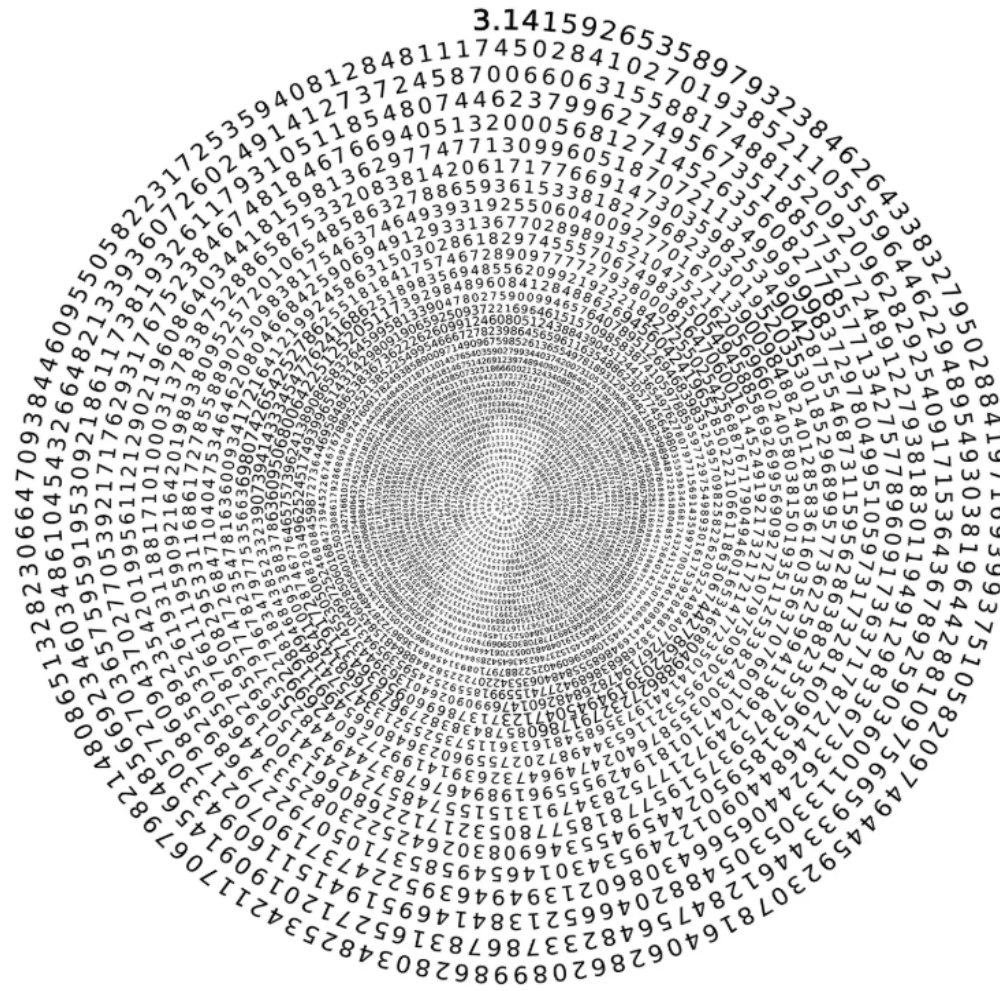
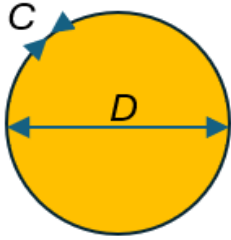


Mādhava's Infinite Series for π [1400 AD]



Mādhava's Infinite Series for π [1400 AD]



$$\pi \equiv \frac{C}{D}$$

$$C = \left[\frac{4D}{1} - \frac{4D}{3} + \frac{4D}{5} - \frac{4D}{7} + \frac{4D}{9} + \dots \dots \right] = 4D \sum_{i=1}^{\infty} \frac{-1^{(i-1)}}{2i-1}$$

Finite Term Estimate: $C(n) = 4D \sum_{i=1}^n \frac{-1^{(i-1)}}{2i-1}$

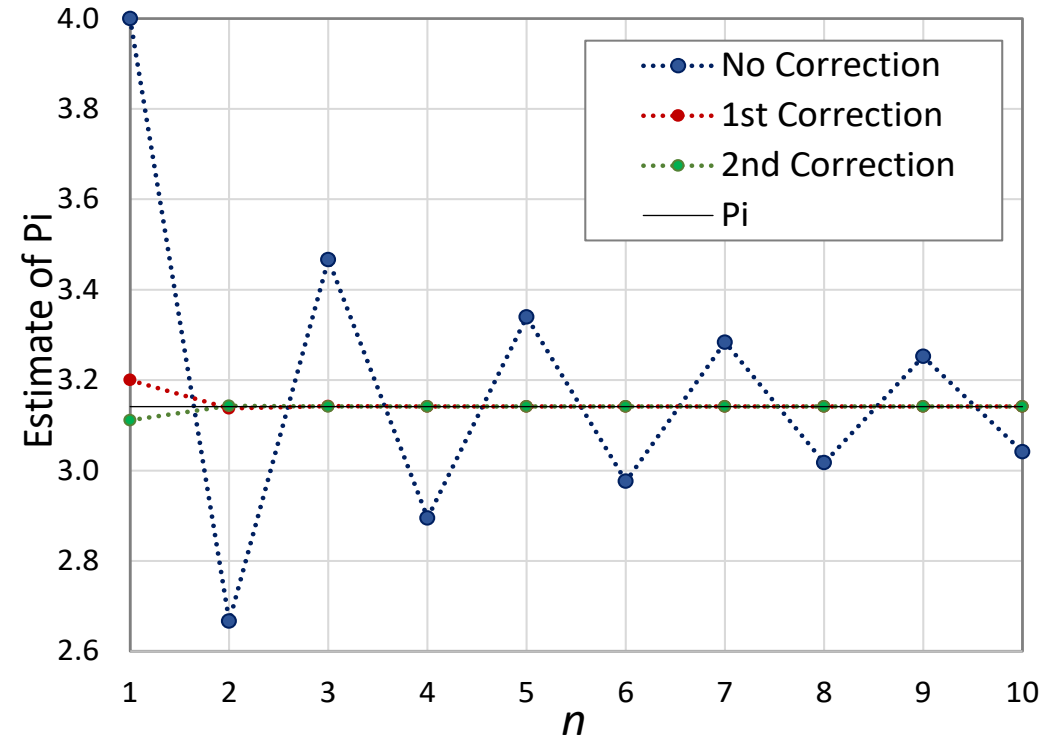
Corrected Estimate: $C \cong C(n) + 4D(-1)^n R_j(n); j = 1, 2$

