

MACHINE

DESIGN

01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

M

WWW.GATENOTES.IN

Design - is a process plan to satisfy a human need.

It is a mechanism to convert an idea into reality
It is a solution to the problem

It is a solution to the problem.

Machine - A m/c is a combination of resistant bodies with successfully constrained motion (relative motion) to other body or of energy into mechanical energy.

To transmit and modify the available energy to do some useful work.

Machine design involve the selection of suitable material and optimum dimensions to withstand the load without failure.

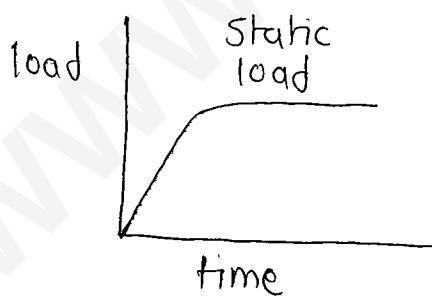
Loads - Externally applied force or moment acting on the member

```

graph TD
    loads --> Static
    loads --> Variable_load[Variable load]

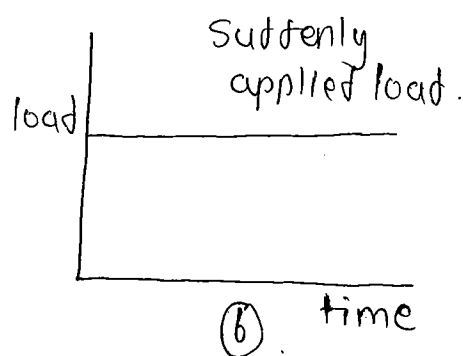
```

Conb. w. & to time { Magnitude
Direction
Point of application



(a)

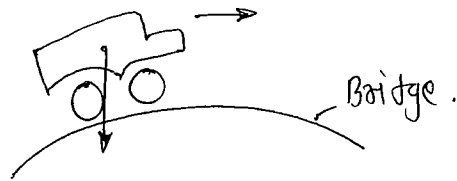
Gradually applied load
can also called as static load.



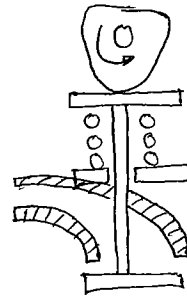
WWW.GATENOTES.IN

① Magnitude changes - magnitude is fluctuating but direction remain same

Ex:-



the direction of force is cont. but magnitude is changing while vehicle crossing the bridge.

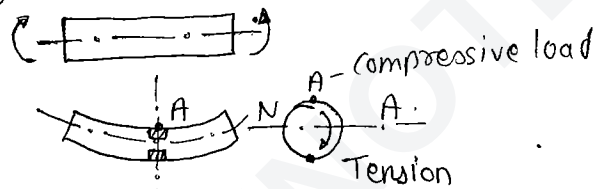


valve spring.

② Direction Changes - Constant magnitude

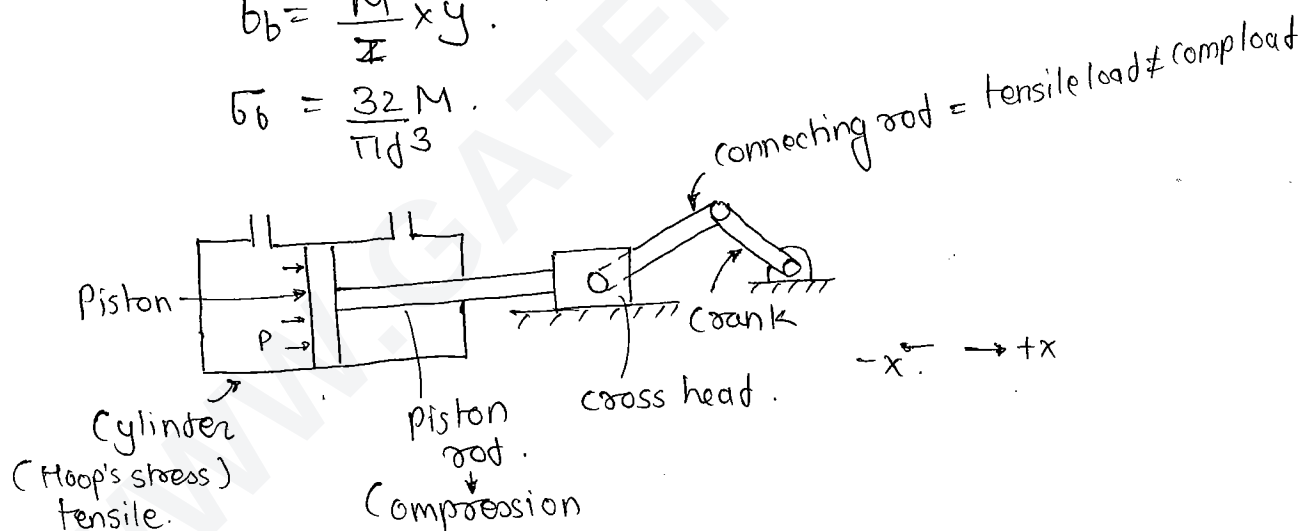
Ex:- Rack pinion., gear m/csm.

A rotating shaft causing a const. bending movement.



$$\sigma_b = \frac{M}{I} \times y$$

$$\sigma_b = \frac{32 M}{\pi d^3}$$



Pressure P

$$\vec{F} = P_{\text{res}} \times \text{Area}$$

$$= P \times \pi/4 D^2$$

$$\vec{F} = P \times \frac{\pi}{4} (D^2 - d^2)$$

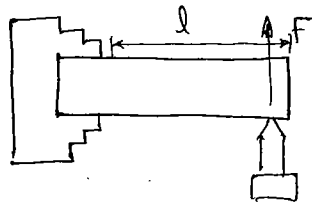
$$d \ll D$$

$$\vec{F} = \vec{F}$$

Piston rod = $\vec{F}$ - tensile load $\vec{F}$ - comp. load.

hence piston rod come under this category.

③ Point of application.

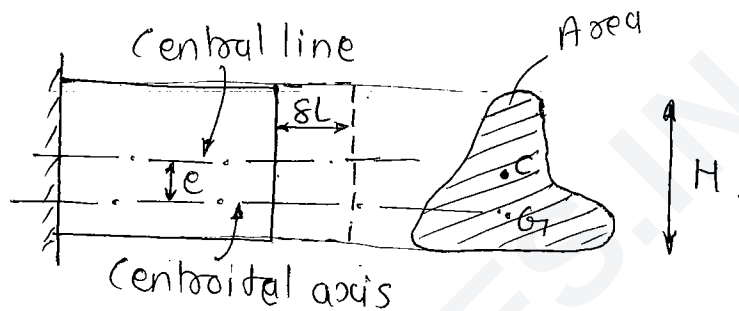


$$M_{\max} = F \times l$$

$$M_{\min} = F \times 0$$

Stress (σ)

$$\sigma_{\text{central axis}} = \frac{\text{load}}{\text{area}} = \frac{F}{A}$$



$$\sigma_{\text{central line}} = \frac{F}{A} + \frac{M}{I} y$$

Strain (ϵ)

$$\epsilon = \frac{\delta L}{L}$$

Hook's law

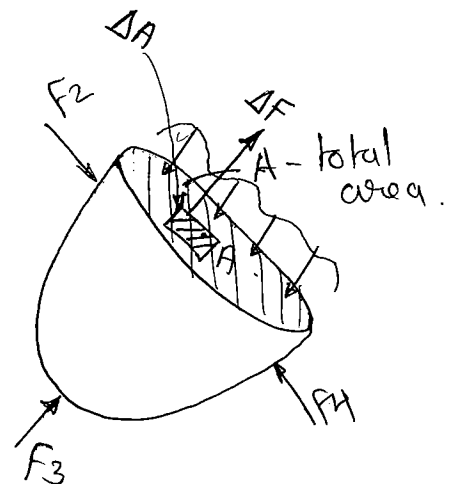
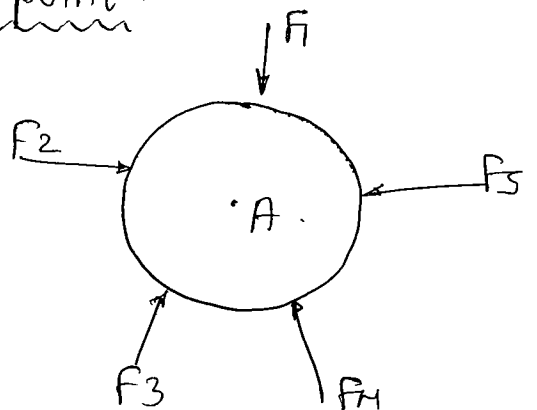
$\sigma \propto \epsilon$ (within the proportional limit) (No elastic limit).

Stiffness (K)

$$K = \frac{\text{load}}{\text{deflection}}$$

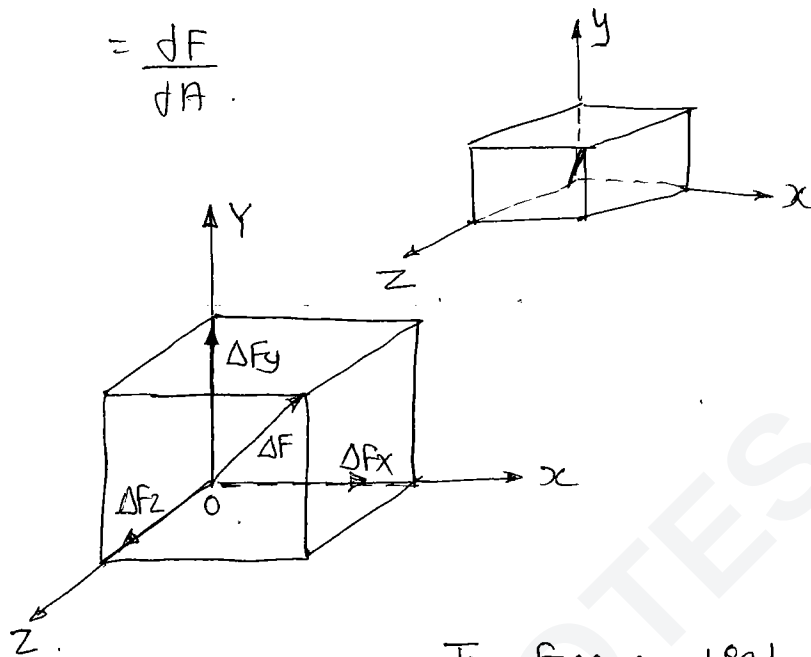
$$= \frac{F}{\frac{FL}{AE}} = \frac{AE}{L}$$

Stress at a point.



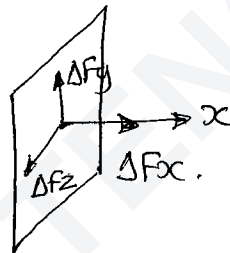
$$\sigma_t = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

$$= \frac{dF}{dA}$$



→ Force $\perp$ to x axis

Two faces are $\perp$ to x axis.



Tensor
(vector quant) $\sigma_{xx} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_x}{\Delta A}$

Force Normal to co-ordinate Force direction

$$\sigma_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_y}{\Delta A}$$

$$\sigma_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_z}{\Delta A}$$

→ Force $\perp$ to y axis (same)
" " " z axis " "

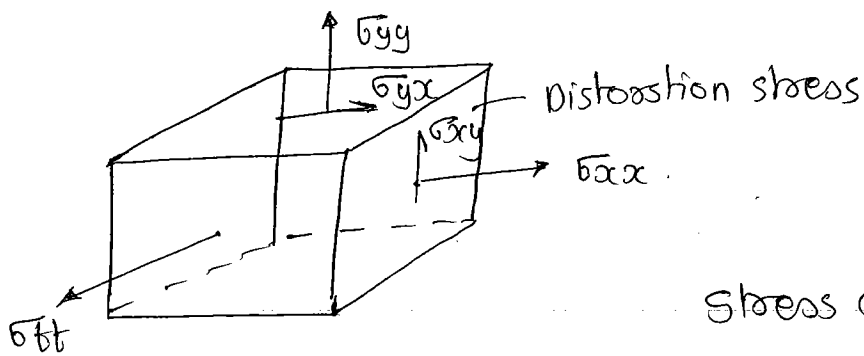
Stress tensor

$$\sigma = \begin{bmatrix} x & \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ y & \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ z & \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Direct stress
Normal stresses, (volume change)

Distortion stress

Normal stress - It changes the size (volume changes)
It is called as direct stress (No shape change)



stress component - 9.

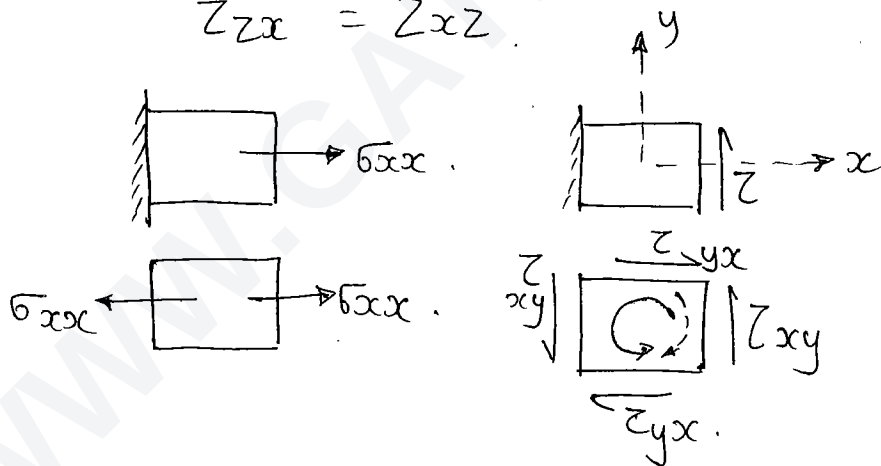
$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Complementary shear stress are

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$



6 stress components are req to define the total stress at a point in the space.

$$\begin{matrix} \sigma_{xx} & \tau_{xy} \\ \sigma_{yy} & \tau_{yz} \\ \sigma_{zz} & \tau_{zx} \end{matrix}$$

Two dimension stress, (Plane stress)

Ex:- ① Thin sheets (laminar)



② Thin cylindrical shell

$\bar{\sigma}_\theta, \bar{\sigma}_z$ and p (pressure) is neglected.

$$\bar{\sigma} = \begin{bmatrix} \bar{\sigma}_{xx} & \tau_{xy} \\ \tau_{yx} & \bar{\sigma}_{yy} \end{bmatrix}$$

3 stress components are required to define the stress $\bar{\sigma}_{xx}, \bar{\sigma}_{yy}, \tau_{xy} = \tau_{yx}$.

Principle Stress.

Principle plane - A plane carrying zero ^{shear} ~~principle~~ stress.

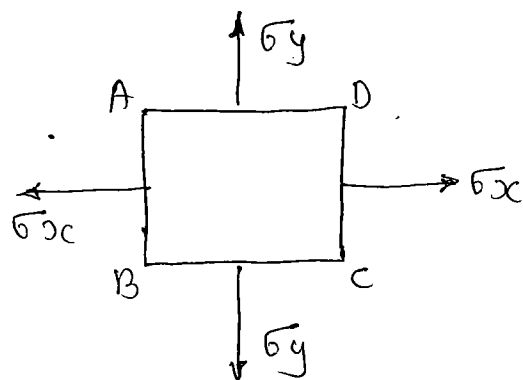
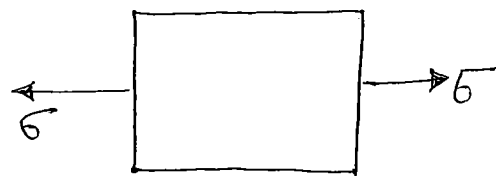
Principle stress - The stress (normal) acting on the principle plane.

Principle planes and stresses.

Principle plane.

zero shear stress

$$\begin{cases} AB \text{ or } CD & \bar{\sigma}_1 = \bar{\sigma} \\ AD \text{ or } BC & \bar{\sigma}_2 = 0 \end{cases}$$



$$\bar{\sigma}_y > \bar{\sigma}_x$$

Biaxial stress

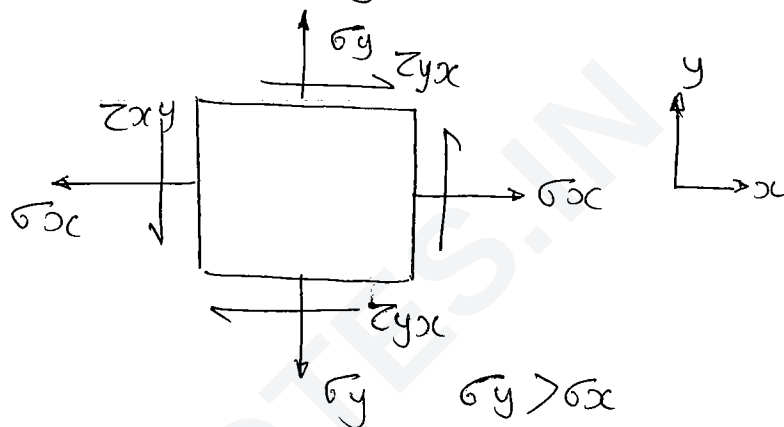
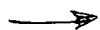
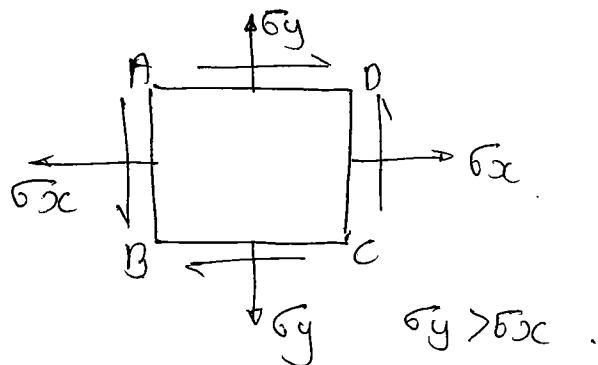
Principle plane.

$$AB \text{ or } CD \quad \bar{\sigma}_1 = \bar{\sigma}_y \quad \text{Major}$$

$$AD \text{ or } BC \quad \bar{\sigma}_2 = \bar{\sigma}_x \quad \text{Minor}$$



$AB \propto CD \quad \sigma_2 = \sigma_x$
 $AD \propto BC \quad \sigma_1 = \sigma_y$

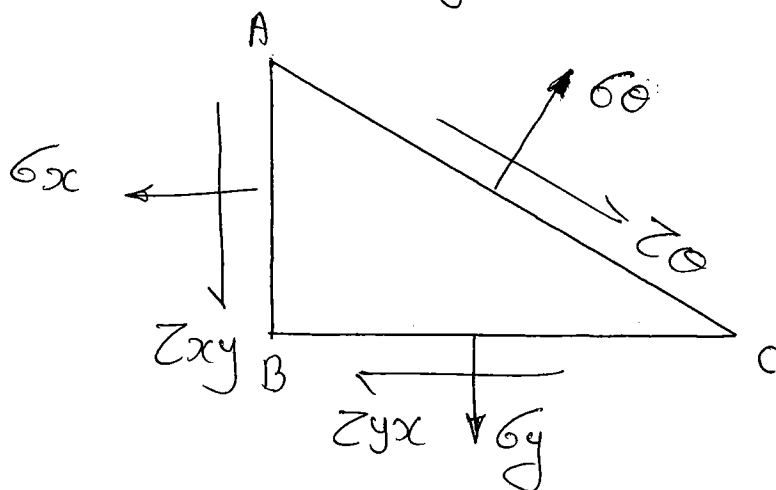
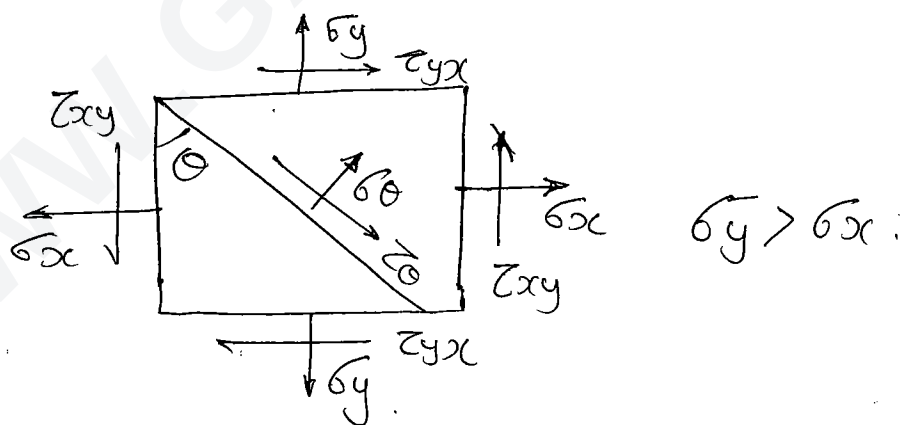


Principle ~~planes~~ stresses

$$\sigma_1 \text{ \& \& } \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2} \right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

Principal planes

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



$$\sigma_{\theta} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{\theta} = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - \tau_{xy} \cos 2\theta$$

downward shear stress
anti direction

Mohr's Circle.

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

Assume

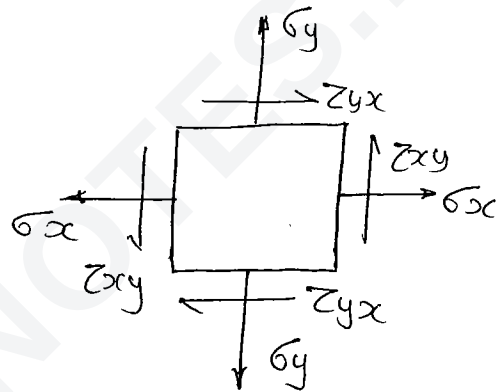
$$\sigma_x > \sigma_y$$

$$OA = \sigma_x$$

$$OB = \sigma_y$$

$$AD = -\tau_{xy}$$

$$BE = +\tau_{yx}$$



Sign Convention

→ Normal stress - abscissa

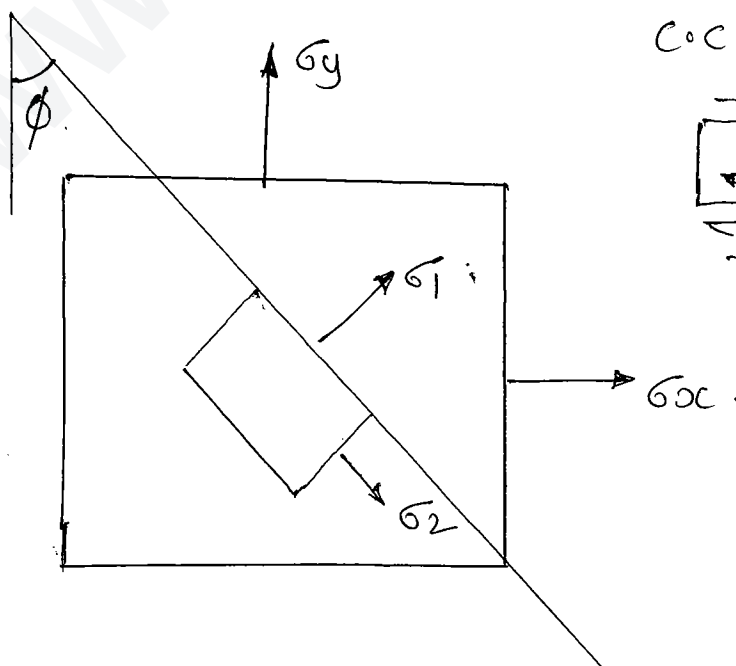
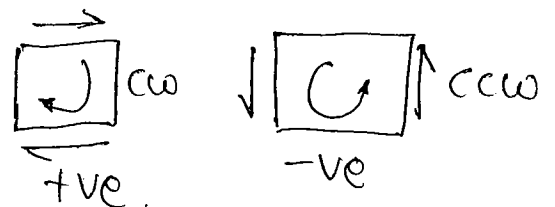
Tensile - - - +ve

Compressive - - -ve

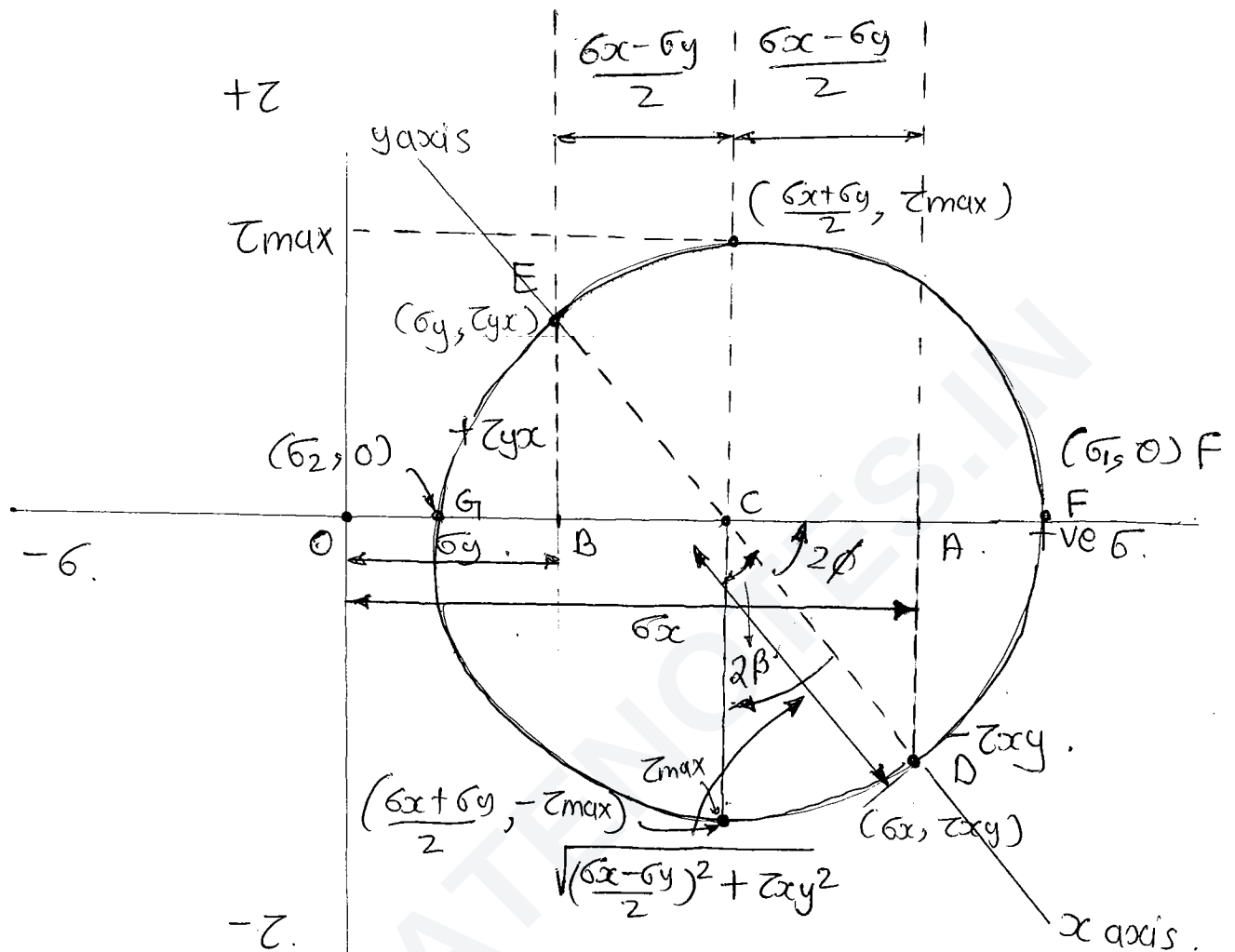
→ Shear stress - ordinate

Clockwise +ve

C.C.W -ve



2ϕ - ~~half~~ twice the actual value.



$\sigma_1 = OF$ - Maximum stress with zero shear stress. Hence Max ~~shear~~ principle stress.

$\sigma_2 = OG$ - Min stress with zero shear stress. Hence Min principle stress.

$$AC = BC = \frac{AB}{2} = \frac{\sigma_x - \sigma_y}{2}$$

$$AB = \sigma_x - \sigma_y$$

$$OC = \sigma_y + \frac{\sigma_x - \sigma_y}{2} = \frac{\sigma_x + \sigma_y}{2} \quad (OB + OC)$$

$$OF = \sigma_1 = OC + CF \quad (CF = CD = R)$$

$$= OC + CD.$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \sigma_x + \sigma_y - R.$$

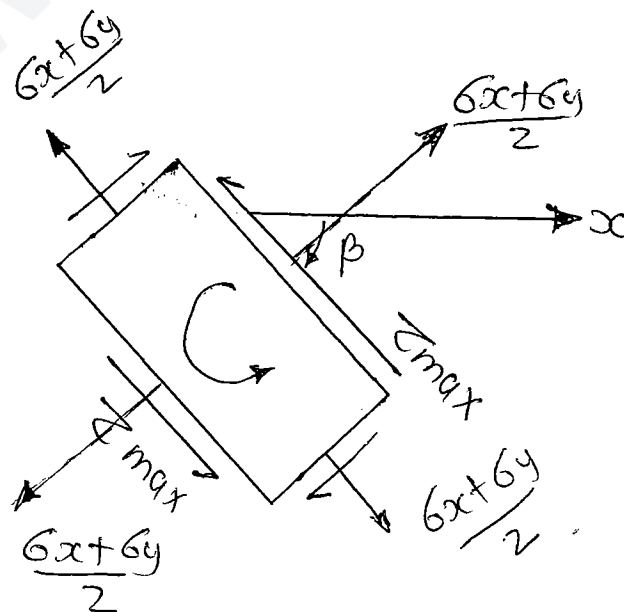
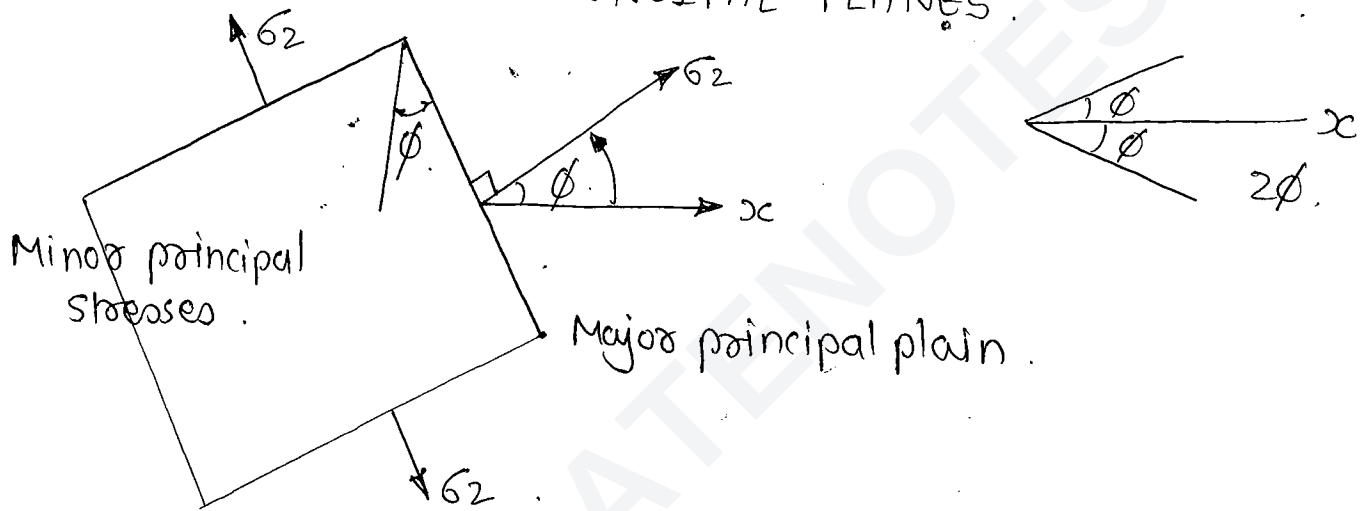
$\tau_{max} = \text{Radius of Mohr's circle} = R.$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \tau_{max}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \tau_{max}$$

ORIENTATION OF PRINCIPAL PLANES.



Design for Static Strength.

Q1] The strength of stress at a point is given, in Mpa is

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & -30 \end{bmatrix} \text{ The maximum shear stress.}$$

a] 20 MPa b] 30 MPa c] 35 d] 40 MPa.

$$\sigma_1 = 40 \quad \sigma_2 = -30 \quad \sigma_3 = 0$$

$$\tau_{\max} = \max \left\{ \begin{array}{l} \frac{|\sigma_1 - \sigma_2|}{2} \\ \frac{|\sigma_2 - \sigma_3|}{2} \\ \frac{|\sigma_3 - \sigma_1|}{2} \end{array} \right\} = \left\{ \begin{array}{l} \frac{40 - (-30)}{2} \\ \frac{+40}{2} \\ \frac{-30}{2} \end{array} \right\} \begin{array}{l} \frac{70}{2} = 35 \\ \frac{40}{2} = 20 \\ \frac{-30}{2} = 15 \end{array}$$

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & 30 \end{bmatrix} \quad \tau_{\max} = \frac{40 - 30}{2} = 10/2 = 5$$

$$\frac{40}{2} = 20 \quad \checkmark$$

$$\frac{30}{2} = 15$$

2] The stress at a point in Mpa is. $\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix}$
The principle stress.

$$\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = 50 \quad \sigma_2 = 30 \quad \sigma_3 = 0 \quad \text{Principal stress.}$$

$$\sigma_A = \frac{50 + 30}{2} + \sqrt{\left(\frac{50 - 30}{2}\right)^2 + 10^2} = 54.1421 \text{ MPa.}$$

$$\sigma_B = \frac{50 + 0}{2} + \sqrt{\left(\frac{50}{2}\right)^2 + 10^2} = 51.92 \text{ MPa.}$$

$$\sigma_C = \frac{30}{2} + \sqrt{\left(\frac{30}{2}\right)^2 + 10^2} = 33.02 \text{ MPa}$$

Shear stress

$$\sigma_A = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = 14.14$$

$$\sigma_B = 26.92 \text{ MPa. } \checkmark$$

③ The stress tensor at a point is given by (in MPa)

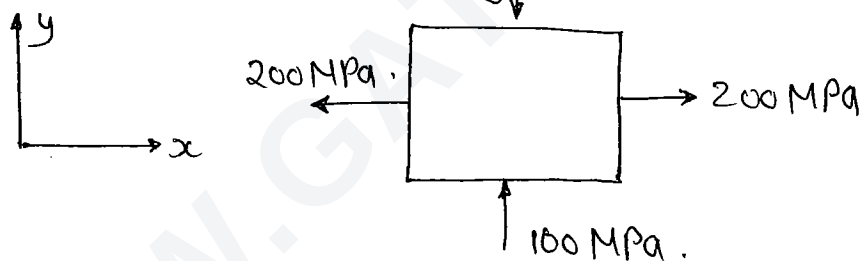
$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix} \quad \text{The principal stresses are}$$

$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix} \quad \sigma_3 = -20 \text{ MPa.}$$

$$\sigma_1 = \frac{10+20}{2} + \sqrt{\left(\frac{10-20}{2}\right)^2 + 5^2} = 22.071$$

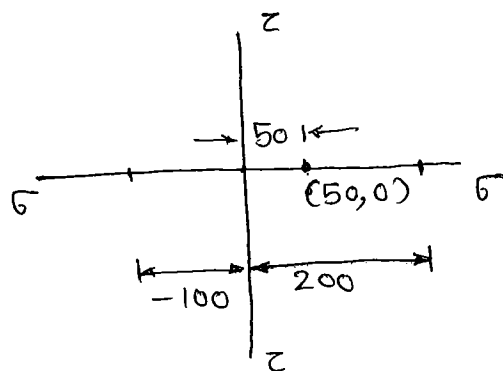
$$\sigma_2 = \frac{10+20}{2} - \sqrt{\left(\frac{10-20}{2}\right)^2 + 5^2} = 7.93 \text{ MPa.}$$

④ Consider a two dimension state of stress given from element as shown in Fig. 100 MPa.



What are the co-ordinate centre of Mohr's circle.

- a) (0,0) b) (100,200) c) (200,100) d) (50,0) ✓



⑤ In a strained material one of the principal stresses is twice the other. The maximum shear stresses in the same case is τ_{max} , then what is the max value of principal stress

a] τ_{max} b] $2\tau_{max}$ c] $4\tau_{max}$ d] $8\tau_{max}$

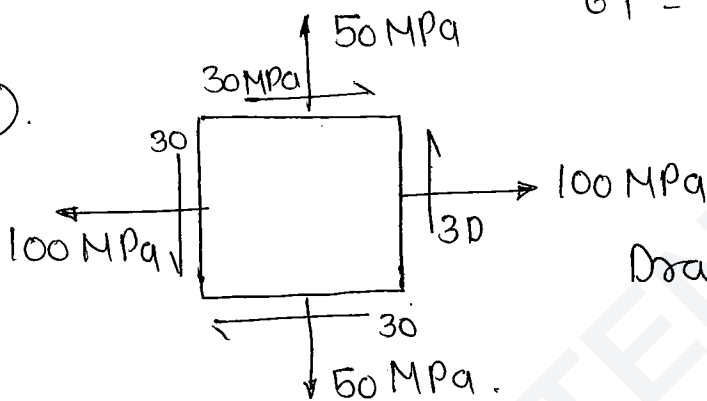
$$\sigma_1 = 2\sigma_2$$

$$\tau_{max} = \frac{2\sigma_1 - \sigma_2}{2} = \frac{2\sigma_2 - \sigma_2}{2}$$

$$\sigma_2 = 2\tau_{max}$$

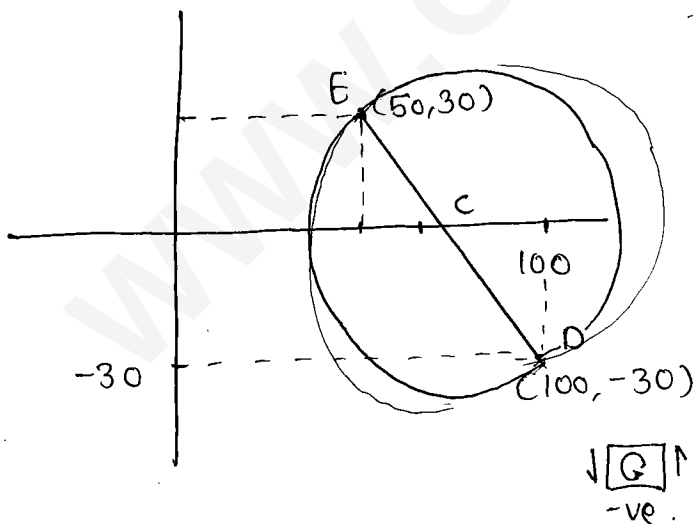
$$\sigma_1 = 4\tau_{max} \checkmark$$

⑥



Draw the Mohr's circle.

$$1 \text{ cm} = 10 \text{ MPa}$$



$$\tau_{max} = \max \left\{ \begin{array}{l} \frac{\sigma_1 - \sigma_2}{2} \\ \frac{\sigma_1}{2} \\ \frac{\sigma_2}{2} \end{array} \right\}$$

$$= \frac{2\sigma_2 - \sigma_2}{2} = \frac{\sigma_2}{2}$$

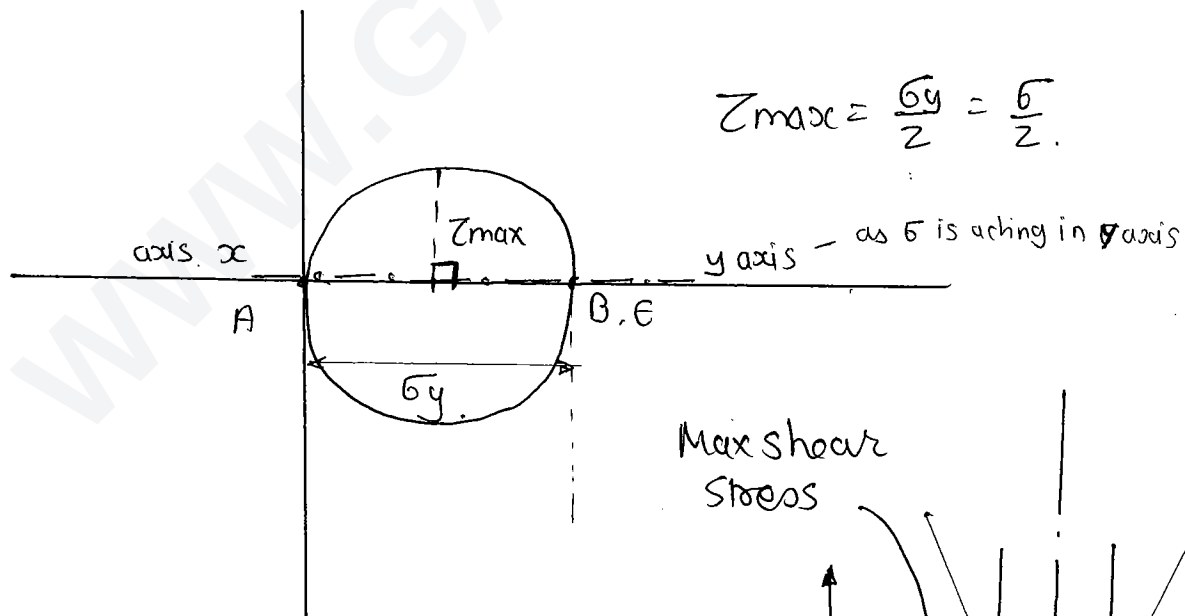
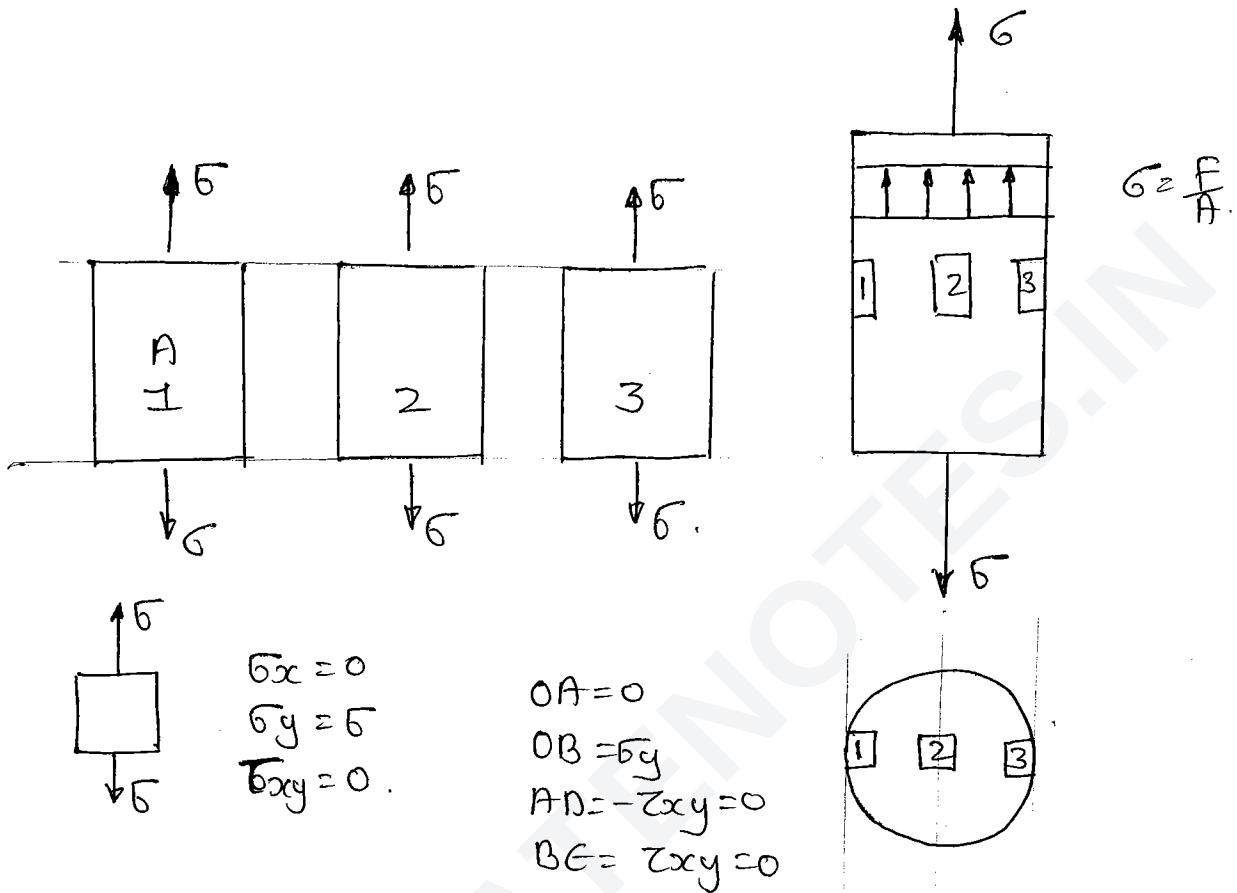
$$= \frac{2\sigma_2}{2} = \sigma_2 = \tau_{max}$$

$$= \frac{\sigma_2}{2}$$

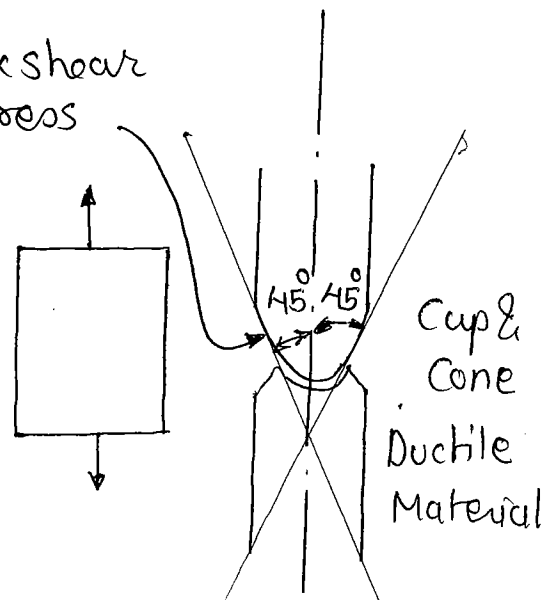
$$\sigma_1 = 2\tau_{max}$$

⑦. Draw a Mohr's circle under uniaxial state of stress.

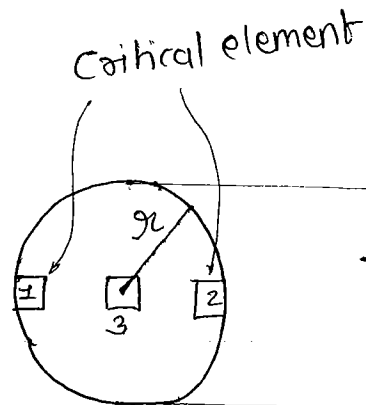
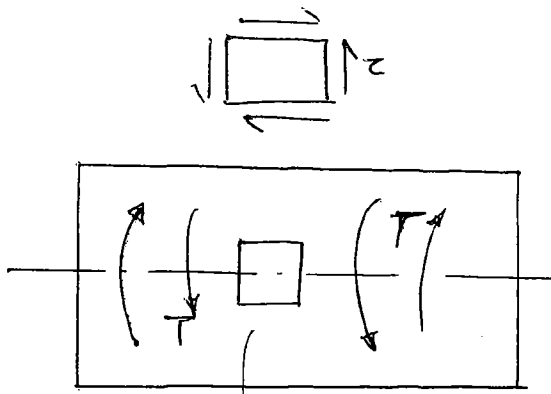
Mohr's circle for simple tension



Max shear stress



Mohr's Circle For Torsion stresses.



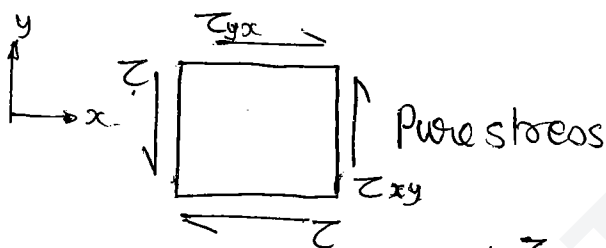
$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau \propto R$$

$$\tau \uparrow \quad R \uparrow$$

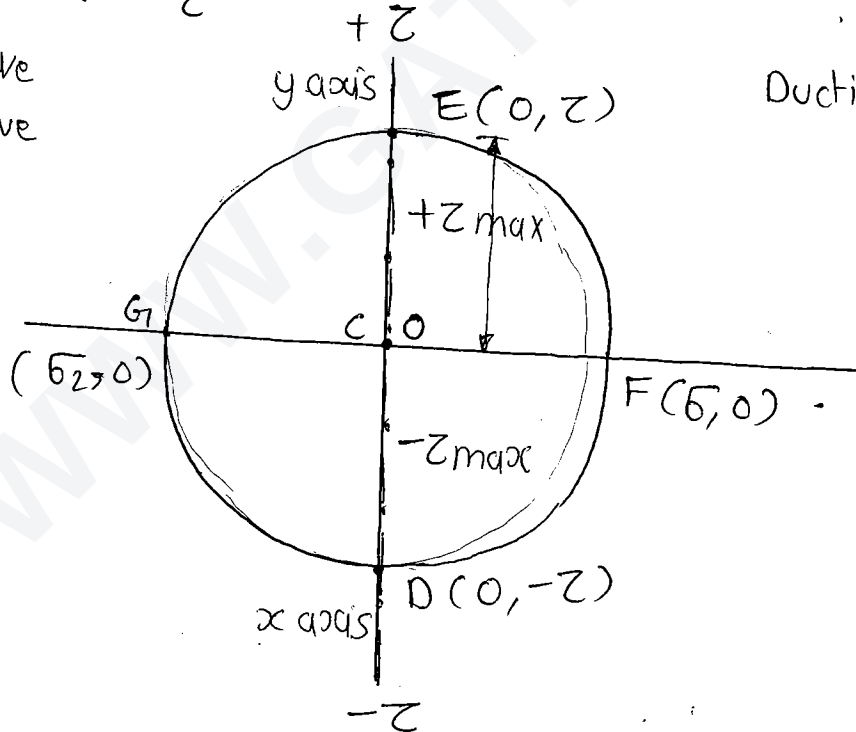
$$R=0 \quad \tau=0$$

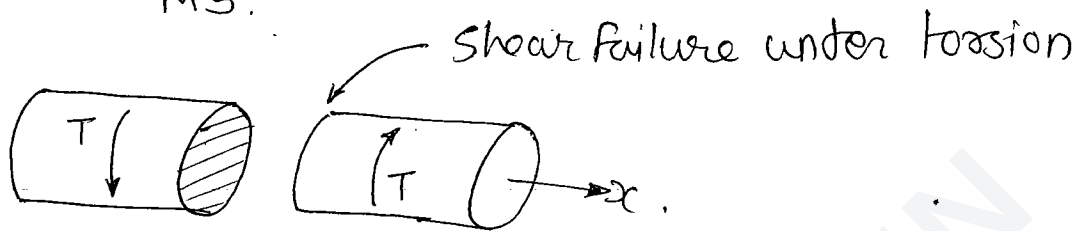
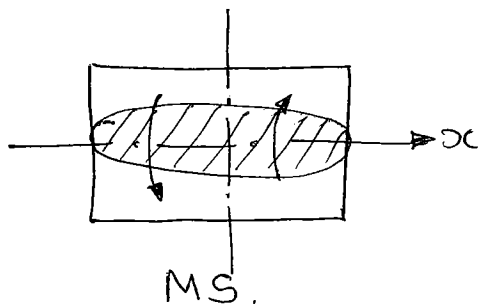
Direction of shear stress is depends upon the direction of torque we applied.



