About the set theory model of categorical syllogisms

Abstract
One discusses valid syllogisms (VS), sorites, "distribution conservation", empty set constraints (ESC), and one compares the Classic Categorical Syllogistic (CCS) with a Set Theoretical Model (STM). A valid categorical argument (VCA) is a pair of categorical premises (PCP) together with its entailed logical consequence (LC). CCS defines a VS as a VCA which satisfies the supplementary conditions that the PCP be formulable using only the "positive" S,P,M terms, and that the LC - obtained via existential import (ei) or not - be one of the A(S,P), O(S,P), E(S,P) or I(S,P) statements. Therefore, in CCS, the P term has to be the predicate of the LC. Both the CCS moods and figures notation, and the STM PCP matrix notation use the convention that in a PCP one firstly lists the P-premise. STM interprets the S,P,M terms as sets, and also allows the "negative" terms, non-S, non-P, non-M, (denoted by S',P',M' – the complementary sets in a universal set U to S,P,M, respectively), to appear in the PCPs and LCs. To characterize a VCA, STM does not use syllogistic figures, the condition of P term being the predicate of the LC, the automatic removal of the middle term from any entailed LC, nor the rules of valid syllogisms (RofVS). In STM any LC pinpoints to just one and only one of the eight subsets partitioning U, as either being definitely not empty, (Darii type LC), or, as being the only one subset possibly non empty of either one of the sets S,P,S',P', (Barbara/Barbari type LC), or of one of the sets M,M' (Darapti type ei LC). A tree like method immediately finds the LC of a any VCA.

Keywords: *categorical syllogisms • categorical premises • Karnaugh map • valid categorical argument • valid syllogism• set relabelings • set theory model of categorical syllogisms*

1. Introduction

The universal set U contains the S,P,M sets, and S',P',M' – the complements in U of S,P,M. Thus, U is partitioned into 8 subsets: MSP:=M∩S∩P, MS'P, MSP', MS'P', M'SP, M'S'P, M'SP', M'S'P'; one observes that each of the S,P,M,S',P',M' sets is partitioned only into 4 subsets out of the 8 subsets listed above.
There are six distinct P-premises, (resp. S-premises), worded using only the "positive" terms M,P, (resp. M,S). Thus a "PCP matrix", with P-premises listed firstly – as rows, and S-premises listed secondly – as columns, has 36 distinct premises, even if, by using syllogistic figures one wrongly counts 64 PCPs as being distinct. By removing the CCS "positive terms only" restriction, i.e., allowing all the S,P,M,S',P',M' terms to appear in the premises, and by using conversions and obversions, one can write the resulting eight distinct P-premises as follows: E(M*,P*) will denote the 4 universal P-premises, and I(M*,P*) will denote the 4 particular P-premises. Similarly, one gets eight S-premises, E(M*,S*), I(M*,S*), where $M^* \in \{M, M'\}$, $P^* \in \{P,P'\}$, $S^* \in \{S, S'\}$. For example, E(M',P')=:A', meaning All M' is P, and, I(M',P')=:O', meaning Some M' is not P, are the two extra premises added to the six P-premises formulable using only positive terms. Note that the other two P-premises "acting on" M' but formulable via positive terms are A(P,M)=E(M',P)=:E', and O(P,M)=I(M',P)=:I'. This way one gets four P-

premises acting on M, E,A,I,O, and four P-premises acting on M', E',A',I',O', for a total of eight distinct P-premises. Note that A,E,I,O form a square of opposition containing M and P, and A',E',I',O' form another square of opposition containing M' and P. Together they form a cube of opposition, with, e.g., the universal statements A,E,E',A' on the top face of the cube.Similarly there are eight distinct S-premises which form another cube of opposition.

Pairing a P-premise from the "P-cube" with an S-premise from the "S-cube" one gets now 64 distinct PCPs. By convention, the P-premise will be always listed firstly. Thus AI are Darii's premises and E'I' are Baroco's premises. One may also ask if one can find sets of VCAs which might be simultaneously sound, taking into account that, e.g., A(M,P) and E(M,P) being both true as P-premises in two different VCAs would imply M being empty, M=Ø. Other relationships implied by two universal P-premises being simultaneously true: E&E' imply P=Ø, A'&E' imply M'=Ø, A&A' imply P'=Ø, A&E' imply P=M, A'&E imply P=M', and similar relationships hold for the top face of the S-cube. This shows, e.g., that Barbara, AE', and Camestres, E'E, can both be sound, but uninteresting, since in that universal set, P=M and S=Ø. (More about ESC in Section 6.)

In STM, the task of finding if a PCP entails an LC, or not, was already solved by George Boole and Lewis Carroll: there are three types of PCPs, such that each PCP entails at least one LC, and another two PCP types, each type containing 16 PCPs, which do not entail any LC. These latter two types contain the following PCPs: I(M*,P*)I(M*,S*) and I(M*,P*)I(M*',S*) - they are the PCPs made of two particular premises, and, E(M*,P*)I(M*',S*) and I(M*,P*) E(M*',S*) - i.e., the PCPs made of one universal and one particular premises, acting one on M and the other on M'. (Note that M*'=M if M*=M', etc.) Each of the eight type (1), (or type Barbara), E(M*,P*)E(M*',S*), PCPs, entails two LCs – since these PCPs are in fact sorites with just one middle term. By handling these PCPs using a tree like method, (similar to Lewis Carroll's method of subscripts), and reading the sorite in the Aristotelian way, i.e. starting with the S* term of the PCP, one gets: S*=S*M*+S*M*'=S*M*= S*M*P*+ S*M*P*'= S*M*P*', where S*M*P*':= M*∩S*∩P*', etc., and the + sign denotes union of disjoint sets. Reading the sorite in the Goclenian way, i.e. starting with the P* term of the PCP, one gets the 2nd LC: P*=P*M*+P*M*'= P*M*'= P*M*'S*+P*M*'S*'=P*M*'S*'. It is even easier to find the LC of the eight type (2), or type Darapti PCPs, E(M*,P*)E(M*,S*). They entail just one LC: M*=M*P* +M*P*'= M*P*'= M*P*'S*+ M*P*'S*'=M*P*'S*'. Thus the middle term is very much part of the LC; it can be eliminated only via ei on M*. (Aristotle's definition, (Striker 2009: p.20), "A syllogism is an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so", provides a justification for the elimination of the middle term from the LC. But this elimination weakens the LC, which instead of asserting something about a unique subset of U, will now assert the same thing, less precisely, about two subsets of U – namely that they might be non-empty or that at least one of them is definitely non-empty. Moreover, the contradictory statement of the weakened LC is stronger than the contradictory statement of the initial, stronger LC. Thus the usual indirect reduction proofs of validity prove weakened LCs using stronger than necessary premises. See Section 3.) If one chooses M*=M, P*=P', S*=S' and M≠Ø one obtains the PCP and the ei LC, I(S,P), of the ei VS Darapti. But if one chooses M*=M, P*=P, S*=S and M≠Ø, then the PCP, E(M,P) E(M,S), contradicts the RofVS - two negative premises are not allowed - and its ei LC, I(S',P'), is not among the four ei VS LCs admitted by the CCS. Nevertheless E(M,P)E(M,S): M=MS'P' is a VCA, (with the LC completely specified after the column sign), and E(M,P)E(M,S): I(S',P'), M≠Ø is an ei VCA. Similarly, the VCA, E(M',P) E(M',S)= A(P,M)

