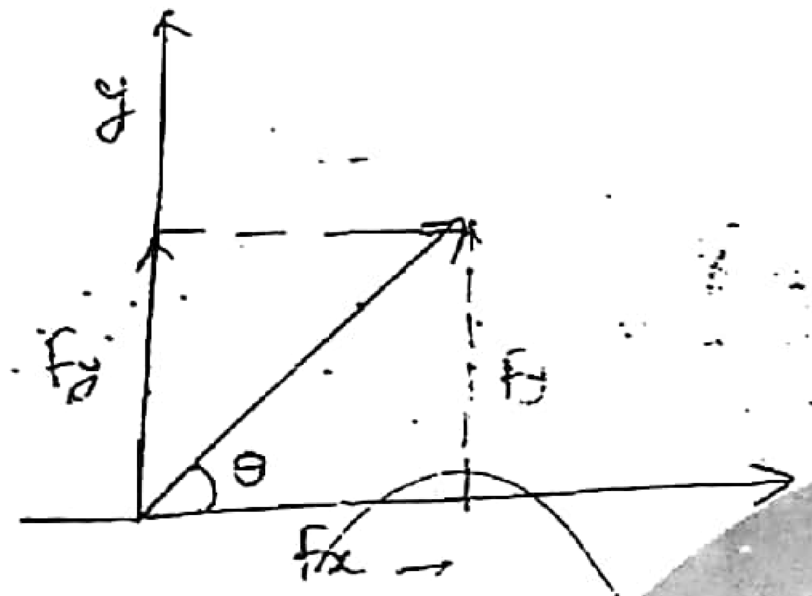
— Engineering Mechanics —

1st topic →

Component of forces —
- → (a) Rect. compo. of a force → Angle btw compo. is 90°
- → (b) oblique component of forces → Angle btw compo. is not 90°

a) Rectangular components →



$F_x \Rightarrow F\cos\theta$

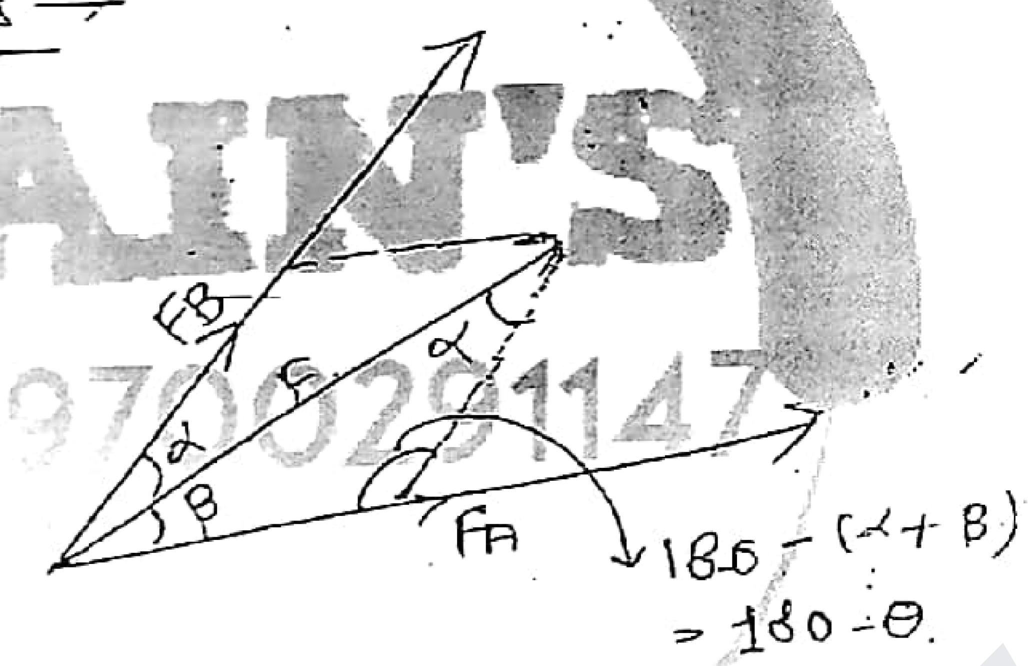
$F_y \Rightarrow F\sin\theta$

$\boxed{F = \sqrt{F_x^2 + F_y^2}}$

$\boxed{\tan\theta \Rightarrow F_y/F_x}$

b) Oblique components →

$\theta = \alpha + \beta$



using cos Rule →

$F^2 = F_A^2 + F_B^2 - 2F_A F_B \cos(180-\theta)$ ($\cos(180-\theta) = -\cos\theta$)

$$\boxed{F = \sqrt{F_A^2 + F_B^2 + 2F_A F_B\cos\theta}}$$

find α ← $F_A^2 = F^2 + F_B^2 - 2F \times F_B \times \cos\alpha \Rightarrow$ ~~find~~ α find

$F_B^2 \Rightarrow F^2 + F_A^2 - 2 \times F \times F_A \cos\beta \Rightarrow$ find β

← Using sine Rule →

$$\frac{F_A}{\sin\alpha} \Rightarrow \frac{F_B}{\sin\beta} \Rightarrow \frac{F}{\sin\{180-(\alpha+\beta)\}}$$

Concept (3) Type of Equillibrium →

① Stable equillibrium →

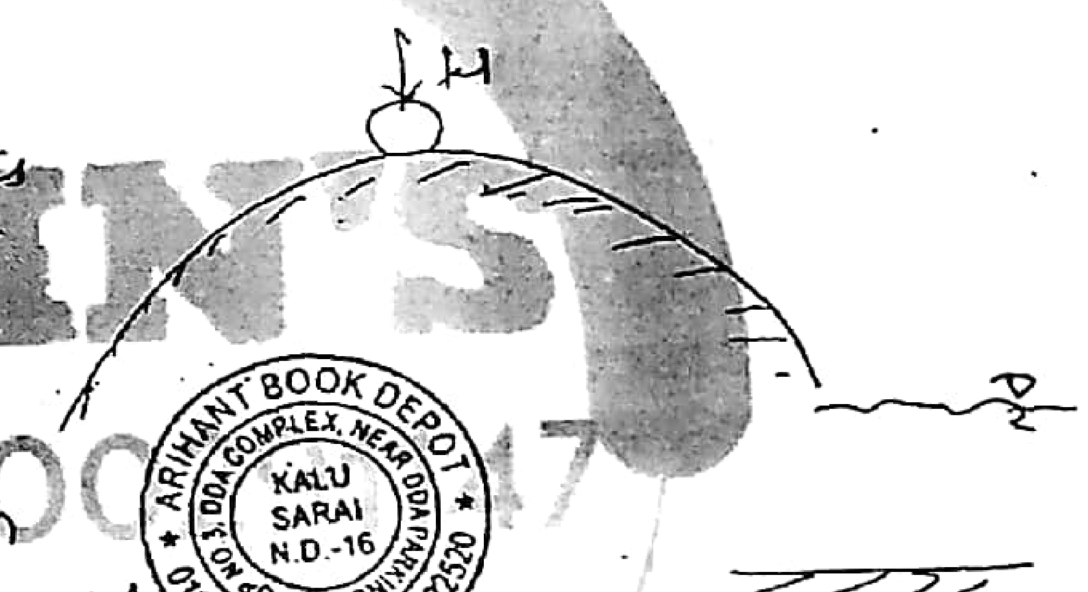
→ if a body is displaced from its Equillibrium position then if it comes back to its original position, then it is called stable equillibrium.

→ for stable equillibrium potential energy is minm.

2) Neutral Equillibrium →
(colum theory is derived)

if a body is displaced from its equillibrium position then if it takes up another equi. position, then it is called neutral equillibrium.

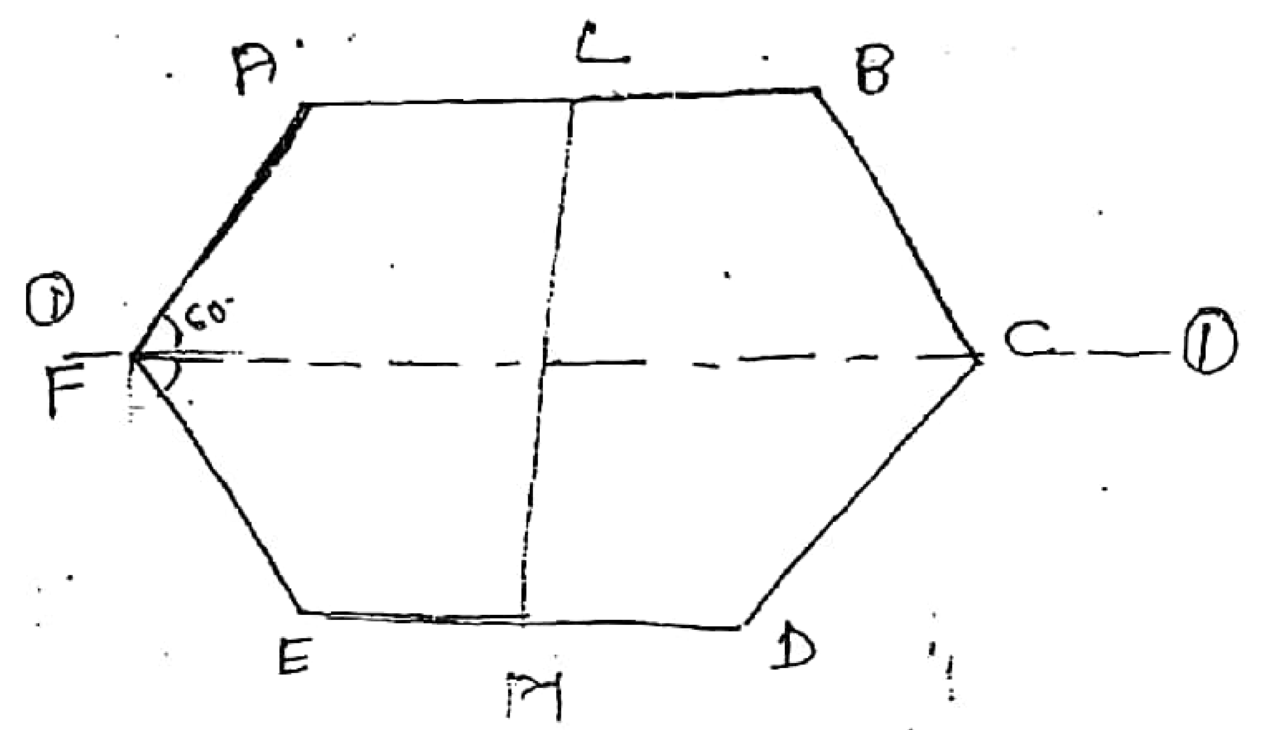
3) Unstable Equillibrium →

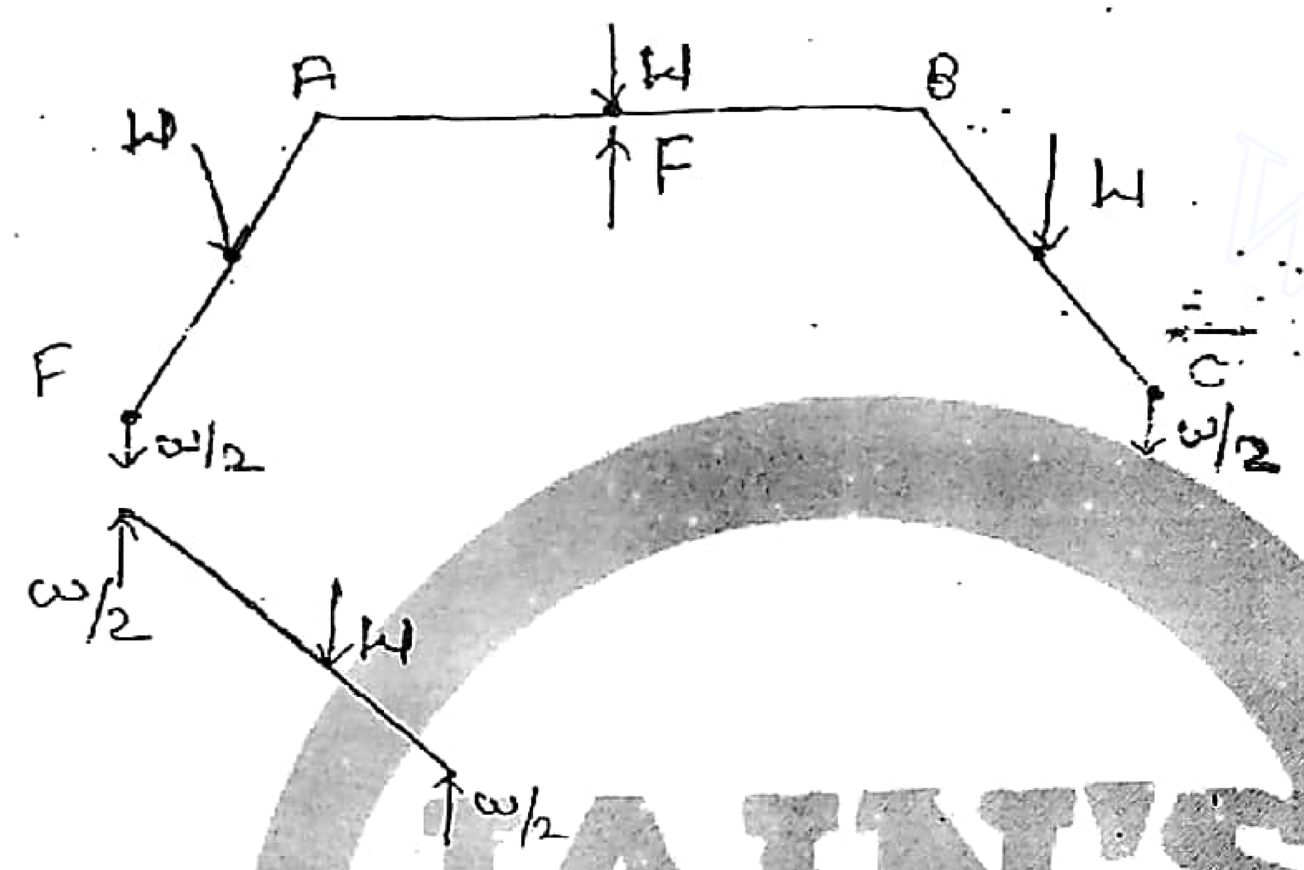


→ if a body is displaced from its equil position, then if it does not come back to its original position nor takes up another useful equilibrium position then it is called unstable Equillibrium.

Que → 6 identical uniform Rods AB, BC, CD, DE, EF and FA each weighing 'W' are freely ~~jointed~~ joined at their extremities, so that they form a regular hexagon. the Rod AB is fixed in horizontal position and middle points L and M of AB, and DE are connected by weightless Rod. the force induced in the connecting Rod LM is ?

A) 6ω B) 4ω
C) 3ω D) 2.5ω





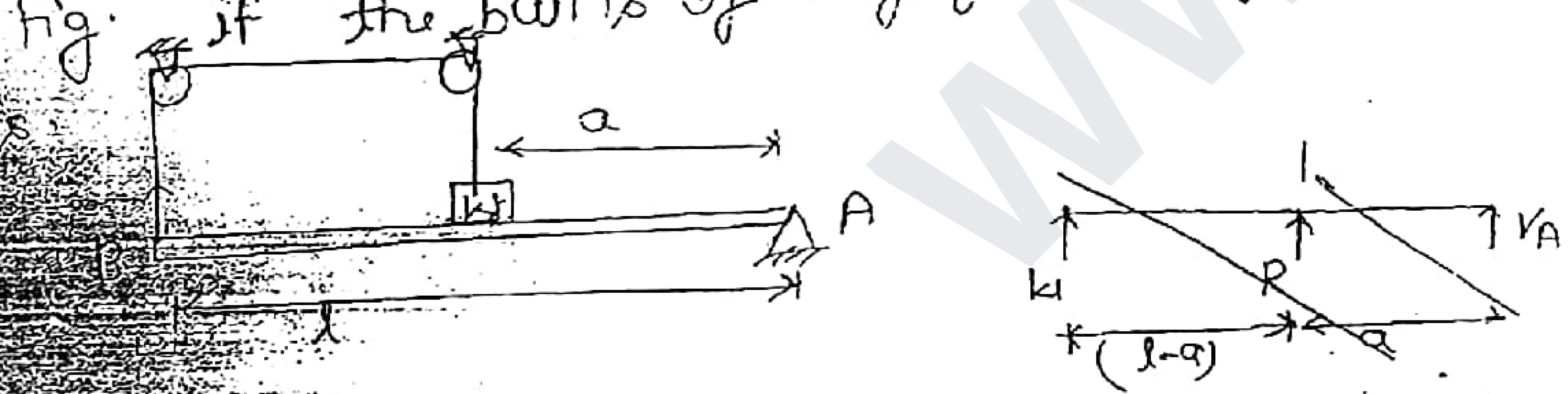
F.B.D. of upper portion

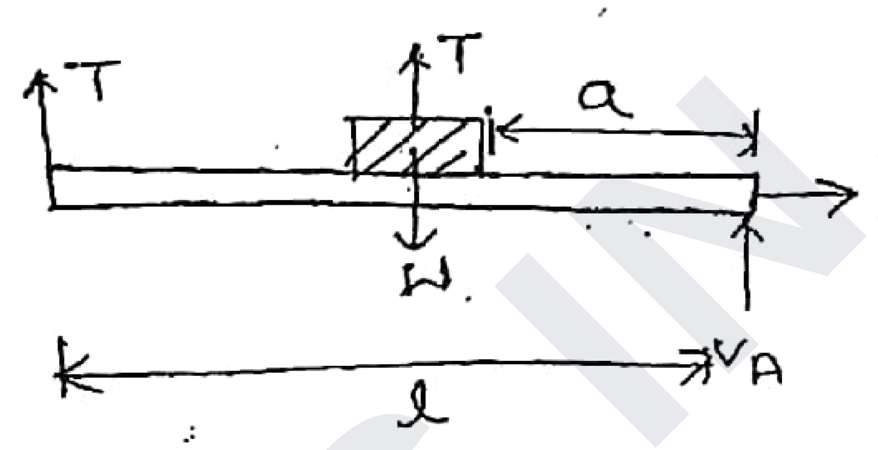
$\Sigma y = 0$

$F - (\omega - \omega - \omega) - \omega/2 - \omega/2 = 0$

$\boxed{F = 4\omega}$ (comp.) Ans

Que- A Bar AB is supported at A and connected by a wire passing over two frictionless pullies as shown in fig. if the bar is of negligible weight, Reaction at A is





FBD of bar AB

$\Sigma M_A = 0$

$T \times l + T \times a - \omega \times a = 0$

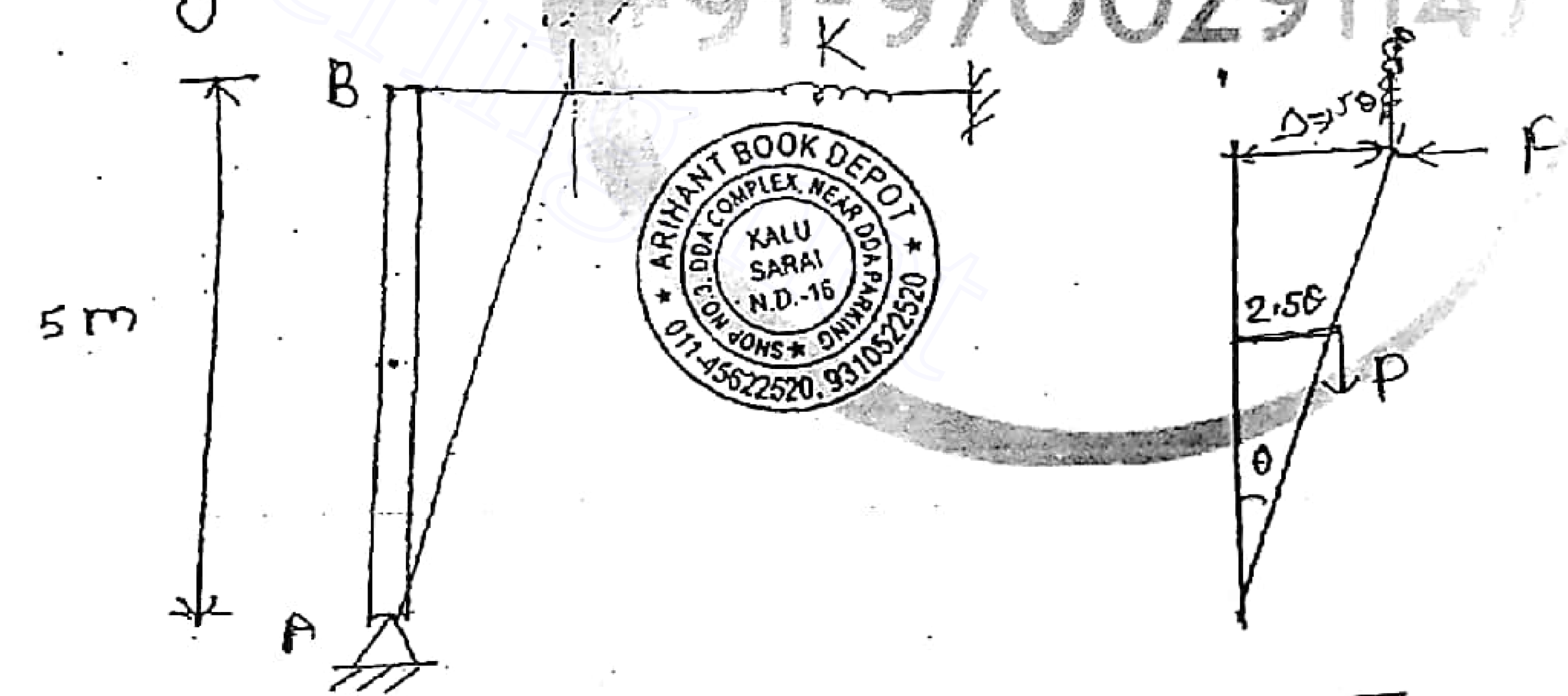
$\boxed{T = \frac{\omega a}{l + a}}$

$\Sigma y = 0$

$T + T - W + V_A = 0$

$V_A = W - 2T = \left(\omega - \frac{2\omega a}{l+a}\right) = \boxed{\frac{\omega(l-a)}{(l+a)}} \Rightarrow V_A$

Que- A column AB of length 5m and weight 6KN is hinged at A and supported by a spring of stiffness k as shown in fig. for the column to be in stable equ. the value of K is.



$P \times \Delta = F \times 5$

$\cancel{6 \times 5 \times \theta} = K \times \theta \times 5$

$\cancel{2.50 \times 6 = K \times 50}$

$\boxed{K = 3/5 = 0.6\ KN/m}$

$F \times \delta = P \times \delta / L$

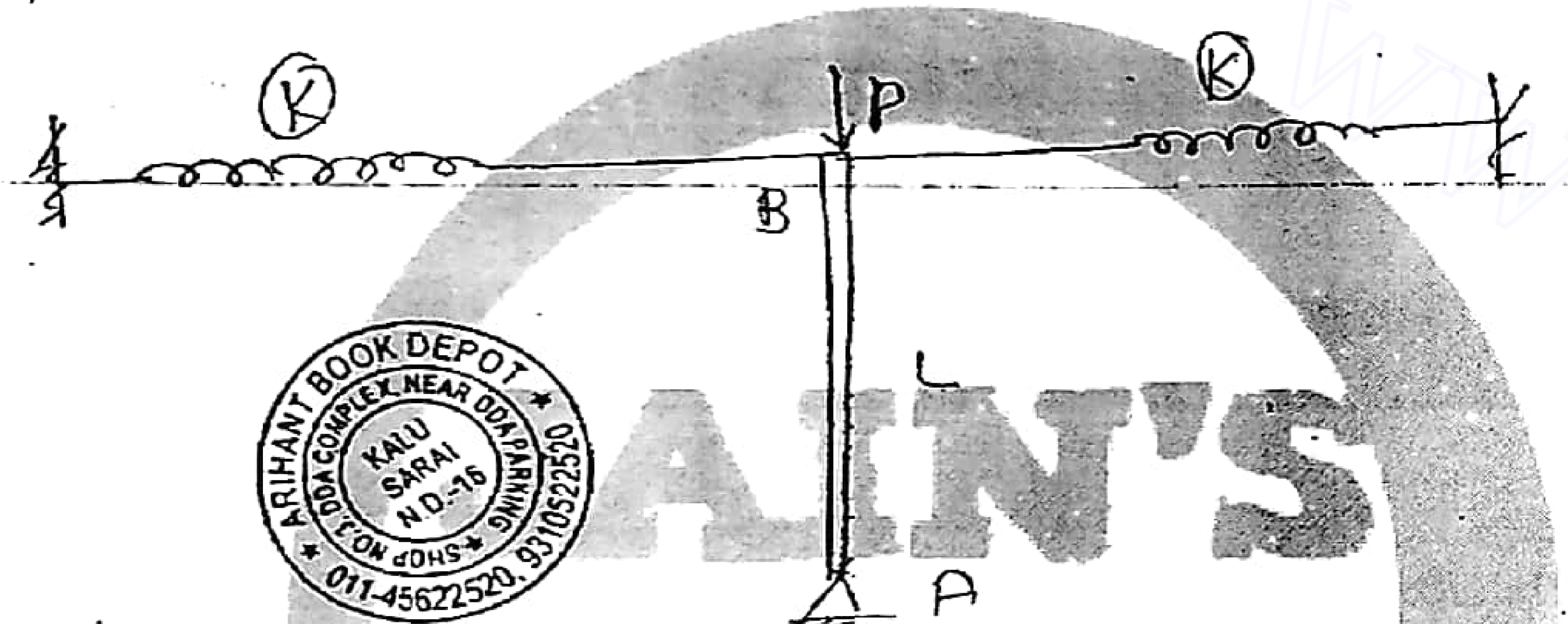
$K\delta \times \delta = P \times \delta / L$

A) $K = 0.6$ KN/m (neutral equillibrium)

B) $K < 0.6$ KN/m (unstable equillibrium)

C) $K > 0.6$ KN/m (stable equillibrium)

Que → A bar AB of negligible weight is hinged at A and supported by 2 springs of stiffness K, as shown in fig. the limiting value of P for the bar to be in stable Equillibrium is ?



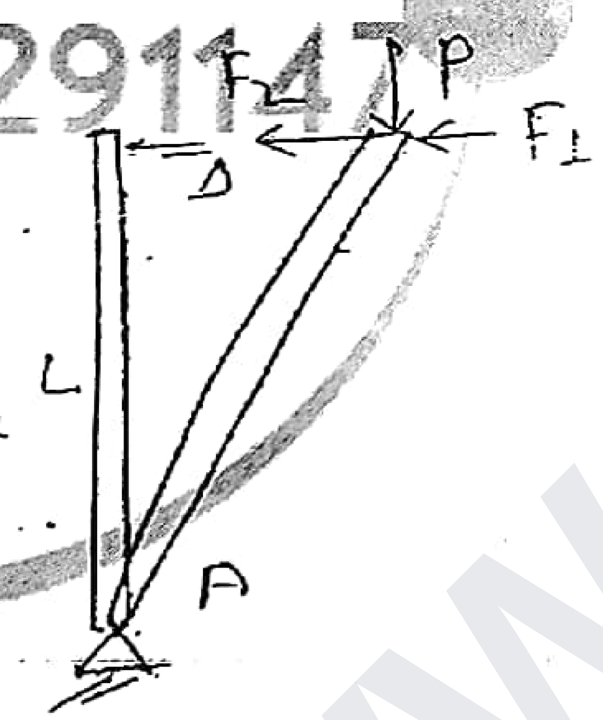
$M_A = 0$

$P \times \Delta = F_1 L + F_2 \times L$

$P \times \Delta = K\Delta L + K\Delta L$

$\boxed{P \Rightarrow 2KL}$ → neutral

$< 2KL$ for stable

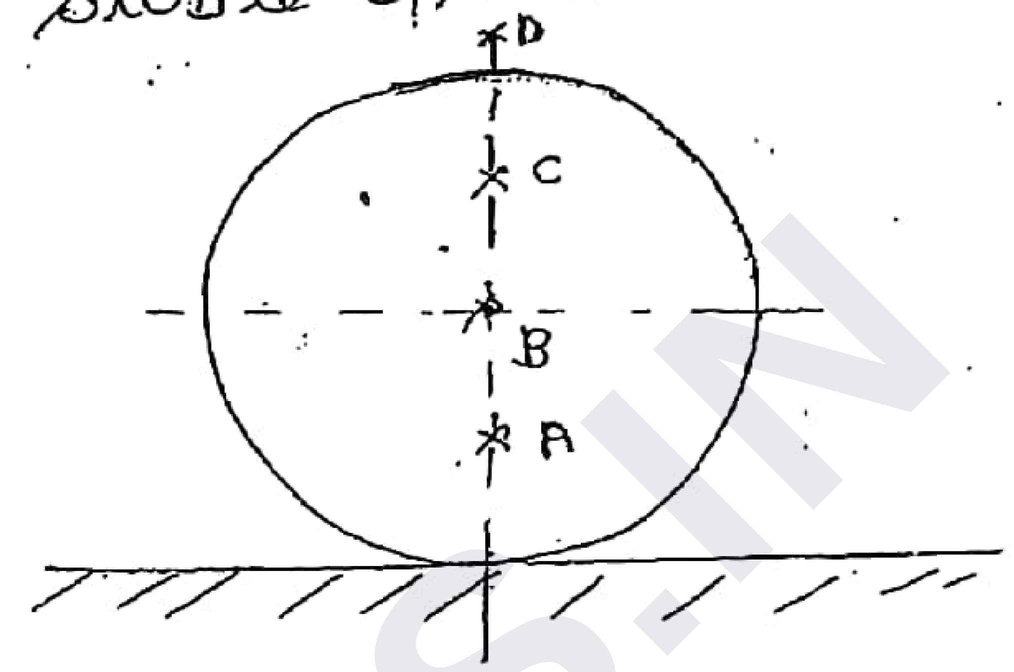


① KL

② 2KL

③ KL/2

Que → for the non homogenous sphere shown in fig. to be in stable Equilibrium. CG must lie at ?



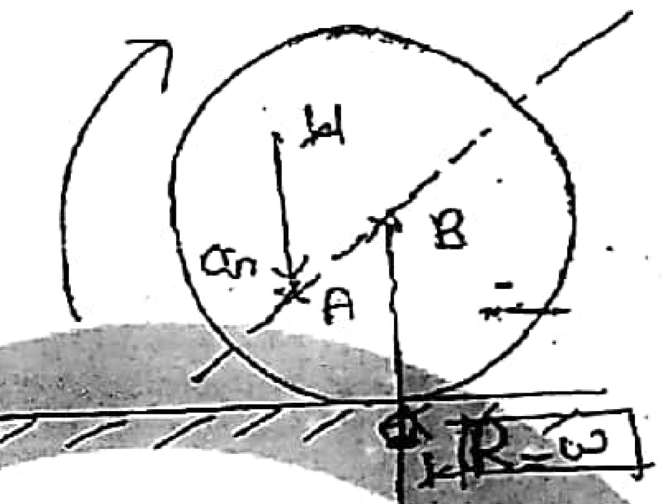
a) A

b) B

c) D

d) C

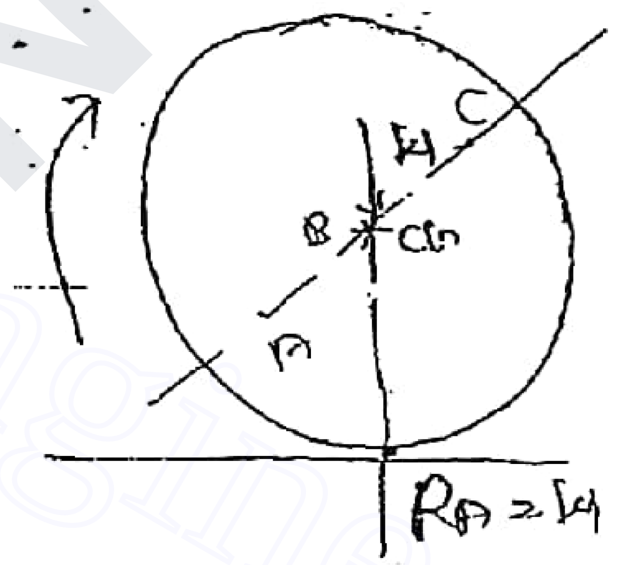
Note → if C.G. at A →



stable Equilibrium.

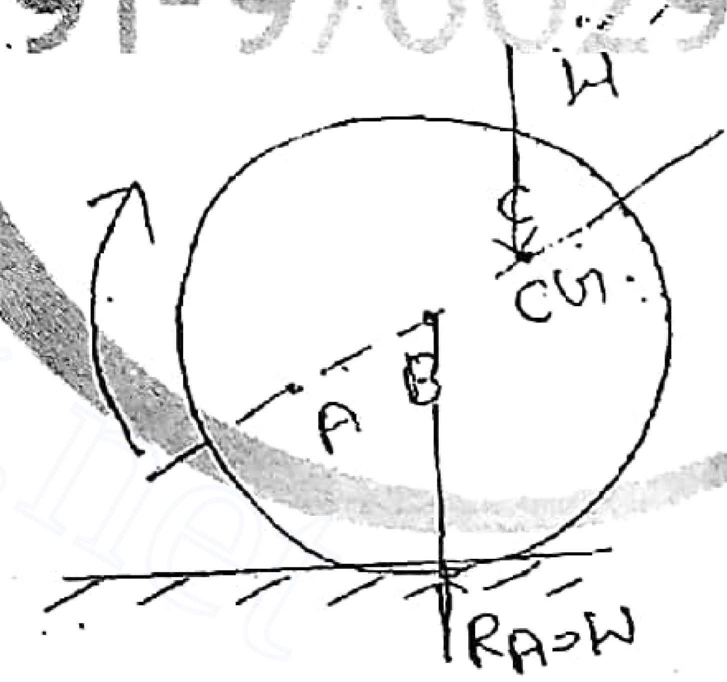
Anticlock-wise couple will stable the sphere back to its original position.

if C.G. at B →



(Neutral Equilibrium)

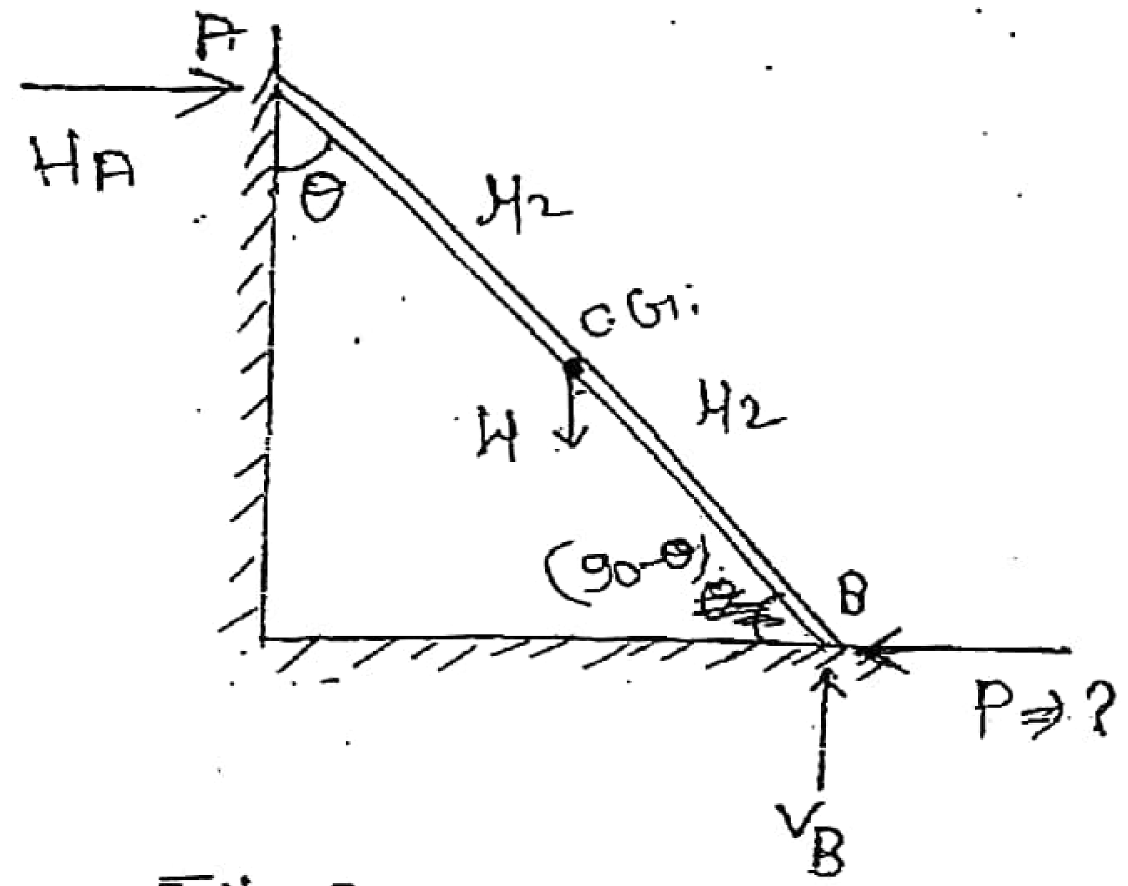
If C.G. at 'C.'



unstable equ.

clockwise couple will rotate the body further.

Q.(i) A ladder AB of weight w and length l is held in equilib by a horizontal force P as shown in fig. Assuming the surfaces are smooth. what is the value of P.



$\Sigma Y = 0$

$\boxed{V_B = W}$

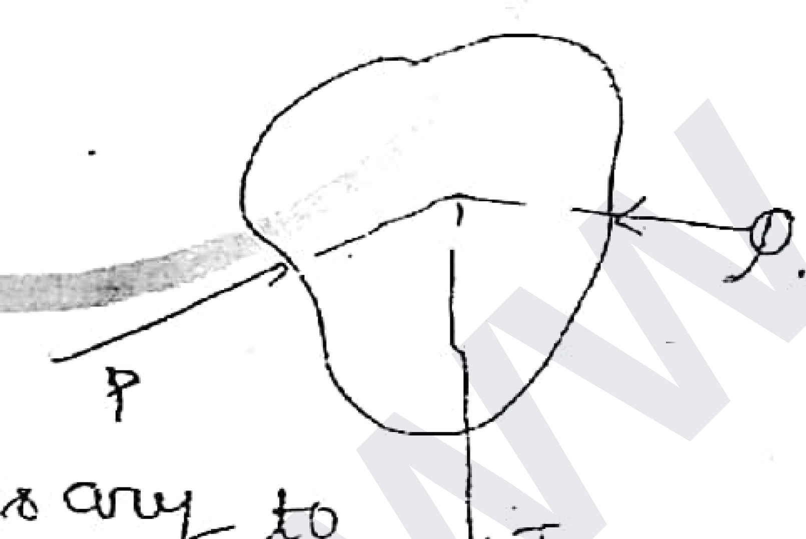
$\boxed{H_A = P}$

$\Sigma M_B = 0 \Rightarrow$

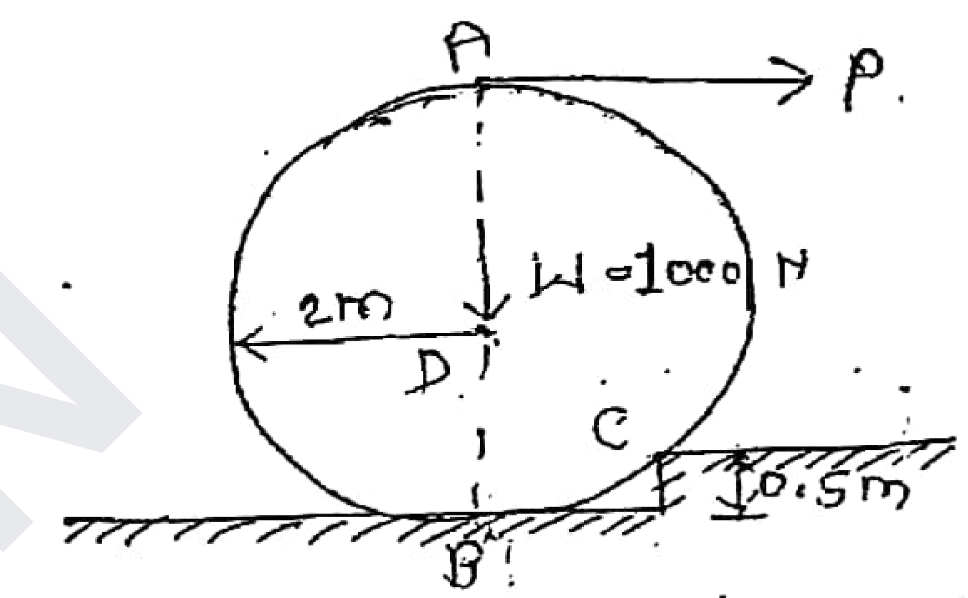
$H_A \times l\cos\theta - W \times \frac{l}{2}\sin\theta = 0$

$$\boxed{P = \frac{W l \sin\theta}{2 l \cos\theta} = \frac{W}{2}\tan\theta}$$

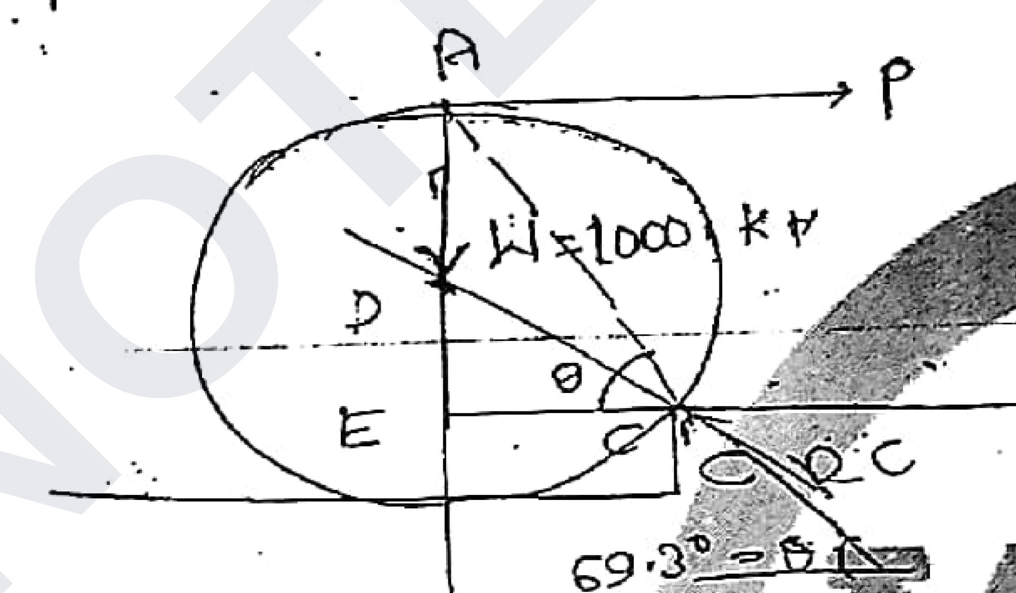
Note ⇒ To keep three forces in Equilib: they must be coplaner and concurrent.



Q.(ii) the horiz. force P necessary to move a cylinder of weight 1000 N out of the ditch shown in fig ⇒



Note (1) if a cylinder is on the verge of moving out of the ditch it looses its point of contact at B. the only point of contact is at C.



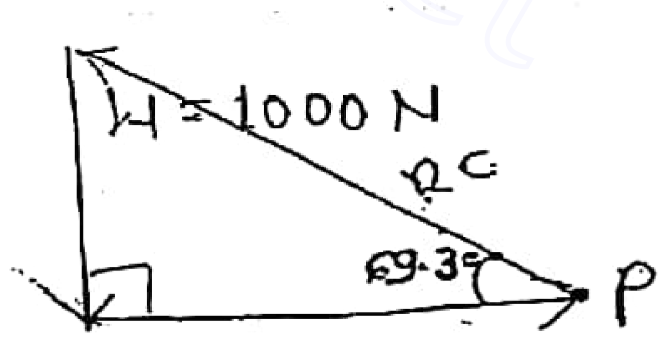
2) to keep 3 forces in equilibr. (i.e. P, W, R_C) they must meet at one point. i.e. at point A.

3) if only 3 forces are acting on a body use force Δ to find the unknowns quickly.

4) from Δ CDE, CD = 2m, DE = 2 − 0.5 = 1.5 m

$EC = \sqrt{2^2 - 0.5^2} \Rightarrow 1.32\,m$

$\tan\theta \Rightarrow \frac{AE}{EC} = \frac{(2+1.5)}{1.32}$; $\boxed{\theta \Rightarrow 69.3^\circ}$



$\tan 69.3^\circ = \frac{1000}{\cancel{R_C}\ P}$

$\boxed{P \Rightarrow \frac{100}{\tan 69.3^\circ}}$ $\boxed{P \Rightarrow 377\ N}$

$\sin 69.3 = \frac{1000}{R_C}$

Note→ to draw force triangle →

→ first draw a force whose magnitude and direction are known. at the head of 1st force draw the tail of the 2nd force. Similarly draw the 3rd force so that a Δ is formed.

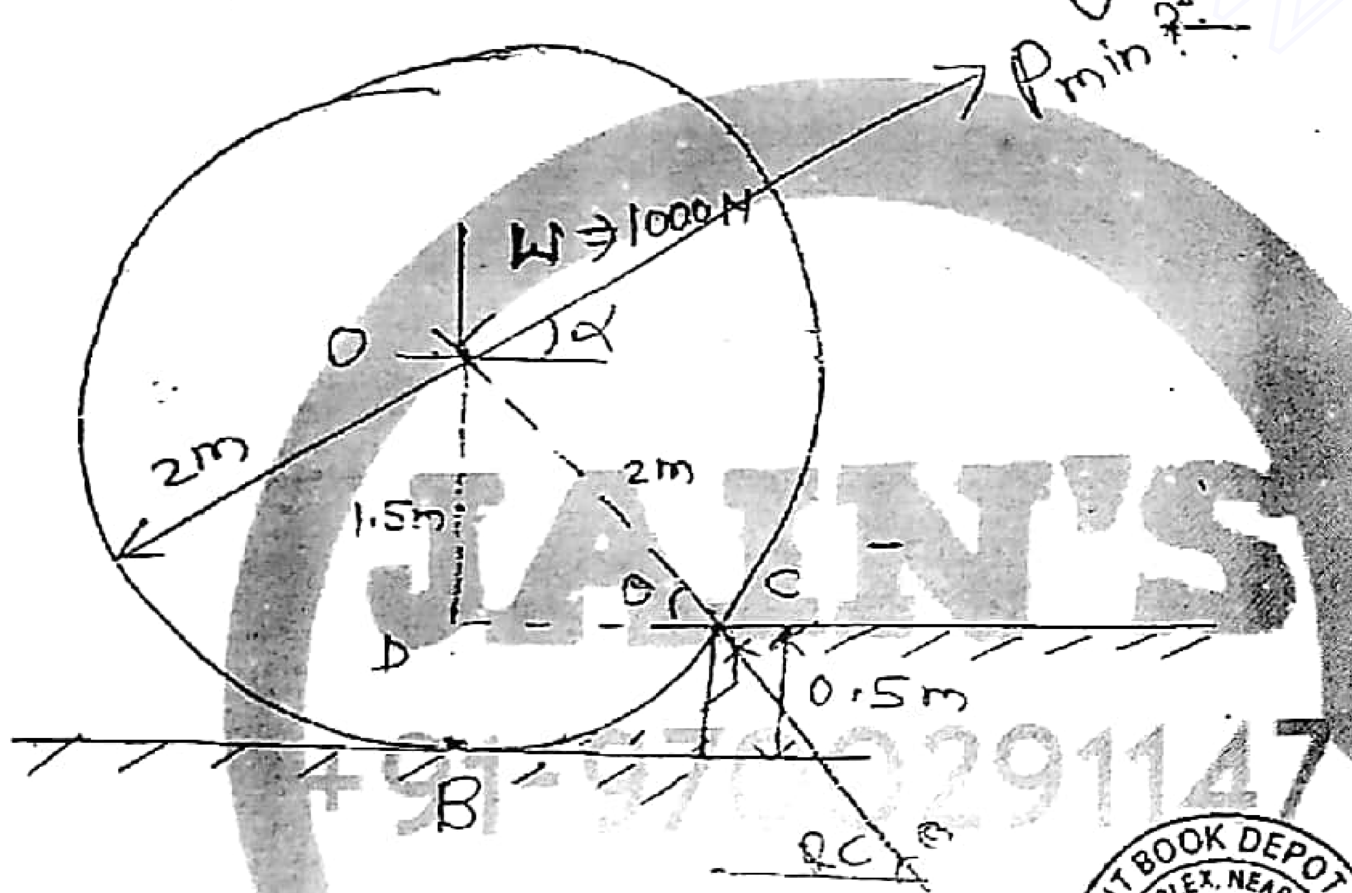
2nd method →

$\Sigma M_c = 0$ (↻ +ve, ↺ −ve)

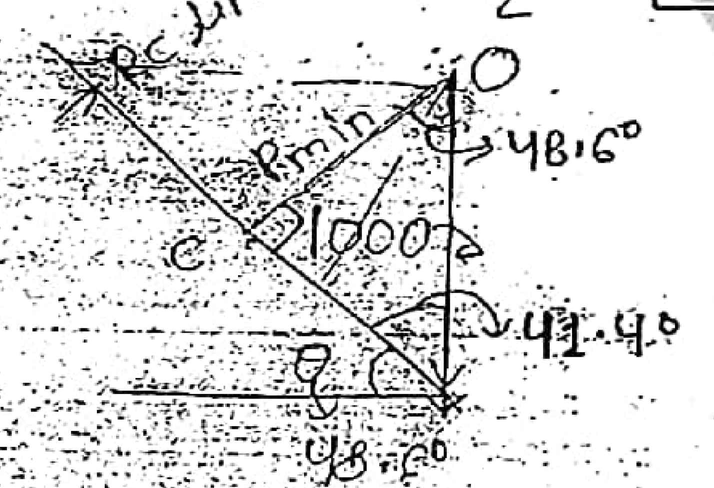
$+P\times 3.5 - \overset{1000}{W}\times 1.32 = 0$

$\boxed{P = 377\ N}$ Ans

Que→ find minm force P necessary to move the cylinder out of the ditch shown in fig.



