A Matricial Vue of Classical Syllogistic and An Extension of the Rules of Valid Syllogism to Rules of Conclusive Syllogisms with Indefinite Terms

To the memory of my beloved ones: Alexandra, Lidia, Constantin and Cristina

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Abstract. One lists the distinct Pairs of Categorical Premises (PCPs) formulable via only the positive terms, S,P,M, by constructing a six by six matrix obtained by pairing the six categorical P-premises, A(P,M), O(P,M), A(M,P*), O(M,P*), where P*∈ {P,P'}, with the six, similar, categorical S-premises. One shows how five Rules of Valid Syllogism (RofVS), select only 15 distinct PCPs that entail Logical Consequences (LCs) belonging to the set L^+:={A(P,S), O(P,S), A(S,P), E(S,P), O(S,P), I(S,P)}. The choice of admissible LCs can be regarded as a condition separated from the conditions (or axioms) contained in the RofVS: the usual eight (Boolean) PCPs that generate Valid Syllogisms are obtained when the only admissible LCs belong to the set L:={A(S,P), E(S,P), O(S,P), I(S,P)} and no existential imports are addressed. A 64 PCP-matrix obtains when both PCPs and LCs may contain indefinite terms - the positive, S,P,M, terms, and their complementary sets, S',P', M', in the universe of discourse, U, called the negative terms. Now one can accept eight LCs: A(S*,P*), I(S*,P*), where P*∈{P,P'}, S*∈{S,S'}, and there are 32 conclusive PCPs, entailing precise, "one partitioning subset of U" LCs. The four Rules of Conclusive Syllogisms (RofCS) predict the less precise LCs, left after eliminating the middle term from the exact LCs. The RofCS also predict that the other 32 PCPs of the 64 PCP-matrix are non-conclusive. The RofVS and the RofCS are generalized, and arguments are given, for also accepting as valid syllogisms the conclusive syllogisms formulable via positive terms which entail the LCs A(P,S) and O(P,S).

Keywords: Categorical Premises, Conclusive Syllogism, Valid Syllogism, Rules of Conclusive Syllogisms, Rules of Valid Syllogism, Term Relabelings.

1 Preliminaries

One uses a set interpretation of syllogistic terms, S,P,M,S',P',M', and one interprets the categorical quantifiers, A, E, I, O, and statements, A(M,P), etc., as expressing set relationships. One may use, interchangeably, the words terms and sets. (For more details one may see Radulescu [1].)

One uses the following notations and abbreviations: U for the universe of discourse of a 3-term syllogism, made of 2^3 partitioning subsets of U, which will be simply called *subsets*. No other set will be called a *subset* except a partitioning subset of U. Juxtaposition of set names/letters will denote set intersections: for example, SM denotes the intersection, S∩M, of the sets S and M. The union of sets will be denoted by a + instead of ∪. PCP will stand for pair of categorical premises; LC for logical consequence or conclusion. Existential Import will be shortened to *ei*. VS stands for valid syllogism(s), recognized as such by the Classical Syllogistic, and the RofVS stands for the Rules of Valid Syllogism as used in Classical Syllogistic, or slightly modified. The RofCS stands for the Rules of Conclusive Syllogisms. ESC stands for Empty Set Constraint. A syllogism contains three categorical statements - two premises and their proposed logical consequence (LC) or conclusion. Each of the two premises contains the middle term, denoted by M, and one of the two other terms, either P or S. By tradition, only the S and P terms, also called *end terms*, will appear again in the LC. The S, P, M terms are called *positive terms* and their complementary sets in U, S', P', M', are the *negative terms*; together they are the *indefinite terms*. From now on, one freely uses the notation S*, P*, M*, as shorthand for $S^*\in\{S, S'\}$, $P^*\in\{P, P'\}$, $M^*\in\{M, M'\}$. The Classical Syllogistic considers premises formulable only via positive terms, and requires, by definition, that the valid syllogisms have only LCs belonging to the set L:={A(S,P), E(S,P), I(S,P), O(S,P)}. By contrast, even after eliminating the middle term from it, the LC of a conclusive syllogism can have any of the eight formats A(S*,P*), I(S*,P*), (where $P^*\in\{P, P'\}$, $S^*\in\{S, S'\}$).

2 Introduction

In Radulescu [1], one argued in favor of LCs satisfying the "one subset LC paradigm", i.e., in favor of LCs that specify a unique partitioning subset of U, out of which, for "precision preservation" the middle term was not eliminated. In this paper, the roles of the RofVS and the RofCS are to predict LCs out of which the middle term (M*) was eliminated. For example, Darapti's PCP, A(M,P)A(M,S) entails the exact LC, M=MSP, as its one subset of U LC. Since $M\subseteq P$ and $M\subseteq S$, an existential import (ei) supplementary condition, $M\neq\emptyset$, can be imposed on (the smallest set) M, thus establishing that I(S,P) holds- the usual Darapti LC, out of which M, the subject of the universal LC, A(M,MSP), was dropped out (or eliminated). Analogously, the Barbara PCP, A(M,P)A(S,M), asserting $S\subseteq M\subseteq P$, entails the exact LC, S=SMP, as its one subset of U LC. Dropping the middle term M, the LC can be written as a universal A(S,P) LC. An ei supplementary condition, $S\neq\emptyset$, can be imposed, again, on the smallest set S, thus establishing that I(S,P) holds- the usual Barbari LC.

The aim of the RofVS and the RofCS is to predict these and other LCs, out of which the middle term was eliminated. In Radulescu [1], (see also Table 1 therein), the 64 PCPs containing indefinite terms were shown to be split into eight types, each type containing eight PCPs. Out of the 64 PCPs, 32 entail LCs, where each LC has one of the following eight formats, A(S*,P*), I(S*,P*), (out of which the middle term was eliminated). The other 32 PCPs do not entail any LCs at all. (This counting of conclusive and non-conclusive syllogisms is not new - see also De Morgan [3], paragraph 41, p.19: "There are 64 possible combinations, of which the 32 enumerated give inference. The remaining 32 may be found by… and in no case does any inference follow." Below one lists the eight types of PCPs, with the LCs, (out of which the middle term was eliminated), written after the column sign - if that type of PCPs entails LCs:

(1) (Type Barbara) A(M*,P*)A(S*,M*): A(S*,P*) [=A(P*',S*')]; I(S*, P*) – after ei on S*; I(P*', S*') - after ei on P*' (for a total of eight ei particular LCs).

(2) (Type Darapti) A(M*,P*)A(M*,S*): I(S*,P*) if $M^*\neq\emptyset$ and after M* is eliminated.

(3i) (Type Darii) A(M*,P*)I(M*,S*): I(S*,P*) - after M* is dropped from the precise, $M^*S^*P^*\neq\emptyset$ one subset of U LC.

(3ii) (Type Disamis) I(M*,P*)A(M*,S*): I(S*,P*) - after M* is dropped from the precise, $M^*S^*P^*\neq\emptyset$ one subset of U LC.

(4i) I(M*,P*) I(M*,S*) No LC. Two particular premises, acting both on M, or both on M'.

(4ii) $I(M^*,P^*)I(M^{*\prime},S^*)$ No LC. Two particular premises, one acting on M, the other on M'.

(5i) $A(M^*,P^*)I(M^{*\prime},S^*)$ No LC. One universal and one particular premises, one acting on M, the other on M'.

(5ii) $I(M^*,P^*)\ A(M^{*\prime},S^*)$ No LC. One particular and one universal premises, one acting on M, the other on M'.

In Section 3 the notions of term distribution and of affirmative and negative categorical statements are extended, in a consistent way, to both terms and their complementary sets and, respectively, to premises and LCs containing negative terms. A new notion of signature of a statement (and LC) and of signature of a PCP is introduced and used in the RofCS #2. Section 4 lists, in a slighted amended form, the (usual) five Rules of Valid Syllogism (RofVS) which apply to all the 36 distinct PCPs formulable via only positive terms. They predict the LCs and ei LCs of the conclusive syllogisms having LCs among the four standard (and traditional) ones, {A(S,P), E(S,P), O(S,P), I(S,P)}=:L, and among A(P,S) and O(P,S), and predict or postulate that the rest of the 36 PCPs either do not entail any LC, or the entailed LCs are not in the above expanded set L^+:={A(S,P) E(S,P), O(S,P), I(S,P), A(P,S), O(P,S)}, because any LC$\in L^+$ one might propose for any of these PCPs, will contradict at least one of the RofVS. Section 5 lists the four Rules of Conclusive Syllogisms (RofCS) which apply to all 64 distinct PCPs containing indefinite terms. They predict the LCs and ei LCs of all 32 conclusive syllogisms and predict that the other, non-conclusive, 32 PCPs, are indeed non-conclusive since any LC one might propose for any one of them, contradicts at least one of the four RofCS. (This time around, when a RofCS is contradicted, there is no LC at all. Compare this with the two RofVS saying that “the middle term has to be distributed in at least one premise” and that “two negative premises are not allowed”: we’ll interpret the admonitions “has to be distributed“ and “not allowed” as asserting that a PCP not satisfying one of these two RofVS, either does not entail any LC at all, or that, if an LC is entailed, it is not one belonging to the expanded set L^+ of allowable LCs. As said, the RofCS can predict, for each conclusive syllogism or ei conclusive syllogism, its LC – out of which the middle term was eliminated. One recognizes that the whole purpose of the RofVS and the RofCS - to be able to quickly find the LC of any PCP, can still be accomplished, probably with a little less ease, by using the above formulas, (1) to (5ii), describing the four types of the conclusive syllogisms and their LCs, and the four types of non-conclusive PCPs. The Sections 6 and 7 examine empty set constraints (ESCs) and how many simultaneously sound conclusive syllogisms one may obtain out of three given terms.