A(S,M): M'=M'S'P', does not satisfy the RofVS, “the middle term has to be distributed in at least one premise”, or, equivalently, its LC and ei LC, I(S',P'), M≠Ø, fail to be one of the four admitted VS LCs. Note also that after ei is imposed on any of S,S',P,P',M,M' sets left with only one possible non-empty subset (out of four) - as a consequence of the two universal premises of types either (1) or (2) having been taken into account, the resulting ei LC is identical with an LC of a PCP of type (3): one of the 8 subsets of U is definitely not empty. There are 16 distinct PCPs of type (3), split into two subtypes of eight PCPs each: (3a), E(M*,P*)I(M*,S*), (or Darii/Datisi subtype), and, (3b), I(M*,P*) E(M*,S*), (or Disamis/Dimaris subtype). The very short trees “revealing” the LCs are: Ø≠M*S*=M*S*P*+ M*S*P*'=M*S*P*', and, resp., Ø≠M*P*= M*S*P*+M*S*'P*= M*S*'P*. Types (1), (2), (3a) and type (3b), together, account for all the 32 PCPs which entail at least one LC. (As mentioned, the other 32 PCPs do not entail any LC.)
In STM, the relabelings p:= P↔P', s:=S↔S', m:=M↔M' and their compositions transform VCAs into VCAs of the same (sub)type. The relabeling S↔P obviously maps subtype (3a) VCAs onto the subtype (3b) VCAs and vice versa. If one adheres to the convention that the premise firstly listed contains the P term, the relabeling S↔P amounts, when specific terms are substituted for the letters S and P, to a switch of the premises' order. The rest of the paper develops the VCA focused STM, and compares it to the VS focused CCS.

2. STM and the Karnaugh map for n=3 (drawn for subsets instead of truth values)

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

Fig. 1

The universal set U is graphed as a rectangle – but the left and right borders of the rectangle are glued together to generate a cylinder, so that S'PM and S'P'M are adjacent, and S'PM' and S'P'M' are adjacent, too – as in the usual 3-circle Venn diagram. Since the 8 subsets of Figure 1 are the “elementary” subsets of U, one calls them just subsets; no other set will be a “subset”. On this (Marquand-Veitch-)Karnaugh map one can see that any universal premise empties two “horizontal subsets” located either on the M or the M' row, and its contradictory particular premise places set elements in at least one of the same two “horizontal subsets” emptied by the contradictory universal premise. This makes it clear that a PCP made of two particular premises does not entail any LC (beyond a possible verbatim repetition of premises' content). Also, there is no LC entailed by any PCP made of one universal premise emptying two subsets on M', and a particular premise “acting on M”, i.e. placing set elements into two subsets of M, (or the other way around). As mentioned, in STM, any LC is characterized by the fact that it singles out just one of the 8 subsets partitioning U, and only three types of LCs are possible: (1) One of the sets S or S' and one of the sets P or P', have each three empty subsets and one possibly not empty subset (hence each type (1) PCP entails two LCs), (2) One of M or M' has three empty subsets and one possibly not empty subset, (3) One of the subsets of U is definitely not empty. These three types of LCs are entailed by the following three types of PCPs, respectively: (1) PCPs made of two universal premises, emptying a total of four subsets on different rows - M and M'. When, e.g., Celarent's premises empty three subsets of S, they also empty three subsets of P – hence the two LCs entailed by only one PCP, (2) PCPs made of two universal premises both emptying subsets on the same row – either M or M', (3) PCPs made of one universal plus one particular premises, one emptying, and one placing elements on subsets of the same row.

As also mentioned, the 8 distinct P-premises, may be expressed via only the quantifiers E and I: E(M,P), A(M,P) =E(M,P'), A(P,M)=E(M',P), A(M',P)=E(M',P'), I(M,P), O(M,P)=I(M,P'), O(P,M)=I(M',P), O(M',P)= I(M',P'). In shorthand notation, these may be written either as E,A,E',A',I,O,I',O' or as E(M*,P*), I(M*,P*) where $M^* \in \{M, M'\}$, $P^* \in \{P,P'\}$. The "prime signs" on E',A',I',O' mean that one of their arguments has to be M'; the other argument, either P or S is determined by the position in which the quantifiers appear in a PCP: namely, since one agrees to always firstly list the premise containing the P term, Barbara's premises are AE', Bramantip PCP is E'A, etc. In STM, being part of the premise listed first, is the only condition that defines the P term. The CCS adds the condition that the P term should also be able to serve as the predicate of the LC, (if any is entailed by the PCP), since the only accepted LCs of a VS are the A(S,P), O(S,P), E(S,P) or I(S,P) statements.

To recap - there are three types of PCPs which generate each one or two VCAs:

(1) E(M*, P*)E(M*', S*), $S^* \in \{S, S'\}$, $M^* \in \{M, M'\}$, $P^* \in \{P,P'\}$. For each of the eight PCPs, to get the two LCs, start two trees with S*, resp. P*: S*=S*M*+S*M*'=S*M*= S*M*P*+S*M*P*'=S*M*P*' and P*=P*M*'+P*M*=P*M*'=S*M*'P*+S*'M*'P*= S*'M*'P*, where the + sign denotes union of disjoint sets. As mentioned, Barbara, A(M, P)A(S, M)=E(M, P')E(S, M')=AE', has two LCs: S=SM'+SM =SM= SMP'+SMP=SMP and P'=P'M'+P'M=P'M'= SM'P' + S'M'P' = S'M'P' . These distinct LCs, S=SMP and P' = S'M'P', after being weakened via discarding the middle terms, turn out, by contraposition, to become identical: A(S,P)=A(P',S')=E(S,P'). Existential import (ei) on S leads to the ei LC I(S,P), Barbari, and ei on P' leads to a different ei LC I(S',P').