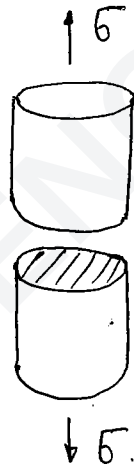
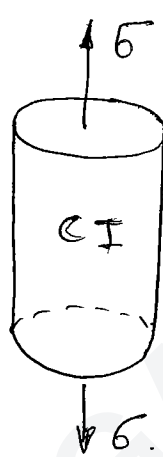
τ_{xy} -ve
 τ_{yx} +ve

Brittle material - Separation of layer
Ductile " - Sliding





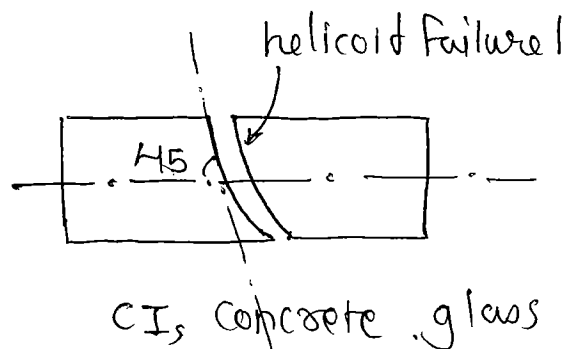
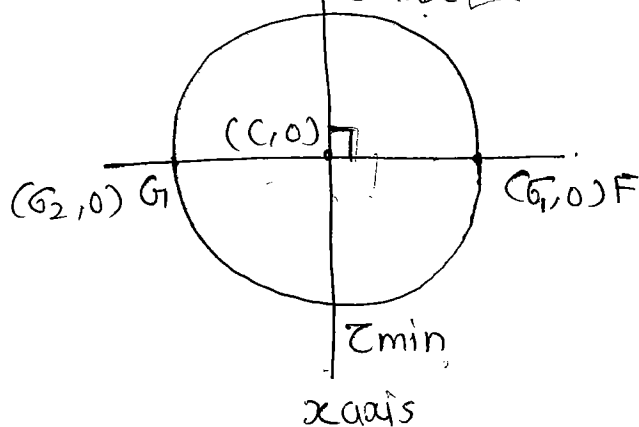
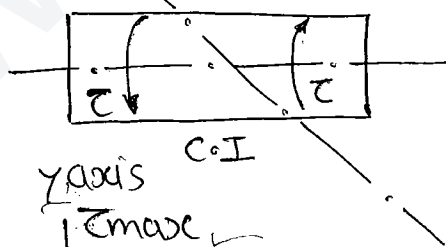
Resistance is less. Hence Failure take place at a plane $\perp$ to x axis.



under Normal stress
Brittle material

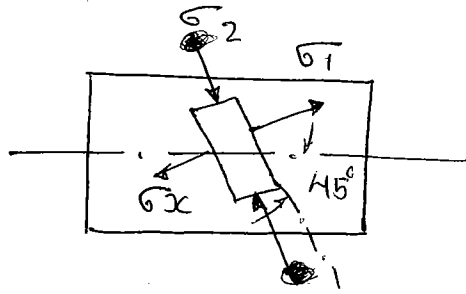
Fractured.
Separated.

plane of max Normal stress at an angle of 45°



$$\bar{\sigma} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} +2 & 0 \\ 2 & -2 \end{bmatrix}$$

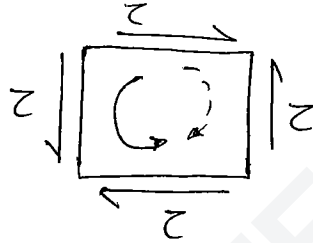
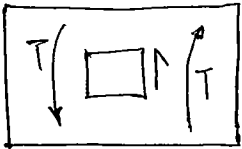
Normal Shear stress



$$\bar{\sigma} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & -\sigma_2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

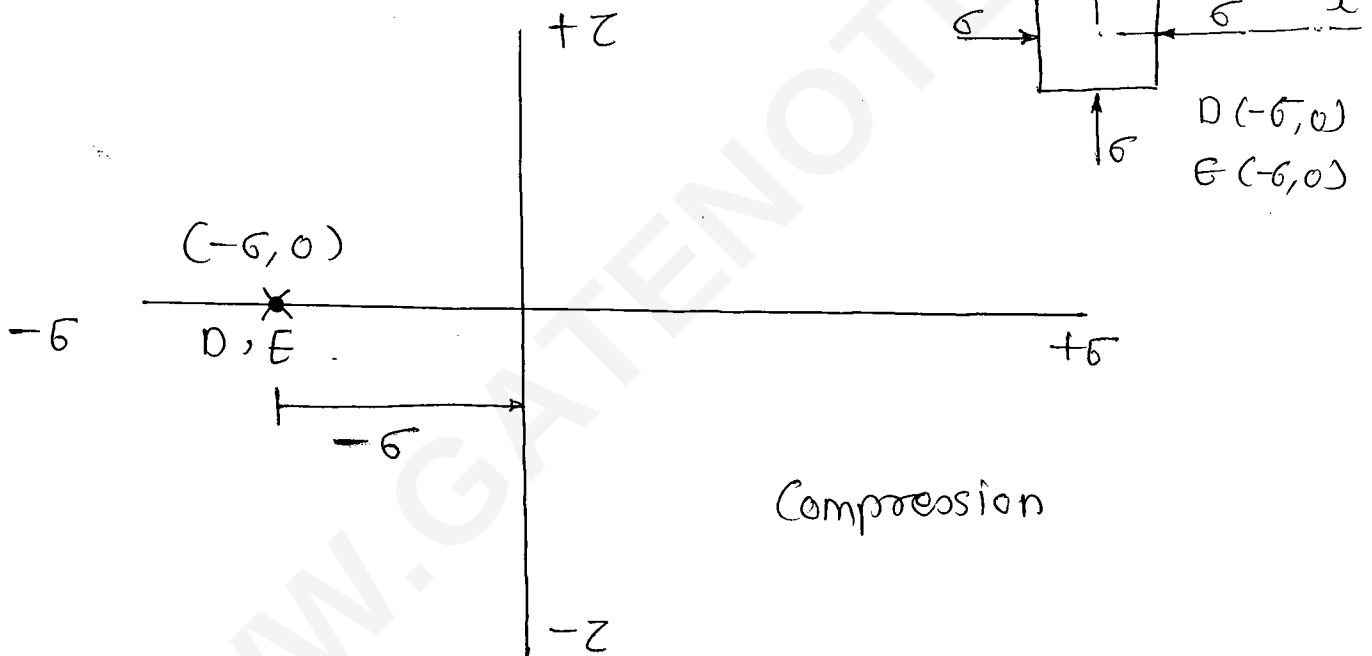
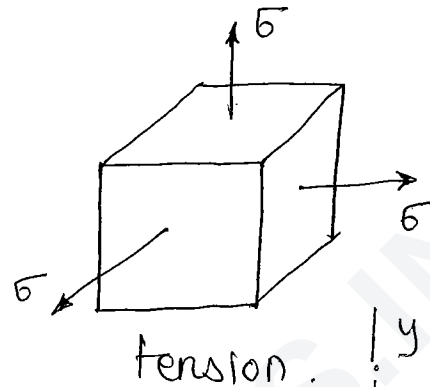
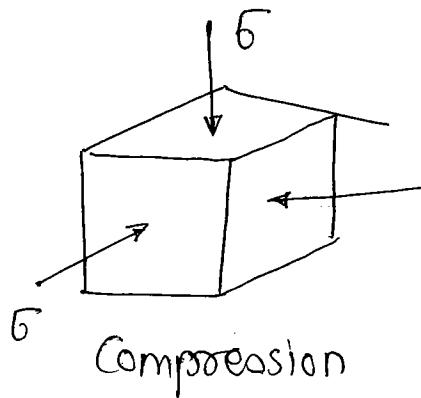
$\sigma_1 = 5$
 $\sigma_2 = 5$



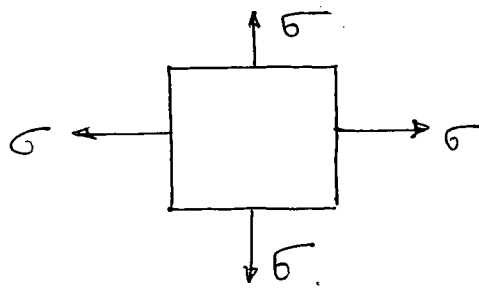
$$\bar{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

Mohr's circle for Hydrostatic stress.

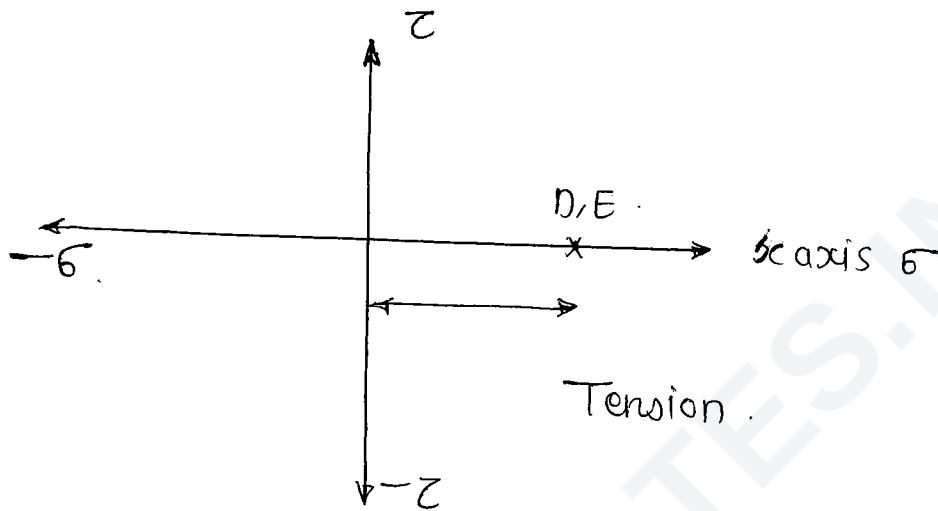
The stress is same in all direction. in compression or in tension.



A point -ve side of abscissa is called as hydrostatic stress under compression



$D(\sigma, 0) - x \text{ plane}$
 $E(\sigma, 0) - y \text{ plane}$



A point on the +ve side of the x axis is called as Hydrostatic stress under tension.

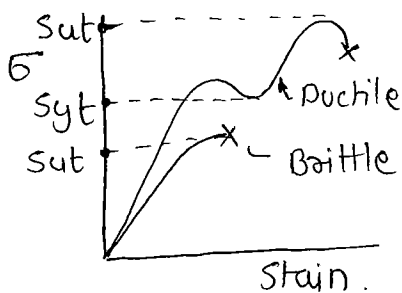
Ductile Material

Failure mode

yielding
(Slipping of plane under shear)

Strengths

$S_{yt} = S_{yc}$
 Strength at yield point under tension or comp.



(Isotropic)

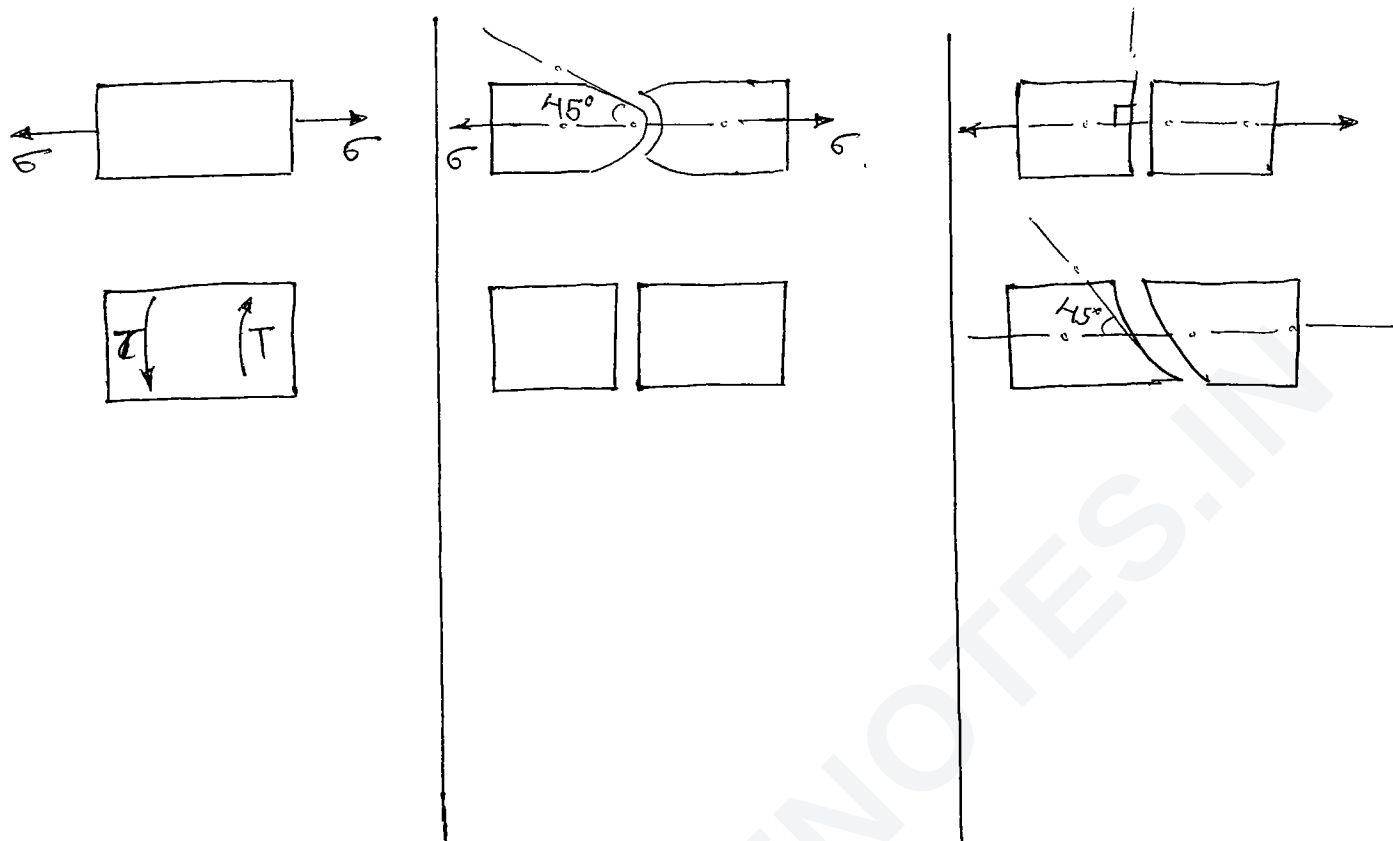
Brittle Material.

Separation or fracture.

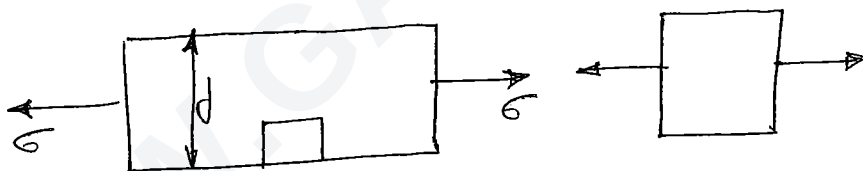
~~S_{yt}~~ = ~~S_{yc}~~ $S_{ut} \neq S_{uc}$
 (Not isotropic)

WWW.GATENOTES.IN

WWW.GATENOTES.IN



Theories of failure.



$$\sigma = \frac{F}{\pi/4 d^2} = \text{strength.}$$

The component may get fail because of S_{ut} , S_{uc} , S_{yt} , S_z and the max value out of this is consider.

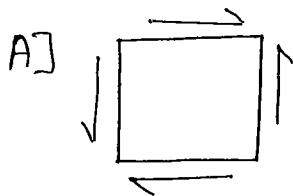
Establishes a relationship between principal stresses, together with some elastic constants and the strength of the component determine by simple torsion.

$E, G \propto C \propto N, K, \mu$ - cont.

List 1

List 2

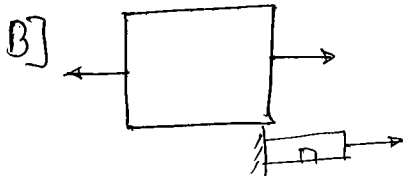
Theories of failure.



Combine bending & torsion.

a)

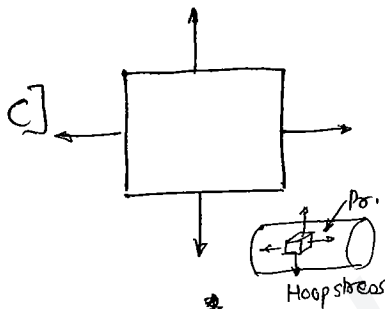
	A	B	C	D
a)	2	1	3	4



Torsion.

b)

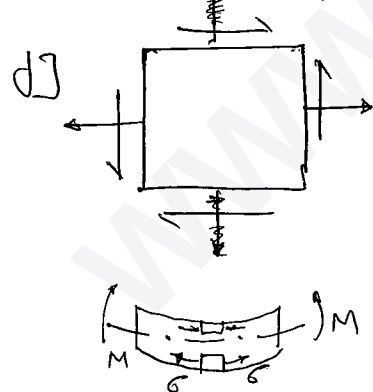
	A	B	C	D
b)	3	4	2	1



Thin cylindrical shells under pressure

c)

	A	B	C	D
c)	2	4	3	1



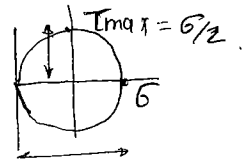
Tie bar subjected to tensile force

d)

	A	B	C	D
d)	3	1	2	4

Q1. The maximum shear stress in a bar of c/s area $A \text{ mm}^2$ and subjected to a tensile load of P newtons is

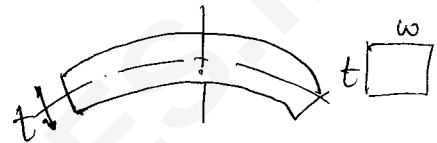
- a] 0 b] P/A ☒ c] $P/2A$ d] $P/4A$



Q2. A steel saw blade 1mm thick is bent into an arc of circle of 500mm radius. The width of the blade is 15mm and $E = 210 \text{ GPa}$.

1. Flexural stress induced is.

- a] $\frac{P}{110}$ b] $\frac{P}{210}$ ☒ c] $\frac{P}{250}$ d] $\frac{P}{350}$



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = E \times \frac{y}{R} = 210 \times 10^3 \times \frac{t}{2} \times \frac{1}{500}$$

$$\sigma = 210 t \cdot t = 1$$

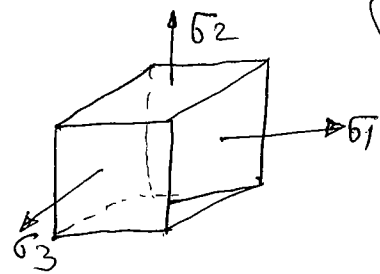
$$\boxed{\sigma = 210}$$

Theory of Failures.

① Rankine's theory (or) Maximum normal stress theory.

$$\text{If } \sigma_1 > \sigma_2 > \sigma_3$$

Then $\sigma_1 = \text{Max normal stress}$



If $\sigma_1 > \sigma_{yt}$ - ductile.

$\sigma_2 > \sigma_{ut}$ - brittle. } Component of Failure.

$$\sigma_1 \leq \sigma_{ut} - \text{Safe}$$

$$\sigma_1 \leq \sigma_{yt} - \text{Safe}$$

Design Equation.

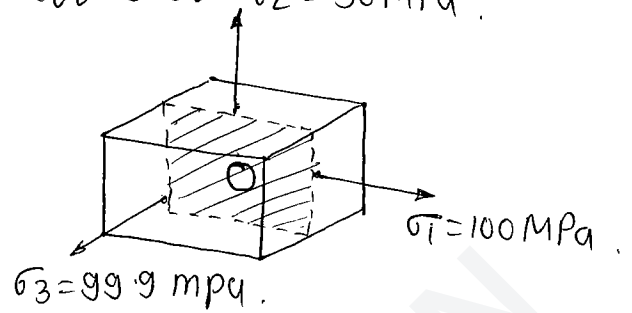
$$\sigma_1 = \frac{\sigma_{yt}}{F.S} - \text{Ductile}$$

F.S = factor of safety.

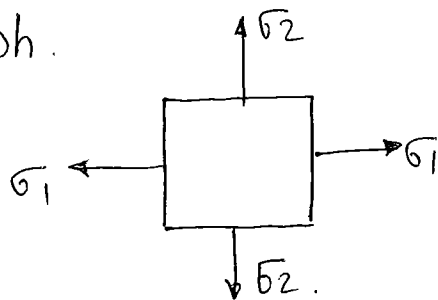
$$\sigma_2 = \frac{\sigma_{ut}}{F.S} - \text{Brittle}$$

Validities - Valid for brittle material and not valid for ductile material.

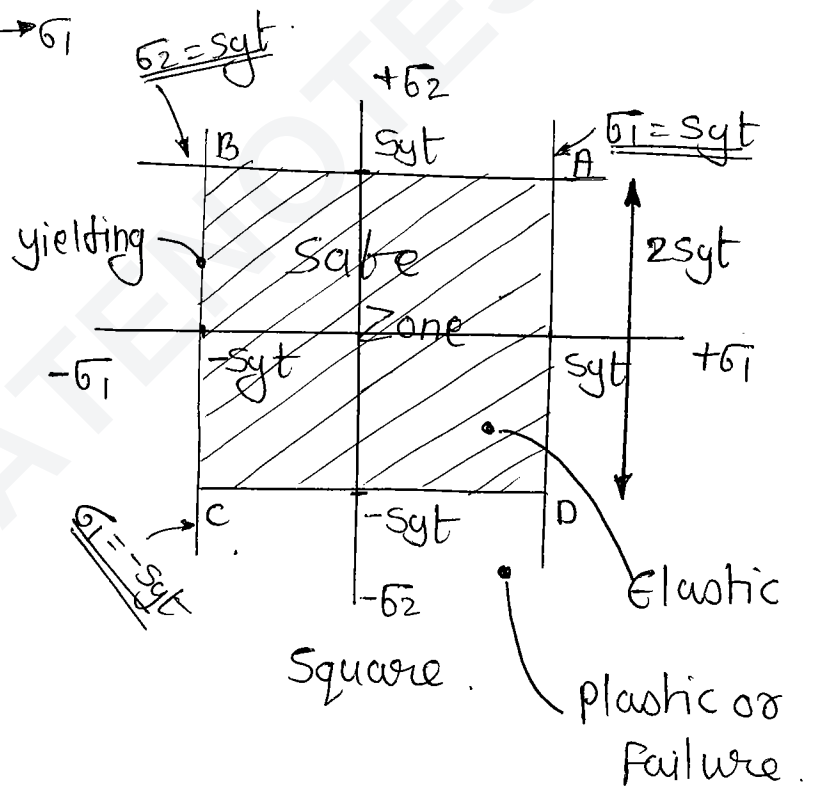
- Disregards the effect of other stresses.



Graph.



For Ductile material



a) If $\sigma_1 > \sigma_2$

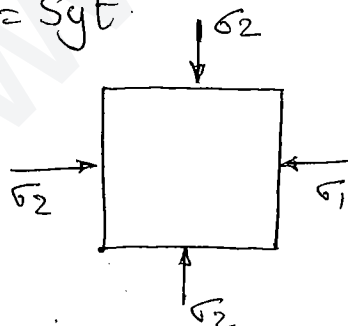
$$\sigma_1 = \frac{S_{yt}}{F.S = 1}$$

$$\sigma_1 = S_{yt}$$

$$x = a$$

b) If $\sigma_2 > \sigma_1$

$$\sigma_2 = S_{yt}$$



c) If $\sigma_1 > \sigma_2$

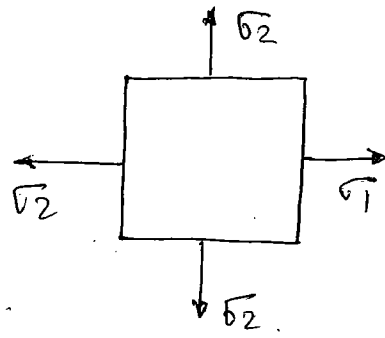
$$\sigma_1 = S_{ye} = -S_{yt} \text{ (isotropic material)}$$

d) If $\sigma_2 > \sigma_1$

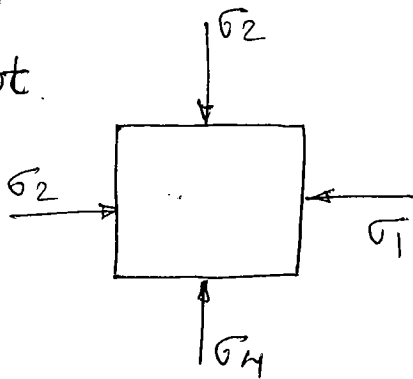
$$\sigma_2 = -S_{yt}$$

Graph - Brittle Material.

- a) If $\sigma_1 > \sigma_2$
 $\sigma_1 = S_{ut}$

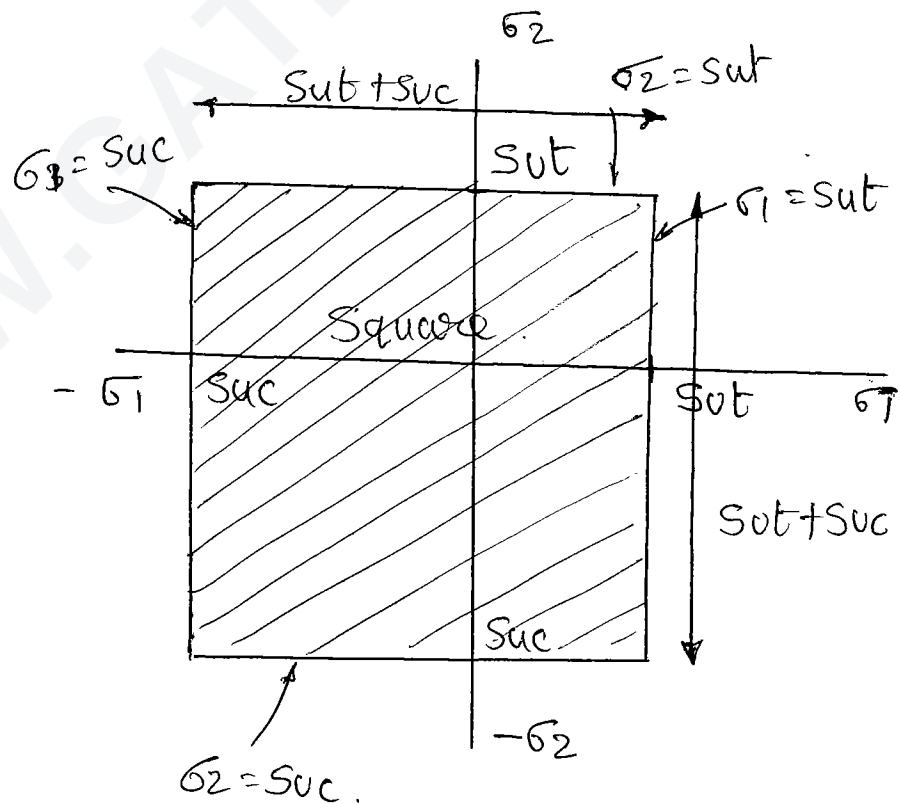


- b) If $\sigma_2 > \sigma_1$
 $\sigma_2 = S_{ut}$



- c) If $\sigma_1 > \sigma_2$
 $\sigma_1 = S_{uc}$

- d) If $\sigma_2 > \sigma_1$
 $\sigma_2 = S_{uc}$



A component subjected to complex state of stress is said to be failed when the maximum normal stress at a point in the material exceeds the strength at yield point under simple tension.

② Guest's theory or Tresca's theory or Maximum shear stress theory.

Maximum shear stress.

$$\tau_{max} > S_{sy} - \text{Failed.}$$

S_{sy} = shear strength at yield point.

Design

$$\tau_{max} = \frac{S_{sy}}{F.S}$$

If Maximum shear stress

$$\tau_{max} \leq S_{sy} - \text{Safe.}$$

$$\tau_{max} = \max \left\{ \begin{array}{l} \frac{\sigma_1 - \sigma_2}{2} \\ \frac{\sigma_1 - \sigma_3}{2} \\ \frac{\sigma_2 - \sigma_3}{2} \end{array} \right\} = \max \left\{ \begin{array}{l} \left| \frac{\sigma_1 - \sigma_2}{2} \right| \\ \left| \frac{\sigma_1 - \sigma_3}{2} \right| \\ \left| \frac{\sigma_2 - \sigma_3}{2} \right| \end{array} \right\}$$

If $\sigma_1 > \sigma_2 > \sigma_3$ and all are like stresses

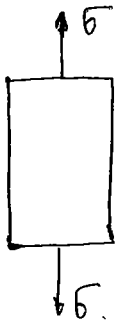
then,

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{S_{yt}}{2 \times F.S.}$$

$$\boxed{\sigma_1 - \sigma_3 = \frac{S_{yt}}{F.S.}}$$

$$\begin{array}{l} \tau_{max} = \frac{\sigma}{2} \\ S_{sy} = \frac{S_{yt}}{2} \end{array}$$


$$\begin{aligned} \tau_{max} &= \frac{S_{sy}}{F.S} \\ &= \frac{S_{yt}}{2 \times F.S.} \end{aligned}$$

* Validity

① Valid for only ductile material

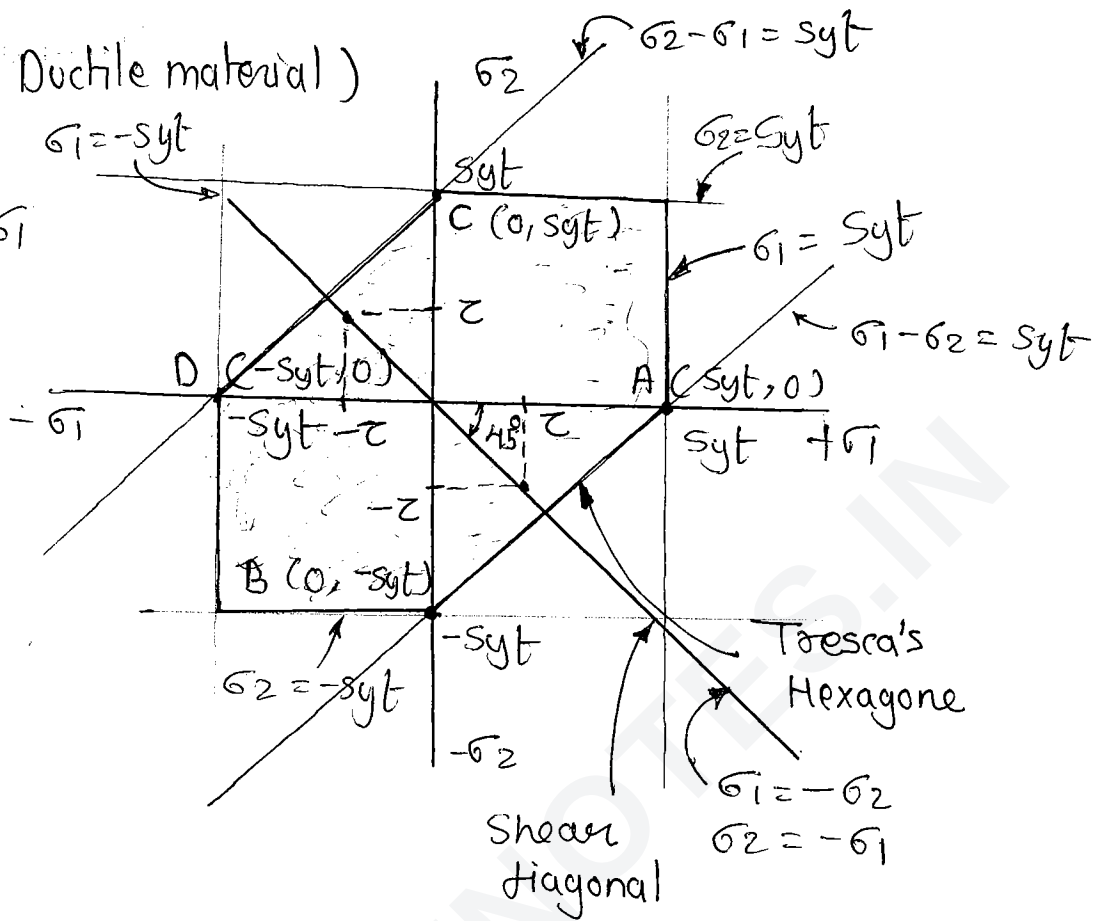
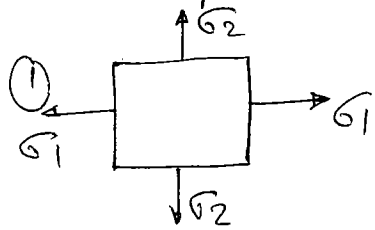
② Not valid for brittle material

③ Not explained the failure phenomenon under hydrostatic stress



$\tau_{max} = \frac{\sigma}{2}$

Graph. (Ductile material)



a] If $\sigma_1 > \sigma_2$

Then, $\tau_{max} = \frac{\sigma_1}{2}$

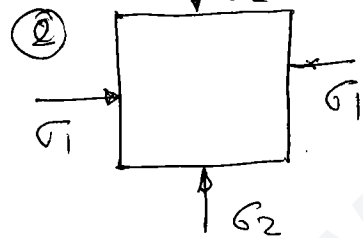
$$\tau_{max} = \frac{\sigma_1}{2} = \frac{syt}{2}$$

$$\sigma_1 = syt$$

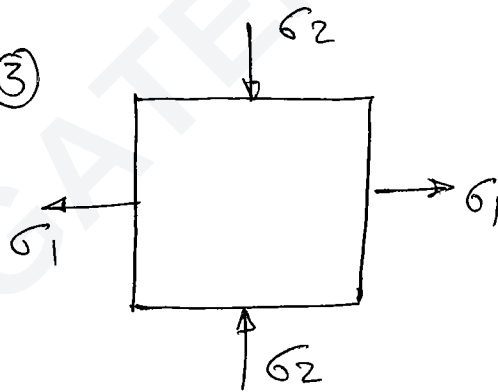
b] If $\sigma_2 > \sigma_1$

$$\tau_{max} = \frac{\sigma_2}{2} = \frac{syt}{2}$$

$$\sigma_2 = syt$$



③



$$\tau_{max} = \max \left\{ \begin{array}{l} \frac{\sigma_1 - \sigma_2}{2} \\ \frac{\sigma_1 - \sigma_3}{2} \\ \frac{\sigma_2 - \sigma_3}{2} \end{array} \right\}$$

c] If $\sigma_1 > \sigma_2$

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{-syt}{2}$$

$$\sigma_1 = -syt$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{syt}{2}$$

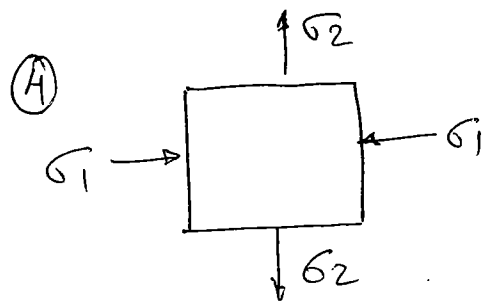
$$\sigma_1 - \sigma_2 = syt$$

$$\text{If } \sigma_2 = 0, A(syt, 0)$$

$$\sigma_1 = 0, B(0, -syt)$$

d] If $\sigma_2 > \sigma_1$

$$\tau_{max} = \frac{\sigma_2}{2} = \frac{-syt}{2}$$



$$\tau_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{\sigma_{yt}}{2}$$

$$\sigma_2 - \sigma_1 = \sigma_{yt}$$

$$\text{If, } \sigma_1 = 0 \quad C(0, \sigma_{yt})$$

$$\sigma_2 = 0 \quad D(0, -\sigma_{yt}, 0)$$

* State of pure shear

$$\tau = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1 & \tau_{xy} \\ \tau_{xy} & \sigma_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\tau = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sigma_1}{2} & 0 \\ 0 & \frac{\sigma_2}{2} \end{bmatrix}$$

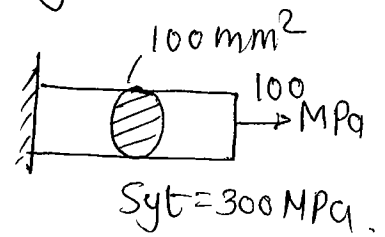
A component subjected to complex state of stress is said to be failed when the maximum shear stress at a point in the material exceeds the shear strength at yield point under simple tension.

Q A component is loaded as shown in fig the factor of safety according to

1] Maximum normal stress theory.

a) 1 b) 2 c) 3 d) 4.

$$\sigma_{yt} = \frac{\sigma}{F.S}$$



2] Maximum shear stress theory

a) 1 b) 2 c) 3 d) 4

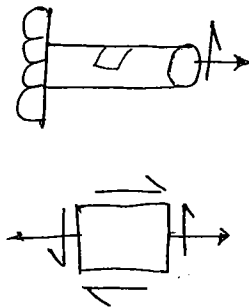
$$\tau_{max} = \frac{\sigma}{2} \leq \frac{\sigma_{yt}}{2 \times F.S}$$

Q A steel bolt is subjected to a direct tensile load of 8.4 kN and a shear load of 48 kN. The cross section area of bolt is 600 mm² and σ_{yt} for bolt material is 200 MPa, the F.S according to maximum shear stress theory is

a) 1 b) 2 c) 3 d) 4.

$$\sigma_1 = \frac{8.4 \times 10^3}{600} = 14 \text{ MPa}$$

$$\tau = \frac{48 \times 10^3}{600} = 80 \text{ MPa}$$



$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$= 90 \text{ MPa}$$

$$\sigma_2 = -10 \text{ MPa}$$

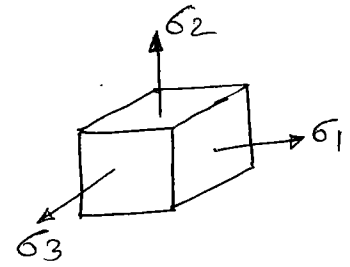
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{90 - (-10)}{2} = 50 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_{yt}}{2 F.S}$$

$$50 = \frac{200}{2 F.S} \quad \boxed{F.S = 2}$$

③ Saint Venant's theory (or) Maximum Principal strain theory.

If $\sigma_1 > \sigma_2 > \sigma_3$
 Then $\epsilon_1 > \epsilon_2 > \epsilon_3$.



$\epsilon_1 \rightarrow$ Maximum principal strain

$\epsilon_1 > \frac{\sigma_{yt}}{E}$ - Failed.

$\mu = \frac{-\text{lateral strain}}{\text{longitudinal strain}}$

$\mu = 0$ for cork

$\mu = 0.25$ to 0.33
for steel.

$= 0.47$ for rubber

Design Equation

$\epsilon_1 = \frac{\sigma_{yt}}{E \times F.S.}$ - (1)

$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$ - (2)

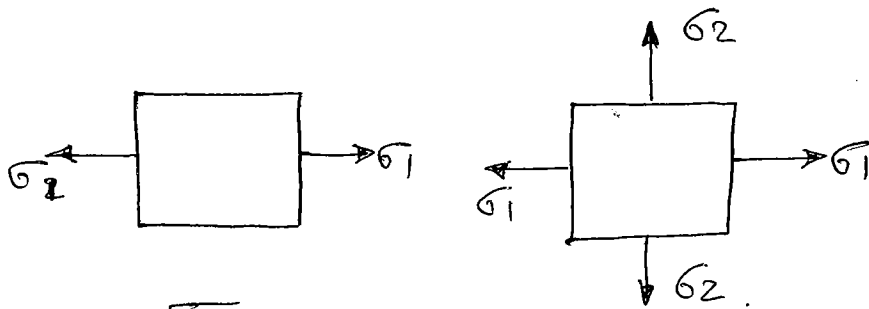
$\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} = \frac{\sigma_{yt}}{E \times F.S.}$

$\sigma_1 - \mu \sigma_2 - \mu \sigma_3 = \frac{\sigma_{yt}}{F.S.}$

For 2D $\sigma_3 = 0$

$\sigma_1 - \mu \sigma_2 = \frac{\sigma_{yt}}{F.S.}$

Not valid for Brittle material or for Ductile material.

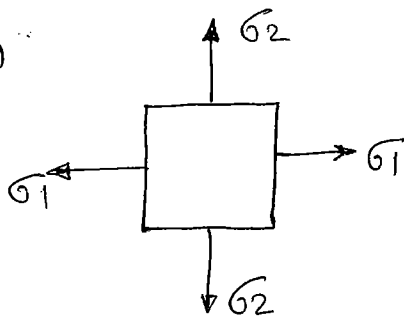


$$\epsilon_{uni} = \frac{\sigma_1}{E}$$

$$\epsilon_{biaxial} = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\epsilon_{uni} > \epsilon_{bi}$$

Graph
(Ductile)



a) If $\sigma_1 > \sigma_2$

$$\epsilon = \frac{\sigma_1 - \mu \sigma_2}{E} = \frac{\sigma_{yt}}{E}$$

$$\sigma_1 - \mu \sigma_2 = \sigma_{yt}$$

If $\sigma_1 = 0$ $A(0, -\frac{\sigma_{yt}}{\mu})$

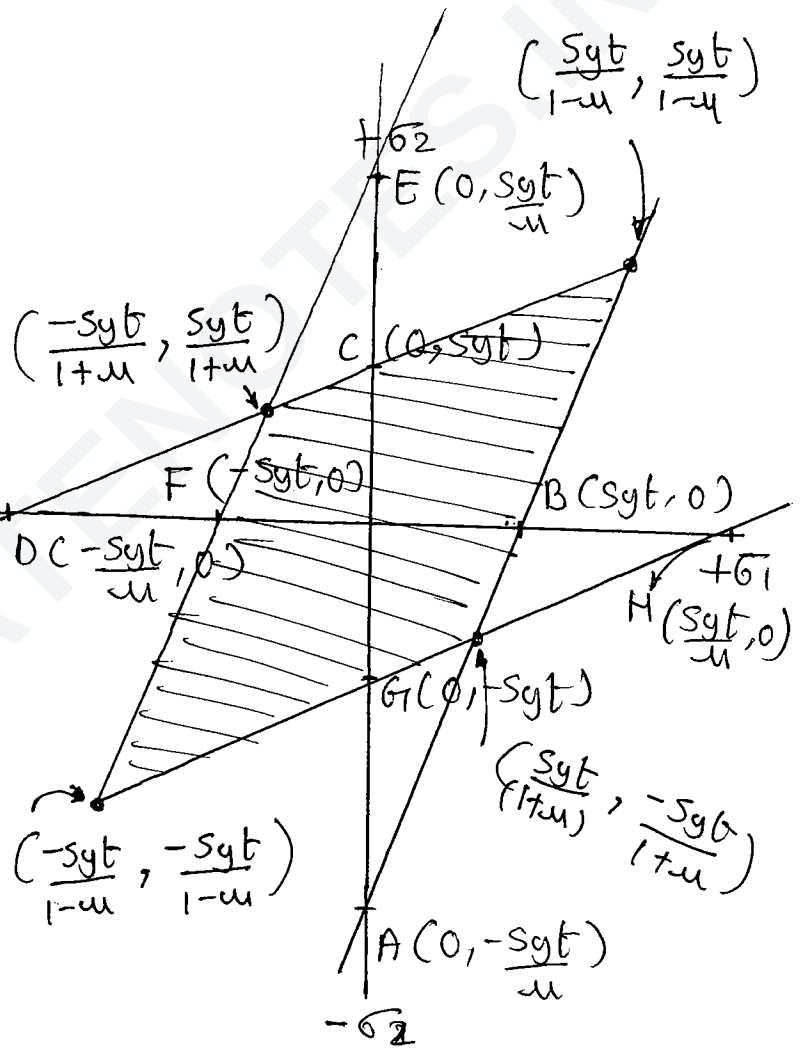
$\sigma_2 = 0$ $B(\sigma_{yt}, 0)$

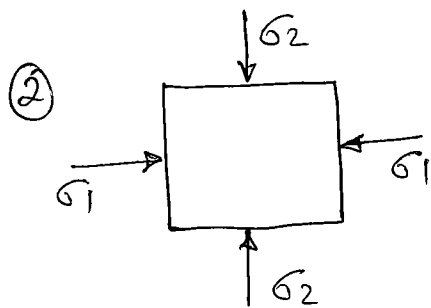
b) If $\sigma_2 > \sigma_1$

$$\sigma_2 - \mu \sigma_1 = \sigma_{yt}$$

If $\sigma_1 = 0$ $C(0, \sigma_{yt})$

$\sigma_2 = 0$ $D(-\frac{\sigma_{yt}}{\mu}, 0)$





C] If $\sigma_1 > \sigma_2$.

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} = \frac{\sigma_{yc}}{E}$$

$$\sigma_1 - \mu \sigma_2 = -\sigma_{yt}$$

$$\text{If } \sigma_1 = 0 \quad E(0, \frac{\sigma_{yt}}{\mu})$$

$$\sigma_2 = 0 \quad F(-\sigma_{yt}, 0)$$

D] If $\sigma_2 > \sigma_1$

$$\sigma_2 - \mu \sigma_1 = -\sigma_{yt}$$

$$\text{If } \sigma_1 = 0 \quad G(0, -\sigma_{yt})$$

$$\sigma_2 = 0 \quad H(\frac{\sigma_{yt}}{\mu}, 0)$$

A component subjected to complex state of stress is said to be failed when the maximum principal strain at a point in the material exceeds the strain at yield point under simple tension.

(A) Haigh's Theory (or) Maximum strain energy theory.

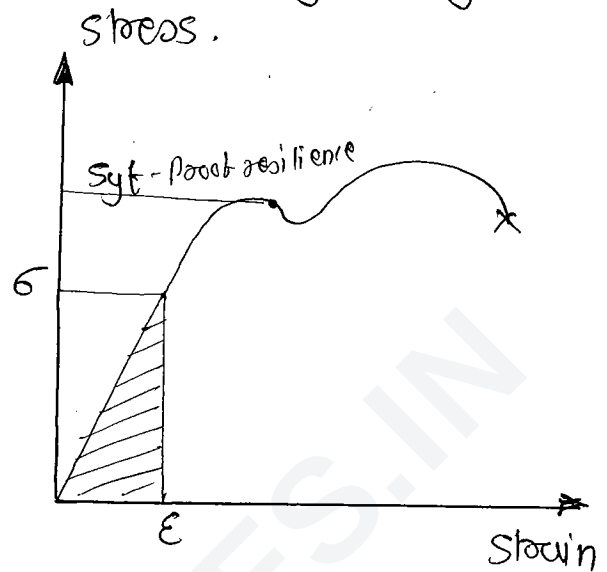
Strain Energy

$$U = \frac{1}{2} \epsilon \times \sigma$$

$$= \frac{1}{2} \sigma \epsilon$$

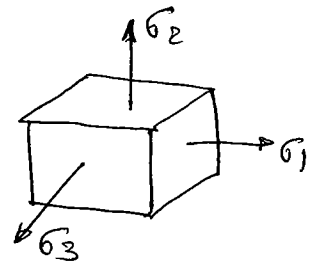
$$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E}$$

$$= \frac{\sigma^2}{2E}$$



$$U_{\text{at yield point}} = \frac{\sigma_{yt}^2}{2E}$$

Strain energy density under complex loading.



$$U_{\text{complex}} = \frac{1}{2} \sigma_1 \epsilon_1 + \frac{1}{2} \sigma_2 \epsilon_2 + \frac{1}{2} \sigma_3 \epsilon_3$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$U_{\text{complex}} = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$U_{\text{complex}} \leq U_{\text{simple tension}} - \text{safe.}$

$$\frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] = \left(\frac{\sigma_{yt}}{F.S} \right)^2 \times \frac{1}{2E}$$

$$\frac{\sigma_{yt}}{F.S} = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

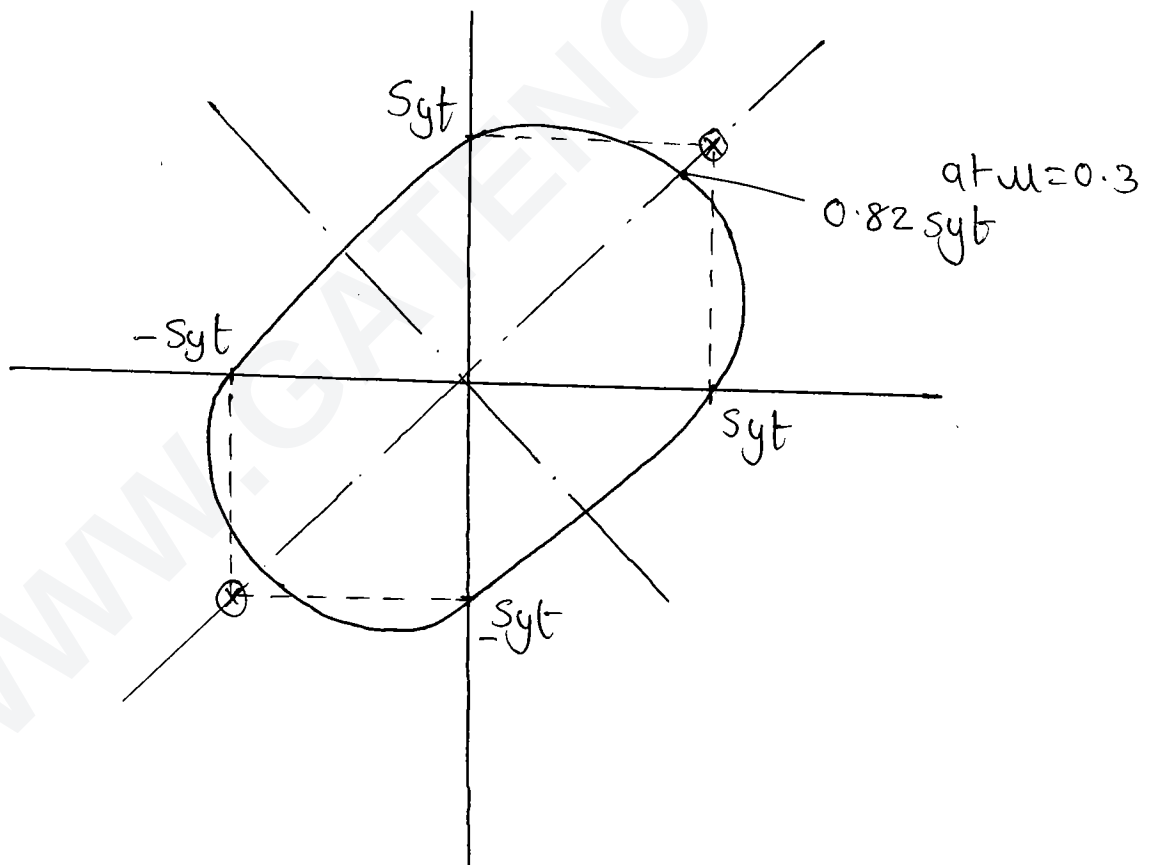
For 2D

$$\frac{S_{yt}}{F.S} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}$$

$$x^2 + y^2 - 2bxy = a^2.$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = S_{yt}^2.$$

$$\begin{aligned} \text{If } \sigma_1 = 0 \quad \sigma_2 &= \pm S_{yt} \\ \sigma_2 = 0 \quad \sigma_1 &= \pm S_{yt} \end{aligned}$$



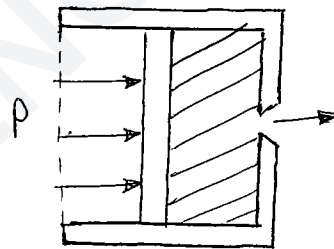
Validity - Valid for Ductile material.

Q If the principal stresses at point are $\sigma_1 = +2f$, $\sigma_2 = +f$, $\sigma_3 = -\frac{1}{2}f$, and $\sigma_{yt} = 200 \text{ N/mm}^2$, $\mu = 0.3$. The value of 'f' according to Haish's theorem

a) 100 N/mm^2 b) 75 N/mm^2 c) 90 N/mm^2 d) 150 N/mm^2

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right]^{1/2} = \frac{\sigma_{yt}}{f \cdot S}$$

5] Von Mises and Henky's theory (or) Shear strain Energy theory (or) Distortion Energy theory.

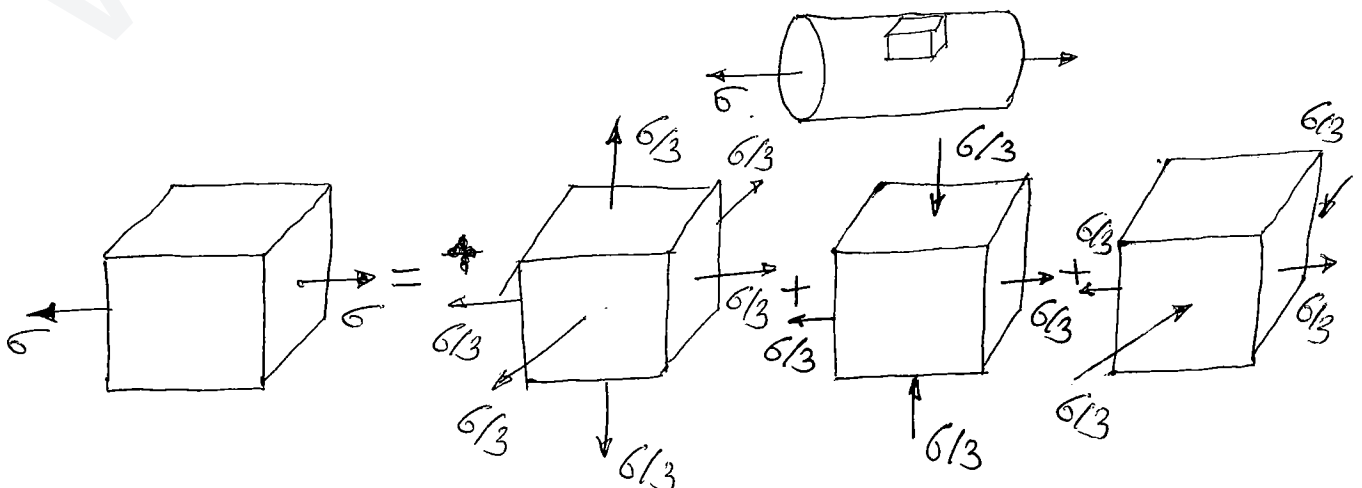


Total strain Energy (U_t)

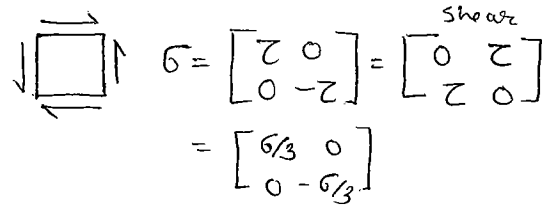
$$U_t = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right]$$

$$U_t = U_v + U_d$$

= Volume change + Change of shape.



= Volume change + Distortion + Distortion



$$\sigma = \begin{bmatrix} \tau & 0 \\ 0 & -\tau \end{bmatrix} = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix} \quad \text{shear}$$

$$= \begin{bmatrix} \tau/3 & 0 \\ 0 & -\tau/3 \end{bmatrix}$$

$$E = 3K(1-2\mu)$$

$$= 3 \frac{\sigma}{\epsilon_v} (1-2\mu)$$

$$\epsilon_v = 3 \frac{\sigma}{E} (1-2\mu) \quad \dots \quad \epsilon_v = \frac{\delta v}{V}$$

$$\delta v = \frac{3\sigma}{E} (1-2\mu) V$$

If $\mu = 0.5$, $\delta v = 0$.

$$U_t = \frac{1}{2E} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

If $\mu = 0.5$ $U_t = U_d$.

$$U_d = \frac{1}{2E} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$= \frac{(1+\mu)}{3E} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$E = 2G(1+\mu)$$

$$= \frac{1}{6G} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

For simple tension.

$$\sigma_1 = \sigma, \quad \sigma_2 = \sigma_3 = 0.$$

$$U_d = \frac{1}{6G} \sigma^2$$

$$E = 2G(1+\mu)$$

$$U_d = \frac{1+\mu}{3E} \sigma^2$$

Distortion energy under
Complex loading $>$ Distortion energy under
Simple tension at yield point
- Failed

$$= \frac{1}{6G} S_{yt}^2 \dots \dots \text{At yielded point}$$

$$\frac{1}{6G} [(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)] = \frac{1}{6G} \left(\frac{S_{yt}}{F.S} \right)^2$$

$$\frac{S_{yt}}{F.S} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)}$$

For 2D stress.

