

5/6
FM is branch of science which deals with property of fluids (liquids and gases), behaviour of fluids at rest, under motion with and without forces causing motion.

App of FM

1. Hydraulic Analysis of turbine, pumps, Hydraulic equipment
Flow Analysis in pipes and Hydraulic losses in pipes for optimisation of the pipe size etc.

unit 1

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1. Properties of Fluids
2. Fluid statics
3. Fluid kinematics
4. Fluid dynamics
5. Laminar ^{Flow of} Incompressible fluids through pipes and in between plates
6. Turbulent Flow thru pipes
7. Boundary Layer theory concepts
8. Turbo machinery (Hydraulic turbines)

PROPERTIES OF FLUIDS

- Property of fluids
- Newton's law of viscosity
- Compressibility and Bulk modulus

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Pressure intensity of fluid and vapour pressure

→ classification of fluid Based on fluid power Law

[Rheological eqn of Fluids]

PROPERTIES

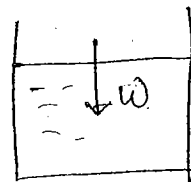
there are 4 properties

1. Mass of the matter
2. weight of the fluid
3. Area
4. volume
5. Specific mass [mass density]
6. weight density [unit weight or sp weight]
7. Specific Gravity or relative density
8. Specific volume
9. Dynamic viscosity
10. Kinematic viscosity
11. compressibility of liquids [Bulk Modulus]
12. surface tension [surface Energy]
13. Pressure Intensity
14. Vapour pressure

Weight

A force due to gravitational pull.

$$W = mg$$



Newton's 2nd Law

2

$$F = ma$$

$$1N = 1kg \times \frac{1m}{sec^2}$$

$$N = kg \cdot m/sec^2$$

$$W = mg$$

$$kg \cdot f = m \times 9.81$$

$$kg \left[\frac{m}{sec^2} \right]$$

$$\boxed{kgf = 9.81 N}$$

$$= \underline{\underline{10 N}}$$

③

Area

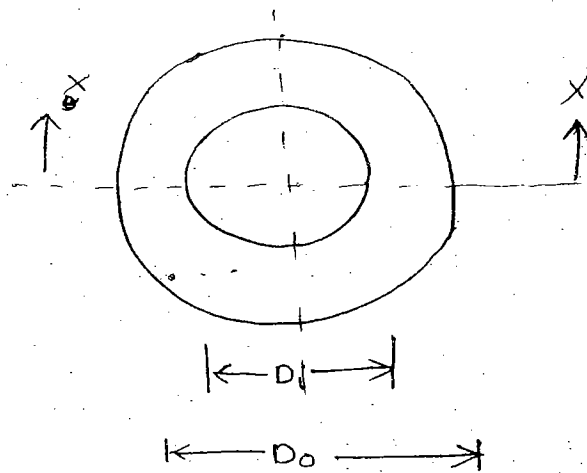
Area is the space occupied by the matter in 2D.

- Normal Area ✓
- cls Area
- plane Area ✓
- Resisting Area
- surface Area ✓
- shear Area
- crushing Area
- tearing Area
- projected Area ✓
- Flow Area ✓
- ~~for FM~~

Normal Area

without touching the body

2 dimensions visible



$$A_n = \frac{\pi}{4} [D_o^2 - D_i^2]$$



$$A_{cls} = \frac{\pi \times d^2 \times 2}{4}$$

Flow Area

Depending on the fluid touching the sides

Volume

It is the space occupied by the matter in 3D

units

m^3 , mm^3 , cm^3

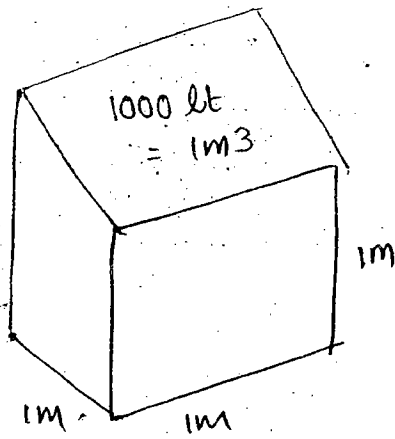
$$V = (A \times l)$$

$$V = l \times b \times h$$

→ litres (lt)

For water

$$1m^3 = 1000 \text{ lit}$$



$$1000 \text{ cm}^3 = 1 \text{ lit}$$

$$\text{cm}^3 = \frac{1}{1000} \text{ lit}$$

$$= 10^{-3} \text{ lit}$$

$$= \text{milli lit}$$

$$\boxed{\text{cm}^3 = \text{ml}}$$

Specific Mass or Mass density

— Property of matter

— matter occupied
1 volume

$$\rho = \frac{m}{v} \left[\frac{\text{kg}}{\text{m}^3} \right]$$

Ex:

$$\begin{aligned} \rho_{\text{air}} &= 1.2 \text{ kg/m}^3 \\ \rho_{\text{water}} &= 1000 \text{ kg/m}^3 \\ \rho_{\text{Hg}} &= 13600 \text{ kg/m}^3 \end{aligned}$$

$$\rho_{\text{gold}} = 19200 \text{ kg/m}^3$$

$$\rho_{\text{Pt}} = 22500 \text{ kg/m}^3$$

↘ Highest

$$\rho_{\text{oil}} < \rho$$

1) A steel plate 10 mm thick, 1 m width and length 2 m. cost of 1 tonne steel is Rs 30,000. Estimate cost of plate.

$$\text{Volume} = 0.01 \times 1 \times 2$$

$$= 0.02 \text{ m}^3$$

$$= \cancel{20} \cancel{20} \text{ m}$$

$$\rho = \frac{m}{v}$$

$$7850 \left(\frac{\text{kg}}{\text{m}^3} \right) = \frac{m \text{ (kg)}}{\cancel{20} 0.02}$$

$$m = 7850 \times \cancel{20} 0.02$$

$$\underline{\underline{157 \text{ kg}}}$$

$$1 \text{ Ton} = 1000 \text{ kg}$$

$$\cancel{1 \text{ kg} = 30}$$

$$1000 \text{ kg} = 30,000$$

$$1 \text{ kg} = 30 \text{ Rs}$$

$$\text{Cost} = 157 \times 30$$

$$= \underline{\underline{4710 \text{ Rs}}}$$

2) A water ~~pond~~ sump of inner dimensions 6 ft, 10 ft, 5 ft, Estimate how many lit of

~~Volume~~ = 10 A family consists 4 people and
Consumption per Head 150 lit/day. How many day
the sump can serve 3

Volume

$$\rho_w = \frac{m_w}{\text{Volume}}$$

$$1000 = \frac{m}{5\text{ft} \times 6\text{ft} \times 10\text{ft}}$$

$$m = \frac{1000 \times (5 \times 6 \times 10) \text{ft}^3}{35} \\ = \frac{1000 \times 300}{35} \left(\frac{\text{kg}}{\text{m}^3} \right) \\ m = \underline{\underline{8570 \text{ kg}}}$$

$$8570 \text{ kg} = 8750 \text{ lb}$$

$$\frac{8750}{600} = \underline{\underline{15 \text{ days}}}$$

weight density or sp weight

It is the weight of the matter occupied per unit volume.

$$\frac{\text{weight}}{\text{volume}} = \frac{w}{V} = \frac{N}{\text{m}^3}$$

Volume =

$$1 \text{ft} = 12 \text{ in}$$

$$\text{Inch} = 25.4 \text{ mm}$$

$$= 12 \times 25.4 \text{ mm}$$

$$= \frac{12 \times 25.4}{1000}$$

$$= \underline{\underline{0.3048}}$$

$$35 \text{ft}^3 = 1 \text{m}^3$$

$$\left[\frac{\text{kg}}{\text{m}^3} \right]$$

$$\boxed{w = \gamma = \rho g \text{ (N/m}^3\text{)}}$$

$$\gamma_{\text{water}} = \rho_{\text{water}} \times g$$

$$= 1000 \times 9.81$$

$$= 9810 \frac{\text{N}}{\text{m}^3}$$

$$= \cancel{98} \ 9.81 \frac{\text{kN}}{\text{m}^3}$$

Relative density or Sp gravity [S]

It is the Ratio of mass density of any matter

" " " std fluid -

↓
water

$$\boxed{S = R.D = \frac{\rho_x}{\rho_{\text{water}}}}$$

$$S_{\text{steel}} = \frac{\rho_{\text{steel}}}{\rho_{\text{water}}} = \frac{7850}{1000}$$

$$= \underline{\underline{7.85}}$$

$$\text{ex: } S_{\text{mercury}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} = \frac{13600}{1000}$$

$$= \underline{\underline{13.6}}$$

$$S_{\text{air}} = \frac{\rho_{\text{air}}}{\rho} = \frac{1.2}{1000} = \underline{\underline{0.001}}$$

Specific Volume $[v_s]$

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Volume of matter for unit mass

→ very useful for gases

$$v_s = \frac{\text{Volume}}{\text{Mass}} \quad \frac{\text{m}^3}{\text{kg}}$$

$$= \frac{V}{m} = \frac{1}{\rho}$$

$$v_{s \text{ water}} = \frac{1}{1000} = 0.001 \frac{\text{m}^3}{\text{kg}}$$

$$v_{s \text{ air}} = \frac{1}{1.2} = 0.8733 \frac{\text{m}^3}{\text{kg}}$$

a) 1 lit of an oil weighing 7N. Det

1. mass of oil in kg
2. vol of oil in m^3
3. mass density of oil in kg/m^3 , g/cc , g/m^3
4. ~~sp~~ sp weight of oil in $\frac{\text{kN}}{\text{m}^3}$
5. sp gravity of oil
6. sp vol of oil. m^3/kg , $\frac{\text{lit}}{\text{kg}}$, $\frac{\text{ml}}{\text{gm}}$

$$W = 7\text{N}$$

$$W = mg$$

$$7 = m \times 9.81$$

$$m = \frac{7}{9.81} = 0.713 \text{ kg}$$

$$\rho = \frac{m}{\text{volume}}$$

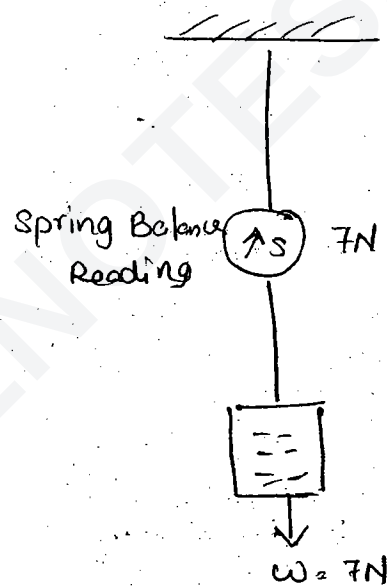
$$\text{Vol} = \frac{0.713}{1 \text{ lit}} = \frac{10^{-3} \text{ m}^3}{1 \text{ m}^3}$$

$$1 \text{ m}^3 = 1000 \text{ lit}$$

$$\text{Vol} = 0.713 \text{ lit}$$

$$0.713 \times 10^{-3} \text{ m}^3$$

PHYSICAL REPRESENTATION



3. Mass density

$$\rho = \frac{m}{V}$$

$$= \frac{0.713}{10^{-3}}$$

$$= 0.713 \times 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$\text{in g/cc} = 0.713 \times 10^6 \frac{\text{g}}{\text{m}^3}$$

$$= \frac{0.713 \times 10^6}{10^6}$$

$$= 0.714 \text{ g/cc}$$

$$\text{in g/ml} = \frac{0.713 \times 10^3 \times 10^3}{10^6 \text{ ml}}$$

4) weight density (γ)

$$\gamma = \frac{W}{\text{Volume}} = \rho g$$

$$= 714 \times 9.81$$

$$= \underline{\underline{7000 \text{ N/m}^3}}$$

5) specific Gravity (S)

$$S_{\text{oil}} = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} = \frac{714 \text{ (kg/m}^3\text{)}}{1000 \text{ (kg/m}^3\text{)}} \\ = \underline{\underline{0.714}}$$

6) sp volume (v_s)

$$v_s = \frac{V_{\text{oil}}}{m_{\text{oil}}} = \frac{1}{\rho_{\text{oil}}}$$

$$= \frac{1}{714} = 1.400 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}$$

$$= \underline{\underline{1.4 \frac{\text{lt}}{\text{kg}}}}$$

Viscosity

It is a property of fluids [liquids and gases only] which offers resistance to the flow of fluids.

forces and adhesive forces

cohesive - attraction b/w same molecules

adhesive - attracting with other molecules

• ie water with oil etc

mercury - cohesive forces

It never wets any substance

- calibration fluid

Energy	Nm
unit weight	N
	= m

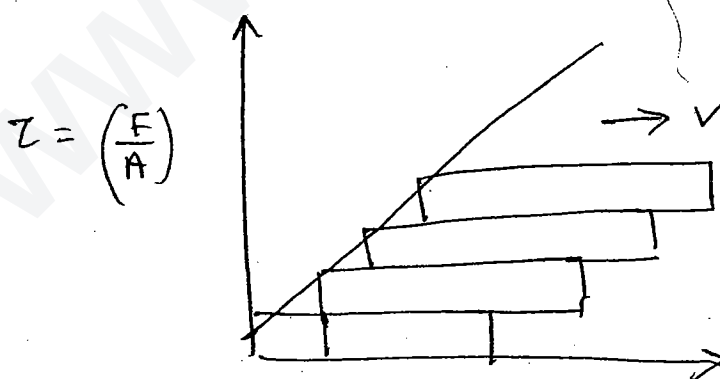
$$\frac{100 \text{ J}}{1000 \text{ N}}$$

$$= 0.1 \text{ Nm}$$

- Imp for pump selection

lift
against
gravity

NEWTON'S LAW OF VISCOSITY



$\tau \propto \left(\frac{du}{dy} \right) \rightarrow \text{vel gradient}$

$$\tau = \mu \frac{du}{dy}$$

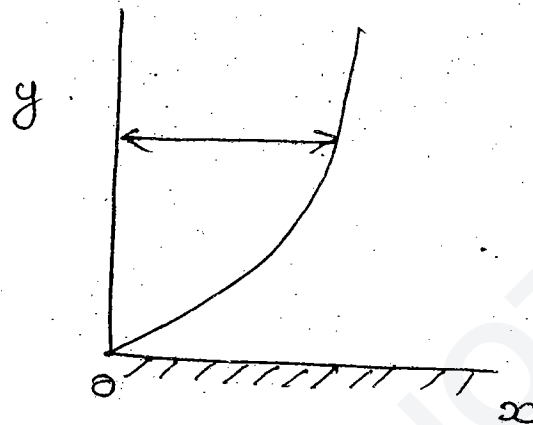
μ = dynamic viscosity

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$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

→ vel Gradient or

Rate of shear strain



~~SHEAR STRESS~~

[SHEAR FORCE IS
MEASURED BY
FRUSTUM TUBE]

→ Shear stress on fluid layer $\propto$ vel gradient [Rate of deformation or Rate of shear strain]

$$\text{Unit} = \frac{\frac{N}{m^2}}{\frac{m/sec}{m}} = \frac{Ns}{m^2}$$

$$\mu = \frac{\text{Force}}{\text{Area}} \cdot \text{time}$$

Dynamic viscosity - study under forces.

KINEMATIC VISCOSITY

$$\nu = \frac{\mu}{\rho}$$

Ratio of dynamic viscosity to density

← No force involved

Ⓢ Ⓢ

$$\eta = \frac{Ns \times m^3}{m^2 \times kg}$$

$$N = \frac{kg \cdot m}{s^2}$$

$$m \cdot \frac{Ns}{kg} = \frac{kg}{s} \cdot \frac{kg \cdot m}{s^2} \times \frac{1}{kg} \cdot m$$

$$\text{Unit} = \frac{m^2}{sec}$$

Ex:

$$\mu_{\text{water}} = \frac{1}{1000}$$

Water

$$\frac{N \cdot sec}{m^2} = \frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2}$$

$$= 1 \times 10^{-3} \frac{Ns}{m^2}$$

$$\eta = \frac{1}{1000} \frac{m^2}{sec}$$

$$= 1 \times 10^{-6} \frac{Ns}{m^2}$$

AIR

$$\mu_{\text{AIR}} = 1.8 \times 10^{-5} \frac{N \cdot sec}{m^2}$$

$$1.8 \times 10^{-6} \frac{N \cdot sec}{m^2}$$

[18475
10 kN/m^2]

$$\eta_{\text{AIR}} = \frac{\mu}{\rho} = \frac{1.8 \times 10^{-6}}{1.2}$$

$$= 1.5 \times 10^{-6} \frac{m^2}{sec}$$

$$= 1.5 \times 10^{-6} \frac{m^2}{sec}$$

(1)

$$\mu_{\text{water}} > \mu_{\text{AIR}}$$

$$v_{\text{WATER}} < v_{\text{AIR}}$$

NEWTON'S LAW OF VISCOSITY

made into 5 models

$$\tau = \mu \frac{du}{dy}$$

$$F = \mu A \frac{du}{dy}$$

A - surface Area

Model 1

(3)
An incompressible fluid of sp gravity 0.8 and kinematic viscosity $7.4 \times 10^{-7} \frac{\text{m}^2}{\text{s}}$ is subjected to tensional force of 1 unit Area. the fluid is moving b/w 2 fixed plates of gap 1mm with velocity $30 \frac{\text{m}}{\text{min}}$. Estimate the τ caused on the plate 2

$$\tau = \mu \frac{du}{dy}$$

$$S = 0.8$$

$$v = \frac{\mu}{\rho}$$

$$0.8 = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}}$$

$$\mu = 7.4 \times 10^{-7} \times 800$$

$$\mu = v \times \rho$$

$$\rho = 0.8 \times 1000$$

$$dA = 1 \text{ m}^2$$

$$= \underline{\underline{800 \frac{\text{kg}}{\text{m}^3}}}$$

$$dy = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$v = \frac{30}{60} = 0.5 \text{ m/s}$$

$$Z = \frac{7.4 \times 10^{-7} \times 0.5 \times 800}{10^{-3}}$$

$$Z = \underline{\underline{3.7 \times 10^{-4} \text{ N/m}^2}} \quad 0.296 \text{ N/m}^2$$

A Newtonian fluid filled b/w a shaft and sleeve Bearing. A force 800N is applied on shaft. The shaft gains 1.5 cm/s velocity. If force is increased to 2.4 kN then new speed will be

- a) 1.5 cm/s
- b) 13.5 cm/s
- c) 0.5 cm/s
- ☒ d) 4.5 cm/s

Case 1

$$F = 800 \text{ N}$$

$$V = \frac{1.5 \text{ cm}}{\text{s}} = \frac{1.5 \times 10^{-2} \text{ m}}{\text{s}} = \frac{1.5 \text{ cm}}{\text{s}}$$

$$\tau = \mu \frac{du}{dy}$$

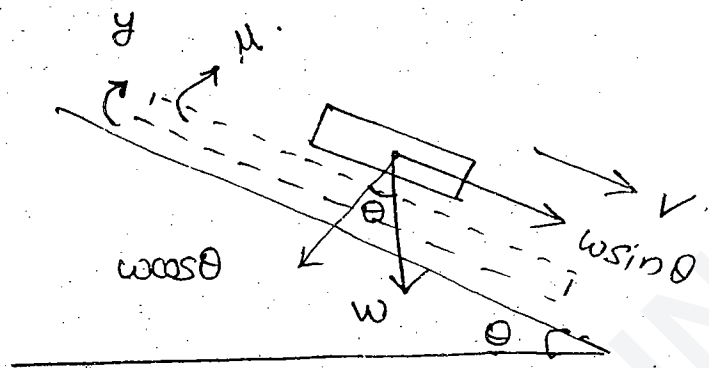
$$\frac{F}{A} \propto \frac{du}{dy}$$

$$\frac{F_1}{F_2} = \frac{V_1}{V_2}$$

$$\frac{800}{2400} = \frac{1.5}{V_2} \quad V_2 = \frac{2400 \times 1.5}{800}$$

Inclined plane Model

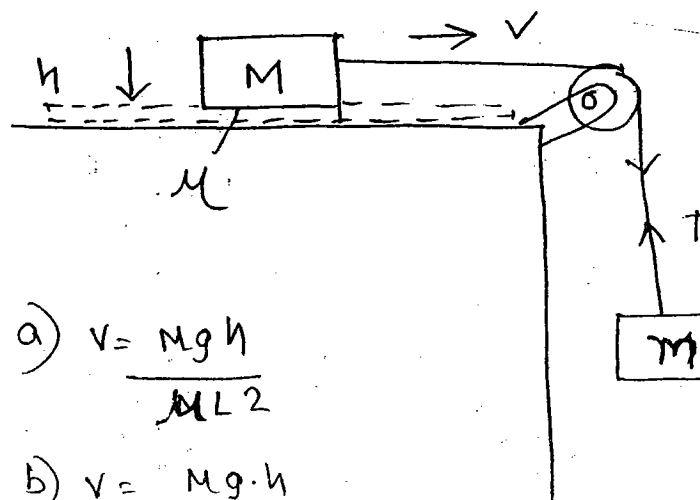
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$$F = \mu A \cdot \frac{v}{y}$$

$$w \sin \theta = \mu \cdot A \cdot \frac{v}{y}$$

- Q.) A cubic Block of each side 'L' and mass 'm' is pulled over an oil film across a table shown in fig. A small mass 'm' is connected to a string which passes over a smooth pulley. What is the steady velocity of the Block (M) against the Newtonian fluid resisting force.



a) $v = \frac{Mgh}{\mu L^2}$

b) $v = \frac{Mg \cdot h}{\mu L}$

d) $v = mgh$

$$\sum F = ma$$

$$= m \times 0$$

$$= 0$$



$$F = \mu A \times \frac{v}{y}$$

$$T = \mu \times L^2 \times \frac{v}{h}$$

tension

D'Alembert's principle

$$\Sigma F = ma$$

$$mg - T = 0$$

$$\underline{T = mg}$$

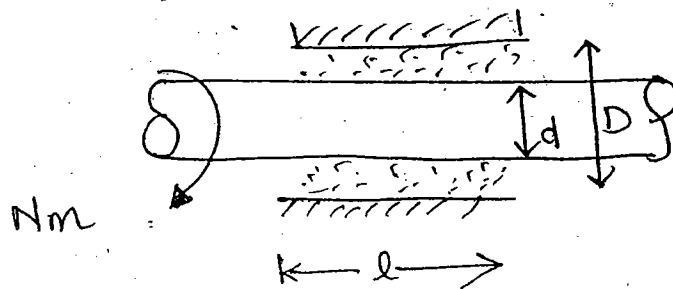
$$mg = \mu \times L^2 \times \frac{v}{h}$$

$$\boxed{v = \frac{mgh}{\mu L^2}}$$

Model - III

Power lost in viscosity

Subjected to a
torque



$$\text{Power Lost} = \frac{2\pi NT}{60}$$

$$T = F \times \underline{d}$$

T = Torque

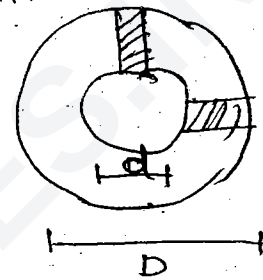
N = Speed in rpm

$$T = \mu A \frac{v}{y} \cdot \frac{d}{2} \quad (\text{Nm})$$

surface Area

$$T = \frac{\mu \times \pi d \times l \times \left(\frac{\pi d N}{60} \right) \cdot \frac{d}{2}}{\left(\frac{D-d}{2} \right)}$$

$$P = \frac{2 \pi N}{60} \left[\mu \cdot \pi d l \cdot \frac{\pi d N}{60} \right] \cdot \frac{d}{2} \quad \text{watts}$$



Radial gap is only reqd

A loaded Bearing has shaft dia 50mm

Bush length 20mm & Bush Bore 50.05mm

Subjected to 1200 rpm with a Newtonian fluid

of viscosity 0.03 Pa sec. what is the power lost due to viscosity in watts?

- | | |
|------------|--------|
| a) 37 watt | |
| b) 74 | |
| c) 118 | d) 237 |

$$P = \frac{2 \pi N T}{60}$$

$$D = 50.05$$

$$d = 50$$

$$T = \mu \times \pi \times 1200 \times$$

$$\frac{D-d}{2} = \frac{0.025}{2}$$

$$\mu = 0.03 \frac{\text{N}}{\text{m}^2} \text{ sec}$$

$$F = \mu A \frac{v}{y}$$

$$v = \frac{\pi d N}{60}$$

$$T = F \times \frac{d}{2} = \mu \left(\frac{\pi d N}{60} \right) \cdot \frac{d}{2}$$

$$= \frac{\pi \times 50 \times 120}{60}$$

$$v = 3141.59 \text{ r}$$

$$\pi \times r \quad 2 \times \pi \times 1200 \times 0.03 \times 3141.59 \times 50$$

$$\frac{\text{sec}}{\text{J/sec}^2 \cdot \text{rad}}$$

$$= \text{watts} \cdot \text{rad} = \text{watts}$$

POINTS:-

Fluids at Rest: Fluid statics

Fluids in motion: Fluid kinematics

Here pressure forces not considered

Fluid dynamics:

study of Fluids in motion when pressure forces are also considered

DENSITY:-

For liquids
 $\rho = \text{const}$

$$\rho = \frac{m}{V}$$

$$\rho_{\text{WATER}} = 1000 \text{ kg/m}^3 \\ = 1 \text{ gm/cm}^3$$

Specific weight or weight density

$$\frac{10^3 \times 10^3}{10^6}$$

$$W = \gamma = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Vol of fluid}} = \underline{\underline{\rho g}}$$

specific volume:- (V_s)

$$V_s = \frac{\text{Vol}}{\text{Mass}} = \underline{\underline{\frac{1}{\rho}}}$$

unit

$$\frac{\text{m}^3}{\text{kg}}$$

[Applicable to Gases]

Specific Gravity

$$S = \frac{\rho_{\text{liq}}}{\rho_{\text{std liquid}}}$$

std liquid $\rightarrow$ water

$$S = \frac{\rho_{\text{liq}}}{1000}$$

$$S = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

For mercury $S = 13.6$

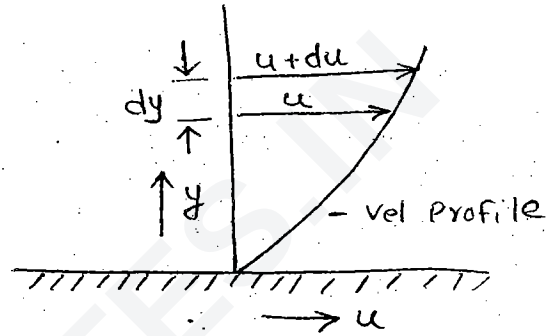
Viscosity

dy - dist b/w layers

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \left(\frac{du}{dy} \right)$$

Rate of shear
Strain or vel Gradient



unit

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{N/m^2}{\frac{m \times 1}{s \cdot m}} = \frac{Ns}{m^2} \text{ - SI unit}$$

MKS unit

$$\mu = \frac{\text{kgf} \cdot \text{sec}}{m^2}$$

CGS unit

$$\mu = \frac{\text{dyne} \cdot \text{sec}}{cm^2} = \text{Poise}$$

$$1 \text{ POISE} = \frac{1}{10} \frac{Ns}{m^2}$$

$$\frac{1}{CP} = 1 \text{ CP} = \frac{1}{100} \text{ POISE}$$

KINEMATIC VISCOSITY

$$\nu = \frac{\text{viscosity}}{\rho} = \frac{\mu}{\rho} = \frac{Ns \times m^3}{m^2 \times kg}$$

$$= \frac{\cancel{kg} \cdot m \times \cancel{s} \times m^3}{m^2 \cdot s^2 \cdot \cancel{kg}} = \frac{m^2}{s}$$

Cgs unit - STOKES - cm^2/s

$$1 \text{ STOKES} = 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$1 \text{ CS} = \frac{1}{100} \text{ STOKES}$$

FLUIDS

NEWTONIAN

OBEY NEWTON'S LAW
OF VISCOSITY

NON NEWTONIAN

DOESN'T OBEY NEWTON'S
LAW OF VISCOSITY

VARIATION OF VISCOSITY WITH TEMPERATURE

VISCOSITY $\downarrow$ WITH $\uparrow$ in TEMP

$$\mu = \mu_0 \left[\frac{1}{1 + \alpha t + \beta t^2} \right]$$

μ_0 = viscosity of liq at 0°C in poise

μ = " " " at $t^\circ\text{C}$ in poise

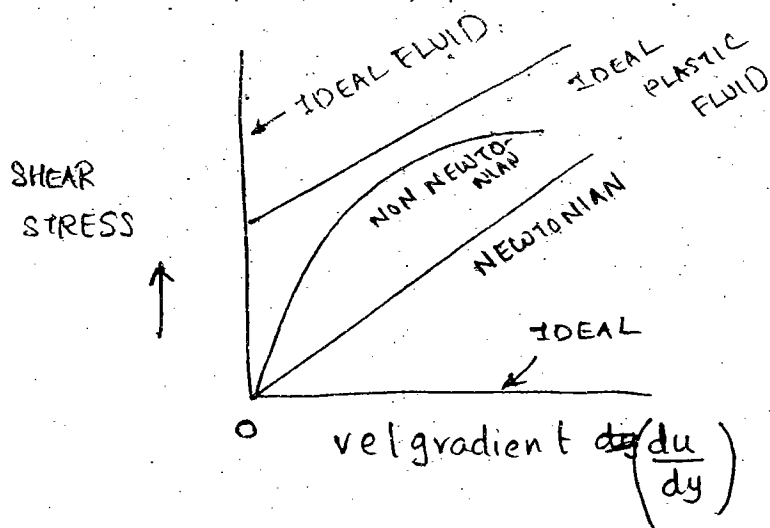
α, β = constants

$$\mu_{\text{air}} = 18 \times 10^{-6} \frac{\text{N s}}{\text{m}^2}$$

$$\mu_{\text{water}} = \frac{1}{1000} \frac{\text{Ns}}{\text{m}^2}$$

Fluid types

1. Ideal Fluid = Incompressible, No viscosity, only imaginary
2. Real " = Possess viscosity, All fluids in Practice are real.
3. Newtonian " - obeys Newton law of viscosity
4. Non Newtonian - doesn't obey Newton's law of viscosity



PROBLEMS

HN 1.3
TEXT

$$u = \frac{2}{3}y - y^2$$

u = velocity in m/s

$$\tau = ?$$

$$\text{At } y = 0, y = 0.15 \text{ m}$$

$$\mu = 8.63 \text{ Poise} = \frac{8.63}{10} \frac{\text{Ns}}{\text{m}^2} = 0.863 \frac{\text{Ns}}{\text{m}^2}$$

$$\text{we have } \tau = \mu \left[\frac{du}{dy} \right]$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\text{At } y = 0, \quad \frac{du}{dy} = \frac{2}{3}$$

$$\therefore \tau = 0.863 \times \frac{2}{3} = \underline{\underline{0.5756 \text{ N/m}^2}}$$

$$\begin{aligned} \text{At } y = 0.15, \quad \frac{du}{dy} &= \frac{2}{3} - 2 \times 0.15 \\ &= \underline{\underline{0.367}} \end{aligned}$$

$$\tau = 0.863 \times 0.367 = \underline{\underline{0.3167 \text{ N/m}^2}}$$

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$$\begin{array}{c} \text{---} \rightarrow 60 \text{ cm/s} \\ \text{---} \updownarrow 0.025 \text{ mm} \text{---} \\ \text{---} \end{array}$$

$$\tau = \frac{2 \text{ N}}{\text{m}^2}$$

$$\mu = ?$$

$$V = 60 \text{ cm/s} = \underline{\underline{0.6 \text{ m/s}}}$$

$$du = \dots$$

$$\tau = \mu \left(\frac{du}{dy} \right)$$

$$\frac{F}{A} = \mu \left(\frac{du}{dy} \right), \quad \frac{2N}{m^2} = \mu \left[\frac{0.6}{0.025 \times 10^{-3}} \right]$$

$$\therefore \mu = 8.33 \times 10^{-5} \frac{Ns}{m^2}$$

$$(1.5) \quad A = 1.5 \times 10^6 \text{ mm}^2 \therefore 1.5 \times 10^{-3} \text{ m}^2$$

$$du = 0.4 \text{ m/s}$$

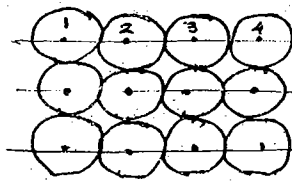
$$dy = 0.15 \text{ mm} = 0.15 \times 10^{-3} \text{ m}$$

J - Energy

$$W = F \times S$$

$$\text{N} \times \text{m}$$

J



Fluids made of sphere molecule centre doesn't change

- a) The velocity distribution in a laminar flow at given section is given by $u = 5 \sin(5\pi y)$ where u is in m/s and y is in m. The viscosity of the fluid is 0.5 Pa.s. Then the shear stress at $y = 5\text{cm}$ is in N/m^2 is

(a) ☒ 39.27 N/m^2 (b) 27.76 N/m^2 (c) 38.9 N/m^2 (d) None

Ans:

$$\tau = \mu \left[\frac{du}{dy} \right]$$

$$\mu = 0.5 \frac{\text{Ns}}{\text{m}^2}$$

$$u = 5 \sin(5\pi y)$$

$$\frac{du}{dy} = 5 \times \cos(5\pi y) \times 5\pi$$

$$= 25 \cos(5\pi y) \times \pi$$

$$\tau = 0.5 \times 25 \cos(5\pi y) \times \pi$$

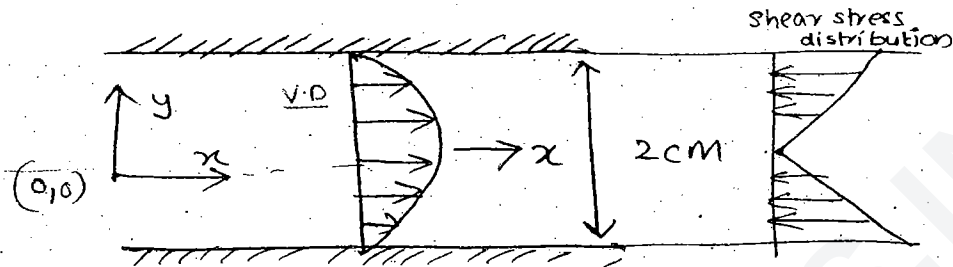
$$\tau \Big|_{y=0.05} = 0.5 \times 25 \times \cos(5 \times \pi \times 0.05) \times \pi$$



in Radians

$$= \underline{\underline{39.27 \text{ N/m}^2}} \quad \underline{\underline{27.768 \text{ N/m}^2}}$$

Q) The velocity profile for Newtonian fluid in laminar flow b/w 2 parallel plates shown in fig is given by $u = 0.01 [1 - 10000y^2]$ where u is in m/s and y is in m.



$$\frac{du}{dy} = 0.01 [-20,000y]$$

The viscosity of fluid is assumed to be water whose value is 1×10^{-3} Pa.s. Then shear stress on the plates will be.

$$\tau = \mu \left[\frac{du}{dy} \right]$$

$$y = 1 \text{ cm}$$

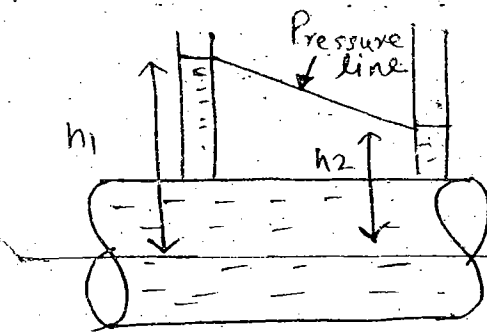
$$\tau = 10^{-3} \times 0.01 \left[-20,000 \times 1 \right] \times 10^{-2}$$

$$\tau = 0.002 \text{ N/m}^2$$

NOTE

In Laminar flow velocity distribution is parabolic

shear stress distribution is linear

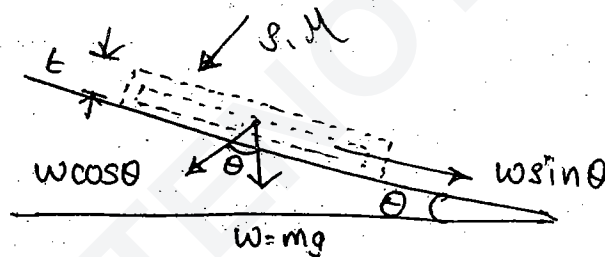


$$P_a = \frac{N}{m^2}$$

14

$$(-ve) \quad \frac{dp}{dx} \downarrow \rightarrow$$

- Q.) A liquid of density ρ , dynamic viscosity μ moves steadily down the inclined plane in a thin sheet of const thickness t . Neglect friction, and the shearsstress at the bottom of liquid surface due to flow parallel to the inclined plane will be



- a) $\tau = \rho g t \sin \theta$ b) $\tau = \rho g t \cos \theta$ c) $\tau = \rho g \tan \theta$
 d) $\tau = \mu \sqrt{\frac{g}{t}}$

$$\tau = \mu \times \left[\frac{du}{dy} \right]$$

~~It is neglected~~

~~$\tau = \mu$~~

$$\tau = \frac{F}{\text{Area}}$$

Area — surface area

$$\tau = \frac{W \sin \theta}{A} = \frac{mg \sin \theta}{A}$$

$$\rho = \frac{m}{V}$$

$$= \frac{\rho V g \sin \theta}{A}$$

$$\frac{\rho \times (A \times t) g \sin \theta}{A}$$

As

$$\rho t g \sin \theta$$

$$\frac{\rho \times (A \times t) \times g \sin \theta}{A}$$

A

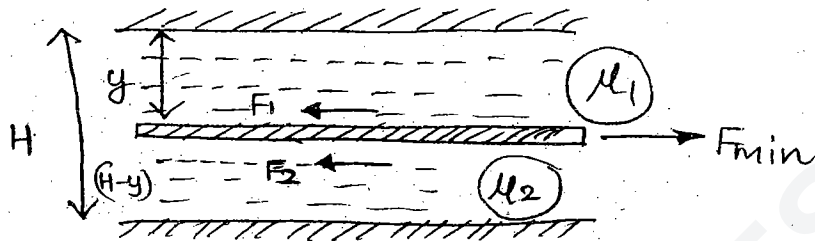
$$= \rho g t \sin \theta$$

(a)

V

Multi Fluid Flow

(1.)



- a) A Thin plate is placed b/w 2 flat surfaces which are 'H' metres apart such that the viscosity of liquids on the top and Bottom of plates are μ_1 and μ_2 respectively. what is the position of the thin plate such that the viscous resistance to have the uniform motion velocity v of the plate is minimum.

$$a) \frac{y}{H-y} = \sqrt{\frac{\mu_1}{\mu_2}} \quad b) \frac{H-y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$c) \frac{y}{H} = \sqrt{\frac{\mu_1}{\mu_2}} \quad d) \frac{y}{H} = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$F = F_1 + F_2$$

$$= \mu_1 A_1 \frac{v}{y} + \mu_2 A_2 \frac{v}{(H-y)}$$

to minimise F

$$\frac{dF}{dy} = 0$$

$$\mu_1 A_1 v \left[-\frac{1}{y^2} \right] + \mu_2 A_2 v \left[\frac{1 \times 1}{(H-y)^2} \right]$$

At 20°C

$$\eta = \frac{\mu}{P} = 1.6 \times 10^{-5}$$

AIR - GAS

At 70°C =

$$\mu \uparrow =$$

so when
temp ↑

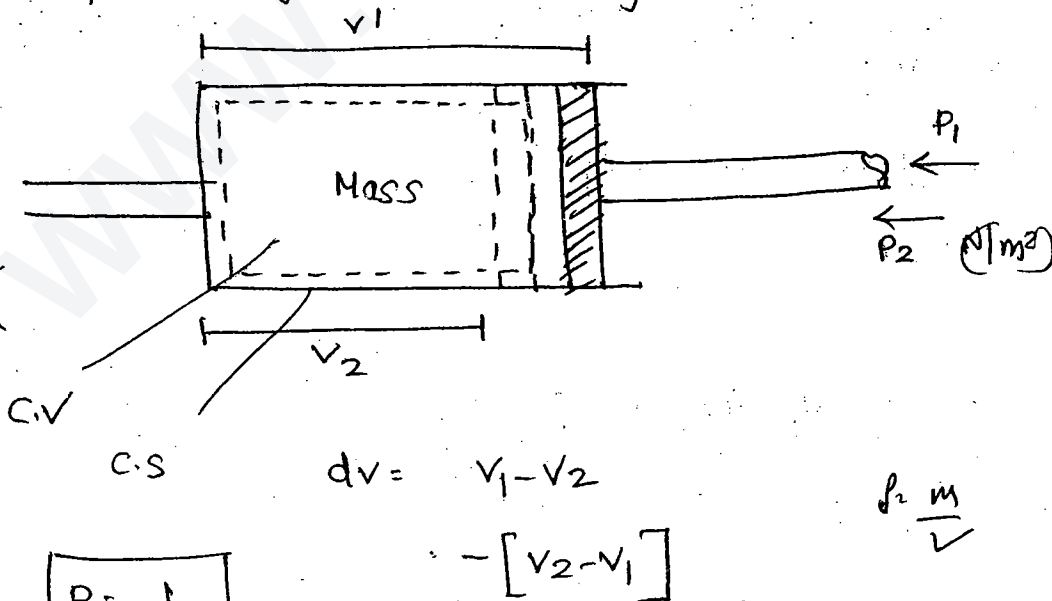
viscosity ↑

$$\text{so } \eta > 1.6 \times 10^{-5}$$

so (a)

→ Compressibility and Bulk Modulus of elasticity

In case of hydraulic systems the working fluid is incompressible ($\rho = \text{const}$) fluid [water, oils, Mercury etc] subjected to the external pressures causing the change in volumetric strain [decreased] or increased density strain due to the fluid behaviour compressibility as shown in fig



$$\beta = \frac{1}{K}$$

K = Bulk Modulus of elasticity of liquid

$$= \frac{\Delta P}{\Delta \rho / \rho}$$

$$\mu_1 A \cdot V \times \frac{-1}{y^2} + \mu_2 A \cdot V \times \frac{1}{(H-y)^2} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2} = 0$$

$$\underline{AV = 0}$$

$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(H-y)^2}$$

$$\frac{\mu_1}{\mu_2} = \frac{y^2}{(H-y)^2}$$

$$\boxed{\sqrt{\frac{\mu_1}{\mu_2}} = \frac{y}{H-y}}$$

VARIATION OF VISCOSITY WITH RESPECT TO TEMPERATURE

NOTE:

$$\mu = \mu_0 \left[\frac{1}{1 + \alpha t + \beta t^2} \right]$$

1. FOR liquids viscosity decreases with the increase of temp
2. For gases viscosity increases with the increases of temp

1. the viscosity of air at 20° is given to be

99 GATE 13

$1.6 \times 10^{-5} \text{ m}^2/\text{s}$ - Its ν at 70° will vary approximately

- a) $2.2 \times 10^{-5} \text{ m}^2/\text{sec}$
- b) $1.6 \times 10^{-5} \text{ m}^2/\text{sec}$
- c) $1.2 \times 10^{-5} \text{ m}^2/\text{sec}$
- d) $1.6 \times 10^{-5} \text{ m}^2/\text{sec}$

At 20°C

$$\eta = \frac{\mu}{\rho} = 1.6 \times 10^{-5}$$

AIR - GAS

At 70°C =

$\mu \uparrow =$

so when temp $\uparrow$

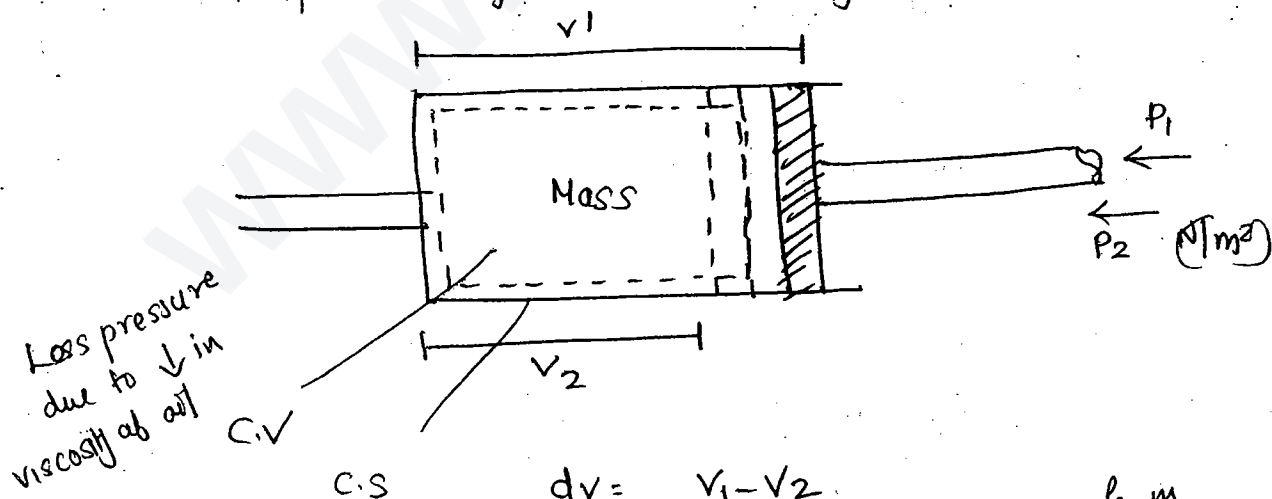
viscosity $\uparrow$

$$\text{so } \underline{\underline{\eta > 1.6 \times 10^{-5}}}$$

so (a)

→ Compressibility and Bulk Modulus of elasticity

In case of Hydraulic systems the working fluid is incompressible ($\rho = \text{const}$) fluid [water, oils, Mercury etc] subjected to the external pressures causing the change in volumetric strain [decreased] or increased density strain due to the fluid behaviour compressibility as shown in fig



$$dv = V_1 - V_2$$

$$\rho = \frac{m}{V}$$

$$\beta = \frac{1}{K}$$

$$- [V_2 - V_1]$$

$K =$ Bulk Modulus of elasticity of liquid

$$= \Delta P$$

where $uv = v_2 - v_1$

UNIT

$dp = p_2 - p_1$

$\frac{N/m^2}{\frac{m^3}{m^3}}$

Unit = $\frac{N}{m^2}$

DRDO

The increase in pressure required to decrease unit volume of mercury fluid by 0.1% is

- (a) 285 GPa (b) 285 MPa (c) 2.85 MPa (d) 28.5 MPa

Bulk modulus of mercury is 28.5 MPa

Ans:

$K = 28.5 \frac{N}{mm^2}$

$K = \frac{\Delta P}{-\frac{dv}{v}}$

$v = \phi$

Final Volume

$v = \frac{0.1 \times 28}{100}$

$28.5 MPa = \frac{\Delta P}{-[-0.1\%]}$

$\Delta P = \frac{0.999}{28.5}$

$10^6 \times 28.5 = \frac{\Delta P}{-[-\frac{0.1}{100}]}$

$\Delta P = 28.5 \times 10^6 \times \frac{0.1}{100}$

$= 28.5 \times 10^3 \times \phi$

$= 28.5 \times 10^3 N/m^2$

$= 28.5 kPa$

9)

when the pressure p on a given mass of liquid is increased from 30 bar to 35 bar, the density of the liquid increases from 500 kg/m^3 to 501 kg/m^3 . what is the value of Bulk modulus of the given tested liquid for the given range of pressure?

- (a) 700 MPa (b) 600 MPa (c) 500 MPa
(d) 250 MPa

Ans:

$$P \uparrow \quad 30 \rightarrow 35 \text{ Bar} \quad dP = 5 \text{ bar}$$

$$M = \text{const} \quad d\rho = 1 \text{ kg/m}^3$$

$$\rho \uparrow = 500 \rightarrow 501$$

$$K = \frac{P_2 - P_1}{\frac{\rho_2 - \rho_1}{\rho_1}} = \frac{5 \text{ bar}}{\frac{501 - 500}{500}}$$

$$= \frac{5 \times 10^5}{\frac{1}{500}}$$

$$= \underline{\underline{250 \text{ MPa}}}$$

(9)

which one of the following is represented 97-P.1 by Bulk modulus of a fluid (symbols have the usual meaning)

a) $\frac{d\rho}{\rho \cdot dP}$

b) $\frac{P \cdot d\rho}{dP}$

$K = \left(\frac{dP}{d\rho/\rho} \right)$

c) $\frac{dP}{\rho \cdot d\rho}$

d) $\rho \cdot \frac{dP}{d\rho}$

Water to that of Mercury.

Bulk Modulus of Mercury = 30 MPa

Bulk " " Water = ~~2~~²⁶ GPa $K_{\text{MERCURY}} = 30 \text{ MPa}$

$$\frac{2 \text{ MPa}}{30 \text{ MPa}} =$$

$$K_{\text{WATER}} = 26 \text{ GPa}$$

$$\frac{2 \text{ GPa}}{30 \text{ MPa}} = \frac{2 \times 10^9 \text{ Pa}}{30 \times 10^6 \text{ Pa}} = \frac{200}{3} = \underline{\underline{66.67}}$$

$$K_W = (60-70) \text{ Km}$$

SURFACE ENERGY OF FLUIDS OR SURFACE TENSION

All the fluids in Nature made from tiny molecules and the surface of the fluids exhibits a tensile force ~~or~~ in energy form which makes the fluid to hold other matter easily. This is due to the forces of molecules. [cohesive forces and adhesive forces]

Cohesive forces are those forces caused b/w own molecules attraction.

ex: mercury - liquid

Adhesive forces are those forces ~~cause~~ which are attraction b/w 2 diff matter molecules.

[water and other similar liquids]

Surface Tension of Liquids > that of gases

It is represented by σ

$$\sigma = \frac{\text{Force (F)}}{\text{unit length surface of liquid}}$$

Unit

$$\frac{N}{m}$$

EIL

Hygrometer
is the device
used to measure
surface tension

Specific Gravity

$$\sigma = \frac{\text{Energy possessed (Joules)}}{\text{unit surface Area (m}^2\text{)}}$$

$$\frac{Nm}{m^2} = \frac{Nm}{m^2} = \frac{N}{m}$$

units of surface tension

$$\frac{N}{m} = \frac{kg \cdot m}{s^2 \cdot m} = \frac{kg}{s^2}$$

$$= M^1 L^0 T^{-2}$$

$$\sigma_{\text{water - air}} = 0.073 \frac{N}{m}$$

$$\sigma_{\text{mercury}} = 0.44$$

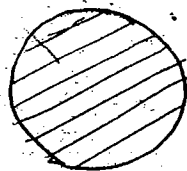
[6 times Higher]

$$\sigma_{\text{mercury}} > \sigma_{\text{water}}$$

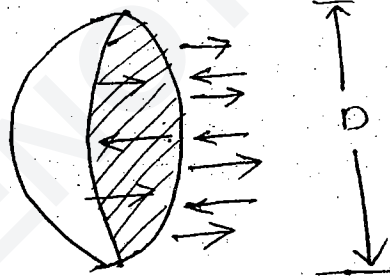
$$\sigma_M = 600$$

1. Droplet formation
2. Bubble formation
3. Jet travelling
4. Capillarity phenomena.

Sol :- D = dia of liquid droplet (metres)
 σ = Surface tension of liquid (N/m)
 P = Diff in pressure of liquid exerted (N/m²)



Droplet of liquid



Pressure exerted by - fluid due to surface tension

(a) droplet of liquid

$$\Delta P = \frac{4\sigma}{D} \left(\frac{\text{N/m}}{\text{m}} \right) = \frac{\text{N}}{\text{m}^2}$$

$$\Delta P = \frac{\text{Force}}{\text{Area (Normal)}} = \frac{F}{\frac{\pi \times D^2}{4}}$$

$F = \Delta P \times \frac{\pi \times D^2}{4}$

①

σ = surface tension of liquid formed in drop

$$\sigma = \frac{F}{\text{unit length}} = \frac{F}{\pi D}$$

$$F = \sigma \cdot \pi D \quad \text{--- (2)}$$

$$(1) = (2)$$

$$\Delta P \times \frac{\pi}{4} \times D^2 = 2 \sigma \cdot \pi \cdot D$$

$$\Delta P = \frac{4\sigma}{D}$$

AIR BUBBLE

$$\Delta P = \frac{F \times \pi \times D^2}{4}$$

$$\sigma = \frac{F}{\pi D}$$

$$\sigma \times \pi D = \Delta P \times \frac{\pi \times D^2}{4}$$

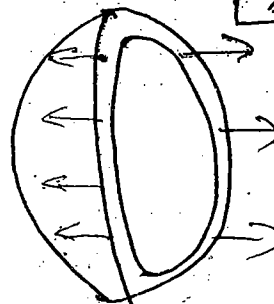
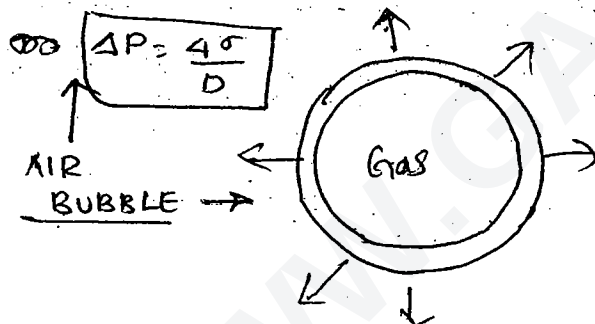
$$\Delta P = \frac{4\sigma}{D}$$

LIQUID BUBBLE

$$F = \Delta P / \pi \times D^2$$

$$\sigma = \frac{F}{2\pi D}$$

$$\Delta P = \frac{8\sigma}{D}$$

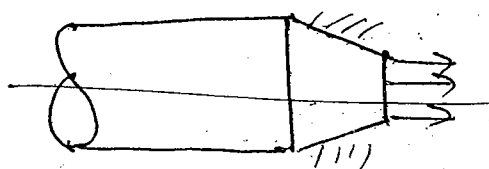


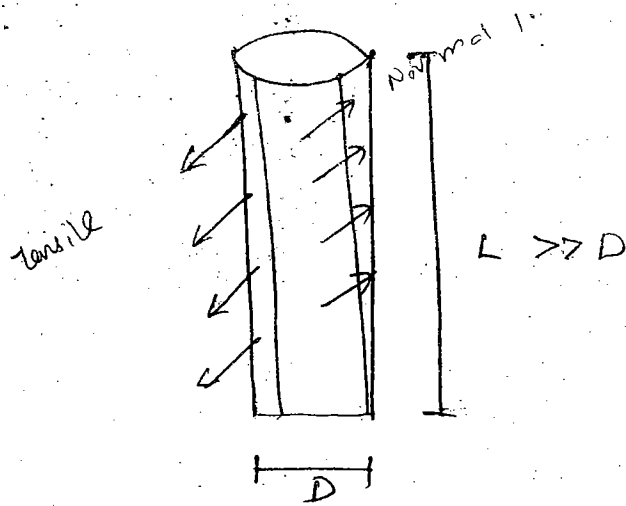
For
Liquid
Bubble

$$\Delta P = \frac{8\sigma}{D}$$

$$\Delta P_{\text{Bubble}} = 2 \times \Delta P_{\text{droplet}}$$

Jet of Liquid





Pressure difference (ΔP) = $\frac{F}{\text{Area Normal}}$

Surface tension = $\sigma = \frac{F}{\text{unit length}}$

$F = \Delta P \cdot A_N = \sigma \cdot \text{UNIT LENGTH}$

$\Delta P \times [D \times L] = \sigma \cdot [D + L + D + L]$

$\Delta P \times (D \times L) = \sigma (2D + 2L)$

$L \gg D$

$2L \gg 2D$

$\Delta P (D \times L) = \sigma (2L)$



$\Delta P = \frac{2\sigma}{D}$

$\Delta P_{\text{jet}} = \frac{\Delta P_{\text{DROPLET}}}{2}$

Q) The surface tension of water with air interface is 0.073 N/m . The Gauge pressure inside a rain droplet of diameter 1 mm will be.

- (a) 0.146 N/m^2 (b) 73 N/m^2 (c) 146 N/m^2
(d) 292 N/m^2

Ans: —

$$\sigma = 0.073$$

$$D = 1 \text{ mm}$$

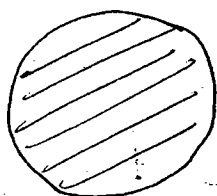
$$P = \frac{4\sigma}{D}$$

$$= \frac{4 \times 0.073}{10^{-3}}$$

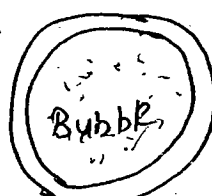
$$= \underline{\underline{292 \text{ N/m}^2}}$$

Q) If 'P' is the Gauge pressure within a spherical droplet then the gauge pressure within a Bubble of same fluid and of same size will be

- (a) $P/4$
(b) $P/2$
(c) P
(d) $2P$



$$P_1 = \frac{4\sigma}{D}$$



$$P_2 = \frac{8\sigma}{D}$$

$$= 2 \times 4\sigma$$

2 surfaces
so

$$\frac{8\sigma}{D}$$

what is the S.T. of soap film?

$$D = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$$

$$P = 25 \text{ N/m}^2$$

$$P = 25 \text{ N/m}^2$$

$$\sigma = ?$$

Bubble

$$P = 2 \times \frac{4\sigma}{D}$$

$$\sigma = \frac{P \times D}{8} = \frac{25 \times 50 \times 10^{-3}}{8} = \underline{\underline{0.156 \text{ N/m}^2}}$$

A water jet issued from a nozzle of diameter 2mm and length 25mm. The surface tension at water with air interface = $0.075 \frac{\text{N}}{\text{m}}$. What is the Pressure inside the jet above the atmosphere in N/m^2 ?

$$\sigma = 0.075 \text{ N/m}$$

$$l = 25 \text{ mm} = 25 \times 10^{-3} \text{ m}$$

$$D = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\Delta P = \frac{2\sigma}{D} = \frac{2 \times 0.075}{2 \times 10^{-3}}$$

$$= \underline{\underline{75 \text{ N/m}^2}}$$

$$\sigma = \text{Nm}$$

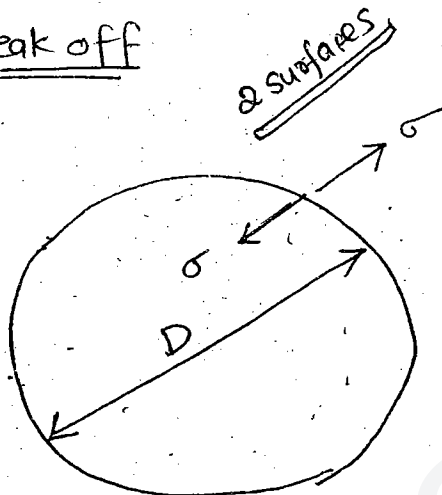
$$\frac{\sigma}{s} = \frac{\text{work}}{s}$$
$$\frac{\text{Nm}}{s}$$

1. Bubble Break off

2. Large size Bubble convert into no of Bubbles

→ Bubble Break off

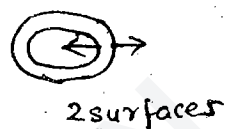
LIQUID BUBBLE



$$D_o = D_i = D$$

So thin film

RING



Work Reqd to Break a liquid Bubble

$$\sigma = \frac{F}{\text{unit length}} : \frac{\text{Energy Possessed}}{\text{unit Area}}$$

$$\sigma = \frac{\text{Energy}}{\text{Area}}$$

$$\text{Energy} = \sigma \times \text{Area}$$

$$\frac{J}{m^2} \times m^2 = \underline{J}$$

$$E = \sigma \left[\cancel{4\pi R^2} + \cancel{4\pi R^2} \right]$$

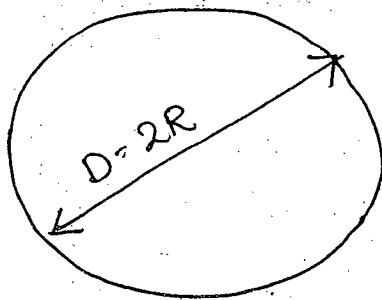
$$\boxed{E = \sigma (8\pi R^2)} \quad \text{Joules}$$

→
$$\boxed{E = \sigma [8\pi R^2]}$$

$$S.A = 4\pi R^2 \text{ (m}^2\text{)}$$

$$V = \frac{4}{3}\pi R^3 \text{ (m}^3\text{)}$$

Break off into small Bubbles



$$= \textcircled{1} \textcircled{2} \textcircled{3} \dots \textcircled{n}$$

LAW OF CONSERVATION OF MASS

mass Before collapse = mass After collapse

$$\cancel{\rho} V = n \cdot \cancel{\rho} v$$

$$V = n v$$

Mass Big Bubble = $n \times$ Mass small Bubbles

$$\frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$$

$$R^3 = n \cdot r^3$$

$$r = \frac{R}{\sqrt[3]{n}}$$

Work reqd to Break 'n' Bubbles

$$\sigma = \frac{\text{Energy}}{\Delta A}$$

ΔA change in area

$$\Delta A = n \cdot a - A$$

Area of small Bubbles

Big Bubble Area

no. of Bubbles

$$\text{Energy} = \sigma \times (\Delta A)$$



$$= \sigma \left[n \cdot \frac{4}{3}\pi r^2 - \frac{4}{3}\pi R^2 \right] = \sigma \left[n \cdot \frac{4}{3}\pi \left(\frac{R}{\sqrt[3]{n}} \right)^2 - \frac{4}{3}\pi R^2 \right]$$

① What is the energy reqd to Break the Soap liquid Bubble of dia 100 mm.

S.T of soap soln is $0.05 \frac{N}{m}$ is

- a) 0.00157 Joule (N-m)
- b) 0.00314 Joule
- c) 0.00628 Joule
- d) None

$$\frac{J}{m^2} = \frac{N \cdot m}{m^2}$$

$$\sigma = \frac{\text{Energy}}{\text{Area}}$$

$$\text{Energy} = \sigma \times [4\pi R^2 + 4\pi R^2]$$

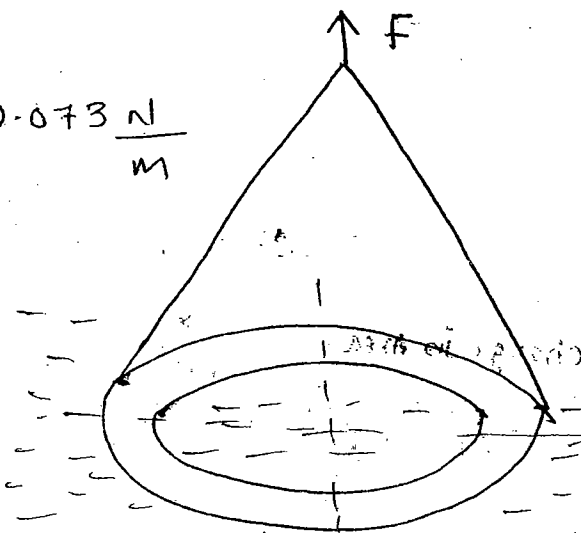
$$= 8\pi R^2 \times \sigma$$

$$= 8 \times \pi \times \left[\frac{100 \times 10^{-3}}{2} \right]^2 \times 0.05$$

$$= \underline{\underline{3.14 \times 10^{-3} \text{ Joules}}}$$

② What is the force necessary to lift a Platinum wire from water surface shown in fig. Neglect self weight of Ring and Buoyancy force.

$$\sigma = 0.073 \frac{N}{m}$$



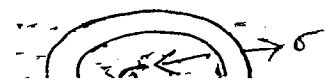
a) 0.00685 N

b) 0.0137 N

c) 0.0274 N

d) 0.0175 N

Like a Ring



$$\gamma = \frac{\text{force applied}}{\text{unit length}} \quad \frac{1}{m}$$

$$0.073 = \frac{\text{Force applied}}{2 \cdot \pi D}$$

$$D = 0.04 \text{ m}$$

$$0.073 = \frac{F}{2 \pi \times 0.04}$$

$$F = 2 \pi \times 0.04 \times 0.073$$

$$= \underline{\underline{0.0175 \text{ (N)}}}$$

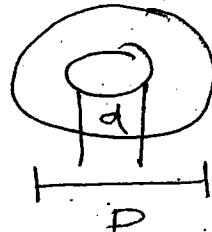
2 - inside and outside surface

③

PLANE SHEET

[LIKE METAL WASHER]

$$= \pi D$$



unit length = πD

→ surface 2

③

A spherical liquid Bubble of Radius 'r' is blown into 64 small bubbles of equal size and volume - what is the work reqd to convert large size into small n's in terms of S.T.

$$\text{Work} = \sigma [n \cdot a - A]$$

$$\sigma \left[n \frac{4\pi r^2}{3} - \frac{4\pi R^2}{3} \right]$$

a) $12\pi R^2 \sigma$

b) $16\pi R^2 \sigma$

c) $4\pi R^2 \sigma$

d) None

$$\delta = R$$

$$\delta = R$$

$$\sigma \left[64 \times \frac{4\pi \times R^2}{16} - \frac{4\pi \times R^2}{16} \right]$$

$$= \underline{12 \pi R^2 \cdot \sigma}$$

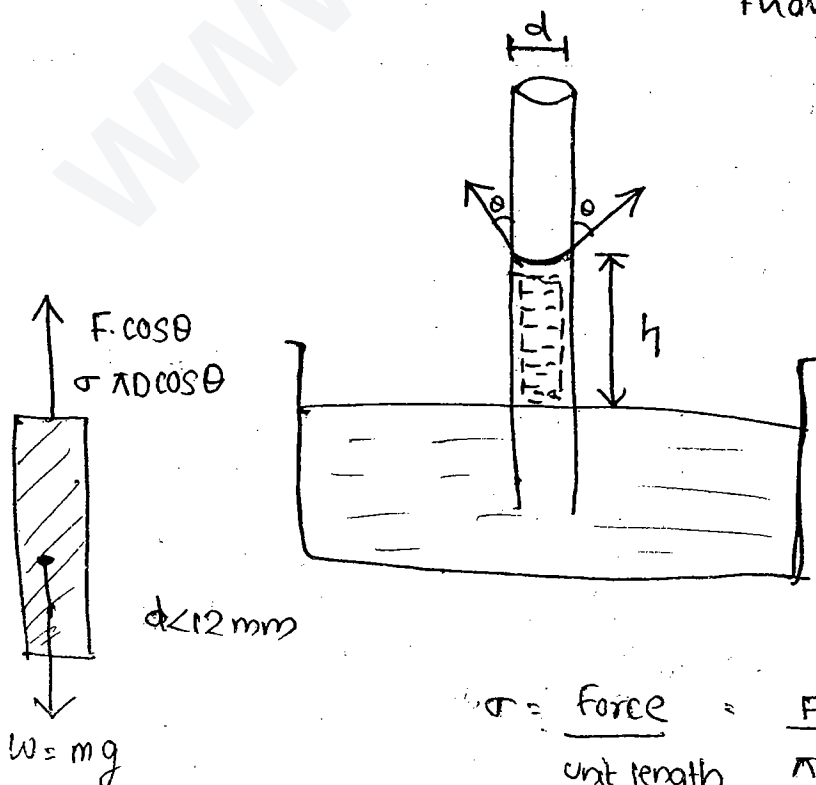
CAPILLARITY PHENOMENA

[SAP - plants receiving minerals from Beneath]

CAPILLARITY is the phenomena of liquid due to which the liquid rises their level in small narrow conduit [tube, channel] expect mercury where the capillarity effect is the depression or fall of mercury column. This phenomenon is observed in narrow tubes

$$\text{ie } D < 12 \text{ mm}$$

Expression for Capillary Rise of liquid other than mercury



$$\sigma = \frac{F}{L}$$

$$= \frac{F}{\pi D}$$

$$\sigma = \frac{\text{force}}{\text{unit length}} = \frac{F}{\pi D}$$

$$F \cos \theta = mg$$

$$\sigma \times \pi d \cos \theta = \rho \cdot V \cdot g$$

$$= \rho \cdot A \cdot h \cdot g$$

$$\sigma \times \frac{\pi d^2}{4} \cdot h \cdot g$$

$$\sigma \times \pi \cos \theta = \rho \times \frac{\pi d}{4} \cdot h \cdot g$$

$$h = \frac{4 \sigma \cos \theta}{\rho g d}$$

$$\frac{4 \sigma \cos \theta}{\gamma d}$$

$$h = \frac{4 \sigma \cos \theta}{\gamma d}$$

$$\frac{\frac{N}{m}}{\frac{N}{m^3} \times m}$$

[if High dia tube is used
Rise will be very small]

unit

$$= \underline{m}$$

$$w = \rho g$$

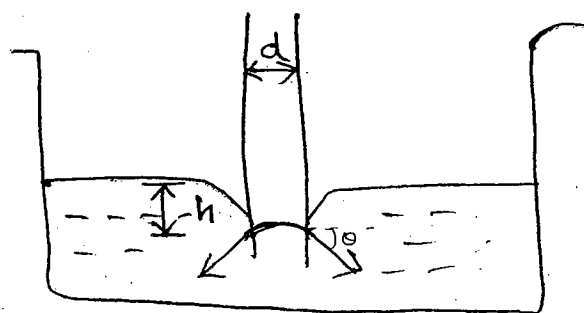
$$\frac{kg \times \frac{m}{s^2}}{m^3}$$

if $d \uparrow$ then $h \downarrow$
capillary fall.

Capillary action of Mercury.

θ = angle of contact

capillary fall



$$\theta > 90$$

$$(135^\circ)$$

- 1) A capillary Rise in 3mm dia of Tube immersed in a liquid is 1.5 cm. If another tube of dia 4mm is immersed in same liquid then capillary Rise would be in mm.

$$d_1 = 3 \text{ mm}$$

$$h_1 = 1.5 \times 10 \text{ mm}$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

$$h \propto \frac{1}{d}$$

$$h_1 d_1 = h_2 d_2$$

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}$$

~~4~~ ~~1.5~~

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}$$

$$\frac{15}{h_2} = \frac{4}{3}$$

$$h_2 = \frac{15 \times 3}{4} = \frac{45}{4} = \underline{\underline{11.25 \text{ mm}}}$$

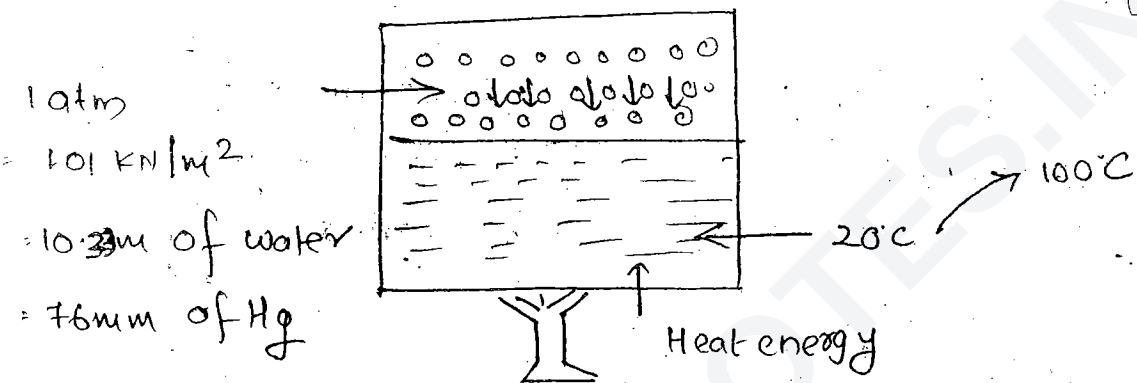
VAPOUR PRESSURE

It is a property of fluid due to which the fluid surface exerts additional Pressure intensities (Partial Pressures) due to which bubbles in liquid are formed. The Bubbles rises from the liquid and collapse on the metal surfaces due to which the metal surface eroded. This

the sharp bends of the pipes

Every liquid and gas has its own vapour pressure.

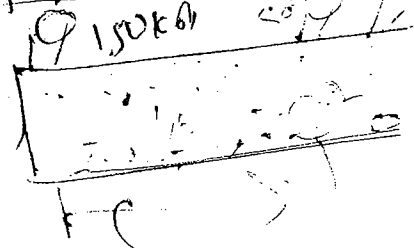
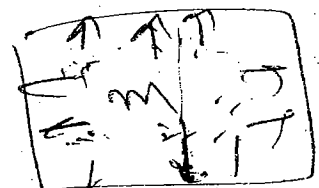
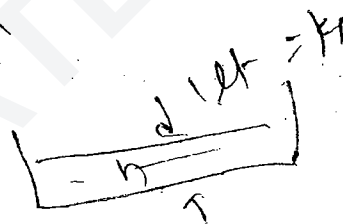
ex: Water = 25 kPa [2.5 m of water column]



$$p = \rho gh$$

Boyles Law

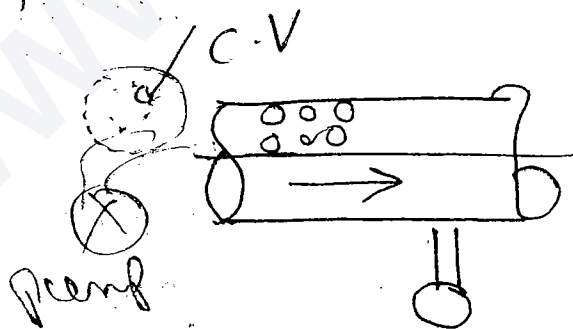
$$P \propto \frac{1}{V} \quad T \uparrow$$



At low pressure, liquid mole. can't exist, so they jump into higher state, i.e. vapour.

$$P \propto T$$

temp remains constant



During state change temp will not change

In fluid mechanics the vapour pressure is concerned with the liquids where the region pressure is

Below atmos pressure or reaches the fluid self

liquid pressure which leads to Bubble formation

Thermodynamic properties

Gases - compressible fluids

$$PV = mRT$$

$$PV_s = RT$$

→ V_s = specific volume

$$= \frac{V}{m} = \frac{1}{\rho}$$

T - absolute temp in °K

R = Gas const

ρ = density

$$\rightarrow \frac{P}{\rho} = RT$$

[When temp and Pressure changes ' ρ ' varies]

SO

$$\underline{R}$$

$$\frac{P}{\rho} = RT$$

in SI

$$R = 287 \text{ J/kgK}$$

R-unit

UNIT

$$R = \frac{P}{\rho T} = \frac{N}{m^2 \times \frac{kg}{m^3} \times K}$$

Processes

→ Isothermal

$$\frac{P}{\rho} = \text{const}$$

→ Adiabatic

$$\frac{P}{\rho^k} = \text{const}$$

$$\frac{N m^3}{m^2 kg K} = \frac{Nm}{kg K} = \underline{\underline{J/kgK}}$$

$$k = \frac{c_p}{c_v} = 1.4 \text{ for air}$$

$$= M \times R$$

M = Molecular Mass

= 8314 J/kg-mole in SI unit

M = mass of Gas molecules

Mass of a Hydrogen atom

Compressibility

$$\beta = \frac{1}{K}$$

$$K = \frac{\text{Increase in Pressure}}{\text{volumetric strain}} = \frac{dP}{-\frac{dV}{V} \text{ or } \frac{dP}{\beta}}$$

Relationship b/w K and Pressure of Gas

Isothermal process

$$\frac{P}{\rho} = \text{const}$$

$$P V_s = \text{const}$$

$$\text{Diff } P dV_s + V_s dP = 0$$

$$P dV_s = -V_s dP$$

$$P = -V_s \frac{dP}{dV_s} = \frac{dP}{-\frac{dV_s}{V_s}}$$

$$\rightarrow \boxed{K = P}$$

Adiabatic

$$\frac{P}{\rho^k} = \text{const}$$

$$\rightarrow \boxed{K = P^k}$$

$$P dV^k + V^k dP = 0$$

$$P dV^k = -V^k dP$$

$$P = -\frac{V^k dP}{dV^k}$$

$$P \rho_s^k = \text{const}$$

diff

$$dv_s P.K v_s^{k-1} = -v_s dp$$

$$P.K = \frac{-v_s dp}{v_s^{k-1}}$$

$$P.K = - \frac{dp}{v_s^k}$$

$$v_s^k$$

$$P.K dv_s^{k-1} \cdot dv_s = -v_s^k dp$$

$$P.K = \frac{-v_s^k dp}{dv_s^{k-1} \cdot dv_s} = \frac{dp}{-v_s^k}$$

PROBLEMS

TEXT 1.5

$$A = 1.5 \times 10^8 \text{ mm}^2$$

$$v = 0.4 \text{ m/s}$$

$$y = 0.15 \text{ mm}$$

$$F = 2$$

$$\text{Power } P = 2$$

$$\mu = 1 \text{ POISE} = \frac{1 \text{ NS}}{10 \text{ m}^2}$$

$$\frac{F}{A} = \mu \frac{v}{y}$$

$$F = \frac{1 \times 1.5 \times 0.4}{10 \times 0.15 \times 10^{-3}}$$

$$\text{SHEAR FORCE} = \frac{400 \text{ N}}{1}$$

(ii) Power required to maintain this speed

$$= F \times \text{velocity} = 160$$

$$= 400 \times 0.4$$

$$\frac{\text{N} \times \text{m}}{\text{s}}$$

$$\frac{\text{J}}{\text{s}} = \underline{\underline{\text{W}}}$$

$$= \underline{\underline{160 \text{ W}}}$$

16

$$\mu = 1 \text{ POISE} = \frac{1 \text{ NS}}{10 \text{ m}^2}$$

$$d = 10 \text{ cm} = 10 \times 10^{-2} \text{ m} = 0.1 \text{ m}$$

$$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$N = 150 \text{ rpm}$$

$$v = \pi DN = \pi \times 0.1 \times 150 = 0.785 \text{ m/s}$$

$$\tau = \mu \times \frac{du}{dy} = \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}}$$

$$= \underline{\underline{52.33 \text{ N/m}^2}}$$

1.7 $\mu = 2$

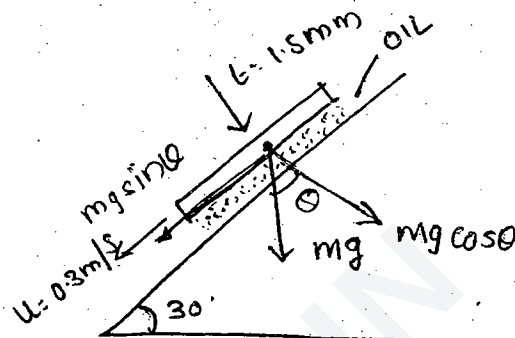
$$A = 0.8 \times 0.8$$

$$\theta = 30^\circ$$

$$W = 300 \text{ N}$$

$$V = 0.3 \text{ m/s}$$

$$y = 1.5 \times 10^{-3} \text{ m}$$



$$\tau = \mu \times \frac{du}{dy}$$

$$\mu = \tau \times \frac{dy}{du}$$

$$\tau = \frac{F}{A} = \frac{mg \sin \theta}{0.64}$$

$$= \frac{300 \sin 30}{0.64} = \frac{150}{0.64}$$

$$= \underline{\underline{234.375 \text{ N/m}^2}}$$

$$\mu = 1.17 \times \frac{\text{Ns}}{\text{m}^2}$$

$$= \underline{\underline{1.17 \text{ Poise}}} \quad 11.7 \text{ POISE}$$

$$1 \text{ Poise} = \frac{1 \text{ N s}}{10 \text{ m}^2}$$

1.8 $y = 1.25 \times 10^{-2} \text{ m} ; 0.0125 \text{ m}$

$$\text{oil } \mu = 14 \text{ poise} = \frac{14}{10} \frac{\text{Ns}}{\text{m}^2} = \underline{\underline{1.4 \text{ N s/m}^2}}$$

$$\tau = 3$$

$$V = 2.5 \text{ m/s}$$

$$\tau = \mu \cdot \frac{du}{dy} = \underline{\underline{280 \text{ N/m}^2}}$$

1.9 Area = $60 \times 60 = 3600 \text{ cm}^2$
oil.

$$y = l = 12.5 \text{ mm}$$

$$du = 2.5 \text{ m/s}$$

$$F = 98.1 \text{ N}$$

$$\mu = ?$$

$$V = ?$$

$$\frac{F}{A} = \mu \cdot \frac{du}{dy}$$

$$\mu = \frac{F \times dy}{A \times du}$$

$$\mu = \frac{98.1}{3600 \times 10^{-4}} \times \frac{12.5 \times 10^{-3}}{2.5}$$

$$= \underline{\underline{1.36 \text{ N s} / \text{m}^2}} = \underline{\underline{13.6 \text{ Poise}}}$$

$$S_{\text{po}} \frac{\rho_{\text{liq}}}{\rho_{\text{wo}}} = \frac{\rho_{\text{oil}}}{1000}$$

$$\rho_{\text{oil}} = 1000 \times 0.95$$

$$= \underline{950 \text{ kg/m}^3}$$

$$\nu = \frac{\mu}{\rho} = \underline{\underline{\text{m}^2/\text{s}}}$$

(1.10)

$$\rho = 981 \text{ kg/m}^3$$

$$\nu = 3$$

$$Z = 0.2452$$

$$\frac{du}{dy} = 0.2$$

$$Z = \mu \frac{du}{dy}$$

$$\nu = \frac{\mu}{\rho}$$

$$\underline{12.5 \text{ cm}^2/\text{s}} = \underline{12.5 \text{ stoke}}$$

(1.11)

$$S = 3$$

$$\mu =$$

$$\mu = 0.05 \text{ Poise}$$

$$= 0.005 \frac{\text{N s}}{\text{m}^2}$$

$$\nu = 0.035 \text{ stoke}$$

$$= 0.034 \times 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$\rho_{\text{fluid}} = 3$$

$$\nu = \frac{\mu}{\rho_{\text{fluid}}}$$

$$S = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}$$

$$S = \underline{\underline{\quad}}$$

(1.12)

$$\nu = 6 \text{ stokes} = 6 \times 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$S = 1.9$$

$$\nu = \frac{\mu}{\rho}$$

$$u \text{ in m/s}$$

$$y \text{ in m}$$

$$z = 0$$

$$y = 0.15$$

$$\frac{du}{dy} = \frac{3}{4} - 2y$$

$$\tau = \mu \times \frac{du}{dy}$$

$$\tau = \frac{8.5}{10} \times \frac{3}{4} - 2y$$

$$\tau = 0.86 \times \frac{3}{4} - 2 \times 0.15$$

$$= \underline{\underline{0.3825 \frac{N}{m^2}}}$$

1.14

$$\mu = 6 \text{ poise}$$

$$= \frac{6}{10} \frac{Ns}{m^2}$$

$$d = 0.4 \text{ m}$$

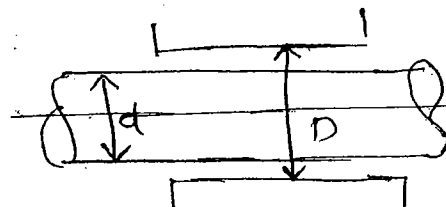
$$N = 190 \text{ r.p.m}$$

$$P = 3$$

$$l = 90 \text{ mm} = 9 \times 10^{-3} \text{ m}$$

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$$\frac{F}{\frac{\pi}{4} \times D^2} = 0.6 \times$$



$$\frac{F}{A} = \mu \times$$

$$\mu = 0.01 \text{ N s/m} = \frac{0.01}{10} \frac{Ns}{m}$$

$$\tau = \mu \times \frac{du}{dy}$$

$$\tau \times dy$$

$$\frac{\tau \times dy}{du}$$

$$\frac{N \times m}{m^2 \times m^2}$$

$$\frac{N \times m}{m^2 \times 1}$$

$$\frac{N}{m^2} \times \frac{m \times s}{m^2}$$

$$\underline{\underline{\frac{Ns}{m^2}}}$$

$$P = \frac{2\pi NT}{60}$$

$$T = F \times r_L$$

$$= F \times \frac{d}{2}$$

$$V = \frac{\pi D N}{60}$$

$$= \frac{\pi \times 0.4 \times 190}{60}$$

$$= \underline{\underline{3.98 \text{ m/s}}}$$

$$\frac{2 \times \pi \times N \times F \times \frac{D}{2}}{60}$$

$$60$$

$$y = \frac{D-d}{2}$$

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$$A = \pi dL$$

$$P = \frac{2 \times \pi \times N \times \mu \times \pi \times dL \times \frac{\pi D N \times 2}{60}}{60 \times D-d}$$

1.14

$$\mu = 0.6 \frac{Ns}{m^2}$$

$$d = 0.4m$$

$$N = 120 \text{ rpm}$$

$$P = 2$$

$$L = 90 \times 10^{-3} m$$

$$t = 1.5 \times 10^{-3} m$$

$$P = \frac{2 \pi N t}{60}$$

$$T = F \times \frac{D}{2}$$

$$V = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 120}{60}$$

$$F = \mu \times A \times \frac{du}{dy}$$

$$= 0.6 \times \frac{\pi \times (0.4)^2}{4} \times \frac{3.98}{1.5 \times 10^{-3}} \times 90 \times 10^{-3}$$

$$= \frac{2.51 m/s}{3.98 m/s}$$

$$\text{Area} = \pi dL$$

1.15

$$V = 120 \frac{cm}{sec} = \frac{120 \times 10^{-2} m/s}{1}$$

$\frac{du}{dy}$

NOTE:-

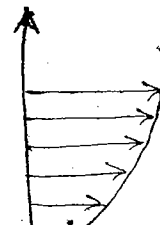
EQUATION OF VELOCITY PROFILE
(PARABOLIC) IS GIVEN BY -

$$u = ay^2 + by + C$$

a, b, c are constants

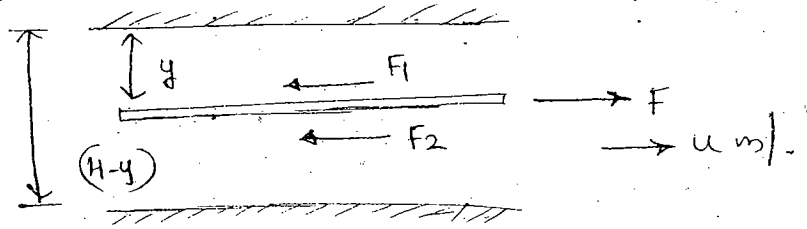
These values determined by
Boundary conditions

ie at: $y=0, u=0$



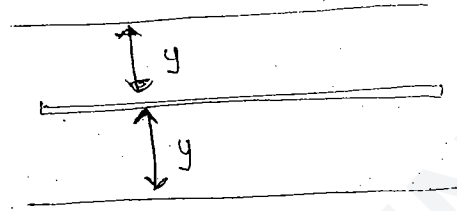
NOTE:-

If plate is very thin then thickness can be neglected.

