A Venn diagram analysis of the logical conclusions entailed by all pairs of categorical premises

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Abstract

One separates the validity of categorical syllogisms from the requirement that in their entailed conclusion one of the operators A,O,E,I has to be applied to the ordered pair (S,P). The 24 classically valid syllogisms (CVS), are valid syllogistic arguments, i.e., are made from pairs of categorical statements which contain the terms S,P,M, and entail a logical consequence of an "(S,P) type". Using a "cylindrical Venn diagram" (CVD), we graphically prove that the 36 distinct pairs of categorical premises split into 5 classes: 3 classes entail a logical conclusion, (thus generating 3 classes of valid syllogisms (VS)), and 2 classes do not. Each one of the 3 VS classes, contains CVS and also VS whose logical conclusions are not of the (S,P) type; some VS do not satisfy "the middle term has to be distributed in at least one premise" or "two negative premises are not allowed" . After the paper was written, I realized that the CVD is a Karnaugh map with n=3; these maps replace the "Venn circles" with squares, and show graphically and obviously which types of pairs of premises entail a logical conclusion. A new notation for the operators A and O is introduced, which makes the syllogistic figures unnecessary. Using contraposition and obversion in the same way as Aristotle used them to reduce syllogisms from the 2nd and 3rd figures to the first figure, one can argue that the syllogisms inside each of the 3 different VS (and CVS) classes are logically equivalent.

Keywords: categorical syllogisms • categorical premises • cylindrical Venn diagram • Karnaugh map

1. Notations

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

Fig. 1

For easier drawing, the universal set U is graphed as a rectangle – but please imagine that the left and right borders of the rectangle are glued together, so that S'PM and S'P'M are adjacent, and S'PM' and S'P'M' are adjacent, too – as in the usual 3-circle Venn diagram. On this "cylindrical Venn diagram", no inference rules and no axioms are needed to prove any of the syllogistic conclusions: it is self-evident that the 36 distinct pairs of categorical premises split into 5 classes - 2 classes do not entail any logical conclusion, but 3 classes do, and thus generate valid syllogisms, VS. We do not impose the "(S,P) conclusion" restriction which characterizes the 24 classically valid syllogisms, CVS. (P is "just a set called P", which can also be relabeled as S or P', etc., and is not necessarily the predicate of the conclusion.) Since the 8 subsets of Figure 1 are the "special/elementary" subsets we'll be dealing with all the time, we'll refer to them as just subsets; no other set will be a "subset".

As known, A(M,P) means "All M is P", i.e., the set P'M :=P' ∩M is empty. Thus A(M,P) acts on the M row, by emptying (two "horizontally adjacent" subsets) P'M = SP'M + S'P'M. Compare the above to A(P,M), which means "All P is M", i.e., the set PM'=P∩M' =Ø. Thus A(P,M) acts on the M' row, by emptying, two other horizontally adjacent subsets: PM' = SPM' + S'PM'. It follows that A(M,P) and A(P,M) empty subsets not only on different rows, but also on totally different/complementary columns.

We follow three conventions concerning the pairs of categorical {P, S} premises:

1. Always list a pair of categorical premises with the P-premise first, and the S-premise second. (P won't necessarily be the “predicate of the conclusion”; it's “just a set called P”.)
2. Since A(M,P) ≠ A(P,M), and O(M,P) ≠ O(P,M), the operators A and O will receive an index: 1 or 2, depending on the position of M inside the ordered pair on which they act.
3. Namely, define A_1 o{*, M}:= A(M,*), and A_2 o{*, M}:= A(*, M), where * is either S, or P. This way, when A_1, (resp. A_2), is applied to an unordered pair {*, M}, it will pick up M as the first, (resp. second), set for A to act upon. One can now use a one letter indexed categorical operators to symbolize an S or P premise: the meaning of A_1A_2 will be, (using the convention to firstly list the P-premise), A(M,P) A(S,M) – the premises of the syllogism Barbara. Same notation rule will be applied to the O operator. O(M,P) means “Some M is not P”, i.e., the set P'M ≠Ø, and O(S,M) will mean “Some S is not M ”, i.e., the set SM' ≠Ø. Analogously, O_1 o{*, M}:= O(M,*), and O_2 o{*, M}:= O(*, M). The E and I operators do not need indices since they are symmetric. E(S,M) means SM = Ø and I(S,M) means SM ≠Ø; they act on the M row, as A_1, O_1 do. Thus, a no index, or an index 1 operator, acts on the M row.

