



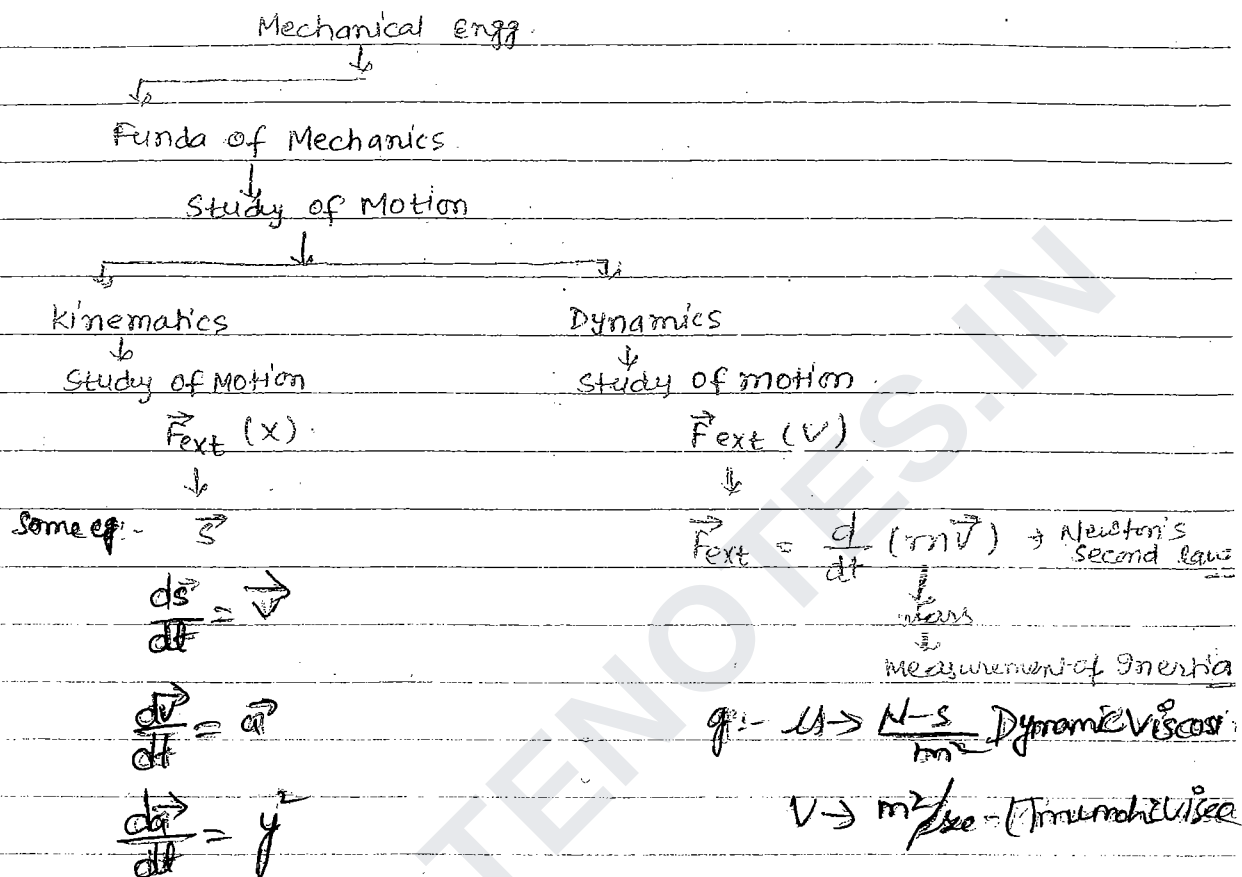
29/08/19

Simple Mechanism

CLASSMATE

Date 03/08/19

Page 01

KomSimple Mechanism:-

Kinematic Link:-
or, element

Every Part of a Machine which is having some relative motion with respect to some other Part is known as Kinematic link or element.

It is not necessary for the link to be perfectly rigid but it is necessary for the link to be a resistant body ^{so} that the power ~~at~~ or motion can be transmitted.

Types of link:-

- ① Rigid link.
- ② flexible link.
- ③ fluid link.

Rigid :- deformation is negligible.

Flexible :- deformation is in permissible limit. eg:- belt, rope etc.

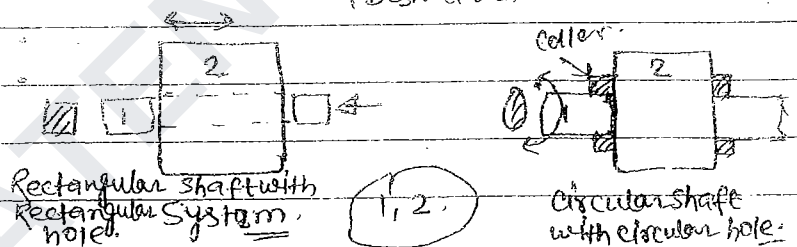
* Fluid :- When power is transmitted through the fluid pressure then fluid is known as fluid link.
eg:- all hydraulic devices.

Types of Relative motion:-

- ① Completely constrained motion:-] → Constrained.
- ② Successfully constrained motion.
- ③ Incompletely constrained motion.] → Unconstrained.

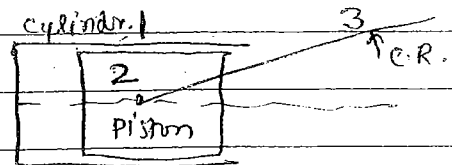
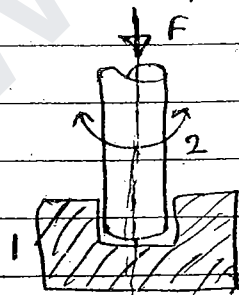
Constrained:- only one type of motion.

Completely :- self.



Successfully :- constrained with help of surrounding or other.

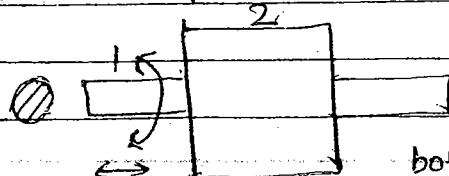
eg:- Piston/cylinder. →



Unconstrained:- undesired
more than one are present.

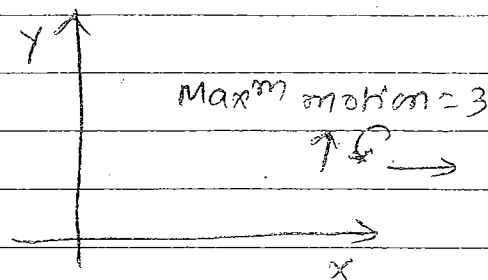
eg:-

Circular Shaft with Circular hole.



both rotation & reciprocating.

2D :- (Planar mechanism) :-



$$F = 3(l-1) - 2P_1 - 1P_2$$

No. of

Binary joints

No. of higher pairs

$$F = 3(l-1) - 2j - h \Rightarrow \text{Kutzbach eqn.}$$

Binary joint

higher pair

Grubler's equation :-

→ Mechanism with $F=1$, $h=0$.

i.e., $1 = 3(l-1) - 2j - h$

$$\Rightarrow [3 - 4 - 2j = 0] \Rightarrow \text{Grubler's eqn.}$$

$\downarrow$ even $\downarrow$ even $\downarrow$ even

Minimum No. of link required for Mechanism ($l_{\min} = 4$).

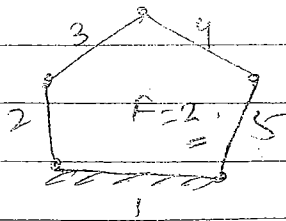
$F = 0 \rightarrow$ Frame.

$F < 0 \rightarrow$ super structure.

$F = 1 \rightarrow$ kinematic chain.

$\left. \begin{matrix} > 1 \\ 2 \\ 3 \end{matrix} \right\} \rightarrow$ unconstrained chain.

##



→ How to give the Constraint motion with unconstraint chain.

⇒ No. of Input required to give the constraint motion = No. of DOF.

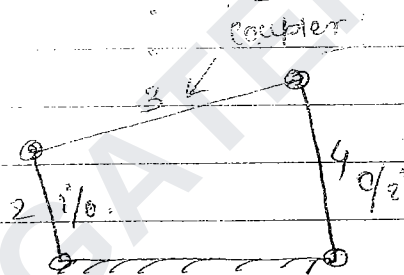
Degree of freedom is the No. of required input to give the desired constraint output.

Four bar mechanism:- (4 links connected by four turning pair)

or Quadric cycle:- (4 link + 4TP)

→ When the 4 links are connected by four turning pairs then the mechanism is known as four bar mechanism.

→ Second name is quadric cycle.



↑ frame or, fixed link.

→ adjacent link → input or, output.

→ Connecting of adjacent link is known as coupler.

i/o :
 → Rotate → Crank
 → Oscillation → Rocker or, Lever.

Inversion:-

① Double Crank or, Crank-Crank.

② Crank Rocker or, Rocker Crank.

③ Rocker-Rocker or, Double Rocker.

Kinematic Pair:-

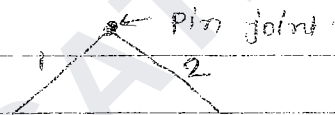
The connection between two links is always joint or a pair. But this pair will be a kinematic pair if the relative motion between them is a constrained motion.

Classification of kinematic pairs:-

① A/c to the type of relative motion there are five types of kinematic pairs:-

- ① Turning pair or (Revolute pair)
- ② Sliding pair or (Prismatic pair)
- ③ Rolling pair or
- ④ Screw pair
- ⑤ Spherical pair

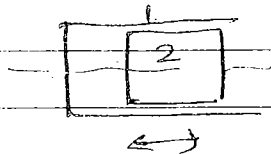
→ when relative motion is pure turning then turning pair



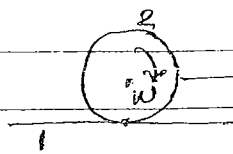
eg:- wrist pin, crank pin etc

→ when relative motion is purely ~~sliding~~ ^{sliding} the sliding pair.

eg:- Piston inside the cylinder.



→ when relative motion is purely rolling without skidding



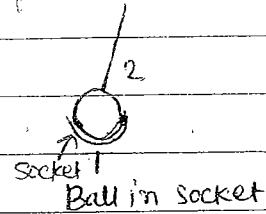
eg:- Ball bearing.

$V_{c.m.}$ → velocity of centre of mass
 $V_{c.m.} = R\omega$
 ↑ dependent motion
 ↑ independent motion
 is directly dependent on rolling (ω) and is independent parameter

→ when relative motion is purely screw thread motion.

eg:- Lead in nut & bolt.

- three dimensional partial rotation motion with surface to surface contact.



① A/c to the type of Contact:-

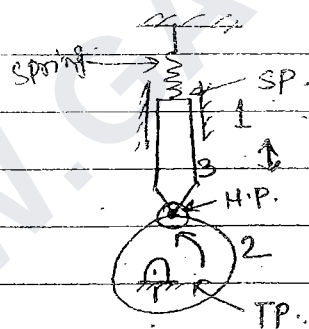
- ① Lower pair → surface contact
- ② Higher pair → point or line contact
- ③ Wrapping pair → discontinuous surface contact one over another with one having tight side & other slack side.

Lower:- Turning, Sliding, spherical, screw

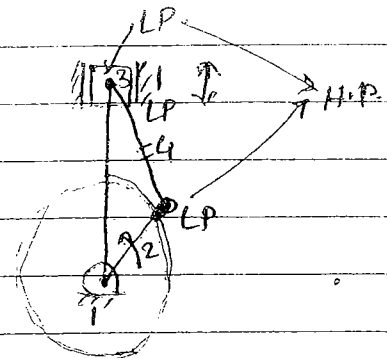
Higher:- Rolling pair.

→ Wrapping pair is closed to Higher pair.

② 1 Higher pair $\equiv$ 2 Lower pair.



$\equiv$



③ A/c to the type of closed:-

- ① Self closed → closed. → two links connected with a pin.
- ② Force closed → unclosed. → cam follower with spring.
→ Automatic clutch.
↓
Because of external spring force.

Kinematic chain:-

If all the links are connected in such a way such that first link is connected with the last link directly or indirectly to form a ^{closed} chain and the relative motion between any two links is a constrained motion then the closed chain will be known as kinematic chain.

Conditions for the kinematic chain:-

two methods for checking:-

① $L = (2P - 4)$

No. of
links

No. of kinematic
pairs

② $L = \left(\frac{3}{2} J - 2 \right)$

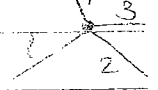
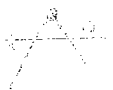
No. of
Binary
joint

No. of links

Binary

Turning

Quaternary



$1T \equiv 2B$

$1Q \equiv 3B$

(1,2)
(2,3)

(1,2)
(2,3)

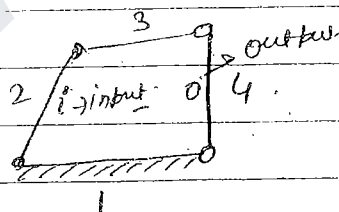
(1,3) (x)

(3,4)

already
with two
upper Binary.

Then chain is Kinematic Chain.

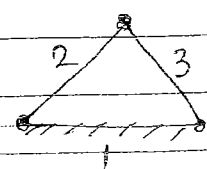
eg:-



$L = 4$
 $J = 4$ $\Rightarrow 4 = 4 \Rightarrow$ Kinematic Chain

$\Rightarrow$ when $J > \left(\frac{3}{2} L - 2 \right)$

then, No relative motion. $\Rightarrow$ that is frame.



$L = 3$

$J = 3$

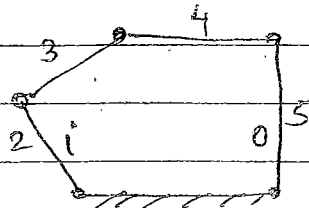
i.e., $3 > 2.5$

i.e., Frame of structure.

⇒ when $j < \left(\frac{3}{2}l - 2\right)$

then unconstrained motion.

eg:-



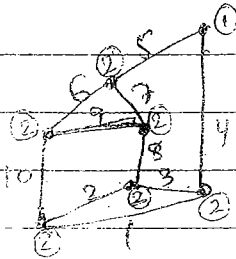
$$l = 5$$

$$j = 5$$

$$\text{then, } 5 < 5.5$$

that is output is not dependent on input.

Q:-



$$l = 10$$

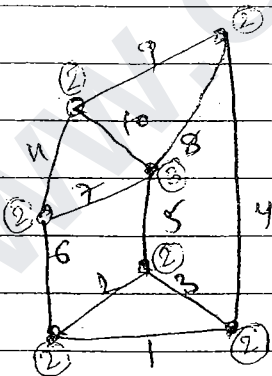
$$j = 13$$

$$\frac{3 \times 10}{2} - 2 = 13$$

$$j = 13 - 2 = 13$$

$j = 13$ Ans ⇒ Kinematic chain.

Q:-



$$l = 11$$

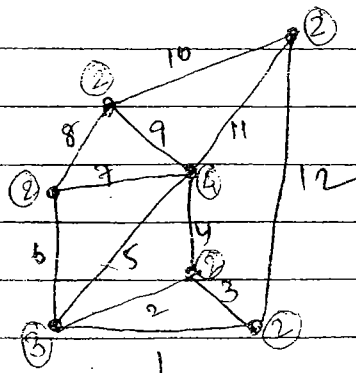
$$j = 15$$

$$j = \frac{3 \times 11}{2} - 2$$

$$15 \geq 14.5 \Rightarrow \text{Frame.}$$

Ans

Q:-



$$l = 12$$

$$j = 17$$

$$17 > 16$$

Frame

Super structure or, indeterminate structure:-

$$j > \left(\frac{3}{2}l - 2\right)$$

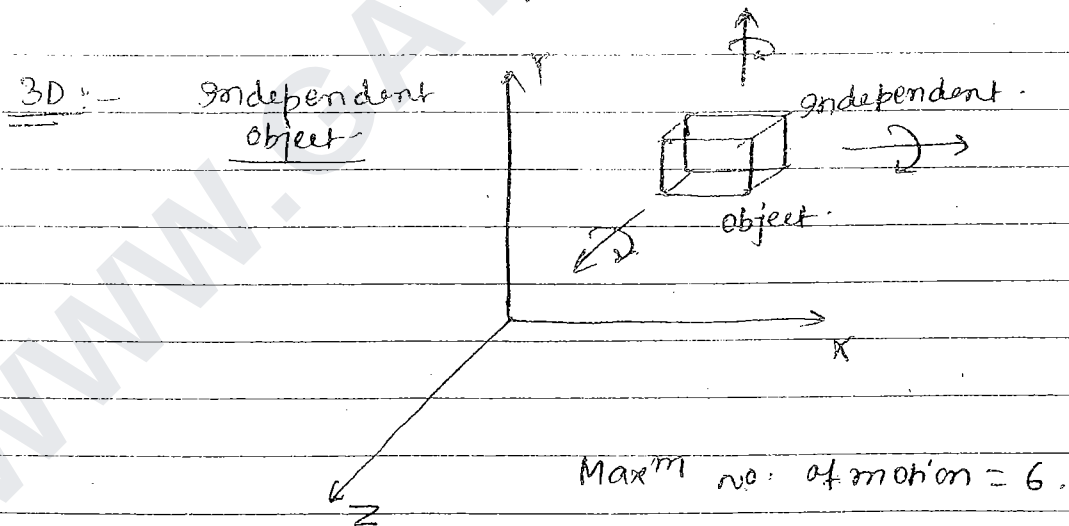
Such structures are known as super structures.

$\Rightarrow$ No. of extra link for superstructure = $\frac{\text{margin for frame to extra}}{\text{frame required for no. of links (minimum times margin)}}$

Degree of freedom:-

The minimum No. of independent variables required to define the ^{position of} motion of the system is known as degrees of freedom of a system.

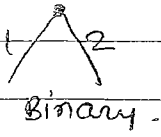
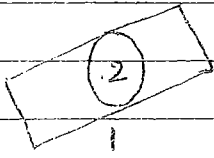
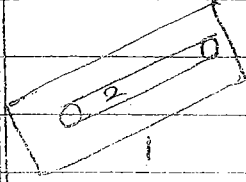
motion / position
 $\swarrow \searrow$
analogous



For dependent object,

$$F = \text{DOF} = 6 - \text{Restrains.}$$

$$\left. \begin{array}{l} \text{LP} = 1 \text{ DOF} \\ \text{HP} = 2 \text{ DOF} \end{array} \right\}$$

Pair	Restraints	Dof.
 Binary.	$3T + 2R$ $= 5$	$6 - 5 = 1$
	$1T = 1$	$6 - 1 = 5$
	$1T + 1R$ $= 2$	$6 - 2 = 4$

Mechanism:-

$$Dof = F = ?$$

No. of links = 1
 one link is fixed.

$$F = 6(1-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

Where, P_1 = No. of those pairs with $Dof = 1$,

$$P_2 = \dots \dots \dots = 2$$

$$P_3 = \dots \dots \dots = 3$$

$$P_4 = \dots \dots \dots = 4$$

$$P_5 = \dots \dots \dots = 5$$

Grashof's law:-

continuous motion

continuous motion

For the continuous relative motion,

$$(S+1) \leq (P+2)$$

other two

1, 4, 3, 2

uncombine motion

2, 2, 3, 3

$$(S+1) < (P+2)$$

$$(S+1) = (P+2)$$

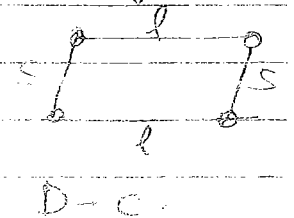
Not having pair

$$(S+1) = (P+2)$$

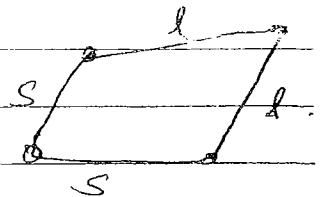
having pair

① $S \rightarrow$ fixed
D-C② $S \rightarrow$ adjacent to fixed
C-R③ $S \rightarrow$ coupler
D-R

① Parallelogram linkage



② Deltoid linkage

(a) $S \rightarrow$ fixed
then D-C(b) $l \rightarrow$ fixed
then C-RNote:- If

$$(S+1) > (P+2)$$

then only R-R. or D-R can possible only.

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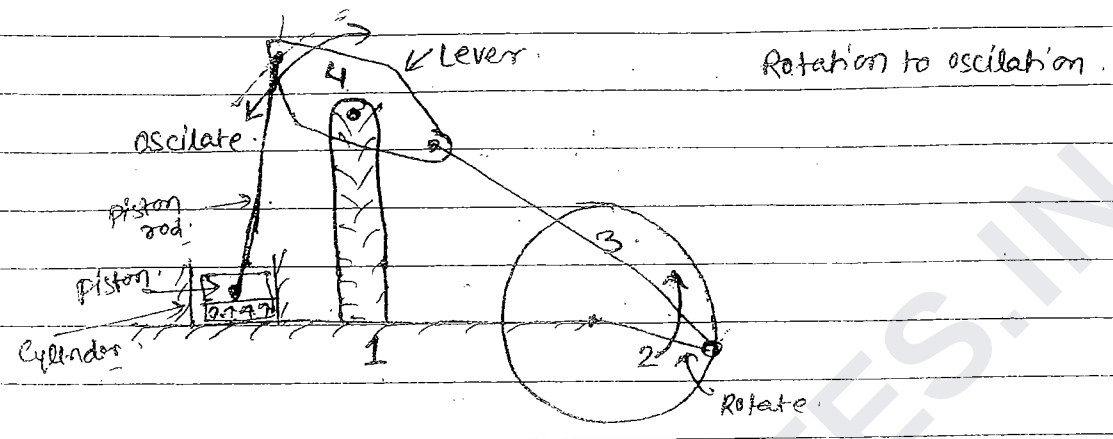
TCM

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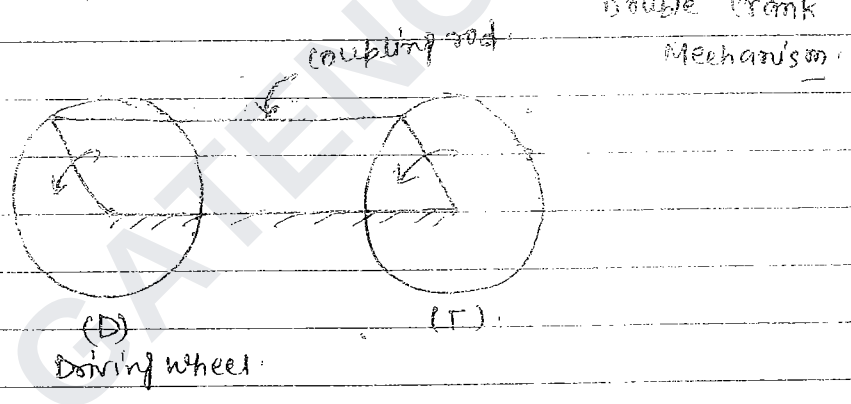
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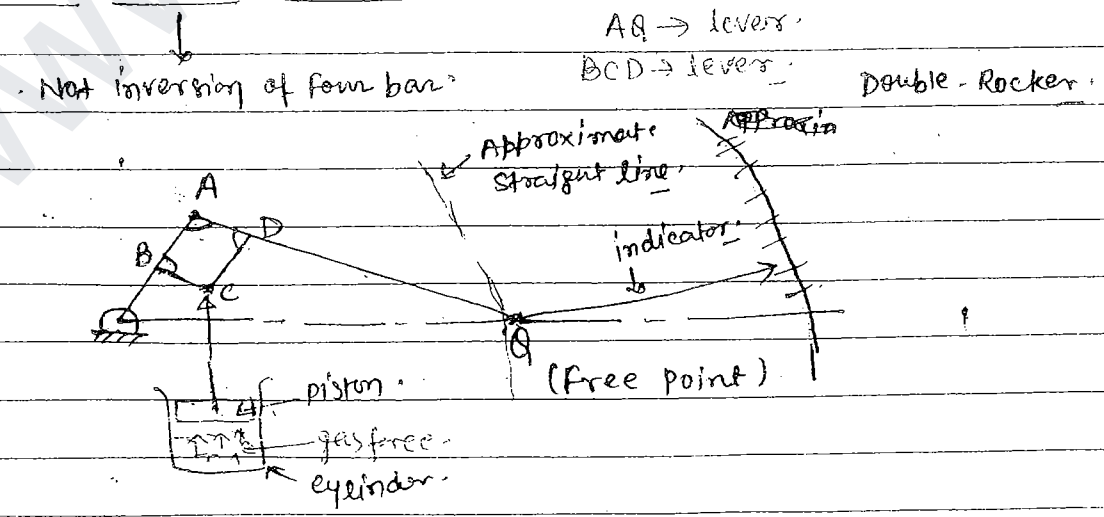
① Beam Engine Mechanism: - (C-R Mechanism)



② Coupling Rod of the locomotive: -



③ Watt's Indicator Mechanism: -



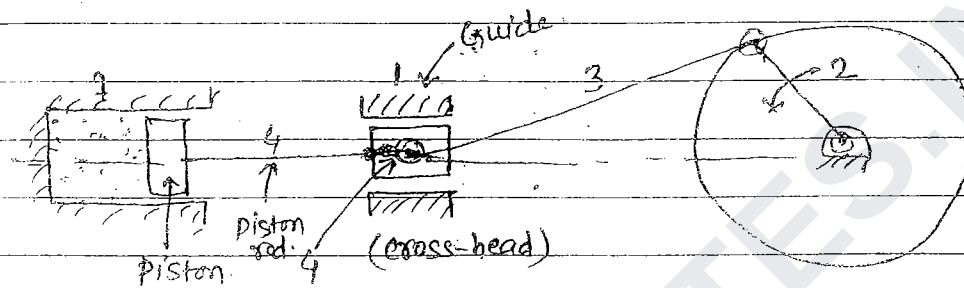
→ Due to angle at B AD all point not changes so there is not the turning pair.
so it is not the inversion of four bar.

Single Slider Crank Mechanism:-

(4 links + 3 Turning pair + 1 sliding pair)

O → Turning pair

14 → Sliding pair



Figure

① Cylinder Fixed:-

Rotation to Reciprocation → Reciprocating compressor.

Reciprocation to Rotation → Reciprocating engine.

② Crank 2 fixed:-

Whithworth Quick return mechanism.

Rotary I.C. Engine Mechanism. (Gnome Engine)

③ Connecting rod fixed:-

Crank & slotted lever & RMM.

Oscillation cylinder engine mechanism.

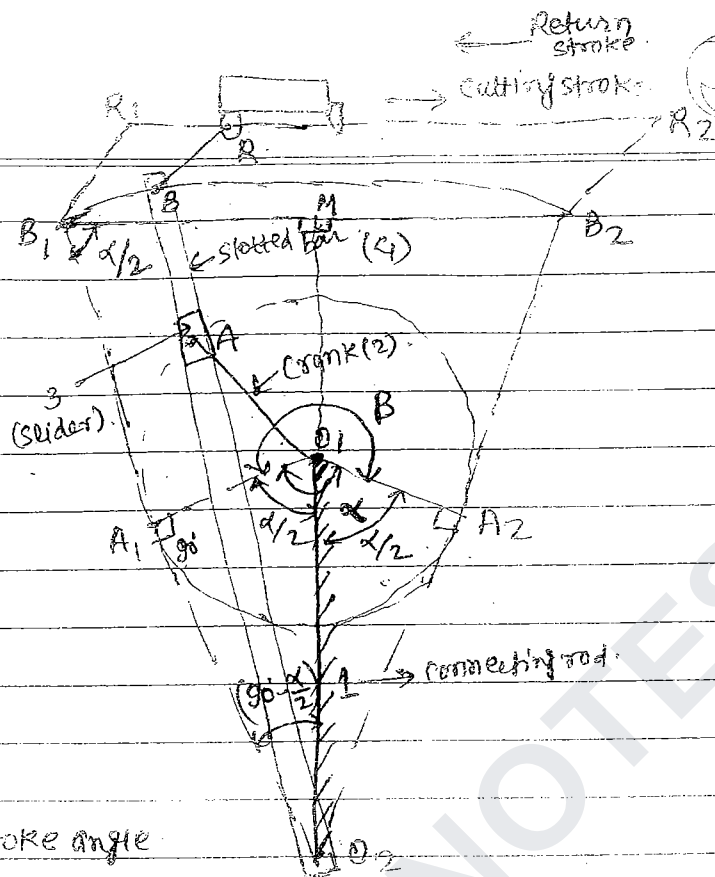
④ Slider fixed:-

Hand Pump. (Pendulum pump)

or, (Bull engine).

QRM

Crank & slotted lever Mechanism:-



$B \rightarrow$ cutting stroke angle

$\alpha \rightarrow$ Return stroke angle

and $\alpha < B$

$$\text{Quick Return ratio} = \frac{B}{\alpha}$$

always greater than 1.

$$\therefore B > \alpha$$

$$\text{Stroke} = R_1 R_2 = B_1 B_2$$

$$= 2 B_1 M \quad (\text{By symmetry}).$$

$$= 2 (O_2 B_1) \cos \frac{\alpha}{2}$$

$$= 2 (O_2 B_1) \frac{(O_1 A_1)}{O_1 O_2}$$

$$\text{Stroke} = 2 (O_2 B) \times \frac{(O_1 A)}{(O_1 O_2)}$$

$O_1 O_2 \rightarrow$ C.R.
 $O_2 B \rightarrow$ slotted bar
 $O_1 A \rightarrow$ crank

$\Rightarrow$ By managing $O_1 O_2$ i.e. C.R. length we can change the stroke of the shaper & planner.

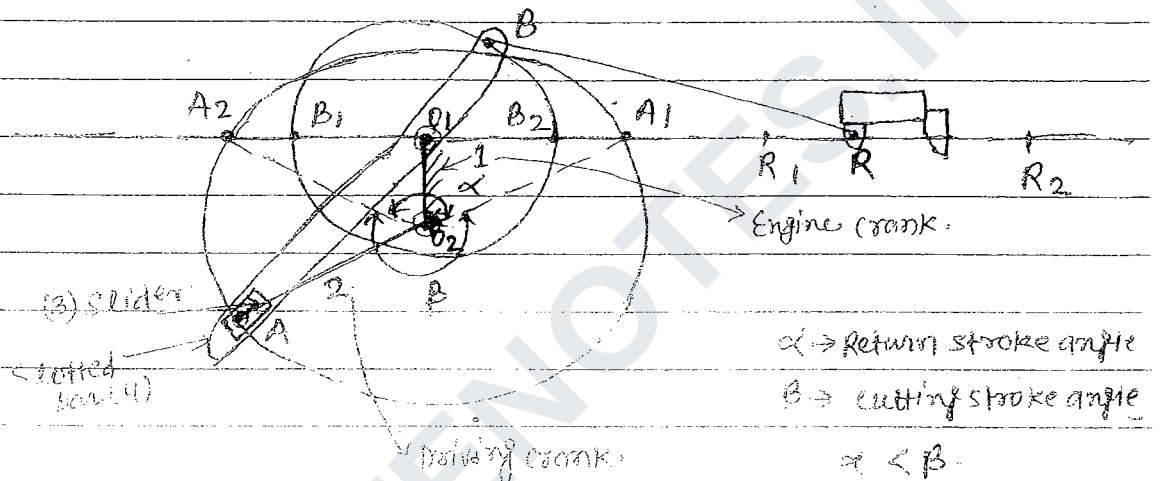
$\rightarrow$ slotted bar is in oscillation.

$\rightarrow$ Three parameter for changing stroke length.

Whitworth Quick Return Motion Mechanism:-

(Crank)
It is the shortest link of this Mechanism.

① → Turning for



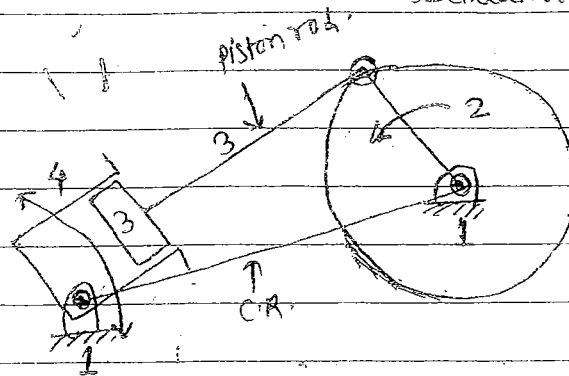
$$\text{Stroke} = R_1 R_2 = B_1 B_2 = 2 O_1 B$$

By varying O_1 Point
Varying the stroke
length.

- Slotted bar is in complete rotation.
- Rotation to Rotation are only possible when crank is the shortest link.
- only one parameter for changing stroke length.

Oscillating cylinder engine mechanism:-

Oscillation to Rotation or vice versa.



O → Turning

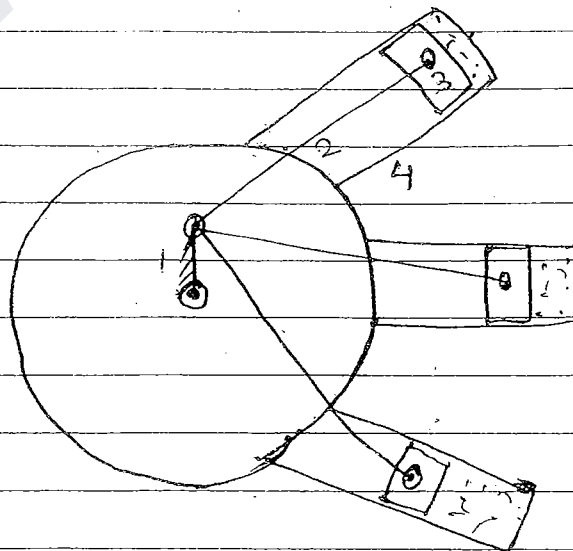
- Due to the limitation of the crank size for movement of piston in cylinder it is not possible for medium or high power transmission.

Rotary Internal Combustion Engine:-

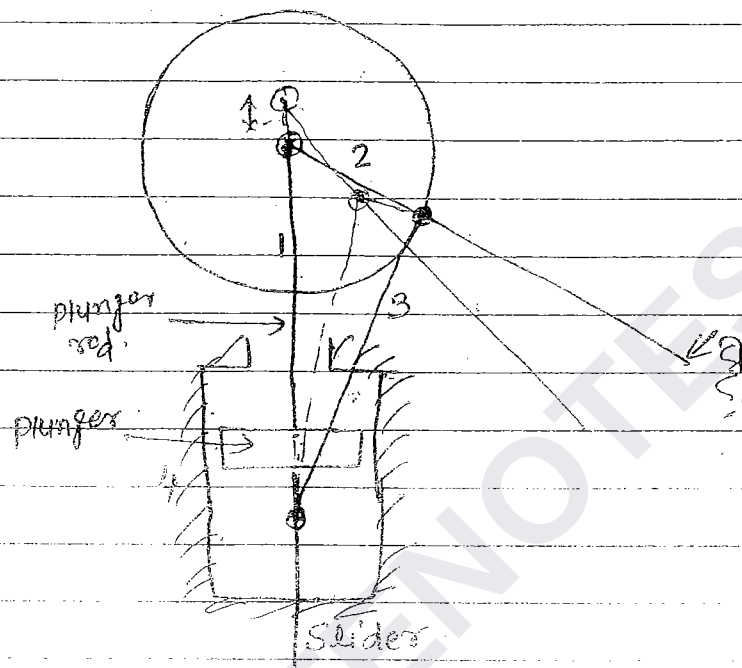
(Cochrane Engine)

- 7 or 9 cylinders arrangement → Reciprocating to Rotary motion.
- Crank is fixed.
- All cylinder are sliders.
- Radial engine in Aircraft.

O → Turning Pair.



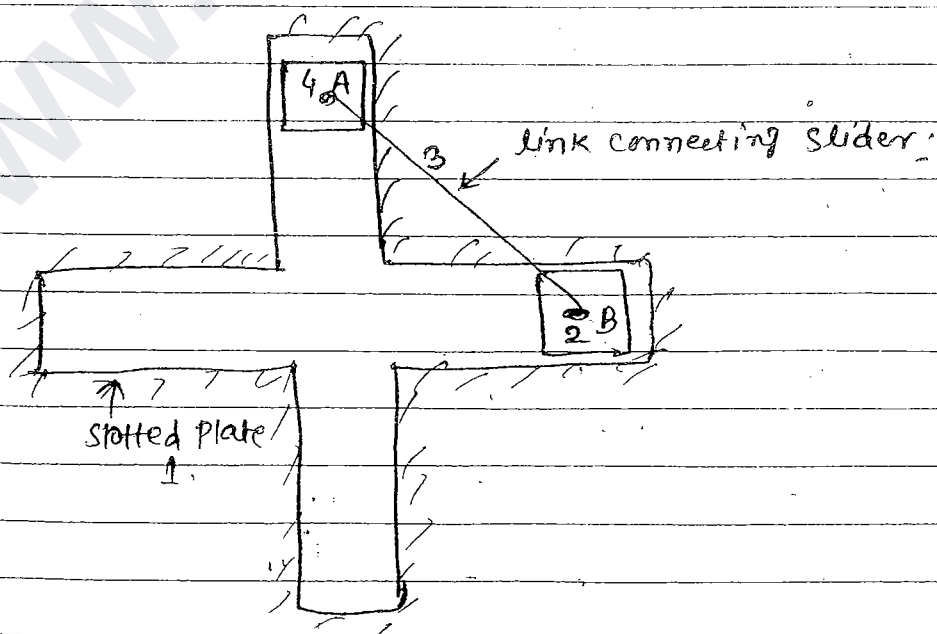
Hand Pump or, Bull engine or, Pendulum engine:-



Hand Pump

Double Slider Crank chain:-

4 links $\rightarrow (2TP + 2SP)$

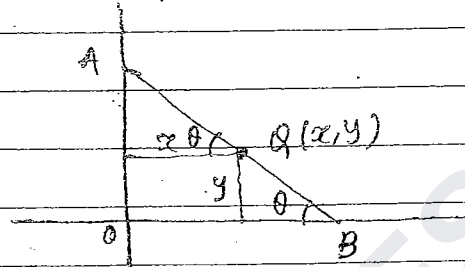


elliptical trammel

① when slotted plate is fixed



inversion is Elliptical Trammels ⇒



~~in~~ in small triangle,

$$\sin \theta = \frac{y}{BQ}$$

and,

$$\frac{x^2}{AQ^2} + \frac{y^2}{BQ^2} = 1 \rightarrow \text{eqn of ellipse}$$

$$\cos \theta = \frac{x}{AQ}$$

⇒ If $AQ = BQ$ i.e. midpoint is Q,

then,

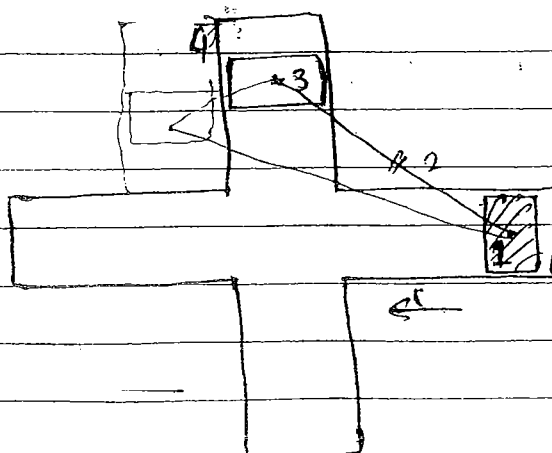
$$x^2 + y^2 = AQ^2 = BQ^2 = \text{equation of circle}$$

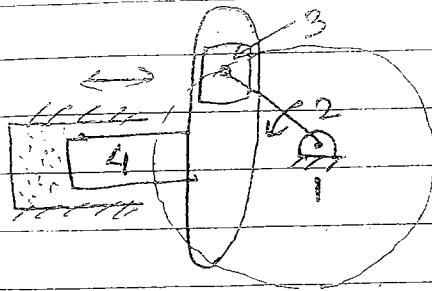
② Any of the slider is fixed then:



Rotation → to Reciprocation

⇒ Scotch-yoke Mechanism -





→ Demand two slider are used, which produces more wear & tear

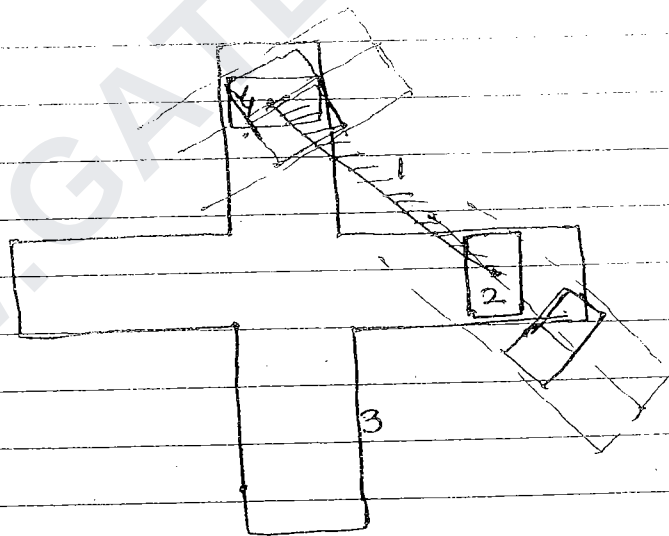
③ Link connecting sliders is fixed:-



Oldham's Coupling:-

→ This coupling is basically used to connect the two shafts which have lateral misalignment.

→ Angular shafts are connected by universal coupling or Hook's joint.



Oldham's Coupling.

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2. Velocity Analysis

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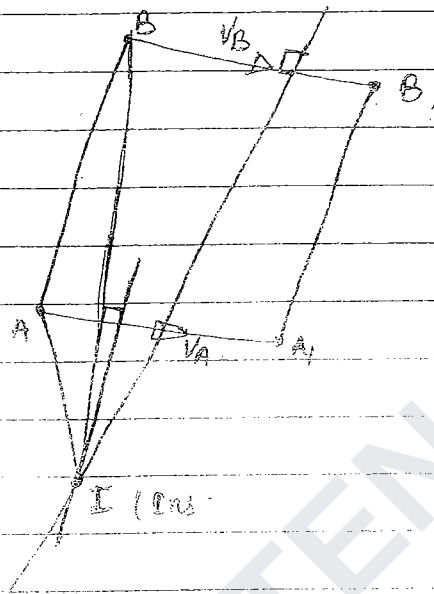
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Theories of Instantaneous Centres:-

(In Many many thanks to the god of this field a Kenedy).

Instantaneous centre of rotation:-



$$\Rightarrow \frac{ds}{dt}$$

(At) $t = t$,
 $t = t + dt$.

$$BB_1 \rightarrow 0$$

$$AA_1 \rightarrow 0$$

In general the motion of any link in the mechanism is neither ~~purely~~ ^{purely} translation nor purely rotation. It is the combination of translation & rotation. But

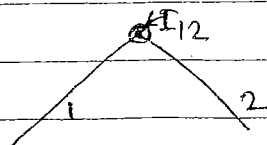
"The Point in the space with respect to which the whole link can be assumed to be in perfect rotation is known as Instantaneous centre of rotation".
Centrode and Axode:-

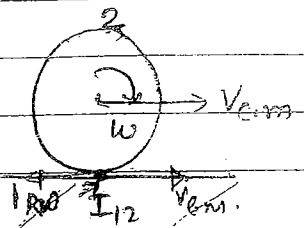
→ The locus of the I-centre it is known as centrode.

→ Axode is the locus of I-axis of the rotation.

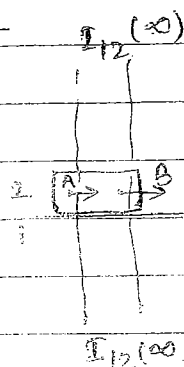
Basic instantaneous centres:-

① Turning Pair:-



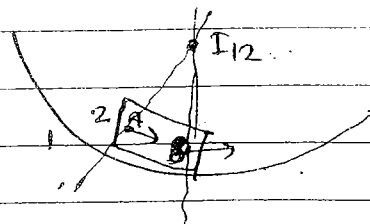
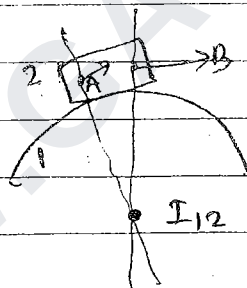
② Rolling pair:-

both V_{cm} & $R\omega$ can be canceled by one another.

③ sliding pair:-(a) plane surface:-

⇒ I-centre is on

centre of curvature of the curved surface in case of sliding pairs.

(b) Curved surface:Number of instantaneous centres in a Mechanism:-

No. of links = 1.