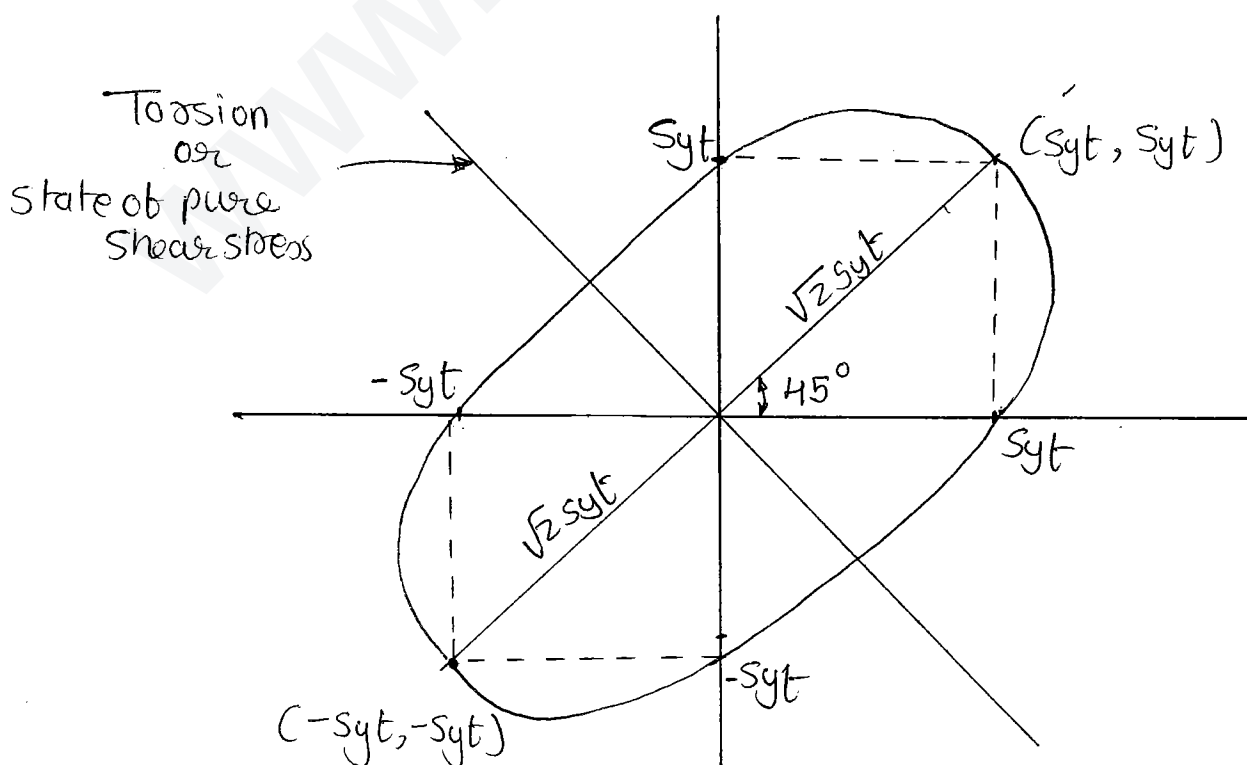
$$\frac{S_{yt}}{F.S} = \sqrt{(\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)}$$

$$x^2 + y^2 - xy = a^2 \dots \dots \text{Ellipse eqn.}$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = S_{yt}^2$$

$$\sigma_1 = 0 \quad \sigma_2 = \pm S_{yt}$$

$$\sigma_2 = 0 \quad \sigma_1 = \pm S_{yt}$$



Validity - This theory gives very good results for

$$\textcircled{1} \quad \sigma_1 = \frac{\sigma_{yt}}{F.S} \quad - \text{Rankine}$$

$$\textcircled{2} \quad \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_{yt}}{F.S} \quad - \text{Guest}$$

$$\textcircled{3} \quad \sigma_1 - \mu \sigma_2 = \frac{\sigma_{yt}}{F.S} \quad - \text{St Venant}$$

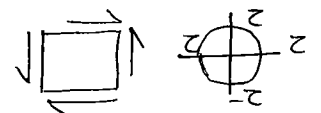
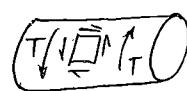
$$\textcircled{4} \quad \sigma_1^2 + \sigma_2^2 - 2\mu \sigma_1 \sigma_2 = \left(\frac{\sigma_{yt}}{F.S} \right)^2 \quad - \text{Haigh's}$$

$$\textcircled{5} \quad \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \left(\frac{\sigma_{yt}}{F.S} \right)^2 \quad - \text{Von Moises}$$

Q A component is subjected to torsion, T . Estimate the permissible value of shear stress τ according to different theories of failure in terms of yield strength, σ_{yt} . Take $\mu = 0.3$ $E = 2.1 \times 10^5 \text{ N/mm}^2$

Ans

$$\textcircled{1} \quad \text{Rankine} \quad \sigma_1 = \frac{\sigma_{yt}}{F.S} =$$



$$\sigma = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix} = \begin{bmatrix} \tau & 0 \\ 0 & -\tau \end{bmatrix}$$

$$\sigma_1 = \sigma_{yt}$$

$$\tau = \sigma_{yt}$$

$$\frac{\tau}{\sigma_{yt}} = 1$$

$$\text{Exp. value} \rightarrow S_{sy} = 0.57 \text{ to } 0.6 \text{ of } \sigma_{yt}$$

$\textcircled{2}$ Guest

(most conservative theory)

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_{yt}}{F.S}$$

$$\tau - (-\tau) = \sigma_{yt}$$

$$2\tau = \sigma_{yt} \quad \tau = \frac{\sigma_{yt}}{2} = 0.5 \sigma_{yt}$$

③ St Venant's Theory

$$\sigma_1 - \mu \sigma_2 = S_{yt}$$

$$Z - 0.3 \times (-Z) = S_{yt}$$

$$Z = \frac{S_{yt}}{1.3}$$

$$Z = 0.769 S_{yt}, \quad \frac{Z}{S_{yt}} = 0.769$$

④ Haigh's Theory

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = S_{yt}^2$$

$$Z + (-Z)^2 - 2\mu(Z)(-Z) = S_{yt}^2$$

$$Z = 0.63 S_{yt}$$

⑤ Von Mises

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_{yt}^2$$

$$Z^2 + (-Z)^2 - Z(-Z) = S_{yt}^2$$

$$2Z^2 + Z^2 = S_{yt}^2 \quad (\text{more economical})$$

$$Z = 0.577 S_{yt}$$

Note:-

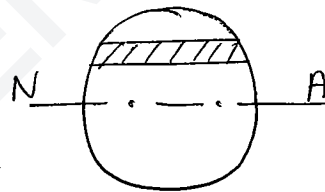
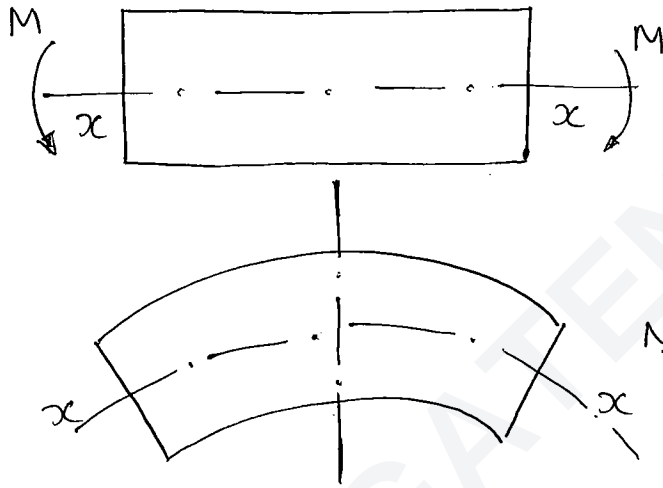
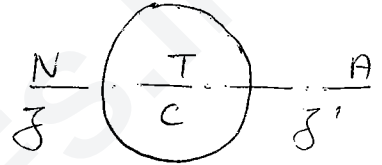
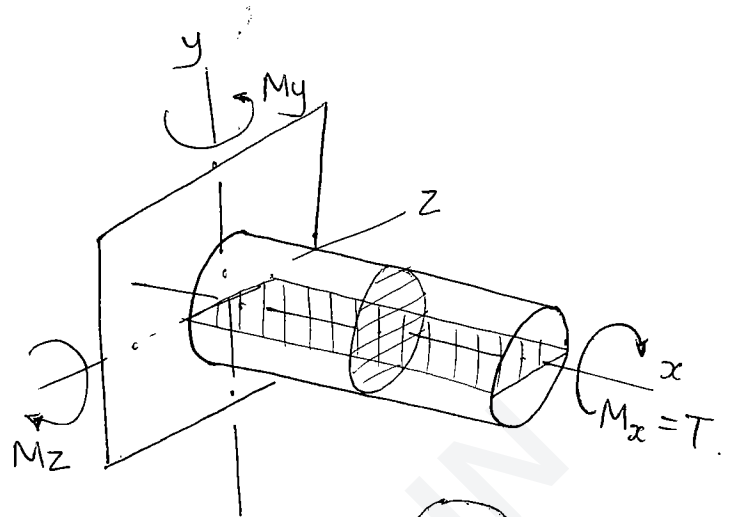
- 1. Use of guest theory increase size hence increase more cost and create bulky parts

Bending Equation .

$$M = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

$\downarrow$
 Torsion . $\underbrace{\hspace{10em}}$ Bending .

$$M = M_x \hat{i} + M_z \hat{k}$$



$$\frac{M}{I} = \frac{\sigma_y}{y} = \frac{E}{R}$$

M = Bending moment -

I = M.I of c/s about neutral axis

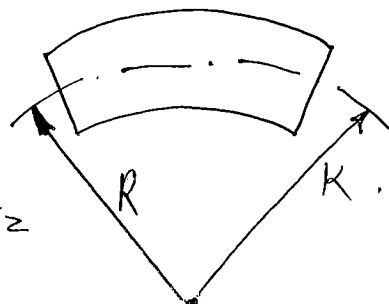
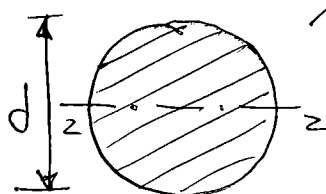
σ_y = Bending stress

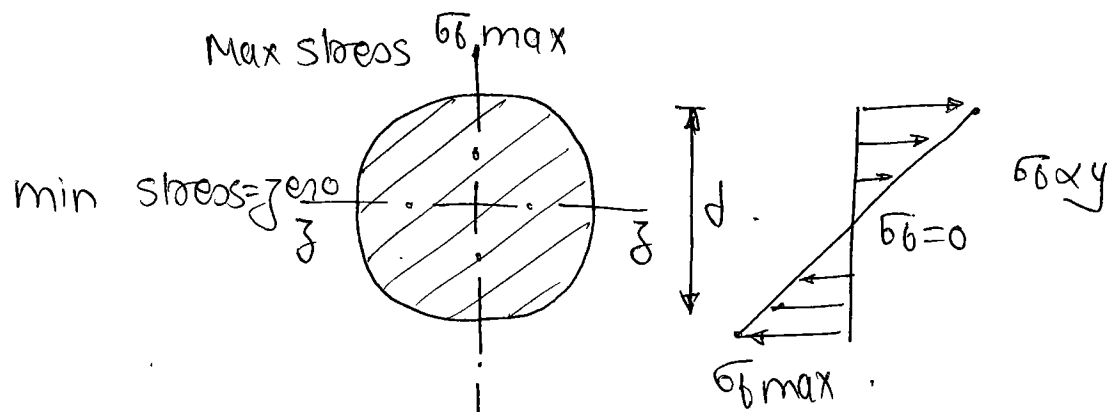
y = Distance measured from Neutral axis .

R = Radius measured from neutral axis .

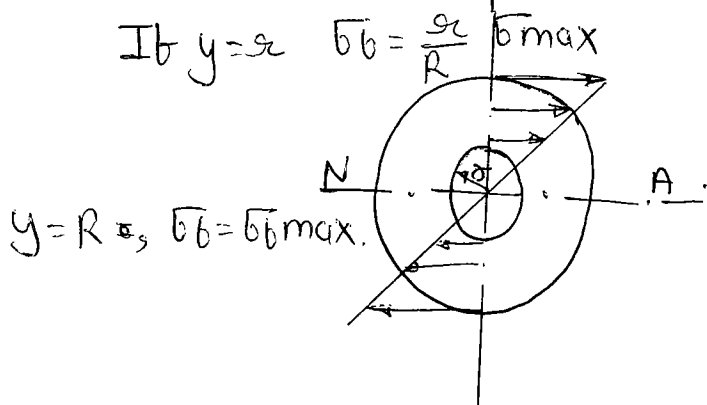
$$I = \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (d_o^4 - d_i^4)$$





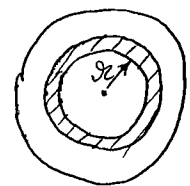
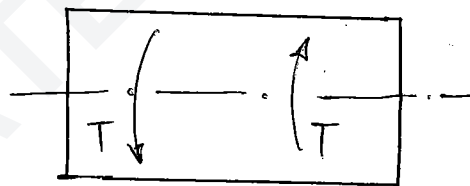
$$\text{If } y = r \quad \sigma_b = \frac{r}{R} \sigma_{b \max}$$



Along the N-A. Hence $\sigma_{b \min}$ is zero.

Torsion Equation.

$$\tau \propto r \quad \left(\frac{T}{J} = \frac{\tau}{r} \right) = \frac{G\theta}{l}$$



T = Torsion

J = Polar moment of Inertia.

$$= I_{xx} + I_{yy} = \frac{\pi}{32} d^4$$

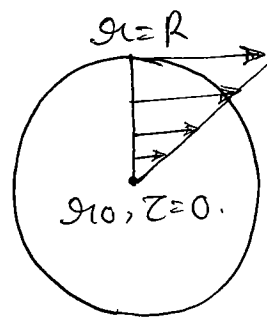
$$= \frac{\pi}{32} (D^4 - d^4)$$

τ = Shear stress at radius R

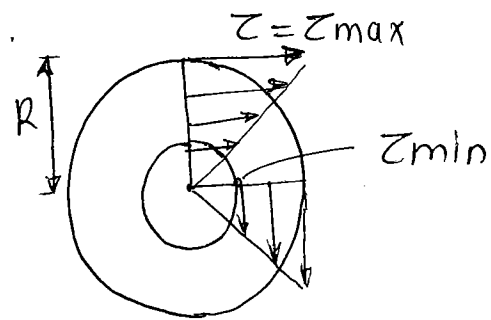
G = Modulus of rigidity

θ = Angle of twist.

l = length of member under torsion



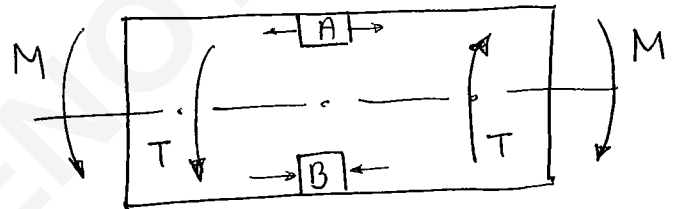
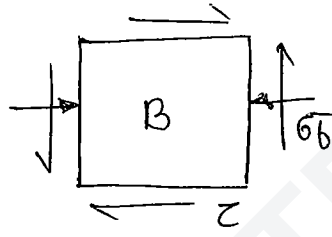
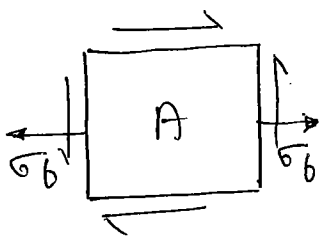
For solid.



$$\tau_{min} = \frac{r}{R} \tau_{max}$$

For hollow.

Von mises eqⁿ for Axial and torsion loading.
(Bending)



$$[\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2]^{1/2} = \frac{\sigma_{yt}}{F.S} = \sigma_{equivalent}$$

for A.

$$\sigma_x = \sigma$$

$$\sigma_y = 0$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \sigma/2 + R$$

$$\sigma_2 = \sigma/2 - R$$

put in eqⁿ

$$[(\sigma/2 + R)^2 + (\sigma/2 - R)^2 - (\sigma/2 + R)(\sigma/2 - R)]^{1/2} = \sigma_{equivalent}$$

$$[(\sigma/2)^2 + 3R^2]^{1/2} = \sigma_{eq}$$

$$\left[\left(\frac{\sigma}{2} \right)^2 + 3 \left\{ \sqrt{\left(\frac{\sigma}{2} \right)^2 + \tau^2} \right\}^2 \right]^{1/2} = \sigma_{eq}$$

$$\left[\left(\frac{\sigma}{2} \right)^2 + 3 \left(\frac{\sigma}{2} \right)^2 + 3 \tau^2 \right]^{1/2} = \sigma_{eq}$$

$$\left[4 \left(\frac{\sigma}{2} \right)^2 + 3 \tau^2 \right]^{1/2} = \sigma_{eq}$$

$$2 \frac{\sigma}{2} + \tau (1 + 3)^{1/2} = \sigma_{eq}$$

$$\boxed{\sigma_{eq} = \sqrt{\sigma^2 + 3 \tau^2}}$$

Q: The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

where the stress component value are in MPa. using Von mises yield criterion.

a] 9.50 b] 16.07 c] 28.52 d] 19.41

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$= \frac{10 + 20}{2} + \sqrt{\left(\frac{10 - 20}{2} \right)^2 + 5^2}$$

$$= 22.071$$

$$\sigma_2 = \frac{\sigma_y + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_z}{2} \right)^2 + \tau_{yz}^2}$$

$$= \frac{20 + (-10)}{2} + \sqrt{15^2 + 0^2}$$

$$= 7.93$$

$$\sigma_3 = -10 \text{ mpa}.$$

$$\frac{S_{yt}}{F_s} = \sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)}$$

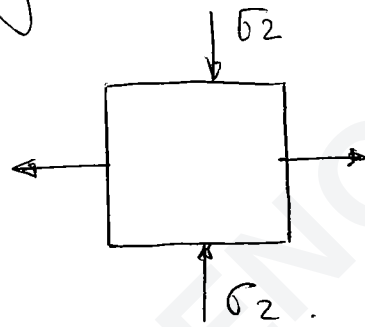
$$= 27.83.$$

$$Z = 0.557 \sigma_{eq}.$$

$$= 0.557 \times 27.83$$

$$= 16.057.$$

⑥. Mohr's Theory (Coulomb - Mohr's Theory)



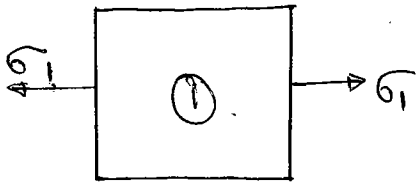
For Ductile $S_{yt} = S_{yc}$ (isotropic - Assumption)

$$\sigma_1 - \sigma_2 = \frac{S_{yt}}{F_s}.$$

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yt}} = \frac{1}{F_s}$$

For Brittle Material ($S_{yt} \neq S_{yc}$)

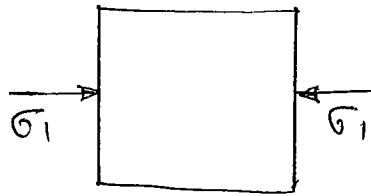
$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yc}} = \frac{1}{F_s}.$$



$$\frac{\sigma_{1su}}{S_{ut}} - \frac{\sigma_2}{S_{ut}} = \frac{1}{F.S.}$$

$$\sigma_2 = 0.$$

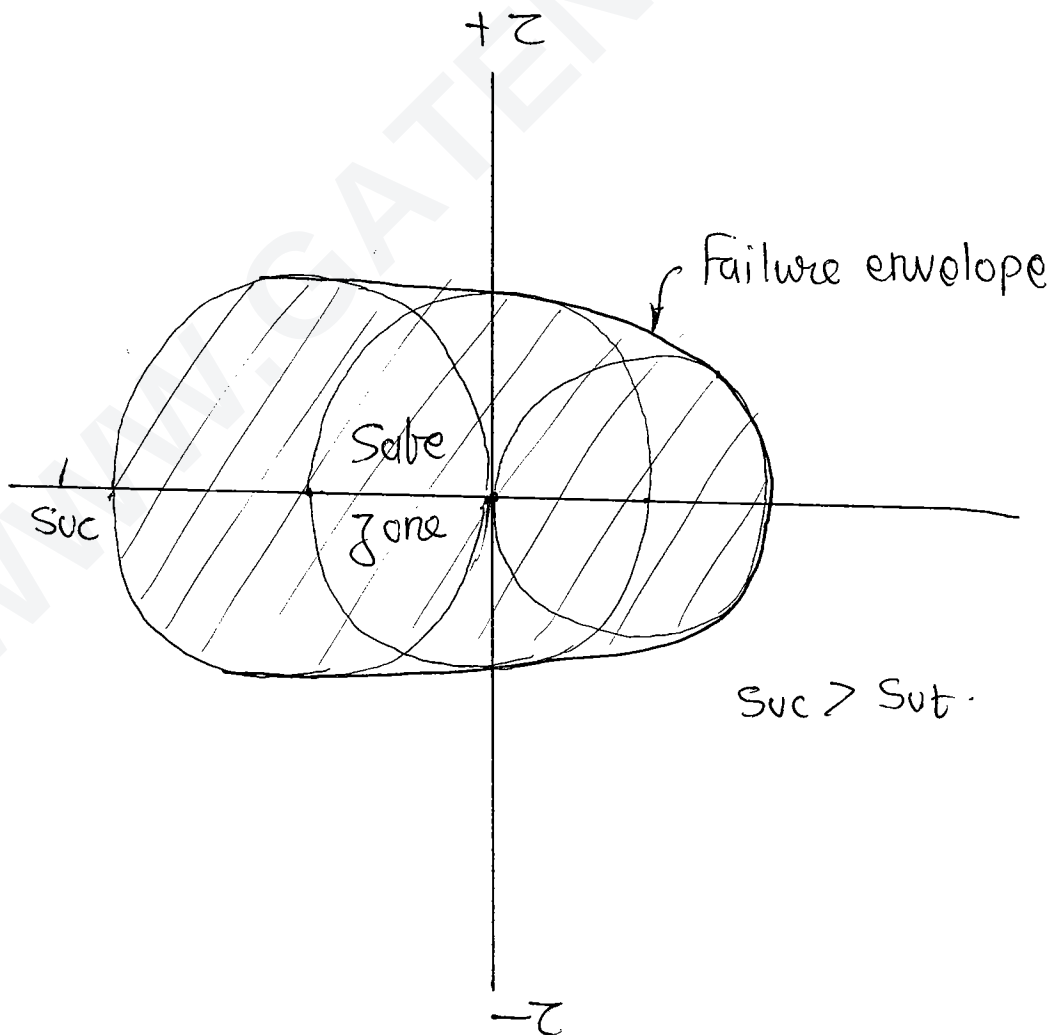
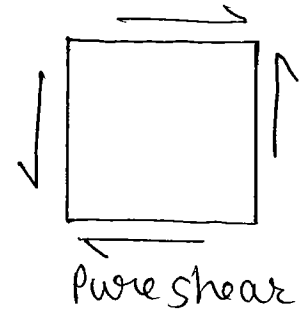
$$\sigma_1 = S_{ut}.$$



$$\frac{-\sigma_1}{S_{uc}} - \frac{\sigma_2}{S_{uc}} = \frac{1}{F.S.}$$

$$\sigma_2 = 0.$$

$$S_{uc} = -\sigma_1.$$



Design for Brittle material

Q. A steel pin subjected to a shear force 25 kN and the direct compression of 41 kN. The permissible compressive stresses in pin material is 50 MPa, then the suitable dia of pin according to distortion energy theory is

a] 75 mm b] 38 mm c] 20 mm d] 45 mm.

$$T = \frac{\pi}{16} F_s d^3$$

$$F_s = 25 \text{ kN}$$

$$F_t = 41 \text{ kN}$$

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$$

$$50 = \sqrt{\left(\frac{41}{A}\right)^2 + 3\left(\frac{25}{A}\right)^2} = 59.63$$

$$= \sqrt{\frac{41^2 + 3 \times 25^2}{A^2}}$$

$$A = \frac{2\sqrt{889}}{50}$$

$$A = 1192.64$$

$$\frac{\pi}{4} d^2 = 1192.64$$

$$\boxed{d = 38 \text{ mm}}$$

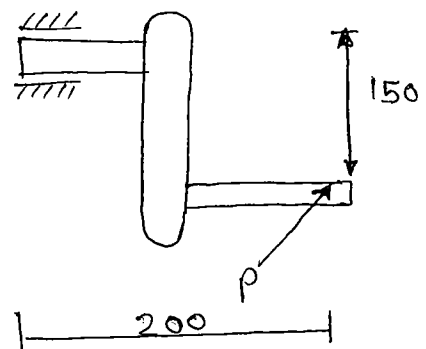
Q. A load P 45 kN is applied to a crank shaft of dia. 90 mm at a distance of 200 mm from bearing. The material is 30C4 with $S_{yt} = 315 \text{ MPa}$. Factor of safety according to Guest's theory is

a] 1 b] 2 c] 3 d] 4.

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{S_{yt}}{2FS}$$

$$\begin{aligned} \text{(Torsion)} T &= P \times 150 \\ &= 45 \times 150 \\ &= 6750 \text{ kN-mm} \end{aligned}$$

$$\begin{aligned} \text{(Bending)} M &= P \times 200 \\ &= 45 \times 200 \\ &= 9000 \text{ kN-mm} \end{aligned}$$



load P is applied $\perp$ to plane of paper.

$$T = \frac{\pi}{16} F_s d^3$$

$$0.047$$

$$= 4.244 \text{ kN-mm}$$

$$\begin{aligned}
 \bar{\sigma}_b &= \frac{32M}{\pi d^3} \\
 &= \frac{32 \times 9000}{\pi \times 90^3} \\
 &= 0.125 \text{ KN/mm}^2 \\
 &= 125.75 \text{ N/mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \tau_{\max} &= \sqrt{\left(\frac{\bar{\sigma}_b}{2}\right)^2 + \tau^2} \\
 &= \sqrt{\left(\frac{125.75}{2}\right)^2 + (47.15)^2} \\
 &= 78.59
 \end{aligned}$$

$$\tau_{\max} = \frac{S_{yb}}{2 \times F \cdot S}$$

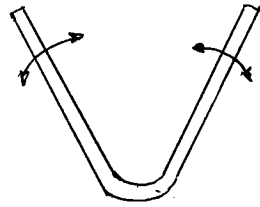
$$78.59 \times 2 \times F \cdot S = 315$$

$$\boxed{F \cdot S = 2}$$

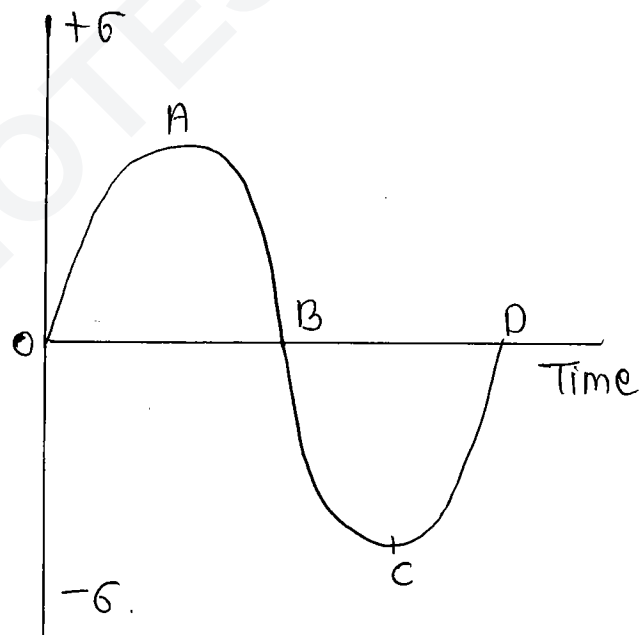
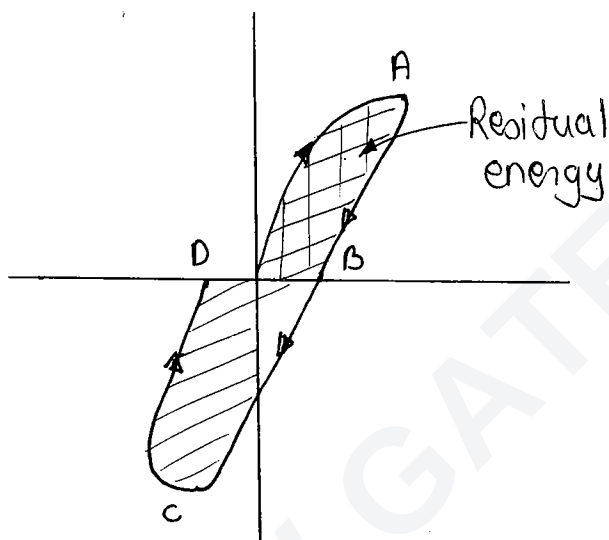
§ A component subjected to Fluctuating stresses

Designing for Fatigue.

A component is subjected to a fluctuating stresses fails at a stress much below their static strength. This phenomenon is called as Fatigue.

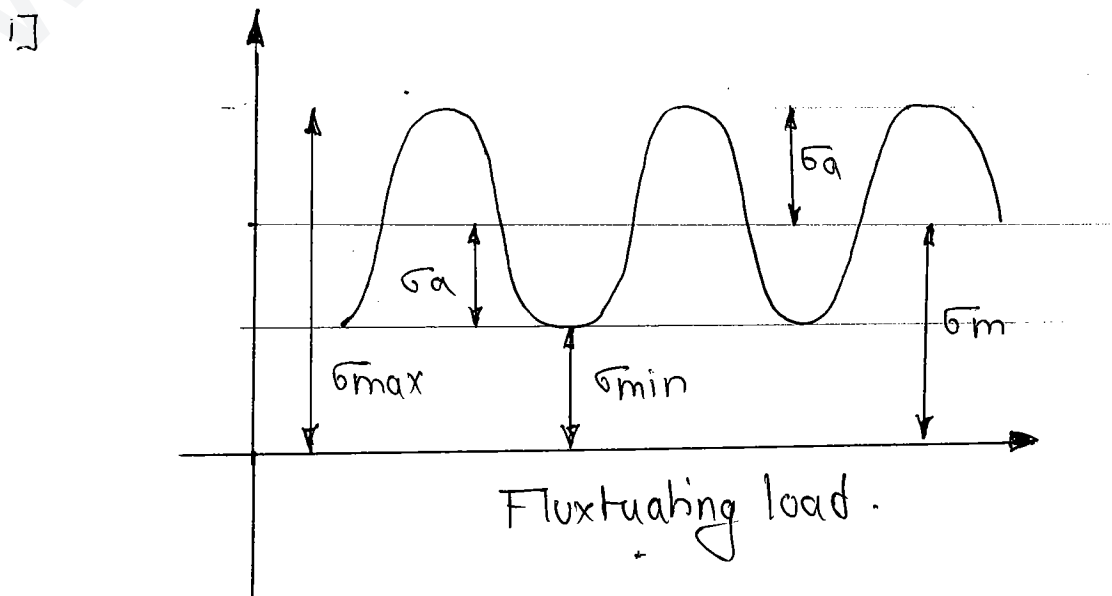


Wohler - Fatigue.



Ex:- 100 kN - strength.
10 kN - Residual $\uparrow$ then strength become 90 kN $\downarrow$

Terminology.

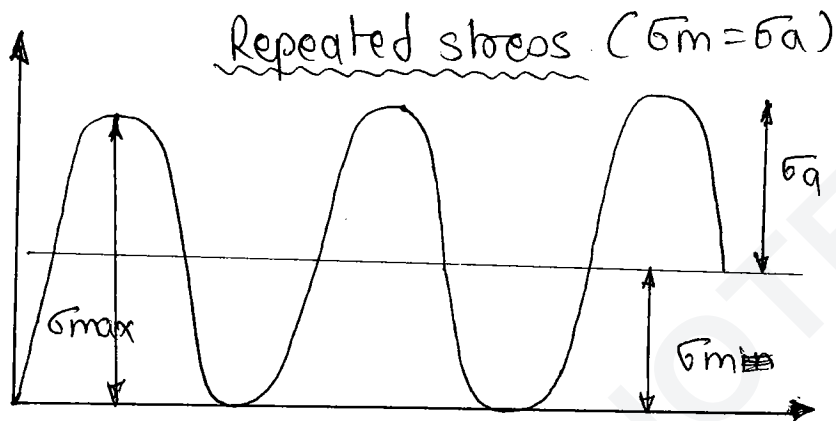


mean stress, $\bar{\sigma}_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$

Stress amplitude $\bar{\sigma}_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$

Stress Ratio, $R = \frac{\sigma_{\min}}{\sigma_{\max}}$

2]

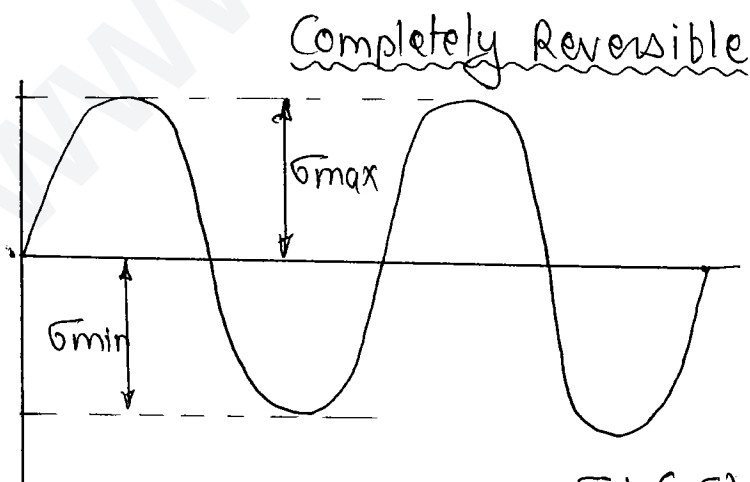


$$\bar{\sigma}_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{\sigma + 0}{2} = \sigma/2$$

$$\bar{\sigma}_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma + 0}{2} = \sigma/2$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = 0$$

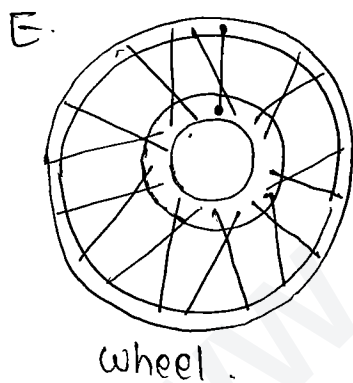
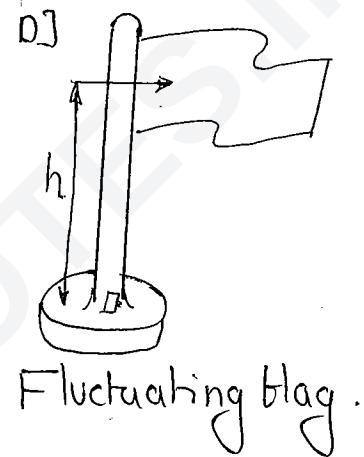
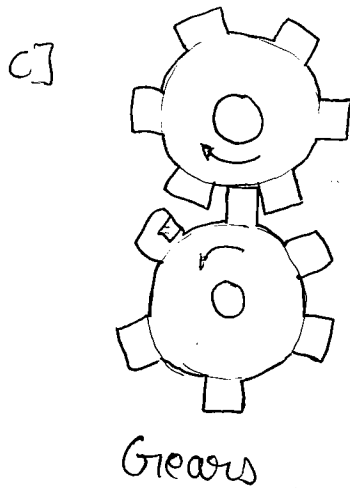
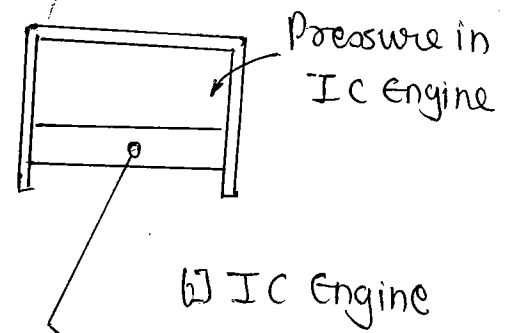
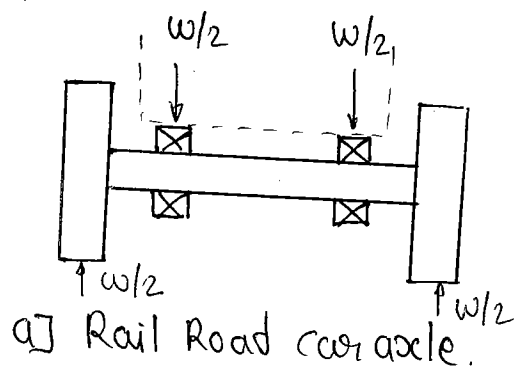
3]



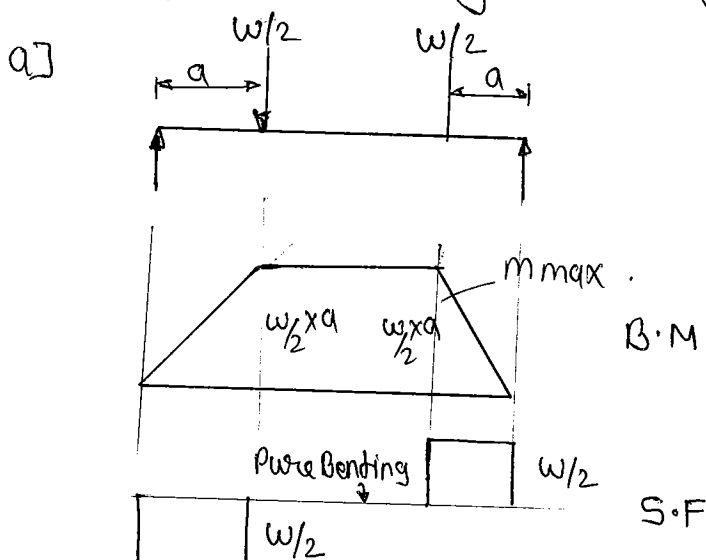
$$\bar{\sigma}_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{\sigma + (-\sigma)}{2} = 0$$

$$\bar{\sigma}_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{\sigma - (-\sigma)}{2} = \frac{2\sigma}{2} = \sigma$$

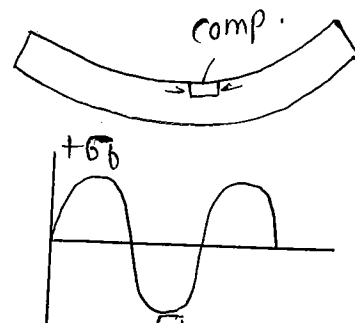
$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{-\sigma}{\sigma} = -1$$



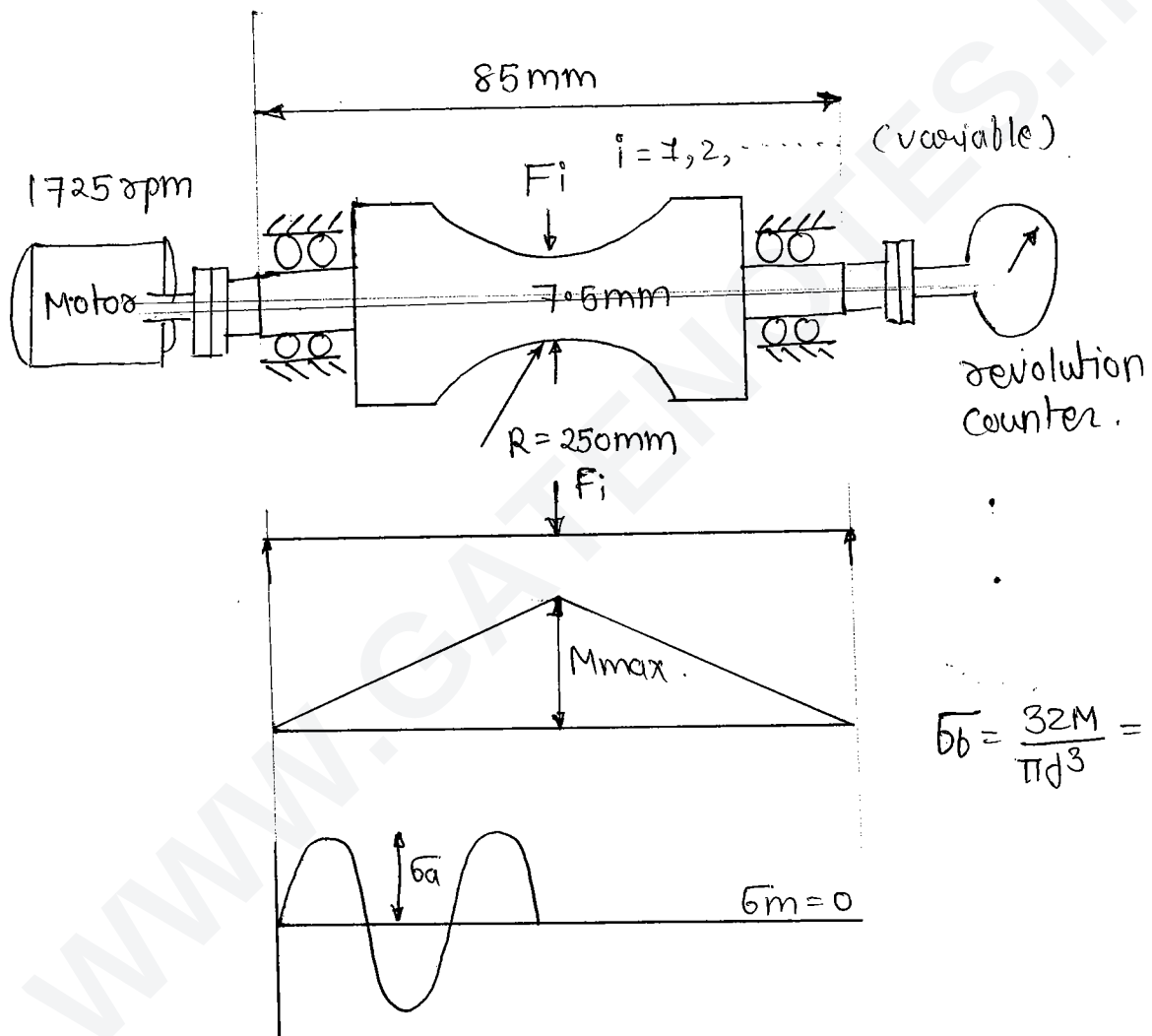
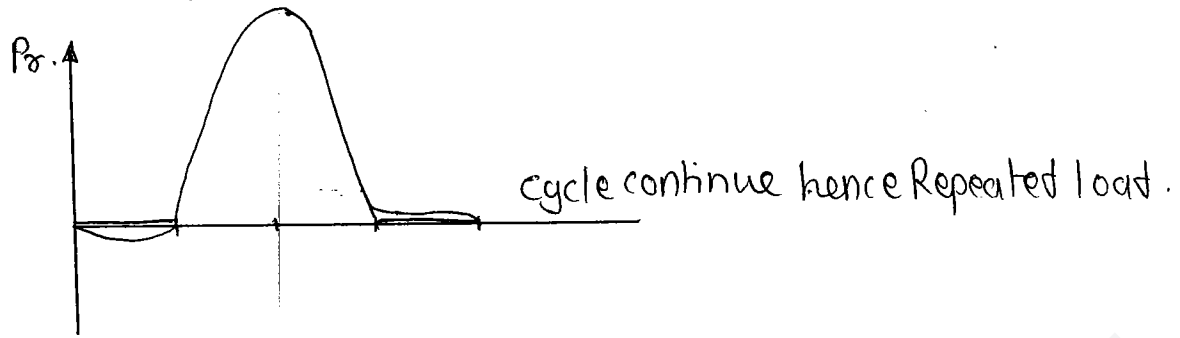
Axle $\rightarrow$ stationary or rotating member and is loaded in bending.



$$\sigma_b = \frac{32 M_{max}}{\pi d^3}$$

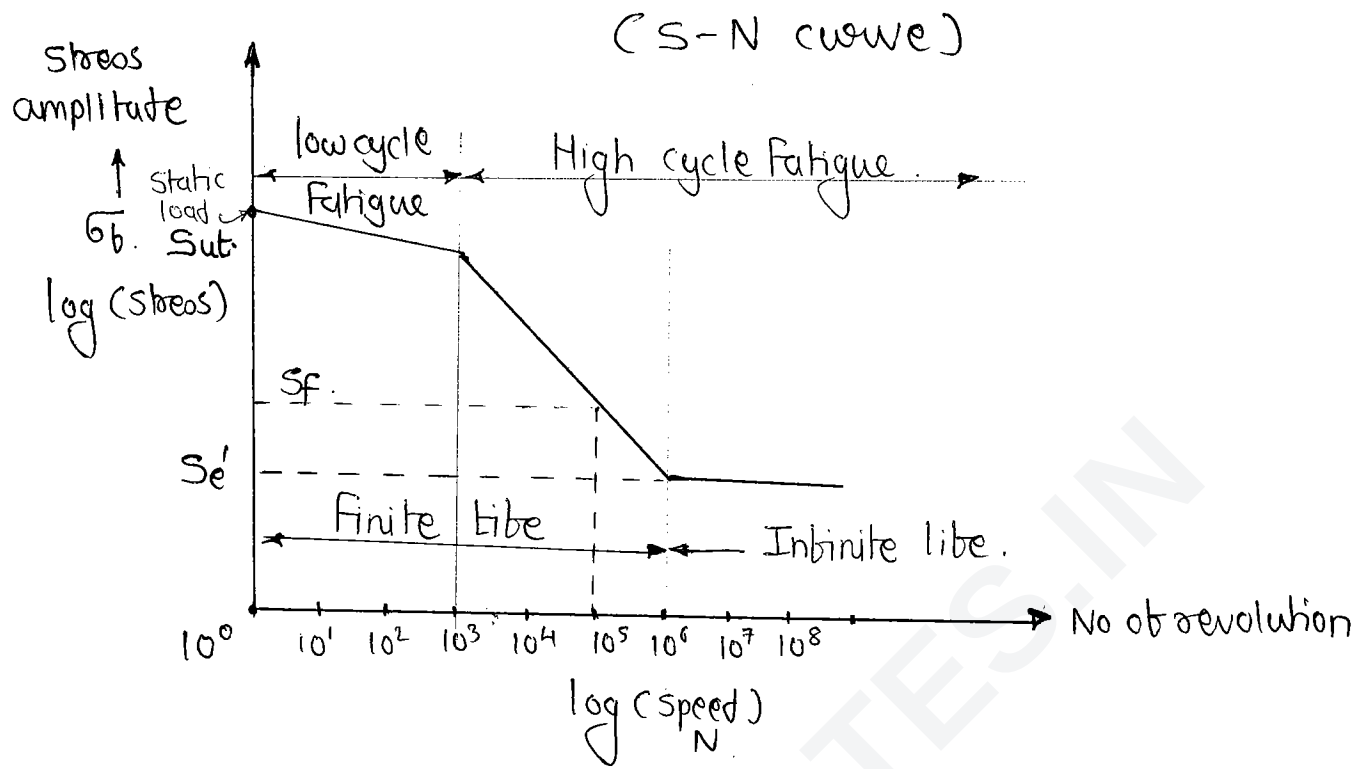


B] I C Engine.



$$\bar{\sigma}_b = \frac{32M}{\pi d^3} = \bar{\sigma}_a.$$

Load	Speed rpm
$\bar{\sigma}_{a1} = F_1$	N_1
$\bar{\sigma}_{a2} = F_2$	N_2
$\bar{\sigma}_{a3} = F_3$	N_3
$\vdots$	$\vdots$

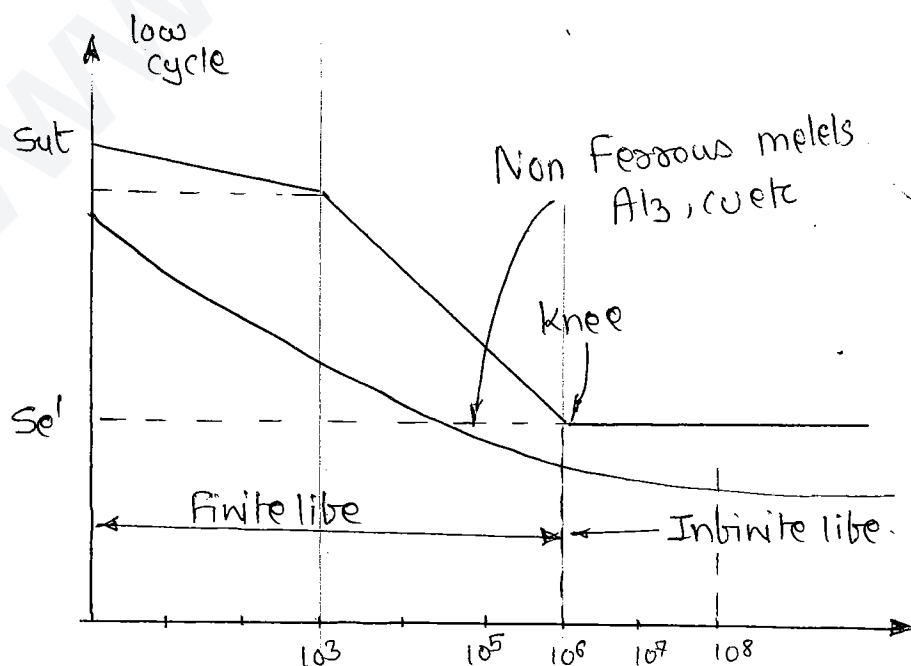


Endurance strength ($S_{e'}$)

It is the maximum value of a completely reversible bending stress which a standard polished specimen can withstand for a minimum life of one million revolution.

Stress of finite life expressed as Fatigue strength

Stress of infinite life expressed as endurance limit



$$\begin{aligned}
 10^6 \text{ rev} & - 9.6 \text{ hr.} \\
 10^8 \text{ rev} & - 9.6 \times 100 \\
 & = \frac{960 \text{ hr}}{24} = 40.2 \text{ days} \\
 & = 1 \frac{1}{2} \text{ months.}
 \end{aligned}$$

Empirical relation

For steel - $S_e' = 0.5 S_{ut}$ if $S_{ut} < 1400 \text{ MPa}$
 $= 700 \text{ MPa}$ if $S_{ut} \geq 1400 \text{ MPa}$.

For C.I $S_e' = 0.4 S_{ut}$

For non ferrous materials.

$$S_e' = 0.3 S_{ut}$$

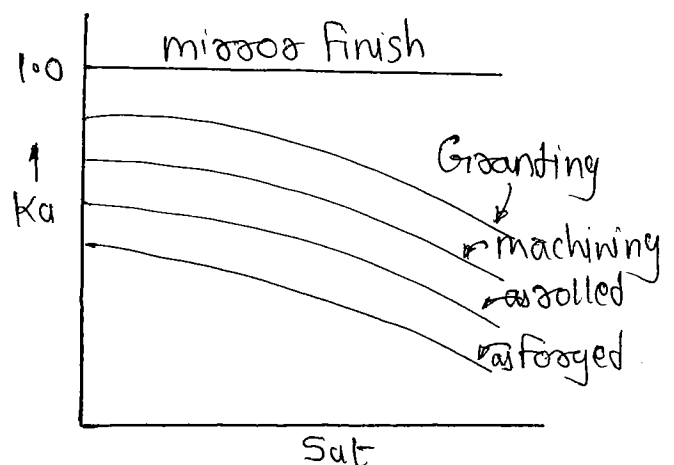
Factors affecting the endurance strength of components

Corrected endurance strength

$$S_e = K_a K_b K_c K_d K_e K_f \dots S_e'$$

1) Surface finish factor (K_a)

$K_a = 1$ for polished specimen.



2] $K_b = \text{Size Factor}$

$K_b = 1 \quad d \leq 8 \text{ mm}$

$= 1.189 d^{-0.097}$

As dia of component increases the value of size factor reduces

$d \uparrow \quad K_b \downarrow$

3] $K_e = \text{Reliability Factor}$

$= 1 \quad 50\% \text{ reliability}$

$= 0.897 \quad 90\% \quad "$

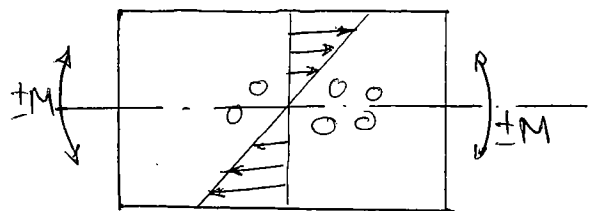
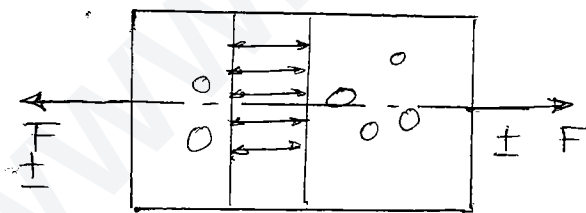
4] $K_e = \text{load Factor}$

$= 1 \quad \text{Completely reversed bending. } \left(\square \right) \pm M$



$= 0.7 \text{ to } 0.8 \quad (\text{It has two values limit is given \& select list one})$

Ex:- Connecting rod



5] $K_g = \text{Temperature factor}$

$= 1 \quad T < 300^\circ\text{C}$

$= 0.5 \quad T > 300^\circ\text{C}$

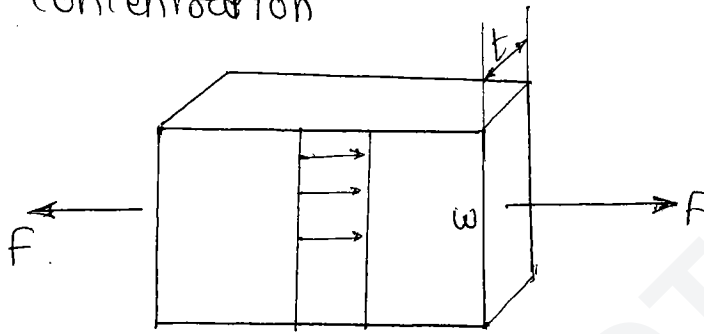
$K_t =$ Modifying stress Concentration Factor

$$= \frac{1}{K_f}$$

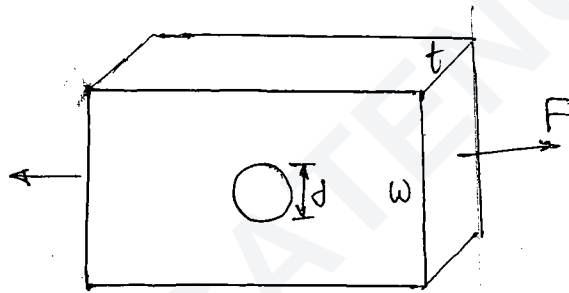
$K_f =$ Actual stress concentration Factor

Stress Concentration

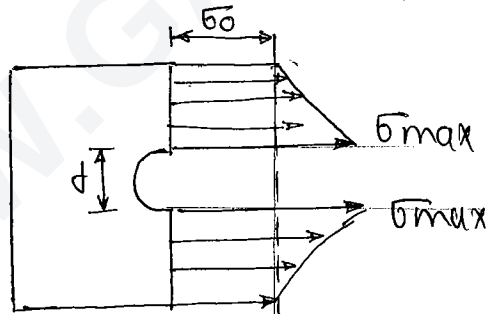
Localised high stress near the discontinuity is called stress concentration



$$\begin{aligned}\sigma_{\text{uniform}} &= F \\ &= \frac{F}{w \times t}\end{aligned}$$



$$\sigma_0 = \frac{F}{(w-d)t}$$

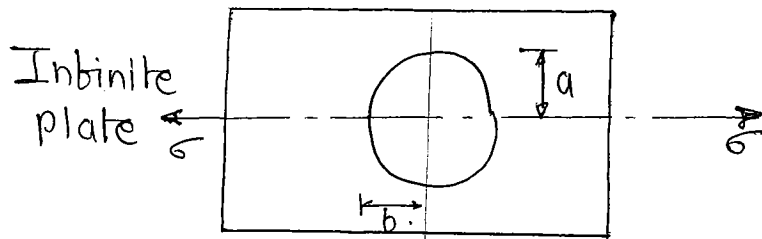


Theoretical stress concentration Factor,

$K_t = \frac{\text{Maximum stress near the discontinuity}}{\text{Nominal stress obtained by min area of C/S.}}$

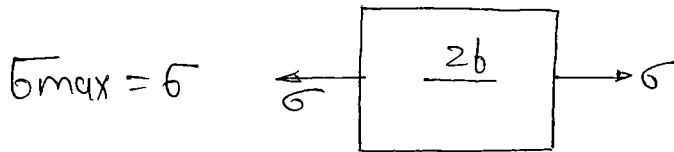
$$K_t = \frac{\sigma_{\text{max}}}{\sigma_0}$$

$$\sigma_0 = \sigma_{\text{uniform}} \quad (d \ll w)$$

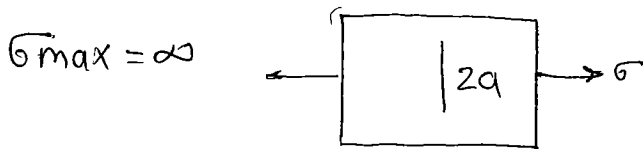


$$\sigma_{\max} = \sigma \left(1 + \frac{2a}{b} \right) \quad (\sigma_0 = \sigma_{\infty})$$

i] If $a=0$.

