

# An old-new presentation of categorical syllogisms

To the memory of my sister Cristina Popa, 1948-2018.

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## **Abstract**

*One recognizes five types of pairs of categorical premises (PCPs) – only three out of the five types entail logical conclusions (LCs) and thus generate valid categorical arguments (VCAs), some of which are the usual valid syllogisms (VSs). (George Boole and Lewis Carroll obtained these results.) One argues, (using set relabelings), that all VCAs of the same type are equivalent. (This result was stressed out by L. Carroll in his Symbolic Logic book.) As a consequence, Barbara, Darii and Darapti may be chosen as the representatives of the three types of VCAs. One generalizes the three PCP types to sorites that entail an LC. (Again, L. Carroll dealt with such generalizations.) All the above results are obtained on a set model for the terms appearing in the PCPs or sorites. In the case of  $n-1$  premises containing  $n$  terms, the modeling universal set  $U$  is partitioned in  $2^n$  subsets, and any LC pinpoints to just one of these  $2^n$  subsets. Thus, in particular, for categorical syllogisms,  $n=3$ , (where the three syllogistic terms are denoted  $S,P,M$ , and their complementary sets in  $U$  are  $S',P',M'$ ), the middle term  $M$ , (or  $M'$ ), is very much part of the LC, and eliminating the middle term from the LC weakens the conclusion which will then refer to two of the 8 subsets partitioning  $U$ . To make  $M$  and  $M'$  appear in a symmetric way in the PCPs, one extends the number of PCPs to 64 from the usual 36 - where the  $A,E,O,I$  quantifiers are applied only to the terms  $S,P,M$ . Consequently, the eight  $M-P$  premises form a cube of opposition, and the eight  $M-S$  premises form another cube of opposition.*

**Keywords:** *categorical syllogisms • categorical premises • Karnaugh map • valid syllogistic argument • valid syllogism • set relabelings*

## **1. Introduction**

The syllogistic terms  $S,P,M$ , non- $S$  ( $S'$ ), non- $P$  ( $P'$ ), non- $M$  ( $M'$ ) are modeled as sets, and any universal, (resp. particular), premise empties, (resp. lays set elements into), two subsets out of the 8-subset partition of a universal set  $U$ :  $U=MSP+MS'P+MSP'+MS'P'+M'SP+M'S'P+M'S'P'+M'S'P'$ , (where the union of disjoint sets is denoted by a plus sign,  $S',P',M'$  are the complementary sets in  $U$  of the  $S,P,M$  terms respectively, and  $MSP:=M\cap S\cap P$ , etc.) By definition a categorical syllogism is

made of a PCP to which one tacks a 3rd statement, an (S,P)-conclusion, i.e., one of the categorical operators, (or quantifiers), A,O,E,I applied to the ordered pair (S,P). If the conclusion is truly entailed by the PCP one has a VS, otherwise the syllogism is invalid. There are 36 distinct PCPs expressed via only the S,P,M terms and the categorical operators A,O,E,I applied to pairs of these 3 terms. For example four of the six M-S premises are  $A(M,S)=\text{All } M \text{ is } S$ , meaning  $MS'=\emptyset$ ,  $O(M,S)=\text{Some } M \text{ is not } S$ , meaning  $MS'\neq\emptyset$ ,  $E(M,S)=\text{No } M \text{ is } S$ , meaning  $MS=\emptyset$ ,  $I(M,S)=\text{Some } M \text{ is } S$ , meaning  $MS\neq\emptyset$ . One can say that these 4 premises are “acting on M”. Interestingly, the next two premises are “acting on M'”:  $A(S,M)=\text{All } S \text{ is } M$ , meaning  $SM'=\emptyset$ , and  $O(S,M)=\text{Some } S \text{ is not } M$ , meaning  $SM'\neq\emptyset$ . One sees that  $A(S,M)$  and  $O(S,M)$  act on M' exactly how  $E(M,S)$  and  $I(M,S)$  act on M. Indeed, by obversion  $A(S,M)=E(M',S)$ , (one may use the shorthand E' for  $E(M',S)$ , see below), and  $O(S,M)=I(M',S)$ , (one may use the shorthand I'). If one wants symmetry between the M and M' “actions”, one has to introduce two more S-premises:  $A(M',S)=E(M',S')$ , (one may use the shorthand A'), and  $O(M',S)=I(M',S')$ , (one may use the shorthand O'), for a total of eight M-S premises which may be grouped in two squares of opposition, one acting on M,  $A(M,S)$ ,  $E(M,S)$ ,  $I(M,S)$ ,  $O(M,S)$ , (shorthand A,E,I,O), and the other one acting on M',  $A(M',S)$ ,  $E(M',S)$ ,  $I(M',S)$ ,  $O(M',S)$ , (shorthand A',E',I',O'). The two squares together make a cube of opposition, with universal premises on the top face of the cube, and the particular premises on the bottom face. The M-P premises are treated analogously, and by always listing the P-premise first, one can understand that a PCP such as AE' means  $A(M,P)E(M',S)=A(M,P)A(S,M)$  which are Barbara's premises. We'll use this extended set of 64 PCPs to distinguish, as George Boole and Lewis Carroll did before, only three types of VCAs, some of which are the usual VSs. Via set relabelings, which are similar to syllogism reduction, we'll show that within each type, all the VCAs are equivalent.