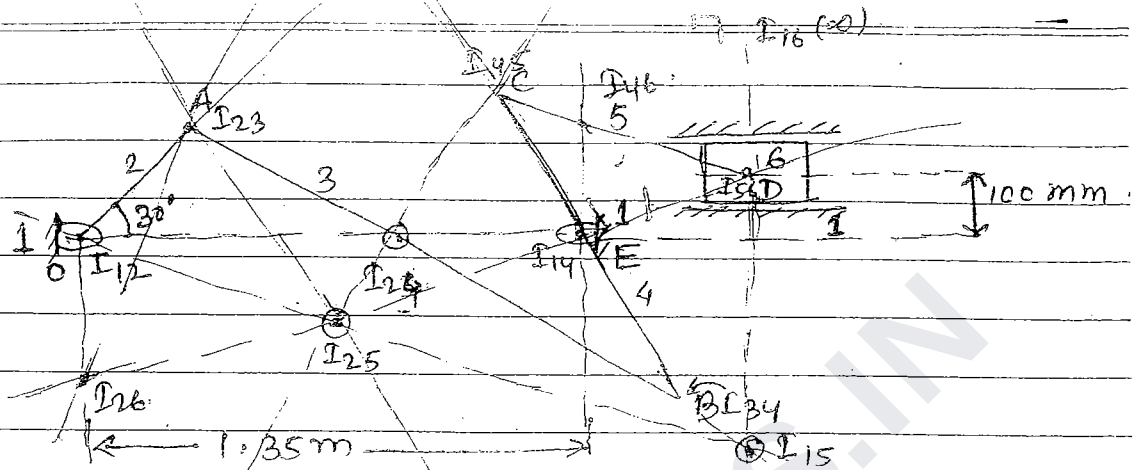
one I-centre gives by two links.

∴ No. of I-centres in a Mechanism

$$= I_{c_2} = \frac{1(1-1)}{2}$$

$$\boxed{\text{No. of I-centres} = \frac{1(1-1)}{2}}$$

Q1 ⇒ ①



$$OA = 200 \text{ mm}$$

$$AB = 1.5 \text{ m}$$

$$BC = 600 \text{ mm}$$

$$CD = 500 \text{ mm}$$

$$BE = 400 \text{ mm}$$

Given, $\omega_{OA} \rightarrow 120 \text{ r.p.m. (clockwise)}$

$$\text{get! - } V_B = ? \quad 3.2$$

$$\omega_{AB} = ? \quad 2.99$$

$$V_C = ? \quad 1.6$$

$$\omega_{BC} = ? \quad 8$$

$$V_D = ? \quad 0.8$$

$$\omega_{CD} = ? \quad 2.16$$

$$\text{Soln: } V_A = (OA) \cdot \omega_{OA} = \frac{200}{1000} \times \left(\frac{2\pi \times 120}{60} \right) = \frac{4\pi}{5} \text{ m/sec}$$

Scale used, $200 \text{ mm} = 1 \text{ cm}$

$$\omega_2 (I_{25} I_{18}) = \omega_5 (I_{25} I_{13})$$

$$\omega_2 (I_{26} I_{12}) = \omega_5 (I_{25} I_{13})$$

I_{35} $\swarrow$ I_{36}, I_{65}
 $\swarrow$ I_{34}, I_{45}
 $\swarrow$ I_{32}, I_{25}
 $\swarrow$ I_{31}, I_{15}

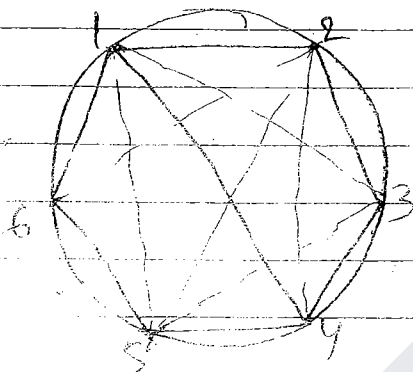
I_{36} $\swarrow$ I_{35}, I_{56}
 $\swarrow$ I_{31}, I_{16}
 $\swarrow$ I_{32}, I_{26}

I_{46} $\swarrow$ I_{45}, I_{56}
 $\swarrow$ I_{14}, I_{16}

No. of links = $l = 6$.

$$\text{No. of IC} = \frac{l(l-1)}{2} = \frac{6(6-1)}{2} = 15.$$

(12) 13 (14) 15 (16)
 (23) 24 25 26
 (34) 35 36
 (45) 46
 (56)



12, 23

I_{13}

14, 43

I_{15}

14, 45

16, 65

I_{24}

23, 34

21, 14

I_{25}

24, 45

21, 15

(A,B) link, link-3.

$\hookrightarrow I_{13}$

$$\frac{V_A}{I_{13A}} = \frac{V_B}{I_{13B}} = \omega_3 = \omega_{AB}.$$

I_{26}

I_{25}, I_{56}

I_{21}, I_{16}

(B,C) link, link-4.

$\hookrightarrow I_{14}$

$$\frac{V_B}{I_{14B}} = \frac{V_C}{I_{14C}} = \omega_4 = \omega_{BC}.$$

(C,D) link, link-5.

$\hookrightarrow I_{15}$

$$\frac{V_C}{I_{15C}} = \frac{V_D}{I_{15D}} = \omega_5 = \omega_{CD}.$$

Sol. $\Rightarrow$

Scale used:- 200 mm = 1 cm.

 $\therefore \omega_{OA} = 120 \text{ rpm (clockwise)}$ Now, $OA = 200 \text{ mm} = 1 \text{ cm}$.

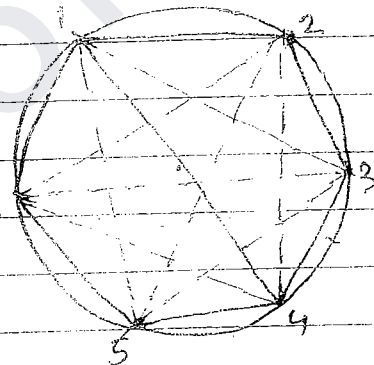
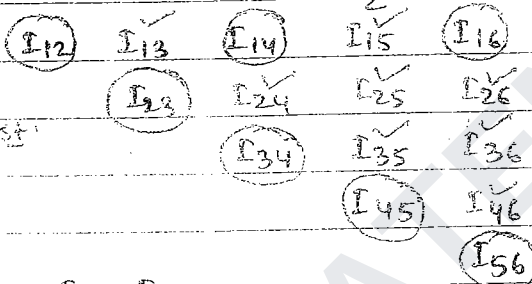
$$= \frac{2\pi \times 120}{60}$$

 $AB = 1.5 \text{ m} = 1500 \text{ mm} = 7.5 \text{ cm}$. $\angle AOE = 30^\circ$ $BE = 600 \text{ mm} = 3 \text{ cm}$. $CD = 500 \text{ mm} = 2.5 \text{ cm}$. $DE = 400 \text{ mm} = 2 \text{ cm}$.

$$\text{Now, } V_A = (OA) \times \omega_{OA} = \frac{200}{1000} \times \left(\frac{2\pi \times 120}{60} \right) = \frac{4\pi}{5} \text{ m/sec} = 2.5132$$

No. of links = 6.

$$\text{No. of I centers} = \frac{6(6-1)}{2} = 15.$$



$$I_{13} \begin{cases} I_{12}, I_{23} \\ I_{14}, I_{43} \end{cases}$$

$$I_{24} \begin{cases} I_{23}, I_{34} \\ I_{21}, I_{14} \end{cases}$$

$$I_{35} \begin{cases} I_{34}, I_{45} \\ I_{32}, I_{25} \\ I_{31}, I_{15} \end{cases}$$

$$I_{15} \begin{cases} I_{16}, I_{65} \\ I_{14}, I_{45} \end{cases}$$

$$I_{25} \begin{cases} I_{24}, I_{45} \\ I_{12}, I_{15} \end{cases}$$

$$I_{36} \begin{cases} I_{35}, I_{56} \\ I_{31}, I_{16} \\ I_{32}, I_{26} \end{cases}$$

$$I_{26} \begin{cases} I_{25}, I_{56} \\ I_{21}, I_{16} \end{cases}$$

$$I_{46} \begin{cases} I_{45}, I_{56} \\ I_{14}, I_{16} \end{cases}$$

For link AB, link-3.

$$\hookrightarrow I_{13}. \quad \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B} = \omega_3 = \omega_{AB}$$

$$I_{13}A = 3.8 \text{ cm} \\ = 38 \text{ mm} \\ = 0.038 \text{ m}$$

$$\Rightarrow \frac{4\pi/5}{0.038} = \frac{V_B}{1.12} \Rightarrow V_B = \frac{4\pi}{5} \times \frac{1.12}{0.038} = 3.403 \text{ m/sec}$$

$$I_{13}B = 5.5 \text{ cm} \\ = 55 \text{ mm} \\ = 0.055 \text{ m}$$

$$\omega_{AB} = \frac{V_B}{AB} = \frac{3.403}{1.5} = 2.268 \text{ rad/sec}$$

ONLY GRAPH

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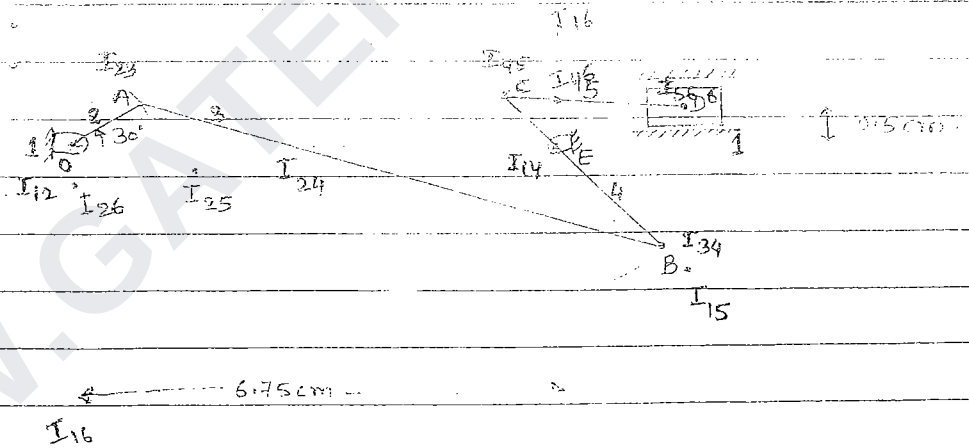
Scale used - 200mm = 1cm

I₁₆

I₁₆ (∞)

I₃₆

I₁₃



For link BC, link-4.

↳

$$I_{4B}, \frac{V_B}{I_{4B}} = \frac{V_C}{I_{4C}} = \omega_4 = \omega_{BC}$$

$$\Rightarrow \frac{3.603}{0.4} = \frac{V_C}{0.2}$$

$$\Rightarrow V_C = 9.0075 \times 0.2 = 1.8015 \text{ m/sec.}$$

$$\omega_4 = \omega_{BC} = 9.0075 \text{ r.p.m. } \int \underline{A_{u1}}$$

For link CD, link-5.

↳

$$\frac{V_C}{I_{5C}} = \frac{V_D}{I_{5D}} = \omega_5 = \omega_{CD}$$

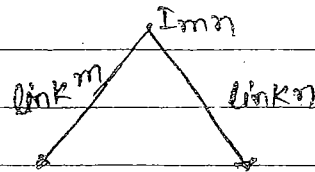
$$\Rightarrow \frac{1.8015}{0.72} = \frac{V_D}{0.46} \quad \begin{aligned} I_{5C} &= 3.6 \text{ cm} \\ &= 720 \text{ mm} \\ &= 0.72 \text{ m} \end{aligned}$$

$$\Rightarrow V_D = 2.502 \times 0.46 = 1.1509 \text{ m/sec. } \int \underline{A_{u1}} \quad \begin{aligned} I_{5D} &= 2.3 \text{ cm} \\ &= 460 \text{ mm} \\ &= 0.46 \text{ m} \end{aligned}$$

$$\omega_{CD} = 2.502 \text{ r.p.m.}$$

11/08/10

Theorem of Angular Velocities:-

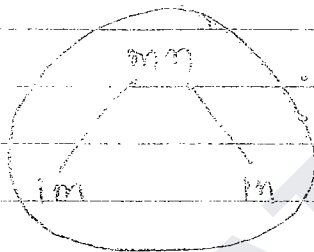


$$V_{Imn} = \omega_m (I_{mn} I_m) = \omega_n (I_{mn} I_n) = V_{Imn}$$

for eg:- for I_{25} ,

$$\omega_2 (I_{25} I_2) = \omega_5 (I_{25} I_5)$$

i.e.



$I_m, I_n \rightarrow$ in a straight line

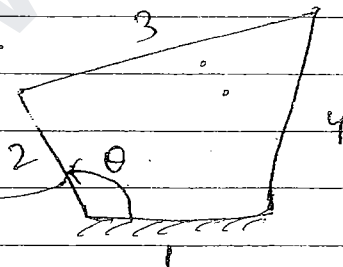
$\rightarrow$ When we sheet on m and I_m and I_n are on same side then the direction of angular velocity is same and when on opposite side then the direction is opposite

Q:- Q:-

$\omega_2 = 2 \text{ rad/sec}$
Clocks

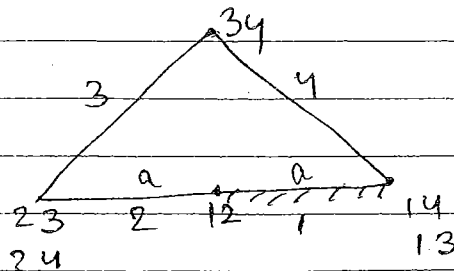
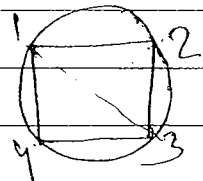
$\omega_3 = ?$

Soln:-



When $\theta = 180^\circ$

calculate:-

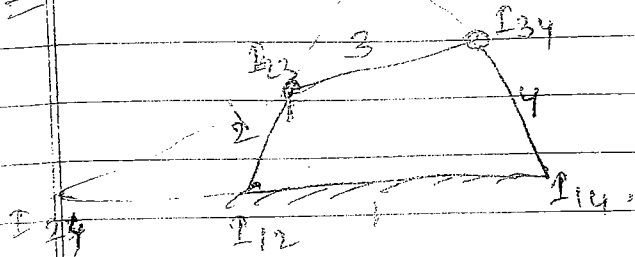


$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$2 (\text{Clock}) = \omega_3 (2 \text{ Clock})$$

$\omega_3 = 1 \text{ rad/sec}$ same side so Clock $\omega_3 =$

Q.1)

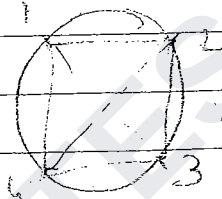
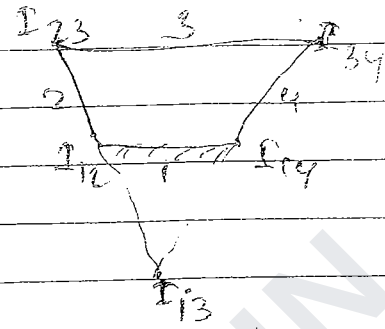


$$\omega_2 = 2 \text{ rad/sec (clockwise)}$$

$$\omega_3 = 3 \text{ rad/sec (—)}$$

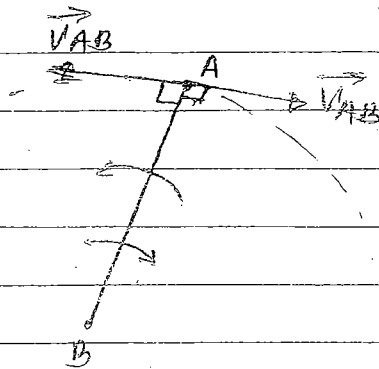
$$\begin{aligned} \vec{\omega}_{23} &= \vec{\omega}_2 - \vec{\omega}_3 \\ &= (+2) - (+3) \end{aligned}$$

Q.2)



$$\begin{aligned} I_{13} &= 32, 21 \\ I_{34} &= 34, 14 \\ I_{24} &= 21, 14 \\ I_{12} &= 23, 37 \end{aligned}$$

Relative velocity Method:-



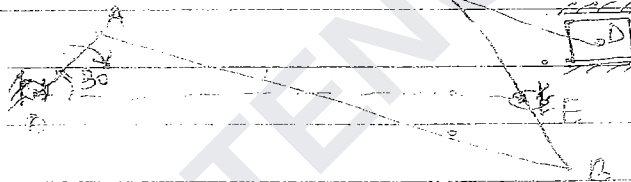
→ Point A will move w.r.t. the point B will be in the direction $\perp$ to the link AB.

Configuration diagram:

Problem 1

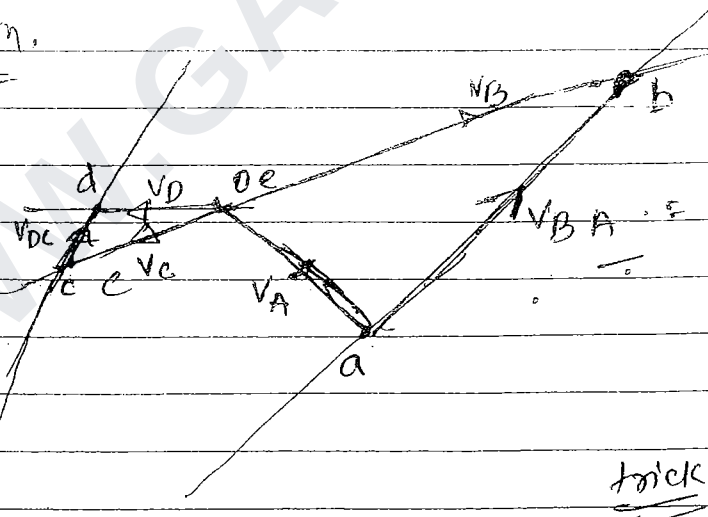
Given,

$$V_A = 4 \text{ m/sec}$$



Soln: Velocity $\rightarrow$ scale used.

Soln.



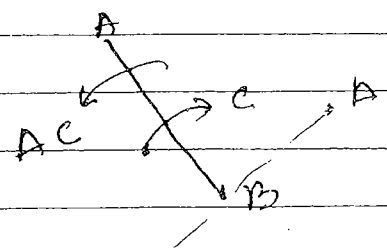
Velocity of B w.r.t. A = Velocity of first w.r.t. last.

Trick:

$$\omega_{AB} = \frac{V_{AB}}{AB}$$

$$\omega_{BC} = \frac{V_{BC}}{BC}$$

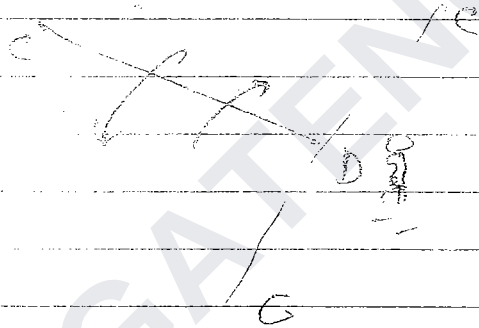
$$\omega_{CD} = \frac{V_{CD}}{CD}$$



for sign check

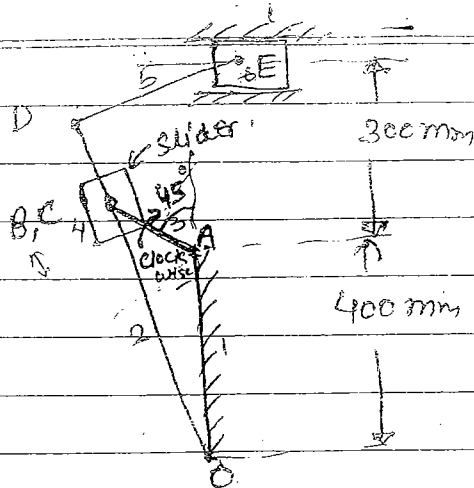
Same sign on opposite side

Point	W.r.t.	Procedure
A	O	line $\perp^r$ to OA
B	A	line $\perp^r$ to BA
B	E	line $\perp^r$ to BE
C	$\frac{bc}{be} = \frac{BC}{BE}$	$\rightarrow$ be??
D	C	line $\perp^r$ to CD
D	Fixed	line $\parallel$ to the motion of slider.



Check with Velocity
diagram Point
is above or below.
and find the
sign.

Q: 1)



$$AB = 150 \text{ mm}$$

$$OD = 700 \text{ mm}$$

$$DE = 200 \text{ mm}$$

Given,

Crank AB $\rightarrow$ 120 rpm (clock)

$$\text{Get, } V_E = ?$$

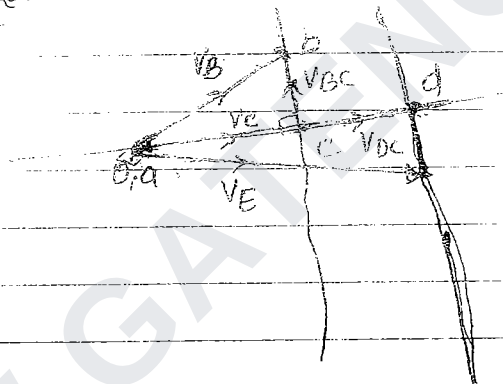
$$W_{OD} = ?$$

Solⁿ:B $\rightarrow$ sliderC $\rightarrow$ Coincident point on slotted bar
OBD. $V_B \rightarrow$ Given

$$V_B = (AB) \cdot \omega_{AB} = \left(\frac{150}{1000} \right) \times \left(\frac{2\pi \times 120}{60} \right) \text{ m/sec}$$

$$= 1.884 \text{ m/sec}$$

Assumed scale


 V_{BC} = sliding velocity
of slider

Point

w.r.t.

procedure

C

O

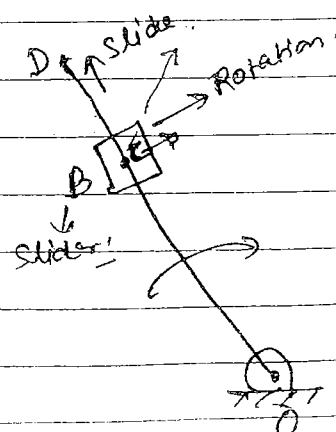
line $\perp$ to OC ✓

C

B

line $\parallel$ to slotted bar OC.

$$W_{OD} = \frac{V_{OD}}{OD}$$



V = 1.884

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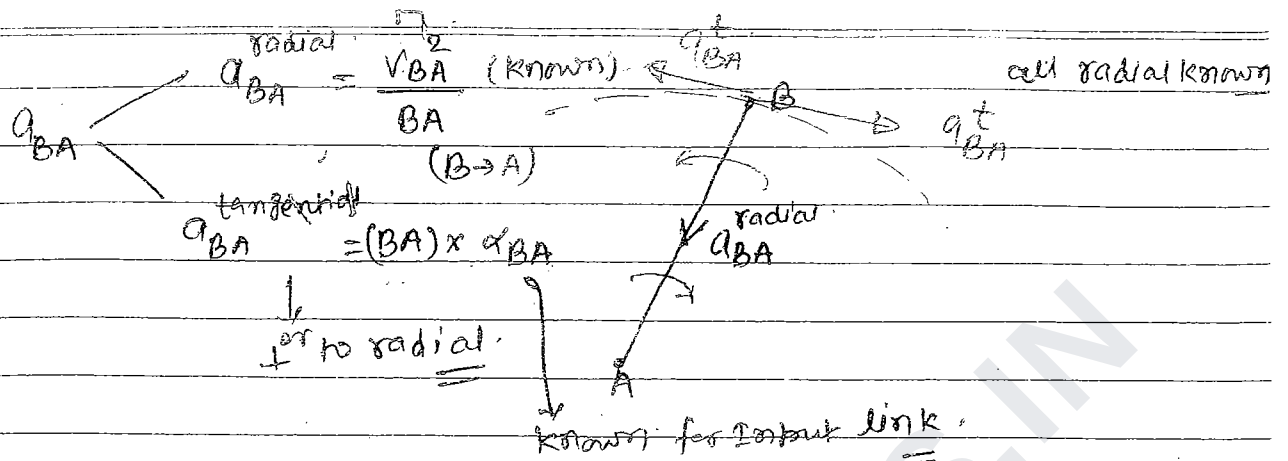
3.

Acceleration Analysis

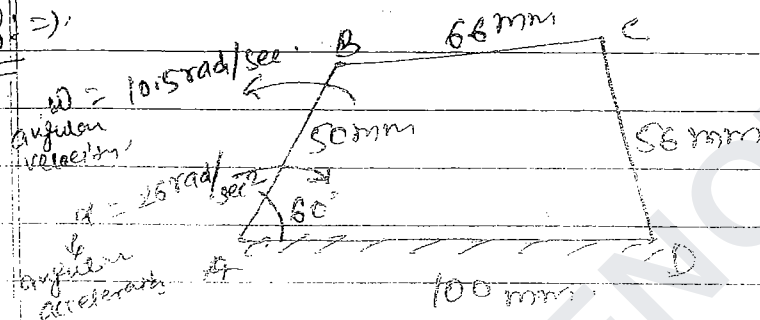
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Q1 =)



Ans: -

for $\alpha_{BC} = ?$ 34 rad/sec

$\alpha_{CD} = ?$ 79 rad/sec

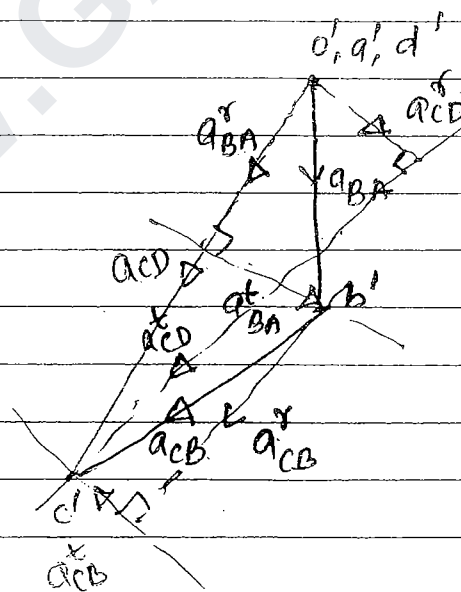
Q2 =)

Acceleration diagram
used scale

Configuration
Velocity diagram
Acceleration diagram

$$\omega_{AB} = \frac{V_{AB}}{AB} \Rightarrow$$

$$\Rightarrow V_{AB} = \omega_{AB} (AB) = 10.5 \times$$



Point

Point

Procedure

B

A

$$a_{BA}^r = \frac{v_{BA}^2}{BA} \text{ (Given)} \rightarrow (B \rightarrow A)$$

$$a_{BA}^t = (BA) \times \alpha_{BA} \text{ (Given)}$$

↓
t^r to radial

C

B

$$a_{CB}^r = \frac{v_{CB}^2}{CB} \text{ (Known)} (C \rightarrow B)$$

$$a_{CB}^t = CB \times \alpha_{CB} \text{ (t^r to radial)}$$

↓
?

C

D

$$a_{CD}^r = \frac{v_{CD}^2}{CD} \rightarrow \text{(Known)} (C \rightarrow D)$$

$$a_{CD}^t = (CD) \times \alpha_{CD} \text{ (t^r to radial)}$$

↓
?

Coriolis acceleration: — Albert Einstein $\rightarrow$ Theory of Relativity

(Real)

Slider which is sliding on rotating object.

$2 \times V_{\text{slider}} \times \omega_{\text{rotating}}$

In Crank & slotted lever Mechanism,

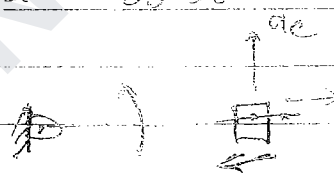
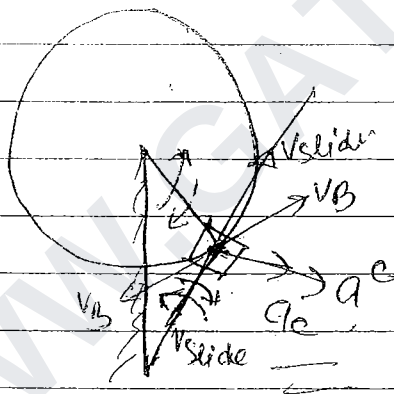
two extremes $\rightarrow \omega = 0$

Top & bottom $\rightarrow V_{\text{sliding}} = 0$

} other
Coriolis seen

(i) Take the sense of rotating

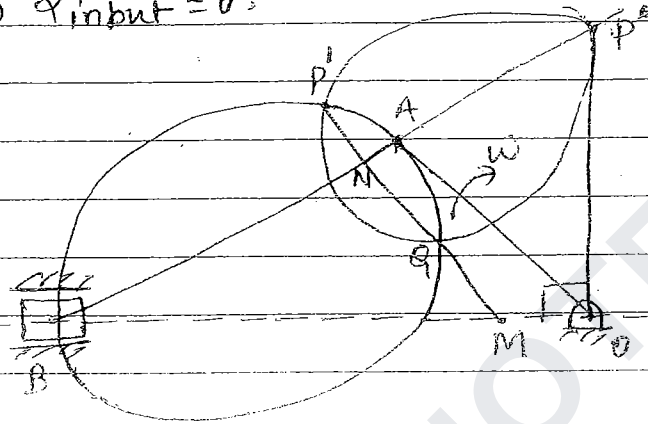
(ii) Rotate the V_{slider} for that sense by 90°



Klein's Construction:-

Limitation:-

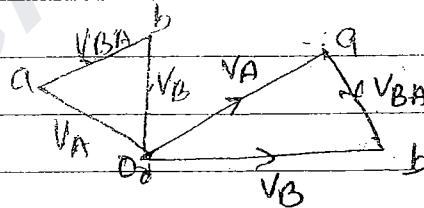
- (i) It can only apply single slider crank mechanism.
- (ii) $\alpha_{input} = 0$.



For ΔOAP ,

Velocity:

$$\frac{V_A}{OA} = \frac{V_B}{OB} = \frac{V_{BA}}{AP} = \omega \rightarrow \text{given.}$$

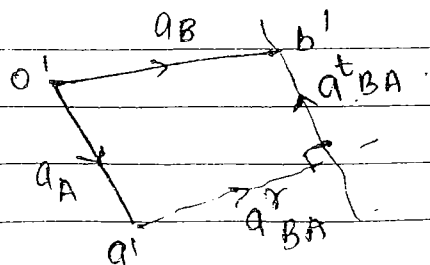


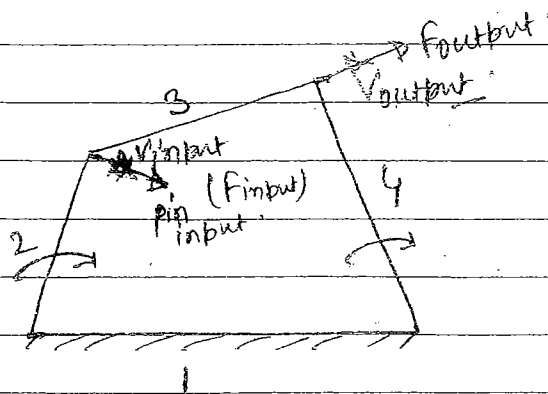
Rotate by 90°

Acceleration: Quadrilateral

OANM,

$$\frac{a_A}{OA} = \frac{a_{BA}}{NM} = \frac{a_B}{MO} = \frac{a_{BA}}{NA} = \omega^2 \quad (\text{given}).$$



Mechanical Advantage of a Mechanism:-

Real, $M.A. = \frac{F_{output}}{F_{input}}$

Ideal:

Conservation of power

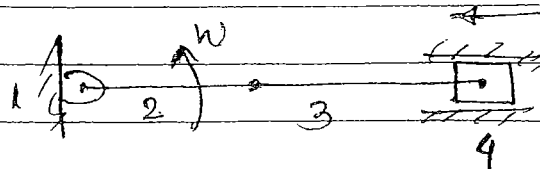
$$P_i = P_f$$

$$\Rightarrow F_{input} \times V_{input} = F_{output} \times V_{output}$$

$$\Rightarrow \frac{F_{output}}{F_{input}} = \frac{V_{input}}{V_{output}}$$

$$(M.A.)_{ideal} = \frac{V_{input}}{V_{output}}$$

$$\therefore (M.A.)_{Real} = \frac{V_{input}}{V_{output}} \times \eta_{mechanism}$$



$$\left. \begin{aligned} \omega_{input} &= \omega \\ \omega_{output} &= 0 \end{aligned} \right\}$$

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4. Gears.

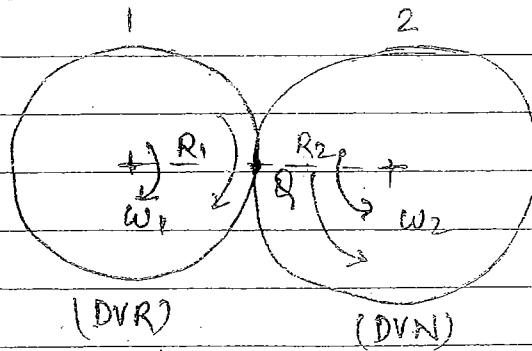
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Q → point of contact



at Q. For Disc 1.

$$V_Q = R_1 w_1$$

at Q. For Disc 2.

$$V_Q = R_2 w_2$$

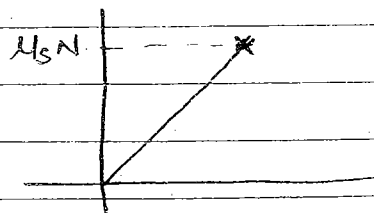
$$\text{If } R_1 w_1 = R_2 w_2$$

For no slipping:-

$$\frac{w_1}{w_2} = \frac{R_2}{R_1} \quad \left\{ \begin{array}{l} = \text{constant} \\ \rightarrow \text{Condition} \end{array} \right.$$

$$0 \leq F_s \leq \mu_s N$$

↓
static friction



For slipping:-

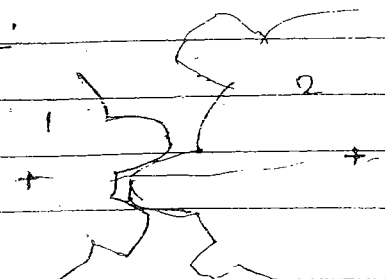
$$R_1 w_1 \neq R_2 w_2$$

$$\frac{w_1}{w_2} \neq \text{constant}$$

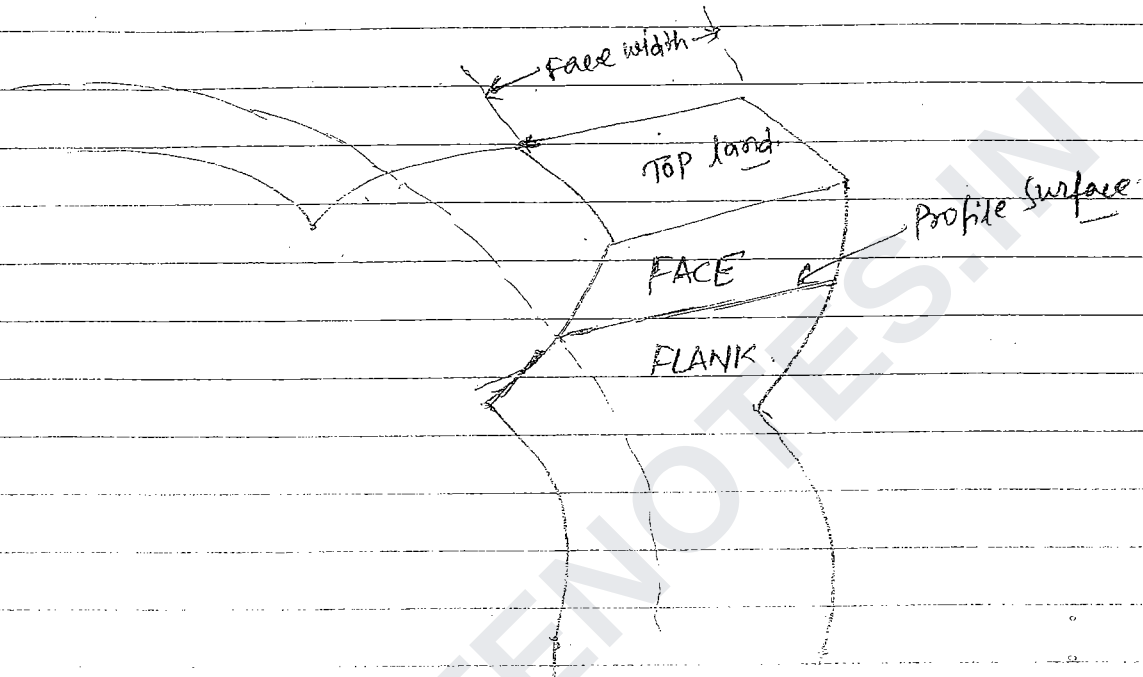
Note

→ All the power transmission device in which slip is possible are known as Negative drives.
with no slip → Positive drives.

eg:- Gears.



Classification of Gears:-



Gear Profile:-

(i) A/c to the position of Axes of shafts:-

PARALLEL SHAFTS:-

Spur Gears

(a) Teeth (Top land) are straight and Parallel to the Axis of rotation.

↓
Spur gear. → Impact stresses.

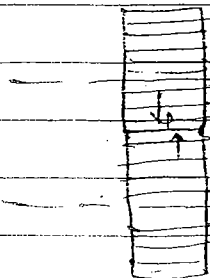
↓

Problem of Impact on Gear profile so almost not in use.

used when very low speed. (1000 RPM).

↓

used in first gear & Reverse gear in Cars & bikes, in gear box.



Axial Thrust = Zero.

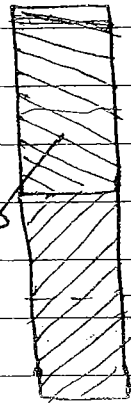
Spur gears

(b) Inclined to Axis (i.e., teeth are straight but inclined to the Axis of the disc) :-

Helical Gears

Axial thrust

F



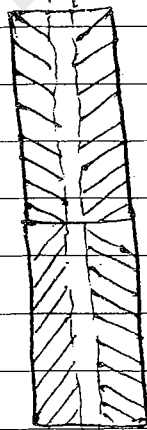
Helical Gear

used is other than
First & Reverse Gear
But ^{some} Axial thrust



Not used in ^{very} high speed
Gears.

Tool run
out



Double Helical
Gears

Balance
No
Axial
thrust

F

F



Double Helical

used is High speed
Gears and No
Axial thrust so
it is used in high speed
engines.

Herringbone Gears

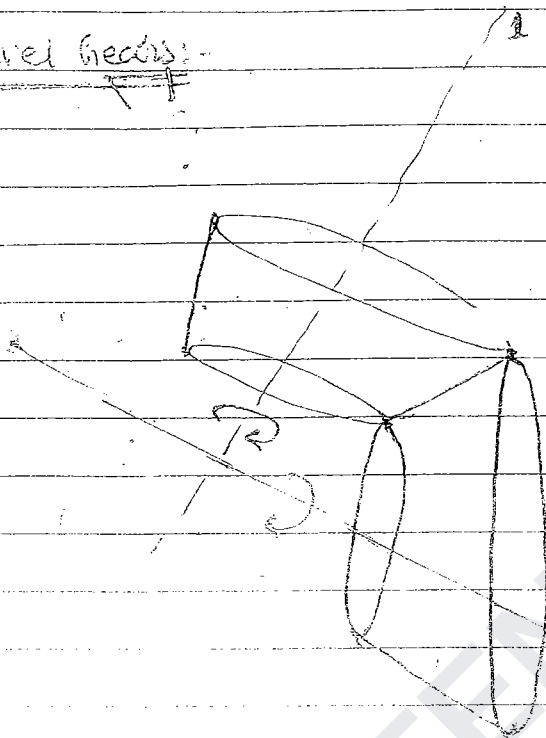
with zero tool run
out

Herringbone
Gears

INTERSECTING SHAFTS:-

② If the shaft axes are intersecting:-

Bevel Gears:-



Bevel Gears

Bevel Gears

Straight Bevel
Gears.



Low speed engine.



Problem of Impact
Stresses.



eg:- Drill Machine.

Helical
Bevel Gears.



Very very High Speed engines.

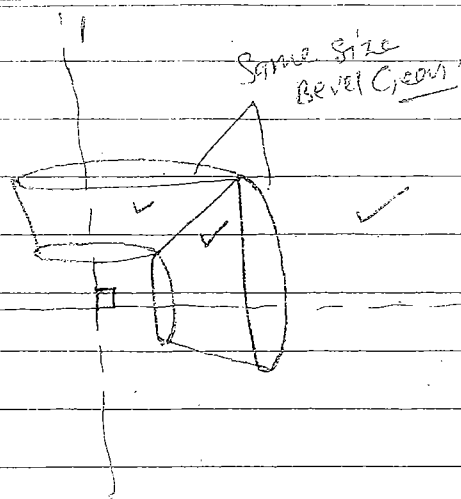


No Impact stresses.



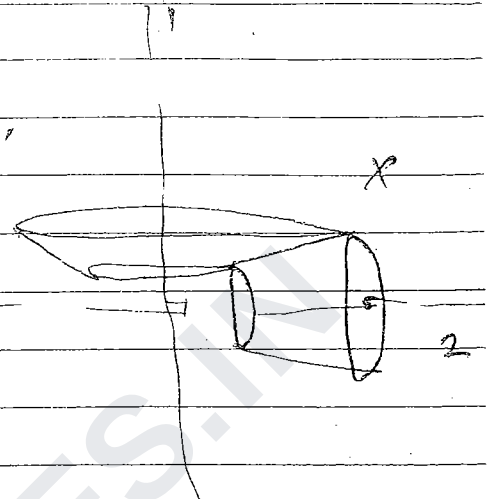
used in differential of the
Automobiles.

(cars, Trucks etc).



Mitre Gears

two bevel Gears having
mutually Perpendicular
Shafts having same size and
same speed.



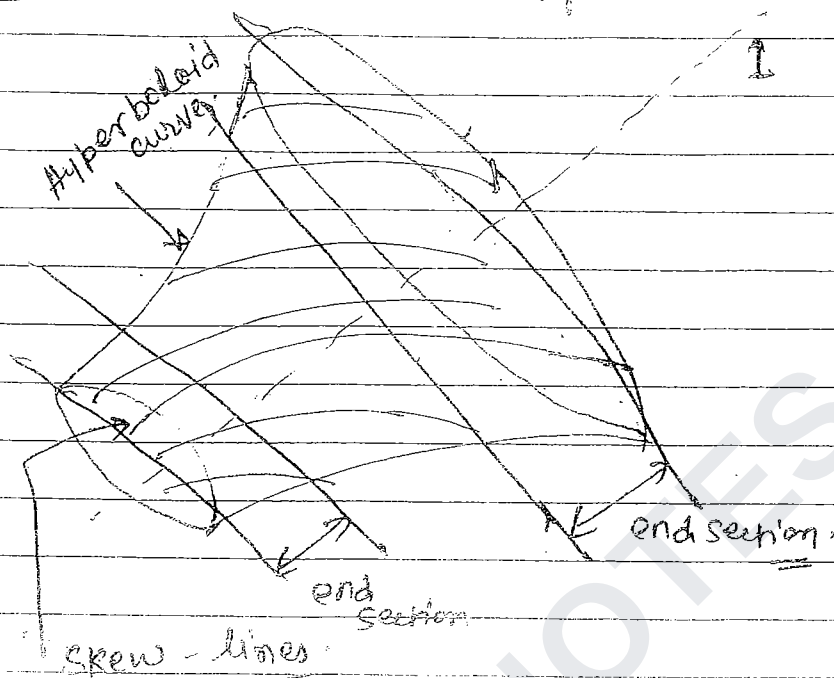
this is not similar
and 1 or
but not the
Mitre Gears.

③ If the shaft axes are neither Parallel nor intersecting: -
(Non coplanar)

⇒ If we want to connect two shafts which are
neither Parallel nor intersecting any kind of
Pure rolling motion is impossible.

⇒ Any kind of motion which is possible between
these shafts is a rolling motion having some
Partial Sliding.

Skew Bevel Gears or Spiral Gears:-



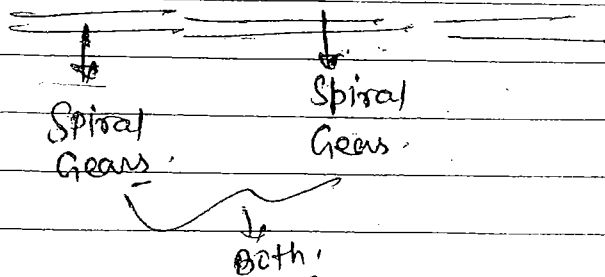
Neither Parallel nor Intersecting

Skew Bevel Gears
or Spiral Gears

Hypoid Gears:-

If the end section of Hyperboloid is used to form the spiral gear such type of spiral gear are known as Hypoid gears.

Worm & worm wheels:-



⇒ Very High reduction ratio (starting 10:1)

⇒ In this type reduction ratio (1000:1)

Starting from, 100:1, 300:1, 1000:1

→ Company name:- AUDI

↓
Torque in muridies

↓
Used in Differential of muridies

Worm



Spiral Gears



Very less dia.



Very high spiral angle



Driver



$F \sin \psi_1$

Worm:-

Worm wheel



Spiral Gears



Very large dia.



