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Aug 14' 2010

THEORY OF MACHINES

ANALYSIS OF PLANE MECHANISMS

- MECHANISM :-
  Converts available motion into desired motion.
  Eg: Slider Crank
- MACHINE :-
  Converts energy from one form to another.
  Machine consists of a no. of individual mechanisms.

If all the links are constrained to move in a single plane or parallel plane it is called a PLANE MECHANISM.

ANALYSIS
- Forward (Routine)
- Reverse (Inverse)

MECHANISM : Inter connected rigid bodies.

MECHANICS :- study of rigid bodies under forces.

MECHANICS
- STATICS (Body under rest)
- DYNAMICS (Body in motion)

STATICS :- Body in equilib.
Net force acting = 0.

DYNAMICS :-

1) Kinematics : It deals with the Geometric variables of motion without considering the causes of motion.

Geometric variable : vel, acc^n, position, disp, jerk etc...

2) Kinetics : study along with forces that cause motion.

LINKS :-
Kinematic elements which are the individual parts of a mechanism.

In a kinematic pair, there should be constrained relative motion b/w the 2 bodies.

CLASSIFICATION OF Kinematic pair :-

(1) Based on nature of the relative motion
a) Turning pair / Revolute / Rotary / Hinge
b) Sliding pair / prismatic pair / Translation.

Combination of turning & sliding pair gives cylindrical pair.

- Spherical pair.

(2) Based on Nature of contact
(a) Lower pairs - surface to surface contact
(b) Higher pairs - point or line contact.

Higher pairs { Eg: Pairs of gear meshed ← Line contact
Cam & follower ← Point contact

(3) Based on nature of closure
(a) Form closure
(b) Force closure.

Contact by ~~form/shape is called form closure~~
If contact is by an external contact,

which is responsible for maintaining contact, is called force contact.

(4) Based on Constraint.

(5) Based on the Degrees of freedom.

For any given system, the no. of individual coord's reqd the specify the location of the body is called D.O.F.

DOF for a rigid body in space = 6 ...

Body in a plane, DOF = 3 (2 translation + 1 rotation

SPATIAL KINEMATIC PAIRS

When 2 links form a pair, it loose some of the individual DOF.

CONSTRAINED DOF	ALLOWED DOF	
5	1	(CLASS-I)
4	2	(CLASS-II)
3	3	(CLASS III)
2	4	(CLASS IV)
1	5	(CLASS V)

PLANAR KINEMATIC PAIRS

C·DOF	A·DOF	
2	1	(class I)
1	2	(class II)

KINEMATIC CHAIN

- Mechanism is obtained by fixing any one link on a kinematic chain.
- So mechanism is derived from kinematic chain.

Closed kinematic chains are most stable than open " ".

Eg: Mechanism of Robot manipulator is an open kinematic chain.

DEGREES OF FREEDOM OF A SPATIAL KINEMATIC CHAIN.

$J_1 \rightarrow$ No: of class I pairs
$J_2 \rightarrow$ " " II "
$J_3, J_4 \rightarrow$ corresp class III, IV pairs.
l = links.

$$DOF = (6 \times l) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5$$

Each class I pair looses 5 degrees of freedom.
class II " " 4 degrees " " etc..

dof for a spatial Mechanism

$$dof = 6(l-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5.$$

dof for a PLANAR KINEMATIC CHAIN.

$$dof = \underbrace{3l}_{l} - \underbrace{2J_1}_{J_1} - \underbrace{1J_2}_{J_2}.$$

~~dof for a PLANAR MECHANISM = $3(l-1) - 2J_1 - J_2$~~

MECHANISM WITH SINGLE dof.

l, $J_1 = J$, $J_2 = 0$.

$1 = 3(l-1) - 2J$.

$J = \frac{3}{2}l - 2$.

If "l" is odd, J becomes fractional.
So "l" has to be even, then "J" becomes a whole no.

- For $l=2$, $J=1$
- $l=4$, $J=4$.
- $l=6$, $J=7$
- $l=8$, $J=10$.

FOUR BAR CHAIN OR QUADRIC CYCLE CHAIN.

(1) Length of longest link < Sum of the other 3

↳ NECCESSARY condition for a 4 bar chain.

(2) Also,
sum of longest + smallest link < sum of other 2.

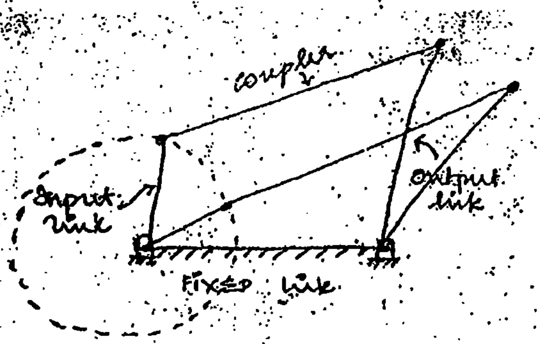
ie $l + s < p + q$.

The above is called the **GRASHOFF's CONDITION**

If $l+s > p+q$: Non Grashoff's kinematic chain.
If $l+s = p+q$: Special " " " .

INVERSION

Process of obtaining diff mechanisms by fixing diff links of a kinematic chain.



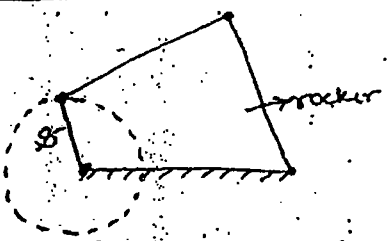
If input link is capable of complete rotation its called a CRANK.

If output link isnt capable of complete revolution, its called a Rocker / oscillatory link / lever.

	Input	output
1.	Crank	Crank
2.	Crank	Rocker
3.	Rocker	Crank
4.	Rocker	Rocker

Inversions (Grashoff's kinematic chain)

(1)



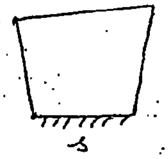
s → shortest link

By fixing the link on ~~other~~ right side of the shortest link, we get the shortest link as crank and the opp link as rocker.

If we fix the link on the left of shortest link we get ~~crank~~ "s" as rocker & opp as crank.

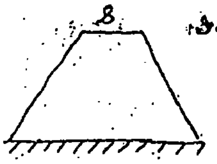
So 2 mechanisms: crank-rocker & Rocker-crank.

(2)



Crank - crank

(3)



If s = coupler.

Rocker - Rocker

INVERSIONS {NON - GRASHOFF's}

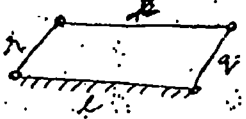
All inversions are double rockers {R-R's}.

INVERSIONS { special Grashoff's }

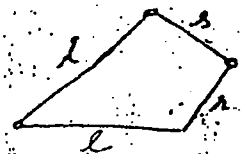
$l + s = p + q$

If $l = p$, $s = q$: Parallelogram linkage

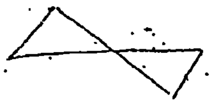
In a ||gm linkage all inversions are double cranks.



} parallelogram



} DELT



} Anti parallelogram

Fix s : C-C
Fix "l" : R-R

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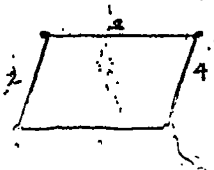
EQUIVALENT CHAIN

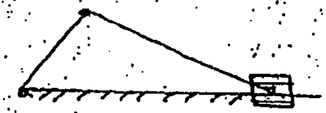
Replacing a turning pair with a sliding pair in a 4-bar chain we get a Equivt chain.

We get a SLIDER crank chain.



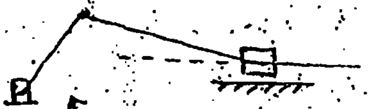
INVERSIONS OF SLIDER CRANK

(1)



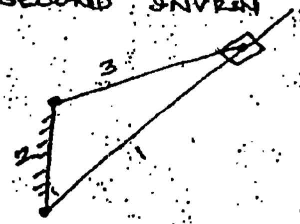
FIRST INV.

Appln of 1st inversn i.e I.C engine



OFFSET SLIDER CRANK MECHANISM

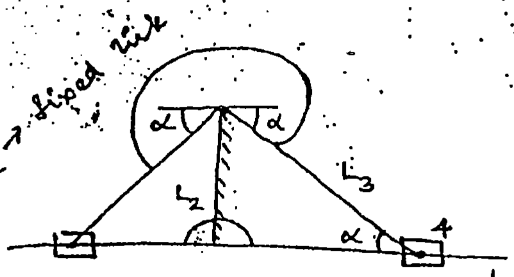
(2) SECOND INVRSN (double crank)



As Time α Angular disp

$180 + 2\alpha$ → Time for FORWARD STROKE

$180 - 2\alpha$ → Time for REVERSE STROKE



Quick Return Ratio,

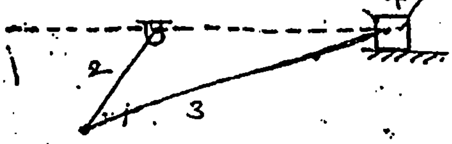
$$QRR = \frac{180 + 2\alpha}{180 - 2\alpha}$$

$$\alpha = \sin^{-1} \frac{l_2}{l_3}$$

↳ Quick return mechanism

Applicn : In shaper, planing and cutting operations

Cutting tool

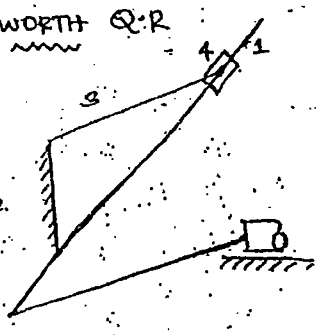


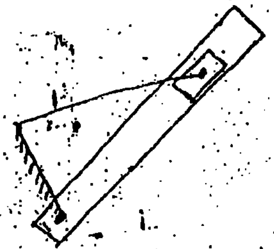
This could have been used as the quick return mechanism but drive cannot be given on link 2. so we combine 1st inv & 2nd inversion.

1st In

I.C engine

WHITWORTH Q.R





ROTARY CYLINDER MECHANISM.

Whitworth Q.R mechanism is a combination of 2nd inversion and 1st inversion.

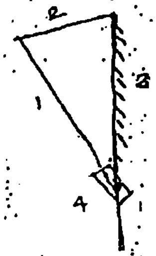
Whitworth & Rotary → application of 2nd inversion

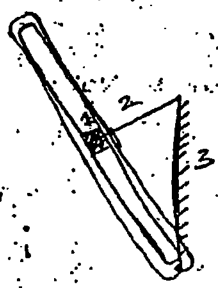
(3) 3rd INVERSION

ORWARD
OKE

REVERSE
OKE

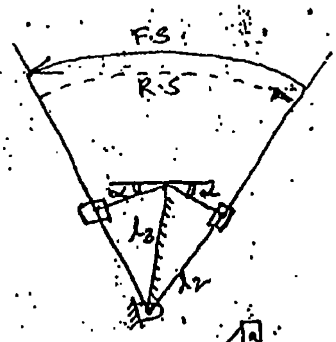
(*)





SLOTTED LEVER MECHANISM.

(*)

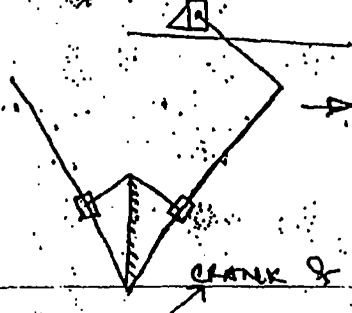


$$\frac{T_{FS}}{T_{RS}} = \frac{180+2\alpha}{180-2\alpha}$$

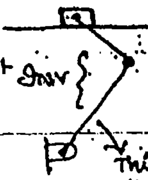
$\alpha = \sin^{-1} l_2/l_3$

l_3 → Fixed link.

(*)



→ combination of 3rd inversion and the offset slider crank (1st) inversion.



CRANK & SLOTTED LEVER type

This is ... in this mechanism ... a combination of 3rd & 1st.

(4) FOURTH INVERSION

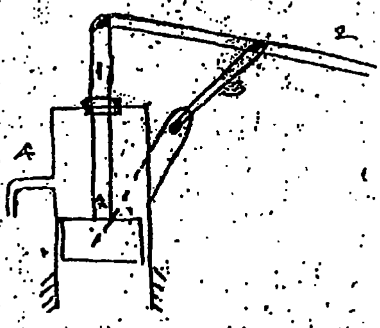
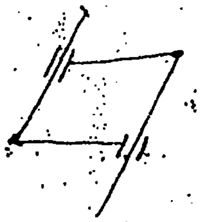
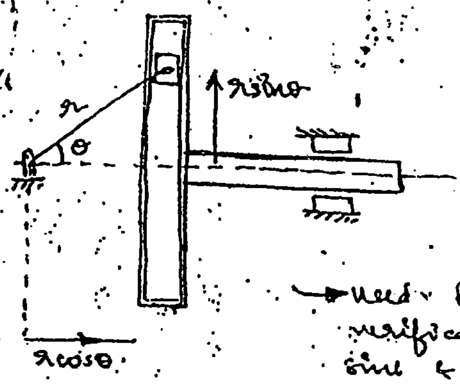


fig: Applicatn. of 4th inversion in a hand-pump.

DOUBLE SLIDER CRANK CHAIN f/re 4-bar

(1) 2 sliding pairs, 2 turning pairs & a CRANK
↳ called scotch Yoke mechanism



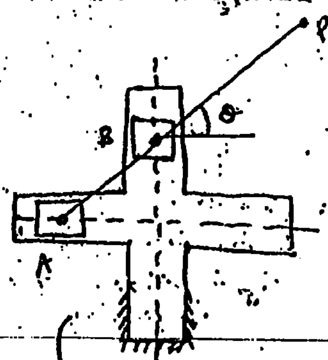


To generate a Harmonic motion mechanically ⟹

→ Used for the verification of sine & cosine fn in olden times.

To loca- the

(2) ELLIPTICAL TRAMMEL



$x_p = BP\cos\theta$

$y_p = AP\sin\theta$

$$\left(\frac{x_p}{BP}\right)^2 + \left(\frac{y_p}{AP}\right)^2 = 1$$

If AP = BP → Circle is generated.

An ellipse is generated}

(B) OLDHAM'S COUPLING

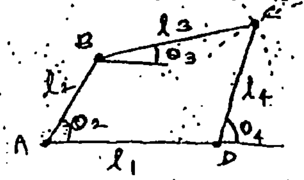
Its another inversion of Double slider double crank 4-bar mechanism.

- used to join non-parallel, i.e. mis-aligned shafts.
- used for small mis-alignments, low speed applications.

POSITION, VEL, ACCLN ANALYSIS

POSITION ANALYSIS

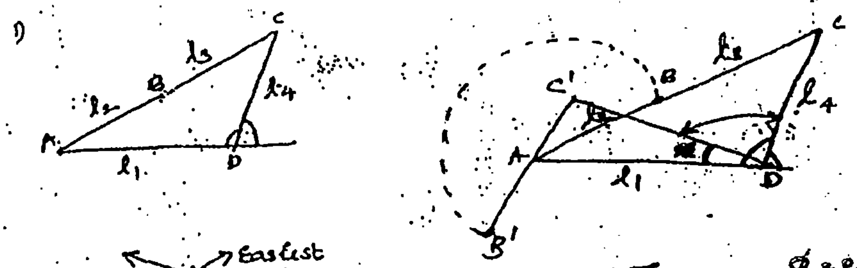
4-bar mechanism :-

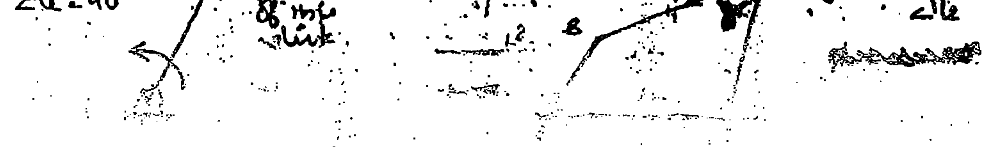


To locate the pt "C"

$$\begin{cases} x_C = l_2\cos\theta_2 + l_3\cos\theta_3 = l_1 + l_4\cos\theta_4 \\ y_C = l_2\sin\theta_2 + l_3\sin\theta_3 = l_4\sin\theta_4 \end{cases}$$

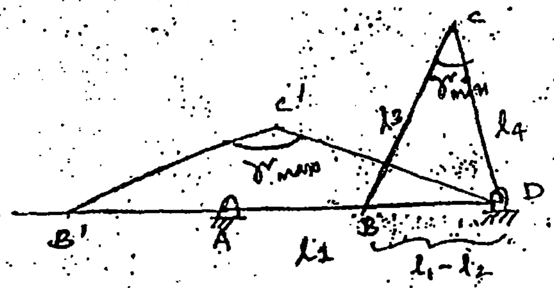
Extreme position : (for the rocker or lever)



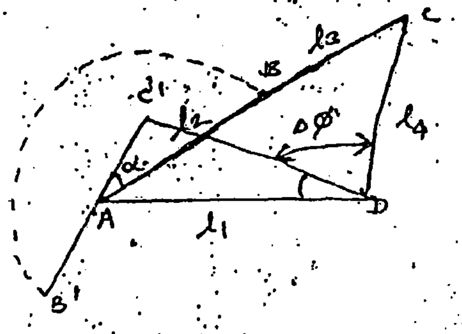


$\gamma_{transmission\ \angle le} = 90° \pm 30°$ is best $\angle$le

∴ when l_1 is aligned with l_2 : we get max transmission $\angle$le.



This is also a quick return mechanism

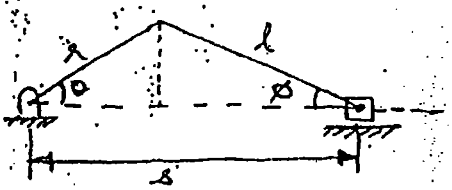


$$\frac{180+\alpha}{180-\alpha} = QRR$$

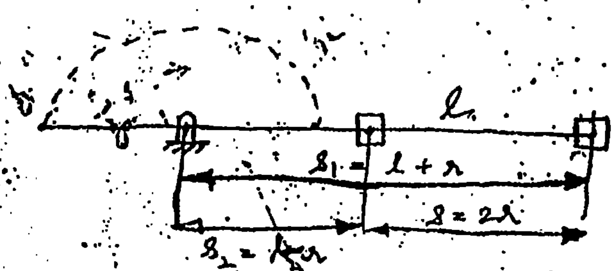
$\alpha \rightarrow$ Angle that indicates the angular displacement at the i/p crank.