3 The notions of distribution and of affirmative and negative categorical statements extended to indefinite terms

“A term is said to be distributed when reference is made to all the individuals denoted by it; it is said to be undistributed when they are only referred to partially, i.e., information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class.” Keynes [3]. One may expand the definition of distribution, by agreeing that whatever distribution the two terms appearing in a categorical statement may have due to their position inside the statement, then their complementary terms in U, are automatically assigned an opposite distribution. Thus since in I(M,P), the terms M and P are undistributed, the terms M', P' are distributed in the same I(M,P) statement. This is in agreement with the obversion and contraposition rules and the standard definition of distribution: from I(M,P)=O(M,P')= O(P,M') one realizes that M' and P' are indeed distributed since M,P were not. The same definition of distribution, extended to indefinite terms, was used, e.g., by Alvarez and Correia [4] and was cited in Alvarez-Fontecilla and Lungenstrass [5] as having been used in 1932 by Wilkinson [6]. Therefore, the positive and negative terms S,P,M, S',P',M' can "be taken universally" (see Alvarez-Fontecilla [7]), i.e., be distributed, or can "be taken particularly", i.e., be undistributed, depending on their position inside the A,E,I,O statements. Note also that the arguments of a statement and of its contradictory one, have opposite distributions – in E(M,P) both M and P are distributed, while in the contradictory statement, I(M,P), both M and P are undistributed, (while M', P' are distributed). Similarly, in A(M,P), M is distributed and P is not, while in the contradictory statement, O(M,P), the term distributions are reversed.

As usually, one assigns to the A and E, (resp. I and O), quantifiers and statements (or propositions) the attributes of being universal, (resp. particular). One can define, for both universal and particular premises and LCs, when they are considered as affirmative statements (or affirmative propositions), and when they are considered as negative statements. The universal negative premises are E(M,P), E(M,S), E(M',P'), E(M',S'), and the only particular negative premises are O(P,M), O(S,M), O(M,P), and O(M,S). Denoting $h\in\{S,P\}$, $h'\in\{S',P'\}$, the universal negative premises are E(M,h), E(M',h'), the particular negative premises are I(M',h), I(M,h'), the universal affirmative premises are E(M',h)=A(h,M), E(M,h')=A(M,h), the particular affirmative premises are I(M,h), I(M',h'). These definitions of affirmative and negative propositions put on the same footing the positive and negative terms – for example, E(M,P), E(M',P')= A(M',P) are all negative statements/propositions. Compare this with A and I, (resp. E and O), being affirmative, (resp. negative) quantifiers - or statement symbols. The switch M↔M', (resp. h↔h'), transforms affirmative premises into negative premises and vice versa. [One can see that the switch E↔I while the arguments are left unchanged transforms universal negative premises into particular affirmative premises (and vice versa), and transforms universal affirmative premises into particular negative premises (and vice versa).] One can define the negativity or signature, s, of a statement symbol (or quantifier), s(A)=s(I)=0, s(E)= s(O)=1, the signature of a term, s(M)=s(P)=s(S)=0, s(M')=s(P')=s(S')=1, and, the signature of a whole statement as the sum of the signatures modulo 2 of the statement's symbol and of all of its terms. Then a statement is affirmative, if its signature is zero, and is negative if its signature is 1. Thus, e.g., s(A(M,P))= s(E(M,P'))= s(A(P',M')) =0, s(A(M',P))=s(E(M',P'))=1; therefore the first three statements are affirmative, and the last two are negative; these definitions are invariant under inversions and obversions. Importantly, one defines the signature of a whole PCP as the sum of the signatures of each of its two statements (premises). One uses the latter definition to formulate the RofCS #2 from Section 5.

4 The RofVS

For newer takes on the RofVS, see, e.g., Alvarez and Correia [4], Alvarez-Fontecilla [7], Alvarez-Fontecilla and Lungenstrass [5], Correia [8].

One notes that the RofVS, like the Classical Syllogistic, suppose, at a minimum, that the PCPs are formulable using only positive terms, and that, if a PCP entails an LC, then the LC is formulable using only positive terms, too. The usual RofVS suppose in fact that the admissible LCs are only those in the set L:={A(S,P), E(S,P), O(S,P), I(S,P)}. One argues – at the end of this Section – that it makes more sense to admit as generating valid syllogisms all the PCPs whose LCs belong to the set L^+:={A(S,P) E(S,P), O(S,P), I(S,P), A(P,S) , O(P,S)}, i.e., to admit as generating valid syllogisms all the PCPs whose LCs are formulable via using only the positive terms S and P.

The aim of this Section is to find an expression for the RofVS that allows them to single out the conclusive PCPs whose LCs and/or ei-LCs have one of the formats in L^+, and to predict these LCs and/or ei-LCs.

· One already knows, (Radulescu [1]), that two universal premises, (which split into two types – either Barbara or Darapti), always entail LCs via establishing that one of the sets S,M,P,S',M' or P', was reduced to only one partitioning subset of the universe of discourse U (in short subset of U), since all its three other subsets are guaranteed to be emptied by the action of the two universal premises. Thus, as already mentioned, one of Barbara's LCs is S=SMP (the other one being P'=P'S'M'), and the Darapti premises' unique LC is M=MSP. One may then impose an existential import (ei) condition on the positive term S, (resp. M), to obtain the ei LCs Barbari, (resp. Darapti), of this format: I(S,P). The ei condition is always imposed on the "smallest" set – the one included in the other two; afterward any one of the three sets can be dropped out of the ei-LC. For example, Barbara's premises, A(M,P)A(S,M), imply: S=SM+SM'=SM=SMP+ SMP'=SMP. (Cf. Jevons' [11] substitution method). The "one subset LC paradigm" is simply this: a PCP entails an LC if and only if its two premises pinpoint a unique subset of U. As just said, the paradigm applies to syllogisms of type Darii and Disamis, too. For example, Darii's premises, A(M,P)I(M,S), establish that M=MP and MS $\neq$Ø; therefore, (cf. Jevons [11] substitution method), one knows that the MPS subset of U is non-empty, from where the usual LC, I(S,P), follows by dropping the "undesirable" middle term M. The 36-element PCP matrix is inserted below; all the LCs of one of the eight formats A(S*,P*, I(S*,P*), where P*$\in${P, P'}, S*$\in${S, S'}, are recorded – these LCs will be all predicted by the RofCS – in the next Section.

One lists the RofVS, close to their formulation in Stebbing [12]. (See also Copi [9] Hurley [10] and Keynes [3]). One will start the RofVS list with the rules which are similar to the RofCS:

RofVS #1 - The distribution of the end terms, P and S, is conserved in all non-ei and ei valid syllogisms, except in the type Barbara ei valid syllogisms, where ei on S (resp. P) changes S, (resp. P), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P, (resp. S), remains the same as it was in the PCP. (As mentioned, the ei condition has to be imposed on the smallest set – the one included in the other two terms of a syllogism of types Barbara or Darapti – afterward the smallest set, or any other term which includes it, can be dropped/eliminated to obtain an ei LC.) [The usual formulation of this Rule of Valid Syllogism is: "any term distributed in the LC must be distributed in the PCP", i.e., the distribution of the end terms cannot "increase" from undistributed in the premises to distributed in the LC, but can, conceivable, decrease from distributed in the premises to undistributed in the LC. This allows, later on, a discussion of the ei-valid syllogisms: Bramantip, Barbari, Celaront/Cesaro, Camestros/Camenos, Darapti, Felapton/Fesapo. But since the end terms distributions are conserved even in the ei-LCs of Darapti type syllogisms, one prefers to explicitly list the cases when an ei-LC decreases the distribution of an end term.]

RofVS #2 - "if only one premise is negative, the LC, if any, is also negative".

RofVS #3 - "if both premises are affirmative, the LC, if any, is affirmative".

Table 1 The 36-element matrix of the PCPs formulable only via positive terms.