$\sin\theta = \frac{1.5}{2} \Rightarrow \boxed{\theta = 48.6°}$

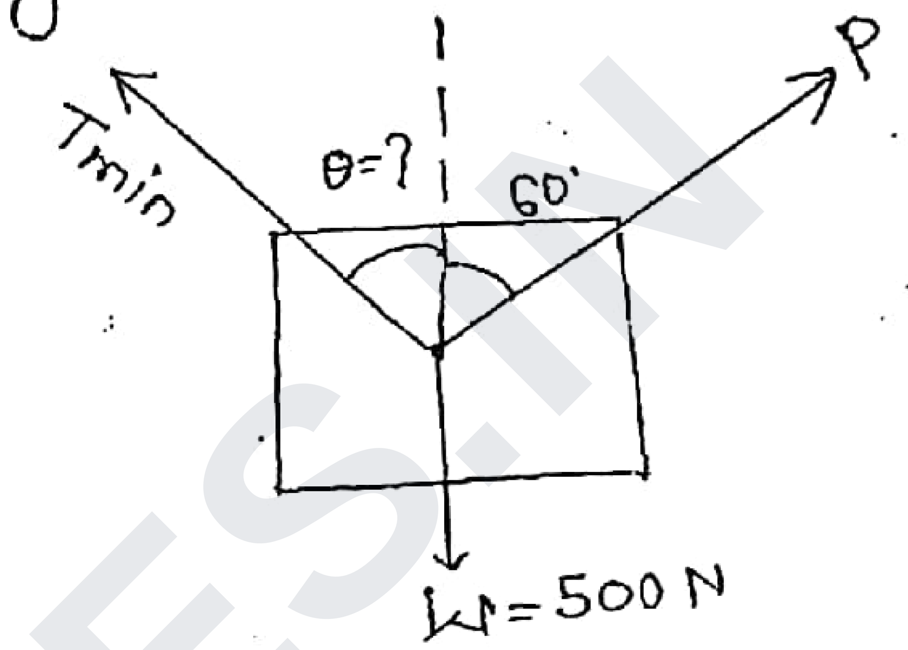


$\cos 48.6° = \frac{P_{min}}{1000}$

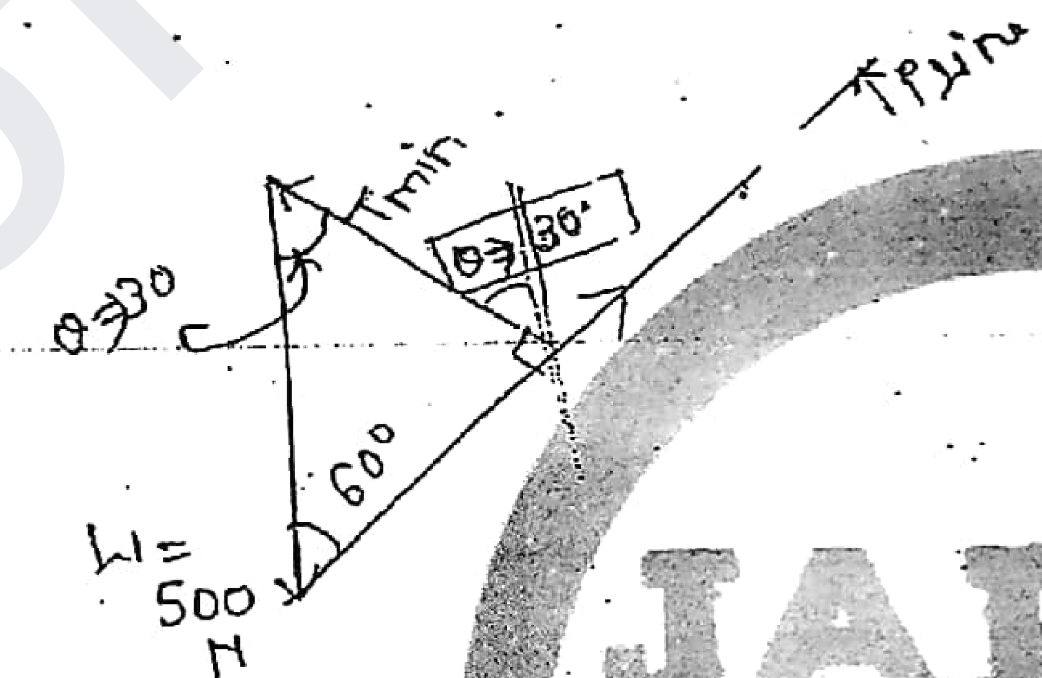
$P_{min} = 1000 \cos 48.6°$
$= 661.4\ KN$

to get minm value of P_{min} it must act ⊥ to R_c line (i.e. OC line.

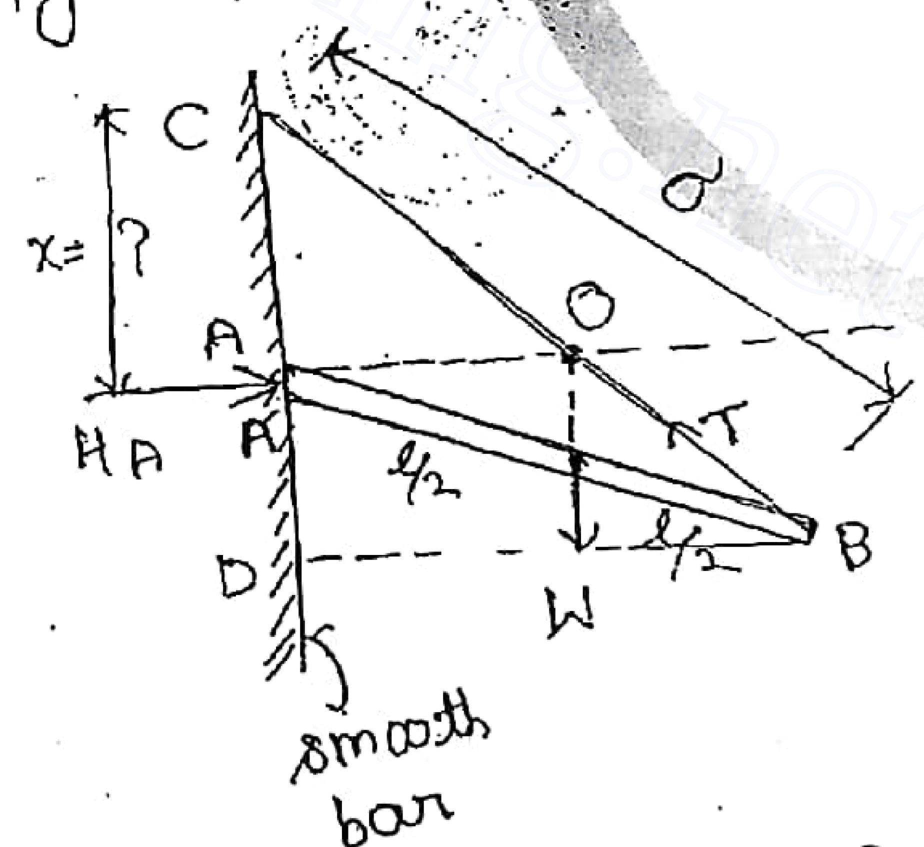
Que→ A block of weight W = 500N is supported as shown in fig. for T to be min, θ is



a) θ = 90°
b) θ = 60°
c) θ = 30°
D) θ = 45°



Que→ A bar of length l is supported against a smooth vertical wall and by a wire of length a as shown in fig. For the bar to be in equ. the value of x is



Note-1 to keep 3 forces in equilibrium (W, T, HA) they must meet at 1 point (i.e. at O) from Δ^n ABD, $AB^2 = AD^2 + BD^2$

$l^2 = x^2 + BD^2$

$\boxed{BD^2 = l^2 - x^2}$ —— (1)

Δ BCD, $CB^2 = CD^2 + BD^2$

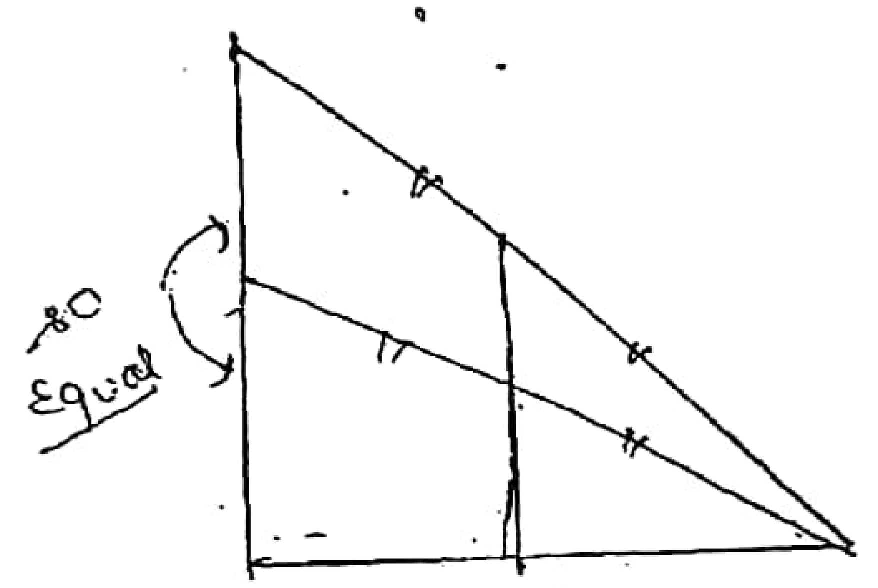
$q^2 = (2x)^2 + (l^2 - x^2)$

$q^2 = 4x^2 + l^2 - x^2$

$x^2 = \left(\frac{q^2 - l^2}{3}\right)$

~~$x = l^2 - q^2$~~

$\boxed{x = \sqrt{\frac{q^2 - l^2}{3}}}$ Ans.



Que ⇒ 3 forces acting at a point O R $P_1 \Rightarrow (3i + 6j)\,N$
$P_2 \Rightarrow (-1.5i + 4.5j)\,N$
$P_3 \Rightarrow (-10.5i + 1.5j)\,N$

if a fourth force P_4 is added such that a point O is in equilibrium. then P_4 will be.

$P_4 \Rightarrow (P_x i + P_y \hat{j})$

$\Sigma X \Rightarrow 0$

$0 = 3i - 1.5i - 10.5 + P_x \Rightarrow (+9i)\,N$

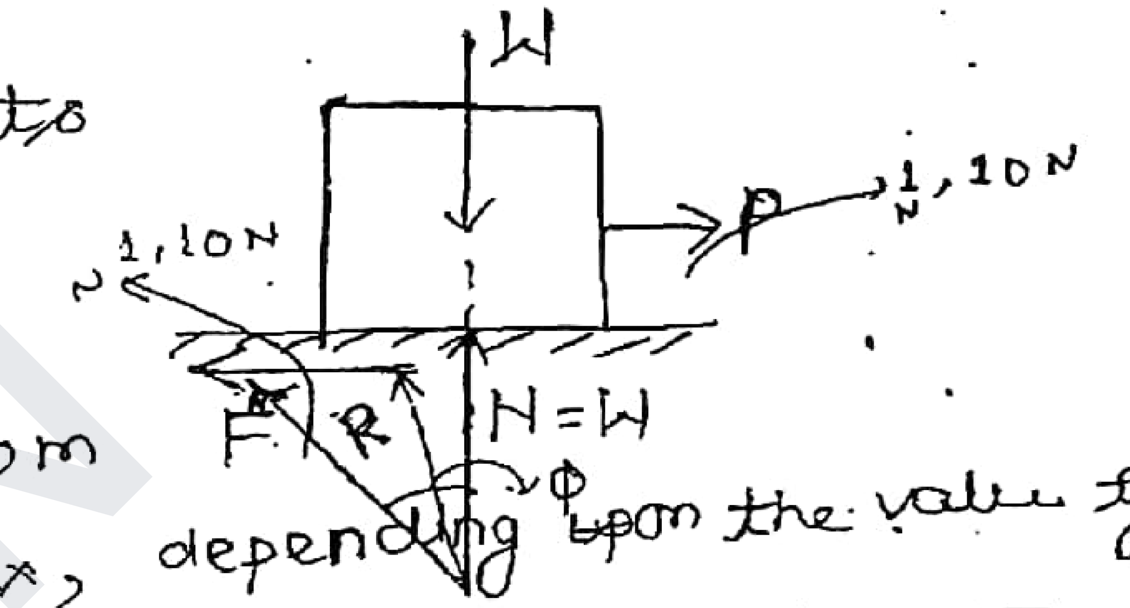
$\Sigma y = 0$

$0 = 6 + 4.5 + 1.5 + P_y \Rightarrow -12\,N$

$P_4 = (+9i - 12\hat{j})\,N$ Ans

← Friction →

Concept (1)



1) → frictional force always acts opposite to relative motion. of the given F.B.D.

2) frictional force varies from 0 to a max. value Fmax, depending upon the value of P.

3) When F is at its Fmax. value $\boxed{F_{max} \propto \text{Normal Rxn}}$

$\boxed{F_{max} \Rightarrow \mu N}$ $\mu \Rightarrow$ coefficient of static friction.

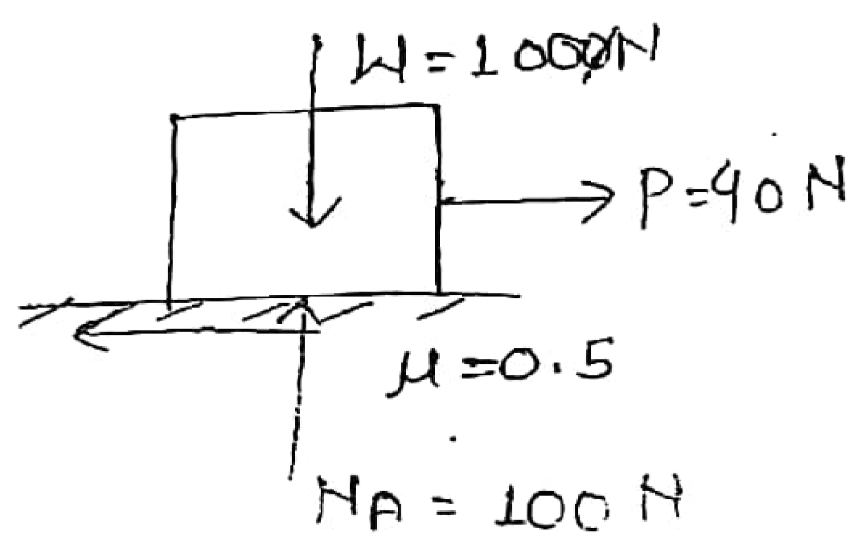
(4) When F is at its Fmax. value the angle btw the resultant and the normal Rxn is called Angle of friction.

5) $\boxed{\tan\phi \Rightarrow \frac{F_{max}}{N} \Rightarrow \mu}$ coefficient of friction.

6) When Resultant is Rotated around a normal Rxn the line, we get a cone called cone of friction. If any other Resultant lies within this cone, it means that frictional force is less than value. it means that the body is at rest.

7) Coefficient of Kinetic friction (μ_K) exists only when the body is in motion, is always less than μ_s (coefficient of static friction)

Que → frictional force developed at the contact surface shown in fig is.



A) 50N
B) 60 N
C) 1000N
d) 40N

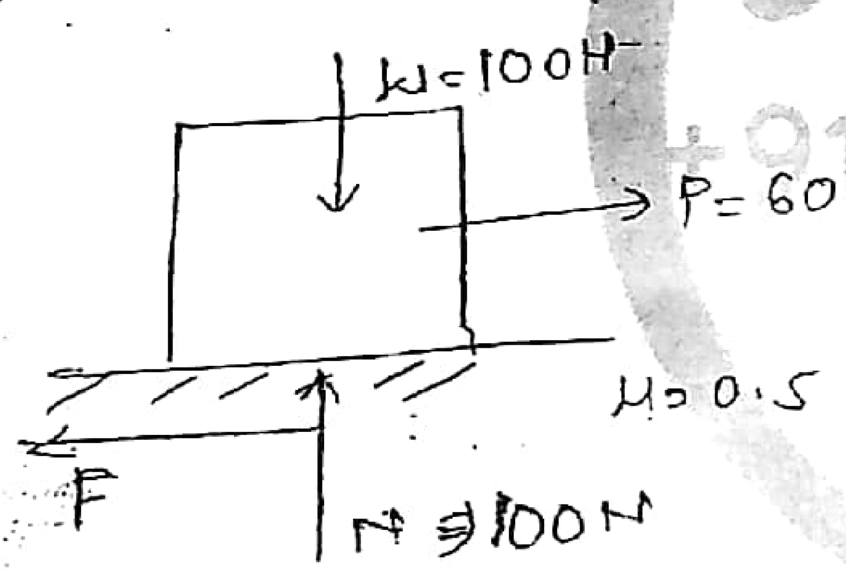
F_{max} = Max. friction ⇒ force that can be developed $= 0.5 \times 100 = 50N$

but actual frictional force ⇒ $P \Rightarrow$ ~~0.5×~~ $40N$

so actual force develop at contact surface is also 40 N. also

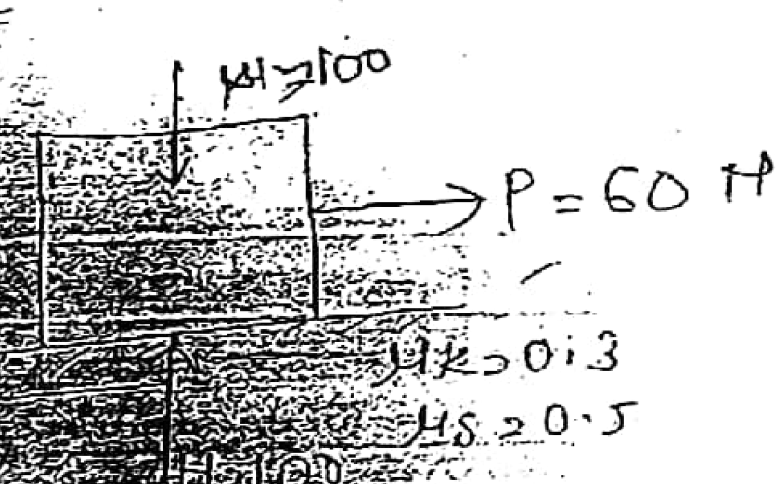
$\boxed{40 - F \geq 0}$ $\boxed{F \geq 40N}$ Ans

Que → find frictional force developed at the contact surface



$F_{max} = \mu N$
$= 0.5 \times 100 = \underline{50N}$

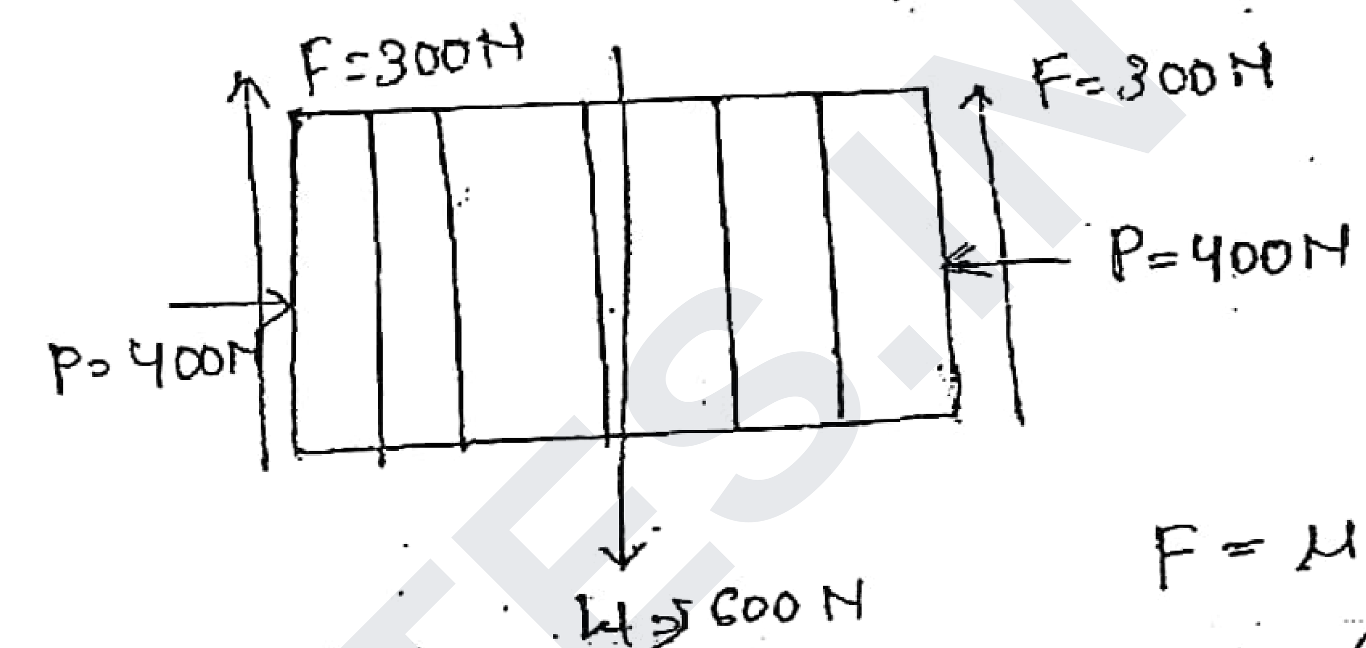
Que → find the frictional force developed at contact surface.

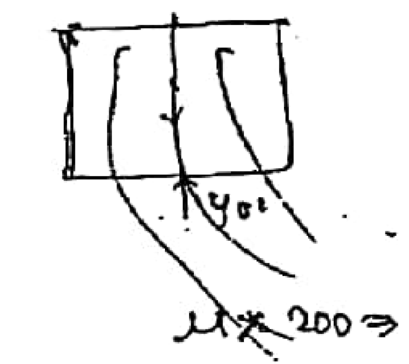


Note → due to 60N force, the block moves. so μ_k comes into picture.

$F_{max} \Rightarrow \mu_k \cdot N = 0.3 \times 100 = \underline{30N}$ Ans

Que → 6 books each weighing 100N are lifted with hand by applying a comp. force of 400N. if the books are on the verge of slipping, μ between books and hand is. ?

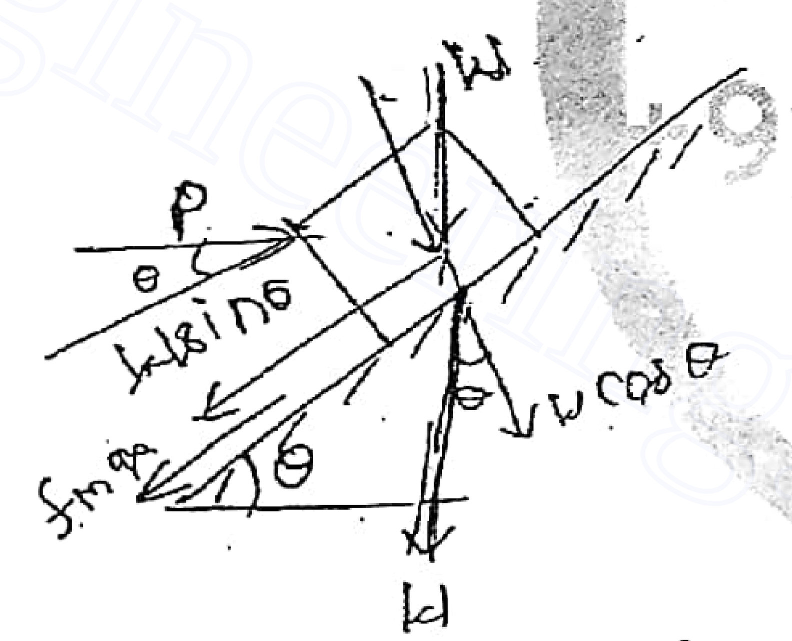




$F = \mu P$

$\mu = \left(\frac{F}{P}\right) = \frac{300}{400} = 0.75$ Ans

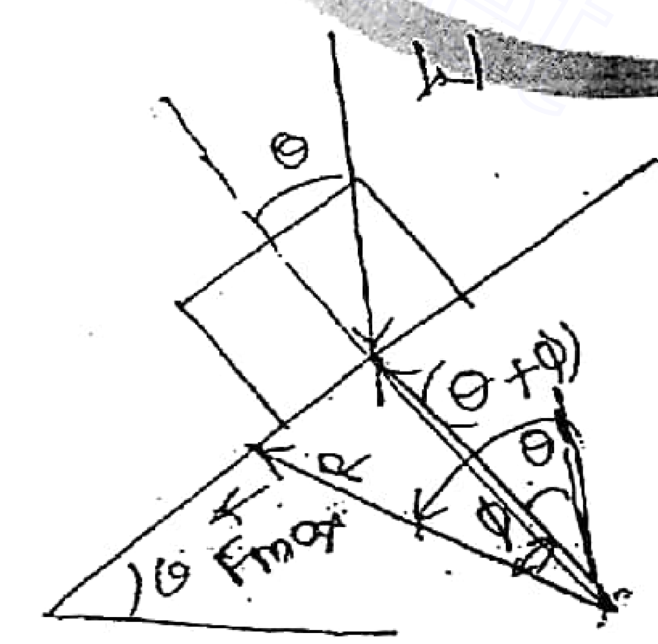
Que → A block of weight W is subjected to a horizontal force P as shown in fig. if the block is on the verge of moving up the inclined (ϕ is the angle of friction) then the value of P is.

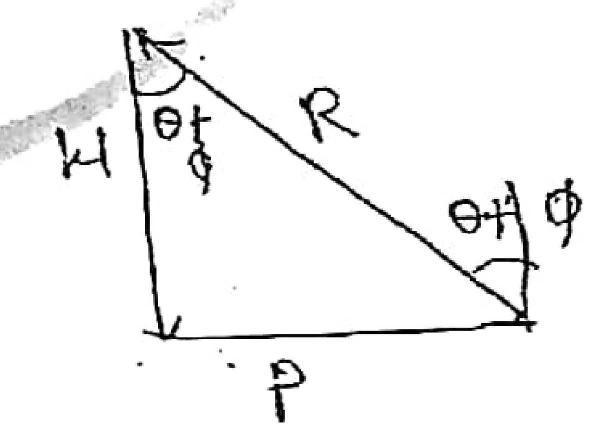


$W\sin\theta - P\cos\theta - f \geq 0$

$W\sin\theta - \mu W\cos\theta = P\cos\theta$

$\boxed{P = W(\tan\theta - \mu W)}$



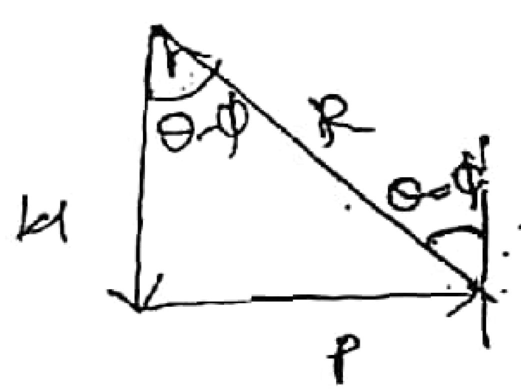


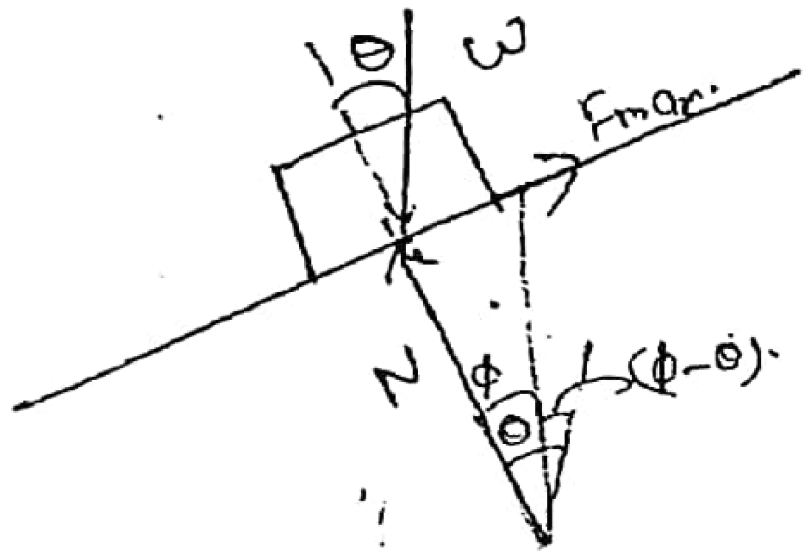
$\tan(\theta+\phi) = \frac{P}{W}$

$\boxed{P = W\tan(\theta+\phi)}$

Ans

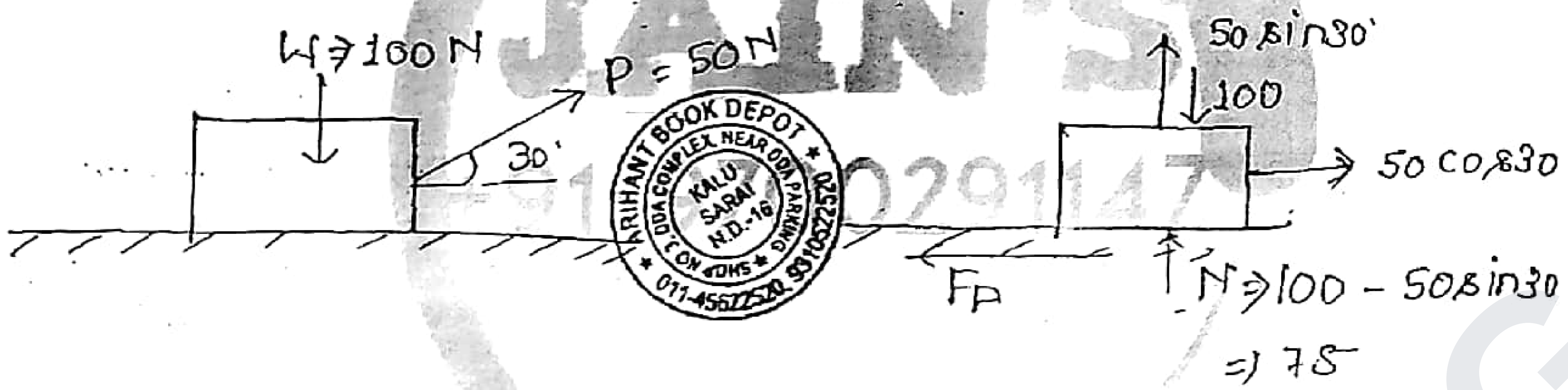
in the above problem, the Horizontal force P necessary to prevent slipping down the inclined is=?





$\tan(\theta-\phi) \Rightarrow \frac{P}{W}$

Que→ if 50N force is necessary to move the block right side as shown in fig. Coefficient of friction btw block and ground is. ?



$\Sigma Y = 0 \quad N_A + 50\sin 30° - 100 \Rightarrow 0$

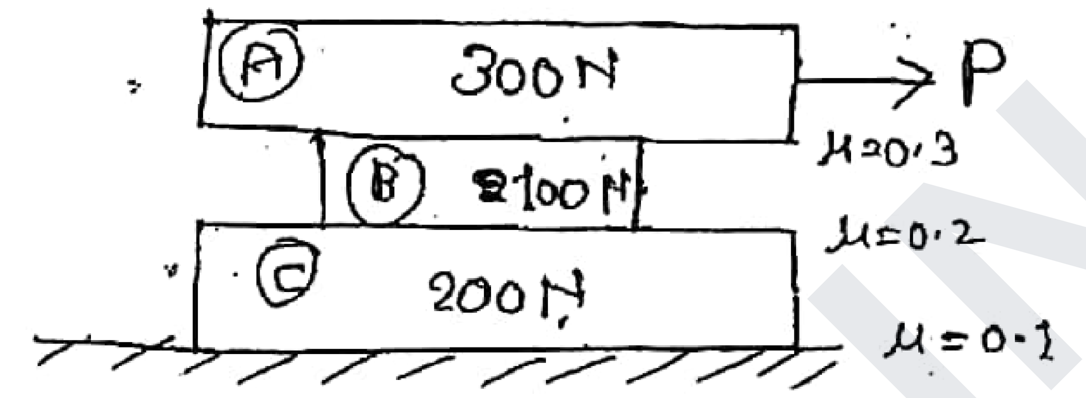
$\boxed{N_A = 75N}$

$\Sigma X = 0 \quad -F_A + 50\cos 30 = 0$

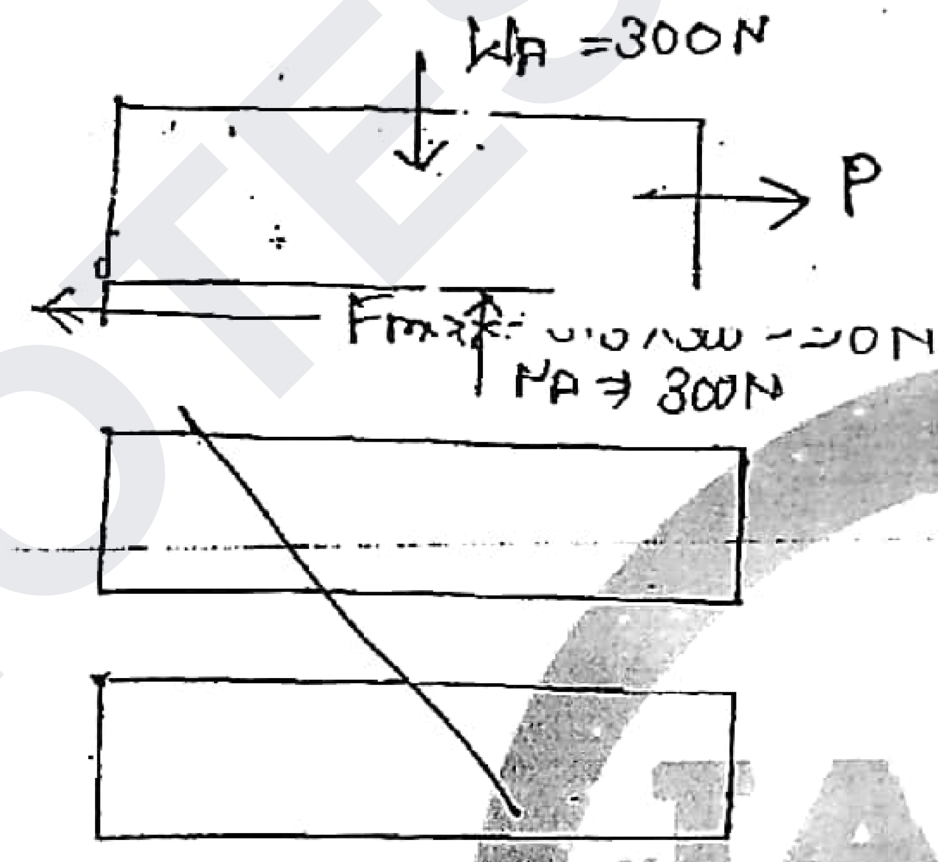
$\boxed{F_A = 50\cos 30°\ N}$

$\mu = \frac{F_A}{N_A} = \frac{50\cos 30°}{75} \Rightarrow 0.577$ Ans

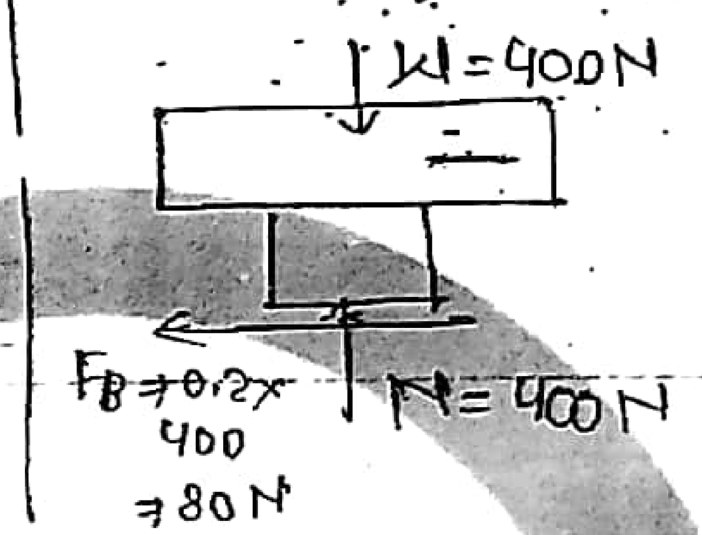
Que→ Find the force P necessary to move 1 or more blocks shown in fig?



FBD of A



FBD of A & B

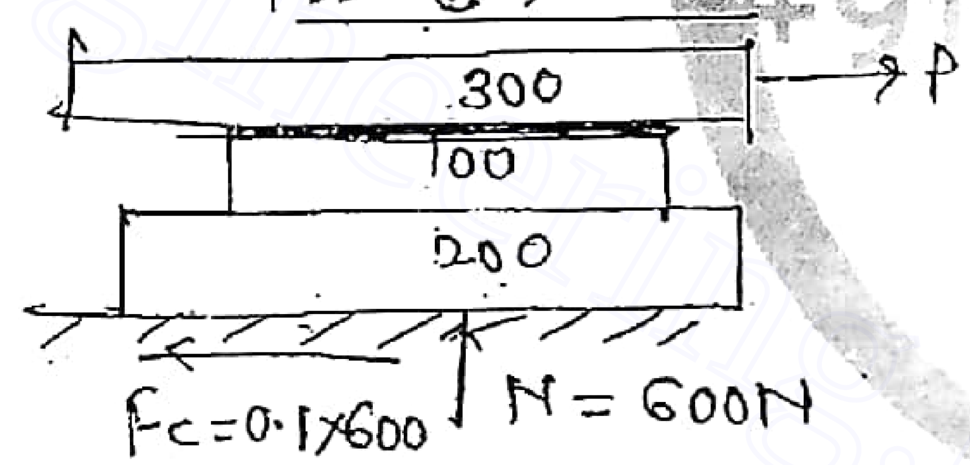


$\Sigma X = 0$

$+P - 80 = 0$

$\boxed{P = 80N}$

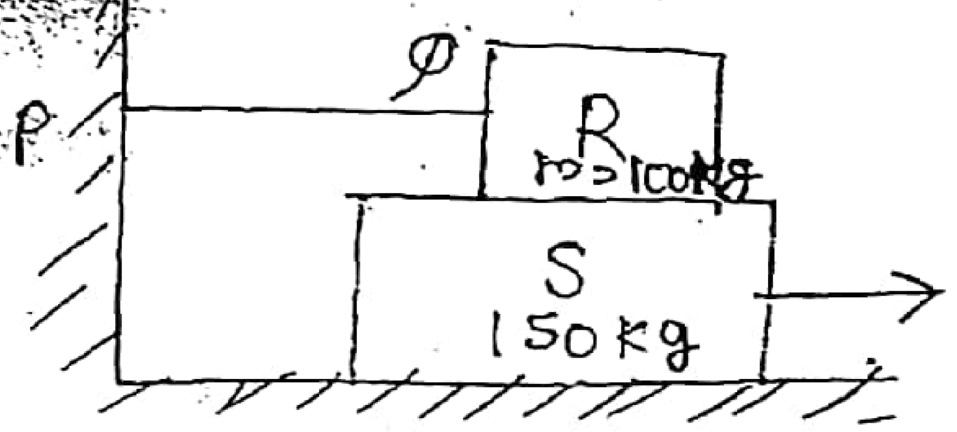
FBD of A, B & C



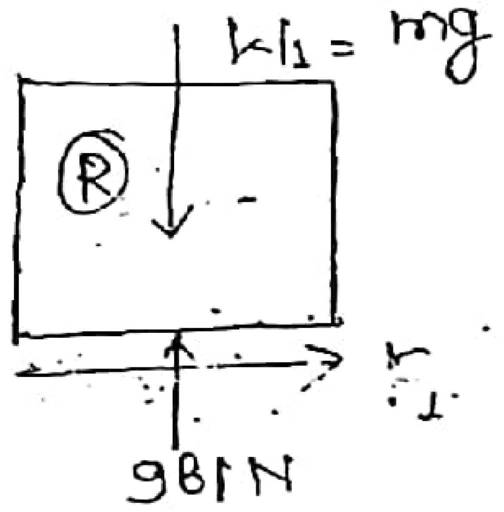
$\boxed{P = 60N}$

$\boxed{P_{min} \Rightarrow 60N}$

Que→ A block of mass 100 kg. is placed on a block of S' of mass 150kg as shown in fig. Block R is ~~tight~~ tied to wall by a massless inextensible string PQ. if $\mu \Rightarrow 0.4$ at all contact surfaces. the minm force P in KN needed to move the block S is.



FBD of 'R'

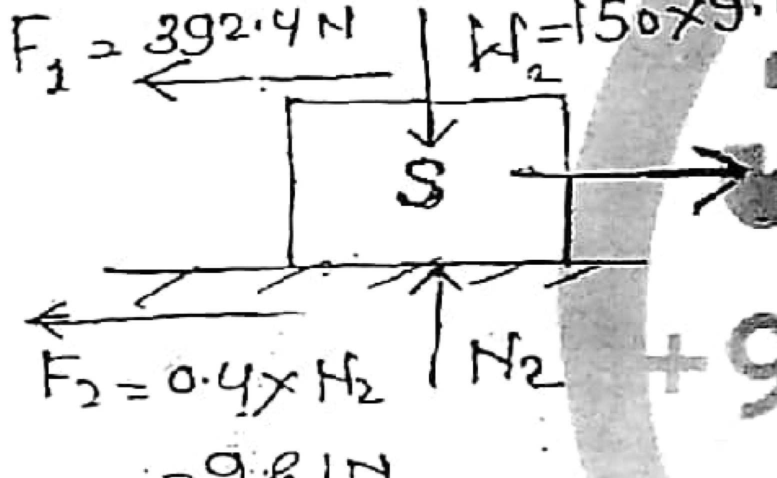


$N_1 = mg \Rightarrow 100 \times 9.81 \Rightarrow 981$ N

[w.r.t S block R is moving leftward so frictional force acts rightwards on FBD of 'R'.

$F_1 = 0.4 \times 981$
$= 392.4$ N ⇒ Tension

F.B.D of S



$F_1 = 392.4$ N

$N_2 = 150 \times 9.81 \Rightarrow 1471.5$ N

$F_2 = 0.4 \times N_2 \Rightarrow 981$ N

$\Sigma Y = 0$

$N_2 - 1471.5 - 981 = 0$

$N_2 = 2452.5$ N

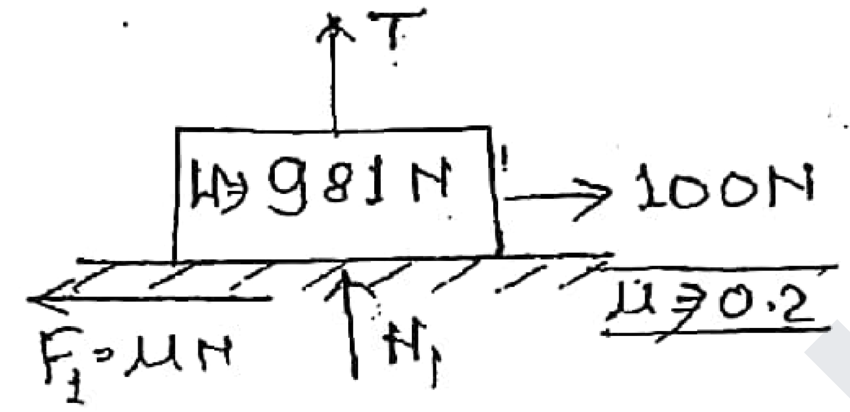
$\Sigma X = 0$

$F - F_1 - F_2 = 0$

$F \Rightarrow 392.4 + 981 \Rightarrow 1373.4$

$\Rightarrow 1.38$ KN Ans

Que ⇒ A block weighing 981N is resting on a horizontal surface. $\mu \Rightarrow 0.2$. A vertical cable attached to the block provides partial support as shown in fig. A man can pull horizontally with a force of 100N. What will be the tension T in the cable if the man is just able to move the block to the right.



$F_1 = 100$N

$F_1 = 0.2 N_1$

$\boxed{N_1 + T - 981 = 0} \Rightarrow \boxed{N_1 \Rightarrow 981 - T}$

$F_1 = 0.2 N_1 = 100$N

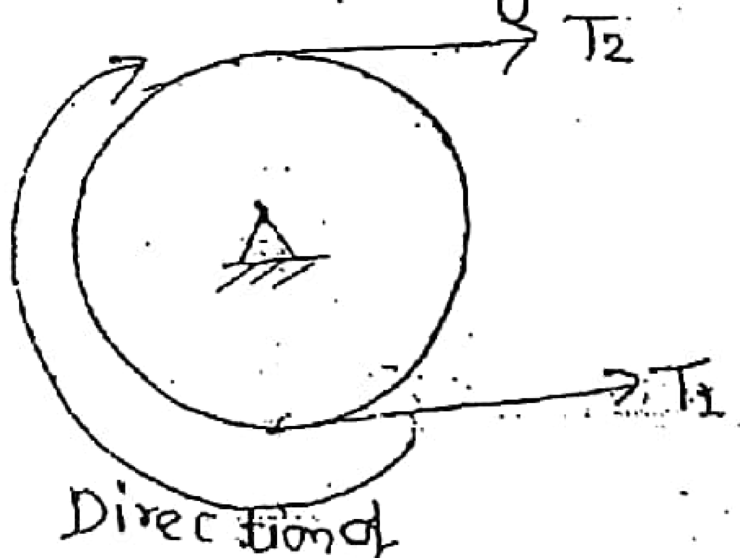
$0.2(981 - T) = 100$

$\boxed{T \Rightarrow 481 \text{ N}}$ Ans

← Belt friction →

Concept-1 →

FBD of driven pulley.



T_2 = Tension in the belt leaving the pulley.

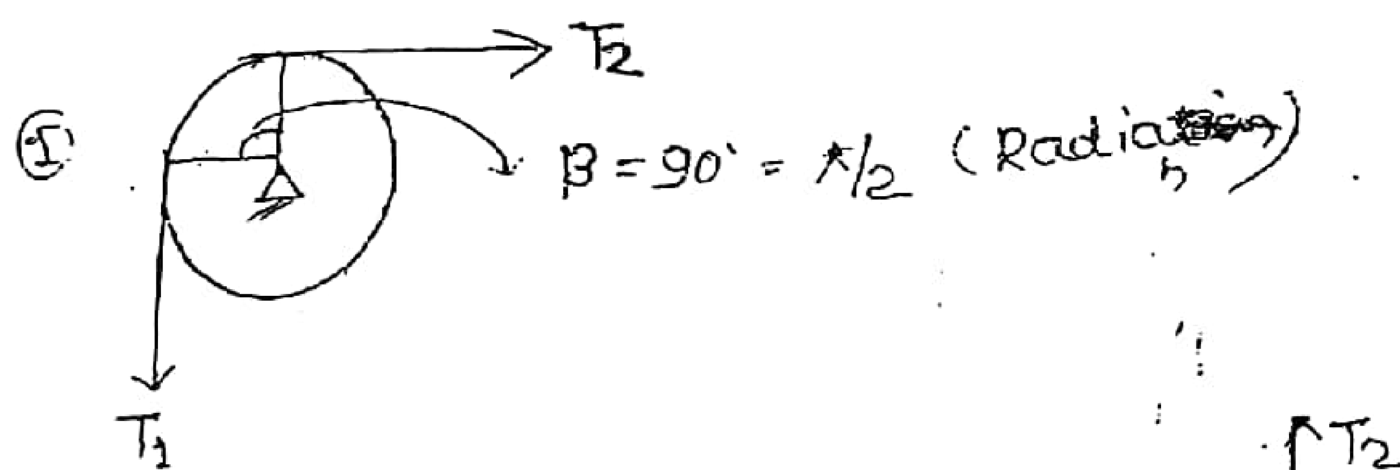
$T_1 \Rightarrow$ Tension in the belt approaching the pulley.

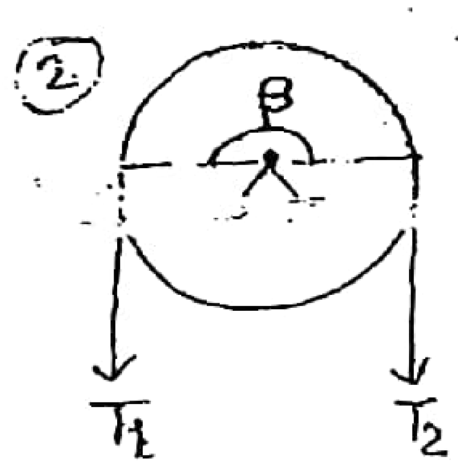
$\boxed{T_2 \Rightarrow T_1 e^{\mu\beta}}$

$\mu \Rightarrow$ coefficient of friction btw belt & pulley.