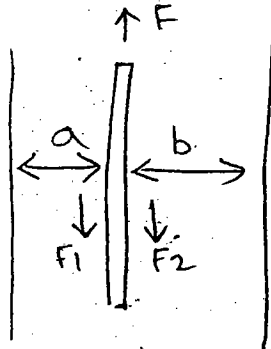


~~For~~

Vertical plates



forces
upward thrust
weight acting downwards



Force required to pull the plate upwards

Here the upthrust is also to be considered since weight of plate is acting in opposite direction

upward thrust = weight of fluid displaced
= weight density of fluid $\times$ volume of plate

$$\rho \times g \times \text{volume}$$

This upward thrust is due to the weight
the net force acting in downward direction due to weight and upward thrust

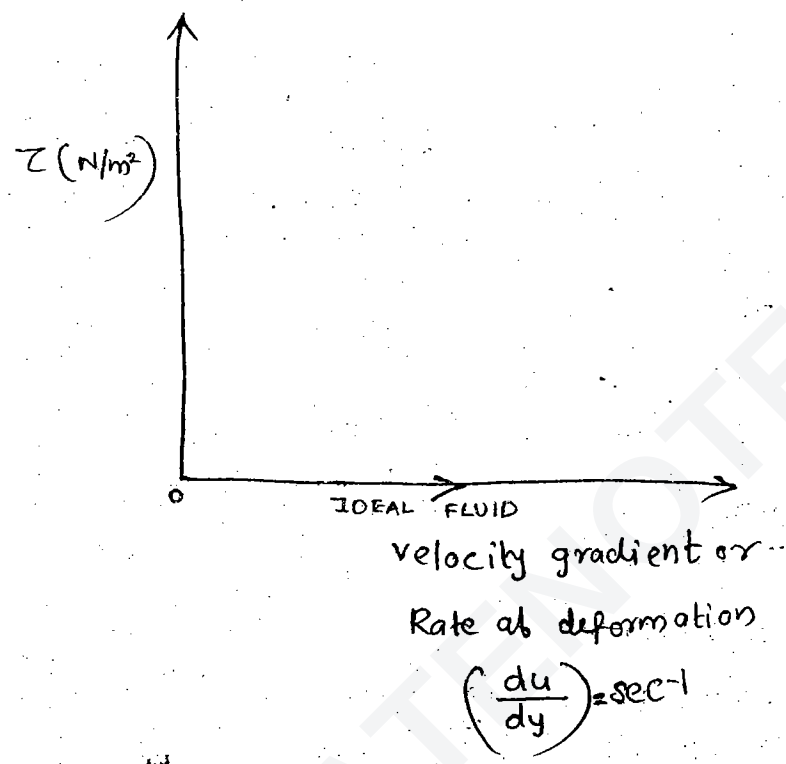
= weight - upward thrust

Total Force Reqd to lift plate = shear force + net force
($F = F_1 + F_2$) + w - upward thrust

due to viscosity

$$\text{Total Force} = (F_1 + F_2) + [mg - \text{upward thrust}]$$

Classification of Fluid Based on "Fluid Power Law" or Rheological Eq of Fluid



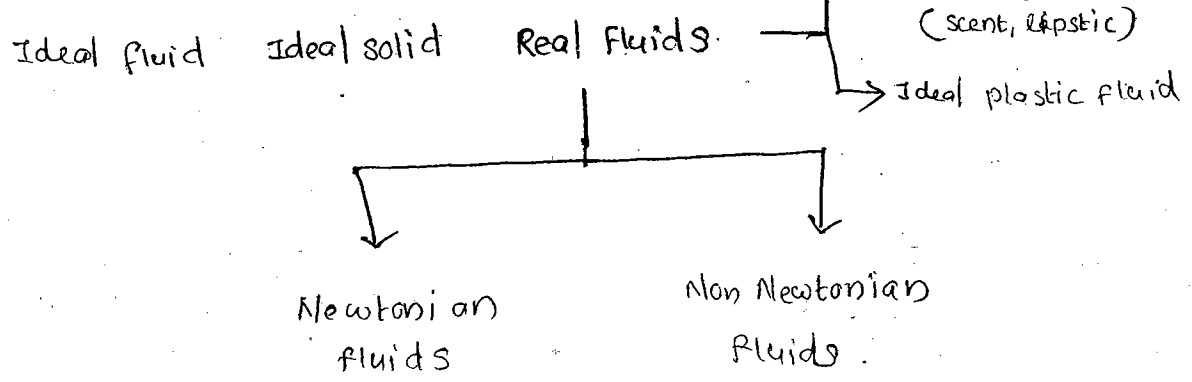
$$\tau = \tau_{yp} + \left(\frac{du}{dy}\right)^n \cdot \mu$$

$\tau = B + A \left(\frac{du}{dy}\right)^n$

 $\tau = B + A \left(\frac{du}{dy}\right)^n$

where n = flow behaviour index

Classification:-



$$\tau = \tau_{yp} + \left(\frac{du}{dy}\right)^n \cdot \mu$$

theoretical fluid. Never exists in Nature

$$\tau = B + A \left(\frac{du}{dy} \right)^n$$

$$B = 0$$

$$A = \mu = 0$$

$$\Rightarrow \tau = 0$$

2) Ideal solid:-

$$\tau \rightarrow \infty \text{ [Large]}$$

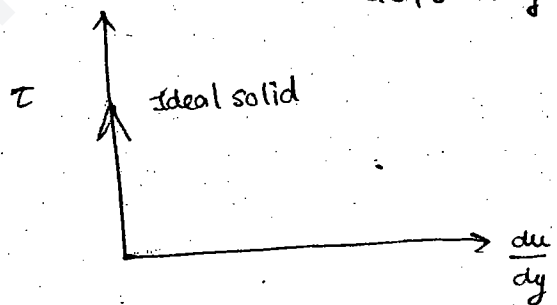
$$\frac{du}{dy} = 0 \Rightarrow$$

$$B \rightarrow \infty$$

$\tau_{yp} = \infty$ [very large forces needs will be applied for deforming]

3) Newtonian Fluids

→ Fluids which follows Newton's law of viscosity



$$\tau = B + A \left(\frac{du}{dy} \right)^n$$

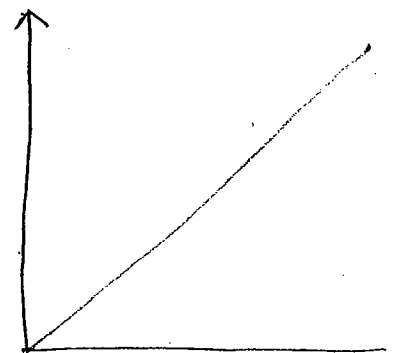
$$\text{ie } \tau \propto \frac{du}{dy}$$

$$B = 0, \tau_y = 0$$

$$\phi A = \mu$$

$$n = 1$$

$$\tau = \mu \frac{du}{dy}$$

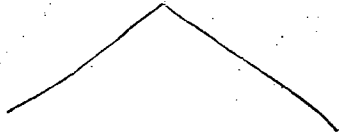


$$\text{slope } \mu = \frac{du}{dy} = \text{const}$$

1) Air

Non Newtonian Fluids

Fluids which does not obeys Newton's Law



Dilatent Fluid

$$B = \tau = 0$$

γ_p

$$A = \mu$$

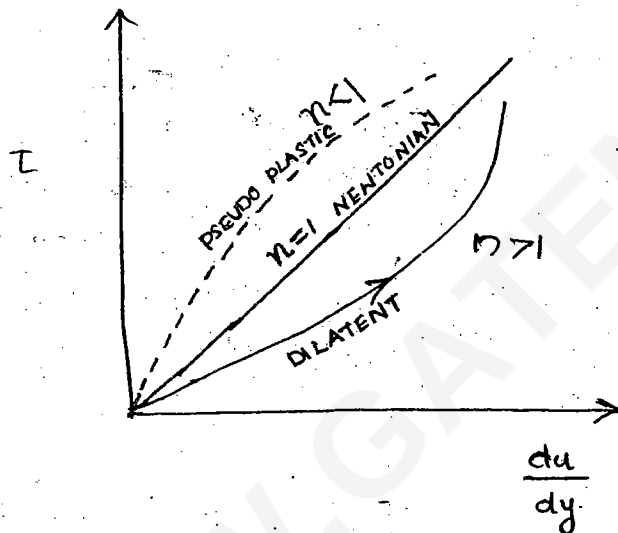
$$n > 1$$

DILATENT FLUID

$$n > 1$$

$$B = \tau_{yp} = 0$$

$$A = \mu$$



Ex: sugar water soln

salt water soln

Ex
imp

Polymer solution [Rubber solutions]

⊕ Butter

Rice starch

Two diff Fluids mixed to
form a new fluid

PSEUDO FLUIDS:-

$$n < 1$$

$$B = 0$$

$$A = \mu$$

ex: Blood

Milk

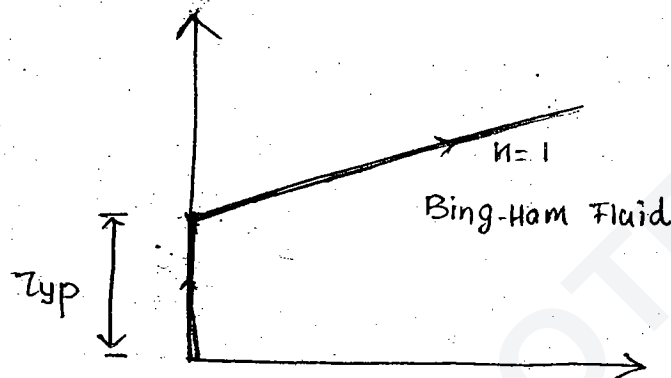
Suspension Fluids

$$\tau = B + A \left[\frac{du}{dy} \right]^n$$

$B = \tau_{yp} = \text{yield shear stress}$

$$A = \mu$$

$$n = 1$$



→ Tooth paste

Greases in tube

GREASES

Sewage sludge

Drilling mud

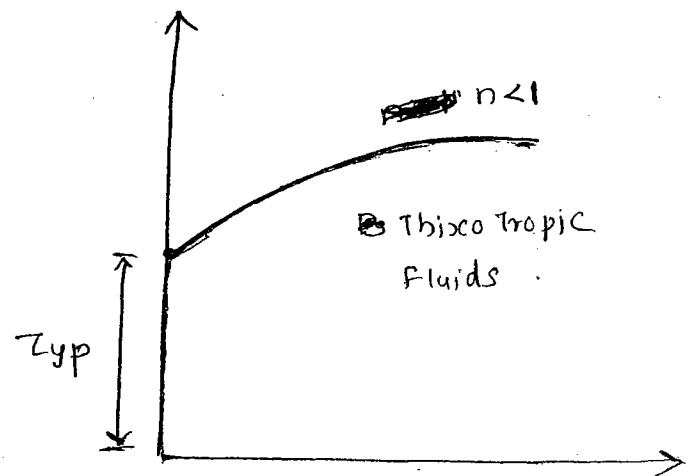
Thixotropic Fluids:-

$$\tau = B + A \left(\frac{du}{dy} \right)^n$$

$$B = \tau_{yp}$$

$$A = \mu$$

$$n < 1$$



— Fluids which are in
evaporative nature

Lipsticks solutions

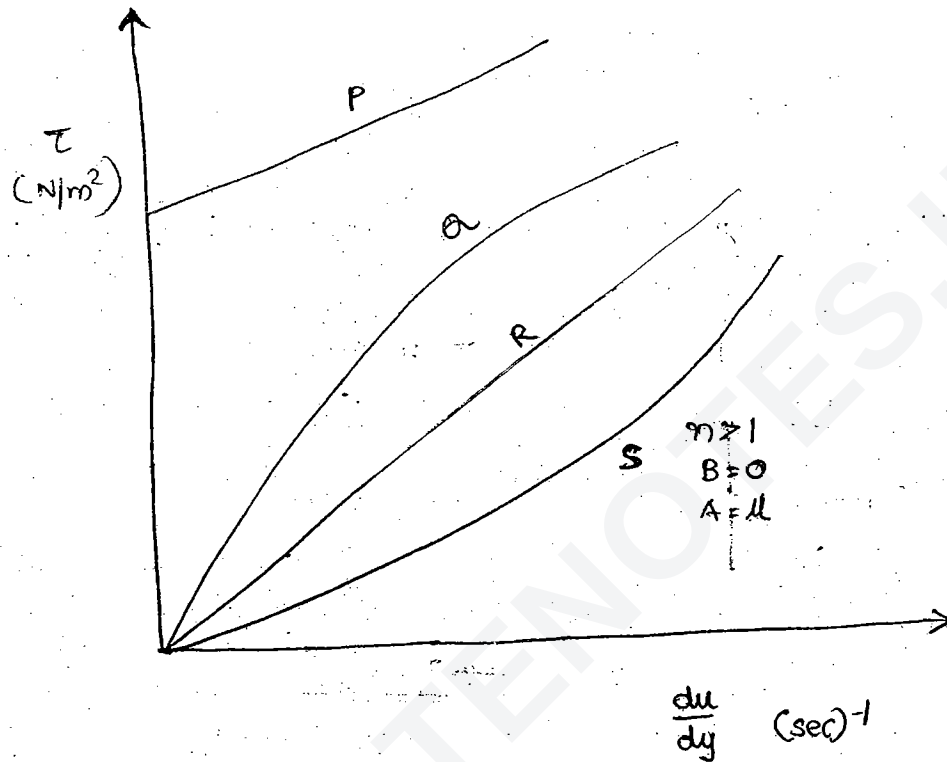
Print

Scant

Scant deodorants

2010

The Rheological diagram, the Relation b/w shear stress and shear strain rate for diff Types of Fluids shown below.



For S:

$$\tau = B + A \left(\frac{du}{dy} \right)^n$$

$$B = 0$$

$$A = \mu$$

$$n > 1$$

For R:

Newtonian Fluid

$$B = 0$$

$$A = \mu$$

$$n = 1$$

$$\tau = \mu \frac{du}{dy}$$

pseudoplastic fluid:

$$B = 0$$

$$A = \mu$$

$$n < 1$$

ex: Blood, Milk, Paint,

For P

$$B = \tau_{xy}$$

$$A = \mu$$

$$n = 1$$

Bingham Fluids

Tooth paste

Sewage sludge

Drilling mud

$$\tau = B + A \left(\frac{du}{dy} \right)^n$$

[Dry Fluids / Force has to be applied]

- Q. The most suitable Relation for the flow of tooth paste being squeezed out of the Tube is given by the curve

Ans:

ie Bingham Fluids

[so curve P]

- Q. Match List I with List II and select correct code given below.

List I (Fluid Power eqn)

(A) $\tau = \mu \left[\frac{du}{dy} \right]^n, n=1$

(B) $\tau = \mu \left[\frac{du}{dy} \right]^n, n < 1$

(C) $\tau = \mu \left[\frac{du}{dy} \right]^n, n > 1$

(D) $\tau = \mu \left(\frac{du}{dy} \right)^n, n=1$

List II (Fluid)

(1) Bingham Fluids

(2) Dilatant Fluids

(3) Newtonian Fluids

(4) Pseudo-Fluids

List I

- A) Ideal Fluid
- B) Newtonian Fluid
- C) Non-Newtonian
- D) Bingham Fluid

Ans:

B-1

C-2

D-3

A-4

List II

[Variation of shear stress]

- 1) shear stress varies linearly with rate of strain
- 2) shear stress doesn't vary linearly with rate of strain
- 3) Fluid behaves like a solid until a min yield stress beyond which it exhibits a linear relationship b/w shear stress and rate of strain
- 4) shear stress is zero

x ————— x

~~not~~

- Q. In an expt conducted to determine the Rheological behaviour of a material, it is observed that the Relation b/w shear stress τ and rate of shear strain $\frac{du}{dy}$ is given as

$$\tau = \tau_0 + c \left[\frac{du}{dy} \right]^{0.5}$$

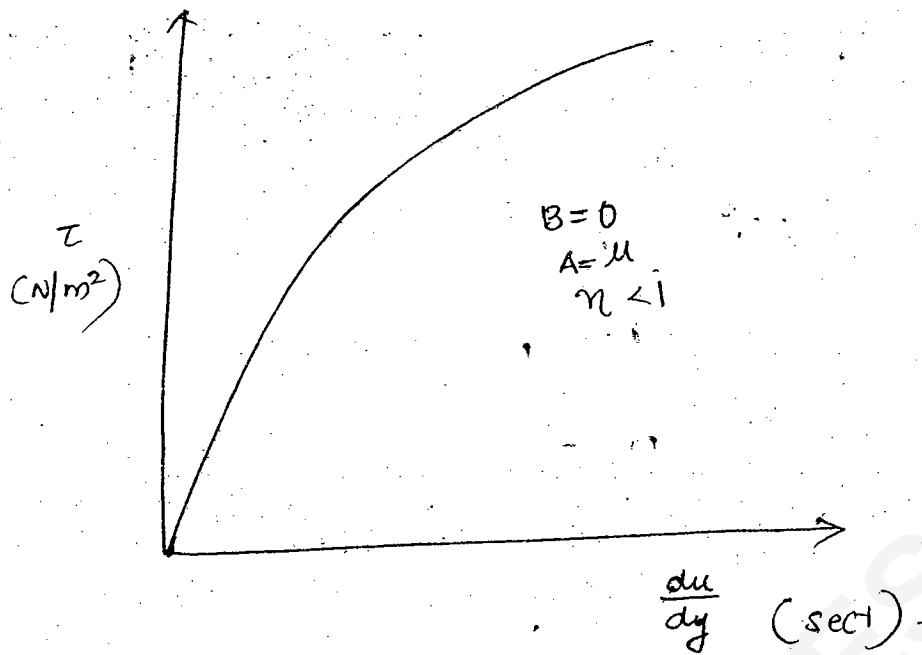
The material is

- a) Newtonian fluid
- b) Bingham plastic fluid
- c) An ideal plastic fluid
- d) thixotropic fluid

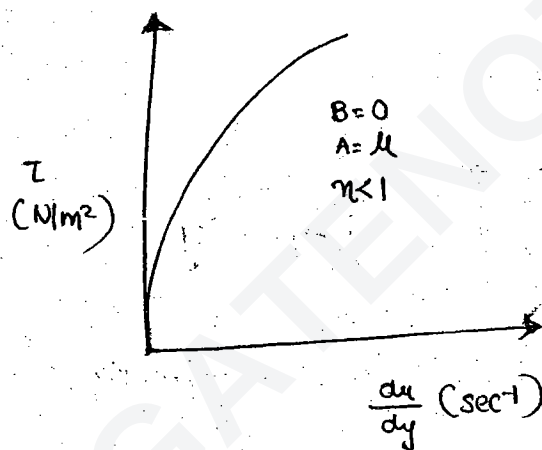
Ans:

D

$\tau_0 > 0$ $n < 1$



$$n < 1$$



POISE

$$1 \text{ POISE} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$$

$$1 \text{ STROKE} = 10^{-4} \frac{\text{m}^2}{\text{s}}$$

List I

a) A sp Gravity

B coeff of viscosity

C kinematic viscosity

D stress

List II

1) $\text{M}^0 \text{L}^2 \text{T}^{-1}$

2) $\text{M}^0 \text{L}^0 \text{T}^0$

3) $\text{M} \text{L}^{-1} \text{T}^{-1}$

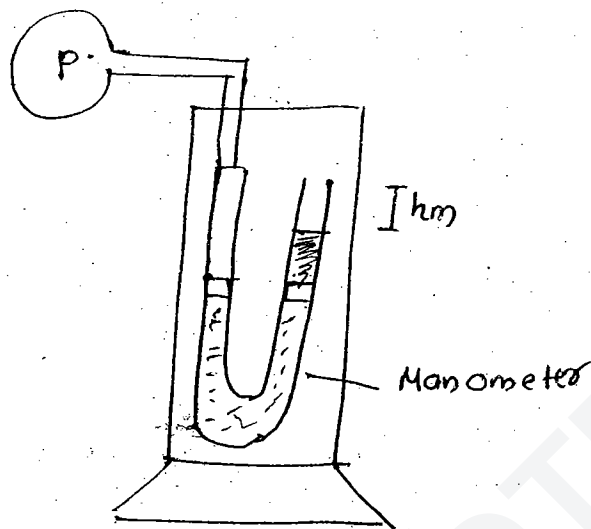
4) $\text{M} \text{L}^{-1} \text{T}^{-2}$

$$n = \frac{\log \mu}{\log \mu}$$

$$\frac{F}{A} = \frac{N}{\text{m}^2}$$

CAPILLARITY EFFECT

$$h_{\text{actual}} = h_m \pm h_{\text{capillarity}}$$



If mercury is manometer fluid

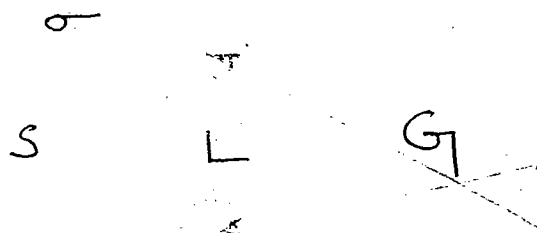
~~hact = h_m~~

$$h_{\text{act}} = h_m + h_{\text{capillarity}}$$

If other than mercury is manometer fluid

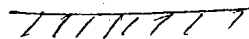
$$h_{\text{act}} = h_m - h_{\text{capillarity}}$$

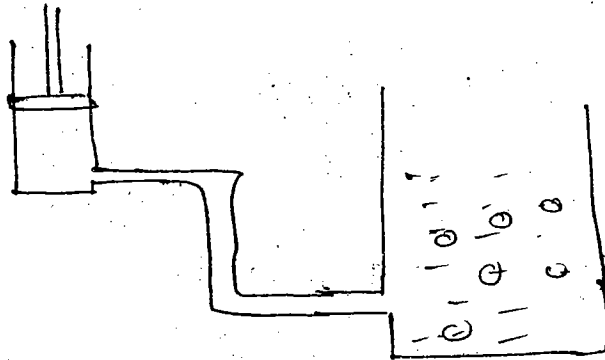
oil drop falling on solid surface and liquid surface



$\sigma_{GG} = \text{neglected}$

Liquid.  Gas


Solid



PAIR BUBBLE = $\frac{K}{D} \frac{q}{9}$

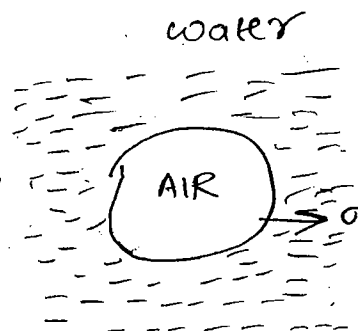
$K = 4$

air
water

PAIR BUBBLE

PAIR BUBBLE

$P_{\text{droplet}} = \frac{4\sigma}{D}$
 $P_{\text{Bubble}} = \frac{8\sigma}{D}$
 $P_{\text{jet}} = \frac{2\sigma}{D}$

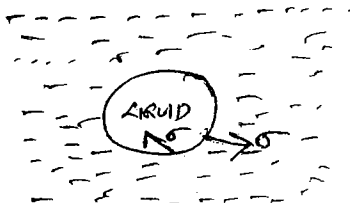


only one surface

σ_{GG} - ignore

AK JAIN

{ SOM & BISWAS }



2 surfaces



UNIT-II

FLUID STATICS

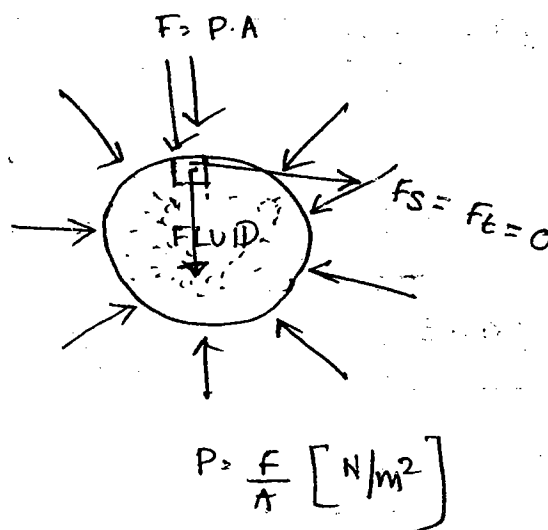
- 1.) Buoyancy force
- 2.) Hydrostatic force
- 3.) Pressure measurement

Fluid statics deals with fluid Behaviour and its effects when it is at Rest.

A fluid is said to be at Rest if no shear stresses acting on fluid particle
ie no viscosity.

only 2 forces involved
(gravity force)

1. Body force [self wt of fluid matter]
2. Normal Force on the surface due to Pressure intensity of the fluid.



Pressure $\rightarrow$ scalar qty

compressive stress.

$$M = \rho V$$

Bouyancy principle

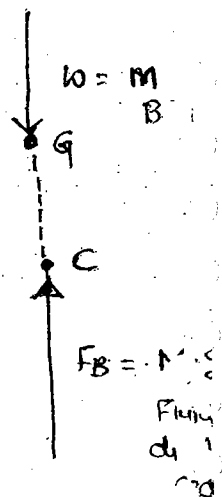
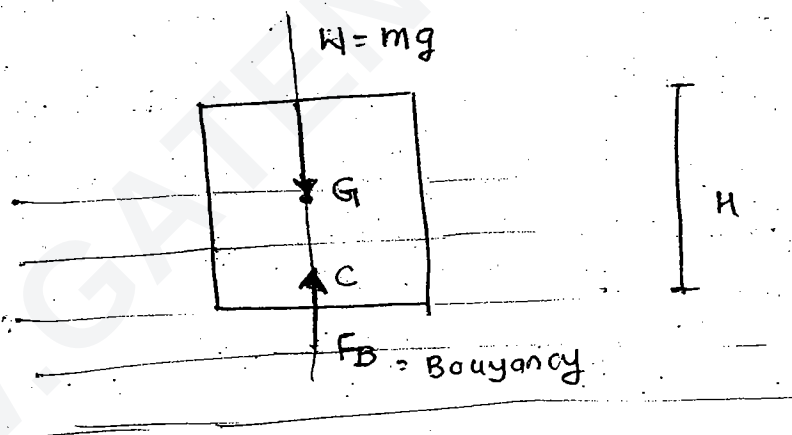
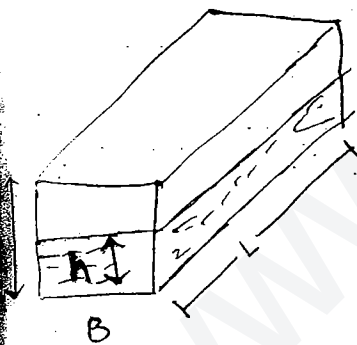
Bouyancy is a phenomena which is observed when a solid body interact with any liquid. There are 3 possibilities.

- 1) the body can float (partially submerged)
- 2) the Body can sink or (submerged fully)
- 3) Body top surface coincide with liquid surface

E/C QUES
Centre of Bouyancy

G = centre of Gravity

C = centre of Bouyancy



$$W = M_{\text{body}} \cdot g$$

$$= \rho_{\text{Body}} \cdot V \cdot g = \rho_{\text{Body}} \cdot A \cdot H \cdot g \text{ (Newton)}$$

$$F_B = M_{\text{fluid displaced}} \cdot g$$

$$\rho_{\text{fluid}} \cdot V_{\text{displaced}} \cdot g$$

$$\rho_{\text{fluid}} \cdot A \cdot h \cdot g \text{ (Newton)}$$

Under Equilibrium

35

$$F_B = W$$

$$\rho_{\text{fluid}} \cdot A \cdot h \cdot g = \rho_{\text{body}} \cdot A \cdot H \cdot g$$

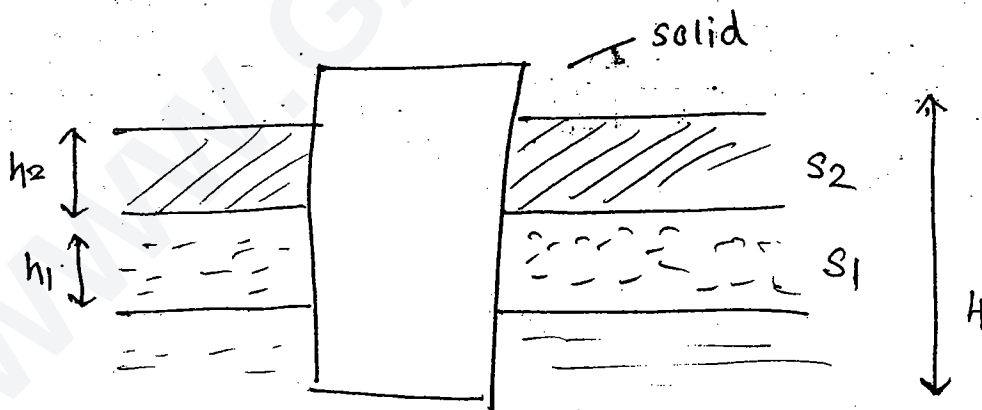
$$\frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} \cdot h = \frac{\rho_{\text{body}}}{\rho_{\text{water}}} \cdot H$$

$$S_{\text{fluid}} \cdot h = S_{\text{body}} \cdot H$$

$$\frac{S_{\text{FLUID}}}{S_{\text{BODY}}} = \frac{H}{h}$$

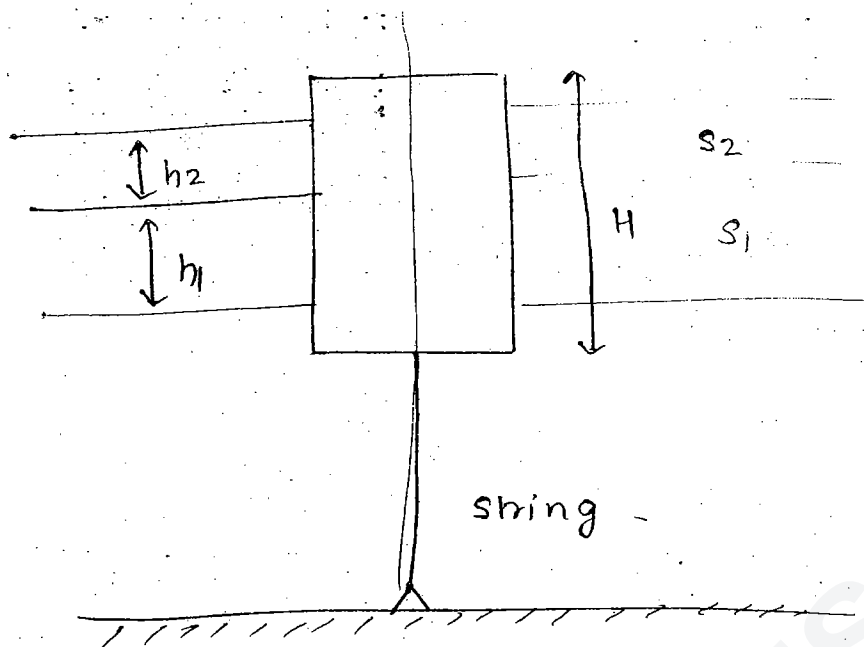
h = depth of immersion

Model 2

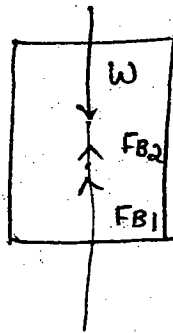


$$S_1 \cdot h_1 + S_2 \cdot h_2 = S_{\text{SOLID}} \cdot H$$

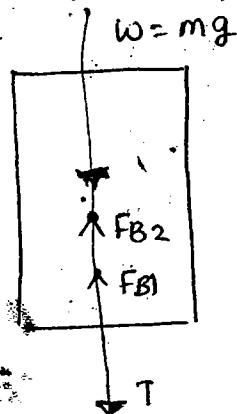
$$S_{\text{matter}} = \frac{\rho_{\text{MATTER}}}{\rho_{\text{WATER}}}$$



Free Body diagram



Let T = Tension in string



$$\sum F_y = 0$$

$$T + mg = F_{B1} + F_{B2}$$

$$\underline{T = ?}$$

$$T + m_{\text{body}} \cdot g = \rho_{\text{fluid1}} \cdot V_1 \cdot g + \rho_{\text{fluid2}} \cdot g \cdot V_2$$

$$V_1 = A \cdot h_1$$

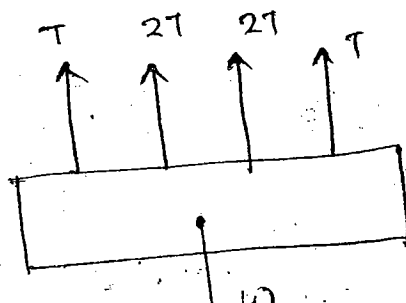
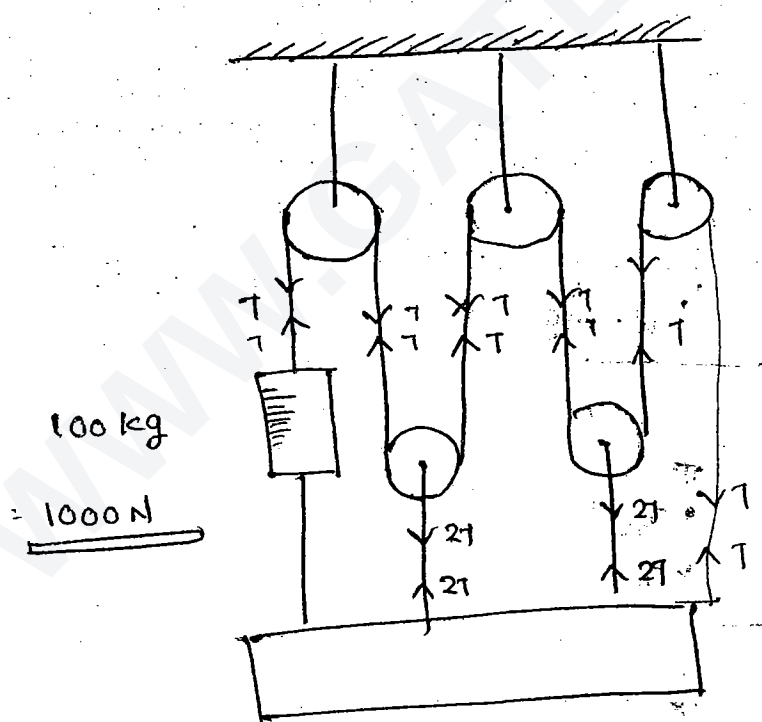
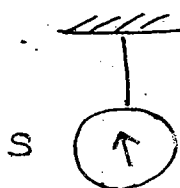
wetted volume of
fluid 1

$$V_2 = A \cdot h_2$$

wetted volume of
fluid 2

Model 4:-

Spring Balance



$$6 \times 1000 = W$$

$$W = 6000$$

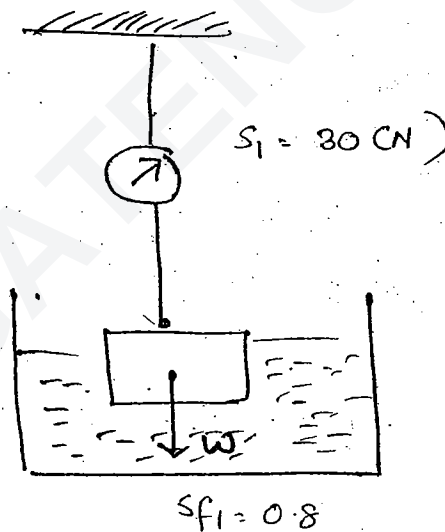
NOTE:

Spring Balance reading (S)

= Tension in string connected to Balance (T)

$$S = T$$

Model-4



Ans:

Case 1

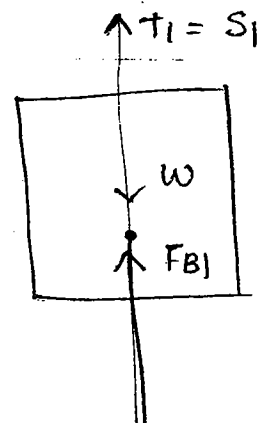
Free Body diagram

$$\text{So } F_{B1} + T_1 = W$$

$$\left[\rho_{\text{fluid}} \cdot V \cdot g + S_1 = M_{\text{body}} \cdot g \right]$$

$$800 \times V \times 10 + 30 = M_{\text{body}} \times 10$$

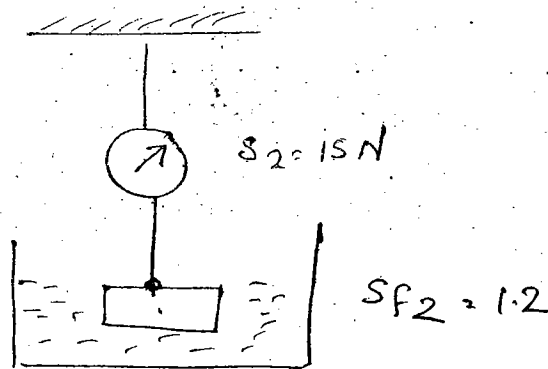
$$30 + F_{B1} = W \quad \text{--- (1)}$$



$$S_{f1} = 0.8 = \frac{\rho_{\text{fluid}}}{1000}$$

$$\rho_{\text{fluid}} = 800 \text{ kg/m}^3$$

Same
Body



- Find
- (1) Volume of Body in m^3 , in Lts
 - (2) weight of Body
 - (3) ~~sp~~ ^{wt} ~~g~~ of Body
 - (4) Buoyancy forces offered by Fluids

Case 2

$$15 + F_{B2} = W \quad \text{--- (2)}$$

$$30 + F_{B1} = W$$

$$15 + F_{B2} = W$$

$$15 + F_{B1} - F_{B2} = 0$$

$$F_{B1} - F_{B2} = -15$$

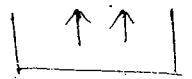
$$\rho_{\text{fluid1}} \times V_{\text{fluid1}} \times g - \rho_{\text{fluid2}} \times V_{\text{fluid2}} \times g = -15$$

$$800 \times \underline{V_{\text{fluid1}}} \times 10 - 1200 \times \underline{V_{\text{fluid2}}} \times 10 = -15$$

$$V_{\text{fluid}} [8000 - 12000] = -15$$

$$V_{\text{fluid}} = \frac{-15}{-4000} = \underline{3.75 \times 10^{-3} \text{ m}^3}$$

$$15 + F_{B2} = W$$



$$F_{B1} = \rho_{F1} \cdot V_1 \times g$$

$$= 800 \times 3.75 \times 10^{-3} \times 10$$

$$\underline{30 \text{ N}}$$

iii y

$$F_{B2} = \rho_{F2} \cdot V_2 \times g$$

$$= 1200 \times 3.75 \times 10^{-3} \times 10$$

$$= \underline{45 \text{ N}}$$

[Low density fluid - Low F_B]

$$\frac{F_{B2}}{F_{B1}} = \frac{45}{30} = \frac{3}{2}$$

$$30 + F_{B1} = W$$

$$30 + 30 = W$$

$$\underline{W = 60 \text{ N}}$$

③ Sp wt. $\gamma = \frac{W}{\text{Volume}}$

$$\frac{60}{3.75 \times 10^{-3}} = \underline{15.7 \text{ kN/m}^3}$$

④ Specific gravity = $\frac{W_{\text{body}}}{W_{\text{water}}} = \frac{15.7 \times 1000}{1000 \times 10} = \underline{1.57}$

Q. Two ~~spheres~~ Bodies one is solid sphere and another Body is hollow cube which are in equal Surface Area. what is the Ratio of the Buoyancy force by the fluid on the sphere to that on the cube ?

$$\frac{F_{B \text{ sphere}}}{F_{B \text{ cube}}} = ?$$

Solid sphere

$$\rightarrow F_B = \rho_{\text{fluid}} \cdot A \cdot h \cdot g$$

$$F_B \propto h$$

$$\frac{F_{B \text{ sphere}}}{F_{B \text{ cube}}} = \frac{\cancel{4\pi R^2} \cdot h_{\text{sphere}}}{\cancel{6a^2} \cdot h_{\text{cube}}}$$

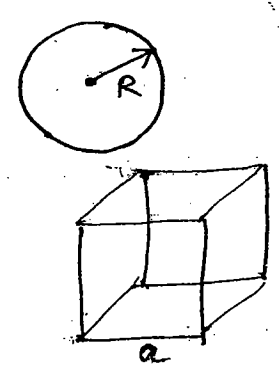
Given

$$A_{\text{Surface Area}} = A_{\text{cube}}$$

$$4\pi R^2 = 6a^2$$

$$R^2 = \frac{6a^2}{4\pi}$$

$$R = \frac{\sqrt{6}a}{2\sqrt{\pi}}$$



$$\frac{F_B \text{ sphere}}{F_B \text{ cube}} = \frac{W_{\text{fluid sphere}}}{W_{\text{fluid cube}}} = \frac{M_{\text{f sphere}}}{M_{\text{f cube}}}$$

$$= \frac{\cancel{\rho_{\text{fluid}}} \times V_{\text{sphere}}}{\cancel{\rho_{\text{fluid}}} \times V_{\text{cube}}}$$

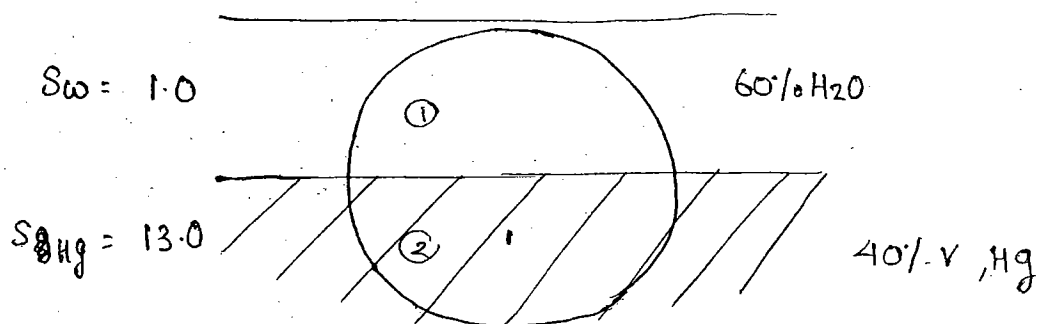
$$= \frac{\frac{4}{3} \pi R^3}{a^2 \cdot a}$$

$$\frac{\frac{4}{3} \pi R^3}{\frac{4 \pi R^2}{6} \cdot \sqrt{\frac{4 \pi R^2}{6}}}$$

$$\frac{F_B \text{ sphere}}{F_B \text{ cube}} = \sqrt{\frac{6}{\pi}}$$

a.)

An unknown sp gravity of Body of uniform cls is allowed to submerge in two fluid system shown in fig. 40% of Body volume submerged in mercury fluid and remaining 60% volume of Body in water. Then specific gravity of Body is 3



$$S_w \cdot h_1 + S_{Hg} \cdot h_2 = S_{\text{solid}} \times H$$

~~S_{oil} V₁~~

$$S_w \cdot V_1 + S_{Hg} \cdot V_2 = S_{solid} \cdot V$$

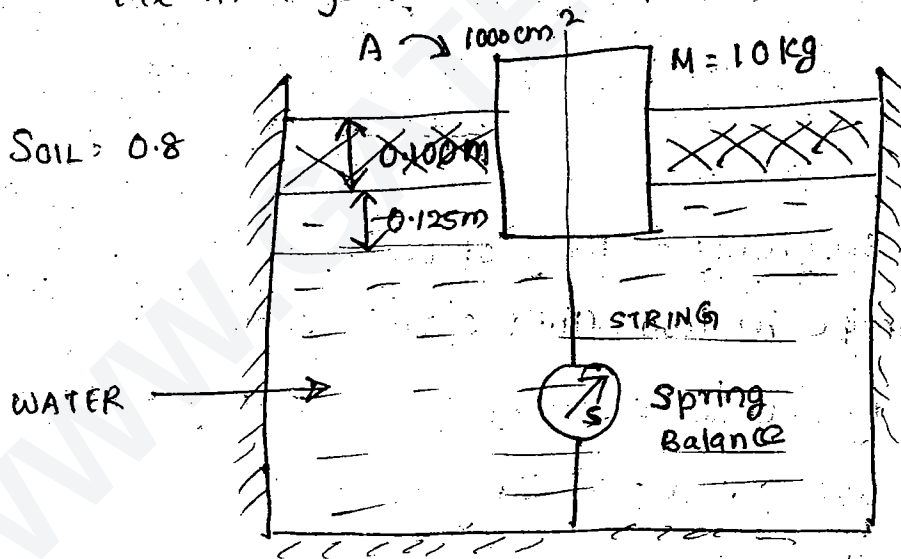
~~13.6 x 0.4 V~~

$$1 \times 0.6 V + 13.6 \times 0.4 V = S_{solid} \times V$$

$$\underline{S_{solid} = 6.04}$$

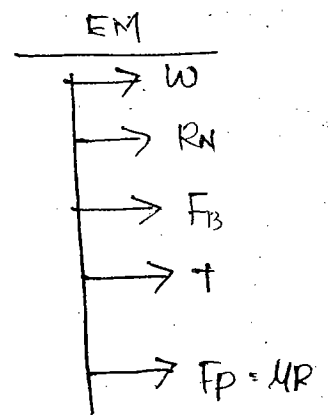
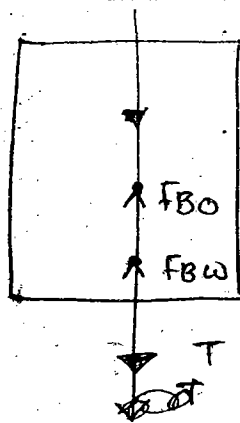
a.)

A 10 kg Body; 1000 cm^2 Area is kept in a Layered fluid system and connected to a string of the spring balance at the bottom of the container. The arrangement shown in figure.



- Determine
- ① Spring Balance Reading
 - ② Gauge pressure at the Bottom of Body
 - ③ Ratio of the Buoyancy forces offered by water to that of oil.

Ans: $S_{oil} \times 0.100 + S_{water} \times 0.125 = S_{solid} \times H$



$$F_{B1} + F_{B2} = W + T$$

$$\rho_w \times A \times h_{1g} + \rho_{oil} \times A \times h_{2g} = W + T$$

FREE SURFACE OF liquids



NO SHEAR STRESS

$$9.81 \times 1000 \times 1000 \times 10^{-4} \times 0.125 + 0.8 \times 1000 \times 1000 \times 10^{-4} \times 0.100 \times 9.81$$

$$205 = 10 \times 9.81 + T$$

$$T = 103.0 \text{ N}$$

(3.)

$$\frac{F_{B \text{ water}}}{F_{B \text{ oil}}} = \frac{\rho_w \times A_w \times h_w \times g}{\rho_{oil} \times A_{oil} \times h_{oil} \times g}$$

~~Gauge pressure~~

$$= \frac{1000 \times 10 \times 0.125 \times 9.81}{0.8 \times 10^3 \times 0.1 \times 0.100 \times 9.81}$$

$$= 1.56$$

(a) Gauge pressure

$$P_2 = \rho_{oil} + \rho_{water}$$

$$\rho_w h_w + \rho_o h_o = \rho_w h_w + \rho_o h_o$$

$$= 1000 \times 9.81 \times 0.125 + 800 \times 9.81 \times 0.1$$

$$= 2011 \text{ N/m}^2$$

$$= \underline{\underline{2.011 \text{ KPa}}}$$

Q

A cubical steel Body [$s = 7.85$] of each side 10cm is immersed in liquid mercury.

Water is added to top of solid Body till

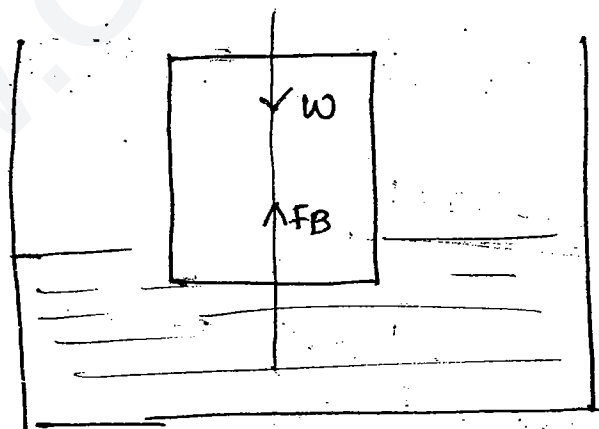
submerged. Determine the following

① Depth of mercury when water is not included.

② Depth of water

Ans:

Case I

~~A~~ $S_{\text{steel}} = 7.85$ ~~$S = 7.85$~~ $\rho_{\text{steel}} = 7850 \text{ kg/m}^3$ 

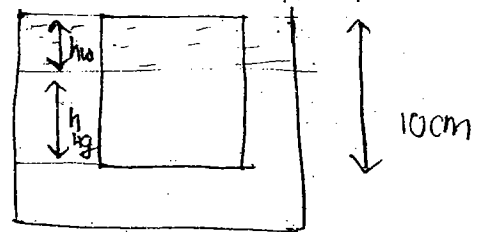
$$S_{\text{Hg}} \times h_{\text{Hg}} = S_{\text{solid}} \times H$$

$$13.6 \times h = 7.85 \times 10$$

$$h = \frac{7.85 \times 10}{13.6}$$

After adding water

$$[h_{Hg1} = \cancel{5.5} \cdot 5.772 \text{ cm}]$$



$$S_w \times h_w + S_{Hg} \times h_{Hg} = S_{solid} \times H$$

$$1 \times h_w + 13.6 \times \cancel{5.772}^{(10-h_w)} = 7.85 \times 10$$

$$h_w = \underline{\hspace{2cm}}$$

$$H = h_g + h_w$$

$$h_g = H - h_w$$

$$h_w + 13.6 - 13.6 h_w = 78.5$$

$$10 - h_w$$

$$13.6 - 12.6 h_w = 78.5$$

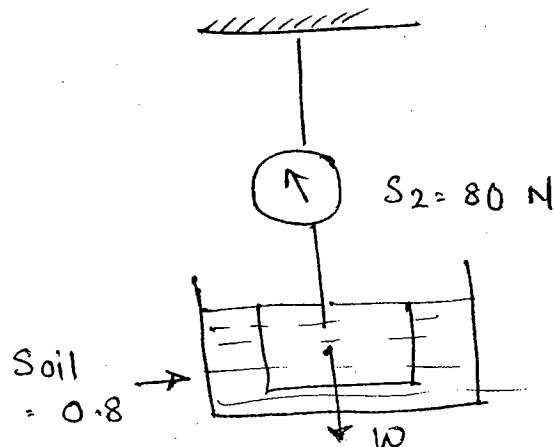
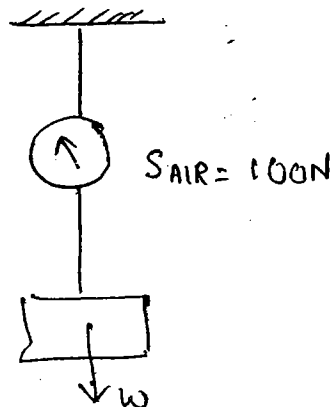
$$h_w = \underline{4.56 \text{ cm}}$$

$$\underline{h_{Hg} = 5.44 \text{ cm}}$$

Note:
When H_2O is added to Hg the height of immersion decreases.

Model No: 5

Finding specific gravity of solids and liquids



$$S_{\text{SOLID}} = \left[\frac{W_{\text{AIR}}}{W_{\text{AIR}} - W_{\text{OIL}}} \right] \times S_{\text{OIL}}$$

$$= \frac{100}{100 - 80} \times 0.8$$

$$= \frac{100 \times 0.8}{20}$$

$$= 5 \times 0.8$$

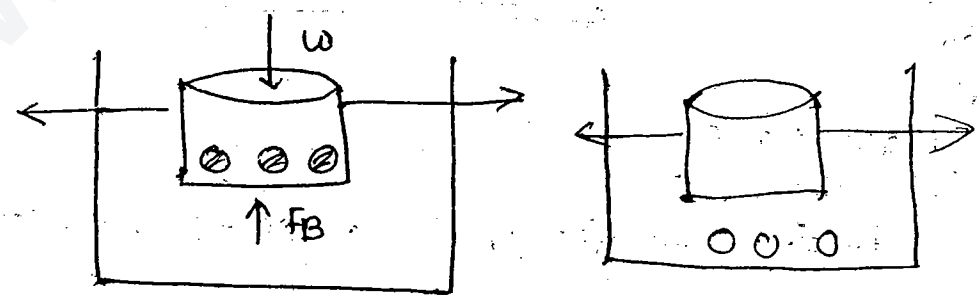
$$= \underline{\underline{4}}$$

a)

S ↑ ρ ↑
F_B ↑

So h is less.

b)



h same

cube having same surface Area

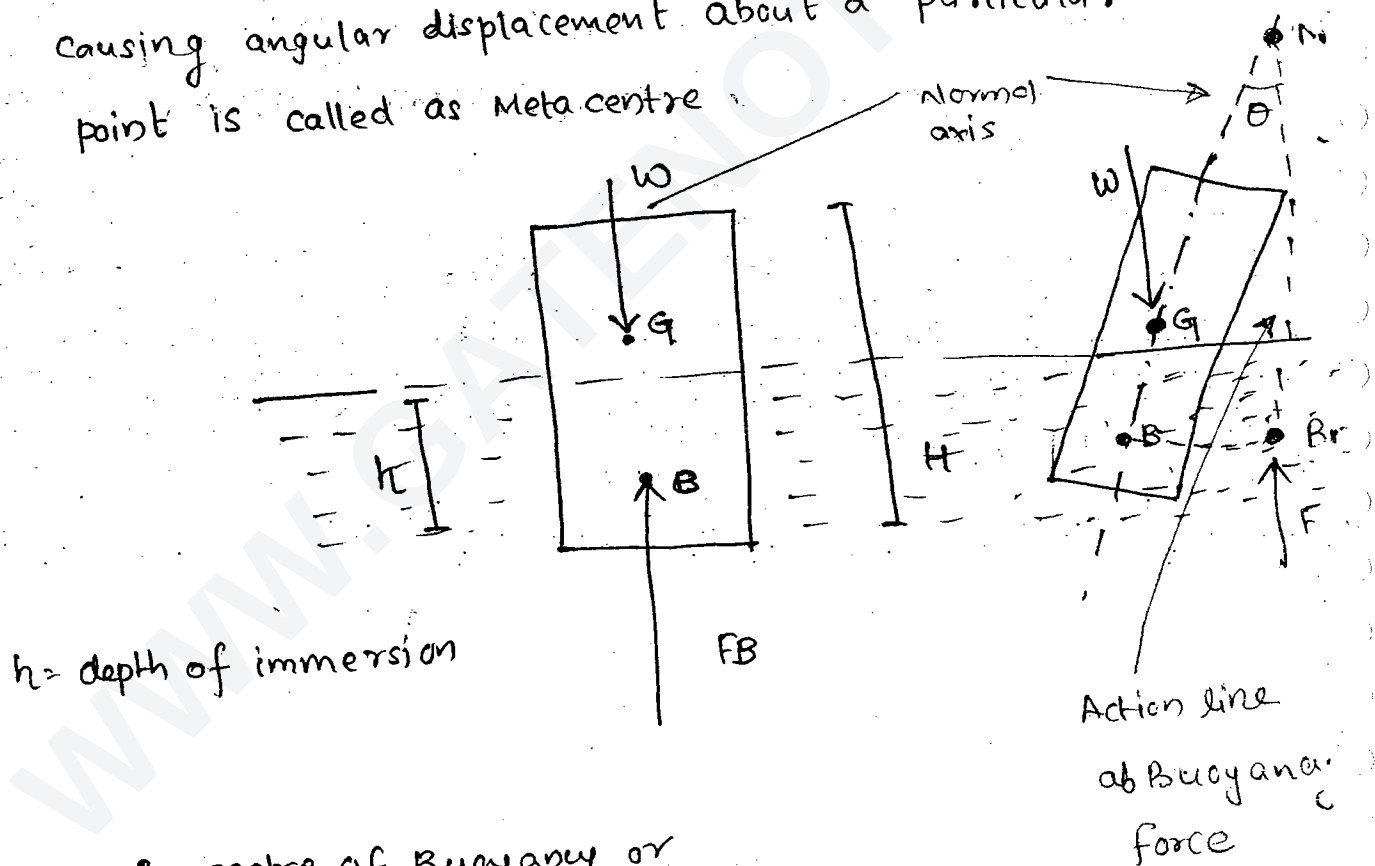
STABILITY OF FLOATING BODIES

When a Body is allowed to float in liquid freely, then two forces are involved

→ self wt of Body

→ Buoyancy force offered by the fluid

When small disturbance is applied to the body causing angular displacement about a particular point is called as Metacentre



B = centre of Buoyancy or
centre of gravity of volume of fluid
displaced

→ Metacentre point M is obtained by intersecting the Normal axis of the Body ~~with~~ and the action line of Buoyancy force.

Conditions of Eqbm

1. stable Eqbm
2. unstable equilibrium
3. Neutral Equilibrium

Stable Equilibrium

After the floating Body disturbed and gaining its initial position without over turning is said to be in stable equilibrium. This happens due to the Restoring Couple of the Bouyancy force against the disturbed couple formed by self wt 'W'.

It is possible if M is ^{located} above 'G' i.e.

GM = Metacentric Height

$$GM > 0$$

More ~~ex~~ the Metacentric Height, More the Stability of Floating Body

unstable Equilibrium

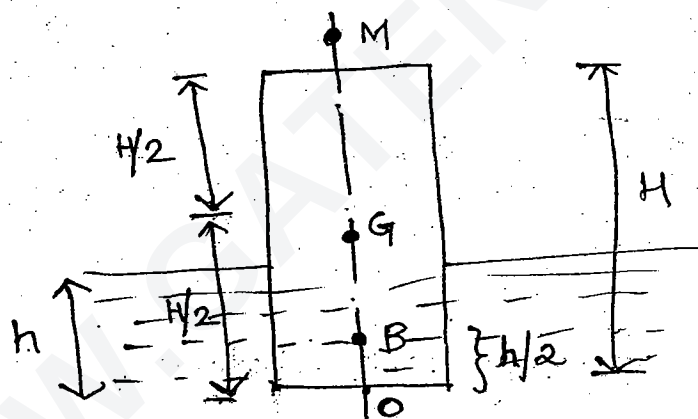
When a floating Body is disturbed and the Body can't return its original position, then Body is said to be in unstable Eqbm. The Result is over turning. This happens if M is located Below 'G'.

If M and G coincide, then Metacentric height $GM = 0$, then Body is said to be in Neutral eqbm. It means that Body will not turn, instead it moves in translational motion ($\theta = 0^\circ$)

Methods to find Metacentric Height (GM)

- ① Analytical Method
- ② Experimental Method

Analytical Method



$$GM = BM - BG$$

$$= BM - [OG - OB]$$

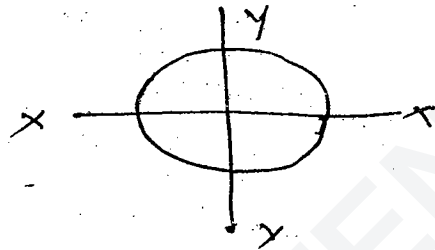
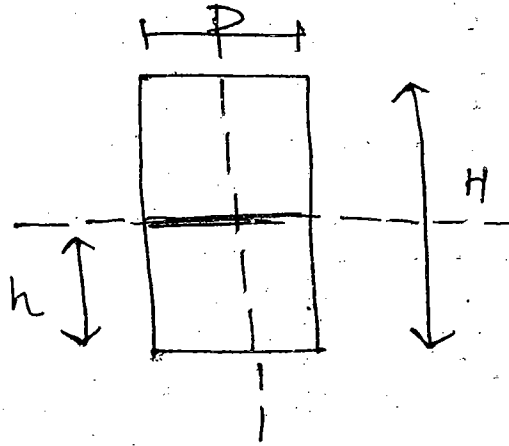
$$BM = \left[\frac{H}{2} - \frac{h}{2} \right]$$

$BM = \frac{I}{A \cdot h}$ $\rightarrow$ MOI of ~~top~~ plane Area ab
Top view (PLAN) of floating
Body at liquid level

$$\text{wetted volume} = A_{cl} \times h$$

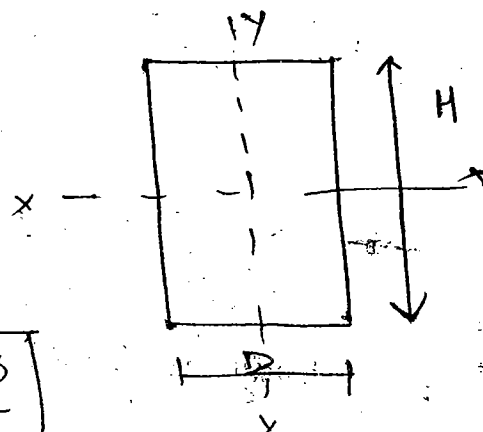
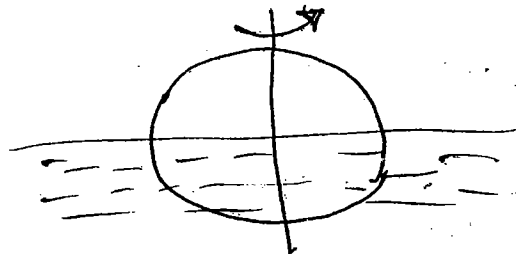
Calculation of I

- ① A cylinder floating vertically

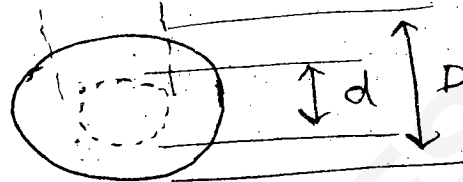
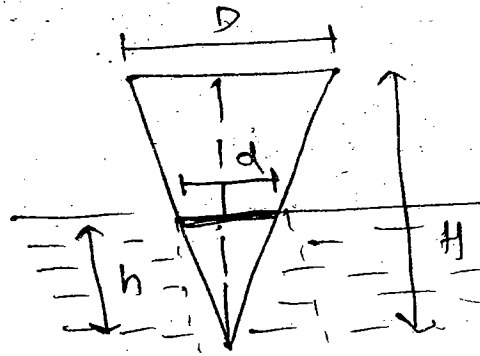


$$I_{yy} \text{ at liquid level} = \frac{\pi d^4}{64}$$

- ② A cylinder floating horizontally



$$I_{yy} = \frac{H D^3}{12}$$



$$\frac{D}{H} = \frac{d}{h}$$

$$I_{yy} = \frac{\pi}{64} [d^4]$$

Frequencies of Floating Bodies in Rolling

$$\frac{d^2\theta}{dt^2} + \omega_n^2 \theta = 0$$

$$t_p = 2\pi \sqrt{\frac{K^2}{g \cdot G M}}$$

$$f_n = \frac{1}{t_p} \text{ C.P.S}$$

Hertz

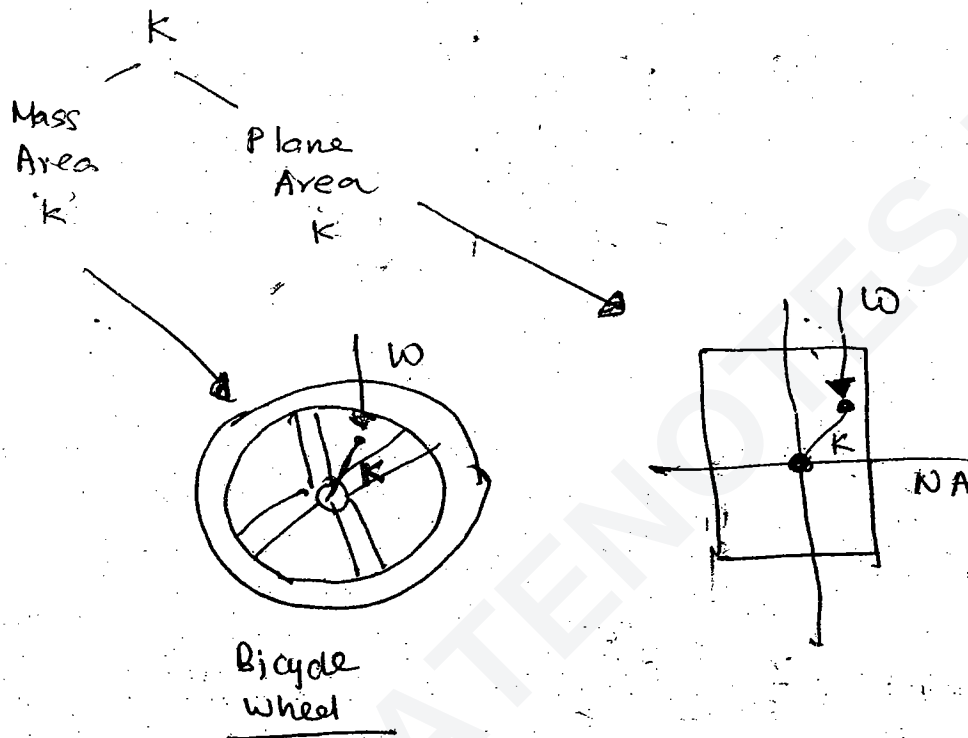
Rev/sec

(cps)

$$\omega_n = \frac{2\pi}{t_p} \text{ (rad/sec)}$$

$$K = \sqrt{\frac{I}{A}}$$

K : Radius of Gyration



Q) A cylinder made of unknown material of diameter 4m and height also 4cm allowed to float in a fluid of relative density 1.4. It is observed that the depth of immersion is 2.4m. Determine the following -

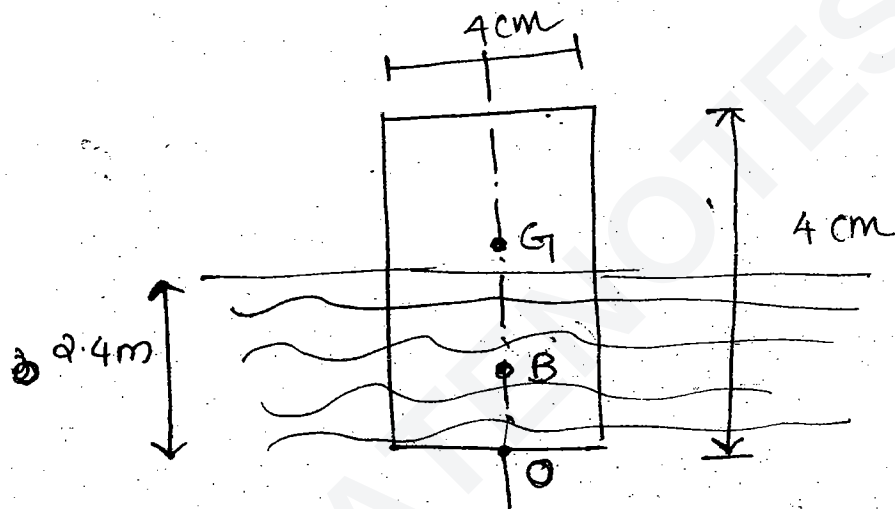
1. specific gravity of cylinder material
2. volume of fluid displaced (∇)
3. Metacentric Height.
4. Check for stability: [stable or unstable]

of the cylinder in terms of dia to achieve stability

S_{solid}

$$S_{\text{fluid}} = 1.4$$

$$\rho_{\text{fluid}} = 1.4 \times 1000 \frac{\text{kg}}{\text{m}^3} = \underline{1400 \text{ kg/m}^3}$$



$$\textcircled{1} \quad S_{\text{fluid}} : h = S_{\text{solid}} \times H$$

$$1.4 \times 2.4 = S_{\text{solid}} \times 4$$

Non met-

$$S_{\text{solid}} = \frac{1.4 \times 2.4}{4} = \underline{0.84}$$

$$\textcircled{2} \quad V = A_{\text{cs}} \times h$$

$$\text{or } \frac{\pi}{4} \times D^2 \times 2.4$$

$$\frac{\pi}{4} \times (4)^2 \times 2.4$$

$$\underline{30.16 \text{ m}^3}$$

Buoyancy force offered by fluid ($s=1.4$)

45

$$F_B = \rho \times V$$

$$(1400 \times 9.81) \times 30.16$$

$$\text{KN}$$

$$\begin{aligned} F_B &= m_{\text{fluid}} \cdot g \\ &= \rho_{\text{fluid}} \times V \cdot g \\ &= \rho \times V \end{aligned}$$

$$\frac{5}{2} \times \frac{10}{2}$$

② Metacentric Height (GM)

$$GM = BM - BG$$

$$\frac{\pi}{4} - \left[\frac{H}{2} - \frac{h}{2} \right]$$

$$= \frac{\pi}{64} [4]^4 - \left[\frac{4}{2} - \frac{2.4}{2} \right]$$

$$GM = -0.384 \text{ m}$$

Below G

$GM < 0$ [means M is below G]

Body is under unstable equilibrium

x

$$BM - BG > 0$$

$$\frac{I}{A} - \left[\frac{H}{2} - \frac{h}{2} \right] > 0$$

$$\frac{I}{A} > \frac{H}{2} - \frac{h}{2}$$

$$\frac{\pi \times D^4}{64} \times \frac{1}{\frac{\pi \times D^2}{4} \times h} > \frac{H}{2} - \frac{h}{2}$$

$$\frac{D^2}{16h} > \frac{H}{2} - \frac{h}{2}$$

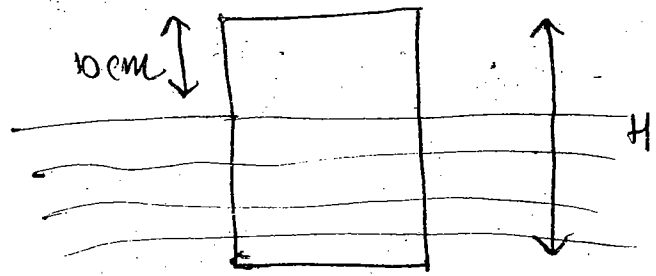
$$\frac{D^2}{8h} > H - h$$

$$H = \text{---}$$

$$\text{Replace } h = \frac{S_{\text{solid}} \times H}{S_{\text{liquid}}} =$$

$$\therefore H =$$

- Q An ice block is floating in seawater shown in figure



Specific gravity of ice = 0.925

Sp gravity of sea water = 1.025. The exposed height is 10 cm. determine height of ice block?

Ans:-

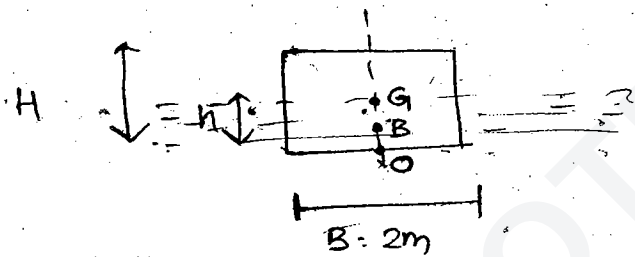
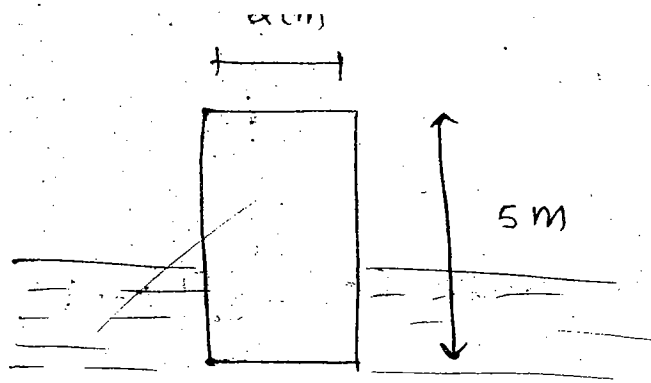
$$S_{ice} \times H = S_{seawater} \times h$$

$$0.925 \times H = 1.025 \times (H - 10)$$

$$H = 102.5 \text{ cm}$$

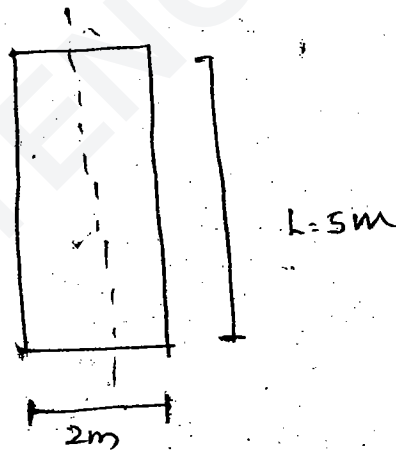
A Rectangular Block 5 m long and 2 m width is floating in water. The water line is 0.4 m above the bottom and centre of gravity is 0.8 m from bottom then the body's metacentric height will be

- a) 0.83 m b) 0.6 m c) 0.46 m d) 0.23 m



$$h = 0.4 \text{ m}$$

$$OG = 0.8 \text{ m}$$



$$I_{yy} = \frac{5 \times (2)^3}{12}$$

$$GM = \frac{I}{A} - \left[\frac{H}{2} - \frac{h}{2} \right]$$

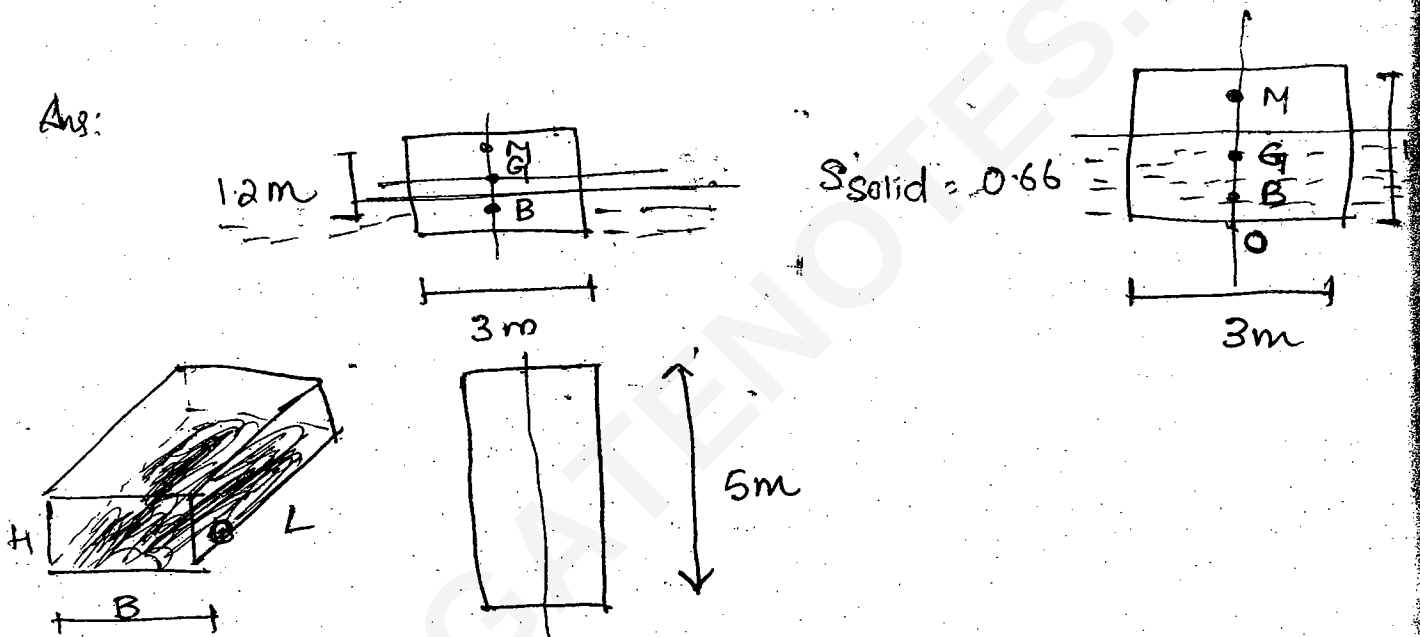
$$\frac{\frac{5 \times (2)^3}{12}}{A \times h} - \left[\right]$$

$$GM = \frac{5 \times (2)^3}{12} - \left[0.8 - \frac{0.4}{2} \right]$$

$$\underline{GM = 0.23 \text{ m}}$$

- Q. A Rectangular Block 5m long 3m width and 1.2m depth [$S = 2/3$] kept horizontally in a liquid [$S = 1.025$]. State the condition of eqbm.

Ans:



$$GM = \frac{I}{V} - \left[\frac{H}{2} - \frac{h}{2} \right]$$

$$\frac{I}{V} = S_{\text{solid}} \times H = S_{\text{fluid}} \times h$$

$$h = \frac{0.66 \times 1.2}{1.025} = \underline{0.78}$$

$$I_{yy} = \frac{5 \times 3^3}{12} = \underline{\hspace{2cm}}$$

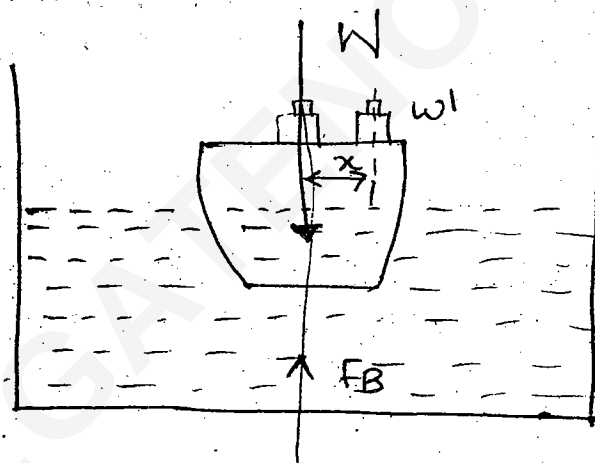
$$GM = \frac{5 \times 3^3}{12} - \left[\frac{H}{2} - \frac{h}{2} \right]$$

$$GM = 0.75 \text{ m} > 0$$

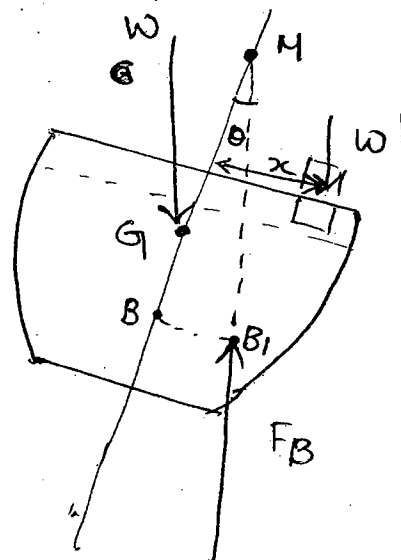
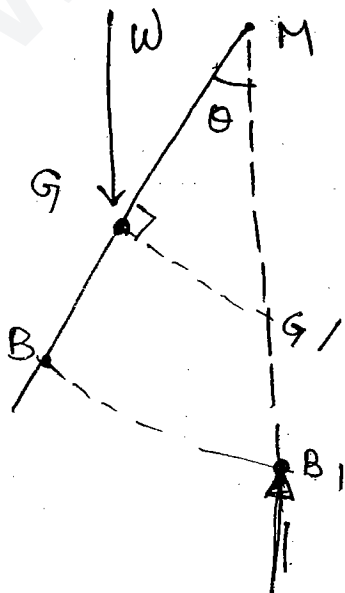
Stable equilibrium

Experimental method to find metacentric height

In practice like stability of large sized ships and vessels it is easy to check the stability along with the model testing analysis for which the given object is scaled down and applied in ideal conditions of fluid -



Moment about M



$$\tan \theta = \frac{GG_1}{GM}$$

$$GG_1 = \tan \theta \cdot GM$$

$$W' \cdot x = W \cdot GG_1$$

$$W' \cdot x = W \times GM \tan \theta$$

$$GM = \frac{W' \cdot x}{W \tan \theta}$$

It will give accuracy since θ can't be accurately measured.

STABILITY OF SUBMERGED BODIES IN FLUIDS.

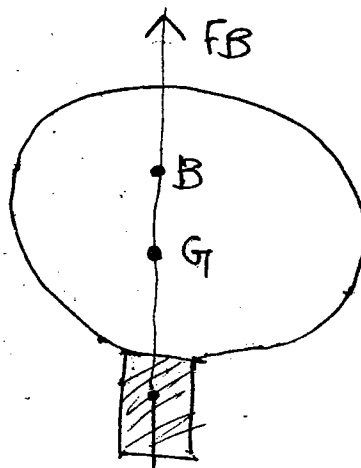
NOTES:-

When a body is submerged fully in any fluid then no meta centre is involved, then the stability is obtained by locating the point B with reference to G.

Ex: 1:-

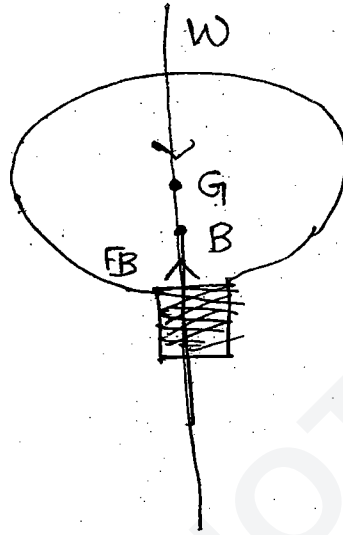
Stable equilibrium for Submerged Body

B is above G
it makes body
return to its original
position



Unstable Equilibrium for submerged Body

B is below G. It cannot return to its original position



B and G coincides

$$\theta = 0$$

Moves in Translational motion

Q) For a Body Completely submerged in a fluid, the centre of Gravity is G and centre of Buoyancy is O which are known. The Body is considered to be in stable equilibrium if

- ☒ a) O doesnot coincide with centre of mass of the displaced fluid
- b) G coincides with centre of mass of fluid displaced.
- c) O lies Below the G
- ☒ d) O lies above the G

9.) A Floating Body with its centre of Gravity at 'G', centre of Buoyancy at B and metacentre at M is stable when, ~~G lies~~

~~above~~

a) G lies above B

b) B lies above M

c) B lies below M

✓ d) G lies below M

WWW.GATENOTES.IN

Hydrostatic forces on fluid contact Surfaces:-

1. Introduction
2. Hydrostatic force, magnitude and its point of application (Centre of pressure)
3. Hydrostatic force on horizontal plane surface, vertical plane surface, inclined plane surface, curved surface

4) Moments of Hydrostatic forces

When a fluid ^{is} in contact with any surface

→ pressure is exerted (Normal) which depends on sp wt of fluid and depth of fluid. The Hydrostatic force is the product of Normal pressure and the wetted surface Area.

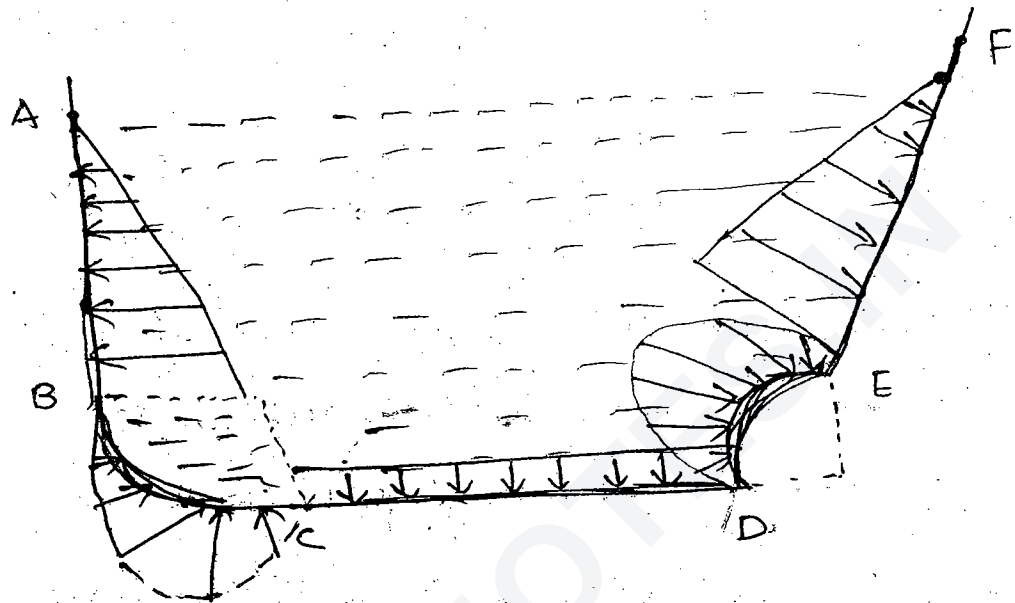
Hydro-static force (F_H)

$$= P \times A$$

$$= \rho g h \times A$$

$$= (\gamma h) \times A$$

to centroid of the plane surface in
contact with fluid.