Very less spiral angle



Driven

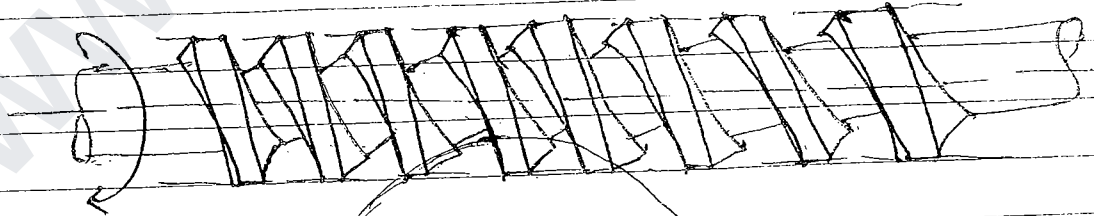


$\Rightarrow F \sin \psi_2 \leq F \sin \psi_1$

So the worm wheel not rotate in worm

sliding force

Worm



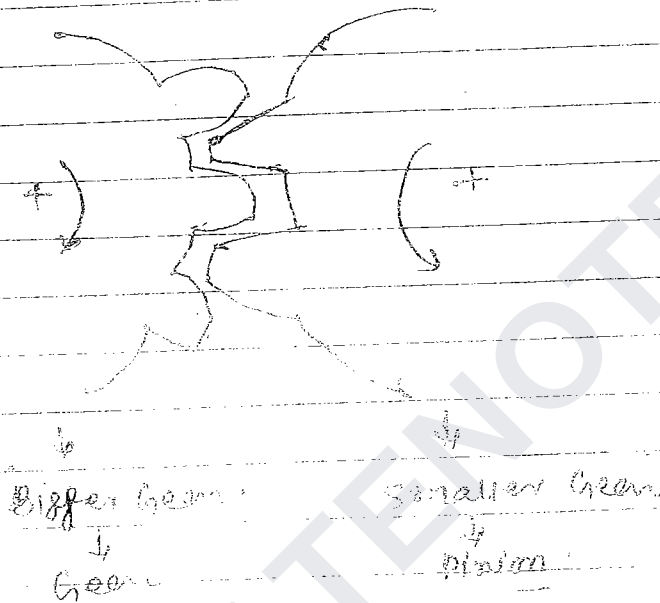
Worm wheel

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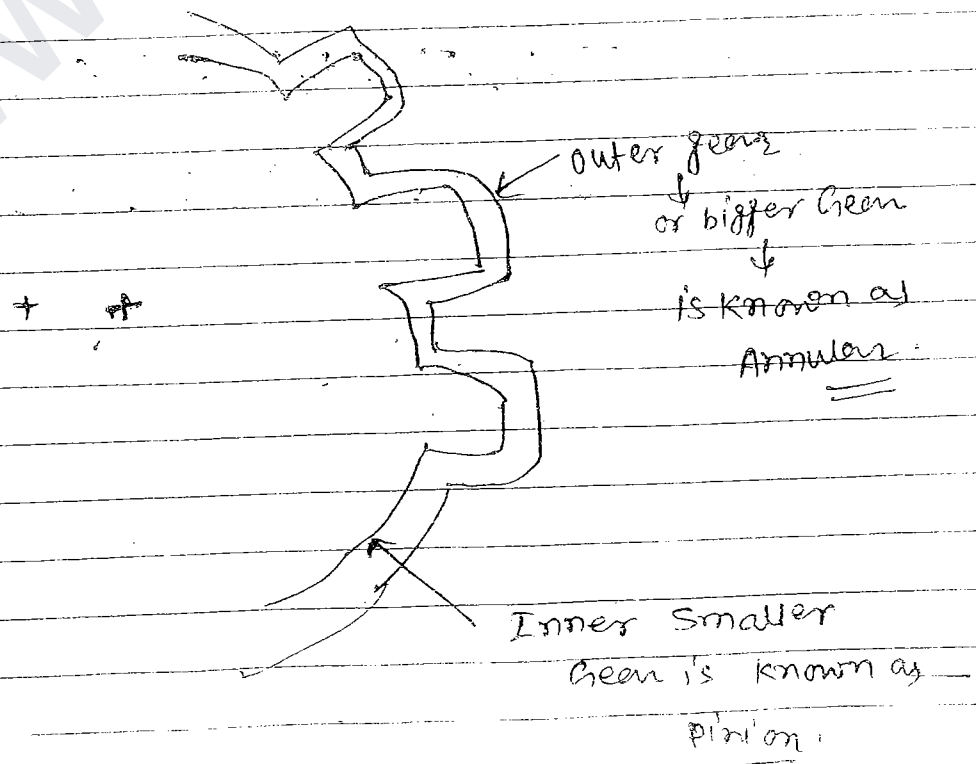
WWW.GATENOTES.IN

② A/c to the type of Gearing classification of Gear:-

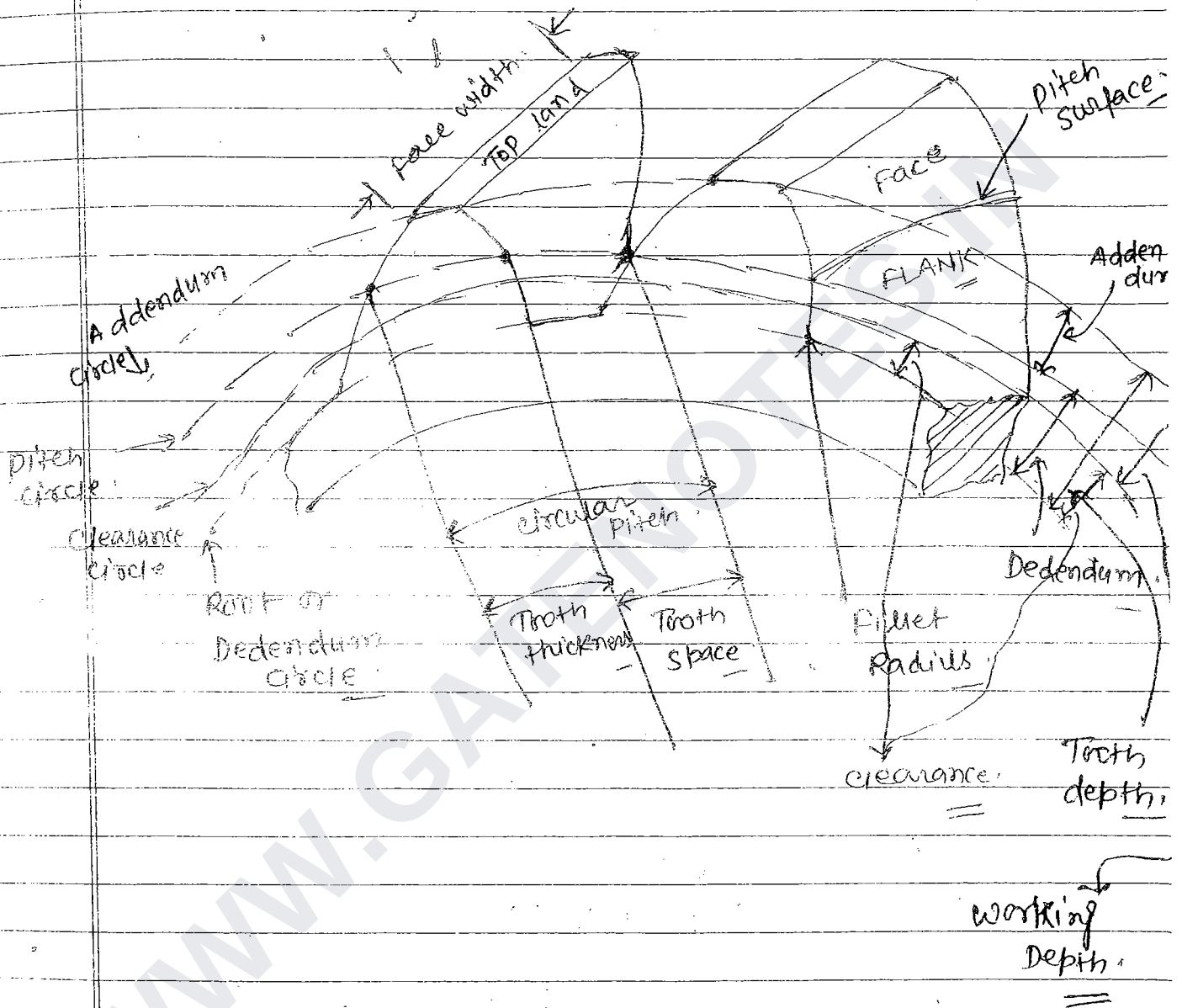
① External Gearing:-



② Internal Gearing:-



Gear Terminology:-



Pitch circle:-

It is an Imaginary circle on which the pure rolling motion gives the same motion as the motion transmitted between the matching Gears.

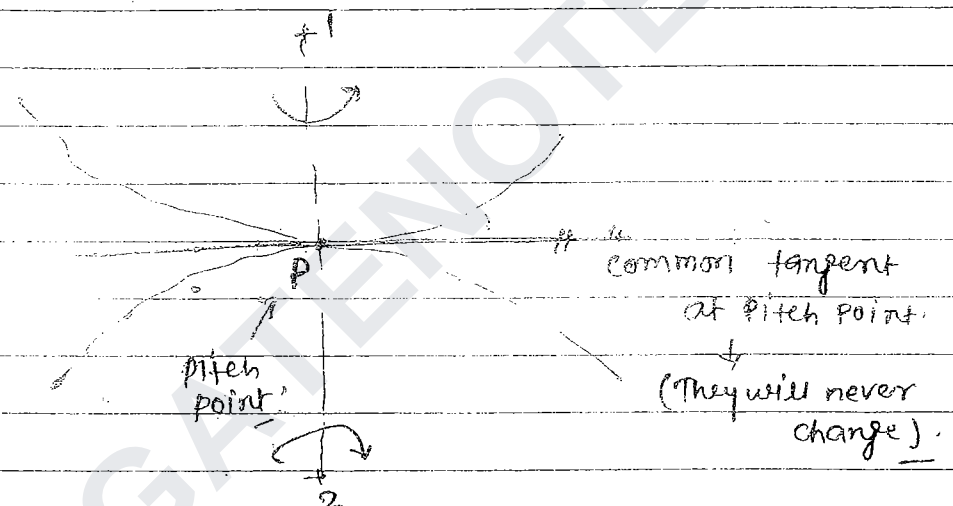
↓
Size of the gear is define by this circle.

↓
It is not the Physical characteristics of the

gear because it is an imaginary circle ~~but~~
~~circle~~ but because of most important circle
it is the one of the most biggest specification
of the gear.

⇒ Size of the gear is specified by Pitch circle dia.

⇒ Two pitch circle Never intersect they will always touch.

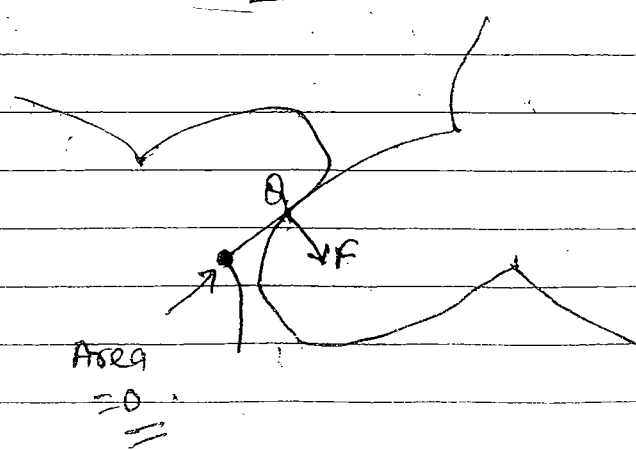


→ Face → part of the pitch surface above it.

→ Flank → part of the pitch surface below it

→ To Avoid the stress concentration fillet radius
can be provide.

$$Area = 0.$$



→ Circular pitch (p_c) = $\frac{\pi D}{T}$

→ (1,2) ~~Mating~~ Mating Gears,

$$p_{c1} = p_{c2}$$

$$\Rightarrow \boxed{\frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}}$$

$$\frac{D}{T} = \text{same}$$

$$\boxed{\text{Module (m)} = \frac{D}{T}} \quad \text{where } D \text{ in mm}$$

→ $\frac{1}{m} = P_d = \text{Diametral Pitch} = \frac{T}{D}$

$$\Rightarrow \boxed{P_c \cdot P_d = \pi}$$

Backlash:-

$$\boxed{\text{Tooth space} - \text{Tooth thickness} = \text{Backlash}}$$

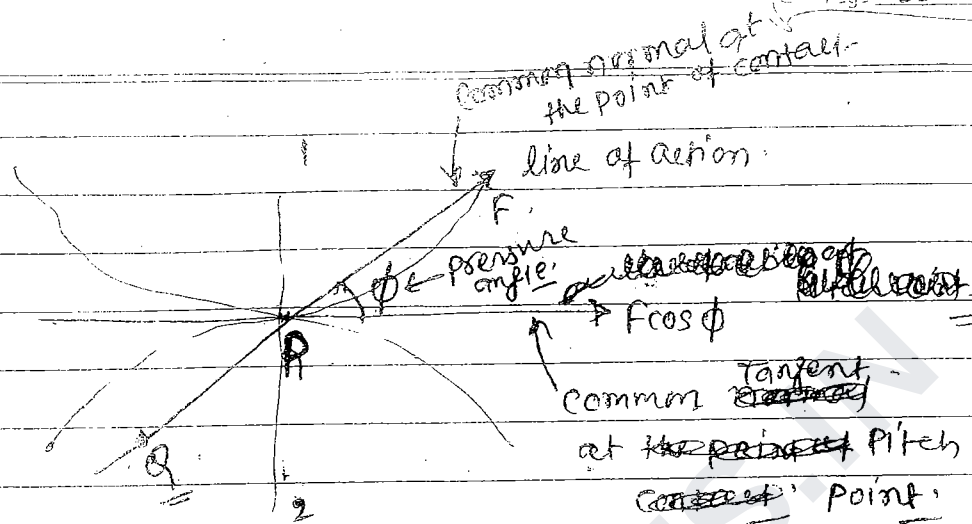
→ To avoid the jamming because of thermal expansion due to excessive sliding we give the clearance sufficient which is known as Backlash.

Pressure angle:- (ϕ),

It is the angle between the pressure line or (line of action or, common normal) and common tangent to the pitch circle at pitch point.

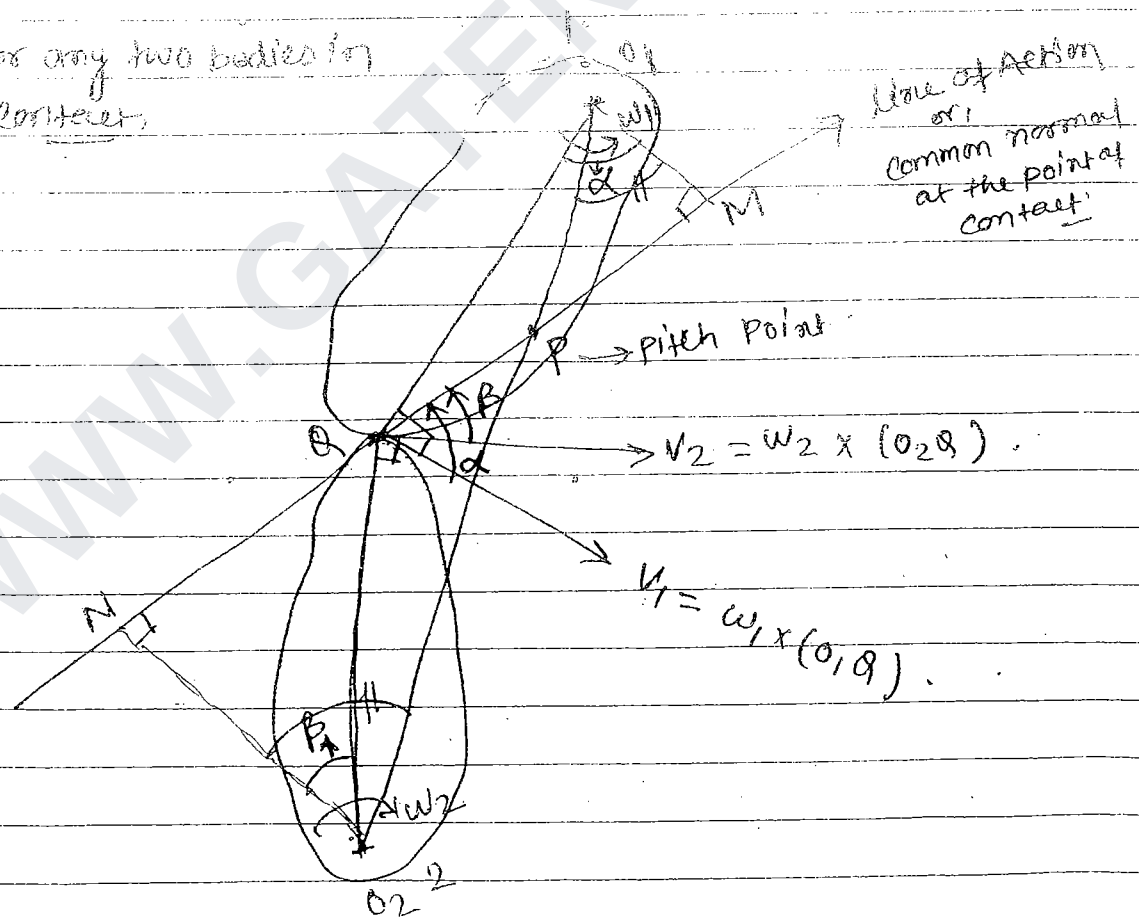
→ For more power transmission lesser pressure angle are used.

Standard pressure angle are 20° & 25° .



Law of Gearing:

For any two bodies in contact,



To be in contact:-

$$V_1 \cos \alpha = V_2 \cos \beta$$

$$\Rightarrow \omega_1 (\cancel{O_1 Q}) \times \frac{O_1 M}{\cancel{O_1 Q}} = \omega_2 (\cancel{O_2 Q}) \frac{O_2 N}{\cancel{O_2 Q}}$$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}} \quad \text{--- (1)}$$

For similar triangle, $O_1 P M$ & $O_2 P N$.

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{PN}{PM} = \frac{O_2 P}{O_1 P}}$$

If these bodies are Gears, then,

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{PN}{PM} = \frac{O_2 P}{O_1 P} = \text{Constant}}$$

$$\Rightarrow \frac{O_2 P}{O_1 P} = \text{const.}$$

O_1 & O_2 are fixed, so O_1, O_2 are Constant.

So Point P should not change,



"Common normal at the point of contact in the two mating gears should always pass through a fixed point on the line joining the centres of the gear and this fixed point is known as pitch point."

Velocity of sliding:-

$$V_{\text{sliding}} = |V_1 \sin \alpha - V_2 \sin \beta|$$

$$= \left| \omega_1 \times (\cancel{O_1Q}) \times \frac{QM}{\cancel{QO_1Q}} - \omega_2 \times (\cancel{O_2Q}) \cdot \frac{QN}{\cancel{QO_2Q}} \right|$$

$$= | \omega_1 (QP + PM) - \omega_2 (PN - QP) |$$

$$= | \omega_1 \cdot QP + \cancel{\omega_1 PM} - \omega_2 PN + \omega_2 QP |$$

$$\left(\frac{\omega_1}{\omega_2} = \frac{PN}{PM} \right)$$

$$\Rightarrow \boxed{V_{\text{sliding}} = (\omega_1 + \omega_2) \cdot QP}$$

For Q at point P, i.e. at pitch point and point at contact will same then $QP = 0$.

$$\therefore (V_{\text{sliding}})_{\text{at pitch point}} = 0.$$

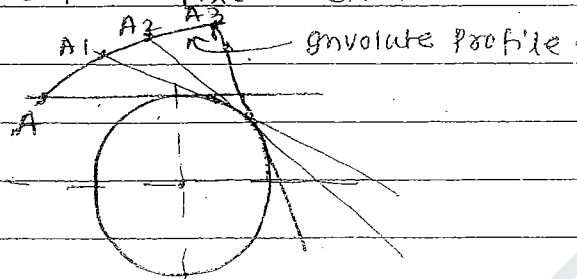
i.e., purely rolling motion.

13/08/10

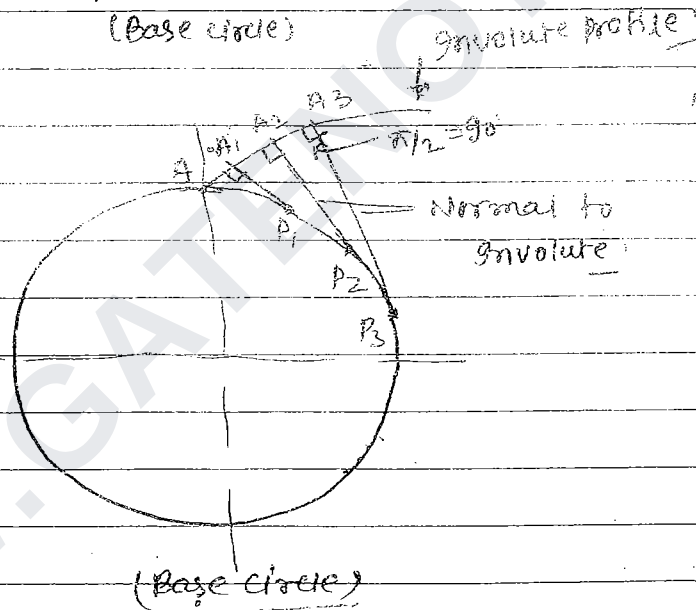
Involute Profile:- (By Nature Conjugate).

(~~Math~~ Blessing in the field of maths)

It is a locus of a point on the line which rolls without slipping on a fixed circle.



fixed circle
(Base circle)



$$\text{Arc } (AP_1) = P_1 A_1$$

$$|| (AP_2) = P_2 A$$

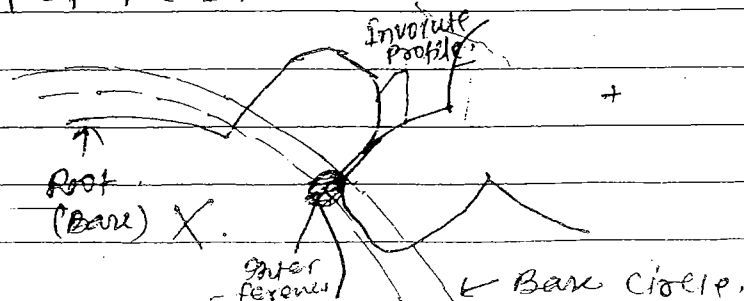
$$|| (AP_3) = P_3 A$$

Normal to
Involute

NOTE:-

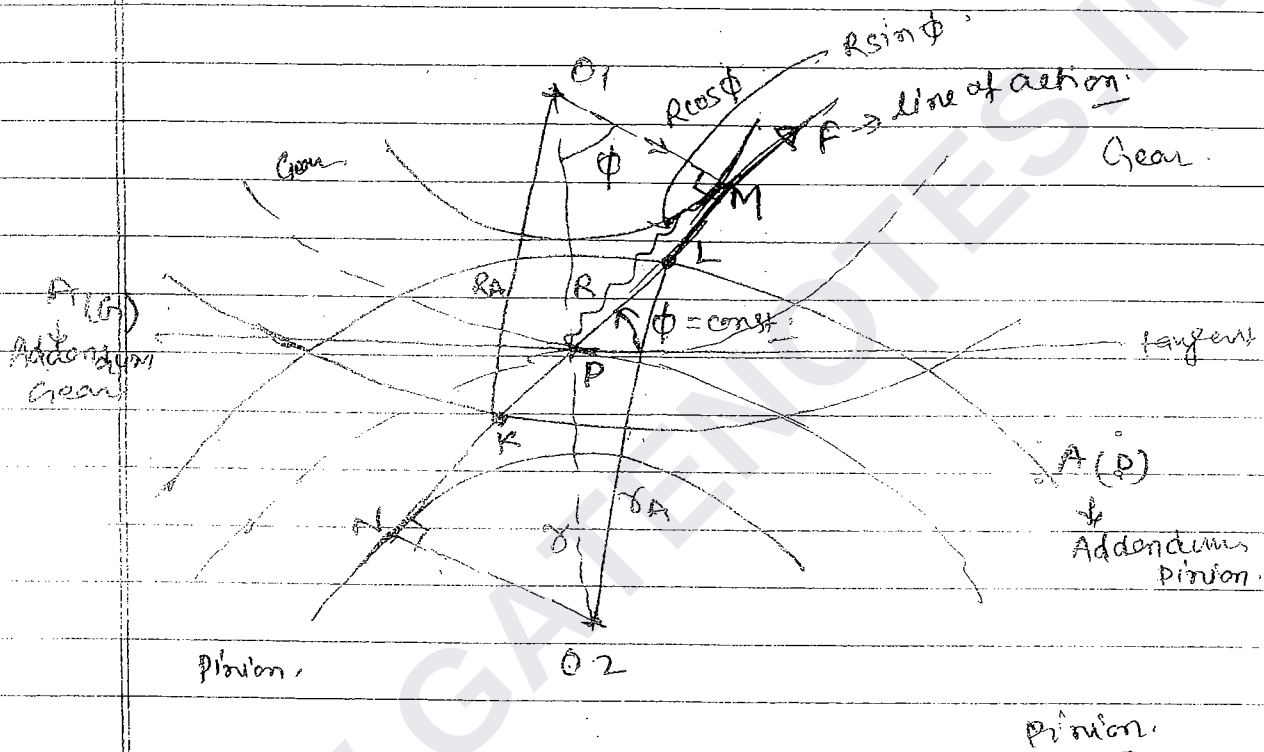
Normal drawn at any point of involute curve will become tangent to its base circle.

→ Base Circle can not be dedendum circle. It is a physical property of the Involute Gear.

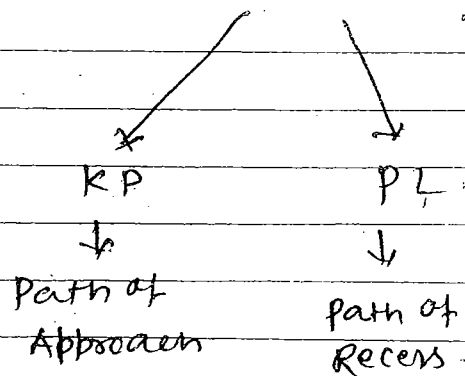


Power transmission between the involute Gears:-

- Pressure angle for involute gear is constant.
- Point of contact is always travel along the same line of action.



$KL = \text{Total travel} = \text{Path of contact.}$



gn Δ o, km:-

$$R_A^2 = R^2 \cos^2 \phi + (KP + R \sin \phi)^2$$

$$\Rightarrow \boxed{KP = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi}$$

$$\boxed{PL = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi}$$

and, Path of Contact = $KP + PL$

Arc of contact:-

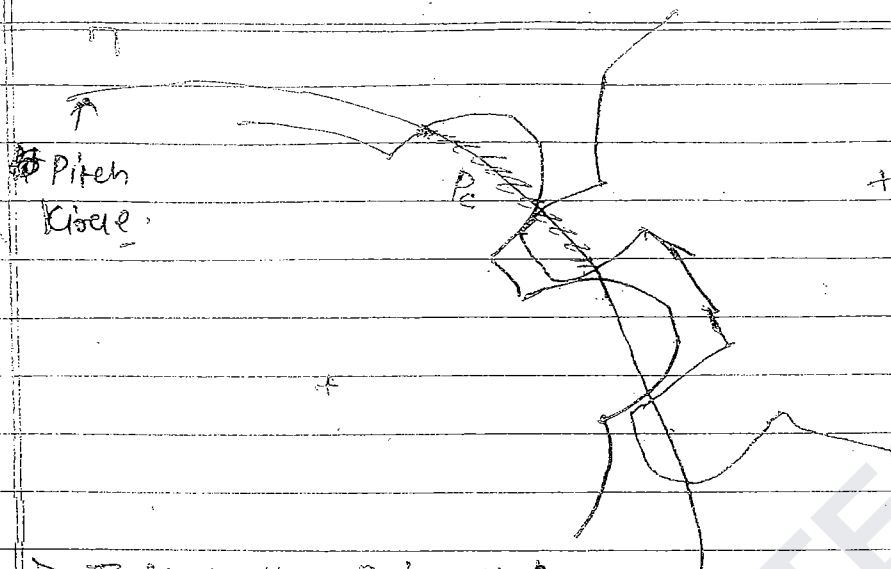
It is ~~the~~ analog of the distance of the Path of Contact but measured along the pitch circle.

$$\text{Arc of approach} = \frac{\text{Path of Approach}}{\cos \phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of recess}}{\cos \phi}$$

$$\text{Arc of Contact} = \frac{\text{Path of Contact}}{\cos \phi}$$

$$\boxed{\text{Contact ratio} = \frac{\text{Arc of Contact}}{p_c \text{ (Circular Pitch)}}$$



⇒ To find the Pair at Engagement by Arc of Contact this is something called contact ratio.
It's value is always greater than 1.

~~Given~~ No. of Teeth = 1201

↓
This fraction teeth ~~contact ratio~~ can not possible, and minimum requirement is 1201 so we can take the higher value. So Ans is ~~1201~~ 13. so we take either 1 or 2

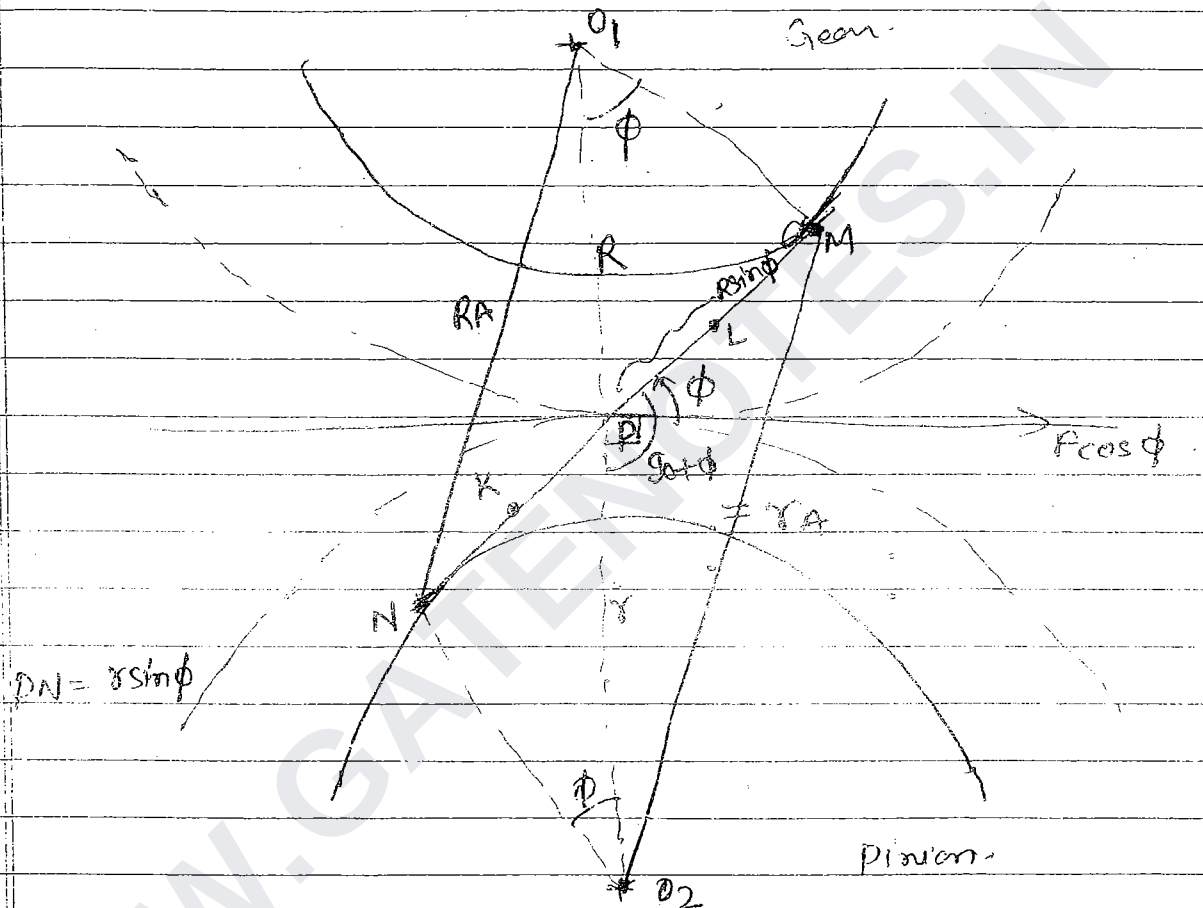
Contact ratio = 1.21

↓
It means the one pair of teeth are in contact and the second pair of teeth are only 21% time in contact.

↓
It is the average value.
↓
So we can assume two pairs are engaged.

Interference:-

The biggest Abuses In the history of Gears.



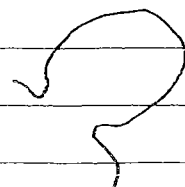
- } (a) $r_A > O_2M$.
 } (b) $r_A > O_1N$.
- M, ? → interference point
 M } (critical point)

Methods of elimination interference:-

① Undercut tooth :-

Generating process.

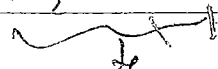
↓
By M/c tool.



Undercut tooth.

② Pressure angle (ϕ):-

$14\frac{1}{2}^\circ$ to 20°



Standard,

$20^\circ \rightarrow$ Best.

Some time $\rightarrow 25^\circ$.

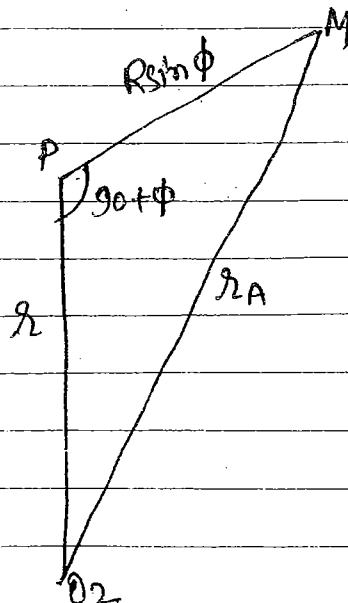
$\rightarrow$ When we increase the pressure angle then the driving force $F \cos \phi$ will decrease due to increase in ϕ . So we can not increase the pressure angle upto a certain limit at $14\frac{1}{2}^\circ$ to 20° . Otherwise the power transmission can be reduces.

$\rightarrow$ By increasing No. of teeth we eliminate the problem of interference.

③ Minimum No. of teeth on Pinion to avoid interference.

$\rightarrow$ By increasing the No. of teeth ~~we~~ then decrease the Addendum circle of Pinion so we avoid the interference.

By Cosine Rule,



$$r_A^2 = r^2 + R^2 \sin^2 \phi - 2r(R \sin \phi) \cdot \cos(90^\circ + \phi)$$

$$= r^2 + (R^2 + 2rR) \sin^2 \phi$$

$$r_A = r \sqrt{1 + \frac{R}{r} \left(\frac{R}{r} + 2 \right) \sin^2 \phi}$$

$$\Rightarrow r_A = r \sqrt{1 + G(G+2) \sin^2 \phi}$$

$$\text{where } G = \text{Gear ratio} = \frac{R}{r} = \frac{T}{t}$$

$$\begin{aligned} \text{Addendum of pinion} &= r \left[\sqrt{1 + G(G+2) \sin^2 \phi} - 1 \right] \\ &= \frac{m t_{\min}}{2} \left[\sqrt{1 + G(G+2) \sin^2 \phi} - 1 \right] \end{aligned}$$

where,

A_p = Fractional Addendum of Pinion for one module in order to avoid interference.

module = m .

$$\text{Addendum of Pinion} = m \times A_p \quad \text{--- (2)}$$

For (1) and (2), will be same so,

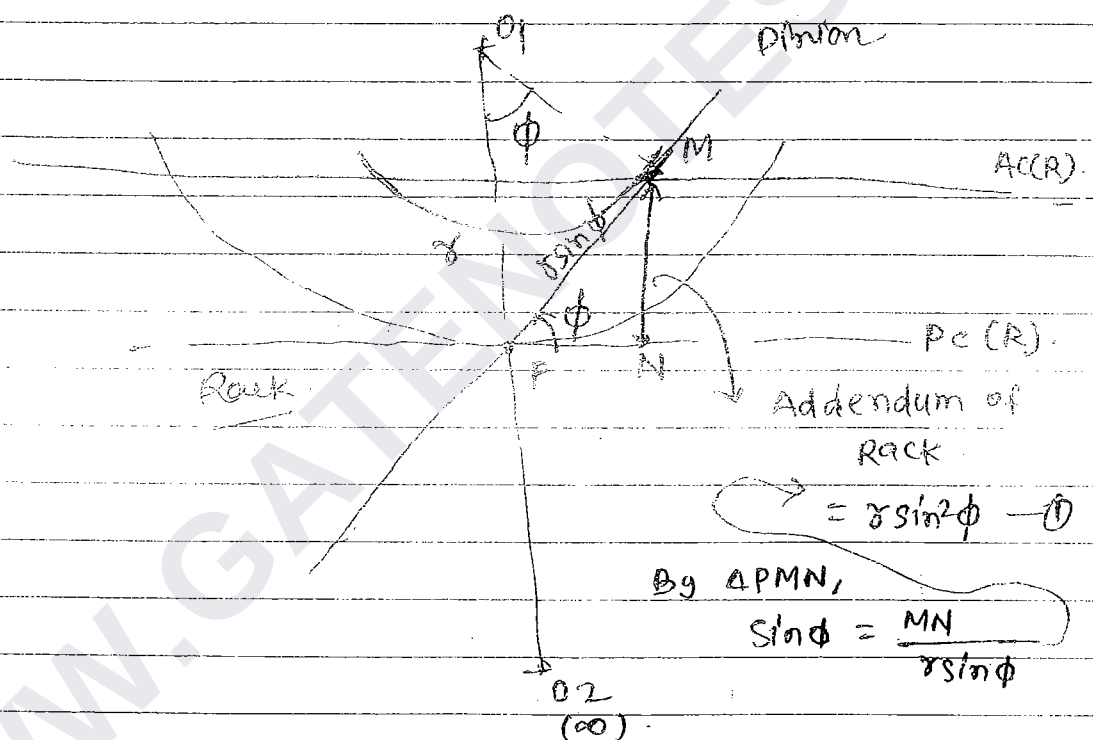
$$\cancel{m} A_p = \cancel{m} \times \frac{t_{\min}}{2} \left[\sqrt{1 + G(G+2) \sin^2 \phi} - 1 \right]$$

$$\Rightarrow t_{\min} = \frac{2 A_p}{\sqrt{1 + G(G+2) \sin^2 \phi} - 1}$$

$$\Rightarrow T_{\min} = \frac{2 A_R}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$$



Minimum No. of teeth on Pinion to avoid interference for involute Rack & Pinion Arrangement:



Where A_R = Fractional Addendum of Rack for one module to avoid interference.

module = m .

$$\text{Addendum of Rack} = m \cdot A_R \quad \text{--- (2)}$$

From (1) and (2),

$$m A_R = r \sin^2 \phi$$

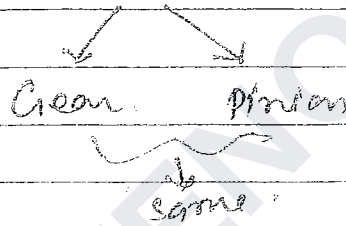
$$2) m \cdot AR = \frac{m \cdot t_{min}}{2} \times \sin^2 \phi$$

$$\Rightarrow t_{min} = \frac{2AR}{\sin^2 \phi}$$

NOTE:- Check for interference in No. of teeth:-

① To find t_{min} ?

When Addendum

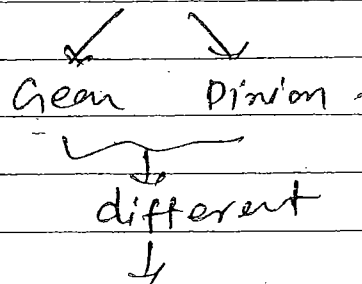


Then we check for Gear only,
and calculate $T_{min} = ?$

$$\text{Then, By, } G = \frac{T_{min,1}}{t_{min}}$$

and find $t_{min,1} = ?$ By above Relation

② When Addendum



Then we check for both Gear &
pinion for No. of teeth to avoid
interference.

IES 2009

Q.1) (3)

$$t_{min} = ?$$

$$\text{Addendum} = 0.84 \text{ mm} = \eta A_P = \eta A_G$$

$$A_P = A_G = 0.84$$

$$G = 3$$

$$\cos \phi = 0.95$$

$$\sin \phi = ?$$

$$T_{min} = ?$$

$$t_{min} = \frac{T_{min}}{G}$$

IES 2007

Q.2) (3)

$$G = 3$$

$$\text{Addendum} = 1 \text{ mm} = \eta A_P = \eta A_G$$

$$0.777$$

$$A_P = A_G = 1$$

$$\phi = 20^\circ$$

$$T_{min} = \frac{2 A_G}{\left\{ \sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1 \right\}} = 44.98 \approx 45$$

$$t_{min} = \frac{45}{3} = 15$$

New:

$$t_{min} = 15 - 3 = 12$$

$$T_{min} = 12 \times 3 = 36$$

$$26 = \frac{2 A_G}{\left\{ \sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1 \right\}}$$

$$\Rightarrow A_G = 0.8$$

3

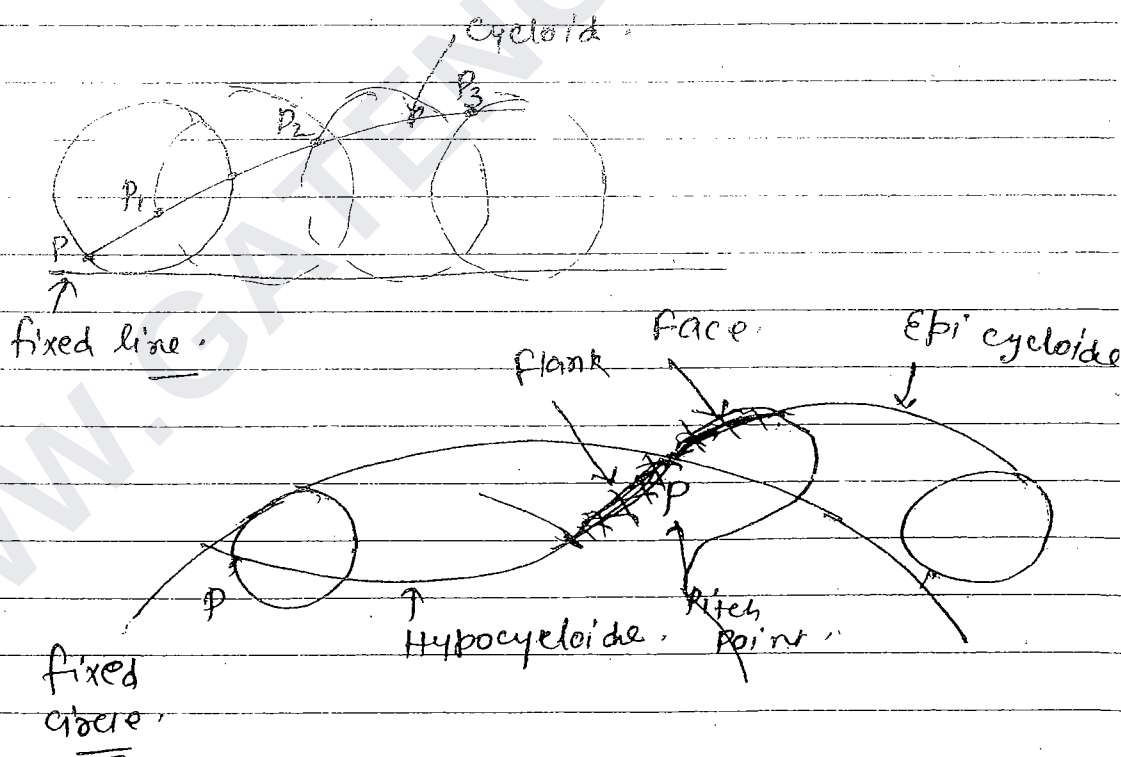
(2)

Cycloidal Profile:-

It is the locus of a point on the circumference of the circle which rolls without slipping ~~along~~ on a fixed straight line.

Advantage:-

- Absence of interference.
- Flank ~~are~~ are wider so higher strength.
- life is more due to less wear & smooth running.



Disadvantage:-

- Pressure angle is not constant.

It is maximum at the beginning of engagement which reduces to zero at the pitch point and again becoming maximum at the end of engagement. Which is something non-smooth (less smooth) running of the gear as compare to involute gear.

Note - $\Rightarrow$ Due to the vibration in cycloidal gear ~~gear~~ velocity ratio change due to their change of central distance. So they are not used in general.

But in case of involute gear having vibration their central distance also varies but the base circle of the profile does not change so its velocity ratio remains constant and this is the very big advantage of involute gear over the cycloidal gear. So they can be used in practical life.

