Chapter 2 Solved Exercises

Question 1. (Exercise 2.3 part (c)) Prove that the product of two odd integers is odd.

Question 2. (Nearly Exercise 2.5(a)) Suppose that n is an integer. Prove that if n is odd, then $n^2 + 6n + 5$ is even.

Question 3. (Nearly Exercise 2.8(b)) Give an example of the following property. Then, prove that it is true.

- If n is an integer, then $5n^2 + n + 3$ is odd.

Question 4. (Exercise 2.10 (a) and (c)) Prove the following. For each, m, n and t are integers.

(a) If $m \mid n$, then $m^2 \mid n^2$.

(c) If $m \mid n$ and $m \mid t$, then $m \mid (n + t)$.

Question 5. (Exercise 2.15)

(a) Prove that if n is a positive integer, then 4 divides $1 + (-1)^n(2n - 1)$.

(b) Prove that every multiple of 4 is equal to $1 + (-1)^n(2n - 1)$ for some positive integer n.

Question 6. (Nearly Exercise 2.16) For each pair of integers, find the unique quotient and remainder when a is divided by q.

(a) $a = 17$, $m = 5$ (b) $a = 5$, $m = 17$ (c) $a = -10$, $m = 3$

Question 7. (Nearly Exercise 2.20) Determine the remainder when 4^{301} is divided by 17, and show how you found your answer (without a calculator!).

Question 8. (Exercise 2.21) Assume that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$. Prove the following.

(a) $a - c \equiv b - d \pmod{n}$. (b) $a \cdot c \equiv b \cdot d \pmod{n}$.

Question 9. (Exercise 2.22) Assume that a is an integer and p and q are distinct primes. Prove that if $p \mid a$ and $q \mid a$, then $pq \mid a$. Also, before your proof, give three examples of this property.

Question 10. (Exercise 2.25) Prove that for every integer n, either $n^2 \equiv 0 \pmod{4}$ or $n^2 \equiv 1 \pmod{4}$.