Horizontal surface = CD

Vertical surface = AB

$$\frac{dp}{dz} = -\gamma$$

$$\frac{dp}{dh} = -\gamma$$

$$P = \gamma \bar{h}$$

$$\frac{P}{\bar{h}} = \gamma$$

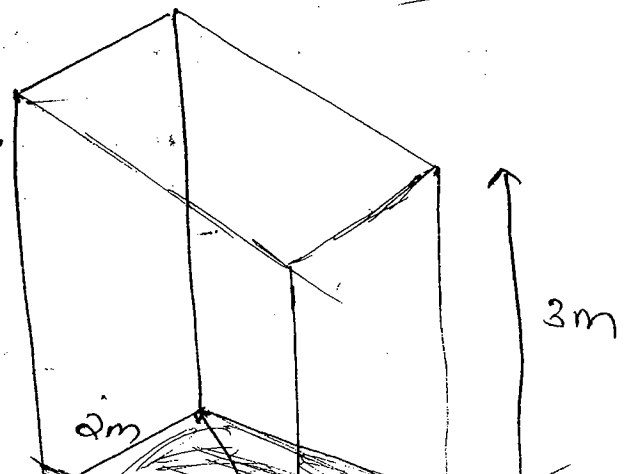
[Free surface - free ~~surface~~ from shear]

Example:-

$$F_{\text{horizontal}} = \rho g \bar{h} \cdot A$$

$$= 1000 \times 10 \times 3 \times (2 \times 1)$$

$$= 60 \text{ kN}$$



$$F_{\text{vertical}} = \rho g h \times A$$

$$= 1000 \times 10 \times \frac{4}{2} \times [3 \times 2]$$

$$= 1000 \times 10 \times \frac{3}{2} \times [3]$$

$$= \underline{\underline{90 \text{ kN}}}$$

- Q.) A tank with 4 equal vertical planes of width w and depth h is completely filled with liquid. If the hydrostatic force of any vertical plane surface is equal to that force at the bottom, then ratio of H/w will be

- a) 2 b) $\sqrt{2}$ c) 1 d) 0.5

ie

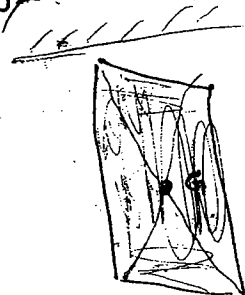
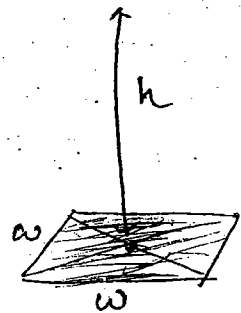
$$F_{\text{vertical}} = F_{\text{horizontal}}$$

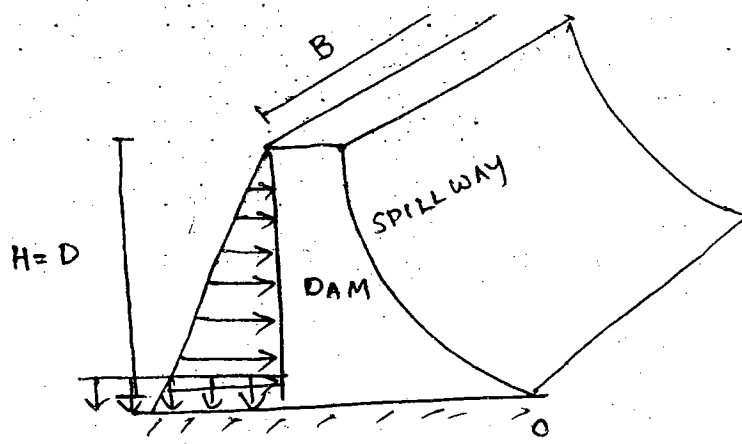
~~$$\rho \times g \times h \times \frac{w \times h}{2} = \rho \times g \times w \times h$$~~

~~$$\rho \times g \times \frac{h}{2} \times h \times w = \rho \times g \times h \times w$$~~

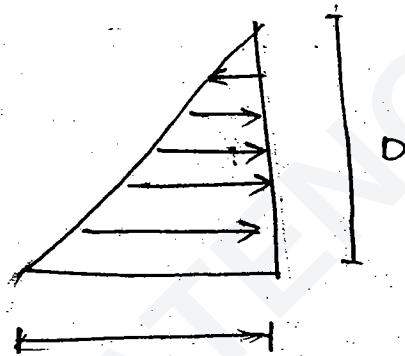
$$h = 2w$$

$$\boxed{\frac{H}{w} = 2}$$





SPILL WAY DAM



$$P = \gamma \cdot D$$

$$= \rho g \cdot D$$

Case 1

Pressure intensity at Bottom = $\gamma \cdot D$

$$= \gamma \cdot H$$

$$= \rho \cdot g \cdot H \text{ N/m}^2$$

Hydrostatic Force (N)

or

Total Pressure (N)

$$F = \rho \cdot g \cdot H \cdot A_{\text{horizontal}}$$



Force normal to horizontal floor

Hydrostatic force on vertical plane surface of dam

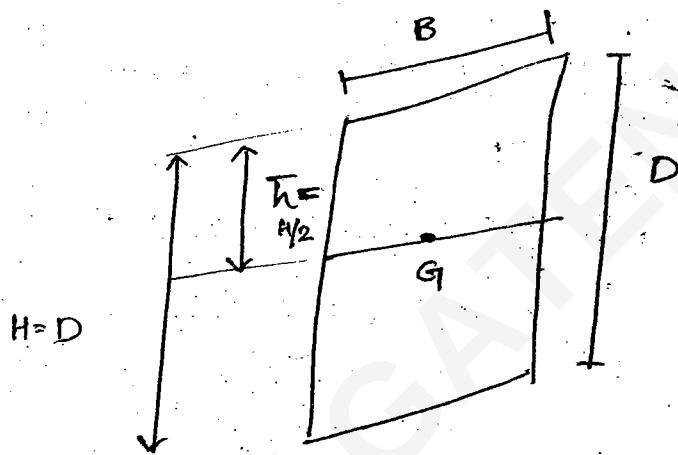
$$P = \rho g \bar{h} = \gamma \bar{h}$$

$$= \gamma \cdot \frac{D}{2}$$

$$\gamma \frac{H}{2}$$

where $\bar{h}$ = distance from free surface level

to centroid of vertical plane surface.

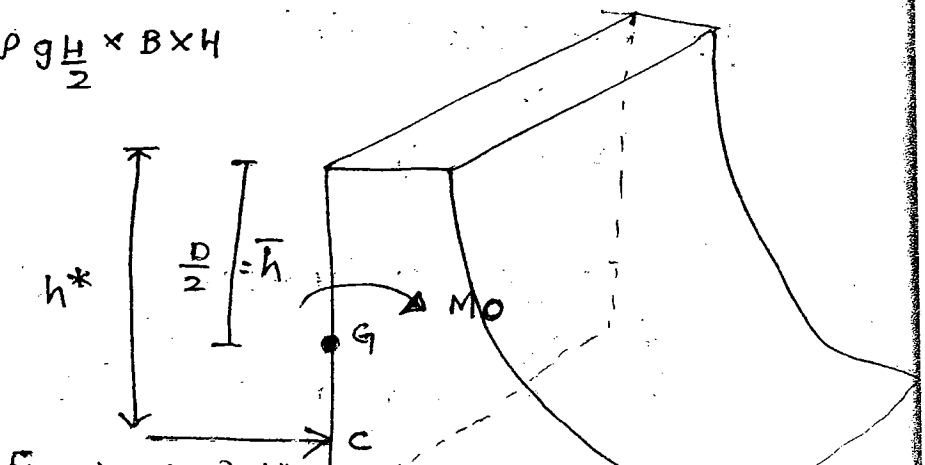


Total pressure force

$$F = P \times A_{\text{normal}}$$

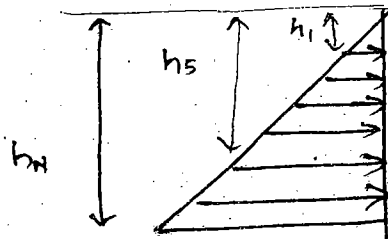
$$= \rho g \bar{h} \times A$$

$$= \rho g \frac{H}{2} \times B \times H$$



level to point where hydrostatic force acts

h^* calculation



$$P_1 = \rho g h_1$$

$$P_s = \rho g h_s$$

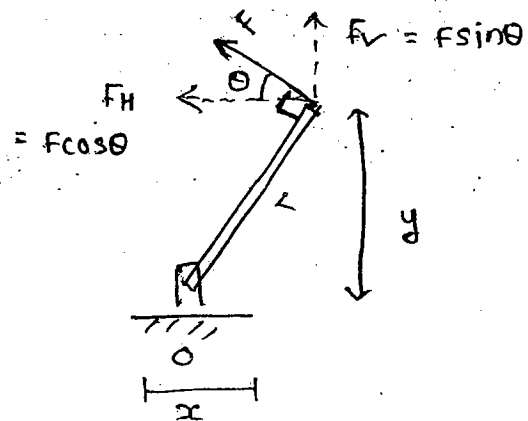
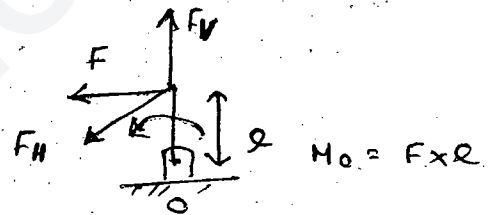
PRESSURE DIAGRAM

Principle of Moments

Moment of Force components

F_v and F_H about O =

$$M_O = -F_v \times x = -F_H \times y$$



$$M_O = F \times L$$

Moment of Hydrostatic forces about free liquid surface

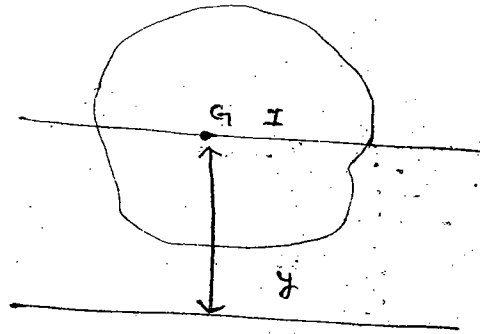
$$\sum M_F = F_1 \times h_1 + F_2 \times h_2 + \dots + F_n \times h_n$$

$$\sum M_F = F_H \times h^*$$

$$h^* = \frac{F_1 \times h_1 + F_2 \times h_2 + \dots + F_n \times h_n}{F_H}$$

$$h^* = \frac{\sum_{i=1}^N F_i h_i}{F_H}$$

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$$I_{xx} = I_{CG} + A y^2$$

$$h^* = \bar{h} + \frac{I_{CG}}{A \cdot \bar{h}}$$

Lever Arm (Moment Arm) of for F_H about

$$O = \frac{H}{3}$$

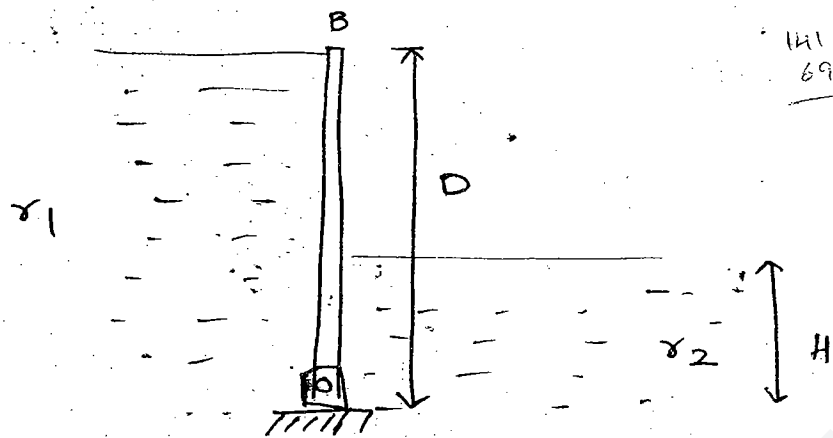
Moment of F_H about O

$$= F_H \times \frac{H}{3}$$

$$= \left[\gamma \times \frac{H}{2} \times \frac{BH}{1} \right] \times \frac{H}{3}$$

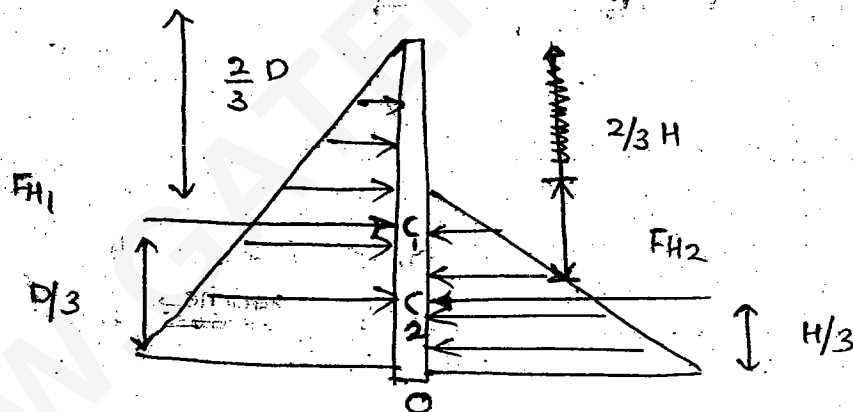
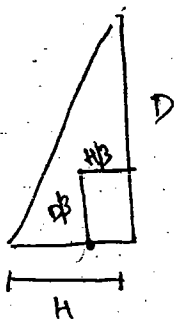
Area

A Rectangular plate of dimensions $B \times D$ (width $\times$ Depth) kept vertically and supporting a fluid of specific wt γ_1 on one side and supporting another



$$F_{H1} = \gamma_1 \times \frac{D}{2} \times (B \cdot D) \text{ N} \quad \text{wetted surface Area}$$

$$F_{H2} = \gamma_2 \times \frac{H}{2} \times (B \cdot H) \text{ N}$$



Moment of forces about O

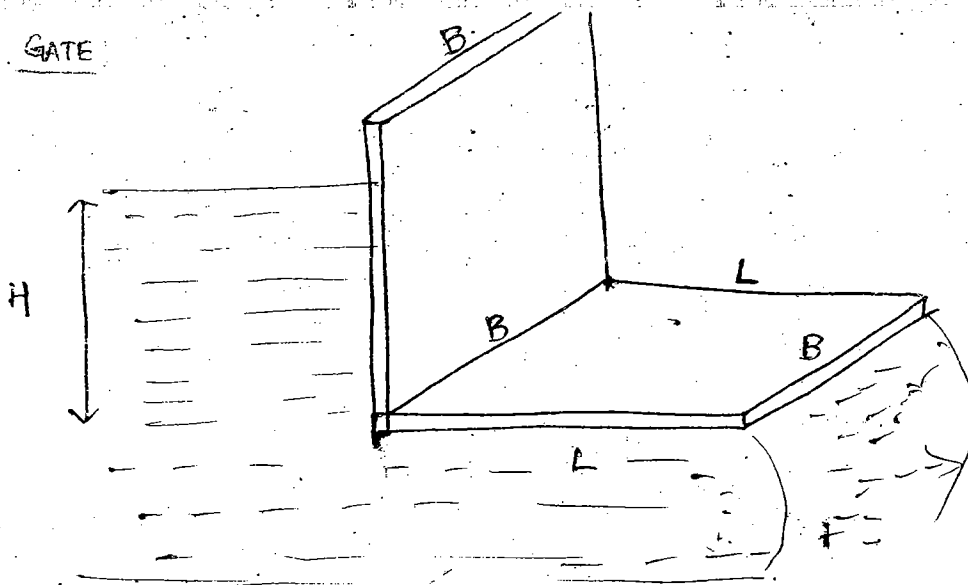
$$F_{H1} \times \frac{D}{3} = F_{H2} \times \frac{H}{3}$$

$$\gamma_1 \times \frac{BD^2}{2} \cdot \frac{D}{3} = \gamma_2 \times \frac{BH^2}{2} \times \frac{H}{3}$$

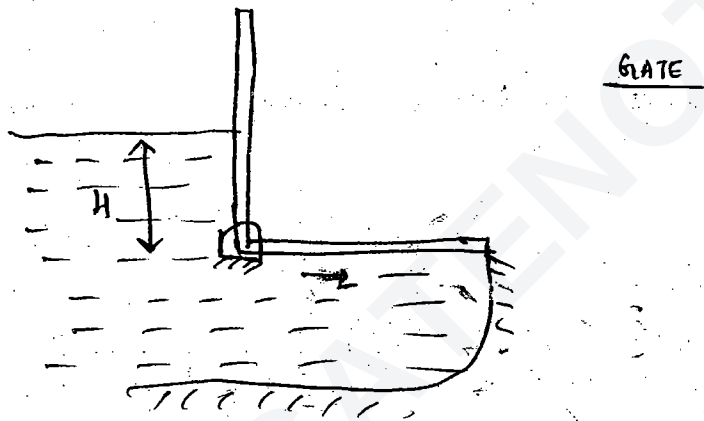
$$\gamma_1 \cdot \frac{BD^3}{6} = \gamma_2 \cdot \frac{BH^3}{6}$$

$\frac{D \times BD}{2}$

$$F_H = \gamma \times h \times \text{Area} \times \frac{D}{3}$$



At what depth system will turn

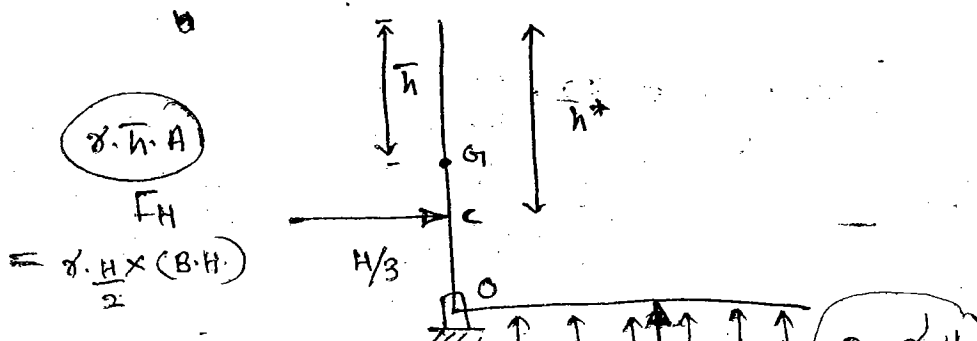


Find depth of fluid in terms of B & L such that at particular depth H' , ~~reaches~~ ^{reaches}, then gate turns.

Ans:

$$F_{H1} = \gamma \times \left(\frac{BH^3}{6} \right)$$

$F_{H2} \rightarrow$ Horizontal



Moment of Horizontal = Moment of Vertical

$$M_H = M_V$$

$$\cancel{\rho} \cdot \cancel{H} \cdot (L \cdot B) \times \frac{L}{2} = \cancel{\rho} \cdot \frac{\cancel{H}}{2} \times (B \cdot H) \times \frac{H}{3}$$

$$\frac{H^2}{3} = \frac{L^2}{2}$$

$$H = \sqrt{3} L$$

a) What is the total pressure on a circular plate of 2m dia is immersed horizontally in water at a depth of 1m.



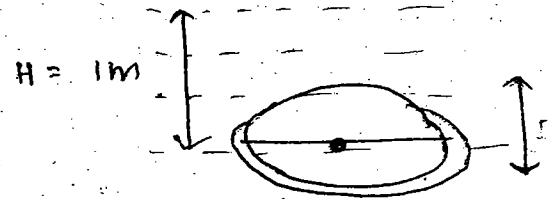
- a) 10 kN b) 4 kN c) 2 kN d) ~~31.4 kN~~

Ans:- $F = \rho \times g \times \bar{h} \times \text{Area}$

$$= 1000 \times 10 \times 1 \times \frac{\pi}{4} \times (2)^2$$

$$= 10 \pi \text{ kN}$$

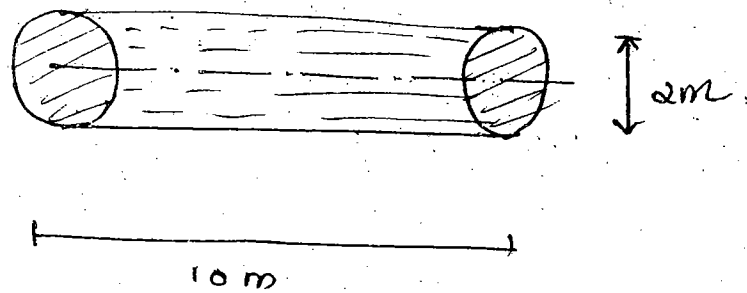
$$= 31.4 \text{ kN}$$



$$\bar{h} = H = h^*$$

a) A cylindrical tank of 2m dia and 10m length is laid with its axis parallel to the horizontal plane and cylinder is filled with water just to its top what is the Hydrostatic force on one of its end plates

- a) 123 kN b) 61.51 kN c) 30.76 kN d) 19.58 kN



$$F_H = (\gamma \times \bar{h}) \times \text{Area}$$

$$= \gamma \times \frac{D}{2} \times \frac{\pi}{4} \times (2)^2$$

$$= 1000 \times 9.81 \times \frac{2}{2} \times \frac{\pi}{4} \times 4$$

$$= 30.819 \text{ kN}$$



$$h^* = \bar{h} + \frac{I}{A \times \bar{h}}$$

$$= \frac{\frac{\pi}{64} D^4 + \frac{D}{2}}{\frac{\pi}{4} D^2 \times \frac{D}{2}} = \frac{\frac{D}{8} + \frac{D}{2}}{\frac{D+4D}{8}}$$

$$= \frac{\frac{D}{2} + \frac{D}{8}}{\frac{5D}{8}} = \frac{5D}{8} = 1.25$$

h^*

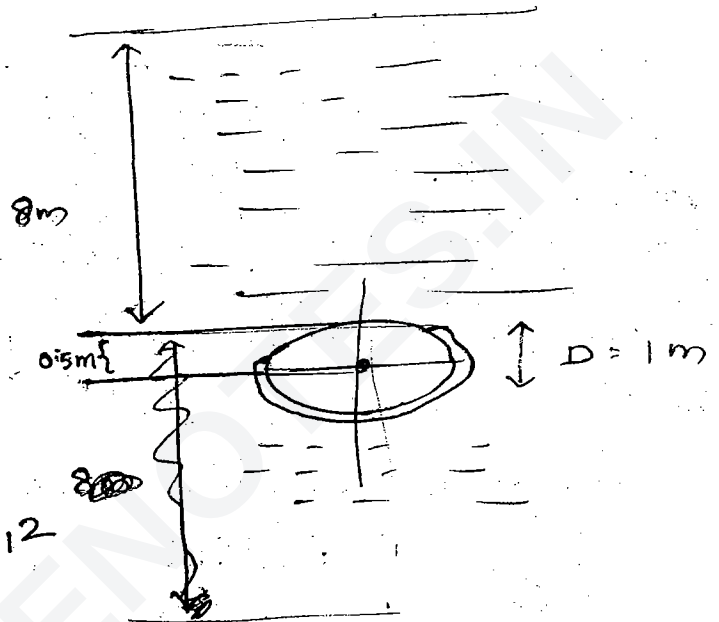
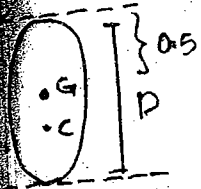
$$= \frac{1}{16} D^2$$

$$= \frac{1}{16} \times 2^2$$

$$= \frac{4}{16} = \frac{1}{4} = 0.25$$

in water such that its upper edge is 8 m below the free water surface what is the total hydrostatic pressure force on one side of plate.

- a) 6.70 kN b) 65.40 kN c) 45.00 kN d) 77 kN



$$F_H = (\rho \times g \times \bar{h}) \times \text{Area}$$

$$= 1000 \times 9.81 \times 8.5 \times \frac{\pi}{4} \times 1^2$$

$$= 65.40 \text{ kN}$$

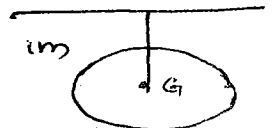
$$\bar{h} = 8 + \frac{D}{2}$$

$$= 8 + \frac{1}{2} = 8.5 \text{ m}$$

$$8 + \frac{1}{2} = 8.5 \text{ m}$$

The Magnitude of Hydrostatic pressure force on one side of a circular surface, $\pi \text{ m}^2$ Area with its centroid is 4 m below the free water surface depends on the orientation of the Area of circular plate.

(2) is the p.d.t of spl wt of water, vertical distance from free surface to center of pressure and its Normal Area.



(3) 10 kN

(4) 1000 N

56

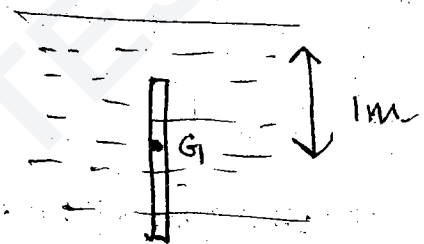
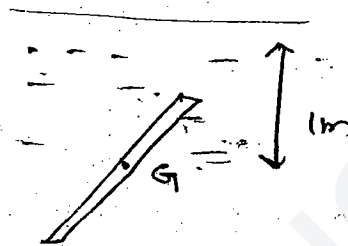
$$F_H = 1000 \times 9.81 \times 1 \times (12)$$

$$1000 \times 10$$

$$= 10000 \text{ N}$$

~~9.81 N~~ 10 kN
~~1000 N~~

Note:-



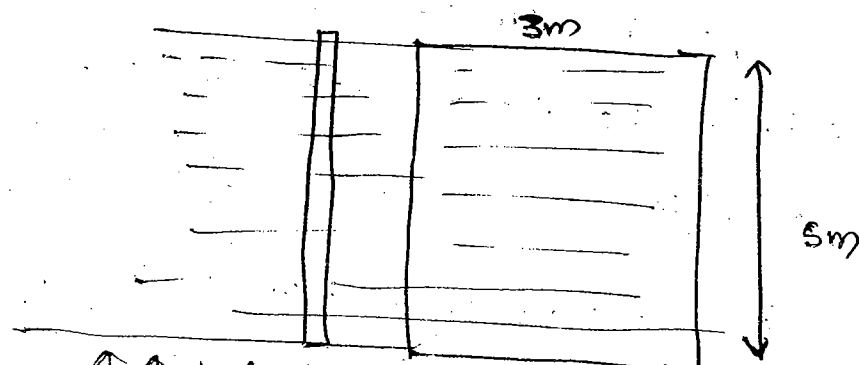
ORIENTATION IS INDEPENDANT

Q)

A vertical ~~gate~~ Gate closes a horizontal tunnel of 3m width and 5m height running under full water. The pressure at the bottom of the gate is $2 \times 10^4 \text{ kg/m}^2$. The magnitude of hydrostatic force on the gate is would be in tonnes

(a) 192 T (b) 250 T (c) 262.5 T

(d) 225.5 T



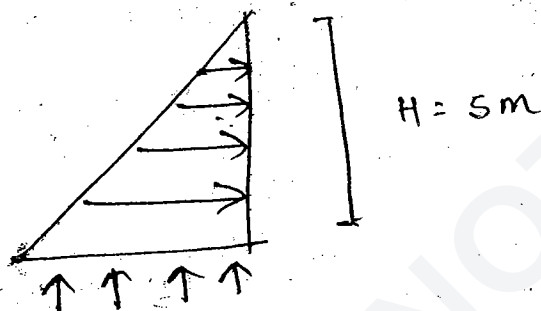
$$F_H = (\rho \times g) \times \bar{h} \times \text{Area}$$

$$= 1000 \times 10 \times \frac{H}{2} \times 5 \times 3$$

$$= 1000 \times 10 \times \frac{5}{2} \times 15$$

$$= 75 \times 5000$$

$$= 375\,000$$



$$P = \rho g H$$

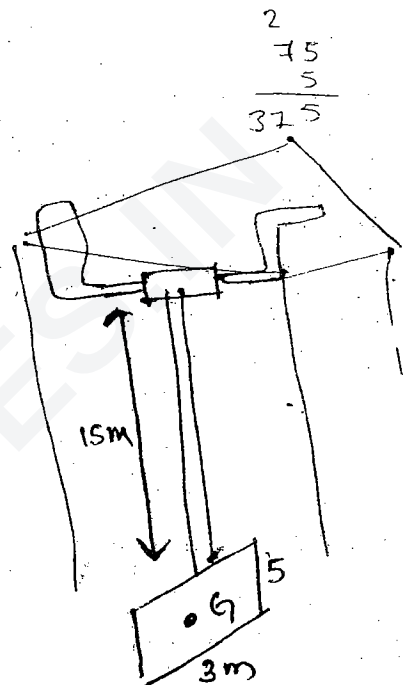
$$= 2 \times 10^4 \text{ kg/m}^2$$

$$= 2 \times 10^4 \times 9.81 \frac{\text{N}}{\text{m}^2}$$

$$196,200 \text{ N/m}^2$$

$$196200 = 1000 \times 9.81 \times H$$

$$H = 20\text{m}$$



$$F_H = 1000 \times 9.81 \times \left[15 + \frac{5}{2} \right] \times (5 \times 3)$$

$$\frac{2575125}{9.81} = 262500$$

$$262500$$

$$= \text{kg}$$

$$262500$$

$$\text{Tonnes}$$

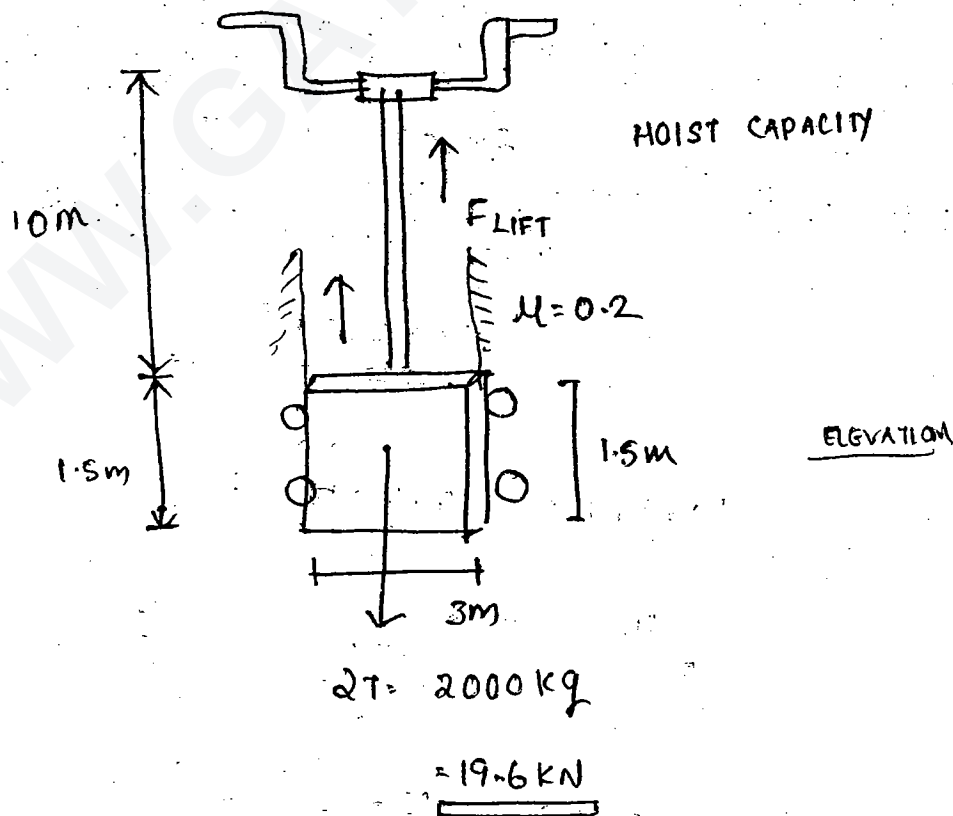
Q.

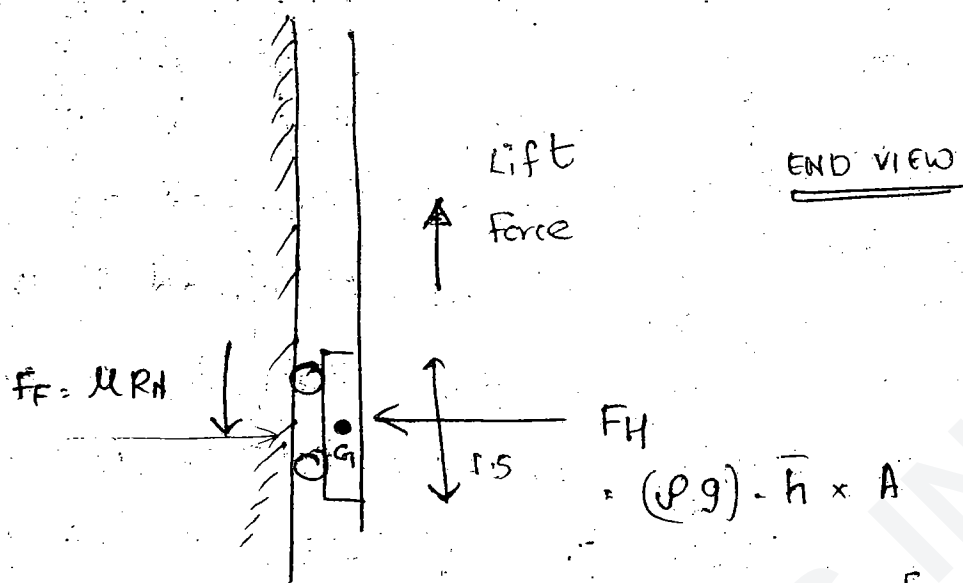
A sliding gate 3m width and 1.5m height lies in a vertical plane against the frictional surface which has coeff of friction 0.2 b/w.

The gate roller and vertical rubbing surface. The self wt is 2T. (19.6 kN). The upper edge of gate is 10m below the free water surface. Determine the vertical force reqd to lift the gate against self wt and frictional force. Neglect buoyancy effects on the gate.

- a) 115 kN b) 494 kN c) 95 kN d) 125 kN

Ans:





$$F_H = 1000 \times 9.81 \times \left[10 + \frac{1.5}{2} \right] \times [3 \times 1.5]$$

$$= 474.56 \times 10^3 \text{ N}$$

$$R_N = F_H$$

$$F_f = 0.2 \times 474.56 \times 10^3$$

$$= 94911.175 \text{ N}$$

$$= 94.91 \text{ kN}$$

Rolling friction

less

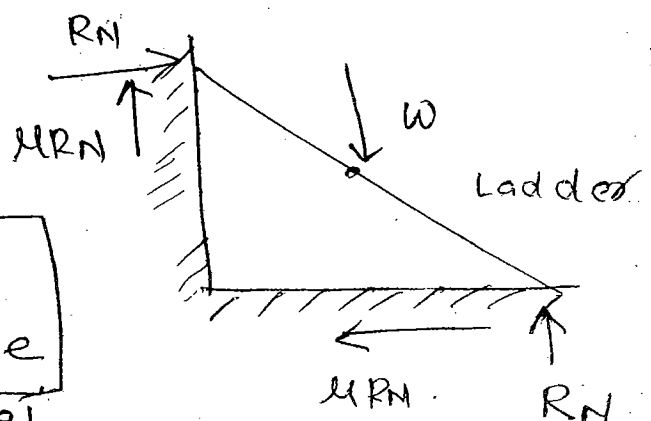
when compared to sliding force

$$F_f = \text{Force applied}$$

$$= 94.91 \text{ kN}$$

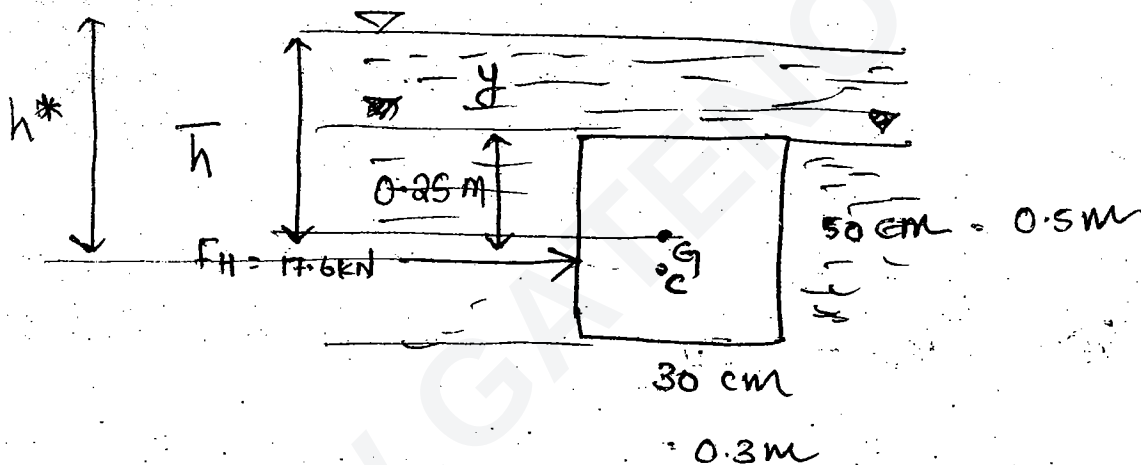
$$\text{Lift force} = w_{\text{self}} + \text{Friction force}$$

$$= 19.04 + 94.91$$



58
 9) A Rectangular plate 30 cm X 50 cm is immersed vertically in water with its longer side edge vertical. The total hydrostatic force on one side of the plate is estimated as 17.6 kN. Now the plate is turned in the vertical plane about its centroid by 90 degrees and keeping other parameters remains same then the New Total force on one side of the plate would be -

- (a) 8.8 kN (b) 15.6 kN (c) 17.6 kN (d) 19.6 kN



$$F_N = F_H = 17.6 \text{ kN}$$

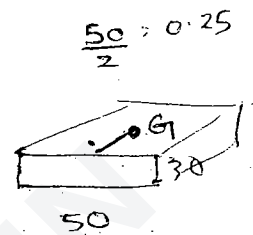
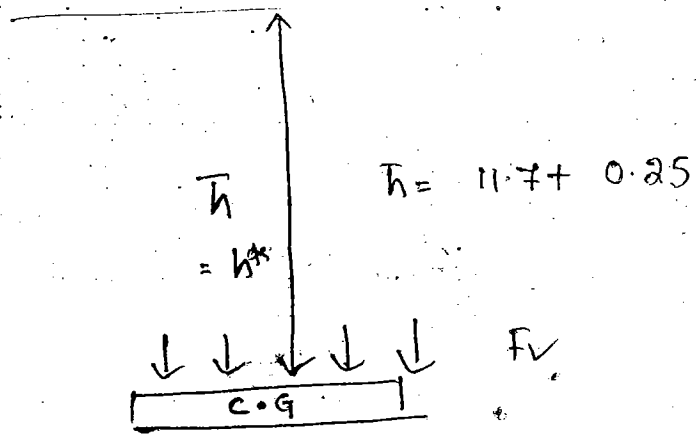
$$= \rho g \cdot \left[y + \frac{D}{2} \right] \times \text{Area}$$

$$= 17.6 \times 10^3 = 1000 \times 9.81 \left[y + \frac{0.5}{2} \right] \times (0.3 \times 0.5)$$

$$\underline{\underline{y = 11.71 \text{ m}}}$$

Case 2

Tilting

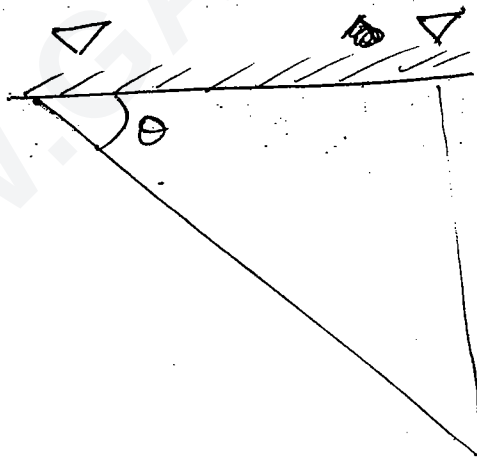


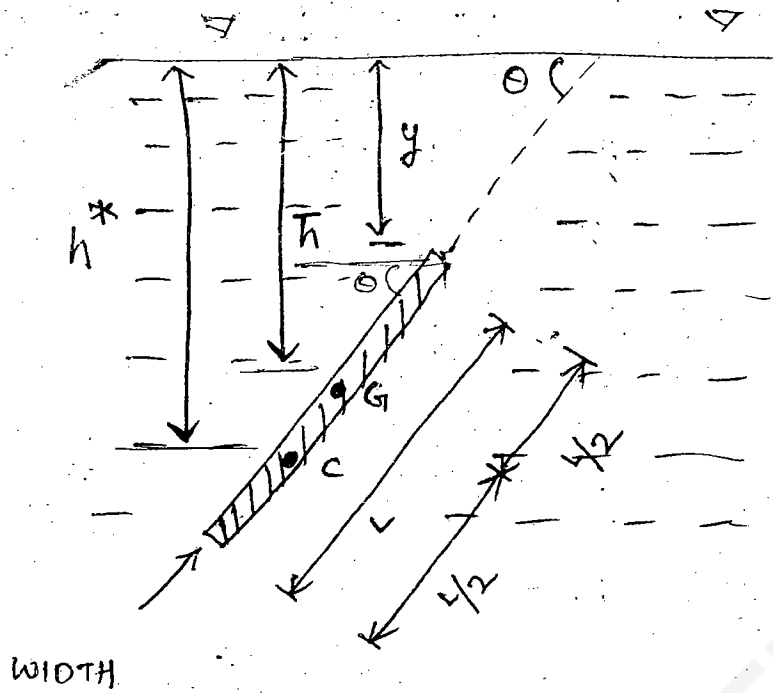
$$F_v = \rho \cdot g \cdot \bar{h} \times \text{Area}$$

$$= 1000 \times 9.81 \times [11.7 + 0.25] \times (0.3 \times 0.5)$$

$$= 17.6 \times 10^3 \text{ N}$$

Inclined PLANE SURFACE





$$P = \rho g \bar{h}$$

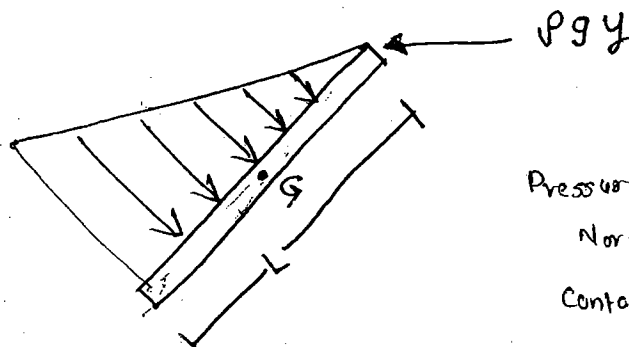


$$F_N = \rho \times g \times \bar{h} \times \text{Area}$$

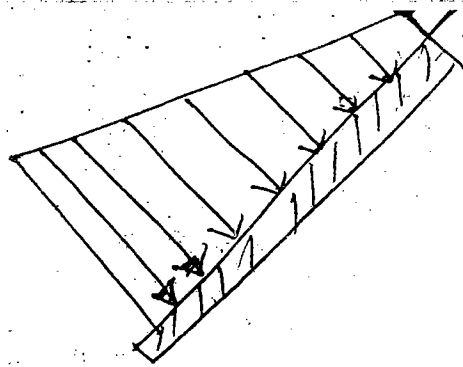
$$\rho \times g \left[y + \frac{L}{2} \sin \theta \right] \times BL$$

$$F_N = \rho g \left[y + \frac{L}{2} \sin \theta \right] \times BL$$

Pressure diagram



Pressure
Normal to
Contact surface



$$h^* = \bar{h} + \frac{I \cdot \sin^2 \theta}{A \cdot \bar{h}}$$

If $y=0$

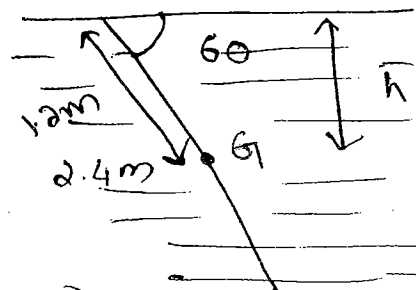
- Q.) A Rectangular plate $0.75 \text{ m} \times 2.4 \text{ m}$ is immersed in a liquid of relative density 0.8 with its ~~0.75 m~~ 0.75 m side horizontal and just at the liquid surface. The plane of the plate makes an angle 60° with the horizontal. The total pressure force on one side of plate in kN and centre of pressure in metres are

- a) 15.6 kN and 1.39 m
- b) 7.8 kN and 1.2 m
- c) 24.0 kN and 1.39 m
- d) 18.0 kN and 1.2 m

$$F_N = \rho \cdot g \cdot \bar{h} \cdot A$$

$$F_N = 0.8 \times 1000 \times 9.81 \times [1.2 \sin 60] \times$$

$$[0.75 \times 2.4]$$



$$\underline{F = 15.6 \text{ kN}}$$

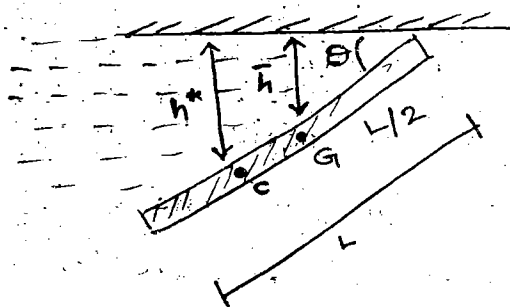
$$\begin{aligned}
 h^* &= \bar{h} + \frac{I_{CG} \times \sin^2 \theta}{A \cdot \bar{h}} \\
 &= \left(\frac{L}{2} \sin 60 \right) + \frac{\frac{0.75 \times (2.4)^3}{12} \sin^2 60}{0.75 \times \left(\frac{L}{2} \sin 60 \right)} \\
 &= 1.04 + \frac{\frac{0.75 \times (2.4)^3}{12} \times (\sin 60)^2}{0.75 \times 2.4 \times 1.04}
 \end{aligned}$$

$$h^* = \underline{\underline{1.39 \text{ m}}}$$

$$= \left(\frac{2.4}{3} \right) \sin 60$$

$$= \frac{2 \times 2.4 \times \sin 60}{3}$$

$$h^* = \underline{\underline{1.39 \text{ m}}}$$



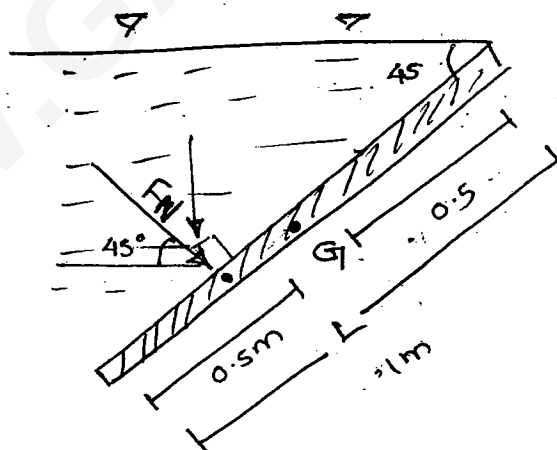
$$\sin \theta = \frac{\bar{h}}{L/2}$$

$$\bar{h} = L/2 \sin \theta$$

$$h^* = \bar{h} + \frac{I_{CG} \sin^2 \theta}{A \bar{h}}$$

- Q) A Rectangular plate of unit width and length 1m shown in the figure. Determine the Hydrostatic force for unit width. The density of liquid is ρ .

Ans:



a) $\sqrt{2} \rho g$

b) $\rho g/2$

c) $\frac{\rho g}{2\sqrt{2}}$

d) $\frac{2}{\rho g}$

$$F_H = F_N \cos 45$$

$$= \frac{\rho g}{\sqrt{2} \times 2\sqrt{2}}$$

$$= \frac{\rho g}{4}$$

$$F_H = \rho \times g \times 0.5 \sin 45 \times \text{Area}$$

$$= \rho \times 9.81 \times 0.5 \times \sin 45 \times [1 \times 1]$$

$$F_H = \frac{1}{2\sqrt{2}} \rho g$$

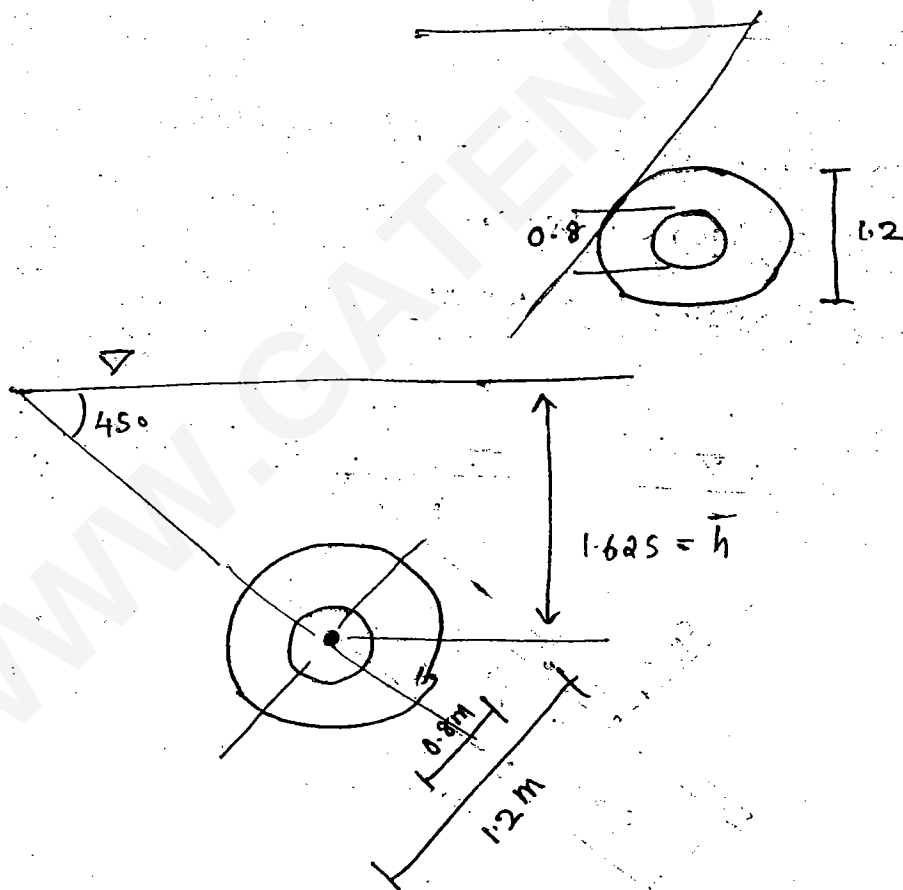
$$F_H = \frac{1}{2\sqrt{2}} \rho g$$

$$\bar{h} = \frac{L}{2} \sin \theta$$

- Q) A circular plate with concentric circles of dia 1.2m and 0.8 m is immersed in water with its plane inclined at an angle 45° with horizontal. If the centre of the circle is 1.625m below the free water surface. Then total pressure force on one side of the circular Ring plate in kN is

- a) 70.7 kN $\left(\frac{10}{\sqrt{2}}\right)$
 b) 10 kN
 c) 14.14 kN $(10 \times \sqrt{2})$
 d) 18.0 kN

Ans:-

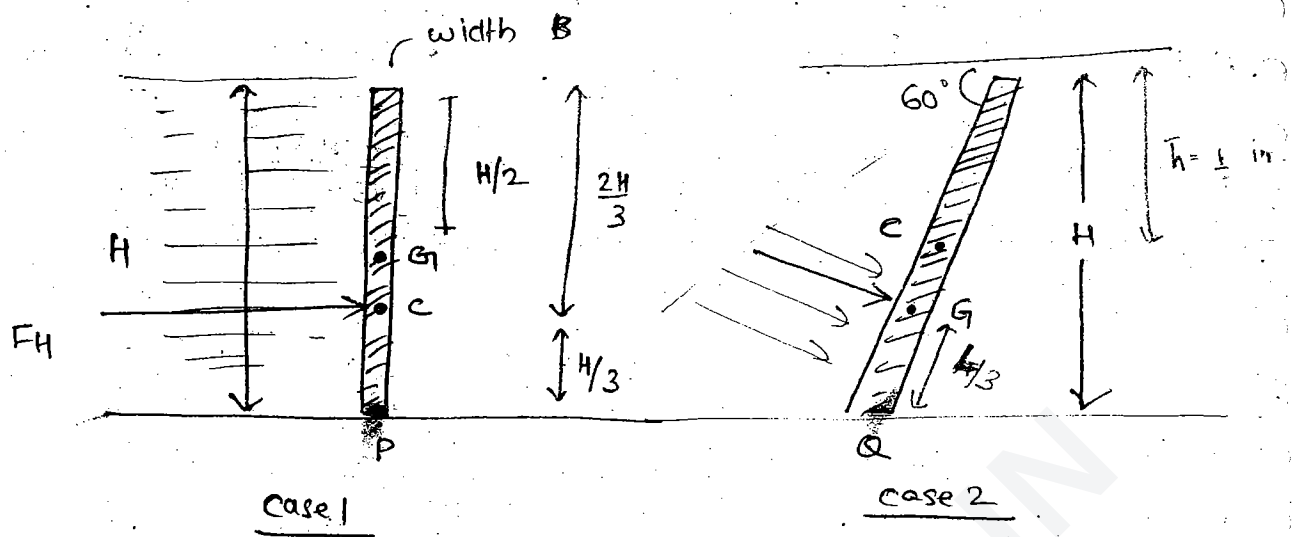


$$F_H = \rho g \bar{h} \cdot A$$

$$= 1000 \times 9.81 \times 1.625 \times \frac{\pi}{4} \left[(1.2)^2 - (0.8)^2 \right]$$

$$= 10000 \text{ N}$$

$$= \underline{\underline{10 \text{ kN}}}$$



Find moment of Hydrostatic force difference

ie $M_Q - M_P$

$$F_H \times \frac{H}{3} - F_H \times \frac{H}{3}$$

$$\frac{\rho g B H^3}{6} - \rho g$$

$$\rho g \times \bar{h} \times \text{Area} \times \frac{H}{3} - \rho g \times \bar{h} \times \frac{H}{3} \times \text{Area}$$

$$\rho g \times \frac{H}{2} \times [B \times H] \times \frac{H}{3} - \left(\rho g \times \frac{L \sin 60}{2} \right) \times \frac{L}{3} \times (L \times B)$$

Area

$$\rho g \frac{B H^3}{6} - \rho g \times \frac{B L^3}{6} \times \frac{\sqrt{3}}{2}$$

$$\rho g \frac{B H^3}{6} - \rho g \frac{\sqrt{3} B L^3}{12}$$

$$\rho g \frac{B H^3}{6} - \rho g \frac{\sqrt{3} B L^3}{4 \times \sqrt{3} \times \sqrt{3}}$$

$$\rho g \frac{B H^3}{6} - \rho g \frac{B L^3}{4 \sqrt{3}}$$

$$\rho g \frac{B H^3}{6} - \rho g \times$$

$$\sin 60 = \frac{H}{L}$$

$$H = L \sin 60$$

$$L = \frac{H}{\sin 60}$$

$$L = \frac{2}{\sqrt{3}} H$$

$$\rho g \frac{BH^3}{6} - \rho g \times \frac{L}{2} \sin 60^\circ \times [L \times B] \times \frac{L}{3}$$

$$\rho g \frac{BH^3}{6} - \rho g \times \frac{2}{\sqrt{3}} H \sin 60^\circ \left[\frac{2}{\sqrt{3}} H \times B \right] \times \frac{2}{\sqrt{3}} H$$

$$\rho g \frac{BH^3}{6} - \rho g \times \frac{H \times \sqrt{3}}{\sqrt{3} \times 2} \left[\frac{2 \times BH}{\sqrt{3}} \right] \times \frac{2}{\sqrt{3}} H$$

$$\frac{\rho g BH^3}{6} - 2 \frac{\rho g BH^3}{3 \times 3}$$

$$= \frac{\rho g BH^3 - 4 \rho g BH^3}{6}$$

$$\frac{\rho g BH^3}{6} - \frac{2}{9} \rho g BH^3$$

$$= \frac{-3 \rho g BH^3}{6}$$

$$= \frac{1}{18} \rho g BH^3$$

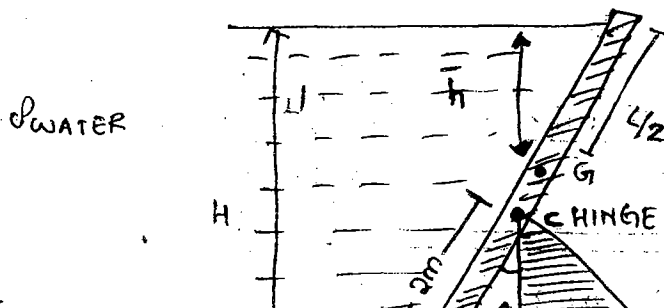
$$= \frac{1}{18} \rho g BH^3$$

Q)

A plate subjected to hydrostatic force shown in Figure. determine the

① At what depth of water the plate to tilt about hinge point

② What is the hinge reaction



Rectangular
flap Board
plate

(FLAP GATE)

$$\frac{\rho \times g \times L \times B}{2}$$

$$\rho \times g \times \frac{L}{2} \sin 50 \times [B \times L]$$

$$= \frac{1000 \times 9.81 \times BL^2}{2}$$

$$= \frac{4905 BL^2}{2}$$

$$1000 \times 9.81 \times \sin 50 \times \frac{BL^2}{2}$$

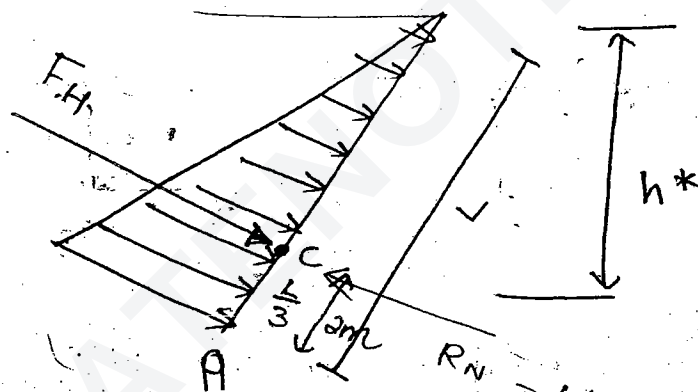
$$BC = 2 \sin 50$$

$$= 1.532$$

$$= \frac{3757.44 BL^2}{2}$$

R_N is reaction of Hinge

R_N is Balanced by Hydrostatic force



$$R_N = F_H$$

Moment About A

$$F_H \times 2 = R_N \times 2$$

$$F_H = R_N$$

Net
needed

$$\frac{L}{3} = 2$$

$$L = 6 \text{ m}$$

$$\sin 50 = \frac{H}{L}$$

$$H = 6 \sin 50$$

$$= 4.6 \text{ m}$$

$$R_N = F_H$$

$$= \frac{4905 \times B \times 6^2}{2}$$

$$= 3757.44 \times 1 \times 36$$

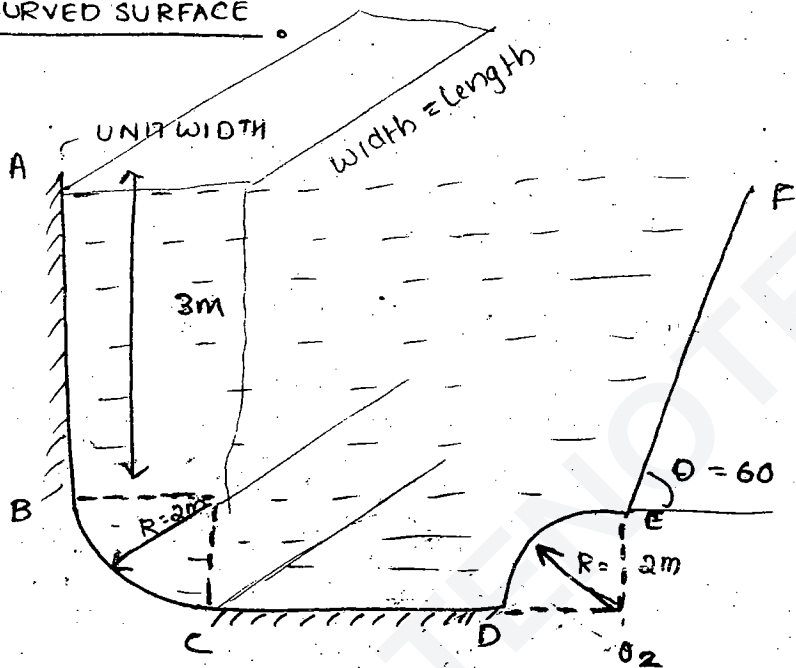
$$= 135,268.12 \text{ N}$$

$$= 135.268 \text{ kN}$$

Take $B = 1 \text{ m}$

A Hinge Support is necessary to withstand wherever
Hydrostatic force acts
ie at centre of pressure

CURVED SURFACE



BC and DE are curved Surfaces

NOTE:-

Curved Surface subjected to two hydrostatic components
or force-

1. Horizontal component (F_H)
2. Vertical Component (F_V)

F_H

It is the Normal force due to hydrostatic pressure

$$F_H = \rho g h \times A \quad \text{Newton}$$

where h = vertical depth from free water surface to
the centroid of the projected plane of

F_v = vertical component

It is the weight of the fluid supported by the curved surface.

$$F_v = mg = \rho \times V \times g$$

$$\rho \times (\text{Area} \times w) \times g$$

$\swarrow$
c/s Area

$$= \rho g (\Sigma A) \times w \rightarrow \begin{matrix} \text{width} \\ \text{or} \\ \text{length} \end{matrix}$$

Q. Determine the following for the curved surface BC and DE shown in the above figure

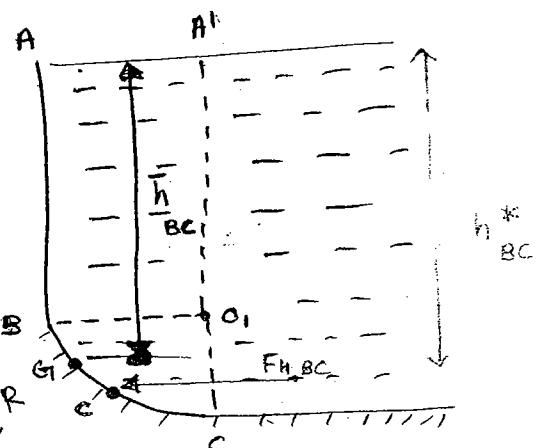
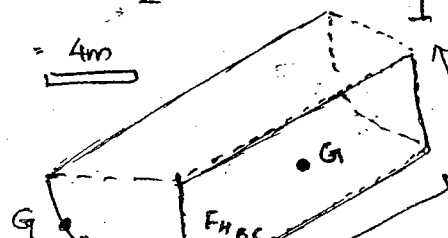
- ① centroid of the curved surfaces
- ② Horizontal hydrostatic force
- ③ vertical component of Hydrostatic force
- ④ Resultant of Hydrostatic force and its position
OR Total pressure
- ⑤ Moment of the vertical Hydrostatic force component about the plane ~~BC~~ at C and D.
- ⑥ Moment of horizontal Hydrostatic forces.

Case 1:

Curved surface BC

$$\bar{h}_{BC} = \left[y = (AB) \right] + \frac{2}{2} \quad y = 3m$$

$$3 + \frac{2}{2} = 4m$$



Projected is

b.)

Horizontal force on BC

$$F_{H_{BC}} = \rho g \bar{h}_{BC} \cdot A_{\text{projected}}$$

$$1000 \times 10 [3+1] \cdot [L \times R]$$

$$g = 10$$

$$1000 \times 10 [4] \times [1 \times 2]$$

$$w = L = 1$$

unit width

$$F_{H_{BC}} = \underline{80 \text{ kN/metre width}}$$

c.)

$F_v =$ w_{water}
supported by
BC curved
portion



$$= M g = \rho V \cdot g$$

$$F_v = \rho g \left[A_{\text{rect}} + A_{\text{quad}} \right] \times L (\text{or } w)$$

$$1000 \times 10 \left[2 \times 3 + \frac{\pi R^2}{4} \right] \times 1$$

$$1000 \times 10 [6 + \pi]$$

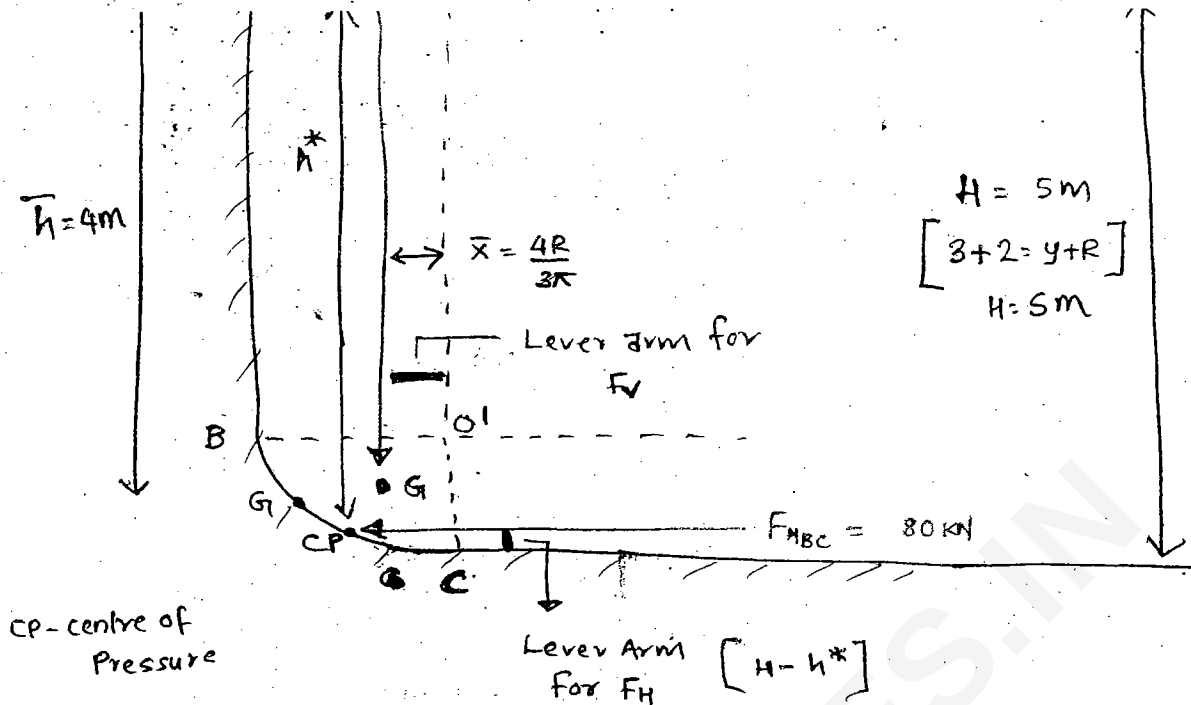
$$F_v = \underline{91.4 \text{ kN/metre}}$$

x ————— x

Note:

$$\begin{aligned} &\text{Area of QUADRANT} \\ &= \frac{\pi R^2}{4} \end{aligned}$$





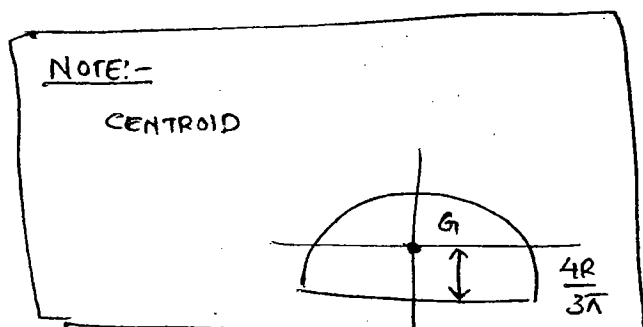
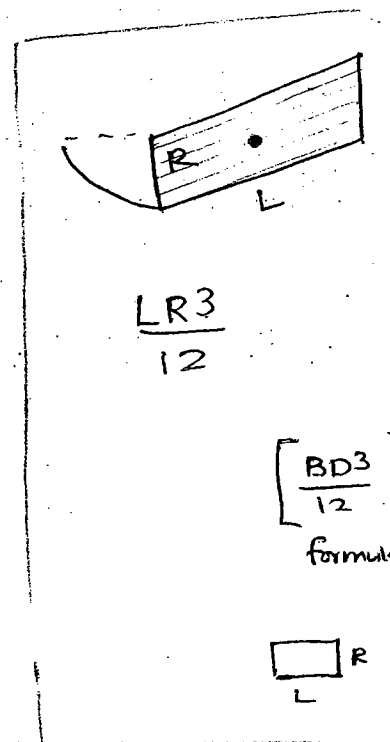
Lever arm for F_v } $\bar{x} = \frac{4R}{3\pi} = \frac{4 \times 2}{3\pi} = \frac{8}{3\pi}$ metres

$$h^* = \bar{h} + \frac{I}{A \times \bar{h}}$$

$$= 4 + \frac{\left[\frac{1 \times 2^3}{12} \right]}{(2 \times 1) \times 4}$$

$$= 4 + \frac{1}{12} = \frac{49}{12}$$

$$= \underline{\underline{4.08\text{m}}}$$



d) Moment of Hydrostatic force components

$$M_{E_1} = F_v \times 4R$$

$$(91.4) \left(\frac{4 \times 2}{3\pi} \right)$$

KN m

$$M_{(F_H)_C} = f_{H_{BC}} \times \left[h^* - h \right] \left[H - h^* \right]$$

Lever Arm

$$= 80 \times \left[(3+2) - 4.08 \right]$$

KN m

d) Resultant Force on curved Surface

$$F = \sqrt{F_H^2 + F_V^2}$$

$$\sqrt{80^2 + 91.42^2}$$

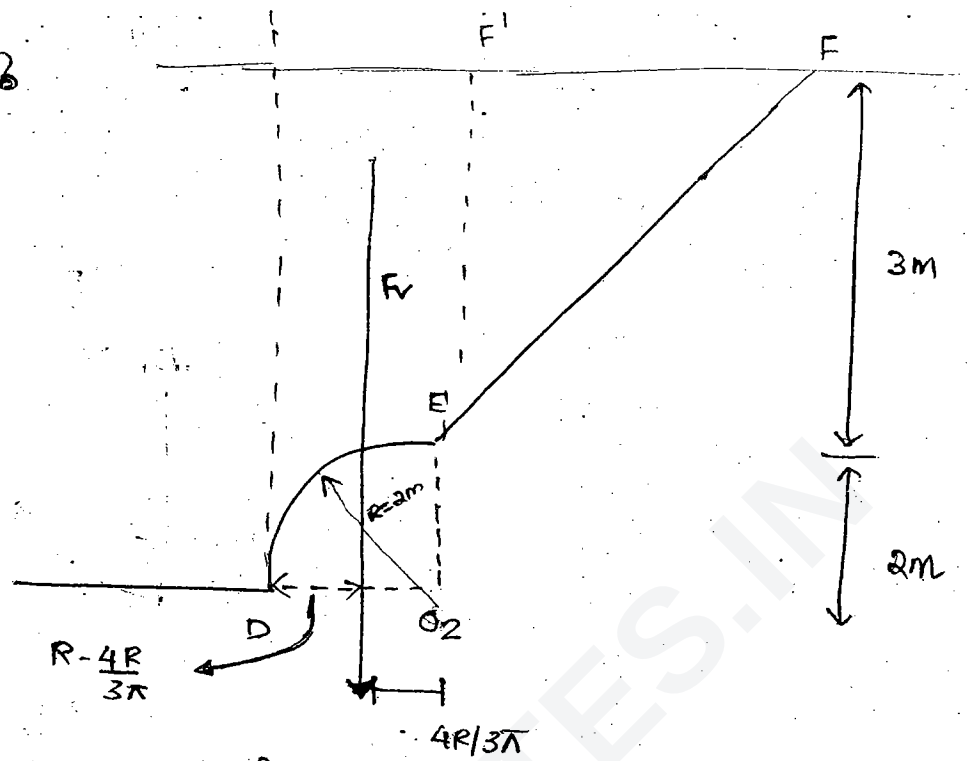
KN

$$\tan \alpha = \frac{F_V}{F_H}$$

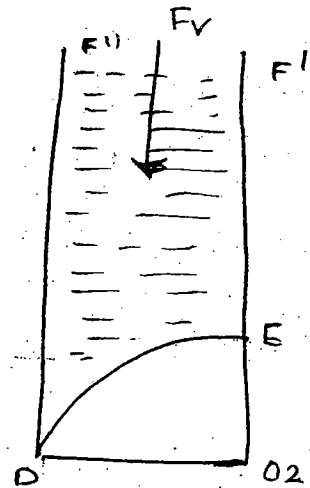
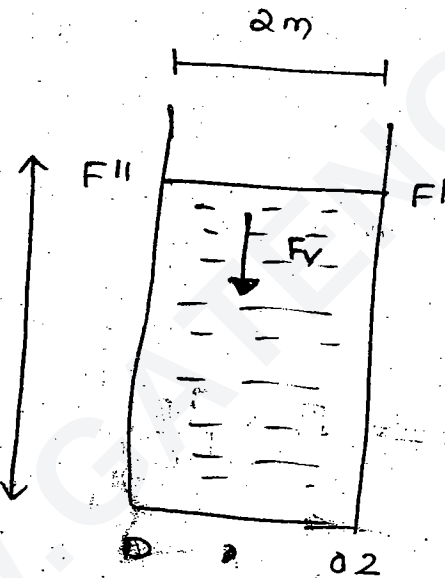
$$\alpha = \tan^{-1} \left[\frac{F_V}{F_H} \right]$$

Direction

$$F_{VDE} = 8$$



$$3+2=5m$$



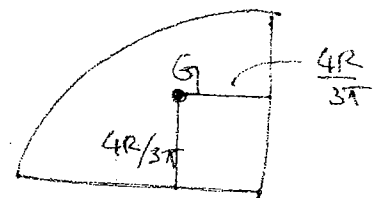
$F_v = \text{weight of fluid supported by curved surface}$

$$F_v = \rho g \left[\frac{A}{2} \times 5 - \frac{A}{4} \times 2 \right] L$$

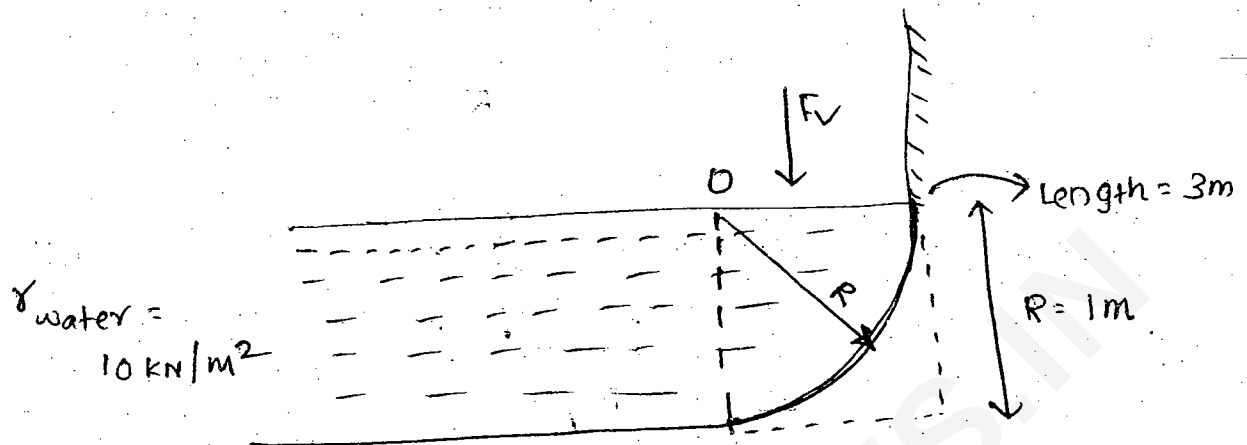
$L = \text{unit length}$

$$= \rho g \left[(2 \times 5) - \frac{\pi \times [2]^2}{4} \right] \times 1$$

$=$



- Q.) Determine moment of vertical component of hydrostatic thrust on one quadrant of cylinder as shown in fig



- a) 7.854 kN-m
 b) 10.000 kN-m
 c) 11.781 kN-m
 d) 15.708 kN-m

one quadrant of cylinder diameter 2m and 3m length

Ans:-

$$(M)_{F_v} = \cancel{\rho \times g} F_v \times \left[\frac{4R}{3\pi} \right]$$

$$= \gamma \times V \times \frac{4R}{3\pi}$$

$$V = \frac{\pi R^2 \times L}{4}$$

$$\gamma \times \left[\frac{\pi R^2 \times L}{4} \right] \times \frac{4R}{3\pi}$$

$$= \gamma \cdot \left[\frac{\pi R^2 \times 3}{4} \right] \left[\frac{4R}{3\pi} \right]$$

$$= \gamma \cdot R^3$$

$$= 10 \times 1^3$$

$$= \underline{10 \text{ kN-m}}$$

with fill with water; then what happens?

Ans:

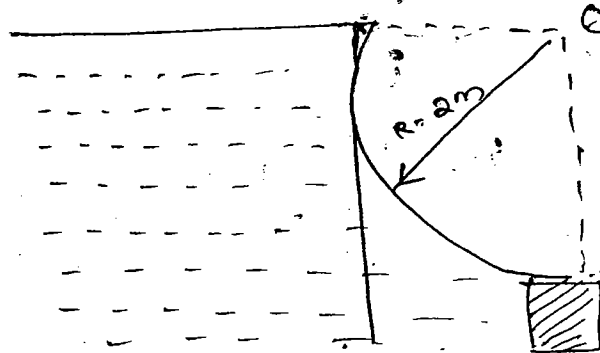
A Quadrant of cylinder diameter = 4m; width is unity

find the (i) horizontal hydrostatic force (kN)

(ii) vertical " " " (kN)

(iii) Resultant " " " (kN) and its orientation

(iv) Moment of vertical component of water force



(i) $F_H = \rho \times g \times \bar{h} \times (\text{Area})$

$1000 \times 9.81 \times \frac{4 \times 2}{3} \times [2 \times 1]$

$$1000 \times 9.81 \times \frac{h}{2} \times [h \times B]$$

$$9810 \times \frac{2}{2} \times [2 \times 1]$$

$$= \underline{19.62 \text{ kN}}$$

Vertical Force (F_v)

F_v = weight of water displaced

$$= m \cdot g$$

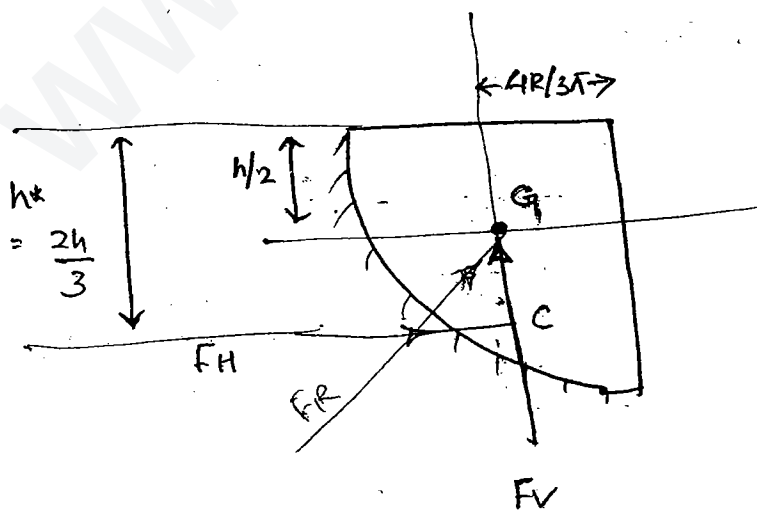
$$= \rho_{\text{water}} \times V \times g$$

$$\rho_{\text{water}} \times A_N \times g$$

$$1000 \times 9.81 \times \frac{\pi R^2}{4} \times (1)$$

$$= 1000 \times 9.81 \times \frac{\pi \times 2^2}{4} \times (1)$$

$$= \underline{30.81 \text{ kN}}$$



$$F_R = \sqrt{F_H^2 + F_V^2}$$

$$= \underline{\underline{36.5 \text{ kN}}}$$

$$\tan \alpha = \frac{F_V}{F_H}$$

$$\underline{\underline{\alpha = 52.5^\circ}}$$

Moment of vertical force about O

$$F_V \times \frac{4R}{3\pi}$$

$$= 30.81 \times \frac{4 \times 2}{3\pi}$$

$$= \underline{\underline{\quad \quad \quad \text{Nm} \quad \quad \quad}}$$

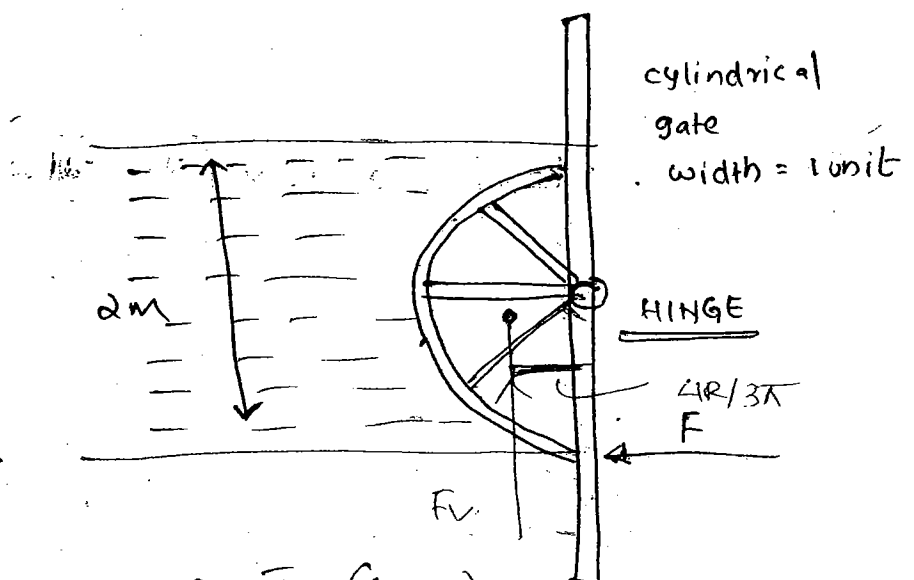
- Q) Fig shows a semi circular cylindrical gate supporting water load. Determine the force F read to keep the gate under equilibrium.

a) 0.00 kN

b) 9.81 kN

c) 19.62 kN

d) None of these



$$F_H = \rho g \times h \times (\text{Area})_{\text{proj}}$$

Ans

$$F_v = w = Mg$$

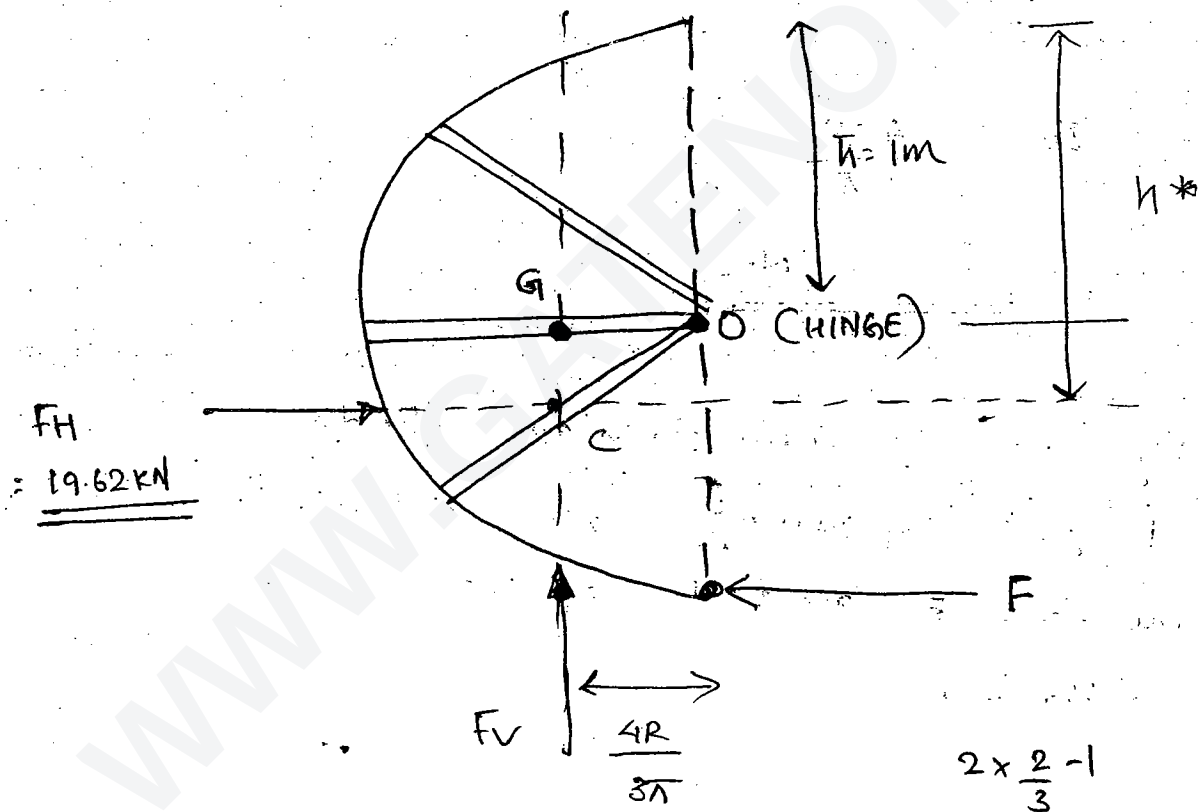
~~$F_v \times \frac{h}{2}$~~

$$= \rho \cdot g \cdot A \times h$$

$$= 1000 \times 9.81 \times \left(\frac{\pi R^2}{2} \right) \times (w)$$

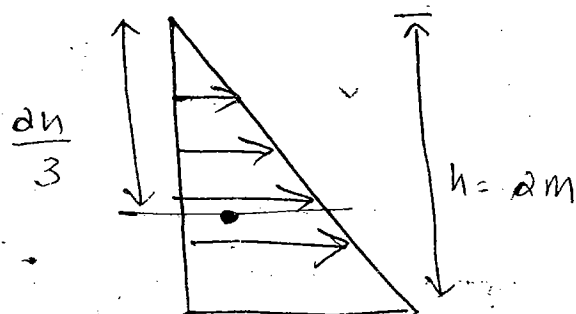
$$= 1000 \times 9.81 \times \frac{\pi (1)^2 \times 1}{2}$$

$$= \frac{9.81 \pi \text{ kN}}{2}$$



$F_H = 19.62 \text{ kN}$

~~$F_H \times (1) + F_H \times \left[2 \times \frac{2}{3} - 1 \right] + F_v \times \left[\frac{4 \times 1}{3\pi} \right] = 0$~~



$$h^* = \frac{h}{2} + \frac{w \cdot h^3}{12 A \cdot h}$$

$$= \frac{h}{2} + \frac{h}{6}$$

$$h^* = \frac{2h}{3}$$

~~$$F = 19.62 \times \left[\frac{4-1}{3} \right] - \frac{9.81 \pi}{2} \times \frac{4}{3\pi}$$~~

~~$$F = 19.62 \text{ kN}$$~~

~~$$F = 6.54$$~~

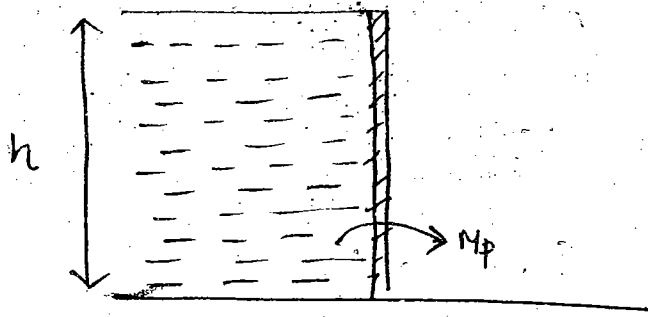
$$F \times 1 - F_H \times \left[\frac{2 \times \frac{2}{3} - 1}{3} \right] + F_V \times \frac{4R}{3\pi}$$

$$F \times 1 - 19.62 \times \left[\frac{4}{3} - 1 \right] + \frac{9.81 \pi}{2} \times \frac{4 \times 1}{3\pi}$$

$$F = 0 \text{ N}$$

Q.