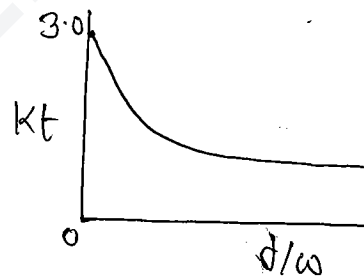
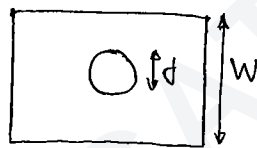


ii] If $b=0$



iii] If $a=b$. A circular hole

$$\sigma_{\max} = 3\sigma$$

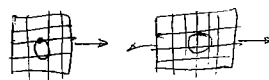


Methods of determining K_t .

1] Mathematical analysis
Theory of elasticity

2] Experimental method

- a] Photoelasticity
- b] Brittle Coating
- c] Strain gauge method
- d] Grid method

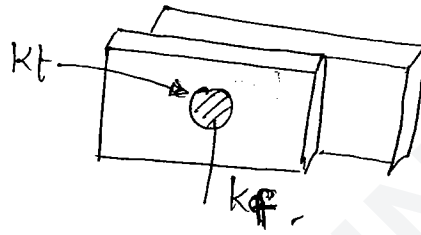


3] Computer based

Notch Sensitivity Factor (q)

The degree to which the effect of theoretical stress concentration is actually reached

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase theoretical stress over nominal stress}}$$



$$K_t = \frac{\sigma_{\max \text{ actual}}}{\sigma_0}$$

$$K_t = \frac{\sigma_{\max}}{\sigma_0}$$

$$\sigma_{\max \text{ actual}} = K_t \sigma_0$$

$$\sigma_{\max} = K_t \sigma_0$$

$$\text{Increase} = K_t \sigma_0 - \sigma_0$$

$$\text{Increased} = K_t \sigma_0 - \sigma_0$$

$$q = \frac{K_t \sigma_0 - \sigma_0}{K_t \sigma_0 - \sigma_0} = \frac{(K_t - 1) \sigma_0}{(K_t - 1) \sigma_0}$$

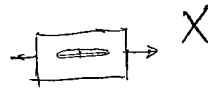
$$= \frac{K_t - 1}{K_t - 1}$$

$$K_t = 1 + q(K_t - 1) \quad q \leq 1$$

$$\text{If } q = 1$$

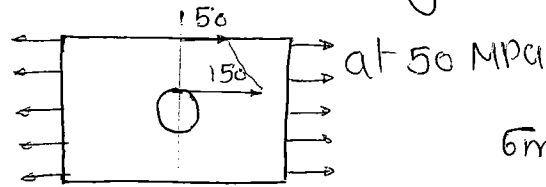
$$K_t = K_t$$

Effect of K_t on type of Material

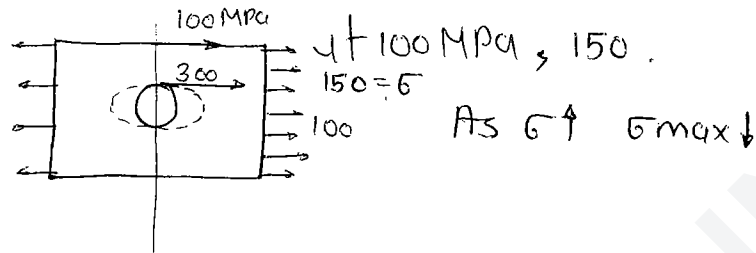
Type.	Static load	Fluctuating load
Ductile Material	 X	✓ K_t
Brittle Material	✓ K_t	✓ K_t

Ductile material.

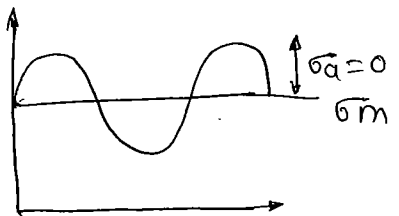
$$S_{yt} = 300 \text{ MPa}$$



$$\sigma_{\max} = \sigma \left(1 + \frac{2a}{b}\right)$$

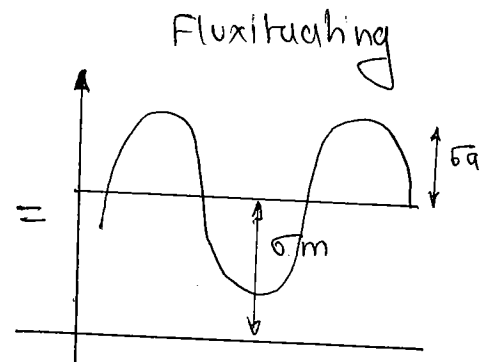
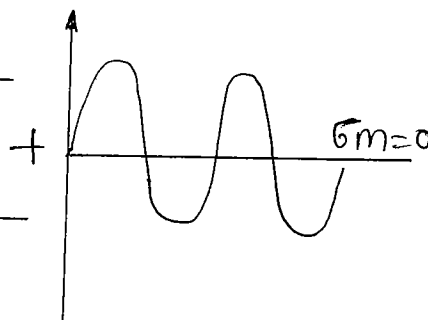
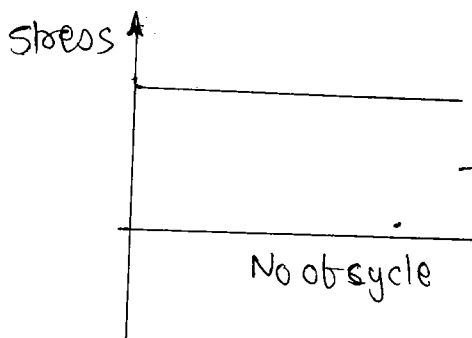
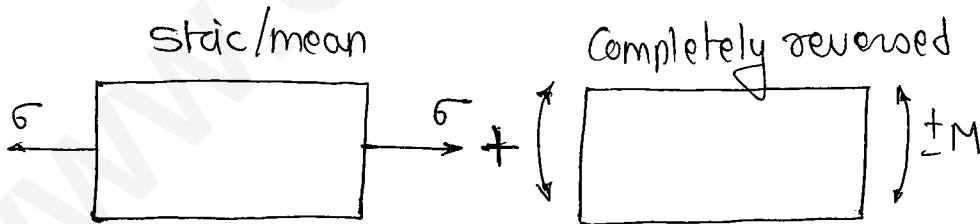
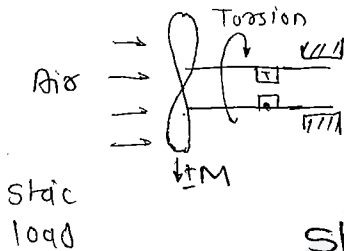


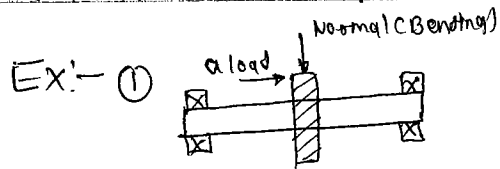
Effect of Mean stress on endurance limit.



we have considered completely reverse condition

$\sigma_a = 0$ static load.

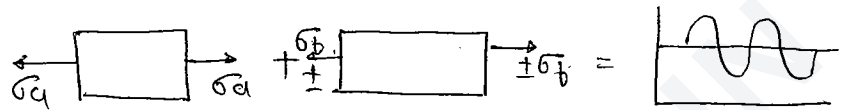
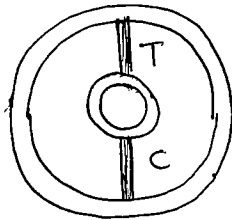




②. IC Engine connecting rod

③ Preloaded head bolt of IC engine steam engine

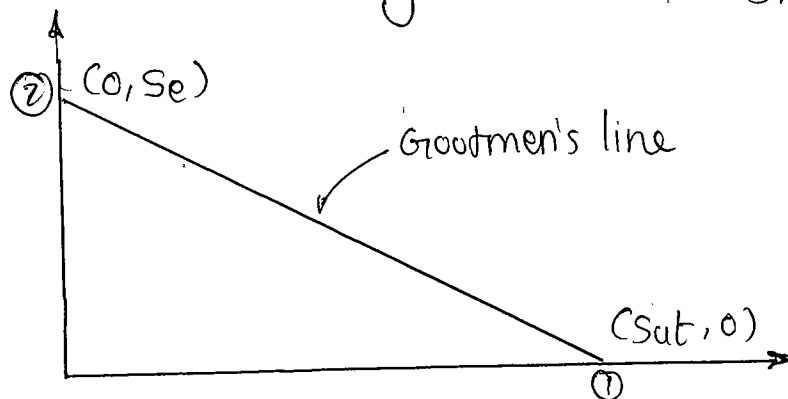
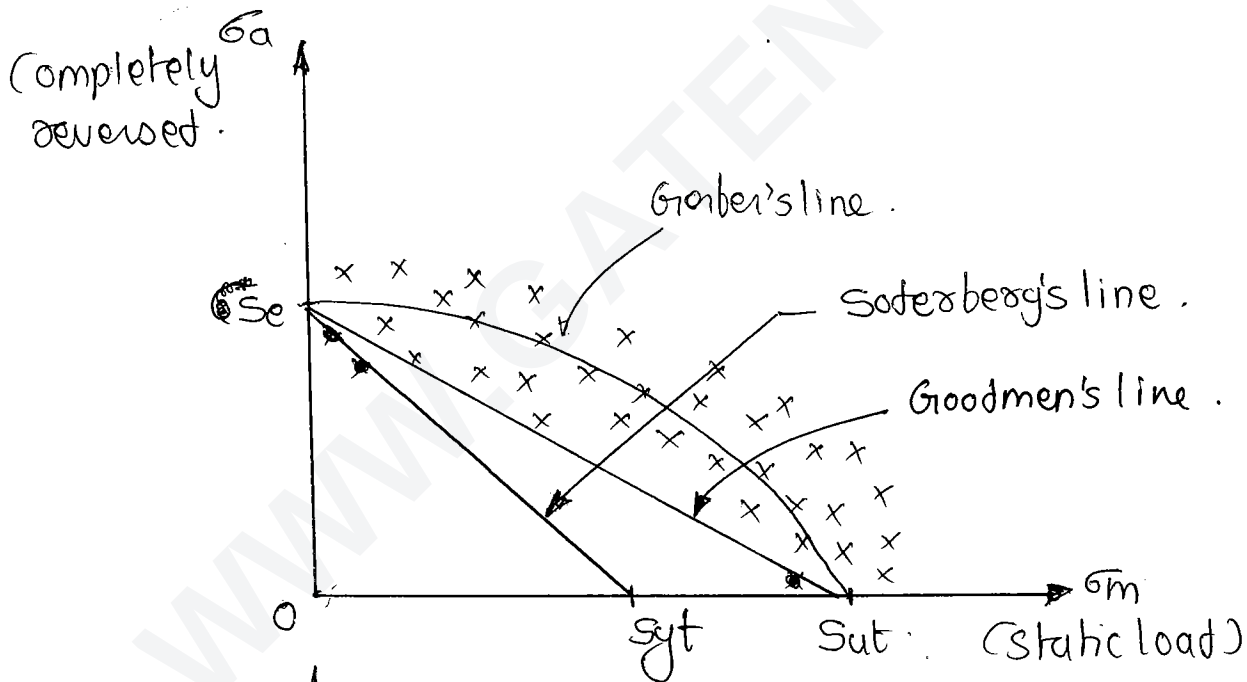
④ Cycle spokes.



$$\sigma_{max} = \sigma_a + \sigma_b$$

$$\sigma_{min} = \sigma_a - \sigma_b$$

Effect of mean stress on Endurance strength.



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\sigma_a - 0 = \frac{0 - \sigma_{se}}{\sigma_{st} - 0} (\sigma_m - \sigma_{st})$$

$$\frac{\sigma_a}{\sigma_{me}} = -\frac{\sigma_m}{\sigma_{ut}} + \frac{\sigma_{ut}}{\sigma_{ut}}$$

$$\frac{\sigma_a}{\sigma_{se}} + \frac{\sigma_m}{\sigma_{ut}} = 1$$

$$\frac{\sigma_a}{\sigma_{se}} + \frac{\sigma_m}{\sigma_{ut}} = \frac{1}{F.S.}$$

2] Soderberg's equations.

$$\frac{\sigma_a}{\sigma_{se}} + \frac{\sigma_m}{\sigma_{yt}} = \frac{1}{F.S.}$$

3] Gerber's equation.

$$\frac{\sigma_a}{\sigma_{se}} + \left(\frac{\sigma_m}{\sigma_{ut}}\right)^2 = 1$$

$$\frac{\sigma_a}{\sigma_{se}/F.S.} + \left(\frac{\sigma_m}{\sigma_{ut}/F.S.}\right)^2 = 1$$

Q :- A steel plate shown in fig is subjected to completely reversed load of 16 kN. The plate is made of steel ($\sigma_{ut} = 440 \text{ MPa}$), $q = 0.8$, $k_a = 0.67$, $k_b = 0.85$, $k_c = 0.9$, $k_d = 0.897$, $k_t = 2.35$ and $F.S. = 1.5$

a] Then the plate thickness for infinite life is

a] 6mm b] 12mm c] 20mm d] 30mm

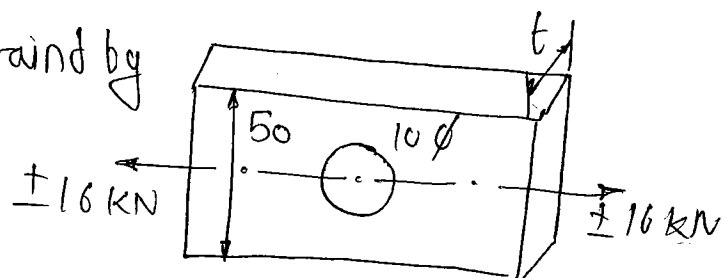
b] Optimum thickness is obtained by

a] Gerber's Method

b] Goodman eqn

c] Soderberg's eqn

d] Both Goodman & Soderberg's method



$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{S_{ut}} = \frac{1}{F_s} \quad \text{Goodman}$$

$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{S_{yt}} = \frac{1}{F_s} \quad \text{Soderberg}$$

Ans d]

$$\frac{\bar{\sigma}_a}{S_e} = \frac{1}{F \cdot S}$$

$$\begin{aligned} S_e &= k_a k_b k_c k_d \dots S_e' \\ &= 0.67 \times 0.85 \times 0.9 \times 0.897 \times \frac{1}{2.08} \times 220 \\ &= 48.62 \text{ MPa} \end{aligned}$$

$$\begin{aligned} S_e' &= 0.5 S_{ut} \\ &= \frac{440 \times 0.5}{1} \quad \text{if } < 1400 \text{ MPa} \\ &= 220 \text{ MPa} \end{aligned}$$

$$F = \pm 10 \text{ kW}$$

$$S_{ut} = 440 \text{ MPa}$$

$$q = 0.8$$

$$k_a = 0.67$$

$$k_b = 0.85$$

$$k_c = 0.9$$

$$k_d = 0.897$$

$$t = ?$$

$$F_s = 1.5$$

$$K_t = 2.35 \text{ - If given then}$$

k_e - Modifying stress concentration factor.

$$\begin{aligned} k_e &= \frac{1}{k_f} = \frac{1}{1 + q(K_t - 1)} \\ &= \frac{1}{1 + 0.8(2.35 - 1)} = \frac{1}{2.08} \end{aligned}$$

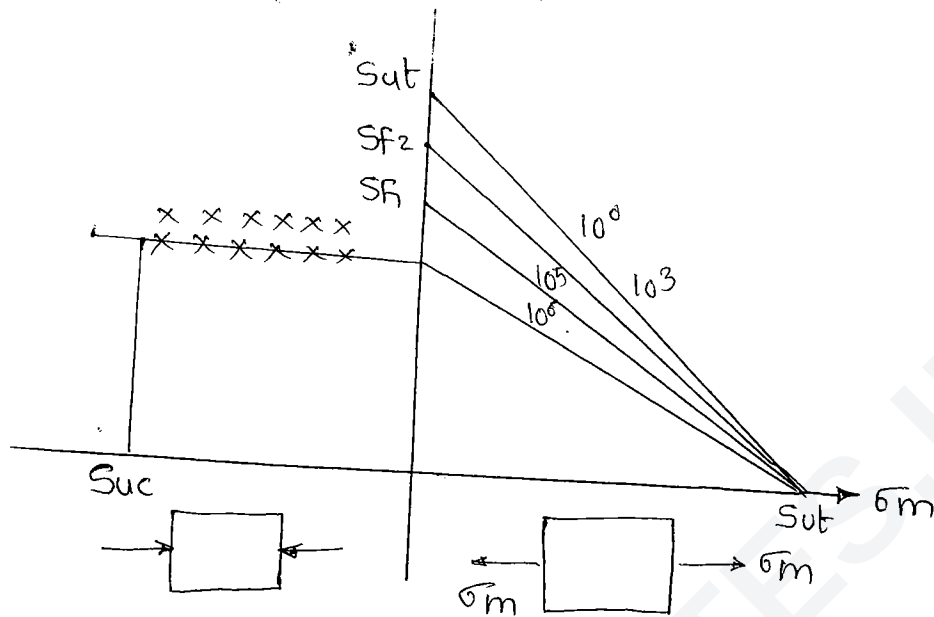
$$\bar{\sigma}_a = \frac{10 \times 10^3}{(w-d)t} = \frac{16 \times 10^3}{(50-10)t} =$$

$$\frac{\bar{\sigma}_a}{S_e} = \frac{1}{F \cdot S}$$

$$\frac{16 \times 10^3}{40 \times t} = \frac{48.62}{1.5}$$

$$t \approx 12 \text{ mm}$$

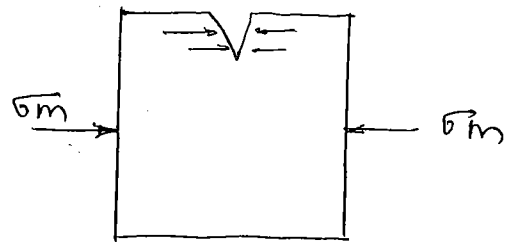
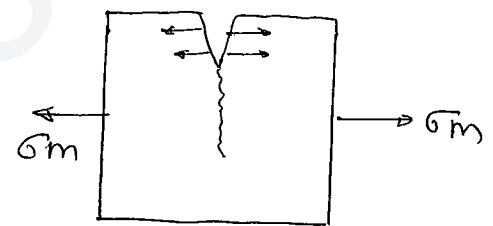
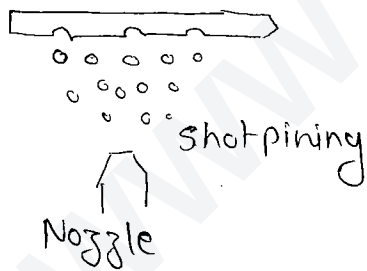
Compressive strength of material



To improve fatigue strength

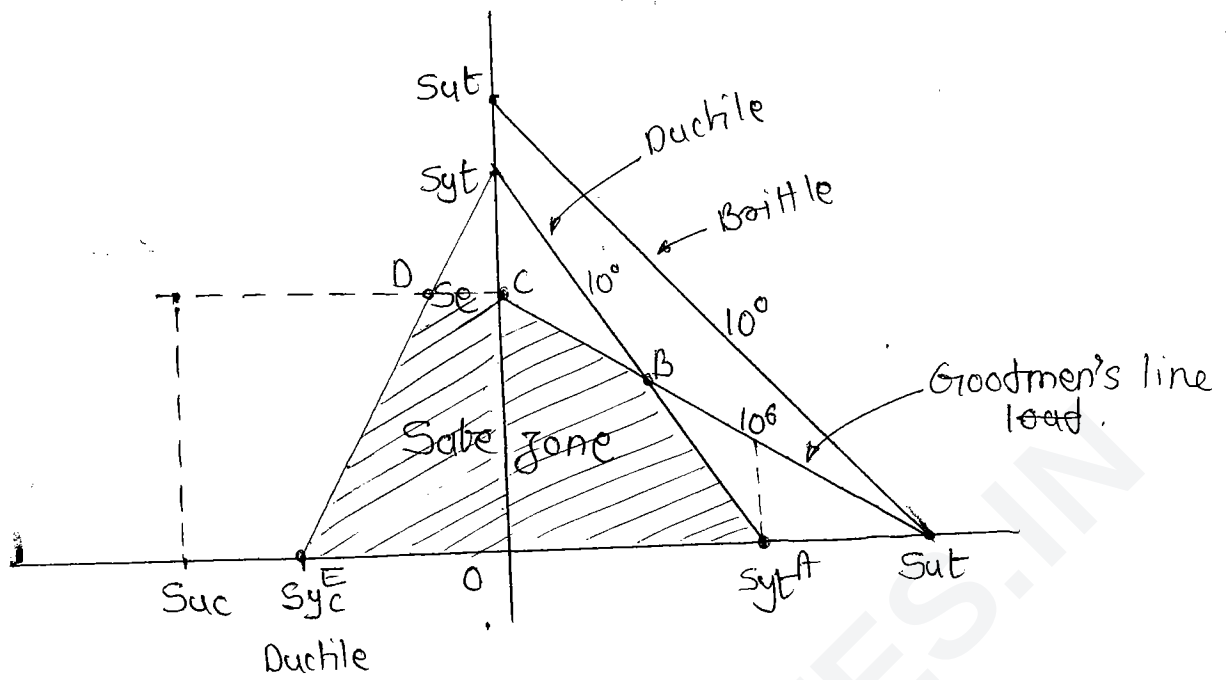
- 1] Cold rolling
- 2] Burnishing
- 3] Shot pining
- 4] Case hardening
- 5] Coining

Residual stress or Preloading



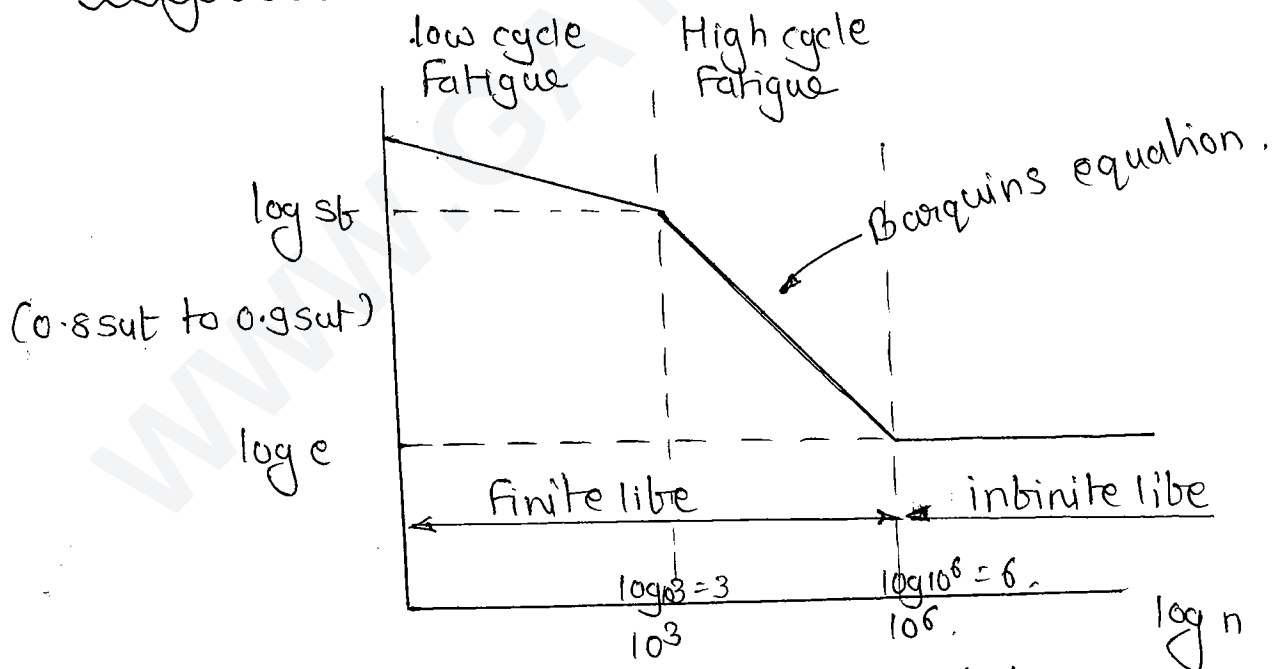
Decrease the effects of surface notches

Beoz. of intention the strength is increased in case of sand casting surface scratches produce which reduces surface fatigue strength.



Assuming material will fail at syt line here give name 10^0

Design box finite life



Design box finite life and high cycle fatigue

S_b corresponding to 10^3 rev is $0.8s_{yt}$ to $0.9s_{yt}$.

Basquin's Equations.

$$A = S_b L^B$$

A and B are constants

S_b - fatigue strength

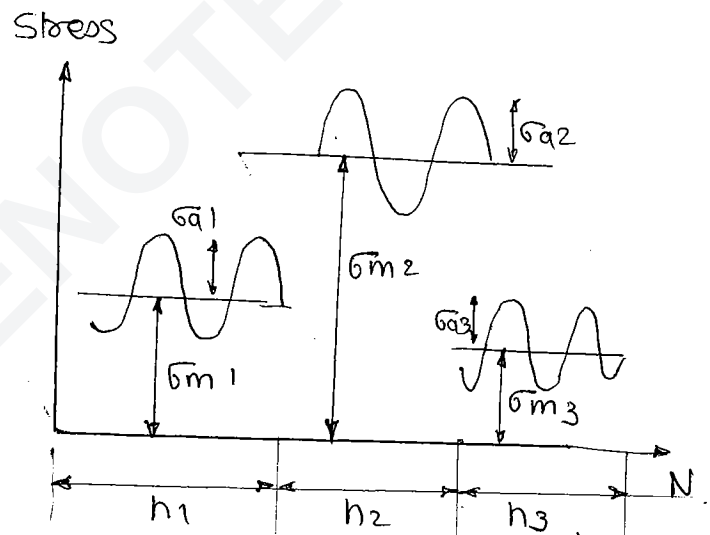
L - Life in revolution.

$$A = S_e (10^6)^B \quad - (1)$$

$$A = S_b (10^3)^B \quad - (2)$$

Find the value of A & B then, using above (1) & (2) eqⁿ.

Cumulative fatigue.



Ex:- Pulley, Bearings

Cumulative fatigue - Miner's equation

Let, a component is subjected to

Stress	σ_1	box	n_1	no of cycle.
	σ_2	"	n_2	"
	σ_3	"	n_3	"
	σ_4	"	n_4	"

If the component is subjected to only stress σ_1 then, let its life = L_1 revolution

$$\text{Life completed in one revolution} = \frac{1}{L_1} \leftarrow \text{total life}$$

$$\text{Life completed during } n_1 \text{ revolution} = \frac{n_1}{L_1}$$

$$\text{Total life consumed} = \frac{n_1}{L_1} + \frac{n_2}{L_2} + \frac{n_3}{L_3} + \dots + \frac{n_x}{L_x} = 1$$

$$\text{Let, } n_1 = \alpha_1 L$$

$$n_2 = \alpha_2 L \quad \begin{array}{l} L \rightarrow \text{total life} \\ \alpha_1 \rightarrow \text{part of the total life at } \sigma_1 \end{array}$$

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \dots + \frac{\alpha_x}{L_x} = \frac{1}{L}$$

Q1982 A machine part is made of steel with $S_{ut} = 630 \text{ MPa}$, $S_e = 0.22 S_{ut}$, Fatigue strength for 10^3 cycles is $0.9 S_{ut}$. The life of part is 25000 cycles. The load for 50% of time is $\pm 225 \text{ MPa}$ and 30% of time is $\pm 145 \text{ MPa}$.

The lives of part at 225 MPa and 145 MPa respectively are (in revolution)

- a] 9.29×10^4 , 8.01×10^5 c] 8×10^4 , 4×10^5
b] 9.7×10^4 , 6×10^5 d] 3×10^5 , 5×10^5 .

Ans $S_{ut} = 630 \text{ MPa}$ $L = 25000$
 $S_e = 0.22 S_{ut}$ $S_f = 10^3 \text{ cycles is } 0.9 S_{ut}$
 $A = S_f L^B$

$$S_{f1} = \pm 225 \text{ MPa} \quad \alpha_1 = 0.5$$

$$S_{f2} = \pm 145 \text{ MPa} \quad \alpha_2 = 0.3$$

$$S_e = 0.22 \times 630 = 138.6 \quad \dots \dots \dots 10^6 \text{ rev}$$

$$S_f = 0.9 \times 630 = 567 \quad \dots \dots \dots 10^3 \text{ rev}$$

$$A = 138.6 \times 10^6 B$$

$$A = 567 \times 10^3 B$$

$$138.6 \times 10^6 B = 567 \times 10^3 B \quad 10^{6B-3B} = \frac{567 \times 10^3}{138.6 \times 10^6}$$

$$B = 0.203$$

$$A = 2304$$

$$2304 = S_b L^{0.203}$$

$$2304 = 225 L_1^{0.203}$$

$$L_1 = 9.32 \times 10^4$$

$$L_2 = 8.03 \times 10^5$$

The load during the remaining time.

a) 250 MPa b) 375 MPa c) 395 MPa d) 415 MPa.

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \frac{\alpha_3}{L_3} = \frac{1}{L}$$

$$\frac{0.5}{9.32 \times 10^4} + \frac{0.3}{8.03 \times 10^5} + \frac{0.2}{L_3} = \frac{1}{25000}$$

$$L_3 = 8756.15 \text{ } 5837.46 \text{ cycle.}$$

$$2304 = S_b L^{0.203}$$

$$2304 = S_b \times 5837.46^{0.203}$$

$$S_b = 396 \text{ MPa.}$$

29] The life of a machine component under heavy test load was 10 hrs. The life under combine loading was 18 hours of which ^{9.8 hr} at heavy load and 8.2 hours was at the normal load. What is the expected life under normal operating load

a) 54.5 hr b) 6 hrs c) 410 hr d) 490 hr.

$$\frac{n_1}{L_1} + \frac{n_2}{L_2} = 1$$

$$\frac{9.8}{10} + \frac{8.2}{L_2} = 1$$

$$L_2 = 410$$

OR

$$\alpha_1 = \frac{9.8}{18}, \alpha_2 = \frac{8.2}{18}$$

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} = \frac{1}{L} \quad \frac{0.511}{10} + \frac{0.455}{L_2} = \frac{1}{18}$$

Welding

Welding is a process of joining metals by the application of heat with and without the application of pressure.

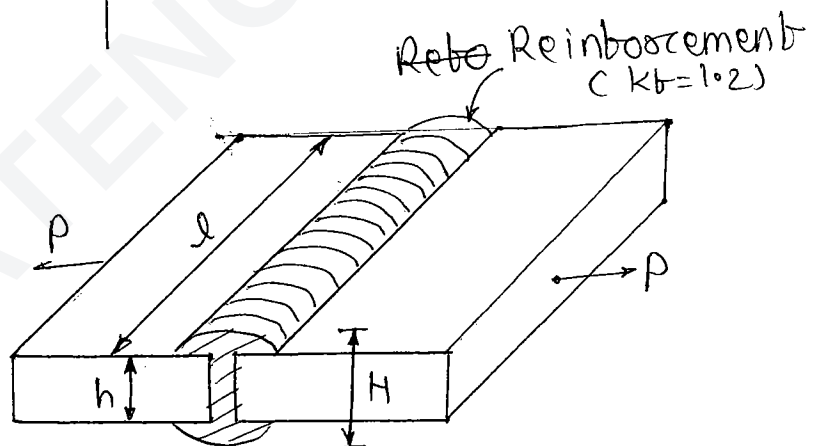
Advantages

- 1] Highest Joint efficiency
- 2] Lighter in weight
- 3] Easy to weld any part of the structure at any point.
- 4] Easy to make alteration

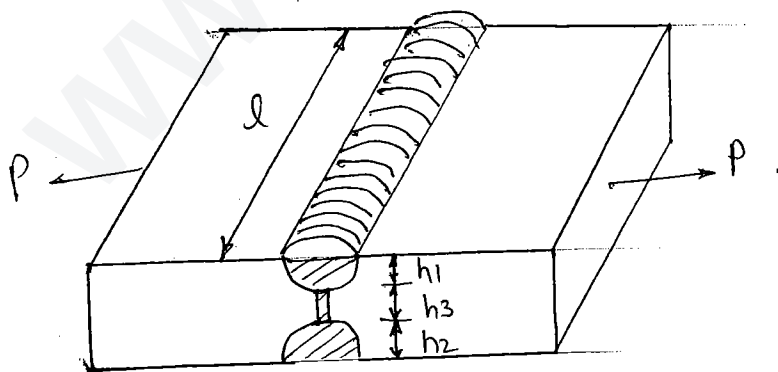
Disadvantages

- 1] Due to non uniform heating and cooling thermal stresses will be induced it result in distortion of component.

Joints
 ↗ Butt
 ↘ Fillet

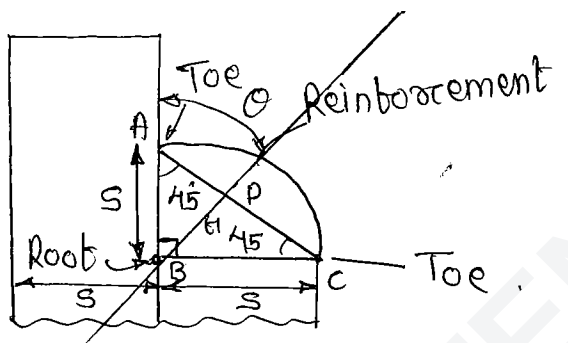
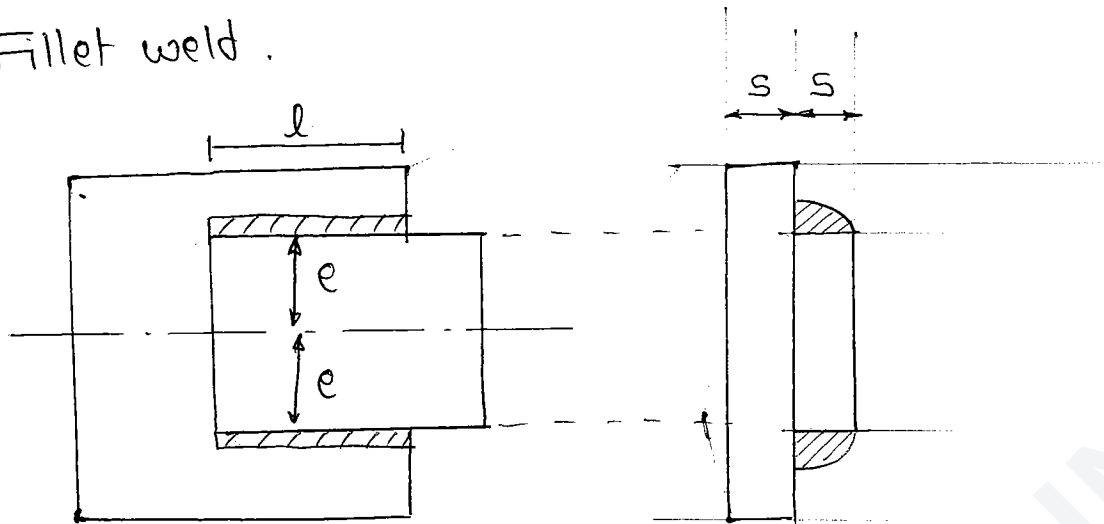


$$\sigma_t = \frac{P}{h \times l}$$

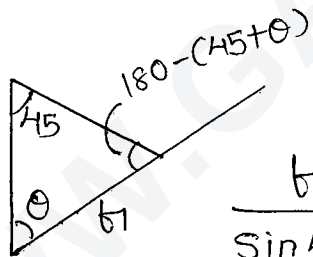


$$\sigma_t = \frac{P}{(h_1 + h_2) l}$$

Fillet weld.



Toe and Root are the weakest point because of high stress concentration



$$\frac{b_1}{\sin 45^\circ} = \frac{S}{\sin(180^\circ - (45^\circ + \theta))}$$

$$b_1 = \frac{S \sin 45^\circ}{\sin(45^\circ + \theta)}$$

$$= \frac{S \sin 45^\circ}{\sin 45^\circ (\cos \theta + \cos 45^\circ \sin \theta)}$$

$$= \frac{S}{\cos \theta + \sin \theta}$$

$$b_1 = \frac{S}{\sin \theta + \cos \theta}$$

Shear stress on the plane at an angle θ

$$\tau_{\theta} = \frac{P}{l \times l}$$

$$\tau_{\theta} = \frac{P(\sin \theta + \cos \theta)}{S \times l}$$

Max^m shear stress

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$\frac{P}{Sl} \neq 0$$

$$\frac{\partial}{\partial \theta} (\sin \theta + \cos \theta) = 0$$

$$(\cos \theta - \sin \theta) = 0$$

$$\cos \theta = \sin \theta = 45^\circ$$

$$h = \frac{S}{\sin \theta + \cos \theta} \quad \theta = 45^\circ$$

$$[h = 0.707 S] \quad - \text{Throat thickness.}$$

$$\tau_{45^\circ} = \tau_{\max} = \frac{P(\sin 45 + \cos 45)}{Sl}$$

$$\tau_{\max} = \frac{\sqrt{2}}{Sl} P$$

$$[P = \frac{1}{\sqrt{2}} Sl \tau_{\max}]$$

Load carrying capacity of single parallel fillet weld

$$P = 0.707 Sl \left(\frac{S_{sy}}{F.S} \right) -$$

$$P = 1.414 Sl \left(\frac{S_{sy}}{F.S} \right) - \text{Double fillet weld.}$$

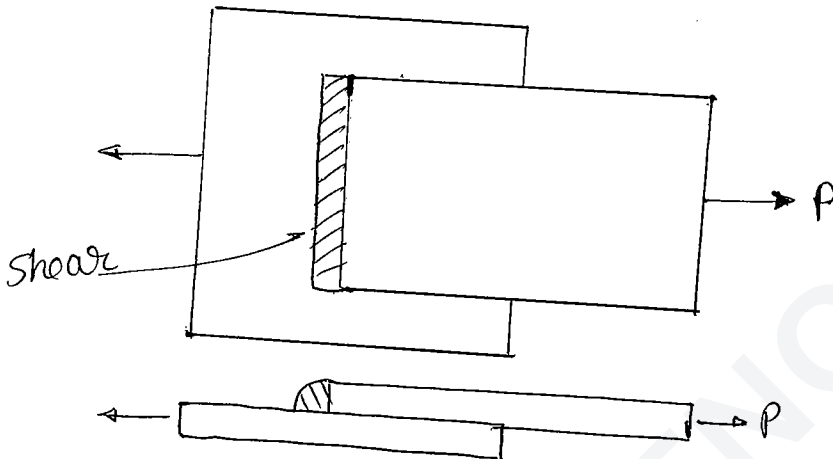
$$\tau_{90} = \frac{P}{S_l}$$

$$\tau_0 = \frac{P}{S_l}$$

$$\frac{\tau_{90}}{\tau_{45}} = 0.707$$

Transverse billet weld.

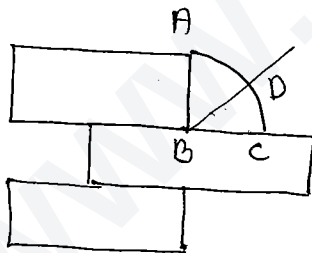
$$\tau_{max} = \tau_{67.5}$$



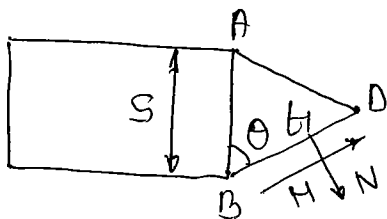
$$\tau_{max} = \tau_{67.5}$$

$$P = 0.707 S_l \tau_{max} \text{ — Transverse billet weld}$$

$$\frac{\tau_{67.5}}{\tau_{45}} < 1$$



$$H =$$



$$H = P \sin \theta$$

$$N = P \cos \theta$$

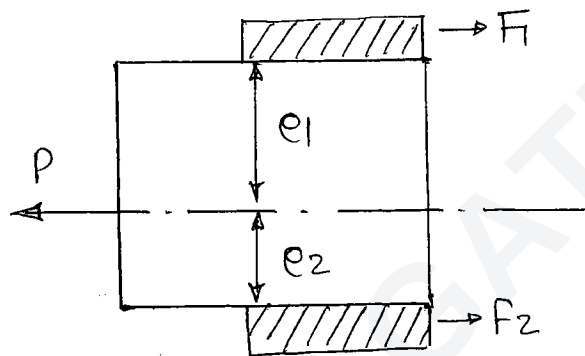
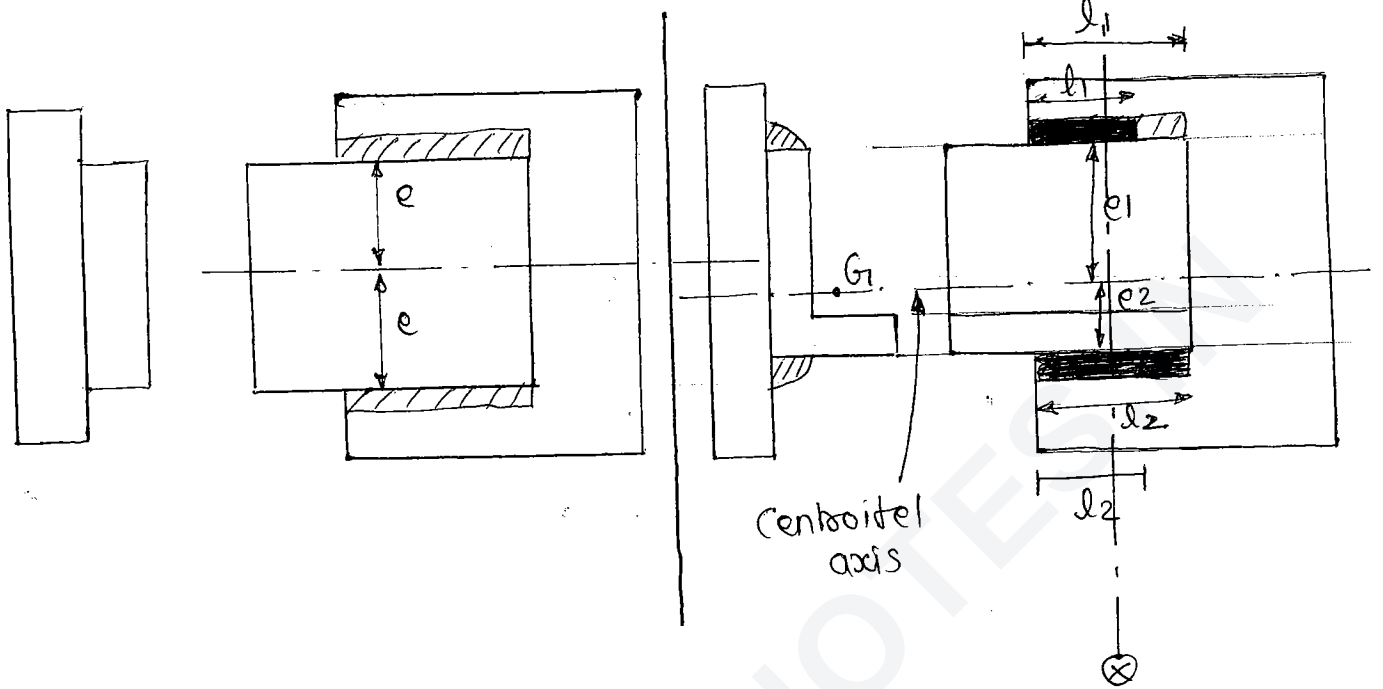
$$\tau = \frac{H}{S_l}$$

$$= \frac{P \sin \theta (\sin \theta + \cos \theta)}{S_l}$$

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$2\theta = 135^\circ$$

Axially loaded unsymmetrical welds.



$$P = F_1 + F_2$$

$$\sum M_{\text{centroidal axis}} = 0$$

$$F_1 \times e_1 - F_2 \times e_2 = 0$$

$$F_1 e_1 = F_2 e_2$$

$$\frac{F_1}{e_2} = \frac{F_2}{e_1} = \frac{F_1 + F_2}{e_1 + e_2} = \frac{P}{e_1 + e_2}$$

$$F_1 = \frac{e_2}{e_1 + e_2} \times P$$

$$F_2 = \frac{e_1}{e_1 + e_2} \times P$$

Ex:-

$$P = 0.707 S l \left(\frac{S_{sy}}{F \cdot S} \right)$$

$$l = l_1 + l_2$$

$$F_1 = 0.707 S \times l_1 \left(\frac{S_{sy}}{F \cdot S} \right)$$

$$F_2 = 0.707 S \times l_2 \left(\frac{S_{sy}}{F \cdot S} \right)$$

$$P = F_1 + F_2$$

$$l = l_1 + l_2$$

$$P \propto l$$

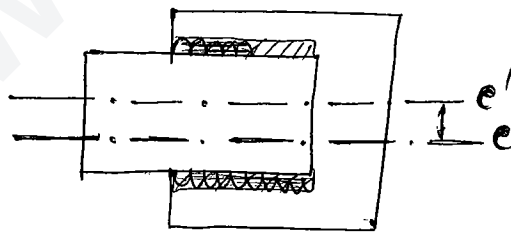
$$F_1 \propto l_1$$

$$F_2 \propto l_2$$

$$F = \frac{e_2}{e_1 + e_2} P$$

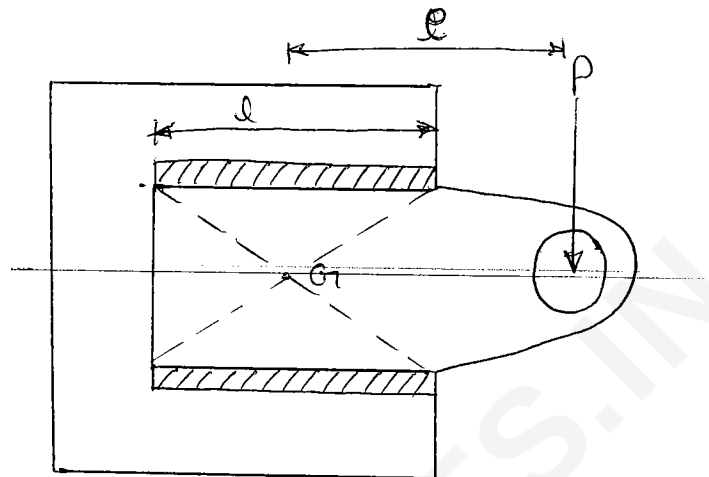
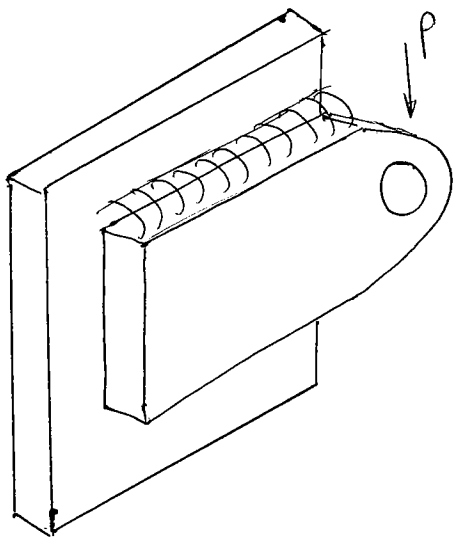
$$l_1 = \frac{e_2}{e_1 + e_2} \times l$$

$$l_2 = \frac{e_1}{e_1 + e_2} \times l$$

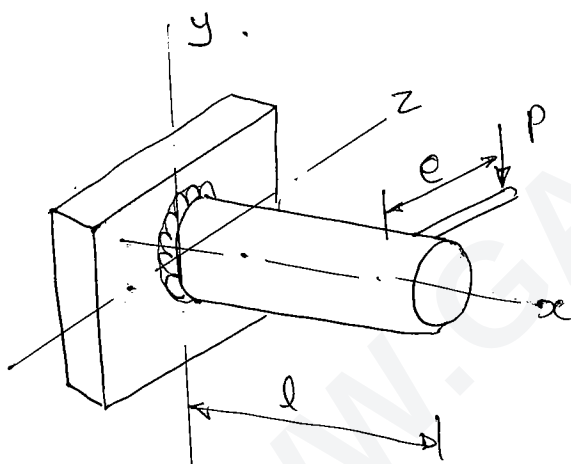


Bending movement will take place because of change in centroidal axis.

Eccentrically loaded Welded Joint.



eccentric load cause a movement stress it may be bending or torsion according to the application of load



$$M_x = P \times e = T.$$

$$M_y = 0$$

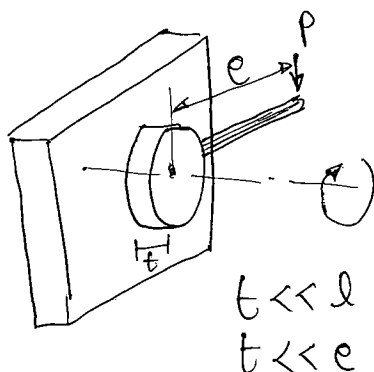
$$M_z = P \times l$$

$$M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

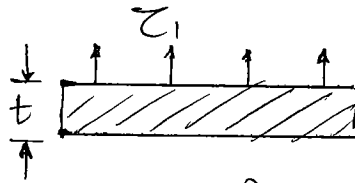
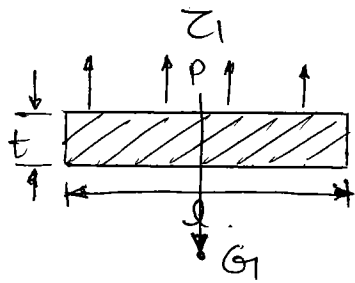
$$T + 0 + B$$

Any member trying to move member about its own axis is called Torsion.

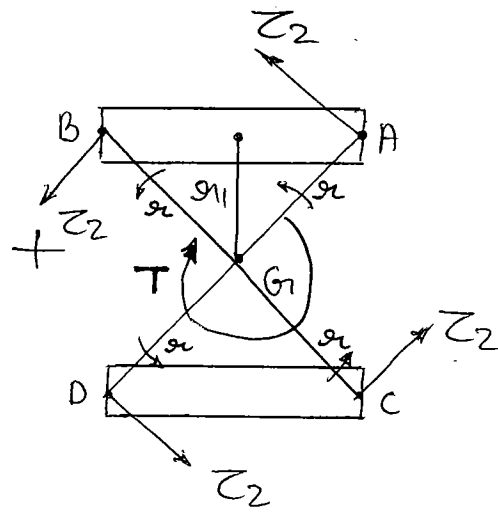
If $l=0$. there is only torque.



Hence it causes a torsional movement.



$$\tau_1 = \frac{P}{2t \times l}$$

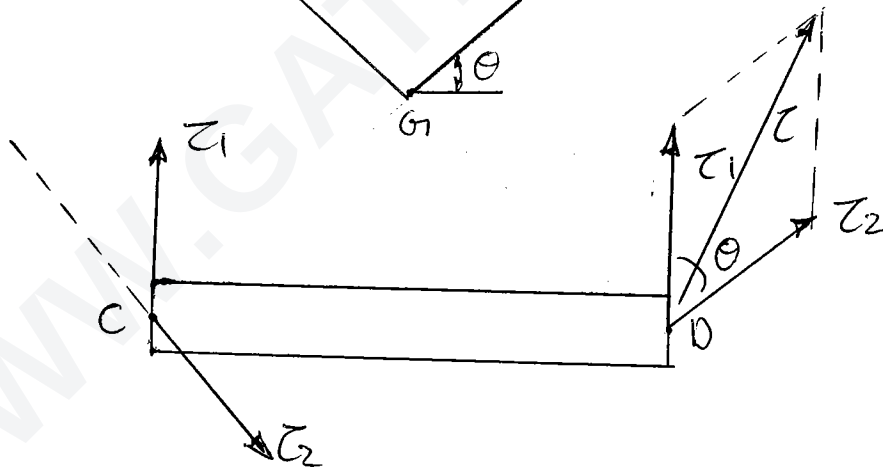
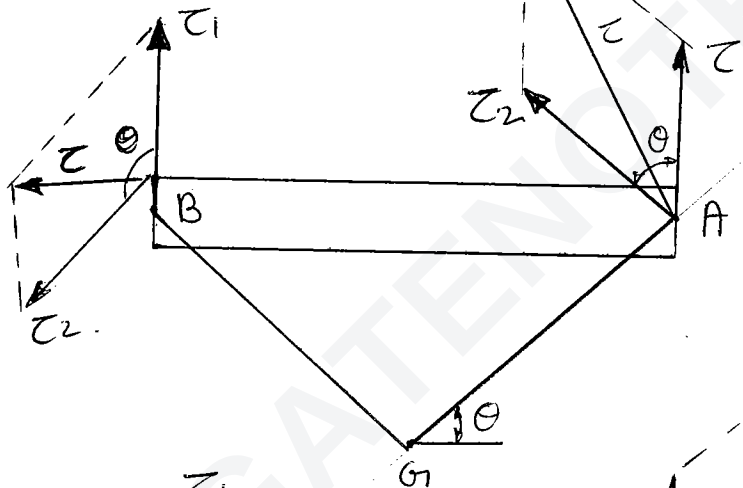


$$\tau_2 = \frac{T}{J} \times r$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{J} \times r$$

$$\tau \propto r$$



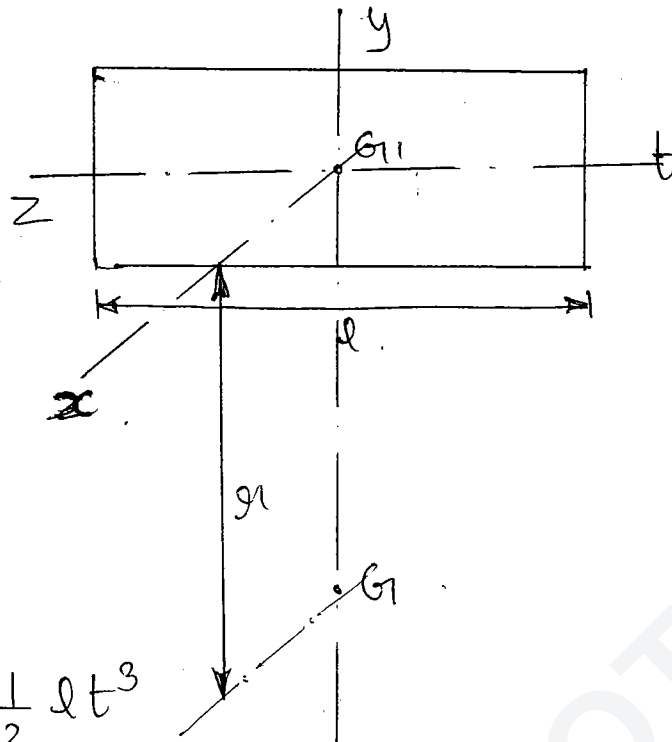
$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos \theta}$$

$$\tau = \phi(\tau_1, \tau_2, \theta)$$

$$\theta \downarrow \cos \theta \uparrow$$

Hence object will first fail from point A & D

Polar movement of Inertia.



$$I_z = \frac{1}{12} l t^3$$

$$I_y = \frac{1}{12} t l^3$$

$$I_x = J_{G_1} = I_z + I_y$$

$$= \frac{1}{12} l t^3 + \frac{1}{12} t l^3 \quad t \lll l$$

$$I_{G_1} = \frac{1}{12} t l^3$$

$$I_G = I_{G_1} + A x_1^2$$

$$= \frac{1}{12} t l^3 + t \times l \cdot x_1^2$$

$$I_G = A \left[\frac{l^2}{12} + x_1^2 \right]$$

For both side

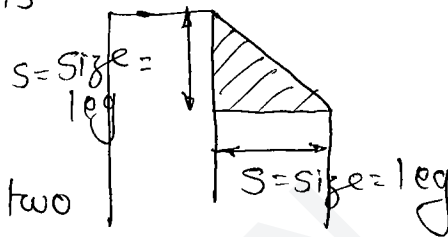
$$I_G = 2A \left[\frac{l^2}{12} + x_1^2 \right]$$

Note:- If two thickness value is given then take the lowest value box weld design.

