

# *Fluid Mechanics and Fluid Machines*

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## **Note**

*If you think there should be a change in option, don't change it by yourself send me a mail at [swapan\\_mondal\\_01@yahoo.co.in](mailto:swapan_mondal_01@yahoo.co.in) I will send you complete explanation.*

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*S K Mondal*

# **1. Properties of Fluids**

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## **Contents of this chapter**

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## Theory at a Glance (for IES, GATE, PSU)

### Definition of Fluid

A fluid is a substance which deforms continuously when subjected to external shearing forces.

### Characteristics of Fluid

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.
3. It is interesting to note that a solid suffers strain when subjected to shear forces whereas a fluid suffers **Rate of Strain** i.e. it flows under similar circumstances.

### Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure, velocity etc.) and fluid properties vary continuously from one point to another. Mathematical descriptions of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established.

For example density at a point is normally defined as

$$\rho = \lim_{\Delta V \rightarrow 0} \left( \frac{m}{\Delta V} \right)$$

Here  $\Delta V$  is the volume of the fluid element and  $m$  is the mass

If  $\Delta V$  is very large  $\rho$  is affected by the inhomogeneities in the fluid medium. Considering another extreme if  $\Delta V$  is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of  $\Delta V$ , before statistical fluctuations become significant. This is called continuum limit and is denoted by  $\Delta V_c$ .

$$\rho = \lim_{\Delta V \rightarrow \Delta V_c} \left( \frac{m}{\Delta V} \right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is

characterised by mean free path ( $\lambda$ ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analysed by the molecular theory.

A dimensionless parameter known as Knudsen number,  $K_n = \lambda / L$ , where  $\lambda$  is the mean free path and  $L$  is the characteristic length. It describes the degree of departure from continuum.

Usually when  $K_n > 0.01$ , the concept of continuum does not hold good.

Beyond this critical range of Knudsen number, the flows are known as

slip flow ( $0.01 < K_n < 0.1$ ),

transition flow ( $0.1 < K_n < 10$ ) and

free-molecule flow ( $K_n > 10$ ).

However, for the flow regimes considered in this course,  $K_n$  is always less than 0.01 and it is usual to say that the fluid is a continuum.

Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good.

In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

## Ideal and Real Fluids

### 1. Ideal Fluid

An ideal fluid is one which has

no viscosity

no surface tension

and incompressible

### 2. Real Fluid

An Real fluid is one which has

viscosity

surface tension

and compressible

Naturally available all fluids are real fluid.

## Viscosity

**Definition:** Viscosity is the property of a fluid which determines its resistance to shearing stresses.

**Cause of Viscosity:** It is due to cohesion and molecular momentum exchange between fluid layers.

**Newton's Law of Viscosity:** It states that the shear stress ( $\tau$ ) on a fluid element layer is directly proportional to the rate of shear strain.

The constant of proportionality is called the co-efficient of viscosity.

When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du.

Velocity gradient =

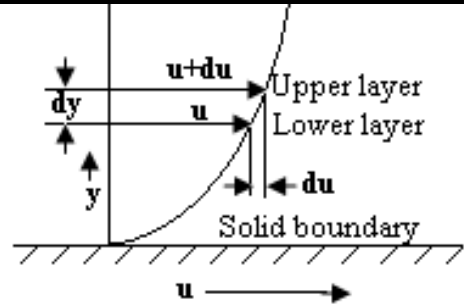
$$\frac{du}{dy}$$

According to Newton's law

$$\tau \propto \frac{du}{dy}$$

or

$$\tau = \mu \frac{du}{dy}$$



Velocity Variation near a solid boundary

Where  $\mu$  = constant of proportionality and is known as co-efficient of Dynamic viscosity or only Viscosity

As  $\mu = \frac{\tau}{\frac{du}{dy}}$  Thus viscosity may also be defined as the shear stress required

$$\mu = \frac{\tau}{\left[ \frac{du}{dy} \right]}$$

producing unit **rate of shear strain**.

## Units of Viscosity

S.I. Units: Pa.s or N.s/m<sup>2</sup>

C.G.S Unit of viscosity is Poise= dyne-sec/cm<sup>2</sup>

One Poise= 0.1 Pa.s

1/100 Poise is called centipoises.

Dynamic viscosity of water at 20°C is approx= 1 cP

## Kinematic Viscosity

It is the ratio between the dynamic viscosity and density of fluid and denoted by  $\nu$   
Mathematically

$$\nu = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho}$$

## Units of Kinematic Viscosity

S.I units: m<sup>2</sup>/s

C.G.S units: stoke = cm<sup>2</sup>/sec

One stoke = 10<sup>-4</sup> m<sup>2</sup>/s

Thermal diffusivity and molecular diffusivity have same dimension, therefore, by analogy, the kinematic viscosity is also referred to as the *momentum diffusivity* of the fluid, i.e. the ability of the fluid to transport momentum.

# Classification of Fluids

## 1. Newtonian Fluids

These fluids follow Newton's viscosity equation.

For such fluids viscosity does not change with *rate of deformation*.

## 2. Non- Newtonian Fluids

These fluids does not follow Newton's viscosity equation.

Such fluids are relatively uncommon e.g. Printer ink, blood, mud, slurries, polymer solutions.

Non-Newtonian Fluid ( )		
$\tau \neq \mu \frac{du}{dy}$		
Purely Viscous Fluids		Visco-elastic Fluids
Time - Independent	Time - Dependent	Visco-elastic Fluids
<b>1. Pseudo plastic Fluids</b>  $\tau = \mu \left( \frac{du}{dy} \right)^n; n < 1$  <b>Example:</b> Blood, milk  <b>2. Dilatant Fluids</b>  $\tau = \mu \left( \frac{du}{dy} \right)^n; n > 1$  <b>Example:</b> Butter  <b>3. Bingham or Ideal Plastic Fluid</b>  $\tau = \tau_o + \mu \left( \frac{du}{dy} \right)^n$  <b>Example:</b> Water suspensions of clay and flash	<b>1.Thixotropic Fluids</b>  $\tau = \mu \left( \frac{du}{dy} \right)^n + f(t)$  <i>f(t) is decreasing</i> <b>Example:</b> Printer ink; crude oil  <b>2. Rheopectic Fluids</b>  $\tau = \mu \left( \frac{du}{dy} \right)^n + f(t)$  <i>f(t) is increasing</i> <b>Example:</b> Rare liquid solid suspension	

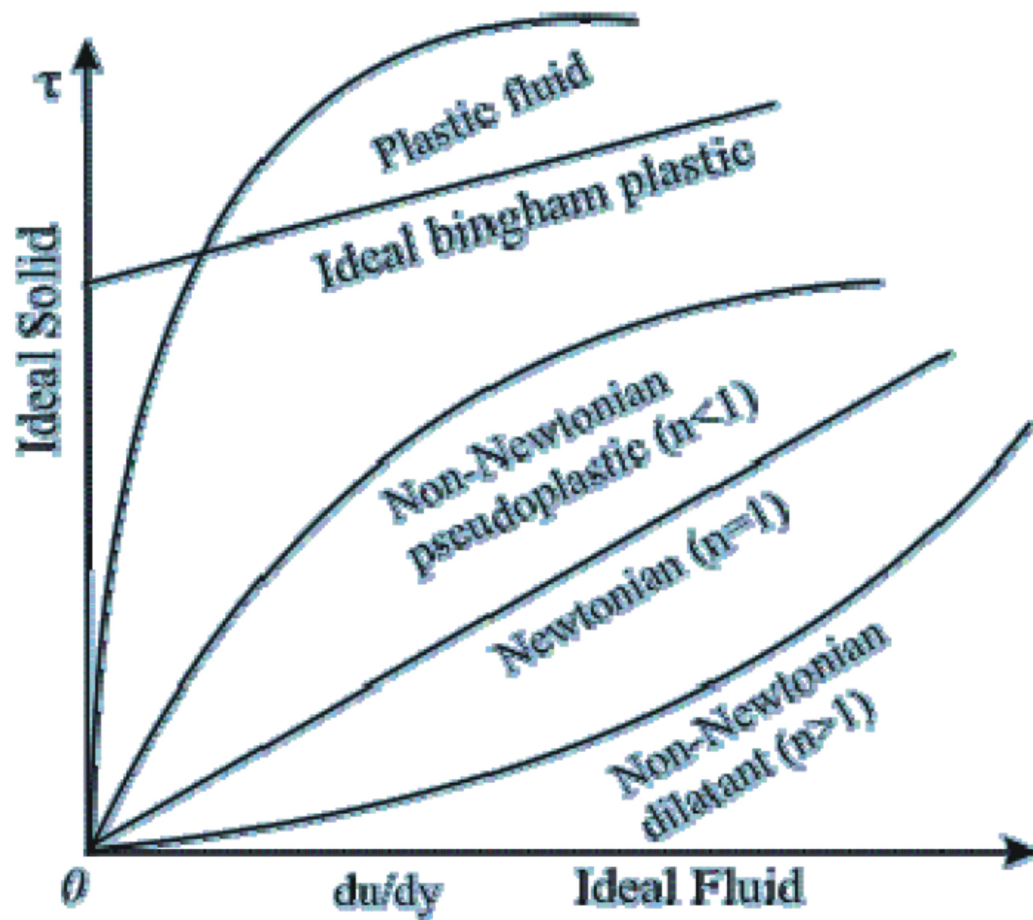


Fig. Shear stress and deformation rate relationship of different fluids

## Effect of Temperature on Viscosity

With increase in temperature  
 Viscosity of liquids decrease  
 Viscosity of gasses increase

- Note:** 1. Temperature responses are neglected in case of Mercury.  
 2. The lowest viscosity is reached at the critical temperature.

## Effect of Pressure on Viscosity

Pressure has very little effect on viscosity.

But if pressure increases intermolecular gap decreases then cohesion increases so viscosity would be increase.

## Surface tension

Surface tension is due to cohesion between particles at the surface.  
 Capillarity action is due to both cohesion and adhesion.

### Surface tension

The tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.



## Pressure Inside a Curved Surface

For a general curved surface with radii of curvature  $r_1$  and  $r_2$  at a point of interest

$$\Delta p = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

a. Pressure inside a water droplet,

$$\Delta p = \frac{4\sigma}{d}$$

b. Pressure inside a soap bubble,

$$\Delta p = \frac{8\sigma}{d}$$

c. Liquid jet.

$$\Delta p = \frac{2\sigma}{d}$$

## Capillarity

A general term for phenomena observed in liquids due to inter-molecular attraction at the liquid boundary, e.g. the rise or depression of liquids in narrow tubes. We use this term for capillary action.

Capillary rise and depression phenomenon depends upon the surface tension of the liquid as well as the material of the tube.

1. General formula,

$$h = \frac{4\sigma \cos \theta}{\rho g d}$$

2. For water and glass  $\theta = 0^\circ$ ,

$$h = \frac{4\sigma}{\rho g d}$$

3. For mercury and glass  $\theta = 138^\circ$ ,

$$h = -\frac{4\sigma \cos 42^\circ}{\rho g d}$$

(h is negative indicates capillary depression)

**Note:** If adhesion is more than cohesion, the wetting tendency is more and the angle of contact is smaller.

## Derive the Expression for Capillary Rise

Let us consider a glass tube of small diameter 'd' opened at both ends and is inserted vertically in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let, d = diameter of the capillary tube.

h = height of capillary rise.

$\theta$  = angle of contact of the water

$\sigma$

surface.

$\rho$  = surface tension force for

$\sigma$

unity length.

$\rho$  = density of liquid.

$\rho$

g = acceleration due to gravity.

Under a state of equilibrium,

Upward surface tension force (lifting force) = weight of the water column in the tube (gravity force)

or

$$\pi d \cdot \sigma \cos \theta = \frac{\pi d^2}{4} \times h \times \rho \times g$$

or

$$h = \frac{4 \sigma \cos \theta}{\rho g d}$$

If  $\theta > \pi/2$ , h will be negative, as in the case of mercury in glass capillary

$$\theta > \pi/2$$

$$\theta = 138^\circ$$

depression occurred.

**Question:** A circular disc of diameter 'd' is slowly rotated in a liquid of large viscosity ' $\mu$ ' at a small distance 't' from the fixed

surface. Derive the expression for torque required to maintain the speed ' $\omega$ '.

$\omega$

**Answer:**

Radius,  $R = d/2$

Consider an elementary circular ring of radius r and thickness dr as shown.

Area of the elements ring =

The shear stress at ring,

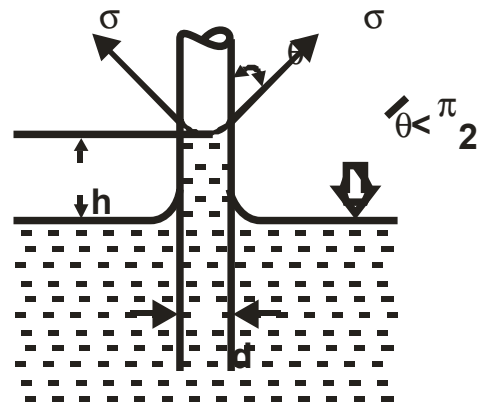
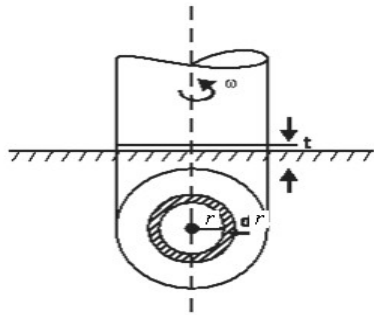


Fig. Capillary rise (As in water)



$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \mu \frac{r\omega}{t}$$

Shear force on the elements ring

$$dF = \tau \times \text{area of the ring} = \tau \times 2\pi r dr$$

Torque on the ring = dF × r

$$\therefore dT = \mu \frac{r\omega}{t} \times 2\pi r dr \times r$$

Total torque, T =

$$\therefore \int dT = \int_0^R \frac{\mu r\omega}{t} \times 2\pi r dr$$

$$= \frac{2\pi\mu\omega}{t} \int_0^R r^3 dr = \frac{\pi\mu\omega}{2t} R^4 = \frac{\pi\mu\omega}{2t} \left(\frac{d}{2}\right)^4$$

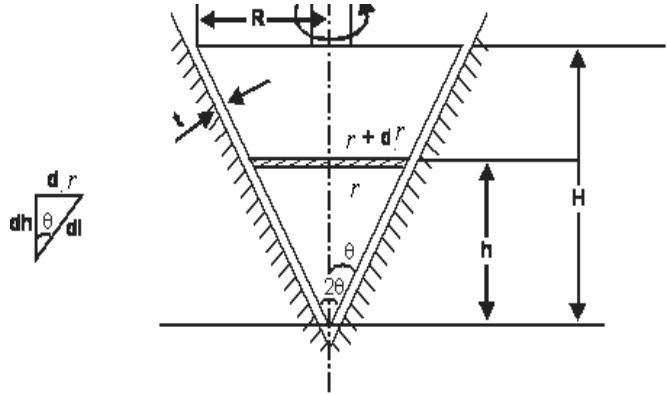
$$T = \frac{\pi\mu\omega d^4}{32t}$$

**Question:** A solid cone of radius R and vortex angle 2θ is to rotate at an angular velocity, ω. An oil of dynamic viscosity μ and thickness 't' fills the gap between the cone and the housing.

**Determine the expression for Required Torque. [IES-2000; AMIE (summer) 2002]**

**Answer:** Consider an elementary ring of bearing surface of radius r. at a distance h from the apex. and let r + dr is the radius at h + dh distance

Bearing area =



$$2\pi r dl$$

$$=$$

$$2\pi r \cdot \frac{dr}{\sin\theta}$$

Shear stress

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \mu \frac{r\omega}{t}$$

Tangential resistance on the ring

∴

$$dF = \text{shear stress} \times \text{area of the ring}$$

$$=$$

$$\mu r \frac{\omega}{t} \times 2\pi r \frac{dr}{\sin \theta}$$

Torque due to the force dF

∴

$$dT = dF \cdot r$$

$$dT = \frac{2\pi\mu\omega}{t\sin\theta} \times r^3 dr$$

Total torque

∴

$$T = \int dT = \int_0^R \frac{2\pi\mu\omega}{t\sin\theta} \times r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta} \times \frac{R^4}{4} = \frac{\pi\mu\omega R^4}{2t\sin\theta}$$

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Viscosity

**GATE-1.** The SI unit of kinematic viscosity (  $\nu$  ) is: **[GATE-2001]**

- (a)  $\text{m}^2/\text{s}$                       (b)  $\text{kg}/\text{m}\cdot\text{s}$                       (c)  $\text{m}/\text{s}^2$                       (d)  $\text{m}^3/\text{s}^2$

**GATE-1. Ans. (a)**

**GATE-2.** Kinematic viscosity of air at  $20^\circ\text{C}$  is given to be  $1.6 \times 10^{-5} \text{m}^2/\text{s}$ . Its kinematic viscosity at  $70^\circ\text{C}$  will be vary approximately **[GATE-1999]**

- (a)  $2.2 \times 10^{-5} \text{m}^2/\text{s}$     (b)  $1.6 \times 10^{-5} \text{m}^2/\text{s}$     (c)  $1.2 \times 10^{-5} \text{m}^2/\text{s}$     (d)  $3.2 \times 10^{-5} \text{m}^2/\text{s}$

**GATE-2. Ans. (a)** Viscosity of gas increases with increasing temperature.

#### Newtonian Fluid

**GATE-3.** For a Newtonian fluid **[GATE-2006; 1995]**

- (a) Shear stress is proportional to shear strain  
 (b) Rate of shear stress is proportional to shear strain  
 (c) Shear stress is proportional to rate of shear strain  
 (d) Rate of shear stress is proportional to rate of shear strain

**GATE-3. Ans. (c)**

#### Surface Tension

**GATE-4.** The dimension of surface tension is: **[GATE-1996]**

- (a)  $\text{ML}^{-1}$                       (b)  $\text{L}^2\text{T}^{-1}$                       (c)  $\text{ML}^{-1}\text{T}^1$                       (d)  $\text{MT}^{-2}$

**GATE-4. Ans. (d)**

**GATE-5.** The dimensions of surface tension is: **[GATE-1995]**

- (a)  $\text{N}/\text{m}^2$                       (b)  $\text{J}/\text{m}$                       (c)  $\text{J}/\text{m}^2$   
 (d)  $\text{W}/\text{m}$

**GATE-5. Ans. (c)** The property of the liquid surface film to exert a tension is called the surface tension. It is the force required to maintain unit length of the film in equilibrium. In SI units surface tension is expressed in

$$\text{N} / \text{m} \left( \frac{\text{J}}{\text{m}^2} \right).$$

In metric gravitational system of units it is expressed in  $\text{kg(f)}/\text{cm}$  or  $\text{kg(f)}/\text{m}$ .

**Previous 20-Years IES Questions**

## Fluid

**IES-1. Assertion (A): In a fluid, the rate of deformation is far more important than the total deformation itself. Reason (R): A fluid continues to deform so long as the external forces are applied. [IES-1996]**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-1. Ans. (a)** Both A and R correct and R is correct explanation for A

**IES-2. Assertion (A): In a fluid, the rate of deformation is far more important than the total deformation itself. [IES-2009]**

**Reason (R): A fluid continues to deform so long as the external forces are applied.**

- (a) Both A and R are individually true and R is the correct explanation of A.
- (b) Both A and R are individually true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

**IES-2. Ans. (a)** This question is copied from  
**Characteristics of fluid**

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.
3. It is interesting to note that a solid suffers strain when subjected to shear forces whereas a fluid suffers Rate of Strain i.e. it flows under similar circumstances.

## Viscosity

**IES-3. Newton's law of viscosity depends upon the [IES-1998]**

- (a) Stress and strain in a fluid      (b) Shear stress, pressure and velocity
- (c) Shear stress and rate of strain      (d) Viscosity and shear stress

**IES-3. Ans. Newton's law of viscosity**

**(c)** 
$$\tau = \mu \frac{du}{dy} \quad \text{where, } \tau \rightarrow \text{Shear stress}$$

$$\frac{du}{dy} \rightarrow \text{Rate of strain}$$

**IES-4. What is the unit of dynamic viscosity of a fluid termed 'poise' equivalent to? [IES-2008]**

<b>Properties of Fluids</b> <b>S K Mondal's</b> <span style="float: right;"><b>Chapter 1</b></span>
--

- (a) dyne/cm<sup>2</sup>                      (b) gm s/cm                      (c) dyne s/cm<sup>2</sup>  
 (d) gm-cm/s

**IES-4. Ans. (c)**

**IES-5. The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 1 cm apart and moving with relative velocity of 2 m/s is: [IES-2001]**

- (a) 20 N/m<sup>2</sup>                      (b) 196.2 N/m<sup>2</sup>                      (c) 29.62 N/m<sup>2</sup>                      (d) 40 N/m<sup>2</sup>

**IES-5. Ans. (b)**  $du = 2 \text{ m/s}$ ;  $dy = 1 \text{ cm} = 0.01 \text{ m}$ ;  $\mu = 9.81 \text{ poise} = 0.981 \text{ Pa.s}$

$$\text{Therefore } \tau = \mu \frac{du}{dy} = 0.981 \times \frac{2}{0.01} = 196.2 \text{ N/m}^2$$

**IES-6. What are the dimensions of kinematic viscosity of a fluid? [IES-2007]**

- (a)  $\text{LT}^{-2}$                       (b)  $\text{L}^2\text{T}^{-1}$                       (c)  $\text{ML}^{-1}\text{T}^{-1}$                       (d)  $\text{ML}^{-1}\text{T}^{-2}$

**IES-6. Ans. (b)**

**IES-7. An oil of specific gravity 0.9 has viscosity of 0.28 Stokes at 38°C. What will be its viscosity in Ns/m<sup>2</sup>? [IES-2005]**

- (a) 0.2520                      (b) 0.0311                      (c) 0.0252                      (d) 0.0206

**IES-7. Ans. (c)** Specific Gravity = 0.9 therefore Density =  $0.9 \times 1000 = 900 \text{ kg/m}^3$   
 One Stoke =  $10^{-4} \text{ m}^2/\text{s}$   
 Viscosity ( $\mu$ ) =  $\rho \nu = 900 \times 0.28 \times 10^{-4} = 0.0252 \text{ Ns/m}^2$

**IES-8. Decrease in temperature, in general, results in [IES-1993]**

- (a) An increase in viscosities of both gases and liquids  
 (b) A decrease in the viscosities of both liquids and gases  
 (c) An increase in the viscosity of liquids and a decrease in that of gases  
 (d) A decrease in the viscosity of liquids and an increase in that of gases

**IES-8. Ans. (c)** The viscosity of water with increase in temperature decreases and that of air increases.

**IES-9. Assertion (A): In general, viscosity in liquids increases and in gases it decreases with rise in temperature. [IES-2002]**

**Reason (R): Viscosity is caused by intermolecular forces of cohesion and due to transfer of molecular momentum between fluid layers; of which in liquids the former and in gases the later contribute the major part towards viscosity.**

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-9. Ans. (d)**

## Non-Newtonian Fluid

IES-10. If the Relationship between the shear stress  $\tau$  and the rate of

shear strain is expressed as then the fluid with

$$\frac{du}{dy} \quad \tau = \mu \left( \frac{du}{dy} \right)^n$$

exponent  $n > 1$  is known as which one of the following? [IES-2007]

- (a) Bingham Plastic (b) Dilatant Fluid  
(c) Newtonian Fluid (d) Pseudo plastic Fluid

IES-10. Ans. (b)

IES-11. Match List-I (Type of fluid) with List-II (Variation of shear stress) and select the correct answer: [IES-2001]

- | List-I                 | List-II   |
|------------------------|---|
| A. Ideal fluid         | 1. Shear stress varies linearly with the rate of strain   |
| B. Newtonian fluid     | 2. Shear stress does not vary linearly with the rate of strain  |
| C. Non-Newtonian fluid | 3. Fluid behaves like a solid until a minimum yield stress beyond which it exhibits a linear relationship between shear stress and the rate of strain |
| D. Bingham plastic     | 4. Shear stress is zero   |

Codes:	A	B	C	D	A	B	C	D
(a)	3	1	2	4	(b)	4	2	1
(c)	3	2	1	4	(d)	4	1	2
							3	

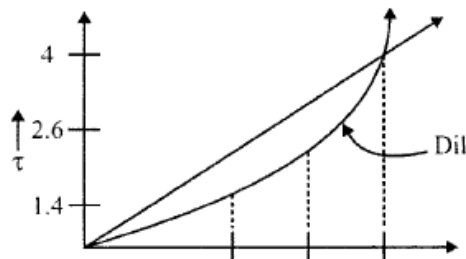
IES-11. Ans. (d)

IES-12. In an experiment, the following shear stress - time rate of shear strain values are obtained for a fluid: [IES-2008]

Time rate of shear strain (1/s):	0	2	3	4
Shear stress (kPa):	0	1.4	2.6	4

- (a) Newtonian fluid (b) Bingham plastic  
(c) Pseudo plastic (d) Dilatant

IES-12. Ans. (d)



IES-13. Match List-I (Rheological Equation) with List-II (Types of Fluids) and select the correct the answer: [IES-2003]

- | List-I                           | List-II            |
|----------------------------------|--------------------|
| A. $\tau = \mu(du/dy)^n$ , $n=1$ | 1. Bingham plastic |



- B.  $\tau = \mu (du/dy)^n$ ,  $n < 1$       2. Dilatant fluid
- C.  $\tau = \mu (du/dy)^n$ ,  $n > 1$       3. Newtonian fluid
- D.  $\tau = \tau_0 + \mu (du/dy)^n$ ,  $n = 1$       4. Pseudo-plastic fluid

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a) 3	2	4	1		(b) 4	1	2	3	
(c) 3	4	2	1		(d) 4	2	1	3	

IES-13. Ans. (c)

IES-14. **Assertion (A): Blood is a Newtonian fluid.** [IES-2007]  
**Reason (R): The rate of strain varies non-linearly with shear stress for blood.**

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-14. Ans. (d) A is false but R is true.

IES-15. **Match List-I with List-II and select the correct answer.** [IES-1995]  
**List-I (Properties of fluids)**      **List-II (Definition/ Results)**

- |  |   |
|--|---|
| <b>A.</b> Ideal fluid<br><br><b>B.</b> Newtonian fluid<br><b>C.</b> $\mu / \rho$<br><br><b>D.</b> Mercury in glass | <b>1.</b> Viscosity does not change with rate of deformation<br><b>2.</b> Fluid of zero viscosity<br><b>3.</b> Dynamic viscosity<br><br><b>4.</b> Capillary depression<br><b>5.</b> Kinematic viscosity<br><b>6.</b> Capillary rise |
|--|---|

<b>Code:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a) 1	2	4	6		(b) 1	2	3	4	
(c) 2	1	3	6		(d) 2	1	5	4	

IES-15. Ans. (d)

## Surface Tension

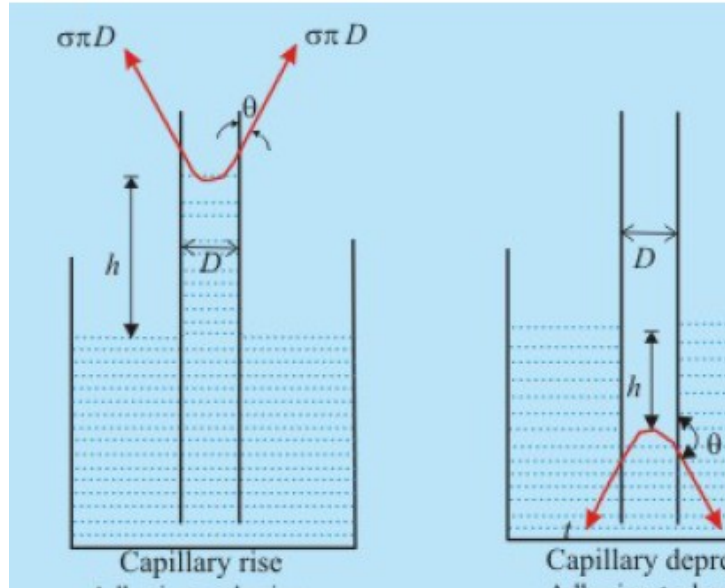
IES-16. **Surface tension is due to** [IES-1998]

- (a) Viscous forces      (b) Cohesion  
 (c) Adhesion      (d) The difference between adhesive and cohesive forces

IES-16. Ans. (b) Surface tension is due to cohesion between liquid particles at the surface, where as capillarity is due to both cohesion and adhesion. The property of cohesion enables a liquid to resist tensile stress, while adhesion enables it to stick to another body.

IES-17. **What is the pressure difference between inside and outside of a droplet of water?** [IES-2008]





- IES-22.** A capillary tube is inserted in mercury kept in an open container.  
**Assertion (A):** The mercury level inside the tube shall rise above the level of mercury outside. [IES-2001]  
**Reason (R):** The cohesive force between the molecules of mercury is greater than the adhesive force between mercury and glass.  
 (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true
- IES-22. Ans. (d)** Mercury shows capillary depression.
- IES-23.** What is the capillary rise in a narrow two-dimensional slit of width 'w'? [IES-2009]  
 (a) Half of that in a capillary tube of diameter 'w'  
 (b) Two-third of that in a capillary tube of diameter 'w'  
 (c) One-third of that in a capillary tube of diameter 'w'  
 (d) One-fourth of that in a capillary tube of diameter 'w'
- IES-23. Ans. (a)**
- IES-24.** **Assertion (A):** A narrow glass tube when immersed into mercury causes capillary depression, and when immersed into water causes capillary rise. [IES-2009]  
**Reason (R):** Mercury is denser than water.  
 (a) Both A and R are individually true and R is the correct explanation of A.  
 (b) Both A and R are individually true but R is not the correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.
- IES-24. Ans. (b)** Causes of capillary depression: Adhesion is less than cohesion, the wetting tendency is less and the angle of contact is high.
- IES-25.** Consider the following statements related to the fluid properties:  
 1. Vapour pressure of water at 373 K is  $101.5 \times 10^3 \text{ N/m}^2$ .

2. Capillary height in cm for water in contact with glass tube and air is (tube dia)/0.268.

3. Blood is a Newtonian fluid

Which of the statements given above is/are correct? [IES-2008]

(a) 1 only (b) 1 and 3 (c) 1 and 2 (d) 2 only

IES-25. Ans. (a) Vapour pressure of water at 373 K means 100°C is one atmosphere = 1.01325 bar = 101.325 × 10<sup>3</sup> N/m<sup>2</sup>.

Capillary height in cm for water in contact with glass tube =

$$\frac{0.3}{d}$$

For water and glass

$$\theta = 0^\circ, h = \frac{4\sigma}{\rho g d}$$

Blood is a pseudoplastic fluid.

Where

$$\tau = \mu \left( \frac{du}{dy} \right)^n; n < 1$$

## Compressibility and Bulk Modulus

IES-26. Which one of the following is the bulk modulus K of a fluid? (Symbols have the usual meaning) [IES-1997]

(a) (b) (c) (d)

$$\rho \frac{dp}{d\rho}$$

$$\frac{dp}{\rho d\rho}$$

$$\frac{\rho d\rho}{dp}$$

$$\frac{dp}{\rho d\rho}$$

IES-26. Ans.

(a)

Bulk modulus

$$K = -\frac{dp}{\frac{dv}{v}} \quad \text{and} \quad v = \frac{1}{\rho}$$

$$\therefore K = -\frac{dp}{-\frac{d\rho}{\rho^2}} \quad \therefore dv = -\frac{d\rho}{\rho^2}$$

$$K = \frac{\rho d\rho}{d\rho}$$

IES-27. When the pressure on a given mass of liquid is increased from 3.0 MPa to 3.5 MPa, the density of the liquid increases from 500 kg/m<sup>3</sup> to 501 kg/m<sup>3</sup>. What is the average value of bulk modulus of the liquid over the given pressure range? [IES-2006]

(a) 700 MPa (b) 600 MPa (c) 500 MPa (d) 250 MPa

IES-27. Ans. (d)

$$\frac{500 \times (3.5 - 3.0)}{(501 - 500)} = 250 \text{ MPa}$$

## Vapour Pressure

**IES-28. Which Property of mercury is the main reason for use in barometers?**

- (a) High Density (b) Negligible Capillary effect  
(c) Very Low vapour Pressure (d) Low compressibility

**[IES-2007]**

**IES-28. Ans. (c)**

**IES-29. Consider the following properties of a fluid:**

**[IES-2005]**

1. Viscosity 2. Surface tension  
3. Capillarity 4. Vapour pressure

**Which of the above properties can be attributed to the flow of jet of oil in an unbroken stream?**

- (a) 1 only (b) 2 only (c) 1 and 3 (d) 2 and 4

**IES-29. Ans. (b)** Surface tension forces are important in certain classes of practical problems such as,

1. Flows in which capillary waves appear
2. Flows of small jets and thin sheets of liquid injected by a nozzle in air
3. Flow of a thin sheet of liquid over a solid surface.

Here the significant parameter for dynamic similarity is the magnitude ratio of the surface tension force to the inertia force. And we must use Weber number for similarity. Therefore the answer will be surface tension.

And you also know that Pressure inside a Liquid jet.

$$\Delta p = \frac{2\sigma}{d}$$

**IES-30. Match List-I with List-II and select the correct answer using the code given below the lists:**

**[IES-2008]**

**List-I (Variable Expression)**

**List-II**

**(Dimensional)**

- A.** Dynamic Viscosity  
**B.** Moment of momentum  
**C.** Power  
**D.** Volume modulus of elasticity

1.  $M L^2 T^{-3}$
2.  $M L^{-1} T^{-2}$
3.  $M L^{-1} T^{-1}$
4.  $M L^2 T^{-2}$
5.  $M L^2 T^{-1}$

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	4	2	3	(b)	3	5	1	2
(c)	1	5	2	3	(d)	3	4	1	2

**IES-30. Ans. (b)**

## Previous 20-Years IAS Questions

### Fluid

**IAS-1. Which one of the following sets of conditions clearly apply to an ideal fluid?**

**[IAS-1994]**

- (a) Viscous and compressible (b) Non-viscous and incompressible  
(c) Non-viscous and compressible (d) Viscous and incompressible

**IAS-1. Ans. (b)**

## Viscosity

**IAS-2.** When a flat plate of  $0.1 \text{ m}^2$  area is pulled at a constant velocity of  $30 \text{ cm/sec}$  parallel to another stationary plate located at a distance  $0.01 \text{ cm}$  from it and the space in between is filled with a fluid of dynamic viscosity  $= 0.001 \text{ Ns/m}^2$ , the force required to be applied is: **[IAS-2004]**

- (a)  $0.3 \text{ N}$  (b)  $3 \text{ N}$  (c)  $10 \text{ N}$   
(d)  $16 \text{ N}$

**IAS-2. Ans. (a)** Given,  $\mu = 0.001 \text{ Ns/m}^2$  and  $du = (V - 0) = 30 \text{ cm/sec} = 0.3 \text{ m/s}$   
and distance  $(dy) = 0.01 \text{ cm} = 0.0001 \text{ m}$   
Therefore, Shear stress  $(\tau) =$

$$\tau = \mu \frac{du}{dy} = \left( 0.001 \frac{\text{Ns}}{\text{m}^2} \right) \times \frac{(0.3 \text{ m/s})}{(0.0001 \text{ m})} = 3 \text{ N/m}^2$$

$$\text{Force required (F)} = \tau \times A = 3 \times 0.1 = 0.3 \text{ N}$$

## Newtonian Fluid

**IAS-3.** In a Newtonian fluid, laminar flow between two parallel plates, the ratio  $(\tau)$  between the shear stress and rate of shear strain is

given by

- (a)  $\mu \frac{d^2 \mu}{dy^2}$  (b)  $\mu \frac{du}{dy}$  (c)  $\mu \left( \frac{du}{dy} \right)^2$  **[IAS-1995]**  
(d)  $\mu \left( \frac{du}{dy} \right)^{1/2}$

**IAS-3. Ans. (b)**

**IAS-4.** Consider the following statements: **[IAS-2000]**

1. Gases are considered incompressible when Mach number is less than  $0.2$
  2. A Newtonian fluid is incompressible and non-viscous
  3. An ideal fluid has negligible surface tension
- Which of these statements is /are correct?

- (a) 2 and 3 (b) 2 alone (c) 1 alone (d) 1 and 3

**IAS-4. Ans. (d)**

## Non-Newtonian Fluid

**IAS-5.** The relations between shear stress  $(\tau)$  and velocity gradient for

ideal fluids, Newtonian fluids and non-Newtonian fluids are given below. Select the correct combination. **[IAS-2002]**

- (a)  $\tau = 0$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^2$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^3$
- (b)  $\tau = 0$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^2$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^3$
- (c)  $\tau = \mu \left( \frac{du}{dy} \right)^2$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^3$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^3$
- (d)  $\tau = \mu \left( \frac{du}{dy} \right)^2$ ;  $\tau = \mu \left( \frac{du}{dy} \right)^3$ ;  $\tau = 0$

**IAS-5. Ans. (b)**

**IAS-6. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are known as [IAS-1997]**

- (a) Rhedoplectic fluids (b) Thixotropic fluids  
(c) Pseudoplastic fluids (d) Newtonian fluids

**IAS-6. Ans. (a)** where  $f(t)$  is increasing

$$\tau = \mu \left( \frac{du}{dy} \right)^n + f(t)$$

## Surface Tension

**IAS-7. At the interface of a liquid and gas at rest, the pressure is: [IAS-1999]**

- (a) Higher on the concave side compared to that on the convex side  
(b) Higher on the convex side compared to that on the concave side  
(c) Equal to both sides  
(d) Equal to surface tension divided by radius of curvature on both sides.

**IAS-7. Ans. (a)**

## Vapour Pressure

**IAS-8. Match List-I (Physical properties of fluid) with List-II (Dimensions/Definitions) and select the correct answer: [IAS-2000]**

**List-I**

- A.** Absolute viscosity  
**B.** Kinematic viscosity  
**C.** Newtonian fluid  
**D.** Surface tension

**List-II**

- 1.**  $du/dy$  is constant  
**2.** Newton per metre  
**3.** Poise  
**4.** Stress/Strain is constant  
**5.** Stokes

Codes:	A	B	C	D		A	B	C	D
(a) 5	3	1	2		(b) 3	5	2	4	
(c) 5	3	4	2		(d) 3	5	1	2	

**IAS-8. Ans. (d)**

## Answers with Explanation (Objective)

### Problem

1. A circular disc of diameter  $D$  is slowly in a liquid of a large viscosity ( $\mu$ ) at a small distance ( $h$ ) from a fixed surface. Derive an expression of torque ( $T$ ) necessary to maintain an angular velocity ( $\omega$ )

1. Ans.  $T =$

$$\frac{\pi \mu D^4 \omega}{32h}$$

2. A metal plate  $1.25 \text{ m} \times 1.25 \text{ m} \times 6 \text{ mm}$  thick and weighting  $90 \text{ N}$  is placed midway in the  $24 \text{ mm}$  gap between the two vertical plane surfaces. The Gap is filled with an oil of specific gravity  $0.85$  and dynamic viscosity  $3.0 \text{ N.s/m}^2$ . Determine the force required to lift the plate with a constant velocity of  $0.15 \text{ m/s}$ .

2. Ans.  $168.08 \text{ N}$

3. A  $400 \text{ mm}$  diameter shaft is rotating at  $200 \text{ rpm}$  in a bearing of length  $120 \text{ mm}$ . If the thickness of oil film is  $1.5 \text{ mm}$  and the dynamic viscosity of the oil is  $0.7 \text{ Ns/m}^2$  determine:  
(i) Torque required overcoming friction in bearing;  
(ii) Power utilization in overcoming viscous resistance;

3. Ans. (i)  $58.97 \text{ Nm}$  (ii)  $1.235 \text{ kW}$

4. In order to form a stream of bubbles, air is introduced through a nozzle into a tank of water at  $20^\circ\text{C}$ . If the process requires  $3.0 \text{ mm}$  diameter bubbles to be formed, by how much the air pressure at the nozzle must exceed that of the surrounding water? What would be the absolute pressure inside the bubble if the surrounding water is at  $100.3 \text{ kN/m}^2$ ? ( $\sigma = 0.0735 \text{ N/m}$ )

$\sigma$

4. Ans.  $P_{\text{abs}} = 100.398 \text{ kN/m}^2$  (Hint. Bubble of air but surface tension of water).

5. A U-tube is made up of two capillaries of diameters  $1.0 \text{ mm}$  and  $1.5 \text{ mm}$  respectively. The U tube is kept vertically and partially filled with water of surface tension  $0.0075 \text{ kg/m}$  and zero contact angles. Calculate the difference in the level of the menisci caused by the capillarity.

5. Ans.  $10 \text{ mm}$



6. If a liquid surface (density  $\rho$ ) supports another fluid of density,  $\rho_b$

above the meniscus, then a balance of forces would result in capillary rise  $h =$

$$\frac{4\sigma \cos \theta}{(\rho - \rho_b)gd}$$

## 2. Pressure and Its Measurements

### Contents of this chapter

1. Pressure of a Fluid
2. Hydrostatic Law and Aerostatic Law
3. Absolute and Gauge Pressures
4. Manometers
5. Piezometer
6. Mechanical Gauges

**Question:** *Find out the differential equations of aerostatic in isothermal and adiabatic state and find out the pressure at an altitude of  $z$  taking the pressure, density and altitude at sea level to be*

$p_0$ ,  $\rho_0$  and  $z_0 = 0$

**[AMIE - Winter 2002]**

**Answer:** We know that, Hydrostatic law  
, when  $z$  is measured vertically upward

$$\frac{\partial p}{\partial z} = -\rho g$$

[Symbols has usual meanings]

In case of compressible fluids the density (  $\rho$  ) changes with the change  
 $\rho$

of pressure and temperature.

For gases equation of state

$$\frac{p}{\rho} = RT$$

or

$$\rho = \frac{p}{RT}$$

$$\therefore \frac{\partial p}{\partial z} = - \frac{pg}{RT}$$

$$\text{or } \frac{\partial p}{p} = - \frac{g}{RT} dz$$

**Question:** *Stating the underlying assumptions show that the temperature variation in an atmosphere can expressed by*

<b>Pressure and Its Measurements</b> <b>S K Mondal's</b> <span style="float: right;"><b>Chapter 2</b></span>
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*[the value of  $\gamma$  varies with height]* [AMIE- WINTER]

$$\frac{dT}{dz} = -\frac{g}{R} \cdot \left( \frac{\gamma-1}{\gamma} \right)$$

**Answer:** **-2002]**

Assumptions:

(i) Air is an Ideal fluid.

(ii) Temperature variation follows adiabatic process.

We know that temperature at any point in compressible fluid in an adiabatic process.

$$T = T_0 \left[ 1 - \frac{\gamma-1}{\gamma} \cdot \frac{gz}{RT_0} \right]$$

$$\therefore \frac{dT}{dz} = -\frac{g}{R} \cdot \frac{\gamma-1}{\gamma}$$

**Note:** If allotted marks is high for the question then more then proof from Aerostatic law also.

$$T = T_0 \left( 1 - \frac{\gamma-1}{\gamma} \cdot \frac{gz}{RT_0} \right) \quad \left( \frac{\partial p}{\partial z} = -\rho g \right)$$

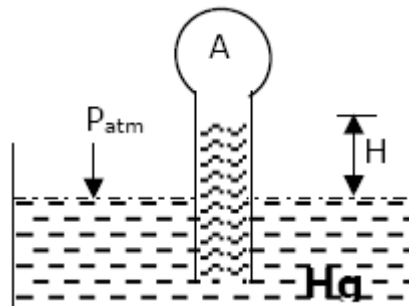
## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Absolute and Gauge Pressures

**GATE-1.** In given figure, if the pressure of gas in bulb A is 50 cm Hg vacuum and  $P_{\text{atm}} = 76$  cm Hg, then height of column H is equal to

- (a) 26 cm                      (b) 50 cm  
(c) 76 cm                      (d) 126 cm



[GATE-2000]

**GATE-1. Ans. (b)** If the pressure of gas in bulb A is atm.  $H = \text{zero}$ .

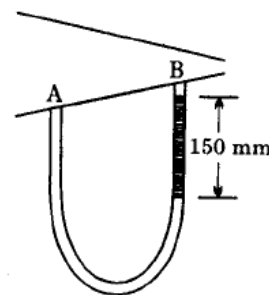
If we create a pressure of gas in bulb A is 1 cm Hg vacuum then the vacuum will lift 1 cm liquid,  $H = 1$  cm.

If we create a pressure of gas in bulb A is 2 cm Hg vacuum then the vacuum will lift 2 cm liquid,  $H = 2$  cm.

If we create a pressure of gas in bulb A is 50 cm Hg vacuum then the vacuum will lift 50 cm liquid,  $H = 50$  cm.

#### Manometers

**GATE-2.** A U-tube manometer with a small quantity of mercury is used to measure the static pressure difference between two locations A and B in a conical section through which an incompressible fluid flows. At a particular flow rate, the mercury column appears as shown in the figure. The density of mercury is  $13600 \text{ Kg/m}^3$  and  $g = 9.81 \text{ m/s}^2$ . Which of the following is correct?



- (a) Flow Direction is A to B and  $P_A - P_B = 20 \text{ KPa}$   
(b) Flow Direction is B to A and  $P_A - P_B = 1.4 \text{ KPa}$   
(c) Flow Direction is A to B and  $P_B - P_A = 20 \text{ KPa}$   
(d) Flow Direction is B to A and  $P_B - P_A = 1.4 \text{ KPa}$

[GATE-2005]

**GATE-2. Ans. (a)**

and as

$$P_B + 150 \text{ mm} - Hg = P_A \text{ Or } P_A - P_B = 0.150 \times 13600 \times 9.81 \approx 20 \text{ kPa}$$

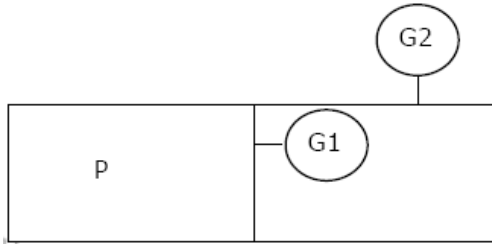
$P_A$  is greater than  $P_B$  therefore flow direction is A to B.

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**GATE-3.** The pressure gauges  $G_1$  and  $G_2$  installed on the system show pressures of  $P_{G1} = 5.00$  bar and  $P_{G2} = 1.00$  bar. The value of unknown pressure  $P$  is? (Atmospheric pressure 1.01 bars)

- (a) 1.01 bar  
(b) 2.01 bar  
(c) 5.00 bar  
(d) 7.01 bar

[GATE-2004]



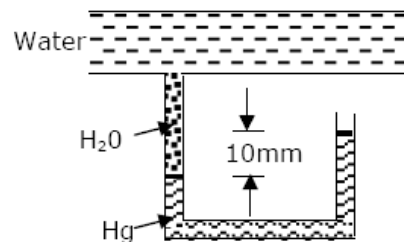
**GATE-3. Ans. (d)** Pressure in the right cell =  $P_{G_2}$  + Atmospheric pressure

$$= 1.01 + 1.0 = 2.01 \text{ bar}$$

Therefore  $P = P_{G_1}$  + Pressure on right cell = 5 + 2.01 = 7.01 bar

**GATE-4.** A mercury manometer is used to measure the static pressure at a point in a water pipe as shown in Figure. The level difference of mercury in the two limbs is 10 mm. The gauge pressure at that point is

- (a) 1236 Pa (b) 1333 Pa  
(c) Zero (d) 98 Pa



[GATE-1996]

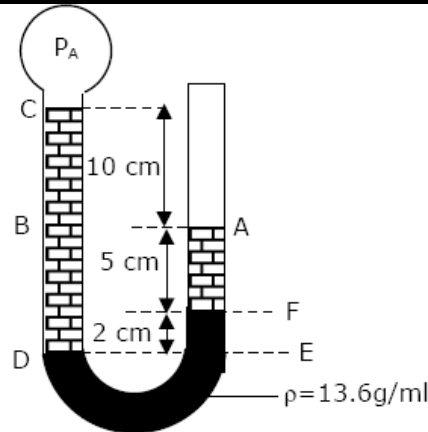
**GATE-4. Ans. (a)**

$$h = y \left( \frac{s_h}{s_l} - 1 \right) \text{ m of light fluid or } h = 0.010 \left( \frac{13.6}{1} - 1 \right) = 0.126 \text{ m of water column}$$

$$\text{or } P = h\rho g = 0.126 \times 1000 \times 9.81 = 1236 \text{ N/m}^2 = 1236 \text{ Pa}$$

**GATE-5.** Refer to Figure, the absolute pressure of gas A in the bulb is:

- (a) 771.2 mm Hg
- (b) 752.65 mm Hg
- (c) 767.35 mm Hg
- (d) 748.8 mm Hg



[GATE-1997]

**GATE-5. Ans. (a)** Use 'hs' formula;

$$H_A + 170 \times 1 - 20 \times 13.6 - 50 \times 1 = h_{atm.} (760 \times 13.6) \text{ [All mm of water]}$$

$$\text{Or } H_A = 10488 / 13.6 \text{ mm of Hg} = 771.2 \text{ mm of Hg (Abs.)}$$

## Mechanical Gauges

**GATE-6.** A siphon draws water from a reservoir and discharges it out at atmospheric pressure. Assuming ideal fluid and the reservoir is large, the velocity at point P in the siphon tube is: [GATE-2006]

- (a)  $\sqrt{2gh_1}$
- (b)  $\sqrt{2gh_2}$
- (c)  $\sqrt{2g(h_2 - h_1)}$
- (d)  $\sqrt{2g(h_2 + h_1)}$

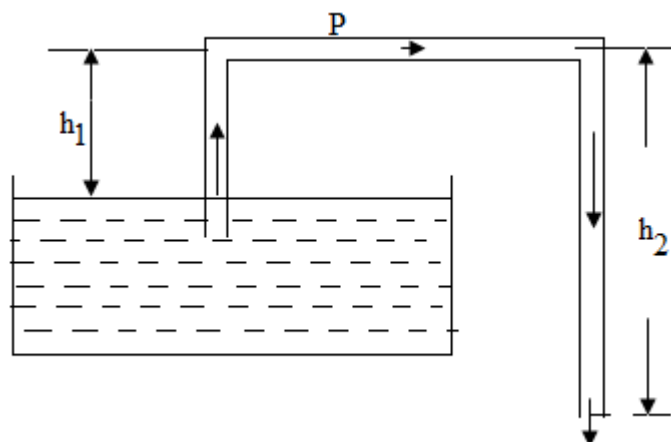
**GATE-6. Ans. (c)** By energy conservation, velocity at point Q =

$$\sqrt{2g(h_2 - h_1)}$$

As there is a continuous and uniform flow, so velocity of liquid at point Q and P is same.

$$V_p =$$

$$\sqrt{2g(h_2 - h_1)}$$



<b>Pressure and Its Measurements</b> <b>S K Mondal's</b> <b>Chapter 2</b>
<b>Previous 20-Years IES Questions</b>

## Pressure of a Fluid

**IES-1.** A beaker of water is falling freely under the influence of gravity. Point B is on the surface and point C is vertically below B near the bottom of the beaker. If  $P_B$  is the pressure at point B and  $P_C$  the pressure at point C, then which one of the following is correct? [IES-2006]

- (a)  $P_B = P_C$                       (b)  $P_B < P_C$                       (c)  $P_B > P_C$                       (d) Insufficient data

**IES-1. Ans. (a)** For free falling body relative acceleration due to gravity is zero  
 $P = \rho gh$  if  $g=0$  then  $p=0$  (but it is only hydrostatic pr.) these will be  
 $\therefore \rho$

atmospheric pressure through out the liquid.

**IES-2.** Assertion (A): If a cube is placed in a liquid with two of its surfaces parallel to the free surface of the liquid, then the pressures on the two surfaces which are parallel to the free surface, are the same. [IES-2000]

Reason (R): Pascal's law states that when a fluid is at rest, the pressure at any plane is the same in all directions.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-2. Ans. (d)**

**IES-3.** In an open U tube containing mercury, kerosene of specific gravity 0.8 is poured into one of its limbs so that the length of column of kerosene is about 40 cm. The level of mercury column in that limb is lowered approximately by how much? [IES-2008]

- (a) 2.4 cm                      (b) 1.2 cm                      (c) 3.6 cm                      (d) 0.6 cm

**IES-3. Ans. (b)**  $0.8 \times 40 = 13.6 \times (2h) \Rightarrow h = 1.2 \text{ cm}$

## Hydrostatic Law and Aerostatic Law

**IES-4.** Hydrostatic law of pressure is given as [IES 2002; IAS-2000]

- (a)  $\frac{\partial p}{\partial z} = \rho g$                       (b)  $\frac{\partial p}{\partial z} = 0$                       (c)  $\frac{\partial p}{\partial z} = z$                       (d)  $\frac{\partial p}{\partial z} = \text{const.}$

**IES-4. Ans. (a)**

# Pressure and Its Measurements

## S K Mondal's Chapter 2

IES-5. If  $z$  is vertically upwards,  $\rho$  is the density and  $g$  gravitational acceleration (see figure) then the

$$\partial p / \partial z$$

pressure in a fluid at rest due to gravity is given by

- (a)  $\rho g z^2 / 2$       (b)  $-\rho g$   
 (c)  $-\rho g z$       (d)  $-\rho g / z$

[IES-1995; 1996]

IES-5. Ans. (b) Pressure at any point at depth  $z$  due to gravitational acceleration is,  $p = \rho g h$ . Since  $z$  is vertically upwards,  $\partial p / \partial z = -\rho g$

$$\partial p / \partial z$$

## Absolute and Gauge Pressures

IES-6. The standard atmospheric pressure is 762 mm of Hg. At a specific location, the barometer reads 700 mm of Hg. At this place, what does at absolute pressure of 380 mm of Hg correspond to? [IES-2006]

- (a) 320 mm of Hg vacuum      (b) 382 of Hg vacuum  
 (c) 62 mm of Hg vacuum      (d) 62 mm of Hg gauge

IES-6. Ans. (a)

## Manometers

IES-7. The pressure difference of two very light gasses in two rigid vessels is being measured by a vertical U-tube water filled manometer. The reading is found to be 10 cm. what is the pressure difference? [IES-2007]

- (a) 9.81 kPa      (b) 0.0981 bar      (c) 98.1 Pa  
 (d) 981 N/m<sup>2</sup>

IES-7. Ans. (d)  $p = \rho g h = 1000 \times 9.81 \times 0.1 = 981 \text{ N/m}^2$

$$\Delta \Delta \times \times \times \times$$

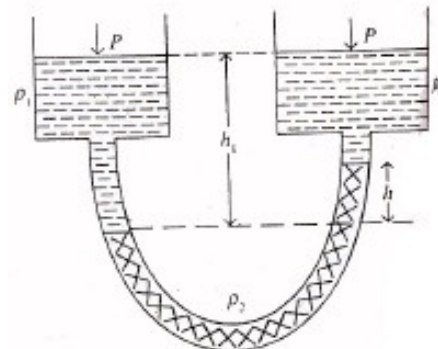
IES-8. The balancing column shown in the diagram contains 3 liquids of different densities,  $\rho_1$ ,  $\rho_2$ , and  $\rho_3$ . The

$$\rho_1 \quad \rho_2 \quad \rho_3$$

liquid level of one limb is  $h_1$  below the top level and there is a difference of  $h$  relative to that in the other limb.

What will be the expression for  $h$ ?

- (a)      (b)



[IES-2004]



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(c)

$$\frac{\rho_1 - \rho_3}{\rho_2 - \rho_3} h_1$$

(d)

$$\frac{\rho_1 - \rho_2}{\rho_2 - \rho_3} h_1$$

**IES-8. Ans. (c)**

$$h_1 \rho_1 = h \rho_2 + (h_1 - h) \rho_3$$

**IES-9. A mercury-water manometer has a gauge difference of 500 mm (difference in elevation of menisci). What will be the difference in pressure? [IES-2004]**

(a) 0.5 m

(b) 6.3 m

(c) 6.8 m

(d) 7.3 m

**IES-9. Ans. (b)**

$$h = y \left( \frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.5 \left( \frac{13.6}{1} - 1 \right) = 6.3 \text{ m of water.}$$

**IES-10. To measure the pressure head of the fluid of specific gravity  $S$  flowing through a pipeline, a simple micro-manometer containing a fluid of specific gravity  $S_1$  is connected to it. The readings are as indicated as the diagram. The pressure head in the pipeline is:**

(a)  $h_1 S_1 - h S - \frac{\Delta h}{2} (S_1 - S)$

$\Delta$

(b)  $h_1 S_1 - h S + \frac{\Delta h}{2} (S_1 - S)$

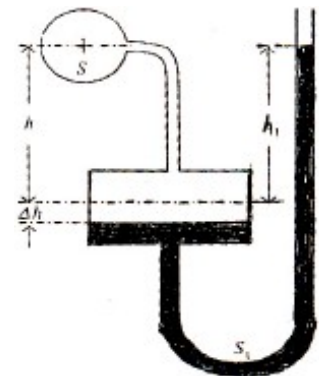
$\Delta$

(c)  $h S - h_1 S_1 - \frac{\Delta h}{2} (S_1 - S)$

$\Delta$

(d)  $h S - h_1 S_1 + \frac{\Delta h}{2} (S_1 - S)$

$\Delta$



**[IES-2003]**

**IES-10. Ans. (a)** Use 'hs' rules;

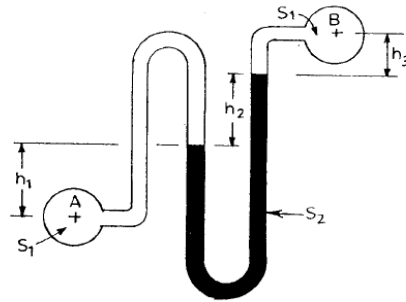
The pressure head in the pipeline ( $H_p$ )

$$H_p + h S + \frac{\Delta h S}{2} - \frac{\Delta h S_1}{2} - h_1 S_1 = 0 \text{ or } H_p = h_1 S_1 - h S - \frac{\Delta h}{2} (S_1 - S)$$

# Pressure and Its Measurements

## S K Mondal's Chapter 2

**IES-11.** Two pipe lines at different pressures,  $P_A$  and  $P_B$ , each carrying the same liquid of specific gravity  $S_1$ , are connected to a U-tube with a liquid of specific gravity  $S_2$  resulting in the level differences  $h_1$ ,  $h_2$  and  $h_3$  as shown in the figure. The difference in pressure head between points A and B in terms of head of water is:



[IES-1993]

- (a)  $h_1 S_2 + h_2 S_1 + h_3 S_1$  (b)  $h_1 S_1 + h_2 S_2 - h_3 S_1$   
 (c)  $h_1 S_1 - h_2 S_2 - h_3 S_1$  (d)  $h_1 S_1 + h_2 S_2 + h_3 S_1$

**IES-11. Ans. (d)** Using hS formula:  $P_A$  and  $P_B$  (in terms of head of water)

$$P_A - h_1 S_1 - h_2 S_2 - h_3 S_1 = P_B \text{ or } P_A - P_B = h_1 S_1 + h_2 S_2 + h_3 S_1$$

**IES-12.** Pressure drop of flowing through a pipe (density  $1000 \text{ kg/m}^3$ ) between two points is measured by using a vertical U-tube manometer. Manometer uses a liquid with density  $2000 \text{ kg/m}^3$ . The difference in height of manometric liquid in the two limbs of the manometer is observed to be 10 cm. The pressure drop between the two points is:

[IES-2002]

- (a)  $98.1 \text{ N/m}^2$  (b)  $981 \text{ N/m}^2$  (c)  $1962 \text{ N/m}^2$   
 (d)  $19620 \text{ N/m}^2$

**IES-12. Ans. (b)**

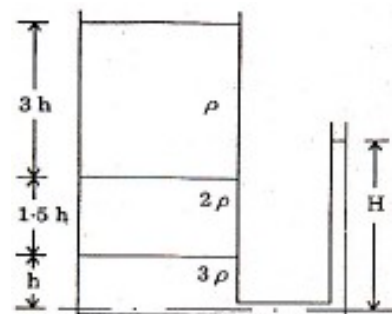
$$h = y \left( \frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.1 \left( \frac{2}{1} - 1 \right) = 0.1 \text{ m of light fluid}$$

$$\text{The pressure drop between the two points is } = h \rho g = 0.1 \times 9.81 \times 1000 = 981 \text{ N/m}^2$$

**IES-13.** Three immiscible liquids of specific densities  $\rho$ ,  $2\rho$  and  $3\rho$  are kept in a

jar. The height of the liquids in the jar and at the piezometer fitted to the bottom of the jar is as shown in the given figure. The ratio  $H/h$  is :

- (a) 4 (b) 3.5  
 (c) 3 (d) 2.5



[IES-2001]

**IES-13. Ans. (c)** Use 'hs' formula

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$$3h \times \rho + 1.5h \times 2\rho + h \times 3\rho - H \times 3\rho = 0 \text{ Or } H/h = 3$$

**IES-14. Differential pressure head measured by mercury oil differential manometer (specific gravity of oil is 0.9) equivalent to a 600 mm difference of mercury levels will nearly be: [IES-2001]**

- (a) 7.62 m of oil (b) 76.2 m of oil  
(c) 7.34 m of oil (d) 8.47 m of oil

**IES-14. Ans. (d)**

$$h = y \left( \frac{S_h}{S_l} - 1 \right) \text{ m of light fluid or } h = 0.600 \left( \frac{13.6}{0.9} - 1 \right) = 8.47 \text{ m of oil}$$

**IES-15. How is the difference of pressure head, "h" measured by a mercury-oil differential manometer expressed? [IES-2008]**

- (a)  $h = x \left[ 1 - \frac{S_g}{S_o} \right]$  (b)  $h = x [S_g - S_o]$   
(c)  $h = x [S_o - S_g]$  (d)  $h = x \left[ \frac{S_g}{S_o} - 1 \right]$

Where x = manometer reading;  $S_g$  and  $S_o$  are the specific gravities of mercury and oil, respectively.

**IES-15. Ans. (d) Measurement of h using U tube manometer**

**Case 1.** When specific gravity of manometric liquid is more than specific gravity of liquid flowing

$$h = y \left( \frac{S_g}{S_o} - 1 \right)$$

In m of liquid flowing through pipe ( i.e. m of light liquid)

**Case 2.** When specific gravity of manometric fluid is less than the specific gravity of liquid flowing.

$$h = y \left( 1 - \frac{S_g}{S_o} \right)$$

In m of liquid flowing through pipe (i.e. m of heavy liquid)

**IES-16. The differential manometer connected to a Pitot static tube used for measuring fluid velocity gives [IES-1993]**

- (a) Static pressure (b) Total pressure  
(c) Dynamic pressure (d) Difference between total pressure and dynamic pressure

**IES-16. Ans. (c)** Fig. 6 shows a Pitot static tube used for measuring fluid velocity in a pipe and connected through points A and B to a differential manometer.

Point A measures velocity head

$$\frac{V^2}{2g}$$

+ static pressure.

Whereas point B senses static pressure.

In actual practice point B is within the tube and not separate on the pipe. Thus manometer reads only dynamic pressure ( )

$$\frac{V^2}{2g}$$

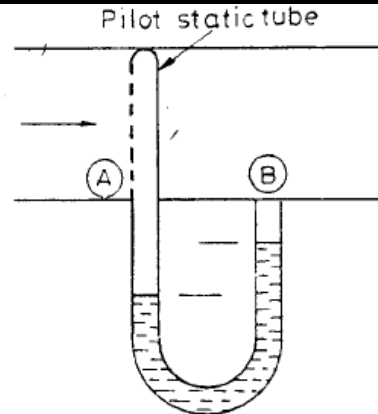


Fig. 6

**IES-17. Assertion (A): U-tube manometer connected to a venturimeter fitted in a pipeline can measure the velocity through the pipe.**

**[IES-1996]**

**Reason (R): U-tube manometer directly measures dynamic and static heads.**

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**IES-17. Ans. (a)**

**IES-18. Pressure drop of water flowing through a pipe (density 1000 kg/m<sup>3</sup>) between two points is measured by using a vertical U-tube manometer. Manometer uses a liquid with density 2000 kg / m<sup>3</sup>. The difference in height of manometric liquid in the two limbs of the manometer is observed to be 10 cm. The pressure drop between the two points is:**

**[IES-2002]**

(a) 98.1 N/m<sup>2</sup>

(b) 981 N/m<sup>2</sup>

(c) 1962 N/m<sup>2</sup>

(d) 19620 N/m<sup>2</sup>

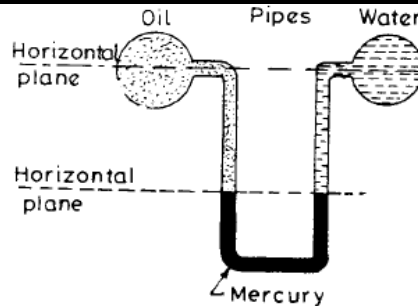
**IES-18. Ans.**

**(b)** 
$$\frac{P_1 - P_2}{\rho g} = h \left( \frac{S_m}{S} - 1 \right)$$

$$(P_1 - P_2) = 1000 \times 0.1(2 - 1) = 981 \text{ N / m}^2$$

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**IES-19.** The manometer shown in the given figure connects two pipes, carrying oil and water respectively. From the figure one



- (a) Can conclude that the pressures in the pipes are equal.
- (h) Can conclude that the pressure in the oil pipe is higher.
- (c) Can conclude that the pressure in the water pipe is higher.
- (d) Cannot compare the pressure in the two pipes for want of sufficient data.

[IES-1996]

**IES-19. Ans. (b)** Oil has density lower than that of water. Thus static head of oil of same height will be lower. Since mercury is at same horizontal plane in both limbs, the lower static head of oil can balance higher static head of water when oil pressure in pipe is higher.

**IES-20.** In order to increase sensitivity of U-tube manometer, one leg is usually inclined by an angle  $\theta$ . What is the sensitivity of inclined

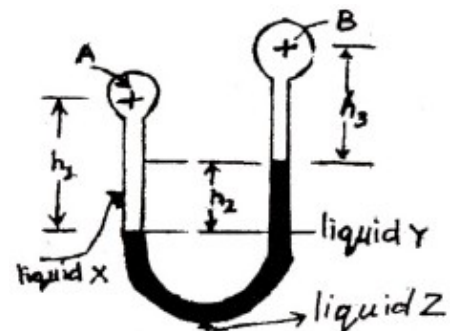
tube compared to sensitivity of U-tube?

[IES-2009]

- (a)  $\sin \theta$
- (b)  $\frac{1}{\sin \theta}$
- (c)  $\frac{1}{\cos \theta}$
- (d)  $\tan \theta$

**IES-20. Ans. (b)**

**IES-21.** A differential manometer is used to measure the difference in pressure at points A and B in terms of specific weight of water,  $W$ . The specific gravities of the liquids X, Y and Z are respectively  $s_1$ ,  $s_2$  and  $s_3$ . The correct difference is given by :



$$\left( \frac{P_A}{W} - \frac{P_B}{W} \right)$$

[IES-

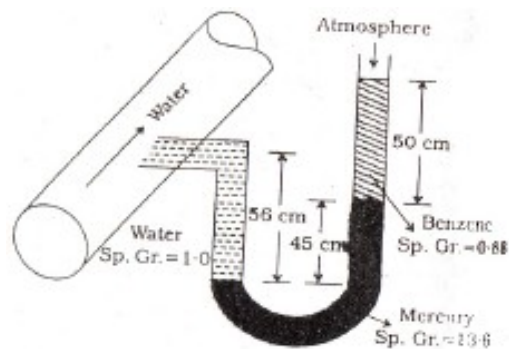
1997]

- (a)  $h_3 s_2 - h_1 s_1 + h_2 s_3$
- (b)  $h_1 s_1 + h_2 s_3 - h_3 s_2$
- (c)  $h_3 s_1 - h_1 s_2 + h_2 s_3$
- (d)  $h_1 s_1 + h_2 s_2 - h_3 s_3$

**IES-21. Ans (a)** Use 'hs' formula

$$\frac{P_A}{W} + h_1 s_1 - h_2 s_3 - h_3 s_2 = \frac{P_B}{W} \text{ Or } \frac{P_A}{W} - \frac{P_B}{W} = h_3 s_2 - h_1 s_1 + h_2 s_3$$

- IES-22.** A U-tube manometer is connected to a pipeline conveying water as shown in the Figure. The pressure head of water in the pipeline is  
 (a) 7.12 m (b) 6.56 m  
 (c) 6.0 m (d) 5.12 m

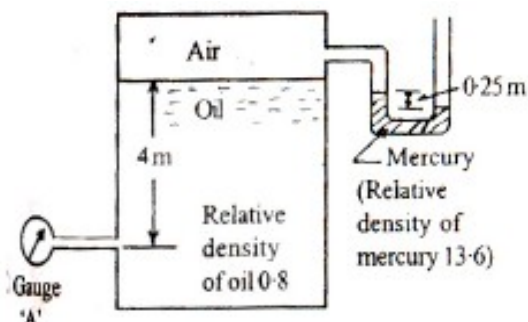


[IES-2000]

- IES-22. Ans. (c)** Use 'hs' formula;

$$H + 0.56 \times 1 - 0.45 \times 13.6 - 0.5 \times 0.88 = 0$$

- IES-23.** The reading of gauge 'A' shown in the given figure is:  
 (a) -31.392 kPa  
 (b) -1.962 kPa  
 (c) 31.392 kPa  
 (d) +19.62 kPa



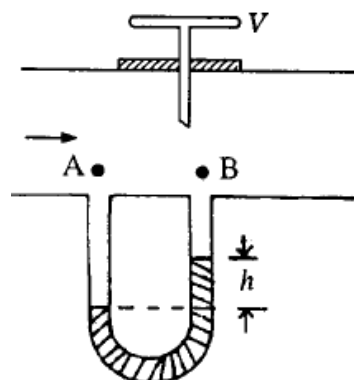
[IES-1999]

- IES-23. Ans. (b)** Use 'hs' formula;

$$H_A - 4 \times 0.8 + 0.25 \times 13.6 = 0 \text{ Or } H_A = -0.2 \text{ m of water column}$$

$$= -0.2 \times 9.81 \times 1000 \text{ N/m}^2 = -1.962 \text{ kPa}$$

- IES-24.** A mercury manometer is fitted to a pipe. It is mounted on the delivery line of a centrifugal pump. One limb of the manometer is connected to the upstream side of the pipe at 'A' and the other limb at 'B', just below the valve 'V' as shown in the figure. The manometer reading 'h' varies with different valve positions.



**Assertion (A):** With gradual closure of the valve, the magnitude of 'h' will go on increasing and even a situation may arise when mercury will be sucked in by the water flowing around 'B'.

**Reason (R):** With the gradual closure of the valve, the pressure at 'A' will go on increasing.

[IES-1998]

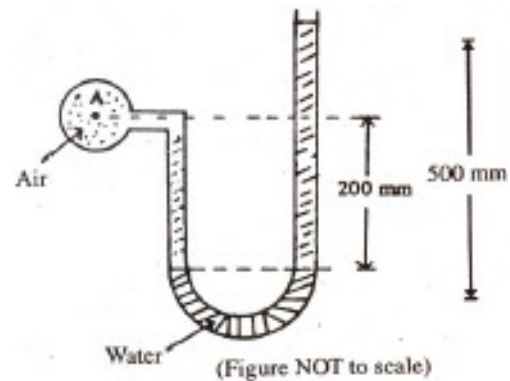
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- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-24. Ans. (a)** With gradual closure the valve, the valve will be restricted the flow of liquid. Then pressure at A will be increased.

**IES-25. In the figure shown below air is contained in the pipe and water is the manometer liquid. The pressure at 'A' is approximately:**

- (a) 10.14 m of water absolute  
 (b) 0.2 m of water  
 (c) 0.2 m of water vacuum  
 (d) 4901 pa



**[IES-1998]**

**IES-25. Ans. (d)** Use 'hs' formula;

$$H_{air} + 0.2 \times S_{air} (1.3/1000) - 0.5 \times 1 = 0 \text{ or } H_{air} = 0.49974 \text{ m of water column (Gauge)}$$

$$= 0.49974 \times 9.81 \times 1000 \text{ Pa} = 4902 \text{ Pa (gauge)}$$

It is not 10.14 m of absolute because atmospheric pressure = 10.33 m of water column if we add 0.49974 m gauge with atmosphere it will give us 10.83 m of absolute pressure. Without any calculation we are able to give the same answer as elevation of point A is lower than right limb then pressure at point A will be more than atmospheric (10.33m of water column).

**IES-26. A manometer is made of a tube of uniform bore of 0.5 cm<sup>2</sup> cross-sectional area, with one limb vertical and the other limb inclined at 30° to the horizontal. Both of its limbs are open to atmosphere and, initially, it is partly filled with a manometer liquid of specific gravity 1.25. If then an additional volume of 7.5 cm<sup>3</sup> of water is poured in the inclined tube, what is the rise of the meniscus in the vertical tube?**

**[IES-2006]**

- (a) 4 cm                                      (b) 7.5 cm                                      (c) 12 cm                                      (d) 15 cm

**IES-26. Ans. (a)** Let 'x' cm will be rise of the meniscus in the vertical tube. So for this 'x' cm rise quantity of 1.25 s.g. liquid will come from inclined limb. So we have to lower our reference line =  $x \sin 30^\circ = x/2$ . Then Pressure balance gives us

$$\left(x + \frac{x}{2}\right) \times 1250 \times 9.81 = \left(\frac{7.5}{0.5}\right) \sin 30^\circ \times 1000 \times 9.81 \quad \text{or } x = 4$$

**IES-27. The lower portion of a U-tube of uniform bore, having both limbs vertical and open to atmosphere, is initially filled with a liquid of**

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specific gravity 3S. A lighter liquid of specific gravity S is then poured into one of the limbs such that the length of column of lighter liquid is X. What is the resulting movement of the meniscus of the heavier liquid in the other limb? [IES-2008]

- (a) X (b) X/2 (c) X/3  
(d) X/6

**IES-27. Ans. (d)**  $(s) \times (x) = (3s) \times (y)$

$$\therefore y = \frac{x}{3}$$

Resulting movement of meniscus =

$$\frac{x}{6}$$

## Piezometer

**IES-28.** A vertical clean glass tube of uniform bore is used as a piezometer to measure the pressure of liquid at a point. The liquid has a specific weight of 15 kN/m<sup>3</sup> and a surface tension of 0.06 N/m in contact with air. If for the liquid, the angle of contact with glass is zero and the capillary rise in the tube is not to exceed 2 mm, what is the required minimum diameter of the tube? [IES-2006]

- (a) 6 mm (b) 8 mm (c) 10 mm (d) 12 mm

**IES-28. Ans. (b)**

$$h = \frac{4\sigma \cos \theta}{\rho g d} \leq 0.002 \quad \text{or} \quad d \geq \frac{4 \times 0.06 \times \cos 0^\circ}{15000 \times 0.002} = 8 \text{ mm}$$

**IES-29.** When can a piezometer be not used for pressure measurement in pipes?

- (a) The pressure difference is low [IES-2005]  
(b) The velocity is high  
(c) The fluid in the pipe is a gas  
(d) The fluid in the pipe is highly viscous

**IES-29. Ans. (c)**

## Mechanical Gauges

**IES-30.** In a pipe-flow, pressure is to be measured at a particular cross-section using the most appropriate instrument. Match List-I (Expected pressure range) with List-II (Appropriate measuring device) and select the correct answer: [IES-2002]

- | List-I  | List-II                          |
|---|----------------------------------|
| <b>A.</b> Steady flow with small positive gauge pressure              | <b>1.</b> Bourdon pressure gauge |
| <b>B.</b> Steady flow with small negative and positive gauge pressure | <b>2.</b> Pressure transducer    |
|   | <b>3.</b> Simple piezometer      |



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- C. Steady flow with high gauge pressure      4. U-tube manometer  
D.

Unsteady flow with fluctuating pressure

Codes:	A	B	C	D		A	B	C	D
(a)	3	2	1	4	(b)	1	4	3	2
(c)	3	4	1	2	(d)	1	2	3	4

IES-30. Ans. (c)

## Previous 20-Years IAS Questions

### Pressure of a Fluid

- IAS-1.** The standard sea level atmospheric pressure is equivalent to  
 (a) 10.2 m of fresh water of  $\rho = 998 \text{ kg/m}^3$  [IAS-2000]  
 (b) 10.1 m of salt water of  $\rho = 1025 \text{ kg/m}^3$   
 (c) 12.5 m of kerosene of  $\rho = 800 \text{ kg/m}^3$   
 (d) 6.4 m of carbon tetrachloride of  $\rho = 1590 \text{ kg/m}^3$

**IAS-1. Ans. (b)**  $\rho gh$  must be equal to  $1.01325 \text{ bar} = 101325 \text{ N/m}^2$   
 For (a)

$$998 \times 9.81 \times 10.2 = 99862 \text{ N/m}^2$$

(b)  $1025 \times 9.81 \times 10.1 = 101558 \text{ N/m}^2$

(c)  $800 \times 9.81 \times 12.5 = 98100 \text{ N/m}^2$

(d)  $1590 \times 9.81 \times 6.4 = 99826 \text{ N/m}^2$

### Hydrostatic Law and Aerostatic Law

- IAS-2.** Hydrostatic law of pressure is given as [IES 2002; IAS-2000]  
 (a)  $\frac{\partial p}{\partial z} = \rho g$       (b)  $\frac{\partial p}{\partial z} = 0$       (c)  $\frac{\partial p}{\partial z} = z$       (d)  $\frac{\partial p}{\partial z} = \text{const.}$

**IAS-2. Ans. (a)**

- IAS-3.** Match List-I (Laws) with List-II (Phenomena) and select the correct answer using the codes given below the lists: [IAS-1996]

**List-I**

**A.** Hydrostatic law

**List-II**

**1.** Pressure at a point is equal in all directions in a fluid at rest

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- |   |  |
|---|--|
| <b>B.</b> Newton's law<br><br><b>C.</b> Pascal's law<br><br><b>D.</b> Bernoulli's law | <b>2.</b> Shear stress is directly proportional to velocity gradient in fluid flow<br><b>3.</b> Rate of change of pressure in a vertical direction is proportional to specific weight of fluid |
|---|--|

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	3	-	1	(b)	3	2	1	-
(c)	2	-	1	3	(d)	2	1	-	3

**IAS-3. Ans. (b)**

## Absolute and Gauge Pressures

- IAS-4.** The reading of the pressure gauge fitted on a vessel is 25 bar. The atmospheric pressure is 1.03 bar and the value of  $g$  is  $9.81 \text{ m/s}^2$ . The absolute pressure in the vessel is: [IAS-1994]  
 (a) 23.97 bar (b) 25.00 bar (c) 26.03 bar (d) 34.84 bar

**IAS-4. Ans. (c)** Absolute pressure = Atmospheric pressure + Gauge Pressure  
 =  $25 + 1.03 = 26.03 \text{ bar}$

- IAS-5.** The barometric pressure at the base of a mountain is 750 mm Hg and at the top 600 mm Hg. If the average air density is  $1 \text{ kg/m}^3$ , the height of the mountain is approximately [IAS-1998]  
 (a) 2000 m (b) 3000 m (c) 4000 m (d) 5000 m

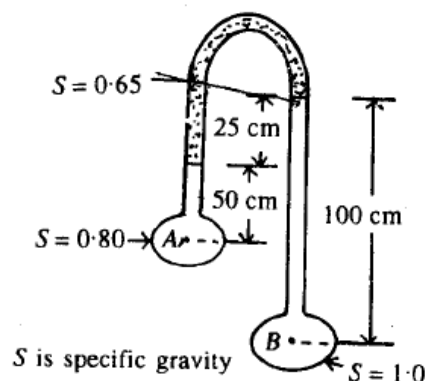
**IAS-5. Ans. (a)** Pressure difference =  $(750 - 600) = 150 \text{ mmHg}$  H m of air column  

$$= \text{height of mountain.} \quad \text{Therefore}$$

$$0.150 \times (13.6 \times 10^3) \times g = H \times 1 \times g \quad \text{or} \quad H = 2040 \text{ m} \approx 2000 \text{ m}$$

## Manometers

- IAS-6.** The pressure difference between point B and A (as shown in the above figure) in centimeters of water is:  
 (a) -44 (b) 44  
 (c) -76 (d) 76



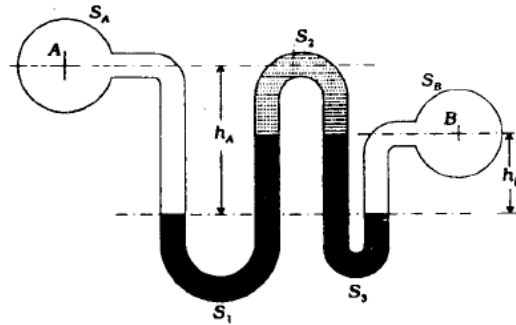
[IAS-2002]

**IAS-6. Ans. (b)** Use 'hs' formula

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$$h_A - 50 \times 0.8 - 25 \times 0.65 + 100 \times 1 = h_B \text{ or } h_B - h_A = 43.75 \text{ cm of water column}$$

**IAS-7.** A double U-tube manometer is connected to two liquid lines A and B. Relevant heights and specific gravities of the fluids are shown in the given figure. The pressure difference, in head of water, between fluids at A and B is



[IAS-2001]

- (a)  $S_A h_A + S_1 h_B - S_3 h_B + S_B h_B$   
 (b)  $S_A h_A - S_1 h_B - S_2 (h_A - h_B) + S_3 h_B - S_B h_B$   
 (c)  $S_A h_A + S_1 h_B + S_2 (h_A - h_B) - S_3 h_B + S_B h_B$   
 (d)

$$h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$$

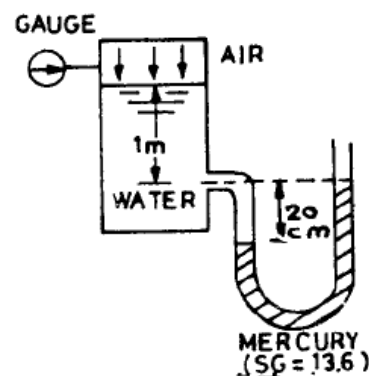
**IAS-7. Ans. (d)** Use 'hs' formula

$$H_A + h_A S_A - (h_A - h_B) S_1 + (h_A - h_B) S_3 - h_B S_B = H_B$$

$$\text{or, } H_B - H_A = h_A S_A - (h_A - h_B)(S_1 - S_3) - h_B S_B$$

**IAS-8.** The pressure gauge reading in meter of water column shown in the given figure will be

- (a) 3.20 m                      (b) 2.72 m  
 (c) 2.52 m                      (d) 1.52 m



[IAS-1995]

**IAS-8. Ans. (d)** Use 'hs' formula;

$$H_G + 1 \times 1 + 0.2 \times 1 - 0.2 \times 13.6 = 0 \text{ or } H_G = 1.52 \text{ m of water column}$$

## Mechanical Gauges

**IAS-9.** Match List-I with List-II and select the correct answer using the codes given below the lists: [IAS-1999]

**List-I (Device)**

- A.** Barometer
- B.** Hydrometer
- C.** U-tube manometer
- D.** Bourdon gauge

**Codes:**

- |            | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> |
|------------|----------|----------|----------|----------|
| <b>(a)</b> | <b>2</b> | <b>3</b> | <b>1</b> | <b>4</b> |
| <b>(c)</b> | <b>3</b> | <b>2</b> | <b>4</b> | <b>1</b> |

**List-II (Use)**

- 1.** Gauge pressure
- 2.** Local atmospheric pressure
- 3.** Relative density
- 4.** Pressure differential

- |            | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> |
|------------|----------|----------|----------|----------|
| <b>(b)</b> | <b>3</b> | <b>2</b> | <b>1</b> | <b>4</b> |
| <b>(d)</b> | <b>2</b> | <b>3</b> | <b>4</b> | <b>1</b> |

**IAS-9. Ans. (d)**

## **3. Hydrostatic Forces on Surfaces**

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### **Contents of this chapter**

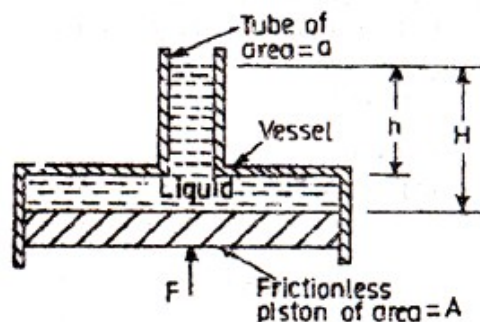
1. Hydrostatic forces on plane surface
  2. Hydrostatic forces on plane inclined surface
  3. Centre of pressure
  4. Hydrostatic forces on curved surface
  5. Resultant force on a sluice gate
  6. Lock gate
-

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

**GATE-1.** The force  $F$  needed to support the liquid of density  $\rho$  and the vessel on top is:

- (a)  $\rho g d [h a - (H - h) A]$   
 (b)  $\rho g d H A$   
 (c)  $\rho g d H a$   
 (d)  $\rho g d (H - h) A$



[GATE-1995]

**GATE-1. Ans. (b)**

**GATE-2.** A water container is kept on a weighing balance. Water from a tap is falling vertically into the container with a volume flow rate of  $Q$ ; the velocity of the water when it hits the water surface is  $U$ . At a particular instant of time the total mass of the container and water is  $m$ . The force registered by the weighing balance at this instant of time is:

[GATE-2003]

- (a)  $mg + \frac{QU}{\rho}$       (b)  $mg + 2 \frac{QU}{\rho}$       (c)  $mg + \frac{QU^2/2}{\rho}$       (d)  $\frac{QU^2/2}{\rho}$

$$\frac{QU^2/2}{\rho}$$

**GATE-2. Ans. (a)** Volume flow rate =  $Q$

$$\text{Mass of water strike} = \frac{Q}{\rho}$$

Velocity of the water when it hit the water surface =  $U$

Force on weighing balance due to water strike

$$= \text{Initial momentum} - \text{final momentum}$$

$$= \frac{QU}{\rho} - 0 = \frac{QU}{\rho}$$

$$\frac{QU}{\rho}$$

(Since final velocity is perpendicular to initial velocity)

$$\text{Now total force on weighing balance} = mg + \frac{QU}{\rho}$$

### Previous 20-Years IES Questions

**IES-1.** A tank has in its side a very small horizontal cylinder fitted with a frictionless piston. The head of liquid above the piston is  $h$  and the piston area  $a$ , the liquid having a specific weight  $\gamma$ . What is

$\gamma$

the force that must be exerted on the piston to hold it in position against the hydrostatic pressure?