There are 2 ~~blades~~ Rectangular Blades Subjected to water thrust in two diff cases shown in figure. Determine diff of the moments of the hydrostatic forces about their base points. shown ?



- Q. A Hollow Hemispherical object of dia 'D' is immersed in water with its plane surface coinciding with the free surface of the water. What is the vertical component of Hydrostatic force on curved surface of the given object ?
- (a) $\frac{3}{8} \gamma \pi D^3$ (b) $\frac{1}{12} \gamma \pi D^3$ (c) $\frac{1}{24} \gamma \pi D^3$ (d) 0



$$F_v = F_B = W_{\text{fluid displaced}}$$

$$= M \cdot g$$

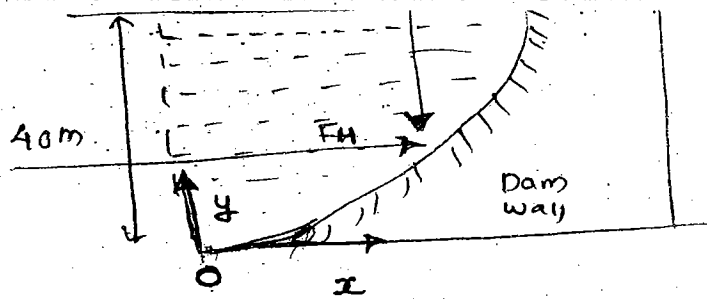
$$= \rho \cdot V \times g$$

$$= \gamma \times \frac{4}{3} \pi R^3 / 2$$

$$= \gamma \times \frac{4}{6} \pi \times R^3$$

$$= \gamma \times \frac{4}{6} \times \pi \times \frac{D^3}{8}$$

$$= \gamma \times \pi D^3$$



A Reservoir dam spillway shown in fig subjected to water load - Determine

- ① eqn of curved surface
- ② Horizontal force
- ③ vertical component of water load
- ④ Resultant force

Ans:-

~~F_H~~ ~~3x^2 = 4y~~

$$y = mx^2$$

At y = 40

$$x = 15$$

$$40 = m \times 15^2$$

$$m = \frac{40}{225}$$

$$m = \underline{\underline{0.177}}$$

F_H

$$F_H = \rho \times g \times h \times (\text{Area})$$

$$= 1000 \times 9.81 \times \frac{40}{2} \times [40 \times 1]$$

$$\underline{\underline{392400}} \times \underline{\underline{8 \text{ MN}}}$$

F_v

$$F_v = W$$

$$= m \times g = \rho \times V \times g$$

$$F_R = \sqrt{F_H^2 + F_V^2}$$

$$\underline{F_H = 8 \text{ MN}}$$

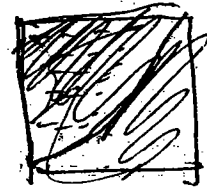
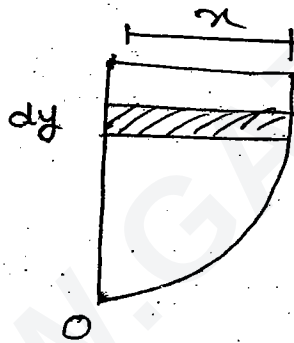
$$F_V = \rho \times V \times g$$

$$1000 \times V \times g$$

$$\rho g \times [\text{Area} \times w]$$

$$1000 \times 10 \times [\text{Area} \times 1]$$

: A = Area of curved surface



$$\text{Area} = x \cdot dy$$

$$y = 0.18 x^2$$

$$\text{Area} = \int_0^{40} \sqrt{\frac{y}{0.18}} dy$$

$$= \frac{1}{\sqrt{0.18}} \int_0^{40} y^{1/2} dy$$

$$= \frac{1}{\sqrt{0.18}} \left[\frac{y^{1/2+1}}{\frac{1}{2}+1} \right]_0^{40}$$

$$F_v = 1000 \times 10 \times 397.5 \times 1$$

$$= \underline{3975 \text{ KN}}$$

$$F_R = \sqrt{F_v^2 + F_H^2}$$

$$= \underline{\hspace{2cm} \text{KN} \hspace{2cm}}$$

2-5:30 (408)

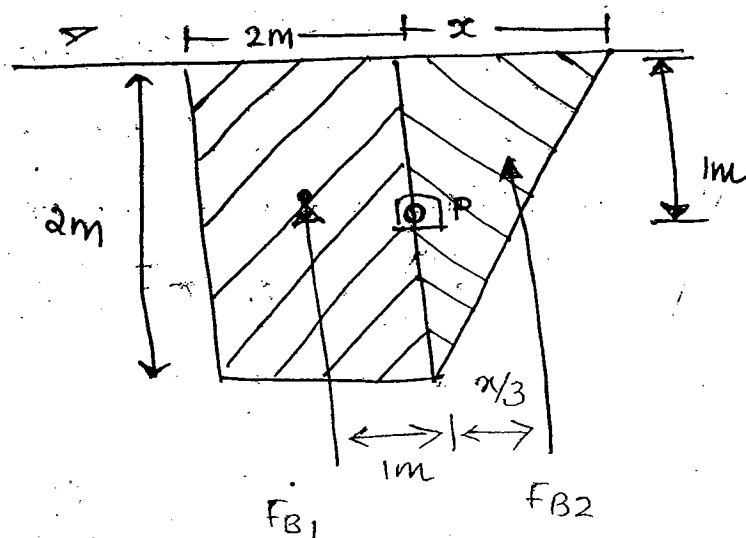
6-8:30

{ Laplace Transform }

Q. A cross section of an object (having same section normal to plane of the paper) is submerged into a fluid consist of a square cross section of each side 2m and a triangle shown in the figure. The submerged object is hinged at point P shown. object is to be kept in that position, then the value of x dimension should be.

- a) $2\sqrt{3} \text{ m}$ b) $4\sqrt{3} \text{ m}$ c) 4 m d) 8 m

Ans:



Moment abt P

$$F_B \times 1 = F_B \times \frac{\pi}{3}$$

$$\cancel{\rho} \times \cancel{g} \times \cancel{20} \times 1 = \cancel{\rho} \times \cancel{g} \times \cancel{20} \times \frac{x}{3}$$

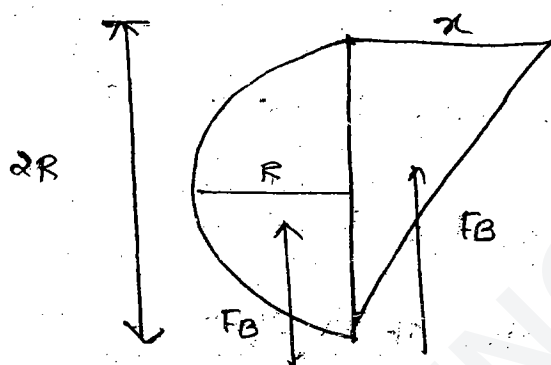
$$\cancel{A} \times \cancel{w} \times 1 = \cancel{A} \times \cancel{w} \times 1 \times \frac{x}{3}$$

$$x = \frac{20}{3}$$

$$\underline{x = 2\sqrt{3}}$$

$$\cancel{x = \frac{20}{3}}$$

a.)



$$A_{\text{semicircle}} \times \frac{4R}{3\pi} = A_{\text{triangle}} \times \frac{x}{3}$$

$$\frac{\pi R^2}{2} \times \frac{4R}{3\pi} = \frac{1}{2} \times x \times 2R \times \frac{x}{3}$$

$$\underline{x = \sqrt{2} \cdot R}$$

a.)

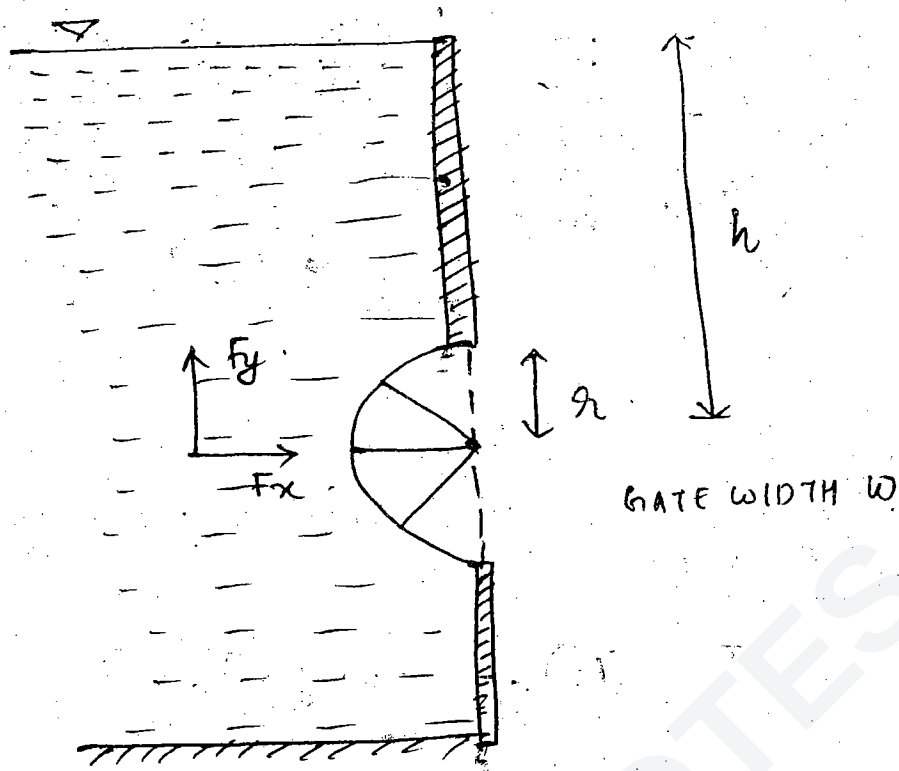
the horizontal and vertical forces F_x and F_y on semicircular gate having a width w into the plane of paper are

a) $F_x = \rho g h \cdot x w$ and $F_y = 0$

b) $F_x = 2 \rho g \cdot h \cdot x w$ and $F_y = 0$

c) $F_x = 2 \rho g \cdot h \cdot x \cdot w$ and $F_y = \rho g \cdot w \cdot x^2$

d) $F_x = 2 \rho g \cdot h \cdot x \cdot w$ and $F_y = \pi \rho g \cdot w \cdot x^2$



$$F_x = (\rho \times g) \times \bar{h} \times \text{Area}$$

$$= \rho \times g \times h \times (r \times 2 \times w)$$

$$= \underline{\underline{2 \rho g h r w}}$$

$$F_v = M \cdot g$$

$$= \rho \times V \times g$$

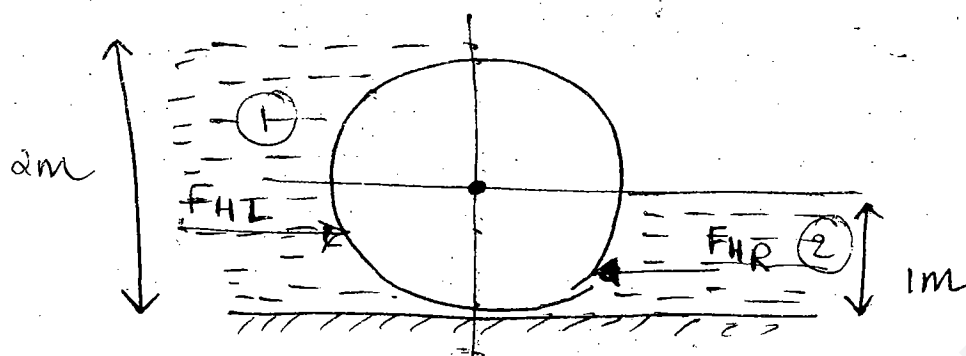
$$= \rho \times (A \times w) \times g$$

$$= \cancel{\frac{\pi r^2}{2}} \rho \times g \left(w \times \frac{\pi r^2}{2} \right)$$

$$F_v = \underline{\underline{\frac{\pi \rho g w r^2}{2}}}$$

Q) A solid cylinder of dia $2m$ and length $10m$ supporting the water shown in fig. Det

- ① Net Horizontal force and Net vertical component.



ΣF_H

$$F_{HL} - F_{HR}$$

$$\rho_1 \times g \times h_1 \times (A_1) - \rho_2 \times g \times h_2 \times (A_2)$$

$$\cancel{\rho_1 \times g \times h_1 \times (A_1)}$$

$$1000 \times 9.81 \times \left[\frac{2}{2} \right] \times [2 \times 10] -$$

$$1000 \times 9.81 \times \left(\frac{1}{2} \right) \times [1 \times 10]$$

$$= 1000 \times 9.81 \left[1 \times 2 \times 10 - \frac{1}{2} \times 10 \right]$$

$$= 15 \times 9.81 \text{ kN}$$

$$= \underline{150 \text{ kN}}$$

$$= \underline{15 \text{ tonnes}} \text{ (towards right)}$$

ΣF_V

$$F_{V1} + F_{V2}$$

$$= w_1 + w_2$$

$$= m \cdot a + m \cdot a$$

as Force/unit Area. It is as many directions

$$1000 \times 9.81 \times 10 \left[\frac{\pi R^2}{2} + \frac{\pi R^2}{4} \right]$$

$$= 1000 \times 9.81 \times 10 \left[\frac{\pi (1)^2}{2} + \frac{\pi (1)^2}{4} \right]$$

Q. state true or false

1. the centre of pressure on plane surface is

a) depends upon the type of fluid

b) ~~is~~ contact with the surface (x)

$$h^* = \bar{h} + \frac{I}{A \bar{h}}$$

[depends only on geometric properties] & depth of centroid (h)

centre of pressure is independent upon type of fluid in contact with surface

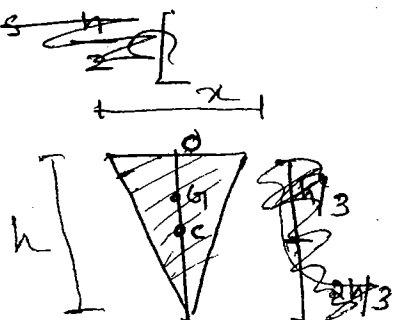
Q. A vertical component of the hydrostatic force on curved surface is equal to weight of the liquid vertically above the curved surface (✓)

Q. A triangular plate immersed in water with vertex downwards. the altitude of the vertical triangular face is h . the centre of pressure is

$$OC = \frac{h}{2}$$

[True]

$$h^* = \bar{h} + \frac{I}{A \bar{h}}$$



$$\frac{h}{3} + \frac{\frac{1}{36} \times x \times h \times 3}{\frac{1}{2} \times x \times h}$$

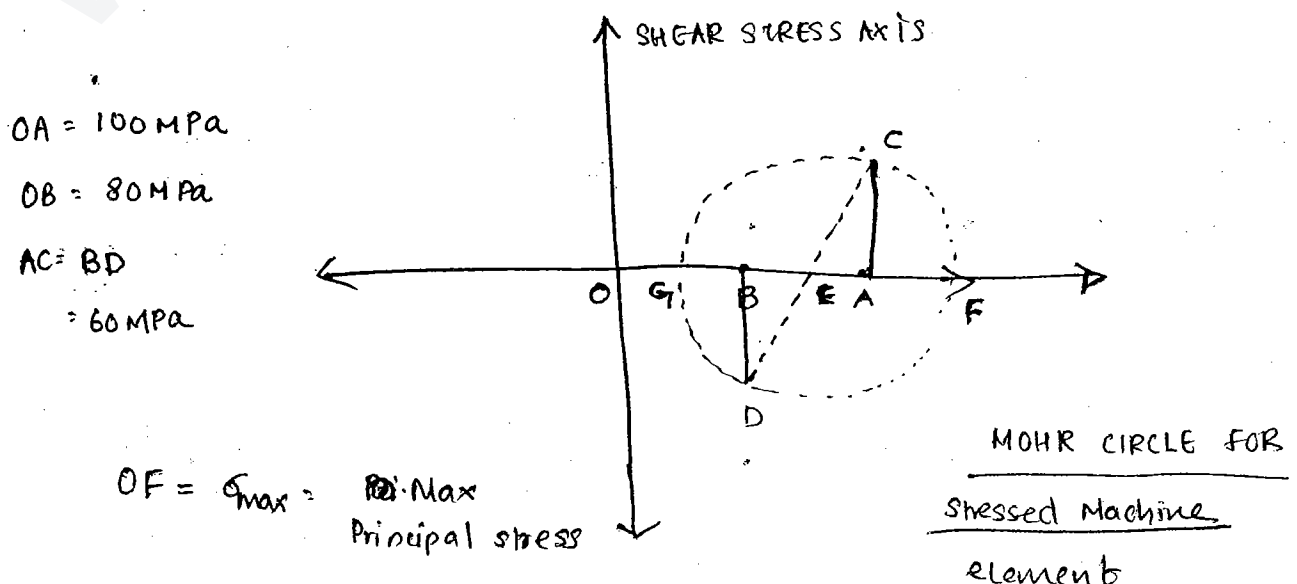
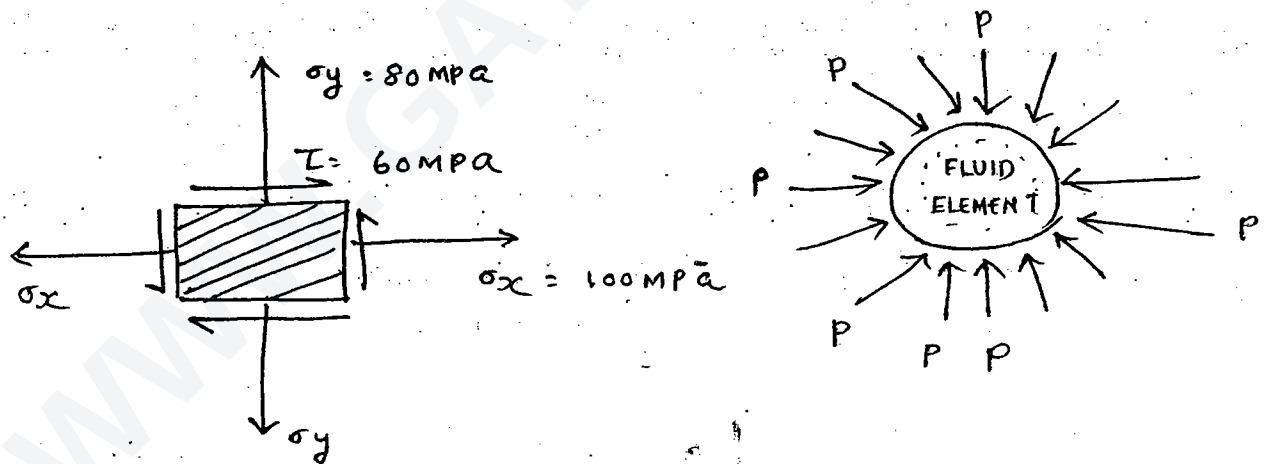
$$\frac{h}{3} + \frac{h}{6}$$

$$= \underline{\underline{\frac{h}{2}}}$$

Pressure is defined as Force/unit Area. It is a scalar qty. Because it has many directions of action on the given Area.

In FM pressure is due to the fluid contact with the surface. when fluid at ^{Rest} no tangential force involved. only Normal forces involved. It can be studied through Mohr's circle concept also.

When fluid element in space subjected to normal pressures which are in compressive nature with same magnitude. ie $P_x = P_y$.



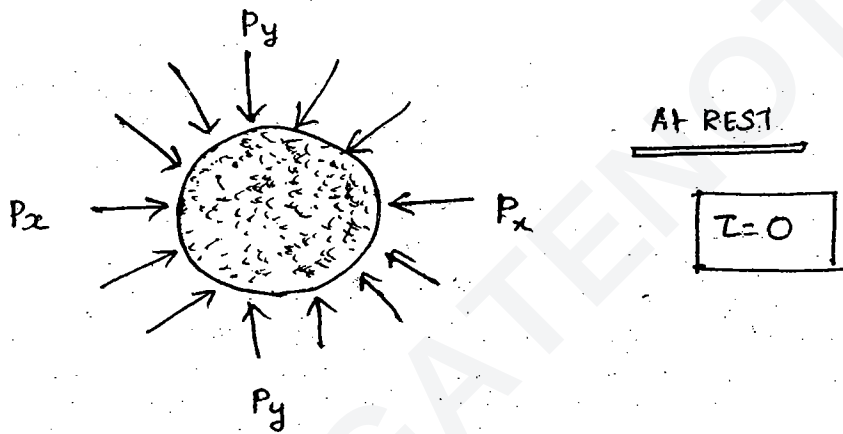
$$\sigma_f = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sigma_{\max}$$

$$\text{OGr} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sigma_{\min}$$

$EC = ED = \text{Radius of Mohr's circle}$

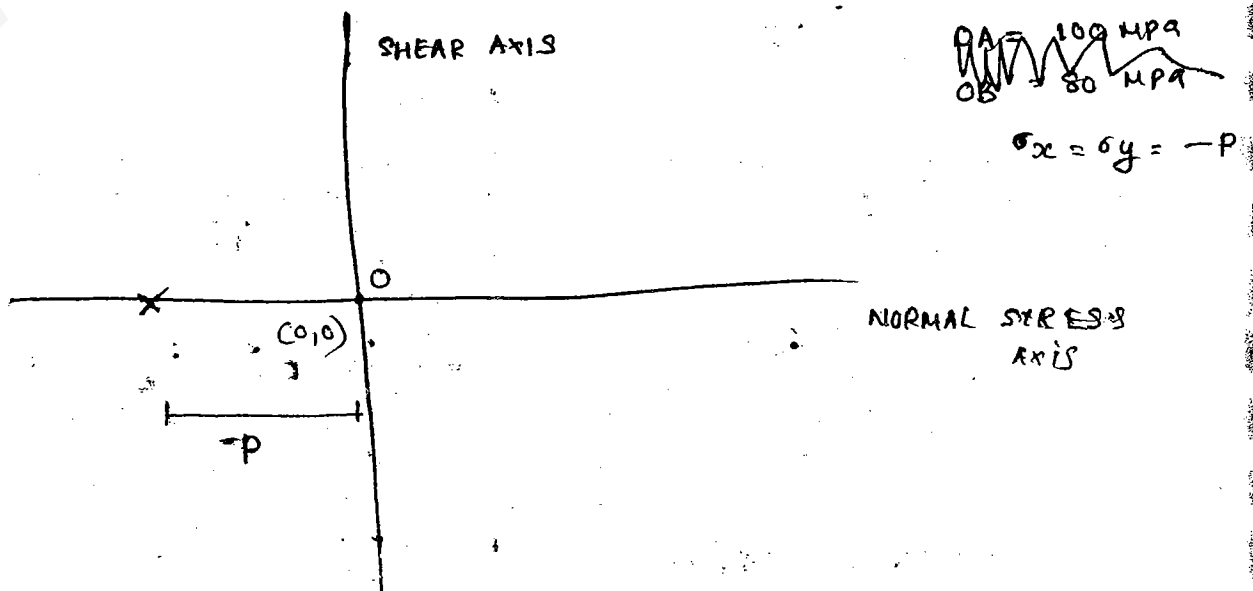
$$I_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$= \sqrt{\left(\frac{6x - 5y}{2}\right)^2 + 1^2}$$



$$P = P_x = P_y = \dots = \sigma_c$$

$$\left. \begin{aligned} \sigma_x = \sigma_y = \sigma_z = -P \\ E = 0 \end{aligned} \right\} \begin{array}{l} \text{Fluid element} \\ \text{At Rest} \end{array}$$



on the -negative normal stress axis.

Q) If a Mohr's circle is drawn for a fluid element which is at Rest then the Mohr's circle would be

- a) A circle touching the circle
- b) A point on shear stress axis
- c) A circle ~~not~~ touching the origin
- d) A point on the Normal stress Axis

Exactly:

A point on -ve normal stress Axis

NOTE:-

The Normal stresses of same magnitude in all directions at a point in a fluid possible when fluid has no shear stress.

PRESSURE INTENSITY AT A POINT DUE TO FLUID

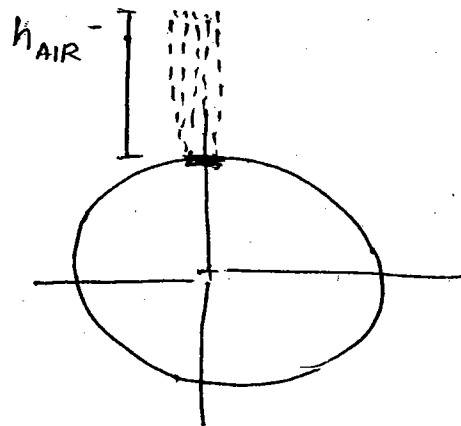
$$P = \frac{F}{A} = \frac{W}{A}$$

$$= \frac{mg}{A}$$

$$= \frac{\rho \times V \cdot g}{A}$$

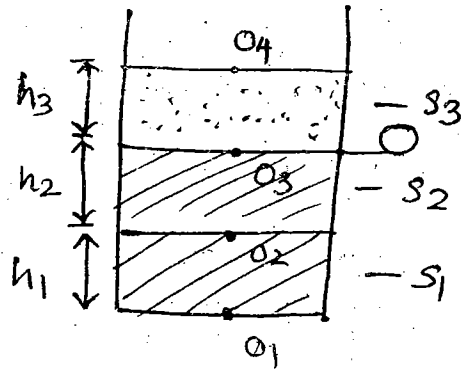
$$= \frac{\rho \times A \times h \cdot g}{A}$$

$$\boxed{P = \rho gh}$$



$$P = \rho h$$
$$\frac{\rho}{\rho} = 1$$

Pressure Intensity in Multi Fluids:-



$$P_{O4} = P_{atm} = 101.3 \text{ kPa}$$

$$= 100 \text{ kN/m}^2 \approx \underline{\underline{107 \text{ kN/m}^2}}$$

$$P_{O3} \text{ (gauge)} = \rho_3 g \cdot h_3$$

$$P_{O2} \text{ (gauge)} = \rho_3 g \cdot h_3 + \rho_2 g \cdot h_2$$

$$P_{O1} \text{ (gauge)} = \rho_3 g h_3 + \rho_2 g h_2 + \rho_1 g h_1$$

$$P = \rho_{AIR} \cdot g \cdot h_{AIR} = \rho_{Hg} \cdot g \cdot h_{Hg} = \rho_{H_2O} \cdot g \cdot h_{H_2O} = \rho_{oil} \cdot g \cdot h_{oil}$$

(N/m²)

Pressure at a point can be Represented in terms of
Columns: [Air, water, oil, ...]

$$P = \frac{F}{A} \left[\frac{\text{N}}{\text{m}^2} \right]$$

↓
Pascal (Pa)

cm²

cm → m

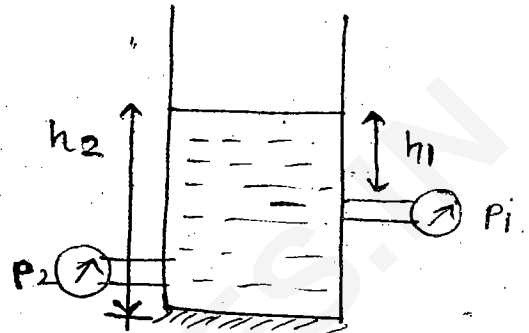
$P =$ metres of water or Hg or oil

Difference in Pressure b/w 2 points! -

$$P_1 = \rho \cdot g \cdot h_1$$

$$P_2 = \rho \cdot g \cdot h_2$$

$$P_2 - P_1 = \rho g (h_2 - h_1)$$

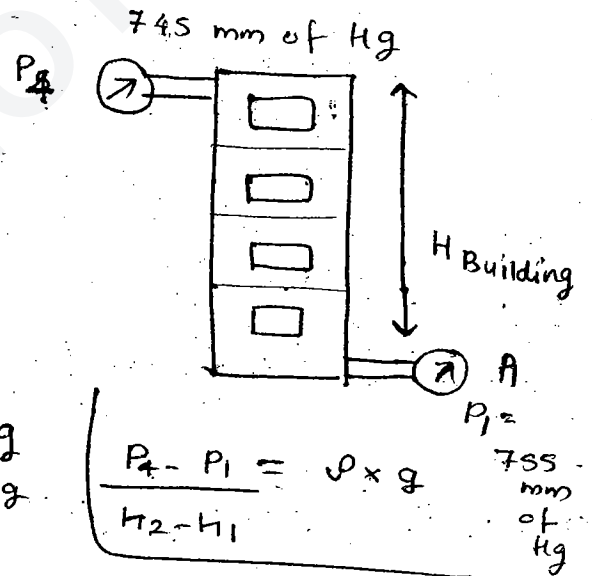


$$\frac{P_2 - P_1}{h_2 - h_1} = \rho g$$

$$\Delta P = P_2 - P_1 = \rho g (h_2 - h_1)$$



$$\rho_{Hg} \cdot g [h_{Hg} - h_{Hg}] = \rho_{AIR} \cdot h_{Building} \cdot g$$



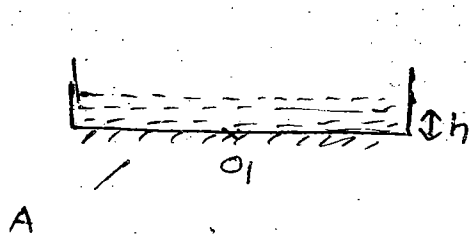
$$\frac{P_4 - P_1}{h_2 - h_1} = \rho \times g$$

$$13600 \times 9.81 [0.755 - 0.745] = 1.2 \times 9.81 \times H_{Building}$$

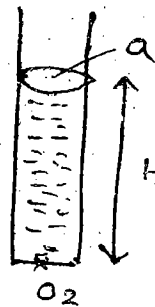
$$H_{Building} = 120 \text{ m}$$

NOTE: —

1 kg of water = 1 lt of water



$$P_{O1} = \rho g h$$



$$P_{O2} = \rho g H$$

$$P_{O2} > P_{O1}$$

2011

Pressure intensity is dependant on depth only.

XE

a) Two tanks A and B with same height are filled with water till the top. The volume of Tank A is 10 times the vol of Tank B. What can you say about the pressure P_A and pressure P_B at the bottom of the tanks A and B resp.

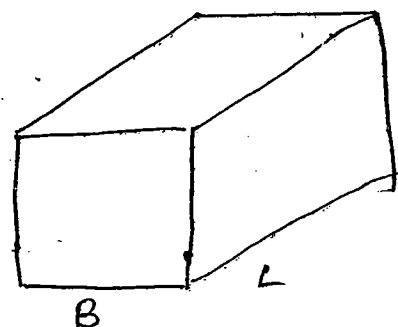
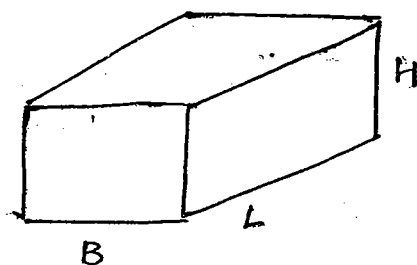
a) $P_A = 10 P_B$

b) $P_B = 10 P_A$

c) $P_A = P_B$

d) Additional data is reqd to compare the two pressures.

$$H_A = H_B$$



$$V_A = 10 V_B$$

$$A_A \cdot H_A = (10 A_B) \cdot H_B$$

resp at heights 8m, and 5m resp. Gauges connected on side of the Tank filled with a liquid - what is the specific gravity of that liquid in the Tank.

$$P_1 = 57.4 \text{ kPa}$$

$$h_1 = 8 \text{ m}$$

$$P_2 = 80 \text{ kPa}$$

$$h_2 = 5 \text{ m}$$

$$\frac{P_1 - P_2}{h_2 - h_1} = \rho \cdot g$$

$$\frac{P_2 - P_1}{h_2 - h_1} = \rho \cdot g$$

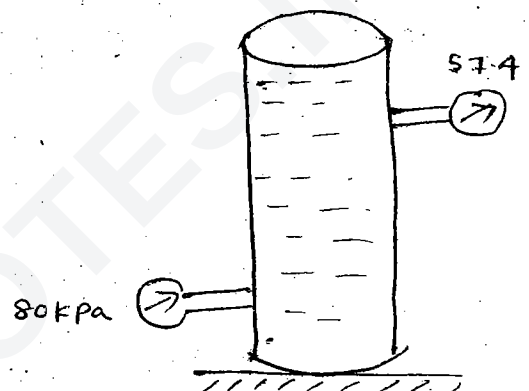
$$\frac{10^3 (80 - 57.4)}{8 - 5} = \rho \times 9.81$$

$$\rho = \frac{76768}{9.81}$$

$$S = \frac{\rho_{\text{liq}}}{\rho_{\text{H}_2\text{O}}} = \frac{768}{1000}$$

$$S = 0.768$$

$$S = 0.76$$

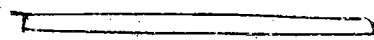


- ① A 6m deep tank contains 4m of water at Bottom and 2m of oil (S.R.D = 0.9). Above the water the Pressure at Bottom of tank in KN/m^2 is Unit wt of water = 10 KN/m^3

$$P = \rho_w \cdot h_w \cdot g + \rho_{\text{oil}} \cdot h_{\text{oil}} \cdot g$$

$$56.898 \text{ N/m}^2$$

$$56.898 \text{ kN/m}^2$$



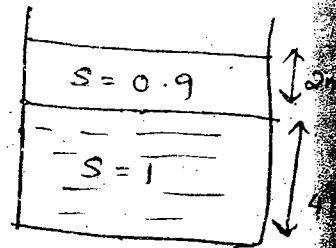
OR

$$\gamma_w \cdot h_w + \gamma_{oil} \cdot h_{oil}$$

$$10 \times 4 + (\gamma_{oil} \times \gamma_{water}) \cdot h_{oil}$$

$$10 \times 4 + (0.9 \times 10) \cdot 2$$

$$\frac{58 \text{ kN}}{\text{m}^2}$$



Q) ~~⊗~~ which of the following pressure units represents least pressure units

a) millibar

b) mm of Hg

c) N/mm²

d) kgf/cm²

$$\text{Millibar} = 10^{-3} \text{ Bar}$$

$$= 10^{-3} \times 10^5 \text{ Pa} = \underline{100 \text{ Pa}}$$

mm of Hg

$$760 \text{ mm of Hg} = 1 \text{ bar}$$

$$\text{mm of Hg} = \frac{1}{760} \text{ Bar} = \frac{1}{760} \times 10^5 \text{ Pa} = \underline{131.51 \text{ Pa}}$$

$$P = \rho_{Hg} \cdot g \cdot H_{Hg}$$

$$= 13600 \times 10 (1 \times 10^{-3}) = \underline{\underline{136 \text{ PaS}}}$$

PUT IN ORDER of Least unit

$$(A) < (B) < (D) < (C)$$

Given that Specific gravity of mercury = 13.6,

Intensity of pressure = 40 kPa

express the intensity of pressure (gauge) in various units [S.I]

~~40 kPa~~

a) 0.3 bar, 3.077 m of H₂O;
0.15 m of Hg

b) 0.5 bar, 5.077 m of H₂O, 0.339
m of H₂O

☒ c) 0.4 bar, 4.077 m of H₂O, 0.229 m
of Hg

d) None of these

$$40 \times 10^3 \text{ Pa}$$

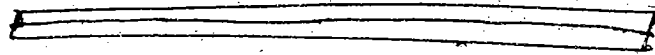
= 40

$$\frac{40 \times 10^3}{10^5} \text{ bar} = \underline{\underline{0.4 \text{ Bar}}}$$

$$P = \rho g h$$

$$40 \times 10^3 = 13600 \times 9.81 \times h_{Hg}$$

$$h_{Hg} = 0.229 \text{ m of Hg}$$



Q) The pressure in metres of oil ($S_{oil} = 0.8$)

equivalent to 80 m of water is

- a) 64 m
- b) 88 m
- c) 80 m
- d) 100 m

$$P = 80 \text{ m of water}$$

$$\rho_w \cdot h_w \cdot g = \rho_{oil} \cdot h_{oil} \cdot g$$

$$1000 \cdot 80 = 0.8 \times 1000 \cdot h_{oil}$$

$$h_{oil} = \frac{80}{0.8} = \underline{\underline{100 \text{ m of oil}}}$$

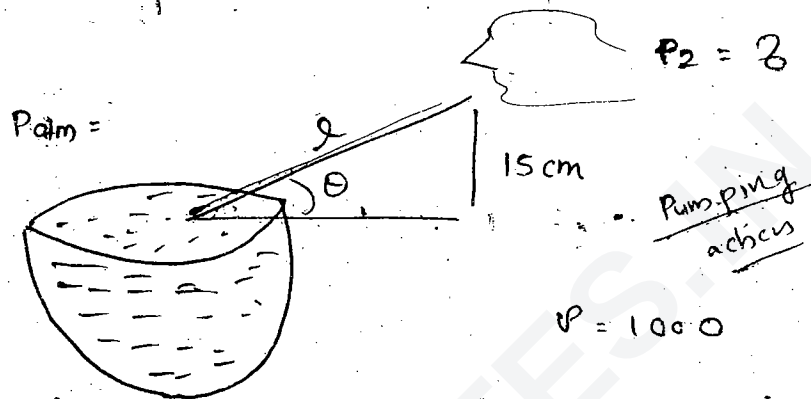
Q) Specific Gravity of a liquid is 0.75. what height of that liquid is needed to provide a pressure difference of 2 bar?

- a) 27.18 m
- b) 17.98 m
- c) 9.81 m
- d) 15 m

$$2 \times 10^5 = (0.75 \times 1000) \times 9.81 \times h$$

$$h = \underline{\underline{27.183 \text{ m. of water}}}$$

Q.



How much pressure is to be borne by lungs.

$$P_2 - P_1 = \rho g \Delta h$$

$$= 1000 \times 9.81 \times (0.15)$$

$$= 1.5 \times 10^3 \text{ N/m}^2$$

$$= \underline{\underline{1.5 \text{ kPa}}}$$

$$P_2 - P_1 = -1.5 \text{ kPa}$$

$$P_2 = -1.5 + 101$$

$$P_2 = \underline{\underline{99.5 \text{ kPa}}}$$

$$\text{work done} = \underline{\underline{P \cdot V}}$$

Q) Specific wt of sea water may be taken to vary according to the expression $(\gamma = \gamma_0 + C\sqrt{h})$

where γ_0 = sp wt of the sea water at datum
(free surface)

h = depth below the sea surface.

The pressure at a depth h metres in excess of $\gamma_0 h$ is a) $\frac{1}{3} C h^{3/2}$

b) $\frac{2}{3} C h^{3/2}$

c) $\frac{1}{3} C h^{2/3}$

d) $\frac{1}{2} C h^{2/3}$

$$\frac{dp}{dz} = -\gamma$$

$$\frac{dp}{dh} = \gamma$$

$$dp = \gamma dh$$

$$dp = (\gamma_0 + C\sqrt{h}) dh$$

$\int$

$$p = \int \gamma_0 dh + C \int \sqrt{h} dh$$

$$= \gamma_0 h + C \frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \gamma_0 h + C \cdot \frac{2}{3} h^{3/2}$$

$$P = \rho g h$$

$$\frac{P}{h} = \rho g$$

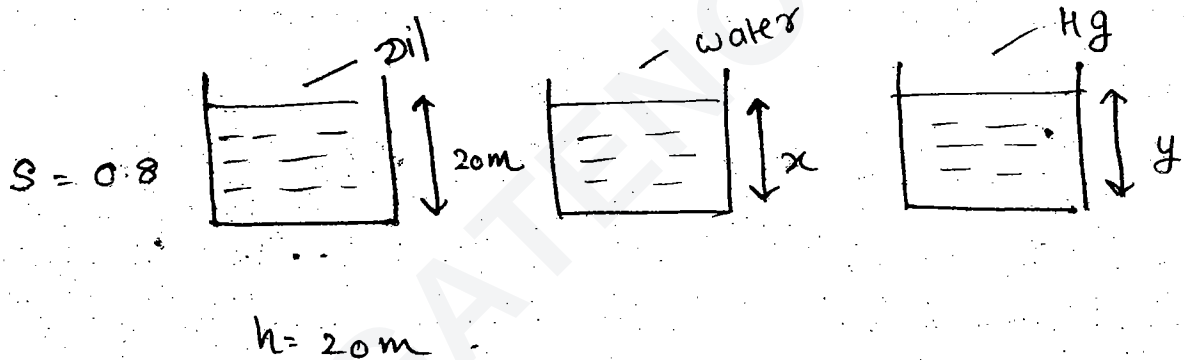
$$\frac{P}{h} = \gamma$$

$$\frac{dp}{dh} = -\gamma$$

$$h \uparrow \quad P \uparrow$$

$$\frac{dp}{dh} = \gamma$$

Q) Figure shows an open tank of diff fluids shown 3 tanks at same height



$$P = \rho_{oil} \cdot g \cdot h_{oil} = \rho_w \cdot g \cdot h_w = \rho_{Hg} \cdot h_{Hg} \cdot g$$

$$P = \cancel{0.8 \times 10^3 \times 9.81 \times 20}$$

$$\cancel{8000 \times 20}$$

$$\rho_{oil} \cdot g \cdot h_{oil} = \rho_w \cdot g \cdot h_w$$

$$\cancel{10^3 \times 156.960}$$

$$\cancel{10^3} \times 0.8 \times 20 = \cancel{10^3} \times x$$

$$x = 20 \times 0.8$$

$$= 16m$$

$$\rho_{oil} \cdot g \cdot h_{oil} = \rho_{Hg} \cdot h_{Hg} \cdot g$$

$$0.8 \times \cancel{10^3} \times 20 = 13.6 \times \cancel{10^3} \cdot y$$

Q) Match list 1 and list 2

List 1

[Pressure due to a column of 0.4m]

- a) water
- b) Mercury
- c) Air
- d) oil ($S=0.9$)

List 2

[Pressure Intensity (units)]

0.353 N/cm²

5.337 N/cm²

0.392 N/cm²

4.709 N/m²

$$P_w = \rho_w \cdot h_w \cdot g$$

$$= 103 \cdot 0.4 \times 9.81$$

$$= \underline{0.392}$$

$$P_{Hg} = \rho_{Hg} \cdot h_{Hg} \cdot g$$

$$= 13600 \times 0.4 \times 9.81$$

$$= \underline{5.337}$$

$$P_{air} = \rho_{air} \cdot h_{air} \cdot g$$

$$= 1.2 \times 0.4 \times 9.81$$

$$= \underline{4.709 \text{ N/m}^2}$$

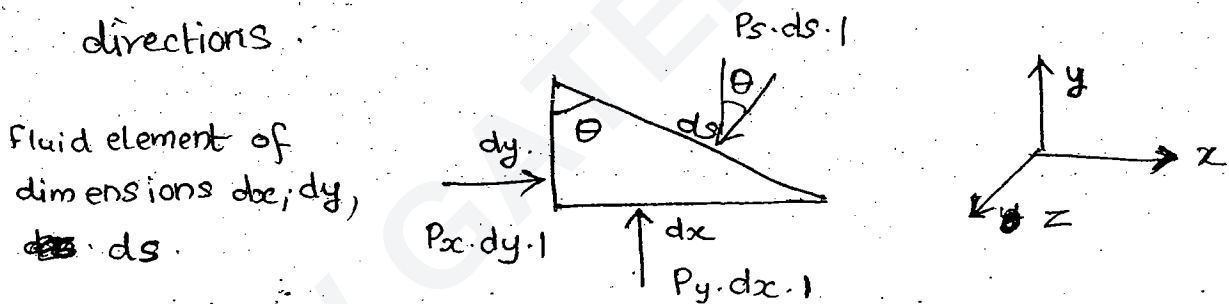
$$P_{oil} = \rho_{oil} \cdot h_{oil} \cdot g$$

$$= \underline{0.35 \text{ N/cm}^2}$$

1. Barometer
2. Piezometer
3. Simple Manometer
4. U tube Manometer
5. Differential Manometer
6. Inverted Differential Manometer
7. Bourdon gauge

PASCALS LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all the directions.



According to pascals Law

$$P_x = P_y = P_z$$

HYDROSTATIC LAW

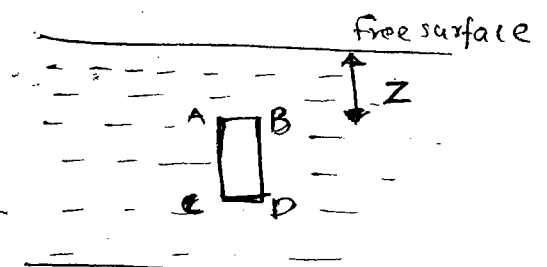
It states that the increase in pressure of a fluid in vertical direction is equal to the weight density of the fluid at that point.

$$\frac{dP}{dz} = \rho g = \gamma$$

$$\text{Integrating } dP = \rho g dz$$

$$P = \rho g z$$

$$P = \rho g h$$

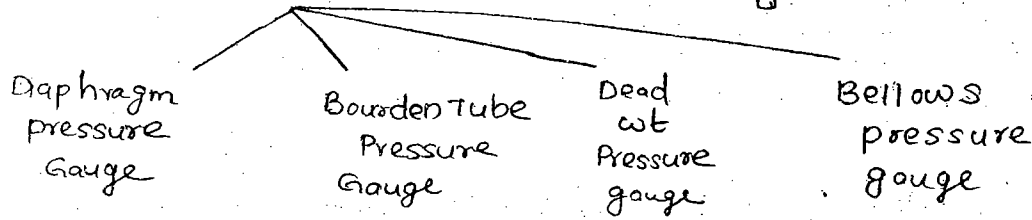


$$h = \frac{P}{\rho g}$$

MEASUREMENT OF PRESSURE:-

81

1. Manometers: → Measure Pressure at a point by balancing column of fluid by same or another column of fluid
2. Mechanical Gauges

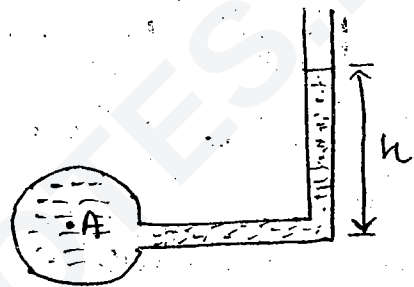


Manometers:-

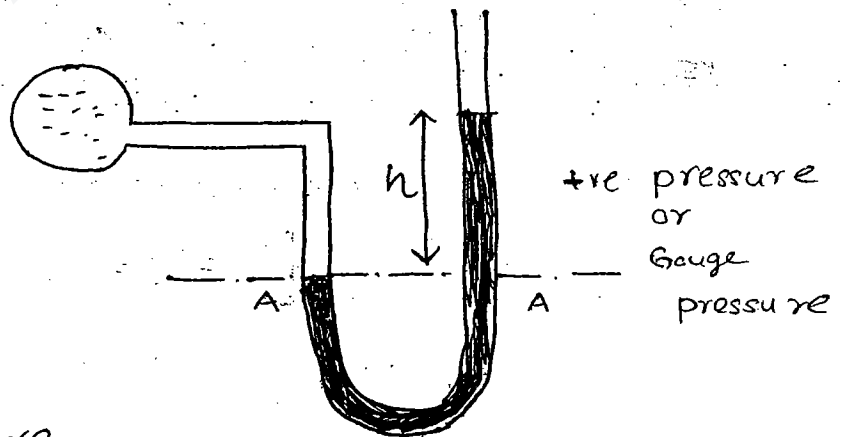
1. Simple Manometer:-

$$P_A = \rho g h \frac{N}{m^2}$$

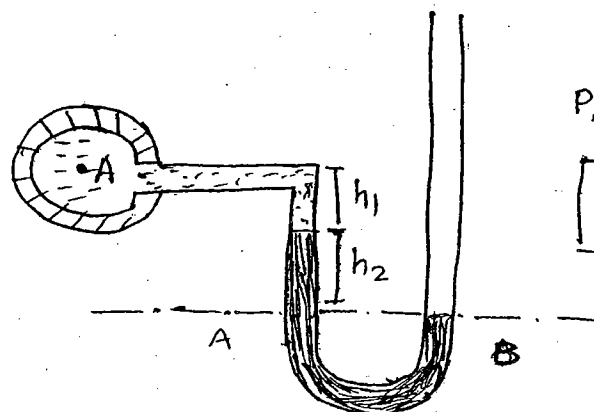
PIEZOMETER →



U tube Manometer showing Negative pressure or Vacuum pressure.



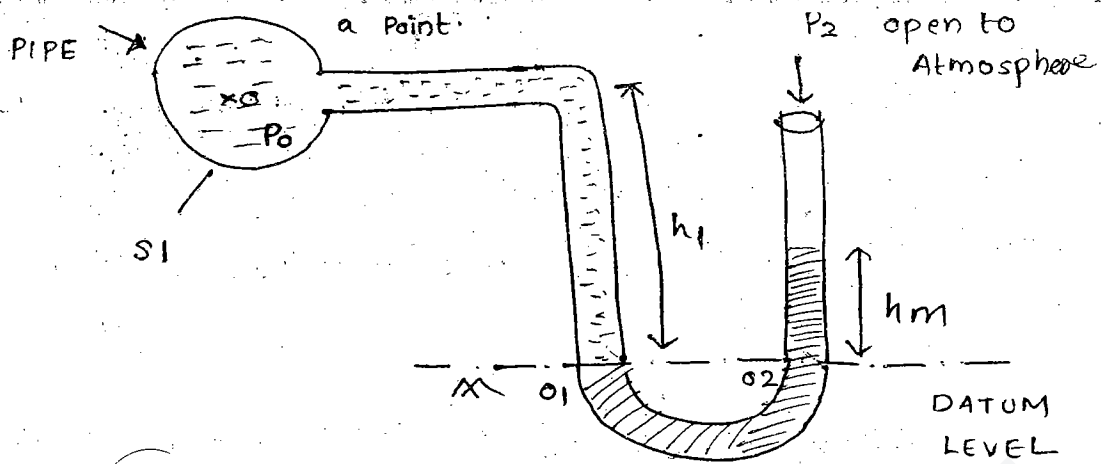
~~So~~ Vacuum pressure



$$P_A + \rho_m g h_2 + \rho g h_1 = 0$$

$$P_A = -(\rho_m g h_2 + \rho g h_1)$$

N/m^2



Mercury - Manometric fluid

According to principle of Manometry
(Pascals Law)

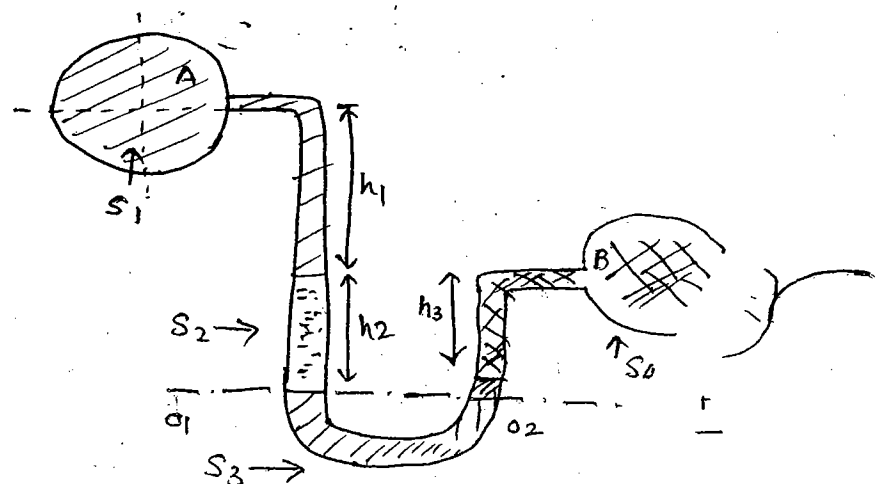
$$P_{o1} = P_{o2}$$

$$(P_o)_{\text{gauge}} + \rho \cdot g \cdot h_1 = \rho_m g h_m$$

$$(P_o)_{\text{gauge}} = \rho_m g \cdot h_m - \rho \cdot g \cdot h_1 \quad \text{N/m}^2$$

4.) Differential Manometer

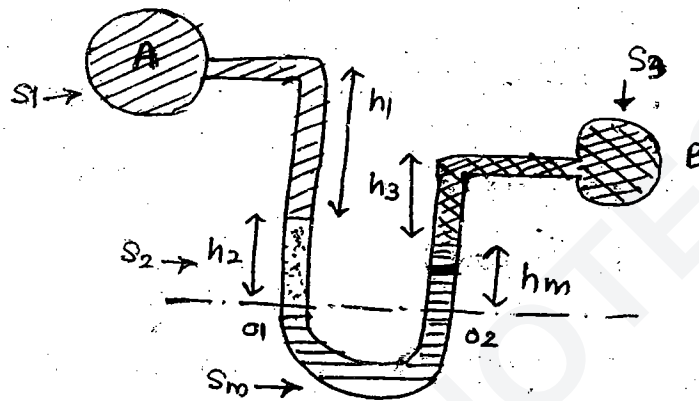
It measures pressure b/w 2 points of the fluids under pressure.



$$P_{01} = P_{02}$$

$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = \rho_m g h_m + \rho_3 g h_3 + P_B$$

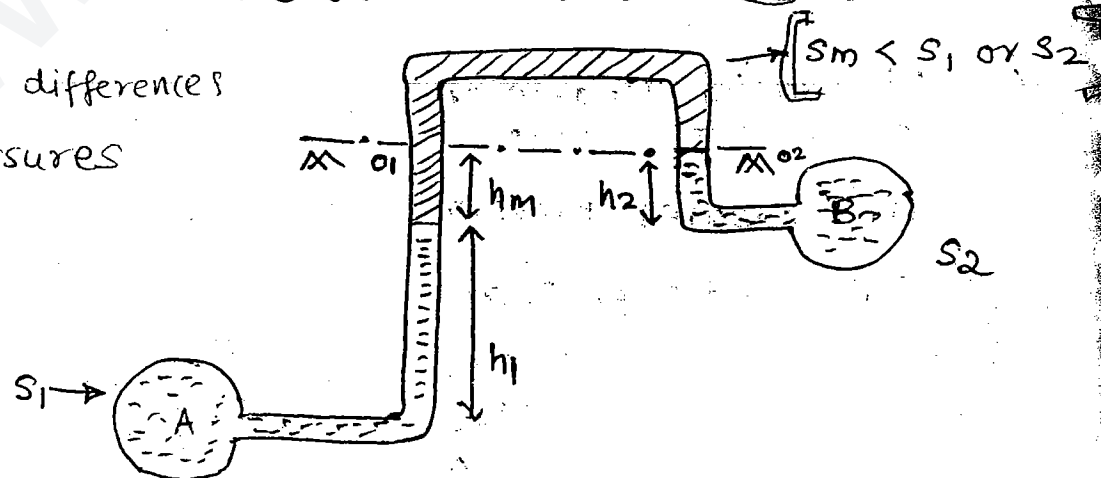
$$P_A - P_B = \rho_m g h_m + \rho_3 g h_3 - \rho_1 g h_1 - \rho_2 g h_2$$



Inverted U-tube Manometer

This meter consists of Manometric fluid of specific Gravity lower than the fluids in pipes where Pressure diff is to be Measured - this meter is used to measure Pressure differences in fractions

measures differences
at low pressures

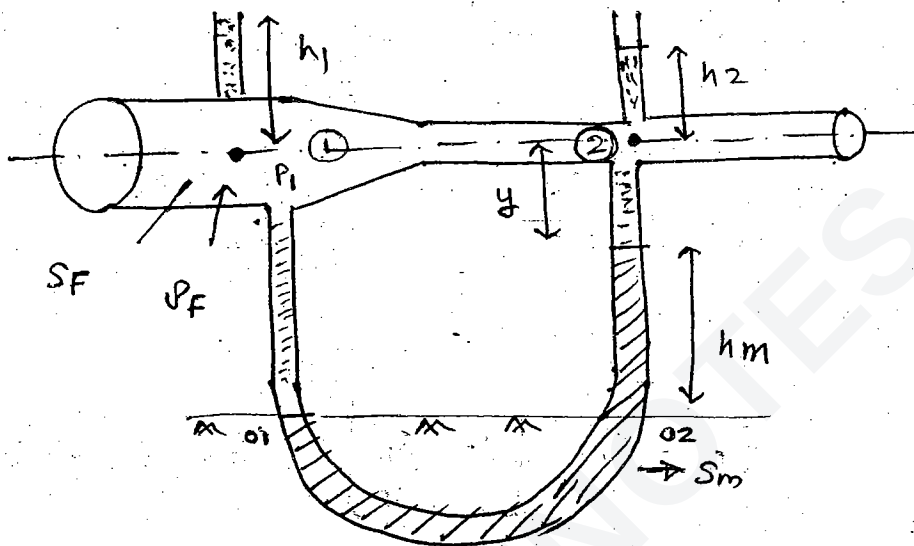


Identification

$$S_m < S_1 \text{ or } S_2$$

$$P_A - P_B = -\rho_2 g h_2 + \rho_1 g h_1 + \rho_m g h_m$$

VENTURIMETER: To measure flow Rate



Derive an expression of pressure difference $\bullet$ $(P_1 - P_2)$ or Pressure difference head $(h_1 - h_2 = \Delta h)$ in terms of (S_F, S_m, h_m)

$$\Delta h = h_1 - h_2 = h_F = \frac{P_1 - P_2}{\rho_F \cdot g} = h_m \left[\frac{S_m - 1}{S_F} \right]$$

According to Principle of Manometer

$$P_{01} = P_{02}$$

$$P_1 + \rho_1 g [y + h_m] = P_2 + \rho_m g h_m + \rho_1 g y$$

$$P_1 - P_2 = \rho_m g h_m + \cancel{\rho_1 g y} - \cancel{\rho_1 g y} - \rho_1 g h_m$$

$$P_1 - P_2 = \rho_m g \cdot h_m - \rho_1 g h_m$$

$$P_1 - P_2 = [\rho_m - \rho_1] g h_m$$

$$\frac{P_1 - P_2}{\rho_1 g} = h_m \left[\frac{\rho_m}{\rho_1} - 1 \right]$$

$$h_1 - h_2 = h_m \left[\frac{\frac{\rho_m}{\rho_{\text{water}}}}{\rho_1 / \rho_{\text{water}}} - 1 \right]$$

$$h_1 - h_2 = h_m \left[\frac{S_m}{S_f} - 1 \right]$$

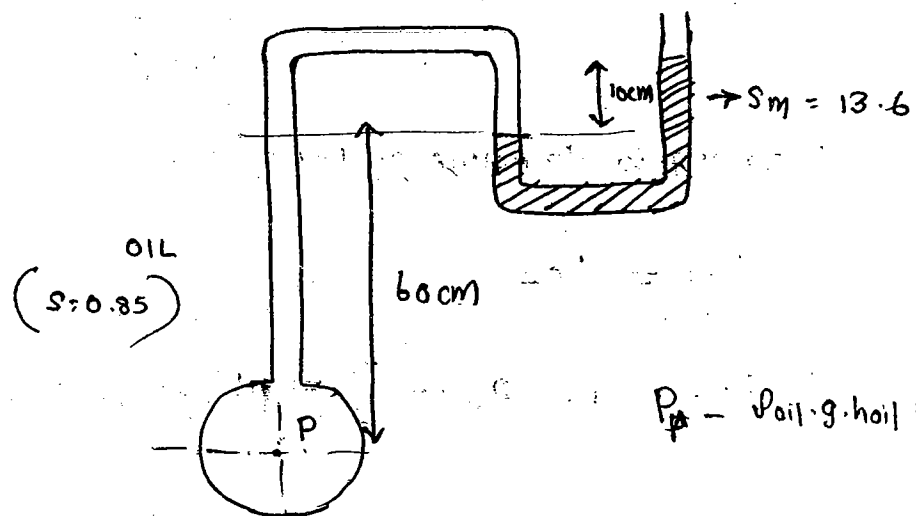
$$\Delta h_F = h_m \left[\frac{S_m}{S_f} - 1 \right]$$

$$S_m > S_f$$

Head in terms
of m of water

$$(\Delta h)_F = h_m \left[1 - \frac{S_m}{S_f} \right] \text{ if } S_m < S_f$$

- a.) Fig shows pressure measurement Arrangement. Determine Pressure at the Point P (gauge Pressure).



$$P_P - \rho_{\text{oil}} \cdot g \cdot h_{\text{oil}} = \rho_m g h$$

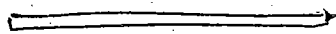
$$P_P = \rho_m \cdot g \cdot h_m + \rho_{\text{oil}} \cdot g \cdot h_{\text{oil}}$$

$$= 13600 \times 9.81 \times 0.1 + 850 \times 9.81 \times 0.6$$

$$P_p = \rho_{oil} \cdot g \cdot H_{oil}$$

$$18344.7 = 850 \times 9.81 \times H_{oil}$$

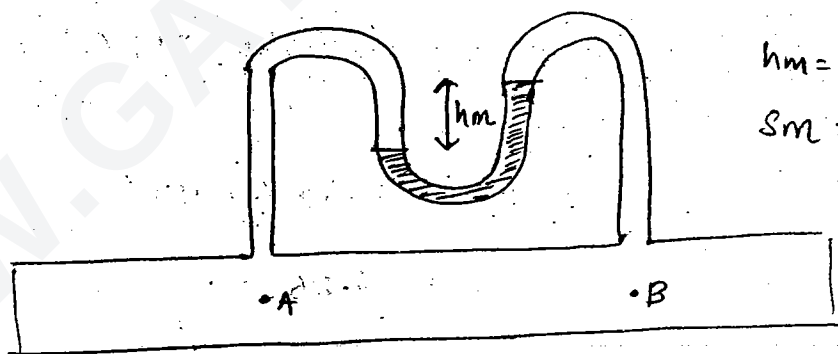
$$H_{oil} = 2.2 \text{ m}$$



A pipe carrying water, a diff manometer connected to pipe at 2 points A and B and the deflection of the mercury level is 60 cm and the unit weight of mercury is 13.6 g/ml.

$$\left[\rho_{Hg} = 13.6 \text{ g/ml} = 13.6 \text{ g/cm}^3 = 13600 \text{ kg/m}^3 \right]$$

$$\left[\text{g/ml} \rightarrow \frac{\text{g}}{\text{cm}^3} \right]$$



$$h_m = 60 \text{ cm}$$

$$S_m = 13.6 \text{ g/ml}$$

find $P_A - P_B$ in terms of m of water

- (a) 7.56 m of H_2O
- (b) 75.76
- (c) 8.16
- (d) 16.32

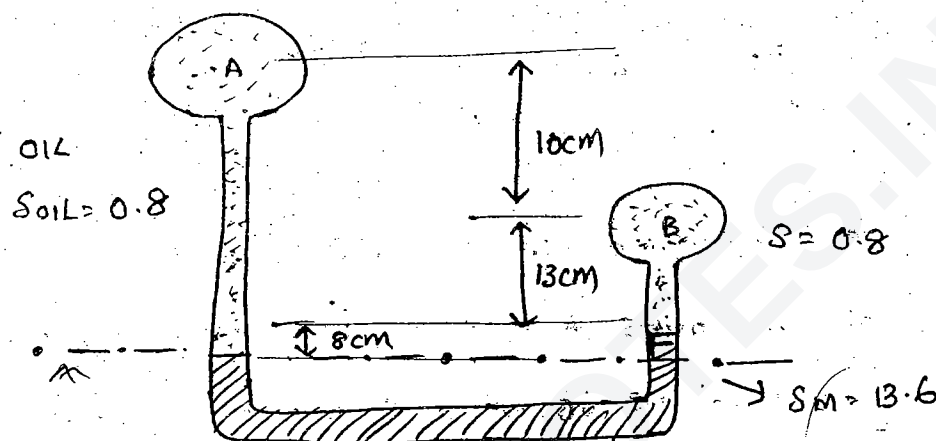
$$h_A - h_B = (\Delta h)_{AB} = h_m \left[\frac{S_m - 1}{S_{\text{water}}} \right]$$

$$= 0.6 \left[\frac{13.6 - 1}{1} \right]$$

$$= 0.6 \times 12.6$$

$$= 7.56 \text{ metres of water}$$

a) what is the Pressure diff b/w Points A and B shown in the figure



$$P_A + \rho_{oil} \cdot g \cdot h_{oil} = P_B + \rho_m \cdot g \cdot h_m + \rho_{oil} \cdot g \cdot h_{oil}$$

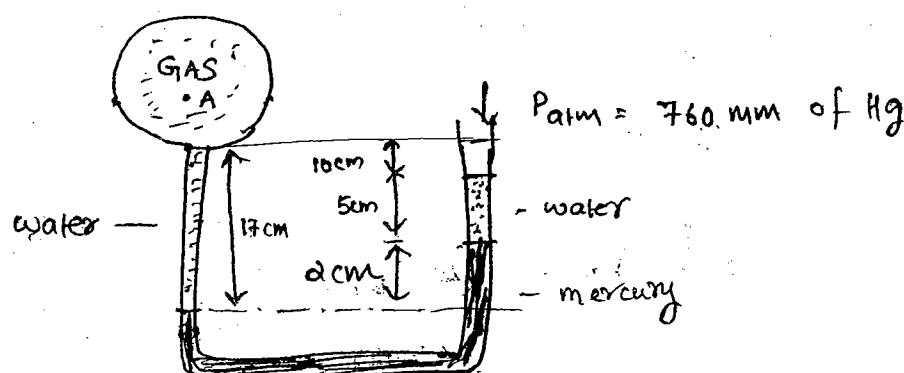
$$P_A - P_B = (13.6 \times 1000) \times 9.81 \times 0.08 + 800 \times 9.81 \times 0.13$$

$$800 \times 9.81 \times 0.08$$

$$9260.64 \text{ N/m}^2$$

$$= 9.26 \text{ kPa}$$

b) Refer to the figure the Absolute pressure of the Gas in the pipe at A is



$$P_A + \rho_w \cdot g \cdot h_w = \rho_m \cdot g \cdot h_m + \rho_w \cdot g \cdot h_w + \cancel{\rho_g \cdot g \cdot h_g}$$

$$(P_A)_{\text{gauge}} = 13600 \times 9.81 \times 0.02 + 1000 \times 9.81 \times 0.05 + \cancel{12000 \times 9.81 \times 0.17}$$

$$- 1000 \times 9.81 \times 0.17$$

$$(P_A)_{\text{gauge}} = \frac{3157.99}{105} \text{ N/m}^2 = 0.031 \text{ mm of Hg}$$

$$(P_{\text{abs}}) = (P_A)_{\text{gauge}} + P_{\text{atm}}$$

$$3157.99 \text{ N/m}^2$$

$$P_{\text{abs}} = 760 + 0.031$$

$$= 760.031$$

$$1 \text{ bar} = 760 \text{ mm of Hg}$$

$$105 \text{ Pa} = 760$$

$$1 \text{ Pa} = \frac{760}{105} \text{ mm}$$

$$(P_A)_{\text{gauge}} = \underline{1491.12 \text{ N/m}^2}$$

$$(P_A)_{\text{gauge}} = \rho_{\text{Hg}} \cdot g \cdot H_{\text{Hg}}$$

$$1491.12 = 13600 \times 9.81 \times H_{\text{Hg}}$$

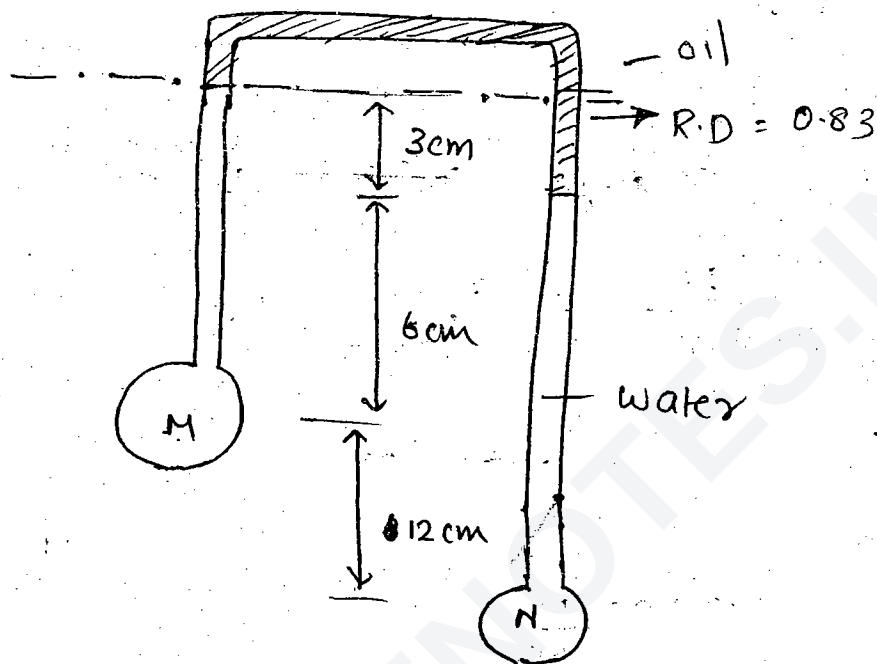
$$H_{\text{Hg}} = \frac{1491.12}{13600 \times 9.81} = \underline{0.0117 \text{ mm of Hg}}$$

$$= \underline{11.7 \text{ mm of Hg}}$$

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$= 760 \text{ mm} + 11.7 \text{ mm} = \underline{771.7 \text{ mm}}$$

- c) Pressure diff b/w M and N for the manometer shown in fig in terms of cm of Hg oil



$$P_M - \rho_w \cdot g \cdot h_{w1} = P_N - \rho_{\text{fluid}} \cdot h_{f1} \cdot g - \rho_w \cdot h_{w2} \cdot g$$

$$P_M - P_N = \rho_w \cdot g \cdot h_{w1} - \rho_{\text{fl}} \cdot h_{f1} \cdot g - \rho_w \cdot h_{w2} \cdot g$$

$$1000 \times 9.81 \times 0.09 - 830 \times 9.81 \times 0.03 - 1000 \times 0.18 \times 9.81$$

$$P_M - P_N = -1127.169 \text{ N/m}^2$$

$$(P_M - P_N) = \rho_{\text{oil}} \cdot h_{\text{oil}} \cdot g$$

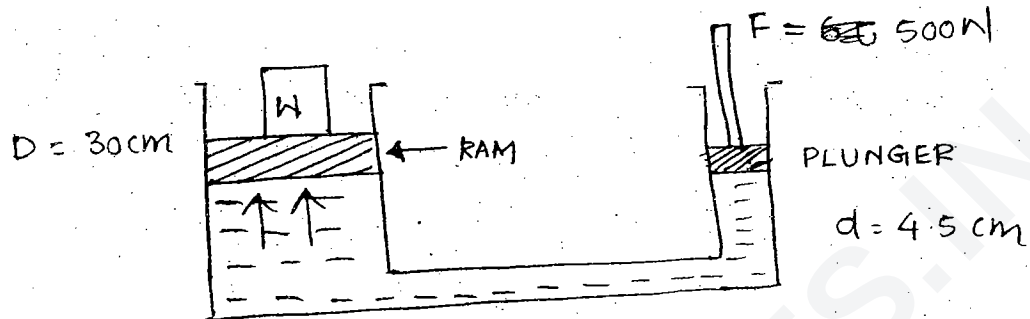
$$1127.169 = \frac{13600}{830} \cdot h_{\text{oil}} \cdot 9.81$$

$$h_{\text{oil}} = 8.44 \times 10^{-3} \text{ m of Hg}$$

$$= 0.84$$

PROBLEMS BASED ON HYDROSTATIC LAW

Q.1 A hydraulic press has a ram of 30cm dia



$$P = \frac{F}{A} = \frac{500}{\frac{\pi \times (0.045)^2}{4}} = 314465.4 \frac{\text{N}}{\text{m}^2}$$

$$P_{\text{RAM}} = P_{\text{PLUNGER}}$$

$$\frac{W}{\frac{\pi \times (0.3)^2}{4}} = 314465.4$$

$$W = 22.222 \text{ kN}$$

V =

Q.2 D = 20cm = 0.2m

d = 3cm = 0.03m

W = 30 kN

$F_p = ?$

$$P_{\text{RAM}} = P_{\text{PLUNGER}}$$

$$F_{\text{RAM}} = F_{\text{PLUNGER}}$$

$$\frac{\pi \times D^2}{4} \times W = \frac{\pi \times d^2}{4} \times F_p$$

$$0.2^2 \times 30 \times 10^3 = (0.03)^2 \times F_p$$

$$\frac{W}{\frac{\pi \times D^2}{4}} = \frac{F_p}{\frac{\pi \times d^2}{4}}$$

$$F_p =$$

$$F_p = \frac{30 \times 10^3 \times (0.03)^2}{0.2^2} = 675 \text{ N}$$

(23)

$P = 2$

0.3 m of water

86

$$P = \rho_w \times g \times h_w$$

$$= 1000 \times 9.81 \times 0.3$$

$$= \underline{2943 \text{ N/m}^2}$$

due to oil

$S = 0.8$

$$P = 0.8 \times 10^3 \times 9.81 \times 0.3$$

$$= \underline{2354.4}$$

Mercury

$S = 13.6$

$$P = 13600 \times 9.81 \times 0.3$$

$$= \underline{40024.8 \text{ N/m}^2}$$

(24)

$P = 3.924 \text{ N/cm}^2$

$$\frac{3.924 \text{ N}}{10^{-4} \text{ m}^2}$$

$h = 2$

(a) water

(b) oil of $S = 0.9$

$3.924 = \rho_w \times g \times h_w$

$3.924 \times 10^4 = 1000 \times 9.81 \times h_w$

$\underline{h_w = 4 \text{ m}}$

(b)

oil

$3.924 \times 10^4 = 900 \times 9.81 \times h_{oil}$

$\underline{h_{oil} = 4.44 \text{ m of oil}}$

(25)

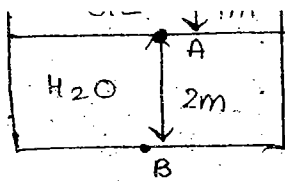
$S = 0.9$

$h = 40 \text{ m}$
oil

$h_w = 2$

$$P_{oil} = 0.9 \times 1000 \times 9.81 \times 40$$

$$= \underline{353160 \text{ N/m}^2}$$



$$P_A = \rho_{oil} \times g \times h_{oil}$$

$$P_A = 0.9 \times 1000 \times 9.81 \times 1m$$

$$= \underline{8829 N/m^2}$$

$$P_B = \rho_{oil} \times g \times h_{oil} + \rho_w \times g \times h_w$$

$$= 8829 + 1000 \times 9.81 \times 2$$

$$= \underline{28449 N/m^2}$$

2.7.

$$d = 3cm$$

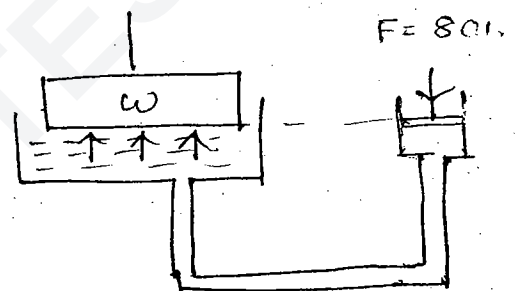
$$D = 10cm$$

Pistons at same level

$$P_{small} = P_{large}$$

$$\frac{80}{\frac{\pi}{4} \times (0.03)^2} = \frac{W}{\frac{\pi}{4} \times (0.1)^2}$$

$$W = \underline{888.89 N}$$



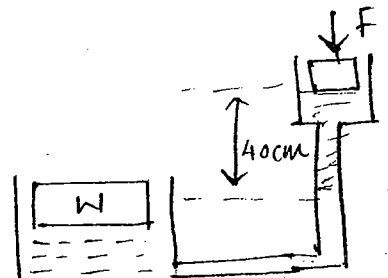
Case 2:

Small piston 40 cm above large piston

$$P_{large} = P_{small} + P_{due \text{ to } 40cm}$$

$$\frac{W}{\frac{\pi}{4} \times (0.1)^2} = \frac{80}{\frac{\pi}{4} \times (0.03)^2} + \rho_w \cdot h_w \cdot g$$

$$= \frac{80}{\frac{\pi}{4} \times 0.1^2} + 1000 \times 0.4 \times 9.81$$



Text

P 43

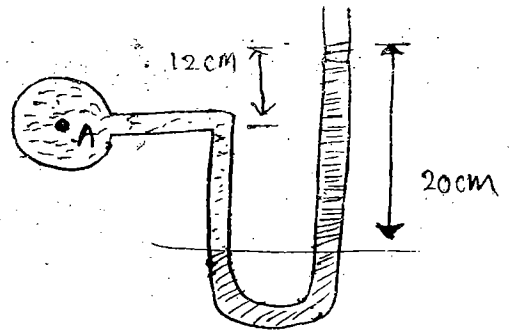
2.9

$$P_A = \rho_m g h_m - \rho_{oil} g h_{oil} \quad [S = 0.9]$$

$$13600 \times 9.81 \times 0.2 -$$

$$900 \times 9.81 \times 0.18$$

$$\underline{25093.98 \text{ N/m}^2}$$

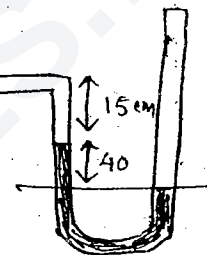


2.10

Case of vacuum pressure PIPE

S = 0.8

Vacuum pressure



cm's

$$P_A + 13600 \times 9.81 \times 0.4 +$$

$$800 \times 9.81 \times 0.15 = 0$$

$$P_A = - [54543.6] \text{ N/m}^2$$

$$\underline{- 5.45 \text{ N/cm}^2}$$

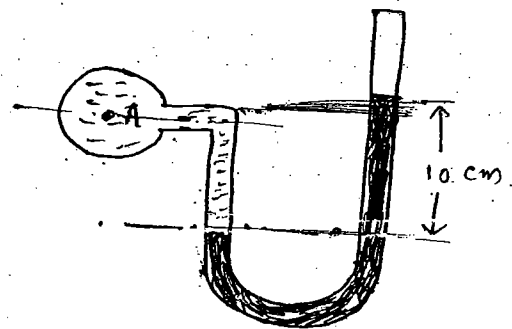
2.11

Case 1:

$$P_A + 1000 \times 9.81 \times 0.10$$

$$= 13600 \times 9.81 \times 0.10$$

$$\underline{P_A = 12360.6 \text{ N/m}^2}$$



Case 2:

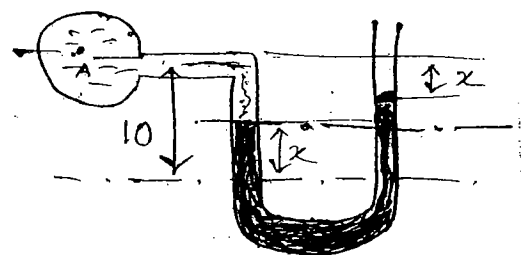
$$P_A = 9810 \text{ N/m}^2$$

$$9810 + 1000 \times 9.81 \times \left(\frac{10-x}{100} \right)$$

$$+ 13600 \times 9.81 \times \left(\frac{10-x}{100} \right)$$

$$= 13600 \times 9.81 \times \left(\frac{10-2x}{100} \right)$$

$$9810 + 9810 (10-x) = 133416 [10-2x]$$



100

$$-257022x =$$

$$981000 + 98100 =$$

$$1334160$$

$$-257022x = -255060$$

$$x = 0.99 \approx \underline{1m}$$

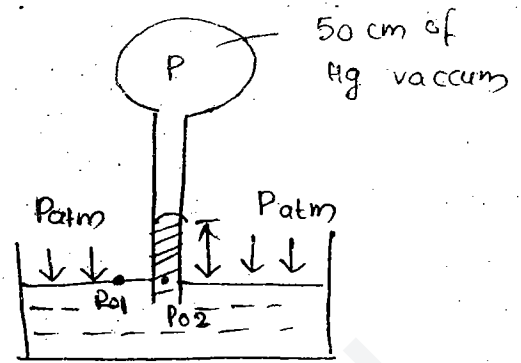
$$\text{Diff in mercury level} = 10 - 2x$$

$$= 10 - 2 \times 0.99$$

$$= \underline{8.02m}$$

8) Figure the pressure reading Find H

- a) 50 cm
- b) 76 cm
- c) 126 cm
- d) None of these



$$(P_{01})_{\text{gauge}} = (P_{02})_{\text{gauge}}$$

$$0 = \rho_{\text{Hg}} \cdot g \cdot H + P$$

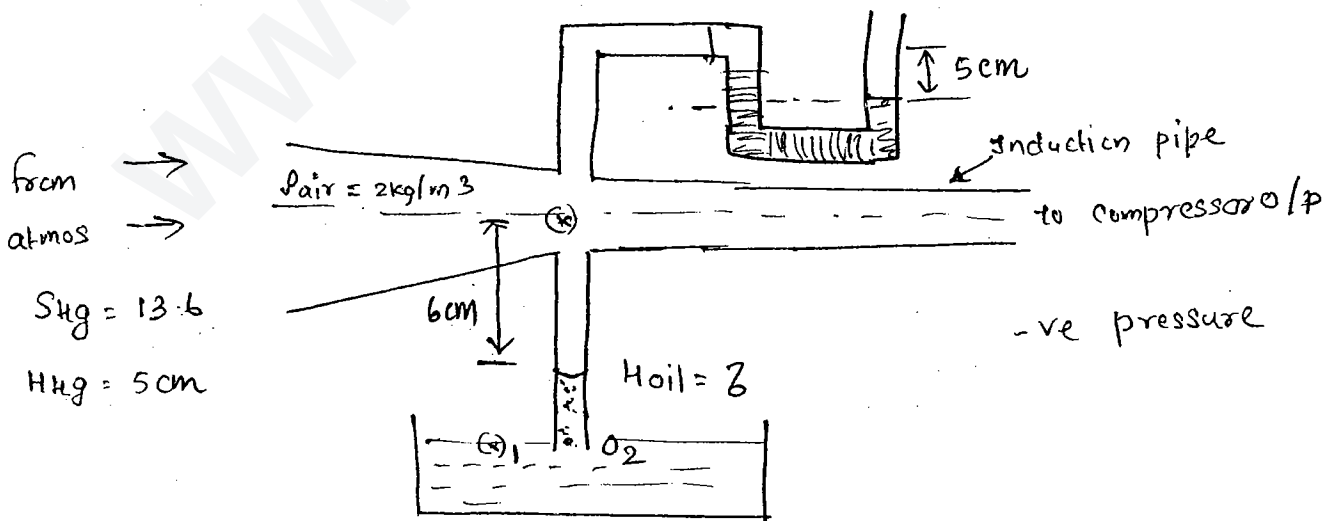
$$= \rho_{\text{Hg}} \cdot g \cdot H + \rho_{\text{Hg}} \cdot g \cdot 50 \text{ cm of vacuum}$$

$$0 = \rho_{\text{Hg}} \cdot g [H - 50]$$

$$H - 50 = 0$$

$$\underline{H = 50 \text{ cm}}$$

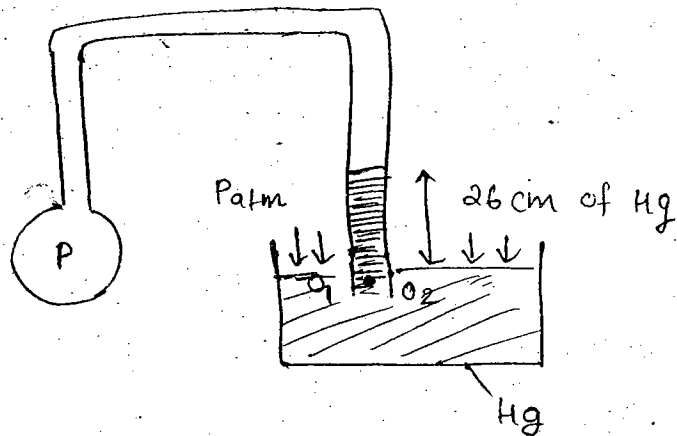
9) An Induction pipe to a Compressor (vacuum cleaner is connected) to a pressure gauge show in fig. Determine the height of oil Rised



$$(P_{01})_{\text{gauge}} = (P_{02})_{\text{gauge}}$$

$$0 = \rho_{\text{oil}} \times g \times h_{\text{oil}} + \rho_{\text{air}} \cdot g \cdot H_{\text{air}} + P$$

Hoil = 0.835 m of oil



- a) 26 cm of Hg (abs)
- b) 50 cm of Hg (abs)
- c) 76 " "
- d) 102 " "

What is Pressure at P ?

$$(P_{O1})_{\text{gauge}} = (P_{O2})_{\text{gauge}}$$

$$0 = \underbrace{\rho_{\text{Hg}} \cdot g \cdot h_{\text{Hg}}}_{26 \text{ cm of Hg}} + \text{Gas column} + P_p$$

neglected

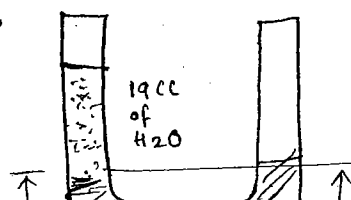
$$\underline{P_p = -26 \text{ cm of Hg}}$$

$$(P_p)_{\text{abs}} = -26 + 76 \text{ cm of Hg}$$

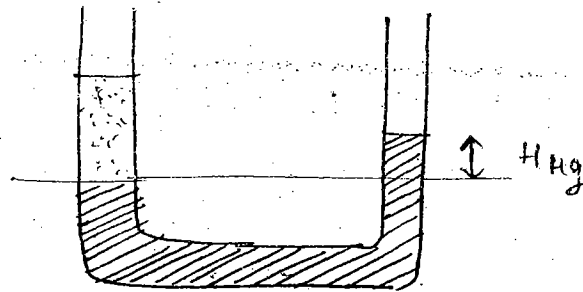
$$= \underline{50 \text{ cm of Hg}}$$

- a) A U bend tube Both ends are open and filled with Hg and observed 10 cm above Horizontal plane. The inner dia of glass tube is 8mm, if 19cc of H₂O is added to one side of U bend then what is the diff in Hg level at eqbm ?