Correction1: $R_1(n) = \frac{n}{4n^2+1}$

Correction2: $R_2(n) = \frac{n^2+1}{n[4(n^2+1)+1]}$

tantra-saṃgrahaḥ, nīlakanṭha-somayājī [1444 AD - 1545 AD]

yukti-dīpika-laghu-vivṛtti, ṣaṅkara vāriyar [1500 AD - 1560 AD]



व्यासे वारिधि-निहते रूप-हते व्यास-सागर-अभिहते त्रि-शर-आदि-विषम-संख्या-भक्तम् ऋणम् स्वम् पृथक् क्रमात् कुर्यात् २७१
vyāse vāridhi-nihate rūpa-hṛte vyāsa-sāgara-abhihate tri-śara-ādi-viṣama-saṁkhyā-bhaktam ṛṇam svam pṛthak kramāt kuryāt 271
On the diameter multiplied by four divided by one, the diameter multiplied by four divided by the odd numbers three, five, etc, is to be separately debit, credit from sequence.
यत् संख्यया अत्र हरणे कृते निवृत्ता हतिः तु जामितया तस्याः ऊर्ध्व-गतायाः सम-संख्या तत्-दलम् गुणः अन्ते स्यात् २७२
yat saṁkhyayā atra haraṇe kṛte nivṛttā hṛtiḥ tu jāmitayā tasyāḥ ūrdhva-gatāyāḥ sama-saṁkhyā tat-dalam guṇaḥ ante syāt 272
That number by which here when performed division ceased, indeed by repetition, the half of that one higher even number would be the multiplier in the end.
तत्-वर्गः रूप-युतः हारः व्यास-अब्धि-घाततः प्राग्वत् ताभ्याम् आसम् स्वम् ऋणे कृते धने क्षेपः एव करणीयः २७३
tat-vargaḥ rūpa-yutaḥ hāraḥ vyāsa-abdhi-ghātataḥ prāgvat tābhyām āptam svam ṛṇe kṛte dhane kṣepaḥ eva karaṇīyaḥ 273
That square incremented by one is divisor. As before from four multiplying the diameter. By these two obtained, indeed should add/subtract when [last] having subtracted/added.
लब्धः परिधिः सूक्ष्मः बहु-कृत्वः हरणतः अति-सूक्ष्मः स्यात् २७४
labdhaḥ paridhiḥ sūkṣmaḥ bahu-kṛtvaḥ haraṇataḥ ati-sūkṣmaḥ syāt 274
The obtained circumference is refined. From dividing many times it would be very refined.
एभ्यः सूक्ष्मतरः अन्यः विलिख्यते कश्चन अपि संस्कारः अन्ते सम-संख्या-दल-वर्गः स-एकः गुणः स एव पुनः २९५
ebhyaḥ sūkṣmataraḥ anyaḥ vilikhyate kaścana api saṁskāraḥ ante sama-saṁkhyā-dala-vargaḥ sa-ekaḥ guṇaḥ saḥ eva punaḥ 295
More refined than these also is a certain other arrangement delineated. Now indeed the multiplier in the end is square of half of the even number together with one
युग-गुणितः रूप-युतः सम-संख्या-दल-हतः भवेत् हारः त्रि-शर-आदि-विषम-संख्या-हरणात् परम् एतत् एव वा कार्यम् २९६
yuga-guṇitaḥ rūpa-yutaḥ sama-saṁkhyā-dala-hataḥ bhavet hāraḥ tri-śara-ādi-viṣama-saṁkhyā-haraṇāt param etat eva vā kāryam 296
Multiplied by four incremented by one, multiplied by half of the even number should be the divisor. Besides from division by odd numbers three, five, etc, this indeed is performed alternatively.

tantra-saṁgrahaḥ, nīlakanṭha-somayājī [1444 AD - 1545 AD]

yukti-dīpika-laghu-vivṛtti, ṣaṅkara vāriyar [1500 AD - 1560 AD]

$$C = \left[\frac{4D}{1} - \frac{4D}{3} + \frac{4D}{5} - \frac{4D}{7} + \dots \right]$$

$$= 4D \times \sum_{i=1}^{\infty} \frac{-1^{(i-1)}}{2i-1}$$

$$C(n) = 4D \sum_{i=1}^n \frac{-1^{(i-1)}}{2i-1}$$

$$C \cong C(n) + 4D(-1)^n \frac{n}{4n^2 + 1}$$

$$C(n) = 4D \sum_{i=1}^n \frac{-1^{(i-1)}}{2i-1}$$

$$C \cong C(n) + 4D(-1)^n \frac{n^2 + 1}{n[4(n^2 + 1) + 1]}$$

m Loc Sg m Loc Sg m Loc Sg m Loc Sg
vyāse vāridhi-nihate rūpa-hṛte vyāsa-sāgara-abhihate |
-a a=-ā-
m Acc Sg m Acc Sg m Acc Sg Adverb m Abl Sg 3rd Sg Opt
tri-śara-ādi-viṣama-saṃkhyā-bhaktam ṛṇam svam pṛthak kramāt kuryāt ||271||
-a ā=-ā- -m s=-ṃ s- -m p=-ṃ p-

On the diameter multiplied by four divided by one, the diameter multiplied by four divided by the odd numbers three, five, etc, is to be separately debit, credit from sequence.

Element	Type	Meaning	Formation
vyāsa- व्यास-	Noun	the diameter of a circle	Object Noun of vi-(as);vi-['separated from'];(as) IV,I 'to throw'
vāridhi- वारिधि-	Noun	the number 4	vāri-dhi-;vāri-[Noun:'water'];dhi-[Noun:'receptacle']
niyata- निहत-	Adjective	multiplied	ni-(han)-ta;ni-['down','inside'];(han) II 'to strike';-ta-[Past Pass Pcpl]
rūpa- रूप-	Noun	the number 1	(rūp)-a-;(rūp) X 'to express';-a-[Noun]
hṛta- हृत-	Adjective	divided	(hṛ)-ta-;(hṛ) I,III 'to take';-ta-[Past Pass Pcpl]
sāgara- सागर-	Noun	the number 4	vṛddhi[sa-(gṛ)-a-];sa-['with'];(gṛ) VI 'to swallow';-a-[Object Noun]
abhiyata- अभिहत-	Adjective	multiplied	abhi-(han)-ta-;abhi-[intensifier];(han) II 'to kill';-ta-[Past Pass Pcpl]
tri- त्रि-	Number	three	
śara- शर-	Noun	the number 5	guṇa(śṛ)-a-;(śṛ) IX 'to shatter';-a-[Noun]
ādi- आदि-	Adjective	and so no, et caetera	ā-guṇa(dā)-i-;ā-['towards'];(dā) III 'to give';-i-[Adj/Noun]
viṣama- विषम-	Adjective	odd	vi-sama-;vi-['separate from'];sama-[Noun:'even']
saṃkhyā- संख्या-	Noun	number, count	sam-(khyā)-;sam-['together'];(khyā) II 'to look'
bhaktā- भक्त-	Adjective	divided	(bhaj)-ta-;(bhaj) I 'to divide';-ta-[Past Pass Pcpl]
ṛṇa- ऋण-	Noun	minus, debit	
sva- स्व-	Noun	plus, credit	
pṛthak- पृथक्	Adverb	distinctly	Based on (pṛth) X 'to extend'
krama- क्रम-	Noun	order, succession, series, sequence	(kram)-a-;(kram) I 'to step';-a-[Adj/Noun]
(kṛ) (कृ)	Verb VIII	to do	