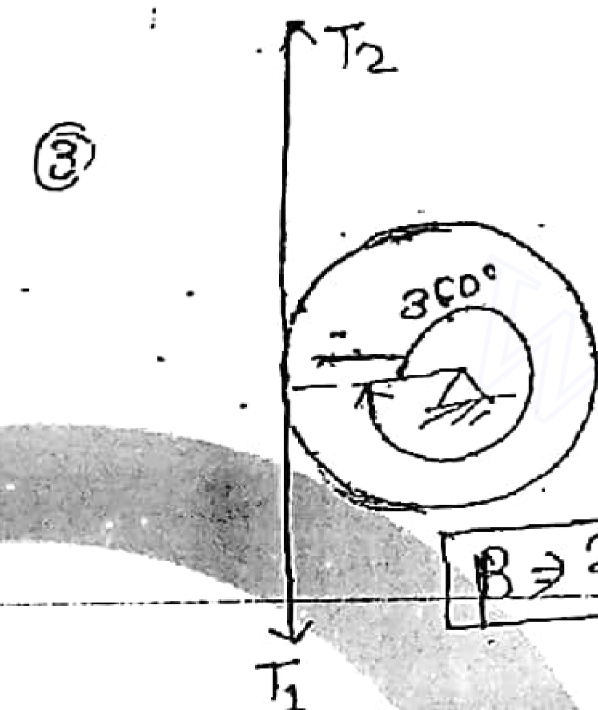
$\beta \Rightarrow$ Angle of contact betⁿ pulley & belt (always measured in radian)

Ex→ calculation of β →



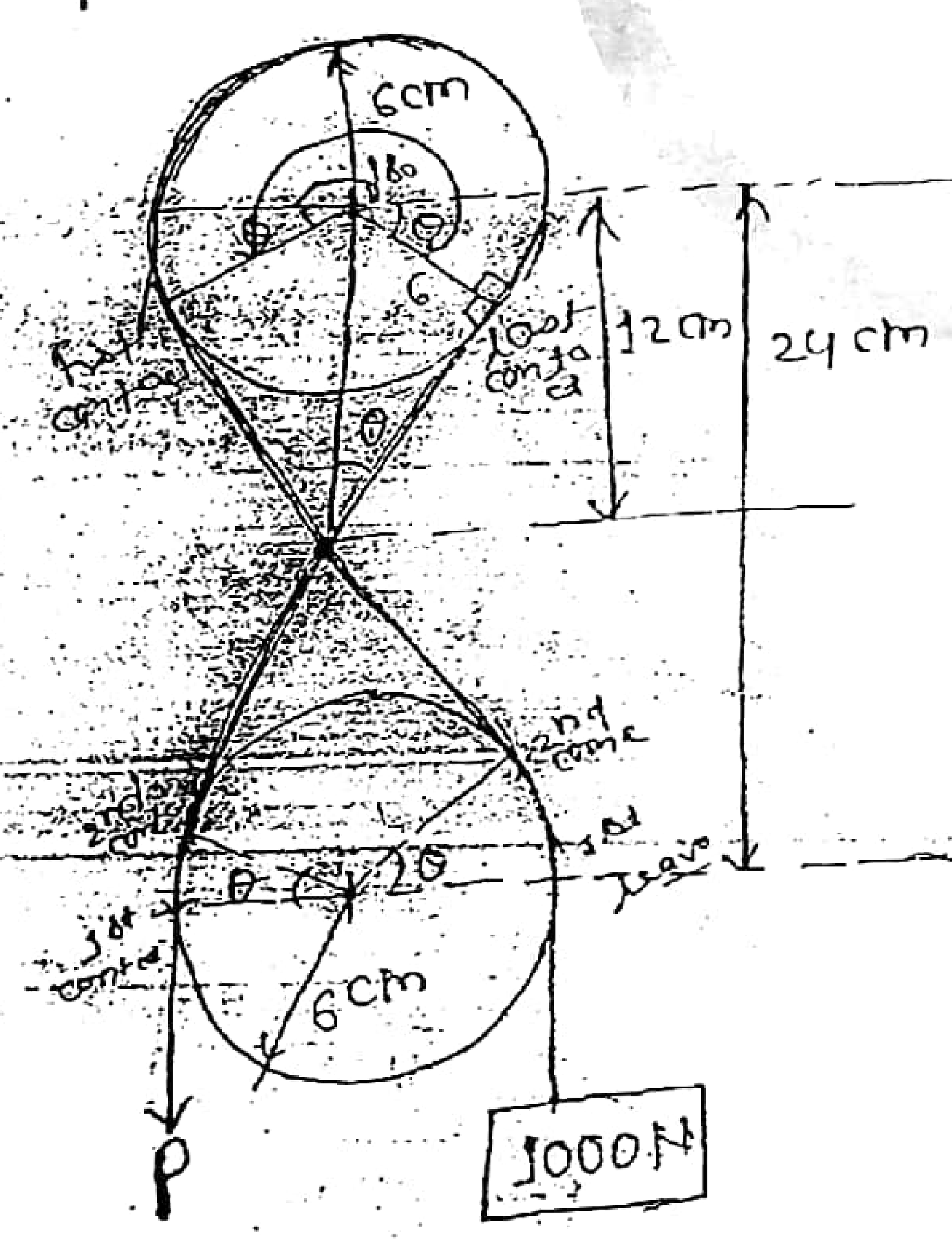


$\beta = 180 \Rightarrow \pi$ Radian.



$\boxed{\beta \Rightarrow 2\pi \text{ Rad}}$ Ans

Que→ A rope is looped over two fixed drums and connected to weights as shown in fig. if $\boxed{\mu \Rightarrow 0.3}$ at all contact surfaces. the minm force P Required to keep 1000N block in equilibrium is ⇒?



from F.B.D.

$\sin\theta = 6/12$

$\boxed{\theta \Rightarrow 30°}$

total angle of contact ⇒ β

$= (180 + \theta + \theta) + (\theta + \theta)$

upper drum → $> 300°$; lower drum

$\Rightarrow \boxed{\dfrac{300 \times \pi}{180} \text{ rad}}$

$\boxed{\beta = \dfrac{300\pi}{180}}$

T_2 = tension in the belt leaving the pulley = 1000N

T_1 = tension " " approaching " " = P_1

$T_2 \Rightarrow T_1 e^{\mu\beta}$ → $P \Rightarrow P_{min} \Rightarrow P e^{\left(\frac{0.3 \times 300\pi}{180}\right)}$

$1000 = P e^{(\;\;)}$

$\boxed{P = P_{min} = 208\text{N}}$ Ans

Note⇒ As the angle of contact $\beta \uparrow$ ∴ frictional force ↑ exponentially.

Que→ in the above problem find max. value of the force P to keep 1000N block in equilibrium. (loose block approaching)

$T_2 \Rightarrow P \uparrow$ so leave.

T_2 = Tension in the belt leaving the pulley

T_1 = tension in the belt approaching the pulley = 1000

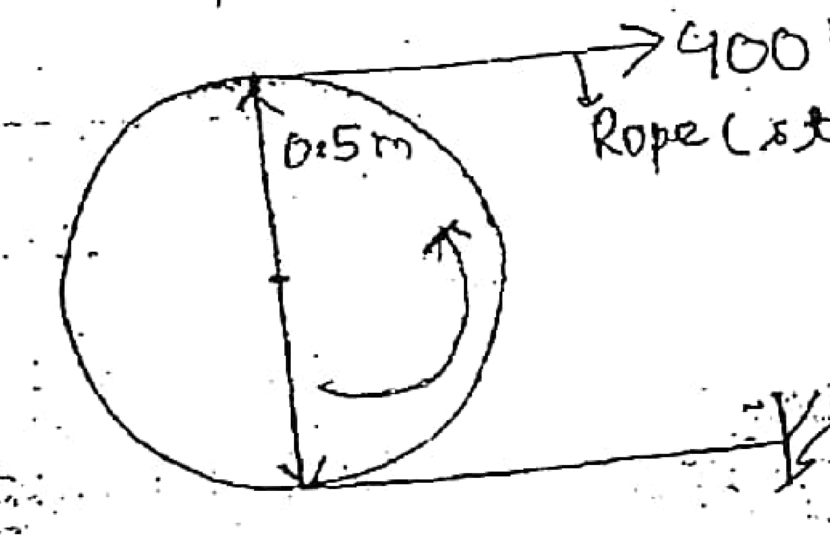
$T_2 = T_1 e^{\mu\beta}$

$P = P_{max} \Rightarrow 1000\, e^{(0.3 \times 300\pi/180)}$

$= 4810\text{N}$ Ans

Que→ A force of 400N is applied to brake drum of 0.5m diameter in a brake band system as shown in fig. & where the ~~roping~~ wrapping angle is 180°. If $\mu \Rightarrow 0.25$ betⁿ the drum and belt, the breaking torque applied is?

→ (system of vehicle to stop)



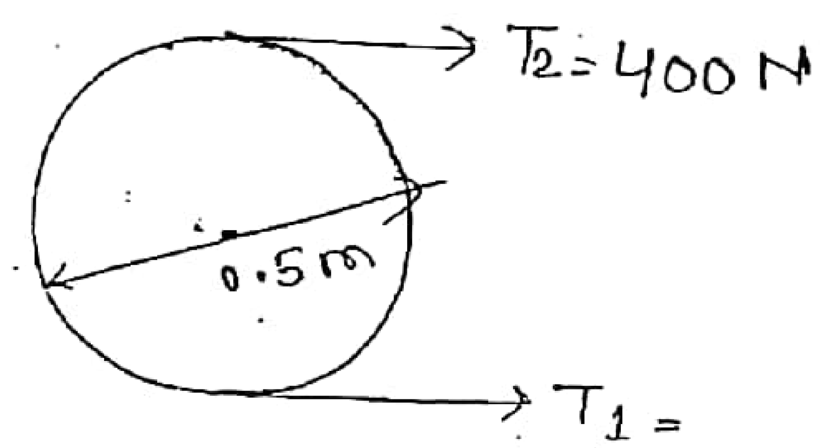
$T_2 = T_1 e^{\mu\beta}$

$400 \Rightarrow T_1 e^{(0.25 \times \pi)}$

$\boxed{T_1 \Rightarrow 182.34\text{N}}$

T_2 = tension in the belt leaving the pulley or drum = 400 N

T_1 ⇒ Tension in the belt approaching the pulley.



$\beta \Rightarrow \pi$ Radian

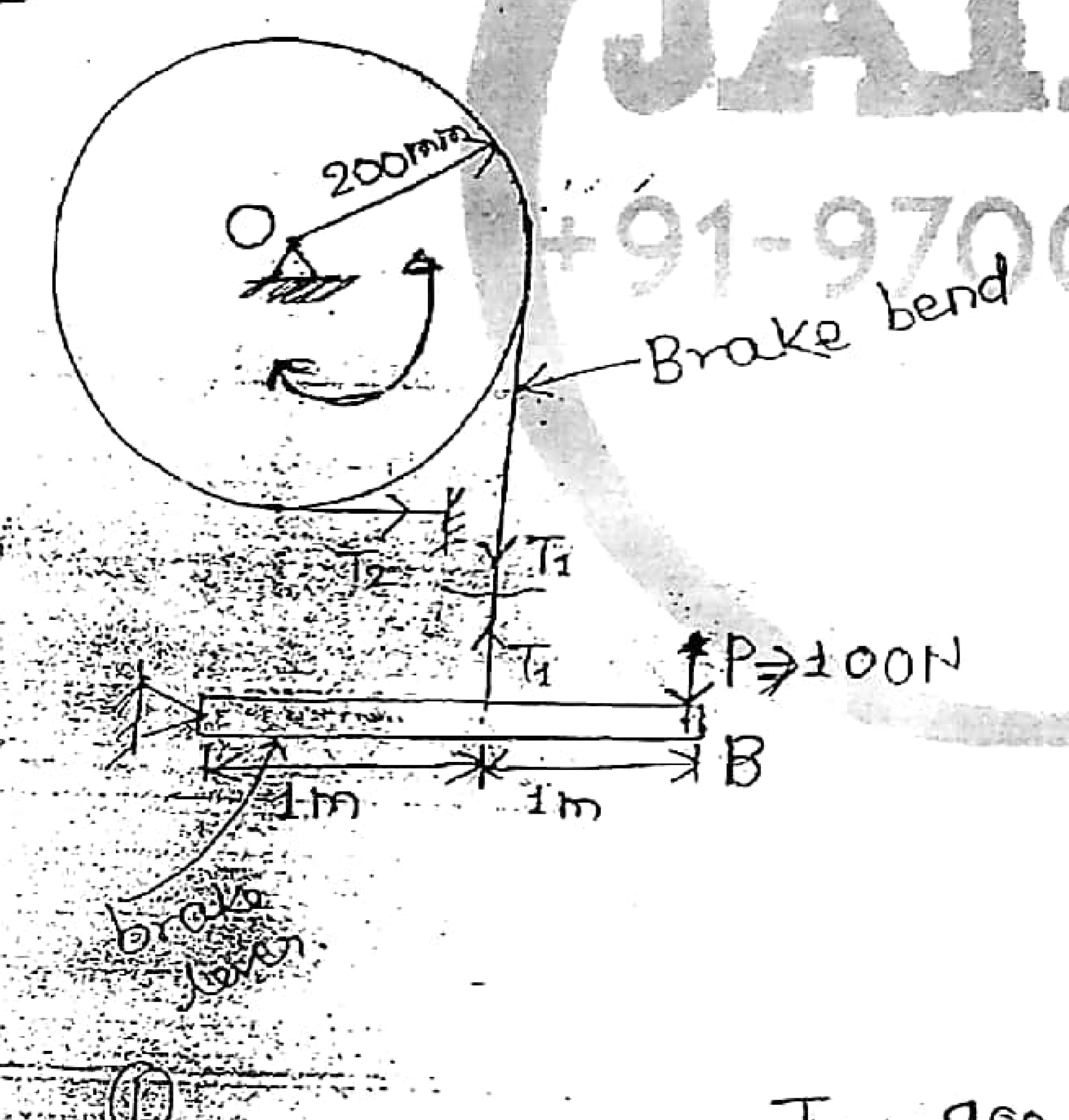
Braking Torque ⇒ $(T_2 \times 0.25) - (T_1 \times 0.25)$

(moment required to stop the rotation of drum.)

$\Rightarrow 0.25(400 - 182.37)$

$\Rightarrow 54.4$ KN·m

Que ⇒ A Force P ⇒ 100 N is applied on the brake lever (like rod) AB as shown in fig. max tension that can be developed in the brake bend is ($\mu \Rightarrow 0.3$)



from F.B.D. of brake lever AB

$\Sigma M_A \Rightarrow 0 \quad P \times 2 - T_1 \times 1 = 0$

$\boxed{T_1 \Rightarrow 200\,N}$

F.B.D. of pulley

$T_2 = T_1 e^{\mu\beta}$

$\beta = 270° \Rightarrow \frac{3}{2}\pi$ Rad

$\mu = 0.3$

$T_2 \Rightarrow 200 \times e^{(0.3 \times 3\pi/2)}$

$\boxed{T_2 = 822.2\ KN}$ Ans

in the above problem braking torque:-

$\Rightarrow -T_2 \times 200 + T_1 \times 200 \Rightarrow 200(-822.24 + 200)$

$\Rightarrow -124$ N-m

Ans

Principle of Virtual Work

Concepts → 1) It states " when a body is in equilibrium, the virtual work done by all forces is zero.

Note - principle of virtual work is applicable only when the body is ... i.e. when the body is in equilibrium.

2) procedure → take any fixed point in the problem as origine and fix co-ordinate axes. and find co-ordinates of all the points where forces are acting.

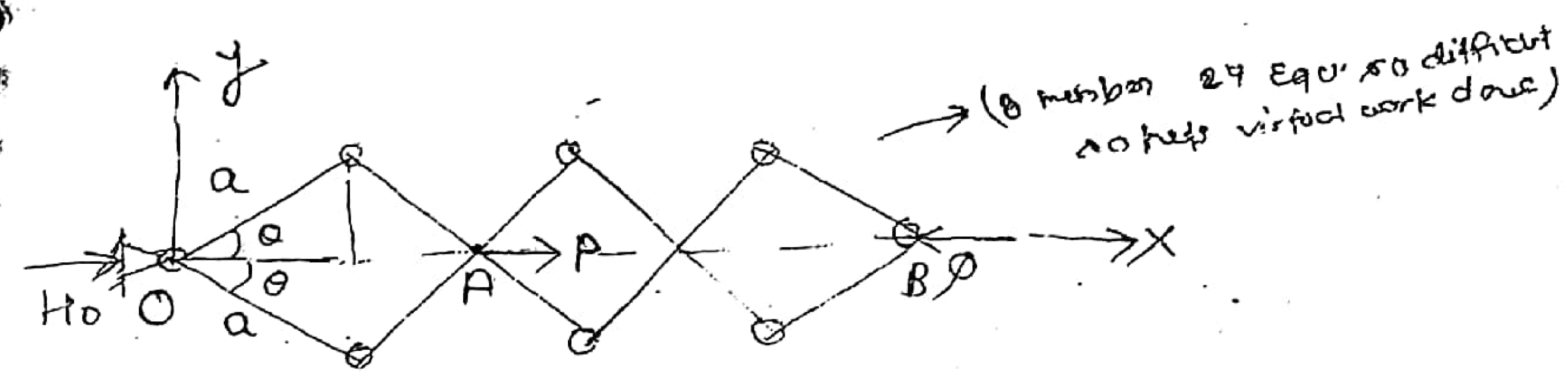
(II) step - find virtual displacements.

IIIrd step - Use principle of virtual work to find unknown forces.

Note → 1) if any force is acting along (+) x axis or (+) y axis then take that force as positive. if any force acts along (-) x axis or (-) y axis, then take that force as (-).

2) the sign convention for the quardant of the point will depend on quardant in which the point is ~~lying~~ lying.

Que 2 mark → for the lazy tong mechanism shown in fig the Relationship btw P and ϕ to keep it in Equilibrium is.



A) $P=\phi$ B) $P=3\phi$ C) $\phi=3P$ D) $P=2\phi$

Note → No. of unknowns are 2 (H_o, P), ϕ is given.

No of equillibrium equation available ⇒ 1 ($\Sigma x=0$)

→ with 1 equation 2 unknowns are not found.

so use principle of virtual work.

1st step → Take any fixed point (Refrence) in the problem ~~as origin~~ and fix coordinate axis. and find co.ordinate of all points where force are acting.

→ Take O. as origin and fix co-ordinate axis as shown in fig →

$x_A \Rightarrow 2a\cos\theta$, $x_B = 6a\cos\theta$

IInd step → find virtual displacement →

$\delta x_A \Rightarrow -2a\sin\theta\,\delta\theta$

$\delta x_B \Rightarrow -6a\sin\theta\,\delta\theta$

Use principle of virtual work to find unknown.

$U \Rightarrow 0$

imaginary — 2 mark problem.

$\boxed{+P\times\delta x_A - \phi\,\delta x_B \Rightarrow 0}$

$\Rightarrow P\times(-2a\sin\theta\,\delta\theta) - \phi(-6a\sin\theta\,\delta\theta) \Rightarrow 0$

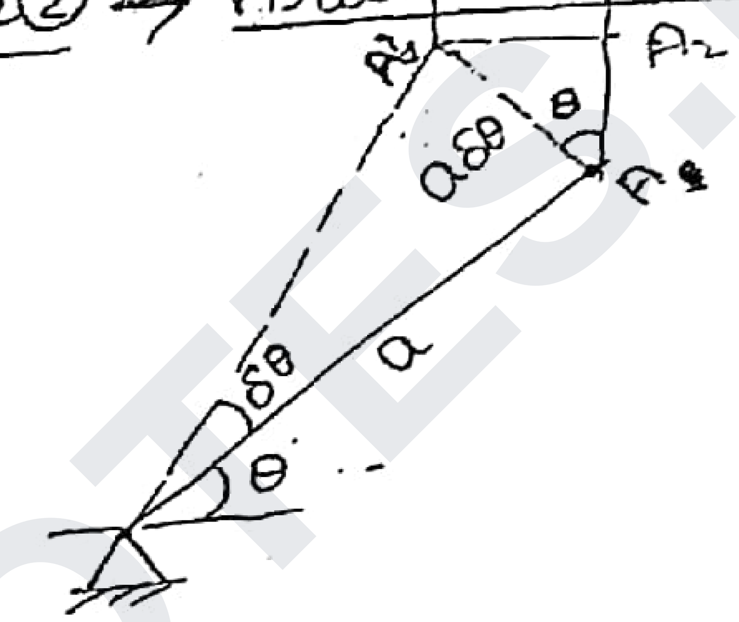
$\boxed{P=3\phi}$ Ans

Note ① $\Sigma x \Rightarrow 0$ ($\overset{\rightarrow}{+}$ $\overset{\leftarrow}{-}$) $H_o + P - \phi = 0$ (P → 3ϕ)

$\boxed{H_o = -2\phi}$

(−) sign implies that H_o acts left.

Note ② → Virtual displacement →



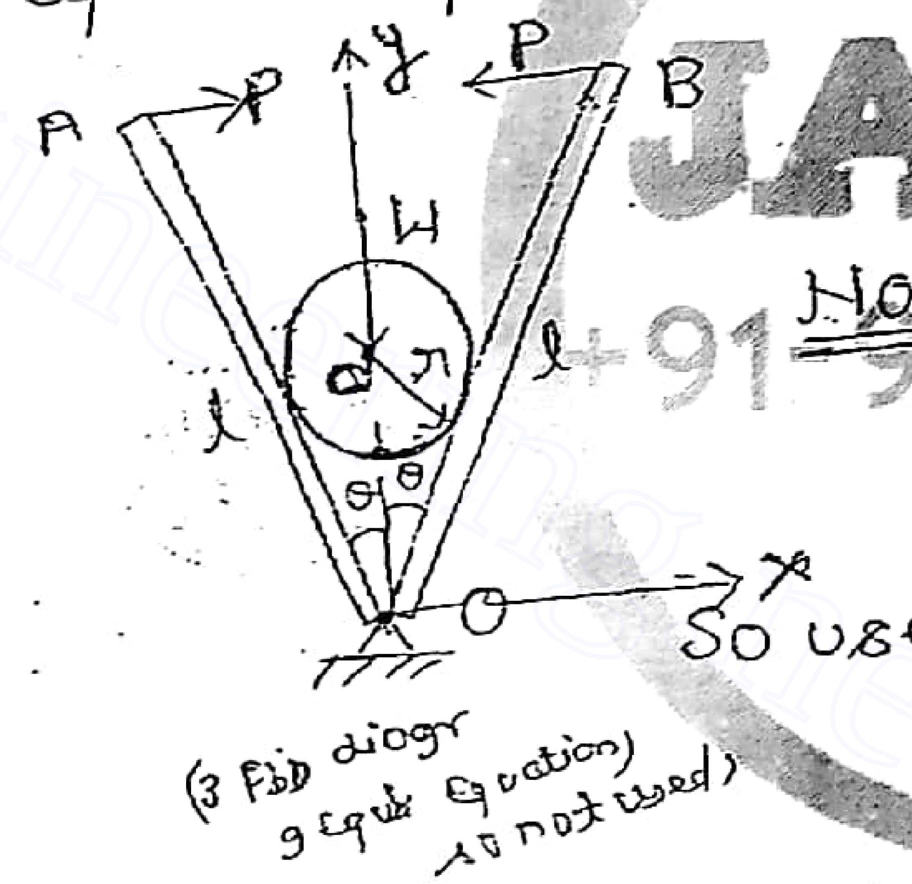
$\sin\theta \Rightarrow \frac{A_1A_2}{a\cdot\delta\theta}$

$A_1A_2 \Rightarrow -(a\cdot\delta\theta\sin\theta)$

$\boxed{\delta x_A = -a\sin\theta\,\delta\theta}$

Que → A sphere of weight 'W' is supported by two Rods as shown in fig. the value of P required to keep it in Equillibrium is?

(if P inside w upward)
(if P outside w inward)



Note → $\Sigma M_O = 0$ ⇒ ($\overset{\curvearrowright}{+}$ $\overset{\curvearrowleft}{-}$) $+P\times l\cos\theta - P\times l\cos\theta + W\times 0 = 0$

$\boxed{0=0}$

So use principle of virtual work.

1st step → $x_A \Rightarrow -l\sin\theta$ (because in 2nd quardant so (−))

$x_B \Rightarrow l\sin\theta$

$y_C \Rightarrow \frac{r}{\sin\theta} = r\,\mathrm{cosec}\,\theta$

IInd step → $\delta x_A \Rightarrow -l\cos\theta\,\delta\theta$

$\delta x_B = +l\cos\theta\,\delta\theta$

$\delta y_C \Rightarrow -r\,\mathrm{cosec}\,\theta\cdot\cot\theta\cdot\delta\theta$

III^{rd} step → $U \Rightarrow 0$

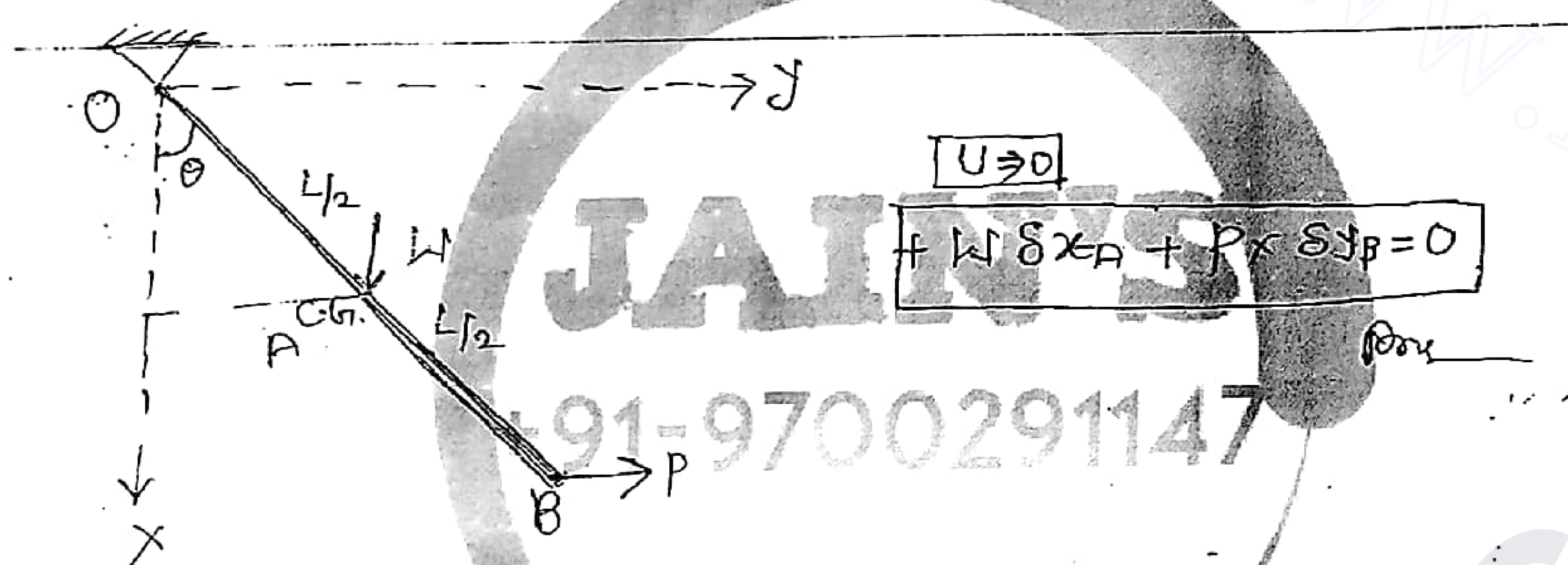
$$\Rightarrow +\underbrace{P\times\delta x_A}_{\text{at } A} - \underbrace{P\times\delta x_B}_{\text{at } B} - \underbrace{\omega\times\delta y_c}_{\text{at } C} \Rightarrow 0$$

$$\Rightarrow +P(-l\cos\theta\,\delta\theta) - P(l\cos\theta\,\delta\theta) - \omega(-r\,\text{cosec}\,\theta\cot\theta\,\delta\theta) \Rightarrow 0$$

$$P = \frac{\omega r\,\text{cosec}\,\theta\cot\theta}{2l\cos\theta} \Rightarrow \frac{\omega r\times\frac{1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}}{2l\cos\theta}$$

$$\boxed{P \Rightarrow \frac{\omega r}{2l\sin^2\theta}}$$ ***

Que → a bar is located as shown in fig. the virtual work expression is ?

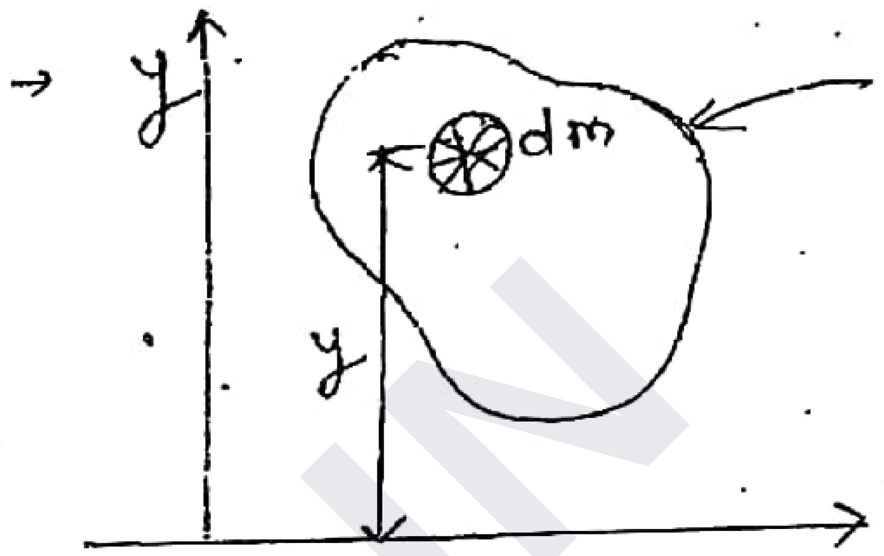


$U \Rightarrow 0$

$$\boxed{+W\,\delta x_A + P\times\delta y_B = 0}$$

← Mass Moment of Inertia → (M.M.I.)

Concept 1 →



A plate of thickness = t
mass = t
Density = γ
area = A

$y\,dm$ is called first moment of mass.

$y^2\,dm$ is called second moment of mass or mass moment of Inertia

M.M.I ⇒ $I_{xx} = \int y^2\,dm$ $I_{yy} = \int x^2\,dm$ $\boxed{I_{zz} = I_{xx} + I_{yy}}$

② $dm = \gamma\,(t\cdot dA)$ (gamma)

$$I_{xx} \Rightarrow \int y^2\,dm = \int y^2(\gamma\,t\,dA) \Rightarrow \gamma\cdot t\times\underbrace{\int y^2\,dA}_{\text{(Area M.M.I.)}}$$

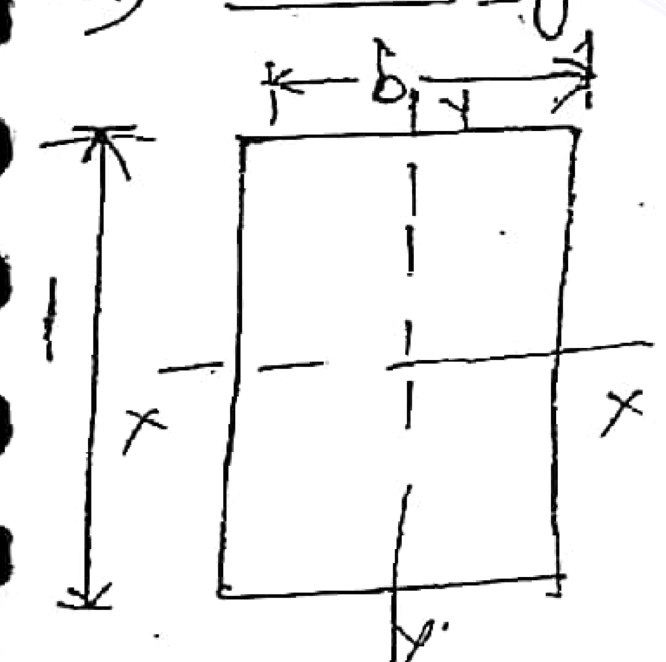
$$\boxed{I_{xx} = \gamma\cdot t\cdot I_x}$$ → Area M.I.

double — single in Area

||| similarly $I_{yy} \Rightarrow$ M.M.I. $\Rightarrow \gamma\cdot t\cdot I_y$; $I_{zz} \Rightarrow$ M.M.I. $\Rightarrow \gamma\cdot t\cdot I_z$ } valid only plates.

③ M.M.I. for different plates →

a) Rectangular plate →



Mass = m = Density × volume = $\gamma\times(b\times d\,t)$

thickness of plate ⇒ t

Density = γ (gamma)

I_{xx} = MMI ⇒ $\gamma\cdot t\cdot I_x \Rightarrow \gamma\cdot t\times\frac{bd^3}{12}$

$$I_{xx} \Rightarrow \gamma\cdot t\times\frac{bd^3}{12}\times\frac{m}{\gamma\cdot(b\cdot d\,t)}$$

$$\boxed{I_{xx} \Rightarrow \frac{md^2}{12}}$$ **

$I_{yy} \Rightarrow \frac{mb^2}{12}$ $I_{zz} = I_{xx} + I_{yy} = \boxed{\frac{md^2}{12} + \frac{mb^2}{12}}$

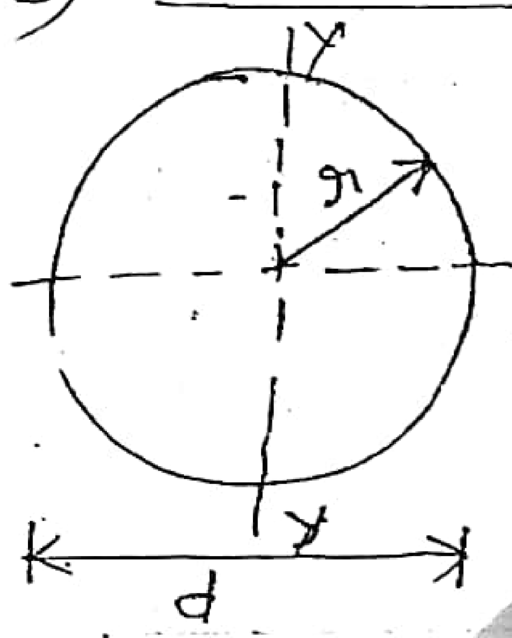
Note → for a square plate of size $d \times d$,

$$I_{xx} \Rightarrow \frac{md^2}{12}$$

$$I_{yy} = \frac{md^2}{12}$$

$$\boxed{I_{zz} \Rightarrow \frac{md^2}{6}} \rightarrow \text{(to be Remembered)}$$

b) Circular Plate ⇒ or pulley or Disc



Thickness of plate ⇒ t

Density ⇒ γ

$$\text{mass} = m = \gamma \cdot t \cdot \left(\frac{\pi d^2}{4}\right)$$

$$I_{xx} = \text{M.M.I} \Rightarrow \gamma \cdot t \cdot \left(\cancel{I_x}\right) \rightarrow \text{area M.I.}$$

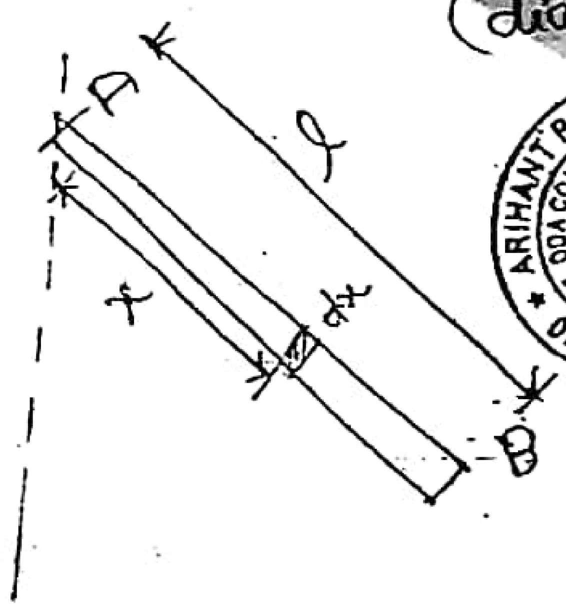
$$I_{xx} \Rightarrow \left(\gamma \cdot t \times \frac{\pi d^4}{64}\right)$$

$$I_{xx} \Rightarrow \left[\gamma \cdot t \times \frac{\pi d^4}{64}\right] \times \frac{m}{\left(\gamma \cdot t \cdot \frac{\pi d^2}{4}\right)} \Rightarrow \left(\frac{md^2}{16}\right) \Rightarrow \left(\frac{mr^2}{4}\right)$$

from symmetry → $\boxed{I_{yy} \Rightarrow \frac{mr^2}{4}}$

$\boxed{I_{zz} \Rightarrow I_{xx} + I_{yy} \Rightarrow \frac{mr^2}{2}}$ to be Remembered

③ M.M.I of a thin ~~cylinder~~ slender Rod ;—
(dia less)



c/s area of Rod ⇒ A

mass of rod = γ.A.L.

Density = γ

$$I_{yy} \Rightarrow \left[\int x^2 \cdot dm\right] = \int (x \sin\theta)^2 \, (A\,dx \cdot \gamma)$$

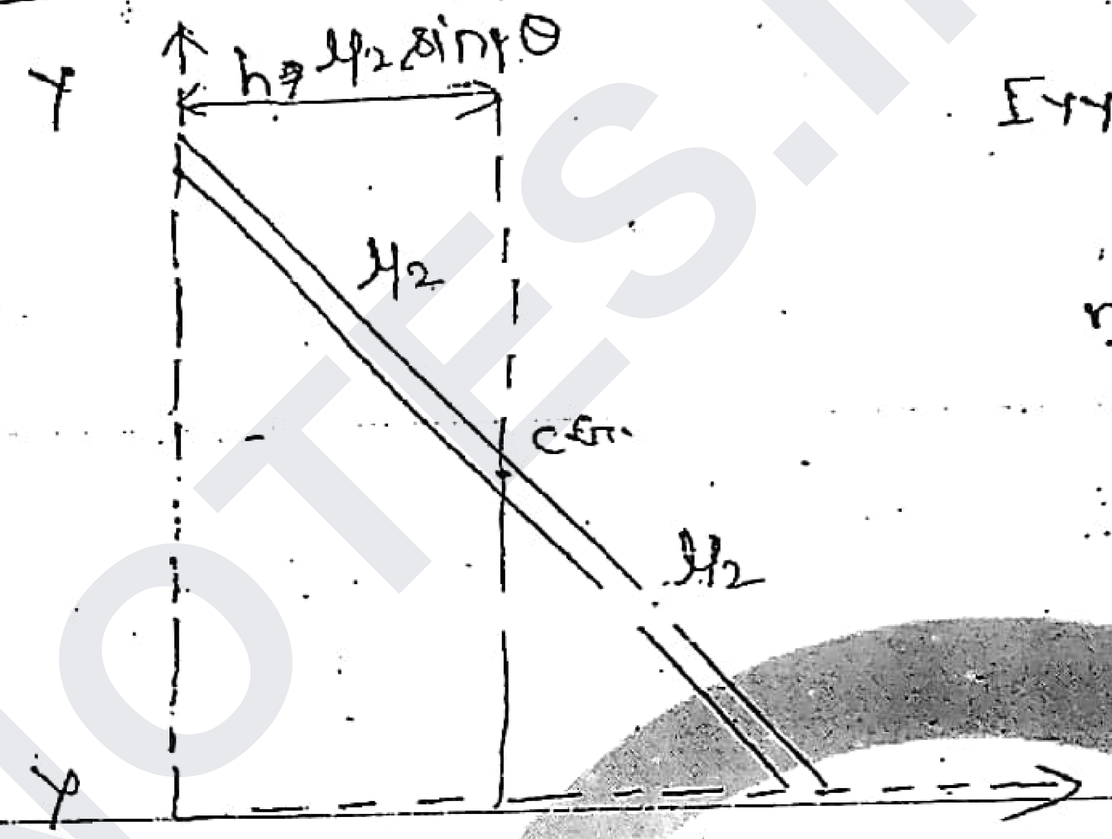
Std formula

$$\Rightarrow \int_0^l (x \sin\theta)^2 (A \cdot dx \cdot \gamma)$$

$$\Rightarrow \gamma \cdot A \sin^2\theta \int_0^l x^2 dx = \gamma \cdot A \cdot \sin^2\theta \left(\frac{l^3}{3}\right)$$

$$I_{yy} \Rightarrow \left[\gamma A \sin^2\theta \frac{l^3}{3}\right] \times \frac{m}{(\gamma \cdot A \cdot l)} \Rightarrow \boxed{\left(\frac{ml^2 \sin^2\theta}{3}\right)}$$ Remember

Note 1 → $I_{y_0y_0}$



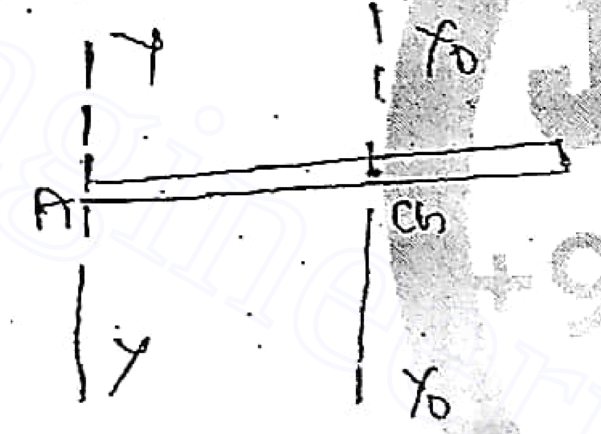
$I_{yy} \Rightarrow I_{y_0y_0} + mh^2$

⇒ transfer formula

$$\frac{ml^2 \sin^2\theta}{3} = I_{y_0y_0} + m\left(\frac{l}{2}\sin\theta\right)^2$$

$$\boxed{I_{y_0y_0} \Rightarrow \frac{ml^2 \sin^2\theta}{12}}$$

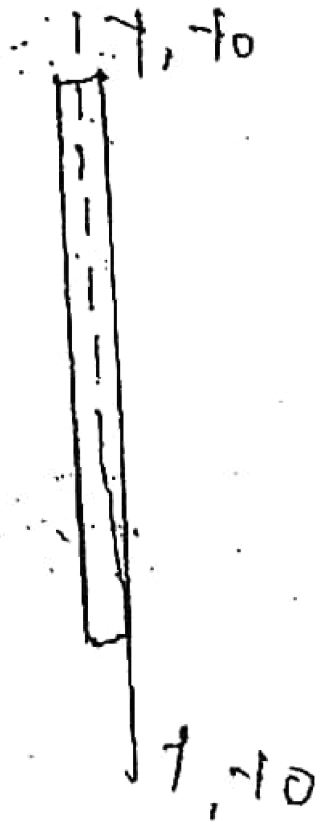
Note → when $\theta \Rightarrow 90°$



$$I_{yy} = \frac{ml^2}{3}\sin^2 90° \Rightarrow \boxed{\frac{ml^2}{3}}$$

$$I_{y_0y_0} \Rightarrow \frac{ml^2}{12} \sin^2 90° \Rightarrow \boxed{\frac{ml^2}{12}}$$

Note (3) if $\boxed{\theta = 0°}$



$$\boxed{I_{yy} = I_{y_0y_0} \Rightarrow 0}$$

Note → M.M.I of a thin slender rod about its longitudinal axis is zero.

← Dynamics →

Kinematics: dealing with s, v, a without considering the forces causing them.

Kinetics: Deal with s, v, a considering the forces causing them.

$\vec{R} = \left(\frac{w}{g}\right) \times a$ → Mass, linear acceleration

Resultant force

Result moment $\vec{M} = (I)\,\alpha$ → M.M.I., Angular acceleration

[if R and M = 0 so in Equ.]

Note-1 → mass ⇒ $\frac{w}{g}$ is a measure of resistance to translation (because for a given resultant force R, if mass is more then acceleration a will be less.)

2) M.M.I. (I) is a measure of Resistance to Rotation. (because for a given moment, if I is more, then α will be less)

← Types of Motion →

1) Rectilinear Motion → if there is only a Resultant force acting on a body whose direction is const then we get Rectilinear motion. it is a straight line motion. (the body will not rotate because resultant moment is zero.)

2) Curvilinear Motion → if there is only a resultant force whose direction is not const; we get curvilinear motion. (the body will not rotate because Resultant moment is zero in this case also)

3) Rotation → if there is only a Resultant moment acting on a body, we get rotation. (the body will not translate because resultant force is zero.)

4) Plane Motion → if there is a Result. force and moment acting on a body, then it will rotate and translate this type of motion is called plane motion.

Ex → Motion of a wheel on the ground.

← Kinematics →

I topic → Rectilinear motion - Kinematics →

concept-(1) Avg. Velocity = V ⇒ $\left(\frac{S_2 - S_1}{t}\right)$ → final displ., initial displacement, total time

2) Instantaneous velocity → $V = ds/dt$ = Rate of change of displace

3) Avg. Acceleration → $\frac{V_2 - V_1}{t}$ → final, initial

4) Instantaneous Accel. → $\left(a = \frac{dv}{dt}\right)$ ⇒ Rate of change of velocity

5) Jerk ⇒ $\frac{da}{dt}$ ⇒ Rate of change of acceleration.

6) $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$

$dt = \left(\frac{ds}{v}\right) = \left(\frac{dv}{a}\right)$

$\boxed{a\,ds = v\,dv}$ $\boxed{a = v\frac{dv}{ds}}$

Note → the above expression is useful when acceleration is given as a function of displacement.

* If 'a' is constant → $a = \frac{dv}{dt}$, $\int a\,dt = \int_{v_0} dv$ ⇒ $at = v - v_0$ $\boxed{v = v_0 + at}$

$$\begin{cases} V = V_0 + at \\ S = V_0 t + \frac{1}{2} a t^2 \\ V^2 = V_0^2 + 2as \end{cases}$$

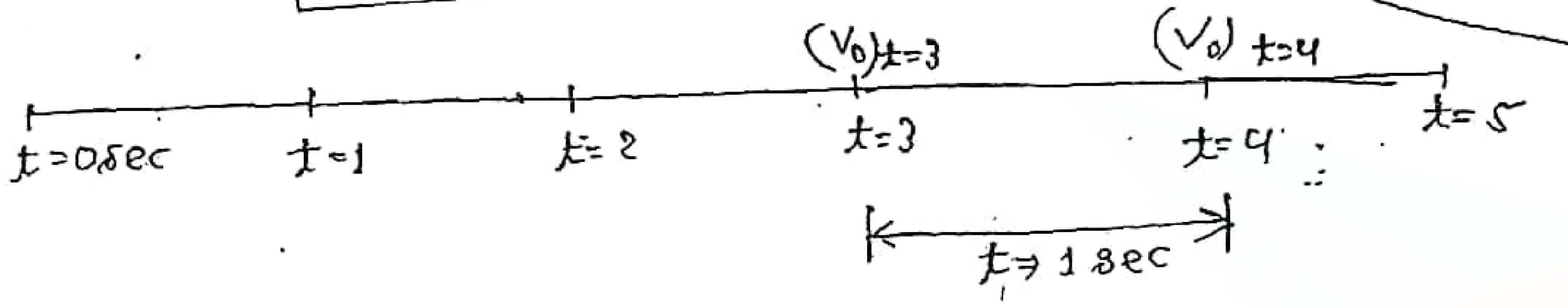
(useful when we don't want to deal with time)

Que→(4) if $s \Rightarrow \left(\frac{t^3}{3} - 36t\right)$ the avg. acceleration during 4th second is.

$V = \frac{ds}{dt} = t^2 - 36$

$a = \frac{dv}{dt} = (2t) \rightarrow$ instan

$$\boxed{V \Rightarrow \frac{ds}{dt} \Rightarrow t^2 - 36}$$



avg. acc.l ⇒ $a \Rightarrow \left(\frac{V_2 - V_1}{t}\right)$

$(V_2)_{t \Rightarrow 4sec} - 4^2 - 36 = 20$ m/sec.

$(V_1)_{t=3sec} = (3)^2 - 36 = -27$ m/sec

$a = \frac{V_2 - V_1}{t} = \frac{(-20)-(-27)}{1} \Rightarrow 7$ m/sec^2 Ans

Que in the above problem instans. acceleration at 4th second is⇒?

$a = \frac{dv}{dt} \Rightarrow (2t)$

$(a)_{4sec} \Rightarrow 2 \times 4 \Rightarrow 8$ m/sec^2 Ans

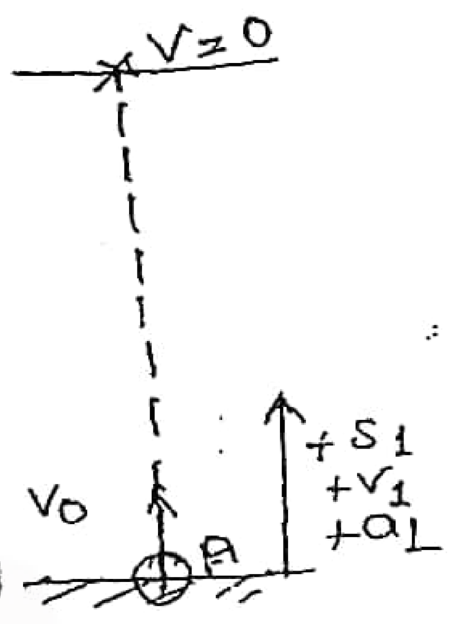
Que. in the above probl. instant acceleration when the particle reverses its direction is ⇒?

Note - in the inst. of reversing the direction, its velocity must be zero.

$V = t^2 - 36 \Rightarrow 0$ $\boxed{t = 6 \text{ sec}}$

$(a)_{t=6sec} \Rightarrow 2t \Rightarrow 12$ m/sec^2 Ans

Que a stone is thrown vertically upward and it reaches the ground after 5 seconds. the max. height it travelled is?



Ist method→

$a \Rightarrow -9.81$ m/s^2 (-ve because it acts opposite to initial direc. of motion)

$t_1 \Rightarrow t_2 \Rightarrow 2.5$ sec

$V = V_0 + at$

$\Rightarrow 0 = V_0 + (-9.81 \times 2.5)$ $\quad V_0 \Rightarrow 24.5$ m/sec

$s = V_0 t + \frac{1}{2} a t^2 \Rightarrow 24.5 \times 2.5 + \frac{1}{2}(-9.81) \times 2.5^2$

$\Rightarrow 30.6$ m Ans

IInd method → take B as starting point.

B ↓ +s +v +a

$V_0 = 0$

$a = +9.81$ m/s^2 (same direction)

$s = V_0 t + \frac{1}{2} a t^2$

$s = 0 + \frac{1}{2} \times 9.81 \times (2.5)^2 \Rightarrow 30.6$ m

Que-5 if $a \Rightarrow -8s^{-2}$, velocity of point particle at $s \Rightarrow 16$ m is?