$s = r\cos\theta + l\cos\phi$

$r\sin\theta = l\sin\phi$

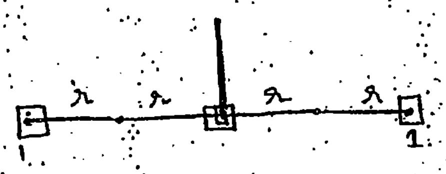


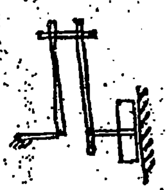
Extreme positions :-



Q: What is the length of stroke if the length of connecting rod is equal to length of crank?

a) r   b) 2r   c) 3r   d) 4r ✓



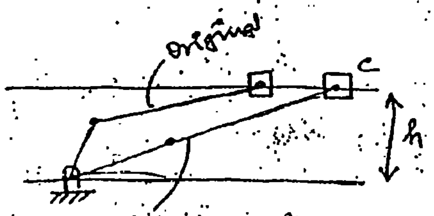


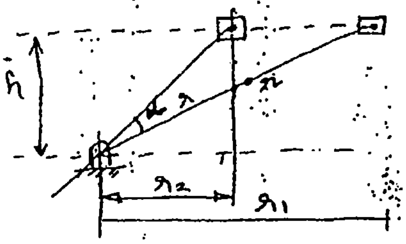
OFFSET SLIDER CRANK MECHANISM

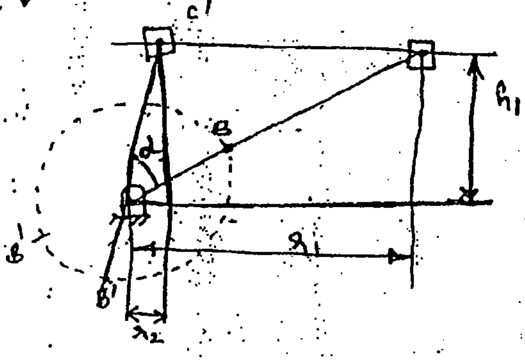
forward stroke

F.S → 180 + α

B.S → 180 − α







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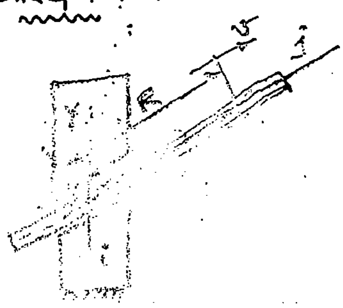
VELOCITY AND ACCELERATION ANALYSIS

SLIDING PAIRS



$v = \frac{dr}{dt} = \dot{r}$

$f = \frac{dv}{dt} = \ddot{r}$

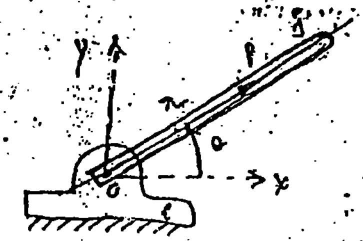
- If position vector inc
  ie posn vector radially outwards } +ve vel

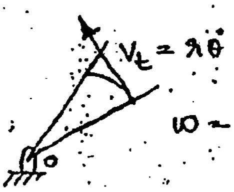
  If positn vector dec
  ie p.v. radially inwards } −ve vel.

- If link "i" is STATIONARY → the vel is ABSOLUTE vel

  If link "i" is MOVING → the vel is RELATIVE vel.

TURNING PAIR





$\omega = \frac{d\theta}{dt}$

vel → Always tangential wrt O.

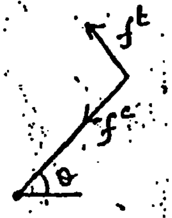
Accelerations :

(1) - Tangential accn
  - parallel to tangential vel
  - ⊥'r to position vector

$$f^t = r\alpha \qquad \alpha = \frac{d\omega}{dt} = \ddot{\theta}$$

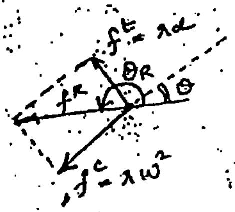
(2) Centripetal accn
  - always acts towards the centre of rotation
  - responsible for tilt or change in direcn of vel.

$$f^c = r\omega^2.$$



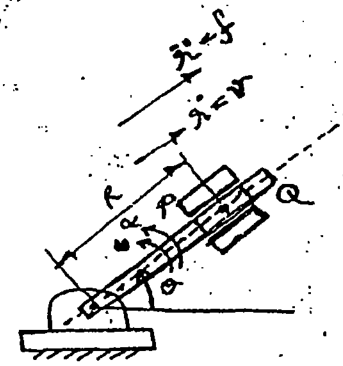
- If ω and α have the same sign {ie its magnitude inc with resp to time}, they are in the same direction.
- If ω and α have same sign & its accelerato
- If ω and α have diff sign : then it is decceleration.

RESULTANT acceleration :-



f^R - resultant accn
θ_R - angle the resultant makes.

TURN - SLIDE PAIR

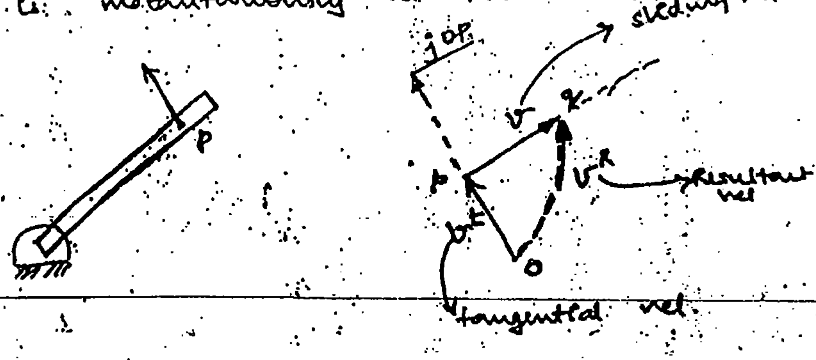


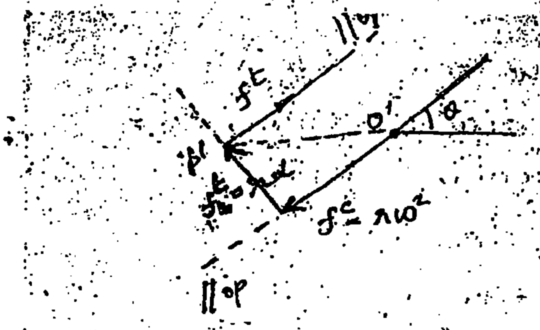
combination of turning and sliding.

"Q" has some relative motion wrt "P".

Instantaneously P & Q coincides and hence they are called COINCIDENT POINTS.

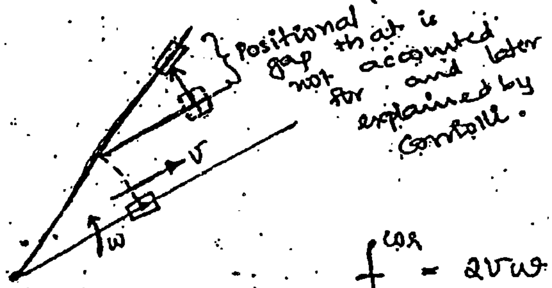
Instantaneous ang vel = ω.
Instantaneous ang accn = α
Block is instantaneously at "R".





CORRIOLLI'S COMPONENT

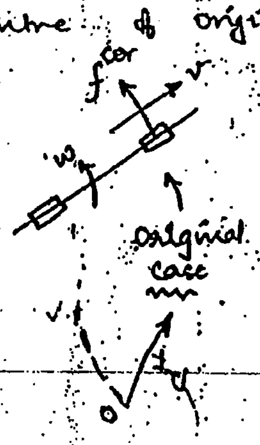
Whenever axis of the sliding pair undergoes rotation → Corriolli's component of acceleration

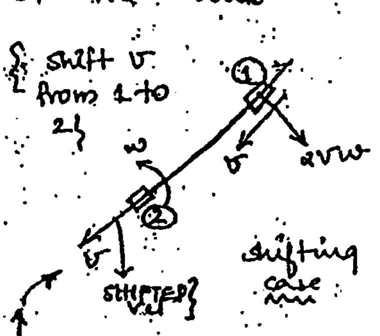


$$f^{cor} = 2v\omega$$

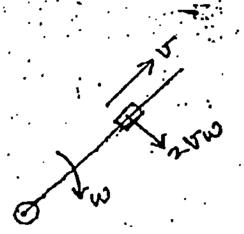
By rotating the vel vector in the directn of angular vel by 90° → To get DIRECTION of Corriolli's acceln

- vel vector has to be drawn from the centre of origin of the link

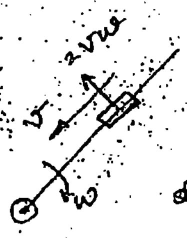




Here the vel has to be shifted from 1 to the pt of origin of the link ie:(2).

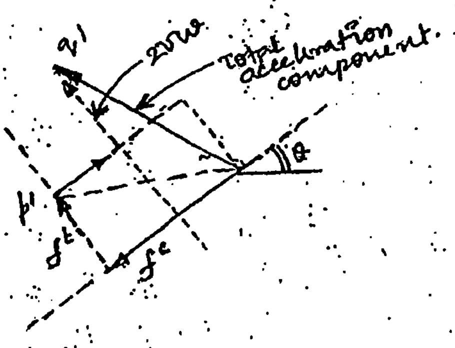


shifting of vel to origin of link

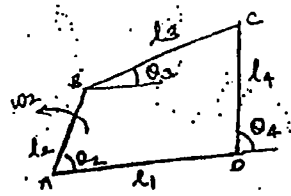


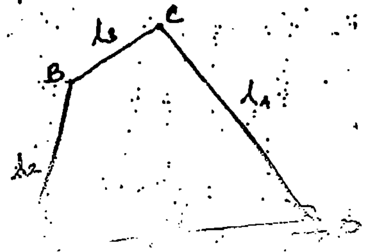
shifting of vel to the origin of link.

- Using corriolli's for getting TOTAL acceleration:



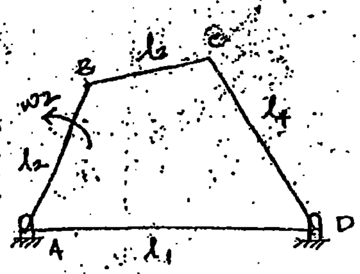
- VELOCITY ANALYSIS OF A FOUR BAR MECHANISM





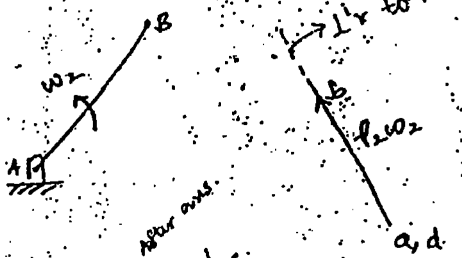
A & B are the reference pts (with absolute zero velocity) for the entire velocities.

Analysis:

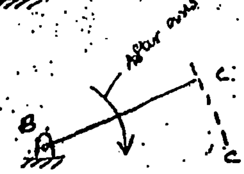


$\omega_3 = ?$
$\omega_4 = ?$

For AB:



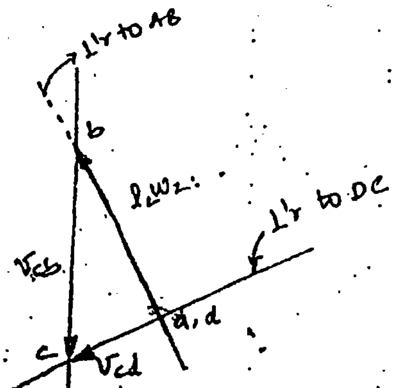
For BC:



: Not possible to arrive at a conclusion

For CD:

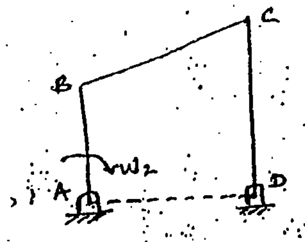
: Vel. of c wrt , $\perp$'r to DC.

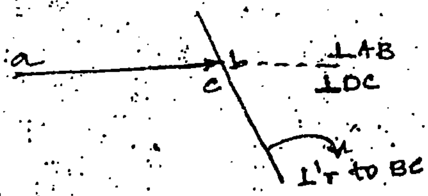


$$v_{cb} = l_3\omega_3 \implies \omega_3 = \frac{v_{cb}}{l_3}$$

$$v_{cd} = l_4\omega_4 \implies \omega_4 = \frac{v_{cd}}{l_4}$$

CASE 1 : AB and DC are parallel to each other





b and c coincides

$V_{cb} = 0$

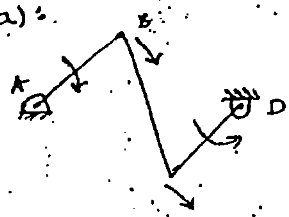
$\omega_3 = 0$

$V_{cd} = V_{ba}$

$l_4 \omega_4 = l_2 \omega_2$

$V_b = V_c$

CASE 1(a) :

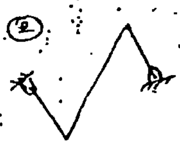


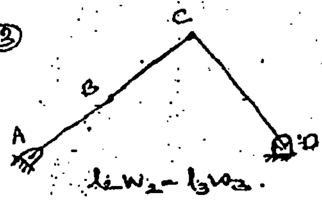
Here i/p and o/p will move in opp directns.

Here also AB || CD

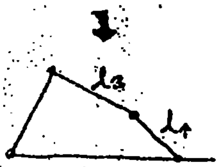
Rotate the fig : (to get diff configuratns)

① 

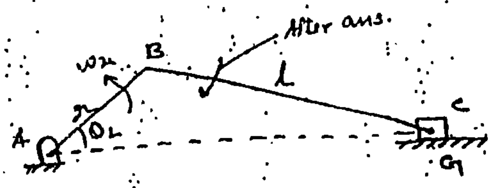
② 

③ 

$l_2 W_2 - l_3 \omega_3$.



SLIDER - CRANK MECHANISM

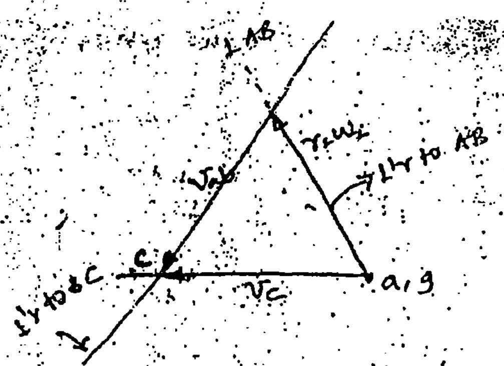


$\omega_3 = ?$ $V_g = ?$

Fixed pts → A and G.

AB is crank.

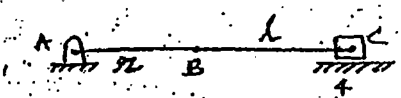
$V_{c\,wrt\,b}$ → ⊥'r to BC etc....

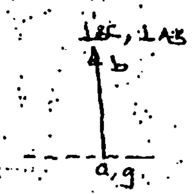


$$V_{cb} = l\omega_3 \Rightarrow \omega_3 = \frac{V_{cb}}{l} = \frac{\vec{bc}}{BC}$$

$$V_{slider} = V_c = \vec{gc} \text{ or } \vec{ac}$$

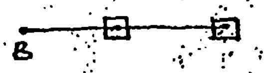
Ques: 180° configuration:





{ v = 0 for piston
so DEAD centre position

$V_c = 0$.

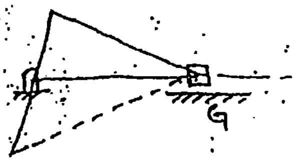


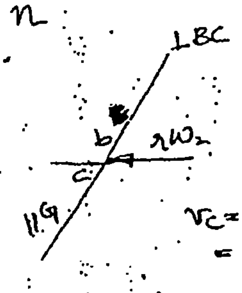
$V_{cb} = -V_{ba}$

$l\omega_3 = -r\omega_2$

$\omega_3 = -\frac{r\omega_2}{l} = \frac{-\omega_2}{n}$

{ $V_{slider} = V_{tip}$ of crank

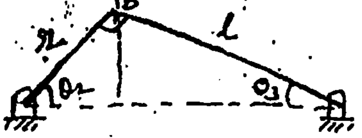




$V_c = V_b$
$= r\omega_2$.

$V_{slider} = V_{tip}$ of the crank

Ques:



$\theta_3 = 90 - \theta_2$.

$r\sin\theta_2 = l\sin\theta_3$
$= l\cos\theta_2$

$= \frac{\vec{bc}}{BC}$

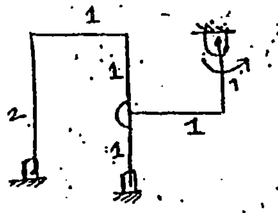
ston
nbe
D

$= V_{tip}$ of crank

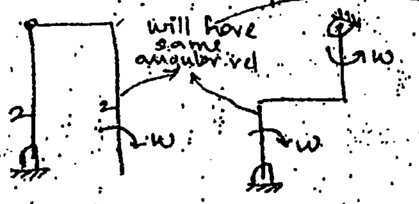
of the
ink

$\therefore \tan\theta_2 = l/r$

Qns :



Seperate the figure into 2 so that it becomes more simpler...



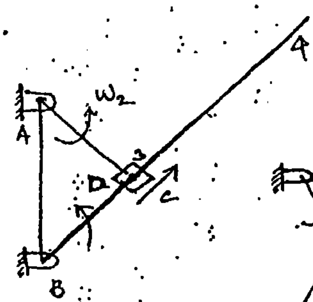
These 2 links are linked ∴ They are together (adjacent)
{ so angular vel of one is same as that on another }

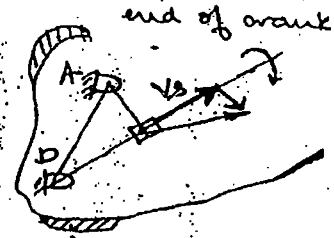
2-5
5.30-1

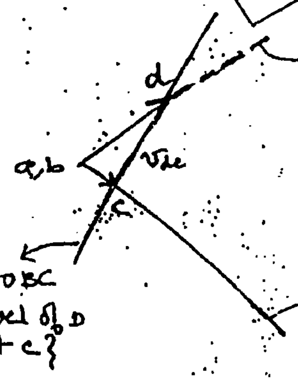
NOV 13' 2010

VELOCITY ANALYSIS FOR INVERSIONS OF SLIDER CRANK

D → a pt on the block, at the end of crank







vel of D wrt C is $||^l$ to BC.

{ ~~C is a pt on link BC, At the moment C & D~~ coincides, D slides along the link }

$V_{da} = AD \cdot \omega_2$

$\omega_4 =$

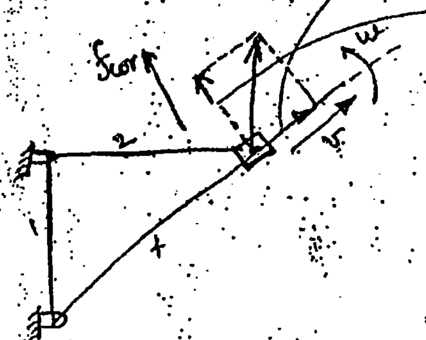
$V_{cb} = \omega_4 \times BC$

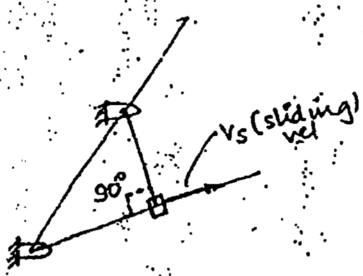
$\omega_4 = \dfrac{V_{cb}}{BC}$

$V_{sliding} = \overline{cd}$

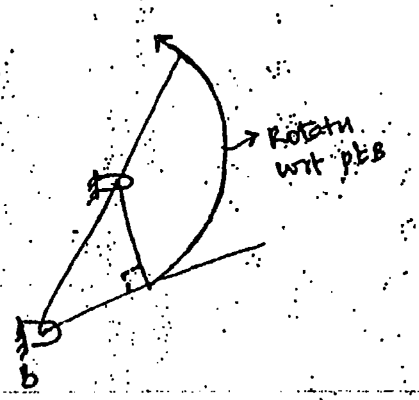
$f_{cr} = 2v\omega = 2V_{sliding} \times \omega_4$

↳ corriolli's component of accn





Vs is the sliding vel.
Here in this case,
Vs exists,
but tangential component
is zero.
Hence the corriolli's
accn is zero in this
case when the angle
included is 90°

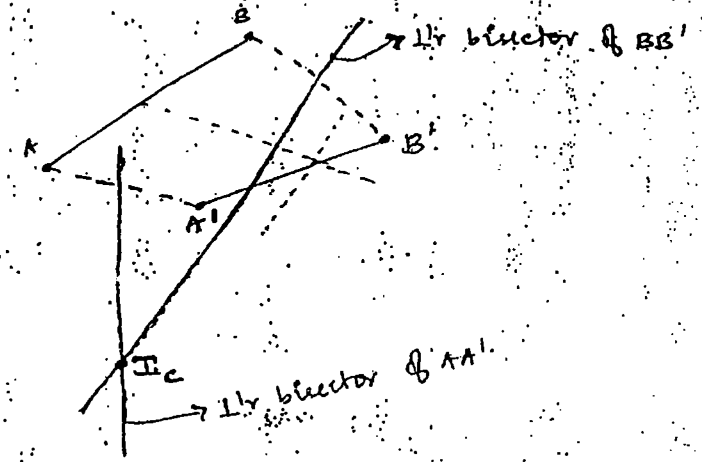


$l_4 \omega_4$ = vel. of the link
when there is a
rotation wrt point 'B'.

