of the Prime Minister.

Codes :

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) Both (A) and (R) are correct, but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

27. **Assertion (A) :** The crux of Development administration is societal change in tune with modernity.

Reason (R) : Its focus is essentially on indigenous development which is sustainable.

Codes :

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(B) Both (A) and (R) are correct, but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true