$a\,ds = v\,dv$

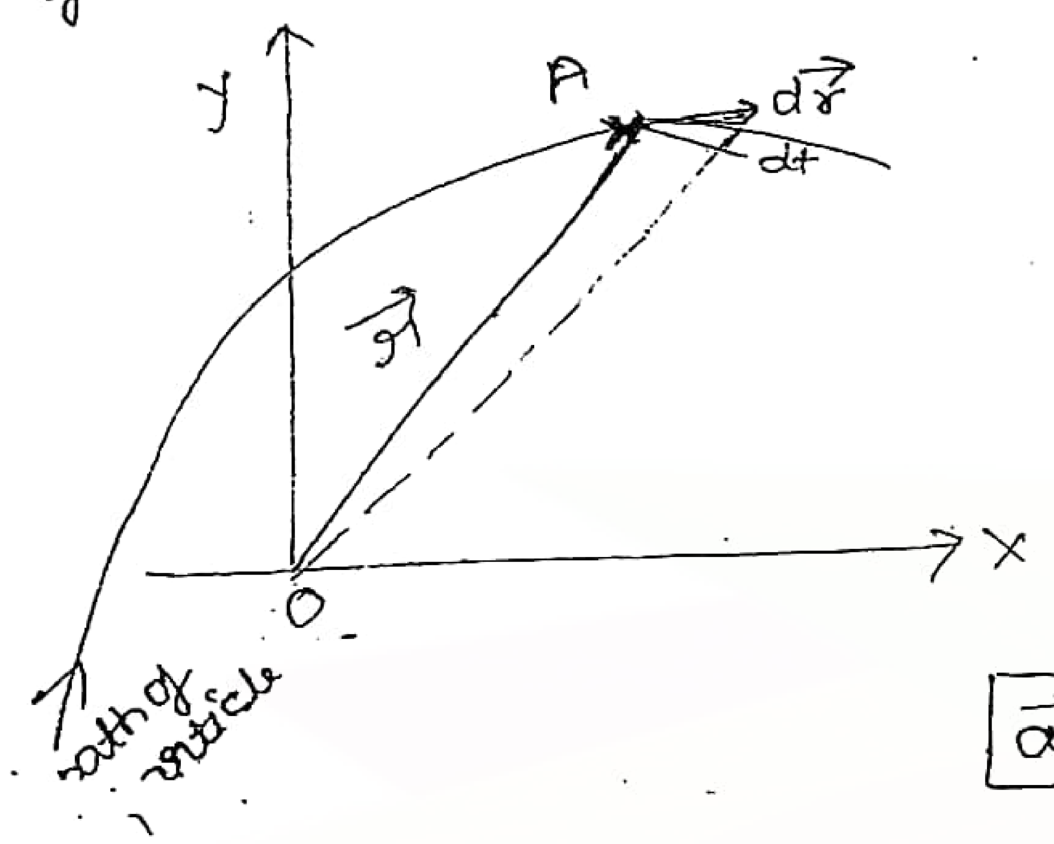
$\int -8 s^{-2} ds = \int v\,dv$

$\frac{-8 s^{-1}}{-1} = \left(\frac{v^2}{2}\right)$

$\frac{+8}{s} = \frac{v^2}{2} \Rightarrow \boxed{v^2 = 16/s}$ $\quad \boxed{v^2 \Rightarrow \frac{16}{16} \Rightarrow 1 \text{ m/s}}$

IInd topic → Curvilinear Motion

concept ① Curvilinear motion is nothing but the summation of two or three Rectilinear motions.



$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}} \quad ①$$

$$\vec{V} \Rightarrow \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

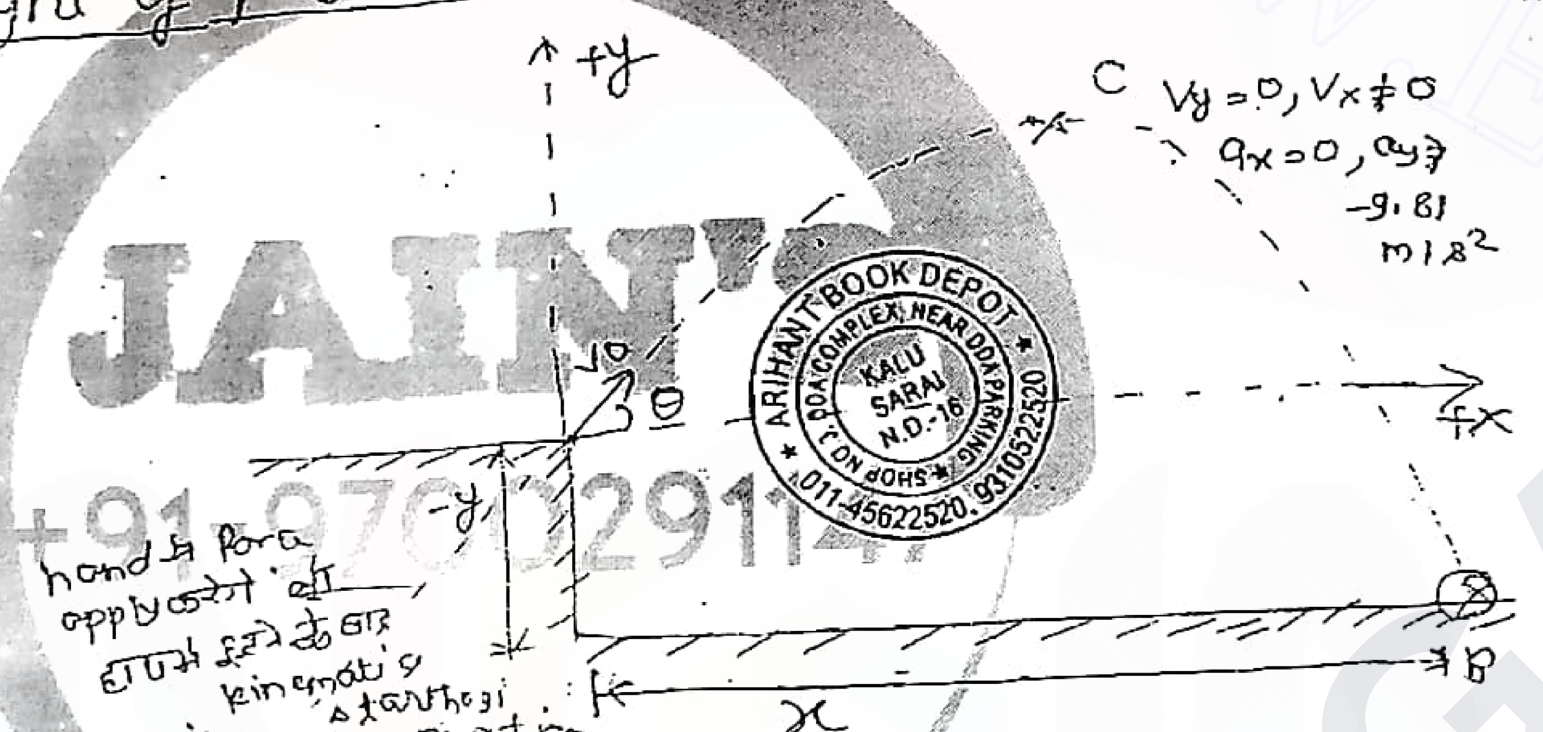
$$\boxed{\vec{V} \Rightarrow V_x\hat{i} + V_y\hat{j} + V_z\hat{k}} \quad ②$$

$$\vec{a} \Rightarrow \frac{d\vec{V}}{dt} \Rightarrow \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k}$$

$$\boxed{\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}} \quad ③$$

~~nothing~~ but 2 or more Rectilinear motion summation

concept - ② Flight of projections →



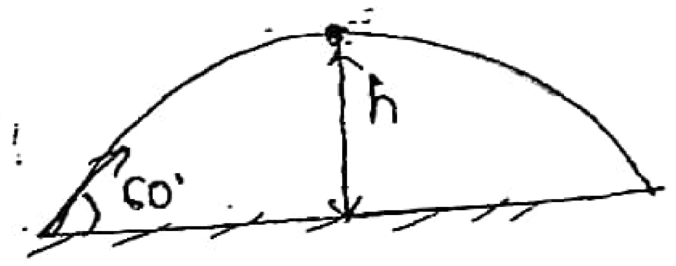
Rectilinear motion with const. a.	Curvilinear motion in x dir.	in y-direction
1) $V = V_0 + at$	1) $V_x = V_{0x} + a_x \cdot t$	1) $V_y = V_{0y} + a_y \cdot t$
2) $S = V_0 t + \frac{1}{2}at^2$	2) $x = V_{0x}t + \frac{1}{2}a_x t^2$	2) $y = V_{0y}t + \frac{1}{2}a_y t^2$
3) $V^2 = V_0^2 + 2aS$	$V_x^2 = V_{0x}^2 + 2a_x \cdot x$	3) $V_y^2 \Rightarrow V_{0y}^2 + 2a_y \cdot y$

$V_{0x} = V_0 \cos\theta$
$V_{0y} = V_0 \sin\theta$

if $a_x \Rightarrow 0$, $\boxed{V_x \Rightarrow V_{0x}}$ it means that velocity in x direction constant throughout its flight.

Que → 1) A stone is thrown with a velocity of 10 m/sec. by making an angle of 60° with the horizontal. the max. height it travelled is. (y direction height)

Note → since we don't want to deal with time.



$$V_y^2 = V_{0y}^2 + 2a_y \cdot y$$

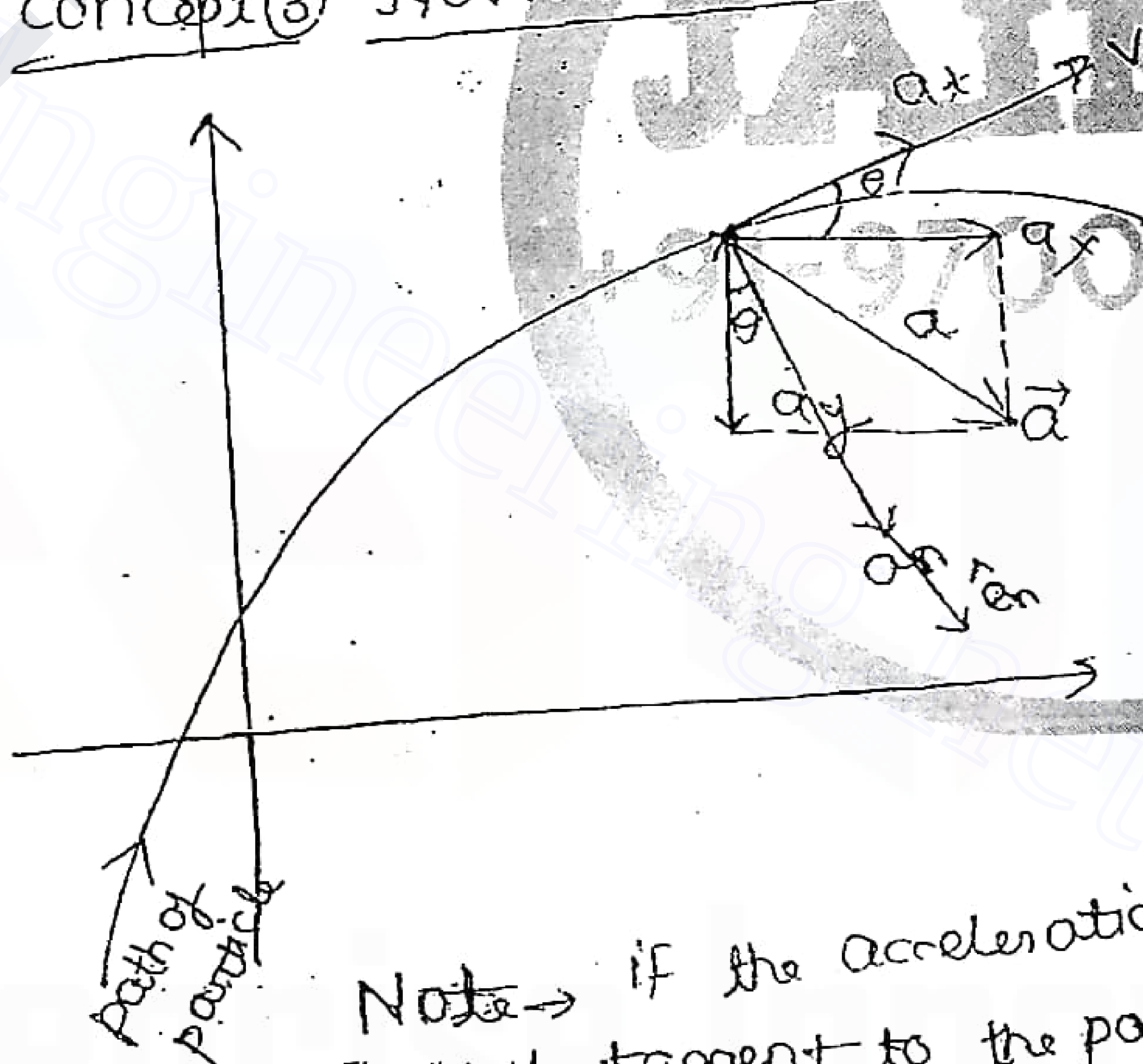
$$0 \Rightarrow V_{0y}^2 + 2a_y \cdot y$$

$$0 = (10\sin 60)^2 + 2(-9.81) \times h$$

$$\therefore \frac{(10\sin 60)^2}{2 \times 9.81} \Rightarrow 3.8\ m \text{ Ans}$$

$$\boxed{h = \frac{u^2\sin^2\theta}{2g}}$$

concept ③ Normal and tangential components of accel. → in curvilinear motion



$$\boxed{a \Rightarrow \sqrt{a_x^2 + a_y^2}}$$

$$\boxed{a \Rightarrow \sqrt{a_t^2 + a_n^2}}$$

Note → if the acceleration is resolved normal to the path and tangent to the path, then the components are called normal acceleration and tangential acceleration.

$\vec{V} = V \cdot \hat{e}_t$ (unit vector)

$\vec{a} \Rightarrow \frac{d\vec{V}}{dt}$

$\vec{a} = \frac{d}{dt}(v.\hat{e}_t) = \frac{dv}{dt}(\hat{e}_t) + v\left(\frac{d\hat{e}_t}{dt}\right) \rightarrow \frac{v}{r}.\hat{e}_n$

$\vec{a} \Rightarrow \frac{dv}{dt}.\hat{e}_t + \frac{v^2}{r}(\hat{n})$

$\boxed{\vec{a} \Rightarrow a_t.\hat{e}_t + a_n\hat{e}_n}$

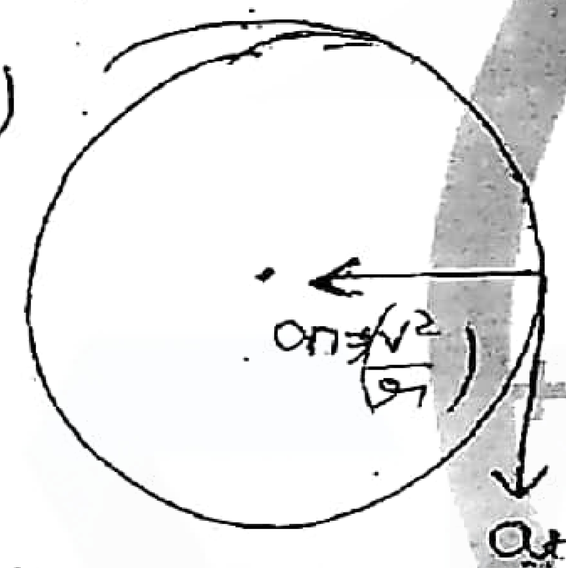
where $a_t = \frac{dv}{dt}$ = tang. accelaration {It is developed due to change in magnitude of velocity}

$a_n = \frac{v^2}{r}$ = Normal accl. (It is developed due to change in direction of velocity)

$\hat{e}_t, \hat{e}_n$ are unit vectors. tangent to the path and normal to path

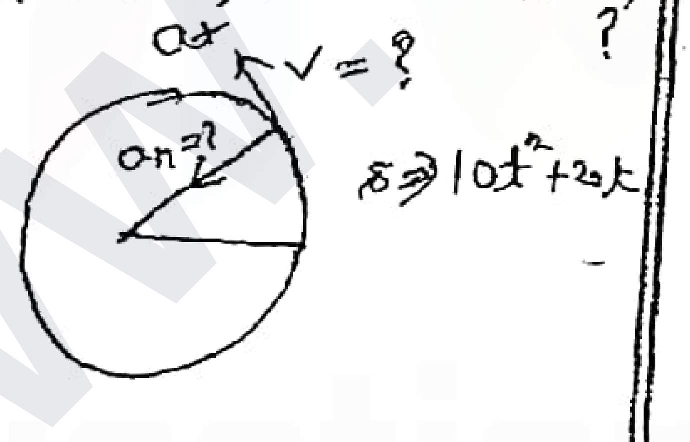
r = Radious of curvature of path

Note-1 if a particle is moving along a straight line it normal accelration is zero. ($r = \infty$, $a_n = \frac{v^2}{r} = \frac{v^2}{\infty} = 0$) but tangential accelaration may exist.

2) 

→ if a particle is moving in a circular path with const velocity v. then $a_t = \frac{dv}{dt} = 0$ bt ($a_n \Rightarrow v^2/r$) will exist

Que-1 A particle moves in a circular path so that arch $s \Rightarrow 10t^2 + 20t$. radious of circle $\Rightarrow$ 400m. at t = 3 sec, $a_n \Rightarrow$?



$v = \frac{ds}{dt} = 20t + 20$

at t = 3 sec = $20 \times 3 + 20 = 80$ m/s

$a_n = \left(\frac{v^2}{r}\right) = \frac{(80)^2}{400} = \frac{6400}{400} = 16$ m/sec² R

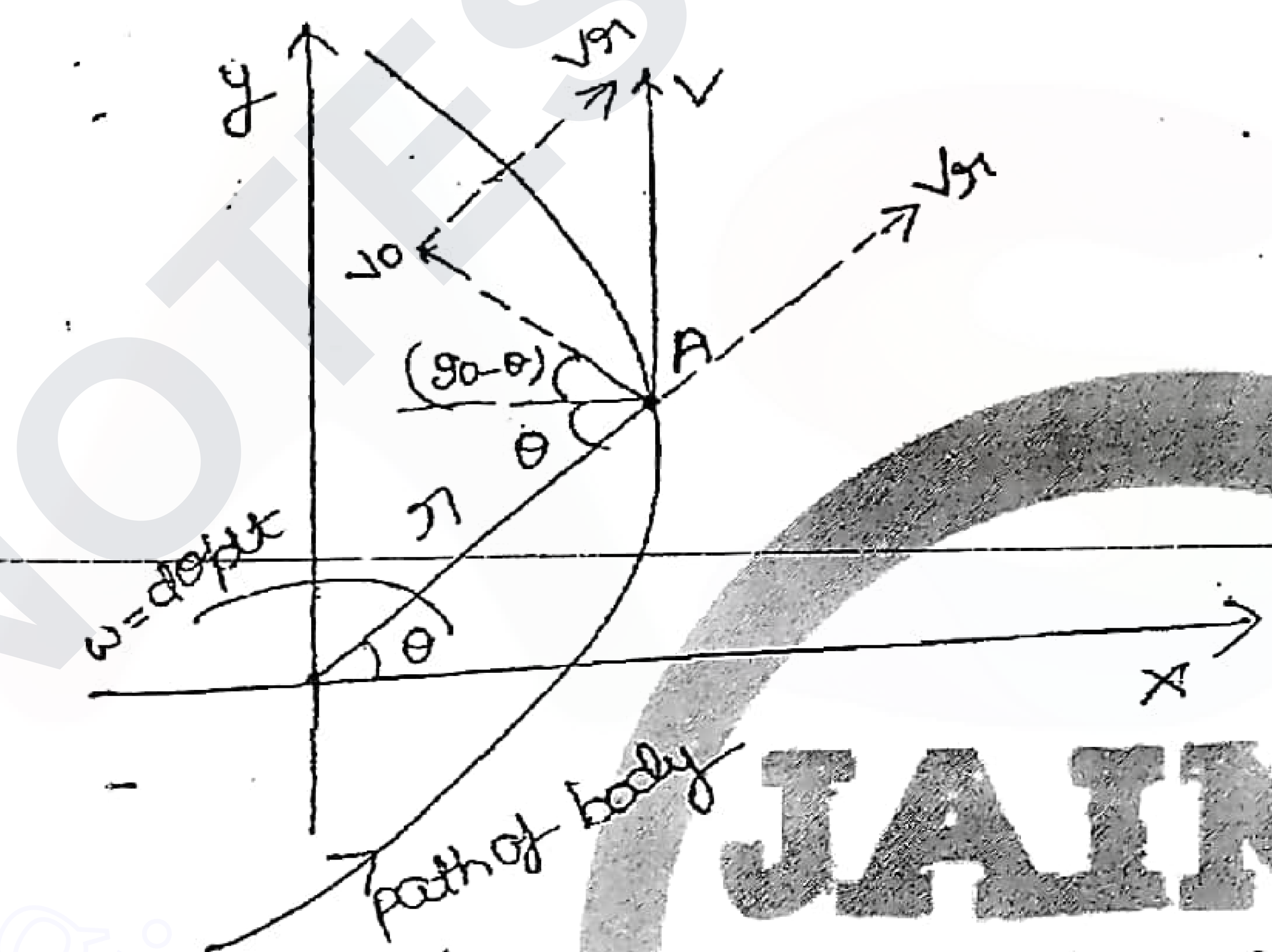
2) In the above problem tangential accl. at $t \Rightarrow 3$ sec is?

$a_t = \frac{dv}{dt} \Rightarrow 20$ m/sec² R

③ in the above problem accelaration of the particle at $t \Rightarrow 3$ sec. is ?

$a = \sqrt{a_n^2 + a_t^2} \Rightarrow \sqrt{(20)^2 + (16)^2} \Rightarrow 25.6$ m/sec² Ans

Concept-4 → Radial and transverse components of velocity →



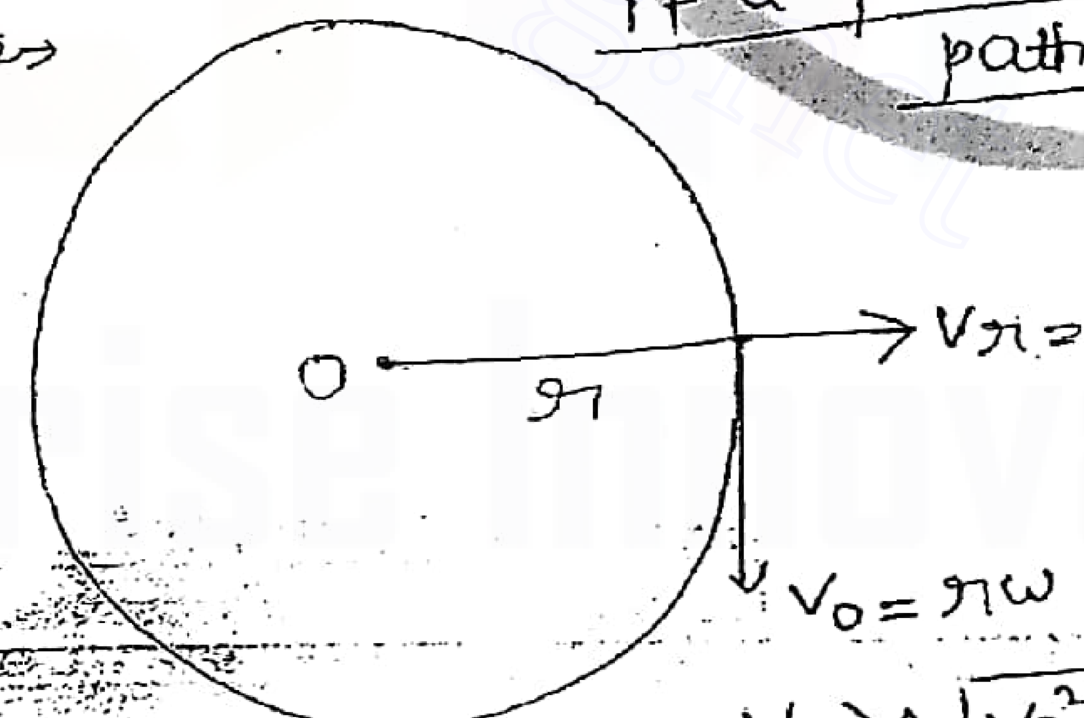
$\boxed{V = \sqrt{V_r^2 + V_\theta^2}}$

$V_r \Rightarrow \frac{dr}{dt}$

$V_\theta \Rightarrow r\omega$ {$\omega \Rightarrow$ Angular velocity}

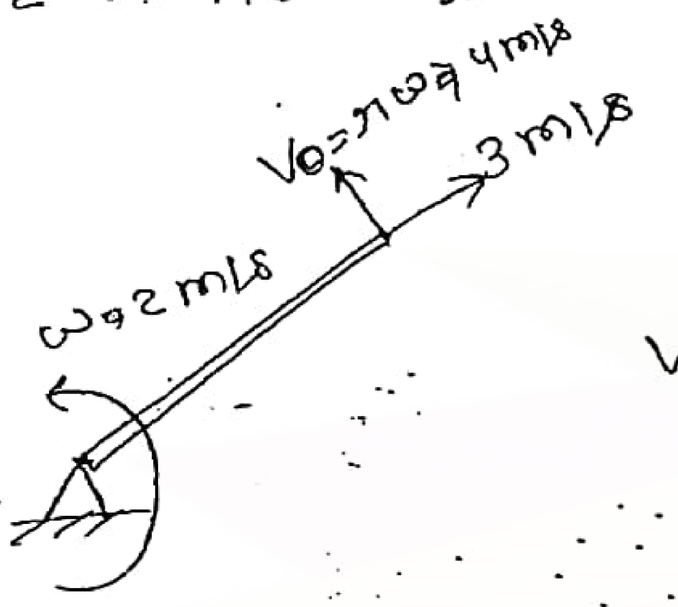
if the velocity is resolved in the radial direction and ⊥ to the radial direction, then they are called Radial velocity and transverse velocity.

Note → if a particle is moving in a circular path.



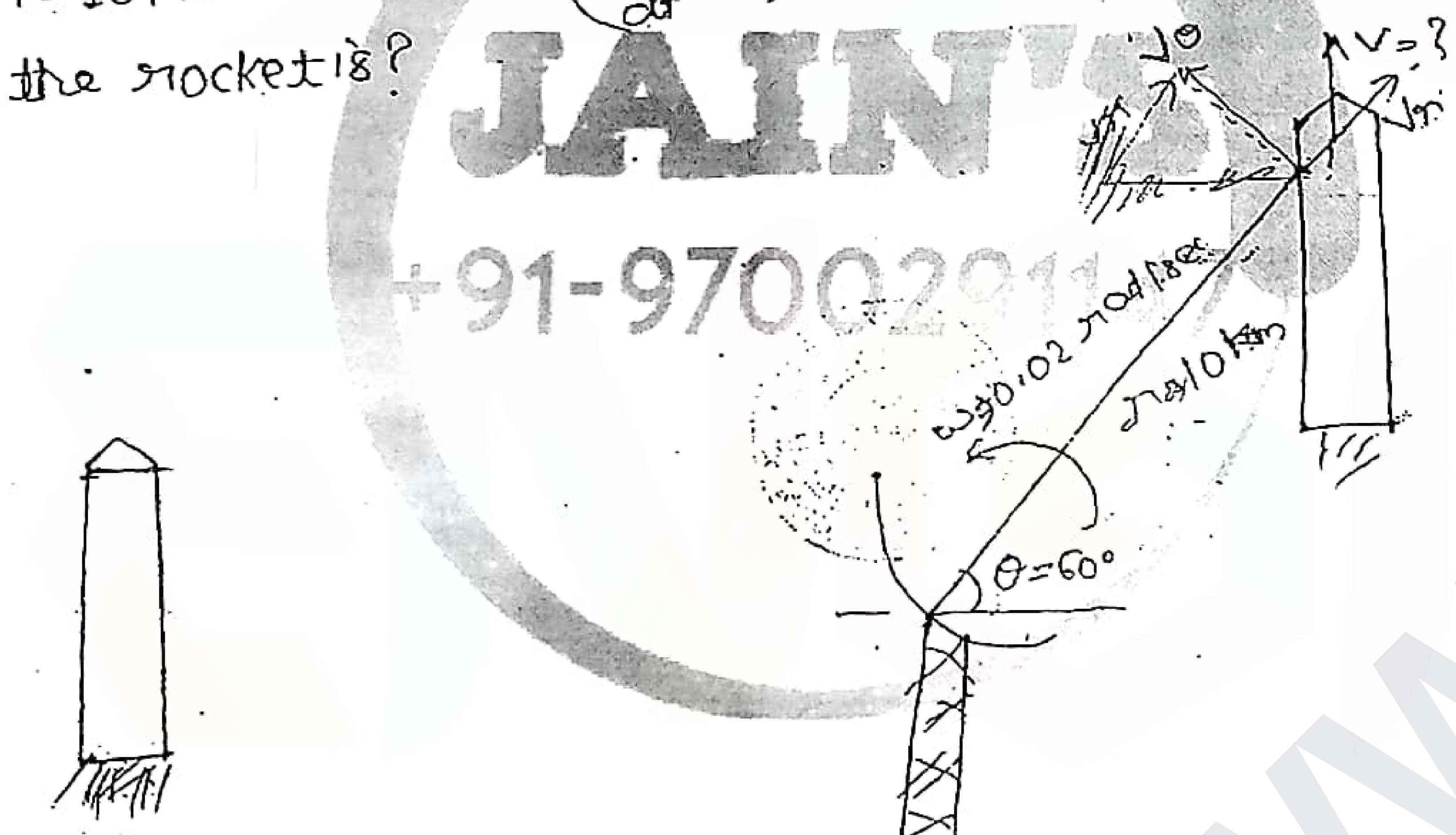
$V \Rightarrow \sqrt{V_r^2 + V_\theta^2} \Rightarrow r\omega$

Que. A shell is fired from a cannon. at the instant the shell is just about to leave the barrel its velocity relative to barrel is 3m/sec. while the barrel is swinging upwards with const angular velocity of 2 rad/sec. the mag of absolute velocity of the shell is?



$V \Rightarrow \sqrt{V_r^2 + V_\theta^2} \Rightarrow \sqrt{3^2+4^2} \Rightarrow 5\,m/sec$ Ans

(2) A rocket is fired vertically and tracked by a radar as shown in fig. at the instant $\boxed{\theta = 60^\circ}$ it is known that $r = 10\,km$. and $\theta' \left(\frac{d\theta}{dt} = \omega\right) = 0.02\,rad/sec$, then the velocity of the rocket is?



$V_\theta \Rightarrow r.\omega \Rightarrow (10,000)\,m \times 0.02$

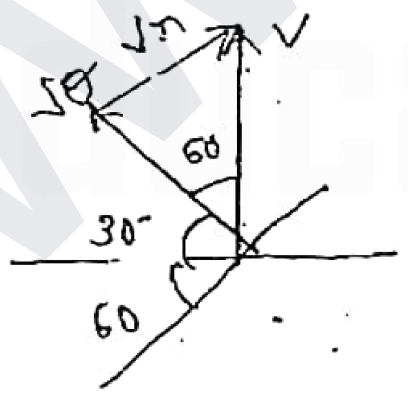
$\Rightarrow 200\,m/sec$

from FBD ⇒

$V_\theta = V\cos 60^\circ$

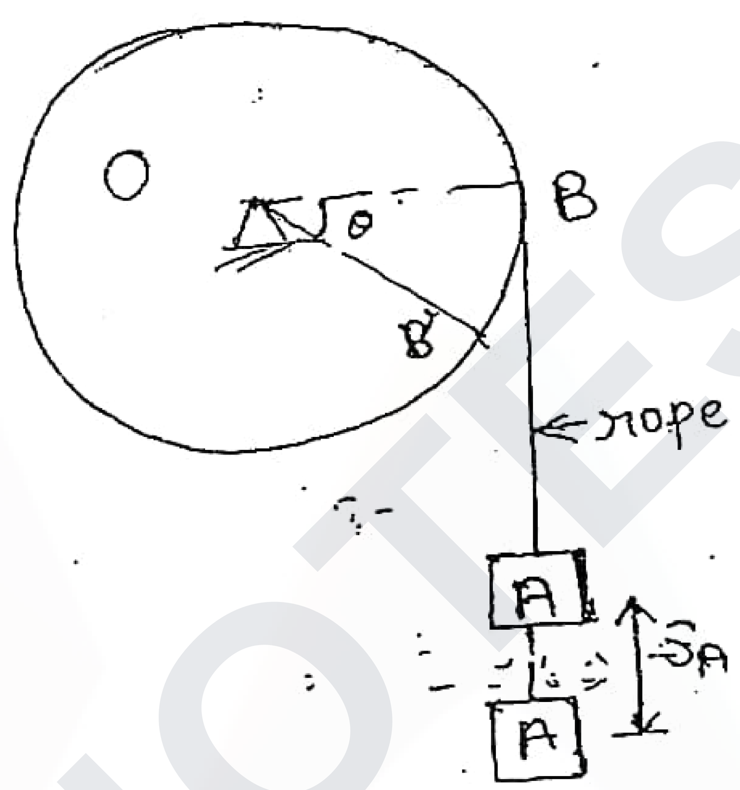
$200 = V\cos 60^\circ \Rightarrow$

$V = \frac{200}{\cos 60} \Rightarrow 400\,m/sec$



IIIrd topic ← Rotation → Kinematics →

Concepts -(1) Relation btw linear and Angular displacements ⇒



$S_A = BB' = r\theta$

Diff ⇒ $V_A = \frac{dS_A}{dt} = r.\frac{d\theta}{dt} \Rightarrow r\omega$

$\boxed{V_A \Rightarrow r.\omega}$ —(2)

Diff $a_A \Rightarrow$ Tang. acceleration ⇒

$\frac{dV_A}{dt} \Rightarrow r.\frac{d\omega}{dt} \Rightarrow r\alpha$ {$\alpha \Rightarrow$ angular acceleration}

$\boxed{a_t \Rightarrow r\alpha}$

Note ⇒ 1) Point B on the rope can only have tangential acceleration but point B on the pulley will have tangential acceler. and normal acceleration.

2) if $\alpha \Rightarrow$ const, then

Rectilinear motion with a const	Rotation with α const
① $V \Rightarrow V_0 + at$	① $\omega = \omega_0 + \alpha t$
② $s = V_0 t + \frac{1}{2}at^2$	2) $\theta \Rightarrow \omega_0 t + \frac{1}{2}\alpha t^2$
③ $V^2 = V_0^2 + 2as$	③ $\omega^2 = \omega_0^2 + 2\alpha\theta$

③ if α is not const.

Rectilinear motion	Rotation
① $V = ds/dt$	① $\omega = d\theta/dt$
② $a = dv/dt$	② $\alpha = d\omega/dt$
③ $a\,ds = v\,dv$	③ $\alpha\,d\theta = \omega\,d\omega$

Que> A flywheel rotating at 3 rev/sec. as its speed ↑ at a const. rate of 45 Rev/minute each second during an interval of 10 sec. the no. of revolutions it has made during this time is ?

Note- Speed is ↑ at a const. rate means angular acel. α is const.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$\omega_0 \Rightarrow$ initial angular velocity

$\alpha \Rightarrow 45$ rev/min/sec $\Rightarrow \frac{45}{60}$ rev/sec/sec $= \frac{45}{60}$ rev/sec^2

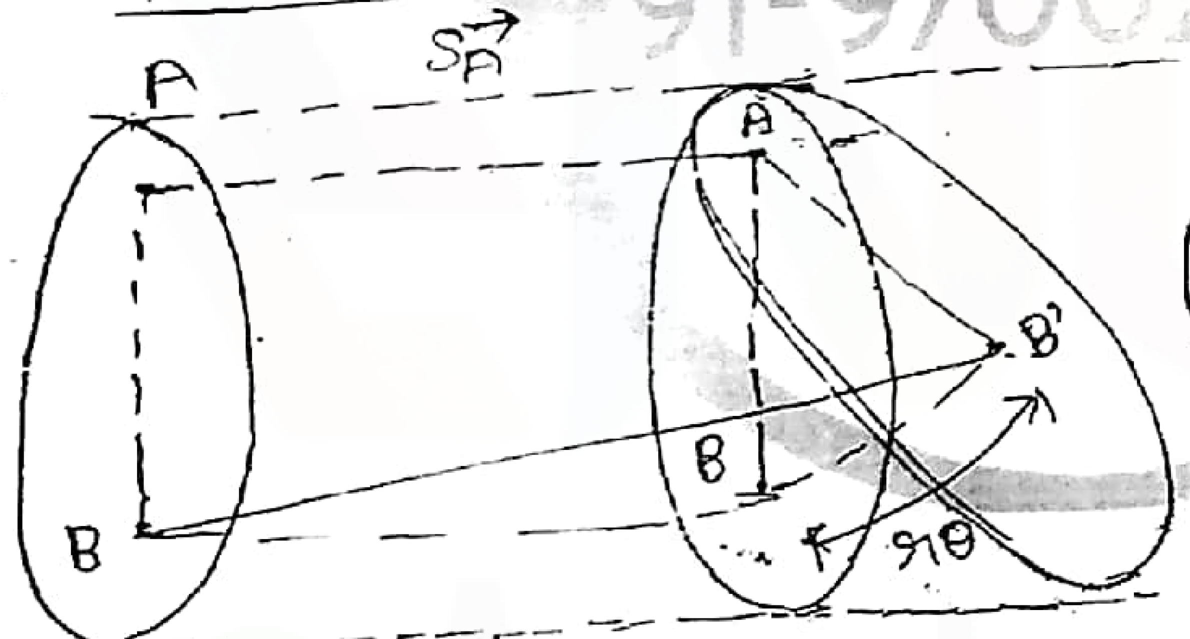
$t \Rightarrow 10$ sec

$$\theta \Rightarrow (3\times10) + \frac{1}{2}\times\frac{45}{60}\times(10)^2 \Rightarrow 67.5 \text{ rev. Ans}$$

IV topic → Plane motion → Kinematics →

concepts-1 > if the body rotates and translates, then it is called plane motion.

2- **Kinematic Equation for plane motions →**



$\vec{S_B} = \vec{S_A} + \vec{S}_{B/A}$

$\vec{S_B} = \vec{S_A} + \vec{S}_{B/A} \; (= \vec{r\theta})$ —(1)

Diff $\vec{V_B} = \vec{V_A} + \vec{V}_{B/A} \; (= \vec{r\omega})$ —(2)

Diff $\vec{a_B} = \vec{a_A} + \vec{a}_{B/A} \; (= \vec{r\alpha} + \vec{r\omega^2})$ —(3)

} Kinematics equ for plane motions. or CHASLE's theorm

$a_t = \alpha r$, $a_n = \frac{V^2}{r} = r\omega^2$

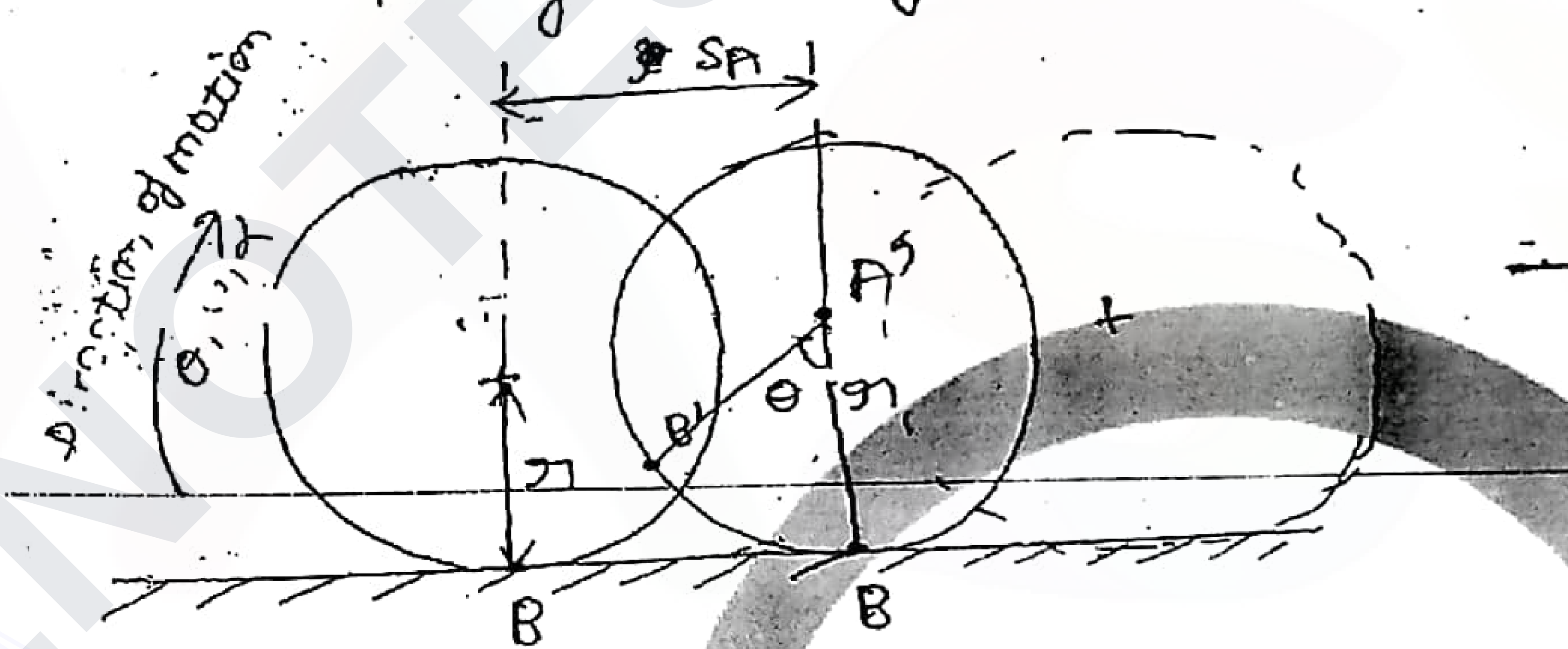
$S_{B/A}$ read as - Displ of 'B' assumed to be rotating about A.

$\vec{V}_{B/A}$ = velocity of B assumed to be rotating about A

$\vec{a}_{B/A}$ = " " accel. of B " "

Note> Somehow find S_A, V_A, a_A then S_B, V_B, a_B of any other point on the plane motion can be found from the above 3 equations.

concept-3 Application of plane motion equ. to freely ~~rotating~~ Rolling bodies →



$S_A = r\theta$ —(1)

Diff $V_A = r\omega$ —(2)

$a_A = r\alpha$ —(3)

Note (1) After finding S_A, V_A, a_A we can find S_B, V_B, a_B of any other point from the plane motion equation.

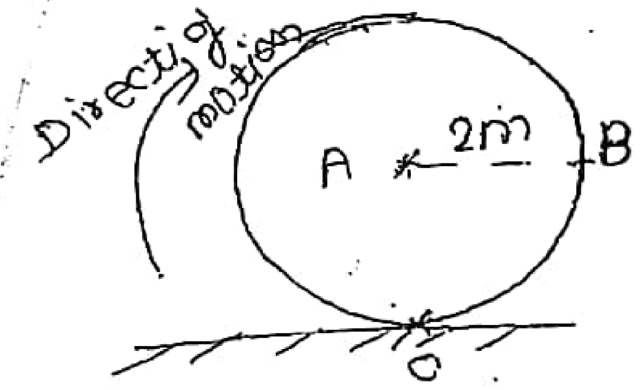
the above Equ. are valid only when the wheel rolls freely without slipping and skidding.

(3) **Slipping** → If rotation > translation → $\{S_A < r\theta\}$ then it is called slipping.

4) **Skidding** - If translation > rotation $(S_A > r\theta)$ then it is called skidding.

5) If $S_A = r\theta$ then it means that there is no slipping or skidding and it rolls freely.

Que> A wheel shown in fig rolls freely. at the given position velocity of point A is 6 m/sec. the velocity of B is?



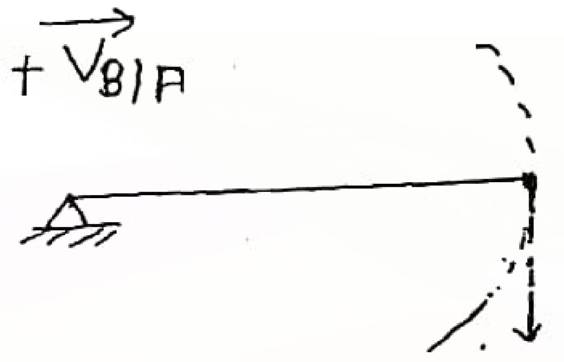
$V_A = r\omega \Rightarrow 2\times\omega \Rightarrow 6\ m/s$

ω = Angular velocity of wheel $= 6/2 \Rightarrow 3\ rad/s$

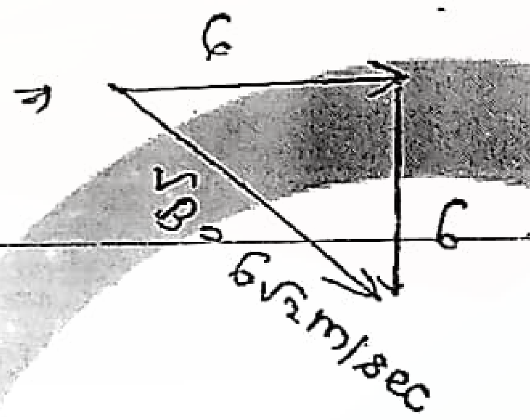
Velocity of B ⇒

$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

$\Rightarrow \vec{6} +$



$V_{B/A} = r\omega \Rightarrow 2\times3 \Rightarrow 6\ m/sec$

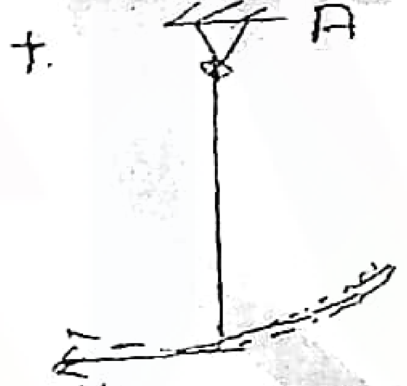


$\vec{V_B} = \sqrt{6^2+6^2} \Rightarrow 6\sqrt{2}\ m/sec$ Ans

Que in the above problem velocity of point C is?

$\vec{V_C} = \vec{V_A} + \vec{V_{C/A}}$

$= \vec{6} +$



$V_{C/A} \Rightarrow r\omega = 2\times3 = 6\ m/s$

$\vec{V_C} = \xrightarrow{6\,m/s} + \xleftarrow{6\,m/s}$

$\Rightarrow 0\ m/sec$

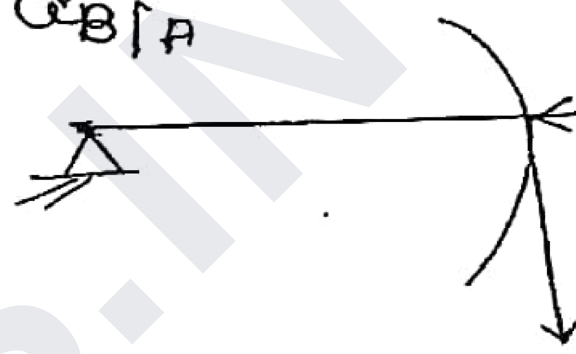
Note ⇒ Instant center or instantaneous center of rotation → gives a point in plane motion whose velocity is momentarily zero. with respect to instant center a plane motion problem can be treated as pure rotation problem.

Que in the above problem if the accel of point A is 20 m/s², accel. of point B is?

$a_A = r\alpha \Rightarrow \cancel{2\times20=40}\quad 2\times\alpha = 20 \quad \boxed{\alpha = 10\ rad/sec^2}$

$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$

$\Rightarrow \vec{20} +$



$r\omega^2 = 2\times3^2 \Rightarrow 18\ m/s^2$

$a_t \Rightarrow r\alpha \Rightarrow 2\times10 \Rightarrow 20\ m/s^2$

⇒ 20 → 18 ← 20 ↓ ⇒ 2, 20

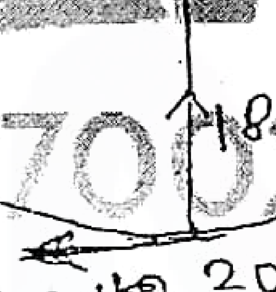
$a_{Bx} = \vec{20} + \overleftarrow{18} \Rightarrow \vec{2}\ m/sec^2$ $\quad \sqrt{2^2+20^2} = \sqrt{404}\ m/sec^2$

$a_{By} \Rightarrow 0 + 20(\downarrow) \Rightarrow 20\ m/sec^2$ $\quad \Rightarrow 20.1\ m/s^2$ Ans

Que in the above problem acceln. of point C is

$\vec{a_C} = \vec{a_A} + \vec{a_{C/A}}$

$= \xrightarrow{20}$



$a_t \Rightarrow r\alpha \Rightarrow 2\times10 \Rightarrow 20\ m/sec^2$

$20 = a_n = r\omega^2 \Rightarrow 2\times3^2 \Rightarrow 18$

$\Rightarrow \xrightarrow{20} \xleftarrow{20} \uparrow 18$

$\vec{a_C} = 18\ m/sec^2 (\uparrow)$

$a_{cx} = \vec{20} + \overleftarrow{20} = 0\ m/sec^2$

$a_{cy} = 0 + 18\ m/sec^2 \Rightarrow 18\ m/sec^2$ Ans

(if accel. $a_c = 0$ wheel will be rest)

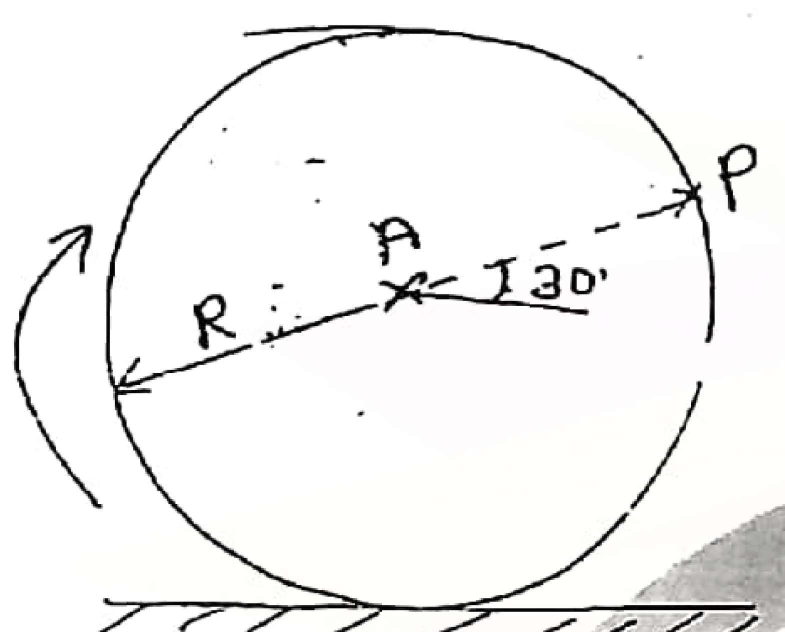
mark Conclusion: instant center are instantaneous center of rotation has zero velocity but not zero acceleration.