ANALYSIS USING INSTANTANEOUS CENTRES :-

General rigid body ~~motion~~ motion = Translation + Rotation.

For a particular pt the motion of the link can be pure rotation ⇒ this pt is called instantaneous centre.



→ For "n" rigid bodies in motion, there are $^{n}C_2$ instantaneous centres

Eg: for a 4 bar mechanism

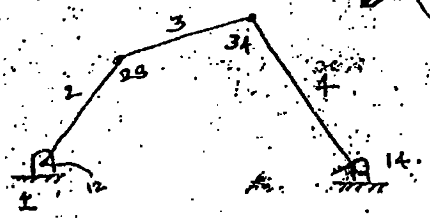
4 links ⇒ No. of IC's = $^{4}C_2 = \frac{4\times(4-1)}{2} = 6$

→ For a disc rotating w/o slipping: IC will lie at the contact pt.

→ If the disc is slipping, IC will lie on the axis of rotation and

the IC will lie on the ~~[illegible]~~ [illegible] common straight line (ARNOLD / KENNEDY's theorem)

Vel Analysis 4-Bar mechanism using Instantaneous centres

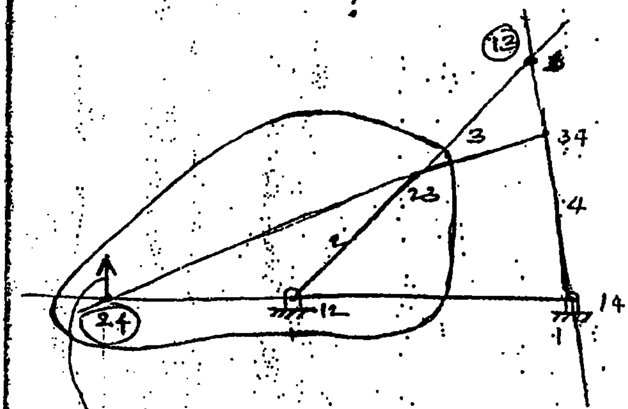


when 2 pairs are joined directly, the joint itself is an instantaneous centre.

IC for 1-3:

1-3 not directly connected, {so use the Kenedy's theorem or three centre theorem}.

13		24	
123	143	124	324
12 23 13	13 14 34	12 14 24	23 34 24
	?	?	?



ω_{input} known $\Rightarrow$ $\omega_{output} = ?$

$V_{24} = (12-24)\,\omega_2 = (14-24)\,\omega_4$

If the body is a part of link 2

If the body is a part of link 4

$V_{23} = (12-23)\,\omega_2$

$= (13-23)\,\omega_3$

Eg:

Instantaneous centre at infinity

so $V_{23} = (12-23)\,\omega_2$

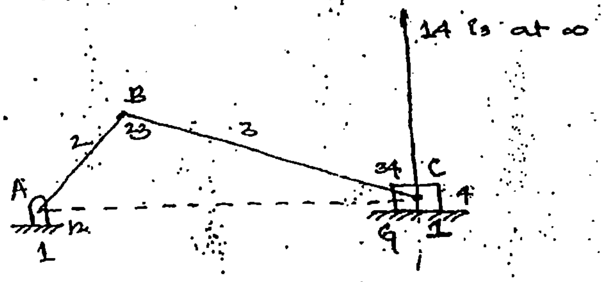
$V_{23} = (13-23)\,\omega_3$

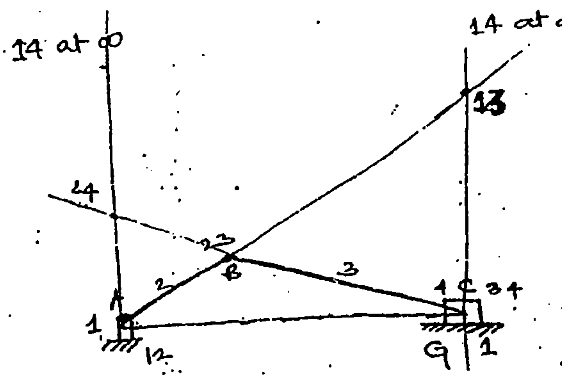
If this dist $= \infty$ ω_3 has to be zero.

Ans: $\underline{\omega_3 = 0}$

day 5

INSTANTANEOUS CENTRE METHOD FOR A SLIDER CRANK MECHANISM.





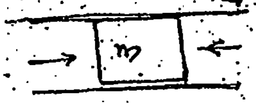
$V_{24} = (12-24)\,\omega_2 = V_4'$ (sliding vel)

$V_{23} = (12-23)\,\omega_2 = (13-23)\,\omega_3$

$V_{34} = (13-34)\,\omega_3 = V_4$

DYNAMIC ANALYSIS

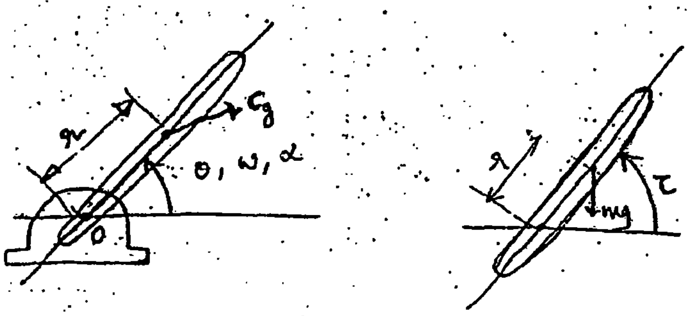
SLIDING PAIR :-



$\Sigma F = ma$

Net force along the axis of reciprocation.

TURNING PAIR :-



$$M_o = \tau - mgr\cos\theta = I_o \alpha.$$

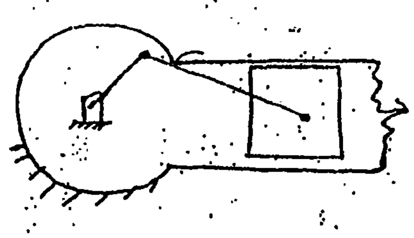
$$\alpha = \ddot{\theta} \quad \{\text{angle } \ddot{\theta}\}.$$

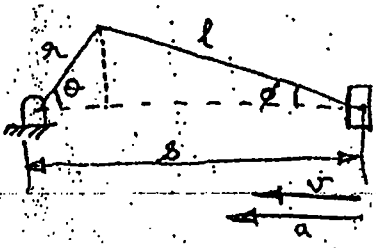
$$I_o \ddot{\theta} + mgr\cos\theta = \tau$$ Torque.

DYN ANALYSIS OF SLIDER CRANK BASED m/c

3 turning pairs }
1 sliding pair }

Slider crank mechanism based m/c.





$s = r\cos\theta + l\cos\phi$

$r\sin\theta = l\sin\phi$, $w = \dot{\theta}$

$v = r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$

$a = r\omega\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$\Big\}$ $n = \frac{l}{r}$ ratio

Net force on the piston = PISTON EFFORT.

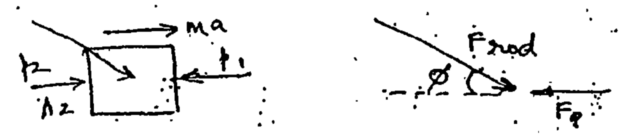
Pressure on the connecting rod = Thrust

Thrust leads to turning moment

Piston effort consists of :

$F_p = F_g - ma$ → force due to gas

m = mass of

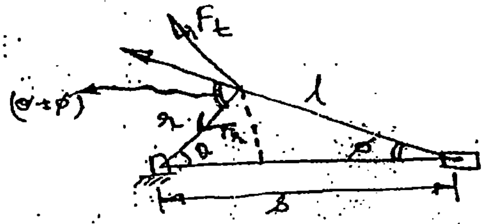


$F_p = F_g - ma$

$F_{rod} = F_p / \cos\phi$

F_t = Tangential force on the crank

(F_t = CRANK Effort)



→ we have ignored the mass of the connecting rod.

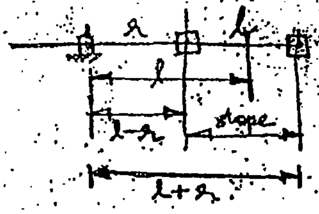
- $F_t = F_{rod} \times \sin(\theta+\phi)$
- $F_r = F_{rod} \times \cos(\theta+\phi)$
- $F_t = \dfrac{F_p \sin(\theta+\phi)}{\cos\phi}$

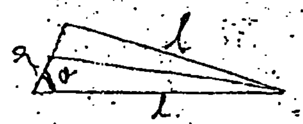
Turning moment, TM = $F_t \times r$

$$\boxed{(TM) = \frac{r F_p \sin(\theta+\phi)}{\cos\phi}}$$

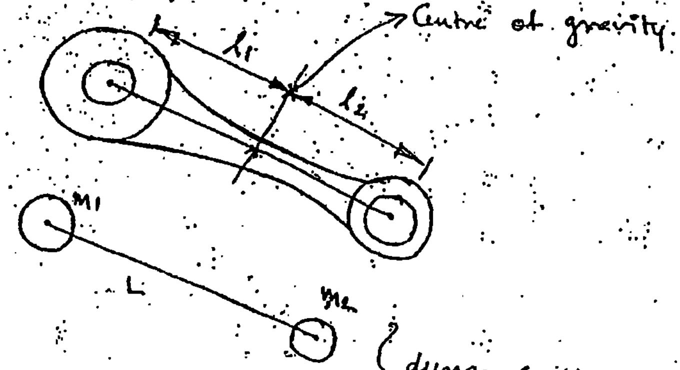
Crank effort
turning moment } o/p of the engine

eg:





DYNAMICALLY EQUIVALENT LINK (DEL)



{ dynamically equivalent link

conditions for DEL:-

1) $m_1 + m_2 = m$.

2. $m_1 . l_1 = m_2 l_2$

3) $I_{eq}(cg) = m_1 l_1 + m_2 l_2$.

3 eqns cant be satisfied

↓ so we use the first 2 eqns
But then we mention that the moment of inertias are not the same.

so to account for the diff in MOI b/n the actual link & the DEL { Error of moment of inertia }, we introduce a compensation for this error.

(GT) (18)

Pg 5 [CRO]

1 (c) $d = (l-1)3 - J \cdot 2$ $\quad \left\} \begin{matrix} l = 8 \\ J = 9 \end{matrix} \right.$

$= 21 - 18 = 3$

2 (b) $AB = 265 \quad \{\sqrt{240^2 - 100^2}\}$

Its hence a Grashoff's linkage.

Also its a crank rocker mechanism

(link adj to shortest link is fixed)

3 (b) I/p link and o/p link are parallel.

$V_A = V_B$

$OA \times \omega_2 = OB \times \omega_4$

$60 \times 8 = 160 \times \omega_4$

$\omega_4 = \frac{60 \times 8}{160} = 3 \text{ rad/s}$

$\frac{60 \times 1}{160}$

4 (b) since planar → no gravity terms in eqn

$(mg_2 L \sin\theta) = 0$.

$\alpha = 0$ → No inertial torque.

{Axis of radius & centre of mass coincides, hence no centrifugal force}

since along AB 30 N acts,

At O_2 also 30 N should act.

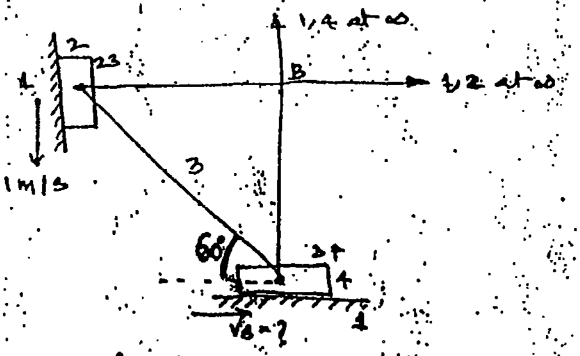
5 ( ) $f_{coriolis} = 2V\omega$

$= 2 \times 12 \times \left(\frac{2\pi \times 120}{60}\right) \omega$

$\left\{ \omega = \frac{2\pi N}{60} \right\}$

$\omega = 2 \times \pi \times \frac{120}{60}$

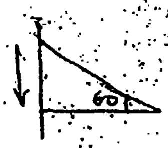
6 (d) AND 22



24
12 4
12 14 24

I_{24} is at ∞.

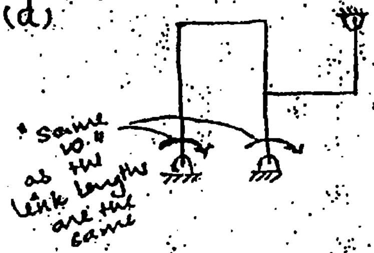
22 ( ) same fig as 6



7 (c) R-2

8 (c) P-8 { Allow 1; 5 are constrained for revolute pair }.

9 (d)



OD = 100 mm

ω = 2

v = rω = 200 mm/s

Centripetal accn = 400 mm/s²

10 (c)

11 ( ) Vel polygon is a rt ∠'led Δ'le.

Imp

{∠'le b/n crank and connecting rod = 90°}

Use pythogaras

12 (c) $v_{BA} = ?$

$v_{BA} = v_B - v_A$

$v_B = OB \times \omega$, $v_A = OA \times \omega$

$v_{BA} = (r_b \times \omega) - (r_a \times \omega) = (r_b - r_a)\omega$

13 (d) ω uniform
so only centripetal accn.

$f_{BA} = f_B - f_A = (r_b - r_a)\omega^2$

14 (b) Higher pair - class II

If only lower pairs are allowed, the min no. of links is 4.

{ If higher is allowed, then the no. of links is 3. }

15(b) Only when axis of the slider undergoes rotation so it the 2^{nd} or 3^{rd} inversion of slider crank.

⇒ SHAPER

16(d) Hooke's / universal ⟶ Large misalignment
Oldham's ⟶ small misalignment.

18( ) $V_B = EB \times \omega_3 = 0.5$ m/s.

$\omega_3 = $ —.

$V_c = EC \times \omega_3$

19(b) Pure rolling ⟶ Its at the centre of disc ⟶ (1)
Pure sliding ⟶ Its at centre of surface ⟶ (2)
Both rolling + sliding ⟶ Its the line joining (1) & (2).

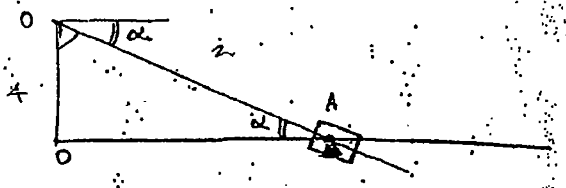
↳ so common N^l at the contact pt.

20(c) 4 links, 3 turning pairs ⟶ 12 dof

12 dof − (5 × 2) = 2 //

21(b) LINK fixed is SHORTEST ⟶ double CRANK mechanism.

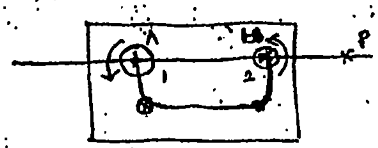
reason



WHITWORTH mechanism

$\alpha = \sin^{-1}\left(\frac{OO'}{OA}\right)$

23( )



$AP(\omega_1) = BP(\omega_2)$

$r_1\omega_1 = r_2\omega_2$

$\omega_2 = \frac{r_1}{r_2}\omega_1 = 2\omega_1$

Try ↓
2 pulleys connected to a belt drive.

ite

ulss

24 ( ) x coord , $x_B = p$

y coord , $y_B = p\tan\theta$

vel in x dir $= \frac{dp}{dt} = 0$

vel in y dir $(V_y) = \frac{d(p\tan\theta)}{dt} = p\sec^2\theta \cdot \dot{\theta}$

$= \omega p \sec^2\theta$ //

$\left\{ \dot{\theta} = \frac{d\theta}{dt} = \omega \right.$

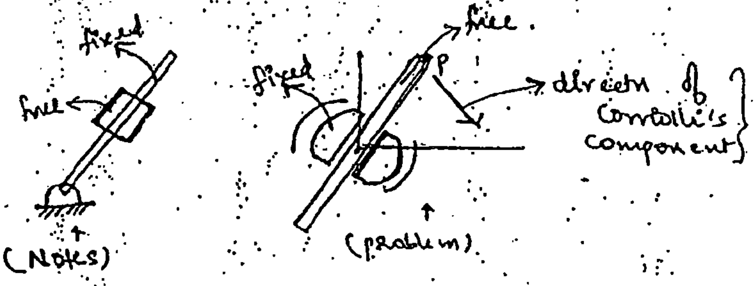
$a_x = 0$

$a_y = \frac{d(V_y)}{dt} = -2\cos^{-3}\theta\,(-\sin\theta)\,p\,\dot{\theta}\,\dot{\theta}$

$= 2p\omega^2 \frac{\sin\theta}{\cos^3\theta}$ //

Note: If $v_x \neq 0$,
then find resultant of v_x and v_y

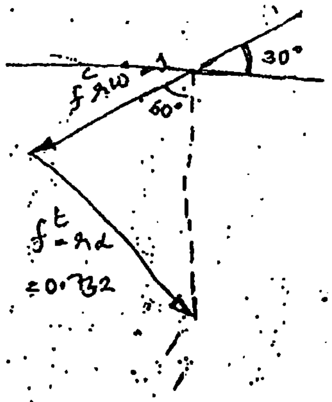
25( )



No radial acceln

v is -ve, ω^2 is +ve.

$f^{coriolis} = 2v\omega_2$
$= 1\,m/s^2$



26 (d)

Mechanical adv $= \frac{1}{\text{vel ratio}}$

vel ratio $= \frac{\text{vel output member}}{\text{vel of input member}}$

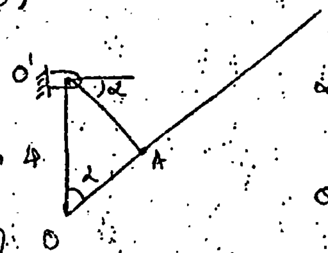
∴ Mech adv $= \infty$

$= \frac{d\theta}{dt} = \omega$

27(c)

28(b)

21 (b)



$\alpha = \sin^{-1}\left(\frac{O'A}{OO'}\right)$

$30°$,

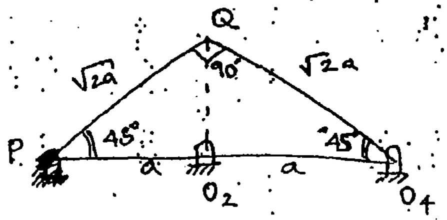
$QRR = \frac{180+2\alpha}{180-2\alpha} = 2$.