The only operators acting on the M' row are A_2, O_2. Their respective actions on the M' row are similar to the actions of E, resp. I, on the M row: for example, A(P,M) empties PM', E(P,M) empties PM, etc. Note that giving indices to A and O replaces the use of the 4 Figures into which the two premises' terms can be arranged. Keeping up with the “4 figures”, resulted, e.g., in a quadruple naming - Ferio, Festino, Ferison, Fresison, denote one and the same syllogism: EI:O(S,P). Getting rid of the rest of “double naming”, (Celarent/Cesare, Celaront/Cesaro, Disamis/Dimaris, etc., etc.), reduces the number of classically valid syllogisms, (CVS), from 24 to only 14 (with only 6 out of 14 – instead of 9 out of 24 – based on existential import (ei).

One more notation:

The “emptying operators” A_1, A_2, and E appear in universal premises (All..., No...), and the “element laying” operators O_1, O_2 , and I appear in particular (Some..., Some... not) premises. We'll order all six possible P-premises, (resp. all six possible S-premises), as vector components: $\mathbb{P}_i$ ={ A_1, E, A_2, O_1, I, O_2} o{P, M}, resp. $\mathbb{S}_i$ ={ A_1, E, A_2, O_1, I, O_2} o{S, M}. All the possible pairs of categorical premises are the components of the direct product of these two vectors $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,..,6. So, the total number of distinct pairs of premises is 36. As it will be noticed below, 17 pairs do not entail any logical conclusion(s), 15 do each entail exactly one logical conclusion, and each of the 4 pairs of “2-row action” universal premises, entail 3 independent logical conclusions each, for a total number of 19 pairs of premises that entail a total of 27 logical conclusions. Thus, according to our definitions, starting with categorical pairs of premises in the S,P,M variables, (i.e., A,O,E,I applied to the terms/sets S,P,M), one obtains 27 valid syllogisms, (VS), out of which, 14 - the CVS - have familiar names (even more than one familiar name per each CVS). Counting each set of two and the one set of four equivalent CVS as just one distinct syllogism per set, aka disregarding figures for equivalent, (or identical content), syllogisms, one gets just 8 CVS without ei, and 6 ei CVS.

2. Conclusions' shape

As one can see from the below discussion of all the possible pairs of premises, each and every one of the entailed logical conclusions falls in one of the following two categories:

(α) one, or even two, of the sets S, P, M, S', P', M' is reduced, via two universal, (aka emptying), premises to only one of its 4 subsets

(β) one of the 8 subsets in Figure 1 is shown to be ≠ Ø (possibly via an existential import (ei) supposition). [I think a computer may be programmed to choose three random terms/nouns from the English dictionary and then search for the most insightful valid syllogism one can build using those 3 terms. When the extensions of the 3 terms are mutually exclusive – as in cats, dogs, insects – I think the best syllogism one can build with such terms starts with the premises EE. When two of the terms do not intersect, but are both included in the third – as in cats, dogs, animals, I think the best syllogism one may come up with starts from the premises A_2A_2. If the intersection of two terms contains the third one, then the best premises are A_1A_1 – as in beautiful humans, uneducated, babies. Below we'll show in which sense these three pairs of premises and their respective conclusions are equivalent.]

When ei is used, the conclusion is reached in two stages: first one of S, P, M, S' , P', or M' is reduced to just one subset out of 4 (stage **(α)**), then, the ei makes/declares that subset ≠Ø. Thus each logical conclusion, is "bound" to one and only one particular subset (from Figure's 1 eight subsets). Since a CVS requires an "(S,P) conclusion", i.e., that, in the conclusion, one of the operators A,O,E,I be applied to the ordered pair (S, P), all CVS conclusions are bound on SPM, or SP'M, or SP'M'! Any VS, bound on another subset, has no name. (But, for example, logical conclusions "bound on SPM" are A(S,P) (Barbara), I(S,P) (Barbari, Bramantip, Darapti, Darii/Datisi, Disamis/Dimaris), A(P,S). The last one, originates from the VS A_2A_1: P=SPM, S'= S'P'M'. Then one gets A(P, SPM), and thus A(P,S), which has no name, even if the conclusion is bound to SPM, because A_2A_1 empty the set P except for SPM, and this does not fit the CVS requirement for an "(S,P) conclusion". But the ei, (P≠Ø), conclusion, I(S,P), gives the CVS Bramantip.

When one premise is universal and the other one is particular, then the logical conclusion, if any, is reached in one stage: one of the 8 subsets in Figure 1, uniquely determined, turns out to be ≠Ø. (The particular premise will have available only one subset, not two, to lay an element on, since the other horizontally adjacent subset was "just" emptied by the universal premise: only this arrangement can make both premises TRUE **and** the syllogism valid. See Fact #1 below.)

Note that any subset relabeling, such as, for example, P'↔M, S↔S', does not change the immediate neighbours of any of the subsets, and does not change the conclusions of any of the premises' pairs: the conclusion of "All P is M, All M is S" =A_2A_1, on the new, "relabeled Figure 1", will still be P=SPM, S'= S'P'M'.