16/08/20

5. Gear Trains:-

classmate

Date 16/08/20

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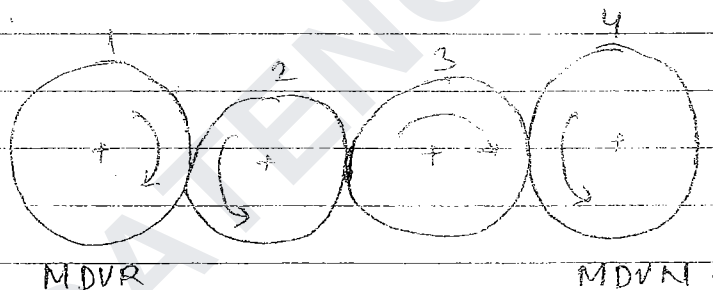
- Combination of Gears.
- Need → Large central distance.
- Main Driver — Main driven

$$\text{Speed ratio} = \frac{\text{Main Driver (angular speed)}}{\text{Main Driven (angular speed)}}$$

$$\text{Train value (T.V.)} = \frac{1}{\text{speed ratio}}$$

Simple Gear Trains:-

- Every shaft is only one gear in use is known as simple gear train.



$$\text{for 1-2, } \frac{W_1}{W_2} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

$$\text{2-3, } \frac{W_2}{W_3} = \frac{T_3}{T_2} \quad \text{--- (2)}$$

$$\text{3-4, } \frac{W_3}{W_4} = \frac{T_4}{T_3} \quad \text{--- (3)}$$

from, (1) × (2) × (3),

$$\boxed{\frac{W_1}{W_4} = \frac{T_4}{T_1}}$$

- Gts a worst gear train have possible intermediate gears are not contribute in velocity ratio, so these intermediate gears are known as Idler Gears.

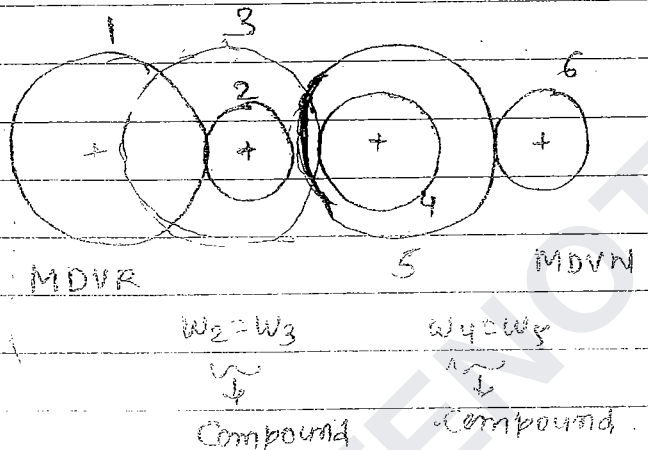
No. of Idler odd → same direction

" " " even → opposite direction

→ modules of all gears should be same. performance is poor.

Compound gear train:-

- If any intermediate shafts having more than one gears in use then the train is compound gear train.
- Such a gears on the same shaft are known as Compound Gears.



$$\left. \begin{array}{l} m_1 = m_2 \\ m_3 = m_4 \\ m_5 = m_6 \end{array} \right\} \rightarrow \text{for meshing Gears.}$$

$$\frac{1-2}{\frac{w_1}{w_2}} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

2, 4, 6
↓
DVN.

$$\frac{3-4}{\frac{w_3}{w_4}} = \frac{T_4}{T_3} \quad \text{--- (2)}$$

1, 3, 5
↓
DVR.

$$\frac{5-6}{\frac{w_5}{w_6}} = \frac{T_6}{T_5} \quad \text{--- (3)}$$

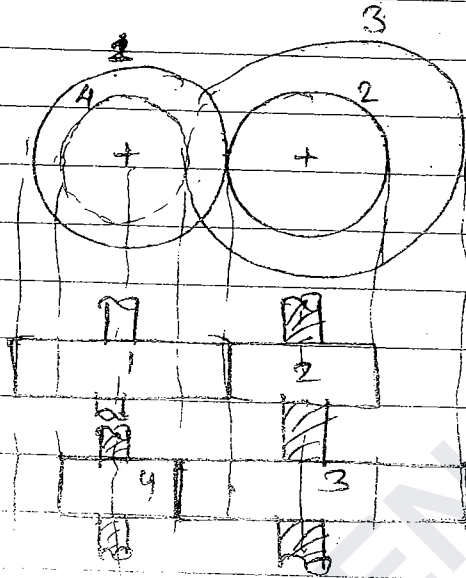
From ① x ② x ③,

$$\frac{w_1}{w_2} \times \frac{w_3}{w_4} \times \frac{w_5}{w_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\Rightarrow \frac{w_1}{w_6} = \frac{\text{Product of No. of teeth on DVN's}}{\text{Product of No. of Teeth on DVR's}}$$

* Reverted Gear trains:-

It's a compound Gear train in which input & output shafts are co-axial.



2-3 $\rightarrow$ compound.

1 $\rightarrow$ MDVR

4 $\rightarrow$ MDVN.

$$m_1 = m_2 = m$$

$$m_3 = m_4 = m'$$

DVR: - 1, 3

DVN: - 2, 4

$$\frac{\omega_1}{\omega_4} = \left(\frac{T_2 \times T_4}{T_1 \times T_3} \right)$$

Now, $(r_1 + r_2 = r_3 + r_4)$

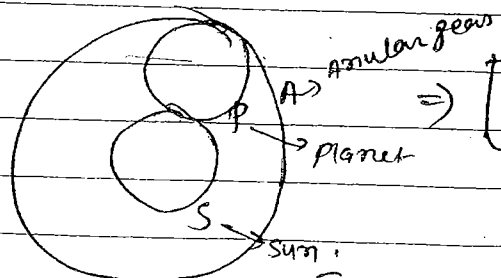
$$2) \frac{m T_1}{2} + \frac{m T_2}{2} = \frac{m' T_3}{2} + \frac{m' T_4}{2}$$

$$\Rightarrow m(T_1 + T_2) = m'(T_3 + T_4)$$

If all the gears are having same module,

$$T_1 + T_2 = T_3 + T_4$$

eg:-



Sun & Planet Gears

$\Rightarrow$

$$T_A = (T_S + 2T_P)$$

→ when not given then pinion is always the driver.

IES-2006

Q. 3)

$$T_1 = T_3 = 24$$

$$T_2 = ?$$

$$m = 2$$

$$T_4 = ?$$

$$m' = 3$$

$$\omega_4 = \frac{1}{12} \omega_1$$

$$\frac{\omega_1}{\omega_4} = 12$$

$$\Rightarrow \frac{\omega_1}{\omega_4} = 12 = \frac{T_2 \times T_4}{T_1 \times T_3} = \frac{T_2 \times T_4}{24 \times 24} \quad (1)$$

$$T_2 \times T_4 = 8912 \quad (1) \quad T_2 = \frac{8912}{T_4}$$

$$\Rightarrow 2(T_1 + T_2) = 3(T_3 + T_4)$$

$$\Rightarrow 2(24 + T_2) = 3(24 + T_4)$$

$$\Rightarrow 48 + 2T_2 = 72 + 3T_4 \quad (2)$$

$$\Rightarrow 2T_2 - 3T_4 = 24 \quad (2)$$

$$T_2 = 108$$

$$T_4 = 64$$

Ans

$$(\omega_1 + \omega_2) = \frac{m'(T_1 + T_2)}{2}$$

$$= \frac{2}{2} \times (T_1 + T_2) = (T_1 + T_2)$$

$$= 24 + 108 = 132 \text{ mm Ans}$$

$$\Rightarrow 2 \times \frac{8912}{T_4} - 3T_4 = 24$$

$$\Rightarrow 3T_4^2 + 24T_4 - 13824 = 0$$

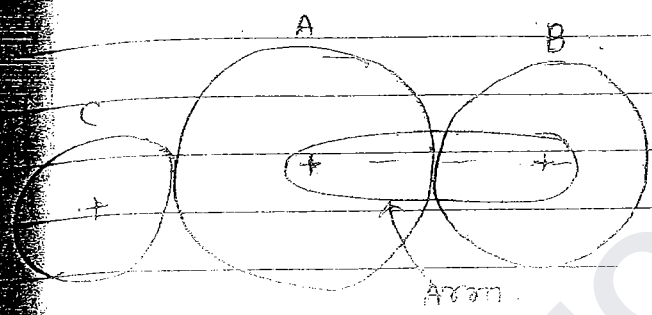
$$\Rightarrow T_4 = 64$$

$$T_2 = \frac{8912}{64} = 108$$

Epicyclic Gear train: - (Best)

↓
Nature
Epi cyclic
↓
Axis Rotation

↓
Rotation of Axis.



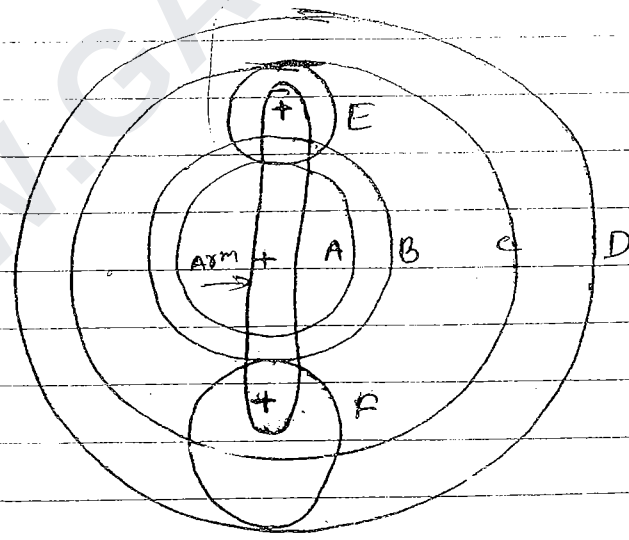
① Arm Fixed

↓
Not Epicyclic.

② Arm rotate.

↓
Epicyclic

③



A-B → compound

$$T_A = 20$$

$$T_B = 30$$

$$T_E = T_F = 10$$

(All gears are having same module)

Find:- If $N_D = 20$

$$N_{arm} = 200 \text{ rpm}$$

$$N = ?$$

$$=$$

→ +ve } Assume
 ↺ -ve } Assume

	Motion	Arm	A/B ($\frac{20}{30}$) Compound	E ($\frac{10}{10}$)	C ($\frac{40}{10}$)	F ($\frac{10}{10}$)	D ($\frac{100}{50}$)
1.	Arm fixed. Given A rotates +x revolution.	0	+x	$-x \left(\frac{20}{10} \right)$	$-x \left(\frac{20}{10} \right) \times \left(\frac{10}{40} \right)$	$-x \times \frac{30}{10}$	$-x \times \frac{30}{10} \times \left(\frac{10}{5} \right)$
2.	Arm free.	+y	y+x	y-2x	$y - \frac{x}{2}$	y-3x	$y - \frac{3x}{5}$

Now, $y - \frac{3x}{5} = 0$ — (1)

$y = +200$ — (2)

From (1) & (2),

$$200 = \frac{3x}{5}$$

$$\Rightarrow x = \frac{1000}{3} = 333.33$$

$$N_A = y + x = 200 + 333.33 = 533.33$$

$$N_E = y - 2x = 200 - 2 \times 333.33 = -466.66$$

= 466.66 (anti-clockwise)

$$N_C = y - \frac{x}{2} = 200 - \frac{333.33}{2} = 33.33$$

$$N_F = y - 3x = 200 - 3 \times 333.33$$

$$= 200 - 1000 = -800$$

= 800 (anti-clockwise)

$$N_D = 0$$

3

$$T_C = T_A + 2T_E$$

$$= 20 + 2 \times 10$$

$$= 40$$

$$T_D = T_B + 2T_F$$

$$= 30 + 2 \times 10$$

$$= 50$$

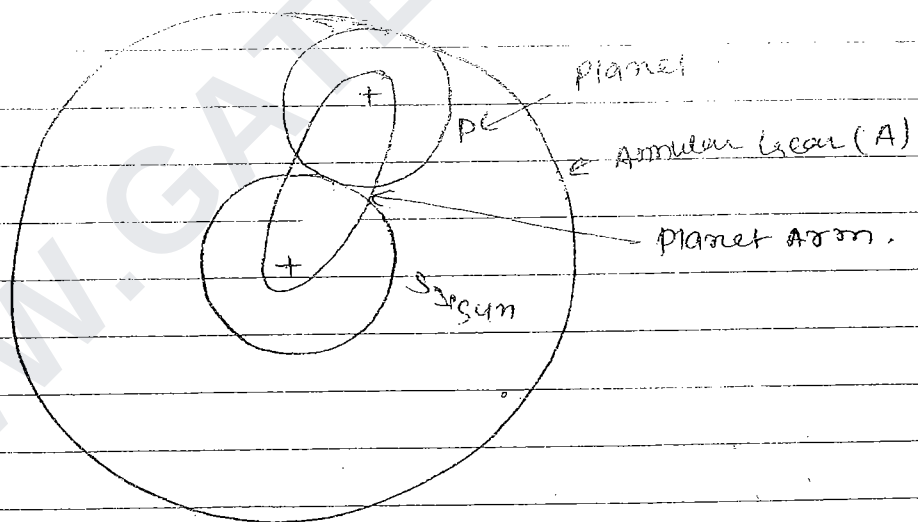
$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\Rightarrow N_2 = \frac{N_1 T_1}{T_2}$$



1 & 2 → Internal Gear

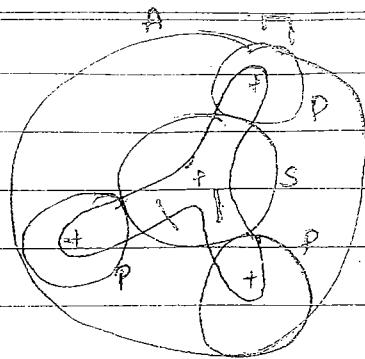
Planetary Gear trains: -



Driver $\left\{ \begin{array}{l} \text{Sun} \rightarrow \text{fixed} \rightarrow \checkmark \\ \text{Annular} \rightarrow \checkmark \rightarrow \text{fixed} \end{array} \right.$

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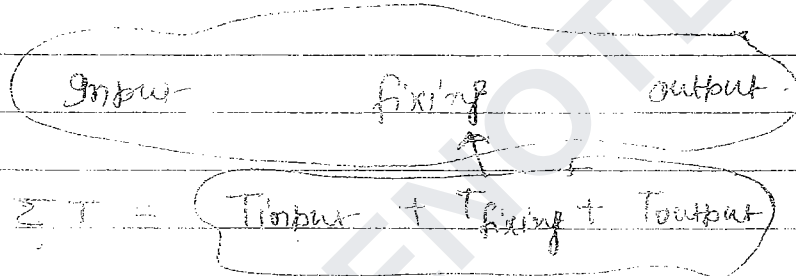
Q. 2) (9)



$$N_A = 0$$

$$\frac{N_s}{N_{arm}} = \frac{5}{1}$$

Fixing torque in a Epicyclic Gear train:-



$$\Sigma T = T_{input} + T_{fixing} + T_{output} = 0$$

$$\Sigma T = 0$$

↓ Reason:-

$$T = I \alpha$$

↓

$$\alpha = 0$$

↓

gears having constant velocity ratio

$$T_{fixing} = -(T_{input} + T_{output}) \quad \text{--- (1)}$$

$$E_f = E_i \quad \text{Energy Conservation}$$

$$E_f - E_i = 0$$

$$\Rightarrow T_{input} \cdot \omega_{input} + T_{output} \cdot \omega_{output} = 0 \quad \text{--- (2)}$$

Find fixing torque

6 GOVERNORS

CLASSMATE

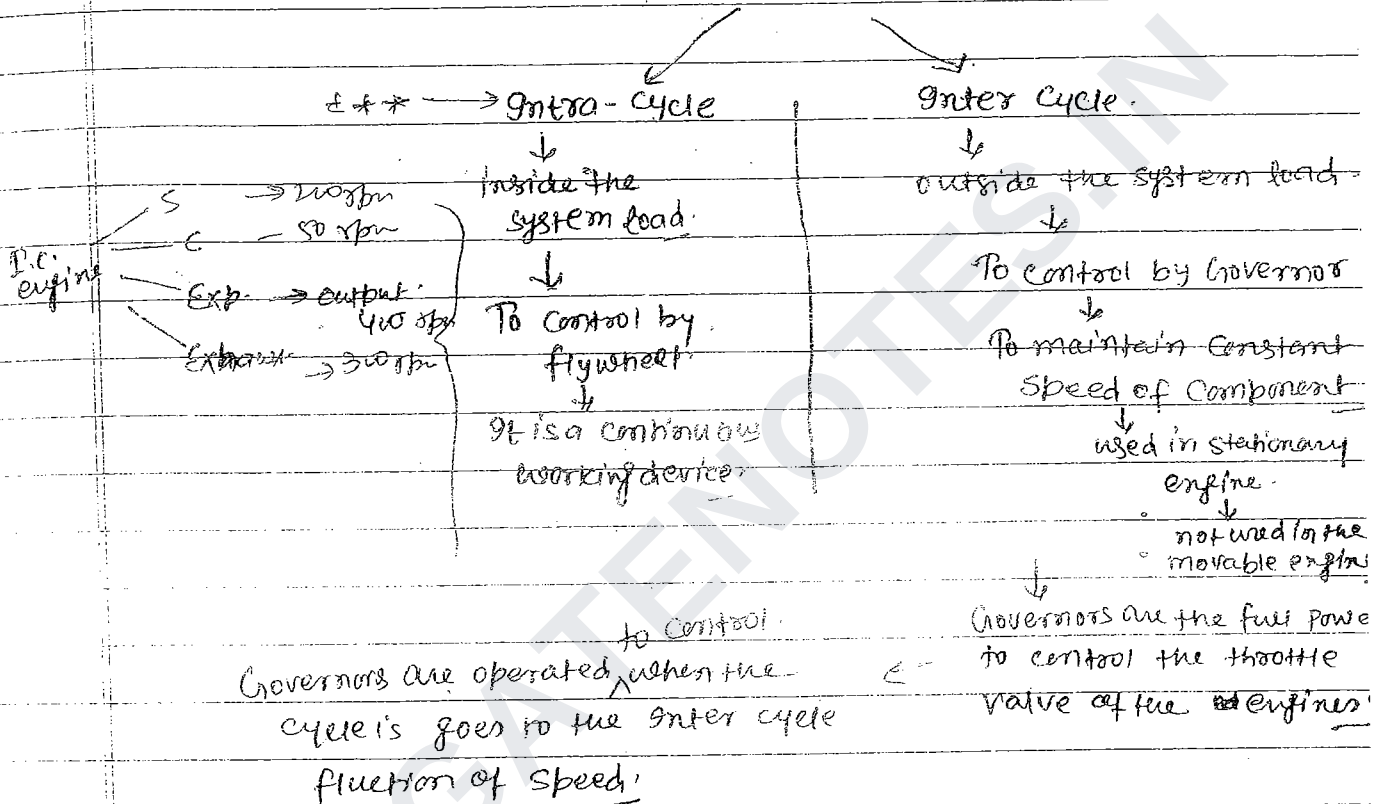
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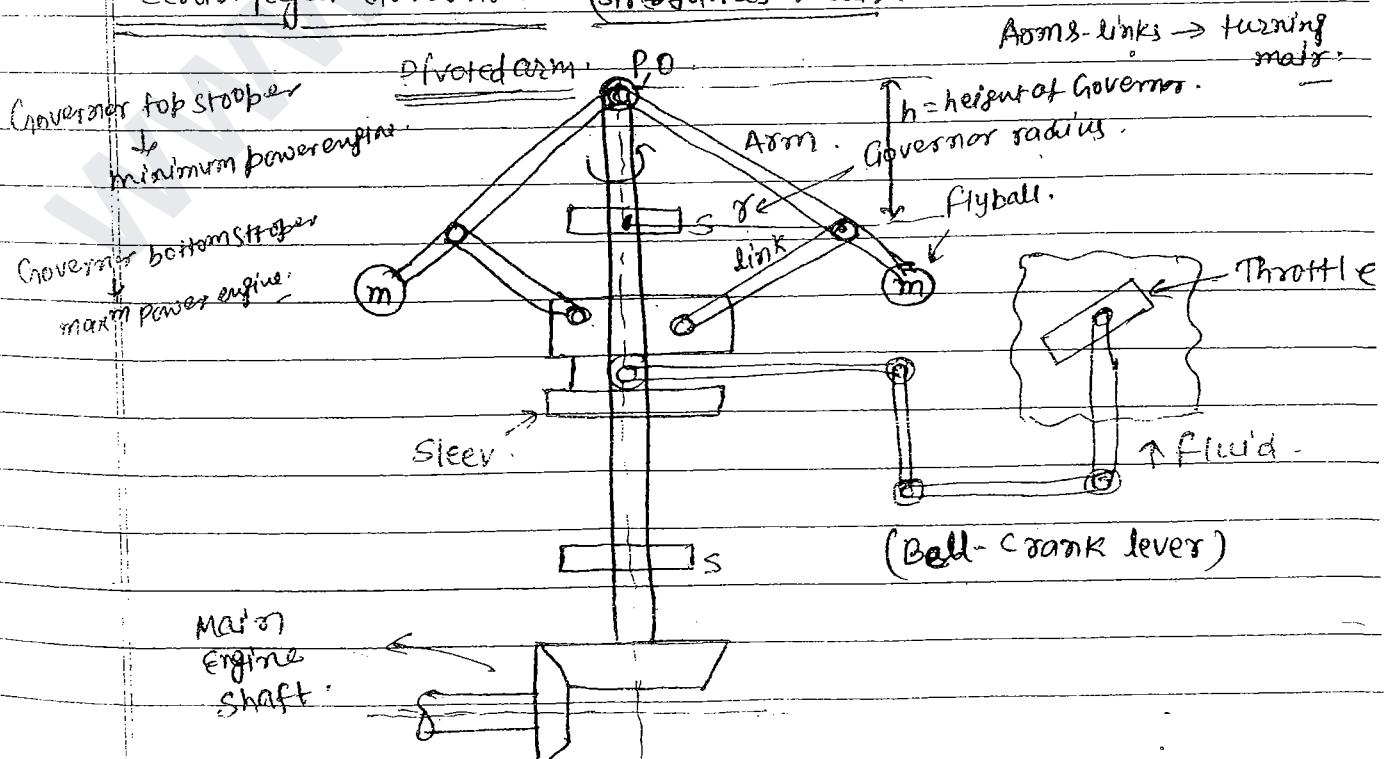
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Instantaneous: — For very small time in denominator.

Instantaneous fluctuation in Speed :-

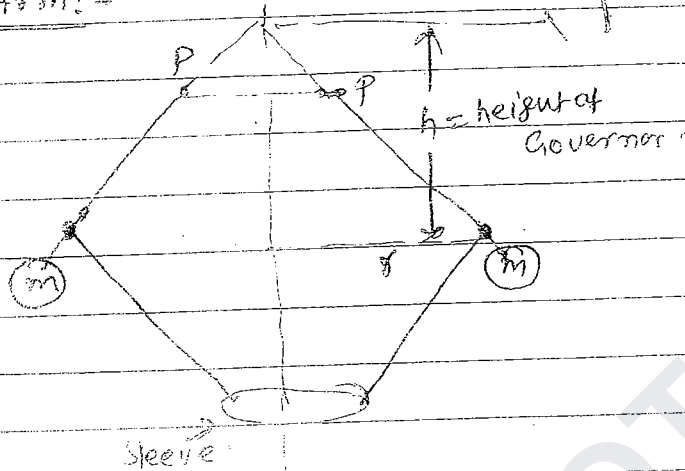


Centrifugal Governor :- (Sir James Watt) :-

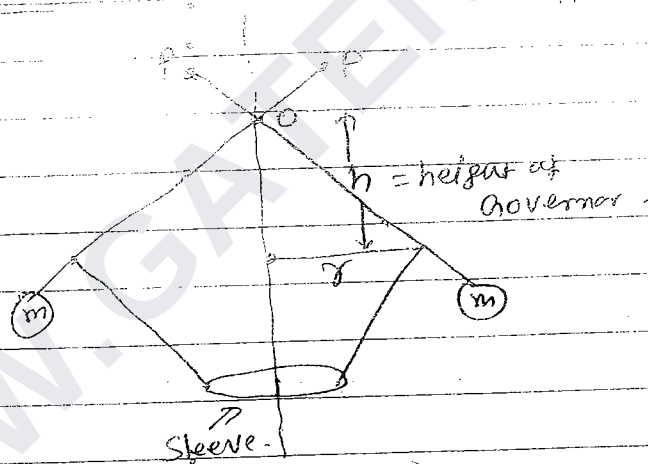


Gover Arm Intersection to radial line distance $\Rightarrow h$ (height of Governor).

Open Arm:-



Closed Arm:-



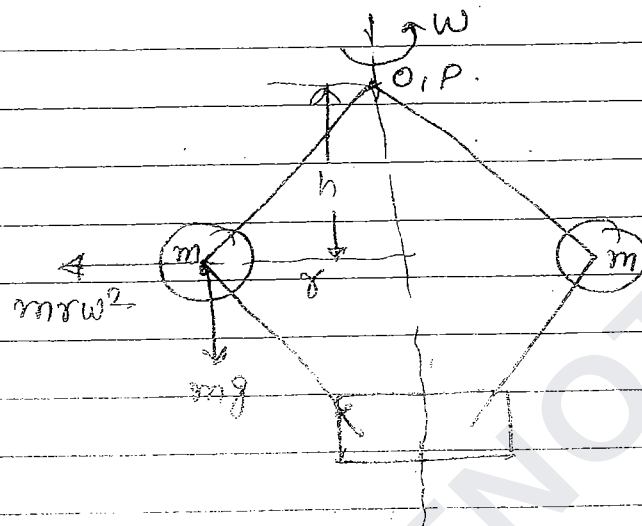
decimal upto fourth place

not use approximation

(a) Pendulum type of Governor Not other than

Watt Governor:-

Sleeve $\rightarrow$ massless comparison to flyball.



$m \rightarrow$ Mass of flyball

Taking Moment about O

$$(mrw^2) \cdot h = mg(x)$$

$$\Rightarrow w^2 = \frac{g}{h}$$

$$\Rightarrow \left(\frac{2\pi N}{60} \right)^2 = \frac{g}{h}$$

$$\Rightarrow N^2 = \left(\frac{60}{2\pi} \right)^2 \cdot \frac{g}{h}$$

Taking $g = 9.81 \text{ m/sec}^2$

$$\Rightarrow \boxed{N^2 = \frac{895}{h(\text{meters})}}$$

$\leftarrow$ Watt's governor

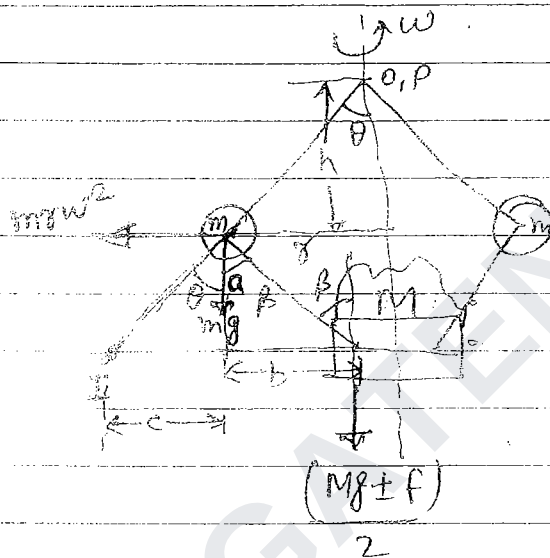
$\Rightarrow$ These Governors are insensitive in high speed engines.

$\rightarrow$ failed beyond 60 rpm.

Porter Governor: —

In walt's Governor sleeve $\rightarrow$ massless

On porter Governor mass of steerer is comparatively very big as compare to mass of ball. i.e; ($M \gg \gg m$).



f - static friction

$m \rightarrow$ Mass of fly ball.

M → Mans of sleeve >>> m

Taking moment about I ,

$$(mrv^2) \cdot a = mg \cdot c \pm \frac{(Mg \pm f)(b+c)}{2}$$

$$\Rightarrow N^2 = \left(\frac{60}{2\pi}\right)^2 \cdot \frac{g}{h} \left\{ \frac{2mg + (Mg \pm f)(1+k)}{2mg} \right\}$$

where $k = \frac{\tan \beta}{\tan \theta}$

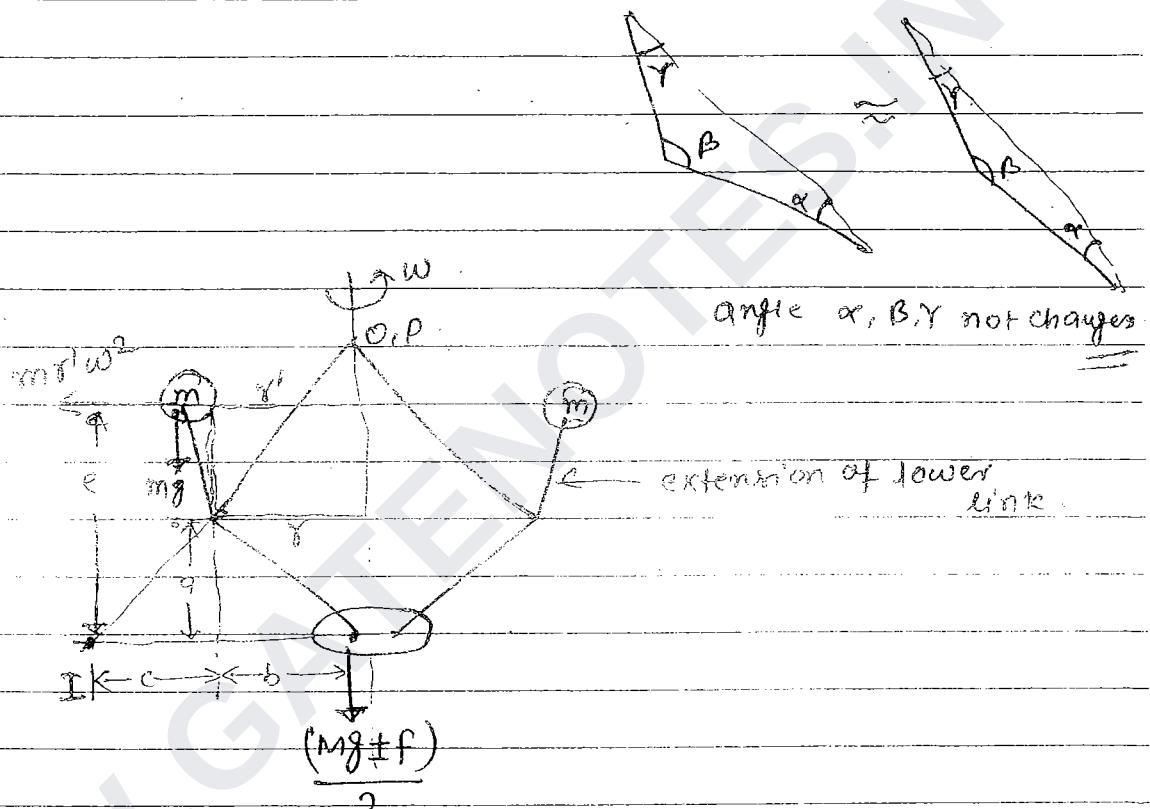
$$\Rightarrow N^2 = \frac{895}{h} \left\{ \frac{2mg + (Mg \pm f)(1+k)}{2mg} \right\}$$

$$1 + \frac{(Mg \pm f)(1+k)}{2mg}$$

$$M \gg m$$

Proell Governor:-

Extension of link



Taking moment about I:-

$$\text{If } r \neq r' \Rightarrow (mrw^2) \cdot e = mg(c+r-r') + \frac{(Mg \pm f)(b+c)}{2}$$

$$\Rightarrow \text{If } r' = r \text{ i.e., extension arm is purely vertical}$$

$$\Rightarrow (mrw^2) \cdot e = mg(c) + \frac{Mg \pm f(b+c)}{2}$$

$$\Rightarrow \underbrace{mrw^2 \cdot a}_{\downarrow \text{Porter}} = \frac{a}{e} \left[\underbrace{mg \cdot c + \frac{(Mg \pm f)(b+c)}{2}}_{\downarrow \text{Porter}} \right]$$

$$N^2 = \frac{895}{h} \times \left(\frac{q}{e}\right) \times \left(\frac{2mg + (Mg \pm f)(1+k)}{2mg} \right)$$

NOTE: $\rightarrow$ mass of the flywheel ball in Proell governor is comparatively less than for Porter Governor for same output.

$\downarrow$
By explanation,
ratio of $\left(\frac{q}{e}\right)$ is less than 1. Same ratio

We increase the $\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg} \right)$.

$\downarrow$
Here we maintain by increasing M or, decreasing m .

$\downarrow$
We cannot increase M by inertia.

So we decrease the mass of flyball (m).

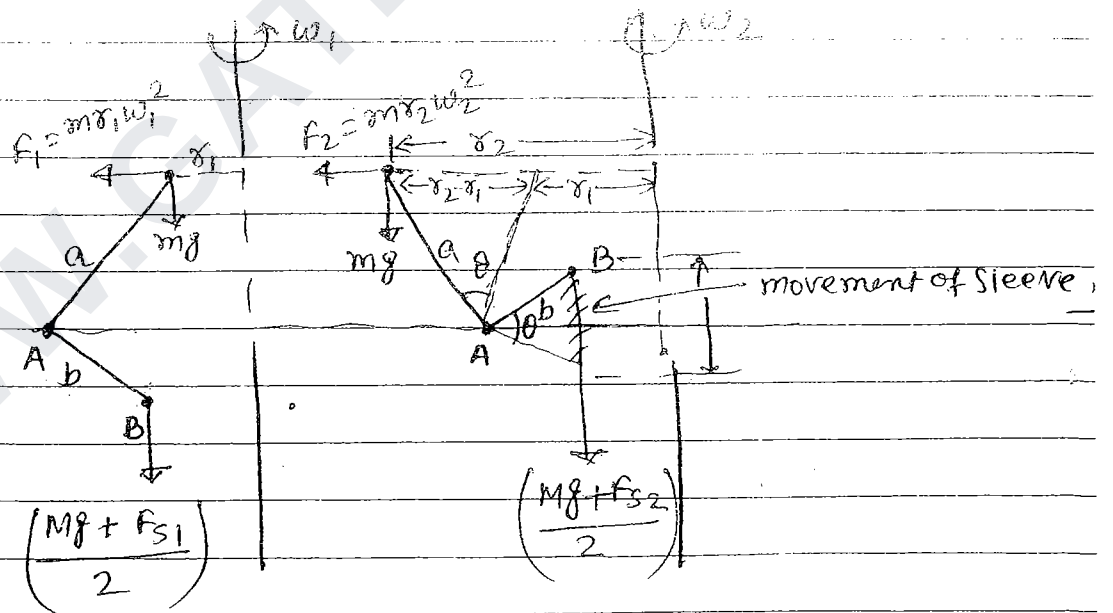
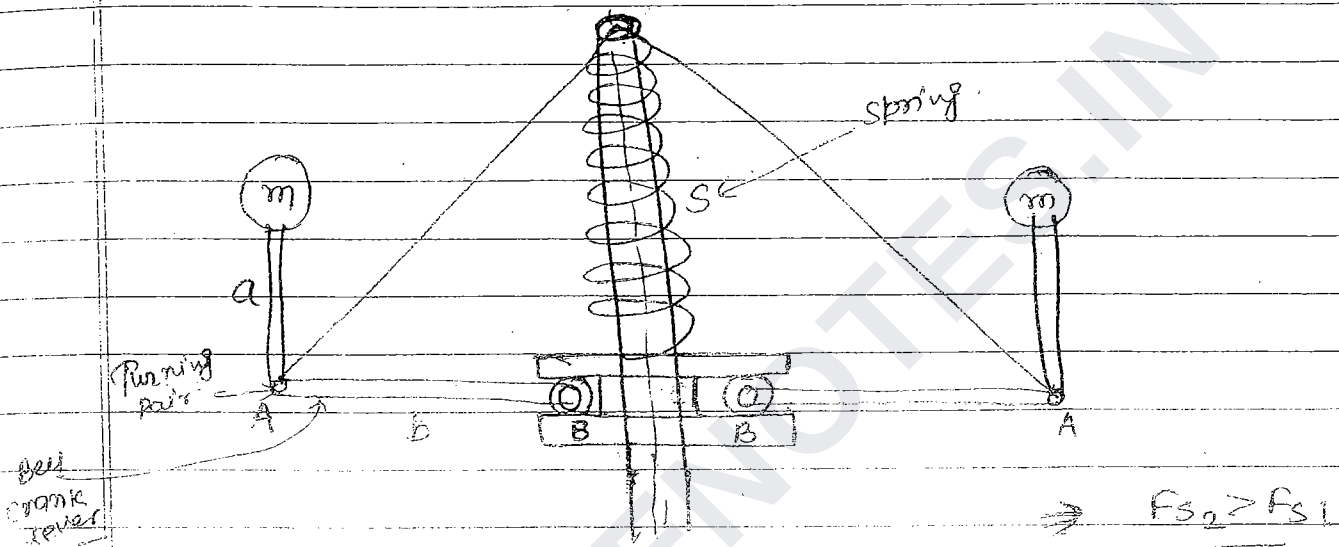
$\downarrow$
and reduce the inertia of the mechanism.

$\rightarrow$ decrease the mass m when increase the r by r .

Hartnell Governor! - Best performed Governor

Central
→ Spring always in compression.

✓ Approximation needed.



Approx! - Neglecting obliquity,

at A, Moment, $F_1 \cdot a = \frac{(Mg + F_{s1}) \cdot b}{2}$ — ①

$F_2 \cdot a = \frac{(Mg + F_{s2}) \cdot b}{2}$ — ②

From ② -- ①,

$$2(F_2 - F_1) \cdot \frac{q}{b} = (F_{S1} - F_{S2})$$

→ Movement of sleeve = $b\theta$

$$= \frac{b(r_2 - r_1)}{a} \quad \left. \vphantom{\frac{b(r_2 - r_1)}{a}} \right\} \rightarrow \text{Ist objective}$$

change in spring force -

← $(F_{S1} - F_{S2}) = \frac{b(r_2 - r_1)}{a} \times S \quad \left. \vphantom{\frac{b(r_2 - r_1)}{a} \times S} \right\} \rightarrow \text{IInd objective}$

$\downarrow$ $\downarrow$
movement of sleeve stiffness

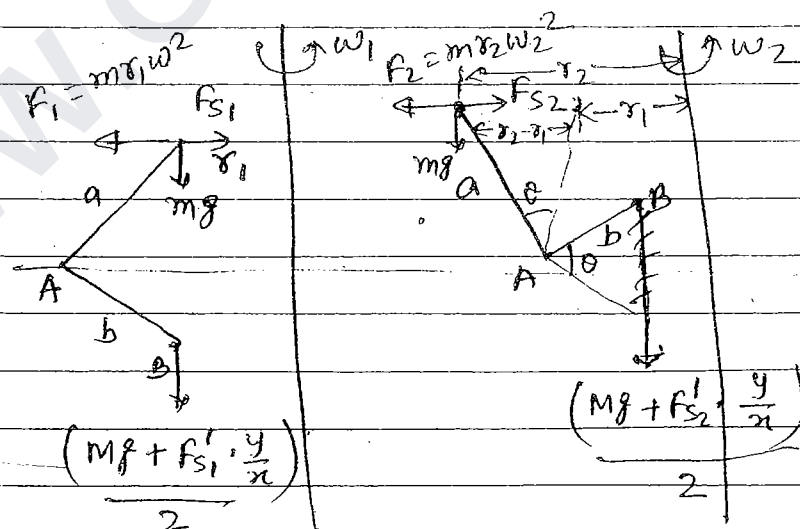
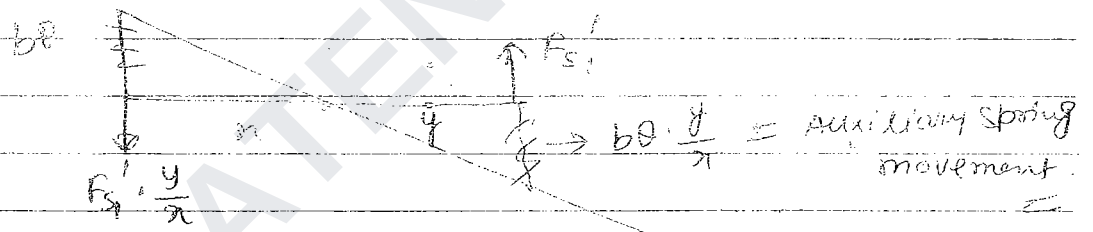
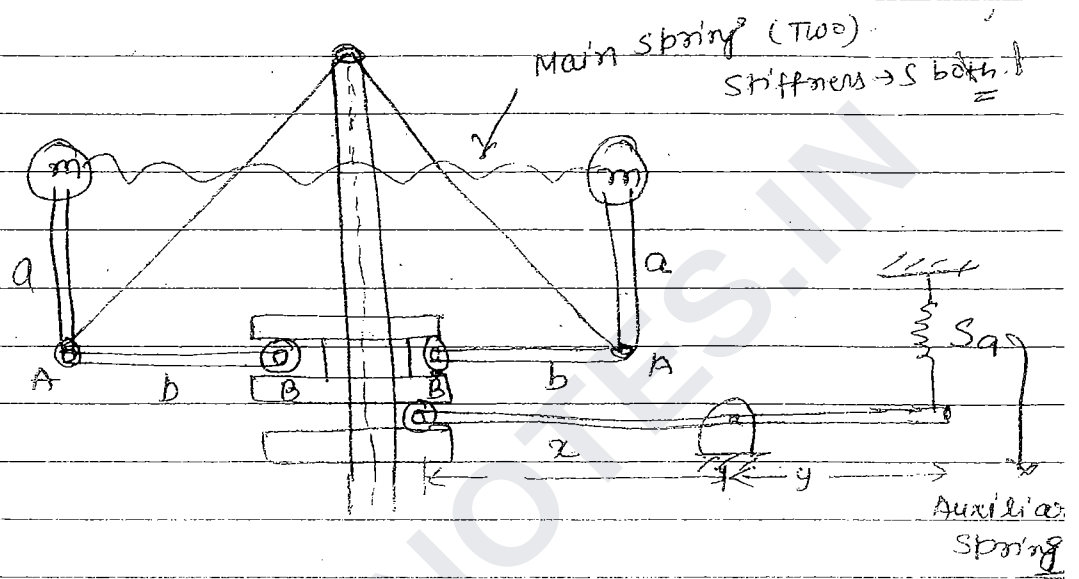
or
compression
of
spring

→ Stiffness!

$$2(F_2 - F_1) \times \frac{q}{b} = \frac{b(r_2 - r_1)}{a} \times S$$

$$\frac{2(F_2 - F_1)}{(r_2 - r_1)} = \left(\frac{q}{b}\right)^2 \quad \left. \vphantom{\frac{2(F_2 - F_1)}{(r_2 - r_1)}} \right\} \rightarrow \text{IIIrd objective}$$

Wilson-Hartnell Governor : — → two springs one back to other.



By Approximation:-

Neglecting obliquity,

$$(F_1 - F_{s1}) \cdot a = \frac{(Mg + F_{s1}' \cdot \frac{y}{x})}{2} \times b \quad \text{--- (1)}$$

$$(F_2 - F_{s2}) \cdot a = \frac{(Mg + F_{s2}' \cdot \frac{y}{x})}{2} \times b \quad \text{--- (2)}$$

From (2) - (1),

$$\frac{2a}{b} \left\{ (F_2 - F_1) - (F_{s2} - F_{s1}) \right\} = (F_{s2}' - F_{s1}') \times \frac{y}{x}$$

$$\Rightarrow (F_{s2} - F_{s1}) = 2(x_2 - x_1) \cdot S \times 2 = 4S(x_2 - x_1) \rightarrow \text{1st objective}$$

$$\Rightarrow (F_{s2}' - F_{s1}') = \left(b \cdot a \cdot \frac{y}{x} \right) \times S_a = \frac{b \cdot a \cdot (x_2 - x_1)}{a} \times \frac{y}{x} \times S_a \rightarrow \text{2nd objective}$$

$\downarrow$ movement of Auxiliary spring $\rightarrow$ Stiffness of Auxiliary spring

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Prob - (A)

$\rightarrow$ 3rd objective

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Stability of a Governor:-

A Governor is said to be in stable equilibrium if :-

- It is having unique equilibrium speed at unique radius.
- The restoring forces are quite dominate as compare to disturbing forces.

i.e; $\gamma \uparrow = N_{eqm} \uparrow$ } stable eqm
 $\gamma \downarrow = N_{eqm} \downarrow$ }



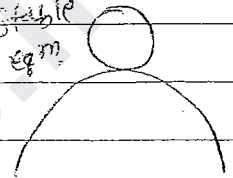
stable equilibrium

restoring > disturbing force



Neutral equilibrium

isochronous Governor



Highly unstable equilibrium

N_{max}
 insensitive zone $\rightarrow$ There is no motion of sleeve in this zone.
 N_{min}

due to $(Mg \pm f) \rightarrow N_{min} \rightarrow f$

NOTE:- (a) $\gamma \uparrow \Rightarrow N_{eqm} \downarrow$
 $\gamma \downarrow \Rightarrow N_{eqm} \uparrow$ } unstable equilibrium

(b) $\gamma \uparrow \Rightarrow N_{eqm} = \text{Const.}$ } Neutral eqm
 $\gamma \downarrow \Rightarrow$

Sensitivity :-

For the same sleeve movement,

$$\rightarrow \text{Sensitivity} \propto \frac{1}{(N_1 - N_2)} = \frac{N_{mean}}{(N_1 - N_2)}$$

$\rightarrow$ for same change in speed having high sleeve movement.