vyāsa-m	Singular	Dual	Plural	abhihata-m	Singular	Dual	Plural	sva-m	Singular	Dual	Plural
Nominative	vyāsaḥ	vyāsau	vyāsāḥ	Nominative	abhihataḥ	abhihatau	abhihatāḥ	Nominative	svaḥ	svau	sve svāḥ
Vocative	vyāsa	vyāsau	vyāsāḥ	Vocative	abhihata	abhihatau	abhihatāḥ	Vocative	sva	svau	svāḥ
Accusative	vyāsam	vyāsau	vyāsān	Accusative	abhihatam	abhihatau	abhihatān	Accusative	svam	svau	svān
Instrumental	vyāsena	vyāsābhyām	vyāsaiḥ vyāsebhiḥ	Instrumental	abhihatena	abhihatābhyām	abhihataiḥ abhihatebhiḥ	Instrumental	svena	svābhyām	svaiḥ
Dative	vyāsāya	vyāsābhyām	vyāsebhyaḥ	Dative	abhihatāya	abhihatābhyām	abhihatebhyaḥ	Dative	svasmai	svābhyām	svebhyaḥ
Ablative	vyāsāt	vyāsābhyām	vyāsebhyaḥ	Ablative	abhihatāt	abhihatābhyām	abhihatebhyaḥ	Ablative	svāt svasmāt	svābhyām	svebhyaḥ
Genitive	vyāsasya	vyāsayoḥ	vyāsānām	Genitive	abhihatasya	abhihatayoḥ	abhihatānām	Genitive	svasya	svayoḥ	sveṣām
Locative	vyāse	vyāsayoḥ	vyāseṣu	Locative	abhihate	abhihatayoḥ	abhihateṣu	Locative	sve svasmin	svayoḥ	sveṣu
nihata-m	Singular	Dual	Plural	bhakta-m	Singular	Dual	Plural	krama-m	Singular	Dual	Plural
Nominative	nihataḥ	nihatau	nihatāḥ	Nominative	bhaktaḥ	bhaktau	bhaktāḥ	Nominative	kramaḥ	kramau	kramāḥ
Vocative	nihata	nihatau	nihatāḥ	Vocative	bhakta	bhaktau	bhaktāḥ	Vocative	krama	kramau	kramāḥ
Accusative	nihatam	nihatau	nihatān	Accusative	bhaktam	bhaktau	bhaktān	Accusative	kramam	kramau	kramān
Instrumental	nihatena	nihatābhyām	nihataiḥ nihatebhiḥ	Instrumental	bhaktena	bhaktābhyām	bhaktaiḥ bhaktebhiḥ	Instrumental	krameṇa	kramābhyām	kramaiḥ
Dative	nihatāya	nihatābhyām	nihatebhyaḥ	Dative	bhaktāya	bhaktābhyām	bhaktebhyaḥ	Dative	kramāya	kramābhyām	kramebhyaḥ
Ablative	nihatāt	nihatābhyām	nihatebhyaḥ	Ablative	bhaktāt	bhaktābhyām	bhaktebhyaḥ	Ablative	kramāt	kramābhyām	kramebhyaḥ
Genitive	nihatasya	nihatayoḥ	nihatānām	Genitive	bhaktasya	bhaktayoḥ	bhaktānām	Genitive	kramasya	kramayoḥ	kramāṇām
Locative	nihate	nihatayoḥ	nihateṣu	Locative	bhakte	bhaktayoḥ	bhakteṣu	Locative	krame	kramayoḥ	krameṣu
ḥṛta-m	Singular	Dual	Plural	ṛṇa-m	Singular	Dual	Plural	(kṛ)	VIII	Primary	Optative
Nominative	ḥṛtaḥ	ḥṛtau	ḥṛtāḥ	Nominative	ṛṇaḥ	ṛṇau	ṛṇāḥ	Active	Singular	Dual	Plural
Vocative	ḥṛta	ḥṛtau	ḥṛtāḥ	Vocative	ṛṇa	ṛṇau	ṛṇāḥ	First	kuryām	kuryāva	kuryāma
Accusative	ḥṛtam	ḥṛtau	ḥṛtān	Accusative	ṛṇam	ṛṇau	ṛṇān	Second	kuryāḥ	kuryātām	kuryāta
Instrumental	ḥṛtena	ḥṛtābhyām	ḥṛtaiḥ ḥṛtebhiḥ	Instrumental	ṛṇena	ṛṇābhyām	ṛṇaiḥ ṛṇebhiḥ	Third	kuryāt	kuryātām	kuryuḥ
Dative	ḥṛtāya	ḥṛtābhyām	ḥṛtebhyaḥ	Dative	ṛṇāya	ṛṇābhyām	ṛṇebhyaḥ				
Ablative	ḥṛtāt	ḥṛtābhyām	ḥṛtebhyaḥ	Ablative	ṛṇāt	ṛṇābhyām	ṛṇebhyaḥ				
Genitive	ḥṛtasya	ḥṛtayoḥ	ḥṛtānām	Genitive	ṛṇasya	ṛṇayoḥ	ṛṇānām				
Locative	ḥṛte	ḥṛtayoḥ	ḥṛteṣu	Locative	ṛṇe	ṛṇayoḥ	ṛṇeṣu				

n Nom Sg	f Inst Sg	Adverb	m Loc Sg	m Loc Sg	f Nom Sg	f Nom Sg Conj	f Instr Sg
yat saṃkhyayā atra haraṇe kṛte nivṛttā hr̥tiḥ tu jāmitayā							
f Gen Sg	f Gen Sg	-ā a--ā-	f Nom Sg	n Nom Sg	-ḥ t--st-	m Nom Sg	m Loc Sg 3rd Sg Opt
tasyāḥ ūrdhva-gatāyāḥ sama-saṃkhyā tat-dalam guṇaḥ ante syāt 272							
-āḥ u--ā u-			-t d--d d-	-m g--ṃ g-	-aḥ a--o'-		

That number by which here when performed division ceased, indeed by repitition, the half of that one higher even number would be the multiplier in the end.