(2) E(M*,P*)E(M*,S*), $S^* \in \{S, S'\}$, $M^* \in \{M, M'\}$, $P^* \in \{P,P'\}$. These eight PCPs, entail each just one LC: M*=M*P*+M*P*'= M*P*'= M*P*'S*+ M*P*'S*'=M*P*'S*'.

For type (2) PCPs, choosing M*=M, S*=S', P*=P and M≠Ø the ei LC is I(S,P')=O(S,P), i.e., one gets the ei VS Felapton and/or Fesapo. But for S*=S and P*=P and M* being either M or M', the ei LC becomes I(S',P'), which is unacceptable as an ei VS LC. Equivalently, one may argue that the ei VCA generated by, E(M,P)E(M,S), (resp. E(M',P)E(M',S)), is not a VS because it does not satisfy the RofVS,"two negative premises are not allowed", (resp., "M has to be distributed in at least one premise").

Finally, the 16 PCPs made of one particular plus one universal premises, both acting on the same row, split into two sets of 8 PCPs, each PCP entailing one LC:

(3a) E(M*, P*)I(M*,S*): S*M*=S*M*P* + S*M*P*' = S*M*P*'≠Ø. (One starts the very short tree with the non-empty set and then one removes its empty subset/part.)

(3b) I(M*,P*)E(M*, S*): P*M*=S*M*P* + S*'M*P* = S*'M*P*≠Ø

One can notice that, e.g., a (3a) PCP, E(M, P')I(S, M), Darii/Datisi, (obtained when choosing S*=S, M*=M, P*=P' in (3a)), and a (3b) PCP, I(M, P)E(S', M), Disamis/Dimaris, (choose S*=S', M*=M, P*=P in (3b)), have the same LC: SMP≠Ø. Analogously Bocardo, O(M,P) A(M,S)=I(M,P')E(M,S') and Ferio/Festino/Ferison/Fresison, E(M,P)I(M,S), have the same LC: SMP'≠Ø. Note that all the (3a) VCAs become the (3b) VCAs via a relabeling S↔P, i.e., on specific VCA examples, one just have to switch the premises' order to obtain a (3b) VCA from a type (3a) VCA, and vice versa, and to keep up with the convention of listing firstly the P-premise.

By working with 64 distinct PCPs, STM "raises" on an equal footing any of the 8 subsets partitioning U. Because of this, each one of the 8 subsets of U, S*P*M*, appears as the "subject" of an LC exactly eight times: three times as a "possibly non empty subset" LC of types (1) and (2), S*=S*P*M*, P*=S*P*M*, (these are LCs of the 8 PCPs of type (1), each entailing two LCs per PCP), and M*=S*P*M*, (this is the LC of one of the eight type (2) PCP), and five times as a " definitely non empty subset" LC, (twice as the LC S*P*M*≠Ø of PCPs of types (3a) and (3b), and 3 times as the ei LC generated by the previous three "universal LC" using ei on S*,P* and resp. M*. This shows that STM "likes equally" each of the U's eight subsets.
By contrast, CCS "likes only three subsets", SPM, SP'M, and SP'M': any VS LC asserts that either one of the above three subsets is not empty, (if the PCP contains one particular and one universal premises, both acting on the same set - either M or M'), or that one of the above three subsets is the only part of either S,P or M which is possibly left non empty by a PCP containing two universal premises. More precisely, SPM appears as LC of Barbara, Barbari, Bramantip, (but there is no "Bramanta"), Darapti, Darii/Datisi, Disamis/Dimaris; SP'M appears as LC of Celarent/Cesare, Celaront/Cesaro, Felapton/Fesapo, Bocardo, Ferio/Festino/Ferison/ Fresison; SP'M' appears as LC of Camestres/Camenes, Camestros/Camenos and Baroco. Out of the 64 distinct PCPs, the ones which do not entail any LC are the 16 PCPs containing two particular premises and the 16 PCPs containing one universal premise plus one particular premises acting on different rows, M and M'. Out of the 36 distinct PCPs formulable using only the "positive" S,P,M terms, the ones which do not entail any LC are the 9 PCPs containing two particular premises and the 8 PCPs containing one universal premise plus one particular premises acting on different rows, M and M'.

3. Inside each of the three types of VCA, any VCA can be re-written, without changing its PCP nor LC "content", as any other VCA of the same type

One can define a "relabeling group" acting inside each of the types (1), (2), (3a), and (3b) VCA sets. Let p:= P↔P', s:=S↔S', m:=M↔M'. One can see that compositions of s,p,m generate a commutative group G with eight distinct elements:1,s,p,m,sp,sm,pm, spm. Obviously $1=s^2=p^2=m^2=(spm)^2=(ms)^2=(ps)^2=(pm)^2$. Each of the above 4 sets of VCAs of types (1), (2), (3a) and (3b) respectively, is invariant under the action of G. For example, the m relabeling transforms Barbara into a Barbara' and vice-versa, (and Darapti into a Darapti' and vice-versa), p(Celarent/Cesare)=Barbara, m(Celarent/Cesare) =(Camestres/Camenes), etc. Since, via set relabelings, all variability inside each of the VCA sets (1), (2), (3a) and (3b) is accounted for, all the VCA from each set are logically equivalent. The S↔P relabeling, when combined with the convention to firstly list the P-premise, amounts to changing the premises' order. CCS uses it for syllogism reduction to the 1st figure; it transforms the type (3a) VCAs into type (3b) VCAs and vice versa.
For example, spm(E'E')=spm(E(M',P)E(M',S))=E(M,P')E(M,S') =A(M,P) A(M,S)=AA which are Darapti's premises with the LC M=MSP. This means that E'E'=E(M',P)E(M',S)=A(P,M) A(S,M) become Darapti's premises after an spm relabeling. Equivalently one may read E'E', i.e., All P is M, All S is M, as All non P' is non M', All non S' is non M' which by contraposition means All M' is P', All M' is S', which now reads like a Darapti in the variables M',P',S', with the LC M'=M'S'P', which is still the LC of E'E'.