Que→ A wheel is rolling freely on the ground, the point of contact of the wheel with the ground will have.

(A) only velocity B) only tangential acceleration

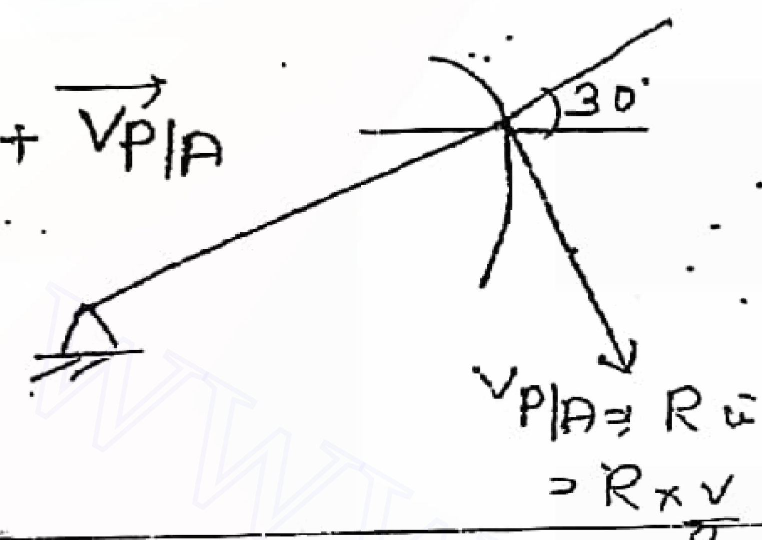
(C) only normal acceler. D) velocity and normal acceleration

Que→ A circular disc of Radius R rolls without slipping at a velocity V. the magnitude of velocity at point P is



$\vec{V}_P = \vec{V}_A + \vec{V}_{P/A}$

$= \vec{V} +$



$V_{P/A} = R\omega$

$= R \times \frac{V}{R}$

$= V$

$V_A \Rightarrow r\omega = R \times \omega = V$

ω = angular velocity $= (V/R)$

$V_{Px} = V + V\cos 60°$

$V_{Py} = 0 + V\sin 60 (\downarrow)$

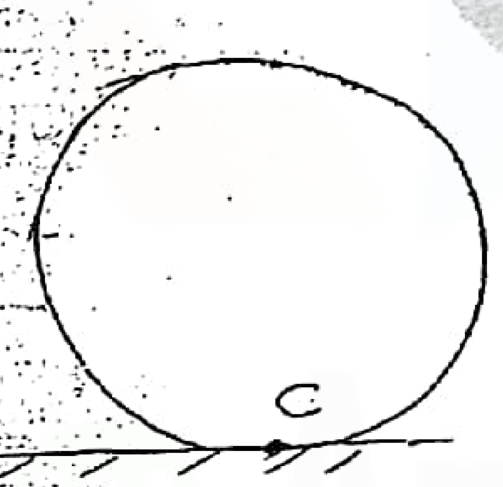
$V_P = \sqrt{(V + V\cos 60)^2 + (V\sin 60)^2}$

$\Rightarrow \sqrt{3}V$ Ans

2 mark

Concept-4 Location of instant centre (2 mark)

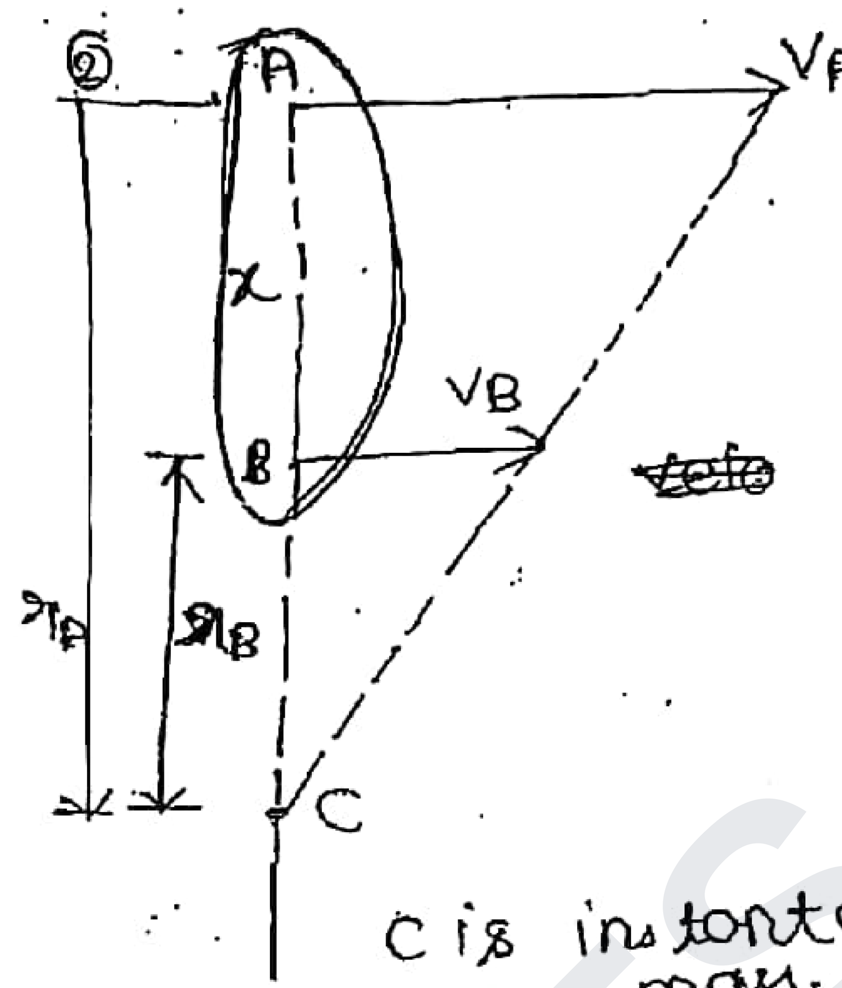
Ex (1)



C is instant center.

(2)

(2)

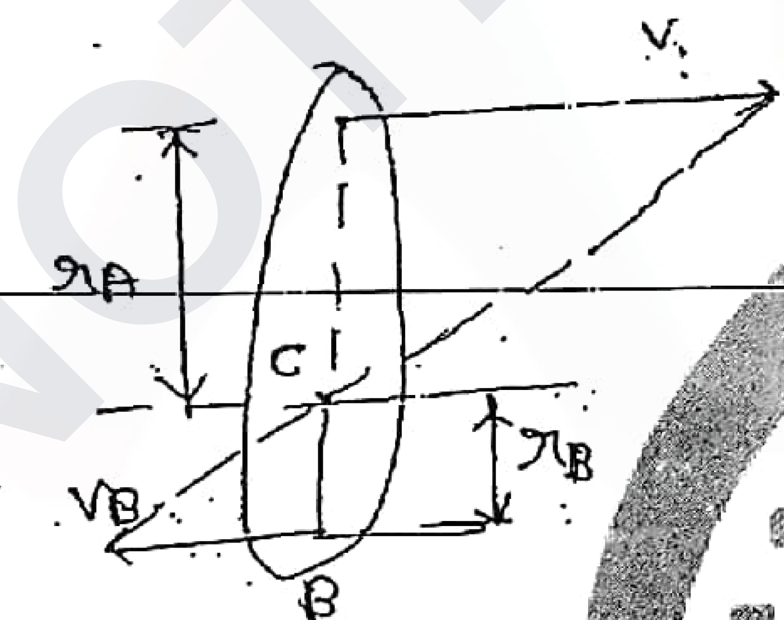


C is instant center of mass.

w.r.to instant center of

$V_B = r_B \cdot \omega$ {velocity is always ⊥ to radius line}

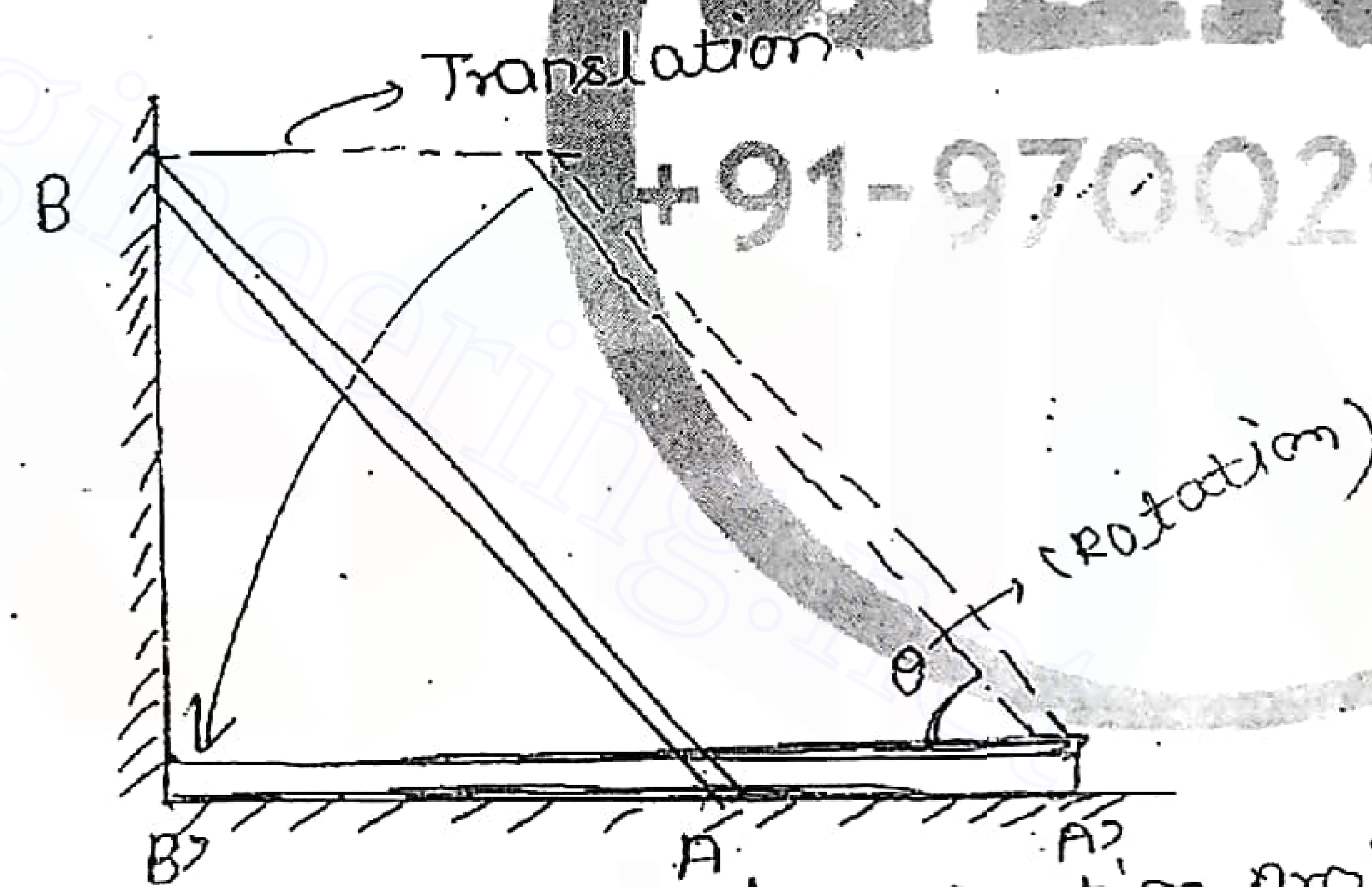
(3)



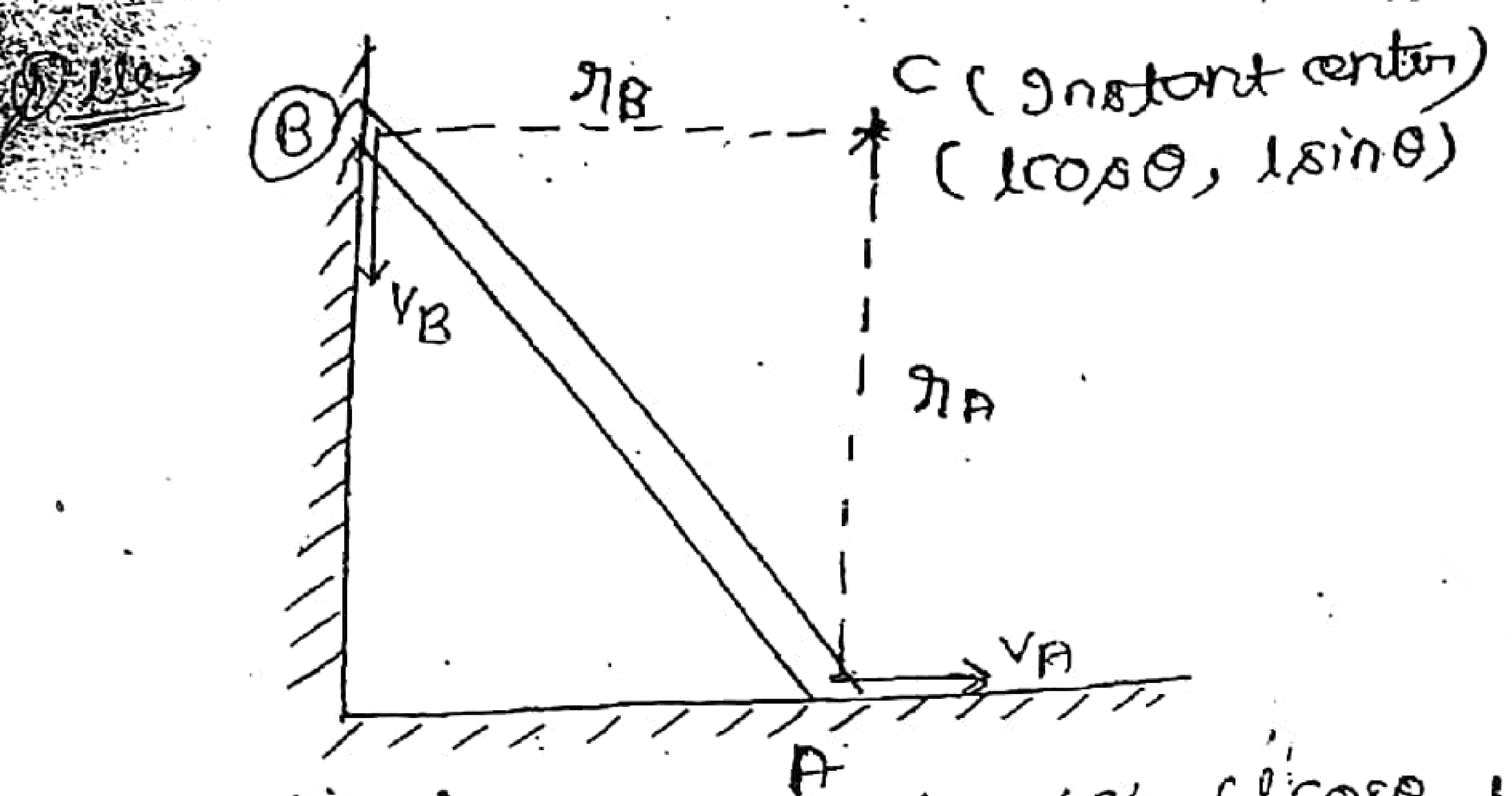
C is instant center

w.r.t. C, $\left[V_B = r_B \cdot \omega, \; V_A = r_A \cdot \omega \right]$

(4)



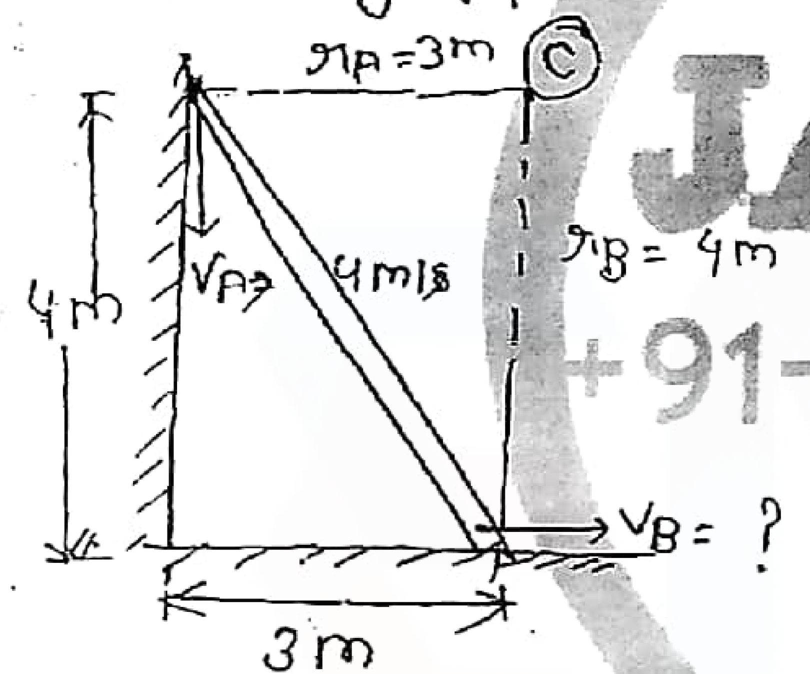
→ so it is a plane motion problem.



Co-ordinate of inst. center 'C' ⇒ ($l\cos\theta$, $l\sin\theta$)
w.r.to. instant center 'C' plane motion problem can be treated as a plane rotation problem

$$\boxed{\begin{array}{l} V_A \Rightarrow r_A \cdot \omega \\ V_B \Rightarrow r_B \cdot \omega \end{array}}$$

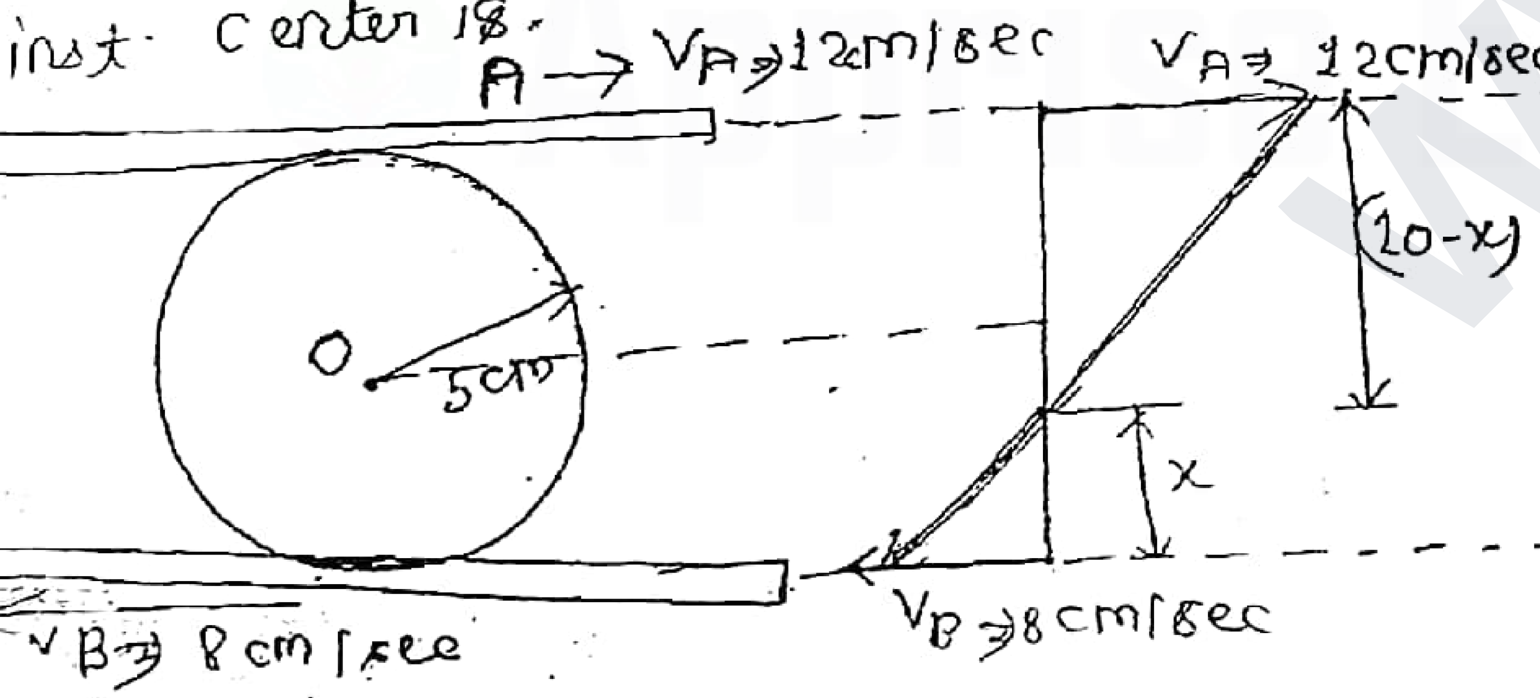
~~for the ladder shown in fig~~ if velocity of point is 4 m/s. velocity of point B is?



w.r.to 'C'
$V_A = r_A \cdot \omega = 4$ m/s.
⇒ ω = angular velocity of ladder at given inst. $= \frac{4}{3}$ rad/sec

$V_B = r_B \cdot \omega \Rightarrow 4 \times 4/3 \Rightarrow \frac{16}{3}$ m/sec. {2 mark} Ans

Que ⇒ A roller of radious 5 cm rolles freely btw 2 horizontal plates whose velocities are as shown in fig. the location of inst. center is.



$\frac{x}{8} \Rightarrow \frac{(10-x)}{12}$

$10x \Rightarrow 80 - 8x$
$x \Rightarrow$ (4 cm from bottom)

Que ① in the above problem angular velocity of the roller is
Que ② in the above problem linear velocity of mass center O is

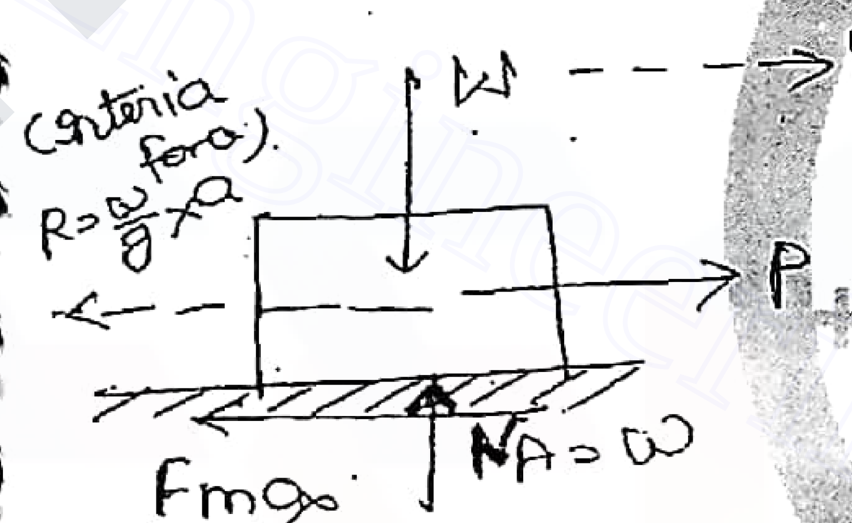
Sol.
① $V_A = r_A \omega \Rightarrow$
$12 = 6 \times \omega$ $\quad \boxed{\omega = 2 \text{ rad/sec}}$

② $V_O = r_O \omega \Rightarrow 1 \times 2 \Rightarrow 2$ m/sec

← Kinetics →

1st topic ⇒ Rectilinear motion ⇒ kinetics

concept-① → Kinetic Equation for Rectilinear motion →



$\Rightarrow \Sigma F_x \Rightarrow P - F_{max} \Rightarrow R \Rightarrow \frac{W}{g} \times a$

Kinetic equ. for rectilinear motion

concept ② → Dynamic equilibrium - D'alembert's principle

if a body is moving rightwards with an acceleration 'a' then it means that there must be a resultant force of $R = \frac{W}{g} \times a$ acting rightward.

⇒ Now if we apply a force $R \Rightarrow \left(\frac{W}{g} \times a\right)$, at the c.o.g of the body acting leftwards, then the body will be in equilibrium called dynamic equilibrium.

Note ① the imaginary force we applied leftwards is called inertia force. the resultant force that and (P - F_{max}) is called effective force.

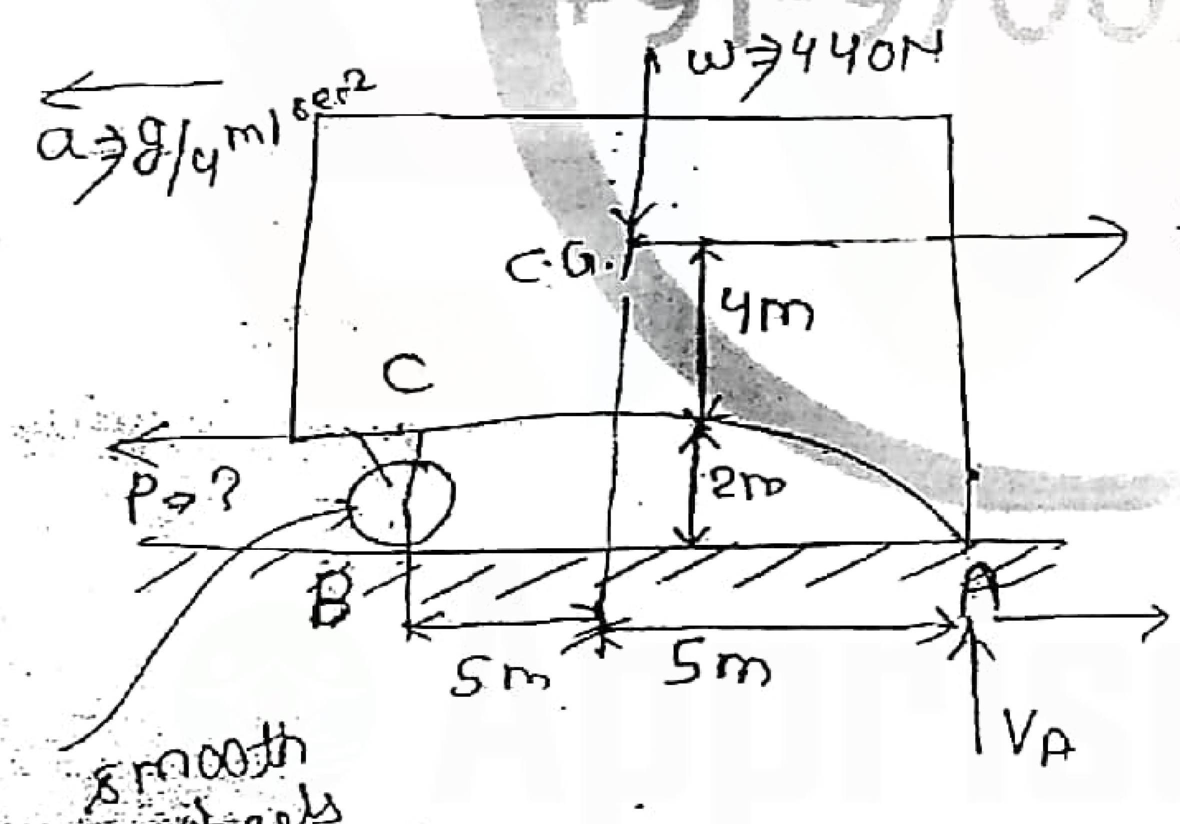
2 mark
② D' Alembert's principle → It states - "under the action of inertia force and effective force, the body will be in dynamic equilibrium.

2 mark
3) D' alember. principle converts a dynamic problem into a static problem.

4) if acceleration is known, keep the body in dynamic equilibrium and use $\Sigma x = 0$, $\Sigma y = 0$, $\Sigma M = 0$. to find the unknown quickly.

5) inertia force always act opposite to the direction of acceleration but not opposite to the dir of motion.

Que→① the 440N body shown in fig is supported by wheels at B which rolls freely ~~without friction~~ and by a skid at A under which $\mu = 0.5$. the value of P is to cause an acceleration of $g/4$ m/sec² is →



$P = \frac{w}{g} \times a = \frac{440}{g} \times \frac{g}{4} = 110N$ (inertia force)

⇐ Body is in dynamic Equilibrium →

$\Sigma M_c = 0$

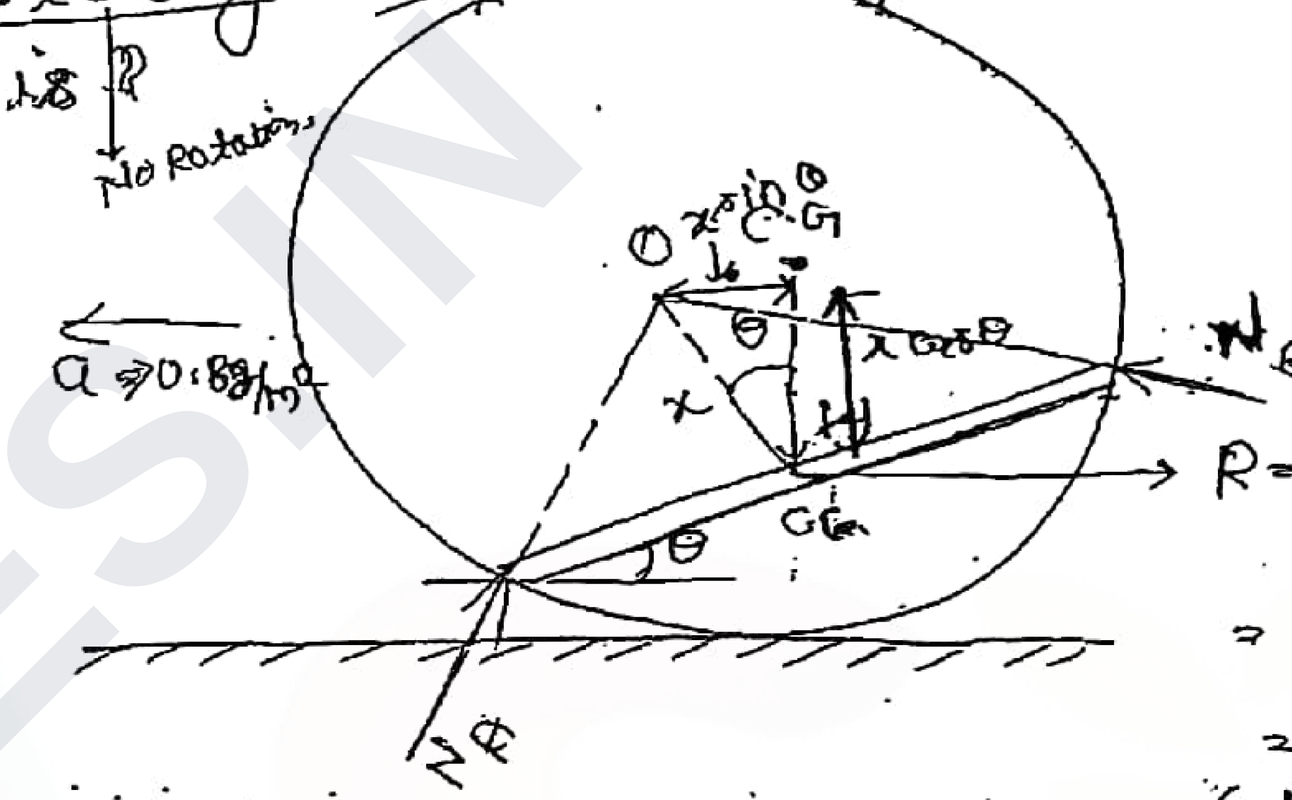
$-N_A \times 10 - 0.5 \times N_A \times 2 + 440 \times 5 + 110 \times 4 = 0$

$\boxed{N_A = 240N}$

$\Sigma x = 0$

$-P + f_A + 110 = 0$ $\boxed{P = 120 + 110 = 230N}$

② the angle 'θ' at which a uniform bar of weight w can be maintained inside the smooth surface of a cylinder drum translating and accelerating leftward at 0.8g m/sec². is ?



$R = \frac{w}{g} \times a = \frac{w}{g} \times 0.8g = 0.8w$ (inertia force)

Bar in dynamic Equilibrium →

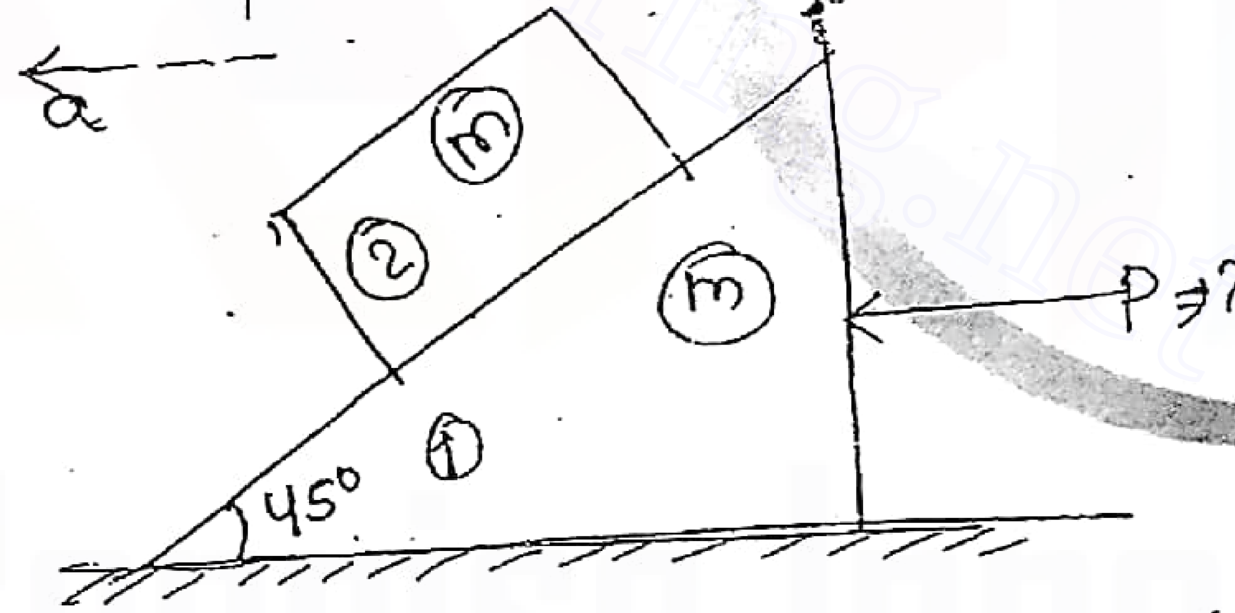
$\Sigma M_O = 0 \Rightarrow$ $wx \sin\theta - (0.8 w x \cos\theta) = 0$

+ve -ve

$\tan\theta = 0.8$

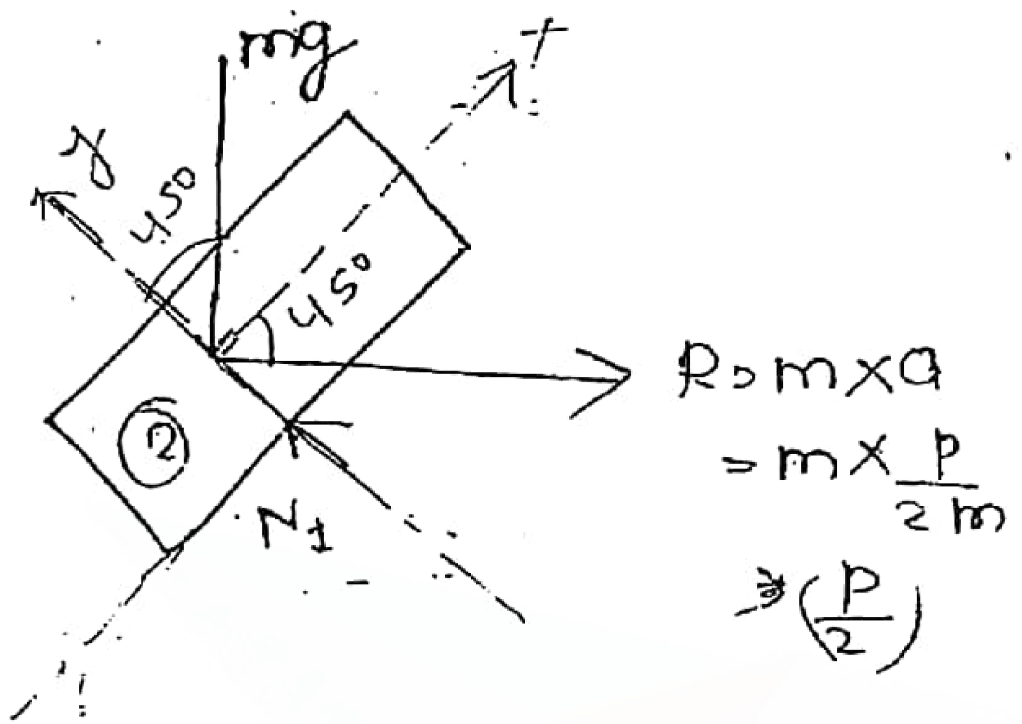
$\boxed{\theta = 38.6^\circ}$

Que→ Blocks ① and ② shown in fig have equal mass. all the surfaces are smooth. value of P required to prevent sliding of block 2 on block ① is ?



$\Sigma x = 0$ $\frac{w}{g} \times a = P = \left(\frac{2m}{g} \times a\right)$

a = Accl. for Rest 2 block system $\boxed{a = P/2m}$



Block ② in dynamic equill..

$\Sigma x = 0$ $R\cos 45° - mg\sin 45° = 0$ [→+ ←(-)]

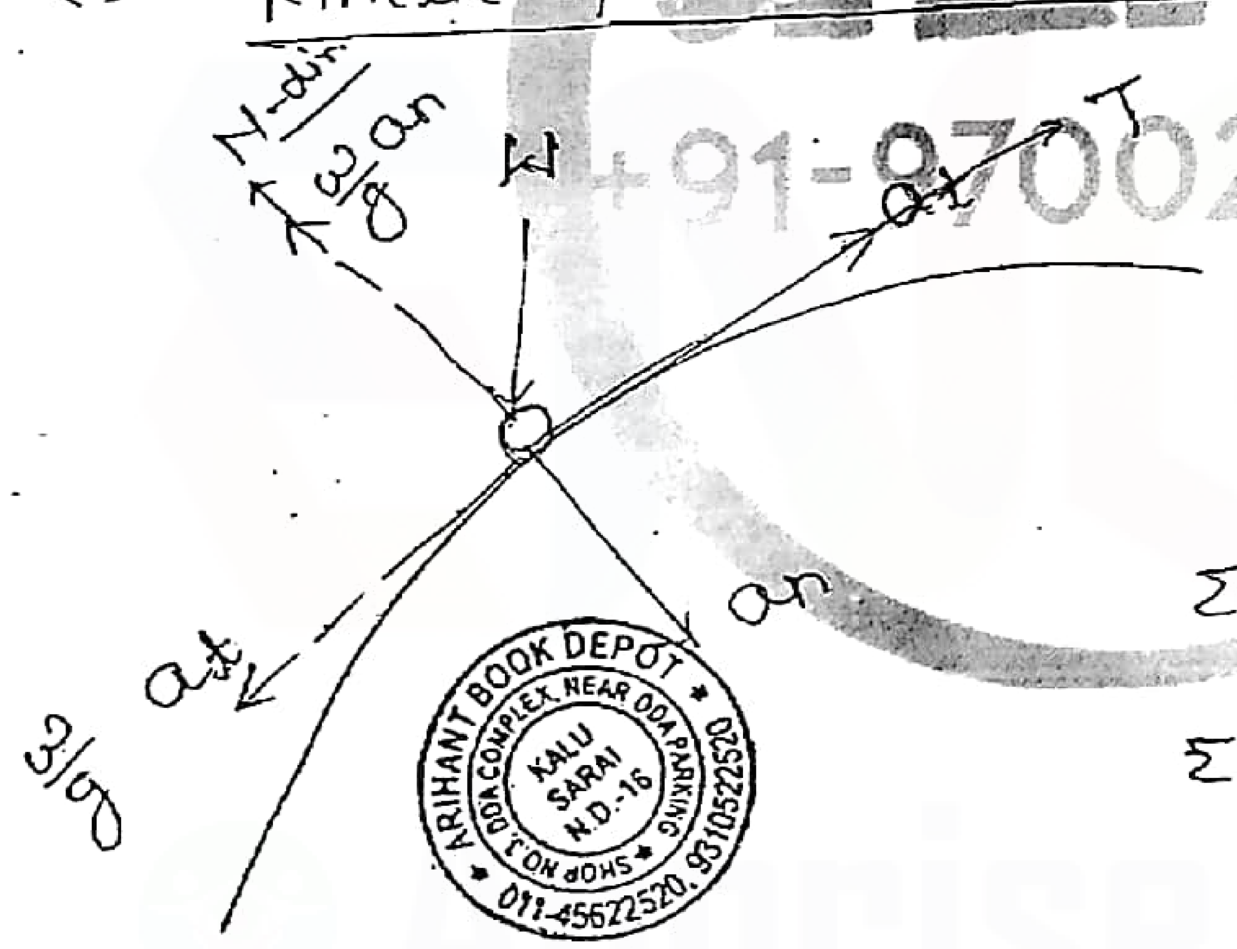
$R \to P/2$

$\boxed{P = 2mg}$ ** A

IInd topic → Curvilinear motion – Kinetics →

Concepts-1. A curvilinear motion is the summation of 2 or 3 Rectilinear motions.

(2) Kinetic Equations for curvilinear motion →



Body in dynamic equill

$\Sigma T = \frac{\omega}{g} \times a_T$ —— ①

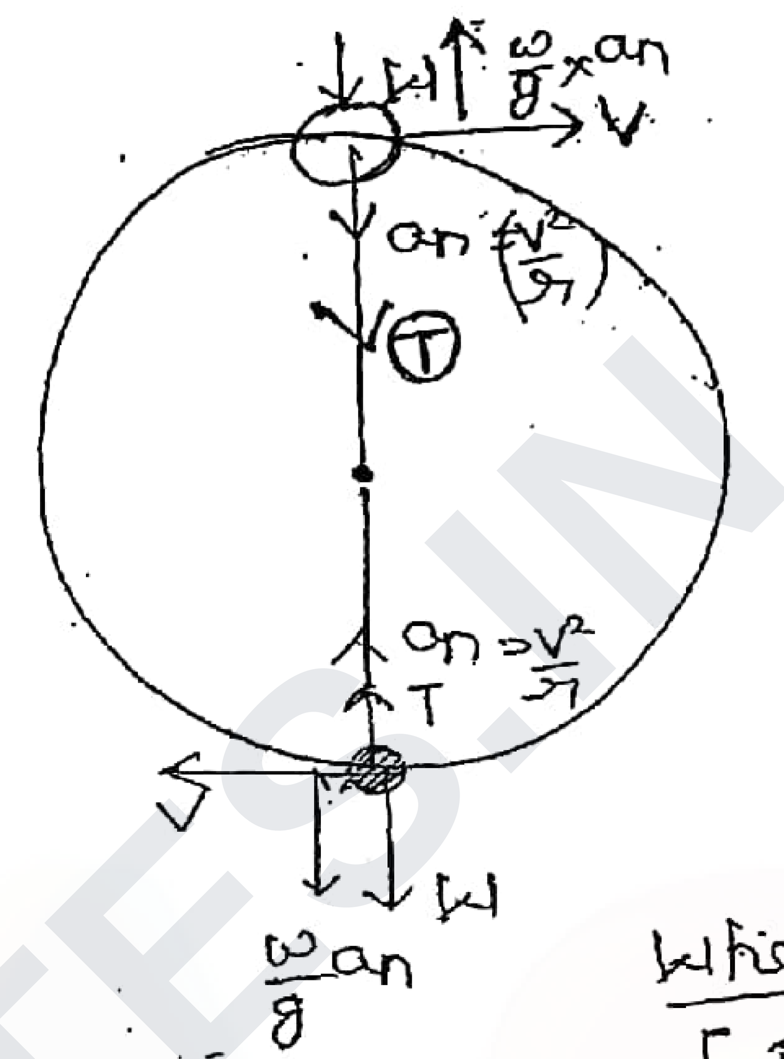
$\Sigma N = \frac{\omega}{g} \times a_n$ —— ②

} kinetics for curvilinear motion.

Que → A stone is tied to a string and is whirled in a vertical circle of radius ~~R~~ 'r'. the tension in the string will be the max. when the stone is - (?)

① at the top of the circular path ② at the bottom of circular

3) half way btw top and bottom 4) none)



When stone is at top

$\Sigma y = 0 \Rightarrow$ [↑+ ↓(-)]

$-W - T + \frac{\omega}{g} \times a_n = 0$

$\boxed{T = \frac{W}{g} \times \frac{V^2}{r} - W}$ —— ①

When stone is at bottom

[↑+ ↓(-)]

$-\omega - \frac{\omega}{g} a_n + T = 0$

$\boxed{T = \omega + \frac{\omega}{g} \times \frac{V^2}{r}}$ —— ②

Que in the above problem the min^m velocity of the stone to maintain circular path is ?

Note - at min velocity the tension in the string becomes 0 when the stone is at top position.

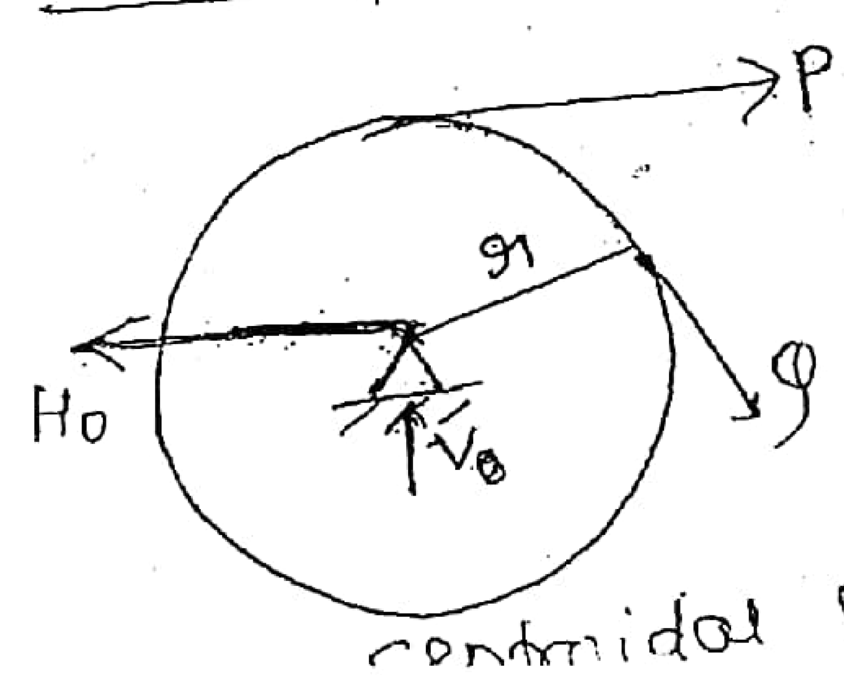
$T = 0 \Rightarrow \frac{\omega V^2}{g \cdot r} - \omega = 0$

$\cancel{V^2 = rg}$ $V^2 = rg$

$\boxed{V = \sqrt{rg}}$

IIIrd topic → Rotation - Kinetics →

Concepts → 1) Kinetics Equation for Rotation →



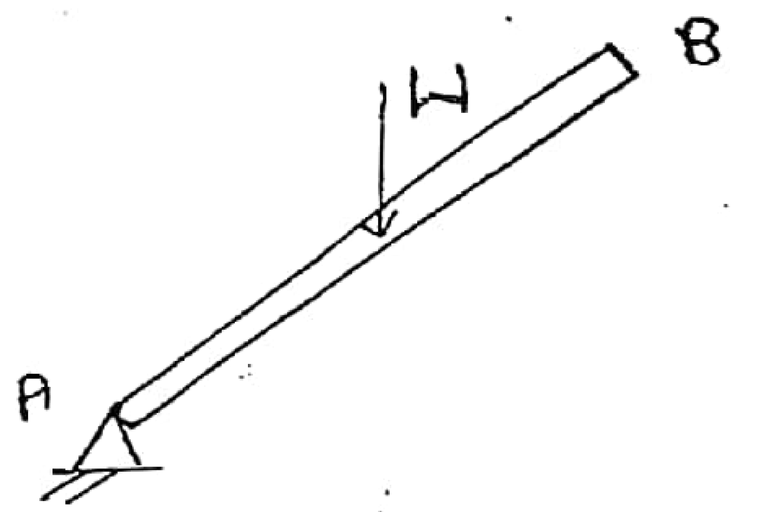
Centroidal Rotation

$\Sigma x = 0$ —— ①
$\Sigma y = 0$ —— ②
$\Sigma M = I_0 \alpha$ —— ③
} Kinetics equ for rotation

$I_0 \Rightarrow$ M.M.I. about the axis of rotation

$\alpha =$ Angular acceleration

Note→ Non centroidal Rotation →

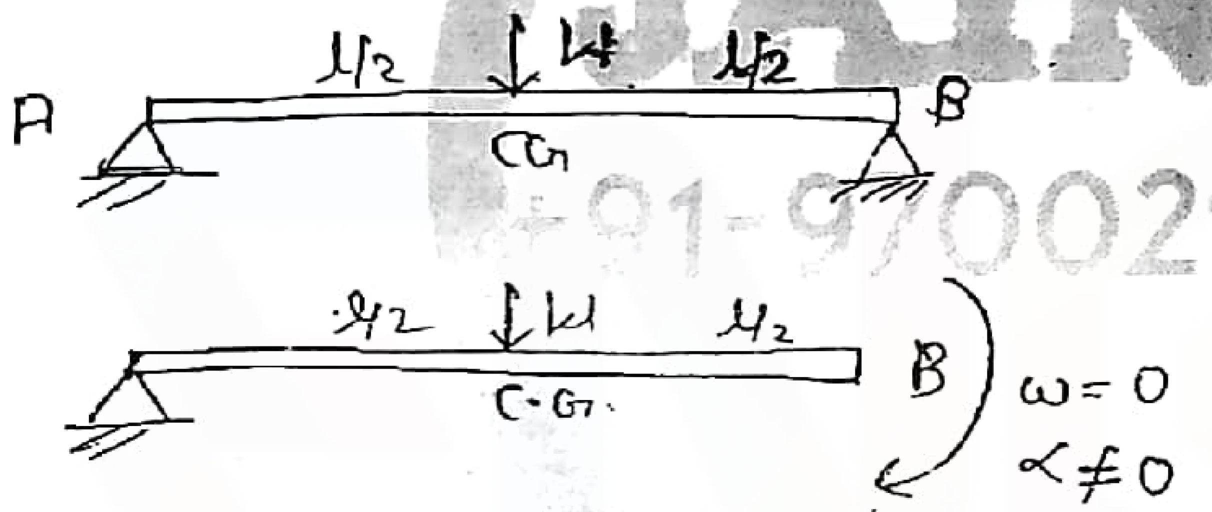


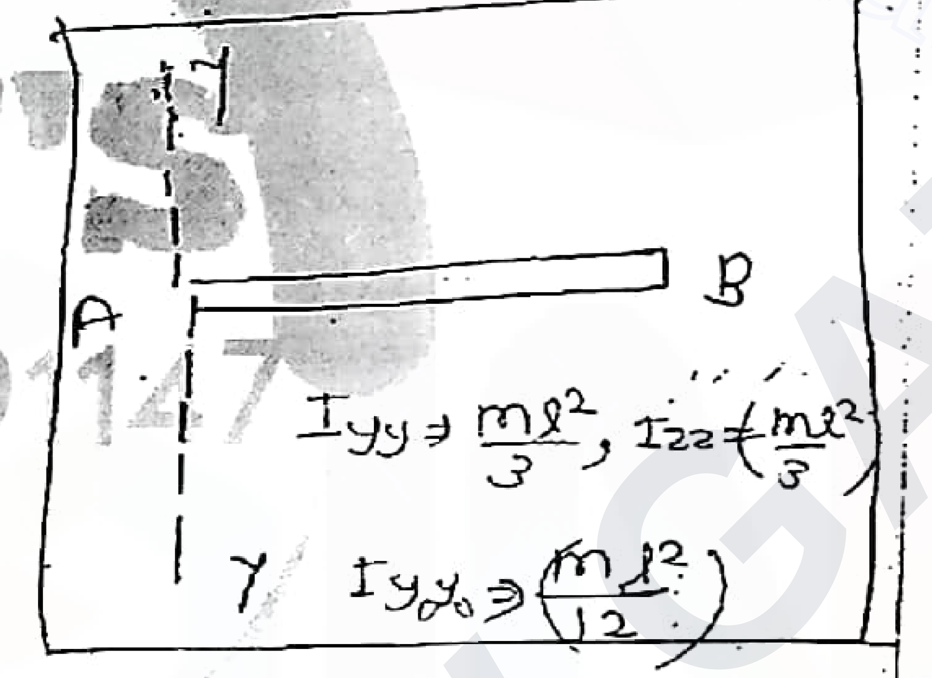
$$\boxed{\Sigma M_A \Rightarrow I_A \cdot \alpha}$$ **

ΣM_A = Sum of moments of all forces about A.