0.8cm



H₂O is added



$$\rho_w \times g \times h_w = \rho_{Hg} \times g \times H_{Hg}$$

$$H_{Hg} = \frac{\rho_w \times h_w}{\rho_{Hg}}$$

$$= 1000 \times$$

$$\underline{h_w = 8}$$

$$V = A \times h$$

$$19 \text{ cm}^3 = \frac{\pi \times d^2 \times h}{4}$$

$$19 = \frac{\pi \times (0.8)^2 \times h}{4}$$

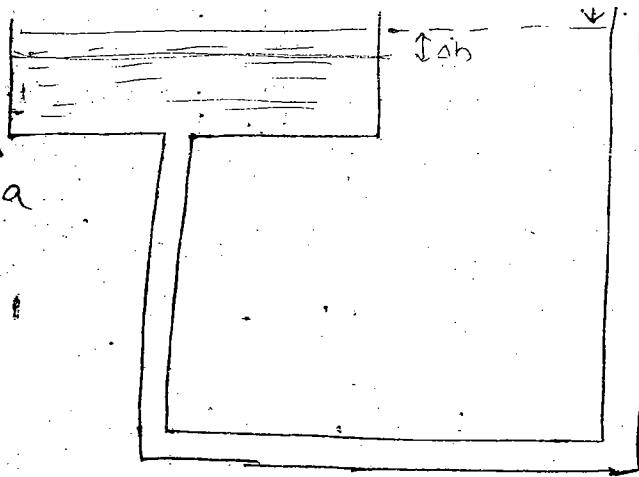
$$\underline{h = 37.79 \text{ cm}}$$

$$H_{Hg} = \frac{1000}{13600} \times 37.79$$

$$\underline{= 2.77 \text{ cm of Hg}}$$

- Q.) The c/s Area of manometer shown in figure -
made 500 times larger area than narrow
tube. Then reading in figure. Det %
error in measurement.

$$A = 500a$$



Law of conservation of mass

mass remains const

$$M = m$$

$$\rho \times V = \rho \times V$$

$$A \times \Delta h = a \times H$$

$$a \times 500 \times \Delta h = a \times H$$

$$500 \times \Delta h = H$$

$$\frac{\Delta h}{H} = \frac{1}{500} = 2 \times 10^{-3}$$

$$= 0.2\%$$

Mass Risen in
tube = mass fallen
in tank
when Area is
increased

Single Column Manometer

- Modified version of u tube Manometer
- large c/s Area (180 times) the c/s of tube
Reservoir

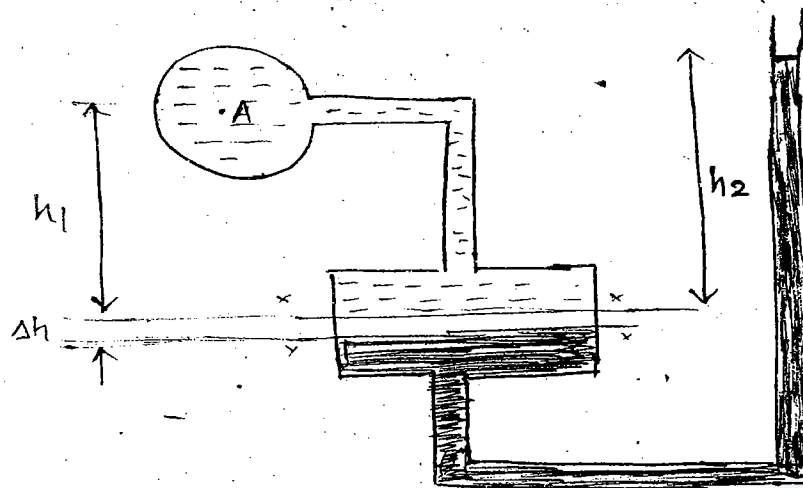
Types

1. vertical Single Column Manometer
2. Inclined single column Manometer

Reservoir
having large
area

VERTICAL SINGLE COLUMN MANOMETER

90



Fall of Heavy liquid in the Reservoir

= Rise of Light liquid in the Tube

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

$$P_A = \Delta h \cdot [\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1$$

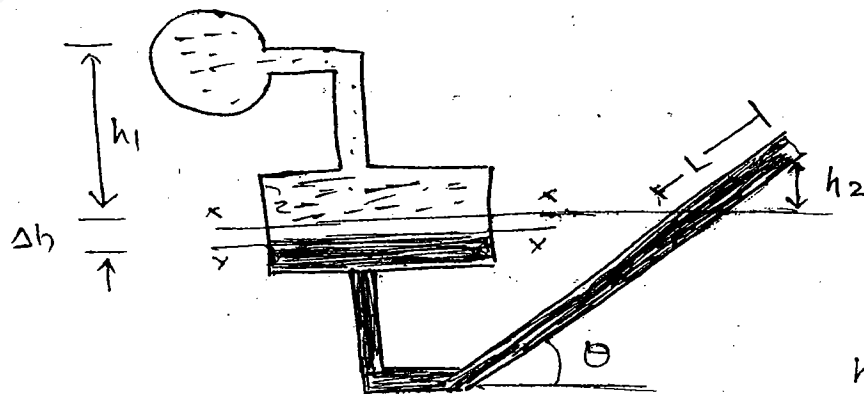
If A is very large compared to a

$\left[\frac{a}{A} \text{ can be neglected} \right]$

then

$$P_A = \rho_2 g h_2 - \rho_1 g h_1$$

Inclined single column Manometer



$$h_2 = L \sin \theta$$

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

$$P_A = L \sin \theta \rho_2 g - h_1 \rho_1 g$$

- Forces involved in fluid motion
- Assumptions in fluid dynamics
- Fluid motion equations
- Derivation of Euler's eqn of motion
- Statement and eqn of energy for a fluid under motion (Bernoulli's eqn)
- Applications of Bernoulli's eqn
- 5 models
- Momentum theorem (Impulse-Momentum eqn) and its application (4 models).

Fluid dynamics is a Branch of fluid science which deals motion of the fluid under influence of external forces. fluid dynamics focus on flow analysis through pipes, design of turbines, pumps and forces involved in fluid motion.

Diff. forces on fluid dynamics

The following are the three forces involved in fluid dynamics.

1. Body force (self wt of fluid matter)
2. Normal forces (due to pressure of the fluid)
3. external forces acting on the fluid matter.

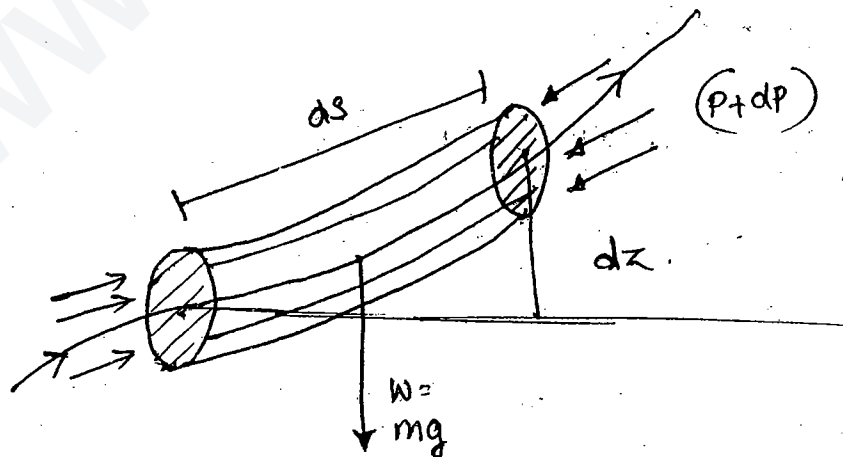
Equation of motion.

1. Euler's eqn Based on law of conservation of mass
2. Energy eqn (Bernoulli's eqn)
3. Momentum eqn

Assumptions:-

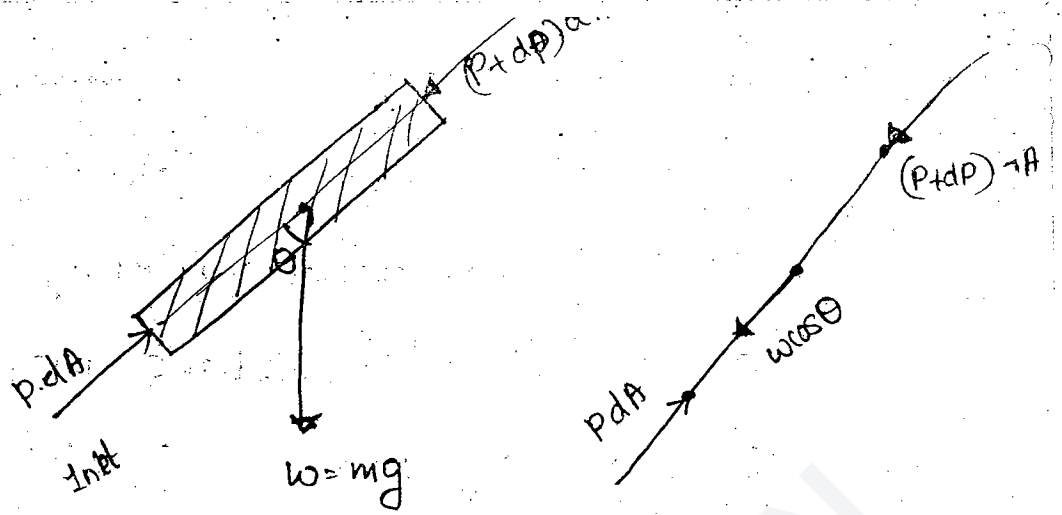
1. fluid flow is steady
2. fluid flow is non uniform
3. fluid is non-viscous
4. flow is through streamlines
5. flow is continuous
6. forces ~~due to~~ surface tension, Bulk modulus (due to compressibility) due to turbulence and influence of magnetism etc are ignored.

Expression for Euler's motion eqn



Consider a fluid element of length ds and normal area dA (one dimensional flow).

In Actual process decrease in pressure occurs.



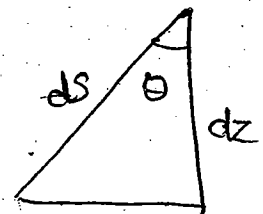
$$\sum \text{Forces} = \text{mass} \times \text{Accn}$$

$$p \cdot dA - w \cos \theta - (p + dp) dA = m \times \frac{dv}{dt}$$

$$p dA - w \cos \theta - p dA - dp dA = m \times \frac{dv}{ds} \times \frac{ds}{dt}$$

$$mg \cos \theta - dp dA = m \cdot \frac{dv}{ds} \cdot v$$

$$\underline{mg} \cdot \frac{dz}{ds} - dp \cdot dA = \rho \cdot dA ds \cdot \frac{dv}{ds} \cdot v$$



$$\rho \cdot dA ds \cdot \frac{dz}{ds} \cdot g - dp dA = \rho dA ds \cdot \frac{dv}{ds} \cdot v$$

$$\cancel{dp} \cdot \cancel{dA} - \rho g dz - dp = \rho \cdot v \cdot dv$$

$$dp + \rho g dz + \rho v dv = 0$$

Divide by ρ

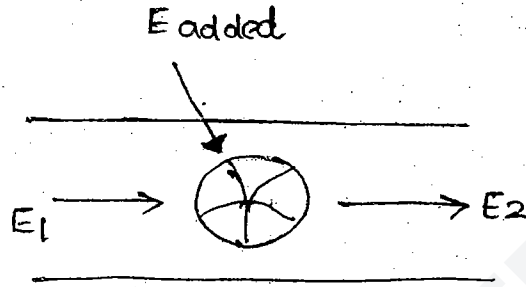
$$\boxed{\frac{dp}{\rho} + g dz + v dv = 0} \quad \text{EULER'S EQN}$$

Apply Bernoulli's equation

$$\frac{P_1}{\rho g} + Z_1 + \frac{v_1^2}{2g} - h_L + E_{\text{added}} = \frac{P_2}{\rho g} + Z_2 + \frac{v_2^2}{2g}$$

Head Rise

• Turbine •



$$E_1 + E_{\text{added}} = E_2$$

(J) ← E

E_{added}
Kwhr

$$Q = 50 \text{ l/s}$$

$$P = 7.5 \text{ kW}$$

$$H = 2$$

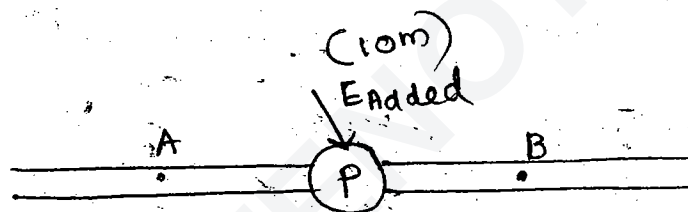
$$1 \text{ m}^3 = 1000 \text{ l}$$

$$\eta = \frac{O/P}{I/P} = \frac{m g H}{P_{\text{shaft}}}$$

$$100 = \frac{\rho \cdot Q \cdot g \cdot H}{P_{\text{shaft}}} = \frac{1000 \times 50 \times 10^{-3} (\text{m}^3/\text{s}) \times 9.81}{7.5 \times 10^3}$$

$$H = 15.32 \text{ m}$$

Q7.



$$h_L = 3.0$$

$$P_A = 2$$

$$P_B = 120 \text{ kPa}$$

$$H = 10 \text{ m}$$

$$V_A = V_B$$

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A - h_L + E_{\text{added}} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{P_A}{\rho g} + (-3) + 10 = \frac{120 \times 10^3}{\rho g}$$

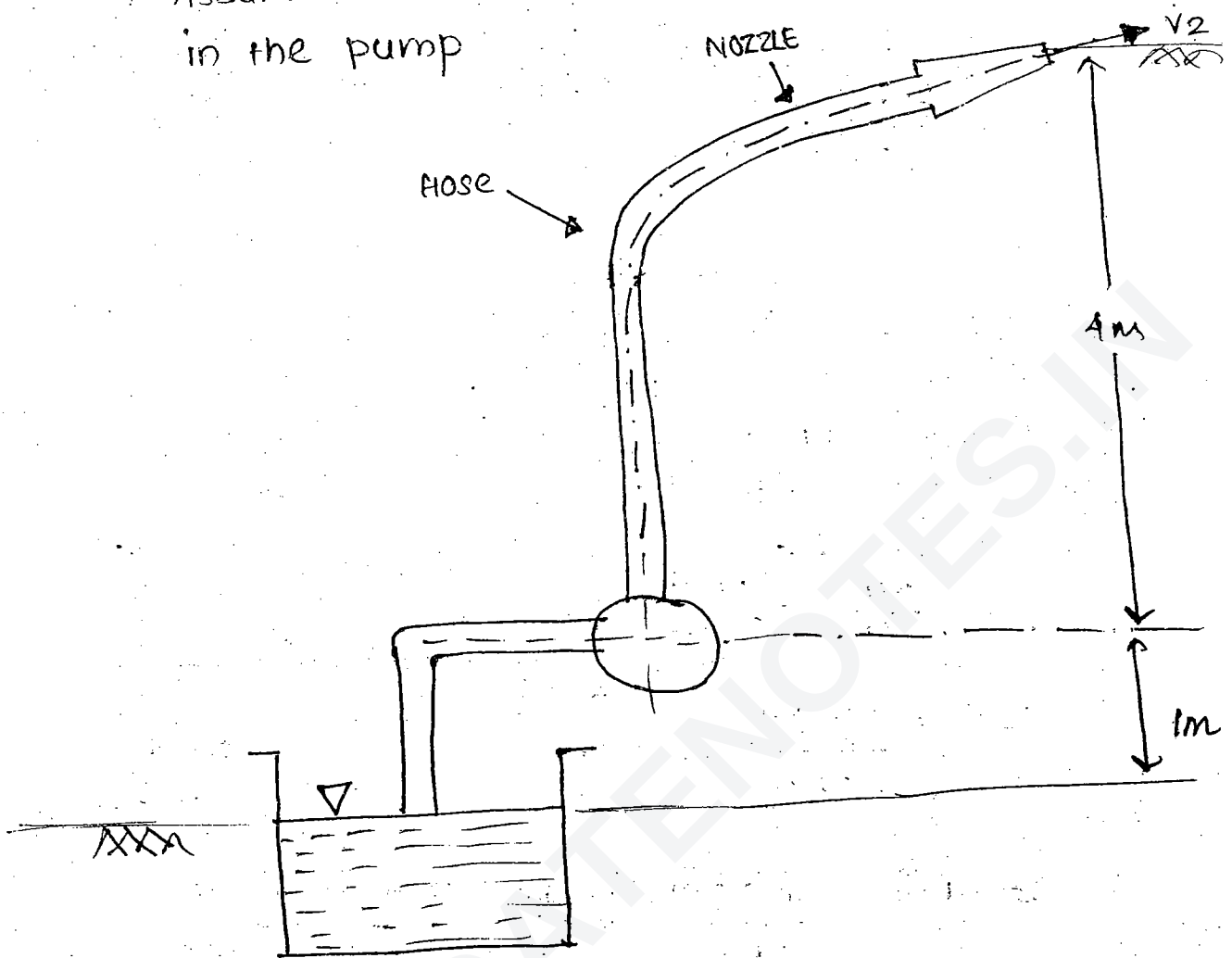
$$P_A = 120$$

$$\frac{P_A}{\rho g} + 7 = \frac{120 \times 10^3}{1000 \times 9.81}$$

$$\frac{P_A}{\rho g} = \frac{120 \times 10^3}{1000 \times 9.81} - 7$$

(b) Power Req'd to Run the pump.

Assume there are no losses in the pipeline and in the pump



$$P_2 = P_{atm}$$

$$P_{gauge} = 0$$

$$V_2 = ?$$

$$Z_2 = 5m$$

$$Z_1 = 0$$

$$P_1 = 0$$

$$\frac{V_1^2}{2g} = 0$$

$$0 + 0 + 0 - 0 + E_{added} = 0 + 5 + \frac{V_2^2}{2g}$$

$$Q = A_2 V_2$$

$$\frac{3000 \text{ m}^3}{\text{min}} = \frac{\pi}{4} \times (50 \times 10^{-3})^2 \cdot V_2$$

$$\frac{(\text{m}^3)}{\text{sec}} \quad \frac{3000 \times 10^{-3}}{60} = \frac{\pi}{4} \times (50 \times 10^{-3})^2 \cdot V_2$$

$$E_{added} = \frac{(25.46)^2}{2 \times 10}$$

H developed by: = 37.42m of water
Pump

⑥ $P = 3$

$$\eta = \frac{O/P}{I/P} = \frac{m g H}{P_{shaft}}$$

$$\text{Power} = m g H$$

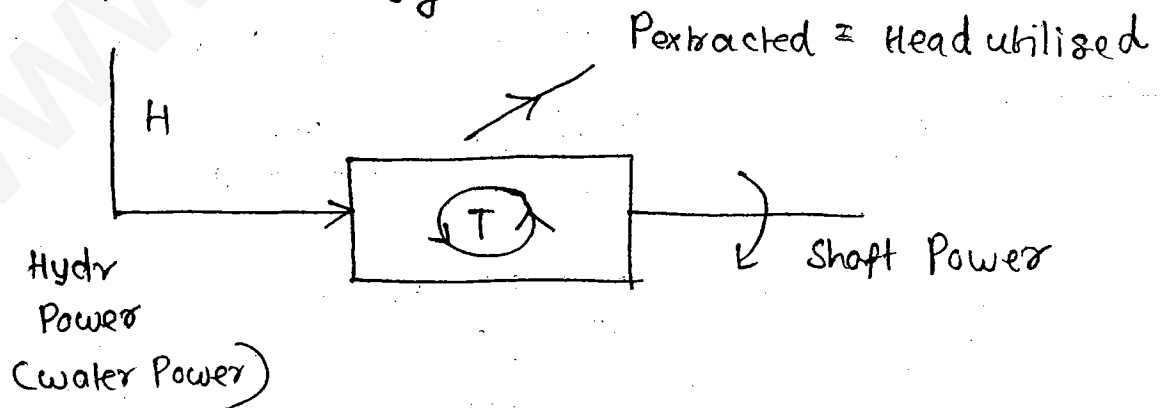
$$= \rho Q g H$$

$$= 1000 \times 10 \times \left(\frac{3600 \times 10^3}{60} \right) \times 37.42$$

$$= \underline{\underline{\quad\quad\quad}} \text{ kW}$$

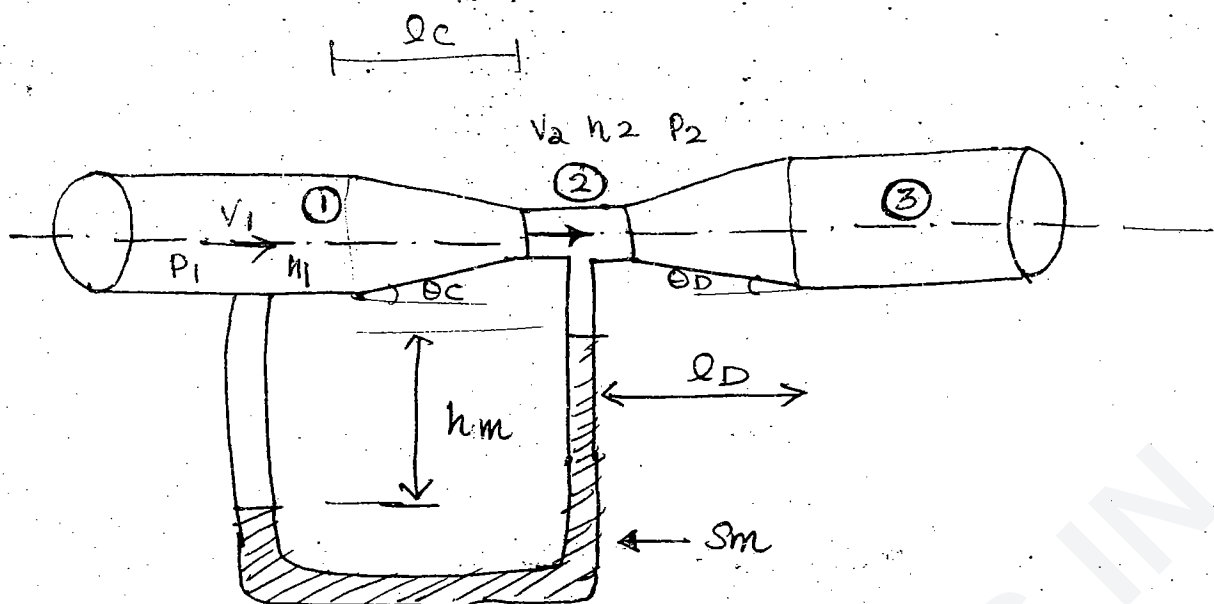
TURBINE.

It is a machine which converts Hydraulic energy into mechanical energy



$$\eta = \frac{O/P}{I/P} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{m g \cdot H}$$

$$\eta = \frac{P_{shaft}}{\rho \cdot g \cdot Q \cdot H}$$



① $l_D \gg l_c \rightarrow$ To avoid flow separation

② $\theta_D < \theta_c$ -

③ $\frac{d_{throat}}{d_{pipe}} = \frac{1}{4}$ to $\frac{1}{2}$

Eqs

1. $Q = A_1 v_1 = A_2 v_2$

2. Apply Bernoulli's eqn

$$\frac{P_1}{\rho g} + \cancel{z_1} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \cancel{z_2} + \frac{v_2^2}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$h_1 - h_2 = \frac{v_2^2 - v_1^2}{2g}$$

$$\Delta h_{12} = h = \left[\frac{Q}{A_2} - \frac{Q}{A_1} \right]^2$$

$$Q = \frac{\sqrt{2gh}}{\sqrt{\frac{1}{A_2^2} - \frac{1}{A_1^2}}}$$

$$Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

$$Q_{act} = C_d \times Q_{th}$$

$$[C_d = 0.96 \text{ to } 0.99]$$

NOTE:-

Differential manometer connected to venturimeter. The deflection of manometer will not change by changing orientation of pipe bcoz Diff manometer always measures pressure b/w 2 points

Venturimeter is Best

disadv

Consume space

(15)

$$A_2 = 3 \text{ cm}^2$$

$$A_1 = 5 \text{ cm}^2$$

$$h = 5 \text{ cm of water}$$

(*)

$$h_{fluid} = h_m \left[\frac{s_m - 1}{s_{fluid}} \right]$$

~~h_{manometer}~~

(16)

Deflection remains same in material at angle of inclination

$$\underline{h = 25 \text{ cm}}$$

(15)

$$A_2 = 3 \text{ cm}^2$$

$$A_1 = 5 \text{ cm}^2$$

$$h = 5 \text{ cm of H}_2\text{O}$$

$$Q_{water} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh_{water}}$$

$$\frac{5 \times 3}{\sqrt{5^2 - 3^2}} \times \sqrt{2 \times 9.81 \times 5 \text{ cm}} \times \frac{\text{cm}}{\text{s}^2}$$

$$= \underline{370 \text{ cm}^3/\text{sec}}$$

Q. Determine the velocity of water at section ① shown in figure

SMANTHA
BISWAS
GOOD PROBLEMS

Ans:-

$$h_w = h_m \left[\frac{S_m}{S_w} - 1 \right]$$

$$h_w = 1.5 \times 10^{-2} \left[\frac{11}{1} - 1 \right]$$

$$= (\text{m})$$

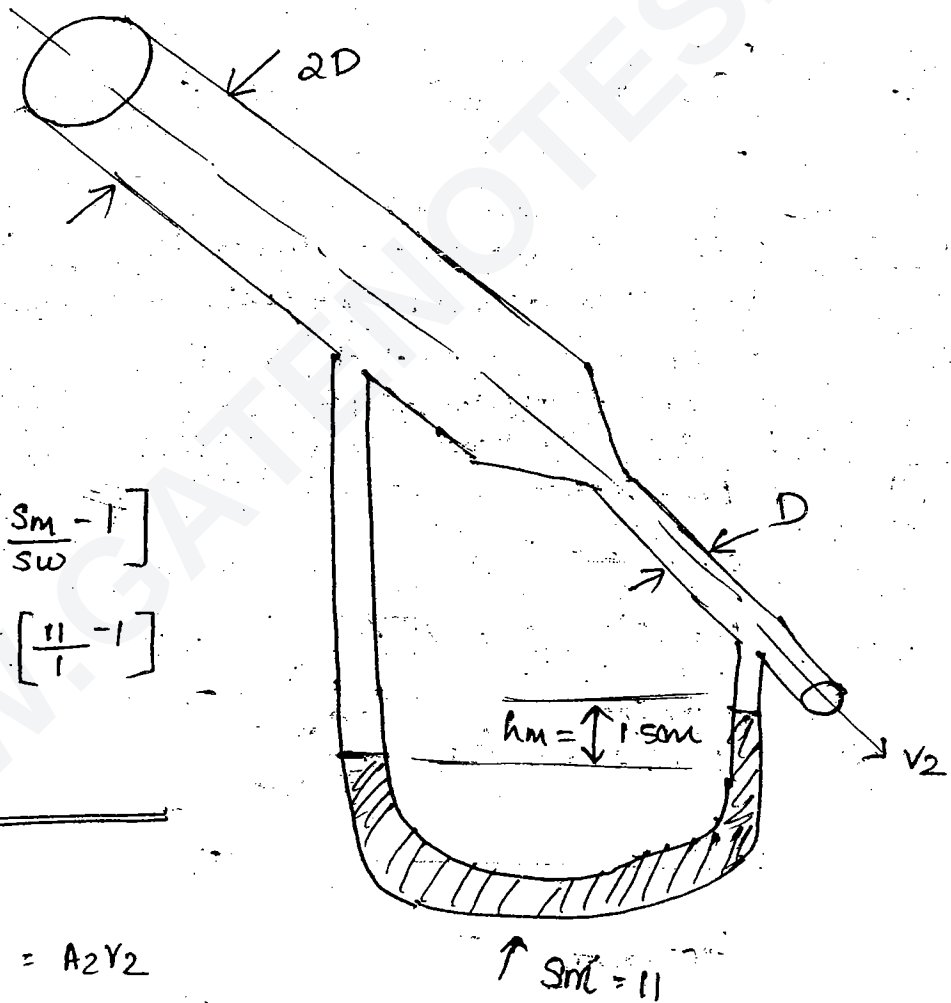
$$Q_{\text{water}} = A_1 V_1 = A_2 V_2$$

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_{\text{water}}}$$

$$A_1 V_1 = \frac{A_1 A_2 \sqrt{2gh_w}}{\sqrt{A_1^2 - A_2^2}}$$

$$V_1 = \frac{\frac{\pi}{4} \times D^2}{\sqrt{\frac{\pi^2}{4} (20)^2 - \frac{\pi^2}{4} D^2}} \times \sqrt{2 \times 10 \times 1.5 \times 10^{-2} \times \left(\frac{11}{1} - 1 \right)}$$

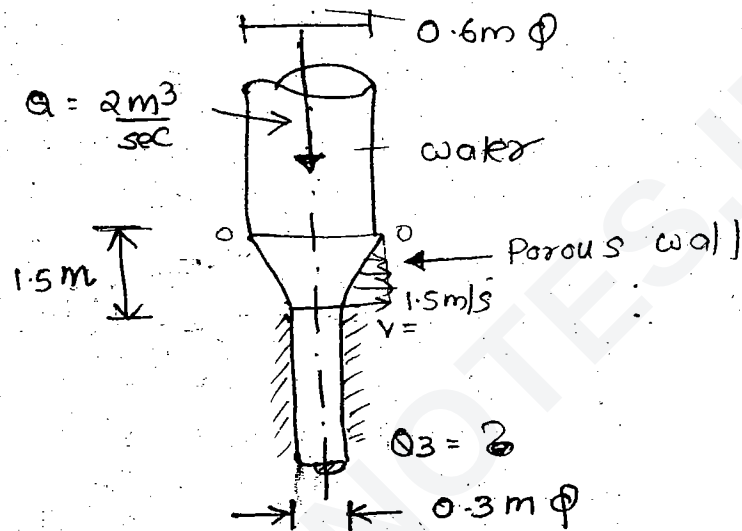
$$\frac{\text{m}}{\text{sec}^2}$$



$$\frac{V}{\sqrt{1604 - D^4}} \times \sqrt{\left(2 \times 10 \times \frac{1.5 \times 10}{100}\right)}$$

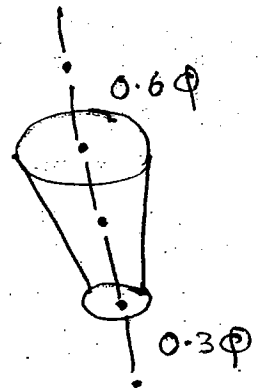
$$= \underline{\underline{\sqrt{0.2} \text{ m/s}}}$$

Q) Find discharge through a pipe at ③



$$\underline{\underline{0.587 \text{ m/s}}}$$

→ velocity of fluid through closed pipes



ENERGY EQUATION

(For fluid in motion)

92

OR

BERNOULLI'S EQUATION

It is Based on law of conservation of energy. i.e. energy is neither created nor destroyed.

For the Above Assumptions total energy of fluid at any point in a continuous matter remains constant. It is obtained by integrating the Euler's eqn

$$\int \frac{dp}{\rho} + \int g dz + \int v \cdot dv = \int 0$$

For Incompressible flow

$$\rho_1 = \rho_2 = \text{const}$$

ENERGY/
UNIT MASS

$$\boxed{\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}}$$

$$\frac{N}{m^2 \cdot kg} \cdot m^3$$

$$\frac{m \times m}{s^2}$$

$$\left(\frac{m}{\text{sec}}\right)^2$$

↙

$$\frac{kg \cdot m}{m^2 s^2} \cdot \frac{m^3}{kg} = \frac{m^2}{s^2}$$

$$\frac{Nm}{kg \cdot s^2}$$

$$\frac{kg \cdot m \cdot m}{kg \cdot s^2}$$

Energy (Nm)
unit (Mass)

$$\frac{kg \cdot m}{s^2} \cdot m$$

$$\frac{m^2}{s^2}$$

~~mass~~
Mass

$$P + \rho g Z + \frac{\rho v^2}{2} = C$$

ENERGY / UNIT VOLUME.

$$\left[\frac{N}{m^2} \right]$$

$$\left[\frac{N}{m^2} \right]$$

Dynamic pressure

$$= \frac{\rho v^2}{2} = \frac{kg}{m^3} \times \frac{m^2}{s^2}$$

$$= \frac{kg \cdot m}{m^3 \cdot s^2} \cdot m = \frac{N \cdot m}{m^3}$$

$$= \frac{\text{Joule}}{\text{Volume}}$$

$$\frac{P}{\rho g}$$

Pressure

Head → datum Head

$$\frac{P}{\rho g} + Z + \frac{v^2}{2g} = C$$

m

velocity Head

$$\frac{N}{m^2} \times \frac{m^3}{kg \times m} \times \frac{s^2}{m}$$

$$\frac{m^2}{s^2} \times \frac{s^2}{m}$$

$$= \underline{\underline{m}}$$

$$\frac{kg \cdot m}{m^2 \cdot s^2} \times \frac{m^3}{kg \cdot m} \times \frac{s^2}{m}$$

$$\boxed{\frac{P}{\rho g} + Z + \frac{v^2}{2g} = C}$$

ENERGY / UNIT WEIGHT.

$$\frac{\text{Energy}}{\text{weight}} = \frac{N \cdot m}{kg \cdot N} = \frac{m}{kg}$$

Applications of Bernoulli's eqn

93

- ① Energy of fluid at a point
- ② Identify the direction of fluid flow
- ③ Bernoulli's eqn applied to Hydraulic Machines
Pumps and Turbines
- ④ Measurement of discharge [venturimeter, orifice meter
nozzle meter, Bend meter, Rotameter]
- ⑤ vel. of the flow flow.
[pitot tube]

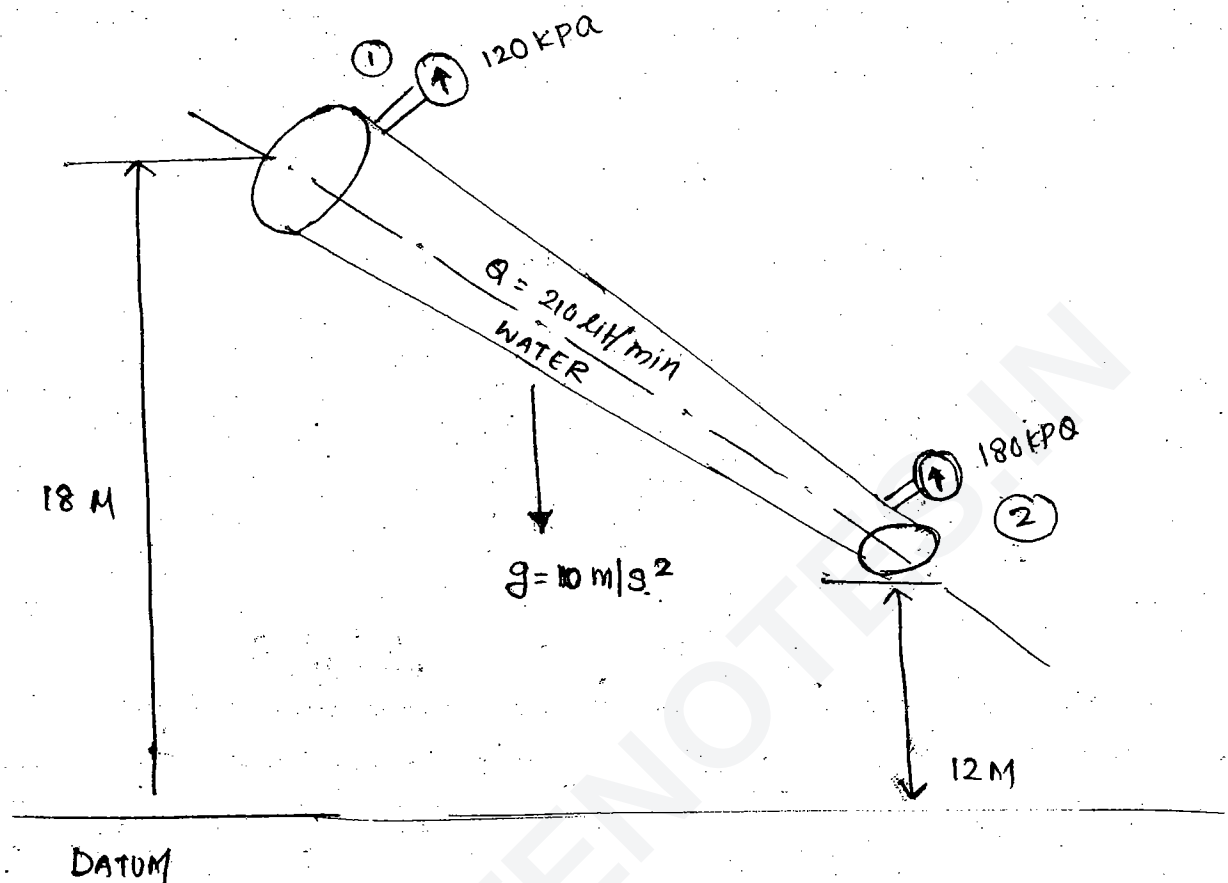
ex 1

MODEL 1
MODEL 2

A tapered pipe of dia at section 1, 300mm reduces to 100mm at section 2. The elevation [datum] of section 1 is 18m and the section 2 is 12m. The gauge pressure at section 1 is 120kPa and at section 2 is 180kPa. The fluid flow is water and Rate of flow is 210 lit/min. Take $g = 10 \text{ m/s}^2$. Assume flow is steady, one dimensional and determine the following

- ① Volume flow Rate of the water in $\frac{\text{m}^3}{\text{s}}$
- ② Mass flow Rate of water in $\frac{\text{kg}}{\text{s}}$ (~~discharge~~)
- ③ vel of water flow at section 1 and at section 2
- ④ total energy Head of the water at section 1 and at section 2.

⑦ Loss of power during the flow in kw and in H.P



[Fluid Flow due to energy diff]

$$Q = \dot{V} = 210 \text{ l P M}$$

$$= \frac{210 \text{ lit}}{60 \text{ sec}}$$

$$= \frac{3.5 \text{ lit}}{\text{sec}}$$

$$= 3.5 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$= \underline{\underline{0.0035 \text{ cumec}}}$$

$$\dot{m} = \rho A V = \rho \times Q$$

$$= 1000 \times 0.0035$$

$$= \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{\text{sec}}$$

$$= \underline{\underline{3.5 \text{ kg/sec}}}$$

$$\left(1 \times 10^{-4}\right) \times \left(10 \times 10^{-2}\right) = \left(1 \times 10^{-6}\right) \left(v_2\right)$$

m² m/s m² m/s

96

$$v_2 = \underline{10 \text{ m/s}}$$

$$v_1 = 0.1 \text{ m/s}$$

$$v_2 = \underline{10 \text{ m/s}}$$

$$v_2 = \underline{10 \text{ m/s}}$$

$$M \cdot A =$$

Apply Bernoulli's eqn

GAUGE
PRESSURE

$$\frac{p_1}{\rho g} + \cancel{z_1} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \cancel{z_2} + \frac{v_2^2}{2g}$$

$$\underline{p_2 = 0}$$

$$\frac{p_1}{\rho g} + \frac{0.1^2}{2g} = 0 + \frac{10^2}{2g}$$

$$\frac{p_1}{\rho g} = \left[\frac{10^2}{2} - \frac{0.1^2}{2} \right] \times \frac{1}{g}$$

$$\frac{p_1}{\rho} = 49.995$$

$$p_1 = 49.995 \times 1000$$

$$= 50,000$$

$$\frac{50 \times 10^3 \text{ N}}{\text{m}^2}$$

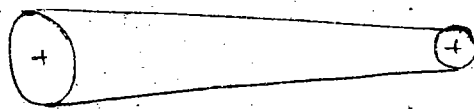
$$\underline{50 \text{ kPa}}$$

$$F = P \times \text{Area}$$

$$= 50 \times 1 \times 10^{-4}$$

$$= 5 \text{ kN}$$

$$\underline{5 \text{ N}}$$



5 cm

$$\frac{P}{\rho g} = 10 \text{ m}$$

$$\frac{v^2}{2g} = 1 \text{ m}$$

$$\text{Total Head} = \frac{P}{\rho g} + \frac{v^2}{2g} + 1$$

$$= \underline{\underline{11 \text{ m}}}$$

$$\underline{\underline{Z_1 = Z_2 = 0}}$$

Ans: 11

② Pressure at section 2 is negative

$$\frac{P_1}{\rho g} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{v_2^2}{2g}$$

$$10 + 1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$Z_1 = Z_2 = 0$$

$$A_1 v_1 = A_2 v_2$$

$$\frac{\pi \times (0.1)^2 \times v_1}{4} = \frac{\pi \times (0.05)^2 \times v_2}{4}$$

$$\frac{\pi \times (0.1)^2 \times v_1}{4} = \frac{\pi \times (0.05)^2 \times v_2}{4}$$

$$\left(\frac{0.1}{0.05} \right)^2 \times v_1 = v_2$$

$$\underline{\underline{4v_1 = v_2}}$$

$$(4v_1)^2 = (v_2)^2$$

$$16v_1^2 = v_2^2$$

EQUATION OF CONTINUITY

PH

$$\dot{m} = \text{constant}$$

$$\rho Q = \text{const}$$

$$\rho A V = \text{const}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For Incompressible fluids

$$\rho_1 = \rho_2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times (0.3)^2 \times V_1 = \frac{\pi}{4} \times (0.1)^2 \times V_2 = 0.0035$$

$$V_1 = \underline{0.0495 \text{ m/s}} \approx \underline{0.05 \text{ m/s}}$$

$$V_2 = \underline{0.45 \text{ m/s}}$$

(4) Total energy at section 1

$$E_1 = H_1$$

H = Head

$$= \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

ALWAYS USE THIS
EQN ONLY

$$= \frac{120 \times 10^3}{1000 \times 10} + 18 + \frac{0.05^2}{2 \times 10}$$

$$E_1 = \underline{30.000125}$$

(5)

Energy at Point 2

$$E_2 = H_2$$

$$= \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$E_2 = \underline{180 \times 10^3} + 12 + 0.45^2$$

$$ie \quad H_2 > H_1$$

Hence flow from 2 $\rightarrow$ 1 [Pump think]

⑥ Losses

$$H_2 - h_L = H_1$$

$$30.010125 - h_L = 30.000125$$

$$h_L = H_2 - H_1$$

or

$$= E_2 - E_1$$

$$= \underline{0.01 \text{ m of Hg H}_2\text{O}}$$

$$= \frac{0.01}{13.6} \text{ m of Hg}$$

$$P \quad \rho_w \cdot \beta \cdot h_w = \rho_{Hg} \times g \cdot h_{Hg}$$

$$h_{Hg} = \frac{1000 \times 0.01}{13600}$$

⑦ Loss of Power $\rightarrow$ Rate of energy

$$\text{Power} = \text{Rate of fluid energy}$$

$$= \dot{m} g h_L$$

$$= 3.5 \times 10 \times 0.01$$

=

$$\underline{0.35 \text{ watt}}$$

$$\boxed{1 \text{ HP} = 736 \text{ watt}} = \underline{0.35} = \underline{\hspace{2cm}} \text{ HP}$$

1.

C

$$P = \frac{N}{m^2}$$

Kg

3

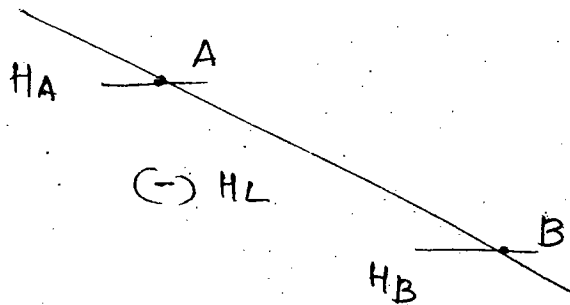
$$E = \frac{NM}{m^3}$$

$$= \frac{N}{m^2}$$

2.

C

3.



If $H_A > H_B$

Flow is $H_A - H_L = H_B$

A — B flow occurs

$$H_A = H_B + H_L$$

If $H_B > H_A$

Flow is from B — A

$$H_B - H_L = H_A$$

$$= H_B = H_A + H_L$$

Ans (A) (B)

4.

GATE 2010

D = 200 mm

S1

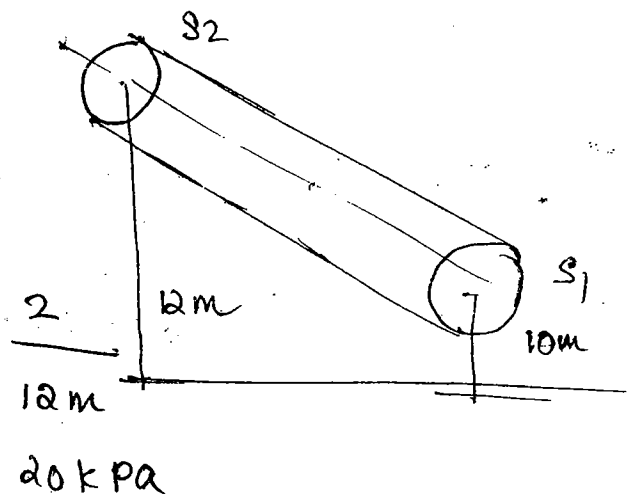
10 m

~~P1 = 50 kPa~~

P1 = 50 kPa

V = 2 m/s

$\alpha = 9.8 \text{ m/s}^2$



$$\frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$V_1 = V_2$$

$$= \frac{20 \times 10^3}{9.81 \times 1000} + 12 + \frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 2 \text{ m/s}$$

$$14.2448$$

$$E_1 = H_1$$

$$= \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

$$V_1 = 2 \text{ m/s}$$

$$= \frac{50 \times 10^3}{1000 \times 9.81} + 10 + \frac{V_1^2}{2 \times 9.81}$$

$$15.300$$

from S_1 to S_2 flow will occur

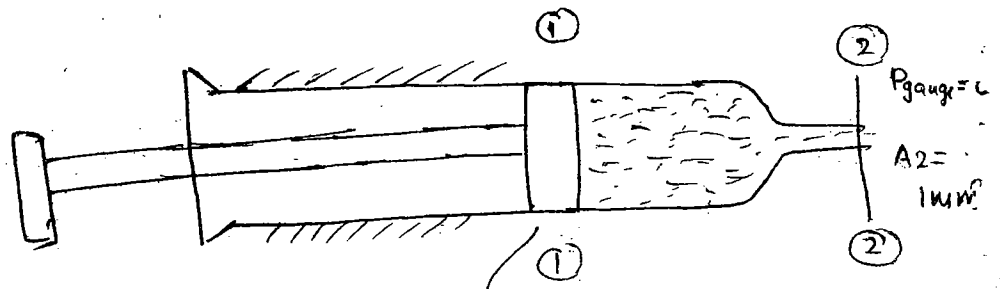
$$H_1 - H_L = H_2$$

$$15.300 - H_L = 14.248$$

$$H_L = 1.06 \text{ m}$$

Ans (C)

(7)
(5)



$$A_1 = 10 \text{ cm}^2$$

$$V_1 = 10 \text{ cm/s}$$

$$A_1 V_1 = A_2 V_2$$

$$10 \times 10 = 1 \times V_2$$

$$\text{cm/s}$$

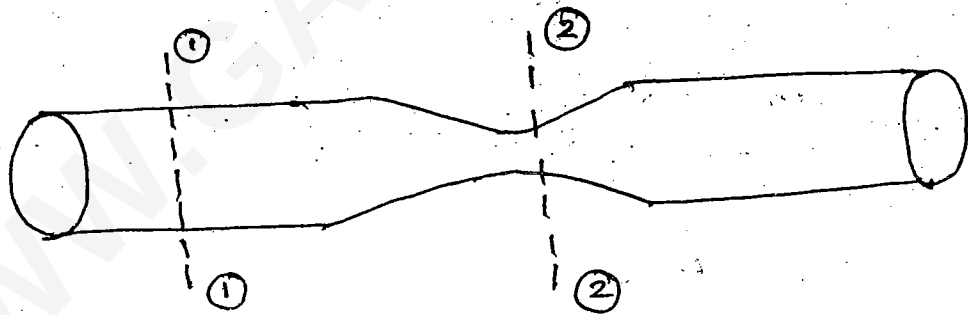
$$16 \frac{v_1^2}{2g} = \frac{v_2^2}{2g}$$

$$16 \times 1 = \frac{v_2^2}{2g}$$

$$10 + 1 = \frac{P_2}{\rho g} + 16$$

$$\frac{P_2}{\rho g} = \underline{\underline{-5 \text{ metres of water}}}$$

- Q.) At section 1 of a horizontal pipe the fluid pressure head is 32 cm and velocity head is 4 cm. The reduction in area at section 2 is such that the pressure head dropped down to zero value. What is the Ratio of velocity at section 2 to that at section 1?



$$\frac{P_1}{\rho g} = 32 \text{ cm}$$

$$\frac{P_2}{\rho g} = 0$$

$$\frac{v_1^2}{2g} = 4 \text{ cm}$$

$$\frac{v_2^2}{2g} = ?$$

Apply Bernoulli's eqn

$$\frac{P_1}{\rho g} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{v_2^2}{2g}$$

$$32 + 0 + 4 = 0 + 0 + \frac{v_2^2}{2g}$$

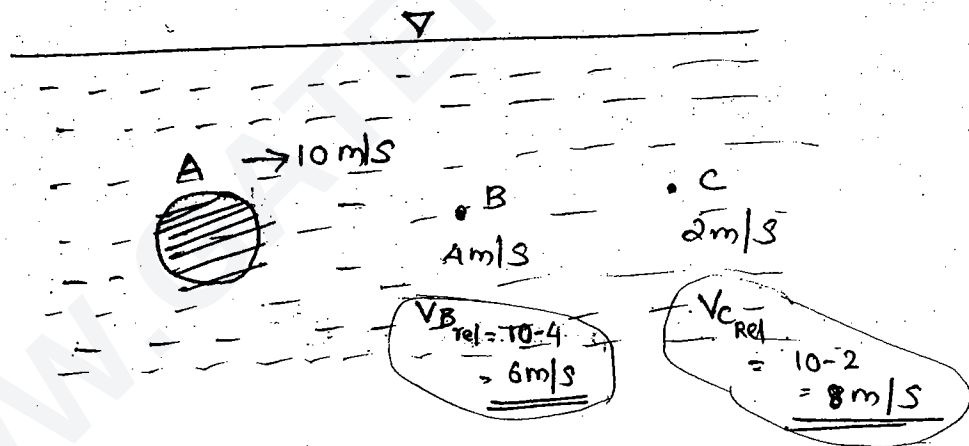
1. ~~1.1~~

$$\frac{v_2^2}{2g} \quad \frac{v_1^2}{2g}$$

$$\frac{v_2^2}{2g} = \frac{36}{2g} \Rightarrow \frac{v_2^2}{v_1^2} = 9$$

$$\frac{v_2}{v_1} = 3$$

- Q. In a static fluid a body is thrown with const vel 10m/s which gives velocity to the water at point B 4m/sec ; At point C 2m/sec. Then what is the value of $P_B - P_C = ?$
- a) 12 kPa b) 14 kPa c) 24 kPa d) 28 kPa



$$\frac{P_A}{\rho g} + \frac{v_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{v_B^2}{2g} + Z_B$$

$$\frac{P_B}{\rho g} + Z_B + \frac{v_B^2}{2g} = \frac{v_C^2}{2g} + \frac{P_C}{\rho g} + \frac{v_C^2}{2g} + Z_C$$

$$\frac{P_B - P_C}{\rho g} = \frac{v_C^2 - v_B^2}{2g}$$

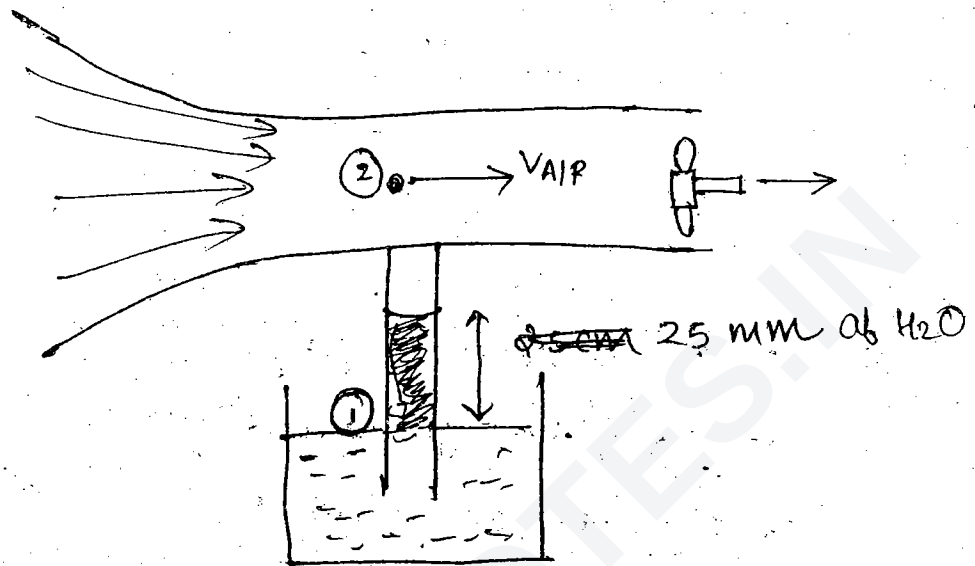
$$P_B - P_C = \frac{\rho}{2} [v_C^2 - v_B^2]$$

$$= 1000 [8^2 - 6^2] = 500 [64 - 36]$$

$$\underline{14000 \frac{\text{N}}{\text{m}^2}} = \underline{14 \text{ kPa}}$$

98

97



$$\rho_{\text{AIR}} = 1.2 \text{ kg/m}^3$$

$$\frac{P}{\rho g} + Z + \frac{V_2^2}{2g} = \text{const}$$

$$P + \rho g Z + \frac{\rho V_2^2}{2} = \text{const}$$

$$\frac{P}{\rho} + gZ + \frac{V_2^2}{2} = \text{const}$$

$$\frac{P}{\rho} + gZ + \frac{V_2^2}{2} = \text{const}$$

DYNAMIC
HEAD

$$\frac{\rho V^2}{2} = \frac{1.2 V^2}{2} =$$

$$\frac{1.2 V^2}{2} = \rho_w \cdot H_w \cdot g$$

$$\frac{1.2 V^2}{2} = \frac{1000 \times 9.81 \times 25 \times 10^{-3}}{2}$$

$$V = \underline{20.21 \text{ m/s}}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + 0 = 0 + 25 \text{ mm of water} + \frac{V_2^2}{2g}$$

At point

$P_2 = P_{atm}$

= 0 gauge pressure

$$= -25 \text{ mm of water} = - \frac{V_{air}^2}{2g}$$

$$\rho_w \cdot g \cdot h_w = \rho_a \cdot g \cdot h_a$$

$$1000 \times h_{25} = 1.2 \times h_a$$

$$h_a = \frac{1000 \times 25 \times 10^{-3}}{1.2} = \underline{\underline{20.833}}$$

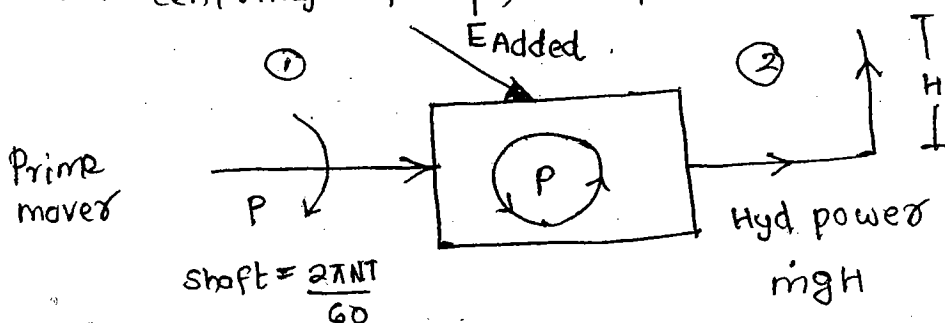
$$20.833 = \frac{V_{air}^2}{2g}$$

$$\underline{\underline{V_{air} = 20.21 \text{ m/s}}}$$

Pump

Pump is a device which converts mechanical energy into hydraulic energy.

ex: centrifugal pump, reciprocating pumps.



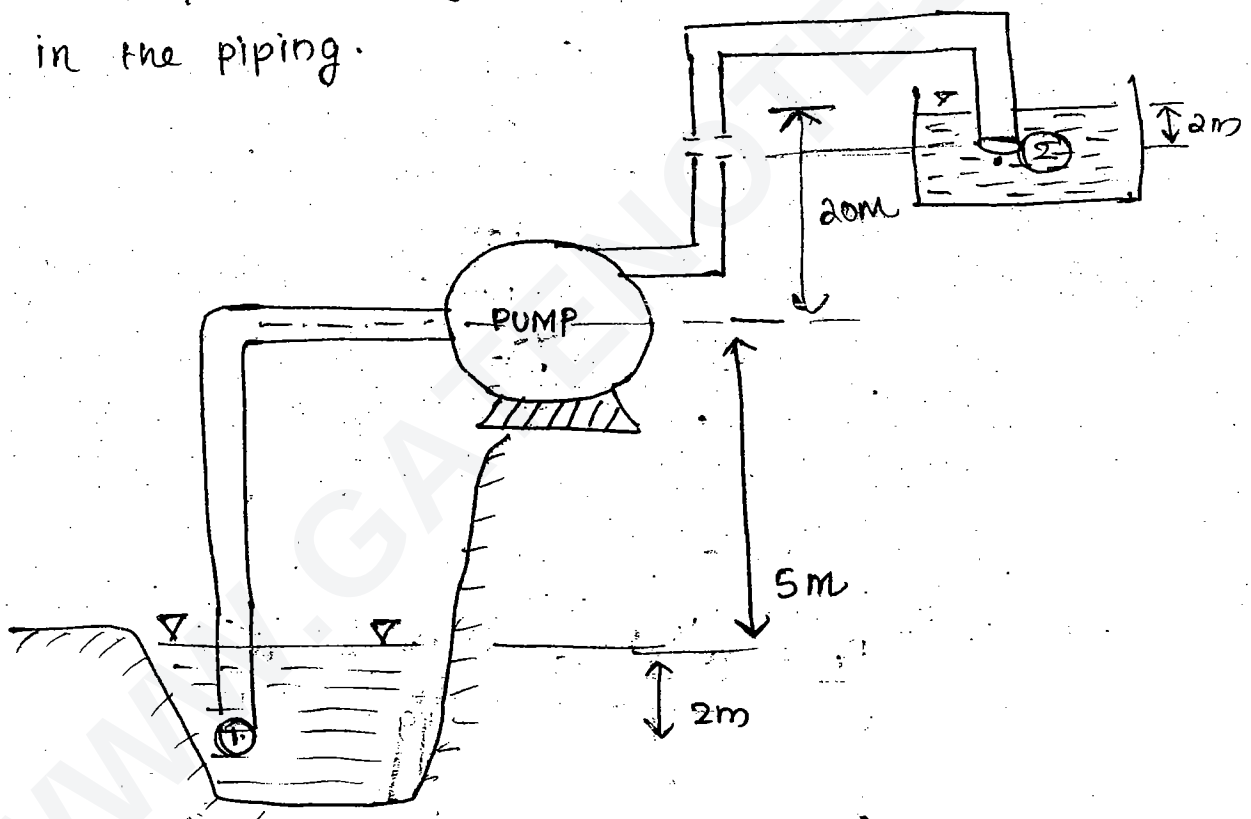
$$\eta = \frac{Q/P}{I/P} = \frac{\text{Fluid Power}}{\text{Shaft Power}} = \frac{mgh}{P_{shaft}} = \frac{\rho \cdot g \cdot Q \cdot H}{P_{shaft}}$$

$$P_A = 51.5 \times 10^3 \text{ N/m}^2$$

$$P_A = 51.5 \text{ kPa}$$

- Q) A pump draws water from a reservoir and discharges it to an overhead tank as shown in figure. The Area of the outlet pipe is 20 cm^2 & the average velocity in the outlet pipe is 3 m/s . Neglecting the minor & major losses in the piping.

Ans:-



- Q1. Total Head developed by the pump in metres of ~~water~~ water?
- (a) 27m (b) 25m (c) 24m (d) 22m
- Q2. If Combined efficiency of the pump and motor is 75%. then power reqd to run the pump in kW is ~~1.76 kW~~
- a) 1.76 kW b) 1.92 kW c) 2 kW d) 2.16 kW

$$v_2 = 3 \text{ m/s}$$

$$\text{So } A_2 v_2 = 20 \times 10^{-4} \times 3$$

$$= 60 \times 10^{-4}$$

$$= \underline{\underline{60 \times 10^{-4} \text{ m}^3/\text{s}}}$$

continuity eqn

$$A_1 v_1 = A_2 v_2 = Q$$

$$A_1 v_1 = 60 \times 10^{-4} \text{ m}^3/\text{s}$$

$$Q = \underline{\underline{60 \times 10^{-4} \text{ m}^3/\text{s}}}$$

$$= 6 \times 10^{-3} \text{ m}^3/\text{s}$$

$$= \underline{\underline{0.006 \text{ m}^3/\text{s}}}$$

Mass flow Rate

$$\dot{m} = \rho \cdot Q$$

$$\dot{m} = 1000 \times 0.006$$

$$= \underline{\underline{6 \frac{\text{kg}}{\text{s}}}}$$

Apply Bernoulli's eqn

[General eqn applied to pump]

$$\left(\frac{P_1}{\rho g} \right)_{\text{gauge}} + z_1 + \frac{v_1^2}{2g} + E_{\text{added}} = \left(\frac{P_2}{\rho g} \right)_{\text{gauge}} + z_2 + \frac{v_2^2}{2g}$$

$$P_1 = \rho \cdot g \cdot H_1$$

$$= \left(\frac{P_1}{\rho g} \right) = 2 \text{ m}$$

~~2.2 m~~

At optimum

$$\text{Head} = 25 \text{ m}$$

So it should be more than that since E is added

$$2 + 0 + 0 + E_{\text{added}} = 2 + (7+18) + \frac{3^2}{2 \times 10}$$

$$E_{\text{added}} = 25 + \frac{3^2}{2 \times 10}$$

$$= 25 + 0.45 = 25.45 \text{ m}$$

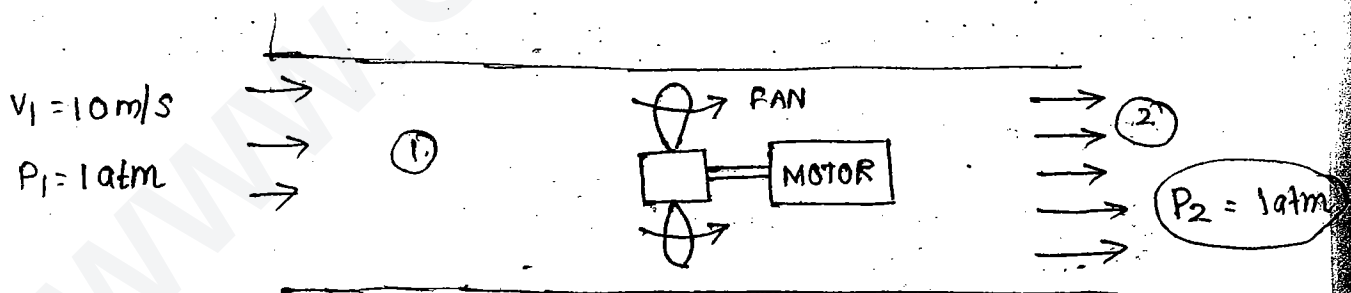
b) $\eta = 75\%$
 $P = ?$

$$\eta = \frac{m g H}{P_{\text{shaft}}} = \frac{\rho \times Q \times g \times H}{P_{\text{shaft}}}$$

$$P_{\text{shaft}} = \frac{6 \times 10 \times 27}{0.75}$$

$$= \underline{\underline{2.16 \text{ kW}}}$$

- Q. Consider one dimensional incompressible flow of water thru a constant c/s Area 1 sqm pipe shown in fig. An electric motor imparts 1 MW power to the fluid via a fan. Neglect energy losses and fan doesn't produce any swirl of fluid flow.



1) what is the velocity v_2 in m/s ?

- a) -10 m/s b) 17.32 m/s c) insufficient data d) More than 17.32 m/s .

2) what is the pressure P_2 ?

- a) 1 atm b) 2 atm c) 0.5 atm d) 3 atm

$$\frac{\rho \cdot 10^2}{2 \times 10} - 0 + (E_{\text{added}}) = \frac{v_2^2}{2g}$$

$$5 + E_{\text{added}} = \frac{v_2^2}{2g}$$

$$E_{\text{added}} = 1 \text{ MW} \\ = 1 \times 10^6 \text{ W}$$

$$\eta = \frac{\text{o/p}}{\text{i/p}} \frac{(\text{watt})}{(\text{watt})}$$

$$100\% = \frac{\dot{m} g H}{1 \text{ MW}}$$

$$A_1 V_1 =$$

$$Q = A \cdot V$$

$$10 \times 1 = \frac{10 \text{ m}^3}{\text{s}}$$

$$1 = \frac{\rho Q g H}{1 \text{ MW}} = \frac{1000 \times 10 \times (10) \times H}{1 \times 10^6}$$

$$\underline{\underline{H = 10 \text{ m}}}$$

$$5 + 10 = \frac{v^2}{2 \times 10}$$

$$15 \times 20 = v^2$$

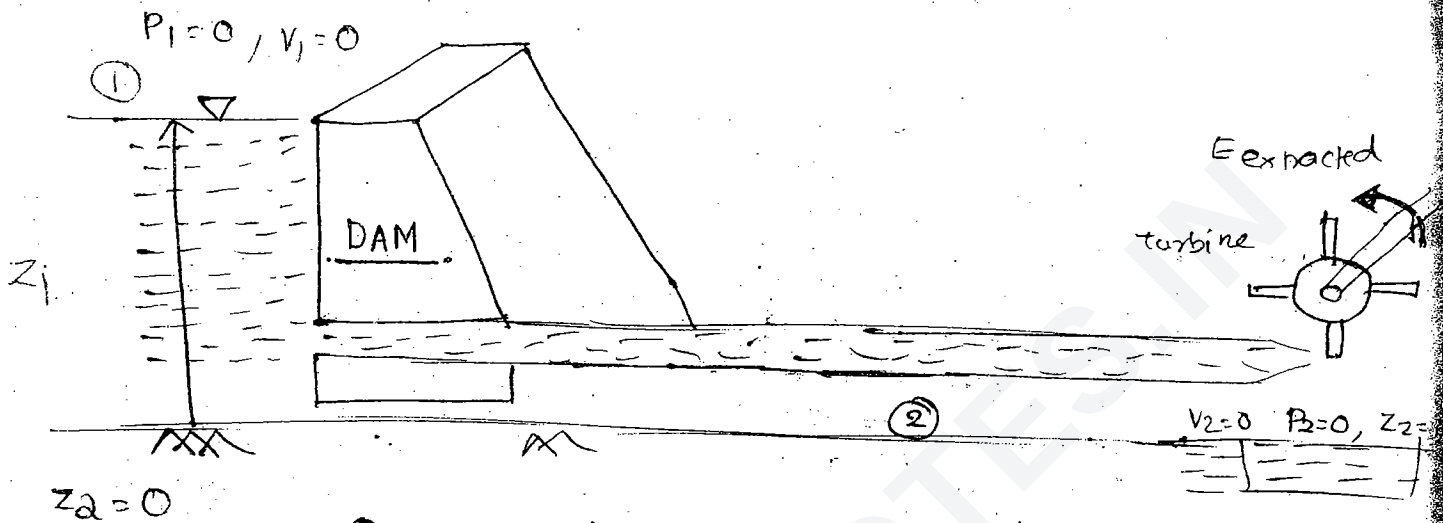
$$v = \sqrt{300} = \underline{\underline{17.32 \text{ m/s}}}$$

- a) A fire fighting hose with an exit nozzle dia = 50 mm discharges water at the rate of 3000 ^{lit} / min. The height of the nozzle exit from the pump is 4 m and water level in the sump is 1 m below the pump shown in the fig. Calculate the head developed.

Apply Bernoulli's eqn.

103

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + H_{\text{extracted or } (E_{\text{extracted}})}$$



$$0 + z_1 + 0 - h_L = 0 + 0 + 0 + H_{\text{extracted}}$$

$$z_1 - h_L = H_{\text{extracted}}$$

Q10.

$$Q = 1.2 \text{ m}^3/\text{s}$$

$$H = 120 \text{ m}$$

$$\eta = 90\%$$

$$\eta = \frac{Q P}{\dot{W}_P} = \frac{P_{\text{shaft}}}{\rho \cdot Q \cdot H g}$$

$$P_{\text{shaft}} = 0.9 \times 1000 \times 1.2 \times 120 \times (10) \rightarrow g$$

$$= 1271 \times 10^3 \text{ watt}$$

$$= 1271 \text{ kW}$$

Q8.

$$H = 50 \text{ m}$$

$$Q = 3 \text{ m}^3/\text{s}$$

$$h_L = 5 \text{ m}$$

$$P = 1000 \text{ kW}$$

$$\eta_{\text{TURBINE}} = \frac{\text{O/P}}{\text{I/P}} = \frac{1000 \times 10^3 \text{ watt}}{1000 \times 9.81 \times 3 \times H_{\text{extracted}}}$$

$$1 = \frac{1 \times 10^6 \text{ watt}}{1000 \times 9.81 \times 3 \times H_{\text{extracted}}}$$

$$H_{\text{extracted}} = 33.98 \text{ m}$$

$$Z_1 - h_L = H_{\text{extracted}}$$

$$50 - h_L = H_{\text{extracted}}$$

$$50 - 5 = h_{\text{residual}} = 33.98$$

$$45 - h_{\text{residual}} = 33.98$$

$$h_{\text{residual}} = 45 - 33.98$$

$$= \underline{\underline{10.95 \text{ m}}}$$

Discharge ($Q = \dot{V}$) Measurement

1. venturimeter
2. orificemeter
3. Nozzlemeter
4. Bend meter
5. Rotameter

Discharge Flow Rate
meter

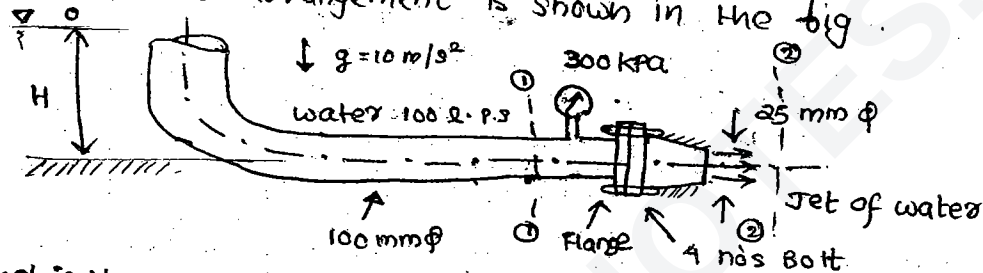
VENTURIMETER

It is a device used to measure volume flow rate
ie discharge through closed pipes or conduits; not

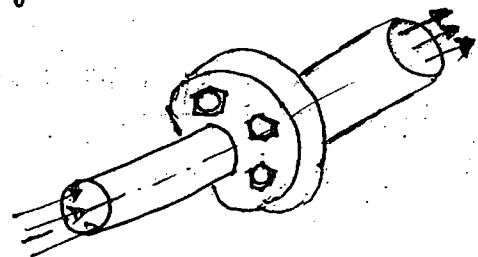
1. Force Analysis in pipe flow:-
2. Forces on lifted Bodies:-
3. Force exerted by moving fluid on fixed plates, moving plates
4. power developed by moving fluid.

Model 1

- a) A smooth pipe of inner dia 100mm is subjected to water flow at the rate of 100 lit/s. At the end of the pipe a nozzle is connected with the help of a flange and no. of bolts. The arrangement is shown in the fig.



1. What is the mass flow rate in pipe system? (kg/s)
2. avg velocity in pipe line?
3. velocity of jet?
4. what is the possible head from which pipe is receiving water?
5. What type of force acting on flange bolts?
6. What is the total force on the flange?
7. What is the force per each bolt?
8. Determine size of bolt for the data given?
9. Hydraulic losses in the nozzle?
10. Power lost in the nozzle?
11. efficiency of the pipe system?



Ans:

1. mass flow rate $m = \rho Q$
 $= 1000 \times 100 \times 10^{-3} \text{ kg/sec} = \underline{100 \text{ kg/sec}}$

2. By continuity eqn

$$A_1 V_1 = A_2 V_2$$

$$100 \times 10^{-3} = \frac{\pi}{4} \times (0.1)^2 V_1 = \frac{\pi}{4} \times (0.025)^2 V_2$$

$$\therefore V_1 = \underline{12.73 \text{ m/s}}$$

$$V_2 = \underline{203.7 \text{ m/s}} \approx 204 \text{ m/s}$$

4. Apply Bernoulli's eqn at 0 & 1

$$\frac{P_0}{\rho g} + Z_0 + \frac{V_0^2}{2g} - h_{0-1} = \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g}$$

$$0 + H + 0 - 0 = \frac{300 \times 10^3}{1000 \times 10} + 0 + \frac{12.73^2}{2 \times 10}$$

$$H = \underline{38.45 \text{ m}}$$

$$100 \times (v_2 - v_1)$$

$$100 (204 - 13) = 19100 \text{ N}$$

$$= \underline{19.1 \text{ kN}}$$

Total load on Bolted Flange 19.1 kN

⑦ Load Per Bolt = $\frac{19.1 \text{ kN}}{4} = \underline{4.78 \text{ kN}}$

⑧ Diameter of the Bolts.

$$\sigma_{\text{Bolt}} = \frac{F_{\text{Bolt}}}{A_{\text{cls}}}$$

$$\left(\frac{\text{N}}{\text{mm}^2} \right) \quad \frac{250}{4} = \frac{4.78 \times 10^3 \text{ (N)}}{\frac{\pi \times d^2}{4} \text{ (mm}^2\text{)}} \quad d = \text{dia of Bolt}$$

$$\therefore d_{\text{Bolt}} = \underline{9.86 \text{ mm}} \approx \underline{10 \text{ mm}}$$

⑨ Hydraulic losses b/w 1 & 2.

$$\frac{P_1}{\rho g} + Z_1 + \frac{v_1^2}{2g} - h_L = \frac{P_2}{\rho g} + Z_2 + \frac{v_2^2}{2g}$$

$$\frac{300 \times 10^3}{1000 \times 10} + 0 + \frac{13^2}{2 \times 10} - h_L = 0 + 0 + \frac{204^2}{2 \times 10}$$

G.M. - G.M.

$$h_L = - \underline{\underline{2042}}$$

⑩ Power lost

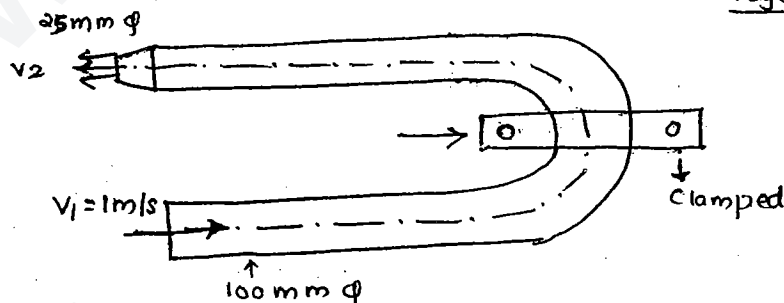
$$P = \dot{m} g h_L$$

$$= 100 \times 10 \times 2042$$

⑪ $\eta = \frac{\dot{m} g h_L}{\frac{1}{2} \dot{m} v_1^2}$

⑫ A U Bend pipe of 100 mm kept on the Horizontal plane ab dia 100 mm and water flow at vel 1 m/s. At the end of the pipe a nozzle is fitted with exit dia 25 mm. determine the force necessary to ~~clamp~~ hold the pipe in that position

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$$\sum F = \dot{m} (v_2 - v_1)$$

$$= \rho Q (v_2 - v_1)$$

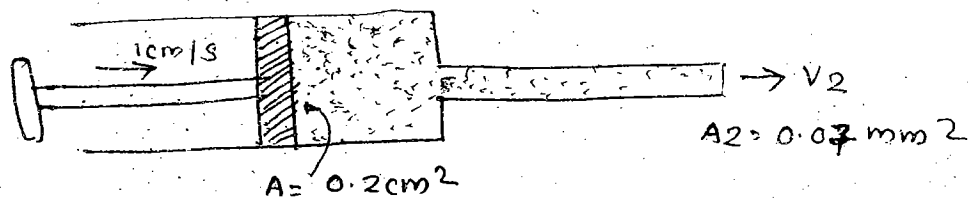
$$\sum F = 1000 \times 0.00785 (16 - 1)$$

$$\underline{\underline{\sum F = 118 \text{ N}}}$$

$$Q = A_1 v_1 = A_2 v_2 = 0.00785$$

$$\frac{\pi \times (0.1)^2 \times 1}{4} = \frac{\pi \times (0.025)^2 \times v_2}{4}$$

$$\underline{\underline{v_2 = 16 \text{ m/s}}}$$



$$\Sigma F = \dot{m}(\Delta V)$$

$$= \rho Q (V_2 - V_1)$$

$$= 1000 \times 2.7 \times 10^{-7} (2.85 - 0.001)$$

$$= 5.68 \times 10^{-4} \text{ N}$$

$$= \frac{5.68 \times 10^{-4}}{9.81} \text{ (kgf)}$$

$$= 5.8 \times 10^{-5} \text{ (kgf)}$$

$$Q = A_1 V_1 = (0.2 \times 10^{-4}) \times (1 \times 10^{-2})$$

$$= 2 \times 10^{-7} \frac{\text{m}^3}{\text{sec}}$$

$$A_2 V_2 = 2 \times 10^{-7}$$

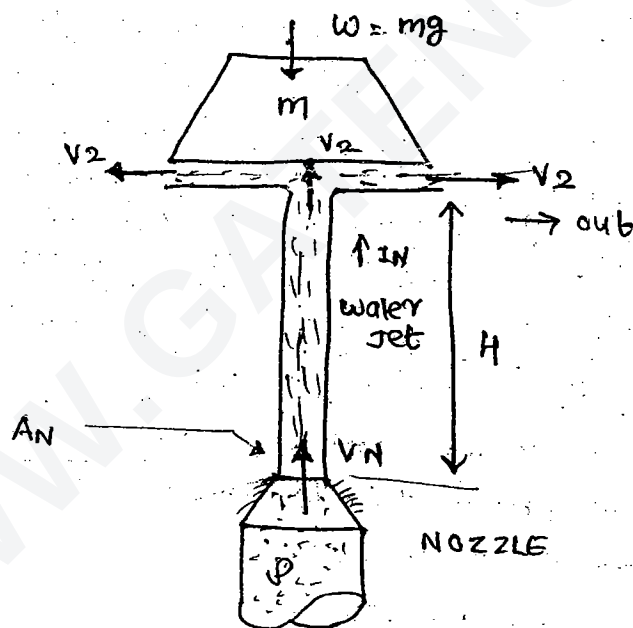
$$(0.07 \times 10^{-6}) \times V_2 = 2 \times 10^{-7}$$

$$V_2 = 2.85 \text{ m/s}$$

Model 2

Force exerted by Fluid on lifting Body

MOVING
FLUID



Parameters Available

A_N, ρ, V_N, m, H

Tool

① Continuity eqn

$$Q = A_N V_N$$

② Bernoulli's eqn

Apply Bernoulli's eqn at 1 & 2

$$\left(\frac{P_1}{\rho g} \right) + z_1 + \frac{V_1^2}{2g} - h_L = \left(\frac{P_2}{\rho g} \right) + z_2 + \frac{V_2^2}{2g}$$

$$0 + 0 + \frac{V_N^2}{2g} - 0 = 0 + H + \frac{V_2^2}{2g}$$

$$\sum F_y = m(\Delta v_y)$$

$$= m(v_{y \text{ final}} - v_{y \text{ initial}})$$

$$-mg = m(0 - v_2)$$

$$-mg = -\rho Q v_2$$

$$mg = \rho Q v_2$$

$$mg = \rho A_N v_N v_2$$

$$v_2 = \frac{mg}{\rho \cdot A_N v_N}$$

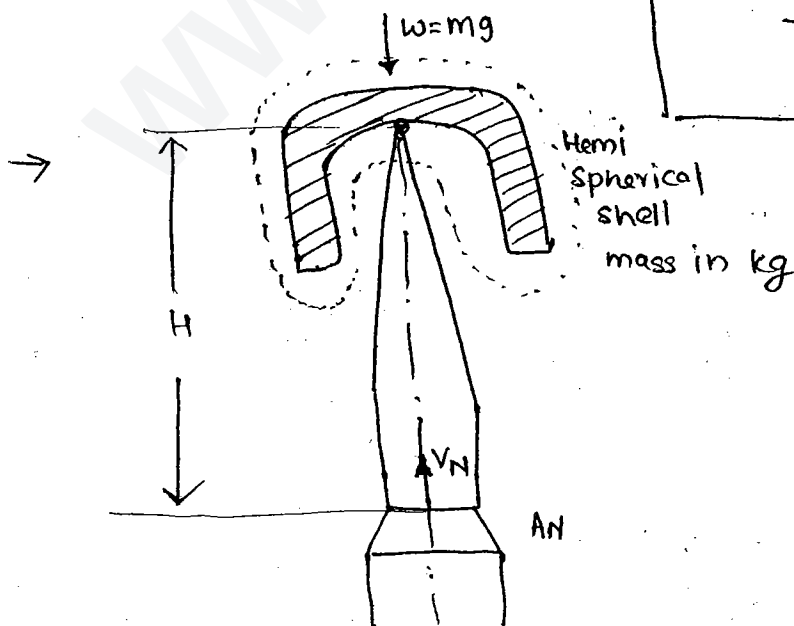
But $\textcircled{8}$ $v_2 = \sqrt{v_N^2 - 2gH}$

$$\sqrt{v_N^2 - 2gH} = \frac{mg}{\rho \cdot A_N v_N}$$

Squaring $v_N^2 - 2gH = \left[\frac{mg}{\rho \cdot A_N v_N} \right]^2$

$$H = \frac{v_N^2 - \left[\frac{mg}{\rho \cdot A_N v_N} \right]^2}{2g}$$

$$H = \frac{v_N^2 - \left[\frac{mg}{\rho \cdot A_N v_N} \right]^2}{2g}$$



$\textcircled{Q.}$ A vertical jet of water is able to keep a hemispherical

nozzle of Area A_N with velocity V_N at the nozzle exit. using Bernoulli's eqn and momentum theorem develop the expressions for jet velocities V_2 at the shell point & expression for height H at which the shell is balanced. shell mass is m and water density is ρ . Assume ideal flow conditions.