## 2. The Karnaugh map for n=3 (drawn for subsets instead of truth values) and the five types of PCPs

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

*Fig. 1*

The universal set U is graphed as a rectangle – but the left and right borders of the rectangle are glued together to generate a cylinder, so that S'PM and S'PM are adjacent, and S'PM' and S'PM' are adjacent, too – as in the usual 3-circle Venn diagram. Since the 8 subsets of Figure 1 are the “elementary” subsets of U, one calls them just subsets; no other set will be a “subset”. On this Karnaugh map one can see that any universal premise empties two “horizontal subsets” located either on the M or the M' row, and its contradictory particular premise places set elements in at least one of the same two “horizontal subsets” emptied by the universal

premise. For example, Barbara's PCP,  $A(M,P)A(S,M)$ , contains two universal premises, acting one on  $M$  and the other on  $M'$ . This is the first type of PCP (according to both Boole and Carroll). The premises mean  $MP' = \emptyset$ ,  $SM' = \emptyset$ , i.e., 4 subsets are emptied on Fig. 1, thus reducing  $S$  to  $S = SPM$  and  $P'$  to  $P' = S'P'M'$  – which, after eliminating  $M$ , translate to the LCs  $A(S,P)$  and resp.  $A(P',S')$  – the latter being equal, via contraposition, to  $A(S,P)$ . Using existential import (ei) on  $S$ , and resp.  $P'$  one gets the ei LC  $I(S,P)$  – Barbari, and the no name ei LC  $I(S',P')$ . The same results are easily found via a “tree like method” which eliminates, (i.e., closes), any subset (i.e., branch), emptied by a universal premise:  $S = SM + SM' = SM = SPM + SP'M = SPM$  and  $P' = P'M + P'M' = P'M' = S'P'M' + S'P'M' = S'P'M'$ . This amounts - for all 8 PCPs containing two universal premises acting one on  $M$  and the other on  $M'$  - to the general rule of starting two trees, one for each letter other than the middle terms  $M$  and  $M'$ . Note that after finding the LC in either the graphical or tree like way, the middle term elimination consists in “just do not mention the middle term” – and by so doing, the usual LC wording refers to a column of the Karnaugh map instead of only one subset of it. (The above “tree method” is similar with Carroll's “Method of subscripts”; he solved sorites either by the subscripts' method or by a “tree method”.)

The second type of PCP that entails an LC contains 16 PCPs, with one universal premise plus one particular premise, both acting on the same row (either  $M$  or  $M'$ ). Darii's PCP,  $A(M,P)I(M,S)$ , is an example. The LC is  $SPM \neq \emptyset$ , and results either from Fig. 1, since  $I(M,S)$ ,  $MS \neq \emptyset$ , places set elements on either  $SP'M$  or/and  $SPM$ , but  $A(M,P)$ ,  $MP' = \emptyset$ , by emptying  $S'P'M$  and  $SP'M$  “forces”  $I(M,S)$  to definitely place its element(s) only on  $SPM$ . Or, one may start a (very short) tree with the non-empty set specified by the particular premise  $MS = MSP + MSP' = SPM \neq \emptyset$ .