Element	Type	Meaning	Formation
yad-	यद्-	Pronoun	Relative: who, that conj. so that
saṃkhyā-	संख्या-	Noun	number, count
atra	अत्र	Adverb	here towards here in this case
haraṇa-	हरण-	Adj/Noun	division
kṛta-	कृत-	Adjective	done, accomplished, finished
nivṛtta-	निवृत्त-	Adjective	ceased
hr̥ti-	हृति-	Noun	division, quotient
tu	तु	Conjunction	but, however also, indeed therefore
jāmitā-	जामिता-	Noun	repetition, repetitiveness
tad-	तद्-	Pronoun	Third Person: it, this, that
ūrdhva-	उर्ध्व-	Adjective	upwards
gata-	गत-	Adjective	gone
sama-	सम-	Adjective	even
dala-	दल-	Noun	half
guṇa-	गुण-	Noun	multiplier, coefficient
anta-	अन्त-	Noun	end
(as)	(अस्)	Verb II	to be

yad-n	Singular	Dual	Plural	nivṛtta-f	Singular	Dual	Plural	gata-f	Singular	Dual	Plural	(as)	II	Primary	Optative
Nominative	yat	ye	yāni	Nominative	nivṛttā	nivṛtte	nivṛttāḥ	Nominative	gatā	gate	gatāḥ	Active	Singular	Dual	Plural
Vocative	-	-	-	Vocative	nivṛtte	nivṛtte	nivṛttāḥ	Vocative	gate	gate	gatāḥ	First	syām	syāva	syāma
Accusative	yat	ye	yāni	Accusative	nivṛttām	nivṛtte	nivṛttāḥ	Accusative	gatām	gate	gatāḥ	Second	syāḥ	syātam	syāta
Instrumental	yena	yābhyām	yaiḥ	Instrumental	nivṛttayā	nivṛttābhyām	nivṛttābhiḥ	Instrumental	gatayā	gatābhyām	gatābhiḥ	Third	syāt	syātām	syuḥ
Dative	yasmai	yābhyām	yebhyaḥ	Dative	nivṛttāyai	nivṛttābhyām	nivṛttābhyaḥ	Dative	gatāyai	gatābhyām	gatābhyaḥ				
Ablative	yasmāt	yābhyām	yebhyaḥ	Ablative	nivṛttāyāḥ	nivṛttābhyām	nivṛttābhyaḥ	Ablative	gatāyāḥ	gatābhyām	gatābhyaḥ				
Genitive	yasya	yayoḥ	yeśām	Genitive	nivṛttāyāḥ	nivṛttayoḥ	nivṛttānām	Genitive	gatāyāḥ	gatayoḥ	gatānām				
Locative	yasmin	yayoḥ	yeṣu	Locative	nivṛttāyām	nivṛttayoḥ	nivṛttāsu	Locative	gatāyām	gatayoḥ	gatāsu				
saṃkhyā-f	Singular	Dual	Plural	hṛti-f	Singular	Dual	Plural	dala-n	Singular	Dual	Plural				
Nominative	saṃkhyā	saṃkhye	saṃkhyāḥ	Nominative	hṛtiḥ	hṛti	hṛtayaḥ	Nominative	dalam	dale	dalāni				
Vocative	saṃkhye	saṃkhye	saṃkhyāḥ	Vocative	hṛte	hṛti	hṛtayaḥ	Vocative	dala	dale	dalāni				
Accusative	saṃkhyām	saṃkhye	saṃkhyāḥ	Accusative	hṛtim	hṛti	hṛtiḥ	Accusative	dalam	dale	dalāni				
Instrumental	saṃkhyayā	saṃkhyābhyām	saṃkhyābhiḥ	Instrumental	hṛtyā	hṛtibhyām	hṛtibhiḥ	Instrumental	dalena	dalābhyām	dalaiḥ				
Dative	saṃkhyāyai	saṃkhyābhyām	saṃkhyābhyaḥ	Dative	hṛtyai	hṛtibhyām	hṛtibhyaḥ	Dative	dalāya	dalābhyām	dalebhyaḥ				
Ablative	saṃkhyāyāḥ	saṃkhyābhyām	saṃkhyābhyaḥ	Ablative	hṛtyāḥ	hṛtibhyām	hṛtibhyaḥ	Ablative	dalāt	dalābhyām	dalebhyaḥ				
Genitive	saṃkhyāyāḥ	saṃkhyayoḥ	saṃkhyānām	Genitive	hṛtyāḥ	hṛtyoḥ	hṛtīnām	Genitive	dalasya	dalayoḥ	dalānām				
Locative	saṃkhyāyām	saṃkhyayoḥ	saṃkhyāsu	Locative	hṛtyām	hṛtau	hṛtiṣu	Locative	dale	dalayoḥ	daleṣu				
haraṇa-m	Singular	Dual	Plural	jāmitā-f	Singular	Dual	Plural	guṇa-m	Singular	Dual	Plural				
Nominative	haraṇaḥ	haraṇau	haraṇāḥ	Nominative	jāmitā	jāmite	jāmitāḥ	Nominative	guṇaḥ	guṇau	guṇāḥ				
Vocative	haraṇa	haraṇau	haraṇāḥ	Vocative	jāmite	jāmite	jāmitāḥ	Vocative	guṇa	guṇau	guṇāḥ				
Accusative	haraṇam	haraṇau	haraṇān	Accusative	jāmitām	jāmite	jāmitāḥ	Accusative	guṇam	guṇau	guṇān				
Instrumental	haraṇena	haraṇābhyām	haraṇaiḥ haraṇebhiḥ	Instrumental	jāmitayā	jāmitābhyām	jāmitābhiḥ	Instrumental	guṇena	guṇābhyām	guṇaiḥ				
Dative	haraṇāya	haraṇābhyām	haraṇebhyaḥ	Dative	jāmitāyai	jāmitābhyām	jāmitābhyaḥ	Dative	guṇāya	guṇābhyām	guṇebhyaḥ				
Ablative	haraṇāt	haraṇābhyām	haraṇebhyaḥ	Ablative	jāmitāyāḥ	jāmitābhyām	jāmitābhyaḥ	Ablative	guṇāt	guṇābhyām	guṇebhyaḥ				
Genitive	haraṇasya	haraṇayoḥ	haraṇānām	Genitive	jāmitāyāḥ	jāmitayoḥ	jāmitānām	Genitive	guṇasya	guṇayoḥ	guṇānām				
Locative	haraṇe	haraṇayoḥ	haraṇeṣu	Locative	jāmitāyām	jāmitayoḥ	jāmitāsu	Locative	guṇe	guṇayoḥ	guṇeṣu				
kṛta-m	Singular	Dual	Plural	tad-f	Singular	Dual	Plural	anta-m	Singular	Dual	Plural				
Nominative	kṛtaḥ	kṛtau	kṛtāḥ	Nominative	sā	te	tāḥ	Nominative	antaḥ	antau	antāḥ				
Vocative	kṛta	kṛtau	kṛtāḥ	Vocative	-	-	-	Vocative	anta	antau	antāḥ				
Accusative	kṛtam	kṛtau	kṛtān	Accusative	tām	te	tāḥ	Accusative	antam	antau	antān				
Instrumental	kṛtena	kṛtābhyām	kṛtaiḥ	Instrumental	tayā	tābhyām	tābhiḥ	Instrumental	antena	antābhyām	antaiḥ				
Dative	kṛtāya	kṛtābhyām	kṛtebhyaḥ	Dative	tasyai	tābhyām	tābhyaḥ	Dative	antāya	antābhyām	antebhyaḥ				
Ablative	kṛtāt	kṛtābhyām	kṛtebhyaḥ	Ablative	tasyāḥ	tābhyām	tābhyaḥ	Ablative	antāt	antābhyām	antebhyaḥ				
Genitive	kṛtasya	kṛtayoḥ	kṛtānām	Genitive	tasyāḥ	tayoḥ	tāsām	Genitive	antasya	antayoḥ	antānām				
Locative	kṛte	kṛtayoḥ	kṛteṣu	Locative	tasyām	tayoḥ	tāsu	Locative	ante	antayoḥ	anteṣu				