	A(S,M) (Undistri-buted M)	A(M,S)	E(M,S)	I(M,S) (Undistri-buted M)	O(M,S) (Undistri-buted M)	O(S,M)
A(P,M) (Undis-tributed M)	M double un-distributed (Darapti type premises) LC: I(S’,P’), via ei on M’, i.e.,M’≠ Ø	Bramantip PCP Satisfies all the RofVS (Barbara type premises) LCs: A(P,S); I(S,P) via ei on P,i.e., P≠ Ø	Camestres/ Camenes (Barbara type premises) LCs: E(S,P)= A(S,P’); O(S, P) via ei on S, i.e.,S≠ Ø	M double un-distributed (5i) type PCP No LC en-tailed	M double un-distributed (5i) type PCP No LC en-tailed	Baroco (Darii type premises) LC: O(S,P)
A(M,P)	Barbara (Barbara type premises) LC: A(S,P); I(S, P) via ei on S, i.e.,S≠ Ø	Darapti (Darapti type premises) LC: I(S,P), via ei on M, i.e., M≠ Ø	Satisfies all the RofVS (Darapti type premises) LC: O(P,S), via ei on M, i.e., M≠ Ø	Darii (Darii type premises) LC: I(S,P)	Satisfies all the RofVS (Darii type premises) LC: O(P,S)	One negative premise so a negative LC which distri-butes an un-distributed premise term (5i) type PCP No LC
E(M, P)	Celarent/Ce-sare; Barbara type premises LCs: E(S,P)= A(S,P’); O(S, P) via ei on S, i.e.,S≠ Ø	Felapton/Fe-sapo. (Darapti type premises) LC: O(S,P), via ei on M, i.e., M≠ Ø	Two negative premises (Darapti type premises) LC: I(S’,P’), via ei on M, i.e., M≠ Ø	Ferio/Festino/ Ferison/Fresi-son (Darii type premises) LC: O(S,P)	Two negative premises (Darii type premises) LC: I(S’,P’)	Two negative premises (5i) type PCP No LC en-tailed
I(M,P) (Undis-tributed M)	M double un-distributed (5ii) type PCP No LC en-tailed	Disamis (Disamis type premises) LC: I(S,P)	Satisfies all the RofVS (Disamis type premises) LC: O(P,S)	M double un-distributed (4i) type PCP No LC en-tailed	M double un-distributed (4i) type PCP No LC en-tailed	Two particu-lar premises (4ii) type PCP No LC
O(M,P) (Undis-tributed M)	M double un-distributed (5ii) type PCP No LC en-tailed	Bocardo (Disamis type premises) LC: O(S,P)	Two negative premises (Disamis type premises) LC: I(S’,P’)	M double un-distributed (4i) type PCP No LC en-tailed	M double un-distributed (4i) type PCP No LC en-tailed	Two negative premises (4ii) type PCP No LC en-tailed
O(P,M)	Satisfies all the RofVS (Disamis type premises) LC: O(P,S)	One negative premise so a negative LC which distri-butes an un-distributed premise term (5ii) type PCP No LC	Two negative premises (5ii) type PCP No LC en-tailed	Two particu-lar premises (4ii) type PCP No LC	Two negative premises (4ii) type PCP No LC en-tailed	Two negative premises (4i) type PCP No LC en-tailed

RofVS #4 - “the middle term has to be distributed in at least one premise.
RofVS #5 - “two negative premises are not allowed”.

The other two habitual Rules of Valid Syllogism are unnecessary, since they are consequences of the previously listed ones: RofVS #6 - “if one premise is particular, then the LC is particular”, RofVS #7 - “from two universal premises, no particular LC may be drawn unless existential import (ei) is imposed as a supplementary condition”.

As Stebbing [12], Corollary (i) p.57, shows, using the RofVS #1, (in its weaker form – “any term distributed in the LC must be distributed in the PCP”), and the RofVS #2 to #5, it results that no PCPs made of two particular premises, may entail an LC belonging to the set L^+ and such that the LC is compatible with the RofVS. Indeed, PCPs made of two particular and affirmative premises – in which, therefore, no term is distributed, (resp. two particular and negative premises – part of the “not allowed” PCPs), are already discarded, by RofVS #4, (resp. RofVS #5). The PCPs made of one affirmative and one negative particular premises, should have an LC which is negative - according to RofVS #2. But this means that at least one end term will be distributed in the LC, without being distributed in the PCP – since, according to the distribution's definition and RofVS #4, only the middle term will be distributed in the negative particular premise: contradiction. Thus, by postulating, via RofVS #4 and #5, two new classes of PCPs that either do not entail any LCs, or, if they do, then the entailed LCs are of an inadmissible format, (how else one might interpret the admonitions “has to be distributed” and “are not allowed” from RofVS #4 and #5?), one was able to prove that any PCP made of two particular premises should be excluded from the set of PCPs that entail LCs of one of the formats listed in the set L^+. As one can see from the Table 1 above and Table 2 inserted bellow, there are nine PCPs made of two particular premises which are formulable only via positive terms. Five, (resp. four), of these PCPs are examples of PCPs built according to the formula I(M*,P*)I(M*,S*), (resp. I(M*,P*)I(M*',S*), where $M^* \in \{M, M'\}$, $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$. In Radulescu [1] these formulas are assigned type (4i), (resp. (4ii)): the two particular premises of type (4i), (resp. (4ii)), are either both “acting” on M, or, are both “acting” on M', (resp. are acting one on M and the other on M'). The RofCS replace the RofVS #4 and #5 with the more naturally sounding RofCS #3, “If one premise is particular, the LC, if any, is particular”, and the RofCS #4: “The PCPs of type (4i) are non-conclusive, i.e., they do not entail any LCs”. (One can see from the Table 1 above and Table 3 below, that there are PCPs made of two negative premises, or made of premises where M is double undistributed, which all entail the LC I(S',P') – which does not belong to the set L^+.) One can also see from these Tables, that there are eight PCPs made of one particular premise and one universal premise, one acting on M and the other on M’ (or vice versa). They are examples of PCPs obtained from the formulas of types (5i), A(M*,P*)I(M*',S*), and (5ii), I(M*,P*)A(M*',S*).

Interestingly, as in Stebbing [12], one can now prove as a Theorem – useful to remember when one tries to predict LCs using the RofVS – that indeed, “if one premise is particular, then an LC, such that $LC \in L^+$, if it exists, is particular” (Corollary (ii), Stebbing [12], p.58): Since the other premise has to be universal, the possibilities are: the universal premise could be affirmative or negative; the particular premise could be affirmative or negative. If both premises are affirmative, only the middle term is distributed in the universal premise – the LC, if any, has to be affirmative and particular – so as to not distribute any of the end terms – cf. RofVS #1. Both premises being negative “is not allowed” cf. RofVS #5. If the universal premise is affirmative and the particular premise is negative, then the LC, if any, is negative cf. RofVS #2. In this case the premises distribute two terms, one of them being the middle term - cf. RofVS #4; thus, the LC being negative could not be universal – since then it would distribute both end

Table 2 The 36-element matrix of the PCPs. PCPs eliminated by the RofVS.

	A(S,M) (Undistri-buted M)	A(M,S)	E(M,S)	I(M,S) (Undistri-buted M)	O(M,S) (Undistri-buted M)	O(S,M)
A(P,M) (Undis-tributed M)	M double un-distributed (Darapti type premises) LC: I(S’,P’), via ei on M’, i.e.,M’≠ Ø			M double un-distributed (5i) type PCP No LC en-tailed	M double un-distributed (5i) type PCP No LC en-tailed	
A(M,P)						One negative premise so a negative LC which distri-butes an un-distributed premise term (5i) type PCP No LC
E(M, P)			Two negative premises (Darapti type premises) LC: I(S’,P’), via ei on M, i.e., M≠ Ø		Two negative premises (Darii type premises) LC: I(S’,P’)	Two negative premises (5i) type PCP No LC en-tailed
I(M,P) (Undis-tributed M)	M double un-distributed (5ii) type PCP No LC en-tailed			M double un-distributed (4i) type PCP No LC en-tailed	M double un-distributed (4i) type PCP No LC en-tailed	Two particu-lar premises (4ii) type PCP No LC
O(M,P) (Undis-tributed M)	M double un-distributed (5ii) type PCP No LC en-tailed		Two negative premises (Disamis type premises) LC: I(S’,P’)	M double un-distributed (4i) type PCP No LC en-tailed	M double un-distributed (4i) type PCP No LC en-tailed	Two negative premises (4ii) type PCP No LC en-tailed
O(P,M)		One negative premise so a negative LC which distri-butes an un-distributed premise term (5ii) type PCP	Two negative premises (5ii) type PCP No LC en-tailed	Two particu-lar premises (4ii) type PCP No LC	Two negative premises (4ii) type PCP No LC en-tailed	Two negative premises (4i) type PCP No LC en-tailed

Table 3 The 36-element PCP matrix. Only the PCPs satisfying all the RofVS are displayed.

	A(S,M) (Undistri-buted M)	A(M,S)	E(M,S)	I(M,S) (Undistri-buted M)	O(M,S) (Undistri-buted M)	O(S,M)
A(P,M) (Undis-tributed M)		Bramantip PCP Satisfies all the RofVS (Barbara type premises) LCs: A(P,S); I(S,P) via ei on P,i.e., P≠ Ø	Camestres/ Camenes (Barbara type premises) LCs: E(S,P)= A(S,P'); O(S, P) via ei on S, i.e.,S≠ Ø			Baroco (Darii type premises) LC: O(S,P)
A(M,P)	Barbara (Barbara type premises) LC: A(S,P); O(S, P) via ei on S, i.e.,S≠ Ø	Darapti (Darapti type premises) LC: I(S,P), via ei on M, i.e., M≠ Ø	Satisfies all the RofVS (Darapti type premises) LC: O(P,S), via ei on M, i.e., M≠ Ø	Darii (Darii type premises) LC: I(S,P)	Satisfies all the RofVS (Darii type premises) LC: O(P,S)	
E(M, P)	Celarent/Ce-sare; Barbara type premises LCs: E(S,P)= A(S,P'); O(S, P) via ei on S, i.e.,S≠ Ø	Felapton/Fe-sapo. (Darapti type premises) LC: O(S,P), via ei on M, i.e., M≠ Ø		Ferio/Festino/ Ferison/Fresi-son (Darii type premises) LC: O(S,P)		
I(M,P) (Undis-tributed M)		Disamis (Disamis type premises) LC: I(S,P)	Satisfies all the RofVS (Disamis type premises) LC: O(P,S)			
O(M, P) (Undis-tributed M)		Bocardo (Disamis type premises) LC: O(S,P)				
O(P,M)	Satisfies all the RofVS (Disamis type premises) LC: O(P,S)					

terms – which would contradict RofVS #1. If the universal premise is negative and the particular premise is affirmative, then the LC, if any, is negative cf. RofVS #2. In this case the premises distribute again two terms, (the universal premise distributes two, the particular premise being affirmative distributes none), one of them being the middle term - cf. RofVS #4; thus, the LC being negative could not be universal – since then it would distribute both end terms – which would contradict RofVS #1.