The 32, (out of the distinct 64), PCPs which entail at least one LC per PCP, were classified as being of types (1), (2), (3a), and (3b) – each of the these types containing 8 PCPs. One may arrange these 32 PCPs into 8 sets of four PCPs each, such that the variables in each set are (almost) the same. One numbers the sets from 1 to 8 and one lets the relabeling group G act on these eight sets of four PCPs each. Below, the first PCP in such a set of four PCPs always belongs to type (2) PCP, the following two belong to type (3a) and (3b) PCP and the last one to type (1) PCP – this latter PCP entails two LCs and thus generates two VCAs, from which two ei VCAs can further be inferred. We'll say that each of such a four PCPs set is "bound to" a same subset of U: the four PCPs do not act at all on the subset of U on which they are all "bound", but act on some of its "neighbours" in the Karnaugh map. Thus to each of the 8 subsets of U one "attaches" a set of four PCPs "bound" to it. Listing on one column the four PCPs, and on the second column their LCs, these eight sets of PCPs and LCs are:

1. VCAs "bound to" the subset S'P'M:

EE=E(M,P)E(M,S) M=S'P'M. If M≠Ø: I(S',P'), No name
IE=I(M,P)E(M,S) S'PM≠Ø or O(P,S), No name
EI=E(M,P)I(M,S) SP'M≠Ø or O(S,P), Ferio/Festino/Ferison/Fresison
EE'=E(M,P)E(M',S) S=SP'M, P=S'PM', E(S,P), Celarent/Cesare
O(S,P) if S≠Ø, Celaront/Cesaro; O(P,S) if P≠Ø, No name

2. VCAs bound to the subset SP'M:

EA=E(M,P)E(M,S') M=SP'M. If M≠Ø: O(S,P), Felapton/Fesapo
IA=I(M,P)E(M,S') SPM≠Ø or I(S,P), Disamis/Dimaris
EO=E(M,P)I(M,S') S'P'M≠Ø or I(S',P'), No name
EA'=E(M,P)E(M',S') P=SPM', S'=S'P'M, A(P,S)=A(S',P'),
I(S,P) if P≠Ø, Bramantip' (the prime refers to M' in P=SPM'); I(S',P') if S'≠Ø, No name

3. VCAs bound to the subset S'PM:

AE=E(M,P')E(M,S) M=S'PM. If M≠Ø: O(P,S), No name
OE=I(M,P')E(M,S) S'P'M≠Ø or I(S',P'), No name
AI=E(M,P')I(M,S) SPM≠Ø or I(S,P), Darii/Datisi
AE'=E(M,P')E(M',S) S=SPM, P'=S'P'M', A(S,P), Barbara
I(S,P) if S≠Ø, Barbari; I(S',P') if P'≠Ø, No name

4. VCAs bound to the subset SPM:

AA=E(M,P')E(M,S') M=SPM. If M≠Ø: I(S,P), Darapti
OA=I(M,P')E(M,S') SP'M ≠Ø or O(S,P), Bocardo
AO=E(M,P')I(M,S') S'PM≠Ø or O(P,S), No name
AA'=E(M,P')E(M',S') S'=S'PM, P'=SP'M', E(S',P'), No name
O(P,S) if S'≠Ø, No name; O(S,P) if P'≠Ø, No name

M' row VCAs:

5. VCAs bound to the subset S'P'M':

E'E'=E(M',P)E(M',S) M'=S'P'M'. If M'≠Ø: I(S',P'), No name
I'E'=I(M',P)E(M',S) S'PM'≠Ø or O(P,S), No name
E'I'=E(M',P)I(M',S) SP'M'≠Ø or O(S,P), Baroco

E'E=E(M',P)E(M,S) S=SP'M', P=S'PM, E(S,P), Camestres/Camenes
O(S,P) if S≠Ø, Camestros/Camenos; O(P,S) if P≠Ø, No name

6. VCAs bound to the subset SP'M':
E'A'=E(M',P)E(M',S') M'=SP'M'. If M'≠Ø: O(S,P), Felapton'/Fesapo'
I'A'=I(M',P)E(M',S') SPM'≠Ø or I(S,P), Disamis'/Dimaris'
E'O'=E(M',P)I(M',S') S'P'M≠Ø or I(S',P'), No name
E'A=E(M',P)E(M,S') S'=S'P'M', P=SPM, E(S',P)=A(P,S), No name
I(S,P) if P≠Ø, Bramantip, I(S',P') if S'≠Ø, No name

7. VCAs bound to the subset S'PM':
A'E'=E(M',P')E(M',S) M'=S'PM'. If M'≠Ø: O(P,S), No name
O'E'=I(M',P')E(M',S) S'P'M'≠Ø or I(S',P'), No name
A' I'=E(M',P')I(M',S) SPM'≠Ø or I(S,P), Darii'/Datisi'
A'E=E(M',P')E(M,S) S=SPM', P'=S'P'M, A(S,P)=A(P',S'), Barbara'
I(S,P) if S≠Ø, Barbari'; I(S',P') if P'≠Ø, No name

8. VCAs bound to the subset SPM':
A'A'=E(M',P')E(M',S') M'=SPM'. If M'≠Ø: I(S,P), Darapti'
O'A'=I(M',P')E(M',S') SP'M' ≠Ø or O(S,P), Bocardo'
A'O'=E(M',P')I(M',S') S'PM≠Ø or O(P,S), No name
A'A=E(M',P')E(M,S') S'=S'PM', P'=SP'M, E(S',P'), No name
O(P,S) if S'≠Ø, No name; O(S,P) if P'≠Ø, No name

One can check that the group G acts on the above PCPs and LCs sets 1,2,...,8, as follows: p(1)=3, p(2)=4, p(5)=7, p(6)=8; s(1)=2, s(3)=4, s(5)=6, s(7)=8; m(1)=5, m(2)=6, m(3)=7, m(4)=8.
One can also check that {G(1)}= {G(2)}= …={G(8)}= {1,2,3,...,8}. This shows that any VCA from any of the four VCA (sub)types can be recast as any other VCA of the same (sub)type. (A change of the order of premises, together with the convention of firstly listing the P-premise, completes the transformation of a subtype (3a) VCA into a subtype (3b) VCA and vice versa.)
To “shrink” the list of 36 PCPs to only those that will generate the eight Boolean non ei VS, plus 6 ei VS, (see Hurley p. 291, Copi, pp. 240-248), one only needs to observe that, by definition, a VCA is acceptable as a VS only if its LC is an A,O,E, or I statement applied to the ordered pair (S,P). If an O(P,S) or an A(P,S) statement results as the LC of a VCA, then such a VCA is repurposed, (even if it already satisfies all the RofVS!), as a VS via a P↔S relabeling, followed by listing its premises in reverse order. – then, from the two VS candidates having “switched S, P terms”, one retains at most one - the one, (if any), having two true premises. A VCA whose LC is I(S',P') is not counted as a VS since its LC contains negative terms, or, equivalently, because it does not satisfy the RofVS. All that is left from the 36 PCPs after also discarding those PCPs without an LC, are the 11 PCPs generating the 8 non ei VS and the 3 ei VS. They correspond, when syllogistic figures are taken into account, to the 15 non ei VS plus 4 ei VS: Bramantip, (obtained via ei on P from a PCP which without ei generates a VCA whose LC is A(P,S), and thus that PCP is repurposed as Barbara's PCP), Darapti and Felapton/Fesapo, (obtained via ei on M). For these 4 ei VS with particular LCs, there are no corresponding VS