1. continuity eqn

$$Q = A_N V_N$$

② Bernoulli's eqn

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$0 + 0 + \frac{V_N^2}{2g} = 0 + H + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{V_N^2 - 2gH}$$

③ Momentum eqn

$$\sum F_y = \dot{m}(\Delta v_y) = \dot{m}(v_{\text{final}} - v_{\text{initial}})$$

$$-mg = \dot{m}(-V_2 - V_2)$$

$$-mg = -\rho Q 2V_2$$

$$mg = \rho Q 2V_2$$

$$mg = \rho \cdot A_N V_N \cdot 2V_2$$

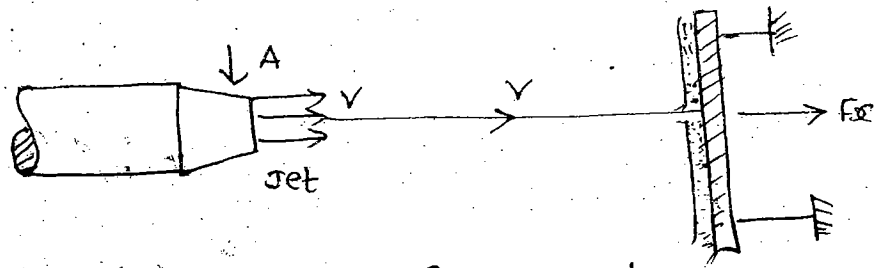
$$\left[\frac{mg}{\rho Q A_N V_N} \right] = V_2$$

$$\frac{mg}{2\rho \cdot A_N \cdot V_N} = \sqrt{V_N^2 - 2gH}$$

$$\left[\frac{mg}{2\rho \cdot A_N \cdot V_N} \right]^2 = V_N^2 - 2gH$$

Since mg is in downward direction

①



$$F_x = \rho A V^2 = \text{Reactive force offered by plate}$$

$$\downarrow$$

$$\frac{\text{kg}}{\text{m}^3} \times \text{m}^2 \times \frac{\text{m}}{\text{s}^2} = \text{kg} \frac{\text{m}}{\text{s}^2} = \text{N}$$

$$F_x = \cancel{ma} \text{ ma}$$

$$= \dot{m} (\Delta V)_x$$

$$\dot{m} (v_{\text{final}} - v_{\text{initial}})$$

$$= \dot{m} (0 - v)$$

$$F_x = -\dot{m}v$$

-ve sign means energy lost by fluid

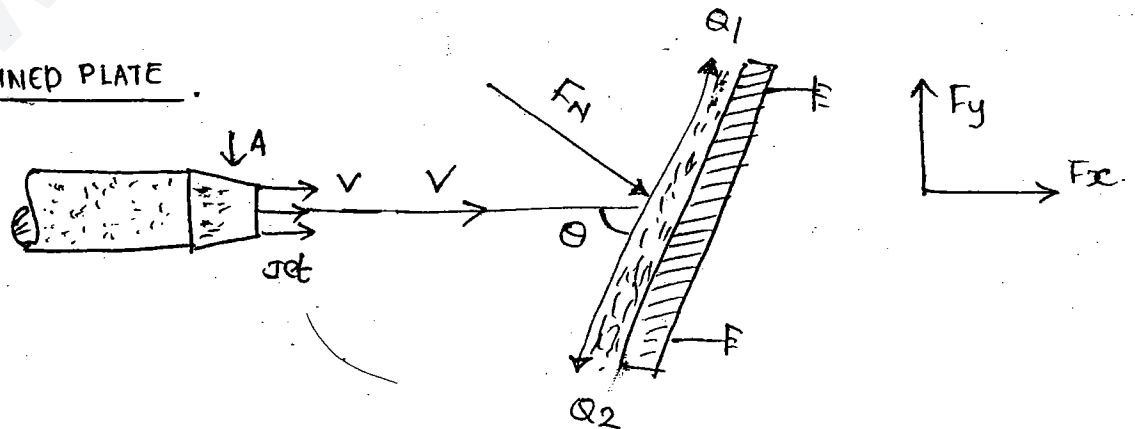
ie ~~the~~ force exerted by jet of fluid on plate
(Reaction of plate)

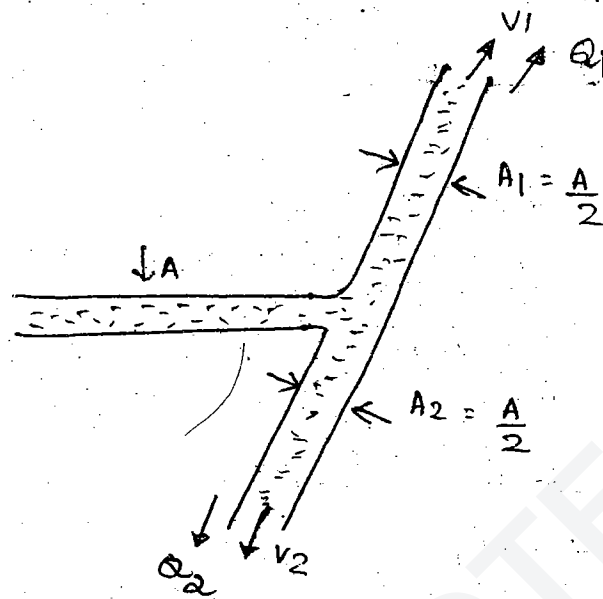
$$F_x = \dot{m}v$$

$$= \rho Q v$$

$$\boxed{F_x = \rho A V^2}$$

② INCLINED PLATE





$$Q = Q_1 + Q_2 \quad \text{--- By continuity}$$

$$\rho A V = \rho A_1 v_1 + \rho A_2 v_2$$

$$A V = \frac{A}{2} v_1 + \frac{A}{2} v_2$$

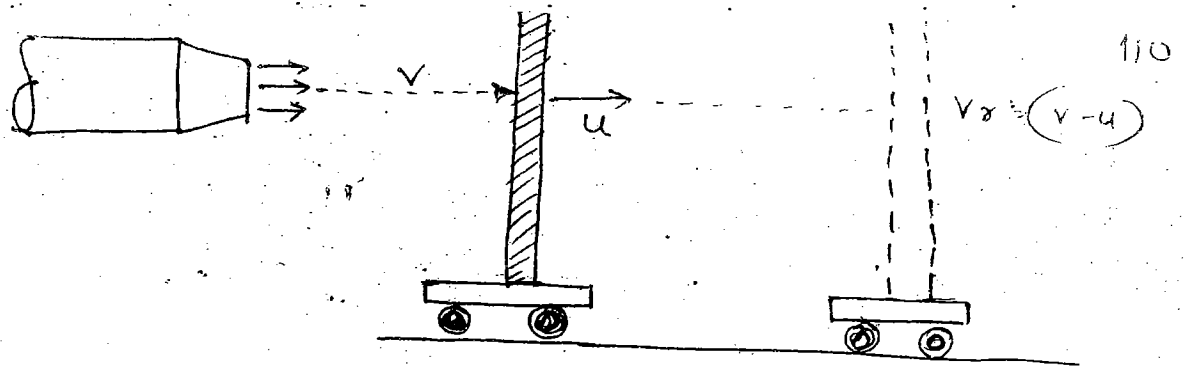
$$V = \frac{v_1}{2} + \frac{v_2}{2}$$

$$V = \frac{v_1}{2} + \frac{v_1}{2} \quad \text{or} \quad \frac{v_2}{2} + \frac{v_2}{2}$$

$$V = \frac{2v_1}{2} \quad \text{or} \quad \frac{2v_2}{2}$$

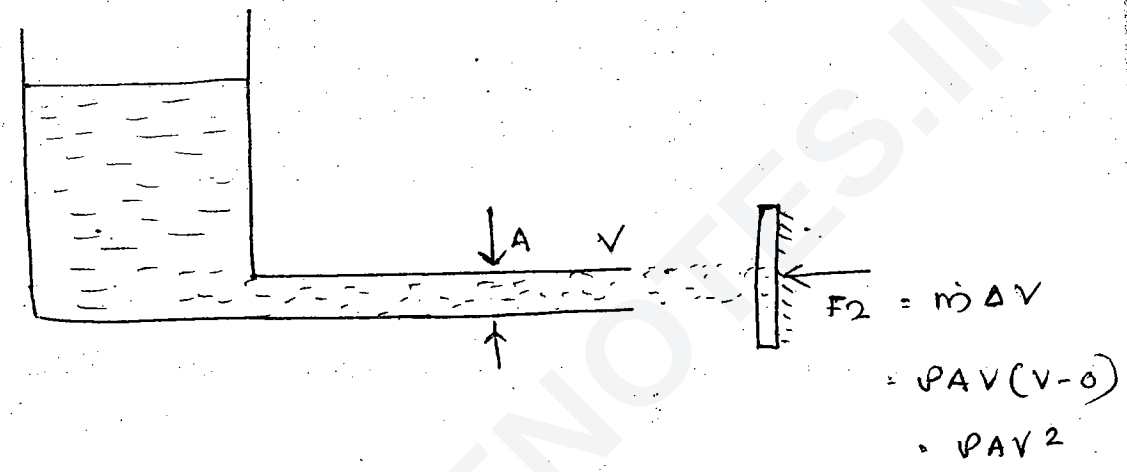
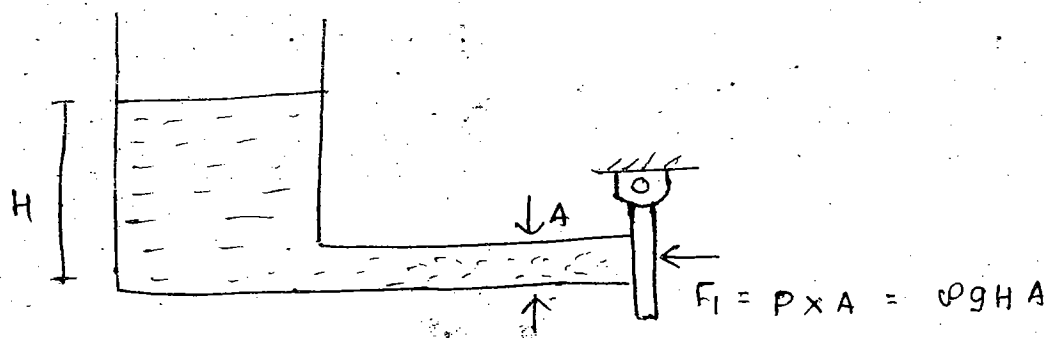
$$\boxed{V = v_1 = v_2}$$

www.gatenotes.in



2009 Q.)

static
fluid

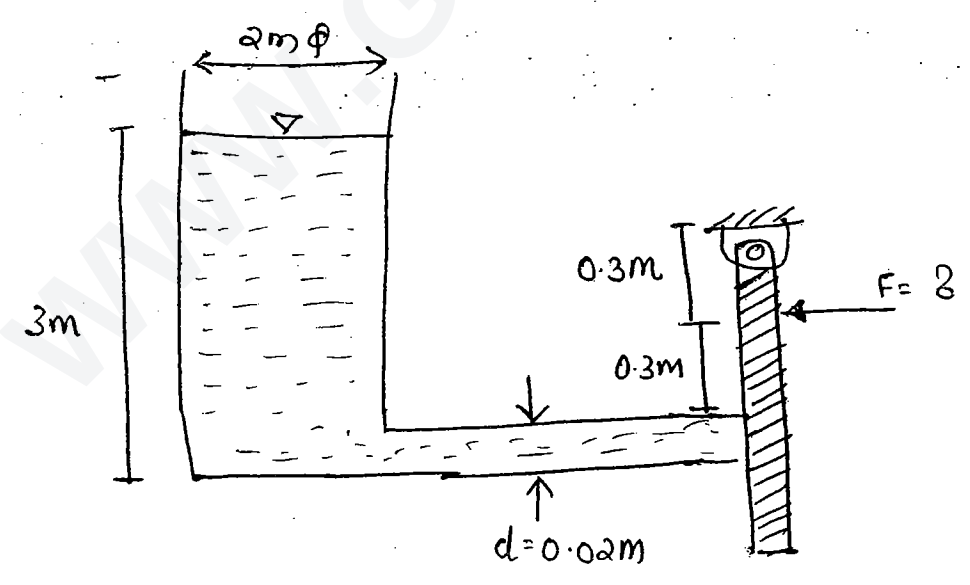


$$\frac{F_1}{F_2} = \frac{\rho g H A}{2 \rho g H A} = \underline{\underline{2}}$$

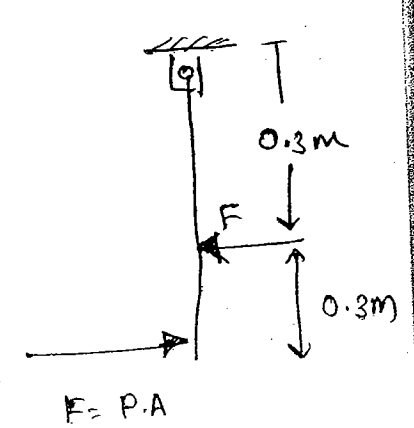
$$F_2 = \rho A (\sqrt{2gH})$$

$$\underline{\underline{F_2 = \rho A \times 2gH}}$$

Q.)



- a) 18.5 N
- b) 37 N
- c) 48.5 N
- d) 74 N



$$F \times 0.3 = 9.24 \times 0.6$$

$$\underline{F = 18.48 \text{ N}}$$

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Q1.

$$F = \rho A (v-u)^2$$

$$= 1000 \times 0.01 (20-10)^2$$

$$= \underline{1000 \text{ N}}$$

Q2.

$$\text{Power developed} = F \times v$$

$$= 1000 \times 10$$

$$= 10,000 \text{ Watt}$$

$$= \underline{10 \text{ kW}}$$

Q3.

force developed

$$F = \rho A (v - (-u))^2$$

$$= \rho A (v+u)^2$$

$$= 1000 \times 0.01 (20+10)^2$$

$$= \underline{9000 \text{ N}}$$

Q4.

~~V~~ Relative velocity

$$v - (-\frac{v}{2})$$

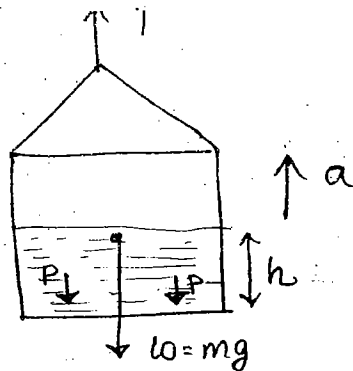
$$= \underline{\underline{\frac{3v}{2} = 1.5v}}$$

$$Q_2 = A \cdot v \cdot \rho$$

$$\underline{\underline{\rho A (v+u)}}$$

$$= A (v + \frac{v}{2})$$

5



$$\Sigma F = m a$$

$$T - W = m a$$

$$T - mg = m a$$

$$T = mg + m a$$

$$T = m(g + a)$$

$$\text{Pressure} = \frac{T}{A} = \frac{m(g + a)}{A}$$

$$= \frac{\rho V(g + a)}{A}$$

$$= \frac{\rho \cdot h \cdot A(g + a)}{A}$$

Fluid at Rest

$$P = \rho g h$$

Fluid moving up =

~~$$P = \rho g h$$~~

$$P = \rho h(g + a)$$

Fluid moving down

$$P = \rho h(g - a)$$

$$P = 1200 \times 10 (9.81 + 5 \times 9.81)$$

$$706320$$

~~$$70632$$~~

$$N/m^2$$

$$706320$$

$$9.81$$

~~$$70632$$~~

~~$$9.81$$~~

~~$$kg/cm^3$$~~

$$kgf$$

$$m^2$$

PRAVEEN-T.P 112

PM IS

MECHANICAL

PH NO: 7207237642

FLUID MECHANICS - II

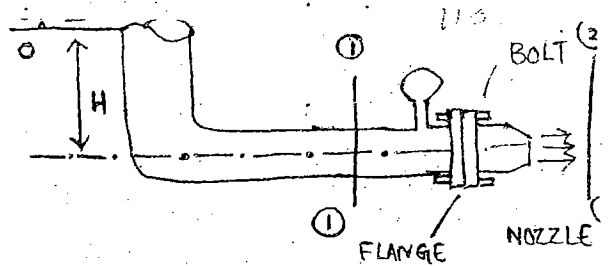
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1. Efficiency of piping system = $\frac{\rho g h_L}{\frac{1}{2} \dot{m} v_1^2}$

- Force on Bolts

= S_{yt} of Bolt $\times$ Area of c/s

$S_{yt} \times \frac{\pi}{4} \times d^2$



2. Force on Bolts
 $F = \dot{m} \Delta v$

$\Delta v = v_2 - v_1$

v_2 = Nozzle exit vel

v_1 = vel in piping system

- Force per Bolt = $\frac{F}{\text{No of Bolts}}$

- Power Lost = $\dot{m} g h_L$

3. Mass flow Rate.

$\dot{m} = \rho A V$

= ρQ

MODEL 2

Force Analysis in Lifting Bodies

Parameters Available:

v_2, A_N, m, H, ρ

Tool:

(1) Continuity eqn:

$Q = A_N V_N$

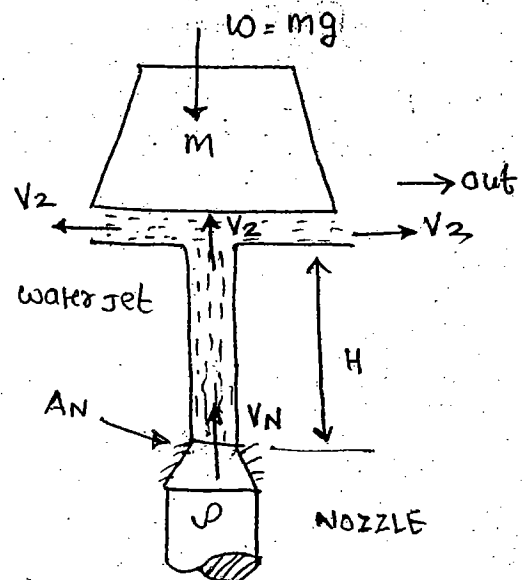
(2) Bernoulli's eqn:

$\left[\frac{P_1}{\rho g} \right]_{\text{gauge}} + Z_1 + \frac{v_1^2}{2g} = \left[\frac{P_2}{\rho g} \right]_{\text{gauge}} + Z_2 + \frac{v_2^2}{2g}$

$0 + 0 + \frac{V_N^2}{2g} = 0 + H + \frac{V_2^2}{2g}$

$\frac{V_2^2}{2g} = \frac{V_N^2}{2g} - H$

$V_2 = \sqrt{V_N^2 - 2gH}$



③ Momentum eqn:

$$\sum F_y = \dot{m}(\Delta v_y) = \dot{m}(v_{final} - v_{initial})$$

$$-mg = \dot{m}(0 - v_2)$$

$$-mg = -\rho Q v_2$$

$$v_2 = \frac{mg}{\rho Q}$$

$$v_2 = \frac{mg}{A_N v_N \rho}$$

But $v_2 = \sqrt{v_N^2 - 2gH}$

$$\sqrt{v_N^2 - 2gH} = \frac{mg}{\rho A_N v_N}$$

$$v_N^2 - 2gH = \left[\frac{mg}{\rho A_N v_N} \right]^2$$

$$H = \frac{v_N^2 - \left[\frac{mg}{\rho A_N v_N} \right]^2}{2g}$$

MODEL-3

Force exerted by moving fluid on a fixed plate

① Force exerted by jet on plate

= Rate of change of momentum in direction of force

If force

exerted on = $\dot{m} [\Delta v]$

Jet is to

be calculated

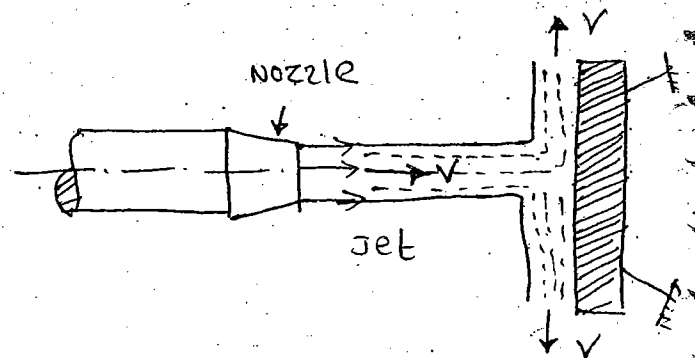
we will

take

$$\left[\begin{array}{l} \text{final vel} - \\ \text{initial vel} \end{array} \right]$$

$$\dot{m} [v - 0]$$

$$\rho A v \cdot v = \underline{\underline{\rho A v^2}}$$



$$F_x = \rho A v^2$$

② Inclined plate

$$F_H = \rho A v^2 \sin \theta$$

$$F_n = \frac{\text{mass}}{\text{time}} [\text{Initial vel.} - \text{Final vel.}]$$

$$F_n = \rho A V [v \sin \theta - 0]$$

$$F_n = \rho A V^2 \sin \theta$$

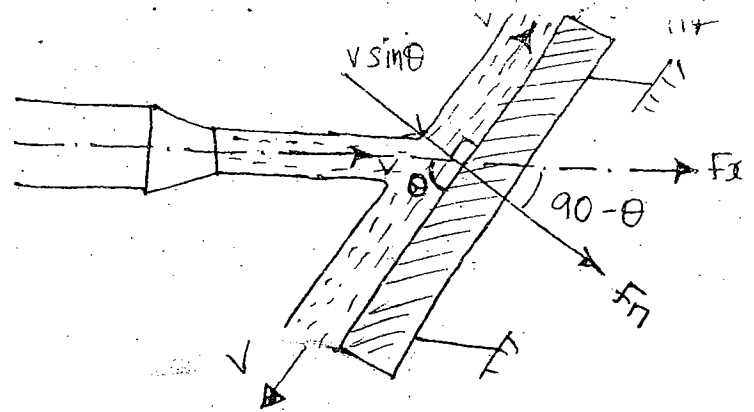
$$F_x = F_n \cos(90 - \theta)$$

$$F_x = \rho A V^2 \sin^2 \theta$$

$$F_y = F_n \sin(90 - \theta)$$

$$= F_n \cos \theta$$

$$F_y = \rho A V^2 \sin \theta \cos \theta$$

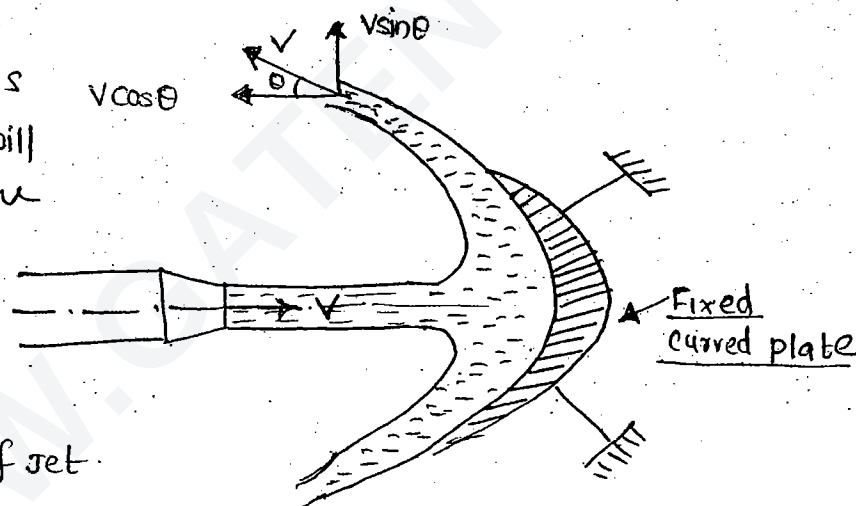


Initial vel in the direction of n

Final vel = 0 in dir of n

3. stationary curved plate:

If there is no loss of energy the jet will come out with same vel in the tangential direction.



$V \cos \theta$ = opp dir of jet

Component in the direction of jet = $-V \cos \theta$

" " " " of jet = $V \sin \theta$

$$F_x = \frac{\text{Mass}}{\text{sec}} [V_{1x} - V_{2x}]$$

$$\rho A V [V - (-V \cos \theta)]$$

$$F_x = \rho A V^2 [1 + \cos \theta]$$

$$F_x = 2 \rho A V^2 \cos^2 \theta / 2$$

Initial
 V_{1x} = vel in the direction of jet

V_{2x} = final vel in the direction of jet

$$F_y = \rho A V [V_{1y} - V_{2y}]$$

$$= \rho A V [0 - V \sin \theta]$$

$$F_y = -\rho A V^2 \sin \theta$$

-ve indicates force is acting in downward direction.

Angle of Deflection means $180 - \theta$

Jet striking Tangentially

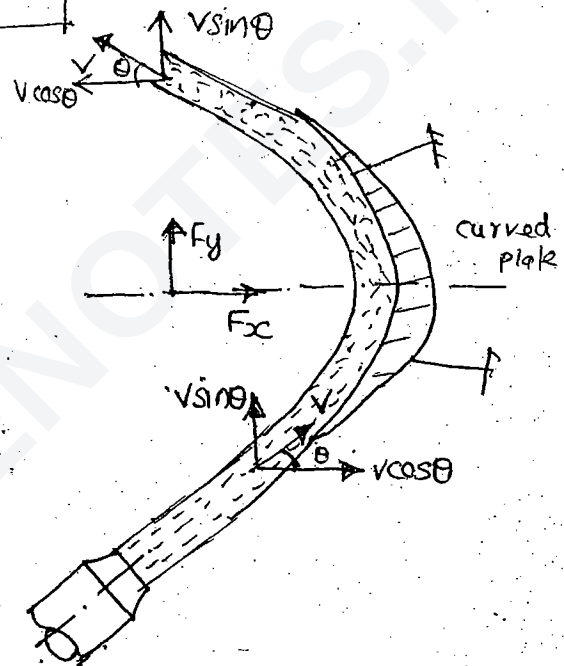
$$F_{2x} = \rho A V [V_{1x} - V_{2x}]$$

$$= \rho A V [V \cos \theta - (-V \cos \theta)]$$

$$F_{2x} = 2 \rho A V^2 [\cos \theta]$$

$$F_y = \rho A V [V \sin \theta - V \sin \theta]$$

$$F_y = 0$$



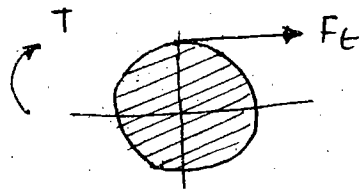
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Water sprinkler.

It works on Momentum-Momentum equation

Eqn:

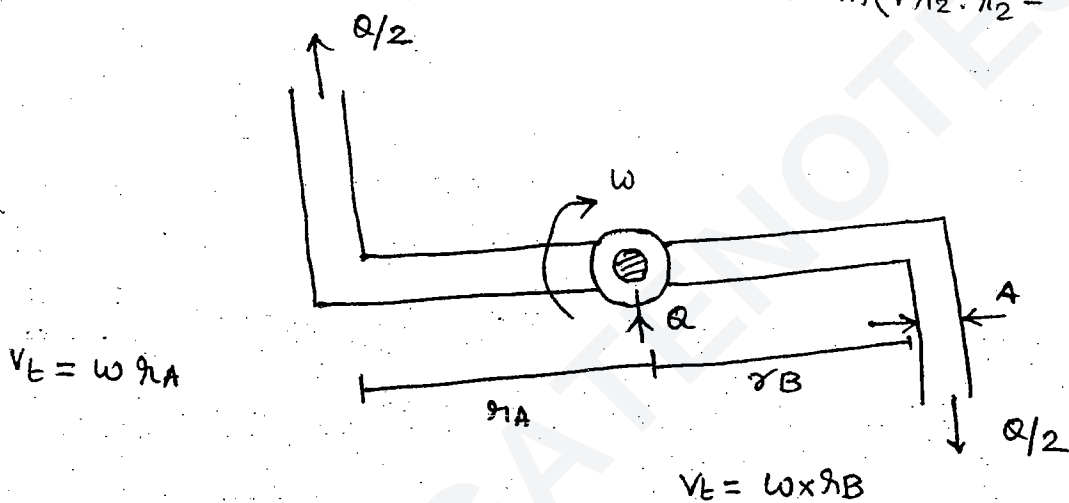
$$\text{Torque} = F \times r$$



$$T = \dot{m}(\Delta V) \cdot r$$

$$= \dot{m}(V_{r2} - V_{r1}) \cdot r$$

$$= \dot{m}(V_{r2} \cdot r_2 - V_{r1} \cdot r_1)$$



$$\boxed{\frac{Q}{2} = AV}$$

$$V = \frac{\left(\frac{Q}{2}\right)}{A}$$

$$V_{abs} = V_r \pm V_t$$

$$= V_r \pm \omega \times r$$

$$V_{abs} = \frac{Q/2}{\text{Area pipe}} \pm \omega \times r$$

Q11. $V_{abs \text{ at } A} = V_{rA} - V_{tA}$

$$= (Q) - \omega \times r$$

$$= \left[\frac{\frac{2.4 \times 10^{-3}}{2}}{1.2 \times 10^{-4}} \right] = [2.69 \times 0.2]$$

$$= 10 - 1.538$$

$$= \underline{8.462 \text{ m/s}}$$

Q12. $V_{\text{absolute}} = V_{AB} + V_{EB}$

$$= 10 + (7.69 \times 0.3)$$

$$= \underline{12.307 \text{ m/s}}$$

→ 5. LAMINAR FLOW ←

* Viscous Flow / Layer Flow / Laminar flow through pipes and parallel plates of incompressible fluids

5.1 Introduction

5.2 Reynold No and its Importance

5.3 Analysis of laminar flow (viscous flow through a circular pipe (shear stress distribution, velocity distribution, Pressure Head Loss, Power lost due to viscosity and Ratio of Maximum velocity to Mean velocity in pipe flow & b/w two parallel fixed plates flow).

* Laminar flow is a Type of flow which Rarely observed in real life. ex: ① Blood flow through veins.

② Rain water collected in reservoir / Percolation of water (seepage of water)

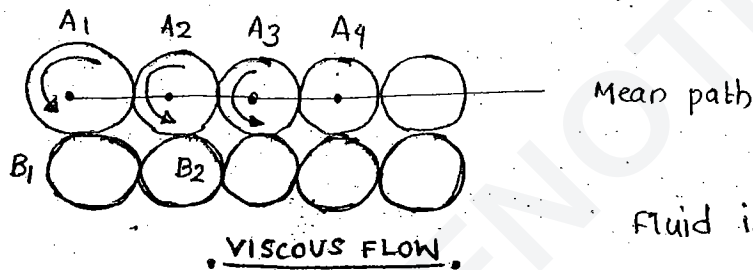
③ Lubrication for Hydrostatic Bearings

7. Sap of the plants

Make a drop of water on a dry leaf and notice change in color due to Laminar flow. slowly only this is a color change.

The following are the conditions under which flow is said to be a Laminar flow:

1. Each molecule of the fluid moves in its mean path.
2. Fluid flow is a Rotational Flow.
3. Flow is steady.
4. Flow is 2-D.
5. Fluid is viscous in Nature.
6. Reynold's No of Fluid Flow ≤ 2000



5.2 → REYNOLD NUMBER ←

It is the Ratio of Inertia Force to Viscous force

$$Re = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{m a}{L \cdot A} = \frac{m \left(\frac{v-u}{t} \right)}{\mu \frac{v}{y} \cdot A}$$

$$= \frac{m \left(\frac{v-0}{t} \right)}{\mu \frac{vA}{y}} = \frac{\rho \cdot A \cdot v}{\mu \frac{vA}{y}} = \frac{\rho A v}{\mu \frac{A}{y}}$$

$$= \frac{\rho v}{\frac{\mu}{y}} = \frac{\rho v y}{\mu}$$

Pipe flow $y = D$
Characteristic

dimension
= diameter

$$\boxed{Re_{\text{pipe}} = \frac{\rho v D}{\mu} = \frac{\rho D}{\eta}}$$

$$Re = \frac{VD}{\mu} \quad Re = \frac{VD}{\nu} \leq 2000$$

Laminar flow

$$Re_{critical} = 2000$$

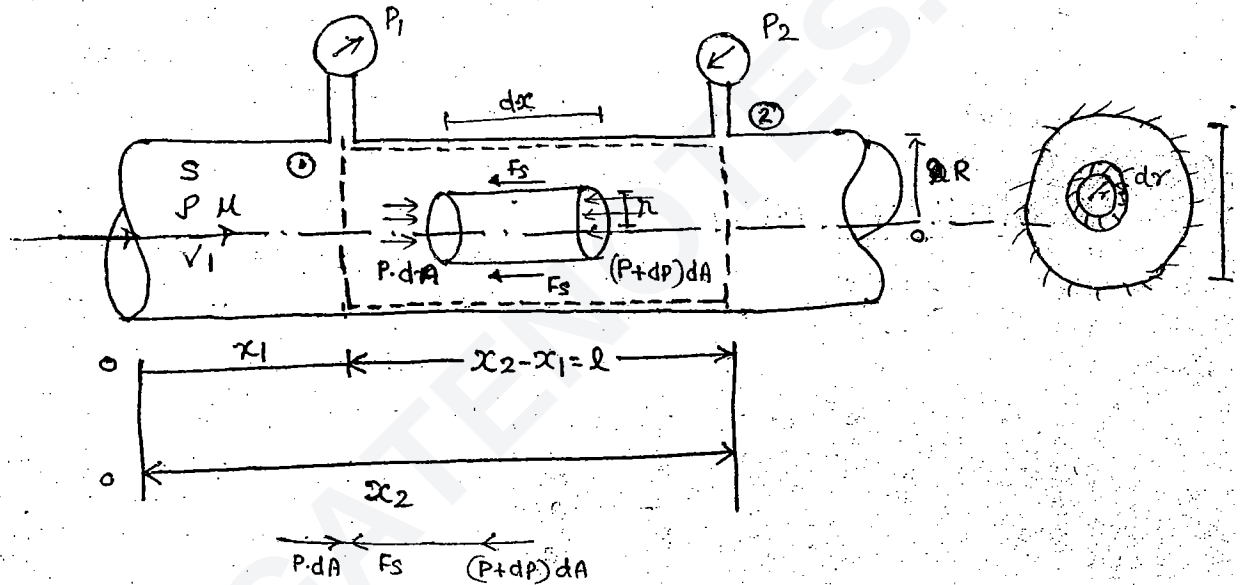
Flow through porous media (soil)

$$Re < 500$$

Flow through open channels

Viscous Flow Analysis:-

Consider a const dia of pipe 'D' and length 'l' subjected to viscous fluid flow shown in figure.



$D = d$	Physical characteristic
l	
S	Fluid self properties
ρ	
γ	
μ	
ν	

$\dot{m}$
 a
 v

$$\sum F = ma$$

Fluid is steady
so $a = 0$

$$P \cdot dA - F_s - (P + dp) \cdot dA = m \cdot 0$$

dx = length of the element

$$-F_s - dp \cdot dA = 0$$

$$-\tau \cdot dA_s - dp \cdot dA = 0$$

$$\tau \cdot dA_s = -dp \cdot dA$$

$$\tau \cdot \frac{2\pi r \cdot dx}{\text{shear}} = -dp \cdot \frac{\pi r^2}{\text{Normal Area}}$$

shear

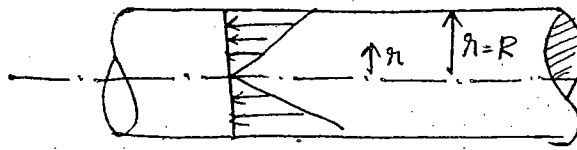
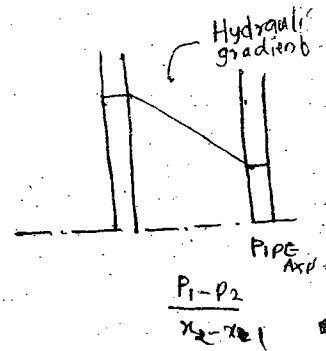
Normal Area

$$\tau = -\frac{dp}{dx} \cdot \frac{r}{2}$$

Pressure Gradient



LINEAR VARIATION



$$\tau_{\text{wall}} = -\frac{dp}{dx} \cdot \frac{R}{2}$$

τ is Max at the WALLS

$$\tau_{\text{centre}} = -\frac{dp}{dx} \times 0 = 0$$

$\tau = 0$ at centre since $r = 0$

Velocity distribution

we have

$$\tau_r = -\frac{dp}{dx} \cdot \frac{r}{2}$$

{ shear stress at any Radius }

we have Newtons law of viscosity

$$\tau = \mu \frac{du}{dy}$$

where $y = R - r$

y = dist. b/w the 2 layers or thickness of Layer

Diff y

$$dy = 0 - dr$$

$$dy = -dr$$

$$\tau_r = -\mu \frac{du}{dr}$$

$$-\mu \frac{du}{dr} = -\frac{dp}{dx} \cdot \frac{r}{2}$$

$$\mu \frac{du}{dr} = \frac{dp}{dx} \cdot \frac{r}{2}$$

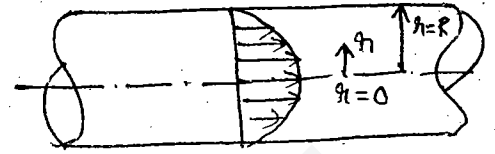
$$du = \frac{1}{\mu} \frac{dp}{dx} \cdot \frac{r}{2} dr$$

$$\int du = \frac{1}{2\mu} \frac{dp}{dx} \int r dr$$

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$$u_r = \frac{1}{2\mu} \frac{dp}{dx} \left(\frac{r^2}{2} \right) + \text{const}$$

$$u_r = \frac{1}{4\mu} \left(\frac{dp}{dx} \right) r^2 + C \quad \text{--- (2)}$$



using boundary conditions

At $r=R$,

$$u_r = 0$$

$$0 = \frac{1}{4\mu} \left(\frac{dp}{dx} \right) R^2 + C$$

$$C = -\frac{1}{4\mu} \left(\frac{dp}{dx} \right) R^2$$

velocity distribution

$$u_r = \frac{1}{4\mu} \left(\frac{dp}{dx} \right) r^2 - \frac{1}{4\mu} \left(\frac{dp}{dx} \right) R^2$$

$$u_r = \frac{1}{4\mu} \left(-\frac{dp}{dx} \right) \left[R^2 - r^2 \right]$$

At $r=0$

$u_r = u_{\max}$ (centre line velocity)

$$u_{\max} = \frac{1}{4\mu} \left(-\frac{dp}{dx} \right) \left[R^2 - 0 \right]$$

$$u_{\max} = \frac{1}{4\mu} \left(-\frac{dp}{dx} \right) R^2$$

$$u_r \propto r^2$$

PARABOLIC
VARIATION

III → Pressure drop

we have

$$u_{\max} = \frac{1}{4\mu} \left(-\frac{dp}{dx} \right) \cdot R^2$$

$$\int_{P_1}^{P_2} -dp = \frac{4\mu \cdot u_{\max}}{R^2} \int_{x_1=0}^{x_2=l} dx$$

$$-(P_2 - P_1) = \frac{4\mu u_{\max}}{R^2} [l]$$

$$P_1 - P_2 = \frac{4\mu u_{\max} \cdot l}{R^2}$$

$$P_1 - P_2 = \frac{16\mu \cdot u_{\max} \cdot l}{D^2}$$

IV. In Pipe Flow.

$$u_{\max} = 2V$$

Max vel = 2x Avg velocity

V

Parallel Plate flow

$$u_{\max} = 1.5V$$

Loss of Head due to viscosity

$$h_L = \frac{P_1 - P_2}{\rho g} = \frac{32 \mu V l}{\rho g D^2}$$

Hagen Poiseuille's Eqn

$$\text{Power Lost} = \dot{m} g h_L$$

A 50 mm internal dia of pipe of length 0.5 km is subjected to an oil flow ($S = 0.9$), $\mu = 1$ poise at the rate 210 l.p.m.

1. Mass density of oil in kg/m^3 ; g/cc or g/ml .

2. Weight density of oil in KN/m^3 .

3. dynamic viscosity of oil in S.I unit.

4. kinematic viscosity of oil in S.I unit and C.G.S unit.

5. Volume flow Rate of oil in the pipe in m^3/s .

6. Mass flow Rate of oil in kg/s .

7. Avg vel of oil flow in m/s .

8. Reynolds no. of oil flow in pipe.

9. state type of the oil flow (whether Laminar or Not)

10. Maximum velocity of the oil flow in pipe in lit/s .

11. Pressure drop due to viscosity in MPa .

12. Exit pressure at outlet of pipe in MPa .

13. Pressure Gradient (Hydraulic Gradient) in $\frac{\text{N}}{\text{m}^3}$ or $\frac{\text{kPa}}{\text{m}}$.

14. Loss of Head in 'm' of oil due to viscosity.

15. Loss of Head in m of water and in metres of mercury.

16. Power lost due to viscosity in kW .

17. Power Reqd to Run the pump to maintain the flow in H.P.

18. Velocity of the oil flow at 8mm from the surface of the pipe.

19. shear stress at the surface of pipe in N/m^2 .

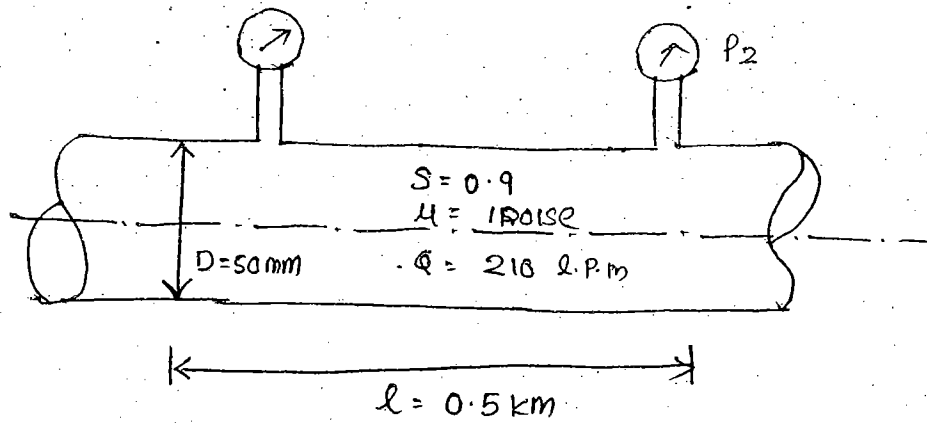
20. shear stress at 12mm from the centreline of the pipe.

21. Shear force on the surface of the pipe.

22. Draw the shear stress and velocity distribution.

23. At what Radial distance Avg vel occurs.

$$P_1 = 2 \text{ MPa}$$



$$(1) \quad S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$$

$$\rho_{oil} = 0.9 \times 1000 = \underline{900 \text{ kg/m}^3}$$

$$(2) \quad \gamma_{oil} = \rho_{oil} \cdot g$$

$$= 900 \times 9.81 \text{ N/m}^3$$

$$= \underline{883 \text{ kN/m}^3}$$

$$(3) \quad \mu = 1 \text{ poise} = \underline{0.1 \frac{\text{N} \cdot \text{s}}{\text{m}^2}}$$

$$(4) \quad \nu = \frac{\mu}{\rho} = \frac{0.1}{900} = \underline{1.11 \times 10^{-4} \text{ m}^2/\text{sec}}$$

$$= 1.11 \text{ cm}^2/\text{sec}$$

$$= \underline{1.11 \text{ stokes}} \quad (\text{c.g.s})$$

$$(5) \quad \dot{V} = Q = 210 \text{ l.p.m.}$$

$$= \frac{210}{60} \frac{\text{l}}{\text{s}}$$

$$= 3.5 \text{ lt/sec}$$

$$= 3.5 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$= \underline{0.0035 \text{ m}^3/\text{s}}$$

$$= 900 \times 0.0035 = \underline{3.15 \text{ kg/sec}}$$

(7)

$$Q = A \cdot V$$

$$0.0035 = \frac{\pi \times (0.05)^2}{4} \cdot V$$

$$\underline{V = 1.78 \text{ m/s}}$$

(8)

$$Re = \frac{V \cdot D}{\nu} = \frac{1.78 \times 0.05}{1.11 \times 10^{-4}}$$

$$= \underline{809} < 2000$$

(9)

Flow is Laminar

(10)

$$\left(\frac{U_{\max}}{V} \right)_{\text{pipe flow}} = 2$$

$$U_{\max} = 2 \times V = 2 \times 1.78$$

$$= \underline{3.56 \text{ m/s}}$$

(11)

$$P_1 - P_2 = \frac{32 \mu V L}{D^2} = \frac{32 \times 0.1 \times 1.78 \times 500}{(0.05)^2}$$

$$= 1.14 \times 10^6 \text{ N/m}^2 = \underline{1.14 \text{ MPa}}$$

(12)

Exit Pressure

$$P_1 - P_2 = 1.14 \text{ MPa}$$

$$2 - P_2 = 1.14$$

$$\therefore P_2 = 2 - 1.14 = \underline{0.86 \text{ MPa}}$$

(13)

Pressure Gradient (Hydraulic Gradient)

$$\frac{-dp}{dx} = \frac{P_1 - P_2}{x_2 - x_1} = \frac{1.14 \times 10^6}{500}$$

$$2.28 \times 10^3 \text{ N/m}^2 = \underline{2.28 \text{ kPa/m}}$$

(14)

$$h_1 - h_2 = \frac{P_1 - P_2}{\rho g} = \frac{1.14 \times 10^6}{1000 \times 9.81} = \underline{129 \text{ m}}$$

(15)

Loss of head in metres

$$P_1 - P_2 = \rho_{\text{oil}} \cdot g \cdot h_{\text{oil}} = \rho_{\text{water}} \cdot g \cdot h_w = \rho_{\text{mer}} \cdot g \cdot h_{\text{mer}}$$

$$1.14 \times 10^6 = 1000 \times 9.81 \times h_w$$

$$h_w = \underline{116.2 \text{ metres of water}}$$

$$1.14 \times 10^6 = 13600 \times 9.81 \times h_{\text{mercury}}$$

$$h_{\text{mercury}} = \underline{8.5 \text{ m of water}}$$

(16)

Power lost

$$P_{\text{lost}} = m \cdot g \cdot h$$

$$= 3.15 \times 9.81 \times 129$$

$$= 3990 \text{ watt}$$

$$= 3.99 \text{ kW} = \underline{4 \text{ kW}}$$

(17)

$$1 \text{ HP} = 0.736 \text{ kW}$$

Power required to run pump to maintain flow

$$= \frac{4 \text{ kW}}{0.736} = \underline{5.43 \text{ HP}}$$

(18)

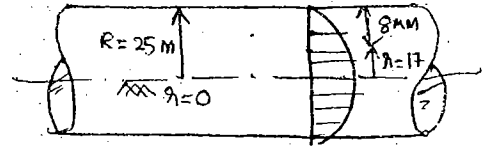
$$u_r = \frac{1}{4\mu} \left(\frac{-dp}{dx} \right) (R^2 - r^2)$$

$$u_{\text{max}} = \frac{1}{4\mu} \left(\frac{-dp}{dx} \right) R^2$$

$$u_r = R^2 - r^2 = 1 - r^2$$

$$\frac{u_r}{3.56} = 1 - \left(\frac{17}{25} \right)^2$$

$$\underline{u_r = 1.91 \text{ m/s}}$$



$$\text{Q19)} \quad \tau_r = - \left(\frac{dp}{dx} \right) \cdot \frac{r}{2}$$

$$\tau_{\text{wall}} = - \left(\frac{dp}{dx} \right) \cdot \frac{R}{2}$$

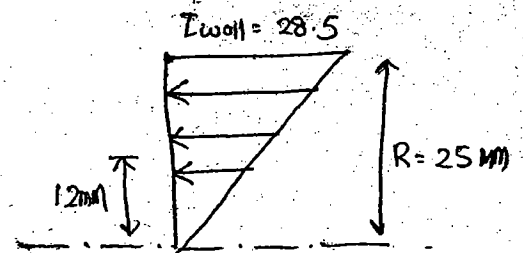
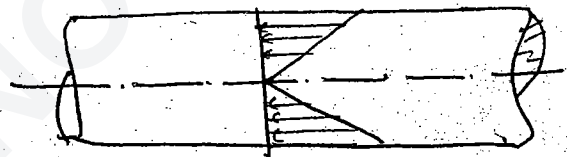
$$= (-2.28 \times 10^3) \left(\frac{0.05}{2} \right) = \underline{\underline{28.5 \text{ N/m}^2}}$$

20)

$$\tau_r = - \frac{dp}{dx} \cdot \frac{r}{2}$$

$$= \frac{28.5}{25} = \frac{\tau}{12}$$

$$\tau = \underline{\underline{13.68 \text{ N/m}^2}}$$



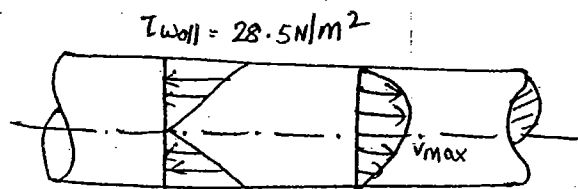
21)

$$F_{\text{S wall}} = \tau_{\text{wall}} \cdot A$$

$$= 28.5 \times \pi D L = 28.5 \times \pi \times 0.05 \times 500$$

$$= \underline{\underline{2238 \text{ N}}} = \underline{\underline{2.24 \text{ kN}}}$$

22)



S.S
distribution

Vel
distribution

23)

$$V_r = \frac{1}{4\mu} \left(- \frac{dp}{dx} \right) (R^2 - r^2)$$

$$\underline{u_{\text{max}}} = \frac{1}{4\mu} \left(- \frac{dp}{dx} \right) (R^2 - r^2)$$

$$\boxed{r_{\text{Av}} = 0.707 R}$$

$$\frac{1}{8\mu} \left(\frac{-dp}{dx} \right) R^2 = \frac{1}{4\mu} \left(\frac{-dp}{dx} \right) (R^2 - y^2)$$

$$\frac{1}{2} R^2 = R^2 - y^2$$

$$y^2 = R^2 - \frac{1}{2} R^2$$

$$y^2 = \frac{1}{2} R^2$$

$$y = \frac{1}{\sqrt{2}} R$$

Mean
vel occurs at
a distance of

$$y = 0.707R$$

$$\frac{u_r}{u_{max}} = 1 - \left(\frac{y}{R} \right)^2$$

$$\frac{1}{2} = \left(\frac{y}{R} \right)^2$$

$$y = \frac{R}{\sqrt{2}}$$

Key class work

Page No: 82

4.) B

5.) B (3, 4 & 5) correct

6.) C

7.) C

10.) C

11.) a

12.) C

13.) B

9.) C

8.) d

10.)

$$1) Q = A \cdot V$$

$$= \frac{\pi \times d^2}{4} \times V = \frac{\pi \times (d)^2}{4} \times V$$

For pipe flow

V_{max} = centre line velocity

$$Max\ vel = 1.5\ m/s$$

$$V_{max} = 2V$$

$$V = \frac{V_{max}}{2} = \underline{0.75\ m/s}$$

$$Q = \frac{\pi}{4} \times (4 \times 10^{-2})^2 \times 0.75$$

$$= \frac{3\pi}{10000}\ m^3/s$$

2) → FRICTION FACTOR

For Laminar flow

$$f = \frac{64}{Re}$$

$$Re = \frac{\rho V D}{\mu}$$

For Laminar

take $Re = 2000$

$$f = \frac{64}{2000} = \underline{0.032}$$

ISRO

Q8)

$$D = 100\ mm$$

$$l = 10\ m$$

$$V = 5\ m/s$$

$$\tau_{wall} = 250\ N/m^2$$

$$\tau_r = -\frac{dp}{dx} \times \frac{r}{2}$$

$$\tau_{wall} = -\left(\frac{dp}{dx}\right) \cdot \frac{R}{2}$$

$$dx = 10$$

$$250 = -\left[\frac{dp}{dx}\right] \times \frac{50 \times 10^{-3}}{2}$$

$$250 = \frac{-dp}{10} \times \frac{50 \times 10^{-3}}{2}$$

$$\underline{dp = 1 \times 10^5 \text{ N/m}^2}$$

$$(9) \quad \frac{dp}{dx} = \frac{N}{m^2 \times m} = \frac{\text{kg} \cdot \text{m}}{s^2 \times m^3}$$

$$= \frac{ML}{T^2 \times L^3}$$

$$= \underline{ML^{-2}T^{-2}} \quad (C)$$

$$(10) \quad \rho = 1000 \text{ kg/m}^3$$

$$Q = 800 \text{ mm}^3/s$$

$$L = 2 \text{ m} = 2000 \text{ mm}$$

$$D = 0.5 \text{ mm}$$

$$\mu = 3$$

$$\frac{dp}{dx} = 2 \text{ MPa}$$

$$\tau = \mu \frac{du}{dy} \quad dp = \frac{16 \mu \cdot u_{max} L}{D^2}$$

$$u_{max} = 2 \times V \text{ for pipe flow}$$

$$Q = A \times V =$$

$$V = \frac{800 \times 4}{\pi \times (0.5)^2} = \frac{4074.001771}{\pi} = 4.07 \text{ m/s} \quad 125$$

$$dP = \frac{32 \mu V L}{D^2}$$

$$Q = \frac{2}{10^{-6}} = \frac{32 \times 4 \times 4.07 \times 2}{(0.5 \times 10^{-3})^2}$$

$$\frac{N}{\text{mm}^2}$$

$$\frac{N}{10^{-6} \text{ m}^2}$$

$$\mu = \frac{\frac{2}{10^{-6}} \times (0.5 \times 10^{-3})^2}{32 \times 4.07 \times 2}$$

$$= 0.001919 \frac{\text{Ns}}{\text{m}^2}$$

(11)

$$V = 8 \omega$$

$$V = \frac{h \omega y}{h}$$

$$\omega y = \frac{V}{h}$$

$$\omega = 0 \quad \left\{ \frac{2}{D} \text{ flow} \right\}$$

→ SHEAR VELOCITY ←

$$V_* = \sqrt{\tau_0 / \rho} = V \sqrt{f/8}$$

SHEAR STRESS VARIATION

$$\tau = \tau_0 \frac{r}{R}$$

$$\tau_0 = \frac{8 \mu V}{D}$$

→ Pressure drop or Head loss due to viscosity

$$h_L = \frac{32 \mu V L}{\rho g D^2}$$

(X) Hagen Poiseles eqn

→ Darcy's Head loss:-

$$(h_f) = \frac{4 f L v^2}{2 g D}$$

$$\frac{4 f L v^2}{2 g D} = \frac{32 \mu V \cdot L}{\rho g \cdot D^2}$$

$$(4f) = \frac{32 \cdot \mu V \cdot L}{\rho \cdot g \cdot D^2} \cdot \frac{2 g D}{L v^2}$$

↓
Darcy's friction factor

$$4f = \frac{64 \mu}{\rho v D}$$

$$4f = \frac{64}{\frac{\rho v D}{\mu}}$$

$$4f = \frac{64}{Re}$$

(X)

Q. A 20cm dia pipe 20 km long laid horizontally carries oil ($s=0.9$) $\mu = 0.8$ poise. at the rate of 100 lit/sec. calculate the power reqd to run the pump in H.P to maintain flow against viscosity?

Ans: $P = \dot{m} g \cdot h_L$ or $\dot{m} g h_f$

$$h_f = \frac{4 f L v^2}{2 g D}$$

$$h_L = \frac{32 \mu V L}{\rho \cdot g \cdot D^2}$$

$$\mu = \frac{0.8}{10} \frac{\text{Ns}}{\text{m}^2}$$

$$S = 0.9$$

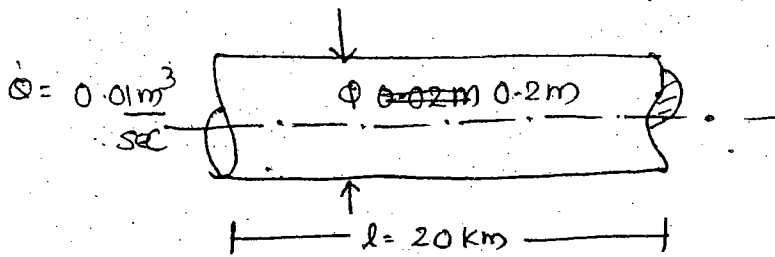
$$\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$$

oil

$$L = 20 \times 1000 \text{ m}$$

$$D = 0.2 \text{ m}$$

$$Q = 10 \frac{\text{lit}}{\text{sec}} = 10 \times 10^{-3} \frac{\text{m}^3}{\text{sec}}$$



$$1 \text{ HP} = 0.746 \text{ kW}$$

$$h_L = \frac{32 \times 0.08 \times 0.318 \times 20000}{900 \times 9.81 \times (0.2)^2}$$

$$45.87 \text{ m}$$

→ Power lost

$$P = \dot{m} g \cdot h_L$$

$$= 9 \times 9.81 \times 45.87 = 4049 \text{ W}$$

$$5 \frac{4 \text{ kW}}{0.746}$$

$$\frac{4}{0.746} = 5.5 \text{ HP}$$

$$\dot{m} = \rho \times Q$$

$$= \frac{900 \times 10 \times 10^{-3}}{9}$$

Classwork

3.

$$\phi: D = 0.2 \text{ m}$$

$$u_{\max} = 1 \text{ m/s}$$

$$u_{r=5} = 8$$

$$u_r = \frac{1}{4\mu} \left[\frac{-dp}{dx} \right] [R^2 - r^2]$$

$$u_{\max} = \frac{1}{4\mu} \left[\frac{-dp}{dx} \right] R^2$$

$$\frac{u_r}{u_{\max}} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R} \right)^2$$
$$= 1 - \left[\frac{5 \times 10^{-2}}{0.2} \right]^2$$

$$\frac{u_r}{1} = 1 - \left(\frac{5 \times 10^{-2}}{0.2} \right)^2$$

$$u_r = \underline{\hspace{2cm}}$$

Q.

A 400 m long horizontal pipe is to deliver 900 kg of oil ($s = 0.9$, $\nu = 2 \text{ stokes}$) per min. if head loss is not to exceed 8 m of oil find the dia of pipe & [friction factor]

$$f = \frac{64}{Re}$$

Ans: $L = 400 \text{ m}$

$$\dot{m} = 900 \frac{\text{kg}}{\text{min}} = \frac{900}{60} \frac{\text{kg}}{\text{s}} = \underline{\underline{15 \frac{\text{kg}}{\text{s}}}}$$

$$s = 0.9$$

$$\nu = 2 \times 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$\rho_{\text{oil}} = 900 \frac{\text{kg}}{\text{m}^3}$$

$$\text{STOKE} = \frac{\text{cm}^2}{\text{s}}$$

$$h_f = \frac{4FLV^2}{2gD}$$

~~80~~ ~~4x~~

$$Re = \frac{V \times D}{2 \times 10^{-4}}$$

$$Re = 0.5 \times 10^4 \times V \times D$$

$$8 = \frac{64}{0.5 \times 10^4 \times VD} \times \frac{400 \times V^2}{2 \times 9.81 \times D}$$

$$8 = \frac{V}{D^2} \quad \text{--- (2)}$$

$$D = \underline{0.162 \text{ m}}$$

5) oil is being pumped through a straight line pipe. The pipe length, diameter and volumetric flow rate are all doubled, for the new

arrangement of pipe. The pipe friction however remains constant. What is the ratio of frictional losses in the new arrangement to that in original pipe configuration.

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 2 d) 4.

$$L \rightarrow 2L$$

$$D \rightarrow 2D$$

$$Q \rightarrow 2Q$$

$$\frac{h_{f2}}{h_{f1}} = \frac{\frac{4FLV_2^2}{2gD}}{\frac{4F \times 2L \times V_2^2}{2g \times 2D}}$$

$$4F = \frac{64}{Re}$$

125

$$Re = \frac{\rho V D}{\mu} \text{ or } \frac{VD}{\nu}$$

$$V = 2$$

$$\dot{m} = \rho \cdot A \cdot V$$

$$15 = 900 \times \frac{\pi}{4} \times (d^2) \cdot V$$

$$V = \frac{60}{900 \times \pi d^2}$$

$$\dot{m} = \rho \cdot Q$$

$$Q = \frac{15}{900} = 0.016$$

$$0.016 = \frac{\pi}{4} \times D^2 \times V$$

$$VD^2 = \frac{0.016 \times 4}{\pi} \quad \text{--- (1)}$$

Solve 1 and 2

$$Q = \frac{A \cdot V}{\frac{\pi \times}{4}}$$

$$\frac{hf_2}{hf_1} = \frac{K \times \frac{4Q}{D^2} \times \frac{L}{D}}{2K \times \frac{Q}{4D^2} \times \frac{L}{2D}}$$

$$\frac{hf_2}{hf_1} = \underline{\underline{\frac{1}{4}}}$$

$$Q = \frac{\pi \times D^2 \times V}{4}$$

or

$$D_2 = 2D$$

$$2Q_2 = \frac{\pi \times 4D^2 \times V}{4}$$

discharge
doubled

Q) Two pipes of uniform c/s but different diameters carries water at same volumetric flow rate. water properties are same in two pipes. The Reynolds no Based on pipe dia. ~~is same in both the pipes~~

- a) same in both the pipes. b) Larger in Narrow pipe
c) smaller in narrow pipe d) depends on pipe material.

$$Re = \frac{\rho V D}{\mu} \quad \text{or} \quad \frac{V D}{\nu}$$

$$Re_1 = \frac{V D_1}{\nu_1} = \frac{\left(\frac{Q}{A}\right) \cdot D_1}{\nu_1}$$

$$Re_1 = \frac{Q \cdot D_1}{\frac{\pi \times D_1^2 \times \nu_1}{4}} = \frac{Q_1}{\frac{\pi \times D_1 \cdot \nu_1}{4}}$$

$$Re \propto \frac{1}{D}$$

so Larger pipes have small. Re

(b)

Q) A farmer uses a long horizontal pipe line to transport water with 1 HP pump and it discharges Q . If farmer replaces 1 HP motor pump by 5 HP water pump for the same pipe line and assuming the friction factor is unchanged then discharge through new arrangement will be -

- a) $5Q$ b) $\sqrt{5}Q$ c) $\sqrt{5} \cdot Q$ d) $\sqrt[3]{5} \cdot Q$

Ans:

$$P = \rho g h_f$$

$$m = \rho \cdot Q$$

$$\frac{P_1}{P_2} = \frac{h_{f1} \cdot Q_1}{h_{f2} \cdot Q_2}$$

$$\frac{1}{5} = \frac{\frac{4FLV_1^2}{2gD}}{\frac{4FLV_2^2}{2gD}} = \frac{V_1^2}{V_2^2}$$

$$\frac{1}{5} = \frac{\left[\frac{Q_1}{A}\right]^2 \cdot Q_1}{\left[\frac{Q_2}{A}\right]^2 \cdot Q_2}$$

$$Q_1 = A \times V_1$$

$$\frac{1}{5} = \frac{Q_1^3}{Q_2^3}$$

$$Q_2^3 = 5Q_1^3$$

$$Q_2^2 = 5 \cdot Q_1^2$$

$$Q_2 = \sqrt{5} \cdot Q$$

$$Q_2 = \sqrt[3]{5 \cdot Q}$$

(d)

5.

$$\frac{h_{f2}}{h_{f1}} = 2$$

$$h_f = \frac{4fLV^2}{2gD}$$

$$h_{f1} = \frac{4fLV^2}{2gD_1}$$

$$h_{f2} = 4f \times 2L \times \left[\frac{Q_2}{A_2}\right]^2$$

New Arrangement

$$L = 2L$$

$$D = 2D$$

$$A = 2A$$

$$h_{f2} = \frac{4 \times F \times 2L \times (2Q_1)^2}{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times 4D^4 \times 16}$$

$$= \frac{4F \times 2L \times 4Q_1^2}{2 \times g \times 2 \times \left[\frac{\pi}{4}\right]^2 \times 4D^5 \times 16}$$

$$\frac{4F \times 2L \times \left[\frac{2Q}{4}\right]^2}{2 \times 2D \times \left[\frac{\pi}{4}\right]^2}$$

$$\frac{h_{f2}}{h_{f1}} \Rightarrow \frac{\cancel{4F} \times \cancel{2L} \times 4Q_1^2}{16 \times \cancel{2} \times \cancel{g} \times \cancel{2} \times \left[\frac{\pi}{4}\right]^2 \times \cancel{4D^5} \times 16} \times \frac{\cancel{2} \times \cancel{g} \times \cancel{2D} \times \left[\frac{\pi}{4}\right]^2 \times \cancel{D^4}}{\cancel{4} \times \cancel{F} \times \cancel{L} \times Q_2^2 \times 16}$$

$$\frac{4Q_1^2}{16Q_2^2} = \frac{\cancel{4}}{\cancel{16}} = \frac{1}{4}$$

x _____ x

New Arrangement

$$v = \frac{2Q}{\frac{\pi}{4} \times (2D)^2} = \frac{2Q}{\frac{\pi}{4} \times 4D^2}$$

$$h_{f2} = \frac{4 \times F \times 2L \times (2Q)^2}{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times (4D^2)^2}$$

$$= \frac{4 \times F \times 2L \times 4Q^2}{2 \times g \times 2D \times \left[\frac{\pi}{4}\right]^2 \times 16D^4}$$

- G.1 Introduction
- G.2 Types of Losses in the pipe flow (Major losses & Minor losses)
- G.3 Power Transmitted by a pipe to carry the water
(Condition for Maximum power transmission and Max Efficiency)
- G.4 Pipe flow Network
(Flow in series and Parallel pipe combinations)

A Flow is said to be Turbulent if the following is observed.

- 1. Flow is fully developed
- 2. Flow is in zig-zag manner
- 3. Fluid molecules never travels in their mean path like Laminar flow
- 4. Turbulency is involved
- 5. viscosity of flow is increased due to turbulency (Eddy viscosity)
- 6. Reynolds No is more than 4000

For every attempt of water flow in a pipe is Bound to be turbulent flow for the following :-

$$Re_{PIPE} = \frac{VD}{\nu}$$

$$Re_{PIPE WATER} = \frac{V \cdot D}{\nu_{WATER}} = \frac{V \cdot D}{1 \times 10^{-6}}$$

$$= VD \times 10^6$$

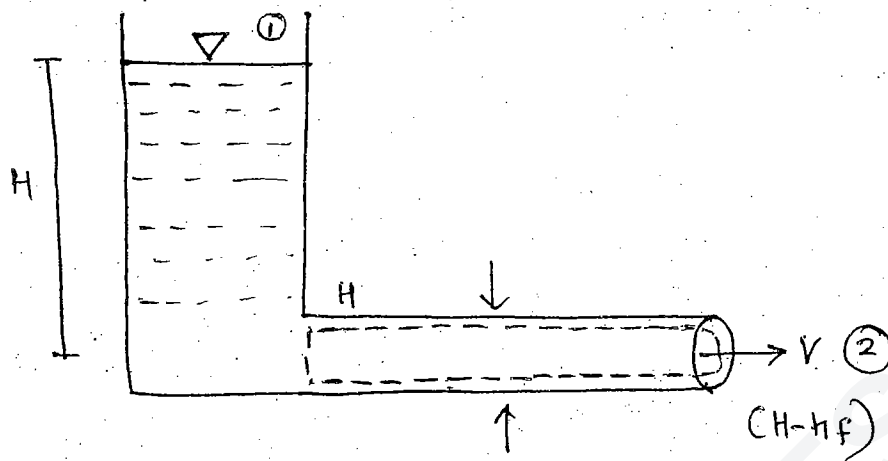
$$\begin{array}{cc} \swarrow & \searrow \\ 0.1 \text{ m/s} & 0.1 \text{ m} \end{array}$$

$$= \underline{\underline{10000}}$$

From the above it is observed that water Flow through pipe line is always a Turbulent flow. Hence a detailed

Analysis for water flow is Required.

→ 6.2 Power transmitted by a pipeline by carrying fluid :-



$$P_{o/p} = \dot{m} g (H - h_f)$$

$$\rho g (H - h_f)$$

$$\rho g \cdot A \cdot V (H - h_f)$$

$$\rho g A (H \cdot V - h_f V)$$

$$\rho g A \left(H \cdot V - \frac{4 f L V^2 \cdot V}{2 g D} \right) = \rho g A \left(H \cdot V - \frac{4 f L V^3}{2 g D} \right)$$

→ To Maximise power Transmission through pipes

$$\frac{d P_{o/p}}{d v} = \rho g A \left[H \cdot 1 - \frac{4 f L 3 V^2}{2 g D} \right]$$

$$= \rho g A [H - 3 h_f]$$

For Maximum Power transmission

$$\frac{d P_{o/p}}{d v} = 0 \Rightarrow$$

$$H - 3 h_f = 0$$

$$3 h_f = H$$

$$\boxed{h_f = \frac{1}{3} H}$$

$$\eta_{\text{PIPE}} = \frac{P_{0/p}}{P_{1/p}} = \frac{mg(H-h_f)}{mgH}$$

$$\eta = \frac{H-h_f}{H}$$

$$\eta_{\text{PIPE}} = 1 - \frac{h_f}{H}$$

$$= 1 - \frac{\frac{1}{3}H}{H}$$

$$\frac{3-1}{3} = \frac{2}{3} = 0.66$$

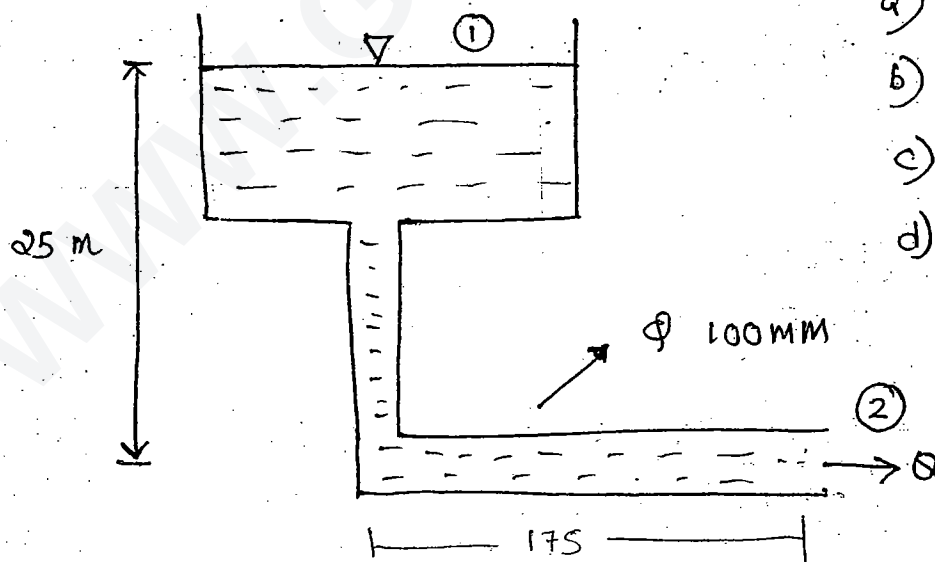
$$\eta_{\text{PIPE}} = 66\%$$

CARNOT PIPE

Q.)

Production

A PIPE ~~Production~~ system supplies water to a bend pipe shown in the figure. The pipe minor losses neglected and consider friction factor 0.03. Estimate maximum discharge Q at the end of the pipe.



- a) $Q = 31.7 \text{ lts}$
- b) $Q = 24 \text{ lps}$
- c) 15.9 lps
- d) $Q = 12 \text{ lps}$

Ans:

$$4f = 0.03$$

$$Q_2 = A_2 V_2$$

Apply Bernoulli's eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - h_f = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$0 + 0 + 25 - h_f = 0 + \frac{V_2^2}{2g} + 0$$

$$25 - \frac{4fLv_2^2}{2gD} = \frac{v_2^2}{2g}$$

$$25 - \frac{0.03 \times 175 \times v_2^2}{2 \times 9.81 \times 0.1} = \frac{v_2^2}{2 \times 9.81}$$

$$\underline{v_2 = 3.028 \text{ m/s}}$$

$$Q_2 = \frac{\pi}{4} \times (0.1)^2 \times 3.028 = \underline{0.024 \text{ m}^3/\text{sec}}$$

Q. Water is to be supplied to a college campus through one single main pipe.

1. Distance b/w Reservoir and college campus is 3km
 2. Loss of Head due to friction 18m
 3. Coeff of friction for the pipe flow = 0.007
 4. Consumption of water per student per day = 180 lit
 5. Daily pump operation is 8hrs - at ~~10%~~ ~~at~~ 50% capacity.
- The dia of the main pipe will be a

- a) 445mm b) 345mm c) 245mm d) 145mm ✓

Ans: $f = 0.007$

$$h_f = \frac{4fLv^2}{2gD}$$

$$D = 3$$

$$h_f = 18$$

$$18 = \frac{4 \times 0.007 \times 3000 \times v^2}{2 \times 9.81 \times D}$$

$$18 = \frac{4 \times 0.007 \times 3000 \times Q^2}{2 \times 9.81 \times \left[\frac{\pi}{4}\right]^2 D^5}$$

$$18 =$$

$$v = \left[\frac{Q}{A}\right]^2$$

$$\frac{Q}{\frac{\pi}{4} D^2}$$

$$v^2 = \frac{Q^2}{\left(\frac{\pi}{4}\right)^2 D^2}$$

$$Q = \frac{180 \text{ lit}}{\text{student in a day}}$$

$$Q = \frac{180 \text{ lit}}{\text{min}} \times 50\%$$

Student - 8hrs

$$\frac{180 \text{ lit}}{\text{min}} \times 0.5 \times 4000 \text{ students}$$

Student $8 \times 3600 \text{ s}$

$$\frac{180 \times 10^{-3} (\text{m}^3) \times 0.5 \times 4000}{8 \times 3600 \text{ s}}$$

$$= \frac{0.0125 \text{ m}^3}{\text{sec}}$$

$$18 = \frac{4 \times 0.007 \times 3000 \times \cancel{0.0125} (0.0125)^2}{2 \times 9.81 \times \left[\frac{\pi}{4} \right]^2 \times D^5}$$

$$D^5 = \underline{\hspace{2cm}}$$

$$D = 0.142 \text{ m say } \underline{145 \text{ mm}}$$

Q.)

2 MARKS

Water is Being pumped at the Rate of 1200 lit/min to an overhead Tank through 150mm dia pipe of length 300m. (out of which 15m above the pump centre line). Darcy's friction factor is 0.03. what is the

(a) ~~the~~ pressure developed by pump.

a) 1.85 bar b) 0.85 bar c) 3.65 bar d) None of these

(b) what is the Power Reqd to Run the pump in kw ?

Ans:

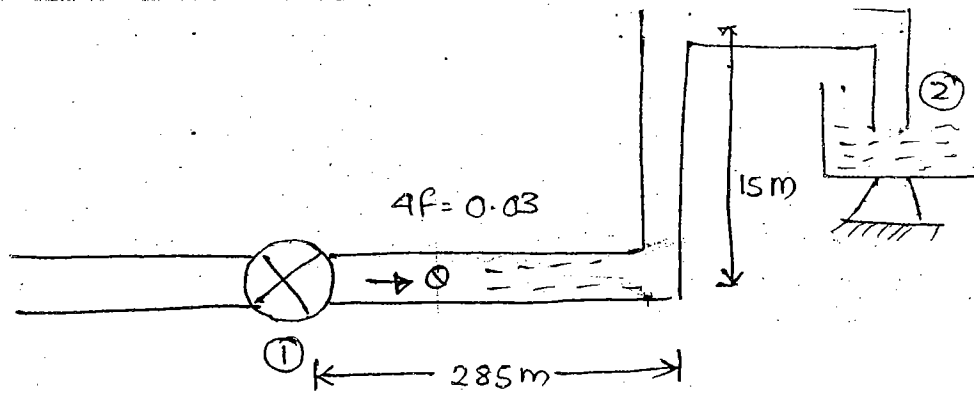
$$Q = \frac{1200 \text{ lit}}{\text{min}}$$

$$= \frac{1200 \times 10^{-3} \text{ m}^3}{60 \text{ s}}$$

Darcy friction factor

$$4f = 0.03$$

$$D = 150 \text{ mm}$$



At $Q = A \cdot V$

$$0.02 = \frac{\pi \times (0.15)^2 \cdot V}{4}$$

$$V = 1.137 \text{ m/sec}$$

$$Q = 1200 \text{ lpm}$$

$$\frac{1200 \times 10^{-3}}{60}$$

$$Q = 0.02 \text{ m}^3/\text{sec}$$

Apply B-E ① and ②

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho g} + 0 + 0 = \frac{4fLV^2}{2gD} = 0 + 15$$

$$\frac{P_1}{1000 \times 9.81} = \frac{0.03 \times 300 \times (1.137)^2}{2 \times 9.81 \times 0.15}$$

$$P_1 = 1.85 \times 10^5 \text{ N/m}^2$$

$$P_1 = 1.85 \text{ bar}$$

$$h_f = \frac{4fLV^2}{2gD} = \frac{0.03 \times 300 \times (1.137)^2}{2 \times 9.81 \times 0.15} = 3.9$$

Power required to run the pump

$$= \dot{m} g (H + h_f)$$

$$= \rho g Q (15 + 3.9)$$

$$= \underline{\underline{\text{Watt}}}$$

Class Work

①

$$Q = 0.4 \text{ cm}^2/\text{sec}$$

$$Q = 0.4 \times 10^{-4} \text{ m}^2/\text{sec}$$

$$D = 8 \text{ cm} = 0.08 \text{ m}$$

for pipe flow $V_{\max} = 2 V_{\text{avg}}$

$$V_{\text{avg}} = \frac{V_{\max}}{2}$$

$$Re = \frac{\rho V D}{\mu} \text{ or } \frac{V D}{\nu}$$

$$2000 =$$

* ~~Flow~~

→ LAMINAR FLOW CONTD. ←

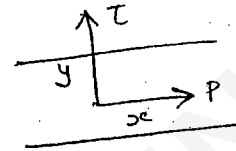
SOME XTRA POINTS

* Navier stoke eqn:-

$$\left(\frac{\partial P}{\partial x}\right) = \left(\frac{\partial \tau}{\partial y}\right)$$

It means Pressure Gradient in the direction of flow is equal to shear Gradient in the direction normal to the flow.

Velocity at a section



$$V = \frac{+1}{4\mu} \left[\frac{-dp}{dx} \right] [R^2 - y^2]$$

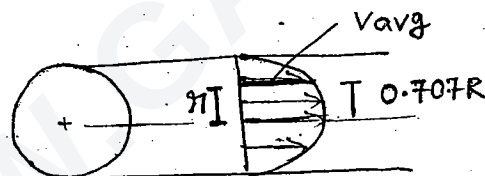
$$V_{max} = \frac{1}{4\mu} \left[\frac{-dp}{dx} \right] R^2$$

$$\frac{V}{V_{max}} = \left[1 - \left(\frac{y}{R} \right)^2 \right]$$

V = local velocity at a point whose Radius is ' y '.

V_{max} = centre line velocity which is maximum.

Since vel distribution is parabolic $V \propto y^2$



velocity is Average : (V_{mean}) at a distance $= \frac{R}{\sqrt{2}} = 0.707R$ from the centre of

$$V_{avg} = V_{max} \left[1 - \left(\frac{R}{\sqrt{2}R} \right)^2 \right]$$

$$= V_{max} \left[1 - \frac{1}{2} \right]$$

$$V_{avg} = \frac{V_{max}}{2}$$

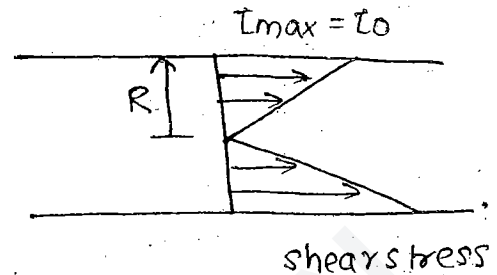
Shear stress distribution : —

Assertion: — vel is Maximum at centre, Zero at the boundary

the Boundary

$$\text{Velocity} \propto \frac{1}{\text{Shear stress}}$$

$$\tau_0 = \tau_{\max} = -\frac{dp}{dx} \left(\frac{R}{2} \right)$$



$\frac{dp}{dx}$ means change in pressure in a pipe of length dx or simply pressure Gradient

$$V = \frac{1}{4\mu} \left[-\frac{dp}{dx} \right] [R^2 - r^2]$$

If $r = 0$, $V = V_{\max}$

$$\text{So } V_{\max} = \frac{1}{4\mu} \left[-\frac{dp}{dx} \right] R^2$$

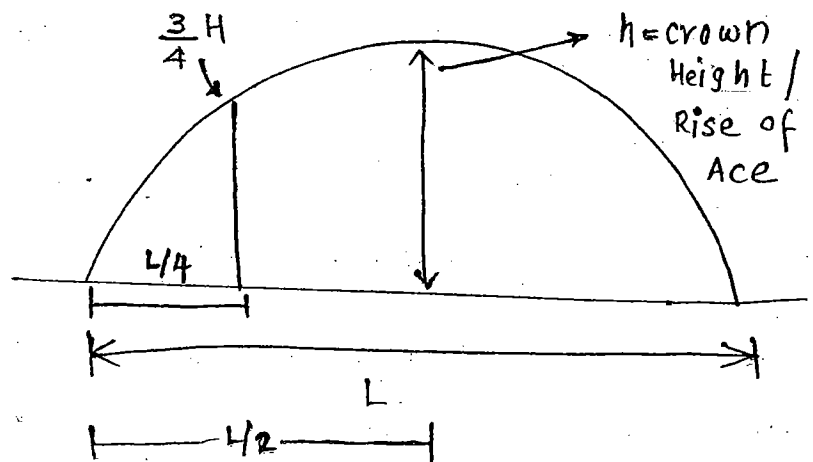
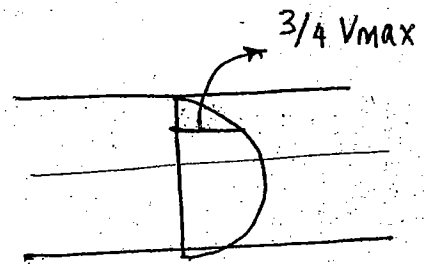
→ Velocity at Quarter Point

$$r = \frac{R}{2}$$

$$V = \frac{1}{4\mu} \left[-\frac{dp}{dx} \right] \left[R^2 - \frac{R^2}{4} \right]$$

$$= \frac{1}{4\mu} \left[-\frac{dp}{dx} \right] \left[\frac{3R^2}{4} \right]$$

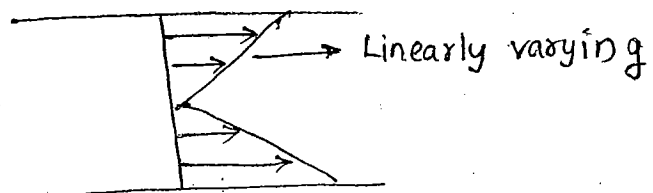
$$V = V_{\max} \times \frac{3}{4}$$



→ VARIATION OF SHEAR STRESS

$$\frac{\tau}{r} = \frac{\tau_{\max}}{R}$$

$$\tau \propto r$$



Shear stress varies linearly with distance from centre of pipe.

$$\tau_0 = \tau_{\max} = \frac{8\mu V}{D} = \frac{\rho f V^2}{8}$$

Laminar

Turbulent flow

* SHEAR VELOCITY (V^*)

$$V^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{8\mu V}{D\rho}} = \sqrt{\frac{8\nu V}{D}}$$

LAMINAR FLOW

$$V^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\rho f V^2}{8\rho}} = V \sqrt{\frac{f}{8}}$$

TURBULENT FLOW

Significance

Shear velocity is a fictional parameter. Shear vel is called so as it uses shear stress in its ~~direction~~ derivation & it has units same as that of velocity [Mathematical significance]

→ Physical significance:-

Shear velocity is used in classifying Turbulent flow into 3 types.

1. Hydrodynamically smooth
2. " Transition
3. " Rough

Shear velocity is used in Analysis of Boundary Layer concepts.