$\rightarrow$ For the same sleeve movement less change in speed then the Governor is more sensitive.

⇒ For Sensitivity of engine, $\rightarrow \frac{N_1 - N_2}{N_{\text{mean}}}$

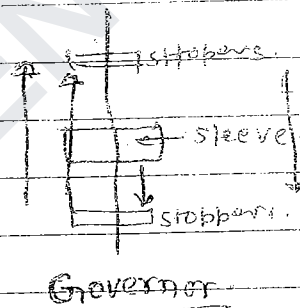
⇒ For Governor clubed in the engine then sensitivity is $\frac{N_1 - N_2}{N_{\text{mean}}}$

Hunting! -

Its a Physical Phenomenon associated with ~~to~~ Excessively high Sensitive Governor.

An excessively fast to-&-fro motion of the sleeve between the stoppers is known as Hunting. It will damage the stoppers of the spindle.

⇒ system will be out of Control due to Hunting of a certain period.



Isocronism or, Isocronous Governor! -

$\begin{matrix} \uparrow \\ \downarrow \end{matrix} \} \Rightarrow \text{Equilibrium} = \text{constant}$

Friction = zero.

i.e. $\boxed{\text{Sensitivity} = \infty}$

→ The Hunting level of Isocronous governor is zero.

→ Not use in practical due to friction of any system can not be zero.

For Porter Governor:-

For Porter Governor as Isochronous,

$$\left. \begin{aligned} N_1 &= N_2 \\ \therefore h_1 &= h_2 \end{aligned} \right\} \begin{aligned} N_1^2 &= \frac{895}{h_1} \left\{ \frac{2mg + Mg(1+k)}{2mg} \right\} \\ N_2^2 &= \frac{895}{h_2} \left\{ \frac{2mg + Mg(1+k)}{2mg} \right\} \end{aligned}$$

i.e; sleeve cannot move, therefore Not isochronous

For Hartnell Governor as Isochronous gov:-

$$(m_1 \omega_1^2) \cdot a = \frac{Mg + F_{s1}}{2} \times b$$

$$(m_2 \omega_2^2) \cdot a = \frac{Mg + F_{s2}}{2} \times b$$

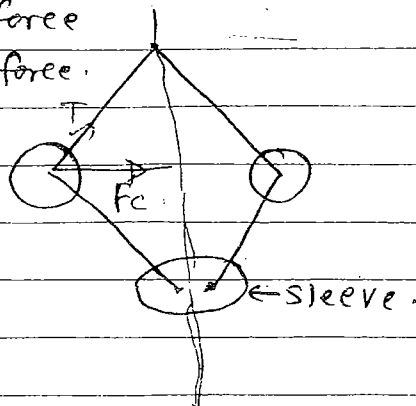
For Isochronous,

$$\omega_1 = \omega_2$$

$$\Rightarrow \left\{ \begin{aligned} r_1 &= \frac{Mg + F_{s1}}{2} \\ r_2 &= \frac{Mg + F_{s2}}{2} \end{aligned} \right\}$$

i.e; when the ratio of these two radii given by the ratio of the $(Mg + F_{s1} / Mg + F_{s2})$ By adjusting the spring force then the sleeve of Hartnell moves and ~~by adjusting the~~ this gives the Isochronous governor. so By this ratio adjusting we made the Hartnell Governor as Isochronous governor.
 Controlling force diagram:-

F_c = Controlling force
= Centripetal force.



Watt

↓
m

Porter
proell

↓
m
M

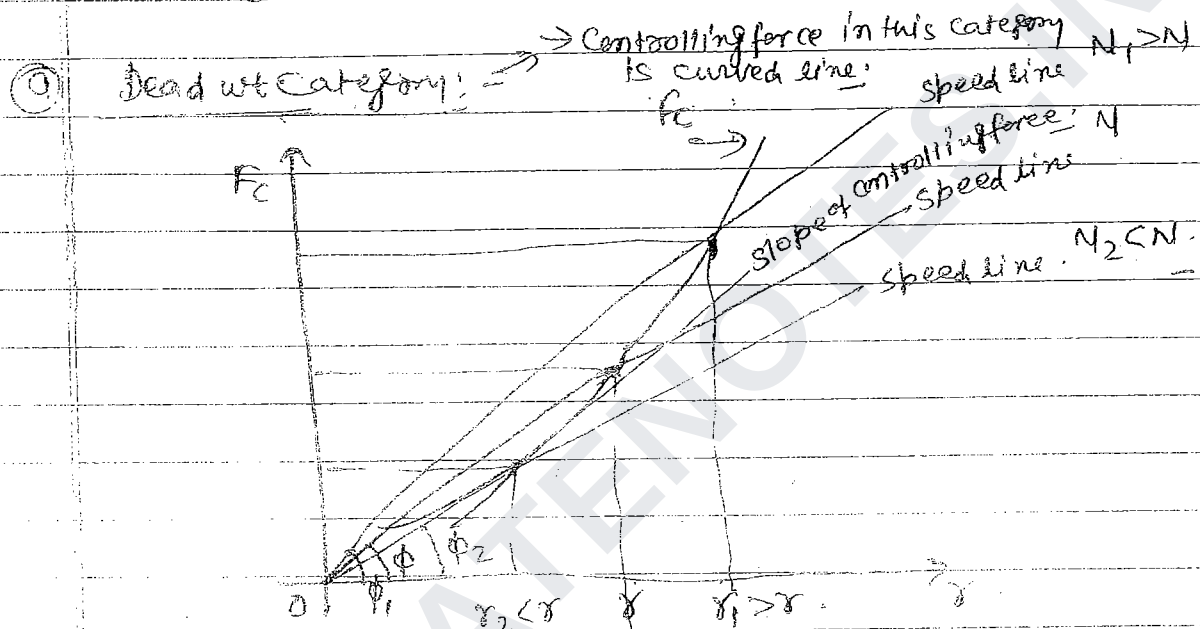
Spring
Controlle.

↓
m
M
Fs

$$F_c = \frac{mv^2}{r}$$

$$\Rightarrow F_c = m r \omega^2$$

$$F_c \propto r$$



Stable equilibrium.

$$F_c = m r \omega^2$$

$$\Rightarrow \omega^2 = \frac{1}{m} \left(\frac{F_c}{r} \right)$$

$$\Rightarrow N^2 = \left(\frac{60}{2\pi} \right)^2 \times \frac{1}{m} \times \left(\frac{F_c}{r} \right)$$

$$\Rightarrow N = \frac{60}{2\pi \sqrt{m}} \sqrt{\frac{F_c}{r}}$$

$$\Rightarrow N = K \sqrt{\frac{F_c}{r}}$$

$$\left(\because K = \frac{60}{2\pi \sqrt{m}} = \text{const. for a governor} \right)$$

$$\Rightarrow N = K \sqrt{\tan \phi}$$

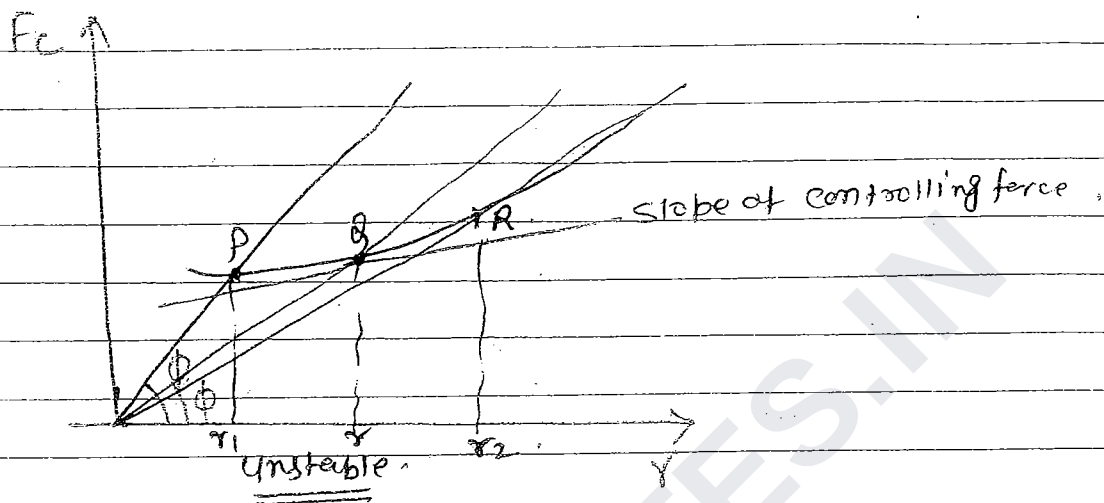
Note:-

→ Slope of controlling force curve is above the speed line then this condition is stable equilibrium otherwise unstable.

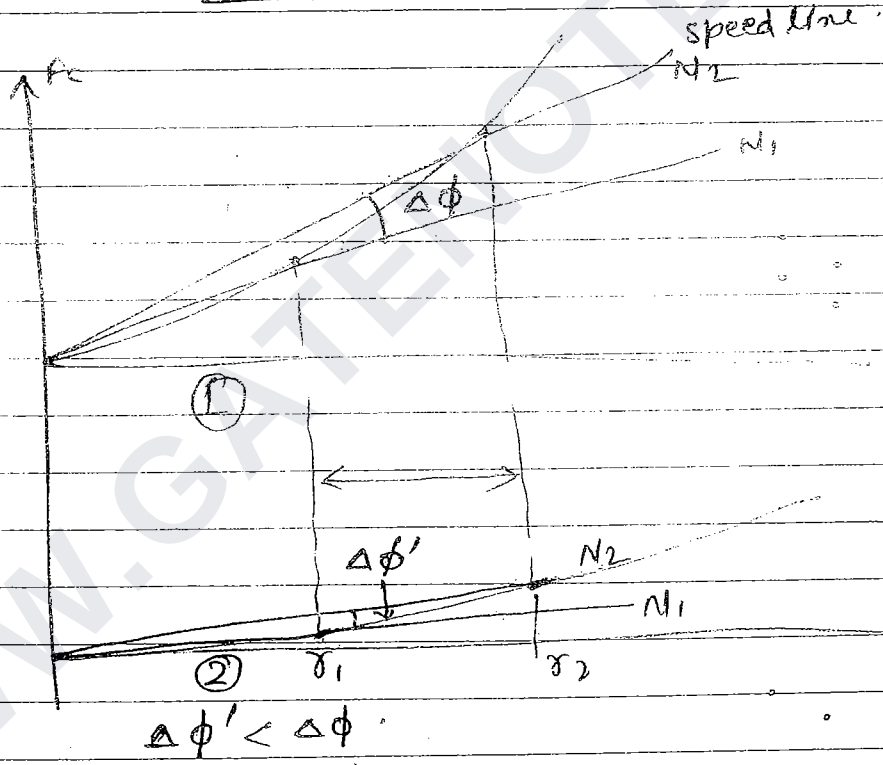
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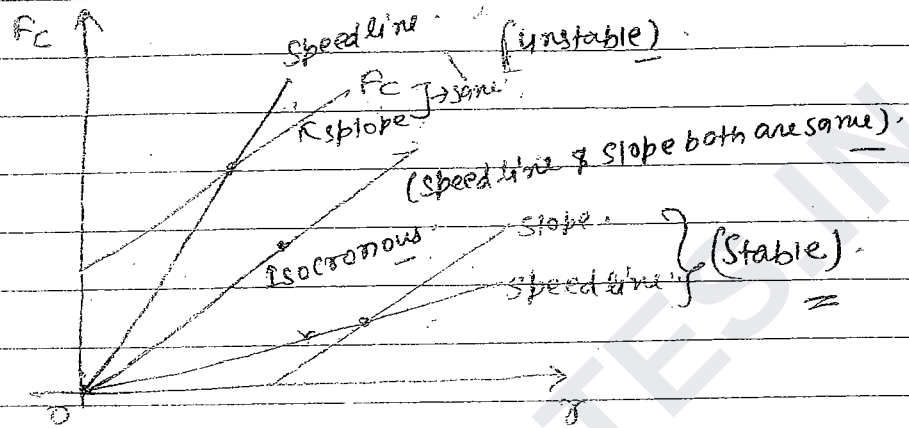


eg:-

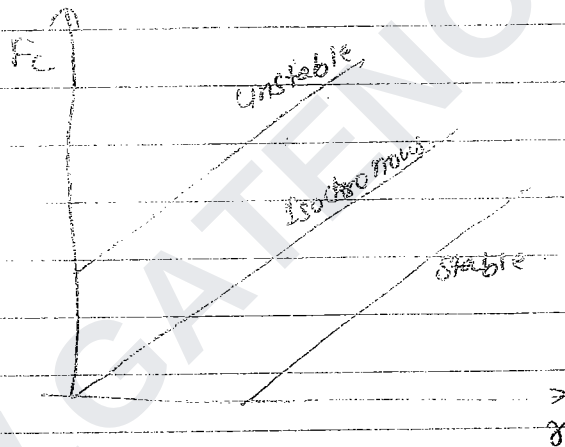


i.e., less change in speed for ② in same sleeve movement so ② is more sensitive.

⑥ Spring controlled category:-



i.e.



Effort of the Governor:-

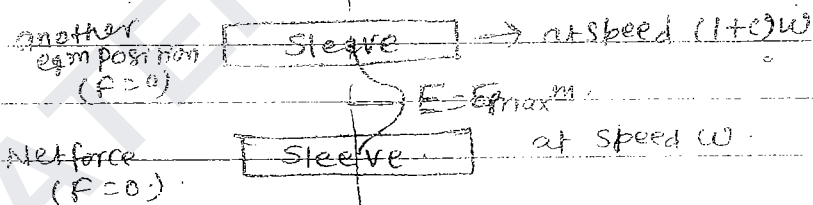
The ~~mean~~ force acting on the sleeve to ~~check~~^{change} its equilibrium position for the slight or fractional change in the speed of the Governor is known as effort of the governor.

Let $c =$ fractional change in speed,

$$\omega_f = (1 \pm c) \omega$$

$c^2 \rightarrow 0$

$$\text{Mean force} = \frac{0 + E}{2} = \frac{E}{2} = \text{Effort}$$



In Porter Governor,

$$\text{Effort} = \frac{E}{2} = \frac{cg}{1+k} [2m + M(1+k)]$$

In Hartnell Governor,

$$\text{Effort} = \frac{E}{2} = c(Mg + F_s)$$

Calculation:-

$$h = \frac{g}{\omega^2} \left[\frac{2mg + Mg(1+k)}{2mg} \right]$$

$$h = \frac{g}{(1+c)^2 \omega^2} \left[\frac{2mg + (Mg + E)(1+k)}{2mg} \right]$$

By calculating these two we find the E.

Power of the Governor: -

The work done at the sleeve to change its equilibrium position is known as Power of the Governor.

Power (P) = Mean force $\times$ movement of the sleeve

$$P = \frac{E}{2} \times \text{Movement of the sleeve}$$

For Porter Gov:-

when All the four arms are equal,

$$K=1$$

$$\text{Effort} = \frac{E}{2} = \frac{Cg}{1+1} \{2m + M(1+1)\} \left\{ \begin{array}{l} h_2 = \frac{g}{w^2} \left[\frac{2mg + Mg(1+1)}{2mg} \right] \\ h_1 = \frac{g}{(1+c)^2 w^2} \left[\frac{2mg + Mg(1+1)}{2mg} \right] \end{array} \right.$$

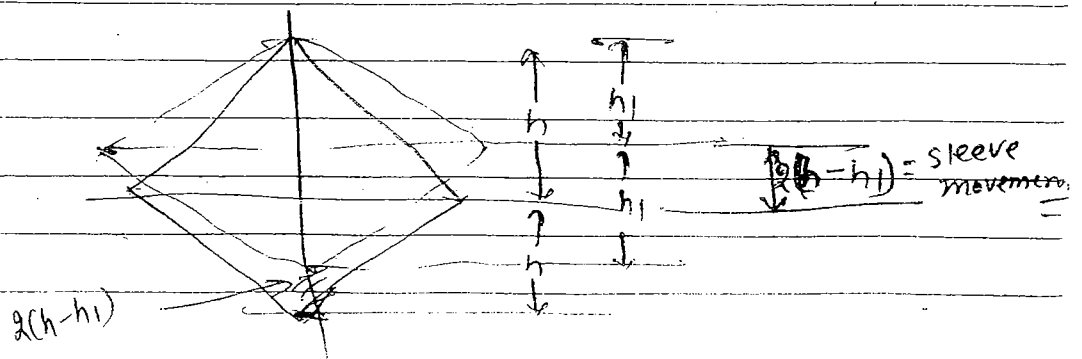
$$= Cg(m+M)$$

$$\text{Sleeve movement} = 2(h-h_1) = 2h \left(1 - \frac{h_1}{h}\right)$$

$$= 2h \left(1 - \frac{1}{1+2c}\right) = \frac{2h \cdot 2c}{(1+2c)}$$

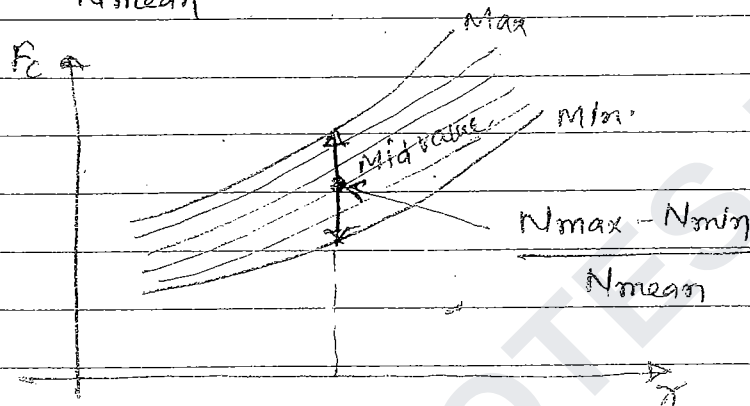
$$\text{Then } P = Cg(m+M) \cdot \frac{2h \cdot 2c}{1+2c}$$

$$\Rightarrow P = gh(m+M) \cdot \frac{4c^2}{1+2c}$$



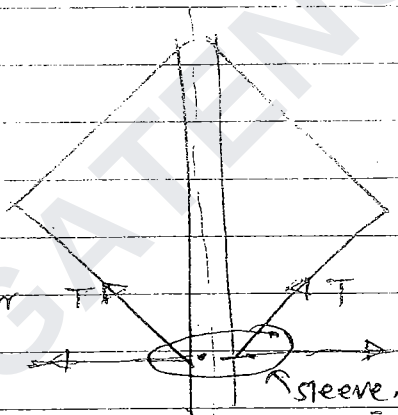
Co-efficient of insensitiveness:-

$$\frac{N_{max} - N_{min}}{N_{mean}} = \text{Co-eff. of insensitiveness}$$



$$F_{s, max} = 21(N)$$

So by adjusting tension we decreased the normal reaction by making governor is symmetric.



QES-2000

Q.1) ②

200 mm

$\theta = \beta$ i.e., $k=1$

$m = 15 \text{ kg}$

$M = 75 \text{ kg}$

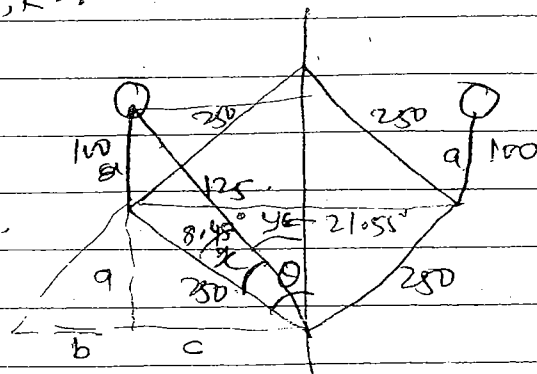
$F = 0$

Minimum

$r_1 = 125 \text{ mm}$, $h_1 = 216.5063 \text{ mm}$

$a = 216.5063$

$e = 316.5063$



$$N_1^2 = \frac{895}{(216.5063)} \left(\frac{308 + 758(1+1)}{308} \right) \times \frac{216.5063}{316.5063}$$

$$= 130.255 \text{ rpm}$$

$$h_2 = 178.5357$$

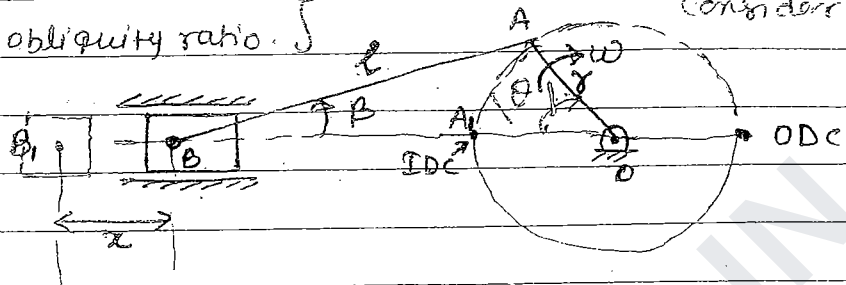
Ans: Range = 40

7. Kinematic Analysis of Single Slider Crank Mechanism

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$l \gg r$
 $\frac{l}{r} = \eta = \text{obliquity ratio.}$ } $\Rightarrow$ inertia of connecting rod is not considered.



$m \rightarrow$ mass of reciprocating parts.

① Piston -

$$x = B_1O - BO = (l+r) - (l\cos\beta + r\cos\theta)$$

$$\eta = \frac{l}{r} \Rightarrow l = \eta r$$

$$\text{and, } l\sin\beta = r\sin\theta$$

$$\Rightarrow \sin\beta = \frac{\sin\theta}{\eta}$$

$$\cos\beta = \sqrt{1 - \frac{\sin^2\theta}{\eta^2}} = \frac{\sqrt{\eta^2 - \sin^2\theta}}{\eta}$$

Put these values then,

$$\text{Displacement} = x = \text{Displacement at piston} = r \left\{ (1 - \cos\theta) + \left(\eta - \sqrt{\eta^2 - \sin^2\theta} \right) \right\}$$

Velocity:

$$V = \frac{dx}{dt} = \frac{dx}{d\theta} \left(\frac{d\theta}{dt} \right) = \omega \frac{dx}{d\theta}$$

$$V = r\omega \left\{ \sin\theta + \frac{\sin^2\theta}{\eta \sqrt{\eta^2 - \sin^2\theta}} \right\}$$

for $\eta \rightarrow \text{large}$.

$$V_{\text{approx}} = r\omega \left\{ \sin\theta + \frac{\sin^2\theta}{\eta} \right\}$$

for $n \rightarrow$ Very very large.

$$V_{\text{approx}} = r\omega \sin\theta$$

Acceleration:

$$a_{\text{approx}} = \frac{dV_{\text{approx}}}{d\theta} \left(\frac{d\theta}{dt} \right) = \omega$$

$$= r\omega^2 \left\{ \cos\theta + \frac{\cos 2\theta}{n} \right\} \Rightarrow \text{for } n \text{ is large,}$$

$$= r\omega^2 \cos\theta \Rightarrow \text{for } n \text{ is very large,}$$

② Connecting rod:-

$$\omega_{C.R.} = \frac{d\beta}{dt}$$

$$\sin\beta = \frac{\sin\theta}{n}$$

$$\Rightarrow \cos\beta \cdot \frac{d\beta}{dt} = \frac{1}{n} \cos\theta \cdot \frac{d\theta}{dt}$$

$$\omega_{C.R.} = \frac{1}{n} \cdot \frac{\cos\theta \cdot \omega}{\sqrt{n^2 - \sin^2\theta}}$$

$$\Rightarrow \boxed{\omega_{C.R.} = \frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2\theta}}}$$

for n is large, $\boxed{(\omega_{C.R.})_{\text{approx}} = \frac{\omega \cos\theta}{n}}$

Angular acceleration,

$$a_{C.R.} = \frac{d\omega_{C.R.}}{dt}$$

$$= \frac{d\omega_{C.R.}}{d\theta} \times \left(\frac{d\theta}{dt} \right) \rightarrow \omega$$

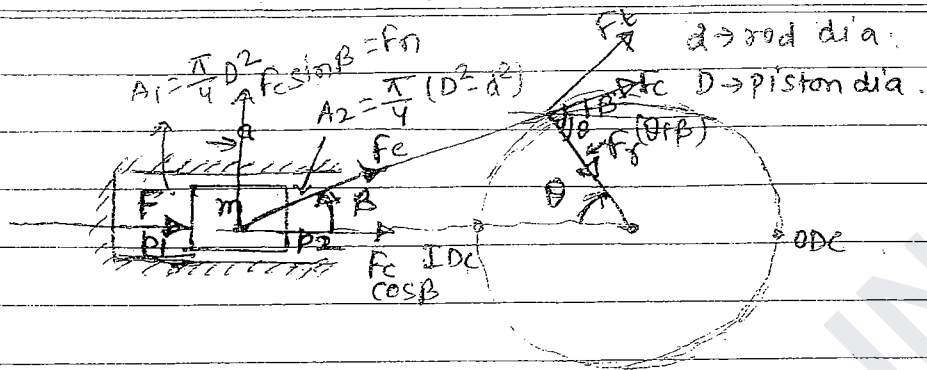
$$a_{C.R.} = -\omega^2 \sin\theta \cdot \left\{ \frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}} \right\}$$

19/08/10

8. Dynamic Analysis of Single Slider Crank Mechanism

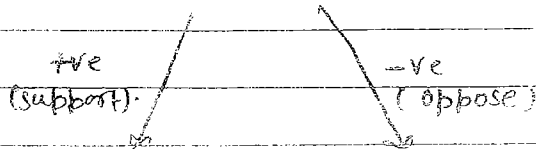
classmate

Date 19/08/10
Page 98



1. Piston effort:-

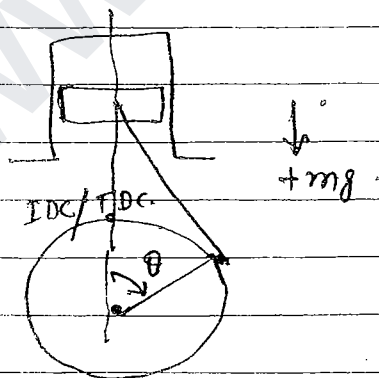
(Effective driving force on the piston):- $(F) =$



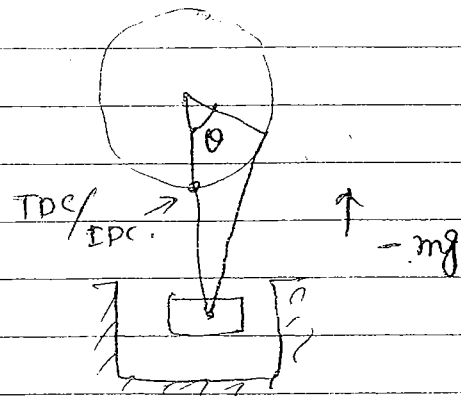
$P_1 A_1 = P_2 A_2$
 $F_{inertia} = m \cdot a$
 $F_{gas} = (P_1 A_1 - P_2 A_2)$
 $F = m r \omega^2 \left\{ \cos \theta + \frac{\cos 2\theta}{n} \right\}$
 (i) $f =$ friction

(a) $\pm mg$

$F = F_{gas} - F_I - f \pm mg$



Vertical engine
(Cylinder up, crank down)



(Cylinder down, crank up)
Vertical engine

→ Vertical engine of this type → max^m speed

↓
Milage → Horizontal engine

② Force along the connecting rod: - (F_c): -

$$F_c \cos \beta = F$$

$$\Rightarrow F_c = \frac{F}{\cos \beta}$$

③ Normal force on the cylinder walls: - (F_n)

$$F_n = F_c \sin \beta$$

$$= \frac{F}{\cos \beta} \cdot \sin \beta$$

$$\Rightarrow F_n = F \tan \beta$$

④ crank effort: - (F_t) = tangential force:-

$\propto$ Tangential force on crank pin:-

$$F_t = F_c \sin(\theta + \beta)$$

$$F_t = \frac{F}{\cos \beta} \cdot \sin(\theta + \beta)$$

⑤ Radial thrust on the crank shaft bearing: - (F_r).

$$F_r = F_c \cos(\theta + \beta)$$

$$\Rightarrow F_r = \frac{F}{\cos \beta} \cdot \cos(\theta + \beta)$$

⑥ Turning Moment on the crank shaft: -

$$T = F_t \cdot r$$

$$T = \frac{F}{\cos \beta} \times \sin(\theta + \beta) \cdot r$$

$$T = I \cdot \alpha$$

$$\omega_f = \omega_i + \alpha t$$

} $\rightarrow$ Flywheel ~~to~~ maintain the fluctuation at constant level not zero.

Turning moment diagram of single cylinder double acting steam engine: - (T- θ diagram)

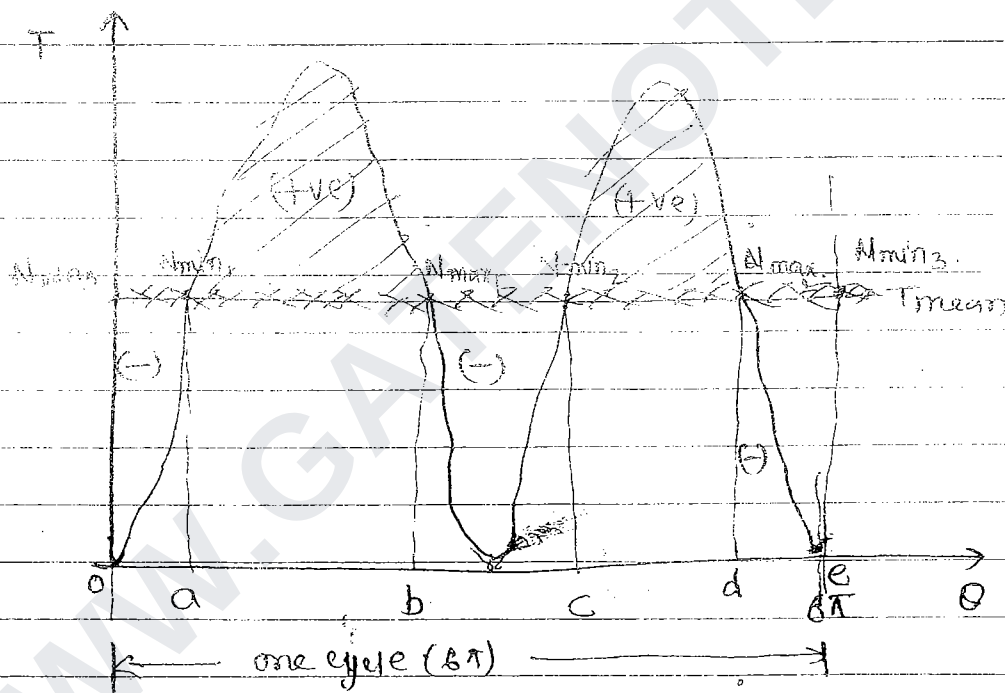
Work done cycle :-

T = Actual torque

$$W.D. \text{ cycle} = T_{\text{mean}} \times 6\pi$$

$$T_{\text{mean}} = \frac{W.D. \text{ cycle}}{6\pi}$$

= Mean torque or, load torque, resisting torque.



min ($N_{\text{min}1}, N_{\text{min}2}, N_{\text{min}3}$)

N_{min}

max ($N_{\text{max}1}, N_{\text{max}2}, N_{\text{max}3}$)

N_{max}

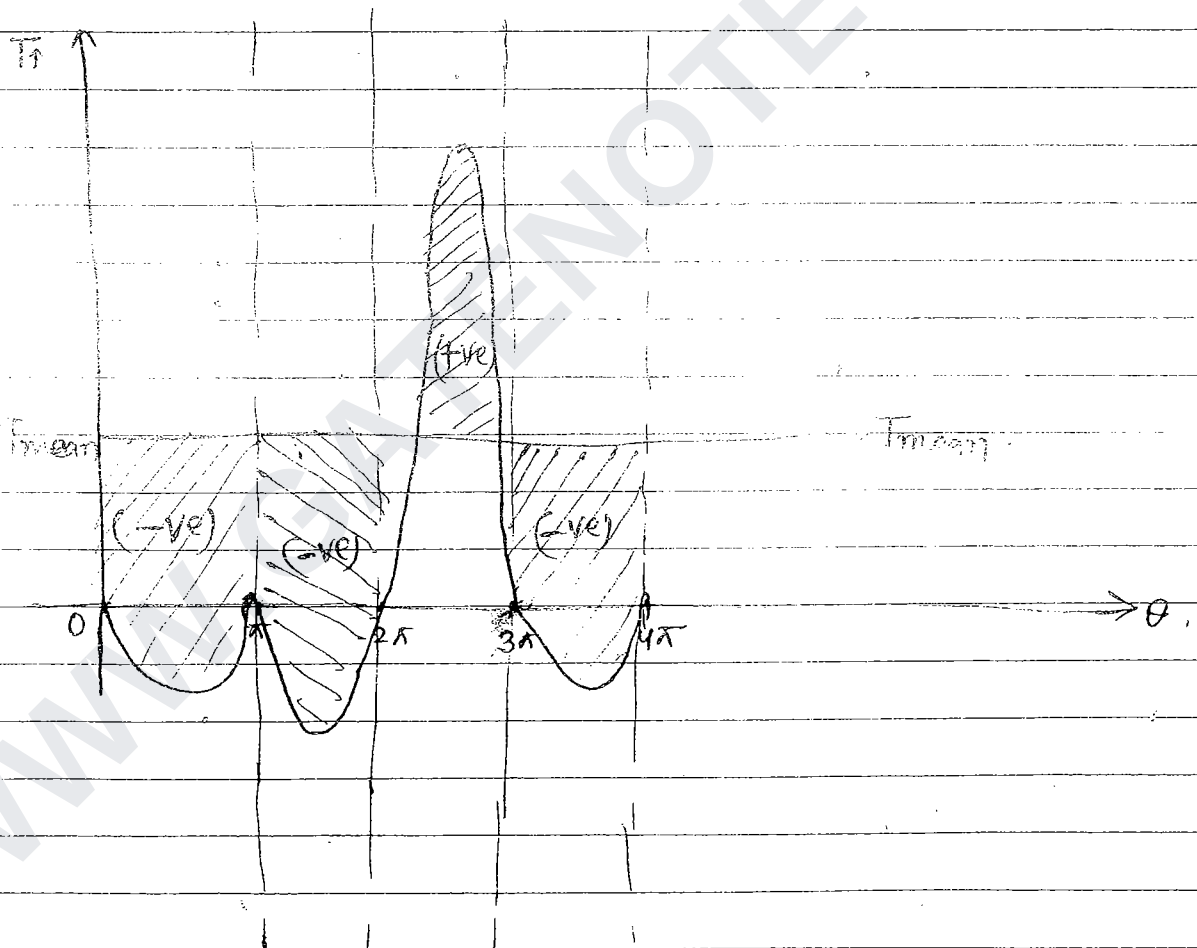
Fluctuation After assembly = $N_{\text{max}} - N_{\text{min}}$
of flywheel

→ High speed engine having lighter flywheel but low speed engine having heavier flywheel.

$$\frac{1}{2} I \cdot \omega^2 = \text{forms of energy storage in flywheel,}$$

↓
moment of Inertia.

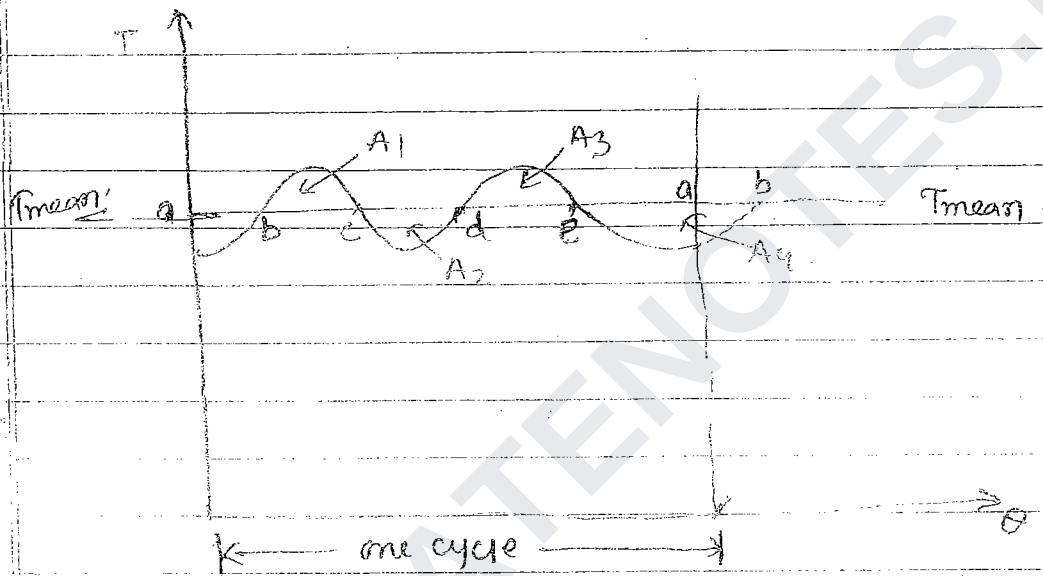
Turning moment diagram of single cylinder four stroke I.C. engine: -



$$T_{\text{mean}} = \frac{W.D. \text{ Cycle}}{4\pi}$$

Concepts of Multicylinders:-

→ Multicylinders are used to eliminate the flywheel by adjusting the firing order and uniform the torque distribution throughout them.



Let E_b = energy at point b = E .

$$E_c = E + A_1$$

$$E_d = E + A_1 - A_2$$

$$E_e = E + A_1 - A_2 + A_3$$

$$E_b = E + A_1 - A_2 + A_3 - A_4$$

For comparing same energy at b,

$$E_b = E_b$$

$$\Rightarrow E = E + A_1 - A_2 + A_3 - A_4$$

$$\Rightarrow \boxed{A_1 - A_2 + A_3 - A_4 = 0}$$



By this find the unknown Area

For energy,

$$E_{\max} = E + 30$$

$$E_{\min} = E - 10$$

$$(\Delta E) = E_{\max} - E_{\min}$$

= Max^m fluctuation = Variation of energy
of energy

$$\Rightarrow \Delta E = (E + 30) - (E - 10) = 40$$

$$\Rightarrow \Delta E = 40 \text{ mm}^2$$

$$\text{Now, } \Delta E = 40 \text{ mm}^2$$

$$= 40 \text{ mm} \times \text{mm}$$

↓ ↓
scale scale on y-axis
on x-axis in N-m
↓ ↓
in radian in N-m

$$= 40 \times \frac{\pi}{3} (\text{rad}) \times 5 (\text{N-m}) = \text{--- Joules}$$

Co-efficient of fluctuation of speed for the flywheel: -

$$\frac{\omega_{\max} - \omega_{\min}}{\omega_{\text{mean}}} = \text{Co-efficient of fluctuation of speed } (C_s)$$

Where

$$\omega_{\text{mean}} = \frac{\omega_{\max} + \omega_{\min}}{2}$$

Co-efficient of fluctuation of energy for the flywheel: -

$$C_E = \text{Co-efficient of fluctuation of energy} = \frac{E_{\max} - E_{\min}}{W.D. \text{ cycle}}$$

Fundamental equation of the flywheel:-

I = Moment of Inertia of flywheel

m = mass of flywheel.

K = Radius of gyration of flywheel.

$$I = mK^2$$

$$\omega_{\max}, \omega_{\min}, \quad \frac{\omega_{\max} + \omega_{\min}}{2} = \omega_{\text{mean}} = \omega.$$

Now,

$$\Delta E = \frac{1}{2} I \omega_{\max}^2 - \frac{1}{2} I \omega_{\min}^2$$

$$= \frac{1}{2} I (\omega_{\max} - \omega_{\min}) \times (\omega_{\max} + \omega_{\min}) \times \omega_{\text{mean}}$$

$$= I \omega^2 \cdot c_s$$

$$\Rightarrow \Delta E = I \omega^2 \cdot c_s = \text{Max}^m \text{ fluctuation or variation in energy}$$

IES-2001

Prob: (2)

$$T = 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta$$

$$T_{\text{mean}} = \text{const.}$$

$$(i) P = ?$$

$$N = 250 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = 26.1799 \text{ rad/sec.}$$

$$(ii) I = ?$$

$$c_s = \pm 0.25\%$$

$$= 0.5\%$$

$$c_s = \frac{0.5}{100} = 0.005$$

$$(iii) \alpha (\theta = 45^\circ) = ?$$

Cycle: $\frac{\pi}{1}, \frac{\pi}{1}$ ← period

$$\frac{\text{L.C.M. of } N^r}{\text{H.C.F. of } D^r} = \frac{\pi}{1} = \pi$$

$$\text{W.D. cycle} = \int_0^{\pi} T d\theta$$

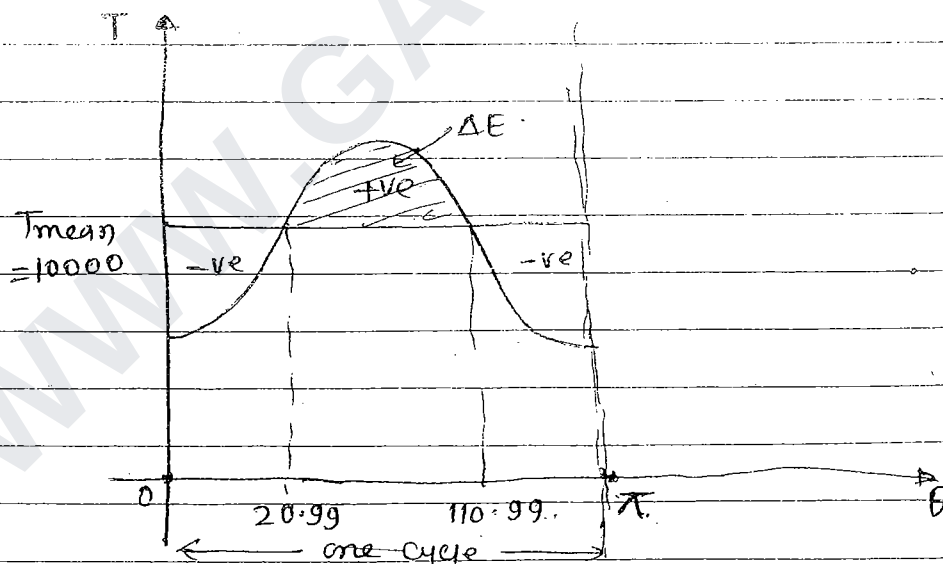
$$= \int_0^{\pi} (10000 + 2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$$

$$= 10000 [\pi] + 2000 \times 0 - 1800 \times 0$$

$$= 10000\pi$$

$$T_{\text{mean}} = \frac{10000\pi}{\pi} = \text{W.D. cycle} / \text{period}$$

$$T_{\text{mean}} = 10000$$



Points where T curve is cutting T_{mean} line.

$$\text{i.e. } T = T_{\text{mean}}$$

$$\Rightarrow 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta = 10000$$

$$\Rightarrow \tan 2\theta = \frac{1800}{2000} = 0.9$$

$$2\theta = 41.98^\circ, \text{ or } 221.98^\circ, 401.98^\circ$$

$$\Rightarrow \theta = 20.99^\circ, 110.99^\circ$$

$$\Delta E = \int_{20.99}^{110.99} (T - T_{\text{mean}}) d\theta = 2690.724 \text{ Joule}$$

$$= \int_{20.99}^{110.99} (2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$$

$$= \left[-2000 \cos 2\theta - \frac{1800}{2} \sin 2\theta \right]_{20.99}^{110.99}$$

$$= -2000 \cos 221.98^\circ - 900 \sin 221.98^\circ - \left(-2000 \cos 41.98^\circ - 900 \sin 41.98^\circ \right)$$

$$= 2690.724 \text{ Joule}$$

$$(i) P = T_{\text{mean}} \cdot \omega$$

$$(ii) \Delta E = I \cdot \omega^2 \cdot c_s$$

$$\Rightarrow 2690.724 = I \times (26.179)^2 \times 0.005$$

$$\Rightarrow \boxed{I = 785.22 \text{ kg-m}^2} \text{ Ans}$$

$(T - T_{\text{mean}})$ is responsible for acceleration,

i.e.,

$$(T - T_{\text{mean}})_{\theta=45^\circ} = I \cdot \alpha$$

$$\Rightarrow \boxed{\alpha = 2.54 \text{ rad/s}^2} \text{ Ans}$$

Flywheel in Power Presses:-

PAS-2003

Prob (3)
from material

$$\text{Cycle time} = 30 \text{ holes/min} \\ = 1 \text{ hole/2 sec}$$

$$\frac{2}{1} \times \frac{1}{6} \text{ sec}$$

Exact punching time $\rightarrow$ to operate the punch for material removal $= \frac{1}{6} \text{ sec}$.

$$P = 1.5 \text{ kW}$$

$$C_s = \frac{20}{100} = 0.2$$

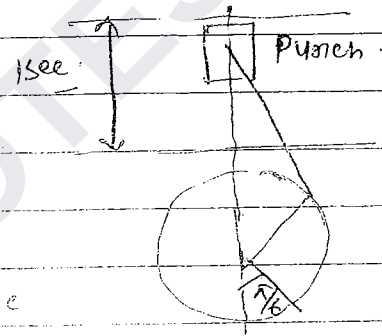
$$W = \pi \text{ 500 J/sec}$$

$$P_{\text{motor}} = \text{Energy required/sec}$$

$$\Rightarrow 1500 = E_{\text{hole}} \times \text{No. of hole/sec}$$

$$\Rightarrow 1500 = E_{\text{hole}} \times \frac{30}{60}$$

$$E_{\text{hole}} = 3000 \text{ Joule}$$



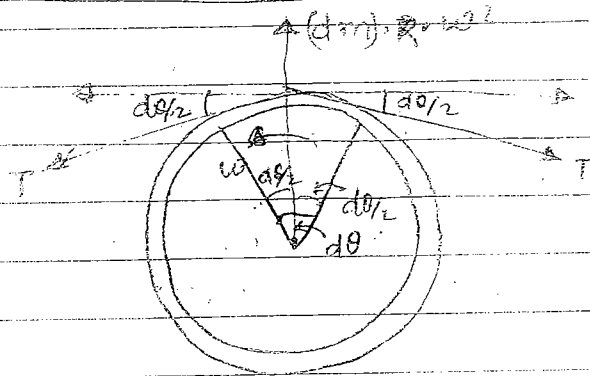
For punching, $E_{\text{available}} = (1500 \times \frac{1}{6}) \text{ Joule} = 250 \text{ Joule}$.

$$E_{\text{hole}} = 3000 \text{ Joule}$$

$$\Rightarrow (3000 - 250) = I \omega^2 \cdot C_s$$

$$\Rightarrow I = 1393.16 \text{ kg-m}^2$$

Designing of Flywheel:



$$\frac{2T \sin \frac{d\theta}{2}}{2} = (dm) \cdot R \cdot \omega^2$$

$$\Rightarrow \frac{2T d\theta}{2} = (dm) \cdot R \cdot \omega^2$$

$$\Rightarrow T d\theta = (V) \cdot \rho \cdot R \omega^2 \quad (\rho = \text{volume density})$$

$$\Rightarrow T d\theta = A(R d\theta) \cdot \rho \cdot R \omega^2$$

$$\Rightarrow \frac{T}{A} = \rho (R \omega)^2 = \rho \cdot v^2 \quad \text{periferal velocity}$$

$$\Rightarrow \sigma = \rho \cdot v^2$$

$$\Rightarrow v = \sqrt{\frac{\sigma}{\rho}} \quad \Rightarrow \quad v_{\max} = \sqrt{\frac{\sigma_b}{\rho}}$$

Balancing

↓
 Furbalance
 (vibration).