[IES-2009]

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(a)  $2 \gamma ha$  (b)  $\gamma ha$  (c)  $\frac{2 \gamma ha}{3}$  (d)  $\gamma \frac{ha}{2}$  —

**IES-1. Ans. (b)** Pressure of the liquid above the piston =  $\gamma h$   
 Force exerted on the piston to hold it in position =  $\gamma ha$

**IES-2. Which one of the following statements is correct? [IES-2005]**  
**The pressure centre is:**

- (a) The cycloid of the pressure prism
- (b) A point on the line of action of the resultant force
- (c) At the centroid of the submerged area
- (d) Always above the centroid of the area

**IES-2. Ans. (b)**

**IES-3. A semi-circular plane area of diameter 1 m, is subjected to a uniform gas pressure of 420 kN/m<sup>2</sup>. What is the moment of thrust (approximately) on the area about its straight edge? [IES-2006]**

- (a) 35 kNm (b) 41 kNm (c) 55 kNm (d) 82 kNm

**IES-3. Ans. (a)** Force (P) = p.A =

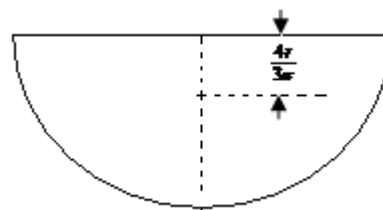
$$420 \times \frac{\pi \cdot 1^2}{4 \times 2}$$

Moment (M) =

$$P \times \bar{h}$$

=

$$420 \times \frac{\pi \times 1^2}{4 \times 2} \times \frac{4 \times (1/2)}{3 \times \pi} = 35 \text{ kNm}$$



**IES-4. A horizontal oil tank is in the shape of a cylinder with hemispherical ends. If it is exactly half full, what is the ratio of magnitude of the vertical component of resultant hydraulic thrust on one hemispherical end to that of the horizontal component? [IES-2006]**

- (a)  $\frac{2}{\pi}$  (b)  $\frac{1}{2}$  (c)  $\frac{4}{3\pi}$  (d)  $\frac{3}{4\pi}$

**IES-4. Ans. (b)** =

$$P_H = \rho g A \bar{x} = \rho g \left( \frac{\pi \cdot r^2}{4 \times 2} \right) \frac{4r}{3\pi} = \frac{2}{3} \rho g r^3$$

$$P_V = \rho g \nabla = \rho g \cdot \frac{1}{4} \left( \frac{4}{3} \pi r^3 \right) \therefore \frac{P_V}{P_H} = \frac{\pi}{2}$$

**IES-5. A rectangular water tank, full to the brim, has its length, breadth and height in the ratio of 2: 1: 2. The ratio of hydrostatic forces at the bottom to that at any larger vertical surface is: [IES-1996]**

- (a) 1/2 (b) 1 (c) 2 (d) 4

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**IES-5. Ans. (b)** Hydrostatic force at bottom = (length =  $2x$ ;

$$\rho g A \bar{z} = \rho g (2x \times 1x) \times 2x$$

breadth =  $1x$ ; height =  $2x$ ) =

$$4\rho g x^3$$

Hydrostatic force at larger vertical surface =  $\rho g (2x \times 2x) \times 2x/2 =$   
 $\times \times 4\rho g x^3$

$\therefore$  Ratio of above two forces = 1

**IES-6. A circular plate 1.5 m diameter is submerged in water with its greatest and least depths below the surface being 2 m and 0.75 m respectively. What is the total pressure (approximately) on one face of the plate?**

**[IES-2007, IAS-2004]**

- (a) 12kN                      (b) 16kN                      (c) 24kN                      (d) None of the above

**IES-6. Ans. (c)**

$$P = \rho g A \bar{x} = \rho g \left( \frac{\pi \times 1.5^2}{4} \right) \times \left( \frac{0.75 + 2}{2} \right) = 24 \text{ kN}$$

**IES-7. A tank with four equal vertical faces of width  $l$  and depth  $h$  is**

**filled up with a liquid. If the force on any vertical side is equal to the force at the bottom, then the value of  $h/l$  will be: [IAS-2000,**

**IES-1998]**

- (a) 2                      (b)  $\sqrt{2}$                       (c) 1                      (d) 1/2

**IES-7. Ans. (a)** or

$$P_{\text{bottom}} = P_{\text{side}} \quad h \rho g l t = \rho g t h (h/2) \text{ or } \frac{h}{t} = 2$$

**IES-8. The vertical component of the hydrostatic force on a submerged curved surface is the [IAS-1998, 1995, IES-1993, 2003]**

- (a) Mass of liquid vertically above it  
 (b) Weight of the liquid vertically above it  
 (c) Force on a vertical projection of the surface  
 (d) Product of pressure at the centroid and the surface area

**IES-8. Ans. (b)**

**IES-9. What is the vertical component of pressure force on submerged curved surface equal to? [IES-2008]**

- (a) Its horizontal component  
 (b) The force on a vertical projection of the curved surface  
 (c) The product of the pressure at centroid and surface area  
 (d) The gravity force of liquid vertically above the curved surface up to the free surface

**IES-9. Ans. (d)** The vertical component of the hydrostatic force on a submerged curved surface is the weight of the liquid vertically above it.



# Hydrostatic Forces on Surfaces

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[IAS-1998, 1995, IES-1993, 2003]

**IES-10. Resultant pressure of the liquid in case of an immersed body acts through which one of the following? [IES-2007]**

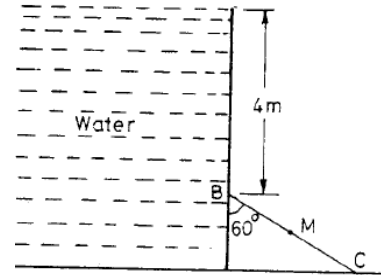
- (a) Centre of gravity (b) Centre of pressure  
(c) Metacentre (d) Centre of buoyancy

**IES-10. Ans. (b)**

**IES-11. In the situation shown in the given figure, the length BC is 3 m and M is the mid-point of BC. The hydrostatic force on BC measured per unit width (width being perpendicular to the plane of the paper) with 'g' being the acceleration due to gravity, will be**

- (a) 16500 g N/m passing through M  
(b) 16500 g N/m passing through a point between M and C  
(c) 14250 g N/m passing through M  
(d) 14250 g N/m passing through a point between M and C

**[IES-1993]**



**IES-11. Ans. (d)** The hydrostatic force on BC = . Vertical component ( $F_v$ )

$$\sqrt{F_v^2 + F_H^2}$$

= weight over area BC (for unit width)

$$= \left( \frac{4 + 5.5}{2} \right) \times \frac{3\sqrt{3}}{2} \times 10^3 g$$

$$= 12.34 \times 10^3 g \text{ N/m}$$

$F_H$  = Projected area of BC, i.e. BD  $\times$  depth upon centre of BD  $\times 10^3 g$

$$= 1.5 \times 4.75 \times 10^3 g \text{ (for unit width)}$$

$$= 7.125 \times 10^3 g \text{ N/m}$$

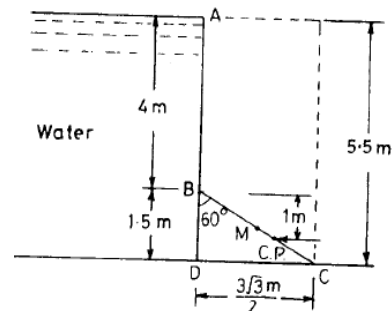
Resultant =

$$10^3 g \sqrt{12.34^2 + 7.125^2} = 14250 g \text{ N/m}$$

This resultant acts at centre of pressure, i.e., at of BD or between M

$$\frac{2}{3}$$

and C.



**IES-12. A circular annular plate bounded by two concentric circles of diameter 1.2m and 0.8 m is immersed in water with its plane making an angle of  $45^\circ$  with the horizontal. The centre of the circles is 1.625m below the free surface. What will be the total pressure force on the face of the plate? [IES-2004]**

- (a) 7.07 kN (b) 10.00 kN (c) 14.14 kN (d) 18.00 kN

# Hydrostatic Forces on Surfaces

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IES-12. Ans. (b) =

$$\rho g A \bar{x} = 1000 \times 9.81 \times \frac{\pi}{4} (1.2^2 - 0.8^2) \times 1.625 \approx 10 \text{ kN}$$

IES-13. A plate of rectangular shape having the dimensions of 0.4m x 0.6m is immersed in water with its longer side vertical. The total hydrostatic thrust on one side of the plate is estimated as 18.3 kN. All other conditions remaining the same, the plate is turned through 90° such that its longer side remains vertical. What would be the total force on one of the plate? [IES-2004]

- (a) 9.15 kN (b) 18.3 kN (c) 36.6 kN (d) 12.2 kN

IES-13. Ans. (b)

IES-14. Consider the following statements about hydrostatic force on a submerged surface: [IES-2003]

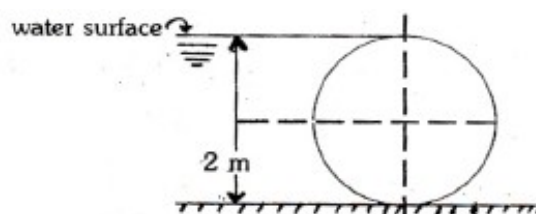
1. It remains the same even when the surface is turned.
2. It acts vertically even when the surface is turned.

Which of these is/are correct?

- (a) Only 1 (b) Only 2 (c) Both 1 and 2 (d) Neither 1 nor 2

IES-14. Ans. (a)

IES-15. A cylindrical gate is holding water on one side as shown in the given figure. The resultant vertical component of force of water per meter width of gate will be:



[IES-1997]

- (a) Zero (b) 7700.8 N/m (c) 15401.7 N/m (d) 30803.4 N/m

IES-15. Ans. (c) Vertical component of force = Weight of the liquid for half cylinder portion

$$= \text{Volume} \times \rho g = \left( \frac{\pi D^2}{4} \times \frac{L}{2} \right) \times \rho g = \left( \frac{\pi 2^2}{4} \times 1 \right) \times 1000 \times 9.81 = 15409.5 \text{ N/m}$$

IES-16. The vertical component of force on a curved surface submerged in a static liquid is equal to the [IES-1993]

- (a) Mass of the liquid above the curved surface
- (b) Weight of the liquid above the curved surface
- (c) Product of pressure at C.G. multiplied by the area of the curved surface
- (d) Product of pressure at C.G. multiplied by the projected area of the curved surface

IES-16. Ans. (b) The correct choice is (b) since the vertical component of force on a curved surface submerged in a static liquid is the weight of the liquid above the curved surface.

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**IES-17. The depth of centre of pressure for a rectangular lamina immersed vertically in water up to height 'h' is given by: [IES-2003]**

- (a)  $h/2$  (b)  $h/4$  (c)  $2h/3$   
 (d)  $3h/2$

**IES-17. Ans. (c)**

**IES-18. The point of application of a horizontal force on a curved surface submerged in liquid is: [IES-2003]**

- (a) (b) (c) (d)

$$\frac{I_G}{Ah} - \bar{h}$$

$$\frac{I_G + A\bar{h}^2}{A\bar{h}}$$

$$\frac{A\bar{h}}{I_G} + \bar{h}$$

$$\frac{I_G}{\bar{h}} + A\bar{h}$$

Where A = area of the immersed surface  
 $\bar{h}$  = depth of centre of surface immersed

$$\bar{h}$$

$I_G$  = Moment of inertia about centre of gravity

**IES-18. Ans. (b)**

**IES-19. What is the vertical distance of the centre of pressure below the centre of the plane area? [IES-2009]**

- (a)  $\frac{I_G}{A\bar{h}}$  (b)  $\frac{I_G \cdot \sin \theta}{A\bar{h}}$  (c)  $\frac{I_G \cdot \sin^2 \theta}{A\bar{h}}$  (d)  $\frac{I_G \cdot \sin^2 \theta}{A\bar{h}^2}$

**IES-19. Ans. (c)**

**IES-20. What is the depth of centre of pressure of a vertical immersed surface from free surface of liquid equal to? [IES-2008]**

- (a)  $\frac{I_G}{Ah} + \bar{h}$  (b)  $\frac{I_G A}{h} + \bar{h}$  (c)  $\frac{I_G \bar{h}}{A} + \bar{h}$  (d)  $\frac{A\bar{h}}{I_G} + \bar{h}$

**(Symbols have their usual meaning)**

**IES-20. Ans. (a)** Without knowing the formula also you can give answer option (b), (c) and (d) are not dimensionally homogeneous.

**IES-21. Assertion (A): The centre of pressure for a vertical surface submerged in a liquid lies above the centroid (centre of gravity) of the vertical surface. [IES-2008]**

**Reason (R): Pressure from the free surface of the liquid for a vertical surface submerged in a liquid is independent of the density of the liquid.**

- (a) Both A and R are true and R is the correct explanation of A  
 (b) Both A and R are true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

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**IES-21. Ans. (d)** Centre of pressure lies always below the centre of gravity of vertical surface . Therefore the distance of centre of pressure

$$\bar{h} = \bar{x} + \frac{I_{CG}}{A\bar{x}}$$

from free surface is independent of density of liquid. So, (d) is the answer.

**IES-22. Assertion (A):** A circular plate is immersed in a liquid with its periphery touching the free surface and the plane makes an angle  $\theta$  with the free surface with different values of  $\theta$ , the position of centre of pressure will be different. [IES-2004]

**Reason (R):** Since the center of pressure is dependent on second moment of area, with different values of  $\theta$ , second moment of area for the circular plate will change.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**IES-22. Ans. (c)**

$$\frac{I_G \cdot \sin \theta}{A \cdot \bar{h}}$$

**IES-23. A rectangular plate 0.75 m × 2.4 m is immersed in a liquid of relative density 0.85 with its 0.75 m side horizontal and just at the water surface. If the plane of plate makes an angle of 60° with the horizontal, what is the approximate pressure force on one side of the plate?**

[IES-2008]

(a) 7.80 kN

(b) 15.60 kN

(c) 18.00 kN

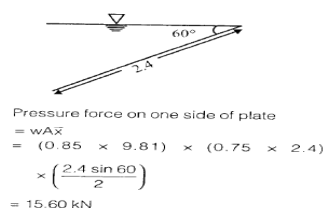
(d) 24.00 kN

**IES-23. Ans. (b)** Pressure force on one side of plate

$$= wA\bar{x}$$

$$= (0.85 \times 9.81) \times (0.75 \times 2.4) \times \left( \frac{2.4 \sin 60^\circ}{2} \right)$$

$$= 15.60 \text{ kN}$$



**IES-24. The hydrostatic force on the curved surface AB shown in given figure acts.**

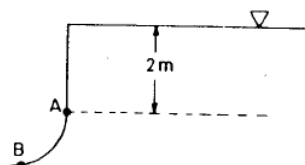
(a) Vertically downwards

(b) Vertically upwards

(c) Downwards, but at an angle with the vertical plane.

(d) Upwards, but at an angle with the vertical plane

[IES-1994]



**IES-24. Ans. (d)** Total hydrostatic force on the curved surface is upwards, but at an angle with the vertical plane.

# Hydrostatic Forces on Surfaces

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**IES-25.** A dam is having a curved surface as shown in the figure. The height of the water retained by the dam is 20m; density of water is  $1000\text{kg/m}^3$ . Assuming  $g$  as  $9.81\text{ m/s}^2$ , the horizontal force acting on the dam per unit length is: [IES-2002]

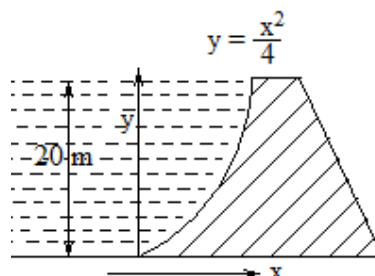
- (a)  $1.962 \times 10^2\text{ N}$  (b)  $2 \times 10^5\text{ N}$   
 (c)  $1.962 \times 10^6\text{ N}$  (d)  $3.924 \times 10^6\text{ N}$

**IES-25. Ans. (c)**

$$P_H = \rho g A \bar{x}$$

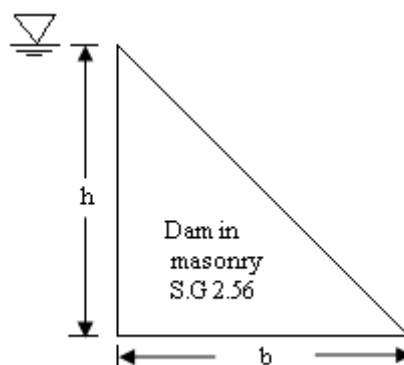
$$= 1000 \times 9.81 \times (20 \times 1) \times (20/2)$$

$$= 1.962 \times 10^6\text{ N}$$



**IES-26.** A triangular dam of height  $h$  and base width  $b$  is filled to its top with water as shown in the given figure. The condition of stability

- (a)  $b = h$  (b)  $b = 2.6h$   
 (c)  $b = \sqrt{3}h$  (d)  $b = 0.425h$



[IES-1999]

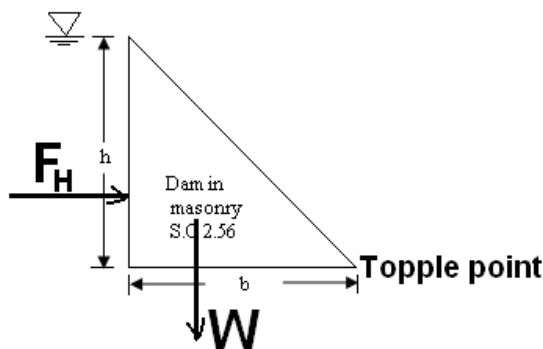
**IES-26. Ans. (d)** Taking moment about topple point

$$F_H \times \frac{h}{3} = W \times \frac{2b}{3}$$

$$\left( \rho_w g h \times 1 \times \frac{h}{2} \right) \times \frac{h}{3}$$

$$= \rho_w \times 2.56 \times g \times \left( \frac{bh}{2} \times 1 \right) \times \frac{2b}{3}$$

or  $b = 0.442h$   
 Nearest answer (d)



**IES-27.** What acceleration would cause the free surface of a liquid contained in an open tank moving in a horizontal track to dip by  $45^\circ$ ? [IES-2008]

- (a)  $g/2$  (b)  $2g$  (c)  $g$  (d)  $(3/2)g$

**IES-27. Ans. (c)**

**IES-28.** A vertical sludge gate, 2.5 m wide and weighting 500 kg is held in position due to horizontal force of water on one side and associated friction force. When the water level drops down to 2 m above the bottom of the gate, the gate just starts sliding

# Hydrostatic Forces on Surfaces

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down. The coefficient of friction between the gate and the supporting structure is: [IES-1999]

- (a) 0.20 (b) 0.10 (c) 0.05 (d) 0.02

IES-28. Ans. (b) or

$$\mu P = W \quad \mu \rho g A \bar{x} = mg \quad \mu = \frac{m}{\rho A \bar{x}} = \frac{500}{1000 \times (2 \times 2.5) \times (2/2)} = 0.1$$

## Previous 20-Years IAS Questions

IAS-1. A circular plate 1.5 m diameter is submerged in water with its greatest and least depths below the surface being 2 m and 0.75 m respectively. What is the total pressure (approximately) on one face of the plate?

[IES-2007, IAS-2004]

- (a) 12kN (b) 16kN (c) 24kN (d) None of the above

IAS-1. Ans. (c)

$$P = \rho g A \bar{x} = \rho g \left( \frac{\pi \times 1.5^2}{4} \right) \times \left( \frac{0.75 + 2}{2} \right) = 24 \text{ kN}$$

IAS-2. A tank with four equal vertical faces of width  $l$  and depth  $h$  is

filled up with a liquid. If the force on any vertical side is equal to the force at the bottom, then the value of  $h/l$  will be: [IAS-2000, IES-1998]

- (a) 2 (b)  $\sqrt{2}$  (c) 1 (d) 1/2

IAS-2. Ans. (a) or

$$P_{\text{bottom}} = P_{\text{side}} \quad h \rho g l t = \rho g t h (h/2) \text{ or } \frac{h}{t} = 2$$

IAS-3. The vertical component of the hydrostatic force on a submerged curved surface is the [IAS-1998, 1995, IES-1993, 2003]

- (a) Mass of liquid vertically above it  
(b) Weight of the liquid vertically above it  
(c) Force on a vertical projection of the surface  
(d) Product of pressure at the centroid and the surface area

IAS-3. Ans. (b)

IAS-4. What is the vertical component of pressure force on submerged curved surface equal to? [IAS-1998, 1995, IES-1993, 2003]

- (a) Its horizontal component  
(b) The force on a vertical projection of the curved surface  
(c) The product of the pressure at centroid and surface area  
(d) The gravity force of liquid vertically above the curved surface up to the free surface

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**IAS-4. Ans. (d)** The vertical component of the hydrostatic force on a submerged curved surface is the weight of the liquid vertically above it.

**IAS-5. Consider the following statements regarding a plane area submerged in a liquid:** **[IAS-1995]**

1. The total force is the product of specific weight of the liquid, the area and the depth of its centroid.
2. The total force is the product of the area and the pressure at its centroid.

**Of these statements:**

- |                            |                              |
|----------------------------|------------------------------|
| (a) 1 alone is correct     | (b) 2 alone is correct       |
| (c) Both 1 and 2 are false | (d) Both 1 and 2 are correct |

**IAS-5. Ans. (d)**

**IAS-6. A vertical dock gate 2 meter wide remains in position due to horizontal force of water on one side. The gate weights 800 Kg and just starts sliding down when the depth of water upto the bottom of the gate decreases to 4 meters. Then the coefficient of friction between dock gate and dock wall will be:** **[IAS-1995]**

- |          |         |          |
|----------|---------|----------|
| (a) 0.5  | (b) 0.2 | (c) 0.05 |
| (d) 0.02 |         |          |

**IAS-6. Ans. (c)**

or

or

$$\mu P = W \quad \mu \cdot g(4 \times 2) \cdot (4/2) = 800 \times g \quad \mu = 0.05$$

**IAS-7. A circular disc of radius 'r' is submerged vertically in a static fluid up to a depth 'h' from the free surface. If  $h > r$ , then the position of centre of pressure will:** **[IAS-1994]**

- |                                   |                                    |
|-----------------------------------|------------------------------------|
| (a) Be directly proportional to h | (b) Be inversely proportional of h |
| (c) Be directly proportional to r | (d) Not be a function of h or r    |

**IAS-7. Ans. (a)**

**IAS-8. Assertion (A): The total hydrostatic force on a thin plate submerged in a liquid, remains same, no matter how its surface is turned.** **[IAS-2001]**

**Reason (R): The total hydrostatic force on the immersed surface remains the same as long as the depth of centroid from the free surface remains unaltered.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-8. Ans. (d)** If  $\bar{x}$  changes it also change.  $P = \rho g A \bar{x}$

$\bar{x}$

$\bar{x}$

**IAS-9. A rectangular tank of base 3 m × 3 m contains oil of specific gravity 0.8 upto a height of 8 m. When it is accelerated at 2.45 m/s<sup>2</sup> vertically upwards, the force on the base of the tank will be:** **[IAS-1999]**

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) 29400 N | (b) 38240 N | (c) 78400 N | (d) 49050 N |
|-------------|-------------|-------------|-------------|

**IAS-9. Ans. (c)**





## 4. Buoyancy and Flotation

### Contents of this chapter

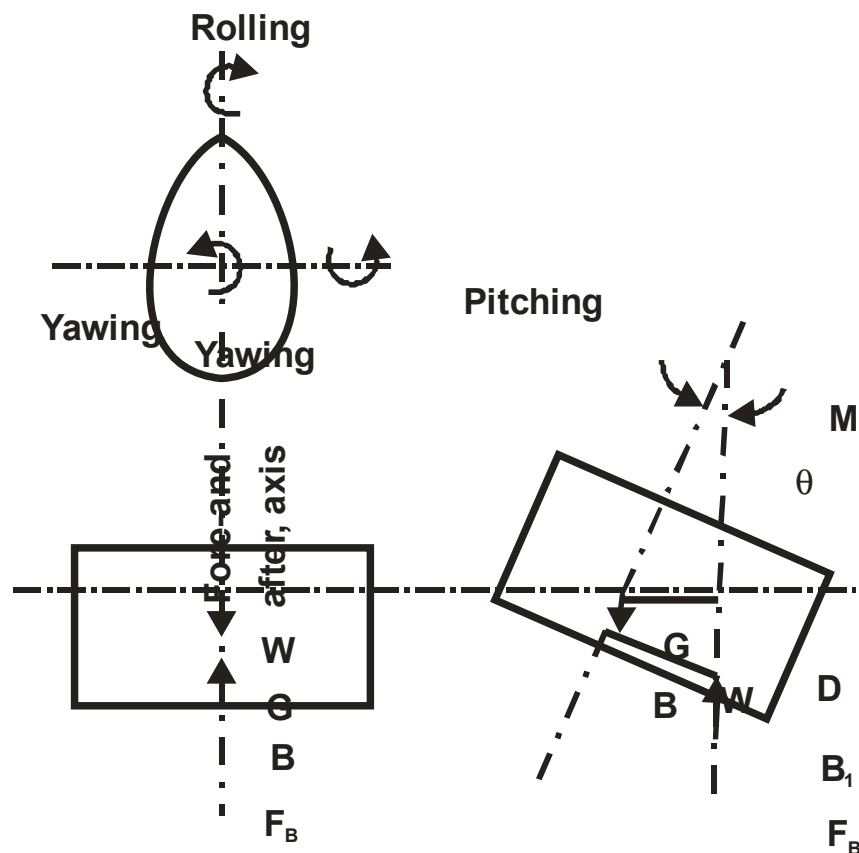
1. Buoyancy
2. Equilibrium of floating bodies
3. Equilibrium of floating bodies
4. Metacentric height
5. Time period of rolling of a floating body

**Question:** *Show that, for small angle of tilt, the time period of oscillation of a ship floating in stable equilibrium in water is given by*

$$T = \frac{2\pi k}{\sqrt{GM \cdot g}}$$

[AMIE (WINTER)-2001]

**Answer:** Consider a floating body in a liquid is given a small angular displacement  $\theta$  at an instant of time  $t$ . The body starts oscillating about its metacenter (M).



Let,  $W$  = Weight of floating body.

$\theta$  = Angle (in rad) through which the body is depressed.

$\theta$

$\alpha$  = Angular acceleration of the body in rad/

$\alpha$

$s^2$

T = Time of rolling (i.e. one complete oscillation) in seconds.

k = Radius of gyration about G.

I = Moment of inertia of the body about its center of gravity

G.

=

$$\frac{W}{g} \cdot k^2$$

GM = Metacentric height of the body.

When the force which has caused angular displacement is removed the body is acted by an external restoring moment due to its weight and buoyancy forces; acting parallel to each other GD apart. Since the weight and buoyancy force must be equal and distance GD equals GM sin  $\theta$ . The external moment equals W. GM sin  $\theta$  and acts so as to

$\theta$

$\theta$

oppose the tilt  $\theta$ .

$\theta$

$$- W.GM \sin \theta =$$

$$\therefore \theta I \frac{d^2 \theta}{dt^2}$$

$$\text{or, } W.GM \sin \theta =$$

$$\theta \frac{W}{g} k^2 \frac{d^2 \theta}{dt^2}$$

$$\text{or, } \frac{d^2 \theta}{dt^2} + \frac{g}{k^2} GM \sin \theta = 0$$

$$\frac{d^2 \theta}{dt^2} + \frac{g}{k^2} GM \sin \theta$$

For  $\theta$  is very small sin  $\theta \approx \theta$

$\theta$

$\theta \approx \theta$

$$= 0$$

$$\therefore \frac{d^2 \theta}{dt^2} + \frac{g.GM}{k^2} \cdot \theta$$

The solution of this second order linear differential

eqn

$$\theta = C_1 \sin \left( \sqrt{\frac{g.GM}{k^2}} \cdot t \right) + C_2 \cos \left( \sqrt{\frac{g.GM}{k^2}} \cdot t \right)$$

Boundary conditions (applying)

$$(i) \theta = 0 \text{ at } t = 0$$

$\theta$

$$\therefore 0 = C_1 \sin 0 + C_2 \cos 0$$

$$\therefore C_2 = 0$$

(ii) When

$$t = \frac{T}{2}, \theta = 0$$

$$\therefore 0 = C_1 \sin \left( \sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} \right)$$

$$\text{As } C_1 \neq 0$$

$$\therefore \sin \left( \sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} \right) = 0$$

$$\text{or } \sqrt{\frac{g \cdot GM}{k^2}} \cdot \frac{T}{2} = \pi$$

$$\text{or } T = 2\pi \times \sqrt{\frac{k^2}{g \cdot GM}}$$

$$\text{or } \boxed{T = \frac{2\pi k}{\sqrt{GM \cdot g}}} \text{ required expression.}$$

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

**GATE-1. Bodies in flotation to be in stable equilibrium, the necessary and sufficient condition is that the centre of gravity is located below the.....** **[GATE-1994]**

- |                       |                        |
|-----------------------|------------------------|
| (a) Metacentre        | (b) Centre of pressure |
| (c) Centre of gravity | (d) Centre of buoyancy |

**GATE-1. Ans. (a)**

### Previous 20-Years IES Questions

**IES-1. What is buoyant force?** **[IES-2008]**

- (a) Lateral force acting on a submerged body
- (b) Resultant force acting on a submerged body
- (c) Resultant force acting on a submerged body
- (d) Resultant hydrostatic force on a body due to fluid surrounding it

**IES-1. Ans. (d)** When a body is either wholly or partially immersed in a fluid, a lift is generated due to the net vertical component of hydrostatic pressure forces experienced by the body. This lift is called the buoyant force and the phenomenon is called buoyancy.

**IES-2. Assertion (A): The buoyant force for a floating body passes through the centroid of the displaced volume.** **[IES-2005]**

**Reason (R): The force of buoyancy is a vertical force & equal to the weight of fluid displaced.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-2. Ans. (a)** When a solid body is either wholly or partially immersed in a fluid, the hydrostatic lift due to net vertical component of the hydrostatic pressure forces experienced by the body is called the buoyant force. The buoyant force on a submerged or floating body is equal to the weight of liquid displaced by the body and acts vertically upward through the centroid of displaced volume known as centre of buoyancy.

The  $x$  coordinate of the center of the buoyancy is obtained as

$$x_g = \frac{1}{V} \iiint_V x dV$$

Which is the centroid of the displaced volume. It is due to the buoyant force is equals to the weight of liquid displaced by the submerged body of volume and the force of buoyancy is a vertical force.

**IES-3. Which one of the following is the condition for stable equilibrium for a floating body?** **[IES-2005]**

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- (a) The metacenter coincides with the centre of gravity
- (b) The metacenter is below the center of gravity
- (c) The metacenter is above the center of gravity
- (d) The centre of buoyancy is below the center of gravity

**IES-3. Ans. (c)**

**IES-4. Resultant pressure of the liquid in case of an immersed body acts through which one of the following? [IES-2007]**

- (a) Centre of gravity
- (b) Centre of pressure
- (c) Metacenter
- (d) Centre of buoyancy

**IES-4. Ans. (b)** For submerged body it acts through centre of buoyancy.

**IES-5. Assertion (A): If a boat, built with sheet metal on wooden frame, has an average density which is greater than that of water, then the boat can float in water with its hollow face upward but will sink once it overturns. [IES-1999]**

**Reason (R): Buoyant force always acts in the upward direction.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-5. Ans. (b)** Both A and R are true, but R does not give sufficient explanation for phenomenon at A. Location of Metacentre and centre of buoyancy decide about floating of a body.

**IES-6. Which of the following statement is true? [IES-1992]**

- (a) For an ideal fluid  $\mu = 0$ ,  $\rho = \text{constant}$ ,  $K = 0$
- (b) A floating body is in stable, unstable or neutral equilibrium according to as the metacentric height zero, positive or negative.
- (c) The exact analysis of viscous flow problems can be made by Euler's equation
- (d) The most economical diameter of a pipe is the one for which the annual fixed cost and annual power cost (to overcome friction) are minimum.

**IES-6. Ans. (d)** Here (a) is wrong: For an ideal fluid  $\mu = 0$ ,  $\rho = \text{constant}$ ,  $K = \text{const.}$

(b) is wrong: A floating body is in stable, unstable or neutral equilibrium according to as the metacentric height positive or negative or zero respectively. (c) is wrong: The exact analysis of viscous flow problems can be made by Navier stroke equations. Euler's equation is valid for non-viscous fluid.

**IES-7. A hydrometer weighs 0.03 N and has a stem at the upper end which is cylindrical and 3 mm in diameter. It will float deeper in oil of specific gravity 0.75, than in alcohol of specific gravity 0.8 by how much amount? [IES-2007]**

- (a) 10.7 mm
- (b) 43.3 mm
- (c) 33 mm
- (d) 36 mm

**IES-7. Ans. (d)** and Now

$$V_{oil} = \frac{W}{\rho_{oil}g} \quad V_{al} = \frac{W}{\rho_{al}g} \quad V_{oil} - V_{al} = \frac{W}{g} \left( \frac{1}{\rho_{oil}} - \frac{1}{\rho_{al}} \right) = \frac{\pi \cdot (0.003)^2 h}{4}$$

**IES-8.** A wooden rectangular block of length  $l$  is made to float in water

with its axis vertical. The centre of gravity of the floating body is  $0.15$  above the centre of buoyancy. What is the specific gravity

of the wooden block?

(a) 0.6

(b) 0.65

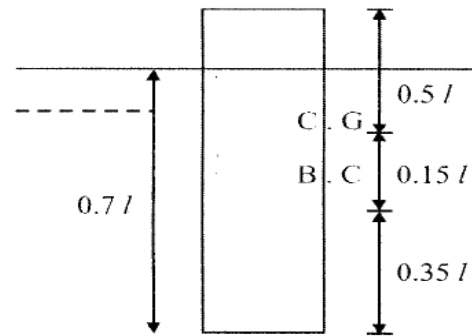
(c) 0.7

(d) 0.75

**IES-8. Ans. (c)**

$$0.7 / A \rho_w g = l A \rho_b g$$

$$\therefore \rho_b = 0.7 \rho_w \Rightarrow \frac{\rho_b}{\rho_w} = 0.7$$



$$0.7 / A \rho_w g = l A \rho_b g$$

$$\therefore \rho_b = 0.7 \rho_w$$

**IES-9.** A body weighs 30 N and 15 N when weighed under submerged conditions in liquids of relative densities 0.8 and 1.2 respectively. What is the volume of the body? **[IES-2009]**

(a) 12.50

(b) 3.82

(c) 18.70

(d) 75.50

**IES-9. Ans. (d)**

$$W - V \rho_1 g = 30 \quad \dots (i)$$

$$W - V \rho_2 g = 15 \quad \dots (ii)$$

$$\Rightarrow \frac{W - V (800) (9.8)}{W - V (1200) (9.8)} = 2$$

$$\Rightarrow W - V (800) (9.8) = 2W - 2V (1200) (9.8)$$

$$\Rightarrow W = 15680V$$

Putting the value of W in Equation (i)

$$15680 V - V(800)(9.8) = 30$$

$$\Rightarrow V = 3.82 \times 10^{-3} \text{ m}^3 = 3.82 \text{ liters}$$

**IES-10.** If B is the centre of buoyancy, G is the centre of gravity and M is the Metacentre of a floating body, the body will be in stable equilibrium if

(a)  $MG = 0$

(b) M is below G

(c)  $BG = 0$

(d) M is above G

**[IES-1994; 2007]**

**IES-10. Ans. (d)**

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**IES-11. For floating bodies, how is the metacentric radius defined? [IES-2009]**

- (a) The distance between centre of gravity and the metacentre.
- (b) Second moment of area of plane of flotation about centroidal axis perpendicular to plane of rotation/immersed volume.
- (c) The distance between centre of gravity and the centre of buoyancy.
- (d) Moment of inertia of the body about its axis of rotation/immersed volume.

**IES-11. Ans. (a)** Metacentric Radius or Metacentric Height is the distance between Centre of Gravity and the Metacentre.

**IES-12. The metacentric height of a passenger ship is kept lower than that of a naval or a cargo ship because [IES-2007]**

- (a) Apparent weight will increase
- (b) Otherwise it will be in neutral equilibrium
- (c) It will decrease the frequency of rolling
- (d) Otherwise it will sink and be totally immersed

**IES-12. Ans. (c)**

**IES-13. Consider the following statements: [IES-1996]**

**The metacentric height of a floating body depends**

1. Directly on the shape of its water-line area.
2. On the volume of liquid displaced by the body.
3. On the distance between the metacentre and the centre of gravity.
4. On the second moment of water-line area.

**Of these statements correct are:**

- (a) 1 and 2
- (b) 2 and 3
- (c) 3 and 4
- (d) 1 and 4

**IES-13. Ans. (b)** The metacentric height of a floating body depends on (2) and (3), i.e. volume of liquid displaced by the body and on the distance between the metacentre and the centre of gravity.

**IES-14. A cylindrical vessel having its height equal to its diameter is filled with liquid and moved horizontally at acceleration equal to acceleration due to gravity. The ratio of the liquid left in the vessel to the liquid at static equilibrium condition is: [IES-2001]**

- (a) 0.2
- (b) 0.4
- (c) 0.5
- (d) 0.75

**IES-14. Ans. (c)**

**IES-15. How is the metacentric height, GM expressed? [IES-2008]**

- |                       |                       |
|-----------------------|-----------------------|
| (a) $GM = BG - (I/V)$ | (b) $GM = (V/I) - BG$ |
| (c) $GM = (I/V) - BG$ | (d) $GM = BG - (V/I)$ |

Where  $I$  = Moment of inertia of the plan of the floating body at the water surface

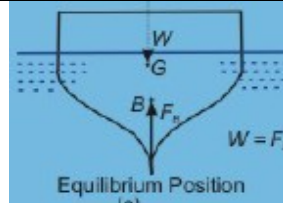
$V$  = Volume of the body submerged in water

$BG$  = Distance between the centre of gravity ( $G$ ) and the centre of buoyancy ( $B$ ).

**IES-15. Ans. (c)**

$$GM = BM - BG \text{ and } BM =$$

$$\frac{I}{V}$$



**IES-16. Stability of a floating body can be improved by which of the following?**

1. Making its width large
2. Making the draft small
3. Keeping the centre of mass low
4. Reducing its density

[IES-2008]

**Select the correct answer using the code given below:**

- |                   |                     |
|-------------------|---------------------|
| (a) 1, 2, 3 and 4 | (b) 1, 2 and 3 only |
| (c) 1 and 2 only  | (d) 3 and 4 only    |

**IES-16. Ans. (b)** Stability of a floating body can be improved by making width large which will increase.

$I$  and will thus increase the metacentric height and keeping the centre of mass low and making the draft small.

**IES-17. The distance from the centre of buoyancy to the meta-centre is given by  $I/V_d$  where  $V_d$  is the volume of fluid displaced. [IES-2008] What does  $I$  represent?**

- (a) Moment of inertia of a horizontal section of the body taken at the surface of the fluid
- (b) Moment of inertia about its vertical centroidal axis
- (c) Polar moment of inertia
- (d) Moment of inertia about its horizontal centroidal axis

**IES-17. Ans. (a)**  $BM$  = distance between centre of buoyancy to metacentre is given by  $\frac{I}{V_d}$ , where  $V_d$  is volume of fluid displaced.  $I$  represents the

$$\frac{I}{V_d}$$

Moment of Inertia of a horizontal section of a body taken at the surface of the fluid.

**IES-18. A 25 cm long prismatic homogeneous solid floats in water with its axis vertical and 10 cm projecting above water surface. If the same solid floats in some oil with axis vertical and 5 cm projecting above the liquid surface, what is the specific gravity of the oil?** [IES-2006]

- |          |          |          |          |
|----------|----------|----------|----------|
| (a) 0.60 | (b) 0.70 | (c) 0.75 | (d) 0.80 |
|----------|----------|----------|----------|

**IES-18. Ans. (c)**

$$\text{Let Area is } A \text{ cm}^2 \therefore A \times 25 \times s_s \times g = A \times 15 \times s_w \times g \text{ or } s_s = \frac{15}{25}$$

$$A \times 25 \times s_s \times g = A \times 20 \times s_{oil} \times g \text{ or } s_{oil} = \frac{25}{20} s_s = \frac{25}{20} \times \frac{15}{25} = 0.75$$

**IES-19. Consider the following statements:** [IES-1997, 1998]  
**Filling up a part of the empty hold of a ship with ballasts will**

1. Reduce the metacentric height.
2. Lower the position of the centre of gravity.



3. Elevate the position of centre of gravity.

4. Elevate the position of centre of buoyancy.

Of these statements

(a) 1, 3 and 4 are correct

(b) 1 and 2 are correct

(c) 3 and 4 are correct

(d) 2 and 4 are correct

**IES-19. Ans. (d)** Making bottom heavy lowers the c.g. of ship. It also increases displaced volume of water and thus the centre of displaced water, i.e. centre of buoyancy is elevated.

**IES-20. Assertion (A):** A circular plate is immersed in a liquid with its periphery touching the free surface and the plane makes an angle  $\theta$  with the free surface with different values of  $\theta$ , the

position of centre of pressure will be different.

[IES-2004]

**Reason (R):** Since the centre of pressure is dependent on second moment of area, with different values of  $\theta$ , second moment of

area for the circular plate will change.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**IES-20. Ans. (c)**

**IES-21. An open rectangular box of base 2m × 2m contains a liquid of specific gravity 0.80 up to a height of 2.5m. If the box is imparted a vertically upward acceleration of 4.9 m/s<sup>2</sup>, what will the pressure on the base of the tank?** [IES-2004]

(a) 9.81 kPa

(b) 19.62 kPa

(c) 36.80 kPa

(d) 29.40 kPa

**IES-21. Ans. (d)**

$$p = h\rho(g + a)$$

**IES-22. Assertion (A):** For a vertically immersed surface, the depth of the centre of pressure is independent of the density of the liquid.

**Reason (R):** Centre of pressure lies above the centre of area of the immersed surface. [IES-2003]

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**IES-22. Ans. (c)**

**IES-23. Match List-I with List-II and select the correct answer:[IES-1995, 2002]**

**List-I (Stability)**

**A.** Stable equilibrium of a floating body

**B.** Stable equilibrium of a submerged body

**List-II (Conditions)**

**1.** Centre of buoyancy below the centre of gravity

**2.** Metacentre above the centre of gravity

- C. Unstable equilibrium of a floating body    3. Centre of buoyancy above the centre of gravity  
 D. Unstable equilibrium of a submerged body    4. Metacentre below the centre of gravity

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	4	3	2	1	(b)	2	3	4	1
(c)	4	1	2	3	(d)	2	1	4	3

**IES-23. Ans. (b)**

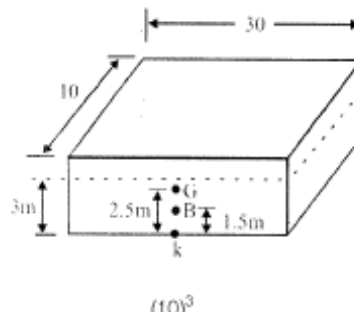
**IES-24. A barge 30m long and 10m wide has a draft of 3m when flowing with its sides in vertical position. If its centre of gravity is 2.5m above the bottom, the nearest value of metacentric height is: [IES-2001]**

- (a) 3.28m                      (b) 2.78m                      (c) 1.78m                      (d) Zero

**IES-24.                      Ans.                      (c)**

$$BM = \frac{1}{\Delta} = \frac{30 \times \frac{10^3}{12}}{30 \times 10 \times 3} = 2.778 \text{ m}$$

where  $I$  = least moment of inertia  
 and  $\Delta$  = displacement  
 $k_B = 3/2 = 1.5 \text{ m}$



$$k_M = k_B + BM = 2.778 + 1.5 = 4.278 \text{ m}$$

$$GM = k_M - k_G = 4.278 - 2.5 = 1.778 \text{ m} \approx 1.78 \text{ m}$$

**IES-25. A block of aluminium having mass of 12 kg is suspended by a wire and lowered until submerged into a tank containing oil of relative density 0.8. Taking the relative density of aluminium as 2.4, the tension in the wire will be (take  $g=10 \text{ m/s}^2$ ) [IES-2001]**

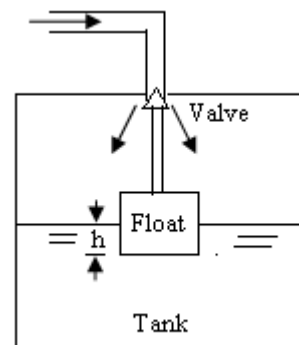
- (a) 12000N                      (b) 800 N                      (c) 120 N                      (d) 80N

**IES-25. Ans. (d)**

$$T = mg - v\rho g$$

**IES-26. A float of cubical shape has sides of 10 cm. The float valve just touches the valve seat to have a flow area of  $0.5 \text{ cm}^2$  as shown in the given figure. If the pressure of water in the pipeline is 1 bar, the rise of water level  $h$  in the tank to just stop the water flow will be:**

- (a) 7.5 cm                      (b) 5.0 cm  
 (c) 2.5 cm                      (d) 0.5 cm



**[IES-2000]**

**IES-26. Ans. (b)**

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- IES-27. Stability of a freely floating object is assured if its centre of**  
 (a) Buoyancy lies below its centre of gravity [IES-1999]  
 (b) Gravity coincides with its centre of buoyancy  
 (c) Gravity lies below its metacenter  
 (d) Buoyancy lies below its metacenter

**IES-27. Ans. (c)**

- IES-28. Match List-I with List-II regarding a body partly submerged in a liquid and select answer using the codes given below: [IES-1999]**

List-I				List-II				
A.	Centre of pressure			1.	Points of application of the weight of displace liquid			
B.	Centre of gravity			2.	Point about which the body starts oscillating when tilted by a small angle			
C.	Centre of buoyancy			3.	Point of application of hydrostatic pressure force			
D.	Matacentre			4.	Point of application of the weight of the body			
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a) 4	3	1	2	(b)	4	3	2	1
(c) 3	4	1	2	(d)	3	4	2	1

**IES-28. Ans. (c)**

- IES-29. A cylindrical piece of cork weighting 'W' floats with its axis in horizontal position in a liquid of relative density 4. By anchoring the bottom, the cork piece is made to float at neutral equilibrium position with its axis vertical. The vertically downward force exerted by anchoring would be:**  
 (a) 0.5 W (b) W (c) 2W [IES-1998]  
 (d) 3 W

**IES-29. Ans. (d)** Due to own weight of cylinder, it will float upto  $1/4^{\text{th}}$  of its height in liquid of relative density of 4. To make it float in neutral equilibrium, centre of gravity and centre of buoyancy must coincide, i.e. cylinder upto full height must get immersed.  
 For free floating: Weight (W) = Buoyancy force (i.e. weight of liquid equal to  $1/4^{\text{th}}$  volume cork)  
 The vertically downward force exerted by anchoring would be weight of liquid equal to  $3/4^{\text{th}}$  volume cork = 3W.

- IES-30. If a piece of metal having a specific gravity of 13.6 is placed in mercury of specific gravity 13.6, then**  
 (a) The metal piece will sink to the bottom [IES-1999]  
 (b) The metal piece simply float over the mercury with no immersion  
 (c) The metal piece will be immersed in mercury by half  
 (d) The whole of the metal piece will be immersed with its top surface just at mercury level.

**IES-30. Ans. (d)**

- IES-31. A bucket of water hangs with a spring balance. if an iron piece is suspended into water from another support without touching the sides of the bucket, the spring balance will show**  
 (a) An increased reading [IES-1999]  
 (b) A decreased reading  
 (c) No change in reading

(d) Increased or decreased reading depending on the depth of immersion

**IES-31. Ans. (c)**

**IES-32. A large metacentric height in a vessel [IES-1997]**

- (a) Improves stability and makes periodic time to oscillation longer
- (b) Impairs stability and makes periodic time of oscillation shorter
- (e) Has no effect on stability or the periodic time of oscillation
- (d) Improves stability and makes the periodic time of oscillation shorter

**IES-32. Ans. (d)** Large metacentric height improves stability and decreases periodic time of oscillation.

**IES-33. The least radius of gyration of a ship is 9 m and the metacentric height is 750 mm. The time period of oscillation of the ship is: [IES-1999]**

- (a) 42.41 s
- (b) 75.4 s
- (c) 20.85 s
- (d) 85 s

**IES-33. Ans. (c)** = 20.85 s

$$T = 2\pi \sqrt{\frac{k^2}{GM \cdot g}} = 2\pi \sqrt{\frac{9^2}{0.750 \times 9.81}}$$

**IES-34. What are the forces that influence the problem of fluid static?**

**[IES-2009]**

- (a) Gravity and viscous forces
- (b) Gravity and pressure forces
- (c) Viscous and surface tension forces
- (d) Gravity and surface tension forces

**IES-34. Ans. (b)** Gravity and pressure forces influence the problem of Fluid statics.

## Previous 20-Years IAS Questions

**IAS-1. A metallic piece weighs 80 N air and 60 N in water. The relative density of the metallic piece is about [IAS-2002]**

- (a) 8
- (b) 6
- (c) 4
- (d) 2

**IAS-1. Ans. (c)**

**IAS-2. Assertion (A): A body with wide rectangular cross section provides a highly stable shape in floatation. [IAS-1999]**

**Reason (R) The center of buoyancy shifts towards the tipped end considerably to provide a righting couple.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-2. Ans. (a)**

**IAS-3. Match List-I (Nature of equilibrium of floating body) with List-II (Conditions for equilibrium) and select the correct answer using the codes given below the Lists: [IAS-2002]**

List-I (Nature of equilibrium of floating body)	List-II (Conditions for equilibrium)
A. Unstable equilibrium	1. $MG = 0$
B. Neutral equilibrium	2. M is above G

<b>Buoyancy and Flotation</b> <b>S K Mondal's</b> <b>Chapter 4</b>
--

C. Stable equilibrium

3. M is below G

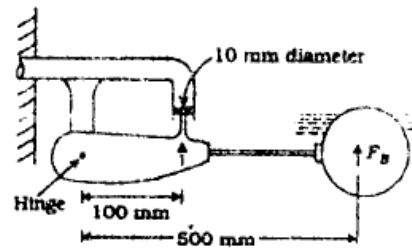
4. BG = 0

(Where M, G and B are metacenter, centre of gravity and centre of buoyancy respectively)

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>A</b>	<b>B</b>	<b>C</b>
(a)	1	3	2	(b)	3	1
(c)	1	3	4	(d)	4	2
					2	3

IAS-3. Ans. (b)

IAS-4. A float valve of the 'ball-clock' type is required to close an opening of a supply pipe feeding a cistern as shown in the given figure.



The buoyant force  $F_B$  required to be exerted by the float to keep the valve closed against a pressure of 0.28 N/mm<sup>2</sup> is:

- (a) 4.4 N                      (b) 5.6 N  
(c) 7.5 N                      (d) 9.2 N

[IAS-2000]

IAS-4. Ans. (a)

$$\text{Pressure force on valve } (F_v) = \text{Pressure} \times \text{area} = 0.28 \times \frac{\pi \times 10^2}{4} N = 22 N$$

$$\text{Taking moment about hinge, } F_v \times 100 = F_B \times 500 \text{ or } F_B = 4.4 N$$

IAS-5. **Assertion (A): A body with rectangular cross section provides a highly stable shape in floatation.** [IAS-1999]

**Reason (R): The centre of buoyancy shifts towards the tipped end considerably to provide a righting couple.**

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-5. Ans. (a)

IAS-6. A weight of 10 tonne is moved over a distance of 6m across the deck of a vessel of 1000 tonne floating in water. This makes a pendulum of length 2.5m swing through a distance of 12.5cm horizontally. The metacentric height of the vessel is: [IAS-1997]

- (a) 0.8m                      (b) 1.0m                      (c) 1.2m                      (d) 1.4m

IAS-6. Ans. (c)

IAS-7. The fraction of the volume of a solid piece of metal of relative density 8.25 floating above the surface of a container of mercury of relative density 13.6 is: [IAS-1997]

- (a) 1.648                      (b) 0.607                      (c) 0.393                      (d) 0.352

IAS-7. Ans. (c)

**IAS-8. If a cylindrical wooden pole, 20 cm in diameter, and 1m in height is placed in a pool of water in a vertical position (the gravity of wood is 0.6), then it will: [IAS-1994]**

- (a) Float in stable equilibrium (b) Float in unstable equilibrium  
(c) Float in neutral equilibrium (d) Start moving horizontally

**IAS-8. Ans. (b)**

**IAS-9. An open tank contains water to depth of 2m and oil over it to a depth of 1m. If the specific gravity of oil is 0.8, then the pressure intensity at the interface of the two fluid layers will be: [IAS-1994]**

- (a) 7848 N/m<sup>2</sup> (b) 8720 N/m<sup>2</sup> (c) 9747 N/m<sup>2</sup>  
(d) 9750 N/m<sup>2</sup>

**IAS-9. Ans. (a)**

**IAS-10. Consider the following statements [IAS-1994]  
For a body totally immersed in a fluid.**

- I. The weight acts through the centre of gravity of the body.  
II. The up thrust acts through the centroid of the body.**

**Of these statements:**

- (a) Both I and II are true (b) I is true but II is false  
(c) I is false but II is true (d) Neither I nor II is true

**IAS-10. Ans. (b)**

**IAS-11. Consider the following statements regarding stability of floating bodies: [IAS-1997]**

- 1. If oscillation is small, the position of Metacentre of a floating body will not alter whatever be the axis of rotation**
- 2. For a floating vessel containing liquid cargo, the stability is reduced due to movements of gravity and centre of buoyancy.**
- 3. In warships and racing boats, the metacentric height will have to be small to reduce rolling**

**Of these statements:**

- (a) 1, 2 and 3 are correct (b) 1 and 2 are correct  
(c) 2 alone is correct (d) 3 alone is correct

**IAS-11. Ans. (c)**

**IAS-12. Assertion (A): To reduce the rolling motion of a ship, the metacentric height should be low. [IAS-1995]**

**Reason (R): Decrease in metacentric height increases the righting couple.**

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IAS-12. Ans. (c) A is true but R is false**

Since high metacentric height will result in faster restoring action, rolling will be more. Thus to reduce rolling metacentric height should be low. However reason (R) is reverse of true statement.



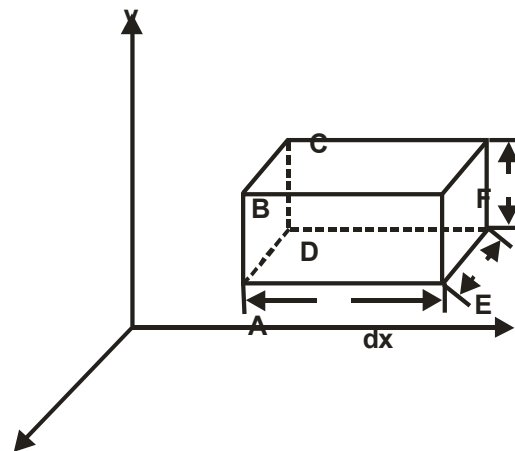
## 5. Fluid Kinematics

### Contents of this chapter

1. Velocity
2. Acceleration
3. Tangential and Normal Acceleration
4. Types of Flow
5. Stream Line
6. Path Line
7. Streak Line
8. Continuity Equation
9. Circulation and Vorticity
10. Velocity Potential Function
11. Stream Function
12. Flow Net
13. Acceleration in Fluid Vessel

**Question:** Derive three dimensional general continuity equations in differential form and extend it to 3-D in compressible flow (Cartesian Coordinates).

**Answer:** Consider a fluid element (control volume) - Parallelepiped with sides  $dx$ ,  $dy$  and  $dz$  as shown in Fig. Let  $\rho$  = mass density of the fluid at a particular instant  $t$ .  $u, v, w$  = components of velocity of the flow entering the three faces  $x, y$  and  $z$  respectively, of the parallelepiped.



Rate of mass of fluid  
 $\therefore$

entering the face ABCD (i.e. fluid influx)

=  $\rho$   $u$  velocity in  $x$ -direction  $\times$

Area ABCD =  $\rho u dy dz$

and Rate of mass of fluid leaving the face EFGH (i.e. fluid efflux)

$$= \rho u dy dz + \frac{\partial}{\partial x}(\rho u dy dz) dx$$



The gain in mass per unit time due to flow in the x-direction is given by the difference between the fluid influx and fluid efflux.

Mass accumulated per unit time due to flow in x-direction

$$\begin{aligned} \therefore &= \rho \int_V u \, dy \, dz - \rho \int_V u \, dy \, dz - \frac{\partial}{\partial x} (\rho u \, dy \, dz) \, dx = - \frac{\partial}{\partial x} (\rho u) \, dx \, dy \, dz \end{aligned}$$

Similarly, the gain in fluid mass per unit time in the parallelepiped due to flow in Y and z-directions.

$$= \quad \text{(in Y-direction)}$$

$$- \frac{\partial}{\partial y} (\rho v) \, dx \, dy \, dz$$

$$= \quad \text{(in z-direction)}$$

$$- \frac{\partial}{\partial z} (\rho w) \, dx \, dy \, dz$$

The total (or net) gain in fluid mass per unit time for fluid flow along 3 co-

$$\therefore \text{ordinate axes} = \text{Rate of change of mass of the parallelepiped (c. v.)} = \frac{\partial}{\partial t} (\rho \, dx \, dy \, dz)$$

$$\therefore - \frac{\partial}{\partial x} (\rho u) \, dx \, dy \, dz - \frac{\partial}{\partial y} (\rho v) \, dx \, dy \, dz - \frac{\partial}{\partial z} (\rho w) \, dx \, dy \, dz = \frac{\partial}{\partial t} (\rho \, dx \, dy \, dz)$$

or General continuity in 3-D

$$\boxed{\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) + \frac{\partial \rho}{\partial t} = 0} \quad \text{eq. 5.1}$$

For incompressible fluid

$$\rho = \text{const} \quad \therefore \quad \frac{\partial \rho}{\partial t} = 0$$

For 3-D incompressible flow.

$$\therefore \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

### Acceleration

**GATE-1.** In a two-dimensional velocity field with velocities  $u$  and  $v$  along the  $x$  and  $y$  directions respectively, the convective acceleration along the  $x$ -direction is given by: [GATE-2006]

(a)  $u$  (b)  $u$

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y}$$

(c)  $u + v$

$$\frac{\partial v}{\partial x} + u \frac{\partial u}{\partial y}$$

(d)  $v$

$$\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y}$$

**GATE-1. Ans. (a)**

**GATE-2. For a fluid flow through a divergent pipe of length L having inlet and outlet radii of  $R_1$  and  $R_2$  respectively and a constant flow rate of Q, assuming the velocity to be axial and uniform at any cross-section, the acceleration at the exit is:** [GATE-2004]

(a)

$$\frac{2Q(R_1 - R_2)}{\pi L R_2^3}$$

(b)

$$\frac{2Q^2(R_1 - R_2)}{\pi L R_2^3}$$

(c)

$$\frac{2Q^2(R_1 - R_2)}{\pi^2 L R_2^5}$$

(d)

$$\frac{2Q^2(R_2 - R_1)}{\pi^2 L R_2^5}$$

**GATE-2. Ans (c)** At a distance x from the inlet radius ( $R_x$ ) =

$$\left( R_1 + \frac{R_2 - R_1}{L} x \right)$$

Area

$$\therefore A_x = \pi R_x^2$$

$$u = \frac{Q}{A_x}$$

$$\therefore u = \frac{Q}{\pi \left( R_1 + \frac{R_2 - R_1}{L} x \right)^2}$$

Total acceleration  $a_x = u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$  for constant flow rate i.e. steady flow

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial t} = 0$$

$$a_x = u \frac{\partial u}{\partial x}$$

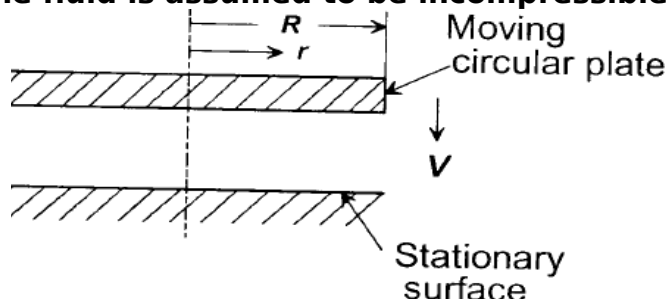
$$\therefore a_x = \frac{\partial u}{\partial x} \times \frac{Q}{\pi \left( R_1 + \frac{R_2 - R_1}{L} x \right)^2} \times \frac{-2Q \frac{R_2 - R_1}{L}}{\pi \left( R_1 + \frac{R_2 - R_1}{L} x \right)^3}$$

at  $x = L$  it gives

$$\frac{2Q^2(R_1 - R_2)}{\pi^2 L R_2^5}$$

**Statement for Linked Answers & Questions Q3 and Q4:**

The gap between a moving circular plate and a stationary surface is being continuously reduced. as the circular plate comes down at a uniform speed  $V$  towards the stationary bottom surface, as shown in the figure. In the process, the fluid contained between the two plates flows out radially. The fluid is assumed to be incompressible and inviscid.



**GATE-3. The radial velocity  $v$ , at any radius  $r$ , when the gap width is  $h$ , is:** [GATE-2008]

- (a)  $v_r = \frac{Vr}{2h}$       (b)  $v_r = \frac{Vr}{h}$       (c)  $v_r = \frac{2Vr}{h}$       (d)  $v_r = \frac{Vr}{h}$

**GATE-3. Ans. (a)** At a distance  $r$  take a small strip of  $dr$ .

volume of liquid will pass through of length within one second.

$$\pi r^2 V$$

$$2\pi r$$

$$\therefore v_r = \frac{\pi r^2 V}{2\pi r h} = \frac{rV}{2h}$$

**GATE-4. The radial component of the fluid acceleration at  $r = R$  is:** [GATE-2008]

- (a)  $\frac{3V^2 R}{4h^2}$       (b)  $\frac{V^2 R}{4h^2}$       (c)  $\frac{V^2 R}{2h^2}$       (d)  $\frac{V^2 h}{4R^2}$

**GATE-4. Ans. (c)** Acceleration ( $a_r$ ) =

$$\frac{\partial(v_r)}{\partial t} = \frac{\partial}{\partial t} \left( \frac{RV}{2h} \right) = \frac{RV}{2} \cdot \frac{\partial}{\partial t} \left( \frac{1}{h} \right) = -\frac{RV}{2h^2} \cdot \frac{\partial h}{\partial t}$$

( given) Therefore ( $a_r$ ) =

$$\frac{\partial h}{\partial t} = -V$$

$$\frac{V^2 R}{2h^2}$$

## Tangential and Normal Acceleration

**GATE-5. For a fluid element in a two dimensional flow field (x-y plane), if it will undergo** [GATE-1994]

- (a) Translation only      (b) Translation and rotation  
(c) Translation and deformation      (d) Deformation only

**GATE-5. Ans. (b)**

## Types of Flow

**GATE-6. You are asked to evaluate assorted fluid flows for their suitability in a given laboratory application. The following three flow choices, expressed in terms of the two-dimensional velocity fields in the xy-plane, are made available. [GATE-2009]**

**P.  $u = 2y, v = -3x$**

**Q.  $u = 3xy, v = 0$**

**R.  $u = -2x, v = 2y$**

**Which flow(s) should be recommended when the application requires the flow to be incompressible and irrotational?**

(a) P and R

(b) Q

(c) Q and R

(d) R

**GATE-6. Ans. (d)**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{continuity}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \quad \text{irrational}$$

## Stream Line

**GATE-7. A two-dimensional flow field has velocities along the x and y directions given by  $u = x^2t$  and  $v = -2xyt$  respectively, where t is time. The equation of streamlines is: [GATE-2006]**

(a)  $x^2y = \text{constant}$

(b)  $xy^2 = \text{constant}$

(c)  $xy = \text{constant}$

(d) not possible to determine

or

integrating both side

or

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad \frac{dx}{x^2t} = \frac{dy}{-2xyt}$$

$$\int \frac{dx}{x} = -\frac{1}{2} \int \frac{dy}{y}$$

$$\ln(x^2y) = 0$$

**GATE-7. Ans. (a)**

**GATE-8. In adiabatic flow with friction, the stagnation temperature along a streamline [GATE-1995]**

(a) Increases (b) Decreases

(c) Remains constant

(d) None

**GATE-8. Ans. (c)**

## Streak Line

**GATE-9. Streamlines, path lines and streak lines are virtually identical for [GATE-1994]**

(a) Uniform flow

(b) Flow of ideal fluids

(c) Steady flow

(d) Non uniform flow

**GATE-9. Ans. (c)**

## Continuity Equation

**GATE-10.** The velocity components in the x and y directions of a two dimensional potential flow are u and v, respectively. Then  $\frac{\partial u}{\partial x}$  is

$$\frac{\partial u}{\partial x}$$

equal to:

[GATE-2005]

(a)

$$\frac{\partial v}{\partial x}$$

(b)

$$\frac{\partial v}{\partial x}$$

(c)

$$\frac{\partial v}{\partial y}$$

(d)

$$-\frac{\partial v}{\partial y}$$

**GATE-10. Ans. (d)** From continuity eq.  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$  or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

**GATE-11.** The velocity components in the x and y directions are given by:

$$u = \lambda xy^3 - x^2y \quad \text{and} \quad v = xy^2 - \frac{3}{4}y^4$$

The value of  $\lambda$  for a possible flow field involving an incompressible fluid is: [GATE-1995]

(a) -3/4

(b) -4/3

(c) 4/3

(d) 3

**GATE-11. Ans. (d)** Just use continuity eq.  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

**GATE-12.** The continuity equation in the form  $\Delta \vec{V} = 0$  always represents

$$\Delta \vec{V} = 0$$

an incompressible flow regardless of whether the flow is steady or unsteady. [GATE-1994]

**GATE-12. Ans.** True General continuity equation, if

$$\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0 \quad \rho = \text{const} \quad \Delta \vec{V} = 0$$

**GATE-13.** If  $\vec{V}$  is velocity vector of fluid, then  $\Delta \vec{V} = 0$  is strictly true for

$$\vec{V}$$

$$\Delta \vec{V} = 0$$

which of the following?

[IAS-2007; GATE-2008]

(a) Steady and incompressible flow

(b) Steady and irrotational flow

(c) Inviscid flow irrespective of steadiness

(d) Incompressible flow irrespective of steadiness

**GATE-13. Ans. (d)**

$$\nabla \cdot \vec{V} = 0 \quad \text{Or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

## Circulation and Vorticity

**GATE-14. Circulation is defined as line integral of tangential component of velocity about a ..... [GATE-1994]**

- (a) Closed contour (path) in a fluid flow
- (b) Open contour (path) in a fluid flow
- (c) Closed or open contour (path) in a fluid flow
- (d) None

**GATE-14. Ans. (a)**

## Velocity Potential Function

**GATE-15. Existence of velocity potential implies that [GATE-1994]**

- (a) Fluid is in continuum
- (b) Fluid is irrotational
- (c) Fluid is ideal
- (d) Fluid is compressible

**GATE-15. Ans. (b)**

## Stream Function

**GATE-16. The 2-D flow with, velocity  $\vec{v} = (x + 2y + 2)\mathbf{i} + (4 - y)\mathbf{j}$  is:**

**[GATE-2001]**

- (a) Compressible and irrotational
- (b) Compressible and not irrotational
- (c) Incompressible and irrotational
- (d) Incompressible and not irrotational

**GATE-16. Ans. (d)** Continuity equation satisfied but

$$\omega_z \neq 0$$

## Flow Net

**GATE-17. In a flow field, the streamlines and equipotential lines [GATE-1994]**

- (a) Are Parallel
- (b) Are orthogonal everywhere in the flow field
- (c) Cut at any angle
- (d) Cut orthogonally except at the stagnation points

**GATE-17. Ans. (d)**

## Previous 20-Years IES Questions

## Acceleration

**IES-1. The convective acceleration of fluid in the x-direction is given by:**

(a)

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial z}$$

(b)

$$\frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t}$$

[IES-2001]

(c)

$$u \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial y} + u \frac{\partial w}{\partial z}$$

(d)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

**IES-1. Ans. (d)**

**IES-2. For a steady two-dimensional flow, the scalar components of the velocity field are  $V_x = -2x$ ,  $V_y = 2y$ ,  $V_z = 0$ . What are the components of acceleration?** [IES-2006]

(a)  $a_x = 0$ ,  $a_y = 0$ (b)  $a_x = 4x$ ,  $a_y = 0$ (c)  $a_x = 0$ ,  $a_y = 4y$ (d)  $a_x = 4x$ ,  $a_y = 4y$ **IES-2. Ans (d)**  $a_x = u$ Given  $u = V_x = -2x$ ;  $v = V_y = 2y$  and  $w =$ 

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$V_z = 0$$

$$a_y = u$$

$$\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

**IES-3. A steady, incompressible flow is given by  $u = 2x^2 + y^2$  and  $v = -4xy$ . What is the convective acceleration along x-direction at point (1, 2)?**

(a)  $a_x = 6$  unit(b)  $a_x = 24$  unit

[IES-2008]

(c)  $a_x = -8$  unit(d)  $a_x = -24$  unit**IES-3. Ans. (c)** Convective acceleration along  $x$  direction at point (1, 2)

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (2x^2 + y^2)(4x) + (-4xy)(2y)$$

$$= (2 + 4)(4) + (-4 \cdot 1 \cdot 2)(2 \cdot 2) = 24 - 32 = -8 \text{ unit}$$

**IES-4. The area of a 2m long tapered duct decreases as  $A = (0.5 - 0.2x)$  where 'x' is the distance in meters. At a given instant a discharge of  $0.5 \text{ m}^3/\text{s}$  is flowing in the duct and is found to increase at a rate of  $0.2 \text{ m}^3/\text{s}^2$ . The local acceleration (in  $\text{m/s}^2$ ) at  $x = 0$  will be:** [IES-2007]

(a) 1.4  
0.667

(b) 1.0

(c) 0.4

(d)

**IES-4. Ans.(c)**  $u =$   $=$  local acceleration at  $x =$ 

$$\therefore \frac{Q}{A_x} = \frac{Q}{(0.5 - 0.2x)} \quad \frac{\partial u}{\partial t} = \frac{1}{(0.5 - 0.2x)} \times \frac{\partial Q}{\partial t}$$

0

$$=0.4$$

$$\frac{\partial u}{\partial t} = \frac{1}{(0.5)} \times 0.2$$

**IES-5. Assertion (A): The local acceleration is zero in a steady motion. Reason (R): The convective component arises due to the fact that a fluid element experiences different velocities at different locations.**

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

**[IES-2009]**

(d) A is false but R is true

**IES-5. Ans. (b)** • A flow is said to be steady when conditions do not change with time at any point.

• In a converging steady flow, there is only convective acceleration.

• Local acceleration is zero in steady flow.

**IES-6. The components of velocity in a two dimensional frictionless incompressible flow are  $u = t^2 + 3y$  and  $v = 3t + 3x$ . What is the approximate resultant total acceleration at the point (3, 2) and  $t = 2$ ?**

**[IES-2004]**

(a) 5

(b) 49

(c) 59

(d) 54

**IES-6. Ans. (c)**  $a_x = u$

and  $a_y = v$

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}$$

$$\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

or  $a_x = (t^2 + 3y).(0) + (3t + 3x).(3) + 2t$  and  $a_y = (t^2 + 3y).(3) + (3t + 3x).(0) + 3$

at  $x = 3, y = 2$  and  $t = 2$

$a = \quad = \quad = 59.08$

$$\sqrt{a_x^2 + a_y^2} \quad \sqrt{49^2 + 3^2}$$

**IES-7. Match List-I (Pipe flow) with List-II (Type of acceleration) and select the correct answer:**

**[IES-1999]**

**List-I**

**List-II**

**A.** Flow at constant rate passing through a bend **1.** Zero acceleration

**B.** Flow at constant rate passing through a straight uniform diameter pipe **2.** Local and convective acceleration

**C.** Gradually changing flow through a bend **3.** Convective acceleration

**D.** Gradually changing flow through a straight pipe **4.** Local acceleration

**Codes:**

**A**

**B**

**C**

**D**

**A**

**B**

**C**

**D**

(a)

3

1

2

4

(b)

3

1

4

2

(c) 1

3

2

4

(d)

1

3

4

2

**IES-7. Ans. (a)**



## Types of Flow

**IES-8. Match List-I (Flows Over or Inside the Systems) with List-II (Type of Flow) and select the correct answer: [IES-2003]**

**List-I**

- A. Flow over a sphere
- B. Flow over a long circular cylinder
- C. Flow in a pipe bend
- D. Fully developed flow in a pipe at constant flow rate

**List-II**

- 1. Two dimensional flow
- 2. One dimensional flow
- 3. Axisymmetric flow
- 4. Three dimensional flow

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	1	2	4	(b)	1	4	3	2
(c)	3	1	4	2	(d)	1	4	2	3

**IES-8. Ans. (c)**

**IES-9. Match List-I with List-II and select the correct answer using the code given below the lists: [IES-2007]**

**List-I (Condition)**

- A. Existence of stream function
- B. Existence of velocity potential
- C. Absence of temporal Variations
- D. Constant velocity vector

**List-II (Regulating Fact)**

- 1. Irrotationality of flow
- 2. Continuity of flow
- 3. Uniform flow
- 4. Steady flow

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	4	3	2	1	(b)	2	1	4	3
(c)	4	1	2	3	(d)	2	3	4	1

**IES-9. Ans. (b)**

**IES-10. Irrotational flow occurs when: [IES-1997]**

- (a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.
- (b) Streamlines are curved.
- (c) There is no net rotation of the fluid element about its mass center.
- (d) Fluid element does not undergo any change in size or shape.

**IES-10. Ans. (c)** If the fluid particles do not rotate about their mass centres while moving in the direction of motion, the flow is called as an irrotational flow.

**IES-11. Which one of the following statements is correct? [IES-2004]**

**Irrotational flow is characterized as the one in which**

- (a) The fluid flows along a straight line
- (b) The fluid does not rotate as it moves along
- (c) The net rotation of fluid particles about their mass centres remains zero.
- (d) The streamlines of flow are curved and closely spaced

**IES-11. Ans. (c)**

**IES-12. In a two-dimensional flow in x-y plane, if \_\_\_\_\_, then the fluid**

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

**element will undergo**

**[IES-1996]**

- (a) Translation only
- (b) Translation and rotation
- (c) Translation and deformation
- (d) Rotation and deformation.

**IES-12. Ans. (a)** In a two-dimensional flow in x-y plane, if i.e. irrotational,

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

then the fluid element will undergo translation and deformation.

Remember: Any fluid element can undergo translation, rotation and RATE OF deformation. As i.e. irrotational flow, rotation is not there but

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

not translation and RATE OF deformation must be possible.

**IES-13. Two flows are specified as** **[IES-2008]**

**(A)  $u = y, v = - (3/2) x$**  **(B)  $u = xy^2, v = x^2y$**

**Which one of the following can be concluded?**

- (a) Both flows are rotational
- (b) Both flows are irrotational
- (c) Flow A is rotational while flow B is irrotational
- (d) Flow A is irrotational while flow B is rotational

**IES-13. Ans. (c)**

$$(A) \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \left( \frac{\partial}{\partial x} \left( -\frac{3}{2}x \right) - \frac{\partial}{\partial y} (y) \right)$$

$$= \frac{1}{2} \left( -\frac{3}{2} - 1 \right) = \frac{1}{2} \times \left( -\frac{5}{2} \right) = -\frac{5}{4} \neq 0$$

∴ Flow A is rotational

$$(B) \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \left( \frac{\partial}{\partial x} (x^2y) - \frac{\partial}{\partial y} (xy^2) \right) = \frac{1}{2} (2xy - 2xy) = 0$$

∴ Flow (b) is irrotational

**IES-14. Match List-I with List-II and select the correct answer: [IES-2002]**

**List-I (Example)**

**A.** Flow in a straight long pipe with varying flow rate

**B.** Flow of gas through the nozzle of a jet engine

**C.** Flow of water through the hose of a fire fighting pump

**D.** Flow in a river during tidal bore

**Codes:**

**(a)**

**(c)**

**A**

**1**

**1**

**B**

**4**

**2**

**C**

**3**

**3**

**D**

**2**

**4**

**(b)**

**3**

**(d)**

**3**

**List-II (Types of flow)**

**1.** Uniform, steady

**2.** Non-uniform, steady

**3.** Uniform, unsteady

**4.** Non-uniform, unsteady

**A**

**3**

**3**

**B**

**2**

**2**

**C**

**1**

**1**

**D**

**4**

**2**

**IES-14. Ans. (b)**

## Stream Line

**IES-15. A streamline is a line:**

**[IES-2003]**

- (a) Which is along the path of the particle
- (b) Which is always parallel to the main direction of flow
- (c) Along which there is no flow
- (d) On which tangent drawn at any point given the direction of velocity

**IES-15. Ans. (d)**

**IES-16. Assertion (A): Stream lines are drawn in the flow field such that at a given instant of time there perpendicular to the direction of flow at every point in the flow field. [IES-2002]**

**Reason (R): Equation for a stream line in a two dimensional flow is given by  $V_x dy - V_y dx = 0$ .**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-16. Ans. (d)** A streamline in a fluid flow is a line tangent to which at any point is in the direction of velocity at that point at that instant.

**IES-17. Assertion (A): Streamlines can cross one another if the fluid has higher velocity. [IES-2003]**

**Reason (R): At sufficiently high velocity, the Reynolds number is high and at sufficiently high Reynolds numbers, the structure of the flow is turbulent type.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-17. Ans. (d)**

**IES-18. The streamlines and the lines of constant velocity potential in an inviscid irrotational flow field form. [IES-1994]**

- (a) Parallel grid lines placed in accordance with their magnitude.
- (b) Intersecting grid-net with arbitrary orientation.
- (c) An orthogonal grid system
- (d) None of the above.

**IES-18. Ans. (c) Flow net:**

Streamline  $\psi = \text{const.}$

$\psi$

Velocity potential line  $\phi = \text{const.}$

$\phi$

- The streamlines and velocity potential lines form an orthogonal net work in a fluid flow.
- Observation of a flow net enables us to estimate the velocity variation.
- Streamline and velocity potential lines must constitute orthogonal net work except at the stagnation points.

**IES-19. A velocity field is given by  $u = 3xy$  and  $v = \frac{3}{2}(x^2 - y^2)$ . What is the**

**relevant equation of a streamline?**

**[IES-2008]**

(a) (b) (c) (d)

$$\frac{dx}{dy} = \frac{(x^2 - y^2)}{xy}$$

$$\frac{3xy}{(x^2 - y^2)}$$

$$\frac{2xy}{(x^2 - y^2)}$$

$$\frac{dx}{dy} = \frac{(x^2 - y^2)}{2xy}$$

**IES-19. Ans. (c)**

$$u = 3xy \quad v = \frac{3}{2}(x^2 - y^2)$$

Equation of streamline is given by

$$vdx = udy$$

$$\Rightarrow \frac{dx}{dy} = \frac{u}{v}$$

$$\Rightarrow \frac{dx}{dy} = \frac{3xy}{\frac{3}{2}(x^2 - y^2)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{2xy}{(x^2 - y^2)}$$

## Path Line

**IES-20. Consider the following statements regarding a path line in fluid flow:**

1. A path line is a line traced by a single particle over a time interval.
2. A path line shows the positions of the same particle at successive time instants. [IES-2006]
3. A path line shows the instantaneous positions of a number of a particle, passing through a common point, at some previous time instants.

**Which of the statements given above are correctly?**

- (a) Only 1 and 3 (b) only 1 and 2  
(c) Only 2 and 3 (d) 1, 2 and 3

**IES-20. Ans. (b)** 3 is wrong because it defines Streak line.

- (i) A path line is the trace made by a single particle over a period of time. i.e. It is the path followed by a fluid particle in motion.

$$\text{Equation } x = \int u dt; y =$$

$$\int v dt; z = \int w dt$$

- (ii) Streak line indicates instantaneous position of particles of fluid passing through a point.

## Streak Line

**IES-21. Which one of the following is the correct statement? [IES-2007]**  
**Streamline, path line and streak line are identical when the**

- (a) Flow is steady
- (b) Flow is uniform
- (c) Flow velocities do not change steadily with time

(d) Flow is neither steady nor uniform

**IES-21 Ans (a)**

## Continuity Equation

**IES-22. The differential form of continuity equation for two-dimensional flow of fluid may be written in the following form in**

$$\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0$$

**which u and v are velocities in the x and y-direction and ρ is the density. This is valid for [IES-1995]**

(a) Compressible, steady flow

(b) Compressible, unsteady flow

(c) Incompressible, unsteady flow

(d) Incompressible, steady flow

**IES-22. Ans. (d)** The equation is valid for incompressible steady

$$\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0$$

flow.

**IES-23. The general form of expression for the continuity equation in a Cartesian coordinate system for incompressible or compressible flow is given by: [IES-1996]**

(a)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(b)

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

(c)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

(d)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 1$$

**IES-23. Ans. (c)** general form valid for

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

incompressible or compressible flow, steady and unsteady flow.

valid for incompressible or compressible flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

but steady flow.

valid for incompressible flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

**IES-24. Which of the following equations are forms of continuity equations?**

( ' is the velocity and ' ' is volume)

**[IES-1993]**

$\frac{1}{V}$

$\nabla$

$$1. A_1 V_1 = A_2 V_2$$

$$2. \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$3. \int_s \rho \vec{V} \cdot d\vec{A} + \frac{\partial}{\partial t} \int_V \rho dV = 0$$

$$4. \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0$$

**Select the correct answer using the codes given below:**

**Codes:**

(a) 1, 2, 3 and 4

(b) 1 and 2

(c) 3 and 4

(d) 2, 3 and 4

**IES-24. Ans. (b)** const. In case of compressible fluid.

$$\rho A V =$$

AV = const. In case of incompressible fluid.

Differential form of continuity equation in Cartesian co-ordinates system.

, Vector form , for incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla \cdot \vec{V} = 0$$

General form

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

Vector form .(

$$\nabla \cdot (\rho \vec{V}) + \frac{\partial \rho}{\partial t} = 0$$

General form valid for

Viscous or Inviscid; steady or unsteady; uniform or non-uniform; compressible or incompressible.

Integral form: .dA+

$$\int_s \rho \vec{V} \cdot d\vec{A} + \frac{\partial}{\partial t} \int_V \rho dV = 0$$

Differential form of continuity equation in cylindrical co-ordinate system

, for incompressible flow.

$$\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

**IES-25. Consider the following equations:**

**[IES-2009]**

$$1. A_1 V_1 = A_2 V_2$$

$$2. \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$3. \oint_s \vec{v} \cdot d\vec{A} + \frac{\partial}{\partial t} \left( \oint_V dV \right) = 0$$

$$4. \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0$$

**Which of the above equations are forms of continuity equations? (Where u, v are velocities and V is volume)**

(a) 1 only

(b) 1 and 2

(c) 2 and 3

(d) 3 and 4

**IES-25. Ans. (b)** General continuity in 3-D:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

For incompressible fluid

$$\rho = \text{const} \quad \therefore \frac{\partial \rho}{\partial t} = 0$$

For 3-D incompressible flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For 2-D incompressible flow.

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

**IES-26. In a two-dimensional incompressible steady flow, the velocity component  $u = Ae^x$  is obtained. What is the other component  $v$  of velocity?** **[IES-2006]**

- |                   |                |                          |                        |
|-------------------|----------------|--------------------------|------------------------|
| (a) $v = Ae^{xy}$ | (b) $v = Ae^y$ | (c) $v = -Ae^x y + f(x)$ | (d) $v = -Ae^x + f(y)$ |
|-------------------|----------------|--------------------------|------------------------|

**IES-26. Ans. (c)** From continuity eq.  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$  or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -Ae^x$$

$$v = -Ae^x y + f(x)$$

**IES-27. For steady incompressible flow, if the  $u$ -component of velocity is  $u = Ae^x$ , then what is the  $v$ -component of velocity?** **[IES-2008]**

- |            |               |                |             |
|------------|---------------|----------------|-------------|
| (a) $Ae^y$ | (b) $Ae^{xy}$ | (c) $-Ae^{xy}$ | (d) $-Ae^x$ |
|------------|---------------|----------------|-------------|

**IES-27. Ans. (c)** Using continuity equation

$$\begin{aligned} \Rightarrow \frac{du}{dx} + \frac{dv}{dy} &= 0 & \Rightarrow \frac{d(Ae^x)}{dx} + \frac{dv}{dy} &= 0 \\ \Rightarrow Ae^x + \frac{dv}{dy} &= 0 & \Rightarrow v &= -Ae^x y \end{aligned}$$

**IES-28. Which one of the following stream functions is a possible irrotational flow field?** **[IES-2003]**

- |                    |                  |                       |                        |
|--------------------|------------------|-----------------------|------------------------|
| (a) $\psi = x^3 y$ | (b) $\psi = 2xy$ | (c) $\psi = Ax^2 y^2$ | (d) $\psi = Ax + By^2$ |
|--------------------|------------------|-----------------------|------------------------|

$$\psi = Ax + By^2$$

**IES-28. Ans. (b)** Use continuity equation

**IES-29. Which one of the following stream functions is a possible irrotational flow field ? [IES-2007]**

- (a) (b) (c) (d)

$$\psi = y^2 - x^2$$

$$\psi = A \sin(xy)$$

$$\psi = Ax^2y^2$$

$$\psi = Ax + By^2$$

**IES-29 Ans. (a)** Satisfy Laplace Equation.

**IES-30. The continuity equation for a steady flow states that [IES-1994]**

- (a) Velocity field is continuous at all points in flow field  
 (b) The velocity is tangential to the streamlines.  
 (c) The stream function exists for steady flows.  
 (d) The net efflux rate of mass through the control surfaces is zero

**IES-30. Ans. (c)** It is a possible case of fluid flow therefore the stream function exists for steady flows.

## Circulation and Vorticity

**IES-31. Which of the following relations must hold for an irrotational two-dimensional flow in the x-y plane? [IAS-2003, 2004, IES-1995]**

- (a)  $\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 0$  (b)  $\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$   
 (c)  $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$  (d)  $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

**IES-31. Ans. (d)** i.e.

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

**IES-32. Irrotational flow occurs when: [IES-1998]**

- (a) Flow takes place in a duct of uniform cross-section at constant mass flow rate.  
 (b) Streamlines are curved.  
 (c) There is no net rotation of the fluid element about its mass center. [IES-1998]  
 (d) Fluid element does not undergo any change in size or shape.

**IES-32. Ans. (c)** If the fluid particles do not rotate about their mass centre and while moving in the direction of motion. The flow is called as an irrotational flow.

**IES-33. The curl of a given velocity field indicates the rate of [IES-**

$$(\nabla \times \vec{V})$$

**1996]**

- (a) Increase or decrease of flow at a point. (b) Twisting of the lines of flow.  
 (c) Deformation (d) Translation.



**IES-33. Ans. (c)** The curl of a given velocity field  $\left(\nabla \times \vec{V}\right)$  indicates the rate of

angular deformation.