3rd inversion → Crank & slotted lever mechanism

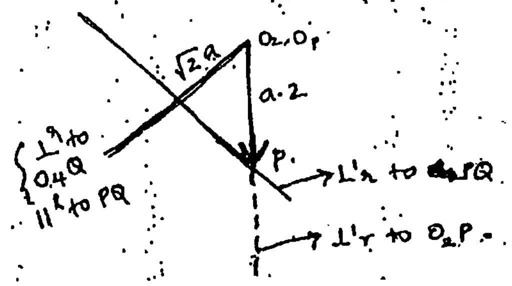
29(d)

30(a) Atleast one of the links must be a CRANK
↳ so Grashoff's or special Grashoffs.

31(c)



Relative vel method :-

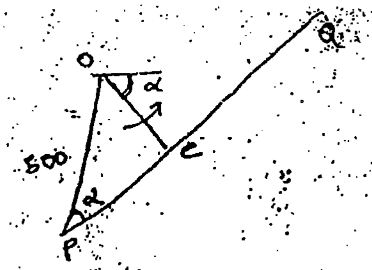


$V_{qp} = pq = \sqrt{2}a = l_3 w_3$

$w_3 = \frac{\sqrt{2}a}{\sqrt{2}a} = 1$ rad/sec

$v_{30_2} = \sqrt{2}a$

33 (c.)



$$QRR = \frac{180 + 2\alpha}{180 - 2\alpha} = 2$$

$\alpha = 30°$

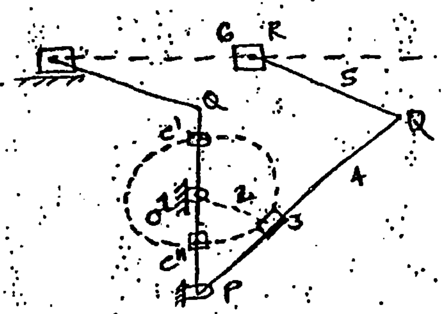
$\sin\alpha = \frac{OC}{OP}$; $\sin 30 = 1/2$

$OC = OP/2 = 250$ mm //

34 (b) Max speed is when the angle is 90°

i.e. when i/p link is ⊥r to line of stroke, slider ~~[illegible]~~ will have max vel.

{This can happen in forward and return stroke}



$W_4 \, PC' = W_2 \, OC'$

$W_4 (750) = 2(\pi/3)\, 210$

$$W_4 = \frac{2 \times 210}{750} = 2/3 \text{ rev per sec} //$$

During return stroke:

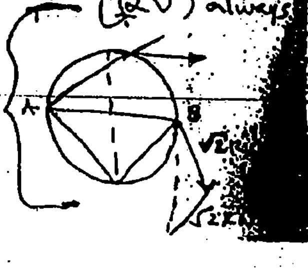
$W_4 \, PC'' = W_2 \, OC''$

$\omega_1 = \omega_2$

So return stroke ω and forward stroke ω are different.

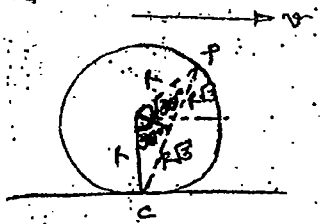
full vel b/w the ends of diam (2V) always

35 (b) Ans: 1



31 Qw
A:

36( )



$v = R\omega$

$\omega = v/R$

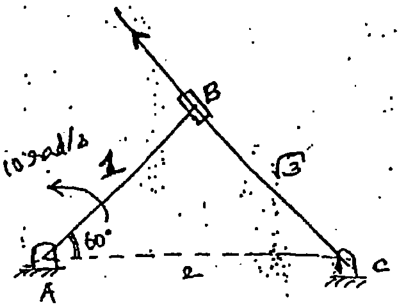
$CP = 2R\frac{\sqrt{3}}{2} = \sqrt{3}R$

$V_p = \sqrt{3}R\omega = \sqrt{3}R\frac{v}{R} = \sqrt{3}v$

$V_p = CP.\omega$

Ques: For the config shown, the angular vel of link AB is 10 rad/s counter clockwise. The magnitude of the relative sliding vel in m/s of slider B wrt link AB is...?

Ans:

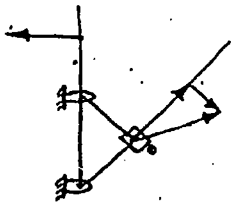


AB = 250

BC = $250\sqrt{3}$

AC = 500

slider vel = AB. ω_2.



Horiz comp of s shown balance this side. Hence this is also s

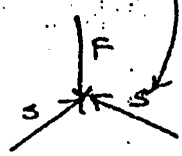
Pg 11 [Additional objectives].

1.(d) $P = S\cos\alpha$.

$S\sin\alpha + S\sin\alpha = F$

$F = 2S\sin\alpha$

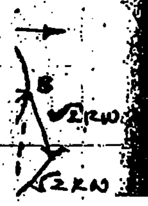
$= \frac{2P}{\cos\alpha}\sin\alpha = 2P\tan\alpha$ //



Note

2

1 know the J) always



2·(b) $v_B = v_C$

$AB\ \omega_2 = DC(\omega_4)$

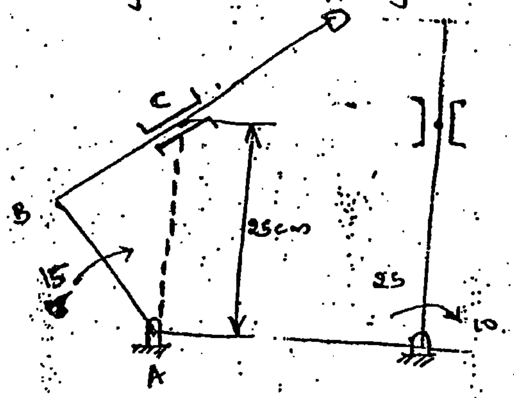
$CD = AB + 30$

$AB\ (5) = (AB + 30)\,2.$

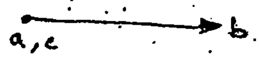
$AB = 20$ cm

3·(a) Rolling w/o slipping → contact pt. So A.

4·(d)



$v_{cb} = v_{ba}$



$BC\ \omega_3 = AB\ \omega_2$

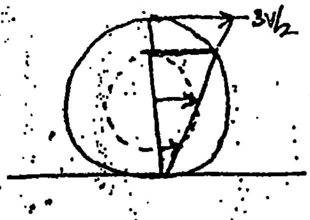
$\omega_3 = \frac{AB}{BC}\ \omega_2 = \frac{15}{10} \times 10 = 15$ rad/s

5(c) Rubbing vel = Radius × (Relative radial vel)

$= r(\omega_1 + \omega_2)$

6(c)

7(c)



$\omega = V/r.$

$\xrightarrow{r\,\omega} v$

13(b) 1 → No 2 pts have same vel.

15( ) $v_{BA} = v_B - v_A = r_b\omega - r_a\omega$

$= (AB)\ \omega$

$140 - 80 = 300\ \omega$

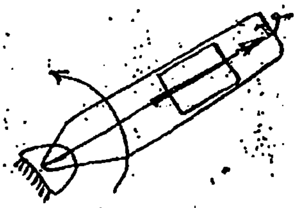
$\frac{60000}{300} = 200$ rad/sec //

$V_B = OB\,\omega = 140$

$OB = \frac{140}{\omega} = 700$ mm

DIAM = $2 \times 700 = 1400$ mm (Diam = 2 OB)

8 (a)



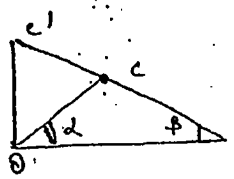
Uniform rotation } so No tangential

{ piston reciprocating & cylinder rotating : so Corrcolis's accn is present.

9 (a)

They have to form a kinematic pair first. otherwise there is no question of constrain

10 (a)
11 (b)



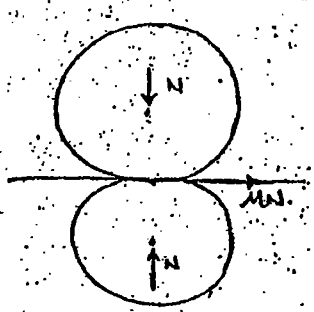
vel = oc'(ω)

vel)

12 (d)
14 (d)

GEARS

- Toothed wheel to transmit power from one shaft to another.



FRICTION DISC

- Used only for slow power transmission
- Torque transmitted is low.

$\tau = MN \times r$

- By traditional methods involute is easier to generate than cycloid but with the advent of NC m/c's both are equally easy.

- Involute gears : Contact b/n convex region
  Cycloidal gears : Contact b/n convex & concave regions

- Contact stress is more for 2 convex regions
  As contact stress ↑ , wear ↑

- Hence higher contact stress for involute, hence higher wear for involute

- ϕ = pressure angle.
  ϕ for involute

  $r_b = r_p \cos\phi$ (remains constant)

- For cycloidal, pressure angle keeps on varying inducing variable transmission loads. (disadv)

- In case of cycloidal gears, pr an...
and hence variation in centre di...
the vel ratio (disadv)
Hence they are no longer conjugate a...
- Interference : contact b/n 2 Non-conjug... regions.
- Below base circle, involute generation is not possible. so we use circular arc, this region comes into contact with the tip of the other tooth (which is a conjugate part)
Circular arc : its not a conjugate region
Region which is not involute can be removed by UNDER-CUTTING.
- Undercutting reduces the strength of the gear
- If $\phi \uparrow$, T_{min} (min no. of teeth) can be reduced
- composite teeth :-
wherever a non-involute exists, make a cycloidal.

$\left.\begin{array}{l}\text{Hypocycloidal at bottom}\\ \text{Involute at middle}\\ \text{Epi cycloid at top}\end{array}\right\}$ composite teeth profile.

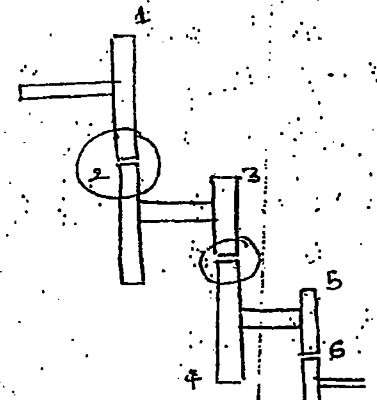
- Arc of contact = $\dfrac{\text{path of contact}}{\cos\phi}$
- No. of pairs of teeth in contact = contact ratio

CONTACT RATIO :-
Its the ratio of arc of contact to circular pitch
Recommended value = 1.4 for contact ratio
(1.2 to 1.35)

- For 20% of time load is shared by the prev pair of contact teeth.
  For 60% of time load is taken by the present pair of contact teeth
  For the last 20% of the time the load is taken by the next pair of teeth.
- Min contact ratio is desired.

COMPOUND GEAR TRAINS

Gear trains with one or more compound wheel.
- If 2 or more gears are keyed on to the same shaft → its called COMPOUND.



$$VR = \frac{N_1}{N_4} = \frac{N_1}{N_2}\frac{N_2}{N_3}\frac{N_3}{N_4}$$

$$= -\frac{T_2}{T_1}\cdot 1 \cdot -\frac{T_4}{T_3}$$

$$VR = \left(\frac{T_2}{T_1}\right)\left(\frac{T_4}{T_3}\right)$$

$$\left.\frac{N_1}{N_6} = \left(\frac{T_2}{T_1}\right)\left(\frac{T_4}{T_3}\right)\left(\frac{T_6}{T_5}\right)\right\}$$ 3 stages so -ve...

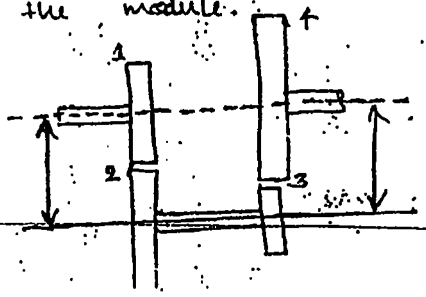
Even no. of stages → +ve vel ratio
odd no. of stages → -ve vel ratio

- With lesser no. of gears, we get larger VR's in case of compound G.T's

RIVETED GT's

In a compound GT, if input shaft and o/p shafts are coaxial, its called a riveted G.T.

- In a SGT, all gears should have a common module
- In a CGT, 1 & 2, 3 & 4 etc should have same module {i.e meshed gears should have same module}.
- High speed gear ; less tangential force ; smaller the module.



$$r_1 + r_2 = r_3 + r_4$$

$$\frac{m_1(T_1+T_2)}{2} = m_2(T_3+T_4)$$

$$VR = \left(\frac{T_2}{T_1}\right)\left(\frac{T_4}{T_3}\right)$$

In a compound G.T meshed gears should have the same module.

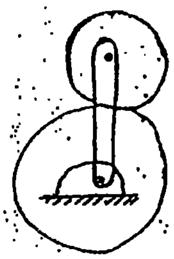
In Reverted both the gears can have different modules.

CYCLOIDAL GEAR TRAINS

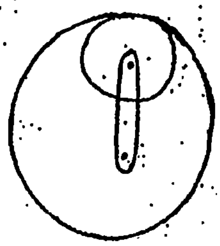
In any of the gear trains (simple, compound or reverted), if one of the axis is FLOATING it is called a cycloidal G.T.

- So any type of G.T can be converted to a cycloidal type if one of the axis is floating / moving (not fixed).

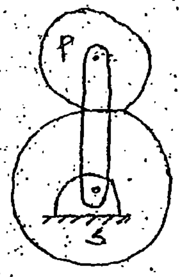
EPICYCLOIDAL GEAR TRAIN :-



HYPOCYCLOIDAL GEAR TRAIN :



Epicycloidal & Hypocycloidal gear trains have been together called as the CYCLOIDAL G.T's.



$$\frac{N_{sa}}{N_{pa}} \qquad \frac{N_{sa}}{N_{pa}} = -\frac{T_p}{T_s}$$

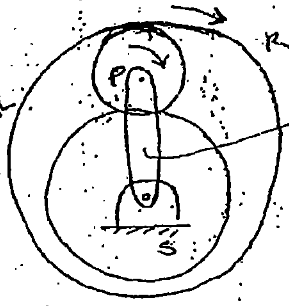
$$N_s = N_a + N_{sa} \longrightarrow N_{sa} = N_s - N_a$$

$$N_p = N_a + N_{pa} \longrightarrow N_{pa} = N_p - N_a$$

$$\boxed{\frac{N_s - N_a}{N_p - N_a} = -\frac{T_p}{T_s}} \qquad \left.\begin{matrix} N_{Ra} \\ N_{Ra} = N_R - N_a \end{matrix}\right\}$$

Eqn based on relative vel concept.

ANALYTICAL method



For one more element R: {ring gear}

$$\frac{N_p - N_a}{N_R - N_a} = \frac{T_R}{T_p}$$

$$\frac{N_{pa}}{N_{Ra}} = \frac{T_R}{T_p}$$

TABULAR Method:

S No.	Action	Arm	S (T_s)	P (T_p)	R (T_R)
1)	Arm sliding H to S	0	+1	$-\frac{T_s}{T_p}$	$-\frac{T_s}{T_R}$
2)	Rel vel of wheel (unknown) = x	0	+x	$-\frac{T_s}{T_p}x$	$-\frac{T_s}{T_R}x$

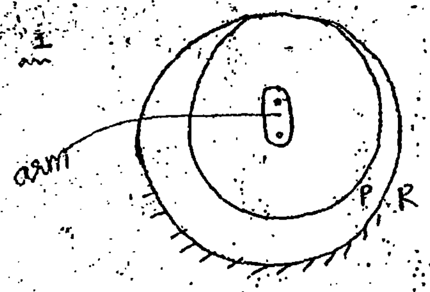
{so specifying the rel vel of all the wheels in terms of unknown vel wheel}.

3)	+y to arm	y	x+y	$-\frac{T_s}{T_p}x+y$	$-\frac{T_s}{T_R}x+y$

so at the end we get the absolute vel.

pg 23 {Numericals - GEAR TRAINS}

1



Another view of the same arrangement.

$N_1 = 0$
$T_1 = 104$
$T_2 = 96$
$N_2 = ?$
$N_3 = 60$

N_p can be got directly, so we take the vel rel to arm,

So $N_p - N_a$ is taken.

$$\frac{\overset{\text{unknown}}{N_p} - \overset{60}{N_a}}{\underset{0}{N_R} - \underset{60}{N_a}} = \frac{\overset{104}{T_R}}{\underset{96}{T_p}}$$

$N_p =$ ___ {N_p is the only unknown}

so $\dfrac{N_2 - N_3}{N_1 - N_3} = \dfrac{T_1}{T_2}$] 1st stage

3

$N_S = 100$, $N_R = 0$

$T_S = 20$, $T_P = 30$, $T_R = 80$

$R_s + 2R_p = R_R$ {rad of ring gear}

so, $T_s + 2T_p = T_R$

$$\frac{N_s - N_a}{N_R - N_a} = -\frac{T_R}{T_s} \quad \text{---(1)}$$

OR we have $\frac{N_s - N_a}{N_p - N_a} = -\frac{T_p}{T_s}$ & $\frac{N_p - N_a}{N_R - N_a} = \frac{T_R}{T_p}$

xply the 2 eqns,

we get $\frac{N_s - N_a}{N_R - N_a} = -\frac{T_R}{T_s}$ → same eqn

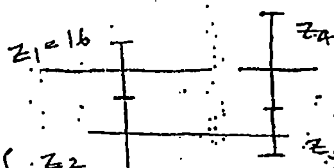
so $\frac{N_s - N_a}{N_R - N_a} = -\frac{T_R}{T_s}$

$\frac{100 - N_a}{0 - N_a} = -\frac{80}{20}$

$N_a = 20$ rpm //

3. I/p & o/p shafts are collinear → so RIVETED.

Overall gear ratio = 12.
First stage gear ratio = $g_1 = 4$
Second " " " = $g_2 = 3$
1st module, $m_1 = 3$ mm.
2nd module, $m_2 = 4$ mm.



$Z_3 = 15$, $m = 4$, $g = ?$ } 2nd stage

1st stage { Z_2, $m = 3$ mm, $g_1 = 4$

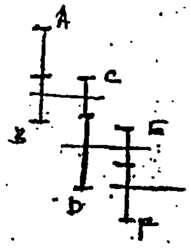
$r_3 + r_4 = \frac{m Z_3}{2} + \frac{m Z_4}{2}$

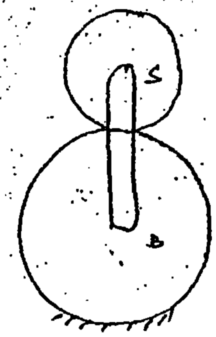
$= \frac{3}{2}[15 + 45] = 90$

{ POSSIBLE Extension. $m_1(Z_1+Z_2) = m_2(Z_3+Z_4)$ so find module of the 2nd stage

5 b)
6 b)

7. $\frac{N_F}{N_A} = \left(-\frac{T_A}{T_B}\right)\left(-\frac{T_C}{T_D}\right)\left(-\frac{T_E}{T_F}\right) = -\frac{T_A T_C T_E}{T_B T_D T_F}$ //





$N_B = 0$

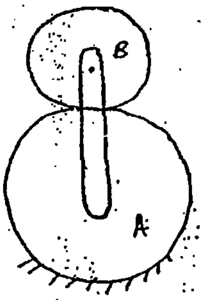
$N_a = 1$

$N_S = ?$

$$\frac{N_S - N_a}{N_B - N_a} = -\frac{T_B}{T_S}$$

$$\frac{N_S - 1}{0 - 1} = -\frac{100}{25}$$

$N_S = 5$ //

9. 