Rotating
 unbalance

+ Reciprocating
 unbalance..

Rotating unbalance: -

on Rotating Balancing: -

(a) $\sum \vec{F} = 0$

(b) $\sum \vec{M} = 0$

Rotating Balancing

Static Balancing

↓
 $\sum \vec{F}_{ext} = 0$

Dynamic Balancing

$\sum \vec{F}_{ext} = 0$

$\sum \vec{M} = 0$

Static Balancing: - Analytical Method: -

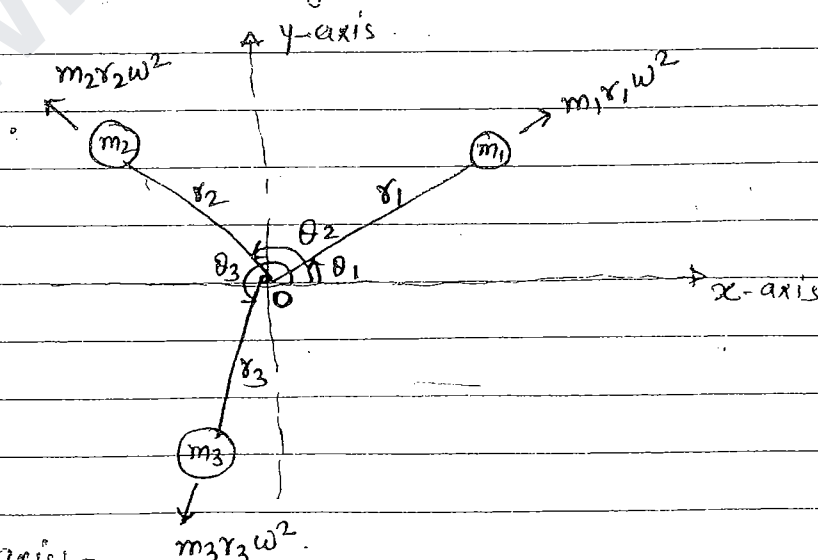
→ All the masses are rotating in the same plane.

For Balanced
 mass,

$m_B = ?$

$r_B = ?$

$\theta_B = ?$



Net force on $\vec{x}$ -axis: -

$$\sum \vec{F}_x = m_1 r_1 \omega^2 \cos \theta_1 + m_2 r_2 \omega^2 \cos \theta_2 + m_3 r_3 \omega^2 \cos \theta_3$$

$$= \sum (m r \omega^2) \cos \theta$$

For horizontal balancing,

$$\sum (mr \cancel{\omega^2} \cos \theta) + m_B r_B \cancel{\omega^2} \cos \theta_B = 0.$$

$$\Rightarrow m_B r_B \cos \theta_B = -\sum (mr \cos \theta) \quad \text{--- (1)}$$

Similar for
Vertical
balancing,

$$\text{and, } m_B r_B \sin \theta_B = -\sum (mr \sin \theta) \quad \text{--- (2)}$$

Now, $(1)^2 + (2)^2$

$$\Rightarrow m_B^2 r_B^2 = \{ -\sum mr \cos \theta \}^2 + \{ -\sum mr \sin \theta \}^2$$

$$\Rightarrow m_B r_B = \sqrt{\{ -\sum mr \cos \theta \}^2 + \{ -\sum mr \sin \theta \}^2}$$

and, $(2) \div (1)$, then,

$$\tan \theta_B = \frac{-\sum (mr \sin \theta)}{-\sum (mr \cos \theta)}$$

Note - If we cancel signs in Numerator & denominator

our selection ~~at~~ will be automatically cancelled;

irrespective of our examination.

Signs of the Numerator & denominator decides the quadrant of the balanced mass.

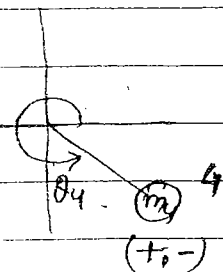
for eg:-

$$\sum mr \sin \theta = +4$$

$$\sum mr \cos \theta = -3.$$

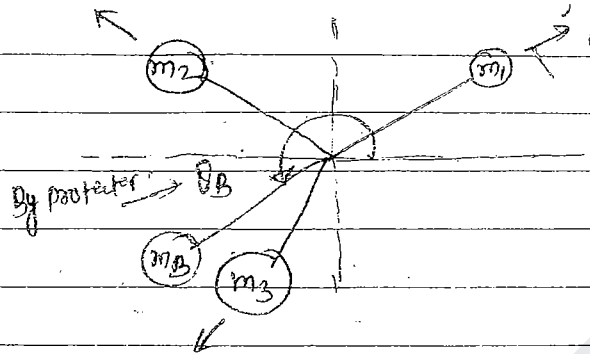
$$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\tan \theta_B = \frac{-(+4)}{-(-3)} = \frac{-4}{3} = \frac{4}{3}$$



Graphical Method:-

(a) Configuration:-



(b) Force:-

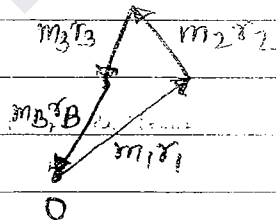
$$m_1 r_1 =$$

$$m_2 r_2 =$$

$$m_3 r_3 =$$

then find scale:-

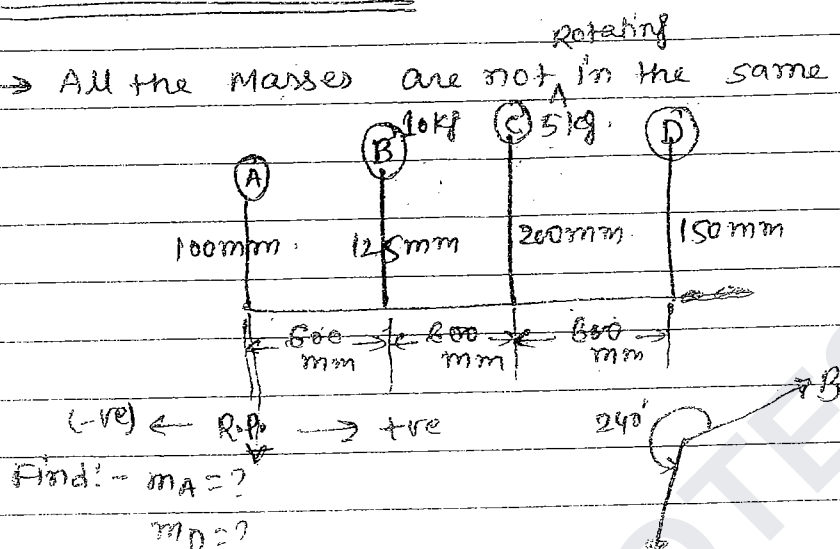
$m_B r_B \rightarrow$ Convert
By scale
and find
magnitude



Dynamic Balancing:-

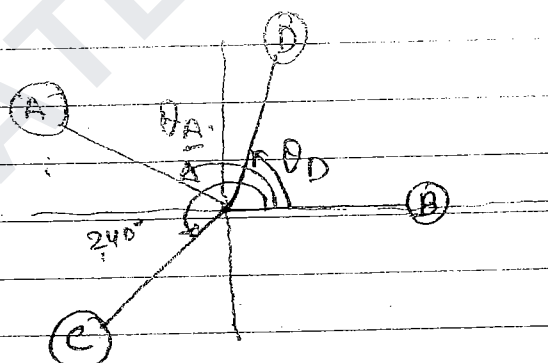
→ All the masses are not in the same plane.

eg:-
Prob:-



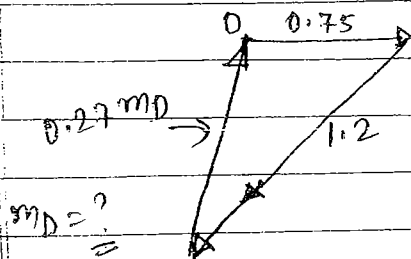
Sol:- Graphical

(a) Configuration:-



Moment:-

Scale used



Force:-

Scale used:-

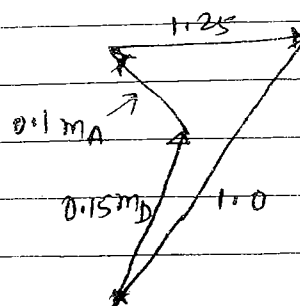




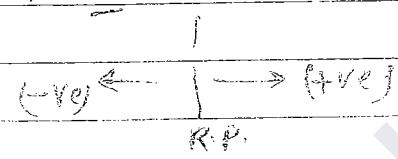
Table 1 -

Reference plane

Planes	m	x	$m \cdot x$	l	$m \cdot x \cdot l$
A	m_A	0.1	$0.1 m_A$	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1.0	1.2	1.2
D	m_D	0.15	$0.150 m_D$	1.8	$0.27 m_D$

NOTE:

⇒ Taking sign convention for distance measurement from the Reference Plane:

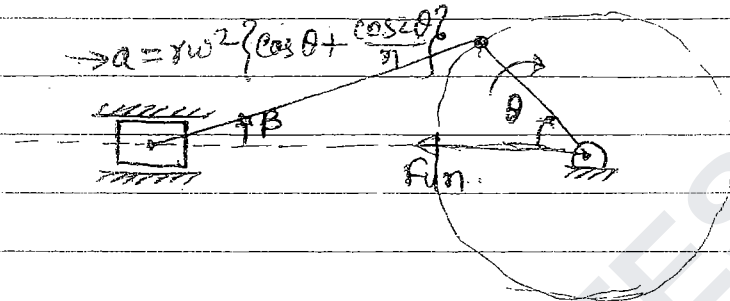


⇒ Draw moment diagram

Draw +ve moment → in direction of $\vec{F}$

Draw -ve moment → in opposite direction of $\vec{F}$

Balancing of Reciprocating Masses: -



$$F_{unbalance} = m r \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

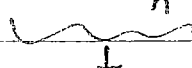


$$F_{unbalance} = m r \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

$$= m r \omega^2 \cos \theta + m r \omega^2 \frac{\cos 2\theta}{n}$$



$F_{primary}$

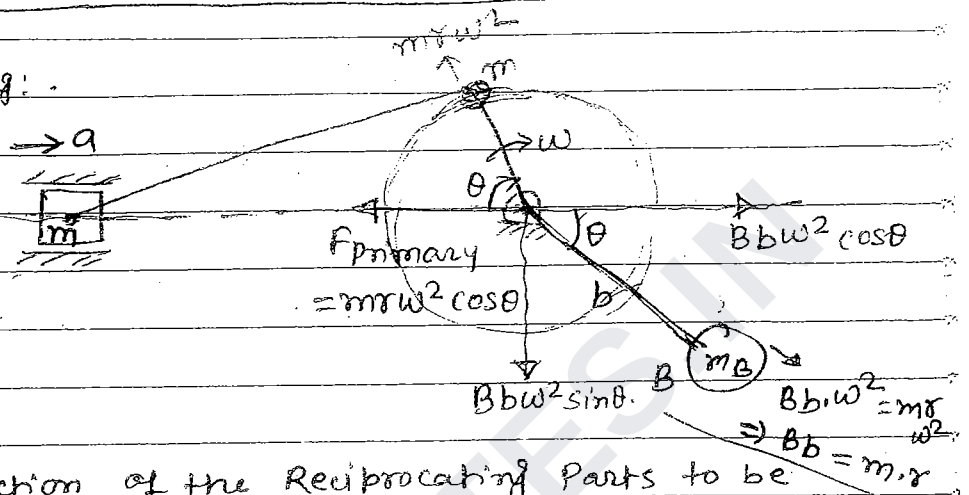


$F_{secondary}$

$$F_{secondary} \ll F_{primary}$$

Primary Balancing of the Reciprocating masses:-

↓
partial Balancing:-



Let c = fraction of the Reciprocating Parts to be Balanced. ($c < 1$).

where ($c < 1$).

$$B \cdot b = c m \cdot r$$

$$F_{un} \text{ (Along the line of stroke)} = (1-c) mr\omega^2 \cos \theta$$

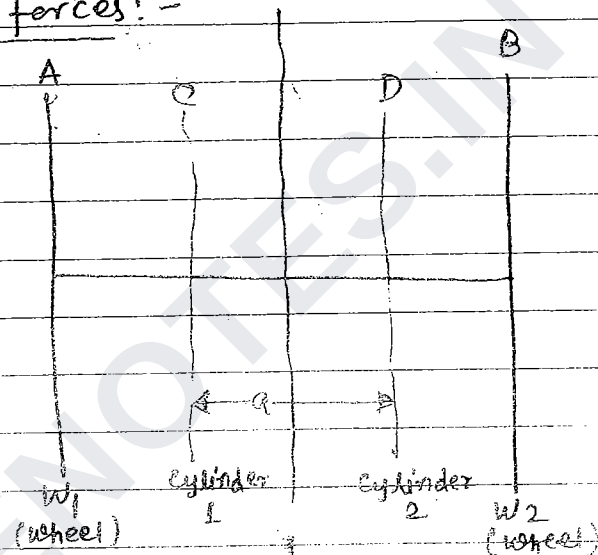
$$F_{un} \text{ (Perp to the line of stroke)} = c m r \omega^2 \sin \theta$$

Two cylinder's locomotive's :-

Effects of Partial Balancing in two cylinders locomotive :-
 → along the line of motion (traction).

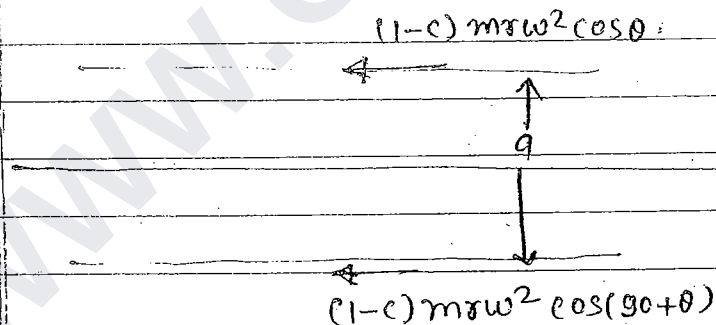
(1) Variation in Tractive forces :-

↓
 Ultimate Negative
 effects of partial Balancing)



Note:-

→ firing order in this engine is at 90° sets.



$$F_{unbalanced} = (1-c)mrw^2 \cos \theta + (1-c)mrw^2 \cos(90+\theta)$$

$$F_{unbalanced} = (1-c)mrw^2 \{ \cos \theta - \sin \theta \}$$

For max^m $F_{unbalanced}$ force,

$$\frac{d}{d\theta} (\cos \theta - \sin \theta) = 0$$

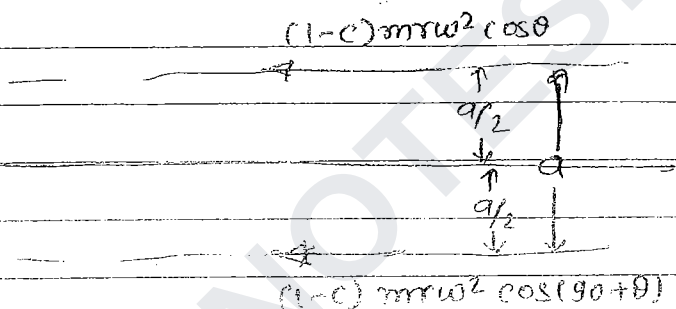
$$\Rightarrow \tan \theta = -1$$

$$\theta = 135^\circ, 315^\circ$$

$$\text{Variation in tractive forces} = \pm \sqrt{2} (1-c) m r \omega^2$$

② Swaying couple:-

(ultimate Negative effect of Partial Balancing):-



$$\text{Swaying couple} = \left\{ (1-c) m r \omega^2 \cos \theta - (1-c) m r \omega^2 \cos(90+\theta) \right\} \frac{a}{2}$$

$$= \frac{a}{2} (1-c) m r \omega^2 \{ \cos \theta + \sin \theta \}$$

For max^m swaying couple,

$$\frac{d}{d\theta} (\cos \theta + \sin \theta) = 0$$

$$\Rightarrow \tan \theta = +1$$

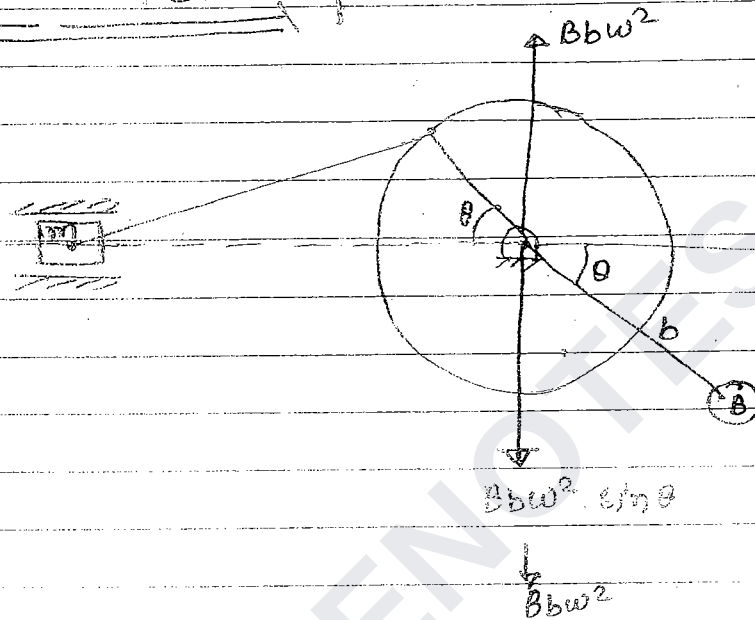
$$\theta = 45^\circ, 225^\circ$$

$$\text{Variation in swaying couple} = \pm \frac{a}{\sqrt{2}} (1-c) m r \omega^2$$

→ Swaying action inside or outside the tracks.

Biggest abuse in the history of locomotive :-

③ Hammer Blow



For Max^m value of $Bbw^2 \sin \theta$,

$$\sin \theta = 1 \quad (\text{Max}^m)$$

$$= Bbw^2$$

$$\Rightarrow \boxed{\text{Hammer Blow} = B \cdot b \cdot w^2}$$

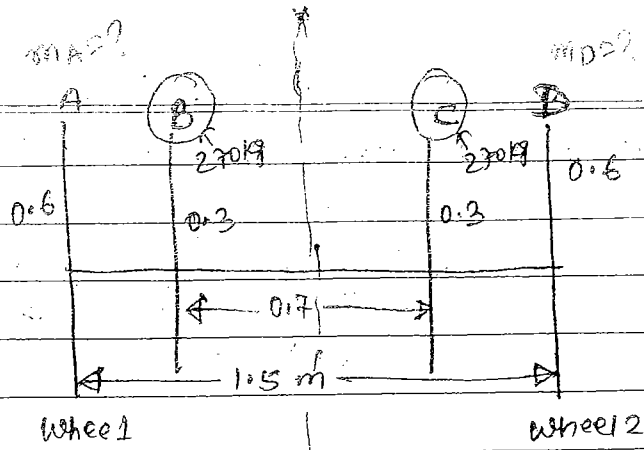
Balance mass

required to balance

c fraction of Reciprocating
mass of the engine.

$$\Rightarrow \boxed{Bbw^2 \leq \text{Static wt.}}$$

Prob: $Q \Rightarrow$



Stroke = 0.6 m

$\Rightarrow r = 0.3 \text{ m}$

Rotating masses/cylinder = 150 kg

Reciprocating masses/cylinder = 180 kg

distance between wheel = 1.5 m

$m_A = ?$ } Same
 $m_D = ?$ } = 105 kg

$\theta_A = ?$

$\theta_D = ?$

Balance: $(\text{Rotating} + \frac{2}{3} \text{ Reciprocating}) = 150 + \frac{2}{3} \times 180 = 270 \text{ kg}$

when $b = 0.6 \text{ m}$

$N = 300 \text{ rpm}$

Solⁿ: $\Rightarrow$

Table: -

Plane	m	r	mr	θ	mr θ
-------	---	---	----	----------	-------------

Variation of Inertive force

Radius of crank

$$= \pm \sqrt{2} (1-c) m r \omega^2$$

$$= \pm \sqrt{2} (1-\frac{2}{3}) \times 180 \times 0.3 \times \sqrt{1}$$

Sweating couple:-

$$\pm \frac{q}{\sqrt{2}} (1-c) m r \omega^2$$

Hammer blow:-

$$B \cdot b \cdot \omega^2$$

$$Bb = cmr$$

$$\Rightarrow 60 \times 0.6 \times \left(\frac{2\pi \times 30}{60} \right)^2$$

$$\Rightarrow B \times 0.6 = \frac{2}{3} \times 180 \times 0.3$$

$$= 1130.97$$

$$\Rightarrow B = 60$$

$$35530.57$$

$$270 \rightarrow 105$$

$$1 \rightarrow 105$$

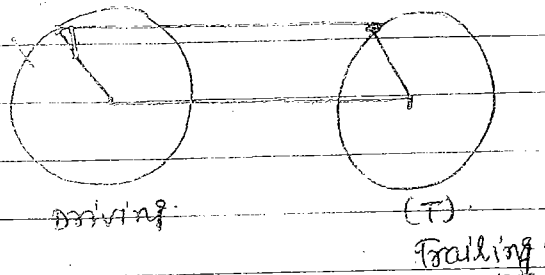
$$270$$

$$120 \rightarrow \frac{105}{270} \times 120 = 46.6718$$

$$= 46.67 \times 0.6 \times \left(\frac{2\pi \times 30}{60} \right)^2$$

$$= 27636.86$$

$$120 \times (986.96) \\ 120 \times 986.96 \\ 118435.2$$

Coupled wheels locomotives :-

prob) $\Rightarrow$ Stroke = 650 mm

Mass of Revolving Parts/cy = 200 kg

Mass of Reciprocating Parts/cylinder = 240 kg

Mass of each Coupling rod = 250 kg

Radius of centre of coupling rod pin = 65 mm

Distance b/w cylinders = 0.6 m

Distance b/w wheels = 1.5 m

Distance b/w coupling rod = 1.8 m

The main cranks are right angles and the coupling rods pins are 180° to their respective main cranks. The balanced masses are to be placed in the wheels and the mean radius of 675 mm in order to balance complete rotating and $3/4$ to the reciprocating. The Balanced mass for the reciprocating masses is to be divided equally between the driving wheel & trailing wheel. Speed of rotation is 300 rpm.

(i) Balance the system.

(ii) Find the value of Hammer blow per wheel.

Solⁿ $\Rightarrow$

Rotating/cylinder = 200 kg

$\downarrow$
Balance on driving wheel.

Reciprocating/cylinder = 240 kg

$$= \frac{3}{4} \times 240$$

$$= 180 \text{ kg}$$

$\swarrow \quad \searrow$
90 kg (Driving) 90 kg (Trailing)

Coupling rod $\rightarrow$ 250 kg

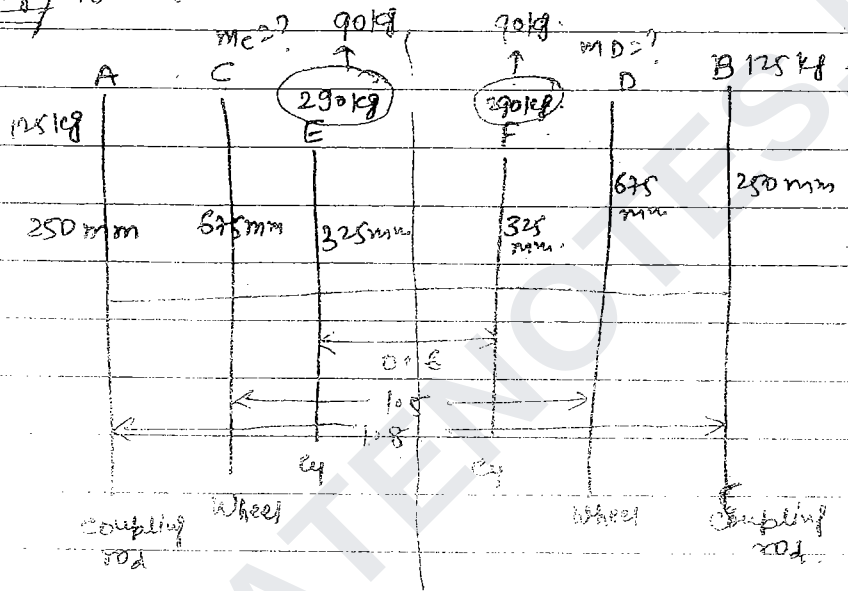
125 kg

(Driving)

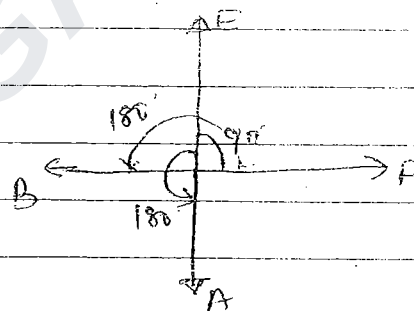
125 kg

(Trailing)

Driving / Trailing



Polygon.



$$\begin{matrix} m_C = ? & \theta_C = ? \\ m_D = ? & \theta_D = ? \end{matrix}$$

$\rightarrow$ Hammer blow Can be calculated by without considering the coupling rod & it's 4 plane problem and then solve it & it's similar for trailing & driving wheel.

$$B = cm.$$

Driving:-

$$m_C = m_D = 66.7 \text{ kg}$$

θ_C

θ_D

Trailing:-

$$m_C = m_D = 27.2 \text{ kg}$$

Hammer blow

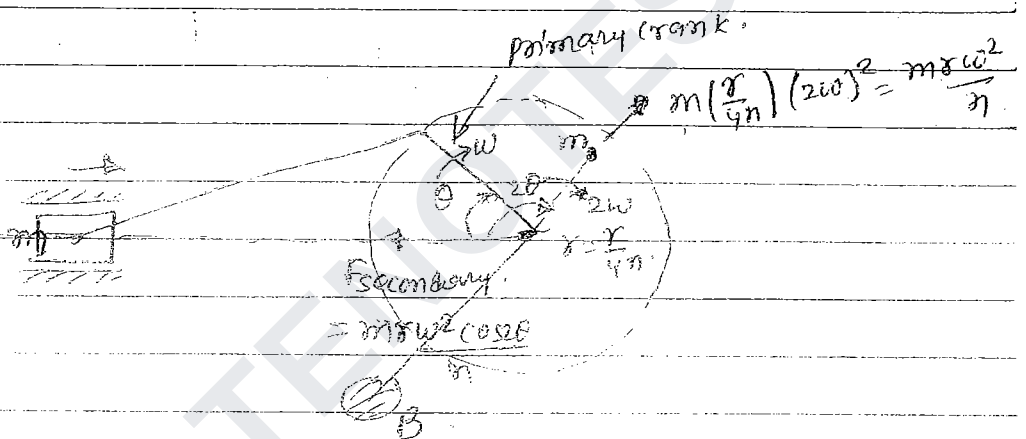
~~max. max~~

$$B = 33 \text{ kg}$$

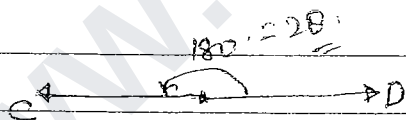
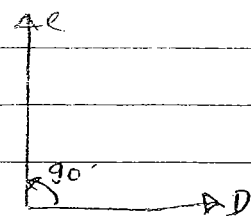
Secondary Balancing:-

$$\frac{m r \omega^2 \cos \theta}{m r \omega^2 \frac{\cos 2\theta}{n}}$$

$$= m \left(\frac{r}{4n} \right) (2\omega)^2 \cos 2\theta$$



Polygon of secondary:

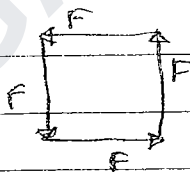
Secondary crank
diagram.primary crank
diagram

Role of firing order in Balancing:-

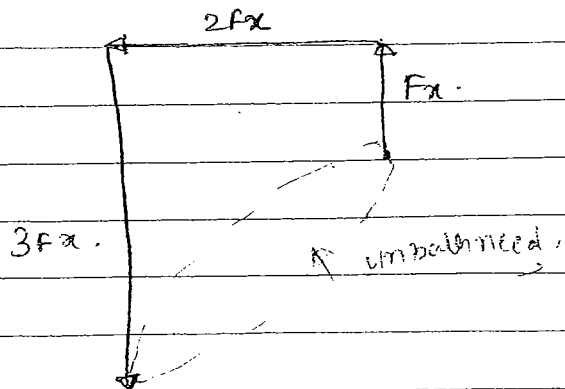
	1	2	3	4
				
Force:-	F	F	F	F
Moment:-	0	Fx	$2Fx$	$3Fx$

Firing order

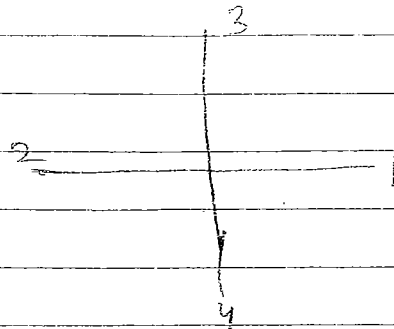
Force:-



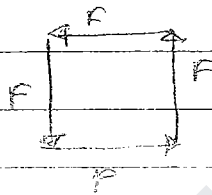
Moment:-



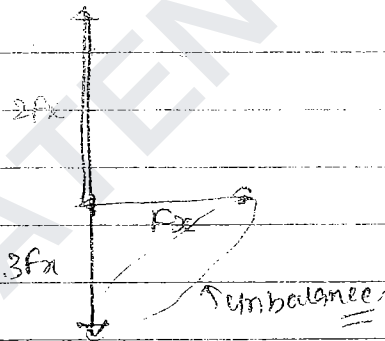
Firing order:-



Force:-

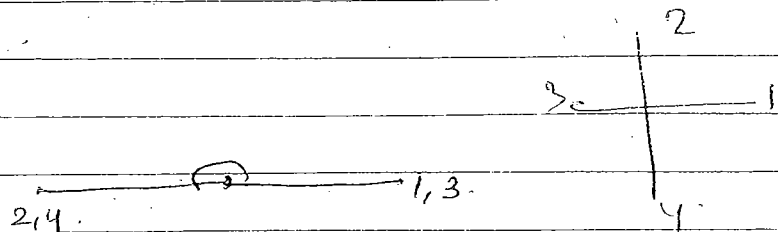


Moment:-



Four cylinders are not fully balanced but six cylinders are fully balanced primary as well as secondary.

Secondary balancing:-

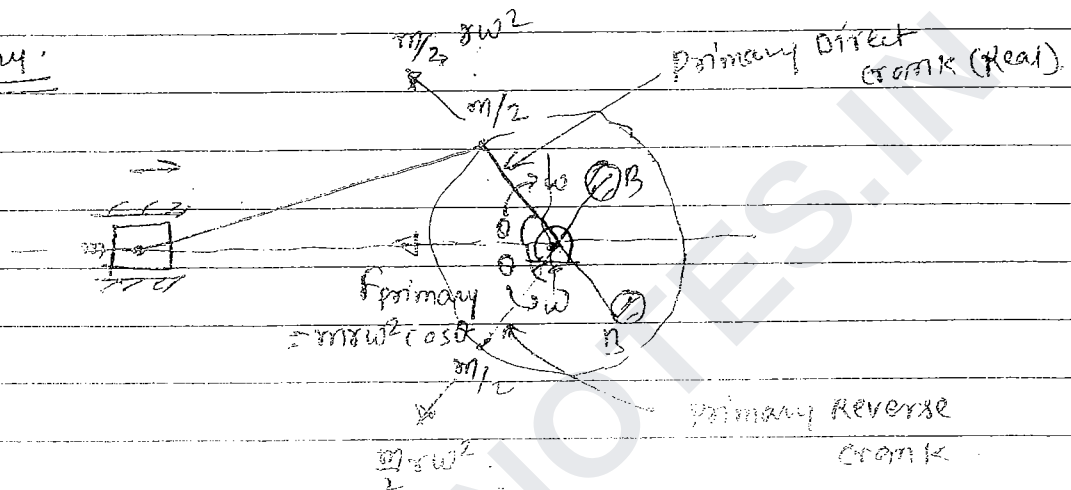


Secondary (SRC)

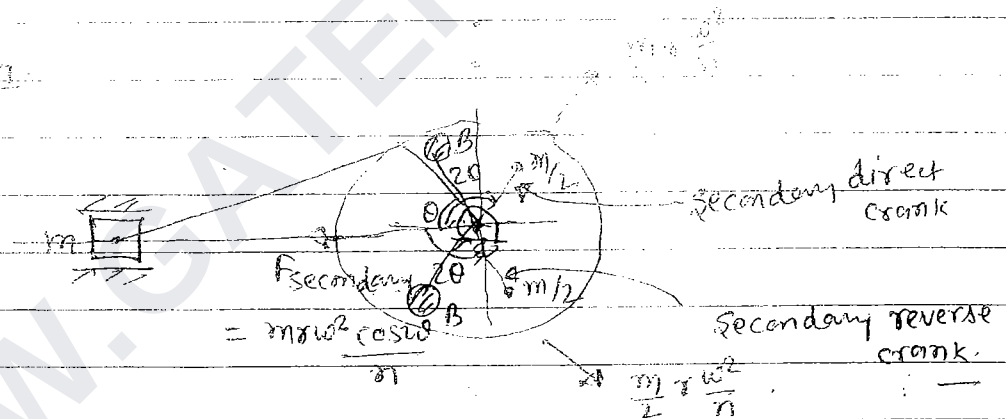
Direct & Reverse Crank Technique! -

→ Mostly applied in Radial engines.

Primary



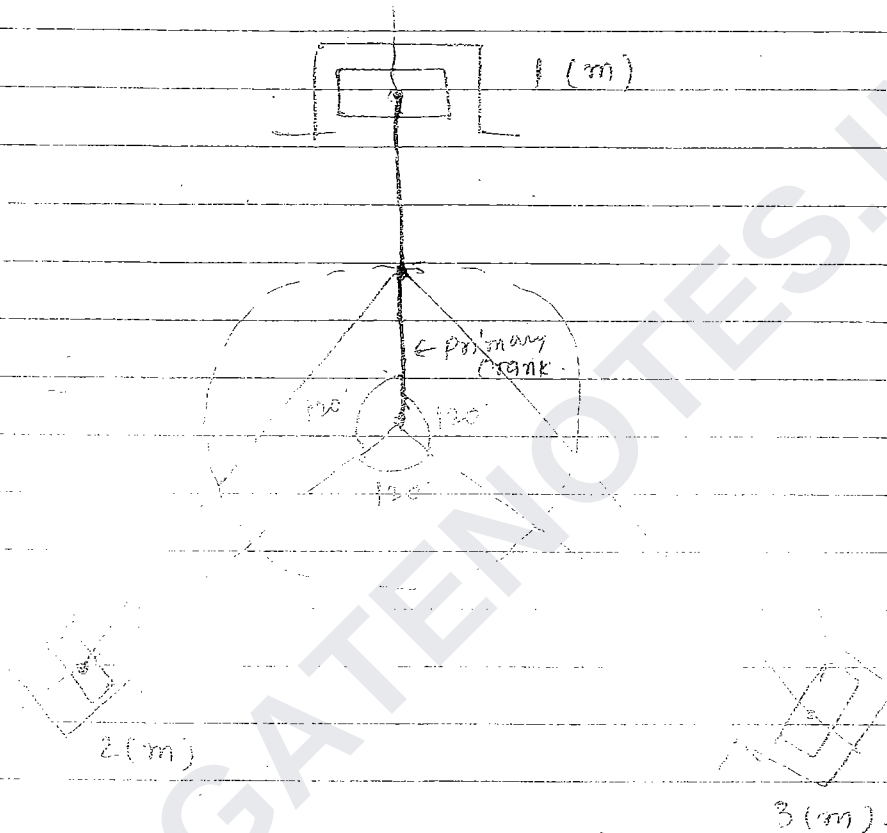
Secondary



→ In this case five Rotating masses two for each primary & secondary & one crank and system are fully balanced.

Three cylinder radial engine:-

→ Radial engine ^{are} single crank or only one crank.