Fact #1 For any pair of premises, {P-premise, S-premise}, both acting on the same row, there will always be one and only one subset "acted upon twice"; for any pair {P-premise, S-premise}, acting on two rows, there will always be one and only one column whose two subsets are both acted upon.

Proof: Cf. Fig. 1, two of the sets S, P, S', P', unless they are complementary sets, always have one and only one common column. Consider first the "M-row operators" A_1, O_1, E, I. In a P-premise, the operators A_1, O_1 act on the two P' columns and the E,I operators act on the two P columns. In an S-premise, the operators A_1, O_1 act on the two S' columns and the E,I operators act on the two S columns. Thus a pair (P-premise, S-premise), both acting on the M row, may act either on {P', S'}, or on {P', S}, or on {P, S'}, or on {P, S}, in which cases, respectively, either the subset S'P'M, or SP'M, or S'PM, or SPM is acted upon twice, and, respectively, either the subset SPM, or S'PM, or SP'M, or S'P'M is **not** acted upon at all. Thus two universal premises acting on the same row will empty 3 subsets, (of M or M'), and one universal and one particular premise acting on the same row will always place a set element on precisely one subset.

Since the A_2, O_2 operators - which act on the M' row - behave similarly to the E,I operators which act on M row - i.e., in a P-premise, the operators A_2, O_2 act on the two P columns, (exactly as E,I do on the M row), and in an S-premise, the operators A_2, O_2 act on the two S columns, (exactly as E,I do on the M row), it follows, as above, that a "2-row acting" pair of premises will always "act upon a column twice" either emptying both column's subsets, (and this is the only interesting case!), or possibly laying set elements in both column's subsets, or emptying one of the column's subset and laying a set element on the other column's subset – all these latter variants correspond to pairs of premises that do not entail any logical conclusion. (See below the paragraphs (i) and (ii2).) The four 2-row acting pairs of universal premises will thus empty one column, plus two other subsets, located on two different rows, on each side of that emptied column.(See the paragraph (ii1) below.) QED. (An examination of the 36 cases below makes the proof of Fact #1 clear, too.)

3. A more detailed discussion of the matrix L_{ij} , i,j = 1,..,6

The matrix $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,6 naturally splits into four 3 by 3 sub matrices: $L^{(1)}:=L_{ij}$, i,j = 1,2,3, contains only, (and they are the only ones), pairs of two universal premises; $L^{(2)}:=L_{ij}$, i=4,5,6, j=1,2,3, contains pairs of one particular P-premise, [gotten from replacing in $L^{(1)}$ the universal P-premise with the corresponding, (and contradictory), particular P-premise], and one universal S-premise (left unchanged from $L^{(1)}$); $L^{(3)}:=L_{ij}$, i=1,2,3, j=4,5,6, contains pairs of one universal P-premise, (unmodified from $L^{(1)}$), and one particular S-premise, [gotten from replacing in $L^{(1)}$ the universal S-premise with the corresponding, (and contradictory), particular S-premise]; and the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6 which contains only, (and they are the only ones), pairs of two particular premises.

(i) **$L^{(4)}$:** The pairs of premises in the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6, do not entail any logical conclusion. The two particular premises will "lay set elements" either on three subsets of the same row (M or *M'*), or on 4 subsets on different rows. Since, any conclusion of such a pair would just relist one or two of its premises, there is no way to satisfy Aristotle's insight, (Striker 2009: 20), that "A syllogism is an argument in which, certain things being posited, something **other than what was laid down** results by necessity because these things are so." Thus, per Aristotle's insight, these pairs will not generate any valid syllogism, VS; this means nine pairs of premises on the no conclusion/discarded list.

(ii) **$L^{(1)}$:** contains two sorts of universal premises pairs:

(ii0) The 5 "1-row acting" pairs of universal premises. Four pairs act on the M row only, $L_{11} = A_1A_1$, $L_{12} = A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, and, one pair acts on the M' row only, $L_{33} = A_2A_2$. As the Fact #1 has shown, the M subsets SPM, or S'PM, or SP'M, or S'P'M are **not** emptied by $L_{11} = A_1A_1$, $L_{12} = A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, respectively, and the S'P'M' subset of M' is **not** emptied by $L_{33} = A_2A_2$. Existential imports on M, resp., M', will produce 5 VS, each respectively "bound" on one of the above **not** emptied subsets. (2 out of 5 are the CVS Darapti and Felapton/Fesapo, bound on SPM and SP'M, respectively.) Thus the 5 "1-row acting" pairs of universal premises each produces one ei conclusion or VS, since we get one conclusion if ei is used each time one of the sets M*,* or M'*,* is reduced, via two "1-row acting" universal premises, to only one of its 4 subsets.

