Chapter 5 Solved Exercises

Question 1. (Exercise 5.1) Explain why the following is logically correct.

1. Everyone loves my baby;

2. My baby loves only me;

3. Therefore, I am my own baby.

Question 2. (Nearly Exercise 5.3) Determine which of the following are statements. Among those that are, determine whether it is true or false.

(a) $\sin(\pi) = 0$.

(b) If x is an integer, then x is positive.

(c) The sets $\mathbb{Z}$ and $\mathbb{Q}$.

(d) The sets $\mathbb{Z}$ and $\mathbb{Q}$ both contain $\sqrt{2}$.

Question 3. (Exercise 5.5) Consider this open sentence:

$$\frac{2n^2 + 5 + (-1)^n}{2} \text{ is prime.}$$

Give an n-value for which this becomes a true statement, and an n-value for which this becomes a false statement.

Question 4. (Exercise 5.8 parts (f)-(h)) Rewrite each of the following sentences to be of the form "If P, then Q." Make sure your new wording does not change its meaning. (You don't need to understand what all these are saying to answer this question. However, you may learn all of these before you earn your degree!)

(f) Whenever a tree has m edges, it has $m + 1$ vertices.

(g) An integer is even provided it is not odd.

(h) A geometric series with ratio r diverges whenever $|r| \geq 1$.

Question 5. (Exercise 5.9 part (d)) Rewrite each of the following sentences to be of the form "P if and only if Q." Make sure your new wording does not change its meaning.

(d) If N is a normal subgroup of G, then $Ng = gN$ for all $g \in G$, and conversely.

Question 6. (Exercise 5.10 parts (g)-(h)) Negate the following sentences.

(g) For all $\epsilon > 0$ there exists some $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \epsilon$.

(h) If I pass Algebra I and Analysis I this semester, then I will take Algebra II or Analysis II next semester.

Question 7. (Nearly Exercise 5.18 part (d)) Given statements P and Q, write the truth table for the statement $\sim(\sim P \vee Q)$.

Question 8. (Exercise 5.18 part (f)) Given statements P, Q and R, write the truth tables for $(P \wedge Q) \vee \sim R$.

Question 9. (Exercise 5.22 part (g)) Construct truth tables to prove that $(P \Rightarrow Q) \Leftrightarrow \sim P \vee Q$, for statements P and Q.

Question 10. (Exercise 5.26 part (d)) A *tautology* is a statement which is guaranteed to be true. By finding its truth table, determine whether $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is a tautology.