$I_A \Rightarrow$ M.M.I about the axis of rotation passing through A.

Que→(1) A bar AB of length l is supported at both ends as shown in fig. If one of the supports is suddenly removed, angular acceleration of the bar at the instant is.





$I_{yy} \Rightarrow \frac{ml^2}{3}$, $I_{zz} = \frac{ml^2}{3}$

$I_{y_0y_0} \Rightarrow \left(\frac{ml^2}{12}\right)$

Note - After the support at B is removed the bar rotate about an axis passing through A, so it is a kinetics Rotation problem.

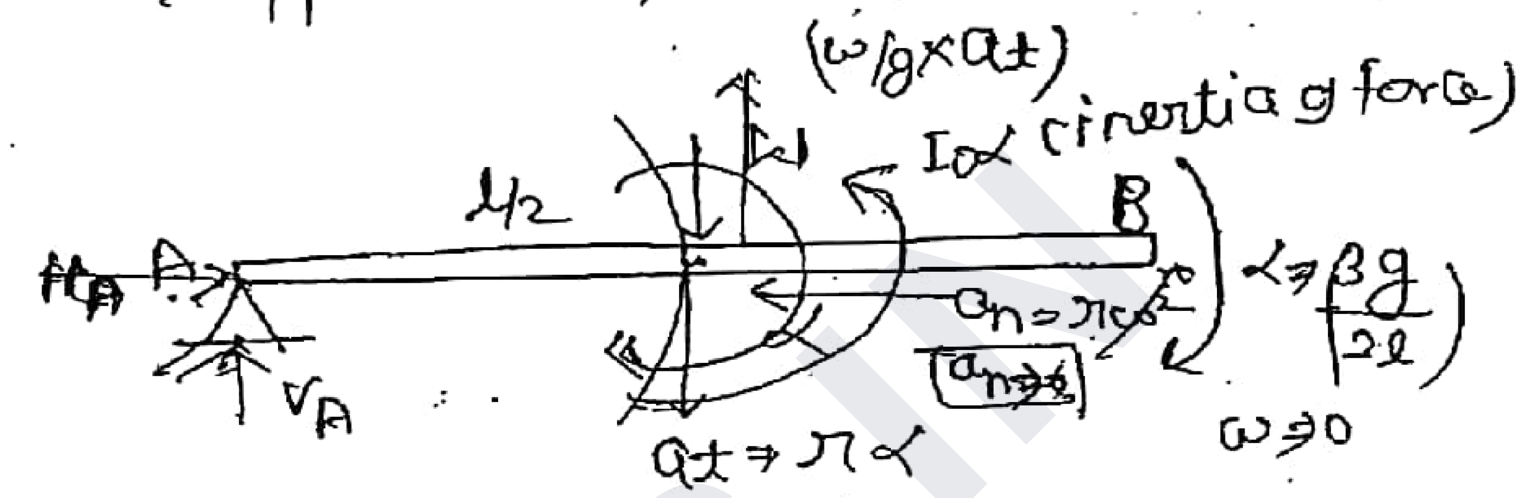
$$\left[I_A \Rightarrow \frac{ml^2}{3} = \frac{\omega}{g}\cdot\frac{l^2}{3}\right]$$

so, $\Sigma M_A \Rightarrow I_A\alpha$

$W l/2 \Rightarrow \left(\frac{\omega}{g}\cdot\frac{l^2}{3}\right) \times \alpha$

$$\boxed{\alpha = \frac{3g}{2l}}$$

→ in the above problem at the instant of removing the support at B, the Rxn at A is ?



Apply Inertia force / Inertia moment and stop the bar so apply equ. eqn

$a_t \Rightarrow r\alpha$

$= \frac{l}{2} \times \frac{3g}{2l}$

$= (3g/4)$

$\Sigma y \Rightarrow 0$

$V_A + \frac{\omega}{g} \times a_t - \omega = 0$

$V_A = \omega - \frac{\omega}{g} \times \frac{3g}{4}$

$$\boxed{V_A \Rightarrow \omega/4}$$ A

← IInd method → $\Sigma M_{C.G} \Rightarrow 0$

(+ve) (-ve) $V_A \times l/2 - I_0\alpha \Rightarrow 0$

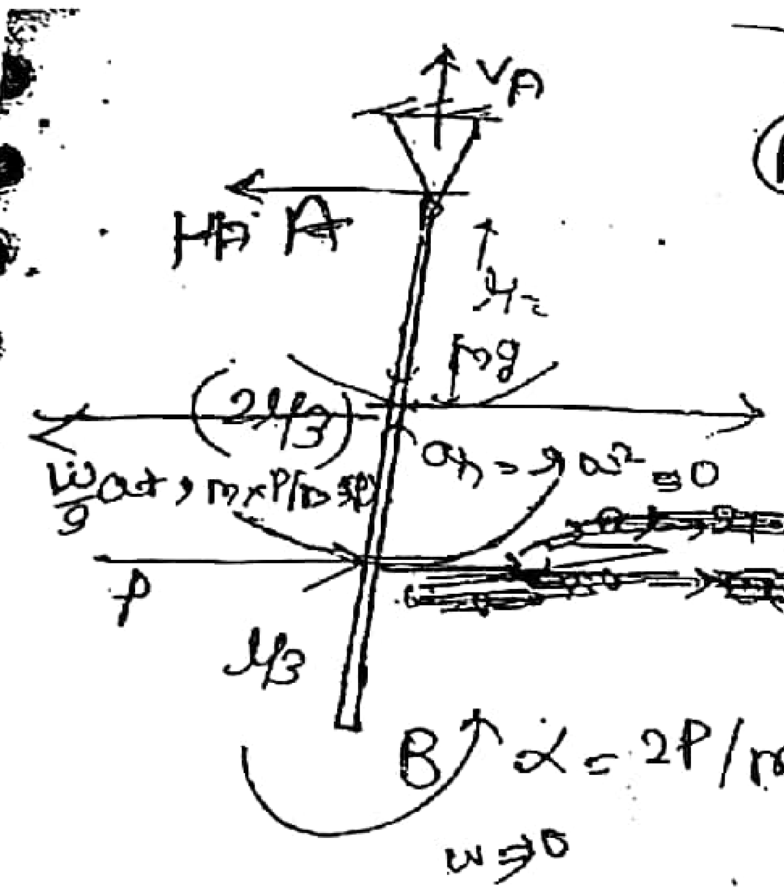
$$\left[I_0 = \frac{\omega l^2}{12 g}\right]$$

$V_A \times l/2 - \frac{\omega l^2}{12 g} \times \frac{3g}{2l} = 0$

$$\boxed{V_A \Rightarrow \omega/4}$$

Que A uniform Rigid bar of mass m and length l is hinged at one end as shown in fig. a force P is applied at a dist of $2l/3$ from the hinge so that the rod swings to the right. the Rxn at the hinge is ?

(A) $-P$ B) $2P/3$ C) 0 D) $P/3$



$a_t = r \cdot \alpha = \frac{l}{2} \times \frac{2P}{ml} \Rightarrow (P/m)$

Note ⇒ Since the bar rotates due to the force P, it is a kinetics rotation problem.

$\Sigma M_A = I\alpha$

$(P \times 2l/3) \Rightarrow \frac{ml^2}{3} \times \alpha$

$\boxed{\alpha \Rightarrow \frac{2P}{ml}}$

the bar is in dynamic Equilibrium

$m a_t$

$= \cancel{m} \times \frac{2P}{\cancel{m}}$

$\Sigma x \Rightarrow +P - P + H_A = 0$

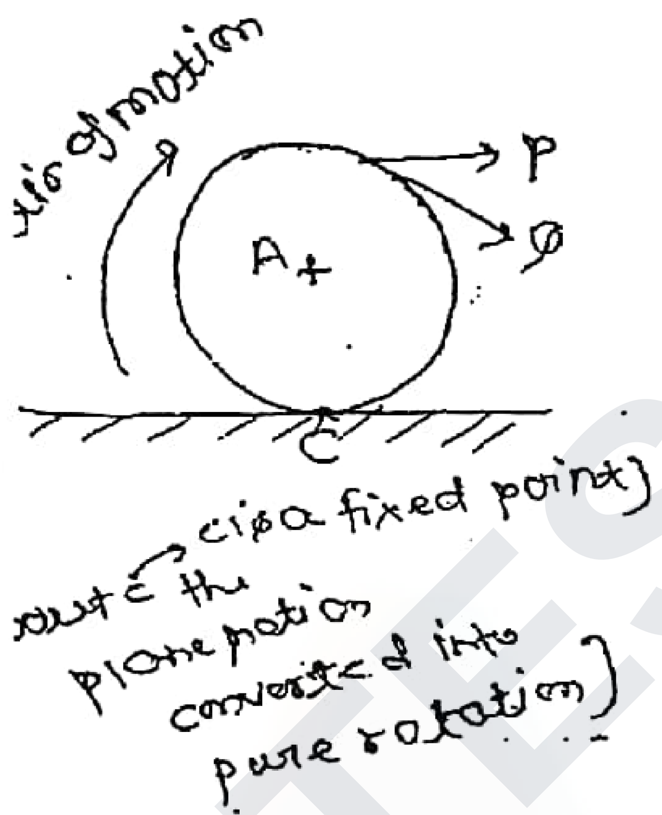
$\boxed{H_A \Rightarrow 0}$

$-mg + V_A \Rightarrow 0$

$\boxed{V_A = mg}$

V topic → Plane Motion → Kinetics →

Concepts-1 → kinetic equation for plane motion →



$\Sigma X = \frac{W}{g} \times a_A$ —— ①

[→ ←]

$\Sigma Y = 0$ (because vertical dir equ.) = 0 —— ②

$\boxed{\Sigma M_A = I_A \cdot \alpha}$ —— ③ Kinetic Equ. for plane motion.

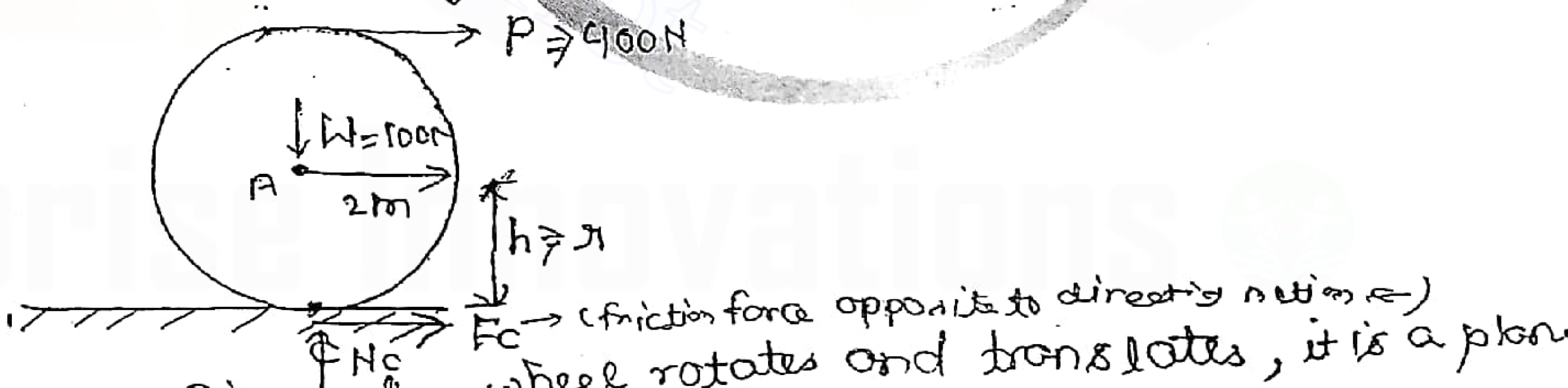
I_A = M.M.I. about the axis of rotation passing through A.

Note ⇒ in plane motion problem ~~can be treated~~ as a pure rotation problem w.r.to instant center C. then kinetic equ. for plane motion can be written as ~~ΣMc~~ $\boxed{\Sigma M_C \Rightarrow I_C \alpha}$

{ where I_C ⇒ M.M.I. w.r.to 'C' ⇒ $I_A + mh^2 \Rightarrow \frac{mr^2}{2} + mr^2 \Rightarrow \frac{3}{2}mr^2$
axis passing through 'C' $\Rightarrow 3/2\, mr^2$

I_A = M.M.I. about A ⇒ $\left(\frac{mr^2}{2}\right)$

Que. A wheel of Radius 2m and weight 100 N is subjected to a horizontal force P as shown in fig. angular acceler. of the wheel at the given instant is



Note ⇒ Since the wheel rotates and translates, it is a plane motion problem. it can be treated as pure rotation problem w.r.to instant center C.

$\boxed{\Sigma M_C \Rightarrow I_C \cdot \alpha}$

$(400 \times 4) + (100 \times 0) = \left(\frac{3 \omega r^2}{2g}\right) \times \alpha$

$1600 \Rightarrow \frac{3\times100\times2^2}{2\times9.81}\times\alpha$

$\boxed{\alpha \Rightarrow 26.16\ \text{Rad/sec}^2}$

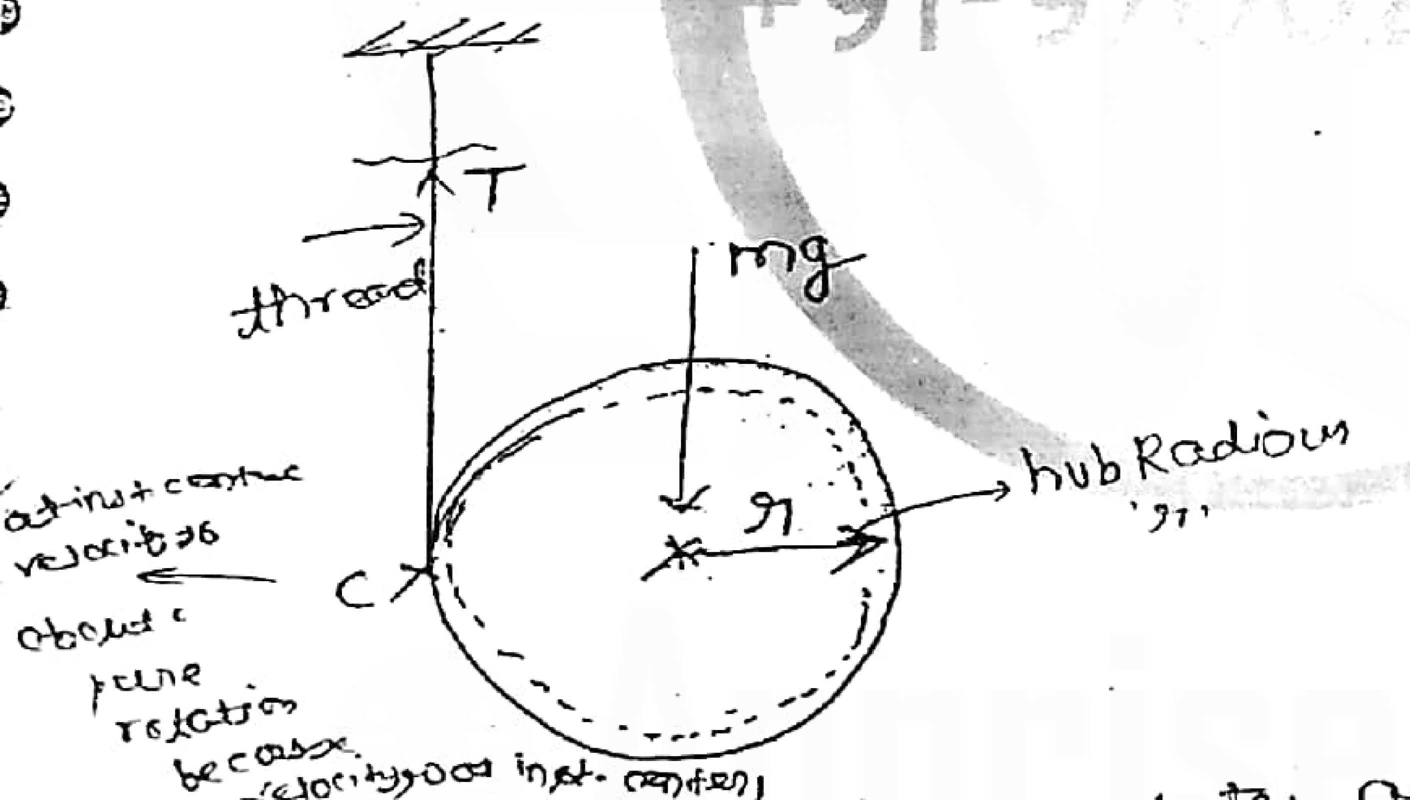
Que→ the frictional force developed at the contact surface so that it rolls freely.

$\Sigma M_A = I_A \cdot \alpha$

$400\times2-(F_c\times2) = \left(\frac{100}{g}\times\frac{2^2}{2}\right)\times26.16$

$\boxed{F_c \Rightarrow 133.03\,N}$ Ans

Que→ A wheel of mass 'm' and radius of gyration 'k' is rolling down smoothly from rest with one end of the thread bound on it held in the ceiling as shown in fig. ~~constant~~ consider the thickness of thread, and its mass negligible in comparison with radius r of the hub and mass 'm'. the linear acceleration of the wheel 'C'.



Note→ Since the wheel rotates and translates, it is a plane motion problem, it can be treated as a pure rotation problem w.r.to instant center 'C'.

$\Sigma M_c \Rightarrow I_c \cdot \alpha$

$(T\times0)+(mg\times r) = (mk^2+mr^2)\alpha$

$I_c \Rightarrow I_{cm}+mh^2 \Rightarrow mk^2+mr^2$

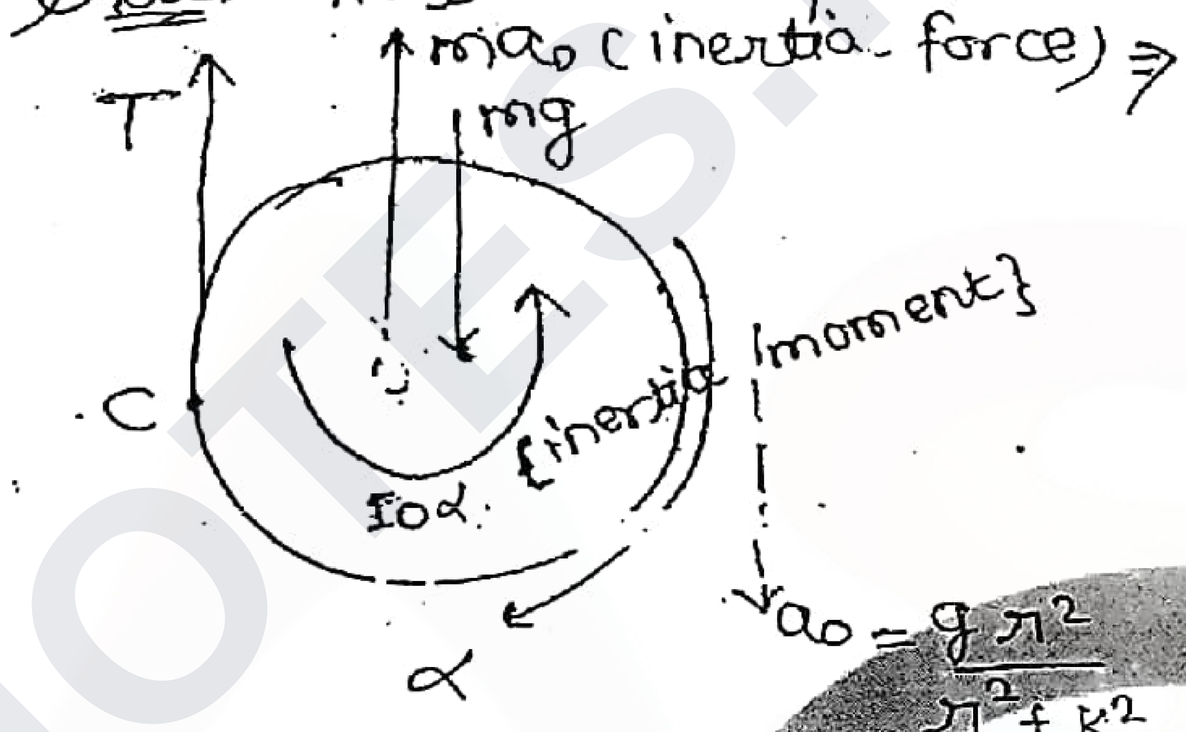
{$h \Rightarrow \perp$ dist btw c.G and inst center}

$\alpha \Rightarrow \frac{g r}{(r^2+k^2)} \Rightarrow$ Angular accl of wheel.

Linear acceleration of wheel $= a_0 \Rightarrow r_0\alpha$ {r_0 = Dist btw instant centre & 'O' = r}

$= a_0 = \left(\frac{r\times g\times r}{r^2+k^2}\right) \Rightarrow \boxed{\frac{g r^2}{k^2+r^2}}$ **

Que→ in the above problem tension in the thread is ⇒?



cylinder is in dynamic equilibrium

[↑ +ve ↓ −ve] ~~T=mg~~

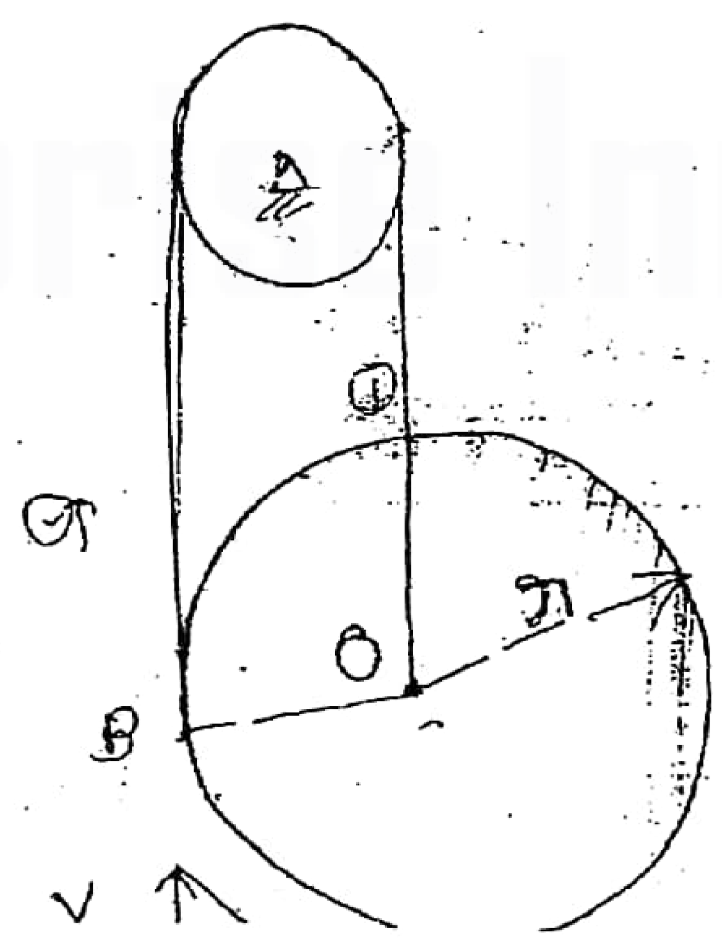
$T + ma_0 - mg = 0$

$T = mg - m\times\frac{g r^2}{(r^2+k^2)}$

$T \Rightarrow \left[\frac{mg(r^2+k^2)-mg r^2}{r^2+k^2}\right]$

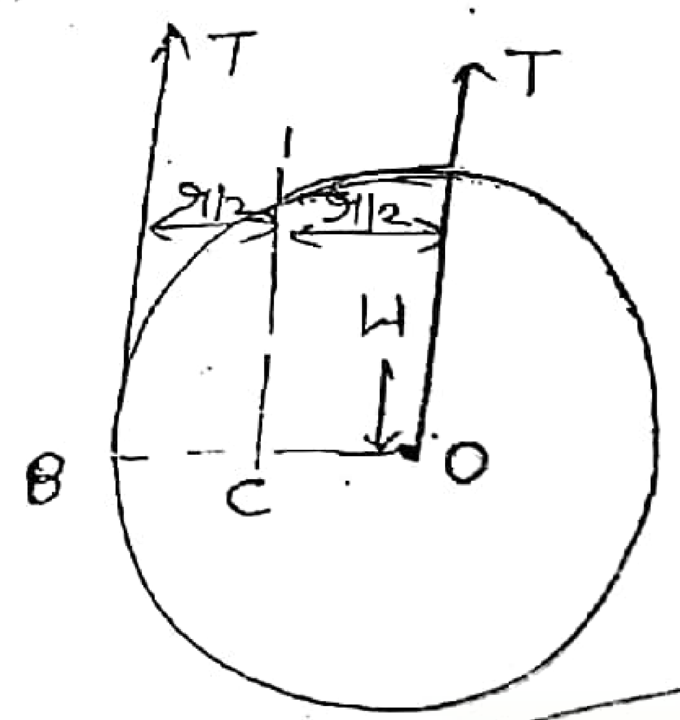
$\boxed{T \Rightarrow \frac{mg k^2}{r^2+k^2}}$ **

Que→ A cylinder of radius r and weight 'w' is connected as shown in fig. at the given instant, instant center is located at



c is located ~~btw~~ midway btw B and O as shown in fig.

Que: In the above problem the linear accel^n of mass center 'O' is = ?



w.r.to instant center 'c', plane motion can be treated as plane rotation problem.

$\Sigma M_c = I_c \cdot \alpha$

$T \times r/2 - T \times r/2 + W \times r/2 = I_c \alpha$ $\quad \frac{3wr^2}{4g}$

$\boxed{\alpha \Rightarrow \frac{2g}{3r}}$

$I_c = I_{CG} + mh^2$

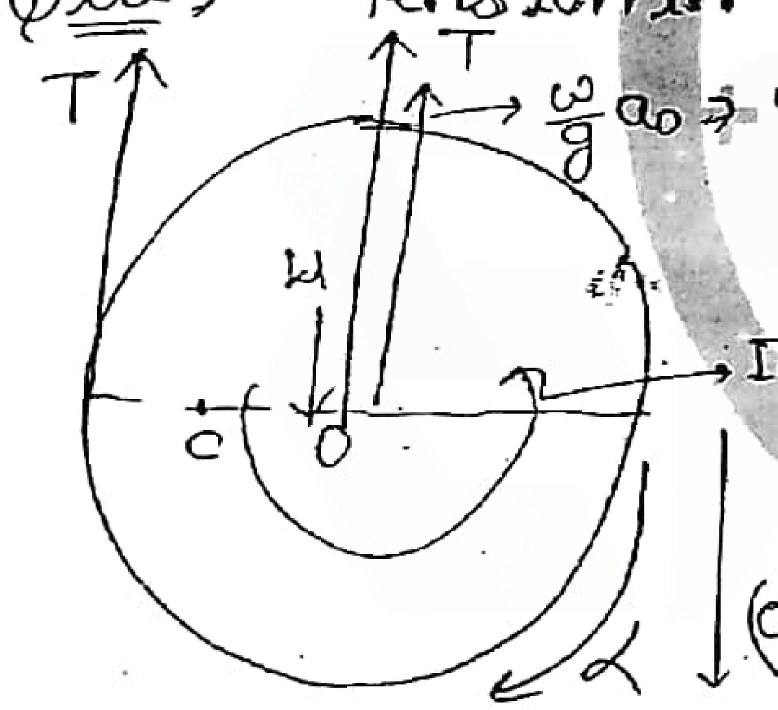
$= \left(\frac{w}{g} \times \frac{r^2}{2}\right) + \frac{w}{g}(r/2)^2$

$\Rightarrow \left(\frac{wr^2}{2g} + \frac{wr^2}{4g}\right) \Rightarrow \left(\frac{3wr^2}{4g}\right)$

Linear acceleration of 'O', $\Rightarrow a_0 \Rightarrow r_0 \alpha$ { r_0 - Dist btw instant centre at cond 'O' $= r/2$)

$\boxed{a_0 \Rightarrow r/2 \left(\frac{2g}{3r}\right) \Rightarrow g/3}$ ***

Que: Tension in the thread is ⇒ ?



$\frac{w}{g} a_0 \Rightarrow \frac{w}{g}(g/3)$ → inertia force.

$I\alpha$ (inertia moment) to keep the body in equi so apply $\Sigma y = 0$

$(a_0 \Rightarrow g/3)$

Cylinder is in dynamic Equil^n

$\Sigma F_y = 0 \Rightarrow T + T - W + \frac{w}{g} a_0 = 0$
[↑ +ve ↓(-)]

$2T = W - \frac{w}{g} \times g/3$

$\boxed{T \Rightarrow w/3}$ **

← Work - Energy Method → (No accel is involved) so apply this

concept ⇒ 1) Work Energy Equation - Rectilinear motion

$R = \frac{w}{g} \times a$

Multiply both sides by 'ds'

$R \cdot ds \Rightarrow \left(\frac{w}{g} \cdot a \cdot ds\right)$ $\{a \cdot ds = v \cdot dv\}$

$\int R \cdot ds = \int \frac{w}{g} \times a \cdot ds = \int \frac{w}{g} \cdot v \cdot dv$

$= \int \frac{w}{g} v \cdot dv \Rightarrow \frac{w}{g} \left|\frac{v^2}{2}\right|_{v_0}^{v}$

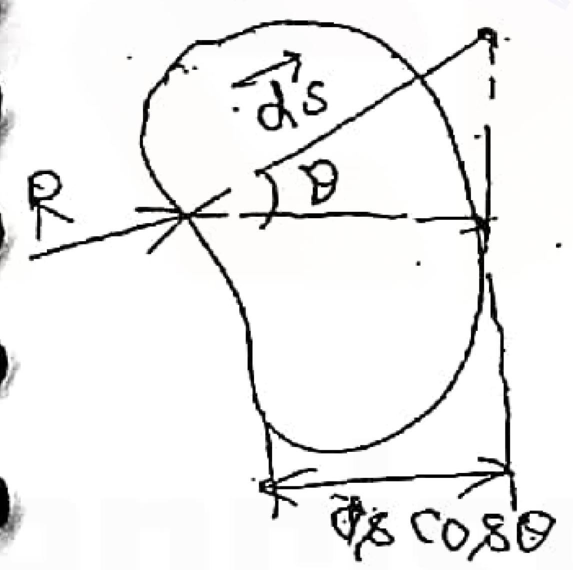
$\int R \cdot ds \Rightarrow \frac{w}{2g}(v^2 - v_0^2)$ → it is called Resultant workdone (R.W)

$\boxed{R.W \Rightarrow \left(\frac{w}{2g} \times (v^2 - v_0^2)\right)}$ *** (change in kinetic energy)

Result workdone

$\boxed{\text{Result. workdone} \Rightarrow \frac{w}{2g} \times (v^2 - v_0^2)}$ *** W.E. Eq^n for Rectilinear motion.

Note → Work done & kinetic energy are scalar quantities.



Work done ⇒ $R \cdot ds \cdot \cos\theta = \vec{R} \cdot \vec{ds}$

{ Dot product of two vectors is a scalar quantity

Note: if force, velocity & displacement are involved in a problem (without accelerations) then it means that it is a work - energy problem.

Concept② K.E. Equation — ← Curvilinear motion →

Since curvilinear motion is summation of two rectilinear motion, work energy equation can be written as →

$$R.W \Rightarrow \underbrace{\frac{W}{2g}\left(V_x^2 - V_{0x}^2\right)}_{\text{change in K.E in x direction}} + \underbrace{\frac{W}{2g}\left(V_y^2 - V_{0y}^2\right)}_{\text{change in K.E in y direction}}$$

Rearranging terms →

$$\boxed{R.W \Rightarrow \frac{W}{2g}\left(V^2 - V_0^2\right)}$$

$V^2 = V_x^2 + V_y^2 \rightarrow \quad V_0^2 = V_{0x}^2 + V_{0y}^2$

Concept 3 . Work energy - Rotation

$\Sigma M = I\alpha$

Multiply both sides by "dθ"

$(\Sigma M)\,d\theta = I\alpha\,d\theta \quad \{\alpha\,d\theta \Rightarrow \omega\,d\omega\}$

$$\int (\Sigma M)\,d\theta = \int_{\omega_0}^{\omega} I\omega\,d\omega \Rightarrow I\left(\frac{\omega^2}{2} - \frac{\omega_0^2}{2}\right)$$

where $\int(\Sigma M).d\theta$ = Resul. work done.

$$\boxed{R.W \rightarrow \frac{I}{2}\left(\omega^2 - \omega_0^2\right)}$$

** learn - w.k. equ for Rotation

where $I \Rightarrow$ M.M.I of the body about the axis of rotation.

Note → the term $\frac{1}{2}I\omega^2$ represents Kinetic energy in Rotation (or) Rotational kinetic energy.

Concept - 4 → K.E. Equation of plane motion

Since the body rotates and translates W.E equation for plane motion can be written as →

$$R.W \Rightarrow \underbrace{\frac{W}{2g}\left(V^2 - V_0^2\right)}_{\text{change in K.E due to transl.}} + \underbrace{\frac{I_0}{2}\left(\omega^2 - \omega_0^2\right)}_{\text{change in K.E due to rotation}}$$

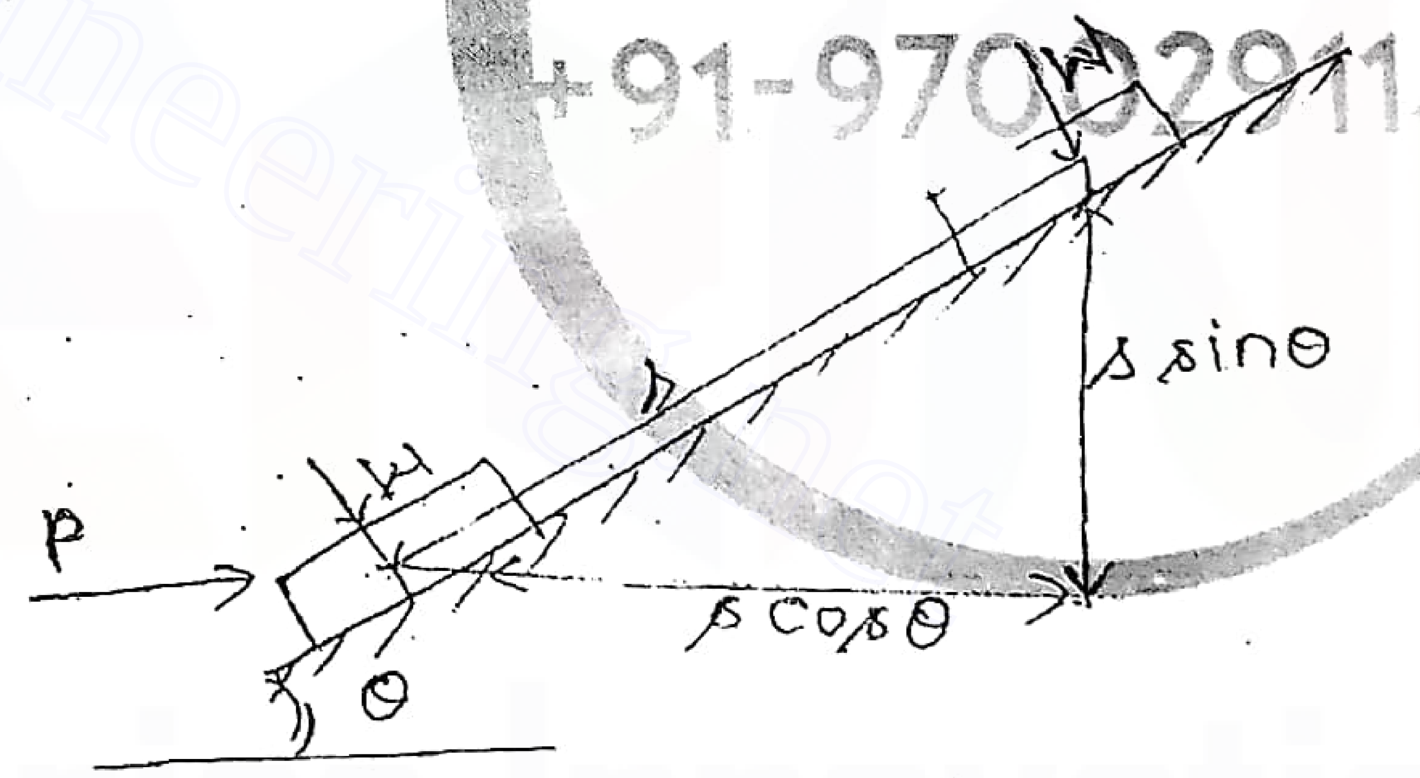
Note → plane motion can be treated as pure rotation w.r.t instant center 'c'. then work energy equation can be written as →

$\boxed{R.W. \Rightarrow \frac{I_c}{2}\left(\omega^2 - \omega_0^2\right)}$ w.E equ for plane motion

$I_c \Rightarrow$ M.M.I. w.r.t instant center 'c'.

Concept - '5' calculation of resultant workdone →

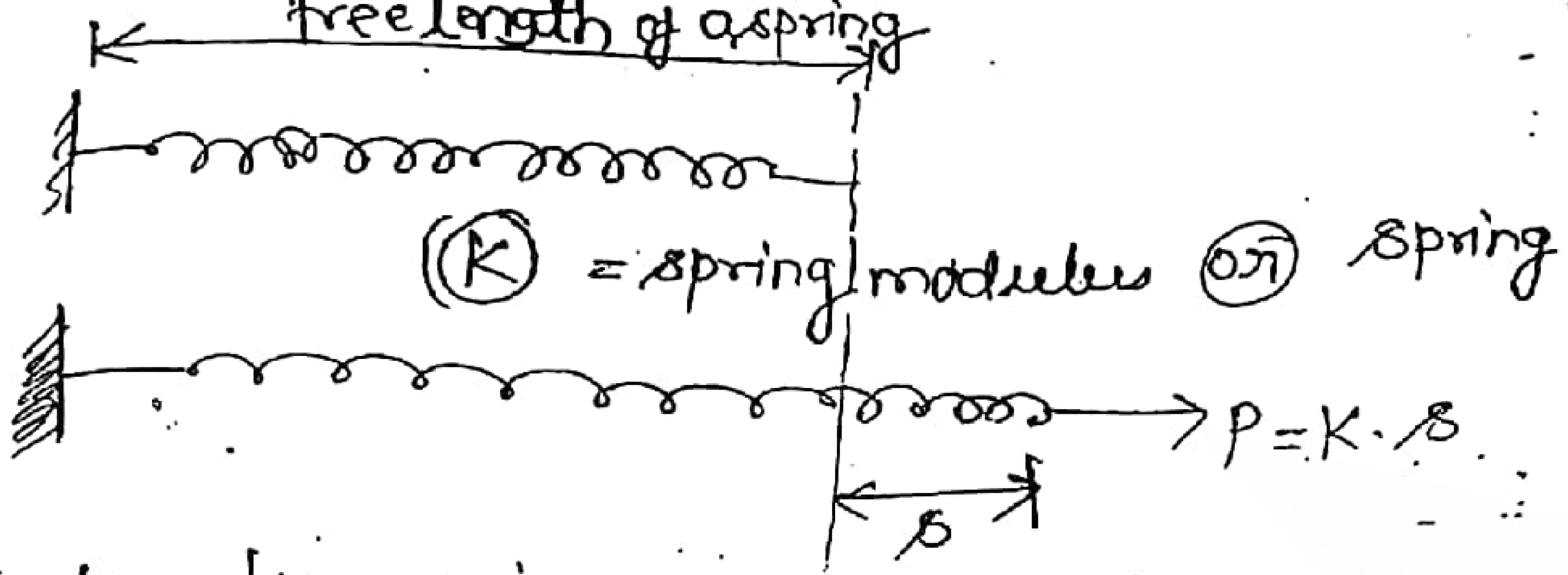
Ex- ①



Work done by $P \Rightarrow P \times s\cos\theta$

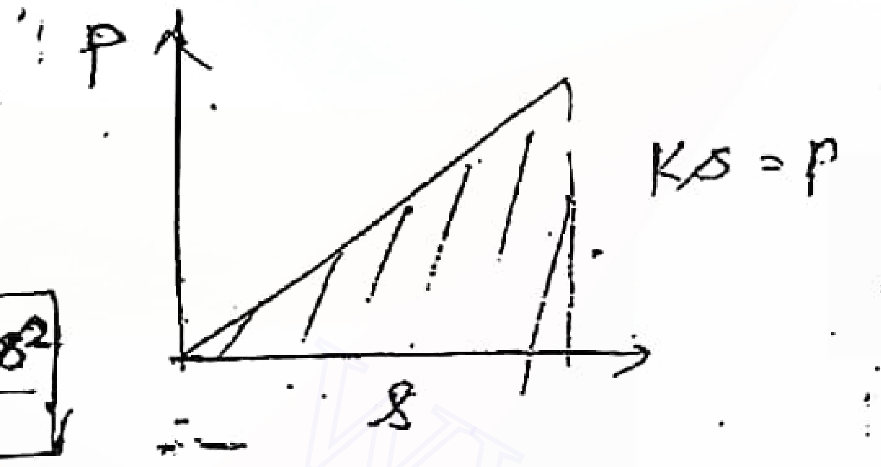
Work done by $W \Rightarrow -W \times s\sin\theta$ {-ve because force & displ. are oppo. to each other}

② Work done by springs



Work done by spring

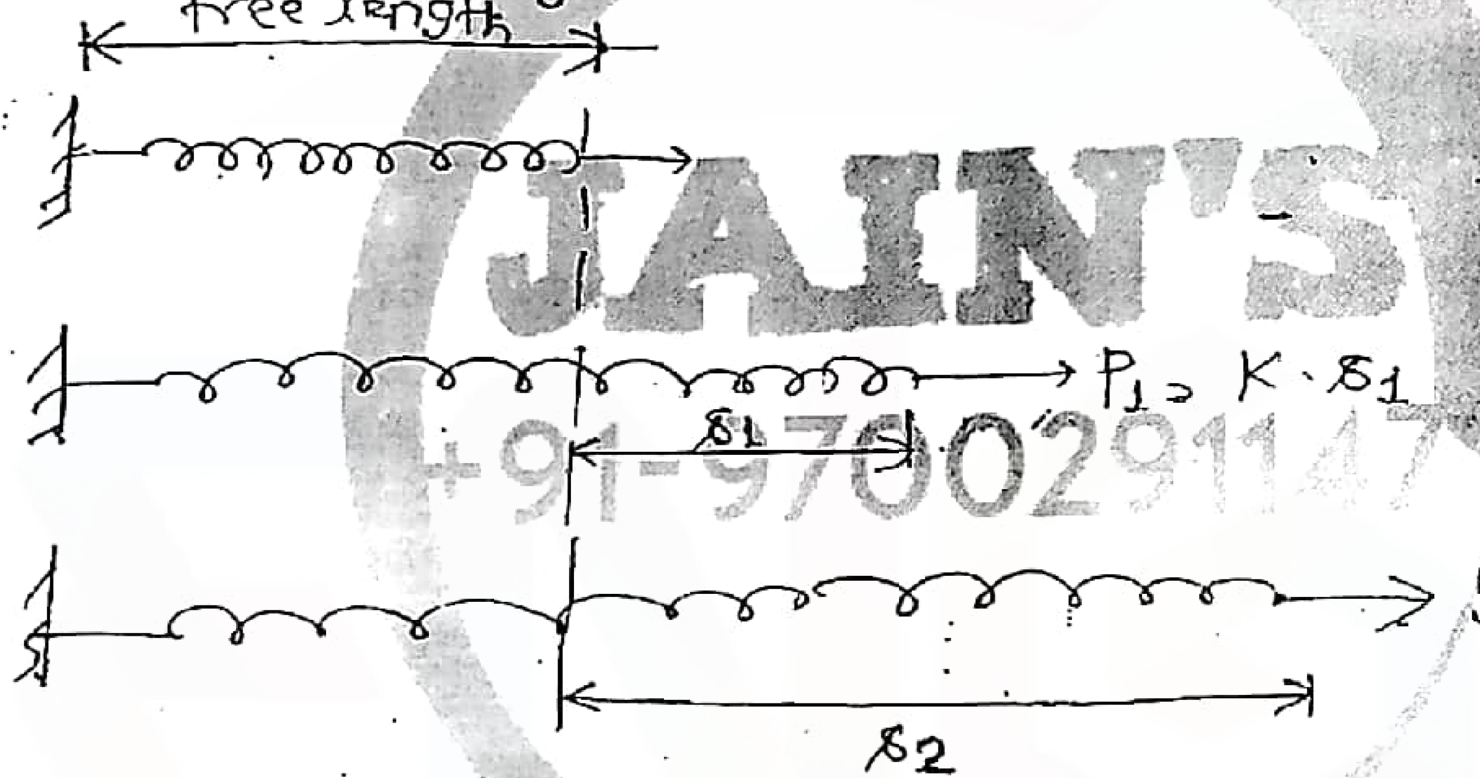
U = Area of Load v/s deflection curve



$= \frac{1}{2}\times P\times\delta \Rightarrow \frac{1}{2}\times\delta\times K\cdot\delta \Rightarrow \boxed{\frac{K\cdot\delta^2}{2}}$

IInd method → $U = \int P\cdot d\delta \Rightarrow \int (K\cdot\delta)\,d\delta \Rightarrow \frac{1}{2}K\delta^2$

(b)

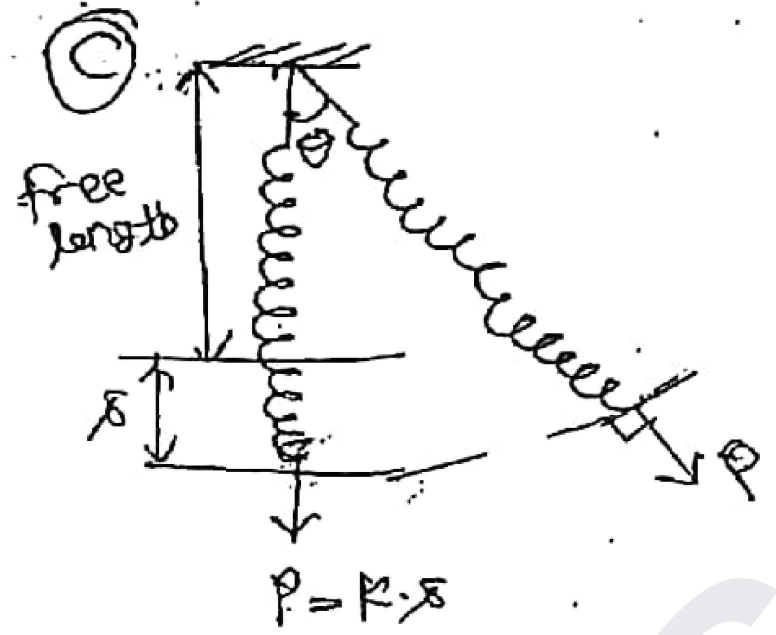


Work done on the spring during elongation from δ_1 to δ_2 only

$$\boxed{U \Rightarrow \left(\frac{1}{2}K\cdot\delta_2^2 - \frac{1}{2}K\cdot\delta_1^2\right)}$$

δ_1 & δ_2 are measured from ~~force~~ free length of spring.