m Nom Sg	m Nom Sg	m Nom Sg	Adverb	Adverb				
tat-vargaḥ rūpa-yutaḥ hāraḥ vyāsa-abdhi-ghātataḥ prāgvat								
-t v=-d v-	-aḥ r=-o r-	-aḥ h=-o h-	-a a=-ā-	-s p->-ḥ p-				
m Inst Du	n Nom Sg	n Nom Sg	n Loc Sg	n Loc Sg	n Loc Sg	m Nom Sg	Adverb	m Nom Sg
tābhyām āptam svam ṛṇe kṛte dhane kṣepaḥ eva karaṇīyaḥ 273								
	-m s=-ṁ s-							

That square incremented by one is divisor. As before from four multiplying the diameter.
By these two obtained, indeed should add/subtract when [last] having subtracted/added.

Element	Type	Meaning	Formation
tad-	तद्-	Pronoun	Third Person: it, this, that
varga-	वर्ग-	Noun	square
rūpa-	रूप-	Noun	the number 1
yuta-	युत-	Adjective	increased
hāra-	हार-	Noun	divisor
vyāsa-	व्यास-	Noun	the diameter of a circle
abdhi-	अब्धि-	Noun	the number four
ghātatas	घाततस्	Adverb	from multiplying
prāgvat	प्राग्वत्	Adverb	as before
āpta-	आप्त-	Adverb	obtained, quotient
sva-	स्व-	Noun	plus
ṛṇa-	ऋण-	Noun	minus
kṛta-	कृत-	Adjective	having been done
dhana-	धन-	Noun	positive quantity, addition
kṣepa-	क्षेप-	Noun	subtraction
eva	एव	Particle	indeed
karaṇīya-	करणीय-	Adjective	to be done

varga-m	Singular	Dual	Plural	tad-m	Singular	Dual	Plural	ṛṇa-n	Singular	Dual	Plural	kṣepa-m	Singular	Dual	Plural
Nominative	vargaḥ	vargau	vargāḥ	Nominative	saḥ sa	tau	te	Nominative	ṛṇam	ṛṇe	ṛṇāni	Nominative	kṣepaḥ	kṣepau	kṣepāḥ
Vocative	varga	vargau	vargāḥ	Vocative	-	-	-	Vocative	ṛṇa	ṛṇe	ṛṇāni	Vocative	kṣepa	kṣepau	kṣepāḥ
Accusative	vargam	vargau	vargān	Accusative	tam	tau	tān	Accusative	ṛṇam	ṛṇe	ṛṇāni	Accusative	kṣepam	kṣepau	kṣepān
Instrumental	vargeṇa	vargābhyām	vargaiḥ vargebhiḥ	Instrumental	tena	tābhyām	taiḥ	Instrumental	ṛṇena	ṛṇābhyām	ṛṇaiḥ	Instrumental	kṣepeṇa	kṣepābhyām	kṣepaiḥ kṣepebhiḥ
Dative	vargāya	vargābhyām	vargebhyaḥ	Dative	tasmai	tābhyām	tebhyaḥ	Dative	ṛṇāya	ṛṇābhyām	ṛṇebhyaḥ	Dative	kṣepāya	kṣepābhyām	kṣepebhyaḥ
Ablative	vargāt	vargābhyām	vargebhyaḥ	Ablative	tasmāt	tābhyām	tebhyaḥ	Ablative	ṛṇāt	ṛṇābhyām	ṛṇebhyaḥ	Ablative	kṣepāt	kṣepābhyām	kṣepebhyaḥ
Genitive	vargasya	vargayoḥ	vargāṇām	Genitive	tasya	tayoḥ	teṣām	Genitive	ṛṇasya	ṛṇayoḥ	ṛṇānām	Genitive	kṣepasya	kṣepayoḥ	kṣepāṇām
Locative	varge	vargayoḥ	vargeṣu	Locative	tasmin	tayoḥ	teṣu	Locative	ṛṇe	ṛṇayoḥ	ṛṇeṣu	Locative	kṣepe	kṣepayoḥ	kṣepeṣu
yuta-m	Singular	Dual	Plural	āpta-n	Singular	Dual	Plural	ṛṛta-n	Singular	Dual	Plural	karaṇīya-m	Singular	Dual	Plural
Nominative	yutaḥ	yutau	yutāḥ	Nominative	āptam	āpte	āptāni	Nominative	ṛṛtam	ṛṛte	ṛṛtāni	Nominative	karaṇīyaḥ	karaṇīyau	karaṇīyāḥ
Vocative	yuta	yutau	yutāḥ	Vocative	āpta	āpte	āptāni	Vocative	ṛṛta	ṛṛte	ṛṛtāni	Vocative	karaṇīya	karaṇīyau	karaṇīyāḥ
Accusative	yutam	yutau	yutān	Accusative	āptam	āpte	āptāni	Accusative	ṛṛtam	ṛṛte	ṛṛtāni	Accusative	karaṇīyam	karaṇīyau	karaṇīyān
Instrumental	yutena	yutābhyām	yutaiḥ yutebhiḥ	Instrumental	āptena	āptābhyām	āptaiḥ	Instrumental	ṛṛtena	ṛṛtābhyām	ṛṛtaiḥ	Instrumental	karaṇīyena	karaṇīyābhyām	karaṇīyaiḥ karaṇīyebhiḥ
Dative	yutāya	yutābhyām	yutebhyaḥ	Dative	āptāya	āptābhyām	āptebhyaḥ	Dative	ṛṛtāya	ṛṛtābhyām	ṛṛtebhyaḥ	Dative	karaṇīyāya	karaṇīyābhyām	karaṇīyebhyaḥ
Ablative	yutāt	yutābhyām	yutebhyaḥ	Ablative	āptāt	āptābhyām	āptebhyaḥ	Ablative	ṛṛtāt	ṛṛtābhyām	ṛṛtebhyaḥ	Ablative	karaṇīyāt	karaṇīyābhyām	karaṇīyebhyaḥ
Genitive	yutasya	yutayoḥ	yutānām	Genitive	āptasya	āptayoḥ	āptānām	Genitive	ṛṛtasya	ṛṛtayoḥ	ṛṛtānām	Genitive	karaṇīyasya	karaṇīyayoḥ	karaṇīyānām
Locative	yute	yutayoḥ	yuteṣu	Locative	āpte	āptayoḥ	āpteṣu	Locative	ṛṛte	ṛṛtayoḥ	ṛṛteṣu	Locative	karaṇīye	karaṇīyayoḥ	karaṇīyeṣu
hāra-m	Singular	Dual	Plural	sva-n	Singular	Dual	Plural	dhana-n	Singular	Dual	Plural				
Nominative	hāraḥ	hārau	hārāḥ	Nominative	svam	sve	svāni	Nominative	dhanam	dhane	dhanāni				
Vocative	hāra	hārau	hārāḥ	Vocative	sva	sve	svāni	Vocative	dhana	dhane	dhanāni				
Accusative	hāram	hārau	hārān	Accusative	svam	sve	svāni	Accusative	dhanam	dhane	dhanāni				
Instrumental	hāreṇa	hārābhyām	hāraiḥ hārebhiḥ	Instrumental	svena	svābhyām	svaiḥ	Instrumental	dhanena	dhanābhyām	dhanaiḥ				
Dative	hārāya	hārābhyām	hārebhyaḥ	Dative	svasmai	svābhyām	svebhyaḥ	Dative	dhanāya	dhanābhyām	dhanebhyaḥ				
Ablative	hārāt	hārābhyām	hārebhyaḥ	Ablative	svāt svasmāt	svābhyām	svebhyaḥ	Ablative	dhanāt	dhanābhyām	dhanebhyaḥ				
Genitive	hārasya	hārayoḥ	hārāṇām	Genitive	svasya	svayoḥ	sveṣām	Genitive	dhanasya	dhanayoḥ	dhanānām				
Locative	hāre	hārayoḥ	hāreṣu	Locative	sve svasmin	svayoḥ	sveṣu	Locative	dhane	dhanayoḥ	dhaneṣu				