(ii1) The 4 "2-row acting" pairs of universal premises. They have to contain A_2 as a premise - since this is the only universal operator acting on the 2nd row M'. These 4 pairs are: $L_{13} = A_1A_2$, $L_{23}=EA_2$, $L_{31}=A_2A_1$, $L_{32}=A_2E$. They empty four subsets on two different rows and three different columns, located, cf. Fact #1, as follows: two empty subsets are on the same column, and the other two empty subsets are on different rows and on different sides of the empty column. These pairs are responsible for 12 different conclusions:

1. The pair of premises $L_{13} = A_1A_2$=A(M,P) A(S,M)= E(M,P')E(M',S) empties the column SP' and the subsets S'P'M and SPM', and, out of the 3 columns SP', S'P' and SP, occupied by the sets S and P', (whose intersection is SP'), only the subsets SPM out of S, and S'P'M' out of P' "survive". Logical conclusions are therefore aplenty: A(S, SPM), A(P', S'P'M'), E(S,P'), from which it follows A(S, P), A(P', S'), E(S, P'), A(S, M), A(P', M'). But the last two conclusions are exactly the premises – so they do not count, (as new knowledge), and the first three, via set theory, (or contraposition and obversion), are equivalent: A(S, P)=A(P',S')= E(S,P'). We'll keep just A(S,P) as the only one universal conclusion, out of the three independent conclusions entailed by the "Barbara pair of premises" $L_{13} = A_1A_2$. The other two independent conclusions involve ei: on S, i.e., supposing S≠Ø, one gets I(S,P), Barbari, and, via ei on P', one gets the no name I(P', S'), for a total of three independent conclusions entailed by the pair $L_{13} = A_1A_2$=A(M,P) A(S, M). Any other conclusions, such as I(S,M) or I(P,M) are not independent: they follow directly from the premises and S≠Ø. Moreover, P'= S'P'M' follows from S=SPM: if we list, (now, for simplicity, on one row), from left to right, the adjacent/neighbouring subsets that were not emptied by Barbara's premises, they are SPM, S'PM, S'PM', S'P'M'. This reads, from left to right, (resp. from right to left), precisely as $S \subseteq M \subseteq P$, and, resp., $P' \subseteq M' \subseteq S'$ – which is also how the transitivity of the inclusions A(S, M), A(M,P), or the Euler diagrams, would have represented Barbara's premises.

2. Analogously, the premises A_2A_1 =A(P,M) A(M,S)= E(M',P)E(M,S'), empty 4 subsets out of 6 from the columns S'P, S'P' and SP, occupied by the sets S' and P, (whose intersection is S'P). Only the subsets

SPM out of P and S'P'M' out of S' will again "survive". Thus, same "survivors" but now as parts of other "big sets" S', P instead of S,P'. The independent conclusions are the no name A(P,S), and, via ei on P, I(S,P) - Bramantip. Via ei on S', one gets (again) a no name I(P', S'). One can also see, that via a simple relabeling transformation, M→M, S→P, P→S, A_2A_1 becomes A_1A_2: A_2A_1 =A(P,M) A(M,S)→ A(S,M)A(M,P)=E(M,P')E(M',S). One can also see, that via another relabeling transformation, M→M', S→S', P→P', A_2A_1 also becomes A_1A_2: A_2A_1 =A(P,M) A(M,S)→A(P', M') A(M',S')=E(M,P')E(M',S), [or one may use contraposition on A(P', M') to get A(M,P), and on A(M',S') to get A(S,M)]. The difference between the two relabeling transformations is that the first one also maps the conclusions of A_2A_1 onto the conclusions of A_1A_2.

3. The EA_2 = E(M,P) E(M',S) and A_2E= E(M',P) E(M,S) are even more similar than A_1A_2 and A_2A_1 are. Each of EA_2 and A_2E, empty 4 subsets out of the 6 subsets of same 3 columns SP', SP' and SP. The two subsets that survive are: SP'M and S'PM' if the premises are EA_2, and SP'M' and S'PM if the premises are A_2E. The type (α), two entailed logical conclusions per pair of premises, are thus, for EA_2: A(S, SP'M), A(P, S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), plus, via ei on S, O(S,P), (Celaront/Cesaro).

4. Initial conclusions for A_2E are: A(S, SP'M'), A(P, S'PM). One chooses, as independent conclusion E(S,P) (=A(S,P')=A(P, S')), (Camestres/Camenes). And, via ei on P, the no name O(P,S), plus, via ei on S, O(S,P), (Camestrop/Camenop). This way, we get again to three independent conclusions when ei is used each time one of the sets S, P, S' , P' is reduced, via two "2-row acting" universal premises, to only one of its 4 subsets.