**IES-34. If the governing equation for a flow field is given by  $\vec{V}^2 \phi = 0$  and the velocity is given by  $\vec{V} = \nabla \phi$ , then** **[IES-1993]**

(a)  $\nabla \times \vec{V} = 0$       (b)  $\nabla \times \vec{V} = 1$       (c)  $\nabla^2 \times \vec{V} = 1$       (d)  $\left(\vec{V} \cdot \nabla\right) \times \vec{V} = \frac{\partial \vec{V}}{\partial t}$

**IES-34. Ans. (a)**

## Velocity Potential Function

**IES-35. The relation  $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$  for an irrotational flow is known as which**

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

**one of the following?**

- (a) Navier - Stokes equation  
(c) Reynolds equation

- (b) Laplace equation  
(d) Euler's equation

**[IES-2007]**

**IES-35. Ans. (b)**

**IES-36. Consider the following statements:** **[IES-1994]**  
**For a two-dimensional potential flow**

1. Laplace equation for stream function must be satisfied.
2. Laplace equation for velocity potential must be satisfied.
3. Streamlines and equipotential lines are mutually perpendicular.
4. Stream function and potential function are not interchangeable.

- (a) 1 and 4 are correct      (b) 2 and 4 are correct  
(c) 1, 2 and 3 are correct      (d) 2, 3 and 4 are correct

**IES-36. Ans. (c)**

**IES-37. Which of the following functions represent the velocity potential in a two-dimensional flow of an ideal fluid?** **[IES-2004, 1994]**

1.  $2x + 3y$       2.  $4x^2 - 3y^2$       3.  $\cos(x - y)$       4.  $\tan^{-1}(x/y)$

**Select the correct answer using the codes given below:**

- (a) 1 and 3      (b) 1 and 4      (c) 2 and 3      (d) 2 and 4

**IES-37. (a)** Checking  $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$  for all the above.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

## Stream Function

**IES-38.** If for a flow, a stream function  $\psi$  exists and satisfies the Laplace equation, then which one of the following is the correct statement?

- (a) The continuity equation is satisfied and the flow is irrotational. **[IES-2005]**
- (b) The continuity equation is satisfied and the flow is rotational.
- (c) The flow is irrotational but does not satisfy the continuity equation.
- (d) The flow is rotational.

**IES-38. Ans (a)** if a stream function  $\psi$  exists means a possible case of flow, if it satisfies the Laplace equation then flow is irrotational.

**IES-39.** In a two-dimensional flow, the velocity components in x and y directions in terms of stream function  $\psi$  are: **[IES-1995]**

$$(\psi)$$

- |   |  |
|---|--|
| <p>(a) <math>u = \partial \psi / \partial x, v = \partial \psi / \partial y</math></p> <p>(c) <math>u = (-) \partial \psi / \partial y, v = \partial \psi / \partial x</math></p> | <p>(b) <math>u = \partial \psi / \partial y, v = \partial \psi / \partial x</math></p> <p>(d) <math>u = \partial \psi / \partial x, v = -\partial \psi / \partial y</math></p> |
|---|--|

**IES-39. Ans. (c)** The stream function  $\psi$  is defined as a scalar function of space and time, such that its partial derivative with respect to any direction given the velocity component at right angles (in the counter clockwise direction) to this direction. Hence,

$$\frac{\partial \psi}{\partial y} = -U \quad \text{and} \quad \frac{\partial \psi}{\partial x} = V.$$

**IES-40.** Of the possible irrotational flow functions given below, the incorrect relation is (where  $\psi$  = stream function and  $\phi$  = velocity potential).

**potential).**

**[IES-1995]**

- |   |   |
|---|---|
| <p>(a) <math>\psi = xy</math></p> <p>(c) <math>\phi = ur \cos \theta + \frac{u}{r} \cos \theta</math></p> | <p>(b) <math>\psi = A(x^2 - y^2)</math></p> <p>(d) <math>\phi = \left( r - \frac{2}{r} \right) \sin \theta</math></p> |
|---|---|

**IES-40. Ans. (d)** Equation at (d) is not irrotational. If the stream function satisfy the Laplace equation the flow is irrotational, otherwise rotational.  
If (Laplace equation)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Then,

$$\omega_z = 0$$

A.

$$\psi = xy$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 (xy)}{\partial x^2} + \frac{\partial^2 (xy)}{\partial y^2} = 0 + 0 = 0$$

Satisfy the Laplace equation therefore flow is irrotational.

B.

$$\psi = A(x^2 - y^2)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 [A(x^2 - y^2)]}{\partial x^2} + \frac{\partial^2 [A(x^2 - y^2)]}{\partial y^2} = 2A - 2A = 0 + 0 = 0$$

Satisfy Laplace equation, therefore flow is rotational.

C.

$$\nabla_\phi^2 = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 [A(x^2 - y^2)]}{\partial x^2} + \frac{\partial^2 [A(x^2 - y^2)]}{\partial y^2} = 2A - 2A = 0 + 0 = 0$$

$$V_r = \frac{\partial \phi}{\partial r} \quad \text{and} \quad V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

$$2w_z = \frac{1}{r} \frac{\partial}{\partial r} (V_\theta) - \frac{1}{r^2} \frac{\partial}{\partial \theta} (V_r)$$

$$2w_z = \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial \phi}{\partial \theta} \right] - \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ \frac{\partial \phi}{\partial r} \right]$$

$$\phi = ur \cos \theta + \frac{u}{r} \cos \theta$$

$$V_r = -\frac{\partial \phi}{\partial r} = -u \cos \theta + \frac{u}{r^2} \cos \theta$$

$$V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{1}{r} \left[ -ur \sin \theta - \frac{u}{r} \sin \theta \right]$$

$$V_\theta = u \sin \theta + \frac{u}{r^2} \sin \theta$$

$$\begin{aligned} 2w_z &= \frac{1}{r} \frac{\partial}{\partial r} \left[ u \sin \theta + \frac{u}{r^2} \sin \theta \right] - \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[ u \cos \theta + \frac{u}{r^2} \cos \theta \right] \\ &= \frac{1}{r} \left( -\frac{2u}{r^3} \sin \theta \right) - \frac{1}{r^2} \left[ -u \sin \theta + \frac{u}{r^2} \sin \theta \right] \\ &= -\frac{2u \sin \theta}{r^4} - \frac{u \sin \theta}{r^2} + \frac{u}{r^4} \sin \theta \end{aligned}$$

D.

$$\phi = \left(r - \frac{2}{r}\right) \sin \theta \quad \text{and} \quad V_t = -\frac{\partial \phi}{\partial r} = -\left[1 + \frac{2}{r^2}\right] \sin \theta$$

$$V_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{1}{r} \left[r - \frac{2}{r}\right] \cos \theta = -\left[1 - \frac{2}{r^2}\right] \cos \theta$$

$$\begin{aligned} 2D : \frac{1}{r} \frac{\partial}{\partial r} (rV_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (V_\theta) \\ = \frac{1}{r} \frac{\partial}{\partial r} \left[ \left(-r - \frac{2}{r}\right) \sin \theta \right] + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \left(-1 - \frac{2}{r}\right) \cos \theta \right] \\ = \frac{1}{r} \left(-1 - \frac{2}{r}\right) \sin \theta + \frac{1}{r} \times \left(1 - \frac{2}{r}\right) \sin \theta \\ = \frac{2}{r} \left[-1 + \frac{2}{r^2}\right] = 0 \end{aligned}$$

**IES-41. The velocity potential of a velocity field is given by  $= x^2 - y^2 + \text{const.}$  Its stream function will be given by:** [IES-2002]

- (a)  $-2xy + \text{constant}$  (b)  $+2xy + \text{constant}$   
(c)  $-2xy + f(x)$  (d)  $-2xy + f(y)$

**IES-41. Ans. (b)** Use Cauchy - Riemann equation

**IES-42. The stream function in a 2-dimensional flow field is given by,  $\psi = xy$ .**

**xy.**

**The potential function is:**

[IES-2001]

- (a)  $\frac{x^2 + y^2}{2}$  (b)  $\frac{x^2 - y^2}{2}$  (c)  $xy$  (d)  $x^2 y + y^2 x$

**IES-42. Ans (b)** and therefore

$$u = \frac{\partial \psi}{\partial y} = x = -\frac{\partial \phi}{\partial x} \quad v = -\frac{\partial \psi}{\partial x} = -y = -\frac{\partial \phi}{\partial y} \quad d\phi =$$

$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

**IES-43. A stream function is given by  $(x^2 - y^2)$ . The potential function of the flow will be:** [IES-2000]

- (a)  $2xy + f(x)$  (b)  $2xy + \text{constant}$  (c)  $2(x^2 - y^2)$  (d)  $2xy + f(y)$

**IES-43. Ans. (b)**

**IES-44. The velocity components for a two dimensional incompressible flow of a fluid are  $u = x - 4y$  and  $v = -y - 4x$ . It can be concluded that** [IES-1992]

- (a) The flow does not satisfy the continuity equation (b) The flow is rotational  
(c) The flow is irrotational (d) None of the above

**IES-44. Ans. (c)**

Since,  $u = x - 4y$ ;  $\therefore \frac{\partial u}{\partial x} = 1$

and,  $v = -y - 4x$ ;  $\therefore \frac{\partial v}{\partial y} = -1$

$\therefore \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = -1 - 1 = 0$

Hence, the flow satisfies the continuity equation

Also,  $\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = -4 - (-4) = 0$

Hence, the flow is irrotational.

**IES-45. The stream function  $\psi = x^3 - y^3$  is observed for a two dimensional flow field. What is the magnitude of the velocity at point (1, -1)?**

**[IES-2004; IES-1998]**

- (a) 4.24                                      (b) 2.83                                      (c) 0                                      (d) -

**IES-45. Ans. (a)**                                      and                                      = -3

$u = \frac{\partial \psi}{\partial y} = -3y^2 = -3$                                        $v = -\frac{\partial \psi}{\partial x} = -3x^2$                                        $\therefore$

= 4.24

$\sqrt{(-3)^2 + (-3)^2}$

**IES-46. The stream function in a flow field is given by  $\psi = 2xy$ . In the same flow field, what is the velocity at a point (2, 1)?** **[IES-2008]**

- (a) 4 unit                                      (b) 5.4 unit                                      (c) 1.73 unit                                      (d) 4.47 unit

**IES-46. Ans. (d)**

$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$

$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y$

$u(2, 1) = -4$  and  $v(2, 1) = 2$

$\therefore$  Velocity at point (2, 1) =  $\sqrt{(-4)^2 + 2^2} = \sqrt{20} = 4.47$  unit

**IES-47. For irrotational and incompressible flow, the velocity potential and stream function are given by,                                      respectively. Which one**

$\phi$  and  $\psi$

**of the following sets is correct?**

**[IES-2006]**

(a)

$$\nabla^2\phi = 0, \nabla^2\psi = 0$$

(b)

$$\nabla^2\phi \neq 0, \nabla^2\psi = 0$$

(c)

$$\nabla^2\phi = 0, \nabla^2\psi \neq 0$$

(d)

$$\nabla^2\phi \neq 0, \nabla^2\psi \neq 0$$

**IES-47. Ans. (a)**

**IES-48. Which one of the following statements is true to two-dimensional flow of ideal fluids? [IES-1996]**

- (a) Potential function exists if stream function exists.
- (b) Stream function may or may not exist.
- (c) Both potential function and stream function must exist for every flow.
- (d) Stream function will exist but potential function may or may not exist.

**IES-48. Ans. (d)** For a possible case of fluid flow Stream function will exist, but potential function will exist only for irrotational flow. In this case flow may be rotational or irrotational.

**IES-49. The realisation of velocity potential in fluid flow indicates that the**

- (a) Flow must be irrotational [IES-1993]
- (b) Circulation around any closed curve must have a finite value
- (c) Flow is rotational and satisfies the continuity equation
- (d) Vorticity must be non-zero

**IES-49. Ans. (a)** The realisation of velocity potential in fluid flow indicates that the flow must be irrotational.

## Flow Net

**IES-50. For an irrotational flow, the velocity potential lines and the streamlines are always. [IES-1997]**

- (a) Parallel to each other
- (b) Coplanar
- (c) Orthogonal to each other
- (d) Inclined to the horizontal.

**IES-50. Ans. (c)** Slope of velocity potential =

$$\left(\frac{dy}{dx}\right)_1 = -\frac{v}{u}$$

Slope of stream line

$$\left(\frac{dy}{dx}\right)_2 = \frac{v}{u}$$

$$\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -\frac{v}{u} \times \frac{v}{u} = -1$$

Hence, they are orthogonal to each other.

## Acceleration in Fluid Vessel

**IES-51. A cylindrical vessel having its height equal to its diameter is filled with liquid and moved horizontally at acceleration equal to acceleration due to gravity. The ratio of the liquid left in the vessel to the liquid at static equilibrium condition is: [IES-2001]**

- (a) 0.2
- (b) 0.4
- (c) 0.5
- (d) 0.75

**Previous 20-Years IAS Questions**

### Tangential and Normal Acceleration

**IAS-1. Which one of the following statements is correct? [IAS-2004]**

**A steady flow of diverging straight stream lines**

- (a) Is a uniform flow with local acceleration
- (b) Has convective normal acceleration
- (c) Has convective tangential acceleration
- (d) Has both convective normal and tangential accelerations

**IAS-1. Ans. (c)**

### Stream Line

**IAS-2. In a two-dimensional flow, where  $u$  is the  $x$ -component and  $v$  is the  $y$ -component of velocity, the equation of streamline is given by [IAS-1998]**

- (a)  $u dx - v dy = 0$
- (b)  $v dx - u dy = 0$
- (c)  $u v dx + dy = 0$
- (d)  $u dx + v dy = 0$

**IAS-2. Ans. (b)** or

$$\frac{dx}{u} = \frac{dy}{v} \qquad v dx - u dy = 0$$

### Streak Line

**IAS-3. Consider the following statements: [IAS-2001]**

1. Streak line indicates instantaneous position of particles of fluid passing through a point.
2. Streamlines are paths traced by a fluid particle with constant velocity.
3. Fluid particles cannot cross streamlines irrespective of the type of flow.
4. Streamlines converge as the fluid is accelerated, and diverge when retarded.

**Which of these statements are correct?**

- (a) 1 and 4
- (b) 1, 3 and 4
- (c) 1, 2 and 4
- (d) 2 and 3

**IAS-3. Ans. (b)** 2 is wrong.

### Continuity Equation

**IAS-4. Which one of the following is the continuity equation in differential form? (The symbols have usual meanings) [IAS-2004; IAS-2003]**

- (a)  $\frac{dA}{A} + \frac{dV}{V} + \frac{dp}{\rho} = \text{const.}$
- (b)  $\frac{dA}{A} + \frac{dV}{V} + \frac{dp}{\rho} = 0$

(c)

$$\frac{A}{dA} + \frac{V}{dV} + \frac{\rho}{d\rho} = \text{const.}$$

(d)  $AdA + VdV + \frac{d\rho}{\rho} = 0$

**IAS-4. Ans. (b)**  $= 0$

$$\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho}$$

Integrating, we get  $\log A + \log V + \log \rho = \log C$

$\therefore$

or,  $\log\left(\frac{AV\rho}{\rho}\right) = \log C$

$$AV\rho = C$$

$\therefore \rho$

which is the continuity equation

**IAS-5. Which one of the following equations represents the continuity equation for steady compressible fluid flow? [IAS-2000]**

(a)

$$\Delta \cdot \rho V + \frac{\partial \rho}{\partial t} = 0$$

(b)

$$\Delta \cdot \rho V + \frac{\partial \rho}{\partial t} = 0$$

(c)

$$\Delta \cdot V = 0$$

(d)

$$\Delta \cdot \rho V = 0$$

**IAS-5. Ans. (d)** General continuity equation

$$\nabla \cdot \rho V + \frac{\partial \rho}{\partial t} = 0$$

For steady flow  $\frac{\partial \rho}{\partial t} = 0$  and for compressible fluid the equation  $\nabla \cdot \rho V = 0$

$$\frac{\partial \rho}{\partial t} = 0,$$

$$\nabla \cdot \rho V$$

For steady, incompressible flow  $\frac{\partial \rho}{\partial t} = 0$  and  $\rho = \text{const.}$  So the equation

$$\frac{\partial \rho}{\partial t} = 0 \quad \rho$$

$$\nabla \cdot V = 0$$

**IAS-6. The continuity equation for 3-dimensional flow  $\nabla \cdot V = 0$  is**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

**applicable to:**

**[IAS-1999; IAS-1998, 1999]**

(a) Steady flow

(b) Uniform flow

(c) Ideal fluid flow

(d) Ideal as well as viscous flow

**IAS-6. Ans. (c)**

**IAS-7. In a steady, incompressible, two dimensional flow, one velocity component in the x-direction is given by  $u = cx^2/y^2$ . The velocity component in the y-direction will be: [IAS-1997]**

(a)  $V = -c(x + y)$

(b)  $v = -cx/y$

(c)  $v = -xy$

(d)  $v = -cy/x$

**IAS-7. Ans. (b)**



**IAS-8.** The components of velocity  $u$  and  $v$  along  $x$ - and  $y$ -direction in a 2-D flow problem of an incompressible fluid are: [IAS-1994]

1.  $u = x^2 \cos y$ ;  $v = -2x \sin y$       2.  $u = x + 2$ ;  $v = 1 - y$

3.  $u = xy$ ;  $v = x^3 - y^2 t / 2$       4.  $u = \ln x + y$ ;  $v = xy - y/x$

Those which would satisfy the continuity equation would include

- (a) 1, 2 and 3 (b) 2, 3 and 4      (c) 3 and 4      (d) 1 and 2

**IAS-8. Ans. (a)** Checking  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$  for all cases.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

**IAS-9.** If  $\vec{V}$  is velocity vector of fluid, then  $\Delta \vec{V} = 0$  is strictly true for

which of the following? [IAS-2007; GATE-2008]

- (a) Steady and incompressible flow  
 (b) Steady and irrotational flow  
 (c) Inviscid flow irrespective of steadiness  
 (d) Incompressible flow irrespective of steadiness

**IAS-9. Ans. (d)**

$$\nabla \cdot \vec{V} = 0 \quad \text{Or} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

## Circulation and Vorticity

**IAS-10.** Which one of the following is the expression of the rotational component for a two-dimensional fluid element in  $x$ - $y$  plane?

- (a)      (b) [IAS-2004; IAS-2003]

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

(c)

(d)

$$\omega_z = \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)$$

$$\omega_z = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

**IAS-10. Ans. (a)**

**IAS-11.** Which of the following relations must hold for an irrotational two-dimensional flow in the  $x$ - $y$  plane? [IAS-2003, 2004, IES-1995]

(a)

(b)

$$\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

(c)

(d)

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

**IAS-11. Ans. (d)** i.e.

$$\omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

**IAS-12. A liquid mass readjusts itself and undergoes a rigid body type of motion when it is subjected to a** **[IAS-1998]**

- |                               |                                   |
|-------------------------------|-----------------------------------|
| (a) Constant angular velocity | (b) Constant angular acceleration |
| (c) Linearly varying velocity | (d) Linearly varying acceleration |

**IAS-12. Ans. (a)**

## Velocity Potential Function

**IAS-13. The velocity potential function in a two dimensional flow fluid is given by  $\phi = x^2 - y^2$ . The magnitude of velocity at the point (1, 1) is:**

$$\phi =$$

**[IAS-2002]**

- |       |       |                 |                 |
|-------|-------|-----------------|-----------------|
| (a) 2 | (b) 4 | (c) $2\sqrt{2}$ | (d) $4\sqrt{2}$ |
|-------|-------|-----------------|-----------------|

**IAS-13. Ans. (c)**  $u =$

$$-\frac{\partial \phi}{\partial x} = -2x, \quad v = -\frac{\partial \phi}{\partial y} = +2y$$

$$V = \quad \text{unit}$$

$$\sqrt{u^2 + v^2} = \sqrt{(2x)^2 + (2y)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

## Stream Function

**IAS-14. For a stream function to exist, which of the following conditions should hold?** **[IAS-1997]**

1. The flow should always be irrotational.
  2. Equation of continuity should be satisfied.
  3. The fluid should be incompressible.
  4. Equation of continuity and momentum should be satisfied.
- Select the correct answer using the codes given below:

**Codes:**

- |                         |                |             |       |
|-------------------------|----------------|-------------|-------|
| (a) 1, 2, 3 and 4 alone | (b) 1, 3 and 4 | (c) 2 and 3 | (d) 2 |
|-------------------------|----------------|-------------|-------|

**IAS-14. Ans. (d)**

**IAS-15. Consider the following statements:** **[IAS-2002]**

1. For stream function to exist, the flow should be irrotational.
2. Potential functions are possible even though continuity is not satisfied.
3. Streamlines diverge where the flow is accelerated.
4. Bernoulli's equation will be satisfied for flow across a cross-section.

**Which of the above statements is/are correct?**

- |                        |                |             |       |
|------------------------|----------------|-------------|-------|
| (a) 1, 2, 3 and 4 only | (b) 1, 3 and 4 | (c) 3 and 4 | (d) 2 |
|------------------------|----------------|-------------|-------|

**IAS-15. Ans. (c)** 1. Stream function is exist for possible case of fluid flow i.e. if continuity is satisfied but flow may be rotational or irrotational, 1 is wrong.

2. Potential function will exist for possible and irrotational flow so both continuity and irrotational must be satisfied, 2 is wrong.

**Flow Net**

**IAS-16.** Consider the following statements for a two dimensional potential flow:

1. Laplace equation for stream function must be satisfied. [IAS-2002]
2. Laplace equation for velocity potential must be satisfied.
3. Streamlines and equipotential lines are mutually perpendicular.
4. Streamlines can intersect each other in very high speed flows.

**Which of the above statements are correct?**

- (a) 1 and 4                      (b) 2 and 4                      (c) 1, 2 and 3                      (d) 2, 3 and 4

**IAS-16. Ans. (c)** Streamlines never intersect each other.



## Fluid Dynamics

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### **Contents of this chapter**

1. Bernoulli's Equation
  2. Euler's Equation
  3. Venturimeter
  4. Orifice Meter
  5. Pitot Tube
  6. Free Liquid Jet
  7. Impulse Momentum Equation
  8. Forced Vortex
  9. Free Vortex
-

**Question:** Derive from the first principles the Euler's equation of motion for steady flow along a streamline. Obtain Bernoulli's equation by its integration. State the assumptions made. [IES – 1997] [Marks-10]

**Answer:** Consider steady flow of an ideal fluid along the stream tube. Separate out a small element of fluid of cross-sectional area  $dA$  and length  $dS$  from stream tube as a free body from the moving fluid.

Fig (below) shows such a small element LM of fluid of cross section area  $dA$  and length  $dS$ .

Let,  $p$  = Pressure on the element at L.

$p + dp$  = Pressure on the element at M and

$V$  = velocity of fluid along stream line.

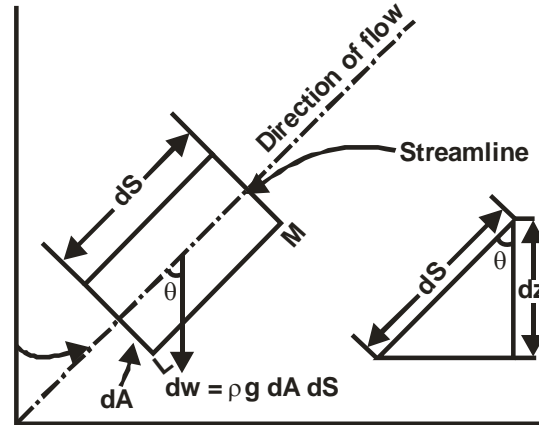


Fig: Force on a fluid element

The external forces tending to accelerate the fluid element in the direction of stream line are as follows.

1. Net pressure force in the direction of flow is  
 $p \cdot dA - (p + dp) dA = -dp dA$
2. Component of weight of the fluid element in the direction of flow is  
 $= -\rho g dA dS \cos \theta$   
 $= -\rho g dA dS \cdot \left( \frac{dz}{dS} \right) \quad \left[ \because \cos \theta = \frac{dz}{dS} \right]$   
 $= -\rho g dA dS dz$

Mass of the fluid element =  $\rho dA dS$

The acceleration of a fluid element

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \cdot \frac{dv}{ds} \quad (v \text{ along the direction of streamline})$$

$\therefore$  According to Newton second law of motion

Force = mass  $\times$  acceleration

$$-dp dA - \rho g dA dz = \rho dA dS \cdot v \frac{dv}{ds}$$

$$\text{or } -dp dA - \rho g dA dz = \rho dA \cdot (v \cdot dv)$$

Dividing both side by  $\rho dA$

$$\frac{-dp}{\rho} - g dz = v \cdot dv$$

or,  $\boxed{\frac{dp}{\rho} + v \cdot dv + g dz = 0}$  Euler's equation of motion for steady flow along a stream line.

**Question:** *Derive Bernoulli's Equation*

**Answer:** Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{const.}$$

∴ For incompressible flow ( $\rho = \text{const.}$ )

$$\frac{p}{\rho} + gZ + \frac{v^2}{2} = \text{const}$$

$$\text{or } \frac{p}{\rho g} + Z + \frac{v^2}{2g} = \text{const.}$$

$$\text{or } \boxed{\frac{p}{w} + \frac{v^2}{2g} + z = \text{const.}} \quad \text{where } w = \text{unit weight (specific weight)}$$

**For compressible flow**  $\left(\frac{p}{\rho^\gamma} = c\right)$  **undergoing adiabatic flow.**

$$p = c \cdot \rho^\gamma$$

$$\text{or } dp = c \cdot \gamma \cdot \rho^{\gamma-1} d\rho \quad \text{and} \quad \int \frac{c \cdot \gamma \cdot \rho^{\gamma-1} d\rho}{\rho g} + \frac{1}{g} \int v dv + \int dz = \text{const.}$$

$$\text{or } \frac{\gamma \cdot c}{g} \int \rho^{\gamma-2} d\rho + \frac{v^2}{2g} + z = \text{const.}$$

$$\text{or } \frac{\gamma}{g} \cdot \frac{p}{\rho^\gamma} \cdot \frac{\rho^{\gamma-1}}{\gamma-1} + \frac{v^2}{2g} + z = \text{const.}$$

$$\text{or } \boxed{\frac{\gamma}{\gamma-1} \cdot \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{const.}} \quad \text{For compressible flow undergoing adiabatic process.}$$

**For compressible flow**  $\left(\frac{p}{\rho} = c\right)$  **undergoing isothermal process**

$$\rho = \frac{p}{c}$$

$$\therefore c \int \frac{d\rho}{\rho} + g \int dz + \int v dv = \text{const}$$

$$\text{or } \frac{p}{\rho} \cdot \ln \rho + gz + \frac{v^2}{2} = \text{const.}$$

$$\text{or } \boxed{\frac{p \ln p}{\rho g} + \frac{v^2}{2g} + z = \text{const.}} \quad \text{For compressible flow undergoing isothermal process.}$$

**Question:** *Define Bernoulli's theorem and explain what corrections are to be made in the equation for ideal fluid, if the fluid is a real fluid.*

[AMIE (Win) 2002; AMIE MAY 1974]

**Answer:** **Statement of Bernoulli's Theorem:** "It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant".

The total energy consists of

(i) Pressure energy =  $\frac{p}{\rho g}$

(ii) Kinetic energy =  $\frac{v^2}{2g}$ , and

(iii) Datum or potential energy =  $z$

Thus Mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

**Correction:**

(i) Bernoulli's equation has restriction of frictionless flow. For real fluid this is accommodated by introducing a loss of energy term ( $h_L$ ) i.e.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

(ii) Restriction of irrotational flow is waived in most of the cases.

**Question:** *Velocity distribution in a pipe is given by  $\frac{u}{u_{\max}} = 1 - \left(\frac{r}{R}\right)^n$*

Where,  $u_{\max}$  = Maximum velocity at the centre of pipe.

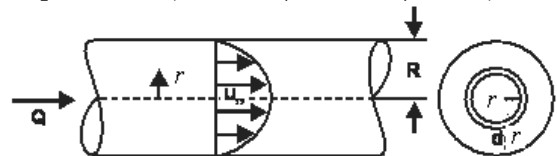
$u$  = velocity at a distance  $r$

$R$  = radius of the pipe.

Obtain an expression for mean velocity in terms of  $u_{\max}$  and  $n$ .

[IES-1997; AMIE (summer)-1998, 2001]

**Answer:** Consider an elementary strip at a distance  $r$  from the center and thickness  $dr$ .  
 $\therefore$  Area,  $dA = 2\pi r dr$



As velocity is  $u$  at that point so discharge then the elementary ring

$$dQ = dA \times u = 2\pi r dr \cdot u$$

$$\therefore dQ = 2\pi r u_{\max} \left(1 - \frac{r^n}{R^n}\right) dr$$

$\therefore$  Total flow  $Q$  is

$$\begin{aligned} Q &= \int dQ = \int_0^R 2\pi u_{\max} r \left(1 - \frac{r^n}{R^n}\right) dr = 2\pi u_{\max} \int_0^R \left(r - \frac{r^{n+1}}{R^n}\right) dr \\ &= 2\pi u_{\max} \left[ \frac{R^2}{2} - \frac{R^{n+2}}{(n+2)R^n} \right] = 2\pi u_{\max} R^2 \left( \frac{1}{2} - \frac{1}{n+2} \right) \\ &= \pi u_{\max} R^2 \left( \frac{n}{n+2} \right) \end{aligned}$$

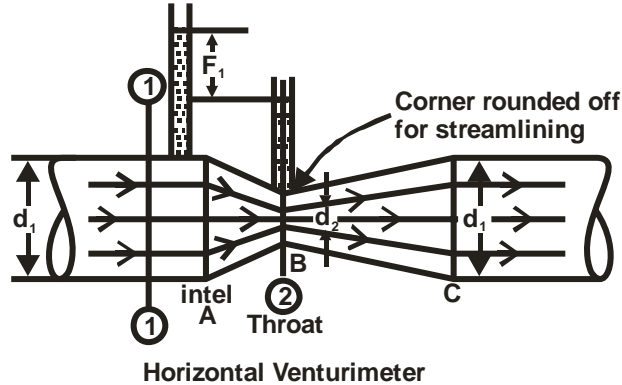
If mean velocity is  $\bar{U}$  then flow  $Q = \pi R^2 \bar{U}$

$$\therefore \pi R^2 \bar{U} = \pi u_{\max} R^2 \frac{n}{(n+2)}$$

$$\therefore \bar{U} = u_{\max} \times \frac{n}{(n+2)}$$

**Question:** Derive an expression for rate of flow through Horizontal Venturimeter. What changes have to be made for vertical & inclined Venturimeter? [IES-2003]

**Answer:** A venturimeter consists of the following three parts  
 (i) A short converging part (AB)  
 (ii) Throat, B, and  
 (iii) Diverging part, BC  
 Fig (below) shows a venturimeter fitted in horizontal pipe through which a incompressible fluid is flowing.



Let,  $d_1$  = diameter at inlet

$$A_1 = \text{Area at inlet} \left( \frac{\pi d_1^2}{4} \right)$$

$p_1$  = Pressure at inlet

$V_1$  = Velocity at inlet

and,  $d_2$ ,  $A_2$ ,  $p_2$  and  $V_2$  are the corresponding values at throat.

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\text{or } \frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\text{or } h = \frac{V_2^2 - V_1^2}{2g} \quad [\text{where } h = \text{difference of pressure head from manometer}]$$

$$\text{or } V_2^2 - V_1^2 = 2gh$$

Applying continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\text{or } V_1 = \frac{A_2}{A_1} V_2$$

$$\therefore V_2^2 - V_1^2 = 2gh$$

$$\text{or } V_2^2 - \left( \frac{A_2}{A_1} \right)^2 V_2^2 = 2gh$$

$$\text{or } V_2^2 \{ A_1^2 - A_2^2 \} = A_1^2 \times 2gh$$



$$\text{or } V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$$\therefore \text{Discharge (Q)} = A_2 V_2 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

If co-efficient of discharge is  $C_d$  the actual discharge

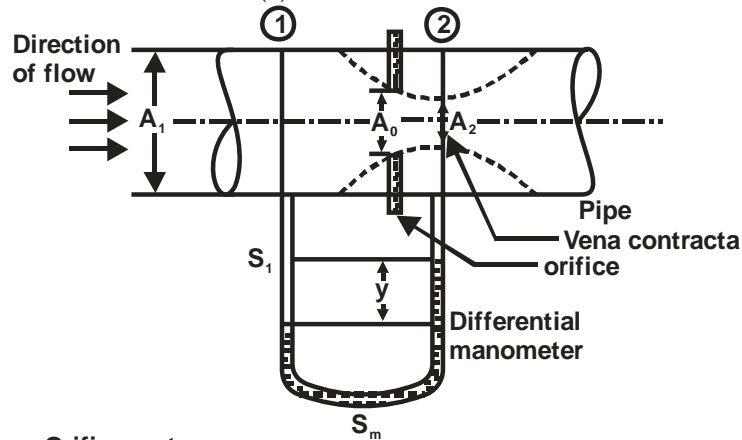
$$Q_{\text{actual}} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

**Question:**

**Derive an expression for discharge through orifice meter.**

**Answer:**

It consists of a flat circular plate having a circular sharp edged hole (called orifice) and a differential manometer is connected between section (1) and vena contracta section (2)



**: Orifice meter :**

Let,  $A_1$  = Area of pipe

$A_2$  = Area of vena-contracta

$A_0$  = Orifice Area

$p_1$  = Pressure at section (1)

$p_2$  = Pressure at vena contracta, section (2)

$V_1$  = Velocity at section (1)

$V_2$  = Velocity at section (2)

$$C_c = \text{Co-efficient of contraction} \left( \frac{A_2}{A_0} \right)$$

$$C_d = \text{Co-efficient of discharge} \left[ C_d = C_c \times \frac{\sqrt{1 - \left( \frac{A_0}{A_1} \right)^2}}{\sqrt{1 - \left( \frac{A_0}{A_1} \right)^2 \cdot C_c^2}} \right]$$

Applying Bernoulli's equation between section (1) and (2) we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \left( \frac{p_1}{\rho g} + Z_1 \right) - \left( \frac{p_2}{\rho g} + Z_2 \right) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\text{or } h = \frac{V_2^2 - V_1^2}{2g} \left[ \text{where } h \text{ is the differential head (h)} = y \left\{ \frac{S_m}{S_l} - 1 \right\} \right]$$

Using continuity equation  $A_1 V_1 = A_2 V_2$  and  $A_2 = A_0 \cdot C_c$

$$V_1 = \frac{A_2}{A_1} \cdot V_2 = \frac{A_0 C_c}{A_1} \cdot V_2$$

$$\therefore V_2^2 - \frac{A_0^2 C_c^2}{A_1^2} V_2^2 = 2gh$$

$$\text{or } V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} \cdot C_c^2}}$$

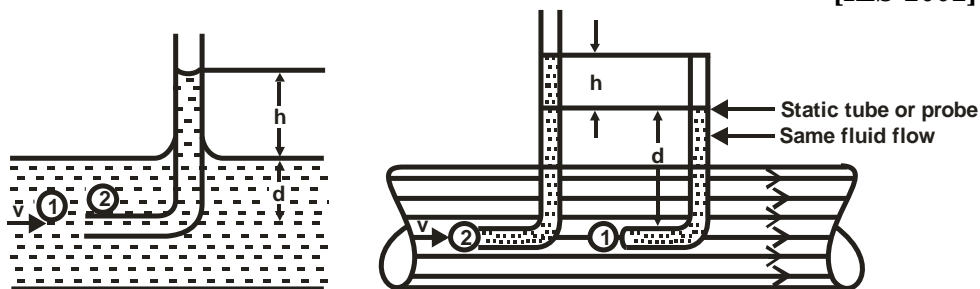
$$\therefore \text{Discharge (Q)} = A_2 V_2 = A_0 C_c \cdot V_2$$

$$= \frac{A_0 \cdot C_c \cdot \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} \cdot C_c^2}} = A_0 \cdot \frac{C_d}{\sqrt{1 - \frac{A_0^2}{A_1^2}}} \cdot \sqrt{2gh} = \frac{C_d \cdot A_1 \cdot A_0 \cdot \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

$$\therefore Q = \frac{C_d A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

**Question:** Describe the working principal of a pitot-static tube with the help of neat sketch and explain how it can be used to measure the flow rate.

[IES-2002]



A. Pitot-tube in a open channel

B. Pitot-tube in Pipe flow

The 'stagnation pressure' at a point in a fluid flow is the total pressure which would result if the fluid were brought to rest isentropically. In actual practice, a stagnation point is created by bringing the fluid to rest at the desired point and the pressure at the point corresponds to the stagnation pressure as shown is above fig. A & B, 'h'. A simple device to measure the stagnation pr. is a tube with a hole in the front inserted in the flow such that the velocity is normal to the plane of the hole.

Stagnation Pressure = static Pressure + dynamic Pressure

$$\therefore p_0 = p + \frac{\rho}{2} v^2$$

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Here  $z_1 = z_2$  and  $v_2 = 0$  and  $\frac{p_1}{\rho g} = d$  and  $\frac{p_2}{\rho g} = (d+h)$

$$\therefore d + \frac{v_1^2}{2g} = d+h$$

$$\text{or } v_1 = \sqrt{2gh}$$

Here  $v_1$  is theoretical velocity  $\therefore v_{\text{actual}} = c_v \cdot c_1 = c_v \sqrt{2gh}$

Where  $c_v$  = co-efficient of velocity depends on tube.

$\therefore$  If velocity is known then

$$\text{Discharge (Q)} = A \cdot v_{\text{actual}} = c_v A \sqrt{2gh}$$

**Question:** *What is vortex flow? What is the difference between Free vortex flow and forced vortex flow?*

**Answer:** **Vortex flow:** A flow in which the whole fluid mass rotates about an axis. In vortex flow streamlines are curved.

**Forced vortex flow:** Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency.

Here angular velocity,  $\omega = \frac{v}{r} = \text{constant}$

**Example:** Rotation of water through the runner of a turbine.

**Free vortex flow:** Free vortex flow is one in which the fluid mass rotates without any external impressed contact force.

The whole fluid mass rotates either due to fluid pr. itself or the gravity or due to rotation previously imparted.

Here Moment of Momentum = const.

i.e.  $V \times r = \text{constant}$

**Example:** A whirlpool in a river.

# OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

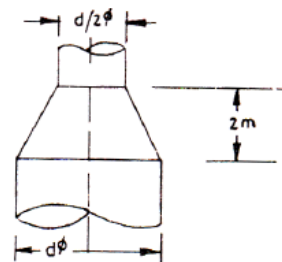
### Bernoulli's Equation

GATE-1. Bernoulli's equation can be applied between any two points on a streamline for a rotational flow field. [GATE-1994]

GATE-1. Ans. True

GATE-2. Water flows through a vertical contraction from a pipe of diameter  $d$  to another of diameter  $d/2$ . The flow velocity at the inlet to the contraction is  $2\text{ m/s}$  and pressure  $200\text{ kN/m}^2$  if the height of the contraction measures  $2\text{ m}$ , the pressure at the exit of the contraction will be very nearly

- (a)  $168\text{ kN/m}^2$  (b)  $192\text{ kN/m}^2$   
(c)  $150\text{ kN/m}^2$  (d)  $174\text{ kN/m}^2$



[GATE-1999]

GATE-2. Ans. (c)

GATE-3. Consider steady, incompressible and irrotational flow through a reducer in a horizontal pipe where the diameter is reduced from  $20\text{ cm}$  to  $10\text{ cm}$ . The pressure in the  $20\text{ cm}$  pipe just upstream of the reducer is  $150\text{ kPa}$ . The fluid has a vapour pressure of  $50\text{ kPa}$  and a specific weight of  $5\text{ kN/m}^3$ . Neglecting frictional effects, the maximum discharge (in  $\text{m}^3/\text{s}$ ) that can pass through the reducer without causing cavitation is: [GATE -2009]

- (a)  $0.05$  (b)  $0.16$  (c)  $0.27$  (d)  $0.38$

GATE-3. Ans. (b)  $\rho g = 5000\text{ N/m}^3$

$$A_1 V_1 = A_2 V_2 \quad \dots\dots(1)$$

$$\text{or } V_2 = \left(\frac{20}{10}\right)^2 \times V_1 = 4V_1$$

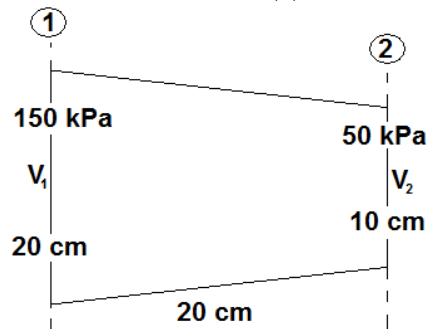
$$\frac{150 \times 10^3}{5000} + \frac{V_1^2}{2g} = \frac{50 \times 10^3}{5000} + \frac{V_2^2}{2g} \quad \dots\dots(2)$$

$$\text{or } 30 + \frac{V_1^2}{2g} = 10 + \frac{V_2^2}{2g} \quad 16 \frac{V_1^2}{g}$$

$$\text{or } 20 = 15 \frac{V_1^2}{2g}$$

$$\text{or } V_1 = \sqrt{\frac{20 \times 2 \times g}{15}}$$

$$\text{Then } Q = A_1 V_1 = 0.16$$



## Euler's Equation

GATE-4. Navier Stoke's equation represents the conservation of [GATE-2000]

- (a) Energy (b) Mass (c) Pressure (d) Momentum

GATE-4. Ans. (d)

## Venturimeter

GATE-5. In a venturimeter, the angle of the diverging section is more than that of converging section. [GATE-1994]

- (a) True (b) False (c) Insufficient data (d) Can't say

GATE-5. Ans. (b) The angle of diverging section is kept small to reduce the possibility of flow separation. Due to this the angle of converging section is more as compared to its diverging section.

GATE-6. A venturimeter of 20 mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40 mm diameter. If the pressure difference between the pipe and throat sections is found to be 30 kPa then, neglecting frictional losses, the flow velocity is:

[GATE-2005]

- (a) 0.2 m/s (b) 1.0 m/s (c) 1.4 m/s (d) 2.0 m/s

GATE-6. Ans. (d) We know,  $A_1 V_1 = A_2 V_2$

$$\Rightarrow V_2 = \frac{D_1^2}{D_2^2} V_1 = \frac{16}{4} V_1$$

$$\therefore V_2 = 4V_1$$

Applying Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

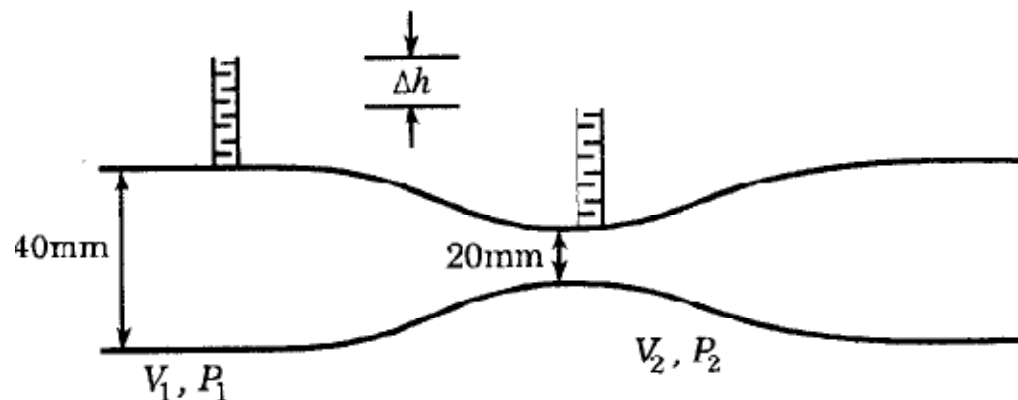
$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow \frac{15V_1^2}{2} = \frac{30 \times 10^3}{1000}$$

$$\Rightarrow V_1^2 = 4$$

$$\Rightarrow V_1 = 2.0 \text{ m/s}$$

So velocity of flow is 2.0 m/sec.



GATE-7. Air flows through a venture and into atmosphere. Air density is  $\rho$ ; atmospheric pressure  $P_a$ ; throat diameter is  $D_t$ ; exit diameter is  $D$  and

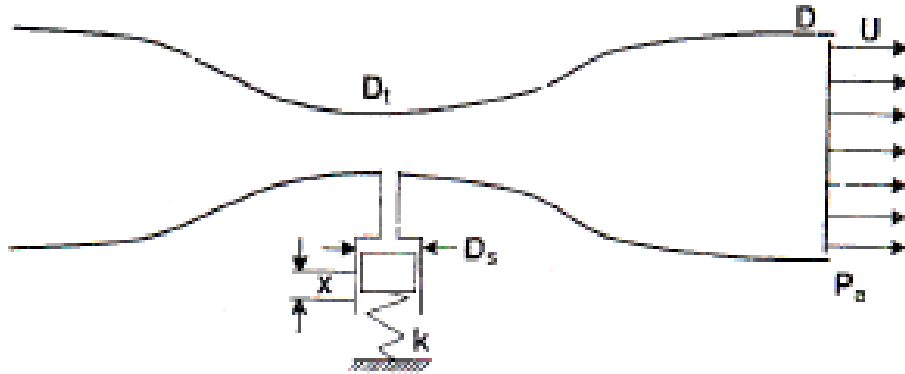
exit velocity is  $U$ . The throat is connected to a cylinder containing a frictionless piston attached to a spring. The spring constant is  $k$ . The bottom surface of the piston is exposed to atmosphere. Due to the flow, the piston moves by distance  $x$ . assuming incompressible frictionless flow,  $x$  is: [GATE-2003]

(a)  $(\rho U^2 / 2k) \pi D_s^2$

(b)  $(\rho U^2 / 8k) \left( \frac{D^2}{D_t^2} - 1 \right) \pi D_s^2$

(c)  $(\rho U^2 / 2k) \left( \frac{D^2}{D_t^2} - 1 \right) \pi D_s^2$

(d)  $(\rho U^2 / 8k) \left( \frac{D^4}{D_t^4} - 1 \right) \pi D_s^2$



**GATE-7. Ans. (d)** Applying Bernoulli's equation at points (1) and (2), we have

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Since venturi is horizontal  $z_1 = z_2$

$$\text{Now, } \left( \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow (P_1 - P_2) = \frac{\rho g}{2g} (v_2^2 - v_1^2) = \frac{\rho}{2} (v_2^2 - v_1^2)$$

Since  $P_2 = P_a = \text{atmospheric pressure}$

$$\therefore (P_1 - P_a) = \frac{\rho}{2} (v_2^2 - v_1^2) \quad \text{----- (i)}$$

Applying continuity equation at points (i) and (ii), we have

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_1 = \left( \frac{A_2}{A_1} \right) v_2 \quad \text{since } V_2 = U$$

$$v_1 = \left( \frac{\frac{\pi D^2}{4}}{\frac{\pi D_t^2}{4}} \right) U \quad \Rightarrow \quad v_1 = \left( \frac{D}{D_t} \right)^2 U$$

From equation (i),

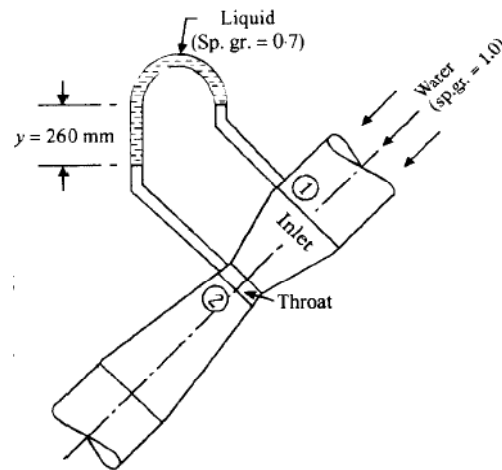
$$P_1 - P_a = \frac{\rho}{2} \left[ v^2 - \left( \frac{D}{D_t} \right)^2 U^2 \right] = \frac{\rho}{2} U^2 \left[ 1 - \frac{D^4}{D_t^4} \right]$$

At point P: Spring force = pressure force due air

$$-kx = \frac{\pi}{4} D_s^2 \times \frac{\rho U^4}{2} \left[ 1 - \frac{D^4}{D_t^4} \right]$$

$$\Rightarrow x = \frac{\pi D_s^2 \rho U^2}{8k} \left[ 1 - \frac{D^4}{D_t^4} \right]$$

**GATE-8.** Determine the rate of flow of water through a pipe 300 mm diameter placed in an inclined position where a Venturimeter is inserted having a throat diameter of 150 mm. The difference of pressure between the main and throat is measured by a liquid of sp. gravity 0.7 in an inverted V-tube which gives a reading of 260 mm. The loss of head between the main and throat is 0.3 times the kinetic head of the pipe.



[GATE-1985]

(a) 0.0222 m<sup>3</sup>/s(b) 0.4564 m<sup>3</sup>/s(c) 1.m<sup>3</sup>/s(d) m<sup>3</sup>/s

**GATE-8. Ans. (a)**

## Free Liquid Jet

**GATE-9.** Two balls of mass  $m$  and  $2m$  are projected with identical velocities from the same point making angles  $30^\circ$  and  $60^\circ$  with the vertical axis, respectively. The heights attained by the balls will be identical.

[GATE-1994]

(a) True

(b) False

(c) None

(d) Can't say

**GATE-9. Ans. (b)**

## Forced Vortex

**GATE-10.** Which combination of the following statements about steady incompressible forced vortex flow is correct? [GATE-2007]

**P:** Shear stress is zero at all points in the flow.

**Q:** Vorticity is zero at all points in the flow

**R:** Velocity is directly proportional to the radius from the centre of the vortex.

**S:** Total mechanical energy per unit mass is constant in the entire flow field.

(a) P and Q

(b) R and S

(c) P and R

(d) P and S

**GATE-10. Ans. (b)**

**GATE-11.** A closed cylinder having a radius  $R$  and height  $H$  is filled with oil of density  $\rho$ . If the cylinder is rotated about its axis at an angular velocity of  $\omega$ , then thrust at the bottom of the cylinder is: [GATE-2004]

(a)  $\pi R^2 \rho g H$ (b)  $\pi R^2 \frac{\rho \omega^2 R^2}{4}$

(c)  $\pi R^2(\rho \omega^2 R^2 + \rho g H)$

(d)  $\pi R^2 \left( \frac{\rho \omega^2 R^2}{4} + \rho g H \right)$

GATE-11. Ans. (d) We know that

$$\frac{\partial P}{\partial r} = \frac{\rho v^2}{r} = \frac{\rho \omega^2 r}{r} = \rho \omega^2 r.$$

$$[\because v = \omega \times r]$$

$$\therefore \int_0^p \partial p = \int_0^r \rho \omega^2 r dr \quad [p = \frac{\rho}{2} \omega^2 r^2]$$

Area of circular ring =  $2\pi r dr$

Force on elementary ring =  
Intensity of pressure  $\times$  Area of ring

$$= \frac{\rho}{2} \omega^2 r^2 2\pi r dr$$

$\therefore$  Total force on the top of the cylinder

$$= \int_0^R \frac{\rho}{2} \omega^2 r^2 2\pi r dr = \frac{\rho}{2} \omega^2 2\pi \int_0^R r^3 dr$$

$$= \frac{\rho}{2} \omega^2 2\pi \frac{R^4}{4} = \frac{\rho}{4} \omega^2 \times \pi R^4$$

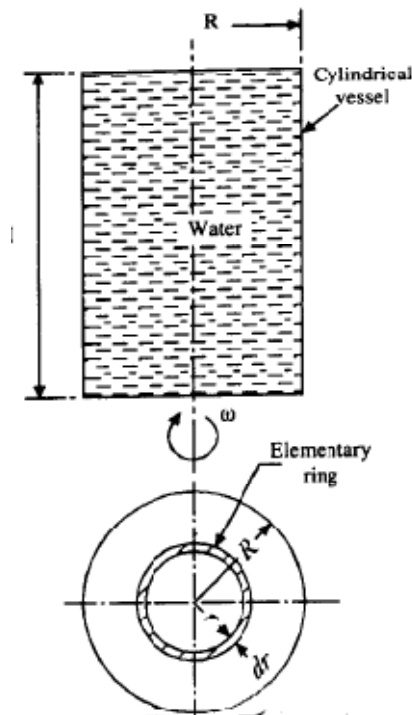
Thrust at the bottom of the cylinder

= Weight of water in cylinder +

Total force on the top of cylinder

$$= \rho g \times \pi R^2 \times H + \frac{\rho}{4} \omega^2 \times \pi R^4$$

$$= \pi R^2 \left[ \frac{\rho \omega^2 R^2}{4} + \rho g h \right]$$



## Previous 20-Years IES Questions

### Bernoulli's Equation

IES-1. Bernoulli's equation represents the [IES-1994]

- Forces at any point in the flow field and is obtained by integrating the momentum equation for viscous flows.
- Energies at any point in the flow field and is obtained by integrating the Euler equations.
- Momentum at any point in the flow field and is obtained by integrating the equation of continuity.
- Moment of momentum and is obtained by integrating the energy equation.

IES-1. Ans. (b)

IES-2. When is Bernoulli's equation applicable between any two points in a flow fields? [IES-2009]

- The flow is steady, incompressible and rotational
- The flow is steady, compressible and irrotational
- The flow is unsteady, incompressible and irrotational
- The flow is steady, incompressible and irrotational

IES-2. Ans. (d) The assumptions made for Bernoulli's equation.



- (i) The liquid is ideal (viscosity, surface tension is zero and incompressible)
- (ii) The flow is steady and continuous
- (iii) The flow is along the streamline (it is one-dimensional)
- (iv) The velocity is uniform over the section and is equal to the mean velocity.
- (v) The only forces acting on the fluid are the gravity force and the pressure force

**The assumptions NOT made for Bernoulli's equation**

- (i) The flow is uniform
- (ii) The flow is irrotational

**IES-3. Assertion (A): Two table tennis balls hang parallelly maintaining a small gap between them. If air is blown into the gap between the balls, the balls will move apart. [IES-1994]**

**Reason (R): Bernoulli's theorem is applicable in this case.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-3. Ans. (c)**

**IES-4. Which of the following assumptions are made for deriving Bernoulli's equation? [IES-2002]**

- 1. Flow is steady and incompressible
- 2. Flow is unsteady and compressible
- 3. Effect of friction is neglected and flow is along a stream line.
- 4. Effect of friction is taken into consideration and flow is along a stream line.

**Select the correct answer using the codes given below:**

- (a) 1 and 3
- (b) 2 and 3
- (c) 1 and 4
- (d) 2 and 4

**IES-4. Ans. (a)**

**IES-5. The expression  $(p + \rho gz + \rho v^2/2)$  commonly used to express Bernoulli's equation, has units of [IES-1995]**

- (a) Total energy per unit mass
- (b) Total energy per unit weight
- (c) Total energy per unit volume
- (d) Total energy per unit cross - sectional area of flow

**IES-5. Ans. (c)** The expression  $p + \rho gz + \rho v^2/2$ , has units of  $\frac{N}{m^2}$  or  $\frac{Nm}{m^3} \left( \frac{\text{energy}}{\text{volume}} \right)$

**IES-6. The expression: [IES-2003]**

$$\frac{\partial \phi}{\partial t} + \int \frac{\partial p}{\rho} + \frac{1}{2} |\Delta \phi|^2 + gz = \text{constant}$$

**represents :**

- (a) Steady flow energy equation
- (b) Unsteady irrotational Bernoulli's equation
- (c) Steady rotational Bernoulli's equation
- (d) Unsteady rotational Bernoulli's equation

IES-6. Ans. (b) 
$$\frac{\partial \phi}{\partial t} + \int \frac{\partial \rho}{\rho} + \frac{1}{2} (\nabla \phi)^2 + gz = \text{const.}$$

$\downarrow$   
Unsteady

$\downarrow$   
irrotational

IES-7. Which one of the following statements is correct? While using boundary layer equations, Bernoulli's equation [IES-2006]

- (a) Can be used anywhere
- (b) Can be used only outside the boundary layer
- (c) Can be used only inside the boundary layer
- (d) Cannot be used either inside or outside the boundary layer

IES-7. Ans. (b)

IES-8. Assertion (A): Bernoulli's equation is an energy equation. [IES-1997]  
Reason (R): Starting from Euler's equation, one can arrive at Bernoulli's equation.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-8. Ans. (b) Starting from Euler's equation, one can arrive at Bernoulli's equation. And we know that Euler equation is a momentum equation and integrating Euler equation we can arrive at Bernoulli's equation.

IES-9. Assertion (A): After the fluid has re-established its flow pattern downstream of an orifice plate, it will return to same pressure that it had upstream of the orifice plate. [IES-2003]

Reason (R): Bernoulli's equation when applied between two points having the same elevation and same velocity gives the same pressure at these points.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-9. Ans. (d) There is a loss of energy due to eddy formation and turbulence. This is the reason for that pressure is less than that it had upstream of the orifice plate.

## Euler's Equation

IES-10. Consider the following assumptions: [IES-1998]

1. The fluid is compressible
2. The fluid is inviscid.
3. The fluid is incompressible and homogeneous.
4. The fluid is viscous and compressible.

The Euler's equation of motion requires assumptions indicated in :

- (a) 1 and 2
- (b) 2 and 3
- (c) 1 and 4
- (d) 3 and 4

IES-10. Ans. (b)

IES-11. The Euler's equation of motion is a statement of [IES-2005]

- (a) Energy balance
- (b) Conservation of momentum for an inviscid fluid

- (c) Conservation of momentum for an incompressible flow  
 (d) Conservation of momentum for real fluid

IES-11. Ans. (b)

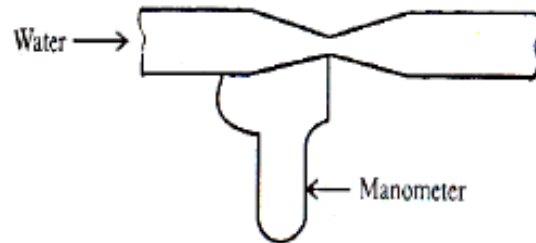
IES-12. The Euler equations of motion for the flow of an ideal fluid is derived considering the principle of conservation of [IES-1994]

- (a) Mass and the fluid as incompressible and inviscid.  
 (b) Momentum and the fluid as incompressible and viscous.  
 (c) Momentum and the fluid as incompressible and inviscid.  
 (d) Energy and the fluid as incompressible and inviscid.

IES-12. Ans. (c) For inviscid flows, the steady form of the momentum equation is the Euler equation. For an inviscid incompressible fluid flowing through a duct, the steady flow energy equation reduces to Bernoulli equation.

## Venturimeter

IES-13. A horizontal pipe of cross-sectional area  $5 \text{ cm}^2$  is connected to a venturimeter of throat area  $3 \text{ cm}^2$  as shown in the below figure. The manometer reading is equivalent to  $5 \text{ cm}$  of water. The discharge in  $\text{cm}^3/\text{s}$  is nearly:



- (a) 0.45 (b) 5.5 (c) 21.0 (d) 370 [IES-1998]

IES-13. Ans. (d) Just use the Venturimeter formula:  $Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$

Here  $C_d = 1.0$ ;  $A_1 = 5 \text{ cm}^2$ ;  $A_2 = 3 \text{ cm}^2$ ;  $g = 981 \text{ cm/s}^2$  and  $h = 5 \text{ cm}$

$$Q_{act} = \frac{3 \times 5}{\sqrt{5^2 - 3^2}} \times \sqrt{2 \times 981 \times 5} = 371.42 \text{ cm}^3/\text{s}$$

IES-14. An orifice meter with  $C_d = 0.61$  is substituted by Venturimeter with  $C_d = 0.98$  in a pipeline carrying crude oil, having the same throat diameter as that of the orifice. For the same flow rate, the ratio of the pressure drops for the Venturimeter and the orifice meter is: [IES-2003]

- (a)  $0.61 / 0.98$  (b)  $(0.61)^2 / (0.98)^2$  (c)  $0.98 / 0.61$  (d)  $(0.98)^2 / (0.61)^2$

IES-14. Ans. (b)

IES-15. A Venturimeter in an oil (sp. gr. 0.8) pipe is connected to a differential manometer in which the gauge liquid is mercury (sp.gr.13.6). For a flow rate of  $0.16 \text{ m}^3/\text{s}$ , the manometer registers a gauge differential of  $20 \text{ cm}$ . The oil-mercury manometer being unavailable, an air-oil differential manometer is connected to the same venturimeter. Neglecting variation of discharge coefficient for the venturimeter, what is the new gauge differential for a flow rate of  $0.08 \text{ m}^3/\text{s}$ ? [IES-2006]

- (a) 64 cm (b) 68 cm (c) 80 cm (d) 85 cm

IES-15. Ans. (c)

## Orifice Meter

IES-16. An orifice meter, having an orifice of diameter  $d$  is fitted in a pipe of diameter  $D$ . For this orifice meter, what is the coefficient of discharge  $C_d$ ? [IES-2007]

- (a) A function of Reynolds number only
- (b) A function of  $d/D$  only
- (c) A function of  $d/D$  and Reynolds number
- (d) Independent of  $d/D$  and Reynolds number

IES-16. Ans. (b)  $C_d = C_c \times \frac{\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2}}{1 - C_c^2 \times \left(\frac{A_0}{A_1}\right)^2}$  or,  $C_A = f\left(\frac{A_0}{A_1}\right) = F\left(\frac{d}{D}\right)$

IES-17. A tank containing water has two orifices of the same size at depths of 40 cm and 90 cm below the free surface of water. The ratio of discharges through these orifices is: [IES-2000]

- (a) 1 : 1
- (b) 2 : 3
- (c) 4 : 9
- (d) 16 : 81

IES-17. Ans. (b)

IES-18. How is the velocity coefficient  $C_v$ , the discharge coefficient  $C_d$ , and the contraction coefficient  $C_c$  of an orifice related? [IES-2006]

- (a)  $C_v = C_c C_d$
- (b)  $C_c = C_v C_d$
- (c)  $C_d = C_c C_v$
- (d)  $C_c C_v C_d = 1$

IES-18. Ans. (c)

## Pitot Tube

IES-19. The velocity of a water stream is being measured by a L-shaped Pilot-tube and the reading is 20 cm. Then what is the approximate value of velocity? [IES-2007]

- (a) 19.6 m/s
- (b) 2.0 m/s
- (c) 9.8 m/s
- (d) 20 cm/s

IES-19. Ans. (b)  $\frac{V^2}{2g} = h$  or,  $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.2} = 1.981 \text{ m/s}$

IES-20. A Prandtl Pilot tube was used to measure the velocity of a fluid of specific gravity  $S_1$ . The differential manometer, with a fluid of specific gravity  $S_2$ , connected to the Pitot tube recorded a level difference as  $h$ . The velocity  $V$  is given by the expression. [IES-1995]

- (a)  $\sqrt{2gh(S_1/S_2 - 1)}$
- (b)  $\sqrt{2gh(S_2/S_1 - 1)}$
- (c)  $\sqrt{2gh(S_1 - S_2)}$
- (d)  $\sqrt{2gh(S_2 - S_1)}$

IES-20. Ans. (b)  $\frac{P_1}{\rho g} + y + \frac{hS_2}{S_1} = \frac{P_2}{\rho g} + (h + y)$

$$\therefore \frac{P_2 - P_1}{\rho g} = h \left( \frac{S_2}{S_1} - 1 \right) = \frac{V_1^2}{2g}$$

$$\therefore V_1 = \sqrt{2gh \left( \frac{S_2}{S_1} - 1 \right)}$$

**IES-21.** A Pitot-static tube ( $C = 1$ ) is used to measure air flow. With water in the differential manometer and a gauge difference of 75 mm, what is the value of air speed if  $\rho = 1.16 \text{ kg/m}^3$ ? [IES-2004]

- (a) 1.21 m/s                      (b) 16.2 m/s                      (c) 35.6 m/s                      (d) 71.2 m/s

**IES-21. Ans. (c)**  $\frac{V^2}{2g} = h$  or  $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 64.58} = 35.6 \text{ m/s}$

$$h = y \left( \frac{S_h}{S_l} - 1 \right) = 0.075 \left( \frac{1000}{1.16} - 1 \right) = 64.58 \text{ m of air column}$$

**IES-22.** What is the difference in pressure head, measured by a mercury-oil differential manometer for a 20 cm difference of mercury level?

(Sp. gravity of oil = 0.8) [IES-2009]

- (a) 2.72 m of oil    (b) 2.52 m of oil                      (c) 3.40 m of oil                      (d) 2.00 m of oil

**IES-22. Ans. (c)** Difference in pressure head in m of oil (i.e. light liquid) =

$$= \left( \frac{S_h}{S_l} - 1 \right) \times y = \left( \frac{13.6}{0.8} - 1 \right) \times 0.2 = 3.2 \text{ m}$$

**IES-23.** Match List-I (Measuring Devices) with List-II (Measured Parameter) and select the correct answer using the codes given below: [IES-2004]

List-I					List-II				
A.	Pitot tube				1.	Flow static pressure			
B.	Micro-manometer				2.	Rate of flow (indirect)			
C.	Pipe band meter				3.	Differential pressure			
D.	Wall pressure tap				4.	Flow stagnation pressure			
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	3	2	4	(b)	4	3	2	1
(c)	1	2	3	4	(d)	4	2	3	1

**IES-23. Ans. (b)**

**IES-24.** The instrument preferred in the measurement of highly fluctuating velocities in air flow is: [IES-2003]

- (a) Pitot-static tube                      (b) Propeller type anemometer  
(c) Three cup anemometer                      (d) Hot wire anemometer

**IES-24. Ans. (d)**

**IES-25.** If a calibration chart is prepared for a hot-wire anemometer for measuring the mean velocities, the highest level of accuracy can be: [IES-1996]

- (a) Equal to accuracy of a Pitot tube  
(b) Equal to accuracy of a Rotameter  
(c) Equal to accuracy of a venturimeter  
(d) More than that of all the three instruments mentioned above

**IES-25. Ans. (d)** Hot wire anemometer is more accurate than Pitot tube, rotameter, or venturi meter.

**IES-26.** Which one of the following is measured by a Rotameter? [IES-2006]

- (a) Velocity of fluids                      (b) Discharge of fluids  
(c) Viscosity of fluids                      (d) Rotational speed of solid shafts

**IES-26. Ans. (b)**

**IES-27.** In a rotameter as the flow rate increase, the float [IES-1992]

- (a) Rotates at higher speed                      (b) Rotates at lower speed  
(c) Rises in the tube                                (d) Drops in the tube

**IES-27. Ans. (c)**

**IES-28.** A Pitot static tube is used to measure the velocity of water using a differential gauge which contains a manometric fluid of relative density 1.4. The deflection of the gauge fluid when water flows at a velocity of 1.2 m/s will be (the coefficient of the tube may be assumed to be 1) **[IES-2000]**

- (a) 183.5 mm                      (b) 52.4 mm                      (c) 5.24 mm                      (d) 73.4 mm

**IES-28. Ans. (a)** Use  $V = \sqrt{2gh}$                       where,  $h = y \left( \frac{s_h}{s_l} - 1 \right)$

Given  $y = ?$  and  $V = 1.2$  m/s,  $\frac{s_h}{s_l} = 1.4$

**IES-29.** Match List-I with List-II and select the correct answer using the codes given below the lists: **[IES-1993]**

**List-I**  
**(Discharge measuring device)**  
A. Rotameter  
B. Venturimeter  
C. Orifice meter  
D. Flow nozzle

**List-II**  
**(Characteristic feature)**  
1. Vena contracta  
2. End contraction  
3. Tapering tube  
4. Convergent – divergent  
5. Bell mouth entry

Codes:	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	3	4	1	5
(c)	5	4	2	1	(d)	3	5	1	2

**IES-29. Ans. (b)**

**IES-30.** A glass tube with a 90° bend is open at both the ends. It is inserted into a flowing stream of oil,  $S = 0.90$ , so that one opening is directed upstream and the other is directed upward. Oil inside the tube is 50 mm higher than the surface of flowing oil. The velocity measured by the tube is, nearly, **[IES-2001]**

- (a) 0.89 m/s                      (b) 0.99 m/s                      (c) 1.40 m/s                      (d) 1.90 m/s

**IES-30. Ans. (b)**

**IES-31.** The speed of the air emerging from the blades of a running table fan is intended to be measured as a function of time. The point of measurement is very close to the blade and the time period of the speed fluctuation is four times the time taken by the blade to complete one revolution. The appropriate method of measurement would involve the use of **[IES-1993]**

- (a) A Pitot tube    (b) A hot wire anemometer  
(c) High speed photography                              (d) A Schlieren system

**IES-31. Ans. (b)** A Pitot tube is used for measuring speed in closed duct or pipe. Hot wire anemometer is used for measuring fluctuation of speed. High speed photography may be useful to measure blade speed but not of air.

## Free Liquid Jet

**IES-32.** A constant-head water tank has, on one of its vertical sides two identical small orifices issuing two horizontal jets in the same vertical

plane. The vertical distance between the centres of orifices is 1.5 m and the jet trajectories intersect at a point 0.5 m below the lower orifice. What is the approximate height of water level in the tank above the point of intersection of trajectories? [IES-2004]

(a) 1.0 m

(b) 2.5 m

(c) 0.5 m

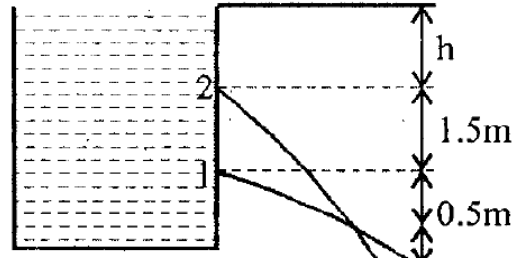
(d) 2.0 m

IES-32. Ans. (b)  $C_{v1} = \sqrt{\frac{x^2}{4HY}}$

$$\therefore \frac{x^2}{4 \times 0.5 \times (h + 1.5)} = \frac{x^2}{4 \times 2 \times h}$$

$$\therefore h = 0.5 \text{ m}$$

$$\text{Total height} = 0.5 + 1.5 + 0.5 = 2.5 \text{ m}$$



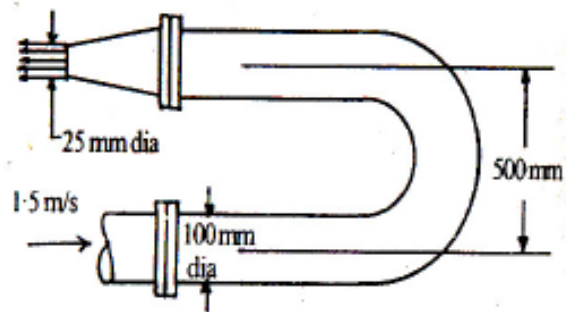
IES-33. The elbow nozzle assembly shown in the given figure is in a horizontal plane. The velocity of jet issuing from the nozzle is:

(a) 4 m/s

(b) 16 m/s

(c) 24 m/s

(d) 30 m/s



[IES-1999]

IES-33. Ans. (c)

## Impulse Momentum Equation

IES-34. Which one of the following conditions will linearize the Navier-Stokes equations to make it amenable for analytical solutions? [IES-2007]

(a) Low Reynolds number ( $Re \ll 1$ )(b) High Reynolds number ( $Re \gg 1$ )(c) Low Mach number ( $M \ll 1$ )(d) High Mach number ( $M \gg 1$ )

IES-34. Ans. (c)

## Forced Vortex

IES-35. Which one of the statements is correct for a forced vortex? [IES-2009]

(a) Turns in an opposite direction to a free vortex

(b) Always occurs in conjunction with a free vortex

(c) Has the linear velocity directly proportional to the radius

(d) Has the linear velocity inversely proportional to the radius

IES-35. Ans. (c) Forced vortex flow: Forced vortex flow is one in which the fluid mass is made to rotate by means of some external agency. Where  $(v) = \omega \times r$  As  $\omega$  is constant linear velocity  $(v)$  is directly proportional to the radius  $(r)$ .

IES-36. A right circular cylinder, open at the top is filled with liquid of relative density 1.2. It is rotated about its vertical axis at such a speed that half liquid spills out. The pressure at the centre of the bottom will be:

(a) Zero

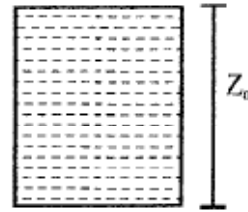
(b) One-fourth of the value when the cylinder was full

(c) Half of the value when the cylinder was full

(d) Not determinable from the given data

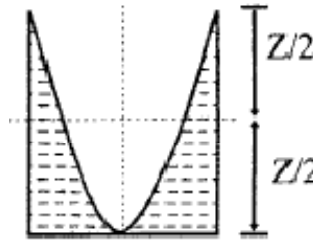
[IES-1998]

IES-36. Ans. (a) When cylinder is rotated such that half of the liquid spills out. Then liquid left in cylinder at height  $\frac{z}{2}$ ,



initial condition

and liquid will rise at the wall of the cylinder by the same amount as it falls at the centre from its original level at rest.



Final condition

IES-37. Assertion (A) : A cylinder, partly filled with a liquid is rotated about its vertical axis. The rise of liquid level at the ends is equal to the fall of liquid level at the axis. [IES-1999]

Reason (R) : Rotation creates forced vortex motion.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-37. Ans. (b)

IAS-38. For a real fluid moving with uniform velocity, the pressure [IES-1993]

- (a) Depends upon depth and orientation
- (b) Is independent of depth but depends upon orientation
- (c) Is independent of orientation but depends upon depth
- (d) Is independent of both depth and orientation

IAS-38. Ans. (d) In case of a real fluid moving with uniform velocity, the velocity head and pressure head are dependent on each other and their total sum remains constant. The pressure is thus independent of both depth and orientation, but in case of fluids under static condition, the pressure would depend on depth.

IES-39. A right circular cylinder is filled with a liquid upto its top level. It is rotated about its vertical axis at such a speed that half the liquid spills out then the pressure at the point of intersection of the axis and bottom surface is: [IES-2001]

- (a) Same as before rotation
- (h) Half of the value before rotation
- (c) Quarter of the value before rotation
- (d) Equal to the atmospheric pressure

IES-39 Ans. (d)

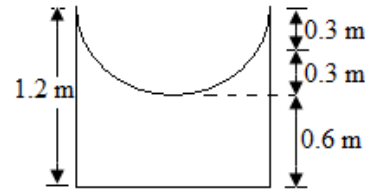
IES-40. An open circular cylinder 1.2 m high is filled with a liquid to its top. The liquid is given a rigid body rotation about the axis of the cylinder and the pressure at the centre line at the bottom surface is found to be 0.6 m of liquid. What is the ratio of Volume of liquid spilled out of the cylinder to the original volume? [IES-2007]

- (a)  $1/4$
- (b)  $3/8$
- (c)  $1/2$
- (d)  $3/4$



IES-40. Ans. (a)  $\frac{\text{Volume of paraboloid}}{\text{Total volume}}$

$$= \frac{(1/2) \times A \times 0.6}{A \times 1.2} = \frac{1}{4}$$



## Free Vortex

IES-41. Both free vortex and forced vortex can be expressed mathematically as functions of tangential velocity  $V$  at the corresponding radius  $r$ . Free vortex and forced vortex are definable through  $V$  and  $r$  as [IES-1993]

**Free vortex**

**Forced vortex**

(a)  $V = r \times \text{const.}$

$Vr = \text{const.}$

(b)  $V \times r = \text{const.}$

$V^2 = r \times \text{const.}$

(c)  $V \times r = \text{const.}$

$V = r \times \text{const.}$

(d)  $V^2 \times r = \text{const.}$

$V = r \times \text{const.}$

IES-41. Ans. (c) Free vortex can be expressed mathematically as  $V \times r = \text{constant}$  and the forced vortex as  $V = r \times \text{constant}$ .

IES-42. An incompressible fluid flows radially outward from a line source in a steady manner. How does the velocity in any radial direction vary? [IES-2008]

(a)  $r$

(b)  $r^2$

(c)  $1/r^2$

(d)  $1/r$

IES-42. Ans. (d) For an incompressible fluid flow radially outward from a line source in a steady manner. Angular momentum is conserved.

$$\Rightarrow mvr = \text{constant} \quad \Rightarrow vr = \text{constant} \quad \Rightarrow v = \frac{\text{constant}}{r}$$

$$\therefore v \propto \frac{1}{r}$$

IES-43. In a cylindrical vortex motion about a vertical axis,  $r_1$  and  $r_2$  are the radial distances of two points on the horizontal plane ( $r_2 > r_1$ ). If for a given tangential fluid velocity at  $r_1$ , the pressure difference between the points in free vortex is one-half of that when the vortex is a forced one, then what is the value of the ratio ( $r_2/r_1$ )? [IES-2007]

(a)  $\sqrt{3/2}$

(b)  $\sqrt{2}$

(c)  $3/2$

(d)  $\sqrt{3}$

IES-43. Ans. (b) For free vortex,  $\omega r_1 = \text{const.}(k)$

$$\text{For forced vortex, } V_1 = \text{const.}(k) = \frac{c}{r_1} \text{ Or } c = \omega r_1^2$$

$$(\Delta P)_{\text{forced}} = \frac{\rho \omega^2}{2} [r_2^2 - r_1^2], \quad (\Delta P)_{\text{free}} = \frac{\rho c^2}{2} \left[ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right] \quad \because c = \omega r_1^2$$

$$2 (\Delta P)_{\text{free}} = (\Delta P)_{\text{forced}} \quad \text{Or} \quad \frac{r_2}{r_1} = \sqrt{2}$$

IES-44. An inviscid, irrotational flow field of free vortex motion has a circulation constant  $\Omega$ . The tangential velocity at any point in the flow field is given by  $\Omega/r$ , where,  $r$  is the radial distance from the centre. At the centre, there is a mathematical singularity which can be physically

substituted by a forced vortex. At the interface of the free and forced vortex motion ( $r = r_c$ ), the angular velocity  $\omega$  is given by: [IES-1997]

- (a)  $\Omega / (r_c)^2$  (b)  $\Omega / r_c$  (c)  $\Omega r_c$  (d)  $\Omega r_c^2$

IES-44. Ans. (a) Free vortex,

$$V_r = \text{constant} = \Omega \text{ (given)}$$

$$V = \frac{\Omega}{r}$$

Forced vortex,

$$V = r\omega$$

$$\omega = \frac{V}{r}$$

$$r = r_c$$

$$V = \frac{\Omega}{r_c} \quad (\text{Free vortex})$$

$$\omega = \frac{V}{r_c} \quad (\text{Forced vortex})$$

$$\omega = \frac{\left(\frac{\Omega}{r_c}\right)}{r_c} \Rightarrow \omega = \frac{\Omega}{r_c^2}$$

## Previous 20-Years IAS Questions

### Bernoulli's Equation

IAS-1. The Bernoulli's equation refers to conservation of [IAS-2003]

- (a) Mass (b) Linear momentum  
(c) Angular momentum (d) Energy

IAS-1. Ans. (d)

IAS-2. Bernoulli's equation  $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$ , is applicable to for

- (a) Steady, frictionless and incompressible flow along a streamline [IAS-1999]  
(b) Uniform and frictionless flow along a streamline when  $p$  is a function of  $p$   
(c) Steady and frictionless flow along a streamline when  $p$  is a function of  $p$   
(d) Steady, uniform and incompressible flow along a streamline

IAS-2. Ans. (a)

IAS-3. Bernoulli's theorem  $\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{constant}$  is valid [IAS-1996]

- (a) Along different streamlines in rotational flow  
(b) Along different streamlines in irrotational flow  
(c) Only in the case of flow of gas  
(d) Only in the case of flow of liquid

IAS-3. Ans. (a)

### Venturimeter

IAS-4. Fluid flow rate  $Q$ , can be measured easily with the help of a venturi tube, in which the difference of two pressures,  $\Delta P$ , measured at an

upstream point and at the smallest cross-section and at the smallest cross-section of the tube, is used. If a relation  $\Delta P \propto Q^n$  exists, then n is equal to: [IAS-2001]

- (a)  $\frac{1}{3}$  (b)  $\frac{1}{2}$  (c) 1 (d) 2

IAS-4. Ans. (d)  $Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \therefore Q^2 \propto \Delta h \text{ or } Q^2 \propto \Delta \rho$

IAS-5. Two venturimeters of different area ratios are connected at different locations of a pipeline to measure discharge. Similar manometers are used across the two venturimeters to register the head differences. The first venturimeter of area ratio 2 registers a head difference 'h', while the second venturimeter registers '5h'. The area ratio for the second venturimeter is: [IAS-1999]

- (a) 3 (b) 4 (c) 5 (d) 6

IAS-5. Ans. (b)  $Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{C_d A_1 A_2' \sqrt{2g5h}}{\sqrt{A_1^2 - A_2'^2}} \quad A_1 = 2A_2 \text{ and } A_2 = (A_1/2)$

That gives  $\frac{A_1}{A_2} = 4$

## Orifice Meter

IAS-6. If a fluid jet discharging from a 50 mm diameter orifice has a 40 mm diameter at its vena contracta, then its coefficient of contraction will be: [IAS-1996]

- (a) 0.32 (b) 0.64 (c) 0.96 (d) 1.64

IAS-6. Ans. (b)

IAS-7. What is the percentage error in the estimation of the discharge due to an error of 2% in the measurement of the reading of a differential manometer connected to an orifice meter? [IAS-2004]

- (a) 4 (b) 3 (c) 2 (d) 1

IAS-7. Ans. (d)  $Q = \frac{C_d A_2 A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = \text{const.} \times \sqrt{h}$

or,  $\ln Q = \ln(\text{const.}) + \frac{1}{2} \ln h$

or,  $\frac{dQ}{Q} = 2 \frac{dh}{h} = \frac{1}{2} \times 2 = 1\%$

## Pitot Tube

IAS-8. A simple Pitot tube can be used to measure which of the following quantities? [IAS-1994]

1. Static head      2. Datum head      3. Dynamic head  
4. Friction head      5. Total head

Select the correct answer using the codes given below

Codes:

- (a) 1, 2 and 4 (b) 1, 3 and 5 (c) 2, 3 and 4 (d) 2, 3 and 5

IAS-8. Ans. (b)

**IAS-9.** An instrument which offers no obstruction to the flow, offers no additional loss and is suitable for flow rate measurement is: [IAS-1997]  
 (a) Venturimeter (b) Rotameter (c) Magnetic flow meter (d) Bend meter

**IAS-9. Ans. (c)**

**IAS-10.** The following instruments are used in the measurement of discharge through a pipe: [IAS-1996]

1. Orifice meter                      2. Flow nozzle                      3. Venturimeter

Decreasing order of use:

(a) 1, 3, 2                      (b) 1, 2, 3                      (c) 3, 2, 1                      (d) 2, 3, 1

**IAS-10. Ans. (c)**

**IAS-11.** Match List-I with List-II and select the correct answer: [IAS-2000]

**List-I**

A. Orifice meter  
 B. Broad crested weir  
 C. Pitot tube

D. Rotameter

**List-II**

1. Measurement of flow in a channel  
 2. Measurement of velocity in a pipe/ channel  
 3. Measurement of flow in a pipe of any inclination  
 4. Measurement of upward flow in a vertical pipe

Codes:	A	B	C	D	A	B	C	D
(a)	3	1	4	2	(b)	1	3	2
(c)	3	1	2	4	(d)	1	3	4

**IAS-11. Ans. (c)**

**IAS-12.** Assertion (A): In a rotameter the fluid flows from the bottom of the conical rotameter tube with divergence in the upward direction and the position of the metering float indicated the discharge. [IAS-1996]  
 Reason (R): Rotameter float indicates the discharge in terms of its rotation.

(a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IAS-12. Ans. (c)**

## Free Liquid Jet

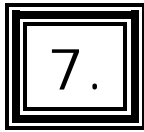
**IAS-13.** A liquid jet issues from a nozzle inclined at an angle of  $60^\circ$  to the horizontal and is directed upwards. If the velocity of the jet at the nozzle is 18m/s, what shall approximately be the maximum vertical distance attained by the jet from the point of exit of the nozzle? [IAS-2004]

(a) 4.2 m                      (b) 12.4 m                      (c) 14.3m                      (d) 16.5m

**IAS-13. Ans. (b)**  $H = u \sin \theta \times t - \frac{1}{2} g t^2$

$$\frac{dH}{dt} = u \sin \theta - gt \quad \text{or } t = \frac{u \sin \theta}{g}$$

$$\therefore H_{\max} = u \sin \theta \times \frac{u \sin \theta}{g} - \frac{1}{2} g \times \left( \frac{u^2 \sin^2 \theta}{2g} \right) = \frac{18^2 \sin^2 60}{2 \times 9.8} = 12.4m$$



## Dimensional & Model Analysis

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  6. Limitations of Dimensional Analysis
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  16. Model (or Similarity) Laws
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  20. Weber Model Law
  21. Mach Model Law
  22. Types of Models (Undistorted models, distorted models)
  23. Scale Effect in Models
  24. Limitations of Hydraulic Similitude
- 

**Question:** *Discuss the importance of Dimensional Analysis.* [IES-2003]

- Answer:**
1. Dimensional Analysis help in determining a systematic arrangement of the variable in the physical relationship, combining dimensional variable to form meaningful non-dimensional parameters.
  2. It is especially useful in presenting experimental results in a concise form.
  3. Dimensional Analysis provides partial solutions to the problems that are too complex to be dealt with mathematically.
  4. Design curves can be developed from experimental data.

**Question:** *Explain clearly Buckingham's  $\pi$ -theorem method and Rayleigh's method of dimensional Analysis.* [IES-2003]

**Answer:** **Buckingham's  $\pi$ -theorem:** statement "If there are  $n$  variable (dependent and independent) in a dimensionally homogeneous equation and if these

variable contain  $m$  fundamental dimensions, then the variables are arranged into  $(n-m)$  dimensionless terms."

These dimensionless terms are called  $\pi$ -terms.

Mathematically, if any variables  $x_1$ , depends on independent variables,  $x_2, x_3, \dots, x_n$ , the functional eq<sup>n</sup> may be written as

$$x_1 = f(x_2, x_3, \dots, x_n)$$

$$\text{or } F(x_1, x_2, \dots, x_n) = 0$$

It is a dimensionally homogeneous eq<sup>n</sup> and contains 'n' variables. If there are  $m$  fundamental dimensions, then According to Buckingham's  $\pi$ -theorem it can be written in  $(n-m)$  numbers of  $\pi$ -terms (dimensionless groups)

$$\therefore F(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0$$

Each dimensionless  $\pi$ -term is formed by combining  $m$  variables out of the total  $n$  variable with one of the remaining  $(n-m)$  variables. i.e. each  $\pi$ -term contain  $(m+1)$  variables.

These  $m$  variables which appear repeatedly in each of  $\pi$ -term are consequently called repeating variables and are chosen from among the variable such that they together involve all the fundamental dimensions and they themselves do not form a dimensionless parameter.

Let  $x_2, x_3$  and  $x_4$  are repeating variables then

$$\pi_1 = x_2^{a_1} x_3^{b_1} x_4^{c_1} \cdot x_1$$

$$\pi_2 = x_2^{a_2} x_3^{b_2} x_4^{c_2} \cdot x_5$$

$$\pi_{n-m} = x_2^{a_{n-m}} x_3^{b_{n-m}} x_4^{c_{n-m}} \cdot x_n$$

Where  $a_1, b_1, c_1; a_2, b_2, c_2$  etc are constant and can be determined by considering dimensional homogeneity.

**Rayleigh's Method:** This method gives a special form of relationship among the dimensional group. In this method a functional relationship of some variables is expressed in the form of an exponential equation which must be dimensionally homogeneous.