The third type of PCP that entails an LC, contains 8 PCPs having two universal premises acting on the same row. Darapti's PCP,  $A(M,P)A(M,S)$ , meaning  $MP' = \emptyset$  and  $MS' = \emptyset$ , is an example of such a PCP. From Fig. 1 it is clear that the LC is  $M = SPM$ , which, via ei on  $M$ , produces the ei LC  $I(S,P)$ . Alternatively, one should start the tree with  $M = MP + MP' = MP = MPS + MPS' = SPM$ . Note that, Boole 1847, on pages 35 to 41, introduces and discusses four classes of PCPs and their LCs. The first three of George Boole's and Lewis Carroll's types, (Carroll, 1977, p.126, Table IX.), can be viewed as the same as the PCP types described above – whose representatives respectively may be chosen Barbara, Darii and Darapti. Boole divided his fourth class into subclasses of PCPs – these subclasses do not entail any LCs and are equivalent with the the following types 4 and 5 PCPs. Type 4: has 16 PCPs containing two particular premises which do not entail any LCs. Type 5: no LCs may be drawn either, from the 16 PCPs containing one universal premise plus one particular premise, acting one on  $M$  and the other one acting on  $M'$ , or vice versa. These are the only five types of PCPs possible. They have been figured out, by and large, by George Boole since 1847. Carroll uses the types 4 and 5 PCPs as examples of fallacies. Carroll, and also Boole 1847, pp.34-35, embrace the existence of VCAs: “The Aristotelian canons, however, beside restricting the *order* of the terms of a conclusion, limit their nature also;—and this limitation is of more consequence than the former. We may, by a change of figure, replace the particular conclusion of *bramantip*, by the general conclusion of *barbara*; but we cannot thus

reduce to rule such inferences,” (aka LCs) “as Some not-Xs are not Ys. Yet there are cases in which such inferences may lawfully be drawn, and in unrestricted argument they are of frequent occurrence. Now if an inference of this, or of any other kind, is lawful in itself, it will be exhibited in the results of our method.” Then, on pages 35 to 41, using equations to eliminate the middle term, Boole notices and discusses his 4 classes of PCPs and their LCs or lack thereof. Here are his conclusions on pp. 39-40: “The lawful cases of the first class comprehend all those in which, from two universal premises, a universal conclusion may be drawn. We see that they include the premises of *barbara* and *celarent* in the first figure, of *cesare* and *camestres* in the second, and of *bramantip* and *camenes* in the fourth. The premises of *bramantip* are included, because they admit of an universal conclusion, although not in the same figure. The lawful cases of the second class are those in which a particular conclusion only is deducible from two universal premises. The lawful cases of the third class are those in which a conclusion is deducible from two premises, one of which is universal and the other particular. The fourth class has no lawful cases.” Carroll has no use for the moods and figures of valid syllogisms: “The writers, and editors, of Logical text-books which run in the ordinary grooves – to whom I refer by the (I hope inoffensive) title “The Logicians”, “elaborately discussed no less than nineteen different forms of Syllogisms – each with its own special and exasperating Rules, while the whole constitutes and almost useless machine, for practical purposes”. And, in the posthumously published Part II of his Symbolic Logic book: “As to syllogisms, I find that their 19 forms, with about a score of others which they have ignored, can all be arranged under three forms, each with a very simple Rule of its own”. In conclusion, the VCA are partitioned into three classes, each class being generated by the PCPs from the types 1, 2, and 3 above. In the next section one will show that inside each of the three VCA types, any VCA may be recast or reformulated, via a relabeling transformation of the sets S,P,M, into, respectively, the sets S',P',M', as any other VCA of the same type, which makes all VCAs equivalent with one of the three representatives chosen one per VCA type. For example Barbara, Darii and Darapti, may be chosen as representatives. In particular, the VCA\VS set whose LCs have one of the formats A(P,S), O(P,S) or I(S',P') may be recast, via a set relabeling as VSs.