As wheel A is fixed,

$N_A = 0$

$T_A = 40$

$T_B = 20$

$N_a = 3$

$N_B = ?$

$$\frac{N_B - N_a}{N_A - N_a} = -\frac{T_A}{T_B}$$

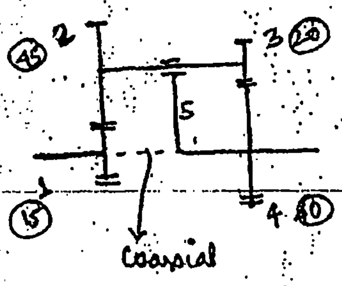
$$\frac{N_B - 3}{0 - 3} = -\frac{40}{20}$$

$N_B = 9$ //

10. (a)

11. dec, inc

13. Its a reverted gear train

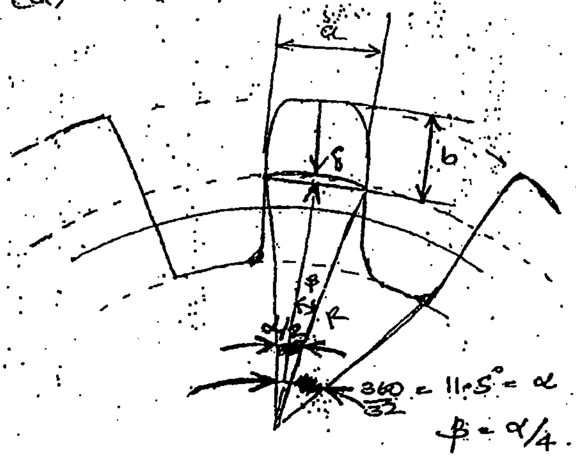


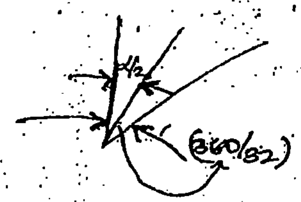
With 5, its made a cycloidal Gear train.

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} = \frac{45}{15} \cdot \frac{40}{20} = 6$$

15 (a) P-2, R-1

16





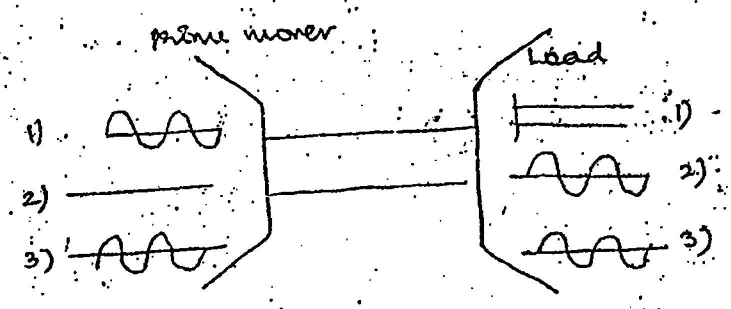
$b = m + \delta$

$a =$

$\left.\begin{array}{l} 32 = T \\ 4 = m \end{array}\right\}$ $\quad \left.\begin{array}{l} a = 2R \sin\beta \\ \delta = R - R\cos\beta \end{array}\right\}$

FLYWHEELS

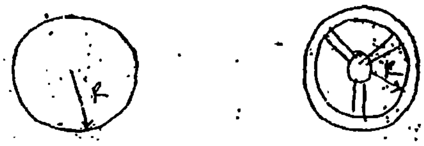
- Reservoir of energy to decrease the fluctuations.
- Useful in IC engines.
- Evene out the fluctuations.



Case 1 : fluctuating i/p & uniform o/p

Case 2 : uniform i/p & fluctuating o/p

Case 3 : fluctuating i/p & fluctuating o/p



$I = \frac{MR^2}{2}$ $\qquad$ $I = MR^2$

$\sigma_c = \frac{\rho v^2}{R} = \rho R \omega^2$ // → centrifugal stress.

ω_1 – max $\qquad$ ω_2 – min

$\omega = \frac{\omega_1 + \omega_2}{2}$ (Mean)

$E_1 = \frac{1}{2} I \omega_1^2 \qquad E_2 = \frac{1}{2} I \omega_2^2$

$\Delta E = E_1 - E_2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$\Delta E = \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$

$$\boxed{\Delta E = I \omega \Delta \omega}$$

$$\Delta E = I\omega^2 \frac{\Delta W}{W} = I\omega^2 C_s$$

C_s → coeff of fluctn of speed.

$\Delta W = W_1 - W_2$

$$C_s = \frac{\Delta W}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$\Delta E = I\omega^2 C_s$

$\Delta E = 2E C_s$

$\frac{\Delta E}{E} = 2C_s$

$\left[C_e = 2C_s \right.$

C_e → coeff. of fluctuation of energy.

pg 18 [NUMERICALS - FLYWHEELS]

I.

80 kW at 300 rpm → $\omega = \frac{2\pi N}{60}$ }

$\Delta E = 0.9\,E$ per cycle

$C_s = 0.02$

$\sigma_c = 6 \times 10^6 \; N/m^2$, $\rho = 7500 \; kg/m^3$

$d = ?$

$I = ?$

$\sigma_c = \rho R \omega^2$

$$R = \frac{\sigma_c}{\rho \omega^2}$$

diam $= 2R$

$$\Delta E = I\omega^2 C_s \implies I = \frac{\Delta E}{\omega^2 C_s}$$

Rate = 80000 J/sec

300 rpm → 5 rps

0.2 sec/rev

o/p

o/p

Inclusions

1 cycle = 2 rev = 0.4 sec

E per cycle = 80000 × 0.4 = 32000 J

$\Delta E = 0.9 \times$ (E per cycle)

$\therefore \Delta E = 0.9 \times (32000)$

$I = \dfrac{\Delta E}{\omega^2 C_s}$ //

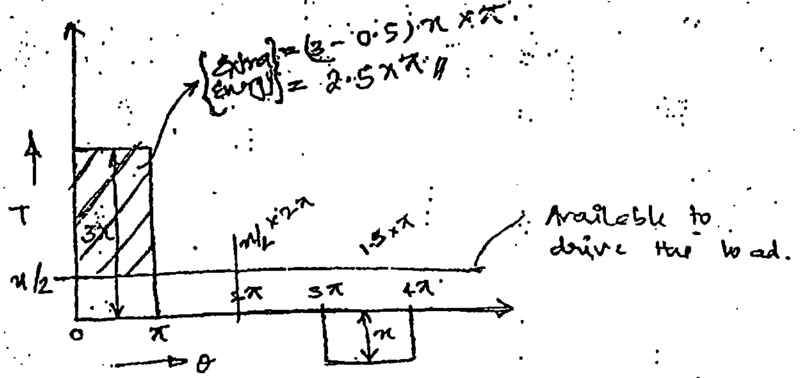
QUESTN CHANGES to make calculation easier:

A: 20 kW at 240 rpm

TM → turning moment

TM is 3 times that of compressn strokes

Avg speed of 240 rpm



20 kW @ 240 rpm

$C_s = 0.01$

$I = ?$

Work done / cycle or Energy developed per cycle = {Net Area under the Turning moment diagram}

$= 3x\pi - x\pi = 2x\pi$

$=$ Mean torque $(T_m) \times 4\pi = 2x\pi$

$T_m = \dfrac{2x\pi}{4\pi} = x/2$ //

$\Delta E = 2.5 x \pi$ Joules

$= 2.5 \times \frac{5000}{\pi} \pi$ J.

$\boxed{I = \frac{\Delta E}{\omega^2 C_s}}$

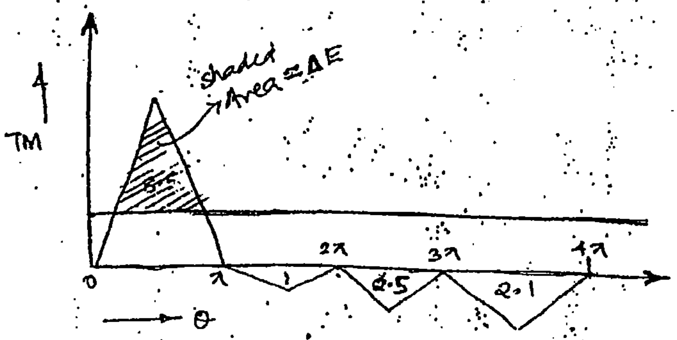
20 kW @ 240 rpm $\longrightarrow$ 4 rps $\rightarrow$ 0.25 rpm

1 cycle = 2 rev = 0.5 sec

$2x\pi = 10000$ J

$x = \frac{5000}{\pi}$

5.



$1\ cm^2 = 1400$ J

98 $\downarrow$ ω_2 – (102) rpm $\downarrow$ ω_1

N = 100 rpm

$C_s = 0.04$

$I = ?$

$I = \frac{2\Delta E}{\omega_1^2 - \omega_2^2}$

$\Delta E =$ —

Energy done per cycle (WDPC) = 8.5 − 0.5 − 0.8 − 2.1

= 5.1 cm^2

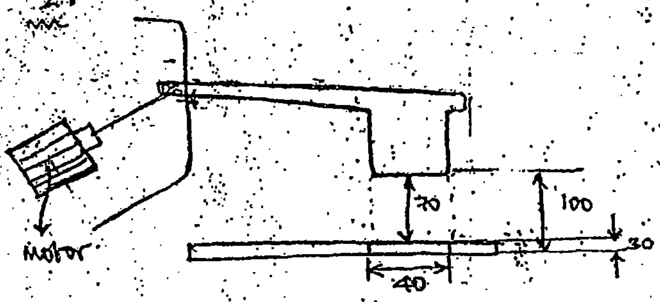
$T_m = \frac{WDPC}{4\pi} = \frac{5.1}{4\pi}$

1 cm × 1 cm = 1400 J

$\downarrow$ 1 rev $\quad$ $\downarrow$ 1400 Nm

ΔE — $cm^2 \times 1400$ — J.

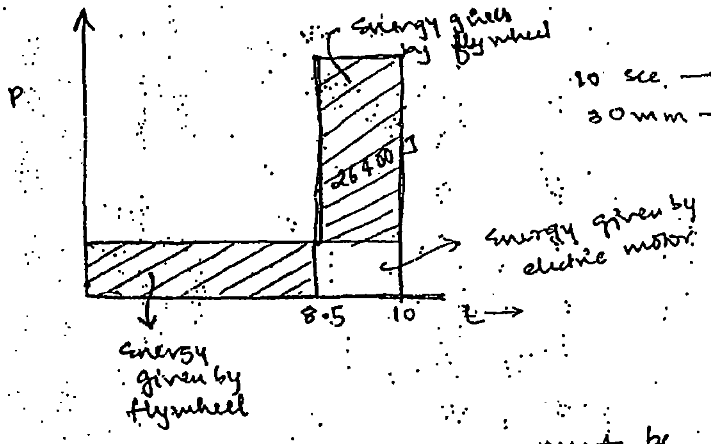
2.



Punch comes down & goes up → one cycle

Energy per hole $= \tau dt$
$= 26400 \text{ J}$

Energy per sec $= \frac{26400}{10} = 2640 \text{ J/s}$
$= 2640 \text{ W}$
$= 2.64 \text{ kW}$



10 sec → 200 mm
30 mm → 2.5 sec

The 2 shaded areas must be equal.

$\Delta E = 8.5\,(2640)$

$26400 - (1.5)(2640)$

$C_s = 0.03$

$v = 25 \text{ m/s} = R\omega$

m

$\Delta E = I\omega^2 C_s$
$= mR^2\omega^2 C_s$
$= m v^2 C_s$

$\Delta E = m v^2 C_s$

$m = \frac{\Delta E}{v^2 C_s}$

Out of 10 secs, 8.5 secs energy is given by the flywheel.

3. [illegible]

600 holes / hr

1 hole in 6 secs → Total cycle time } = 6 secs

1.5 secs ↓ NO WORK & 4.5 secs ↓ 10000 Nm req. } 1.5 + 4.5 = 6

2000 × 4.5 = stored in flywheel.

Last 1.5 s → punching operation.

3000 J given by motor during 1.5 secs }
7000 to be given by flywheels }

80....
7000 & 9000 are both incorrect } Questn: whether to take 7000 or 9000 for the flywheel.

$\frac{10}{6} \times 4.5$ = Energy stored in flywheel.

In last 1.5 secs → { 7.5 given by flywheel.
{ 1.5 given by the motor

whatever is req. is delivered by the motor in 6 secs...

6. Diag gives T.M req to drive the load
Net area = energy for operation.

Pg 11 (Additional Questions)

Q10
300 kW at 90 rpm.
Cycle time = 4/3 sec

$EPC = \frac{4}{3} \times 300 = 400\,kJ$.

Energy per cycle //

out of this $\Delta E = 0.6 \times 400 = 240\,kJ$ //

Ans: (a)

Q11 Ans: d)

Max Energy → Max speed.

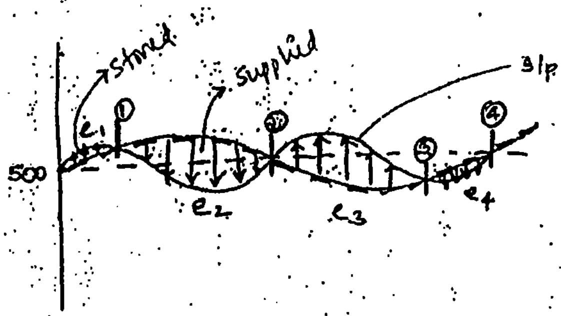
Q9 (d) Negative during major portion
Approaching dead centre, deceleration.

Q12 (d) $\frac{\text{Net area}}{4\pi} \times$ scale factor } 3^{rd} option.

Q13 → $T_1 = T_2$ and $A_1 = A_2$ } correct ans.
NO ANS

Q14 (a) Area of Δ = Area of rect.

Q7
(c)
Load torque = T_L = 500 + 50 sin θ
input torque = T_I = 500 + 50 sin 2θ.



e_1 → i/p more / energy stored
e_2 → o/p more, flywheel energy delivered
e_3 → i/p more, energy stored

e4 → flywheel gives out energy

$T_L = 500 + 50 \sin\theta$

$T_I = 500 + 50 \sin 2\theta$

$\Delta T = T_{in} - T_{out}$

$\int \Delta T \, d\theta$,

$\longrightarrow \Delta T = I\alpha$

At pts ①, ②, ③, ④ value of torque of m/c and the engine will be equal.

{The starting and final pt. can be taken as a single point}

MECHANICAL VIBRATIONS

Mass – spring – Damper → Linear damper

$m \rightarrow kg$
$I \rightarrow kg \cdot m^2$

K (N/m)
K_t (N-m/rad) → torsional stiffness
for rotational springs

C (N-sec/m)
C_t (N-m $\frac{sec}{rad}$) → Torsional damper.

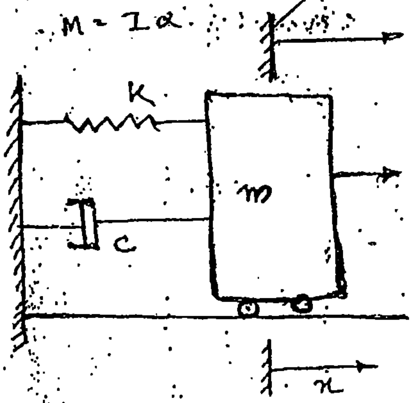
EQUILIBRIUM METHOD

{Newton – Euler method :-}

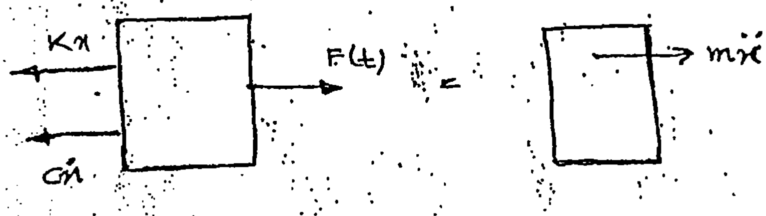
$F = m \cdot a$
$M = I\alpha$

(SEP) STATIC equilib POSITION

$\ddot{x} \rightarrow$ accn
$\dot{x} \rightarrow$ vel
$x \rightarrow$ disp



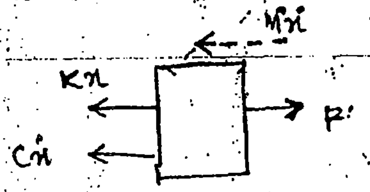
Freebody diag :



$$-Kx - c\dot{x} + F(t) = m\ddot{x}$$

General EQN $\quad m\ddot{x} + c\dot{x} + Kx = F(t)$ → External force

→ mathematical eqn - representation of vibrations



- SPECIFIC EQN

{w/o any external force}

$m\ddot{x} + c\dot{x} + Kx = 0$ {Free vibration}

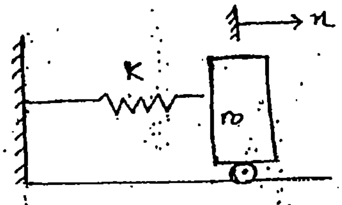
Damped vibration

→ $m\ddot{x} + Kx = 0$

Undamped vibration

Undamped free vibrations {No F(t)}

UNDAMPED FREE VIBRATIONS



$m\ddot{x} + Kx = 0$

$x(0) = x_0$

$\dot{x}(0) = v_0$

} Initial conditions.

Std form : $\ddot{x} + \frac{K}{m} x = 0$

$\ddot{x} + \omega_n^2 x = 0 \qquad \left\{ \omega_n^2 = \frac{K}{m} \right\}$

$D^2 + \omega_n^2 = 0$

$D = \pm \omega_n$

$\frac{K}{m} = \omega_n^2$

$x(t) = A \cos \omega_n t + B \sin \omega_n t$

$x(0) = A = x_0$

$\dot{x}(0) = B\omega_n = v_0$

$\therefore A = x_0$

$B = \frac{v_0}{\omega_n}$

$x(t) = x_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t$

$A = X \cos \phi$

$B = X \sin \phi$

$X = \sqrt{A^2 + B^2} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$

$\phi = \tan^{-1} B/A = \tan^{-1} \frac{v_0}{x_0 \omega_n}$

$x(t) = X_0 \cos(\omega_n t - \phi)$

ω_n = natural freq of system

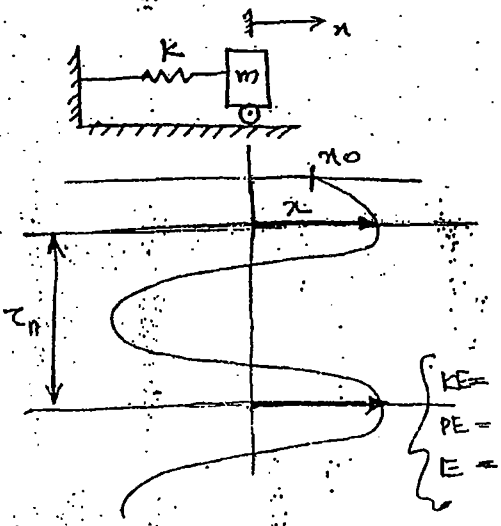
Freq of system depends on K and m

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{m/K}$$

→ Natural time period

τ_n : time period

→ depends on m and K only.



$$\left\{\begin{array}{l} KE = \frac{1}{2} m \dot{x}^2 \\ PE = \frac{1}{2} K x^2 \\ E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} K x^2 \end{array}\right.$$

$$\begin{array}{llll} E = & KE & + \; PE & = \text{Const} \\ & 0 & PE_{max} & \\ & KE_{max} & 0 & \end{array}$$

So $\frac{dE}{dt} = 0$.