Thus, if  $x$  is a dependent variable which depends  $x_1, x_2, x_3, \dots, x_n$  The functional equation can be written as:

$$x = f(x_1, x_2, x_3, \dots, x_n)$$

$$\text{or } x = c x_1^a x_2^b x_3^c \dots x_n^n$$

Where  $c$  is a non-dimensional const and  $a, b, c, \dots, n$ , are the arbitrary powers. The value of  $a, b, c, \dots, n$ , are obtained by comparing the power of the fundamental dimensions on both sides.

**Question:** *What is meant by similitude? Discuss.* **[AMIE-(Winter)-2002]**

**Answer:** Similitude is the relationship between model and a prototype. Following three similarities must be ensured between the model and the prototype.

1. Geometric similarity
2. Kinematic similarity, and
3. Dynamic similarity.

**Question:** *What is meant by Geometric, kinematic and dynamic similarity?*

*Are these similarities truly attainable? If not why ?*

[AMIE- summer-99]

**Answer:** **Geometric similarity:** For geometric similarity the ratios of corresponding length in the model and in the prototype must be same and the include angles between two corresponding sides must be the same.

**Kinematic similarity:** kinematic similarity is the similarity of motion. It demands that the direction of velocity and acceleration at corresponding points in the two flows should be the same.

For kinematic similarity we must have;  $\frac{(v_1)_m}{(v_1)_p} = \frac{(v_2)_m}{(v_2)_p} = v_\gamma$ , velocity ratio

and  $\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = a_\gamma$ , acceleration ratio.

**Dynamic similarity:** Dynamic similarity is the similarity of forces at the corresponding points in the flows.

Then for dynamic similarity, we must have

$$\frac{(F_{\text{inertia}})_m}{(F_{\text{inertia}})_p} = \frac{(F_{\text{viscous}})_m}{(F_{\text{viscous}})_p} = \frac{(F_{\text{gravity}})_m}{(F_{\text{gravity}})_p} = F_\gamma, \text{ Force ratio}$$

**Note:** Geometric, kinematic and dynamic similarity are mutually independent. Existence of one does not imply the existence of another similarity.

No these similarity are not truly attainable.

**Because:**

- (a) The geometric similarity is complete when surface roughness profiles are also in the scale ratio. Since it is not possible to prepare a 1/20 scale model to 20 times better surface finish, so complete geometric similarity can not be achieved.
- (b) The kinematic similarity is more difficult because the flow patterns around small objects tend to be quantitatively different from those around large objects.
- (c) Complete dynamic similarity is practically impossible. Because Reynold's number and Froude number cannot be equated simultaneously.

**Question:**

- (i) Define (IES-03)
- (ii) Significance (IES-02)
- (iii) Area of application (IES-01)  
of Reynolds number; Froude number; Mach number; Weber number; Euler number.

**Answer:**

Sl. No.	Dimension-less no.	Aspects			
		Symbol	Group of variable	Significance	Field of application
1.	Reynolds number	$R_e$	$\frac{\rho v L}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	Laminar viscous flow in confined passages (where viscous effects are significant).

2.	Froude's number	$F_r$	$\frac{v}{\sqrt{Lg}}$	$\sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}}$	Free surface flows (where gravity effects are important)
3.	Euler's Number	$E_u$	$\frac{v}{\sqrt{P/\rho}}$	$\sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$	Conduit flow (where pressure variation are significant)
4.	Weber's Number	$W_e$	$\frac{\rho v^2 L}{\sigma}$	$\frac{\text{Inertia force}}{\text{Surface tension force}}$	Small surface waves, capillary and sheet flow (where surface tension is important)
5.	Mach's number	$M$	$\frac{v}{\sqrt{k/\rho}}$	$\sqrt{\frac{\text{Inertia force}}{\text{elastic force}}}$	High speed flow (where compressibility effects are important)

## Forces

- (i) **Inertia force** =  $\rho A v^2 = \rho L^2 v^2$
- (ii) **Viscous force** =  $\mu \frac{du}{dv} A = \mu \frac{v}{L} A = \mu \frac{v}{L} L^2 = \mu v L$
- (iii) **Gravity force** =  $\rho \times \text{volume} \times g = \rho L^3 g$
- (iv) **Pressure force** =  $PA = PL^2$
- (v) **Surface tension force** =  $\sigma L$
- (vi) **Elastic force** =  $k A = kL^2$

### (a) Reynolds Model Law

- (i) Motion of air planes,
- (ii) Flow of incompressible fluid in closed pipes,
- (iii) Motion of submarines completely under water, and
- (iv) Flow around structures and other bodies immersed completely under moving fluids.

### (b) Froude Model Law

- (i) Free surface flows such as flow over spillways, sluices etc.
- (ii) Flow of jet from an orifice or nozzle.
- (iii) Where waves are likely to be formed on the surface
- (iv) Where fluids of different mass densities flow over one another.

$$V_r = T_r = \sqrt{L_r} \text{ And } Q_r = L_r^{2.5}; F_r = L_r^3$$

### (c) Weber Model Law

Weber model law is applied in the following flow situations:

- (i) Flow over weirs involving very low heads;
- (ii) Very thin sheet of liquid flowing over a surface;
- (iii) Capillary waves in channels;
- (iv) Capillary rise in narrow passages;
- (v) Capillary movement of water in soil.

### (d) Mach Model Law

The similitude based on Mach model law finds application in the following:

- (i) Aerodynamic testing;
- (ii) Phenomena involving velocities exceeding the speed of sound;



- (iii) Hydraulic model testing for the cases of unsteady flow, especially water hammer problems.
- (iv) Under-water testing of torpedoes.

**(e) Euler Model Law**

- (i) Enclosed fluid system where the turbulence is fully developed so that viscous forces are negligible and also the forces of gravity and surface tensions are entirely absent;
- (ii) Where the phenomenon of cavitation occurs.

**Question:** *Considering Froude number as the criterion of dynamic similarity for a certain flow situation, work out the scale factor of velocity, time, discharge, acceleration, force, work, and power in terms of scale factor for length.* [IES-2003]

**Answer:** Let,  $V_m$  = velocity of fluid in model.

$L_m$  = Length (or linear dimension) of the model.

$g_m$  = acceleration due to gravity (at a place where model is tested,

and  $V_p, L_p$  and  $g_p$  are the corresponding value of prototype.

According to Froude model law

$$(F_r)_m = (F_r)_p$$

$$\text{or } \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

As site of model and prototype is same the  $g_m = g_p$

$$\therefore \frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\text{or } \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$\therefore \text{(i) Velocity ratio, } V_r = (L_r)^{1/2}$$

$$\text{(ii) Time scale ratio, } T_r = \frac{T_p}{T_m} = \frac{L_p / V_p}{L_m / V_m} = L_r \cdot (L_r)^{-1/2} = (L_r)^{-1/2} \quad [\because V_r = L_r^{1/2}]$$

$$\text{(iii) Discharge scale ratio, } Q_r$$

$$\text{Discharge (Q)} = AV = L^2 \cdot \frac{L}{T} = \frac{L^3}{T}$$

$$\therefore Q_r = \frac{Q_p}{Q_m} = \frac{L_p^3 / T_p}{L_m^3 / T_m} = \left( \frac{L_p}{L_m} \right)^3 \times \frac{1}{\left( \frac{T_p}{T_m} \right)} = \frac{L_r^3}{L_r^{1/2}} = L_r^{2.5}$$

$$\text{(iv) Acceleration ratio (a}_r\text{)}$$

$$\text{Acceleration, } a = \frac{V}{t}$$

$$\therefore a_r = \frac{a_p}{a_m} = \frac{V_p / T_p}{V_m / T_m} = \frac{V_p}{V_m} \times \frac{1}{\left( \frac{T_p}{T_m} \right)} = L_r^{1/2} \times \frac{1}{L_r^{1/2}} = 1$$

$$\text{(v) Force ratio (F}_r\text{)}$$

$$\text{Force, } F = \text{mass} \times \text{acceleration} = \rho L^3 \times \frac{V}{T}$$

$$\therefore \frac{F_p}{F_m} = \frac{\rho_p L_p^3 V_p / T_p}{\rho_m L_m^3 V_m / T_m} = (L_r)^3 \times L_r^{1/2} \times \frac{1}{L_r^{1/2}} = (L_r)^3$$

(vi) Work done scale ratio or energy scale ratio ( $E_r$ )

$$\text{Energy, } E = \frac{1}{2} m V^2 = \frac{1}{2} \rho L^3 V^2$$

$$\therefore \frac{E_p}{E_m} = \frac{\frac{1}{2} \rho_p L_p^3 V_p^2}{\frac{1}{2} \rho_m L_m^3 V_m^2} = (L_r)^3 \times (L_r^{1/2})^2 = L_r^4$$

(vii) Power scale ratio ( $P_r$ )

$$\text{Power} = \frac{1}{2} \frac{m V^2}{T} = \frac{1}{2} \rho \frac{L^3 V^2}{T}$$

$$\begin{aligned} \therefore \text{Power Ratio, } (P_r) &= \frac{P_p}{P_m} = \frac{\frac{1}{2} \rho_p L_p^3 V_p^2 / T_p}{\frac{1}{2} \rho_m L_m^3 V_m^2 / T_m} \\ &= \left( \frac{L_p}{L_m} \right)^3 \left( \frac{V_p}{V_m} \right)^2 \times \frac{1}{\left( \frac{T_p}{T_m} \right)} = L_r^3 (L_r^{1/2})^2 \times \frac{1}{(L_r)^{1/2}} = L_r^{3.5} \end{aligned}$$

**Question:** Obtain an expression for the length scale of a model, which has to satisfy both Froude's model law and Reynold's model law.

[AMIE (winter) 2000]

**Answer:** (i) Applying Reynold's model law

$$\left( \frac{\rho V L}{\mu} \right)_p = \left( \frac{\rho V L}{\mu} \right)_m$$

$$\text{or} \quad \frac{V_p}{V_m} = \left( \frac{\rho_m \times \mu_p}{\rho_p \times \mu_m} \right) \times \left( \frac{L_m}{L_p} \right)$$

$$\text{or} \quad \frac{V_p}{V_m} = C_1 \times \frac{L_m}{L_p} \quad \dots (i)$$

$C_1$  is unity for the same fluid used in the two flows, but it can be another constant in accordance with the properties of the two fluids.

Applying Froude's model law:

$$\left( \frac{V}{\sqrt{L g}} \right)_p = \left( \frac{V}{\sqrt{L g}} \right)_m$$

$$\text{or} \quad \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} \times \sqrt{\frac{g_p}{g_m}} = \sqrt{\frac{L_p}{L_m}} \quad \dots (ii) \quad [\because g_p = g_m \text{ at same site}]$$

Conditions (i) and (ii) are entirely different showing that the Reynolds number and the Froude's number cannot be equated simultaneously. i.e. complete dynamic similarity is practically impossible.

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

**Buckingham's  $\pi$ -method/theorem**

**GATE-1.** If the number of fundamental dimensions equals 'm', then the repeating variables shall be equal to: [IES-1999, IES-1998, GATE-2002]

- (a) m and none of the repeating variables shall represent the dependent variable.
- (b) m + 1 and one of the repeating variables shall represent the dependent variable
- (c) m + 1 and none of the repeating variables shall represent the dependent variable.
- (d) m and one of the repeating variables shall represent the dependent variable.

**GATE-1. Ans. (c)**

**Reynolds Number (Re)**

**GATE-2.** In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by  $v = u_0(1 + 3 \times x/L)i$ , where x is the distance along the axis of the nozzle from its inlet plane and L is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is: [GATE-2007]

- (a)  $\frac{L}{u_0}$
- (b)  $\frac{L}{3u_0} \ln 4$
- (c)  $\frac{L}{4u_0}$
- (d)  $\frac{L}{2.5u_0}$

**GATE-2. Ans. (b)** Velocity,  $V = \frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = u_0 \left( 1 + \frac{3x}{L} \right) \Rightarrow \frac{dx}{\left( 1 + \frac{3x}{L} \right)} = u_0 dt$$

Integrating both side, we get

$$\begin{aligned} u_0 \int_0^t dt &= \int_0^L \frac{dx}{\left( 1 + \frac{3x}{L} \right)} \\ u_0 t &= \frac{L}{3} \left[ \ln \left( 1 + \frac{3x}{L} \right) \right]_0^L \\ u_0 t &= \frac{L}{3} \ln 4 \\ t &= \frac{L}{3u_0} \ln 4 \end{aligned}$$

**GATE-3.** The Reynolds number for flow of a certain fluid in a circular tube is specified as 2500. What will be the Reynolds number when the tube diameter is increased by 20% and the fluid velocity is decreased by 40% keeping fluid the same? [GATE-1997]

- (a) 1200
- (b) 1800
- (c) 3600
- (d) 200

GATE-3. Ans. (b)  $R_e = \frac{\rho V D}{\mu}$

$$R_{e2} = \frac{\rho V D}{\mu} = \frac{\rho (0.6V)(1.2D)}{\mu} = 0.6 \times 1.2 \times 2500 = 1800$$

## Froude Number ( $Fr$ )

GATE-4. The square root of the ratio of inertia force to gravity force is called  
[GATE-1994, IAS-2003]

- (a) Reynolds number                      (b) Froude number  
(c) Mach number                          (d) Euler number

GATE-4. Ans. (b)

## Mach Number ( $M$ )

GATE-5. An aeroplane is cruising at a speed of 800 kmph at altitude, where the air temperature is 0° C. The flight Mach number at this speed is nearly  
[GATE-1999]

- (a) 1.5                      (b) 0.254                      (c) 0.67                      (d) 2.04

GATE-5. Ans. (c)

GATE-6. In flow through a pipe, the transition from laminar to turbulent flow does not depend on  
[GATE-1996]

- (a) Velocity of the fluid                      (b) Density of the fluid  
(c) Diameter of the pipe                      (d) Length of the pipe

GATE-6. Ans. (d)  $R_e = \frac{\rho V D}{\mu}$

GATE-7. List-I

- (A) Fourier number  
(B) Weber number  
(C) Grashoff number  
(D) Schmidt number

List-II

1. Surface tension  
2. Forced convection  
3. Natural convection  
4. Radiation  
5. Transient heat conduction  
6. Mass diffusion

[GATE-1996]

Codes:	A	B	C	D		A	B	C	D
(a)	1	2	6	4	(b)	4	5	2	1
(c)	5	1	3	6	(d)	4	2	3	1

GATE-7. Ans. (c)

## Previous 20-Years IES Questions

### Dimensions

IES-1. The dimensionless group formed by wavelength  $\lambda$ , density of fluid  $\rho$ , acceleration due to gravity  $g$  and surface tension  $\sigma$ , is: [IES-2000]

- (a)  $\sigma / \lambda^2 g \rho$                       (b)  $\sigma / \lambda g^2 \rho$                       (c)  $\sigma g / \lambda^2 \rho$                       (d)  $\rho / \lambda g \sigma$

IES-1. Ans. (a)

IES-2. Match List-I (Fluid parameters) with List-II (Basic dimensions) and select the correct answer: [IES-2002]

**List-I**

- A. Dynamic viscosity  
B. Chezy's roughness coefficient  
C. Bulk modulus of elasticity  
D. Surface tension ( $\sigma$ )

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	2	4	1
(c)	3	4	2	1

**List-II**

1.  $M / t^2$   
2.  $M / L t^2$   
3.  $M / L t$   
4.  $\sqrt{L/t}$

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(b)	1	4	2	3
(d)	1	2	4	3

IES-2. Ans. (c)

IES-3. In M-L-T system. What is the dimension of specific speed for a rotodynamic pump? [IES-2006]

- (a)  $L^{\frac{-3}{4}} T^{\frac{3}{2}}$  (b)  $M^{\frac{1}{2}} L^{\frac{1}{4}} T^{\frac{-5}{2}}$  (c)  $L^{\frac{3}{4}} T^{\frac{-3}{2}}$  (d)  $L^{\frac{3}{4}} T^{\frac{3}{2}}$

IES-3. Ans. (c)

IES-4. A dimensionless group formed with the variables  $\rho$  (density),  $\omega$  (angular velocity),  $\mu$  (dynamic viscosity) and  $D$  (characteristic diameter) is: [IES-1995]

- (a)  $\rho\omega\mu / D^2$  (b)  $\rho\omega D^2 / \mu$  (c)  $\rho\omega\mu D^2$  (d)  $\rho\omega\mu D$

IES-4. Ans. (b) Let  $\phi = \rho^a D^b \mu^c \omega$

$$M^0 L^0 T^0 = [ML^{-3}]^a [L]^b [ML^{-1}T^{-1}]^c [T^{-1}]$$

$$a + c = 0 \quad (1)$$

$$-3a + b - c = 0 \quad (2)$$

$$-c - 1 = 0 \quad (3)$$

Hence,  $a = 1$ ,  $b = 2$ , and  $c = -1$

$$\therefore \phi = \frac{\rho\omega D^2}{\mu}$$

Alternate solution: check the dimensions individually.

IES-5. Which of the following is not a dimensionless group? [IES-1992]

- (a)  $\frac{\Delta p}{\rho N^2 D^2}$  (b)  $\frac{gH}{N^2 D^2}$  (c)  $\frac{\rho\omega D^2}{\mu}$  (d)  $\frac{\Delta p}{\rho V^3}$

IES-5. Ans. (d)  $\frac{\Delta p}{\rho V^3} = \frac{[ML^{-1}T^{-2}]}{[ML^{-3}][LT^{-1}]^3} = L^{-1}T$ , hence dimensionless.

IES-6. What is the correct dimensionless group formed with the variable  $\rho$ -density,  $N$ -rotational speed,  $d$ -diameter and  $\pi$  coefficient of viscosity?

- (a)  $\frac{\rho N d^2}{\pi}$  (b)  $\frac{\rho N d}{\pi}$  [IES-2009]  
(c)  $\frac{Nd}{\rho\pi}$  (d)  $\frac{Nd^2}{\rho\pi}$

IES-6. Ans. (a)

IES-7. Match List-I (Fluid parameters) with List-II (Basic dimensions) and select the correct answer: [IES-2002]

**List-I**

**List-II**

A. Dynamic viscosity	1. $M/t^2$
B. Chezy's roughness coefficient	2. $M/Lt^2$
C. Bulk modulus of elasticity	3. $M/Lt$
D. Surface tension (o)	4. $\sqrt{L/t}$
<b>Codes:</b>	<b>A      B      C      D</b>
(a)	3      2      4      1      (b) 1      4      2      3
(c)	3      4      2      1      (d) 1      2      4      3

IES-7. Ans. (c)

## Rayleigh's Method

IES-8. Given power 'P' of a pump, the head 'H' and the discharge 'Q' and the specific weight 'w' of the liquid, dimensional analysis would lead to the result that 'P' is proportional to: [IES-1998]

- (a)  $H^{1/2} Q^2 w$       (b)  $H^{1/2} Q w$       (c)  $H Q^{1/2} w$       (d)  $HQ w$

IES-8. Ans. (d)

IES-9. Volumetric flow rate Q, acceleration due to gravity g and head H form a dimensionless group, which is given by: [IES-2002]

- (a)  $\sqrt{\frac{gH^5}{Q}}$       (b)  $\frac{Q}{\sqrt{gH^5}}$       (c)  $\frac{Q}{\sqrt{gH^3}}$       (d)  $\frac{Q}{\sqrt{g^2H}}$

IES-9. Ans. (b)

## Buckingham's $\pi$ -method/theorem

IES-10. If the number of fundamental dimensions equals 'm', then the repeating variables shall be equal to: [IES-1999, IES-1998, GATE-2002]

- (a) m and none of the repeating variables shall represent the dependent variable.  
 (b)  $m + 1$  and one of the repeating variables shall represent the dependent variable  
 (c)  $m + 1$  and none of the repeating variables shall represent the dependent variable.  
 (d) m and one of the repeating variables shall represent the dependent variable.

IES-10. Ans. (c)

IES-11. The time period of a simple pendulum depends on its effective length I and the local acceleration due to gravity g. What is the number of dimensionless parameter involved? [IES-2009]

- (a) Two      (b) One      (c) Three      (d) Zero

IES-11. Ans. (b)  $m = 3$  (time period, length and acceleration due to gravity);  $n = 2$  (length and time). Then the number of dimensionless parameter =  $m - n$ .

IES-12. In a fluid machine, the relevant parameters are volume flow rate, density, viscosity, bulk modulus, pressure difference, power consumption, rotational speed and characteristic dimension. Using the Buckingham pi ( $\pi$ ) theorem, what would be the number of independent non-dimensional groups? [IES-1993, 2007]

- (a) 3      (b) 4      (c) 5      (d) None of the above

IES-12. Ans. (c) No of variable = 8

No of independent dimension ( $m$ ) = 3

$\therefore$  No of  $\pi$  term =  $n - m = 8 - 3 = 5$

**IES-13.** The variable controlling the motion of a floating vessel through water are the drag force  $F$ , the speed  $v$ , the length  $l$ , the density  $\rho$ , dynamic viscosity  $\mu$  of water and gravitational constant  $g$ . If the non-dimensional groups are Reynolds number ( $Re$ ), Weber number ( $We$ ), Prandtl number ( $Pr$ ) and Froude number ( $Fr$ ), the expression for  $F$  is given by: [IES-1997]

- |  |  |
|--|--|
| (a) $\frac{F}{\rho v^2 l^2} = f(Re)$     | (b) $\frac{F}{\rho v^2 l^2} = f(Re, Pr)$ |
| (c) $\frac{F}{\rho v^2 l^2} = f(Re, We)$ | (d) $\frac{F}{\rho v^2 l^2} = f(Re, Fr)$ |

**IES-13. Ans. (d)** To solve this problem we have to use Buckingham's  $\pi$ -Theory.

**IES-14.** Consider the following statements: [IES-2003]

1. Dimensional analysis is used to determine the number of variables involved in a certain phenomenon
2. The group of repeating variables in dimensional analysis should include all the fundamental units.
3. Buckingham's  $\pi$  theorem stipulates the number of dimensionless groups for a given phenomenon.
4. The coefficient in Chezy's equation has no dimension.

Which of these are correct?

- (a) 1, 2, 3 and 4                      (b) 2, 3 and 4                      (c) 1 and 4                      (d) 2 and 3

**IES-14. Ans. (d)** 1 and 4 are wrong, coefficient in Chezy's equation has dimension  $[L^{1/2}T^{-1}]$

## Reynolds Number ( $Re$ )

**IES-15.** Match List-I with List-II and select the correct: [IES-1996]

- | List-I             | List-II   |
|--------------------|---|
| A. Reynolds Number | 1. Film coefficient, pipe diameter, thermal conductivity  |
| B. Prandtl Number  | 2. Flow velocity, acoustic velocity                       |
| C. Nusselt Number  | 3. Heat capacity, dynamic viscosity, thermal conductivity |
| D. Mach Number     | 4. Flow velocity, pipe diameter, kinematic viscosity      |

- |       |   |   |   |   |     |   |   |   |   |
|-------|---|---|---|---|-----|---|---|---|---|
| Code: | A | B | C | D |     | A | B | C | D |
| (a)   | 4 | 1 | 3 | 2 | (b) | 4 | 3 | 1 | 2 |
| (c)   | 2 | 3 | 1 | 4 | (d) | 2 | 1 | 3 | 4 |

**IES-15. Ans. (b)** As.  $Re = \frac{\rho V l}{\mu}$        $Pr = \frac{\mu C_p}{k}$        $N_u = \frac{h l}{k}$        $M = \frac{V}{V_a}$

## Euler Number ( $Eu$ )

**IES-16.** Euler number is defined as the ratio of inertia force to: [IES-1997]

- |                    |                   |
|--------------------|-------------------|
| (a) Viscous force  | (b) Elastic force |
| (c) Pressure force | (d) Gravity force |

**IES-16. Ans. (c)** Euler number

$$Eu = \left( \frac{\text{Inertia force}}{\text{Pressure force}} \right)^{1/2} = \frac{V}{\sqrt{P/\rho}}$$

**Weber number**

$$W = \left( \frac{\text{Inertia force}}{\text{Surface force}} \right)^{1/2} = \frac{V}{\sqrt{\sigma/\rho L}}$$

**Mach number**

$$M = \left( \frac{\text{Inertia force}}{\text{Elastic force}} \right)^{1/2} = \frac{V}{\sqrt{K/\rho}}$$

## Mach Number (*M*)

**IES-17. Match List-I (Dimensionless number) with List-II (Nature of forces involved) and select the correct answer using the code given below the lists:** **[IES-2008]**

**List-I**

- A. Euler number
- B. Weber number
- C. Mach number
- D. Froude number

**List-II**

- 1. Surface tension
- 2. Gravity
- 3. Pressure
- 4. Elastic

<b>Code:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	1	4	2
(b)	3	4	1	2
(c)	4	1	2	3
(d)	4	2	1	3

**IES-17. Ans. (a)**

1.	<b>Reynolds number</b>	$\frac{\text{Inertia force}}{\text{Viscous force}}$
2.	<b>Froude's number</b>	$\frac{\text{Inertia force}}{\text{Gravity force}}$
3.	<b>Euler's number</b>	$\frac{\text{Inertia force}}{\text{Pressure force}}$
4.	<b>Weber's number</b>	$\frac{\text{Inertia force}}{\text{Surface tension}}$
5.	<b>Mach's number</b>	$\frac{\text{Inertia force}}{\text{Elastic force}}$

**IES-18. Match List-I (Dimensionless numbers) with List-II (Definition as the ratio of) and select the correct answer:** **[IES-2001]**

**List-I**

- A. Reynolds number
- B. Froude number
- C. Weber number
- D. Mach number

**List-II**

- 1. Inertia force and elastic force
- 2. Inertia force and surface tension force
- 3. Inertia force and gravity force
- 4. Inertia force and viscous force

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	2	3	4	(b)	4	3	2	1
(c)	1	3	2	4	(d)	4	2	3	1

**IES-18. Ans. (b)**



**IES-19. Which one of the dimensionless numbers identifies the compressibility effect of a fluid? [IES-2005]**

- |                  |                   |
|------------------|-------------------|
| (a) Euler number | (b) Froude number |
| (c) Mach number  | (d) Weber number  |

**IES-19. Ans. (c)**

**IES-20. It is observed in a flow problem that total pressure, inertia and gravity forces are important. Then, similarly requires that [IES-2006]**

- (a) Reynolds and Weber numbers be equal  
 (b) Mach and Froude numbers be equal  
 (c) Euler and Froude numbers be equal  
 (d) Reynolds and Mach numbers be equal

**IES-20. Ans. (c)**

**IES-21. Match List-I (Flow/Wave) with List-II (Dimensionless number) and select the correct answer: [IES-2003]**

List-I					List-II				
A. Capillary waves in channel					1. Reynolds number				
B. Testing of aerofoil					2. Froude number				
C. Flow around bridge piers					3. Weber number				
D. Turbulent flow through pipes					4. Euler number				
					5. Mach number				
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	5	4	3	2	(b)	3	5	4	1
(c)	5	4	2	1	(d)	3	5	2	1

**IES-21. Ans. (d)**

**IES-22. Match List-I (Predominant force) with List-II (Dimensionless numbers) and select the correct answer [IES-1996]**

List-I					List-II				
A. Compressibility force					1. Euler number				
B. Gravity force					2. Prandtl number				
C. Surface tension force					3. Mach number				
D. Viscous force					4. Reynolds number				
					5. Weber number				
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	2	3	4	(b)	3	2	5	4
(c)	3	1	4	5	(d)	2	3	5	1

**IES-22. Ans. (b)** When compressibility force is predominant, mach number is used; when gravity force predominates, Froude number is adopted. Similarly for surface tension force and viscous force, Weber number and Reynolds number are considered.

**IES-23. Match List-I (Forces) with List-II (Dimensionless groups) and select the correct answer. [IES-1994]**

List-I		List-II	
A. Viscous force		1. Reynolds number	
B. Elastic force		2. Froude number	
C. Surface tension		3. Waber number	

D. Gravity

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	4	2	3
(c)	3	4	1	2

4. Mach number

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(b)	1	2	4	3
(d)	1	4	3	2

IES-23. Ans. (d)

IES-24. List-I gives 4 dimensionless numbers and List-II gives the types of forces which are one of the constituents describing the numbers. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-1993]

**List-I**

- A. Euler number
- B. Froude number
- C. Mach number
- D. Webber number

**List-II**

- 1. Pressure force
- 2. Gravity force
- 3. Viscous force
- 4. Surface tension
- 5. Elastic force

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	3	4	5	(b)	3	2	4	5
(c)	2	1	3	4	(d)	1	2	5	4

IES-24. Ans. (d) Euler number is concerned with pressure force and this choice is available for A in code (d) only. If one is confident, then there is no need to look for items B, C and D. However a cross checks will show that Froude number is concerned with gravity force, Mach number with elastic force, and Weber number with surface tension. Hence the answer is (d) only.

IES-25. Match List-I (Type of Model) with List-II (Transference Ratio for Velocity) and select the correct answer: [IES-2004]

**List-I**

- A. Reynolds model
- B. Froude model
- C. Weber model
- D. Mach model

**List-II**

- 1.  $\sqrt{K_r / \rho_r}$
- 2.  $\sqrt{\sigma_r / (\rho_r l_r)}$
- 3.  $\mu_r / (\rho_r l_r)$
- 4.  $\sqrt{g_r l_r}$

(Where symbols g,  $\mu$ ,  $\rho$ ,  $\sigma$  and k have their usual meanings and subscript r refers to the ratio)

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	1	2	4	(b)	3	4	2	1
(c)	2	1	3	4	(d)	2	4	3	1

IES-25. Ans. (b)

## Model (or Similarity) Laws

IES-26. Consider the following statements: [IES-2005]

- 1. For achieving dynamic similarity in model studies on ships, Froude numbers are equated.

2. Reynolds number should be equated for studies on aerofoil for dynamic similarity.
3. In model studies on a spillway, the ratio of width to height is equated for kinematic similarity.

What of the statements given above are correct?

- (a) 1, 2 and 3                      (b) 1 and 2                      (c) 2 and 3                      (d) 1 and 3

**IES-26. Ans. (d)** Mach number should be equated for studies on aerofoil for dynamic similarity.

**IES-27. Kinematic similarity between model and prototype is the similarity of**  
[IES-1996]

- (a) Shape                      (b) Discharge                      (c) Stream line pattern                      (d) Forces

**IES-27. Ans. (c)** Kinematic similarity between a model and its prototype is said to exist if the flow patterns are in geometric i.e. velocity, acceleration etc are similar. Remember discharge is not related with kinematic similarity.

## Reynolds Model Law

**IES-28. Assertion (A): Reynolds number must be same for the model and prototype immersed in subsonic flows.** [IES-2003]

**Reason (R): Equality of Reynolds number for the model and prototype satisfies the dynamic similarity criteria.**

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-28. Ans. (b)**

**IES-29. A model test is to be conducted in a water tunnel using a 1: 20 model of a submarine, which is to travel at a speed of 12 km/h deep under sea surface. The water temperature in the tunnel is maintained, so that is kinematic viscosity is half that of sea water. At what speed is the model test to be conducted to produce useful data for the prototype?**

[IES-2002]

- (a) 12 km/h                      (b) 240 km/h                      (c) 24 km/h                      (d) 120 km/h

**IES-29. Ans. (d)** Apply Reynolds Model law.

**IES-30. A sphere is moving in water with a velocity of 1.6 m/s. Another sphere of twice the diameter is placed in a wind tunnel and tested with air which is 750 times less dense and 60 times less viscous than water. The velocity of air that will give dynamically similar conditions is:**

[IES-1999]

- (a) 5 m/s                      (b) 10 m/s                      (c) 20 m/s                      (d) 40 m/s

**IES-30. Ans. (b)**

**IES-31. The model of a propeller, 3 m in diameter, cruising at 10 m/s in air, is tested in a wind tunnel on a 1: 10 scale model. If a thrust of 50 N is measured on the model at 5 m/s wind speed, then the thrust on the prototype will be:**

[IES-1995]

- (a) 20,000 N                      (b) 2,000 N                      (c) 500 N                      (d) 200 N

IES-31. Ans. (a) Force ratio  $= \frac{\rho_m}{\rho_p} \times \frac{L_m^2}{L_p^2} \times \frac{V_m^2}{V_p^2}; \quad \frac{F_m}{F_p} = 1 \times \left(\frac{1}{10}\right)^2 \times \left(\frac{5}{10}\right)^2$   
 or  $\frac{50}{F_p} = \frac{1}{100} \times \frac{1}{4}; \quad F_p = 50 \times 400 = 20000 \text{ N}$

## Froude Model Law

IES-32. A 1.0 m log model of a ship is towed at a speed of 81 cm/s in a towing tank. To what speed of the ship, 64 m long does this correspond to?

[IES-2004]

- (a) 7.20 m/s                      (b) 6.48 m/s                      (c) 5.76 m/s                      (d) 3.60 m/s

IES-32. Ans. (b) Apply Froude Model law  $(F_r)_m = (F_r)_p$  or  $\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$

or  $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \Rightarrow \frac{0.81}{V_p} = \sqrt{\frac{1}{64}}$

IES-33. A ship model 1/60 scale with negligible friction is tested in a towing tank at a speed of 0.6 m/s. If a force of 0.5 kg is required to tow the model, the propulsive force required to tow the prototype ship will be:

[IES-1999]

- (a) 5 MN                      (b) 3 MN                      (c) 1 MN                      (d) 0.5 MN

IES-33. Ans. (c)

IES-34. A1:256 scale model of a reservoir is drained in 4 minutes by opening the sluice gate. The time required to empty the prototype will be: [IES-1999]

- (a) 128 min                      (b) 64 min                      (c) 32 min                      (d) 25.4 min

IES-34. Ans. (b)

IES-35. A ship whose full length is 100 m is to travel at 10 m/s. For dynamic similarity, with what velocity should a 1: 25 model of the ship be towed?

[IES-2004]

- (a) 2 m/s                      (b) 10 m/s                      (c) 25 m/s                      (d) 250 m/s

IES-35. Ans. (a) For ship Froude Model law is used.

$\therefore \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} \text{ or } V_m = V_p \times \sqrt{\frac{L_m}{L_p}} = 10 \times \sqrt{\frac{1}{25}} = 2 \text{ m/s}$

IES-36. A  $\frac{1}{25}$  model of a ship is to be tested for estimating the wave drag. If the speed of the ship is 1 m/s, then the speed at which the model must be tested is:

[IES-1992, IAS-2002]

- (a) 0.04 m/s                      (b) 0.2 m/s                      (c) 5.0 m/s                      (d) 25.0 m/s

IES-36. Ans. (b) Apply Froude Model law  $(F_r)_m = (F_r)_p$  or  $\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{g \cdot L_p}}$

or  $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{25}} = \frac{1}{5} \text{ or } V_m = \frac{1}{5} = 0.2 \text{ m/s.}$

**IES-37.** In a flow condition where both viscous and gravity forces dominate and both the Froude number and the Reynolds number are the same in model and prototype; and the ratio of kinematic viscosity of model to that of the prototype is 0.0894. What is the model scale? [IES-2004]

- (a) 1: 3.3                      (b) 3.3: 1                      (c) 5: 1                      (d) 1:5

**IES-37. Ans. (c)**  $(R_e)_{\text{model}} = (R_e)_{\text{prototype}}$  gives  $\frac{V_m}{V_p} \times \frac{L_m}{L_p} \times \frac{\nu_p}{\nu_m} = 1 \dots (i)$

and  $(F_r)_{\text{model}} = (F_r)_{\text{prototype}}$  gives  $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \dots (ii)$

(i) and (ii) gives  $\left(\frac{L_m}{L_p}\right)^{3/2} = \frac{\nu_p}{\nu_m} = 0.0894$  or  $\frac{L_m}{L_p} = 0.2$

$$L_m : L_p = 5 : 1$$

**IES-38.** A 1:20 model of a spillway dissipates 0.25 hp. The corresponding prototype horsepower dissipated will be: [IES-1998]

- (a) 0.25                      (b) 5.00                      (c) 447.20                      (d) 8944.30

**IES-38. Ans. (d)**  $P_r = L_r^{3.5} = 20^{3.5}$  Therefore  $P_p = 0.25 \times 20^{3.5} = 8944 \text{ hp}$

**IES-39.** A ship with hull length of 100 m is to run with a speed of 10 m/s. For dynamic similarity, the velocity for a 1: 25 model of the ship in a towing tank should be: [IES-2001]

- (a) 2 m/s                      (b) 10 m/s                      (c) 20 m/s                      (d) 25 m/s

**IES-39. Ans. (a)** Use  $V_r = \sqrt{L_r}$

**IES-40.** A ship's model, with scale 1: 100, has a wave resistance of 10 N at its design speed. What is the corresponding prototype wave resistance in kN? [IES-2007]

- (a) 100                      (b) 1000                      (c) 10000  
(d) Cannot be determined because of insufficient data

**IES-40. Ans. (c)** We know that  $F_r = L_r^3$

$$\text{or, } \frac{F_p}{F_m} = \left(\frac{L_p}{L_m}\right)^3 \text{ or } F_p = F_m \times \left(\frac{L_p}{L_m}\right)^3 = 10 \times (100)^3 \text{ N} = 10000 \text{ kN}$$

**IES-41.** A model test is to be conducted for an under water structure which each likely to be exposed for an under water structure, which is likely to be exposed to strong water currents. The significant forces are known to be dependent on structure geometry, fluid velocity, fluid density and viscosity, fluid depth and acceleration due to gravity. Choose from the codes given below, which of the following numbers must match for the model with that of the prototype: [IES-2002]

1. Mach number                      2. Weber number  
3. Froude number                      4. Reynolds number.

- (a) 3 alone                      (b) 1, 2, 3 and 4                      (c) 1 and 2                      (d) 3 and 4

**IES-41. Ans. (d)**

## **Types of Models (Undistorted models, distorted models)**

**IES-42.** Consider the following statements:

[IES-2003]

1. Complete similarity between model and prototype envisages geometric and dynamic similarities only.
2. Distorted models are necessary where geometric similarity is not possible due to practical reasons.
3. In testing of model of a ship, the surface tension forces are generally neglected.
4. The scale effect takes care of the effect of dissimilarity between model and prototype.

Which of these statements are correct?

- (a) 1 and 3                      (b) 1, 2 and 4                      (c) 2 and 3                      (d) 2 and 4

**IES-42. Ans. (c)** 1 is wrong. Complete similarity between model and prototype envisages geometric, kinematic and dynamic similarities only.  
4 is also wrong. The scale effect takes care of the effect of dissimilarity (size difference) between model and prototype.

## Previous 20-Years IAS Questions

### Similitude

**IAS-1.** The drag force  $D$  on a certain object in a certain flow is a function of the coefficient of viscosity  $\mu$ , the flow speed  $v$  and the body dimension  $L$  (for geometrically similar objects); then  $D$  is proportional to: [IAS-2001]

- (a)  $L \mu V$                       (b)  $\frac{\mu^2 V^2}{L^2}$                       (c)  $\mu^2 v^2 L^2$                       (d)  $\frac{\mu L}{V}$

**IAS-1. Ans. (a)**

**IAS-2.** For a 1:  $m$  scale model of a hydraulic turbine, the specific speed of the model  $N_{sm}$  is related to the prototype specific speed  $N_{sp}$  as [IAS-1997]

- (a)  $N_{sm} = N_{sp}/m$                       (b)  $N_{sm} = m N_{sp}$                       (c)  $N_{sm} = (N_{sp})^{1/m}$                       (d)  $N_{sm} = N_{sp}$

**IAS-2. Ans. (d)**

### Froude Number ( $Fr$ )

**IAS-3.** The square root of the ratio of inertia force to gravity force is called [GATE-1994, IAS-2003]

- (a) Reynolds number                      (b) Froude number  
(c) Mach number                      (d) Euler number

**IAS-3. Ans. (b)**

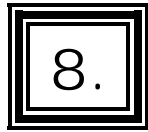
### Froude Model Law

**IAS-4.** A  $\frac{1}{25}$  model of a ship is to be tested for estimating the wave drag. If the speed of the ship is 1 m/s, then the speed at which the model must be tested is: [IES-1992, IAS-2002]

- (a) 0.04 m/s                      (b) 0.2 m/s                      (c) 5.0 m/s                      (d) 25.0 m/s

**IAS-4. Ans. (b)** Apply Froude Model law  $(Fr)_m = (Fr)_p$  or  $\frac{V_m}{\sqrt{g L_m}} = \frac{V_p}{\sqrt{g L_p}}$

$$\text{or } \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{25}} = \frac{1}{5} \text{ or } V_m = \frac{1}{5} = 0.2 \text{ m/s.}$$



## Boundary Layer Theory

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### Contents of this chapter

1. Boundary Layer Definitions and Characteristics
  2. Boundary Layer Thickness ( $\delta$ )
  3. Displacement Thickness ( $\delta^*$ )
  4. Momentum Thickness ( $\theta$ )
  5. Energy thickness ( $\delta_e$ )
  6. Momentum Equation for Boundary Layer by Von-karman
  7. Laminar Boundary Layer
  8. Turbulent Boundary Layer
  9. Total Drag Due to Laminar and Turbulent Layers
  10. Boundary Layer Separation and its Control
  11. Thermal Boundary Layer
-



## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

**Momentum Equation for Boundary Layer by Von-karman**

**GATE-1.** For air flow over a flat plate, velocity ( $U$ ) and boundary layer thickness ( $\delta$ ) can be expressed respectively, as [GATE-2004]

$$\frac{U}{U_a} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3; \quad \delta = \frac{4.64x}{\sqrt{\text{Re}_x}}$$

If the free stream velocity is 2 m/s, and air has kinematic viscosity of  $1.5 \times 10^{-5} \text{ m}^2/\text{s}$  and density of  $1.23 \text{ kg/m}^3$ , then wall shear stress at  $x = 1 \text{ m}$ , is

- (a)  $2.36 \times 10^2 \text{ N/m}^2$  (b)  $43.6 \times 10^{-3} \text{ N/m}^2$   
(c)  $4.36 \times 10^{-3} \text{ N/m}^2$  (d)  $2.18 \times 10^{-3} \text{ N/m}^2$

**GATE-1. Ans. (c)** Given:  $\rho = 1.23 \text{ kg/m}^3$ ,  $\nu = \frac{\mu}{\rho} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $u = 2 \text{ m/s}$ ,  $x = L = 1 \text{ m}$ .

$$\text{Re}_x = \frac{\rho u L}{\mu} = \frac{2 \times 1}{\left( \frac{\mu}{\rho} \right)} = \frac{2 \times 1}{1.5 \times 10^{-5}} = 1.34 \times 10^5$$

$$\text{Now, shear stress, } \tau_0 = \mu \left( \frac{du}{dy} \right)_{y=0}$$

$$\text{Where, } \frac{u}{U} = \frac{3}{2} \frac{y}{\delta} - \frac{y^3}{2\delta^3} \text{ or } \frac{du}{dy} = U \left[ \frac{3}{2\delta} - \frac{3}{2} \frac{y^2}{\delta^3} \right]$$

$$\text{Hence, } \left( \frac{du}{dy} \right)_{y=0} = \frac{3U}{2\delta}$$

$$\text{Given: } \delta = \frac{4.64x}{\sqrt{\text{Re}_x}} = \frac{4.64 \times x}{\sqrt{\frac{\rho U x}{\mu}}}$$

$$\text{Putting } x = 1, (\delta)_{x=1} = \frac{4.64}{\sqrt{\frac{2 \times 1}{1.5 \times 10^{-5}}}} = 0.0127$$

$$\therefore \tau_0 = \mu \cdot \frac{du}{dy} = \frac{3}{2} \frac{\mu U}{\delta} = \frac{3}{2} \times \frac{(1.5 \times 10^{-5} \times 1.23) \times 2}{0.0127} = 4.355 \times 10^{-3} \text{ N/m}^2$$

**Laminar Boundary Layer**

**GATE-2.** The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]

- (a)  $X$  (b)  $X^{\frac{1}{2}}$  (c)  $X^{\frac{1}{5}}$  (d)  $X^{\frac{4}{5}}$

**GATE-2. Ans. (b)**  $\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re}_x}}$  or  $\delta \propto \frac{5x}{\sqrt{\frac{\rho \nu x}{\mu}}}$  or  $\delta \propto \sqrt{x}$

## Total Drag Due to Laminar and Turbulent Layers

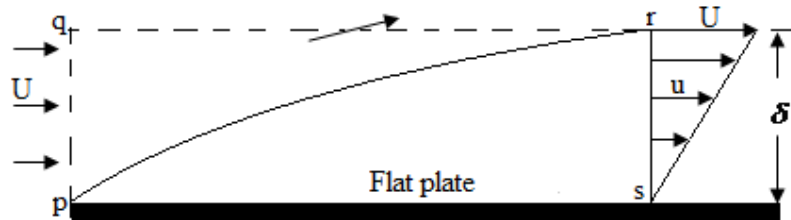
**GATE-3.** Consider an incompressible laminar boundary layer flow over a flat plate of length  $L$ , aligned with the direction of an oncoming uniform free stream. If  $F$  the ratio of the drag force on the front half of the plate to the drag force on the rear half, then [GATE-2007]

- (a)  $F < 1/2$  (b)  $F = 1/2$  (c)  $F = 1$  (d)  $F > 1$

**GATE-3. Ans. (d)**  $F_D = \text{some Const} \times \int_{l_1}^{l_2} x^{-1/2} dx$ . Therefore ratio  $= \frac{\sqrt{L/2} - 0}{L - \sqrt{L/2}} = \frac{1}{\sqrt{2} - 1} > 1$

### Statement for Linked Answer and Questions Q4–Q5:

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at  $U = 10$  m/s. The thickness of the boundary layer at section r-s is 10 mm, the breadth of the plate is 1 m (into the paper) and the density of the gas  $\rho = 1.0$  kg/m<sup>3</sup>. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution,  $u = U(y/\delta)$ , at the section r-s, where  $y$  is the height from plate.



**GATE-4.** The mass flow rate (in kg/s) across the section q – r is: [GATE-2006]

- (a) Zero (b) 0.05 (c) 0.10 (d) 0.15

**GATE-4. Ans. (b)** Mass entering from side q – p = Mass leaving from side q – r + Mass leaving the side r – s.

**GATE-5.** The integrated drag force (in N) on the plate, between p – s, is:

- (a) 0.67 (b) 0.33 (c) 0.17 (d) Zero [GATE-2006]

**GATE-5. Ans. (c)** By momentum equation, we can find drag force.

## Boundary Layer Separation and Its Control

**GATE-6.** Flow separation is caused by: [IAS-1996; IES-1994, 1997; 2000; GATE-2002]

- (a) Reduction of pressure to local vapour pressure  
(b) A negative pressure gradient  
(c) A positive pressure gradient  
(d) Thinning of boundary layer thickness to zero.

**GATE-6. Ans. (c)** i.e. an adverse pressure gradient.

When the pressure goes increasing  $\left(\frac{\partial P}{\partial x} > 0\right)$  in the direction of flow, the

pressure force acts against the direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickens the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as “Boundary Layer Separation”.

**GATE-7.** The necessary and sufficient condition which brings about separation of boundary layer is  $\frac{dp}{dx} > 0$  [GATE-1994]

**GATE-7. Ans.** False because Separation takes place where  $\frac{dp}{dx} > 0$  and  $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

## Thermal Boundary Layer

**GATE-8.** The temperature distribution within thermal boundary layer over a heated isothermal flat plate is given by  $\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$ , where  $T_w$  and  $T_\infty$  are the temperature of plate and free stream respectively, and  $y$  is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness  $\delta_t$  is given by [GATE-2007]

- (a) 1.33                      (b) 1.50                      (c) 2.0                      (d) 4.64

**GATE-8. Ans. (b)**  $Nu = \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$

where,  $T^* = \frac{T - T_w}{T_\infty - T_w}$  and  $y^* = \frac{y}{\delta}$

Where  $T_w$  is surface temperature and  $T_\infty$  is free-stream temperature.

$$\Rightarrow Nu = \frac{3}{2} - \frac{3}{2} \left( \frac{y}{\delta_t} \right) \Big|_{y=0} = \frac{3}{2} - \frac{3}{2} y^* \Big|_{y^*=0} = 1.5$$

**GATE-9.** Consider a laminar boundary layer over a heated flat plate. The free stream velocity is  $U_\infty$ . At some distance  $x$  from the leading edge the velocity boundary layer thickness is  $\delta_v$ . If the Prandtl number is greater than 1, then [GATE-2003]

- (a)  $\delta_v > \delta_T$                       (b)  $\delta_T > \delta_v$                       (c)  $\delta_v$

**GATE-9. Ans. (a)** Prandtl number =  $\frac{\text{Molecular diffusivity of mom}}{\text{Molecular diffusivity of heat}}$

From question, since Prandtl number  $> 1$

$\therefore$  Velocity boundary thickness ( $\delta_v$ )  $>$  thermal boundary thickness.

**GATE-10.** A fluid flowing over a flat plate has the following properties:

Dynamic viscosity:  $25 \times 10^{-6}$  kg/ms

[GATE-1992]

Specific heat: 2.0 kJ/kgK

Thermal conductivity: 0.05 W/mK

The hydrodynamic boundary layer thickness is measured to be 0.5 mm.

The thickness of thermal boundary layer would be:

- (a) 0.1 mm                      (b) 0.5 mm                      (c) 1.0 mm                      (d) None of the above

**GATE-10. Ans. (b)**  $\frac{\delta}{\delta_{th}} = (Pr)^{1/3}$  if  $Pr = \frac{\mu C_p}{k} = \frac{25 \times 10^{-6} \text{ kg/ms} \times 2000 \text{ J/kgK}}{0.05 \text{ kg/mK}} = 1$ . So,  $\delta = \delta_{th}$

**GATE-11.** For flow of fluid over a heated plate, the following fluid properties are known: viscosity = 0.001 Pa.s ; specific heat at constant pressure = 1 kJ/kg.K; thermal conductivity = 1 W/m.K. [GATE-2008]

The hydrodynamic boundary layer thickness at a specified location on the plate is 1 mm. The thermal boundary layer thickness at the same location is:

- (a) 0.001 mm                      (b) 0.01 mm                      (c) 1 mm                      (d) 1000 mm

**GATE-11. Ans. (c)** We know that  $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$ .

$$\text{Here Prandtl number } (P_r) = \frac{\mu c_p}{k} = \frac{0.001 \times 1000}{1} = 1. \text{ So, } \delta = \delta_{th}.$$

**GATE-12.** For air near atmosphere conditions flowing over a flat plate, the laminar thermal boundary layer is thicker than the hydrodynamic boundary layer. [GATE-1994]

**GATE-12. Ans. False**

## Previous 20-Years IES Questions

### Boundary Layer Definitions and Characteristics

**IES-1. Boundary layer is defined as** [IES-1998]

- (a) A thin layer at the surface where gradients of both velocity and temperature are small
- (b) A thin layer at the surface where velocity and velocity gradients are large
- (c) A thick layer at the surface where velocity and temperature gradients are large
- (d) A thin layer at the surface where gradients of both velocity and temperature are large

**IES-1. Ans. (b)**

- The **boundary layer** of a flowing fluid is **the thin layer close to the wall**
- In a flow field, **viscous stresses are very prominent within this layer.**
- Although the layer is thin, it is very important to know the details of flow within it.
- The **main-flow velocity** within this layer **tends to zero** while approaching the wall (**no-slip condition**).
- Also the gradient of this velocity component in a direction normal to the surface is large as compared to the gradient in the stream wise direction.

**IES-2. In the boundary layer, the flow is:** [IES-2006]

- (a) Viscous and rotational                      (b) Inviscid and irrotational
- (c) Inviscid and rotational                      (d) Viscous and irrotational

**IES-2. Ans. (a)**

**IES-3. Assertion (A): In the boundary layer concept, the shear stress at the outer edge of the layer is considered to be zero.** [IES-2008]

**Reason (R): Local velocity is almost equal to velocity in potential flow.**

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-3. Ans. (a)

IES-4. In the region of the boundary layer nearest to the wall where velocity is not equal to zero, the viscous forces are: [IES-1993, 1995]

- (a) Of the same order of magnitude as the inertial forces
- (b) More than inertial forces
- (c) Less than inertial forces
- (d) Negligible

IES-4. Ans. (c) Reynold's number = Inertia force / Viscous force  
and it is more than one therefore the viscous forces are less than inertial forces.

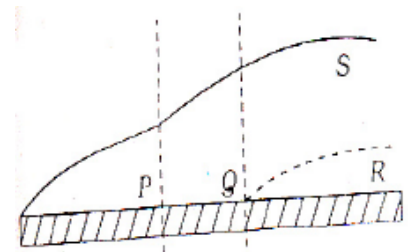
IES-5. The critical value of Reynolds number for transition from laminar to turbulent boundary layer in external flows is taken as: [IES-2002]

- (a) 2300
- (b) 4000
- (c)  $5 \times 10^5$
- (d)  $3 \times 10^6$

IES-5. Ans. (c)

IES-6. The development of boundary layer zones labeled P, Q, R and S over a flat plate is shown in the given figure.

Based on this figure, match List-I (Boundary layer zones) with List-II (Type of boundary layer) and select the correct answer:



[IES-2000]

List-I

- A. P
- B. Q
- C. R
- D. S

List-II

- 1. Transitional
- 2. Laminar viscous sub-layer
- 3. Laminar
- 4. Turbulent

Codes:	A	B	C	D		A	B	C	D
(a)	3	1	2	4	(b)	3	2	1	4
(c)	4	2	1	3	(d)	4	1	2	3

IES-6. Ans. (a)

IES-7. The transition Reynolds number for flow over a flat plate is  $5 \times 10^5$ . What is the distance from the leading edge at which transition will occur for flow of water with a uniform velocity of 1 m/s? [For water, the kinematic viscosity,  $\nu = 0.858 \times 10^{-6} \text{ m}^2/\text{s}$ ] [IES-1994]

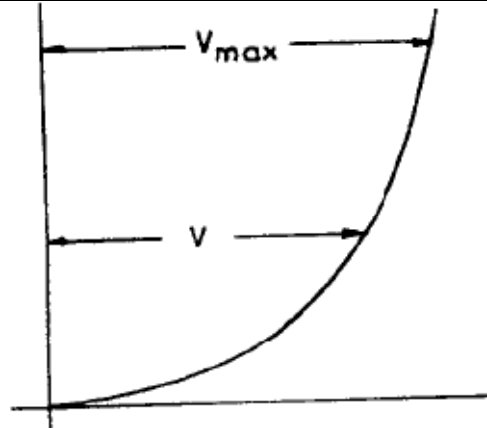
- (a) 1 m
- (b) 0.43 m
- (c) 43 m
- (d) 103 m

IES-7. Ans. (b)  $R_N = 5 \times 10^5$ ,  $R_N = \frac{Vx}{\nu}$ , or  $x = \frac{R_N \times \nu}{V} = \frac{5 \times 10^5 \times 0.858 \times 10^{-6}}{1} = 0.43 \text{ m}$

IES-8. The 'velocity defect law' is so named because it governs a [IES-1993]

- (a) Reverse flow region near a wall
- (b) Slip-stream flow at low pressure
- (c) Flow with a logarithmic velocity profile a little away from the wall
- (d) Re-circulating flow near a wall

IES-8. Ans. (c) Figure shows the logarithmic velocity profile a little away from the wall. Velocity difference ( $V_{\max} - V$ ) is known as velocity defect. So velocity defect law occurs due to occurrence of flow with a logarithmic velocity profile a little away from the wall.



IES-9. Which one of the following velocity distributions of  $u/u_\alpha$  satisfies the boundary conditions for laminar flow on a flat plate? (Here  $u_\alpha$  is the free stream velocity,  $u$  is velocity at any normal distance  $y$  from the flat plate,  $\eta = y/\delta$  and  $\delta$  is boundary layer thickness) [IES-1996]

- (a)  $\eta - \eta^2$  (b)  $1.5\eta - 0.5\eta^3$  (c)  $3\eta - \eta^2$  (d)  $\cos(\pi\eta/2)$

IES-9. Ans. (b) The relation  $\frac{u}{u_\alpha} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3$  satisfies boundary condition for laminar flow on a flat plate.

IES-10. The predominant forces acting on an element of fluid in the boundary layer over a flat plate placed in a uniform stream include [IES-1996]

- (a) Inertia and pressure forces (b) Viscous and pressure forces  
(c) Viscous and body forces (d) Viscous and inertia forces

IES-10. Ans. (d) That so why we are using Reynold's number for analysis.

## Boundary Layer Thickness ( $\delta$ )

IES-11. The hydrodynamic boundary layer thickness is defined as the distance from the surface where the [IES-1999]

- (a) Velocity equals the local external velocity  
(b) Velocity equals the approach velocity  
(c) Momentum equals 99% of the momentum of the free stream  
(d) Velocity equals 99% of the local external velocity

IES-11. Ans. (d)

IES-12. Assertion (A): The thickness of boundary layer is an ever increasing one as its distance from the leading edge of the plate increases.

Reason (R): In practice, 99% of the depth of the boundary layer is attained within a short distance of the leading edge. [IES-1999]

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-12. Ans. (c) Why A is true? For laminar boundary layer:

$$\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re } x}} \quad \text{or} \quad \delta \propto \frac{5x}{\sqrt{\rho \nu x}} \quad \text{or} \quad \delta \propto \sqrt{x}$$

For turbulent boundary layer:

$$\frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}} \text{ or } \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5} \text{ or } \delta \propto x^{4/5}$$

Therefore for both the cases if  $x$  increases  $\delta$  will increase.

Why R is false? There is no term in the boundary layer like “depth of the boundary layer”. Till today we never heard about depth of the boundary layer.

**IES-13. For the velocity profile  $u / u_{\infty} = \square$ , the momentum thickness of a laminar boundary layer on a flat plate at a distance of 1 m from leading edge for air (kinematic viscosity =  $2 \times 10^{-5} \text{ m}^2/\text{s}$ ) flowing at a free stream velocity of 2 m/s is given by: [IES-2001]**

- (a) 3.16 mm                      (b) 2.1 mm                      (c) 3.16 m                      (d) 2.1 m

**IES-13. Ans. (b)** Thickness of Boundary layer,  $\delta = \frac{5x}{\sqrt{\text{Re}_x}} = \frac{5x}{\sqrt{Ux/\nu}} = \frac{5 \times 1}{\sqrt{2 \times 2 / 2 \times 10^{-5}}} =$

$$0.01118 \text{ m and for such velocity distribution } \theta = \frac{\delta}{6} = 1.863 \text{ mm nearest ans. (b)}$$

**IES-14. A flat plate,  $2\text{m} \times 0.4\text{m}$  is set parallel to a uniform stream of air (density  $1.2\text{kg/m}^3$  and viscosity 16 centistokes) with its shorter edges along the flow. The air velocity is 30 km/h. What is the approximate estimated thickness of boundary layer at the downstream end of the plate? [IES-2004]**

- (a) 1.96 mm                      (b) 4.38 mm                      (c) 13.12 mm                      (d) 9.51 mm

**IES-14. Ans. (b)** Thickness of Boundary layer,

$$\delta = \frac{5x}{\sqrt{\text{Re}_x}} = \frac{5L}{\sqrt{\frac{UL}{\nu}}} = \frac{5 \times 0.4}{\sqrt{\frac{30 \times (5/18) \times 0.4}{16 \times 10^{-6}}}} = 4.38 \text{ mm}$$

**IES-15. A laminar boundary layer occurs over a flat plate at zero incidence to the flow. The thickness of boundary layer at a section 2 m from the leading edge is 2 mm. The thickness of boundary layer at a section 4 m from the leading edge will be: [IES-1995]**

- (a)  $2 \times (2)^2 \text{ mm}$                       (b)  $2 \times (2)^{1/2} \text{ mm}$                       (c)  $2 \times (2)^{4/5} \text{ mm}$                       (d)  $2 \times (2)^{1/5} \text{ mm}$

**IES-15. Ans. (b)** Thickness of boundary layer at 4 mm from leading edge =  $2 \times (4/2)^{1/2} = 2 \times 2^{1/2}$

## Displacement Thickness ( $\delta^*$ )

**IES-16. If the velocity distribution in a turbulent boundary layer is given by**

$\frac{u}{u_{\infty}} = \left(\frac{y}{\delta}\right)^{1/9}$  then the ratio of displacement thickness to nominal layer thickness will be: [IAS-1998; IES-2006]

- (a) 1.0                      (b) 0.6                      (c) 0.3                      (d) 0.1

**IES-16. Ans. (d)** Displacement thickness ( $\delta^*$ ) =  $\delta \int_0^1 (1 - z^{1/9}) dz = 0.1\delta$

IES-17. For linear distribution of velocity in the boundary layer on a flat plate, what is the ratio of displacement thickness ( $\delta^*$ ) to the boundary layer thickness ( $\delta$ )? [IES-2005]

- (a)  $\frac{1}{4}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{5}$

IES-17. Ans. (c) Remember it.

## Momentum Thickness ( $\theta$ )

IES-18. If  $U_\infty$  = free stream velocity,  $u$  = velocity at  $y$  and  $\delta$  = boundary layer thickness, then in a boundary layer flow, the momentum thickness  $\theta$  is given by: [IES-1997; IAS-2004]

- (a)  $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy$  (b)  $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$   
 (c)  $\theta = \int_0^\delta \frac{u^2}{U_\infty^2} \left(1 - \frac{u}{U_\infty}\right) dy$  (d)  $\theta = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

IES-18. Ans. (a)

IES-19. Given that [IES-1997]

$\delta$  = boundary layer thickness,  $\delta^*$  = displacement thickness

$\delta_e$  = energy thickness  $\theta$  = momentum thickness

The shape factor  $H$  of a boundary layer is given by

- (a)  $H = \frac{\delta_e}{\delta}$  (b)  $H = \frac{\delta^*}{\theta}$  (c)  $H = \frac{\delta}{\theta}$  (d)  $H = \frac{\delta}{\delta^*}$

IES-19. Ans. (b) Shape factor =  $\frac{\text{Displacement thickness}}{\text{Momentum thickness}}$

IES-20. The velocity distribution in the boundary layer is given as  $u / u_s = y / \delta$ , where  $u$  is the velocity at a distance  $y$  from the boundary,  $u_s$  is the free stream velocity and  $\delta$  is the boundary layer thickness at a certain distance from the leading edge of a plate. The ratio of displacement to momentum thicknesses is: [IES-2001; 2004]

- (a) 5 (b) 4 (c) 3 (d) 2

IES-20. Ans. (c) Remember it.

IES-21. The velocity profile in a laminar boundary layer is given by  $u/U = y/\delta$ . The ratio of momentum thickness to displacement thickness for the boundary is given by which one of the following? [IES-2008]

- (a) 2 : 3 (b) 1 : 2 (c) 1 : 6 (d) 1 : 3

IES-21. Ans. (d) Velocity profile of laminar boundary layer is given

$$\frac{u}{U} = \frac{y}{\delta}$$

Displacement thickness:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \frac{\delta}{2}$$

Momentum thickness:



$$\theta = \int_0^{\delta} \left(1 - \frac{u}{U}\right) \frac{u}{U} dy = \int_0^{\delta} \left(\frac{y}{\delta}\right) \left(1 - \frac{y}{\delta}\right) dy = \frac{\delta}{6}$$

$$\therefore \frac{\theta}{\delta^*} = \frac{1}{3}$$

### Energy Thickness ( $\delta_e$ )

IES-22. The energy thickness for a laminar boundary layer flow depends on local and free stream velocities within and outside the boundary layer  $\delta$ . The expression for the energy thickness is given by (symbols have the usual meaning). [IES-1994]

(a)  $\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) dy$  (b)  $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u}{U_{\infty}}\right) dy$

(c)  $\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right)^2 dy$  (d)  $\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u^2}{U_{\infty}^2}\right) dy$

IES-22. Ans. (d)

IES-23. Which one of the following is the correct relationship between the boundary layer thickness  $\delta$ , displacement thickness  $\delta^*$  and the momentum thickness  $\theta$ ? [IAS-2004; IES-1999]

(a)  $\delta > \delta^* > \theta$  (b)  $\delta^* > \theta > \delta$  (c)  $\theta > \delta > \delta^*$  (d)  $\theta > \delta^* > \delta$

IES-23. Ans. (a)  $\delta > \delta^* > \theta > \delta^{**}$

IES-24. List-I give the different items related to a boundary layer while List-II gives the mathematical expressions. Match List-I with List-II and select the correct answer using the codes given below the lists: (symbols have their usual meaning). [IES-1995]

List-I					List-II			
A.	Boundary layer thickness				1.	$y = \delta, u = 0.99 U_{\infty}$		
B.	Displacement thickness				2.	$\int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) dy$		
C.	Momentum thickness				3.	$\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u}{U_{\infty}}\right) dy$		
D.	Energy thickness				4.	$\int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u^2}{U_{\infty}^2}\right) dy$		
Code:	A	B	C	D	A	B	C	D
(a)	1	2	3	4	(b)	2	1	4
(c)	2	1	3	4	(d)	1	2	4

IES-24. Ans. (a)

### Momentum Equation for Boundary Layer by Von-karman

IES-25. According to Blasius law, the local skin friction coefficient in the boundary-layer over a flat plate is given by: [IES-2001, 1994]

(a)  $0.332/\sqrt{R_e}$  (b)  $0.664/\sqrt{R_e}$  (c)  $0.647/\sqrt{R_e}$  (d)  $1.328/\sqrt{R_e}$

IES-25. Ans. (b)

**IES-26. Match List-I (Variables in Laminar Boundary layer Flow over a Flat Plate Set Parallel to the Stream) with List-II (Related Expression with usual notations) and select the correct answer using the codes given below: [IES-2004; IAS-1999]**

**List-I**

- A. Boundary layer thickness
- B. Average skin-friction coefficient
- C. Shear stress at boundary
- D. Displacement thickness

**List-II**

- 1.  $1.729 / \sqrt{Ux / \nu}$
- 2.  $0.332 \rho U^2 / \sqrt{Ux / \nu}$
- 3.  $5\sqrt{\nu x / U}$
- 4.  $0.664 \sqrt{\nu / Ux}$
- 5.  $1.328 / \sqrt{UL / \nu}$

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	5	4	2	(b)	2	4	1	3
(c)	3	5	2	1	(d)	5	4	1	2

**IES-26. Ans. (c)**

**IES-27. The equation of the velocity distribution over a plate is given by  $u = 2y - y^2$  where  $u$  is the velocity in m/s at a point  $y$  meter from the plate measured perpendicularly. Assuming  $\mu = 8.60$  poise, the shear stress at a point 15 cm from the boundary is: [IES-2002]**

- (a) 1.72 N/m<sup>2</sup>      (b) 1.46 N/m<sup>2</sup>      (c) 14.62 N/m<sup>2</sup>      (d) 17.20 N/m<sup>2</sup>

**IES-27. Ans. (b)**

## Laminar Boundary Layer

**IES-28. The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]**

- (a)  $X$       (b)  $X^{\frac{1}{2}}$       (c)  $X^{\frac{1}{5}}$       (d)  $X^{\frac{4}{5}}$

**IES-28. Ans. (b)**  $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$  or  $\delta \propto \frac{5x}{\sqrt{\frac{\rho \nu x}{\mu}}}$  or  $\delta \propto \sqrt{x}$

**IES-29. For laminar flow over a flat plate, the thickness of the boundary layer at a distance from the leading edge is found to be 5 mm. The thickness of the boundary layer at a downstream section, which is at twice the distance of the previous section from the leading edge will be: [IES-1994]**

- (a) 10 mm      (b)  $5\sqrt{2}$  mm      (c)  $5\sqrt{2}$  mm      (d) 2.5 mm

**IES-29. Ans. (b)** Thickness of boundary layer for laminar flow over a flat plate is proportional to square root of ratio of distances from the leading edge. Thus new thickness =  $5 \times \sqrt{2}$  mm.

**IES-30. The laminar boundary layer thickness,  $\delta$  at any point  $x$  for flow over a flat plate is given by:  $\delta / x =$  [IES-2002]**

- (a)  $\frac{0.664}{\sqrt{Re_x}}$       (b)  $\frac{1.328}{\sqrt{Re_x}}$       (c)  $\frac{1.75}{\sqrt{Re_x}}$       (d)  $\frac{5.0}{\sqrt{Re_x}}$

**IES-30. Ans. (d)**

## Turbulent Boundary Layer

**IES-31. The velocity profile for turbulent layer over a flat plate is: [IES-2003]**

(a)  $\frac{u}{U} = \sin\left(\frac{\pi}{2} - \frac{y}{\delta}\right)$

(b)  $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

(c)  $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

(d)  $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

IES-31. Ans. (b)

IES-32. The velocity distribution in a turbulent boundary layer is given by  $u/U = (y/\delta)^{1/7}$ . [IES-2008]

What is the displacement thickness  $\delta^*$ ?

(a)  $\delta$

(b)  $\delta/7$

(c)  $(7/8)\delta$

(d)  $\delta/8$

IES-32. Ans. (d)  $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy = \left(y - \frac{7}{8} \frac{y^{8/7}}{\delta^{1/7}}\right) \Big|_0^\delta = \delta - \frac{7}{8}\delta = \frac{\delta}{8}$

IES-33. The thickness of turbulent boundary layer at a distance  $x$  from the leading edge over a flat plate varies as

[IAS-2003; 2004; 2007; IES-1996, 1997; 2000]

(a)  $x^{4/5}$

(b)  $x^{1/2}$

(c)  $x^{1/5}$

(d)  $x^{3/5}$

IES-33. Ans. (a)  $\frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}} \text{ or } \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5} \text{ or } \delta \propto x^{4/5}$

IES-34. For turbulent boundary layer low, the thickness of laminar sublayer ' $\delta$ ' is given by: [IES-1999]

(a)  $\nu / u^*$

(b)  $5 \nu / u^*$

(c)  $5.75 \log \nu / u^*$

(d)  $2300 \nu / u^*$

IES-34. Ans. (b)

IES-35. Assertion (A): The 'dimples' on a golf ball are intentionally provided.

Reason (R): A turbulent boundary layer, since it has more momentum than a laminar boundary layer, can better resist an adverse pressure gradient. [IES-2009]

(a) Both A and R are individually true and R is the correct explanation of A.

(b) Both A and R are individually true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

IES-35. Ans. (a)

## Boundary Layer Separation and Its Control

IES-36. The boundary layer flow separates from the surface if [IES-1995, 2002]

(a)  $du/dy = 0$  and  $dp/dx = 0$

(b)  $du/dy = 0$  and  $dp/dx > 0$

(c)  $du/dy = 0$  and  $dp/dx < 0$

(d) The boundary layer thickness is zero

IES-36. Ans. (b)

IES-37. In a boundary layer developed along the flow, the pressure decreases in the downstream direction. The boundary layer thickness would:

(a) Tend to decrease

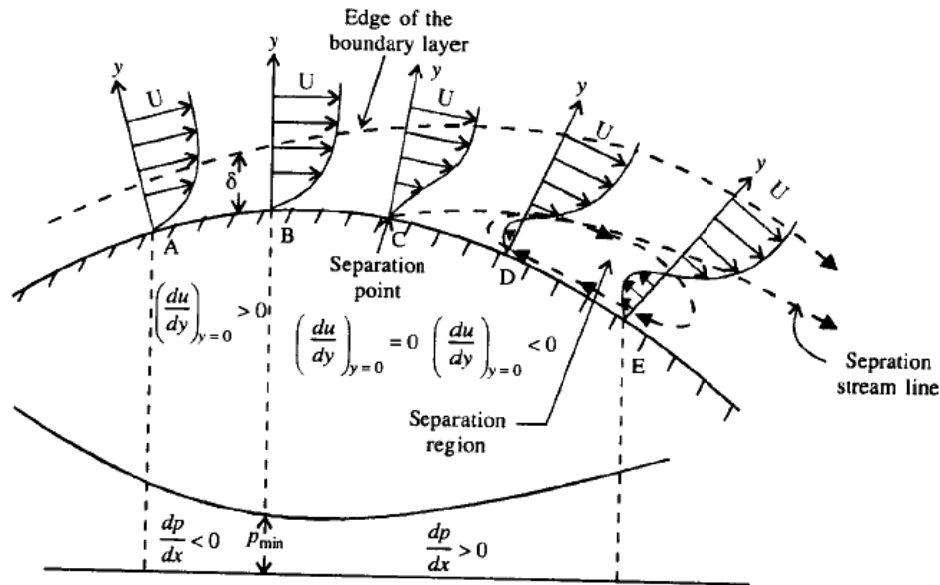
(b) Remain constant

[IES-1998]

(c) Increase rapidly

(d) Increase gradually

IES-37. Ans. (d) Consider point A to B where pressure decreases in the downstream direction but boundary layer thickness increases.



IES-38. Flow separation is caused by: [IAS-1996; IES-1994, 1997;2000; GATE-2002]

- (a) Reduction of pressure to local vapour pressure
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Thinning of boundary layer thickness to zero.

IES-38. Ans. (c) i.e. an adverse pressure gradient.

When the pressure goes increasing  $\left(\frac{\partial P}{\partial x} > 0\right)$  in the direction of flow, the pressure force acts against the direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickening the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as "Boundary Layer Separation".

IES-39. At the point of boundary layer separation [IES-1996]

- (a) Shear stress is maximum
- (b) Shear stress is zero
- (c) Velocity is negative
- (d) Density variation is maximum.

IES-39. Ans. (b) At the verge of separation  $\left(\frac{\partial u}{\partial y}\right)_{y=0}$  is zero

$\therefore$  Shear stress,  $\tau = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$  is also zero.

IES-40. Laminar sub-layer may develop during flow over a flat-plate. It exists in [IES-1992]

- (a) Laminar zone
- (b) Transition zone
- (c) Turbulent zone
- (d) Laminar and transition zone

IES-40. Ans. (c)

IES-41. Consider the following statements regarding laminar sublayer of boundary layer flow: [IES-2004]

1. The laminar sublayer exists only in a region that occurs before the formation of laminar boundary layer
2. The laminar sublayer is a region next to the wall where the viscous force is predominant while the rest of the flow is turbulent
3. The laminar sublayer occurs only in turbulent flow past a smooth plate

Which of the statements given above is/are correct?

- (a) 1, 2 and 3                      (b) 1 and 2                      (c) Only 2                      (d) 1 and 3

**IES-41. Ans. (c)**

**IES-42. Consider the following statements pertaining to boundary layer:**

1. Boundary layer is a thin layer adjacent to the boundary where maximum viscous energy dissipation takes place.
2. Boundary layer thickness is a thickness by which the ideal flow is shifted.
3. Separation of boundary layer is caused by presence of adverse pressure gradient.

Which of these statements are correct?

**[IES-2003]**

- (a) 1, 2 and 3                      (b) 1 and 2                      (c) 1 and 3                      (d) 2 and 3

**IES-42. Ans. (c)** 2 is wrong it defines displacement thickness.

Boundary layer thickness: The thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity reaches 99 percent of the velocity of the free stream. It is denoted by the symbol  $\delta$

Displacement thickness:  $\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$

It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.

or,

It is an additional "wall thickness" that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation

**IES-43. Drag on cylinders and spheres decreases when the Reynolds number is in the region of  $2 \times 10^5$  since** **[IES-1993]**

- (a) Flow separation occurs due to transition to turbulence
- (b) Flow separation is delayed due to onset of turbulence
- (c) Flow separation is advanced due to transition to turbulence
- (d) Flow reattachment occurs

**IES-43. Ans. (d)** In the region of  $2 \times 10^5$  (Reynolds number), the boundary layer on the cylinders and sphere begins to become unstable and thus boundary layer is said to reattach and the separation point moves back along the cylinder. Due to flow reattachment, a pressure recovery takes place over the back side and thus the drag force decreases.

**IES-44. During the flow over a circular cylinder, the drag coefficient drops significantly at a critical Reynolds number of  $2 \times 10^5$ . This is due to**

- (a) Excessive momentum loss in the boundary layer **[IES-1996]**
- (b) Separation point travelling upstream
- (c) Reduction in skin-friction drag
- (d) The delay in separation due to transition to turbulence

**IES-44. Ans. (d)** The drag co-efficient remains practically constant unit a Reynold's number of  $2 \times 10^5$  is reached. At this stage the  $C_d$  drops steeply by a factor of 5.

This is due to the fact that the laminar boundary layer turns turbulent and stays unseparated over a longer distance, then reducing the wake considerably.

**IES-45. Which one of the following is correct? [IES-2008]**

**For flow of an ideal fluid over a cylinder, from the front stagnation point,**

- (a) Pressure first decreases then increases
- (b) Velocity first decreases then increases
- (c) Pressure remains the same
- (d) Velocity remains the same

**IES-45. Ans. (a)** At the stagnation point pressure is maximum

∴ For flow past ideal fluid over a cylinder from front stagnation point pressure first decreases then increases.

**IES-46. What is the commonly used boundary layer control method to prevent separation? [IES-2009]**

- (a) Use of smooth boundaries
- (b) Using large divergence angle in the boundary
- (c) Suction of accelerating fluid within the boundary layer
- (d) Suction of retarded fluid within the boundary layer

**IES-46. Ans. (d)** Following are some of the methods generally adopted to retard separation:

1. Streamlining the body shape.
2. Tripping the boundary layer from laminar to turbulent by provision roughness.
3. Sucking the retarded flow.
4. Injecting high velocity fluid in the boundary layer.
5. Providing slots near the leading edge.
6. Guidance of flow in a confined passage.
7. Providing a rotating cylinder near the leading edge.
8. Energizing the flow by introducing optimum amount of swirl in the incoming flow.

## Thermal Boundary Layer

**IES-47. Which non-dimensional number relates the thermal boundary layer and hydrodynamic boundary layer? [IES-2008]**

- (a) Rayleigh number
- (b) Peclet number
- (c) Grashof number
- (d) Prandtl number

**IES-47. Ans. (d)** Prandtl number relates the thermal boundary layer and hydrodynamic boundary layer.

$$\frac{\delta}{\delta_t} = \frac{\text{Hydrodynamic boundary layer}}{\text{Thermal boundary layer}} = (\text{Pr})^{1/3}$$

**IES-48. Thermal boundary layer is a region where [IES-1993]**

- (a) Inertia terms are of the same order of magnitude as convection terms
- (b) Convection terms are of the same order of magnitude as dissipation terms
- (c) Convection terms are of the same order of magnitude as conduction terms
- (d) Dissipation is negligible

**IES-48. Ans. (b)**

**IES-49. The thickness of thermal and hydrodynamic boundary layer is equal if (Pr = Prandtl number, Nu = Nusselt number) [IES-1993]**

- (a)  $Pr = 1$                       (b)  $Pr > 1$                       (c)  $Pr < 1$                       (d)  $Pr = Nu$

**IES-49. Ans. (a)**  $\frac{\delta}{\delta_{th}} = (Pr)^{1/3}$  if  $Pr = 1$ ,  $\delta = \delta_{th}$

**IES-50. Hydrodynamic and thermal boundary layer thickness is equal for Prandtl number [IES-1992]**

- (a) Equal                      (b) Less than                      (c) Equal to 1                      (d) More than 1

**IES-50. Ans. (c)**  $\frac{\delta}{\delta_{th}} = (Pr)^{1/3}$  if  $Pr = 1$ ,  $\delta = \delta_{th}$

**IES-51. In a convective heat transfer situation Reynolds number is very large but the Prandtl number is so small that the product  $Re \times Pr$  is less than one in such a condition which one of the following is correct?**

- (a) Thermal boundary layer does not exist [IES-2004, 2007]  
 (b) Viscous boundary layer thickness is less than the thermal boundary layer thickness  
 (c) Viscous boundary layer thickness is equal to the thermal boundary layer thickness  
 (d) Viscous boundary layer thickness is greater than the thermal boundary layer thickness

**IES-51. Ans. (c)**  $\frac{\delta}{\delta_{th}} = (Pr)^{1/3}$  if  $Pr < 1$ ,  $\delta < \delta_{th}$

**IES-52. The ratio of the thickness of thermal boundary layer to the thickness of hydrodynamic boundary layer is equal to (Prandtl number)<sup>n</sup>, where n is: [IES-1994]**

- (a)  $-1/3$                       (b)  $-2/3$                       (c) 1                      (d)  $-1$

**IES-52. Ans. (a)**  $\frac{\text{Thickness of thermal boundary layer}}{\text{Thickness of hydrodynamic layers}} = (Prandtl Number)^{-1/3}$

**IES-53. For flow over a flat plate the hydrodynamic boundary layer thickness is 0.5 mm. The dynamic viscosity is  $25 \times 10^{-6}$  Pa s, specific heat is 2.0 kJ/(kgK) and thermal conductivity is 0.05 W/(m-K). The thermal boundary layer thickness would be: [IES-2001]**

- (a) 0.1 mm                      (b) 0.5 mm                      (c) 1 mm                      (d) 2 mm

**IES-53. Ans. (b)**

**IES-54. Prandtl number of a flowing fluid greater than unity indicates that hydrodynamic boundary layer thickness is: [IES-2002]**

- (a) Greater than thermal boundary layer thickness  
 (b) Equal to thermal boundary layer thickness  
 (c) Greater than hydrodynamic boundary layer thickness  
 (d) Independent of thermal boundary layer thickness

**IES-54. Ans. (a)**

IES-55. Consider the following conditions for heat transfer (thickness of thermal boundary layer is  $\delta_t$ , velocity of boundary layer is  $\delta$  and Prandtl number is  $P_r$ ): [IES-2000]

1.  $\delta_t(t) = \delta(x)$  if  $P_r = 1$       2.  $\delta_t(t) > \delta(x)$  if  $P_r < 1$       3.  $\delta_t(t) < \delta(x)$  if  $P_r > 1$

Which of these conditions apply for convective heat transfer?

- (a) 1 and 2      (b) 2 and 3      (c) 1 and 3      (d) 1, 2, and 3

IES-56. Ans. (d) We know that  $\frac{\delta}{\delta_{th}} = (P_r)^{1/3}$

## Previous 20-Years IAS Questions

### Boundary Layer Definitions and Characteristics

IAS-1. Velocity defect in boundary layer theory is defined as [IAS-2003]

- (a) The error in the measurement of velocity at any point in the boundary layer  
 (b) The difference between the velocity at a point within the boundary layer and the free stream velocity  
 (c) The difference between the velocity at any point within the boundary layer and the velocity nearer the boundary  
 (d) The ratio between the velocity at a point in the boundary layer and the free stream velocity

IAS-1. Ans. (b)

IAS-2. Assertion (A): The thickness of boundary layer cannot be exactly defined. [IAS-1996]

Reason (R): The Velocity within the boundary layer approaches the inviscid velocity asymptotically.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IAS-2. Ans. (a)

IAS-3. Velocity distribution in a turbulent boundary layer follows [IAS-2003]

- (a) Logarithmic law      (b) Parabolic law      (c) Linear law      (d) Cubic law

IAS-3. Ans. (a)

### Displacement Thickness ( $\delta^*$ )

IAS-4. How is the displacement thickness in boundary layer analysis defined?

- (a) The layer in which the loss of energy is maximum [IAS-2007]  
 (b) The thickness up to which the velocity approaches 99% of the free stream velocity.  
 (c) The distance measured perpendicular to the boundary by which the free stream is displaced on account of formation of boundary layer.  
 (d) The layer which represents reduction in momentum caused by the boundary layer.

IAS-4. Ans. (c)



IAS-5. The displacement thickness at a section, for an air stream ( $\rho = 1.2 \text{ kg/m}^3$ ) moving with a velocity of 10 m/s over flat plate is 0.5mm. What is the loss mass rate of flow of air due to boundary layer formation in kg per meter width of plate per second? [IAS-2004]

- (a)  $6 \times 10^{-3}$  (b)  $6 \times 10^{-5}$  (c)  $3 \times 10^{-3}$  (d)  $2 \times 10^{-3}$

IAS-5. Ans. (a)  $Q \text{ (loss per meter)} = \rho \times \delta^* \times \text{velocity} = 1.2 \times \left(\frac{0.5}{1000}\right) \times 10 \text{ kg/ms} = 6 \times 10^{-3} \text{ kg/ms}$

IAS-6. If the velocity distribution in a turbulent boundary layer is given by

$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$  then the ratio of displacement thickness to nominal layer thickness will be: [IAS-1998; IES-2006]

- (a) 1.0 (b) 0.6 (c) 0.3 (d) 0.1

IAS-6. Ans. (d) Displacement thickness ( $\delta^*$ ) =  $\delta \int_0^1 (1 - z^{1/9}) dz = 0.1\delta$

IAS-7. The velocity distribution in the boundary over the face of a high spillway found to have the following from  $\frac{u}{u_a} = \left(\frac{y}{\delta}\right)^{0.25}$ . At a certain section, the free stream velocity  $u_a$  was found to be 20m/s and the boundary layer thickness was estimated to be 5cm. The displacement thickness is: [IAS-1996]

- (a) 1.0 cm (b) 2.0 cm (c) 4.0 cm (d) 5.0 cm

IAS-7. Ans. (a) Displacement thickness ( $\delta^*$ ) =  $\delta \int_0^1 (1 - z^{0.25}) dz = 0.2\delta = 0.2 \times 5 = 1.0 \text{ cm}$

## Momentum Thickness ( $\theta$ )

IAS-8. If  $U_\infty$  = free stream velocity,  $u$  = velocity at  $y$  and  $\delta$  = boundary layer thickness, then in a boundary layer flow, the momentum thickness  $\theta$  is given by: [IES-1997; IAS-2004]

- (a)  $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy$  (b)  $\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$   
 (c)  $\theta = \int_0^\delta \frac{u^2}{U_\infty^2} \left(1 - \frac{u}{U_\infty}\right) dy$  (d)  $\theta = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$

IAS-8. Ans. (a)

## Energy Thickness ( $\delta_e$ )

IAS-9. Which one of the following is the correct relationship between the boundary layer thickness  $\delta$ , displacement thickness  $\delta^*$  and the momentum thickness  $\theta$ ? [IAS-2004; IES-1999]

- (a)  $\delta > \delta^* > \theta$  (b)  $\delta^* > \theta > \delta$  (c)  $\theta > \delta > \delta^*$  (d)  $\theta > \delta^* > \delta$

IAS-9. Ans. (a)  $\delta > \delta^* > \theta$

**Momentum Equation for Boundary Layer by Von-karman**

IAS-10. Match List-I (Variables in Laminar Boundary layer Flow over a Flat Plate Set Parallel to the Stream) with List-II (Related Expression with usual notations) and select the correct answer using the codes given below: [IES-2004; IAS-1999]

List-I					List-II				
A.	Boundary layer thickness				1.	$1.729 / \sqrt{Ux / \nu}$			
B.	Average skin-friction coefficient				2.	$0.332 \rho U^2 / \sqrt{Ux / \nu}$			
C.	Shear stress at boundary				3.	$5\sqrt{\nu x / U}$			
D.	Displacement thickness				4.	$0.664 \sqrt{\nu / Ux}$			
					5.	$1.328 / \sqrt{UL / \nu}$			
Codes:	A	B	C	D	A	B	C	D	
(a)	3	5	4	2	(b)	2	4	1	3
(c)	3	5	2	1	(d)	5	4	1	2

IAS-10. Ans. (c)

IAS-11. The equation of the velocity distribution over a plate is given by  $u = 2y - y^2$  where  $u$  is the velocity in m/s at a point  $y$  meter from the plate measured perpendicularly. Assuming  $\mu = 8.60$  poise, the shear stress at a point 15 cm from the boundary is: [IES-2002]

(a) 1.72 N/m<sup>2</sup>      (b) 1.46 N/m<sup>2</sup>      (c) 14.62 N/m<sup>2</sup>      (d) 17.20 N/m<sup>2</sup>

IAS-11. Ans. (b)

**Laminar Boundary Layer**

IAS-12. The thickness of laminar boundary layer at a distance 'x' from the leading edge over a flat varies as [IAS-1999, IES-1993, GATE-2002]

- (a)  $X$       (b)  $X^{\frac{1}{2}}$       (c)  $X^{\frac{1}{5}}$       (d)  $X^{\frac{4}{5}}$

IAS-12. Ans. (b)  $\frac{\delta}{x} = \frac{5}{\sqrt{\text{Re}_x}}$  or  $\delta \propto \frac{5x}{\sqrt{\frac{\rho \nu x}{\mu}}}$  or  $\delta \propto \sqrt{x}$

IAS-13. For laminar flow over a flat plate, the thickness of the boundary layer at a distance from the leading edge is found to be 5 mm. The thickness of the boundary layer at a downstream section, which is at twice the distance of the previous section from the leading edge will be: [IES-1994]

- (a) 10 mm      (b)  $5\sqrt{2}$  mm      (c)  $5\sqrt{2}$  mm      (d) 2.5 mm

IAS-13. Ans. (b) Thickness of boundary layer for laminar flow over a flat plate is proportional to square root of ratio of distances from the leading edge. Thus new thickness =  $5 \times \sqrt{2}$  mm.