### 3. VCAs EQUIVALENCIES VIA SET RELABELINGS

One way of seeing that the VCAs of the same type are all equivalent, is to write the eight VCAs of type 1 as  $E(S^*, M^*)E(M^*, P^*)$ , where the \* stands for complement or no complement, (i.e., ' or nothing). Thus  $S^* \in \{S, S'\}$ ,  $M^* \in \{M, M'\}$ ,  $P^* \in \{P, P'\}$ , the eight VCAs of type 3 as  $E(S^*, M^*)E(M^*, P^*)$ , and the 16 VCAs of type 2 as  $I(S^*, M^*)E(M^*, P^*)$  and  $E(S^*, M^*)I(M^*, P^*)$ . (One can notice that, e.g.,  $I(S, M)E(M, P')$ , Darii/Datisi, and  $E(S', M)I(M, P)$ , Disamis/Dimaris, have the same LC:  $SMP \neq \emptyset$ .) Since it is arbitrary which set one denotes by M and which one is called M', and similarly which one is called S or S', and resp., which one is called P or P', it follows that all VCAs of the same type are equivalent.

Another, more detailed way of seeing that the VCAs of the same type are all equivalent, is to group the 64 PCPs which entail at least one LC and thus generate VCAs, into 8 sets, numbered from 1 to 8, made of 4 PCPs per set (and generating 6 VCAs per set – including the ei VCAs). Below, the first PCP in such a set of four PCPs always belongs to type 3 PCP, the following two belong to type 2 PCP and the last one to type 1 PCP. We'll say that each of such a four PCPs set is “bound to” a same subset of U: the four PCPs do not act at all on the subset of U on which they are all “bound”, but act on some of its “neighbours” in the Karnaugh map. One may use the shorthand notation for PCPs with the convention that one lists first the P-premise and then the S-premise. Thus to each of the 8 subsets of U one “attaches” a set of four PCPs “bound” to it, and on one column one lists the four PCPs, and on the second column their LCs:

1. VCAs “bound to” the subset S'P'M:

EE=E(M,P)E(M,S)	M=S'P'M. If M≠∅: I(S',P'), No name
IE=I(M,P)E(M,S)	S'PM≠∅ or O(P,S), No name
EI=E(M,P)I(M,S)	SP'M≠∅ or O(S,P), Ferio/Festino/Ferison/Fresison
EE'=E(M,P)E(M',S)	S=SP'M, P=S'PM', E(S,P), Celarent/Cesare

O(S,P) if S≠∅, Celaront/Cesaro; O(P,S) if P≠∅, No name

2. VCAs bound to the subset SP'M:

EA=E(M,P)E(M,S')	M=SP'M. If M≠∅: O(S,P), Felapton/Fesapo
IA=I(M,P)E(M,S')	SPM≠∅ or I(S,P), Disamis/Dimaris
EO=E(M,P)I(M,S')	S'PM≠∅ or I(S',P'), No name
EA'=E(M,P)E(M',S')	P=SPM', S'=S'P'M, A(P,S)=A(S',P'),

I(S,P) if P≠∅, Bramantip' (the prime refers to M' in P=SPM'); I(S',P') if S'≠∅, No name

3. VCAs bound to the subset S'PM:

AE=E(M,P')E(M,S)	M=S'PM. If M≠∅: O(P,S), No name
OE=I(M,P')E(M,S)	S'PM≠∅ or I(S',P'), No name
AI=E(M,P')I(M,S)	SPM≠∅ or I(S,P), Darii/Datissi
AE'=E(M,P')E(M',S)	S=SPM, P'=S'P'M', A(S,P), Barbara

I(S,P) if S≠∅, Barbari; I(S',P') if P'≠∅, No name

4. VCAs bound to the subset SPM:

AA=E(M,P')E(M,S')	M=SPM. If M≠∅: I(S,P), Darapti
OA=I(M,P')E(M,S')	SP'M ≠∅ or O(S,P), Bocardo
AO=E(M,P')I(M,S')	S'PM≠∅ or O(P,S), No name
AA'=E(M,P')E(M',S')	S'=S'PM, P'=SP'M', E(S',P'), No name

O(P,S) if S'≠∅, No name; O(S,P) if P'≠∅, No name

M' row VCAs:

5. VCAs bound to the subset S'P'M':

E'E'=E(M',P)E(M',S)	M'=S'P'M'. If M'≠∅: I(S',P'), No name
I'E'=I(M',P)E(M',S)	S'PM'≠∅ or O(P,S), No name
E'I'=E(M',P)I(M',S)	SP'M'≠∅ or O(S,P), Baroco