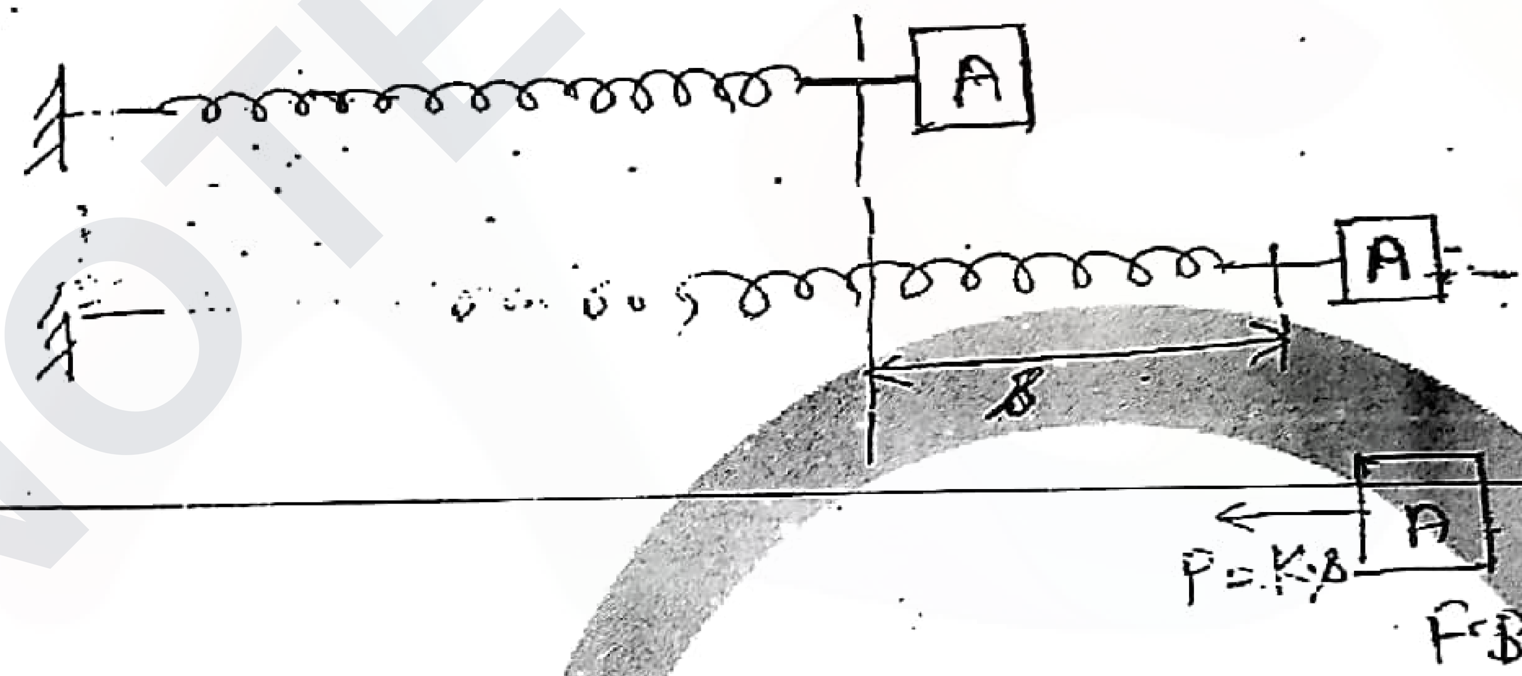
(c)



Work done by the spring during rotation is zero. (because force and displacement are always ⊥ to each other) at any instant as shown in fig.

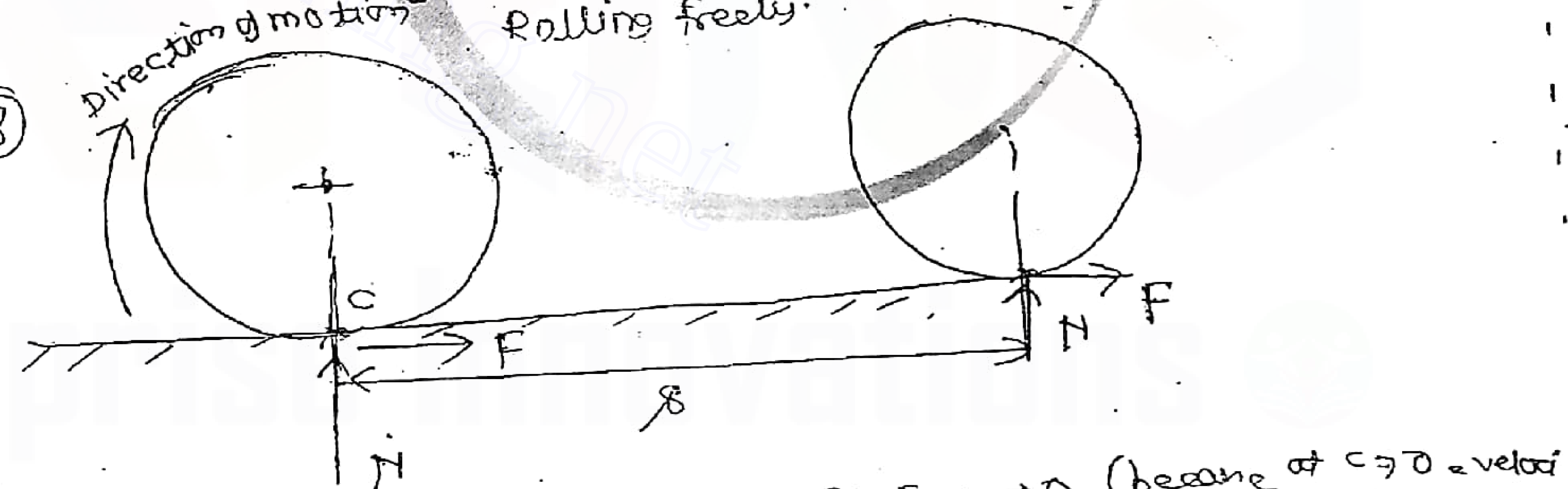
→ Work will be done by the spring only when it is pulled or push.

(d)



Note → if the motion of the block stretches a spring, then spring will do negative work on the block. (because spring force and displ. act in opposite directions as shown in fig.)

③

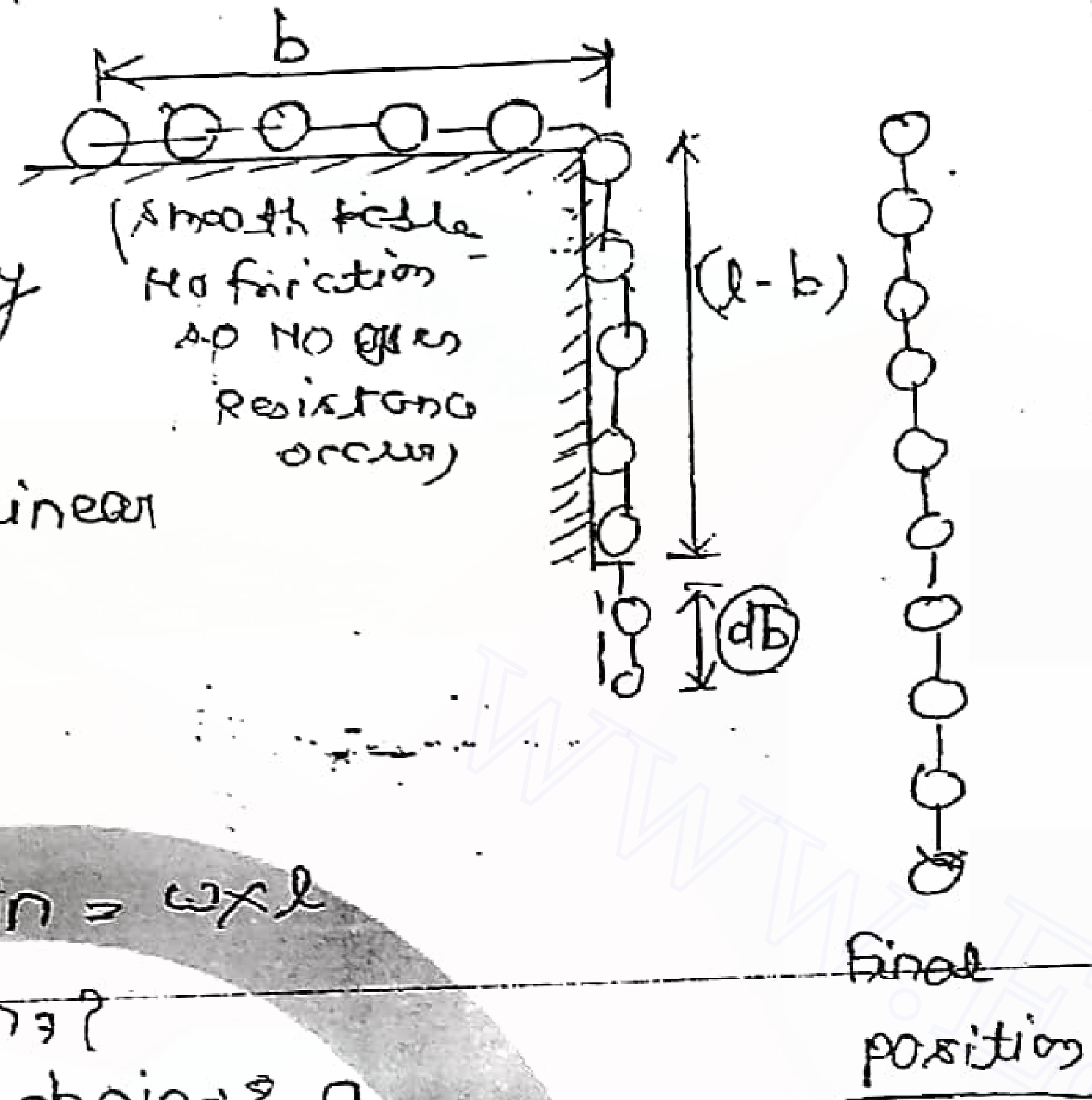


Work done by F is zero ⇒ $F = F\times 0 \Rightarrow 0$ (because at $C \Rightarrow 0$ = velocity always)

if a wheel is rolling freely, then work done by the frictional force is always zero. (because the contact point 'C' is called instant center which is always at given instant)

Ques. A chain of length l and weight w N/m. is ~~dis~~placed on a smooth table as shown in fig. the velocity of the chain as the last link leaves the table is ⇒ ?

Note → Since force displacement, velocity are involved in the problem, it is a Work energy Rectilinear motion problem.



Solution → W.E. Equ. for Rectilinear motion is →

$$\boxed{R.W. \Rightarrow \frac{w}{2g}(V^2 - V_0^2)}$$

W = total weight of chain = w × l

V = final velocity of chain ?

V_0 = initial velocity of chain ⇒ ? 0

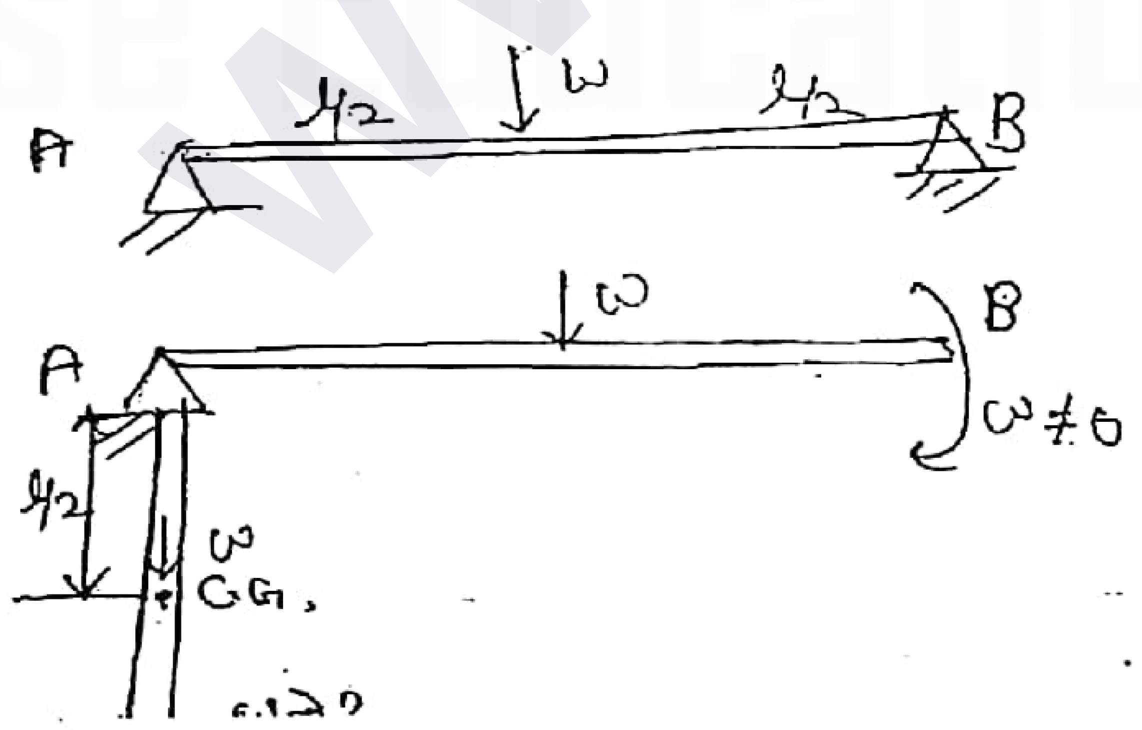
$$R.W. \Rightarrow \text{Resul. workdone} \Rightarrow \int_0^b w(l-b)\,db \Rightarrow \left(wlb - \frac{wb^2}{2}\right)$$

$$\Rightarrow wlb - \frac{wb^2}{2} = \frac{wl}{2g}(V^2 - 0^2)$$

$$V^2 = \frac{(2lb - b^2)g}{l} \Rightarrow \boxed{V = \sqrt{\frac{(2lb - b^2)\cdot g}{l}}}$$

Que. A bar of length is hinged at both ends. if one of the support is removed then its angular velocity after 90 rotation is ⇒ ?

Note → Since the bar rotates after the support is removed and the accl. are not involved in the problem, it is work energy rotation problem.



Note → $R.W. \Rightarrow \frac{I_A}{2}(\omega^2 - \omega_0^2)$

$\omega_0 \Rightarrow$ Initial angular velocity ⇒ 0.

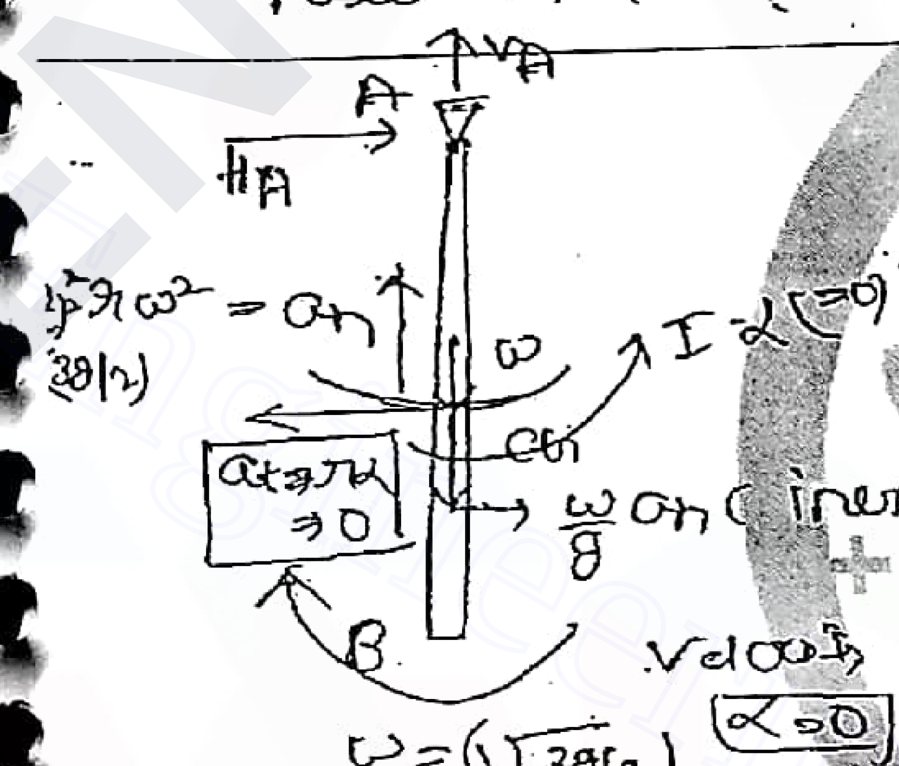
$\omega \Rightarrow$ Final angular velocity ⇒ ?

$I_A \Rightarrow$ M.M.I. of the rod about the axis of rotation passing through A $= \left(\frac{ml^2}{3}\right) \Rightarrow \left(\frac{wl^2}{3g}\right)$

R.W. ⇒ Resultant workdone ⇒ $\underline{wl/2}$

$$\Rightarrow w \times l/2 \Rightarrow \frac{\frac{wl^2}{3g}}{2}\{\omega^2 - 0^2\} \Rightarrow \omega^2 = 3g/l \qquad \boxed{\omega \Rightarrow \sqrt{\frac{3g}{l}}}$$

Que → in the above problem, Rxn at A is ⇒ ? after 90 rotation is ⇒ ?



$\Sigma y = 0$

$$V_A - w - \frac{w}{g} \times a_n = 0$$

$$V_A \Rightarrow \left(w + \frac{w}{g} \times 3g/2\right)$$

$$\boxed{V_A \Rightarrow 5w/2}$$

$\Sigma X = 0 \quad \boxed{H_A = 0}$

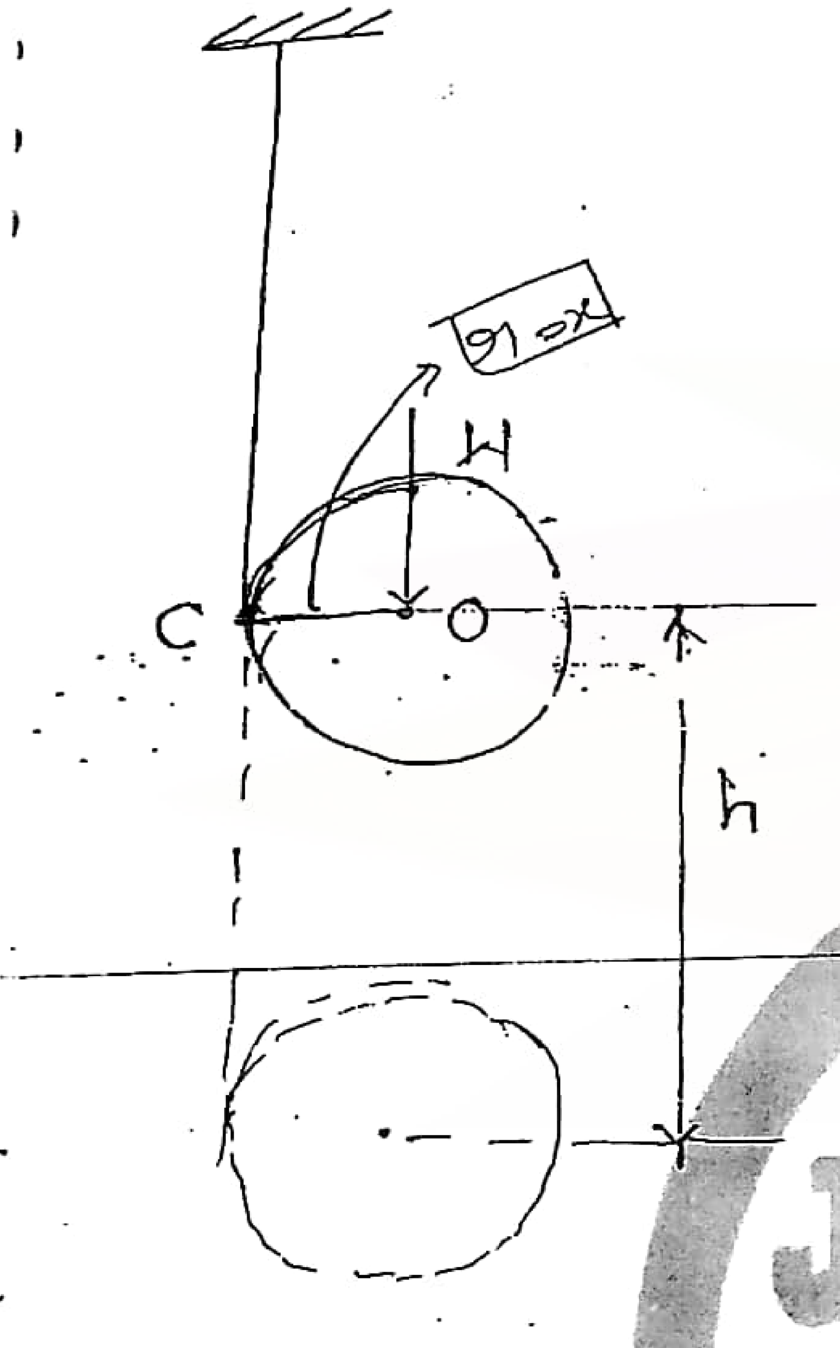
So Reaction $\boxed{V_A = 5w/2}$

$\Sigma y = 0$

Note → after 90 rotation, angular velocity of the bar is max. it means that $\boxed{\alpha = \frac{d\omega}{dt} \Rightarrow 0}$ when the bar is in vertical position.

2) Since acceleration is known, keep the bar in dynamic equilibrium, to find the unknowns quickly.

Que→ A rope is wound round a cylinder and connected to the ceiling as shown in fig. the linear velocity of the cylinder after it has dropped by h meters is=?



Note - since the cylinder rotates and translate and acceleration are not involved, it is a work energy plane motion problem.

(2) A plane motion problem can be treated as a pure rotation problem w.r.to instant centre 'C.'

$$\boxed{R.W \Rightarrow \frac{I_c(\omega^2-\omega_0^2)}{2}}$$

$\boxed{\omega_0=0}$ initial angular velocity

$\omega=?$

$I_c = I_{cm} + mx^2$ { $\perp$ dist btw c.g and inst center C. }

$$= \frac{\omega}{g}\times\frac{r^2}{2} + \frac{\omega}{g}\times r^2 \Rightarrow \left(\frac{3\omega r^2}{2g}\right)$$

Resultant workdone $= \omega\times h$

substituting $\omega\times h = \frac{3\omega r^2}{2g}\times(\omega^2-0)$

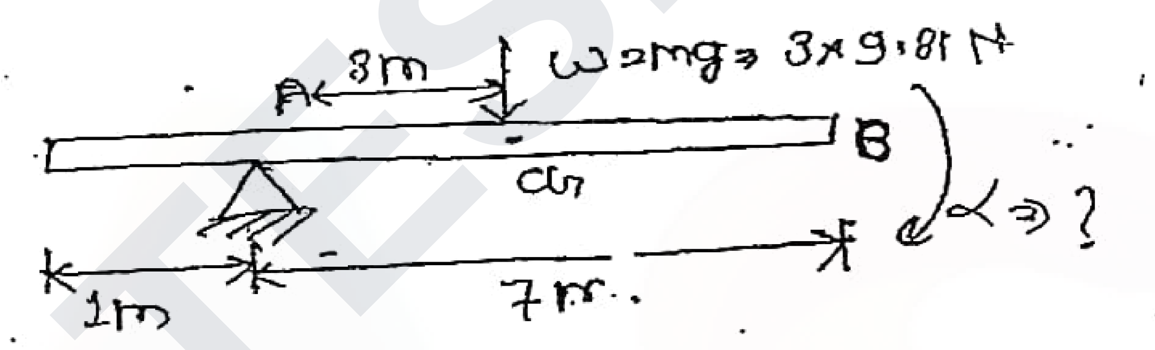
$$\omega^2 = \frac{4gh}{3r^2} \Rightarrow \omega = \frac{2}{r}\sqrt{\frac{gh}{3}}$$

Linear velocity $V = r_0\omega$

$$V = r\times\frac{2}{r}\times\sqrt{\frac{gh}{3}} \Rightarrow \boxed{2\sqrt{\frac{gh}{3}}}$$

$r_0 \Rightarrow$ C and O distance (r)

Que A uniform slender rod 8m length and 3 kg mass rotate in a vertical plane about a horizontal axis 1m from its end as shown in fig. the magnitude of the angular accel. of the rod at the position shown in fig=?



$\Sigma M_A = I_A\alpha$

$(3\times9.81) \Rightarrow 43\times\alpha$

$$\alpha \Rightarrow \left(\frac{3\times9.81\times3}{43}\right)$$

$\boxed{\alpha \Rightarrow 2.053\ rad/sec^2}$ Ans

$I_A = I_{cm} + mh^2$

$$\Rightarrow \frac{ml^2}{12} + m\times3^2$$

$$\Rightarrow\left[\frac{3\times8^2}{12} + 3\times3^2\right] kg\text{-}m^2$$

$\Rightarrow 43\ kg\text{-}m^2$

Only velocity - time) Impulse - Momentum method

Concept (1) → Impulse momentum Equ - Rectilinear motion

$$R = \frac{\omega}{g}\times a$$

$$R = \frac{\omega}{g}\cdot\frac{dv}{dt}$$

$$\int R.dt = \int\frac{\omega}{g}dv$$

where $\int R.dt \Rightarrow$ linear impulse

$\int_{V_0}^{V}\frac{\omega}{g}.dv \Rightarrow \frac{\omega}{g}(V-V_0) \Rightarrow$ change in linear momentum.

$\boxed{\text{impulse} \Rightarrow \frac{\omega}{g}(V-V_0)}$ ** → impulse momentum equ

Note → impulse and momentum are vector quantities.

~~2) Impulse &~~

← Impulse momentum Equ. for curvilinear motion →

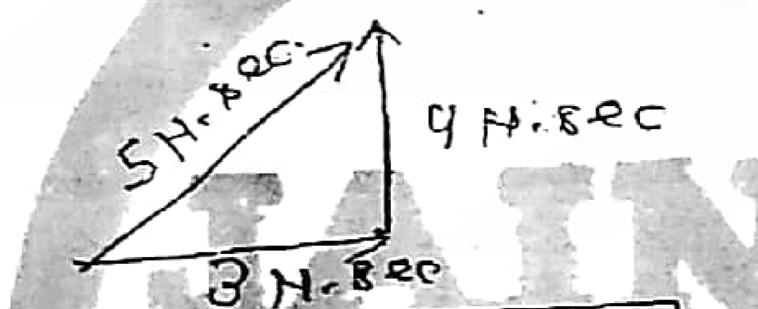
Since curvilinear motion is summation of two Rectilinear motions, impulse momentum Equ. are given by.

$$\int (\Sigma X)\, dt = \frac{w}{g}(V_x - V_{0x})$$

$$\int \Sigma Y \cdot dt = \frac{w}{g}(V_y - V_{0y})$$

Que → if impulse in x dir is 3 N-s and y dir is 4 N-s

Resultant impulse is = ?



Result. impulse ⇒ 5 N-sec

3) Impulse - momentum Equ. - Rotation →

$\Sigma M = I \cdot \alpha$

$\Sigma M = I \cdot \frac{d\omega}{dt}$

$\int \Sigma M \cdot dt = \int I \cdot d\omega$ is called angular impulse.

where $\int \Sigma M \cdot dt$ is called angular impulse.

$$\boxed{\int_{\omega_0}^{\omega} I \cdot d\omega = I(\omega - \omega_0)}$$ ⇒ change in angular momentum

Note → the term $I\omega$ is called angular momentum.

$\int \Sigma M \cdot dt$ = moment of linear momentum
$(F\, dt)$

angular momentum is also called moment of linear momentum.

4) I. M. Equ. → plane motion

→ plane motion can be treated as pure rotation w.r.to instant center.

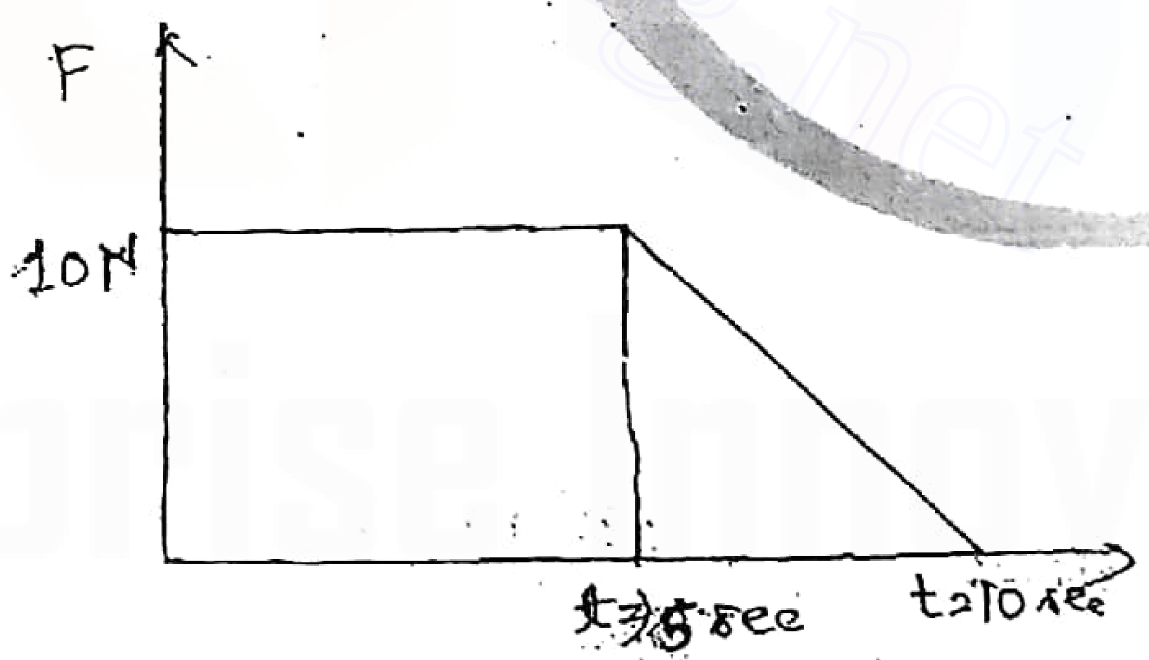
$$\boxed{\int (\Sigma M_c) \cdot dt = I_c(\omega - \omega_0)}$$ **

I. M. Equ for plane motion.

ΣM_c = moments of all forces about instant center

I_c = M.M.I w.r.to inst center

Que. A block of mass 1 kg moving with a velocity of 2 m/sec. is subjected to a force F whose variation with time as shown in fig.? the velocity of the block after 10 sec is?



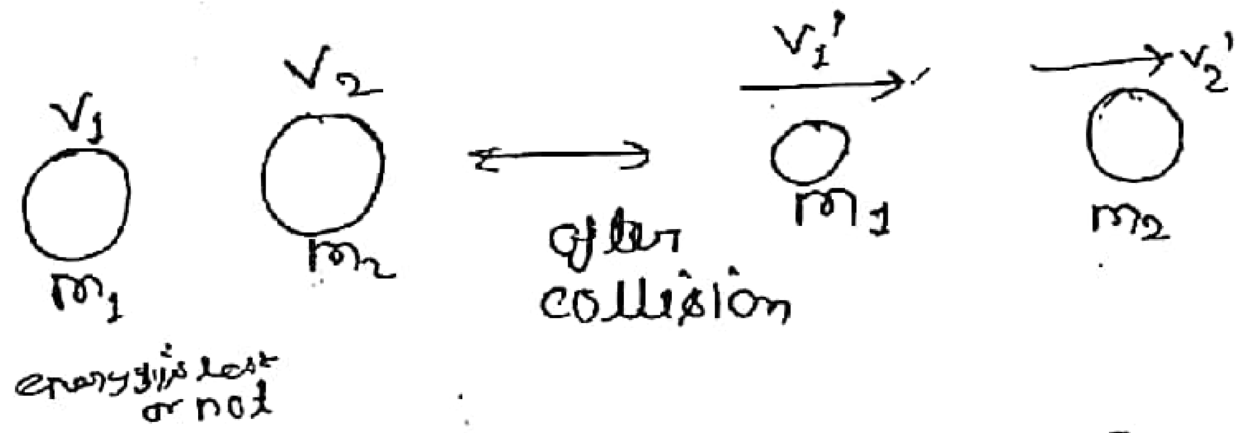
$\int (R)\, dt = \frac{w}{g}(V - V_0)$ → 1 kg mass

Area of F-t curve = $\frac{w}{g}(V - V_0)$

$10 \times 5 + \left(\frac{1}{2} \times 5 \times 10\right) = 1 \times (V - 2)$

Concept-5 → Law of conservation of linear momentum

{ applicable on whether energy is lost or not during impact)



$$\boxed{m_1 V_1 + m_2 V_2 \Rightarrow m_1 V_1' + m_2 V_2'}$$

Linear momentum before impact ⟷ momentum after impact}

Concepts-6 Law of conservation of angular momentum →

** $\boxed{I_1 \omega_1 + I_2 \cdot \omega_2 \Rightarrow I_1 \omega_1' + I_2 \omega_2'}$ **

$\omega_1, \omega_2 =$ ~~M.M.I~~ Angular velocity of both bodies

$I_1, I_2 =$ M.M.I. of the two bodies.

Concepts-7 Law of conservation of energy (no loss of energy)

(only perfectly elastic bodies)

→ it is applicable only when there is no loss of energy during impact. (perfectly elastic bodies do not loose energy during impact. So law of conservation of energy is applicable to perfectly elastic bodies only. For inelastic impact we can not apply law of conservation of energy.

***

$$\boxed{\tfrac{1}{2} m_1 V_1^2 + \tfrac{1}{2} m_2 V_2^2 \Rightarrow \tfrac{1}{2} m_1 V_1'^2 + \tfrac{1}{2} m_2 V_2'^2}$$

for the impact to happen, V_1 must be greater than V_2 after the impact for the sepration to happen $\boxed{V_2' > V_1'}$ must be

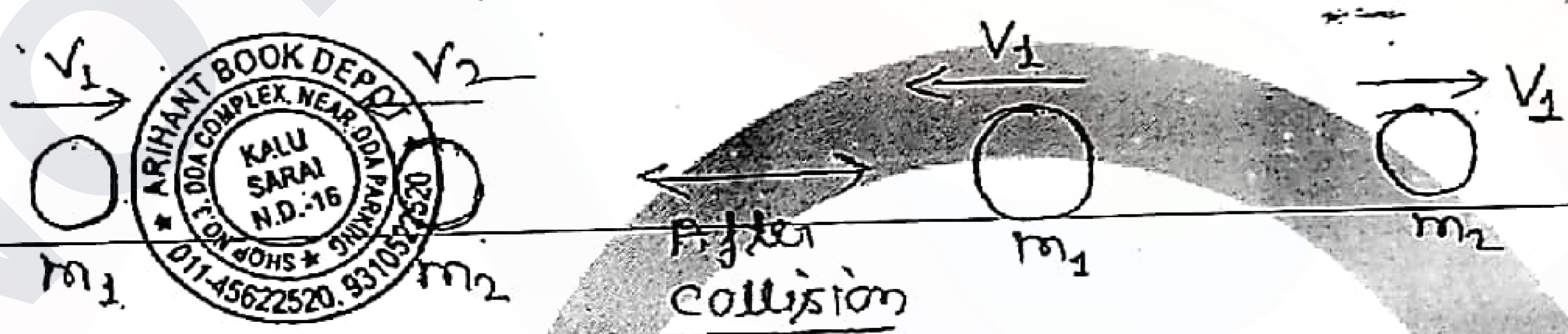
Concept-8 → Impact →

1) Cofficient of Restitution → (e) → Recovery of shape

it is a measure of the elastic properties of a pair of bodies which are under impact.

$$\boxed{e = \frac{\text{Relative velocity after impact}}{\text{Relative velocity before impact}} \Rightarrow \left(\frac{V_2' - V_1'}{V_1 - V_2}\right)}$$

2) perfectly elastic bodies we have some Relative velocity after impact also. (because there is no loss of energy and simply velocity will be exchanged during impact and direction will be some as previous direction.



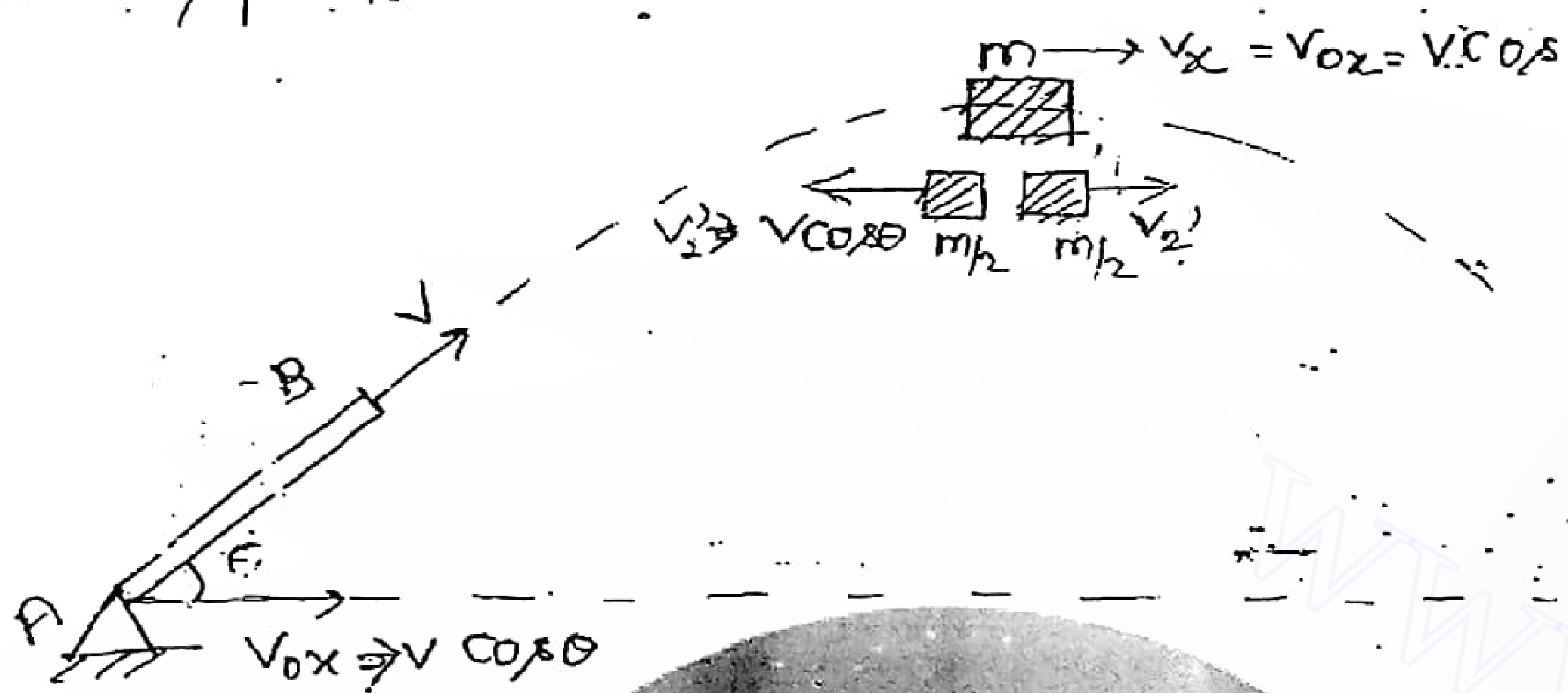
$$\boxed{e = \frac{V_2' - V_1'}{V_1 - V_2} \Rightarrow \left(\frac{V_1 - V_2}{V_1 - V_2}\right) \Rightarrow 1}$$ {for perfectly Elastic bodies}

3) perfectly inelastic bodies (those which stick together and move together} after impact will have some final velocity after impact.

$$e \Rightarrow \frac{V_2' - V_1'}{V_1 - V_2} \Rightarrow \frac{V_2' - V_2'}{V_1 - V_2} \Rightarrow 0$$

$\boxed{\text{Range for } e \text{ is } 0 \text{ to } 1}$

Que → A shell is fired from a cannon with speed 'V' at an angle 'θ' with the horizontal direction. at the highest point in its flight, it explodes into two pieces of equal mass one of the pieces retraces its path to the cannon. the speed of the other piece immediately after explosion is?



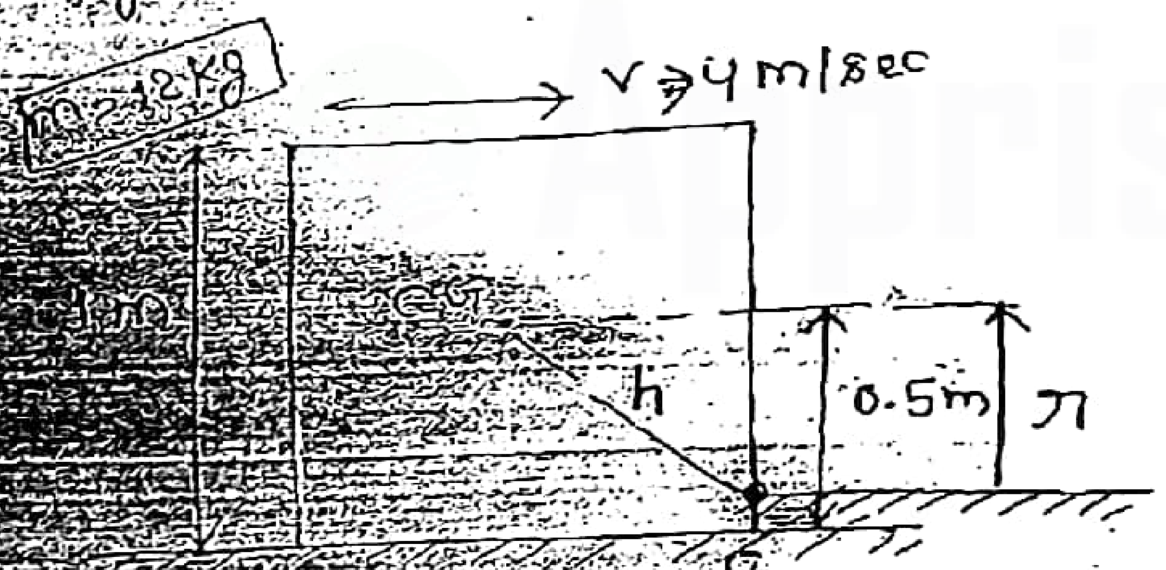
Linear momentum before explosion ⇒ Linear momentum after explosion

$$mV\cos\theta \Rightarrow \left(-\frac{m}{2}V\cos\theta\right) + \frac{m}{2}V_2'$$

$$\frac{m}{2}V_2' \Rightarrow \left(\frac{3mV\cos\theta}{2}\right)$$

$$\boxed{V_2' \Rightarrow 3V\cos\theta}$$ Ans

Que → A box of size 1m×1m×1m and mass 12 kg, moving with the velocity of 4 m/sec strikes and uprises floor tile with a completely inelastic impact. Angular velocity of the box immediately after the box hits the tile? ⇒ ?



Angular momentum before impact = Angular momentum After impact

$$\boxed{(mV)\times r \Rightarrow I_c\cdot\omega \ \{\omega \text{ is angular velocity}\}}$$

I_c ⇒ M.M.I w.r.t axis of rotation at C. ⇒ $I_G + mh^2$

$\{h^2 = 0.5^2 + 0.5^2$
$\Rightarrow 0.5\,m^2$

$$\Rightarrow \frac{md^2}{6} + m(0.5^2+0.5^2)$$

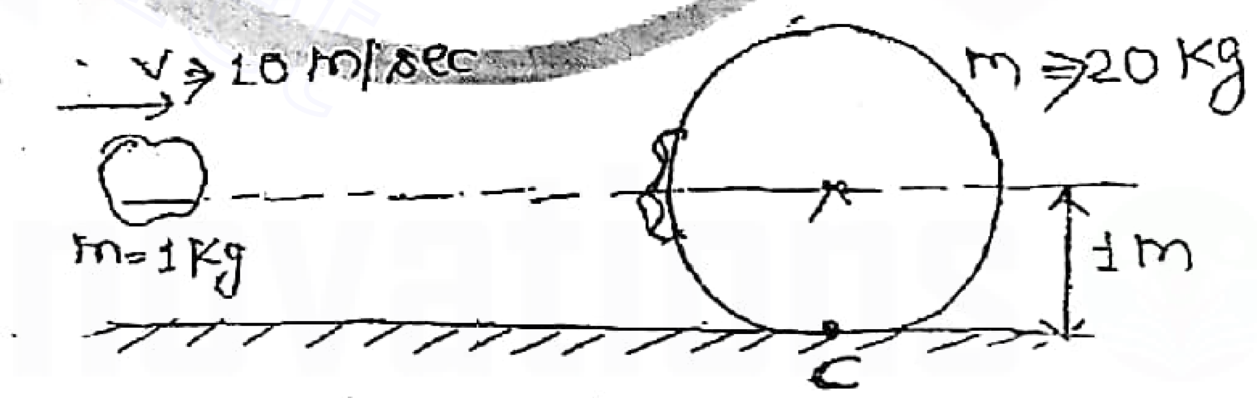
$$\Rightarrow \frac{(12\,kg)\times(1m)^2}{6} + (12\times0.5)$$

$$\boxed{I_c = 8\ kg\cdot m^2}$$

$$\Rightarrow (12)\times4\times(0.5m) \Rightarrow 8\times\omega$$

$$\boxed{\omega = 3\ rad/sec}$$ Ans

Que → A 1 kg mass of clay moving with a velocity of 10 m/sec strikes a stationary wheel and sticks to it. the solid wheel has a mass of 20kg and radious of 1m. assuming that the wheel is set into pure rolling motion, angular velocity of the wheel immediately after impact is approximately →



Note → Since the wheel rotates and translates after impact it is a plane motion problem. it can be treated as a pure rotation problem w.r.t. to instant center C.

clay has linear momentum before impact and the whole has angular momentum after impact. So find moment of linear momentum about c. and equate angular momenta at c after impact.

Angular momentum about c before collision $\Rightarrow$ Angular momentum after impact

$(mv)\,r \Rightarrow I_c \times \omega$

$I_c = I_{cm} + mr^2$ (neglect the I_c of clay small)

$= \frac{mr^2}{2} + mr^2 \Rightarrow \boxed{\frac{3}{2} mr^2} = I_c = \frac{3}{2} \times 20 \times (1)^2$

$= 30\ kg\text{-}m^2$

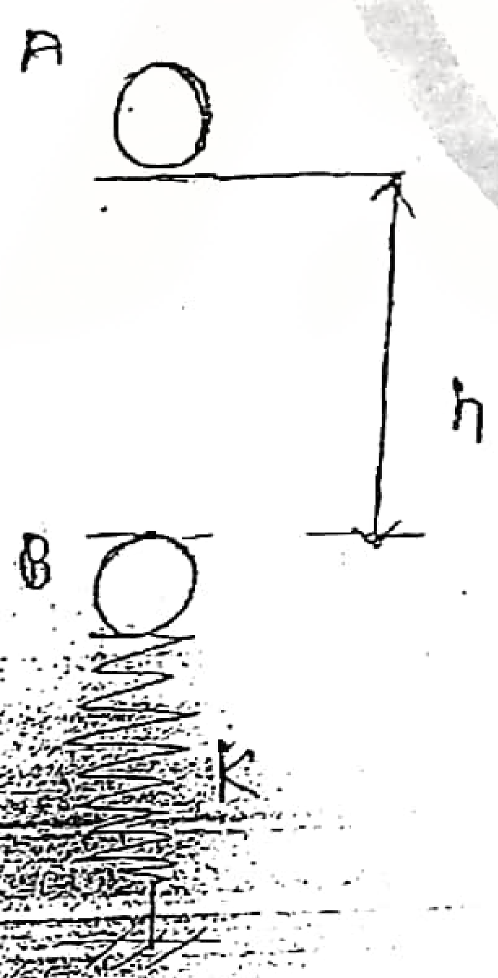
$(1 \times 10 \times 1) \Rightarrow 30 \times \omega$

$\boxed{\omega \Rightarrow 1/3\ rad/sec}$ Ans

Que $\Rightarrow$ A ball 'A', mass m falls under gravity from a height h and strikes another ball 'B' of mass 'm' which is supported at rest on a spring of stiffness K assuming perfectly elastic impact. ?

a) velocity of ball is $\sqrt{\frac{2gh}{2}}$ b) velocity of both ball is $\frac{\sqrt{2gh}}{2}$

c) velocity of ball A is '0' d) none.