**Turbulent Boundary Layer**

IAS-14. The thickness of turbulent boundary layer at a distance  $x$  from the leading edge over a flat plate varies as

[IAS-2003; 2004; 2007; IES-1996, 1997; 2000]

$$\text{IAS-14. Ans. (a) } \frac{\delta}{x} = \frac{0.371}{(\text{Re } x)^{1/5}} \text{ or, } \delta = \frac{0.371}{\left(\frac{\rho V x}{\mu}\right)^{1/5}} = \frac{0.371}{\left(\frac{\rho V}{\mu}\right)^{1/5}} \times x^{4/5} \text{ or, } \delta \propto x^{4/5}$$

**IAS-15. Consider the following statements comparing turbulent boundary layer with laminar boundary layer:** [IAS-1997]

1. Turbulent boundary layers are thicker than laminar boundary layer
2. Velocity in turbulent boundary layers is more uniform
3. In case of a laminar boundary layer, the thickness of the boundary layer increases more rapidly as the distance from the leading edge increases.
4. For the same local Reynolds number. Shear stress at the boundary is less in the case of turbulent boundary layer.

**Of these statements:**

- |                               |                         |
|-------------------------------|-------------------------|
| (a) 1, 2, 3 and 4 are correct | (b) 1 and 3 are correct |
| (c) 3 and 4 are correct       | (d) 1 and 2 are correct |

**IAS-15. Ans. (a)**

## Total Drag Due to Laminar and Turbulent Layers

**IAS-16. In a laminar boundary layer over a flat plate, what would be the ratio of wall shear stresses  $\tau_1$  and  $\tau_2$  at the two sections which lie at distances  $x_1=30$  cm and  $x_2=90$  cm from the leading edge of the plate?** [IAS-2004]

- |                                   |   |   |   |
|-----------------------------------|---|---|---|
| (a) $\frac{\tau_1}{\tau_2} = 3.0$ | (b) $\frac{\tau_1}{\tau_2} = \frac{1}{3}$ | (c) $\frac{\tau_1}{\tau_2} = (3.0)^{1/2}$ | (d) $\frac{\tau_1}{\tau_2} = (3.0)^{1/3}$ |
|-----------------------------------|---|---|---|

**IAS-16. Ans. (c)**  $\tau_o = 0.323 \frac{\mu u}{x} \times \sqrt{\text{Re } x}$  i.e.  $\tau_o \propto \frac{1}{\sqrt{x}}$

$$\therefore \frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}} = \sqrt{\frac{90}{30}} = (3)^{1/2}$$

**IAS-17. Match List-I (Device) with List-II (Use) and select the correct answer using the codes given below the Lists:** [IAS-2002]

**List-I**

- A. Pitot
- B. Preston tube
- C. Flow nozzle
- D. Hot wire anemometer

**Codes:    A        B        C        D**

- |     |   |   |   |   |
|-----|---|---|---|---|
| (a) | 4 | 2 | 3 | 1 |
| (c) | 4 | 1 | 3 | 2 |

**List-II**

1. Boundary shear stress
2. Turbulent velocity fluctuations
3. The total head
4. Flow rate

**A        B        C        D**

- |     |   |   |   |   |
|-----|---|---|---|---|
| (b) | 3 | 1 | 4 | 2 |
| (d) | 3 | 2 | 4 | 1 |

**IAS-17. Ans. (b)**

## Boundary Layer Separation and Its Control

**IAS-18. Assertion (A): In an ideal fluid, separation from a continuous surface would not occur with a positive pressure gradient.** [IAS-2000]

**Reason (R): Boundary layer does not exist in ideal fluid.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-18. Ans. (a)** In Ideal fluid viscosity is zero so no boundary layer is formed.

**IAS-19. Flow separation is caused by: [IAS-1996; IES-1994, 1997;2000; GATE-2002]**

- (a) Reduction of pressure to local vapour pressure
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Thinning of boundary layer thickness to zero.

**IAS-19. Ans. (c)** i.e. an adverse pressure gradient.

When the pressure goes increasing  $\left(\frac{\partial P}{\partial x} > 0\right)$  in the direction of flow, the pressure force acts against the direction of direction of flow thus retarding the flow. This has an effect of retarding the flow in the boundary layer and hence thickenings the boundary layer more rapidly. This and the boundary shear bring the fluid in the boundary layer to rest and causes back flow. Due to this the boundary layer no more sticks to the boundary but is shifted away from the boundary. This phenomenon is called as "Boundary Layer Separation".

**IAS-20. Flow separation is caused by**

**[IAS-2002]**

- (a) Thinning of boundary layer thickness to zero
- (b) A negative pressure gradient
- (c) A positive pressure gradient
- (d) Reduction of pressure to local vapour pressure

**IAS-20. Ans. (c)** Separation takes place where  $\frac{dp}{dx} > 0$  and  $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

**IAS-21. Boundary layer separation takes place when**

**[IAS-2007]**

- |  |  |
|--|--|
| (a) $\left(\frac{du}{dy}\right)_{y=0} = +ve \text{ value}$ | (b) $\left(\frac{du}{dy}\right)_{y=0} = -ve \text{ Value}$ |
| (c) $\left(\frac{du}{dy}\right)_{y=\delta} = 0$            | (d) $\left(\frac{du}{dy}\right)_{y=0} = 0$                 |

**IAS-21. Ans. (d)** But  $\frac{\partial p}{\partial x} > 0$

**IAS-22. Flow separation is likely to take place when the pressure gradient in the direction of flow is:**

**[IAS-1998]**

- |                        |                        |
|------------------------|------------------------|
| (a) Zero               | (b) Adverse            |
| (c) Slightly favorable | (d) Strongly favorable |

**IAS-22. Ans. (b)**

## Question from Conventional Paper

To solve the problems below use “Algorithm” from ‘Highlight’

1. Explain displacement and momentum boundary layer thickness. Assume that the shear stress varies linearly in a laminar boundary layer such that  $\tau = \tau_o \left[ 1 - \frac{y}{\delta} \right]$ . Calculate the displacement and momentum thickness in terms of  $\delta$ . [IES-1998]

2. Derive the integral momentum equation for the boundary layer over a flat plate and determine the boundary layer thickness  $\delta$ , at a distance  $x$  from the leading edge assuming linear velocity profile  $(u/U) = y/\delta$  where  $u$  is the velocity at the location at a distance  $y$  from the plate, and  $U$  is the free stream velocity. [IAS-1998]

3. When a fluid flows over a flat plate, the velocity profile within the boundary layer may be assumed to be  $V_x = U \left[ \frac{3}{2} \left( \frac{y}{\delta} \right) - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right]$  for  $y \leq \delta$ . [IES-1995]

Where  $U$  is a constant and the boundary layer thickness  $\delta$  is a function of  $x$  given by  $\delta = 5 \left( \frac{\mu x}{\rho U} \right)^{1/2}$ . Here  $\mu$  and  $\rho$  denote the viscosity and density of the fluid respectively. Derive an expression for the variation of  $V_y$  across the boundary layer. i.e. calculate displacement thickness.

4. The velocity profile for laminar flow in the boundary layer of a flat plate is given by  $\frac{u}{U} = \sin \left( \frac{\pi}{2} - \frac{y}{\delta} \right)$ . Where  $u$  is the velocity of fluid in the boundary layer at a vertical distance  $y$  from the plate surface and  $U$  is the free stream velocity. Prove that the boundary layer thickness  $\delta$  may be given by the expression,  $\delta = \frac{4.795x}{\sqrt{\text{Re}_x}}$  [IES-1992]

5. Explain briefly the Boundary Layer Theory as propounded by Prandtl. Obtain an expression for the thickness of the boundary layer for laminar flow assuming the velocity distribution law as  $\frac{u}{U} = 2 \left( \frac{y}{\delta} \right) - \left( \frac{y}{\delta} \right)^2$  [IAS-1990]

Where  $U$  = approach velocity of the stream,  $u$  = velocity of the stream in the boundary layer at a distance  $y$  from the boundary and  $\delta$  = thickness of the boundary layer.



## Laminar Flow

### Contents of this chapter

1. Relationship between Shear Stress and Pressure Gradient
2. Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law
3. Flow of Viscous Fluid between Two Parallel Plates

**Question:** *Show that the friction factor is inversely proportional to the Reynold's number in case of laminar flow in circle pipes.*

[IES-2003 AMIE (Summer)-1998]

**Answer:** Let,  $u$  = Velocity of flow at radial distance.  
 $\mu$  = Dynamic viscosity of flow fluid.

$\frac{\partial P}{\partial x}$  = Pressure gradient

$R$  = Radius of pipe

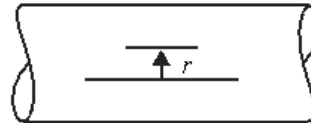
$\bar{U}$  = Average velocity

$$\therefore U_{\max} = -\frac{1}{4\mu} \frac{\partial P}{\partial x} \cdot (R^2 - r^2)$$

$$\bar{U} = \frac{U_{\max}}{2} = \frac{1}{2} \times \left( -\frac{1}{4\mu} \times \frac{\partial P}{\partial x} \times R^2 \right) = -\frac{1}{8\mu} \frac{\partial P}{\partial x} R^2$$

$$-\partial P = \frac{8\mu \bar{U}}{R^2} \partial x$$

Integrating both side we get



$$-\int_{P_1}^{P_2} dP = \int_{x_1}^{x_2} \frac{8\mu\bar{U}}{R^2} dx$$

$$\therefore (P_1 - P_2) = \frac{8\mu\bar{U}}{R^2} (x_2 - x_1) \quad \text{Let } x_2 - x_1 = L \text{ (Length of pipe)}$$

$$\therefore \Delta P = \frac{8\mu\bar{U}L}{R^2}$$

$$\therefore h_f \rho g = \frac{8\mu\bar{U}L}{R^2} \quad \text{where } (h_f) = \text{head loss}$$

$$\text{or, } h_f = \frac{8\mu\bar{U}L}{\rho g \times \left(\frac{D}{2}\right)^2} \quad \text{or, } h_f = \frac{32\mu\bar{U}L}{\rho g \times D^2}$$

$$\text{Comparing with } h_f = \frac{fLV^2}{2Dg} \text{ for } V = \bar{U} \text{ and } f = \text{friction factor}$$

$$\therefore \frac{fLV^2}{2Dg} = \frac{32\mu\bar{U}L}{\rho g \times D^2} \Rightarrow f = \frac{64\mu}{\rho V D} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64}{R_e}$$

$$\therefore f \propto \frac{1}{R_e}$$

i.e. friction factor is inversely proportional to Reynold's number.

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

**GATE-1. In flow through a pipe, the transition from laminar to turbulent flow does not depend on** [GATE-1996]

- (a) Velocity of the fluid (b) Density of the fluid  
(c) Diameter of the pipe (d) Length of the pipe

**GATE-1. Ans. (d)** It is totally depends on Reynolds number =  $\frac{\rho V D}{\mu}$

## Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law

**GATE-2. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by  $u=u_0(1-4r^2/D^2)$ , where  $r$  is the radial distance from the centre. If the viscosity of the fluid is  $\mu$ , the pressure drop across a length L of the pipe is:** [GATE-2006]

- (a)  $\frac{\mu u_0 L}{D^2}$  (b)  $\frac{4\mu u_0 L}{D^2}$  (c)  $\frac{8\mu u_0 L}{D^2}$  (d)  $\frac{16\mu u_0 L}{D^2}$

**GATE-2. Ans. (d)** By Hagen-Poiseuille law, for steady laminar flow in circular pipes

$$\tau = -\mu \frac{\partial u}{\partial r} \Rightarrow \tau = \frac{-\partial P}{\partial x} \cdot \frac{r}{2}$$

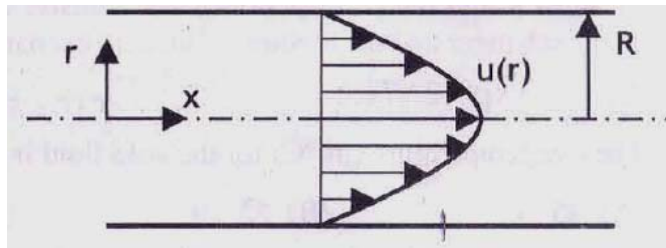
$$\mu \frac{\partial u}{\partial r} = \frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

$$\mu u_0 \left( \frac{-8r}{D^2} \right) = \frac{P}{L} \cdot \frac{r}{2} \quad \text{-----} \left[ \therefore u = u_0 \left( 1 - \frac{4r^2}{D^2} \right) \right]$$

$$P = \frac{-16\mu L u_0}{D^2} \quad [(-) \text{ sign is due to drop}]$$

**GATE-3. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression**

$$u(r) = -\frac{R^2}{4\mu} \left( \frac{dp}{dx} \right) \left( 1 - \frac{r^2}{R^2} \right)$$



[GATE-2009]

Where  $\frac{dp}{dx}$  is a constant. The average velocity of fluid in the pipe is:

- (a)  $-\frac{R^2}{8\mu} \left( \frac{dp}{dx} \right)$  (c)  $-\frac{R^2}{2\mu} \left( \frac{dp}{dx} \right)$  (b)  $-\frac{R^2}{4\mu} \left( \frac{dp}{dx} \right)$  (d)  $-\frac{R^2}{\mu} \left( \frac{dp}{dx} \right)$

**GATE-3. Ans. (a)**



**GATE-4.** A fully developed laminar viscous flow through a circular tube has the ratio of maximum velocity to average velocity as [IES-1994, GATE-1994]  
 (a) 3.0 (b) 2.5 (c) 2.0 (d) 1.5

**GATE-4. Ans. (c)** Ratio =  $\frac{\text{Maximum velocity}}{\text{Average velocity}}$  for fully developed laminar viscous flow through a circular tube has value of 2.0

**GATE-5.** For laminar flow through a long pipe, the pressure drop per unit length increases. [GATE-1996]

- (a) In linear proportion to the cross-sectional area
- (b) In proportion to the diameter of the pipe
- (c) In inverse proportion to the cross-sectional area
- (d) In inverse proportion to the square of cross-sectional area

**GATE-5. Ans. (d)**  $\frac{\Delta P}{L} = \frac{128\mu Q}{\pi D^4} \propto \frac{1}{D^4} \text{ i.e. } \propto \frac{1}{A^2}$

**GATE-6.** In fully developed laminar flow in a circular pipe, the head loss due to friction is directly proportional to..... (Mean velocity/square of the mean velocity). [GATE-1995]

- (a) True (b) False (c) Insufficient data (d) None of the above

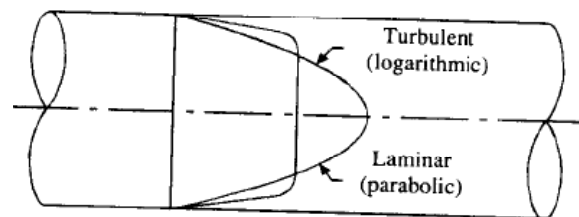
**GATE-6. Ans. (a)**  $h_f = \frac{32\mu u L}{\rho g D^2}$

## Previous 20-Years IES Questions

**IES-1.** Which one of the following statements is correct? [IES-1996]  
**Hydrodynamic entrance length for**

- (a) Laminar flow is greater than that for turbulent flow
- (b) Turbulent flow is greater than that for laminar flow
- (c) Laminar flow is equal to that for turbulent flow
- (d) A given flow can be determined only if the Prandtl number is known

**IES-1. Ans. (a)** Hydrodynamic entrance length for laminar flow is greater than that for turbulent flow.



**Fig. Velocity Distribution curves for laminar and turbulent flow**

**IES-2.** Which one of the following statements is correct for a fully developed pipe flow? [IES-2009]

- (a) Pressure gradient balances the wall shear stress only and has a constant value.
- (b) Pressure gradient is greater than the wall shear stress.
- (c) The velocity profile is changing continuously.
- (d) Inertia force balances the wall shear stress.

**IES-2. Ans. (a)** Relationship between shear stress and pressure gradient

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

This equation indicates that the pressure gradient in the direction of flow is equal to the shear gradient in the direction normal to the direction of flow. This equation holds good for all types of flow and all types of boundary geometry.

## Relationship between Shear Stress and Pressure Gradient

IES-3. Which one of the following is correct? [IES-2008]

In a fully developed region of the pipe flow,

- (a) The velocity profile continuously changes from linear to parabolic shape
- (b) The pressure gradient remains constant in the downstream direction
- (c) The pressure gradient continuously changes exceeding the wall shear stress in the downstream direction
- (d) The pipe is not running full

IES-3. Ans. (b) It can be said that in a fully developed flow, the pressure gradient balances the wall shear stress only and has a constant value at any section.

IES-4. In a steady flow of an oil in the fully developed laminar regime, the shear stress is: [IES-2003]

- (a) Constant across the pipe
- (b) Maximum at the centre and decreases parabolically towards the pipe wall boundary
- (c) Zero at the boundary and increases linearly towards the centre.
- (d) Zero at the centre and increases towards the pipe wall.

IES-4. Ans. (d)  $\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$

IES-5. A 40 mm diameter 2m long straight uniform pipe carries a steady flow of water (viscosity 1.02 centipoises) at the rate of 3.0 liters per minute. What is the approximate value of the shear stress on the internal wall of the pipe? [IES-2004]

- (a) 0.0166 dyne/cm<sup>2</sup>
- (b) 0.0812 dyne/cm<sup>2</sup>
- (c) 8.12 dyne/cm<sup>2</sup>
- (d) 0.9932 dyne/cm<sup>2</sup>

IES-5. Ans. (b)  $\Delta P = \frac{128\mu QL}{\pi D^4}$  or  $\frac{\Delta P}{\Delta x} = \frac{128\mu Q}{\pi D^4} = \frac{128 \times (1.02/100) \times (3000/60)}{\pi \times 4^4} = 0.0812$

$$\tau = -\frac{\Delta P}{\Delta x} \cdot \frac{R}{2} = -\frac{\Delta P}{\Delta x} \times \frac{2}{2} = 0.0812 \text{ dyne/cm}^2$$

IES-6. The pressure drop for a relatively low Reynolds number flow in a 600 mm, 30m long pipeline is 70 kPa. What is the wall shear stress? [IES-2004]

- (a) 0 Pa
- (b) 350 Pa
- (c) 700 Pa
- (d) 1400 Pa

IES-6. Ans. (b)  $-\frac{\partial p}{\partial x} = \frac{\Delta P}{L} = \frac{70 \times 10^3}{30} = 2333$ ;  $\tau_o = -\frac{\partial p}{\partial x} \cdot \frac{R}{2} = 2333 \times \frac{0.6}{4} = 350 \text{ Pa}$

- IES-7. The pressure drop in a 100 mm diameter horizontal pipe is 50 kPa over a length of 10 m. The shear stress at the pipe wall is: [IES-2001]  
 (a) 0.25 kPa (b) 0.125 kPa (c) 0.50 kPa (d) 25.0 kPa

IES-7. Ans. (b)  $\tau = -\frac{\partial p}{\partial x} \cdot \frac{R}{2} = -\frac{(-50 \times 10^3)}{10} \times \frac{0.100/2}{2} = 125 \text{ N/m}^2$

## Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law

- IES-8. Laminar developed flow at an average velocity of 5 m/s occurs in a pipe of 10 cm radius. The velocity at 5 cm radius is: [IES-2001]  
 (a) 7.5 m/s (b) 10 m/s (c) 2.5 m/s (d) 5 m/s

IES-8. Ans. (a) Velocity,  $u = u_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$  and  $\bar{u} = \frac{u_{\max}}{2}$

- IES-9. In a laminar flow through a pipe of diameter D, the total discharge Q, is expressed as ( $\mu$  is the dynamic viscosity of the fluid and  $\left( -\frac{dp}{dx} \right)$  is the pressure gradient). [IES-1994, 1995, 1996]

(a)  $\frac{-\pi D^4}{128\mu} \left( \frac{dp}{dx} \right)$  (b)  $\frac{-\pi D^4}{64\mu} \left( \frac{dp}{dx} \right)$  (c)  $\frac{-\pi D^4}{32\mu} \left( \frac{dp}{dx} \right)$  (d)  $\frac{-\pi D^4}{16\mu} \left( \frac{dp}{dx} \right)$

- IES-9. Ans. (a)

- IES-10. The power consumed per unit length in laminar flow for the same discharge, varies directly as  $D^n$  where D is the diameter of the pipe. What is the value of 'n'? [IES-2008]

(a)  $\frac{1}{2}$  (b)  $-1/2$  (c)  $-2$  (d)  $-4$

- IES-10. Ans. (d)

- IES-11. A fully developed laminar viscous flow through a circular tube has the ratio of maximum velocity to average velocity as [IES-1994, GATE-1994]  
 (a) 3.0 (b) 2.5 (c) 2.0 (d) 1.5

- IES-11. Ans. (c) Ratio =  $\frac{\text{Maximum velocity}}{\text{Average velocity}}$  for fully developed laminar viscous flow through a circular tube has value of 2.0

- IES-12. Velocity for flow through a pipe, measured at the centre is found to be 2 m/s. Reynolds number is around 800. What is the average velocity in the pipe? [IES-2007]

(a) 2 m/s (b) 1.7 m/s (c) 1 m/s (d) 0.5 m/s

- IES-12. Ans. (c)  $Re = 800$  i.e.  $< 2000$  so it is laminar flow and for laminar flow through pipe  $\frac{U_{\max}}{U_{\text{avg}}} = 2$  Or  $U_{\text{avg}} = \frac{U_{\max}}{2} = \frac{2}{2} = 1 \text{ m/s}$

IES-13. The frictional head loss through a straight pipe ( $h_f$ ) can be expressed as

$h_f = \frac{f l v^2}{2 g D}$  for both laminar and turbulent flows. For a laminar flow, 'f' is given by (Re is the Reynolds Number based on pipe diameter)

[IES-1993]

- (a)  $24/Re$  (b)  $32/Re$  (c)  $64/Re$  (d)  $128/Re$

IES-13. Ans. (c) Here 'f' is friction factor.

IES-14. A pipe friction test shows that, over the range of speeds used for the test, the non-dimensional friction factor 'f' varies inversely with Reynolds Number. From this, one can conclude that the [IES-1993]

- (a) Fluid must be compressible (b) Fluid must be ideal  
(c) Pipe must be smooth (d) Flow must be laminar

IES-14. Ans. (d) For the viscous flow the co-efficient of friction is given by,  $f = \frac{16}{Re}$

IES-15. A pipe of 20 cm diameter and 30 km length transports oil from a tanker to the shore with a velocity of 0.318 m/s. The flow is laminar. If  $\mu = 0.1$  N-m/s<sup>2</sup>, the power required for the flow would be: [IES-2000]

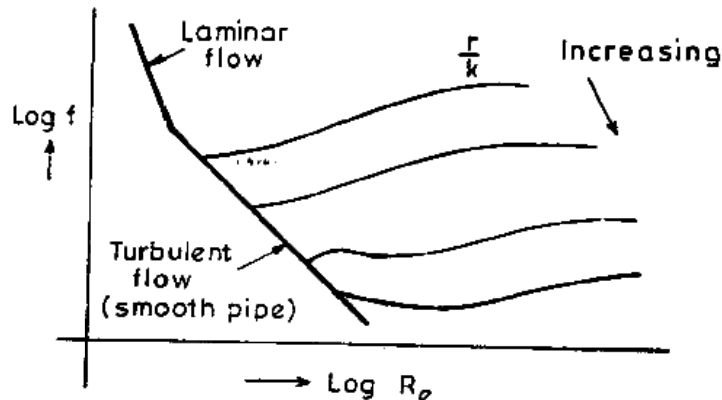
- (a) 9.25 kW (b) 8.36 kW (c) 7.63 kW (d) 10.13 kW

IES-15. Ans. (c)

IES-16. In a rough turbulent flow in a pipe, the friction factor would depend upon [IES-1993]

- (a) Velocity of flow (b) Pipe diameter  
(c) Type of fluid flowing (d) Pipe condition and pipe diameter

IES-16. Ans. (d) Fig. shows a plot of log (friction factor 'f') and log (Reynolds number 'Re'). It would be seen that for smooth turbulent flow, 'f' varies inversely as Re. But in case of rough pipes, behaviour changes depending on value of relative smoothness  $r/k$  (radius/average diameter of sand particles).



Thus friction factor 'f' for rough turbulent flow in a pipe depends upon pipe condition and pipe diameter. Friction factor for laminar flow  $f = \frac{64}{Re}$ , i.e. it is independent of the relative roughness of pipe.

However in the turbulent flow, the friction factor, as observed from several experiments, is a function of the relative roughness, i.e. the pipe condition and pipe diameter. Thus (d) is the correct choice.

## Flow of Viscous Fluid between Two Parallel Plates

IES-17. The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 10 cm apart and moving with relative velocity of 2 m/s is: [IES-2001]

- (a) 20 N/m<sup>2</sup> (b) 19.62 N/m<sup>2</sup> (c) 29.62 N/m<sup>2</sup> (d) 40 N/m<sup>2</sup>

IES-17. Ans. (b)  $\tau = \mu \frac{du}{dy} = \frac{9.81}{10} \times \frac{2}{0.1} = 19.62 \text{ N/m}^2$

## Previous 20-Years IAS Questions

IAS-1. The lower critical Reynold number for a pipe flow is: [IAS-1995]

- (a) Different for different fluids  
(b) The Reynolds number at which the laminar flow changes to turbulent flow  
(c) More than 2000  
(d) The least Reynolds number ever obtained for laminar flow

IAS-1. Ans. (a) The Reynolds number at which the turbulent flow changes to laminar flow is known as lower critical Reynolds number. The lower critical Reynolds number for a pipe flow is different for different fluids.

## Relationship between Shear Stress and Pressure Gradient

IAS-2. Which one of the following is the characteristic of a fully developed laminar flow? [IAS-2004]

- (a) The pressure drop in the flow direction is zero  
(b) The velocity profile changes uniformly in the flow direction  
(c) The velocity profile does not change in the flow direction  
(d) The Reynolds number for the flow is critical

IAS-2. Ans. (c)

IAS-3. The velocity distribution in laminar flow through a circular pipe follows the [IAS-1996]

- (a) Linear law (b) Parabolic (c) Cubic power law (d) Logarithmic law

IAS-3. Ans. (b) Velocity,  $u = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} (R^2 - r^2) = u_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$

IAS-4. For flow through a horizontal pipe, the pressure gradient  $dp/dx$  in the flow direction is: [IAS-1995]

- (a) +ve (b) 1 (c) Zero (d) -ve

IAS-4. Ans. (d) For flow through a horizontal pipe, the pressure gradient  $dp/dx$  in the flow direction is -ve.  $\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$

## Flow of Viscous Fluid in Circular Pipes-Hagen Poiseuille Law

IAS-5. What is the discharge for laminar flow through a pipe of diameter 40mm having center-line velocity of 1.5 m/s? [IAS-2004]

(a)  $\frac{3\pi}{59} \text{ m}^3/\text{s}$       (b)  $\frac{3\pi}{2500} \text{ m}^3/\text{s}$       (c)  $\frac{3\pi}{5000} \text{ m}^3/\text{s}$       (d)  $\frac{3\pi}{10000} \text{ m}^3/\text{s}$

**IAS-5. Ans. (d)** Centre-line velocity =  $U_{\max} = 1.5 \text{ m/s}$

Therefore average velocity ( $\bar{U}$ ) =  $\frac{U_{\max}}{2} = \frac{1.5}{2} \text{ m/s}$

Discharge (Q) = Area  $\times$  Area  $\times$  average velocity

$$= \frac{\pi}{4} \times \left( \frac{40}{1000} \right)^2 \times \frac{1.5}{2} \text{ m}^3/\text{s} = \frac{3\pi}{10,000} \text{ m}^3/\text{s}$$

**IAS-6. The MINIMUM value of friction factor 'f' that can occur in laminar flow through a circular pipe is:** **[IAS-1997]**

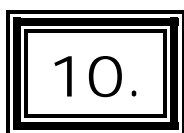
- (a) 0.064                      (b) 0.032                      (c) 0.016                      (d) 0.008

**IAS-6. Ans. (b)** Friction Factor,  $f = \frac{64}{\text{Re}}$  Where Max. Re = 2000.

**IAS-7. The drag coefficient for laminar flow varies with Reynolds number (Re) as** **[IAS-2003]**

- (a)  $\text{Re}^{1/2}$                       (b) Re                      (c)  $\text{Re}^{-1}$                       (d)  $\text{Re}^{-1/2}$

**IAS-7. Ans. (c)** For the viscous flow the co-efficient of friction is given by,  $f = \frac{16}{\text{Re}}$



# Turbulent Flow in Pipes

## Contents of this chapter

1. Characteristics of Turbulent Flow
2. Shear Stresses in Turbulent Flow
3. Prandtl's Mixing Length Theory
4. Resistance to Flow of Fluid in Smooth and Rough Pipes

**Question:** Compare the velocity profiles for laminar flow and turbulent flow in pipe and comment on them. [IES-2003]

**Answer:** Velocity distribution in laminar pipe flow

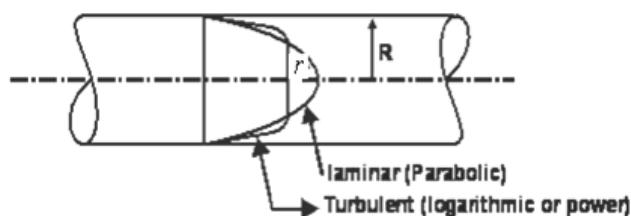
$$u = u_{\max} \left\{ 1 - \left( \frac{r}{R} \right)^2 \right\}$$

Where  $u$  = velocity at  $r$  from center.

$U_{\max}$  = maximum velocity at center line.

$R$  = Radius of pipe.

$r$  = Distance from center.



**Fig. Velocity distribution curves for laminar and turbulent flows in a pipe**

Velocity distribution in turbulent pipe flow,  $u = u_{\max} + 2.5 u_f \log_e \left( \frac{y}{R} \right)$

Where  $u$  = velocity at  $y$  from boundary.

$U_{\max}$  = maximum velocity.

$u_f$  = Shear velocity  $\sqrt{\tau_0/\rho}$

$y$  = distance from boundary

## Comments

- (1) The velocity distribution in turbulent flow is more uniform than in laminar flow.
- (2) In turbulent flow the velocity gradient near the boundary shall be quite large resulting in more shears.
- (3) In turbulent flow the flatness of velocity distribution curve in the core region away from the wall is because of the mixing of fluid layers and exchange of momentum between them.
- (4) The velocity distribution which is paraboloid in laminar flow tends to follow power law and logarithmic law in turbulent flow.

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

## Shear Stresses in Turbulent Flow

GATE-1. As the transition from laminar to turbulent flow is induced a cross flow past a circular cylinder, the value of the drag coefficient drops.

[GATE-1994]

(a) True (b) False (c) May true may false (d) None  
 GATE-1. Ans. (b) For turbulent flow total shear stress ( $\tau$ ) =

$$\tau_{laminar} + \tau_{turbulence} = \mu \frac{du}{dy} + \eta \frac{du}{dy}$$

## Prandtl's Mixing Length Theory

GATE-2. Prandtl's mixing length in turbulent flow signifies [GATE-1994]

- (a) The average distance perpendicular to the mean flow covered by the mixing particles.
- (b) The ratio of mean free path to characteristic length of the flow field.
- (c) The wavelength corresponding to the lowest frequency present in the flow field.
- (d) The magnitude of turbulent kinetic energy.

GATE-2. Ans. (a)

## Previous 20-Years IES Questions

## Characteristics of Turbulent Flow

IES-1. In a turbulent flow,  $\bar{u}$ ,  $\bar{v}$  and  $\bar{w}$  are time average velocity components. The fluctuating components are  $u'$ ,  $v'$  and  $w'$  respectively. The turbulence is said to be isotropic if: [IES-1997]

- (a)  $\bar{u} = \bar{v} = \bar{w}$  (b)  $\bar{u} + u' = \bar{v} + v' = \bar{w} + w'$
- (c)  $(\bar{u})^2 = (\bar{v})^2 = (\bar{w})^2$  (d) None of the above situations prevails

IES-1. Ans. (d) **Isotropic Turbulence:** Turbulence whose properties, especially statistical correlations, do not depend on direction.

Strictly speaking, isotropy, i.e. independence of orientation, implies homogeneity, i.e. independence of position in space. In most situations, all the averaged properties of isotropic turbulence can also be assumed to be invariant under reflection in space.

For Isotropic turbulence  $\overline{(u')^2} = \overline{(v')^2} = \overline{(w')^2}$  and  $\overline{(u_i' u_j')} = 0$  Where  $(i \neq j)$

IES-2. When we consider the momentum exchange between two adjacent layers in a turbulent flow, can it be postulated that if at an instant there is an increase in  $u'$  in the  $x$ -direction it will be followed by a change in  $v'$  in the  $y$  direction? [IES-1994]



- (a) Yes, in such a manner that  $\overline{u'v'} = 0$
- (b) Yes, in such a manner that  $\overline{u'v'} = \text{non-zero and positive.}$
- (c) Yes, in such a manner that  $\overline{u'v'} = \text{non-zero and negative.}$
- (d) No, as  $u'$  and  $v'$  are not dependent on each other.

**IES-2. Ans. (c)**  $\frac{\partial u'}{\partial x} = -\frac{\partial v'}{\partial y}$

It is postulated that if at an instant there is an increase in  $u'$  in the  $x$ -direction, it will be followed by an increase in  $v'$  in the negative  $y$ -direction. In other words,  $\overline{u'v'}$  is **non-zero and negative**.

**IES-3. If**  $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$  **for a turbulent flow then it signifies that** **[IES-1996]**

- (a) Bulk momentum transport is conserved
- (b)  $u'v'$  is non-zero and positive.
- (c) Turbulence is anisotropic
- (d) None of the above is true.

**IES-3. Ans. (a)** As it follow conserve continuity equations.

$$\frac{\partial u'}{\partial x} = -\frac{\partial v'}{\partial y}$$

It is postulated that if at an instant there is an increase in  $u'$  in the  $x$ -direction, it will be followed by an increase in  $v'$  in the negative  $y$ -direction. In other words,  $\overline{u'v'}$  is **non-zero and negative**.

**IES-4. In fully-developed turbulent pipe flow, assuming 1/7th power law, the ratio of time mean velocity at the centre of the pipe to that average velocity of the flow is:** **[IES-2001]**

- (a) 2.0
- (b) 1.5
- (c) 1.22
- (d) 0.817

**IES-4. Ans. (d)**  $U_{avg} = \frac{1}{A} \int u dA = \frac{1}{\pi R^2} \int_0^R u_{max} \left(\frac{r}{R}\right)^{1/7} 2\pi r dr = \frac{14}{15} u_{max}$  or without calculating

this we may say that it must be less than one and option (d) is only choice.

## Shear Stresses in Turbulent Flow

**IES-5. Reynolds stress may be defined as the** **[IES-1994]**

- (a) Stresses (normal and tangential) due to viscosity of the fluid.
- (b) Additional normal stresses due to fluctuating velocity components in a turbulent flow.
- (c) Additional shear stresses due to fluctuating velocity components in a turbulent flow.
- (d) Additional normal and shear stresses due to fluctuating velocity components in the flow field.

**IES-5. Ans. (c)** Reynolds stress may be defined as additional shear stresses due to fluctuating velocity components in a turbulent flow.

**IES-6. In turbulent flow over an impervious solid wall** **[IES-1993]**

- (a) Viscous stress is zero at the wall
- (b) Viscous stress is of the same order of magnitude as the Reynolds stress
- (c) The Reynolds stress is zero at the wall
- (d) Viscous stress is much smaller than Reynolds stress

**IES-6. Ans. (d)**

**IES-7. Shear stress in a turbulent flow is due to:** **[IES-1997]**

- (a) The viscous property of the fluid.
- (b) The fluid
- (c) Fluctuation of velocity in the direction of flow
- (d) Fluctuation of velocity in the direction of flow as well as transverse to it

**IES-7. Ans. (d)**

**IES-8. The pressure drop in a 100 mm diameter horizontal pipe is 50 kPa over a length of 10m. The shear stress at the pipe wall is: [IES-2001]**

- (a) 0.25 kPa
- (b) 0.125 kPa
- (c) 0.50 kPa
- (d) 25.0 kPa

**IES-8. Ans. (b)**  $(p_1 - p_2) \frac{\pi D^2}{4} = \tau_o \pi D L$  or,  $\tau_o = \frac{\Delta P \times D}{4L}$

## Prandtl's Mixing Length Theory

**IES-9. In a turbulent flow, 'l' is the Prandtl's mixing length and  $\frac{\partial \bar{u}}{\partial y}$  is the gradient of the average velocity in the direction normal to flow. The final expression for the turbulent viscosity  $\nu_t$  is given by:**

**[IES-1994, 1997]**

- (a)  $\nu_t = l \left( \frac{\partial \bar{u}}{\partial y} \right)$
- (b)  $\nu_t = l^2 \left( \frac{\partial \bar{u}}{\partial y} \right)$
- (c)  $\nu_t = l^2 \left( \frac{\partial \bar{u}}{\partial y} \right)^2$
- (d)  $\nu_t = l \left| \frac{\partial \bar{u}}{\partial y} \right|$

**IES-9. Ans. (b)**  $\tau = \rho l^2 \left( \frac{d\bar{u}}{dy} \right)^2$  and  $\tau = \eta \frac{d\bar{u}}{dy}$  or  $\nu_t = \frac{\eta}{\rho} = l^2 \left| \left( \frac{d\bar{u}}{dy} \right) \right|$

**IES-10. The universal velocity distribution for turbulent flow in a channel is given by ( $u^*$  is the friction velocity and  $\eta$  is given by  $yu^*/\nu$ . The kinematic viscosity is  $\nu$  and  $y$  is the distance from the wall). [IES-1994]**

- (a)  $\frac{u}{u^*} = 2.5 \ln \eta + 5.5$
- (b)  $\frac{u}{u^*} = \eta$
- (c)  $\frac{u}{u_{\max}} = 5.75 \ln \eta + 5.5$
- (d)  $\frac{d}{u} = 5.75 \ln \eta + 5.5$

**IES-10. Ans. (a)** Universal velocity distribution equation

$$u = u_{\max} + 2.5u_f \log_e \left( \frac{y}{R} \right) \text{ i.e. } u = u_{\max} + 5.75u_f \log_{10} \left( \frac{y}{R} \right) \quad \text{[VIMP]}$$

Where Shear Velocity ( $u_f$ ) =  $\sqrt{\frac{\tau_o}{\rho}}$

- (i) Based on Nikuradse's and Reichardt's experimental data, the empirical constants can be determined for a smooth pipe as

$$\frac{u}{u_f} = 2.5 \log_e \left( \frac{y}{R} \right) + 5.5$$

- (ii) In rough pipes experiments indicate that the velocity profile may be expressed as:

$$\frac{u}{u_f} = 2.5 \log_e \left( \frac{y}{R} \right) + 8.5$$

## Resistance to Flow of Fluid in Smooth and Rough Pipes

**IES-11. Flow takes place and Reynolds Number of 1500 in two different pipes with relative roughness of 0.001 and 0.002. The friction factor [IES-2000]**

- (a) Will be higher in the case of pipe with relative roughness of 0.001.
- (b) Will be higher in the case of pipe having relative roughness of 0.002.
- (c) Will be the same in both the pipes.
- (d) In the two pipes cannot be compared on the basis of data given

**IES-11. Ans. (c)** The flow is laminar (friction factor,  $f = \frac{64}{Re}$ ) it is not depends on roughness but for turbulent flow it will be higher for higher relative roughness.

**IES-12. In a fully turbulent flow through a rough pipe, the friction factor 'f' is (Re is the Reynolds number and  $\xi_s / D$  is relative roughness)**

**[IES-1998; IES-2003]**

- (a) A function of Re
- (b) A function of Re and  $\xi_s / D$
- (c) A function of  $\xi_s / D$
- (d) Independent of Re and  $\xi_s / D$

**IES-12. Ans. (b)**  $\frac{1}{\sqrt{4f}} = 2 \log_{10}(R/K) + 1.74$ ; f is independent of Reynolds number and

depends only on relative roughness (k/D). This formula is widely used. But experimental facts reveals that the friction factor 'f' is also depends on Reynolds number. As many modern handbook uses empirical formula with Reynold's number our answer will be (b).

**IES-13. Consider the following statements:** **[IES-2008]**

1. The friction factor in laminar flow through pipes is independent of roughness.
2. The friction factor for laminar flow through pipes is directly proportional to Reynolds number.
3. In fully turbulent flow, through pipes, friction factor is independent of Reynolds number.

**Which of the statements given above are correct?**

- (a) 1, 2 and 3
- (b) 1 and 3 only
- (c) 2 and 3 only
- (d) 1 and 2 only

**IES-13. Ans. (b)** The friction factor in laminar flow through pipes is given by the expression  $\frac{64}{Re}$  which is independent of roughness.

The friction factor for laminar flow through pipes is inversely proportional to Reynolds number.

In fully turbulent flow through pipes friction factor is independent of Reynolds number.

## Previous 20-Years IAS Questions

### Characteristics of Turbulent Flow

**IAS-1. While water passes through a given pipe at mean velocity V the flow is found to change from laminar to turbulent. If another fluid of specific gravity 0.8 and coefficient of viscosity 20% of that of water, is passed**

through the same pipe, the transition of flow from laminar to turbulent is expected if the flow velocity is: [IAS-1998]

- (a)  $2V$  (b)  $V$  (c)  $V/2$  (d)  $V/4$

IAS-1. Ans. (d)  $Re_w = \frac{\rho_w V_w D_w}{\mu_w} = \frac{0.8 \rho_f \times V_f \times D_f}{0.2 \mu_t} = 4R_{fw}$

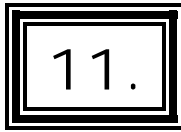
$$V_f = \frac{V_w}{4} = \frac{V}{4}$$

## Shear Stresses in Turbulent Flow

IAS-2. The shear stress in turbulent flow is: [IAS-1994]

- (a) Linearly proportional to the velocity gradient
- (b) Proportional to the square of the velocity gradient
- (c) Dependent on the mean velocity of flow
- (d) Due to the exchange of energy between the molecules

IAS-2. Ans. (b)  $f = \frac{2\tau_o}{\rho V^2}$  or  $\tau_o = \frac{f\rho V^2}{2}$



# Flow Through Pipes

## Contents of this chapter

1. Loss of Energy (or Head) in Pipes
2. Darcy-Weisbach Formula
3. Chezy's Formula for Loss of Head Due to Friction
4. Minor Energy Losses
5. Loss of Head Due to Sudden Enlargement
6. Loss of Head Due to Sudden Contraction
7. Loss of Head Due to Obstruction in Pipe
8. Loss of Head at the Entrance to Pipe
9. Loss of Head at the Exit of a Pipe
10. Loss of Head Due to Bend in the Pipe
11. Loss of Head in Various Pipe Fittings
12. Hydraulic Gradient and Total Energy Lines
13. Pipes in Series or Compound Pipes
14. Equivalent Pipe
15. Pipes in Parallel
16. Syphon
17. Power Transmission through Pipes
18. Diameter of the Nozzle for Transmitting Maximum Power
19. Water Hammer in Pipes
20. Gradual Closure of Valve
21. Instantaneous Closure of Valve, in Rigid Pipes
22. Instantaneous Closure of Valve in Elastic Pipes
23. Time Required by Pressure Wave to Travel from the Valve to the Tank and from Tank to Valve

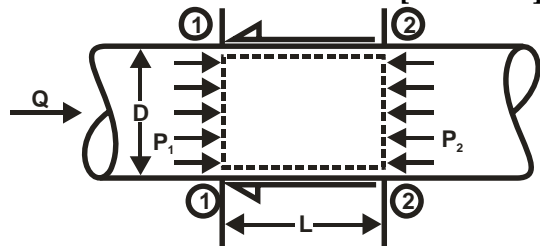
**Question:** (i) *Derive Darcy-Weisbach equation for head loss in pipe due to friction.* [IES-2001]

or

(ii) *Show that the loss of head due to friction in pipe  $(h_f) = \frac{4fLV^2}{2gD}$*  [IES-1999]

**Answer:** When a fluid flows steadily through a pipe of constant diameter, the average velocity at each cross section remains the same. This is necessary from the condition of continuity since the velocity  $V$  is given by,  

$$V = \frac{Q}{A}.$$



The static pressure  $P$  drops along the direction of flow because the stagnation pressure drops due to loss of energy in over coming friction as the flow occurs.

Let,  $P_1$  = intensity of pr. at section 1

$P_2$  = intensity of pr. at section 2

$L$  = length of the pipe, between section 1 and 2.

$D$  = Diameter of the pipe

$C_d$  = co-efficient of drag.

$f$  = co-efficient of friction (whose value depends on type of flow, material of pipe and surface of pipe)

$h_f$  = loss of head due to friction.

Propelling pressure force on the flowing fluid along the flow =  $(P_1 - P_2) \frac{\pi D^2}{4}$

Frictional resistance force due to shearing at the pipe wall =  $C_d \cdot \frac{1}{2} \rho V^2 \cdot \pi DL$

Under equilibrium condition,

Propelling force = frictional resistance force

$$(P_1 - P_2) \frac{\pi D^2}{4} = C_d \cdot \frac{1}{2} \rho V^2 \cdot \pi DL$$

$$\text{or } \frac{P_1 - P_2}{\rho g} = \frac{4 C_d L V^2}{D \cdot 2g}$$

Noting  $\frac{P_1 - P_2}{\rho g}$  is the head loss due to friction,  $h_f$  and  $C_d$  equal the co-efficient of friction.

$$h_f = \frac{4 f L V^2}{D \cdot 2g}$$

This is known as Darcy-Weisbach equation and it holds good for all type of flows provided a proper value of  $f$  is chosen.

**Question:** *Develop an expression for the pressure loss across a conical contraction of diameter  $D_1$  and  $D_2$  in a given length  $L$  for a laminar flow. Explain the physical significant of equation.* [IES-2004]

**Answer:** Let us consider a small section from 'x' apex of the cone and thickness 'dx' therefore. Velocity at that section.

If flow rate is  $Q$

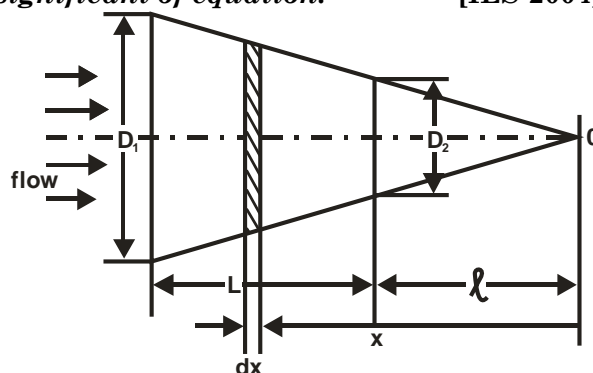
$$\text{then } V = \frac{Q}{\frac{\pi}{4} d^2} \quad [\text{where}]$$

$d$  = diameter At section]

$$\text{and } \frac{D_1}{L + \ell} = \frac{D_2}{\ell} = \frac{d}{x}$$

$$\text{or } \frac{D_1 - D_2}{L} = \frac{d}{x}$$

$$\therefore d = \frac{x}{L} (D_1 - D_2)$$



$$\therefore V = \frac{Q}{\frac{\pi}{4} \times \frac{x^2}{L^2} (D_1 - D_2)^2} = \frac{4QL^2}{\pi(D_1 - D_2)^2 x^2}$$

The Darcy-weisbach equation in differential form can be written as:

$$dh_f = \frac{4f \cdot V^2 \cdot dx}{d \times 2g}$$

$$\therefore dh_f = \frac{4f}{2g} \times \frac{4^2 Q^2 L^4 \times L}{\pi^2 (D_1 - D_2)^2 x^4 \times x (D_1 - D_2)} dx$$

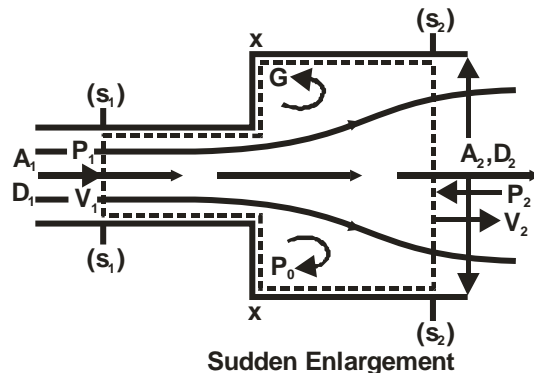
$\therefore$  Frictional loss

$$\begin{aligned} h_f &= \int dh_f = \frac{4f}{2g} \int_{\ell}^{L+\ell} \frac{16Q^2 L^5}{\pi^2 (D_1 - D_2)^5} \times \frac{1}{x^5} dx = \frac{32fQ^2 L^5}{g\pi^2 (D_1 - D_2)^5} \times \left[ -\frac{1}{4x^4} \right]_{\ell}^{L+\ell} \\ &= \frac{8fQ^2 L^5}{g\pi^2 (D_1 - D_2)^5} \left[ \frac{1}{\ell^4} - \frac{1}{(L+\ell)^4} \right] \quad D_2 = \frac{\ell}{L} (D_1 - D_2), D_1 = \frac{L+\ell}{L} (D_1 - D_2) \\ &= \frac{8fQ^2 L^5}{g\pi^2 (D_1 - D_2)} \times \left[ \frac{1}{\left\{ \frac{\ell}{L} (D_1 - D_2) \right\}^4} - \frac{1}{\left\{ \frac{L+\ell}{L} (D_1 - D_2) \right\}^4} \right] \\ &= \frac{8fQ^2 L}{g\pi^2 (D_1 - D_2)} \times \left[ \frac{1}{D_2^4} - \frac{1}{D_1^4} \right] \text{ required expression.} \end{aligned}$$

**Question:** Starting the assumption, deduce an expression for head loss due to sudden expansion of streamline in a pipe. [AMIE (Winter)-1999, 2001]

**Answer:** Fig. shows a liquid flowing through a pipe which has enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy.

Consider two section  $S_1 - S_1$  (before enlargement) and  $S_2 - S_2$  (after enlargement) and taking a fluid control volume  $S_1 S_1 S_2 S_2$



Let,  $A_1$  = area of pipe at section, is equal to  $\frac{\pi D_1^2}{4}$

$P_1$  = intensity of pr. at section  $S_1 - S_1$

$V_1$  = velocity of flow at section  $S_1 - S_1$

and  $A_2$ ,  $P_2$  and  $V_2$  are corresponding values off section  $S_2 - S_2$

$P_0$  = intensity of pr. of liquid eddies on the area  $(A_2 - A_1)$

$h_e$  = head loss due to sudden enlargement.

Applying the continuity equation

$$A_1 V_1 = A_2 V_2 = Q$$

Applying integral momentum equation

$$P_1 A_1 + P_0 (A_2 - A_1) - P_2 A_2 = \rho Q (V_2 - V_1)$$

Assuming that  $P_0$  equals  $P_1$  because there cannot be an abrupt change of pressure across the same section x - x.

$$\therefore (P_1 - P_2) A_2 = \rho Q (V_2 - V_1)$$

$$\text{or } \frac{(P_1 - P_2)}{\rho g} = \frac{1}{g} V_2 (V_2 - V_1) \quad \dots(i) [\because Q = A_2 V_2]$$

Applying Bernoulli's to section  $S_1 - S_1$  and  $S_2 - S_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$\text{or } h_e = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} \quad [\because Z_1 = Z_2 \text{ pipe being horizontal}]$$

$$= \frac{1}{g} [V_2 (V_2 - V_1)] + \frac{V_1^2 - V_2^2}{2g} \quad [\text{from equation (i)}]$$

$$= \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore h_e = \frac{(V_1 - V_2)^2}{2g}$$



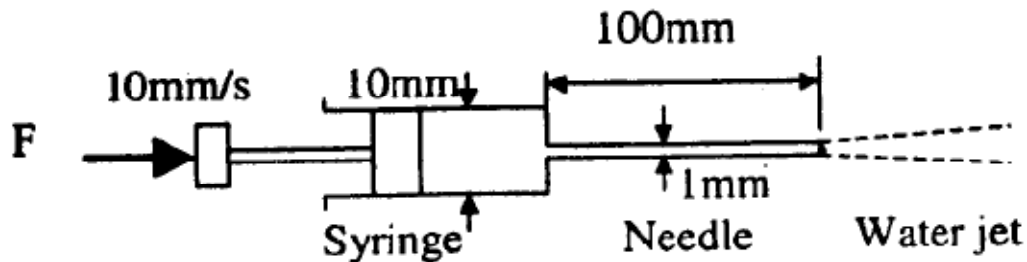
## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

## Darcy-Weisbach Formula

Data for Q1-Q2 are given below. Solve the problems and choose correct answers.

A syringe with a frictionless plunger contains water and has at its end a 100 mm long needle of 1 mm diameter. The internal diameter of the syringe is 10 mm. Water density is  $1000 \text{ kg/m}^3$ . The plunger is pushed in at  $10 \text{ mm/s}$  and the water comes out as a jet.



GATE-1. Assuming ideal flow, the force  $F$  in Newton required on the plunger to push out the water is: [GATE-2003]

- (a) 0 (b) 0.04 (c) 0.13 (d) 1.15

GATE-1. Ans. (b)

GATE-2. Neglect losses in the cylinder and assume fully developed laminar viscous flow throughout the needle; the Darcy friction factor is  $64/\text{Re}$ . Where  $\text{Re}$  is the Reynolds number. Given that the viscosity of water is  $1.0 \times 10^{-3} \text{ kg/s m}$ , the force  $F$  in Newton required on the plunger is:

[GATE-2003]

- (a) 0.13 (b) 0.16 (c) 0.3 (d) 4.4

GATE-2. Ans. (c) Given,  $\nu$  = viscosity of water =  $10 \times 10^{-3} \text{ kg/sm}$

$$\text{Now, } \text{Re} = \frac{\rho v_2 d}{\nu} = \frac{1000 \times 1 \times 0.001}{1 \times 10^{-3}} = \text{Re} = 1000 \text{ ---- since } v_2 = 1$$

$$\text{Darcy's friction factor, } 4f = \frac{64}{\text{Re}} = \frac{64}{1000} = 0.064$$

$$\text{So head loss in needle} = h_f = \frac{f v_2^2}{2gD} = \frac{0.064 \times 0.1 \times (1)^2}{2 \times 9.8 \times 0.001} = 0.3265 \text{ m}$$

Applying Bernoulli's equation at points 1 and 2, we have

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$\therefore \frac{P_1}{\rho g} = \frac{v_2^2 - v_1^2}{2g} + h_L \quad [\text{Since } z_1 = z_2 \text{ and } P_2 = 0]$$

$$P_1 = \frac{\rho}{2} (v_2^2 - v_1^2) + \rho g h_L = \frac{1000}{2} [(1)^2 - (0.01)^2] + 1000 \times 9.8 \times 0.3265$$

Now force required on plunger =  $P_1 \times A_1 = 3699.65 \times \frac{\pi}{4} \times (0.01)^2 = 0.3\text{N}$

**GATE-3.** Fluid is flowing with an average velocity of  $V$  through a pipe of diameter  $d$ . Over a length of  $L$ , the “head” loss is given by  $h_f = \frac{fLV^2}{2g \times D}$ .

The friction factor,  $f$ , for laminar flow in terms of Reynolds number (Re) is..... [GATE-1994]

**GATE-3. Ans.**  $\frac{64}{\text{Re}}$

**GATE-4.** Water flows through a 0.6 m diameter, 1000 m long pipe from a 30 m overhead tank to a village. Find the discharge (in liters) at the village (at ground level), assuming a Fanning friction factor  $f = 0.04$  and ignoring minor losses due to bends etc. [GATE-2001]

**GATE-4. Ans.**  $(0.834 \text{ m}^3/\text{s})$   $h_f = \frac{fLV^2}{2gD} = \frac{0.04 \times 1000 \times V^2}{2 \times 9.81 \times 0.6}$  Therefore  $\Delta H = H - h_f = 30 - h_f$

$$V = \sqrt{2g\Delta H} \text{ Or } \Delta H = \frac{V^2}{2g} = 30 - h_f = 30 - \frac{0.04 \times 1000 \times V^2}{2 \times 9.81 \times 0.6} \Rightarrow V = 2.95 \text{ m/s}$$

$$Q = VA = V \times \frac{\pi D^2}{4} = 2.95 \times \frac{\pi \times (0.6)^2}{4} = 0.834 \text{ m}^3/\text{s}$$

**GATE-5.** Water at 25°C is flowing through a 1.0 km long G.I. pipe of 200 mm diameter at the rate of 0.07 m³/s. If value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 kg/m³, the pumping power (in kW) required to maintain the flow is: [GATE-2009]

- (a) 1.8 (b) 17.4 (c) 20.5 (d) 41.0

**GATE-6. Ans. (b)**

$$\begin{aligned} h_f &= \frac{fLV^2}{2gD} \\ &= \frac{8fLQ_1^2}{\pi^2 g D^5} \\ &= \frac{8 \times 0.02 \times 1000 \times (0.07)^2}{\pi^2 \times 9.81 \times (0.2)^5} = 25.30 \end{aligned}$$

$$\begin{aligned} \text{Power} &= \rho g Q h_f \\ &= 1000 \times 9.81 \times 0.07 \times 25.30 \text{ W} \end{aligned}$$

## Previous 20-Years IES Questions

### Loss of Energy (or Head) in Pipes

**IES-1.** Which one of the following statements is true of fully developed flow through pipes? [IES-1997]

- (a) The flow is parallel, has no inertia effects, the pressure gradient is of constant value and the pressure force is balanced by the viscous force.
- (b) The flow is parallel, the pressure gradient is proportional to the inertia force and there is no viscous effect
- (c) The flow is parallel, the pressure gradient is negligible and inertia force is balanced by the viscous force.
- (d) The flow is not parallel, the core region accelerates and the viscous drag is far too less than the inertia force.

**IES-1. Ans. (a)** For fully developed flow through pipes, the flow is parallel, has no inertia effect. The pressure gradient is of constant value and the pressure force is balanced by the viscous force.

**IES-2. Assertion (A):** For a fully developed viscous flow through a pipe the velocity distribution across any section is parabolic in shape.

**Reason (R):** The shear stress distribution from the centre line of the pipe upto the pipe surface increases linearly. [IES-1996]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-2. Ans. (a)** As  $\tau = \mu \frac{\partial u}{\partial y}$  if  $u$  is second order  $\tau$  must be linear.

For laminar flow through a circular pipe (Radius  $R$ )

Shear stress distribution  $\tau = \left(-\frac{\partial p}{\partial x}\right) \frac{r}{2}$ , i.e.  $\tau \propto r$  [linear variation]

Vel. distribution

$$u = \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x}\right) R^2 \left[1 - \frac{r^2}{R^2}\right]$$

$$u = U_{\max} \left(1 - \frac{r^2}{R^2}\right) \text{ [Parabolic profile]}$$

$$\text{Where, } U_{\max} = \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x}\right) R^2$$

This velocity distribution is derived from linear stress profile.

$$\tau = -\mu \frac{du}{dr} = \left(-\frac{\partial P}{\partial x}\right) \frac{r}{2} \Rightarrow \frac{1}{2\mu} \left(-\frac{\partial P}{\partial x}\right) r dr = du$$

$$\text{Integrating } \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \frac{r^2}{2} + c = u \quad (1)$$

Boundary conditions

At  $r = R$ ,  $u = 0$  [Fixed plate/pipe]

$$\frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \frac{R^2}{2} + c = 0$$

$$c = -\frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \frac{R^2}{2}$$

Putting constant  $c$  in equation (1)

$$\left\{ \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \frac{r^2}{2} \right\} + \left\{ -\frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \frac{R^2}{2} \right\} = u$$

$$\Rightarrow u = \frac{1}{4\mu} \left( -\frac{\partial P}{\partial x} \right) [R^2 - r^2] \quad \Rightarrow u = \frac{1}{4\mu} \left( -\frac{\partial P}{\partial x} \right) R^2 \left[ 1 - \frac{r^2}{R^2} \right]$$

Parabolic distribution of velocity.

**IES-3. Assertion (A): Nature of the fluid flow in a pipe does not depend entirely on average velocity but it actually a function of the Reynolds number.** [IES-1995, 2001]

**Reason (R): Reynolds number depends not only on average velocity but also on the diameter of the pipe and kinematic viscosity of the fluid.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-3. Ans. (a)** Reynold's number decides the fluid flow is laminar or turbulent, i.e.

$$\text{Nature of fluid flow } Re = \frac{\rho V D}{\mu}$$

**IES-4. Aging of pipe implies**

[IES-1992]

- (a) Pipe becoming smoother with time
- (b) Relative roughness decreasing with time
- (c) Increase in absolute roughness periodically with time
- (d) Increase in absolute roughness linearly with time

**IES-4. Ans. (d)**

## Darcy-Weisbach Formula

**IES-5. The head loss in turbulent flow in pipe varies** [IES-2007; IAS-2007]

- (a) Directly as the velocity
- (b) Inversely as the square of the velocity
- (c) Inversely as the square of the diameter
- (d) Approximately as the square of the velocity of the diameter

$$\text{IES-5. Ans. (d)} \quad h_f = \frac{4fLV^2}{D \times 2g}$$

**IES-6. The frictional head loss in a turbulent flow through a pipe varies**

- (a) Directly as the average velocity. [IES-1995]
- (b) Directly as the square of the average velocity.
- (c) Inversely as the square of the average velocity.

(d) Inversely as the square of the internal diameter of the pipe.

**IES-6. Ans. (b)** Frictional head loss in turbulent flow varies directly as the square of average velocity.

**IES-7. How does the head loss in turbulent flow in pipe vary? [IES-2009]**

- (a) Directly as velocity
- (b) Inversely as square of velocity
- (c) Approximately as square of velocity
- (d) Inversely as velocity

**IES-7. Ans. (c)** Head loss in the turbulent flow is Approximately proportional to square of velocity. But Head loss in the Laminar flow is as proportional to velocity.

**IES-8. The value of friction factor is misjudged by + 25% in using Darcy-Weisbach equation. The resulting error in the discharge will be:**

[IES-1999]

- (a) + 25%
- (b) - 18.25%
- (c) - 12.5 %
- (d) +12.5%

**IES-8. Ans. (c)** Correct method  $h_f = \frac{4fLV^2}{2gD}$  Where  $V = \frac{Q}{A}$  or  $V^2 = \frac{16Q^2}{\pi^2 D^4}$

$$\text{or } h_f = \frac{64fLQ^2}{2g\pi^2 D^5} \text{ Or } Q \propto \frac{1}{\sqrt{f}} \text{ or } \frac{Q' - Q}{Q} = \sqrt{\frac{f}{f'}} - 1 = \sqrt{\frac{1}{1.25}} - 1 = -10.55\%$$

Nearest answer is (c)

But Paper setter calculates it in the way given below.

$$\ln(Q) = -\frac{1}{2} \ln(f) \text{ Or } \frac{dQ}{Q} = -\frac{1}{2} \frac{df}{f} = -\frac{1}{2} \times 25 = -12.5\%$$

**Note:** This method is used only for small fluctuation and 25% is not small that so why this result is not correct.

**IES-9. A pipeline connecting two reservoirs has its diameter reduced by 20% due to deposition of chemicals. For a given head difference in the reservoirs with unaltered friction factor, this would cause a reduction in discharge of:**

[IES-2000]

- (a) 42.8%
- (b) 20%
- (c) 17.8%
- (d) 10.6%

**IES-9. Ans. (a)**  $h_f = \frac{4fLV^2}{2gD}$  where  $V = \frac{Q}{A}$  or  $V^2 = \frac{16Q^2}{\pi^2 D^4}$  or  $h_f = \frac{64fLQ^2}{2g\pi^2 D^5}$  or  $Q \propto D^{5/2}$

$$\text{or } \left(1 - \frac{Q_2}{Q_1}\right) = 1 - (0.8)^{2.5} = 42.75\% \text{ (Reduction)}$$

**IES-10. The pressure drop in a pipe flow is directly proportional to the mean velocity. It can be deduced that the**

[IES-2006]

- (a) Flow is laminar
- (b) Flow is turbulent
- (c) Pipe is smooth
- (d) Pipe is rough

**IES-10. Ans. (a)**

## Chezy's Formula for Loss of Head Due to Friction

**IES-11. The hydraulic means depth (where A = area and P = wetted perimeter) is given by:**

[IES-2002]

- (a)  $P / A$
- (b)  $P^2 / A$
- (c)  $A / P$
- (d)  $\sqrt{A / P}$

IES-12. Ans. (c)

IES-13. What is hydraulic diameter used in place of diameter for non-circular ducts equal to? [IES-2008]

- (a)  $A/m$  (b)  $4A/m$  (c)  $A/(4m)$  (d)  $4m/A$

Where  $A$  = area of flow and  $m$  = perimeter.

IES-14. Ans. (b) Hydraulic diameter =  $\frac{4A}{P}$

Note: Hydraulic mean depth =  $\frac{A}{P}$

Hydraulic equivalent diameter =  $\frac{4A}{P}$

Where  $A$  = area of flow and  $p$  = perimeter.

IES-15. Which one of the following is the correct expression for the area of flow for a circular channel? (Where  $\theta$  = half the angle subtended by water surface at the center and  $R$  = radius of the circular channel) [IES-2004]

- (a)  $R^2 \left( 2\theta - \frac{\sin 2\theta}{2} \right)$  (b)  $R^2 \left( \theta - \frac{\sin 2\theta}{2} \right)$   
 (c)  $R^2 (2\theta - \sin 2\theta)$  (d)  $2R^2 (\theta - \sin 2\theta)$

IES-16. Ans. (b)

IES-17. For a circular channel, the wetted parameter (where  $R$  = radius of circular channel,  $\theta$  = half the angle subtended by the water surface at the centre) is given by: [IES-2003]

- (a)  $R\theta / 2$  (b)  $3R\theta$  (c)  $2R\theta$  (d)  $R\theta$

IES-17. Ans. (c)

## Loss of Head Due to Sudden Enlargement

IES-18. Assertion (A): In pipe flow, during sudden expansion, the loss of head is  $\frac{(V_1 - V_2)^2}{2g}$  [IES-1992]

Reason (R): In pipe flow, the loss of head during gradual expansion is

given by  $\int \frac{fV^2}{D2g} dL$

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-18. Ans. (b)

IES-19. If coefficient of contraction at the vena contract is equal to 0.62, then what will be the dynamic loss coefficient in sudden contraction in air-conditioning duct? [IES-2004]

- (a) 0.25 (b) 0.375 (c) 0.55 (d) 0.65

IES-19. Ans. (b)

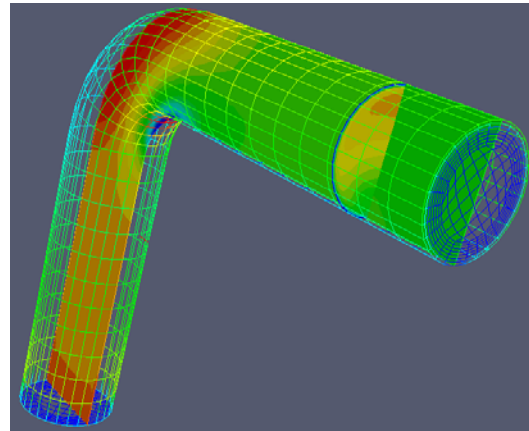
## Loss of Head Due to Bend in the Pipe

**IES-20.** Assertion (A): There will be a redistribution of pressure and velocity from inside of the bend to the outside while a fluid flows through a pipe bend. [IES-1995]

Reason (R): The spacing between stream lines will increase towards the outside wall and decrease towards the inside wall of the bend and thereby create a positive pressure gradient between outside wall to inside wall of the bend.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-20. Ans. (a)** A and R are correct. R is also right reason for A.



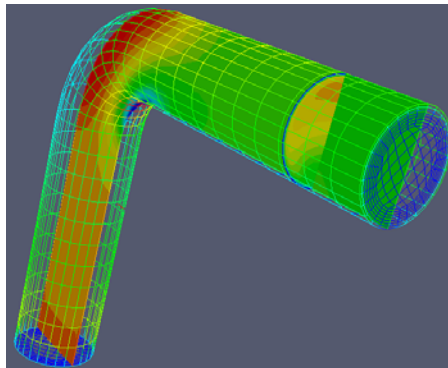
**IES-21.** Assertion (A): In the case of flow around pipe bends, there will be redistribution of pressure and velocity from inside bend to the outside bend. [IES-1997]

Reason (R): Flow will be such that the streamline spacing will decrease towards the inner bend resulting in decrease of pressure head and increase of velocity head at the inner wall.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-21. Ans. (c)** The velocity head at the inner wall will decrease and pressure head increase.

**IES-21. Ans. (a)**







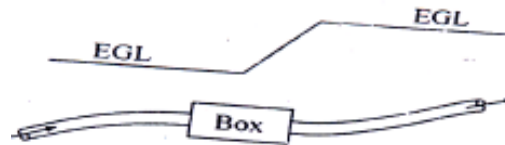
Total head at point A ( $H_A$ ) = Total head at point B ( $H_B$ ) + Loss of head between A and B ( $H_L$ )

If flow is from point B to point A then

Total head at point B ( $H_B$ ) = Total head at point A ( $H_A$ ) + Loss of head between B and A ( $H_L$ )

IES-27. The energy grade line (EGL) for steady flow in a uniform diameter pipe is shown above. Which of the following items is contained in the box?

- (a) A pump
- (b) A turbine
- (c) A partially closed valve
- (d) An abrupt expansion



[IES-2006]

IES-27. Ans. (a) Energy increased so box must add some hydraulic energy to the pipeline. It must be a pump that converts Electrical energy to Hydraulic energy.

IES-28. A 12 cm diameter straight pipe is laid at a uniform downgrade and flow rate is maintained such that velocity head in the pipe is 0.5 m. If the pressure in the pipe is observed to be uniform along the length when the down slope of the pipe is 1 in 10, what is the friction factor for the pipe?

[IES-2006]

- (a) 0.012
- (b) 0.024
- (c) 0.042
- (d) 0.050

IES-28. Ans. (b)  $h_f = \frac{fL}{D} \cdot \frac{V^2}{2g}$  or,  $1 = \frac{f \times 10}{0.12} \times 0.5 \Rightarrow f = 0.024$

## Pipes in Series or Compound Pipes

IES-29. Two pipelines of equal length and with diameters of 15 cm and 10 cm are in parallel and connect two reservoirs. The difference in water levels in the reservoirs is 3 m. If the friction is assumed to be equal, the ratio of the discharges due to the larger diameter pipe to that of the smaller diameter pipe is nearly,

[IES-2001]

- (a) 3.375
- (b) 2.756
- (c) 2.25
- (d) 1.5

IES-29. Ans. (b) Loss of head in larger diameter pipe = Loss of head in smaller diameter pipe

$$h_f = \frac{4fLV^2}{2gD} \text{ Where } V = \frac{Q}{A} \text{ or } V^2 = \frac{16Q^2}{\pi^2 D^4} \text{ or } h_f = \frac{64fLQ^2}{2g\pi^2 D^5} \text{ Or } Q \propto D^{5/2}$$

$$\frac{Q_1}{Q_2} = \left( \frac{15}{10} \right)^{5/2} = 2.756$$

IES-30. A pipe is connected in series to another pipe whose diameter is twice and length is 32 times that of the first pipe. The ratio of frictional head losses for the first pipe to those for the second pipe is (both the pipes have the same frictional constant):

[IES-2000]

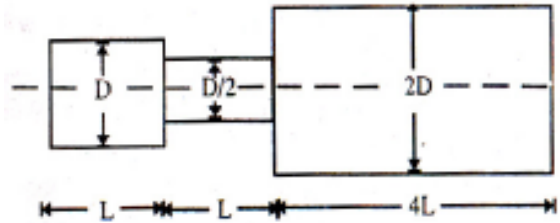
- (a) 8
- (b) 4
- (c) 2
- (d) 1

IES-30. Ans. (d)  $h_f = \frac{4fLV^2}{2gD} \text{ Where } V = \frac{Q}{A} \text{ or } V^2 = \frac{16Q^2}{\pi^2 D^4} \text{ or } h_f = \frac{64fLQ^2}{2g\pi^2 D^5}$

$$\frac{h_{f1}}{h_{f2}} = \left( \frac{L_1}{L_2} \right) \left( \frac{D_1}{D_2} \right)^5 = 32 / 32 = 1$$

IES-31. The equivalent length of stepped pipeline shown in the below figure, can be expressed in terms of the diameter 'D' as:

- (a) 5.25 L (b) 9.5 L  
(c)  $33\frac{1}{32}$  L (d)  $33\frac{1}{8}$  L



[IES-1998]

IES-31. Ans. (d)  $\frac{Le}{D^5} = \frac{L}{D^5} + \frac{L}{(D/2)^5} + \frac{4L}{(2D)^5} = 33\frac{1}{8}L$

IES-32. Three identical pipes of length I, diameter d and friction factor f are connected in parallel between two reservoirs. what is the size of a pipe of length I and of the same friction factor f equivalent to the above pipe?

[IES-2009]

- (a) 1.55d (b) 1.4d (c) 3d (d) 1.732d

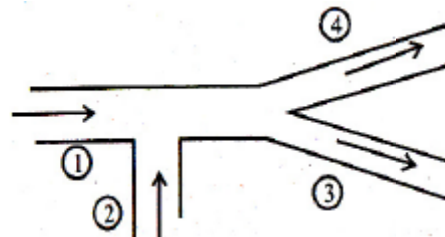
IES-32. Ans. (a)  $\frac{fL \left( \frac{Q}{3} \right)^2}{12d^5} = \frac{fLQ^2}{12d_{eq}^5}$

$$\Rightarrow \frac{1}{9d^5} = \frac{1}{d_{eq}^5}$$

$$\Rightarrow d_{eq} = (9)^{\frac{1}{5}} d = 1.55d$$



IES-33. A pipe flow system with flow direction is shown in the below figure. The following table gives the velocities and the corresponding areas:



Pipe No.	Area (cm <sup>2</sup> )	Velocity (cm/s)
1	50	10
2	50	V <sub>2</sub>
3	80	5
4	70	5

[IES-1998]

The value of V<sub>2</sub> is:

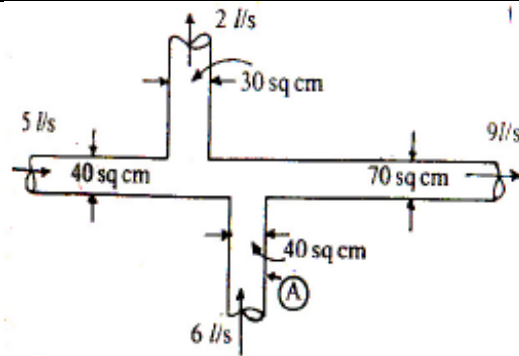
- (a) 2.5 cm/s (b) 5.0 cm/s (c) 7.5 cm/s (d) 10.0 cm/s

IES-33. Ans. (b)  $Q_1 + Q_2 = Q_3 + Q_4$

$$50 \times 10 + 50 \times V_2 = 80 \times 5 + 70 \times 5; \quad V_2 = 5 \text{ cm/sec}$$

IES-34. The pipe cross-sections and fluid flow rates are shown in the given figure. The velocity in the pipe labeled as A is:

- (a) 1.5 m/s (b) 3 m/s  
(c) 15 m/s (d) 30 m/s



[IES-1999]

IES-34. Ans. (a)  $V = \frac{Q}{A} = \frac{6000}{40} = 150 \text{ cm/s} = 1.5 \text{ m/s}$

IES-35. The velocities and corresponding flow areas of the branches labeled ①, ②, ③, ④ and ⑤ for a pipe system shown in the given figure are given in the following table:



Pipe Label	Velocity	Area
①	5 cm/s	4 sq cm
②	6 cm/s	5 sq cm
③	$V_3$ cm/s	2 sq cm
④	4 cm/s	10 sq cm
⑤	$V_5$ cm/s	8 sq cm

The velocity  $V_5$  would be:

[IES-2000]

- (a) 2.5 cm/s (b) 5 cm/s (c) 7.5 cm/s (d) 10 cm/s

IES-35. Ans. (a)  $Q_1 + Q_5 = Q_4$  or  $5 \times 4 + V_5 \times 8 = 4 \times 10$  or  $V_5 = 2.5 \text{ cm/s}$

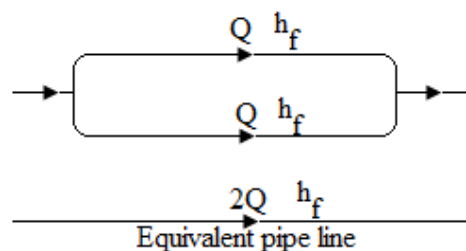
IES-36. A compound pipeline consists of two pieces of identical pipes. The equivalent length of same diameter and same friction factor, for the compound pipeline is  $L_1$  when pipes are connected in series, and is  $L_2$  when connected in parallel. What is the ratio of equivalent lengths  $L_1/L_2$ ?

[IES-2006]

- (a) 32 : 1 (b) 8 : 1 (c) 2 : 1 (d)  $\sqrt{2} : 1$

IES-36. Ans. (b) Pipes connected in series,  $\frac{L_1}{D^5} = \frac{L}{D^5} + \frac{L}{D^5}$  or  $L_1 = 2L$

Pipes connected in parallel,



$$h_f = \frac{4fLV^2}{2gD}$$

$$\text{where } V = \frac{Q}{A}$$

$$\text{or } V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$\text{or } h_f = \frac{64fLQ^2}{2g\pi^2 D^5} = \frac{64fL_2(2Q)^2}{2g\pi^2 D^5}$$

$$L_2 = \frac{L}{4} \quad \therefore \frac{L_1}{L_2} = \frac{2L}{L/4} = 8$$

## Power Transmission through Pipes

IES-37. For maximum transmission of power through a pipe line with total head  $H$ , the head lost due to friction  $h_f$  is given by: [IAS-2007; IES-2001]

- (a)  $0.1 H$  (b)  $H/3$  (c)  $H/2$  (d)  $2H/3$

IES-37. Ans. (b)

IES-38. Assertion (A): The power transmitted through a pipe is maximum when the loss of head due to friction is equal to one-third of total head at the inlet. [IES-2007]

Reason (R): Velocity is maximum when the friction loss is one-third of the total head at the inlet.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-38. Ans. (c) Velocity is optimum when the friction loss is one-third of the total head at the inlet.

IES-39. If  $H$  is the total head at inlet and  $h_1$  is the head lost due to friction, efficiency of power transmission, through a straight pipe is given by: [IES-1995]

- (a)  $(H - h_1)/H$  (b)  $H/(H+h_1)$  (c)  $(H - h_1)/(H+h_1)$  (d)  $H/(H - h_1)$

IES-39. Ans. (a) Efficiency of power transmission through a pipe =  $\frac{H - h_1}{H}$

## Diameter of the Nozzle for Transmitting Maximum Power

IES-40. A 20 cm diameter 500 m long water pipe with friction factor  $u_f = 0.025$ , leads from a constant-head reservoir and terminates at the delivery end into a nozzle discharging into air. (Neglect all energy losses other than those due to pipe friction). What is the approximate diameter of the jet for maximum power? [IES-2004]

- (a) 6.67 mm (b) 5.98 mm (c) 66.7 mm (d) 59.8 mm

IES-40. Ans. (d)  $d = \left( \frac{D^5}{2fL} \right)^{1/4} = \left( \frac{0.20^5}{2 \times 0.025 \times 500} \right)^{1/4} = 0.0598 \text{ m} = 59.8 \text{ mm}$ , Here  $f$  is

friction factor  $d = \left( \frac{D^5}{8fL} \right)^{1/4}$  here,  $f$  is co-efficient of friction.

## Water Hammer in Pipes

IES-41. Water hammer in pipe lines takes place when [IES-1995]

- (a) Fluid is flowing with high velocity
- (b) Fluid is flowing with high pressure
- (c) Flowing fluid is suddenly brought to rest by closing a valve.
- (d) Flowing fluid is brought to rest by gradually closing a valve.

IES-41. Ans. (c) Water hammer in pipe lines takes place when flowing fluid is suddenly brought to rest by closing a valve. When the water flowing in a long pipe is suddenly brought to rest by closing the valve or by any similar cause, there will be sudden rise in pressure due to the momentum or the moving water being destroyed. This causes a wave of high pressure to be transmitted along the pipe which creates noise known as knocking. This phenomenon of sudden rise in pressure in the pipe is known as water hammer or hammer blow.

IES-42. Velocity of pressure waves due to pressure disturbances imposed in a liquid is equal to: [IES-2003]

- (a)  $(E / \rho)^{1/2}$
- (b)  $(E \rho)^{1/2}$
- (c)  $(\rho / E)^{1/2}$
- (d)  $(1 / \rho E)^{1/2}$

IES-42. Ans. (a)

IES-43. Which phenomenon will occur when the valve at the discharge end of a pipe connected to a reservoir is suddenly closed? [IES-2005]

- (a) Cavitation
- (b) Erosion
- (c) Hammering
- (d) Surging

IES-43. Ans. (c)

## Instantaneous Closure of Valve, in Rigid Pipes

IES-44. Which one of the following statements relates to expression ' $\rho v c$ '?

- (a) Pressure rise in a duct due to normal closure of valve in the duct [IES-2009]
- (b) Pressure rise in a duct due to abrupt closure of valve in the duct
- (c) Pressure rise in a duct due to slow opening of valve in the duct
- (d) Pressure rise in a duct due to propagation of supersonic wave through the duct

IES-44. Ans. (d)

## Previous 20-Years IAS Questions

## Loss of Energy (or Head) in Pipes

IAS-1. Two identical pipes of length 'L', diameter 'd' and friction factor 'f' are connected in parallel between two points. For the same total volume flow rate with pipe of same diameter 'd' and same friction factor 'f', the single length of the pipe will be: [IAS-1999]

(a)  $\frac{L}{2}$

(b)  $\frac{L}{\sqrt{2}}$

(c)  $\sqrt{2} L$

(d)  $\frac{L}{4}$

IAS-1. Ans. (d)  $h_f = \frac{4fLV^2}{2g \times D}$  for same diameter Velocity, V will be (V/2)  $\Delta P$  Will be  $\frac{1}{4}$  times.

IAS-2. The loss of head in a pipe of certain length carrying a rate of flow of Q is found to be H. If a pipe of twice the diameter but of the same length is to carry a flow rate of 2Q, then the head loss will be: [IAS-1997]

(a) H

(b) H/2

(c) H/4

(d) H/8

IAS-2. Ans. (d)  $H = \frac{4fLV^2}{2gD}$

where  $V = \frac{Q}{A}$  or  $V^2 = \frac{16Q^2}{\pi^2 D^4}$  or  $H = \frac{64fLQ^2}{2g\pi^2 D^5}$

or  $H_2 = \frac{64fL(2Q)^2}{2g\pi^2 (2D)^5} = \frac{2^2}{2^5} H = \frac{H}{8}$

IAS-3. The coefficient of friction 'f' in terms of shear stress ' $\tau_0$ ' is given by

[IAS-2003]

(a)  $f = \frac{\rho v^2}{2\tau_0}$

(b)  $f = \frac{\tau_0}{\rho v^2}$

(c)  $f = \frac{2\tau_0}{\rho v^2}$

(d)  $f = \frac{2\rho v^2}{\tau_0}$

IAS-3. Ans. (c)

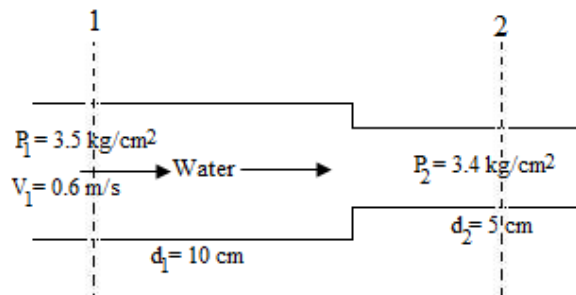
IAS-4. The energy loss between sections (1) and (2) of the pipe shown in the given figure is:

(a) 1.276 Kg-m

(b) 1.00 Kg-m

(c) 0.724 Kg-m

(d) 0.15 Kg-m



[IAS-1995]

IAS-4. Ans. (c). Energy loss between sections 1 and 2

$$= \frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \quad \text{Also } V_1 \times A_1 = V_2 \times A_2$$

$$\text{or } 0.6 \times \frac{\pi}{4} \times (0.1)^2 = V_2 \times \frac{\pi}{4} (0.05)^2 \quad \text{or } V_2 = 0.6 \times 4 = 2.4 \text{ m/s}$$

$$\text{Energy loss} = \frac{(3.5 - 3.4) \times 10000 \times 9.81}{9.81 \times 1000} + \frac{0.6^2}{2g} (1 - 16)$$

$$= 0.1 \times 10 - \frac{15 \times 0.36}{2 \times 9.81} = 1 - 0.266 = 0.724$$

IAS-5. A laminar flow is taking place in a pipe. Match List-I (Term) with List-II (Expression) and select the correct answer using the codes given below the Lists: [IAS-2002]

List-I

A. Discharge, Q

B. Pressure drop,  $\frac{\Delta P}{L}$

C. Friction factor, f

List-II

1.  $\frac{16\mu}{\rho VD}$

2.  $\frac{\pi l^2 \Delta p}{128 \mu L}$

3.  $\frac{32 \mu V}{D^2}$

4.  $\frac{\pi l^4 \Delta p}{128 \mu L}$

Codes:	A	B	C	A	B	C
(a)	2	3	4	(b)	4	3
(c)	4	1	3	(d)	1	2

IAS-5. Ans. (b) Here 'C' is wrong. Friction factor =  $\frac{64}{Re}$  and

Coefficient of friction =  $\frac{16}{Re}$  so 'C' would be co-efficient of friction.