$E'E=E(M',P)E(M,S)$                        $S=SP'M', P=S'PM, E(S,P),$  *Camestres/Camenes*  
 $O(S,P)$  if  $S \neq \emptyset$ , *Camestros/Camenos*;  $O(P,S)$  if  $P \neq \emptyset$ , *No name*

6. VCAs bound to the subset  $SP'M'$ :

$E'A'=E(M',P)E(M',S')$                        $M'=SP'M'$ . If  $M' \neq \emptyset$ :  $O(S,P)$ , *Felapton'/Fesapo'*  
 $I'A'=I(M',P)E(M',S')$                        $SPM' \neq \emptyset$  or  $I(S,P)$ , *Disamis'/Dimaris'*  
 $E'O'=E(M',P)I(M',S')$                        $S'P'M' \neq \emptyset$  or  $I(S',P')$ , *No name*  
 $E'A=E(M',P)E(M,S')$                        $S'=S'P'M', P=SPM, E(S',P)=A(P,S),$  *No name*  
 $I(S,P)$  if  $P \neq \emptyset$ , *Bramantip*,  $I(S',P')$  if  $S' \neq \emptyset$ , *No name*

7. VCAs bound to the subset  $S'PM'$ :

$A'E'=E(M',P')E(M',S)$                        $M'=S'PM'$ . If  $M' \neq \emptyset$ :  $O(P,S)$ , *No name*  
 $O'E'=I(M',P')E(M',S)$                        $S'P'M' \neq \emptyset$  or  $I(S',P')$ , *No name*  
 $A' I'=E(M',P')I(M',S)$                        $SPM' \neq \emptyset$  or  $I(S,P)$ , *Darii'/Datisi'*  
 $A'E=E(M',P')E(M,S)$                        $S=SPM', P'=S'P'M, A(S,P)=A(P',S'),$  *Barbara'*  
 $I(S,P)$  if  $S \neq \emptyset$ , *Barbari'*;  $I(S',P')$  if  $P' \neq \emptyset$ , *No name*

8. VCAs bound to the subset  $SPM'$ :

$A'A'=E(M',P')E(M',S')$                        $M'=SPM'$ . If  $M' \neq \emptyset$ :  $I(S,P)$ , *Darapti'*  
 $O'A'=I(M',P')E(M',S')$                        $SP'M' \neq \emptyset$  or  $O(S,P)$ , *Bocardo'*  
 $A'O'=E(M',P')I(M',S')$                        $S'PM' \neq \emptyset$  or  $O(P,S)$ , *No name*  
 $A'A=E(M',P')E(M,S')$                        $S'=SPM', P'=SP'M, E(S',P'),$  *No name*  
 $O(P,S)$  if  $S' \neq \emptyset$ , *No name*;  $O(S,P)$  if  $P' \neq \emptyset$ , *No name*

One can now define a “relabeling group” acting on the above VCAs sets 1,2,..., 8. Let  $p:= P \leftrightarrow P'$ ,  $s:=S \leftrightarrow S'$ ,  $m:=M \leftrightarrow M'$ . One can see that compositions of  $s,p,m$  generate a commutative group  $G$  with eight distinct elements:  $1,s,p,m,sp,sm,pm,spm$ . Obviously  $1=s^2=p^2=m^2=(spm)^2=(ms)^2=(ps)^2=(pm)^2$ . This group acts on the above VCAs sets 1,2,...,8, as follows:

$p(1)=3, p(2)=4, p(5)=7, p(6)=8; s(1)=2, s(3)=4, s(5)=6, s(7)=8; m(1)=5, m(2)=6, m(3)=7, m(4)=8$ . (The  $m$  relabeling transforms *Barbara* into *Barbara'* and vice-versa, *Darapti* into *Darapti'* and vice-versa, etc.)

One can check that  $\{G(1)\} = \{G(2)\} = \dots = \{G(8)\} = \{1,2,3,\dots,8\}$ . This shows that any VCA from any of the three VCA types can be recast as any other VCA of the same type. For example,  $spm(E'E)=AA=A(M,P) A(M,S)=E(M,P)E(M,S')$  which are *Darapti'*'s premises. This means that the premises  $E'E=E(M',P)E(M',S)=A(P,M) A(S,M)$ , become *Darapti'*'s premises after an  $spm$  relabeling.