So $E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} K x^2$

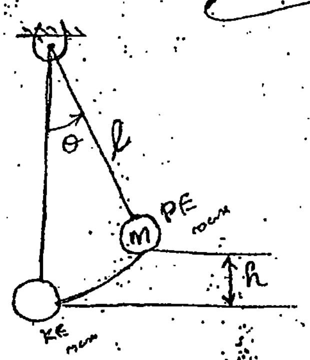
So $\frac{dE}{dt} = \frac{1}{2} m \, 2\dot{x} \ddot{x} + \frac{1}{2} K \, 2x \, \dot{x} = 0$

$(m\ddot{x} + Kx)\,\dot{x} = 0$

$\dot{x}$ cannot be equal to 0

Hence $m\ddot{x} + Kx = 0$.

SIMPLE PENDULUM



$K = \frac{1}{2} m l^2 \dot{\theta}^2$

$P = mgh$

$P = mgl(1-\cos\theta)$

$\{h = l(1-\cos\theta)\}$

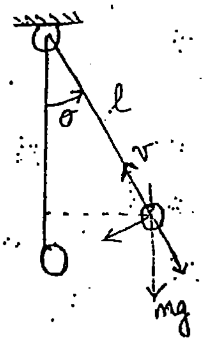
$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1-\cos\theta)$

$\frac{dE}{dt} = \frac{1}{2} m l^2 2\dot{\theta}\ddot{\theta} + mgl(\sin\theta)\dot{\theta} = 0$

$(ml^2)\ddot{\theta} + mgl\,\theta = 0$

$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{g/l}$

- ENERGY METHOD :



Mean position → Net force = 0

Any other position → $mg\sin\theta$.

$ml^2\ddot{\theta} + mgl\sin\theta = 0$.

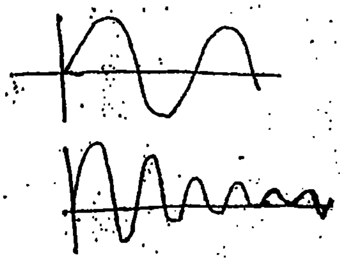
$\{ \text{As } -mgl\sin\theta = I\ddot{\theta} = ml^2\ddot{\theta} \}$

DAMPED FREE VIBRATION

$\tau_d > \tau_n$ → time for natural oscillation

τ_d → time for damped oscillations

$\omega_d < \omega_n$



- If system is disturbed, it comes back to the original system : CRITICAL damping.

$$C_c = 2\sqrt{Km}$$

- $c < C_c \rightarrow$ Under damping system
- $c = C_c$ with critical ~~xxxx~~ damping system
- $c > C_c \rightarrow$ Over damped system.

$$\xi = \frac{c}{C_c} = \frac{c}{2\sqrt{Km}}$$

DAMPED FREE VIBRATIONS

$$m\ddot{x} + c\dot{x} + Kx = 0.$$

$$x(0) = x_0 \ , \qquad \dot{x}(0) = V_0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{K}{m}x = 0.$$

$$\frac{K}{m} = \omega_n^2$$

$$\frac{c}{m} = \frac{c}{2\sqrt{K}}\frac{2}{\sqrt{m}}\frac{\sqrt{K}}{\sqrt{m}} = \frac{2c}{2\sqrt{Km}}\sqrt{\frac{K}{m}} = 2\xi\omega_n$$

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$$

$$x(t) = e^{-\xi\omega_n t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$$

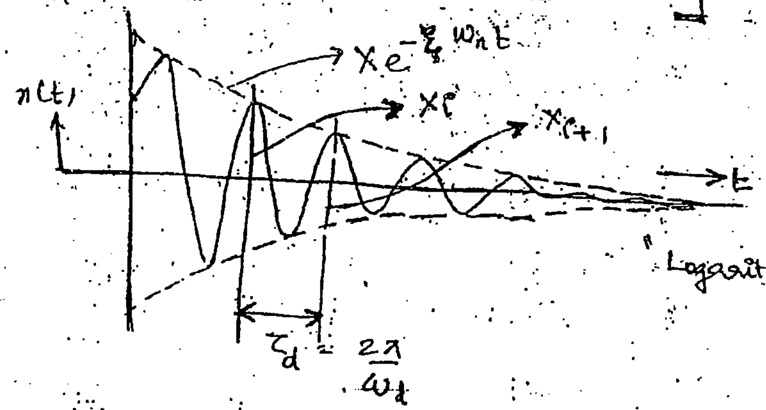
$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

k.to

g

(8)

$$\boxed{x(t) = X e^{-\xi \omega_n t} \cos(\omega_d t - \phi)}$$



"Logarithmic decay"

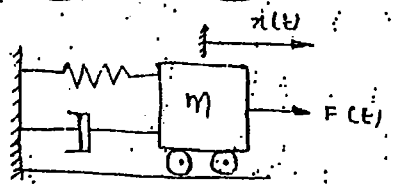
δ = logarithmic decrease or log-dec.

* → $\delta = \ln \dfrac{X_i}{X_{i+1}}$ } logarithmic decrement.

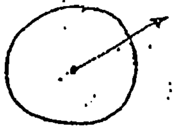
$$\delta = \frac{1}{n} \ln \frac{X_i}{X_{i+n}}$$

* → $\xi = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

FORCED VIBRATIONS.



$$m\ddot{x} + c\dot{x} + kx = F(t) \begin{cases} \to F\cos\omega t \\ \to F\sin\omega t \end{cases}$$



m = 10 kg

e = 1 cm

N = 60.000 rpm

force due to vibration

$\omega = \dfrac{2\pi N}{60} \approx 4000\pi$ rad/sec.

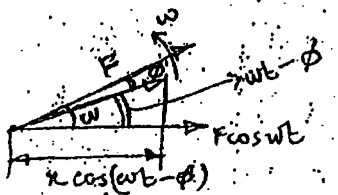
$me\omega^2 = 10 \times 10^{-2} \times (4000\pi)^2$

$= 16000000$ N //

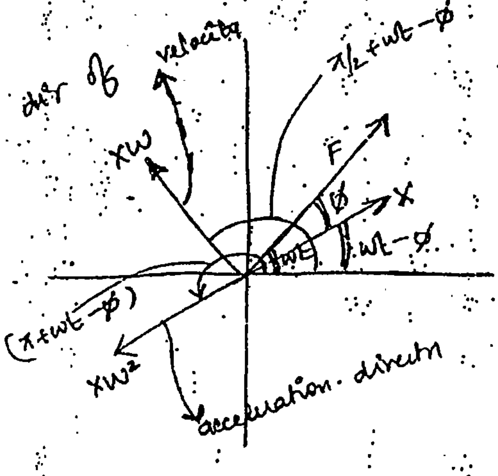
$$x(t) = \underbrace{C.F}_{\text{free}} + \underbrace{P.I}_{\text{forced}}$$

Transient   Steady state

$$x(t) = X\cos(\omega t - \phi)$$



For force at an angle $\omega t - \phi$, the magnitude of Horiz component is $x\cos(\omega t - \phi)$.



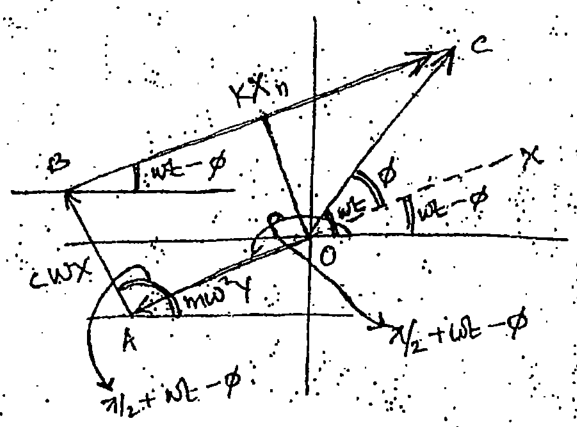
$$x = X\cos(\omega t - \phi)$$

$$\dot{x} = -X\omega\sin(\omega t - \phi) = X\omega\cos(\pi/2 + \omega t - \phi)$$

$$\ddot{x} = -X\omega^2\cos(\omega t - \phi) = X\omega^2\cos(\pi + \omega t - \phi)$$

$$m\ddot{x} + c\dot{x} + kx = F\cos(\omega t - \phi)$$

$$m\omega^2 X\cos(\pi + \omega t - \phi) + c\omega X\cos(\pi/2 + \omega t - \phi) + KX\cos(\omega t - \phi) = F\cos\omega t.$$



Rotation vector diag that represents forced vibratio

All vectors rotate with an angular vel "ω"

$\Delta ODC \rightarrow$ Right angled a'le.

$DC^2 + OD^2 = OC^2$

$(BC - BD)^2 + (OD)^2 = OC^2$

$BD = AD,$

$(KX - m\omega^2 X)^2 + (c\omega X)^2 = F^2$

$$X = \frac{F}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{K - m\omega^2}\right)$$

$$K - m\omega^2 = K\left(1 - \frac{m}{K}\omega^2\right)$$

$$= K\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)$$

$$c\omega = K\left[\frac{c\omega}{K}\right]$$

$$= K\left[\frac{c\omega}{m\omega_n^2}\right] = K\left[\frac{c}{m}\left(\frac{\omega}{\omega_n}\right)^2\right]$$

$$\frac{F}{K} = X_{st}$$

$$X = \frac{F/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$X = \frac{X_{st}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}}$$

Non dimensionalised eqns

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\,\omega/\omega_n\right)^2}}$$

X static

$$MF = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

→ $X = X_{st}\,(MF)$

Magnification factor

Freq response Fn (FRF)

- Freq ratio & damping ratio decides the amplitude of the vibration.

Also, $\phi = \tan^{-1}\frac{c\omega}{K - m\omega^2} = \tan^{-1}\frac{2\xi\,\omega/\omega_n}{1-(\omega/\omega_n)^2}$

- MF is a fn of freq ratio and damping ratio.

- $MF = \sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$

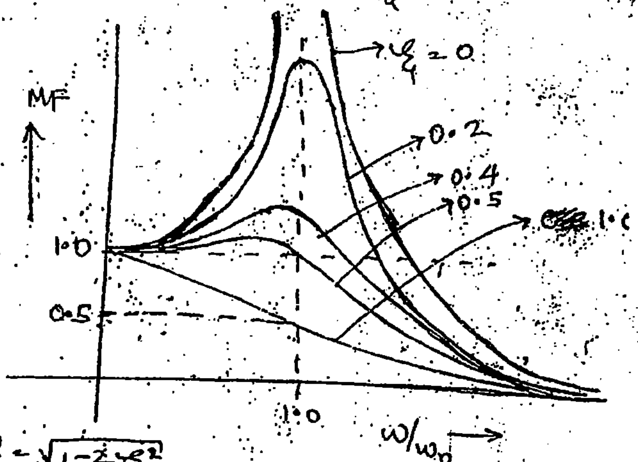
small force leading to large amplitude, $MF = \infty$ } Resonance

→ For undamped system, $\xi = 0$, $MF = \infty$

When there is damping $\rightarrow \xi =$ finite value & $\xi \neq 0$

$\frac{\omega}{\omega_n} = 1$

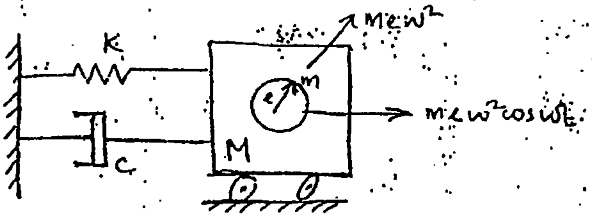
$MF = \frac{1}{2\xi}$



MF is max at $\frac{\omega}{\omega_n} = \sqrt{1-2\xi^2}$

$\phi = 0+$	$\omega/\omega_n < 1$
$\phi = 90°$	$\omega/\omega_n = 1$
$\phi = 180°$	$\omega/\omega_n > 1$

ROTATING OR RECIPROCATING UNBALANCES



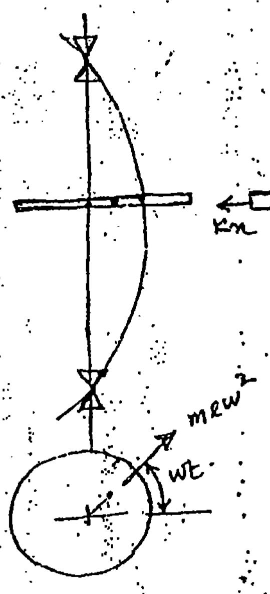
$$M\ddot{x} + C\dot{x} + kx = me\omega^2 \cos\omega t$$

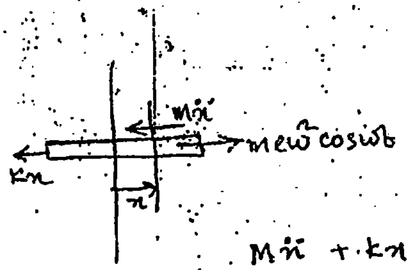
$$x(t) = X\cos(\omega t - \alpha)$$

$$X = \frac{me\omega^2}{\sqrt{(K-M\omega^2)^2 + (C\omega)^2}}$$

WHIRLING OF SHAFTS / CRITICAL SPEEDS

- shaft rotates about its own axis
- shaft which is deformed rotates about the unbend axis → its called WHIRLING





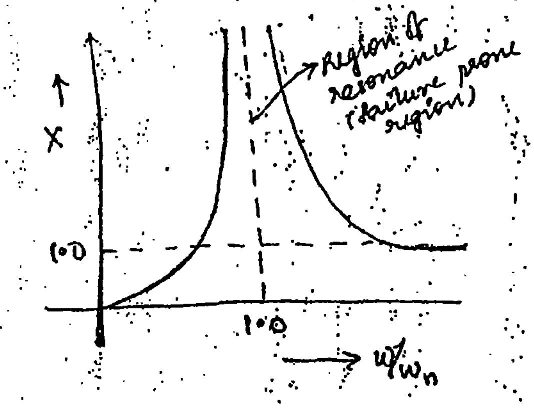
$$M\ddot{x} + kx = me\omega^2 \cos\omega t$$

$$x(t) = X\cos(\omega t - \phi)$$

$$X = \frac{me\omega^2}{k - m\omega^2}$$

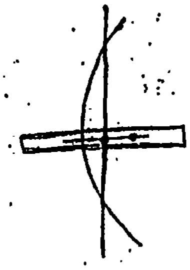
$$X = \frac{e(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$$

X = whirling radius or amplitude of whirling.



$$\omega = \omega_n = \sqrt{\frac{K}{m}}$$ → critical speed

$$X = e\,\frac{(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$$



when $\omega/\omega_n = 50$

$$X = \frac{e(50)^2}{1-(50)^2} = -e$$

shaft bends opp to the direcn of force

At high speeds it deflects backwards and it rotates around its centre of gravity.

If it can't bend → it will undergo failure.

NG

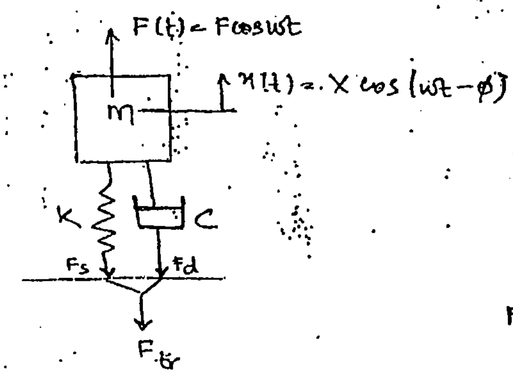
It brings back the { CG on its centre of rotation → so escapes the failure.

→ coasting up and coasting down procedure is to avoid shaft failure while crossing the resonance region / critical speed region.

TRANSMISSIBILITY

Transmissibility

- Force transmission
- motion / isolation / support motion

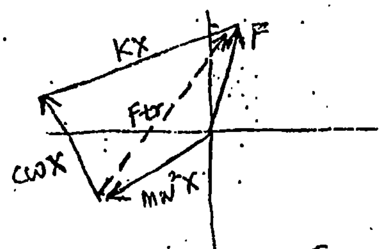


$$\vec{F}_{tr} = \vec{F_s} + \vec{F_d}$$

($\vec{F_s}$: spring force; $\vec{F_d}$: damping force)

$$F_{tr} = \sqrt{(KX)^2 + (c\omega X)^2}$$

$$F_{tr} = X\sqrt{K^2 + (c\omega)^2}$$

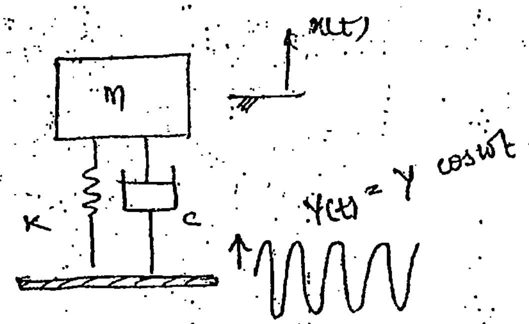


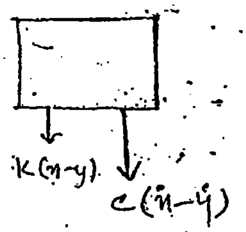
$$F_{tr} = \frac{\sqrt{K^2 + (c\omega)^2}\; F}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$$

$$\frac{F_{tr}}{F} = \sqrt{\frac{K^2 + (c\omega)^2}{(K - m\omega^2)^2 + (c\omega)^2}}$$

Transmissibility ratio

$$\frac{F_{tr}}{F} = \sqrt{\frac{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$





Both ends in motion
so rel disp for spring
rel vel for damper.

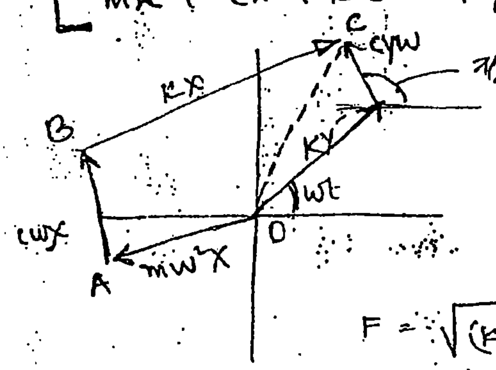
$m\ddot{x} + c(\dot{x}-\dot{y}) + K(x-y) = 0.$

$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$

$y(t) = Y\cos\omega t$

$\dot{y}(t) = -Y\omega\sin\omega t = Y\omega\cos(\pi/2 + \omega t)$

$m\ddot{x} + c\dot{x} + kx = kY\cos\omega t + cY\omega\cos(\pi/2 + \omega t)$

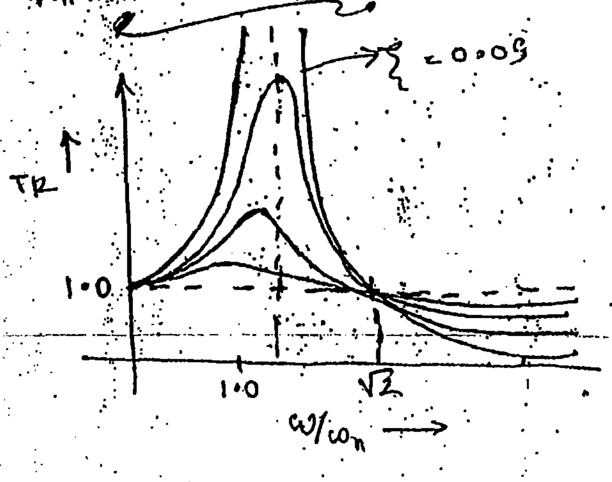


pictorial representn of the above eqn

$F = \sqrt{(KY)^2 + (cY\omega)^2}$

$X = \dfrac{\sqrt{(KY)^2 + (cY\omega)^2}}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}}$

VARIATION of transmissibility ratio



$= e\frac{\omega}{\omega_n} = 0$, TR for all ξ

TR is max at $\frac{\omega}{\omega_n} = 1^-$

$(x-y) = 0$

$0 \le \omega/\omega_n \le 1^-$

TR $\uparrow$ $\omega/\omega_n \uparrow$

TR $\downarrow$ $\xi \uparrow$

$1^- \le \omega/\omega_n \le \sqrt{2}$ TR > 1

TR $\downarrow$ $\omega/\omega_n \uparrow$

TR $\downarrow$ $\xi \uparrow$

At $\omega/\omega_n = \sqrt{2}$ → TR $= 1$ for all ξ

$v(t)$

$\frac{\omega}{\omega_n} > \sqrt{2}$

TR < 1

TR $\downarrow$ $\omega/\omega_n \uparrow$

TR $\uparrow$ $\xi \uparrow$

$\frac{\omega}{\omega_n} \ge 4$, Transmissibility is less, so better isolation

$\frac{\omega}{\omega_n} \approx 10$, Good isolation.