m Nom Sg

m Nom Sg

m Nom Sg

Adverb

Adverb

m Nom Sg

3rd Sg Opt

labdhaḥ paridhiḥ sūkṣmaḥ bahu-kṛtvaḥ haraṇataḥ ati-sūkṣmaḥ syāt ||274||

-ḥ b=-o b-

-as a->-aḥ a=-o' - -as a->-aḥ a=-o' -

The obtained circumfrence is refined. From dividing many times it would be very refined.

Element		Type	Meaning	Formation
labdha-	लब्ध-	Adjective	obtained	Past Passive Participle of (labh) I 'to obtain'
paridhi-	परिधि-	Noun	circumfrence	pari-dhi-;pari-['around'];dhi-[Noun:'receptacle']
sūkṣma-	सूक्ष्म-	Adjective	refined	From sūci-[f Noun:'needle']
bahu-	बहु-	Adjective	many	(baṃh)-u-;(baṃh) I 'to grow';-u-[Adjective]
kṛtvas	कृत्वस्	Adverb	times, -fold	(kr)-tvas;(kr) V 'to do';-tvas[Adverb of frequency]
haraṇatas	हरणतस्	Adverb	from dividing	haraṇa-tas;haraṇa-[Noun:'removal'];-tas[Ablative Adverb]
ati-	अति-	Adverb	very, too	
(as)	(अस्)	Verb II	to be	

labdha-m	Singular	Dual	Plural	sūkṣma-m	Singular	Dual	Plural
Nominative	labdhaḥ	labdhau	labdhāḥ	Nominative	sūkṣmaḥ	sūkṣmau	sūkṣmāḥ
Vocative	labdha	labdhau	labdhāḥ	Vocative	sūkṣma	sūkṣmau	sūkṣmāḥ
Accusative	labdham	labdhau	labdhān	Accusative	sūkṣmam	sūkṣmau	sūkṣmān
Instrumental	labdhena	labdhābhyām	labdhaiḥ labdhebhīḥ	Instrumental	sūkṣmeṇa	sūkṣmābhyām	sūkṣmaiḥ
Dative	labdhāya	labdhābhyām	labdhebhyaḥ	Dative	sūkṣmāya	sūkṣmābhyām	sūkṣmebhyaḥ
Ablative	labdhāt	labdhābhyām	labdhebhyaḥ	Ablative	sūkṣmāt	sūkṣmābhyām	sūkṣmebhyaḥ
Genitive	labdhasya	labdhayoḥ	labdhānām	Genitive	sūkṣmasya	sūkṣmayoḥ	sūkṣmānām
Locative	labdhe	labdhayoḥ	labdheṣu	Locative	sūkṣme	sūkṣmayoḥ	sūkṣmeṣu
paridhi-m	Singular	Dual	Plural	(as)	II	Primary	Optative
Nominative	paridhiḥ	paridhī	paridhayaḥ	Active	Singular	Dual	Plural
Vocative	paridhe	paridhī	paridhayaḥ	First	syām	syāva	syāma
Accusative	paridhim	paridhī	paridhīn	Second	syāḥ	syātām	syāta
Instrumental	paridhinā	paridhibhyām	paridhibhiḥ	Third	syāt	syātām	syuḥ
Dative	paridhaye	paridhibhyām	paridhibhyaḥ				
Ablative	paridheḥ	paridhibhyām	paridhibhyaḥ				
Genitive	paridheḥ	paridhyoḥ	paridhīnām				
Locative	paridhau	paridhyoḥ	paridhiṣu				

m Abl Pl	m Nom Sg	m Nom Sg	3rd Sg Pres Pass	Indeclinable	Part	m Nom Sg
ebhyaḥ	sūkṣmatarah	anyaḥ	vilikhyate	kaścana	api	saṃskārah
	-aḥ a--o-	-ḥ v--o v-		-a a--ā-		
m Loc Sg	m Nom Sg	m Nom Sg	m Nom Sg	Particle	Adverb	
ante	sama-saṃkhyā-dala-vargaḥ	sa-ekah	guṇaḥ	saḥ	eva	punaḥ 295
	-a e--ai-	-ḥ g--o g-		-aḥ e--a e-		-r > -ḥ

More refined than these also is a certain other arrangement delineated.
Now indeed the multiplier in the end is square of half of the even number together with one.