Pg no 44

1] The permissible shear stress in a billet used is 100 MPa. The billet has equal leg length of 15 mm each. The allowable shear load on the weldment per mm is

- a] 2.25 kN b] 1.15 kN c] 1.06 kN d] 0.15 kN



Size of weld = least thickness of two plates

$$P = 0.707 S l \left(\frac{S_{sy}}{F_s} \right)$$
$$= 0.707 \times 15 \times 1 \times 100$$
$$= 1.06 \text{ kN}$$

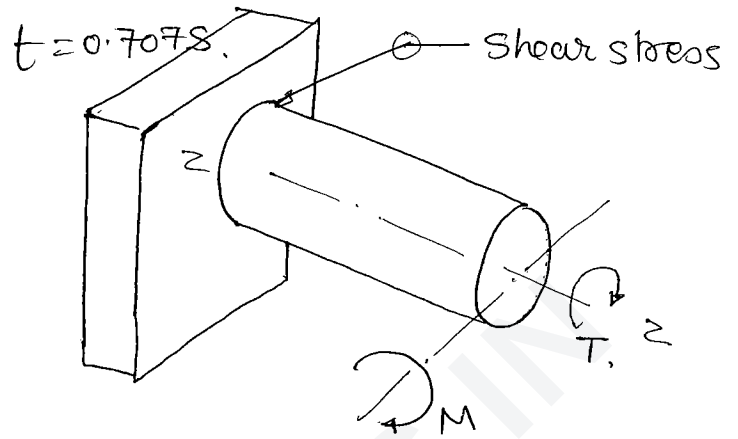
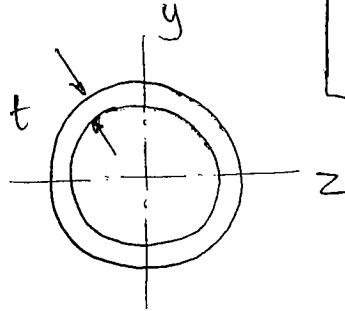
2] The allowable load per linear centimeter of the billet weld for the 10 mm size, assume shear stress intensity to be 80 MPa is

- a] 2.8 kN b] 5.6 kN c] 8 kN d] 11.3 kN

$$P = 0.707 S l \left(\frac{S_{sy}}{F_s} \right)$$
$$= 0.707 \times 10 \times 10 \times 80$$
$$= 5.6 \text{ kN}$$

Estimate the shear stress due to Torsion (T)

$$\tau = \frac{T}{J} \times r$$



$$I_z = \pi t r^3$$

$$I_y = \pi t r^3$$

$$I_x = \int_0^{2\pi} \int_0^t y^2 dA$$

$$= \int_0^{2\pi} r^2 \sin^2 \theta \times r d\theta \times t$$

$$= r^3 t \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta$$

$$= \frac{r^3 t}{2} \int_0^{2\pi} 1$$

$$= r^3 t \pi$$

$$= \pi t r^3$$

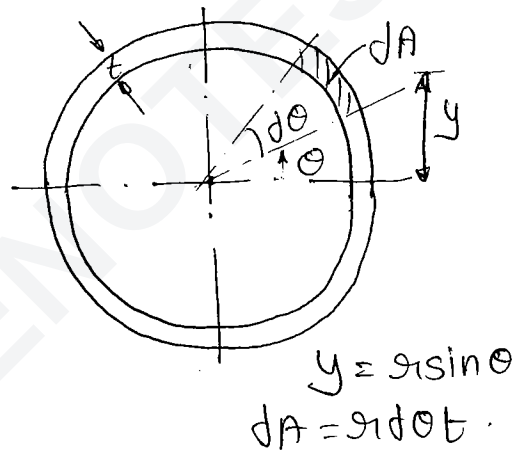
$$I_y = \pi t r^3$$

$$J = I_x = I_z + I_y = 2\pi t r^3$$

$$\tau = \frac{T}{J} \times r$$

$$= \frac{T}{2\pi t r^3} \times r = \frac{T}{2\pi t r^2}$$

$$\boxed{\tau = \frac{T}{2\pi t r^2}}$$



Bending stress (σ_b) due to B.M

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y$$
$$= \frac{M}{\pi t r^3} \times r$$

$$\boxed{\sigma_b = \frac{M}{\pi t r^2}}$$

Ratio betn the Bending and Shear stress due to M & T.

$$\frac{\sigma_b}{\tau} = \frac{M/\pi t r^2}{T/2\pi t r^2}$$

$$\frac{\sigma_b}{\tau} = \frac{M}{T/2}$$

$$\boxed{\frac{\sigma_b}{\tau} = \frac{2M}{T}}$$

Q:- The moment on the circular billet weld whose axis coincide with x axis is given by.

$M = 40 \hat{j} + 20 \hat{k}$ The ratio of bending & Shear stress is

a) 2 b) 4 c) 3 d) 1

$$\frac{\sigma_b}{\tau} = \frac{2 \times 20}{40} = 1$$

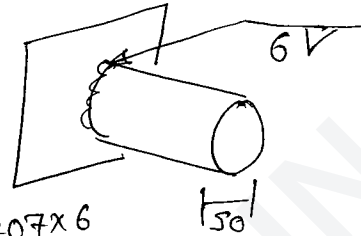
$$\frac{\sigma_b}{\tau} = \frac{2M}{T}$$

Q The torque T , that can be applied, if the permissible shear stress is 140 MPa , is

- a] 2332.5 N-m b] 1452.3 N-m c] 1252 N-m
d] None

$$\tau = \frac{T r}{2 \pi t r^2}$$

$$\begin{aligned} T &= \tau \cdot 2 \pi t r^2 = 0.707 \times 6 \\ &= 140 \times 2 \pi \times t \times 25^2 \\ &= 2332.5 \text{ N-m} \end{aligned}$$



Q25 - The billet weld weld of length 100 mm each are used to support flat 'b' on the bracket 'a'. Flat 'b' carries only tensile load. The leg length of both billet weld is 5 mm . The properties of weld metal are

$S_{ut} = 480 \text{ MPa}$, $S_{yt} = 410 \text{ MPa}$, $S_{sy} = 230 \text{ MPa}$ and reduction in area 40% . Find the load P .

- a] 230 kN b] 162.6 kN c] 480 kN d] 80.8 kN

$$t = 0.707 S = 0.707 \times 5$$

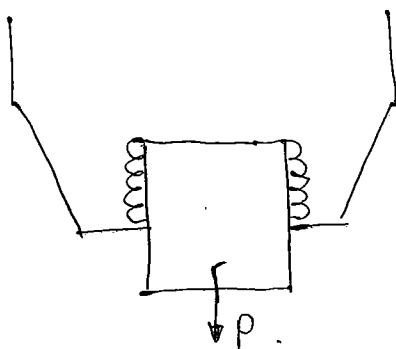
$$P = 2 \times l \times t \times S_{yt} \times 1.4 \quad P = 0.707 \times 5 \times 2 \left(\frac{S_y}{f_s} \right) \times 2$$

=

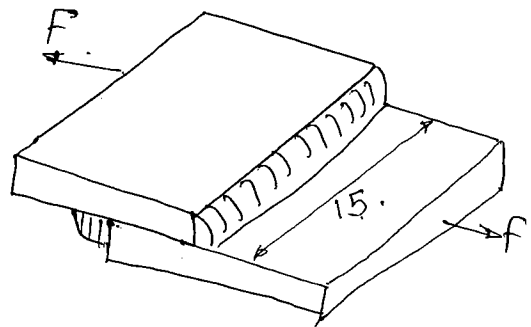
$$= 0.707 \times 5 \times 100 (230) \times 2$$

$$= 162.61 \times 10^3$$

$$= 162.61 \text{ MPa}$$



Q. A billet weld joint is subjected to transverse loading 'F' as shown in fig. Both legs of the billets are of 10 mm size and the weld depth is 30 mm. If the allowable shear stress of the weld metal is 94 MPa. Considering the minimum throat area of the weld, the maximum allowable transverse load in kN is



A] 14.44

B] 17.92

C] 19.93

D] 22.16

$$\begin{aligned}
 P &= l \times 0.707 \times s \times \tau \\
 &= 0.707 \times 10 \times 30 \times 94 \\
 P &= 19.93 \times 10^3 \text{ N} \\
 &= 19.93 \text{ kN}
 \end{aligned}$$

Q) 26 & 27 An angle plate is welded as shown in fig. The allowable stress for weld is 75 N/mm²

26.] The length of weld at top in mm is

a] 103.7 b] 96.3 c] 90 d] 110

27.] The force carried by top weld is

a] 55 kN b] 60 kN c] 65 kN d] 50 kN

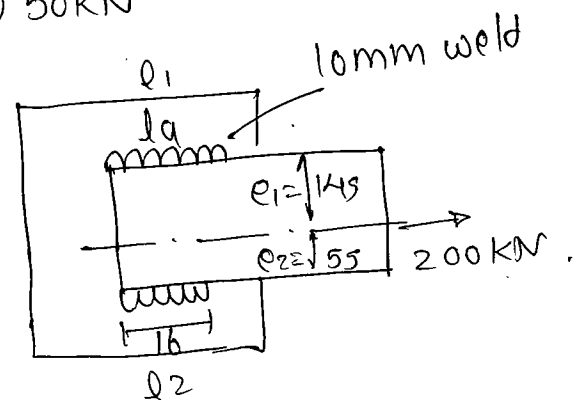
$$l_1 = \frac{e_2}{e_1 + e_2} l$$

$$l_1 = \frac{55}{155 + 45} (210)$$

$$l_1 = 103.7 \text{ mm}$$

$$l_2 = \frac{145}{200} (210)$$

$$l_2 = 150.75 \text{ mm}$$



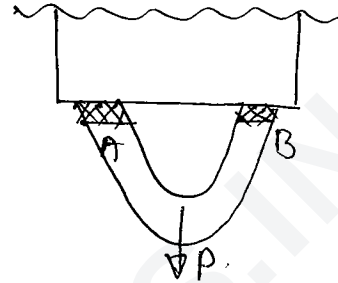
$$\begin{aligned}
 F_1 &= \frac{e_2}{e_1 + e_2} P \\
 &= \frac{55}{200} \times 200 \\
 &= 55 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 F_1 &= 0.707 S l_1 \left(\frac{s_y}{F_s} \right) \\
 55 &= 0.707 \times 10 l_1 (75) \\
 &= 10.3 \text{ m} \\
 l_1 &= 103.72
 \end{aligned}$$

48]

Q 30. In the welded joint shown in fig. If the weld B has thicker fillets than at A then the load carrying capacity P of the joint will be.

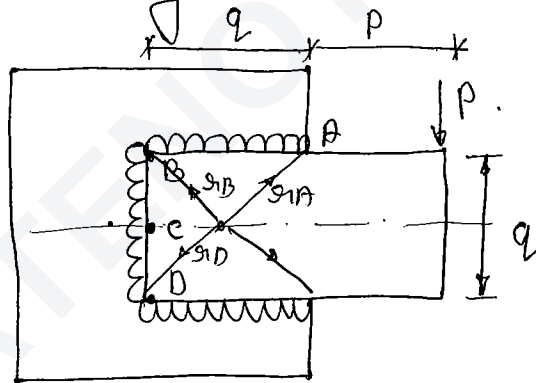
- a] Increases
- ☒ b] Decreases.
- c] Unaffected
- d] exactly got ~~double~~ double



Pg 53

Q 10] Compound fillet welding joint is loaded by P as shown. The maximum shearing stress occurs at a point

- a] A and B
- b] B & D
- ☒ c] only A
- d] only c



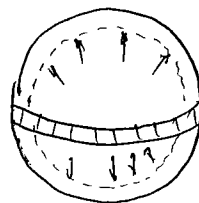
(12) A spherical pressure vessel made of 10mm steel plate hemispheres butt welded together and the tank is to be 15m diameter. The tensile strength of weld metal is 85 MPa. The allowable internal pressure inside the tank is

- ☒ a] 0.227 MPa
- b] 0.452 MPa
- c] 0.161 MPa
- d] 0.752 MPa

$$\sigma_h = \frac{PD}{4t}$$

$$85 = \frac{P \times 15 \times 10^3}{4 \times 10}$$

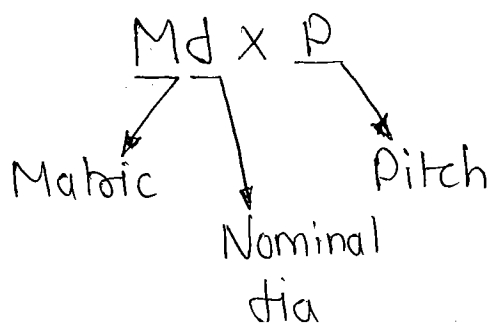
$$P = 0.226 \text{ MPa}$$



Screwed Fasteners.

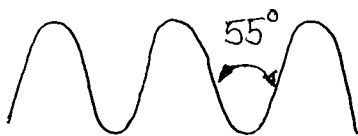
WWW.GATENOTES.IN

Designation



Screw Forms

① BSW



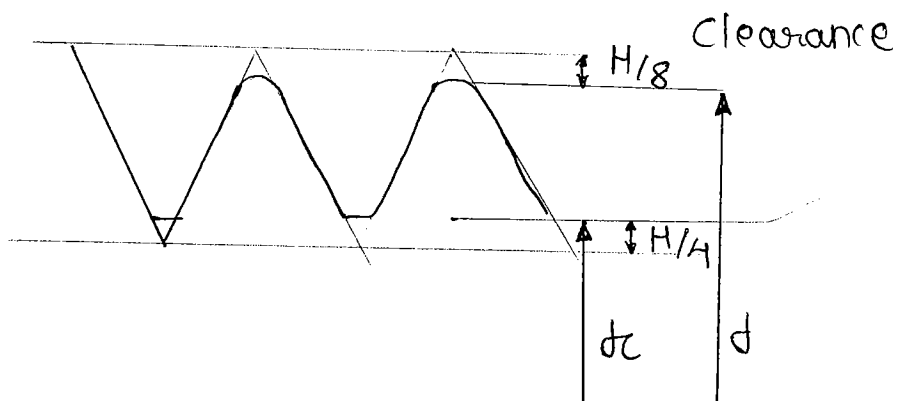
② BA



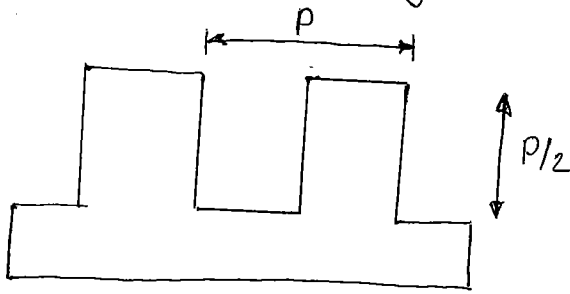
③. ANS



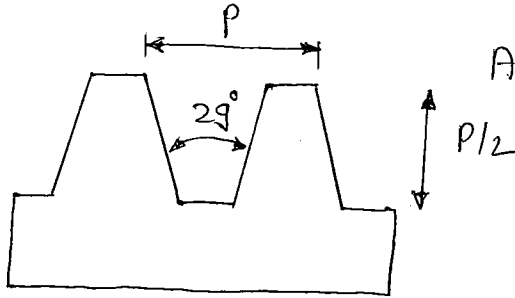
④ Isometric.



⑤ Square Thread angle = 0°

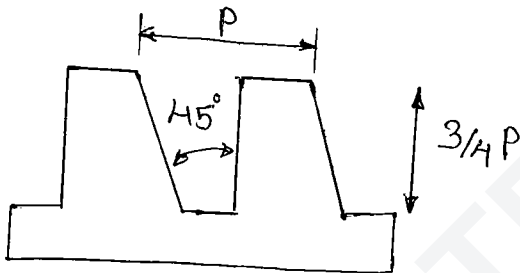


⑥



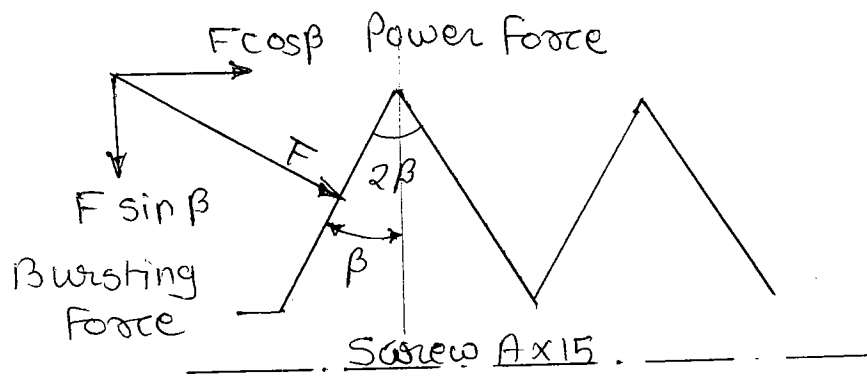
ACME Thread

⑦



Bolted Joint

Power Transmission.



Locking force/Bursting Force
 $= F \sin \beta$

$\beta \uparrow \quad F \sin \beta \uparrow \uparrow$

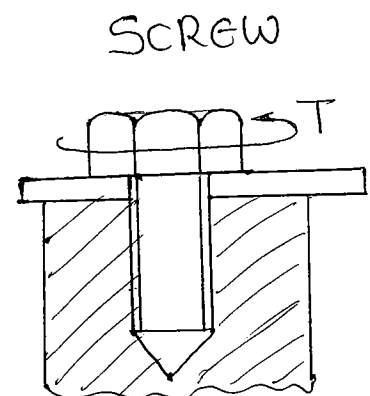
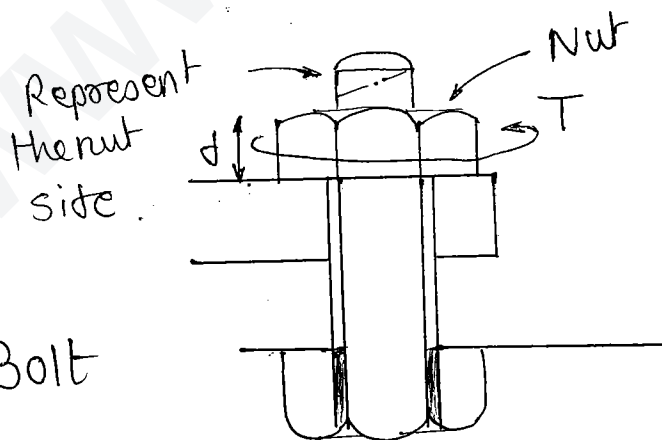
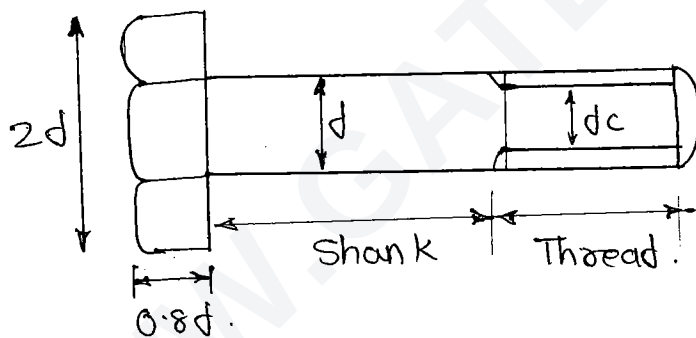
Power Force

$= F \cos \beta$

$\beta \downarrow \quad \text{Power} \uparrow$

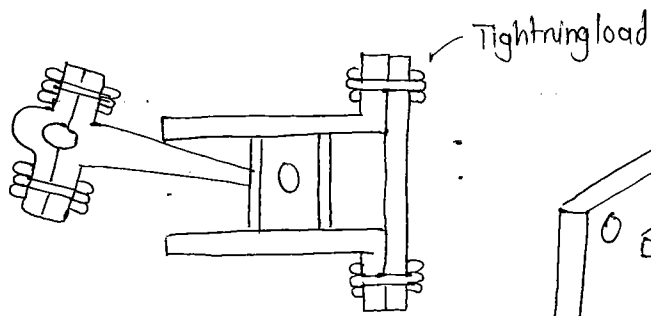
For Max power $\beta = 0$. (square thread)

Stresses in screw

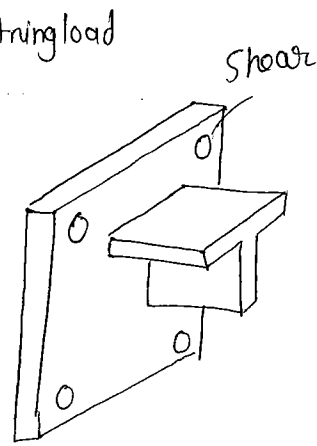


- Bolt is always having a nuts & torque given to nut
- Screw is made for tapered hole. It don't have nut and torque given to head

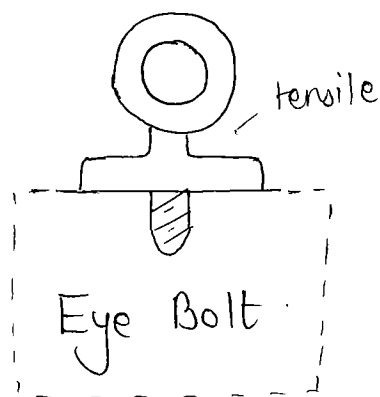
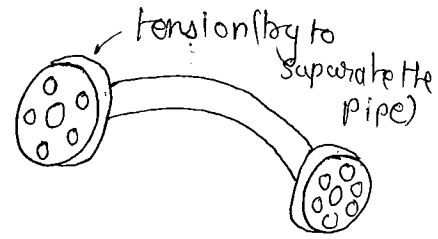
Stresses in screw



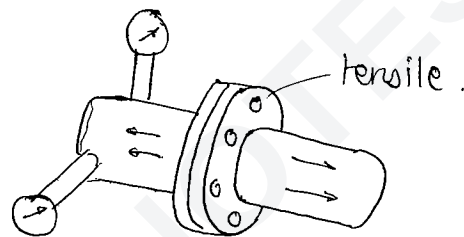
I.C Engine



Wall Brackets.



Eye Bolt



Power vessel

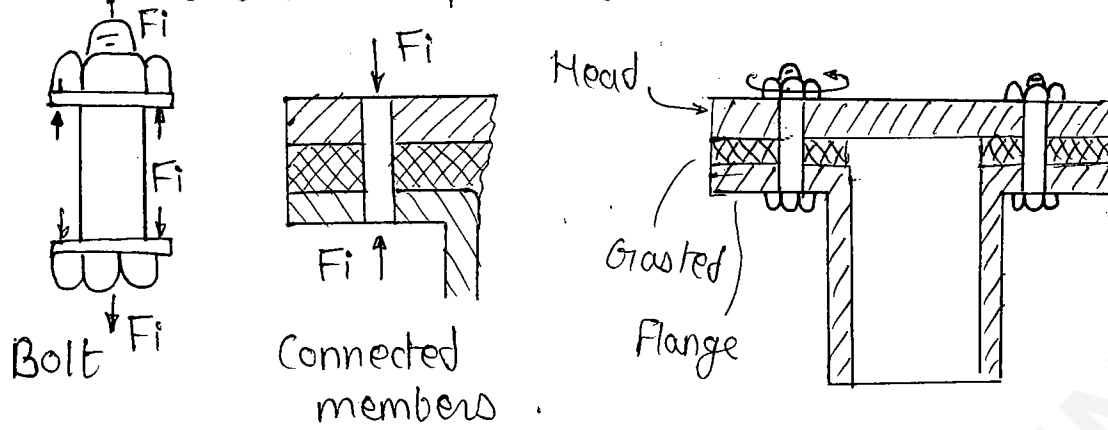
We are designing the ~~load~~ Bolt for tensile force.

- ① Preload (To tight the bolts)
- ② External force (load to which bolt is subjected)

Stresses in Screw

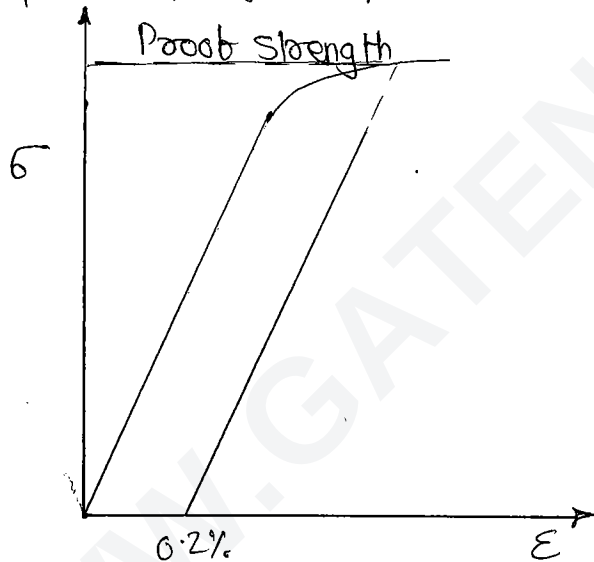
- ① Stresses due to preload / Initial tightening
- ② Stresses due to external load
- ③ Stresses due to combine preload and external load.

Stresses due to pre load.



$F_i = 2840d$ N For leak proof joint based on the proof strength (without permanent deformation)

$F_i = 1420d$ N For ordinary joint



Initial load is expressed in term of dia of bolt or in term of load applied

$$F_i = 2840d \quad \text{or} \quad F_i = 1.5 P.$$

Stresses due to Preload

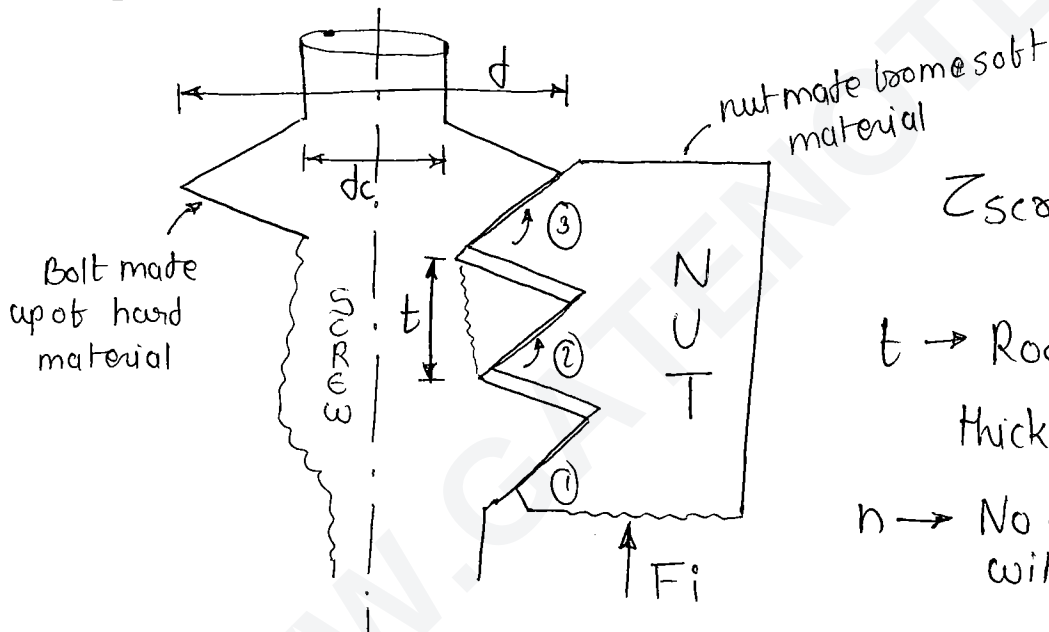
① Tensile stresses.

$$\sigma_t = \frac{F_i}{\frac{\pi}{4} d_c^2}$$

② Torsional shear stress

$$\tau = \frac{16T}{\pi d_c^3}$$

③ Shear stress across thread.



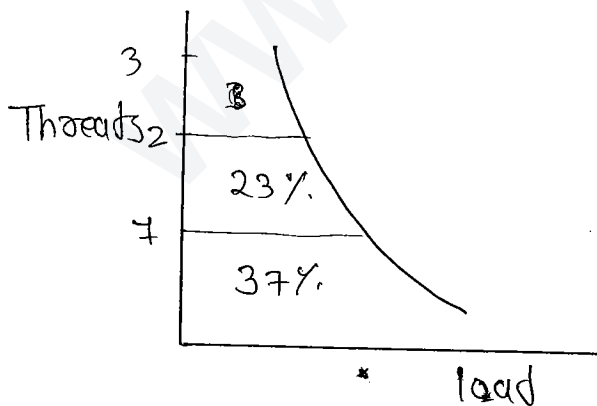
$$\tau_{\text{screw}} = \frac{F_i}{\pi d_c t x n}$$

$t \rightarrow$ Root thickness or thickness of thread

$n \rightarrow$ No of thread in contact with nut.

$$\tau_{\text{nut}} = \frac{F_i}{\pi d t x n}$$

$d \rightarrow$ external dia

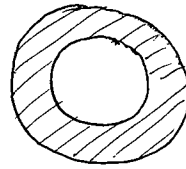


Increasing the size of nut will not give advantage because last thread take zero load

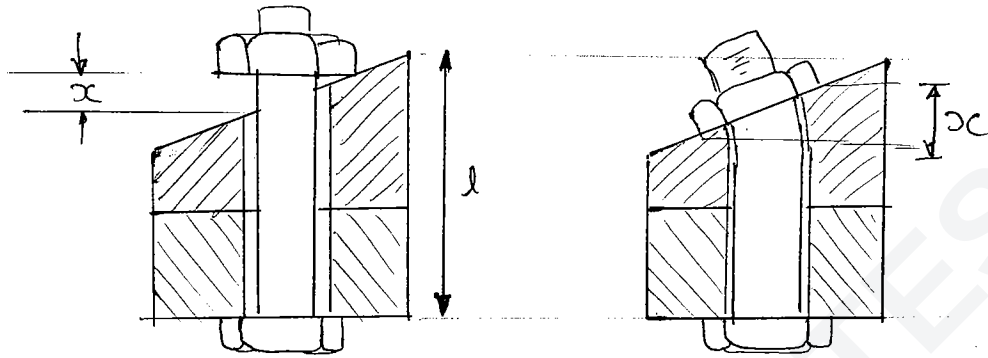
Bearing pressure - When two threads are in contact they transmit the load called as bearing pressure or crushing stress

④ Crushing stress

$$\sigma_c = \frac{F_i}{\frac{\pi}{4}(d^2 - d_c^2)n}$$



⑤ Bending stress



$$\sigma_b = \frac{x \times E}{2l}$$

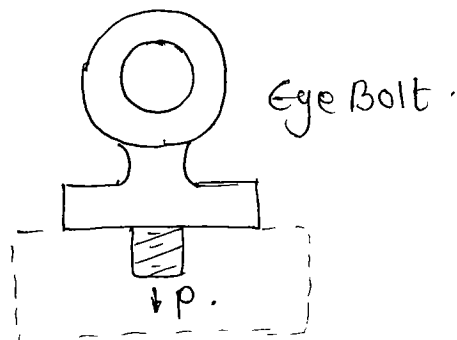
x - distance betⁿ the external corner of nut
 l - holding length
 E - Young's Modulus of elasticity.

Stresses due to external load.

① Tensile stress

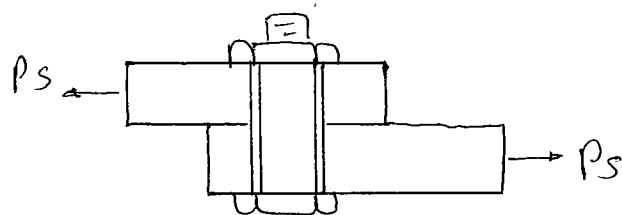
$$\sigma_t = \frac{P}{\frac{\pi}{4}d_c^2}$$

P - External load.



② Shear stress.

$$\tau = \frac{P}{\frac{\pi}{4}d^2}$$



Note - Due to shear & tension combinedly acting

① Guest's Theory.

$$\tau_{max} = \left[\sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2} \right] \frac{1}{A}$$

$$\sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

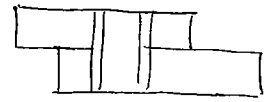
$$\frac{1}{A} \sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2}$$

② Distortion energy theory.

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2}$$

$$= \sqrt{\sigma_t^2 + 3\tau^2}$$

Note:- Take τ in both shear and tension to avoid confusions.



Stresses due to combined external and preloads

Let, p_r = pressure
 D = bore dia.

$$\text{External load } P = p_r \times \frac{\pi}{4} D^2 \cdot N$$

P = Total load.

$$P = P_b + P_m$$

P_b = External load shared by bolt

P_m = Part of external load taken by connected member.

F_b = Resultant load on bolt

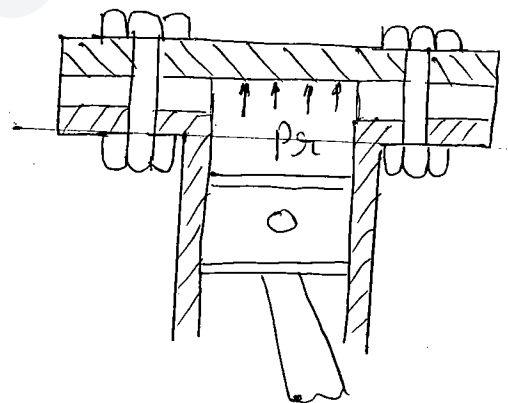
$$= P_b + F_i$$

F_m = Resultant load on connected member

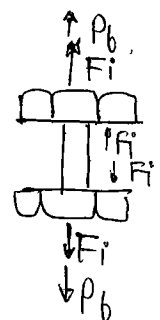
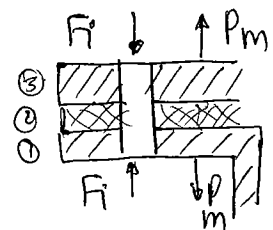
$$= P_m - F_i$$

Δb = Elongation of bolt due to F_i

Δm = Compression of connected member due to F_i



$$F_i = 1.5P$$



$$\delta_b = \frac{\text{External and Preload}}{\text{Increase in elongation of bolt due to } P_b}$$

$$\delta_m = \text{decrease in compression of conned members due to } P_m$$

For compatibility

$$\boxed{\delta_b = \delta_m}$$

$$k_b = \text{Stiffness of bolt}$$

$$= \frac{P_b}{\delta_b}$$

$$k_m = \text{Combined stiffness of connected members (Series)}$$

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$k_m = \frac{P_m}{\delta_m}$$

$$\delta_b = \delta_m$$

$$\frac{P_b}{k_b} = \frac{P_m}{k_m}$$

$$P_m = \frac{k_m}{k_b} \times P_b$$

$$\rightarrow P = P_b + P_m$$

$$P = P_b + \frac{k_m}{k_b} \times P_b$$

$$P = P_b \left[\frac{k_b + k_m}{k_b} \right]$$

$$P_b = \left[\frac{k_b}{k_b + k_m} \right] P$$

$$P_m = \left(\frac{k_m}{k_b + k_m} \right) P$$

$C = \text{stiffness factor.}$

$$= \frac{k_b}{k_b + k_m}$$

$$P_b = C P.$$

$$P_m = (1 - C) P.$$

$$\boxed{F_b = P_b + F_i}$$

$$F_m = P_m - F_i$$

$$\boxed{F_m = (1 - C) P - F_i}$$

i] Size of bolt.

$$F_b = \frac{\pi}{4} d^2 \times \frac{S_{yt}}{F.S.}$$

2] Leakage is prevented.

$F_m \leq 0$. - Resultant load must be compression

$$(1 - C) P - F_i \leq 0.$$

$$\boxed{(1 - C) P \leq F_i}$$

Pno. 34

Q.] The ratio of load required to open the bolt joint to the initial tightening load with stiffness of mating part is equal to four times stiffness of bolt is

a] 0.2 b] 0.8 c] 1.25 d] 2.5.

$$k_m = 4 k_b.$$

$$\frac{P}{F_i}$$

$$(1 - C) P - F_i = 0.$$

$$\begin{aligned} \frac{P}{F_i} &= \frac{1}{1 - C} \\ &= \frac{1}{1 - 0.2} \\ &= \frac{1}{0.8} \end{aligned}$$

$$\frac{P}{F_i} = 1.25$$

$$\begin{aligned} C &= \frac{k_b}{k_b + k_m} \\ &= \frac{k_b}{5 k_b} = \frac{1}{5} = 0.2 \end{aligned}$$

③ A bolted joint connects two members with an axial tightening force F kN. The bolt has metric threads of 4 mm pitch. The torque req. to achieve this force F is 1.4 kN-mm. What is the value of ' F '?

a] 0.35 kN b] 0.7 kN c] 1.05 kN d] 2.2 kN.

$$p = 4 \text{ mm}$$

$$T = 1.4 \text{ kN-mm}$$

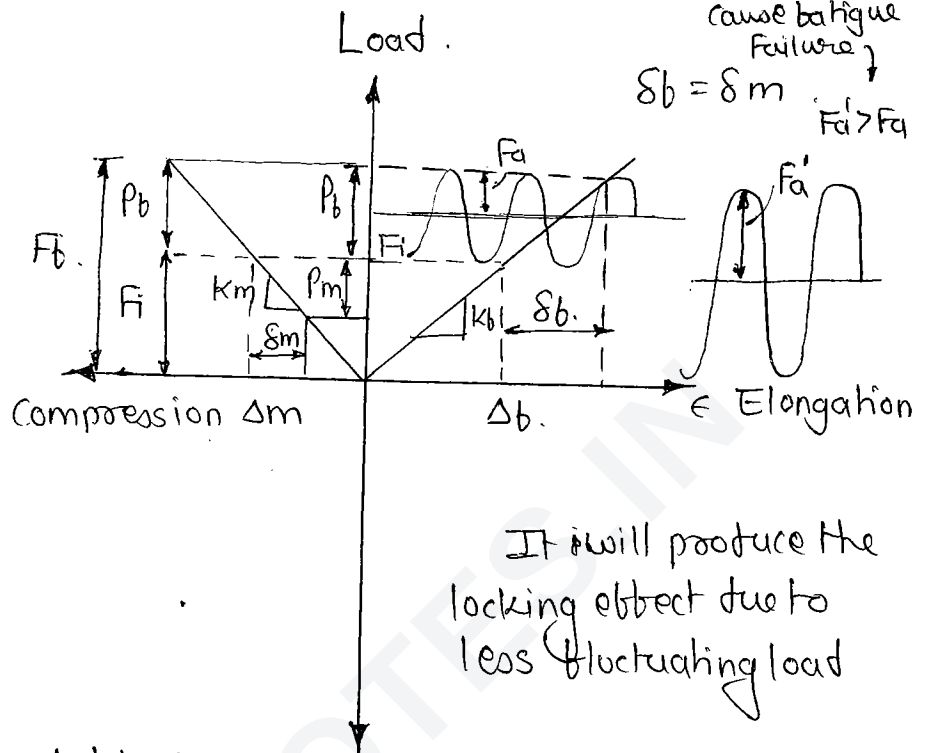
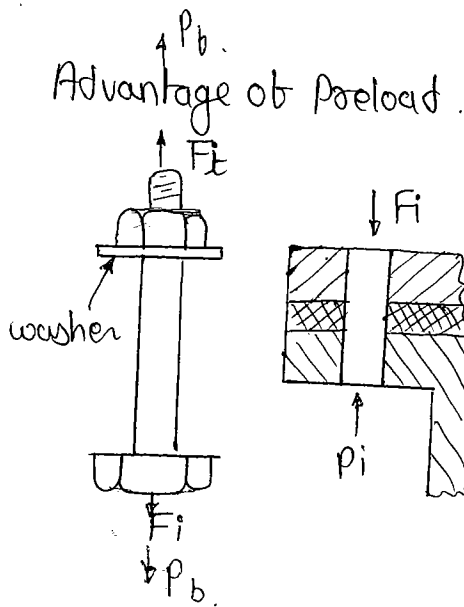
Work done

$$W = \text{Force} \times \text{distance} = \text{Torque} \times \theta$$

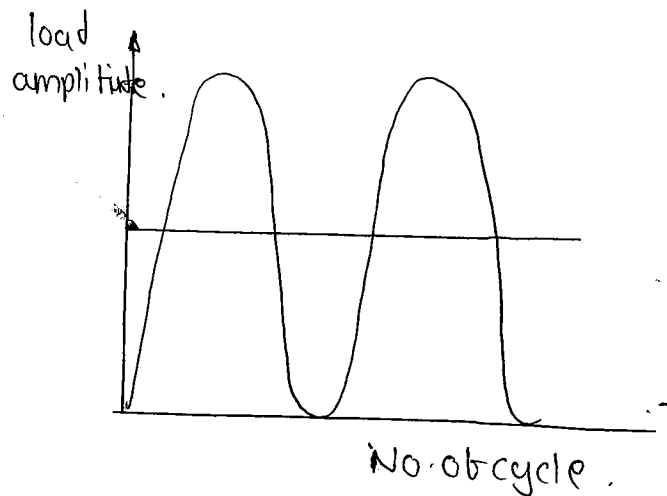
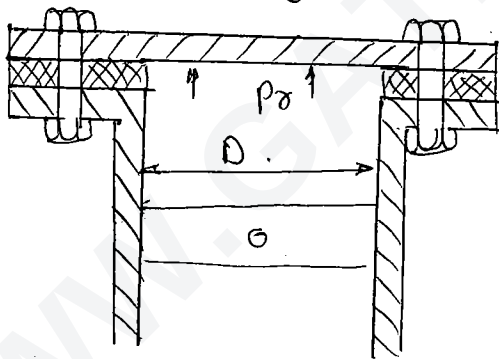
$$F \times 4 = 1.4 \text{ kN-mm} \times 2\pi$$

$$F = 2.2 \text{ kN}$$

Bolted Joints.



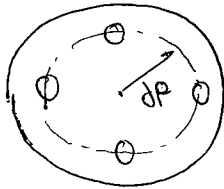
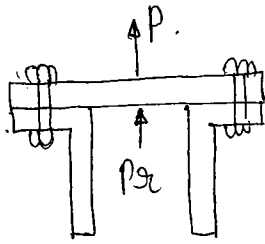
- ① To prevent leakage of fluid.
- ② Increases the fatigue strength of bolt
- ③ Increases the locking effect.



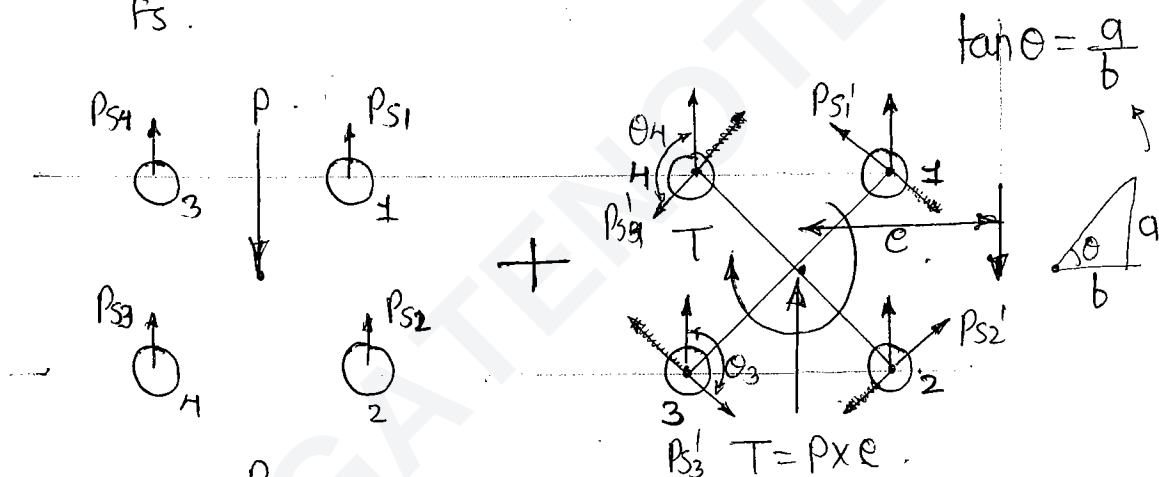
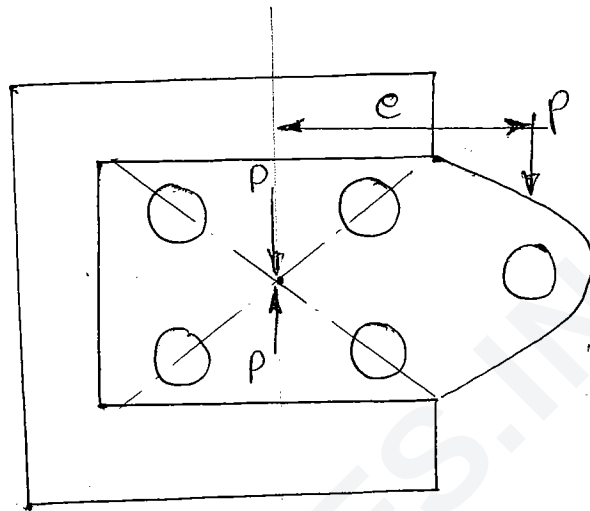
$$P = P_A \times \frac{\pi}{4} D^2$$

On the bolt the P is varying from min to maximum value.

Eccentric load on bolt Joint.



$$P = \frac{\pi}{4} d_c^2 \times n \times \frac{S_{yt}}{F_s}$$



$$P_{S1} = \frac{P}{4}$$

$$P_{S1} = P_{S2} = P_{S3} = P_{S4} = \frac{P}{4}$$

Primary shear load
OR

Direct shear load.

$$\frac{T}{J} = \frac{Z_2}{r}$$

$$Z_2 \propto r$$

Secondary shear load.

Resultant shear loads on the bolt

$$P_s = \sqrt{P_{S1}^2 + P_{S1}'^2 + 2 P_{S1} P_{S1}' \cos \theta}$$

$$P_s \angle = \angle (P_s, P_{S1}', \angle)$$

$$\theta \downarrow \cos \theta \uparrow P_s \uparrow$$

$$(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$$

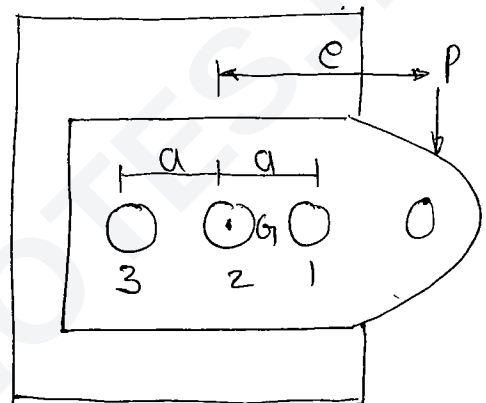
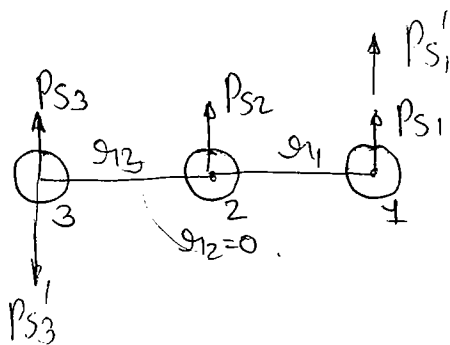
Q1.1) The weakest bolt. — only one.

2) Total shear force on bolt (2) — $P/3$.

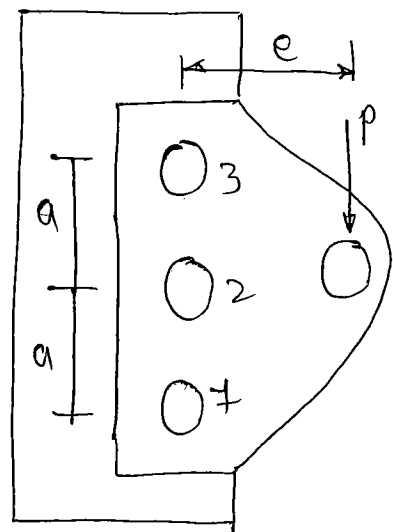
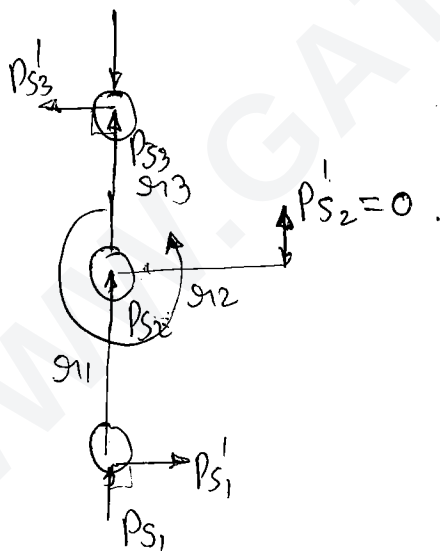
3) Least loaded bolt — 3

4) Primary shear load on bolt 3 — $P/3$

5] Secondary shear load on bolt ②. — 0.



1]



Particular

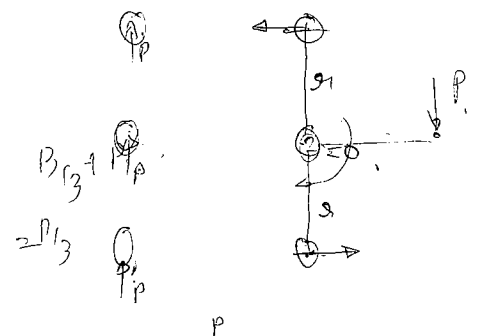
1] ~~97~~ 1 and 3

2] $P/3$.

3] 2.

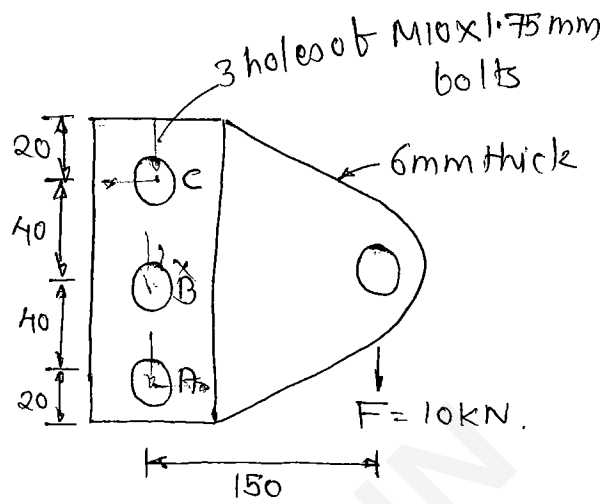
4] $P/3$

5] 0.



③ A bolt joint is shown in the fig

The maximum shear stress in MPa in bolts A and B respectively are.



$$Z = \frac{10}{\frac{\pi}{4} d^2} + \frac{16 T}{\pi d^3}$$

$$= \frac{10 \times 10^3}{3 \times \frac{\pi}{4} \times 10^2} + \frac{16 \times 10 \times 10^3 \times 150}{\pi \times 10^3}$$

$$= 127.32 \text{ KN} + 7639.43$$

$$= 63.66$$

$$Z = 3333.33 \text{ N/mm}^2$$

$$\frac{T}{J} = \frac{F_s}{R}$$

$$T = \frac{\pi}{16} F_s d^3$$

$$F_s = \frac{16 T}{\pi d^3}$$

$$= \sqrt{p^2 + q^2 + 2pq \cos \theta}$$

=

$$T = P_s A \times r_A + P_s B \times r_B + P_s C \times r_C$$

$$= K r_A \times r_A + K r_B \times r_B + K r_C \times r_C$$

$$P \times l = K (r_A^2 + r_B^2 + r_C^2)$$

$$K = \frac{P \times l}{r_A^2 + r_B^2 + r_C^2}$$

$$K = 0.468$$

$$P_s A = 0.468 \times 40$$

$$= 18.75 \text{ KN}$$

$$P_A = \frac{10}{3} = 3.33$$

$$P_B = \frac{10}{3} = 3.33$$

$$P_C = \frac{10}{3} = 3.33$$

$$P = \sqrt{(P_s A)^2 + (P_s B)^2}$$

$$= \sqrt{(3.33)^2 + (18.75)^2}$$

$$P = 19.04 \text{ KN}$$

$$Z = \frac{19.04}{\frac{\pi}{4} \times 10^2}$$

$$Z = 0.24246 \text{ KN}$$

$$Z = 242.46 \text{ N}$$

$$\tau_{SB} = \frac{P_{SB}}{\frac{\pi}{4} 10^2} = \frac{3.33 \times 10^3}{\frac{\pi}{4} (10)^2}$$

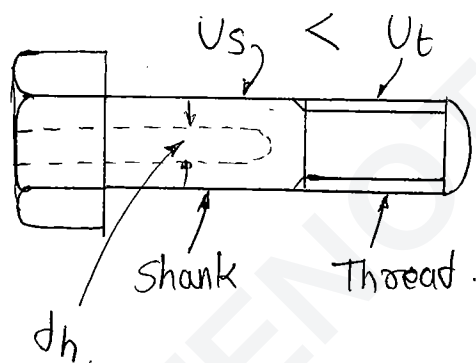
$$\tau_{SB} = 42.39 \text{ N/mm}^2$$

a] 2H2.6, H2.5 b] H2.5, 2H2.6 c] H2.5, H2.5

d] 2H2.6, 2H8.6.