Finally, as Stebbing [12], p. 91, notices, the universal premises assert that set intersections are empty, while particular premises assert that set intersections are non-empty - therefore the particular premises assert existence of set elements, while the universal premises do not; it follows that RofVS #7 always holds: one needs ei as a supplementary condition in order to infer a particular LC from universal premises.

Then, one may prove that PCPs of types (5i) and (5ii) do not entail any LCs, either: one can see on Table 2 that out of the four PCPs of type (5i) and four PCPs of type (5ii) which are formulable only via positive terms, two PCPs contain only negative premises, in four of the PCPs the middle term is not distributed at all, and in another two PCPs, one of the premises is particular, one is negative, and in each of these latter two PCPs both end terms are undistributed. Since the LC should be negative and particular, (cf. RofVS #2 and Stebbing's [12] Corolary (ii)), and the middle term has to be distributed at least once, therefore, one end term would be distributed in the LC, without being distributed in the premises - thus contradicting RofVS #1. [The complete details are as follows. Two PCPs, E(M,P)O(S,M) – formula (5i) for M*=M', S*=S and P*=P, and O(P,M)E(M,S) – formula (5ii) for M*=M', S*=S and P*=P, are made only of negative premises, and therefore, in accordance with RofVS #5, their LCs, if any, are not in L^+, i.e., are not formulable via using only positive terms. The formula (5i), E(M*',P*)I(M*,S*), leads, for M*=M, P*=P and S*∈{S, S'} to two PCPs, (containing only positive terms), where M is nowhere. Similarly, the formula (5ii), I(M*,P*)E(M*',S*), leads, for M*=M, S*=S and P*∈{P, P'} to two PCPs, (containing only positive terms), where M is again nowhere distributed. According to RofVS #4, all the above four PCPs will not entail any LC$\in L^+$. In A(M,P)O(S,M) – which is formula (5i) for M*=M', S*=S and P*=P', and in O(P,M)A(M,S) – which is formula (5ii) for M*=M', S*=S' and P*=P, the middle term is distributed in both premises, but S and P are nowhere distributed; since the LC should be particular and negative, (due to one premise being negative and particular), the LC would distribute either S or P, thus contradicting RofVS #1.]

Once all the non-conclusive PCPs and the PCPs whose LCs $\notin L^+$ were removed from Table 1, (see them all in Table 2), one can check that all the 15 remaining PCPs displayed in Table 3 have their LCs or ei LCs predicted by the RofVS. To take a concrete example, let's find the LC of Carroll's [13] p. 240 syllogism: "None of my boys are conceited; None of my girls are greedy". Before declaring that this PCP contains two negative premises, one needs to have the same middle term in both premises: choose it to be (my) boys; so Girls=non-Boys=Boys'. Thus, Carroll's PCP becomes "No B are C; No B' are G(reedy)". As such this PCP does not contain only positive terms, but it can be transformed into one, via a conversion and an obversion: No B are C; All G are B. Now one can recognize that this is a Celarent syllogism whose LC is therefore No C are G, that can be translated back into words as "No conceited child of mine is greedy". Or, one can use the RofVS #1 to predict these two LCs which both conserve that the C and G terms are distributed in the PCP: E(C,G), or I(C',G'). Since one premise is negative one has to choose, cf. RofVS #2, the solution E(C,G). Alternatively, one can directly apply the RofCS to the (conclusive) syllogism "No B are C; No B' are G": see Section 5. Carroll's example of a conclusive syllogism with two negative premises does not use only positive terms – when it does – one can see that its premises are Celarent' premises. On Table 2 one can see three examples of conclusive PCPs, made truly of negative premises that are formulated using only positive terms: E(M,P)O(M,S), E(M,P)O(M,S), and O(M,P)E(M,S); each one of these conclusive syllogisms has the LC I(S',P') $\notin L^+$. Because they are made of two negative premises, cf. RofVS #5, these PCPs are eliminated as valid syllogism candidates. One can say, maybe, that agreeing that Girls=(Boys) '=non-Boys=the complementary set of the term (or set) Boys, is a case of fuzzy syllogistic, since the subset of (possible) intersex kids is neglected. As another example of “fuzzy syllogistic”, if one agrees that good people - bad people, wise people - foolish people, intelligent people - dumb people, are three pairs of complementary terms in the universe of people, (thus denying, e.g., that there are people who are neither wise nor foolish), then any syllogism one might construct using these six terms, can be expressed without using the “non” prefix, and any O or E statement can be naturally expressed as an I or A statement (again without using the “non” prefix). Carroll used this “trick” to express an A statement as an E statement without using the “non” prefix in front of the term Boys. As another example, Barbara’s syllogism, A(M,P)A(S,M) can have, cf. RofVS #1, either the LC A(S,P), or the LC I(S',P) – as distribution preserving LCs. But I(S',P)=O(P,S) is a negative statement while both premises are affirmative: therefore, the second LC would contradict the RofVS #3. Thus, the correct LC prediction is A(S,P) – as it should be.

On Table 3 one can notice that any two PCPs - among the four universal PCPs of type Barbara, and among the three universal PCPs of type Darapti – being sound, i.e., made of true premises, will put constraints on U: for example both Barbara and Bramantip premises being true implies S=M=P; both premises of Barbara and Felapton being true implies M=S=Ø; both Barbara and Darapti premises being true implies S=M; both Barbara and Camestres premises being true implies S=Ø and M=P. One may not be interested in such specially defined universes of discourse.

One may say that the six P-premises are the vertices of a triangular prism, with the three universal premises, A(P,M), A(M,P), E(M,P) being the vertices of the top triangular face and their contradictory, particular P-premises being placed directly under them as vertices of the bottom triangular face, such that the P-prism has three “edges of contradiction” A(P,M)—O(P,M), A(M,P)—O(M,P), E(M,P)-- I(M,P). Similarly, one can “build” the S-prism of six S-premises formulable using only positive terms. The 36 PCPs from Table 1 were obtained by pairing up each premise from the P-prism with an S-premise from the S-prism. Importantly, the pairing on Table 1 was done in the following order: in the upper left corner of the PCP matrix, one has a 3 by 3 sub-matrix made of nine elements resulting from pairing the three universal P-premises with the three universal S-premises – these nine PCPs are the Barbara and Darapti type PCPs. In the upper right corner of the PCP matrix, one has a 3 by 3 sub-matrix made of nine elements resulting from pairing the three universal P-premises with the three particular S-premises – these nine PCPs are the five Darii type PCPs and the four PCPs of type (5i) whose premises “act” one on M and the other on M'. Similarly, in the lower left corner of the PCP matrix, one has a 3 by 3 sub-matrix made of nine elements resulting from pairing the three particular P-premises with the three universal S-premises – these nine PCPs are the five Disamis type PCPs and the four PCPs of type (5ii) whose premises “act” one on M and the other on M'. Finally, in the lower right corner of the PCP matrix, one has a 3 by 3 sub-matrix made of nine elements resulting from pairing the three particular P-premises with the three particular S-premises – as one will see in Section 5, these PCPs do not entail any LCs. In the present Section one just found that, according to RofVS #4 and #5, these PCPs do not entail any LCs $\notin L^+$.

On Table 3 one can also notice, (the easiest way by using Karnaugh maps with n=3, see, e.g., https://osf.io/x5dhc), that, for example, if Bramantip's PCP and Baroco's PCP which is on the same matrix row as Bramantip's PCP, plus Bocardo's PCP which is on the same matrix column as Bramantip's PCP, having all premises which are simultaneously true, would not impose restrictions on the structure of U: Baroco's and Bocardo's universal premises are the same as one of Bramantip's premises, and Baroco's and Bocardo's particular premises assert that SM'P'≠Ø and SMP'≠Ø, respectively, and, about these two latter subsets of U, Bramantip's premises do not assert anything. Thus, Bramantip, Baroco and Bocardo are all "compatible" – they can be simultaneously sound without imposing a special structure on U. Similarly, the four PCPs of Barbara, Darii, one more PCP containing a particular premise from the same row matrix as Barbara, and another no name PCP from the same column matrix as Barbara, the latter two PCPs entailing the O(P,S) LCs are all compatible, i.e., all four of the above PCPs (and syllogisms) being sound, will not impose a special structure on U: the particular premises of Darii and of the other two syllogisms whose LCs are O(P,S), assert that MSP≠Ø, MS'P≠Ø, and M'S'P≠Ø, while nothing is asserted, by Barbara's premises, about any one of the above, non-empty, subsets of U. (Note that when looking for PCPs compatible with a PCP made of two universal premises, (e.g., Barbara, A(M,P)A(S,M)), one has to avoid PCPs whose particular premises, (in Barbara's case, O(M,P) and O(S,M)), contradict the universal premises of the PCP made of two universal premises.)