#### 4. Categorical syllogisms' squares and cubes of opposition

In general, one may say that the purpose of listing all immediate inferences between the  $A,O,E,I$  statements from the opposition square is to know how to optimally use more than one of these statements as premises: for example one shouldn't try to use both  $A$  and  $O$  when they contradict each other (which is always), and it is not ideal or economical to list both  $A$  and  $I$  as premises if  $A$  already implies  $I$  (which happens,

e.g., if both terms appearing in A are non-empty). The statements usually listed at the vertices of the square of opposition are A(S,P), E(S,P), I(S,P), O(S,P), **Beziau** (2012, 2017). In an enlarged,  $M \leftrightarrow M'$  symmetrical version, the categorical syllogisms use eight P-premises A(M,P), E(M,P), I(M,P), O(M,P), A(M',P), E(M',P), I(M',P), O(M',P), and eight S-premises, A(M,S), E(M,S), I(M,S), O(M,S), A(M',S), E(M',S), I(M',S), O(M',S), (which will both be shortened to A,E,I,O,A',E',I',O'). One sees that the P-premises, (resp. S-premises), form two squares of opposition, one acting on M and the other one acting on M'. Equivalently, one may replace the eight P-premises, (resp. S-premises), by a cube of opposition, the front face of the cube being the square of opposition acting on M, and the back face of the cube being the square of opposition acting on M'. The top, (resp. bottom), face of each cube contains the universal, (resp. particular) premises. As in the previous section, we'll consider only the modern squares and cubes of opposition – the Aristotelian ones having the unwanted feature that, e.g., the contradictory, (aka the negation), of A(M,P) &  $M \neq \emptyset$ , (where  $M \neq \emptyset$  adds from the start an existential import (ei) extra premise about M), is not O(M,P) but  $[O(M,P) \vee M = \emptyset]$  – see Westerstahl 2009. Thus in the modern square of opposition A(M,P), E(M,P) are not contraries any more (unless one adds  $M \neq \emptyset$ !); instead when both are true it results that  $M = \emptyset$  (but not that  $P = \emptyset$ !). Analogously, when E(M,P) and E(M',P) are both true then  $P = \emptyset$ ; when A(M',P) and E(M',P) are both true then  $M' = \emptyset$ ; when A(M,P) and A(M',P) are both true then  $P' = \emptyset$ . One may replace P by S in the previous phrase and obtain, respectively, conditions for M, S, M' and S' being empty sets. This shows that on each edge of the top faces of the premises' two cubes of oppositions one has two universal premises which when both true amount to an empty set constraint (ESC). Vice-versa, when  $M = \emptyset$ , then both A(M,P) and E(M,P) are true, (since the empty set is included in every other set, and its intersection with any other set is empty, too), and their respective contradictory statements, O(M,P), I(M,P), are both false. E(M,P) means  $M \subseteq P'$ , or  $P \subseteq M'$ , or, equivalently,  $MP := M \cap P = \emptyset$ . The classical square of opposition imposes the condition that both M and P be non empty, and this validates all the square's well known immediate implications, Beziau (2012, 2017). By contrast, in the “modern square of opposition” one does not commit to  $M \neq \emptyset$  and/or  $P \neq \emptyset$  and thus one knows only that A and O, (resp. E and I), are contradictory statements. Also if A(M,P) is true, then  $P = \emptyset$  implies  $M = \emptyset$ , but not vice-versa.

Note that one can work with a hexagon of opposition, Beziau (2012, 2017), by adding to the square the statements  $Y := I \wedge O$ , and  $W := A \vee E$ . When M is empty, then A,E,W are true and I,O,Y are false. When the classical hexagon of opposition applies, i.e.,  $M \neq \emptyset$  and  $P \neq \emptyset$ , then, A being true implies that I and W are also true, while E,O,Y are all false. Vice-versa, if E is true, then O and W are also true, while A,I,Y are all false. Both cases mean that existential import is warranted since A implies I and E implies O. Starting with any particular statement in  $\{Y,I,O\}$ , being true, just implies that its respective contradictory statement from  $\{W,E,A\}$  is false, while the sets appearing both in the particular statement, and in its contradictory universal statement, are non empty.