—X—X—

Bolts of Uniform strengths.



$$U = \frac{\sigma^2}{2E}$$

$$\sigma_t > \sigma_s$$

$$\sigma_t = \frac{P}{\frac{\pi}{4} d_t^2}, \quad \sigma_s = \frac{P}{\frac{\pi}{4} d^2}$$

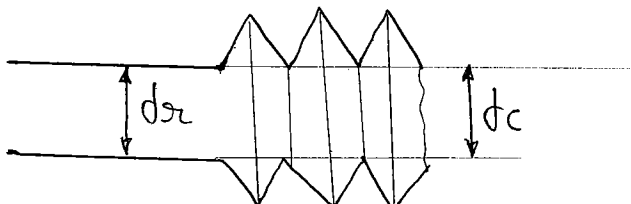
$\sigma_t \uparrow \quad \sigma_s \downarrow$

If the energy stored in shank and bolt is same the bolt is called bolt of uniform strength.

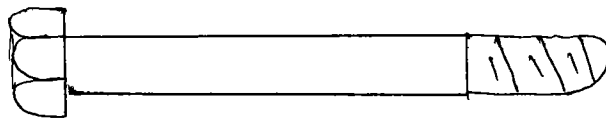
$$U_s = U_t$$

$$\frac{\pi}{4} d_h^2 = \frac{\pi}{4} d^2 - \frac{\pi}{4} d_c^2$$

$$d_h = \sqrt{d^2 - d_c^2}$$



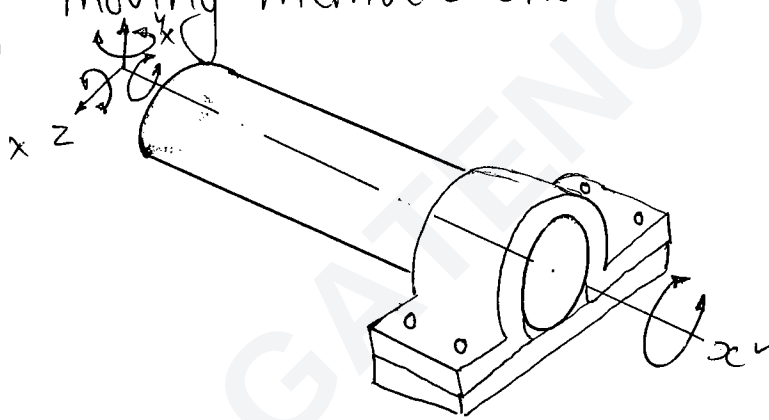
Total strain energy



$$U = \frac{\sigma^2}{2E} \times A \times l.$$

DESIGN OF BEARINGS.

A bearing is a machine element which supports a moving member and confines its motion.



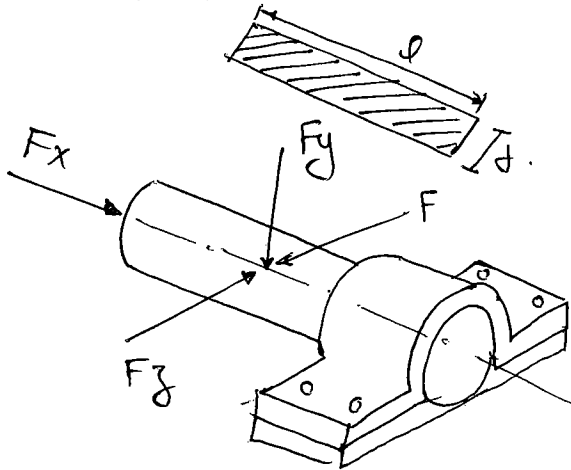
Bearings

- 1] Sliding Contact bearings
One surface slides over the other surface
- 2] Rolling contact Bearing
Point or line contact.

Sliding Contact Bearing.

1] Journal Bearing

→ Radial loads



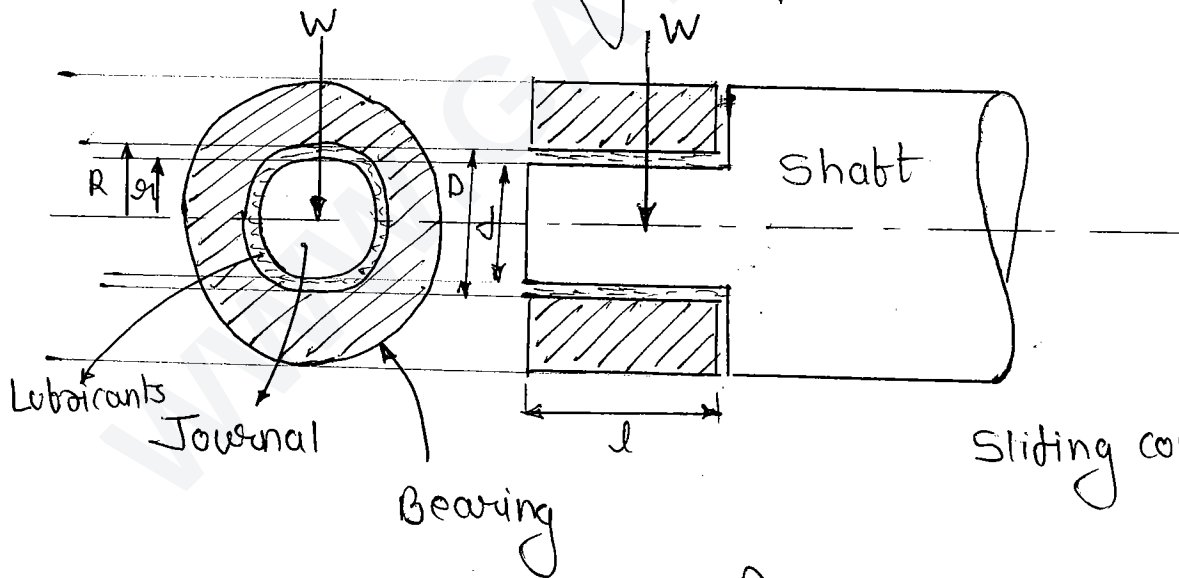
→ F_y, F_z - Radial loads

2] Collar bearings or Thrust Bearings

→ Axial loads.

→ F_x axial load

1] Journal Bearing



$d \rightarrow$ dia of Journal

$D \rightarrow$ dia of Bearing.

Pressure

$$p = \frac{\text{load}}{\text{Projected area}} \\ = \frac{W}{ld}$$

Note :- load is always $\perp$ to bearing

Clearance used

1] Radial clearance

$$C = R - r$$

2] Diametrical clearance

$$C_d = D - d$$
$$= 2C$$

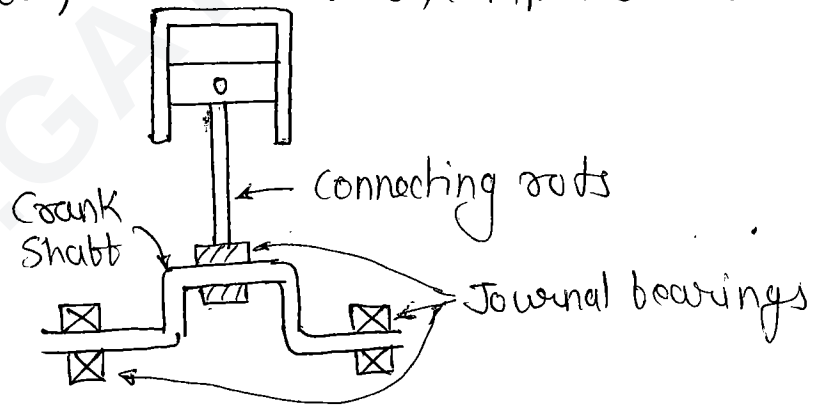
Types of Journal Bearing

① Full Journal Bearing

- Angle of contact 360°
- Supports load in all radial directions



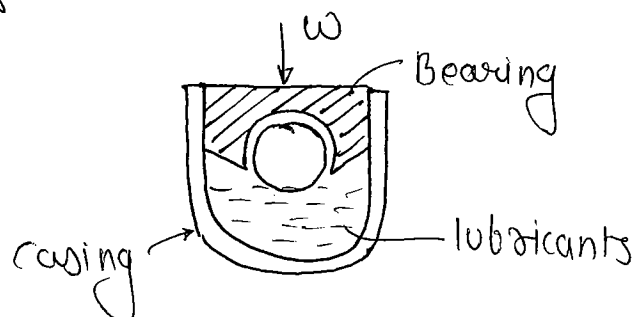
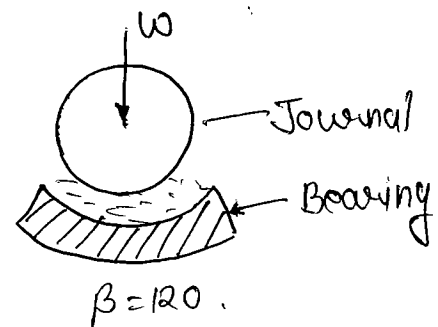
Ex:- I.C engines, steam turbine, compressors etc.



② Partial Journal Bearing

- self cooled bearing.
- load carry only one direction
- $\beta < 180^\circ$

Ex:- Rail road car axles.

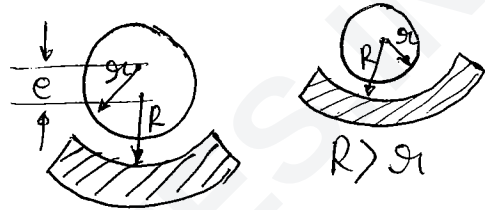


Fitted Journal bearing.

Radius of Journal is always equal to the radius (inner radius) of Bearing.

$$r_1 = R$$

Some type of eccentric alignment is taken into account.

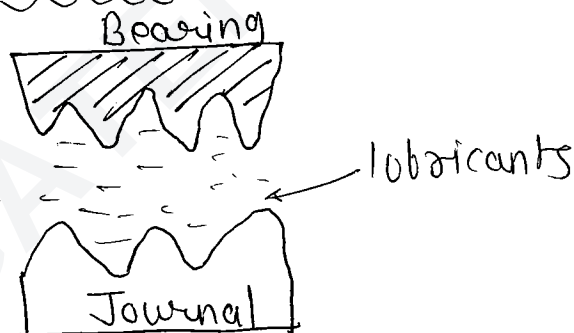


Methods of lubrications

a] Thick Film lubrication

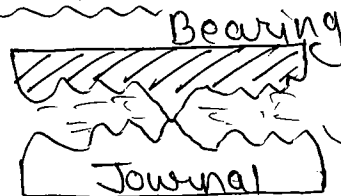
- Surface finish do not affect the performance hence independent of surface finish.

- No contact.



b] Thin Film lubrication

- Some contact is there
- High surface finish



Thin Film lubricants.

c] Solid lubrication

d] Elasto hydrodynamic lubrication

used in point contact
type of location

$$p = \frac{W}{A}$$

$A \downarrow \quad p \uparrow$

Thick Film lubrication.

- 1] Hydrodynamic lubrication
- 2] Hydrostatic lubrication

Hydrodynamic lubrication

The load supporting fluid film is created by action of journal

→ No pump is used.

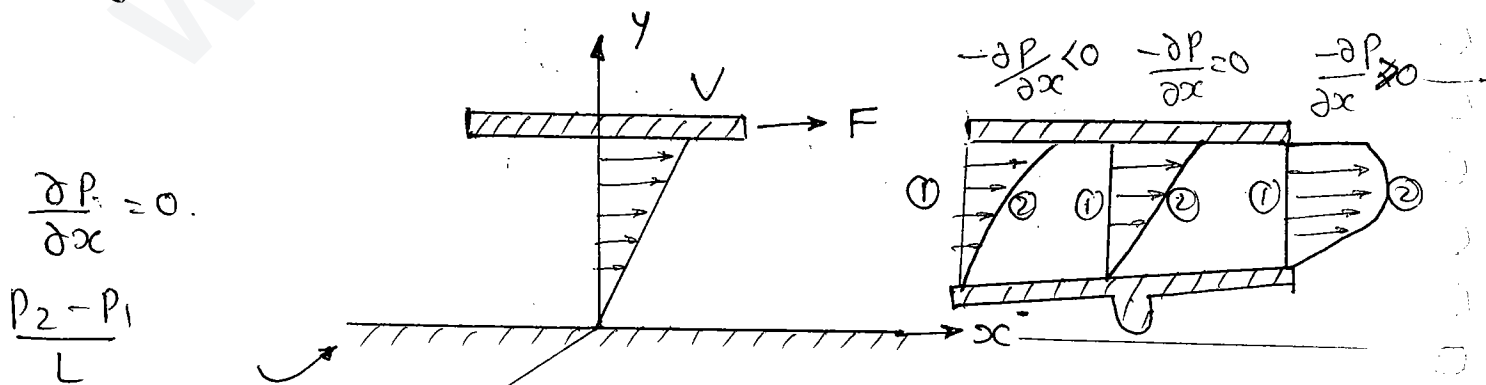
Hydrostatic Bearing

Load supported fluid film is generated by external pressure source.

→ Pump is used to generate the pressure.

Ex:- M/c bead

① working speed is low hence pressure is not generated by bearing



Pressure gradient is zero in x direction

$$\tau = \mu \frac{du}{dy}$$

μ = absolute viscosity

N-s/m²
Pa-s or
kg/m-s

Kinematic viscosity

$$\nu = \frac{\mu}{\rho} \quad \text{m}^2/\text{s} \quad \rho = \text{mass density}$$

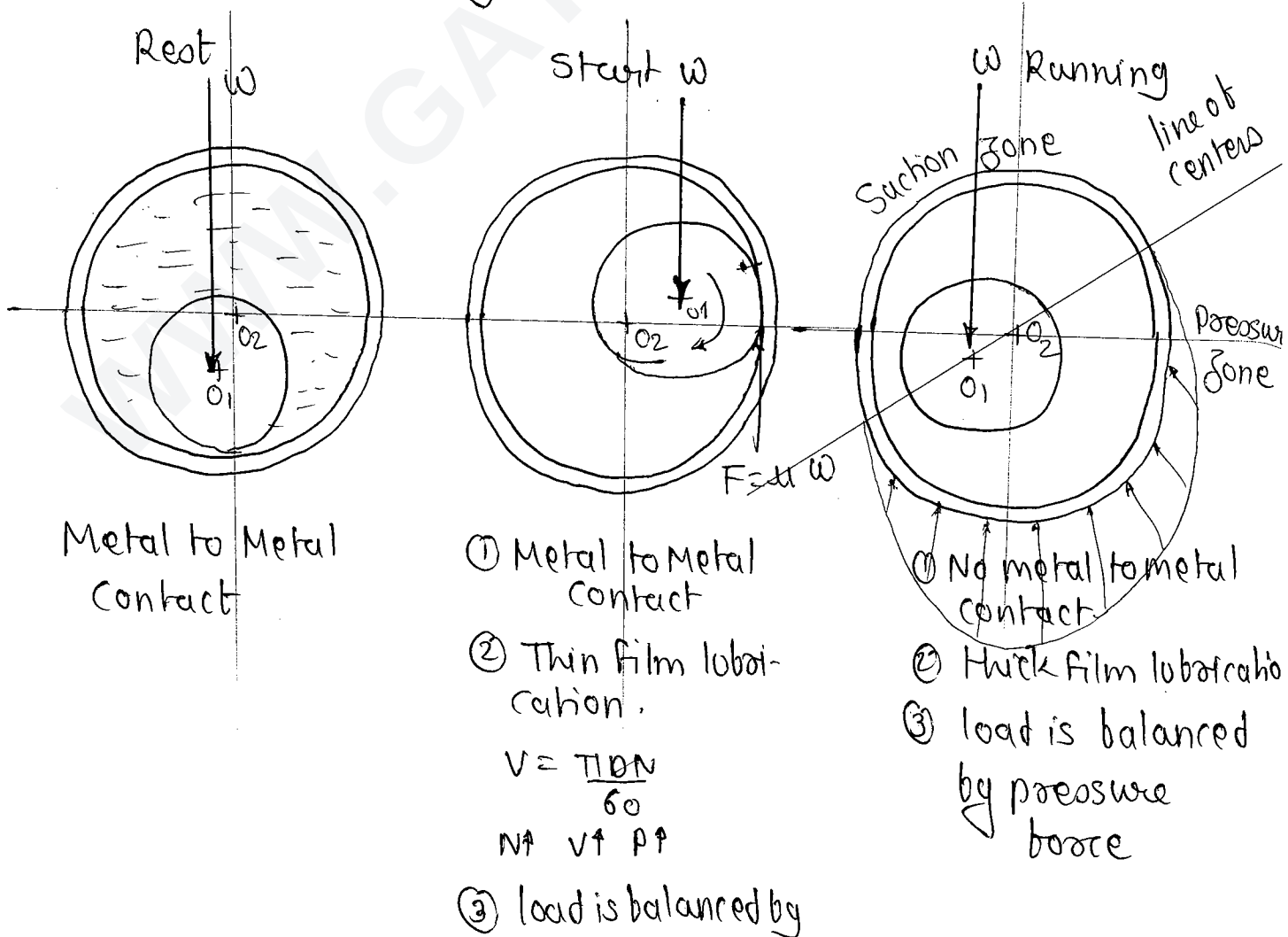
Because of plate velocity and adhesive force take fluid from low pressure to high pressure

$$-\frac{\partial p}{\partial x} < 0 \Rightarrow -\frac{\partial p}{\partial x} = 0 \Rightarrow -\frac{\partial p}{\partial x} > 0$$

$$\frac{P_1 - P_2}{L} > 0$$

→ Conditions for generating pressure

- ① Wedging action converging & diverging zones.
- ② Proper lubricant
- ③ Relative velocity



Performance of Journal Bearing.

→ Coefficient of friction is main to determine the performance

Petroff's equation 1883

Torque $T = \text{Force} \times \text{Radius}$

$$= \text{Stress} \times \text{area} \times r$$

$$= \tau \times 2\pi r l \times r$$

$$= \mu \frac{du}{dy} \times 2\pi r l \times r$$

$$T = \frac{\mu 2\pi r l N_s}{c} \times 2\pi r l \times r$$

$$T_f = F \times W \times r$$

Pressure

$$p = \frac{W}{\text{Projected area}}$$

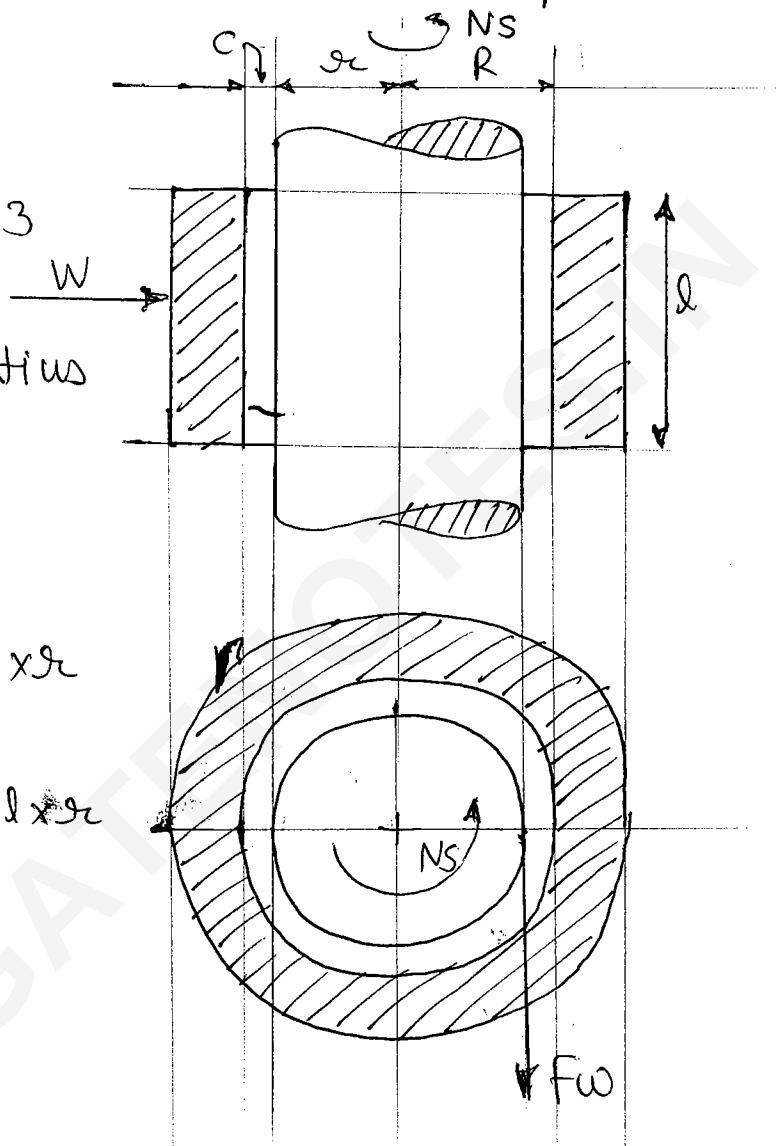
$$p = \frac{W}{2r l}$$

$$W = p \times 2r l$$

Friction torque

$$T_f = F_w \times r$$

$$T_f = f \times p \times 2r l \times r$$



Performance of journal bearing

$$F_p \times \cancel{r}^2 l = \mu \times \frac{2\pi \cancel{r} N_s}{c} \times 2\pi \cancel{r} l \times r$$

$$f = 2\pi^2 \left(\frac{\mu N_s}{p} \right) \left(\frac{r}{c} \right)$$

Assumption in this

① Lighter load is applied.

② Journal rotates concentrically

} Not fit for Bearings

$\frac{\mu N_s}{p} \Rightarrow$ Bearing characteristic Number

$\frac{c}{r} \Rightarrow$ Radial clearance ratio.

2] McKee's Equation.

→ At condition D.

Heat generated $\uparrow$

temp $\uparrow$

$\mu \downarrow$

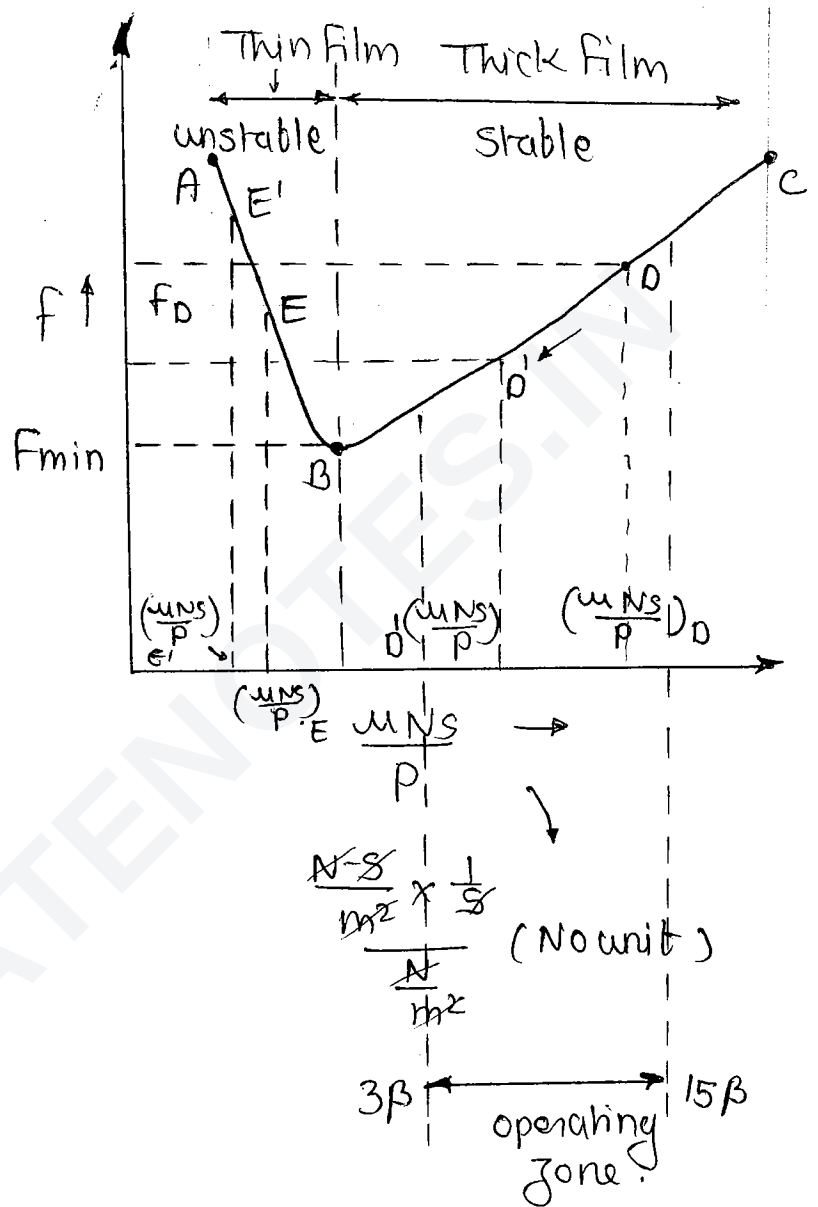
$(\frac{\mu N s}{p}) \downarrow$

→ At condition D'

$f \downarrow$ $H_g \downarrow$ $T \downarrow$ $\mu \uparrow$ $(\frac{\mu N s}{p}) \uparrow$

→ At condition E

$H_g \uparrow$ $T \uparrow$ $\mu \downarrow$ $(\frac{\mu N s}{p}) \downarrow$ $f \uparrow$



Compound Bearing

HSB + HDB

Hydrostatic + Hydrodynamic
Bearing Bearing

$\frac{\mu N s}{p}$ corresponding to minimum coefficient of friction is called bearing modulus, β .

Bearing modulus only deal with f_{min}

Operating zone

$$3\beta \leq (\frac{\mu N s}{p})_{operating} \leq 15\beta$$

$$f = \underbrace{0.326}_{0.33} \left(\frac{\mu N}{p} \right) \left(\frac{d}{c_d} \right) + K$$

N - revolution per min.

$$\mu = \text{N-s/m}^2$$

$$N = \text{rpm}$$

$$p = \text{N/m}^2$$

K - Leakage factor.

$$K = 0.002$$

$$0.75 \leq \frac{l}{d} \leq 2$$

Friction Torque

$$T = F \mu \times r$$

Power loss = Heat generated, H_g .

$$= \frac{2 \pi N T}{60}$$

$$= f W \cdot v$$

$$v = \frac{2 \pi d N}{60}$$

H_d = Heat dissipated

$$= C A (t_b - t_a)$$

C = Heat dissipation coefficient.

A = Projected area of beam

t_b = bearing temp.

t_a = Ambient air temp.

Pg no 54.

1] $\phi = 40 \text{ mm}$.

$W = 73 \text{ kN}$.

$p = 1.3 \text{ MPa}$.

$$p = \frac{\text{load}}{\text{Projected area}}.$$

$$1.3 = \frac{3 \times 10^3}{2\phi L}$$

$$2 \cdot L = \frac{3 \times 10^3}{2 \times 1.3}$$

$$L = \frac{3 \times 10^3}{40 \times 1.3}$$

$$L = 57.69.$$

$$\frac{L}{D} = \frac{57.69}{40} = 1.44. \quad \text{Ans b] .}$$

$$\frac{L}{D} > 1 \quad \text{long bearing}$$

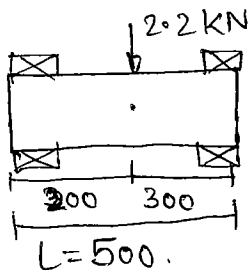
Design for less friction under thin film lubrication
Length is more, less leakage.

$$\frac{L}{D} < 1 \quad \text{Short Bearing}$$

Thick film lubrication
more leakage

$$\frac{L}{D} = 1 \quad \text{Square bearing.}$$

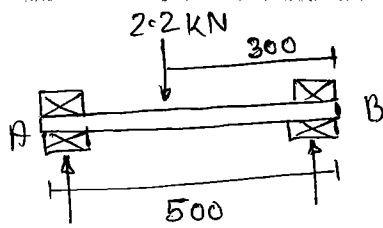
4] $D = 25 \text{ mm}$



$$25 \text{ mm}$$

$$p = \frac{\text{load}}{\text{Projected area}}.$$

$$\frac{L}{D} = 1.5$$



$$R_A + R_B = 2.2 \text{ kN}$$

$$R_A \times 500 + 2.2 \times 300 = 0$$

$$R_A = 1.32 \text{ kN}$$

$$R_B = 0.88 \text{ kN}$$

$$\frac{L}{D} = 1.5$$

$$L = 1.5D$$

$$p = \frac{\text{load}}{Ld} = \frac{1.32 \times 10^3}{1.5 \times 25 \times 25} = 1.408$$

⑤

$$d = 75 \text{ mm}$$

$$N_1 = 300 \text{ rpm}$$

$$\mu_1 = 0.06 \text{ Pa-sec}$$

$$C_1 = 0.2 \text{ mm}$$

$$p_1 = 1.4 \text{ MPa}$$

$$N_2 = 400 \text{ rpm}$$

Bearing contact number

$$\begin{aligned} &= \frac{\mu N_1}{p_1} \\ &= \frac{0.06 \times 300}{1.4} \\ &= 12.85 \end{aligned}$$

To stabilize the bearing

$$\frac{\mu_1 N_1}{p_1} = \frac{\mu_2 N_2}{p_2}$$

$$12.85 = \frac{0.06 \times 400}{p_2}$$

$$p_2 = 186 \text{ MPa}$$

⑥

$$d = 150 \text{ mm}$$

$$l = 225 \text{ mm}$$

$$W = 9 \text{ kN}$$

$$N = 1000 \text{ rpm}$$

$$c = 0.075 \text{ mm}$$

$$h_d = 90 \text{ kJ/min}$$

$$\mu = ?$$

$$h_d = cA(T_b - T_a)$$

$$h_g = FwV \text{ J/s. or J/min}$$

$$h_g = h_d$$

$$F \times 9 \text{ kN} \times \pi \times 150 \times 10^{-3} \times 1000 = 90 \text{ kJ/min}$$

$$f = 0.02$$

$$f = 0.326 \left(\frac{\mu N}{P} \right) \left(\frac{d}{C_d} \right) + K$$

$$(0.02) f = 0.326 \times \left(\frac{\mu \times 1006}{9 \times 10^3 N} \right) \times \frac{150}{2 \times 0.075} + 0.002$$

$\frac{0.150}{m} \times \frac{0.22}{m}$

$$\mu = \frac{0.0035 \text{ Pa-s}}{0.0157}$$

⑨

$$d = 50 \text{ mm}$$

$$L = 75 \text{ mm}$$

$$P = 2 \text{ MPa}$$

$$N = 500 \text{ rpm}$$

$$C = 11.6 \text{ W/m}^2\text{-K}^\circ\text{C}$$

$$A = 8 \times Ld$$

$$t_a = 28^\circ\text{C}$$

$$f = 0.0015$$

$$H_d = C A (t_b - t_a)$$

$$H_g = F \omega \times V$$

$$= 0.0015 \times P \times A \times Ld \times \pi d N$$

$$= 0.0015 \times 2 \times 75 \times 50 \times \pi \times 50 \times \frac{500}{60}$$

$$= 883.57 \times 10^3$$

$$H_d = H_g$$

$$\frac{883.57 \times 10^3}{1000} = 11.6 \times 8 \times (75 \times 10^{-3} \times 50 \times 10^{-3}) \times (t_b - 28)$$

$$H_g = 0.0015 \times 2 \times \frac{N}{\text{mm}^2} \times \frac{50 \times 75}{\text{mm}^2} \times \pi \times 50 \times 10^{-3} \times \left(\frac{500}{60} \right)$$

$$= 14.72$$

$$H_g = H_d$$

$$14.72 = C A (t_b - t_a)$$

$$14.72 = 11.6 \times 8 \times 50 \times 10^{-3} \times 75 \times 10^{-3} (t_b - 28)$$

$$t_b = 70.31^\circ\text{C}$$

$$\boxed{t_b = 343.31 \text{ K}}$$

Sommerfeld Number / Bearing number.

$$S = \left(\frac{\mu N S}{p} \right) \left(\frac{r^2}{c} \right)$$

$\downarrow$ Radial clearance

Pg 60

Pno 55]

$W = 50 \text{ kN}$

$L = 150 \text{ mm}$

$d = 150 \text{ mm}$ } square

$c = 0.15 \text{ mm}$

$b_{\min} = 0.03 \text{ mm}$

$\mu = 8 \text{ cP} = 8 \times 10^{-3}$

$S = 0.0446$

$N = ?$

$p = \frac{\text{load}}{\text{Projected area}}$

$= \frac{50 \times 10^3}{150 \times 10^{-3} \times 150 \times 10^{-3}}$

$= 6666.6 \times 10^3$

$$S = \left(\frac{\mu N S}{p} \right) \left(\frac{r^2}{c} \right)$$

$$0.0446 = \left(\frac{8 \times 10^{-3} \times N S}{6666.6 \times 10^3} \right) \times \left(\frac{150}{2 \times 0.15} \right)^2$$

$0.0446 = 3 \times 10^{-4} N S$

$N S = 49.55$

$N = N S \times 60$

$N = 2973.33 \text{ rpm}$

Attitude or Eccentricity ratio (ϵ)

h_o - min oil film thickness

$$R = e + r + h_o$$

$$R - r = e + h_o$$

$$c = e + h_o$$

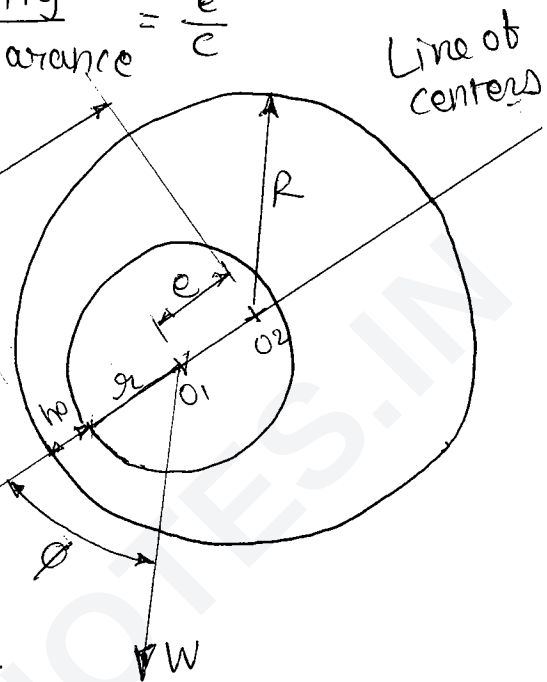
$$\text{If } e = 0, c = h_o$$

(Not possible)

ϕ = attitude angle.

- If the ϕ maintain constant then that type of bearing is stable bearing.
- If the ϕ not maintain constant oil whirl can balance fluid forces created due to vibration.

$$\begin{aligned}\epsilon &= \frac{\text{Eccentricity}}{\text{Radial clearance}} = \frac{e}{c} \\ &= \frac{c - h_o}{c} \\ &= 1 - \frac{h_o}{c}\end{aligned}$$



Oil whirl

unbalanced fluid forces

Oil whip.

Fluid forces frequency matches with frequency bearing material.

Pg no.

54

63.

$$\beta = 0.8$$

$$\frac{h_0}{c}$$

$$\beta = 1 - \frac{h_0}{c}$$

$$0.8 = 1 - \frac{h_0}{c}$$

$$\frac{h_0}{c} = 0.2$$

WWW.GATENOTES.IN

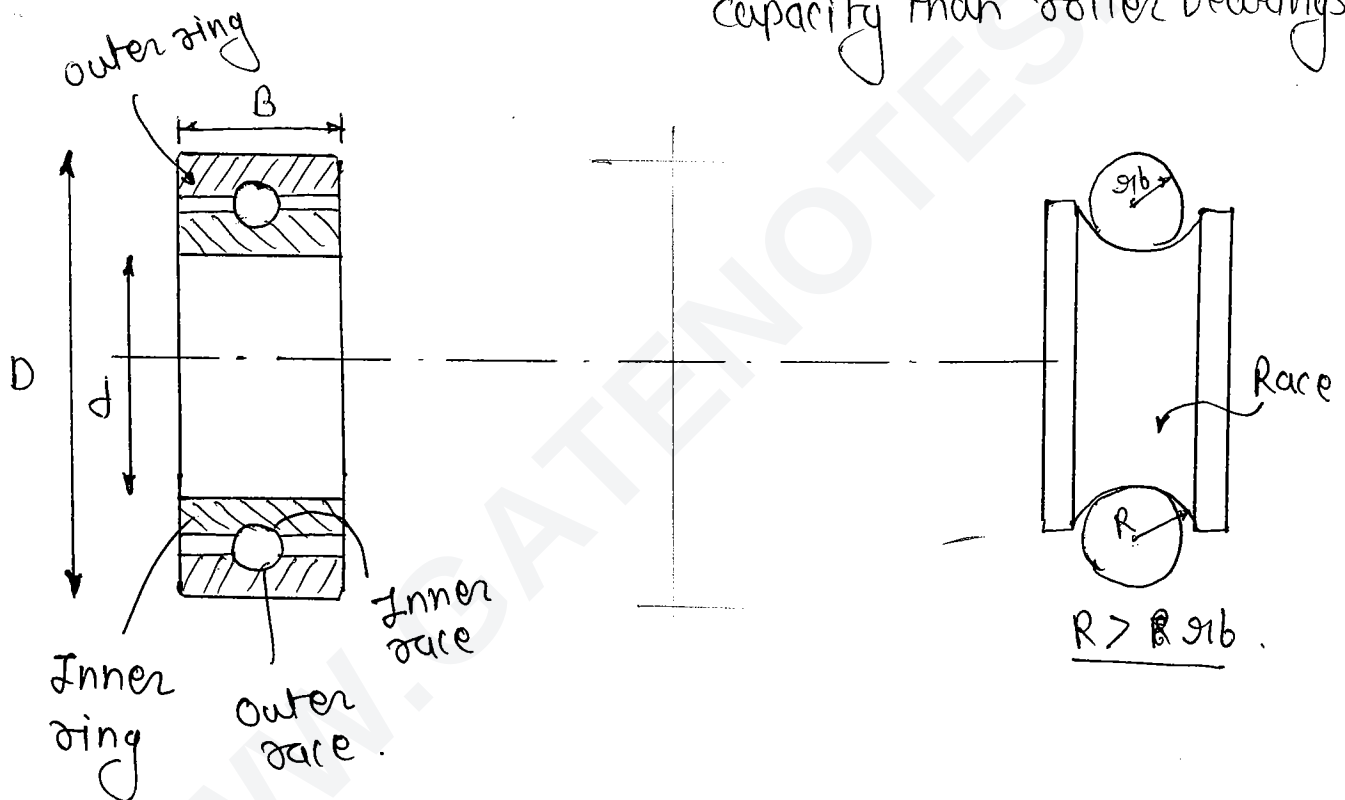
Rolling Contact Bearings.

Rolling contact Bearings (Antifriction bearings)

① Ball Bearing
→ Point Contact

② Roller Bearings
→ line contact

More load carrying capacity than roller bearings



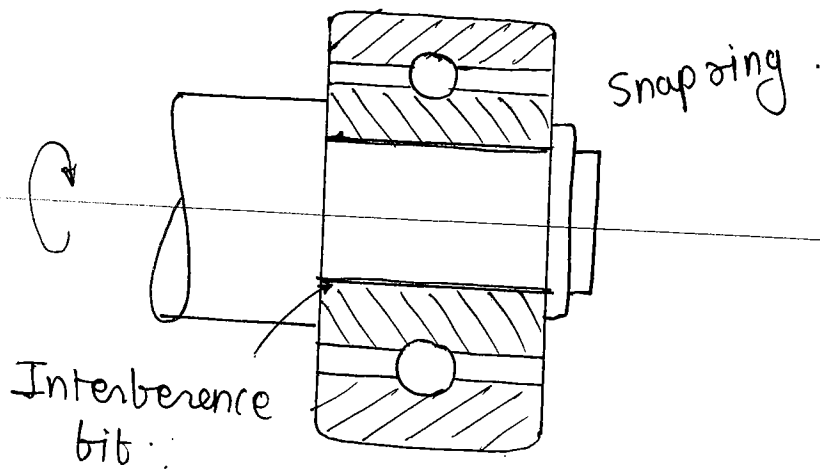
d = Inner dia or
dia of shaft

D = Outer dia

B = axial width.

High chromium steel -
outer ring

More rigidity



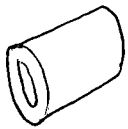
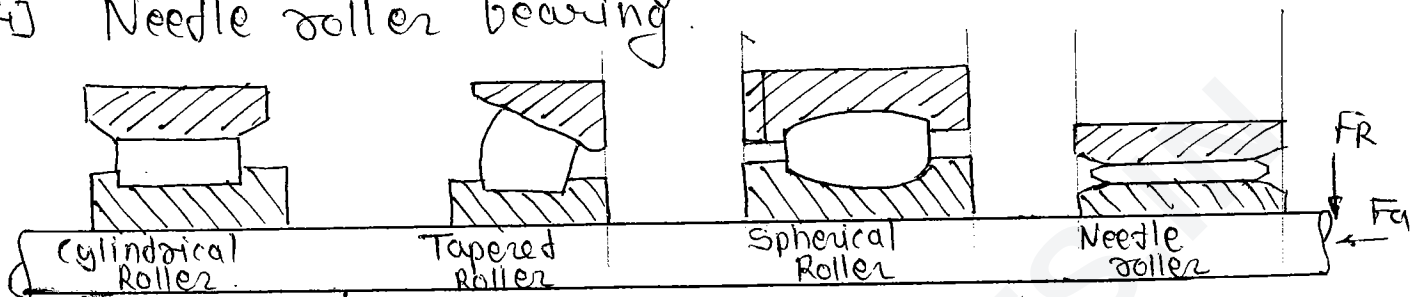
Types of ball bearings

- ① Deep groove ball bearings
- ② Angular contact ball bearings
- ③ Self aligning " "
- ④ Thrust ball bearings

Deep groove	Angular contact B.	Self align	Thrust ball bearing
1/4 deep both side Radial & axial load axial load Radial load	In this case side is use to insert balls Design to carry more axial & Radial load. $\frac{F_a}{F_{rc}} > 1$ Radial load axial load hence two bearing is used in pairs side by side	least Cobb of friction Radial load and axial load It will work in angular misalignment angular misalignment	only support axial load axial load in both direction

Roller contact bearing

- 1] Cylindrical Roller bearing
- 2] Tapered Roller bearing
- 3] Spherical Roller bearing
- 4] Needle roller bearing



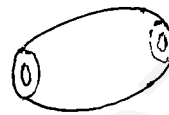
Only medium to heavy radial load



Both radial and axial load

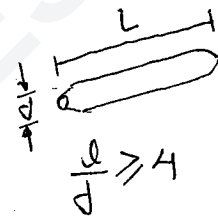
Ex- turbine, automobile axle

Magnitude is heavy.



Radial and axial loads

Angular misalignment upto $\alpha = 30^\circ$ it sustain



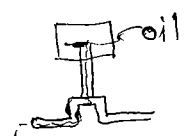
less radial space but more axial space is required
perfect alignment is req. otherwise it is bent.

Heavy radial load and No axial load can sustain

oscillating load can be sustain by this

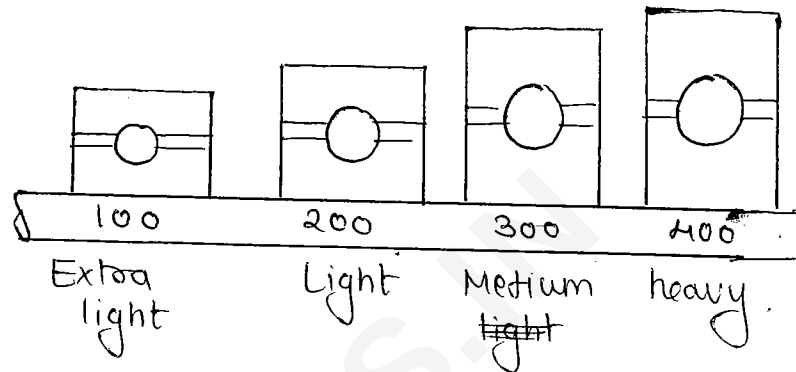
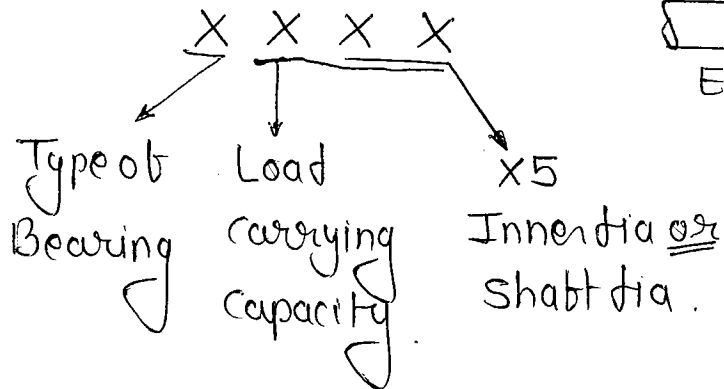
Ex:- Heavy duty Diesel engine

Piston pin bearing can be replaced by this



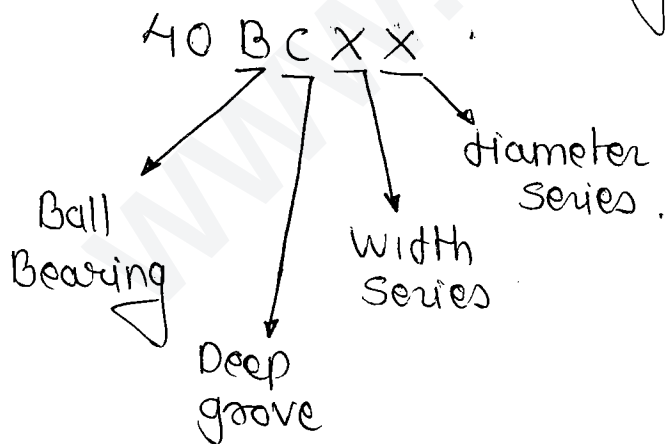
Rolling Contact bearing.

Designation
 SKF -



Ex:- 6307
 7x5 = 35 mm dia
 Medium duty

2) BIS - Antifriction Bearing manufacturing association (AFBMA)

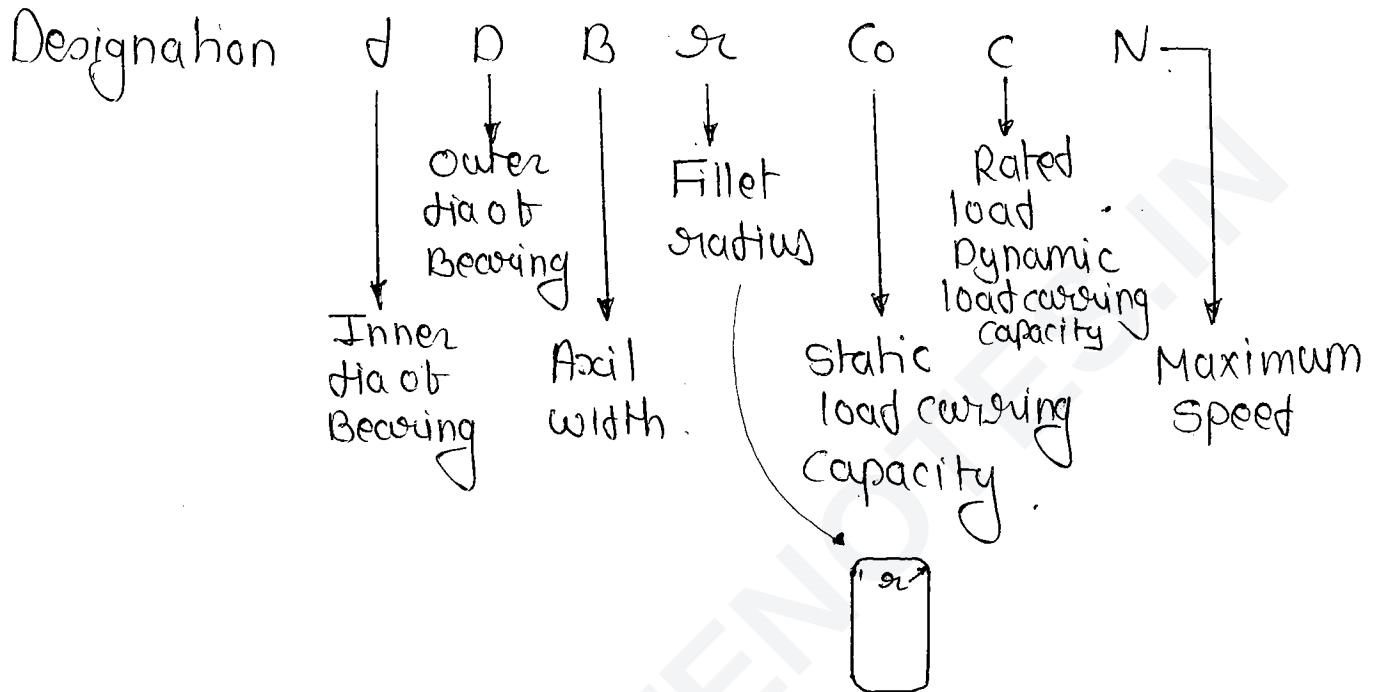


Ex:- ① 6307
 BIS 35 BC 03 (07x5)

② 6410
 BIS 50 BC 04

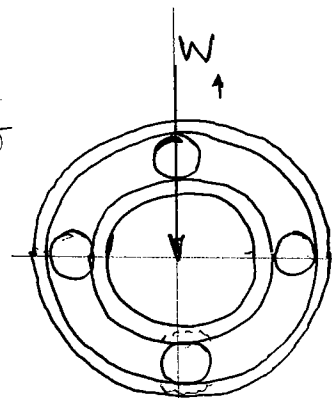
3) SKF BIS 100 series Extra

6 1 1 2	12x5 BC 01	= 60 BC 01 - light
6 2 1 2		= 60 BC 02 - light
6 3 1 2		60 BC 03 - Medium
6 4 1 2		60 BC 04 - heavy



Static load carrying capacity $[C_0]$

Load required to produce permanent deformation in the balls and races at the most heavy stress point of contact equals to $0.0001 D$, where D is the dia of ball in mm.



Bainellings Failure

$$C_0 = \frac{K D^2 Z}{5} \text{ - For ball bearing.}$$

K = Constant depends on ball dia.

D = Dia of ball

Z = No of balls.

$$C_0 = \frac{K D Z L}{5} \text{ - For roller bearing}$$

L = length of roller.

Rated Life.

Life $\rightarrow$ No. of revolution completed before the evidence of fatigue crack.

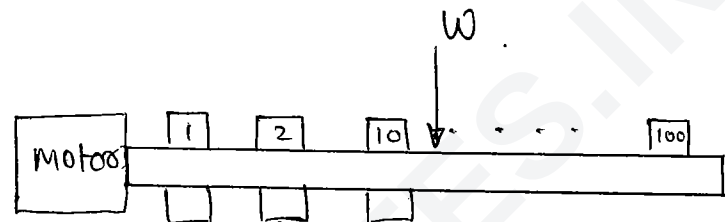
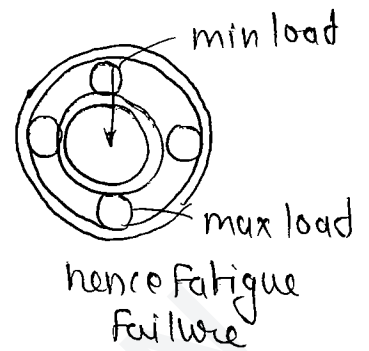
Rated life

$L_{10} \rightarrow$ 90% are in good condition

$L_{50} \rightarrow$ 50% are in good condition.

$\downarrow$
avg life.

$$L_{50} = 5 \times L_{10}$$



vibration sensing device

1 bearing is fail then 2

upto 10 bearing is fail that is called L_{10} life.

The load is applied and calculated the rev. upto 10th bearing fail and the the load value is reduce to cover 10^6 rev. and corresponding load is calculated

Rating Life (L_{10})

This is the life of group of apparently identical Ball or roller bearings, in number of revolution or hours at a given speed that 90% of bearings will complete or exceed before any indication of failure

Basic load Rating (C)

It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

$$L = \left(\frac{C}{P} \right)^K$$

L = Rated life

C = Basic load rating

P = Equivalent load

$K = 3$ For Ball bearing

$K = 10/3$ For Roller Bearing

Equivalent load

$$P = (X V F_r + Y F_a) C_s$$

X = radial load factor

V = Race Rotation Factor

= 1 For inner race rotation

= 1.2 For outer race rotation

F_r = radial load

Y = Axial load factor

F_a = Axial load

C_s = Service factor

Life in million Revolution

$$L = \frac{60 n L_H}{10^6}$$

n = Speed rpm

L_H = Life in hours

Pg no 66

9]

$$\frac{6205}{\downarrow} \rightarrow 10.8 \text{ kN,}$$

200 series

$$\frac{6305}{\downarrow} \rightarrow 16.2 \text{ kN (rest are less than 10.8 kN)}$$

300 series

Ans d]

10]. 6210 bearing.

3

$$C = 22.5 \text{ kN}$$

$$L = 27 \text{ million rev}$$

$$P = ?$$

$$L = \left(\frac{C}{P} \right)^3$$

$$27 = \left(\frac{22.5}{P} \right)^3$$

$$P = 7.5 \text{ kN}$$

$$L_{50} = 5 \times L_{10}^{\text{Rated life.}}$$

$$L = K \left(\frac{C}{P} \right)^K$$

$$P = X \sqrt{F_H} + Y F_a.$$

$$L = \frac{60 n L_H}{10^6}.$$

Pg no 67.

13] $F_H = 2.5 \text{ kN.}$

$F_a = F_t = 1.5 \text{ kN.}$

$C_s = 1.5.$

40 hrs/week for 5 years

$n = 1000 \text{ rpm}$

$X = 0.56$

$Y = 1.4$

$$t = \left(\frac{C}{P} \right)^K$$

$$= \left(\frac{1.5}{2.5} \right)^3$$

$$=$$

$$L = \frac{60 n L_H}{10^6} = 60 \times 1000 \times$$

$$n_H = 40 \times \frac{\text{hr}}{\text{week}} \times 52 \text{ week} \times 5 \text{ years}$$

$$= 10400 \text{ hr}$$

$$L = \frac{60 \times 1000 \times 10400}{10^6}$$

$$L = 624$$

$$L = \left(\frac{C}{P} \right)^K$$

$$P = (X \sqrt{F_H} + Y F_a) C_s$$

$$= (0.56 \times 1 \times 2.5 + 1.5 \times 1.4) 1.5$$

$$= 5.25 \text{ kN.}$$

$$624 = \left(\frac{C}{5.25} \right)^3$$

$$C = 44.86$$

(12)

$$C = 48.545 \text{ kN}.$$

$$LH = 6000 \text{ hrs (Avg life)}. P = (X V F_{a1} + Y F_{a2}) G$$

$$n = 500 \text{ rpm}.$$

$$L = \frac{60 \times 500 \times 6000}{10^6}$$

$$L = 180 \text{ million rev.}$$

$$L = \left(\frac{C}{P}\right)^K$$

$$\frac{180}{5} = \left(\frac{48.545}{P}\right)^3 = L_{10}.$$

$$P = 14.7 \text{ kN}.$$

Bearing Subjected to a cyclic loads.