Finally, one may say that the term P plays a "more universal role" in the matrix elements, i.e., PCPs, above the matrix diagonal, while the S term plays a "more particular role" there, and the situation is reversed for the PCPs below the matrix diagonal. A relabeling transformation P↔S will relabel the PCPs above the diagonal, (such as Baroco in the upper right corner of the matrix), with the names had by PCPs symmetric to them with respect to the diagonal – so Baroco will be labeled (without a change in content – this is only a relabeling!) as the no name PCP entailing an O(P,S) LC from the lower left hand side corner of the matrix in Table 1 – which instead will get a new Baroco label. Thus, a P↔S relabeling, will relabel the PCPs above the matrix diagonal with the names appearing on the PCPs below the diagonal, and vice-versa. It follows that the changing of the listing order of a syllogism's premises resulting in an implicit P↔S relabeling, (according to the convention that the firstly listed premise contains the P term), will transform all O(P,S) LC syllogisms into valid syllogisms entailing O(S,P) LCs, but at the same time will relabel the previously valid syllogisms (having O(S,P) as their LCs), as no name PCPs entailing an O(P,S) LC.

As another "cluster of PCP compatibility", note that three PCPs that entail an O(S,P) LC, Felapton, Ferio, Bocardo, are compatible, because all of them assert that SP'M≠Ø while their universal premises do not assert that SP'M is empty. Similarly the four no name PCPs that entail the O(P,S) LC - call them, for the purpose of this comparison – Falepton, Fireo, Bacordo and Boraco, and whose premises pairwise contradict the premises of Felapton, Ferio, Bocardo and Baroco, respectively, are also compatible. More precisely, any PCP from the compatibility cluster Felapton, Ferio, Bocardo, contradicts any PCP from the compatibility cluster Falepton, Fireo, Bacordo, and each of the two clusters is compatible with either Baroco or Boraco, but not both, since the latter two have PCPs that contradict each other, too. The same P↔S relabeling, will relabel Bramantip's PCP as Barbara and vice-versa, Camestres as Celarent and vice-versa, Darii as Disamis and vice-versa. One has already mentioned a few "clusters of PCP compatibility". Any two of the nine PCPs containing only universal premises being simultaneously sound, i.e., containing a total of four true premises, out of which at least three are different, imposes a special structure on the universe of discourse U.

Given three concrete terms, S,P,M, where M is assigned the middle term role, the compatibility cluster examples from the previous paragraph show that, when one builds sound syllogisms with the three terms, (as in the Table 3), some of these sound syllogisms might acquire or lose their valid syllogisms' names after a P↔S relabeling; these sound syllogisms satisfy the RofVS, except that their LCs belonging to the set L depends on the P↔S relabeling. For example, suppose that Boraco and Ferio are sound. If one wants to relabel Boraco as Baroco, then Ferio becomes Fireo. Therefore, a "fairer approach" would be to declare that the valid syllogisms are all the syllogisms from the Table 3, i.e., all the syllogisms with LCs $\in L^+$, instead of only those syllogisms with LCs $\in$ L. In other words, the syllogisms whose LCs are A(P,S) and O(P,S) should be admitted as valid syllogisms, under names like Bramanta, Falepton, Fireo, Bacordo and Boraco - or similar. (Examples of other names: "Hence Buridan's Tifesno, Robaco, Carbodo, Lapfeton, and Rifeson reduce to Festino, Baroco, Bocardo, Felapton, and Ferison, respectively" - see Lagerlund, Henrik, https://plato.stanford.edu/entries/medieval-syllogism).

Note that one can use the following shorthand notations, E':=E(M',x)=A(x,M), I':= I(M',x)= O(x,M), A':=A(M',x), O':= O(M',x), where x∈{S, P}; these notations underline that the above statements are "acting on" M'; A' and O' are the two premises that cannot be written using only positive terms. In accord with the convention that in a PCP the P-premise is always listed firstly, one may use a shorthand notation for any PCP: for example, one can write Barbara's premises as AE' and Celarent premises as EE' – without mentioning, or keeping track, of the (unnecessary) syllogistic figures. This notation helps if one wants to attach three squares of opposition (in fact, rather of compatibility) to the lateral faces of the above triangular P-prism (and S-prism) by replacing the three "edges of contradiction" with three triplets of compatibility, {A; I, I'}, {E; O, I'}, {E'; I, O}, via replacing each particular premise from an "edge of contradiction" with the two particular premises from the other two edges of contradiction. Then, on each of the three lateral prism faces one can independently draw a square of opposition (or rather compatibility), without any two squares having any side in common. Note that the square of opposition is derived from a less sophisticated "square of contradiction" where on the vertical side under A (resp. E) one places the contradictory particular premise O (resp. I); then one switches the positions of O and I: now A(M,P), i.e., M⊆P or MP'=Ø, has under it the subalternate I(M,P), i.e., MP≠Ø. This way the more primitive and irrelevant "square of contradiction" becames a square of opposition or rather compatibility since MP'=Ø and MP≠Ø can both be true if M≠Ø (then P≠Ø, too). The same applies to the pair E(M,P), (i.e., M⊆P' or MP=Ø), and O(M,P), i.e., MP'≠Ø: they can be both true if M≠Ø (then P'≠Ø, too). But if both A(M,P) and E(M,P) are true, i.e., if M= Ø, then both particular statements (or premises) I(M,P) and O(M,P) are false.

An analogous construction will replace the cube having the eight P-premises as vertices and four vertical "edges of contradiction", with a "tent of compatibility", such that on each of the four lateral faces of the tent, and on the two diagonal sections crisscrossing through the inside of the tent, one can draw a total of six squares of opposition – or compatibility – based now on four quadruplets of compatibility: {A; I, I', O'}, {E; O, I',O'}, {E'; I, O, O'}, {A'; I, O, I'}, and without any two squares having any side in common. The six squares (of opposition or compatibility) would be AEOI, EE'II', E'A'I'O', AA'OO', AE'OI', EA'IO' (the latter two crisscross "inside the tent"; the squares containing, e.g., A as a vertex, also contain the compatible particular statements I, O', I' which were brought to replace the particular statement O - the contradictory of A statement on the A—O initial "edge of contradiction" which was thus replaced by the quadruplet of compatibility {A; I, I', O'}.)

But no matter how many squares of opposition one draws on, or attach to, the triangular prism's faces or to the cube's faces, I do not think that one can draw any other useful conclusions than the existence of the three triplets of compatibility for the prism, and the four quadruplets of compatibility for the cube. On the other hand, the same compatibility triplets and quadruplets can be easily obtained from the 36 and the 64 PCP matrices: any universal PCP is compatible with the particular premises from its column and its row which are different from the particular premises that directly contradict the universal premises of that PCP – this ensued from the clusters of compatibility found above for the 36 PCP matrix.

5 The Rules of Conclusive Syllogisms (RofCS) as derived from the 64-element matrix of the PCPs containing indefinite (i.e., positive and negative) terms

One can use the formulas (1) to (5ii) from the Introduction, to formulate the RofCS which can be used to predict all the LCs of the 32 conclusive syllogisms, and to postulate that some of the 32 non-conclusive syllogisms do not entail any LCs, and then to prove that the remaining non-conclusive PCPs are indeed non-conclusive since any LCs that they might predict would contradict at least one of the RofCS. The formulas (1)-(3ii) have the advantage that their LCs contain the same type of statements, (A,E,I,O), as the ones appearing in the premises, but these simplified LCs lose the precision of the "one subset LCs" from the similar formulas (1)-(3ii) in Radulescu [1]. The present formulas (1) - (3ii), which list the LCs from which the middle term was eliminated, agree with the four Rules of Conclusive Syllogisms (RofCS) listed below:

RofCS #1 (almost identical to the RofVS #1): The distribution of the end terms is conserved in all non-ei and ei conclusive syllogisms, except in the type Barbara ei conclusive syllogisms, where ei on S*, (resp. P*) changes S*, (resp. P*), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P*, (resp. S*), remains the same as it was in the PCP. (The ei condition has to be imposed on the smallest set – the one included in the other two sets (or terms) of a syllogism of types Barbara or Darapti – afterwards the smallest set (or term), or any other term which includes it, can be dropped/eliminated to obtain an ei LC.)

RofCS #2: The signatures of a PCP and of its entailed LC (if any) are the same. This implies that if the two premises are affirmative or the two premises are negative, then the LC, if any, is affirmative. It also implies that from one affirmative and one negative premises a negative LC follows – if any. This rule is valid even for LCs obtained after an ei condition was imposed.

RofCS #3: If one premise is particular, the LC, if any, is particular.

RofCS #4: Any two particular premises "acting" both either on M or on M', I(M*,P*) I(M*,S*), do not entail any LCs.

The formulas (1) - (3ii) verify, by inspection, the RofCS #1 to #4. The precise definitions of affirmative and negative statements and PCPs have already been given in Section 3.