For eg: Laminar Sub Layer:

$$V_{1/2} = (F + \mu V_{1/2})$$

1 cc → 16

→ Relation b/w Friction Factor & Reynold No:

$$h_f = \frac{4fL V^2}{2gD} = \frac{32\mu V L}{\rho g D^2} \rightarrow \text{LAMINAR}$$

↓

TURBULENT

Equate Head loss due to friction of laminar and turbulent flow

$$h_f = \frac{4FLv^2}{2gD} = \frac{32\mu VL}{gD^2}$$

$$4F \times \frac{v}{2} = \frac{32\mu}{gD}$$

$$\frac{4FvD}{4} = \frac{64}{4F}$$

$$4F = \frac{64}{Re}$$

$4F$ = Darcy Friction factor

Coeff of Friction = f

$$F = \frac{64}{4Re}$$

$$F = \frac{16}{Re}$$

** In Laminar flow friction factor depends on Reynold No only. It is independent of Surface ~~st~~ Roughness.

* In Laminar flow $F \propto \frac{1}{Re}$

The Max possible Re is 2000. it is called critical Re for Laminar flow

$$\therefore f = \frac{64}{Re} = \frac{64}{2000} = \underline{0.032}$$

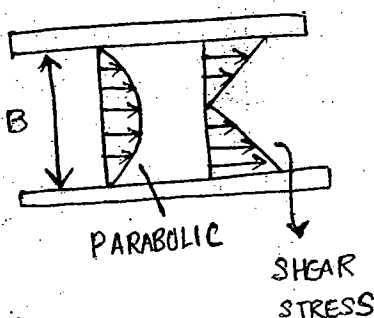
→ for Flow through circular pipes

Max permissible friction factor = 0.032

→ LAMINAR FLOW B/W PLATES ←

Both plate Fixed

eg: cracks in the wall and fluid moving (thick)

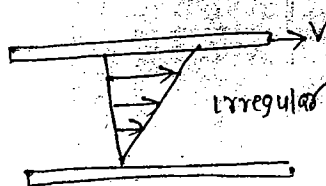


$$V_{avg} = \frac{2}{3} V_{max}$$

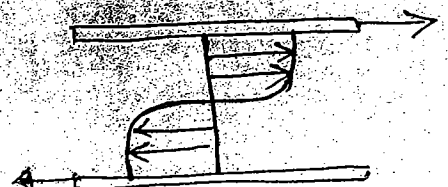
$$h_f = \frac{12\mu VL}{gD^2} \rightarrow \text{Gap b/w}$$

one plate Fixed and other Moving

⊗ Couette Flow



Both plate Moving



* Printing Press

$$\tau_0 = \tau_{\max} = \frac{G \mu V}{D}$$

→ Measurement of viscosity →

(1)

$$\tau = -\mu \frac{du}{dy}$$

Coaxial cylinder viscometer uses the concept of Newton's law of viscosity for measuring viscosity

(2) Hagen Poiseuille's eqn:-

$$h_f = \frac{32 \mu V L}{\rho g D^2}$$

** Saybolt viscometer [capillary type viscometer] uses Hagen Poiseuille's eqn

(3)

Stokes eqn:-

$$V = \frac{D^2}{18\mu} (\gamma_s - \gamma_f)$$

V = terminal velocity

γ_s = sp wt of settling particle

γ_f = sp " " fluid

" Falling sphere viscometer "

uses Stokes eqn for measuring viscosity

→ CREeping MOTION ←

If Re is < 1 such flow is called Creeping motion.

Q10) $\frac{dQ}{Q} = 25\%$

$\frac{d(4f)}{4f} = 2$

$$h_f = \frac{4f l v^2}{2gD} = \frac{4f \cdot l \left(\frac{Q}{A}\right)^2}{2 \times g \times D}$$

$$= \frac{4f l Q^2}{2 \times g \left[\frac{\pi}{4}\right]^2 \cdot D^2}$$

$$4f Q^2 = C$$

$$f \cdot Q^2 = \text{constant}$$

differentiate

$$f \cdot 2Q \cdot dQ + Q^2 \cdot d(f) = 0$$

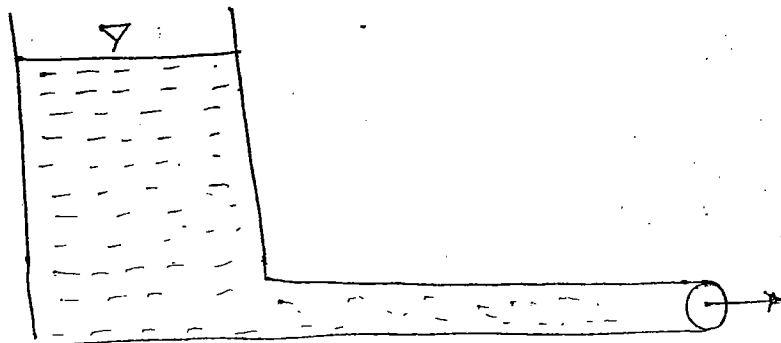
$$\frac{df}{f} = - \frac{2Q \cdot dQ}{Q^2}$$

$$\frac{df}{f} = -2 \left(\frac{dQ}{Q} \right)$$

$$\frac{df}{f} = -2 \times 25\% = \underline{\underline{-50\%}}$$

Q14

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$$P_{o/p} = \rho Q (H - h_f)$$

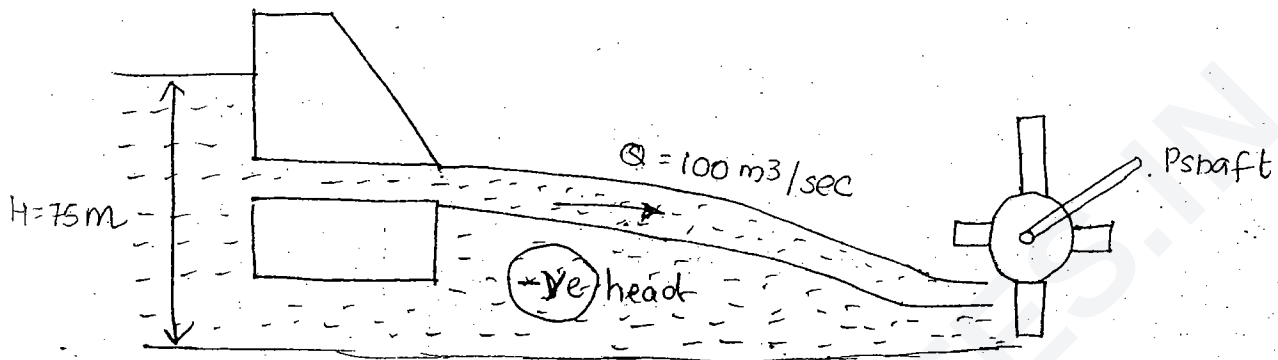
Maximum power ~~developed~~ transmitted by pipe

Condition is $h_f = \frac{H}{2}$

$$P_o/p = 1000 \times 9.81 \times 1 \times \left[99 - \frac{99}{3} \right]$$

$$= \underline{660 \times 10^3 \text{ W}} = \underline{660 \text{ kW}}$$

Q15



$$\eta_{\text{Turbine}} = \frac{P_{\text{shaft}}}{P_{\text{water}}} = \frac{P_{\text{shaft}}}{\rho g Q (H - h_f)}$$

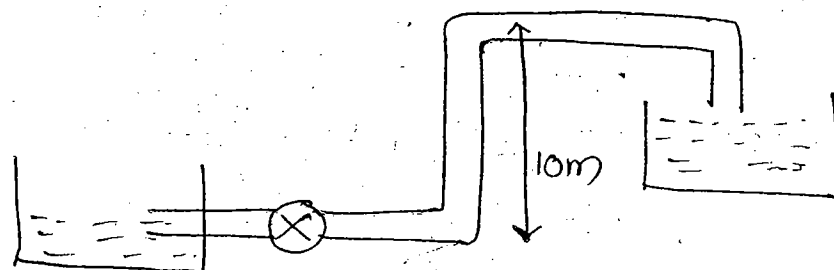
$$100\% = 1 = \frac{(P_{\text{shaft}})_{\text{theoretical}}}{1000 \times 10 \times 100 (H - 0)}$$

$$(P_{\text{shaft}})_{\text{theo}} = 1 \times 1000 \times 10 \times 1000 (75 - 0)$$

$$= 75000 \times 10^3 \text{ W}$$

$$= \underline{75000 \text{ kW}}$$

Q16



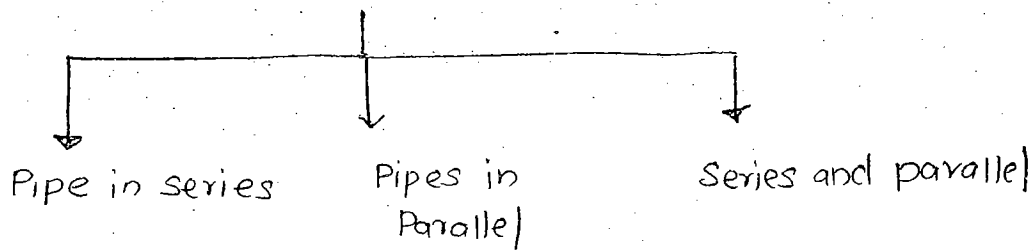
$$\eta_{\text{pump}} = \frac{P_{\text{water}}}{P_{\text{shaft}}} = \frac{\rho g Q (H + h_f)}{P_{\text{shaft}}}$$

$$100\% = 1 = 1000 \times 9.81 \times 0.1 \times (10 + 5)$$

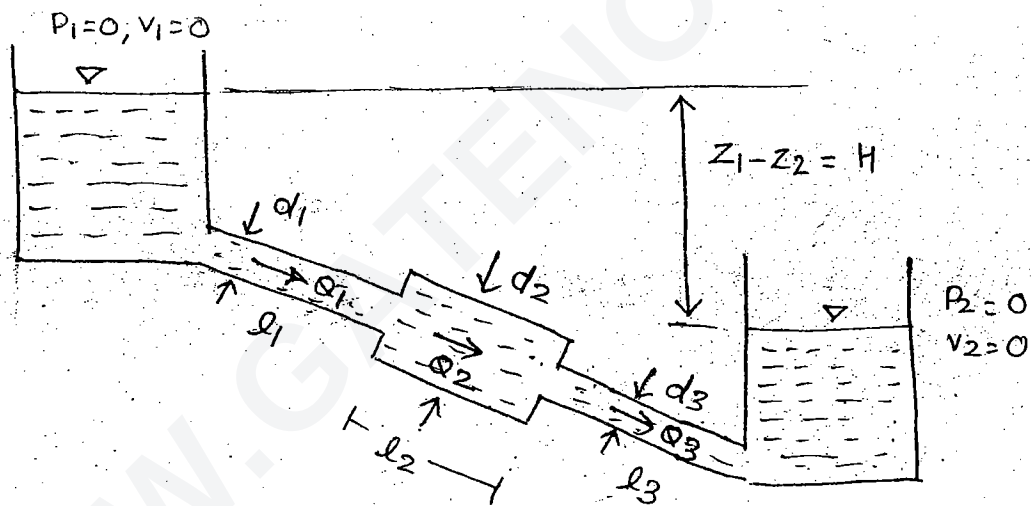
$$P_{\text{shaft}} = 1 \times 1000 \times 9.81 \times 0.1 \times 15 = 14.715 \text{ kW}$$

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PIPE NETWORK (Neglect Minor Losses)



When No of pipes are connected end to end, discharge in each pipe is same and loss of Head for all the pipe is the summation of the individual pipe head losses. Purpose is to carry the Fluid from one plate to another plate without changing discharge.



$$\textcircled{1} Q = Q_1 = Q_2 = Q_3$$

PIPES IN SERIES

Same discharge

STEP - I

By Continuity eqn,

$$Q = A_1 V_1 = Q_2 = A_2 V_2 = A_3 V_3$$

$$d_1^2 V_1 = d_2^2 V_2 = d_3^2 V_3$$

STEP - II

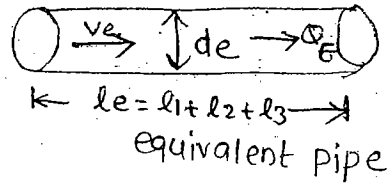
By Bernoulli's equation

$$-h_f + \frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

$$Z_1 - h_f = Z_2$$

$$Z_1 - Z_2 = h_f$$

$$H = h_f$$



STEP-III

$$H = h_f$$

$$= h_{f1} + h_{f2} + h_{f3}$$

$$H = h_{fe} = \frac{4f_1 l_1 v_1^2}{2g d_1} + \frac{4f_2 l_2 v_2^2}{2g d_2} + \frac{4f_3 l_3 v_3^2}{2g d_3}$$

$$\frac{4f_e l_e v_e^2}{2g d_e} = \frac{4f_1 l_1 v_1^2}{2g d_1} + \frac{4f_2 l_2 v_2^2}{2g d_2} + \frac{4f_3 l_3 v_3^2}{2g d_3}$$

Assume friction factor is same

$$4f_e = 4f_1 = 4f_2 = 4f_3$$

$$\frac{l_e v_e^2}{d_e} = \frac{l_1 v_1^2}{d_1} + \frac{l_2 v_2^2}{d_2} + \frac{l_3 v_3^2}{d_3}$$

$$v_1 = \frac{d_e^2 v_e}{d_1^2}$$

$$\frac{d_e^4 v_e^2}{d_1^4}$$

eliminate velocities v_1, v_2 and v_3 in terms of v_e

$$\boxed{\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}}$$

Equivalent Pipe (d_e)

& same

H is same

$$l_1 + l_2 = 6$$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5}$$

$$l_e = l_1 + l_2 = 2L$$

$$\frac{2L}{d_e^5} = \frac{L}{(10)^5} + \frac{L}{(20)^5}$$

$$\frac{2L}{d_e^5} = L \left[\frac{1}{10^5} + \frac{1}{20^5} \right]$$

$$d_e^5 = \frac{2 \times (10)^5 \times (20)^5}{(20)^5 + (10)^5} =$$

$$d_e = \underline{11.4} \text{ cm}$$

Q2.

$$R_A = R_B$$

$$d_A = 20\% \text{ more than } d_B$$

$$d_A = 1.2 d_B$$

$$h_f = \frac{4 f L v^2}{2 g d} = \frac{4 f L \left(\frac{Q}{A} \right)^2}{2 \times g \times d}$$

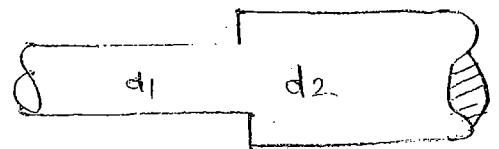
$$h_f = \frac{4 f L \cdot Q^2}{2 g \left[\frac{\pi}{4} \right]^2 \times d^5}$$

$$H = h_f = \frac{4 f L}{12.1} \frac{Q^2}{d^5}$$

~~at B~~

$$h_f \cdot d^5 = \text{const}$$

$$\frac{h_{fA}}{h_{fB}} = \left(\frac{d_B}{d_A} \right)^5 = \left(\frac{d_B}{1.2 d_B} \right)^5$$



$$4 f_1 = 4 f_2$$

$$l_1 = l_2$$

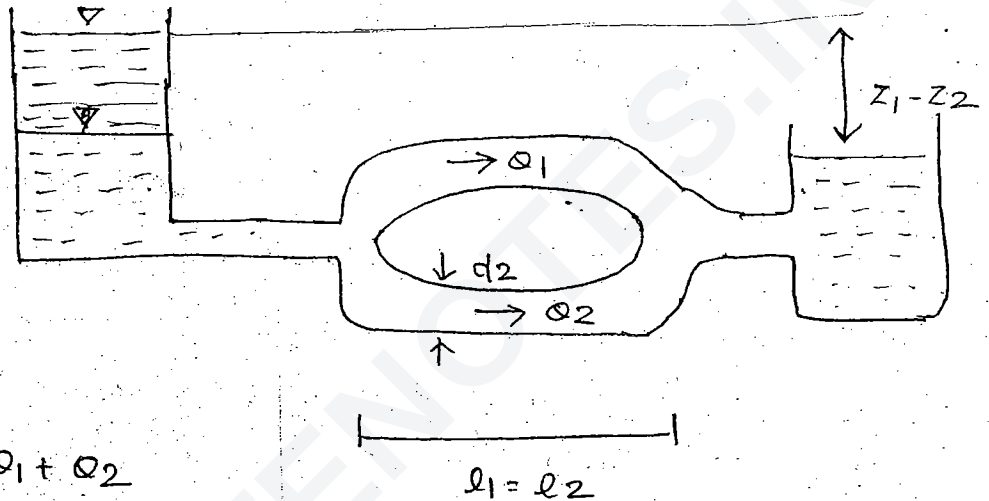
$$d_2 = 1.2 d_1$$

$$\frac{h_{fA}}{h_{fB}} = \left(\frac{1}{1.2}\right)^5$$

$$= \underline{\underline{0.402}}$$

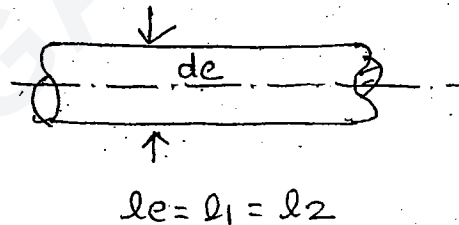
PIPES IN PARALLEL

When No of Pipes Connected in such a way that the discharge is distributing and head loss is same across each pipe



$$1) Q = Q_1 + Q_2$$

$$Q_e = A_1 V_1 + A_2 V_2$$



$$h_{f1} = h_{f2} = h_{fe}$$

$$\frac{4 f_e l_e V_e^2}{2 g d_e} = \frac{4 f_1 l_1 V_1^2}{2 g d_1} = \frac{4 f_2 l_2 V_2^2}{2 g d_2} \dots$$

$$\frac{l_e V_e^2}{d_e} = \frac{l_1 V_1^2}{d_1} = \frac{l_2 V_2^2}{d_2} \dots \text{--- (2)}$$

$$Q_e = A_1 V_1 + A_2 V_2$$

$$d_e^2 \cdot V_e = d_1^2 \cdot V_1 + d_2^2 \cdot V_2 \text{ --- (1)}$$

eliminate V_e from eq (1) and (2)

$$V_2 = \frac{d_e^2 v_e}{d_2^2} \quad V_1 = \frac{d_e^2 v_e}{d_1^2}$$

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$$d_e^{5/2} = d_1^{5/2} + d_2^{5/2}$$

conditions friction factor
same, length are equal

$$\text{If } d_1 = d_2 = d$$

$$(d_e)^{5/2} = 2(d)^{5/2}$$

$$d_e = (2)^{2/5} d$$

$$d_e = (2)^{0.4} d$$

for n pipes
of
equal dia

$$d_e = (n)^{0.4} d$$

Eg: $\rightarrow$ suppose

$x = 25\%$ more discharge

$$Q_e = 1.25 \times Q$$

$$d_e = \left(\frac{n}{1.25} \right)^{0.4} \times d$$

$$Q3.) \quad d_A = d_B$$

$$L_A = L_B$$

$$4f_A = 4 \times 4f_B$$

$$\frac{Q_A}{Q_B} = ?$$

for parallel pipes

$$h_{fA} = h_{fB}$$

$$\frac{4f_A \cdot Q_A / Q_A^2}{12.1 \cdot D_A^2} = \frac{4f_B \cdot Q_B / Q_B^2}{12.1 \cdot D_B^2}$$

$$\left(\frac{Q_A}{Q_B} \right)^2 = \frac{1}{4}$$

4:00
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Energy losses in pipes

Major losses (h_f)

This is due to friction

It can be calculated by

1. Darcy Weisbach Formula
2. Chezy's Formula

Darcy Weisbach Formula

$$h_f = \frac{4 f L V^2}{2 g D}$$

Chezy's Formula

$$V = C \sqrt{m i}$$

$$C = \text{Chezy's const} = \sqrt{\frac{49}{4 f}}$$

i = loss of head per unit length

$$i = \frac{h_f}{L}$$

m = Hydraulic mean depth

$$= \frac{\text{Area}}{\text{Perimeter (wetted)}} = \frac{\frac{\pi \times D^2}{4}}{\pi D} = \left(\frac{D}{4} \right) \text{ for pipes}$$

☆ Flow Through pipes:-

It is also called Pressure flow. Because in pipes the pressure

is other than atmospheric. It may be positive or negative

$$\text{Reynold No: } \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$\text{Inertia force} = m \times a$$

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$$= \rho \times V \left(\frac{v-u}{t} \right)$$

$$\rho L^3 \frac{v}{t} = \rho L^2 \times \frac{L}{t} \times v$$

$$\boxed{\text{Inertia force} = \rho L^2 v^2}$$

$$\text{Viscous force} = \tau \times A$$

$$= \mu \frac{du}{dy} \times A$$

$$= \mu \times \frac{v}{L} \times L^2$$

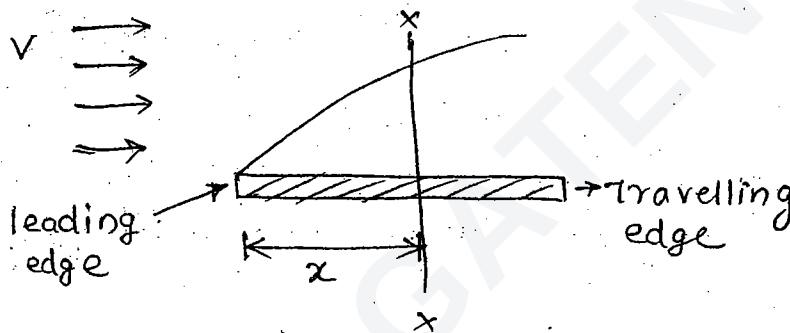
$$\boxed{\text{Viscous force} = \mu v L}$$

$$\text{Reynold No} = \frac{\rho L^2 v^2}{\mu v L} = \frac{\rho v D}{\mu}$$

L = characteristic dimension

for pipes it is diameter

= x for flow over the plates



L = characteristic dimension

$$Re_{(x-x)} = \frac{\rho v x}{\mu} = \frac{v x}{\nu}$$

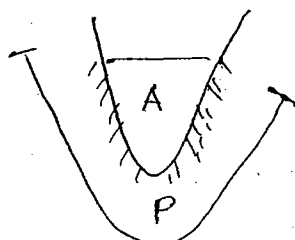
{ Reynolds No from leading edge }

$$L = 4 R_m$$

$L = 4 R_m$ in case of non-circular ~~plates~~ pipes

R_m = hydraulic mean Radius

$$R_m = \frac{\text{Wetted Area}}{\text{wetted perimeter}} = \frac{A}{P}$$

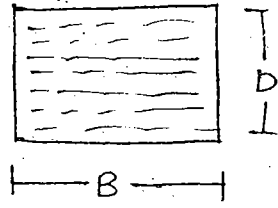


the Boundary length in contact with liquid is wetted perimeter



$$\text{For pipes } R_m = \frac{\pi/4 D^2}{\pi D} = \frac{D}{4}$$

Q) A Rectangular duct of width B and height 'D' is running full of water. cal. the Reynolds no. of flow in terms of ρ, V, μ .



Ans: Hydraulic mean Radius $R_m = \frac{A}{P} \Rightarrow \frac{\text{wetted Area}}{\text{wetted Perimeter}}$

$$= \frac{BD}{2(B+D)}$$

$$Re = \frac{\rho V L}{\mu} = \frac{\rho V (4R_m)}{\mu}$$

$$= \frac{\rho V 4 \cdot \frac{BD}{2(B+D)}}{\mu} = \frac{2 \rho V \cdot BD}{\mu (B+D)}$$

$$= \frac{2000 VBD}{\mu (B+D)}$$

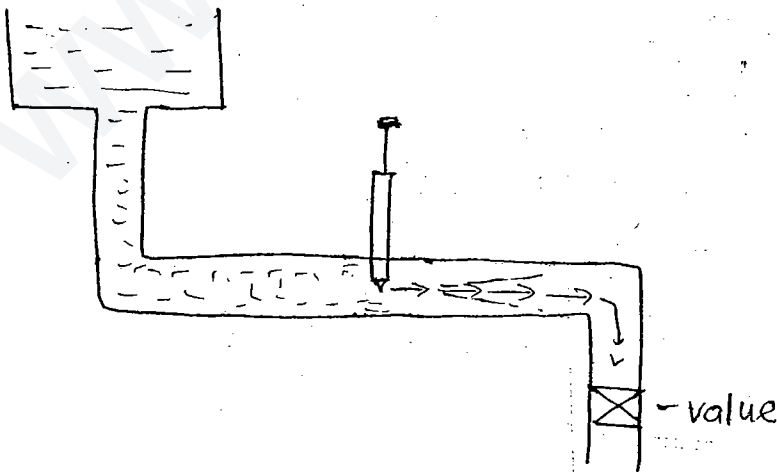
Reynolds Experiment:-

Aniline is used. It has same specific weight as that of water

Layered motion = laminar

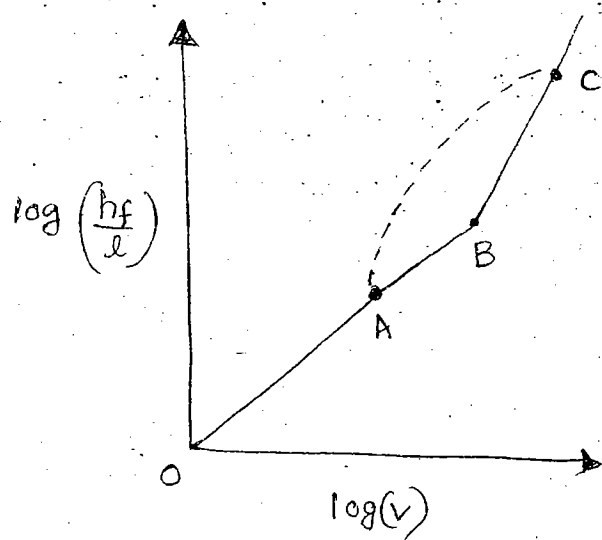
Intermixing = turbulent (zigzag)

transitional = slightly dispersed



Reynold chart:-

log-log plot ★★



A = Lower critical point

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B = upper critical point

C = Limit of transition

OA = Laminar flow

AB = Transition

BC = Turbulent

- * In Engg purpose Lower critical point is of greater importance used for defining Laminar flow for flow through circular pipe

if $Re \leq 2000$ laminar

$2000 < Re < 4000$ Transitional

$Re \geq 4000$ Turbulent

From Reynold chart

$$h_f \propto v^n$$

$n=1$ for Laminar flow $h_f \propto v$

$n=2$ for Turbulent (Rough pipes)

$n=1.75$ for smooth pipes Turbulent

General Formula:-

$$h_f \propto v^n$$

$n=1.75$ to 2 For Turbulent flow

* Miner's losses

* They are those losses whose Contribution is less than 5% of total losses.

* elongation ductile materials have more than 15% elongation.

Brittle materials have less than 5% elongation. If percentage

is ~~in~~ of elongation is in b/w 5-15% it is called

material of intermediate ductility

N.B! →

Major ~~loss~~ elongation of M.S = 35%

→ Major loss [frictional loss]

$$h_f = \frac{32 \mu V L}{\rho g D^2} \quad \left[\begin{array}{l} \text{LAMINAR FLOW THROUGH} \\ \text{OILY PIPES} \end{array} \right]$$

* In Laminar flow viscous friction (μ) is the Reason for Friction loss

$$h_f = \frac{128 \mu Q L}{\pi D^4}$$

$$\left\{ V = \frac{4Q}{\pi D^2} \right\}$$

$$Q = A \cdot V$$

$$V = \frac{Q}{A}$$

i) $h_f \propto V^n$, $n=1$ [Laminar]

ii) $h_f \propto \mu$ [h_f proportional to viscosity of liquid]

∴ as temp ↑ μ ↓ ∴ Hence frictional losses ↓

iii) Frictional losses of laminar flow is independent of nature of surface of inner pipe Boundary.

iv) for a given discharge

$$h_f \propto \frac{1}{D^4}$$

As dia/area of pipe ↑
Frictional loss ↓

v) $h_f \propto L$ is when length ↑ Frictional loss ↑

vi) $\frac{h_f}{L}$ = hydraulic gradient

$$\frac{h_f}{L} = \frac{32 \mu V}{\rho g D^2}$$

$$\frac{\Delta P}{\gamma} = \frac{32 \mu V L}{\rho g D^2}$$

$$\Delta P = \frac{32 \mu V L}{D^2} = \frac{128 \mu Q L}{\pi D^4} = \underline{\underline{P_2 - P_1}}$$

$$P = \rho g h_f$$

$$P = \rho Q g h_f$$

$$P = \gamma Q [h_f] = \gamma Q \times \frac{128 \mu Q L}{\pi D^4}$$

$$= \frac{128 \mu Q^2 L}{\pi D^4} = Q \Delta P$$

$$= \frac{Q(P_1 - P_2)}{\gamma D^2}$$

→ Head loss due to friction in Turbulent flow

$$h_f = \frac{4fLV^2}{2gD} \quad \text{Darcy weisbach eqn}$$

$$\Rightarrow h_f \propto V^n \quad \text{where } n = 2$$

actually n varies from 1.75 to 2

$n = 1.75$ smooth pipe

$n = 2$ Rough pipe

- $f \propto V^n$
- independent of Pressure
- depend on Nature of surfaces in contact

ii) h_f depends on Nature of surfaces (f). i.e. roughness protrusion

With Age the inside boundary of pipe gets collected with dust particles, debris etc - so Roughness $\uparrow$. so Friction factor increases.

iii) h_f is independent of temperature because it is independent of viscosity.

$$iv) h_f \propto L$$

$$v) h_f \propto \frac{1}{D^5} \quad \text{As Dia } \uparrow \quad h_f \downarrow$$

$$vi) \text{ Hydraulic Gradient } \frac{H_f}{L} \Rightarrow \frac{4fV^2}{2gD}$$

$$\Rightarrow \frac{4fQ^2}{2 \times 9.81 \times \frac{\pi^2}{4} D^5} \Rightarrow \frac{8fQ^2}{\pi^2 D^5 \times 9.81}$$



2 MARKS

- 3.1 Introduction
- 3.2 classification of fluid flows
- 3.3 continuity eqn
- 3.4 Fluid particle motion in space coordinate (Cartesian coordinates)
- 3.5 Equation of fluid motion (stream line eqn, velocity and acceleration of fluid)
- 3.6 Fluid ~~flow~~ field functions [stream fn and vel potential fn]
- 3.7 Angular speed of components of fluid flow in space
- (vorticity, discharge blow & points, shear strain rate of fluid flow)

Introduction:-

Fluid kinematics is a branch of fluid science which deals with motion of fluid in space which considering forces causing motion. Hence fluid kinematics is restricted to study of displacement, velocity, acceleration, mass flow rates and discharge in given space.

Types of fluid flows:-

1. steady and unsteady flow
2. uniform and non uniform flow
3. laminar and turbulent flow
4. compressible and incompressible fluid flow
5. 1-D, 2-D, 3-D fluid flows
6. Rotational and irrotational flow.

→ Steady flow:-

It is the one in which the fluid characteristics (flow or self properties) remains const. w.r.t time

$$\frac{\partial \psi}{\partial t} = 0 \quad \text{ie}$$

$$\frac{\partial p}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \mu}{\partial t} = 0$$

(b) unsteady Flow:-

The properties change w.r.t time

$$\frac{\partial v}{\partial t} \neq 0, \frac{\partial p}{\partial t} \neq 0 \dots$$

2a) uniform flow:-

Fluid properties w.r.t space coordinates remains const

$$\frac{\partial v}{\partial s} = 0$$

$\rightarrow$ Space coordinates

$$s = f(x, y, z)$$

eg: flow through a const dia pipe

2b)

Non uniform flow:-

Flow properties change w.r.t space coordinates

$$\frac{\partial v}{\partial s} \neq 0$$

eg: Flow through variable dia pipe

3a)

Compressible Flow:-

Flow in which density of fluid change at every location of flow.

eg: Air & other gas flow.

3b)

Incompressible Flow:-

Flow in which the density ρ of fluid remains

(ch)

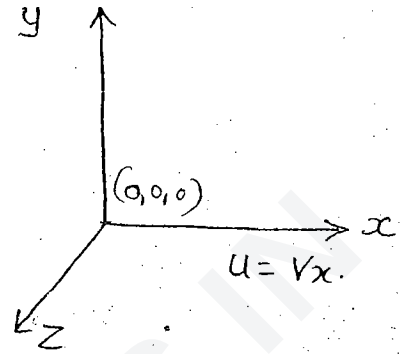
const at all locations. eg: oil, water, mercury etc.

7.4) 1-D Flow:-

In this flow the velocity of fluid flow in one direction is considered. all the other directions are ignored.

$$\text{velocity} = f(x, y, z, t)$$

$$\begin{aligned}\vec{V} &= (\vec{u})\hat{i} + (\vec{v})\hat{j} + (\vec{w})\hat{k} \\ &= (\vec{V}_x)\hat{i} + (\vec{V}_y)\hat{j} + (\vec{V}_z)\hat{k}\end{aligned}$$



for 1D flow:-

$$V = f(x, 0, 0) \rightarrow \text{1D steady flow}$$

$$V = f(x, 0, 0, t) \rightarrow \text{1D unsteady flow}$$

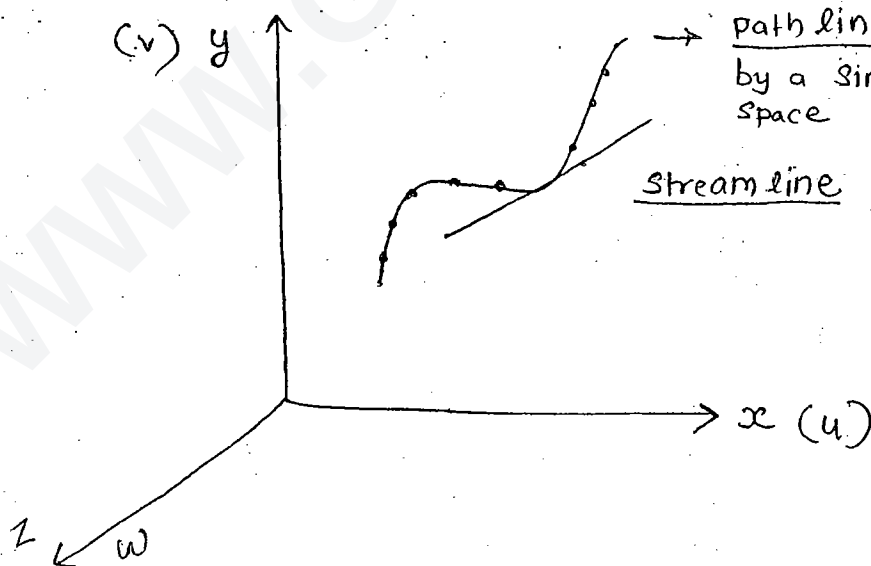
for 2D flow:-

$$V = f(x, y, 0, 0) \rightarrow \text{2D steady flow}$$

$$V = f(x, y, 0, t) \rightarrow \text{2D unsteady flow}$$

3.3

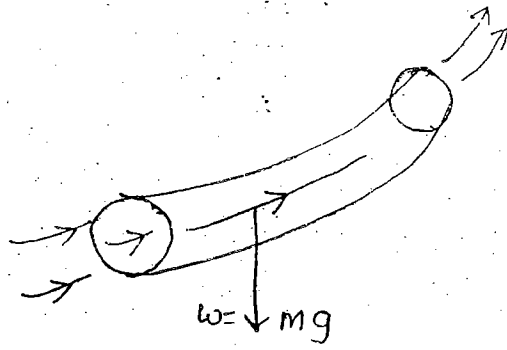
Fluid particle in space



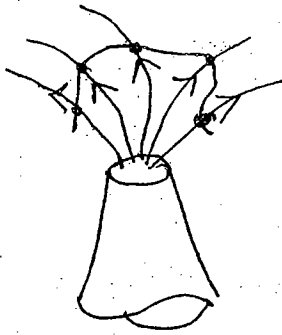
→ path line:- A line traced by a single fluid particle in space

Streamline : A line tangent at a point in flow

Stream Tube : combination of streamlines across which no flow takes place.



→ Streakline: - It is a line which traces the different fluid particles at given time



Streakline

Along x direction : $u = \frac{dx}{dt}$ — (1)

Along y direction : $v = \frac{dy}{dt}$ — (2)

$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

for 3-D

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

→ Acceleration: ↓

$$\vec{a} = (a_x)\hat{i} + (a_y)\hat{j} + (a_z)\hat{k}$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \left(\frac{\partial u}{\partial t} \right)$$

Local Acceleration

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Page 42

Q NO1

$$u = -x$$

$$v = 2xy$$

(1,1)

$$\text{we have } u = u_x = \frac{dx}{dt} = -x$$

$$v = v_y = \frac{dy}{dt} = 2y$$

$$\frac{u}{v} = \frac{dx}{dy}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{-x}{dx} = \frac{2y}{dy}$$

$$+\frac{dx}{x} = -\frac{dy}{2y}$$

$$+\log_e x = -\frac{1}{2} \log y + \text{constant}$$

$$\log_e x + \frac{1}{2} \log_e y = \text{const}$$

$$\log_e x + \log_e \sqrt{y} = \log_e C$$

$$\log x \sqrt{y} = \log_e C$$

$$\underline{x\sqrt{y} = C}$$

$$\text{at } (1,1) = \underline{C=1}$$

Q No: 2 $\vec{V} = u\hat{i} + v\hat{j}$

$$|\vec{V}| = \sqrt{u^2 + v^2}$$

$$V = 4\sqrt{x^2 + y^2}$$

$$V = 4\sqrt{16+9} = \underline{\underline{4 \times 5 = 20}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt} 4\sqrt{x^2 + y^2}$$

$$V = \sqrt{(4x)^2 + (-4y)^2}$$

$$V = \sqrt{u^2 + v^2}$$

$$u = 4x$$

$$v = -4y$$

continuity eqn: conservation of mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\vec{a} = (\vec{a_x})\hat{i} + (\vec{a_y})\hat{j}$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$$

$$= 4x \cdot 4 + (-4y) \cdot (0) + 0 = \underline{\underline{16x}}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$= 4x \times 0 + (-4y) \times (-4) + 0 = \underline{\underline{16y}}$$

$$\vec{a} = (\vec{a_x})\hat{i} + (\vec{a_y})\hat{j}$$

$$= 16x\hat{i} + 16y\hat{j}$$

$$\text{At } (x, y) = (4, 3)$$

$$= (16 \times 4) \hat{i} + (16 \times 3) \hat{j}$$

$$= 64 \hat{i} + 48 \hat{j}$$

$$|a| = \sqrt{64^2 + 48^2}$$

$$\text{Q} \quad \vec{v} = (u) \hat{i} + (v) \hat{j}$$

$$\vec{v}_{(x,y)} = (4x) \hat{i} + (4y) \hat{j}$$

$$= 16 \hat{i} + 12 \hat{j}$$

$$|v| = \sqrt{16^2 + 12^2}$$

$$= \underline{\underline{20 \text{ m/sec}}}$$

$$\textcircled{3} \quad v = 2x \hat{i} + y \hat{j}$$

$$u = 2x$$

$$v = y$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 2x \times 2 + 0 = 4x$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 0 + y = \underline{\underline{y}}$$

$$a = (a_x) \hat{i} + (a_y) \hat{j}$$

$$= 4x \hat{i} + y \hat{j}$$

$$\text{At } (1,1) = 4 \hat{i} + \hat{j}$$

$$|a| = \sqrt{16 + 1} = \underline{\underline{\sqrt{17}}}$$

(28)

$$\vec{V} = 2\hat{i} + 3\hat{j}$$

~~$u = 2$~~

~~$v = 3$~~

~~$\frac{u}{v} = \frac{dx}{dy}$~~

$$\frac{2}{3} = \frac{dx}{dy}$$

$$\underline{3dx - 2dy = 0}$$

(29)

(26)

$$V = 2y\hat{i} + 3x\hat{j}$$

$$u = 2y$$

$$v = 3x$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}$$

$$= \cancel{2y \times 0} + \cancel{3x \times 0} = 0$$

$$2y \times 0 + 3x \times 2$$

$$= \underline{6x}$$

$$\text{At } (1,1) = \underline{6 \text{ m/s}^2}$$

(27)

$$u = 1.5x$$

$$v = 0$$

$$\vec{V} = 1.5x\hat{i}$$

$$\text{At } (x,y) = (1,0)$$

$$v_y = v = 0$$

$$u = \frac{dx}{dt}$$

$$\frac{u}{v} = \frac{dx}{dy}$$

$$v = \frac{dy}{dt}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

In order to have fluid flow condition is

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

$$\frac{\partial}{\partial x}(1.5x) = -\frac{\partial}{\partial y}(v)$$

$$1.5 = -\frac{\partial v}{\partial y}$$

$$1.5 = -\frac{\partial v}{\partial y}$$

$$1.5 dy = -\partial v$$

Integrating

$$1.5y = -v + \text{constant}$$

$$v = -1.5y + \text{constant}$$

$$v = -1.5y + 0$$

$$\underline{v = -1.5y}$$

Boundary conditions

$$(x, y) = (1, 0)$$

$$0 = -1.5 \times 0 + C$$

$$\underline{C = 0}$$

$$(14) u = 6xy - 2x^2$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

$$6y - 4x = -\frac{\partial v}{\partial y}$$

$$dv = -(6y - 4x) dy$$

$$\text{Integrating } v = -\left[\frac{6y^2}{2} - 4xy\right]$$

$$\underline{v = 4xy - 3y^2}$$

(12)

$$V \propto \frac{1}{r^2}$$

$$V = \frac{C}{r^2}$$

$$a = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial r} \cdot \frac{\partial r}{\partial t}$$

$$= \frac{\partial v}{\partial r} \cdot r$$

$$a = v \cdot \frac{\partial v}{\partial x}$$

$$a = \frac{c}{x^2} \cdot \frac{\partial}{\partial x} \left[\frac{c}{x^2} \right]$$

$$= \frac{c}{x^2} \cdot c \frac{\partial}{\partial x} [x^{-2}] = \frac{c^2}{x^2} \cdot -2x^{-3}$$

$$= \frac{-2c^2}{x^2 x^3} = \frac{-2c^2}{x^5}$$

$$a = \frac{-2c^2}{x^5}$$

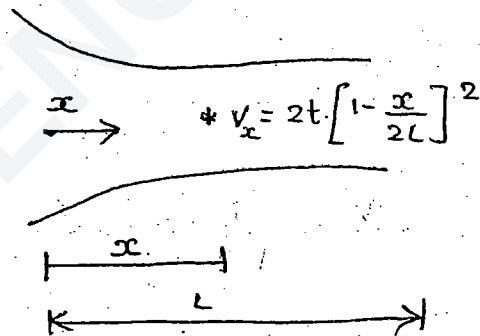
$$\boxed{a \propto \frac{1}{x^5}}$$

KE

$$(8) \quad u = 2t \left[1 - \frac{x}{2L} \right]^2$$

Total Acceleration

= convective Acc'n + Local Acc'n.



$$a = a_x + a_t$$

$$= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

Conv Acc'n

Local Acc'n

$$\text{Local Acc'n} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left[2t \left(1 - \frac{x}{2L} \right)^2 \right]$$

$$= 2 \times 1 \cdot \left[1 - \frac{x}{2L} \right]^2$$

$$= 2 \left[1 - \frac{x}{2L} \right]^2$$

$$= 2 \times \left[1 - \frac{0.5}{2 \times 0.8} \right]^2 = \underline{\underline{0.94}}$$

9.) Convective accn

$$a_x = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$= \frac{\partial u}{\partial x} \cdot u$$

$$= u \cdot \frac{\partial (u)}{\partial x}$$

$$= 2t \left[1 - \frac{x}{2L} \right]^2 \times \frac{\partial}{\partial x} \left[2t \left(1 - \frac{x}{2L} \right)^2 \right]$$

$$= 2t \left[1 - \frac{x}{2L} \right]^2 \times 2t \times \left(2 \left(1 - \frac{x}{2L} \right) \times \left(-\frac{1}{2L} \right) \right)$$

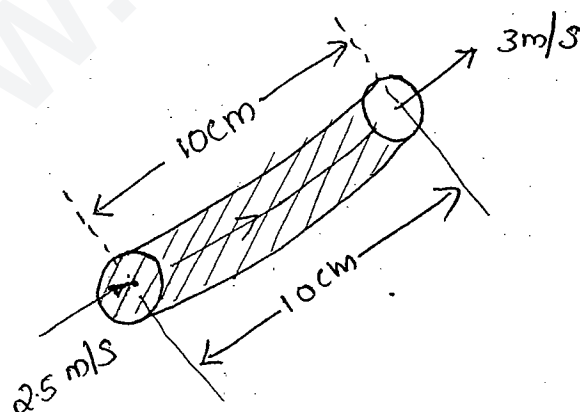
$$= -2t \left[1 - \frac{x}{2L} \right]^2 \times \frac{4t}{2L} \times \left[1 - \frac{x}{2L} \right]$$

$$a_x = -\frac{4t^2}{L} \left[1 - \frac{x}{2L} \right]^3$$

At $\left. \begin{array}{l} t = 3 \text{ sec} \\ L = 0.8 \text{ m} \\ x = 0.5 \text{ m} \end{array} \right\}$

$$a = \frac{-4(3)^2}{0.8} \cdot \left[1 - \frac{0.5}{2 \times 0.8} \right]^3 = \underline{\underline{14.6 \text{ m/sec}^2}}$$

17.



$$a_{\text{convection}} = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$a_{\text{conv}} = u \cdot \frac{\partial u}{\partial x}$$

$$u \times \left(\frac{u_2 - u_1}{x_2 - x_1} \right)$$

$$\left(\frac{2.5+3}{2} \right) \left(\frac{3-2.5}{0.1} \right)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t}$$

$$= 2.75 \times 5$$

$$= \underline{13.75 \text{ m/sec}^2}$$

Q3.

(d)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\lambda y^3 - 2xy + 2xy - \left(\frac{3}{4} \right) 4y^3 = 0$$

$$\cancel{\lambda y^3} = \frac{3 \times \cancel{y^3} \times 4}{4}$$

$$\underline{\lambda = 3}$$

LAW OF CONSERVATION OF MASS:-

CONTINUITY EQUATION

In fluid flow field, it is a constrained particle

for which the velocity components

differentiation w.r.t their direction (u, v, w) must be

equal to zero. i.e. ~~mass~~ total mass flow rate remains

constant at every point in the continuous fluid flow path.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

for 3-D steady flow
incompressible fluid
flow

$$\boxed{\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) + \frac{d\rho}{dt} = 0}$$

3-D compressible flow

Principle

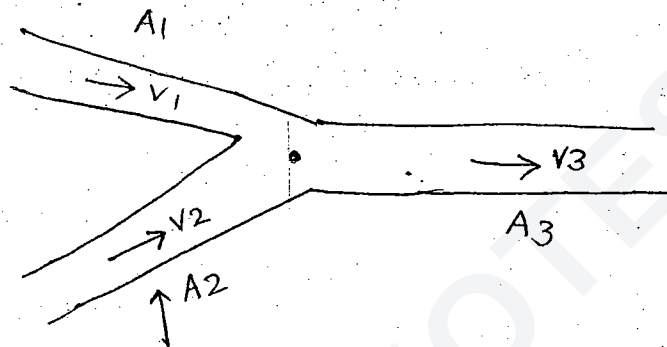
$$\dot{m} = \text{const}$$

$$\rho Q = \text{const}$$

$$\rho A V = \text{const}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad (\text{compressible fluid})$$

$$Q = A_1 V_1 = A_2 V_2 = \dots \quad [\text{Incompressible flow}]$$



$$\sum \dot{m} = 0$$

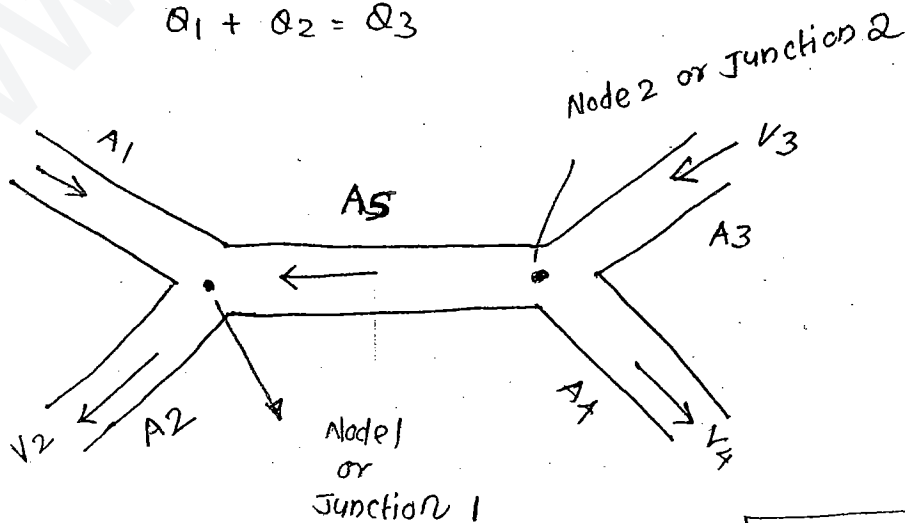
$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

for incompressible fluid

$$\rho_1 A_1 V_1 + \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$

$$A_1 V_1 + A_2 V_2 = A_3 V_3$$

$$Q_1 + Q_2 = Q_3$$



$$Q_3 = Q_4 + Q_5$$

$$Q_1 + Q_5 = Q_2$$

$$A_1 V_1 + A_5 V_5 = A_2 V_2$$

Entering +ve
Leaving -ve

$$A_3 V_3 = A_4 V_4 + A_5 V_5$$

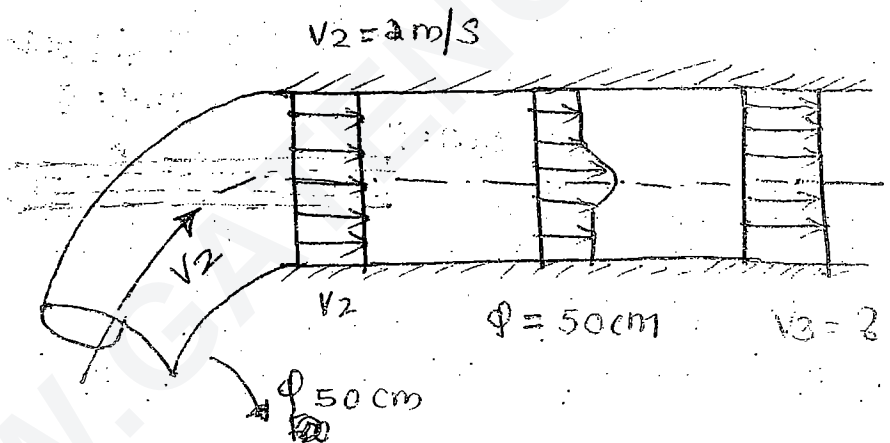
PIPE NO.	Area	Velocity (m/sec)
①	1	10
②	2	V_2
③	2.5	8
④	A_4	5
⑤	3	3

$$A_4 = 2$$

$$V_2 = 9.5 \text{ m/s}$$

$$A = 2.2 \text{ m}^2$$

Mixing of Fluids

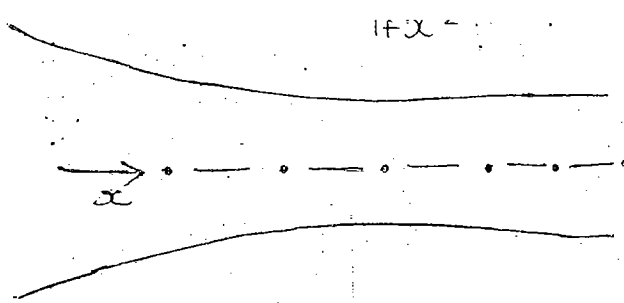


$$A_1 V_1 + A_2 V_2 = A_3 V_3$$

$$\frac{\pi}{4} \times (0.1)^2 \times 5 + \frac{\pi}{4} \times (0.5^2 - 0.1^2) \times 2 = \frac{\pi}{4} \times (0.5)^2 \times V_3$$

$$V_3 = \underline{2.12 \text{ m/s}}$$

- Q) Figure shows a nozzle through which water flowing at the rate of $300 \text{ m}^3/\text{min}$ steadily. The ~~Area~~ ^{velocity} of the Nozzle is given as $\frac{1}{1+x^2}$



Determine the Acceleration of the fluid

- a) $-50(1+x^2)$ b) $-50(x+x^3)$

$$a_x = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t}$$

$$= \frac{\partial u}{\partial x} \cdot u$$

$$a_x = u \cdot \frac{\partial u}{\partial x}$$

We have

$$Q = A V$$

$$300 \frac{\text{m}^3}{\text{min}} = \frac{1}{1+x^2} \cdot u$$

$$\frac{300}{60} = \frac{1}{1+x^2} \cdot u$$

$$u = 5(1+x^2)$$

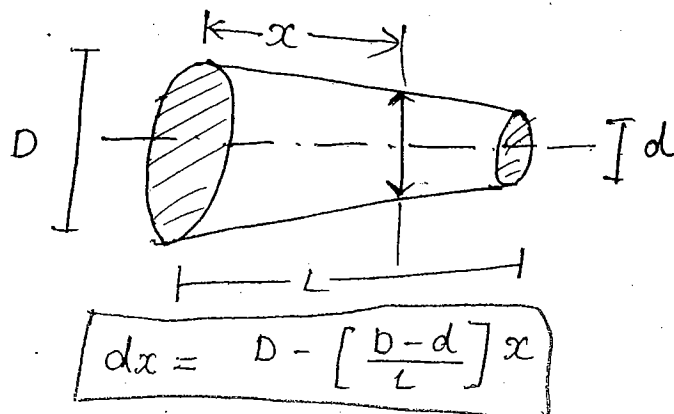
$$a_x = 5(1+x^2) \times \left[\frac{\partial}{\partial x} (5(1+x^2)) \right]$$

$$= 5(1+x^2) \times [5(2x)]$$

$$= 50x(1+x^2)$$

$$a_x = 50(x+x^3)$$

NOTE:-



$$Ax = A_1 - \left(\frac{A_1 - A_2}{L} \right) x$$

P42

⑥

$$L = 2m$$

$$A = (0.4 - 0.1x)$$

$$Q = 0.48 \text{ m}^3/\text{s}$$

$$\frac{dQ}{dt} = \frac{0.12 \text{ m}^3/\text{sec}}{\text{sec}}$$

$$a_{x=0}$$

Local Acceleration

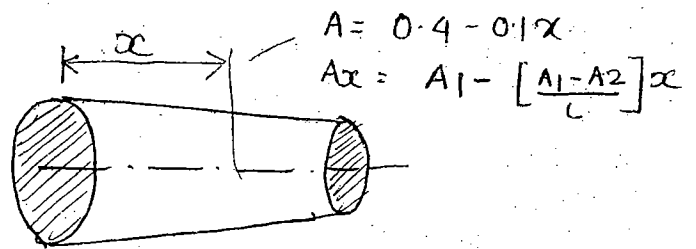
$$a = \frac{\partial u}{\partial t}$$

$$= \frac{\partial}{\partial t} \left(\frac{Q}{A} \right)$$

$$a = \frac{1}{A} \frac{\partial Q}{\partial t}$$

$$a_{x=0} = \frac{1}{0.4} \times 0.12$$

$$= 0.3 \text{ m/s}^2$$



$$\text{So } A_1 = 0.4$$

$$\frac{A_1 - A_2}{L} = 0.1$$

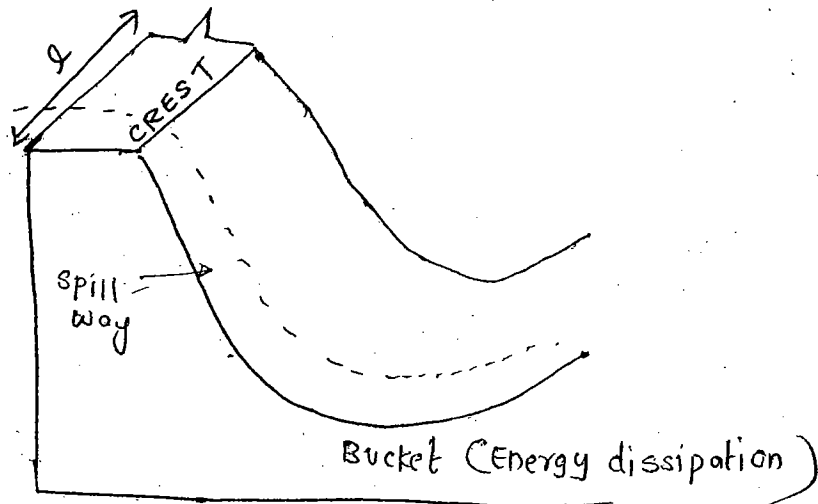
$$\frac{0.4 - A_2}{2} = 0.1$$

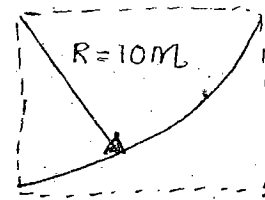
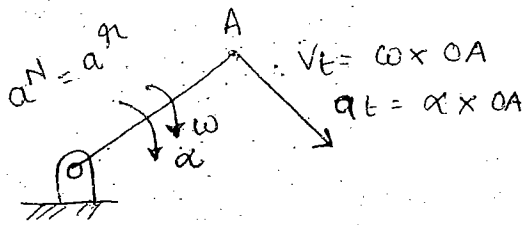
$$\therefore A_2 = 0.2 \text{ m}$$

$$\frac{\text{m}^3}{\text{s}} \times \frac{1}{\text{s}}$$

④

Spill way: A concrete structure constructed across the dam:-





$$a^N = a^n = a^{\text{centripetal}}$$

$$= \frac{V_{Ao}^2}{OA}$$

$$a^N = a^n = \frac{V^2}{R}$$

Soln:-

$$Q = AV$$

$$\frac{5m^3}{\text{sec}} / \text{unit length}$$

$$= (L \times B) \times V$$

$$5 = (1 \times 0.5) \times V$$

unit length. So
1m taken.

$$\therefore 5 = 0.5V$$

$$V = \frac{5}{0.5} = \underline{\underline{10m/s}}$$

$$a^N = \frac{V^2}{R} = \frac{10^2}{10} = \underline{\underline{10 \frac{m}{s^2}}}$$

STREAK LINE:-

time

A streak line at a fixed t is the line connecting locus of all fluid particles, which have or will pass through a fixed location $\vec{y}$ at time t_1 .

WWW.GATENOTES.IN

Fluid kinematics is the Branch of science which deals with motion of particles without considering forces causing motion

→ Methods of describing fluid motion

1. Lagrangian Method

2. Eulerian Method

In Lagrangian method a single fluid particle is followed ~~and~~ during its motion and its vel, accln, ρ etc are described. But in Eulerian method, the vel, accln and ρ are described at a point in fluid flow field.

→ Steady Flow:-

Fluid chara like vel, Pressure, ρ etc do not change with time

$$\text{ie } \left(\frac{\partial v}{\partial t} \right)_{x_0, y_0, z_0} = 0 \quad \left(\frac{\partial p}{\partial t} \right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t} \right)_{x_0, y_0, z_0} = 0$$

→ Unsteady flow:-

Fluid chara change with respect to time

Mathematically

$$\left(\frac{\partial v}{\partial t} \right)_{x_0, y_0, z_0} \neq 0 \quad \left(\frac{\partial p}{\partial t} \right)_{x_0, y_0, z_0} \neq 0 \text{ etc}$$

→ Uniform Flow:-

It is type of flow in which velocity at any ~~time~~ given time does not change with respect to space (ie length of direction of flow)

$$\left(\frac{\partial v}{\partial s} \right)_{t=\text{const}} = 0$$

∂v = change in velocity

∂s = Length of flow in the direction s

→ Non uniform Flow:-

Type of flow in which velocity at any given time changes w.r.t space

$$\left(\frac{\partial v}{\partial s} \right)_{t=\text{const}} \neq 0$$

→ LAMINAR FLOW:-

Type of flow in which fluid particles move along well defined paths or streamline and all the streamlines are straight and parallel

Here the fluid particles move in laminae or they are gliding smoothly over the adjacent layers. This type of flow is also called streamline flow or viscous flow

In terms of Reynold NO:

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} \quad \left[\text{PIPE FLOW} \right]$$

If $Re < 2000$, flow is called Laminar

→ TURBULENT FLOW ←

It is the type of flow in which fluid particles move in zig-zag pattern. Here due to zig-zag movement of particles eddies formation takes place which result in high energy loss

If $Re > 4000$ [Turbulent]

[If $2000 < Re < 4000$]

Flow is Laminar or Turbulent

→ Compressible Flow ←

$$\rho \neq \text{const}$$

→ Incompressible Flow ←

$$\rho = \text{const}$$

→ Rotational Flow ←

Type of flow in which fluid particles move along streamlines and also rotate w.r.t its own axis

→ Irrotational Flow ←

Type of flow in which fluid particles while flowing along streamlines do not rotate about their axis

→ 1-D FLOW ←

Type of flow in which flow parameter such as velocity is a function of time and one space coordinate only. say x

The variation of velocities in other directions are assumed to be negligible

For steady 1-D flow

$$u = F(x) \quad v = 0, \quad \omega = 0$$

u, v, ω are velocity components in x, y and z directions

→ 2D Flow ←

$$u = f_1(x, y) \quad \omega = 0$$
$$v = f_2(x, y)$$

→ 3D Flow ←

mutually $\perp$

Velocity is a function of three space coordinates and time

$$u = f_1(x, y, z)$$

$$v = f_2(x, y, z)$$

$$w = f_3(x, y, z)$$

Rate of Flow: or Discharge:-

Quantity of fluid flowing per second through a section of pipe or channel.

For Liquids

$$Q \text{ is in } \frac{m^3}{s} \text{ or } \frac{\text{litres}}{s}$$

For Gases

$$Q \text{ is } \frac{kgf}{s} \text{ or } \frac{\text{Newton}}{s}$$

$$Q = A \times V$$

→ CONTINUITY EQN: ←

Based on Law of Conservation of Mass

For a fluid flowing through a pipe at ~~ex~~ all cross section the qty of fluid per second is constant.

Rate of flow at section 1

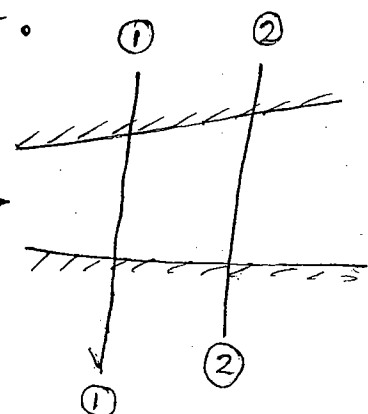
$$= \rho_1 A_1 V_1$$

Rate of flow at section 2

$$= \rho_2 A_2 V_2$$

DIR OF

FLOW



Rate of flow at Section 1 = Rate of flow at Section 2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho A V = \text{const}$$

$$\boxed{\rho Q = \text{const}}$$

For Incompressible fluids

$$Q = \text{const}$$

$$\boxed{A V = \text{const}}$$

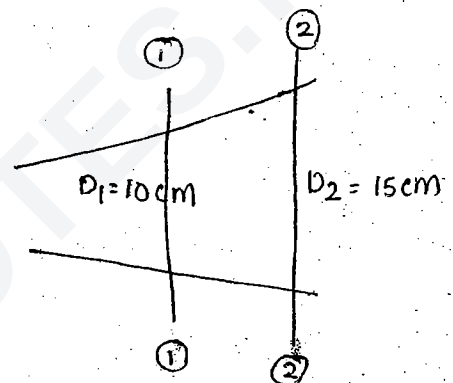
$$A_1 V_1 = A_2 V_2$$

~~FM~~

FM TEXT

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5.1) $D_1 = 0.1 \text{ m}$
 $D_2 = 0.15 \text{ m}$
 $V_1 = 5 \text{ m/s}$



$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times D_1^2 \cdot V_1 = \frac{\pi}{4} \times D_2^2 \cdot V_2$$

$$\frac{\pi}{4} \times (0.1)^2 \times 5 = \frac{\pi}{4} \times (0.15)^2 \times V_2$$

$$V_2 = \frac{(0.1)^2 \times 5}{(0.15)^2} = \underline{\underline{2.22 \text{ m/s}}}$$

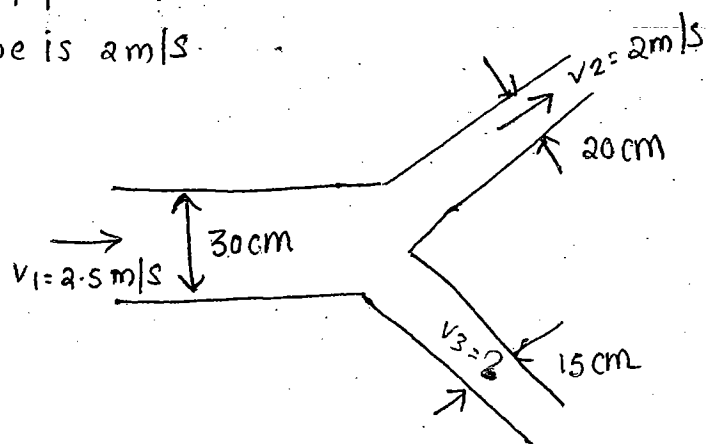
- Q. A 30 cm dia pipe conveying water, Branches into two pipes of dia 20 cm and 15 cm. If Average vel in the 30 cm pipe is 2.5 m/s. find the discharge in pipe. Also find vel in 15 cm pipe if average vel in 20 cm pipe is 2 m/s.

Sol:-

$$Q_1 = A_1 V_1$$

$$= \frac{\pi}{4} \times (0.30)^2 \times 2.5$$

$$= \underline{\underline{0.1767 \text{ m}^3/\text{s}}}$$



$$Q_1 = Q_2 + Q_3$$

$$0.1767 = 0.062 + Q_3$$

$$\underline{\underline{Q_3 = 0.11 \text{ m}^3/\text{s}}}$$

$$Q_2 = \frac{\pi}{4} \times (0.2)^2 \times 2$$

$$= 0.062 \text{ m}^3/\text{s}$$

$$v_3 = \frac{1}{4} \times (0.15)^2 \times 3$$

$$v_3 = \frac{4 \times 0.11}{\pi \times (0.15)^2} = \underline{\underline{6.22 \text{ m/s}}}$$

- Q.) Water flows through a pipe AB 1.2 dia at 3m/s and then passes through pipe BC 1.5 m dia. At c, the pipe branches. Branch CD is 0.8m in dia and carries one third flow of AB. The flow vel in branch CE is 2.5 m/s. Find vol rate of flow in AB, vel in BC, vel in CD and dia of CE.

$$Q_{CD} = \frac{1}{3} Q_1$$

$$Q_1 = \frac{\pi \times (1.2)^2 \times 3}{4}$$

$$Q_1 = \underline{\underline{3.39 \text{ m}^3/\text{s}}}$$

$$Q_1 = Q_2 \quad Q_2 = Q_3 + Q_4$$

$$Q_2 = \frac{\pi \times (1.5)^2 \times v_2}{4}$$

$$3.39 = \frac{\pi \times (1.5)^2 \times v_2}{4}$$

$$v_2 = \underline{\underline{1.92 \text{ m/s}}}$$

$$Q_1 = Q_3 + Q_4$$

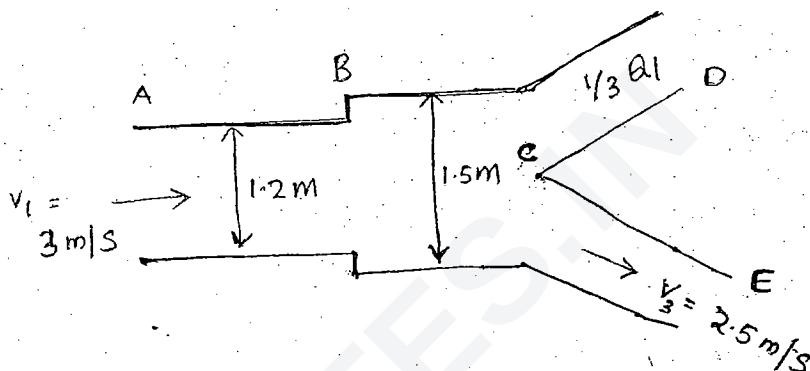
$$3.39 = Q_3 + \frac{3.39}{3}$$

$$Q_3 = \underline{\underline{2.26 \text{ m}^3/\text{s}}}$$

$$Q_3 = \frac{\pi \times d_3^2 \times v_3}{4}$$

$$2.26 = \frac{\pi \times d_3^2 \times 2.5}{4}$$

$$d_3 = \underline{\underline{1.07 \text{ m}}}$$



$$Q_1 = Q_2$$

By continuity eqn

$$A_1 v_1 = A_2 v_2$$

$$Q_4 = \frac{1}{3} Q_1$$

(CD)

- Q.) A 25 cm dia pipe carries oil of sp gr 0.9 at a vel of 3 m/s. At another section the dia is 20 cm. Find the vel at this section and also mass rate of flow.

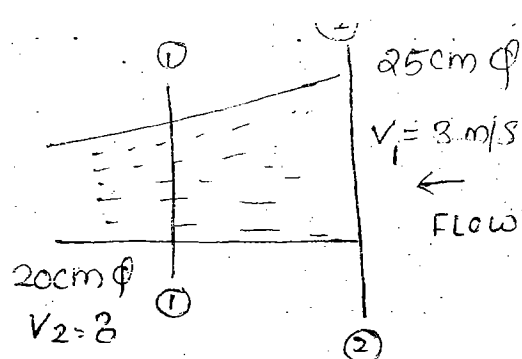
Soln: $S = 0.9$

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times [0.25]^2 \cdot 3 = \frac{\pi}{4} \times [0.20]^2 \times V_2$$

$$V_2 = \left[\frac{0.25}{0.20} \right]^2 \cdot 3$$

$$V_2 = 4.6875 \text{ m/s}$$



Mass flow Rate ($\dot{m}$) = $\rho_1 A_1 V_1$

$$= (0.9 \times 1000) \times \left[\frac{\pi}{4} \times (0.25)^2 \right] \cdot 3$$

$$= 132.53 \text{ kg/s}$$

Q. A jet of water from a 25 mm dia nozzle is directed vertically upwards. Assuming that jet remains circular and neglecting any loss of energy, that will be the dia at point 4.5 m above the nozzle. If the vel with which jet leaves nozzle is 12 m/s.

Apply Bernoulli's eqn

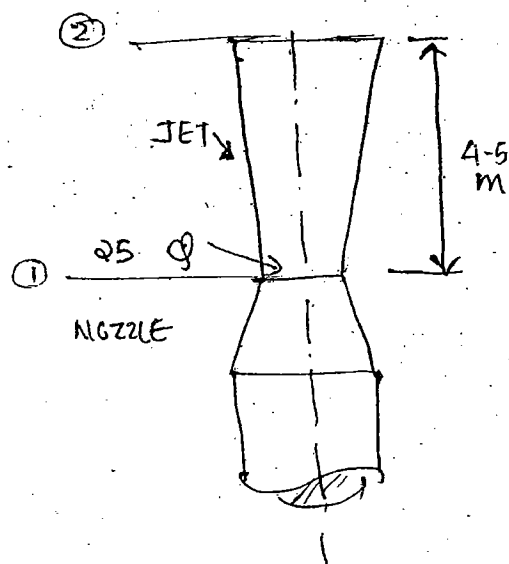
$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{0}{\rho g} + 0 + \frac{12^2}{2 \times 9.81} = \frac{0}{\rho g} + 4.5 + \frac{V_2^2}{2 \times 9.81}$$

$$\frac{12^2}{2 \times 9.81} = 4.5 + \frac{V_2^2}{2 \times 9.81}$$

$$2.83 = \frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 7.46$$



$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \times [0.025]^2 \times 12 = \frac{\pi}{4} \times [d_2]^2 \times 7.46$$

$$d_2 = 31.70 \text{ mm}$$

$$\frac{\partial}{\partial x} [\rho u] + \frac{\partial}{\partial y} [\rho v] + \frac{\partial}{\partial z} [\rho w] + \frac{\partial \rho}{\partial t} = 0$$

Continuity eqn in cartesian coordinates

BASIC EQN

This eqn is applicable to

- (i) steady and unsteady flow
- (ii) uniform and Nonuniform flow
- (iii) Compressible and incompressible flow

Case 1: Steady Flow

so continuity eqn becomes

$$\frac{\partial}{\partial x} [\rho u] + \frac{\partial}{\partial y} [\rho v] + \frac{\partial}{\partial z} [\rho w] = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} = 0}$$

Case 2: Incompressible flow:

$$\underline{\rho = C}$$

$$\text{so } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For 2-D flow $\frac{\partial w}{\partial z} = 0$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

→ CONTINUITY EQN IN POLAR COORDINATES: ←

$$\boxed{\frac{\partial}{\partial r} (r \cdot u_r) + \frac{\partial}{\partial \theta} (u_\theta) = 0}$$

2D steady flow
incompressible

$$\frac{\partial}{\partial r} (r \cdot u_r) = u_r + r \cdot \frac{\partial u_r}{\partial r}$$

General eqn:-

$$\boxed{\frac{\partial \rho}{\partial t} + \rho \frac{\partial}{\partial r} (r \cdot u_r) + \rho \cdot \frac{\partial}{\partial \theta} (u_\theta) = 0}$$

→ VELOCITY AND ACCELERATION

u, v and w are vel components in x, y and z directions

V = Resultant velocity at a point in fluid flow

$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

$$w = f_3(x, y, z, t)$$

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

$$= \sqrt{u^2 + v^2 + w^2}$$

Resultant velocity

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$
$$V = \sqrt{u^2 + v^2 + w^2}$$

a_x, a_y and a_z are total accln in x, y & z directions

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x} \frac{dx}{dt} + \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial v}{\partial z} \frac{dz}{dt} + \frac{\partial v}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \rightarrow \text{Local Accln}$$

Convective Accln

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} + \frac{\partial w}{\partial t}$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow $\frac{\partial V}{\partial t} = 0$

V is Resultant velocity

$$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$$

→ Acceleration vector (A) ←

$$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$A = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

→ Local Acceleration ←

It is defined as Rate of increase of velocity ~~at~~ with respect to time at ~~any~~ a point in a flow field

$$\text{ie } \frac{\partial u}{\partial t}, \frac{\partial v}{\partial t} \text{ or } \frac{\partial w}{\partial t}$$

→ Convective Acceleration ←

Terms other than $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ in the a_x, a_y, a_z are

Called Convective Acceleration.

(5.6) The velocity vector in a fluid flow is given by

$$\vec{V} = 4x^3 \hat{i} + (-10x^2y) \hat{j} + 2t \hat{k} \cdot \text{Find Accn and}$$

velocity at (2, 1, 3) at time $t = 1$.

Ans:- $u = 4x^3, v = -10x^2y, z = 2t$

$$\vec{V}_{(2,1,3)} = 4 \times 2^3 \hat{i} - 10 \times 2^2 \hat{j} + 2 \times 3 \hat{k}$$

$$\vec{V}_{(2,1,3)} = 32 \hat{i} - 40 \hat{j} + 6 \hat{k}$$

$$|\vec{V}| = \sqrt{(32)^2 + (-40)^2 + 6^2}$$

$$= \sqrt{2660}$$

$$|\vec{V}| = \underline{\underline{51.57 \text{ m/s}}}$$

$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 4x^3 [12x^2] + (-10x^2y)(0) + 2t \times 0 + 0$$

$$w = 2t$$

$$u = 4x^3$$

$$v = -10x^2y$$

$$a_x = 48x^5$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$4x^3 [-20x] + (-10x^2y)(-10x^2) + 2t \times 0$$

$$a_y = -80x^4 + 100x^4y$$

$$a_z = u \frac{dw}{dx} + v \frac{dw}{dy} + w \frac{dw}{dz} + \frac{dw}{dt}$$

$$= 4x^3(0) + (-10x^2y) \times 0 + 2t \times 0 + 2$$

$$a_z = 2$$

$$\vec{A} = 48x^5 \hat{i} + (-80x^4 + 100x^4y) \hat{j} + 2 \hat{k}$$

$$\text{At } (2, 1, 3) = 48 \times (2)^5 \hat{i} + [(-80) \times 2^4 + 100 \times 2^4 \times 1] \hat{j} + 2 \hat{k}$$

$$= 1536 \hat{i} + [320] \hat{j} + 2 \hat{k}$$

$$|\vec{A}| = \sqrt{(1536)^2 + (320)^2 + 2^2} = 1568.98 \text{ m/s}^2$$

$$\text{Resultant Acceleration} = 1568.98 \text{ m/s}^2$$

② Determine the third Component of velocity such that it satisfies continuity eqn:-

$$(i) u = x^2 + y^2 + z^2$$

$$v = xy^2 - yz^2 + xz$$

$$(ii) v = 2yz, w = 2xyz$$

Soln - we have continuity eqn $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$(2x+0+0) + (2xy) + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -2x - 2xy - z^2 + x = 0$$

$$\partial w = (-2x - 2xy - z^2 + x) dz$$

$$\int dw = \int (-2x - 2xy - z^2 + x) dz$$

$$w = -2xz - 2xyz - \frac{z^3}{3} + xz + C$$

C is not a fn of z . It can be $f(x, y)$

Soln:-

$$(2x+0+0) + (2xy - z^2 + x) + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} + 2x + 2xy - z^2 + x = 0$$

$$\frac{\partial w}{\partial z} = -2x - 2xy + z^2 - x$$

$$\partial w = (-3x - 2xy + z^2) dz$$

$$\int \partial w = \int (-3x - 2xy + z^2) dz$$

$$w = \left[-3xz - 2xyz + \frac{z^3}{3} \right] + \text{constant}$$

$\downarrow f(x, y)$

(ii) $v = 2y^2$, $w = 2xyz$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial v}{\partial y} = 4y$$

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

$$\frac{\partial w}{\partial z} = 2xy$$

$$\frac{\partial u}{\partial x} = -4y - 2xy$$

$$\int \partial u = \int (-4y - 2xy) dx$$

$$u = -4xy - \frac{2x^2y}{2} + \text{const}$$

$$\underline{u = -4xy - x^2y + \text{const}}$$

Q. A flow field is given by

$$\vec{V} = x^2y \hat{i} + y^2z \hat{j} - (2xyz + yz^2) \hat{k}$$

P.T it is a
case of steady
incompressible
Flow

Soln:-

$$u = x^2y$$

$$v = y^2z$$

$$w = -(2xyz + yz^2)$$

$$\frac{\partial u}{\partial x} = 2xy$$

$$\frac{\partial v}{\partial y} = 2yz$$

$$\frac{\partial w}{\partial z} = -(2xy + 2yz)$$

Continuity equation:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2xy + 2yz - 2xy - 2yz = 0$$

Q2. From the above Given vel field vector $\vec{V}$, calculate velocity and acceleration at point (2, 1, 3)

$$\vec{V}_{(2,1,3)} = (2^2 \cdot 1) \hat{i} + (1^2 \cdot 3) \hat{j} - (2 \times 2 \times 1 \times 3 + 1 \times 3^2) \hat{k}$$

$$\vec{V}_{(2,1,3)} = 4 \hat{i} + 3 \hat{j} - 15 \hat{k}$$

$$|\vec{V}| = \sqrt{16 + 9 + 225}$$

$$|\vec{V}| = \sqrt{250}$$

$$|\vec{V}| = 15.81 \text{ m/s}$$

Acceleration at (2, 1, 3)

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= x^2y (2xy) + y^2z (x^2) - (2xyz + yz^2) \times 0 + 0$$

$$= 2x^3y^2 + x^2y^2z$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$= x^2y \times 0 + y^2z (2yz) - (2xyz + yz^2) (y^2) + 0$$

$$= 2y^4z^2 - 2x^2y^3z - y^3z^2$$

$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$= \underline{\underline{28\hat{i} - 3\hat{j} + 105\hat{k}}}$$

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5 Q31.)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

3D ^{un}steady compressible fluid flow

$$\rho = \rho_0 e^{-2t}$$

$$u = 5x + 6y + 7z$$

$$v = 6x + 5y + 9z$$

$$w = 3x + 2y + \lambda z$$

$$\frac{\partial u}{\partial x} = 5$$

$$\frac{\partial v}{\partial y} = 5$$

$$\frac{\partial w}{\partial z} = \lambda$$

$$\frac{\partial \rho}{\partial t} = \rho_0 e^{-2t} \times (-2)$$

$$-\rho_0 e^{-2t} \times 2 + \rho [5] + \rho [5] + \rho [\lambda] = 0$$

$$\rho [5+5+\lambda] = \rho_0 e^{-2t} \times 2$$

$$\rho [10+\lambda] = \rho_0 e^{-2t} \times 2$$

$$\cancel{\rho_0} \times \cancel{e^{-2t}} [10+\lambda] = \cancel{\rho_0} \cancel{e^{-2t}} \times 2$$

$$10+\lambda = 2$$

$$\lambda = 2-10$$

$$\lambda = -8$$

(c)

Q) A velocity field is given by

$$\vec{V} = (0.3x)\hat{i} + (-0.3y)\hat{j}$$

or

$$\vec{V} = (kx)\hat{i} + (-ky)\hat{j}$$

① - Is the following steady or ~~flow~~ unsteady?

② - Is it 2D or 3D?

⑤ Accn vector

$$u = 0.3x$$

$$v = -0.3y$$

$$\frac{\partial u}{\partial x} = 0.3$$

$$\frac{\partial v}{\partial y} = -0.3$$

Continuity eqn:-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$= +0.3 + (-0.3) = 0$$

∴ It Satisfies continuity eqn.

So ~~steady~~ yes

⑥ Total Accn at $(-2, -2)$

⑦ obtain the stream line Equation

Soln:- flow is steady because time doesnot appear in velocity components

2. flow is 2D because $u, v \neq 0$ and $w = 0$

3. $u = 0.3x$
 $v = -0.3y$

$$\frac{\partial u}{\partial x} = 0.3$$

$$\frac{\partial v}{\partial y} = -0.3$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\underline{0.3 - 0.3 = 0}$$

so it satisfies continuity eqn

Hence it obeys Law of conservation of energy.

④

$$u(1,1) = 0.3$$

$$v(1,1) = -0.3$$

$$\vec{V} = 0.3\hat{i} + (-0.3)\hat{j}$$

$$|\vec{V}| = \sqrt{0.3^2 + (-0.3)^2}$$

$$|\vec{V}| = \underline{0.3\sqrt{2} \text{ m/s}}$$

⑤

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$a_x = 0.3x(0.3) + (\cancel{0})(-0.3y) + 0$$

$$\cancel{a_x} \quad a_y = 0.3x(0) + (-0.3y)(-0.3) + 0$$

$$\vec{A} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{A} = (0.09x)\hat{i} + (0.09y)\hat{j}$$

$$|\vec{A}| = \sqrt{(0.09)^2 + (0.09)^2} = 0.18\sqrt{2} \text{ m/s}^2$$

Given $u = 0.3x \Rightarrow \frac{dx}{dt} = 0.3x$ — (1)

$v = -0.3y \Rightarrow \frac{dy}{dt} = -0.3y$ — (2)

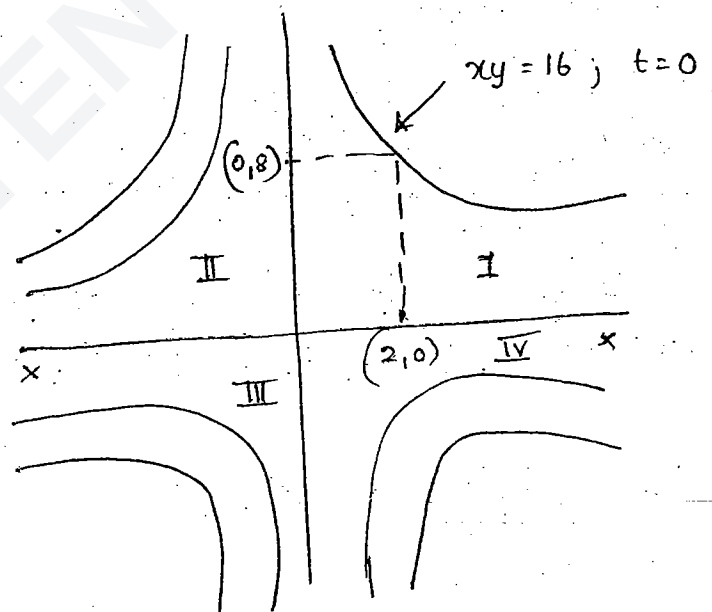
$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{+0.3x}{-0.3y}$$

$$\frac{dx}{dy} = -\frac{x}{y}$$

Separating variables and integrating

$$\int \frac{dx}{x} = - \int \frac{dy}{y}$$

$$\boxed{xy = C}$$



$t = 0 \quad x_0 = 2$
 $y_0 = 8$

$t = 6 \text{ sec} \quad x_6 = 2$
 $y_6 = 8$

$$u = 0.3x$$

$$\frac{dx}{dt} = 0.3x$$

$$\frac{dx}{x} = 0.3 dt$$

$$x \int_{x_0}^x \frac{dx}{x} = 0.3 \int_{t=0}^{t=6} dt$$

$$[\log_e x]_{x_0}^x = 0.3 [t]_0^6$$

$$\log \left(\frac{x}{x_0} \right) = 0.3(t-0)$$

$$\frac{x}{x_0} = e^{0.3t}$$

$$\boxed{x = x_0 e^{0.3t}}$$

Q9) $\vec{V} = (0.3 \times 2)\hat{i} + (-0.3 \times 8)\hat{j}$
 $(2, 8)$
 $= 0.6\hat{i} - 2.4\hat{j}$

$$x = x_0 e^{-0.3t}$$

$$x = 2 \times e^{-0.3 \times 6}$$

$$= 2e^{-1.8} = \underline{12.1 \text{ m}}$$

$$y = 8 \times e^{-0.3 \times 6}$$

$$= 8e^{-1.8} = \underline{1.32 \text{ m}}$$

$$\vec{V} = (-0.3x)\hat{i} + (-0.3y)\hat{j}$$

$$\vec{V}_{(12.1, 1.32)} = (-0.3 \times 12.1)\hat{i} + (-0.3 \times 1.32)\hat{j}$$

$$= \sqrt{(\quad)^2 + (\quad)^2} = \underline{3.65 \text{ m/sec}}$$

Q9) $\vec{V} = (0.3 \times 2)\hat{i} + (-0.3 \times 8)\hat{j}$
 $2, 8$
 $= (0.6)\hat{i} + (-2.4)\hat{j}$
 $= \sqrt{(0.6)^2 + (-2.4)^2}$
 $= \underline{\quad}$

Representation of Flow field through functions:-

Velocity potential function and streamline function

→ Velocity potential function (ϕ) ←

It is a scalar function and it represents the fluid particle in space through functions.

$$\phi = f(x, y, z)$$

It is valid for 3-D and it has the following properties.