Assume a bearing is subjected to load.

P_1 at n_1 rpm for α_1 % of time.

P_2 at n_2 rpm for α_2 % of time.

$\vdots$

P_x at n_x rpm for α_x % of time.

If the bearing is subjected to only load P_1 , then its life is, $L = \left(\frac{C}{P_1}\right)^3 \times 10^6$ revolution

Life consumed during one revolution, $\frac{1}{L_1} = \frac{P_1^3}{C^3 \times 10^6}.$

At load

P_1 Speed n_1 , rpm = α % of time

2000 4 min.

Total revolution $N_1 = \alpha_1 n_1 \text{ rev.}$

$$N_1 \text{ revolution, } \frac{N_1}{L_1} = \frac{N_1 P_1^3}{C^3 10^6}$$

Like consume for N_2 revolution = $\frac{N_2}{L_2} = \frac{N_2 P_2^3}{C^3 10^6}$

$$\frac{N_1 P_1^3}{C^3 10^6} + \frac{N_2 P_2^3}{C^3 10^6} + \dots + \frac{N_x P_x^3}{C^3 10^6} = \frac{N P_e^3}{C^3 10^6}$$

$$N = N_1 + N_2 + N_3 + \dots + N_x$$

$$P_e = \left[\frac{N_1 P_1^3 + N_2 P_2^3 + \dots + N_x P_x^3}{N_1 + N_2 + \dots + N_x} \right]^{1/3}$$

(Equivalent load)

P_e = Equivalent load or Cubic mean load.

Pg no. 67.
Q 19 and 20.

Element no.	% of element time	load (kN)	Speed (rpm)
1	30	5	900
2	40	7	1440
3	30	3	720

$$N_1 = \alpha_1 n_1 = 30 \times 900$$

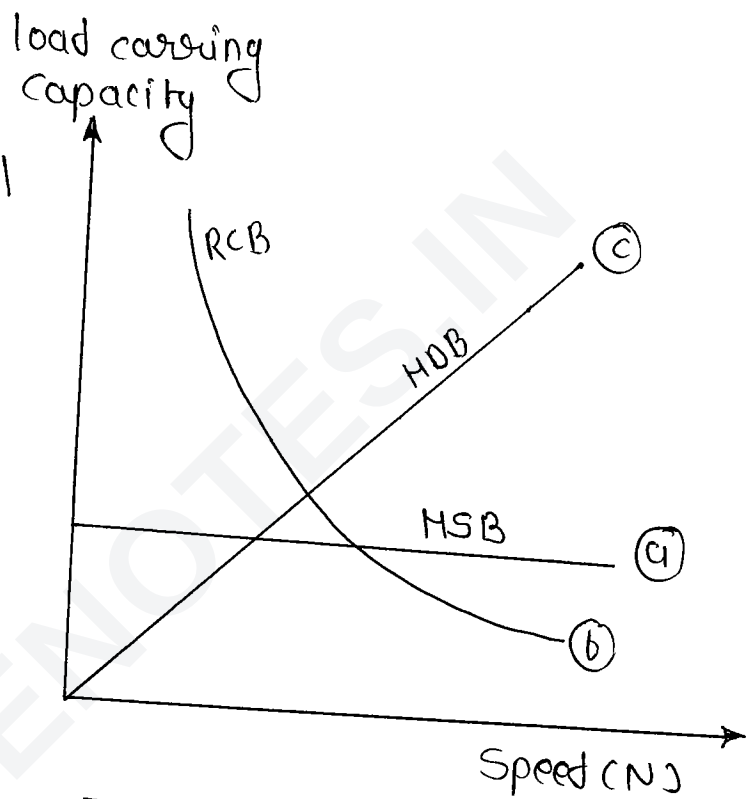
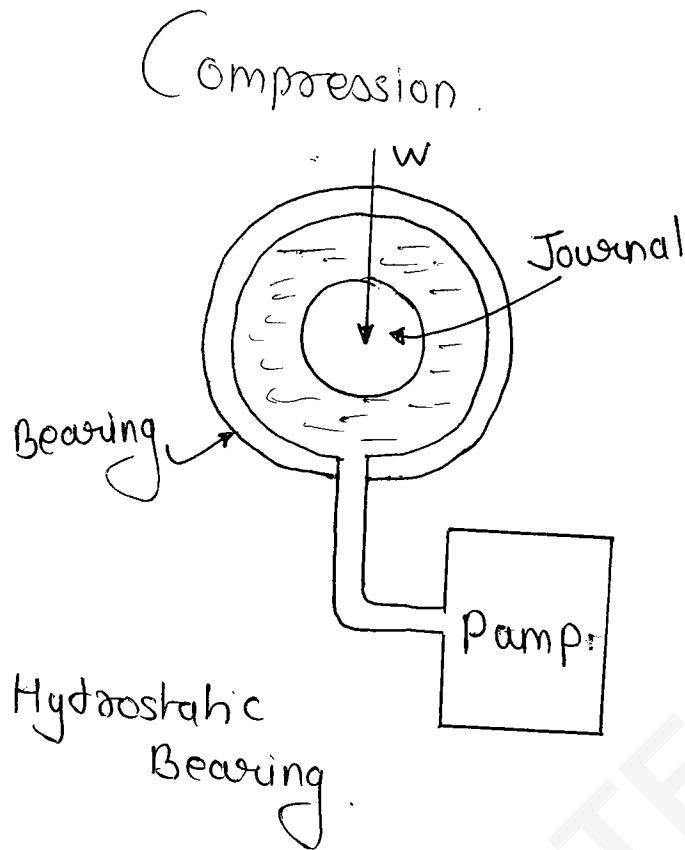
$$N_2 = \alpha_2 n_2 = 1440 \times 40$$

$$N_3 = 720 \times 30$$

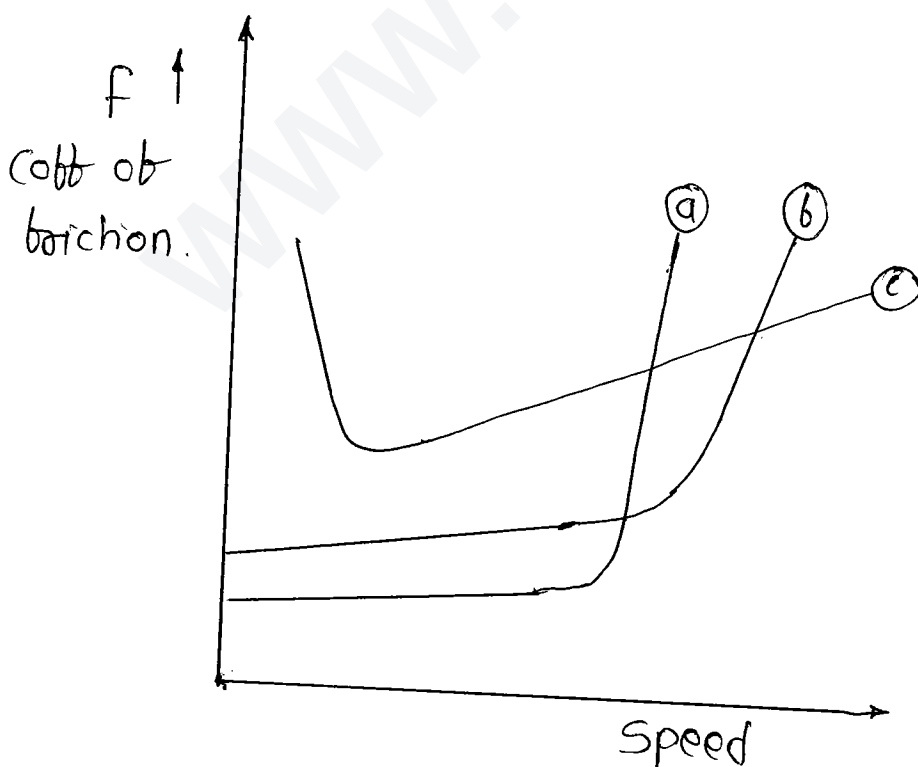
$$P_e = \left[\frac{30 \times 900 \times (5)^3 + 1440 \times 40 \times (7)^3 + 720 \times 30 \times (3)^3}{30 \times 900 + 1440 \times 40 + 720 \times 30} \right]^{1/3}$$

$$L_{10} = \left(\frac{c}{p_e} \right)^3 = \left(\frac{16.6}{6.066} \right)^3$$

$$= 20.46 \text{ million rev}$$



- a] Hydrostatic
- b] Rolling contact
- c] Hydrodynamic



- a - Ball
- b - Roller
- c - Hydrodynamic

3] Starting Torque.

Hydrodynamic bearing (HDB)

- High - because of high starting friction.

Rolling contact bearing (RCB)

- low - because of low starting friction.

4] Space.

→ Hydrodynamic B

More axial space, less radial

→ Rolling C.B

More radial and less axial.

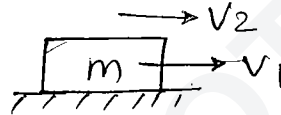
DESIGN OF BRAKES.

A brake is a device used to control the speed of a moving member by:

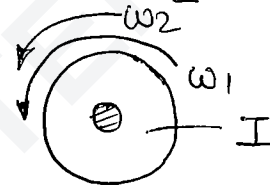
- 1] Providing suitable deceleration
- 2] Absorbing Energy.

Energies.

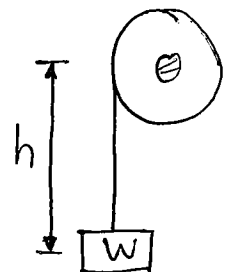
- 1] Kinetic energy of translation, $K.E_T = \frac{1}{2} m(v_1^2 - v_2^2)$



- 2] K.E of Rotation, $K.E_R = \frac{1}{2} I(\omega_1^2 - \omega_2^2)$



- 3] Potential Energy, $PE = wh$



Classifications of Mechanical Brakes.

Brakes

Radial Force is applied

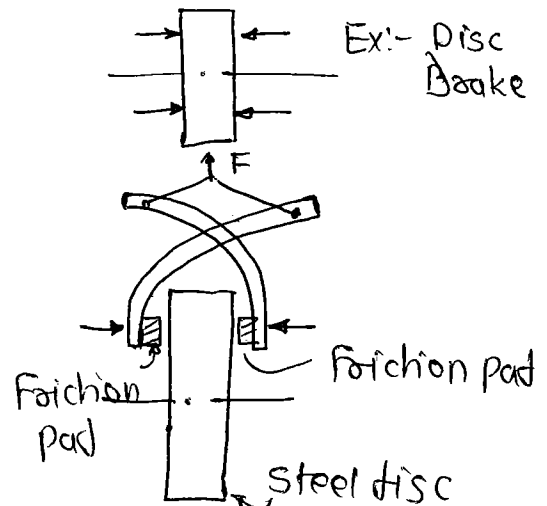
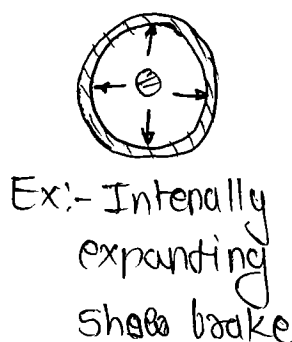
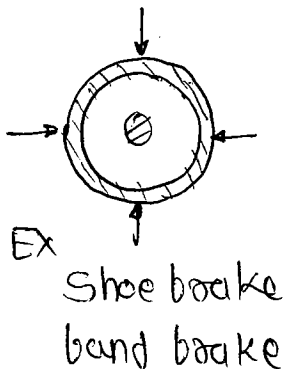
Radial

Axial

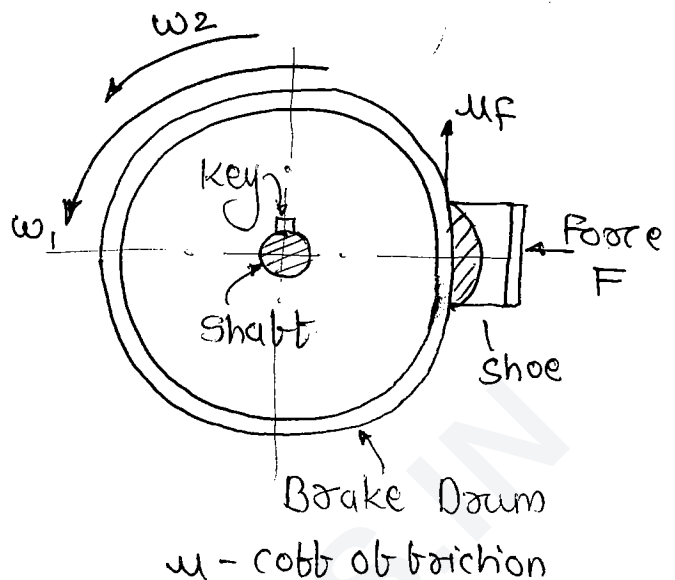
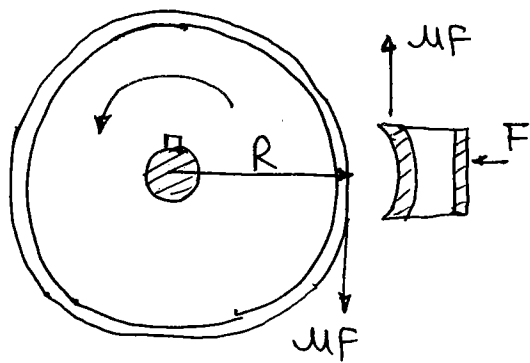
Axial Force is applied

External

Internal



Principle of Shoe Brake.



→ Friction Torque.

$$T_B = \mu F \times R.$$

→ Energy dissipated.

$$\Delta E = T_B \times \theta.$$

① $v = u + at$

$$\omega_2 = \omega_1 + \alpha t$$

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

② $v^2 = u^2 = 2as$

$$\omega_2^2 - \omega_1^2 = 2\alpha\theta$$

$$\omega_2^2 - \omega_1^2 = 2 \times \frac{\omega_2 - \omega_1}{t} \times \theta$$

$$\theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t$$

→ Heat generated.

$$H_g = T_B \times \left(\frac{\omega_1 + \omega_2}{2} \right) t$$

$$= \mu F \times R \omega \times t$$

$$= \mu F v \times t$$

$$H_g / \text{sec} = \mu F v$$

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

$$v = R\omega$$

p - pressure.

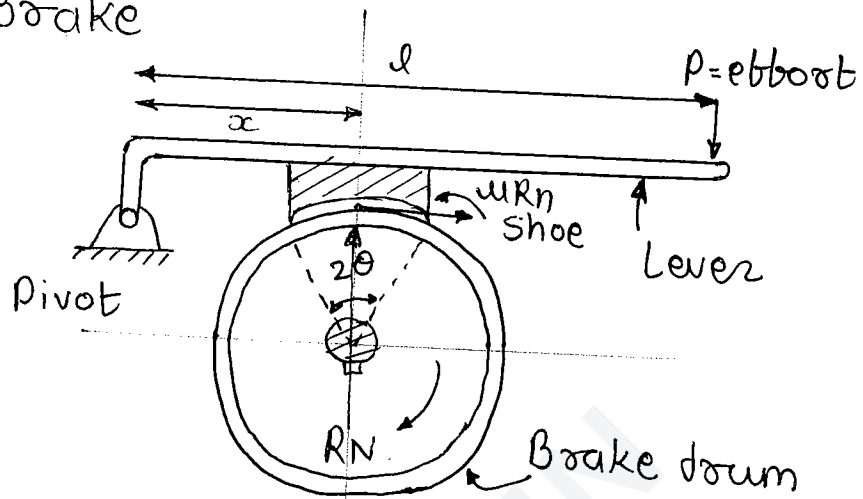
Shoe or Block Brake

Case I.

$$\sum M_{pivot} = 0$$

$$P \times l - R_N \times x = 0$$

$$P = \frac{R_N x}{l}$$



Case II

$$\sum M_{pivot} = 0$$

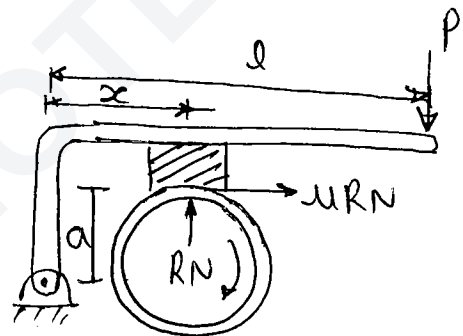
$$P \times l - R_N x + \mu R_N x a = 0$$

AM

Self energising

Movement of $P \times l$ and μR_N are in same direction.

$$P = \frac{R_N (x - \mu a)}{l}$$



If $\mu a > x$

P -ve.

$R_N < 0$ self locking Brake.

To avoid self locking

$$x \geq \mu a$$

P is +ve

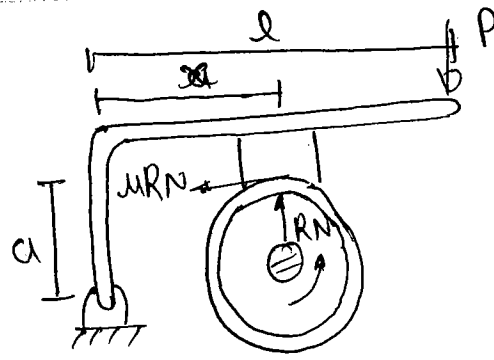
AM - Applied movement
FM - Frictional movement

AM = FM = same direction
If same magnitude
self locking.

Case III

$$P \times l - R_N x - \mu R_N x a = 0$$

$\curvearrowleft$ AM $\curvearrowright$ $\curvearrowright$ FM



$$P = \frac{R_N(x + \mu a)}{l}$$

→ Not self energising

If $\mu a \geq x$ P is +ve

$P \leq 0$ - self locking brake.

Q:- If $\mu = 0.4$ and $x = 500\text{mm}$ what should be the value of a to get self locking.

$$x = \mu a$$

$$P = \frac{R_N(x - \mu a)}{l}$$

$$a = \frac{x}{\mu} = \frac{500}{0.4}$$

$$a = 1250\text{mm}$$

Short Shoe.

$2\theta \leq 45^\circ \rightarrow$ Pressure is uniform

Long Shoe

$2\theta \geq 45^\circ \rightarrow$ Pressure is non uniform

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

μ' = Virtual coeff of friction

If $\theta = 0^\circ$.

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$$

$\sin\theta$ is very small $\sin\theta = \theta$

$$= \frac{4\mu \theta}{2\theta + 2\theta} = \mu$$

$$\boxed{\mu' = \mu}$$

If $\theta = \pi/2$.

$$\mu' = \frac{4\mu \sin \pi/2}{2 \times \frac{\pi}{2} + \sin 2 \times \frac{\pi}{2}}$$

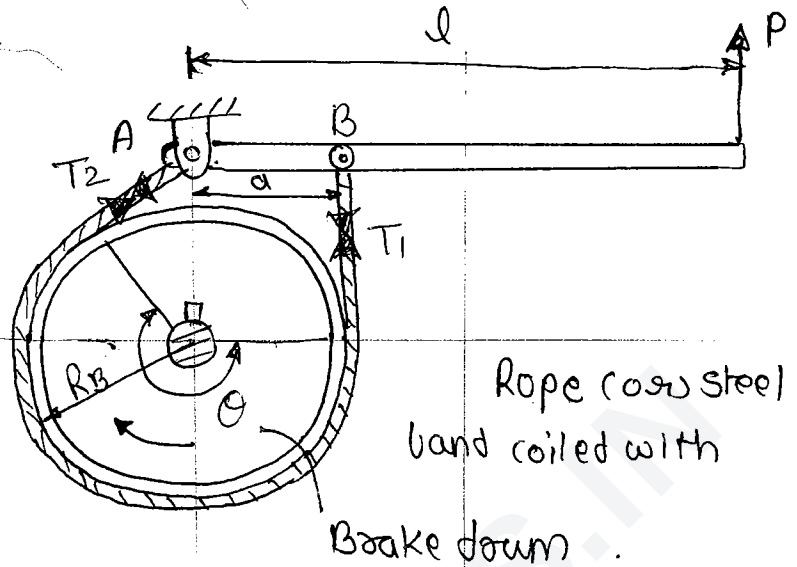
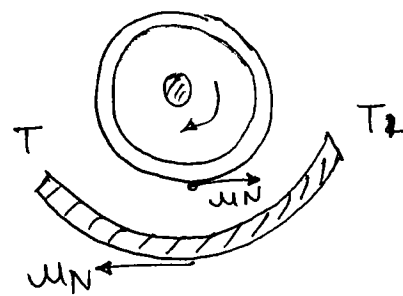
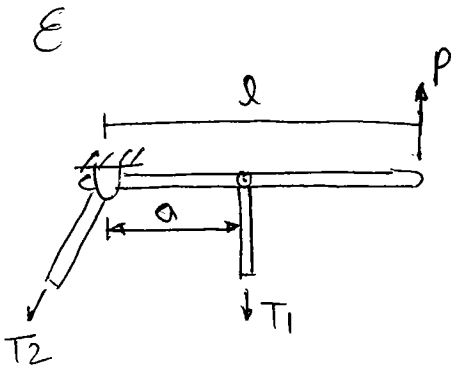
$$= \frac{4\mu}{\pi}$$

$$\mu' = 1.273\mu$$

$$\boxed{\mu' = 1.3\mu}$$

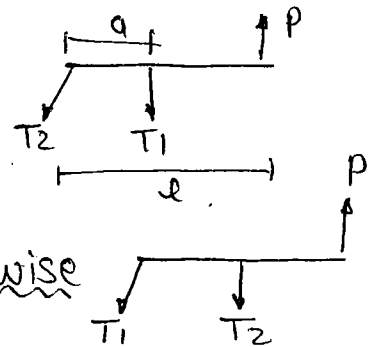
$$1 \leq \frac{\mu'}{\mu} \leq 1.3$$

Band Brake



$$\sum M_{\text{pivot}} = 0.$$

case i) $P \times l - T_1 \times a = 0$ (1) clockwise



case ii) $P \times l - T_2 \times a = 0$ (2) counterclockwise

case (i)

$$P = \frac{T_1 \times a}{l}$$

case (ii)

$$P = \frac{T_2 \times a}{l}$$

Not self locking type.

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

(Braking torque) $T_B = (T_1 - T_2) R_B$.

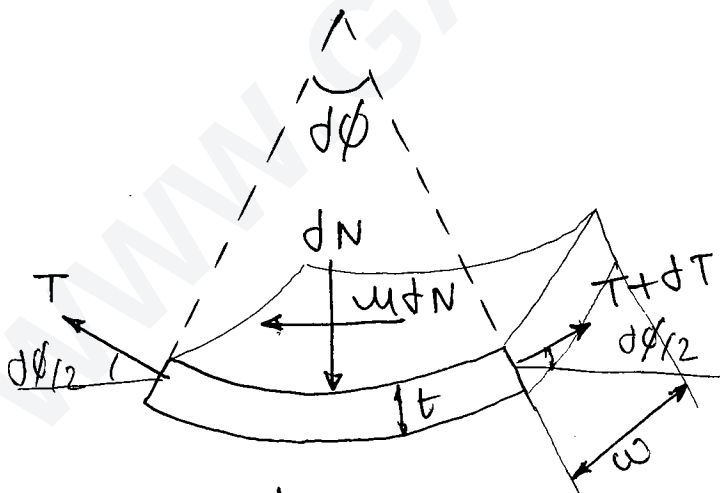
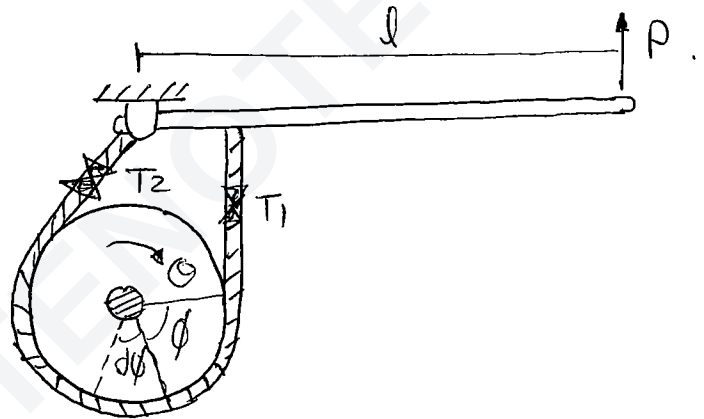
Simple band Brake.

One end of band is connected to pivot and the other end is connected to lever.

- Non self energising brake
- Not self locking brake.

In case of band brake

Pressure is not uniform hence local wear takes place in case of shoe brake, wear is uniform hence shoe brake is not used in automotive.



$$T \sin \frac{d\phi}{2} + (T + dT) \sin \frac{d\phi}{2} = dN$$

$d\phi$ is very small

$$\sin \frac{d\phi}{2} = \frac{d\phi}{2}$$

$$T \frac{d\phi}{2} + \frac{dT}{2} = dN$$

t = thickness
 w = width

$$T d\phi = dN \cdot - (1)$$

Normal force = pressure $\times$ area

$$dN = P \times (R d\phi) \omega$$

$$dN = P R d\phi \times \omega$$

$$T d\phi = P R d\phi \times \omega$$

$$P = \frac{T}{\omega R}$$

P - pressure.

Max pressure

$$P_{\max} = \frac{T_1}{\omega R}$$

Min pressure

$$P_{\min} = \frac{T_2}{\omega R}$$

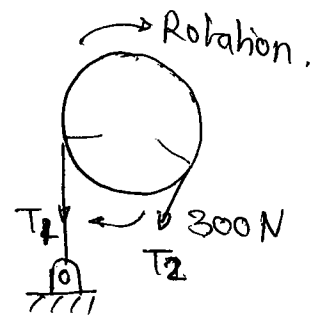
Pg no 88

Q no 28 and 29.

$$R = 500 \text{ mm}$$

$$F = 300 \text{ N}$$

$$\mu = 0.45$$



$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$T_B = 800 \text{ N}$$

$$T_B = (T_1 - T_2) R_B$$

$$800 = \frac{(300 - T_2) \times 500 \times 10^{-3}}{T_1 - 300}$$

$$T_1 = 301.6 \cdot 1900$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{1900}{300} = e^{0.45 \theta}$$

$$\ln \frac{1900}{300} = 0.45 \theta$$

$$\text{and } 4.101 = \theta$$

$$\boxed{\theta = 235.01^\circ}$$

$$P_{\max} = \frac{T_1}{\omega R} = \frac{1900}{30 \times 10^{-3} \times 500 \times 10^{-3}}$$

$$= 126.66 \times 10^3 \text{ N/m}^2$$

$$= 126.66 \text{ kN/m}^2$$

Q no. 8 and 2

Pg no 84

$$m = 1500 \text{ kg}$$

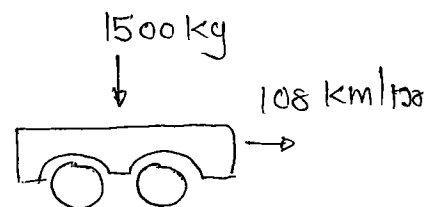
$$v = 108 \text{ km/hr}$$

$$S = 100 \text{ m}$$

$$D = 80 \text{ cm}$$

$$S = ut + \frac{1}{2} at^2$$

$$v^2 - u^2 = 2as$$



$$T_b = ?$$

$$T_b = (T_1 - T_2) R$$

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1500 \times \left(\frac{108 \times 1000}{60 \times 60} \right)^2$$

$$= 0.675 \text{ kg } \frac{\text{km}^2}{\text{s}^2}$$

$$W \cdot D = F_t \times S$$

$$0.675 = F_t \times S$$

$$10^6 \times 0.675 = F_t \times 100$$

$$F_t = 6.75 \times 10^3 \text{ N}$$

$$T_b = F_t \times d/2 = 6.75 \times 10^3 \times \frac{80 \times 10^{-2}}{2}$$

$$= 2.7 \times 10^3 \times 100 = 2.7 \text{ kN-m}$$

②

$$F_t = \mu R$$

$$6.75 \times 10^3 = \mu \times 1500 \times 9.81$$

$$\mu = 0.458$$

$$\frac{F_t}{F} = \mu \frac{R}{F}$$

Q no 3 and 4

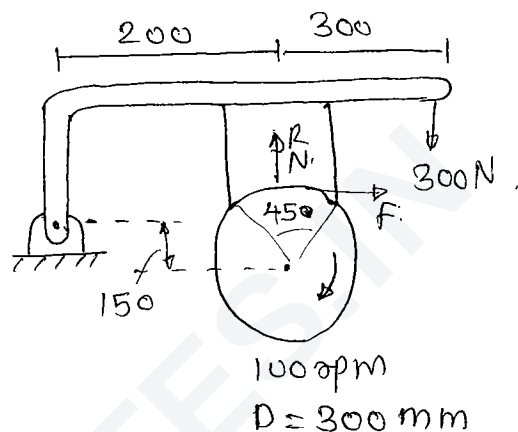
$$F \times 500 + -R_N \times 200 + \mu R_N \times 150 = 0$$

$$F \times 500 = R_N \times 200$$

$$300 \times 500 = R_N \times 200$$

$$R_N = 750 \text{ N}$$

$$T = \mu R_N \frac{D}{2} = 0.24 \times 750 \times \frac{300 \times 10^{-3}}{2} = 27 \text{ N-m}$$



④ Heat generated

$$H_g = T \times \theta$$

$$= T \times \left(\frac{\omega_1 + \omega_2}{2} \right) \times t$$

$$= 27 \left(\frac{\frac{2\pi \times 100}{60} + 0}{2} \right) \times 5$$

$$= 706.85 \text{ Joules}$$

⑤

$$\mu' = \frac{4\mu \sin \theta}{2\theta + \sin 2\theta}$$

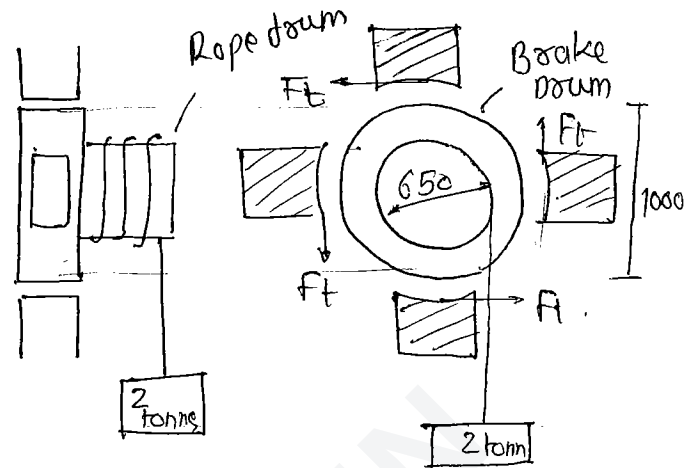
$$= \frac{4 \times 0.3 \sin 45}{2 \times 45 + \sin 90}$$

do not put 45
put in rad.

$$\mu' = 0.330$$

Q 6 to 9

$D_r = 650 \text{ mm}$
 $D_b = 1000 \text{ mm}$
 $\text{load} = 2 \text{ tonnes}$
 $V = 9 \text{ kmph}$
 $S = 2.75 \text{ m}$
 $\mu = 0.2 \text{ (Four shoe)}$
 $p = 0.3 \text{ MPa}$



⑥ $u = \frac{9 \times 1000}{60 \times 60} = 2.5 \text{ m/s}$

$v^2 - u^2 = 2as$

$0 - (2.5)^2 = 2 \times a \times 2.75$

$a = -1.13 \text{ m/s}^2$

$F_{\text{total}} = W - (ma)$

$= 2000 \times 9.81 - (2000 \times -1.136)$

$= 21892.72 \text{ N}$

$= 21.89 \text{ kN}$

$F = \frac{\mu N}{4}$

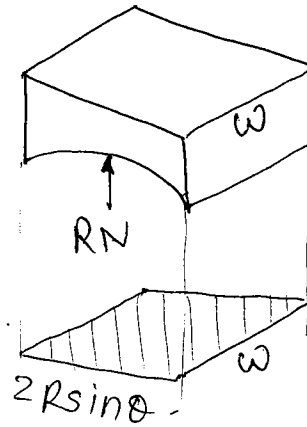
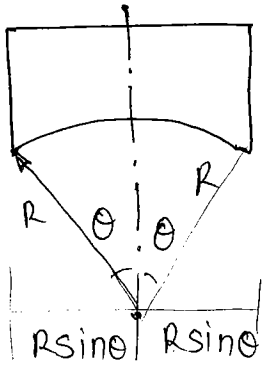
⑦ $\text{torque} = F_{\text{total}} \times \frac{D_r}{2} = F_t \times \frac{D_b}{2} \times 4$

$F_{\text{total}} \times \frac{650}{2} \times 10^{-3} = F_t \times \frac{1000}{2} \times 10^{-3} \times 4$

$21.89 \times \frac{650 \times 10^{-3}}{2} = F_t \times \frac{1000 \times 10^{-3} \times 4}{2}$

$F_t = 3.55 \text{ kN}$

⑧



Pressure,

$$p = \frac{\text{Normal reaction}}{\text{Projected area}}$$

$$= \frac{RN}{w \times 2R \sin \theta}$$

$$F_t = \mu RN$$

$$\begin{aligned} 0.3 \times 10^6 &= \frac{F_t}{\mu} \\ &= \frac{17.175 \times 10^3}{w \times 2 \times \frac{1000 \times 10^{-3}}{2} \times \sin 45/2} \end{aligned}$$

$$\frac{N}{m^2} = \frac{N \times 10^6}{1000 \times 1000}$$

$$w = 0.1546 \text{ m}$$

$$w = 154.6 \text{ mm}$$

⑨

Heat generated

$$\begin{aligned} H_g &= T_b \times \theta \\ &= F_t \times \frac{D}{2} \times 4 \times \frac{\pi}{H} \\ &= 3.55 \times \frac{1000 \times 10^{-3}}{2} \times 4 \times \frac{\pi}{82} \end{aligned}$$

$$H_g / \text{sec} = \underbrace{F_t}_{\mu RN} \times V \quad \text{or } T w$$

time req. for stopping.

$$v = u + at$$

$$0 = 2.5 - 1.136t$$

$$t = 2.200 \text{ sec}$$

$$H_g = \mu R N \times v \times t$$

$$T = F_{\text{total}} \times \frac{0.650}{2}$$

$$= 7.11 \text{ kN-m}$$

$$H_g = T \times \theta$$

$$= 7.11 \times \left(\frac{\omega_1 + \omega_2}{2} \right) t$$

$$= 7.11 \times \left(\frac{7.69 + 0}{2} \right) \times 2.2$$

$$= 60.14 \text{ kJ}$$

$$\times \text{ Torque (T)} = F_{\text{total}} \times R \times \omega \times t$$

$$= 21.892 \times \frac{1000 \times 10^{-3}}{2} \times 7.69 \times 10^{-3} \times 2.2$$

Q 10 and 11

$$T = 450 \text{ N-m (clockwise)}$$

$$\sum M_{\text{pivot}} = 0$$

$$220 \times 200 + T_1 \times 50 = T_2 \times 100$$

$$T_1 \times 50 - T_2 \times 100 = -220 \times 200 \quad (1)$$

$$T = (T_1 - T_2) R$$

$$450 = (T_1 - T_2) \times \frac{150}{2}$$

$$\frac{450 \times 2}{150} = T_1 - T_2$$

$$T_1 - T_2 = \frac{450 \times 2 \times 10^3}{150} = 6 \times 10^3 \text{ N-m} \quad (2)$$

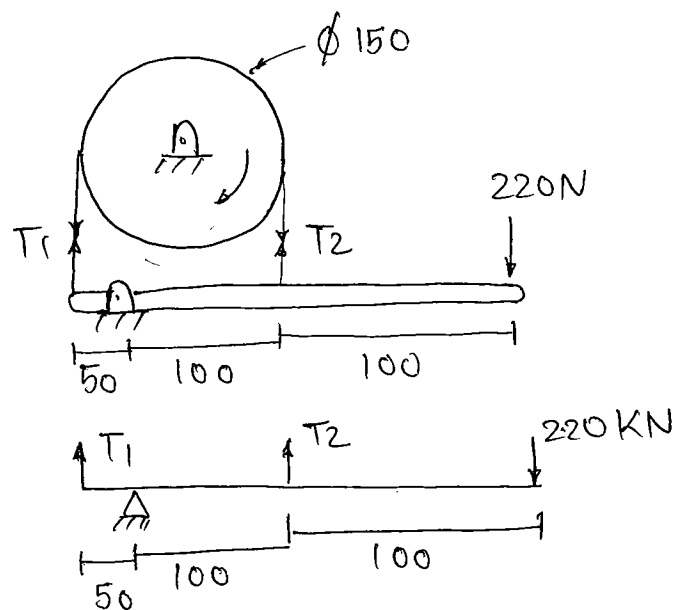
$$\theta = \left(\frac{\omega_1 + \omega_2}{2} \right) t$$

$$v = \frac{D\theta}{2} \times \omega \quad \boxed{v = R\omega}$$

$$\frac{m}{\text{sec}} \times 2.5 = \frac{650}{2} \times \omega$$

$$\omega_1 = 7.69 \text{ rad/sec}$$

$$\omega_2 = 0$$



$$T_1 = 12800$$

$$T_2 = 6800$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

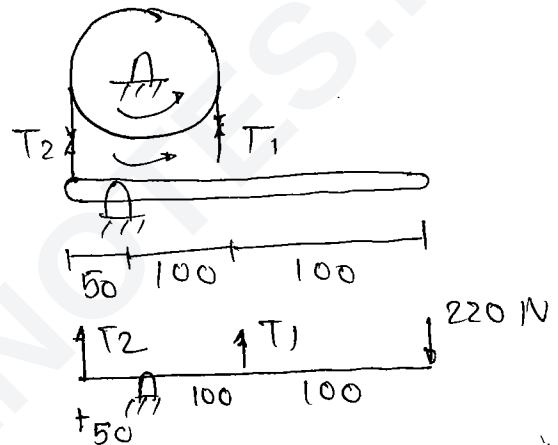
$$\frac{12800}{6800} = e^{\mu \times \frac{\pi}{180} \times 180}$$

$$\boxed{\mu = 0.29}$$

① $\mu = 0.4$

$$220 \times 200 + T_2 \times 50 = T_1 \times 100$$

$$T_1 \times 100 - T_2 \times 50 = 220 \times 200 \quad \text{--- (1)}$$



$$\mu = 0.4$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{T_1}{T_2} = e^{0.4 \times \pi}$$

$$T_1 = 3.51 T_2$$

$$T_2 = -146.179$$

$$T_1 = 513.089$$

$$T = (T_1 - T_2) \frac{150}{2}$$

$$= (513.089 - 146.179) \frac{150}{2}$$

$$= 27518.30 \text{ N-mm}$$

$$T = 27.51 \text{ N-m}$$

Q:- A force of 400 N is applied to the brake drum of 0.5 m diameter in a band brake system as shown in Fig when the wrapped angle is 180° . If the coefficient of friction is 0.25 the braking torque applied in N-m is

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{T_1}{T_2} = e^{0.25 \times 180 \times \frac{\pi}{180}}$$

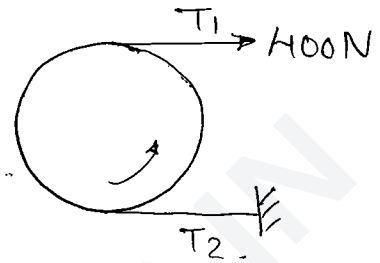
$$T_1 = 2.19 T_2$$

$$T_2 = 182.37$$

$$T_b = (T_1 - T_2) \cdot D/2$$

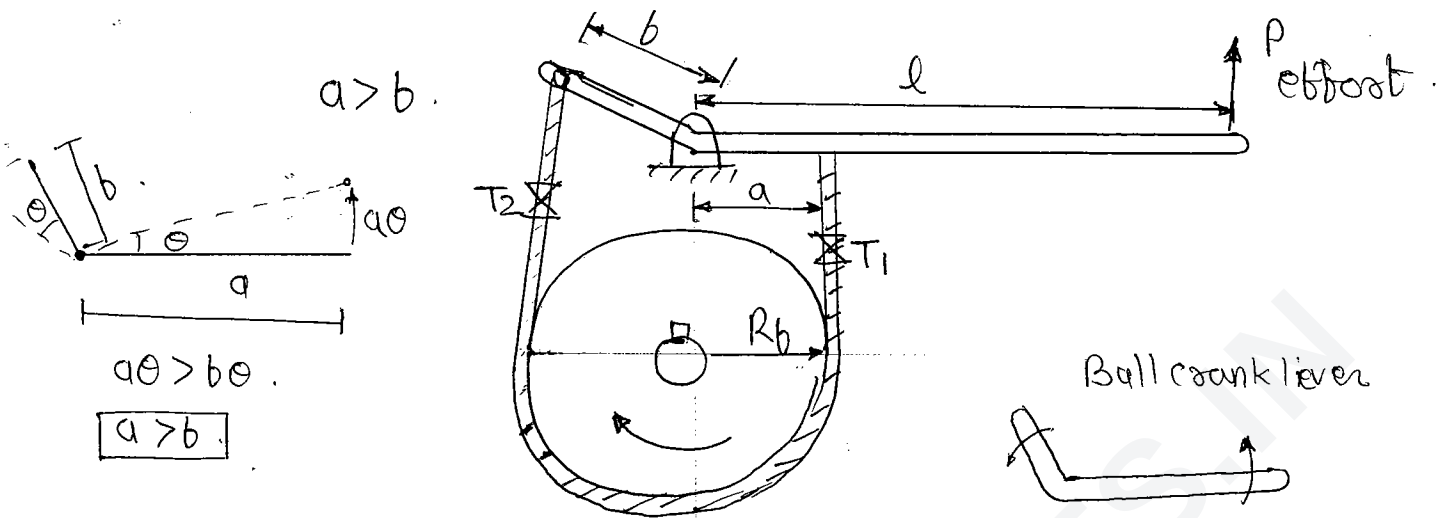
$$= (400 - 182.37) \times \frac{0.5}{2}$$

$$T_b = \frac{870.49}{54.40} \text{ N-m}$$



Direction of arrow
in braces shows
 T_1 direction

Differential Band Brakes.



$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Torque, $T = (T_1 - T_2) R_b$.

$\Sigma M_{\text{pivot}} = 0$.

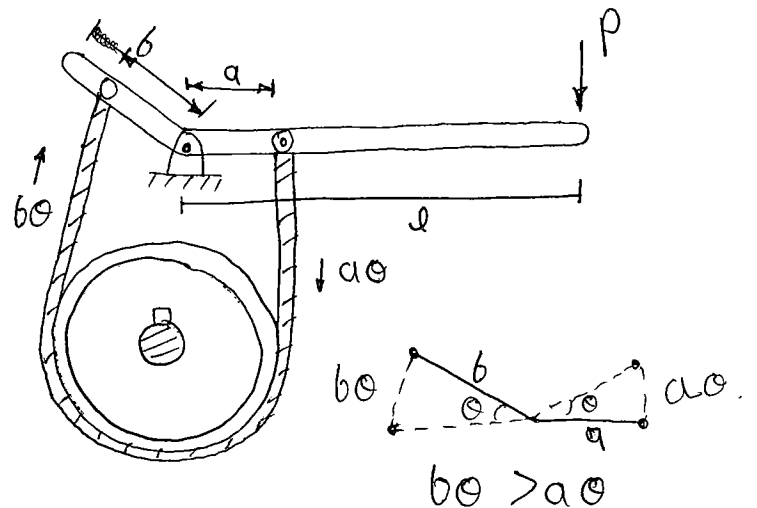
$$P \times l - T_1 \times a + T_2 \times b = 0$$

$$T_1 a - T_2 b = Pl$$

For self locking $P \leq 0$.

$$T_1 a - T_2 b = 0$$

$$\frac{T_1}{T_2} = \frac{b}{a} = e^{\mu\theta}$$



$b > a$
 $b\theta$ is $>$ than $a\theta$ hence more

Internal Expanding Shoe Brake (Drum Brake).

$$\sum M_{O_1} = 0 \quad (\text{leading shoe})$$

$$F_x l - Nxy + F_t x = 0$$

Applied movement
AM

Friction movement
FM

The direction of AM and FM is same hence it is self energizing brake.

Take moment about O_2 .

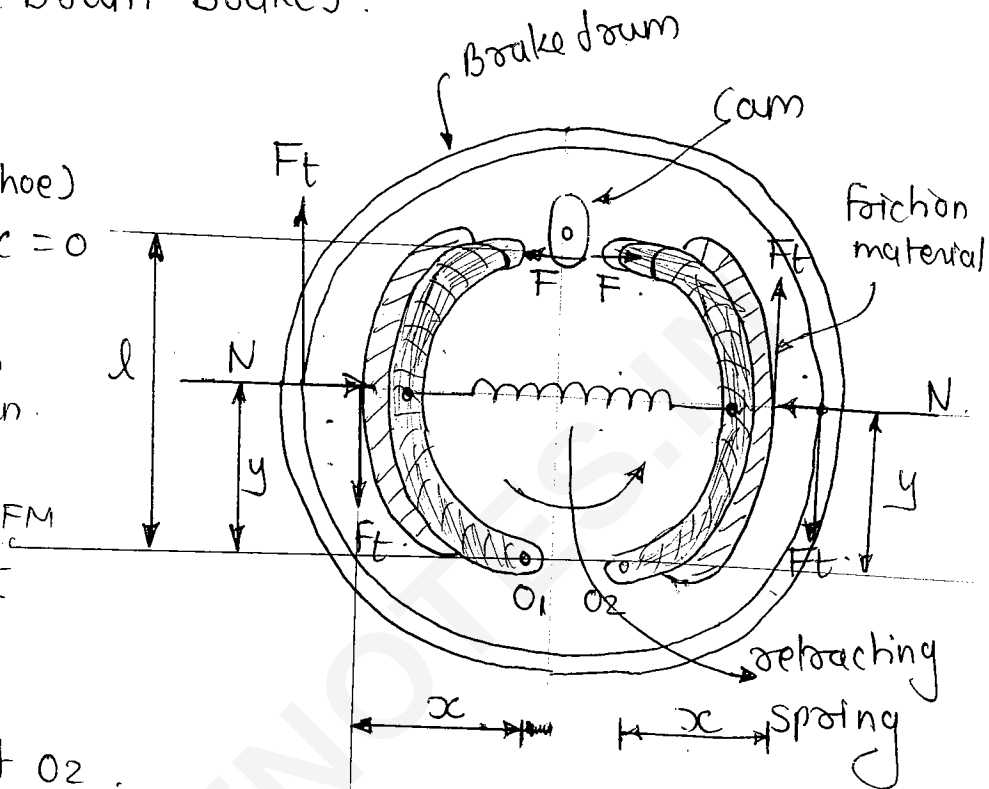
$$\sum M_{O_2} = 0.$$

$$F_x l - Nxy - F_t x = 0$$

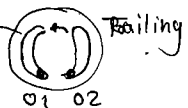
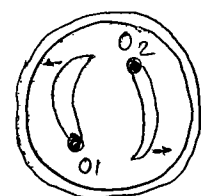
AM

FM

AM and FM are in opposite direction
(Trailing shoe).



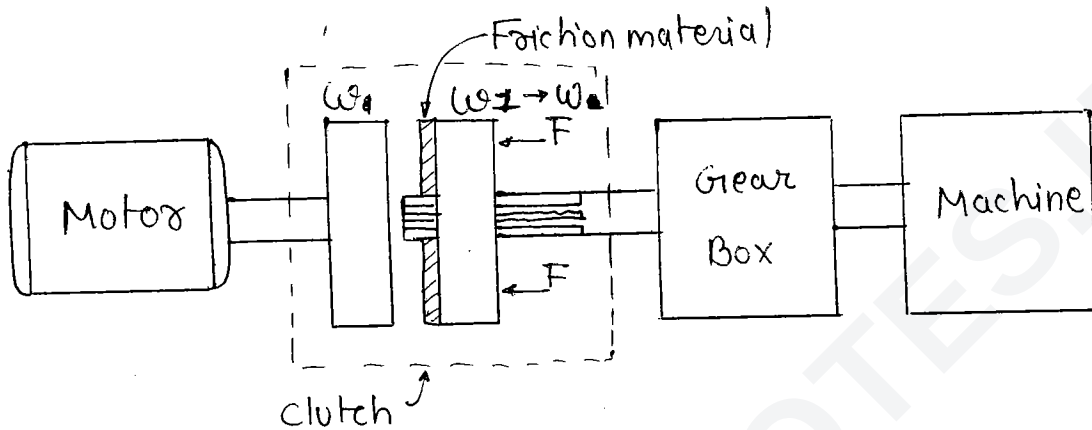
The direction of arrow between two shoe indicate leading or trailing shoe. arrow direction show the leading shoe direction between two shoe space.



Both self energizing

DESIGN OF CLUTCHES

A clutch is used to connect or disconnect the source of power from the remaining part of power transmission at the will of operator.



Amount of Heat generated (work done)

$$H_g = T \times \theta$$

$$\omega_2 = \omega_1 + \alpha t \quad \leftarrow v = u + \alpha t$$

$$\omega = 0 + \alpha t$$

$$\alpha = \frac{\omega}{t}$$

$$F = m \cdot a$$

Torque

$$T = I \alpha$$

$$= I \times \frac{\omega}{t}$$

$$v^2 - u^2 = 2as$$

$$\omega^2 - 0^2 = 2 \times \frac{N}{t} \times \theta$$

$$I = mk^2 \quad - \quad k - \text{Radius of gyration}$$

$$= \frac{D}{2} \quad \text{For a rimmed Flywheel}$$

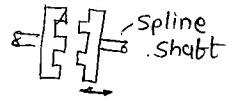
$$= \frac{D}{2\sqrt{2}} \quad \text{For a solid disc}$$

Types of clutches.

- ① Plate clutch
- ② Conical clutch
- ③ Centrifugal clutch.

Clutches $\begin{cases} \rightarrow \text{Friction} \rightarrow \text{Friction force is used and there is slip.} \\ \rightarrow \text{Positive} \rightarrow \text{No Friction force is used and no slip.} \end{cases}$

$\rightarrow$ ① Square Jaw clutch ② Spiral Jaw clutch.



1] $\rightarrow$ Plate clutch

- ① Single plate

Ex:- Used in tractors, Buses, lorries and cars.

- ② Multiplate clutch.

Ex:- Machine tools, two wheelers.

2] $\rightarrow$ Conical clutch

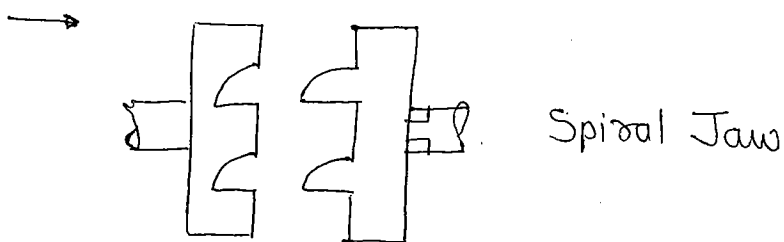
Ex:- Feed drives of m/c tool.

3] $\rightarrow$ Centrifugal clutch

Ex:- Mopeds, washing m/c, coal crushers.



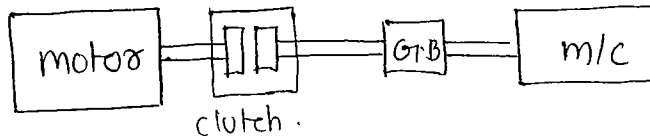
very slow speed.



Bit high speed we can use this clutch.

2000rpm (High speed side)

500rpm

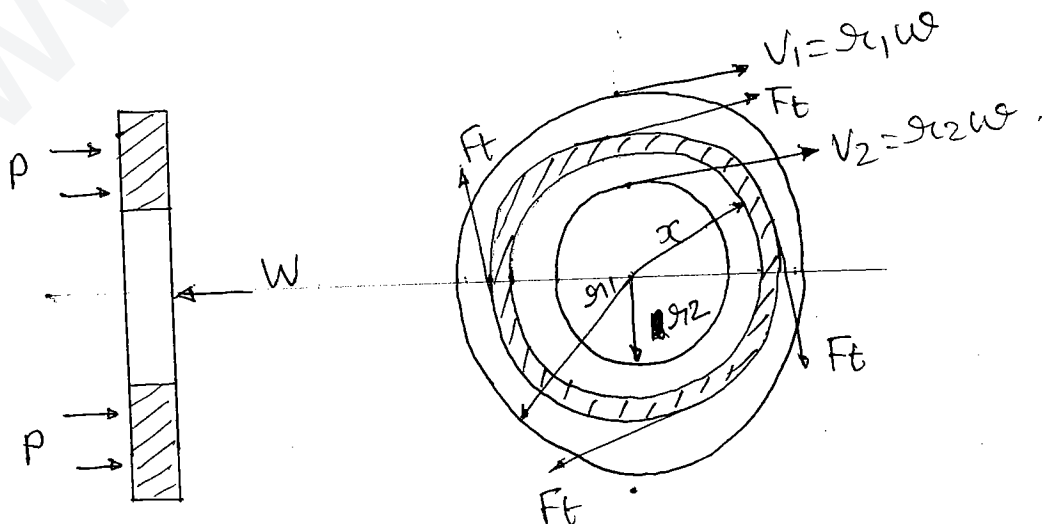
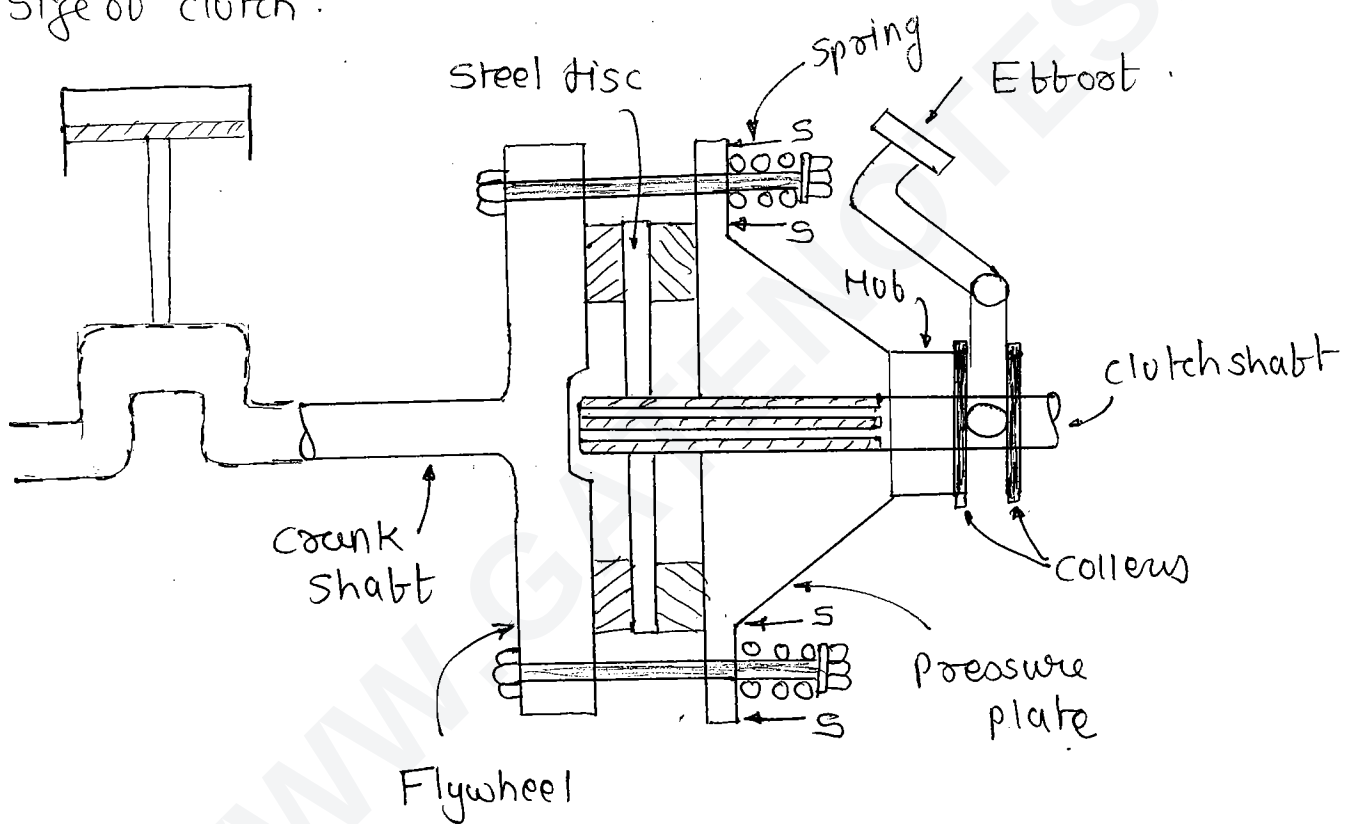


$$P = \frac{2\pi NT}{60}$$

$$P \propto NT$$

$$N \uparrow T \downarrow$$

As we req. more torque it we place it after H.G.B. the size of the clutch will increase ($T \propto \text{Radius of clutch}$) Hence we place clutch at high speed side ($N \uparrow T \downarrow$) which reduces the size of clutch.



$$\delta w = \text{Pressure} \times \text{Area} \\ = p \times 2\pi x dx$$

Friction Force

$$\delta F = \mu \delta w \\ = \mu p \times 2\pi x dx$$

Torque

$$\delta T = \delta F \times x \\ = \mu p \times 2\pi x^2 dx$$

Pressure is not uniform or may be uniform according to clutch condition or wear position.