having universal, “stronger” LCs. Another 3 Boolean ei VS, resp. 5 ei VS when figures are counted, are obtained via ei on S, by “weakening” universal LCs: Barbari, Celaront/Cesaro and Camestros/Camenos. This brings the total to the usual 24 VS and ei VS. Note that, e.g., Keynes, Stebbing, Copi, Hurley prove that a VCA whose LC is one of the statements A(S,P), O(S,P),E(S,P) or I(S,P), necessarily satisfies the RofVS, (but a VCA whose LC is A(P,S) or O(P,S), also satisfies the RofVS), and that, vice versa, the only VCAs having as LC one of the A(S,P), O(S,P),E(S,P) or I(S,P) statements, and, moreover, satisfy the RofVS, are the eight Boolean VS Barbara, Celarent/Cesare, Camestres/Camenes, Bocardo, Disamis/Dimaris, Darii/Datisi, Ferio/Festino/Ferison/Fresison, and Baroco. Thus, in CCS, for the proofs that RofVS are necessary and sufficient conditions for a VS, the universe of discourse is, by hypothesis, restricted to the PCPs formulable using only positive terms, and which, moreover, entail one of the LCs A(S,P), O(S,P),E(S,P) or I(S,P), i.e., the universe of discourse is restricted to the VS to begin with. Since in STM the explicit LCs are easily found via a “tree like method”, all the RofVS discussions become unnecessary. In STM, a Boolean VS, (or ei VS), is by definition a VCA, whose PCP is formulable using only positive terms, and whose entailed LC, (or ei LC), is one of the A(S,P), O(S,P), E(S,P), or I(S,P) statements.

4. Comments on distribution and the rules of valid syllogisms

“A term is said to be distributed when reference is made to all the individuals denoted by it; it is said to be undistributed when they are only referred to partially, i.e., information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class.” (Keynes, 1887). One may agree that an E(M,P) statement refers to both sets in their totality and thus in E(M,P) both M and P are distributed. Then one can see that the usual definition of distribution can be recovered just by using obversion and negation of statements, and by proclaiming that in any statement the complement of a set has a distribution opposite to the set itself and the negation of a categorical statement changes the distribution of all its terms. According to the latter definition, in E(M,P), M' and P' are undistributed, (indeed M' includes P, etc.), and in I(M,P), both M and P are undistributed, (since they were distributed in the contradictory E statement), and M' and P' are distributed; in an A(M,P)=E(M,P') statement, M and P' are distributed while P and M' are not, and in its contradictory O(M,P) statement, M and P' are thus undistributed, while P and M' are distributed.

One may now see, from the very way in which the LCs are obtained for each of the three types of VCAs, that “distribution is conserved”, i.e., the S* and P* terms have the same distribution in the premises and in the LC(s):

(1) The LC, S* = S*M*P*', (see Section 2), can be expressed as All S* is P*'. Thus, in the LC, S* is distributed and P*' is not – the same term distributions as in the E(M*,P*) E(M*',S*) PCP of type (1), where both S* and P* were distributed. In the second LC of the same PCP, P*=S*'M*'P* translates to All P* is S*', and thus the distribution is again conserved. Imposing the ei conditions, which assure the validity of immediate sub-alternation inferences, one gets the ei LCs I(S*,P*') and I(P*,S*') which, as expected, do not conserve anymore the distributions that S* and P* had in the PCP.

(2) The LC, M* = S*'M*P*', leads, after ei on M to the ei LC I(S*',P*') - the

distributions of S* and P* are again the same in the LC, as they were in the E(M*, P*)E(M*, S*) PCP of type (2).

(3a) and (3b) Their respective LCs can be written I(S*,P*') and I(P*,S*'). One sees that in the I(S*, M*) E(M*, P*) and I(P*, M*)E(M*, S*) PCPs, and in their LCs, the S* and P* terms have the same distributions, i.e., the distribution is conserved.

The above proves that, with the exception of VCA obtained via ei on S* or P*, the S* and P* terms are distributed in the PCP of a VCA if and only if they are distributed in its LC. This adds more precision to the syllogistic rule "Any term distributed in the conclusion must be distributed in the premises", by pointing out the only cases of ei - on S* or P* - when an "end term", S* or P*, can be distributed in the premises without being distributed in the conclusion. It also turns this RofVS, into an if and only if theorem applicable to any VCA, with the exception of the ei VCA obtained via ei on S* or P*.
Note that, except for the four PCPs of type (2) acting only on M', E(M',P*)E(M',S*), in any PCP entailing at least one LC, the middle term M is distributed at least once. (For example, if the PCPs (3a) and (3b) contain M, then M is distributed in the the universal premise, if they contain M', then M is distributed in the the particular premise.) In the case of the E(M',P*) E(M',S*) PCPs, one may argue that the real middle term is M' not M, or, in classical fashion, one may discard a PCP similar to "All dogs are animals", "All cats are animals", as not entailing an LC, (meaning, in fact, an LC formulable with positive terms only), because the middle term does not satisfy the RofVS "The middle term has to be distributed in at least one premise". As mentioned above, the LC of the above PCP with undistributed middle is actually M'=M'S*'P*', where M=animals, P*=dogs, S*=cats, which translates to "All non-animals are non-dogs and non-cats". Using ei on M' (the non-animals), one finds out that "Some non-cats are non-dogs", take, e.g., mosquitoes. Thus the rule that the middle term M has to be distributed in at least one premise eliminates, out of the 36 PCPs containing just positive terms, the only one PCP of type (2) acting on M': A(P,M) A(S,M). Note also that Baroco, All P is M, Some S is not M, (or E'I'), is of type (3a), and acts on M' – with M undistributed in the universal premise and M' distributed in the particular premise. But I'E':O(P,S), which also satisfies all RofVS, is nevertheless discarded due to the "wrong O(P,S) LC". In general, if an αβ PCP has an A(S,P) or O(S,P) LC, then the LC of the βα PCP will be A(P,S) resp. O(P,S). This syllogism, "Some barking creatures are not dogs", "No cats are barking creatures", Therefore "Some not dogs are not cats" is invalid – since two negative premises are not allowed. Equivalently, what this syllogism does not satisfy is the VS definition that the LC has to be one of the operators A,E,I,O applied to the ordered pair (S,P); S and P are cats and dogs, but the LC contains negative terms – it can not be written as one of A,E,I,O applied to the ordered pair (cats,dogs) or (dogs, cats). Thus the RofVS, ban negative term syllogisms – which brought the ire of Lewis Carroll. See Carroll 1986, p.240 or Carroll, 1958, p.173. Note that only 15 VS do exist under the RofVS used by Copi 2009, by Hurley 2008, and by Keynes 1887: "the middle term has to be distributed in at least one premise", "two negative premises are not allowed", (these two syllogistic rules refer to the PCPs only – the following three rules refer to the entire syllogism, PCP and LC), "any term distributed in the LC must be distributed in the PCP", "if either premise is negative, the LC must be negative", "from two universal premises, no particular LC may be drawn" (unless ei is used – one might add!). Using the above definition of a VS, (PCP formulable with only positive terms and LC one of the A(S,P), O(S,P), E(S,P) or I(S,P) statements, one may show that a VS necessarily satisfies all of RofVS, and, vice versa, supposing that a VS is defined as above, the RofVS are also sufficient to show