(iii) **$L^{(2)}$ and $L^{(3)}$**. Firstly, observe that the "2-row acting", 1-particular, 1-universal pairs of premises from $L^{(2)}$: $L_{43}=O_1A_2$, $L_{53}=IA_2$, $L_{61}=O_2A_1$, $L_{62}=O_2E$, and from $L^{(3)}$: $L_{16}=A_1O_2$, $L_{26}=EO_2$, $L_{34}=A_2O_1$, $L_{35}=A_2I$, do not entail any conclusion. These 8 pairs are gotten from the 4 (ii1) pairs, by substituting a particular premise in place of an universal premise. But by doing this, the emptying, and the element laying, happen now on two different rows. Any logical conclusion would just relist the premises. Thus, as per Aristotle's insight, the 8 pairs of 1-particular, 1-universal premises, acting on 2 rows, M **and** M', span the 2nd class of pairs that do not entail any logical conclusion. This adds up to a total of 9+8=17 of such pairs. Out of the other 36-17=19 pairs, we already saw 4 pairs of premises, (ii1), that entail 3 independent conclusions per pair, and 5 pairs of premises, (ii0), that entail one conclusion per pair. The rest of 10 pairs from $L^{(2)}$ and $L^{(3)}$, originate from the 5 "1-row acting" pairs of universal premises in $L^{(1)}$, by replacing one universal premise with its contradictory particular premise, and thus, cf. Fact #1, each such pair results in one precise subset being ≠Ø, and entails exactly one logical conclusion per pair, for a total of 27 valid syllogisms, (VS), 14 out of which - the classically valid syllogisms, (CVS), have names [even multiple names for one and the same syllogism, (or pair of premises), when the premises' terms can be switched around without changing the premises' meaning]. More precisely, the five $L^{(2)}$ pairs, (which were obtained from $L^{(1)}$'s five "1-row acting" universal pairs, by changing an universal P-premise into its contradictory, particular P-premise): $L_{41}=O_1A_1$, $L_{42}=O_1E$, $L_{51}=IA_1$, $L_{52}=IE$, $L_{63}=O_2A_2$, lead to, in order, the following (β) type, conclusions: SP'M≠Ø (or O(S,P), Bocardo), S'P'M ≠Ø (or I(S',P'), no name), SPM≠Ø (or I(S,P), Disamis/Dimaris), S'PM≠Ø (or O(P,S) no name), S'PM'≠Ø (or O(P,S) no name). For the last 5 out of 10, one substitutes the contradictory particular S-premise for the universal S-premise of the $L^{(1)}$'s five "1-row acting" universal pairs, to obtain: $L_{14}=A_1O_1$, $L_{24}=EO_1$, $L_{15}=A_1I$, $L_{25}=EI$, $L_{36}=A_2O_2$. The conclusions of these pairs are, in order: S'PM≠Ø (or O(P,S)), S'P'M ≠Ø (or I(S',P')), SPM≠Ø (or I(S,P), Darii/Datisi), SP'M≠Ø (or O(S,P), Ferio/Festino/Ferison/Fresison), SP'M'≠Ø (or O(S,P), Baroco). One can notice that A_1O_1 and IE have the same conclusion S'PM≠Ø, O_1A_1 and EI have the same conclusion SP'M≠Ø, IA_1 and A_1I have the same conclusion SPM≠Ø, O_1E and EO_1 have the same conclusion S'P'M ≠Ø (since on the M row there are only 4 subsets and one has 8 pairs of premises which place/lay at least one set element in exactly one subset of M).

4. Classes of equivalent syllogisms

The premises' action is easier to follow if we uniformly express any premise as either an E or I operator, acting firstly on M, or M', as the case may be. Consider for example the pairs: A_1A_1, O_1A_1, A_1O_1. Write:

A_1A_1=E(M,P')E(M,S')

O_1A_1=I(M,P')E(M,S')

A_1O_1=E(M,P')I(M,S'). All three pairs use the same variables M,P',S'. This is because, as was observed in Fact #1's proof, A_1A_1 acts twice on S'P'M, not at all on SPM, (we'll say that Darapti is bound not on the subset on which the premises' pair acts twice, but on SPM on which it doesn't act at all, and thus allows the conclusion M= SPM, out of which, via ei, the Darapti's conclusion follows. Equally important is that A_1A_1 acts once on SP'M, and once on S'PM, the subsets next to S'P'M on the "cylindrical Venn diagram", and these are exactly the subsets assured to be ≠Ø by O_1A_1, (Bocardo), and A_1O_1, respectively.