IAS-6. From a reservoir, water is drained through two pipes of 10 cm and 20 cm diameter respectively. If frictional head loss in both the pipes is same, then the ratio of discharge through the larger pipe to that through the smaller pipe will be: [IAS-1998]

- (a)  $\sqrt{2}$  (b)  $2\sqrt{2}$  (c) 4 (d)  $4\sqrt{2}$

IAS-6. Ans. (d)  $h_f = \frac{4fLV^2}{D \times 2g}$   $\frac{V_R^2}{D_R} = \frac{V_S^2}{D_S}$  or  $\frac{V_l}{V_s} = \sqrt{\frac{D_l}{D_s}} = \sqrt{\frac{20}{10}} = \sqrt{2}$

$$\frac{Ql}{Qs} = \frac{A_l V_l}{A_s V_s} = \frac{D_l^2}{D_s^2} \times \frac{V_l}{V_s} = \frac{D_l^2}{D_s^2} \times \sqrt{\frac{D_l}{D_s}} = 4\sqrt{2}$$

or, we may directly use  $Q \propto D^{5/2}$

$$h_f = \frac{4fLV^2}{2gD} \text{ Where } V = \frac{Q}{A} \text{ or } V^2 = \frac{16Q^2}{\pi^2 D^4} \text{ or } h_f = \frac{64fLQ^2}{2g\pi^2 D^5} \text{ Or } Q \propto D^{5/2}$$

IAS-7. Which one of the following expresses the hydraulic diameter for a rectangular pipe of width b and height a? [IAS-2007]

- (a)  $\frac{ab}{2(a+b)}$  (b)  $\frac{ab}{(a+b)}$  (c)  $\frac{2ab}{(a+b)}$  (d)  $\frac{a+b}{2ab}$

IAS-7. Ans. (c) Hydraulic diameter =  $\frac{4A}{P} = \frac{2ab}{2(a+b)}$

Note: Hydraulic mean depth =  $\frac{Ac}{P}$

Hydraulic equivalent diameter =  $\frac{4Ac}{P}$

## Loss of Head Due to Sudden Contraction

- IAS-8. Assertion (A): Head loss for sudden expansion is more than the head loss for a sudden contraction for the same diameter ratio. [IAS-2003]  
Reason (R): Head loss varies as the square of the upstream and downstream velocities in the pipe fitted with sudden expansion or sudden contraction.
- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-8. Ans. (c)

- IAS-9. Assertion (A): Energy grade line lies above the hydraulic grade line and is always parallel to it. [IAS-2003]  
Reason (R): The vertical difference between energy grade line and hydraulic grade line is equal to the velocity head.
- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-9. Ans. (a)

- IAS-10. Two pipelines of equal lengths are connected in series. The diameter of the second pipe is two times that of the first pipe. The ratio of frictional head losses between the first pipe and the second pipe is: [IAS-1996]
- (a) 1:32 (b) 1:16 (c) 1:8 (d) 1:4

IAS-10. Ans. (a)  $h_f = \frac{4fLV^2}{2gD}$  Where  $V = \frac{Q}{A}$  or  $V^2 = \frac{16Q^2}{\pi^2 D^4}$  or  $h_f = \frac{64fLQ^2}{2g\pi^2 D^5}$  Or  $h_f \propto \frac{1}{D^5}$

$$\frac{h_{f1}}{h_{f2}} = \left(\frac{D_2}{D_1}\right)^5 = (2)^5 = 32$$

## Equivalent Pipe

- IAS-11. A pipeline is said to be equivalent to another, if in both [IAS-2007]
- (a) Length and discharge are the same  
(b) Velocity and discharge are the same  
(c) Discharge and frictional head loss are the same  
(d) Length and diameter are the same

IAS-11. Ans. (c)  $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \dots$

- IAS-12. A stepped pipelines with four different cross-sections discharges water at the rate of 2 litres per second. Match List-I (Areas of pipe in sq cm) with List-II (Velocities of water in cm/s) and select the correct answer using the codes given below the Lists: [IAS-2001]

List-I

- A. 500  
B. 100  
C. 400

List-II

1. 4  
2. 5  
3. 10



D. 200

4. 15

5. 20

Codes: A B C D

(a) 5 1 2 3

(c) 1 5 3 4

A B C D

(b) 1 5 2 3

(d) 3 2 5 1

IAS-12. Ans. (b) Volume flow rate =  $A.V = 2000 \text{ cm}^3/\text{sec}$

$$A_1V_1 = A_2V_2 = A_3V_3 = A_4V_4 = 2000$$

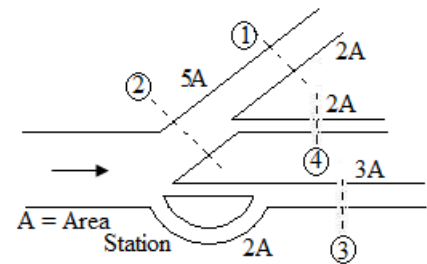
IAS-13. A branched pipeline carries water as shown in the given figure. The cross-sectional areas of the pipelines have also been indicated in the figure. The correct sequence of the decreasing order of the magnitude of discharge for the four stations is:

(a) 2, 3, 1, 4

(b) 3, 2, 1, 4

(c) 3, 2, 4, 1

(d) 2, 3, 4, 1

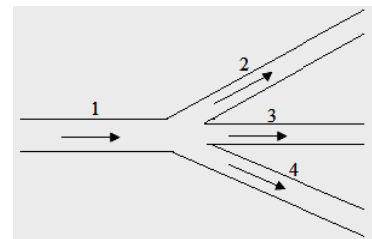


[IAS-1996]

IAS-13. Ans. (d) Don't confuse with section 1 and section 4 both has area = '2A' as it is vertically up so discharge will be less.

IAS-14. Pipe '1' branches to three pipes as shown in the given figure. The areas and corresponding velocities are as given in the following table.

Pipe	Velocity (cm per second)	Area (sq cm)
1.	50	20
2.	$V_2$	10
3.	30	15
4.	20	10



The value of  $V_2$  in cm per second will be:

(a) 15

(b) 20

(c) 30

(d) 35

[IAS-1995]

IAS-14. Ans. (d)  $Q_1 = Q_2 + Q_3 + Q_4$

$$50 \times 20 = V_2 \times 10 + 30 \times 15 + 20 \times 10; \text{ or } 1000 = 10V_2 + 450 + 200$$

$$10V_2 = 1000 - 650 = 350 \text{ and } V_2 = 35 \text{ cm/sec.}$$

## Power Transmission through Pipes

IAS-15. For maximum transmission of power through a pipe line with total head  $H$ , the head lost due to friction  $h_f$  is given by: [IAS-2007; IES-2001]

(a)  $0.1 H$

(b)  $H/3$

(c)  $H/2$

(d)  $2H/3$

IAS-15. Ans. (b)

IAS-16. What will be the maximum efficiency of the pipeline if one-third of the available head in flow through the pipeline is consumed by friction?

(a) 33.33%

(b) 50.00%

(c) 66.66%

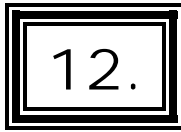
(d) 75.00%

[IAS-2004]

IAS-16. Ans. (c)  $h_f = \frac{H}{3} \therefore \eta = \frac{H - h_f}{H} \times 100 = \frac{2}{3} \times 100 = 66.66\%$

IAS-17. In a pipe flow, the head lost due to friction is 6 m. If the power transmitted through the pipe has to be the maximum then the total head at the inlet of the pipe will have to be maintained at [IAS-1995]  
(a) 36 m (b) 30 m (c) 24 m (d) 18 m

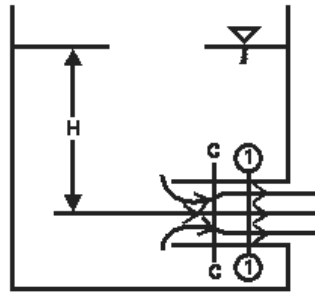
IAS-17. Ans. (d) Head lost due to friction is 6 m. Power transmitted is maximum when friction head is  $\frac{1}{3}$  of the supply head.  
 $\therefore$  Supply head should be 18 m.



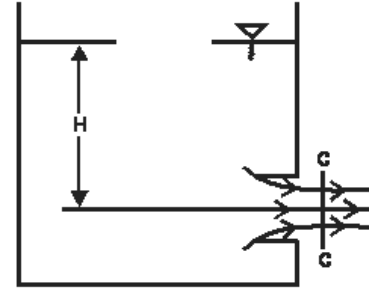
## Flow Through Orifices and Mouthpieces

**Question:** Prove that an internal mouthpiece running full discharges around 41.4 % more water than that when it runs free. [AMIE (Winter) 2001]

**Answer:**



**Mouthpiece running full**



**Mouthpiece running free**

Let,  $a_c$  = area at vena-contracta,

$a$  = area of orifice or mouthpiece.

$V_c$  = velocity of the liquid at C-C (vena-contracta)

$V_1$  = velocity of the liquid at I-I (or outlet), and

$H$  = height of liquid above mouthpiece.

**For mouthpiece running full:**

Applying continuity equation  $a_c V_c = a_1 V_1$

$$\text{or } V_c = \frac{a_1 V_1}{a_c} \dots(i)$$

We know that co-efficient of contraction of an internet mouthpiece is 0.5

$$\therefore C_c = \frac{a_c}{a_1} = 0.5 \quad \therefore \text{From equation (i);} \quad V_c = 2 V_1 \dots(ii)$$

The jet of liquid after passing through C-C suddenly enlarges at section I-I. Therefore, there will be loss of head due to sudden en/assent

$$h_2 = \frac{(V_c - V_1)^2}{2g} = \frac{(2V_1 - V_1)^2}{2g} = \frac{V_1^2}{2g}$$

Applying Bernoulli's eq<sup>n</sup> to free water surface in tank and section I-I and assuming datum line passing through the centre line of mouth piece

$$0 + 0 + H = 0 + \frac{V_1^2}{2g} + 0 + h_2$$

$$\text{or } H = \frac{V_1^2}{2g} + \frac{V_1^2}{2g}$$

$$\text{or } H = \frac{V_1^2}{2g} \quad \therefore V_1 = \sqrt{gH} \dots(iii)$$

But theoretical velocity at (1)  $V_{th,1} = \sqrt{2gH}$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{V_1}{V_{th,1}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}}$$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence co-efficient of contraction = 1.0

$$\therefore C_d = C_c \times C_v = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\text{Discharge, } Q_{F1} = C_d \times a \times \sqrt{2gH} = 0.707 a \sqrt{2gH} \quad \dots(iv)$$

### Mouthpiece Running Free:

Pressure of the liquid on the mouthpiece

$$P = \rho g H$$

And force acting on the mouthpiece =  $P \times A = \rho g H A$

Mass of liquid flowing per second =  $\rho a_c V_c$

Momentum of flowing fluid/see = mass x velocity =  $\rho a_c V_c \times V_c = \rho a_c V_c^2$

Since the water is initially at rest, therefore initial momentum = 0

$$\therefore \text{ Change of momentum} = \rho a_c V_c^2$$

$\therefore$  As per Newton's second law of motion.

$$\rho g H \times a = \rho a_c V_c^2 \quad \therefore H = \frac{V_c^2}{2g}$$

$$\text{or } \rho g \times \frac{V_c^2}{2g} a = \rho a_c V_c^2$$

$$\therefore \frac{a_c}{a} = \frac{1}{2} = 0.5$$

$$\therefore \text{ Co-efficient of contraction } C_c = \frac{a_c}{a} = 0.5$$

Since there is no loss of head, co-efficient of velocity,  $C_v = 1.0$

$$\therefore \text{ Co-efficient of discharge } C_d = C_v \times C_c = 1 \times 0.5 = 0.5$$

$$\therefore \text{ Discharge } Q_{F_e} = 0.5 a \sqrt{2gH}$$

$$\therefore \frac{Q_{F1} - Q_{F_e}}{Q_{F_e}} = \frac{0.707 - 0.5}{0.5} = 0.414 \text{ around 41.1\% more water than that when it runs free.}$$

## OBJECTIVE QUESTIONS (IES, IAS)

### Previous Years IES Questions

#### Flow through an Orifice

**IES-1.** Match List-I (Measuring device) with List-II (Parameter measured) and select the correct answer using the codes given below the Lists:

List-I	List-II	[IES-1997]
A. Anemometer	1. Flow rate	
B. Piezometer	2. Velocity	
C. Pitot tube	3. Static pressure	
D. Orifice	4. Difference between static and stagnation pressure	

Codes:	A	B	C	D		A	B	C	D
(a)	1	3	4	2	(b)	1	2	3	4
(c)	2	3	4	1	(d)	2	4	3	1

**IES-1. Ans. (c)** Anemometer is an instrument for measuring wind force and velocity.

#### Discharge through an External Mouthpiece

**IES-2.** Given,  $H$  = height of liquid,  $b$  = width of notch,  $a$  = cross-sectional area,  $a_1$  = area at inlet,  $A_2$  = area at the throat and  $C_d$  = coefficient of drag.

Match List-I with List-II and select the correct answer using the codes given below the Lists: [IES-1997]

List-I	List-II
A. Discharge through Venturimeter	1. $\frac{2}{3} C_d b \sqrt{2g} H^{3/2}$
B. Discharge through an external mouthpiece	2. $\frac{8}{15} C_d b \sqrt{2g} H^{5/2}$
C. Discharge over a rectangular notch	3. $\frac{C_d a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gH}$
D. Discharge over right angled notch	4. $0.855 a \sqrt{2gH}$

Code:	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	3	4	1	2
(c)	2	1	3	4	(d)	2	3	1	4

**IES-2. Ans. (b)**

**IES-3.** In a submerged orifice flow, the discharge is proportional to which one of the following parameters? [IES-2009]

- (a) Square root of the downstream head
- (b) Square root of the upstream head
- (c) Square of the upstream head
- (d) Square root of the difference between upstream and downstream heads

**IES-3. Ans. (d)** A drowned or submerged orifice is one which does not discharge into open atmosphere, but discharge into liquid of the same kind.

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} \sqrt{2g(h - h_L)}$$

∴ In submerged orifice flow discharge is proportional to square root of the difference b/w upstream and downstream heads.

### Previous Years IAS Questions

#### Co-efficient of Discharge ( $C_d$ )

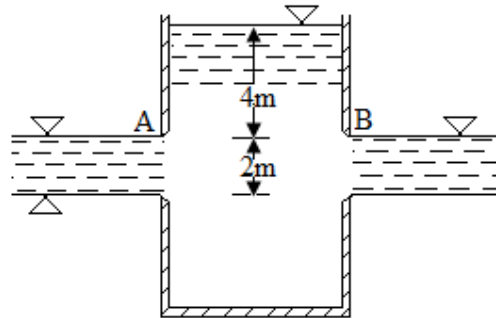
- IAS-1. A fluid jet is discharging from a 100 mm nozzle and the vena contracta formed has a diameter of 90 mm. If the coefficient of velocity is 0.95, then the coefficient discharge for the nozzle is: [IAS-1994]
- (a) 0.855                      (b) 0.81                      (c) 0.9025                      (d) 0.7695

IAS-1. Ans. (d)  $C_c = \frac{A_v}{A} = \frac{\frac{\pi}{4}(90)^2}{\frac{\pi}{4}(100)^2} = 0.81$ ,  $C_v = 0.95$  ∴  $C_d = C_c \times C_v = 0.81 \times 0.95 = 0.7695$

#### Discharge through a Large Rectangular Orifice

- IAS-2. Water discharges from a two-dimensional rectangular opening into air as indicated at A in the given figure. At B water discharge from under a gate onto the floor. The ratio of velocities  $V_A$  to  $V_B$  is:

(a)  $\frac{\sqrt{5}}{2}$     (b)  $\frac{1}{2}$     (c) 2    (d)  $\frac{1}{\sqrt{2}}$



[IAS-1996]

- IAS-2. Ans. (a) It is to be noted that the side of the reservoir having rectangular opening into air (as denoted by two triangles) should have a average theoretical velocity of water given by,  $V_A = \sqrt{2gH}$  where  $H = \frac{H_1 + H_2}{2}$

Theoretical velocity of water from under a gate onto the floor (see the figure) is given by ✗

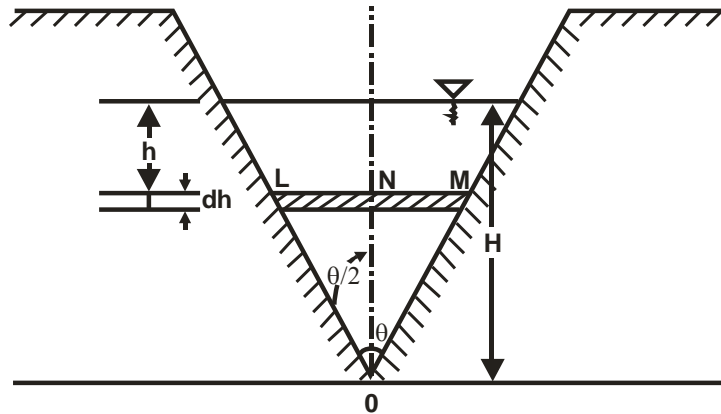
Velocity ratio,  $\frac{V_A}{V_B} = \sqrt{\frac{H_1 + H_2}{2H_1}}$

# 13. Flow Over Notches and Weirs

## Theory at a Glance (for IES, GATE, PSU)

**Question:** Prove that the error in discharge due to error in the measurement of head over a triangular notch or weir is given by  $\frac{dQ}{Q} = \frac{5}{2} \frac{dH}{H}$

**Answer:**



Let,  $H$  = head of water above the apex of the notch.

$\theta$  = angle of the notch.

$C_d$  = Co-efficient of discharge.

Consider a horizontal strip of water of thickness  $dh$ , and at a depth  $h$  from the water surface.

$$\begin{aligned} \therefore \text{Area of the strip (dA)} &= LM \, dh \\ &= 2 \, LN \, dh && \text{From } \triangle LNO \text{ Triangle} \\ &= 2 (H - h) \tan \frac{\theta}{2} dh && [LN = (H - h) \tan \frac{\theta}{2}] \end{aligned}$$

We know that theoretical velocity of water through the strip =  $\sqrt{2gh}$

$$\begin{aligned} \therefore \text{Discharge through the strip, } dQ &= C_d \times \text{area of strip} \times \text{velocity} \\ &= C_d \times 2(H-h) \times \tan \frac{\theta}{2} \times dh \times \sqrt{2gh} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total discharge, } Q &= \int_0^H dQ = 2C_d \sqrt{2g} \times \tan \frac{\theta}{2} \int_0^H (H\sqrt{h} - h^{3/2}) dh \\ &= 2C_d \sqrt{2g} \times \tan \frac{\theta}{2} \left[ H \times \frac{h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right] \\ &= 2C_d \sqrt{2g} \times \tan \frac{\theta}{2} \times H^{5/2} \left[ \frac{2}{3} - \frac{2}{5} \right] \\ &= \frac{8}{15} C_d \sqrt{2g} \times \tan \frac{\theta}{2} \times H^{5/2} \end{aligned}$$

$$\text{For a given notch } \frac{8}{15} C_d \sqrt{2g} \times \tan \frac{\theta}{2} = \text{const (k)}$$

$$\therefore Q = kH^{5/2}$$

Taking  $\log_e$  both side

$$\ln Q = \ln k + \frac{5}{2} \ln H$$

Differentiating both side

$$\frac{dQ}{Q} = 0 + \frac{5}{2} \frac{dH}{H}$$

$$\therefore \frac{dQ}{Q} = \frac{5}{2} \frac{dH}{H} \quad \text{Proved.}$$



## OBJECTIVE QUESTIONS (IES, IAS)

## Previous Years IES Questions

## Discharge over a Rectangular Notch or Weir

IES-1. Which of the following is/are related to measure the discharge by a rectangular notch? [IES-2002]

1.  $2/3 C_d b \sqrt{2g} H^2$                       2.  $2/3 C_d b \sqrt{2g} H^{3/2}$   
 3.  $2/3 C_d b \sqrt{2g} H^{5/2}$                       4.  $2/3 C_d b \sqrt{2g} H^{1/2}$

Select the correct answer using the codes given below:

- (a) 1 and 3                      (b) 2 and 3                      (c) 2 alone                      (d) 4 alone

IES-1. Ans. (c)

IES-2. Match List-I (Measuring Instrument) with List-II (Variable to be measured) and select the correct answer using the code given below the lists: [IES-2007]

## List-I

- A. Hot-wire anemometer  
 B. Pitot-tube  
 C. V-notch weir  
 D. Tachometer

## List-II

1. Discharge  
 2. Rotational speed  
 3. Velocity fluctuations  
 4. Stagnation pressure

Code:	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	2	1
(c)	4	3	1	2	(d)	3	4	1	2

IES-2. Ans. (d)

IES-3. A standard  $90^\circ$  V-notch weir is used to measure discharge. The discharge is  $Q_1$  for heights  $H_1$  above the sill and  $Q_2$  is the discharge for a height  $H_2$ . If  $H_2 / H_1$  is 4, then  $Q_2 / Q_1$  is: [IES-2001]

- (a) 32                      (b)  $16\sqrt{2}$                       (c) 16                      (d) 8

IES-3. Ans. (a) We know that for a V-notch,  $Q = \frac{8}{15} \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$

$$\text{i.e. } Q \propto H^{5/2} \therefore \frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{5/2} = 4^{5/2} = 32$$

## Previous Years IAS Questions

## Discharge over a Triangular Notch or Weir

IAS-1. A triangular notch is more accurate measuring device than the rectangular notch for measuring which one of the following? [IAS-2007]

- (a) Low flow rates                      (b) Medium flow rate  
 (c) High flow rates                      (d) All flow rates

IAS-1. Ans. (a)



# Flow Around Submerged Bodies- Drag and Lift

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## Contents of this chapter

1. Force Exerted by a Flowing Fluid on a Body
  2. Expressions for Drag and Lift
  3. Stream-lined and Bluff Bodies
  4. Terminal Velocity of a Body
  5. Circulation and Lift on a Circular Cylinder
  6. Position of Stagnation Points
  7. Expression for Lift Co-efficient for Rotating Cylinder
  8. Magnus Effect
  9. Lift on an Airfoil
-

## OBJECTIVE QUESTIONS (IES, IAS)

### Previous Years IES Questions

#### Force Exerted by a Flowing Fluid on a Body

**IES-1. Whenever a plate is submerged at an angle with the direction of flow of liquid, it is subjected to some pressure. What is the component of this pressure in the direction of flow of liquid, known as? [IES-2007]**

- (a) Stagnation pressure      (b) Lift      (c) Drag      (d) Bulk modulus

**IES-1. Ans. (c)**

**IES-2. The drag force exerted by a fluid on a body immersed in the fluid is due to: [IES-2002]**

- (a) Pressure and viscous forces      (b) Pressure and gravity forces  
(c) Pressure and surface tension forces      (d) Viscous and gravity forces

**IES-2. Ans. (a)** Total drag on a body = pressure drag + friction drag

**IES-3. The drag force exerted by a fluid on a body immersed in the fluid is due to: [IES-2002]**

- (a) Pressure and viscous forces      (b) Pressure and gravity forces  
(c) Pressure and surface tension forces      (d) Viscous and gravity forces.

**IES-3. Ans. (a)**

**IES-4. Whenever a plate is submerged at an angle with the direction of flow of liquid, it is subjected to some pressure. What is the component of this pressure in the direction of flow of liquid, known as? [IES-2007]**

- (a) Stagnation pressure      (b) Lift  
(c) Drag      (d) Bulk modulus

**IES-4. Ans. (c)**

**IES-5. An automobile moving at a velocity of 40 km/hr is experiencing a wind resistance of 2 kN. If the automobile is moving at a velocity of 50 km/hr, the power required to overcome the wind resistance is: [IES-2000]**

- (a) 43.4 kW      (b) 3.125 kW      (c) 2.5 kW      (d) 27.776 kW

**IES-5. Ans. (a)** Power,  $P = F_D \times V = C_D \times \frac{\rho V^2}{2} \times A \times V$  Or  $P \propto V^3$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^3 \text{ or } P_2 = (F_{D1} \times V_1) \times \left(\frac{V_2}{V_1}\right)^3 = \left(2 \times 40 \times \frac{5}{18}\right) \times \left(\frac{50}{40}\right)^3 = 43.4 \text{ kW}$$

**IES-6. Which one of the following causes lift on an immersed body in a fluid stream? [IES-2005]**

- (a) Buoyant forces.  
(b) Resultant fluid force on the body.  
(c) Dynamic fluid force component exerted on the body parallel to the approach velocity.  
(d) Dynamic fluid force component exerted on the body perpendicular to the approach velocity.

IES-6. Ans. (d)

## Stream-lined and Bluff Bodies

IES--7. Which one of the following is correct? [IES-2008]

**In the flow past bluff bodies**

- (a) Pressure drag is smaller than friction drag
- (b) Friction drag occupies the major part of total drag
- (c) Pressure drag occupies the major part of total drag
- (d) Pressure drag is less than that of streamlined body

IES-7. Ans. (c) In the flow past bluff bodies the pressure drag occupies the major part of total drag. During flow past bluff-bodies, the desired pressure recovery does not take place in a separated flow and the situation gives rise to pressure drag or form drag.

IES-8. Improved streaming produces 25% reduction in the drag coefficient of a torpedo. When it is travelling fully submerged and assuming the driving power to remain the same, the crease in speed will be:

[IES-2000]

- (a) 10%
- (b) 20%
- (c) 25%
- (d) 30%

IES-8. Ans. (a)  $C_{D1} \times V_1^3 = C_{D2} \times V_2^3$  or  $\frac{V_2}{V_1} = \sqrt[3]{\frac{C_{D1}}{C_{D2}}} = \sqrt[3]{\frac{100}{75}} = 1.10$

IES-9. Match List-I with List-II and select the correct answer: [IES-2001]

**List-I**

- A. Stokes' law
- B. Bluff body
- C. Streamline body
- D. Karman Vortex Street

**List-II**

- 1. Strouhal number
- 2. Creeping motion
- 3. Pressure drag
- 4. Skin friction drag

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	3	1	4	(b)	3	2	4	1
(c)	2	3	4	1	(d)	3	2	1	4

IES-9. Ans. (c) In non-dimensional form, the vortex shedding frequency is expressed as as the Strouhal number named after V. Strouhal, a German physicist who experimented with wires singing in the wind. The Strouhal number shows a slight but continuous variation with Reynolds number around a value of 0.21.

IES-10. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-2009]

**List-I**

- A. Singing of telephone wires
- B. Velocity profile in a pipe is initially parabolic and then flattens
- C. Formation of cyclones
- D. Shape of rotameter tube

**List-II**

- 1. Vortex flow
- 2. Drag
- 3. Vortex shedding
- 4. Turbulence

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	1	4	2	(b)	2	1	4	3
(c)	3	4	1	2	(d)	2	4	1	3

IES-10. Ans. (c)

## Terminal Velocity of a Body

**IES-11.** A parachutist has a mass of 90 kg and a projected frontal area of 0.30 m<sup>2</sup> in free fall. The drag coefficient based on frontal area is found to be 0.75. If the air density is 1.28 kg/m<sup>3</sup>, the terminal velocity of the parachutist will be: [IES-1999]

- (a) 104.4 m/s                      (b) 78.3 m/s                      (c) 25 m/s                      (d) 18.5 m/s

**IES-11. Ans. (b)** Total Drag ( $F_D$ ) = Weight ( $W$ ) or  $C_D \times \frac{\rho V^2}{2} \times A = mg$

$$\text{or } V = \sqrt{\frac{2mg}{C_D \times \rho \times A}} = \sqrt{\frac{2 \times 90 \times 9.81}{0.75 \times 1.28 \times 0.3}} = 78.3 \text{ m/s}$$

**IES-12.** For solid spheres falling vertically downwards under gravity in a viscous fluid, the terminal velocity,  $V_1$  varies with diameter 'D' of the sphere as [IES-1995]

- (a)  $V_1 \propto D^{1/2}$  for all diameters  
 (b)  $V_1 \propto D^2$  for all diameters  
 (c)  $V_1 \propto D^{1/2}$  for large D and  $V_1 \propto D^2$  for small D  
 (d)  $V_1 \propto D^2$  for large D and  $V_1 \propto D^{1/2}$  for small D.

**IES-12. Ans. (b)** Terminal velocity  $V_1 \propto D^2$  for all diameters. Stokes' formula forms the basis for determination of viscosity of oils which consists of allowing a sphere of known diameter to fall freely in the oil. After initial acceleration. The sphere attains a constant velocity known as Terminal Velocity which is reached when the external drag on the surface and buoyancy, both acting upwards and in opposite to the motions, become equal to the downward force due to gravity.

## Circulation and Lift on a Circular Cylinder

**IES-13.** The parameters for ideal fluid flow around a rotating circular cylinder can be obtained by superposition of some elementary flows. Which one of the following sets would describe the flow around a rotating circular cylinder? [IES-1997]

- (a) Doublet, vortex and uniform flow    (b) Source, vortex and uniform flow  
 (c) Sink, vortex and uniform flow        (d) Vortex and uniform flow

**IES-13. Ans. (a)**

**IES-14.** Assertion (A): When a circular cylinder is placed normal to the direction of flow, drag force is essentially a function of the Reynolds number of the flow. [IES-1993]

Reason (R): As Reynolds Number is about 100 and above, eddies formed break away from either side in periodic fashion, forming a meandering street called the Karman Vortex street.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-14. Ans. (a)** Both A and R are true and R provides a correct explanation of A.

# Flow Around Submerged Bodies-Drag & Lift

## S K Mondal's Chapter 14

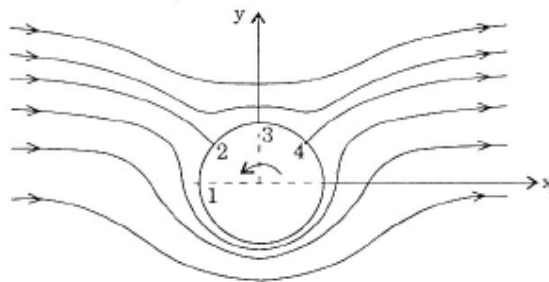
IES-15. Match List-I (Types of flow) with List-II (Basic ideal flows) and select the correct answer: [IES-2001, IAS-2003]

List-I					List-II				
A.	Flow over a stationary cylinder				1.	Source + sink + uniform flow			
B.	Flow over a half Rankine body				2.	Doublet + uniform flow			
C.	Flow over a rotating body				3.	Source + uniform flow			
D.	Flow over a Rankine oval				4.	Doublet + free vortex + uniform flow			
Codes:	A	B	C	D		A	B	C	D
(a)	1	4	3	2	(b)	2	4	3	1
(c)	1	3	4	2	(d)	2	3	4	1

IES-15. Ans. (d)

## Position of Stagnation Points

IES-16. A cylindrical object is rotated with constant angular velocity about its symmetry axis in a uniform flow field of an ideal fluid producing streamlines as shown in the figure given above. At which point(s), is the pressure on the cylinder surface maximum?



[IES-2007]

- (a) Only at point 3  
 (b) Only at point 2  
 (c) At points 1 and 3  
 (d) At points 2 and 4

IES-16. Ans. (d)

IES-17. A circular cylinder of 400 mm diameter is rotated about its axis in a stream of water having a uniform velocity of 4 m/s. When both the stagnation points coincide, the lift force experienced by the cylinder is: [IES-2000]

- (a) 160 kN/m      (b) 10.05 kN/m      (c) 80 kN/m      (d) 40.2 kN/m

IES-17. Ans. (d) For single stagnation point, Circulation

$$(\Gamma) = 4\pi VR = 4\pi \times 4 \times \frac{0.400}{2} = 10.05 \text{ m}^2 / \text{s}$$

$$\text{And Lift force } (F_L) = \rho L V \Gamma = 1000 \times L \times 4 \times 10.05 \text{ N} \Rightarrow \frac{F_L}{L} = 40.2 \text{ kN/m}$$

**Caution:** In this question “both the stagnation points coincide” is written therefore you can't just simply use  $\frac{1}{2} \rho A v^2$

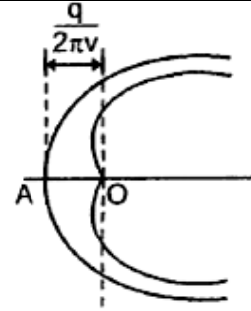
IES-18. Flow over a half body is studied by utilising a free stream velocity of 5 m/s superimposed on a source at the origin. The body has a maximum width of 2 m. The co-ordinates of the stagnation point are: [IES-1995]

- (a)  $x = 0.32 \text{ m}, y = 0$       (b)  $x = 0, y = 0$   
 (c)  $x = (-) 0.32 \text{ m}, y = 0$       (d)  $x = 3 \text{ m}, y = 2 \text{ m}$

IES-18. Ans. (c) A is the stagnation point and O is the origin.

$$-x = \frac{q}{2\pi v} = \frac{2 \times 5}{2\pi \times 5} = 0.32 \text{ m}$$

$$x = -0.32 \text{ m} \quad \text{and} \quad y = 0$$



## Expression for Lift Co-efficient for Rotating Cylinder

IES-19. Which one of the following sets of standard flows is superimposed to represent the flow around a rotating cylinder? [IES-2000]

- (a) Doublet, vortex and uniform flow    (b) Source, vortex and uniform flow  
 (c) Sink, vortex and uniform flow    (d) Vortex and uniform flow

IES-19. Ans. (a)

IES-20. How could 'Magnus effect' be simulated as a combination? [IES-2009]

- (a) Uniform flow and doublet  
 (b) Uniform flow, irrotational vortex and doublet  
 (c) Uniform flow and vortex  
 (d) Uniform flow and line source

IES-20. Ans. (b) Magnus Effect: Flow about a rotating cylinder is equivalent to the combination of flow past a cylinder and a vortex. As such in addition to superimposed uniform flow and a doublet, a vortex is thrown at the doublet centre which will simulate a rotating cylinder in uniform stream. The pressure distribution will result in a force, a component of which will culminate in lift force. The phenomenon of generation of lift by a rotating object placed in a stream is known as Magnus effect

IES-21. Consider the following statements: [IES-2007]

1. The phenomenon of lift produced by imposing circulation over a doublet in a uniform flow is known as Magnus effect.
2. The path-deviation of a cricket ball from its original trajectory is due to the Magnus effect.

Which of the statement given above is/are correct?

- (a) 1 only    (b) 2 only    (c) Both 1 and 2    (d) Neither 1 nor 2

IES-21. Ans. (c)

## Lift on an Airfoil

IES-22. Consider the following statements: [IES-1999]

1. The cause of stalling of an aerofoil is the boundary layer separation and formation of increased zone of wake.
2. An aerofoil should have a rounded nose in supersonic flow to prevent formation of new shock.
3. When an aerofoil operates at an angle of incidence greater than that of stalling, the lift decreases and drag increase.
4. A rough ball when at certain speeds can attain longer range due to reduction of lift as the roughness induces early separation.

Which of these statements are correct?

<b>Flow Around Submerged Bodies-Drag &amp; Lift</b> <b>S K Mondal's</b> <span style="float: right;"><b>Chapter 14</b></span>
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(a) 3 and 4                      (b) 1 and 2                      (c) 2 and 4                      (d) 1 and 3

**IES-22. Ans. (d)**

- IES-23. Which one of the following is true of flow around a submerged body?** **[IES-1998]**
- (a) For subsonic, non-viscous flow, the drag is zero
  - (b) For supersonic flow, the drag coefficient is dependent equally on Mach number and Reynolds number
  - (c) The lift and drag coefficients of an aerofoil is independent of Reynolds number
  - (d) For incompressible flow around an aerofoil, the profile drag is the sum of from drag and skin friction drag.

**IES-23. Ans. (d)** Profile drag comprises two components. Surface friction drag and normal pressure drag (form drag).

- IES-24. When pressure drag over a body is large as compared to the friction drag, then the shape of the body is that of:** **[IES-2000]**
- (a) An aerofoil
  - (b) A streamlined body
  - (c) A two-dimensional body
  - (d) A bluff body

**IES-24. Ans. (d)**

- IES-25. Assertion (A): Aircraft wings are slotted to control separation of boundary layer especially at large angles of attack.** **[IES-2003]**  
**Reason (R): This helps to increase the lift and the aircraft can take off from, and land on, short runways.**
- (a) Both A and R are individually true and R is the correct explanation of A
  - (b) Both A and R are individually true but R is not the correct explanation of A
  - (c) A is true but R is false
  - (d) A is false but R is true

**IES-25. Ans. (c)**

- IES-26. The critical value of Mach number for a subsonic airfoil is associated with sharp increase in drag due to local shock formation and its interaction with the boundary layer. A typical value of this critical Mach number is of the order of:** **[IES-1995]**
- (a) 0.4 to 0.5
  - (b) 0.75 to 0.85
  - (c) 1.1 to 1.3
  - (d) 1.5 to 2.0

**IES-26. Ans. (c)** The critical Mach number is defined as free stream Mach number at which Sonic flow ( $M = 1$ ) is first achieved on the airfoil surface. Critical value of mach number for a subsonic airfoil is 1.1 to 1.3.

## Previous Years IAS Questions

### Expressions for Drag and Lift

- IAS-1. Assertion (A): A body with large curvature causes a larger pressure drag and, therefore, larger resistance to motion.** **[IAS-2002]**  
**Reason (R): Large curvature diverges the streamlines, decreases the velocity resulting in the increase in pressure and development of adverse pressure gradient leading to reverse flow near the boundary.**
- (a) Both A and R are individually true and R is the correct explanation of A
  - (b) Both A and R are individually true but R is not the correct explanation of A
  - (c) A is true but R is false



<b>Flow Around Submerged Bodies-Drag &amp; Lift</b> <b>S K Mondal's</b> <span style="float: right;"><b>Chapter 14</b></span>
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(d) A is false but R is true

IAS-1. Ans. (a)

IAS-2. **Assertion (A): In flow over immersed bodies.** **[IAS-1995]**

**Reason(R): Drag can be created without life. Life cannot be created without drag.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-2. Ans. (b). Both the statements of A and R are true, but R is not necessarily the explanation for A.

IAS-3. **Consider the following coefficients: ( $Re$  = Reynolds number)** **[IAS-1999]**

- |  |  |
|--|--|
| 1. $1.328 Re_{(0.5)}$ for laminar flow   | 2. $2.072 Re_{(0.2)}$ for turbulent flow |
| 3. $0.072 Re_{(0.2)}$ for turbulent flow | 4. $1.028 Re_{(0.5)}$ for laminar flow   |

**The coefficient of drag for a flat would include**

- (a) 1 and 2
- (b) 2 and 4
- (c) 1 and 3
- (d) 3 and 4

IAS-3. Ans. (c)

IAS-4. **Match List-I (Types of flow) with List-II (Basic ideal flows) and select the correct answer:** **[IES-2001, IAS-2003]**

List-I	List-II																								
A. Flow over a stationary cylinder	1. Source + sink + uniform flow																								
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(a) 1	4	3	2																						
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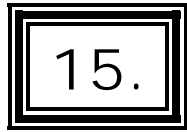
IAS-4. Ans. (d)

## Magnus Effect

IAS-5. **The Magnus effect is defined as** **[IAS-2002]**

- (a) The generation of lift per unit drag force
- (b) The circulation induced in an aircraft wing
- (c) The separation of boundary layer near the trailing edge of a slender body
- (d) The generation of lift on a rotating cylinder in a uniform flow

IAS-5. Ans. (d)



# Compressible Flow

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## Contents of this chapter

1. Compressible Flow
  2. Basic Thermodynamic Relations
  3. Sonic Velocity
  4. Mach Number
  5. Propagation of Disturbance in Compressible Fluid
  6. Stagnation Properties
  7. Area-Velocity Relationship and Effect of Variation of Area for Subsonic, Sonic and Supersonic Flows
  8. Flow of Compressible Fluid through a Convergent Nozzle
  9. Flow through Laval Nozzle (Convergent-Divergent Nozzle)
  10. Normal Shock Wave
  11. Oblique Shock Wave
  12. Fanno Line
  13. Fanno Line Representation of Constant Area Adiabatic Flow
  14. Rayleigh Line
  15. Steam Nozzle
  16. Supersaturated Flow
-

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous Years GATE Questions

## Compressible Flow

**GATE-1. Net force on a control volume due to uniform normal pressure alone**

- (a) Depends upon the shape of the control volume [GATE-1994]  
 (b) Translation and rotation  
 (c) Translation and deformation  
 (d) Deformation only

**GATE-1. Ans. (c)**

**GATE-2. List-I**

- (A) Steam nozzle  
 (B) Compressible flow  
 (C) Surface tension  
 (D) Heat conduction

**List-II**

1. Mach number  
 2. Reaction Turbine  
 3. Biot Number  
 4. Nusselt Number  
 5. Supersaturation  
 6. Weber Number

**[GATE-1997]**

**GATE-2. Ans. (A) -5, (B) -1, (C) -6, (d) -3**

## Basic Thermodynamic Relations

**GATE-3. Match List-I and List-II for questions below. No credit will be given for partial matching in each equation. Write your answers using only the letters A to D and numbers 1 to 6.**

**[GATE-1997]**

**List-I**

- (a) Steam nozzle  
 (b) Compressible flow  
 (c) Surface tension  
 (d) Heat conduction

**List-II**

1. Mach Number  
 2. Reaction Turbine  
 3. Biot Number  
 4. Nusselt Number  
 5. Supersaturation  
 6. Weber Number

**GATE-3. Ans. (a) -5, (b) -1, (c) -6, (d) -3**

## Sonic Velocity

**GATE-4. For a compressible fluid, sonic velocity is:**

**[GATE-2000]**

- (a) A property of the fluid  
 (b) Always given by  $(\gamma RT)^{1/2}$  where  $\gamma$ , R and T are respectively the ratio of specific heats, gas constant and temperature in K  
 (c) Always given by  $(\partial p / \partial \rho)_s^{1/2}$ . Where p,  $\rho$  and s are respectively pressure, density and entropy.  
 (d) Always greater than the velocity of fluid at any location.

**GATE-4. Ans. (a)**  $(\gamma RT)^{1/2}$  only when the process is adiabatic and  $(RT)^{1/2}$  when the process is isothermal.

**GATE-5. What is the speed of sound in Neon gas at a temperature of 500K (Gas constant of Neon is 0.4210 kJ/kg-K)?**

**[GATE-2002]**

(a) 492 m/s

(b) 460 m/s

(c) 592 m/s

(d) 543 m/s

GATE-5. Ans. (c)  $C = \sqrt{\gamma RT} = \sqrt{1.67 \times (0.4210 \times 10^3) \times 500} = 592 \text{ m/s}$

$$\gamma = 1 + \frac{2}{N} \text{ for monoatomic gas } N = 3, \gamma = 1 + 2/3 = 1.6$$

GATE-6. An aeroplane is cruising at a speed of 800 kmph at an altitude, where the air temperature is 0°C. The flight Mach number at this speed is nearly [GATE-1999]

(a) 1.5

(b) 0.254

(c) 0.67

(d) 2.04

GATE-6. Ans. (c)  $V = 800 \text{ km/hr} = \frac{800 \times 1000}{3600} \text{ m/s} = 222.22 \text{ m/s}$

$$C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 273} = 331.2 \text{ m/s}$$

$$\text{Mach Number (M)} = \frac{V}{C} = \frac{222.22}{331.2} = 0.67$$

## Stagnation Properties

GATE-7. In adiabatic flow with friction, the stagnation temperature along a streamline..... (increases/decreases/remains constant)

[GATE-1995]

GATE-7. Ans. Remains constant.

GATE-8. Subsonic and supersonic diffusers have the following geometry

(a) Divergent and convergent respectively

[GATE-1992]

(b) Both divergent

(c) Both convergent

(d) Convergent and divergent respectively

GATE-8. Ans. (a)

GATE-9. In a steady flow through a nozzle, the flow velocity on the nozzle axis is given by  $v = u_0(1 + 3x/L)$ , where  $x$  is the distance along the axis of the nozzle from its inlet plane and  $L$  is the length of the nozzle. The time required for a fluid particle on the axis to travel from the inlet to the exit plane of the nozzle is:

[GATE-2007]

(a)  $\frac{L}{u_0}$ (b)  $\frac{L}{3u_0} \ln 4$ (c)  $\frac{L}{4u_0}$ (d)  $\frac{L}{2.5u_0}$ 

GATE-9. Ans. (b)  $dt = \frac{dx}{V} \text{ or } T = \int dt = \int_0^L \frac{dx}{u_0 \left(1 + \frac{3x}{L}\right)} = \frac{L}{3u_0} \ln 4$

GATE-10. A correctly designed convergent-divergent nozzle working at a designed load is: [GATE-2002]

(a) Always isentropic

(b) Always choked

(c) Never choked

(d) Never isentropic

GATE-10. Ans. (b) Divergent portion will be act as nozzle only if flow is supersonic for that it must be choked.

GATE-11. A small steam whistle (perfectly insulated and doing no shaft work) causes a drop of 0.8 kJ/kg in the enthalpy of steam from entry to exit. If the kinetic energy of the steam at entry is negligible, the velocity of the steam at exit is: [GATE-2001]

(a) 4 m/s (b) 40 m/s (c) 80 m/s (d) 120 m/s  
 GATE-11. Ans. (b)  $V = \sqrt{2000 \times \Delta h}$  m/s =  $\sqrt{2000 \times 0.8}$  m/s = 40 m/s

## Previous Years IES Questions

### Compressible Flow

- IES-1. Acoustic velocity in an elastic gaseous medium is proportional to: [IES-2003]  
 (a) Absolute temperature  
 (b) Stagnation temperature  
 (c) Square root of absolute temperature  
 (d) Square root of stagnation temperature
- IES-1. Ans. (c)

- IES-2. The concentration of pressure pulses created by an object moving at Mach number of 0.5 is: [IES-2007]  
 (a) Larger ahead of the object (b) Larger behind the object  
 (c) Uniform within Mach cone (d) Uniform outside Mach cone

IES-2. Ans. (a) Pressure pulses moves with sonic velocity but object moves with half of sonic velocity so concentration of pressure pulse become larger ahead of the object.

- IES-3. Assertion (A): In a supersonic nozzle, with sonic condition at the throat, any reduction of downstream pressure will not be felt at the inlet of the nozzle. [IES-2004]  
 Reason (R): The disturbance caused downstream of supersonic flows travels at sonic velocity which cannot propagate upstream by Mach cone.  
 (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-3. Ans. (a)

- IES-4. If the velocity of propagation of small disturbances in air at 27°C is 330 m/s, then at a temperature of 54°C, its speed would be: [IES-1993]  
 (a) 600 m/s (b)  $330 \times \sqrt{2}$  m/s (c)  $330 / \sqrt{2}$  m/s (d)  $330 \times \sqrt{\frac{327}{300}}$  m/s

IES-4. Ans. (d) Velocity of propagation of small disturbance is proportional to  $\sqrt{T}$

So, new velocity of propagation will be  $330 \times \sqrt{\frac{327}{300}}$  m/s

### Mach Number

- IES-5. If a bullet is fired in standard air at 15°C at the Mach angle of 30°, the velocity of the bullet would be: [IES-2000]  
 (a) 513.5 m/s (b) 585.5 m/s (c) 645.5 m/s (d) 680.5 m/s

IES-5. Ans. (d) For Mach angle  $\alpha$ ,  $\sin \alpha = \frac{C_t}{V_t} = \frac{C}{V} = \frac{1}{M}$

$$\text{Where } C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times (273 + 15)} = 340 \text{ m/s}$$

$$\therefore V = \frac{C}{\sin \alpha} = \frac{340}{\sin 30} = 680 \text{ m/s}$$

**IES-6.** The stagnation temperature of an isentropic flow of air ( $k = 1.4$ ) is 400 K. If the temperature is 200K at a section, then the Mach number of the flow will be: [IES-1998]

- (a) 1.046 (b) 1.264 (c) 2.236 (d) 3.211

**IES-6. Ans. (c)**  $\frac{T_0}{T} = 1 + \frac{(\gamma - 1)}{2} M^2$  or,  $\frac{400}{200} = 1 + \frac{(1.4 - 1)}{2} M^2$  or  $M = \sqrt{5} = 2.236$

**IES-7.** An aero plane travels at 400 km/hr at sea level where the temperature is 15°C. The velocity of the aero plane at the same Mach number at an altitude where a temperature of -25°C prevailing, would be: [IES-2000]

- (a) 126.78 km/hr (b) 130.6 km/hr (c) 371.2 km/hr (d) 400.10 km/hr

**IES-7. Ans. (c)** For same Mach number

$$\frac{V_1}{C_1} = \frac{V_2}{C_2} \Rightarrow V_2 = V_1 \times \frac{C_2}{C_1} = V_1 \times \sqrt{\frac{T_2}{T_1}} = 400 \times \sqrt{\frac{(273 - 25)}{(273 + 15)}} = 371.2 \text{ km/hr}$$

**IES-8.** An aircraft is flying at a speed of 800 km/h at an altitude, where the atmospheric temperature is -20°C. What is the approximate value of the Mach number of the aircraft? [IES-2004]

- (a) 0.653 (b) 0.697 (c) 0.240 (d) 0.231

**IES-8. Ans. (b)** Sonic velocity (a) corresponding to temperature -20°C or 253 K or,

$$a = \text{standard velocity} \times \sqrt{\frac{T}{T_0}} \quad \therefore C = \sqrt{\frac{\gamma RT}{M}}$$

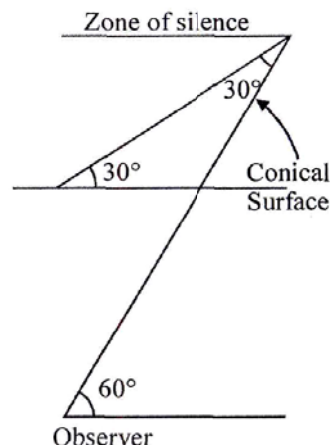
$$\text{or } a = 331 \times \sqrt{\frac{253}{273}} \text{ m/s} = 331 \times \sqrt{\frac{253}{273}}$$

$$\text{velocity of aircraft (V)} = 800 \times \frac{5}{18} \text{ m/s} = 222.2 \text{ m/s}$$

$$\text{Therefore Mach Number (M)} = \frac{V}{C} = \frac{222.2}{331 \times \sqrt{\frac{253}{273}}} = 0.697$$

**IES-9.** A supersonic aircraft is ascending at an angle of 30° to the horizontal. When an observer at the ground hears its sound, the aircraft is seen at an elevation of 60° to the horizontal. The flight Mach number of the aircraft is Zone of silence

- (a)  $2/\sqrt{3}$  (b)  $\sqrt{3}/2$   
(c) 1/2 (d) 2



[IES-1995]

IES-9. Ans. (a)  $\alpha = \sin^{-1}\left(\frac{1}{M_a}\right); \quad \therefore M_a = \frac{2}{\sqrt{3}}$

IES-10. Match angle  $\alpha$  and Mach number  $M$  are related as:

[IES-1999]

(a)  $M = \sin^{-1}\left(\frac{1}{\alpha}\right)$  (b)  $\alpha = \cos^{-1}\left(\frac{\sqrt{M^2 - 1}}{M}\right)$   
 (c)  $\alpha = \tan^{-1}\left(\sqrt{M^2 - 1}\right)$  (d)  $\alpha = \operatorname{cosec}^{-1}\left(\frac{1}{M}\right)$

IES-10. Ans. (b) To confuse student this type of question is given.

See,  $\sin \alpha = \frac{1}{M}$  or  $\alpha = \sin^{-1}\left(\frac{1}{M}\right)$  But it is not given.

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{1}{M}\right)^2 = \frac{M^2 - 1}{M^2}$$

$$\text{or } \cos \alpha = \frac{\sqrt{M^2 - 1}}{M} \quad \text{or } \alpha = \cos^{-1}\left(\frac{\sqrt{M^2 - 1}}{M}\right)$$

IES-11. The Mach number for nitrogen flowing at 195 m/s when the pressure and temperature in the undisturbed flow are 690 kN/m abs and 93°C respectively will be: [IES-1992]

- (a) 0.25 (b) 0.50 (c) 0.66 (d) 0.75

IES-11. Ans. (b)  $c = \sqrt{KRT} = \sqrt{1.4 \times (297) \times (273 + 93)} = 390 \text{ m/s}$

$$\therefore M = \frac{195}{390} = 0.5$$

## Propagation of Disturbance in Compressible Fluid

IES-12. An aircraft flying at an altitude where the pressure was 35 kPa and temperature -38°C, stagnation pressure measured was 65.4 kPa. Calculate the speed of the aircraft. Take molecular weight of air as 28.

[IES-1998]

IES-12. Ans. (349 m/s) Here  $\gamma$  is not given so compressibility is neglected

$$p_s = p + \frac{\rho V^2}{2} \quad \text{where, } \rho = \frac{m}{V} = \frac{pM}{RT} = \frac{35 \times 28}{8.314 \times (273 - 38)} = 0.5 \text{ kg/m}^3$$

$$\text{Therefore } V = \sqrt{\frac{2(p_s - p)}{\rho}} = \sqrt{\frac{2(65.4 - 35) \times 10^3}{0.5}} = 349 \text{ m/s}$$

IES-13. The eye of a tornado has a radius of 40 m. If the maximum wind velocity is 50 m/s, the velocity at a distance of 80 m radius is: [IES-2000]

- (a) 100 m/s (b) 2500 m/s (c) 31.25 m/s (d) 25 m/s

IES-13. Ans. (d) In a tornado angular momentum must conserve.

$$mr_1V_1 = mr_2V_2 \quad \text{or } r_1V_1 = r_2V_2$$

IES-14. While measuring the velocity of air ( $\rho = 1.2 \text{ kg/m}^3$ ), the difference in the stagnation and static pressures of a Pitot-static tube was found to be 380 Pa. The velocity at that location in m/s is: [IES-2002]

(a) 24.03

(b) 4.02

(c) 17.8

(d) 25.17

**IES-14. Ans. (d)**  $p_o = p + \frac{\rho V^2}{2}$ , when compressibility effects are neglected

**IES-15. Match List-I (Property ratios at the critical and stagnation conditions) with List-II (Values of ratios) and select the correct answer using the codes given below the Lists: [IES-1997]**

**List-I**

**A.**  $\frac{T^*}{T_0}$

**B.**  $\frac{\rho^*}{\rho_0}$

**C.**  $\frac{p^*}{p_0}$

**D.**  $\frac{S^*}{S_0}$

**List-II**

**1.**  $\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$

**2.**  $\frac{2}{\gamma+1}$

**3.** 1

**4.**  $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>(a)</b>	<b>2</b>	<b>1</b>	<b>4</b>	<b>3</b>	<b>(b)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>(c)</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>4</b>	<b>(d)</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>3</b>

**IES-16. Ans. (a)** (a)  $\frac{T^*}{T_0} \rightarrow \frac{2}{\gamma+1}$  (b)  $\frac{\rho^*}{\rho_0} \rightarrow \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$   
 (c)  $\frac{p^*}{p_0} \rightarrow \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$  (d)  $\frac{S^*}{S_0} \rightarrow 1$

**IES-17. In isentropic flow between two points, the stagnation: [IES-1998; IAS-2002]**

- (a) Pressure and stagnation temperature may vary
- (b) Pressure would decrease in the direction of the flow.
- (c) Pressure and stagnation temperature would decrease with an increase in velocity
- (d) Pressure, stagnation temperature and stagnation density would remain constant throughout the flow.

**IES-17. Ans. (d)** Stagnation temperature cannot vary.

**IES-18. Consider the following statements pertaining to isentropic flow:**

- 1. To obtain stagnation enthalpy, the flow need not be decelerated isentropically but should be decelerated adiabatically.
- 2. The effect of friction in an adiabatic flow is to reduce the stagnation pressure and increase entropy.
- 3. A constant area tube with rough surfaces can be used as a subsonic nozzle.

**Of these correct statements are:**

**[IES-1996]**

- (a) 1, 2 and 3
- (b) 1 and 2
- (c) 1 and 3
- (d) 2 and 3

**IES-18. Ans. (d)** To obtain stagnation enthalpy, the flow must be decelerated isentropically.



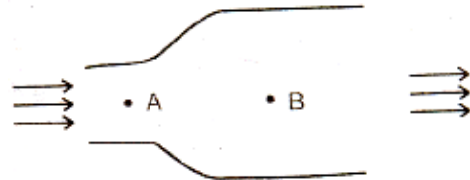
IES-19. For adiabatic expansion with friction through a nozzle, the following remains constant: [IES-1992]

- (a) Entropy (b) Static enthalpy  
(c) Stagnation enthalpy (d) Stagnation pressure

IES-19. Ans. (c)

## Area-Velocity Relationship and Effect of Variation of Area for Subsonic, Sonic and Supersonic Flows

IES-20. A compressible fluid flows through a passage as shown in the above diagram. The velocity of the fluid at the point A is 400 m/s.



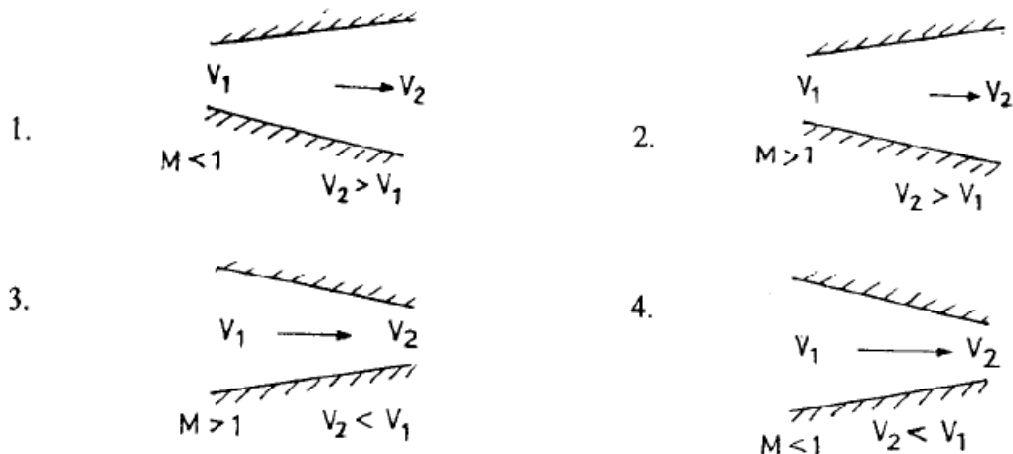
Which one of the following is correct? At the point B, the fluid experiences [IES-2004]

- (a) An increase in velocity and decrease in pressure  
(b) A decrease in velocity and increase in pressure  
(c) A decrease in velocity and pressure  
(d) An increase in velocity and pressure.

IES-20. Ans. (a) Velocity at A is very high we may say it is supersonic so above diagram is a divergent nozzle.

IES-21. Which of the following diagrams correctly depict the behaviour of compressible fluid flow in the given geometries? [IES-1993]

Select the correct answer using the codes given below:



Codes:

- (a) 1 and 4 (b) 2 and 4 (c) 2 and 3 (d) 1 and 3

IES-21. Ans. (c)

IES-22. Match List-I (Names) with List-II (Figures) given below the lists:

## List-I (Names)

## List-II (Figures)

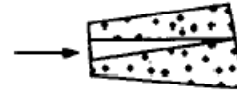
A. Subsonic nozzle

1.



B. Supersonic nozzle

2.



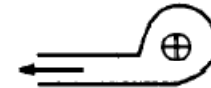
C. Subsonic diffuser

3.

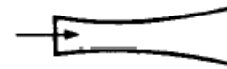


D. Centrifugal compressor

4.



5.



Codes:

[IES-2001]

	A	B	C	D		A	B	C	D
(a)	3	4	2	5	(b)	1	5	3	4
(c)	3	5	2	4	(d)	1	4	3	5

IES-22. Ans. (b)

IES-23. The Mach number at inlet of gas turbine diffuser is 0.3. The shape of the diffuser would be: [IES-1992]

- (a) Converging (b) Diverging  
(c) Stagnation enthalpy (d) Stagnation pressure

IES-23. Ans. (b)

IES-24. For one-dimensional isentropic flow in a diverging passage, if the initial static pressure is  $P_1$  and the initial Mach number is  $M_1$  ( $M_1 < 1$ ), then for the downstream flow [IES-1993]

- (a)  $M_2 < M_1$ ;  $p_2 < p_1$  (b)  $M_2 < M_1$ ;  $p_2 > p_1$   
(c)  $M_2 > M_1$ ;  $p_2 > p_1$  (d)  $M_2 > M_1$ ;  $p_2 < p_1$

IES-24. Ans. (b) For down stream flow,  $M_2 < M_1$  and  $P_2 > P_1$  for diverging section and subsonic flow conditions.

IES-25. Consider the following statements:

[IES-1999]

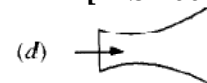
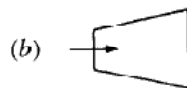
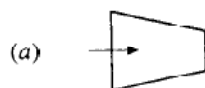
1. De Laval nozzle is a subsonic nozzle.
2. Supersonic nozzle is a converging passage.
3. Subsonic diffuser is a diverging passage.

Which of these statements is/are correct?

- (a) 1 and 2 (b) 2 and 3 (c) 1 alone (d) 3 alone

IES-25. (d) Only third statement is correct, i.e. subsonic diffuser is a diverging passage.

IES-26. Which one of the following diagrams depicts, correctly the shape of a supersonic diffuser? [IES-1999]



IES-26. Ans. (a) Supersonic diffuser has converging section.

IES-27. It is recommended that the diffuser angle should be kept less than  $18^\circ$  because [IES-2006]

- (a) Pressure decreases in flow direction and flow separation may occur
- (b) Pressure decreases in flow direction and flow may become turbulent
- (c) Pressure increases in flow direction and flow separation may occur
- (d) Pressure increases in flow direction and flow may become turbulent

IES-27. Ans. (c)

IES-28. If the cross-section of a nozzle is increasing in the direction of flow in supersonic flow, then in the downstream direction [IES-2005]

- (a) Both pressure and velocity will increase
- (b) Both pressure and velocity will decrease
- (c) Pressure will increase but velocity will decrease
- (d) Pressure will decrease but velocity will increase

IES-28. Ans. (d)

IES-29. In a subsonic diffuser [IES-2007]

- (a) Static pressure increases with Mach number
- (b) Mach number decreases with increasing area ratio
- (c) Static pressure decreases with Mach number
- (d) Area ratio decreases in the flow direction

IES-29. Ans. (a)

IES-30. Which one of the following is the correct statement? [IES-2005]

To get supersonic velocity of steam at nozzle exit with a large pressure drop across it, the duct must:

- (a) Converge from inlet to exit
- (b) Diverge from inlet to exit
- (c) First converge to the throat and then diverge till exit
- (d) Remain constant in cross-section

IES-30. Ans. (c)

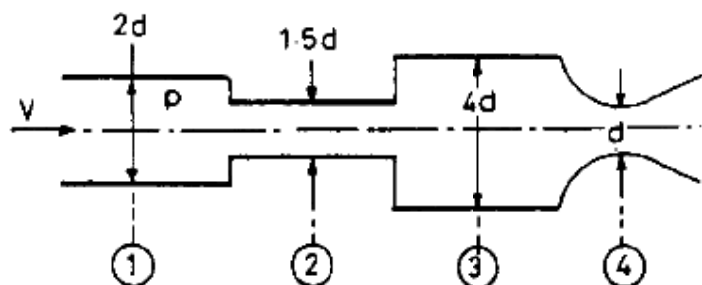
IES-31. Assertion (A): Gas and steam nozzles are shaped at inlet in such a way that the nozzle converges rapidly over the first portion of its length.

Reason(R): This shape is provided so that velocity at inlet to the nozzle is negligibly small in comparison with the exit velocity. [IES-2005]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-31. Ans. (c)

IES-32. Water flow through a pipeline having four different diameters at 4 stations is shown in the given figure. The correct sequence of station numbers in the decreasing order of pressures is:



[IES-1996]

- (a) 3, 1, 4, 2      (b) 1, 3, 2, 4      (c) 1, 3, 4, 2      (d) 3, 1, 2, 4
- IES-32. Ans. (d)** It must be remembered that velocity and pressure head behave in reverse fashion. Thus velocity will be maximum for least area and pressure will be minimum. Pressure is least where velocity is highest and at larger cross-section for constant discharge, velocity lowers.

## Flow of Compressible Fluid through a Convergent Nozzle

- IES-33.** The critical pressure ratio  $\left(\frac{P_2}{P_1}\right)$  for maximum discharge through a nozzle is: [IAS-1999; IES-2002, 2004]

(a)  $\left(\frac{2}{n+1}\right)^{\left(\frac{n-1}{n}\right)}$       (b)  $\left(\frac{2}{n+1}\right)^{\left(\frac{n}{n-1}\right)}$       (c)  $\left(\frac{2}{n-1}\right)^{\left(\frac{n-1}{n}\right)}$       (d)  $\left(\frac{2}{n-1}\right)^{\left(\frac{n+1}{n}\right)}$

- IES-33. Ans. (b)**

- IES-34.** What is the critical pressure ratio for isentropic nozzle flow with ratio of specific heats as 1.5? [IES-2004]

(a)  $(0.8)^3$       (b)  $(0.8)^{0.6}$       (c)  $(1.25)^{0.33}$       (d)  $(1.25)^3$

- IES-34. Ans. (a)** Just use  $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

- IES-35.** The index of expansion of dry saturated steam flowing through a nozzle is equal to 1.135, and then what is the critical pressure ratio for this flowing steam in the nozzle? [IES-2009]

(a) 0.96      (b) 0.58      (c) 0.33      (d) 0.15

- IES-35. Ans. (b)** Critical pressure ratio =  $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} = \left(\frac{2}{1.135+1}\right)^{\left(\frac{1.135}{0.135}\right)} = 0.577$

- IES-36.** The critical pressure ratios for the flow of dry saturated and superheated steam through a nozzle are respectively. [IES-1994]

(a) 0.5279 and 0.528      (b) 0.577 and 0.550  
(c) 0.577 and 0.546      (d) 0.5279 and 0.546

- IES-36. Ans. (c)** Critical pressure ratio for dry saturated and superheated steam through a nozzle are 0.577 and 0.546.

- IES-37.** In a De Laval nozzle expanding superheated steam from 10 bar to 0.1 bar, the pressure at the minimum cross-section will be: [IES-1993]

(a) 3.3 bar      (b) 5.46 bar      (c) 8.2 bar      (d) 9.9 bar

- IES-37. Ans. (b)** The isentropic index for superheated steam is 1.3 and throat pressure

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \text{ or } \frac{p_2}{10} = \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{1.3-1}} \text{ or } p_2 = 5.46 \text{ bar}$$

- IES-38.** If the cross-section of a nozzle is increasing in the direction of flow in supersonic flow, then in the downstream direction. [IES-2005]

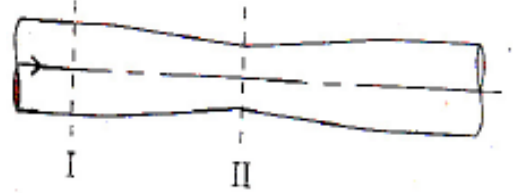
(a) Both pressure and velocity will increase.  
(b) Both pressure and velocity will decrease.  
(c) Pressure will increase but velocity will decrease.

(d) Pressure will decrease but velocity will increase.

IES-38. Ans. (d)

### Flow through Laval Nozzle (Convergent-Divergent Nozzle)

IES-39. At location-I of a horizontal line, the fluid pressure head is 32 cm and velocity head is 4 cm. The reduction in area at location II is such that the pressure head drops down to zero. The ratio of velocities at location -II to that at location-I is:



[IES-2001]

- (a) 3 (b) 2.5 (c) 2 (d) 1.5

IES-39. Ans. (a)  $32 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$  or  $\frac{V_2}{V_1} = \sqrt{\frac{32}{V_1^2/2g} + 1} = \sqrt{8+1} = 3$

IES-40. Consider the following statements:

[IES-1996]

A convergent-divergent nozzle is said to be choked when

1. Critical pressure is attained at the throat.
2. Velocity at the throat becomes sonic.
3. Exit velocity becomes supersonic.

Of these correct statements are

- (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-40. Ans. (b) A convergent divergent nozzle is said to be choked when critical pressure is attained at the throat and velocity at the throat becomes sonic.

IES-41. Consider the following statements:

[IES-2009]

Choked flow through a nozzle means:

1. Discharge is maximum
2. Discharge is zero
3. Velocity at throat is supersonic
4. Nozzle exit pressure is less than or equal to critical pressure.

Which of the above statements is/are correct?

- (a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1 and 4

IES-41. Ans. (d)

IES-42. Steam pressures at the inlet and exit of a nozzle are 16 bar and 5.2 bar respectively and discharge is 0.28 m<sup>3</sup>/s. Critical pressure ratio is 0.5475. If the exit pressure is reduced to 3.2 bar then what will be the flow rate in m<sup>3</sup>/s?

[IES-2009]

- (a) 0.280 (b) 0.328 (c) 0.356 (d) 0.455

IES-42. Ans. (a) Flow Ratio remains constant for choked condition. Therefore flow rate will remain same even when there is decrease in the pressure to 3.2 bar.

$\therefore$  Flow rate in m<sup>3</sup> / sec = 0.280 m<sup>3</sup> / sec

IES-43. Assertion (A): A correctly designed convergent divergent nozzle working at designed conditions is always choked. [IES-2005]

Reason(R): In these conditions the mass flow through the nozzle is minimum.

IES-43. Ans. (c)

- IES-44. Consider the following statements in relation to a convergent-divergent steam nozzle operating under choked conditions: [IES-2002]
1. In the convergent portion steam velocity is less than sonic velocity
  2. In the convergent portion steam velocity is greater than sonic velocity
  3. In the divergent portion the steam velocity is less than sonic velocity
  4. In the divergent portion the steam velocity is greater than sonic velocity
- Which of the above statements are correct?
- (a) 1 and 3                      (b) 1 and 4                      (c) 2 and 3                      (d) 2 and 4

IES-44. Ans. (b)

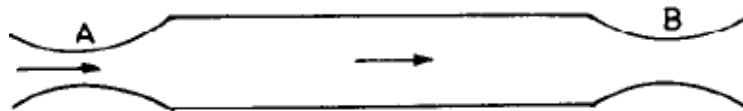
- IES-45. Which one of the following is correct? [IES-2008]  
For incompressible flow a diverging section acts as a diffuser for upstream flow which is:
- (a) Subsonic only                      (b) Supersonic only  
(c) Both subsonic and supersonic                      (d) Sonic

IES-45. Ans. (c) For incompressible flow a diverging section acts as a diffuser for both subsonic and supersonic. For compressible flow a diverging section acts as a diffuser for subsonic flow only.

- IES-46. Assertion (A): For pressure ratio greater than the critical pressure ratio, a convergent-divergent nozzle is required. [IES-1996]  
Reason (R): Divergent portion increases the flow area which increases the mass flow rate.
- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-46. Ans. (c) A is correct but R is wrong.

- IES-47. Two identical convergent divergent nozzles A and B are connected in series, as shown in the given figure, to carry a compressible fluid. Which one of the following statements regarding the velocities at the throats of the nozzles is correct? [IES-1994]
- (a) Sonic and supersonic velocities exist at the throats of nozzles A and B respectively.  
(b) Sonic velocity can exist at throats of both nozzles A and B.  
(c) Sonic velocity will always exist at the throat of nozzle A while subsonic velocity will exist at throat of nozzle B.  
(d) Sonic velocity exists at the throat of nozzle B while subsonic velocity exists at throat of nozzle A.



IES-47. Ans. (b)

- IES-48. A convergent divergent nozzle is designed for a throat pressure 2.2 MPa. If the pressure at the exit is 2.3 MPa, then the nature of the flow at the exit will be: [IES-1992]
- (a) Supersonic                      (b) Sonic

- (c) Subsonic (d) Not predictable with the available data  
 IES-48. Ans. (c)

- IES-49. Assertion (A): A convergent-divergent nozzle may give supersonic or subsonic flow at the exit even if the throat is choked. [IES-2001]  
 Reason (R): Depending on the back pressure ratio  $P_b/P_o$  the divergent part of the nozzle may act as a supersonic nozzle or a subsonic diffuser.  
 (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-49. Ans. (a)

- IES-50. Consider the following statements for compressible flow through a varying area passage: [IES-2008]  
 1. For a convergent nozzle, if the exit pressure is less than critical, external flow will not be isentropic.  
 2. Supersonic-subsonic diffuser would appear similar to nozzle and works without irreversibility.  
 Which of the statements given above is/are correct?  
 (a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

IES-50. Ans. (c)

## Normal Shock Wave

- IES-51. Across a normal shock wave in a converging-diverging nozzle for adiabatic flow, which of the following relations are valid? [IES-2007]  
 (a) Continuity and energy equations, equation of state, isentropic relation  
 (b) Energy and momentum equations, equation of state, isentropic relation  
 (c) Continuity, energy and momentum equations, equation of state  
 (d) Equation of state, isentropic relation, momentum equation, mass-conservation Principle.

IES-51. Ans. (d)

- IES-52. Which one of the following is proper for a normal shock wave?  
 (a) Reversible (b) Irreversible [IES-2009]  
 (c) Isentropic (d) Occurs in a converging tube

IES-52. Ans. (b)

- IES-53. In a normal shock in a gas, the: [IES-1998; 2006]  
 (a) Upstream shock is supersonic  
 (b) Upstream flow is subsonic  
 (c) Downstream flow is sonic  
 (d) Both downstream flow and upstream flow are supersonic

IES-53. Ans. (a)

- IES-54. Which of the following parameters decrease across a normal shock wave? [IES-2006]  
 1. Mach number 2. Static pressure  
 3. Stagnation pressure 4. Static temperature  
 Select the correct answer using the codes given below:  
 (a) Only 1 and 3 (b) Only 2 and 4 (c) 1, 2 and 3 (d) 2, 3 and 4

IES-54. Ans. (a)

- IES-55. If the upstream Mach number of a normal shock occurring in air ( $k = 1.4$ ) is 1.68, then the Mach number after the shock is: [IES-2000]  
 (a) 0.84 (b) 0.646 (c) 0.336 (d) 0.546

IES-55. Ans. (b)  $M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)} = \frac{(1.4-1) \times 1.68^2 + 2}{2 \times 1.4 \times 1.68^2 - (1.4-1)} = 0.417$   
 or  $M_2 = \sqrt{0.417} = 0.646$

- IES-56. Air from a reservoir is to be passed through a supersonic nozzle so that the jet will have a Mach number of 2. If the static temperature of the jet is not to be less than  $27^\circ\text{C}$ , the minimum temperature of air in the reservoir should be: [IES-1999]  
 (a)  $48.6^\circ\text{C}$  (b)  $167^\circ\text{C}$  (c)  $267^\circ\text{C}$  (d)  $367^\circ\text{C}$

IES-56. Ans. (c)  $\frac{T_R (\text{temperature of air in reservoir})}{T_j (\text{static temperature of jet})} = 1 + \left(\frac{\gamma-1}{2}\right) M_N^2 = 1 + \left(\frac{1.4-1}{2}\right) \times 2^2 = 1.8$   
 $\therefore T_R = 300 \times 1.8 = 540 \text{ K} = 267^\circ\text{C}$

- IES-57. In a normal shock in a gas: [IES-2002]  
 (a) The stagnation pressure remains the same on both sides of the shock  
 (b) The stagnation density remains the same on both sides of the shock.  
 (c) The stagnation temperature remains the same on both sides of the shock  
 (d) The Mach number remains the same on both sides of the shock.

IES-57. Ans. (c)

- IES-58. A normal shock: [IES-2002]  
 (a) Causes a disruption and reversal of flow pattern  
 (b) May occur only in a diverging passage  
 (c) Is more severe than an oblique shock  
 (d) Moves with a velocity equal to the sonic velocity

IES-58. Ans. (b)

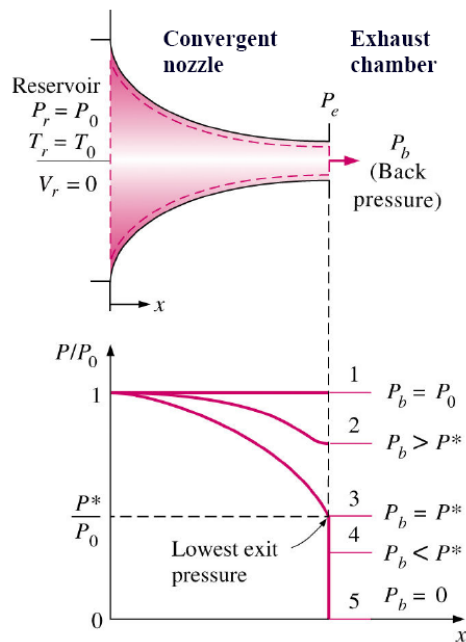
- IES-59. The fluid property that remains unchanged across a normal shock wave is: [IES-2003]  
 (a) Stagnation enthalpy (b) Stagnation pressure  
 (c) Static pressure (d) Mass density

IES-59. Ans. (a)

- IES-60. Consider the following statements: [IES-2005]  
 In the case of convergent nozzle for compressible flow,  
 1. No shock wave can occur at any pressure ratio.  
 2. No expansion wave can occur below a certain pressure ratio.  
 3. Expansion wave can occur below a certain pressure ratio  
 4. Shock wave can occur above a certain pressure ratio.  
 Which of the following statements given above are correct?  
 (a) 1 and 2 (b) 3 and 4 (c) 1 and 3 (d) 2 and 4

IES-60. Ans. (d)





- **State 1:**  $P_b = P_0$ , there is no flow, and pressure is constant.
- **State 2:**  $P_b < P^*$ , pressure along nozzle decreases. Also  $P_b > P^*$  (critical).
- **State 3:**  $P_b = P^*$ , flow at exit is sonic ( $M = 1$ ), creating maximum flow rate called choked flow. For the above states, nozzle exit  $p_r$  is same as exhaust chamber pressure.
- **State 4:**  $P_b < P^*$ , there is no change in flow or pressure distribution in comparison to state 3.

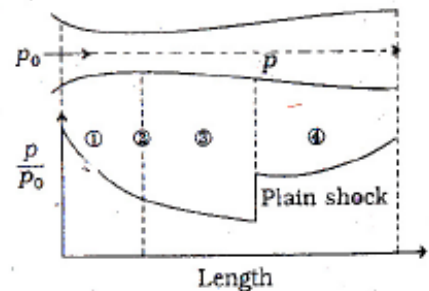
Here, nozzle exit pressure  $P_e$  does not decrease even when  $P_b$  is further reduced below critical. This change of pressure from  $P_e$  to  $P_b$  takes place outside the nozzle exit through a expansion wave.

- **State 5:**  $P_b = 0$ , same as state 4.

IES-61. The plot for the pressure ratio along the length of convergent-divergent nozzle is shown in the given figure. The sequence of the flow condition labeled ①, ②, ③ and ④ in the figure is respectively.

- Supersonic, sonic, subsonic and supersonic
- Sonic, supersonic, subsonic and supersonic
- Subsonic, supersonic, sonic and subsonic
- Subsonic, sonic, supersonic and subsonic

IES-61. Ans. (d)



[IES-2000]

IES-62. Consider the following statements pertaining to one-dimensional isentropic flow in a convergent-divergent passage [IES-2003]

1. A convergent-divergent passage may function as a supersonic nozzle or a venturi depending on the back pressure.
2. At the throat, sonic conditions exits for subsonic or supersonic flow at the outlet.
3. A supersonic nozzle discharges fluid at constant rate even if the exit pressure is lower than the design pressure.
4. A normal shock appears in the diverging section of the nozzle if the back pressure is above the design pressure but below a certain minimum pressure for venturi operation.

Which of these statements are correct?

- (a) 1, 2, 3 and 4
- (b) 1, 3 and 4
- (c) 2, 3 and 4
- (d) 1 and 2

IES-62. Ans. (a) At the throat, sonic conditions not exists for subsonic flow when it is venture.

IES-63. Which one of the following is correct? [IES-2008]

In a normal shock wave in one dimensional flow,

- (a) The entropy remains constant
- (b) The entropy increases across the shock
- (c) The entropy decreases across the shock
- (d) The velocity, pressure and density increase across the shock

IES-63. Ans. (b) Entropy increases across a shock with consequent decrease in stagnation pressure and stagnation density across the shock.

IES-64. Consider the following statements: [IES-1996]

Across the normal shock, the fluid properties change in such a manner that the

- 1. Velocity of flow is subsonic
- 2. Pressure increases
- 3. Specific volume decreases
- 4. Temperature decreases

Of these correct statements are

- (a) 2, 3 and 4
- (b) 1, 2 and 4
- (c) 1, 3 and 4
- (d) 1, 2 and 3

IES-64. Ans. (d)

IES-65. Assertion (A): A normal shock wave can occur at any section in a convergent-divergent nozzle. [IES-2008]

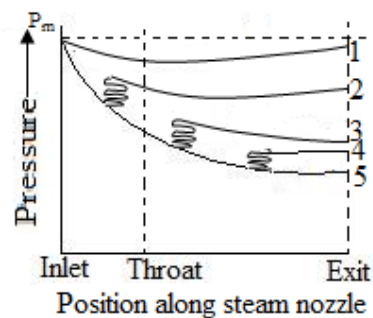
Reason (R): A normal shock wave occurs only when the flow of the fluid is supersonic and the subsequent flow after the shock is subsonic.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-65. Ans. (d) A shock wave takes place in the diverging section of a nozzle, in a diffuser, throat of a supersonic wind tunnel, in front of sharp-nosed bodies.

IES-66. Consider the curves in the sketch shown below (indicates normal shock) Out of these curves, those which are not correctly drawn will include

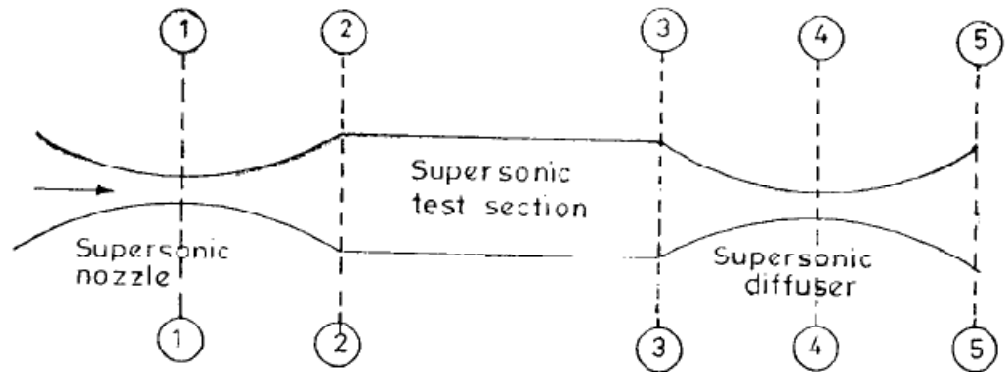
- (a) 1 and 2
- (b) 3 and 4
- (c) 2 and 4
- (d) 1 and 5



[IAS-1998]

IES-66. Ans. (c)

IES-67. The given figure represents a schematic view of the arrangement of a supersonic wind tunnel section. A normal shock can exist without affecting the test conditions. [IES-1993]



- (a) Between sections 4 and 5  
(b) At section 4  
(c) Between sections 4 and 3  
(d) Between sections 1 and 2

**IES-67. Ans. (d)** A normal shock can exist between 1 and 2 without affecting the test conditions, as it can be swallowed through the second throat by making it larger than the first.

**IES-68.** In a perfect gas having ratio of specific heats as 1.4 what is the strength of a normal shock with upstream Mach number equal to 5.0? [IES-2004]  
(a) 27 (b) 28 (c) 29 (d) 24

**IES-68. Ans. (b)** Strength of normal shock =  $\frac{\text{pressure rise across the shock } (\Delta P)}{\text{upstream pressure } (P_1)}$

$$= \frac{2\gamma}{\gamma+1} [M_1^2 - 1] = \frac{2 \times 1.4}{1.4 + 1} \times [5^2 - 1] = 28$$

**IES-69.** Assertion (A): A normal shock always makes a supersonic flow of a compressible fluid subsonic, but an oblique shock may not ensure subsonic flow after the shock. [IES-2003]

Reason (R): A normal shock reduces the stagnation pressure and stagnation enthalpy considerably whereas the loss at oblique shock is minimized.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IES-69. Ans. (c)**

**IES-70.** Which one of the following is correct for tangential component of velocities before and after an oblique shock? [IES-2009]

- (a) Unity (b) Equal (c) Unequal (d) None of the above

**IES-70 Ans. (b)**

**IES-71.** Introduction of a Pitot tube in a supersonic flow would produce [IES-1994]

- (a) Normal shock at the tube nose  
(b) Curved shock at the tube nose  
(c) Normal shock at the upstream of the tube nose  
(d) Curved shock at the upstream of the tube nose

**IES-71. Ans. (a)**

**IES-72.** In flow through a convergent nozzle, the ratio of back pressure to the

inlet pressure is given by the relation  $\frac{p_B}{p_1} = \left[ \frac{2}{\gamma + 1} \right]^{-\gamma/(\gamma-1)}$  [IES-1996]

If the back pressure is lower than  $P_B$  given by the above equation, then

- (a) The flow in the nozzle is supersonic.
- (b) A shock wave exists inside the nozzle.
- (c) The gases expand outside the nozzle and a shock wave appears outside the nozzle.
- (d) A shock wave appears at the nozzle exit.

**IES-72. Ans. (c)**

**IES-73.** At which location of a converging - diverging nozzle, does the shock-boundary layer interaction take place? [IES-1995]

- (a) Converging portion
- (b) Throat
- (c) Inlet
- (d) Diverging portion

**IES-73. Ans. (d)** Shock-boundary layer interaction takes place in diverging portion of nozzle.

**IES-74.** A converging diverging nozzle is connected to a gas pipeline. [IES-2006]  
At the inlet of the nozzle (converging section) the Mach number is 2. It is observed that there is a shock in the diverging section. What is the value of the Mach number at the throat?

- (a)  $< 1$
- (b) Equal to 1
- (c)  $> 1$
- (d)  $\geq 1$

**IES-74. Ans. (b)**

**IES-75.** Shock waves in nozzles would occur while turbines are operating

- (a) At overload conditions
- (b) At part load conditions
- (c) Above critical pressure ratio
- (d) At all off-design conditions

**IES-75. Ans. (d)**

## Oblique Shock Wave

**IES-76.** For oblique shock, the downstream Mach number: [IES-1997]

- (a) Is always more than unity
- (b) Is always less than unity
- (c) May be less or more than unity
- (d) Can never be unity.