that only 15 VS do exist - if syllogistic figures are taken into account. From a Boolean point of view only 8 VS are distinct (e.g., Ferio, Festino, Ferison, Fresison are 4 different names corresponding to the same pair of premises, “No M is P”, “Some M is S” - add the LC “Some S is not P” and they generate just one VS). (Hurley p.291, mentions this Boolean point of view.)

When three specific terms are given, writing down all 36 or 64 PCPs means trying to see what sound VS may one get out of the three terms. In the above example one saw that IE has true premises when M=barking creatures, P=dogs, S=cats. This PCP leads to a true A(P,S) LC. But any EI, i.e., Ferio type VS with the same terms has false premises. If one switches the IE premises one gets a Ferio type VS with true premises. In this case, repurposing, via switching the premises' order, the IE PCP, as a Ferio's EI PCP, could not produce confusion since at most one PCP out of the two may have true premises. Contradictory or contrary relationships hold as well between the other four PCPs having A(P,S) or O(P,S) as LCs, and their corresponding VS with A(S,P), O(S,P) LCs: E'A, implies $P\subseteq S$, and AE' (Barbara), implies $S\subseteq P$, i.e., unless $P\equiv S$, the PCPs contradict each other “pairwise”. AE=A(M,P) E(M,S) and EA=E(M,P)A(M,S) (Felapton/Fesapo) have premises contraries to one another. Thus, unless P, or S, or both, are empty sets, if AE premises are true, those of Felapton/Fesapo are false and vice versa. AO and OA (Bocardo) have contradictory premises, and I'E' and E'I' (Baroco) also have contradictory premises.

Not admitting negative terms in the LC is what justifies discarding the I(S',P') LC and the corresponding PCPs that entail it: EE, E'E', OE, EO. Using the above shorthand notation and the convention of listing the P-premise first, EE means E(M,P)E(M,S) and E'E' means A(P,M)A(S,M), etc. Equivalently, one may use the RofVS “two negative premises are not allowed” and “the middle term has to be distributed in at least one premise” to justify removing these 4 PCPs from the list of VS candidates.

As noted above, if the LC is an E or I statement, CCS recognizes, by accepting the validity of the conversions, E(S,P)=E(P,S) and I(S,P)=I(P,S), that it can not pinpoint/define anymore which one is the (“true”) predicate. This explains why Bramantip is an accepted ei VS, but “Bramanta” has to be repurposed as Barbara.

If one compares E'E (the premises of Camestres/Camenes) with EE' (the premises of Celarent/Cesare) one sees that the two pairs of premises are contraries to each other – even if both are recognized as distinct VS, and no switching around of the premises is required. The same direct reduction transforms one VS into the other one via a switch of premises' order and/or a relabeling/re-lettering $P\leftrightarrow S$. If one works in abstracto, with term variables, the re-lettering has to be explicitly performed; if one works with specific terms, the re-lettering is implicitly performed as soon as the premises' order is switched - the old subject becomes predicate and vice versa – in accord with the convention that the premise listed firstly contains the P term. Thus starting with a concrete example of a Camestres/ Camenes' PCP, All barking animals are dogs, No cats are dogs, the LC is, (fairly enough), No barking animals are cats. But just by firstly uttering No cats are dogs, and, then, All barking animals are dogs, and following the convention that the premise listed first contains the P term, one gets Celarent/Cesare's PCP, having the same LC, and, in the CCS view, what was done above is a direct reduction of Camestres/Camenes to Celarent/Cesare, and no PCP repurposing is implied or necessary. Note also that instead of switching the premises' order and performing an implicit re-lettering $S\leftrightarrow P$ in accord with the convention that the premise listed first contains the P term, one may perform the transformation of Camestres/Camenes to Celarent/Cesare and vice versa via a re-lettering

M↔M', or, equivalently, one may leave the Camestres/Camenes premises' order unchanged, but re-express them using M'=non-dogs instead of M=dogs: thus All barking animals are dogs becomes All barking animals are not non-dogs= No barking animals are non-dogs, and No cats are dogs becomes No cats are not non dogs=All cats are non-dogs. One thus again obtains, from a Camestres/ Camenes PCP, a Celarent/Cesare PCP where the middle term is now non-dogs, but still the "content" and LC of the "new" Celarent/Cesare are the same as the content and LC of the Celarent/Cesare obtained via classical/Aristotelian direct reduction of the initial Camestres/ Camenes. In general, without changing its content or its LC, any VCA may be re-written as any other VCA of the same type. Since there are three different types of VCA, (and VS), any VCA may be written as either a Barbara/Barbari, a Darapti, or a Darii VS– see below. By contrast, the Celarent/Cesare PCP with the same P= barking animals and S=cats, has false premises: No barking animals are dogs, All cats are dogs.
Since the LC is the symmetric E(S,P), the premises of Celarent/Cesare, E(M,P)A(S,M), and of Camestres/ Camenes, A(P,M)E(M,S) are both accepted by CCS as generating distinct VS even if their premises are contraries. As are accepted the premises of both Barbari and Bramantip, whose LCs are the same, I(S,P) - after ei on S and resp. P. But "Bramanta"'s PCP – which is the same as Bramantip's PCP – is repurposed as Barbara's PCP, with no implied confusion, since if its premises are true, Barbara's premises, built with the same P and S as Bramanta's before the switch S↔P, are false. In STM, P and S are just term names; by convention, the term which appears in both premises is denoted by M, the other term in whatever premise is listed firstly is denoted by P, and the third term is denoted by S; A(P,S) and O(P,S) are accepted LCs, and no PCP is repurposed via its premises' order being switched around. Instead, STM discards redundant syllogistic figure information: Ferio/Festino/Ferison/Fresison are "lumped together" as a VS whose PCP is E(M,P)I(M,S) and LC is O(S,P). But STM has "nothing against" a VCA whose PCP is, e.g., I(M,P)E(M,S) and whose LC is thus O(P,S). The VS Barbara may be written as a Celarent/Cesare VS, or vice versa, via a P↔P' relabeling, and the whole set of 8 VCA of the same type as Barbara, Celarent/Cesare, Camestres/Camenes, "Bramanta", etc., transform into one another via the relabelings p:= P↔P', s:=S↔S', m:=M↔M' and their compositions – see Section 3 above.