Element	Type	Meaning	Formation
idam-	इदम्-	Pronoun	Demonstrative: this, that, it
sūkṣmatara-	सूक्ष्मतर-	Adjective	finer, more refined sūkṣma-tara-;sūkṣma-[Noun:'refined'];tara-[comparative:'-er','more']
anya-	अन्य-	Pronoun	other, another
(vi-likh)	(वि-लिख)	Verb VI	to delineate vi-(likh);vi-['separated from'];(likh) VI 'to write'
kaścana	कश्चन	Pronoun	Indefinite: some, certain ka-cana-;ka-[Pronoun:'who',which'];cana[Particle:indefinite value to Pronoun]
api	अपि	Particle	also
saṃskāra-	संस्कार-	Noun	arrangement sam-vṛddhi(kṛ)-a-;sam-['together'];(kṛ) V,VIII 'to do';-a-[Noun/Adj]
anta-	अन्त-	Noun	end, boundary, edge
sama-	सम-	Adjective	even
saṃkhyā-	संख्या-	Noun	number, count sam-(khyā)-;sam-['together'];(khyā) II 'to look'
dala-	दल-	Noun	half Agent Noun of (dal) I 'to split'
varga-	वर्ग-	Noun	square Agent Noun of (vrj) VII 'to set aside'
tad-	तद्-	Pronoun	Third Person: it, this, that
eka-	एक-	Number	one
guṇa-	गुण-	Noun	multiplier, coefficient
eva	एव	Particle	indeed ā-iva;ā-['towards'];-iva['as if']
punar	पुनर्	Adverb	now

idam-m	Singular	Dual	Plural	saṃskāra-m	Singular	Dual	Plural	eka-m	Singular	Dual	Plural	(vi-likh)	VI	Primary	Present
Nominative	āyam	imau	ime	Nominative	saṃskāraḥ	saṃskārau	saṃskārāḥ	Nominative	ekaḥ	-	-	Passive	Singular	Dual	Plural
Vocative	-	-	-	Vocative	saṃskāra	saṃskārau	saṃskārāḥ	Vocative	-	-	-	First	vilikhye	vilikhyāvahe	vilikhyāmahe
Accusative	imam	imau	imān	Accusative	saṃskāram	saṃskārau	saṃskārān	Accusative	ekam	-	-	Second	vilikhyase	vilikhyethe	vilikhyadhve
Instrumental	anena	ābhyām	ebhiḥ	Instrumental	saṃskāreṇa	saṃskārābhyām	saṃskāraiḥ	Instrumental	ekena	-	-	Third	vilikhyate	vilikhyete	vilikhyante
Dative	asmai	ābhyām	ebhyaḥ	Dative	saṃskārāya	saṃskārābhyām	saṃskārebhyaḥ	Dative	ekasmai	-	-				
Ablative	asmāt	ābhyām	ebhyaḥ	Ablative	saṃskārāt	saṃskārābhyām	saṃskārebhyaḥ	Ablative	ekasmāt	-	-				
Genitive	asya	anayoḥ	eṣām	Genitive	saṃskārasya	saṃskārayoḥ	saṃskārāṇām	Genitive	ekasya	-	-				
Locative	asmin	anayoḥ	eṣu	Locative	saṃskāre	saṃskārayoḥ	saṃskāreṣu	Locative	ekasmin	-	-				
sūkṣmatara-m	Singular	Dual	Plural	anta-m	Singular	Dual	Plural	guṇa-m	Singular	Dual	Plural				
Nominative	sūkṣmataraḥ	sūkṣmatarau	sūkṣmatārāḥ	Nominative	antaḥ	antau	antāḥ	Nominative	guṇaḥ	guṇau	guṇāḥ				
Vocative	sūkṣmatara	sūkṣmatarau	sūkṣmatārāḥ	Vocative	anta	antau	antāḥ	Vocative	guṇa	guṇau	guṇāḥ				
Accusative	sūkṣmataram	sūkṣmatarau	sūkṣmatārān	Accusative	antam	antau	antān	Accusative	guṇam	guṇau	guṇān				
Instrumental	sūkṣmatareṇa	sūkṣmatārābhyām	sūkṣmataraiḥ sūkṣmatarebhiḥ	Instrumental	antena	antābhyām	antaiḥ	Instrumental	guṇena	guṇābhyām	guṇaiḥ				
Dative	sūkṣmatārāya	sūkṣmatārābhyām	sūkṣmatarebhyaḥ	Dative	antāya	antābhyām	antebhyaḥ	Dative	guṇāya	guṇābhyām	guṇebhyaḥ				
Ablative	sūkṣmatārāt	sūkṣmatārābhyām	sūkṣmatarebhyaḥ	Ablative	antāt	antābhyām	antebhyaḥ	Ablative	guṇāt	guṇābhyām	guṇebhyaḥ				
Genitive	sūkṣmatarasya	sūkṣmatarayoh	sūkṣmatārāṇām	Genitive	antasya	antayoḥ	antānām	Genitive	guṇasya	guṇayoḥ	guṇānām				
Locative	sūkṣmatare	sūkṣmatarayoh	sūkṣmatareṣu	Locative	ante	antayoḥ	anteṣu	Locative	guṇe	guṇayoḥ	guṇeṣu				
anya-m	Singular	Dual	Plural	varga-m	Singular	Dual	Plural	tad-m	Singular	Dual	Plural				
Nominative	anyaḥ	anyau	anye	Nominative	vargaḥ	vargau	vargāḥ	Nominative	saḥ sa	tau	te				
Vocative	-	-	-	Vocative	varga	vargau	vargāḥ	Vocative	-	-	-				
Accusative	anyam	anyau	anyān	Accusative	vargam	vargau	vargān	Accusative	tam	tau	tān				
Instrumental	anyena	anyābhyām	anyaiḥ	Instrumental	vargeṇa	vargābhyām	vargaiḥ vargebhiḥ	Instrumental	tena	tābhyām	taiḥ				
Dative	anyasmai	anyābhyām	anyebhyaḥ	Dative	vargāya	vargābhyām	vargebhyaḥ	Dative	tasmai	tābhyām	tebhyaḥ				
Ablative	anyasmāt	anyābhyām	anyebhyaḥ	Ablative	vargāt	vargābhyām	vargebhyaḥ	Ablative	tasmāt	tābhyām	tebhyaḥ				
Genitive	anyasya	anyayoḥ	anyeṣām	Genitive	vargasya	vargayoḥ	vargāṇām	Genitive	tasya	tayoḥ	teṣām				
Locative	anyasmin	anyayoḥ	anyeṣu	Locative	varge	vargayoḥ	vargeṣu	Locative	tasmin	tayoḥ	teṣu				