From a syllogistic perspective the immediate inferences available in a cube of

opposition do not matter much since any syllogism or poly-syllogism (or sorite) chooses just one statement from the eight available.

## 5. Sorites

Using the same notations as those from the beginning of Section 3, the three types of PCPs which generate VCAs may be immediately extended to sorites of the same type:

Type 1. The premises of the “Barbara type” Aristotelian/Goclenian sorite:

$$\text{SOR1} := E(S^*, M_1^*)E(M_1^*, M_2^*)E(M_2^*, M_3^*) \dots E(M_n^*, P^*) = A(S^*, M_1^*)A(M_1^*, M_2^*)A(M_2^*, M_3^*) \dots A(M_n^*, P^*)$$

LC<sub>1</sub>:  $S^* = S^*M_1^*M_2^*M_3^* \dots M_n^*P^*$  (the Aristotelian logical conclusion)

LC<sub>2</sub>:  $P^* = P^*M_n^* \dots M_3^*M_2^*M_1^*S^* = P^*M_1^*M_2^*M_3^* \dots M_n^*S^*$  (the Goclenian logical conclusion of the same premises. This explains why, e.g., Barbara has two different LCs.)

Type 2. The premises of the “Darii type” sorite are:

$$\text{SOR2} := E(S^*, M_x^*)E(M_1^*, M_y^*)E(M_2^*, M_y^*) \dots I(M_x^*, M_y^*) \dots E(M_i^*, M_y^*)E(M_{i+1}^*, M_x^*) \dots E(M_n^*, M_y^*)E(M_x^*, P^*)$$

LC:  $M_x^*M_y^*S^*M_1^*M_2^*M_3^* \dots M_i^*M_{i+1}^* \dots M_n^*P^* \neq \emptyset$

Type 3. The premises of the “Darapti type” sorite are:

$$\text{SOR3} := E(S^*, M^*)E(M_1^*, M^*)E(M_2^*, M^*) \dots E(M_i^*, M^*)E(M_{i+1}^*, M^*) \dots E(M_n^*, M^*)E(M^*, P^*)$$

LC:  $M^* = M^*S^*M_1^*M_2^*M_3^* \dots M_i^*M_{i+1}^* \dots M_n^*P^*$  with, if  $M^* \neq \emptyset$ , some of the many ei LCs being  $S^*M_1^*M_2^*M_3^* \dots M_i^*M_{i+1}^* \dots M_n^*P^* \neq \emptyset$ ,  $I(S^*, P^*)$ , etc.

Note that each of the three types sorites admits “Darapti decorations”, i.e., a Darapti type sequence of premises may be added using any term/letter as the “base” - exactly as the term (or letter) M was used as the “base” of SOR3 - the Darapti type 3 sorite above.

For example, if to SOR1 one adds Darapti decorations of length=1 on  $S^*$ ,  $M_1^*$ , and  $P^*$ , respectively, and a Darapti decoration of length=3 on  $M_2^*$ , i.e., if one adds these premises:

$$E(S^*, M_{n+1}^*)E(M_1^*, M_{n+2}^*)E(M_2^*, M_{n+3}^*)E(M_2^*, M_{n+4}^*)E(M_2^*, M_{n+5}^*)E(P^*, M_{n+6}^*),$$

then one obtains a sorite which has just one LC not two. This LC is an “extension” of LC<sub>1</sub>:

$$S^* = S^*M_1^*M_2^*M_3^* \dots M_n^*P^*M_{n+1}^*M_{n+2}^*M_{n+3}^*M_{n+4}^*M_{n+5}^*M_{n+6}^*.$$

For the Goclenian reading of SOR1 one needs to add, as Darapti decorations of the same lengths as above, e.g., these premises:

$$E(S^*, M_{n+1}^*)E(M_1^*, M_{n+2}^*)E(M_2^*, M_{n+3}^*)E(M_2^*, M_{n+4}^*)E(M_2^*, M_{n+5}^*)E(P^*, M_{n+6}^*),$$

and then, again, one obtains a sorite which has just one LC not two. This LC is an “extension” of LC<sub>2</sub>:

$$P^* = P^*M_1^*M_2^*M_3^* \dots M_n^*S^*M_{n+1}^*M_{n+2}^*M_{n+3}^*M_{n+4}^*M_{n+5}^*M_{n+6}^*.$$