5. Sorites

Using the same notations as those from the beginning of Section 3, the three types of PCPs which generate VCAs may be immediately extended to sorites of the same type:

Type (1). The premises of the "Barbara type" Aristotelian/Goclenian sorite:

SOR1:=$E(S^*, M_1^*)E(M_1^{*\prime}, M_2^*)E(M_2^{*\prime}, M_3^*)...E(M_n^{*\prime}, P^*)=A(S^*, M_1^{*\prime})A(M_1^{*\prime}, M_2^{*\prime})\,A(M_2^{*\prime}, M_3^{*\prime})...A(M_n^{*\prime}, P^{*\prime})$
LC_1: $S^*=S^*M_1^{*\prime}M_2^{*\prime}M_3^{*\prime}...M_n^{*\prime}P^{*\prime}$ (the Aristotelian logical consequence)
LC_2: $P^*=P^*M_n^*...M_3^*M_2^*M_1^*S^{*\prime}=P^*M_1^*M_2^*M_3^*...M_n^*S^{*\prime}$ (the Goclenian logical consequence of the same premises. This explains why, e.g., Barbara has two different LCs.)

Type (2). The premises of the "Darapti type" sorite are:

SOR2:=$E(S^*, M^*)E(M_1^*, M^*)E(M_2^*, M^*)...E(M_i^*, M^*)E(M_{i+1}^*, M^*)...$
$E(M_n^*, M^*)E(M^*, P^*)$

LC: $M^*=M^*S^{*'}M_1^{*'}M_2^{*'}M_3^{*'}...M_i^{*'}M_{i+1}^{*'}...M_n^{*'}P^{*'}$ with, if $M^*\neq\emptyset$, some of the many ei LCs being $S^{*'}M_1^{*'}M_2^{*'}M_3^{*'}...M_i^{*'}M_{i+1}^{*'}...M_n^{*'}P^{*'}\neq\emptyset$, $I(S^{*'},P^{*'})$, etc.

Type (3). The premises of the "Darii type" sorite are:

SOR3:=$E(S^*, M_x^*)E(M_1^*, M_y^*)E(M_2^*, M_y^*)...I(M_x^*, M_y^*)...E(M_i^*, M_y^*)E(M_{i+1}^*, M_x^*)... E(M_n^*, M_y^*)E(M_x^*, P^*)$

LC: $M_x^*M_y^*S^{*'}M_1^{*'}M_2^{*'}M_3^{*'}...M_i^{*'}M_{i+1}^{*'}...M_n^{*'}P^{*'}\neq\emptyset$

Note that there is just one particular premise and the rest are universal premises, all "related" to the particular premise.

Also note that each of the three type sorites admits "Darapti decorations", i.e., a Darapti type sequence of premises may be added using any term/letter as the "base" - exactly as the term (or letter) M was used as the "base" of SOR2 - the Darapti type (2) sorite above.
For example, if to SOR1 one adds Darapti decorations of length=1 on S^*, $M_1^{*'}$, and $P^{*'}$, respectively, and a Darapti decoration of length=3 on $M_2^{*'}$, i.e., if one adds these premises:
$E(S^*, M_{n+1}^*)E(M_1^{*'}, M_{n+2}^*)E(M_2^{*'}, M_{n+3}^*)E(M_2^{*'}, M_{n+4}^*)E(M_2^{*'}, M_{n+5}^*)E(P^{*'}, M_{n+6}^*)$, then one obtains a sorite which has just one LC not two. This LC is an "extension" of LC_1:
$S^*=S^*M_1^{*'}M_2^{*'}M_3^{*'}...M_n^{*'}P^{*'}M_{n+1}^{*'}M_{n+2}^{*'}M_{n+3}^{*'}M_{n+4}^{*'}M_{n+5}^{*'}M_{n+6}^{*'}$.
For the Goclenian reading of SOR1 one needs to add, as Darapti decorations of the same lengths as above, e.g., these premises:
$E(S^{*'}, M_{n+1}^*)E(M_1^*, M_{n+2}^*)E(M_2^*, M_{n+3}^*)E(M_2^*, M_{n+4}^*)E(M_2^*, M_{n+5}^*)E(P^*, M_{n+6}^*)$, and then, again, one obtains a sorite which has just one LC not two. This LC is an "extension" of LC_2:
$P^*=P^*M_1^*M_2^*M_3^*...M_n^*S^{*'}M_{n+1}^{*'}M_{n+2}^{*'}M_{n+3}^{*'}M_{n+4}^{*'}M_{n+5}^{*'}M_{n+6}^{*'}$.
One may add Darapti decorations to a Darapti type sorite, SOR2, via adding, e.g., these premises:
$E(S^{*'}, M_{n+1}^*)E(M_1^{*'}, M_{n+2}^*)E(M_2^{*'}, M_{n+3}^*)E(M_2^{*'}, M_{n+4}^*)E(M_2^{*'}, M_{n+5}^*)E(P^{*'}, M_{n+6}^*)$. Then one obtains a sorite whose LC is an "extension" of the above SOR2 LC:
$M^*=M^*S^{*'}M_1^{*'}M_2^{*'}M_3^{*'}...M_i^{*'}M_{i+1}^{*'}...M_n^{*'}P^{*'}M_{n+1}^{*'}M_{n+2}^{*'}M_{n+3}^{*'}M_{n+4}^{*'}M_{n+5}^{*'}M_{n+6}^{*'}$
Analogously, Darapti decorations of the same lengths as above may be added to a Darii type sorite, SOR3, via adding these premises:
$E(S^{*'}, M_{n+1}^*)E(M_1^{*'}, M_{n+2}^*)E(M_2^{*'}, M_{n+3}^*)E(M_2^{*'}, M_{n+4}^*)E(M_2^{*'}, M_{n+5}^*)E(P^{*'}, M_{n+6}^*)$. Then one obtains a sorite whose LC is an "extension" of the SOR3 LC: $M_x^*M_y^*S^{*'}M_1^{*'}M_2^{*'}M_3^{*'}...M_i^{*'}M_{i+1}^{*'}...M_n^{*'}P^{*'}M_{n+1}^{*'}M_{n+2}^{*'}M_{n+3}^{*'}M_{n+4}^{*'}M_{n+5}^{*'}M_{n+6}^{*'}$.