□ New clutch — Uniform pressure

$$T = \int \delta T \\ = \int_{r_2}^{r_1} \mu p 2\pi x^2 dx \\ = \mu p 2\pi \left[\frac{x^3}{3} \right]_{r_2}^{r_1} \\ = 2\mu p \pi \left[\frac{r_1^3 - r_2^3}{3} \right] \\ = 2\pi \mu p \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$\text{If } \delta w = p 2\pi x dx$$

$$W = \int_{r_1}^{r_2} p 2\pi x dx$$

$$W = p \pi [r_1^2 - r_2^2]$$

$$W = 2\pi p \left[\frac{r_1^2 - r_2^2}{2} \right]$$

$$p = \frac{W}{\pi [r_1^2 - r_2^2]}$$

$$= 2\pi \mu \times \frac{W}{\pi [r_1^2 - r_2^2]} \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$= \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

② Used clutches - Uniform wear.

(Heat generated) $H_g = T \times \theta$.

$$H_g / \text{sec} = T \times \frac{\theta}{\text{time}}$$

$$'' = T \times \omega$$

$$= \mu F \times r \times \omega$$

$$H_g / \text{sec} / \text{mm}^2 = \mu F V$$

$$= \mu \times \text{pressure} \times \text{velocity}.$$

$$H_g / \text{sec} / \text{mm}^2 = \mu P V$$

As wear $\propto H_g / \text{sec} / \text{mm}^2$
 $\propto \mu P V$
 $\propto P V$

$\therefore$ If $PV = \text{constant}$ wear is uniform.

$$PV = \text{constant}$$

$$V = r \omega$$

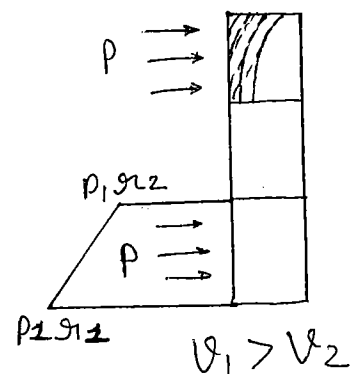
$$= r \omega$$

$$V \propto r$$

$$Pr = \text{constant}$$

$$P_1 r_1 = P_2 r_2 = \text{constant}$$

$$P_{\min} r_{\max} = P_{\max} r_{\min}$$



Initially

$$P_1 = P_2 \quad r_1 > r_2$$

$$P_1 r_1 > P_2 r_2$$

After initial wear, wearing
 take place uniform

$$P_1 r_1 = P_2 r_2 = C$$

$$\delta T = \mu p 2\pi x^2 dx$$

$$T = \int_{r_2}^{r_1} \delta T$$

$$= \mu 2\pi \int_{r_2}^{r_1} p x^2 dx$$

$$= \mu 2\pi C \int_{r_2}^{r_1} x dx$$

$$T = \mu 2\pi C \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$\delta W = p \times 2\pi x dx$$

$$\delta W = \frac{C}{x} 2\pi x dx$$

$$W = 2\pi C \int_{r_2}^{r_1} dx$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

$$T = \mu \times 2\pi \times \frac{W}{2\pi(r_1 - r_2)} \times \left[\frac{r_1^2 - r_2^2}{2} \right]$$

$$T = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

$$T = \mu W r_m$$

$$r_m = \frac{r_1 + r_2}{2}$$

$$T_{\text{uniform pres.}} = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$T_{\text{uniform wear}} = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

For shorter period of clutch life it subjected to uniform pr. after that its wear is uniform hence consider uniform wear when condition is not given

Uniform pressure

$$T_{up} = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

Uniform wear

$$T_{uw} = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

$$T_{up} > T_{uw}$$

$$(r_f)_{up} > (r_f)_{uw}$$

$$\frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] > \frac{r_1 + r_2}{2}$$

Multiplate clutch.

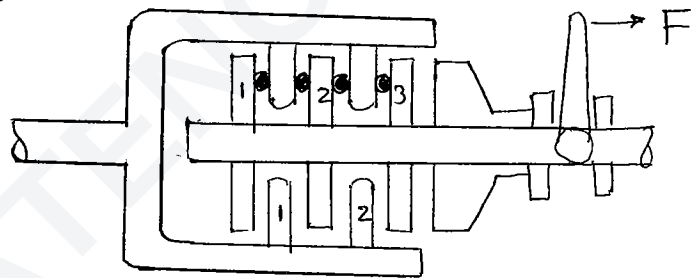
$$T_{wear} = \mu W \left(\frac{r_1 + r_2}{2} \right) n$$

n = no. of pairs of surface in contact

$$= n_1 + n_2 - 1$$

$$= 3 + 2 - 1$$

$$= 4$$



Pno 76.

Q no 6 and 7

$$r_f = 100 \text{ mm}$$

$$\mu = 0.1$$

$$N = \frac{600}{\pi} \text{ rpm}$$

$$W = 3 \text{ kN}$$

$$T_{up} = ?$$

$$p = ?$$

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \mu W R_f$$

$$= 0.1 \times 3 \times 100$$

$$= 30 \text{ N-mm}$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 600/\pi \times 30}{60}$$

$$P = 600 \text{ W}$$

Pg no 77

Q no 12

$$n_1 = 5$$

$$n = n_1 - 1 = 4$$

$$T = 0.4 \text{ KN-m}$$

$$= 0.4 \times 10^3 \text{ N-m}$$

$$\mu = 0.5$$

$$r_1 = 150 \text{ mm}$$

$$r_2 = \frac{200 \text{ mm}}{2} = 100 \text{ mm}$$

load/spring

$$T = \mu \omega \left(\frac{r_1 + r_2}{2} \right) n$$

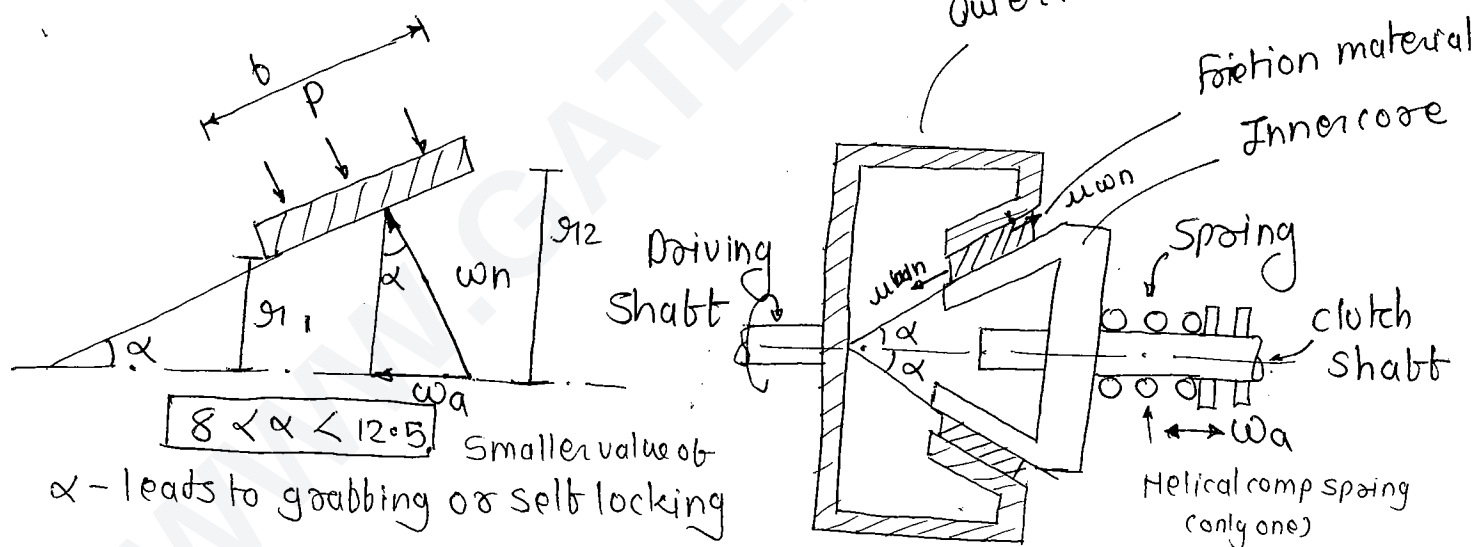
$$0.4 \times 10^3 = 0.5 \times \omega \left(\frac{150 \times 10^{-3} + 200 \times 10^{-3}}{2} \right) 4$$

$$\omega = \frac{0.4 \times 10^3}{0.35}$$

$$\omega = \frac{1142.85 \text{ N}}{1600}$$

$$\frac{\text{load}}{\text{spring}} = \frac{\omega}{\text{no. of spring}} = \frac{\omega}{4} = 400 \text{ N}$$

Cone / Conical Clutch



→ Uniform pressure

$$T = \frac{2}{3} \mu \omega n \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \frac{2}{3} \mu \frac{\omega n}{\sin \alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

→ Uniform wear

$$T = \mu \omega n \left(\frac{r_1 + r_2}{2} \right) =$$

$$= \mu \omega n (r_1 + r_2)$$

$$\sin \alpha = \frac{r_1 - r_2}{b}$$

$$\sin \alpha = \frac{\omega_a}{\omega_n}$$

$$\omega_a = \omega_n \sin \alpha$$

ω_a = Axial force applied during running.

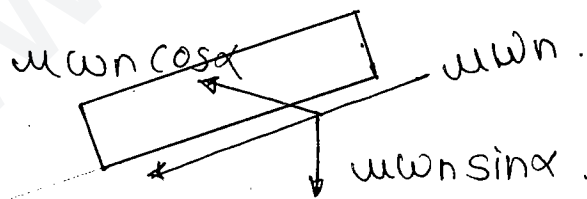
$$T \propto \frac{1}{\sin \alpha}$$

$$\alpha \downarrow \sin \alpha \downarrow T \uparrow \dots \dots \dots$$

Note -

- ① Driver and driven shaft must be collinear otherwise uneven wear take place.
- ② Only one spring is used to balance the force even at reduced stiffness.

Axial force required to engage the clutch.



$$\omega_{ae} = \omega_a + \mu \omega_n \cos \alpha$$

$$= \omega_n \sin \alpha + \mu \omega_n \cos \alpha$$

$$\omega_{ae} = \omega_n (\sin \alpha + \mu \cos \alpha)$$

Pg no 77

Q 13 to 14.

$$P = 20 \text{ kW}$$

$$N = 1000 \text{ rpm}$$

$$r_b = \frac{r_1 + r_2}{2} = 150 \text{ mm}$$

$$2\alpha = 24^\circ$$

$$\alpha = 12^\circ$$

$$\mu = 0.2$$

$$p = 70 \times 10^3 \text{ N/m}^2$$

$$T = \mu \omega_n r_b$$

$$= 0.2 \times \omega_n \times 150 \times 10^{-3} \quad \text{--- (1)}$$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N} \quad \text{--- (2)}$$

$$\text{(1) = (2)}$$

$$0.2 \times \omega_n \times 150 \times 10^{-3} = \frac{20 \times 10^3 \times 60}{2 \times \pi \times 1000}$$

$$\omega_n = 6366.19 \text{ N}$$

$$W_{ae} = \omega_n (\sin \alpha + \mu \cos \alpha)$$

$$= 6366.19 (\sin 12^\circ + 0.2 \cos 12^\circ)$$

$$W_{ae} = 2569.023$$

$$= 2.56 \text{ kN.} \quad \text{Ans b]}$$

$$\text{Pressure} = \frac{\text{Normal force}}{\text{mean area of surface}}$$

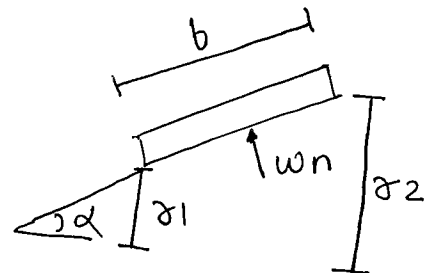
$$= \frac{W_n}{2\pi r_b}$$

$$70 \times 10^3 = \frac{6366.19}{2\pi \times 150 \times 10^{-3} \times b}$$

$$b = 0.0964 \text{ m}$$

$$b = 96.49 \text{ mm}$$

Ans b]}



$$r_b = r_1 = \frac{r_1 + r_2}{2}$$

Pg no 79

Q 38

$$T_b = 140 \text{ Nm}$$

$$T_b = (T_1 - T_2) R$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$140 = (T_2 k - T_2) R$$

$$\frac{T_1}{T_2} = e^{0.2 \times 0}$$

$$140 = T_2 (k - 1) R$$

$$T_1 = k T_2 \quad -x-$$

$$T = \mu \omega \left(\frac{r_1 + r_2}{2} \right) = \mu \times 2\pi C (r_1 - r_2) \times \left(\frac{r_1 + r_2}{2} \right)$$

(Case 1) 27.

$$C = P_{\max} r_{\min} = P_{\min} r_{\max}$$

$$= P_{\max} r_2 = P_{\min} r_1$$

$$T = \mu \times 2\pi \times P_{\max} r_2 \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$= \mu \pi P_{\max} r_2 (r_1^2 - r_2^2)$$

(Case 1) $OD = 220 \text{ mm}$, $ID = 150 \text{ mm}$, $P_{\max} = 0.25 \text{ MPa}$ $\mu = 0.3$

$$T_b = 0.3 \times \pi \times 0.25 \times \frac{150}{2} \left(\left(\frac{220}{2} \right)^2 - \left(\frac{150}{2} \right)^2 \right)$$

$$= 114.42 \times 10^3 \text{ N-mm}$$

$$= 114.42 \text{ N-m} < T$$

Not recommended

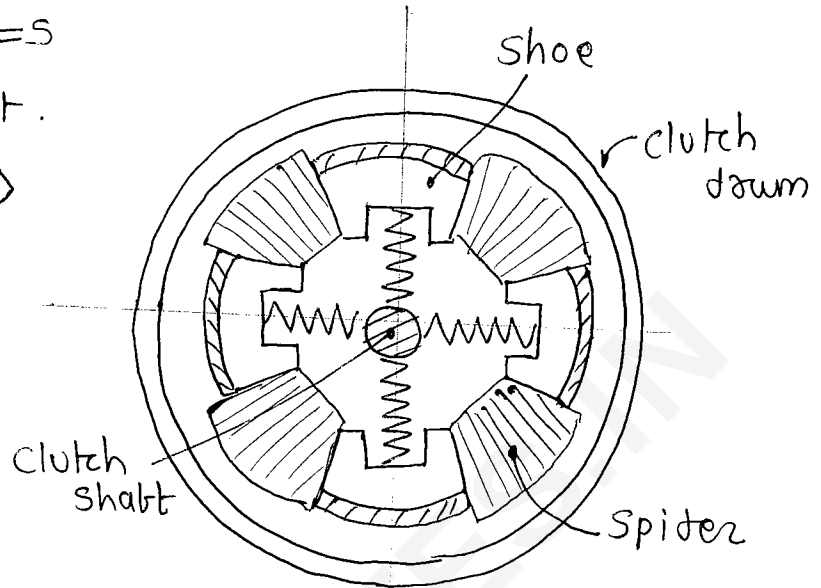
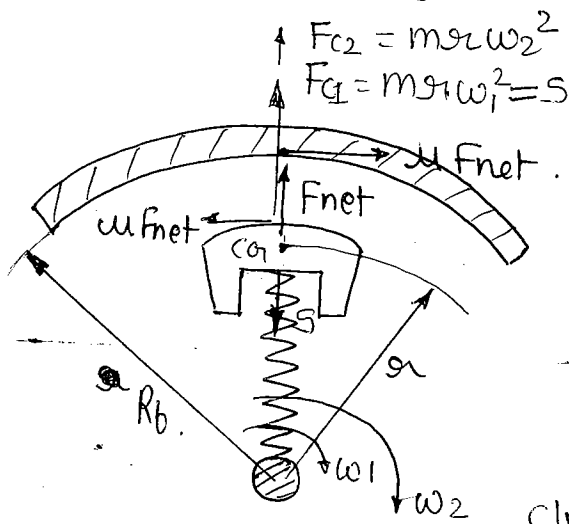
(Case 2] $OD = 250 \text{ mm}$, $ID = 150$, $P_{\max} = 0.21 \text{ MPa}$ $\mu = 0.3$

$$T_b = 148 \text{ N-m} > T \text{ recommended (very good)}$$

(Case 3] $OD = 275 \text{ mm}$, $ID = 150 \text{ mm}$, $P_{\max} = 0.185$ $\mu = 0.3$

$$T_b = 172 \text{ N-m} > T \text{ recommended}$$

Centrifugal Clutch



Provides time delay between starting of an engine and engagement of load

ω = Cobb of traction

m = mass of each shoe

ω_1 = engaging speed

ω_2 = running speed

r = distance between center of shaft to C.G. of mass

R_b = Inner radius of clutch drum.

$$F_{net} = F_{c2} - S.$$

$$= F_{c2} - F_{c1}$$

$$= m \cdot r (\omega_2^2 - \omega_1^2)$$

Torque

$$T = \text{Force} \times \text{radius}$$

$$= \omega F_{net} \times R_b.$$

$$= \omega \times m \cdot r (\omega_2^2 - \omega_1^2) \times R_b \times n$$

n = no. of shoes.

Pg no 78

Q. no 28

$$T = 255 \text{ N-m}$$

$$n = 4$$

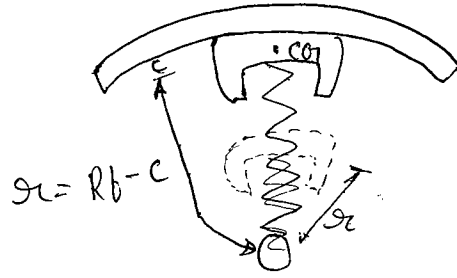
$$\mu = 0.35$$

$$R_b = \frac{330}{2} = 165$$

$$r_1 = 140 \text{ mm}$$

$$\omega_2 = 80 \text{ rad/sec}$$

$$\omega_1 = 60 \text{ rad/sec}$$



$$F_c = m r_1 \omega_1^2$$

$$F_{c2} = m r_2 \omega_2^2$$

$$r_2 \gg \gg \gg c$$

$$T = \mu m r_1 (\omega_2^2 - \omega_1^2) \times R_b \times n$$

$$255 = 0.35 \times m \times 140 \times 10^{-3} (80^2 - 60^2) \times 165 \times 10^{-3} \times 4$$

$$m = 2.48 \text{ kg}$$

GEARS

A gear is a toothed wheel used to transmit power at constant angular velocity ratio.

$$P = \frac{2\pi NT}{60}$$

- Positive Drive (No slip)

- Short center distance.

$$P \propto NT$$

$$N \downarrow T \uparrow$$

Types of Gears

1] Spur gear

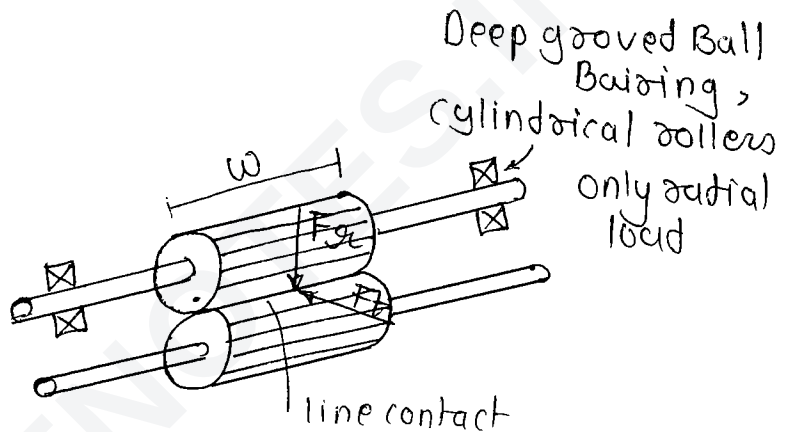
① Tooth are cut parallel to axis

② Joins coplaner, parallel shaft

③ Line contact, sudden engagement and disengagement

④ Only radial loads.

⑤ Velocity ratio 3:1 to 6:1



2] Helical gear

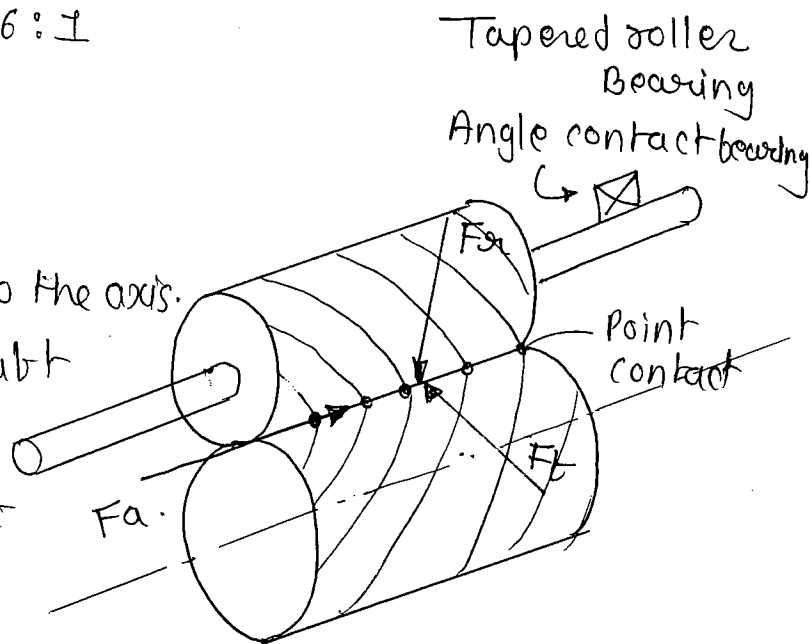
- Tooth are cut at an angle to the axis.

- Joins coplaner parallel shaft

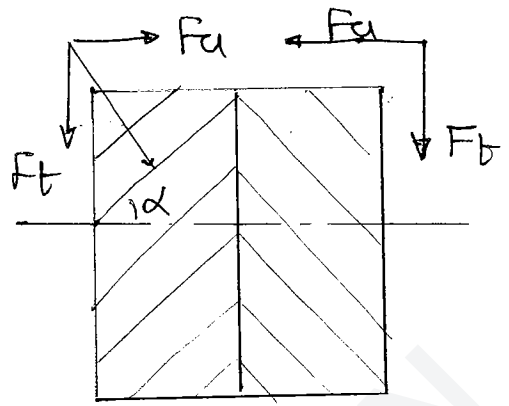
- Initially point contact but gradually become line contact

- Velocity ratio 6:1 to 10:1

- loads both axial and radial



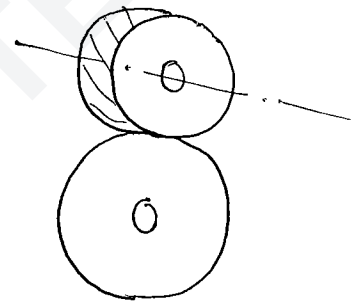
3] Herringbone gears .



axial Force = 0 .

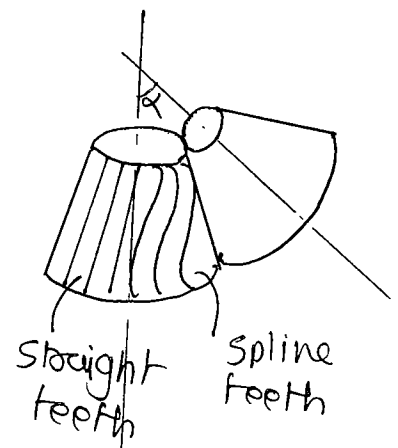
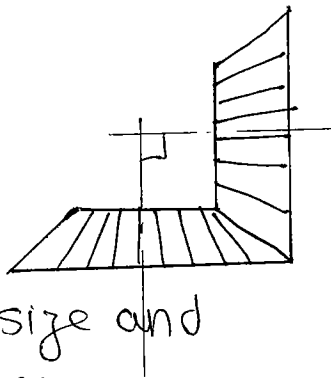
3] Spiral gear / crossed helical gear .

- Joins non-coplanar and non intersecting shafts (skew shaft)
- Always point contact
- Suitable for light loads.



4]

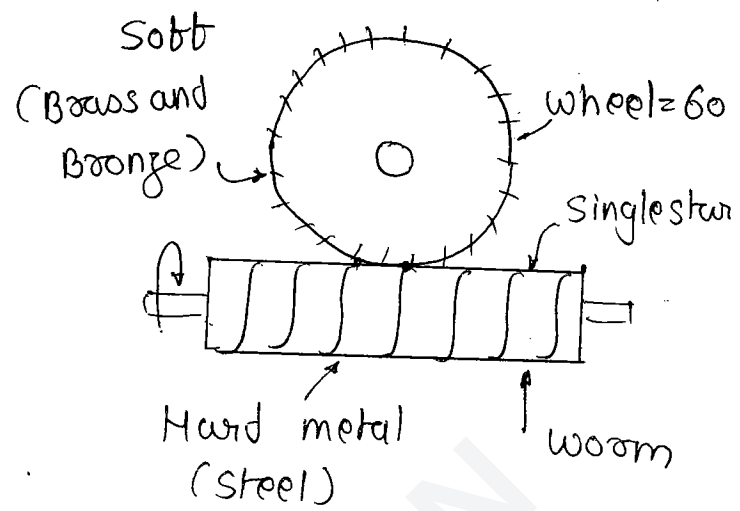
- Transmit power at angle
- Radial and axial loads
- Two bevel gear of equal size and at 90° are called miter gear
- A line contact



5] Worm gear

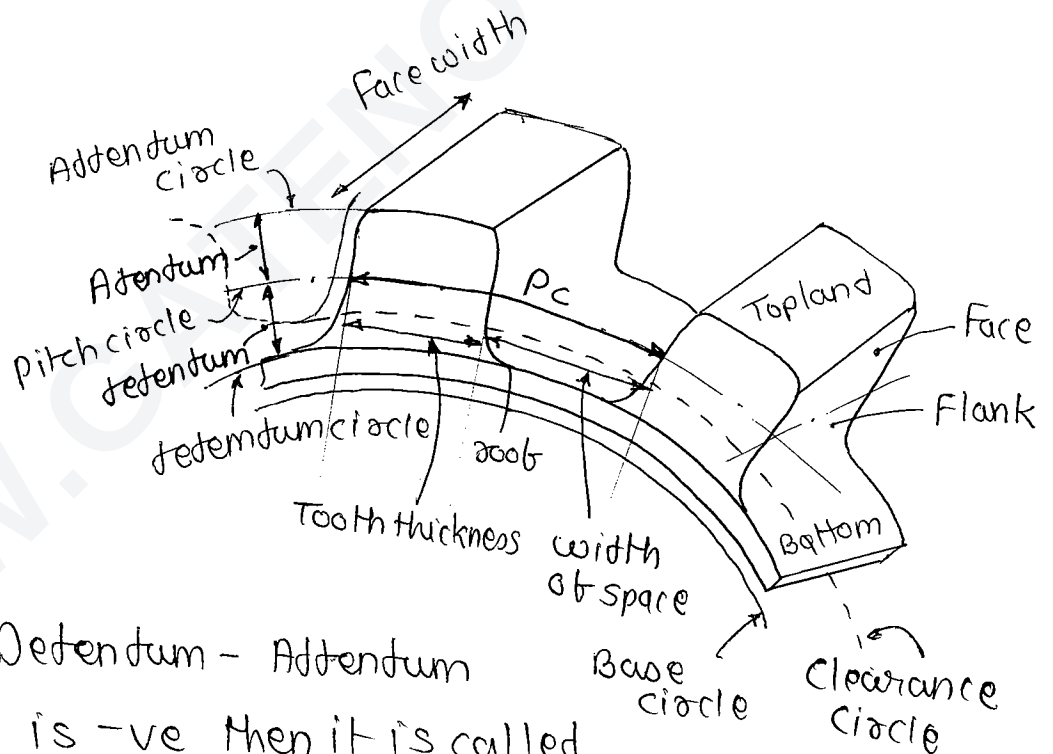
- Velocity ratio

$$= \frac{1}{\text{No of teeth of wheel}}$$
- Skew shaft
- Velocity ratio upto 100:1
- Initially point contact than line contact.



SPUR GEAR.

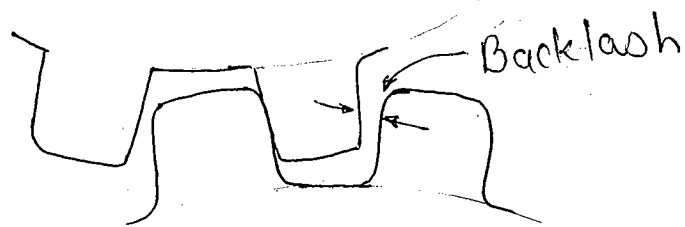
Gear Nomenclature.



$$\text{Clearance} = \text{Detendum} - \text{Addendum}$$

If clearance is -ve then it is called interference.

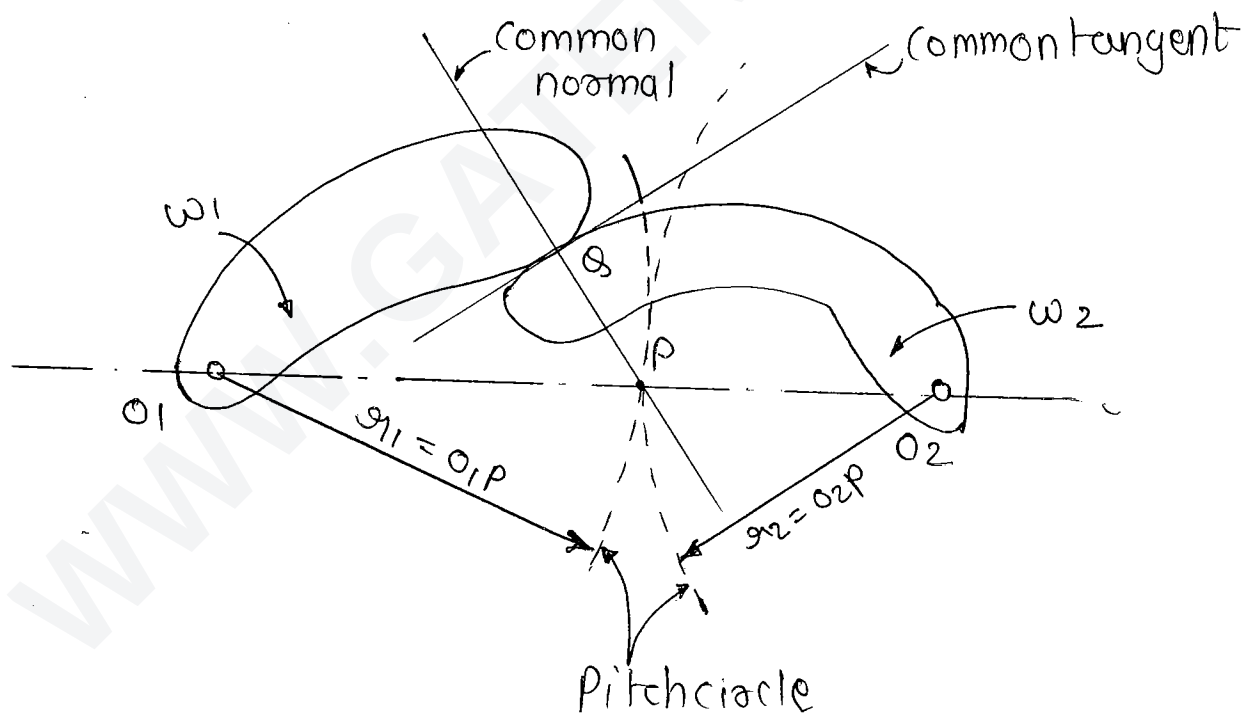
Backlash



Backlash
 = Width of space - Tooth Thickness

Law of gearing.

The common normal at the point of contact always passes through a fixed point on the line joining centers and divides the line joining centers in the inverse ratio of their angular velocities.



$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{O_2P}{O_1P}$$

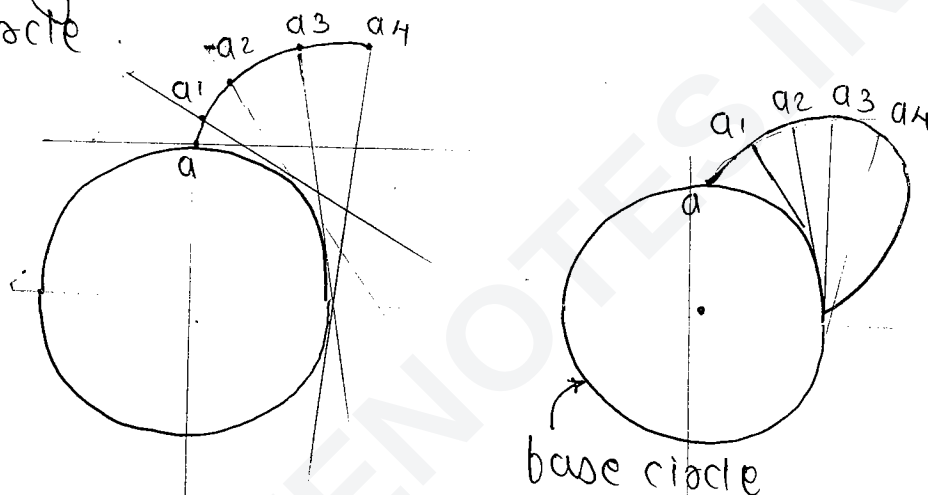
Pitch line velocity, $V = r_1 \omega_1 = r_2 \omega_2$

Conjugate Problem $\rightarrow$ (Involute, cycloid).

- Satisfy law of gearing.
- Maintain constant angular velocity ratio.

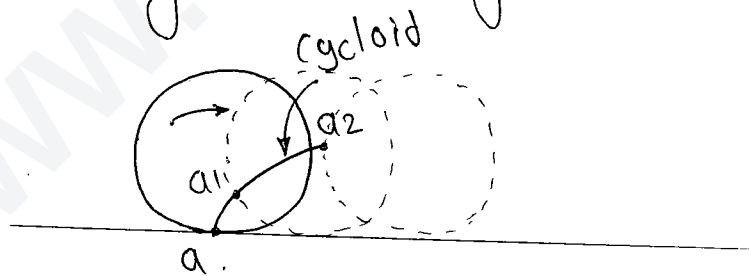
Involute

The locus of a point on a straight line which rolls without slipping on a circle is called involute circle is called base circle.

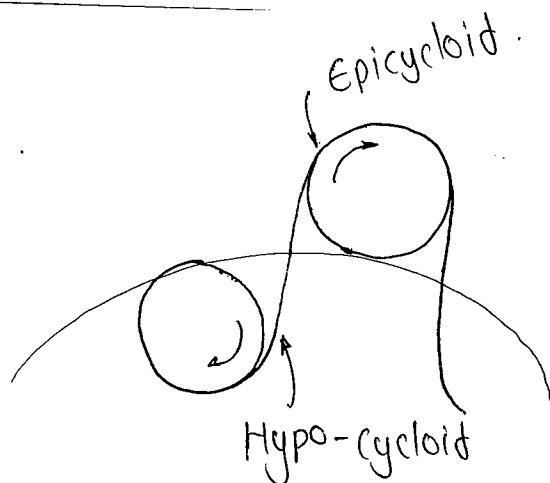


Cycloid

The locus of a point on a circle which rolls without slipping on a straight line is called cycloid.



- More strength.
- zero interference.
- Less wear.
- Difficult to cut.



Advantages of involute

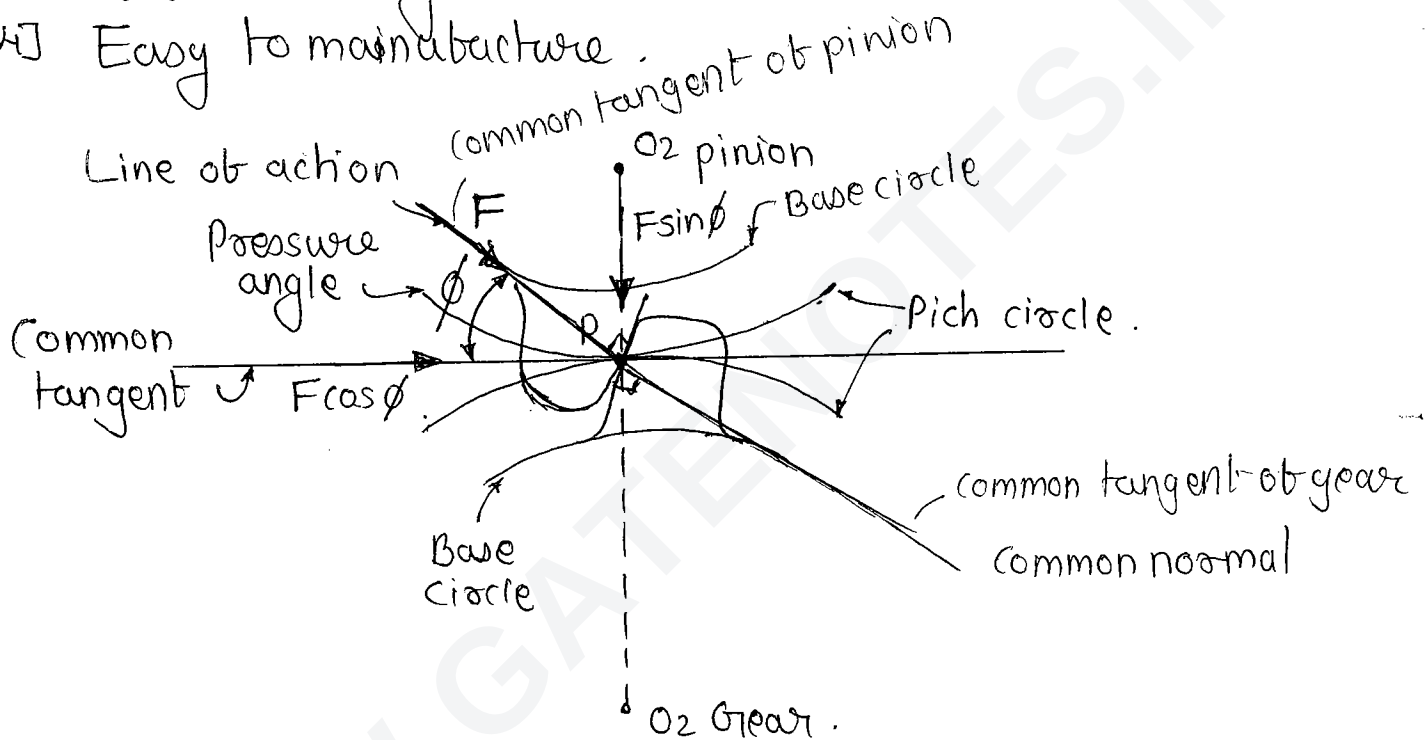
1] Conjugate in action

2] Constant pressure angle (ϕ)

(Pressure angle is zero at the pitch point and is maximum at tips for cycloid)

3] Small variation in the center distance will not effect velocity ratio.

4] Easy to manufacture.



Pressure angle (ϕ)

- Angle between common normal and common tangent
- Angle between common tangent to base circle and common tangent to pitch circle
- Angle between line of action and common tangent is pressure angle.

$$F_t = F \cos \phi$$

$$F_r = F \sin \phi$$

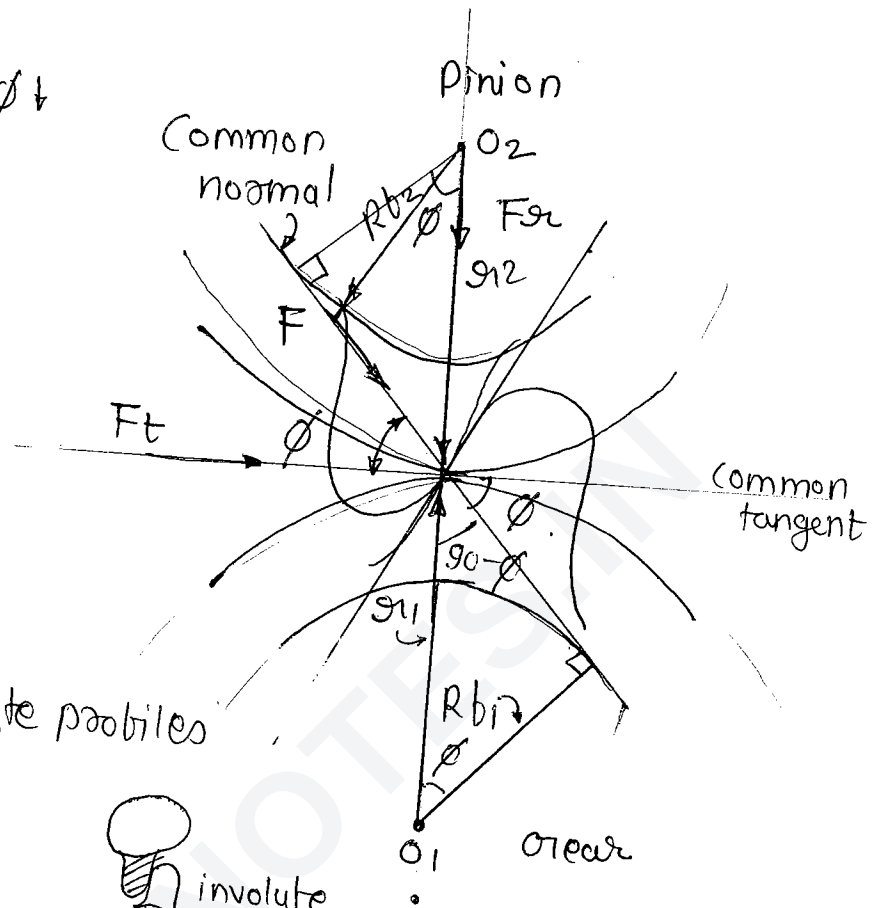
$$\text{Torque, } T = F_t \times R$$

$$T \uparrow F_t \uparrow \cos \phi \uparrow \phi \downarrow$$

$$\begin{aligned} \phi &= 14\frac{1}{2}^\circ \text{ absolute} \\ &= 20^\circ - \text{widely used} \\ &= 25^\circ \end{aligned}$$

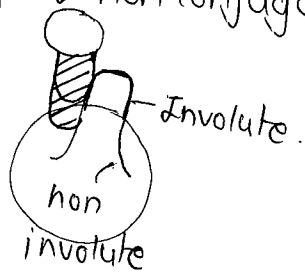
$$F_R = F \sin \phi$$

$$F_R \downarrow \sin \phi \downarrow \phi \downarrow$$

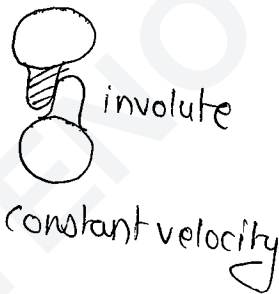


Interference

- ① Negative clearance
- ② Meshing of non conjugate profiles



velocity ratio is not constant



constant velocity

$$F_t = F \cos \phi$$

$$\phi \downarrow F_t \uparrow$$

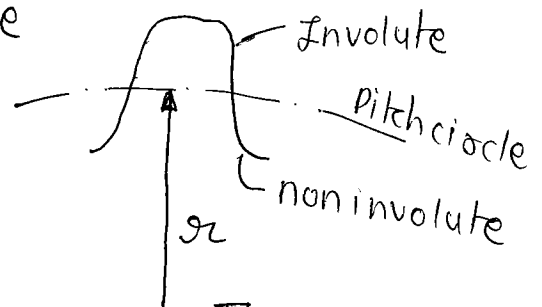
$$r_b = r \cos \phi$$

$$r_b \quad \phi \uparrow \cos \phi \downarrow$$

we cant give or put $\phi = 0$

Then $r_b = r$ Then profile

become



$$\phi = 20^\circ$$

$$r_b = r \cos 20^\circ$$

$$r_b = 0.9397r$$

This cause Interference hence velocity ratio is not constant

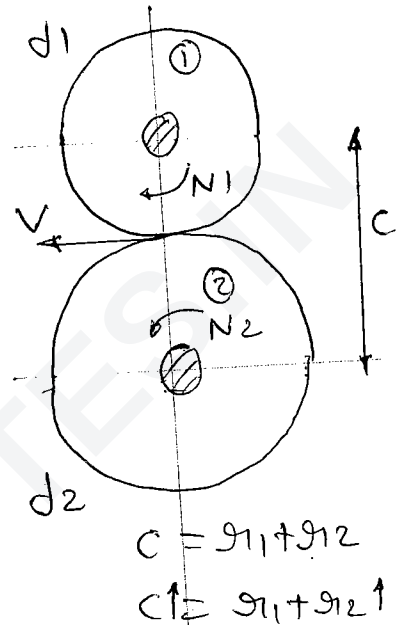
$$I \pm \phi = c$$

- ③ Small variation in the center distance doesn't change v.p

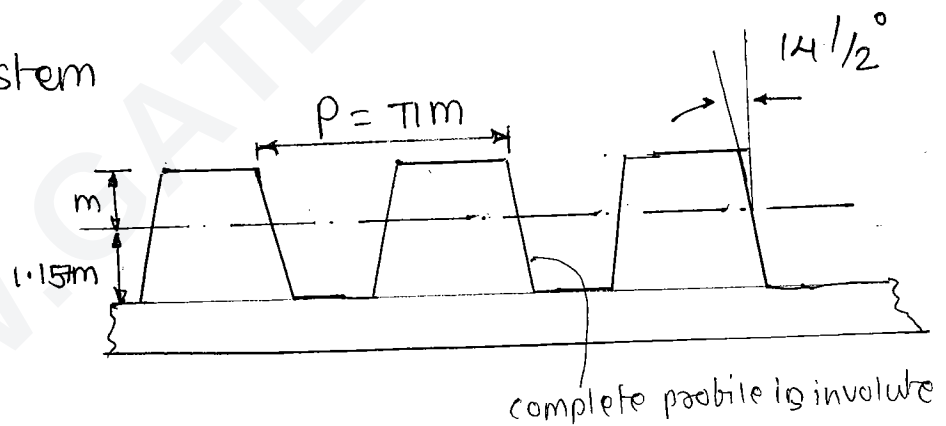
$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$$V \cdot R = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{r_1}{r_2}$$

Ex:- $C = 100 \text{ mm}$ - design
but actual assembly time
 $C = 98 \text{ mm}$



Gear System



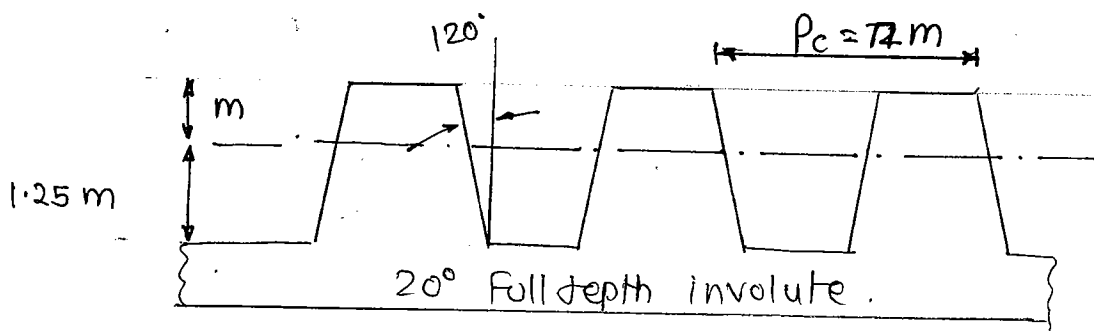
① $14\frac{1}{2}$ Full depth involute

② Pitch of gear

① Diametral pitch, $P_d = \frac{\text{No. of teeth}}{\text{pitch circle dia.}} = \frac{T}{D}$ is difficult

② Circular pitch, $P = \frac{\pi D}{T}$ difficult to express
D - pitch circle dia
T - no. of teeth

③ Module, $m = \frac{D}{T} \text{ mm}$
 $= \frac{1}{P_d}$

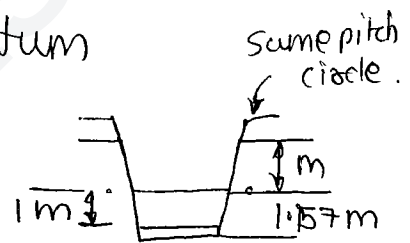


① $1\frac{1}{2}$ Full depth involute

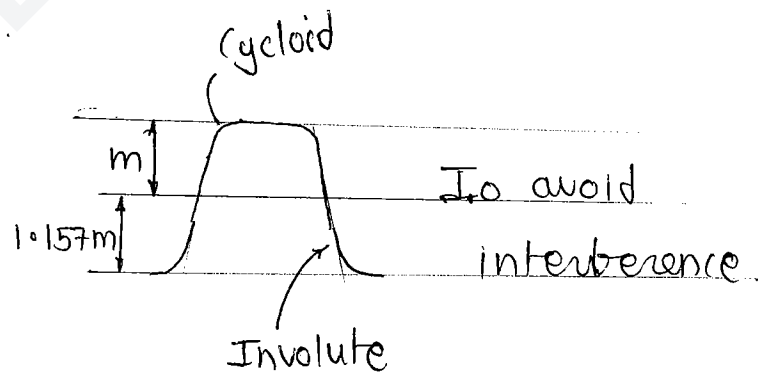
$$\begin{aligned} \text{Total depth} &= \text{Addendum} + \text{Dedendum} \\ &= m + 1.25m = 2.25m \end{aligned}$$

$$\begin{aligned} \text{Working depth} &= \text{addendum} + \text{addendum} \\ &= 2m \end{aligned}$$

$$\text{clearance, } e = 1.157m - 1m = 0.157m.$$



② $1\frac{1}{2}$ Composite teeth.



③ 20° Full depth involute

$$\text{only clearance} = 0.25m$$

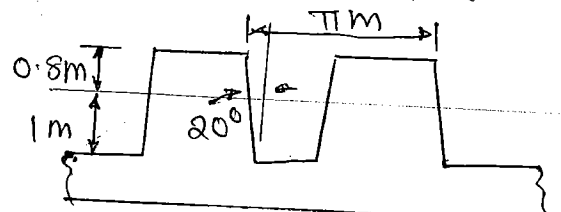
④ 20° Stub involute

To reduce the bending stress because of high tooth height in 20° full depth we use 20° stub involute.

There is height limit to maintain the contact surface.

$$\begin{aligned} &= 1 - 0.8 \\ \text{Clearance} &= 0.2m \end{aligned}$$

$$\text{working depth} = 1.8m$$



To avoid interference the minimum no of teeth on pinion

$$T_{\min} = \frac{2}{\sin^2 \phi}$$

$$\text{For } 14\frac{1}{2}^\circ \rightarrow T_{\min} = 31.9 \\ \approx 32$$

$$20^\circ \rightarrow T_{\min} = 17.09 \\ \approx 18$$

$$25^\circ, \rightarrow T_{\min} = 12$$

Tooth Thickness

$$t = \frac{P_c}{2} = \frac{\pi m}{2} = \frac{\pi}{2} \times \frac{D}{T}$$

$$t = \frac{\pi D}{2T}$$

$$t \uparrow \quad T \downarrow \quad \phi \uparrow \quad \dots \dots \dots T_{\min} \propto \frac{2}{\sin \phi}$$

WWW.GATENOTES.IN

WWW.GATENOTES.IN

6-9-11
9:30-1-11
English
HT.

SAJIT CHINTAMKAR

Accepted that

NOT FALL IN LOVE WITH
ANY GIRL IN LIFE

I HATE GIRLS*

*** I LOVE BOYS ONLY

Sajit
3/6/2012
8:10 AM

Dhanyawatam - Thanks.

Anshul.

9028372813

① Ni peranti waks your name?

Adhi ② A Wat Pen - My pen

③ Dine xute ~~an~~ anda - How was the cost

④ Bhagunnara ~~Bhannara~~ - How you are fine

⑤ Mink ~~ekko~~ ^{ekkada} untana - You where are you ~~leaving~~ leaving

⑥ Mitro Friendship ~~chaycha~~ - I want to be your friend.

⑦ Mezu a college ~~olo chatchatu~~ ^{make} ~~vu~~ ^{thunna} ru.

6-9 - calculus

9-30-1 - M-S

2-5-30 - Production

6-8-30 thermal

$$Z_0 = \frac{P \sin \theta (\cos \theta + \sin \theta)}{SL}$$

$$\frac{\partial Z}{\partial \theta} = 0$$

$$\frac{\partial}{\partial \theta} (\sin \theta (\cos \theta + \sin \theta))$$

6-9 - PI

9-11 - TM

2-5-30 - TOM

6-PI - PI