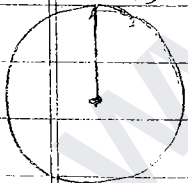


$$F_{\text{primary}} = ?$$

$$\uparrow \left(\frac{m}{2} r \omega^2 \right) \times 3$$

$$\frac{m}{2} \quad \frac{m}{2} \quad \frac{m}{2}$$

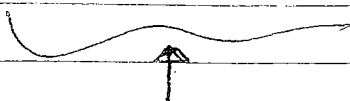
$$1, 2, 3$$



Primary Direct



$$\frac{3}{2} m r \omega^2$$



$$\frac{3}{2} m r \omega^2$$

Net Resultant = zero

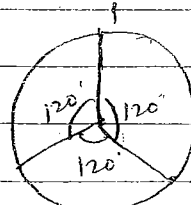
→ Similarly proceed for v-engines.

$$F_{\text{secondary}} = ?$$

$$\uparrow \left(\frac{m}{2} \left(\frac{r}{4n} \right) (2\omega)^2 \right) \times 3$$

$$\frac{m}{2} \quad \frac{m}{2} \quad \frac{m}{2}$$

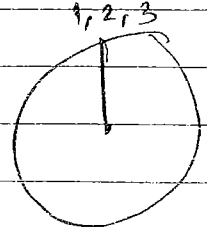
$$1, 2, 3$$



Secondary Direct



Net resultant = zero



Secondary Reverse

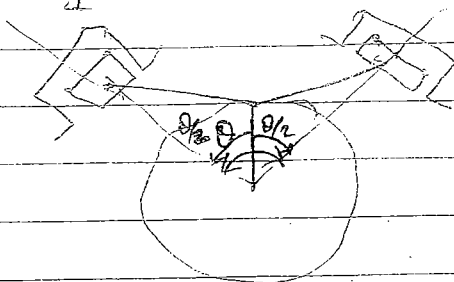
$$\uparrow \frac{3m}{2} r \frac{\omega^2}{n}$$



$$\frac{3}{2} m r \frac{\omega^2}{n}$$

V-Engines:-

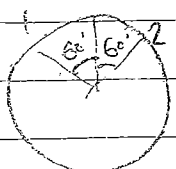
Prob. of:-



D-V engine

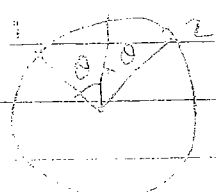
$$\theta = ?$$

where

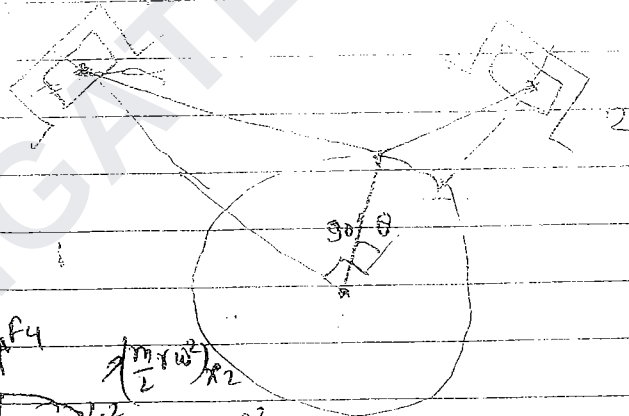


PRE

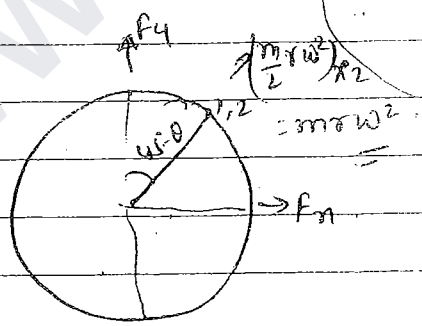
Soln:-



$\frac{d}{d\theta}$ max min



=>



Posmay direct

(Independent of theta)

Ans. for max & minimum

11. Vibrations

classmate

Date 23/08/10
Page 129

Vibration:-

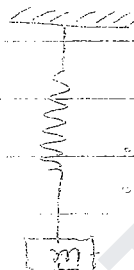
↓
System

Natural Vibration

- { (i) K.E. storing device ($m \rightarrow m$),
(ii) P.E. storing device (stiffness $\rightarrow S$),
(iii) Friction,
(iv) Harmonic unbalanced force }

for
vibrating
system

Natural Vibrations:-



mean position

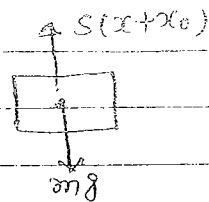
or equilibrium position

$$mg = Sx_0$$

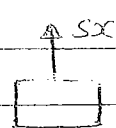


Displaced position

By Newton's Second law
(in the direction of displacement),



⇒



$$(0: mg = Sx_0)$$

$$0 - Sx = ma$$

$$\Rightarrow ma + Sx = 0$$

$$\Rightarrow a + \frac{S}{m}x = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{S}{m}\right)x = 0$$

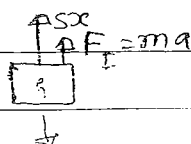
By D'Alembert's Principle:-

Inertia force is opposite to the displacement,

$$ma + Sx = 0$$

$$\Rightarrow a + \frac{S}{m}x = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{S}{m}\right)x = 0$$



Harmonic $\Rightarrow$ Toss for periodic

Now, $\ddot{x} + \left(\frac{s}{m}\right)x = 0$

$\Rightarrow \frac{d^2x}{dt^2} + \left(\frac{s}{m}\right)x = 0$

$x = R \sin\left(\sqrt{\frac{s}{m}}t + \phi\right)$ is the solution of the equation

(i) $t=0 \begin{cases} x=x_0 \\ \dot{x}=0 \end{cases} \left. \begin{matrix} R, \phi = ? \end{matrix} \right\} \text{(Initial conditions)}$

(ii) $t=0 \begin{cases} x=0 \\ \dot{x}=v_0 \end{cases} \left. \begin{matrix} R, \phi = ? \end{matrix} \right\}$

(iii) $t=0 \begin{cases} x=x_0 \\ \dot{x}=v_0 \end{cases} \left. \begin{matrix} R, \phi = ? \end{matrix} \right\}$

$x = R \sin\left(\sqrt{\frac{s}{m}}t + \phi\right)$ is solⁿ

Harmonic function

$\downarrow$
Vibration

Natural frequency $= \omega_n = \sqrt{\frac{s}{m}}$

$\downarrow$
equation, $\boxed{\ddot{x} + (\omega_n)^2 x = 0}$

Energy method:-

$$\text{Time period (T)} = \frac{2\pi}{\omega_n}$$

$$E = \frac{1}{2} mv^2 + \frac{1}{2} sx^2 = \text{constant}$$

$$f = \frac{1}{T}$$

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \frac{1}{2} m (2v) \frac{dv}{dt} + \frac{1}{2} s (2x) \frac{dx}{dt} = 0$$

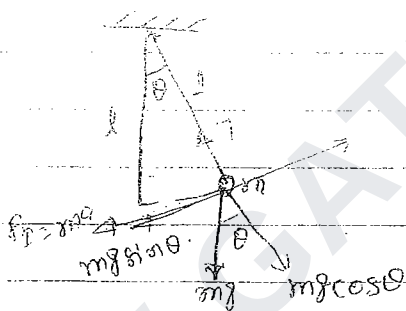
$$\Rightarrow m \cdot \frac{dv}{dt} + sx = 0$$

$$\Rightarrow m \cdot a + sx = 0$$

$$\Rightarrow \ddot{x} + \frac{s}{m} x = 0$$

eg:-

①



$$ma + mg \sin \theta = 0$$

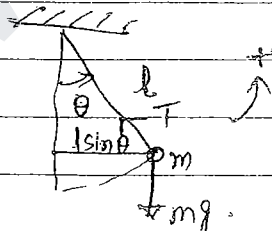
$$\Rightarrow a + g \sin \theta = 0$$

$$\Rightarrow a + g \cdot \frac{x}{l} = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{g}{l} \right) x = 0$$

Torque method:-

②

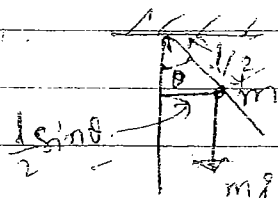


$$I \cdot \alpha + mg \cdot l \sin \theta = 0$$

$$(ml^2) \cdot \ddot{\theta} + mg l \cdot \theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(\frac{g}{l} \right) \theta = 0$$

③

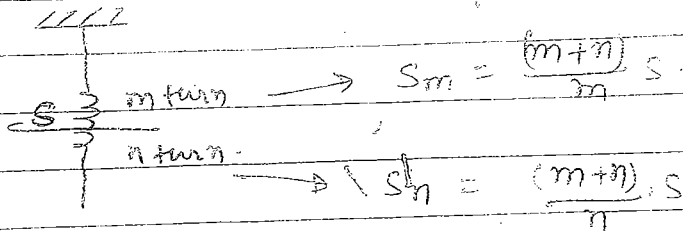


$$I \alpha + mg \cdot \frac{l}{2} \sin \theta = 0$$

$$\Rightarrow \frac{ml^2}{3} \cdot \ddot{\theta} + mg \cdot \frac{l}{2} \theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(\frac{3g}{2l} \right) \theta = 0 \Rightarrow \omega_n = \sqrt{\frac{3g}{2l}}$$

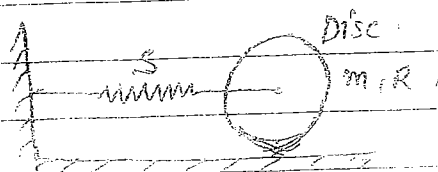
Note:-



series:- $\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2}$

parallel:- $S_{eq} = S_1 + S_2$

Prob:-



$$E = \frac{1}{2} S x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{m R^2}{2} \right) \left(\frac{v}{R} \right)^2$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} \left(\frac{3m}{2} \right) v^2$$

$$\omega_n = \sqrt{\frac{S}{\frac{3m}{2}}} = \sqrt{\frac{2S}{3m}} \underline{\underline{Ans}}$$

or,

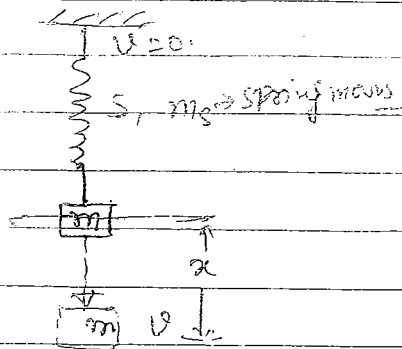
$$E = \frac{1}{2} S x^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} \left(\frac{3}{2} M R^2 \right) \times \frac{v^2}{R^2}$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} \left(\frac{3}{2} m \right) v^2$$

$$\omega_n = \sqrt{\frac{2S}{3m}} \underline{\underline{Ans}}$$

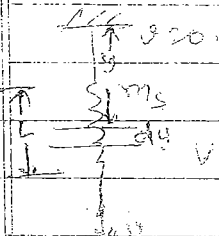
Q. 22)



$$E = \frac{1}{2} Sx^2 + \frac{1}{2} mv^2 + \frac{1}{2} \left(\frac{ms}{3} \right) v^2$$

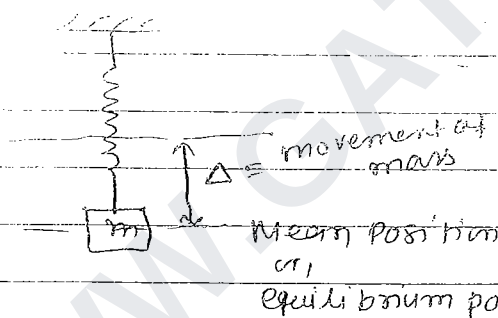
$$E = \frac{1}{2} Sx^2 + \frac{1}{2} \left(m + \frac{ms}{3} \right) v^2$$

$$\omega_n = \sqrt{\frac{S}{m + \frac{ms}{3}}}$$



$$v = \left(\frac{v}{L} y \right) = \int_0^L \frac{1}{2} \left(\frac{ms}{L} \right) dy \left(\frac{v}{L} y \right)^2$$

Method of static Deflection of Mass:



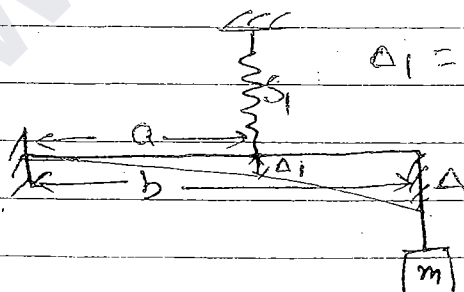
$$\Delta = \frac{mg}{S}$$

$$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{\frac{mg}{S}}} = \sqrt{\frac{S}{m}} = \omega_n$$

$$\Rightarrow \omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{Sg}{m}}$$

Seq. find for all types of problem which are either series or parallel

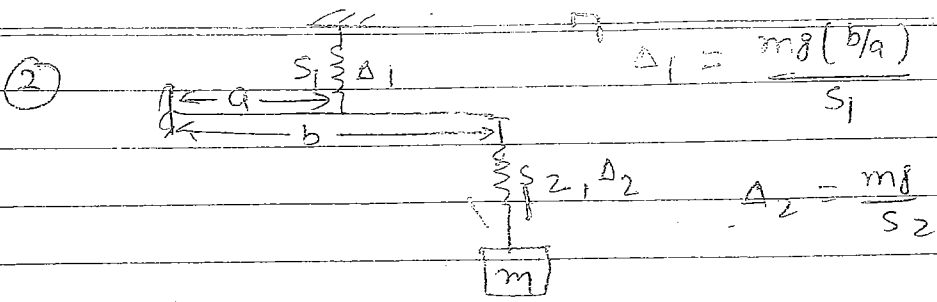
①



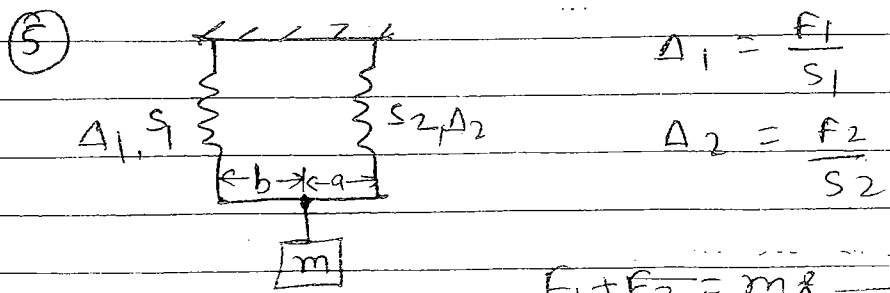
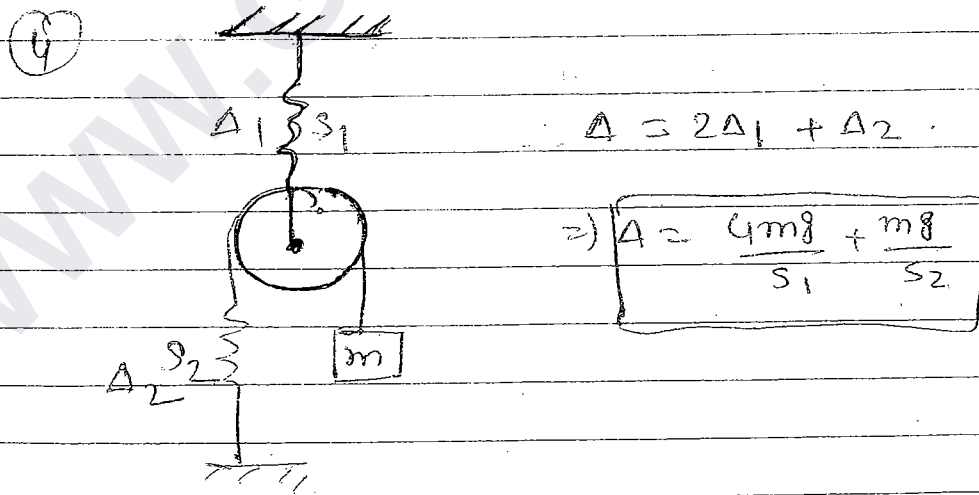
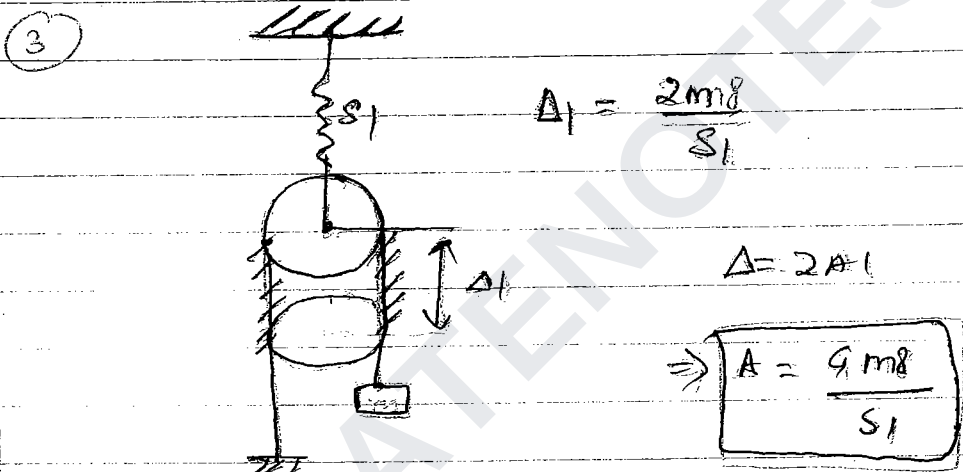
$$\Delta_1 = \frac{mg(b/a)}{S_1}$$

$$\Delta = \left(\Delta_1 \cdot \frac{b}{a} \right)$$

$$\Rightarrow \Delta = \frac{mg}{S_1} \left(\frac{b}{a} \right)^2$$



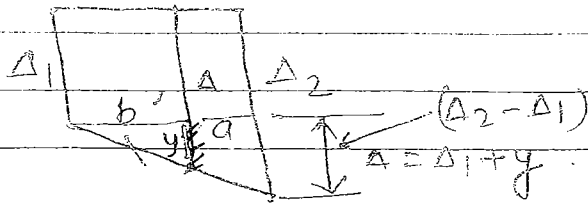
$$\Delta = \Delta_1 \left(\frac{b}{a} \right) + \Delta_2$$



$F_1 + F_2 = mg$ — ①

$F_1 \cdot b = F_2 \cdot a$ — ②

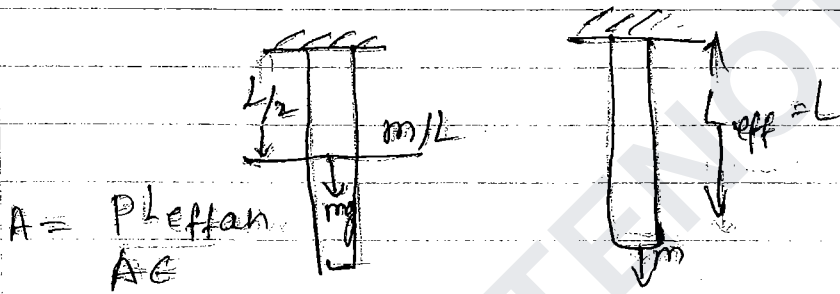
By these two find F_1 & F_2



$$\Delta = \Delta_2 + y$$

y can find by similar triangle.

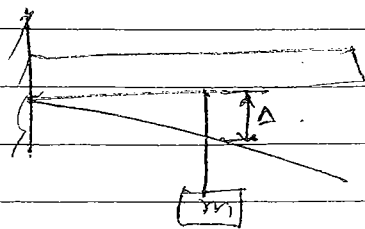
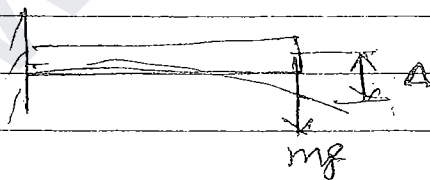
Longitudinal & Transverse Vibrations of the Beam: -



$$A = \frac{P_{\text{eff}}}{A E}$$

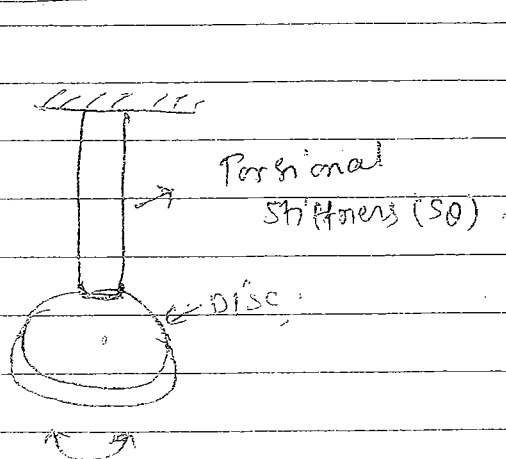
$$\omega_n = \sqrt{\frac{g}{A}}$$

$$\omega_n = \sqrt{\frac{g}{A}} = \sqrt{\frac{g_{\text{eq}}}{m}}$$



By this we can find the equivalent stiffness of the beam.

Torsional Vibration:-



$$I \ddot{\theta} + S_0 \theta = 0$$

$$I \ddot{\theta} + S_0 \theta = 0$$

$$\Rightarrow I \ddot{\theta} + S_0 \theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(\frac{S_0}{I} \right) \theta = 0$$

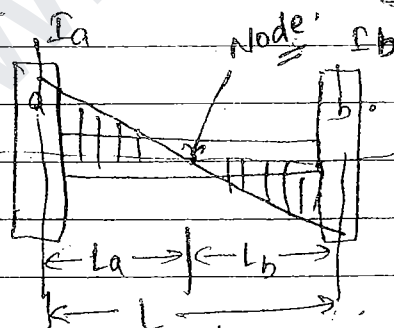
$$\omega_n = \sqrt{\frac{S_0}{I}}$$

If shaft is also having I_{shaft} & mass of shaft then

$$\omega_n = \sqrt{\frac{S_0}{I + \frac{I_{shaft}}{3}}}$$

Two Rotor:-

Shaft is assumed to be massless.



When No. of rotor = n.

Node point = (n-1)

$$L_a + L_b = L \quad \text{--- (1)}$$

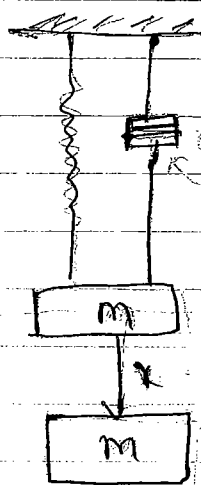
$$\sqrt{\frac{S_0 a}{I_a}} = \sqrt{\frac{S_0 b}{I_b}} \Rightarrow \frac{S_0 a}{I_a} = \frac{S_0 b}{I_b}$$

$$\Rightarrow \frac{GJ}{I_a \cdot L_a} = \frac{GJ}{I_b \cdot L_b} \Rightarrow \frac{I_a}{L_b} = \frac{I_b}{L_a} \quad \text{--- (2)}$$

Ex ① & ② find Ia & Ib.

Damped Vibration:-

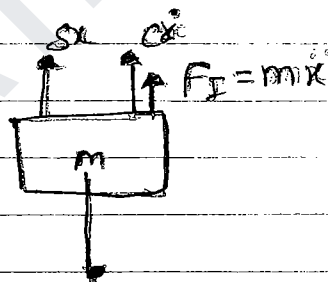
Friction $\neq 0$.



c = Damping coefficient

Damper (used for forerun symbol)

mean on equilibrium position



A/c to D'Alembert,

$$m\ddot{x} + c\dot{x} + Sx = 0$$

$$\Rightarrow \ddot{x} + \left(\frac{c}{m}\right)\dot{x} + (W_n)^2 x = 0$$

Soln:

$$x = (Ae^{\alpha_1 t} + Be^{\alpha_2 t}) \quad \text{where } (\alpha_1 \neq \alpha_2)$$

$$= (A + Bt)e^{\alpha t} \quad \text{where } (\alpha_1 = \alpha_2 = \alpha)$$

where, α_1 & α_2 are the roots of Auxiliary equation

$$\alpha^2 + \left(\frac{c}{m}\right)\alpha + (W_n)^2 = 0$$

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}}{2}$$

$$= \frac{-\frac{c}{2m} \pm \sqrt{\underbrace{\left(\frac{c}{2m}\right)^2}_{\text{damping}} - \underbrace{(\omega_n^2)}_{\text{spring dominating force}}}}$$

$$\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2} = \text{degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2}} = \zeta = \text{Damping factor or, Damping ratio (denoted by } \zeta \text{)}$$

$$\Rightarrow \zeta = \sqrt{\frac{\frac{c^2}{4m^2}}{\frac{8}{m}}} = \frac{c}{2\sqrt{8m}}$$

$$\Rightarrow 2\zeta\omega_n = 2 \times \frac{c}{2\sqrt{8m}} \times \sqrt{\frac{8}{m}} = \frac{c}{m}$$

Then the equation comes out,

$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = 0$$

Now,

$$\alpha_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1})\omega_n$$

Remarks: -

- ① when $\zeta > 1$ then → over damped system.
↓
No vibration
- ② when $\zeta < 1$, then → under damped system.
↓
vibration.
- ③ when $\zeta = 1$, then → Critically damped system.
↓
No vibration condition or limit.

Over damped systems: -

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Over Damped System: -

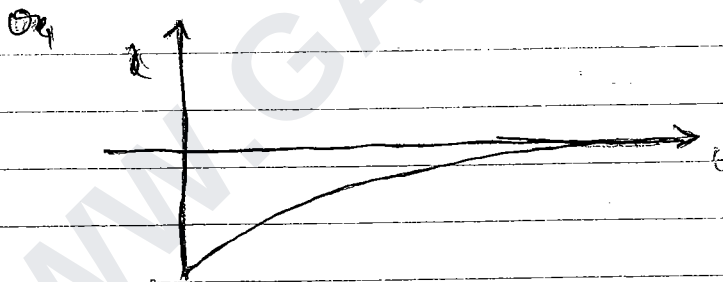
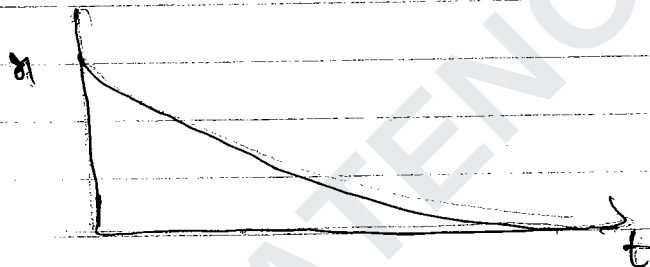
$$\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = 0$$

$$\alpha_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1})\omega_n$$

Case-I. over Damped system ($\zeta > 1$)

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = Ae^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + Be^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t}$$



No vibration.

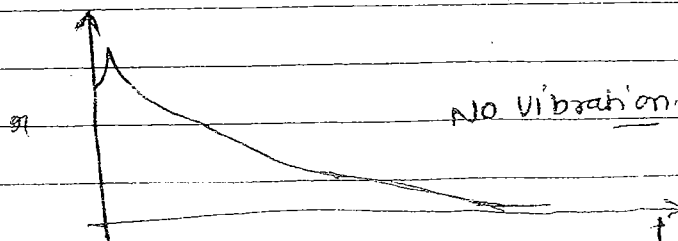
No vibration zone

Critically Damped system: -

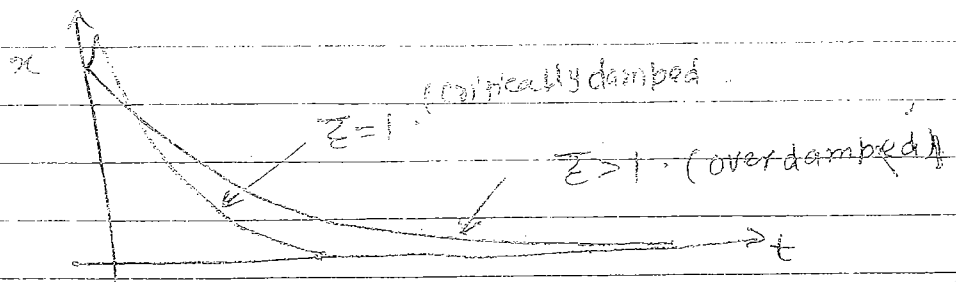
$$\zeta = 1$$

$$\alpha_1 = \alpha_2 = \alpha = -\omega_n$$

$$\Rightarrow x = (A + Bt)e^{-\omega_n t}$$



No vibration.



combined diagram for critically damped & over damped.

Under - Damped system ($\zeta < 1$)

$$\alpha_{1,2} = -\zeta\omega_n \pm i \sqrt{1-\zeta^2} \omega_n$$

↓
 ω_d (Subbase)

$$\Rightarrow \alpha_{1,2} = (-\zeta\omega_n \pm i\omega_d)$$

$$\Rightarrow x = Ae^{(-\zeta\omega_n + i\omega_d)t} + Be^{(-\zeta\omega_n - i\omega_d)t}$$

$$= e^{-\zeta\omega_n t} \left[\underbrace{(A+B) \cos \omega_d t}_{X \sin \phi} + i \underbrace{(A-B) \sin \omega_d t}_{X \cos \phi} \right]$$

$$\Rightarrow x = X e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$$

$$\Rightarrow x = \underbrace{X e^{-\zeta\omega_n t}}_{\text{Amplitude}} \underbrace{\sin(\omega_d t + \phi)}_{\text{vibration}}$$

Amplitude

$f_n(t)$

$t \uparrow \Rightarrow A \downarrow$

vibration

frequency damped

$$\omega_d = (\sqrt{1-\zeta^2}) \omega_n$$

$$= \text{const.} < \omega_n$$

$$\text{Time period } (T_d) = \frac{2\pi}{\omega_d}$$

~~Decrement~~
Decrement
table

Logarithmic decrement (δ): -

$$\delta = \ln_e e^{\xi \omega_n T_d}$$

$$= \frac{\xi \omega_n \cdot 2\pi}{\sqrt{1-\xi^2} \cdot \cancel{\omega_n}}$$

$$\Rightarrow \boxed{\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}}} = \text{logarithmic decrement}$$

Q. - when $\delta = 2$

then, $x_0 = 2$
 x_5

$$\therefore \frac{x_0}{x_5} = \frac{x_0}{x_1} \times \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5}$$

$$= \left(\frac{x_0}{x_1}\right)^5 = (e^2)^5$$

$$= e^{10} \text{ Ans.}$$

Conventional
Work Book

Q. -

(5)

$$\eta = 7.5 \text{ kg}$$

$$T_d = \frac{35}{60} \text{ sec.} \Rightarrow \frac{2\pi}{\omega_d} = \frac{35}{60} \Rightarrow \omega_d = 10.7711 \text{ rad/s.}$$

$$\Rightarrow 10.7711 = \underline{\omega_d}$$

$$\frac{x_0}{x_7} = 2.5$$

$$\Rightarrow \left(\frac{x_0}{x_1}\right)^7$$

$$\Rightarrow \frac{x_0}{x_1} = (2.5)^{1/7}$$

$$\Rightarrow \delta = \ln (2.5)^{1/7} = \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow (0.13089)^2 =$$

$$\Rightarrow 0.01713 = \frac{4\pi \xi^2}{1-\xi^2}$$

$$\Rightarrow \xi = 0.02082$$

$$\xi = 0.03690$$

$$\Rightarrow 1.3635 \times 10^{-3}$$

PLEASE CALL RE SWIFT
BY MISTAKE

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ALL CAN BE SWEEP
BY MISTAKE

WWW.GATENOTES.IN

$$\omega_d = \sqrt{1 - \xi^2} \cdot \omega_n$$

$$\Rightarrow \omega_n = \frac{\omega_d}{\sqrt{1 - \xi^2}} = \frac{10.7711}{\sqrt{1 - 0.0208^2}} = 10.7734 \text{ rad/sec.}$$

$$\Rightarrow \omega_n = \sqrt{\frac{s}{m}}$$

$$\Rightarrow s = \omega_n^2 \times m = 870.5 \text{ N/m}$$

$$\text{and } 2\xi\omega_n = \frac{c}{m}$$

$$c = 2\xi\omega_n \times m = 2.36 \text{ N/m/s}$$

$$\text{Now, } 2\xi\omega_n = \frac{c}{m}$$

$$2 \times 1 \times \omega_n = \frac{c_c}{m}$$

$$\Rightarrow \xi = \frac{c}{c_c} = \frac{\text{Actual damping Co-efficient}}{\text{Critical Damping Co-efficient}}$$

Definition of ξ

$$\Rightarrow c_c = 161.38 \text{ N/m/s}$$

For velocity & Acceleration calculation:

at $t=0$

$$x=0$$

$$\dot{x} = 120 \text{ mm/s.}$$

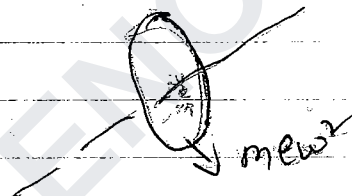
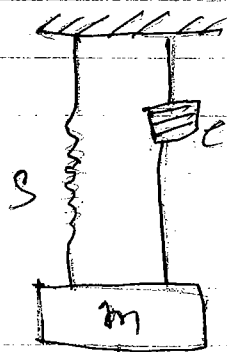
$$= 0.120 \text{ m/s.}$$

$$\Rightarrow x = x e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$$

By initial condition we can find x and ϕ and then we calculate velocity & acceleration at any time.

Forced Damped vibration:-

(Perfect Reality associated with vibrating system)!



unbalanced force:
due to
Rotation!

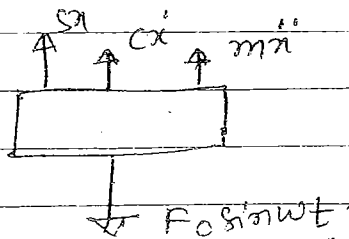
$$F_0 = m e \omega^2$$

Same procedure for
Reciprocation for
find F_0 & ω in any
system

$F_0 \sin \omega t$ } unbalance
Harmonic force

ω = force frequency

FBD (Free body diagram)



$$\Rightarrow (m\ddot{x} + c\dot{x} + Sx - F_0 \sin \omega t) = 0$$

$$\Rightarrow \ddot{x} + \frac{c}{m} \dot{x} + \frac{S}{m} x = \left(\frac{F_0}{m} \right) \sin \omega t$$

$$\Rightarrow \ddot{x} + (2\zeta \omega_n) \dot{x} + \omega_n^2 x = (F_0/m) \sin \omega t$$

$$x = CF + PI$$

$$CF = \begin{cases} \xi < 1 \\ \xi = 1 \\ \xi > 1 \end{cases} = \frac{1}{\omega_n} e^{-\xi \omega_n t} \sin(\omega_d t + \phi_1) \quad (\text{for } \xi < 1)$$

PI

$$PI = \frac{(F_0/m) \sin \omega t}{D^2 + (2\xi \omega_n)D + \omega_n^2} \quad \left\{ \begin{array}{l} D \rightarrow \text{operator} \\ D^2 = -\omega^2 \end{array} \right.$$

$$= \frac{(F_0/m) \sin \omega t}{(\omega_n^2 - \omega^2) + (2\xi \omega_n)D}$$

$$= \frac{(F_0/m) \sin \omega t}{(\omega_n^2 - \omega^2) + (2\xi \omega_n)D} \cdot \frac{(\omega_n^2 - \omega^2) - 2\xi \omega_n D}{(\omega_n^2 - \omega^2) - 2\xi \omega_n D}$$

$$= \frac{(F_0/m)}{(\omega_n^2 - \omega^2)^2 + (2\xi \omega_n \omega)^2} \left\{ \underbrace{(\omega_n^2 - \omega^2) \sin \omega t}_{\omega \cos \phi} - \underbrace{(2\xi \omega_n \omega) \cos \omega t}_{\omega \sin \phi} \right\}$$

$$= \frac{(F_0/m) \sin(\omega t - \phi)}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi \omega_n \omega)^2}}$$

$$\because \omega_n = \sqrt{\frac{s}{m}}$$

$$= \frac{(F_0/s)}{\sin(\omega t - \phi)}$$

$$\sqrt{\left\{ 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right\}^2 + \left\{ \frac{2\xi \omega}{\omega_n} \right\}^2}$$

$\frac{\partial x}{\partial t}$ space $\rightarrow$ velocity
 $\frac{\partial^2 x}{\partial t^2}$ space $\rightarrow$ acceleration

$\frac{\partial P}{\partial t}$ space $\rightarrow$ steady state

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Amplitude $\approx$ const

$$x = x e^{-\zeta \omega_n t} \sin(\omega_n t + \phi_1) +$$

(F₀/s)

$$\sin(\omega t - \phi)$$

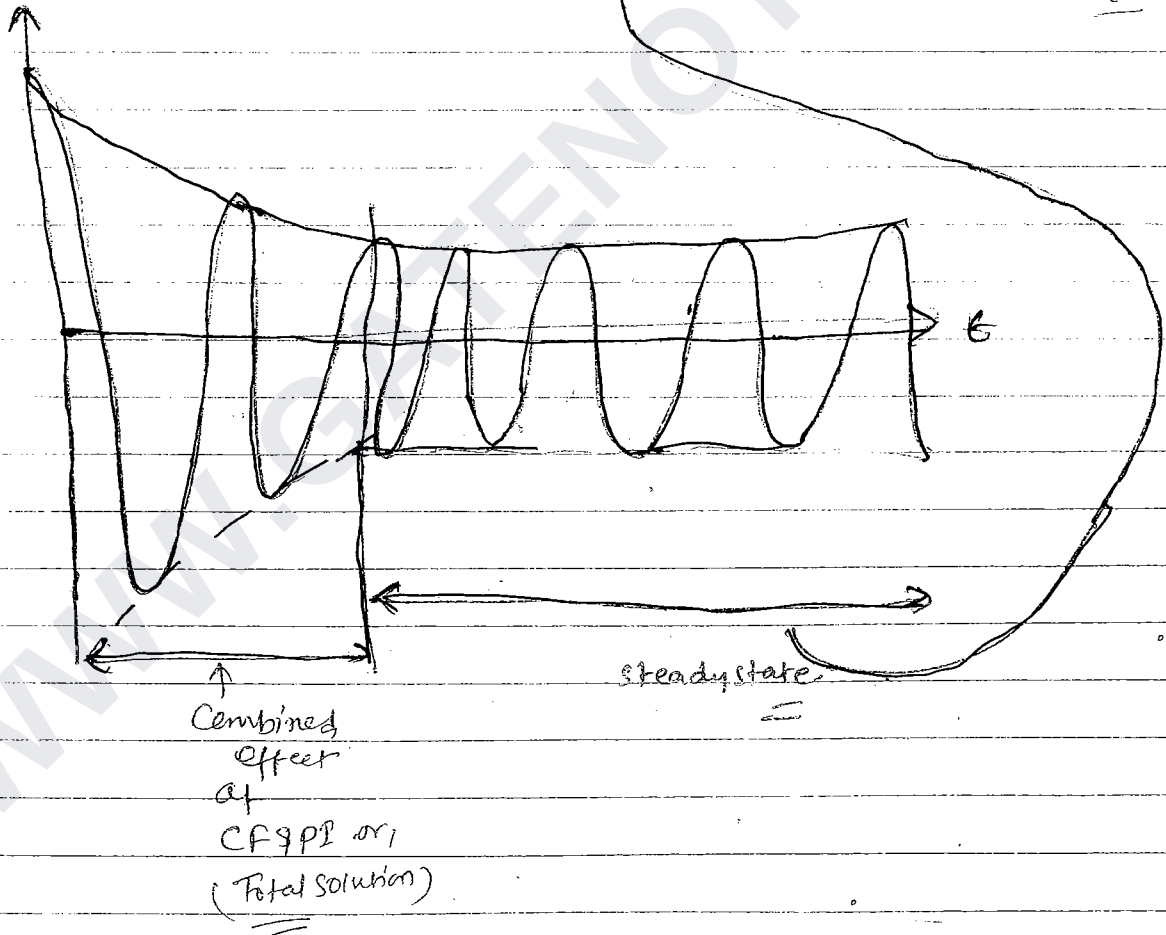
$$\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}$$

Damped - Free Response

Steady State Response

Amplitude of Steady state

(Because of the Amplitude is constant so it's called steady state & this vibration of running system cannot be zero)



Now,

$$x = A \sin(\omega t - \phi)$$

where A = Amplitude of steady state response,

$$= \frac{(F_0/s)}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}$$

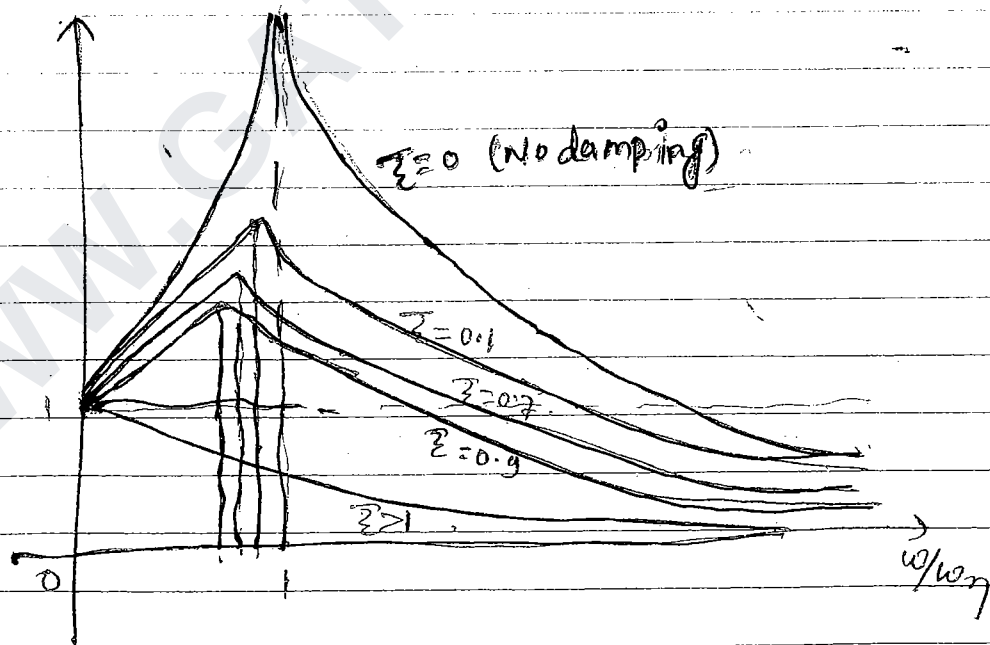
Magnification Factor :-

$$MF = \frac{A}{(F_0/s)}$$

$$\Rightarrow MF = \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

Steady state
Measurement

∴ MF dependson :- (i) $\frac{\omega}{\omega_n}$
(ii) ζ



For the No damping
max amplitude
of the steady state
Response at $\omega = \omega_n$
i.e. at the
Resonance

- (i) $\omega = \omega_n$
- (ii) $\omega < \omega_n$ (✓)
- (iii) $\omega > \omega_n$
- (iv) None of these

For the underdamping
max Amplitude of
the steady state Response
($\omega < \omega_n$)

Phaser diagram:-

$$x = A \sin(\omega t - \phi)$$

$$x = A \omega \cos(\omega t - \phi) = A \omega \sin \left\{ \frac{\pi}{2} + (\omega t - \phi) \right\}$$

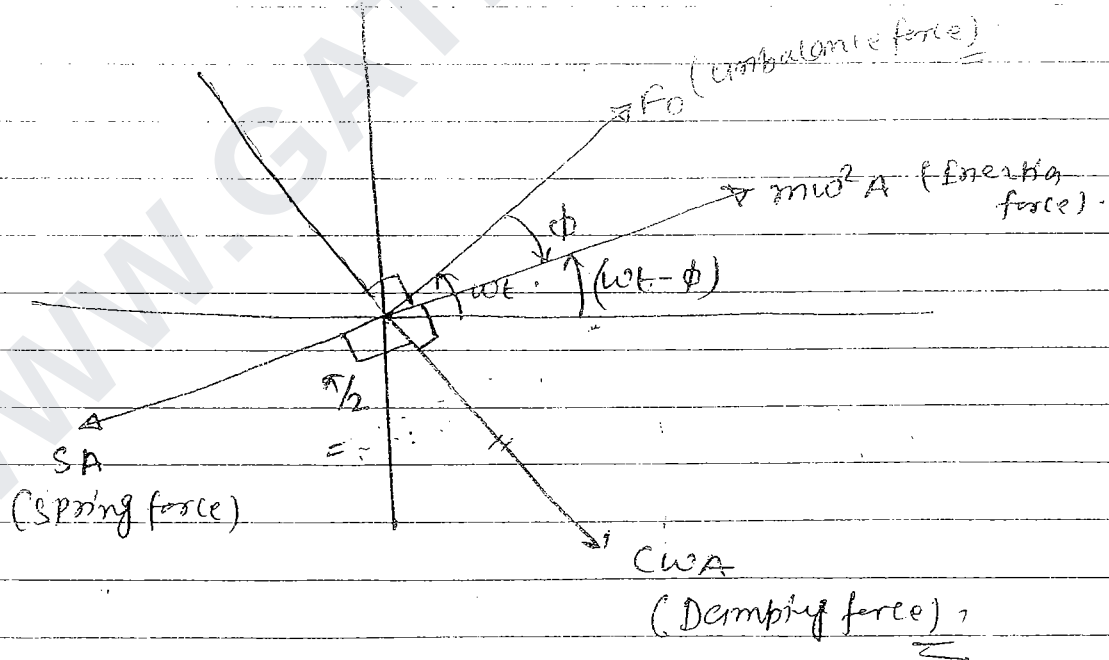
$$x'' = -A\omega^2 \sin(\omega t - \phi)$$

equation of forced damped vibration,

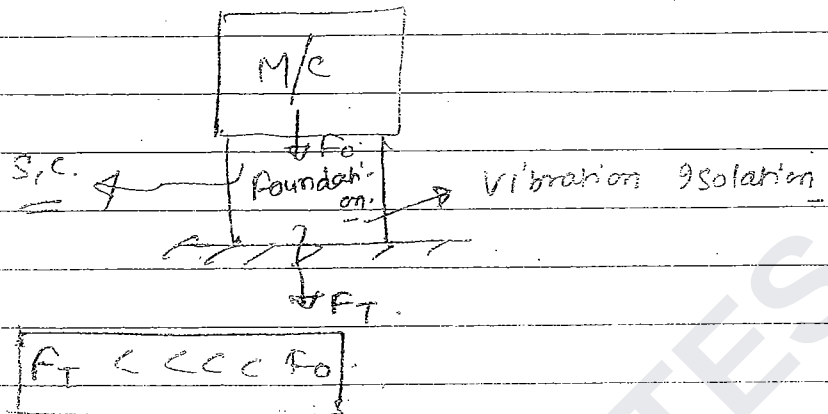
$$m\ddot{x} + c\dot{x} + sx = F_0 \sin \omega t$$

$$\Rightarrow f_0 \text{ ist mit } -m\ddot{x} = c\dot{x} - s\ddot{x} = 0$$

$$\Rightarrow F_0 \sin \omega t + m \omega^2 A \sin(\omega t - \phi) - \{CWA \sin\left\{\frac{\pi}{2} + (\omega t - \phi)\right\} - SA \sin(\omega t - \phi)\} = 0$$



Vibration Isolation & Transmissibility:



Transmissibility, $(E) = \frac{F_T}{F_0}$

$$F_T = \sqrt{(SA)^2 + (C\omega A)^2}$$

$$= SA \sqrt{1 + \left(\frac{C\omega A}{SA}\right)^2}$$

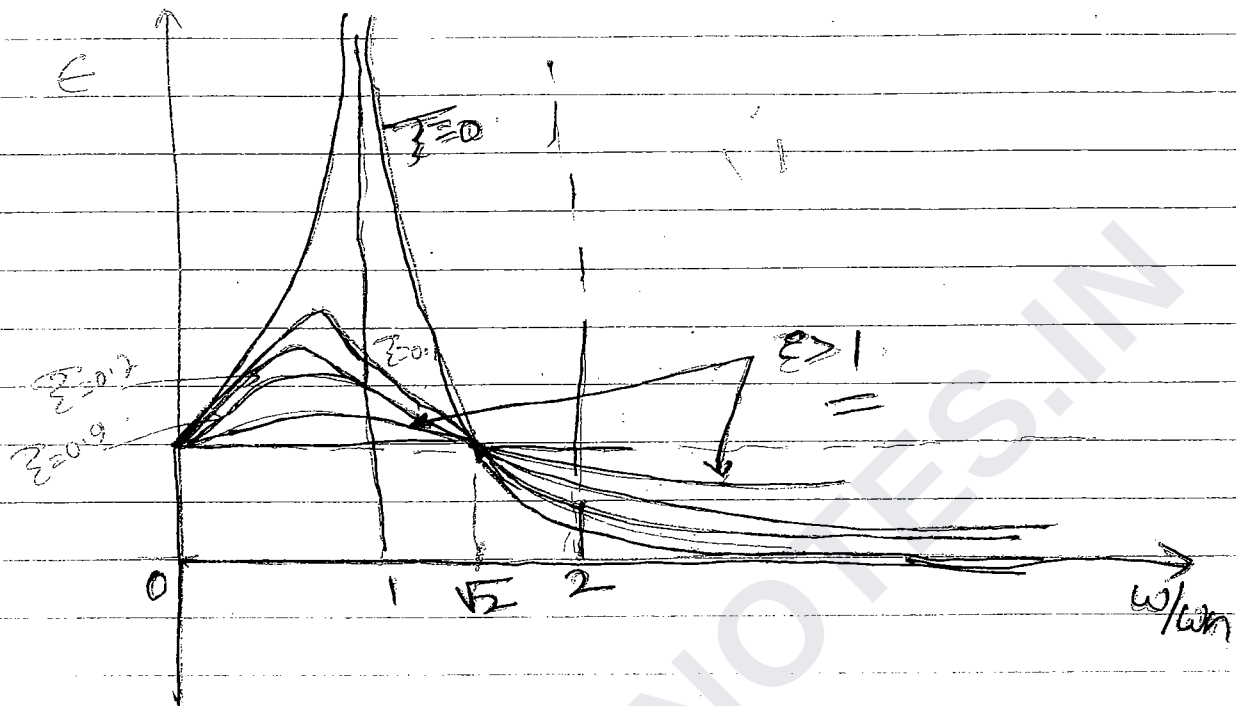
$$\Rightarrow F_T = SA \sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

$$F_0 = SA \sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}$$

$$\Rightarrow E = \frac{\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

E can depends on : - (i) $\frac{\omega}{\omega_n}$

(ii) ζ



Transmissibility Curve

for no damping,

$$\zeta = 0$$

then,

$$E_{\text{no damping}} = \left| 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right|$$

Cases -

$$(i) \frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow E < 1 \Rightarrow F_T < F_0$$

$$(ii) \frac{\omega}{\omega_n} = \sqrt{2} \Rightarrow E = 1 \Rightarrow F_T = F_0$$

$$(iii) \frac{\omega}{\omega_n} < \sqrt{2} \Rightarrow E > 1 \Rightarrow F_T > F_0$$

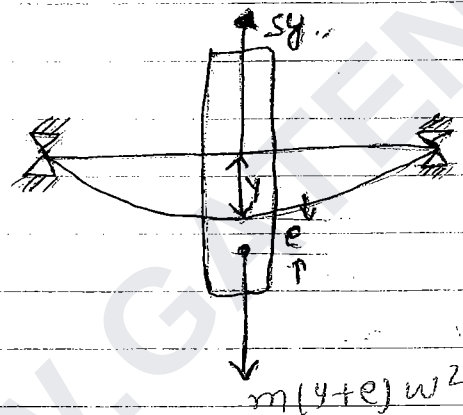
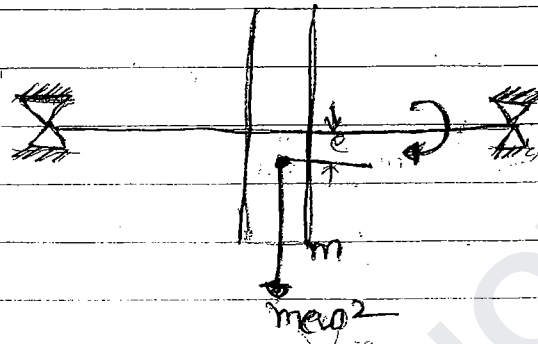
of Resonance effective zone $\frac{\omega}{\omega_n} > \sqrt{2}$

$$c) \frac{\omega_n}{\omega} < \frac{1}{\sqrt{2}} \Rightarrow \left[\omega_n < \frac{\omega}{\sqrt{2}} \right]$$

Failure of Shafts:-

⇒ Whirling of Shafts:- @ Whipping

Whirling speed or Critical Speed:



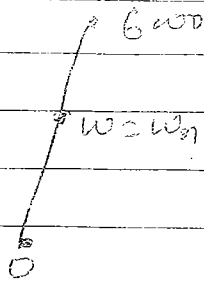
$y = \text{eccentricity}$

$$S \cdot y = m(y+e)w^2$$

$$\Rightarrow y(S - mw^2) = me w^2$$

$$\Rightarrow y = \frac{me w^2}{mw^2 \left(\frac{S}{mw^2} - 1 \right)}$$

$$\Rightarrow \boxed{y = \frac{e}{\left(\frac{\omega_n}{\omega} \right)^2 - 1}}$$



$$\omega = \omega_n = \sqrt{\frac{S}{m}}$$

→ Condition of resonance,

$$\omega_{\text{whirling}} = \omega_{\text{whirling}} = \omega_{\text{critical}} =$$

Conversion:

Q.11) Clear 10 marks

(10)

$$m = 17 \text{ kg}$$

$$S = 1 \text{ N/mm} = 1000 \text{ N/mm}$$

$$m_p = 2 \text{ kg}$$

$$r = \frac{75}{2000} \text{ m}$$

$$\omega = \frac{2\pi \times 500}{60}$$

$$F_0 = m_p \omega^2$$

$$= 2 \times \left(\frac{75}{2000}\right) \times \left(\frac{2\pi \times 500}{60}\right)^2$$

$$= 205.616 \text{ N}$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{1000}{17}} = 7.669 \text{ rad/sec}$$

$$\omega = 52.359 \text{ rad/sec}$$

$$\frac{\omega}{\omega_n} = 6.8267$$

$$A = \frac{(F_0/S)}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

$$\zeta = 0.20$$

$$\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

$$= \frac{205.616/1000}{\sqrt{\left(1 - (6.8267)^2\right)^2 + (2 \times 0.20 \times 6.8267)^2}}$$

$$= 0.0040 \text{ m}$$

$$= 4.51 \text{ mm}$$

=

$$\epsilon = \frac{\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\xi\omega}{\omega_n}\right\}^2}}$$

$$= 0.0636 = \frac{F_T}{F_0}$$

$$\Rightarrow F_T = F_0 \times 0.0636$$

$$= 13.09 \text{ N}$$

prob: 6

$$m = 10 \text{ kg}$$

$$S = 10,000 \text{ N/m}$$

$$n = \frac{n_0}{10}$$

$$\Rightarrow \frac{n_0}{n_4} = 10 \Rightarrow \left(\frac{n_0}{n_4}\right)^4 = 10$$

$$\Rightarrow \delta = 10^{1/4} = 0.57564$$

$$\Rightarrow \delta = \frac{2\xi\epsilon}{\sqrt{1-\epsilon^2}}$$

$$1.04580 = \frac{\epsilon^2}{1-\epsilon^2}$$

$$\epsilon = 0.912$$

check

$$\Rightarrow 0.02636 = \frac{\epsilon^2}{1-\epsilon^2} = 0.0256$$

$$0.209$$

$$F = 150 \cos 5t$$

$$F_0 = 150$$

$$\omega = 5$$

$$\omega_n = ?$$

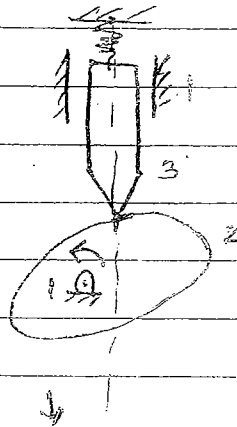
Cams & Followers

classmate

Date 26/08/10

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↓
Higher Pair



→ Any kind of motion is design
by single parameter which
is cam profile. So it is good.