One thus sees that to any term/letter appearing in the LC of a sorite of types 1, 2 or 3, one may add a "Darapti decoration", i.e., a (no matter how long) Darapti type sequence of premises with that letter as the "base", exactly as the term (or letter) M was the "base" of SOR2 - the Darapti type (2) sorite listed above.

If represented on a subsets' Karnaugh map with enough variables, the LC of the above SOR1, (resp. SOR2), say that S^*, P^*, (resp. M^*), were reduced by the universal premises to just one, possibly not empty, partition subset of the universal set U. The LC of SOR3 affirms about just one partition subset of the universal set U that it is not empty. (How to construct Karnaugh maps for any number of variables, starting from an n=2 Karnaugh map and then repeatedly using mirror images – first towards the right, then towards the bottom of the page, is shown very clearly on Figures 3.1-3.4 at davidbonal.com.)

6. About empty sets

The above three types of VCA may also be used to settle which VCA are compatible with some of the sets S,P,M,S',P',M' being empty. In the modern square of opposition A(M,P), E(M,P) are not contraries anymore - unless one adds the condition M≠Ø. Instead when both A(M,P), E(M,P) are true it results that M=Ø. This empty set constraint (ESC) – which empties all four subsets of M- is compatible with the universal premises of the VCA of type 1 and 2 - but not with the ei on M. Nor is the M=Ø ESC compatible with the type (3a) and (3b) premises, I(S*, M) and I(P*, M). In fact, since the VCA of the same type are all equivalent, it results that a complete discussion of the compatibility of various ESCs and VCA may be reduced to examining just three or four representative cases:

Darii's PCP, A(M,P)I(S,M), means MP'=Ø, SM≠Ø, and the LC is SM= SMP+ SMP'=SMP≠Ø. From the LC SMP≠Ø, one may, with some loss of information, eliminate M, and re-express the LC as I(S,P)="Some S is P". Thus the PCP is incompatible with the S=Ø, M=Ø, and P=Ø ESCs, but is compatible with the S'=M'=P'=Ø ESCs, (which imply S=M=P=U; thus in this latter, extreme, case Darii's PCP and LC just affirm that U is non-empty).
Darapti's PCP, A(M,P)A(M,S), means MP'=Ø, MS'=Ø, and the LC is M=MP+ MP'=MP=MPS+MPS'=MPS, which may be written as A(M,SP). This time around one may eliminate M only via the ei hypothesis M≠Ø, then re-express the LC as I(S,P). Thus the ei hypothesis is incompatible with the M=Ø, S=Ø and P=Ø ESCs, but is compatible with the S'=M'=P'=Ø ESCs, (which imply S=M=P=U; thus in this latter, extreme, case Darapti's PCP plus ei LC affirms that U is non-empty). Note that Darapti's PCP without the added ei condition is compatible even with U=Ø, in which case the PCP is just "chatter about empty sets".
Barbara's PCP, A(M,P)A(S,M), means MP'=Ø, SM'=Ø, and the LCs are S=SM+ SM'=SM=SMP+SMP'=SMP, and P'=P'M+ P'M'=P'M'= P'M'S+P'M'S'=P'M'S'. The first LC may be written as A(S,MP), or, with some loss of information, one may eliminate M, and write A(S,P) =E(S,P'), which now refers to an entire column of U instead of just one of the eight subsets of U. (A "precise" LC always pinpoints to just one of the eight subsets of U.) The second LC may be written as A(P', S'M'), or, with some loss of information, one may eliminate M', and write A(P',S) =E(S,P') – the same as the first LC. Since Barbara's PCP contains only universal premises, the PCP is compatible even with U= Ø in which case all the deductions and the LCs – either "precise" or "classically expressed", are just "chatter about empty sets". One may then add an ei hypothesis, S≠Ø, to the 1st LC, and a different ei hypothesis, P'≠Ø, to the 2nd LC, to obtain, after the M, resp. M', elimination the new ei LCs: I(S,P), (Barbari), and resp., (the un-named), I(S',P'). The S≠Ø ei hypothesis means, since S=SPM, that also P≠Ø and M≠Ø, while the compatible ESCs are S'= Ø, or/and, P'= Ø, or/and, M'=Ø. The S'=P'=M'=Ø constraint amounts to Barbari affirming U≠Ø. The P'≠Ø ei hypothesis means that also S'≠Ø and M'≠Ø. If both ei hypotheses are true then all the sets M, M', S,S',P,P' are non-empty, and there are no ESCs compatible with both ei hypotheses.
In conclusion any universal premise is compatible with any ESC. But any ei hypothesis or any LC of a VCA of type (3), (containing one universal and one particular premise - both acting on either M or M'), specifies three sets that are non-empty, and thus pinpoints to three ESCs with which the ei hypothesis or the "type (3) LC" is incompatible.

7. **Conclusions**

The set model shows very intuitively what an LC is, and what a lack of LC means. For each of the three types of PCP entailing at least one LC, the pointing to a unique subset of U is key: if a PCP singles out one of the 8 subsets of U, then one has an LC – if not, there is no LC.

STM also clearly proves the distribution conservation of the "end terms" S and P.

For classification purposes, the P-premise is listed firstly in both STM and the CCS. The latter recognizes two equivalence classes of VCAs: the VS having A(S,P) or O(S,P) statements as LCs, and the VCA with A(P,S) or O(P,S) LCs: the VCA from one class are transformed into the other class VCA via a relabeling S↔P and a switch of the premises' order. This equivalence is taken as a sign that the VCA from the second class may be discarded since they can be recasted as VS. Thus the VCA whose LCs are A(P,S) or O(P,S) statements are weeded out by the very definition of a VS, not by RofVS, which the discarded VCA still satisfy. Note though that the same S↔P relabeling and a switch of the premises' order, transforms Camestres/ Camenes into Celarent/ Cesare, but nevertheless neither Camestres/ Camenes nor Celarent/Cesare are discarded since both have as LC the E(S,P)= E(P,S) statement in which either term may be used as the predicate. This may suggest that the RofVS are rather ad hoc rules meant to weed out VCA containing negative terms in the PCP, or the LC, and, as an afterthought, to weed out PCPs which entail no LC. (Stebbing, (1961) pp.57-58.)
The Karnaugh (-Veitch) map for n=3, (idea started by Marquand, 1881), matches "close enough" the adjacency displayed by its 8 subsets on the 3-circle Venn diagram (idea expounded by Venn, 1880). "Close enough", means, e.g., that after Barbara's premises empty 4 subsets out of 8, the other 4 subsets left would be disconnected on a rectangular diagram, but are still connected on the Karnaugh cylindrical map, and still satisfy $S \subseteq M \subseteq P$.
For other takes on categorical syllogisms one may read, e.g., Frank Thomas Sautter, (2019), or Stephen Read, (2017).

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