Let's now consider another similar group of 3 pairs of premises:

EE=E(M,P)E(M,S)

IE=I(M,P)E(M,S)

EI=E(M,P)I(M,S). All three pairs use the same variables M,P,S. This is because, as was observed in Fact #1's proof, EE acts twice on SPM, not at all on S'P'M, (we'll say that the no name EE:M=S'P'M is bound not on the subset on which the premises' pair acts twice, but on S'P'M on which the pair doesn't act at all, and thus allows the conclusion M= S'P'M, out of which, via ei, the no name I(S',P') conclusion follows. Equally important is that EE acts once on SP'M, and once on S'PM, and these are exactly the subsets assured to be ≠Ø by EI, (Ferio/Festino/Ferison/Fresison), and IE, respectively.

<u>Fact #2</u>: if we relabel P'→P, S'→S, then the first group of 3 pairs of premises is transformed in the 2nd group of 3 pairs of premises, and, the 3 conclusions from the 1st group of pairs, via this relabeling, become the 3 conclusions of the 2nd group of pairs. This happens because the subsets on which A_1A_1 acted twice, resp. not at all, are mapped into subsets on which EE acts twice, resp. not at all. The same is true about the subsets on which A_1A_1 acted once – they are transformed into subsets on which EE acts once. This way not only pairs of premises are mapped onto pairs of premises, but their conclusions are mapped into respective conclusions, too. There are 5 different groups of 3 pairs of premises each, and 4 relabeling transformations that map the first set of 3 pairs of premises to the other 4 and back to the 1st groups of 3 pairs of premises. One can argue that only one set of 3 pairs of premises is independent and the rest represent just what one would have gotten by a relabeling of the variables S,P,M. The final conclusion is that the 5 pairs of two universal premises acting on the same row, A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and all the other 10 pairs of premises, one universal and one particular, are equivalent, too. This is so because the two strains of 5 equivalent syllogisms each, which start with O_1A_1 and A_1O_1, and continue with IE and resp. EI, etc. are in fact equivalent, too: one can see this, for the above mentioned pairs, via a relabeling S↔P. Thus we have 10 pairs that generate equivalent syllogisms: O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2. The set of 4 "2-row acting" pairs of universal premises can be transformed, by relabeling, among themselves, too. Thus we found 3 different types of pairs of premises, easily characterized as being: 4 pairs of 2 universal premises acting on **two** rows, M **and** M', 5 pairs of 2 universal premises acting on **one** row, M **or** M', 10 pairs of one universal and one particular premises, acting on **one** row, M **or** M'. So one has 3 types of syllogisms' generating pairs, and the pairs of premises belonging to each type generate equivalent syllogisms.

Below one lists the syllogisms from two of the VS classes, grouped by the subset they do not act upon, and to which we say that they are "bound" to. These syllogisms use, (or act on), the complementary variables to the variables characterizing the subset these syllogisms are bound to.

1. Bound to the subset SPM:

A_1A_1=E(M,P')E(M,S') M=SPM. If M≠Ø: I(S,P), Darapti

O_1A_1=I(M,P')E(M,S') SP'M ≠Ø or O(S,P), Bocardo

A_1O_1=E(M,P')I(M,S') S'PM≠Ø or O(P,S), No name

2. Bound to the subset SP'M:

EA_1=E(M,P)E(M,S') M=SP'M. If M≠Ø: O(S,P), Felapton/Fesapo

EO_1=E(M,P)I(M,S') S'P'M≠Ø or I(S',P'), No name

IA_1=I(M,P)E(M,S') SPM≠Ø or I(S,P), Disamis/Dimaris

3. Bound to the subset S'P'M:

EE=E(M,P)E(M,S) M=S'P'M. If M≠Ø: I(S',P'), No name

IE=I(M,P)E(M,S) S'PM≠Ø or O(P,S), No name

EI=E(M,P)I(M,S) SP'M≠Ø or O(S,P), Ferio/Festino/Ferison/Fresison

4. (M' row) Bound to the subset S'P'M':

A_2A_2=E(M',P)E(M',S) M'=S'P'M'. If M'≠Ø: I(S',P'), No name

O_2A_2=I(M',P)E(M',S) S'PM'≠Ø or O(P,S), No name

A_2O_2=E(M',P)I(M',S) SP'M'≠Ø or O(S,P), Baroco

5. Bound to the subset S'PM:

A_1E=E(M,P')E(M,S) M=S'PM. If M≠Ø: O(P,S), No name

O_1E=I(M,P')E(M,S) S'P'M≠Ø or I(S',P'), No name

A_1I=E(M,P')I(M,S) SPM≠Ø or I(S,P), Darii/Datisi

One sees that the 5 groups of 3 VS each, [which include 7 distinct CVS, (two of them based on ei on M)], are, modulo a relabeling of S,P,M, equivalent.