**IES-76. Ans. (c)** If  $M > 1$ , then weak shock wave,  
If  $M < 1$ , then strong shock wave.

## Fanno Line

**IES-77.** Assertion (A): In the case of Fanno line flow, in the subsonic region friction causes irreversible acceleration. [IES-1997]

Reason (R): In the case of Fanno line, flow, decrease in entropy is not possible either for supersonic or subsonic flows.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-77. Ans. (c)**

**IES-78.** During subsonic, adiabatic flow of gases in pipes with friction, the flow properties go through particular mode of changes. Match List-I (Flow

properties) with List-II (Mode of changes) and select the correct answer: [IES-2002]

## List-I

- A. Pressure  
B. Density  
C. Temperature  
D. Velocity

## List-II

1. Increase in flow direction  
2. Decreases with flow direction

Codes:	A	B	C	D		A	B	C	D
(a)	1	1	2	2	(b)	2	2	2	1
(c)	2	2	1	2	(d)	2	1	1	2

IES-78. Ans. (d)

IES-79. The prime parameter causing change of state in a Fanno flow is:

[IES-1998]

- (a) Heat transfer (b) Area change (c) Friction (d) Buoyancy

IES-79. Ans. (c)

IES-80. Fanno line flow is a flow in a constant area duct:

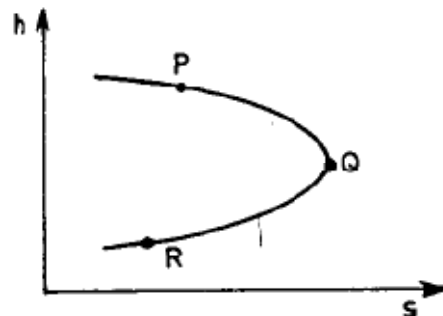
[IES-1997]

- (a) With friction and heat transfer but in the absence of work.  
(b) With friction and heat transfer and accompanied by work.  
(c) With friction but in the absence of heat transfer or work.  
(d) Without friction but accompanied by heat transfer and work.

IES-80. Ans. (c) Fanno line flow is the combination of momentum and continuity equation. For constant area duct fanno line flow with friction but in the absence of heat transfer and work.

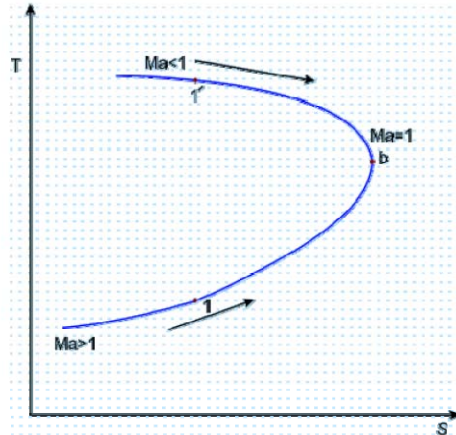
IES-81. In the Fanno line shown in the given figure

- (a) Subsonic flow proceeds along PQR.  
(b) Supersonic flow proceeds along PQR.  
(c) Subsonic flow proceeds along PQ and supersonic flow proceeds along RQ.  
(d) Subsonic flow proceeds along RQ and supersonic flow proceeds along PQ.



[IES-1994]

IES-81.Ans. (c)



### Fanno Line Representation of Constant Area Adiabatic Flow

IES-82. Which one of the following statements is correct about the Fanno flow? [IES-2007]

- (a) For an initially subsonic flow, the effect of friction is to decrease the Mach number towards unity
- (b) For an initially supersonic flow, the effect of friction is to increase the Mach number towards unity
- (c) At the point of maximum entropy, the Mach number is unity
- (d) Stagnation pressure always increases along the Fanno line

IES-82. Ans. (c)

IES-83. Which one of the following is the correct statement? [IES-1999]

- (a) The Mach number is less than 1 at a point where the entropy is maximum whether it is Rayleigh or Fanno line.
- (b) A normal shock can appear in subsonic flow
- (c) The downstream Mach number across a normal shock is more than one
- (d) The stagnation pressure across a normal shock decreases

IES-83. Ans. (d)

IES-84. The effect of friction on flow of steam through a nozzle is to:

- (a) Decrease the mass flow rate and to increase the wetness at the exit
- (b) Increase the mass flow rate and to increase the exit temperature
- (c) Decrease the mass flow rate and to decrease the wetness of the steam
- (d) Increase the exit temperature, without any effect on the mass flow rate

IES-84. Ans. (c) The effect of friction of flow of steam through a nozzle is to decrease the mass flow rate and to decrease the wetness of the steam.

### Rayleigh Line

IES-85. Rayleigh line flow is a flow in a constant area duct: [IES-1997]

- (a) With friction but without heat transfer
- (b) Without friction but with heat transfer
- (c) With both friction and heat transfer
- (d) Without either friction or heat transfer

IES-85. Ans. (b) Rayleigh line flow in a constant area duct without friction but with heat transfer. Fanno line flow is a flow in a constant area duct with friction but in the absence of heat transfer and work.

IES-86. Which of the following assumptions/conditions are true in the case of Rayleigh flow? [IES-2005]

1. Perfect gas
2. Constant area duct
3. Steady one-dimensional real flow
4. Heat transfer during the flow

Select the correct answer using the code given below:

- (a) 1, 2 and 3      (b) 2, 3 and 4      (c) 1, 3 and 4      (d) 1, 2 and 4

IES-86. Ans. (d) Assumption of Rayleigh flow

- i. Perfect gas
- ii. Constant area duct
- iii. Heat transfer during flow
- iv. Without friction during flow

IES-87. Air at 2 bar and 60°C enters a constant area pipe of 60 mm diameter with a velocity of 40 m/s. During the flow through the pipe, heat is added to the air stream. Frictional effects are negligible and the values of  $C_p$  and  $C_v$  are that of standard air. The Mach number of the flow corresponding to the maximum entropy will be: [IES-1999]

- (a) 0.845      (b) 1      (c) 0.1212      (d) 1.183

IES-87. Ans. (b)

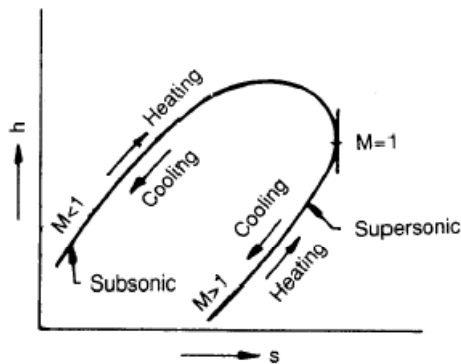


Fig. Rayleigh line on h-s plot

IES-88. Given  $k$  = ratio of specific heats, for Rayleigh line, the temperature is maximum at a Mach number of: [IES-1994]

- (a)  $\frac{1}{\sqrt{k}}$       (b)  $\sqrt{k}$       (c)  $1/k$       (d)  $k$

IES-88. Ans. (a)

## Steam Nozzle

IES-89. In a steam nozzle, to increase the velocity of steam above sonic velocity by expanding steam below critical pressure [IES-2006]

- (a) A vacuum pump is added
- (b) Ring diffusers are used
- (c) Divergent portion of the nozzle is necessary
- (d) Abrupt change in cross-section is needed

IES-89. Ans. (c)

IES-90. Which of the following statement(s) is/are relevant to critical flow through a steam nozzle? [IES-2001]

1. Flow rate through the nozzle is minimum

2. Flow rate through the nozzle is maximum
3. Velocity at the throat is super sonic
4. Velocity at the throat is sonic

Select the correct answer using the codes given below:

**Codes:**

- (a) 1 alone                      (b) 1 and 3                      (c) 2 and 4                      (d) 4 alone

**IES-90. Ans. (c)**

**IES-91. In a steam nozzle, inlet pressure of superheated steam is 10 bar. The exit pressure is decreased from 3 bar to 1 bar. The discharge rate will:**

**[IES-1999]**

- (a) Remain constant                      (b) Decrease                      (c) Increase slightly  
(d) Increase or decrease depending on whether the nozzle is convergent or convergent-divergent.

**IES-91. Ans. (a)** Since exit pressure is less than that corresponding to critical pressure, discharge rate remains unchanged with further decrease in exit pressure. Statement at (d) is also not correct.

**IES-92. The total and static pressures at the inlet of a steam nozzle are 186 kPa and 178 kPa respectively. If the total pressure at the exit is 180 kPa and static pressure is 100 kPa, then the loss of energy per unit mass in the nozzle will be:**

**[IES-1997]**

- (a) 78 kPa                      (b) 8 kPa                      (c) 6 kPa                      (d) 2 kPa

**IES-92. Ans. (c)** Loss = Total pressure of inlet – Total pressure at exit = 186 – 180 = 6 kPa.

**IES-93. Under ideal conditions, the velocity of steam at the outlet of a nozzle for a heat drop of 400 kJ/kg will be approximately.**

**[IES-1998]**

- (a) 1200 m/s                      (b) 900 m/s  
(c) 600 m/s                      (d) The same as the sonic velocity

**IES-93. Ans. (b)**  $V = \sqrt{2 \times \text{enthalpy drop (in J/kg)}} = \sqrt{2 \times 400 \times 1000} = 900 \text{ m/s}$

**IES-94. Consider the following statements:**

**[IES-1997]**

When dry saturated or slightly superheated steam expands through a nozzle,

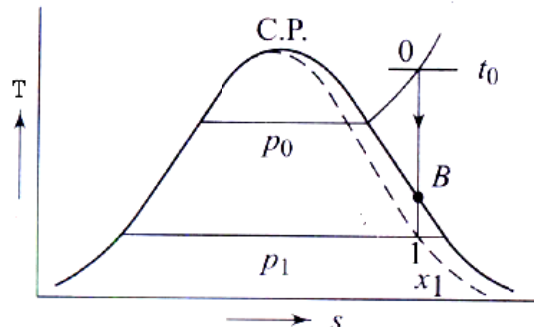
1. The coefficient of discharge is greater than unity.
2. It is dry upto Wilson's line
3. Expansion is isentropic throughout.

**Of these statements**

- (a) 1, 2 and 3 are correct                      (b) 1 and 2 are correct  
(c) 1 and 3 are correct                      (d) 2 and 3 are correct

**IES-94. Ans. (b)** It is supersaturated flow.

When steam expands from superheated state to the two-phase wet region in a nozzle, the expansion occurs so rapidly that the vapour does not condense immediately as it crosses the dry saturated line, but somewhat later.



**Fig. Slow and isentropic expansion of steam from the superheated into the two phase region**



(at  $x = 0.96$  to  $0.97$ ),

when all the vapour suddenly condenses into liquid. Beyond the dry saturation line till the state when the vapour condenses, the flow is said to be *supersaturated* and the system is in *metastable equilibrium*, which means that it is stable to small disturbances but unstable to large disturbances. *Wilson line* ( $x = 0.96$  to  $0.97$ ) is the locus of states below the dry saturation line where condensation within the vapour occurs at different pressures.

**IES-95. A nozzle has velocity head at outlet of 10m. If it is kept vertical the height reached by the stream is:** [IES-1992]

- (a) 100 m                      (b) 10 m                      (c)  $\sqrt{10}$  m                      (d)  $\frac{1}{\sqrt{10}}$

**IES-95. Ans. (b)**

**IES-96. The following lists refer to fluid machinery. Match List-I with List-II and select the correct answer.** [IES-1994]

**List-I**

- A. Draft tube  
B. Surging  
C. Air vessel  
D. Nozzle

**List-II**

1. Impulse turbine  
2. Reciprocating pump  
3. Reaction turbine  
4. Centrifugal pump

Codes:	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	2	1
(c)	3	4	1	2	(d)	4	3	1	2

**IES-96. Ans. (b)** The correct matching is Draft tube-reaction turbine, surging-centrifugal pump, air vessel-reciprocating pump, and nozzle - impulse turbine.

**IES-97. Consider the following statements** [IES-2000]

**For supersaturated flow through a steam nozzle, the**

1. Enthalpy drop reduces further      2. Exit temperature increases  
3. Flow rate increases

**Which of these statements are correct?**

- (a) 1, 2 and 3                      (b) 1 and 2                      (c) 2 and 3                      (d) 1 and 3

**IES-97. Ans. (a)**

**IES-98. When compared to stable flow, for supersaturated flow of steam through a nozzle the available enthalpy drop.** [IES-1994]

- (a) Remains the same                      (b) Increases  
(c) Decreases                      (d) Is unpredictable

**IES-98. Ans. (c)**

**IES-99. Wilson line is associated with which one of the following?** [IES-2006]

- (a) Total steam consumption with respect to power output.  
(b) Supersonic flow of steam through a nozzle  
(c) Nozzle flow with friction  
(d) Supersaturated flow of steam through a nozzle

**IES-99. Ans. (d)**

## Previous Years IAS Questions

**IAS-1. The velocity of sound in an ideal gas does not depend on** [IAS-2002]

- (a) The specific heat ratio of the gas      (b) The molecular weight of the gas

(c) The temperature of the gas

(d) The density of the gas

IAS-1. Ans. (d)  $V = \sqrt{\frac{\gamma RT}{M}}$  [R is universal gas constant]

IAS-2. The velocity of sound in an ideal gas does not depend on [IAS-2002]

(a) The specific heat ratio of the gas

(b) The molecular weight of the gas

(c) The temperature of the gas

(d) The density of the gas

IAS-2. Ans. (d):  $C = \sqrt{\frac{\gamma RT}{M}}$

IAS-3. In isentropic flow between two points [IES-1998; IAS-2002]

(a) The stagnation pressure decreases in the direction of flow.

(b) The stagnation temperature and stagnation pressure decrease with increase in the velocity.

(c) The stagnation temperature and stagnation pressure may vary.

(d) The stagnation temperature and stagnation pressure remain constant.

IAS-3. Ans. (d) Pressure and temperature may varied but Stagnation pressure and Stagnation temperature remains constant.

IAS-4. Steam enters a diffuser at Mach number 2.5 and exit at Mach number 1.8. The shape of the diffuser is: [IAS-2000]

(a) Divergent

(b) Convergent

(c) Convergent divergent

(d) Divergent convergent

IAS-4. Ans. (b) Supersonic diffuser = subsonic nozzle.

IAS-5. Consider the following statements: [IAS-2001]

A tube, with the section diverging in the direction of flow, can be used as

1. Supersonic nozzle

2. Subsonic nozzle

3. Supersonic diffuser

4. Subsonic diffuser

Select the correct answer using the codes given below:

Codes:

(a) 1 alone

(b) 3 alone

(c) 1 and 4

(d) 2 and 3

IAS-5. Ans. (c)

IAS-6. Match List-I (Variable area devices) with List-II (Name of device) and select the correct answer using the codes given below the lists:[IAS-1995]

List-I

A. Pr increases

B. Pr increases

C Pr decreases

D. Pr decreases

List-II

1. Supersonic nozzle

2. Supersonic diffuser

3. Rayleigh flow device

4. Subsonic nozzle

5. Subsonic diffuser

Codes:	A	B	C	D		A	B	C	D
--------	---	---	---	---	--	---	---	---	---

	(a)	5	1	4	2	(b)	5	4	3	1
--	-----	---	---	---	---	-----	---	---	---	---

	(c)	2	4	1	3	(d)	5	2	4	1
--	-----	---	---	---	---	-----	---	---	---	---

IAS-6. Ans. (d)

## Flow of Compressible Fluid through a Convergent Nozzle

IAS-7. The critical pressure ratio  $\left(\frac{P_2}{P_1}\right)$  for maximum discharge through a nozzle is: [IAS-1999; IES-2002, 2004]

- (a)  $\left(\frac{2}{n+1}\right)^{\left(\frac{n-1}{n}\right)}$  (b)  $\left(\frac{2}{n+1}\right)^{\left(\frac{n}{n-1}\right)}$  (c)  $\left(\frac{2}{n-1}\right)^{\left(\frac{n-1}{n}\right)}$  (d)  $\left(\frac{2}{n-1}\right)^{\left(\frac{n+1}{n}\right)}$

IAS-7. Ans. (b)

IAS-8. Assertion (A): In convergent-divergent nozzle, once sonic conditions are established at the throat. Any amount of reduction of pressure at the exit will not be effective in increasing the flow rate. [IAS-1995]  
Reason (R): The reduction of upstream pressure caused by the depletion of the reservoir compensates for the acceleration of flow due to lowering of back pressure.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-8. Ans. (a) Both A and R are true and R is the correct explanation of A

IAS-9. A nozzle is said to have choked flow when [IAS-2002]

1. Discharge is maximum.
2. Throat velocity is sonic.
3. Nozzle exit pressure is more than the critical pressure.
4. Discharge is zero.

Select the correct answer using the codes given below:

- (a) 1 only (b) 1 and 2 (c) 4 only (d) 1, 2 and 3

IAS-9. Ans. (b)

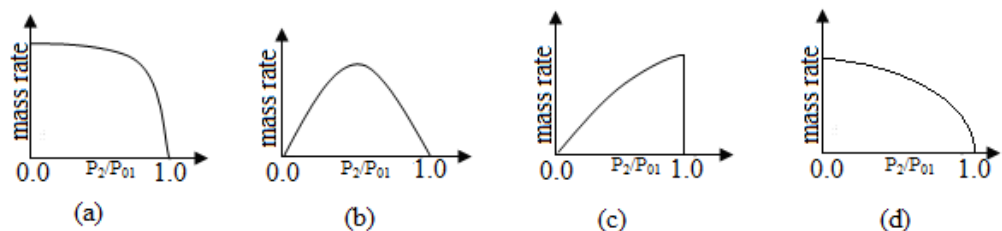
IAS-10. Assertion (A): When the pressure ratio ( $p_2/p_1$ ) in a nozzle reaches critical pressure ratio, the discharge becomes zero. [IAS-2004]

Reason (R): The nozzle gets choked.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-10. Ans. (d) Discharge become maximum.

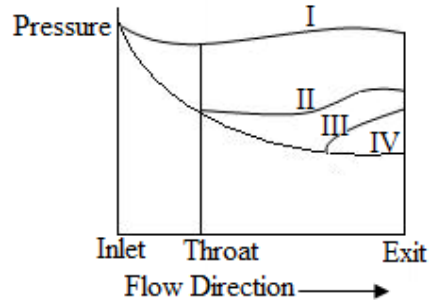
IAS-11. The variation of flow through a convergent-divergent nozzle with variation in exit pressure is represented as [IAS-1999]



IAS-11. Ans. (a)

IAS-12. Out of the four curves (I, II, III and IV) shown in the given figure, the one which represents isentropic flow through a convergent-divergent nozzle is:

- (a) I (b) II  
(c) III (d) IV



[IAS-1996]

IAS-12. Ans. (d)

IAS-13. In a convergent-divergent steam nozzle, the flow is supersonic at which location? [IAS-2007]

- (a) Entrance of the nozzle (b) Throat  
(c) Converging portion of the nozzle (d) Diverging portion of the nozzle

IAS-13. Ans. (d)

IAS-14. Assertion (A): A convergent-divergent nozzle is used to deliver steam to the turbine or blades at high pressure. [IAS-2000]

Reason (R): A stream nozzle is a passage of varying cross-sectional area in which the energy of steam is converted into kinetic energy.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IAS-14. Ans. (d) Nozzle is used for high velocity not high pressure.

IAS-15. In a normal shock wave in one-dimensional flow [IAS-2003]

- (a) Pressure, density and temperature increase  
(b) Velocity, temperature and density increase  
(c) Pressure, density and temperature decrease  
(d) Velocity, pressure and density decrease

IAS-15. Ans. (a)

IAS-16. Which of the following is caused by the occurrence of a normal shock in the diverging section of a convergent-divergent nozzle? [IAS-2004]

- (1) Velocity jump (2) Pressure jump  
(3) Velocity drop (4) Pressure drop

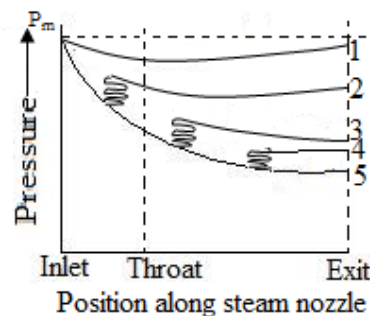
Select the correct answer using the codes given below:

- (a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1 and 4

IAS-16. Ans. (c)

IAS-17. Consider the curves in the sketch shown below (indicates normal shock) Out of these curves, those which are not correctly drawn will include

- (a) 1 and 2 (b) 3 and 4  
(c) 2 and 4 (d) 1 and 5



[IAS-1998]

IAS-17. Ans. (c)

IAS-18. In a convergent-divergent nozzle, normal shock can generally occur

(a) Along divergent portion and throat (b) Along the convergent portion

(c) Anywhere along the length (d) Near the inlet [IAS-1996]

IAS-18. Ans. (a)

IAS-19. Consider the following statements:

[IAS 1994]

1. Almost all flow losses take place in the diverging part of a nozzle.

2. Normal shocks are likely to occur in the converging part of a nozzle.

3. Efficiency of reaction turbines is higher than that of impulse turbines.

Of these statements

(a) 1, 2 and 3 are correct

(b) 2 and 3 are correct

(c) 1 and 2 are correct

(d) 1 and 3 are correct

IAS-19. (d)

IAS-20. Steam flows at the rate of 10 kg/s through a supersonic nozzle. Throat diameter is 50 mm. Density ratio and velocity ratio with reference to throat and exit are respectively 2.45 and 0.8. What is the diameter at the exit?

[IAS-2004]

(a) 122.5 mm

(b) 58 mm

(c) 70 mm

(d) 62.5 mm

IAS-20. Ans. (c)  $W = \rho_o A_o V_o = \rho_e A_e V_e$  or  $\frac{A_e}{A_o} = \frac{\rho_o V_o}{\rho_e V_e} = 2.45 \times 0.8$ 

$$\text{or } \frac{d_e}{d_o} = \sqrt{1.96} = 1.4 \quad \text{or } d_e = 1.4 \times d_o = 70 \text{ mm}$$

IAS-21. Total enthalpy of stream at the inlet of nozzle is 2800 kJ while static enthalpy at the exit is 2555 kJ. What is the steam velocity at the exit if expansion is isentropic?

[IAS-2004]

(a) 70 m/s

(b) 245 m/s

(c) 450 m/s

(d) 700 m/s

IAS-21. Ans. (d)  $V = \sqrt{2000 \times \Delta h} = \sqrt{2000 \times (2800 - 2555)} = 700 \text{ m/s}$ 

IAS-22. If the enthalpies at the entry and exit of a nozzle are 3450 kJ/kg and 2800 kJ/kg and the initial velocity is negligible, then the velocity at the exit is:

[IAS-2002]

(a) 806.2 m/s

(b) 25.5 m/s

(c) 36 m/s

(d) 1140.2 m/s

IAS-22. Ans. (d)  $V = \sqrt{2000(h_1 - h_2)} = \sqrt{130} \times 100$  As  $11^2 < 130 < 12^2$  Therefore answer must be between 1100 and 1200 so choice is (d).

IAS-23. At the inlet of a steam nozzle, stagnation enthalpy of steam is 787.30 kcal/kg at exit, the static enthalpy of steam is 738.00 kcal/kg. For a frictionless adiabatic flow, the velocity of steam at the exit will be (assume 1 kcal = 4.18 kJ)

[IAS-1997]

(a) 582.2 m/s

(b) 600.6 m/s

(c) 620 m/s

(d) 640.5 m/s

IAS-23. Ans. (d)  $V = \sqrt{2000 \times (\Delta h)}$   $[\Delta h = (787.30 - 738) \times 4.18 \text{ kJ} = 206.074 \text{ kJ}]$   
 $= \sqrt{2000 \times 206.074} = 642 \text{ m/s nearest}$

**IAS-24.** An isentropic nozzle is discharging steam at the critical pressure ratio. When the back pressure is further decreased, which one of the following will take place? [IAS-2007]

- (a) Nozzle flow decreases
- (b) Nozzle flow increases
- (c) Nozzle flow will first increase, reach a maximum and then decrease
- (d) Nozzle flow will remain unaltered

**IAS-24. Ans. (d)** It is a case of choked flow. If Back pressure decreases then velocity of steam will increase but mass flow rate will be constant.

**IAS-25.** Consider the following statements: [IAS-1998]

1. Actual mass flow rate of steam will be less than the theoretical value in all cases of flow through a nozzle.
2. Mass flow of steam cannot be increased beyond a certain value in the case of flow through a nozzle.
3. Actual velocity of steam at the exit of a nozzle will always be less than the theoretical value.

Of these statements:

- (a) 1, 2 and 3 are correct
- (b) 1 and 2 are correct
- (c) 2 and 3 are correct
- (d) 1 and 3 are correct

**IAS-25. Ans. (c)**

**IAS-26.** A nozzle is discharging steam through critical pressure ratio. When the back pressure is further decreased, the nozzle flow rate will: [IAS-2001]

- (a) Decrease
- (b) Increase
- (c) Remain unaltered
- (d) First increase to a maximum and then will decrease

**IAS-26. Ans. (c)**

**IAS-27.** Assertion (A): The steam discharge through a nozzle can be increased only after the pressure at throat attains a value equal to critical pressure. [IAS-1995]

Reason (R): A maximum discharge is obtained at the critical pressure ratio.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-27. Ans. (d)** A is false but R is true

**IAS-28.** Consider the following statements in respect of nozzles: [IAS-2007]

1. The nozzle efficiency is defined as the ratio of the actual enthalpy drop to the isentropic enthalpy drop between the same pressures.
2. Velocity coefficient is defined as the ratio of exit velocity when the flow is isentropic, to the actual velocity.
3. The included angle of divergence is usually kept more than  $20^\circ$ .

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 1 and 2 only
- (c) 3 only
- (d) 1, 2 and 3

**IAS-28. Ans. (a)**

## Supersaturated Flow

**IAS-29.** In flow through steam nozzles, the actual discharge will be greater than the theoretical value when [IAS-1996]

(a) Steam at inlet is superheated

(b) Steam at inlet is saturated

(c) Steam gets supersaturated

(d) Steam at inlet is wet

IAS-29. Ans. (c)

IAS-30. With reference to supersaturated flow through a steam nozzle, which of the following statements are true? [IAS 1994]

1. Steam is subcooled

2. Mass flow rate is more than the equilibrium rate of flow.

3. There is loss in availability

4. Index of expansion corresponds to wet steam conditions.

Select the correct answer using the codes given below:

Codes:

(a) 1, 2 and 3

(b) 1 and 2

(c) 1 and 4

(d) 2, 3 and 4

IAS-30. Ans. (a)

IAS-31. Assertion (A): The actual discharge from the nozzle is slightly greater than the theoretical value. [IAS-2007]

Reason (R): The converging part of the nozzle is so short and steam velocity is so high that the molecules do not have sufficient time to collect and form droplets. Hence normal condensation does not take place.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IAS-31. Ans. (a) This condition is called supersaturated flow through nozzle.

IAS-32. During an expansion process, steam passes through metastable conditions (supersaturated flow). Which of the following would DECREASE as result of the effects of super-saturation? [IAS-1997]

1. Enthalpy drop

2. Specific volume at exit

3. Final dryness fraction

4. Entropy

Select the correct answer using the codes given below:

Codes:

(a) 1 and 2

(c) 3 and 4

(b) 2 and 3

(d) 1 and 4

IAS-32. Ans. (a)

IAS-33. How does entropy change when the supersaturation occurs in a steam nozzle? [IAS-2007]

(a) The entropy decreases

(b) The entropy increases

(c) The entropy decreases and then increases

(d) The entropy increases and then decreases

IAS-33. Ans. (b) For any spontaneous process entropy will increase as large amount of irreversibility is present there.

IAS-34. In the case of supersaturated steam flow through a nozzle, which of the following statements are correct? [IAS-2003]

1. Availability increases.

2. Mass flow coefficient is greater than unity.

3. Nozzle velocity coefficient is less than unity.

4. A flexible layout is preferred.

Select the correct answer using the codes given:

**Codes:**

(a) 1, 2 and 3

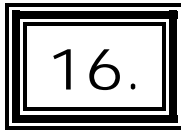
(b) 1 and 2

(c) 2 and 3

(d) 1 and 3

**IAS-34. Ans. (a)**





# 16. Flow Through Open Channel

## Contents of this chapter

1. Laminar Flow and Turbulent Flow
2. Sub-critical Flow, Critical Flow and Supercritical Flow
3. Most Economical Section of Channel
4. Most Economical Trapezoidal Channel Section
5. Hydraulic Jump or Standing Wave

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous Years GATE Questions

## Laminar Flow and Turbulent Flow

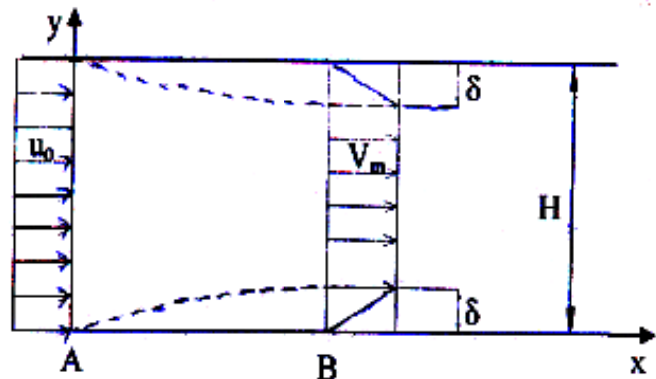
Statement for linked answer Question: 1 to 2:

[GATE-2007]

Consider a steady incompressible flow through a channel as shown below:

The velocity profile is uniform with a value of  $u_0$  at the inlet section A. The velocity profile at section B downstream is:

$$u = \begin{cases} V_m \frac{y}{\delta} & 0 \leq y \leq \delta \\ V_m & \delta \leq y \leq H - \delta \\ V_m \frac{H-y}{\delta} & H - \delta \leq y \leq H \end{cases}$$



GATE-1. The ratio  $V_m/u_0$  is:

- (a)  $\frac{1}{1-2(\delta/H)}$       (b) 1      (c)  $\frac{1}{1-(\delta/H)}$       (d)  $\frac{1}{1+(\delta/H)}$

GATE-1. Ans. (c) Continuity equation gives,  $u_0 \times b \times H = V_m \times b \times (H - 2\delta) + \frac{1}{2} \times b \times 2\delta$

$$\text{or } \frac{V_m}{u_0} = \frac{H}{H - \delta} = \frac{1}{1 - \delta/H}$$

GATE-2. The ratio  $\frac{p_A - p_B}{\frac{1}{2} \rho u_0^2}$  (where  $p_A$  and  $p_B$  are the pressures at section A and

B, respectively, and  $\rho$  is the density of the fluid) is:

$$(a) \frac{1}{(1 - (\delta / H))^2} - 1$$

$$(b) \frac{1}{[1 - (\delta / H)]^2} - 1$$

$$(c) \frac{1}{(1 - (2\delta / H))^2} - 1$$

$$(d) \frac{1}{1 + (2\delta / H)} - 1$$

GATE-2. Ans. (a)

## Previous Years IES Questions

IES-1. Chezy's formula is given by (m, i, C and V are, respectively, the hydraulic mean depth, slope of the channel, Chezy's constant and average velocity of flow) [IES-1993]

$$(a) V = i\sqrt{mC} \quad (b) V = C\sqrt{im} \quad (c) V = m\sqrt{iC} \quad (d) V = \sqrt{miC}$$

IES-1. Ans. (b)

## Sub-critical Flow, Critical Flow and Supercritical Flow

IES-2. Match List-I (Flow depth) with List-II (Basic hydraulic condition associated there with) and select the correct answer: [IES-2004]

### List-I

- A. Conjugate depth
- B. Critical depth
- C. Alternate depth
- D. Normal depth

### List-II

- 1. Uniform flow
- 2. Same specific energy
- 3. Minimum specific energy
- 4. Same specific force
- 5. Same bed slope

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>(a)</b>	<b>3</b>	<b>5</b>	<b>4</b>	<b>2</b>	<b>(b)</b>	<b>2</b>	<b>4</b>	<b>1</b>	<b>3</b>
<b>(c)</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>	<b>(d)</b>	<b>5</b>	<b>4</b>	<b>1</b>	<b>2</b>

IES-2. Ans. (c) Only one matching (B with 3) will give us ans. (c) The depth of flow at which specific energy is minimum is called critical depth.

IES-3. An open channel of symmetric right-angled triangular cross-section is conveying a discharge Q. Taking g as the acceleration due to gravity, what is the critical depth? [IES-2006]

$$(a) \left( \frac{Q^2}{g} \right)^{\frac{1}{3}} \quad (b) \left( \frac{2Q^2}{g} \right)^{\frac{1}{3}} \quad (c) \left( \frac{Q^2}{g} \right)^{\frac{1}{5}} \quad (d) \left( \frac{2Q^2}{g} \right)^{\frac{1}{5}}$$

IES-3. Ans. (a) Note: Here Q = discharge per unit width (m<sup>2</sup>/s) and not m<sup>3</sup>/s.

IES-4. The critical depth of a rectangular channel of width 4.0 m for a discharge of 12 m<sup>3</sup>/s is, nearly, [IES-2001]

$$(a) 300 \text{ mm} \quad (b) 30 \text{ mm} \quad (c) 0.972 \text{ m} \quad (d) 0.674 \text{ m}$$

IES-4. Ans. (c) Discharge per unit width,  $q = \frac{Q}{b} = \frac{12}{4} = 3.0 \text{ m}^2 / \text{s}$

$$\text{Critical depth, } y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{3^2}{9.81} \right)^{1/3} = 0.972 \text{ m}$$

**Most Economical Section of Channel**

IES-5. How is the best hydraulic channel cross-section defined? [IES-2005]

- (a) The section with minimum roughness coefficient.
- (b) The section that has a maximum area of a given flow.
- (c) The section that has a minimum wetted perimeter
- (d) The section that has a maximum wetted area.

IES-5. Ans. (c)

IES-6. Velocity of air passing through a rectangular duct and a circular duct is same. Which one of the following is the correct expression for the equivalent diameter of the circular duct in respect of a rectangular duct for the same pressure loss per unit length? (a and b are the length and breadth of the rectangular duct cross-section) [IES-2004]

- (a)  $\frac{a+b}{ab}$
- (b)  $\frac{2ab}{a+b}$
- (c)  $\frac{2a}{a-b}$
- (d)  $\frac{2b}{a+b}$

IES-6. Ans. (b) We know that Hydraulic mean depth ( $R_m$ ) =  $\frac{A}{P} = \frac{ab}{2(a+b)}$

$$\text{Hydraulic mean depth of circular cross section } (R_m) \frac{\pi d^2}{4 \times \pi d} = \frac{d}{4}$$

$$\text{Equivalent diameter } (d_e) = 4 \times \frac{ab}{2(a+b)} = \frac{2ab}{(a+b)}$$

Alternatively: Dimensional analysis gives answer (b) only 'b's dimension is [L]

**Most Economical Trapezoidal Channel Section**

IES-7. For hydraulically most efficient symmetric trapezoidal section of an open channel, which one of the following is the false characterization? [IES-2008]

- (a) Wetted perimeter is minimum for a given area of flow section
- (b) Hydraulic radius is half the flow depth
- (c) Width at top liquid is twice the hydraulic depth
- (d) Discharge is maximum for given area of flow, bed slope and roughness

IES-7. Ans. (c) Trapezoidal section

(i) Half top width = sloping side or  $\frac{b+2ny}{2} = y\sqrt{n^2+1}$

(ii) Hydraulic radius,  $m = \frac{y}{2}$

- (iii) A semi-circle drawn from the mid-point of the top width with radius equal to depth of flow will touch the three side of the channel. Best side slope for most economical trapezoidal section is

$$\theta = 60^\circ \text{ or } n = \frac{1}{\sqrt{3}} = \frac{1}{\tan \theta}$$

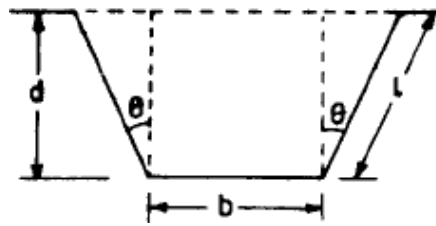
IES-8. Assertion (A): To have maximum hydraulic efficiency, the trapezoidal section of an open channel should be a half-hexagon. [IES-1999]

Reason (R): For any cross-section, a hexagon has the lest-perimeter.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-8. Ans. (c) We all know that for any cross-section, a circular section has the least-perimeter. So R is false.

IES-9. A trapezoidal open channel has the cross-section as shown in the given figure. In order to have maximum hydraulic efficiency, the hydraulic radius,  $R$  and the length of the side,  $L$  should be.



[IES-1994]

- (a)  $\frac{d}{4}$  and  $\frac{2}{\sqrt{3}}d$  respectively      (b)  $\frac{d}{4}$  and  $\frac{\sqrt{2}}{3}d$  respectively  
 (c)  $\frac{d}{2}$  and  $\frac{2}{\sqrt{3}}d$  respectively      (d)  $\frac{d}{2}$  and  $\frac{\sqrt{2}}{3}d$  respectively

IES-9. Ans. (c) For trapezoidal open channel, for maximum hydraulic efficiency, the hydraulic radius  $R = d/2$  and length of side  $L = 2d/\sqrt{3}$ .

## Hydraulic Jump or Standing Wave

IES-10. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 12 m/s and depth of flow of 1 m. What is the depth of flow after the hydraulic jump? [IES-2009]

- (a) 6.42 m      (b) 5.84 m      (c) 4.94 m      (d) 2.21 m

IES-10. Ans. (c)  $\frac{v}{\sqrt{gy}} = F \Rightarrow F = \frac{12}{\sqrt{9.8 \times 1}} = 3.83$

$$\therefore y_1 = 1 \text{ m}$$

$$\frac{y_2}{y_1} = \frac{1}{2} \left[ -1 + \sqrt{1 + 8F^2} \right] \Rightarrow y_2 = 4.94 \text{ m}$$

IES-11. A hydraulic jump occurs in a channel

[IES-1997]

- (a) Whenever the flow is supercritical  
 (b) If the flow is controlled by a sluice gate  
 (c) If the bed slope changes from mild to steep  
 (d) If the bed slope changes from steep to mild

IES-11. Ans. (a) If the flow changes from supercritical to sub-critical. The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state.

IES-12. Consider the following statements regarding a hydraulic jump:

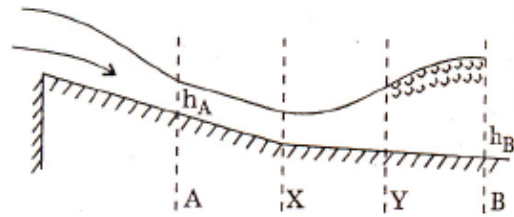
1. There occurs a transformation of super critical flow to sub-critical flow. [IES-1999]
  2. The flow is uniform and pressure distribution is due to hydrostatic force before and after the jump.
  3. There occurs a loss of energy due to eddy formation and turbulence.
- Which of these statements are correct?

- (a) 1, 2 and 3      (b) 1 and 2      (c) 2 and 3      (d) 1 and 3

IES-12. Ans. (a)

IES-13. An open channel flow encounters a hydraulic jump as shown in the figure. The following fluid flow conditions are observed between A and B: [IES-2001]

1. Critical depth
2. Steady non-uniform flow
3. Unsteady non-uniform flow
4. Steady uniform flow.



The correct sequence of the flow conditions in the direction of flow is:

- (a) 1, 2, 3, 4                      (b) 1, 4, 2, 3                      (c) 2, 1, 4, 3                      (d) 4, 2, 3, 1

IES-13. Ans. (b)

IES-14. Consider the following statements: [IES-2003]

A hydraulic jump occurs in an open channel

1. When the Froude number is equal to or less than one.
2. At the toe of a spillway.
3. Downstream of a sluice gate in a canal.
4. When the bed slope suddenly changes.

Which of these are correct?

- (a) 1, 2, 3 and 4                      (b) 1, 2 and 3                      (c) 2, 3 and 4                      (d) 1 and 4

IES-14. Ans. (c) Only 1 is wrong so (a), (b) and (d) out.

IES-15. Match the following: [IES-1992]

**List-I**

- A. Circular sewer maximum discharge  
B. Maximum velocity in circular sewer  
C. Triangular Channel  
D. Rectangular Channel

**List-II**

1.  $y = 0.938 D$
2.  $y = 0.81 D$
3.  $y_c = \frac{4}{5} E$ ,  $\frac{V_c^2}{2g} = \frac{y_c}{4}$
4.  $y_c = \frac{2}{3} E$ ,  $\frac{V_c^2}{2g} = \frac{y_c}{2}$

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	4	3	2	1	(b)	3	4	1	2
(c)	2	3	1	4	(d)	1	4	2	3

IES-15. Ans. (d)

## Previous Years IAS Questions

IAS-1. A hydraulic jump is formed in a 5.0 m wide rectangular channel with sequent depths of 0.2 m and 0.8 m. The discharge in the channel, in  $\text{m}^3/\text{s}$ , is: [IAS-1998]

- (a) 2.43                      (b) 3.45                      (c) 4.43                      (d) 5.00

IAS-1. Ans. (c)  $q = \sqrt{gy_1 y_2 \frac{(y_2 + y_1)}{2}} = 0.8854$ ;

$$Q = qL = 0.8854 \times 5 = 4.43 \text{ m}^3/\text{s}$$

17.

## Force Exerted on Surfaces

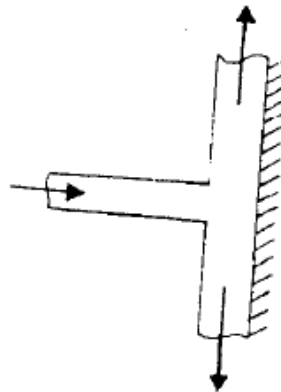
## OBJECTIVE QUESTIONS (IES, IAS)

## Previous Years IES Questions

## Introduction

IES-1. A circular jet of water impinges on a vertical flat plate and bifurcates into two circular jets of half the diameter of the original. After hitting the plate

- (a) The jets move at equal velocity which is twice of the original velocity
- (b) The jets move at equal velocity which is 3 times of the original velocity



[IES-2006]

- (c) Data given is insufficient to calculate velocities of the two outgoing jets
- (d) The jets move at equal velocity which is equal to the original velocity

IES-1. Ans. (d)

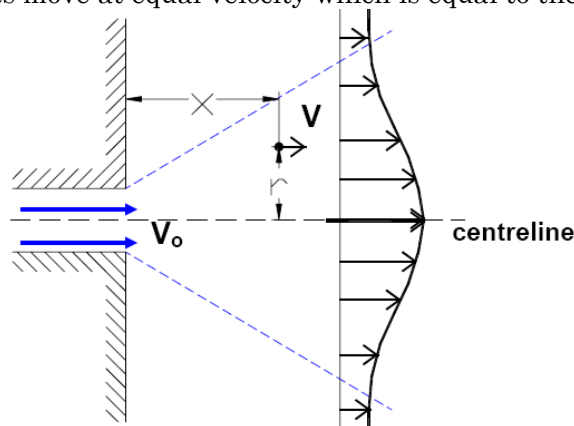


Fig. Velocity distribution through a circular jet

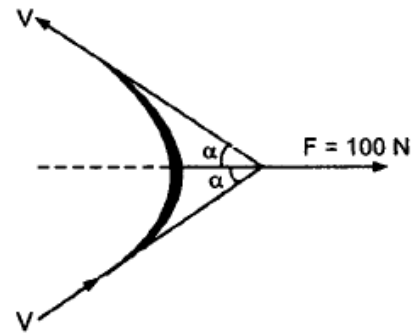
## Force Exerted on a Curved Vane when the Vane is Moving in the Direction of Jet

IES-2. The force of impingement of a jet on a vane increases if: [IES-2002]

- (a) The vane angle is increased
- (b) The vane angle is decreased
- (c) The pressure is reduced
- (d) The vane is moved against the jet

IES-2. Ans. (d)

- IES-3. A symmetrical stationary vane experiences a force 'F' of 100 N as shown in the given figure, when the mass flow rate of water over the vane is 5 kg/s with a velocity 'V' 20 m/s without friction. The angle ' $\alpha$ ' of the vane is:
- (a) Zero (b)  $30^\circ$   
(c)  $45^\circ$  (d)  $60^\circ$



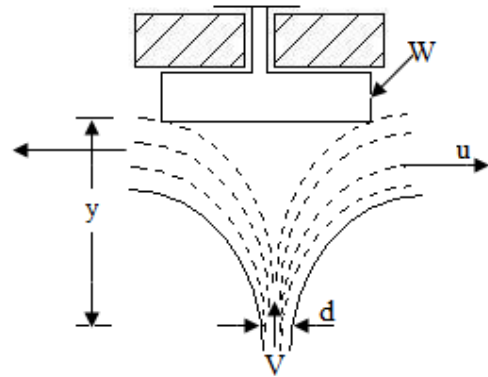
[IES-2001]

IES-3. Ans. (d)

## Previous Years IAS Questions

### Force Exerted on a Stationary Flat Plate Held Normal to the Jet

- IAS-1. A vertical jet of water 'd' cm in diameter leaving the nozzle with a velocity of V m/s strikes a disc weighing 'W' kgf as shown in the figure. The jet is then deflected horizontally. The disc will be held in equilibrium at a distance 'y' where the fluid velocity is 'u', when 'y' is equal to:



- (a)  $(V^2 - u^2) / 2g$  (b)  $V^2 / 2g$   
(c)  $W / V^2$  (d)  $W / u^2$

[IAS-1996]

IAS-1. Ans. (a)

- IAS-2. A jet of water issues from a nozzle with a velocity of 20 m/s and it impinges normally on a flat plate moving away from it at 10 m/s. If the cross-sectional area of the jet is  $0.02 \text{ m}^2$  and the density of water is taken as  $1000 \text{ kg/m}^3$ , then the force developed on the plate will be:

- (a) 10 N (b) 100 N (c) 1000 N (d) 2000 N

[IAS 1994]

IAS-2. Ans. (d) Force on plate =  $\rho a (V - u) (V + u) = 1000 \times 0.02 \times (20 + 10)^2 = 2000 \text{ N}$ .

- IAS-3. A jet of water issues from a nozzle with a velocity of 20 m/s and it impinges normally on a flat plate moving away from it at 10 m/s. If the cross-sectional area of the jet is  $0.02 \text{ m}^2$  and the density of water is taken as  $1000 \text{ kg/m}^3$ , then the force developed on the plate will be:

- (a) 10 N (b) 100 N (c) 1000 N (d) 2000 N

[IAS-1994]

IAS-3. Ans. (d)



# Hydraulic Turbine

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## Contents of this chapter

1. Introduction
  2. Classification of Hydraulic Turbines
  3. Impulse Turbines – Pelton Wheel
  4. Work Done and Efficiency of a Pelton Wheel
  5. Definitions of Heads and Efficiencies
  6. Design Aspects of Pelton Wheel
  7. Reaction Turbine
  8. Design of a Francis Turbine Runner
  9. Propeller Turbine
  10. Kaplan Turbine
  11. Draft Tube
  12. Specific Speed
  13. Model Relationship
  14. Runaway Speed
  15. Cavitation
  16. Surge Tanks
  17. Performance Characteristics
-



## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous Years GATE Questions

#### Introduction

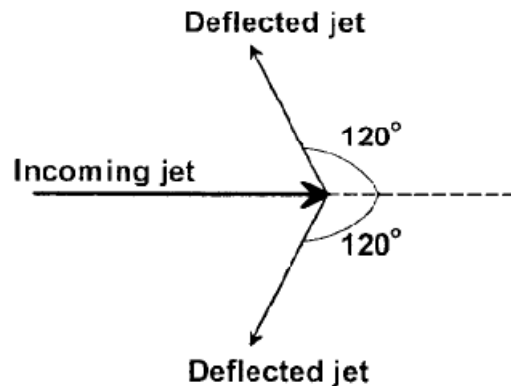
**GATE-1.** In a Pelton wheel, the bucket peripheral speed is 10 m/s, the water jet velocity is 25 m/s and volumetric flow rate of the jet is  $0.1 \text{ m}^3/\text{s}$ . If the jet deflection angle is  $120^\circ$  and the flow is ideal, the power developed is:

[GATE-2006]

- (a) 7.5 kW      (b) 15.0 kW      (c) 22.5 kW      (d) 37.5 kW

**GATE-1. Ans. (c)** From velocity triangle, Power developed =  $\int Q(V_{w1} + V_{w2}) \times u = 22.5 \text{ KW}$

**GATE-2.** Water, having a density of  $1000 \text{ kg/m}^3$ , issues from a nozzle with a velocity of 10 m/s and the jet strikes a bucket mounted on a Pelton wheel. The wheel rotates at 10 rad/s. The mean diameter of the wheel is 1 m. The jet is split into two equal streams by the bucket, such that each stream is deflected by  $120^\circ$ , as shown in the figure. Friction in the bucket may be neglected. Magnitude of the torque exerted by the water on the wheel, per unit mass flow rate of the incoming jet, is:



[GATE-2008]

- (a) 0 (N.m) / (kg/s)      (b) 1.25 (N.m) / (kg/s)  
(c) 2.5 (N.m) / (kg/s)      (d) 3.75 (N.m) / (kg/s)

**GATE-2. Ans. (d)**  $\dot{\omega} = \frac{W}{mg} = \frac{(V_{w1} + V_{w2})u}{g}$

$$V_{w1} = V_1 = 10 \text{ m/s}, u = rw = 0.5 \times 10 = 5 \text{ m/s}$$

$$V_{w2} = (V_1 - u) \cos \phi - u = (10 - 5) \cos(180 - 120) - 5 = -2.5 \text{ m/s}$$

$$\dot{\omega} = \frac{(10 - 2.5)}{9.81} \times 5 = 3.82$$

**GATE-3.** Kaplan turbine is:

[GATE-1997]

- (a) A high head mixed flow turbine      (b) A low axial flow turbine  
(c) An outward flow reaction turbine      (d) An impulse inward flow turbine

**GATE-3. Ans. (b)**

**GATE-4.** The specific speed of an impulse hydraulic turbine will be greater than the specific speed of a reaction type hydraulic turbine. [GATE-1995]

- (a) True      (b) False      (c) Can't say      (d) None

**GATE-4. Ans. (b)** Specific speed of impulse hydraulic turbine 10 – 35 rpm  
Specific speed of a reaction hydraulic turbine 300 – 1000 rpm

**GATE-5.** At a hydro electric power plant site, available head and flow rate are 24.5 m and 10.1 m<sup>3</sup>/s respectively. If the turbine to be installed is required to run at 4.0 revolution per second (rps) with an overall efficiency of 90%, then suitable type of turbine for this site is:

[GATE-2004]

- (a) Francis                      (b) Kaplan                      (c) Pelton                      (d) Propeller

**GATE-5. Ans. (a)** Given:  $H = 24.5$  m,  $Q = 10.1$  m<sup>3</sup>/s;  $N = 4$  rev/s =  $4 \times 60 = 240$  r.p.m.

$$\eta_0 = 0.90 \quad \therefore \text{Power generated} = \rho gQH \times 0.9$$

$$= 1000 \times 9.81 \times 10.1 \times 24.5 \times 0.9 = 2184.7 \text{ kW}$$

$$\text{Again, } N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{240\sqrt{2184.7}}{(24.5)^{5/4}} = 205.80; \quad 51 < N_s < 255, \text{ hence turbine is}$$

Francis.

**GATE-6.** In a hydroelectric station, water is available at the rate of 175 m<sup>3</sup>/s under a head of 18 m. The turbines run at speed of 150 rpm with overall efficiency of 82%. Find the number of turbines required if they have the maximum specific speed of 460 ..... 2 (two) [GATE-1996]

**GATE-6. Ans.** Total Power generated =  $\rho gQH \times 0.9 = 1000 \times 9.81 \times 175 \times 18 \times 0.82 = 25313$  kW

$$\text{Again, } N_s = \frac{N\sqrt{P}}{H^{5/4}} = 460 = \frac{150\sqrt{P}}{(18)^{5/4}} \text{ or } P = 12927 \text{ kW}; \text{ So no of Turbine} = \frac{25313}{12927} \approx 2$$

**GATE-7.** Specific speed of a Kaplan turbine ranges between [GATE-1993]

- (a) 30 and 60                      (b) 60 and 300                      (c) 300 and 600                      (d) 600 and 1000

**GATE-7. Ans. (d)**

## Model Relationship

**GATE-8.** A large hydraulic turbine is to generate 300 kW at 1000 rpm under a head of 40 m. For initial testing, a 1: 4 scale model of the turbine operates under a head of 10 m. The power generated by the model (in KW) will be: [GATE-2006; 1992]

- (a) 2.34                      (b) 4.68                      (c) 9.38                      (d) 18.75

$$\text{GATE-8. Ans. (a)} \quad \frac{H}{N^2 D^2} = \text{const. and } \frac{P}{N^3 D^5} = \text{const. gives } \frac{P}{H^3 D^2} = \text{const. so, } \left( \frac{P}{H^3 D^2} \right)_m = \left( \frac{P}{H^3 D^2} \right)_p$$

$$\text{or } P_m = P_p \left( \frac{H_m}{H_p} \right)^{3/2} \left( \frac{D_m}{D_p} \right)^2 = 300 \times \left( \frac{10}{40} \right)^{3/2} \times \left( \frac{1}{4} \right)^2 = 2.34$$

## Cavitation

**GATE-9.** Cavitation in a hydraulic turbine is most likely to occur at the turbine [GATE-1993]

- (a) Entry                      (b) Exit                      (c) Stator exit                      (d) Rotor exit

**GATE-9. Ans. (d)**

**GATE-10.** Match List-I (Phenomena) with List-II (Causes) and select the correct answer: [GATE-1996]

**List-I**

**List-II**

- A. Shock wave  
B. Flow separation  
C. Capillary rise  
D. Cavitation
1. Surface tension  
2. Vapour pressure  
3. Compressibility  
4. Adverse pressure gradient

Codes:	A	B	C	D		A	B	C	D
(a)	3	1	2	4	(b)	4	2	1	3
(c)	3	4	1	2	(d)	4	1	2	3

GATE-10. Ans. (c)

## Previous Years IES Questions

IES-1. Of all the power plants, hydel is more disadvantageous when one compares the [IES-1996]

- (a) Nearness to load centre (b) Cost of energy resource  
(c) Technical skill required (d) Economics that determine the choice of plant.

IES-1. Ans. (a) Of all the power plants, hydel is more disadvantageous when one compares the nearness to load centre because it is in hilly areas.

IES-2 Euler equation for water turbine is derived on the basis of [IES-1995]

- (a) Conservation of mass  
(b) Rate of change of linear momentum  
(c) Rate of change of angular momentum  
(d) Rate of change of velocity

IES-2. Ans. (c) Eulers equation for water turbine is:

$$H = \underbrace{\frac{U_2^2 - U_1^2}{2g}}_{\text{Centrifugal head}} + \underbrace{\frac{U_2^2 + U_1^2}{2g}}_{\text{Kinetic head}} + \underbrace{\frac{U_{r1}^2 - U_{r2}^2}{2g}}_{\text{Relative velocity head}}$$

IES-3. If H is the head available for a hydraulic turbine, the power, speed and discharge, respectively are proportional to: [IES-2002]

- (a)  $H^{1/2}$ ,  $H^{1/2}$ ,  $H^{3/2}$  (b)  $H^{3/2}$ ,  $H^{1/2}$ ,  $H^{1/2}$  (c)  $H^{1/2}$ ,  $H^{3/2}$ ,  $H^{1/2}$  (d)  $H^{1/2}$ ,  $H^{1/2}$ , H

IES-3. Ans. (b)

IES-4. Assertion (A): In turbomachines, stalling is a local phenomenon while surging affects the whole machine. [IES-1993]

Reason (R): Stalling occurs when flow breaks away from the blades while surging causes complete breakdown of the flow.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-4. Ans. (a) Both A and R are true and R provides satisfactory explanation for A.

IES-5. A Francis turbine is coupled to an alternator to generate electricity with a frequency of 50 Hz. If the alternator has 12 poles, then the turbine should be regulated to run at which one of the following constant speeds? [IES-2004]

- (a) 250 rpm (b) 500 rpm (c) 600 rpm (d) 1000 rpm

IES-5. Ans. (b)  $n = \frac{120f}{P} = \frac{120 \times 50}{12} = 500 \text{rpm}$

IES-6. The gross head on a turbine is 300 m. The length of penstock supplying water from reservoir to the turbine is 400 m. The diameter of the penstock is 1 m and velocity of water through penstock is 5 m/s. If coefficient of friction is 0.0098, the net head on the turbine would be nearly [IES-2001]

- (a) 310 m                      (b) 295 m                      (c) 200 m                      (d) 150 m

IES-6. Ans. (b)

IES-7. Consider the following statements: [IES-2000]

A water turbine governor

1. Helps in starting and shutting down the turbo unit
2. Controls the speed of turbine set to match it with the hydroelectric system
3. Sets the amount of load which a turbine unit has to carry

Which of these statements are correct?

- (a) 1, 2 and 3                      (b) 1 and 2                      (c) 2 and 3                      (d) 1 and 3

IES-7. Ans. (a)

IES-8. Match List-I with List-II and select the correct answer using the codes given below the lists: [IES-2009]

**List-I**

- A. Pelton turbine  
B. Francis turbine  
C. Propeller turbine  
D. Kaplan turbine

**List-II**

1. Specific speed from 300 to 1000 axial flow with fixed runner vanes
2. Specific speed from 10 to 50 Tangential flow
3. Specific speed from 60 to 300 mixed flow
4. Specific speed from 300 to 1000 axial flow with adjustable runner vanes

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	1	3	4	(b)	4	1	3	2
(c)	2	3	1	4	(d)	4	3	1	2

IES-8. Ans. (c)

IES-9. Match List-I with List-II and select the correct answer [IES-1996]

**List-I**

- A. Pelton wheel (single jet)  
B. Francis Turbine  
C. Kaplan Turbine

**List-II**

1. Medium discharge, low head
2. High discharge, low head
3. Medium discharge, medium head
4. Low discharge, high head

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>		<b>A</b>	<b>B</b>	<b>C</b>
(a)	1	2	3	(b)	1	3	4
(c)	4	1	3	(d)	4	3	2

IES-9. Ans. (d) Pelton wheel:

- i. Impulse turbine
- ii. High head turbine (300 – 2000 m)
- iii. Low specific discharge
- iv. Axial flow turbine
- v. Low specific speed turbine (4 – 70 rpm)

**Francis turbine:**

- i. Reaction turbine
- ii. Medium head turbine (30 – 500 m)

- iii. Medium specific discharge
- iv. Radial flow turbine, but modern francis turbine are mixed flow turbine
- v. Medium specific speed turbine (60 – 400 rpm)

**Kaplan turbine:**

- i. Reaction turbine (2 – 70 m)
- ii. Low head turbine
- iii. High specific discharge
- iv. Axial flow
- v. High specific speed (300 – 1100 rpm)

**IES-10. Match List-I (Types of turbines), List-II (Characteristics of turbines) and select the correct answer. [IES-1994]**

**List-I**

- A. Propeller
- B. Francis
- C. Kaplan
- D. Pelton

**List-II**

- 1. Inward flow reaction
- 2. Tangential flow impulse
- 3. Axial flow reaction with fixed vanes
- 4. Axial flow reaction with adjustable vanes

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	4	1	3	(b)	3	4	1	2
(c)	2	1	4	3	(d)	3	1	4	2

**IES-10. Ans. (d)**

**IES-11. Match List-I (Flow parameter) with List-II (Type of turbine) and select the correct answer: [IES-2004]**

**List-I**

- A. High head
- B. Axial flow
- C. Mixed flow
- D. High specific speed

**List-II**

- 1. Francis turbine
- 2. Pelton wheel
- 3. Kaplan turbine

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	2	3	1	(b)	1	2	3	2
(c)	2	3	1	3	(d)	2	2	1	3

**IES-11. Ans. (c)**

**IES-12. Consider the following statements: [IES-2002]**

- 1. Pelton wheel is a tangential flow impulse turbine
- 2. Francis turbine is an axial flow reaction turbine
- 3. Kaplan turbine is a radial flow reaction turbine

**Which of the above statements is/ are correct?**

- (a) 1 and 3
- (b) 1 alone
- (c) 2 alone
- (d) 3 alone

**IES-12. Ans. (b)**

**IES-13. Consider the following types of water turbines: [IES-2003]**

- 1. Bulb
- 2. Francis
- 3. Kaplan
- 4. Pelton

**The correct sequence of order in which the operating head decreases while developing the same power is**

- (a) 4 – 2 – 3 – 1
- (b) 3 – 4 – 1 – 2
- (c) 2 – 1 – 4 – 3
- (d) 1 – 3 – 2 – 4

**IES-13. Ans. (a)**

**IES-14. Which of the following types of turbine is/are suitable for tidal power plants? [IES-2005]**

- 1. Tubular turbine
- 2. Kaplan turbine

**3. Bulb turbine**

**4. Francis turbine**

**Select the correct answer using the code given below:**

- (a) 1 only                      (b) 1 and 3                      (c) 2 and 4                      (d) 4 only

**IES-14. Ans. (b)** See theory of bulb and tubular turbine in the theory section.

**IES-15. Which one of the following turbines is used in underwater power stations? [IES-1998]**

- (a) Pelton turbine                      (b) Deriaz turbine  
(c) Tubular turbine                      (d) Turgo-impulse turbine

**IES-15. Ans. (c)**

**IES-16. Assertion (A): For the same power, the rotor of an impulse turbine need not be as large as that of a reaction turbine. [IES-2004]**

**Reason (R): In the case of a reaction turbine, water has to be admitted to the runner around its entire circumference.**

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IES-16. Ans. (d)**  $\frac{D_{\text{impulse}}}{D_{\text{Reaction}}} = \frac{3}{\sqrt{n}}$  where n = no. of jets.

For Pelton wheel n is generally 4 to 6 so  $D_{\text{impulse}} > D_{\text{Reaction}}$

**IES-17. A Pelton wheel is ideally suited for [IES-1998]**

- (a) High head and low discharge                      (b) High head and high discharge  
(c) Low head and low discharge                      (d) Medium head and medium discharge

**IES-17. Ans. (a)**

**IES-18. Consider the following energies associated with a Pelton turbine:**

- 1. Mechanical energy                      2. Kinetic energy                      3. Potential energy**

**The correct sequence of energy conversion starting from the entry of fluid is: [IES-2003]**

- (a) 1 – 2 – 3                      (b) 2 – 3 – 1                      (c) 3 – 2 – 1                      (d) 1 – 3 – 2

**IES-18. Ans. (c)**

**IES-19. The maximum number of jets generally employed in an impulse turbine without jet interference is: [IES-2001]**

- (a) 4                      (b) 6                      (c) 8                      (d) 12

**IES-19. Ans. (b)**

**IES-20. Which one of the following statements regarding an impulse turbine is correct? [IES-1997]**

- (a) There is no pressure variation in flow over the buckets and the fluid fills the passageway between the buckets  
(b) There is no pressure variation in flow over the buckets and the fluid does not fill the passageway between the buckets  
(c) There is pressure drop in flow over the buckets and the fluid fills the passageway between the buckets  
(d) There is pressure drop in flow over the buckets and the fluid does not fill the passageway between the buckets

**IES-20. Ans. (b)**

## Work Done and Efficiency of a Pelton Wheel

IES-21. Euler equation of turbine giving energy transfer per unit mass  $E_0$  (where  $U$ ,  $V_w$ ,  $V_r$  and  $V$  represents the peripheral, whirl, relative and absolute velocities respectively. Suffix 1 and 2 refer to the turbine inlet and outlet respectively) is given by: [IES-2003]

- (a)  $E_0 = U_1 V_{w1} - U_2 V_{w2}$  (b)  $E_0 = U_1 V_{r1} - U_2 V_{r2}$   
(c)  $E_0 = U_1 V_1 - U_2 V_2$  (d)  $E_0 = V_1 V_{w1} - V_2 V_{w2}$

IES-21. Ans. (a)

IES-22. The speed ratio of a Pelton wheel operating under a head of 900 m is 0.45. What is the peripheral velocity of the turbine wheel? [IES-2009]

- (a) 28 m/s (b) 96 m/s (c) 42 m/s (d) 60 m/s

IES-22. Ans. (d) Speed Ratio =  $\frac{u}{\sqrt{2gH}}$

IES-23. A Pelton wheel with single jet rotates at 600 rpm. The velocity of the jet from the nozzle is 100m/s. If the ratio of the vane velocity to jet velocity is 0.44, what is the diameter of the Pelton wheel? [IES-2005]

- (a) 0.7 m (b) 1.4 m (c) 2.1 m (d) 2.8 m

IES-23. Ans. (b) Vane velocity =  $0.44 \times 100 = 44 \text{ m/s} = \frac{\pi DN}{60}$

$$\text{or } D = \frac{44 \times 60}{\left(\frac{22}{7}\right) \times 600} = 1.4 \text{ m}$$

IES-24. If  $\alpha$  is the blade angle at the outlet, then the maximum hydraulic efficiency of an ideal impulse turbine is: [IAS-1999; IES-2005]

- (a)  $\frac{1 + \cos \alpha}{2}$  (b)  $\frac{1 - \cos \alpha}{2}$  (c)  $\frac{1 - \sin \alpha}{2}$  (d)  $\frac{1 + \sin \alpha}{2}$

IES-24. Ans. (a)

IES-25. The maximum efficiency in the case of Pelton wheel is (angle of deflection of the jet =  $180 - \beta$ ) [IES-2002]

- (a)  $\frac{1 - \cos \beta}{2}$  (b)  $\frac{1 + \cos \beta}{2}$  (c)  $\frac{\cos \beta}{2}$  (d)  $\frac{1 + \cos \beta}{4}$

IES-25. Ans. (b)

IES-26. What should be the ratio of blade speed of jet speed for the maximum efficiency of a Pelton wheel? [IES-2002, 2005]

- (a)  $\frac{1}{4}$  (b)  $\frac{1}{2}$  (c)  $\frac{3}{4}$  (d) 1

IES-26. Ans. (b)

## Definitions of Heads and Efficiencies

IES-27. The overall efficiency of a Pelton turbine is 70%. If the mechanical efficiency is 85%, what is its hydraulic efficiency? [IES-2007]

- (a) 82.4% (b) 59.5% (c) 72.3% (d) 81.5%

IES-27. Ans. (a)  $\eta_o = \eta_m \times \eta_h$  Or  $\eta_h = \frac{\eta_o}{\eta_m} = \frac{0.70}{0.85} = 0.8235$

IES-28. The gross head available to a hydraulic power plant is 100 m. The utilized head in the runner of the hydraulic turbine is 72 m. If the hydraulic efficiency of the turbine is 90%, the pipe friction head is estimated to be: [IES-2000]

- (a) 20 m                      (b) 18 m                      (c) 16.2 m                      (d) 1.8 m

IES-28. Ans. (a)

IES-29. A reaction turbine discharges 30 m<sup>3</sup>/s of water under a head of 10 m with an overall efficiency of 92%. The power developed is: [IES-1997]

- (a) 2952 kW                      (b) 2870 kW                      (c) 2760 kW                      (d) 2652 kW

IES-29. Ans. (c)

$$\eta = \frac{\text{Power(kW)}}{\frac{\rho Q g H}{1000}} \quad \text{or} \quad \text{Power(kW)} = \eta \times \frac{\rho Q g H}{1000} = 0.92 \times \frac{1000 \times 30 \times 9.81 \times 10}{1000} = 2707 \text{ kW}$$

IES-30. As water flows through the runner of a reaction turbine, pressure acting on it would vary from: [IES-1997]

- (a) More than atmospheric pressure to vacuum  
(b) Less than atmospheric pressure to zero gauge pressure  
(c) Atmospheric pressure to more than atmospheric pressure  
(d) Atmospheric pressure to vacuum

IES-30. Ans. (a) Pressure of water in reaction turbine runner varies from more than atmospheric to vacuum. At runner inlet the pressure is more than atmospheric pressure and at runner outlet pressure is less than atmospheric (vacuum).

## Design of a Francis Turbine Runner

IES-31. Which of the following advantages is/are possessed by a Kaplan turbine over a Francis turbine? [IES-2006]

1. Low frictional losses.
2. Part load efficiency is considerably high.
3. More compact and smaller in size.

Select the correct answer using the codes given below

- (a) Only 1                      (b) Only 1 and 2                      (c) Only 2 and 3                      (d) 1, 2 and 3

IES-31. Ans. (d)

IES-32. A Francis turbine working at 400 rpm has a unit speed of 50 rpm and develops 500 kW of power. What is the effective head under which this turbine operates? [IES-2009]

- (a) 62.5 m                      (b) 64.0 m                      (c) 40.0 m                      (d) 100 m

IES-32. Ans. (b) Unit speed of the turbine is given by

$$\frac{N}{\sqrt{H}} = 50 \quad \Rightarrow \quad \frac{400}{\sqrt{H}} = 50 \quad \Rightarrow \quad H = 64 \text{ m}$$

## Kaplan Turbine

IES-33. Consider the following statements in respect of Kaplan Turbine:

1. It is a reaction turbine. [IES-2008]



2. It is a mixed flow turbine.

3. It has adjustable blades.

Which of the statements given above are correct?

(a) 1, 2 and 3      (b) 2 and 3 only      (c) 1 and 3 only      (d) 1 and 2 only

**IES-33. Ans. (c)** Kaplan turbine is axial flow reaction turbine. It has got adjustable blades. Whereas propeller turbines have fixed blades.

**IES-34. Assertion (A):** A Kaplan turbine is an axial flow reaction turbine with its vanes fixed to the hub. [IES-2001]

**Reason (R):** Water flows parallel to the axis of rotation of the turbine and a part of the pressure energy gets converted to kinetic energy during its flow through the vanes.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**IES-34. Ans. (a)**

**IES-35. What is the range of the speed ratio for Kaplan turbine for its most efficient operation?** [IES-2004]

(a) 0.10 to 0.30      (b) 0.43 to 0.65      (c) 0.85 to 1.20      (d) 1.40 to 2.00

**IES-35. Ans. (d)** Speed ratio,  $K_u = \frac{u}{\sqrt{2gH}}$ ;  $K_u$  ranges from 1.40 to 2.0.

## Draft Tube

**IES-36. The use of a draft tube in a reaction type water turbine helps to:**

(a) Prevent air from entering

[IES-1996; 2002; 2007]

(b) Increase the flow rate

(c) Convert the kinetic energy to pressure energy

(d) Eliminate eddies in the downstream

**IES-36. Ans. (c)**

**IES-37. The level of runner exit is 5 m above the tail race, and atmospheric pressure is 10.3 m. The pressure at the exit of the runner for a divergent draft tube can be:** [IES-2001]

(a) 5 m

(b) 5.3 m

(c) 10 m

(d) 10.3 m

**IES-37. Ans. (b)**

**IES-38. Which of the following water turbines does not require a draft tube?**

(a) Propeller turbine

(b) Pelton turbine

[IES-2006]

(c) Kaplan turbine

(d) Francis turbine

**IES-38. Ans. (b)**

**IES-39. Consider the following statements:**

[IES-1999]

1. A draft tube may be fitted to the tail end of a Pelton turbine to increase the available head.

2. Kaplan turbine is an axial flow reaction turbine with adjustable vanes on the hub.

3. Modern Francis turbine is a mixed flow reaction turbine.

Which of these statements are correct?

(a) 1, 2 and 3

(b) 1 and 2

(c) 2 and 3

(d) 1 and 3

**IES-39. Ans. (c)** In a Pelton turbine, draft tube can't help to increase the available head since the jet from nozzle gets exposed to atmospheric air.

**IES-40. Which one of the following forms of draft tube will NOT improve the hydraulic efficiency of the turbine? [IES-1998]**

- (a) Straight cylindrical (b) Conical type  
(c) Bell-mouthed (d) Bent tube

**IES-40. Ans. (a)**

**IES-41. A hydraulic power station has the following major items in the hydraulic circuit: [IES-1995]**

1. Draft tube                      2. Runner                      3. Guide wheel  
4. Penstock                      5. Scroll case

**The correct sequence of these items in the direction of flow is:**

- (a) 4,2,3,1,5                      (b) 4,3,2,5,1                      (c) 1,2,3,5,4                      (d) 1,3,2,4,5

**IES-41. Ans. (b)** The sequence of various items in hydraulic plant is penstock, guide wheel, runner, scroll case, draft tube.

**IES-42. Match the following [IES-1992]**

**List-I**

A. Wicket gates

B. Volute tube

C. Draft tube

D. Axial flow pumps

**List-II**

1. Gradually converts the velocity head to pressure head

2. Pressure head recovery

3. High value of N and Ns

4. Adjustable by governor according to load

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	4	1	2	3	(b)	1	3	2	4
(c)	2	3	4	1	(d)	2	3	1	4

**IES-42. Ans. (a)**

**IES-43. The movable wicket gates of a reaction turbine are used to: [IES-1995]**

- (a) Control the flow of water passing through the turbine.  
(b) Control the pressure under which the turbine is working.  
(c) Strengthen the casing of the turbine  
(d) Reduce the size of the turbine.

**IES-43. Ans. (a)** The movable wicket gates of a reaction turbine are used to control the flow of water passing through the turbine.

**IES-44. A hydraulic reaction turbine working under a head of 16 m develops 640 kW of power. What is the unit power of the turbine? [IES-2009]**

- (a) 10 kW                      (b) 40 kW                      (c) 60 kW                      (d) 160 kW

**IES-44. Ans. (a) Unit power of the turbine**

$$= \frac{P}{H^{\frac{5}{2}}} = \frac{640}{(16)^{\frac{5}{2}}} = \frac{640}{64} = 10 \text{ kW}$$

## Specific Speed

**IES-45. Consider the following statements pertaining to specific speed of turbo machines: [IES-2008]**

1. Specific speed varies with shape of the runner and other parts of the machine.  
2. Machines with higher specific speeds are limited to low heads.

3. Specific speed is dimensionless and is independent of variation of type of fluid used.

Which of the statements given above are correct?

- (a) 1, 2 and 3      (b) 1 and 2 only      (c) 2 and 3 only      (d) 1 and 3 only

IES-45. Ans. (b) (i) Specific speed varies with shape of the runner and other parts of the machine.

- (ii) Machines with higher specific speeds have low heads

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

- (iii) Specific speed is not a dimensionless quantity. It is different in MKS and SI units also.

IES-46. The specific speed ( $N_s$ ) of a water turbine is expressed by which one of the following equations? [IES-1997, 2007; IAS-1996]

- (a)  $N_s = \frac{N\sqrt{P}}{H^{5/4}}$       (b)  $N_s = \frac{N\sqrt{P}}{H^{3/4}}$       (c)  $N_s = \frac{N\sqrt{Q}}{H^{5/4}}$       (d)  $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$

IES-46. Ans. (a)

IES-47. Specific speed of a pump and specific speed of a turbine are (symbols have the usual meaning). [IES-1994]

- (a)  $\frac{N\sqrt{Q}}{H^{3/4}}$  and  $\frac{N\sqrt{P}}{H^{5/4}}$  respectively      (b)  $\frac{N\sqrt{Q}}{H^{3/4}}$  and  $\frac{N\sqrt{P}}{H^{3/4}}$  respectively  
(c)  $\frac{N\sqrt{Q}}{H^{5/4}}$  and  $\frac{N\sqrt{P}}{H^{5/4}}$  respectively      (d)  $\frac{N\sqrt{Q}}{H^{5/4}}$  and  $\frac{N\sqrt{P}}{H^{3/4}}$  respectively

IES-47. Ans. (a)

IES-48. Assertion (A): For higher specific speeds, radial flow pumps have the greatest efficiency. [IES-2004]

Reason (R): Pumps having larger discharge under smaller heads have higher specific speeds.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-48. Ans. (a)  $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$  if  $Q \uparrow$ ;  $H \downarrow$  and  $N_s \uparrow$

IES-49. In the statement, "in a reaction turbine installation, the head of water is decreased and the rpm is also decreased at a certain condition of working. The effect of each of these changes will be to X power delivered due to decrease in head and to Y power delivered due to

decrease in rpm",  $\frac{N\sqrt{Q}}{H^{3/4}}$ , X and Y stand respectively for [IES-1993]

- (a) Decrease and increase      (b) Increase and increase  
(c) Decrease and decrease      (d) Increase and decrease

IES-49. Ans. (a) We have to find the effect of decrease of head and decrease of speed on power developed.

For hydraulic reaction turbines,  $P \propto H^{3/2}$

Thus decrease of head would result in decrease of power delivered.

The speed  $N \propto \frac{1}{\sqrt{P}}$  or  $P \propto \frac{1}{N^2}$

Thus decrease in speed will result in increase of power.

**IES-50. Which one of the following is correct? [IES-2008]**

**If the number of jets in a Pelton turbine is  $n$ , then the specific speed is:**

- (a)  $\propto n^2$  (b)  $\propto n$  (c)  $\propto n^{1/2}$  (d) Independent of  $n$

**IES-50. Ans. (c)** Specific speed ( $N_s$ ): of a turbine is defined as the speed of a geometrically turbine which would develop unit power when working under a unit head. It is given by the relation,  $N_s = \frac{N\sqrt{P}}{H^{5/4}}$  where,  $P$  = shaft power, and  $H$  = net head on the turbine. In shaft if number of jet increases then power will increase. Shaft power ( $P$ ) =  $n.p$

$$N_s = \frac{N\sqrt{np}}{H^{5/4}}$$

**IES-51. The specific speed of a turbine is defined as the speed of a member of the same homologous series of a such a size that it [IES-1996]**

- (a) Delivers unit discharge at unit head.  
(b) Delivers unit discharge at unit power.  
(c) Delivers unit power at unit discharge.  
(d) Produces unit power under a unit head.

**IES-51. Ans. (d)** The specific speed of a turbine is defined as the speed of member of the same homologous series of such a size that it produces unit power under a unit head.

**Specific speed:** It is defined as the speed of a similar turbine working under a head of 1 m to produce a power output of 1 kW. The specific speed is useful to compare the performance of various type of turbines. The specific speed differs per different types of turbines and is same for the model and actual turbine.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

**IES-52. Which one of the following is the correct statement? [IES-2007]**  
**Specific speed of a fluid machine**

- (a) Refers to the speed of a machine of unit dimensions.  
(b) Is a type-number representative of its performance?  
(c) Is specific to the particular machine.  
(d) Depends only upon the head under which the machine operates.

**IES-52. Ans. (c)** It is not depends only on head.

**IES-53. An impulse turbine operating with a single nozzle has a specific speed of 5. What will be the approximate specific speed of the turbine if the turbine is operated with one more additional nozzle of the same size? [IES-2004]**

- (a) 4 (b) 6 (c) 7 (d) 10

**IES-53. Ans. (c)** Power will be double with additional nozzle.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \text{ as } N \text{ \& } H \text{ are const}$$

$$N_s \propto \sqrt{P} \text{ or } \frac{(N_s)_2}{(N_s)_1} = \sqrt{\frac{P_2}{P_1}} \text{ or } (N_s)_2 = (N_s)_1 \times \sqrt{\frac{P_2}{P_1}} = 5 \times \sqrt{2} = 7$$

**IES-54.** Two Pelton wheels *A* and *B* have the same specific speed and are working under the same head. Wheel *A* produces 400 kW at 1000 rpm. If *B* produces 100 kW, then its rpm is: [IES-2003]

- (a) 4000 (b) 2000 (c) 1500 (d) 1250

**IES-54. Ans. (b)**  $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

$$N_A \sqrt{P_A} = N_B \sqrt{P_B} \Rightarrow 1000 \sqrt{400} = N_B \sqrt{100} \Rightarrow N_B = 2000 \text{ rpm}$$

**IES-55.** Match List-I (Specific speed) with List-II (Expression/Magnitude) and select the correct answer: [IES-2004]

List-I					List-II				
A. Specific speed of turbine					1.	$N\sqrt{Q} / H^{3/4}$			
B. Specific speed of pump					2.	$N\sqrt{P} / H^{5/4}$			
C. Specific speed of Pelton wheel					3.	50 – 250			
D. Specific speed of Francis turbine					4.	10 – 50			
Codes:	A	B	C	D		A	B	C	D
(a)	3	4	1	2	(b)	3	2	1	4
(c)	2	1	4	3	(d)	1	2	4	3

**IES-55. Ans. (c)**

**IES-56.** Assertion (A): The specific speed of a Pelton turbine is low. [IES-2002]  
Reason (R): Pelton turbine works under a high head and handles low discharge.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IES-56. Ans. (b)**

**IES-57.** Consider the following turbines/wheels [IES-1998, 1999, 2000, 2001]

- Francis turbine
- Pelton wheel with two or more jets
- Pelton wheel with a single jet
- Kaplan turbine

The correct sequence of these turbines/wheels in increasing order of their specific speeds is:

- (a) 2, 3, 1, 4 (b) 3, 2, 1, 4 (c) 2, 3, 4, 1 (d) 3, 2, 4, 1

**IES-57. Ans. (b)** Specific speed of propeller is around 300 – 1000, Francis 60 – 300, and Pelton 10 – 60.

**IES-58.** If the full-scale turbine is required to work under a head of 30 m and to run at 428 r.p.m., then a quarter-scale turbine model tested under a head of 10 m must run at: [IES-2000]

- (a) 143 r.p.m. (b) 341 r.p.m. (c) 428 r.p.m. (d) 988 r.p.m.

**IES-58. Ans. (d)**  $\frac{H}{N^2 D^2} = \text{const. or } \left( \frac{H}{N^2 D^2} \right)_m = \left( \frac{H}{N^2 D^2} \right)_p \text{ or } N_m = N_p \sqrt{\left( \frac{H_m}{H_p} \right)} \times \left( \frac{D_p}{D_m} \right)$

$$N_m = 428 \sqrt{\left( \frac{10}{30} \right)} \times \left( \frac{4}{1} \right) = 988 \text{ rpm}$$



IES-64. Ans. (c)

IES-65. Consider the following statements: [IES-1995]

Cavitation in hydraulic machines occurs at the

1. Exit of a pump      2. Entry of the pump      3. Exit of a turbine

Of these correct statements are:

- (a) 1 and 2      (b) 1 and 3      (c) 1, 2 and 3      (d) 2 and 3

IES-65. Ans. (d)

IES-66. In the phenomenon of cavitation, the characteristic fluid property involved is: [IES-2002]

- (a) Surface tension      (b) Viscosity  
(c) Bulk modulus of elasticity      (d) Vapour pressure

IES-66. Ans. (d)

IES-67. Match the following [IES-1992]

**List-I**

- A. Wave drag of a ship  
B. Pressure coefficient  
C. Thoma number  
D. Stokes law

**List-II**

1. Cavitation in pumps and turbines  
2.  $\rho r.L^3 r$   
3.  $Re \approx 0.1$   
4.  $\frac{\Delta p}{\rho V^2 / 2}$

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	2	4	3	(b)	4	3	1	2
(c)	1	3	4	2	(d)	2	4	1	3

IES-67. Ans. (d)

IES-68. Match List-I (Fluid properties) with List-II (Related terms) and select the correct answer. [IES-1996]

**List-I**

- A. Capillarity  
B. Vapour pressure  
C. Viscosity  
D. Specific gravity

**List-II**

1. Cavitation  
2. Density of water  
3. Shear forces  
4. Surface tension

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	4	2	3	(b)	1	4	3	2
(c)	4	1	2	3	(d)	4	1	3	2

IES-68. Ans. (d) It may be noted that capillarity is related with surface tension, vapour pressure with cavitation, viscosity with shear forces, and specific gravity with density of water.

IES-70. Which one of the following is correct? [IES-2008]

Water-tube boilers are preferred for

- (a) High pressure and high output  
(b) High pressure and low output  
(c) Low pressure and high output  
(d) Low pressure and low output

IES-70. Ans. (a)

IES-71. Consider the following statements: [IES-2001]

A surge tank provided on the penstock connected to a water turbine

1. Helps in reducing the water hammer

2. Stores extra water when not needed
  3. Provides increased demand of water
- Which of these statements are correct?

(a) 1 and 3      (b) 2 and 3      (c) 1 and 2      (d) 1, 2 and 3

IES-71. Ans. (d)

## Performance Characteristics

IES-72. Which of the following water turbines maintain a high efficiency over a long range of the part load? [IES-2006]

1. Francis turbine
2. Kaplan turbine
3. Pelton turbine
4. Propeller turbine

Select the correct answer using the codes given below

(a) 1 and 4      (b) Only 2 and 3      (c) 1, 2 and 3      (d) 2, 3 and 4

IES-72. Ans. (b)

IES-73. Which one of the following turbines exhibits a nearly constant efficiency over a 60% to 140% of design speed? [IES-2005]

- (a) Pelton turbine
- (b) Francis turbine
- (c) Deriaz turbine
- (d) Kaplan turbine

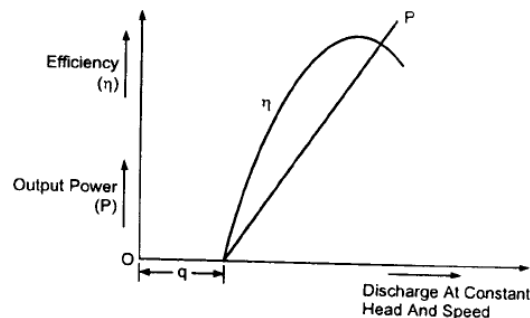
IES-73. Ans. (d)

IES-74. When a hydraulic turbine is operated, it is found that it has high design efficiency and this efficiency remains constant over a wide range of regulation from the design condition. What is the type of this turbine? [IES-2005]

- (a) Pelton
- (b) Francis
- (c) Kaplan
- (d) Propeller

IES-74. Ans. (c)

IES-75. For a water turbine, running at constant head and speed, the operating characteristic curves in the given figure show that upto a certain discharge 'q' both output power and efficiency remain zero. The discharge 'q' is required to:

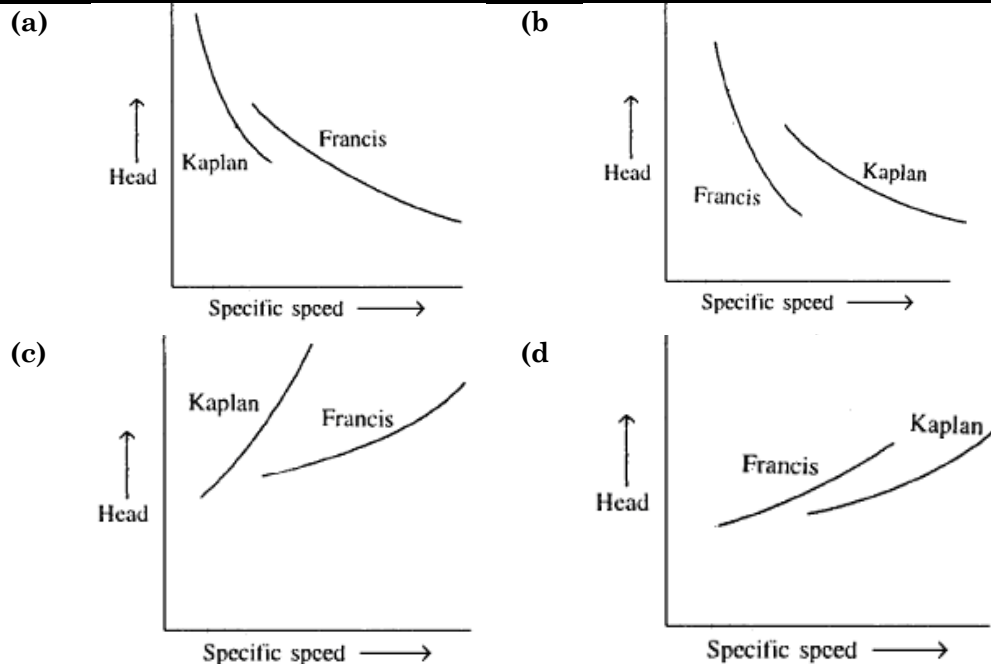


- (a) Overcome initial inertia
- (b) Overcome initial friction [IES-2001]
- (c) Keep the hydraulic circuit full
- (d) Keep the turbine running at no load

IES-75. Ans. (b)

IES-76. Which one of the following graphs correctly represents the relations between Head and Specific speed for Kaplan and Francis turbine? [IES-2003]





IES-76. Ans. (b)

IES-77. What does Euler's equation of turbo machines relate to? [IES-2008]

- (a) Discharge and power
- (b) Discharge and velocity
- (c) Head and power
- (d) Head and velocity

IES-77. Ans. (a) Euler's equation of turbo machines relate to discharge and power.

