

Sum of Squares of Arithmetic Progression

$$S(n) \stackrel{\text{def}}{=} a^2 + (a + d)^2 + (a + 2d)^2 + \cdots + (a + (n - 1)d)^2$$

a: first term d: increment n: number of terms

द्विगुण-एकोन-पद-उत्तर-कृति-हति-षष्ठ-अंश-मुख-चय-हत-युतिः ।
व्येक-पद-घ्ना मुख-कृति-सहिता पद-ताडित-इष्ट-कृति-चितिका ॥

*dviguṇa-ekona-pada-uttara-kṛti-hati-ṣaṣṭha-aṃśa-mukha-caya-hata-yutiḥ |
vyeka-pada-ghnā mukha-kṛti-sahitā pada-tāḍita-iṣṭa-kṛti-citikā ||*

Twice the number of terms minus one, multiplied by the squared increment, divided by six, together with the first term multiplied by the increment, multiplied by the number of terms minus one, together with the squared first term, multiplied by the number of terms, is the desired sum of squares.

gaṇita-sāra-saṅgrahaḥ, miśra-kāvya-vyavahāraḥ, Chapter 6, ślokaḥ 298, mahāvīracāryaḥ [850 AD]



$$S(n) = \left[\left\{ \frac{(2n-1)d^2}{6} + ad \right\} (n-1) + a^2 \right] n$$

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Chapter 6, ślokaḥ 298, mahāvīrācāryaḥ [850 AD]

dviguṇa-ekona-pada-uttara-kṛti-hati-ṣaṣṭha-aṃśa-mukha-caya-hata-yutiḥ		f Nom Sg
-a e--ai-	-a u--o-	-a a--ā-
f Nom Sg	f Nom Sg	f Nom Sg
vyeka-pada-ghnā mukha-kṛti-sahitā pada-tāḍita-iṣṭa-kṛti-citikā		-a i--e-

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Element	Type	Meaning	Formation
dviguṇa- द्विगुण-	Adjective	double, twice	dvi-guṇa-;dvi-[Number:'two'];guṇa-[Noun:'multiplier']
ekona- एकोन-	Adjective	minus one	eka-ūna-;eka-[Number:'one'];ūna-[Adj:'deficient']
pada- पद-	Noun	step	(pad)-a-;(pad) IV 'to go';-a-[Noun]
uttara- उत्तर-	Adjective	increment	ut-tara-;ut-['out'];tara-[form a comparative]
kṛti- कृति-	Noun	squared	Action Noun of (kr) V,VIII 'to do'
hati- हति-	Noun	multiplied	Action Noun of (han) II 'to strike'
ṣaṣṭha- षष्ठ-	Adjective	sixth	ṣaṣ-tha-;ṣaṣ-[Number:'six'];tha-[ordinal Adjective]
aṃśa- अंश-	Noun	share, part, portion, division	Action Noun of (aś) V 'to obtain'
mukha- मुख-	Noun	front, first, beginning	
caya- चय-	Noun	common difference	Action Noun of (ci) V 'to accumulate'
hata- हत-	Adjective	struck, multiplied	*(han)-ta-;(han) II 'to kill';ta-[Past Pass Pcpl:'having been x-ed']
yuti- युति-	Noun	union, addition, totality	Action Noun of (yu) II, VI, IX 'to join'
vyeka- व्येक-	Adjective	predecessor, minus one	vi-eka-;vi-['separated from'];eka-[Number:'one']
ghna- घ्न-	Adjective	that which strikes, multiplier	Agent Noun of (han) II 'to strike,kill'
sahita- सहित-	Adjective	together with, added	sa-hita-;sa-['together'];hita-[Past Pass Pcpl of (dhā) III 'to place']
tāḍita- ताडित-	Adjective	struck, multiplied	Past Pass Pcpl of Causative of (taḍ) X 'to strike'
iṣṭa- इष्ट-	Adjective	desired	(iṣ)-ta-;(iṣ) VI 'to desire';-ta-[Past Pass Pcpl]
citika- चितिक-	Noun	pile, sum	(ci)-ti-kā-;(ci) V 'to collect';-ti-[Action Noun];-ka-[Descriptive Agent]

yuti-f	Singular	Dual	Plural	ghna-f	Singular	Dual	Plural
Nominative	yutiḥ	yutī	yutayaḥ	Nominative	ghnā	ghne	ghnāḥ
Vocative	yute	yutī	yutayaḥ	Vocative	ghne	ghne	ghnāḥ
Accusative	yutim	yutī	yutiḥ	Accusative	ghnām	ghne	ghnāḥ
Instrumental	yutyā	yutibhyām	yutibhiḥ	Instrumental	ghnayā	ghnābhyām	ghnābhiḥ
Dative	yutyai yutaye	yutibhyām	yutibhyaḥ	Dative	ghnāyai	ghnābhyām	ghnābhyaḥ
Ablative	yutyāḥ yuteḥ	yutibhyām	yutibhyaḥ	Ablative	ghnāyāḥ	ghnābhyām	ghnābhyaḥ
Genitive	yutyāḥ yuteḥ	yutyoḥ	yutīnām	Genitive	ghnāyāḥ	ghnayoḥ	ghnānām
Locative	yutyām yutau	yutyoḥ	yutiṣu	Locative	ghnāyām	ghnayoḥ	ghnāsu
sahita-f	Singular	Dual	Plural	citika-f	Singular	Dual	Plural
Nominative	sahitā	sahite	sahitāḥ	Nominative	citikā	citike	citikāḥ
Vocative	sahite	sahite	sahitāḥ	Vocative	citike	citike	citikāḥ
Accusative	sahitām	sahite	sahitāḥ	Accusative	citikām	citike	citikāḥ
Instrumental	sahitayā	sahitābhyām	sahitābhiḥ	Instrumental	citikayā	citikābhyām	citikābhiḥ
Dative	sahitāyai	sahitābhyām	sahitābhyaḥ	Dative	citikāyai	citikābhyām	citikābhyaḥ
Ablative	sahitāyāḥ	sahitābhyām	sahitābhyaḥ	Ablative	citikāyāḥ	citikābhyām	citikābhyaḥ
Genitive	sahitāyāḥ	sahitayoḥ	sahitānām	Genitive	citikāyāḥ	citikayoḥ	citikānām
Locative	sahitāyām	sahitayoḥ	sahitāsu	Locative	citikāyām	citikayoḥ	citikāsu

द्विगुणैकोनपदोत्तरकृतिहतिषष्ठांशमुखचयहतयुतिः ।
व्येकपदघ्ना मुखकृतिसहिता पदताडितेष्टकृतिचितिका ॥

dviguṇaikonapadottarakṛtihatīṣṣṭhāṃśamukhacayahatayutiḥ ।
vyekapadaghnā mukhakṛtisahitā padatāḍiteṣṭakṛticitikā ॥

gaṇita-sāra-saṅgrahaḥ, miśra-kāvya-vyavahāraḥ
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