

Contents

1	Intuitive Proofs	1
1.1	Chessboard Problems	2
1.2	Naming Results	10
1.3	The Pigeonhole Principle	13
1.4	Bonus Examples	23
	Exercises	31
	<i>Introduction to Ramsey Theory</i>	41
2	Direct Proofs	47
2.1	Working From Definitions	48
2.2	Proofs by Cases	53
2.3	Divisibility	54
2.4	Greatest Common Divisors	59
2.5	Modular Arithmetic	62
2.6	Bonus Examples	74
	Exercises	81
	<i>Introduction to Number Theory</i>	89
3	Sets	97
3.1	Definitions	97
3.2	Proving $A \subseteq B$	101
3.3	Proving $A = B$	105
3.4	Set Operations	106
3.5	Bonus Examples	119
	Exercises	125
	<i>Introduction to Topology</i>	137
4	Induction	147
4.1	Dominoes, Ladders and Chips	147
4.2	Examples	149
4.3	Strong Induction	164
4.4	Non-Examples	173

4.5 Bonus Examples	175
Exercises	188
 <i>Introduction to Sequences</i>	199
 5 Logic	207
5.1 Statements	207
5.2 Truth Tables	214
5.3 Quantifiers and Negations	219
5.4 Proving Quantified Statements	227
5.5 Paradoxes	228
5.6 Bonus Examples	232
Exercises	242
 <i>Introduction to Real Analysis</i>	253
 6 The Contrapositive	261
6.1 Finding the Contrapositive of a Statement	263
6.2 Proofs Using the Contrapositive	264
6.3 Counterexamples	269
6.4 Bonus Examples	273
Exercises	278
 <i>Introduction to Big Data</i>	285
 7 Contradiction	293
7.1 Two Warm-Up Examples	295
7.2 Examples	298
7.3 The Most Famous Proof in History	300
7.4 The Pythagoreans	304
7.5 Bonus Examples	311
Exercises	320
 <i>Introduction to Game Theory</i>	325
 8 Functions	331
8.1 Approaching Functions	331
8.2 Injections, Surjections and Bijections	335
8.3 The Composition	347
8.4 Invertibility	352
8.5 Bonus Examples	356
Exercises	362
 <i>Introduction to Cardinality</i>	371

9 Relations	379
9.1 Equivalence Relations	379
9.2 Abstraction and Generalization	391
9.3 Bonus Examples	395
Exercises	402
<i>Introduction to Group Theory</i>	413
Appendices	421
A Other Proof Methods	423
A.1 Probabilistic Method	424
A.2 Linear Algebra Method	429
A.3 Combinatorial Method	434
A.4 Computer-Assisted Proofs	440
A.5 Proofs by Picture	448
B Proofs From The Book	453
B.1 Merry Madness from March	454
B.2 Significant Sets of Shifting Shapes	455
B.3 A Flow of Factors From Fermat	460
B.4 A Pinpointed Proof Pausing Prussian Parades	462
B.5 Cleverly Cutting the Cruising Coins	464
B.6 An Antisocial Ant Avalanche	468
B.7 A Pack of Pretty (Book) Proofs by Picture	472
B.8 An Image's Insightful Illusion	476
B.9 Monotone Marches through Muddled Marks	479
B.10 Zigging Zeniths and Zagging Zones	484
C Writing Advice	487
C.1 Writing Proofs	487
C.2 Writing in L ^A T _E X	493
Index	497