$V^2 = \cancel{V_0^2} + 2gs$

$\boxed{V \Rightarrow \sqrt{2gh}}$

Before impact

$V_A \Rightarrow \sqrt{2gh}$

$V_B \Rightarrow 0$

Immediately after impact

$\boxed{V_A \Rightarrow 0}$

$\boxed{V_B \Rightarrow \sqrt{2gh}}$

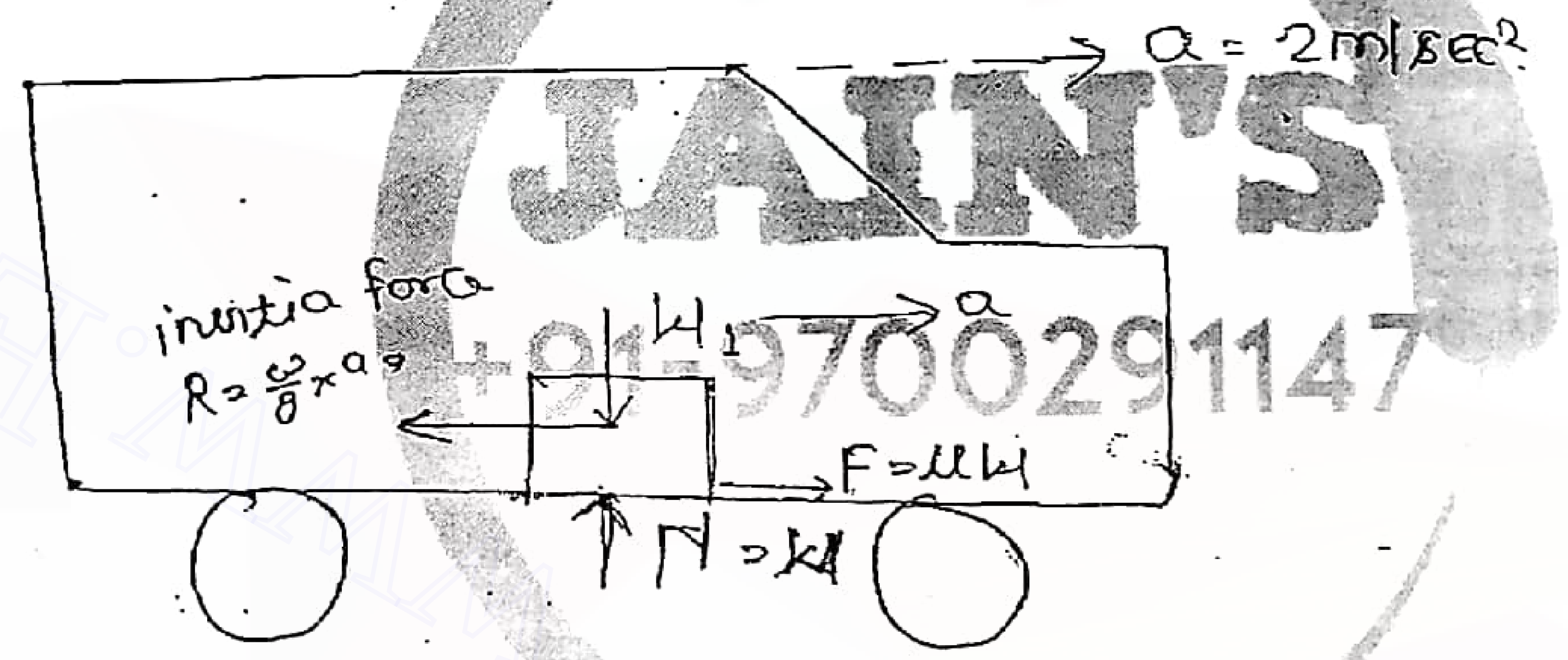
Que two perfectly inelastic sphere of equal masses moving in a same dir with their velocities in the ratio 2:1 have an impact. After the impact they will —

a) move in the same direction with the same velocity ratio 2:1

b) have a velocity ratio 1:2 c) moving the same direction with velocity ratio (1) d) none.

Que $\Rightarrow$ A box rests in a rear of a truck moving with an acceleration of $2\ m/sec^2$. to prevent box from sliding the approximate value of static coefficient of friction b/w the box and the bed of the truck should :

(A) 0 B) 0.2 C) 0.3 D) 0.4



$R = \frac{\omega}{g} \times a \Rightarrow (0.2\omega)$

block in dynamic Equilibrium

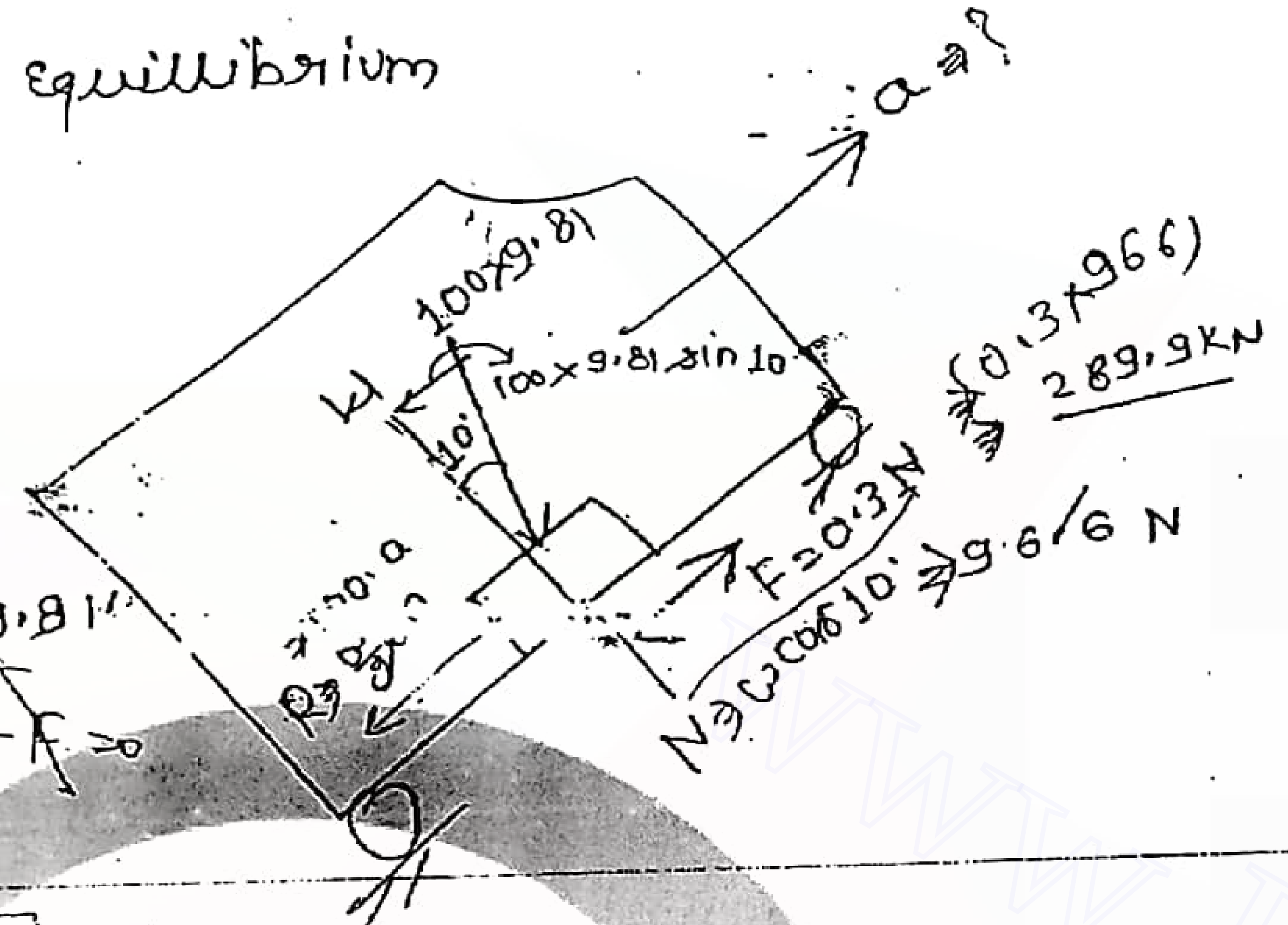
$\Sigma X \Rightarrow 0$ $[\overset{+}{\rightarrow}\ \circlearrowleft]$

$-0.2\omega + F = 0$

$\boxed{F = 0.2\omega = \mu N} \Rightarrow \boxed{\mu = 0.2}$ Ans

Que A truck accelerates up a 10° inclined with a crate of 100 kg. μ btw crate and truck surface is 0.3. max value of acceleration of the truck such that the crate does not slide down is.

Weight is in dynamic equillibrium



$\Sigma X = 0$

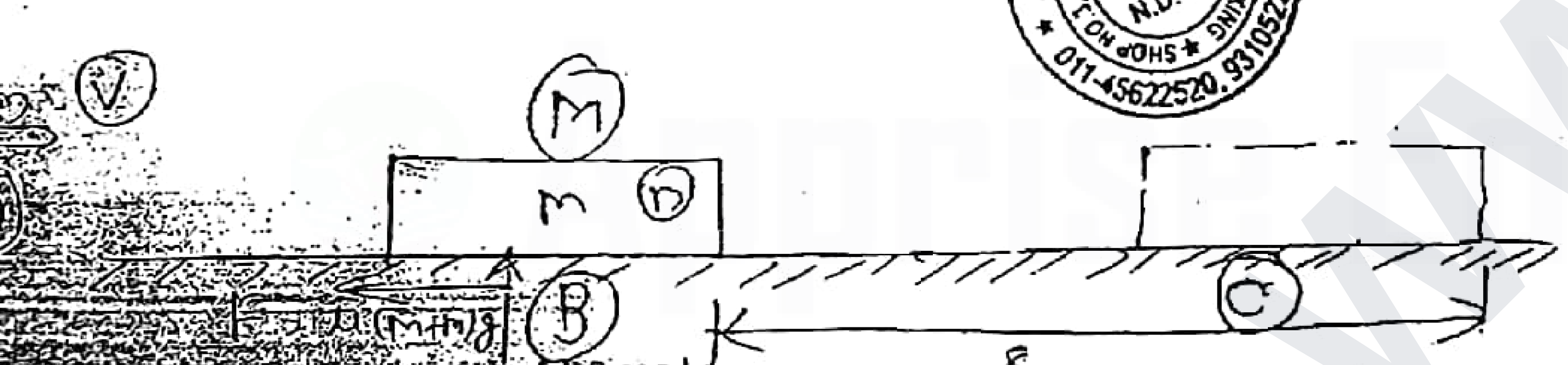
$(\overset{+}{\rightarrow} \leftarrow) \Rightarrow$ ~~100a~~

$-100a - 981 \sin 10 + F = 0$

$F \Rightarrow$

$\boxed{a \Rightarrow 1.19 \ m/sec^2}$

Que- A bullet of mass m travels at a very high velocity 'V' and gets embaded inside the block of mass 'M' initally at rest on a rough horizontal floor, the block with the bullet is seen to move a dist. s, along the floor assuming μ ⇒ as a coefficient of kinetic friction and g ⇒ accel. due to gravity, what is the velocity of bullet?



1) Apply law of conservation of linear momentum btw (A) and (B)

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V'$

$mV + M(0) = (m + M) V'$

$$\boxed{V' \Rightarrow \left(\frac{mV}{m+M}\right)}$$

2) Apply Work Energy Equ. btw B and (C)

$R.w = \frac{w}{2g}(V^2 - V_0^2)$

R.w = Resultant work done $\Rightarrow -\mu (M+m) g \times s$

$V \Rightarrow$ Final velocity at 'C' $\Rightarrow 0$

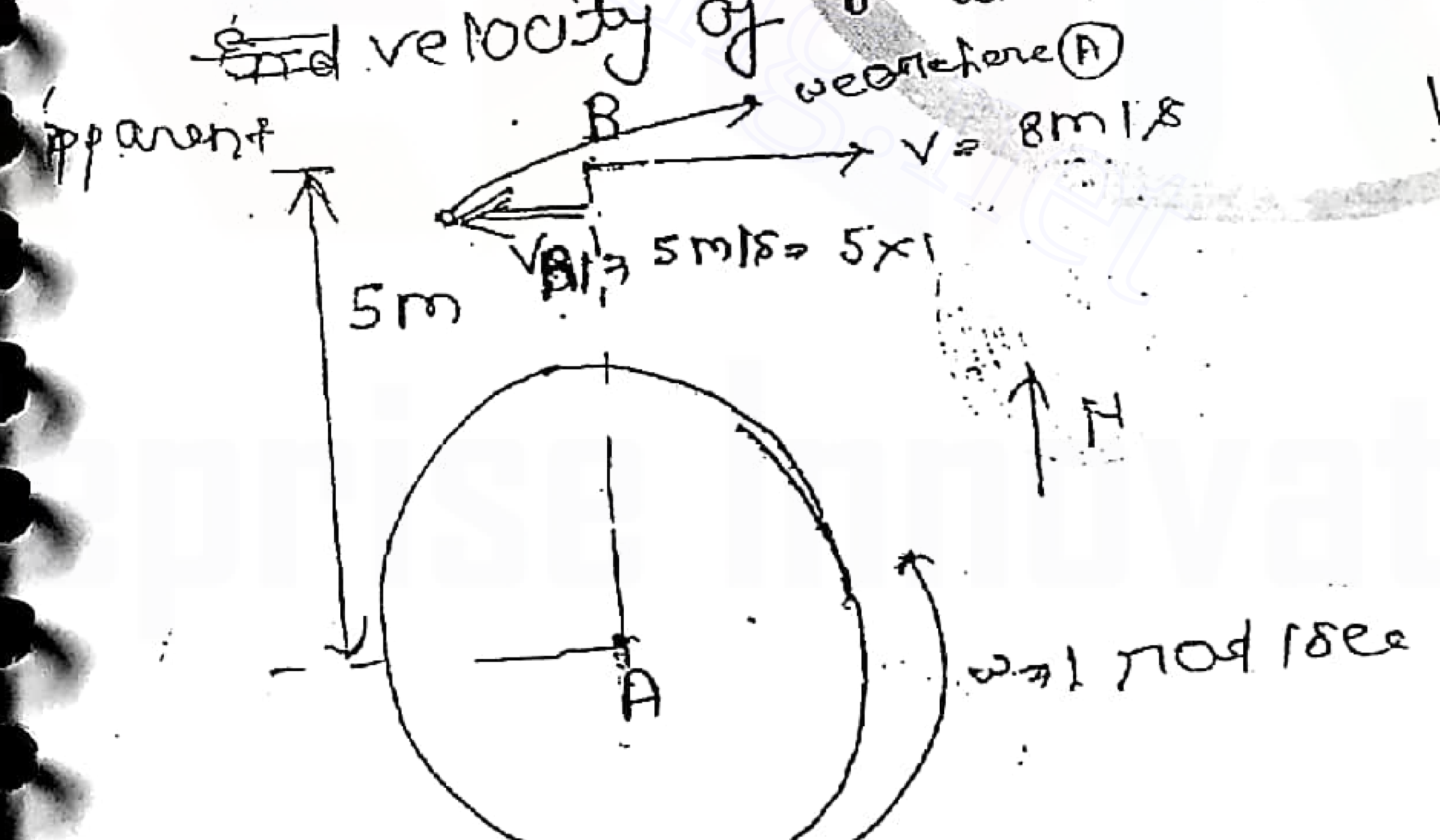
$V_0 \Rightarrow$ initial velocity at B $= \frac{mV}{m+M}$

$$-\mu g s (M+m) = \left(\frac{M+m}{2}\right)\left[\left(0^2 - \left(\frac{mV}{m+M}\right)^2\right)\right]$$

$$V^2 = \frac{\mu g s (m+M) \times 2}{(m+M)} = 2 \mu g s \frac{(m+M)^2}{m^2}$$

$$\boxed{V = \frac{M+m}{m}\sqrt{2\mu g s}}$$ Ans

Que- As shown in fig, a person 'A' is standing at the centre of a rotating platform facing heading east. person B who is riding a bicycle, the relevant speed and dist are shown in given fig? the relevant speed under consideration, what is the upper at the instant under consideration, what is the apparent velocity of B as seen by A.



V_B as seen by A $\Rightarrow 8 + 5$

$\Rightarrow 13$ m/sec

heading (E)

Ans

angular velocity ↑ from 60rpm to 180rpm ...
m.m.I. of the fly wheel?

$$\boxed{R.W \Rightarrow \frac{I}{2}(\omega^2 - \omega_0^2)}$$

$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = (2\pi)$

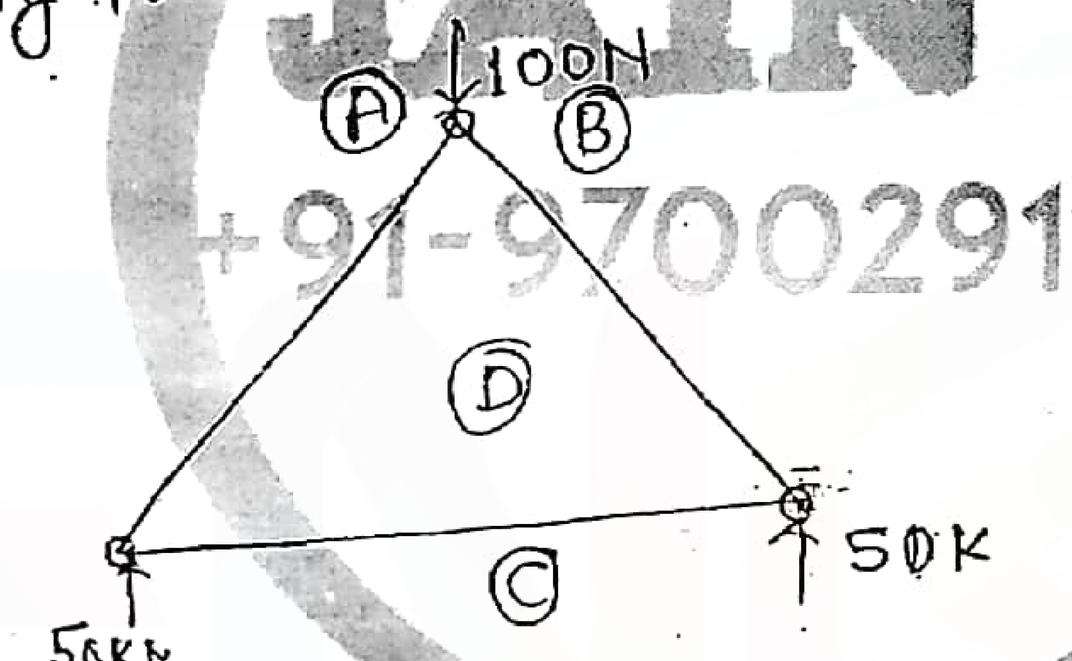
$100 \text{ N-m} \Rightarrow \frac{I}{2}((6\pi)^2 - (2\pi)^2)$

$\omega_1 = 3 \times 2\pi = 6\pi$

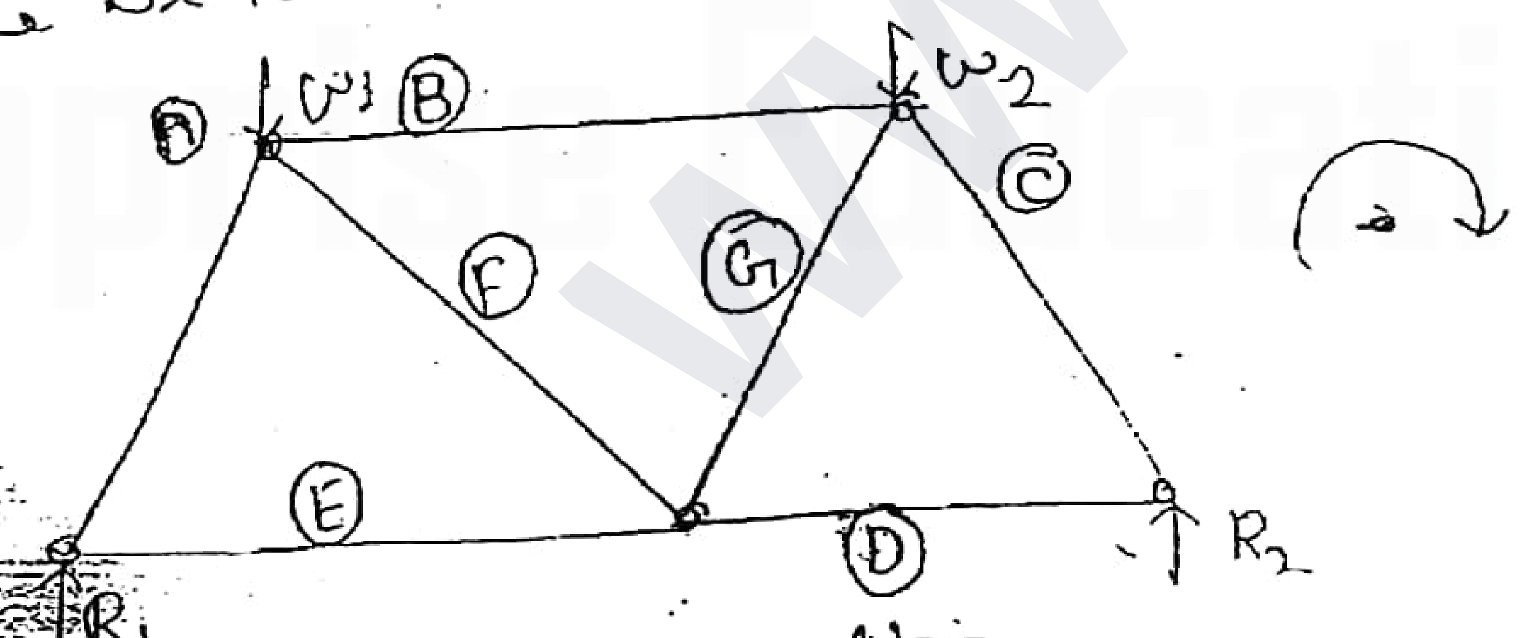
$I = \frac{2 \times 100}{(4\pi)^2} \Rightarrow 0.63 \text{ kg.m}^2$ Ans

Graphic Statics

Concepts (1). Naming the forces → Bow's notation →
for graphical construction, each force is designated by two letters which appear on either side of it. this system of naming forces is called bow's notation.



2) Space diagram → it is a diag representing the physical relationship btw the structural member., it is drawn to a convinient scale bt forces are not represented in any scale

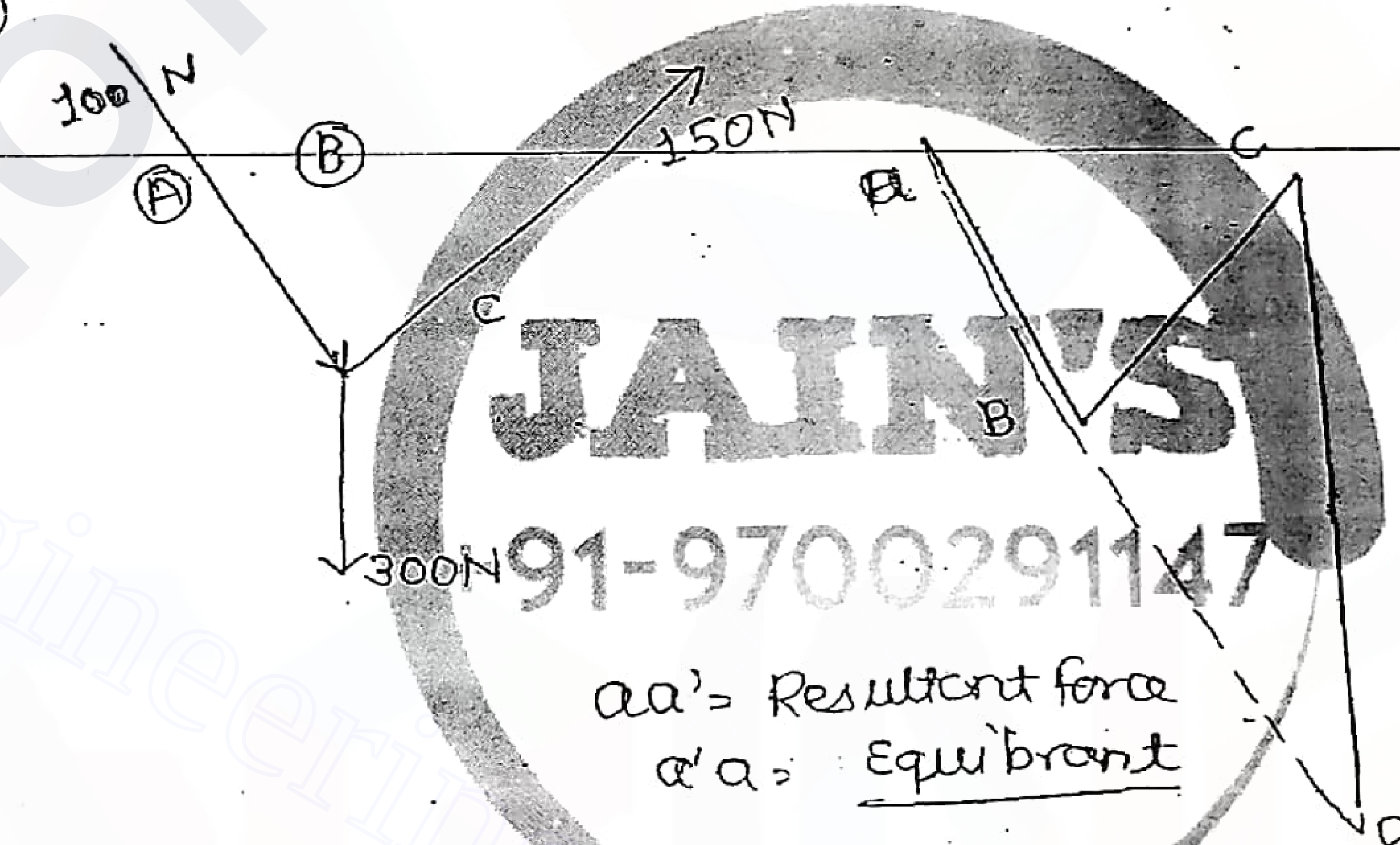


space diag.

3) Force diagram or force polygon
it is a diagr. composed of force vectors drawn to some scale.

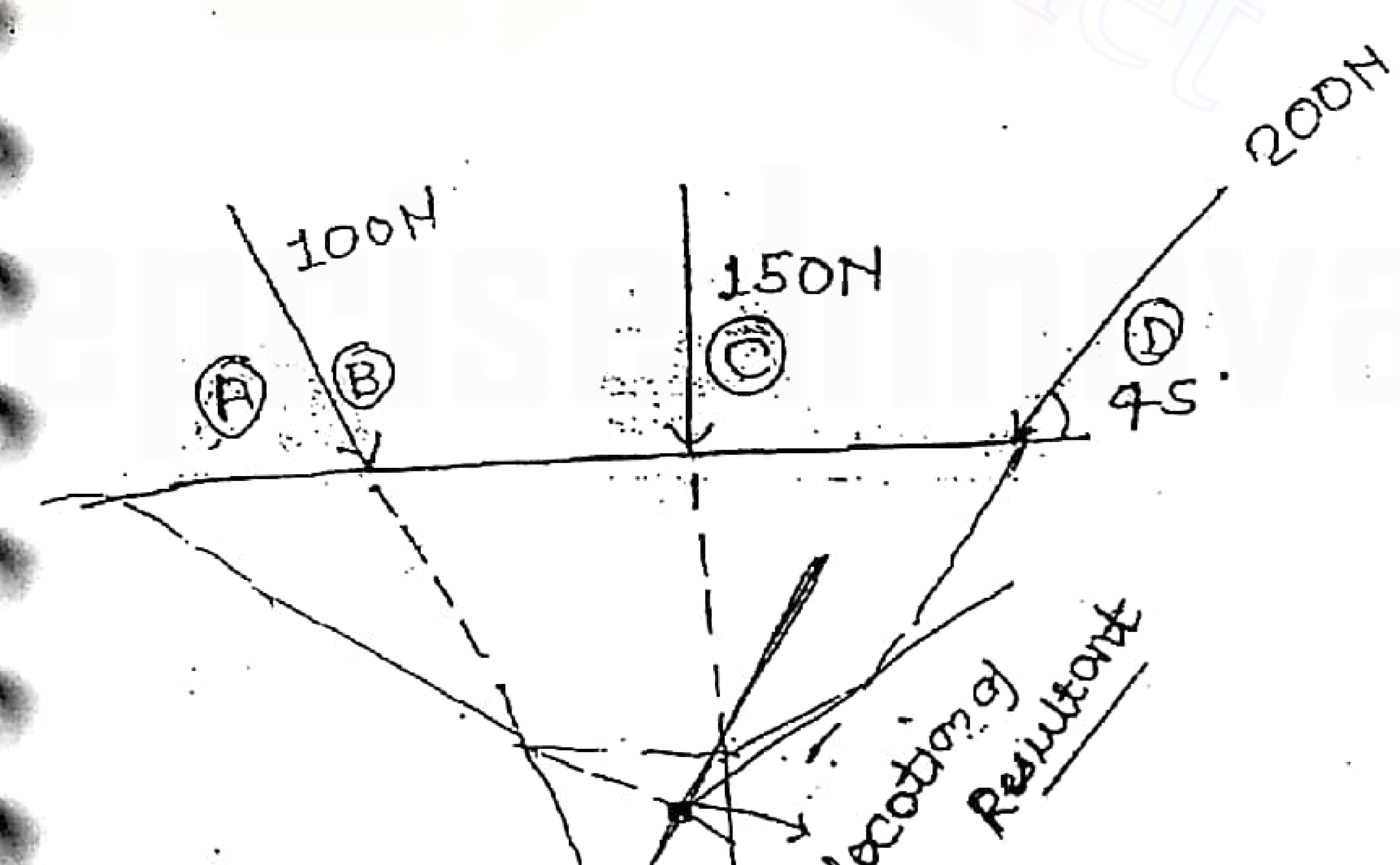
4) Funicular Polygon (string polygon)
it is a shape of freely suspended string when loaded by forces. the purpose of drawing a funicular polygon it is to find the resultant of a given force system, and finding the support R×n for any stru. under the given loading conditions.

Ex (1)



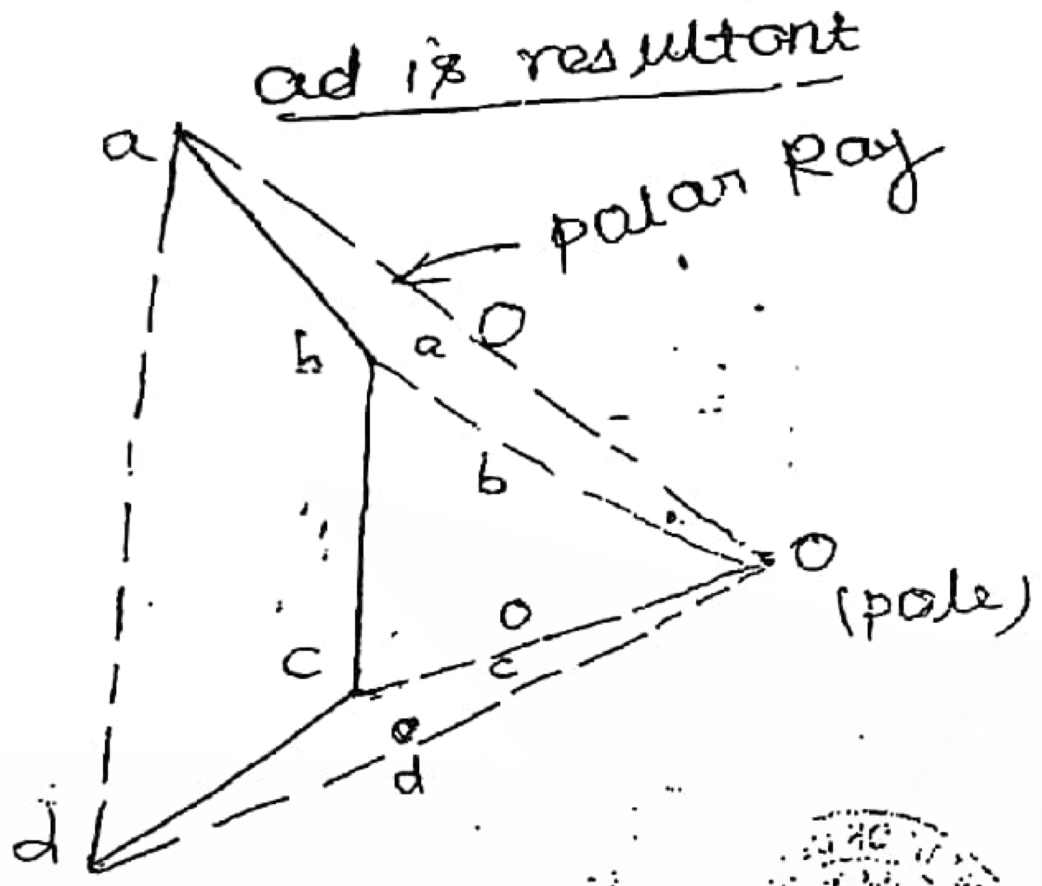
aa' = Resultant force
a'a = Equibrant

(2) find resultant and its location for non concurrent forces



1st step - Space diagram.

2nd step → force diagram.



force diagram

is called polar diagram

Que→ Determination of forces in the members of a truss.

Graphical method→

procedure→ I step - draw the space diagram showing all loads at various joints.

2nd step→ acc. to Bows notation name all the forces. ~~Bowing~~ going round the str. in clock wise direction. by naming 1st the External forces and then internal forces.

3rd step→ draw the force diagr

4th step - obtain support Rns.

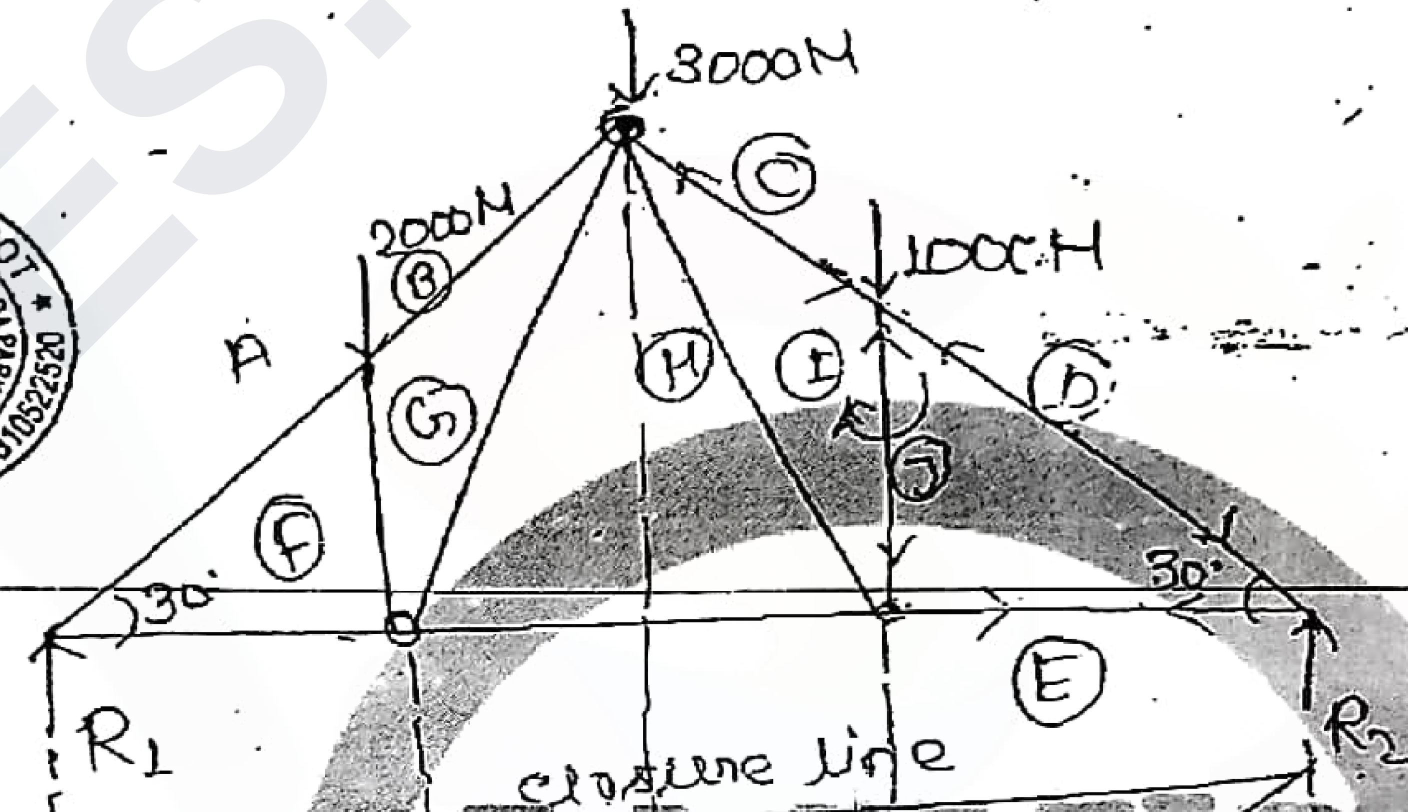
Vst step→ consider a joint where only two unknowns are available and draw the force diag. for each joint. directions are to be given in the members at the joint. Read the letters at the joints in clockwise direction.

See from the vector diagram how corresponding vectors drawn. by so reading mark the arrows at the joints accordingly.

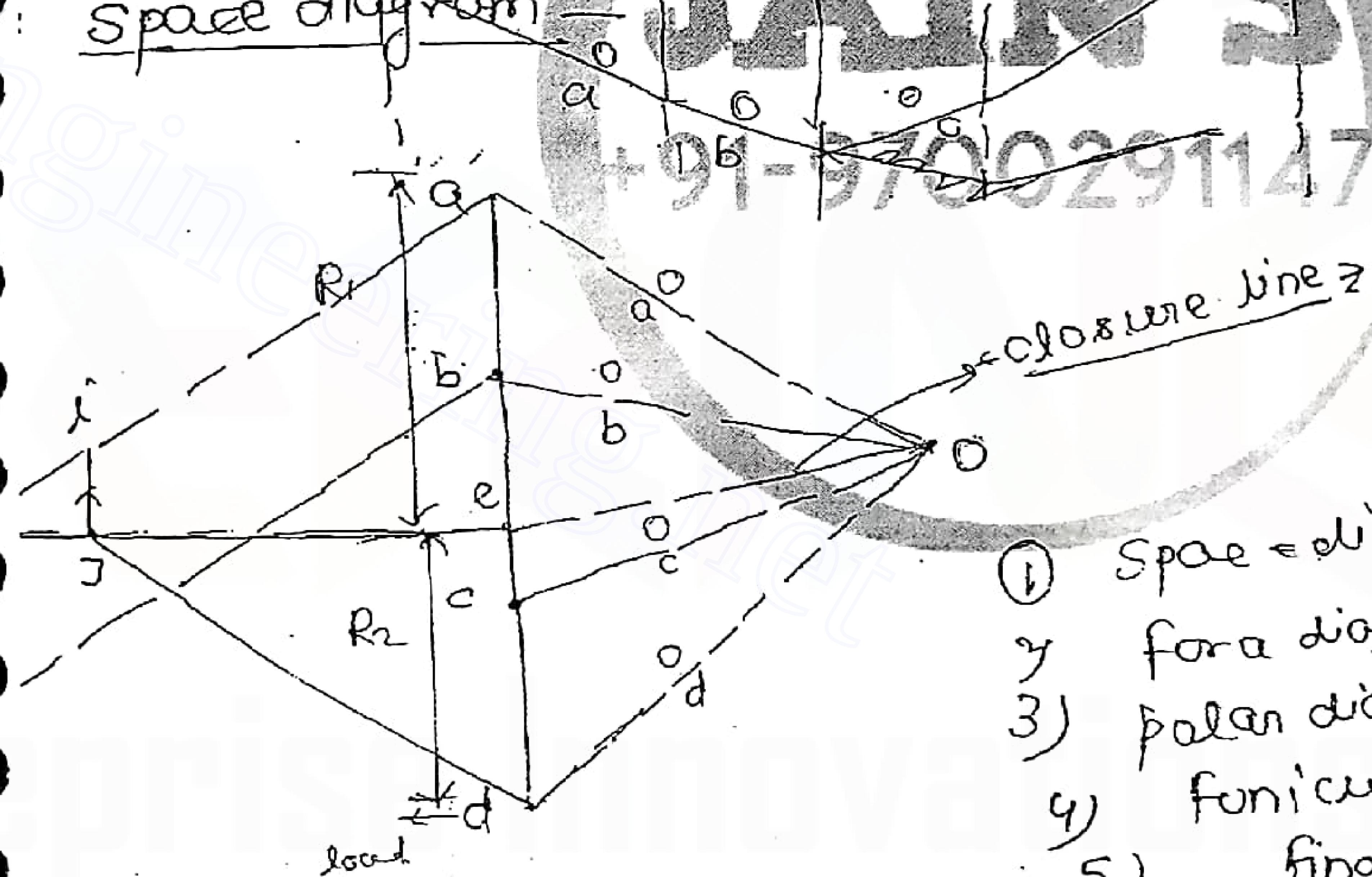
Que- the fig shows a pin jointed str. which is simply supported at R_1 and R_2. determine.

① magnitude of support Rns. R_1 and R_2.

② magnitude of the force in each member of the truss.



Space diagram

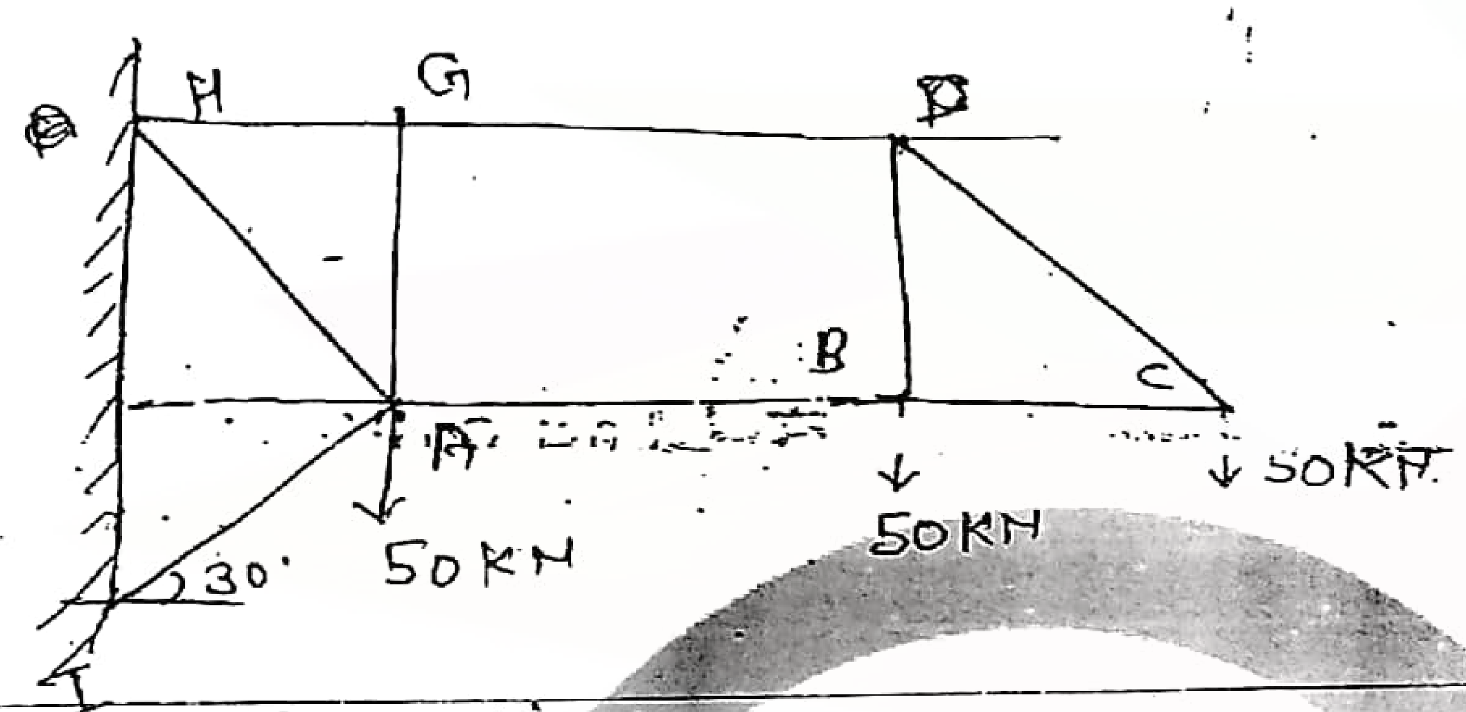


1) Space dia
2) force diagr
3) polar dia
4) funicular polygon
5) find Rns
6) member forces

Note 1 if the force diag. or force polygon is a closed diag then it means that resultant force is zero.

2) if funicular polygon is a closed diagram, then it means that result moment is zero.

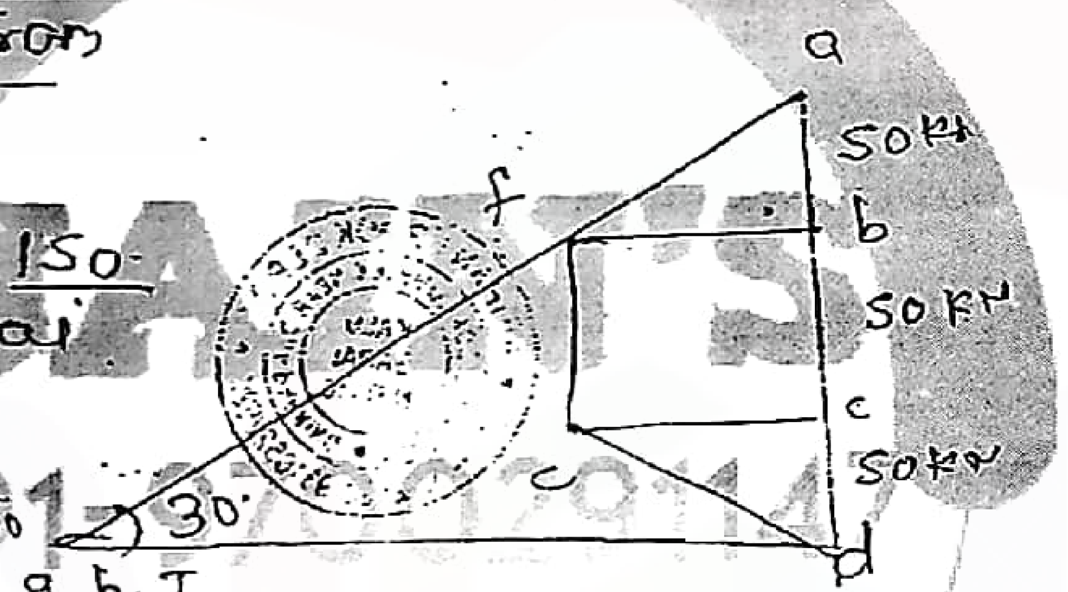
Que→ force in the member AI is (5.3/284)



force diagram

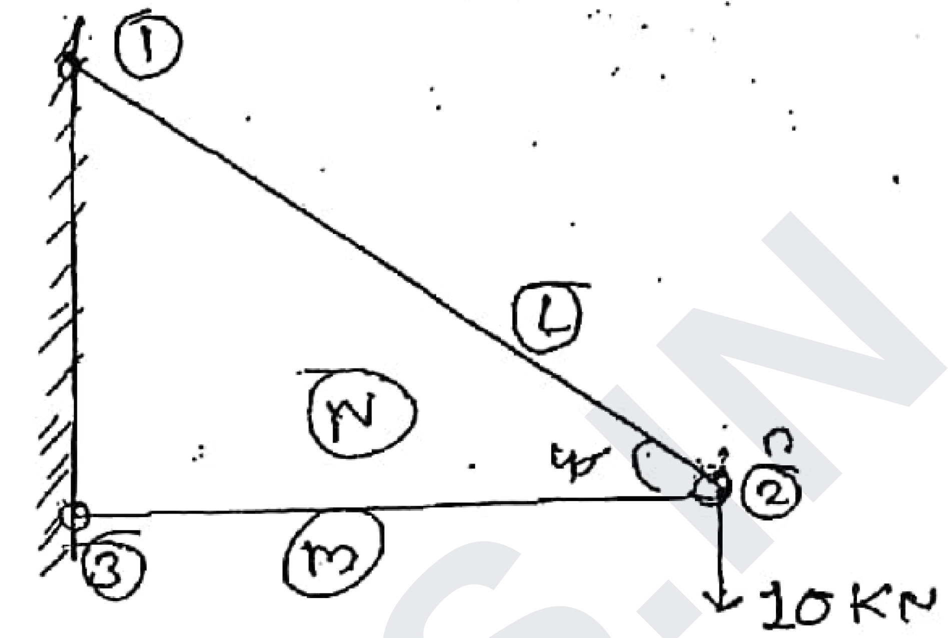
$\sin 30° = \frac{ad}{ai} \Rightarrow \frac{150}{ai}$

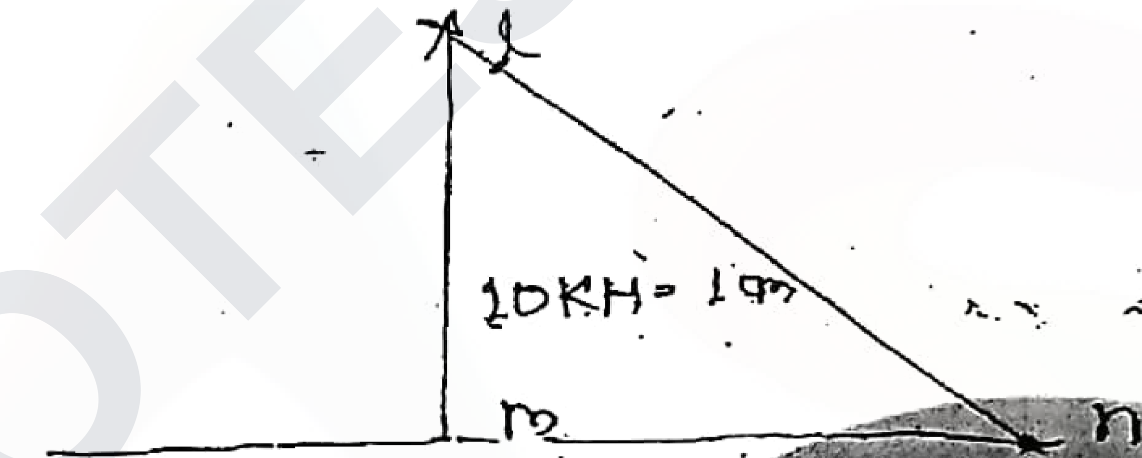
$ai = F_{AI} = \frac{150}{\sin 30} \Rightarrow 300\ KN$ (Comp)



5.4/284

Que→ (2 mark) A cantilever truss with joints leveled as ①, ②, ③ as shown in fig. it carries a vertical load of 10 KN. at joint ② the force diagr. draw at joint ② is represented as→





force diagram at joint ②

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