guṇita-m	Singular	Dual	Plural	hāra-m	Singular	Dual	Plural	kārya-n	Singular	Dual	Plural
Nominative	guṇitaḥ	guṇitau	guṇitāḥ	Nominative	hāraḥ	hārau	hārāḥ	Nominative	kāryam	kārye	kāryāṇi
Vocative	guṇita	guṇitau	guṇitāḥ	Vocative	hāra	hārau	hārāḥ	Vocative	kārya	kārye	kāryāṇi
Accusative	guṇitam	guṇitau	guṇitān	Accusative	hāram	hārau	hārān	Accusative	kāryam	kārye	kāryāṇi
Instrumental	guṇitena	guṇitābhyām	guṇitaiḥ guṇitebhiḥ	Instrumental	hāreṇa	hārābhyām	hāraiḥ hārebhiḥ	Instrumental	kāryeṇa	kāryābhyām	kāryaiḥ
Dative	guṇitāya	guṇitābhyām	guṇitebhyaḥ	Dative	hārāya	hārābhyām	hārebhyaḥ	Dative	kāryāya	kāryābhyām	kāryebhyaḥ
Ablative	guṇitāt	guṇitābhyām	guṇitebhyaḥ	Ablative	hārāt	hārābhyām	hārebhyaḥ	Ablative	kāryāt	kāryābhyām	kāryebhyaḥ
Genitive	guṇitasya	guṇitayoḥ	guṇitānām	Genitive	hārasya	hārayoḥ	hārāṇām	Genitive	kāryasya	kāryayoḥ	kāryāṇām
Locative	guṇite	guṇitayoḥ	guṇiteṣu	Locative	hāre	hārayoḥ	hāreṣu	Locative	kārye	kāryayoḥ	kāryeṣu
yuta-m	Singular	Dual	Plural	haraṇa-m	Singular	Dual	Plural	(bhū)	I	Primary	Optative
Nominative	yutaḥ	yutau	yutāḥ	Nominative	haraṇaḥ	haraṇau	haraṇāḥ	Active	Singular	Dual	Plural
Vocative	yuta	yutau	yutāḥ	Vocative	haraṇa	haraṇau	haraṇāḥ	First	bhaveyam	bhaveva	bhavema
Accusative	yutam	yutau	yutān	Accusative	haraṇam	haraṇau	haraṇān	Second	bhaveḥ	bhavetam	bhaveta
Instrumental	yutena	yutābhyām	yutaiḥ yutebhiḥ	Instrumental	haraṇena	haraṇābhyām	haraṇaiḥ haraṇebhiḥ	Third	bhavet	bhavetām	bhaveyuh
Dative	yutāya	yutābhyām	yutebhyaḥ	Dative	haraṇāya	haraṇābhyām	haraṇebhyaḥ				
Ablative	yutāt	yutābhyām	yutebhyaḥ	Ablative	haraṇāt	haraṇābhyām	haraṇebhyaḥ				
Genitive	yutasya	yutayoḥ	yutānām	Genitive	haraṇasya	haraṇayoḥ	haraṇānām				
Locative	yute	yutayoḥ	yuteṣu	Locative	haraṇe	haraṇayoḥ	haraṇeṣu				
hata-m	Singular	Dual	Plural	etad-n	Singular	Dual	Plural				
Nominative	hataḥ	hatau	hatāḥ	Nominative	etat	ete	etāni				
Vocative	hata	hatau	hatāḥ	Vocative	-	-	-				
Accusative	hatam	hatau	hatān	Accusative	etat	ete	etāni				
Instrumental	hatena	hatābhyām	hataiḥ hatebhiḥ	Instrumental	etena	etābhyām	etaiḥ				
Dative	hatāya	hatābhyām	hatebhyaḥ	Dative	etasmai	etābhyām	etebhyaḥ				
Ablative	hatāt	hatābhyām	hatebhyaḥ	Ablative	etasmāt	etābhyām	etebhyaḥ				
Genitive	hatasya	hatayoḥ	hatānām	Genitive	etasya	etayoḥ	eteṣām				
Locative	hate	hatayoḥ	hateṣu	Locative	etasmīn	etayoḥ	eteṣu				

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते । त्रिशरादिविषमसंख्याभक्तमृणं स्व पृथक्क्रमात् कुर्यात् ॥२७१॥
vyāse vāridhinhate rūpahṛte vyāsaśāgarābhigate । triśarādiviṣamasamkhyābhaktamṛṇaṃ svaṃ pṛthakkrāmāt kuryāt ॥271॥
यत्संख्ययात्र हरणे कृते निवृत्ता हृतिस्तु जामितया । तस्या ऊर्ध्वगतायाः समसंख्या तदलं गुणोऽन्ते स्यात् ॥२७२॥
yatsamkhyayātra haraṇe kṛte nivṛttā hṛtistu jāmitayā । tasyā ūrdhvagatāyāḥ samasamkhyā taddalaṃ guṇo'nte syāt ॥272॥
तद्वर्गो रूपयुतो हारो व्यासाब्धिघाततः प्राग्वत् । ताभ्यामाप्तं स्वमृणे कृते धने क्षेप एव करणीयः ॥२७३॥
tadvargo rūpayuto hāro vyāsābhighātataḥ prāgvat । tābhyāmāptaṃ svamṛṇe kṛte dhane kṣepa eva karaṇīyaḥ ॥273॥
लब्धः परिधिः सूक्ष्मो बहुकृत्वो हरणतोऽतिसूक्ष्मः स्यात् ॥२७४॥
labdhaḥ paridhiḥ sūkṣmo bahukṛtvo haraṇato'tisūkṣmaḥ syāt ॥274॥
एभ्यः सूक्ष्मतरोऽन्यो विलिख्यते कश्चनापि संस्कारः । अन्ते समसंख्यादलवर्गः सैको गुणः स एव पुनः ॥२९५॥
ebhyaḥ sūkṣmataro'nyo vilikhyate kaścanāpi saṃskāraḥ । ante samasamkhyādalavargaḥ saiko guṇaḥ sa eva punaḥ ॥295॥
युगगुणितो रूपयुतः समसंख्यादलहतो भवेद हारः । त्रिशरादिविषमसंख्याहरणात् परमेतदेव वा कार्यम् ॥ २९६॥
yugagūṇito rūpayutaḥ samasamkhyādalahato bhaved hāraḥ । triśarādiviṣamasamkhyāharaṇāt parametadeva vā kāryam ॥ 296॥

tantra-saṃgrahaḥ, nīlakaṇṭha-somayājī [1444 AD - 1545 AD]

yukti-dīpika-laghu-vivṛtti, śaṅkara vāriyar [1500 AD - 1560 AD]