One may verify the transitivity of the equivalences using the following relabeling maps:

1↔2: P'↔P

1↔3: S'↔S, P'↔P

1↔4: M↔M', S'↔S, P'↔P

1↔5: S'↔*S*

2↔3: S↔S'

2↔4: M↔M', S'↔S

2↔5: P'↔P, S↔S'

3↔4: M↔M'

3↔5: P↔P'

4↔5: M↔M', P'↔P

Because there are only 4 subsets per each row, (M or M'), when, by relabeling, one maps one "binding subset" into another "binding subset", one also map subsets on which the group of syllogisms, bound to the 1st

subset, do not act, act once, or act twice, into subsets on which the 2nd group of syllogisms, bound to the 2nd subset, do not act, act once, or act twice, respectively. This ensures that not only the pairs of premises of the 1st group of syllogisms transform into the pairs of premises of the 2nd group of syllogisms, but the conclusions from the 1st group of syllogisms, transform into the conclusions of the 2nd group of syllogisms.

Another way to show that the 5 groups of 3 syllogisms each are equivalent, is to start with 3 pairs of premises written in the variables A,B,C instead of the usual S,P,M:

Group 0. All B is A, All B is C

Some B is not A, All B is C

All B is A, Some B is not C

Choosing B=M, A=P, C=S we get the group 1 syllogisms' pairs of premises.

Choosing B=M, A=P, C=S' we get the group 2 syllogisms' pairs of premises.

Choosing B=M, A=P', C=S' we get the group 3 syllogisms' pairs of premises.

Choosing B=M', A=P', C=S', we get the group 4 syllogisms' pairs of premises.

Finally, choosing B=M, A=P', C=S we get the group 5 syllogisms' pairs of premises.

It is as if we represented Group 0, in 5 different system of coordinates: the number of distinct premise pairs, and syllogisms, is at most 3 not 15. We can further notice that the 5 syllogisms generated by "Some B is not A, All B is C", are equivalent to the 5 syllogisms generated by "All B is A, Some B is not C", via the relabeling A↔C.

This way one can see that the same generic wording of the premises can be represented in different ways, leading to different syllogisms, with different conclusions, but in fact the 5 groups are equivalent: the 5 syllogisms generated by the pairs of premises A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and the 10 syllogisms generated by the pairs of premises O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2 are equivalent, too.

The above equivalences show again that if a pair of premises entails a logical conclusion, it should be admitted as a valid syllogism, VS, even if that conclusion does not have the standard, classical "(S,P) format".

Note that M is not distributed in the VS A_2A_2: M'=S'P'M'→I(S',P'), (via ei on M'), and that A_2A_2 turns out to be equivalent to A_1A_1: M=SPM→I(S,P), (via ei on M, Darapti). Also, there are pairs of two negative premises in three of the VS - EE, O_1E, EO_1: EE generates a syllogism equivalent to Darapti, (or Felapton/Fesapo), and O_1E, EO_1 generate syllogisms equivalent to Darii. Thus there are pairs of premises that entail a logical conclusion but do not satisfy the usual "valid syllogisms rules", "the middle term has to be distributed in at least one premise", and, "no valid syllogism has 2 negative premises". One can start with the premises of Darapti and Darii, (i.e., A_1A_1, and resp., A_1I), re-write them using obversion and contraposition as the premises A_2A_2, (resp. O_1E), written in other variables, get the conclusions of A_2A_2, (resp. O_1E), in those variables, then realize that those conclusions can be re-written, (via appropriate "back relabelings"), as the usual Darapti, M=SPM, and Darii, SPM≠Ø, conclusions. This way one can use VS which do not satisfy the usual "rules of valid syllogisms" to "bear the burden" of inferring all the conclusions of the CVS from the two VS classes which contain Darapti and resp. Darii.

The "2-row acting" syllogisms:

EA_2=E(M,P)E(M',S) SP'M, S'PM'="survive" as the only subsets of S, resp. P, which are not emptied by the premises EA_2. Thus: A(S,SP'M), A(P,S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), and, via ei on

S, O(S,P), (Celaront/Cesaro).