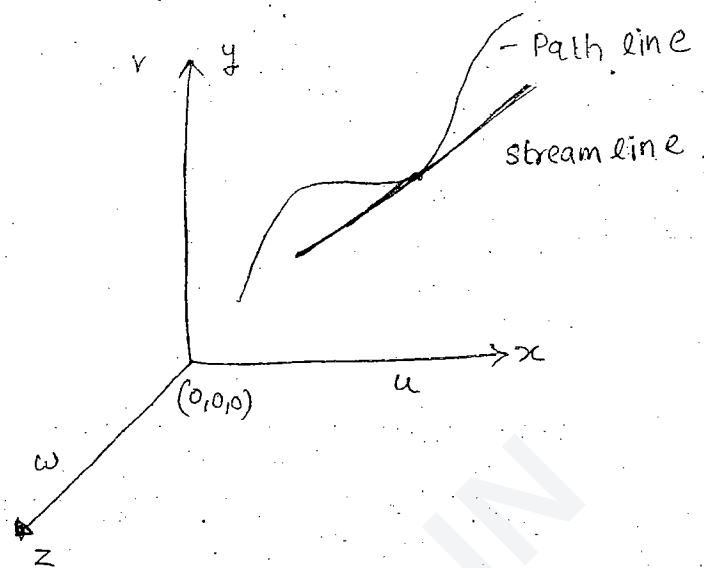
- ① velocity potential fn satisfies Law of conservation of mass
- ② It provides the condition of flow for irrotation.
- ③ It satisfies Laplace Transform

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$$

$$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 w}{\partial y \partial z}$$

$$\frac{\partial^2 w}{\partial z \partial x} = \frac{\partial^2 u}{\partial z \partial x}$$



$$\phi = F(x, y, z)$$

$$\frac{\partial \phi}{\partial x} = -u$$

$$\frac{\partial \phi}{\partial y} = -v$$

$$\frac{\partial \phi}{\partial z} = -w$$

-ve sign indicates that one value increases, other parameter decreases

ex: pressure change b/w 2 points.

Stream function (ψ)

Stream function is also a scalar function in space which provides 2-D fluid flow velocity components into a function w.r.t to normal planes / normal axis.

It is represented by ψ .

Stream function exists which means that law of conservation of mass is satisfied.

Stream function is valid for rotational and irrotational flow.

Laplace transform should be valid.

Physical Meaning:-

Provide discharge b/w 2 points of flow.

Mathematically :

$$\frac{\partial \psi}{\partial y} = -u$$

$$\frac{\partial \psi}{\partial x} = +v$$

Summary

$\frac{\partial \phi}{\partial x} = -u$	$\frac{\partial \psi}{\partial y} = -u$
$\frac{\partial \phi}{\partial y} = -v$	$\frac{\partial \psi}{\partial x} = v$
$\frac{\partial \phi}{\partial z} = -w$	ϕ —

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$$\psi = x^2 - y^2$$

$$\frac{\partial \psi}{\partial y} = -2y$$

$$\frac{\partial \psi}{\partial x} = 2x$$

$$\frac{\partial \psi}{\partial x} = 2x$$

$$u = - \frac{\partial}{\partial y} (x^2 - y^2)$$

$$\underline{u = 2y}$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} (x^2 - y^2)$$

$$= \underline{2x}$$

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\vec{V} = (2y)\hat{i} + (2x)\hat{j}$$

$$\vec{A} = \text{Local Accn} + \text{Convective Accn}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$= 0 + 2x \times 2$$

$$= \underline{4x}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

$$= 2y \times 0 + 2x \times 0$$

$$\vec{A} = a_x \hat{i} + a_y \hat{j}$$

$$\vec{A} = 4x \hat{i} + 4y \hat{j}$$

$$|\vec{A}| = \sqrt{4^2 + 4^2}$$

$$|\vec{A}| = \sqrt{16 + 16} = \sqrt{32} = \sqrt{4 \times 8} \\ = 2\sqrt{4 \times 2} \\ = \underline{4\sqrt{2} \text{ m/s}^2}$$

$$\begin{array}{r} 21 \\ 1732 \\ \underline{4} \\ 6928 \end{array}$$

(19)

$$\psi = 2xy$$

$$\frac{\partial \psi}{\partial y} = 2x = -u$$

$$\boxed{u = -2x}$$

$$\frac{\partial \psi}{\partial x} = 2y$$

$$\boxed{v = 2y}$$

$$\vec{V} = (-2x) \hat{i} + (2y) \hat{j}$$

$$|\vec{V}| = \sqrt{4 + 4} = \underline{4\sqrt{2}}$$

$$\vec{V} = (-4) \hat{i} + (2 \times 2) \hat{j} \\ (+2, -2)$$

$$\vec{V} = -8 \hat{i} + 8 \hat{j} \\ (2, -2)$$

$$|\vec{V}| = \sqrt{4^2 + 4^2} = \underline{4\sqrt{2}}$$

$$\left[(-2 \times 2) \hat{i} + (2 \times 2) \hat{j} \right]$$

(23)

(24)

$$\phi = x^3 - y^3$$

(24)

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0}$$

Laplace eqn

$$\frac{\partial^2 \phi}{\partial x^2} = 6x$$

$$\frac{\partial^2 \phi}{\partial y^2} = -6y$$

$$\underline{x = y}$$

$$\begin{array}{l} -3y^2 \\ -6y \end{array}$$

$$\phi = x^2 - y^2$$

$$-2y$$

$$\frac{\partial^2 \phi}{\partial x^2} = 2 \quad \vee \quad \frac{\partial^2 \phi}{\partial y^2} = -2$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\textcircled{a} \quad \underline{\underline{2-2=0}}$$

⑥

②⑨. Flow Rate = Discharge (Q)

$$\text{Discharge} = |\partial \psi|$$

$$Q = |\psi_A - \psi_B| \quad \therefore |0 - 4| = \underline{\underline{4 \text{ m}^3/\text{sec}}}$$

$$\psi_A (3, 0) = 2 \times 3^2 \times 0 + (3+1) \times 0 = 0$$

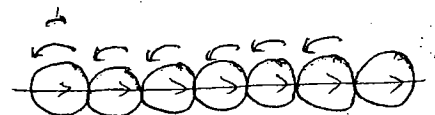
$$\psi_B (0, 2) = 2 \times 0 \times 2 + (0+1) \times 2^2 = \underline{\underline{4}}$$

→ Rotational and Irrotational Flow: ←

Rotational Flow is a type of fluid flow in which the fluid particles move along the stream line or stream path with its own axis rotation also

ex!

→ Irrotational Flow:



Type of flow in which fluid molecules move in stream line without rotating about its own axis



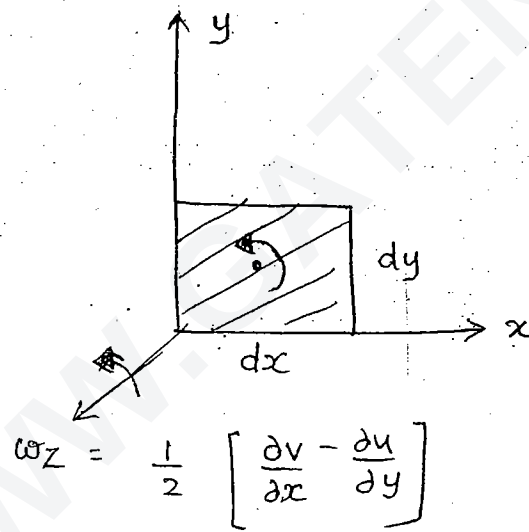
$$\text{circulation} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$\downarrow$
 2ω

$$\omega_x = \frac{|\text{circulation}|}{2} = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$\omega_y = \frac{|\text{circulation}|}{2} = \frac{1}{2} \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right)$$

$$\omega_z = \frac{|\text{circulation}|}{2} = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$



$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\vec{\omega} = (\omega_x)\hat{i} + (\omega_y)\hat{j} + (\omega_z)\hat{k}$$

If $\omega_z = 0$ { irrotational flow }

$\omega_z \neq 0$ { rotational flow }

Q.18

$$\vec{V} = (x+2y+2)\hat{i} + (4-y)\hat{j}$$

$$u = x+2y+2$$

$$v = 4-y$$

$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\frac{\partial v}{\partial x} = 0, \quad \frac{\partial u}{\partial y} = 2$$

$$\omega_z = \frac{1}{2} \begin{bmatrix} 0 & -2 \end{bmatrix}$$

$$= \underline{-1} \quad \text{So rotational}$$

$$-1 \neq 0 \quad (d)$$

(17)

$$u = ax + by$$

$$v = cx + dy$$

$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\frac{\partial v}{\partial x} = c$$

$$\frac{\partial u}{\partial y} = b$$

$$\frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] = 0 \quad \text{for irrotation}$$

$$\frac{1}{2} [c - b] = 0$$

$$\underline{c = b}$$

2001
GATE

(18)

A 2-D flow velocity vector is given as

$$\vec{V} = (x + 2y + 2)\hat{i} +$$

→ SHEAR STRAIN RATE ←

$$\tau_x = \mu \frac{\partial u}{\partial y}$$

→ Rate of shear strain

$$\tau_y = \mu \frac{\partial v}{\partial x}$$

$$\epsilon_{xy} = \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]$$

$$\epsilon_{zx} = \left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right]$$

(23)

$$\vec{V} = x^2y \hat{i} + 2xy^2z \hat{j} - yz^3 \hat{k}$$

$$\epsilon_{yz} = \frac{1}{2} \left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right]$$

$$\frac{\partial w}{\partial y} =$$

$$u = x^2y$$

$$v = 2xy^2z$$

$$w = yz^3$$

$$\frac{\partial v}{\partial z} = 2xy^2$$

$$\frac{1}{2} [2xy^2 - z^3]$$

$$x = -2$$

$$At y = -1$$

$$z = 2$$

$$\epsilon_{yz} = \frac{1}{2} [2x(-2)(-1)^2 - (2)^3]$$

$$\frac{1}{2} [-4 - 8]$$

$$\epsilon_{yz} = \frac{1}{2} [-12] = -6$$

$$\underline{\underline{\epsilon_{yz} = -12}}$$

Page 84

(11)

$$w_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\frac{1}{2} \times \left[0 - \frac{v}{h} \right]$$

$$= \underline{\underline{-\frac{v}{2h}}}$$

$$u_x = \frac{v}{h}$$

$$w_y = \frac{1}{2} \left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right]$$

$$= 0$$

Conclusions of kinematics of fluid:-

In fluid rotational system, the particle traces the path is expressed in terms of circulation and vorticity. Circulation will have 3 components about abt 3 different planes namely ω_x , ω_y and ω_z . A particle in xy plane can rotate about 3rd plane ω_z . Similarly remaining 2 planes also provide two angular speed components. Circulation is the fluid flow in a closed curve which is mathematically line integral of tangential velocity component.

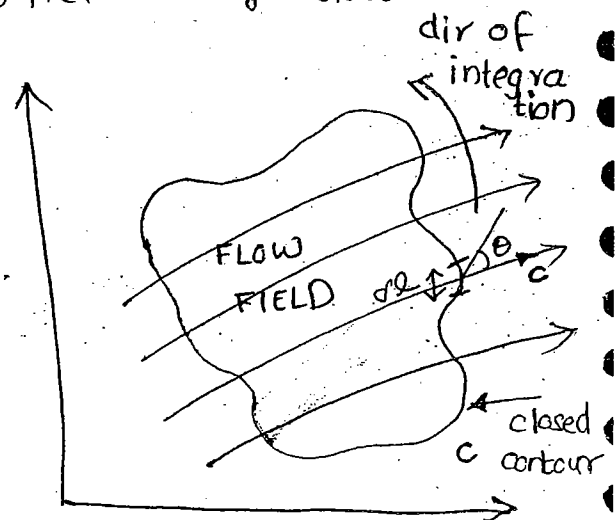
CIRCULATION

Circulation is defined as the line integral of velocity around a closed contour in a flow field. Fig Below shows a closed contour C.

velocity vector c at an elemental length " dl " is inclined at an angle θ to the curve. The line integral is $c \cos \theta \cdot dl$ and circulation

$$\Gamma = \oint_C c \cos \theta \cdot dl$$

$\oint_C$ represents integration around closed contour C.



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Vorticity (ξ)

It is the limiting value of fluid circulation and the contour covered by the fluid particle tends to be zero. Hence vorticity is given by circulation per unit Area.

$$\text{Vorticity } \xi = \frac{\text{circulation}}{\text{Area}}$$

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Introduction:-

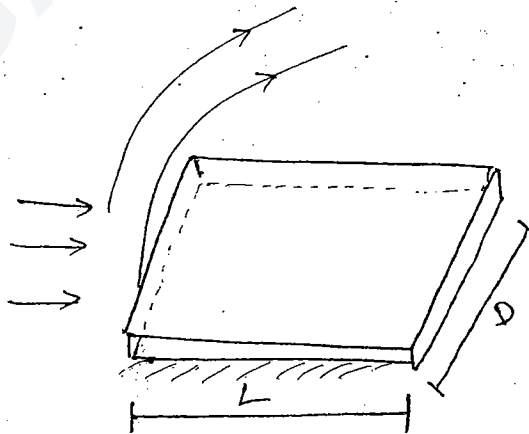
In Nature two observations - one is a fluid passing over the stationary body. Next is a body moves in static fluid. In the above cases the pressure gradient is zero which means flow is external.

eg: Aeroplane motion in motion, submarine in water, Gas turbine blade in chamber, steam turbine blades, High speed Rotary equipment.

When fluid is passing over the body a thin layer is formed around the body due to which they bound to be mass flow rate loss, momentum loss, and kinetic energy of the moving fluid effects the surface of the body under consideration. eg: Nose of the Aeroplane.
wings of space vehicle.
Blade tips of turbine.

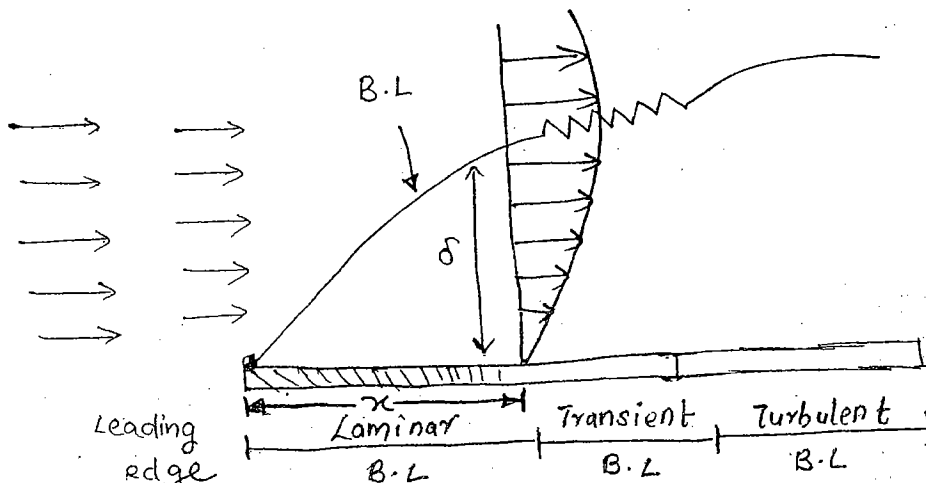
$T = \text{const}$

ρ
 S
 μ



B.L

Boundary Layer



$$Re = 5 \times 10^5$$

$$Re_{x \text{ (critical)}} = \frac{u_{\infty} \cdot x}{\nu} = 5 \times 10^5$$

The Boundary Layer is a Thin layer around the solid surface which takes place due to viscosity, Resistive force and to some extent due to ~~the~~ pressure forces (Circular & Surface). Boundary Layer is measured from Leading edge in 2D & till the fluid reach its original ~~to~~ uniform velocity. within the Boundary Layer the velocity profile exists ~~at~~ ⁱⁿ different forms, at particular distance along the plate, the ~~for~~ ^{or} distance is named as Boundary Layer thickness. Boundary Layer thickness (δ) is a fn of distance 'x' and Reynolds No. within the ~~Boundary layer thickness~~

Laminar Layer, $\delta = \frac{5x}{\sqrt{Re_x}}$

$$\delta = \frac{5x}{\sqrt{\frac{u_{\infty} \cdot x}{\nu}}}$$

$$\delta \propto \frac{x}{\sqrt{x}}, \quad \delta \propto (x)^{1/2}$$

$$\boxed{\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}}$$

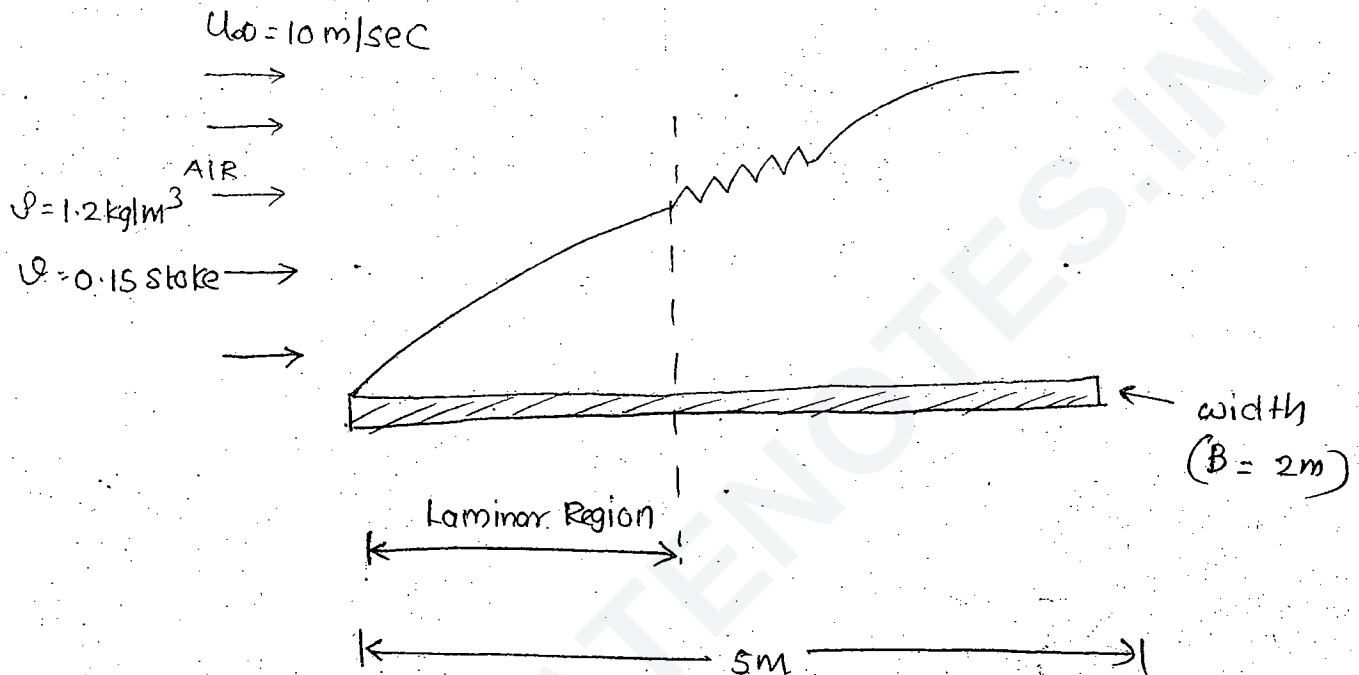
- Q) A flat plate of length 5m and width 2m subjected to an external air flow at 10m/s. the properties of air are $\rho = 1.2 \text{ kg/m}^3$ and $\nu = 0.15 \text{ stoke}$. Determine the following
- (i) the distance at which laminar flow is seized.

② Boundary Layer thickness $\delta = 0.5m$

③ Boundary Layer thickness downstream of the point where the initial Boundary thickness estimated ($0.15m$) down ④ the mass flow rate through the specified Boundary Region in kg/s .

⑤ shear stress at wall of the plate

⑥ shear force at the surface of the laminar Region.



①

$$Re_x = \frac{U_{\infty} \cdot x}{\nu}$$

$$5 \times 10^5 = \frac{10 \times x}{0.15 \times 10^{-4}}$$

$$\therefore x = 0.75 m$$

②

NOTE:

$$Re = \frac{V \cdot L_c}{\nu}$$

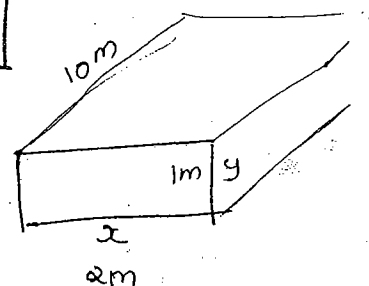
L_c = characteristic length

$$Re_{plate} = \frac{V \cdot x}{\nu}$$

$$L_c = \frac{4A}{P}$$

$$L_c = \frac{4(xy)}{2(x+y)}$$

$$L_c = \frac{4(2 \times 1)}{2(2+1)}$$



(b) Boundary Layer thickness at 0.5 m

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

where $x = 0.5 \text{ m}$

$$Re_x = \frac{U_\infty x}{\nu}$$

$$= \frac{10 \times 0.5}{0.15 \times 10^{-4}}$$

$$= 3.33 \times 10^5$$

$$\therefore \delta_{x=0.5 \text{ m}} = \frac{5 \times 0.5}{\sqrt{3.33 \times 10^5}}$$

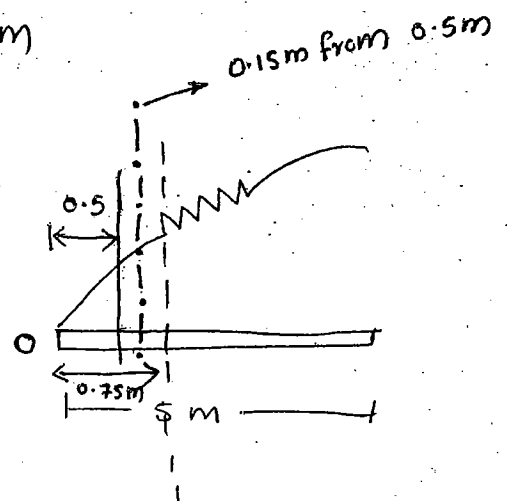
$$\delta_{x=0.5 \text{ m}} = 0.0043 \text{ m} = 4.3 \text{ mm}$$

(c) Boundary layer thickness downstream at 0.5 m

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}$$

$$\frac{4.3}{\delta_2} = \sqrt{\frac{0.5 \text{ (m)}}{(0.5 + 0.15) \text{ m}}}$$

$$\delta_2 = 4.9 \text{ mm}$$

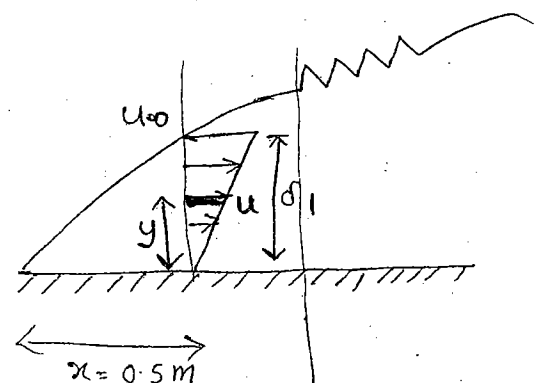


(d) Mass flow Rate ($\dot{m}$)

$$d\dot{m} = \rho dQ$$

$$d\dot{m} = \rho \cdot dA \cdot u$$

$$= \rho \cdot B dy \cdot u$$



$$dm = \rho B \frac{u_\infty}{\delta} y$$

$$\int dm = \rho B \frac{u_\infty}{\delta} \int_0^{\delta_1} y dy$$

$$\dot{m} = \rho B \frac{u_\infty}{\delta} \left[\frac{y^2}{2} \right]_0^{\delta_1}$$

$$\boxed{\dot{m} = \frac{1}{2} \rho B u_\infty \delta}$$

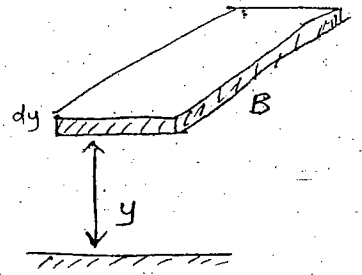
$$\dot{m} = \frac{1}{2} \times 12 \times 2 \times 10 \times 0.0043$$

$$\boxed{\dot{m} = 0.0516 \text{ kg/sec}}$$

LINEAR CURVE

$$\frac{u}{y} = \frac{u_\infty}{\delta_1}$$

$$u = u_\infty \frac{y}{\delta_1}$$



(e) Shear stress (τ) = $\mu \frac{du}{dy}$

$$= \mu \frac{d}{dy} \left[u_\infty \frac{y}{\delta} \right]$$

$$= \mu \frac{u_\infty}{\delta}$$

$$\tau = \frac{\mu}{\rho}$$

$$= 0.15 \times 10^{-4} \times \frac{10}{0.0043}$$

$$= 0.0418 \text{ N/m}^2$$

(f) Drag/shear force For laminar Region

$$F_D = F_S = \tau \times A$$

$$= 0.0418 \times (\text{laminar length} \times \text{width})$$

$$= 0.0418 \times (0.75 \times 2)$$

$$\boxed{F_D = 0.06 \text{ (N)}}$$

Q16

$$U_{\infty} = 280 \text{ kmph}$$

$$= 280 \times \frac{5}{18} = \underline{77.77 \text{ m/sec}}$$

$$\nu = 0.15 \text{ stokes}$$

$$= 0.15 \times 10^{-4} \frac{\text{m}^2}{\text{s}}$$

$$Re_x = 3 \times 10^5$$

① distance at which Laminar layer terminates :

$$Re_x = \frac{U_{\infty} \cdot x}{\nu}$$

$$3 \times 10^5 = \frac{77.77 \times x}{0.15 \times 10^{-4}}$$

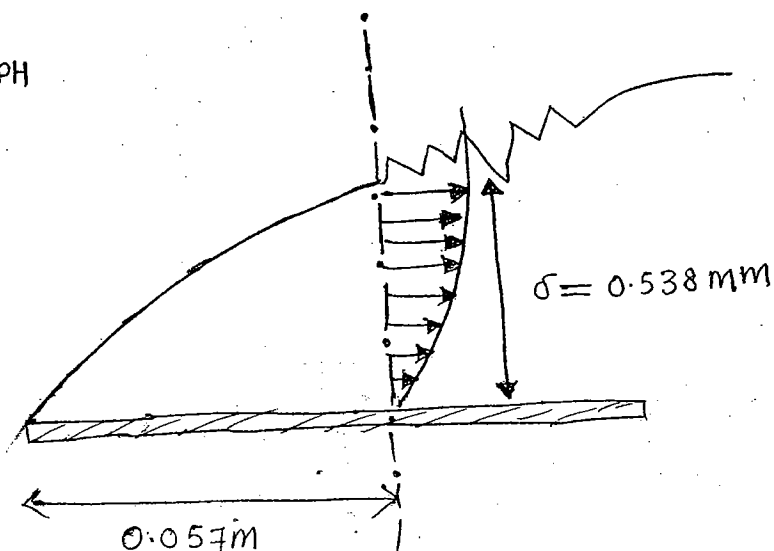
$$\underline{x = 0.057 \text{ m}}$$

② thickness of Boundary Layer

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.057}{\sqrt{3 \times 10^5}}$$

$$= \underline{0.538 \text{ mm}}$$

$$U_{\infty} = 280 \text{ kmph}$$



Q2)

$$Re_x = \frac{Vx}{\nu}$$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$\delta \propto \frac{1}{\sqrt{Re_x}}$$

$$x = \text{const}$$

$$\delta_1 \sqrt{Re_{x1}} = \delta_2 \sqrt{Re_{x2}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{Re_{x2}}{Re_{x1}}} = \sqrt{\frac{256}{100}} = 1.6$$

Q3)

$$\tau = \mu \frac{du}{dy}$$

$$\tau \propto \frac{1}{y}$$

$$\tau \propto \frac{1}{\delta}$$

$$\tau \propto \frac{1}{\sqrt{x}}$$

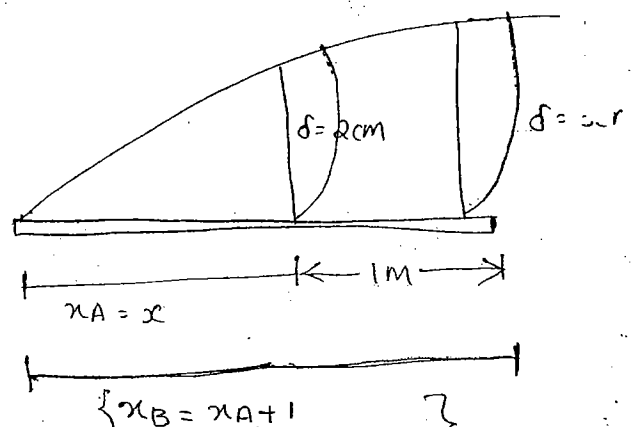
$$\boxed{\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}}$$

Q4) a

Q5) a

Q6) c

Q7)



$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_A}{x_B}}$$

$$\frac{2 \text{ cm}}{3 \text{ cm}} = \sqrt{\frac{x \text{ (metres)}}{(x+1) \text{ metres}}}$$

$$\frac{4}{9} = \frac{x}{x+1}$$

$$\therefore \underline{x = 0.8 \text{ m}}$$

8.) D

$$9.) \tau = \mu \frac{du}{dy}$$

$$\frac{u}{u_m} = 1.5 \times \eta$$

$$= 1.5 \times \frac{y}{\delta}$$

$$\tau = \mu \frac{d}{dy} \left[u_m \cdot 1.5 \times \frac{y}{\delta} \right]$$

$$= \mu \cdot u_m \times \frac{1.5}{\delta} [1]$$

$$1.5 \frac{\mu u_m}{\delta}$$

$$\therefore \tau = K \left(\frac{\mu u_m}{\delta} \right)$$

$$\therefore \underline{K = 1.5}$$

ISRO 9.) The transition Reynold No for flow over a flat plate is 5 lakhs. What is the distance from the leading edge at which transition will occur for the flow of fluid with a uniform vel of 1 m/s?

10.) ~~unifex~~ ($v = 0.86 \times 10^{-6} \text{ m}^2/\text{s}$)

a.) 0.43 m b.) 1 m c.) 43 m d.) 103 m

$$Re_x = \frac{Vx}{\nu} = \frac{1 \times x}{0.86 \times 10^{-6}}$$

$$5 \times 10^5 = \frac{1 \times x}{0.86 \times 10^{-6}}$$

$$\underline{x = 0.43 \text{ m}}$$

Q.
GATE 04
ME

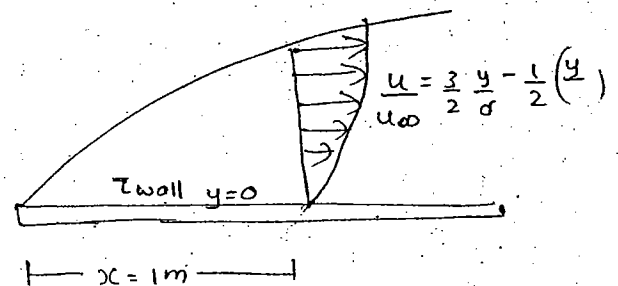
For Air flow over a flat plate the velocity 'u' and Boundary layer thickness δ can be expressed resp as

$$\frac{u}{u_\infty} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \quad \text{and} \quad \delta = \frac{4.64x}{\sqrt{Re_x}}$$

The free stream vel of Air is 2 m/s and air properties are $\rho = 1.23 \text{ kg/m}^3$ and $\mu = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$.

The magnitude of wall shear stress at $x = 1 \text{ m}$.

- (a) $2.36 \times 10^{-2} \text{ N/m}^2$
- (b) $43.6 \times 10^{-3} \text{ N/m}^2$
- (c) $436 \times 10^{-3} \text{ N/m}^2$
- (d) $2.18 \times 10^{-3} \text{ N/m}^2$



Ans:

$$u_\infty = 2 \text{ m/s}$$

$$\rho = 1.23 \text{ kg/m}^3$$

$$\mu = 1.5 \times 10^{-5}$$

$$\tau_{(x=1\text{m})}$$

$$\tau = \mu \frac{d(u)}{dy} = \mu \frac{d}{dy} \left[u_\infty \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) \right]$$

$$= \mu u_\infty \frac{3}{2} \times \frac{1}{\delta} - \frac{1}{2} \times 3 \left(\frac{y}{\delta} \right)^2 \times \frac{1}{\delta}$$

$$\tau_{x=1} = \mu u_\infty \left[\frac{3}{2\delta} - \frac{3}{2} \frac{y^2}{\delta^3} \right]$$

$$y=0$$

$$\mu u_\infty \left[\frac{3}{2\delta} - \frac{3}{2} \frac{0^2}{\delta^3} \right] = \mu u_\infty \frac{3}{2\delta}$$

$$\tau_{x=1} = 1.5 \mu \frac{u_\infty}{\delta}$$

$$\delta_{x=1} = \frac{4.64x}{\sqrt{\frac{u_\infty \cdot x}{\nu}}} = \frac{4.64 \times 1}{\sqrt{\frac{2 \times 1}{1.5 \times 10^{-5}}}}$$

$$\delta = \underline{0.0127 \text{ m}}$$

$$\tau_{x=1} = 1.5 \times (2 \times 10^{-3}) \times \frac{u_\infty}{\delta} \quad \text{res } \mu/\text{s}$$

$$= 1.5 \times (1.5 \times 10^{-5} \times 1.23) \times 2 \times \frac{1}{0.0127}$$

$$= \underline{4.36 \times 10^{-3} \text{ N/m}^2}$$

Q.) A smooth flat plate with a sharp leading edge is placed along a gas stream flow.

$u_\infty = 10 \text{ m/s}$. The thickness of the Boundary layer at the section R-S is 10 mm. The breadth of the plate is 1 m (into the paper of plane). The density of the Gas is 1 kg/m^3 and Assume that the Boundary layer is thin, 2-D flow which follows a linear velocity distribution $\frac{u}{u_\infty} = \frac{y}{\delta}$ at the section R-S; where y is the height measured from plate.

Soln:-

$$u_\infty = 10 \text{ m/s}$$

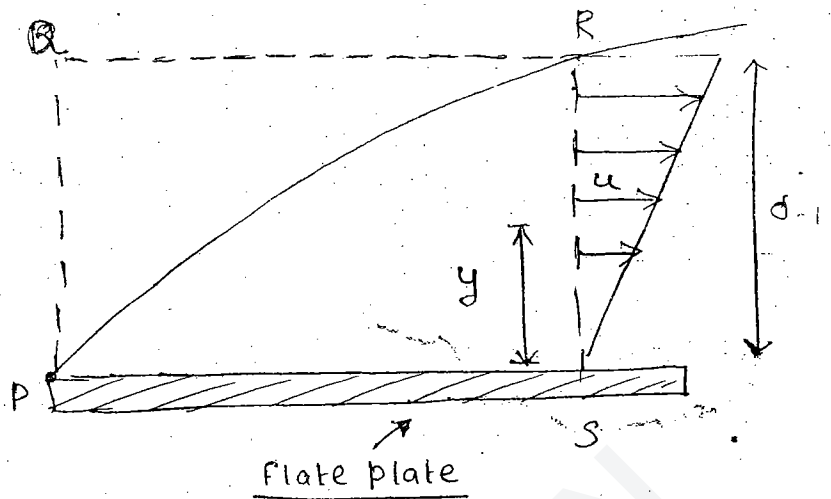
$$\delta = 10 \text{ mm}$$

$$\rho = 1 \text{ kg/m}^3$$

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5x}{\sqrt{\dots}}$$

- ① The Mass flow Rate in (kg/sec) across the section Q-R is

- (a) 0 (b) 0.05 kg/sec
(c) 0.10 kg/sec (d) 0.15 kg/sec



$$\dot{m} = \frac{1}{2} \rho \cdot B \cdot u_0 \cdot \delta$$

$$\frac{1 \times 1 \times 10 \times 1 \times 0.01}{2}$$

$$\dot{m} = 0.05 \text{ kg/sec}$$

- ② Drag force on the plate b/w P and S.

$$F_D = F_s = \tau \times \text{Area}$$

$$= \mu \frac{du}{dy} \times \text{Area}$$

$$F_D = \mu \frac{d}{dy} \left[u_0 \cdot \frac{y}{\delta} \right] \times \text{Area}$$

$$F_D = \mu \cdot \frac{u_0}{\delta} \times [\text{PS} \times \text{Breadth}]$$

$$F_D = 18 \times 10^{-6} \times 10 \left[1 \times 1 \right]$$

$$= 0.018 \frac{\text{N}}{\text{m}^2} \times 1 \times 1$$

$$= 0.018 \frac{\text{N}}{\text{m}}$$

$$\frac{u}{u_0} = \frac{y}{\delta}$$

$$\dot{m} = \rho \times Q$$

$$= \rho \times A \times u$$

$$= \rho \times A \times u_0 \cdot \frac{y}{\delta}$$

$$\rho \frac{u_0}{\delta} B \int_0^\delta y dy$$

$$= \frac{\rho u_0}{\delta} B \left[\frac{y^2}{2} \right]_0^\delta$$

$$= \frac{\rho u_0}{\delta} B \times \frac{\delta^2}{2}$$

$$\dot{m} = \frac{1}{2} \times \rho \times B \times u_0 \times \delta$$

Assume

$$\mu = 18 \times 10^{-6} \frac{\text{N} \cdot \text{sec}}{\text{m}^2}$$

7 ILL EFFECTS OF BOUNDARY LAYER FORMATION

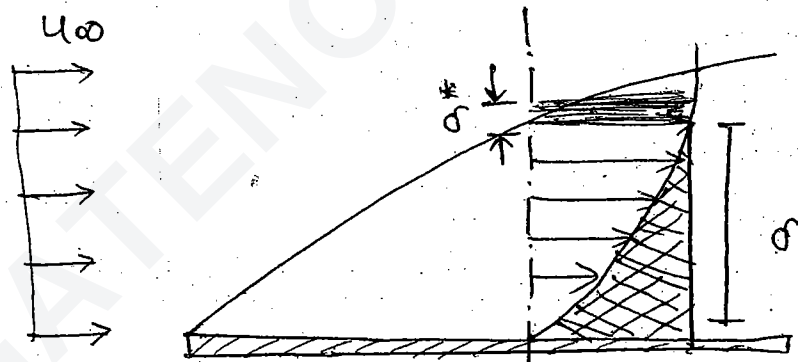
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The following are ill effects

- ① Loss of Mass flow Rate (displacement thickness δ^*)
- ② Loss of Momentum (Momentum thickness θ)
- ③ Loss of Kinetic energy [energy thickness δ^{**}]

→ Displacement Thickness:-

It is the distance measured $\perp$ r to the surface of plate upto which, where the ~~free~~ velocity reaches free stream velocity, the Boundary shifted to compensate loss of mass, due to boundary layer formed.



δ^* = Displacement thickness

(compensation of mass flow Rate)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

→ Momentum thickness:-

It is the distance measured $\perp$ r to flat plate surface upto which the Boundary layer is shifted to compensate loss in momentum due to Boundary layer formation. It is given

by θ = Momentum thickness

$$\theta = \int_0^\delta \frac{u}{u_\infty} \left[1 - \frac{u}{u_\infty}\right] dy$$

→ Energy thickness: δ^{**}

It is the distance $\perp$ to the flat surface upto which the Boundary layer shifted to compensate the loss of K.E of Moving fluid due to Boundary layer formed.

Mathematically it is given by (δ^{**})

$$\delta^{**} = \int_0^{\delta} \frac{u}{u_{\infty}} \left[1 - \left(\frac{u}{u_{\infty}} \right)^2 \right] dy$$

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① ②

② $\frac{u}{u_{\infty}} = \frac{y}{\delta}$

$\frac{u}{u_{\infty}} = \frac{y}{\delta}$

$$\delta^* = \int_0^{\delta} \left[1 - \frac{u}{u_{\infty}} \right] dy$$

$$\delta^* = \int_0^{\delta} \left[1 - \frac{u_{\infty} \cdot y}{u_{\infty} \cdot \delta} \right] dy$$

$$\int_0^{\delta} \left[1 - \frac{y}{\delta} \right] dy = \left[y - \frac{y^2}{2\delta} \right]_0^{\delta}$$

$$\delta - \frac{\delta^2}{2\delta}$$

$$\delta - \frac{\delta}{2} = \underline{\underline{\frac{\delta}{2}}}$$

$$\frac{\delta^*}{\delta} = \frac{\delta}{2\delta} = \underline{\underline{\frac{1}{2}}}$$

$$\frac{\theta}{\delta} = 2$$

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left[1 - \frac{u}{u_{\infty}} \right] dy$$

$$= \int_0^{\delta} \frac{u_{\infty} \cdot y}{u_{\infty} \cdot \delta} \left[1 - \frac{u_{\infty} \cdot y}{\delta u_{\infty}} \right] dy$$

List-II

2. Net main mass flow

$$\int_0^{\delta} \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right) dy$$

$$\int_0^{\delta} \frac{y}{\delta} - \frac{y^2}{\delta^2}$$

$$\left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} \right]_0^{\delta} = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}$$

$$= \frac{\delta}{2} - \frac{\delta}{3}$$

$$= \frac{\delta}{6}$$

$$\frac{\theta}{\delta} = \frac{\delta}{6\delta} = \underline{\underline{\frac{1}{6}}}$$

③

$$\text{SHAPE FACTOR}(H) = \frac{\delta^*}{\theta} = \frac{\text{Displacement thickness}}{\text{Momentum thickness}}$$

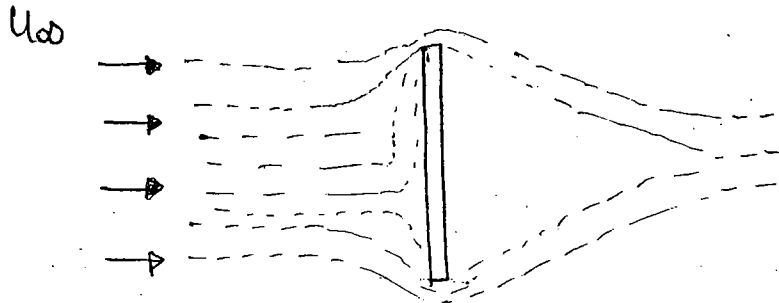
Ratio of displacement thickness to Momentum thickness

$$H = \frac{\frac{\delta}{2}}{\frac{\delta}{6}} = \underline{\underline{3}}$$

④

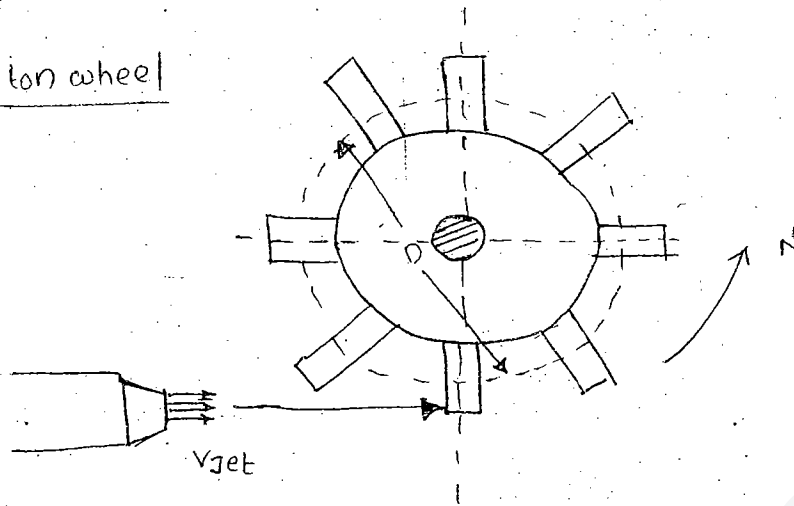
WAKE:-

disturbed Region
downstream of
separation



Tangential Flow

ex: pelton wheel



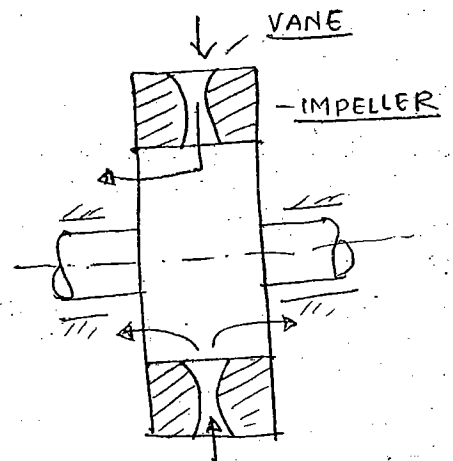
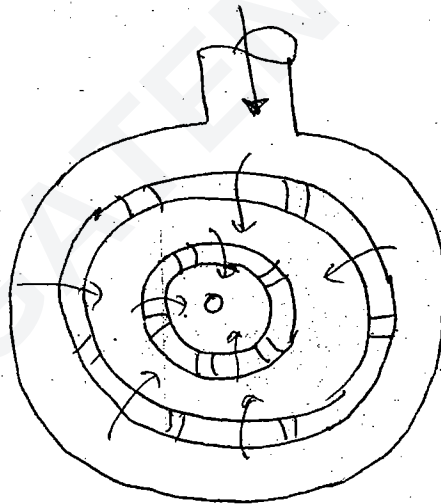
$$V_{jet} = \sqrt{2gH}$$

$$u = \frac{\pi d N}{60} \text{ (m/sec)}$$

$$u = \omega \times R \text{ (m/sec)}$$

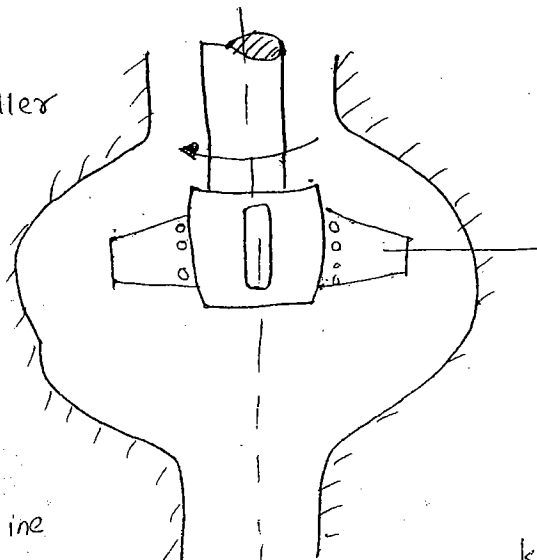
Radial Flow:-

Francis
Turbine
(Inward flow)



Axial Turbine

Kaplan and Propeller



Blades
or
vanes

Note:

① Vanes are adjustable in Kaplan

② Vanes are not adjustable in propeller

High Head
ex: Pelton wheel
Francis turbine

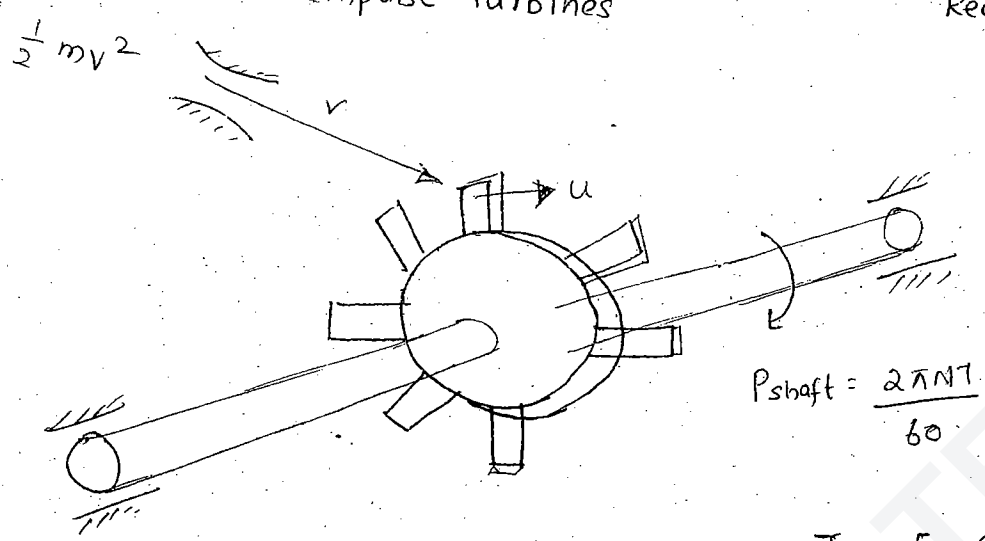
Kaplan and Propeller } Low Head

Impulse

Water Action

Impulse Turbines

Reaction Turbines



③ No draft tube Required

$$T = F \times r$$

$$= m \times a \times r$$

$$= m \times a \times \frac{D}{2}$$

$$T = (\dot{m} \times \Delta v) \times \frac{D}{2}$$

Eulers Turbo Machine
(Turbine)

$$T = \left[\begin{array}{l} \text{Rate of} \\ \text{change of} \\ \text{Momentum} \end{array} \times \begin{array}{l} \text{Radius} \\ \text{of wheel} \end{array} \right]$$

1. works in open atmosphere
2. Input energy pure impulse action
3. No draft tube is Required
4. High Head reqd
5. Low discharge of water
6. Tangential flow

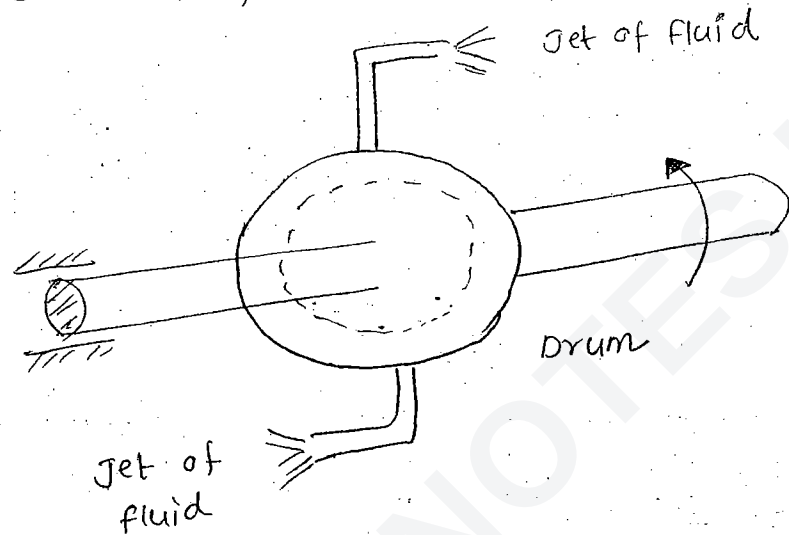
7) Newton's 2nd Law

Reaction Turbine:-

- 1) works under pressure
- 2) Input energy is pure potential or pressure before

Larger the turbine partly Potential and partly kinetic energy

- ③ Draft tube is essential element
- ④ Medium and Low Heads
- ⑤ Medium and large discharge
- ⑥ Newton's 3rd Law of Motion



eg: Francis turbine
Kaplan turbine
propeller turbine

(iv) Based on Specific Speed:- (N_s)

Specific speed is a relative term used in turbines to the use of three operating parameters.

- (i) head on turbine
- (ii) Power produced by turbine
- (iii) Speed of turbines

$$N_s_{\text{turbine}} = \frac{N \sqrt{P}}{(H)^{5/4}}$$

where $N \rightarrow \text{rpm}$

$P \rightarrow \text{kilowatt (kW)}$

$H \rightarrow \text{meters}$

$$Als_{\text{pump}} = \frac{N \sqrt{Q}}{(H)^{3/4}}$$

$N \rightarrow \text{rpm}$

$Q \rightarrow \text{m}^3/\text{sec}$

$H \rightarrow \text{metres}$

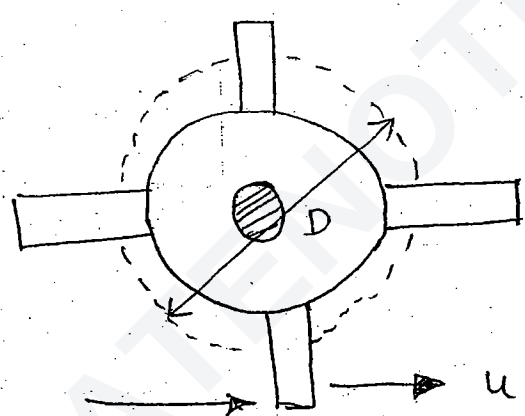
$$P = \rho g H$$

$$P = \rho Q g H$$

$$P \propto Q \cdot H \quad \text{--- (1)}$$

$Q = \text{Area} \times \text{velocity}$
(c/s)

$$Q \propto D^2 \cdot V \quad \text{--- (2)}$$



$$V = \sqrt{2gH}$$

$$u = \frac{\pi D N}{60}$$

$$P = \frac{K H^{5/2}}{N^2}$$

const

$$P \propto D^2 V \cdot H$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$\cancel{P \propto D^2 H} \quad P \propto \frac{H^1}{N^2} \cdot \sqrt{H} \cdot H^1$$

$$P \propto \frac{(H)^{5/2}}{N^2}$$

$$P = \frac{K H^{5/2}}{N^2}$$

$K = \text{const}$

Let $P =$

Dimension

$$\rightarrow M^a L^b T^c$$

$$\rightarrow F^x L^y T^z$$

$$Ns = \frac{N \sqrt{P}}{(H)^{5/4}} = \frac{\text{rpm} \sqrt{kw}}{(m)^{5/4}}$$

$$= \frac{\left(\frac{1}{\text{Time}}\right) \sqrt{\frac{\text{Joule}}{\text{sec}} = \frac{N \cdot m}{\text{sec}} = \frac{kg \cdot \frac{m}{\text{sec}^2} \cdot m}{\text{sec}}}}{(m)^{5/4}}$$

$$= \frac{M^{1/2} \cdot L^1}{T \cdot T^{3/2} \cdot L^{5/4}}$$

$$= M^{1/2} L^{-1/4} T^{-5/2}$$

$$a = 1/2$$

$$b = -1/4$$

$$C = \frac{-5}{2}$$

$$\Rightarrow M^{1/2} L^{-1/4} T^{-5/2}$$

$$\times \text{—————} \times$$

$$= \frac{F^{1/2} L^{1/2}}{\text{—————}}$$

$$T \times T^{1/2} \times (L)^{5/4}$$

$$= \frac{F^{1/2} L^{-3/4} \times T^{-3/2}}{\text{—————}}$$

$$x = 1/2, y = -\frac{3}{4}, z = -\frac{3}{2}$$

$$\Rightarrow \frac{F^{1/2} L^{-3/4} T^{-3/2}}{\text{—————}}$$

Dimensionless specific speed = Shape Number

$$N_s = \frac{N' \sqrt{\frac{P_1}{\rho}}}{(gH)^{5/4}}$$

$$\underline{a = b = c = 0}$$

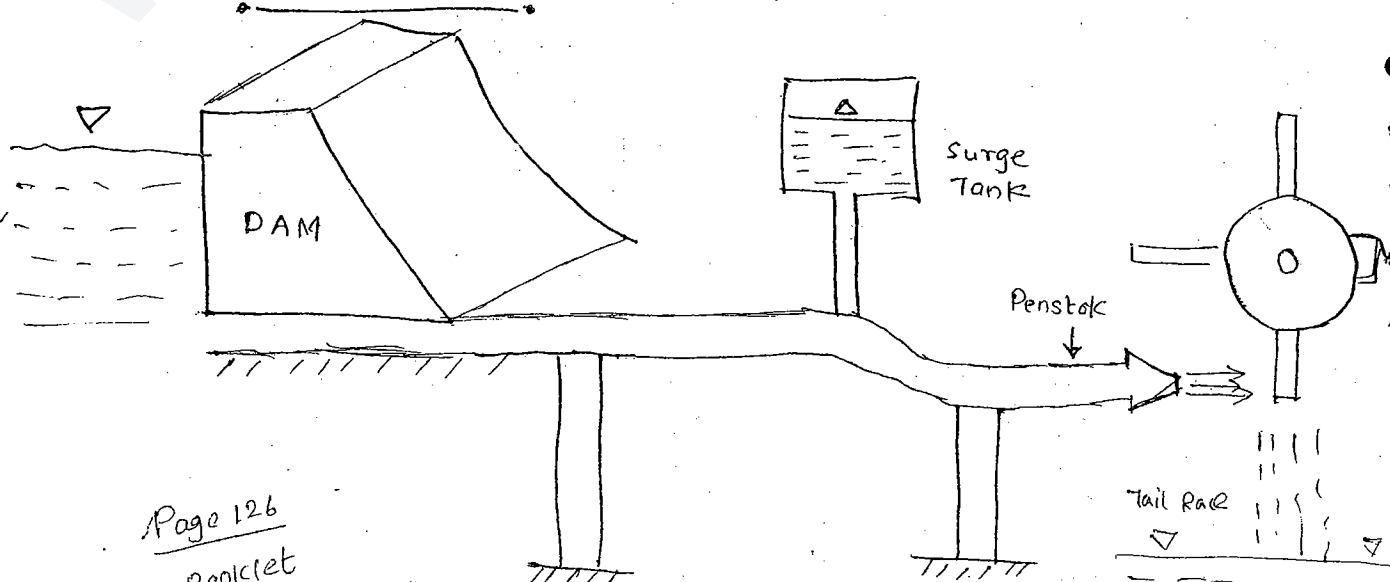
$$N' = \text{rpm} = \frac{N}{60} \text{ rpm}$$

$$P_1 = \text{in watt}$$

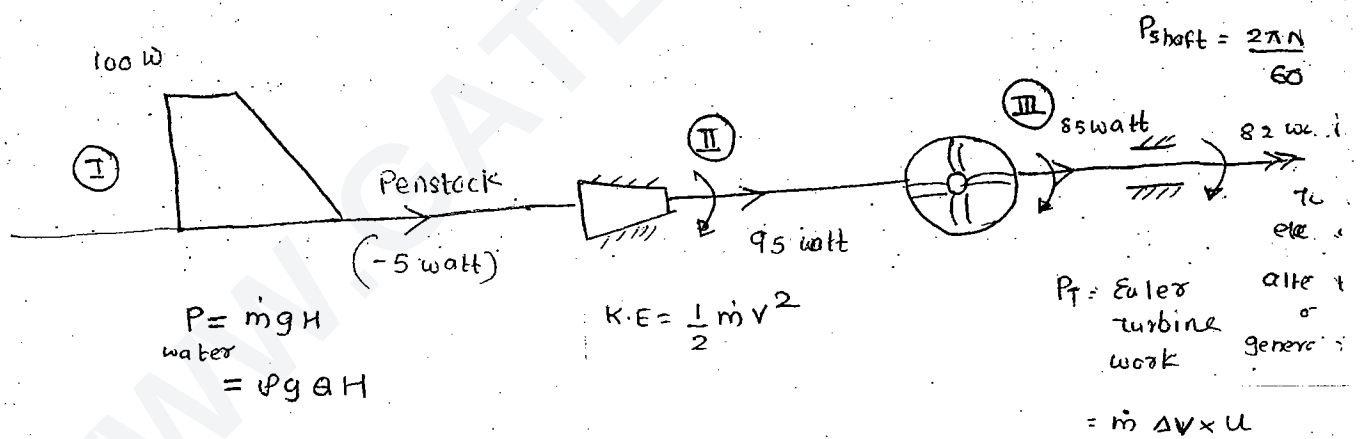
$$= \frac{P}{1000} \text{ kW}$$

$$\rho = \text{density}$$

Line diagram of Hydro electric Power plant



Power flow diagram:-



Water Power

$$P_{water} = \dot{m} g H$$

$$= \rho g Q H$$

Kinetic energy

$$K.E = \frac{1}{2} \dot{m} v^2$$

Turbine power

$$P_T = \dot{m} \Delta V \times u$$

Shaft power:-

$$P_{shaft} = \frac{2\pi N T}{60}$$

$$\eta_{\text{nozzle}} = \frac{\text{II}}{\text{I}} = \frac{\frac{1}{2} \dot{m} V_1^2}{\dot{m} g H}$$

$$\eta_{\text{turbine}} = \frac{\text{III}}{\text{II}} = \frac{P_7}{\frac{1}{2} \dot{m} V_1^2}$$

$$\eta_{\text{hydraulic}} = \eta_{\text{nozzle}} \times \eta_{\text{turbine}}$$

$$\eta_{\text{mechanical}} = \frac{P_{\text{shaft}}}{P_{\text{turbine}}}$$

$$\eta_{\text{overall}} = \frac{P_{\text{shaft}}}{P_{\text{water}}} = \eta_{\text{nozzle}} \times \eta_{\text{turbine}} \times \eta_{\text{mech}}$$

$$= \eta_{\text{hydraulic}} \times \eta_{\text{mechanical}}$$

Model testing :-

It is an experimental approach with due support calculation of the turbine. Before manufacture, erection and commission it is necessary to apply model analysis and model testing. In ~~model~~ model analysis of turbine four numbers are used -

- (i) specific speed
- (ii) Head quantity
- (iii) discharge number or qty
- (iv) Power number or qty

Model turbine :- (small turbine or Laboratory Turbine)

lay turbine.

P- Prototype Turbine (Large scale Turbine)
or

(*) Scale Ratio or Model Ratio = $\frac{u}{D} = \frac{u_{\text{model}}}{D_{\text{prototype}}}$

ex: 1:2
1:4
1:10
1:20

(i) $(Ns)_m = (Ns)_p$

$$\frac{N_m \sqrt{P_m}}{(H_m)^{5/4}} = \frac{N_p (\sqrt{P_p})}{(H_p)^{5/4}}$$

(ii) Head Number

water jet velocity $\propto$ Tangential vel of wheel

$$V_{\text{jet}} \propto u$$

$$\sqrt{2gH} \propto \frac{\pi D N}{60}$$

$$\sqrt{H} \propto DN$$

$$\frac{\sqrt{H}}{DN} = \text{constant}$$

Squaring Both sides

$$\frac{H}{D^2 N^2} = \text{const}$$

$$\frac{\sqrt{H_m}}{D_m \cdot N_m} = \frac{\sqrt{H_p}}{D_p \cdot N_p}$$

(iii) Discharge Number:-

Discharge = Area (c/s) $\times$ vel of fluid

$$Q = D^2 V$$

$$Q = D^2 \sqrt{2gH}$$

$$Q \propto D^2 \sqrt{H}$$

$$\frac{Q}{D^2 \sqrt{H}} = \text{constant}$$

IV , Power Numbers

$$\text{Power} = \rho g Q H$$

$$= \rho g Q H$$

$$P \propto Q H$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$P \propto D^2 H^{3/2}$$

$$\frac{P}{D^2 H^{3/2}} = \text{constant}$$

$$\frac{P_m}{D_m^2 (H_m)^{3/2}} = \frac{P_p}{(D_p)^2 (H_p)^{3/2}}$$

UNIT QUANTITIES :-

they are used for checking the performance of one individual turbine for which scale Ratio is 1 (same ~~size~~ size of turbine)

they are three unit quantities

1. unit ~~head~~ speed (N_u)
2. unit discharge (Q_u)
3. unit Power (P_u)

unit speed (N_u)

$$\sqrt{2gH} \propto \frac{\pi D N}{60}$$

(for same Turbine)

$$\sqrt{H} \propto N$$

$$N \propto \sqrt{H}$$

$$N = N_u \sqrt{H}$$

$$N_u = \frac{N}{\sqrt{H}}$$

$$\text{ie } \boxed{\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}}$$

2.)

$$Q = A \cdot V$$

$$Q = \frac{\pi D^2}{4} \cdot \sqrt{2gH}$$

For same turbine

$$Q \propto \sqrt{H}$$

$$Q = Q_u \sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

$$\boxed{\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}}$$

3.)

$$P = \dot{m} g H$$

$$P = \rho g Q H$$

$$P \propto Q \cdot H$$

$$P = P_u \cdot \sqrt{H} \cdot H$$

$$P_u = \frac{P}{\sqrt{H} \cdot H}$$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

A Hydraulic turbine producing 1000 kW at 40m Head. If Head is reduced to 20m then power delivered by same turbine could be

- (a) 177 kW (b) 354 kW (c) 500 kW (d) 707

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\frac{1000}{(40)^{3/2}} = \frac{P_2}{(20)^{3/2}}$$

$$P_2 = \underline{\hspace{2cm}} \text{ kW}$$

Q.) A hydraulic turbine is Reqd to produce 25 MW at 50m Head and 90 rpm. The laboratory facilities available to test 25 kW model turbine at 50m Head. What should be the model turbine ~~at 50~~ runner speed and the scale Ratio of model to prototype?

Soln: Given

$$P_p = 300 \text{ kW}$$

$$N_p = 100 \text{ rpm}$$

$$H_p = 40 \text{ m}$$

$$\frac{D_m}{D_p} = 1:4 = \frac{1}{4}$$

$$H_m = 10 \text{ m}$$

$$P_m = ?$$

$$\text{we have } \frac{P_m}{H_m^{3/2}} = \frac{P_p}{H_p^{3/2}}$$

$$P_m = \left(\frac{1}{4}\right)^2 \cdot \left(\frac{10}{40}\right)^{-1/2} \cdot 300$$

$$\underline{2.34 \text{ kW}}$$

$$\frac{\sqrt{H_m}}{N_m D_m} = \frac{\sqrt{H_p}}{N_p D_p} \quad , \quad \sqrt{\frac{H_m}{H_p}} \times \frac{D_p}{D_m} \times N_p = N_m$$

$$\sqrt{\frac{5}{50}} \times \frac{6.3}{1} \times 90 = N_m$$

Q) A Model hydraulic turbine is tested at a Head of $\frac{1}{4}$ th of Head with which full scale turbine operates. The Dia of model turbine is 50% of full scale turbine dia. If N is the rpm of the full scale turbine, the rpm of model will be

- (a) $\frac{N}{4}$ (b) $\frac{N}{2}$ (c) N (d) $2N$

Ans: Given

$$H_m = \frac{1}{4} H_p$$

$$D_m = 0.5 D_p$$

$$N_p = N$$

$$N_m = ?$$

Head Ratio

$$N_p = N$$

$$\frac{\sqrt{H_m}}{N_m D_m} = \frac{\sqrt{H_p}}{N_p D_p}$$

$$N_m = ? = \sqrt{\frac{H_m}{H_p}} \times \frac{D_p}{D_m} \times N_p = \sqrt{\frac{1}{4}} \times$$

$$= \underline{N}$$

Q) At a Hydroelectric power plant Available Head and flow Rate are ¹⁸²
~~25.3 m~~ 24.5 m and $10.1 \text{ m}^3/\text{s}$ resp. if the turbine is to
 Run at 4 revolutions per second with overall efficiency of 90% then
 the suitable type of turbine for the given conditions is

- a) Pelton wheel b) Francis Turbine c) Kaplan Turbine d) propeller Turbine

Ans: $NS = \frac{N\sqrt{P}}{(H)^{5/4}}$

where $N = 4 \text{ rev/sec}$

$N = 4 \times 60 \text{ rev/min} = 240 \text{ rpm}$

$H = 24.5 \text{ (m)}$

$P = 2$

$\eta_o = \frac{P_o/p}{P_{\text{water}}} = \frac{P}{\dot{m}gH} = \frac{P}{\rho Q g H}$

$0.9 = \frac{P}{1000 \times 10.1 \times 9.81 \times 24.5}$

$P = 2184 \text{ kW}$

$NS = \frac{240 \sqrt{2154.4 \text{ (kW)}}}{\left(\frac{24.5}{\text{m}}\right)^{5/4}}$

$NS = 205.8 \text{ m}$

$30 < NS < 275 \rightarrow \text{Francis Turbine}$

$NS > 275 \rightarrow \text{Kaplan Turbine}$

$NS < 30$: Low speed Turbine (Pelton wheel)
 with single nozzle

a.) In a Hydroelectric power station water is available at the rate of $175 \text{ m}^3/\text{s}$ and a Head of 18 m . The Turbines run at speed of 150 rpm with overall efficiency 82% . Find the No of Turbines to be installed if they have maximum specific speed of 460

Soln:

$$Q = 175 \text{ m}^3/\text{s}$$

$$H = 18 \text{ m}$$

$$N = ~~0.82~~ 150 \text{ rpm}$$

$$N_s = 460$$

$$\eta_o = 0.82$$

$$\text{No of Turbines} = ?$$

$$N_s = \frac{N \sqrt{P}}{(H)^{5/4}} = \frac{150 \sqrt{P}}{(18)^{5/4}}$$

$$460 = \frac{150 \sqrt{P}}{(18)^{5/4}} \Rightarrow P = 12927 \text{ kW}$$

$$P = \underline{\underline{12.93 \text{ MW}}}$$

$$\eta_o = \frac{P_o/p}{P_{\text{water}}} = \frac{P_{\text{shaft}}}{P_{\text{water}}}$$

$$0.82 = \frac{P_{\text{shaft}}}{\rho g Q H}$$

$$P_{\text{shaft}} = 0.82 \times 1000 \times 9.81 \times 175 \times 18$$

$$= 25.34 \times 10^6 \text{ watt}$$

$$= \underline{\underline{25.34 \text{ MW}}}$$

$$* \text{ No of Turbines Reqd} = \frac{\text{Total Power}}{\text{Power/Turbine}}$$

$$= \frac{25.34}{12.93} = 1.98 \approx \underline{\underline{2 \text{ Turbines}}}$$

9) A water turbine producing 10 MW power is to be tested with a geometrically similar 1:8 model which runs at same speed as that of prototype.

- a) find the Power developed by the Model turbine assuming efficiencies of model and prototype are equal
- b) Find the Ratio of Heads and
- c) find the Ratio of Mass flow Rates b/w prototype and Model turbines.

$$P_p = 10 \text{ MW} = 10,000 \text{ kW}$$

$$\frac{D_m}{D_p} = \frac{1}{8}$$

$$N_m = N_p$$

$$\eta_m = \eta_p$$

a) $P_m = ?$

b) $\frac{H_m}{H_p} = ?$

c) $\frac{\dot{m}_p}{\dot{m}_m} = ?$

$$\frac{\sqrt{H}}{N^2 D} = \text{const}$$

$$\frac{Q}{D^2 \sqrt{H}} = \text{const}$$

$$\frac{P}{D^2 (H)^{3/2}} = \text{const}$$

$$\frac{N \sqrt{P}}{(H)^{3/4}} = \text{const}$$

$$\dot{m} = \rho Q$$

$$\frac{\dot{m}_p}{\dot{m}_m} = \frac{\rho Q_p}{\rho Q_m} = ?$$

$$\frac{\sqrt{H_m}}{N_m^2 D_m} = \frac{\sqrt{H_p}}{N_p^2 D_p}$$

$$\frac{\sqrt{H_m}}{\sqrt{H_p}} = \frac{D_p}{D_m} = 8$$

$$H_p = 64$$

$$\frac{P_m}{D_m^2 (H_m)^{3/2}} = \frac{P_p}{D_p^2 (H_p)^{3/2}}$$

$$P_m = 10,000 \times \left[\frac{1}{8} \right]^2 \times \left[\frac{H_m}{H_p} \right]^{3/2}$$

$$10,000 \times \left[\frac{1}{8} \right]^2 \times \left[\frac{1}{64} \right]^{3/2}$$

$$\underline{P_m = 305 \text{ watt}}$$

c.) $\frac{m_p}{m_m} = \frac{\rho Q_p}{\rho Q_m}$

Discharge No!

$$\frac{Q}{D^2 \sqrt{H}} = \text{const}$$

$$D^2 \sqrt{H}$$

$$\frac{Q_m}{D_m^2 \sqrt{H_m}} = \frac{Q_p}{D_p^2 \sqrt{H_p}}$$

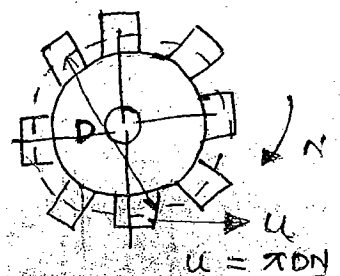
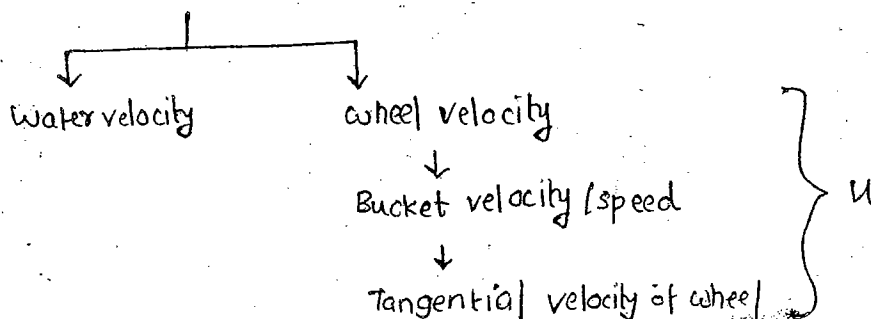
$$\frac{Q_p}{Q_m} = \left[\frac{D_p}{D_m} \right]^2 \times \sqrt{\frac{H_p}{H_m}}$$

$$= [8]^2 \times \sqrt{64}$$

$$\frac{Q_p}{Q_m} = 8^3 = 512$$

Hydraulic Analysis of an Impulse Turbine

Velocity Triangles

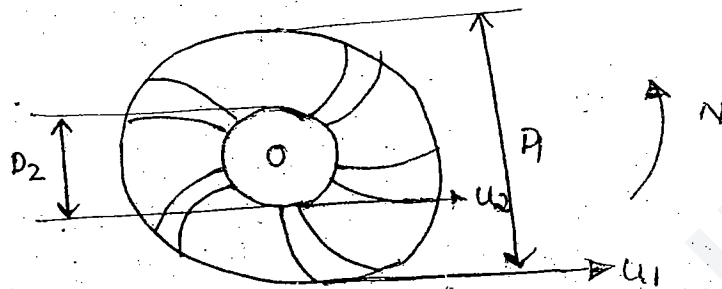


For impulse (Pelton wheel)

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$$u_1 = u_2 = u$$

For ~~other~~ Francis Turbine



$$u_1 = \omega \cdot R_1$$

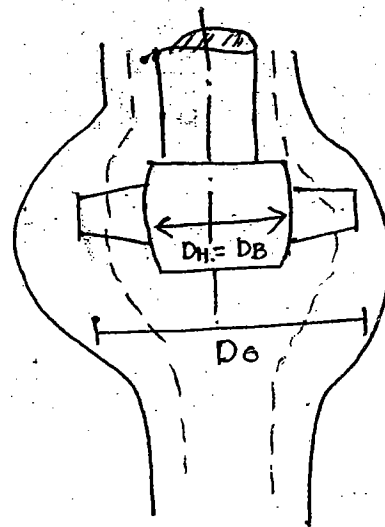
$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_2 = \omega \cdot R_2$$

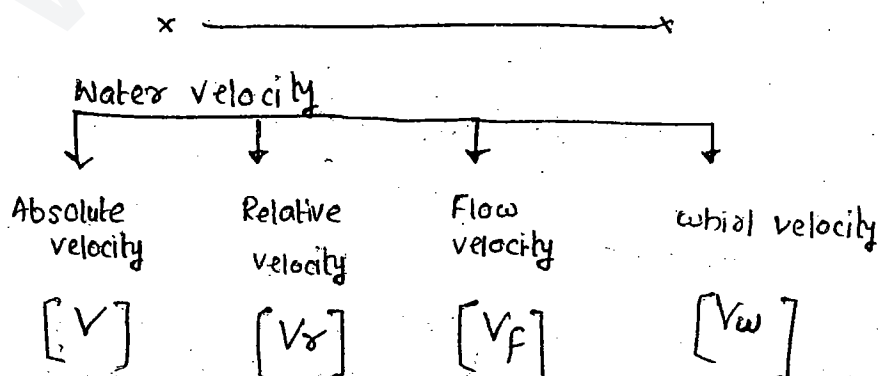
$$u_2 = \frac{\pi D_2 N}{60}$$

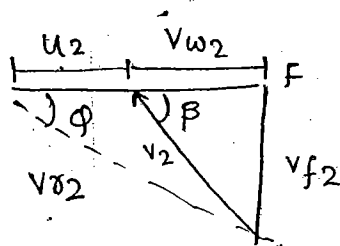
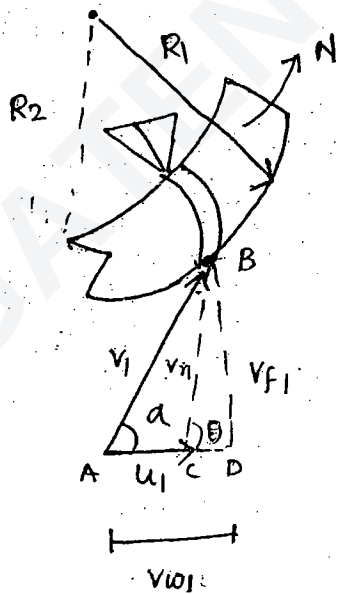
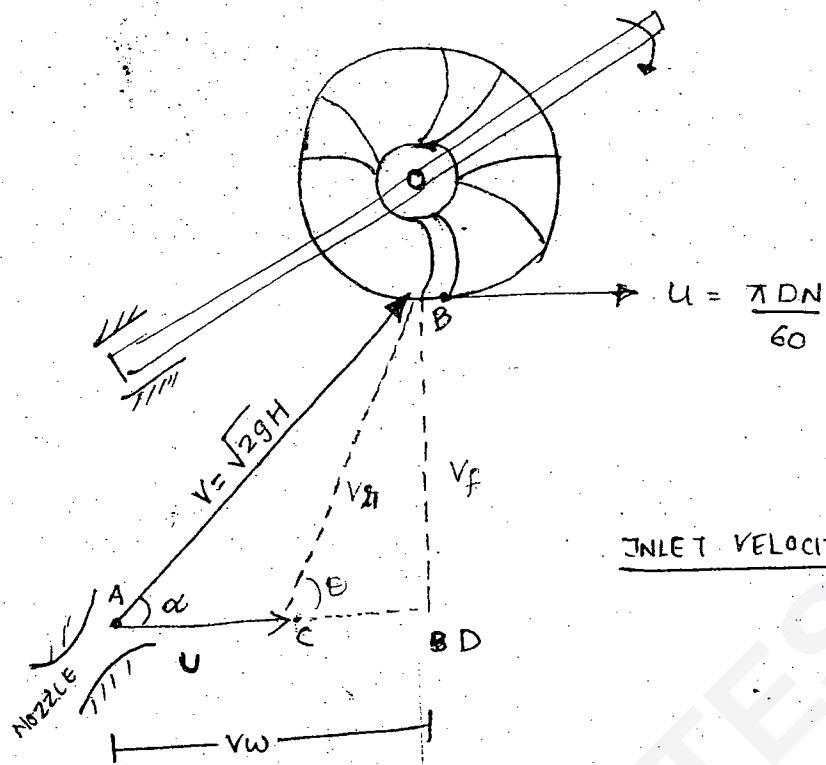
Kaplan Turbine:-

$$u = u_0 = \frac{\pi D_0 N}{60} \text{ (m/sec)}$$



$$D_{hub} = \frac{1}{3} D_0$$





Pelton wheel :-

V_1 = Absolute vel at inlet

u_1 = Tangential vel

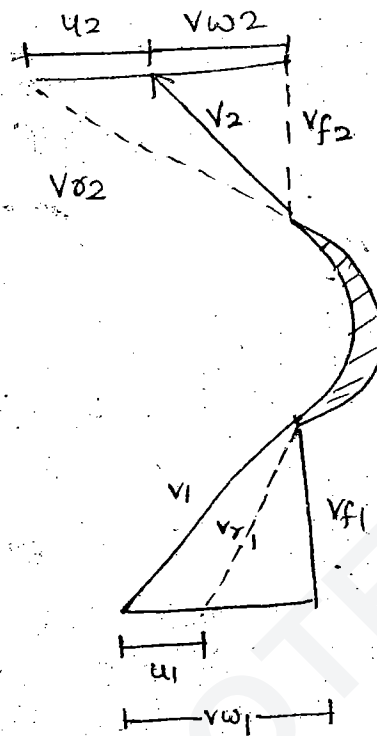
V_{r1} = Relative vel water

V_{f1} = flow vel of water

V_{w1} = whirl vel of water

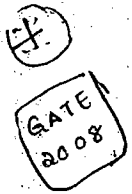
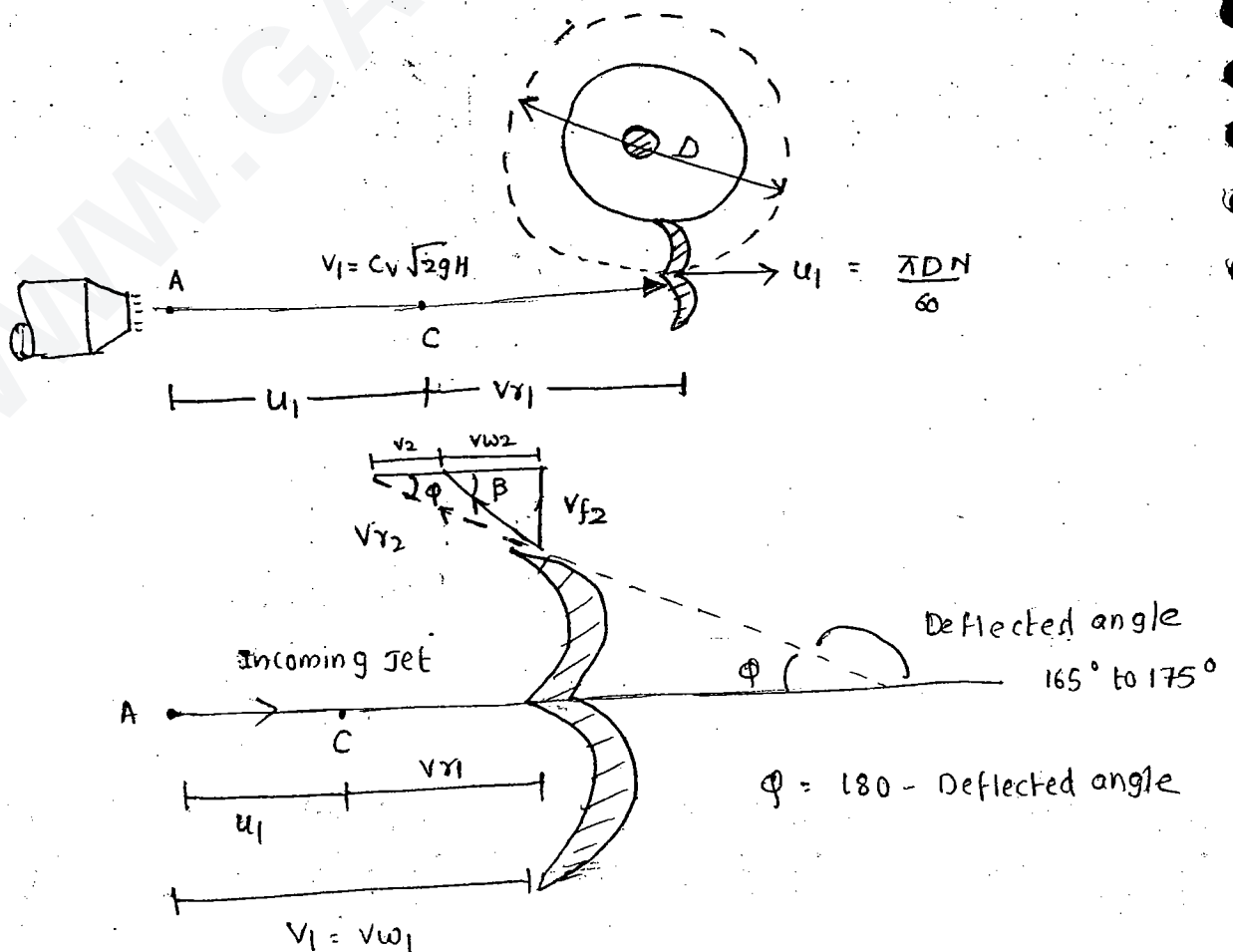
α = Nozzle angle at inlet

θ = Blade angle at inlet



General Turbine

Pelton wheel



WWW.GATENOTES.IN

2011 XE

Q 1

(d)

Q 2

3

4

5

6

7

correction Point B, $1\frac{3}{4}$ down stream

8

Q-10 convention 91

Unit 2

Q No: 1 C
2 B
3 D
4 A
5 A
6 A
7 B
8 C
9 B
10 C
11 A

Q12 - D

13 - A

14 - B

16 B

17 A

18 B

19 C (XE 2011)

Q 01: D

02 C

3 C

4 C

5 A

6 A

7 B

8 C

9 B

10 B

11 D

12 B

13 A

14 A

15 B

Q-1 A

2 D

3 B

4 A

5 C

6 A

7 A

8 B — 2

9 B

10 C

Unit 4Fluid dynamics

Page No: 55

Q No: 1 C

2 C

3 B

4 C

5 Ans: $F = 4.999 \text{ N} \approx 5 \text{ N}$

6 A

7 A

8 B

9 D

10 D

11 D

12 B

13 D

14 C

Q 15 D

Q 16 D

Q 17 A

Q 18 B

Q 19 Ans: 264 (N) Q 20 } (a) $P_A = P_{\text{atm}} - \rho g h_2$ (b) $V_D = \sqrt{2g(h_1 - h_2)}$

Page No 75 & 74

Q-1 A

Q 2 A

3 C

4 C

5 B

6 C

Q 7 C

8 A

9 C

10 D

 5.8×10^{-5}

Q 11 B

Q 12 C