To develop the RofCS into an "axiomatic theory", (see also [4], [5], [7]), one has to prove or postulate that all the non-conclusive PCPs are indeed non-conclusive. One can now see why the RofCS #4 is necessary: the RofCS #1 to #3 do not forbid inferring, from the particular premises I(M*,P*) and I(M*,S*), that the LC is I(S*,P*) – even if the two premises only assert that between one and three partitioning subsets of M* are not empty – in particular, S*P*M* might still remain empty, thus contradicting I(S*,P*) - the "trial" LC. This counterexample shows that the PCPs I(M*,P*)I(M*,S*) are non-conclusive. The rest of 24 non-conclusive PCPs, can then be proved to be non-conclusive: according to RofCS #3, I(M*,P*)I(M*',S*), A(M*,P*)I(M*',S*), and I(M*,P*)A(M*',S*) should entail particular LCs, which will conserve the distributions the S* and P* terms had in the premises, (cf. RofCS #1), but this would contradict the RofCS #2. The above establishes the RofCS as an axiomatic "theory" for the syllogisms containing indefinite terms. Note that, while for the RofVS one was able to prove that a particular premise requires a particular LC, (if any), in the case of the RofCS, one cannot prove the RofCS #3 since the postulates RofVS #4 and #5 have been dropped from the RofCS. Nevertheless, the RofCS #3 is clearly true since a particular premise implies existence of non-empty sets, while a universal LC signifies inclusion between sets that might both be empty.

Note that the RofCS #1 and #2 unambiguously predict the LCs in the case of two universal premises, (see formulas (1) and (2) for the Barbara and Darapti types PCPs and syllogisms): the RofCS #1 will predict either a universal or a particular LCs such that both LC predictions preserve the distributions the end terms had in the PCP. Since there are only two universal statements (or quantifiers) - A and E, an universal LC in which, say, S* and P* should be distributed, can be written either as E(S*,P*) or as A(S*,P*') - but, by obversion, the two LCs are equivalent; analogously, a particular LC in which, say, S* and P* should be distributed, can be written either as I(S*',P*') or as O(S*',P*) - but, by obversion, the two LCs are equivalent. Then, for Barbara type premises, the RofCS #2 is contradicted by the particular RofCS #1 predicted LC, and, for Darapti type premises, the RofCS #2 is contradicted by the universal RofCS #1 predicted LC. Thus, in the case of two universal premises, the RofCS #1 and RofCS #2, together, predict a unique LC. It also means that if the premises are of the Darapti type, (i.e., the same term, M*, appears in both premises), then an existential import condition should be imposed on the middle term, such that a particular LC follows – in accord with RofCS #2. Therefore, for the Darapti type conclusive syllogisms, the RofCS automatically detect that, in the variables S* and P*, only a particular and existential import (ei) LC is possible. So, if the middle term is unwanted in the LC, the RofCS "forces upon us" an ei LC. Similarly, when at least one premise is particular, the RofCS #1 and #3 unambiguously predict the LC: according to RofCS #3, the LC is particular, and if, say, the S* and P* terms should be distributed in the LC, the two possible LC statements are, again, identical: I(S*',P*')=O(S*',P*). Afterward, if RofCS #2 is not satisfied, when added to the "predictive" RofCS #1 and #3, it means instead that the examined PCP is not conclusive. As another example, in Carroll's [13] PCP, "No B are C; No B' are G(reedy)", one recognizes that the two universal negative premises distribute both end terms – therefore, cf. RofCS #1, the only possible LCs are E(C,G) or I(C',G'). But cf. RofCS #2, since the PCP signature is (1+1+1)mod 2=1, while s(E(C,G))=1+0+0=1 and s(I(C',G'))=(0+1+1)mod 2=0, only the universal LC satisfies both RofCS #1 and #2. (It also results, that in the case of Carroll's example of two universal premises, the PCP is of the Barbara's type - if the PCP would have been of the Darapti's type then the LC would have been particular according to RofVS #2, and a supplementary ei condition should have been, in fact, imposed on M.

Since the identically formulated RofCS #1 and RofVS #1 always predict two LCs, one universal and one particular which preserve the distributions of the end terms, one saw that for universal PCPs of types Barbara and Darapti, the RofCS #2 will choose the proper LC. But RofCS #2 cannot choose the correct LC in the cases of a PCP of types (5i) or (5ii). Take for example A(M*,P*)I(M*',S*) – the type (5i) PCPs. The RofCS #1 predicts that the LCs for this type PCPs are either the universal A(S*',P*), or the particular I(S*,P*). The RofCS #2 eliminates the particular LCs whose signature is s(P*)+s(S*) and is thus different from the PCPs signatures, 2s(M*)+ s(P*)+s(S*)+1. But the universal predicted LCs, A(S*',P*), have the same signatures as the PCPs – thus these erroneous LCs are perfectly fine from the point of view of the RofCS #1 and #2: consequently, one sees that the RofCS #3 is necessary. (Although for PCPs of types Darii or Disamis, the RofCS #1 and #2 are enough to predict the correct particular LCs. For the Darii type PCPs, A(M*,P*)I(M*,S*), the RofCS #1 predicts as LCs either A(S*',P*) or I(S*,P*), and RofCS #2 rejects the universal LCs.) As noticed, the RofCS #2 is less successful with the PCPs of types (5i) and (5ii) which do not entail any LCs – there one has the RofCS #3 to require a particular LC, and the RofCS #2 to reject it – thus proving that the PCPs of types (5i) and (5ii) do not entail any LCs.

One could have obtained the above results starting with an eight by eight PCP matrix obtained by pairing up the eight categorical P-premises, A(M*,P*), I(M*,P*), with the eight categorical S-premises, A(M*,S*), I(M*,S*). The 64 PCP matrix contains all the PCPs, conclusive and non-conclusive, appearing in the formulas (1) to (3ii) and (4i) to (5ii) and in Table 1 from Radulescu [1]. One can arrange the eight P-premises as vertices of a cube, with its top square having universal premises as vertices and its bottom square having particular premises as vertices, with the vertices arranged in such a way that the cube's vertical edges are "edges of contradiction": A(M,P)—O(M,P), E(M,P)—I(M,P), E(M',P)—I(M',P), A(M',P)—O(M',P). Then, a 64 PCP Table can be built from four 4 by 4 sub-matrices in the same way and order as the Table 1 from Section 4 was built: by firstly pairing-up the four universal P-premises with the four universal S-premises to obtain the upper left corner 4 by 4 submatrix of the 64 PCP matrix, etc. One can also draw four squares of opposition (or compatibility) on the lateral faces of the cube, and draw two other squares of opposition "hanging down" vertically from the two diagonals A—E' and E—A' of the top face of the cube which has as its vertices the four universal premises A, E, E', A'. This construction evidentiates the four quadruplets of compatibility mentioned in the previous Section: {A; I, I', O'}, {E; O, I',O'}, {E'; I, O,O'}, {A'; I, O, I'}.

Instead of using the RofVS #1 to #5 from Section 4 in order to predict LCs and ei LCs for PCPs formulable via using only positive terms, while only the sets L or L^+ are admissible as LCs, one can instead use the RofCS #1 to #4, to predict the same LCs and ei LCs for PCPs formulable via using only positive terms, by imposing the same supplementary conditions that either the LCs and ei LCs should belong to the set L:={A(S,P), E(S,P), O(S,P), I(S,P)} or to the set L^+:={A(S,P) E(S,P), O(S,P), I(S,P), A(P,S), O(P,S)}. This way, the same set of four RofCS can be used for predicting LC and ei LCs for the class of valid syllogisms as defined by the Classical Syllogistic, or for the class of valid syllogisms extended also to those entailing the A(P,S) and O(S,P) LCs, or, for predicting the LC and ei LCs of the 32 conclusive syllogisms from the class of all the PCPs containing indefinite terms.

6 About empty sets

The four types of conclusive syllogisms may be used to settle which conclusive syllogisms are compatible with some of the sets S, P, M, S', P', M' being empty. In the modern square of opposition A(M,P), E(M,P) are not contraries anymore - unless one adds the condition M≠Ø.

Instead, when both A(M,P), E(M,P) are true, it results that M=MP'+MP=Ø. This empty set constraint (ESC) – which empties all the four subsets of M - is compatible with the universal premises of the conclusive syllogisms of type Barbara and Darapti - but not with the ei on M. Nor is the M=Ø ESC compatible with the type (3i) Darii and type (3ii) Disamis premises, I(S*, M) and I(P*, M). (In a square of opposition, (or in a square of contradiction), both A(M,P), E(M,P) being true, results in both contradictory particular premises becoming false.) In fact, since the conclusive syllogisms of the same type follow the same pattern, it results that a complete discussion of the compatibility of various ESCs and conclusive syllogisms may be reduced to examining just three representative cases (since Darii and Disamis are representatives of the same Darii pattern). Moreover, instead of firstly imposing an ESC, and then finding out the PCPs compatible with it, one can do it the other way around, by listing, for each conclusive syllogism type, the ESCs with which that conclusive syllogism type is compatible or incompatible. Darii's PCP, A(M,P)I(S,M), means MP'=Ø, SM≠Ø, and the LC is SM= SMP+ SMP'= SMP≠Ø. From the LC SMP≠Ø, one may, with some loss of information, eliminate M, and re-express the LC as I(S,P)="Some S is P". Thus Darii's PCP is incompatible with the S=Ø, M=Ø, and P=Ø ESCs, but is compatible with the S'=M'=P'=Ø ESCs, (which imply S=M=P=U; thus in this latter, extreme, case Darii's PCP and LC just assert that U is non-empty).