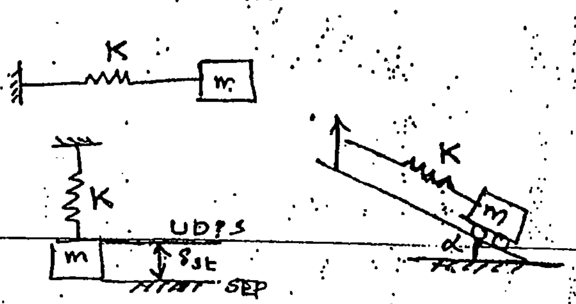
STANDARD PROBLEMS

Q 1) A spring of stiffness K is cut into ½ & joined in parallel. What is stiffness?

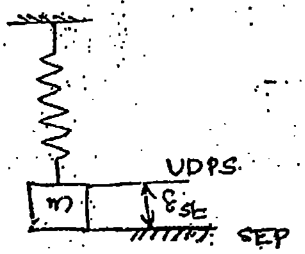
Each piece will have stiffness of 2K

Then joining in parallel, resultant stiffness = 4K

Q 2)



By making static equilb position as the initial position, no need to bother about the wt mg, or the deflection or initial elongation



SEP : static equilb position as reference

UDPS : Undeflected position of spring

δ_{st} : static deflection due to self weight

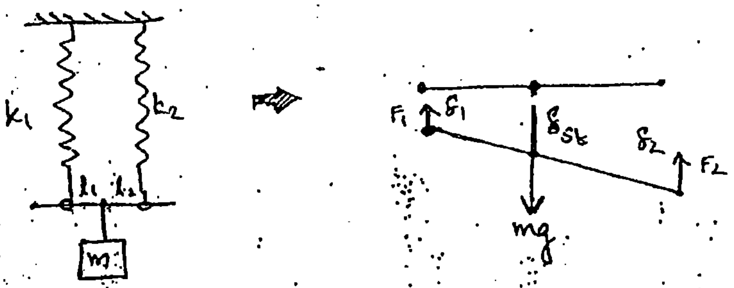
$$\delta_{st} = \frac{mg}{K}$$

$$w_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta_{st}}}$$

$$m\ddot{x} + Kx = 0.$$

$$W_n = \sqrt{g/\delta_{st}}$$

Q3)



$$\delta_{st} = \delta_1 + \frac{(\delta_2 - \delta_1)\, l_1}{l_1 + l_2}$$

$$W_n = \sqrt{g/\delta_{st}}$$

$$K_{eq} = \frac{mg}{\delta_{st}}$$

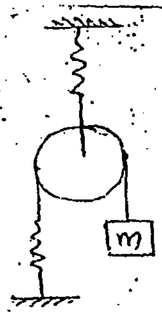
Condition for springs being parallel

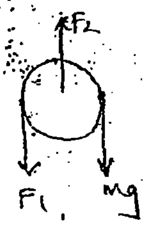
$\delta_1 = \delta_2$

$K_1 l_1 = K_2 l_2$

→ otherwise they are not parallel, its a specific configuration

Q4)





$F_1 = mg$

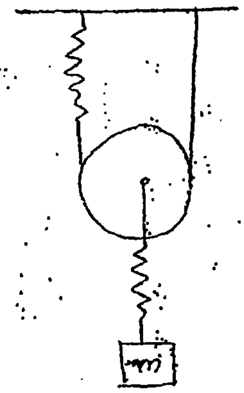
$F_2 = 2mg$

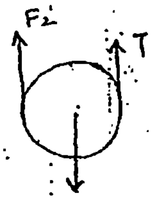
$\delta_1 = \dfrac{F_1}{K_1}$, $\delta_2 = \dfrac{F_2}{K_1}$

$\delta_{st} = \delta_1 + 2\delta_2$

$\omega_n = \sqrt{g/\delta_{st}}$

Q5)





$F_1 = mg$

$F_2 = T = \dfrac{mg}{2}$

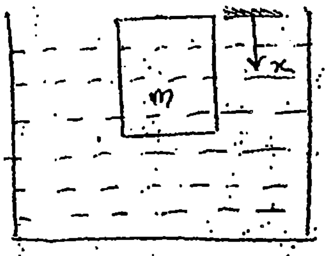
$\delta_1 = \dfrac{mg}{K_1}$

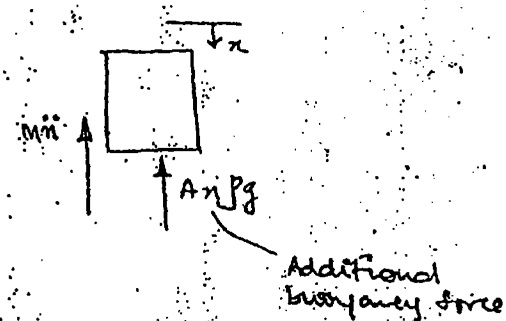
$\delta_2 = \dfrac{mg}{2K_2}$

$\delta_{st} = \delta_1 + \dfrac{\delta_2}{2}$

$\omega_n = \sqrt{g/\delta_{st}}$

Q6) Tub filled with fluid. Block oscillates in fluid. What are the forces acting on the block.





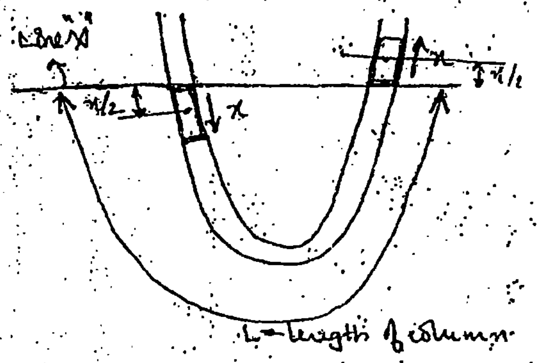
$M\ddot{x} + A\rho g x = 0$

$\omega_n = \sqrt{\dfrac{A\rho g}{m}}$

Q7) U-tube manometer, expose one limb to high pr, release it. Then the column oscillates. Find the freq.

A = Area of c.s

L = length of the column below the line "X"



$PE = Ax\rho g x = \frac{1}{2}\underbrace{(2A\rho g)}_{K_{eq}} x^2$

$KE = \frac{1}{2}\underbrace{(Al\rho)}_{m_{eq}}\dot{x}^2$

$E = ?$

$\frac{dE}{dt} = 0$

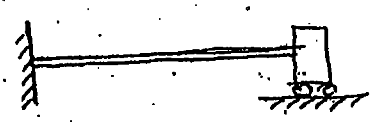
$$m_{eq}\ddot{x} + K_{eq}x = 0$$

$$A\rho l\ddot{x} + 2A\rho g x = 0$$

$$\omega_n = \sqrt{\frac{2A\rho g}{A\rho l}} = \sqrt{2g/l}$$

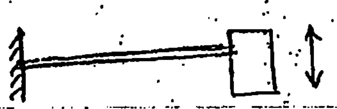
Q8) STRUCTURAL Elements as Springs

Mass oscillates along the axis of the structural member → Axial vibration.



$K = \frac{EA}{L}$ & $K = F/\delta$.

vibration ⊥'r to axis → Transverse / bending vibration.



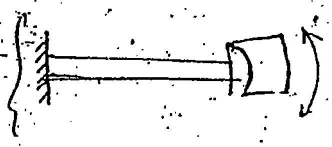
$\delta = \frac{Fl^3}{3EI}$

$K = F/\delta$

$\therefore K = \frac{3EI}{l^3}$ //

$\omega_n = \sqrt{\frac{K_t}{I}}$

case : TORSIONAL VIBRTN



$$\frac{T}{J} = \frac{G\theta}{l}$$

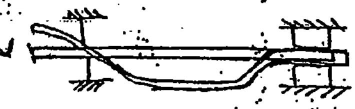
$$K_t = \frac{T}{\theta} = \frac{GJ}{l}$$

case : SHAFT & ~~[illegible]~~ LONG Bearing conditions.

Shaft bearing gives a simply supported end condition.

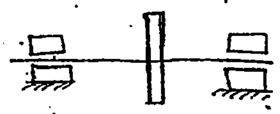
Long bearings gives a fixed end condition.

P.A(Pg) x^3 → K_{eq}



$$\delta = \frac{Fl^3}{48EI}$$

$$K = \frac{48EI}{l^3}$$



$$\delta = \frac{Fl^3}{192EI}$$

$$K = \frac{192EI}{l^3}$$

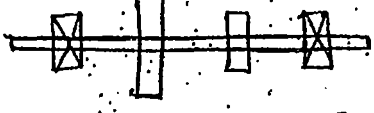
$$\omega_n = \sqrt{\frac{K}{m}}$$

Q7) 2 degree freedom system → 2 freq possesses.

{Lowest freq is called fundamental freq}

↓

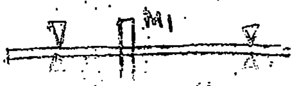
Use Dunkerley's empirical method to estimate the lowest of the 2 natural freq's.



$$\frac{1}{\omega_n^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2}$$

• $f_n = \frac{\omega_n}{2\pi}$ (rad) → In Hertz

ing



• $\frac{1}{[illegible]} = \frac{1}{[illegible]} + \frac{1}{[illegible]}$

$\sqrt{\frac{K_t}{I}}$

spring-mass damper system

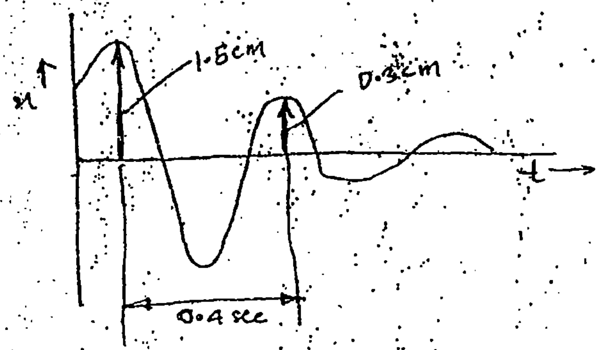
Q(i) - The free vibration curve for a fixed mass system is as shown in the figure.

(LINKED ANSWER TYPE)

If the mass of system = 1kg, determine

(a) spring constant

(b) damping constant



$x_1 = 1.5$ cm, $x_2 = 0.3$ cm

$\delta = \ln \frac{x_1}{x_2}$

$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

If $\delta < 0.7$, only then $\xi \approx \frac{\delta}{2\pi}$ } approx

$\tau_d = 0.4$ secs

$\omega_d = \frac{2\pi}{\tau_d}$

$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}}$, $\frac{K}{m} = \omega_n^2$

so, $K = m\omega_n^2$] — Ans (a)

stiffness calculated.

(b) $C = ?$

$\frac{C}{C_c} = \xi$

$C_c = 2\sqrt{Km}$

$C = \xi C_c$

$C = 2\xi\sqrt{Km}$

Q11) In a spring-mass damper system, the mass completes 5 oscillations in 0.5 secs, during these 5 oscillations the amplitude decays to 5% of the original value. If the stiffness of the spring is 1kN/m, determine the mass and the spring constant.

Ans: $\tau_d = 0.5$ secs for 5 oscilltns $= \frac{0.5}{5} = 0.1$ sec

$\omega_d = \frac{2\pi}{\tau_d} = 20\pi$ rad/sec

Orginal amp $= X_0$

After 5 oscilltns, Amp $= X_5$

$X_0 - X_5 = 0.05$

$\delta = \frac{1}{5} \ln \frac{X_0}{X_5} = \frac{1}{5} \ln \frac{X_0}{0.05 X_0}$

$= \frac{1}{5} \ln 20$

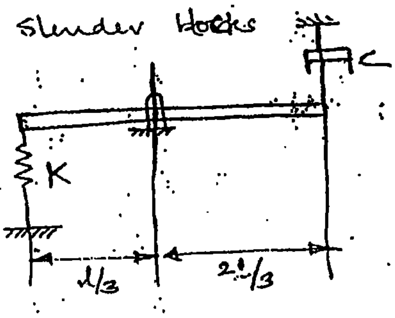
$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$, $\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}}$

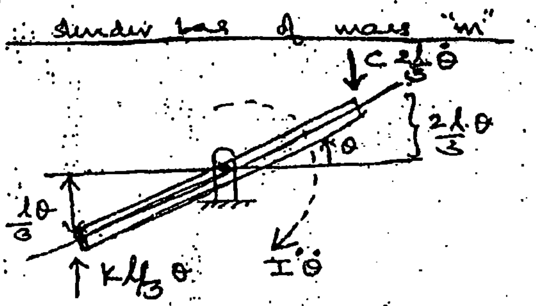
$K = 10{,}00$ N/m

$m = \frac{K}{\omega_n^2}$ —(a)

$C = 2\xi\sqrt{Km}$ —(b)

Q(12) Slender Hooks



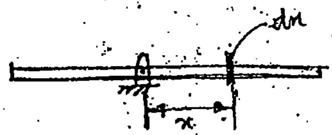


$$I\ddot\theta + C\frac{2l}{3}\dot\theta\,\frac{2l}{3} + K\frac{l}{3}\theta\,\frac{l}{3} = 0$$

$$I\ddot\theta + \frac{4}{9}Cl^2\dot\theta + K\frac{l^2}{9}\theta = 0$$

To find MOI's

(a) About the centre



$\rho = \frac{m}{l}$

$$\int_{-l/2}^{l/2} x^2 \rho \, dx = \rho \frac{x^3}{3}\Big|_{-l/2}^{l/2} = \frac{\rho l^3}{12} = \frac{ml^2}{12}$$

(b) About another pt,

$$\rho \frac{x^3}{3}\Big|_{-l/3}^{2l/3} = \rho l^3 \frac{1}{9} = \frac{ml^2}{9}$$

$$I\ddot{\theta} + c\frac{2l}{3}\dot{\theta}\frac{2l}{3} + k\frac{l}{3}\theta\frac{l}{3} = 0$$

$$\underbrace{\frac{ml^2}{9}\ddot{\theta}}_{M_{eq}} + \underbrace{\frac{4}{9}cl^2\dot{\theta}}_{C_{eq}} + \underbrace{\frac{kl^2}{9}\theta}_{K_{eq}} = 0$$

$$M_{eq} = \frac{ml^2}{9}$$

so $\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{k}{m}}$

Now, DAMPING ratio for the system :-

$$\xi = \frac{C_{eq}}{2\sqrt{K_{eq}M_{eq}}} = \frac{4cl^2/9}{2\sqrt{\frac{kl^2}{9}\frac{ml^2}{9}}} = \frac{2c}{\sqrt{km}}$$

{ Misleading ans as normally you get ξ as $\frac{c}{2\sqrt{km}}$ }

Pg 8 [Prev Gate Objectives]

01 In the mean position, Net force is ZERO

Ans: (A)

02 $\frac{\omega}{\omega_n}$ closer to 1 will have the max vibration

So. Ans: (C) → R with 128

03 Here,

$C = C_c = 2\sqrt{km}$

$K = \frac{3EI}{l^3}$

04

As a simply supported shaft,

So, $K = \frac{48EI}{l^3}$

$M = 20\,kg$

$l = 1\,m$

$25 \times 25\,mm$

$E_{st} = 200 \times 10^9\,N/m^2$

$d = 30 \times 10^{-3}\,m$

$l = 0.5\,m$

$E = 2.1 \times 10^{11}\,pa$

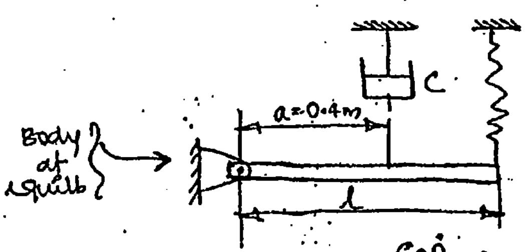
$N_c = ?\ \{Hz\}$

$\omega_n = \sqrt{\frac{K}{m}}$ rad/sec

$N_c = \frac{\omega_n}{2\pi}$ Hz.

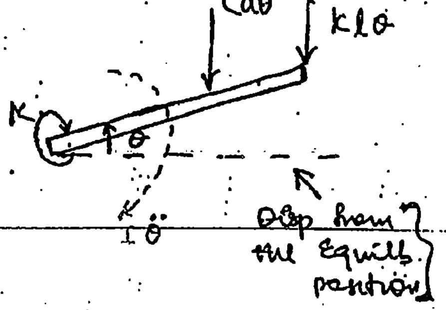
05 (a) slender bar — mass is distributed uniformly

→ $I = \frac{ml^2}{3}$



$I\ddot{\theta} + ca\dot{\theta}a + kl\theta l + K\theta = 0$

$\left(\frac{ml}{3}\right)^2\ddot{\theta} + (ca^2)\dot{\theta} + (K_\theta + kl^2)\theta = 0$



Body in rotational

↓

Take moment equilb about axis of rotation.

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{K_\theta + K l^2}{Ml^2/3}} = 42.43 \text{ rad/sec}$$

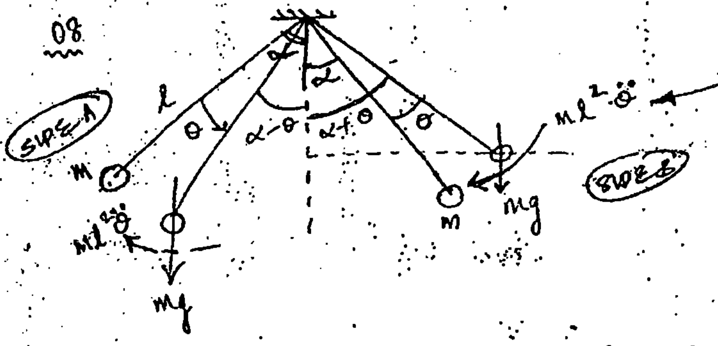
ANS: (a)

06 $C_{eq} = Ca^2$

07 K_1 & K_2 → in series } they are in parallel with K_3

Ans: (b) $\sqrt{\frac{\left(\frac{K_1K_2}{K_1+K_2}\right)+K_3}{J}}$

08



EQUILB method.

$$ml^2\ddot{\theta} + ml^2\ddot{\theta} + mgl\sin(\alpha+\theta) - mgl\sin(\alpha-\theta) = 0$$

$$2ml^2\ddot{\theta} + 2mgl\cos\alpha\ \sin\theta = 0$$

M_{eq} {for small angles, $\sin\theta \approx \theta$ } K_{eq}

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{g\cos\alpha}{l}}$$

ENERGY method:

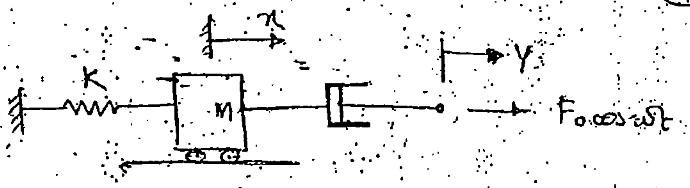
For side A, PE is -ve

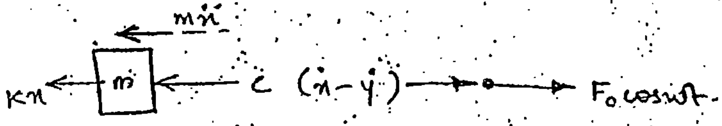
For side B, PE is +ve

so $(-mgl)_A = (mgh)_B$ → As its rising

and then differentiate with respect to time and equate to 0.