Analogously, Darapti decorations of the same lengths as above may be added to SOR2 via adding these premises:

$E(S^*, M_{n+1}^*)E(M_1^*, M_{n+2}^*)E(M_2^*, M_{n+3}^*)E(M_2^*, M_{n+4}^*)E(M_2^*, M_{n+5}^*)E(P^*, M_{n+6}^*)$ . Then one obtains a sorite whose LC is an “extension” of the SOR2 LC:  
 $M_x^*M_y^*S^*M_1^*M_2^*M_3^* \dots M_i^*M_{i+1}^* \dots M_n^*P^*M_{n+1}^*M_{n+2}^*M_{n+3}^*M_{n+4}^*M_{n+5}^*M_{n+6}^*$ .

One may add Darapti decorations to a Darapti type sorite, SOR3, via adding, e.g., these premises:

$E(S^*, M_{n+1}^*)E(M_1^*, M_{n+2}^*)E(M_2^*, M_{n+3}^*)E(M_2^*, M_{n+4}^*)E(M_2^*, M_{n+5}^*)E(P^*, M_{n+6}^*)$ . Then one obtains a sorite whose LC is an “extension” of the above SOR3 LC:

$M^*=M^*S^*M_1^*M_2^*M_3^* \dots M_i^*M_{i+1}^* \dots M_n^*P^*M_{n+1}^*M_{n+2}^*M_{n+3}^*M_{n+4}^*M_{n+5}^*M_{n+6}^*$

One thus sees that to any term/letter appearing in the LC of a sorite of types 1, 2 or 3, one may add a “Darapti decoration”, i.e., a (no matter how long) Darapti type sequence of premises with that letter as the “base”, exactly as the term (or letter) M was the “base” of SOR3 - the Darapti type 3 sorite listed above.

If represented on a subsets' Karnaugh map with enough variables, the LCs of the above SOR1 and, resp., SOR3 say that  $S^*$ ,  $P^*$  and, resp.,  $M^*$  were reduced by the universal premises to just one, possibly not empty, partition subset of the universal set U. The LC of SOR2 affirms about just one partition subset of the universal set U that it is not empty. (How to construct Karnaugh maps for any number of variables, starting from an n=2 Karnaugh map and then repeatedly using mirror images – first towards the right, then towards the bottom of the page, is shown on Figures 3.1-3.4 at davidbonal.com.)

## 5. Conclusions

Discarding the syllogistic moods and figures, syllogistic axioms and inference rules, and valid syllogism rules, in favour of a pure set model of the syllogistic terms, greatly simplifies the categorical syllogisms' presentation. Compare, e.g., with other expositions: Alvarez and Correia 2012, Mineshima, Okada, Takemura 2012, Avarez-Fontecilla 2016, (or Lukasiewicz 1957). One has shown that the middle term M always appears in any LC – since the LC always refers to just one subset out of the eight U subsets. Only by losing some information one may recast the LC in the traditional way as referring to a two subset column. Possible LC examples from each of the three VCA classes are  $S=SPM$ ,  $SPM \neq \emptyset$ ,  $M=SPM$ , as LCs for Barbara, Darii and Darapti, respectively. With some loss of information they translate into the usual  $A(S,P)$ ,  $I(S,P)$ , and, via existential import on M,  $I(S,P)$ . Also, using just set relabelings, instead of syllogism reduction, one has shown that the VCAs of the same type are equivalent: any VCA (or VS) can be recast (or reformulated) as any other VCA of the same type. Thus, e.g., Barbara, Darii and Darapti may be chosen as representatives of the three VCA classes.

This way of presenting the categorical syllogisms was started by George Boole and perfected by Lewis Carroll/Charles Lutwidge Dodgson. I think that it is more than regrettable that “the Logicians” stubbornly filled out the logic textbooks, for

more than a century, with presentations of categorical syllogisms which make less sense, while being much more complicated.

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