

Categorical syllogisms – a set theory do-over

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Abstract

One interprets the terms S,P,M, appearing in the pairs of categorical premises (PCPs) of categorical syllogisms, as subsets of an universal set U, whose other subsets are S',P',M' – the complements in U of S,P,M. Thus, U is partitioned into 8 subsets: MSP, MS'P, MSP', MS'P', M'SP, M'S'P, M'SP', M'S'P'; each of S,P,M,S',P',M' is partitioned only into 4 subsets out of the 8 subsets listed above. A Karnaugh map is used to represent U and its 8 subsets. One reprises results of George Boole, *The mathematical analysis of logic*, 1847, pages 35-41, and Lewis Carroll, *Symbolic Logic*, 1896, pages 74-78, who established that in fact there are only three types of valid categorical arguments (VCAs). The VCAs include the 24 usual valid syllogisms (VSs) – whose entailed logical conclusions (LCs) are of the form I(S,P), A(S,P) and O(S,P) - and also pairs of categorical premises PCPs whose LCs are of the form I(S',P'), A(P,S) and O(P,S). One argues that the VCAs of the same type are all equivalent since any VCA may be re-written, via a term – i.e. set – relabeling, as either a Barbara/Barbari, a Darii, or a Darapti VS. The standard O,I, (resp. A,E), particular, (resp. universal,) quantifiers, lay set elements in, (resp. empty), two out of 8 subsets of U. One may work with 36 PCPs via pairing 6 P-premises with 6 S-premises, or, to have M and M' appear symmetrically in the premises, one may work with 64 PCPs via pairing 8 P-premises with 8 S-premises. Then, e.g., the 8 P-premises form two squares of opposition, or, equivalently, a cube of opposition: A(M,P), E(M,P), I(M,P), O(M,P), and, A(M',P), E(M',P), I(M',P), O(M',P). Here A(M',P), O(M',P) were added to the usual 6 P-premises, and E(M',P) is the same as the usual A(P,M) and I(M',P) is the same as the usual O(P,M).

In short, one way of seeing that the VCAs of the same type are all equivalent, is to write the eight VCAs of type 1 as $E(S^*, M^*)E(M^*, P^*)$, where the * stands for complement or no complement, (i.e., ' or nothing). Thus $S^* \in \{S, S'\}$, $M^* \in \{M, M'\}$, $P^* \in \{P, P'\}$, the 16 VCAs of type 2 as $I(S^*, M^*)E(M^*, P^*)$ and $E(S^*, M^*)I(M^*, P^*)$, and the eight VCAs of type 3 as $E(S^*, M^*)E(M^*, P^*)$. Since it is arbitrary which set one denotes by M and which one is called M', and similarly which one is called S or S', and resp., which one is called P or P', it follows that all VCAs of the same type are equivalent.

One may note that any PCP made of two particular premises, and any PCP made of a particular premise plus a universal premise - one "acting" on M and the other one on M' – do not entail any LC. Also, the LCs of the above 3 types of VCAs always refer to just one subset out the 8 subsets of U:

(1) $E(S^*, M^*)E(M^*, P^*)$: $S^* = S^*M^* + S^*M'^* = S^*M^* = S^*M^*P^* + S^*M^*P'^* = S^*M^*P^*$ and $P^* = P^*M^* + P^*M'^* = P^*M^*$ = $S^*M^*P^* + S^*M^*P'^* = S^*M^*P^*$. For example, Barbara, $A(S, M)A(M, P) = E(S, M')E(M, P')$: $S = SM' + SM = SM = SMP' + SMP = SMP$ and $P' = P'M' + P'M = P'M' = SM'P' + S'M'P' = S'M'P'$, has two LCs $S = SMP$ and $P' = S'M'P'$, which, after discarding the middle terms, prove to say the same thing: $A(S,P) = A(P',S') = E(S,P')$.

(2) $I(S^*, M^*)E(M^*, P^*)$: $S^*M^* = S^*M^*P^* + S^*M^*P'^* = S^*M^*P^* \neq \emptyset$ and $E(S^*, M^*)I(M^*, P^*)$: $P^*M^* = S^*M^*P^* + S^*M^*P'^* = S^*M^*P^* \neq \emptyset$. One can notice that, e.g., $I(S, M)E(M, P')$, Darii/Datisi, and $E(S', M)I(M, P)$, Disamis/Dimaris, have the same LC: $SMP \neq \emptyset$.

(3) $E(M^*, S)E(M^*, P^*)$: $M^* = S^*M^* + S'^*M^* = S^*M^* = S^*M^*P^* + S^*M^*P'^* = S^*M^*P^*$. The LC is $M^* = S^*M^*P^*$, which, after the imposition of the existential import (ei) condition, $M^* \neq \emptyset$, becomes $I(S^*, P^*)$.

The three types of PCPs which generate VCAs may be immediately extended to sorites of the same type.

The set model for categorical syllogisms may be used to "appraise" the rules of valid syllogisms. The fact that M, (or M'), is distributed in at least one premise is clear from the above 3 types of VCAs: for example, in $A(S,M)A(P,M) = E(M',S)E(M',P)$, M is not distributed, but M' is, and it is clear that the middle term of the PCP is in fact M', not M. The rule, "two negative premises do not entail an LC" is invalid – as Boole and Carroll already noticed.

The above 3 types of VCAs may also be used to settle which VCAs are compatible with some of the sets S,P,M,S',P',M' being empty. In the modern square of opposition A(M,P), E(M,P) are not contraries anymore - unless one adds $M \neq \emptyset$; instead when both are true it results that $M = \emptyset$. This empty set constraint is compatible with VCAs of type 1 and 3, but not with ei on M, nor with the type 2 premises $I(S^*, M)$ and $I(P^*, M)$.