→ Space for

→ Rotation to Reciprocation or Reciprocation to Reciprocation

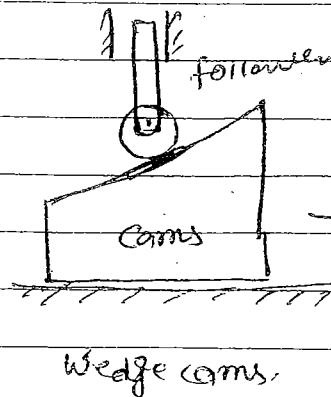
→ This process is not to be reversed.

→ use in power presses, Valve operation of an I.C. engine.

Cams: -

It is a mechanical element which drives another element known as follower in a specified way through the direct contact.

eg: -



Cams

↓
Rotation or Reciprocation

followers

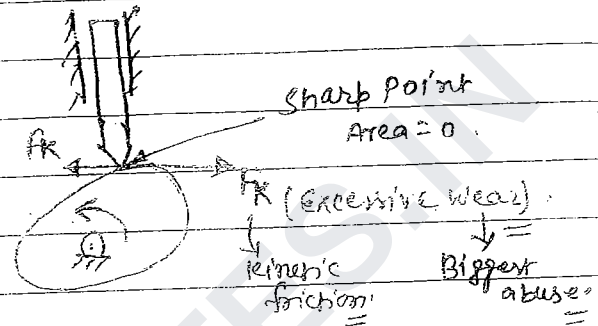
↓

Reciprocation or
oscillations.

Classification of followers:-

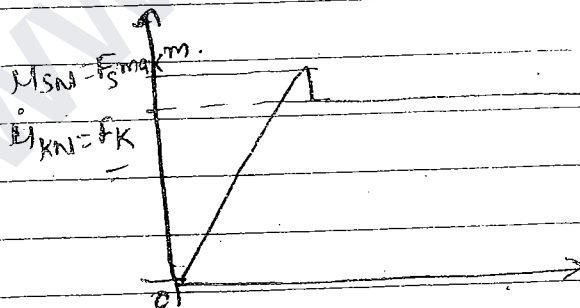
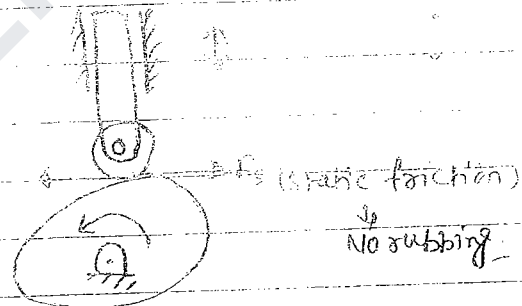
(i) Knife edge follower. (worst follower).

- Excessive wear.
- Excessive side thrust.



(ii) Roller follower (Best follower):-

- wear = zero
- side thrust is reduced but considerable.
- at a point side thrust is greater than knife edge.

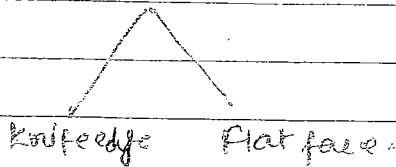


- sometime not used due to space limitation for the roller is three dimensional ~~problem~~ object.
- used in ^{Bifurc} gas engine, Air craft engine valve operation.
- not used in I.C. engine.
- Roller space is higher.

(iii) Flat faced follower: -

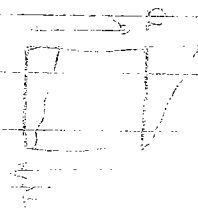
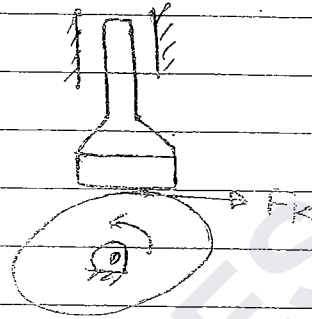
→ Two dimensional flat face.

→ wear highly reduced as compare to knife edge.



Area $\rightarrow 0$
Zero

Area $\rightarrow d \cdot A$
 $da \rightarrow 0$



* surface stresses involved.

NOTE: Mushroom follower:

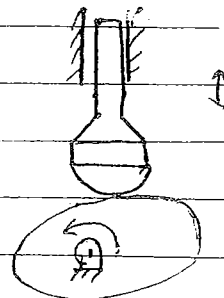
→ when flat face of the follower in circular shape is known as Mushroom follower.

To minimise surface stresses.

↓ Introduce.

(iv) Spherical faced follower: -

→ used in I.C. engine.



Knife edge

Flat face

Area $\rightarrow (1)$

Surface follower

⊙ f_k wear(P)

→ wear can be increased as compare to flat face follower.

→ used for I.C. engine valve operation.

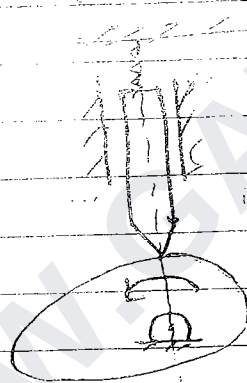
Note: Further slight Reduction in wear can be obtained by giving little bit offset.

Radial followers:

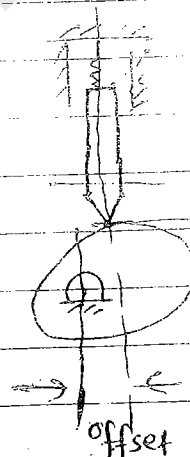
→ follower line of motion is passing through centre of cam.

offset followers:

→ follower line of motion is not passing through centre of the cam.



Radial follower



offset

offset follower

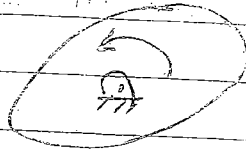
→ stroke can also be less than radial follower.

→ Normal force can also be less by offset.

→ so wear can be reduced.

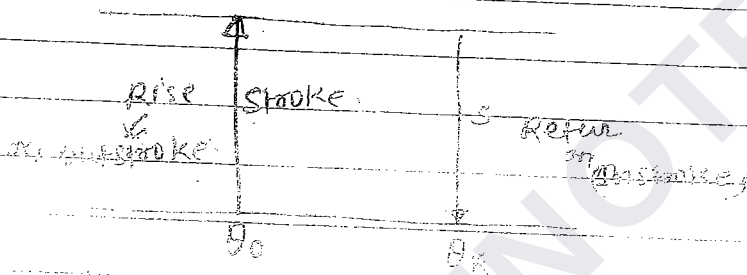
→ purpose of offset follower is to reduce the wear.

↓
ultimate then side thrust also reduced.



$\theta \rightarrow$ Angle Rotation by the cam.

$$\frac{d\theta}{dt} = \omega = \text{const.} \quad (\text{Cam Angular Velocity})$$



Time taken to rise = $\frac{\theta_o}{\omega}$

Time required to return the follower = $\frac{\theta_R}{\omega}$

$$V_o = \frac{s}{\theta_o / \omega} = \frac{\omega \cdot s}{\theta_o}$$

↓
(mean velocity during outstroke)

$$V_R = \frac{s}{\theta_R / \omega} = \frac{\omega \cdot s}{\theta_R}$$

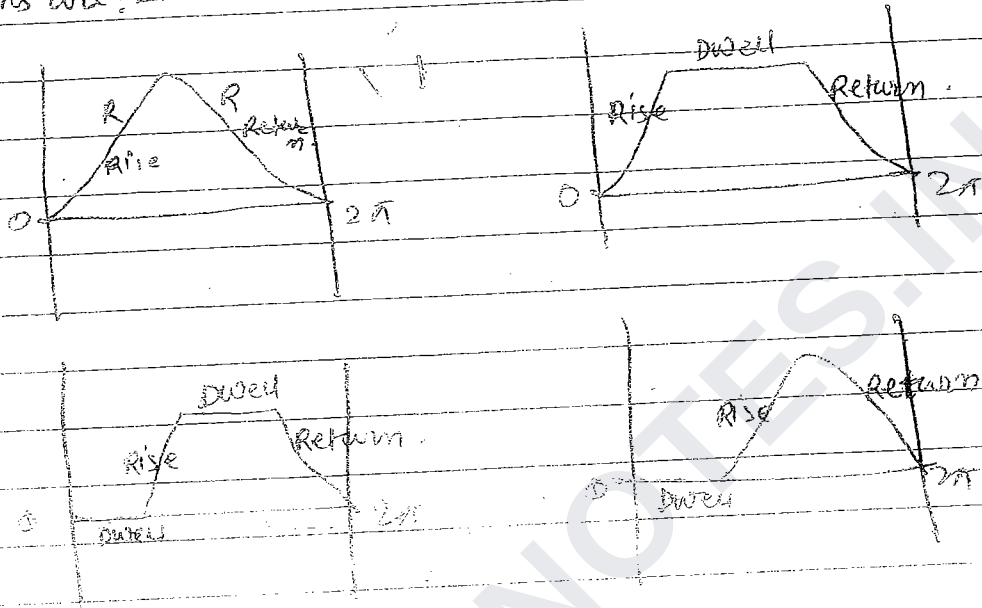
↓
(mean velocity during Return stroke)

Angle of Dwell: - (δ)

It is an angle rotation of the cam in which follower is stationary. It is known as Angle of Dwell.



A/c to the motion Requirement different types of Cams are! -



Following are the notations

$\theta \rightarrow$ Cam Rotation.

$x \rightarrow$ Follower Displacement.

$$\frac{dx}{dt} = \text{velocity}$$

$$\frac{dx}{d\theta} = \frac{dx}{dt} \cdot \frac{dt}{d\theta}$$

velocity $\downarrow$
Parameter is Const.

1st Derivative! -

$$\frac{dx}{d\theta}$$

$x-\theta$ curve:

$\downarrow$
Slope.

$\rightarrow$ Should not be very-very high.

2nd Derivative! - (Radius of curvature).

$$\frac{d^2x}{d\theta^2}$$

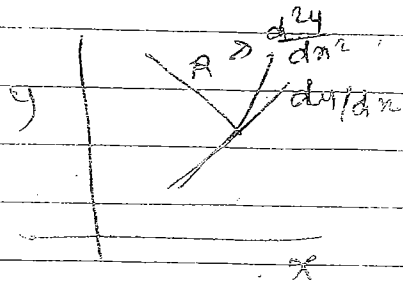
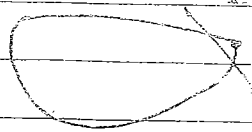
$$\text{If } \frac{d^2x}{d\theta^2} \rightarrow \infty$$

$$R \rightarrow 0$$

$\rightarrow$ Sharp Point.

Should not be very-very high.

undercutting or, Interference
of cam.



Radius of curvature,

$$\frac{1}{R} = \frac{d^2y/dn^2}{\left\{1 + \left(\frac{dy}{dn}\right)^2\right\}^{3/2}}$$

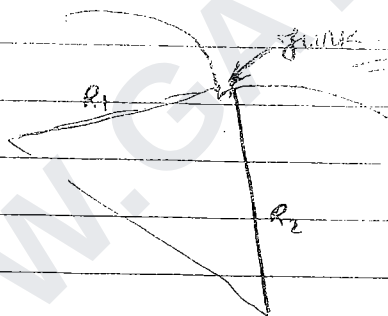
3rd derivative:

$$\frac{d^3y}{dn^3}$$

work:

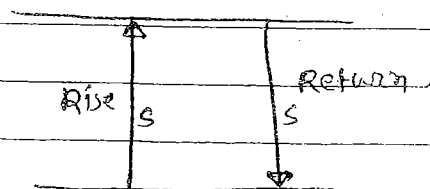


Should not be too high.



Follower motions:

① Uniform Velocity: - or (Linear motion)



$$V_{0 \text{ mean}} = \frac{\omega \cdot S}{\theta_0} = V_0 = V_{0 \text{ max}}$$

$$V_{R \text{ mean}} = \frac{\omega \cdot S}{\theta_R} = V_R = V_{R \text{ max}}$$

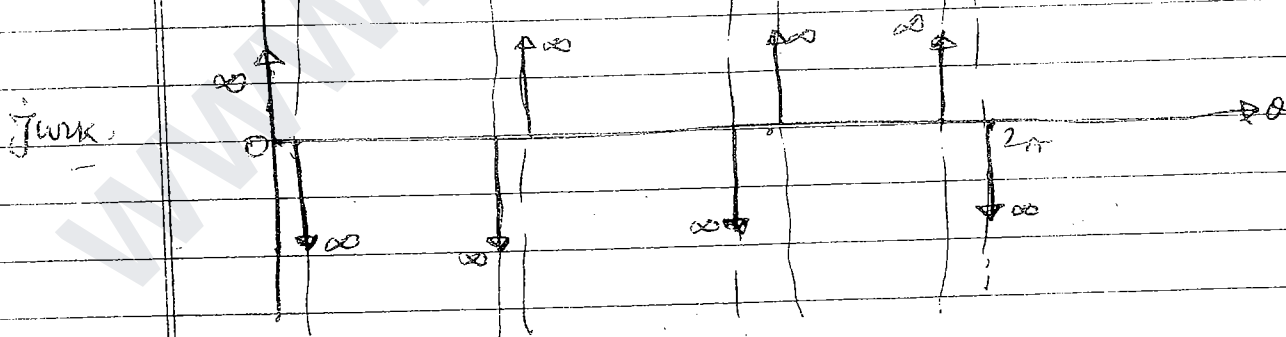
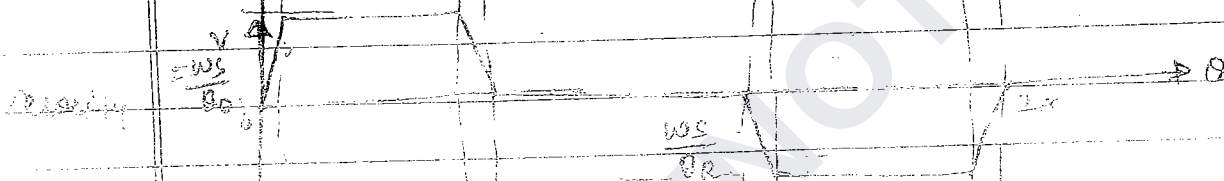
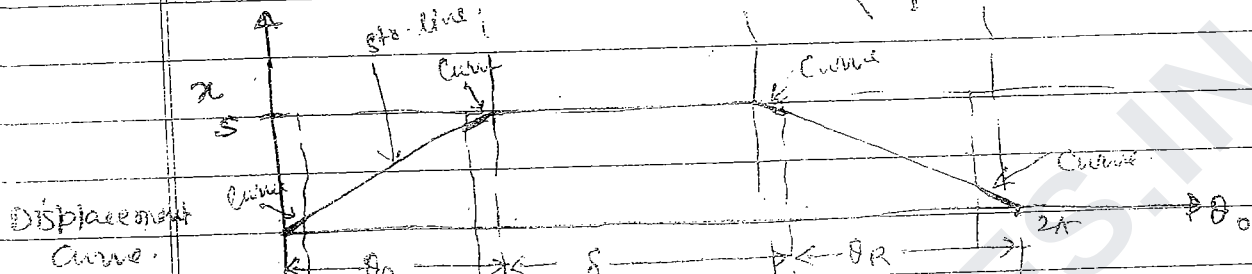
→ No used in practice

→ used in extremely slow speed.

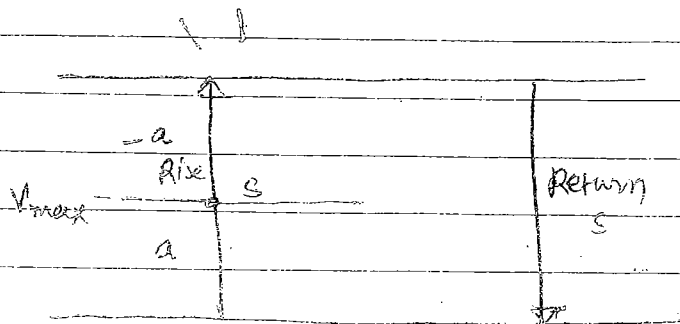
$$\theta = \omega t$$

↓
const

$$\theta = f(t)$$



(2) Uniform Acceleration & Retardation :- i. Parabolic motion.



$$V_{o \text{ mean}} = \frac{w \cdot s}{\theta_0}$$

$$V_{p \text{ mean}} = \frac{w \cdot s}{\theta_R}$$

$$V_{o \text{ max}} = 2 V_{o \text{ mean}}$$

$$V_{o \text{ max}} = \frac{2 \times w \cdot s}{\theta_0}$$

$$V_{o \text{ mean}} = \frac{s}{t_0}$$

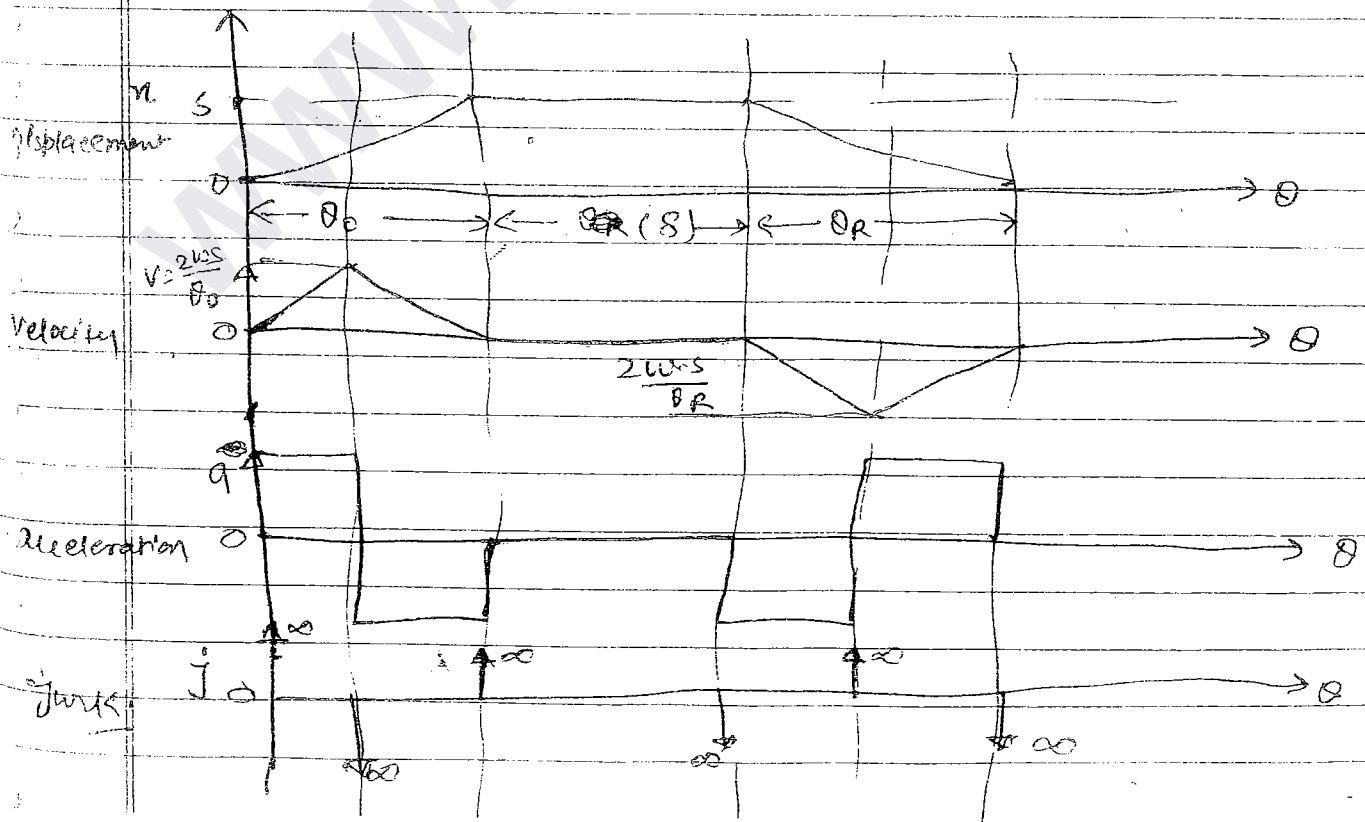
$$= \frac{s}{t_0} = 0 + \frac{1}{2} a \left(\frac{t_0}{2} \right)^2$$

$$s = \frac{a t_0^2}{4}$$

$$V_{o \text{ max}} = \frac{4s}{t_0} = \frac{4s}{2} = \frac{2s}{t_0}$$

$$\therefore V_{o \text{ max}} = 2 V_{o \text{ mean}}$$

→ This motion can be used for slow speed.



③ Simple harmonic Motion (S.H.M): -

→ upto medium speed can be used.

Motion of A' along the dia of the circle when A moves with const velocity along the circular path.

↓
The motion of A' is the simple harmonic motion.

↓
So when we calculate

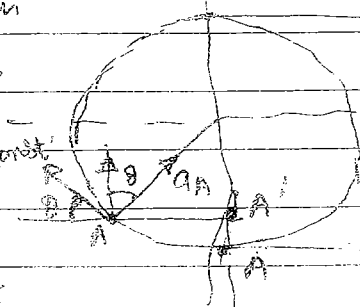
the velocity & acceleration

by using the motion of A and acceleration of A not by motion of A'.

↓
So a_0 & v_0

can be calculated by motion of A.

not by motion of A'.



$S = \text{stroke}$

← follower move on dia. of the circle.

$$v_A = \frac{\pi S}{\frac{2}{\theta_0}} = \frac{\pi \omega S}{2\theta_0}$$

$$\frac{\pi \omega S}{2\theta_0}$$

$$v_{A'} = v_A \sin \theta$$

$$v_{A'} = \frac{\pi \omega S}{2\theta_0} \sin \theta$$

$$a_{A'} = \frac{v_{A'}^2}{(S/2)} = \frac{\pi^2 \omega^2 S^2}{2 \theta_0^2}$$

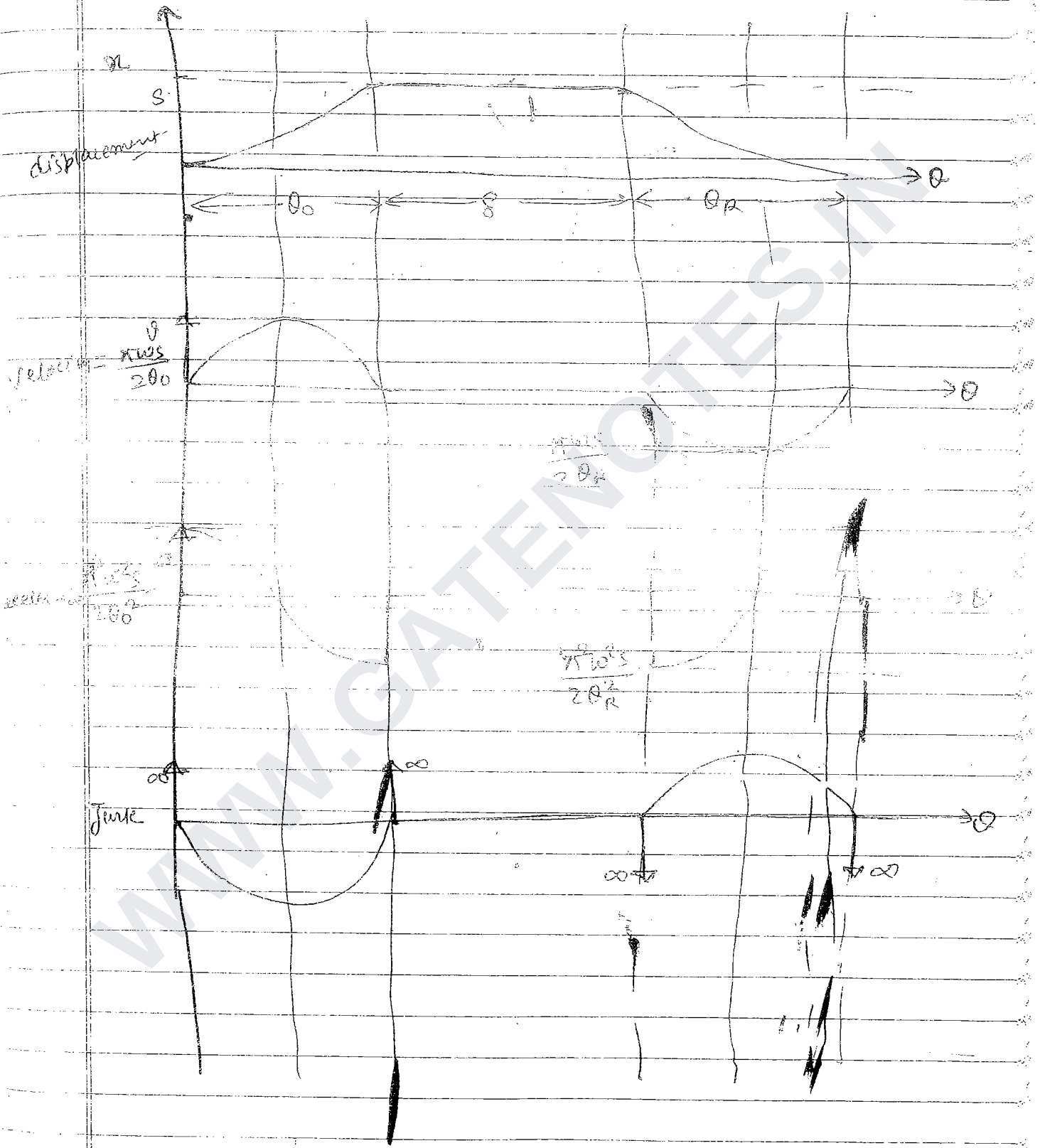
$$= \frac{\pi^2 \omega^2 S^2}{2 \theta_0^2}$$

$$= \frac{\pi^2 \omega^2 S^2}{2 \theta_0^2}$$

$\theta/2$

$$\Rightarrow a_0 = a_{A'} \cos \theta = \frac{\pi^2 \omega^2 S^2}{2 \theta_0^2} \cos \theta$$

$$a_{A'} = -\sin \theta$$



④ Cycloidal motion (Best) :-

$$x_0 = S \left[\frac{\theta^2}{\theta_0} - \frac{1}{2\pi} \sin \left(\frac{2\pi\theta}{\theta_0} \right) \right]$$

$$V_0 = \frac{dx_0}{d\theta} \cdot \left(\frac{d\theta}{dt} \right) \xrightarrow{\omega} = \omega \cdot S \left[\frac{1}{\theta_0} - \frac{1}{2\pi} \cos \left(\frac{2\pi\theta}{\theta_0} \right) \cdot \frac{2\pi}{\theta_0} \right]$$

$$\Rightarrow V_0 = \frac{\omega S}{\theta_0} \left[1 - \cos \left(\frac{2\pi\theta}{\theta_0} \right) \right] \quad \text{fn}(\sin^2 \theta)$$

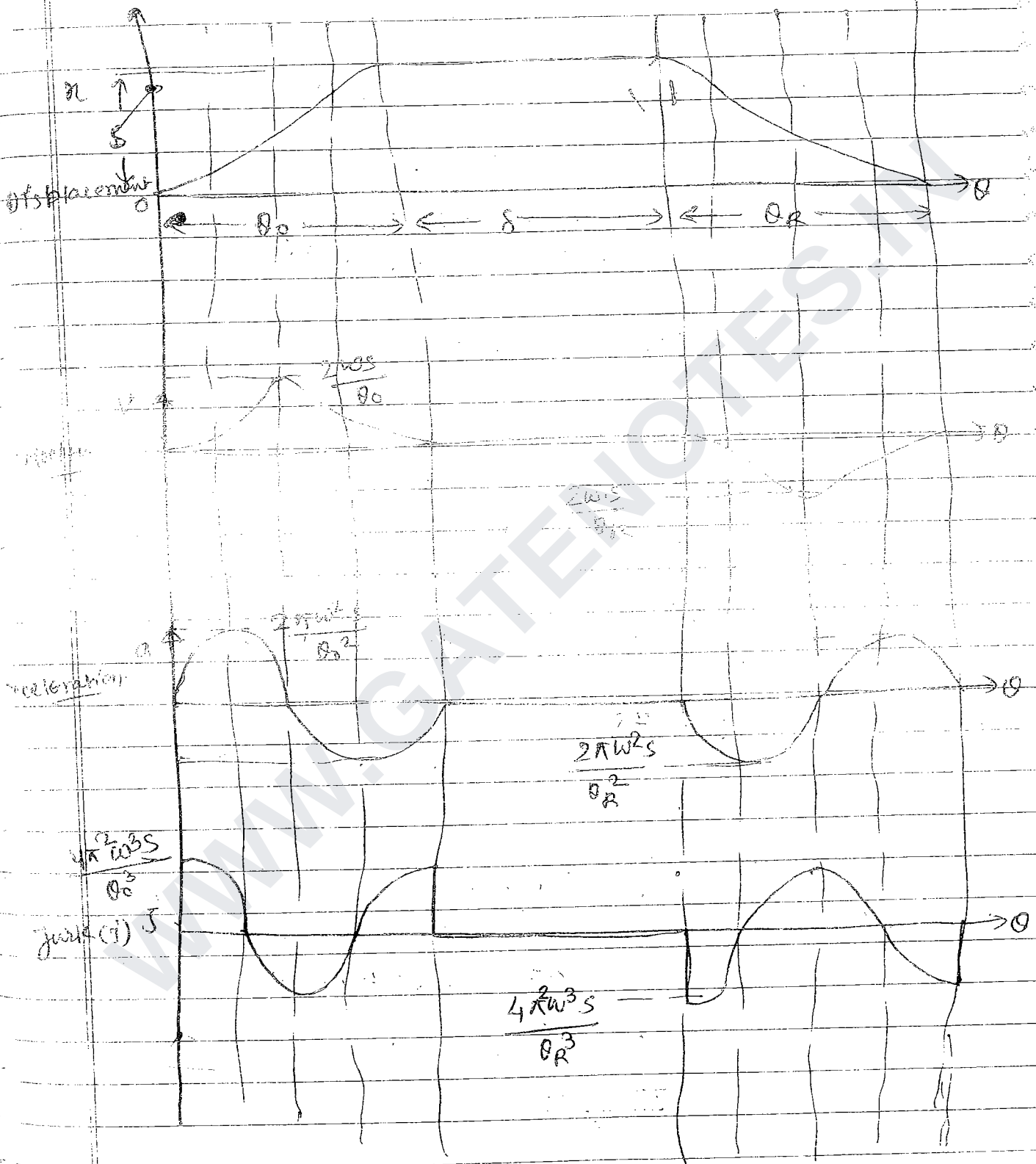
$$\text{at } \theta = \frac{\theta_0}{2} \quad \text{max} = \frac{2\omega S}{\theta_0}$$

$$a_0 = \frac{dV_0}{d\theta} \cdot \left(\frac{d\theta}{dt} \right) \xrightarrow{\omega} = \frac{\omega^2 S}{\theta_0} \left[\sin \left(\frac{2\pi\theta}{\theta_0} \right) \cdot \frac{2\pi}{\theta_0} \right]$$

$$\Rightarrow a_0 = \frac{2\pi\omega^2 S}{\theta_0^2} \sin \left(\frac{2\pi\theta}{\theta_0} \right)$$

$$j_0 = j_{\text{ jerk }} = \frac{da_0}{d\theta} \cdot \left(\frac{d\theta}{dt} \right) \xrightarrow{\omega} = \frac{2\pi\omega^3 S}{\theta_0^3} \cos \left(\frac{2\pi\theta}{\theta_0} \right) \cdot \frac{2\pi}{\theta_0}$$

$$\Rightarrow j_0 = \frac{4\pi^2\omega^3 S}{\theta_0^3} \cos \left(\frac{2\pi\theta}{\theta_0} \right)$$



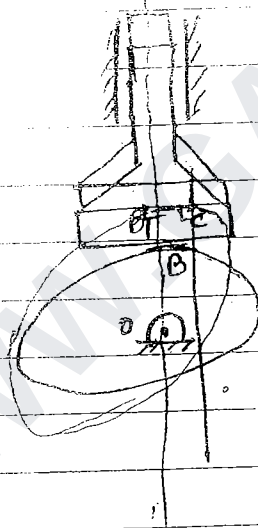
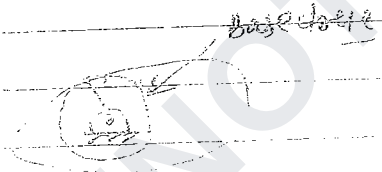
Circular Arc Cam with Flat face follower:-

Circular arc: - When the flanks of the cam are tangential to base circle and nose circle and of convex circular arcs ~~when~~ ^{then} the cam is known as circular arc cam.

→ Minimum radius of the cam is - Base circle radius.

Base circle! -

Minimum radius of the circle which rotate about the same centre of the cam this circle is known as Base circle.

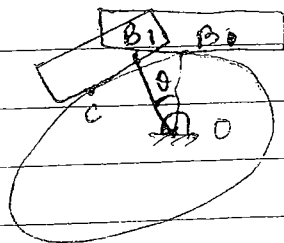


displacement of follower

$$x = 0.81 - 0.8$$

OB₁ line and touching point C.
↓
displacement of follower. Contact point

Normal at touching point c and ~~from~~
OB, line both are Parallel to each
Other.



Let $r_1 =$ Base circle

$r_2 =$ Nose circle

$\alpha =$ Semi angle of action.

$R =$ Flank radius.

$OB = L$.

$\phi =$ angle of action of the flank.

Basic dimension
of the cam.

On Flank:-

$$\theta \in [0, \phi]$$

$x =$ displacement in Flank

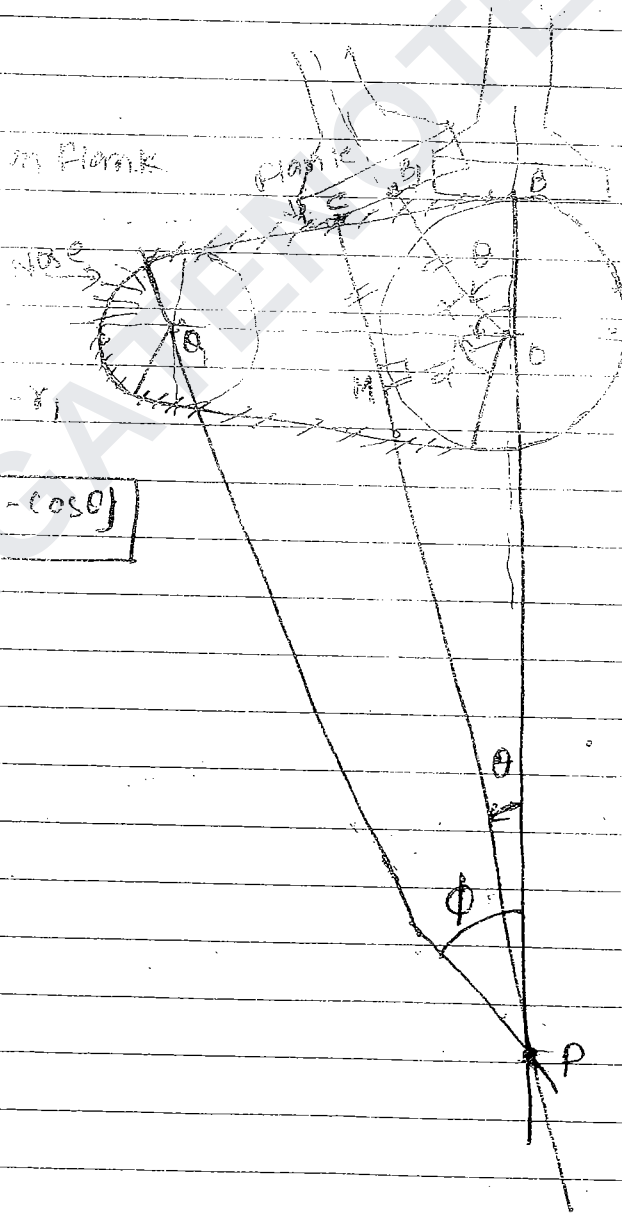
$$= OB - CB$$

$$= CM - OM$$

$$= (R - PM) - r_1$$

$$= R - (R - r_1) \cos \theta - r_1$$

$$\Rightarrow x = (R - r_1) (1 - \cos \theta)$$



on Nose,

$$\theta \in [\phi, \alpha]$$

$$OQ = L$$

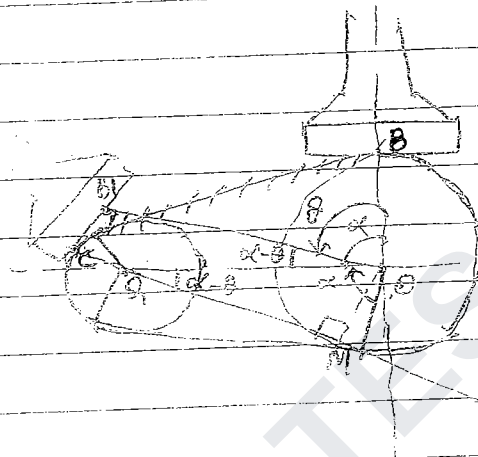
$$x = OB_1 - OB$$

$$= OB_1 - r_1$$

$$= CM - r_1$$

$$= (CQ + QM) - r_1$$

$$= r_2 + L \cos \alpha - r_1$$



Symmetric
Cam.

$$x = (r_2 + L \cos \alpha) - r_1$$

This circle is the minimum radius circle which touches the cam profile but not rotate about the cam centre.

Lift:-

$$\text{at } \theta = \alpha,$$

$$\text{Lift} = (r_2 - r_1) + L$$

Prob:-

$$r_1 = 25 \text{ mm}$$

$$R = ?$$

$$r_2 = 50 \text{ mm}$$

$$\phi = ?$$

$$\alpha = 75^\circ$$

$$L = ?$$

$$\text{Lift} = 20 \text{ mm}$$

$$\Rightarrow \text{Lift} = (r_2 - r_1) + L$$

$$\Rightarrow 20 = (50 - 25) + L$$

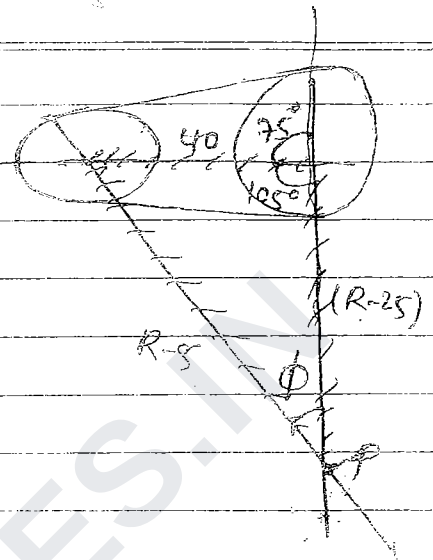
$$\Rightarrow L = 40 \text{ mm}$$

By Cosine Rule

$$R = ?$$

By Sine Rule

$$\phi = ?$$



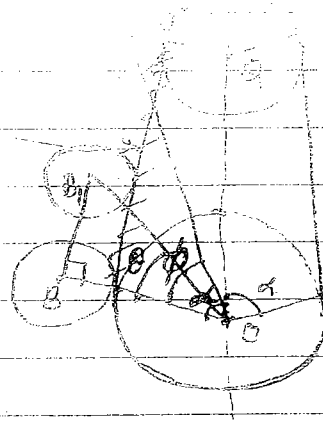
By Sine Rule

$$OB = OP$$

$$\frac{OB}{\cos \theta} = OB$$

$$= OB \left(\frac{1 - \cos \theta}{\cos \theta} \right)$$

$$\Rightarrow x = (r_1 + r_2) \left(\frac{1 - \cos \theta}{\cos \theta} \right)$$



Cam Terminology:

- Trace point.
- Cam profile.
- Pitch curve.
- Pitch point.
- Pressure angle.
- Prime circle.

Open loop :-

- Open loop are highly stable.
- Output not depends on Input.
- any change in input not change the output.

eg: - Flywheel.

↓
design for Nonlinear → ~~Linear~~

↓
when fluctuation in Nonlinear, ϵ of flywheel not change.

Closed loop :-

- Highly unstable.
- ~~Open~~ output is dependent on input.

eg: - Chiller.

Block diagram



TF (OL) = Transfer fun. of open loop.

$$= \frac{\theta_o}{\theta_i}$$

$$= \frac{\theta_1}{\theta_i} \times \frac{\theta_2}{\theta_1} \times \frac{\theta_o}{\theta_2}$$

$$\boxed{TF(OL) = TF(1) \times TF(2) \times TF(3)}$$

Transfer function of closed loop :-

$$\boxed{TF(CCL) = \frac{TF(OL)}{1 + TF(OL)}}$$

The end.