09





$$\left.\begin{aligned} m\ddot{x} + c(\dot{x}-\dot{y}) - kx &= 0 \\ c(\dot{x}-\dot{y}) + F_0 \cos\omega t &= 0 \end{aligned}\right\}$$

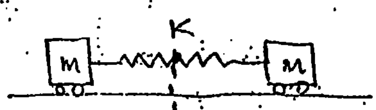
Its a ½ order dof system, Cant exist on its own, so its coupled to another system.
Hence it becomes a 1½ dof system.

10 Natural freq = $\frac{10000}{100}$ = 10 rad/s.
in cycles / sec = $\frac{10}{2\pi} = \frac{5}{\pi}$ //

12 Ans: (C) $10/\pi$ {same as Q 10}

11 $\omega_n = 100$ rad/sec
$\xi = 0.8$
$\omega_d = ?$
$\omega_d = \sqrt{1-\xi^2}\,\omega_n = 60$ rad/s
Ans: (A)

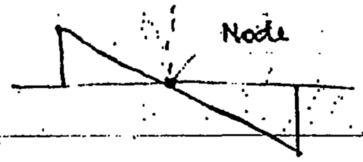
13 Its a 2 degree freedom system.



{ For a 2-mass system, it can move freely in one direction in the condition ~~that~~ there is no tension in the string.

Its a SEMI-DEFINITE System

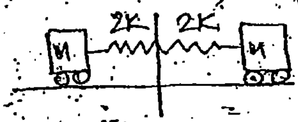
$\omega_1 = 0$



Node : pt with 0 displacement }

If the system consists of equal masses,

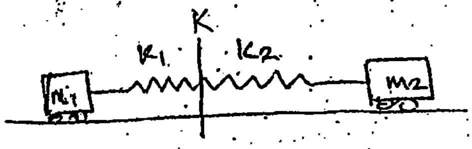
$$\omega_2 = \sqrt{\frac{2K}{m}}$$

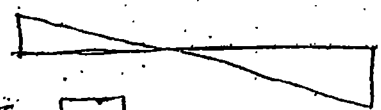


divide the spring into 2 parts with $K_{\text{same}} = 2K$, so $\omega_2 = \sqrt{\frac{2K}{m}}$

→ If system consists of unequal masses,

CASE 2: Then the amplitudes are not equal, the node lent at the centre.





$$\sqrt{\frac{K_1}{m_1}} = \sqrt{\frac{K_2}{m_2}}$$

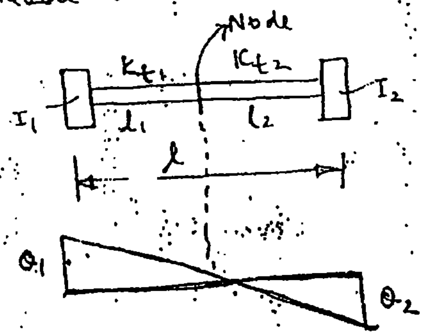
$$K = \frac{K_1 K_2}{K_1 + K_2}$$

The freq's are $0, \sqrt{\frac{K(m_1+m_2)}{m_1 m_2}}$

In this eqn if you put $m_1 = m_2 = m$ we get the equal mass case

EXTRA

NOTE: Node is close to the heavy rotor.

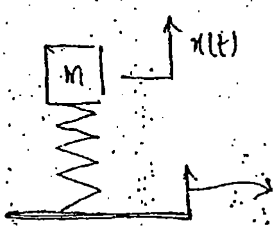


$$K_t = \sqrt{\frac{GJ}{l}}$$

$$K_{t1} = \frac{GJ}{l_1} \qquad K_{t2} = \frac{GJ}{l_2}$$

$$\sqrt{\frac{K_{t1}}{I_1}} = \sqrt{\frac{K_{t2}}{I_2}} \Rightarrow I_1 l_1 = I_2 l_2$$

CASE 1:
divide the spring into 2 parts with each = 2K
so, $\omega_2 = \sqrt{\frac{2K}{m}}$

14 

$x(t)$ {Motion allowed only vertically

The rollers are present to ensure this vertical mov.

$Y(t) = 0.2 \sin 200\pi t$ mm

$Y = 0.2$ mm

$\omega = 200\pi t$ $\quad C = 0$

$X \leq 0.01$ mm, $\quad m = 50$ kgs

$K = ?$

$$\frac{X}{Y} = \sqrt{\frac{K^2}{(K^2 - m\omega^2)^2}}$$

{In Transmissibility eqn, put C=0

$$\frac{X}{Y} = \frac{F}{K^2 - m\omega^2}$$

15. $$\frac{\frac{K_1 * K_2}{(K_1+K_2)} + K_3}{m} = \frac{\left(\frac{3}{4}+2\right) \times 1000}{4}$$

$$\omega_n = \sqrt{\frac{11000}{4}}$$

Ans: (b)

16. $\omega_d < \omega_n$ (~~less~~ greater than)

17. stiffness becomes 4 times

deflection reduces to 1/4

so Ans: D $\quad \left\{ \frac{10}{4} = 2.5 \text{ mm} \right\}$

18. $\tau = 2\pi\sqrt{\frac{m}{k}} = 2$ sec

$\tau' = 2\pi\sqrt{\frac{m+2}{k}} = 3$ sec

$m = ?$ $\quad$ Ans: (B)

{In Q 15 they are in SERIES

↑ Ans: (A)

Q19 is

(similar to Q15 on pg 12)

19. (C) They are in parallel

20. (A) High damping reduces transmissibility.

26. (C)

27. $m = 250$ kg, $\quad K = 100{,}000$ N/m

$F = 350$ N @ 2600 rpm

$\omega = \frac{2\pi N}{60}$

$\xi = 0.15$

Transmissibility ratio (TR) $= \sqrt{\frac{K^2 + (c\omega)^2}{(K - m\omega^2)^2 + (c\omega)^2}}$

if given $\rightarrow (m, c, K, \omega)$ use above formula.

If ω_n, ξ, ω is given using

$\omega_n = \sqrt{K/m}$

$$TR = \sqrt{\frac{1 + (2\xi\, \omega/\omega_n)^2}{(1 - (\omega/\omega_n)^2)^2 + (2\xi \frac{\omega}{\omega_n})^2}}$$

Q8 m, K, c

$c_c = 2\sqrt{Km}$

$\delta = ?$

$\xi = \frac{c}{c_c}$, $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

Pg 19 (Last page)

Q1 (B) Little less than undamped natural freq.

Q2 $K/2$ & $K/2$ in $||^{l}$

K & K in series

Ans: (A)

$K,$
$\frac{1}{K} = \frac{1}{K} + \frac{1}{K}$
$k_{eq} = \frac{K}{2}$

Q3 $x(t) = Ce^{-\xi\omega_n t} \cos(\omega_d t - \phi)$

$e^{-\xi\omega_n t} (A \cos\omega_d t + B \sin \omega_d t)$

$x(0) = X$ $\dot{x}(0) = 0$

find x at $t = n\tau_d$

{Do trial & error for faster solving}

Check and compare exponents:

$-\xi\omega_n t ; = -\xi\omega_n n\tau_d$

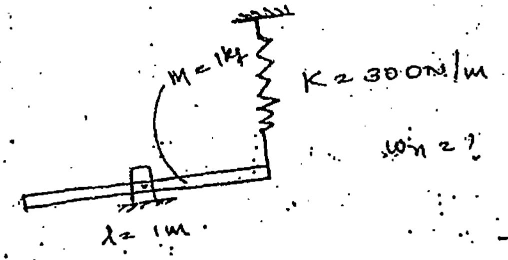
2017 Qs.

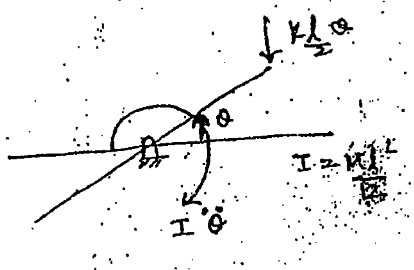
Ans

Note:

$$= \xi\omega_n \, n\frac{2\pi}{\omega_d} = \frac{2\pi n \xi \omega_n}{\sqrt{1-\xi^2}\,\omega_n} = \frac{2\pi n \xi}{\sqrt{1-\xi^2}}$$

Q4.





ω_n = $\sqrt{K/m}$

$I_{rod,\ about\ centre} = \frac{ml^2}{12}$

$$\frac{ml^2}{12}\ddot{\theta} + \frac{Kl^2}{4}\theta = 0$$

$$\omega_n = \sqrt{\frac{Kl^2/4}{ml^2/12}} = \sqrt{\frac{300\times 12}{1\times 4}} = \sqrt{900} = 30\ rad/s$$

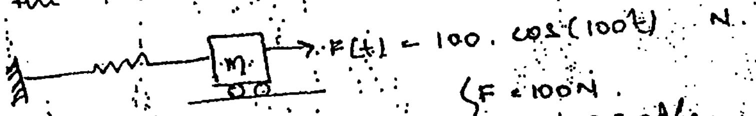
400

Q5 2 springs in parallel

$$\sqrt{\frac{5600}{1.4}} = \sqrt{4000} = \frac{20\times\sqrt{10}}{2\pi} = 10\ Hz$$

2010 GATE

A mass m attached to a spring is subj to a harmonic force as shown. The amplitude of the forced motion is 50 mm. The value of the m is ___.



$\begin{cases} F = 100N \\ \omega = 100\ rad/s \end{cases}$

Ans: $K = 3000\ N/m$

$$X = \frac{F}{k - m\omega^2}$$

$$50\times 10^{-3} = \frac{100}{3000 - m(100)^2}$$

Lift moves up $\rightarrow g+a$
Lift moves down $\rightarrow g-a$ } Natural freq for pendulum in a LIFT,

Notes: Girl $\longrightarrow$ Eiz drum prob

$\alpha = \tan^{-1} \frac{r\omega^2}{g}$

$g_e = \sqrt{g^2 + r^2\omega^2}$

$\omega_n = \sqrt{\frac{g_e}{l}}$

Pg 11 [OBJECTIONS – various competitive]

1 (c) in G.P

2 (b)

3 (a) If equivalent stiffness is +ve
then the system oscillates

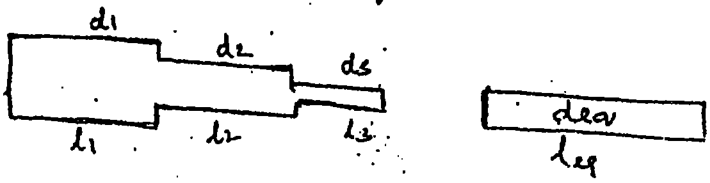
$K \rightarrow$ -ve or 0, then it doesnt oscillate.
UNSTABLE.

4 ( ) $M = 100kg$ $m = 20kg$ $e = 0.5 \times 10^{-3} m$
$K = 85000 N/m$
$C = 0$ { damping negligible}
$\omega = 20\pi$ rad/s.

$X = \frac{me\omega^2}{K - M\omega^2}$ when $c = 0$
so find $X = \underline{?}$.

5 (b)

Stepped shaft can be replaced with an equivalent shaft.



$$\frac{l_{eq}}{d_e^4} = \frac{l_1}{d_1^4} + \frac{l_2}{d_2^4} + \frac{l_3}{d_3^4}$$

6 ( ). $e = 2\,mm$

$\omega_n = 10\ rad/sec$

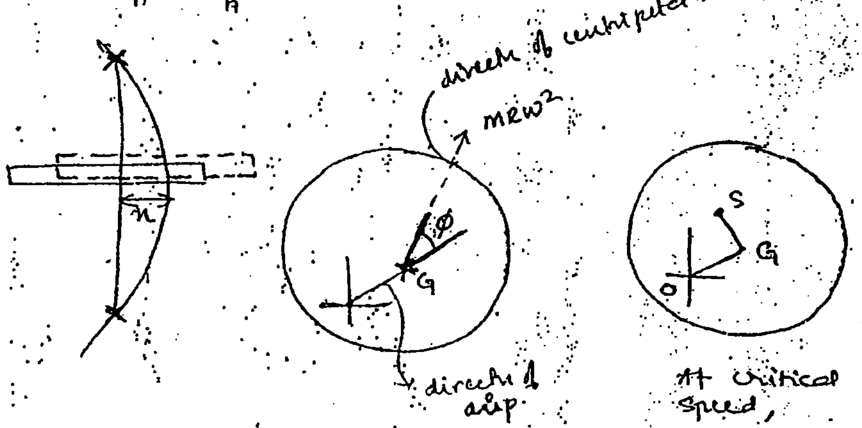
$N = 300\ rpm, \quad \omega = \frac{2\pi N}{60}\ rad/s$

$\therefore \omega = 10\pi\ rad/s$

$$\text{whirling rad} = X = \frac{\overset{F = me\omega^2}{me\omega^2}}{k - m\omega^2} \quad \text{(no damping)}$$

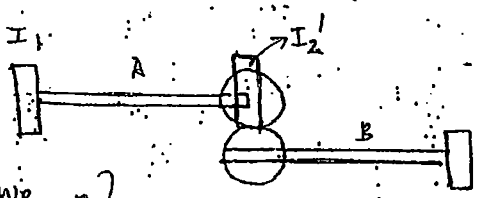
$$X = \frac{e\omega^2}{\omega_n^2 - \omega_n^2} \quad \{ \div \text{ing by } m \}$$

7(c)



At critical speed, phase angle = 90°

8( )



$$\left.\frac{\omega_B}{\omega_A} = n\right\} \qquad \tfrac{1}{2} I_2' \omega_A^2 = \tfrac{1}{2} I_2 \omega_B^2$$

$$I_2' = \left(\frac{\omega_B}{\omega_A}\right)^2 I_2 = n^2 I_2 \,//$$

9(c) Cantilever shaft $\qquad \omega_n = \omega_d \Rightarrow$ Critical speed

Critical speed depends on K & m and NOT on eccentricity

so it remains same

10 (c)

11 (b) $\longrightarrow \sqrt{K/m/2} = \sqrt{2}\sqrt{K/m}$

12 (a) Add equivalent spring mass $\Rightarrow$

$N_c \propto \omega_n \propto \sqrt{K/m}$

$N_c \propto \sqrt{K}$

$N_c \propto \frac{1}{\sqrt{m}}, \; K = \frac{3EI}{l^3}$

~~13 ( ) Sub Dunkerley's eqn~~

$N_c \propto \sqrt{E}$

$N_c \propto l^{-3/2} \qquad N_c \propto \sqrt{I}$

(a) $\frac{\omega}{\omega_n} = 4$, $\omega_n = 1/4 \times 10$

16 ( ) Bell crank lever is given angular disp "θ"

PE $= \frac{1}{2}k_1(c\theta)^2 + \frac{1}{2}k_2(b\theta)^2 + \frac{1}{2}k_3(a\theta)^2$

KE $= \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}m_1(c\dot{\theta})^2 + \frac{1}{2}m_2(b\dot{\theta})^2$

vel $= c\dot{\theta}$
when its given
a disp θ

$\therefore$ PE $= \frac{1}{2}\underbrace{(k_1 l^2 + k_2 b^2 + k_3 a^2)}_{K_{eq}}\theta^2$

KE $= \frac{1}{2}\underbrace{(I + m_1 c^2 + m_2 b^2)}_{M_{eq}}\dot{\theta}^2$

$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}}$

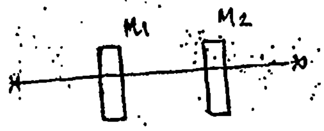
pg 10

Q 21 $m = 2.5$ kg
$k = 3000$ N/m

$\delta = \frac{1}{5}\ln\frac{x_0}{x_0/4} = \frac{1}{5}\ln 4$

$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$; $c = 2\xi\sqrt{km}$

Q22.



$\underbrace{\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}}_{\text{Mass matrix}}$ $\underbrace{\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}}_{\text{stiffness matrix}}$ $\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{\text{flexibility matrix}}$

Flexibility is reciprocal of stiffness.

$\omega_{n_1} = \sqrt{\frac{K_{11}}{M_1}}$ or $\sqrt{\frac{1}{M_1 a_{11}}}$ $\frac{1}{\omega_n^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2}$

So $A = K^{-1}$,

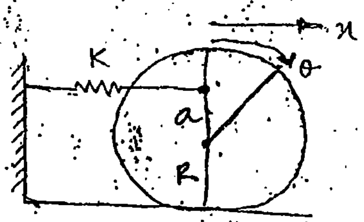
$$\omega_{n_2} = \sqrt{\frac{K_{22}}{m_2}} \quad \text{or} \quad \sqrt{\frac{1}{m_2 a_{22}}}$$

$$\left[\frac{1}{\omega_n^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2}\right.$$

Similar to Q23

★ Std problem :

ENERGY method used



$x = R\theta$ { FOR NO slip — Condition for 1 dof

{ If there is a slip it becomes a 2 dof system

$$KE = \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2}\left(\frac{m l^2}{2}\right)\dot{\theta}^2$$
$$= \tfrac{1}{2} m (R\dot{\theta})^2 + \tfrac{1}{2}\frac{mR^2}{2}\dot{\theta}^2$$
$$= \tfrac{1}{2}\left(\tfrac{3}{2} m l^2\right)\dot{\theta}^2$$

{ With disp x, i.e. relative to the disp x there occurs the angular disp $a\theta$.

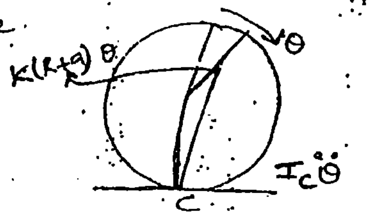
So, $PE = \tfrac{1}{2} K (x + a\theta)^2$

$PE = \tfrac{1}{2}\left(K (R+a)^2\right)\theta^2$ } $\omega_n = \sqrt{\dfrac{K(R+a)^2}{\tfrac{3}{2} m R^2}}$

EQUILB METHOD

Consider moment equilb about instantaneous centre.

Q24



$$I_c\ddot{\theta} + K(R+a)^2\theta = 0$$

Q24 { Bring down speed to (0.7×1400) → safe speed

2 methods { or Once the critical speed

$K = (192)\frac{EI}{l^3}$ → changes critical speed

3	⟶	1	1
48	⟶	16	4
$K = (192)\frac{EI}{l^3}$	⟶	64	8

speed

$w_n \propto \sqrt{E}$

$w_n \propto \sqrt{I} \rightarrow \propto d^2$

$w_n \propto \frac{1}{l^{3/2}}$

	w_n
Increase span →	↓
reduce span →	↑

Q25 $M = 50\,kg$

$K = 35000\,N/m$

$C = C_c = 2\sqrt{Km}$

$f_d = C V_d$ {V_d is given}

prob 3/2 {6am – 8am, 6pm – 8:30pm}