A_1A_2=E(M,P')E(M',S) S=SPM, P'=S'P'M', A(S,P) Barbara, I(S,P) Barbari (S≠Ø), I(S',P') no name (P'≠Ø)

A_2A_1=E(M',P)E(M,S') P=SPM, S'=S'P'M', A(P,S) no name, I(S,P) Bramantip (P≠Ø), I(S',P') no name (S'≠Ø)

A_2E=E(M',P)E(M,S) S=SP'M', P= S'PM. Thus: A(S,SP'M'), A(P,S'PM). One chooses, as independent conclusion E(S,P)(=A(S,P')=A(P, S')), (Camestres/Camenes). And, via ei on P, the no name O(P,S), plus, via ei on S, O(S,P), (Camestrop/Camenop)

The S,P,M relabeling transformations showing that A_1A_2, A_2A_1, A_2E, EA_2 are equivalent:

A_1A_2↔A_2A_1: S↔P

A_1A_2↔A_2E: P↔P', M↔M'

A_1A_2↔EA_2: P↔P'

A_2A_1↔A_2E: P↔P', S↔S'

A_2A_1↔EA_2: M↔M', P↔P', S↔S'

A_2E↔EA_2: M↔M'

Or, one can start with the "generic" pair of premises All B is A, All C is B.

Then, making the obvious choice B=M, A=P, C=S, we get A_1A_2, Barbara's premises.

But choosing B=M, A=P', C=S, we get the EA_2 premises.

And choosing B=M', A=P', C=S, we get the A_2E premises.

Finally choosing B=M, A=S, C=P, we get the A_2A_1 premises.

Thus, no matter what their initial wording is, for any pair of concrete categorical premises presented to us, one can label their 3 terms in such a way, that if the pair entails a logical conclusion, then it can be expressed as either A_1A_2, or A_1A_1, or A_1I, (or any other preferred triplet of representatives from each one of the 3 classes of premises that entail logical conclusions). After the logical conclusion of A_1A_2, or A_1A_1, or A_1I, is written down, one can do a "back relabeling" to re-express the conclusion via the most intuitive term labeling suggested by the initial premises.

5. Conclusion

Instead of the old accounting rules and restrictions imposed on the (classically) valid syllogisms – an (S,P) conclusion, the "syllogistic figures", "In any valid syllogism the middle term is distributed at least once", "No valid syllogism has two negative premises", etc., the **Venn diagram**, (cylindrical or not, but on the usual "3 intersecting circles" Venn diagram, the above facts and interpretations are difficult to see), **approach**, allows simpler rules:

1. The 36 pairs of categorical premises fall into 5 classes: 3 classes entail a logical conclusion and 2 do not.
2. Each logical conclusion is either of type **(α)** or of type **(β)** above, and refers to just one subset, out of the 8 subsets of U.
3. Inside each of the 3 classes of premise pairs entailing a logical conclusion, the pairs and their entailed syllogisms are all equivalent in the sense described above.
4. One may offer two, or even five, "new rules of valid syllogisms". Two negative rules: 1. No two particular premises are allowed (this coincides with one of the old rules). 2. A universal premise and a particular premise, one acting on the middle term M and the other acting on its complementary set M'

are not allowed. (Note that the “old rules of valid syllogisms” were in fact meant to invalidate all but the CVS.) Three positive rules - the rest of the pairs of premises are allowed: two universal premises acting on the “same row” (either M or M'); two universal premises acting on “two rows” (both M and M'); a universal premise and a particular premise acting on the same row (either M or M').

5. As described in Section 3, the logical consequences of the 19 out of 36 possible pairs of premises are as follows: the “(S,P) conclusions” A(S,P), E(S,P), I(S,P), O(S,P) – which are satisfied only by the CVS; A(P,S) entailed only by A_2A_1; I(S',P') and O(P,S). The latter conclusions are entailed by pairs of premises which, via ei or not, generate VS which are not CVS (VS\CVS). If one could logically argue that these I(S',P'), O(P,S), A(P,S) conclusions are not to be admitted, even if logically entailed by the VS\CVS pairs of premises, then, indeed, only the CVS are valid. As most of the logic textbooks do, one can restrict the valid syllogisms, by definition, to only the pairs of premises whose entailed consequences are of the “(S,P) type”; or one can use notions like distribution to help eliminate any pair of premises which does not generate a CVS. I do not see a logical motivation for the (S,P) conclusion restriction, neither for the distribution notion.

Because of its lack of symmetry, the usual “3 intersecting circles Venn diagram” model, was used only to “verify” particular syllogisms' validity, but, as far as I know, never to exhaust the conclusions of all the categorical pairs of premises. See, e.g., Barker (2003). See also, Quine (1982), who proposed as “an hour's pastime” exercise, the Venn diagram checking of all premises' pairs for conclusion entailment.

As a note added after this paper was initially written, let me mention that the “cylindrical Venn diagram” (CVD) is in fact a Karnaugh(-Veitch) map for 3 sets. The “cylinder idea” is used to closer match the adjacency displayed by the subsets of the “3-circle Venn diagram”. For the same adjacency reason a Karnaugh map for 4 sets is represented as a 4 by 4 square with “glued edges” - which thus becomes a torus. (See Marquand (1881), Veitch (1952), Karnaugh (1953), (Wikipedia.org/wiki/Karnaugh_map.)

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