## Previous Years IAS Questions

### Classification of Hydraulic Turbines

IAS-1. Assertion (A): In many cases, the peak load hydroelectric plants supply power during average load as also during peak load, whenever require. Reason (R): Hydroelectric plants can generate a very wide range of electric power, and it is a simple exercise to restart power generation and connecting to the power grid. [IAS-1996]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-1. Ans. (a)

IAS-2. Assertion (A): In many cases, the peak load hydroelectric plants supply power during average load as also during peak load, whenever require. Reason(R): Hydroelectric plants can generate a very wide range of electric power, and it is a simple exercise to restart power generation and connecting to the power grid. [IAS-1996]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-2. Ans. (a)

IAS-3. Match List-I (Water turbines) with List-II (Application) and select the correct answer using the codes given below the lists: [IAS-1999]

List-I				List-II			
A. Pelton				1. High head and low discharge			
B. Francis				2. High head and high discharge			
C. Kaplan				3. Medium head and medium discharge			
				4. Low head and high discharge			
Codes:	A	B	C		A	B	C
(a)	1	3	2	(b)	1	3	4
(c)	2	4	3	(d)	3	2	4

IAS-3. Ans. (b)

## Impulse Turbines – Pelton Wheel

IAS-4. In the case of Pelton turbine installed in a hydraulic power plant, the gross head available is the vertical distance between [IAS-1994]

- (a) Forebay and tail race (b) Reservoir level and turbine inlet  
(c) Forebay and turbine inlet (d) Reservoir level and tail race.

IAS-4. Ans. (b)

IAS-5. In a simple impulse turbine, the nozzle angle at the entrance is  $30^\circ$ . What is the blade-speed ratio ( $u/V$ ) for maximum diagram efficiency? [IAS-2004]

- (a) 0.25 (b) 0.5 (c) 0.433 (d) 0.866

IAS-5. Ans. (c)  $\frac{u}{V} = \frac{\cos \alpha}{2} = \frac{\cos 30}{2} = 0.433 \Rightarrow \frac{u}{\sqrt{2 \times 9.8 \times 900}} = 0.45$   
 $\Rightarrow u = 0.45 \sqrt{2 \times 9.8 \times 900} = 60 \text{ m/sec.}$

IAS-6. If  $\alpha$  is the blade angle at the outlet, then the maximum hydraulic efficiency of an ideal impulse turbine is: [IAS-1999; IES-2005]

- (a)  $\frac{1 + \cos \alpha}{2}$  (b)  $\frac{1 - \cos \alpha}{2}$  (c)  $\frac{1 - \sin \alpha}{2}$  (d)  $\frac{1 + \sin \alpha}{2}$

IAS-6. Ans. (a)

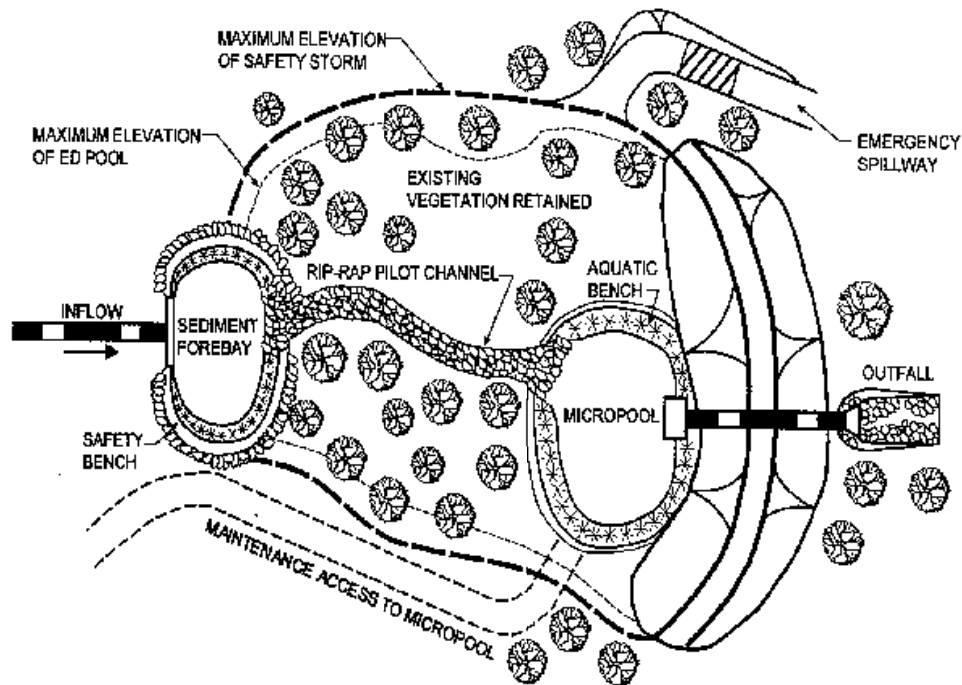
IAS-7. In a hydroelectric power plant, forebay refers to the [IAS-1997]

- (a) Beginning of the open channel at the dam  
(b) End of penstock at the valve house  
(c) Level where penstock begins  
(d) Tail race level at the turbine exit

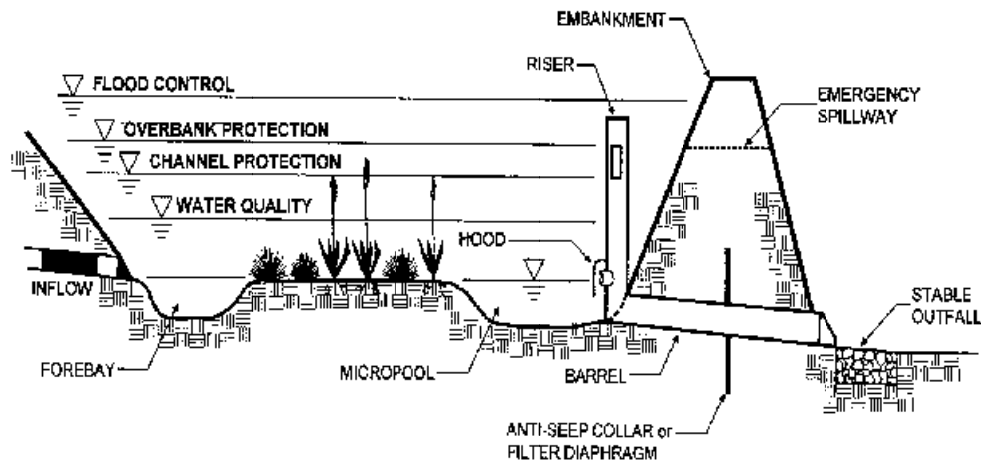
IAS-7. Ans. (c) **What is a sediment forebay:** A sediment forebay is a small pool located near the inlet of a storm basin or other stormwater management facility. These devices are designed as initial storage areas to trap and settle out sediment and heavy pollutants before they reach the main basin. Installing an earth beam, gabion wall, or other barrier near the inlet to cause stormwater to pool temporarily can form the pool area. Sediment forebays act as a pretreatment feature on a stormwater pond and can greatly reduce the overall pond maintenance requirements.

**Why consider a sediment forebay:** These small, relatively simple devices add a water quality benefit beyond what is accomplished by the basin itself.

Forebays also make basin maintenance easier and less costly by trapping sediment in one small area where it is easily removed, and preventing sediment buildup in the rest of the facility.



PLAN view of forebay



Profile of forebay

## Design Aspects of Pelton Wheel

IAS-8. Assertion (A): For high head and low discharge hydraulic power plant, Pelton wheel is used as prime mover. [IAS-2004]

Reason(R): The non-dimensional specific speed of Pelton wheel at designed speed is high.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-8. **Ans. (c)** The non-dimensional specific speed of Pelton wheel at designed speed is low.

## Reaction Turbine

IAS-9. Which one of the following is an example of a pure (100%) reaction machine? [IAS-1998]

- (a) Pelton wheel (b) Francis turbine (c) Modern gas turbine (d) Lawn sprinkler

IAS-9. **Ans. (d)**

IAS-10. In the case of Francis turbine, velocity ratio is defined as  $\frac{V_3}{\sqrt{2gH}}$  where

$H$  is the available head and  $V_3$  is the

- (a) Absolute velocity at the draft tube inlet  
(b) Mean velocity of flow in the turbine  
(c) Absolute velocity at the guide vane inlet  
(d) Flow velocity at the rotor inlet

[IAS-1997]

IAS-10. **Ans. (d)**

## Propeller Turbine

IAS-11. In which of the following hydraulic turbines, the efficiency would be affected most when the flow rate is changed from its design value?

- (a) Pelton wheel (b) Kaplan turbine  
(c) Francis turbine (d) Propeller turbine

[IAS-2007]

IAS-11. **Ans. (d)**

IAS-12. Which one of the following is not correct regarding both Kaplan and propeller turbines? [IAS-1998]

- (a) The runner is axial (b) The blades are wing type  
(c) There are four to eight blades (d) The blades can be adjusted

IAS-12. **Ans. (d)**

IAS-13. Which one of the following is not correct regarding both Kaplan and propeller turbines? [IAS-1998]

- (a) The runner is axial (b) The blades are wing type  
(c) There are four to eight blades (d) The blades can be adjusted

IAS-13. **Ans. (d)**

IAS-14. Based on the direction of flow, which one of the following turbines is different from the other three? [IAS-1998]

- (a) Pelton turbine (b) Kaplan turbine  
(c) De laval turbine (d) Parson's turbine

IAS-14. **Ans. (d)**

IAS-15. The function of the draft tube in a reaction turbine is: [IAS-2002]

- (a) To enable the shaft of the turbine to be vertical  
(b) To transform a large part of pressure energy at turbine outlet into kinetic energy  
(c) To avoid whirl losses at the exit of the turbine  
(d) To transform a large part of kinetic energy at the turbine outlet into pressure energy

IAS-15. **Ans. (d)**

**IAS-16. Assertion (A):** A draft tube is used along with high head hydraulic turbines to connect the water reservoir to the turbine inlet. [IAS-2002]  
**Reason(R):** A draft tube is used to increase both the output and the efficiency of the turbine.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-16. Ans. (d)** A is false. A penstock is used in hydraulic turbine to connect reservoir to the turbine inlet.

**IAS-17. Assertion (A):** Pelton turbine is provided with a draft tube. [IAS-2001]  
**Reason(R):** Draft tube enables the turbine to be set at a convenient height above the tail race without loss of head.

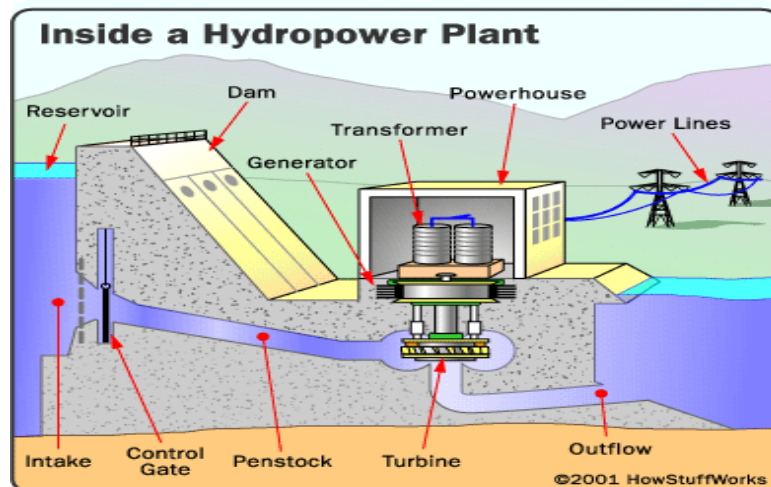
- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IAS-17. Ans. (d)** For Pelton turbine no draft tube needed.

**IAS-18. Match List-I with List-II and select the correct answer using the codes given below the Lists:** [IAS-2002]

List-I				List-II					
A. Head race				1. Channel, tunnel or pipes through which water is carried from reservoir to the turbine					
B. Tail race				2. Reservoir water level					
C. Penstock				3. Diverging tube discharging water from the turbine to the atmosphere					
D. Draft tube				4. The level at which water is discharged at atmospheric pressure					
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	
(a)	1	3	2	4	(b)	2	4	1	3
(c)	1	4	2	3	(d)	2	3	1	4

**IAS-18. Ans. (b)**



**IAS-19. The specific speed ( $N_s$ ) of a water turbine is expressed by which one of the following equations?** [IES-1997, 2007; IAS-1996]

$$(a) N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad (b) N_s = \frac{N\sqrt{P}}{H^{3/4}} \quad (c) N_s = \frac{N\sqrt{Q}}{H^{5/4}} \quad (d) N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

IES-19. Ans. (a)

IAS-20. Match List-I (Turbines) with List-II (Specific speeds in MKS units) and select the correct answer using the codes given below the lists

[IAS-2004]

List-I					List-II				
A.	Kaplan turbine				1.	10 to 35			
B.	Francis turbine				2.	35 to 60			
C.	Pelton wheel with single jet				3.	60 to 300			
D.	Pelton wheel with two or more jets				4.	300 to 1000			
Codes:	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	3	4	2	1
(c)	3	4	1	2	(d)	4	3	2	1

IAS-20. Ans. (a)

IAS-21. Consider the following statements with regard to the specific speeds of different types of turbine:

[IAS-2004]

1. High specific speed implies that it is a Pelton wheel
2. Medium specific speed implies that it is an axial flow turbine
3. Low specific speed implies that it is a Francis turbine

Which of these statements given above is/are correct?

- (a) 1 only      (b) 2 only      (c) 3 only      (d) None

IAS-21. Ans. (d) 1 is wrong. Low specific speed implies that it is a Pelton wheel

2 is wrong, High specific speed implies that it is an axial flow turbine

3 is wrong, Medium specific speed implies that it is a Francis turbine.

IAS-22. The specific speed of a hydraulic turbine is 40. What is the type of that turbine?

[IAS-2007]

- (a) Single jet Pelton turbine      (b) Multiple Pelton turbine  
(c) Francis turbine      (d) Kaplan turbine

IAS-22. Ans. (b) Specific speed of Pelton Turbine: Single Jet 10–30

Multi Jet 30–60

IAS-23. Cavitation damage in the turbine runner occurs near the

[IAS-2001]

- (a) Inlet on the concave side of the blades  
(b) Outlet on the concave side of the blades  
(c) Outlet on the convex side of the blades  
(d) Inlet on the convex side of the blades

IAS-23. Ans. (c)

## Surge Tanks

IAS-24. What is the purpose of a surge tank in high head hydroelectric plants?

[IAS-2007]

- (a) To act as a temporary storage during load changes  
(b) To improve the hydraulic efficiency  
(c) To prevent surges in generator shaft speed  
(d) To prevent water hammer due to sudden load changes

IAS-24. Ans. (d)

**IAS-25. Which one of the following is the purpose of a surge tank in a Pelton Turbine station? [IAS-2004]**

- (a) It acts as a temporary storage during load change
- (b) It prevents hydraulic jump
- (c) It prevents surges at the transformer
- (d) It prevents water hammer due to sudden reduction in load

**IAS-25. Ans. (d)**

**IAS-26. In hydraulic power-generation systems, surge tanks are provided to prevent immediate damage to: [IAS-2001]**

- (a) Draft tube
- (b) Turbine
- (c) Tail race
- (d) Penstocks

**IAS-26. Ans. (d)**

**IAS-27. The location of a surge tank in a high head hydraulic power plant would be: [IAS-1999]**

- (a) Nearer to the dam
- (b) At the powerhouse
- (c) Nearest to the powerhouse
- (d) Immaterial

**IAS-27. Ans. (c)**

**IAS-28. Match List-I (Water turbines) with List-II (Application) and select the correct answer using the codes given below the lists: [IAS-1999]**

List-I				List-II			
A.	Pelton			1.	High head and low discharge		
B.	Francis			2.	High head and high discharge		
C.	Kaplan			3.	Medium head and medium		
				4.	Low head and high discharge		
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>A</b>	<b>B</b>	<b>C</b>	
(a)	1	3	2	(c)	2	4	3
(b)	1	3	4	(d)	3	2	4

**IAS-28. Ans. (b)** There is no any turbine for High head and high discharge.

**IAS-29. Match List-I with List-II and select the correct answer using the codes given below the lists [IAS-1994]**

List-I					List-II				
A.	Propeller turbine				1.	Impulse turbine			
B.	Tangential turbine				2.	Kaplan turbine			
C.	Reaction is zero				3.	Gas turbine			
D.	Reaction turbine				4.	Pelton turbine			
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	
(a)	3	2	1	4	(b)	2	1	4	3
(c)	2	4	1	3	(d)	3	4	2	1

**IAS-29. Ans. (c)**

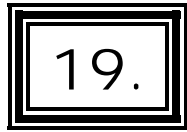
**IAS-30. Consider the following statements: [IAS-1998]**

- 1. A reverse jet protects Pelton turbine from over-speeding
- 2. Runner blades of Francis turbine are adjustable.
- 3. Draft tube is used invariably in all reaction turbine installations.
- 4. Surge shaft is a protective device on penstock.

**Of these statements:**

- (a) 1 and 2 are correct
- (b) 2 and 3 are correct
- (c) 3 and 4 are correct
- (d) 1 and 4 are correct

**IAS-30. Ans. (c)**



## Centrifugal Pump

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  21. Characteristics of Centrifugal Pumps
  22. Net Positive Suction Head (NPSH)
  23. Cavitation in Centrifugal Pumps
  24. Priming of a Centrifugal Pump
  25. Selection of Pumps
  26. Operational Difficulties in Centrifugal Pumps
-



**Question:** Derive the Euler's equation for rotating machines and explain the physical significance of each term. [IES-2002]

**Answer:** Fig. shows a portion of the impellers of a centrifugal pump with the one vane and the velocity triangle at the inlet and the outlet tips of the vane.

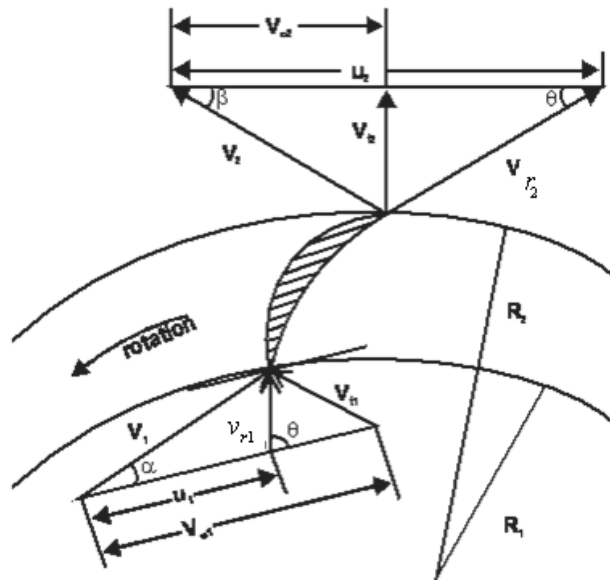


Fig. velocity triangles for an impeller vane

Let,  $D_1$  = diameter of the impeller at inlet ( $R_1 = D_1/2$ )

$u_1$  = tangential velocity of the impeller at inlet.

$$= \frac{\pi D_1 N}{60} = \frac{2\pi R_1 N}{60} = \omega R_1$$

$V_1$  = absolute velocity of water at inlet.

$V_{\omega_1}$  = velocity of whirl at inlet

$V_{r_1}$  = relative velocity of liquid at inlet.

$V_{f_1}$  = velocity of flow at inlet.

$\alpha$  = inlet guide vane angle

$\theta$  = inlet vane angle

$D_2, u_2, V_2, V_{\omega_2}, V_{r_2}, V_{f_2}, \beta$  and  $\phi$  are the corresponding values at outlet, and  $N$  = speed of the impeller in r.p.m

$$\therefore \text{Angular velocity } (\omega) = \frac{2\pi N}{60} \text{ rad/s}$$

→ While passing through the impeller, the velocity of whirl change and there is a change of moment of momentum.

$$\text{Moment of momentum at inlet} = \frac{1}{g}(V_{\omega_1} R_1) \text{ per unit weight of liquid.}$$

$$\text{Moment of momentum at outlet} = \frac{1}{g}(V_{\omega_2} R_2) \text{ per unit weight of liquid.}$$

$$\therefore \text{Work done per second per unit weight of liquid} \\ = \text{Torque} \times \text{angular velocity}$$

= rate of change of moment of momentum x angular velocity

$$= \frac{1}{g} \{V_{\omega_2} R_2 - V_{\omega_1} R_1\} \times \omega = \frac{1}{g} \{V_{\omega_2} (\omega R_2) - V_{\omega_1} (\omega R_1)\}$$

$$= \frac{1}{g} \{V_{\omega_2} u_2 - V_{\omega_1} u_1\}$$

This is known as Euler, momentum eq<sup>n</sup> for centrifugal pump.

The term  $\frac{1}{g} \{V_{\omega_2} u_2 - V_{\omega_1} u_1\}$  is referred to as Euler head (H<sub>e</sub>)

Further from outlet velocity triangle we have

$$V_{f_2}^2 = V_2^2 + (u_2 - V_{\omega_2})^2 \quad \text{or} \quad V_{f_2}^2 = V_2^2 - (u_2 - V_{\omega_2})^2 \quad \text{and} \quad V_{f_2}^2 = V_2^2 - V_{\omega_2}^2$$

$$\therefore V_2^2 - V_{\omega_2}^2 = V_{f_2}^2 - (u_2 - V_{\omega_2})^2 \quad \text{or} \quad V_2^2 - V_{\omega_2}^2 = V_{f_2}^2 - u_2^2 - V_{\omega_2}^2 + 2u_2 V_{\omega_2}$$

$$\therefore u_2 V_{\omega_2} = \frac{1}{2} \{V_2^2 + u_2^2 - V_{f_2}^2\}$$

Similarly from inlet triangle

$$u_1 V_{\omega_1} = \frac{1}{2} \{V_1^2 + u_1^2 - V_{f_1}^2\}$$

$\therefore$  Work done per second per unit weight of liquid (H<sub>e</sub>)

$$H_e = \frac{V_2^2 - V_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} + \frac{V_{f_1}^2 - V_{f_2}^2}{2g}$$

$\Rightarrow$  Above equation indicates that work done on the liquid consists of three terms.

(i) The first term  $\frac{V_2^2 - V_1^2}{2g}$  represent the increase in kinetic energy or dynamic head.

(ii) The second term  $\frac{u_2^2 - u_1^2}{2g}$  represents an increase in static pr.

(iii) The third term  $\frac{V_{f_1}^2 - V_{f_2}^2}{2g}$  indicates the change in kinetic energy due to retardation of flow relative to the impeller.

## OBJECTIVE QUESTIONS (GATE, IES, XIAS)

## Previous Years GATE Questions

- |         |                              |                      |             |
|---------|------------------------------|----------------------|-------------|
| GATE-1. | List-I                       | List-II              | [GATE-1998] |
|         | (A) High head, low flow rate | (1) Streamlined body |             |
|         | (B) Low head, high flow rate | (2) Boundary layer   |             |
|         | (C) Heat transfer            | (3) Orifice meter    |             |
|         | (D) Low drag                 | (4) Centrifugal pump |             |
|         |                              | (5) Axial flow pump  |             |
|         |                              | (6) Nusselt number   |             |

GATE-1. Ans. (A) –4, (B) –5, (C) –6, (d) –1

**Work Done by the Impeller (or Centrifugal Pump) on Liquid**

GATE-2. When the speed of a centrifugal pump is doubled, the power required to drive the pump will: [GATE-2000]

- |                      |                      |
|----------------------|----------------------|
| (a) Increase 8 times | (b) Increase 4 times |
| (c) Double           | (d) Remain the same  |

GATE-2. Ans. (a)

**Heads of a Pump**

**Common Data Question No. 3 & 4.**

A centrifugal pump has an efficiency of 80%. The specifications of the pump are: Discharge = 70 m<sup>3</sup>/hr, head = 7 m, speed = 1450 rpm and diameter = 2000 mm. If the speed of this pump is increased to 1750 rpm.

- GATE-3. Discharge and head developed are given respectively: [GATE-2002]
- |   |                                      |
|---|--------------------------------------|
| (a) 84.48 m <sup>3</sup> /Hr and 10.2 m | (b) 48.8 m <sup>3</sup> /Hr and 20 m |
| (c) 48.8 m <sup>3</sup> /Hr and 10.2 m  | (d) 58.4 m <sup>3</sup> /Hr and 12 m |

GATE-3. Ans. (a)

- GATE-4. Power input required is given by: [GATE-2002]
- |              |              |             |           |
|--------------|--------------|-------------|-----------|
| (a) 1.066 kW | (b) 1.066 kW | (c) 2.12 kW | (d) 20 kW |
|--------------|--------------|-------------|-----------|

GATE-4. Ans. (a)

**Losses and Efficiencies of a Centrifugal Pump****Losses in Centrifugal Pump**

GATE-5. A centrifugal pump is required to pump water to an open water tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01. The average speed of water in the pipe is 2 m/s. If it is to maintain a constant head of 5 m in the tank, neglecting other minor losses, then absolute discharge pressure at the pump exit is: [GATE-2004]

- |               |               |                |                |
|---------------|---------------|----------------|----------------|
| (a) 0.449 bar | (b) 5.503 bar | (c) 44.911 bar | (d) 55.203 bar |
|---------------|---------------|----------------|----------------|

GATE-5. Ans. (b) Given:  $d = 0.2$  m,  $L = 4000$  m  
 $F = 0.01$ ,  $v = 2$  m/s

Head loss due to friction,

$$h_f = \frac{fLv^2}{2gd} = \frac{0.01 \times 4000 \times (2)^2}{2 \times 9.81 \times 0.2} = 40.77 \text{ m}$$

Pressure corresponding to this head =  $\rho g(h_f + h + h_{atm})$

$$= 1000 \times 9.81(40.77 + 5 + 10.3)$$

$$= 5.50 \times 10^5 \text{ N/m}^2 = 5.50 \text{ bar}$$

**GATE-6. In terms of speed of rotation of the impeller (N), discharge (Q) and change in total head through the machine, the specific speed for a pump is.....** [GATE-1994]

**GATE-6. Ans.**  $\frac{N\sqrt{Q}}{H^{3/4}}$

**GATE-7. The following is the arrangement of rotary pumps in descending order of specific speed at their best efficiency:** [GATE-1992]

- (a) Positive displacement, centrifugal, axial
- (b) Centrifugal, positive displacement, axial
- (c) Axial, centrifugal, positive displacement
- (d) Axial, positive displacement, centrifugal

**GATE-7 Ans. (c)**

**GATE-8. A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m at a flow rate of 60 litres per minute. If the rpm is changed to 1000, then the head H in metres and flow rate Q in litres per minute at maximum efficiency are estimated to be:**

- (a) H = 60, Q = 120
- (b) H = 120, Q = 120
- (c) H = 60, Q = 480
- (d) H = 120, Q = 30

[GATE-2003]

**GATE-8. Ans. (b)**  $N_1 = 500 \text{ rpm}$ ,  $H_1 = 30 \text{ m}$   $Q_1 = 60 \text{ l/minute}$

$N_2 = 1000 \text{ rpm}$ ,  $H_2 = ?$  and  $Q_2 = ?$

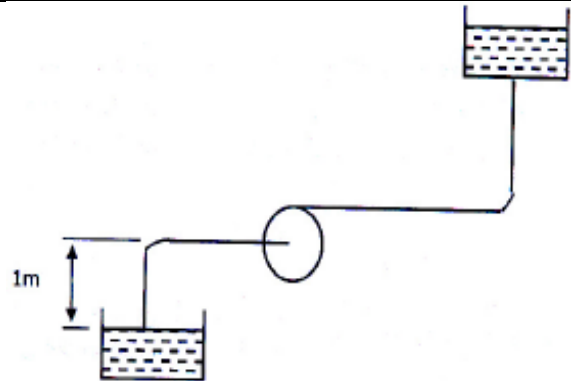
Since,  $\frac{\sqrt{H_1}}{DN_1} = \frac{\sqrt{H_2}}{DN_2}$

$$\therefore H_2 = \left(\frac{N_2}{N_1}\right)^2 \quad \text{and} \quad H_1 = \left(\frac{1000}{500}\right)^2 \times 30 = 120 \text{ m}$$

$$\frac{Q_1}{D^3 N_1} = \frac{Q_2}{D^3 N_2}$$

$$\Rightarrow Q_2 = \left(\frac{N_2}{N_1}\right) Q_1 \quad \text{and} \quad Q_1 = \left(\frac{1000}{500}\right) \times 60 = 120 \text{ l/minute}$$

**GATE-9.** A horizontal-shaft centrifugal pump lifts water at  $65^{\circ}\text{C}$ . The suction nozzle is one meter below pump centerline. The pressure at this point equals 200 kPa gauge and velocity is 3 m/s. Stream tables show saturation pressure at  $65^{\circ}\text{C}$  is 25 kPa, and specific volume of the saturated liquid is  $0.001020 \text{ m}^3/\text{kg}$ . The pump Net Positive Suction Head (NPSH) in meters is:



[GATE-2006]

- (a) 24 (b) 26 (c) 28 (d) 30

**GATE-9. Ans. (a)**

## Priming of a Centrifugal Pump

**GATE-10.** Match the items in columns I and II

[GATE-2007]

Column I					Column II			
P:	Centrifugal compressor				1:	Axial flow		
Q:	Centrifugal pump				2:	Surging		
R:	Pelton wheel				3:	Priming		
S:	Kaplan turbine				4:	Pure impulse		
Codes:	P	Q	R	S	P	Q	R	S
(a)	2	3	4	1	(b)	2	3	1
(c)	3	4	1	2	(d)	1	2	3

**GATE-10. Ans. (a)**

## Previous Years IES Questions

### Classification of Pumps

**IES-1.** Which one of the following is correct? [IES-2004]

In positive displacement pumps, the slip can sometimes be negative when the actual discharge is greater than the theoretical discharge. This happens in

- (a) Small suction pipes coupled with a low delivery head  
 (b) Small suction pipes coupled with a medium delivery head  
 (c) Long suction pipes coupled with a low delivery head  
 (d) Long suction pipes coupled with medium delivery head

**IES-1. Ans. (c)**

**IES-2.** Which one of the following pumps is not a positive displacement pump?

- (a) Reciprocating pump (b) Centrifugal pump [IES 2007]  
 (c) Vane pump (d) Lobe pump

**IES-2. Ans. (b)**

**IES-3.** Assertion (A): The efficiency of a pump is generally less than that of a turbine. [IES-2000]

**Reason (R):** Although the losses in the two types of machines are of the same kind, the losses in pumps are more due to eddy and turbulence.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-3. Ans. (a)** The order of efficiency in pump is 65% but in turbine efficiency is about to 90%. Assertion A is correct. Pump is against nature that so why efficiency is low.

**IES-4. Assertion (A):** Pump lifts water from a lower level to a higher level.

**Reason (R):** In pump, mechanical energy is converted into pressure energy. [IES-1997]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-4. Ans. (a)**

**IES-5. Match List-I (Type of model) With List-II (Liquid handled) and select the correct answer:** [IES-2004]

List-I				List-II			
A. Closed impeller pump				1. Sandy water			
B. Semi-open impeller pump				2. Acids			
C. Open impeller pump				3. Sewage water			
Codes:	A	B	C	A	B	C	
(a)	1	3	2	(b)	3	1	2
(c)	2	3	1	(d)	1	2	3

**IES-5. Ans. (c)**

**IES-6. Match List-I (Industrial needs) with List-II (Type of pump) and select the correct answer using the codes given below the Lists:** [IES-2003]

List-I					List-II				
A. Combustible fluid to be pumped					1. Single stage centrifugal				
B. High head but small discharge needed					2. Multi-stage centrifugal				
C. Low head but large discharge needed					3. Positive displacement				
D. High head and high discharge needed					4. Jet pump				
Codes:	A	B	C	D	A	B	C	D	
(a)	3	2	1	4	(b)	4	3	1	2
(c)	3	1	4	2	(d)	4	3	2	1

**IES-6. Ans. (b)**

## **Advantage of Centrifugal Pump over Displacement (Reciprocating) Pump**

**IES-7. Centrifugal pumps have which of the following advantages?** [IES-2008]

- 1. Low initial cost
- 2. Compact, occupying less floor space
- 3. Easy handling of highly viscous fluids

Select the correct answer using the code given below:

- (a) 1, 2 and 3      (b) 1 and 2 only      (c) 1 and 3 only      (d) 2 and 3 only

IES-7. Ans. (a) Centrifugal pumps has low initial cost and low maintenance cost that so why it is widely used.

## Component Parts of a Centrifugal Pump

IES-8. The correct sequence of the centrifugal pump components through which the fluid flows is: [IES-2002]

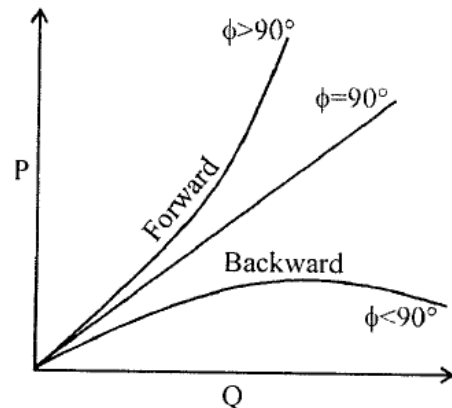
- (a) Impeller, Suction pipe, Foot valve and strainer, Delivery pipe
- (b) Foot valve and strainer, Suction pipe, Impeller, Delivery pipe
- (c) Impeller, Suction pipe, Delivery pipe, Foot valve strainer
- (d) Suction pipe, Delivery pipe, Impeller, Foot valve and strainer

IES-8. Ans. (b)

IES-9. For attaining a non-overloading characteristic in centrifugal pumps.

- (a) Backward bent vanes are preferred over forward bent vanes. [IES-1995]
- (b) Forward bent vanes are preferred over backward bent vanes.
- (c) Forward bent vanes are preferred over vanes radial at outlet.
- (d) Vanes radial at outlet are preferred over backward vanes.

IES-9. Ans. (a) As shown in figure, in forward curved vanes head increases with the discharge. Hence, with forward curves vanes ( $\phi > 90^\circ$ ) power input rises sharply, with the discharge whereas in backward vanes ( $\phi < 90^\circ$ ), the power input decreases steadily with the increases in discharge.



IES-10. What will be the shape of the velocity triangle at the exit of a radial bladed centrifugal impeller, taking into account slip? [IES-1998]

- (a) Right-angled
- (b) Isosceles
- (c) All angles less than  $90^\circ$
- (d) One angle greater than  $90^\circ$

IES-10. Ans. (d)

IES-11. A centrifugal pump driven by a directly coupled 3 kW motor of 1450 rpm speed is proposed to be connected to another motor of 2900 -rpm speed. The power of the motor should be: [IES-2002]

- (a) 6 kW
- (b) 12 kW
- (c) 18 kW
- (d) 24 kW

IES-11. Ans. (d)

IES-12. The power absorbed by a hydraulic pump is directly proportional to which one of the following? [IES-2007]

- (a)  $N$
  - (b)  $N^2$
  - (c)  $N^3$
  - (d)  $N^4$
- (Where  $N$  is the rotational speed of the pump)

IES-12. Ans. (c)

IES-13. The volute casing of a centrifugal pump has which of the following functions? [IES-2008]

- 1. Eliminating loss of head due to change in velocity after exit from impeller

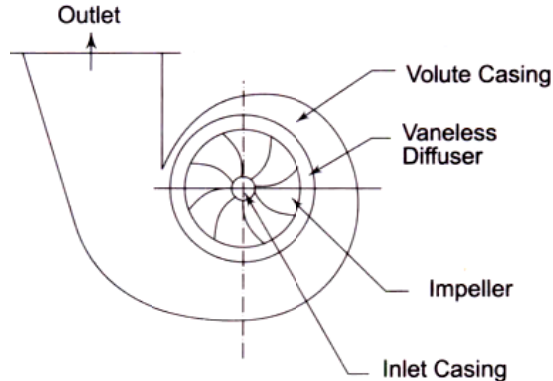
2. Directs the flow towards the delivery pipe
3. Converts a part of the velocity head to pressure head
4. Gives a constant velocity of flow

Select the correct answer using the code given below:

- (a) 1, 2 and 4      (b) 2 and 3 only      (c) 1 and 4 only      (d) 2 and 4 only

IES-13. Ans. (b) The volute casing of a centrifugal pump has following function

- (i) Directs the flow towards the delivery pipe
- (ii) converts a part of the velocity head to pressure head



IES-14. Which of the following are the functions of a volute casing in a centrifugal pump? [IES-1994]

1. To collect water from the periphery of the impeller and to transmit it to the delivery pipe at constant velocity.
2. To increase the discharge of the pump.
3. To increase the efficiency of the pump.
4. To reduce the loss of head in discharge.

Codes:

- (a) 1, 2 and 3      (b) 2, 3 and 4      (c) 1, 3 and 4      (d) 1 and 2

IES-14. Ans. (c)

IES-15. Assertion (A): The volute casing of a centrifugal pump helps in creating the high velocity head necessary for enabling water flow upwards to a higher level. [IES-2004]

Reason (R): The water flows through a diverging passage in the volute chamber.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-15. Ans. (d)

IES-16. Consider the following statements regarding the volute casing of a centrifugal pump: [IES-2000]

1. Loss of head due to change in velocity is eliminated.
2. Efficiency of the pump is increased.
3. Water from the periphery of the impeller is collected and transmitted to the delivery pipe at constant velocity.

Which of these statements are correct?

- (a) 1, 2 and 3      (b) 1 and 2      (c) 2 and 3      (d) 1 and 3

IES-16. Ans. (a)

IES-17. Which of the following purposes are served by the volute casing of a centrifugal pump? [IES-1998]

1. Increase in the efficiency of the pump.



2. Conversion of part of the pressure head to velocity head.  
 3. Giving uniform flow of the fluid coming out of the impeller.  
 Select the correct answer using the codes given below:

**Codes:**

- (a) 1 and 2                      (b) 1 and 3                      (c) 2 and 3                      (d) 1, 2 and 3

**IES-17. Ans. (b)**

**IES-18. At the eye tip of a centrifugal impeller, blade velocity is 200 m/s while the uniform axial velocity at the inlet is 150 m/s. If the sonic velocity is 300 m/s, then the inlet Mach number of the flow will be: [IES-1998]**

- (a) 0.50                      (b) 0.66                      (c) 0.83                      (d) 0.87

**IES-18. Ans. (c)** Velocity of air relative to the fan

$$V_r = \sqrt{V_b^2 + V_t^2} = \sqrt{(200)^2 + (150)^2} = 250 \text{ m/s}$$

$$\text{Mach number} = \frac{V_r}{\text{Sonic velocity}} = \frac{250}{300} = 0.833$$

**IES-19. A centrifugal blower delivering Q m<sup>3</sup>/s against a head of H m is driven at half the original speed. The new head and discharge would be:**

**[IES-2001]**

- (a)  $H$  and  $\frac{Q}{2}$                       (b)  $\frac{H}{4}$  and  $\frac{Q}{2}$                       (c)  $\frac{H}{2}$  and  $\frac{Q}{8}$                       (d)  $H$  and  $\frac{Q}{4}$

**IES-19. Ans. (b)** We Know that  $\frac{H}{D^2 N^2} = \text{constant}$  and  $\frac{Q}{D^3 N} = \text{constant}$

**IES-20. Two centrifugal pumps have impellers with outer dimensions of each equal to twice the inner dimensions. The inner diameter of the second impeller is three times the inner diameter of the first one. What is the speed ratio  $N_2/N_1$ , of pumps, if the pumps are required to develop the same manometric head to start delivery of water? [IES-2004]**

- (a) 9                      (b) 4                      (c) 1/2                      (d) 1/3

**IES-20. Ans. (d)**  $N = \frac{120 \eta_{man} V_{wz} D_o}{\pi (D_o^2 - D_i^2)}$

$$\begin{aligned} \text{or } \frac{N_2}{N_1} &= \frac{(D_o)_2}{(D_o)_2^2 - (D_i)_2^2} \times \frac{(D_o)_1^2 - (D_i)_1^2}{(D_i)_1} = \frac{6 \cdot (D_i)_1}{36 (D_i)_1^2 - 9 (D_i)_1} \times \frac{4 (D_i)_1^2 - (D_i)_1^2}{(D_i)_1} \\ &= \frac{6}{27} \times 3 = (D_o)_1 = 2 (D_i)_1 \\ (D_o)_2 &= 2 \times 3 (D_i)_1 = 2 (D_i)_2 \quad \text{or, } (D_i)_2 = 3 (D_i)_1 \end{aligned}$$

$$D_{o1} = 2 \cdot D_{i1} \quad \text{and} \quad D_{o2} = 2 \cdot D_{i2}$$

$$D_{i2} = 3 D_{i1} \quad \text{so} \quad D_{o2} = 2 \times 3 D_{i1}$$

$$\text{we know } H_{mano} = \frac{U_2^2 - U_1^2}{2g} \quad \text{or} \quad (U_2^2 - U_1^2)_1 = (U_2^2 - U_1^2)_2$$

$$\text{or } \left[ \left( \frac{\pi D_o N}{60} \right)_1^2 - \left( \frac{\pi D_i N}{60} \right)_1^2 \right] = \left( \frac{\pi D_o N}{60} \right)_2^2 - \left( \frac{\pi D_i N}{60} \right)_2^2$$

$$\text{or } \frac{N_2^2}{N_1^2} = \frac{D_{o1}^2 - D_{i1}^2}{D_{o2}^2 - D_{i2}^2} = \frac{4 D_{i1}^2 - D_{i1}^2}{36 D_{i1}^2 - 9 D_{i1}^2} = \frac{3}{27} = \frac{1}{9} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{1}{3}$$

**IES-21.** A centrifugal pump with radial vane tips at the outlet has an impeller of 100 mm outer diameter. If the rotational speed is 3000 rpm and manometric efficiency 0.8 then what is the net head developed?

[IES-2009]

- (a) 10 m                      (b) 20 m                      (c) 30 m                      (d) 40 m

**IES-21. Ans. (b)**  $u_2 = \frac{\pi D_2 N}{60} = \frac{3.14 \times 0.1 \times 3000}{60} = 15.7 \text{ m/sec.}$

For Radial Vane Tips  $V_{\omega 2} = u_2 = 15.7 \text{ m/sec.}$

$$n_H = \frac{gH}{V_{\omega 2} u_2} \quad \text{and} \quad \frac{gH}{V_{\omega 2} u_2} = 0.8$$

$$\Rightarrow H = \frac{0.8 \times u_2^2}{9.8} = \frac{0.8 \times (15.7)^2}{9.8} = 20.12 \text{ metres}$$

**IES-22.** Consider the following statements pertaining to a centrifugal pump:

1. The manometric head is the head developed by the pump. [IES-2001]
2. The suction pipe has, generally, a larger diameter as compared to the discharge pipe.
3. The suction pipe is provided with a foot valve and a strainer.
4. The delivery pipe is provided with a foot valve and a strainer.

Of these statements:

- (a) 1, 2, 3 and 4 are correct                      (b) 1 and 2 are correct  
(c) 2 and 3 are correct                      (d) 1 and 3 are correct

**IES-22. Ans. (c)**

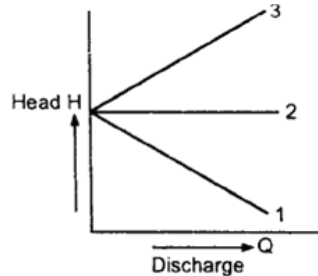
**IES-23.** Match List-I (Outlet vane angle  $\beta_2$ ) with List-II (Curves labelled 1, 2 and 3 in the given figure) for a pump and select the correct answer using the codes given below the Lists:

[IES-2000]

**List-I**

- A.  $\beta_2 < 90^\circ$   
B.  $\beta_2 = 90^\circ$   
C.  $\beta_2 > 90^\circ$

**List-II**



<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>A</b>	<b>B</b>	<b>C</b>
(a)	1	2	3	(b)	1	3
(c)	2	1	3	(d)	3	2
					2	1

**IES-23. Ans. (a)**

**IES-24.** A pump running at 1000 RPM consumes 1 kW and generates head of 10m of water. When it is operated at 2000 RPM, its power consumption and head generated would be:

[IES-2002]

- (a) 4 kW, 50 m of water                      (b) 6 kW, 20 m of water  
(c) 3 kW, 30 m of water                      (d) 8 kW, 40 m of water

**IES-24. Ans. (d)**

**IES-25.** Which one of the following statements is correct?

[IES-2004]

When a fluid passes from the inlet to exit of the rotor in a centrifugal pump, tangential momentum

- (a) Increases and energy increases                      (b) Decreases and energy increases

- (c) Remains unchanged and energy Decreases      (d) Increases and energy remains unchanged

**IES-25. Ans. (a)** Through the rotor we are adding energy to the fluid therefore energy cannot remain unchanged.

**IES-26. A centrifugal pump gives maximum efficiency when its blades are:** **[IES-2002]**

- (a) Bent forward      (b) Bend backward      (c) Straight      (d) Wave shaped

**IES-26. Ans. (b)**

**IES-27. Which one of the following types of impeller vanes are most commonly used in centrifugal type compressors?** **[IES-2008]**

- (a) Forward curved      (b) Radial      (c) Backward curved      (d) Tangential

**IES-27. Ans. (b)** The blades of the compressor are either forward curved or backward curved or radial. Backward curved blades were used in the older compressors, whereas the modern centrifugal compressors use mostly radial blades.

**IES-28. The vanes of a centrifugal pump are generally** **[IES-2007]**

- (a) Radial      (b) Curved backward      (c) Curved forward      (d) Twisted

**IES-28. Ans. (a)**

## Minimum Speed for Starting a Centrifugal Pump

**IES-29. A centrifugal pump is started with its delivery valve kept** **[IES-1997]**

- (a) Fully open      (b) Fully closed      (c) Partially open      (d) 50% open

**IES-29. Ans. (b)** A centrifugal pump is started with delivery valve fully closed.

## Pumps in Series

**IES-30. Assertion (A): Multi-stage centrifugal pumps are only of the radial flow type.** **[IES-2003]**

**Reason (R): In a multi-stage centrifugal pump, two or more impellers are keyed to a single shaft and enclosed in the same casing, the radial inlet to successive impellers being made through guide vanes.**

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IES-30. Ans. (b)**

**IES-31. Why is multi-staging in centrifugal pumps used?** **[IES-2008]**

- (a) For high flow rate      (b) For high head  
(c) For high speed      (d) For high efficiency

**IES-31. Ans. (b)** Centrifugal pumps are able to develop low pressure multi-staging in centrifugal pumps is done for high head.

## Pumps in Parallel

**IES-32. Two pumps can operate independently at heads  $H_1$ ,  $H_2$  and discharge  $Q_1$ ,  $Q_2$ , respectively. If the pumps are connected in parallel, then what are the resulting discharge ( $Q$ ) and head ( $H$ )?** **[IES-2008]**

- (a)  $Q = Q_1 + Q_2$ ,  $H = H_1 + H_2$       (b)  $Q = Q_1 - Q_2$ ,  $H = H_1 - H_2$   
(c)  $Q = Q_1 = Q_2$ ,  $H = H_1 = H_2$       (d)  $Q = Q_1 + Q_2$ ,  $H = H_1 = H_2$

**IES-32. Ans. (d)** If the pumps are connected in parallel  $Q = Q_1 + Q_2$  and if the pumps are connected in series  $H = H_1 + H_2$ .

## Specific Speed

**IES-33. Which one of the following statements is relevant to the specific speed of a centrifugal pump? [IES-2004]**

- (a) Head developed is unity and discharge is unity
- (b) Head developed is unity and power absorbed is unity
- (c) Discharge is unity and power absorbed is unity
- (d) Each of head developed, power absorbed and discharge is equal to unity

**IES-33. Ans. (a)**

**IES-34. On the assumption that a double suction impeller is the equivalent of two single suction impellers placed 'back to back', it is customary to base the specific speed of the double suction pump on [IES-2003]**

- (a) One half of the total capacity
- (b) Three fourth of the total capacity
- (c) Full total capacity
- (d) Double the total capacity

**IES-34. Ans. (c)**

**IES-35. Which one of the following pairs of formulae represents the specific speeds of turbine and pump respectively? (Notations have their usual meanings) [IES-2000]**

- |   |   |
|---|---|
| (a) $\frac{NQ^{1/2}}{H^{3/4}}$ and $\frac{NP^{1/2}}{H^{5/4}}$ | (b) $\frac{NQ^{1/2}}{H^{3/4}}$ and $\frac{NP^{1/2}}{H^{3/4}}$ |
| (c) $\frac{NP^{1/2}}{H^{3/4}}$ and $\frac{NQ^{1/2}}{H^{5/4}}$ | (d) $\frac{NP^{1/2}}{H^{5/4}}$ and $\frac{NQ^{1/2}}{H^{3/4}}$ |

**IES-35. Ans. (d)**

**IES-36. Two centrifugal pumps 'A' and 'B' operate at their maximum efficiencies at 1000 rpm and 500 rpm respectively. Against the same delivery head, pump 'A' discharge 1 m<sup>3</sup>/s and pump B discharge 4 m<sup>3</sup>/s respectively. What is the ratio of specific speeds (N<sub>s</sub>)<sub>A</sub> : (N<sub>s</sub>)<sub>B</sub> ? [IES-2004]**

- (a) 1: 2
- (b) 1: 1
- (c) 1 : 4
- (d) 4: 1

**IES-36. Ans. (b)**  $\frac{(N_s)_A}{(N_s)_B} = \frac{N_A \sqrt{Q_A}}{(H_A)^{3/4}} \times \frac{(H_A)^{3/4}}{N_B \sqrt{Q_B}} = \left(\frac{N_A}{N_B}\right) \times \sqrt{\frac{Q_A}{Q_B}} \times \left(\frac{H_B}{H_A}\right)^{3/4} = \frac{1000}{500} \times \sqrt{\frac{1}{4}} \times 1 = 1$

**IES-37. Consider the following statements regarding the specific speed of a centrifugal pump: [IES-1998]**

1. Specific speed is defined as the speed of a geometrically similar pump developing unit power under unit head.
2. At the same specific speed, the efficiency is greater with larger capacity.
3. The specific speed increases with the increase in outer blade angle.
4. The specific speed varies directly as the square root of the pump discharge.

**Of these statements:**

- (a) 1 and 2 are correct
- (b) 2 and 4 are correct
- (c) 3 and 4 are correct
- (d) 2 and 3 are correct

**IES-37. Ans. (b)** Specific speed is defined as the speed of a geometrically similar turbine developing unit power under unit head. BUT specific speed of a pump is defined as the speed of a geometrically similar pump of such a size that under

corresponding conditions it would deliver unit volume flow of liquid against unit head. That so why statement 1 is wrong.

**IES-38. The specific speed of a hydraulic pump is the speed of geometrically similar pump working against a unit head and** [IES-1993]

- (a) Delivering unit quantity of water (b) Consuming unit power  
(c) Having unit velocity of flow (d) Having unit radial velocity

**IES-38. Ans. (a)** The specific speed of a hydraulic pump is the speed of a geometrically similar pump working against a unit head and delivering unit quantity of water.

It may be noted that specific speed of hydraulic pump =  $\frac{N\sqrt{Q}}{H^{3/4}}$

**IES-39. If, in a pump, the discharge is halved, then, assuming that the speed remains unchanged, what would be the ratio of the heads  $H_1/H_2$ ?**

[IES-2007]

- (a)  $\sqrt{1/3}$  (b)  $\sqrt{2/3}$  (c)  $\sqrt[3]{0.25}$  (d)  $\sqrt[3]{0.5}$

**IES-39. Ans. (c)**  $N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \text{const.}$  or  $H \propto Q^{2/3}$   $\frac{H_1}{H_2} = \left(\frac{Q_1^2}{Q_2^2}\right)^{1/3} = 4^{1/3}$

**IES-40. A mixed flow pump is driven by a 8 kW motor running at 1000 rpm. It delivers water at the rate of 1000 liters/min against a total head of 25 m. What is the specific speed of the pump in meter-minutes?** [IES-2009]

- (a) 90 (b) 50 (c) 45 (d) 75

**IES-40. Ans. (a)** Specific speed of pump =  $\frac{N\sqrt{Q}}{H^{3/4}} = \frac{1000\sqrt{1.0}}{(25)^{3/4}} = 89.44 \approx 90$

## Model Testing and Geometrically Similar Pumps

**IES-41. In utilizing scaled models in the designing of turbo-machines, which of the following relationship must be satisfied?** [IES-2002]

- (a)  $\frac{H}{ND^3} = \text{constant};$   $\frac{Q}{N^2D^2} = \text{constant}$   
(b)  $\frac{Q}{D^2\sqrt{H}} = \text{constant};$   $\frac{Q}{N^3D} = \text{constant}$   
(c)  $\frac{P}{QH} = \text{constant};$   $\frac{H}{N^2D^2} = \text{constant}$   
(d)  $\frac{NQ^{1/2}}{H^{3/2}} = \text{constant};$   $\frac{NP^{1/2}}{N^{3/4}} = \text{constant}$

**IES-41. Ans. (c)**

**IES-42. Which one of the following is the correct statement?** [IES-2005]  
**For a given centrifugal pump,**

- (a) The discharge varies directly as the speed  
(b) The head varies inversely as the speed  
(c) The power varies as the square of the speed  
(d) The discharge varies as the square of the speed

**IES-42. Ans. (a)**  $\frac{Q}{ND^2} = \text{const.}$

**IES-43.** A centrifugal pump needs 1000 W of power when operating at 1500 rpm. What is the power requirement if the speed of the pump is increased to 3000 rpm? [IES-2004]

- (a) 2000 W (b) 4000 W (c) 6500 W (d) 8000 W

**IES-43. Ans. (d)** For A centrifugal pump

$$\frac{P}{N^3 D^5} = \text{const.} \quad \text{as } D = \text{no change}$$

$$P \propto N^3 \text{ or } \frac{P_2}{P_1} = \left( \frac{N_2}{N_1} \right)^3 \rightarrow P_2 = P_1 \times \left( \frac{N_2}{N_1} \right)^3 = 1000 \times \left( \frac{3000}{1500} \right)^3 = 8000 \text{ W}$$

**IES-44.** Consider the following data for the performance of a centrifugal pump: Speed: 1200 rpm, flow rate: 30 l/s, head: 20 m, Power: 5 kW [IES-1999]  
If the speed is increased to 1500 rpm, the power will be nearly equal to:

- (a) 6.5 kW (b) 8.7 kW (c) 9.8 kW (d) 10.9 kW

**IES-44. Ans. (c)** Power is proportional to cube of speed

$$\therefore P = 5 \times \left( \frac{1500}{1200} \right)^3 = 9.8 \text{ kW}$$

**IES-45.** The power ratio of a pump and its  $\frac{1}{4}$  th scale model, if the ratio of the heads is 5: 1, will be: [IES-2003]

- (a) 100 (b) 3.2 (c) 179 (d) 12.8

**IES-45. Ans. (c)**  $\frac{P}{D^5 N^3} = \text{const.}$  and  $\frac{H}{D^2 N^2} = \text{const.}$

$$\text{or } N^6 = \frac{P^2}{D^{10}} = \frac{H^3}{D^6} \quad \text{or } P = D^2 H^{3/2}$$

$$\text{or } \frac{P_p}{P_m} = \left( \frac{D_p}{D_m} \right)^2 \left( \frac{H_p}{H_m} \right)^{3/2} = \left( \frac{4}{1} \right)^2 \left( \frac{5}{1} \right)^{3/2} \approx 179$$

**IES-46.** Assertion (A): With increase in discharge in a single stage centrifugal pump the BHP goes on increasing but beyond a certain discharge the BHP starts decreasing. [IES-1995]

Reason (R): Efficiency of the pump starts decreasing beyond a certain discharge.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

**IES-46. Ans. (d)** A is false and R is true.

## Net Positive Suction Head (NPSH)

**IES-47.** A pump is installed at a height of 5 m above the water level in the sump. Frictional loss on the suction side is 0.6 m. If the atmospheric pressure is 10.3 m of water and vapour pressure head is 0.4 m (abs), the NPSH (Net Positive Suction Head) will be: [IES-2003]

- (a) 3.7 m (b) 4 m (c) 4.3 m (d) 4.6 m

**IES-47. Ans. (c)** NPSH = Barometric head – suction head – vapour pr. head – friction head loss – velocity head = 10.3 – 5 – 0.6 – 0.4 – 0 = 4.3 m.

**IES-48.** Why is a minimum of Net Positive Suction Head required for a hydraulic pump? [IES-2005]

- |                              |                            |
|------------------------------|----------------------------|
| (a) To prevent cavitation    | (b) To increase discharge  |
| (c) To increase suction head | (d) To increase efficiency |

**IES-48. Ans. (a)**

**IES-49. Consider the following statements: [IES-1996]**

**If pump NPSH requirements are not satisfied, then**

1. It will not develop sufficient head to raise water
2. Its efficiency will below
3. It will deliver very low discharge
4. It will be cavitated

**Of these correct statements are:**

- |                |                 |             |                   |
|----------------|-----------------|-------------|-------------------|
| (a) 1, 2 and 3 | (b) 2, 3, and 4 | (c) 1 and 4 | (d) 1, 2, 3 and 4 |
|----------------|-----------------|-------------|-------------------|

**IES-49. Ans. (b)**

**IES-50. Assertion (A): Increase in static suction lift of centrifugal pump may cause cavitation [IES-2009]**

**Reason (R): Available Net Positive Suction Head increase with increase in static suction lift.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-50. Ans. (c)**  $NPSH = \text{Barometric head} - \text{suction head} - \text{vapour pr. head} - \text{friction head loss} - \text{velocity head}$ . Therefore Available Net Positive Suction Head *decreases* with increase in static suction lift.

**IES-51. Priming is necessary in [IES-2003]**

- (a) Centrifugal pumps to lift water from a greater depth
- (b) Centrifugal pumps to remove air in the suction pipe and casing
- (c) Hydraulic turbine to remove air in the turbine casing
- (d) Hydraulic turbine to-increase the speed of turbine and to generate more power

**IES-51. Ans. (b)**

**IES-52. Water is required to be lifted by a 10 kW pump from a depth of 100 m. If the pump is unable to lift the water, then which one of the following is correct? [IES-2004]**

- (a) A greater capacity pump has to be used
- (b) A larger diameter delivery pipe has to be used
- (c) A larger diameter suction pipe has to be used
- (d) A multistage pump has to be used

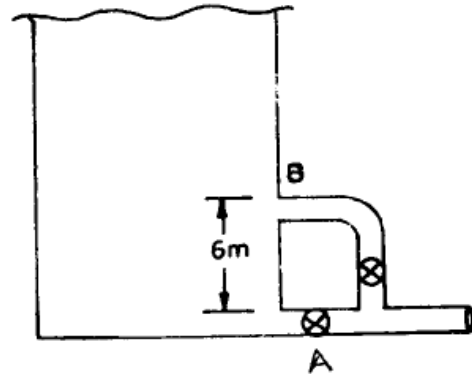
**IES-52. Ans. (d)** Atmospheric pressure = 10.33 m of water column. Suction head can not be more than 10.33 m.

## Previous Years IAS Questions

## Working of a Centrifugal Pump

IAS-1. The water level in an empty vertical cylindrical tank with top open is to be raised by 6 m from a nearby reservoir. The ratio of the cost of pumping through pipes A and B (see given figure) is:

- (a) 1:6 (b) 2:3  
(c) 1:2 (d) 3:5



[IAS-1996]

IAS-1. Ans. (c)

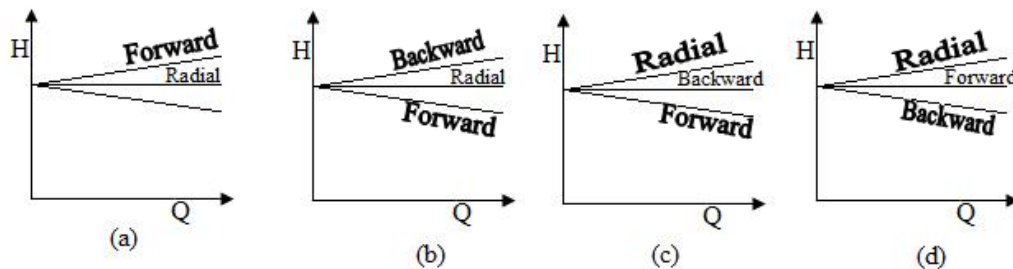
IAS-2. A centrifugal pump delivers water at the rate of 50 litres/s against a total head of 40 meter. Then the power required to drive the pump is:

[IAS-2002]

- (a) 2 kW (b) 15.2 kW (c) 19.6 kW (d) 25.8 kW

IAS-2. Ans. (c)  $\text{Power} = \rho Q g H = 1000 \times \frac{50}{10^3} \times 9.80 \times 40 \text{ W} = 19.6 \text{ kW}$

IAS-3. Which one of the following figures represents theoretical head versus discharge curves for a centrifugal pump with forward radial and backward curved vanes? [IAS-1999]



IAS-3. Ans. (a)

## Efficiencies of a Centrifugal Pump

IAS-4. Manometric efficiency of a centrifugal pump is defined as the ratio of

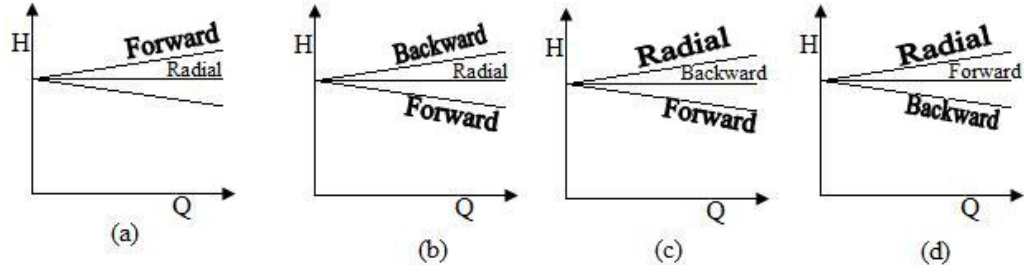
- (a) Suction head to the head imparted by the impeller to water [IAS-1996]  
(b) Head imparted by the impeller to water to the suction head  
(c) Manometric head to the head imparted by the impeller to water  
(d) Head imparted by the impeller to water to the manometric head

IAS-4. Ans. (c)



## Effect of Outlet Vane Angle on Manometric Efficiency

IAS-5. Which one of the following figures represents theoretical head versus discharge curves for a centrifugal pump with forward radial and backward curved vanes? [IAS-1999]



IAS-5. Ans. (a)

IAS-6. Consider the following statements in respect of centrifugal pumps:

1. Head developed is proportional to the square of the speed of rotation
2. Backward curved bladed impellers are generally used in centrifugal pumps
3. These pumps generally do not require priming
4. Multistage pumps would give higher discharge proportional to the number of stages.

Which of these statements are correct? [IAS-2003]

- (a) 1 and 2      (b) 2 and 3      (c) 3 and 4      (d) 1 and 4

IAS-6. Ans. (a)

IAS-7. For discharge 'Q', the specific speed of a pump is 'N<sub>s</sub>'. For half discharge with the same head the specific speed will be: [IAS-1999]

- (a) N<sub>s</sub>      (b)  $\frac{N_s}{\sqrt{2}}$       (c)  $\sqrt{2} N_s$       (d) 2N<sub>s</sub>

IAS-7. Ans. (b)  $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$  or  $N_s \propto \sqrt{Q}$  or  $\frac{N'_s}{N_s} = \sqrt{\frac{Q'}{Q}} = \sqrt{\frac{1}{2}}$  or  $N'_s = \frac{N_s}{\sqrt{2}}$

IAS-8. For discharge 'Q', the specific speed of a pump is 'N<sub>s</sub>'. For half discharge with the same head the specific speed will be: [IAS-1999]

- (a) N<sub>s</sub>      (b)  $\frac{N_s}{\sqrt{2}}$       (c)  $\sqrt{2} N_s$       (d) 2N<sub>s</sub>

IAS-8. Ans. (b)  $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$  or  $N_s \propto \sqrt{Q}$  or  $\frac{N'_s}{N_s} = \sqrt{\frac{Q'}{Q}} = \sqrt{\frac{1}{2}}$

IAS-9. A centrifugal pump having an impeller of 10 cm diameter discharges 40 litre/ second when turning at 1000rpm. The corresponding speed of a geometrically similar pump having an impeller of 40cm diameter and 0.8m<sup>3</sup>/s discharge will be: [IAS-1997]

- (a) 276.4rpm      (b) 298.3rpm      (c) 312.5rpm      (d) 358.2rpm

IAS-9. Ans. (c)

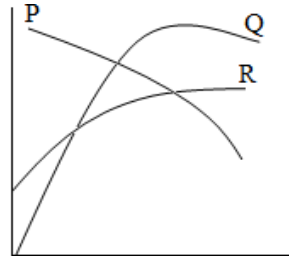
IAS-10. Which one of the following correctly expresses the specific speed of a turbine and a pump, respectively? [IAS-2004]

(a)  $\frac{N\sqrt{Q}}{H^{3/4}}, \frac{N\sqrt{P}}{H^{5/4}}$  (b)  $\frac{N\sqrt{P}}{H^{3/4}}, \frac{N\sqrt{Q}}{H^{5/4}}$  (c)  $\frac{N\sqrt{P}}{H^{5/4}}, \frac{N\sqrt{Q}}{H^{3/4}}$  (d)  $\frac{N\sqrt{P}}{H^{7/4}}, \frac{N\sqrt{Q}}{H^{3/4}}$

IAS-10. Ans. (c)

## Characteristics of Centrifugal Pumps

IAS-11. The characteristics of a pump are as shown in the given figure. Based on this figure, match List-I with List-II and choose the correct answer using the codes given below the lists:



**List-I**

- A. Curve P
- B. Curve Q
- C. Curve R

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>
(a)	2	4	3
(c)	1	4	3

**List II**

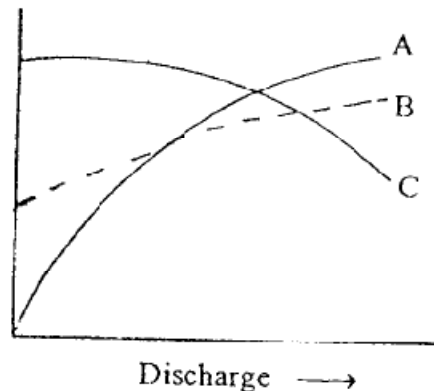
[IAS-1995]

- 1. Discharge versus head
- 2. Head versus discharge
- 3. Power versus Discharge
- 4. Efficiency versus discharge

	<b>A</b>	<b>B</b>	<b>C</b>
(b)	1	3	2
(d)	4	3	1

IAS-11. Ans. (a)

IAS-12. The figure below shows characteristics of three centrifugal pumps A, B and C. If  $E$  = Efficiency,  $H$  = Head and  $P$  = Power, then A, B and C, respectively represent which one of the following?



- (a) E, P, H
- (b) P, E, H
- (c) P, H, E
- (d) H, P, E

[IAS-2004]

IAS-12. Ans. (a) If Discharge '0',  $\eta = 0$ ,  $H = \text{max}$  and  $p = \text{reasonable power is needed}$ .

## Cavitation in Centrifugal Pumps

IAS-13. In the case of a centrifugal pump, cavitation will occur if [IAS-1994]

- (a) It operates above the minimum net positive suction head
- (b) It operates below the minimum net positive suction head
- (c) The pressure at the inlet of the pump is above the atmospheric pressure
- (d) The pressure at the inlet of the pump is equal to the atmospheric pressure

IAS-13. Ans. (b)

IAS-14. Which one of the following helps in avoiding cavitation in centrifugal pumps? [IAS-2004]

- (a) Low suction pressure
- (b) High delivery pressure

(c) Low delivery pressure

(d) High suction pressure

**IAS-14. Ans. (a)****IAS-15. Cavitation in a centrifugal pump is likely to occur at the [IAS-1996]**

(a) Impeller exit (b) Impeller inlet (c) Diffuser exit (d) Involute casing

**IAS-15. Ans. (b)**

## **Selection of Pumps**

### **Operational Difficulties in Centrifugal Pumps**

**IAS-16. Consider the following statements for specific speed: [IAS-2007]**

1. The optimum efficiency of a hydraulic machine depends on its specific speed.
2. For the same power, a turbo machine running at higher specific speed will be smaller in size.
3. Width-diameter ratio of a centrifugal pump increases with the increase in specific speed.

**Which of the statements given above is/are correct?**

(a) 1 only (b) 1 and 2 only (c) 2 and 3 only (d) 1, 2 and 3

**IAS-16. Ans. (d)**

20.

# Reciprocating Pumps

## OBJECTIVE QUESTIONS (GATE, IES, XIAS)

### Previous Years GATE Questions

**GATE-1. Match the following**

[GATE-2004]

- A. Reciprocating pump
- B. Axial flow pump
- C. Micro hydel plant
- D. Backward curved vanes

- 1. Plant with power output below 100 kW
- 2. Plant with power output between 100 kW to 1MW
- 3. Positive displacement
- 4. Draft tube
- 5. High flow rate, low pressure ratio
- 6. Centrifugal pump impeller

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	5	6	2
(c)	3	5	1	6

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(b)	3	5	2	6
(d)	4	5	1	6

**GATE-1. Ans. (c)**

### Previous Years IES Questions

**IES-1. Which one of the following pairs is not correctly matched? [IES-2004]**

- |                        |   |  |
|------------------------|---|--|
| (a) Centrifugal pump   | : | 1. Rotating blades in the rotor create centrifugal head            |
| (b) Reciprocating pump | : | 2. Positive displacement pump                                      |
| (c) Turbine pump       | : | 3. Centrifugal pump with guide vanes                               |
| (d) Gear pump          | : | 4. Gear teeth work like rotating blades to create centrifugal head |

**IES-1. Ans. (d)** External diffuser is creating centrifugal head.

**IES-2. Match List-I (Type of pumps) with List-II (Associated features) and select the correct answer using the codes given below the Lists:**

**List-I**

- A. Centrifugal pump
- B. Gear pump
- C. Reciprocating pump
- D. Turbine pump

**List-II**

- 1. Air vessel
- 2. Draft tube
- 3. Guide vanes pump
- 4. Rotary pump
- 5. Rotor having blades

[IES-2003]

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	4	2	5	3
(c)	4	2	3	1

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(b)	5	4	1	2
(d)	5	4	1	3

**IES-2. Ans. (d)**

**IES-3. Which one of the following is correct?**

[IES-2008]

A turbine pump is basically a centrifugal pump equipped additionally with

- |                            |                            |
|----------------------------|----------------------------|
| (a) Backward curved blades | (b) Vaned diffusion casing |
| (c) Inlet guide blades     | (d) Adjustable blades      |

**IES-3. Ans. (b)** A turbine pump is basically a centrifugal pump equipped additionally with vaned diffusion casing.

**IES-4. Consider the following pumps:** **[IES-1999]**

- |                                   |                                  |
|-----------------------------------|----------------------------------|
| 1. Centrifugal pump, single-stage | 2. Centrifugal pump, multi-stage |
| 3. Reciprocating pump             | 4. Jet pump.                     |

The pump (s) which can be used to lift water through a suction head of 12 m from a well would include

- |             |                |             |             |
|-------------|----------------|-------------|-------------|
| (a) 2 alone | (b) 1, 3 and 4 | (c) 4 alone | (d) 1 and 3 |
|-------------|----------------|-------------|-------------|

**IES-4. Ans. (c)** Since suction head is 12 m, i.e. more than atmospheric pressure, only jet pump can be used to lift water under such a situation.

**IES-5. If a reciprocating pump having a mechanical efficiency of 80% delivers water at the rate of 80 kg/s with a head of 30 m, the brake power of the pump is:** **[IES-2001]**

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) 29.4 kW | (b) 20.8 kW | (c) 15.4 kW | (d) 10.8 kW |
|-------------|-------------|-------------|-------------|

**IES-5. Ans. (a)**

## Air Vessels

**IES-6. Why is an air vessel used in a reciprocating pump?** **[IES-2008]**

- (a) To obtain a continuous supply of water at uniform rate
- (b) To reduce suction head
- (c) To increase the delivery head
- (d) To reduce cavitation

**IES-6. Ans. (a)** An air vessel is a closed chamber containing compressed air in the upper part and liquid being pumped in the lower part. The air vessels are used:

- (i) To get continuous supply of liquid at a uniform rate,
- (ii) To save the power required to drive the pump and
- (iii) To run the pump at a much higher speed without any danger of separation.

**IES-7. Air vessel is used in a reciprocating pump to obtain** **[IES-1992]**

- (a) Reduction of suction head
- (b) Rise in delivery head
- (c) Continuous supply of water at uniform rate
- (d) Increase in supply of water

**IES-7. Ans. (c)**

**IES-8. Which of following are the beneficial effects of air vessel fitted to delivery side of a reciprocating pump?** **[IES-1995]**

- 1. Constant rate of discharge can be ensured
- 2. Power consumption can be reduced
- 3. Discharge can be increased
- 4. Constant velocity of the piston can be ensured

Select the correct answer using the codes given below:

**Codes:**

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) 1 and 4 | (b) 1 and 2 | (c) 2 and 4 | (d) 1 and 3 |
|-------------|-------------|-------------|-------------|

**IES-8. Ans. (b)** The function of air-vessel are:

**1. On suction side**

- i) To reduce accelerating head. This will reduced the total vacuum head, reducing the possibility of separation and saving in power required for supplying accelerating head.
- ii) Pump can be run on higher speed.
- iii) Length of suction pipe below the air vessel can be increased.

**2. On delivery side**

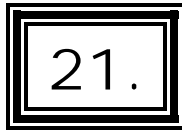
- i) To reduce the accelerating head and affecting in large amount of power consumed in supplying the accelerating head.
- ii) A uniform rate of discharge is ensured.

**Previous Years GATE Questions****Classification of Reciprocating Pumps**

**IAS-1. For pumping molasses, it is preferable to employ [IAS-1994]**

- |                        |  |
|------------------------|--|
| (a) Reciprocating pump | (b) Centrifugal pump with double shrouds |
| (c) Open impeller pump | (d) Multistage centrifugal pump          |

**IAS-1. Ans. (c)**



## Miscellaneous Hydraulic Machines

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### Contents of this chapter

1. The Hydraulic Accumulator
  2. Hydraulic Press
  3. Hydraulic Ram
  4. Hydraulic Coupling
  5. Hydraulic Torque Converter
  6. Air Lift Pump
- 

### OBJECTIVE QUESTIONS (GATE, IES, IAS)

#### Previous Years GATE Questions

- GATE-1. Jet pumps are often used in process industry for their** [GATE-1992]  
(a) High efficiency  
(b) Easy maintenance  
(c) Large capacity  
(d) Capacity to transport gases, liquids and mixtures of both
- GATE-1. Ans. (b, d)**

#### Previous Years IES Questions

### The Hydraulic Accumulator

- IES-1. The function of which of the following hydraulic devices is analogous to that of the flywheel of a reciprocating engine and an electric storage battery?** [IES-2005]  
(a) Hydraulic ram (b) Hydraulic accumulator  
(c) Hydraulic intensifier (d) Hydraulic jack
- IES-1. Ans. (b)**
- IES-2. An accumulator is a device to store** [IES-2003]  
(a) Sufficient quantity of liquid to compensate the change in discharge  
(b) Sufficient energy to drive the machine when the normal energy source does not function  
(c) Sufficient energy in case of machines which work intermittently to supplement the discharge from the normal source  
(d) Liquid which otherwise would have gone to waste
- IES-2. Ans. (c)**

## Hydraulic Press

IES-3. A hydraulic press has a ram of 20 cm diameter and a plunger of 5 cm diameter. The force required at the plunger to lift a weight of  $16 \times 10^4$  N shall be: [IES-2002]

- (a)  $256 \times 10^4$  N (b)  $64 \times 10^4$  N (c)  $4 \times 10^4$  N (d)  $1 \times 10^4$  N

IES-3. Ans. (d)

## Hydraulic Ram

IES-4. Assertion (A): A hydraulic ram is a device used to lift water from deep wells. [IES-2002]

Reason (R): Hydraulic ram works on the principle of water hammer.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-4. Ans. (d)

IES-5. Hydraulic ram is a pump which works on the principle of [IES-1999]

- (a) Water hammer (b) Centrifugal action  
 (c) Reciprocating action (d) Hydraulic press

IES-5. Ans. (a) Hydraulic ram utilizes effect of water hammer to lift water.

## Hydraulic Coupling

IES-6. Which one of the following is correct? [IES-2008]

A hydraulic coupling

- (a) Connects two shafts rotating at about the same speed  
 (b) Connects two shafts running at different speeds  
 (c) Is used to augment the torque to the driven shaft  
 (d) Is used to connect the centrifugal pump and its electrical motor for efficient operation

IES-6. Ans. (b) A hydraulic coupling connects two shafts running at different speeds.

IES-7. In a hydraulic coupling, what is the ratio of speed of the turbine runner to that of the pump impeller to maintain circulatory motion of oil? [IES-2007]

- (a)  $<1$  (b)  $=1$  (c)  $>1$  (d) Can be any value

IES-7. Ans. (a) Efficiency of hydraulic coupling,  $\eta = \frac{\omega_t}{\omega_p}$  should be less than one.

(Where  $\omega_t$  and  $\omega_p$  are the angular speeds of the turbine shaft and pump shaft respectively). The magnitudes of input and output torque are equal.

IES-8. A hydraulic coupling transmits 1 kW of power at an input speed of 200 rpm, with a slip of 2%. If the input speed is changed to 400 rpm, the power transmitted with the same slip is: [IES-2001]

- (a) 2 kW (b) 1/2 kW (c) 4 kW (d) 8 kW

IES-8. Ans. (a)

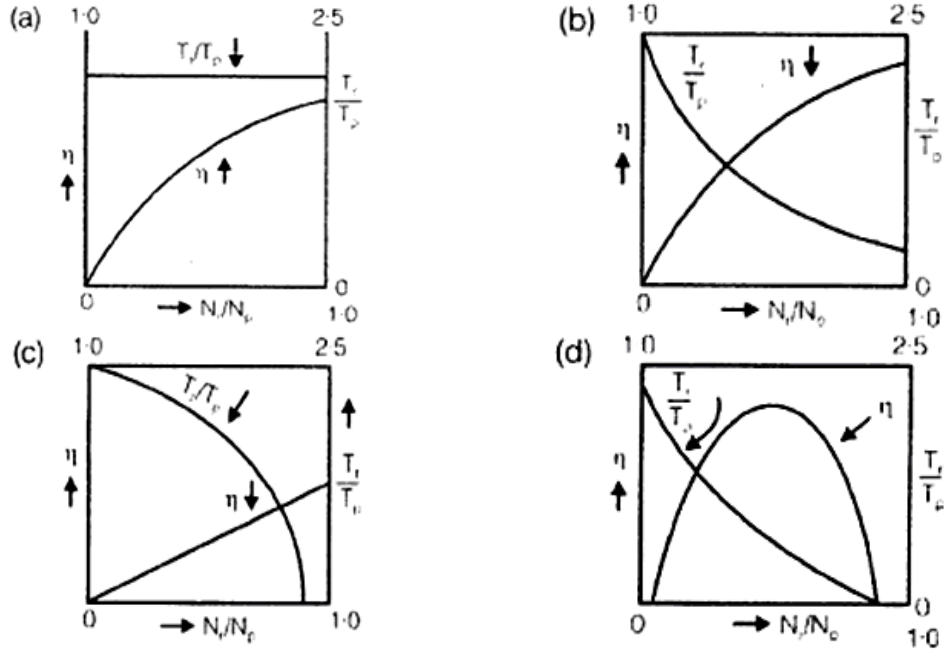
IES-9. Which one of the following graphs represents the characteristics of a torque converter? [IES-2009]



# Miscellaneous Hydraulic Machines

## S K Mondal's Chapter 21

Where suffix  $r$  stands for turbine runner and  $P$  stands for pump impeller.



IES-9. Ans. (b)

IES-10. In a fluid coupling, the torque transmitted is 50 kNm, when the speed of the driving and driven shaft is 900 rpm and 720 rpm respectively. The efficiency of the fluid coupling will be: [IES-2001]

- (a) 20% (b) 25% (c) 80% (d) 90%

IES-10. Ans. (c)

IES-11. Consider the following statements regarding the fluid coupling:

1. Efficiency increases with increase in speed ratio. [IES-2001]
  2. Neglecting friction the output torque is equal to input torque.
  3. At the same input speed, higher slip requires higher input torque.
- Which of these statements are correct?

- (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-11. Ans. (a)

IES-12. If  $\omega_s$  and  $\omega_p$  represent the angular velocities of driver and driving members of a fluid coupling respectively, then the slip is equal to:

[IES-1999]

- (a)  $1 - \frac{\omega_s}{\omega_p}$  (b)  $\frac{\omega_s}{\omega_p}$  (c)  $1 - \frac{\omega_p}{\omega_s}$  (d)  $\frac{\omega_p}{\omega_s}$

IES-12. Ans. (c) Slip =  $1 - \frac{\text{Angular velocity of driving member}}{\text{Angular velocity of driver}}$

IES-13. Assertion (A): In a fluid coupling, hydrodynamic transmission is done by a pump and turbine. [IES-1998]

Reason (R): Fluid coupling is a type of machine in which fluid is used as a means of energy transfer.

- (a) Both A and R are individually true and R is the correct explanation of A

- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-13. Ans. (b)**

**IES-14. Hydraulic transmission through fluid coupling is suitable for [IES-1992]**

- (a) Unsteady operation and increasing torque
- (b) Unsteady operation and increasing speed
- (c) Unsteady operation and low starting torque
- (d) Increasing torque and low starting load

**IES-14. Ans. (c)**

**IES-15. Assertion (A): No solid connection exists between the driving shaft and the driven shaft. [IES-1996]**

**Reason (R): Energy transfer is by the change in moment of momentum.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-15. Ans. (b)** Both A and R are true but R is not correct explanation of A

## Hydraulic Torque Converter

**IES-16. Fluid flow machines are using the principle of either (i) supplying energy to the fluid, or (ii) extracting energy from the fluid. Some fluid flow machines are a combination of both (i) and (ii). They are classified as: [IES-2002]**

- |                       |                        |
|-----------------------|------------------------|
| (a) Compressors       | (b) Hydraulic turbines |
| (c) Torque converters | (d) Wind mills         |

**IES-16. Ans. (c)**

**IES-17. Consider the following statements regarding a torque converter:**

1. Its maximum efficiency is less than that of the fluid coupling.
2. It has two runners and a set of stationary vanes interposed between them.
3. It has two runners.
4. The ratio of secondary to primary torque is zero for the zero value of angular velocity of secondary.

**Which of these statements are correct?**

**[IES-2000]**

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) 1 and 2 | (b) 3 and 4 | (c) 1 and 4 | (d) 2 and 4 |
|-------------|-------------|-------------|-------------|

**IES-17. Ans. (d)**

**IES-18. Consider the following statements regarding torque converter;**

1. It has a stationary set of blades in addition to the primary and secondary rotors.
2. It can be used for multiplication of torques.
3. The maximum efficiency of a converter is less than that of a fluid coupling.
4. In a converter designed to give a large increase of torque, the efficiency falls off rapidly as the speed ratio approaches unity.

**Of these statements**

**[IES-1997]**

- |                               |                            |
|-------------------------------|----------------------------|
| (a) 1, 2, 3 and 4 are correct | (b) 1, 3 and 4 are correct |
| (c) 1, 2 and 4 are correct    | (d) 3 and 4 are correct    |

**IES-18. Ans. (a)** Statement 4 is not correct, but there is no such option.

**Torque converters:** It is a hydrodynamic power transmission device analogous in function to that of a mechanical gear box. It is used to transmit power from the drive shaft to the driven shaft while augmenting the torque on the driven shaft. It is used to multiply or reduce the torque available. It is designed which utilize two or more sets of turbine runners and fixed guide vanes, the fixed vanes being located between the turbine runners. The efficiency of torque converter is better at smaller speed ratio than that of the hydraulic coupling.

**IES-19. In contrast to fluid couplings, torque converters are operated:[IES-1997]**

- (a) While completely filled with liquid (b) While partially filled with liquid  
(c) Without liquid (d) While completely filled with air

**IES-19. Ans. (a)** Torque converters are operated while completely filled with liquid.

## Air Lift Pump

**IES-20. In a jet pump**

**[IES-2006]**

- (a) Kinetic energy of fluid is converted into potential energy  
(b) Energy of high velocity stream is converted into pressure energy  
(c) Energy of high pressure fluid is converted into energy of low pressure fluid  
(d) Potential energy of fluid is converted into kinetic energy

**IES-20. Ans. (b)**

**IES-21. Which one of the following combination represents the power transmission systems? [IES-2009]**

- (a) Pump, hydraulic accumulator, hydraulic intensifier and hydraulic coupling  
(b) Pump, turbine, hydraulic accumulator and hydraulic coupling  
(c) Turbine, accumulator, intensifier and hydraulic coupling  
(d) Accumulator, intensifier, hydraulic coupling and torque converter

**IES-22. Ans. (d)**

## Previous Years IAS Questions

**IAS-1. If a hydraulic press has a ram of 12.5 cm diameter and plunger of 1.25 cm diameter, what force would be required on the plunger to raise a mass of 1 tonne on the ram? [IAS-1998]**

- (a) 981 N (b) 98.1 N (c) 9.81 N (d) 0.98 N

**IAS-1. Ans. (b)** Pressure on the ram = pressure on the plunger

$$\text{or } \left(\frac{F}{A}\right)_R = \left(\frac{F}{A}\right)_P \text{ or } F_R = F_P \times \frac{A_R}{A_P} = 1000 \times 9.81 \times \left(\frac{1.25}{12.5}\right)^2 \text{ N} = 98.1 \text{ N}$$

**IAS-2. A hydraulic coupling belongs to the category of**

**[IAS-1994]**

- (a) Power absorbing machines (b) Power developing machines  
(c) Energy generating machines (d) Energy transfer machines

**IAS-2. Ans. (d)**