Darapti's PCP, A(M,P)A(M,S), means MP'=Ø, MS'=Ø, and the LC is M=MP+MP'=MP =MPS+MPS'=MPS, which may be written as A(M,SP). This time around one may eliminate M only via the ei hypothesis M≠Ø, then re-express the LC as I(S,P). Thus the ei hypothesis is incompatible with the M=Ø, S=Ø and P=Ø ESCs, but is compatible with the S'=M'= P'=Ø ESCs, (which imply S=M=P=U; therefore, in this latter, extreme case, Darapti's PCP plus the ei on M, assert only that U is non-empty). Note that Darapti's PCP without the added ei condition is compatible even with U=Ø, in which case the PCP is just "chatter about empty sets".

Barbara's PCP, A(M,P)A(S,M), means MP'=Ø, SM'=Ø, and the LCs are S=SM+SM'= SM=SMP+SMP'=SMP, and P'=P'M+P'M'=P'M'= P'M'S+P'M'S'=P'M'S'. The first LC may be written as A(S,MP), or, with some loss of information, one may eliminate M, and write A(S,P)=E(S,P'), which now refers to two subsets of U instead of referring to just one of the eight subsets of U. (A "precise" LC always pinpoints just one of the eight subsets of U.) The second LC may be written as A(P', S'M'), or, with some loss of information, one may eliminate M', and write A(P',S') =E(S,P') – the same as the first LC. Since Barbara's PCP contains only universal premises, the PCP is compatible even with U= Ø in which case all the deductions and the LCs – either "precise" or "classically expressed", are just "chatter about empty sets". One may then add an ei hypothesis, S≠Ø, to the 1st LC, and a different ei hypothesis, P'≠Ø, to the 2nd LC, to obtain, after the M, resp. M', elimination the new ei LCs: I(S,P), (Barbari), and resp., (the un-named), I(S',P'). The S≠Ø ei hypothesis means, since S=SPM, that also P≠Ø and M≠Ø, while the compatible ESCs are S'= Ø, or/and, P'= Ø, or/and, M'=Ø. The S'=P'=M'= Ø constraint amounts to Barbari affirming U≠Ø. The P'≠Ø ei condition implies that, also, S'≠Ø and M'≠Ø. If both ei hypotheses are true then all the sets M, M', S, S', P, P' are non-empty, and there are no ESCs compatible with both ei hypotheses.

In conclusion any universal premise is compatible with any ESC. But any ei hypothesis or any LC of a conclusive syllogism of type (3i) Darii, or type (3ii) Disamis, (containing one universal and one particular premise - both acting on either M or M'), specifies three sets that are non-empty, and thus pinpoints three ESCs with which the ei hypothesis or the LCs for the types (3i) or (3ii) syllogisms are incompatible. The above considerations were based on a sort of "temporal commutativity": instead of firstly applying the ESC to obtain a particular universe of discourse, and then searching for the LC in that universe, one writes down the LC in the usual 8-subset universe of discourse U, and one applies an ESC only afterwards, to see if it is compatible with the PCP and its LC.

7 How many sound VS or conclusive syllogisms may one hope to construct out of three given terms, without imposing restrictions on the structure of the universal set U

When three specific terms are given, with one of them already designated as the middle term, one may consider all the 36 or 64 PCPs which can be constructed starting with these three specific terms, (out of which one is designated as the middle term), and one can try to see what sound VS or conclusive syllogisms one may construct out of the three terms. As one shows below, given three terms, with one of them already designated as the middle term, then, at most one sound conclusive syllogism of either type (1) Barbara, or of type (2) Darapti, may be built out of the three terms without restricting U to particular cases. Since that unique conclusive syllogism can be presented either as a Barbara, (or, respectively, a Darapti), syllogism if the terms are appropriately labeled, one may say that given three terms, there exists at most one sound conclusive syllogism of types (1) or (2) – either a Barbara or a Darapti – which can be constructed out of the given three terms, (again, if one of them was already designated as the middle term). If the three given terms generate, (modulo a relabeling), a sound Barbara, then a maximum of two other type (3i) Darii and two other type (3ii) Disamis sound conclusive syllogisms may perhaps be constructed with the same given three terms without restricting U to particular cases: these new sound conclusive syllogisms have their universal premises "stolen" from Barbara and their possible particular premises placing set elements on the four subsets adjacent to the four subsets emptied by Barbara's two universal premises. One of these other possible four conclusive syllogisms is a Darii/Datisi, and the other three have no names since they assert that subsets – other than the three subsets "preferred" as LCs by Classical Syllogistic (SPM, SP'M, SP'M') – are non-empty. If the three given terms generate, (modulo a relabeling), a sound Darapti, then only two other sound conclusive syllogisms, one of type (3i) and one of type (3ii) may be constructed with the same given three terms without restricting U to particular cases: these two new conclusive syllogisms, a Darii/Datisi and a Disamis/Dimaris will have their universal premises "stolen" from Darapti, and the same LC as the ei Darapti (after the middle term is eliminated): SPM≠Ø. When the middle term is pre-determined, then two distinct PCPs of type (1) Barbara, or two distinct PCPs of type (2) Darapti, or one PCP of type (1) and one PCP of type (2), will necessarily contain either two distinct universal P-premises, or two distinct universal S-premises, (i.e., will contain term inclusions), which will impose a particular structure on the universal set U. For example, if A(M,P) and E(M,P), are both true, as P-premises in two different PCPs, that would imply M being empty, M=Ø. The relationships implied by the other five possible combinations of two universal P-premises being simultaneously true: E&E' imply P=Ø, A'&E' imply M'=Ø, A&A' imply P'=Ø, A&E' imply P=M, A'&E imply P=M', and similar relationships hold for the "top face" of the S-cube, (if one places the four universal premises on the top faces of the P and S cubes, and the respective contradictory particular premises on the bottom faces of the two "cubes of contradiction"). (One may represent the eight P-premises as vertices of a cube; similarly for the eight S-premises.) This shows, e.g., that Barbara, AE', and Camestres, E'E, can both be sound, but uninteresting, since in that universal set, P=M and S=Ø. Note that choosing another of the three terms as a middle term, leads to arguments and conclusions similar to the ones above: no two sound and distinct conclusive syllogisms of either type (1), or type (2) may be constructed with the same middle term, unless the universal set has a particular structure; no sound pair of one conclusive syllogism of type (1) and one conclusive syllogism of type (2) may be constructed with the same middle term, unless the universal set has a particular structure.

Thus, the next task is to see if by using a term once as M, and a second time, say, as P, while the term firstly used as P, is afterward used as M, one can produce two distinct PCPs of type (1) without imposing, when all four premises are true, a particular structure on U. By adjoining the "standard" Barbara's PCP, A(M,P)A(S,M), to each of the eight PCPs of type (1), having P as the middle term and S and M as end terms, A(P*,M*)A(P*',S*), one can show that, given three terms, at most one sound conclusive syllogism of type (1) may be constructed with them, without imposing a particular structure on the universal set U, i.e., in short, one may say, that at most one of the terms, (out of three given terms), may be used as the middle term in a type (1) conclusive syllogism. For example, if all the following four premises, A(M,P)A(S,M) and A(P,M)A(S,P), (where the second Barbara's PCP is obtained by switching the roles which M and P played in the first Barbara's PCP), are true, i.e., MP'=SM'=PM'=SP'=Ø, then, M=MP'+ MP= MP= MP+PM'=P. For a complete proof, one may compare, two at a time, the eight PCPs of type (1) having P as the middle term, with the "standard" Barbara's PCP. Expressing Barbara type syllogisms via E statements instead of A statements, one has: From E(M,P')E(M',S) and E(P,M) E(P',S*), it results M=Ø; from E(M,P')E(M',S) and E(P,M') E(P',S*), it results M=MP=P; from E(M,P')E(M',S) and E(P',M) E(P,S*), it results, if S*=S, that S=PS+P'S= P'S=P'SM+P'SM'=Ø+Ø=Ø, and, if S*=S', that M=MP=MPS=S; from E(M,P')E(M',S) and E(P',M') E(P,S*), it results P'= Ø.

But one may easily see that the "standard" Barbara's PCP, E(M,P') E(M',S), and these two type (2) Darapti PCPs, one having S as the middle term, E(S,P')E(S,M'), and one having P' as the middle term, E(S,P')E(M,P'), can be simultaneously sound without imposing a particular structure on U. This reflects the fact that from the chain inclusions, S⊆M⊆P, which characterize the Barbara PCP, one may deduce exactly the two Darapti type chain inclusions whose two Darapti PCPs were written above: S⊆M, S⊆P, and P'⊆M', P'⊆S'.

Even three type (2) Darapti PCPs can be simultaneously sound without imposing a particular structure on U, under the condition that their middle terms are all different. For example, E(M,P')E(M,S), E(S,P')E(S,M), E(P',M)E(P',S), use M, S, and resp. P' as middle terms, and out of their six premises only three are distinct. These three type (2) Darapti PCPs, empty a total of only four subsets of U – the same number as a single PCP of type (1) Barbara empties. Equivalently, the above three PCPs are described by these three sets of Darapti inclusions: M⊆P, M⊆S', correspond to the first Darapti listed above, S⊆P, S⊆M', correspond to the second Darapti listed above, and P'⊆M', P'⊆S' correspond to the third Darapti listed above.

8 Conclusions

In this paper the Rules of Valid Syllogism (RofVS) and the Rules of Conclusive Syllogisms (RofCS) were simplified and generalized, and I hope convincing arguments were given in Section 4 to also accept as valid syllogisms the conclusive syllogisms formulable via positive terms which entail A(P,S) and O(P,S) as logical consequences (LCs): these syllogisms satisfy all the

RofVS #1 to #5, except the supplementary (unnamed) rule that the only admissible LCs for valid syllogisms are the LCs belonging to the set L:={A(S,P), E(S,P), O(S,P), I(S,P)}.

Besides that, one can make the following observations:

(a) Two universal premises always entail at least one universal LC. Namely, if both M and M' appear in the empty set conditions asserted by the two premises, then one deals with a Barbara type PCP which entails two universal LCs: one LC asserts that either the S or the S' set is empty except for, possibly, a uniquely determined partitioning subset of U (the universe of discourse), the other LC asserts that either the P or the P' set is empty except for, possibly, a uniquely determined partitioning subset of U. The above two LCs have "opposing indexes": if, e.g., one LC is $S=S\cap M'\cap P'$, then the second LC has to be $P=P\cap M\cap S'$. (Note that, e.g., $S=S\cap M'\cap P'$, implies $A(S, S\cap M'\cap P')$ which implies $A(S, S\cap P')$, which implies $A(S, P')$.) If the middle terms are eliminated, i.e., just dropped, from the above, precise, LCs, then the two LCs become less precise, but identical: $A(S,P')=A(P,S')= E(S,P)$. Nevertheless, two independent and separate existential import (ei) conditions can be imposed on S and P: $S\neq\emptyset$ and $P\neq\emptyset$. If only M, (resp. M'), does appear in the empty set conditions asserted by the two premises, then one deals with a Darapti type PCP which entails only one universal LC asserting that M, (resp. M'), is empty except for, possibly, a uniquely determined partitioning subset of U. Two examples, out of the possible eight Darapti's type universal LCs are: $M=M\cap S\cap P$ and $M=M\cap S\cap P'$. The existential import (ei) condition has to always be imposed on the smallest set – the one included in all the other sets – in the case of the Darapti type PCPs this set is always M (or M'). Thus if an ei condition is imposed on the precise and universal two LCs above, $M=M\cap S\cap P$ and $M=M\cap S\cap P'$, then a Classical Syllogistic style, less precise, particular LC will result for each of the two examples: I(S,P) (Darapti), and I(S,P')= O(S,P) (Felapton/Fesapo). (For the Darapti type PCPs, there is no universal LC out of which the middle term was eliminated, because an ei condition has to be imposed on the middle term before eliminating it – otherwise, eliminating the "subject" of the LC removes the LC altogether. Note that the precise, one partitioning subset of U, universal LC, uniquely determines the explicit expression of the PCP which entails that precise universal LC. The less precise, Classical Syllogistic style universal LC, obtained for the Barbara type PCPs, determines the explicit expression of the PCP which entails that Classical Syllogistic style LC, up to a replacement $M\leftrightarrow M'$. For example, to the Barbara's Classical Syllogistic style LC, A(S,P), (since only Barbara type PCPs lead to universal LCs out of which the middle term was eliminated, and since cf. RofCS #1 the distributions of the end terms are conserved), correspond the Barbara and Barbara' PCPs: A(M,P)A(S,M) =E(M,P')E(M',S) and A(M',P)A(S,M')= E(M',P')E(M,S). (One defines Barbara' as having the same premises as Barbara up to the substitution $M\rightarrow M'$. Similarly, among all the conclusive syllogisms, there exist a Darapti and a Darapti', a Disamis and a Disamis', etc.)

(b) If the LC is particular, I(S*,P*), in order to recover the PCP entailing the above LC, one needs to know if the LC was obtained via existential import (ei) - and on which term, M*, S*, or P*, the ei condition was imposed, or, one needs to know if the LC is the result of an either Darii type or a Disamis type PCP. In other words, if the given LC is, e.g., I(S,P), then, if the ei was imposed on S, the PCP was the one for either Barbari or Barbari', if the ei was imposed on P, the PCP was the one for either Bramantip or Bramantip', if the ei was imposed on the middle term, the PCP was the one for either Darapti or Darapti', and if one knew that the PCP contained one universal and one particular premises then the possible PCPs are either the ones for Darii or Darii', or, the ones for Disamis or Disamis' – where the PCPs names having a prime sign are obtained from the PCPs names without a prime sign, via replacing in the PCP M by M' and M' by M.

(c) According to the RofCS #1 to #4, or to the formulas (3i) and (3ii), the Darii and Disamis type PCPs entail a particular LC; the latter formulas also mention the precise, one partitioning subset of U which is surely non empty and represents the LC.

(d) The formulas (4i), (4ii), (5i) and (5ii) characterize all the PCPs which do not entail any LCs.

The "one partitioning subset of U" paradigm, i.e., the realization that if the premises pinpoint a unique partitioning subset of U then the premises entail an LC, and otherwise there is no LC, characterizes what logical consequence (LC) means, in a way that differs from the characterization of an LC in the Classical Syllogistic which only asserts that the LC cannot be false if the premises are true. The difference between these two LC characterizations also exposes the difference between the role which the middle term plays in the Classical Syllogistic, (as a facilitator of a direct connection between the end terms S* and P*, the only terms which appear in the LC), and the role the middle term plays in the universe of discourse set model where the terms are "interpreted in extension" only, and where the middle term remains an essential part of the LC – since one cannot uniquely label a partitioning subset of U using only two terms out of the three syllogistic terms.

Note that Aristotle's definition (Striker [14]) "A syllogism is an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so", provides not only a characterization of a syllogism – both premises are necessary to obtain the LC and the LC has to validly follow from the premises - but also a justification, (or a pretext – embodied by the expression "something other than what was laid down"), for the elimination of the middle term from the LC. Nevertheless, this elimination always weakens the LC, which instead of asserting something about a unique subset of U, will now assert the same thing, less precisely, about two subsets of U – namely that two subsets might be non-empty, (e.g., Barbara's LC, A(S,P), means $S \cap P'=Ø$; therefore the LC asserts that $S=S \cap P$=: SP=SPM+SPM', although the premises already assured that SPM'=Ø), or that at least one of the two is definitely non-empty. Moreover, the contradictory statement of the weakened LC is stronger, (since it negates something about a larger number of sets), than the contradictory statement of the initial, stronger LC – which referred to a unique subset of U. (Example: compare "John lives in Miami" with "John lives in Florida": the negation of the less precise information places John out of Florida, while the negation of the stronger info about John, places him only out of Miami. It reflects the fact that negating a multiple "Or" statement produces a multiple "And" statement. Analogously, a negative statement such as "John does not live in Florida" is more powerful than the negative statement "John does not live in Miami"; by negating them, one obtains the affirmative statements "John does live in Florida" and, respectively, "John does live in Miami" – whereby out of the two latter affirmative statements, the last one is the strongest.) Thus, in Classical Syllogistic, when performing an indirect reduction, i.e., a reductio ad absurdum proof of validity, one proves a weakened LC by using stronger than necessary premises. For example, Darii's validity may be proved, by impossibility, from Camestres. But this is unnecessary: suppose, by impossibility, that Darii's precise LC, E(M,P') I(M,S):SPM≠Ø, is false, i.e., SPM=Ø (by the law of excluded middle). Then, from Darii's general premise, A(M,P), i.e., MP'=Ø, it results SM=SMP+SMP'=Ø, which already contradicts Darii's particular premise, I(S,M) or SM≠Ø – no Camestres had to be invoked, and there is no need to suppose, (the stronger), SP=Ø, (the contradictory of Darii's weakened LC, I(S,P), i.e., SP≠Ø), since supposing, by impossibility, that SPM=Ø, suffices. In Classical Syllogistic, one can obtain from Camestres' PCP, A(P,M) E(M,S), via reductio ad absurdum, its Classical Syllogistic conclusion, E(S,P) or S∩P:=SP=Ø, which is a weaker LC than each of the precise LCs provided by the "one subset of U" paradigm LCs: S=S∩M'∩P':=SP'M' and P=P∩M∩S':= PMS'). After eliminating, (i.e., dropping), the middle terms from each of these LCs, the weaker LCs out of which the middle term was eliminated, become identical: A(S,P')=A(P,S')=E(S,P). In Classical Syllogistic the reductio ad absurdum method will prove Camestres' LC by showing that the supposition S∩P≠Ø, (i.e., I(S,P), which negates E(S,P)), when paired up with any of the two of Camestres' premises, A(P,M)E(M,S), will entail an LC which directly contradicts the other of the Camestres' premises. But all this is unnecessary: Camestres premises assert that PM'=SPM'+S'P'M'=Ø, and that SM=SPM+SP'M=Ø, out of which the two precise LCs, S=S∩M'∩P':=SP'M' and P=P∩M∩S':=PMS', and their Classical Syllogistic style (and weaker) LC, A(S,P')=A(P,S')= E(S,P), easily follow.

Author's Declaration:

The submitted paper, "A Matricial Vue of Classical Syllogistic and An Extension of the Rules of Valid Syllogism to Rules of Conclusive Syllogisms with Indefinite Terms", complies with the Ethical standards - it was not submitted anywhere else and any other ethical considerations are not applicable.
The author has no conflicts of interest to declare that are relevant to the content of this article.
The research for this article did not involve asking for an Informed consent from anybody - only the author did the research.
The research for this article did not involve any Ethical approval from anybody; no Ethical approval is applicable for this article.
Thank you.

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